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Long-Term Predictive Maintenance: A Study of Optimal Cleaning of Biomass Boilers

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Abstract

Combustion in a biomass-fired boiler causes build-up of soot, which reduces the heat transfer and decreases the efficiency of operation. In order to mitigate this natural occurrence, cleaning via soot blowing is an important maintenance action. The objective of this study is to develop long-term optimal maintenance strategies, which are model-based and specifically employ the dynamics of boiler efficiency and of anticipated heating demand, both of which are identified from empirical data. An approximate dynamic programming algorithm is set up, resulting in the optimal maintenance actions over time, so that the total operational costs of the biomass boiler plus the cleaning costs are minimised. A practical case study with real data is used to elucidate the benefits of the new approach.

Keywords: optimal maintenance, energy efficiency, biomass boilers, dynamic programming

1. Introduction

Biomass boilers are one of the promising future avenues for heat generation [1], and have been recently deployed significantly in the United Kingdom, due to vigorous governmental subsidies [2]. However, they have arisen several concerns, ranging from hardware design [3] to dynamic control [4]. The main reason is that biomass boilers have longer response times than gas or coal boilers. Poor hardware design and installation, as well as inappropriate and overly reactive control, may result in frequent on/off switching that is very inefficient.

Another difficulty is what is known as fouling. On the surfaces of the heat exchangers, soot is accumulated during operation. It causes worse heat

transfer to water and more heat is lost in the exhaust air. Fouling in biomass boilers may be more problematic than in more traditional types of boilers because the biomass particles are typically more volatile than e.g. coal ones.

This article addresses the problem of mitigating fouling in biomass boilers by deriving optimal maintenance strategies. Specifically, it determines when is the most appropriate moment to clean the heat exchanger against the cost of biomass, the cost of cleaning itself, the dynamics of fouling, and a predicted heating demand. Considering all these aspects explicitly and within the same framework of dynamic programming (as already outlined in [5]) makes the present approach unique and novel.

[6, 7] have examined the option of inference over the system based on expert knowledge, where manually specified rules are applied to current data. Others have dealt with the optimization of the boiler maintenance already: [8] assume the presence of soot blowers and use a set of neural networks to capture the behaviour of the system. The result of these neural networks is evaluated by a set of fuzzy logic rules. More recently, [9] apply more accurate first-principle modelling and the ultimate decision is made about the duration and timing of operation of soot blowers. Similarly [10] focus on soot blowers: they combine neural network modelling with optimisation based on sequential quadratic programming. None of these articles considers long-term prediction of demand as an important factor for the decision making about maintenance actions. The need for that becomes more evident in the case of boilers without soot blowers, where the maintenance (cleaning) requires manual work that is more expensive in contrast to operation of the soot blowers.

This article has the following structure: Section 2 defines the problem formally. Then, model construction is discussed in Section 3. The model is used for optimisation via dynamic programming in Section 4. The algorithms are demonstrated on a case study with real data in Section 5. Finally, Section 6 concludes the article.

2. Problem Formulation

2.1. Description of the Physical System

We consider a system that involves a biomass boiler with a hopper within a building, as shown in Figure 1. We consider three mass flows: the biomass delivery \dot{m}_{fuel} [kg/h], the air flowing through the combustion chamber \dot{m}_{air} [kg/h], and the heated water flowing out of the boiler \dot{m}_{water} [kg/h]. In this

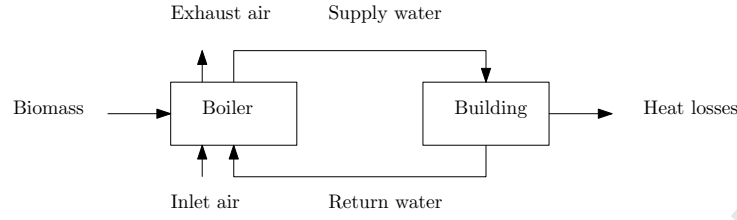


Figure 1: Layout of the physical system.

article, we will assume \dot{m}_{water} and \dot{m}_{air} to be constant. This assumption is valid especially in smaller installations where these two quantities are not subject to control. On the other hand, we consider \dot{m}_{fuel} to be a measurable quantity that is controlled to satisfy the heat demand in the building.

