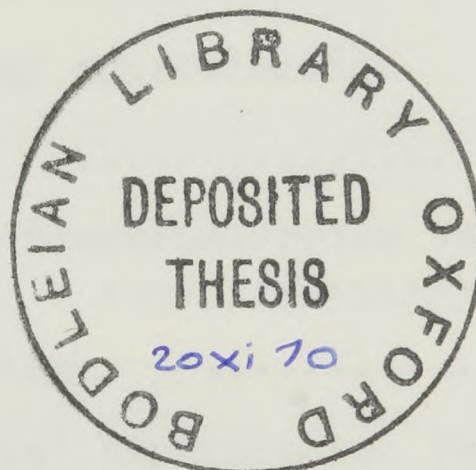


POLARIZATION IN ELECTRON-PROTON SCATTERING

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A B S T R A C T

This thesis is concerned with polarization effects in elastic electron-proton scattering. The principal results are summarized below.

A general polarization analysis of high-energy electron-proton scattering is first presented, and the problem of a 'complete set' of experiments, which allow a unique determination of the elastic scattering matrix at one energy and scattering angle, is discussed. A straightforward reconstruction procedure is shown to be inadequate when an expansion of the strength of interaction in powers of α , the fine structure constant, is reasonable: an alternative procedure for this case involves using both electron and positron scattering experiments. Although the experimental evidence for the one-photon-exchange approximation in electron-proton scattering is generally accepted to be very good, from a strictly logical point of view the analysis is incomplete and does not permit an unambiguous determination of the two-photon-exchange amplitudes. It is shown that measurement of various polarization tensors for both electron and positron scattering could eliminate the ambiguity.

Some interesting polarization experiments are then considered, in the one-photon-exchange approximation. A specific experiment is suggested to measure the electric form factor of the proton G_E , and explicit expressions are derived for the proton depolarization tensor. In the time-like region, the form factors are not restricted to real values and a

'generalized' polarization-asymmetry relation for the two crossed-channel processes, $\bar{P}P \rightarrow e^+e^-$ and $e^+e^- \rightarrow P\bar{P}$, may be derived. Violation of this relation would represent a violation of time-reversal-invariance and γCP -invariance.

The polarization of the recoil proton in elastic electron-proton scattering is strictly zero in the one-photon-exchange approximation: any non-zero polarization must, to lowest order in α , arise from interference of the one-photon-exchange amplitudes with two-photon-exchange amplitudes. The contribution to the polarization of the two-photon-exchange amplitude with an elastic intermediate state, is calculated for arbitrary form factors for the proton. Numerical results are displayed for two interesting parameterizations of the form factors. This elastic effect is expected to be a good approximation to the polarization up to energies where the production of inelastic states becomes important. A rigorous upper bound is derived for the contribution to the polarization, of all two-photon amplitudes involving multi-particle intermediate states: the bound involves measurable inelastic electron-proton cross sections. However, a rough numerical estimate shows the bound has very limited usefulness, since it leads to very large bounds at large momentum-transfers.

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* For a recent review of the subject, see
Electrodynamics, see the article by J. D. Jackson,
"Proceedings of the International Conference on
Photon Interactions at High Energies"

CHAPTER 1

INTRODUCTION

Electron-proton scattering experiments probe the electromagnetic structure of the proton. Electron-photon interactions are well-described by Quantum-Electrodynamics^{*}: the classical limit of Quantum-Electrodynamics as given by the Maxwell equations in fact provides the very basis of measurement in particle physics. To lowest order in α , the fine structure constant ($\alpha \sim 1/137$), the electron-photon vertex can be treated as a point interaction of a Dirac particle. The electromagnetic interactions of mesons and nucleons are influenced by their strong interactions: the existence of an anomalous magnetic moment for the proton is evidence for these strong interaction effects. For the interaction of a virtual photon with a proton, the modifications due to virtual strong interactions may be expressed in terms of two form factors, G_E and G_M , which are functions only of the squared four-momentum transfer. These two structure functions describe the dynamic distribution of electric charge and magnetization respectively in the nucleon. (This decomposition into electric and magnetic distributions is frame dependent: the above interpretation is valid in the proton Breit frame.) An analysis of elastic differential cross sections, in the one-photon-exchange approximation, for electron-proton scattering

^{*} For a recent review of the current status of Quantum-Electrodynamics, see the article by S.J. Brodsky in the "Proceedings of 4th International Symposium on Electron and Photon Interactions at High Energies" (Liverpool, 1969).

therefore yields information on these two structure functions. In the context of Strong Interactions, the problem of the electromagnetic form factors of the nucleon is very interesting since the quantum numbers of the virtual hadronic states are restricted to those of the virtual photon. However, it is probably fair to say that, at present, there exists no wholly satisfactory theory able to predict the experimentally observed behaviour of the two form factors.*

Inelastic electron-proton scattering, observing only the scattered electron in the final state - in the terminology of Feynman [75] an 'Inclusive' experiment - provides additional information on the structure of the proton. Recent experiments seem to indicate a remarkable 'universal' or 'scale invariant' character of the data, and this characteristic has given rise to speculations about a composite nature of the proton [60].

It is relevant to remember that all these considerations are based on an analysis to lowest order in α ; corresponding to a single photon exchanged between the electron and the hadrons. It is therefore worthwhile to examine in detail the basis for this approximation.

In an expansion of the strength of the electron-proton interaction in powers of α , there will be amplitudes involving the exchange of two photons between the electron and proton: such amplitudes are of relative order α with respect to the first Born amplitude. However, in between absorption and emission of the photons, the proton can be 'polarized', i.e.,

* A selection of the early experimental and theoretical work on nucleon form factors may be found in the collection of reprints edited by R. Hofstadter, 'Nuclear and Nucleon Structure' (Benjamin 1963). A review of the current situation is given in reference [12].

excited to arbitrary resonant states of various multipole orders. It is therefore possible that contributions from intermediate states, resonant or otherwise, could partly compensate the small factor α . For example, in proton Compton scattering at photon energies near 300 MeV, the resonance peak in the cross section is about ten times larger than the calculated cross section for a point proton.* Similar effects could enhance the virtual Compton amplitude which is involved in two-photon terms, although previous calculations [38 - 44] suggest that such resonant enhancements lead only to very small deviations from the one-photon-exchange predictions. The effect of the elastic intermediate state can be calculated in terms of the elastic form factors G_E and G_M . For an exact calculation (to this order in α) of all the possible inelastic contributions, the corresponding 'inelastic form factors' would be required. As far as is known, the resonance form factors appear to have a similar momentum transfer dependence to the elastic form factors, apart from a threshold factor** : at large momentum transfers they produce strong damping. However, the inelastic scattering seems to indicate that resonance contributions are not the most important in the deep inelastic region. It may therefore be premature to dismiss the two-photon-exchange contributions as unimportant over the whole range of s and t , (s and t are the usual Mandelstam invariants), on the basis of calculations involving only resonance contributions.

* See the review article by S.D. Drell in "Enrico Fermi School, Course XXVI" (Academic Press 1963).

** For a recent review, see reference [72].

relation is derived.

Experimentally, effects depending on two-photon-exchange have so far remained undetected [12],[18]. However, the experiments involve only differential cross section and polarization measurements and these two results do not admit an entirely unambiguous statement as to the size of the two-photon amplitudes. These two experiments do not constitute a "complete set" for a unique determination of the elastic scattering matrix at a given energy and angle. Measurements of the various polarization tensors could remove this uncertainty. In the one-photon-exchange approximation, measurement of these polarization tensors could provide a better determination of the electric form factor G_E , than is possible from differential cross section measurements.

After this introduction and motivation, we now outline a plan of the thesis.

In Chapter 2, we consider the problem of a 'complete-set' of experiments for high-energy elastic electron-proton scattering. Reconstruction procedures are discussed first for the case when an expansion of the strength of interaction in powers of α is not justifiable, and second for the situation in electron-proton scattering when such an expansion is reasonable.

Chapters 3 and 4 are concerned with some polarization experiments in the one-photon-exchange approximation. A specific experiment which could be used to measure the electric form factor of the proton G_E , is described in Chapter 3. Chapter 4 contains a discussion of polarization effects in the time-like region, and a 'generalized' polarization asymmetry

relation is derived.

In Chapters 5 and 6, we consider the polarization of the recoil proton in elastic e-p scattering from unpolarized initial particles. The lowest order polarization must arise from interference of two-photon-exchange amplitudes with the first Born amplitudes. In Chapter 5, the contribution of the elastic two-photon amplitude is calculated. To illustrate the techniques involved the simpler spin 0 - spin $\frac{1}{2}$ process is first discussed. The methods are then generalized to the complete spin $\frac{1}{2}$ - spin $\frac{1}{2}$ calculation, where one of the particles possesses arbitrary form factors. A covariant formalism is used for all these calculations and the mechanism for the cancellation of the infra-red divergence for the polarization, is explained. Numerical results are displayed for 'dipole' and for 'point-like' form factors.

Finally, in Chapter 6, we consider the effects of inelastic intermediate states in the two-photon-exchange amplitudes. After a brief review of previous calculations of two-photon effects, a rigorous upper bound is derived for the contribution of the inelastic states. A numerical estimate is obtained for the bound, and the results discussed. We conclude with some comments on two possible models for the inelastic intermediate states, and a few general remarks about two-photon-exchange contributions.

Three appendices are included in the thesis. Appendix I contains details of the helicity conventions used in Chapter 2. Our metric and γ -matrices are summarized in Appendix 2, where we also give some useful trace theorems. Appendix 3 contains some details of the integrals required in Chapter 5.

CHAPTER 2

RECONSTRUCTION OF THE ELASTIC ELECTRON-PROTONSCATTERING MATRIX2.1 Introduction

In this chapter we shall consider the problem of reconstructing the elastic scattering matrix, at a fixed energy and scattering angle, for high-energy electron-proton scattering directly from experimental measurements at this energy and angle. The analysis will first be presented in a form independent of the strength of the interaction: it is therefore immediately applicable to the scattering of nucleons, and of polarized proton targets by high-energy electrons by a nucleus of arbitrary charge Z_e , with ground state spin $\frac{1}{2}$. We shall then specialize to electron-proton scattering where an expansion of the strength of interaction in powers of α , the fine structure constant, is meaningful. It will be seen that such an expansion radically affects the reconstruction programme for the scattering matrix.

Our analysis is restricted throughout to high-energy electrons. This results in considerable simplifications, since amplitudes in which the electron helicity changes are of order $\frac{\mu}{\omega}$ with respect to the helicity conserving amplitudes (μ is the electron mass and ω the electron energy.) Thus in this high-energy limit, we need only retain amplitudes in which the electron helicity is conserved. This approximation is justified in detail later in the chapter.

The problem of direct reconstruction of the scattering matrix at one energy and angle has been solved completely for the proton-proton system [1]. (In what follows, we shall often omit to state explicitly that the reconstruction is at fixed energy and scattering angle: this is implied throughout this chapter.) If isospin-invariance is assumed, the proton-proton analysis is trivially extended to the nucleon-nucleon (N-N) system. As a preliminary to our analysis of electron-proton scattering, we review the problem of a unique reconstruction for the N-N system. We shall then discuss the extension of this procedure to the electron-proton (e-p) system, indicating the special features that arise.

Progress in the production of polarized beams of nucleons, and of polarized proton targets has now made the problem of extracting maximum information on the N-N scattering matrix, amenable to direct solution. Such a possibility was first considered in detail by Puzikov, Ryndin and Smorodinski [2], and somewhat later by Schumacher and Bethe [3]. Assuming rotation-, parity-, time-reversal- and isospin-invariance, there are five independent scalar amplitudes for the N-N system [4]. The general method of reconstructing the elastic scattering matrix, free from additional hypotheses about inelastic processes - necessary in a phase-shift analysis, consists in determining these five complex scalar functions directly from the experimental data on elastic scattering at fixed energy and scattering angle. Since the experimental quantities involve the scattering matrix quadratically, this method can only be used because of the non-linear nature of the unitarity condition on the T-matrix.

determine the scattering matrix up to an overall phase.* Therefore for complete knowledge of the five amplitudes up to this overall phase, it is necessary to determine nine quantities. However, since the observables are bilinear in these scalar amplitudes, more than nine measurements are needed to obtain a unique reconstruction of the scattering matrix. Schumacher and Bethe [3] have given a unique solution for these five amplitudes, with an arbitrary common phase, requiring measurements of the differential cross section, polarization, depolarization tensor, polarization-transfer tensor and the polarization correlation tensor - eleven quantities in all. Simplifications of this procedure involving third and fourth rank polarization tensors have also been considered [5].

The analysis for N-N scattering is generally presented in the two-component spinor formalism appropriate for non-relativistic particles. However, Stapp [6] has shown that the simple non-relativistic formulae can be taken over to the fully relativistic case with only minor changes. These modifications and their effect on the reconstruction procedure are considered in detail in a recent paper by Bilenkii, Lapidus and Ryndin [7].

A recent review of the N-N system and of polarization formalism in general is given in reference [1]: this represents a considerable up-dating of the 'classic' review by Wolfenstein [8].

* The unitarity relation involving measurements at all angles, allows this phase to be determined. This is because of the non-linear nature of the unitarity condition on the T-matrix.

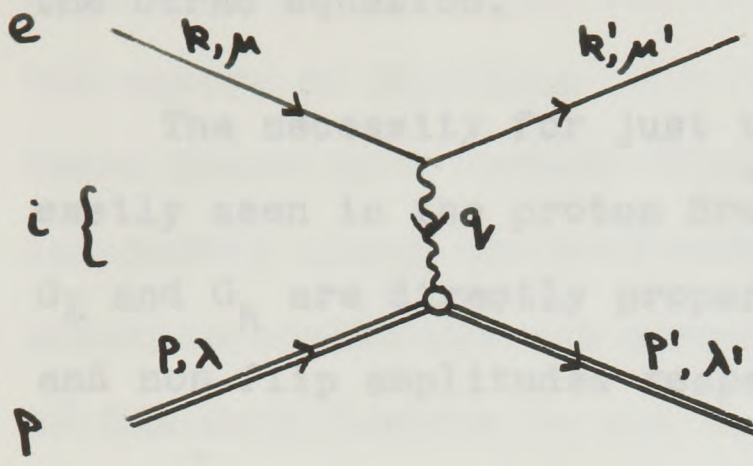
For the general spin $\frac{1}{2}$ - spin $\frac{1}{2}$ elastic scattering problem, assuming rotation-, parity- and time-reversal-invariance as before, there are six independent scalar amplitudes.* The analysis of a complete set of experiments is now a straight-forward extension of that for the N-N system, except that there are now two depolarization tensors, two polarization-transfer tensors etc.[1]. However, when one of the two spin $\frac{1}{2}$ particles is an electron, there are some additional interesting features. Since only amplitudes in which the electron helicity is conserved do not tend to zero at high energy, the number of non-zero amplitudes is reduced to three in this limit. The price for this reduction is that the number of possible 'inequivalent' experiments is also reduced. Furthermore, when we make an expansion of the strength of interaction in powers of α , other features emerge, peculiar to lepton-proton scattering. We shall now discuss the meaning of this expansion in some detail.

In this expansion, the lowest-order term in the amplitude is merely the usual one-photon-exchange approximation. As is well-known, (and will be discussed again in Chapter 3), the one-photon-exchange amplitudes can be parameterized in terms of two unknown real scalar functions - the form factors G_E and G_M - that depend only on the square of the four-momentum-transfer of the virtual photon. Thus the six independent complex scalar amplitudes reduce to two unknown functions in this approximation. This results from the

* In this case, the so-called singlet-triplet transitions are no longer forbidden (as they were for the N-N system).

factorizability of the one-photon amplitude

Figure 2.1
The initial and final electrons are assumed to satisfy the Dirac equation.



k, p, k', p' 4-momenta
 $\mu, \lambda, \mu', \lambda'$ helicities

Figure 2.2

$$T_{fi} \sim J_e^\mu \frac{\epsilon_{\mu\nu}}{q^2} J_p^\nu = \langle e' | J_e^\mu(0) | e \rangle \frac{\epsilon_{\mu\nu}}{q^2} \langle p' | J^\nu(0) | p \rangle \quad (2.1)$$

in an obvious notation.

The electron current is assumed to be given exactly by

$$J_e^\mu = \bar{u}(k', \mu') \gamma^\mu u(k, \mu) \quad (2.2)$$

From Lorentz Invariance, Hermiticity of the Electromagnetic current, and current conservation, the most general form for the proton current can be written in terms of two real independent scalar amplitudes [9].

$$J_p^\mu = \bar{u}(p', \lambda') \Gamma^\mu(q^2) u(p, \lambda) \quad (2.3a)$$

where Γ^μ can be written

$$\Gamma^\mu(q^2) = G_M(q^2) \gamma^\mu - \frac{\{G_M(q^2) - G_E(q^2)\}}{(1 + \tau) 2M} \not{P}^\mu \quad (2.3b)$$

with

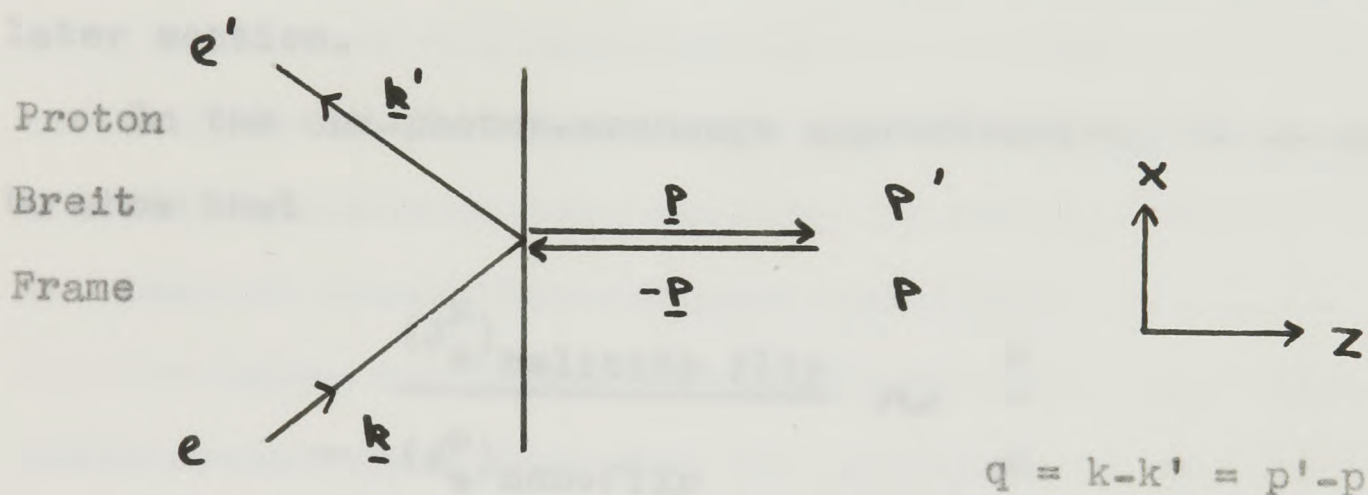
$$\not{P}^\mu = (p + p')^\mu \quad \text{and} \quad \tau = - \frac{q^2}{4M^2}$$

M is the proton mass : $p^2 = p'^2 = M^2$.

The initial and final spinors are assumed to satisfy the Dirac equation.

The necessity for just two form factors can be most easily seen in the proton Breit frame, where the form factors G_E and G_M are directly proportional to the helicity-flip and non-flip amplitudes respectively [10].

Figure 2.2



In this frame :

$$q^\mu = (0, 0, 0, 2p) \quad p = |\underline{p}|$$

i.e., q has no time-like component.

We find, using the appropriate helicity spinors for the proton current, the matrix elements

$$(J_p^\mu)_{\lambda'\lambda} \quad \lambda', \lambda = \pm \text{ for proton helicity } \pm \frac{1}{2}$$

$$(J_p^\mu)_{-+} = G_E(q^2) [1, 0, 0, 0]$$

$$(J_p^\mu)_{++} = \frac{p}{M} G_M(q^2) [0, 1, -1, 0]$$

(2.4)

We remark here that this decomposition - helicity flip proportional to G_E , and non-flip to G_M - is also valid in the LAB frame in which the initial proton is at rest. In the centre of momentum (CM) frame, which is reached from the Breit frame by a Lorentz transformation in the xz -plane but not purely along the z -direction, proton helicity-flip and non-flip amplitudes are proportional to linear combinations of the form factors G_E and G_M . These combinations can be calculated in a straight-forward manner, but the actual expressions are not very illuminating: they are given in a later section.

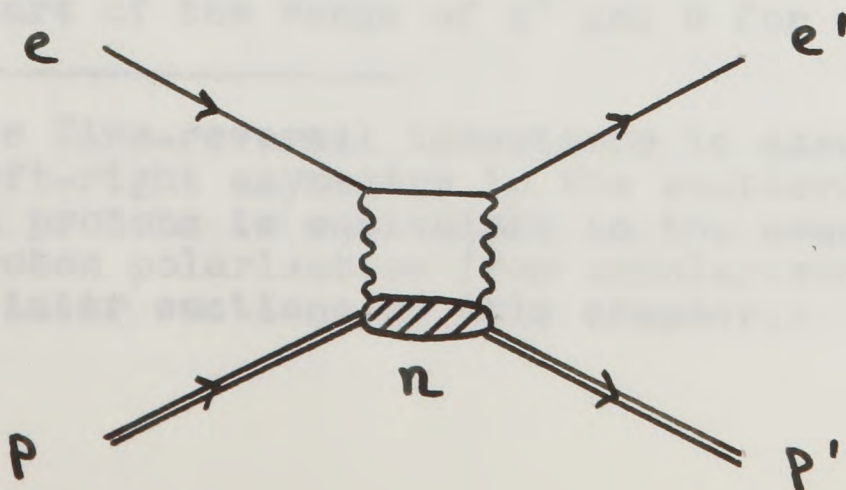
In the one-photon-exchange approximation, it is easy to show that

$$\frac{(J_e^\mu)_{\text{helicity flip}}}{(J_e^\mu)_{\text{non-flip}}} \sim \frac{\mu}{\omega} \quad (2.5)$$

We shall show later that this result remains true for the exchange of any number of photons.

Having discussed the lowest-order term in the expansion of the scattering amplitude in powers of α , in some detail, we now consider terms of order α^2 . These contain the two-photon-exchange diagrams

Figure 2.3



As emphasized by Feynman [11] several years ago, the form factor analysis above is not appropriate for these two-photon-exchange graphs. There will be contributions from all allowed hadronic intermediate states 'n', and clearly it is not possible to factorize the amplitude as in the one-photon case. In the absence of a complete theory for calculating the contributions of all these intermediate states, we must consider six unknown amplitudes of relative order α^2 . In the high-energy limit, conservation of the electron's helicity leaves us with three non-vanishing complex amplitudes.

Because of the smallness of α , we shall ignore higher-order terms in the expansion. Thus we are interested in the one-photon-exchange form factors, G_E and G_M , and in the three independent two-photon-exchange amplitudes.

To date, two types of experiments have been performed, which specifically look for the existence of two-photon-exchange effects. These are the difference in e^-p and e^+p differential cross sections, and the polarization of the recoil proton from an unpolarized initial state^{*}, which depend on the interference with the one-photon amplitudes, of the real and imaginary parts respectively, of these two-photon amplitudes. A recent review [12] concludes that "at present the experimental data show no evidence for the existence of two-photon exchange effects, but, of course, they only cover a small part of the range of q^2 and θ for which cross sections

^{*} Since Time-reversal Invariance is assumed, measurement of the left-right asymmetry in the scattering of electrons by polarized protons is equivalent to the measurement of the recoil proton polarization from unpolarized initial particles. (See the later sections of this chapter.)

have been measured and form factors calculated on the basis of one-photon exchange". From a strictly logical point of view, since there are three unknown two-photon amplitudes, the possibility exists for some cancellations to take place. Therefore the smallness of the experimentally observed effects does not preclude the possibility of sizable two-photon amplitudes. We shall therefore consider what experiments are necessary to separate the three two-photon-exchange contributions, and if indeed, a unique separation is possible.

To summarize, after giving a brief outline of the contents of this chapter, we have reviewed the reconstruction programme for N-N scattering. This is compared with the procedure for e-p scattering, and the expansion of the scattering matrix in powers of α is discussed. Finally, we see that present experiments measuring the two-photon contributions do not constitute a complete set allowing an unambiguous interpretation. This then, provides the motivation for our analysis.

