PLASMA WAVEGUIDES FOR HIGH-INTENSITY LASER PULSES

D. J. Spence
St. Edmund Hall, Oxford

Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford.

Trinity Term 2001

Department of Atomic and Laser Physics
Clarendon Laboratory, Oxford
This thesis documents the development of plasma waveguides for high-intensity laser pulses. Initial work concentrated on the development of the discharge-ablated capillary waveguide, based on the work of A. Zigler (Zigler, A., Y. Ehrlich, C. Cohen, J. Krall and P. Sprangle, J. Opt. Soc. Am. B 13, 68). The waveguide was shown to be capable of guiding picosecond laser pulses with an intensity of $10^{16}$ W cm$^{-2}$ over a length of 10 mm. The pulse energy transmission of the capillary was increased from 48% to 70% when the discharge was fired.

An interferometry-based measurement technique was developed, allowing measurement of the electron density profile formed in the capillary waveguide. These measurements were used as input to a numerical simulation that predicted the propagation of intense laser pulses through partially-ionised plasma waveguides. Numerical simulations accurately reproduced the picosecond pulse guiding results, and gave important insights into the properties and severe drawbacks of partially-ionised waveguides.

Previous work on partially-ionised plasma waveguides has not fully explored the implications of the propagation of intense pulses through the partially-ionised plasma. For polypropylene waveguides, it was shown that for pulses with an intensity of $10^{16}$ W cm$^{-2}$, the waveguide is not capable of high-quality guiding. However, for pulses with an intensity of greater than $10^{17}$ W cm$^{-2}$, high-quality guiding is predicted through the partially-ionised waveguide in a new regime called “quasi-matched guiding”.

A novel gas-filled capillary discharge waveguide was designed and built. The device was shown to form a guiding channel inside a capillary pre-filled with gas. Interferometry measurements of the electron density profile formed in a hydrogen-filled capillary discharge waveguide showed that an approximately parabolic plasma waveguide could be formed in an essentially fully-ionised hydrogen plasma. The device was used to guide femtosecond laser pulses, with an intensity of $10^{17}$ W cm$^{-2}$, over distances of 20 and 40 mm, with a pulse energy transmission of 92% and 82% respectively. For the 20 mm-long waveguide, the peak intensity in the output plane of the waveguide was 70% of that at the waveguide input. These results indicate the lowest coupling and insertion losses of any waveguide published to date.

The gas-filled capillary discharge waveguide is shown to be capable and versatile, and is suited for use as a tool in other applications. The use of the waveguide in the fields of XUV lasers and laser wakefield acceleration is discussed.
To Simon. No one would believe that I was your first student, given your natural leadership and well-judged guidance. You steered me with subtlety, and gave endless enthusiasm for the smallest results. I’m proud of what we achieved together.

To David. Thank you for the time that you let me take from you, for all your ideas, encouragement, and criticism. Your ideas and your thoughts always made me question my work.
[12] ELECTRON ACCELERATION IN A PLASMA WAVEGUIDE

12.1 THEORY

12.2 EXPERIMENTAL DEMONSTRATIONS

12.3 ELECTRON ACCELERATION IN A HYDROGEN-FILLED CAPILLARY DISCHARGE WAVEGUIDE

12.4 PROPOSED GUIDED LWFA EXPERIMENT

12.5 LWFA IN A DISCHARGE-ABLATED CAPILLARY WAVEGUIDE

12.6 TAPERED-CHANNEL PHASE MATCHING

12.7 SUMMARY

[13] CONCLUSIONS

13.1 SUMMARY OF RESULTS AND THEIR IMPLICATIONS

13.2 FUTURE WORK
1.1 Introduction

There is currently considerable interest in the interaction of intense laser pulses with plasmas. Lasers capable of generating pulses with peak powers in the terawatt range have been available for over 20 years at national laboratories. More recently, the advent of the chirped pulse amplification technique (CPA) has allowed smaller “table-top” lasers to reach terawatt powers by producing far shorter amplified pulses than previously achievable (Strickland and Mourou 1985). Indeed off-the-shelf commercial lasers are now available with output peak powers in excess of one terawatt. Using the same technology, national facilities have developed large-scale CPA lasers capable of generating laser pulses with a peak power in excess of a petawatt (Perry et al. 1999), opening the way for research into a new regime of laser-plasma interactions.

The advent of CPA has resulted in the proliferation of high-power lasers, and so research into the interaction between high-intensity laser pulses and plasmas has become of broad interest to the scientific community. While the propagation of intense laser pulses (of intensity \( I \gg 10^{14} \text{ W cm}^{-2} \) ) in vacuum is simple and well understood, the propagation of such pulses in a plasma generates a wealth of complexity, resulting in several branches of research into the diverse physical processes that occur within the plasma. Three major applications of the research are high-harmonic generation, x-ray lasers, and electron acceleration. While each of
these applications relies upon very different physical processes, it is clear that a common desire for all is to increase the length over which intense laser pulses can be made to interact with the plasma. That common goal is the subject of this thesis.

1.1.1 Thesis outline

The outline of the thesis is as follows. In the first chapter, the field of laser-plasma interactions will be introduced, along with several applications that rely on those interactions. The field of guiding high-intensity laser pulses will be discussed in chapter two, detailing the varied approaches that have been used to guide laser pulses over extended distances. Chapters three and four document experiments investigating the properties of a promising type waveguide: the discharge-ablated capillary waveguide. The development of the waveguide is discussed in chapter three, and the results of guiding high-intensity picosecond laser pulses through the guide are presented in chapter four. These results were the first demonstration of guiding of picosecond laser pulses through such a guide.

An interferometric method of measuring the refractive index profile formed by plasma waveguides has been developed, allowing quantitative measurements of the formation and development of the waveguide. A numerical simulation of the propagation of intense laser pulses through ionising plasmas has also been constructed, which allowed prediction of the guiding properties of partially-ionised plasma waveguides. Measurements of the refractive index profile of the waveguide used for the picosecond pulse guiding experiments were used as input for the simulations, and it was possible to accurately reproduce the results of those guiding experiments. The theoretical simulations, presented in chapter six, gave insight into the properties of such partially-ionised waveguides, and highlighted severe drawbacks of the discharge-ablated capillary waveguide.

The results of a systematic optimisation of the refractive index profile formed in the waveguide are presented in chapter seven. All parameters of waveguide were varied, and interferometry was used to quantify the quality of the waveguide, allowing the conditions for optimum guiding to be determined. It was found that the only parameter that significantly affects the quality of the waveguide is the radius of the capillary in which the plasma is confined.

The drawbacks of the discharge-ablated capillary waveguide, which were highlighted by the numerical simulations, are due to the fact that the plasma through which the high-intensity laser pulse is to be guided is only partially-ionised. Ideally, the plasma should be initially fully-ionised. However, it is also possible to achieve high-quality guiding through
partially-ionised waveguides in a regime called ‘quasi-matched’ guiding. The idea of quasi-matching and the regimes in which it may be achieved are discussed in chapter eight. Numerical simulations will be presented showing the onset of quasi-matched guiding as the intensity of the guided laser pulse is increased.

While developing these ideas, it became clear that the best plasma in which to achieve high-quality guiding would be a pure hydrogen plasma. In such a plasma, even if it was only lightly-ionised, high-quality guiding would be possible for pulses with an intensity greater than approximately $10^{16}$ W cm$^{-2}$. Chapter nine documents the development of a new type of waveguide: the gas-filled capillary discharge waveguide. This new device uses a number of novel design features to create a waveguide in a pure hydrogen plasma. Interferometric measurements showed that the waveguide formed an approximately parabolic electron density profile that was stable for hundreds of nanoseconds. Hydrodynamic simulations of the device performed by the group of Bulanov showed excellent agreement with the temporal behaviour of the waveguide.

The results of guiding femtosecond laser pulses with an intensity of $10^{17}$ W cm$^{-2}$ will be presented in chapter ten. These results demonstrate that the hydrogen-filled capillary discharge waveguide is capable of excellent guiding over lengths of 20 and 40 mm. The waveguide was found to have insertion and propagation losses that are the lowest published to date for any waveguide for high-intensity pulses.

Finally, in chapters eleven and twelve, some applications for which the gas-filled capillary discharge waveguide is suited will be discussed. The plasma parameters in the waveguide are ideal for laser wakefield electron accelerators. Furthermore, by doping the plasma with small partial-pressures of other gases, the waveguide becomes suitable for the study of XUV lasers.

In the present chapter, the interaction between intense laser pulses and plasmas will be introduced. The laser pulse can severely perturb the initial state of the plasma, and conversely the state of the plasma strongly affects the propagation behaviour of the laser pulse. The coupling of these two perturbations is responsible for generating what is in general an extremely complex laser-plasma interaction. The applications of laser-plasma interactions in other fields of physics will be introduced.

### 1.2 Plasma perturbations due to intense laser pulses

As a laser pulse propagates through a plasma, the plasma constituents are subjected to strong oscillating electric and magnetic fields. Ions and electrons are strongly accelerated by the electric field of the laser pulse, which also interacts with bound electrons by distorting the
binding potential between the nucleus and the electrons. The interactions between the plasma and the laser pulse can cause radical changes in the ion and electron temperatures, and can create particle velocity distributions and levels of ion excitation that are far from their initial values. In the following sections, the main physical interactions that lead to a perturbation of the state of the plasma will be discussed.

1.2.1 Quiver motion and plasma heating

Free electrons in the plasma are accelerated by the electric field associated with the laser pulse. The electrons undergo a harmonic oscillation at the laser frequency, termed quiver motion. The cycle-averaged energy of the quiver motion is known as the ponderomotive energy $U_p$, given by

$$U_p [eV] = 9.3 \times 10^{-14} I [W cm^2] \lambda^2 [\mu m]$$

in which $I$ is the laser intensity, and $\lambda$ is the laser wavelength. The quiver energy can be extremely large: for example for a Ti:Sapphire laser pulse with a wavelength of 800 nm and an intensity of $10^{18} W cm^2$, $U_p = 0.7$ MeV. It is clear that the ponderomotive energy of the electrons can be several orders of magnitude larger than their thermal energy.

Ions are also driven by the laser field. The cycle-average energy of an ion of charge $Z$ and mass $m_{ion}$ is less than that of an electron by a factor $(Zm_e/m_{ion})^2$, typically of the order $10^{-8}$ ($m_e$ is the electron mass). The cycle-averaged ion velocity is decreased by the square root of the same factor compared to the electron velocity, and so ion quiver motion is normally neglected in comparison to the electron quiver motion.

Consider the passage of a laser pulse through a small region of a plasma. The magnitude of the electric field oscillation in the region is zero before the arrival of the pulse, rises as the leading edge of the pulse propagates through the region, and decreases after the peak of the pulse has passed. The quiver energy of a free electron is also zero before the arrival of the pulse, increases as the laser intensity increases, and decreases to zero as the intensity returns to zero. Thus in a plasma in which no collisions occur, the kinetic energy of the electrons before and after the passage of a laser pulse is the same, and no absorption of laser energy occurs.

The situation is altered by collisions. The plasma electrons all oscillate coherently in the laser field: electron-electron collisions occur only due to the thermal component of their motion, and may be neglected. In contrast, elastic electron-ion collisions occur frequently, and tend to
randomise the quiver motion, resulting in an increase in the random thermal component of electron energy. This heating process is known as inverse-bremsstrahlung heating (IB), and may cause strong absorption of the laser pulse energy in regimes for which the quiver energy is small compared to the thermal velocity of the electrons. This condition for efficient IB heating is for example satisfied for \( I < 10^{14} \, \text{W cm}^{-2} \) for a plasma with an electron temperature of 70 eV.

The average ion stage in the plasma can be strongly perturbed by inelastic electron-ion collisions between the slow moving ions and the energetic electrons undergoing quiver motion. An electron that has not been scattered has a peak kinetic energy of twice the ponderomotive energy of the laser field, and so such quiver ionisation can result in ionisation of the plasma ions to levels far higher than would be expected from the electron temperature. Quiver ionisation rates are often of the order of 1 ps\(^{-1}\), and so quiver ionisation is usually negligible for femtosecond laser pulses.

### 1.2.2 Optical field ionisation

Possibly the most significant interaction between the plasma and the laser field is direct ionisation of the plasma ions by the laser field itself. The ionisation process is characterised by the Keldysh parameter \( K \) (Keldysh 1965), which is given by

\[
K = \frac{E_i}{\sqrt{2U_p}}
\]

in which \( E_i \) is the ionisation energy of the ion in question, and \( U_p \) is given by equation (1.1).

For \( K \gg 1 \), corresponding to low laser intensity, multi-photon ionisation is the dominant mechanism causing ionisation. The rate is typically small for radiation with wavelength around 1 \( \mu \text{m} \), since ionisation energies (typically tens of eV) are many times the single photon energy (~1 eV), and so ionisation requires a large number of photons.

For \( K \leq 1 \) however, corresponding to higher laser intensity, a different description of the ionisation mechanism becomes appropriate (Perelomov, Popov and Terent'ev 1966; Augst and Meyerhofer 1994). For a laser pulse with an intensity of \( 10^{14} \, \text{W cm}^{-2} \), the electric field associated with the laser pulse is comparable to the intra-atomic field experienced by a valence electron. The electron in an atom placed in the laser field experiences a field that is the sum of the original intra-atomic field and the laser field. The resulting distortion of the nuclear field allows the electron to escape the nucleus by tunnelling through the residual potential barrier. This mechanism is known as optical field ionisation (OFI), and is shown
diagrammatically in Figure 1-1. The diagonal grey dotted line shows the potential of an
electron in a laser field. As the laser field oscillates the gradient of this potential also
oscillates. The black dotted line shows the unperturbed atomic potential. The solid line shows
the superposition of the laser potential and the binding potential, representing the potential
experienced by the electron in an atom placed in an intense laser field. For sufficiently high
laser fields, the bound electron can tunnel through the suppressed barrier to become ionised.
Indeed, for laser intensities greater than approximately $10^{14}$ W cm$^{-2}$, OFI is likely to be the
dominant ionisation mechanism in a plasma.

![Figure 1-1: The mechanism of optical field ionisation (OFI). The dotted lines show the
potential of an electron in the laser field, and the unperturbed atomic potential, as a function
of radial distance from the nucleus, in units of the Bohr radius $a_0$. The solid line shows the
distorted atomic binding potential for an atom in an intense laser field.](image)

Optical field ionisation can be modelled in several ways, the most generally applicable of
which is the ac-tunnelling theory (Perelomov et al. 1966; Ammosov, Delone and Krainov
1986). OFI rates are highly non-linear with laser intensity, and a particular ion stage will have a well-defined appearance intensity, above which the ionisation rate to that stage increases rapidly. The appearance intensity for a given species is a function of the ionisation energy, ion charge, and the angular momentum quantum number of the ionising electron.

The electrons liberated by OFI can be left with large residual energies after the passage of the laser pulse. Consider an atom in a linearly polarised laser field, in which the electric field obeys \( E_\hat{x} = E_0 \hat{x} \cos \omega t \), where \( E_0 \) is the peak electric field, \( \omega \) is the angular frequency of the laser, and \( \hat{x} \) is the unit vector in the polarisation direction, perpendicular to the direction of propagation. The peak electric field is related to the laser intensity by \( E_0 = \sqrt{2\mu_0 I_0} \), in which \( \mu_0 \) is the permittivity of free space, and \( c \) is the speed of light. OFI is most likely to occur at the peak of the field oscillation. An electron liberated while the electric field is exactly at its peak will be driven by the laser field with the usual quiver motion. After the passage of the laser pulse the quiver energy will be returned to the laser field and the electron is left with no residual quiver energy. However if the electron is liberated at a time offset from the peak of the field oscillation, the electron oscillates with the usual quiver motion superimposed with a constant velocity \( v \), given by

\[
v = \frac{E_0 e}{m_e \omega} \sin \varphi
\]

in which \( \varphi \) is the phase offset between the moment of liberation of the electron and the peak of the field oscillation, and \( e \) is the electron charge. After the passage of the laser pulse the constant velocity component of the electron motion remains, and so the electron has absorbed energy from the laser field. This heating mechanism is known as above-threshold ionisation (ATI) heating. For linearly-polarised laser pulses, since electrons are predominantly ionised close to the times of peak field (\( \varphi = 0 \)), ATI heating is small and can be as low as just a few eV (Burnett and Corkum 1989; Janulewicz, Grout and Pert 1996).

For circularly polarised pulses, however, the situation is very different. A circularly polarised field may be thought of as a superposition of two orthogonal fields oscillating 90 degrees out of phase. This is written \( E = E_0 (\hat{x} \cos \omega t + \hat{y} \sin \omega t) \), in which \( \hat{x} \) and \( \hat{y} \) are two orthogonal unit vectors, both perpendicular to the propagation direction. The electric field thus has a constant magnitude, \( E_0 = \sqrt{\mu_0 I_0} \), but its direction rotates at the laser frequency. OFI is now equally likely to occur at any time, and may for convenience be considered to occur at the peak of one of the orthogonal field components, and at the zero point of the other component.
While there is no ATI heating due to the first component, the heating due to the other component is maximised since the electron is born maximally out of phase with that field component ($\varphi = \pi / 2$). Electrons can retain energies of the order of keV due to ATI heating by circularly polarised laser pulses.

ATI heating has important implications for the creation of population inversions in OFI-produced plasmas, and will be discussed again in chapter 11.

### 1.2.3 The ponderomotive force

The force on charged plasma constituents due to the laser electric field drives the quiver motion described above. While there is an oscillating component of momentum imparted to the plasma constituents, there is no permanent net momentum transfer and so no bulk plasma motion is caused.

In contrast, the ponderomotive force $F_p$ is capable of imparting a permanent momentum transfer to the plasma, and so bulk motion of the plasma can result. The force is caused by spatial variations in the laser intensity, which in turn cause the ponderomotive energy of the plasma constituents to vary with position. As with any spatially varying energy, so there is an associated force on the plasma constituents in the direction of decreasing ponderomotive energy:

$$F_p = -\nabla U_p$$ (1.4)

The ponderomotive force can cause fluctuations in the local density of the plasma, tending to expel charged particles from regions of high field intensity. The force acts on both electrons and ions, but since the ponderomotive energy is reduced by the factor $Z m_e / m_{ion}$, the ponderomotive force is decreased by this ratio for ions, and the ion acceleration is decreased by the square of the ratio. For this reason, the motion of electrons will dominate the plasma response to an intense laser pulse.

### 1.3 Propagation of laser pulses through a plasma

In this section, the properties of a laser pulse propagating in a plasma will be discussed. First, the simpler problem of propagation of a laser pulse in free space is considered. The behaviour of a laser pulse near a focus is determined by diffraction. Diffraction is a consequence of the limited spatial extent of the laser beam, and tends to increase the beam’s divergence. The effect is only significant near a focus where the beam’s transverse dimensions try to converge to zero. For a Gaussian beam, diffraction is characterised by the Rayleigh range $Z_R$, given by
in which \( W_0 \) is the focal waist, defined as the radius at which the intensity drops to \( 1/e^2 \) of the peak value. For propagation along the \( z \)-axis, the spot size \( W \) of beam focused to a waist \( W_0 \) at an axial position of \( z = 0 \) obeys

\[
W = W_0 \left[ 1 + \left( \frac{z}{Z_R} \right)^2 \right]^{1/4}
\]

Therefore, at a distance \( Z_R \) either side of the focus, the spot size is \( \sqrt{2} W_0 \), and the intensity is half the focal value. To achieve high laser intensities of the order of \( 10^{14} \) to \( 10^{19} \) W cm\(^{-2} \), lasers are often focused to waists of 30 \( \mu \)m or less. For a laser wavelength of 1 \( \mu \)m, \( Z_R = 3 \) mm for \( W_0 = 30 \) \( \mu \)m, and \( Z_R = 300 \) \( \mu \)m for \( W_0 = 10 \) \( \mu \)m.

The propagation of a laser pulse in solids, liquids and plasmas is determined by the refractive index of the medium \( \eta \), which may vary with position in non-uniform media. The refractive index links the frequency of the laser field with wavelength, and determines the phase velocity \( v_p \) of the field and the group velocity \( v_g \) of the envelope of the laser pulse. The refractive index of a medium can be greater or less than its value of unity for vacuum.

The refractive index of a plasma due to free electrons \( \eta(r) \) may be written

\[
\eta(r) = \sqrt{1 - \left( \frac{\omega_p^2(r)}{\omega^2} \right)}
\]

in which \( \omega \) is the angular frequency of the laser, and \( \omega_p(r) \) is the plasma frequency at a general position \( r \), given by

\[
\omega_p(r) = \frac{\sqrt{N(r)e^2}}{\gamma m_e \varepsilon_0}
\]

where \( N(r) \) is the electron density, \( \varepsilon_0 \) is the permittivity of free space. The parameter \( \gamma \) is a function of laser intensity, and allows for the relativistic increase of mass of the plasma electrons resulting if the quiver motion becomes relativistic. Equation (1.7) shows that the refractive index is imaginary for \( \omega_p > \omega \), which implies that radiation with a frequency lower than the plasma frequency cannot propagate through that plasma. Such a plasma is termed over-dense. In this thesis, only propagation in plasmas for which \( \omega \ll \omega_p \) will be considered.
For a plasma in which there are bound electrons (i.e. ions which are not fully-ionised), there is an additional contribution to the refractive index. The refractive index can then be written

\[ \eta(r) = 1 - \left( \frac{\omega_p}{2\omega} \right) + \left( \sum_i N_{\text{ion}}^i (\eta_{\text{ion}}^i - 1) \right) \]

in which an approximation to equation (1.7) has been made, valid for the regime \( \omega \ll \omega_p \).

The third term in equation (1.9) is the refractive index component due to bound electrons, comprising a sum over all ion species present in the plasma, each of which has a density \( N_{\text{ion}}^i \) and refractive index \( \eta_{\text{ion}}^i \). The ion refractive index is a function of laser intensity, due to the non-linear ion polarisation in the laser field. For most cases of interest, the contribution from free electrons dominates the contribution from bound electrons.

There are three distinct processes that can modify a laser pulse as it propagates through a plasma (Mori 1997). Firstly, if the group velocity in the plasma is modulated along the axis of propagation, then the separation of two points on the envelope of the pulse will change with time. This causes a *longitudinal bunching* (or spreading) of the pulse energy. Secondly, a radial variation in the phase velocity in the plasma will cause the wavefronts of the pulse to curve, resulting in *transverse focusing* (or defocusing) of the pulse. Finally, a longitudinal modulation in the phase velocity in the plasma causes a local change in the frequency of the carrier wave of the pulse, and an associated longitudinal shifting of energy in the pulse. This is known as *photon acceleration*.

These three processes distort the distribution of energy within the laser pulse envelope, and are the underlying causes of all pulse propagation instabilities in plasmas. From equation (1.8) and equation (1.9), it can be seen that there are four ways in which the refractive index of a plasma may be caused to vary at different positions in the laser envelope. These are by a spatial variation of the carrier frequency \( \omega \), the relativistic factor \( \gamma \), the electron density \( N(r) \), or the contribution from bound electrons. Spatial variations in these parameters cause further pulse distortion by the three processes described above. The additional distortion can reinforce the original spatial variation and lead to rapid growth of that variation: processes of this type are termed instabilities.

### 1.4 Propagation instabilities in plasmas

In the following sections, individual instabilities are introduced, and their effect on the propagation of pulses under different conditions is described.
1.4.1 Ionisation-induced defocusing

Ionisation-induced defocusing will often dominate the behaviour of a laser pulse propagating through any medium that can be further ionised by the laser pulse (Rae 1993; Mackinnon et al. 1996). The effect of additional ionisation can be seen by considering the way in which the additional electrons alter the plasma refractive index. Figure 1-2 shows schematically a Gaussian laser beam focused into a partially-ionised plasma. The beam has a transverse intensity profile that is peaked on the axis of the beam. Since the ionisation rate generally increases with laser intensity, the additional ionisation will be greatest in the centre of the beam, creating an axially-peaked spatial distribution of electrons.

![Figure 1-2: A Gaussian beam focused into a neutral or partially-ionised plasma may create additional electrons owing to further ionisation of the plasma ions, as shown in the top figure. The radial profile of the extra electrons has an axial maximum, which corresponds to a refractive index with an axial minimum. Such a refractive index profile causes defocusing.](image)

The resulting electron density profile has a local maximum at the beam centre as shown in the cross-section AB in Figure 1-2, which, from equation (1.7), corresponds to an axial minimum in the refractive index. The phase velocity of the wave is thus greatest on axis and decreases towards the wings of the pulse, causing the wavefronts to curve outwards and the pulse to
defocus. Ionisation-induced defocusing, in tandem with diffraction, can curtail the length over which high focal intensities are maintained to much less than the Rayleigh range.

Ionisation-induced defocusing can be so severe that the pulse is defocused long before the position of vacuum focus is reached, with the consequence that the peak intensity generated in the plasma can be orders of magnitude less than would be generated in vacuum (Mackinnon et al. 1996).

For low-intensity laser pulses \( I < 10^{14} \text{ W cm}^{-2} \), additional ionisation will predominantly arise due to collisional ionisation by thermal electrons that have been heated by inverse Bremsstrahlung (IB). Such ionisation rates are generally slow, and so for low-intensity pulses the effects of further ionisation must only be considered for pulses of picosecond duration or longer. For high-intensity pulses, additional ionisation may be caused by quiver ionisation or by optical field ionisation (OFI). OFI can occur during a single cycle of a laser pulse and so the effects of OFI on pulse propagation must be considered for all intense laser pulses, no matter how short.

1.4.2 Ionisation modulation

The ionisation modulation instability is essentially the effect of ionisation-induced defocusing on a guided laser pulse (Sprangle, Esarey and Krall 1996). Ionisation caused by the leading edge of the pulse creates additional electrons near the axis, forming a defocusing radial profile. The trailing edge of the pulse is defocused, but the leading edge is still guided, leading to a decrease of the pulse duration. The intensity of the leading edge of the pulse can increase due to photon acceleration, which creates a strongly blue-shifted spectrum (Rae 1994; Dorchies et al. 1999).

1.4.3 Stimulated Raman scattering (SRS)

Raman scattering can cause a longitudinal break-up of the laser pulse into sub-pulses that have a length scale equal to the plasma wavelength. As an example, for a plasma with an electron density of \( 10^{19} \text{ cm}^{-3} \), the plasma wavelength from equation (1.8) is 10.4 \( \mu \text{m} \), and so beamlets with a duration of 34 fs would be formed. SRS is seeded by longitudinal fluctuations in the plasma density, which modulate \( \nu_p \) and \( \nu_g \), thus causing energy bunching due to longitudinal bunching and photon acceleration. The modulations created in the pulse envelope tend to cause further density perturbations through the ponderomotive force, enhancing the original perturbations and leading to exponential growth of the instability.
The SRS instability can be seeded by random fluctuations in the plasma density. In that case, modulation of the pulse envelope will occur dominantly at the plasma wavelength: the instability growth rate is greatest at the plasma wavelength due to resonant excitation of the plasma wave.

The instability may also be seeded by the plasma wakefield generated by the leading edge of the pulse. In this case, a strong modulation of the bulk of the pulse occurs with a wavelength equal to the plasma wavelength (Sprangle et al. 1992). A large amplitude plasma wave can be generated by this mechanism, with associated electric field strengths of the order of 100 GeV m\(^{-1}\). In this regime, SRS is a useful mechanism for creating wakefield electron accelerators (Tajima and Dawson 1979; Esarey et al. 1996), which will be discussed further in chapter 12.

Stimulated Raman scattering may also be viewed, using a wave formalism rather than a photon formalism, as the generation of a scattered wave that has been frequency shifted by the plasma frequency \(\omega_p\) (Antonsen and Mora 1993; Sakharov and Kirsanov 1994). Interference between the driving and scattered waves results in a modulation of the pulse envelope with the length scale of the plasma wavelength. The corresponding modulation of the ponderomotive potential drives a plasma wave that couples more energy from the driving wave into the Raman scattered wave. This circular mechanism results in stimulated transfer of energy into the scattered wave.

Scattered waves may be generated at all angles to the driving beam. Indeed backwards Raman scattering can have the highest growth rate, although the time available for growth of backwards scattering is limited to a time approximately equal to the pulse duration.

### 1.4.4 Stimulated Brillouin scattering (SBS)

Stimulated Brillouin scattering is similar to SRS in that the driving wave is scattered into frequency-shifted sidebands that can propagate at any angle to the driving wave. Whereas for SRS the driving wave scatters off an electron plasma wave, for SBS the driving wave scatters off ion-acoustic waves in the plasma (McKinstrie et al. 1994; Labaune et al. 1995).

### 1.4.5 Envelope self-modulation

Envelope self-modulation (ESM) is caused by radial oscillations in the plasma density. The oscillations cause periodic transverse focusing or defocusing which create a modulation of the beam spot size along the length of the pulse. ESM can be seeded by radial oscillations associated with the plasma wakefield created by the leading edge of the laser pulse (Esarey et
al. 1997). An associated instability is the laser hose instability (Sprangle, Krall and Esarey
1994), which can result in hose-like propagation in which the transverse position of the laser
envelope fluctuates. The instability can dominate if the position of the transverse centroid of
the laser pulse varies along the length of the pulse.

1.4.6 Relativistic self-focusing
Relativistic effects are characterised by the parameter $\gamma$ in equation (1.8). The onset of
relativistic effects occurs when the quiver motion of the electrons driven by the laser field
becomes relativistic. A useful parameter for estimating the importance of relativistic motion is
the normalised laser potential $a_0$, which may be written in terms of the peak intensity $I$ as

$$a_0^2 = 7.32 \times 10^{-19} \lambda^2 [\text{um}] I [\text{Wcm}^{-2}]$$

(1.10)

Physically, $a_0$ is the quiver momentum of the electron normalised to $m_ec$. When $a_0 > 1$ the
plasma response is highly relativistic, and relativistic instabilities must be considered. For
$\lambda = 800$ nm, this condition corresponds to $I > 2 \times 10^{18}$ W cm$^{-2}$.

Relativistic self-focusing occurs due to the transverse variation in $\eta$ brought about by a
variation of the relativistic factor $\gamma$, caused in turn by the transverse spatial profile of the
laser pulse. For a laser pulse envelope peaked on axis, $\gamma$ decreases with increasing distance
from the axis: $v_0$ therefore also increases, causing the wavefronts to curve inwards.

Self-focusing can balance diffractive defocusing for a laser pulse with a correctly chosen
power, and the pulse can be guided. For higher laser powers, the pulse focuses towards the
axis, and severe oscillations of the spot size of the propagating pulse can result.

1.4.7 Relativistic self-phase modulation
Relativistic self-phase modulation (SPM) causes a longitudinal modulation of the pulse
envelope. Small fluctuations in the pulse envelope cause photon acceleration due to the
modulation of the relativistic contribution to $\eta$. This creates an additional modulation in the
pulse envelope that is out of phase with the original modulation. However, the spatial
oscillation in the carrier frequency caused by the photon acceleration itself modifies $v_0$, and a
longitudinal bunching results, which reinforces the original modulations in the pulse
envelope. Since this is a two-step feedback loop, the growth rate for self-phase modulation is
normally smaller than for SRS.
1.4.8 Atomic modulation instability

Atomic modulation instability (AMI) must be considered when a laser pulse is propagating through a partially-ionised plasma (Sprangle, Esarey and Hafizi 1997; Sprangle, Hafizi and Penano 2000). The instability is due to the contribution to the plasma refractive index from bound electrons, shown in equation (1.7). The bound electrons contribute a group velocity dispersion (GVD) that has the opposite sign to the GVD from the plasma electrons. The interplay between the GVD and the self-phase modulation instability leads to the atomic modulation instability, which can lead to self-focusing, or distortion of the pulse envelope.

1.5 Applications of laser-plasma interactions

1.5.1 XUV lasers

The generation of short wavelength lasers is a problematic owing to the unfavourable scaling with wavelength of the pump power required to produce a population inversion. One of the most versatile ways to achieve lasing at short wavelengths is to use a high-power laser to pump lasant ions in a plasma.

While early research focused on the "large hammer" approach of using a national-laboratory-scale kilojoule laser to create a suitable population inversion, more recently, subtle approaches have demonstrated the possibility of achieving XUV lasing pumped by much smaller scale lasers (Lemoff et al. 1995; Korobkin et al. 1996; Korobkin et al. 1998). Due to the transient nature of the population inversion produced, such systems are usually longitudinally pumped. Presently, the gain lengths achieved are limited by diffraction and ionisation-induced defocusing. By using a waveguide to channel the pump radiation over long distances, the gain and output energy of this type of XUV laser could be significantly enhanced.

1.5.2 Frequency conversion

Frequency conversion is an important method of generating short frequency radiation from longer frequency laser output. Frequency conversion encompasses high-harmonic generation, sum or difference generation, and parametric amplification. Conversion occurs due to the non-linear polarisability of the medium in which the laser fields interact, and is well established in the visible, infra-red, and ultra-violet spectral regions using solid-state non-linear media.
Similar frequency conversion processes may be achieved in plasmas due to the non-linear response of the plasma ions to the laser fields. The use of a plasma as the non-linear medium has several advantages over solid-state crystals. Firstly, higher laser intensities may be used for conversion processes in plasmas, which would damage solid-state media. Increased intensities allow higher-order conversion processes to proceed with significant efficiencies, allowing the generation of shorter wavelength radiation. Secondly, highly-ionised atoms may be used as the source of the required non-linear polarisability, allowing higher energy photons to be generated by the conversion process (Milchberg, Durfee and McIlrath 1995).

Efficient frequency conversion in plasmas requires uniform laser propagation over extended distances. By using a waveguide to channel the laser radiation, the laser-plasma interaction can be increased, allowing higher-efficiency frequency conversion to take place. Furthermore, the mode properties of the waveguide can allow the driving and generated radiation fields to be correctly phase matched, giving further improvements in conversion efficiency (Milchberg et al. 1995; Tamaki et al. 1999).

1.5.3 Electron acceleration

Electron acceleration by conventional means requires long linear accelerators or ring accelerators to accelerate electrons to the multi-GeV level. A long length is required because the maximum accelerating potential that can be used is of the order 0.1 MeV cm\(^{-1}\).

The use of a laser-plasma interaction to accelerate electrons was first proposed in 1979 (Tajima et al. 1979). A high-intensity laser pulse propagating through a plasma can, under the correct circumstances, resonantly drive a plasma wakefield that travels with the group velocity of the laser pulse. The plasma density variations associated with the plasma wave generate extremely large electric fields, which can be as high as 100 GeV m\(^{-1}\). Electrons can become trapped by the plasma wave, and are driven by the accelerating potential. Such an enormous accelerating potential, harnessed correctly, would enable a new generation of electron accelerator to be realised.

Laser wakefield acceleration is a relatively new field. To date electron acceleration has been demonstrated up to energies of only a few hundred MeV (Gordon et al. 1998; Ogata and Nakajima 1998). These results seem disappointing in view of the accelerating potential that is available. However, in all experiments to date, the acceleration distance achieved was limited to just a few millimetres by diffraction and refraction of the driving laser as it propagated through the plasma.
It can be seen that the use of a waveguide to channel the driving laser pulse over increased distances would allow a huge increase in the maximum energy of the accelerated electrons. The combination of optical guiding and laser wakefield acceleration brings the prospect of generating multi-GeV electrons using simple high-repetition-rate table-top lasers available to many university physics laboratories. The resulting increased ease of accessibility of such electron pulses will open many new avenues of research in chemistry and biology. This topic is discussed in detail in Chapter 12.

1.6 Summary

The physics of the interactions of intense laser pulses with plasmas is a rapidly growing field spurred on by the advent of chirped pulse amplification, which has brought terawatt lasers into the realm of university research laboratories.

The interaction of such laser pulses with plasmas is rich with a diverse range of physical processes, which create a complex interdependence between the perturbation of the plasma by the laser and the perturbation of the laser by the plasma.

By carefully selecting a suitable regime of laser intensity and plasma constituents, it is possible to create conditions suitable for high-harmonic generation, XUV lasing and acceleration of electrons to the GeV level. For all these applications, high laser intensities are required, and so the laser pulses must be tightly focused in the plasma. An unavoidable consequence is that the interaction length is fundamentally limited by a short Rayleigh range, and is often further limited by propagation instabilities. All would benefit from an extension of the length over which the desired laser-plasma interaction could be maintained.