There are several energy flows related to the mass flows. First, the heat produced by burning the biomass can be expressed as

$$P_{in} = H_{fuel} \cdot \dot{m}_{fuel}, \quad (1)$$

where H_{fuel} [J/kg], is the amount of heat released by total combustion of unit of fuel, which is assumed to be known. Thus, P_{in} [kW] is observable. A part of the generated heat is delivered to the building, while another is lost with the exhaust air. We will denote them as P_{out} [kW] and P_{loss} [kW] respectively and it holds that

$$P_{in} = P_{out} + P_{loss}. \quad (2)$$

The output power can be calculated as

$$P_{out} = c_{water} \cdot (T_{sw} - T_{rw}) \cdot \dot{m}_{water}, \quad (3)$$

where c_{water} [J/kgK] is the specific heat capacity of water, T_{sw} [K] is the temperature of supply water and T_{rw} [K] is the temperature of return water. All the quantities on the right-hand side are known or observable. Hence, P_{out} can be directly inferred from the measurements.

The proportion of the heat delivered to the building is called combustion efficiency (a dimensionless quantity) and is defined as

$$\eta = \frac{P_{out}}{P_{in}}. \quad (4)$$

The efficiency decreases during the use of the boiler. In other words, more fuel is required to cover a particular heating demand. However, we assume

that the boiler can be subject to cleaning (either manual or automatic by soot blowers), which brings it back to the original state. These cleaning costs p_{cln} are in monetary units, in this work [EUR]. Another price is p_{fuel} [EUR/kg], the price of a unit of biomass, which can be transformed easily to price of a unit of input energy as $p_{en} = p_{fuel}/H_{fuel}$ [EUR/J] .

2.2. General Requirements

Before we define the problem formally, we can intuitively claim some properties that a system for optimal boiler maintenance ought to satisfy:

- The total operational costs, for a mass of fuel m_{fuel} [kg], are $p_{op} = p_{cln} + m_{fuel}p_{fuel}$.
- If $m_{fuel}p_{fuel} \gg p_{cln}$, it is beneficial to perform maintenance as often as possible, such that m_{fuel} is kept at a minimum.
- If $m_{fuel}p_{fuel} \ll p_{cln}$, it is beneficial to do no maintenance.
- If there are almost no heat losses to be compensated in the building, i.e. $P_{out} \approx 0$, no maintenance is required, regardless to the state of boiler.
- If there is extreme (hypothetically infinite) demand on compensation of the heat losses, i.e. $P_{out} \approx \infty$, the maintenance is to be very frequent (hypothetically continuous).

The decisions about maintenance actions depend on the heat demand P_{out} . Specifically, the decisions will be influenced not only by the current heat demand, but also by the forecasted heat demand. This fact motivates the formalisation of the problem as a dynamical system - this is in contrast to the cited state-of-art approaches.

2.3. Problem Formalisation as a Dynamical System

We formalise the process of boiler fouling with a dynamical system. The time step can be arbitrary, but we select rather longer periods such weeks. There are three reasons for that: (i) boiler maintenance actions are considered in long time horizons; (ii) considering short periods such as in the order of hours (the horizon used for thermal dynamics) makes the problem computationally more complex; (iii) the use of longer time intervals can help with

107 aggregation: we can indeed consider the weekly heating demand as some-
 108 thing that can be predicted for months ahead, but we cannot do the same
 109 with hourly consumptions (the precision would be extremely low).

110 We will use $n = 1, \dots, N$ as the indices of time intervals. For example, n is
 111 index of a week and the horizon has N weeks. The length of a time interval
 112 l (e.g. number of seconds in a week) leads to consider the aggregation in
 113 time of the considered quantities. The aggregated input power forms the
 114 aggregated input heat $Q_{in}[n]$ [J] as

$$Q_{in}[n] = \int_{(n-1)l}^{nl} P_{in}(t) dt; \quad (5)$$

115 the aggregated output heat $Q_{out}[n]$ [J] is formed similarly as

$$Q_{out}[n] = \int_{(n-1)l}^{nl} P_{out}(t) dt, \quad (6)$$

116 and the aggregated heat loss is $Q_{loss}[n]$ [J]. Using these quantities, we can
 117 write the aggregated efficiency $\eta[n]$ as

$$\eta[n] = \frac{Q_{out}[n]}{Q_{in}[n]}. \quad (7)$$

118 With respect to the identities (2) and (7), it is sufficient to model the state
 119 as vector $x[n] = (\eta[n], Q_{out}[n])^T$, leaving out $Q_{in}[n] = Q_{out}[n]/\eta[n]$ and
 120 $Q_{loss}[n] = Q_{in}[n] - Q_{out}[n]$.