2.2 Helicity Amplitudes for e-p Scattering

1 The Helicity Formalism

Instead of the two-component spinor formalism usually used in the N-N analysis, we shall use the 'Helicity Formalism' [13]. This has the advantage that the 'relativistic rotations' [6], necessary in the two-component formalism when dealing with relativistic particles, are avoided. This is due essentially to the use of different rest-frames for each

particle, derived by direct Lorentz transformation from the CM frame. For spin 0 - spin $\frac{1}{2}$ scattering a discussion of this formalism has been given by Jacob [14]. The extension to spin $\frac{1}{2}$ - spin $\frac{1}{2}$ scattering is discussed later in this section and also in Appendix I.

Cross sections, polarizations, and the various polarization tensors will be expressed in terms of CM Helicity amplitudes. Our conventions are presented in detail in Appendix I: only a brief outline will be given here.

A one-particle helicity state for a particle with helicity λ , moving with momentum p in the direction (θ, ϕ) is defined

$$| \theta \phi p; \lambda \rangle = R(\phi, \theta, 0) | 0 0 p; \lambda \rangle \quad (2.6)$$

Because of the phase ambiguity, the helicity state for a particle moving in the negative z-direction must be defined. We choose

$$\begin{aligned} | 0 0 -p; \lambda \rangle &= e^{-i\pi s} R(\pi, \pi, 0) | 0 0 p; \lambda \rangle \\ &= (-1)^{s-\lambda} R(0, \pi, 0) | 0 0 p; \lambda \rangle \end{aligned} \quad (2.7)$$

where s is the spin of the particle. The motivation for this choice is discussed in reference [13].

Two-particle states in the CM system are constructed as direct products of one-particle states

$$| \underline{P} = 0; \theta \phi p; \lambda_1 \lambda_2 \rangle = A | \underline{p} \lambda_1 \rangle | -\underline{p} \lambda_2 \rangle \quad (2.8)$$

where A is a convenient normalization factor. Particle 2

has the additional phase factor of our convention (2.7). For electron-proton scattering, the initial state is defined by

$$|i\rangle = |P=0; 0 0 p; \lambda_1 \lambda_2\rangle$$

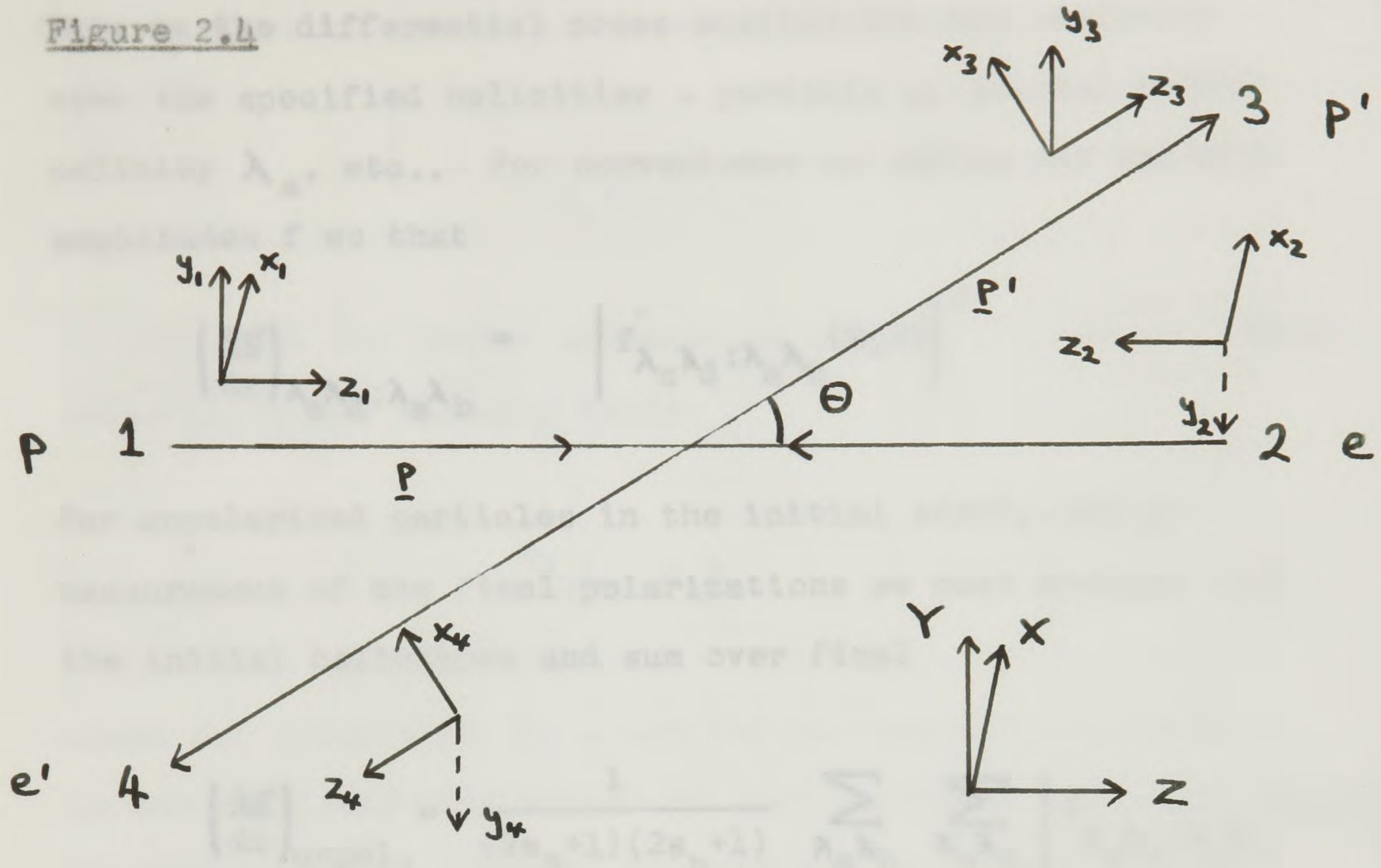
where particle 1 is always chosen to be the proton and particle 2, the electron. For the final state, we shall take from now on, $\theta = 0$, thereby specifying the X-Z plane as the scattering plane.

$$|f\rangle = |P=0; 0 \theta p; \lambda'_1 \lambda'_2\rangle$$

Particle 1' we shall call particle 3 and is always taken to be the recoil proton; particle 2' or 4 is the scattered electron. It is often more convenient to use respectively, the labels a, b, c, d instead of 1, 2, 3, 4.

Since our prescription (2.6) for generating the single-particle states can also be regarded in the passive sense as describing the same state in two different reference systems, our convention amounts to defining the orientation of the rest frame axes. For two-particle helicity states, our conventions (2.7) and (2.8) amount to an additional rotation by π about the z-axis, of the rest frame axes of particle 2, relative to those of particle 1. Thus for ep-elastic scattering in the CM system, we have defined the 'helicity rest frames' for each particle that are shown in Fig. 2.4.

Figure 2.4



CM-system

- 3-axis parallel to \underline{p} Z
- 2-axis parallel to \underline{p} \underline{p}' Y
- 1-axis completes Right-handed (R.H.) set X

Helicity Rest-Frames of the Individual Particles

- 3-axis along direction of motion in CMS z_1
- 2-axis for 1 and 3 parallel to Y } y_1
- 2-axis for 2 and 4 anti-parallel to Y }
- 1-axis completes RH-set x_1

The differential cross section can be written:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\lambda_c \lambda_d: \lambda_a \lambda_b} = \left(\frac{2\pi}{p}\right)^2 \left| \langle 0\theta: \lambda_c \lambda_d | T(W) | 00 \lambda_a \lambda_b \rangle \right|^2$$

p CM momentum; W CM energy.

(The notation \bar{f} is used to denote complex conjugate of f.)

This is the differential cross section for the particles with the specified helicities - particle a, initial proton - helicity λ_a , etc.. For convenience we define our helicity amplitudes f so that

$$\left(\frac{d\sigma}{d\Omega}\right)_{\lambda_c \lambda_d : \lambda_a \lambda_b} = \left| f_{\lambda_c \lambda_d : \lambda_a \lambda_b}(\theta, W) \right|^2 \quad (2.9)$$

For unpolarized particles in the initial state, and no measurement of the final polarizations we must average over the initial helicities and sum over final

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}} = \frac{1}{(2s_a+1)(2s_b+1)} \sum_{\lambda_a \lambda_b} \sum_{\lambda_c \lambda_d} \left| f_{\lambda_c \lambda_d : \lambda_a \lambda_b}(\theta, W) \right|^2 \quad (2.10)$$

In general, the initial spin state is described by a density matrix. (Appendix I contains details of our density matrix formalism.) For the spins in the initial state uncorrelated, we can write

$$\rho_1 = \rho_a \otimes \rho_b \quad (2.11)$$

The differential cross section for no measurement of final polarizations is then given by

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\rho_1} &= \sum_{\lambda_c \lambda_d} \sum_{\lambda_a \lambda_b} \sum_{\lambda'_a \lambda'_b} f_{\lambda_c \lambda_d : \lambda_a \lambda_b} \rho_1_{\lambda_a \lambda_b : \lambda'_a \lambda'_b} \bar{f}_{\lambda_c \lambda_d : \lambda'_a \lambda'_b} \\ &= \text{Tr} [f \rho_1 f^\dagger] \end{aligned} \quad (2.12)$$

(The notation \bar{f} is used to denote complex conjugate of f .)

For unpolarized particles in the initial state:

$$\rho_0^i = \frac{1}{(2s_a+1)(2s_b+1)} \cdot \mathbf{1}$$

We note that the matrix elements of the density matrix are referred to the Helicity basis

$$\rho_{\lambda_c \lambda_d; \lambda_a \lambda_b}$$

where the helicities λ - are the z-component of the spin in the 'helicity rest-frame' of each particle. As explained in more detail in Appendix I, provided we interpret the polarizations \underline{P}_i and the spin operators $\underline{\sigma}_i$ as referring to the helicity rest-frame of each particle 'i', we can expand the density matrix exactly as for particles at rest. For example, if the initial state spins are uncorrelated, we can write

$$\rho^i = \frac{1}{2} (1_a + \underline{P}_a \cdot \underline{\sigma}_a) \otimes \frac{1}{2} (1_b + \underline{P}_b \cdot \underline{\sigma}_b)$$

where $\underline{P}_a, \underline{\sigma}_a$ and $\underline{P}_b, \underline{\sigma}_b$ refer to different rest-frames - the helicity rest-frames of particle a and b respectively.

For measurements of final state polarizations, it is convenient to use the final state density matrix ρ_f defined by

$$\left[\left(\frac{d\sigma}{d\Omega} \right)_{\rho_1} \right] \rho_f = f \rho_1 f^\dagger \quad (2.13)$$

and normalized so that

$$\text{Tr} [\rho_f] = 1.$$

In our helicity basis, the matrix elements of ρ_f are

$$\rho_f^{\lambda_c \lambda_d : \lambda'_c \lambda'_d} = \left[\left(\frac{d\sigma}{d\Omega} \right)_{\rho_i} \right]^{-1} \sum_{\substack{\lambda_a \lambda_b \\ \lambda'_a \lambda'_b}} f^{\lambda_c \lambda_d : \lambda_a \lambda_b} \rho_a^{\lambda_a \lambda_b : \lambda'_a \lambda'_b} \bar{f}^{\lambda'_c \lambda'_d : \lambda_a \lambda_b} \quad (2.14)$$

The expectation value of an observable A referring to the spins of the final particles is :

$$\langle A \rangle = \text{Tr} [\rho_f A] \quad (2.15)$$

where the quantities $\langle A \rangle$ refer to observables in the rest-frames of particles c and d , i.e., referred to the axes (x_3, y_3, z_3) and (x_4, y_4, z_4) of Fig. 2.4.

Thus with this formalism, we can express the results of all possible polarization experiments in e-p scattering, in terms of our CM Helicity amplitudes. Before doing this, we shall consider in the next section, the restrictions imposed by parity- and time-reversal-invariance, and also make an approximation specific to the high-energy limit.

2 Invariance Requirements

(1) Space Inversion

This symmetry relates amplitudes $\{ + \lambda \}$ to $\{ - \lambda \}$

Define:

$$\nu = \lambda_a - \lambda_b \quad \rho = \lambda_c - \lambda_d \quad (2.16)$$

From the behaviour of the helicity states under space-inversion and from the rotational properties of the helicity amplitude $f_{(\lambda)}(\theta, W)$, we find:

$$f_{-\lambda_c - \lambda_d; -\lambda_a - \lambda_b}(\theta, W) = (-1)^{\nu - \rho} f_{\lambda_c \lambda_d; \lambda_a \lambda_b}(\theta, W) \quad (2.17)$$

(2) Time Reversal

Time reversal relates the process $|i\rangle \rightarrow |f\rangle$ to the inverse process $|\tilde{f}\rangle \rightarrow |\tilde{i}\rangle$, where the tilde indicates the time-reversed states. For elastic scattering therefore, we derive a restriction on the amplitude. Using the time-reversal properties of the helicity states and the rotational properties of the helicity amplitudes, we obtain:

$$f_{\lambda_a \lambda_b; \lambda_c \lambda_d}(\theta, W) = (-1)^{\nu - \rho} f_{\lambda_c \lambda_d; \lambda_a \lambda_b}(\theta, W) \quad (2.18)$$

For spin $\frac{1}{2}$ - spin $\frac{1}{2}$ scattering we have at most $2^4 = 16$ independent amplitudes. It is evident that the parity condition halves this number. Application of the time-reversal condition (2.18), in conjunction with the parity restriction reduces the eight remaining amplitudes to six independent amplitudes. These are shown in Table 2.1.

Using the high-energy representations of the three helicity

Table 2.1

		λ_a	+	+	-	-	
		λ_b	+	-	+	-	
λ_c	λ_d						
+	+	f_1	f_2	f_3	f_4		+ stands for helicity + $\frac{1}{2}$
+	-	$-f_2$	f_5	f_6	f_3		
-	+	$-f_3$	f_6	f_5	f_2		
-	-	f_4	$-f_3$	$-f_2$	f_1		

Consider now the 'chirality' operator

3 High-Energy Approximation

At this point, the analysis will be simplified by making an approximation specific to the limit of high-energy electrons: this limit can be represented symbolically by $\mu \rightarrow 0$. We shall now show that all amplitudes that involve a change in electron helicity are of order $(\frac{\mu}{\omega})$ with respect to helicity-conserving amplitudes.

The Dirac equation can be written

$$Hu = (\underline{\alpha} \cdot \underline{p} + \beta\mu)u = \omega u \tag{2.19}$$

where $u = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$ is a four-component spinor
 ϕ, χ two-component spinors.

Using the high-energy representation of the Dirac matrices

$$\beta = \gamma^0 = \rho_1 \quad \underline{a} = \rho_3 \quad \otimes \quad \underline{\sigma}$$

$$\gamma_5 = \rho_3 \quad \underline{\chi} = \beta \underline{a}$$

We obtain :

$$\underline{\sigma} \cdot \hat{p} \phi = \frac{e}{p} \phi - \frac{\mu}{p} \chi \quad (2.20)$$

$$\underline{\sigma} \cdot \hat{p} \chi = -\frac{e}{p} \chi + \frac{\mu}{p} \phi$$

i.e., the coupling between ϕ and χ is only of order $\frac{\mu}{p}$. In the limit $\frac{\mu}{p} \rightarrow 0$, ϕ and χ are eigenstates of the helicity operator and the Dirac equation splits into a pair of two-component equations.

Consider now the 'chirality' operators

$$a = \frac{(1 + \gamma_5)}{2} \quad \bar{a} = \frac{(1 - \gamma_5)}{2} \quad (2.21)$$

These have the property that

$$a u = \begin{pmatrix} \phi \\ 0 \end{pmatrix} \quad \bar{a} u = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

and $a^2 = a$; $a + \bar{a} = 1$; $a \bar{a} = 0$.

Also in our high-energy representation $a = a^\dagger$.

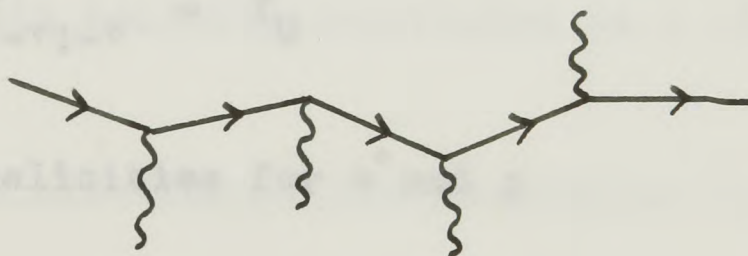
It is easy to see that only vector (V) and axial vector (A) interactions preserve 'chirality'. For example:

$$\begin{aligned} \bar{u} \gamma^\mu u &= u^\dagger (a + \bar{a}) \gamma^0 \gamma^\mu (a + \bar{a}) u \\ &= u^\dagger a \gamma^0 \gamma^\mu a u + u^\dagger \bar{a} \gamma^0 \gamma^\mu \bar{a} u \end{aligned} \quad (2.22)$$

Thus the ψ^+ spinor couples only to ψ , and χ^+ to χ . In the high-energy limit $\frac{\mu}{\omega} \rightarrow 0$, ψ and χ are helicity eigenstates and therefore V and A interactions conserve helicity, in this limit.

In a sequence of V or A interactions (such as Fig. 2.5), if the free mass μ is neglected, the chirality is still preserved. If we assume that an electron only interacts with photons via vector interactions, then in a sequence of interactions like that shown below, even though the intermediate electrons are off mass-shell, the chirality is preserved (up to corrections involving the electron mass μ). Thus the initial and final free electrons have the same chirality, and since both are assumed to be of high energy, consequently, the same helicity.

Figure 2.5



Thus amplitudes which involve a change of electron helicity will be of order $\frac{\mu}{\omega}$, where ω is large, with respect to the helicity-conserving amplitudes in the high-energy limit.

Therefore, in electron-proton scattering, we shall assume that only amplitudes which conserve the electron helicity do not tend to zero at high energy.

i.e. overall phase is undetermined.

However, although the number of amplitudes is reduced, the range of possible experiments. In the limit $\mu \rightarrow 0$ (2.23) these electrons only interact via vector interactions which conserve helicity, detectors are only sensitive to longitudinal

polarizations. Measurement of a transverse polarization Thus in the limit $\frac{\mu}{\epsilon} \rightarrow 0$: involves a measurement of the relative phase between the two helicity states which is not possible in this limit. We must recognize this limitation and restrict the discussion to

$$f_2 \rightarrow 0; \quad f_4 \rightarrow 0 \quad \text{and} \quad f_6 \rightarrow 0 \quad (2.24)$$

experiments involving only longitudinal electron polarizations. We are left with only three non-vanishing amplitudes in this limit. For these we shall adopt the following notation.

4. Summary

$$f_{++;++} = pf_{--;--} = f_L \text{ (reversal-invariant)} \quad (2.25a)$$

there are six independent helicity amplitudes for 'Like' helicities of electron and proton are only vector electromagnetic interactions, which leads to the resulting of $f_{+-;+-} = f_{-+;-+} = f_U$ amplitudes as $\mu \rightarrow 0$ (2.25b)

U.3 'Un-like' helicities for e and p

$$f_{++;-+} = f_{+-;--} = -f_{--;+-} = -f_{-+;++} \quad (2.25c)$$

X 'Exchange' - proton helicity is flipped.

Thus in this limit, for complete knowledge of the scattering matrix, we need only determine three complex amplitudes f_L , f_U and f_X : only five independent quantities

if an overall phase is undetermined.

However, although the number of amplitudes is reduced, so is the range of possible experiments. In the limit $\mu \rightarrow 0$, since electrons only interact via vector interactions which conserve helicity, detectors are only sensitive to longitudinal polarizations. Measurement of a transverse polarization involves a measurement of the relative phase between the two helicity states which is not possible in this limit. We must recognize this limitation and restrict the discussion to experiments involving only longitudinal electron polarizations.

4 Summary

Assuming parity- and time-reversal-invariance (TRI), there are six independent helicity amplitudes for e-p scattering. We also assume that the electron has only vector electromagnetic interactions^{*}, which leads to the vanishing of three of the six helicity amplitudes as $\mu \rightarrow 0$.

2.3 All possible electron-proton scattering experiments

1 Introduction

Table 2.2 shows all possible experiments for elastic electron-proton scattering. A similar table was given for the nucleon-nucleon case in reference [2]. Each experiment is categorized by the initial spin-state of the target protons and electron beam, and by the measured result -

* In principle we could also allow axial-vector interactions. By the Feynman-Gell-Mann [15] V-A theory, a similar analysis could be made for weak interactions.

Table 2.2

All possible e-p Scattering Experiments

Initial Spin State	Unpolarized Beam	Polarized Beam	Unpolarized Beam	Polarized Beam
Result of Experiment	Unpolarized Target	Unpolarized Target	Polarized Target	Polarized Target
Cross section	$\frac{d\sigma}{d\Omega} = I_0$	$\frac{d\sigma}{d\Omega} \Big _e = I_e$	$\frac{d\sigma}{d\Omega} \Big _p = I_p$ $I_p = I_0(1+A)$	$\frac{d\sigma}{d\Omega} \Big _{ep} = I_{ep}$ C'_{ij}
Polarization of Scattered Electron	$\underline{P}_e^{(o)}$	$\underline{D}_{j\ell}^{(e)}$	$\underline{K}_{i\ell}^{(pe)}$	$\underline{T}_{ij\ell}^{(e)}$
Polarization of Recoil Proton	$\underline{P}_p^{(o)}$	$\underline{K}_{jk}^{(ep)}$	$\underline{D}_{ik}^{(p)}$	$\underline{T}_{ijk}^{(p)}$
Correlation of Polarization	C_k	$\underline{P}_{jk\ell}^{(e)}$	$\underline{P}_{ik\ell}^{(p)}$	$\underline{T}_{ijk\ell}$

i, k target and recoil proton polarization direction in HRF
 j, ℓ beam and scattered electron polarization direction in HRF
 (HRF = Helicity Rest Frame)

D Depolarization tensor K Polarization Transfer Tensor
 C Correlation Tensor
 (The notation is explained in detail in the text.)

e.g., differential cross section, polarization, etc.. Our notation for the various tensors is similar to reference [2], and we use the nomenclature of Bethe and Schumacher [3] for the second-rank polarization tensors. One significant difference however, is that our tensor indices refer to directions in the helicity rest-frames (H.R.F.) of each particle: our procedure is entirely relativistic.

The subscripts used are as follows :

- Proton a : i initial proton polarization direction in HRF a
 Electron b: j initial electron polarization direction in HRF b
 Proton c : k final proton polarization direction in HRF c
 Electron d: final electron polarization direction in HRF d.

These tensor indices will be used consistently, and further subscripts, e.g., i_a , will in general be omitted. For specific components, we use the notation x_a, y_b etc., where these refer to directions in the appropriate HRF.

The two-index tensors will be referred to as :

D : Depolarization tensors - of electron and proton

$$D^{(e)} \quad D^{(p)}$$

K : Polarization-transfer tensors - from electron to proton

$$\text{and vice-versa} - K^{(ep)}, K^{(pe)}$$

C : Correlation tensor.

The remaining two-index tensor C_{ij}^i that arises in the differential cross section for scattering of a polarized electron beam by a polarized proton target, will be seen later to be essentially equivalent to the correlation tensor C.

We shall first consider 'simple' experiments involving only tensors up to rank two, and each type of experiment will be briefly discussed. We shall see that the assumption of parity- and time-reversal-invariance relates several tensor components and gives rise to 'equivalent' experiments. The most well-known example is the polarization-asymmetry relation first proved for N-N scattering by Dalitz [4], and by Wolfenstein and Ashkin [4]. However, as a consequence of our limit $\mu \rightarrow 0$, we have additional relations and restrictions on the possible experiments. As discussed above, we restrict ourselves to experiments involving only longitudinal polarizations of the ingoing and outgoing electrons.

We remark here that historically, the parameters measured in N-N scattering using unpolarized targets, were called the 'parameters of single, double and triple scattering' [8]. These names originated from the experimental conditions in the absence of polarized targets, and also of injectors of polarized particles. With the advent of polarized targets, this nomenclature is no longer so appropriate.