This thesis will discuss research into plasma waveguides, which can be used to guide ultra-high intensity pulses over distances far in excess of the limits imposed by diffraction and the many instabilities discussed in this chapter. In the next chapter, techniques for guiding high-intensity laser pulses will be introduced.
References


Keldysh, L. V. (1965).
20: 1307.


"Demonstration of a 10-Hz Femtosecond Pulse-Driven XUV Laser at 41.8-Nm in Xe-IX." Phys. Rev. Lett. 74(9): 1574-1577.


Mori, W. B. (1997).


"Ionization of atoms in an alternating electric field." 23(5): 924-933.


In this chapter, guiding of high-intensity lasers will be discussed. High laser intensities are achieved using a focusing optic to concentrate the laser energy into a small cross-sectional area. In the absence of guiding, the laser pulse expands radially as it propagates away from the focal point. For propagation in vacuum, the length over which the focal intensity is maintained is characterised by the Rayleigh range $Z_R$, as discussed in chapter 1. In a plasma, however, defocusing can be more severe owing to ionisation-induced defocusing, and the length over which the focal intensity is maintained can be dramatically reduced. The aim of a waveguide is to channel the laser pulse with small cross-sectional area after it has reached its focal point, and so maintain the high focal intensity over long distances.

The quality of a waveguide can be quantified in many ways. An ideal waveguide would channel the laser pulse with constant peak intensity along the whole length of the waveguide. This implies that the spot size of the pulse is constant, and that there is no absorption or loss of pulse energy from the waveguide. Loss of pulse energy can occur for many reasons, as discussed below. The energy transmission $T$ of a waveguide of length $l$ may be simply described using the relation $T = T_0 e^{-\alpha l}$, where $T_0$ describes the coupling of the laser into the waveguide, and $\alpha$ is the propagation loss per unit length.

The guiding of very low-intensity laser pulses has been elegantly achieved using graded index optical fibres (Siegman 1986). The solid core of such fibres has a tailored radial refractive index profile that is peaked on axis. Such a profile continually exerts a focusing pressure on
the propagating beam that can cancel diffraction and lead to collimated propagation for hundreds of miles.

For high-intensity laser pulses, which are considered to be pulse with a peak intensity in excess of $10^{14}$ W cm$^{-2}$, guiding through a solid medium is not possible. At such high intensities, the laser pulse would become severely distorted due to the strong non-linear interaction with the solid, and the material may itself be cracked, melted, or vaporised.

Several methods of guiding high-intensity laser pulses have been proposed and demonstrated. In this chapter, these methods will be reviewed, including discussion of the physics underlying each method, and the advantages and limitations of each technique.

2.1 Grazing-incidence guiding

Arguably the simplest method to guide intense laser pulses uses grazing-incidence reflection at the smooth walls of a hollow capillary tube. A hollow cylindrical capillary tube is capable of supporting transverse electric (TE) and transverse magnetic (TM) modes with radial intensity profiles that are Hankel functions (Davies and Mendonca 2000). Alternatively, using a ray picture of the propagation of light, the guiding mechanism may be considered to be multiple reflections of the rays from the wall of the capillary. For rays that impinge on the wall at a grazing angle, the reflectivity of the wall is high. Near a laser focus, the rays propagate at small angles to the axis, and so a laser pulse can be coupled into a capillary and guided with a low propagation loss.

Hollow capillary guiding of high-intensity laser pulses was first achieved by Jackel et al. (Jackel et al. 1995) who demonstrated multi-mode propagation through 266 µm- and 100 µm-diameter capillaries. In that work, picosecond laser pulses with an input intensity of $10^{17}$ W cm$^{-2}$ were guided in two transverse modes through a 30 mm-long 100 µm-diameter capillary, with a pulse energy transmission $T$ of 17%. Since more than one waveguide mode was excited, the transverse intensity profile of the guided laser pulses varied as the pulses propagated through the guide. Other workers have also demonstrated two-mode propagation of laser pulses with an input intensity of $10^{18}$ W cm$^{-2}$ through 7 mm-long 40 µm-diameter capillaries with $T_o = 70\%$ (Borghesi et al. 1998).

Single-mode propagation has been achieved through evacuated capillaries of various diameters. The fundamental capillary mode, TE$_{11}$, has a similar radial profile to a Gaussian beam, and so a laser pulse that is brought to a Gaussian focus at the capillary entrance can effectively couple into the fundamental mode. For a capillary tube of inner radius $a$, the
maximum theoretical coupling of $T_0 = 98\%$ of the energy from a Gaussian input pulse of waist $W_0$ occurs for $W_0 = 0.64a$. The mode propagates through the capillary with a stable axial intensity, with the pulse intensity at the capillary wall equal to approximately $10^4$ times the axial value. Thus it is possible for high-intensity laser pulses to be guided through the capillary with minimal ionisation of the capillary wall.

Pulses with peak input intensities up to $5 \times 10^{16}$ W cm$^{-2}$ have been guided through 70 and 50 $\mu$m-diameter tubes (Dorchies et al. 1999). The group measured the pulse energy transmission through a range of tube lengths up to 100 mm, as shown in Figure 2-1. The fitted curves are of the form $T = T_0 e^{-\alpha l}$. For the 70 $\mu$m-diameter tubes, $T_0 = 0.84$ and $\alpha = 0.056$ cm$^{-1}$, and for the 50 $\mu$m-diameter tubes, $T_0 = 0.78$ and $\alpha = 0.17$ cm$^{-1}$.

![Figure 2-1: Transmission through capillary tubes of radius $a = 35$ and 25 $\mu$m, for lengths of up to 100 mm [taken from (Dorchies et al. 1999)].](image)

Similar work has been carried out for smaller diameter capillary tubes, for which a smaller input spot size was required, allowing higher peak intensities to be reached. At the same intensity of $5 \times 10^{16}$ W cm$^{-2}$, $T = 6\%$ was measured over 10 mm for 30 $\mu$m-diameter tubes (Courtois et al. 2000). Coupling losses of between 60 and 90% were measured, attributed to poor laser beam quality. The beam quality of the input laser pulse is an important issue for this type of guiding. In order to achieve single-mode guiding, the input beam diameter is only slightly smaller than the capillary diameter. As a result, any beam energy that comes to focus significantly offset from the axis will not be coupled into the capillary. Furthermore, the
energy associated with that poor beam quality component of the laser pulse will impinge on the input face of the capillary tube and may cause damage to the capillary.

For most applications of high-intensity laser waveguides, the laser pulse must be guided through a plasma. Grazing-incidence guiding through gas-filled capillaries has been demonstrated: using a 40 mm-long, 50 μm-diameter tube filled with 20 mbar of helium, Dorchies et al. measured a transmission of 5%, compared with 20% when the capillary was evacuated (Dorchies et al. 1999). Since the helium was initially un-ionised, the pulse was strongly blue-shifted and shortened due to defocusing of the trailing edge of the pulse. It is clear that the presence of gas in the capillary greatly increases the propagation loss through this type of guide.

At intensities greater than approximately $10^{17}$ W cm$^{-2}$, there are additional problems associated with the hollow capillary waveguide. Firstly, the capillary tube is destroyed by a single shot. Furthermore, the transmission of a 10 mm-long capillary has been observed to decrease to just $10^{-4}$% at these intensities (Courtois et al. 2000). This was explained due to plasma formed at the capillary entrance by an amplified spontaneous emission (ASE) pedestal several nanoseconds in advance of the main pulse. Because of aberrations in the beam, some of the energy in the pedestal was clipped at the capillary entrance. The pedestal, which was $10^7$ times weaker than the main pulse, was still sufficiently intense to ablate the front wall of the capillary. The ablated material was ionised to form a plasma plume that prevented the main pulse from coupling into the capillary.

While grazing incidence guiding is a promising guiding technique, the difficulties associated with the need for extremely high beam quality and low levels of prepulse and ASE will need to be overcome. At ultra-high intensities, the low shot lifetime of the waveguide will also need to be addressed. For applications of guiding that require laser propagation in a plasma, the capillary must be pre-filled with neutral gas, which greatly increases the propagation losses and distorts the guided laser pulse.

2.2 Relativistic and ponderomotive guiding

Relativistic self-focusing, which was discussed in the previous chapter in terms of an instability, can be exploited and used as a guiding mechanism. An axially-peaked laser pulse creates an axial maximum in the refractive index owing to the relativistic motion of the plasma electrons (i.e. $\gamma > 1$ in equation 1.8). A refractive index profile of this form creates a focusing effect that can balance diffraction, and lead to guided propagation over extended distances. The self-focusing effect becomes stronger than diffraction for incident laser powers
exceeding a critical power (Monot et al. 1995) \( P_c = 17.4 \left( \omega / \omega_p \right)^2 \) GW, where \( \omega_p \) is the plasma frequency, defined in equation 1.8. For laser pulses with powers in excess of \( P_c \) channelling can occur. For a Ti:Sapphire laser pulse with a wavelength of 800 nm, \( P_c = 7.7 \text{ TW} \) for a plasma with an electron density of \( 10^{18} \text{ cm}^{-3} \). It is useful to note that the critical power is inversely proportional to the plasma electron density.

Expulsion of electrons from the axial region by the ponderomotive force also contributes to channelling. An axial minimum develops in the electron density, corresponding to an axial maximum in the refractive index that causes a focusing effect. The ions are not expelled from the axial region during the passage of the laser pulse because the acceleration they feel is reduced by the ratio of the electron and ion masses, which is in excess of \( 10^3 \). The acceleration is too low for the ions to be significantly displaced in the duration of the laser pulse. The additional ponderomotive contribution to relativistic self-focusing reduces the power threshold for guiding to \( P_c = 16.2 \left( \omega / \omega_p \right)^2 \) GW (Sun et al. 1987).

Relativistic and ponderomotive channelling has been observed experimentally by several groups (Borisov et al. 1992; Monot et al. 1995). It is an attractive guiding mechanism since it requires no equipment additional to the laser beam itself. There are however several drawbacks of relativistic guiding. Firstly, it is inherently unstable: any small deviation from the critical power causes excess focusing or defocusing that reinforces the deviation. Instabilities such as relativistic self-phase modulation can also disturb stable guiding. Secondly, due to the finite response time of the plasma, relativistic guiding is not effective in guiding short pulses (Sprangle, Esarey and Ting 1990). Finally, relativistic guiding is of no use for applications in which the required laser power is well above or below the critical power.

### 2.3 Plasma waveguides

A pre-formed plasma channel with an axial minimum in the electron density profile is known as a plasma waveguide. An ideal plasma waveguide has a radial electron density profile which is parabolic:

\[
N(r) = N(0) + \Delta N \left( \frac{r}{r_{ch}} \right)^2
\]  \hspace{1cm} (2.1)

where \( N(r) \) is the electron density at radius \( r \) and \( \Delta N \) is the increase in the electron density at \( r = r_{ch} \) compared to the axial value. The guiding properties of such parabolic plasma waveguides are well known (Esarey et al. 1997): the modes of the waveguide are described
by Hermite-Gaussian functions. The fundamental mode has as simple Gaussian transverse intensity profile \( I(r) \) described by

\[
I(r) = I(0) \exp\left[ -2\left( r/W_M \right)^2 \right] \tag{2.2}
\]

where the spot size \( W_M \) is given by

\[
W_M = \left( \frac{r_e^2}{\pi r_e \Delta N} \right)^{1/4} \tag{2.3}
\]

in which \( r_e \) is the classical electron radius. In the absence of further ionisation of the plasma by the guided laser pulse, and where ponderomotive and relativistic effects can be neglected, a Gaussian laser beam with spot-size \( W_0 = W_M \) can be coupled into the fundamental mode of the waveguide and experience “matched guiding”: propagation with a stable transverse intensity profile. The quantity \( W_M \) is known as the matched spot size of the waveguide and written in convenient units reduces to

\[
W_M^4[\mu m] = 113 \times \frac{r_e^2[\mu m]}{\Delta N[10^{18} \text{ cm}^{-3}]} \tag{2.4}
\]

A laser pulse coupled into the waveguide with waist not equal to \( W_M \) will excite higher waveguide modes, resulting in the transverse intensity profile of the laser pulse changing as the pulse propagates. Specifically, the profile remains Gaussian at all times, with the spot size oscillating between \( W_0 \) and \( W_M^2/W_0 \). The wavelength of the oscillation \( Z_{os} \) is given by

\[
Z_{os} = \frac{\pi^2 W_M^2}{\lambda} \tag{2.5}
\]

Figure 2-2 shows the behaviour of laser pulses, of wavelength 800 nm, propagating through a parabolic plasma waveguide that has a curvature corresponding to \( W_M = 30 \mu m \), for several values of the input spot size \( W_0 \). The pulse focused to \( W_0 = W_M \) experiences matched guiding, and propagates with a constant spot size. The pulses that are not matched into the waveguide exhibit oscillations in their spot size with propagation distance \( z \), with an oscillation wavelength of \( Z_{os} = 11 \mu m \). This smooth ‘scalloping’ behaviour associated with mismatched guiding can lead to ambiguous interpretation of laser guiding experiments, and will be discussed in subsequent chapters.
Figure 2-2: Calculated spot size of Gaussian pulses, of wavelength 800 nm, propagating through a waveguide with a matched spot size $W_m = 30 \mu m$, for several values of input spot size $W_0$.

The matched spot size of a parabolic plasma waveguide is independent of both intensity and wavelength, making the waveguide extremely versatile. Considerable research has gone into investigating a number of techniques for producing suitable plasma channels, as described below.

2.3.1 Expanding cylindrical spark

A suitable channel may be formed during the hydrodynamic expansion of a laser-produced cylindrical plasma. The initial demonstration of this technique by Durfee and Milchberg used 100 ps, 250 mJ pulses from a Nd:YAG laser to create the spark (Durfee and Milchberg 1993). Using an axicon lens, laser pulses were brought to a longitudinal line-focus in a chamber filled with 30 torr of argon. The pulse intensity at the line focus was approximately $3 \times 10^{14}$ W cm$^{-2}$, sufficiently intense to ionise and heat the gas to Ar$^{7+}$. The subsequent hydrodynamic expansion of the cylindrical plasma lead to an electron density profile with a minimum on axis. The channel was shown to guide low-intensity pulses over several millimetres.
Figure 2-3: The radial electron density profile, as a function of time \( t \) after the creation of a cylindrical spark in 33 mbar of argon [taken from (Nikitin et al. 1999)]. The profiles, obtained using transverse interferometry, were measured at the midpoint of a 17 mm-long waveguide. A profile suitable for guiding has developed after 1.4 ns.

The plasma channel created by this technique has been analysed in great detail. The channel develops on a timescale of order a nanosecond. Figure 2-3 shows the measured radial electron density profile, as a function of time \( t \) after the creation of the cylindrical spark in 33 mbar of argon [taken from (Nikitin et al. 1999)]. The profile measured at \( t = 1.4 \) ns is suitable for guiding, and the plasma is estimated to be ionised up to \( \text{Ar}^{7+} \). At higher pressures of several hundred millibars, the plasma was found to be ionised to the closed-shell \( \text{Ar}^{8+} \) ion (Clark and Milchberg 1997; Nikitin et al. 1997). The modes of the waveguide are leaky modes (Durfee, Lynch and Milchberg 1994; Clark and Milchberg 1998), since the walls of the channel are thin: the electron density drops rapidly after peaking at a radius of several spot sizes (see Figure 2-3).

Plasma channels have been formed in an elongated gas jet rather than a gas-filled chamber (Fan, Clark and Milchberg 1998). This approach avoids the presence of neutral gas at either
end of the channel that can defocus laser pulses coupled into the waveguide. Pulses with an intensity of $5 \times 10^{16}$ W cm$^{-2}$ have been guided over 15 mm using this approach in an Ar$^{8+}$ plasma (Nikitin et al. 1999). However the coupling efficiency was limited to 50% due to tapering of the electron density profile at each end of the waveguide.

Guiding at ultra-high intensities through such a guide will be problematic, since the Ar$^{8+}$ ions will be further ionised by guided pulses with intensities greater than approximately $2 \times 10^{18}$ W cm$^{-2}$.

In order to create a more robust channel, researchers have turned towards creating similar channels in fully-stripped helium. In order to correctly generate an expanding cylindrical spark waveguide, the helium gas must first be ionised by OFI, and subsequently strongly heated by IB, thus driving the expansion. Unfortunately, it is difficult to break down and heat a helium plasma to the required temperature with a single laser pulse. The OFI process that initiates the breakdown requires a high-intensity laser pulse, but IB is most efficient for low-intensity pulses. A compromise between high and low intensity exists for argon gas, allowing channels to be created in Ar$^{8+}$. However, there is no suitable compromise for helium gas: the ionisation energy of He$^{+}$ is far greater than that of Ar$^{7+}$, and so OFI requires a much higher laser intensity at which the plasma is not efficiently heated.

To circumvent this problem, the ionisation and heating steps must be separated. Volfbeyn et al. suggested that a fully-ionised helium channel could be formed using two sequential laser pulses to first ionise the gas and then heat the cylindrical plasma created (Volfbeyn, Esarey and Leemans 1999). In an alternative approach, an electrical discharge was used to ionise the plasma, which was subsequently heated by a 400 ps laser pulse (Gaul et al. 2000). The group achieved guiding of pulses with an intensity of $2 \times 10^{17}$ W cm$^{-2}$ over 15 mm, with a pulse energy transmission of 50%.

Expanding cylindrical spark waveguides are a promising solution for guiding high-intensity laser pulses. The guide can be formed for an unlimited number of shots, and the axial density and matched spot size of the waveguide can be tuned by adjusting the initial gas density or changing the delay between the creation of the spark and the injection of the guided laser pulse. One drawback of the technique is that a complex auxiliary laser system is required to generate the channel. Such lasers are expensive, and increase the complexity of the technique. Furthermore, in order to create the guiding channel over longer lengths, a linear increase in the power of the pump laser will be required.

A further disadvantage is that in forming the channel, the plasma is strongly heated on axis. For applications of waveguides such as harmonic generation and XUV lasers, it is often
desirable have relatively cold, low ion-stage dopant species present in the plasma. The laser pulse that creates the channel would also heat these dopants, and so this technique may be unsuitable for those types of applications.

2.3.2 Pre-pulse channel generation

A plasma channel has been created using a low-intensity picosecond laser pulse ($< 10^{16}$ W cm$^{-2}$) focused into a helium gas jet. The channel was formed due to heating and subsequent hydrodynamic expansion of the plasma created in the focal region (Malka et al. 1997). The channel developed on a nanosecond timescale, and so the picosecond pulse that heated the plasma was not guided. However, a trailing coaxial pulse was guided by this channel. Unfortunately, since the pulse that created the channel was unguided, the length over which the channel was created was limited by diffraction and ionisation-induced refraction. It is difficult to see how this mechanism can be extended to long distances (Mackinnon et al. 1998).

A plasma waveguide has been created using a relativistically guided laser pulse (Krushelnick et al. 1997; Sarkisov et al. 1999). The channel was created due to the expulsion of ions from the axial region. During the passage of a relativistically guided laser pulse, electrons are partially expelled from the axial region by the ponderomotive force. The ions are not expelled on a picosecond timescale, and so charge separation forces limit the expulsion of plasma electrons. However, the plasma ions are accelerated by the charge separation forces and move slowly away from the axial region, creating a plasma channel significantly after the passage of pulse. A trailing laser pulse can be guided by the plasma channel.

Using a self-guided pulse to create a waveguide in this way might be useful if the desired characteristics of the trailing pulse were not suitable for it to be itself relativistically guided. To date the longest channels that have been created by this technique were just 1 or 2 mm long.

2.3.3 Z-pinches in capillaries

Transient plasma channels have been observed during the rapid collapse of a helium plasma created in a fast capillary discharge. In this type of discharge, the skin effect ensures that the initial breakdown of the helium gas occurs near the capillary walls. Breakdown is followed by a rapid collapse due to the magnetic pinch effect, driving a plasma shock wave towards the capillary axis.
Hosokai et al. have observed a radial electron density profile suitable for guiding, formed just before the collapse of the shock wave at the capillary axis (Hosokai et al. 2000). In this work, a current pulse with a peak of 4.8 kV rising in 15 ns was driven through a 1 mm-diameter capillary filled with helium gas. The gas was heated sufficiently during the compression phase that the channel was fully-ionised. The group demonstrated guiding of laser pulses with an intensity in excess of $10^{17}$ W cm$^{-2}$ (Hosokai et al. 2000). Transmission through a 2 cm-long capillary was increased from 30% in the absence of the discharge, to 64% when the discharge was fired. Their results indicate that the pulse energy was not strongly to the axial region, however, and so it seems likely that the laser pulse was not guided with a small spot size through the length of the capillary.

Enhancement of the transmission of a CW laser beam has been observed through 14 cm-long z-pinch waveguides, showing that the guiding channel created was transient, capable of guiding during a temporal window of just a few nanoseconds (Fauser and Langhoff 2000).

### 2.3.4 Discharge-ablated capillary waveguides

Finally, plasma channels suitable for guiding may be formed by passing a slow electrical discharge through initially-evacuated capillaries. Known as a discharge-ablated capillary waveguide, the device was pioneered by the group of A. Zigler at Hebrew University (Zigler et al. 1987; Zigler et al. 1996). In this device, a discharge current ablates material from the wall of the capillary, which is usually made of a soft material such as polypropylene ($\text{[CH}_2\text{]}_n$). The ablated material is heated and ionised as it expands to fill the capillary.

In devices of this type, the discharge current is relatively slow: rising to a peak of several hundred amps after several hundred nanoseconds. With such a current pulse, the magnetic pinch effect may be neglected, and so no shock waves are generated in the plasma. The plasma confined within the capillary develops a radial electron density profile that is approximately parabolic: ideal for use as a plasma waveguide.

### 2.4 Summary

A range of waveguides suitable for guiding high-intensity lasers has been discussed. Laser pulses of sufficiently high power may experience self-guiding, although this mode of propagation is prone to instability. Guiding can also be achieved in hollow capillaries, or in preformed plasma channels. In the non-relativistic regime, such guides present the prospect of guiding that is independent of the laser wavelength and intensity. Even in the relativistic...
CHAPTER 2: WAVEGUIDES FOR HIGH-INTENSITY LASER PULSES

regime, the presence of a plasma guide will act to stabilise the instabilities that are inherent to
the relativistic regime (Sprangle et al. 1992).

In 1996, guiding using the slow capillary discharge waveguide was first demonstrated. The
simplicity of the device was a major advantage that would be important if the waveguide was
to be used as a tool for high-intensity laser plasma interaction experiments.

The remainder of this thesis documents the development of plasma waveguides for
high-intensity laser pulses. Initial work centred on development of the discharge-ablated
capillary waveguide, chosen as a base for investigation due to its simplicity: it is a compact
device requiring a small-scale discharge circuit and no other auxiliary components. At the
start of the work described in this thesis, the waveguide has already shown promising guiding
results over a length of 10 mm. In the next chapter, the discharge-ablated capillary waveguide
will be presented in detail.
References


“Generation of a plasma waveguide in an elongated, high repetition rate gas jet.”

“Focussing of laser beams by means of a z-pinch formed plasma guiding system.”

(2000).
“Production and characterization of a fully ionized He plasma channel.” Appl. Phys.

Hosokai, T., M. Kando, H. Dewa, H. Kotaki, S. Kondo, N. Hasegawa, K. Nakajima and K.
Horioka (2000).
“Optical guidance of terrawatt laser pulses by the implosion phase of a fast Z-pinch


Krushelnick, K., A. Ting, C. I. Moore, H. R. Burris, E. Esarey, P. Sprangle and M. Baine
(1997).
“Plasma channel formation and guiding during high intensity short pulse laser plasma

Lett. 80(24): 5349-5352.

Malka, V., E. DeWispelaere, F. Amiranoff, S. Baton, R. Bonadio, C. Coulaud, R.
Lett. 79(16): 2979-2982.

“Experimental Demonstration of Relativistic Self-Channeling of a Multiterawatt

“High efficiency coupling and guiding of intense femtosecond laser pulses in
preformed plasma channels in an elongated gas jet.” Phys. Rev. E 59(4): R3839-
R3842.


"Lasers." California, Mill Valley.


3.1 Overview and history

The discharge-ablated capillary waveguide was developed by the group of A. Zigler at Hebrew University (Zigler et al. 1987). The group initially investigated the properties of a confined discharge struck through a $10 \text{ mm} \times 9 \text{ mm} \times 0.3 \text{ mm}$ slot. Due to ablation of the polypropylene walls of the slot, the discharge generated a high-density plasma, which was expelled from the ends of the slot. It was proposed that the ejected plasma could be heated by a line-focused laser pulse, creating conditions required to achieve XUV laser gain. Later work concentrated on improving the homogeneity and increasing the density of the plasma (Ehrlich et al. 1994).

In 1996, the group changed tack. By using an anode with high thermal conductivity, it was found that the plasma emerging from the slot had a density minimum at its centre. As the plasma passed through the slot in the anode, the plasma was cooled near the walls, resulting in the temperature being greatest on the mid-plane of the slot, decreasing towards the walls. Since the plasma pressure was uniform across the width of the slot, such a temperature profile resulted in an electron density profile with a minimum on axis: a profile suitable for guiding a laser pulse in one-dimension. Guiding was demonstrated in 1996 for laser pulses with an intensity of $10^{16} \text{ W cm}^{-2}$ (Zigler et al. 1996).
In an effort to create a waveguide capable of guiding in two dimensions, the group investigated discharges through cylindrical capillaries. Instead of guiding a laser pulse through the plasma expelled from the device, the guided laser pulse was to be injected along the capillary axis. This geometry is shown in Figure 3-2, in which the waveguide is cylindrically symmetric. In late 1996, the first two-dimensional guiding results were obtained using a slow capillary discharge (Ehrlich et al. 1996). Guiding of laser pulses with an intensity of $10^{16}$ W cm$^{-2}$ was achieved through a 10 mm-long, 350 μm-diameter capillary.

Guiding was attributed to the plasma ablated from the walls of the capillary forming a radial profile with an axial minimum. The results of measurements of the profile of the laser pulse in the exit plane of the capillary are shown in Figure 3-3. It is clear that the transverse dimensions of the pulse in the exit plane of the capillary were significantly reduced in the guided case compared to the unguided dimensions. An increase in pulse energy transmission was also observed from 25% to 75%.
Figure 3-2: Diagram of the discharge-ablated capillary waveguide of Zigler's group, showing the coupling of a high-intensity laser pulse into the waveguide [after (Ehrlich et al. 1996)].

Figure 3-3: Guiding results through a 10 mm-long cylindrical discharge-ablated capillary waveguide. Images a) and b) show the intensity profile in the output of the capillary guided (discharge on) and unguided (discharge off) respectively. A radial cross-section of the same results is shown in c) [Taken from (Zigler et al. 1996)].
3.2 Development of a discharge-ablated capillary waveguide

In this chapter, the design and development of a discharge-ablated capillary waveguide, based on the device invented by Zigler's group, will be presented. The factors that constrained the design of the device will first be discussed, followed by a detailed description of the final device and its properties.

3.2.1 Paschen curve and consequences

The waveguide was designed to operate in an evacuated chamber, because a high-intensity laser pulse coupled into the capillary would suffer from ionisation-induced defocusing in any background gas. The vacuum environment was one of the largest constraints on the design, for the following reason. The voltage needed to strike a discharge across an air gap can be plotted as a function of the air-pressure x gap-length product (PL product): the required voltage depends only on this product, and not on the individual parameters. The resulting curve, known as the Paschen curve, is shown in Figure 3-4. For a PL product of 6 mbar.mm, there is a minimum in the breakdown potential, indicating a preferred breakdown path. At atmospheric pressure, the Paschen minimum corresponds to a gap length of 6 μm. Consequently, any reasonable gap length corresponds to a point on the right of the Paschen minimum, and so the breakdown voltage simply increases approximately linearly with gap length. At $10^{-2}$ mbar, however, the optimum gap length is 60 cm. The counterintuitive significance of this is that a discharge will now not strike along the shortest available path, but will take longer and even curved breakdown paths.

To ensure that a high-intensity beam suffers no distortion or defocusing as it is brought to a focus at the capillary entrance, the background pressure in the vacuum chamber must be maintained at less than 1 mbar. At such pressures, the path length corresponding to the minimum breakdown voltage is greater than 6 mm. Care must be taken to block long paths from high voltage electrodes to ground, for example the path from the anode to the vacuum chamber walls, since such paths may break down at a lower voltage than the desired path through the capillary.
Figure 3-4: The Paschen curve shows the voltage required to break down an air gap as a function of the pressure × gap length product (PL product). The minimum breakdown potential occurs at 6 mbar.mm.

Figure 3-5 shows the optimum pressure for breakdown as a function of path length. The range of capillary lengths that might be desired is indicated, as well as the range of lengths of potential unwanted discharge paths around the device or to the chamber walls. For the required pressure of less than 1 mbar, it is clear that the optimum path length is such that undesirable long breakdown paths are likely to be favourable. It is difficult to block all of these paths from the discharge anode to the chamber walls and round the outside of the capillary, since there must be optical access to the ends of the capillary.

The best solution was found to be to evacuate the chamber using an oil diffusion pump to a pressure of less than $3 \times 10^{-3}$ mbar. At this pressure, the optimum breakdown path was greater than 2 metres, and so all of the available paths, including the path through the capillary, were far shorter than the optimal path length, and so were able to hold off voltages in excess of 3 kV.
The Paschen curve is valid when the electric field in the gap is approximately uniform. However, it must be borne in mind that significant departures from the curve can result owing to field-emission from non-uniform electrodes. Other factors, such as surface contamination, can also become important at low pressures. Furthermore, the desired breakdown path through the capillary would not necessarily be expected to behave precisely in accordance with the Paschen curve. The capillary walls significantly alter the dynamics of the breakdown process, and can reduce the breakdown potential.

The group of Zigler found that a voltage of 10 kV was sufficient to break down a 10 mm-long initially-evacuated capillary. To initiate a discharge through the capillary, the group used a spark gap to connect a charged capacitor across the capillary. This approach has two problems. Firstly, to scale the waveguide to longer lengths would require an increase in the charging voltage of the discharge circuit. If the scaling were linear with waveguide length, the voltage required would soon become prohibitive. Secondly, since the charging voltage of the
main discharge circuit controls the breakdown, it is not possible to change that voltage freely in order to control the properties of the current pulse through the capillary.

An alternative approach was pursued in the present design. The breakdown was initiated using a second circuit to trigger the discharge, thus decoupling the breakdown process from the main discharge circuit. The main discharge circuit was simply a capacitor charged to 1 - 3 kV connected in parallel with the capillary. At such a low voltage, no spontaneous discharge occurred. To initiate the discharge through the capillary a pre-plasma was created at one end of the capillary using the trigger structure discussed below. The presence of the pre-plasma decreased the voltage required to break down the main capillary, allowing the main discharge to strike through the capillary.

After the design of the trigger discharge was completed, Zigler's group published new work that showed that they had independently modified the design of their waveguide to include a trigger discharge (Kaganovich et al. 1997).

### 3.2.2 Device schematic

Figure 3-6 shows a photograph and diagram of the final waveguide that was used for the experiments described in the next chapter. The design was centred around a 20 mm-diameter cylindrical polypropylene insert with a 350 μm-diameter hole drilled on axis. These inserts were disposable, and could be replaced when the capillary diameter had become significantly enlarged. A large cathode was separated from a trigger electrode by a 1 mm-thick replaceable spacer, all of which had a 0.5 mm-diameter hole through their centres. The capillary insert lay between the trigger electrode and the anode, and the whole assembly was held in place by a transparent Perspex casing. The casing had space to house 20 mm-long capillary inserts, or 10 mm-long inserts along with a 10 mm spacer. The Perspex casing and the cathode were carefully designed to block all breakdown paths from the trigger electrode to the chamber walls.

The cores of two BNC cables passed through modified BNC bulkhead connectors mounted on the cathode, and allowed electrical connections from the discharge circuit and the trigger pulse generator to be made to the anode and trigger electrode respectively. Figure 3-7 shows the fully-assembled waveguide, complete with the transparent Perspex casing that held the components in line with one another.
Figure 3-6: Device shown without the Perspex casing (left) and a schematic drawing showing the major parts (right).

Figure 3-7: The assembled waveguide, showing the flying leads ending in vacuum connectors, described in section 3.2.5. The cathode is on the left side of the Perspex housing.
3.2.3 The trigger gap

As discussed in section 3.2.1, the pressure in the vacuum chamber that housed the waveguide was maintained at below $3 \times 10^{-3}$ mbar. This was found to be sufficiently low that when the discharge voltage of up to 3 kV was connected to the anode, no discharge occurred either through the capillary to the cathode, or to the walls of the vacuum chamber. To initiate the discharge, a fast-rising voltage spike was applied to the trigger electrode, causing a small discharge to the cathode. The plasma produced initiated the main discharge through the full length of the capillary.

Several designs for the trigger structure were investigated. The PL product for the trigger gap at a chamber pressure $3 \times 10^{-3}$ mbar was well to the left of the Paschen curve minimum. Indeed it was found that a 16 kV voltage pulse applied to a simple 2 mm trigger gap, as shown in Figure 3-8 a), was not sufficient to cause breakdown. In an attempt to lower the potential required to break down the trigger gap, a second design was investigated, relying on a breakdown along two insulating surfaces shown in Figure 3-8 b). The breakdown potential was lowered by the presence of the surfaces, but was strongly dependent on the condition of the surfaces, which quickly became coated with a carbon deposit. The final design, shown in Figure 3-8 c), included a scalpel blade attached to the inside of the trigger electrode. The blade was found to lower the potential required to break down the trigger gap considerably, an effect attributed to field-emission from the sharp metal edges. The performance of the trigger gap was unaffected by the condition of the surfaces of the trigger washer.

The discharge could be run at a pulse repetition rate of up to 1 Hz, with that limit set by the time required to charge the main capacitor. During series of discharges, it was found that occasionally an unwanted breakdown occurred from the trigger electrode to the cathode around the side of trigger washer. It was postulated that the incorrect breakdown was stimulated by expulsion of the high-density plasma generated by the discharge into the region outside the trigger electrode. By adding o-rings between the inserts and electrodes, as shown in Figure 3-8 c), the expulsion of plasma was prevented, and incorrect breakdowns were no longer observed.
3.2.4 Trigger pulse generation and MPC

The pulse generator used to trigger the main discharge was a critical part of the device, since it determined the jitter of the time of initiation of the main discharge relative to the firing of the trigger circuit. The breakdown of the trigger gap was found to occur when the voltage applied to the trigger electrode was in the range 8-12 kV. The pulse generator was thus designed to meet two requirements. Firstly, the rise-time of the output voltage must be sufficiently short that the jitter in the breakdown voltage of the trigger gap did not cause a severe jitter in the breakdown time. Secondly, the time at which the voltage pulse appeared at the output of the pulse generator was required to have less that 10 ns jitter relative to the timing pulse used to trigger the pulse generator. A jitter significantly larger that this would inhibit accurate timing of the injection of a guided laser pulse relative to the discharge current pulse and the dynamics of the capillary plasma.
Figure 3-9: Schematic diagram of the magnetic pulse compression (MPC) trigger pulse generation circuit. All capacitor were 0.43 nF. Voltage points A, B, and C refer to the traces in Figure 3-10.

Figure 3-10: Voltages measured at various points in the MPC trigger-pulse generation circuit. Voltage probe points A, B, and C are indicated in Figure 3-9. At each stage of compression the rise-time of the voltage pulse decreases. There is a voltage step-up on the last stage due to the decreased capacitance of the final stage.

Figure 3-9 shows diagrammatically the pulse generator that was designed to meet the required specification. A thyristor was used to generate a 400 V voltage step, which was converted
using a transformer to a 13 kV voltage pulse with a rise-time of 1 μs. The technique of
magnetic pulse compression (MPC), which uses capacitor stages separated by saturable
inductors to successively compress a voltage pulse, was used to decrease the rise-time of the
voltage pulse to just 30 ns (Melville 1951). The form of the voltage pulse at various points in
the circuit is shown in Figure 3-10. The first stage of the MPC circuit compressed the pulse
rise-time from 1 μs to approximately 200 ns. The second stage further decreased the 30-70%
rise-time to 30 ns, and increased the pulse voltage from 13 to 16 kV owing to a decrease in
the capacitance of the final capacitor stage. The jitter in the position of the voltage pulse
relative to the timing pulse sent to the thyristor was measured to be less than 10 ns.