121 The dynamical system is subject to actions u (a dimensionless quantity).
 122 At the end of each period n , we can decide to either carry out the maintenance
 123 action, i.e. $u[n] = 1$, or not, i.e. $u[n] = 0$.

124 The so-called single transition costs $C(x[n], u[n])$ [EUR] express the cost
 125 incurred during one period, comprising the cost of operation and of mainte-
 126 nance as

$$C(x[n], u[n]) = p_{en} \cdot \frac{Q_{out}[n]}{\eta[n]} + p_{cln} \cdot u[n]. \quad (8)$$

127 The dynamics of the system is given by the state-evolution model $x[n+1] =$
 128 $f(x[n], u[n])$. The construction of the mapping f is not straightforward and
 129 we deal with it in Section 3.

130 2.4. Problem Statement

131 Based on the available data, we want to schedule maintenance actions
 132 optimally. Considering X to be the set of all possible states and U the set
 133 of all possible actions, we want to calculate decision rules $\pi[n] : X \rightarrow U$ for
 134 all $n = 1, \dots, N - 1$ so that the total cost

$$\sum_{n=1}^N C(x[n], u[n]) \quad (9)$$

135 is minimal, subject to the conditions on the transitions

$$x[n + 1] = f(x[n], u[n]) \quad (10)$$

136 and the application of the action

$$u[n] = \pi[n](x[n]). \quad (11)$$

137 We solve the problem in two steps. First, we identify the state-evolution
 138 model in Section 3. Then, we calculate the optimal strategy by means of
 139 dynamic programming in Section 4.

140 Figure 2 summarises the structure of model and control signals. The
 141 plant/process has states consisting of efficiency and heat demand that are
 142 observed by the controller. The controller applies the current decision rule on
 143 the current observation and decides about next action (input), i.e. whether
 144 the boiler is to be cleaned or not. This influences the state of the plant. The
 145 state of the plant is also influenced by the reference heating demand. Finally,
 146 the cost function quantifies the cost of one-step operations.

147 3. Modeling the System Dynamics

148 In this section, we discuss the model dynamics $x[n + 1] = f(x[n], u[n])$.
 149 The system state has two components: the first one is the efficiency level, the
 150 second one is the heating demand. Both of them capture the most essential
 151 dynamics for the optimal maintenance problem. The construction of the
 152 corresponding models is described in the following.

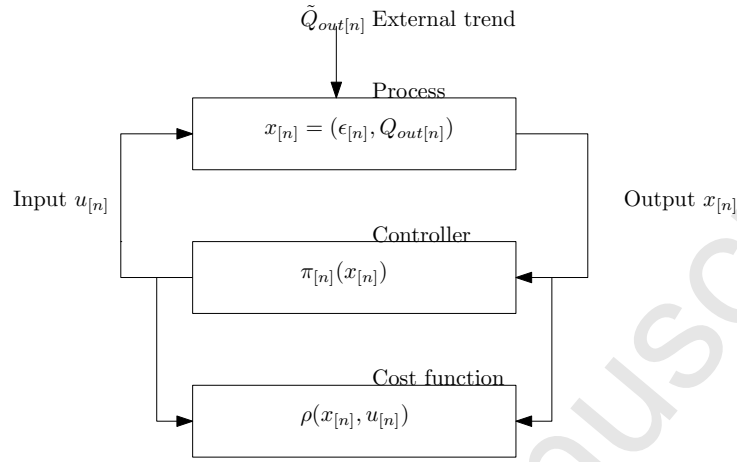


Figure 2: Structure of model and control signals.

3.1. Heating Demand

We discuss how to model the demanded heat supply $Q_{out}[n]$. As mentioned above, $Q_{out}[n]$ is the amount of heat that is required to compensate the heat losses of the building to maintain the indoor comfort at a given level. For the sake of simplicity, we assume that the heating demand is always met.

The heating demand does not depend on the state of the boiler, because it is rather a property of the building and would be the same even if we would equip the building by a completely different heating system. Thus, demand does not depend on the boiler efficiency $\eta[n]$ nor on the maintenance action $u[n]$. However, $Q_{out}[n+1]$ possibly depends on $Q_{out}[n]$, because we can assume that the heating demand will not change rapidly, and on n because the heating demand has a long-term trend, which depends especially on the seasonal influences. We adopt the model

$$Q_{out}[n+1] = \gamma Q_{out}[n] + (1 - \gamma) \tilde{Q}_{out}[n+1], \quad (12)$$

where $\tilde{Q}_{out}[n+1]$ is the long-term trend of Q_{out} and $\gamma \in [0, 1]$ is a parameter. A similar approach combining the autoregressive behaviour and long-term trends was employed also in [11].