2 The 'Simple' Experiments: Tensors with up to Two Indices

(1) Unpolarized Cross section I_0

For an unpolarized electron beam incident on an unpolarized target, the initial density matrix is proportional to the unit operator in the combined spin-spaces.

$$\rho_0^i = \frac{1}{4} I_a \otimes I_b$$

The differential cross section with no measurement of the

final state polarizations is then given by

$$\frac{d\sigma}{d\Omega} \Big|_{\text{unpol.}} \equiv I_0 = \text{Tr} [f \rho_0^i f^\dagger]$$

$$= \frac{1}{4} \sum_{\substack{\lambda_a \lambda_b \\ \lambda_c \lambda_d}} f_{\lambda_c \lambda_d : \lambda_a \lambda_b}^2$$

Using : $f_{\lambda_c \lambda_d : \lambda_a \lambda_b} = \delta_{\lambda_d \lambda_b} f_{\lambda_c \lambda_b : \lambda_a \lambda_b}$

we obtain

$$I_0 = \frac{1}{2} [|f_L|^2 + |f_U|^2] + |f_X|^2 \quad (2.26)$$

This is the 'parameter of single scattering'.

(2) Proton Polarization and Asymmetry Experiments

With unpolarized electrons and a polarized proton target in the initial state, the initial density matrix is :

$$\rho^i = \frac{1}{4} (1_a + \underline{P}_a \cdot \underline{\sigma}_a) \otimes 1_b$$

\underline{p}_a and $\underline{\sigma}_a$ refer to directions in the HRF of the initial proton.

The differential cross section with no measurement of a final-state polarization is then

$$\frac{d\sigma}{d\Omega} \Big|_p \equiv I_p = \text{Tr} [f \rho^i f^\dagger]$$

$$= I_0 (1 + \underline{P}_a \cdot \underline{A}) \quad (2.27)$$

where $I_0 \underline{A} = \frac{1}{4} \text{Tr} [f \underline{\sigma}_a f^\dagger]$.

From parity-invariance, only the component of \underline{P}_a perpendicular to the scattering-plane leads to any asymmetry in the cross-section. is usually determined by measuring the asymmetry

i.e.,
$$\underline{A} = A \underline{n} \tag{2.28}$$

where
$$\underline{n} = \frac{\underline{p} \wedge \underline{p}'}{|\underline{p} \wedge \underline{p}'|}$$

We find

$$I_{0A} = \text{Im} [f_L \bar{f}_X] + \text{Im} [f_U \bar{f}_X] \tag{2.29}$$

Now consider the experiment involving measurement of the polarization of the recoil proton produced in the collision of unpolarized electrons and protons. Parity-invariance restricts the polarization to be in a direction parallel to \underline{n} - the normal to the scattering-plane. The final-state density matrix is given by

$$I_{0\rho^f} = f \rho_0^f f^\dagger = \frac{1}{4} f f^\dagger$$

and the polarization of the recoil proton by

$$\underline{P}_p^{(0)} = \text{Tr} [\rho^f \underline{\sigma}_c 1_d] \tag{2.30}$$

Thus

$$I_{0P_{yc}}^{(0)} = \frac{1}{4} \sum_{\lambda_c \lambda_d} \sum_{\lambda_a \lambda_b} f_{\lambda_c \lambda_d} \bar{f}_{\lambda_a \lambda_b} (\sigma_y)_{\lambda_c \lambda_c} \lambda_d \lambda_d$$

We obtain :

$$I_{0P_{yc}}^{(0)} = \text{Im} [f_L \bar{f}_X] + \text{Im} [f_U \bar{f}_X] = I_{0A} \tag{2.31}$$

This is the polarization-asymmetry relation. A measurement of $P_{yc}^{(0)}$ is often called the 'parameter of double scattering', since it is usually determined by measuring the asymmetry in a second scattering from a target of known analyzing power. However, because of relation (2.31), this parameter can be obtained by a single scattering by a polarized target.

The polarization-asymmetry relation is a consequence of our assumption of TRI [4]. For spin 0 - spin $\frac{1}{2}$ scattering, the relation follows from parity- and rotation-invariance alone, and thus one may wonder whether in our approximation $\mu \rightarrow 0$, TRI is necessary to prove this relation. It is easy to demonstrate why TRI is still required to prove $P_{yc}^{(0)} = A$.

Assuming parity-invariance, but without assuming TRI, we have the following table for the independent helicity amplitudes in e-p scattering.

Table 2.3

		λ_a	+	+	-	-
		λ_b	+	-	+	-
λ_c	λ_d					
+	+	f_1	f_2	f_3	f_4	
+	-	f_7	f_5	f_6	f_8	
-	+	$-f_8$	f_6	f_5	$-f_7$	
-	-	f_4	$-f_3$	$-f_2$	f_1	

TRI gives the additional restrictions

$$f_3 = f_8 \quad \text{and} \quad f_2 = -f_7 \quad (2.32)$$

In the limit $\mu \rightarrow 0$:

$f_2, f_4, f_6, f_7 \rightarrow 0$ (independent of TRI)

leaving four non-zero amplitudes. The polarization and asymmetry are calculated as before.

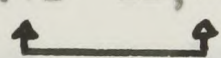
$$I_0^A = \text{Im} [f_8 \bar{f}_1] + \text{Im} [f_3 \bar{f}_5] \quad (2.33a)$$

$$I_0^{P_{y_c}^{(0)}} = \text{Im} [f_3 \bar{f}_1] + \text{Im} [f_8 \bar{f}_5] \quad (2.33b)$$

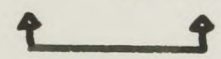
$$\text{with } I_0 = \frac{1}{2} [|f_1|^2 + |f_3|^2 + |f_5|^2 + |f_8|^2]$$

Thus the relation $f_3 = f_8$ is needed for $A = P_{y_c}^{(0)}$.

Physically the necessity for TRI arises in the following manner. For the terms involved in A :


$$f_8 \bar{f}_1 = f_{---:+-} \bar{f}_{---;--}$$


and


$$f_3 \bar{f}_5 = f_{-+:++} \bar{f}_{-+:--}$$


We see that A arises from interference between amplitudes differing only by proton helicity-flip in the initial state.

For the terms in $P_{y_c}^{(0)}$:

$$f_3 \bar{f}_1 = f_{-+:++} \bar{f}_{++:++}$$


and

$$f_8 \bar{f}_5 = f_{---:+-} \bar{f}_{+-:+-}$$


(3) Electron polarization. In unpolarized electron scattering, the interference is between amplitudes differing only by proton helicity-flip in the final state. Time-reversal-invariance is needed to relate :

scattered electron, spin up to scattered electron, spin down by electron helicity $f_{-+; ++}$ to $f_{++; -+}$

approximation. The polarization transfer is zero. For spin 0 - spin $\frac{1}{2}$ scattering, TRI is not needed because $f_{+;-}$ is related to $f_{-;+}$ by parity.

We also note here that

$$P_c^{(0)} = A c \sin \theta \quad (2.34)$$

In the scattering of an unpolarized beam of a polarized proton target, the tensor P_{ij} relates to helicity transfer. This arises because of restrictions due to angular momentum conservation in the forward and backward directions. In general, the singularities of the helicity amplitudes at the boundary of the physical region, can be isolated by defining new amplitudes \hat{f} [16] :

$$f_{\lambda_c \lambda_d : \lambda_a \lambda_b}^{(\nu, \rho)}(\theta, W) = (\cos \frac{\theta}{2})^{|\nu + \rho|} (\sin \frac{\theta}{2})^{|\nu - \rho|} \hat{f}_{\lambda_c \lambda_d : \lambda_a \lambda_b}^{(\nu, \rho)}(\theta, W) \quad (2.35)$$

For scattering in the forward direction $\theta = 0$, angular momentum requires $f_{\lambda_c \lambda_d : \lambda_a \lambda_b}^{(\nu, \rho)}(\theta, W)$ to vanish unless $\nu = \rho$.

Similarly for the backward direction - vanishing unless $\nu = -\rho$.

From Table 2.3 we see that :

$$\text{From parity inv } f_3 \sim \sin \theta \hat{f}_3$$

$$\text{and "y-boost" } f_8 \sim \sin \theta \hat{f}_8$$

Thus: so obtain, as $\hat{A} \sim \sin \theta \hat{A}$. The relation

(3) Electron Polarization and Asymmetry Experiments

An unpolarized electron beam, scattered by an unpolarized target, can give rise to no transverse polarization of the scattered electron, since such a polarization must be caused by electron helicity-flip amplitudes, which are zero in our approximation. The polarization-asymmetry relation shows that we can likewise obtain no information from electron asymmetry experiments.

(4) Proton Depolarization Tensor $D_{ik}^{(p)}$

In the scattering of an unpolarized beam by a polarized proton target, the tensor $D_{ik}^{(p)}$ relates an initial component of polarization P_{ia} to the k^{th} component of recoil proton polarization P_{kc}

$$\rho_i = \frac{1}{4} (1_a + \underline{P}_a \cdot \underline{\sigma}_a) \otimes 1_b$$

$$I_p \rho_f = f \rho_i f^\dagger$$

and $I_p P_{kc} = \text{Tr} [f \rho_i f^\dagger \sigma_k^c]$

$$= I_o (P_{kc}^{(o)} + P_{ia} D_{ik}^{(p)}) \quad (2.36)$$

where $I_o D_{ik}^{(p)} = \frac{1}{4} \text{Tr} [f \sigma_i^a f^\dagger \sigma_k^c]$

$$(\underline{P}^{(o)}) = P_{yc}^{(o)} \underline{n}$$

From parity invariance, all tensor components with only one "y-component" - normal to the scattering plane - are zero. We also obtain, as a result of TRI, the relation

the initial spin matrix is

$$D_{x_a z_c}^{(p)} = - D_{z_a x_c}^{(p)} \quad (2.37)$$

Thus there are four independent components of the tensor $D_{ik}^{(p)}$ corresponding to longitudinally polarized electrons.

(5) Electron Depolarization Tensor $D_{j\ell}^{(e)}$

$D^{(e)}$ is defined in an analogous way to $D^{(p)}$:

$$I_0 D_{j\ell}^{(e)} = \frac{1}{4} \text{Tr} [f \sigma_j^b f^\dagger \sigma_\ell^d]$$

The restrictions from parity and TRI are as for $D^{(p)}$, and our approximation $\mu \rightarrow 0$, gives immediately:

$$I_0 D_{zz}^{(e)} = \frac{1}{4} \text{Tr} [f f^\dagger \sigma_z^b \sigma_z^d]$$

$$D_{xz}^{(e)} = 0$$

of the polarizations of both the final-state particles, produced in the scattering of an unpolarized beam on an unpolarized target. We find:

As we have discussed, the only experiments we shall consider are those involving only longitudinal electron

polarizations. Reflection-invariance requires $D_{yy}^{(e)} = D_{yy}^{(p)}$

[1], but in practice this component can only be measured in our approximation, by measuring the proton tensor component.

As a result of our approximation - essentially that $D_{zz}^{(e)} = 1$ - no information about the scattering amplitude can be obtained from the electron depolarization tensor.

(6) Differential Cross-Section with polarized proton target and polarized electron beam

The initial particles' spins are uncorrelated and

the initial density matrix is

$$\rho^i = \frac{1}{2} (1_a + \underline{P}_a \cdot \underline{\sigma}_a) \otimes \frac{1}{2} (1_b + \underline{P}_b \cdot \underline{\sigma}_b)$$

Only \underline{P}_b corresponding to longitudinally polarized electrons gives non-zero components for the tensor C' , which is defined by :

$$\frac{d\sigma}{d\Omega}_{pe} = I_{pe} = I_0 (1 + \underline{P}_a \cdot \underline{A} + P_{1a} P_{2b} C'_{1a 2b})$$

$$I_0 C'_{1a 2b} = \frac{1}{4} \text{Tr} [f \sigma_i^a \sigma_z^b f^\dagger] \quad (2.38)$$

This tensor that appears in the cross section is related via TRI to the correlation tensor :

$$I_0 C_{j\ell} = \frac{1}{4} \text{Tr} [f f^\dagger \sigma_j^c \sigma_\ell^d] \quad (2.39)$$

of the polarizations of both the final-state particles, produced in the scattering of an unpolarized beam by an unpolarized target. We find :

$$C'_{z_a z_b} = C_{z_c z_d}$$

$$C'_{x_a z_b} = -C_{x_c z_d} \quad (2.40)$$

All other components are zero.

(7) Polarization-Transfer Tensors: $K^{(ep)}$ and $K^{(pe)}$

These polarization-transfer tensors $K^{(ep)}$ and $K^{(pe)}$ relate respectively, the polarization of electron beam to

the polarization of the recoil proton, and the polarization of the target proton to the polarization of the scattered electron. In the N-N case, these two tensors coincide and the one polarization-transfer tensor is useful in the determination of the scattering matrix. However, for e-p scattering, it is a consequence of our approximation that the electron helicity is conserved, that these two tensors are directly related to the tensors considered in the previous section. Since the electron helicity is conserved exactly in our approximation, measuring the polarization of the electrons in the final-state gives no more information than the corresponding experiment with a polarized electron beam. This is evident from the summations:

$$\sum_{\lambda'_d \lambda_d} \sum_{\lambda_b} f_{\lambda_c \lambda_d: \lambda_a \lambda_b} \bar{f}_{\lambda'_c \lambda'_d: \lambda'_a \lambda'_b} (\sigma_z^d)_{\lambda'_d \lambda_d}$$

$$= \sum_{\lambda_d} \sum_{\lambda_b \lambda'_b} f_{\lambda_c \lambda_d: \lambda_a \lambda_b} (\sigma_z^b)_{\lambda_b \lambda'_b} \bar{f}_{\lambda'_c \lambda'_d: \lambda'_a \lambda'_b}$$

Thus we see immediately that :

$$C'_{zz} = K_{zz}^{(pe)} \quad \text{and} \quad C'_{xz} = K_{xz}^{(pe)}$$

$$C_{zz} = K_{zz}^{(ep)} \quad \text{and} \quad C_{xz} = K_{zx}^{(ep)}$$

(2.41)

(8) Determination of the Scattering Matrix

All the experiments discussed so far, involve 'two polarisations' at most: there are yet more difficult

experiments involving 'three polarizations'. In the N-N case, the amplitude can be reconstructed without recourse to these more complicated experiments. From our restricted set of experiments can we determine the complete scattering matrix for high-energy e-p scattering?

In the high-energy limit, there are three amplitudes f_L , f_U and f_X which are complex functions of energy and scattering angle. Since the observed quantities are bilinear in f, \bar{f} an overall phase cannot be determined from measurements of a given angle. It is necessary therefore, to determine five real quantities: the real and imaginary parts of f_L , f_U and f_X with one phase chosen arbitrarily for the reconstruction. The observables contain the three amplitudes bilinearly and in order to determine the scattering matrix uniquely, more than five measurements are needed. (In the N-N case eleven experiments were needed to determine nine quantities.) All the feasible independent measurements were calculated by the methods of the previous sections, and are listed in Table 2.4. (Only up to two index tensors.)

Example of an Explicit Reconstruction of f

Choose the phase of f_L to be zero: f_L real. Using (1), (3) and (8) of Table 2.4 we obtain:

$$|f_L|^2, |f_U|^2 \text{ and } |f_X|^2$$

Thus f_L is determined.

Using $|f_X|^2$ and (5) or (7) we obtain for example:

$$\begin{aligned} I_0 (D_{xx}^{(p)} + \frac{1}{2} - \frac{1}{2} D_{zz}) &= \text{Re} [f_U \bar{f}_L] \\ &= f_L \text{Re } f_U \end{aligned}$$

Table 2.4 Ref_X determined and also Ref_U up to a sign.

(4) and (6) also give
Independent Observables for Tensors up to Two Indices

$$I_o = \frac{1}{2} [|f_L|^2 + |f_U|^2] + |f_X|^2 \quad (1)$$

$$I_o^A = \text{Im}[f_L \bar{f}_X] + \text{Im}[f_U \bar{f}_X] \quad (2)$$

$$I_o^{C'_{z_a z_b}} = \frac{1}{2} [|f_L|^2 - |f_U|^2] \quad (3)$$

$$I_o^{C'_{x_a z_b}} = \text{Re}[f_L \bar{f}_X] - \text{Re}[f_U \bar{f}_X] \quad (4)$$

$$I_o^{D^{(p)}_{x_a x_c}} = \text{Re}[f_U \bar{f}_L] - |f_X|^2 \quad (5)$$

$$I_o^{D^{(p)}_{x_a z_c}} = \text{Re}[f_L \bar{f}_X] + \text{Re}[f_U \bar{f}_X] \quad (6)$$

$$I_o^{D^{(p)}_{y_a y_c}} = \text{Re}[f_U \bar{f}_L] + |f_X|^2 \quad (7)$$

$$I_o^{D^{(p)}_{z_a z_c}} = \frac{1}{2} [|f_L|^2 + |f_U|^2] - |f_X|^2 \quad (8)$$

reasonable to expand the scattering matrix in powers of ω since f_L real. Therefore Ref_U determined and from

$$|f_U|^2 = (\text{Re}f_U)^2 + (\text{Im}f_U)^2 \quad (9.23)$$

we obtain $\text{Im}f_U$ up to a sign.

(4) and (6) give :

$$\begin{aligned} \frac{1}{2} I_o (C'_{xz} + D_{xz}) &= \text{Re}[f_L \bar{f}_X] \\ &= f_L \text{Ref}_X . \end{aligned}$$

Therefore $\text{Re}f_X$ determined and also $\text{Im}f_X$ up to a sign.

(4) and (6) also give :

$$\begin{aligned} \frac{1}{2} I_0 (D_{xz} - C'_{xz}) &= \text{Re}[f_U \bar{f}_X] \\ &= \text{Re}f_U \text{Re}f_X + \text{Im}f_U \text{Im}f_X \end{aligned}$$

Thus the relative sign of $\text{Im}f_U$ and $\text{Im}f_X$ is determined.

The whole scattering matrix has now been determined up to one sign ambiguity. This sign ambiguity is removed by (2) :

$$\begin{aligned} I_0 A &= \text{Im}[f_L \bar{f}_X] + \text{Im}[f_U \bar{f}_X] \\ &= \text{Re}f_X \text{Im}f_U - \text{Im}f_X [f_L + \text{Re}f_U] \end{aligned}$$

so by using seven of the eight relations in Table 2.4, we have been able in principle to determine the scattering matrix completely (up to the common phase).

(9) Reconstruction Procedure for Electron-Proton Scattering

For e-p scattering, the method outlined in the previous section meets a severe difficulty. In this case, it is reasonable to expand the scattering matrix in powers of α - the fine structure constant. To order α^2 :

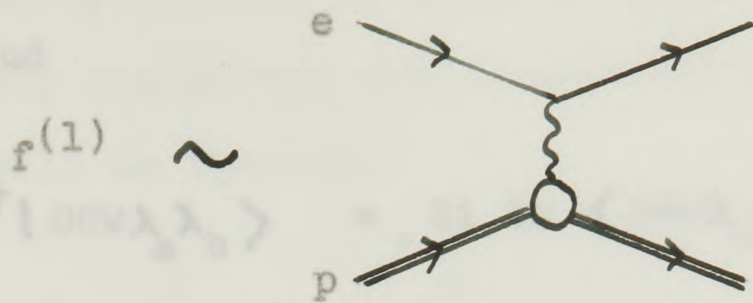
$$f = \alpha f^{(1)} + \alpha^2 f^{(2)} \quad (2.42)$$

$f^{(1)}$ is the one-photon-exchange contribution to e-p

scattering. In the cross section to order α^2 , radiative corrections (rescattering diagrams) must be considered.

$f^{(2)}$ contains these radiative corrections; here we only discuss the two-photon-exchange contribution.

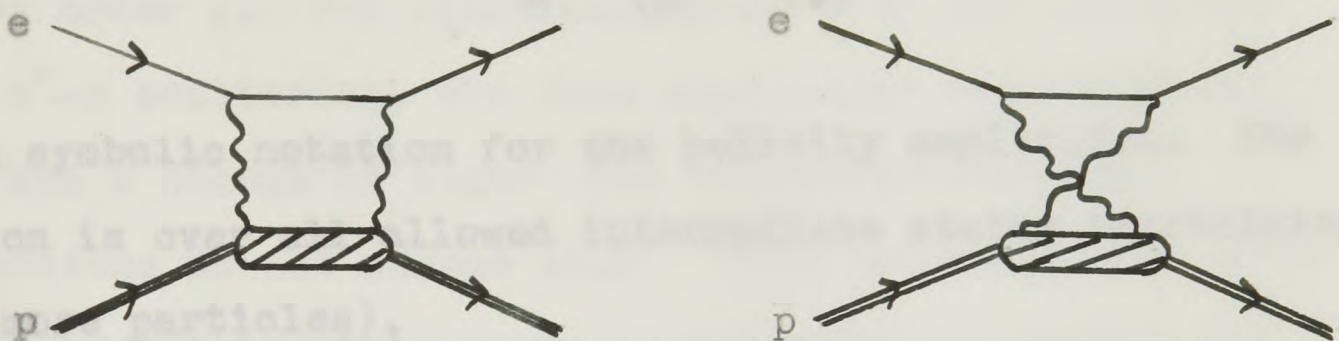
Figure 2.6



and $f^{(2)}$ will contain the two-photon-exchange contributions

*

Figure 2.7



Consider the effect of this expansion in α on the unitarity condition for T-matrix elements.

$$S = \mathbf{1} + i T \quad \text{and} \quad S S^\dagger = \mathbf{1}$$

leads to

$$T - T^\dagger = i T^\dagger T \tag{2.43}$$

in terms of matrix elements between two-particle CM helicity states:

* In the cross section to order α^3 , radiative corrections and soft bremsstrahlung diagrams must be considered. $f^{(2)}$ contains these radiative corrections: here we only discuss the two-photon-exchange contribution.

$$f_{\lambda_c \lambda_d : \lambda_a \lambda_b}(\theta, W) \sim \langle 0 \theta W \lambda_c \lambda_d | T | 0 0 W \lambda_a \lambda_b \rangle$$

Using TRI we find

$$\langle 0 \theta W \lambda_c \lambda_d | T - T^\dagger | 0 0 W \lambda_a \lambda_b \rangle = 2i \operatorname{Im} \langle 0 \theta W \lambda_c \lambda_d | T | 0 0 W \lambda_a \lambda_b \rangle \quad (2.44)$$

Therefore the unitarity condition (2.43) gives a relation of the form:

$$2 \operatorname{Im} f_{(\lambda)} \sim \sum_n f_{(\mu)} \bar{f}_{(\nu)} \quad (2.45)$$

using a symbolic notation for the helicity amplitudes. The summation is over all allowed intermediate states (containing two or more particles).

An expansion of f in powers of α shows immediately that the one-photon-exchange amplitudes are real:

$$2 \operatorname{Im} f_{(\lambda)}^{(1)} = 0 \quad (2.46)$$

independent of angle and energy, and

$$f_{L \text{ and } U}^{(1)} = \bar{f}_{L \text{ and } U}^{(1)}; \quad f_X^{(1)} = \bar{f}_X^{(1)} \quad (2.47)$$

Thus the imaginary part of f is of relative order α with

respect to the real part. It is clear therefore, that the

method outlined in section (8) to determine both the real

and imaginary parts of the amplitudes, is unworkable. A

measurement of $|f|^2$ does not help determine $(\operatorname{Im} f)^2$ since

this is of order a^2 with respect to $(\text{Ref})^2$. Because of the smallness of a , such effects of order of magnitude a^4 are completely negligible and henceforth contributions of this order will be ignored.

$$|f|^2 = (\text{Ref})^2 + (\text{Im}f)^2$$

$$= a^2 |f^{(1)}|^2 + a^3 2f^{(1)}\text{Ref}^{(2)} + O(a^4) \quad (2.48)$$

However, it is possible to extract from $|f|^2$ the term of relative order a . For electron-proton e^-p and positron-proton e^+p scattering, the Born amplitudes are identical apart from a change of sign: the two-photon-exchange contributions do not change sign. Thus combination of e^+p and e^-p scattering data makes it possible to isolate the a^3 term: $f^{(1)}\text{Ref}^{(2)}$. If we determine $|f_L|^2$, $|f_U|^2$ and $|f_X|^2$ for both $e^\pm p$ scattering as outlined before, then addition and subtraction of the two sets of data yields the following quantities:

$$|f_L^{(1)}|^2; |f_U^{(1)}|^2; |f_X^{(1)}|^2$$

(2.49)

and

$$f_L^{(1)}\text{Ref}_L^{(2)}; f_U^{(1)}\text{Ref}_U^{(2)}; f_X^{(1)}\text{Ref}_X^{(2)}$$

Similar considerations apply to the terms of the form $\text{Re}[f\bar{f}']$ which make possible the measurement of the following combinations:

$$f_L^{(1)}f_U^{(1)}; f_L^{(1)}f_X^{(1)}; f_U^{(1)}f_X^{(1)}$$

and

$$f_L^{(1)} \text{Ref}_X^{(2)} + f_X^{(1)} \text{Ref}_L^{(2)} + f_U^{(1)} \text{Ref}_X^{(2)} + f_X^{(1)} \text{Ref}_U^{(2)} + f_U^{(1)} \text{Ref}_L^{(2)} + f_L^{(1)} \text{Ref}_U^{(2)} \quad (2.50)$$

Choosing the sign of one of the Born amplitudes arbitrarily - say $f_L^{(1)}$ positive - then all the six parameters below, can be measured:

$$f_L^{(1)}, f_U^{(1)}, f_X^{(1)}, \text{Ref}_L^{(2)}, \text{Ref}_U^{(2)}, \text{Ref}_X^{(2)} \quad (2.51)$$

This requires six experiments for both e^-p and e^+p scattering - twelve in all.