Figure 3-10: Diagram of the high-voltage connectors used to pass high-voltage signals
through the window of the vacuum chamber.

3.2.5 HV vacuum connectors

Significant problems were encountered with the high-voltage BNC feedthroughs originally
used to pass high-voltage signals through the walls of the vacuum chamber. It became
obvious that the feedthrough used for the trigger pulse was failing due to a breakdown inside
the connector. As a result, the trigger electrode was never exposed to full trigger voltage. The
failure was caused by the vacuum environment, which lowered the breakdown potential of the surface-tracking discharge inside the connector.

Connectors designed to work at atmospheric pressure can prevent unwanted breakdowns occurring by increasing the length of unwanted breakdown paths until the PL product of the path is sufficiently far to the right of the Paschen curve minimum. As the pressure at which the connector is designed to work is reduced, unwanted paths must be made increasingly long to hold off the same voltage. For pressures below 50 mbar, this approach quickly becomes impractical since a path length of greater than 10 cm would be required to hold off only 10 kV.

For pressures below $10^{-1}$ mbar, an alternative approach becomes feasible. Rather than insisting that unwanted breakdown paths have PL products that are well to the right of the Paschen curve minimum, it is possible instead to ensure all such paths are well to the left of the Paschen curve minimum. For a pressure of $10^{-1}$ mbar, a path of length 10 mm or less will be able to hold off 10 kV. As the pressure is reduced, such paths are able to hold off an increased voltage.

Using this approach, a vacuum-compatible feedthrough shown in Figure 3-11 was designed and tested. The core of the cable in the vacuum chamber is mated with a copper pin that connects to the positive pole of a BNC connector outside the chamber. The connection region is closely surrounded by a copper shell, which is earthed to the sheath of the cable in the vacuum chamber, and to the earth of the BNC connector outside the chamber. For the connector to function correctly, the 5 mm gap between the high-voltage copper pins and the earthed casing must be able to hold off up to 18 kV. The connector was found to be suitable for use at pressures less than $10^{-1}$ mbar, at which the PL product of the breakdown path was less than 0.5 mbar.mm. This was sufficiently far to the left of the Paschen curve minimum that breakdown did not occur.

The vacuum connectors were designed at a time when no suitable connectors were commercially manufactured. Since that time, vacuum connectors based on the same principle described here have become available.

### 3.3 Electrical lifetime of the capillary

The electrical properties of the slow capillary discharge were studied as a function of capillary age. The discharge circuit consisted of a 350 µF capacitor charged to between 1 and 3 kV, connected in parallel with the capillary. The residual circuit inductance was found to be 0.907 µH, measured by finding the resonant frequency of the circuit. The profile of the
discharge current was monitored using a Rogowski coil surrounding the wire connecting the main capacitor to the anode.

Correct firing of the discharge was possible for chamber pressures of less than $7 \times 10^{-3}$ mbar. At higher pressures, the plasma created by the trigger pulse initiated a breakdown from the anode to the chamber wall.

The peak discharge current was found to be proportional to the capacitor charging voltage. Figure 3-12 shows the measured discharge current as a function of shot number, for a 10 mm-long capillary with an initial diameter of 350 \( \mu \text{m} \), for a charging voltage of 2 kV. The peak value of the current increased as more shots were fired. This increase was caused by a decrease in the conducting resistance of the capillary discharge, owing to the widening of the capillary diameter by ablation of the walls. After 215 shots, the capillary diameter had been widened from 350 \( \mu \text{m} \) to 440 \( \mu \text{m} \). The lifetime of a capillary was dictated by this rate of widening.

![Figure 3-12: Temporal profile of the discharge current for different shot numbers, for a 350 \( \mu \text{F} \) capacitor charged to 2 kV. The capillary diameter was initially 350 \( \mu \text{m} \) and had increased to 450 \( \mu \text{m} \) after 215 shots.](image)

Theoretical work (Loeb and Kaplan 1989) on confined discharges shows that the resistance \( R \) of the plasma generated in the capillary should obey the scaling law:
in which $I$ is the discharge current, $d$ is the capillary diameter, and $l$ is the capillary length. The strong scaling of resistance with capillary diameter is responsible for the rapid increase in the peak discharge current as the capillary becomes widened. Loeb et al. also derived a scaling law for the plasma electron temperature $T_e$:

$$T_e \text{ [eV]} = 3.3 \frac{I \text{ [kA]}}{350} \left( \frac{d \text{ [μm]}}{350} \right)^{\frac{4}{11}} l \text{ [cm]}$$

yielding an electron temperature of 1 - 2 eV at the peak of the current.

Figure 3-13: Measured discharge current through a 10 mm-long, 450 μm-diameter capillary from a 350 μF capacitor charged to 2 kV. Using the measured inductance of 0.907 μH, the resistance of the capillary inferred from the current pulse is shown, along with a theoretical prediction of the resistance and electron temperature from the work of Loeb et al. (Loeb et al. 1989).
Figure 3-13 shows the resistance of the capillary plasma calculated by modelling the circuit as an LCR circuit with a time-dependent resistance, using the measured current pulse and the known inductance and capacitance of the circuit. The calculated resistance is very sensitive to noise when the current is close to zero, resulting in large random errors in the calculated resistance at late times. Also shown is the resistance and electron temperature calculated from equations (3.1) and (3.2). The good agreement between the measured and predicted resistance lends weight to the theoretical model.

3.4 Summary

A discharge-ablated capillary waveguide was designed, based on the work of the group of A. Zigler from Hebrew University (Zigler et al. 1996). The design was based around disposable polypropylene cylinders with capillaries as small as 350 μm drilled on axis. The inserts could be replaced when the diameter of the capillary became widened by the discharge.

The main discharge circuit was charged to between 1 and 3 kV, with the discharge initiated using a fast-rising trigger pulse applied to a trigger gap at one end of the capillary. The use of a dedicated trigger circuit reduced the jitter of the timing of the discharge to less than 10 ns. Furthermore, since the main discharge circuit played no part in the initiation of the discharge, the charging voltage could be freely changed without altering the timing, or the jitter, of the discharge.

The resistance of a 10 mm-long 350 μm-diameter capillary plasma was found to be between 2 and 3 ohms during the main part of the current pulse, in good agreement with scaling laws derived by Loeb et al. (Loeb et al. 1989). Scaling laws by the same authors predict that the electron temperature in the plasma reached a peak of 2.5 eV.

The capillary plasma created by the discharge has a radial electron density profile suitable for guiding high-intensity laser pulses. In the next chapter, the results of experiments to investigate the ability of the device to guide picosecond laser pulses will be presented.
References


In this chapter, the results of the first study of the channelling of picosecond laser pulses through a discharge-ablated capillary waveguide are presented. The performance of the waveguide was assessed by measuring the energy transmission and the spot size in the exit plane of the waveguide. An energy transmission of up to 65% was achieved through 10 mm-long capillaries for laser pulse energies of approximately 0.5 J, at a peak input intensity of $1 \times 10^{16}$ W cm$^{-2}$. Detailed measurements are presented showing the variation of the pulse energy transmission during the ablation current, and the temporal evolution of the transverse spatial profile of the guided beam in the exit plane of the capillary.

### 4.1 Experimental arrangement

#### 4.1.1 Nd:Glass laser

A Nd:Glass laser system based on the chirped pulse amplification technique was the source of the high intensity optical pulses propagated through the capillary waveguide. The laser system, at Imperial College, London, has been described previously (Luan et al. 1993). The laser system produced a beam with a diameter of 12 mm, at a wavelength of 1.053 nm. The pulse duration was measured to be 2.5 ps, and the average pulse energy was 460 mJ.

Figure 4-1 shows images of the transverse profile of the laser pulse in the plane of best focus, and 5 mm before and after that plane. The images were captured using the imaging system
described below and shown in Figure 4-2. The beam propagated with asymmetric aberrations either side of focus. In the plane of best focus, the average spot size in the $x$- and $y$-directions was estimated to be 32.5 μm (the spot size is defined as the radius at which the intensity has decreased to $1/e^2$ of its peak value). Hence at the average incident beam energy of 460 mJ, and for a pulse duration of 2.5 ps, the peak intensity was approximately $1 \times 10^{16}$ W cm$^{-2}$.

![Figure 4-1: Transverse profiles of the laser pulse in the focal region, taken 5 mm before the focal plane (left), at focus (centre) and 5 mm after the focal plane (right).](image)

Laser pulses rarely focus as tightly as would be expected for an ideal Gaussian beam. Such anomalies are caused by non-Gaussian structure in the transverse intensity profile of the laser, and distortion on the phase of the wavefront across the beam diameter. In many cases, the propagation behaviour of the non-ideal beam can be characterized by the “$M^2$” of the beam, defined as the ratio of the measured focal spot and the ideal diffraction-limited focal spot (Siegman 1986).

The images presented in Figure 4-1 show that the beam quality was clearly not diffraction-limited, indicated by the aberrations that were apparent either side of focus. For an ideal Gaussian beam with diameter of 12 mm and wavelength 1053 nm, a spot size of 10.5 μm would be expected when focused using lens of focal length 500 mm. The measured spot size of 32.5 μm indicates that the beam had an $M^2$ of approximately three.

4.1.2 Optical set-up

Figure 4-2 shows schematically the experimental arrangement employed. Radiation from the Nd:Glass laser was focused into the capillary by an $f/10$ silica lens of 500 mm focal length. Radiation leaving the capillary was collected and rendered approximately parallel to the axis of the optical system by an $f/5$ lens of 250 mm focal length, placed 250 mm from the exit face.
of the capillary. The intensity of the transmitted radiation was reduced by reflections from a series of wedged optical flats, and transmission through thin, optical quality, neutral density filters, before being focused by a second f/5 lens of 250 mm focal length. The image so formed was magnified onto a CCD camera by a ×20 microscope objective, and captured by an 8-bit CCD camera. The magnification of the imaging system was calibrated by recording the image of a wire of 230 μm-diameter, placed in the plane of the exit of the capillary. The axis of the capillary was aligned carefully with that of the incoming laser radiation with the aid of a CW oscillator beam propagated through the laser system at low power.

The pulse energy of the radiation entering the capillary was determined by recording the peak signal from a calibrated photodiode illuminated diffusely by radiation reflected by a thin optical flat placed in the beam path. The transmitted energy was determined from the total signal recorded by the CCD camera, after subtraction of any low-level background signal. The background signal was determined separately for each data point by analysing a portion of the captured image well away from the image of the transmitted beam. The CCD reading was calibrated against the reading of an energy meter placed behind one of the optical flats, its reading having been cross-calibrated against that of the photodiode in the absence of the capillary and its associated structure.

A digital delay generator allowed control over the relative timing of the capillary discharge and the arrival of the laser pulse at the entrance to the capillary. A Rogowski coil placed around the lead to the anode measured the discharge current. The time of arrival of the laser pulse at the entrance of the capillary relative to the onset of the discharge was determined by measuring the onset of the discharge current relative to a temporally calibrated trigger signal from the laser amplifier chain.

4.2 Results

Figure 4-3 shows the measured transmission of the capillary waveguide as a function of the time t of arrival of the laser pulse at the entrance to the capillary after the onset of the discharge current. The error bars were calculated from the standard deviations of the calibrations of the photodiode and CCD signals. The measured transmission of the capillary in the absence of a discharge current was recorded on four occasions during the data run, and the average and standard deviation of these measurements is indicated at t = 0. The observed transmission of approximately 47% at t = 0 is consistent with that expected for a beam focused with an f/10 lens at the capillary entrance and transmitted through an aperture of 350 μm-diameter at the capillary exit.
Figure 4-2: Schematic diagram of the experimental arrangement used to investigate the guiding of high-intensity laser pulses in a discharge-ablated capillary waveguide.
Figure 4-3: The transmission of the waveguide as a function of delay $t$ after the initiation of the discharge current. The current is shown on the right-hand axis.

The measured transmission is seen to remain nearly constant for about 150 ns after the discharge is fired, before increasing and reaching a peak of approximately 65% around $t = 300$ ns. Thereafter the transmission falls, and is almost zero at times greater than 800 ns after the initiation of the discharge.

Figure 4-4 (left column) shows three-dimensional plots of the intensity profile at the exit of the capillary, normalized to the average input pulse energy, and in Figure 4-4 (right column) the same data is shown in greyscale plots, which illustrates the transverse spatial profile more clearly. The transverse dimensions of the beam are seen to be substantially reduced as the current increases, and there is a concurrent increase in the intensity on axis by an order of magnitude. The maximum output intensity is observed for times close to the time of maximum energy transmission of the guide. Once the discharge current is fully established the transverse dimensions of the exiting beam are found to be significantly smaller than the diameter of the capillary.
Figure 4-4: The fluence profiles recorded in the exit plane of the 10 mm-long waveguide for several delays.
At the time of peak transmission, the spot size in the exit plane of the capillary was approximately twice that of the input beam, resulting in a normalized axial intensity of $1.4 \times 10^{15}$ W cm$^{-2}$. One reason for the greater spot size in the exit plane is that, as discussed above, the incident laser beam was not perfectly diffraction-limited. This may have been responsible for the non-cylindrically-symmetric structure observed in the profiles of the exiting beam shown in Figure 4-4, and may have increased the minimum spot size achieved by the waveguide.

4.3 Conclusions

The temporal behaviour of the energy transmission and output pulse intensity profile of intense picosecond laser pulses channelled by a discharge-ablated capillary waveguide has been measured. The peak energy transmission for a 10 mm-long waveguide was measured to be 65% for input pulses with an intensity of $1 \times 10^{16}$ W cm$^{-2}$.

The clear increase in the laser energy transmitted by the capillary shows that the device successfully channelled the laser radiation to some extent. Furthermore, the transverse dimensions of the beam in the exit plane of the capillary were significantly reduced during the discharge current.

An important proviso must be stressed at this point regarding the interpretation of these results. For the above experiment, the spot size of the guided laser beam was known at the entrance and exit of the capillary only: it was not possible to measure the transverse spatial extent of the beam within the waveguide. The fact that the laser pulse energy was confined close to the axis at the capillary exit does not imply that the spot size was maintained at a small value throughout the length of the waveguide.

As discussed in chapter 2, an ideal plasma waveguide is capable of guiding a Gaussian beam with a constant spot size, provided the laser spot size is set equal to the matched spot size of the waveguide. If the laser spot is mismatched, oscillations of the spot size occur as the laser pulse propagates through the guide, shown in Figure 2.2. In this case, a measurement of a small spot size in the exit plane of a waveguide implies merely that the spot size was near a minimum in the oscillation at the waveguide exit.

For the case of a waveguide in which the plasma is not fully-ionised before the passage of the guided laser pulse, further ionisation of the waveguide may severely alter the propagation of the behaviour of the guided pulse. In order to fully and unambiguously interpret the results of the experiment presented in this chapter, it is necessary measure the spatial density distribution and ion stage of the plasma generated in the capillary. In chapter 5, an
interferometric method of probing the plasma formed by the discharge-ablated capillary waveguide is described. Measurements of the electron density profile created in the capillary allowed numerical simulations of the propagation behaviour of intense laser pulses through the waveguide. Those simulations, presented in chapter 6, enabled a full interpretation of the above experimental results, and highlight the severe limitations of the discharge-ablated capillary waveguide.
References


In the previous chapter, the discharge-ablated capillary waveguide was shown to be capable of channelling intense laser pulses over a distance of 10 mm. From this we can infer that an axially-peaked radial refractive index profile must have been created inside the waveguide. In order to optimise the discharge conditions and capillary parameters to obtain the best guiding, it was decided to directly measure the radial electron density profile created inside the waveguide.

5.1 Alternative approaches

For plasma waveguides in which there is transverse optical access along the length of the guide, transverse interferometry may be used to measure the refractive index profile of the waveguide. The radial variation of the refractive index can then be recovered using an Abel transform that assumes the waveguide had cylindrical symmetry. This technique has been used to characterise the radial electron density profile created by the hydrodynamic expansion of plasmas created by either a laser line focus in a gas (Clark and Milchberg 1997; Volfbeyn, Esarey and Leemans 1999; Gaul et al. 2000) or co-linear pre-pulses (Malka et al. 1997; Mackinnon et al. 1998). The technique has also been used to examine waveguides formed by ponderomotive or relativistic channelling (Chen et al. 1998; Sarkisov et al. 1999), and by
interaction with clusters (Ditmire, Smith and Hutchinson 1998). In many cases, the spatial extent of pulses guided through the channel can also be observed using Thompson scattering of the guided radiation (Krushelnick et al. 1997; Chen et al. 1998; Clayton et al. 1998; Sarkisov et al. 1999).

In the case of waveguides for which there is no transverse optical access to the guiding region, such as grazing incidence waveguides (Jackel et al. 1995) and capillary discharge waveguides (Ehrlich et al. 1996), these techniques cannot be used to measure the channel properties, or the degree of confinement of the guided beam within the waveguide. Several other approaches have been investigated by the group of Zigler. Transverse interferometry has been used to observe the profile of the plasma ejected from the ends of a discharge-ablated capillary waveguide (Kaganovich et al. 1999a). That work showed that the plasma extends for a few hundred micrometers beyond the capillary exit. However, measurements could be made no closer that 50 μm to the capillary exit, and it is likely that the profile measured was strongly distorted by end effects. The transverse spatial profile of Thompson scattering of radiation leaving the waveguide has also been recorded (Kaganovich et al. 1999b). However, that measurement does not give insight into the behaviour of the pulse within the guide. Ehrlich et al. have measured the Stark width of the Hα emission from the ends of the capillary, viewed longitudinally and recorded as a function of radius (Ehrlich et al. 1998). From this data, Ehrlich et al. inferred the electron density across the radius of the capillary. Unfortunately, the measurement was again affected by end effects; the electron density that existed in the main body of the waveguide was not probed.

In this chapter, the results of interferometric measurements taken along the axis of a discharge-ablated capillary waveguide are presented. Longitudinal measurements of this type probe the refractive index as a function of radius, averaged along the length of the waveguide. While end effects contribute to the average refractive index they do not dominate, and so information is obtained directly about the guiding profile inside the channel.

### 5.2 Experimental set up

The set up used for longitudinal interferometry is shown schematically in Figure 5-1. The discharge-ablated capillary waveguide was placed in a vacuum chamber evacuated to a pressure of $10^{-3}$ mbar. The waveguide was located in one arm of a Mach-Zehnder interferometer, with the capillary aligned coaxially with the probe beam. The interferometer was illuminated by pulses of approximately 8 ns duration from a Nd:YAG laser, which could be operated at either 532 nm or 355 nm. A lens of 250 mm focal length was placed after the
second beam splitter, forming an image of the exit plane of the capillary. This image was then magnified by a ×20 microscope objective onto a CCD array. The imaging system ensured that the fringe pattern recorded was that which would be observed in the exit plane of the capillary. The interferometer was adjusted to produce high-contrast fringes, with 20 to 30 fringes across the diameter of the capillary. The broadband emission from the discharge plasma was blocked by a band-pass filter with a peak transmission at the probe wavelength.

A digital delay generator was used to control the delay $t$ between the onset of the discharge current and the arrival of the laser probe pulse, as recorded by a photodiode placed after the second beam splitter.

Timing and data collection were computer controlled using the LabVIEW graphical language. A complete set of data for a capillary consisting of a series of measurements for approximately 30 values of $t$ could be taken in under 60 seconds. In this way, widening of the capillary diameter during the experiment was minimised. Fringe patterns were recorded in pairs. First, a ‘background’ pattern was recorded without firing the discharge, and then a ‘data’ pattern was recorded whilst firing the discharge. The delay $t$ was measured automatically by an oscilloscope. A series of fringe pattern pairs were recorded with $t$ ranging from 0 to 600 ns, at both 532 nm and 355 nm. A sample data fringe pattern taken at $t = 340$ ns is shown in Figure 5-2 for a capillary with an diameter of 380 μm.
Figure 5-1: The experimental set up used for measuring the radial electron density profile of a waveguide using longitudinal interferometry.

Figure 5-2: A typical data fringe pattern recorded for a 5 mm-long discharge-ablated capillary waveguide, at $t = 340$ ns.
5.3 Recovery of the phase shift from the fringe patterns

Having recorded a series of fringe patterns, it was necessary to analyse the distortion of the fringe data to recover the desired information. Consider a 2-dimensional fringe pattern that is of the form

\[ g(x,y) = a(x,y) + b(x,y)\cos[2\pi f_0 x + \varphi(x,y)] \]  

(5.1)

where \(a(x,y)\) is a varying background, \(\cos[2\pi f_0 x + \varphi(x,y)]\) is a fringe pattern with a spatial frequency \(f_0\), distorted by a phase shift \(\varphi(x,y)\). The fringes are taken to be parallel to the \(y\)-axis have a varying amplitude given by \(b(x,y)\). The wedge fringes are uniform across the beam when there is nothing placed in the arms of the interferometer. The phase shift \(\varphi(x,y)\) seen when the capillary plasma is placed in one arm is proportional to the change in optical path length experienced by the beam. This change is caused by the refractive index of the capillary plasma, and so \(\varphi(x,y)\) encodes the longitudinally-integrated refractive index of the capillary plasma as a function of transverse position.

To recover the phase shift, a method utilising Fourier transforms was employed (Takeda, Ina and Kobayashi 1982). Equation (5.1) can be rewritten in terms of complex exponentials:

\[ g(x,y) = a(x,y) + [c(x,y)e^{2\pi i f_0 x} + \text{complex conjugate}] \]  

(5.2)

in which the amplitude and phase of the fringe pattern have been combined into a complex amplitude given by

\[ c(x,y) = \frac{1}{2} b(x,y)e^{i\varphi(x,y)} \]  

(5.3)

Taking the 1D Fourier transform of equation (5.2) gives

\[ G(f,y) = A(f,y) + C(f - f_0, y) + C(f + f_0, y) \]  

(5.4)

This is simply the Fourier spectrum of the background level \(a(x,y)\), written \(A(f,y)\), superimposed with two identical sidebands \(C\) at \(f = \pm f_0\): \(C(f,y)\) is the Fourier transform of \(c(x,y)\). If the background level and the desired phase shift both vary slowly on the length-scale of the fringe pattern, there will be no overlap between these three components. One of the sidebands is selected and shifted to \(f = 0\). At this stage, the component from the background level has been filtered out. Finally, the complex amplitude \(c(x,y)\) is recovered by taking the inverse Fourier transform of the shifted sideband, and the phase calculated by taking the imaginary part of the complex logarithm of \(c(x,y)\):

\[ \varphi(x,y) = \arg(c(x,y)) \]
\ln [g(x, y)] = \ln \left[ \sqrt{b(x, y)} \right] + i \varphi(x, y) \tag{5.5}

The recovered phase is indeterminate to within the addition of multiples of $2\pi$. However, for the case of a discharge-ablated capillary waveguide, the phase varies smoothly in space. The unwrapped phase map can thus be recovered by adding multiple of $2\pi$ where required to ensure that the phase map is continuous.

5.3.1 Computation and fast Fourier transforms

A C++ computer code was written to perform the above phase retrieval algorithm. The method of fast Fourier transforms was used, which is a recursive algorithm that is used to evaluate the complex Fourier transform of a discretely sample data set with $Y = 2^X$ elements, where $X$ is an integer. For a $Y$ element array, the Fourier spectrum contains $Y/2$ positive frequencies and corresponding negative frequencies. Since the fringe pattern is real, the transform $G(f, y)$ in equation (5.4) has equal complex coefficients for each positive-negative frequency pair. Hence, the two sidebands $C$ are mirror images of each other. One sideband is selected and translated to $f = 0$ by a discrete number of frequency steps. Taking the inverse fast Fourier transform retrieves $g(x, y)$ from which the phase is recovered by taking a complex logarithm.

An example of the process is shown in Figure 5-3. The fringes in the exit plane of the 5 mm-long capillary are captured using a CCD camera shown in (a). Taking the Fourier transform of a horizontal section through the centre of the image results in the frequency spectrum shown in (b). Notice the strong component at zero frequency corresponding to a DC light level, and the two identical sidebands at around ±75 cycles per 1024 pixels. The structure on these sidebands encodes the distortion of the fringe pattern. The positive sideband is clipped out of the spectrum and translated to zero frequency, shown in (c). The width of the clipped portion determines the resolution of the final fringe shift data obtained. If too wide a portion is taken, the results are susceptible to unwanted noise. The clipping shown is at approximately half the fringe frequency, and so the spatial resolution of the final fringe shift data will be roughly twice the fringe wavelength. By taking the inverse Fourier transform of the shifted sideband, followed by a complex logarithm, the fringe shift as a function of radius is recovered, as shown in (d).
Figure 5-3: Example showing the fringe analysis procedure. Image (a) shows the CCD image of the fringes in the exit plane of the capillary. A horizontal section through the centre of the image is Fourier transformed to obtain the frequency content (b). One sideband is clipped out of the spectrum and translated to zero frequency (c). The inverse Fourier transform and a complex logarithm recovers the fringe shift as a function of radius (d).

5.3.2 Removal of the residual phase error

Unfortunately, the technique described above leaves a residual error on the recovered phase that takes the form of a slope superimposed on the phase map. The error arises if there is not an integer number of fringes in the field of view. In that case, the fringe frequency is not a member of the set of discrete frequencies that make up the Fourier spectrum. Equation (5.4) tells us that sidebands are shifted to $\pm f_0$, and so if $f_0$ is between two of the Fourier frequencies we cannot shift the sideband exactly to zero frequency. The residual offset leaves...
an undesired low frequency component in the recovered function $g(x, y)$, which leads to a residual slope in $\varphi(x, y)$. This residual phase error is not discussed in the paper by Takeda et al. (Takeda et al. 1982) on which the method used here was based.

In order to overcome this problem the data was analysed as follows. As stated above, for each data shot both a background and a data fringe pattern were taken. For both the background and data patterns, a phase map of the beam at the exit of the capillary was calculated. The background shot phase map was subtracted from the data shot phase map, which served to correct for aberrations in the interferometer. The residual slope in $\varphi(x, y)$ was also removed, since the error was the same for the background and data shots. The phase map so obtained represents the unwrapped relative phase profile of the beam after propagating through the discharge plasma.

The phase shift measured at the capillary axis was initially set to lie between 0 and $2\pi$. By ensuring that the phase varied continuously in both the $x$ and $y$ directions, a smooth phase map was obtained. Such phase maps were measured for a range of $t$. The plasma density in the capillary was zero for $t \leq 0$ since the capillary was initially evacuated. Since the plasma density can be assumed to vary smoothly as $t$ was increased, the total phase shift at the capillary axis may be tracked by analysing the time sequence of fringe patterns. Provided the axial phase change between adjacent patterns is much less than $\pi$, multiples of $2\pi$ can be added to phase maps as required to ensure that the axial phase shift is continuous in moving through the time sequence of measurements. In this way, absolute phase shift maps were obtained for a range of $t$.

### 5.4 Calculation of the radial electron density profile

The phase shift $\varphi(x, y)$ caused by the presence of the discharge plasma is given by

$$\varphi(x, y) = \frac{\omega}{c} \int_0^L \Delta \eta(x, y, z) \, dz$$  \hspace{1cm} (5.6)

in which $z$ is measured along the capillary of length $L$, and $\Delta \eta(x, y, z)$ is the change in refractive index due to the plasma and $\omega$ is the angular frequency of the probe laser. The refractive index change $\Delta \eta$ has contributions $\Delta \eta_e$ due to free electrons, and $\sum \Delta \eta_{ion}$, which is a sum of contributions due to each ion species present. For an electron density $N$, we can write (Thorne 1974)
As discussed below, the ion species predominant in the discharge plasma are expected to be C, C\(^+\), C\(^{2+}\), H and H\(^+\). An upper limit for the ion refractive index of each ion species can be obtained by considering only the resonance transition of an ion and setting the oscillator strength to unity, giving (Thorne 1974)

\[
\Delta \eta_{\text{ion}}(x, y, z) = \frac{N_{\text{ion}}(x, y, z) \epsilon^2}{2 \epsilon_0 m \omega^2} \]  

(5.7)

in which \(\omega_{\text{ion}}\) is the angular frequency of the resonance line of the ion and \(N_{\text{ion}}\) is the ion density. For the ions and wavelengths of interest, \(\omega < \omega_{\text{ion}}\), and so \(\sum \Delta \eta_{\text{ion}}\) increases as \(\omega\) is increased, in contrast with \(\Delta \eta_e\). Assuming \(N_{\text{ion}}\) is of the same order as \(N\) and using the known values of \(\omega_{\text{ion}}\), it is found that only un-ionised carbon could potentially contribute significantly to the refractive index change. For a probe wavelength of 355 nm, if the density of neutral carbon were comparable to \(N\), the ion contribution would be expected to be of the same magnitude as \(\Delta \eta_e\).

In order to verify that \(\Delta \eta\) was predominantly due to electrons, the wavelength dependence of \(\varphi(x, y)\) was investigated experimentally. Measurements of the absolute phase shift, for \(t\) ranging from 0 to 200 ns were made at 532 nm and 355 nm, using the discharge conditions described above. Measurements could be made no later than \(t = 200\) ns at 532 nm due to distortion of the probe beam, discussed below. The phase shift measured on the capillary axis was found to be approximately inversely proportional to \(\omega\) at all times, indicating that \(\Delta \eta_e \gg \sum \Delta \eta_{\text{ion}}\). The density of un-ionised carbon in the plasma must therefore be much less than \(N\), and the phase shift maps can be used to calculate the electron density in the capillary.

Combining equation (5.6) with equation (5.7), we can write the electron density \(N(x, y)\) averaged along the capillary of length \(l\) as

\[
\varphi_e(x, y) = \frac{N(x, y) \epsilon^2 l}{2 \epsilon_0 \epsilon_m \omega} \]

(5.9)

This equation can be used to calculate the electron density profile across the capillary radius from the measured phase shifts.
5.4.1 Limitations on capillary length

Since the probe beam travels through a waveguide, it is necessary to consider distortion of the probe beam owing to the focusing properties of the plasma under investigation. Undesired deflection of the probe beam limits the maximum curvature and length of guide that can be probed with longitudinal interferometry. In this section, the maximum length that can be probed without distortion is calculated.

A plane wave injected into a parabolic guide of the form

$$N(r) = N(0) + \Delta N \left( \frac{r}{r_{ch}} \right)^2$$  \hspace{1cm} (5.10)

undergoes periodic focusing (Siegman 1986) with the ray at radius $r_0$ following the locus $r = r_0 \cos Kl$, in which

$$K = \sqrt{\frac{\Delta N e^2}{\varepsilon_0 m 0 r_{ch}^3}}$$  \hspace{1cm} (5.11)

For a waveguide of length $z$, the deflection $\Delta r$ of a ray measured at the capillary exit, for the case $\Delta r \ll r_0$, is given by

$$\frac{\Delta r}{r_0} = \frac{\Delta N e^2 l^2}{2 \varepsilon_0 m 0 r_{ch}^3}$$  \hspace{1cm} (5.12)

Rewriting this results in terms of the electron density gradient at radius $r = r_0$ gives

$$\Delta r = \frac{dN}{dr} \bigg|_{r=r_0} \frac{e^2 l^2}{4 \varepsilon_0 m 0^2}$$  \hspace{1cm} (5.13)

This results allows calculation of the expected distortion of a collimated probe beam passing through a waveguide, in terms of the electron density gradients sampled by the beam. For interferometric measurements to be reliable, the deflection $\Delta r$ must be very much less than the capillary radius.

The deflection $\Delta r$ is proportional to $l^2 \lambda^2$. However, it is also necessary to consider the sensitivity of the measurement of phase shift $\varphi$, which from equation (5.9) is proportional to $l \lambda$. Hence for a given measurement sensitivity, the distortion of the probe beam is the same for all wavelengths. Probe measurements were made with a Nd:YAG laser, and so available wavelengths were 1064 nm, 532 nm, and 355 nm. Due to the importance of minimising end effects, discussed below, it was decided to use a probe beam of wavelength
355 nm which maximises $l$ for a given sensitivity. For the electron density measured in Figure 5-4, the maximum deduced electron density gradient is $5.8 \times 10^{16} \text{ cm}^{-3} \text{ cm}^{-1}$ at a radius $r_0 = 190 \mu\text{m}$. For a 2.5 mm-long capillary probed at 355 nm, equation (5.13) gives $\Delta r = 9.9 \mu\text{m}$: it is concluded that deflection of the probe beam was not significant for the data presented below.

5.5 Measurements for the waveguide used in the picosecond guiding experiments.

Results are now presented for a discharge-ablated capillary waveguide with discharge parameters and capillary material and radius as used in the picosecond pulse guiding experiment, described in chapter 4. Measurements were made using a 2.5 mm-long capillary, limited by the requirement that refractive distortion of the probe beam was at an acceptable level.

Detailed measurements of the time-evolution of the discharge plasma were undertaken using a probe beam of wavelength 355 nm for $t$ ranging from 0 to 600 ns, with a maximum time interval of 75 ns in order to enable tracking of the axial phase. For $t > 600$ ns, distortion of the probe beam prevented unambiguous analysis of the fringe patterns.

Figure 5-4 shows the deduced radial electron density profile, assuming that the plasma was longitudinally uniform and resided within the capillary only. The error bar indicated was estimated by comparing measured electron density profiles obtained under nominally identical conditions. It is seen that during the discharge, the electron density in the capillary increased, and an approximately parabolic well developed. Based on the observed electron density profiles, matched guiding in the waveguide would occur for Gaussian beams with spot sizes of 35 $\mu\text{m}$ and 29 $\mu\text{m}$ at $t = 340$ ns and $t = 578$ ns respectively.

5.5.1 Estimation of end effects

During the discharge, plasma expands into the vacuum chamber leading to a decrease in the electron density near the capillary ends, and the presence of plasma beyond the capillary. To assess the importance of these two end effects, electron density profiles were measured for a 5 mm-long capillary for $t < 210$ ns, this time limit being set by the onset of distortion of the probe beam. The discharge current was the same as for the 2.5 mm measurements. The deduced electron density at $t = 182$ ns, neglecting end effects, is shown in Figure 5-4 (dashed line). Note that the average electron density profile in the 5 mm-long capillary is much higher
that at a similar $t$ in the 2.5 mm capillary, due to the decreased importance of the end effects in the average for the 5 mm capillary.

![Figure 5-4: The deduced radial electron density profile, for various times $t$ during the discharge for a 2.5 mm capillary. The dashed curve shows the electron density profile for a 5 mm-long capillary at $t = 182$ ns.]

The end effects were eliminated as follows. The measured phase shift $\varphi_\ell(r)$ for a capillary of length $L$ may be rewritten from equation (5.9) as

$$\varphi_\ell(r) = \frac{N_\ell(r)e^2[L - 2L_e]}{2ce_0m_0} + 2\varphi_e(r)$$ (5.14)

where $\varphi_e$ is the phase shift due to the non-uniform plasma within and beyond each end of the capillary, $L_e$ is the length of the non-uniform region within the capillary only, and $N_\ell(r)$ is the electron density profile of the uniform central part of the plasma, illustrated in Figure 5-5. If we assume that the end effect is independent of capillary length, then the measurement of $\varphi_\ell(r)$ for two capillary lengths $L_1$ and $L_2$ yields
Hence, we can recover the uniform electron density $N_U$ that exists in the central part of the capillary without assuming the form or extent of the end effect.

Using equation (5.15) and the 2.5 mm and 5 mm data in Figure 5-4, for $t = 179$ ns it is found that $N_U = 3.0 \times 10^{18}$ W cm$^{-2}$ on the capillary axis. Comparison of the two data sets shows that the radial density profiles deduced for the 2.5 mm capillary are approximately 40% of the radial profile in the uniform central region of the plasma, for all $t < 210$ ns. It is assumed that for $t > 210$ ns this ratio is similar. Thus for long capillaries, the electron density profile which exists along the majority of the length of the capillary will have an electron density profile approximately 2.5 times higher than the curves shown in Figure 5-4. This alters the matched spot sizes of the waveguide to 28 μm and 23 μm at $t = 340$ ns and 578 ns respectively.
5.5.2 Average ion stage in the capillary plasma

While the electron density profile alone determines the guiding properties of the plasma channel for low-intensity laser pulses, for high-intensity pulses, it is important to know the ion stage of the plasma constituents, and the plasma temperature.