Given the model structure (12), two steps are to be carried out: (i) to construct the long-term trend $\tilde{Q}_{out}[n+1]$ and (ii) to estimate the parameter γ .

The construction of the long-term trends of heat demand (and energy demand in general) has been addressed by many authors, e.g. [12, 13, 14].

Any of the mentioned methods can be adopted and possibly tailored. Since some of the periods will have no demand (especially in warm season of the year), we construct the auxiliary long-term trend $r[n]$ [J] as follows:

1. First, we create a classifier h that will indicate whether any heat demand is considered for a given n : this function takes value one during the heating season, zero otherwise.
2. Then, the trend r is calculated from data when the heating demand is positive. For that purpose Gaussian process, local regression, or (as in our case) frequency-domain linear regression [15] can be adopted.

The prediction of this trend has been examined extensively [16, 17, 18]. Having identified h and r , the trend is defined as

$$\tilde{Q}_{out}[n] = \begin{cases} r[n] & \text{if } h[n] = 1 \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Further, in order to estimate γ , we can use a de-trending term

$$\Delta[n] = Q_{out}[n] - \tilde{Q}_{out}[n] \quad (14)$$

and employ standard tools for autoregressive modelling. Considering the constraint $\gamma \in [0, 1]$, we can solve an optimisation problem with the objective function

$$\gamma^* = \arg \min_{\gamma \in [0, 1]} \sum_{n=2}^D \left(Q_{out,d}[n] - \gamma Q_{out,d}[n-1] - (1-\gamma) \tilde{Q}_{out,d}[n] \right)^2 \quad (15)$$

considering a data set $\{Q_{out,d}[n], \tilde{Q}_{out,d}[n]\}_{n=1}^D$ where D is number of samples (e.g. observed weeks)¹.

3.2. Fouling Process and Efficiency

In this section, we discuss the dynamics of the efficiency term $\eta[n+1]$, which depends on:

- The most recent efficiency $\eta[n]$. This efficiency corresponds to the level of accumulated soot.

¹Throughout this paper we denote data variables with x_d and model variables as x .

- 196 • The most recent operation. If there is no operation, i.e. the boiler is not
197 used, and no biomass is burned in it, then the efficiency level remains
198 the same. If the operation is intensive, i.e. the boiler burns a large
199 amount of biomass and generates much soot, the efficiency decreases
200 rapidly. The operation can be quantified in terms of delivered heat
201 $Q_{out}[n]$, generated input heat $Q_{in}[n]$, or the number of switches.
- 202 • The most recent action $u[n]$. If the boiler is cleaned and all soot is
203 removed, its efficiency improves, possibly to the original level.

204 We have adopted an approximate model that satisfies these conditions:

$$\eta[n+1] = \begin{cases} \eta[n] + \alpha Q_{out}[n] & \text{if } u[n] = 0 \\ \eta_{max} & \text{otherwise.} \end{cases} \quad (16)$$

205 Since we assume Q_{in} and Q_{out} to be measurable, we can observe η as well. The
206 unknown boiler fouling parameter α is assumed to be negative as it captures
207 the negative impact of the operation $Q_{out}[n]$ on the efficiency $\eta[n+1]$. It can
208 be easily fitted by standard tools for linear regression, considering $\eta[n] -$
209 $\eta[n-1]$ as the output variable and $Q_{out}[n-1]$ as the input variable.

210 There are several other models for efficiency that are based on first prin-
211 ciples [3, 8], however they cope especially with short-term dynamics, which
212 is not the focus of this article.

213 4. Optimal Maintenance via Dynamic Programming

214 Having a specific model for the state evolution $x[n+1] = f(x[n], u[n])$,
215 we can address the problem of the optimal maintenance by means of dynamic
216 programming [19]. This employs a so-called value function V [EUR], which
217 is updated backwards, starting from the very last time interval

$$V[N](x[N]) = \inf_{u[N] \in U} C(x[N], u[N]). \quad (17)$$

218 For the remaining time indices $n = N-1, N-2, \dots, 1$, the value is calculated
219 recursively as

$$V[n](x[n]) = \inf_{u[n] \in U} C(x[n], u[n]) + V[n+1](f(x[n], u[n])). \quad (18)$$