If we now turn to the experimental data on the real part of the two-photon-exchange contribution, only the differential cross sections for e^+p and e^-p have been compared [12].

$$I_0 = \frac{1}{2} [|f_U|^2 + |f_L|^2] + |f_X|^2 \quad (2.52)$$

$$\text{If we define : } \Delta I_0 = (I_0)_{e^+p} - (I_0)_{e^-p} \quad (2.53)$$

then

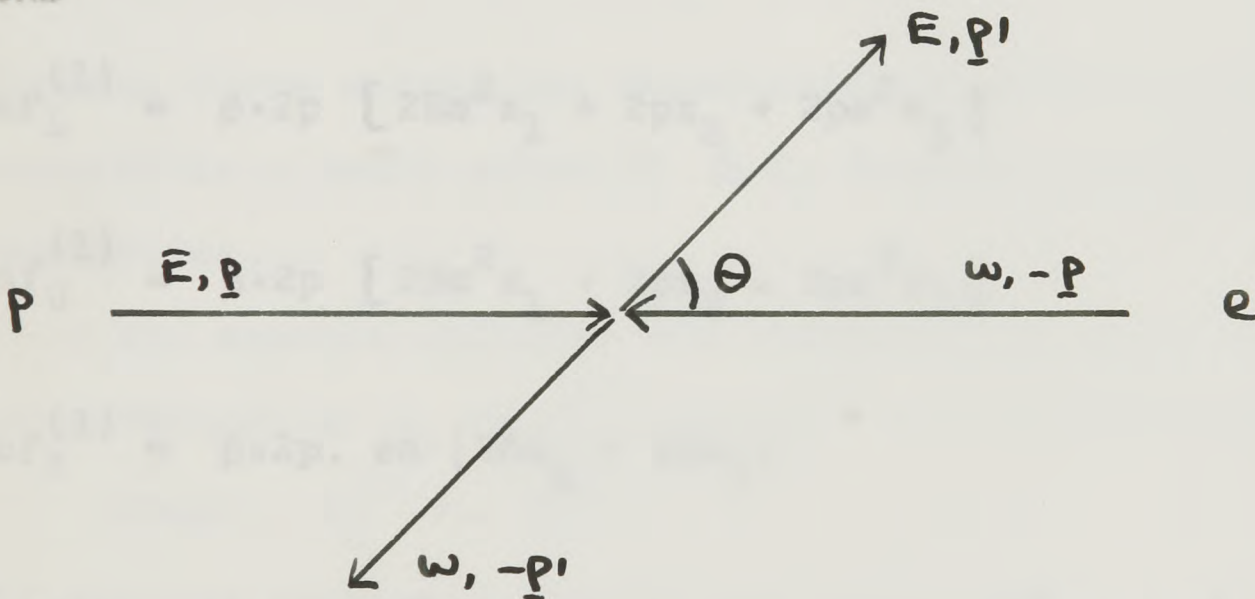
$$\Delta I_0 (a^3) = 2 \left\{ f_L^{(1)} \text{Ref}_L^{(2)} + f_U^{(1)} \text{Ref}_U^{(2)} + 2f_X^{(1)} \text{Ref}_X^{(2)} \right\} \quad (2.54)$$

The first Born amplitudes are isolated by addition of the

$e^+ - p$ and $e^- - p$ data which eliminates the two-photon-exchange term. Using the differential cross sections, only the combination (2.52) is obtained: in terms of the two form factors this contains G_E^2 and G_M^2 and no cross term, (see Chapter 3). In principle, for an accurate determination of two-photon-exchange effects, the form factors G_E and G_M should be extracted by a complete analysis for the three first Born amplitudes, using both $e^+ - p$ and $e^- - p$ data. For completeness we give here the expressions for the one-photon-exchange amplitudes in terms of G_E , G_M and CMS momentum p and scattering angle θ .

Figure 2.8

CMS



$$E^2 = p^2 + M^2 \quad \text{and in the approximation } \mu = 0$$

$$\omega = |p| = |p'| = p$$

Let

$$c = \cos \frac{\theta}{2} \quad s = \sin \frac{\theta}{2} \quad W = E + \omega$$

and

$$\tau = \frac{-q^2}{4M^2} = \frac{p^2}{M^2} s^2$$

We define the following combinations of G_E and G_M :

$$x_1 = \frac{1}{(1 + \tau)} (G_E + \tau G_M)$$

$$x_2 = \frac{1}{(1 + \tau)} \left[G_E \cdot c^2 + s^2 \cdot \frac{E^2}{M^2} G_M \right]$$

$$x_3 = G_M$$

$$x_4 = \frac{1}{(1 + \tau)} \left[\frac{E^2}{M^2} G_M - \frac{P^2}{M^2} c^2 G_M \right]$$

$$x_5 = \frac{1}{(1 + \tau)} \frac{E}{M} [G_E - G_M]$$

$f_L^{(1)}$, $f_U^{(1)}$ and $f_X^{(1)}$ can then be written:

$$af_L^{(1)} = \beta \cdot 2p \{ 2Ec^2 x_1 + 2px_2 + 2ps^2 x_3 \} \quad (2.55a)$$

$$af_U^{(1)} = \beta \cdot 2p \{ 2Ec^2 x_1 + 2px_2 - 2ps^2 x_3 \} \quad (2.55b)$$

$$af_X^{(1)} = \beta \cdot 2p \cdot sc [2Mx_4 + 2px_5] \quad (2.55c)$$

where $\beta = \frac{a}{2WQ^2} \quad (Q^2 = -q^2)$

Note that as expected, $f_X^{(1)} \sim \sin\theta$.

In practice, G_E^2 and G_M^2 are determined just from the e^-p differential cross section, ignoring two-photon-exchange effects. To separate G_E^2 and G_M^2 it is necessary to use measurements at different energies and scattering angles.

The straight-line character of the Rosenbluth plot [12] gives some indication of the validity of this approximation, but it has been shown [17] that not all contributions to the two-photon-exchange amplitude give significant or indeed any deviations from this straight-line behaviour.

As a prelude to Chapter 3, we note that it is clear that measurement of the differential cross section alone, does not determine the relative sign of G_E and G_M .

If we regard G_E and G_M as known to a good approximation, then ΔI_0 (2.54) depends on three independent unknown functions of W and θ , all of the same order of magnitude.

$$\Delta I_0 = 2[f_L^{(1)} \text{Re} f_L^{(2)} + f_U^{(1)} \text{Re} f_U^{(2)} + 2f_X^{(1)} \text{Re} f_X^{(2)}] \quad (2.54)$$

Clearly there exists the possibility of cancellation which results in a small value of ΔI_0 despite sizable two-photon amplitudes.

For example consider the situation at small θ when terms proportional to x_3 in $f_L^{(1)}$ and $f_U^{(1)}$ can be neglected (2.55).

$$\text{Then: } f_L^{(1)} \sim f_U^{(1)}.$$

If the two-photon amplitudes are such that:

$$\text{Re } f_L^{(2)} \sim - \text{Re } f_U^{(2)}$$

$$\text{and } \text{Re } f_X^{(2)} \sim 0$$

$$\text{then } \Delta I_0 \sim 0.$$

The usual analysis for e^+p and e^-p differential cross sections (as given for example in ref. [12]) is as

Nevertheless it would perhaps be desirable to determine whether any of these processes differ appreciably from the one-photon-exchange approximation.

the size of the cross section of the two-photon exchange amplitudes, determined by subtraction of the one-photon and

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 + 4 \operatorname{Re} \left(\frac{A_2}{A_1} \right) \quad (2.56)$$

In our notation: $R \approx 1 + \Delta I_0$

(A_1 and A_2 are one- and two-photon-exchange amplitudes

respectively.) Present experimental data indicates [12]:

$$\left| \operatorname{Re} \left(\frac{A_2}{A_1} \right) \right| \leq 0.02$$

for a range of incident electron-positron LAB energies up to 10 GeV and Q^2 up to 5 (GeV/c)². It is evident that this qualitative analysis in terms of one two-photon-exchange amplitude could be misleading as a criterion for the importance of two-photon effects. However, if the small value of ΔI_0 is a result of cancellation, such a cancellation would have to take place over the whole measured range of s and Q^2 . This is perhaps unlikely, but the measured range of Q^2 and s for the data on ΔI_0 , is very much smaller than the range for the data on e^-p differential cross sections. Furthermore, we have no knowledge of the s and Q^2 dependence of the three two-photon-exchange amplitudes.

The possibility of such a cancellation could of course be eliminated by separation of the three unknown functions. To do this, it is necessary to measure the various polarization tensor components as described above, for both e^-p and e^+p scattering. Such experiments are of course very difficult since effects of relative order α are involved.

then $\Delta \sim 0$.

Nevertheless it would perhaps be possible to determine whether any of these parameters differ appreciably from the one-photon-exchange prediction. It is worth remarking that the size of the interference term of the one- and two-photon amplitudes, determined by subtraction of the electron and positron data, is unaffected by uncertainties in the one-photon-exchange form factor analysis.

So far, the imaginary parts of the two-photon amplitudes have not been considered: experiments measuring $|f|^2$ and $\text{Re}[f\bar{f}']$ terms are not sensitive to these imaginary parts (with neglect of terms $O(\alpha^4)$). The only experiment giving a result of lowest order α^3 , is the asymmetry A: this is directly proportional to the imaginary parts of the two-photon amplitudes.

$$I_0 A = \text{Im}[f_L \bar{f}_X] + \text{Im}[f_U \bar{f}_X] \quad (2.57)$$

Expanding to $O(\alpha^3)$:

$$I_0 A = \alpha^3 \left\{ f_X^{(1)} [\text{Im}f_L^{(2)} + \text{Im}f_U^{(2)}] - \text{Im}f_X^{(2)} [f_L^{(1)} + f_U^{(1)}] \right\} \quad (2.58)$$

Again a null result for A does not necessarily imply negligible two-photon amplitudes. For example, if :

$$\text{Im}f_X^{(2)} \sim 0$$

and

$$\text{Im}f_L^{(2)} \sim -\text{Im}f_U^{(2)}$$

then

$$A \sim 0.$$

Current experimental data is consistent with $A = 0$ [18].

In contrast to the situation with ΔI_0 , with the set of 'simple' experiments the three unknown contributions to A cannot be separated.

To summarize, using polarization tensors of rank less than or equal to two, the following six quantities can be uniquely determined up to an overall sign:

$$f_L^{(1)} \quad f_U^{(1)} \quad f_X^{(1)}$$

$$\text{Ref}_L^{(2)} \quad \text{Ref}_U^{(2)} \quad \text{Ref}_X^{(2)}$$

For this determination it was necessary to use both electron and positron data, and to make use of the fact that the one-photon amplitude changes sign for e^-p to e^+p . Only one of the simple experiments is sensitive to the imaginary parts of the two-photon amplitudes and the three contributions cannot be separated.

The question remaining concerns these three imaginary contributions: is it possible to separate these terms and obtain a unique determination with the use of higher-order tensors?

(10) Equivalent Experiments and Higher rank Tensors

The higher rank tensors involve 'three polarizations' and one involves four. However, just as the polarization-transfer tensor contained no new information because of conservation of the electron helicity, many of these higher order tensors are immediately equivalent to lower ones.

In experiments with polarized electron beams and involving a measurement of the longitudinal polarization of the scattered electron, the following summation is involved:

$$\sum_{\lambda_d \lambda'_d} \sum_{\lambda_b \lambda'_b} f_{\lambda_c \lambda_d : \lambda_a \lambda_b}^{(\sigma_z)} (\sigma_z)_{\lambda_b \lambda'_b} \bar{f}_{\lambda'_c \lambda'_d : \lambda'_a \lambda'_b}^{(\sigma_z)} (\sigma_z)_{\lambda'_d \lambda'_d}$$

with $f_{(\lambda)} = f_{(\lambda)} \delta_{\lambda_b \lambda_d}$, this reduces to the summation with no electron polarizations:

$$\sum_{\lambda_d \lambda'_d} \sum_{\lambda_b \lambda'_b} f_{\lambda_c \lambda_d : \lambda_a \lambda_b} \delta_{\lambda_b \lambda'_b} \bar{f}_{\lambda'_c \lambda'_d : \lambda'_a \lambda'_b} \delta_{\lambda'_d \lambda'_d}$$

For example:

$$\begin{aligned} I_{O D_{zz}}^{(e)} &= \frac{1}{4} \text{Tr} [f \sigma_z^b f^\dagger \sigma_z^d] \\ &= \frac{1}{4} \text{Tr} [f f^\dagger] = I_0 \end{aligned}$$

Thus in our approximation, experiments involving electron polarizations in both initial and final states give the same information as an experiment with no electron polarizations.

For the tensors (see Table 2.2) $T_{ijk\ell}^{(e)}$ and $P_{jk\ell}^{(e)}$ with $j = z_b$; $\ell = z_d$ we have:

$$T_{y_a z_b z_d}^{(e)} = P_{z_b y_c z_d}^{(e)} = A \quad (2.59)$$

The four-index tensor $T_{ijk\ell}$ is equivalent to the proton depolarization tensor $D_{ik}^{(p)}$.

Evidently any new relations must derive from experiments involving two proton polarizations and one electron polarization, i.e. $T_{ijk}^{(p)}$ and $P_{ik}^{(p)}$. The tensor $P_{ik}^{(p)}$ involves measuring the final correlation of polarizations after the scattering of unpolarized electrons by a polarized proton target. As observed for the polarization-transfer tensors, an experiment measuring the polarization of the final electron gives the same results in our approximation, as the corresponding experiment with a polarized beam. The tensors $T_{ijk}^{(p)}$ and $P_{ik}^{(p)}$ are therefore equivalent.

Consequently there is only one independent tensor $T_{ijk}^{(p)}$: its components are evaluated in the standard manner.

$$\rho_i = \frac{1}{2} (1_a + P_a \cdot \sigma_a) \otimes \frac{1}{2} (1_b + P_{zb} \sigma_{zb})$$

and $I_{ep} \rho_f = f \rho_i f^\dagger$

$$\begin{aligned} I_{ep} P_{kc} &= \text{Tr} [f \rho_i f^\dagger \sigma_k^c] \\ &= I_0 [P_{kc}^{(o)} + P_{ia} D_{ik}^{(p)} + P_{zb} K_{zk}^{(ep)}] \\ &\approx |f|^2 [P_{ia} P_{zb} T_{iazbkc}^{(p)}] \end{aligned} \quad (2.60)$$

The non-zero components are :

$$I_0 T_{x_a z_b y_c}^{(p)} = - I_0 T_{y_a z_b x_c}^{(p)} = \text{Im}[f_U \bar{f}_L] \quad (2.61)$$

and

$$I_0 T_{z_a z_b y_c}^{(p)} = I_0 T_{y_a z_b z_c}^{(p)} = \text{Im}[f_L \bar{f}_X] - \text{Im}[f_U \bar{f}_X] \quad (2.62)$$

In an expansion in powers of a , both these components are of order a^3 . Since

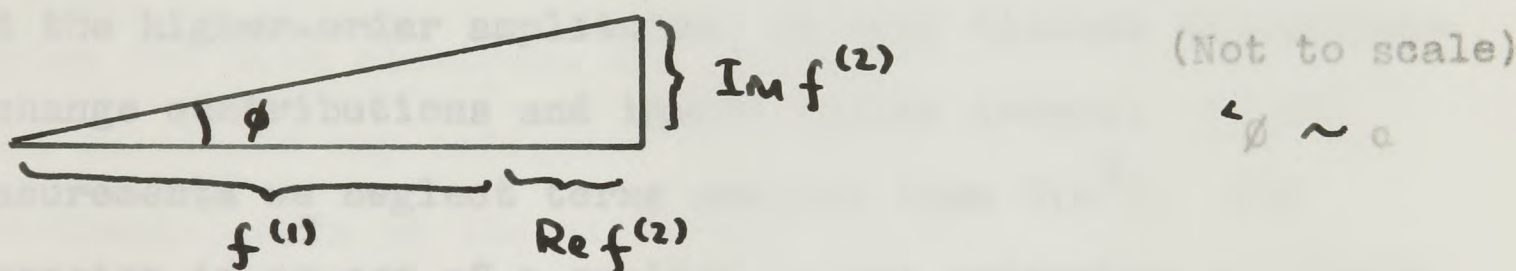
$$I_0 A = \text{Im}[f_L \bar{f}_X] + \text{Im}[f_U \bar{f}_X] \quad (2.29)$$

it is possible to isolate the three quantities:

$$\text{Im}[f_U \bar{f}_L], \quad \text{Im}[f_L \bar{f}_X], \quad \text{Im}[f_U \bar{f}_X] \quad (2.63)$$

Now if we consider each of the amplitudes as a vector in the complex f -plane:

Figure 2.9



$$f = |f| e^{i\phi}$$

$$\text{Im}[f_L \bar{f}_X] = |f_L| |f_X| \sin(\phi_L - \phi_X)$$

$$\approx |f_L| |f_X| (\phi_L - \phi_X)$$

Similarly for $\text{Im}[f_U \bar{f}_X]$ and $\text{Im}[f_U \bar{f}_L]$. Thus an overall phase cannot be determined, although this common phase must be of the order of a . If we take as our convention for the reconstruction:

$$\phi_X = 0 \quad \text{i.e.,} \quad \text{Im} f_X^{(2)} = 0 \quad (2.64)$$

then we obtain a unique solution for ϕ_L and ϕ_U , i.e., $\text{Im } f_L^{(2)}$ and $\text{Im } f_U^{(2)}$.

It is clear therefore, that measurement of the components of the third-rank polarization tensor could remove the possibility of cancellation occurring in the asymmetry A.

2.4 Conclusion

In the high-energy limit, there are three unknown complex amplitudes for electron-proton scattering. Because of the smallness of α , an expansion of these amplitudes in powers of α is reasonable. Physically this corresponds to the separation into the lowest-order one-photon-exchange amplitude and the higher-order amplitudes: we only discuss two-photon-exchange contributions and ignore higher orders. In all measurements we neglect terms smaller than $O(\alpha^3)$. The expansion in powers of α applied to the unitarity condition, gives the result, independent of energy and scattering angle, that the first Born amplitudes are real. There are therefore three real amplitudes of $O(\alpha)$ and three complex amplitudes of $O(\alpha^2)$ to be determined. Because of the smallness of α and the fact that the first Born amplitudes are real, a naive reconstruction programme from the 'simple' experiments, fails. However, we then notice that in changing from e^-p to e^+p scattering, the one-photon-exchange amplitude changes sign, but the two-photon-exchange does not. From the 'simple' experiments for both e^-p and e^+p scattering, it is possible to determine the three first Born amplitudes and the real parts of the three two-photon-exchange amplitudes,

up to an overall sign. The 'simple' experiments cannot separate the imaginary parts of the three two-photon-exchange terms. However, measurement of the one independent third-rank polarization tensor, allows the imaginary parts of the three complex amplitudes to be determined up to an overall phase. If we choose as a convention for the reconstruction:

$\text{Im } f_X = 0$; then the actual complex amplitudes can only differ from the solution so obtained by a common phase of the order of α .

Present experiments to detect two-photon-exchange effects do not form a complete set, and therefore the interpretation of the results of these experiments is not strictly unambiguous. Measurement of the second-rank polarization tensors for both electron- and positron-proton scattering, could eliminate any ambiguity in the interpretation of the experimental data on the difference between the e^- -p and e^+ -p differential cross sections, which is sensitive to the real parts of the two-photon exchange amplitudes. However, for a strictly unambiguous interpretation of the polarization asymmetry data, sensitive to the imaginary parts of the amplitudes, it would be necessary to measure components of the third-rank polarization tensors.

proton - for unpolarized particles in the initial state, is

This is perhaps not immediately obvious in terms of our \mathcal{C}^{\pm} helicity amplitudes of the previous chapter, which are proportional to linear combinations of \mathcal{C}_2 and \mathcal{C}_3 [Equations (2.55)]. However, it is easy to demonstrate, using the Lorentz transformation properties of the helicity amplitudes and the unitarity of the rotation matrices, that the unpolarized cross section contains only \mathcal{C}_2^2 and \mathcal{C}_3^2 . See for example the paper of Dardan, DeLellis and Marr [10].

calculated by the CHAPTER 3

proton current of section 2.1

written PROTON DEPOLARIZATION TENSOR IN THE ONE-PHOTON-

EXCHANGE APPROXIMATION

3.1 Introduction

In this chapter, effects of two-photon-exchange amplitudes are ignored and the one-photon-exchange approximation assumed throughout. As we have seen in section 2.1, the one-photon-exchange amplitudes can be expressed in terms of two structure factors G_E and G_M which, in the initial or final proton rest frames, and in the proton Breit-frame, are proportional to the helicity-flip and helicity-non-flip amplitudes respectively. In the cross section for the scattering of unpolarized electrons by unpolarized protons therefore, there will be no interference between these two amplitudes and the unpolarized differential cross section contains only G_E^2 and G_M^2 .^{*} In the one-photon-exchange approximation, it is simpler to use the covariant Dirac spinor formalism involving two form factors, rather than the helicity formalism of the previous chapter. The cross section in the laboratory system (LAB) - rest frame of the target proton - for unpolarized particles in the initial state, is

* This is the so-called Rosenbluth cross section. This is perhaps not immediately obvious in terms of our CM helicity amplitudes of the previous chapter, which are proportional to linear combinations of G_E and G_M (Equations (2.55)). However, it is easy to demonstrate, using the Lorentz transformation properties of the helicity amplitudes and the unitarity of the rotation matrices, that the unpolarized cross section contains only G_E^2 and G_M^2 . See for example the paper of Durand, DeCelles and Marr [10].

calculated by the standard trace techniques, using the proton current of section 2.1 (equation 2.3). It can be written in the form

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{N.S.} \frac{1}{(1+\tau)} \left[G_E^2 + \tau G_M^2 [1+2(1+\tau)\tan^2 \frac{\theta}{2}] \right] \quad (3.1)$$

with

$$\left(\frac{d\sigma}{d\Omega} \right)_{N.S.} = \frac{a^2}{4\omega^2 \sin^4 \theta/2} \cdot \cos^2 \theta/2 \cdot \left(\frac{\omega'}{\omega} \right)$$

ω, ω' are the initial and final electron energies, and θ is the electron scattering angle in the LAB.

$$q^2 = (k-k')^2 = (p'-p)^2 = -Q^2 \quad (3.2)$$

$$\text{and } \tau = \frac{Q^2}{4M^2} \quad (3.3)$$

where k, k' and p, p' are the four vectors representing the initial and final momenta of the electron and proton, respectively. G_E and G_M are functions of the four-momentum-transfer q^2 .

The electron mass has been neglected in the cross section (3.1).

This is the so-called Rosenbluth cross section. The present notation differs from the original presentation [21] where the cross section was written in terms of the 'effective charge' and 'effective anomalous magnetic moment' of the proton. This is equivalent to the use of the Dirac

and Pauli form factors, F_1 and F_2 respectively. The proton current in terms of these form factors is, in our notation

$$\Gamma^\mu(q^2) = F_1(q^2)\gamma^\mu + \frac{1}{2M} \sigma^{\mu\nu} q_\nu K F_2(q^2) \quad (3.4)$$

K is the anomalous magnetic moment of the proton in nuclear magnetons. ($K = 1.79$) and

$$F_1(0) = 1 \quad F_2(0) = 1$$

F_1 and F_2 are related to G_E and G_M by

$$G_E = F_1 - \tau K F_2 \quad (3.5a)$$

$$G_M = F_1 + K F_2 \quad (3.5b)$$

and $G_E(0) = 1$ $G_M(0) = 1 + K = \mu_p$.

The motivation for the use of G_E and G_M in preference to F_1 and F_2 is discussed in the review article by Hand, Miller and Wilson [22] where references to earlier work are given.