Loeb and Kaplan have developed a model of confined high-pressure electrical discharges, designed for launching projectiles (Loeb and Kaplan 1989), in which a polypropylene capillary of approximately 2 mm-diameter is used to confine discharge of several kiloamps. Loeb and Kaplan calculated scaling laws for the plasma temperature $T_e$, generated in the capillary in terms of the capillary radius $r_{ch}$, length $L$, and the discharge current $I$:

$$T_e [eV] = 3.3 \frac{I [kA]^4 (r_{ch} [\mu m])^{0.6}}{175} L [cm]$$

(5.16)

While the approximations made in deriving equation (5.16) are more appropriate to a larger capillary diameter, higher current regime, it was shown in chapter 3 that a similar scaling law for the resistance of the capillary plasma [equation (3.1)] was in a good agreement with measurements shown in Figure 3.13. Thus, the above scaling law should give a reasonable estimate of the plasma temperature produced in the discharge-ablated capillary waveguide. For the picosecond guiding experiment $r_{ch} = 175 \mu m$, and $L = 1 \text{ cm}$, giving $T_e = 2.3 - 2.9 \text{ eV}$ for $t = 200 - 600 \text{ ns}$, based on the current profile shown in Figure 4.5.

The ion stage of a plasma may be simply calculated from the electron density and temperature alone, provided that the plasma can be shown to be in local thermodynamic equilibrium (LTE). The condition for LTE that radiative transitions between two states can be neglected compared to collisional transitions can be summarised (Thorne 1974) as

$$\Delta E^3 [eV] \ll \frac{5.8 \times 10^{-15} N [\text{cm}^3]}{T_e^{1/2} [eV]}$$

(5.17)

in which $\Delta E$ is the energy separation. For the electron density measurements shown in Figure 5-4 at $t = 340 \text{ ns}$, the axial electron density is $N = 6.5 \times 10^{18} \text{ cm}^{-3}$, and from equation (5.16) it is found that $T_e = 2.7 \text{ eV}$. From equation (5.17), LTE will exist between states provided $\Delta E < 28 \text{ eV}$. This condition is satisfied for all excited states of hydrogen, and is also satisfied for all of the excited states of C, C+, and C2+, in which the largest transition energy is the resonance line of C2+ at 12.7 eV. Thus H and H+ will be in LTE, and the carbon ions will be in LTE up to the ground state of C3+. 
For ions that are in LTE with one another, the Saha equation can be used to calculate the relative abundance of each ion stage (Thorne 1974). Using the Saha equation and the measured values of electron density and temperature, the plasma composition was estimated to be composed predominantly of H\(^+\) and a mixture of C\(^+\) and C\(^2+\). Since no appreciable population of C\(^4+\) and higher ion stages was predicted, the use of the Saha relation was justified even though C\(^4+\) and higher ion stages would not be in LTE with the lower ion stages.

5.6 Comparison with electron density profiles deduced from Stark broadening of H\(_\alpha\)

The radial profile of the electron density in a discharge-ablated capillary waveguide, determined from Stark broadening of the H\(_\alpha\) emission, has been reported by Ehrlich et al (Ehrlich et al. 1998). Their experiment employed a 600 μm-diameter capillary and discharge currents peaking at between 250 and 430 A after 500 ns. Imaging the exit plane of the capillary, the group recorded spatially and spectrally resolved images of the capillary plasma, and recovered the width of the H\(_\alpha\) emission line as a function of radial position.

The electron density profiles measured by Ehrlich et al. are qualitatively similar to those reported here, although they differ in detail because of the differences in experimental conditions. The technique probes the electron density at the capillary exit only, and so is affected strongly by the capillary end effect. A further disadvantage of technique is that time resolution is of the order of 100 ns, compared to 10 ns achieved using interferometry.

The stark width of H\(_\alpha\) is determined predominantly by the ion density in the plasma rather than the electron density (Griem, Kolb and Shen 1959), and so measurement of the stark width probes the ion density profile in the plasma only. It is the electron density profile that determines the refractive index and thus the guiding properties of the plasma channel, and so it is necessary to convert the ion density profile measurement to an electron density profile. Ehrlich et al. did this using an estimate of the radial plasma temperature profile to determine the ion stage of the plasma, introducing unknown errors into the deduced electron density.

5.7 Summary

The temporal evolution of the radial electron density profile in a discharge-ablated capillary waveguide was measured using longitudinal interferometry. This approach allowed the electron density profile in the central region of the capillary to be probed directly. A pronounced axial minimum in the electron density was observed, suitable for guiding high intensity laser pulses with spot sizes of the order of 25 μm.
The capillary plasma was calculated to be only partially-ionised, being composed of H\textsuperscript{+}, C\textsuperscript{+}, and C\textsuperscript{2+}. When considering the propagation of intense laser pulses through such a waveguide, further ionisation of the plasma must be considered. This will be considered in detail in the next chapter.
References


For the case of radially-infinite parabolic plasma waveguides, analytical solutions exist that describe the behaviour of guided Gaussian laser pulses; these solutions were discussed in chapter 2. Those solutions predict stable propagation of the Gaussian pulse shape, with a constant or oscillating spot size as a function of distance, depending on the mismatch between the input spot size and the matched spot size $W_m$ of the parabolic channel.

In the case of real plasma waveguides, several complications make this analytical solution inadequate. The plasma may be perturbed by the laser pulse, either by introducing local variations in the plasma density, or by liberating new electrons through the process of optical field ionisation (OFI) or collisional ionisation. At ultra-high intensity, the refractive index of the plasma is modified due to the relativistic quiver motion of the free electrons. The ponderomotive effect may lead to expulsion of electrons and ions from regions of high laser intensity. All of these effects, discussed in chapter 1, add to the complication of the propagation behaviour. We must resort to numerical simulation in order to give us an indication of how these effects alter the behaviour of intense laser pulses in real plasma waveguides.
A comprehensive code would include a fully-relativistic simulation of the response of the plasma to the laser field, simultaneously solving Maxwell's equation to predict the propagation behaviour of a laser pulse through the perturbed waveguide. While such codes exist (Sprangle et al. 1992; Sprangle, Esarey and Krall 1996), they are extremely complicated and require substantial computing time. In addition, most codes that include plasma motion do not include optical field ionisation, and assume that the plasma is not further ionised by the laser.

In order to help assess the importance of OFI in determining the properties of partially-ionised waveguides, a simplified code was developed, based on a similar code written by Rae (Rae 1993; Grout, Pert and Djaoui 1998), which retained enough of the relevant physics to be applicable to capillary waveguides. This resulted in a streamlined code that was quick to run, and gave great insight into the properties that were required of a real waveguide.

6.1 Solution of the wave equation using finite differences

The code solves the propagation of high-intensity laser pulse through a plasma with an arbitrary cylindrically-symmetric density distribution. The plasma ion density is assumed to be unperturbed by the laser pulse. However, optical field ionisation of the plasma ions is included, and hence the code is capable of describing propagation through partially-ionised waveguides.

The propagation of a linearly-polarised laser pulse through an arbitrary plasma density is predicted by solving the inhomogeneous Helmholtz equation, which describes the evolution of the electric field $E(r,t)$ of an electromagnetic wave, in terms of the plasma current $J(r,t)$:

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial J}{\partial t}$$  \hspace{1cm} (6.1)

6.1.1 Approximations

The code assumes cylindrical symmetry, and so uses cylindrical co-ordinates $(r,z,\theta)$ with no $\theta$-dependences used. We represent the complex laser field $E$ as a product of a complex envelope $U$ and a fast oscillation at the laser frequency and wavelength

$$E(r,z,t) = U(r,z,t)e^{i(kz - \omega t)}$$  \hspace{1cm} (6.2)

Substituting equation (6.2) into equation (6.1), substituting for the plasma current using the free electron plasma conductivity, and expanding the $\nabla^2$ operator, we can eliminate the fast
oscillation. By insisting that the laser envelope is slowly-varying on the length-scale of a wavelength, and making the paraxial approximation that all rays propagate at small angles to the z-axis, we may neglect all remaining second order terms. Using a matrix representation for complex quantities, we are left with

\[
\frac{\partial u}{\partial t} = i \frac{c}{2k} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - c \frac{\partial u}{\partial z} - i \frac{c}{2N_r} u
\]  

(6.3)

in which symbols have their usual meanings, and

\[
u = \begin{pmatrix} \text{Re}(U) \\ \text{Im}(U) \end{pmatrix} \quad \text{and} \quad \mathbf{z} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}
\]  

(6.4)

Moving to speed-of-light co-ordinates using the transform \( \xi = z - ct \) eliminates the remaining spatial derivative in equation (6.3) leaving us to solve a radial partial differential equation independently for each constant-\( \xi \) slice of the laser envelope:

\[
\frac{\partial u}{\partial t} = i \frac{c}{2k} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - i \frac{c}{2N_r} u
\]  

(6.5)

6.1.2 The method of finite differences

We initialise \( u(r, \xi, t) \) at \( t = 0 \) at each point on a \((r, \xi)\) grid defined at points \( r = m \Delta r \) and \( \xi = p \Delta \xi \), where \( 0 \leq m \leq M \) and \( 0 \leq p \leq P \), in which \( m, M, p, \) and \( P \) are positive integers: each grid point is referred to as \( u_m^p \). Recall that \( u_m^p \) is a complex quantity.

Equation (6.5) has no dependence on \( \xi \), so each slice of the laser pulse with a given value of \( p \) may be treated independently. To propagate each \( \xi \)-slice from time \( t \) to \( t + \delta t \), we use the Crank-Nicholson method of finite differences (Strikwerda 1989; Crank and Nicolson 1996). This semi-implicit method allows us to progress from \( u_m^p(t) \) to \( u_m^p(t + \delta t) \), by forming an equation that links the new values of three adjacent radial points with the current values of the same three points. The left hand side of equation (6.5) is replaced with \( \left[ u_m^p(t + \delta t) - u_m^p(t) \right] / \delta t \). Using the six points shown in Figure 6-1, we can represent the value of the spatial derivatives at \( t \) and \( t + \delta t \). Since the temporal derivative is centred at time \( t + \delta t / 2 \), we then substitute into equation (6.5) the spatial derivatives averaged at time \( t + \delta t / 2 \).
CHAPTER 6: SIMULATIONS OF THE PROPAGATION OF INTENSE LASER PULSES THROUGH IONISING PLASMAS

Figure 6-1: Six points are used to solve the partial differential equation (6.5) describing the propagation of each $u_m$ forward in time.

For a typical simulation, the pulse duration is between 50 fs and 2.5 ps, and the propagation distance to be simulated is of order 10 mm. A 1 ps-duration pulse has a spatial extent of just 300 μm. Recall that the evolution of the laser envelope is calculated using moving coordinates. Hence, the spatial grid need not cover the full length of the simulation distance, but only the extent of the laser pulse, allowing the use of a small grid in which to define the laser envelope. However, the local plasma density is stationary in the laboratory frame and so must be moved backwards at the speed of light in the moving coordinate system. This is done simply by insisting that the grid step $\delta \xi$ is set to $c \delta t$: the values in the plasma density grid are then simply stepped back by one $\xi$ grid place during each time step. Accordingly, the value of $N_m u_n$ at time $t$ is $N_m^p(t) u_m^p(t)$, but at time $t + \delta t$, we use $N_{m+1}^p(t + \delta t) u_{m+1}^p(t + \delta t)$.

Combining all the components of equation (6.5), all of which are centred at time $t + \delta t/2$, we are left with

$$
A_{m} \ u_{m-1}^p(t + \delta t) + B_{m} \ u_{m}^p(t + \delta t) + C_{m} \ u_{m+1}^p(t + \delta t) = D_{m} \ u_{m-1}^p(t) + E_{m} \ u_{m}^p(t) + F_{m} \ u_{m+1}^p(t)
$$

(6.6)

in which the matrices $A \ to \ F \ depend only on $N_m^p(t)$ and $N_{m+1}^p(t + \delta t)$. There is a family of equations of the form of equation (6.6), consisting of one equation for each of the possible triplets of points, which can be summarized in the matrix equation.
The right hand side of equation (6.7) has been abbreviated to a known matrix \( \mathbf{d} \) that depends only on \( N_m(t) \) and \( u_m(t) \). Boundary conditions must be defined so that the differential equation can be solved at the radial grid boundaries. At \( r = 0 \), cylindrical symmetry requires that \( w_0 = W_1 \). At \( r = M8r \) the condition \( u_{M+1} = 0 \) is used. These conditions allow the matrix equation (6.7) to be formulated without the need for \( A_0 \) and \( C_m \).

### 6.1.3 Calculating the ion stage of the plasma

In order to determine matrices \( A_m \) to \( F_m \), it is necessary next to calculate the value of \( N_m(t + \delta t) \). For each point on the simulation grid there is stored an ion density \( N_{ion} \) and an average ion charge \( Z^* \), such that \( N = N_{ion}Z^* \). The change in the ion stage at each point is calculated using the rate equation for optical field ionisation derived by Ammosov et al. (Ammosov, Delone and Krainov 1986), and the local value of the laser intensity at time \( t \), which is given by \( |w_m(t)|^2 \). Ideally, the population of every possible ion stage should be tracked by calculating the rates of conversion from each ion stage to the next. However, for OFI the rates of ionisation from one ion stage to the next increase rapidly with intensity, and the ionisation thresholds for adjacent ion stages are sufficiently widely spaced so that at any time there are only two adjacent ion stages that have an appreciable population. Hence, it is possible to simplify the calculation of the electron density by making the approximation that at any time there are only two ion stages with non-zero populations. In this case, only a single rate equation must be solved dealing with the ionisation rate between these two stages.
6.1.4 Final solution

Having calculated \( N_m(t) \) and \( N_m(t + \delta t) \), the matrix equation (6.7) may be solved, using the method of LU decomposition (Press et al. 1996). This is an efficient method of solving matrix equations such as equation (6.7) in which the left-hand matrix is tridiagonal. Having solved the equation, the time evolution of a \( \xi \) -slice of \( u(r, \xi) \) has been obtained. The process is repeated for all of the \( \xi \) -slices until the entire laser envelope has been propagated.

It is necessary to periodically smooth the laser envelope as it is propagated in time. This is because sharp electron density gradients may be formed which create structure in the laser envelope on the scale of the grid: such small-scale structure is not acceptable in finite difference methods and must be suppressed. A Savitsky-Golay smoothing algorithm is used (Press et al. 1996), which preserves peak widths and heights and thus preserves laser pulse energy.

The final addition to the code is to allow for the absorption of laser energy due to the field ionisation process. This is done after each propagation step by looking at the change in the ion stage at each grid point, and reducing the value of \( u_m^p(t) \) by the appropriate amount so that the decrease in the energy stored in the laser field balances the increase in excitation energy of the plasma.

Ideally, this absorption should be solved simultaneously with the propagation step. However, the approximate split-step method described here decreases computation time significantly, and since absorption is generally a small effect, accuracy is not severely compromised.

6.1.5 Self-consistent calculation of the ionisation rate

In the current code, the evolution of \( N_m^p(t) \) to \( N_m^p(t + \delta t) \) is based on the intensity \( |u_m^p(t)|^2 \). Ideally, the average of the intensities at times \( t \) and \( t + \delta t \) should be used, since all other terms used in equation (6.7) are centred at \( t + \delta t / 2 \). Because we are using moving coordinates, in the laboratory frame this intensity is written \((|u_m^p(t)|^2 + |u_m^{p-1}(t + \delta t)|^2)/2\). However, the value of \( N_m(t + \delta t) \) is needed to calculate \( u_m(t + \delta t) \), and the value of \( u_m(t + \delta t) \) is needed to calculate \( N_m(t + \delta t) \). This paradox could most simply be solved using a “self-consistent” approach in which first \( |u_m^p(t)|^2 \) is used to calculate \( N_m(t + \delta t) \), and so equation (6.7) can be solved for \( u_m(t + \delta t) \). Then \( N_m(t + \delta t) \) is calculated correctly using the averaged intensity, before recalculating \( u_m(t + \delta t) \). This loop is continued until the values at \( t + \delta t \) converge on
their self-consistent values. The present code does not use this self-consistent technique because it significantly slows the speed of simulations. However, tests show that the loss of accuracy due to this approximation is not severe.

6.2 Accuracy tests

Several tests were performed to examine the accuracy of the code in simulating the propagation of laser pulses. The $u(r, \xi)$ laser field grid can be initialised with an arbitrary pulse envelope. Initial tests were performed to check that the propagation in free space followed the well know Gaussian beam formulae. Tests for which the envelope was initialised to that of a collimated or focusing Gaussian laser pulse show that the code was able to reproduce the theoretical propagation of the spot size in vacuum to within the accuracy of the grid.

To simulate propagation in plasmas, the ion density grid can be initialised with an arbitrary radial density profile and initial ion stage. With OFI suppressed, simulations of pulse propagation through parabolic electron density profiles correctly reproduced both matched and mismatched guiding discussed in chapter 2, in the latter case with the oscillation length agreeing with theory.

These tests serve to prove that the equations for propagation in non-ionising plasmas have been solved correctly and accurately. To validate the code in the regime in which further ionisation of the plasma occurs, simulations performed by Rae (Rae 1993) were duplicated. These simulations of the propagation of laser pulses focused into hydrogen gas show excellent agreement with the results of Rae.

In some circumstance when sharp defocusing gradients arise in the electron density, the code predicts a partial internal reflection of energy, which can result in a slight increase in axial intensity in situations where it would not be intuitively expected. This feature has been thoroughly investigated, and seems to be a genuine effect, and not an artefact of the propagation algorithm. The effect was later observed by another group (Sergeev et al. 1999). In plasmas densities of the order that are generated in capillary waveguides, only a small amount of energy is diverted by the internal reflection, and the effect is of marginal interest. The effect can be seen as a sharp axial peak at $z = 0.7$ mm in Figure 6-4.

Having successfully demonstrated the accuracy of the code in the circumstances above, the code could now be used to predict the guiding performance of real, non-ideal waveguides.
6.2.1 Wall absorption

The final problem to solve before the propagation can be examined through real waveguides is how to deal with the capillary walls. In reality, the wall will exhibit a combination of specular reflection due to a layer of high-density plasma at the wall boundary, scattering due to irregularities, and absorption. While specular reflection and absorption could be simulated by increasing the plasma density to greater than the critical density near the walls, it is not known to what extent either effect occurs. There is no simple model of the physics of the wall-ablation process, and it was not possible to guess at the plasma profile that exists near the wall.

It was decided to try to model a wall that absorbed all incident radiation without reflection. The best way to do this was to extend the simulation grid out to a radius \( r \) much greater than the radius of the waveguide \( r_{ch} \), and set the plasma density at \( r > r_{ch} \) to be constant, and equal to the value at \( r_{ch} \). Any energy reaching the capillary wall continues to propagate outwards, and is considered to be lost. The energy transmission of the waveguide can then be calculated by calculating how much energy is contained in the region of the grid \( r < r_{ch} \).

6.2.2 Regimes in which the code is applicable

A code applicable in all regimes must solve the exact wave equation along with the relativistic equations describing the fluid response of the plasma to the laser pulse. It must also track the temperature of the plasma constituents, and the population of all potential ion stages, which may become populated thermally or due to OFI.

Several approximations have been made from this ideal case. The main approximation is to neglect plasma motion. Simulations must therefore be limited to regimes in which the response of the plasma to the laser pulse is negligible. Guiding due to the ponderomotive expulsion of electrons from the axial region has been demonstrated (Chen et al. 1998) through a plasma of density of \( 4 \times 10^{19} \) cm\(^{-3} \) at \( I > 6 \times 10^{18} \) W cm\(^{-2} \). Using this intensity as a guide, ponderomotive effects should be negligible for \( I \ll 1 \times 10^{18} \) W cm\(^{-2} \).

The code also neglects relativistic effects associated with the quiver motion of the plasma electrons in the laser field. Such effects can lead to guiding, or in other circumstances instability in the propagation behaviour. The condition that relativistic effect are negligible requires that the laser power is much less than \( P_c = 17(\omega/\omega_p)^2 \) GW. For an electron density of \( 5 \times 10^{18} \) cm\(^{-3} \), \( P_c = 6 \) TW.
The paraxial approximation and the slowly varying envelope approximation are both valid provided the scale of the temporal and spatial structure of the laser envelope is large compared to a wavelength. This is the case provided we simulate pulses that are much longer than 10 fs. Severe pulse break-up into sub-10 fs beamlets during propagation due to Raman scattering could also lead to a violation of these approximations. However, in the regimes with which we deal this is not likely.

The final approximation is that only two ion stages exist in appreciable numbers at any one time. Full simulations of ion stage populations (Rae and Burnett 1992) show that this assumption is reasonable.

### 6.3 Simulation of picosecond pulse guiding experiments

The code has been used to help interpret the results of the picosecond pulse high-intensity guiding experiment carried out at Imperial College, discussed in chapter 4. Interferometric measurements of the electron density profile in the capillary under the discharge conditions used in the experiment have already been presented in chapter 5. Parabolic fits to these profiles were used as the initial electron density profiles for a series of simulations. It was necessary to estimate the initial electron temperature as a function of time through the discharge, since this determines the ion stage of the plasma, and allows calculation of the initial ion density in the plasma. The temperature was obtained using experimentally-verified scaling laws derived by Loeb and Kaplan (Loeb and Kaplan 1989) for discharge-ablated capillaries, which state

\[
T \text{ [eV]} = 3.3 \, I \text{ [kA]} \left( \frac{d \text{ [µm]}}{350} \right)^{8/3} \\
R \text{ [Ω]} = 1.71 \, I \text{ [kA]} \left( \frac{d \text{ [µm]}}{350} \right)^{-2/3} \, l \text{ [cm]} \tag{6.8}
\]

where \( T \) is the electron temperature, \( I \) is the current, \( d \) is the capillary diameter, and \( l \) is the capillary length. The initial ion stage of the hydrogen and carbon ions was calculated assuming that the plasma is in local thermodynamic equilibrium using the Saha equation. While the temperature in the capillary is actually a function of radius, the radial temperature was set constant for the simulations. Since the ion stage is only important near the axis where further ionisation occurs, and since the temperature varies only slowly near the axis, this approximation is valid.
The initial conditions used as input to the simulations are summarised in Table 6-1. Given these conditions, the code could be used to predict the detailed propagation of laser pulses injected into the capillary at various times throughout the discharge. It should be stressed that there are no remaining free parameters in the code. For $t > 638$ ns it was not possible to measure the electron density profile in the capillary. Hence simulations cannot be performed for later times than shown in Table 6-1.

<table>
<thead>
<tr>
<th>$t$ (ns)</th>
<th>Axial electron density ($10^{18}$ cm$^{-3}$)</th>
<th>Matched spot size $W_M$ (µm)</th>
<th>Current (A)</th>
<th>Electron temperature $T$ (eV)</th>
<th>Average carbon ion stage</th>
<th>Average hydrogen ion stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>78</td>
<td>1.5</td>
<td>61.2</td>
<td>180</td>
<td>1.8</td>
<td>0.96</td>
<td>0.84</td>
</tr>
<tr>
<td>176</td>
<td>3.3</td>
<td>37.5</td>
<td>340</td>
<td>2.2</td>
<td>1.06</td>
<td>0.92</td>
</tr>
<tr>
<td>256</td>
<td>4.9</td>
<td>31.6</td>
<td>460</td>
<td>2.5</td>
<td>1.20</td>
<td>0.95</td>
</tr>
<tr>
<td>300</td>
<td>6.0</td>
<td>30.4</td>
<td>510</td>
<td>2.6</td>
<td>1.24</td>
<td>0.96</td>
</tr>
<tr>
<td>340</td>
<td>6.5</td>
<td>28.9</td>
<td>550</td>
<td>2.7</td>
<td>1.31</td>
<td>0.96</td>
</tr>
<tr>
<td>456</td>
<td>9.3</td>
<td>27.0</td>
<td>680</td>
<td>2.9</td>
<td>1.40</td>
<td>0.97</td>
</tr>
<tr>
<td>586</td>
<td>13.3</td>
<td>24.7</td>
<td>740</td>
<td>3.0</td>
<td>1.39</td>
<td>0.96</td>
</tr>
<tr>
<td>638</td>
<td>15.0</td>
<td>24.4</td>
<td>750</td>
<td>3.0</td>
<td>1.36</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 6-1: Initial plasma conditions used for simulations of the picosecond guiding experiment.

The laser pulse was initialised at the waveguide entrance as a Gaussian pulse at focus, with a spot size of 32.5 µm and a peak intensity of $1 \times 10^{16}$ W cm$^{-2}$, as measured in the picosecond guiding experiment. The temporal profile of the pulse was set as a sech$^2$ shape, with a full-width at half-maximum of 2.5 ps. The code was used to simulate the propagation behaviour through a 10 mm-long waveguide. For each simulation the fluence passing each point in the capillary, the electron density in the capillary after the passage of the laser pulse, and the pulse energy transmission were calculated.
6.3.1 Pulse energy transmission

The calculated and measured pulse energy transmission as a function of time is shown in Figure 6-2. It is seen that there is good agreement between the simulated values and experimental measurements. In particular, the temporal peak of the transmission, and the rapid roll-off at late times are reproduced well.

![Graph showing pulse energy transmission](image)

Figure 6-2: The simulated and experimentally-measured transmission through a 10 mm-long polypropylene discharge-ablated waveguide.

6.3.2 Intensity profiles in the exit plane of the capillary

The second indicator used in the experiment to assess the guiding performance of the waveguide was the spot size in the exit plane of the capillary: experimental measurements were presented in Figure 4-4. Figure 6-4 shows a comparison between the measured and calculated pulse profiles in the exit plane of the capillary. Again good agreement is seen between the simulations and the experiment, with the energy being confined near the capillary axis as the discharge develops.
Figure 6-3: Experimentally-measured (A) and calculated (B) intensity profiles in the exit plane of a 10 mm-long discharge-ablated capillary waveguide, under the conditions of the picosecond guiding experiment presented in chapter 4. The spatial scale is in microns, and the vertical scale is fluence in units of $10^3 \text{ J cm}^{-2}$.
There is significantly more structure in the experimental data than the calculated profiles. This is to be expected since the measured profile at the laser focus showed significant structure, as presented in Figure 4-1. Indeed the "times diffraction limit" value was measured in vacuum to be $M^2 \approx 3$, which signifies that the phase and intensity profile of the input laser pulse had unwanted spatial structure. The pulse profile used as input to the code was ideal, with a smooth phase and intensity profile. It might be expected that the lack of similar phase structure in the simulated and experimental pulse would cause a significant divergence in propagation behaviour. However, the phase changes imposed on the pulse by refraction at the capillary entrance are of far greater magnitude, which may explain why the overall good agreement is still achieved.

The good agreement of the simulated transmissions and the exit pulse profiles with the experimental results allows us to be confident that the initial conditions used as inputs for the simulations are appropriate. While the experimental results only give information at the capillary entrance and exit, the simulations may be used to investigate the behaviour of the laser pulse within the waveguide.

6.3.3 Behaviour within the guide and the illusion of guiding

To gain a deeper insight into the quality of the guiding, we must look at the simulated fluence at each point within the capillary. This data, which could not be measured during the experiment, is vital for assessing the quality of guiding.

Figure 6-4 shows the calculated fluence passing each point in the capillary for the case in which the laser pulse is injected at $t = 300$ ns, where $z$ is the distance from the capillary entrance. The simulation clearly shows that the laser energy is sharply defocused at the capillary entrance, with the axial fluence dropping to 10% of the input fluence after propagating just 1 mm into the capillary. However, it is seen that the pulse is refocused at the capillary exit, which leads to the transverse spatial profiles in the exit plane of the capillary showing that the pulse energy is confined to the axial region. This creates the illusion that the pulse energy has been confined near the axial region throughout the length of the capillary.

The reason for this defocusing behaviour is clear. The initial plasma waveguide formed by the discharge is only partially ionised, with the average carbon ion stage between 1 and 2. As the intense laser pulse reaches the entrance to the waveguide, the carbon ions are optically field ionised to a higher ion stage. On axis, the intensity is sufficient to ionise carbon to C$^+$, while in the wings of the pulse, the reduced intensity causes a smaller increase in the ion stage. This
additional ionisation distorts the electron density profile formed by the discharge and creates an axial peak in the electron density profile that strongly defocuses the laser pulse.

Figure 6-4: The simulated fluence passing each point in the 10 mm long capillary, at \( t = 300 \text{ ns} \), under the conditions of the picosecond guiding experiment. Contours are in increments of 10\% of the axial input fluence.

Figure 6-5 shows the calculated radial electron density profile at the capillary entrance after the passage of the laser pulse, for the simulation for \( t = 300 \text{ ns} \). The grey curve shows the radial extent of the laser pulse. The sharp downwards steps in the electron density are responsible for the defocusing of the laser pulse.

After being sharply defocused at the capillary entrance, the pulse diverges, and the pulse intensity decreases rapidly. Consequently, after propagating for a few mm the intensity drops below that required to further ionise the plasma. At this point the pulse experiences an unperturbed guiding profile, which starts to re-collimate and then refocus the laser pulse. Such refocusing results in the high axial intensities in the output plane of the capillary seen in Figure 6-3.
Figure 6-5: The calculated electron density profile at $t = 300$ ns, after the passage of the laser pulse, which has a peak intensity of $10^{16}$ W cm$^{-2}$ and an input spot size of 32.5 µm. The profile of the laser pulse is shown in grey.

Note that at $z = 0.6$ mm, Figure 6-4 shows a peak in the axial fluence. This is due to the total internal reflection of a small amount of the pulse energy off the sharp electron density gradients created by the pulse itself, shown in Figure 6-5. Only a small fraction of the total pulse energy is redirected by this effect, and so it is not important under these conditions.

6.3.4 Pulse energy transmission revisited

The knowledge that there is severe defocusing at the capillary entrance allows an interpretation of the features of the pulse energy transmission curve in Figure 6-2 in terms of the interplay between the defocusing at the capillary entrance and refocusing by the waveguide.

At $t = 0$, there is no plasma in the capillary, and the observed transmission of 49% is in agreement with the theoretical value of 50% transmission through the aperture defined by the exit plane of the capillary. The agreement at $t = 0$ may be taken as a justification of the theoretical treatment of the capillary wall.

For $0 < t < 180$ ns, the simulation predicts a decrease in the transmission. This is not clearly seen in the experimental data due to a lack of data points. For these times, the axial electron
density is significant enough to increase the divergence of the laser pulse at the capillary entrance, while the curvature of the guide is not yet sufficient to prevent the defocused laser pulse from interacting with the capillary walls. Hence, there are increased losses at the capillary walls, and the transmission decreases.

For $150 < t < 500$ ns, the electron density has increased still further, and defocusing at the capillary entrance is more severe. However, the increased curvature of the guide is now sufficient to collimate and refocus the pulse as it propagates through the centre of the waveguide, avoiding significant interaction with the capillary walls. This results in an increased transmission, and a small spot size in the exit plane of the capillary.

For $t > 500$ ns, the increasingly strong defocusing caused by the rising axial electron density starts to outweigh the increase in curvature, and the pulse is no longer re-collimated before it hits the capillary wall: losses therefore increase, and for $t > 600$ ns most of the pulse energy is lost. At such late times, the plasma density is at its highest, and so non-linear effects such as Raman scattering, inverse Bremsstrahlung, and Brillouin scattering may become important. However, the agreement at late times shows that these effects are not necessary to account for the rapid decrease in transmission, and hence they are likely to be negligible.

### 6.4 Simulation of femtosecond pulse guiding experiments

The code has also been used to simulate propagation behaviour of intense laser pulses under the conditions used by Ehrlich et al. (Ehrlich et al. 1996). The group performed guiding experiments through polypropylene discharge-ablated capillary waveguides at intensities of $10^{16}$ W cm$^{-2}$. The pulse duration was 100 fs, and the spot size at the capillary entrance was set to 15 μm. The current profile they used is believed, based on their discharge circuit parameters, to be similar to that presented in chapter 4. The capillary material and dimensions are identical. The group observed a pulse energy transmission of up to 75%, with a spot size in the exit plane of the capillary measured to be 30 μm, and later stated that “it can be assumed that the guided laser beam diameter inside the capillary is less than or equal to this value [in the output plane] (Kaganovich et al. 1999).”

Simulations of their experimental results have been performed using the electron density profiles measured in our device as the initial conditions. Figure 6-6 shows the calculated fluence passing each point in the 10 mm-long capillary, for a laser pulse injected 360 ns after the onset of the discharge. It is clear that the conclusions above relating to picosecond pulses are equally valid for femtosecond pulses. Despite the presence of a small spot in the exit plane
of the capillary that creates the illusion of guiding, the axial intensity is actually extremely low for the majority of the length of the capillary.

Figure 6-6: The simulated fluence passing each point in the 10 mm long capillary, at \( t = 360 \) ns, under the conditions used for the femtosecond pulse guiding experiment reported by Ehrlich et al. (Ehrlich et al. 1996). Contours are in increments of 10% of the axial input fluence.

6.5 Conclusions

The experimental results presented in chapter 4 for picosecond pulse guiding, and those of Ehrlich et al. (Ehrlich et al. 1996) for femtosecond pulse guiding, have shown high pulse energy transmission and a small laser spot at the capillary exit. These results create the illusion of high quality guiding, and it has been tempting for previous authors to infer matched guiding along the whole length of the capillary.

However, the numerical simulations show that a high axial intensity is not maintained over the whole length of the capillary. Indeed for both femtosecond and picosecond pulse guiding, for most of the capillary length the axial fluence is less that 10% of that at the capillary exit.
entrance. It is thus essential to consider the behaviour of the propagating pulse inside the waveguide when gauging the success or failure of a guiding experiment of this type.

For input intensities of the order of $10^{16}$ W cm$^{-2}$, refractive defocusing of the guided pulse limits the performance of the waveguide; such defocusing is unavoidable in polypropylene waveguides that are initially only partially-ionised. Applications of waveguides require a high axial intensity to be maintained over extended distances: by this standard, the polypropylene discharge-ablated capillary waveguide is not well suited for use in applications.
References


"Numerical Recipes in C.", Cambridge University Press.


“Self-guiding and stability of intense optical beams in gases undergoing ionization.”


"Finite difference schemes and partial differential equations." Belmont, Wadsworth and Brooks.
7.1 Desirable plasma characteristics

In chapter 5, the use of longitudinal interferometry to measure the electron density profile formed by a discharge-ablated capillary waveguide was described. This tool allowed investigations into the way in which the profile created in the capillary depended on both the parameters of the discharge, and the properties of the capillary itself.

Three important parameters affect the performance of a plasma waveguide: the axial electron density $N(0)$, the initial average ion stage $Z'$, and the matched spot size of the guide $W_m$. In the previous chapter, the defocusing and refocusing behaviour of high-intensity laser pulses propagating through partially-ionised waveguides was discussed. It was found that peaks in the axial intensity were interspersed with regions of reduced intensity as the pulse was strongly refracted at each focal point. In this chapter, the results of a systematic optimisation of the parameters of the discharge-ablated capillary waveguide will be presented, with the goal of decreasing the strength of the defocusing refraction that occurs near each focal point, and restricting the amplitude of the oscillation of the laser spot size. In this way, the length over which the axial intensity of the laser pulse is maintained close to the peak value at the input of the waveguide may be increased.

The most important parameter that affects the quality of guiding is the matched spot size of the guide $W_m$, which is a measure of the focusing power of the parabolic electron density
profile created by the discharge. For a fully-ionised waveguide, the input spot size of the laser pulse should be set to the $W_M$ in order to achieve laser propagation with a constant spot size. In a partially-ionised guide, however, true matched guiding cannot be achieved since the extra electrons generated by the laser pulse create strong additional defocusing. In this case, the role of the waveguide is to resist that defocusing pressure as the laser pulse diverges, and to refocus the laser pulse once the pulse intensity has decreased to below the threshold at which further ionisation occurs. For this purpose, the matched spot size of the waveguide should be made as small as possible: a guide with a smaller matched spot size will better oppose the defocusing tendency created by the steps in the electron density, and will refocus the pulse more rapidly.

The initial ion stage $Z_0^*$ and electron density $N(0)$ determine the magnitude of the defocusing electron density gradients that are produced at the entrance of the waveguide, and at successive focuses, by the guided laser pulse. The laser pulse further ionises the plasma, and increases the axial electron density to $N(r)Z_f^*(r)/Z_0^*$, in which $Z_f^*(r)$ is the final average axial ion stage as a function of radius $r$. The intensity of the laser decreases with radius, and so the final ion stage drops in a series of sharp steps. Figure 7-1 shows an example of ionisation-induced distortion of a plasma waveguide, duplicated from Figure 6-5. It is clear that the size of the steps in the electron density is proportional to $N(r)$. The magnitude of the defocusing gradients can therefore be reduced by decreasing the axial density $N(0)$: for this to be achieved without changing the curvature of the waveguide, $N$ must be reduced equally at all radii.