220 Having the value functions $V[n]$, the decision rule is defined as

$$\pi[n](x[n]) = \arg \inf_{u[n] \in U} C(x[n], u[n]) + V[n+1](f(x[n], u[n])). \quad (19)$$

221 This optimisation is straightforward because set U has two elements only.
 222 More intuitively, (19) can be qualitatively interpreted as follows:

$$\text{Next action for given } x[n] = \begin{cases} \text{cleaning} & \text{if it is cheaper in the long term} \\ \text{do nothing} & \text{otherwise.} \end{cases} \quad (20)$$

223 To implement a program calculating this optimal strategy, we discretise
 224 the state space $X \subset \mathbb{R}^2$ so the value functions $\hat{V}[n]$ are computed over a
 225 discrete domain [20]. We cover the state space by disjoint set of rectangles
 226 $\bigcup_{i=1}^I X_i = X$. Specifically, we consider the range for the efficiency $[\eta^{\min}, \eta^{\max}]$,
 227 sliced into I_1 intervals, and for the output heat $[Q_{out}^{\min}, Q_{out}^{\max}]$, sliced into I_2
 228 intervals. The number of the points of the discrete domain is thus $I = I_1 \cdot I_2$.

229 Having calculated the value function $\hat{V}[n](x_i)$ at the centre of these rect-
 230 angles $x_i \in X_i$, we approximate the value function for any $x \in X$ by the
 231 value of the corresponding centre, i.e.

$$\hat{V}[n](x) = \hat{V}[n](x_i), \quad (21)$$

232 where $x \in X_i$.

233 The calculation is summarized in Algorithm 1. The algorithm has the
 234 following inputs: the state-evolution models f , discretization of the input
 235 space $(X_i, x_i)_{i=1}^I$, the set of actions U , and the cost function C . On line 2,
 236 the value function approximation is initiated to zeros for an hypothetical
 237 time interval $N + 1$. This initialization assures that the very first step of
 238 the recursion will be carried out according to (17). Lines 3 to 15 describe
 239 the backward recursion. In each step of the recursion, the value function is
 240 calculated for all centers of the rectangles, see lines 4 to 14. Lines 5 to 13
 241 implement the minimization (18), using the approximation (21) implemented
 242 on lines 7 to 9.

243 4.1. Special Case: No Autoregression

244 In Section 3, we discussed the modelling of the heating demand Q_{out} . If
 245 the estimation procedure results in no autoregression term, specifically $\gamma = 0$,
 246 then we can modify the model as follows: $Q_{out}[n] = \tilde{Q}_{out}[n]$ is constant. We
 247 then introduce a modification in the change of the gridding. Namely, the
 248 gridding of $[\eta^{\min}, \eta^{\max}]$ remains the same, and is sliced into I_1 intervals. On
 249 the other hand, the gridding for Q_{out} is implemented so that the second
 250 coordinates of centres of the rectangles are the forecasted values $\tilde{Q}_{out}[n]$.

Algorithm 1 Calculation of value functions

```

1: procedure CALCULATEV( $f, (X_i, x_i)_{i=1}^I, U, C$ )
2:    $\hat{V}[N](x_i) \leftarrow 0 \quad \forall i = 1, \dots, I$ 
3:   for  $n = N - 1, \dots, 1$  do
4:     for  $i = 1, 2, \dots, I$  do
5:        $\hat{V}[n](x_i) \leftarrow \infty$ 
6:       for  $u \in U$  do
7:          $x' \leftarrow f(x_i, u)$ 
8:          $i' \leftarrow$  index of the rectangle where  $x' \in X_{i'}$ 
9:          $V_{tmp} \leftarrow C(x_i, u) + \hat{V}[n+1](x_{i'})$ 
10:        if  $V_{tmp} < \hat{V}[n](x_i)$  then
11:           $\hat{V}[n](x_i) \leftarrow V_{tmp}$ 
12:        end if
13:      end for
14:    end for
15:  end for
16: end procedure

```

251 It then holds that

$$V[n](x[n]) = V[n](\eta[n], Q_{out}[n]) = V[n](\eta[n], \tilde{Q}_{out}[n]). \quad (22)$$

252 This implies that the second argument is fixed. Thus, there is no need to
 253 approximate $V[n]$ for other $Q_{out}[n] \neq \tilde{Q}_{out}[n]$. Therefore, we can modify line 4
 254 so that we iterate not for all $i = 1, 2, \dots, I$, but only for those $i = 1, 2, \dots, I$
 255 where the second component of the centre $x_i \in X_i$ is equal to heat demand
 256 for n , i.e. $x_{i,2} = \tilde{Q}_{out}[n]$.