In the unpolarized cross section we see explicitly from (3.1) that only G_E^2 and G_M^2 can be measured. However, in terms of high-energy helicity analysis of the last chapter, it was seen that all three one-photon-exchange amplitudes could be determined up to an overall sign. In the language of G_E and G_M , it should therefore be possible to measure the relative sign of these two form factors. Experiments sensitive to the interference between electric and magnetic scattering could measure cross terms in $G_E G_M$. Such

measurements would not only establish the relative sign of G_E and G_M but would also be useful for determining the value of G_E at high momentum transfers, where its value is poorly known because of the dominance of magnetic scattering [12]. They could also provide a better determination of the neutron electric form factor which is reported to be close to zero [12]. Previous calculations [23] of the various polarization tensors in terms of F_1 and F_2 , show that a measurement of a cross-term is possible in principle. All spin correlations in elastic electron-nucleon scattering have been calculated by Scofield [24], but it is not emphasized, and not immediately evident from his expression, what experiments are best able to measure cross terms in $G_E G_M$. Dombey [25] has suggested that such a term could be measured in the scattering of polarized lepton beams by polarized nucleons. In this chapter, it is shown that the same information can be obtained from scattering of unpolarized electrons by polarized nucleons, if the polarization of the recoil nucleon is also measured.* Such an experiment should now be feasible, since it does not require the use of polarized electron beams, and polarized proton targets are now available. We therefore discuss the question of the most favourable experimental configuration for the determination of the parameter $\xi = \mu_p G_E/G_M$.

* An account of this work, carried out in collaboration with Dr. P.K. Kabir, has been published [26]. In the proof stage of this paper, we discovered that a closely related discussion had been given previously by Akhiezer and Rekalov [27]. Following the suggestion of Kabir, Dombey has now also considered the same experiment [28].

3.2 Proton Depolarization Tensor

The proton depolarization tensor may be calculated by a straightforward but tedious trace calculation, using the usual covariant spin projection operators. The component of the recoil-nucleon polarization along the direction represented by the four-vector r (chosen to be a spatial unit vector in the rest frame of the recoil nucleon) is given, in the one-photon-exchange approximation, by

$$P_r(j) = \frac{L^{\mu\nu} \text{Tr}[\gamma_5 \not{r}' \Lambda(p') \Gamma_\mu \gamma_5 \not{r} \Lambda(p) \Gamma_\nu]}{L^{\mu\nu} \text{Tr}[\Lambda(p') \Gamma_\mu \Lambda(p) \Gamma_\nu]} \quad (3.6)$$

where

$$L^{\mu\nu} = \text{Tr}[\Lambda(k') \gamma^\mu \Lambda(k) \gamma^\nu]$$

and $\Lambda(p) = (\not{p} + M)/2M$, etc. are the usual positive-energy projection operators. The four-vector j is the covariant target-nucleon polarization, which, in the target rest-frame, has a zero time component and space components equal to the components of the target-nucleon polarization. The result of the calculation may be written

$$P_r(j) = \frac{\mathbb{I}_1 G_E^2 + \mathbb{I}_2 G_E G_M + \mathbb{I}_3 \tau G_M^2}{A G_E^2 + \tau G_M^2 [A + 8M^4 \tau (1 + \tau)]} \quad (3.7a)$$

The functions \mathbb{I}_1 , \mathbb{I}_2 and \mathbb{I}_3 are defined as

$$\mathbb{I}_1 = A \left\{ \frac{1}{2} (p \cdot r)(p' \cdot j) [M^2 (1 + \tau)]^{-1} - (r \cdot j) \right\} \quad (3.7b)$$

$$I_2 = \left\{ [(s-M^2) - 2M^2\tau] \left[(p \cdot r)(k \cdot j) + (p' \cdot j)(k' \cdot r) + (p \cdot r)(k' \cdot j) + (p' \cdot j)(k \cdot r) \right] - (4M^2\tau + A[M^2(1+\tau)]^{-1})(p \cdot r)(p' \cdot j) \right\} \quad (3.7c)$$

$$I_3 = \left\{ (s+M^2) \left[(p \cdot r)(k \cdot j) + (p' \cdot j)(k' \cdot r) \right] + \left[(s-M^2) - 2M^2(1+2\tau) \right] \left[(p \cdot r)(k' \cdot j) + (p' \cdot j)(k \cdot r) \right] - A(r \cdot j) - 4M^2(1+\tau) \left[(k' \cdot r)(k \cdot j) + (k' \cdot j)(k \cdot r) \right] + \left(\frac{1}{2} A [M^2\tau(1+\tau)]^{-1} - 4M^2\tau \right) (p \cdot r)(p' \cdot j) \right\} \quad (3.7d)$$

where target polarization is the scattering plane, in which case $A = [(s-M^2)^2 - 4M^2\tau s]$ must also lie in the plane. If we choose the z-axis along the recoil direction, (this is an obvious choice in the center of mass frame), and the y-axis is the direction of the electron spin, and the z-component of the electron spin is positive, the same

and $p, p',$ and k, k' are the four-vectors representing the initial and final momenta of the nucleon and electron respectively. The electron mass has been neglected and the nucleon electromagnetic vertex taken as

$$\langle p' | J_\mu | p \rangle = \frac{\bar{u}(p')}{(1+\tau)} \left\{ (G_E + \tau G_M) \gamma_\mu + \frac{1}{4} (G_E - G_M) [\gamma_\mu, \gamma_\nu] \frac{(p' - p)^\nu}{M} \right\} u(p) \quad (3.8)$$

where G_E and G_M are functions of τ . (Equation (3.8) is equivalent to equation (2.3)). We use the metric $(a \cdot b) \equiv a_0 b_0 - \underline{a} \cdot \underline{b}$, and γ matrices obeying $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$.

As a consequence of our one-photon-exchange approximation, $P_R(j)$ is proportional to the initial target polarization j ; the general result can be stated in terms of the recoil-nucleon polarizations resulting from scattering off targets polarized along the principal axes. Reflection invariance requires that polarization components in the scattering plane, cannot affect polarization components normal to the plane [26]. Moreover the normal component of the tensor does not involve cross terms in $G_E G_M$: this is evident from (3.7c). Therefore, if one's main interest is in such cross terms, it suffices to consider target polarizations in the scattering plane, in which case the recoil polarization must also lie in the plane. If we choose the z-axis along the recoil direction, (this is an obvious choice in the proton Breit frame), and the x-axis to be in the scattering plane (such that the x-component of the electron momentum is positive), the parameters of interest are

$$P_x(\hat{z}) = -P_z(\hat{x}) = 2Du(1+u^2)^{\frac{1}{2}} \tau^{\frac{1}{2}} G_E G_M \quad (3.9a)$$

$$P_x(\hat{x}) = Du^2 (G_E^2 - \tau G_M^2) \quad (3.9b)$$

$$P_z(\hat{z}) = D [u^2 G_E^2 - (2+u^2) \tau G_M^2] \quad (3.9c)$$

where

$$D^{-1} = u^2 G_E^2 + (2+u^2) \tau G_M^2$$

and

$$u^2 = \frac{1}{4} A [M^4 \tau (1+\tau)]^{-1}$$

while $P_x(\theta) = -P_x(\pi - \theta)$.

$$P_x(\hat{y}) = P_y(\hat{x}) = P_y(\hat{z}) = P_z(\hat{y}) = 0 \quad (3.9d)$$

of the effect to be expected is illustrated in the figures. It will be seen that the cross section for scattering of unpolarized electrons is independent of the nucleon polarization, which is given by the Rosenbluth formula [21] (3.1). In our present notation, this formula can be written in covariant form

$$\frac{d\sigma}{d\tau} = 4\pi\alpha^2 M^2 \tau^{-1} (s-M^2)^{-2} D^{-1}$$

3.3 Discussion

Suppose that the target polarization lies in the scattering plane and makes an angle θ (taken as positive if it is in the same sense as the angle of scattering of the electron) with the recoil direction, measured in the laboratory (target rest frame). The recoil polarization will, in general, be reduced and point in some other direction in the scattering plane. Present methods of detecting recoil polarization, using a second scattering, are sensitive only to the transverse polarization, i.e., to P_x in our case. This is given by

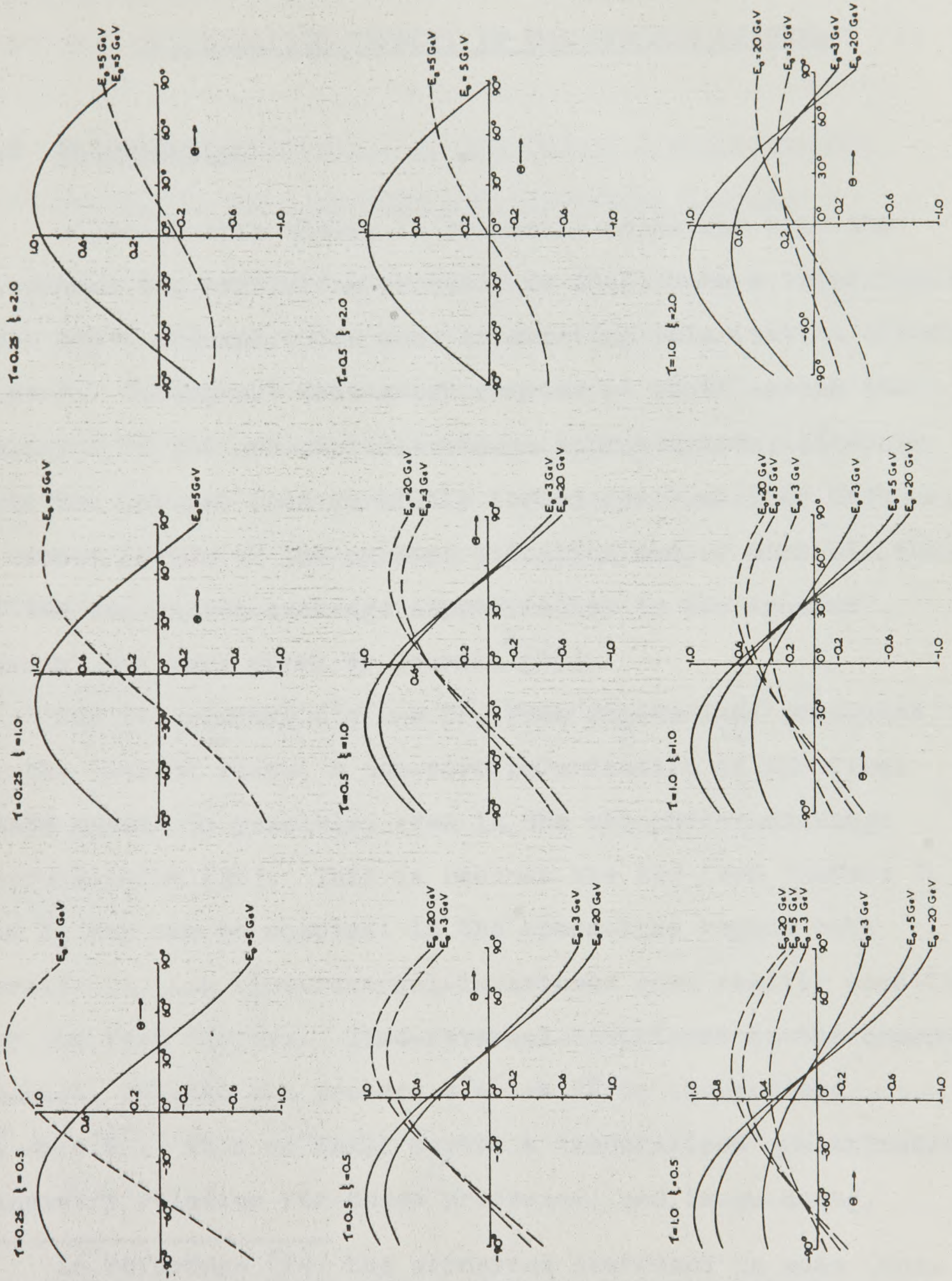
$$P_x(\theta) = P_x(\hat{z})\cos\theta + P_z(\hat{x})\sin\theta \quad (3.10)$$

Equations (3.10), (3.9a) and (3.9b) require that P_x be exactly reversed when θ and $\xi = G_M(0)G_E(\tau)/G_M(\tau)$ are both

reversed. Note also that $P_x(\pi+\theta) = -P_x(\theta)$. The nature of the effect to be expected is illustrated in the graphs of $P_x(\theta)$, which is plotted in Fig. 3.1 for several different values of ξ in the range of interest, with $\tau = 0.25, 0.5$ and 1.0. $P_x(\theta)$ for negative ξ can be obtained by inverting the figures. It will be seen that the transverse recoil polarization can attain values almost equal to the initial target polarization. By reversing the target polarization and measuring the change in the up-down asymmetry (with respect to the electron scattering plane) in the second scattering, the effect can be doubled. The direction of the target polarization for maximum transverse recoil polarization is not, however, the configuration for maximum sensitivity to ξ ; in fact, it is close to the direction for least sensitivity. $dP_x/d\xi$ has also been shown for comparison. Note that

$$\frac{dP_x}{d\xi}(\theta, -\xi) = \frac{dP_x}{d\xi}(-\theta, \xi) \quad (3.11)$$

Figure 3.1



Full lines represent transverse recoil polarization as a function of the target polarization angle θ . For incomplete initial target polarization P_0 , the ordinate is P_x/P_0 . The broken lines represent, on an arbitrary scale, the values of $dP_x/d\xi$. The curves for negative ξ are given by the relation $P_x(\theta, -\xi) = -P_x(\theta, \xi)$ and by Eq. (5). Inversion of the target polarization reverses the recoil polarization: $P_x(\pi + \theta, \xi) = -P_x(\theta, \xi)$. For $\tau = 0.25$, the values of P_x and $dP_x/d\xi$ for $E_0 = 3$ and 20 GeV lie within 5% of the 5-GeV curves over the entire range of θ . For $\tau = 0.5$, the curve for $E_0 = 5 \text{ GeV}$ lies between the corresponding curves for 3 and 20 GeV . E_0 is the energy of the electron in the target rest frame.

display in some detail. CHAPTER 4 Restrictions on the electromagnetic vertex both in the space-like and time-like regions. POLARIZATION EFFECTS IN THE CROSSED CHANNEL

4.1 Introduction

Restrictions on the Proton Electromagnetic Vertex in the Space-like and Time-like Regions

Although this thesis is primarily concerned with the s-channel e-p scattering process, we shall make a brief sortie into the t-channel where some interesting polarization effects appear. Throughout this short chapter we shall assume the validity of the one-photon-exchange approximation; although this has not yet been carefully tested for time-like photons. A recent review of the current situation and of possible tests of the one-photon-exchange approximation in the crossed channel has been given by Gourdin^{*} [29].

For the process $e^+e^- \rightarrow P\bar{P}$, from unpolarized particles in the initial state, a non-zero polarization of the final state proton is possible, even in the one-photon-exchange approximation [30]. This is because the two form factors F_1 and F_2 may now be complex: in the space-like region both hermiticity and time-reversal-invariance gave reality conditions for the form factors. Time-reversal-invariance in the crossed channel, relates the process $e^+e^- \rightarrow P\bar{P}$ to the process $P\bar{P} \rightarrow e^+e^-$. Thus we shall derive a 'generalized' polarization-asymmetry relation for these processes, and in so doing,

* In reference [29] the erroneous statement is made (One-photon-exchange Approximation, Section VII.4) that the processes, virtual $\gamma \rightarrow \rho^0\rho^0, \omega\omega, \phi\phi$ are forbidden by invariance, in contradiction with the explicit counter-examples of references [36],[37]. 36P-

display in some detail the symmetry restrictions on the electromagnetic vertex both in the space-like and time-like regions.

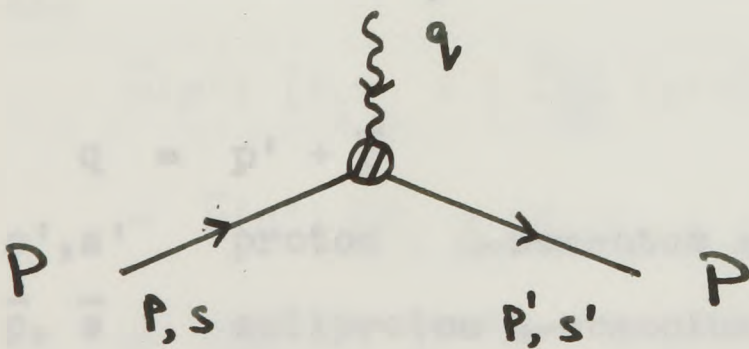
4.2 Symmetry Restrictions on the Proton Electromagnetic Vertex in the time-like and space-like Regions

In both the space-like and time-like regions, Lorentz-invariance and current conservation, together with the fact that the proton spinors satisfy the Dirac equation, allow us to represent the general structure of the proton electromagnetic vertex in terms of two form factors. In the space-like region (see Fig. 4.1) we write (omitting a normalization factor)

$$\langle p's' | J_\mu^\gamma(0) | ps \rangle = \bar{u}(p's') [F_1(q^2)\gamma_\mu + i \frac{\sigma_{\mu\nu}}{2M} q^\nu k F_2(q^2)] u(ps) \quad (4.1)$$

where $q = p' - p$.

Figure 4.1



Proton electromagnetic vertex in the space-like region

Hermiticity of the electromagnetic current J_μ gives the condition

$$\langle a | J_\mu | b \rangle = \langle b | J_\mu | a \rangle^* \quad (4.2)$$

This leads to the result that both F_1 and F_2 are real functions of q^2 .

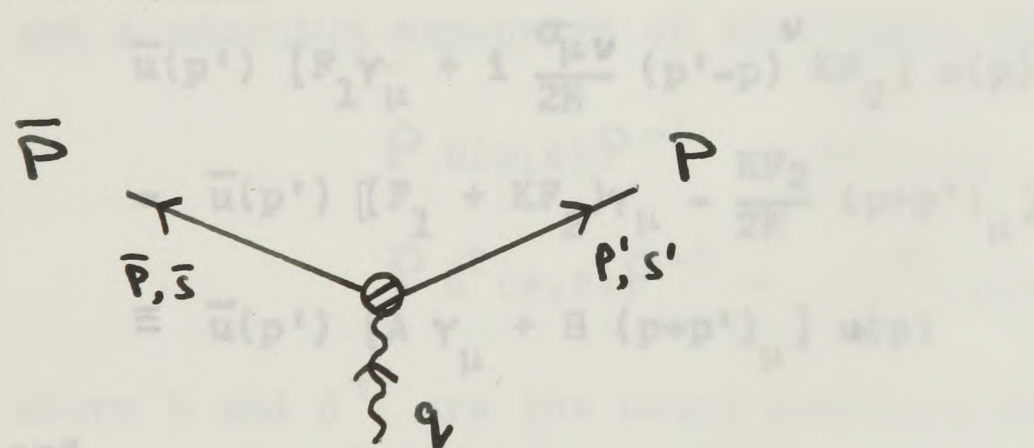
So far, F_1 and F_2 are defined only for negative values of q^2 (space-like). The definitions of F_1 and F_2 can be extended to positive - i.e., time-like - values of q^2 by writing [31]

$$\langle \bar{p}\bar{s}; p's' |_{out} J^\gamma_\mu(0) | 0 \rangle = \bar{u}(p's') [F_1(q^2)\gamma_\mu + i \frac{\sigma_{\mu\nu}}{2M} q^\nu F_2(q^2)] v(\bar{p}\bar{s}) \quad (4.3)$$

where $\langle \bar{p}; p' |_{out} J_\mu | 0 \rangle$ is the amplitude to create a

proton-antiproton pair from a virtual photon (see Fig. 4.2)

(4.1) and (4.3) using the Dirac equation and we obtain



Proton electro-magnetic vertex in the time-like region

$$q = p' + \bar{p}$$

p', s' proton 4-momentum and spin

\bar{p}, \bar{s} antiproton 4-momentum and spin

$\langle \bar{p}\bar{s}; p's' |_{out}$ is an 'Out-state' [32] which gives

a simple description of the physical system at $t \rightarrow \infty$ (i.e., it describes the situation in the final state of a scattering process. Similarly the state $|\bar{p}\bar{s}; p's' \rangle_{in}$ describes the

$\bar{P}P$ system at $t \rightarrow -\infty$, before the scattering process. For interacting fields, we must distinguish between the two-particle state at $t = -\infty$, and the state at $t = +\infty$: the difference between $| \rangle_{in}$ and $| \rangle_{out}$ reflects the possibility that interacting particles can collide and scatter. Only for one-particle states can this distinction be ignored:

$$| p s \rangle_{in} = | p s \rangle_{out}$$

Equation (4.3) defines F_1 and F_2 for $q^2 \gg 4M^2$: it is possible to show that this definition of F_1 and F_2 in the time-like region is the analytic continuation of the form factors defined in Eq. (4.1), treated as a function of the complex variable q^2 [31].

For our purposes, it is convenient to rewrite equations (4.1) and (4.3) using the Dirac equation and we obtain

$$\begin{aligned} & \bar{u}(p') \left[F_1 \gamma_\mu + i \frac{\sigma_{\mu\nu}}{2M} (p'-p)_\nu K F_2 \right] u(p) \\ &= \bar{u}(p') \left[(F_1 + K F_2) \gamma_\mu - \frac{K F_2}{2M} (p+p')_\mu \right] u(p) \\ &\equiv \bar{u}(p') \left[A \gamma_\mu + B (p+p')_\mu \right] u(p) \end{aligned} \quad (4.1')$$

where b and d^\dagger are the usual particle annihilation and antiparticle creation operators, respectively [32].

$$\begin{aligned} & \bar{u}(p') \left[F_1 \gamma_\mu + i \frac{\sigma_{\mu\nu}}{2M} (p'+\bar{p})_\nu K F_2 \right] v(\bar{p}) \\ &= \bar{u}(p') \left[(F_1 + K F_2) \gamma_\mu - \frac{K F_2}{2M} (p'-\bar{p})_\mu \right] v(\bar{p}) \\ &\equiv \bar{u}(p') \left[A^{out} \gamma_\mu + B^{out} (p'-\bar{p})_\mu \right] v(\bar{p}) \end{aligned} \quad (4.3')$$

We have written a superscript 'out' for A and B in Eq. (4.3'), to emphasize that these form factors are defined with respect to the out-states. Note that the current matrix element in

* The transformation $p \rightarrow p - p$ in the argument of F_1 and F_2 applies only to the space components.

the time-like region is obtained from that in the space-like region by the replacement: $p \rightarrow -\bar{p}$ and $u(p) \rightarrow v(\bar{p})$.

We shall now consider the various symmetry restrictions on the current matrix elements [9].

(1) Space-like Region

Parity \mathcal{P}

For Dirac fields [32] :

$$\mathcal{P} \psi(\underline{x}, t) \mathcal{P}^{-1} = \gamma_0 \psi(-\underline{x}, t) \quad (4.4a)$$

Using: the Dirac field

$$\begin{aligned} \gamma_0 u(-p, s) &= u(p, s) \\ \gamma_0 v(-p, s) &= -v(p, s) \end{aligned} \quad (4.4b)$$

and a momentum expansion of the field, we find^{*}

$$\mathcal{P} b(p, s) \mathcal{P}^{-1} = b(-p, s) \quad (4.4c)$$

$$\mathcal{P} d^\dagger(p, s) \mathcal{P}^{-1} = -d^\dagger(-p, s)$$

where b and d^\dagger are the usual particle annihilation, and antiparticle creation operators respectively [32].

For a one-particle state: we find

$$|p, s\rangle = b^\dagger(p, s) |0\rangle$$

we see that :

$$\mathcal{P} |p, s\rangle = | -p, s\rangle \quad (4.4d)$$

where we have chosen the intrinsic parity to be +1.