Increasing the initial ion stage $Z_0^*$ on axis can reduce the number of defocusing steps created by the laser pulse. Ideally, the plasma constituents should be fully-ionised or ionised to highly-stable ions by the discharge: for a polypropylene waveguide, the initial plasma would ideally be composed of $H^+$ and $C^{4+}$, for which no further ionisation of the waveguide would occur for laser pulses with intensities up to $10^{18}$ W cm$^{-2}$. 
Figure 7-1: Example of ionisation of a waveguide by an intense laser pulse, duplicated from Figure 6-5. The black line shows the electron density profile of the waveguide after the passage of an intense laser pulse. The original profile is shown in grey. The ion stage of the plasma changes at well-defined radii, creating steps in the profile.

7.2 Interferometry results

With the importance of $N(0)$, $Z^*$, and $W_m$ borne in mind, an optimisation of the waveguide may now be attempted. A series of interferometric measurements were made under a range of different conditions, to determine those that produced the most effective guiding profile in the capillary. Investigations were performed on the effect of scaling the magnitude and duration of the current pulse, as well as using capillaries of different radii and composed of different materials.

For each set of conditions, measurements of the electron density profile in the waveguide were measured as a function of time. The measured electron density profiles were characterised by fitting a parabola of the form

$$N(r) = N(0) + Cr^2$$

in which $C$ describes the curvature of the parabola. For the purpose of characterising each waveguide, a new parameter was defined, the figure of merit $Q$, given by
$Q \,[\mu m^{-2}] = 10^5 \frac{C \,[cm^3 \mu m^{-2}]}{N(0) \,[cm^3]}$  \hspace{1cm} (7.2)

As discussed above, the best waveguide will have a high value of $C$ for a given $N(0)$, and so $Q$ provides a figure of merit of the plasma channel formed in a particular waveguide. The scaling factor $10^5$ in equation (7.2) sets $Q$ for a typical guide to be of the order unity. In terms of $Q$ and $C$, the matched spot size of the waveguide may be written from equation (2.4) as

$$W_{m}^2 [\mu m] = \frac{113}{C \,[10^{18} \, cm^3 \mu m^{-2}]} = \frac{1.13 \times 10^7}{Q \,[\mu m^{-2}] \, N(0) \,[10^{18} \, cm^3]}$$  \hspace{1cm} (7.3)

For a variety of different capillaries using a range of discharge parameters, electron density profiles were measured as a function of time. For each waveguide, it was found that $Q$ was approximately constant throughout the discharge pulse. While the curvature and the axial electron density evolved over time, the two parameters were approximately proportional to one another at all times. Therefore $Q$ could be used to characterise the guiding properties for each choice of capillary and discharge parameters.

In Table 7-1, temporally-averaged values of $Q$ are tabulated, for a range of different waveguides. The capillary wall material was either polypropylene (PP), or silica. The majority of the radial electron density profiles measured were found to be approximately parabolic and so the parabolic fit employed in calculating $Q$ was a good representation of the data. A few profiles however, notably those generated using 350 nF capacitors, were too steep at the walls to be well fitted by parabolas. In these cases, marked by * in the table, the calculated $Q$ is not an accurate measure of the guiding profile.

The guiding experiments presented in chapter 4 were performed using polypropylene capillaries. To investigate the effect of changing the ablation rate and thermal properties of the wall material, interferometry measurements were made using silica capillaries. Silica was chosen because of its easy availability in capillary form, and because its thermal properties were significantly different from polypropylene: the melting points of polypropylene and silica are approximately 160 and 1600 °C respectively. The behaviour of the two different capillaries was found to be broadly similar. For silica capillaries, the rate of ablation of the capillary wall was significantly slower, and for a period of 10 or 20 ns after the start of the ablation current, the plasma density remained close to zero. The time lag, which was not seen for polypropylene capillaries, may well represent the time taken to heat the walls to their melting point.
### Table 7-1: Summary of the measured figure of merit $Q$ for a variety discharge circuits, capillary materials and capillary diameters.

<table>
<thead>
<tr>
<th>Capillary material</th>
<th>Capillary diameter ($\mu$m)</th>
<th>Peak current (A)</th>
<th>Discharge capacitor (nF)</th>
<th>Figure of merit Q ($\mu$m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica</td>
<td>220</td>
<td>250</td>
<td>4.7</td>
<td>5.85</td>
</tr>
<tr>
<td></td>
<td>220</td>
<td>350</td>
<td>4.7</td>
<td>6.89</td>
</tr>
<tr>
<td></td>
<td>237</td>
<td>580</td>
<td>40</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>248</td>
<td>450</td>
<td>40</td>
<td>5.44</td>
</tr>
<tr>
<td></td>
<td>373</td>
<td>450</td>
<td>350</td>
<td>2.54</td>
</tr>
<tr>
<td>Polypropylene (PP)</td>
<td>380</td>
<td>600</td>
<td>350</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>356</td>
<td>300</td>
<td>350</td>
<td>2.54</td>
</tr>
<tr>
<td></td>
<td>375</td>
<td>650</td>
<td>350</td>
<td>2.84</td>
</tr>
<tr>
<td></td>
<td>356</td>
<td>260</td>
<td>40</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>365</td>
<td>550</td>
<td>40</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>380</td>
<td>550</td>
<td>9.36</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>387</td>
<td>550</td>
<td>4.7</td>
<td>2.51</td>
</tr>
<tr>
<td></td>
<td>424</td>
<td>350</td>
<td>350</td>
<td>3.03 *</td>
</tr>
<tr>
<td></td>
<td>402</td>
<td>550</td>
<td>350</td>
<td>2.76 *</td>
</tr>
<tr>
<td></td>
<td>414</td>
<td>950</td>
<td>350</td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>413</td>
<td>600</td>
<td>40</td>
<td>2.13</td>
</tr>
<tr>
<td></td>
<td>421</td>
<td>300</td>
<td>40</td>
<td>2.21</td>
</tr>
<tr>
<td></td>
<td>280</td>
<td>400</td>
<td>9.36</td>
<td>3.85</td>
</tr>
<tr>
<td></td>
<td>280</td>
<td>700</td>
<td>9.36</td>
<td>4.41</td>
</tr>
</tbody>
</table>
Figure 7-2: Figure of merit $Q$ plotted as a function of peak current for five pairs of experiments. For each pair, all parameters apart from the peak current are the same.

Figure 7-3: Measured temporal variation of the discharge current for a range of values of the discharge capacitor. The charging voltage was set to maintain the peak current at approximately 550 A.
Figure 7-4: Figure of merit $Q$ plotted for a range of values of the discharge capacitor $C$, for polypropylene capillaries of two different diameters.

The parameter $Q$ may be plotted in a number of ways to reveal trends in the data. Figure 7-2 shows a plot of $Q$ versus peak ablation current, for five data pairs. The discharge capacitor, diameter and material for each data pair are listed in the legend. It is seen that varying the peak current over a large range had no systematic effect on $Q$.

Data was also taken for a wide range of values of the discharge capacitor $C$. The discharge current for several values of $C$ is shown in Figure 7-3: the charging voltage was adjusted to keep the peak current approximately constant. The half-period of the current pulse was found to be approximately proportional to $\sqrt{C}$. Figure 7-4 shows a plot of $Q$ versus $C$, for two different capillary diameters. Again, there is no clear trend linking the period of the ablation current with $Q$.

Finally, Figure 7-5 is a plot of $Q$ versus capillary diameter, regardless of peak current and capacitance. The measurements on sub-250 $\mu$m capillaries were made using silica capillaries, which are readily available with small diameters. Measurements on polypropylene capillaries were made down to 260 $\mu$m: capillaries larger than 300 $\mu$m were drilled in polypropylene cylinders, and smaller capillaries were formed using the casing of fine single-core wire.

There is a clear trend: narrower capillaries have a higher $Q$, with the effect becoming more noticeable at smaller diameters. Capillaries with a diameter of 220 $\mu$m have a $Q$ of over twice
that measured for 350 μm capillaries. The solid line in Figure 7-5 is the best-fit power law, which is given by $Q [\mu m^{-2}] = 11664 r^{-1.65} [\mu m]$. The dashed curve is given by $Q [\mu m^{-2}] = 84030 r^{-2} [\mu m]$: in chapter 9, a steady-state model of capillary discharge waveguides will be discussed that predicts a scaling law of this form.

It is apparent that the results for silica and polypropylene (PP) lie on the same curve. Despite their vastly differing melting points, the silica and PP capillaries of the same diameter form very similar electron density profiles. The mechanism responsible for forming the guiding profile is thus not strongly dependant on the thermal properties of the capillary material, or the ease with which it can be ablated.

Figure 7-5: Measured figure of merit $Q$ plotted against capillary diameter, irrespective of the discharge parameters.
7.3 **Expected effect of capillary radius on waveguide properties**

Recall the results of experiments to demonstrate high-intensity picosecond laser pulse guiding, presented in chapter 4. Those experiments employed a 350 µm-diameter polypropylene capillary, and the discharge circuit comprised a 350 nF capacitor charged to up to 2.8 kV. The results presented in section 7.2 show that the waveguide had a $Q$ of approximately 3.

Consider the effect that scaling the capillary radius might have on a high-intensity laser guiding experiment, by comparing the guiding profiles generated in a 350 µm-diameter capillary and a 220 µm-diameter capillary. A 350 µm-diameter capillary has $Q \approx 3$, and so from equation (7.3), an axial electron density of $4.7 \times 10^{18}$ cm$^{-3}$ would be required to create a waveguide with $W_M = 30$ µm.

In contrast, a 220 µm-diameter capillary has $Q = 6$. For an axial electron density of $4.7 \times 10^{18}$ cm$^{-3}$, the matched spot size would be 25.4 µm. Hence, the focusing power of the waveguide would be significantly increased while keeping the axial electron density constant. The defocusing experienced by the pulse at the capillary entrance would thus be unchanged, but the pulse would be refocused more rapidly and the amplitude of the oscillations would be decreased.

Alternatively a guide with $W_M = 30$ µm would be created in a 220 µm-diameter capillary with an axial density of just $2.3 \times 10^{18}$ cm$^{-3}$. The defocusing gradients experienced by the laser pulse at the capillary entrance would be halved while the focusing power of the guide remain unchanged, and the amplitude of the oscillations would again be decreased.

7.4 **Plasma ion stage in the capillary**

The results presented in this chapter demonstrate that the shape of the electron density profile formed in a capillary waveguide depends to a significant extent only on the radius of the capillary that constrains the plasma. For a given profile, however, the guiding properties of that profile are strongly dependent on the ion stage of the plasma. While interferometry measurements give information on the electron density profile, there is no information about the ion density in the plasma, and so it is not possible to calculate the average ion stage in the plasma. Methods by which the ion density could quantitatively be measured, such as the hook method (Pastor, Serdobintsev and Shubin 1985), are complex and time-consuming, and were not undertaken.
In order to increase the ion stage in the plasma, it would be necessary to increase the power deposited in the capillary plasma, while keeping the plasma density constant. Simply raising the peak current passed through the capillary can increase the deposited power. However, such an increase leads to an increased wall ablation rate: the plasma density in the capillary after a given time is larger, and so even though more energy has been deposited in the plasma, it is not clear that the plasma ion stage will have increased. Indeed, for a plasma in local thermodynamic equilibrium (LTE), an increase in plasma electron density tends to lower the average ion stage due to enhanced three-body electron-ion recombination rates.

7.5 Summary

Radial electron density profiles were measured for a wide variety of waveguides. All aspects of the ablation current were altered, and the capillary diameter and capillary wall material were changed. For each waveguide, the electron density profiles measured as a function of time through the current pulse were fitted by parabolas, and a figure of merit $Q$ calculated. It was found that $Q$ was approximately constant in time for a given waveguide: therefore, $Q$ may be used to characterise plasma waveguides formed by discharge-ablation.

Measurements of a wide range of waveguides revealed that $Q$ was insensitive to the form and magnitude of the ablation current. Furthermore, changing the physical properties of the wall material also had little effect. However, a clear correspondence was found between $Q$ and the capillary diameter. Capillaries with a diameter of 220 μm were found to have a $Q$ of more than double that of 350 μm-diameter capillaries. It is suggested, therefore, that the guiding properties of the discharge-ablated capillary waveguide would be significantly improved by working with capillaries of smaller diameter.
References


"Excited Atoms Population Measurement by the Hook Method with Nanosecond
8.1 The concept of quasi-matched guiding

In chapter 5, it was shown that the discharge-ablated capillary waveguide creates an approximately parabolic electron density profile in a partially-ionised plasma. In chapter 6, it was shown that further ionisation of the plasma by a high-intensity laser pulse can prevent matched guiding from occurring.

In order to avoid ionisation-induced defocusing, it is necessary for the initial plasma to be fully-ionised, or ionised to a highly stable ion stage. Due to the relatively low plasma temperature, this is difficult to achieve in a discharge-ablated capillary waveguide. However, for sufficiently high laser intensities, quasi-matched guiding may be achieved in a partially-ionised plasma waveguide if the additional ionisation caused by the propagating laser pulse saturates.

Consider the general case in which a parabolic electron density profile is formed with a curvature corresponding to matched spot size $W_M$. Suppose that initially the ions have an average charge $Z^*$, and the total ion density has the same radial profile as $N$. If further ionisation by the propagating pulse increases the average charge to $Z_f^*$ out to a radius $r_{QM}$, then for $r < r_{QM}$ the electron density profile remains parabolic, but with an increased axial electron density and a reduced matched spot size. The new matched spot size $W_{QM}$ can be calculated from equation 2.3 as
If \( r_{QM} \) is significantly greater than the spot size of the propagating pulse, the main body of the laser pulse experiences a raised parabolic electron density profile and is therefore guided. We call this "quasi-matched guiding", since despite the further ionisation of the waveguide, the laser pulse may be guided over long distances with an approximately constant spot size.

In order that the additional ionisation creates the same ion stage over a wide region around the axis, the ion stage must be created over an extremely large range of field intensity: the ion stage must be highly-stable against ionisation up to the next ion stage. For this reason closed-shell or closed-sub-shell ions are most suitable for quasi-matched guiding.

### 8.2 Quasi-matched guiding in polypropylene waveguides

Numerical simulations were performed to investigate the effects of quasi-matched guiding. As a first example of the concept, simulations were performed for the polypropylene waveguide discussed in chapter 6. In that chapter, simulations of the propagation of laser pulses with intensity \( 1 \times 10^{16} \text{ W cm}^{-2} \) were presented in Figure 6-4, showing severe ionisation-induced defocusing of the pulse at the entrance to the waveguide. Figure 8-1 is a duplicate of Figure 6-5, which shows the electron density at the entrance to the waveguide after the passage of the laser pulse. It is seen that the stable \( \text{C}^{4+} \) ion stage is created near the axis, but only out to \( r_{QM} \approx 23 \mu \text{m} \). The steps in the electron density profile between \( r = 25 \) and \( 50 \mu \text{m} \) sharply defocus the pulse. While a narrow, raised parabolic region has been created near the axis, it does not extend to a large enough radius to prevent defocusing of the wings of the pulse.

In order to extend the raised parabolic plateau to larger radius, it is necessary to increase the input intensity of the laser pulse. Helium-like \( \text{C}^{4+} \) is created by OFI for laser intensities above \( 4.5 \times 10^{15} \text{ W cm}^{-2} \), and it is stable against further ionisation for intensities up to \( 2 \times 10^{18} \text{ W cm}^{-2} \). This large intensity range would allow a wide plateau of \( \text{C}^{4+} \) to be created around the capillary axis by a suitably intense laser pulse. For a pulse of intensity \( 2 \times 10^{18} \text{ W cm}^{-2} \), with a Gaussian radial profile of waist \( W \), \( \text{C}^{4+} \) would be created out to a radius \( r_{QM} = 1.74 \, W \).

Figure 8-2 shows the calculated fluence as a function of propagated distance \( z \) in a 30 mm-long, 350 \( \mu \text{m} \)-diameter polypropylene capillary for the case of an 800 nm, 100 fs laser
pulse focused at the capillary entrance to a peak intensity of $5 \times 10^{17}$ W cm$^{-2}$. The pulse intensity was chosen to be under $1 \times 10^{18}$ W cm$^{-2}$ so that relativistic effects, which are not included in the simulations, would be negligible. The initial electron density profile used for the calculations was that measured for $t = 176$ ns under the conditions of the picosecond pulse guiding experiment discussed in chapters 4, 5, and 6, with $N(0) = 3.3 \times 10^{18}$ cm$^{-3}$ and $W_M = 37.5$ µm.

![Figure 8-1: The calculated electron density profile at the entrance to the waveguide (described in Figure 6.5) after the passage of the laser pulse with a peak intensity of $10^{16}$ W cm$^{-2}$. The grey line shows the radial profile of the laser pulse.](image)

The initial ion stage of the carbon and hydrogen ions in the [CH$_2$] plasma was set to 1.15 and 0.96 respectively, giving an initial average ion stage $Z^*_i = 1.02$. The plasma near the axis is ionised to C$^{4+}$ and H$^+$ by the laser pulse, resulting in a final average ion stage $Z^*_f = 2$. From equation (0.1), the quasi-matched spot size of the guide was expected to be $W_{QM} = 31.7$ µm. The input spot size was therefore set to be equal to 31.7 µm.

It is seen that at an intensity of $5 \times 10^{17}$ W cm$^{-2}$, the sharp defocusing at the channel entrance observed for lower intensities is absent, and the axial fluence is maintained at greater than 40% of the input fluence over more than 16 mm. Figure 8-3 shows the electron density profile
in the entrance plane of the capillary before (dotted line) and after (solid line) the passage of the intense laser pulse. Also shown is the radial profile of the laser pulse (grey line). The leading edge of the guided pulse propagates through partially-ionised plasma and creates the raised parabolic C$^{4+}$ plateau that guides the bulk of the pulse. The leading edge is therefore continually absorbed and defocused, causing a gradual loss of pulse energy as the pulse propagates. Figure 8-4 is a three-dimensional plot of the electron density in the waveguide after the passage of the guided pulse. It is seen that the stable C$^{4+}$ ion is created out to a $r_{QM} = 48 \mu$m. The width of the raised parabolic plateau decreases with $z$ as the pulse intensity decreases.

![Figure 8-2: The calculated fluence as a function of position, for a 800 nm, 100 fs-duration pulse focused to an intensity of $5 \times 10^{17}$ W cm$^{-2}$ at the entrance of a discharge-ablated polypropylene capillary waveguide. The spot-size $W_0$ at the capillary entrance was set to $W_{QM} = 31.7 \mu$m. Contours are in increments of 10% of the input fluence.](image)

Comparison of the simulations presented in Figure 6-4 and Figure 8-2 shows that there is a dramatic change in guiding behaviour when the input pulse intensity is sufficient for quasi-matched guiding to occur. The intensity threshold for quasi-matched guiding may be roughly identified as 100 times the intensity required to ionise the plasma to its stable ion stage. This threshold corresponds to the condition $r_{QM} > 1.5W_0$.
CHAPTER 8: QUASI-MATCHED GUIDING

Figure 8-3: Calculated electron density profile at the entrance to the waveguide of Figure 8-2 before (dotted line) and after (solid line) the passage of the laser pulse. A 48 \( \mu m \)-wide parabolic plateau is created near around the capillary axis as the carbon ions are ionised to \( \text{C}^{4+} \). The grey line shows the radial profile of the laser pulse.

Figure 8-4: The calculated electron density in the waveguide of Figure 8-2 after the passage of the laser pulse. As the pulse propagates through the waveguide, the peak intensity decreases, and the raised plateau becomes narrower.
These conclusions are in qualitative agreement with recent experimental results published by Kaganovich et al. (Kaganovich et al. 1999), who demonstrated guiding of 400 fs pulses through 20 mm-long capillaries with intensities of up to $4 \times 10^{17}$ W cm$^{-2}$. They measured a transmission of 63% for an input intensity of $1 \times 10^{16}$ W cm$^{-2}$, with the transmission increasing to 80% when the laser intensity was increased to $4 \times 10^{17}$ W cm$^{-2}$. The onset of quasi-matched guiding at the higher intensities in Kaganovich's experiment would have had the effect of suppressing the defocusing suffered by the laser pulse at the capillary entrance and so increasing the transmission of the waveguide.

8.3 Quasi-matched guiding in boron waveguides

The quality of quasi-matched guiding may be improved by changing the atomic number of the atoms that form the plasma so that the plasma is ionised to a stable ion stage at a lower intensity. For a given laser intensity, the width of the guiding plateau would increase, decreasing the rate of erosion of the leading edge of the pulse. The constituents of the plasma may be varied by discharge-ablating capillaries formed from different materials, or by filling the capillary with gas.

For example, for a 100-fs laser pulse, boron is field-ionised to the He-like ion at a pulse intensity of only $1 \times 10^{15}$ W cm$^{-2}$, and is stable against further ionisation up to $6.5 \times 10^{17}$ W cm$^{-2}$. Figure 8-6 shows the calculated fluence at each point in a 65 mm-long boron capillary, assuming the same electron density profile and laser parameters as for the polypropylene guide: $N(0) = 3.3 \times 10^{18}$ cm$^{-3}$ and $W_M = 37.5$ μm. The initial boron ion stage was set to 1.05, calculated from an estimate of the plasma temperature of 2.2 eV, discussed in Table 6-1. The final ion stage near the axis after the passage of the laser pulse is B$^{3+}$, and hence with reference to equation (0.1), the input spot size of the laser pulse was set to $W_{QM} = 28.9$ μm. In the simulations for the boron waveguide, the axial fluence remains above 40% of the input fluence over a length of 32 mm, a length significantly greater than the 16 mm predicted for a polypropylene capillary. Figure 8-6 shows the calculated electron density profile as a function of position through the boron capillary after the pulse has passed. The raised parabolic plateau created near the capillary axis extends out to $r_{QM} = 52$ μm, ensuring that the bulk of the pulse experiences an ideal parabolic electron density profile.
Figure 8-5: The calculated fluence as a function of position, for a 100-fs pulse focused to an intensity of $5 \times 10^{17}$ W cm$^{-2}$ at the entrance of a boron waveguide. The spot-size at the capillary entrance was set to $W_{QM} = 28.9$ $\mu$m. Contours are in increments of 10% of the input fluence.

Figure 8-6: The calculated electron density in the boron plasma waveguide of Figure 8-5 after the passage of the laser pulse.
Quasi-matched guiding simulations for boron capillaries show significant improvements compared to polypropylene capillaries. The improvement is due to the lower OFI threshold for the creation of B\(^{3+}\) compared to C\(^{4+}\). Thus, for a given intensity, the radial size of the raised parabolic plateau will be larger for the boron waveguide than for the polypropylene waveguide, and the pulse will be guided with lower losses. Furthermore, for boron capillaries quasi-matched guiding should be possible for intensities as low as \(1 \times 10^{17} \text{Wcm}^{-2}\).

The small oscillations in the axial fluence seen in the polypropylene and boron guides are perhaps surprising given that the bulk of the pulse is matched to the parabolic region of the plateau formed by the ionisation of the initial waveguide. The oscillations arise owing to the raised parabolic region being of finite radial extent, whereas pulses of spot-size \(W_{QM}\) are only perfectly matched for parabolic variations of electron density extending to infinity. However, in the present case the finite radial extent of the plateau region causes the modes of the channel to be lossy and to depart from a purely Gaussian form. After the formation of the plateau region by the leading edge of an intense pulse, the bulk of the pulse excites a small admixture of higher-order modes in addition to the lowest-order mode, and beating between these leads to the observed oscillations in the fluence.

### 8.4 Quasi-matched guiding in hydrogen plasma waveguides

In summary, quasi-matched guiding occurs in any plasma waveguide in which the ions can be further ionised to a stable ion stage by the guided laser. The quality of guiding would improve as the intensity of the laser pulse increases above the threshold for creation of the stable ion stage. For carbon and boron, an upper limit on the intensity that can be used is set by the ionisation of C\(^{4+}\) or B\(^{3+}\) to the next ion stage.

Ideally, we would wish the plasma ions to be fully-ionised by the laser pulse, thereby removing the upper intensity limit. We would also wish the ions to be stripped at lower laser intensities, increasing the quality of quasi-matched guiding at a given intensity. These desired qualities would be best achieved by using a hydrogen plasma waveguide. Table 8-1 compares the laser intensity required to create C\(^{4+}\), B\(^{3+}\) and H\(^{+}\). Hydrogen is fully-ionised at a laser intensity almost an order of magnitude lower than that required for the creation of B\(^{3+}\). For a carbon or boron waveguide, the intensity may only be increased above the ionisation threshold by a factor of 444 and 650 respectively before further ionisation prevents quasi-matched guiding. For a hydrogen waveguide, there would be no such upper intensity limit.
Table 8-1: Intensity threshold (W cm\(^{-2}\)) for ion creation by OFI, for a 100 fs laser pulse.

Carbon, boron and hydrogen are all suitable ions in which to demonstrate quasi-matched guiding. The ratio \(I_2/I_1\) quantifies the stability of the ion required for quasi-matched guiding.

Figure 8-7 shows the calculated fluence at each point in a 270 mm-long hydrogen plasma waveguide, with \(N(0) = 2 \times 10^{18} \text{ cm}^{-3}\) and \(W_m = 39 \text{ \mu m}\). For the estimated plasma temperature of approximately 2 eV, the average ion stage would be expected to be almost 1. However, the initial hydrogen ion stage was set to 0.5, so that the simulations represent a worst-case scenario, and demonstrate the potential of quasi-matched propagation in hydrogen plasmas. The hydrogen ions are fully-stripped by the laser pulse, and so, with reference to equation (0.1), the laser pulse was assumed to be focused to an input spot-size of \(W_{qm} = 32.8 \text{ \mu m}\). The pulse intensity at the waveguide entrance was set to \(5 \times 10^{17} \text{ W cm}^{-2}\), almost 4000 times the intensity required to ionise the plasma to \(\text{H}^+\), sufficient to create the raised parabolic guiding plateau out to a radius of more than twice the spot size.

The simulation predicts extremely high-quality quasi-matched guiding over a distance of 270 mm, with the axial peak intensity decreasing by only 10%. It is clear that guiding would be maintained over a far greater distance. The small ripples in the axial intensity are caused by the interaction of the wings of the pulse with the steps in the electron density at the edge of the raised plateau. However, because the plateau extends out to such a large diameter, the bulk of the beam experiences a perfect parabolic channel.
Figure 8-7: The calculated fluence as a function of position through a hydrogen waveguide.

The waveguide was initially assumed to be composed of $H^{0+}$, with $W_M = 39 \, \mu m$. A 100 fs-duration 800 nm wavelength laser pulse was taken to be focused at the waveguide entrance with a spot size of $W_{QM} = 32.8 \, \mu m$ and a peak intensity of $5 \times 10^{17} \, W \, cm^{-2}$. Contours are in increments of 10% of the input fluence.
8.5 Summary

Quasi-matched guiding allows high-quality guiding to be achieved in partially-ionised plasma waveguides. Simulations have shown that while the discharge-ablated polypropylene capillary waveguide is not capable of matched guiding at $10^{16}\ \text{W cm}^{-2}$, the regime of quasi-matched guiding for laser intensities of $5 \times 10^{17}\ \text{W cm}^{-2}$ allows smooth guiding over tens of millimetres.

Quasi-matched guiding requires the laser intensity to be sufficient to create a wide plateau of a stable ion near the capillary axis. While carbon is suited for achieving quasi-matched guiding, the use of boron as an alternative plasma ion was discussed. Simulations showed that changing to a boron capillary waveguide would significantly improve the properties of the waveguide.

Finally, the concept of quasi-matched guiding was applied to a hydrogen plasma waveguide. Hydrogen is ideally suited for the purpose since it is fully-ionised by laser pulses with an intensity as low as $1.3 \times 10^{14}\ \text{W cm}^{-2}$, and cannot be ionised further. A simulation was presented demonstrating that extremely high-quality guiding could be achieved in partially-ionised hydrogen plasma waveguides. A capillary waveguide in which the plasma is composed of hydrogen would greatly improve the guiding properties of the device. In the next chapter, the design of such a gas-filled capillary waveguide is presented.
REFERENCES


In chapter 8, the concept of quasi-matched guiding was developed. The ideas presented there gave insight into the guiding properties of partially-ionised waveguides, and allowed predictions to be made concerning the regimes in which such a guide would be capable of high-quality guiding. It is clear that the waveguide generated in a discharge-ablated polypropylene capillary is not able to achieve high-quality guiding at intensities of less than $10^{17} \text{ W cm}^{-2}$. Quasi-matched guiding occurs at higher intensities, but for non-relativistic intensities, guiding is limited to a few centimetres. A boron plasma was shown to be capable of improved guiding due to the lower intensity required to ionise boron to the stable He-like $\text{B}^{3+}$ ion.

The logical extension of the trend from carbon to boron is to create a plasma waveguide in hydrogen. Hydrogen is the ideal species in which to form a plasma waveguide, since it requires the lowest temperature to be thermally ionised to a fully-stripped ion and so is most likely to be fully ionised by the discharge. Even if the gas is not initially fully-ionised, since hydrogen is also the species most easily fully-stripped by optical field ionisation, it is the best species in which to achieve quasi-matched guiding.

In this chapter, the design and construction of a gas-filled capillary discharge waveguide is described. This novel device is capable of forming a guiding channel over extended distances in a capillary pre-filled with an arbitrary gas. Discharge-ablation of the capillary wall is minimised, resulting in a long capillary lifetime, essential if the waveguide is to be used as an
element in more complex experiments. Measurements of the radial electron density profile in the waveguide are presented, demonstrating the formation of a parabolic guiding channel in essentially fully-ionised hydrogen.

9.1 The design of a gas-filled capillary discharge waveguide

The gas-filled capillary discharge waveguide aims to overcome the problems associated with discharge-ablated capillary waveguides. The main drawbacks of the discharge-ablated capillary waveguide are as follows. The lifetime of the device is limited to of the order of hundreds of shots, owing to ablation of the capillary walls. Such a short lifetime is not ideal for application for which the waveguide forms part of a more complex experiment. Since the best guiding occurs several hundred nanoseconds after the initiation of the discharge, there is likely to be a significant volume of plasma outside the capillary, which will adversely affect the coupling efficiency of the guide (Kaganovich et al. 1999). At relatively late times, a longitudinal pressure gradient will exist along the length of the capillary leading to non-uniform matched spot size and axial density.

All of these problems are avoided by the gas-filled capillary discharge waveguide. The design features of the guide and the consequences of those features are presented below.

9.1.1 The vacuum couplings

Since a high-intensity laser pulse must be coupled into the waveguide, it is necessary for the surrounding region to be at low pressure (<1 mbar) so that the laser pulse does not experience ionisation-induced defocusing as it approaches the focal region. The discharge-ablated capillary waveguide was placed inside a large chamber that was evacuated to $10^{-3}$ mbar.

The gas-filled capillary discharge waveguide was designed very differently. Instead of placing the waveguide in a vacuum chamber, the device was coupled to vacuum chambers on the input and output sides of the capillary using vacuum connectors and flexible bellows, as shown in Figure 9-1. Using this geometry, a laser pulse could be coupled into and out of the capillary in vacuum, while the outer surfaces of the capillary, and the electrical connections to the electrodes, were in air. The advantages of this approach are two-fold. Firstly, since the electrical connections were not in vacuum, problems associated with electrical feed-throughs were avoided.
Secondly, there were no possible long-path breakdowns around the outside of the capillary. The discharge-ablated capillary waveguide was placed in a larger vacuum chamber, and suppression of unwanted long-path breakdowns required that the chamber pressure be maintained at below $2 \times 10^{-3} \text{ mbar}$. With the new configuration in Figure 9-1, the limits on the chamber pressure were eased. This was essential for successfully building a gas-filled device, since the chamber pressure was found to rise to between $10^{-2}$ and $10^{-1} \text{ mbar}$ as the gas leaked into the vacuum chamber.

The waveguide was mounted on a translation stage, with three linear axes of motion, and two rotational motions, that allowed the pointing of the capillary axis to be altered while keeping the entrance of the capillary fixed. The stages, external to the vacuum system, could be used to align the capillary axis accurately with the axis of the focusing laser beam.

### 9.1.2 Introduction of gas into the capillary

Figure 9-2 shows a detailed cross-sectional view of the gas-filled capillary discharge waveguide. The capillary itself was made from alumina, with an internal diameter of 300 $\mu\text{m}$, and an external diameter of 600 $\mu\text{m}$. The capillary was surrounded by a polypropylene mount that was glued to the capillary at each end. Stainless steel electrodes with 500 $\mu\text{m}$-diameter holes on axis were held at either end of the capillary.
All joints that must be gas-tight had o-rings, which were compressed by an acrylic casing, not shown in Figure 9-2. A reservoir inside the polypropylene mount was filled with gas at a variable pressure. The gas then flowed into the bore of the capillary through two groups of holes, positioned near each end of the capillary. The holes were drilled through the walls of the capillary using the beam from a copper-vapour laser, operated in a “master-oscillator power-amplifier” configuration (CVL MOPA). The output power of the CVL MOPA was approximately 10 W, with a pulse repetition rate of 7 kHz. The MOPA system was configured to output a pulse train consisting of bursts of 255 pulses, with bursts every 0.3 seconds: this mode of operation prevented excess heating of the alumina that would cause local melting. The beam was focused onto the wall of the capillary with a lens of 12 cm focal length. In this configuration, the laser drilled through the wall of the alumina capillary in approximately 1 second.

At each end of the capillary, a group of 6 to 9 holes each with an approximate diameter of 50 μm was drilled through the capillary wall. It was found experimentally that this number of
holes resulted in a suitable gas flow rate into the capillary for a reasonable backing pressure of 0.5 to 5 bar.

Before the discharge was fired, the gas flow was allowed to reach a steady state. Since the gas was introduced at both ends of the capillary at similar pressures, the central part of the capillary reached an approximately uniform gas pressure. Between the gas injection points and the vacuum chamber the pressure decayed to the background pressure of the chamber. The injection points were placed as close to the capillary ends as possible, given the practicalities of fixing the capillary in the polypropylene mount without blocking the injection holes. In practice, the holes were located 1.5 - 2 mm from the capillary ends.

9.1.3 Striking a discharge though the gas

The gas was heated by striking an electrical discharge through the capillary. Unlike the discharge-ablated waveguide, ablation of the capillary wall was undesirable. As stated above, alumina capillaries were employed: the main reason for the choice of alumina was its high melting point, which inhibits ablation by the discharge current.

Although there was a 1.5 - 2 mm-long region of gas of varying pressure at each end of the capillary, these regions were heated and ionised by the discharge, and were constrained by the capillary walls. It is expected that this region of decreasing plasma density would have a guiding profile qualitatively similar to that in the body of the waveguide. Thus rather than having a region of unconstrained expanding plasma outside of the capillary, which was the case for the discharge-ablated capillary waveguide, a structured region of increasing gas pressure was created which allowed high-efficiency coupling into the main section of the waveguide.

Recall the discharge circuit used for the discharge-ablated capillary waveguide. In that case, a capacitor in parallel with the capillary was charged to several kV. The capillary was able to hold off this voltage since it was evacuated. The discharge was initiated by applying to a trigger electrode a fast-rising voltage spike generated using magnetic pulse compression.

The discharge circuit used for the gas-filled capillary discharge waveguide is shown in Figure 9-2. The discharge capacitor was charged through a resistor in parallel with the capillary. The top plate of the capacitor was switched rapidly to ground using a thyratron (EG and G, type HY-35), causing the voltage across the capacitor to reverse and the full charging voltage to appear across the resistor and the capillary. Provided that the charging voltage was sufficient, the gas in the capillary was broken down, and a discharge current developed through the capillary. The charging voltage was normally between 10 and 30 kV.
No trigger circuit was required to pre-ionise the gas. In contrast to the case for an evacuated capillary, the gas-filled capillary had a path length-pressure product which was on the right-hand side of the Paschen curve minimum (see Chapter 3), and the potential required to break down the gap was easily achievable for lengths of up to approximately 40 mm.