257 Treating this case separately will increase the precision because there is
 258 no approximation in terms of Q_{out} . Moreover, it results in faster calculation
 259 because we process only I_1 rectangles in the approximation. The tabular
 260 representation of the optimal actions can also be practically reduced, as the
 261 value functions are not calculated for most of the combinations.

262 5. Case Study

263 For the case study we have used data from a building in Spain, equipped
 264 by a biomass boiler. Based on almost one year of data, we plan to create a

265 maintenance strategy for the following 10 years. Considering the length of a
 266 time interval to be 1 week, the chosen horizon results in $N = 521$ weeks.

267 5.1. Specification of Parameters

268 The general parameters are: specific heat capacity of water $c_{water} = 4180$
 269 [J/kgK], heating value of biomass $H_{fuel} = 2.2 \times 10^7$ [J/kg], price of biomass²
 270 $p_{fuel} = 0.27$ [EUR/kg], price of maintenance³ $p_{cln} = 108.9$ [EUR], mass flow
 271 of the water $\dot{m}_{water} = 2.15 \times 10^4$ [kg/h].

272 5.2. Data Preprocessing

273 The measured data are from October 2013 to September 2014. The avail-
 274 able measurements are: supply water temperature T_{sw} , return water temper-
 275 ature T_{rw} , water mass flow \dot{m}_{water} , biomass consumption \dot{m}_{fuel} . The data are
 276 measured with a sampling time of 15 minutes, i.e. we have $q = 4 \cdot 24 \cdot 7 = 672$
 277 records per week. Let us index the sampled data by l and use temperatures
 278 $T_{rw,d}[l]$ [K], $T_{sw,d}[l]$ [K], which represent the mean value of T_{rw} , T_{sw} within
 279 the sample, and absolute masses $m_{fuel,d}[l]$ [kg], $m_{water,d}[l]$ [kg] each computed
 280 as integral of the corresponding mass flow over sampling interval l .

To obtain the values for $x[n]$ for the construction of state-evolution models
 we have used the following formulas:

$$Q_{out,d}[n] = \sum_{l=(n-1)q+1}^{nq} c_{water} \cdot (T_{sw,d}[l] - T_{rw,d}[l]) \cdot m_{water,d}[l], \quad (23)$$

$$Q_{in,d}[n] = \sum_{l=(n-1)q+1}^{nq} H_{fuel} \cdot m_{fuel,d}[l], \quad (24)$$

$$\eta_d[n] = \frac{Q_{out,d}[n]}{Q_{in,d}[n]}. \quad (25)$$

281

282 5.3. Results of Modelling

283 As described in Section 3, we model the dynamics of the system in two
 284 steps: first the model of the heating demand, then the model of fouling.

²<http://www.avebiom.org/es/noticias/News/show/precios-del-pellet-en-espana-653>

³<http://www.tucalderabarata.es/reparacion-de-calderas/>

285 The first step of heating demand modelling is the construction of the
 286 classifier h . Based on an inspection of available data, we consider the period
 287 from the first week in November to the last week of February as a heating
 288 season, i.e. $h[n] = 1$, with the exception of the last week in December and
 289 first week in January, when the building is not used nor heated. Otherwise,
 290 no heating demand is considered, i.e. $h[n] = 0$. For this classification as well
 291 as for the next calculation, we will use $t[n]$ as the expression of the absolute
 292 time in days, as Matlab implements the date-time values.

For the weeks when the building was heated, the trend $r[n]$ is fitted using frequency-domain linear regression [15] to the model

$$\begin{aligned} r[n] = & \beta_0 + \beta_1 \cos\left(\frac{2\pi t[n]}{365}\right) + \beta_2 \sin\left(\frac{2\pi t[n]}{365}\right) \\ & + \beta_3 \cos\left(\frac{4\pi t[n]}{365}\right) + \beta_4 \sin\left(\frac{4\pi t[n]}{365}\right). \end{aligned} \quad (26)$$

293 The model was fitted by `lscov` Matlab function using 14 data samples. The
 parameters are shown in Table 1.

Table 1: Results of the modelling - parameters

Parameter	β_0	β_1	β_2	β_3	β_4
Value ($\times 10^5$)	-2.8122	-2.1200	3.6246	0.6391	1.0429