For the current matrix element since

$$\mathcal{P} J_\mu(\underline{x}, t) \mathcal{P}^{-1} = J^\mu(-\underline{x}, t) \quad (4.5)$$

* The transformation $p \rightarrow -p$ in the arguments of b and d^\dagger , applies only to the space components.

we find

$$\langle p's' | J_{\mu}(0) | ps \rangle = \langle -p's' | J^{\mu}(0) | -ps \rangle \quad (4.6)$$

This condition is satisfied by our expression

$$\bar{u}(p') \Gamma_{\mu}(p',p) u(p) \quad \text{Eq. (4.1')}.$$

Charge Conjugation \mathcal{C}

For the electromagnetic current we have

$$\mathcal{C} J_{\mu}(x) \mathcal{C}^{-1} = -J_{\mu}(x) \quad (4.7)$$

and for the Dirac field

$$\mathcal{C} \psi_{\alpha}(x) \mathcal{C}^{-1} = C_{\alpha\beta} \bar{\psi}_{\beta}(x) \quad (4.8a)$$

where C is a 4×4 matrix such that

$$C \gamma_{\mu} C^{-1} = -\gamma_{\mu}^T$$

In our representation (see Appendix 2) with only γ_2 imaginary:

$$C = i\gamma_2 \gamma^0 = -C^{-1} = -C^{\dagger} = -C^T$$

On the 4-component spinors we find

$$(C\gamma^0)_{\beta\alpha} u_{\beta}^{\dagger}(p,s) = v_{\alpha}(p,s) e^{i\phi(p,s)} \quad (4.8b)$$

$$(C\gamma^0)_{\alpha\beta} v_{\beta}^{\dagger}(p,s) = u_{\alpha}(p,s) e^{i\phi(p,s)}$$

where $\phi(p,s)$ is a phase factor. A momentum expansion of

ψ yields

where

$$\mathcal{C} d(p,s) \mathcal{C}^{-1} = d(p,s) e^{i\phi(p,s)} \quad (4.8c)$$

$$\mathcal{C} d^\dagger(p,s) \mathcal{C}^{-1} = b^\dagger(p,s) e^{i\phi(p,s)}$$

Thus a one-particle state is transformed by \mathcal{C} into an antiparticle state (as it should be!)

$$\mathcal{C} |P(p,s)\rangle = e^{i\phi(p,s)} |\bar{P}(p,s)\rangle \equiv \sum_s |\bar{P}(p,s)\rangle \quad (4.8d)$$

where $|P(p,s)\rangle$ represents a proton state with momentum p spin s , and $|\bar{P}(p,s)\rangle$ an antiproton state with the same momentum and spin. For the current matrix element, the charge conjugation operation therefore relates the $(P \gamma P)$ vertex to the $(\bar{P} \gamma \bar{P})$ vertex:

$$\langle P(p's') | J_\mu(0) | P(ps) \rangle = - \sum_{s'}^* \sum_s \langle \bar{P}(p's') | J_\mu(0) | \bar{P}(ps) \rangle \quad (4.9)$$

where the u 's are phase factors. If we write $\mathcal{J} = \mathcal{U} \mathcal{J} \mathcal{U}^{-1}$,

where \mathcal{U} is a unitary operator, and \mathcal{J} represents the Time-Reversal \mathcal{J}

instruction to take the complex conjugate of all c -numbers.

The electromagnetic current transforms under time-reversal as follows:

$$\mathcal{U} d^\dagger(p,s) \mathcal{U} \mathcal{J} J_\mu(\underline{x},t) \mathcal{J}^{-1} = J^\mu(\underline{x},t) \quad (4.10)$$

For a one-particle state therefore. Therefore, from the antiunitary nature of the time-reversal operation we obtain:

$$\begin{aligned} \langle p's' | J_\mu(0) | ps \rangle &= \langle \tilde{p} s | J^\mu(0) | \tilde{p}' s' \rangle \\ \text{The restriction } \mu &\text{ given by time-reversal invariance of the} \\ \text{current matrix element then} &= \langle \tilde{p}' s' | J^\mu(0) | \tilde{p} s \rangle^* \end{aligned} \quad (4.11)$$

where

$$|\tilde{p} s\rangle = \gamma |p s\rangle \quad (4.12')$$

The Dirac field transforms under time-reversal as

$$\gamma \psi_{\alpha}(\underline{x}, t) \gamma^{-1} = t_{\alpha\beta} \psi_{\beta}(\underline{x}, t) \quad (4.12a)$$

where the 4×4 matrix t has the properties:

$$t^T = t; \quad t^{\dagger} = t^{-1}; \quad t^{-1} \gamma_{\mu}^T t = \gamma_{\mu}$$

In our representation:

$$t = i\gamma^1 \gamma^3 = t^{\dagger} = t^{-1} = -t^*$$

We have also:

$$\begin{aligned} tu(p, s) &= u^*(-p, -s) e^{i\alpha_+(p, s)} \\ tv(p, s) &= v^*(-p, -s) e^{i\alpha_-(p, s)} \end{aligned} \quad (4.12b)$$

where the α 's are phase factors. If we write $\gamma = uK$, where u is a unitary operator, and K represents the instruction to take the complex conjugate of all c -numbers:

$$\begin{aligned} ub(p, s)u^{-1} &= -b(-p, -s) e^{i\alpha_+(p, s)} \\ ud^{\dagger}(p, s)u^{-1} &= -d^{\dagger}(-p, -s) e^{i\alpha_-(p, s)} \end{aligned} \quad (4.12c)$$

For a one-particle state therefore

$$\langle \gamma | p s \rangle = -e^{-i\alpha_+(p, s)} | -p, -s \rangle \equiv -\gamma_s | -p, -s \rangle \quad (4.12d)$$

The restriction given by time-reversal-invariance on the current matrix element becomes:

$$\langle p's' | J_\mu(0) | ps \rangle = \gamma_{s'} \gamma_s^* \langle -p'-s' | J^\mu(0) | -p,-s \rangle^* \quad (4.11')$$

To obtain the restriction on $\Gamma_\mu(p',p)$ consider the space-components for example:

$$\bar{u}(p's') \Gamma(p',p) u(p,s) = - \gamma_{s'} \gamma_s^* [\bar{u}(-p',-s') \Gamma(-p',-p) u(-p,s)]^*$$

Using the relations in Eq. (4.12b) we find:

$$A\gamma + B(\underline{p} + \underline{p}') = A^*\gamma + B^*(\underline{p} + \underline{p}')$$

$$\text{i.e., } A = A^* \quad \text{and} \quad B = B^* \quad (4.13)$$

Since this result can be obtained from hermiticity alone, time-reversal-invariance gives no additional restrictions for the spin $\frac{1}{2}$ electromagnetic vertex.

(2) Time-like Region

We have treated the space-like case in some detail since we now wish to apply the symmetry conditions to the current matrix element in the time-like region (Eq. 4.3). We notice that the phase factors introduced in the \mathcal{C} and \mathcal{Y} operations do not appear in the final results and for simplicity we may omit them. The matrix element we now consider is:

$$\langle \bar{P}(\bar{p} \bar{s}) P(p's') | J_\mu(0) | 0 \rangle = \bar{u}(p') [A^{\text{out}} \gamma_\mu + B^{\text{out}}(p' - \bar{p})_\mu] v(\bar{p}) \quad (4.3)$$

$\Gamma_\mu^{\text{out}}(p',\bar{p})$ satisfies the requirements of \mathcal{C} -invariance as

* Weak interactions are, of course, ignored.

Parity P

We now need the parity transformation properties of the $|\bar{P}P\rangle_{\text{out}}$ state: since we assume that all the interactions conserve parity*, these are the same as those of the free field states. By the methods outlined before we find

$$P |\bar{P}(\bar{p}, \bar{s}); P(p', s')\rangle = -|\bar{P}(-\bar{p}, \bar{s}); P(-p', s')\rangle \quad (4.14)$$

The minus sign just reflects the property that a proton has the opposite parity to an antiproton in the same orbital state. Again we find the $\Gamma_{\mu}^{\text{out}}(p', \bar{p})$ has well-defined parity.

Charge Conjugation C

For a proton-antiproton state $|\bar{P}P\rangle_{\text{out}}$ formed from the vacuum by the appropriate creation operators, the ordering of the creation operators must be defined since d^{\dagger} and b^{\dagger} anticommute. We can again consider free fields and we define

$$|\bar{P}(\bar{p}, \bar{s}); P(p', s')\rangle = d^{\dagger}(\bar{p}, \bar{s}) b^{\dagger}(p', s') |0\rangle \quad (4.15)$$

Then

$$\begin{aligned} C |\bar{P}(\bar{p}, \bar{s}); P(p', s')\rangle &= b^{\dagger}(\bar{p}, \bar{s}) d^{\dagger}(p', s') |0\rangle \\ &= -|\bar{P}(p', s'); P(\bar{p}, \bar{s})\rangle \end{aligned} \quad (4.16)$$

(where we have omitted the phase factors). Charge conjugation on a $\bar{P}P$ state gives another $\bar{P}P$ state. We find, using the explicit properties of the spinors (Eq. (4.8b)), that $\Gamma_{\mu}^{\text{out}}(p', \bar{p})$ satisfies the requirements of C -invariance on

* Weak interactions are, of course, ignored.

the current matrix element.

Time Reversal and Hermiticity

In the time-like region Hermiticity fails to give a reality condition for the form factors:

$$\langle \bar{P}P |_{\text{out}} J_{\mu}(0) | 0 \rangle = \langle 0 | J_{\mu}(0) | \bar{P}P \rangle_{\text{out}}^* \quad (4.17)$$

The process $\langle \bar{P}P |_{\text{out}} J_{\mu} | 0 \rangle$ is different from the process $\langle 0 | J_{\mu} | \bar{P}P \rangle_{\text{out}}$.

Furthermore, time-reversal-invariance also gives no reality condition. This is because time-reversal changes an 'out-state' to an 'in-state' [33]

$$\begin{aligned} \langle \bar{P}P |_{\text{out}} J_{\mu} | 0 \rangle &= \langle \bar{P}P |_{\text{out}} \gamma^{-1} \gamma J_{\mu} \gamma^{-1} \gamma | 0 \rangle \\ &= \langle 0 | J^{\mu} | \tilde{\bar{P}P} \rangle_{\text{in}} \\ &= \langle \tilde{\bar{P}P} |_{\text{in}} J^{\mu} | 0 \rangle^* \end{aligned} \quad (4.18)$$

Again we may use free fields for $|\tilde{\bar{P}P}\rangle = \gamma |\bar{P}P\rangle$, and we find (omitting phase factors)

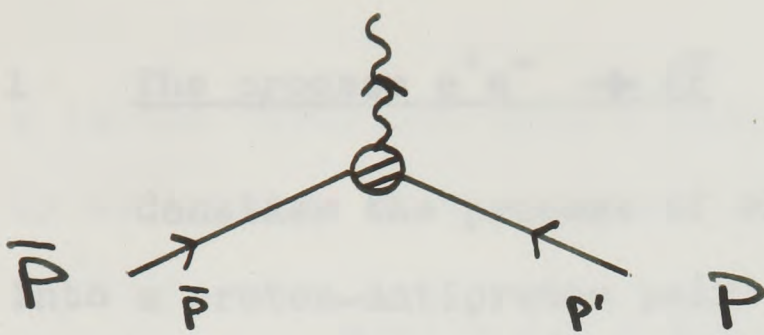
$$\gamma |\bar{p}\bar{s}; p's'\rangle = |-\bar{p}-\bar{s}; -p'-s'\rangle \quad (4.19)$$

Thus:

$$\langle \bar{p}\bar{s}; p's' |_{\text{out}} J_{\mu} | 0 \rangle = \langle 0 | J^{\mu} | -\bar{p}-\bar{s}; -p'-s' \rangle_{\text{in}} \quad (4.18')$$

unitary operations relate the process $\bar{P}P \rightarrow \gamma$ - see Fig. 4.3 - process $\gamma \rightarrow \bar{P}P$.
 However, if we consider the process $\bar{P}P \rightarrow \gamma$, - see Fig. 4.3 - this is described by an amplitude $\langle 0 | J_{\mu} | \bar{P}P \rangle_{\text{in}}$.

Figure 4.3

Vertex $\bar{P}P \rightarrow \gamma$

From Lorentz-invariance and current conservation as before we have:

$$\langle 0 | J_{\mu}(0) | \bar{p} \bar{s}; p' s' \rangle_{in} = \bar{v}(\bar{p} \bar{s}) [A^{\text{in}} \gamma_{\mu} + B^{\text{in}}(p' - \bar{p})_{\mu}] u(p' s') \quad (4.20)$$

where we have defined the form factors A^{in} and B^{in} . Using Eqs. (4.12b), we find

$$\begin{aligned} \langle 0 | \underline{J} | -\bar{p} - \bar{s}; -p' - s' \rangle_{in} &= \bar{v}(-\bar{p} - \bar{s}) [A^{\text{in}} \gamma_{\mu} - B^{\text{in}}(p' - \bar{p})_{\mu}] u(-p' - s') \\ &= -\bar{u}(p' s') [A^{\text{in}} \gamma_{\mu} + B^{\text{in}}(p' - \bar{p})_{\mu}] v(\bar{p} \bar{s}) \end{aligned} \quad (4.21)$$

Thus Eq. (4.18') gives immediately that

$$A^{\text{in}} = A^{\text{out}}; \quad B^{\text{in}} = B^{\text{out}} \quad (4.22)$$

We note here that since the matrix element is invariant under both \mathcal{C} and \mathcal{P} , the \mathcal{CP} condition must be the same as that from \mathcal{J} -invariance (Eq. (4.22)). Both these anti-unitary operations relate the process $\bar{P}P \rightarrow \gamma$ to the inverse process $\gamma \rightarrow \bar{P}P$.

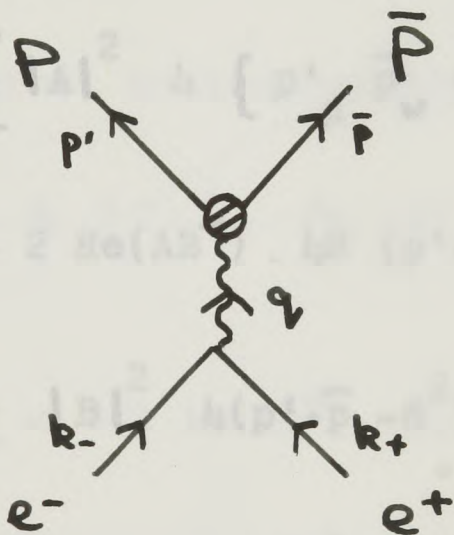
4.3 The Polarization-Asymmetry Relation in the Crossed Channel

1 The process $e^+e^- \rightarrow P\bar{P}$

Consider the process of electron-positron annihilation into a proton-antiproton pair: such a process will shortly be studied in electron-positron colliding beam experiments. In the lowest order perturbation theory (see Fig. 4.4) the amplitude for this process is given by

$$S_{fi} \sim i \frac{e e_p}{q^2} [\bar{v}(k_+) \gamma^\mu u(k_-)] [\bar{u}(p') \Gamma_\mu^{\text{out}} v(\bar{p})] \quad (4.23)$$

Figure 4.4



$$q = k_- + k_+ = p' + \bar{p} \quad (4.24)$$

We wish to calculate the polarization of the final-state proton from the annihilation of unpolarized particles. Parity invariance restricts this polarization to be normal to the reaction plane. We obtain

$$P_{\mu\nu} = \frac{L^{\mu\nu} \text{Tr}[\gamma_5 \not{p}' + M] \Gamma_\mu^{\text{out}}(\bar{p}-M) \bar{\Gamma}_\nu^{\text{out}}]}{L^{\mu\nu} D_{\mu\nu}^{\text{out}}} \quad (4.25)$$

we find:

with

$$L^{\mu\nu} = \text{Tr}[(\not{k}_+ - \mu) \gamma^\mu (\not{k}_- + \mu) \gamma^\nu]$$

and

$$D_{\mu\nu}^{\text{out}} = \text{Tr}[(\not{p}' + M) \Gamma_{\mu}^{\text{out}} (\not{\bar{p}} - M) \bar{\Gamma}_{\nu}^{\text{out}}] \equiv D_{\mu\nu}$$

s is the covariant spin 4-vector of the final-state proton.

By definition of the normal to the reaction plane we have

$$s \cdot p' = s \cdot \bar{p} = s \cdot k_+ = s \cdot k_- = 0$$

The lepton trace is easily evaluated:

$$L^{\mu\nu} = L^{\nu\mu} = 4 [k_+^{\mu} k_-^{\nu} + k_+^{\nu} k_-^{\mu} - g^{\mu\nu} q^2/2] \quad (4.26)$$

The proton trace in the denominator, which is proportional to the cross-section when contracted with $L^{\mu\nu}$, yields

$$D_{\mu\nu} = D_{\nu\mu} = \left\{ |A|^2 4 \left\{ p'_{\mu} \bar{p}_{\nu} + p'_{\nu} \bar{p}_{\mu} - g_{\mu\nu} q^2/2 \right\} \right. \\ \left. - 2 \text{Re}(AB^*) \cdot 4M (p' - \bar{p})_{\mu} (p' - \bar{p})_{\nu} \right. \\ \left. + |B|^2 4(p' \cdot \bar{p} - M^2) (p' - \bar{p})_{\mu} (p' - \bar{p})_{\nu} \right\} \quad (4.27)$$

Therefore, since (We have written : $A \equiv A^{\text{out}}$, $B \equiv B^{\text{out}}$)

The proton trace in the numerator is most easily evaluated using trace theorem 6 of Appendix 2.

Defining

$$N_{\mu\nu} = \text{Tr}[\gamma_5 \not{s} (\not{p}' + M) \Gamma_{\mu}^{\text{out}} (\not{\bar{p}} - M) \bar{\Gamma}_{\nu}^{\text{out}}] \quad (4.28)$$

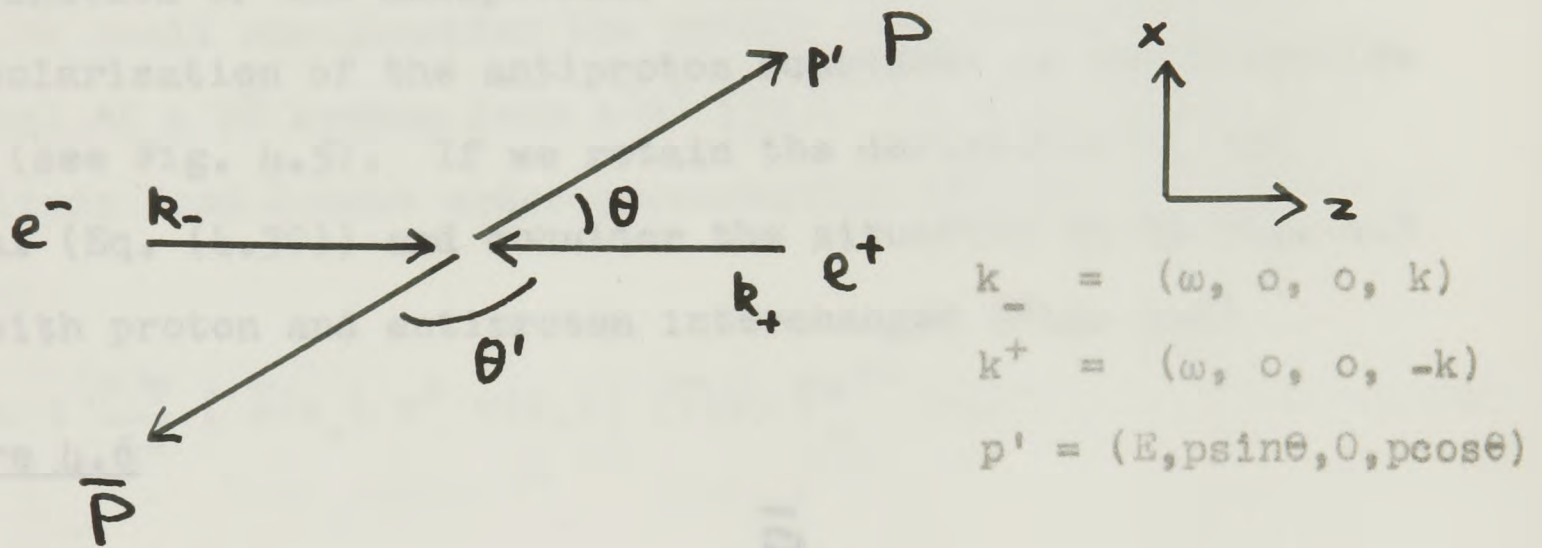
we find:

$$L^{\mu\nu} N_{\mu\nu} = 21 \text{Im}(AB^*) \cdot L^{\mu\nu} (p' - \bar{p})_{\nu} \text{Tr}[\gamma_5 \not{s} \not{p}' \gamma_{\mu} \not{\bar{p}}] \quad (4.29)$$

Thus, even in the one-photon-exchange approximation, there is in general, a polarization of the recoil proton in $e^+e^- \rightarrow P\bar{P}$ from unpolarized leptons [30].

Consider the process in the CMS (Fig. 4.5).

Figure 4.5



We define the normal to the reaction plane:

$$\underline{n} = \underline{k}_- \wedge \underline{p}' \quad (4.30)$$

Then:

$$L^{\mu\nu} N_{\mu\nu} = -2 \text{Im}(AB^*) \cdot 64 k^2 p^2 \omega \sin 2\theta \quad (4.31)$$

Therefore, since

$$\text{Im}(AB^*) = - \frac{K}{2M} \text{Im}[F_1 F_2^*]$$

and $I(\theta)$ is defined by^{*}:

$$I(\theta) = L^{\mu\nu} D_{\mu\nu}$$

the transverse polarization of the final-state proton is

$$I(\theta)P(\theta) = K \text{Im}[F_1 F_2^*] \cdot (64 k^2 p^2 \omega / M) \cdot \sin 2\theta$$

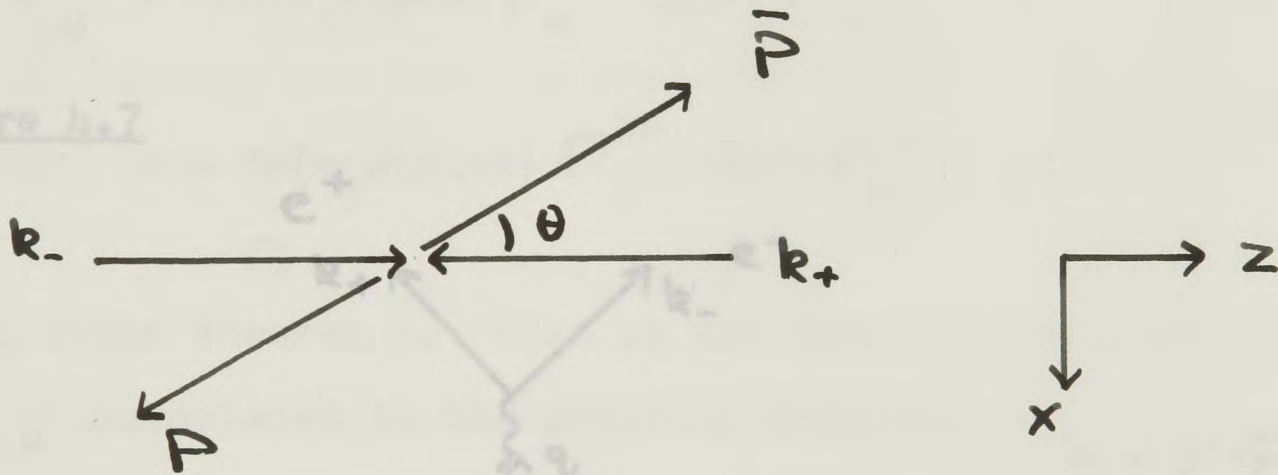
^{*} $I(\theta)$ is of the general form $I(\theta) = \dots$ (see reference [30]).
^{**} $I(\theta)$ is evaluated in the CMS in reference [30].

We note now the following trace property:

$$\text{Tr}[\gamma_5 \not{\epsilon} (\not{p}' + M) \Gamma_\mu (\not{p} - M) \bar{\Gamma}_\nu] = \text{Tr}[(\not{p}' + M) \Gamma_\mu \gamma_5 \not{\epsilon} (\not{p} - M) \bar{\Gamma}_\nu] \quad (4.33)$$

This second form is the trace product required for the polarization of the antiproton. Thus Eq. (4.32), gives also the polarization of the antiproton scattered in the direction θ (see Fig. 4.5). If we retain the definition of the amplitude from lowest order perturbation theory (see Fig. 4.5) and consider the situation as in Fig. 4.5 but with proton and antiproton interchanged (Fig. 4.6)

Figure 4.6



Clearly the scattering angle for the proton is $(\pi - \theta)$ and with these conventions, we have:

$$\bar{P}(\theta) = P(\pi - \theta) = -B(\theta) \quad (4.34)$$

the asymmetry term A in the differential cross section is where \bar{P} indicates the polarization of the antiproton. This relation follows from the form of the proton electromagnetic

* As a consequence of the one-photon-exchange approximation $I(\theta)$ is of the general form $C + D \cos^2 \theta$ and consequently $I(\theta) = I(\pi - \theta)$. (See, for example, Chapter 7 of the book by Gourdin in reference [17].)