The discharge current was approximately half-sinusoidal, with a half-period of between 100 and 200 ns, and a peak current of 100 to 500 A. The magnetic pinch effect for such a slow-rising, low-current discharge is negligible. The timing jitter between the triggering of the thyratron and the onset of the discharge was measured to be as low as 5 ns, made possible by the rapid breakdown of the gas in the capillary coupled with the extremely low jitter associated with the switching of the thyratron.

9.2 Interferometry

Longitudinal interferometry, described in Chapter 5, was used to assess whether a guiding channel could be created by the gas-filled capillary discharge waveguide. Measurements were made for hydrogen-filled capillaries. These measurements were the first demonstration of the formation of a guiding channel in a gas-filled capillary discharge waveguide, and have been published in Physical Review E as a Rapid Communication (Spence and Hooker 2001).

9.2.1 Elimination of end effects

Consider the plasma distribution in the gas-filled capillary discharge waveguide. Before the discharge, the gas pressure is uniform in the central section of the capillary between the gas injection holes. At early times after the initiation of the discharge the plasma has not had sufficient time to be displaced longitudinally, and so the electron density profile in the central section of the capillary was also longitudinally uniform.

The gas injection holes could be placed no closer that 1.5 mm from the ends of the capillary. For a capillary much longer than 10 mm, the effects of the end section of plasma would be negligible. The electron density profile in the central section of the capillary will determine the guiding properties of the device, and so it is this profile that we wish to measure.

The maximum capillary length that may be probed using longitudinal interferometry is limited by distortion of the probe beam as it propagates through the capillary. For the present measurements, a probe beam of 355 nm-wavelength was used to maximise the capillary length that could be probed, as discussed in chapter 5. Unfortunately, the maximum capillary length was limited to approximately 5 mm. For such short capillaries, the contribution of the end effects must be considered.
In order to eliminate the contribution of the end effect from the interferometric measurements, two different capillary length were used, as shown in Figure 9-3. One was a 3 mm-long capillary filled through a gas injection point at the centre of the capillary. The second was a 5 mm-long capillary, filled 1.5 mm from each end. Both capillaries had an internal diameter of 300 \( \mu \text{m} \).

![Figure 9-3: Two capillaries were used for interferometric measurements to enable the contribution of the end effects to be eliminated.](image)

By setting the gas flow rate to be the same for both capillaries, the initial longitudinal pressure variation between each injection point and the end of the capillary was the same in both cases. Hence, provided measurements were made before the plasma had sufficient time to move longitudinally, the fringe shift for the 5 mm capillary would be the same as for the 3 mm capillary, plus an additional shift due to the uniform central 2 mm section of the long capillary.

9.2.2 Results

Measurements were made using an initial hydrogen flow rate of 300 atm cm\(^3\) min\(^{-1}\). The measured fringe shifts as a function of radius for the 3 and 5 mm-long capillaries for \( t = 60 \text{ ns} \) are shown in Figure 9-4.
Figure 9-4: Fringe shifts measured for 3 and 5 mm-long capillaries. The electron density deduced for the central uniform section of the long capillary is shown, with a parabolic fit. The hydrogen flow rate was set to 300 atm cm\(^3\) min\(^{-1}\) in both cases.

Subtracting these two curves leaves the fringe shift due to the 2 mm central section of the 5 mm-long capillary, which may be used to deduce the radial electron density profile for this region, as shown on the right axis in Figure 9-4. The electron density profile was fitted by the parabola

\[ N(r) \times 10^{18} \text{cm}^{-3} = 2.72 + 1.29 \times (r \text{ [\mu m]} / 150)^2 \]  \hspace{1cm} (9.1)

which has an associated matched spot size \(W_M = 37.5 \text{ \mu m}\).

Figure 9-5 shows the average measured axial electron density for a 3 mm-long capillary as a function of \(t\). The hydrogen flow rate was set to 300 atm cm\(^3\) min\(^{-1}\). The measured electron density was seen to decay to zero on a time scale of approximately 600 ns, as the hot plasma was expelled from the capillary. These results imply that for \(t < 100\) ns, longitudinal flow may be neglected and the longitudinal distribution of plasma was unchanged from the initial steady-state distribution. Hence, for the results shown in Figure 9-4, the end effects for the 3 and 5 mm-long capillaries would have been similar, and so the method used to eliminate these end effects is valid for \(t < 100\) ns.
Figure 9-5: The measured axial electron density as a function of time $t$, for a 3 mm-long capillary. The hydrogen flow rate was set to 300 atm cm$^3$ min$^{-1}$.

Measurements of the radial electron density profile as a function of $t$ show that the guiding profile is formed as early as 30 ns after the initiation of the discharge, and is stable for the duration of the discharge current pulse. This is in sharp contrast to z-pinch capillary waveguides, in which a transient guiding channel exists for only a few nanoseconds (Hosokai et al. 2000).

9.2.3 Atomic contribution to the refractive index

In order to check that the change in refractive index is caused by free electrons alone, with no contribution from hydrogen atoms, the measurements for the 5 mm-long capillary were repeated with a probe laser wavelength of 532 nm (see chapter 5). The measured phase shift was found to be proportional to wavelength, for $t < 300$ ns, which confirms that the plasma refractive index was dominated by the contribution from the free electrons, and that it was justified to infer the electron density directly from the measured refractive index of the plasma for these times. Note that it is assumed that the H$_2$ molecules are dissociated at very early times during the discharge.

For $t > 300$ ns the discharge current was approximately zero, and the plasma cools rapidly at the walls. For these times the electron density calculated near the wall becomes negative,
indicating that the H atom contribution to the refractive index, which has the opposite sign to the electron contribution, cannot be neglected for these times.

The significant refractive index contribution from the hydrogen atoms is surprising. In section 5.4, it was concluded that hydrogen atoms could not contribute significantly to the refractive index. The calculation, however, assumed that the majority of the atoms were in the ground state, from which all absorption lines have an energy far greater than the photon energy, leading to a small contribution to the refractive index of the plasma. Excited atoms, however, will have absorption features at a far lower energy that may be in resonance with the laser photons, creating a far larger contribution to the plasma refractive index. If there is a significant number density of excited atoms, then their contribution to the refractive index can become comparable to that of the free electrons.

For a plasma in thermal equilibrium, the excited states populations obey a Boltzmann distribution, and so for hydrogen plasma at a temperature of 6 eV, 85% of H atoms are in the ground state. During the discharge current pulse, the plasma is expected to be in LTE, which implies all states are in thermal equilibrium. Hence the contribution of hydrogen atoms to the refractive index is expected to be low.

However, the discharge current is essentially zero for \( t > 300 \text{ ns} \). It is expected that after \( t = 300 \text{ ns} \) the plasma cools rapidly, followed by recombination of electrons and protons to form excited states of hydrogen. Rapid recombination causes the populations of the excited states to be significantly populated - a considerable departure from the negligible populations predicted by the Boltzmann distribution for a plasma in thermal equilibrium. The significant number of excited H atoms is thought to be responsible for the large bound electron refractive index contribution seen for \( t > 300 \text{ ns} \). The contribution was found to be greatest near the capillary walls, where the plasma cooling and recombination rate will be greatest.

### 9.2.4 Calculation of the average ion stage of the plasma

The electron density profile generated by the hydrogen-filled capillary discharge waveguide was found to be approximately parabolic for early times when the outflow of plasma could be neglected. Thus a plasma waveguide was formed that would be capable of matched guiding of a low-intensity Gaussian beam. However, the performance of a plasma waveguide in the regime of high input laser intensities depends critically on the average ion stage of the plasma constituents. For the hydrogen-filled capillary discharge waveguide, the mean ion charge \( Z^* \) may be deduced by comparing the measured electron density with the initial density of \( \text{H}_2 \) molecules before the initiation of the discharge.
The initial molecular density was measured by longitudinal interferometry. In this case the interferometer was illuminated with a CW helium-neon laser. The capillary was initially evacuated, and then hydrogen was flowed into the capillary by switching a solenoid valve. The fringe shift caused by the gas in the capillary was measured as a function of time after the valve was opened.

The refractive index of 1 atmosphere of H\textsubscript{2} at room temperature is 1.000123. Hence using a probe beam of wavelength 633 nm, the phase shift $S$ is given by

$$S \text{ [radians]} = 1.225 \frac{P \text{ [atm]}}{l \text{ [mm]}}$$

where $l$ is the length of the capillary, and $P$ is the pressure of H\textsubscript{2}. Capillaries with an internal diameter of 300 $\mu$m and $l = 20$ mm were used, with hydrogen injection holes placed 1.5 mm from each end. The long capillary length was required to give a measurable fringe shift since the refractive index of unexcited H\textsubscript{2} is much less than the refractive index of a similar density of electrons.

Figure 9-6 shows one measurement of the H\textsubscript{2} gas pressure as a function of time $t$ after the solenoid valve was opened, for a 20 mm-long capillary and a steady-state H\textsubscript{2} flow rate set to 300 atm cm\textsuperscript{3} min\textsuperscript{-1}. Since the gas pressure in the capillary was uniform for the 17 mm section of capillary between the gas injection points, and decays to the chamber pressure at either end, end effects could be neglected. A steady-state H\textsubscript{2} pressure of 67 mbar was determined from the average of many such measurements. It is seen in Figure 9-6 that after 350 ms the gas pressure had reached 90% of the steady-state value.

It is now possible to calculate $Z_\textsubscript{i}^\ast$ for the plasma channel created by the discharge in the uniform central section of the capillary. From the electron density profile shown in Figure 9-4, the electron density averaged across the capillary diameter was calculated to be $3.33 \times 10^{18}$ cm\textsuperscript{-3}. The initial H\textsubscript{2} pressure in the central section of the capillary was 67 mbar (room temperature), and so the initial H atom density was $3.35 \times 10^{18}$ cm\textsuperscript{3}. Therefore, at $t = 60$ ns, the average ion stage in the capillary was $Z_\textsubscript{i}^\ast = 0.99$. The error associated with this measurement, caused by jitter of the fringe pattern during data collection, and the accuracy to which the hydrogen pressure could be set, was estimated to be $\pm 12\%$. Hence $Z_\textsubscript{i}^\ast$ was deduced to be between 0.87 and 1.
9.2.5 Variation of hydrogen density with flow rate

The flow of hydrogen gas from the injection points, through the capillary and into the vacuum chamber may be estimated theoretically. The mean free path of a H$_2$ molecule at room temperature for a pressure of 67 mbar is less than 10 nm, and so viscous flow theory must be used. The Reynolds number $R$ for a H$_2$ flow rate of 300 atm cm$^3$ min$^{-1}$ through a capillary with an inner diameter of 300 µm is less than 200, and so the flow should be non-turbulent.

Consider the section of capillary between the hydrogen injection point and the capillary exit. Assuming that the pressure at the capillary exit is zero, if the pressure at the injection point positioned at $l = l_0$ is $P(l_0)$, the pressure $P(l)$ at a distance $l$ from the capillary exit may be written:

$$P(l) = P(l_0) \sqrt{\frac{l}{l_0}}$$  \hspace{2cm} (9.3)

The peak pressure $P(l_0)$ that exists in the central section of the capillary obeys the scaling law

$$P(l_0) \propto \frac{\sqrt{F l_0}}{r_{ch}}$$  \hspace{2cm} (9.4)
where $F$ is the mass flow rate of hydrogen through the injection holes, and $r_{ch}$ is the capillary radius.

Figure 9-7 shows how the measured steady-state pressure in the capillary depends on the steady-state hydrogen flow rate. The fitted scaling law for $P_0$ and $F$ is $P_0 \propto F^{0.74}$, which is not in agreement with the theoretical scaling law given in Equation (9.4). The theoretical treatment is appropriate only for long tubes such that $l_0 > 0.1r_{ch}R$. In the present case $l_0 = 0.05r_{ch}R$ and so this condition is not satisfied. However, the main point that is illustrated by the scaling laws is that for a given pressure in the capillary, the flow rate required is a strong function of the capillary radius. For a capillary pressure of 67 mbar and a capillary radius of 150 µm, the flow rate out of the capillary into the vacuum chamber was already sufficient to cause difficulties for the vacuum system for high repetition rates. The flow rate could be drastically reduced by changing to a smaller capillary diameter. Furthermore, high capillary pressures of the order of an atmosphere should be attainable without a significant increase in gas flow rate, by using a capillary with a much smaller radius.

![Figure 9-7: The measured steady-state H$_2$ pressure as a function of flow rate for a 20 mm-long capillary filled with hydrogen at injection points 1.5 mm from either end.](image-url)
9.2.6 The capillary lifetime and the purity of the plasma

The lifetime of a single capillary is an important factor in the suitability of a waveguide for use in more complex experiments. A short lifetime would mean time wasted on replacing and realigning capillaries, and would limit the repetition rate that could sensibly be used.

The lifetime of the 5-mm-long capillary used in the above measurements was determined under the same hydrogen flow conditions and with the same discharge current. After $10^5$ discharge shots, the increase in the capillary diameter was measured to be less than 1 μm. Since an increase of the capillary diameter by 5% is not expected to significantly affect the generated electron density, the capillary was therefore estimated to have a lifetime of greater than $10^6$ shots.

It is possible to use the known lifetime of the capillary to estimate the purity of the hydrogen plasma. If a significant fraction of the plasma was composed of heavier atoms from the wall, which would be only partially-ionised, the guiding properties of the plasma channel would be adversely affected. Assuming the maximum possible capillary ablation rate of 1 μm in $10^5$ shots we can calculate that Al and O atoms ablated from the alumina walls could in total account for at most 0.3% of the plasma ions. These impurities would be ionised to $\text{Al}^{9+}$ and $\text{O}^{9+}$ by a guided laser pulse of intensity of order $10^{18}$ W cm$^{-2}$, and hence could contribute at most 2.5% of the plasma electrons. This is sufficiently small that it may be concluded that impurities will not affect the properties of the waveguide.

9.3 MHD simulations of the creation of the plasma channel

The guiding channel formed by slow discharges through capillaries has long been thought to be caused by quasi-steady state heat flow (Zigler et al. 1996). The plasma is strongly heated by the discharge, and is cooled due to contact with the relatively cold capillary walls. Therefore, the plasma is hottest on the capillary axis and becomes cooler with increasing radial position. Zigler et al. postulated that since the time taken to equalise pressure across the capillary diameter was much shorter than the period of the discharge, the pressure would always be approximately equal at all radii. The radially-decreasing plasma temperature would thus be associated with a radially-increasing plasma density.

Theoretical work on the properties of capillary discharges has been concentrated on z-pinch discharges through gas-filled capillaries, in which the magnetic pressure generated by the fast-rising current pulse rapidly compresses the capillary plasma (Bobrova et al. 2000a; Bobrova et al. 2000b). These devices have been shown to generate XUV laser output due to rapid heating of the collapsing capillary plasma column (Rocca et al. 1994; Rocca 1999).
Theoretical work has also been undertaken on initially-evacuated discharge-ablated capillary waveguides (Kaganovich et al. 1997), discussed in chapters 3 to 6. Simulations have succeeded in reproducing the radial electron density profile generated in such devices, validating the steady-state description of the formation of the plasma channel described above.

9.3.1 Simulations of the development of the capillary plasma

In this section, the results of simulations carried out by Bobrova et al. (Bobrova et al. 2001) will be briefly summarised. The simulations were the first to investigate theoretically the properties of the plasma created by a slow discharge passed through a hydrogen-filled capillary. The work was stimulated by the experimental results presented in Figure 9-4.

Magneto-hydrodynamic (MHD) simulations were performed by Bobrova to predict the radial electron density that would be formed in the hydrogen-filled capillary discharge waveguide. The parameters of the device simulated were those of the device characterised in this chapter: the 300 μm-diameter capillary was initially filled with 67 mbar of H₂. The simulated discharge current was half-sinusoidal with a half-period of 200 ns.

The simulations predict that the gas is heated to a peak axial temperature of 6.5 eV after 100 ns. The hydrogen is fully-ionised for times $t$ greater than approximately 60 ns. Figure 9-8 shows the calculated radial electron density profile and electron temperature predicted for various times $t$ during the discharge.

Three distinct phases of the discharge were identified. For $t < 50$ ns, the electron density and temperature are approximately uniform across the capillary diameter. For $50 < t < 80$ ns, for which times the plasma has been fully-ionised, a redistribution of the plasma occurs as radial temperature gradients become established. An axial minimum in the electron density forms during this phase. For $t > 80$ ns, the plasma is found to be approximately in its steady state condition for the instantaneous value of the discharge current - the plasma thermally-equilibrates on a time scale much faster that the discharge period. Conduction of heat by free electrons radially outwards to the relatively cool capillary walls results in an axial maximum in the electron temperature, and a corresponding minimum in the axial density.

Figure 9-9 shows a direct comparison between the calculated radial profiles and the experimentally-measured profiles of Figure 9-4. Theoretical profiles are presented for $t = 55, 60, \text{ and } 65$ ns. While the calculated electron density profiles are far steeper than the measured profile near the walls, reasonable agreement has been achieved.
The simulations predict that the ablation rate of the alumina capillary wall would be negligible, in agreement with the small measured ablation rate discussed in section 9.2.6.

Figure 9-8: Radial profiles of the electron density $n_e$ (top) and electron temperature $T_e$ (bottom) as a function of radius $r$, calculated from MHD simulations of the hydrogen filled capillary discharge waveguide [taken from (Bobrova et al. 2001)]. The initial hydrogen pressure in the 300 $\mu$m-diameter capillary was 67 mbar. Quantities are plotted for several times $t$ during the discharge.
Bobrova et al. developed a simple model of the plasma, in which the plasma was assumed to be in thermal equilibrium, with the thermal transport properties dominated by free electrons processes. For \( t > 80 \text{ ns} \), the simple model was found to be in excellent agreement with the full simulations. It was found that near the axis, the predicted radial electron density profile was parabolic: precisely as required for matched guiding of Gaussian laser pulses. The matched spot size of the waveguide \( W_M \) was predicted to be

\[
W_M [\mu m] = 4.68 \frac{\sqrt{r_{ch} [\mu m]}}{(Z^* N [10^{18} \text{cm}^{-3}])^\frac{1}{2}}
\]

(9.5)

For \( r_{ch} = 150 \mu m \), and an initial hydrogen pressure of 67 mbar, equation (9.5) gives \( W_M = 42 \mu m \) for a fully-ionised plasma. This is in good agreement with the matched spot size of 37.5 \( \mu m \) for the measured electron density profile.

From the definition of \( Q \) given in equation (7.3), and using equation (9.5) with \( Z^* \) set to unity, the predicted figure of merit \( Q \) can be written
The predicted relationship \( Q \propto r_{ch}^{-2} \) is the same relationship found experimentally for discharge-ablated capillary waveguides: this result was presented in Figure 7-5. The strong predicted scaling of \( W_M \) with \( r_{ch} \) is important, since it will enable waveguides with small matched spot sizes to be generated in the hydrogen-filled capillary waveguide. For example, in a capillary with a radius of 50 \( \mu \)m, for an initial hydrogen pressure of 1000 mbar the matched spot size would be expected to be approximately 13 \( \mu \)m.

9.4 Simulations of guiding in partially-ionised hydrogen waveguides

Numerical simulations were performed to calculate the propagation of laser pulses through a longitudinally-uniform waveguide with the parabolic radial electron density given by Equation (9.1). The initial ionisation \( Z_f^* \) of the waveguide was set to 0.87, which is the lowest value consistent with the calculation presented above. The guiding properties calculated therefore represent the worst case.

Figure 9-10 shows the calculated normalized axial fluence as a function of propagated distance \( z \) for pulses of several different peak intensities. The simulations assumed a pulse with a wavelength of 800 nm, and a sech\(^2\) temporal profile with a full-width at half-maximum of 50 fs. For these parameters, hydrogen is ionised by OFI to \( Z_f^* = 0.99 \) for laser pulse intensity of \( 2.3 \times 10^{14} \) W cm\(^{-2}\).

For the simulations the laser was taken to be focused at the capillary entrance. In order to correctly match the pulse into the waveguide, different waist sizes \( W_0 \) were set depending on the peak intensity of the input pulse. For the pulse of intensity \( 5 \times 10^{13} \) W cm\(^{-2}\), which will not further ionise the plasma channel, \( W_0 \) was set to \( W_M = 37.5 \) \( \mu \)m. For the more intense pulses the channel is ionised to H\(^+\) near the axis and so, as discussed in chapter 8, the spot size was set to

\[
W_{OM} [\mu m] = \left( \frac{Z_f^*}{Z_f^*} \right)^{\frac{1}{4}} W_M [\mu m] = \left( \frac{0.87}{1.0} \right)^{\frac{1}{4}} \times 37.5 \mu m = 36.2 \mu m
\]  

Figure 9-10 shows the predicted axial fluence, normalised to the input fluence, as a function of distance through the waveguide \( z \) for a range of input peak intensities. The results show that the waveguide is capable of matched guiding at \( 5 \times 10^{13} \) W cm\(^{-2}\), since the plasma is unperturbed by the laser pulse. However, at an intensity of \( 1 \times 10^{15} \) W cm\(^{-2}\) the spot size of
the pulse shows strong oscillations as a function of $z$. These oscillations arise due to ionisation-induced defocusing. Quasi-matched guiding is achieved for peak intensities greater than approximately $5 \times 10^{16}$ W cm$^{-2}$, which is a factor of 200 greater than the threshold for ionisation to the stable H$^+$ ion. For example, the simulation for a peak intensity of $5 \times 10^{17}$ W cm$^{-2}$, corresponding to a pulse energy of 450 mJ, shows very stable quasi-matched propagation, with a calculated transmission of 99% over a distance of 100 mm (19.4 Rayleigh ranges).

![Image](image_url)

Figure 9-10: The calculated axial fluence as a function of distance $z$ through a 100 mm-long hydrogen-filled capillary discharge waveguide. The initial radial electron density profile was the parabolic profile given in equation (9.1), with $Z^*$ set to 0.87. Plots are presented for a range of input intensities.

It is seen that in the quasi-matched regime, losses due to ionisation and interaction with the capillary wall are very small, and furthermore, since the majority of the laser pulse experiences a fully ionised plasma, modulational instabilities due to bound electrons should not be significant. As such, the waveguide presented here seems to be an almost ideal plasma waveguide, since any additional losses or instabilities are inherent to plasmas of this density.
Furthermore, as noted above, the simulations assumed that $Z_i^* = 0.87$, the lowest value consistent with the measured value. In fact the MHD simulations predicted that the plasma would be fully ionised for $t > 60$ ns, in which case the guiding properties of the waveguide should be largely independent of intensity, in the absence of non-linear instabilities.

9.5 Summary

The properties of the gas-filled capillary discharge were investigated using time-resolved interferometry. It was shown that an approximately parabolic guiding channel could be formed in a pure hydrogen plasma.

Hydrodynamic simulations of the capillary discharge, performed by Bobrova et al. (Bobrova et al. 2001), show that the channel is formed due to radial conduction of heat to the capillary walls by the plasma electrons. The channel is parabolic near the axis, and so the waveguide naturally forms the profile needed for matched guiding of Gaussian laser pulses. The hydrogen plasma was calculated to be fully ionised for $t > 60$ ns. This is consistent with the interferometric measurements, and has the consequence that the channel will be capable of matched guiding at any intensity, provided non-linear effects can be neglected. Furthermore, even in the regime where relativistic effects and ponderomotive motion of the plasma perturb the propagation, the channel will stabilise the propagation of the pulse, and confine the energy to the axial region (Sprangle et al. 1992).

Simulations of the propagation of intense laser pulses through hydrogen channels demonstrate that even if the waveguide is only partially-ionised to $Z_i^* = 0.87$, high-quality quasi-matched guiding can be achieved for laser intensities greater than $5 \times 10^{16}$ W cm$^{-2}$. If, as expected, the waveguide is initially fully-ionised, then the waveguide will be appropriate for use at all intensities.

The hydrogen-filled capillary discharge waveguide is capable of forming an ideal plasma channel over long lengths. Its simplicity, coupled with the long lifetime make the waveguide a promising candidate for use at the heart of complex laser-plasma interaction experiments.
CHAPTER 9: THE GAS-FILLED CAPILLARY DISCHARGE WAVEGUIDE

References


“MHD simulations of plasma dynamics in pinch discharges in capillary plasmas.”


Hosokai, T., M. Kando, H. Dewa, H. Kotaki, S. Kondo, N. Hasegawa, K. Nakajima and K. Horioka (2000).


In the previous chapter, measurements of the electron density profile created in a hydrogen-filled capillary discharge waveguide were presented, showing that it was possible to form an essentially fully-ionised plasma waveguide in hydrogen. In this chapter, the results of experiments carried out using the Astra laser at the Rutherford Appleton Laboratories are presented. The results demonstrate guiding of high-intensity laser pulses over lengths of up to 40 mm.

10.1 The Astra laser

The Astra laser (Langley 1999) at the Rutherford Appleton Laboratories is a Ti:Sapphire laser, which uses the technique of chirped pulse amplification (Strickland and Mourou 1985). A Kerr-lens mode-locked oscillator outputs pulses of 20 fs-duration at a pulse repetition rate of 76 MHz. The pulses are stretched to 600 ps using a grating stretcher, before being amplified in a Ti:Sapphire multi-pass amplifier. Ten pulses per second are chopped out of the pulse train and further amplified in a second multi-pass amplifier up to a pulse energy of approximately 100 mJ. The pulses are finally amplified up to 1 J per pulse in a power amplifier, pumped using a 5 J, Nd:YAG laser. Recompression is achieved using a two-grating double-pass compressor, with an efficiency of approximately 30%. Compressed pulse
durations as short as 60 fs can be achieved, with a pulse energy on target of up to 300 mJ, corresponding to a peak power of up to 6 TW.

10.2 Experimental set up

Figure 10-1 shows schematically the experimental set up used for demonstrating guiding of high-intensity laser pulses from the Astra laser. Pulses from the compressor were focused at the entrance of the waveguide by a 1.6-m focal length paraboloid used at f/27. During the experiment, the average pulse energy of the laser was 230 mJ. The pulse duration was measured using a second-order autocorrelator, and the full-width at half maximum pulse duration was calculated to be 120 fs, assuming that the pulse had a sech-squared temporal profile. Considerable care was taken to remove small amounts of astigmatism in the Astra beam by reflecting the beam from a mirror with an adjustable radius of curvature in one dimension. As a result, high-quality propagation through the focal region was achieved, with a measured spot size of 29 μm at focus (as shown in Figure 10-3). Measurements of the spatial profile of the beam either side of the waist indicated that the $M^2$ of the beam was approximately 3.

The waveguide structure was mounted on a 5-axis stage external to the vacuum system. Flexible bellows allowed the lateral position and orientation of the capillary axis to be aligned with that of the laser by optimising the transmission of attenuated pulses from the Astra laser.

Radiation leaving the capillary was reflected by a wedged optical flat before being rendered parallel by an f = 700 mm, f/11.7 achromatic lens. After two further reflections from wedged optical flats, and attenuation by neutral density filters, the radiation was re-focused by an f = 500 mm, f/8.3 lens to form an intermediate image which was magnified onto a 8-bit CCD camera by a microscope objective. The object plane of the imaging system could be adjusted by translating the first lens along the axis of the system without changing the transverse magnification. The transverse magnification and the position of the image plane of the complete imaging system were determined by imaging a wire of known diameter placed at the capillary entrance.

The discharge circuit comprised a 1.7 nF capacitor charged to between 10 and 23 kV. A Rogowski coil placed around the cable connecting the charging capacitor to the anode was used to record the temporal profile of the discharge current pulse. A timing photodiode was employed to measure the delay $\tau$ of the arrival of the laser pulse at the capillary entrance after the initiation of the discharge current. The error in $\tau$ was estimated to be ± 5 ns.
Figure 10-1: Schematic diagram of the arrangement for demonstrating guiding of high-intensity pulses from the Astra laser.

The capillary was pre-filled with hydrogen, helium, or argon gas before the onset of the discharge. The rate of flow into the capillary was measured using a gap flow meter calibrated to give the pressure in the uniform central section of the capillary (see section 9.2.4). Gas was either flowed into the capillary continuously, or pulsed by opening a solenoid valve 1 second before each shot. Pulsed gas flow was necessary for higher flow rates, which caused excessively high chamber pressures if gas was flowed continuously: it was necessary to maintain the chamber pressure well below $10^{-1}$ mbar to avoid placing the turbo-molecular pumps under excessive strain.

10.2.1 Measurements of energy transmission and transverse intensity profiles

The input energy of the laser pulses was recorded by an energy meter placed behind a turning mirror located before the paraboloid, and the transmitted pulse energy was recorded by loosely focusing the radiation transmitted by the second optical flat onto the surface of a pyroelectric energy meter. The pulse energy transmission $T$ was calibrated by recording the ratio of the transmitted and input energies as a function of the input laser energy whilst the
waveguide was removed. The error in $T$ was estimated to be $\pm 5\%$, ascertained by calculating the standard deviation of measurements made with the waveguide removed.

The imaging system was used to capture the transverse fluence profiles of the pulses in the exit plane of the capillary. Each image was corrected for any uniform background illumination using outlying areas of the field of view to indicate the background level. The image was then smoothed by averaging blocks of 4 pixels, resulting in a final grid size of 5 µm. Combining the known input energy with the transmission measured for each shot gave the energy contained in each CCD pulse profile. By integrating the total number of counts in each CCD image, the CCD count was converted to a fluence per pixel. The CCD was tested to have a linear response with intensity. Finally, by assuming that the pulse duration was not altered by the waveguide, the transverse intensity profile associated with the CCD image was deduced.

### 10.3 Guiding through 20 mm-long waveguides

Measurements were made of the pulse energy transmission and the transverse intensity profile in the exit plane of the capillary, as a function of the delay $t$, for a variety of capillary configurations and gas flow rates. In the following sections, guiding results are presented for 20 mm-long plasma channels formed in hydrogen-, helium-, and argon-filled capillaries.

#### 10.3.1 Pulse energy transmission

The best guiding results were obtained using a 20 mm-long capillary pre-filled with hydrogen gas. The initial hydrogen pressure in the capillary was measured to be 67 mbar. The temporal variation of the transmission $T$ and the current pulse are plotted in Figure 10-2. The current profile can be seen to have a peak of 240 A at $t = 90$ ns. The small current peak before $t = 0$ is caused by pulse charging of the cable connecting the discharge circuit to the waveguide. As such, this part of the current did not pass through the capillary.

Before the onset of the discharge, the transmission $T$ is seen to be between 10 and 15%. At these times, the laser pulse propagated through neutral hydrogen. The transmission of the capillary when evacuated was measured to be 73%, and so it is clear that the losses at the capillary wall were greatly increased by presence of neutral hydrogen, owing to the ionisation-induced defocusing of the laser pulse at it propagated through the ionising plasma.
Within 50 ns of the onset of the discharge current, \( T \) increased to almost 80%, and for \( 60 < t < 300 \) ns the transmission was maintained at approximately 90%. The plateau in \( T \) implies that there was a long-lived guiding channel that existed throughout the discharge current pulse. This is consistent with the results of interferometric measurements presented in chapter 9 that showed that the guiding channel was stable. The formation of a long-lived stable plasma channel is in sharp contrast with z-pinch capillary waveguides in which the channel is transient, lasting for just several nanoseconds (Fauser and Langhoff 2000; Hosokai et al. 2000).

### 10.3.2 Transverse intensity profiles

Figure 10-3 shows the transverse intensity profiles measured in the exit plane of the 20 mm-long capillary for various delays \( t \). Also shown, for comparison, is the intensity profile measured at the capillary entrance, which has a spot size of 29 \( \mu \)m, and a peak intensity of \( 1 \times 10^{17} \) W cm\(^{-2} \). All plots have the same vertical scale in units of \( 10^{16} \) W cm\(^{-2} \), and spatial scales in \( \mu \)m. Each intensity profile was normalised to the average pulse energy of 230 mJ.
Figure 10-3: Intensity profiles measured in the exit plane of a 20 mm-long capillary for various injection times $t$. The intensity in the capillary entrance plane is included for reference. The spatial scales are in $\mu$m, and the vertical scales are in units of $10^{16}$ W cm$^{-2}$. 
It is apparent that for $t < 0$, the energy in the exit plane of the capillary was spread over the entire diameter of the capillary. As a result, the peak intensity was only $10^{15}$ W cm$^{-2}$ for these times. During the time of high transmission, $60 < t < 300$ ns, we see that the pulse energy was constrained to the axial region at the capillary exit. For the pulse recorded at $t = 206$ ns, the output beam spot size was 33 μm, $T = 92\%$, and the peak axial intensity was $7 \times 10^{16}$ W cm$^{-2}$. For longer delays, as the transmission declined, the spot size in the exit plane of the capillary was found to increase.

The same capillary was used to guide approximately $10^3$ full-power laser shots under these conditions, with no degradation of the guiding performance. This suggests that the lifetime of the capillary when used to guide high-intensity laser pulses may be as high as $10^4$ shots or more.

The intensity profiles were calculated, from fluence profiles recorded by the CCD camera, on the assumption that the pulse duration was not altered by the interaction with the capillary plasma. It is important to consider the validity of this assumption. In chapter 9 it was demonstrated that the plasma is essentially fully-ionised by the discharge, and so it is possible to calculate the amount of pulse stretching that would be expected for the conditions given above. Analytical solutions of the non-paraxial wave equation (Esarey and Leemans 1999), which retain terms responsible for group velocity dispersion, can be used to show that the pulse dispersion length $Z_D$ is given by:

\[
Z_D = \left( \frac{\pi}{\lambda} \right)^3 \left( \frac{W_0^2 \tau(0)}{2 \ln 2} \right)^2 \left( 1 + \frac{Nc^2 W_0^2}{4 \epsilon_0 m_e c^2} \right)^{1/2}
\]

(10.1)

For a waveguide of matched spot size $W_0$, and axial electron density $N$, the pulse length as a function of $z$ obeys $\tau(z) = \tau(0) \times (1 + z^2 / Z_D^2)^{1/2}$. For the waveguide presented in section 10.3, $W_0 = 30 \mu$m, $N = 3 \times 10^{18}$ cm$^{-3}$, and $\tau(0) = 120$ fs. Equation (10.1) then gives $Z_D = 2.0$ m, and hence for 20 mm and 40 mm-long waveguides, pulse stretching may be neglected. Group velocity dispersion can become important for ultra short pulses (< 30 fs) and small spot sizes (< 10 μm), for which $Z_D \leq 6$ cm. The strong wavelength dependence of $Z_D$ means that the dispersion length is almost a factor two longer for a pulse at the Ti:Sapphire wavelength of 800 nm compared with the Nd:Glass laser wavelength of 1 μm.
10.3.3 Pulse energy transmission versus pressure

Figure 10-4: Temporal evolution of the transmission $T$ for a 20 mm-long capillary, as a function of initial hydrogen pressure.

Figure 10-4 shows measurements of the temporal evolution of the pulse energy transmission through a 20 mm-long capillary for a range of initial hydrogen pressures. It is clear that the guide is capable of high energy transmission over a wide range of pressures. The insensitivity to the initial hydrogen pressure is to be expected however. As discussed in chapter 9, MHD simulations of the channel formation process predict that the matched spot size of the guide $W_M$ should scale as $N^{1/2}$ (Bobrova et al. 2001). Hence, for the results of Figure 10-4 above, the pressure range covered corresponds to a change in $W_M$ of just 30%.

For each pressure examined, there was a temporal window for which the spot size in the exit plane of the capillary was small. This time was found to be generally earlier for lower pressures.
10.4 Analysis

10.4.1 Propagation behaviour within the guide

As stressed previously, the appearance of a small spot at the exit of a waveguide does not imply that matched guiding was achieved through the length of the guide. Recall that for partially-ionised waveguides, the existence of a small spot at the waveguide exit can mean that the pulse has been refocused in the exit plane of the capillary, following severe defocusing earlier in its propagation.

For the hydrogen-filled capillary discharge waveguide, the plasma is essentially fully-ionised by the discharge, and hence no ionisation-induced defocusing is expected. Indeed, at $10^{17}$ W cm$^{-2}$, even a partially-ionised waveguide in hydrogen would support high-quality quasi-matched guiding as discussed in chapter 8.