294 The standard deviation from the trend was 3738.8 kWh/week, i.e. 21.69%
 295 of the mean. We tried to explain this by the use of an autoregressive term,
 296 however the result was $\gamma = 0$ which leads to the special case discussed in
 297 Subsection 4.1. The final model as well as the underlying data is shown in
 298 Figure 3.
 299

The boiler fouling parameter α was estimated also by standard `lscov` Matlab function. The result was

$$\hat{\alpha} = -7.4160 \times 10^{-7} [\text{kWh}^{-1}]$$

300 which corresponds to 17.91% decrease of the efficiency for a heating season
 301 without maintenance. This can be seen from Figure 4 which depicts the
 302 measured data versus the resulting model. The fitted original value $\hat{\eta}^{\max} =$
 303 0.6396.

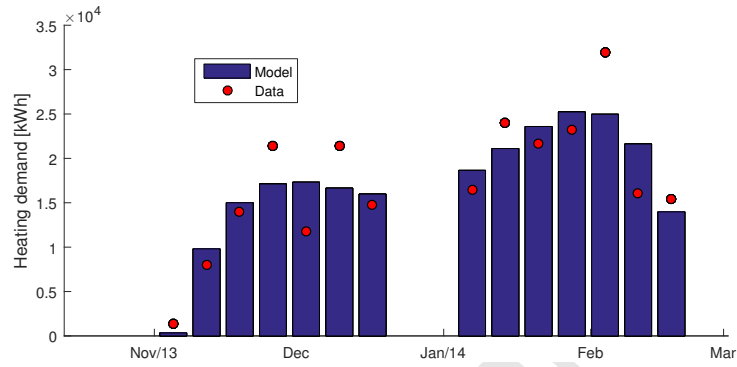


Figure 3: Heating demand (data vs. model).

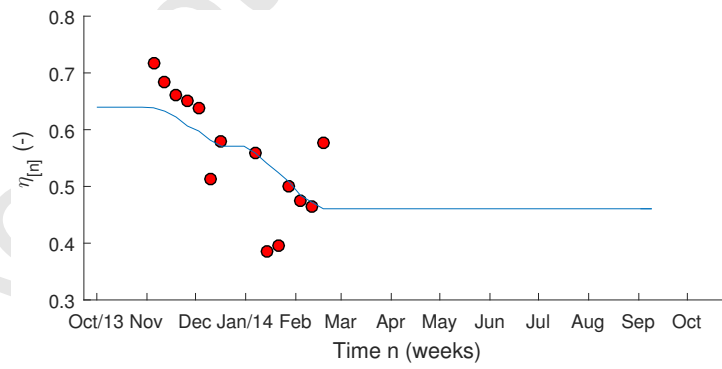


Figure 4: Fouling process and corresponding efficiency model (data vs. model).

304 5.4. Results of Optimization

305 Since the identification resulted in $\gamma = 0$, we adopted the approach dis-
 306 cussed in Subsection 4.1. We used the discretization to $I_1 = 100$ inter-
 307 vals, considering the efficiencies to range from $\eta^{\min} = 0.20$ to the detected
 308 $\eta^{\max} = 0.6396$.

309 The resulting strategy has the shape as shown in Figure 5. For each
 310 time n and efficiency level $\eta[n]$, the optimal action is given. Where the
 311 colour is white, there is no maintenance, i.e. $\pi[n](x[n]) = 0$. Where the
 312 colour is blue (dark), the maintenance action is carried out, i.e. $\pi[n](x[n]) =$
 313 0 . Note that the resulting strategy satisfies the general requirements, as
 314 outlined in Section 2.2: the maintenance depends on the heat demand. We
 315 can also observe decreased willingness to clean the boiler in the last year
 316 of the prediction horizon. This is also an expected behaviour: if there is
 317 no heating assumed anymore then the maintenance does not make sense.
 318 Detailed version is provided in Figure 6. We can observe one interesting
 319 detail: the last month of the prediction horizon is December 2025. The
 320 maintenance actions are carried out in case of very low efficiency only. It
 321 is contrasting to December 2024 where the efficiency can be still high and
 322 the maintenance action is proposed. The reason for this difference is the
 323 following: the strategy decides myopically in the end because in the short-
 324 term perspective the maintenance is relatively more expensive than achieved
 325 savings on fuel.