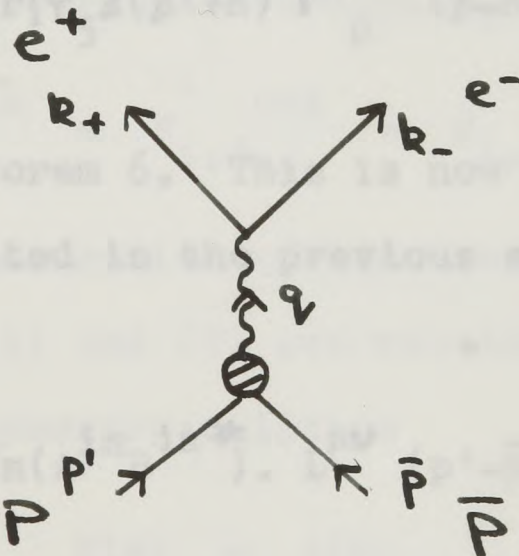
current. Cabibbo and Gatto [30] state that this is a result of γ_5 -invariance, but clearly this has not been used. γ_5 -invariance, like γ -invariance, connects $\gamma \rightarrow P\bar{P}$ to $P\bar{P} \rightarrow \gamma$ because of its antilinear nature.

2 The process $P\bar{P} \rightarrow e^+e^-$

We shall now consider the rather rare annihilation channel of a $P\bar{P}$ system into e^+e^- [34]. In this case, the amplitude from lowest order perturbation theory (see Fig. 4.7) is:

$$S_{fi} \sim i \frac{e e_p}{q^2} [\bar{u}(k_+) \gamma^\mu v(k_-)] [\bar{v}(\bar{p}) \Gamma_\mu^{\text{in}} u(p')] \quad (4.35)$$

Figure 4.7



For $P\bar{P}$ annihilation into e^+e^- from polarized protons the asymmetry term A in the differential cross section is given by

$$A = \frac{L^{\mu\nu} \text{Tr} [(\bar{p}-M) \Gamma_\mu^{\text{in}} \gamma_5 \not{\epsilon} (\not{p}'+M) \bar{\Gamma}_\nu^{\text{in}}]}{L^{\mu\nu} D_{\mu\nu}^{\text{in}}} \quad (4.36)$$

where

$$L^{\mu\nu} = \text{Tr}[(K_{-} + \mu) \gamma^{\mu} (K_{+} - \mu) \gamma^{\nu}]$$

and

$$D_{\mu\nu}^{\text{in}} = \text{Tr}[(\not{p} - M) \Gamma_{\mu}^{\text{in}} (\not{p}' + M) \bar{\Gamma}_{\nu}^{\text{in}}]$$

For the lepton trace we find $L^{\mu\nu} = L^{\mu\nu}$, and the proton trace $D_{\mu\nu}^{\text{in}}$ is of the same form as $D_{\mu\nu}$ in Eq. (4.27), except that A and B are replaced by A^{in} and B^{in} . Thus I^{in} is exactly the same as I except for 'in' labels for the form factors. we obtain

The numerator:

$$N_{\mu\nu}^{\text{in}} = \text{Tr}[\gamma_5 \not{p}' (\not{p}' + M) \bar{\Gamma}_{\nu}^{\text{in}} (\not{p} - M) \Gamma_{\mu}^{\text{in}}]$$

This is exactly the same as for $P(\theta)$ save for the

$$\text{However, } N_{\mu\nu}^{\text{in}} = -\text{Tr}[\gamma_5 \not{p}' (\not{p}' + M) \Gamma_{\mu}^{\text{in}} (\not{p} - M) \bar{\Gamma}_{\nu}^{\text{in}}] \quad (4.37)$$

using trace theorem 6. This is now exactly the same form as $N_{\mu\nu}$ calculated in the previous section.

Thus: processes (1) and (2) are related and we conclude

$$L^{\mu\nu} N_{\mu\nu}^{\text{in}} = -2i \text{Im}(A^{\text{in}} B^{\text{in}*}) \cdot L^{\mu\nu} (p' - p)_{\nu} \text{Tr}[\gamma_5 \not{p}' \gamma_{\mu} \not{p}] \quad (4.38)$$

$$P(\theta) = A(\theta)$$

If we now define the normal by

(with the definitions of the angles θ and the normal as

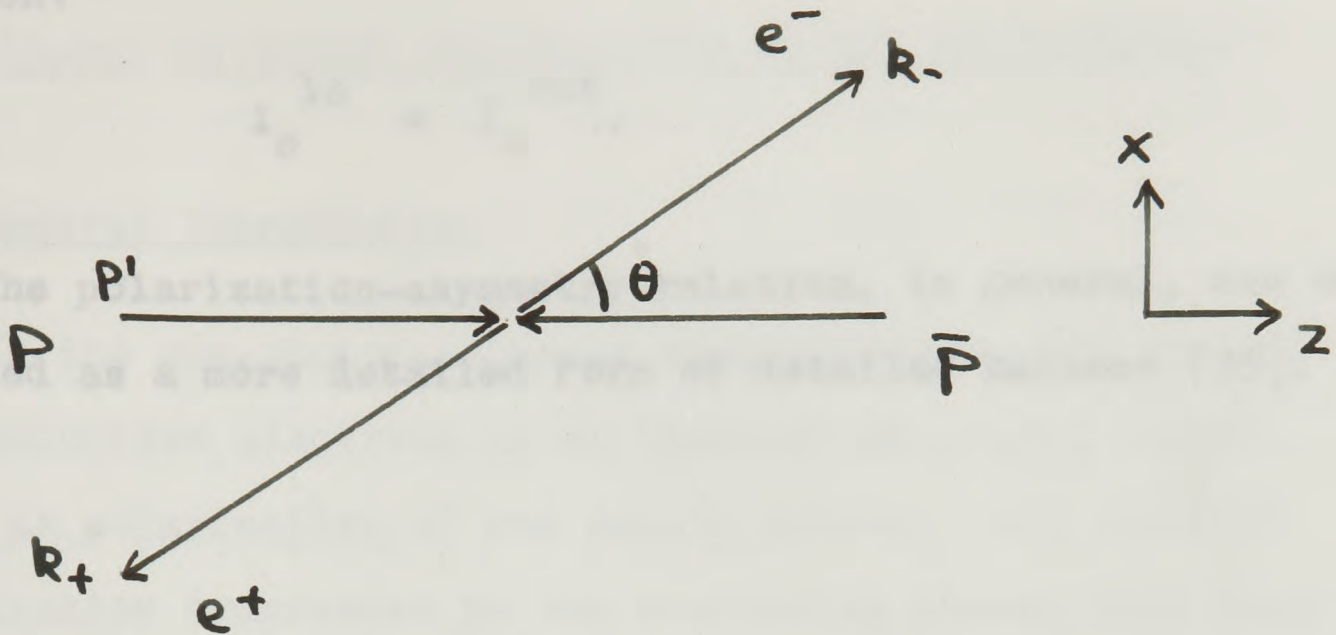
described above), $\underline{n} \propto \underline{p}' \wedge \underline{k}_{-}$ is relative with

imply a violation of time-reversal-invariance, and $\underline{p}' \cdot \underline{k}_{-} = pk \cos \theta$ in the CMS, (as shown in Fig. 4.8), this case since the vertex functions are GF -interactions, we then

would also necessitate a violation of $3P$ -invariance.

$$L^{\mu\nu} N_{\mu\nu}^{\text{in}} = -2i \text{Im}(A^{\text{in}} B^{\text{in}*}) \cdot 64 k^2 p^2 \omega \cdot \sin 2\theta \quad (4.40)$$

Figure 4.8 is given by the spin-averaged detailed balance relation:



Finally we obtain

$$I^{\text{in}}(\theta)A(\theta) = K \text{Im}[F_1^{\text{in}} F_2^{\text{in}*}] \cdot (64 k^2 p^2 \omega/M) \cdot \sin 2\theta \quad (4.41)$$

This is exactly the same as for $P(\theta)$ save for the 'in' labels. However, time-reversal-invariance gives the relation

$$F_{1,2}^{\text{in}} = F_{1,2}^{\text{out}} = F_{1,2} \quad (4.42)$$

Thus with the assumption of time-reversal-invariance the two processes (1) and (2) are related and we obtain a polarization-asymmetry relation

$$P(\theta) = A(\theta) \quad (4.43)$$

(with the definitions of the angles θ and the normals as described above). Violation of this relation would therefore imply a violation of time-reversal-invariance. However, in this case since the vertex functions are CP -invariant, this would also necessitate a violation of CP -invariance.

Needless to say, a more obvious test of time-reversal-

invariance is given by the spin-averaged detailed balance relation:

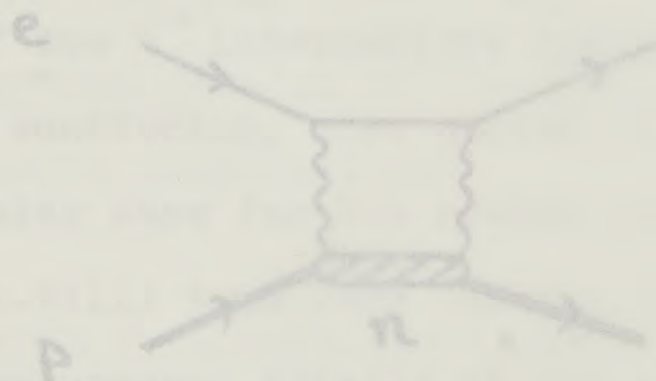
ELASTIC UNITARITY CALCULATIONS OF THE POLARIZATION

$$I_0^{\text{in}} = I_0^{\text{out}}$$

5.1 General Introduction

The polarization-asymmetry relation, in general, can be regarded as a more detailed form of detailed balance [35]. The scattering of unpolarized electrons by an unpolarized proton target gives no polarization of the recoil proton. Any non-zero polarization transverse to the scattering plane, must arise from interference of higher order amplitudes with the one-photon amplitude. We are interested in the contribution from the two-photon-exchange amplitude shown in Fig. 5.1.

Figure 5.1

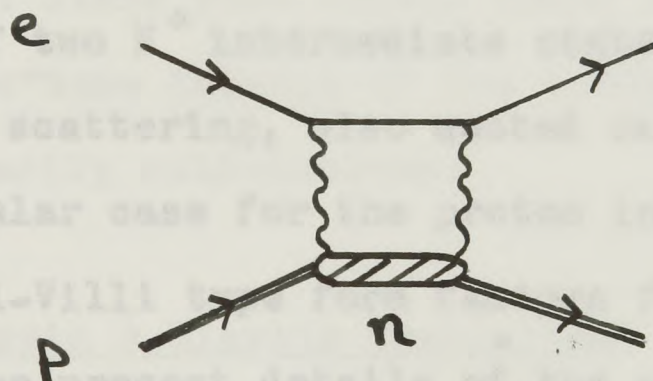


There will be such diagrams for each allowed intermediate hadron state p , N^* , N^{*+} etc. and each of the corresponding amplitudes contributes to the polarization. It is not possible at present, to calculate exactly the total contribution of these diagrams but there have been several interesting attempts to estimate the contribution from these hadronic intermediate states [36 - 43]. These calculations suggest that nuclear effects do not enhance the polarization asymmetry.

CHAPTER 5

ELASTIC UNITARITY CALCULATIONS OF THE POLARIZATION5.1 General Introduction

In the one-photon-exchange approximation, the scattering of unpolarized electrons by an unpolarized proton target gives no polarization of the recoil proton. Any non-zero polarization transverse to the scattering plane, must arise from interference of higher order amplitudes with the one-photon amplitude. We are interested in the contribution from the two-photon-exchange amplitude shown in Fig. 5.1.

Figure 5.1

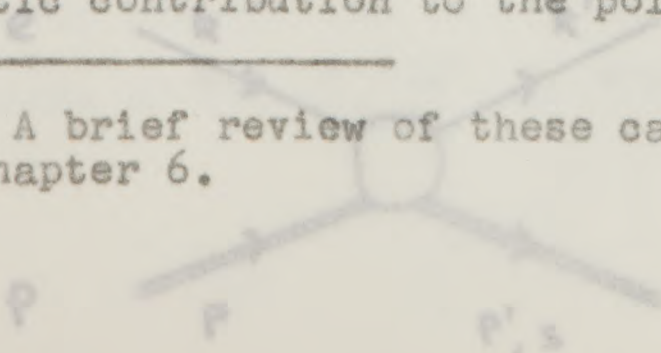
There will be such diagrams for each allowed intermediate hadron state p , N^* , N^{*w} etc. and each of the corresponding amplitudes contributes to the polarization. It is not possible at present, to calculate exactly the total contribution of these diagrams but there have been several theoretical attempts to estimate the contribution from these inelastic intermediate states [38 - 45]. These calculations suggest that hadronic effects do not enhance the polarization significantly above

its intrinsic order of magnitude α .^{*} Despite the smallness of the effect, it is of interest to calculate to lowest order in α , the contribution to the polarization from the elastic intermediate state, using the experimentally measured form factors, G_E and G_M , in the proton electromagnetic current. This will be the only contribution to the polarization at energies below the pion threshold (~ 140 MeV electron LAB. energy), and will remain a good approximation until pion production contributes noticeably to the cross section (~ 340 MeV). At high energies, this elastic contribution will only be one of many similar contributions from all the possible intermediate states.

Barut and Fronsdal [46] have calculated the polarization in μ -e scattering - i.e., for two structureless 'Dirac' particles - and Guérin and Piketty [42], who calculated the contribution of two N^* intermediate states to the polarization in e-p and μ -p scattering, also quoted one numerical result for one particular case for the proton intermediate state, using Clementel-Villi type form factors for the proton. In this chapter, we present details of the calculation of the elastic contribution to the polarization in e-p scattering for arbitrary form factors, and give results for a 'structureless' proton, and for a 'realistic' proton with the experimentally observed dipole form factors.

Towards the end of this calculation, a paper appeared by Arafune and Shimizu [47] which was also concerned with the elastic contribution to the polarization in e-p scattering.

^{*} A brief review of these calculations is included in Chapter 6.



Because of certain approximations made in their calculation, their results do not correspond directly to the physical situation: this is discussed in detail in the later sections of this chapter.

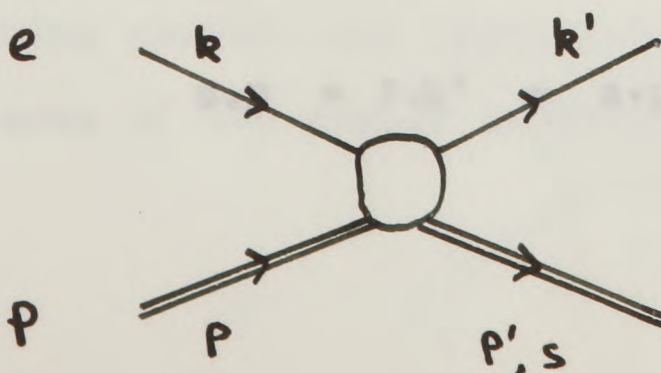
The plan of this chapter is as follows. In section 2, the calculation of the polarization in spin 0 - spin $\frac{1}{2}$ scattering is discussed in some detail, as a preliminary to the electron-proton case. Section 3 contains a detailed explanation of the cancellation of the infra-red divergence in these polarization calculations, and then in Section 4, we sketch the details of the calculation for electron-proton scattering with arbitrary form factors. Our conclusions are summarized in Section 5. Appendices 2 and 3 are relevant to this chapter: the first summarizes our metric and γ -matrices, and lists some useful trace theorems; while the second contains some details of the integrals required for these unitarity calculations.

5.2 Spin 0 - spin $\frac{1}{2}$ Elastic Scattering

1 Theoretical Formalism

We use a covariant formalism similar to that used by Bilen'kii and Semikoz [48] in their calculation of the asymmetry in π -e scattering. Our notation is indicated in Fig. 5.2.

Figure 5.2



$$k^2 = k'^2 = \mu^2$$

$$p^2 = p'^2 = M^2$$

e and p in this section represent a 'spinless electron' and a 'point Dirac proton' respectively.

k, k' electron initial and final 4-momenta respectively
 p, p' proton initial and final 4-momenta respectively
 s is the covariant spin 4-vector of the recoil proton: it reduces to a spatial unit vector in the rest frame.

We define the following 4-vectors:

$$\begin{aligned} Q &= k + p & \Delta &= k - k' \\ P &= p + p' & K &= k + k' \end{aligned} \quad (5.1)$$

The normal to the scattering plane can be defined covariantly as

$$n^\mu = \epsilon^{\mu\nu\rho\sigma} k_\nu k'_\rho p_\sigma \quad (5.2)$$

which reduces in the LAB. to

$$n^0 = 0; \quad \underline{n} = M(\underline{k} \wedge \underline{k}')$$

This direction is taken to be the positive y-direction, and $\underline{k}, \underline{k}', \underline{p}', \underline{p}$ all to be in the xz-plane. For polarization transverse to the scattering plane we can therefore define

$$s^\mu = \frac{n^\mu}{N} \quad (5.3)$$

and since $s^2 = -1$ therefore $n^2 = -N^2$.

By definition

$$s \cdot p = s \cdot k' = s \cdot p' = s \cdot k = 0.$$

Note that this definition of the normal to the scattering plane is opposite to that of the Basle convention, according to which the normal should be defined: $\underline{n}_B \propto \underline{k} \wedge \underline{p}'$.

Here we have defined the normal using the initial and final momenta of the same particle (as used by Powell et al. [18].)

The transition matrix element is defined by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(p+k - p'-k') N_f N_i T_{fi} \quad (5.4)$$

where $\Lambda(p')$, $\Lambda(p)$ are the particles energy projection operators

with $\Lambda(p) = (\not{p} + m) / 2m$, and $\Lambda(p) = \gamma_0 \Lambda(p) \gamma_0$

$$N_f = \left[\frac{R_p R_e}{p_0' k_0'} \right]^{\frac{1}{2}} \quad N_i = \left[\frac{R_p R_e}{p_0 k_0} \right]^{\frac{1}{2}}$$

Unitarity of the S-matrix gives the following conditions

where $R_i = \begin{cases} M_i & \text{for a fermion} \\ \frac{1}{2} & \text{for a boson} \end{cases}$

For our 'spinless electron' we therefore take $R_e = \frac{1}{2}$.

We use the spinor normalizations of Bjorken and Drell [49] and for convenience we have taken the normalization volume $V = 1$.

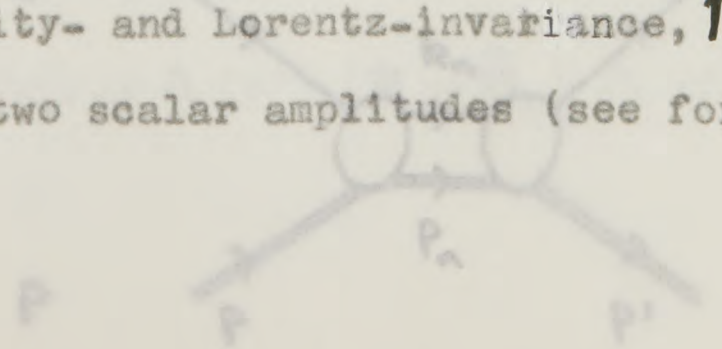
For spin 0 - spin $\frac{1}{2}$ elastic scattering the T matrix element can be written

$$T_{fi} = \bar{u}_f \mathcal{M} u_i \quad (5.5)$$

where \mathcal{M} is a 4 x 4 matrix in the proton spin space.

Assuming parity- and Lorentz-invariance, \mathcal{M} can be written

in terms of two scalar amplitudes (see for example ref. [59]):



For the $m = A(s,t) + \lambda B(s,t)$ (5.6)

The transverse polarization of the recoil proton is given by

$$I_0 P = \text{Tr}[\gamma_5 \not{\epsilon} \Lambda(p') m \Lambda(p) \bar{m}] \quad (5.7)$$

where $\bar{m} = I_0 m = \text{Tr}[\Lambda(p') m \Lambda(p) \bar{m}]$

$\Lambda(p')$, $\Lambda(p)$ are the positive energy projection operators, e.g., $\Lambda(p) = (\not{p} + M) / 2M$, and $\bar{m} = \gamma^0 m^\dagger \gamma^0$.

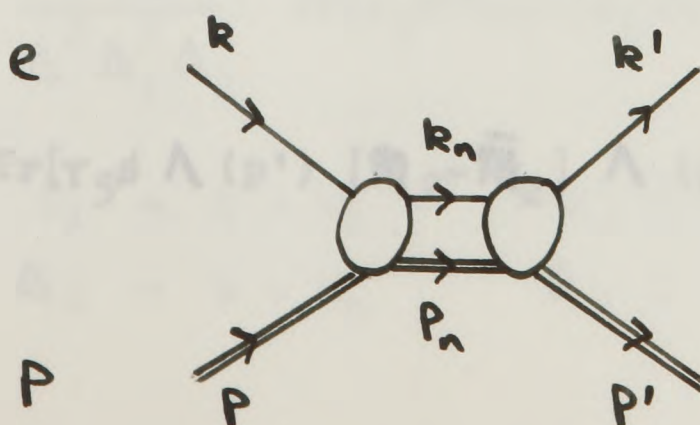
We remark here that with the form of m given by Eq. (5.6), it is trivial to prove the polarization-asymmetry relation in this covariant formalism.

Unitarity of the S-matrix gives the following condition on m :

$$\bar{u}_f [m - \bar{m}] u_i = (+i) \sum_n (2\pi)^4 \delta^4(Q - p_n - k_n) N_n^2 \times \bar{u}_f m \Lambda(p_n) \bar{m} u_i \quad (5.8)$$

where only the elastic intermediate state has been included in the unitarity sum, and the 4-vectors k_n and p_n of the intermediate electron and proton are defined in Fig. 5.3.

Figure 5.3
between M_1 and M_2 . Using the method outlined above, we find



For the two-particle intermediate state

$$\sum_n \rightarrow \int \frac{d^3 k_n}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3}$$

On making an expansion of m in powers of a

$$m = m_1 + m_2 + \dots \quad (5.9)$$

Equation (5.8) gives immediately that $m_1 = \bar{m}_1$. We are now able to prove in our spinor formalism a result of Chapter 2, that there is no polarization in the first Born approximation. Since $[\Lambda(p)]_R = \Lambda(p)$ and $m_R = m$, where the subscript R denotes γ -matrices written in reverse order, thus the T matrix element is we use trace theorem 6 of Appendix 2 to obtain

$$T_{fi} = \bar{u}(p') \text{Tr}[\gamma_5 \not{\epsilon} \Lambda(p') m_1 \Lambda(p) m_1] \quad (5.11)$$

(which is of the form $\text{Tr}[m_1 \Lambda(p) m_1 \Lambda(p') \not{\epsilon} \gamma_5]$)

$$m_1 = \bar{m}_1 \\ = - \text{Tr}[\gamma_5 \not{\epsilon} \Lambda(p') m_1 \Lambda(p) m_1] = 0$$

Therefore the elastic contribution to the polarization is

given by :

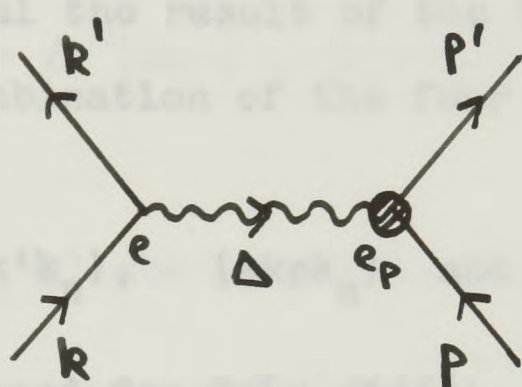
The last step uses the fact that γ_5 anti-commutes with all γ -matrices and $\not{p}' \gamma_5 \not{\epsilon} = \gamma_5 \not{\epsilon} \not{p}'$. Thus the lowest order contribution to the polarization is the result of an interference between m_1 and m_2 . Using the method outlined above, we find

$$\text{where } I_{OP} = \text{Tr}[\gamma_5 \not{\epsilon} \Lambda(p') [m_2 - \bar{m}_2] \Lambda(p) m_1] \quad (5.10)$$

(5.11)

The elastic unitarity relation for $[m_2 - \bar{m}_2]$ (Eq. (5.8)) then allows us to express the polarization entirely in terms of the first Born amplitudes. The one-photon-exchange amplitude for spin 0 - spin $\frac{1}{2}$ scattering (see Fig. 5.4) is given by the Feynman rules: these rules are given for S matrix elements in ref. [49] for example.