For the 20 mm-long hydrogen-filled capillary discharge waveguide, either matched or mismatched guiding was achieved as described in chapter 2. In the matched case, the laser pulse propagates through the waveguide with a constant spot size equal to the spot size $W_M$ at the capillary entrance. However, it is also possible the laser pulse was not matched to the waveguide, meaning that the input spot size $W_0$ was not equal to the matched spot size of the guiding channel $W_M$. In that case the spot size oscillates smoothly between $W_0$ and $W_M^2/W_0$ as the pulse propagates. The wavelength $Z_s$ of the oscillation for a parabolic guide (see chapter 2) is given by

$$Z_s = \frac{\pi^2 W_M^2}{\lambda} \quad (10.2)$$

If the capillary length is an integer multiple of $Z_s$, then the spot size in the entrance and exit planes of the capillary will be equal. For a 20 mm-long capillary, this occurs for $W_M = 40$, $28$, $23$,...μm corresponding to $1$, $2$, $3$... complete oscillations of the spot size within the guide. As an example, consider a laser pulse focused to $W_0 = 29$ μm injected into a waveguide with $W_M = 40$ μm. The laser spot size oscillates between 29 μm and 55 μm, and so the peak intensity oscillates from 100% to 27% of the intensity at the input. In the exit plane of a 20 mm-long capillary, the spot size would be 29 μm. Despite the oscillation, the intensity is maintained at greater than 25% of the input intensity for the length of the waveguide.
There is a key distinction between mismatching oscillations such as these and the ionisation-induced defocusing and refocusing discussed in the case of polypropylene discharge-ablated capillary waveguides. During mismatching oscillations, the magnitude of the oscillation is small, and so the axial intensity does not change too severely. Furthermore, there is no interaction with the capillary wall, and so the pulse can propagate with low losses. In the case of partially-ionised polypropylene waveguides, however, the oscillation has a large magnitude, and significant losses are incurred due to the interaction with the capillary wall.

It is likely we have observed both matched and mismatched guiding in the experiments presented above. However, there was no diagnostic with which to distinguish between the two. For future experiments in which the waveguide is used to extend a laser-plasma interaction, such an OFI-pumped XUV laser, we would ideally want to configure the guide in a regime of matched guiding. It should be possible to use the measurable variation in strength of the desired laser-plasma interaction, while tuning the hydrogen pressure, to optimise the guiding channel to achieve matched guiding.

10.4.2 Plasma flow and plasma heating

Longitudinal flow of plasma along the capillary for $t > 0$ can be important in determining the guiding properties of the waveguide. For the gas-filled capillary discharge waveguides used in the experiments above, the gas was injected at points 2 mm from each end of the capillary. The gas flow was allowed to reach a steady state before the discharge was fired. At $t = 0$, the gas pressure between the two injection points was therefore uniform, whilst between the injection points and the end of the capillary the pressure decayed to the ambient value of the vacuum system. This non-uniform end effect was only 2 mm in length, and by adjusting the alignment and longitudinal position of the laser focus, it was possible to couple into the main, uniform section of the waveguide with an extremely low insertion loss.

Heating of the gas by the discharge disturbs the initial steady state distribution. Interferometric measurements on 3 mm-long capillaries showed that expulsion of the hot plasma from the capillary occurs on a time scale of approximately 600 ns following the initiation of the discharge. Therefore, for a 20 mm-long capillary, the plasma would be expected to be completely expelled on a time scale of 4 μs, with the longitudinal distribution of plasma significantly modified for $t > 400$ ns.

For $t \ll 400$ ns, longitudinal plasma motion may be neglected. As a result the longitudinal variation in plasma density will be the same as the density distribution for $t < 0$, and so the main body of plasma between the capillary injection points will be at a uniform density. This
has the important consequence that the matched spot size of the plasma waveguide will be longitudinally uniform along length of the capillary, essential for achieving matched guiding along the whole length of the waveguide.

The longitudinally uniform waveguide formed by the gas-filled capillary discharge waveguide may be contrasted with the channel formed by the discharge-ablated capillary waveguide. In the latter device, the plasma density in the capillary takes several hundred nanoseconds to build up due to ablation of wall material. By this time, a longitudinal gradient in the plasma density will have formed, and so the guiding channel will have a matched spot size that varies along the length of the guide. Such a guide cannot support matched, oscillation-free guiding.

The guiding properties of the gas-filled capillary discharge waveguide will change for times when longitudinal flow may not be neglected, $t > 400$ ns. While longitudinal flow does not affect the formation of the parabolic radial profile of the channel, the loss of plasma from the capillary will cause the axial density to decline near the capillary ends. This means that the channel parameters will vary along the length of the capillary, adversely affecting the guiding performance. Eventually the axial density and channel curvature will drop so low that the pulse is no longer prevented from interacting with the capillary walls, causing the decrease in transmission seen for late times in Figure 10-4.

More significant is the fact that the discharge current is essentially zero for $t > 300$ ns. The discharge heats the hot axial region of the plasma preferentially, and so removal of the heating contribution will lead to a decrease in the radial temperature gradient in the plasma and an associated decrease in the curvature of the guiding channel. The plasma will cool rapidly, and the channel curvature will decrease further. Recombination of electron and ions, which will be most rapid in the coldest regions near the capillary wall, will also tend to decrease the electron density near the walls. As the focusing power of the channel becomes weaker, the guided pulse will defocus, and propagation losses will increase. The data presented in Figure 10-2 and Figure 10-4 display a marked decrease in transmission and an increase in scatter of the data points for $t > 300$ ns. The decrease occurred at the same delay $t$ regardless of the initial hydrogen pressure, and so it is likely that the decrease is caused solely by the removal of the discharge current rather than considerations of longitudinal flow.

### 10.5 Guiding through helium- and argon-filled waveguides

While a hydrogen plasma is essentially fully-ionised by the discharge, heated to approximately 6 eV (Bobrova et al. 2001), waveguides formed in other gases may not be fully-ionised. For a plasma with an electron density of $3 \times 10^{18}$ cm$^{-3}$ and a temperature of
6 eV, the Saha equation predicts that a helium plasma and an argon plasma would have average ion stages of 1.8 and 3.0 respectively. Hence for helium and argon waveguides, the channel is initially only partially-ionised, and so when considering the properties of the guide we must consider further ionisation of the channel, and the degree to which quasi-matched guiding will occur.

Input intensities of greater than approximately 100 times the ionisation threshold are required to achieve quasi-matched guiding. Helium is fully-stripped by optical field ionisation to He$^{2+}$ for a laser intensity of $6.5 \times 10^{15}$ W cm$^{-2}$. For the peak intensity of $10^{17}$ W cm$^{-2}$ for the experiments presented here, 15 times the ionisation threshold, quasi-matched guiding would not be expected. The laser pulse injected into the guide would be expected to become defocused at the capillary entrance, and then refocused during the transit through the waveguide. Figure 10-5 shows the measured pulse energy transmission through a 20 mm-long helium-filled capillary discharge waveguide. Despite the defocusing at the capillary entrance, the waveguide was capable of high transmission at early times. It can be concluded that the pulse was refocused sufficiently rapidly that it did not interact strongly with the capillary wall.

Figure 10-5: Measured temporal evolution of the discharge current and pulse energy transmission for a 20 mm-long capillary, pre-filled with hydrogen, helium or argon.
Argon is ionised to $\text{Ar}^{8+}$ at an intensity of $2.8 \times 10^{16}$ W cm$^{-2}$. Since $\text{Ar}^{8+}$ is a closed shell ion, it is stable against further ionisation for intensities as high as $1.2 \times 10^{18}$ W cm$^{-2}$. For pulses with an intensity of $10^{17}$ W cm$^{-2}$, just 3.5 times the threshold for creating $\text{Ar}^{8+}$, quasi-matched guiding will not occur through a partially-ionised argon waveguide. Severe defocusing would be expected to occur at the capillary entrance due to the large numbers of additional electron created per ion by the laser pulse. Figure 10-5 shows the transmission through a 20 mm-long argon-filled capillary discharge waveguide. For $t < 0$, the transmission was far less than through hydrogen or helium, and for $t > 0$, the transmission through the argon waveguide never exceeded 10%. This is explained by considering, firstly, that the defocusing gradients are steeper since far more additional electrons are created by OFI in the argon plasma compared to the helium plasma. Secondly, at an input intensity of only 3.5 times the threshold for creating $\text{Ar}^{8+}$, the steps in the radial electron density at the waveguide entrance are close to the axis, causing sharp defocusing of the pulse. These two effects combine to cause severe defocusing and high losses at the capillary wall in the argon waveguide before the pulse can be recollimated.

For the helium waveguide, guiding could be improved by increasing the input laser intensity to above the threshold required for quasi-matched guiding. Since the threshold for creation of $\text{He}^{2+}$ is $6.5 \times 10^{15}$ W cm$^{-2}$, and of course $\text{He}^{2+}$ is stable against further ionisation, quasi-matched guiding would be achieved for intensities greater than approximately $7 \times 10^{17}$ W cm$^{-2}$. Alternatively, if the plasma temperature could be increased from 6 to 8 eV, the average ion stage discharge plasma would increase from 1.78 to 1.98. The helium plasma would then be essentially fully-ionised, and no ionisation-induced defocusing would occur. It is likely that the plasma temperature could be increased by increasing the current density through plasma by either increasing the peak discharge current, or decreasing the capillary diameter.

For the argon waveguide, the situation is more complicated. The threshold for ionisation to the stable $\text{Ar}^{8+}$ ion is $2.8 \times 10^{16}$ W cm$^{-2}$. However $\text{Ar}^{8+}$ is only stable against further ionisation up to $1.2 \times 10^{18}$ W cm$^{-2}$. For pulses with an intensity of $1 \times 10^{18}$ W cm$^{-2}$, a wide plateau of $\text{Ar}^{8+}$ will be created near the axis. However, this intensity is not sufficient to achieve high-quality quasi-matched guiding, although the pulse may be guided over distances of several centimetres before being defocused. Quasi-matched propagation through argon-filled capillary discharge waveguides will be discussed again in chapter 11.
10.6 Guiding through 40 mm-long waveguides

Guiding experiments were performed using a 40 mm-long capillary pre-filled with hydrogen gas. In order to break down the longer length of neutral gas, it was necessary to increase the charging voltage of the discharge capacitor as well as to decrease the hydrogen pressure in the capillary. At a pressure of 34 mbar and a charging voltage of 24 kV, it was possible to strike a discharge through the capillary. Unfortunately, the discharge was unreliable, showing significant fluctuations in the current profile. Owing to limitations of the thyratron, it was not possible to work at a higher charging voltage.

The measured temporal evolution of the pulse transmission for the 40 mm-long capillary is shown in Figure 10-6. The transmission was seen to rise to almost 90% within 50 ns of the onset of the discharge current. However, unlike for the 20 mm waveguide, the transmission began to decrease within 50 ns. The scatter in the data is much greater than for the 20 mm-long capillary results, possibly due to shot-to-shot variations in the discharge current changing the quality of the waveguide.

Unfortunately, the transverse intensity profiles obtained for this experiment were saturated. Figure 10-7 shows the pulse profile recorded at \( t = 64 \) ns. The CCD signal was saturated over
the area of the central contour. The saturation prevents calculation of the intensity scale for this measurement. However, it is clear that the energy has been confined to the axial region of the capillary. The field of view is 300 μm, equal to the capillary diameter.

Figure 10-7: Intensity profile of a guided pulse at the exit of a 40 mm-long waveguide for t = 64 ns. The CCD pixels are saturated within the central contour. The spatial scales are in μm, and the saturated region has a size of approximately 75 × 25 μm.

It is not clear why, for the 40 mm-long waveguide, the transmission was not maintained at a high value for several hundred nanosecond, as was the case for the 20 mm-long capillary. In fact, only one of the 40 mm capillaries tested showed any significant guiding performance at all. For the other capillaries, it is thought that an incorrect breakdown occurred along the outer surface of the capillary, preventing the breakdown and heating of the gas within the capillary. It seems that by working at an increased voltage, the cyano-acrylate seals at the ends of the capillary may have become damaged.

It is possible that during collection of the results presented above, an incorrect breakdown was occurring along side the desired breakdown. Competition between the two discharge paths could cause large fluctuations in the discharge current, and could result in decreased heating of the hydrogen gas, and result in a partially-ionised plasma channel. Note the evolution of the transmission for the 40 mm-long capillary shown in Figure 10-6 is similar to that of the
20 mm-long helium-filled capillary shown in Figure 10-5. For the helium waveguide, the plasma was expected to be only partially-ionised.

Based on the quality of the results obtained with 20 mm-long waveguides, it is likely that the difficulties with 40 mm-long waveguides were due to problems with the construction of the guide, and not intrinsic to the mechanisms of the plasma channel formation. New experiments based on an improved design, in which the extraneous discharge path is blocked, should allow high-quality guiding to be demonstrated over extended distances.

10.7 Analysis of coupling and propagation losses

The transmission $T$ of a waveguide of length $l$ may be described by $T = T_0 e^{-\alpha l}$, where $T_0$ describes the coupling of the laser into the waveguide, and $\alpha$ is the propagation loss per unit length. Using the maximum pulse energy transmission for the 20 and 40 mm-long waveguides of ($92 \pm 3$)% and ($82 \pm 5$)% respectively, it is found that $T_0 = (96.5 \pm 3.5)$% and $\alpha = (0.040 \pm 0.017)$ cm$^{-1}$. These coupling and propagation losses are the smallest reported to date for guiding of laser pulses with peak intensities greater than $10^{16}$ W cm$^{-2}$. The low losses are a result of the radial extent of the plasma channel being much greater than the matched spot size.

10.8 Problems with the Astra Laser

As discussed in section 10.3.2, the measured pulse energy, pulse duration, and spot size at the capillary entrance correspond to a peak input intensity of approximately $10^{17}$ W cm$^{-2}$. However, several months after the completion of the experiments presented in this chapter, it was discovered that, under certain circumstances, the pulses output by the Astra laser system contained significant temporal structure: a train of approximately 8 short pulses, separated in time by roughly 100 ps. If this structure were present during the experiments, given the relatively large interval between the pulses the autocorrelation measurements made would have determined the duration of one sub-pulse. A single sub-pulse would contain approximately only one eighth of the total pulse energy, and so the peak intensity calculated above would be too high by almost an order of magnitude.

It is not known whether this unwanted structure was present during the experiments presented in this chapter. However, even if it had been, the peak intensity of the most intense sub-pulses would have been greater than $10^{16}$ W cm$^{-2}$. It is likely, given the optimal energy transmission of over 90% for the 20 mm-long waveguide, that each sub-pulse was successfully guided.
10.9 Conclusions

In summary, the hydrogen-filled capillary discharge waveguide was demonstrated to be capable of guiding pulses with a peak intensity of $10^{17}$ W cm$^{-2}$ over lengths of 20 mm. The measured peak pulse energy transmission was greater than 90%, with the axial intensity in the output plane of the capillary approximately 70% of that at the input.

The hydrogen-filled capillary discharge waveguide combines many positive features of other waveguides in a single device, and has the lowest coupling and propagation losses reported to date. Unlike plasma waveguides formed by the hydrodynamic expansion of a laser-produced cylindrical plasma (Clark and Milchberg 2000; Gaul et al. 2000), no auxiliary lasers are required. The plasma channel is long-lived, and the discharge circuit is simple, and does not require very low jitter triggering, in contrast to the complex fast discharge circuit of z-pinch waveguides (Hosokai et al. 2000).

The hydrogen-filled capillary discharge waveguide has several advantages over the discharge-ablated capillary waveguide (Kaganovich et al. 1999). The plasma channel is composed of fully-ionised hydrogen rather than low charge states of the wall material, and so ionisation-induced defocusing and temporal and spectral distortion of the laser pulse are avoided (Spence and Hooker 2000). Furthermore, the electrical lifetime of the hydrogen-filled capillary discharge waveguide has been determined to be greater than $10^6$ discharges, whereas that of discharge-ablated capillary waveguides is $10^2 - 10^3$ shots, owing to ablation of the capillary wall.

Mono-mode guiding by grazing incidence reflection in hollow capillaries shows low pulse transmission when the capillary is filled with a target gas (Dorchies et al. 1999). Furthermore, it has been observed that for input laser intensities of $10^{17}$ W cm$^{-2}$ or greater, formation of a plasma at the capillary entrance can decrease the pulse transmission further (Courtois et al. 2000). At such high input intensities, the shot lifetime of the capillary is also very low.

The hydrogen-filled capillary waveguide should be scalable to lengths of hundreds of millimetres, limited only by the length over which electrical breakdown can be achieved. Waveguides with lengths greater than 50 mm are likely to require pre-ionisation of the gas, or a staged discharge design. The MHD analysis suggests that plasma channels with smaller matched spot sizes could be created by using capillaries of smaller diameter and operating at higher hydrogen pressures (Bobrova et al. 2001). For example, an initial hydrogen pressure of 1000 mbar used with a capillary of diameter 200 µm is predicted to create a guiding channel with $W_m \approx 15 \mu m$. 
The characteristics discussed here make the gas-filled capillary discharge well suited for use as a tool for investigating laser-plasma interactions. In chapters 11 and 12, the prospects for using the gas-filled capillary discharge waveguide for XUV lasers and electron acceleration are discussed.
References


Hosokai, T., M. Kando, H. Dewa, H. Kotaki, S. Kondo, N. Hasegawa, K. Nakajima and K. Horioka (2000).


11.1 Introduction

Longitudinal pumping of plasmas using high-intensity lasers is an important approach for achieving gain at XUV wavelengths (Corkum and Burnett 1988; Burnett and Corkum 1989). Highly-ionised species are required for XUV lasing in order that the spacing of the electron energy levels is sufficient. An intense laser pulse propagating though a plasma can ionise the plasma constituents by OFI to a high ion stage, in a time of the order of the pulse duration. Using a femtosecond laser pulse to optically field ionise a plasma, a particular lasant species can be created on a time scale much faster than optical transition rates and electron-ion recombination rates.

The threshold nature of the OFI process means that it is possible for a large proportion of the ions in a plasma to be prepared in the same ion stage, maximising the density of desired lasant ions. The creation of a single ion stage is further facilitated if the lasant ion is stable against further ionisation. The most suitable ions are thus closed-shell ions, such as Xe$^{8+}$, Kr$^{8+}$, Ar$^{8+}$ and Ne$^{6+}$.

Longitudinal pumping is essential for XUV lasers for which the lifetime of the gain is very short. The pumped region moves through the plasma at the group velocity of the pump laser, and an XUV laser pulse is emitted in one direction only, trailing the pump pulse. Longitudinal pumping, compared to pumping using a laser brought to a line focus, also allows higher pump intensities to be reached owing to the smaller area of the laser focus.

The distribution of electron energies in the OFI plasma is strongly dependent on the polarisation of the driving laser, presenting two ways in which a population inversion can be created: collisional excitation for a circularly-polarized pump beam, and three-body electron
recombination for a linearly-polarised pump beam. These mechanisms will be discussed individually below.

Using the longitudinal pumping geometry, the length over which the lasant ion can be created is limited by diffraction and refraction of the pump laser. The use of a waveguide to channel the pump laser and indeed the generated XUV radiation is clearly advantageous for the longitudinal pumping geometry. In this chapter, the applicability of the gas-filled capillary discharge waveguide for these lasers will be investigated using numerical simulations of the propagation of intense femtosecond laser pulses through suitable plasmas.

11.2 Collisionally-pumped XUV lasers

11.2.1 Unguided operation

Collisionally-pumped OFI lasers were first proposed by Corkum and Burnett (Corkum et al. 1988), with three specific schemes being proposed by Lemoff in 1994 (Lemoff, Barty and Harris 1994). The scheme they proposed is as follows. A cell filled with argon, krypton, or xenon gas is pumped longitudinally by an intense laser pulse. The pulse is focused to an intensity sufficient to ionise the gas up to the desired lasant species: the closed-shell, 8-times-ionised ion. The pump laser is circularly polarized, and so due to ATI heating, the ionised electrons retain large energies (of the order of keV) after the passage of the laser pulse. The lasant ions are excited by electron-ion collisions, and the upper levels become populated. For 8-times-ionised Ar, Kr and Xe, the per-state excitation rate of the upper laser level was predicted to be higher than the lower laser level, and so an inversion could be achieved.

XUV lasing at 41.81 nm was observed using this technique using Xe ions in 1995 (Lemoff et al. 1995; Hooker, Epp and Yin 1997), and the work has recently been repeated by Sebban et al. (Sebban et al. 2001). Lemoff et al. showed that the XUV laser output intensity had an approximately exponential dependence on Xe pressure. At a fixed pressure of 12 torr, the XUV output showed an exponential dependence on the length of the gas cell, for lengths of up to 7.5 mm. As the cell length was increased to 8.5 mm, however, the increase in output was lower than predicted by the exponential fit. This was attributed to the intensity of the defocusing pump beam decreasing to below the threshold for the creation of the Xe$^{8+}$ lasant ion: the increase in cell length above 7.5 mm was not accompanied by an increase in gain length. Based on the exponential increase in laser output for lengths of up to 7.5 mm, the group estimated the small-signal gain coefficient to be 13 cm$^{-1}$.
Sebban et al. observed lasing on the same line in Xe\(^8^+\) at a pressure of 15 torr. The group measured an exponential increase in the XUV laser output as the length of the gas cell was increased to 2 mm, followed by saturation of the signal as the length was increased to 5 mm. They inferred a gain coefficient of 67 cm\(^{-1}\) from this behaviour, by assuming that the desired Xe\(^8^+\) ion was created over the whole length of the gas cell. Due to defocusing of the pump laser, this would not necessarily have been the case. Indeed, if the gas cell length had been increased beyond the length over which the desired ion could be created, the lasing signal would show exactly the behaviour that the group observed.

While great care needs to be taken when calculating the gain coefficient for this type of laser, it is clear that collisionally-pumped OFI lasers are a promising type of small-scale XUV laser. Other potential lasers sources pumped by the same mechanism have been predicted in krypton and argon, lasing at 31.9 and 47.8 nm respectively (Lemoff et al. 1994), Be-like ions (Hooker and Harris 1995), and inner-shell transitions (Hooker 2000).

### 11.2.2 Guided operation

In order to increase the output power of collisionally-pumped OFI lasers it is necessary to increase the length over which the lasant ion can be created. The experiments described above were unable to generate the desired ion stage over lengths of more than several millimetres owing to diffractive and refractive defocusing of the pump laser. Using a waveguide to channel the pump laser, diffractive and refractive defocusing can be balanced by the focusing properties of the waveguide, and the laser intensity required to create the desired lasant ion can be maintained over extended distances.

The gas-filled capillary discharge waveguide is well suited for use in XUV laser applications. In chapters 9 and 10, it was shown that a guiding channel can be created in hydrogen, or indeed other gases or mixtures of gases. In order to generate a long column of the Xe\(^8^+\) ion, a waveguide could be created in hydrogen gas doped with a small partial pressure of xenon.

Simulations of the propagation of an intense laser pulse through Xe-doped hydrogen waveguides have been undertaken. A waveguide with the following properties was assumed to have been created. The gas fill was composed of 120 mbar of hydrogen (H\(_2\)) doped with 5 mbar of Xe. Based on an estimated plasma temperature of 6 eV, the initial hydrogen ion stage was set to 1 and the xenon ion stage was set to 3, corresponding to an axial electron density of \(6 \times 10^{18} \text{ cm}^{-3}\). The radial electron density profile was parabola out to a radius of 150 \(\text{\mu m}\), with a curvature corresponding to a matched spot size of 25 \(\text{\mu m}\). Such a matched
spot size would be expected for this plasma confined in a 300 μm-diameter capillary, based on the results of chapter 9.

Calculations were performed to predict the propagation of a 40 mJ laser pulse, of 50 fs-duration and wavelength 800 nm, through the waveguide described above. For comparison, the propagation through a gas cell filled with 5 mbar of neutral Xe was also calculated. The circularly-polarised pulse was assumed to be focused to a spot size of 25 μm at the entrance of the waveguide or gas cell, giving a peak intensity of $8 \times 10^{16}$ W cm$^{-2}$. Note that the minimum laser intensity required to ionise xenon to the closed shell Xe$^{8+}$ ion is approximately $2 \times 10^{16}$ W cm$^{-2}$, and the ion is stable against further ionisation for intensities up to $1 \times 10^{17}$ W cm$^{-2}$.

Figure 11-1 (a) shows the predicted distribution of xenon ion stages generated in a 100 mm-long waveguide after the passage of the pump laser pulse. The required Xe$^{8+}$ ion is predicted to be formed over 84 mm of the guiding channel, out to a radius of 10 - 20 μm. Small oscillations in the peak intensity of the laser pulse create pockets of Xe$^{9+}$ where the pulse intensity is at its greatest.

For comparison, Figure 11-1 (b) shows the predicted distribution of ion stages in a 30 mm-long gas cell filled with 5 mbar of neutral Xe. The lasant ion Xe$^{8+}$ is predicted to form over a length of just 4 mm, which may be compared to the Rayleigh range of 2.5 mm for the incident beam. Propagating in vacuum, the intensity would be sufficiently to create Xe$^{8+}$ over a distance of 6.5 mm: the decrease in length for propagation in 5 mbar of neutral xenon is due to ionisation-induced defocusing of the laser pulse. By positioning the laser focus inside the gas cell, so that the laser pulse is converging as it enters the gas, the lasant ion can be created over a slightly longer length. Simulations based on the experimental conditions of Lemoff show reasonable agreement with their estimated gain length of 7.5 mm (Lemoff et al. 1995).
Figure 11-1: Calculated Xe ion stage generated by a 50 fs-duration laser pulse of intensity \( 8 \times 10^{16} \text{ W cm}^{-2} \) propagating through 5 mbar of neutral xenon (a), and guided through a hydrogen waveguide doped with 5 mbar of Xe (b). The waveguide has \( W_M = 25 \mu\text{m} \).

An un-doped hydrogen waveguide with a matched spot size of 25 \( \mu\text{m} \) would be capable of matched guiding of laser pulses with a 25 \( \mu\text{m} \) spot size at the capillary entrance: the spot size would remain constant as the pulse propagates through the waveguide. The waveguide doped with a small pressure of xenon simulated in Figure 11-1 (a) is not capable of perfect matched guiding, however, because extra electrons created by field ionisation of the xenon distort the original parabolic radial electron density profile in the waveguide. Figure 11-2 shows the radial electron density profile at the entrance of the Xe-doped waveguide after the passage of
the laser pulse. The dotted curve shows the transverse intensity profile of the laser pulse. The series of steps in the electron density between 20 and 40 μm are caused by decreases in the induced ion stage of the Xe ions. The steps defocus the wings of the laser pulse: however, that energy is constantly refocused by the parabolic shape of the electron density at larger radii, and so the laser pulse can propagate over tens of millimetres with an approximately constant peak intensity.

Figure 11-2: The electron density profile at the entrance to the waveguide after the passage of a 50 fs-duration laser pulse of intensity $8 \times 10^{16}$ W cm$^{-2}$. The hydrogen waveguide is doped with 5 mbar of Xe, and has a matched spot size $W_m = 25$ μm.

It is likely that the use of Xe-doped hydrogen-filled capillary discharge waveguides would be able to significantly extend the length over which the lasant ion can be generated. It is necessary, however, to consider whether suitable conditions for creating the population inversion would still exist for the Xe$^{8+}$ ions in the waveguide. Energetic electrons, ionised by OFI and heated by ATI heating, are required to produce the population inversion by electron-ion collisions. A key difference between the unguided and guided case is that in the guided case, those energetic electrons are immersed in a bath of relatively cold electrons formed by the discharge. The cold electrons are not strongly heated by the pump laser, since ATI heating and IB heating would be expected to be small: this is discussed again in section 11.3.2. It is important to show that the presence of the bath of cold electrons does not cool the hot electrons on a time scale faster that the creation of the population inversion.
Fortunately, because of the large energy difference between the two classes of electrons, the time required for the electron temperature to equilibrate is of the order of 1 ns. Since the population inversion is predicted to build up on a time scale of 1 - 10 ps (Lemoff et al. 1994), it is possible to conclude that creation of the population inversion should not be affected by the cold electrons. However, it should be noted that the Stark width of the XUV laser line would be increased by the presence of the cold electrons and so the gain of the system may be decreased.

A doped hydrogen-filled capillary discharge waveguide seems to be a promising device in which to demonstrate collisionally-pumped XUV lasers. There are many proposed lasers of this type, not only in Xe, but also in Ar$^{8+}$, Kr$^{8+}$, and the series of beryllium-like ions. It is also predicted that laser action may be possible on inner-shell transition in Ar (Hooker 2000). By doping the hydrogen waveguide, it should be possible to generate long lengths of any of these lasant ions, creating ideal conditions for demonstrating a broad range of XUV laser wavelengths.

11.2.3 Quasi-matched guided operation

For the case of XUV laser based on Ar$^{8+}$, there is a second possibility for creating a long region of gain. Rather than forming a waveguide in argon-doped hydrogen, it would be of interest to form a waveguide in pure argon. While guiding channels can be formed in any gas, for all gases apart from hydrogen and helium the channel would be only partially-ionised by the discharge. However, since Ar$^{8+}$ is particularly stable, it should possible for a long length of Ar$^{8+}$ to be created by a laser pulse that undergoes quasi-matched guiding through a partially-ionised argon waveguide.

As discussed in chapter 8, a stable ion stage is required for quasi-matched guiding. The input laser intensity must be at least 100 times the intensity required to generate that ion stage, so that a raised parabolic plateau is created out to a radius of several spot sizes around the axis. For a circularly-polarised laser pulse, the intensity required to ionise argon to Ar$^{8+}$ is approximately $5 \times 10^{16} \text{ W cm}^{-2}$, and so an intensity of above $5 \times 10^{18} \text{ W cm}^{-2}$ would be required to achieve high-quality quasi-matched guiding. Unfortunately, Ar$^{8+}$ is only stable up to an intensity of $2.5 \times 10^{18} \text{ W cm}^{-2}$, above which Ar$^{9+}$ is generated, and so the guided laser intensity must be kept below this value. Due to the limitation of the laser intensity, high-quality quasi-matched guiding cannot be achieved. However, even at only 50 times the intensity required to generate Ar$^{8+}$, some benefits of quasi-matching are apparent.
Calculations were performed to predict the propagation of a 50 fs-duration, 800 nm laser pulse through a waveguide formed in 33 mbar of argon. The waveguide was assumed to have a matched spot size of 32 μm, and was composed of Ar$^{3+}$, based on an estimate of the temperature of the plasma of 6 eV. The circularly-polarised laser pulse was taken to be focused to a waist of 30 μm at the entrance of the capillary, resulting in a peak intensity of $2 \times 10^{18}$ W cm$^{-2}$: just below that required to ionise the argon to Ar$^{9+}$.

Figure 11-3 shows the calculated argon ion stage in the capillary after the passage of the laser pulse. It is seen that Ar$^{8+}$ is predicted to be formed over more than 25 mm. Figure 11-4 shows the radial electron density profile at the capillary entrance after the passage of the laser pulse, whose radial intensity profile is shown by the dotted line. After the passage of the laser pulse, the argon is ionised to Ar$^{8+}$ out to a radius of 40 μm, and the matched spot size in this region of the guide is decreased to $W_{QM} = 25$ μm. The input spot size of 30 μm was found to be the optimum for this waveguide: the fact that the optimum spot size is larger than $W_{QM}$ is an indication that the intensity of the laser pulse is a little under that required for full quasi-matched guiding. The wide raised plateau of Ar$^{8+}$ created near the axis is responsible for suppressing ionization-induced defocusing of the laser pulse, but does not extend to large enough radius to achieve high-quality quasi-matched guiding.

While quasi-matched guiding cannot be fully achieved, it is clear that a pure argon waveguide could indeed be used to create an extended column of Ar$^{8+}$ ions, suitable for XUV gain. While the length over which the pump laser could be guided is far longer for an argon-doped waveguide, a pure argon waveguide could have other advantages when used for XUV lasers. The argon pressure can be higher in the case of a pure argon waveguide, which would increase the gain and the saturation intensity of the XUV laser (Pert 2001). Furthermore, the bath of cold electrons associated with background hydrogen is eliminated, avoiding a reduction in the XUV gain owing to an increased Stark width.
Figure 11-3: Calculated Ar ion stage generated in a plasma waveguide by a 50 fs-duration laser pulse focused to a waist of 30 μm, with a peak intensity of $2 \times 10^{18} \text{ W cm}^{-2}$. The waveguide was taken to have $W_M = 32 \mu m$, formed in 33 mbar of Ar$^3$.

Figure 11-4: The calculated electron density profile at the entrance to the waveguide after the passage of the laser pulse, for the conditions described in Figure 11-3.
11.3 Recombination-pumped XUV laser in argon

11.3.1 Unguided operation

OFI-pumped recombination XUV lasers were first proposed by Burnett and Corkum (Burnett et al. 1989). The scheme they proposed is as follows. A desired ion stage is produced via OFI by an intense laser pulse that is focused longitudinally through a gas cell. Recombination of electrons with the ions then occurs: the electrons quickly relax by collisions through the energy levels of the daughter ions and reach low-lying states. If recombination occurs sufficiently rapidly, an inversion can be created between the low-lying levels of the daughter ions, and XUV lasing occurs. Recombination lasers are of particular interest because, in principle, it is possible to achieve lasing to the ground state of the daughter ion. Thus, for a given lasant ion, and hence a given intensity of the driving laser, recombination lasers give access to higher energy transitions.

To achieve rapid recombination, it is important that the electron temperature is sufficiently low. For this reason, linearly-polarised laser pulses should be used, minimising ATI heating of the optically field ionised electrons.

The group of G. Pert at York University has proposed a recombination laser in argon, in which an inversion is created by rapid recombination of electrons with Ar to form sodium-like Ar\(^{7+}\). The group has performed simulations of XUV lasing experiments in which Ar\(^{8+}\) is created by OFI of the neutral gas. They found that by using a low density of argon immersed in a higher density of hydrogen buffer gas, the gain of the recombination laser was greatly enhanced. The optimal conditions that they proposed are an argon density of \(10^{17} \text{ cm}^{-3}\), corresponding to 4 mbar, and a hydrogen atom density of \(3.4 \times 10^{18} \text{ cm}^{-3}\), corresponding to 68 mbar of H\(_2\) (Grout et al. 1997; Pert 2001). The group simulated the propagation of an intense laser pulse through the neutral gas mix, and by calculating excitation and recombination rates, a gain of over 350 cm\(^{-1}\) at a wavelength of 23.2 nm was calculated. For those simulations, the pump laser pulse had a wavelength of 250 nm, a 100 fs-rise time, and 100 fs-duration. The peak intensity was set to \(5 \times 10^{16} \text{ W cm}^{-2}\), sufficient to ionise the hydrogen and argon to H\(^+\) and Ar\(^{8+}\). The length over which the correct argon ion stage was created was predicted to be less than 10 mm, limited by refractive defocusing of the pump laser (Grout, Pert and Djaoui 1998).

The recombination-pumped XUV laser scheme is critically dependent on the availability of cold electrons that recombine rapidly with the ions. For the conditions considered by Pert’s group, the peak plasma temperature was calculated to be 4.1 eV. Plasma heating was due to
above-threshold ionisation (ATI), and inverse bremsstrahlung (IB). Of these, ATI heating was found to dominate.

For a given rate of optical field ionisation, the average ATI energy per electron is proportional to the quiver energy \( E_q \) of the electrons, which is given by

\[
E_q = \frac{e^2 E_0^2}{4m_0\omega_0^2}
\]

in which \( E_0 \) is the peak laser field, and \( \omega_0 \) is the laser frequency. The cycle-averaged rate of inverse bremsstrahlung heating per electron \( P_{ib} \) is given by (Rae and Burnett 1992)

\[
P_{ib} = \frac{e^2 E_0^2 v_{th}^2}{2m_0\omega_0^2} \left[ 1 + \frac{v_q^2}{3v_{th}^2} \right]^{3/2}
\]

Substituting for the characteristic thermal velocity \( v_{th} \) and the quiver velocity \( v_q \), and making the approximation \( v_q \gg v_{th} \), equation (11.2) can be used to write the energy deposition due to IB, \( E_{ib} \), as

\[
E_{ib} = \frac{C_{ib} \omega_0 \tau_p}{E_0}
\]

in which \( \tau_p \) is the pulse duration, and \( C_{ib} \) is a constant that depends on the pulse shape and the plasma parameters. Since ATI was found to be the dominant heating mechanism in the plasma, Pert’s proposed laser parameters were chosen to minimise ATI. The Kr:F laser wavelength of 0.25 \( \mu \text{m} \) was chosen, rather than the more readily available Ti:Sapphire wavelength of 0.8 \( \mu \text{m} \), or the Nd:Glass wavelength of 1 \( \mu \text{m} \), resulting in a reduction in ATI heating of a factor 10 and 16 respectively. The pulse rise time and duration were set to 100 fs, since ATI can be significantly enhanced for sub-100 fs pulses (Janulewicz, Grout and Pert 1996). The choice of a long-duration, short-wavelength pulse reduced ATI heating to an acceptable level, but conversely enhanced IB heating.