326 The visual inspection indicates that the strategy can be interpreted as
 327 follows: if the efficiency is below a threshold, specifically, and if there is a
 328 heating demand in the upcoming period, i.e. $Q_{out}[n+1] > 0$, then the main-
 329 tenance is to be carried out. We can then define an optimal condition-based
 330 maintenance strategy by setting the threshold to a fixed value $\eta^{thr} = 0.58$
 331 based on the visual inspection of Figure 5. Note that this maintenance strat-
 332 egy differs from standard condition based maintenance significantly because
 333 it uses a threshold that is not given by an expert estimate, but as a result
 334 of explicit optimisation that captures all available information. As such, our
 335 procedure could provide an automatic update of such a threshold, which
 336 would otherwise be difficult to set based on expert opinion.

337 Table 2 summarizes the results of the dynamic programming compared
 338 to annual or semi-annual maintenance. We can see that the dynamic pro-
 339 gramming is much better than regular approaches.

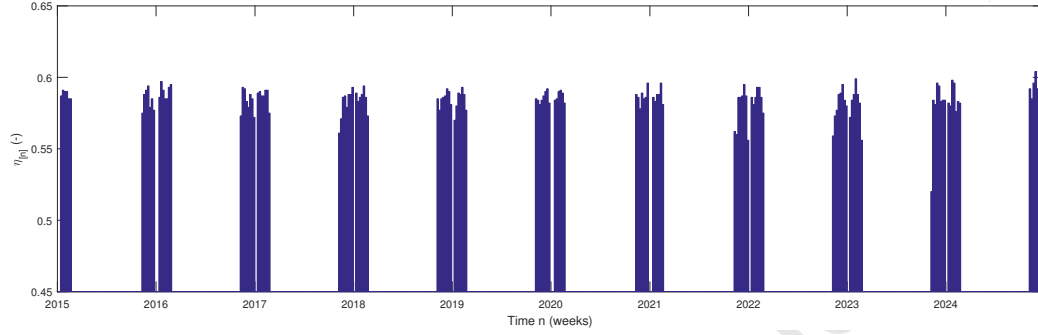


Figure 5: Calculated decision strategy: values of $\eta[n]$ when a maintenance action is to be carried out.

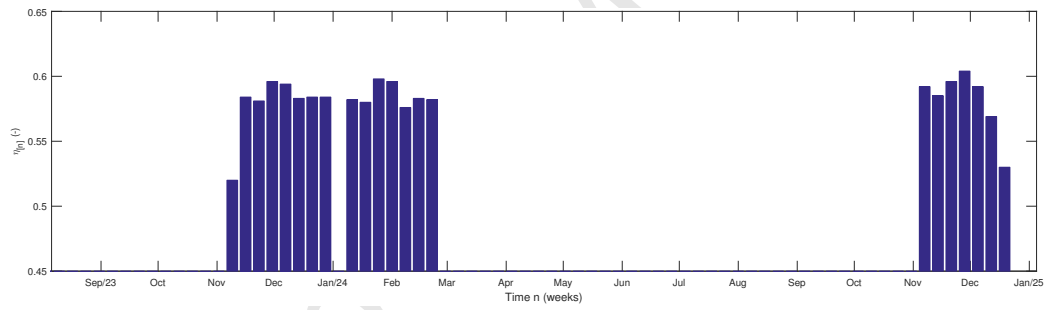


Figure 6: Detail on last two years.

Table 2: Results			
Strategy	Total cost [EUR]	Total actions	Savings by Dynamic Programming
Dynamic programming	5.0675e+04	26	0%
Annual cleaning	5.5385e+04	10	8.50%
6month cleaning	5.6474e+04	20	10.27%
No maintenance	1.2949e+05	0	60.87%

6. Conclusions

The present article has demonstrated the capability of dynamic programming as a tool for the optimal predictive maintenance on practical models obtained from real data. The described methodology is motivated not only on the state of the equipment, but also on the long-term trends of the heating demand, which is novel considering the state-of-art. The approach has been applied to a biomass boiler at a Spanish school and has highlighted possible energy savings when compared to standard maintenance strategies.

There are many open areas for further development. The article has adopted relatively basic tools for the modelling (heating demand, fouling process) as well as for the optimisation (discretised dynamic programming). Advanced modelling and optimisation tools can enrich the approach. These methods can be considered not only in the batch implementation, as described here, but also in an online set-up. It may be also beneficial to explore the possibility to consider retrofit as a possible action to optimise not only the maintenance, but also the procurement of new equipment. A related challenge is to explore the reasonable length of the horizon as the optimal strategy exhibit relatively periodic behaviour. Finally, the cost model may incorporate also discomfort monetisation [21], leading to a number of interesting technical questions.

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