11.3.2 Guided operation

The hydrogen-filled capillary discharge waveguide would seem to be ideally suited to demonstrate the recombination laser described above. The initial hydrogen pressure required by the scheme is exactly the range in which the waveguide has been demonstrated to operate. Furthermore, the hydrogen gas may easily be doped with the small partial pressure of argon
required. A waveguide created in the gas mix would be capable of guiding the intense laser pulse that creates the desired lasant ion, and so a population inversion could be created over long lengths.

Simulations have been performed to predict the propagation of a linearly-polarised, 78 mJ laser pulse with a duration of 100 fs, and a wavelength of 250 nm, through a waveguide with a matched spot size of 22 μm formed in the argon-hydrogen mixture discussed above. It was assumed that the plasma formed by the discharge was ionised to H⁺ and Ar³⁺, based on an estimated plasma temperature of 6 eV. The laser pulse was taken to be focused to a focal spot size of 30 μm at the capillary entrance, corresponding to a peak intensity of 5 \times 10^{16} \text{ W cm}^{-2}.

Figure 11-5 shows the argon ion stage generated in the waveguide following the passage of the laser pulse. It is clear that high-quality guiding is achieved, creating the desired Ar⁸⁺ lasant ion over 30 cm. Suitable conditions for recombination lasing should exist along the entire 30 cm capillary length.

Note that, in this example, guiding is achieved over a far greater distance that the simulation for a Xe-doped waveguide in Figure 11-1. The increase in guided length is largely attributed to the change of wavelength from 800 nm to 250 nm, which reduces the defocusing effect of non-parabolic perturbations of the electron density profile.

Since cold electrons are vital for high-gain recombination lasers, it is important to consider the way in which the waveguide would affect the final electron temperature. In the unguided scheme, the laser ionises both the hydrogen and argon, which are initially neutral. In a guided scheme, however, the initial plasma is not neutral. The hydrogen would be fully-ionised and the argon ionised to approximately Ar³⁺ by the discharge. MHD simulations suggest that the electrons are heated to approximately 6 eV by the discharge (Bobrova et al. 2001). The intense laser pulse will further ionise the argon ions, and the liberated electrons will be heated by ATI. However, for a plasma which was initially composed of Ar³⁺ at a density of 10^{17} \text{ cm}^{-3} and H⁺ at a density of 3.4 \times 10^{18} \text{ cm}^{-3}, the electrons that are heated by ATI comprise just 12% of the total plasma electrons. The majority of the electrons are thus not heated by ATI for the guided case.

If the electron temperature in the guide could be kept as low as 7 eV after the passage of the laser pulse, a gain is 350 cm⁻¹ has been predicted (Grout et al. 1997). The saturation intensity of the laser transition was predicted to be 10^7 \text{ W cm}^{-2}. Although this is relatively small, the gain length that could be achieved using the hydrogen-filled capillary waveguide is predicted to be extremely long, and so a reasonably high XUV output power could be achieved.
Figure 11-5: Calculated Ar ion stage generated by a 100 fs-duration laser pulse of wavelength 250 nm, and intensity $5 \times 10^{16} \text{ W cm}^{-2}$, guided through a hydrogen waveguide doped with 4 mbar of Ar. The waveguide has a matched spot size $W_m = 22 \mu\text{m}$. $\text{Ar}^{8+}$ is generated over more than 30 cm.

Owing to the changes in the way the electrons are heated, it may be possible for the guided XUV laser to be pumped using a Ti:Sapphire laser pulse. Such a change would be advantageous owing to the greater availability of terawatt Ti:Sapphire lasers. As discussed in section 11.3.1, for an initially unionised plasma, ATI heating by a pulse of wavelength 800 nm would heat the plasma to an unacceptably high temperature. However, for the pre-ionised plasma in a waveguide, only a small fraction of the electrons in the final plasma are heated by ATI, and thus it is more important to minimise IB heating. Heating due to IB is far lower for a pump pulse of wavelength 800 nm compared with 250 nm, since the heating rate is proportional to frequency (equation (11.3)). It would also be advantageous to use a pulse with a short duration (<50 fs). It seems likely that moving to a short-pulse Ti:Sapphire laser would decrease the final electron temperature, and lead to increased XUV gain.
11.4 Summary

Collisionally-pumped and recombination-pumped XUV gain following optical field ionisation has previously been predicted in a range of gases such as Xe, Kr, Ar, and Ne. However, XUV output has been experimentally demonstrated in only one of these suggested arrangements: the collisionally-pumped Xe$^8^+$ laser. One of the main limitations of current experiments is that only short gain lengths have been achieved, limited by ionisation-induced refraction. Waveguides offer the possibility of greatly extending the gain length that may be achieved, and the properties of the gas-filled capillary discharge waveguide would seem to be ideally suited to the task.

Simulations have been presented showing that long lengths of Ar$^8^+$ and Xe$^8^+$ can be produced either by doping a hydrogen waveguide with a small partial pressure of the lasant gas, or, in the case of argon, producing a waveguide in the pure gas. Using the gas-filled capillary discharge waveguide, the gain length of longitudinally-pumped XUV lasers could be increased substantially. The corresponding increase in output energy would be important for applications of XUV lasers. Furthermore, it should be possible to saturate the gain in these systems, thereby achieving efficient extraction of the energy associated with the population inversion.

To date, OFI-pumped XUV lasing has been observed only in xenon. In those experiments, the position and alignment of the gas cell and pump laser was found to be critical. The gain length was only a few millimetres after optimisation of the system - a poorly optimised system would not lase at all. Using a waveguide to extend the gain length should make such XUV lasers far easier to align, and hence more robust. Furthermore, laser schemes for which the gain is significantly lower will become much easier to realise, broadening the range of wavelengths available from OFI-pumped XUV lasers.
References

Bobrova, N. A., A. A. Esaulov, J.-I. Sakai, P. V. Sasorov, D. J. Spence, A. Butler, S. M.

“Cold-Plasma Production for Recombination Extreme-Ultraviolet Lasers by Optical-

"Short-wavelength coherent radiation: Generation and applications". OSA proceedings, Optical Society of America 1998.

Commun. 141(3-4): 213-220.


“Demonstration of a 10-Hz Femtosecond Pulse-Driven XUV Laser at 41.8-Nm in Xe-


12.1 Theory

The acceleration of charged particles using a plasma wave was first proposed by Tajima and Dawson in 1979 (Tajima and Dawson 1979). The electric fields associated with a plasma wave can be far in excess of fields attainable by conventional electrical methods. A stable plasma wave can have associated electric field gradients of up to the order of the wave-breaking field, which is given by (Ogata and Nakajima 1998)

\[ E_{WB} = \frac{m_e \omega_p c}{e} \]  

(12.1)

For electron densities of order \(10^{18}\) cm\(^{-3}\), equation (12.1) gives \(E_{WB} = 100\) GV m\(^{-1}\). Accelerating fields in conventional linear or ring accelerators based are limited to the order 100 MV m\(^{-1}\) by electrical breakdown across surfaces. The potential 1000-fold increase in electric field offered by plasma wave accelerators could drastically reduce the scale of future accelerator facilities.

Plasma waves can be excited by the interaction of an intense laser pulse with a plasma. Two distinct methods, known as self-modulated wakefield acceleration (sm-LWFA) (Andreev et al. 1992; Antonsen and Mora 1992; Sprangle et al. 1992) and resonant wakefield acceleration (LWFA) (Tajima et al. 1979), have been proposed.

In LWFA, a plasma wave is excited by the forward ponderomotive force due to the leading edge of the laser pulse and the backward force due to the trailing edge. In order to maximise
the amplitude of the plasma wave, the laser pulse length should approximately equal the plasma wavelength $\lambda_p$, which ensures resonant excitation of the plasma wave. The plasma wavelength $\lambda_p$ can be written from equation (1.8) as

$$\lambda_p = \frac{2\pi c}{\omega_p} = 2\pi c \left( \frac{e \omega_p m_e}{N e^2} \right)^{1/2}$$  \hspace{1cm} (12.2)

Low-energy ($=1\text{ MeV}$) electrons that are injected coaxially with the driving laser pulse can 'surf' on the plasma wave, accelerated by the electric field associated with the plasma wave.

In sm-LWFA, a pulse with a length of many times the plasma wavelength is focused into the plasma. Stimulated Raman scattering, seeded by the plasma wave generated by the pulse's leading edge, modulates the laser pulse into beamlets, each of which has a length approximately equal to the plasma wavelength. The self-modulated pulse then resonantly drives the required plasma wave. The accelerating fields generated by sm-LWFA can be far greater than LWFA. Furthermore, while electrons must be injected into LWFA accelerators, the plasma wave generated by sm-LWFA is sufficiently strong to trap and accelerate free electrons in the plasma. However, undesired propagation instabilities can limit the length over which the plasma wave can be excited.

Each cycle of the driven plasma wave may be divided into an accelerating phase and a decelerating phase, each of which is further divided into a radially focusing and a defocusing part. Since electrons repel one another, an electron bunch must surf on the region of the wave that is both focusing and accelerating. This region has a length of only a quarter of the plasma wavelength. Such a narrow accelerating window leads to the natural production of ultra-short electron bunches: for a typical plasma density of $10^{18} \text{ W cm}^{-2}$, $\lambda_p/4$ is approximately $8\mu\text{m}$, corresponding to a electron pulse duration of only 25 fs.

12.1.1 Acceleration length

The energy that accelerated electrons acquire from the plasma wave is given by the accelerating field multiplied by the acceleration distance. The acceleration distance is limited by one of three effects described below: laser defocusing, dephasing, and pump depletion.

Laser defocusing by diffraction or ionisation-induced defocusing will limit the length over which the laser intensity can be maintained. As the pulse defocuses, the amplitude of the generated plasma wave decreases, and the acceleration gradient decreases correspondingly. The laser defocusing length $L_D$ is defined as the distance after which the laser intensity has
dropped to half of the focal value. In fully-ionised plasmas in the absence of guiding or instabilities, \( L_D \) is given by the Rayleigh range \( Z_R \). In partially-ionised plasmas, \( L_D \) is often reduced significantly below \( Z_R \).

As discussed above, electrons must surf in the appropriate phase of the plasma wave to be correctly accelerated. Dephasing of the plasma wave and the accelerated electrons occurs as the electrons travel through the plasma. The electrons travel with a velocity close to \( c \), whereas the plasma wave propagates at the group velocity of the driving laser pulse. The velocity difference results in the electrons moving out of the accelerating window after propagating a distance known as the dephasing length, given by

\[
L_D = \frac{1}{2} \frac{\lambda_p^2}{\lambda^2}
\]  

(12.3)

where \( \lambda_p \) and \( \lambda \) are the wavelengths of the plasma wave and driving laser respectively (Mora and Amiranoff 1989).

Pump depletion refers to absorption of the driving laser energy by the plasma. In driving the plasma wave, the laser pulse transfers energy to the plasma. As the laser pulse is absorbed, the amplitude of the driven plasma wave decreases. The pump depletion length \( L_{PD} \) is a measure of the distance over which the laser pulse can effectively pump the plasma wave, and is given by \( L_{PD} = L_D / a_0^2 \) (Mora et al. 1989), in which \( a_0 \) is the normalised vector potential of the laser field defined by equation (1.10).

Pump depletion, pump defocusing, and dephasing all occur simultaneously as the laser propagates through the plasma. In practice, the shortest of the lengths that characterise the three processes will set the length over which electrons may be accelerated. The relative sizes of the characteristic lengths depend on the plasma density, laser power and wavelength, and the focusing conditions employed.

### 12.2 Experimental demonstrations

Laser wakefield acceleration has been demonstrated experimentally by several groups. In 1995, self-modulated LWFA was used to accelerate electrons to an energy of 44 MeV (Modena et al. 1995). In this experiment the acceleration length was just over 1 mm. More recently, electrons accelerated up to 100 MeV have been observed from a 4 mm-long gas jet pumped by a 20 TW Nd:Glass laser (Gordon et al. 1998).
Resonant LWFA has also been demonstrated by several groups (Dewa et al. 1998; Kando et al. 1999). Kando et al. demonstrated acceleration of electrons up to 200 MeV. In this work, electrons of energy 17 MeV were injected into a plasma pumped by a 2 TW, 90 fs-duration, laser pulse. The acceleration length was approximately 1 cm, limited by diffraction of the pump laser.

LFWA experiments are strongly limited by the defocusing length of the driving laser pulse. It is clear that plasma waveguides are ideally suited to extend the acceleration distance, allowing far higher electron energies to be reached. While some sm-LWFA experiments have been limited by dephasing (Gordon et al. 1998), there are also regimes in which waveguides could enhance the electron energies achieved. For several years, theoretical work on LWFA has focused on the use of plasma channels in acceleration experiments (Leemans et al. 1996; Leemans et al. 1998; Sprangle et al. 2000): the field of LWFA is currently waiting for the realisation of a waveguide of sufficient quality that the impressive gains offered by guided-LWFA can be obtained in practice.

12.3 Electron acceleration in a hydrogen-filled capillary discharge waveguide

In this section, the design of a guided-LWFA experiment will be considered. The hydrogen-filled capillary discharge waveguide seems ideally suited for this purpose, and so a waveguide with the properties of that device will be assumed. The waveguide is the only guide to date that is capable of low-loss matched guiding over extended distances, through fully-ionised plasmas of an appropriate density. Specifically, a plasma waveguide of the following form will be assumed:

\[
N(r) = N(0) + \frac{\Delta N}{r_c^2} r^2
\]  

(12.4)

The investigation of the discharge-ablated capillary waveguides in chapter 7 showed that the figure of merit of the waveguide, rewritten from equation (7.2) as \( Q \propto \Delta N / r_c^2 N(0) \), was constant for a waveguide formed within a capillary of given radius, regardless of the value of \( N(0) \). This finding was confirmed for hydrogen-filled capillary waveguides by the results of hydrodynamic simulations performed by Bobrova et al. (Bobrova et al. 2001). Furthermore, \( Q \) was found experimentally to depend only on the capillary radius, increasing rapidly as the radius of the capillary was decreased, shown in Figure 7-5. This effect was also predicted by Bobrova, who calculated that \( Q \) should scale inversely as the square of the capillary radius.
Using the measured parameters for a 150 μm-diameter hydrogen-filled capillary waveguide, which had a measured matched spot size of 37.5 μm for an axial electron density of $3.3 \times 10^{18} \text{ W cm}^{-2}$, it is possible to predict the spot size as a function of axial electron density and capillary radius based on the above scaling laws:

$$W_m [\mu m] = 37.5 \times \left( \frac{3.3 \times 10^{18}}{N(0) \text{ [cm}^{-3}] \right)^{\frac{1}{4}} \times \left( \frac{r_\text{cp} \text{ [μm]}}{150} \right)^{\frac{1}{2}}$$  \hspace{1cm} (12.5)

The matched spot size predicted by this equation for a range of capillary radii is shown in Figure 12-1. Note that the matched spot size is independent of the laser wavelength.

![Graph showing calculated matched spot size for a hydrogen-filled capillary waveguide as a function of axial electron density, and capillary radius. The spot size is calculated using experimental data, and the scaling laws summarised in equation (12.5).](image)

**Figure 12-1:** The calculated matched spot size for a hydrogen-filled capillary waveguide as a function of axial electron density, and capillary radius. The spot size is calculated using experimental data, and the scaling laws summarised in equation (12.5).

Using equations (12.4) and (12.5) to model the hydrogen-filled waveguide, it is possible to predict the electron energy that might be achieved in a guided-LWFA experiment. Several variables must be chosen in planning an acceleration experiment, the most important of which is the axial electron density. Since the pulse duration must resonantly excite the plasma wave, the axial electron density determines the pulse length $\tau$: the resonance condition (Dorchies *et al.* 1999) can be written
\[ \omega_p \tau = \left( \frac{N(0)e^2}{\epsilon_0 m_e} \right)^{\frac{1}{2}} \tau = 2 \] (12.6)

The dependence of resonant pulse length on electron density is plotted in Figure 12-2. Note that the resonance condition is again independent of the laser wavelength.

![Figure 12-2: Pulse duration \( \tau \) required for resonant excitation of a plasma wave, and the dephasing length \( L_{DP} \), as a function of the axial electron density.](image)

The choice of axial electron density thus determines the pulse length of the driving laser. The chosen electron density and capillary radius also determine through equation (12.5) the matched spot size of the waveguide. The laser pulse should be focused to this spot size to achieve matched guiding through the waveguide.

It is possible to estimate the maximum electron energy gain that could be achieved using LWFA, assuming that the laser is guided with a constant spot size through the length of the waveguide. The maximum length over which electron may be accelerated is set by the smallest of the laser defocusing length, the dephasing length, and the pump depletion length. For the case of guided-LWFA, the laser defocusing length is infinite for the case of a perfect waveguide. While a real waveguide will have some limitation on the maximum guided length, it seems reasonable that the hydrogen-filled capillary waveguide will be capable of guiding over lengths of at least 10 cm.
In order that relativistic instabilities do not interfere with propagation of the laser pulse through the waveguide, it is preferable to limit the driving laser intensity to values such that the normalised vector potential $a_0$, defined by equation (1.10), is less than unity. In this regime, the pump depletion length is always greater than the dephasing length and so it is the dephasing length that sets the electron acceleration length. The dephasing length $L_D$, defined by equation (12.3), is plotted in Figure 12-2, as a function of electron density.

All that remains is to calculate the magnitude of the accelerating field associated with the plasma wave generated by the laser pulse. A linear model of LWFA has been developed by Gorbunov and Kirsanov (Gorbunov and Kirsanov 1987), which allows calculation of the peak longitudinal accelerating field $E_{z\text{max}}$. The energy gain of an electron is then simply $\Delta W = eE_{z\text{max}}L_D$. While the dephasing length, the resonant pulse duration, and the matched spot size all depend on the choice of axial electron density, $\Delta W$ is in fact independent of $N(0)$, and depends only on the radius of the capillary waveguide, and the pulse energy of the driving laser. The dependence of $L_D$ on $N(0)$ is cancelled by the inverse dependence of $E_{z\text{max}}$ on $N(0)$.

Figure 12-3 shows the dependence of $\Delta W$ on the driving laser pulse energy, which is a simple proportional relationship. It is clear that it is favourable to use narrower capillaries, which allow higher electron energies to be reached using a smaller driving laser. Higher electron energies can be achieved using capillaries of smaller radius owing to their larger figure of merit $Q$. The parameter $Q$ increases as the capillary diameter is decreased, allowing a smaller spot size to be guided through a plasma with a given axial density, resulting in an increase in guided intensity and an associated increase in the magnitude of the plasma wave.
Figure 12-3: Energy gain of accelerated electrons $\Delta W$ as a function of the driving laser pulse energy, for a range of capillary radii. The energy gain is independent of the axial electron density in the waveguide.

There is however a limit to the maximum pulse energy that can be used for any given configuration, set by the requirement that the pulse be non-relativistic. The normalised vector potential $a_0$ can be calculated from the laser parameters and the focal spot size: since the matched focal spot depends on the axial electron density, and the pulse duration must satisfy the resonance condition, it is possible to plot $a_0$ against $N(0)$. Figure 12-4 shows $a_0$ as a function of $N(0)$ for a 1 J, 800 nm laser pulse. The energy of the driving laser thus determines the maximum axial electron density that may be employed: this maximum electron density is plotted as a function of the laser pulse energy in Figure 12-4 (b).
Figure 12-4: (a) The normalised vector potential $a_0$ of a 1 J laser pulse as a function of axial electron density $N(0)$. The spot size of the laser pulse is determined from equation (12.5).

(b): The waveguide must operate in a regime for which $a_0 < 1$, defining a maximum permitted electron density as a function of laser pulse energy.
12.4 Proposed guided LWFA experiment

Consider the following proposed guided-LWFA experiment. The driving laser is a 1.5 J, 800 nm, Ti:Sapphire laser, with a pulse length of 50 fs or greater (the pulse length may be increased by misaligning the compressor). A hydrogen-filled capillary waveguide with a radius of 50 μm will be used. Figure 12-3 immediately reveals that the maximum electron energy gain that can be expected from the experiment is 1.2 GeV.

In order that the guided laser pulse does not suffer from relativistic instabilities, the axial electron density in the waveguide must be chosen so that $a_0$ is less than one. From Figure 12-4 (b), this constrains the axial electron density in the waveguide to less than $7.5 \times 10^{17}$ cm$^{-3}$. The chosen electron density can be anything less than this density, bearing in mind that as the density is decreased, the dephasing length increases and so the length of the waveguide must be increased (to make full use of the accelerating distance). As the electron density is increased, the laser pulse duration required for resonant excitation of the plasma wave decreases. Recall that the final electron energy obtained from the experiment does not depend on $N(0)$. Consulting Figure 12-1 and Figure 12-2, for $N(0) = 6 \times 10^{17}$ cm$^{-3}$, the corresponding values of the dephasing length, the pulse duration, and matched spot size are $L_{dp} = 63$ mm, $\tau = 76$ fs, and $W_m = 26.6$ μm. Since these numbers are reasonable, the chosen value of $N(0)$ is acceptable.

The above choice of axial electron density completes the planning of the acceleration experiment: all parameters are summarised in Table 12-1. Such an electron acceleration experiment would be the first to be carried out inside a waveguide, presenting the first prospect of GeV electron acceleration using LWFA. It is clear that the laser parameters required are not extraordinary. The parameters of the waveguide are also close to those that have already been demonstrated: all that remains is to show that it is possible to operate a hydrogen-filled capillary waveguide with a capillary of 100 μm-diameter.
Laser parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>800 nm</td>
</tr>
<tr>
<td>Energy</td>
<td>1.5 J</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>76 fs</td>
</tr>
<tr>
<td>Focal spot size</td>
<td>26.6 μm</td>
</tr>
</tbody>
</table>

Waveguide parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary radius</td>
<td>50 μm</td>
</tr>
<tr>
<td>Axial density</td>
<td>$6 \times 10^{17}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Initial $H_2$ pressure</td>
<td>12 mbar</td>
</tr>
<tr>
<td>Guiding length required</td>
<td>63 mm</td>
</tr>
</tbody>
</table>

Electron parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron energy attained</td>
<td>1.2 GeV</td>
</tr>
</tbody>
</table>

Table 12-1: Summary of the parameters of a proposed guided-LWFA experiment, based on the properties of the hydrogen-filled capillary discharge waveguide.

12.5 LWFA in a discharge-ablated capillary waveguide

An alternative waveguide that has been proposed for guided-LWFA experiments is the discharge-ablated capillary waveguide. The Zigler group has suggested that electron energy gains of the order of 1 GeV could be obtained using this device (Hubbard et al. 2001).

Unfortunately, there are significant problems with the discharge-ablated capillary waveguide, which could make LWFA experiments prone to failure. The partially-ionised nature of the waveguide has not been addressed in the experiments proposed. The authors assume that high-quality matched guiding would be achieved over several centimetres: simulations presented in chapter 6 have shown however that matched guiding can not be achieved in partially-ionised waveguides of this type. Quasi-matched guiding could be achieved for a
suitable laser intensity, discussed in chapter 8: for non-relativistic laser intensities, however, guiding would be limited to approximately three centimetres, as shown in Figure 8-2.

The group also assumes that the $Q$ of the waveguide would be between 50 and 100: despite current waveguides having $Q$ of the order of 5, they claim that a ten-fold increase in $Q$ “is likely in the future.” The analysis of chapter 7 suggests that such an increase in $Q$ would only be obtained by decreasing the capillary radius to much less than 40 μm.

12.6 Tapered channel phase matching

It has recently been suggested that a plasma channel with an axial density that increases with propagation distance could allow acceleration beyond the de-phasing length (Sprangle et al. 2000). In this approach, the electrons must be accelerated in the correct phase of the plasma wave several wavelengths behind the laser pulse. If the axial electron density increases as the wave propagates, the plasma wavelength decreases and the plasma wave bunches up behind the laser pulse. In this way, the phase velocity of the plasma wave increases with increasing distance behind the laser pulse. If the electron density is correctly tailored, there exists a portion of the plasma wave with a phase velocity approximately equal to $c$. Electrons injected into this part of the plasma wave do not become out of phase as they propagate, and can thus be accelerated for lengths greater than the dephasing length.

It would be possible to create the desired non-uniform pressure distribution in a gas-filled capillary discharge waveguide by tailoring the steady-state gas flow prior to the discharge. Such tailoring could be achieved by altering the location and number of gas injection points, or by tailoring the capillary radius to induce a longitudinal dependence in the flow conductance of the capillary.

12.7 Summary

In conclusion, the hydrogen-filled capillary discharge waveguide would seem to be ideally suited for laser wakefield acceleration: a field that is presently severely limited by ionisation-induced defocusing of the pump laser. Theoretical work to date has focused on guided-LWFA for several years, but until now, no waveguide has been capable of high-quality matched guiding through plasmas of suitable density. In chapter 10, it was shown that the hydrogen-filled capillary waveguide could guide intense laser pulses over distances and plasma densities of interest to LWFA. Optimisation of the device along the lines suggested in section 12.4 offers the prospect of laser-based acceleration to the GeV level for the first time.
Furthermore, the hydrogen-filled capillary discharge waveguide has several features of practical importance. The device is able to operate over a wide range of electron densities, and can be run for thousands of shots without damage. The ability to tune the hydrogen pressure in the capillary during acceleration experiments will enable the resonance condition to be met precisely, thereby maximising the electron energies produced.
References


13.1 Summary of results and their implications

The interaction between high-intensity lasers and underdense plasmas is extremely complex, and is the subject of a broad field of fundamental research. Several specific applications have been developed from this research, including high-harmonic generation, XUV lasers, and electron acceleration. The maturity of chirped pulse amplification technology has seen commercially-available lasers systems producing pulses with terawatt output powers, at costs that make such lasers accessible to a wide range of commercial and academic institutions. The resulting proliferation of high-intensity lasers has strengthened interest in the field of laser-plasma interactions.

This thesis described the development of waveguides suitable for guiding high-intensity laser pulses. In the absence of a waveguide, the distance over which a high-intensity laser pulse interacts with a plasma can be severely limited by diffraction, defocusing, and non-linear effects. A waveguide brings the prospect of greatly increasing that distance: a common goal for applications of laser-plasma interactions.

The initial work in this thesis focused on developing a waveguide based on the discharge-ablated capillary waveguide: a device invented by the group of A. Zigler (Zigler et al. 1996). The new device incorporated an extra trigger electrode at one end of the capillary. A fast-rising voltage pulse, generated using magnetic pulse compression, was applied to the trigger electrode to strike the discharge through the main capillary. In this way, the charging voltage of the discharge capacitor could be freely changed without affecting the breakdown properties of the device.

Experiments were undertaken to investigate the guiding properties of the device for high-intensity picosecond-duration laser pulses. The energy transmission and the spot size of
the laser pulse in the exit plane of the capillary were measured for input laser pulses with an intensity of $10^{16}$ W cm$^{-2}$. During the discharge, the energy transmission through the capillary was seen to be enhanced, and a small spot size was observed at the capillary exit, indicating successful channelling of the laser radiation through the capillary. These results, published in the Journal of the Optical Society of America B. (Hooker, Spence and Smith 2000), were the first to assess the properties of the discharge-ablated capillary waveguide as a function of time through the discharge.

Measurements of the electron density profile generated in the waveguide were required to aid interpretation of the guiding results. A technique using longitudinal interferometry was successfully developed, in which a plane probe beam was propagated along the axis of a short capillary. By making measurements for two different capillary lengths, the influence of end effects was eliminated. These were the first measurements of the electron density profile formed inside the discharge-ablated capillary waveguide, and were published in Optics Letters (Spence, Burnett and Hooker 1999). The electron density profile was found to be approximately parabolic, with the axial electron density and curvature increasing during the discharge.

It was estimated that the plasma generated by the discharge was only partially-ionised. Owing to further ionisation of the plasma by the guided laser pulse, it was clear that the guiding profile would be disturbed to some extent by the propagating pulse. A full interpretation of a guiding experiment therefore required a careful assessment of the properties of partially-ionised plasmas waveguides. For this purpose, a numerical simulation of the propagation of intense laser pulses through plasmas was developed. The code calculated the evolution of a laser pulse envelope propagating through an arbitrary plasma, allowing for further ionisation of the plasma due to optical field ionisation. Using the measured electron density profiles along with estimates of the initial ion stage of the plasma, simulations of the picosecond guiding experiments were performed. Excellent agreement was achieved with the experimental results; a comparison between the two was published in the Journal of the Optical Society of America B (Spence and Hooker 2000). This work was the first detailed study of the guiding properties of partially-ionised plasma waveguides for intense laser pulses. Previous work by the group of Zigler neglected the consequences of the partially-ionised nature of the waveguide. The laser pulse was calculated to defocus sharply near the capillary entrance, before refocusing to a small spot at the capillary exit. While giving the illusion of high-quality matched guiding, in fact the intensity of the guided pulse was maintained at greater than 20% of the input intensity for a total distance of just 1.5 mm.

A systematic optimisation of the discharge-ablated capillary waveguide was performed, quantifying the effect of changing all aspects of the device and the ablating current pulse. It
was discovered that a figure of merit \( Q \) could be assigned according to capillary radius, relating the ratio of the axial electron density and the curvature of the radial profile. The magnitude and period of the ablation current, and the capillary material did not affect the figure of merit. The matched spot size of the waveguide formed in the capillary, for a given axial density, was found to scale approximately inversely as the square of the capillary radius. This result was later theoretically verified by simulations carried out by the Bobrova et al. (Bobrova et al. 2001). The discovery of this simple scaling law allowed realistic predictions of the matched spot size in a range of discharge-ablated capillary waveguides to be made.

For applications requiring waveguides for high-intensity lasers, it is important the axial laser intensity is maintained at close to its peak value for extended distances. At a laser intensity of \( 10^{16} \text{ W cm}^{-2} \), simulations showed that the discharge-ablated waveguide did not satisfy this requirement, maintaining the axial intensity at over 20% of the peak intensity for less than 2 mm. However, a novel regime called quasi-matched guiding was shown to allow high-quality guiding through partially-ionised plasmas (Spence et al. 2000). Simulations showed that for input pulse intensities of \( 5 \times 10^{17} \text{ W cm}^{-2} \), the same waveguide was capable of guiding over almost 25 mm. The stark change of guiding performance as the laser intensity is increased was explained in terms of the creation of a raised parabolic waveguide around the axis, formed because a highly-stable ion stage was generated out to a large radius. For discharge-ablated polypropylene capillary waveguides, the stable ion stages formed near the axis are \( \text{C}^{4+} \) and \( \text{H}^{+} \).

It was suggested that improved guiding performance would be achieved in boron waveguides, in which calculations showed that a laser pulse of intensity \( 5 \times 10^{17} \text{ W cm}^{-2} \) would be guided over 43 mm. The improved guiding is seen because the stable ion stage \( \text{B}^{3+} \) is created at a lower laser intensity than \( \text{C}^{4+} \).

For partially-ionised plasma waveguides, it is vital that the guided pulse has a suitable intensity to achieve quasi-matched guiding: greater than 100 times the threshold intensity for creation of the stable ion stage. The quality of guiding increases as the laser intensity increases, up to the intensity at which the ion stage above the stable ion stage is generated. With this in mind it was suggested that hydrogen would be the perfect ion in which to achieve quasi-matched guiding: it is fully-ionised at approximately \( 10^{14} \text{ W cm}^{-2} \). Simulations showed that exceptionally high-quality guiding could be achieved through partially-ionised hydrogen plasma, over hundreds of millimetres.

With the aim of producing the guiding channel in hydrogen gas, the gas-filled capillary discharge waveguide was invented. Several important design features were incorporated. The capillaries were made from alumina, which was found not to be ablated by the discharge: the
lifetime of a single capillary may be greater than $10^6$ shots. Gas was injected into the capillary through micro-machined holes near the capillary ends, resulting in a uniform plasma along the length of the capillary. Measurements of the electron density profile formed in hydrogen-filled capillary discharge waveguide showed that this simple device formed an approximately parabolic guiding profile in almost fully-ionised hydrogen. Simulations of the propagation of laser pulses through the measured profile suggested that for pulses with intensity greater than $10^{17}$ W cm$^{-2}$, the waveguide would be capable of guiding over hundreds of millimetres, with a performance close to that of an ideal waveguide. These simulations assumed the worst case consistent with the measured results of an average ion stage of 0.87: it is thought that the waveguide may well be fully-ionised, in which case high-quality guiding will be possible for pulse of any intensity.

The promise shown by the simulations was borne out by experiments to demonstrate guiding of femtosecond laser pulses through 20 and 40 mm-long hydrogen-filled capillary discharge waveguides. For input laser pulses with an intensity of $10^{17}$ W cm$^{-2}$, over 90% and 80% energy transmission was observed through 20 and 40 mm-long waveguides respectively, corresponding to the lowest insertion and propagation losses of any waveguide to date. The pulse intensity in the exit plane of the capillary was measured to be 70% of the intensity at the waveguide entrance.

The impressive guiding results obtained using the hydrogen-filled capillary waveguide, along with the long device lifetime associated with the device, make the waveguide among the best for use as a tool for studying laser-plasma interactions. The waveguide seems ideally suited for demonstrating OFI-pumped XUV lasers. The possibilities of doping hydrogen gas with lasant ions, or creating waveguides in other gases, make the waveguide an extremely flexible tool for creating long lengths of XUV lasant ions.

The field of laser wakefield acceleration has been waiting for many years for a waveguide of sufficient quality to advance the state of the art of electron acceleration using lasers. A significant fraction of theoretical work has focused on LWFA in a plasma waveguide. The hydrogen-filled capillary discharge waveguide has been shown to be capable of forming a plasma waveguide with precisely the parameters requested by theorists. Tunability of the waveguide characteristics coupled with the long capillary lifetime would seem to make the device an ideal tool to form the heart of a guided-LWFA experiment, bringing laser-based electron acceleration to GeV energies within reach.
13.2 Future work

Further investigation of the properties of the hydrogen-filled capillary discharge waveguide will form a vital part of future work. It is important to make longer waveguides to explore the limits of the hydrogen-filled capillary discharge waveguide. The use of narrower capillaries, perhaps as small as 100 μm in diameter, would also be of interest for generating waveguides with smaller matched spot sizes, and would create waveguides with properties in the regime of most interest for LWFA.

Measurements of the spot size of the guided laser pulse at all points within the waveguide would be of great use for optimising the guiding properties of the waveguide. Such measurements could be made if there was transverse optical access along the length of the waveguide. While this is not possible for opaque alumina capillaries, it might be possible to image transversely through a transparent capillary material such as sapphire. The construction of a capillary waveguide with transverse optical access would allow a full analysis of the electron density and the guided laser intensity throughout the waveguide, and would give insight into the laser-plasma interaction that took place within the capillary.

The guiding performance of the waveguide demonstrated in this thesis is already of sufficient quality that LWFA and XUV laser experiments can be planned, along the lines discussed in chapters 11 and 12. It is hoped that these experiments will take place in the near future. There are also more radical prospects that a high-quality waveguide brings into the realm of possibility. One of these is the generation of tunable x-ray laser radiation: this could be achieved by creating a miniature free-electron laser (FEL) using electrons accelerated through the waveguide.

It would be possible to create guiding channels with longitudinal profiles that were tailored for particular applications. The axial density and the matched spot size could be tailored as a function of position in the waveguide by tapering or modulating the diameter of the capillary tube. The initial gas pressure in the capillary could be shaped by injecting gas at various pressures through holes placed at various points along the capillary. Such tailored waveguide characteristic could be of use for extending the dephasing length for LWFA, and may be of interest more generally for the studies of laser-plasma interactions.

The hydrogen-filled capillary waveguide is the first waveguide that is simple, robust, and capable enough to allow it to be used as a centrepiece for complex experiments of the type described above. It is hoped that the hydrogen-filled capillary waveguide will lead to waveguides becoming a integral tool for new research in other fields.
 References