Figure 5.4



Thus the T matrix element is

$$T_{fi}^{(1)} = \bar{u}(p') m_1 u(p) = \frac{e e_p}{\Delta} \bar{u}(p') \not{K} u(p) \quad (5.11)$$

(which is of the general form of Eq. (5.6), and clearly

$$m_1 = \bar{m}_1.)$$

Therefore the elastic contribution to the polarization is given by :

$$I_{OP} = (+1) \int \frac{d^3 k_n}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} (2\pi)^4 \delta^4(Q - k_n - p_n) N_n^2 \times \frac{(e e_p)^3}{\Delta_1^2 \Delta_2^2} \text{Tr}[\gamma_5 \not{K}(p') \not{K}_2 \not{K}_1 \not{K}(p)] \quad (5.12)$$

where

$$\begin{aligned} K_1 &= k + k_n & K_2 &= k' + k_n \\ \Delta_1 &= k - k_n & \Delta_2 &= k' - k_n \end{aligned} \quad (5.13)$$

2 Trace Calculations

It is necessary to evaluate the following trace

$$T = \text{Tr}[\gamma_5 \not{s} (\not{p}' + M) \not{K}_2 (\not{p}_n + M) \not{K}_1 (\not{p} + M) \not{K}] \quad (5.14)$$

where: $k + p = k_n + p_n = k' + p'$.

If we take k, p, k_n and k' as the four independent 4-momenta, in general the result of the trace calculation will be a linear combination of the four trace products

$$(skk'p), (skk'k_n), (skpk_n) \text{ and } (spk'k_n)$$

(abcd) is a shorthand for $\text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d}]$. These trace products are not all independent since each of the five 4-vectors s, k, k', p, k_n can be resolved into four orthogonal directions because of the dimensionality of the space. For example, k_n can be resolved onto the following complete orthogonal basis:

$$K; \Delta; p_{\perp} = p - \frac{(p \cdot \Delta)}{\Delta^2} \Delta - \frac{(p \cdot K)}{K^2} K;$$

$$\text{and } n = \epsilon(kk'p).$$

The same relations between the trace products can be obtained, using the identities generated from $\text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d} \not{e} \not{f}]$ when two 4-vectors are the same (see trace theorem 5 of Appendix 2). We obtain for example :

$$(2k \cdot p)(skk'k_n) = (2k \cdot k_n)(skk'p) + (2k \cdot k')(skpk_n)$$

* This is the exact result of the trace calculation. In the expression quoted by Bhabha and Williams (4) as the result of the trace calculation, they have made (without stating)

By straightforward manipulation of such identities the following four important identities can be obtained :

$$K^2 \Delta^2 (skpk_n) = [(2k \cdot k')(2k \cdot k_n) - 2\mu^2(2k' \cdot k_n)] (skk'p) + \Delta^2 [2k \cdot p + 2\mu^2] (skk'k_n) \quad (5.15)$$

3 Integrals

The integrals will be evaluated in the appendix (Fig. 5.5).

$$K^2 \Delta^2 (spk'k_n) = [(2k \cdot k')(2k' \cdot k_n) - 2\mu^2(2k \cdot k_n)] (skk'p) + \Delta^2 [2k \cdot p - 2k \cdot k'] (skk'k_n) \quad (5.16)$$

Figure 5.5

$$2M^2 (skk'k_n) = (2k_n \cdot p)(skk'p) + (2k \cdot p) (spk'k_n) + (2k \cdot p + \Delta^2) (skpk_n) \quad (5.17)$$

$$(skk'k_n) = \frac{\Delta^2 (Q \cdot k) [\Delta_1^2 + \Delta_2^2 - \Delta^2]}{4N^2} (skk'p) \quad (5.18)$$

$P' = (\epsilon, -k')$

with $4N^2 = \Delta^2 (K^2 Q^2 - 4(Q \cdot k)^2)$

the k, k' plane in the scattering plane not is parallel to the not coplaner with k and k' .

$n^2 = -N^2$ is verified by direct calculation and we note also

$$\text{and: } (skk'p) = \frac{1}{N} (nkk'p) = -4iN \quad (5.19)$$

Armed with these results, the trace can be evaluated with some labour, and the result reduced to the form*

* This is the exact result of the trace calculation. In the expression quoted by Bilen'kii and Semikoz [48] as the result of the trace calculation, they have made (without stating)

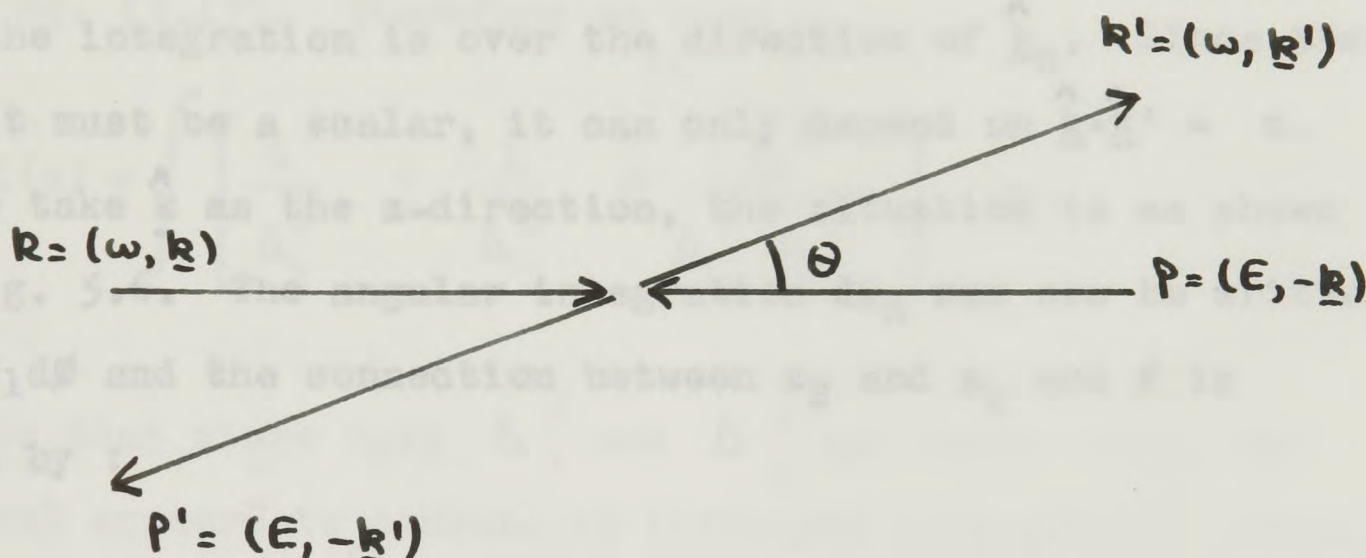
With the aid of $T = -8M [Q^2 - M^2] (skk'k_n)$ (5.20)

It is worth remarking that in this case the use of the trace theorem 5 in Appendix 2 simplifies the evaluation considerably.

3) Integrals

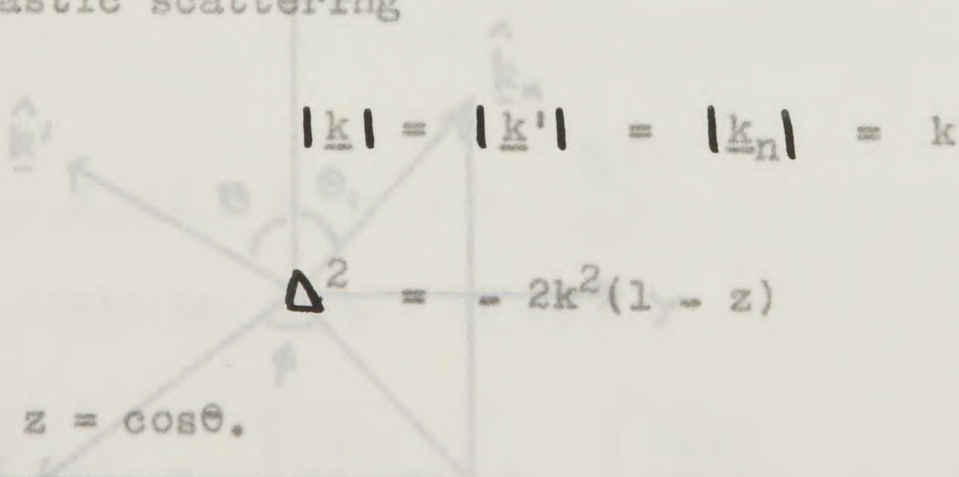
The integrals will be evaluated in the overall CMS (Fig. 5.5).

Figure 5.5



The $\underline{k}, \underline{k}'$ plane is the scattering plane and in general \underline{k}_n is not coplanar with \underline{k} and \underline{k}' .

For elastic scattering



and:

$$\Delta^2 = -2k^2(1 - z)$$

where $z = \cos\theta$.

the approximation $(Q^2 - M^2) \sim Q^2$. This is valid in their case of π -e scattering where M refers to the electron mass.

With the aid of the δ -function the intermediate state integration reduces to an angular integration in the CMS:

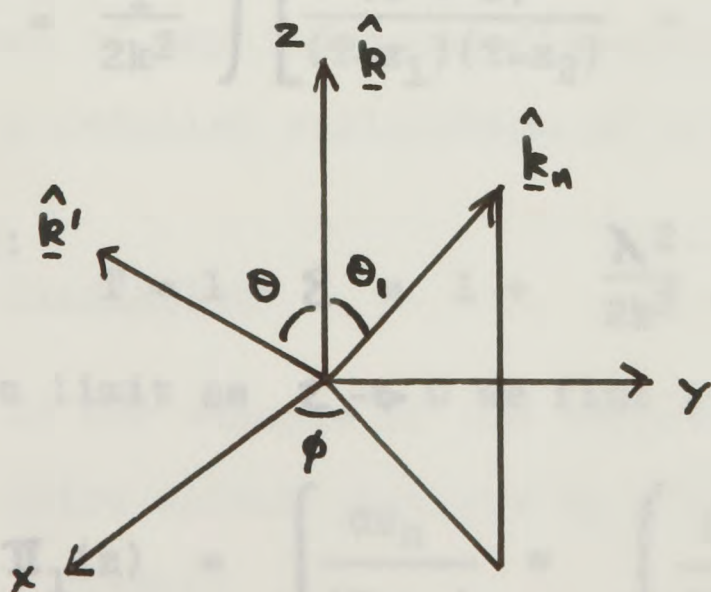
$$\int \frac{d^3 k_n}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} (2\pi)^4 \delta(Q - k_n - p_n) N_n^2 = \frac{M_k}{2W} \frac{1}{(2\pi)^2} \int d\Omega_n \quad (5.22)$$

with $W = E + \omega$.

This is the standard type of unitarity integration (see Appendix 3): there are two fixed directions \hat{k} and \hat{k}' and the integration is over the direction of \hat{k}_n . Since the result must be a scalar, it can only depend on $\hat{k} \cdot \hat{k}' = z$. If we take \hat{k} as the z-direction, the situation is as shown in Fig. 5.6. The angular integration $d\Omega_n$ can now be written $d\cos\theta_1 d\phi$ and the connection between z_2 and z_1 and ϕ is given by :

$$z_2 = \cos\theta_2 = \cos\theta \cos\theta_1 + \sin\theta \sin\theta_1 \cos\phi \quad (5.23)$$

Figure 5.6



$$\hat{k} \cdot \hat{k}_n = z_1 = \cos\theta_1$$

$$\hat{k}' \cdot \hat{k}_n = z_2 = \cos\theta_2$$

$$\hat{k} \cdot \hat{k}' = z = \cos\theta$$

$$d\Omega_n = d\cos\theta_1 d\phi$$

Thus:

$$\Delta_1^2 = -2k^2(1 - z_1)$$

$$\Delta_2^2 = -2k^2(1 - z_2)$$

(5.24)

In our case of spin 0 - spin $\frac{1}{2}$ scattering the required integral takes the form

Thus the divergent term $\int \frac{(s k k' k_n)}{\Delta_1^2 \Delta_2^2} d\Omega_n$ we are left with a finite

$$\int \frac{(\Delta_1^2 + \Delta_2^2 - \Delta^2)}{\Delta_1^2 \Delta_2^2} d\Omega_n$$

$$\sim \int \frac{[\Delta_1^2 + \Delta_2^2 - \Delta^2]}{\Delta_1^2 \Delta_2^2} d\Omega_n$$

using Eq. (5.17). Therefore we have :

$$I(z) = \int \left[\frac{1}{\Delta_1^2} + \frac{1}{\Delta_2^2} - \frac{\Delta^2}{\Delta_1^2 \Delta_2^2} \right] d\Omega_n \quad (5.25)$$

We note that since both Δ_1^2 and Δ_2^2 can reach zero, the integral appears to contain an infra-red divergence. However, if we introduce a small photon mass λ into the propagators, the integral is modified to the form

$$I(z) = \frac{1}{2k^2} \int \left[\frac{(1-z)}{(T-z_1)(T-z_2)} - \frac{1}{(T-z_1)} - \frac{1}{(T-z_2)} \right] d\Omega_n \quad (5.26)$$

where:

$$T = 1 + \varepsilon = 1 + \frac{\lambda^2}{2k^2}$$

In the limit as $\varepsilon \rightarrow 0$ we find :

$$\Pi_1(z) = \int \frac{d\Omega_n}{(T-z_1)} = \int \frac{d\Omega_n}{(T-z_2)} = 2\pi \ln\left(\frac{2}{\varepsilon}\right)$$

and

$$\mathbb{H}_2(z) = \int \frac{d\Omega_n}{(T-z_1)(T-z_2)} = \frac{4\pi}{(1-z)} \ln\left(\frac{1-z}{\epsilon}\right) \quad (5.27)$$

Thus the divergent terms as $\epsilon \rightarrow 0$, explicitly cancel, and we are left with a finite result:

$$I(z) = \frac{1}{2k^2} 4\pi \ln\left(\frac{1-z}{2}\right) \quad (5.28)$$

This cancellation is in fact evident from the form

$$\left[\frac{1}{\Delta_1^2} + \frac{1}{\Delta_2^2} - \frac{\Delta^2}{\Delta_1^2 \Delta_2^2} \right]$$

As $\Delta_1^2 \rightarrow 0$, $\Delta_2^2 \rightarrow \Delta^2$; thus the divergent terms cancel - similarly for $\Delta_2^2 \rightarrow 0$.

The integral (Eq. (5.25)) occurs in the π -e calculation of reference [48] and also in the calculation of the polarization for two point Dirac particles in reference [46]. Thus we see that the polarization (in these examples at least) is free of any infra-red divergence. In Section 5.3 we shall give a detailed explanation of this cancellation.

4. Discussion

In our example of the polarization in the scattering of a point spin-0 particle by a point spin- $\frac{1}{2}$ particle we obtain finally

explanation of η_P $M \omega \sin \theta \tan^2 \theta / 2 \ln(\sin^2 \theta / 2)$
 by considering an electron scattered with polarization arising from the exchange of the photon
 where $\eta_P = +1$ for e^-p
 -1 for e^+p

$$P = \frac{\eta_P M \omega \sin \theta \tan^2 \theta / 2 \ln(\sin^2 \theta / 2)}{2k\omega \left[\cos^2 \theta / 2 + \frac{\omega^2 M^2}{(Q^2 - M^2) k^2} \right]} \quad (5.29)$$

Figure 5.7

This reduces to the result of reference [48] if we make the approximation $Q^2 - M^2 \sim Q^2$.

If we include a form factor in the amplitude

$$T_{fi}^{(1)} = \frac{e e_p}{\Delta^2} F(\Delta^2) \bar{u}(p') \not{\epsilon} u(p) \quad (5.30)$$

the calculation proceeds almost exactly as before except for the unitarity integral

$$I(z) \rightarrow I'(z) = \int F(\Delta_1^2) F(\Delta_2^2) \left[\frac{1}{\Delta_1^2} + \frac{1}{\Delta_2^2} - \frac{\Delta^2}{\Delta_1^2 \Delta_2^2} \right] d\Omega_n$$

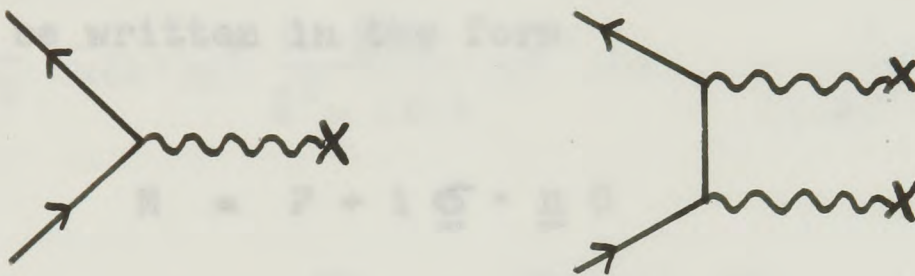
of this, that there is no divergence in the unitarity integral. Since we assume that $F(0)$ and $F(\infty)$ are finite, it is clear in the two-component spinor formalism that the cancellation of the divergence is unaffected. Thus of course the integrals can be carried out explicitly for various types of form factors, e.g., Yukawa, Dipole, and the divergent terms as $\epsilon \rightarrow 0$ cancel, leaving finite expressions. Details of the necessary integrals are given in Appendix 3.

5.3 The Cancellation of the Infra-Red Divergence

As we have shown in the previous section, the infra-red divergent terms explicitly cancel in the a^3 polarization. The

* explanation for this cancellation is most easily illustrated by considering the analogous problem of the polarization of an electron scattered by a potential. Again the lowest order polarization arises from an interference of the first Born amplitude with the two-photon-exchange amplitude (see Fig.5.7).

Figure 5.7



where \hat{q} is the normal to the scattering plane. Consequently Dalitz [50] has shown that the infra-red divergence in the two-photon graph can be isolated into a term proportional to the first Born amplitude. Thus to this order in α , the divergence in the imaginary part acts like a phase factor for the one-photon-exchange amplitude: it is precisely because of this, that there is no divergence in the polarization. In the two-component spinor formalism (totally relativistic of course)

$$T_{fi} \sim \chi_f^\dagger M \chi_i$$

and we make an expansion of M in powers of α :

$$M = \alpha M^{(1)} + \alpha^2 M^{(2)} + \dots$$

The infra-red divergent term in $M^{(2)}$ is purely imaginary

* I thank Professor R.H. Dalitz for a helpful discussion of his paper (reference [50]).

in this case and is proportional to $M^{(1)}$. Thus we may write

$$e^2 M^{(2)} = e^2 M_F^{(2)} + i\alpha^2 \phi(\lambda^2) M^{(1)}$$

where $M_F^{(2)}$ is finite and $\phi(\lambda^2)$ is a scalar function of λ^2 which diverges as $\lambda \rightarrow 0$. Therefore we have

$$M = \alpha M^{(1)} [1 + i\alpha \phi(\lambda^2)] + \alpha^2 M_F^{(2)}$$

Now M can be written in the form

$$M = F + i \underline{\sigma} \cdot \underline{n} G$$

where \underline{n} is the normal to the scattering plane. Consequently

$$M^{(1)} = f^{(1)} + i \underline{\sigma} \cdot \underline{n} g^{(1)}$$

The infra-red divergent part of

$$M_F^{(2)} = f^{(2)} + i \underline{\sigma} \cdot \underline{n} g^{(2)}$$

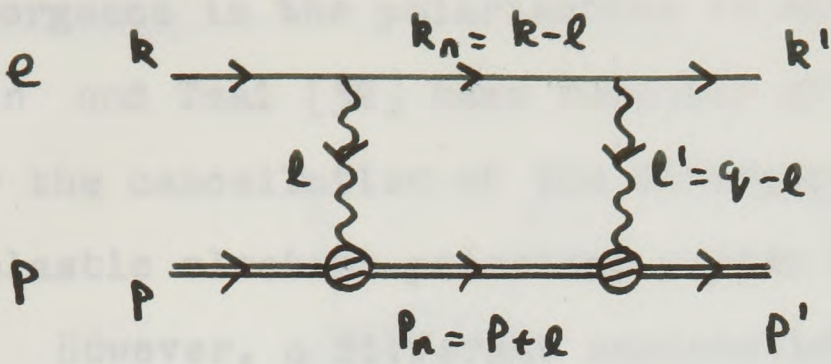
The polarization is sensitive to the relative phase of F and G , and since the divergent phase factor is the same for both $f^{(1)}$ and $g^{(1)}$, there is no divergence in the polarization to order α^3 .

$$P \sim \alpha^3 \text{Im} [f^{(1)} g^{(2)*} + g^{(1)} f^{(2)}]$$

Thus

For the non-static case Tsai [51] has shown that the infra-red divergent part of the two-photon-exchange graph is still proportional to the first Born term. This is easily demonstrated by considering the Feynman amplitude for the process (Fig. 5.8).

Figure 5.8



$$T_2 \sim \int \frac{d^4 l}{(2\pi)^4} \bar{u}(k') \gamma^\nu \frac{(\not{k} - \not{l} + \not{m})}{l^2 - 2l \cdot k} \gamma^\mu u(k) \cdot \frac{1}{(q-l)^2 - \lambda^2} \cdot \frac{1}{l^2 - \lambda^2}$$

$$\times \bar{u}(p') \Gamma_\nu(l') \frac{(\not{l} + \not{p} + \not{M})}{l^2 + 2l \cdot p} \Gamma_\mu(l) u(p)$$

The infra-red divergent part of T_2 can be obtained by letting $l \rightarrow 0$:

$$\begin{aligned} \Gamma_\mu(l) &\rightarrow \gamma_\mu \\ (\not{k} - \not{l} + \not{m}) \gamma^\mu u(k) &\rightarrow 2k^\mu u(k) \\ (\not{p} + \not{l} + \not{M}) \Gamma_\mu(l) u(p) &\rightarrow 2p_\mu u(p) \end{aligned}$$

Thus

$$T_2 \Big|_{IR} \sim \bar{u}(k') \gamma^\nu u(k) \frac{1}{q^2} \bar{u}(p') \Gamma_\nu(q) u(p)$$

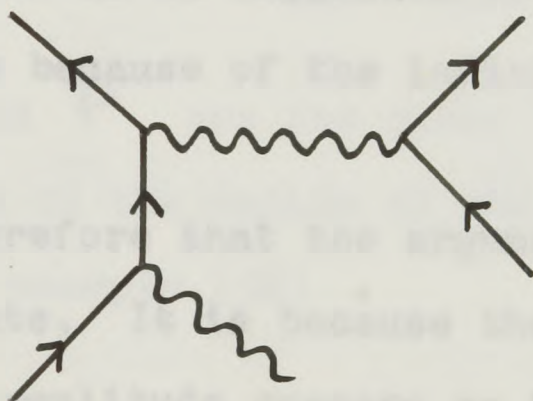
$$\times (4k \cdot p) \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - 2l \cdot k)(l^2 + 2l \cdot p)(l^2 - \lambda^2)}$$

i.e., $T_2 \Big|_{IR} \sim \mathcal{O}(\lambda^2) T_1$

shift: this was first pointed out by [51]. Consequently the argument for the cancellation of the divergence in the polarization is essentially unchanged. Cahn and Tsai [52] have recently given a similar explanation for the cancellation of the divergence in the asymmetry in inelastic electron-polarized proton scattering.

However, a different explanation has been given by Barut and Fronsdal [46] (and repeated in reference [48]). Their argument to show that the result must be free of infra-red divergences runs as follows. "In the calculation of the sixth order scattering cross section, diagrams of the type of Fig. 2 serve to eliminate the infra-red divergence. However, these diagrams have no imaginary parts, and hence do not contribute to the spin dependence of the cross section". Figure 2 of reference [46] is reproduced below as Fig. 5.9.

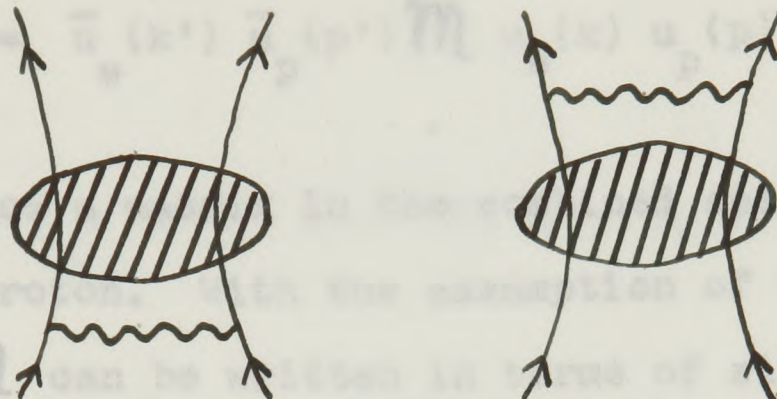
Figure 5.9



Now the soft bremsstrahlung diagrams like that of Fig. 5.9, do indeed cancel the divergences in the α^3 cross-section that arise from the usual radiative corrections [53]. However, the divergence that is encountered in the imaginary part of the two-photon-exchange diagram is not cancelled by these diagrams. In general [54] the divergences in the imaginary parts arising from diagrams such as those in Fig. 5.10 give us the long-range contributions to the familiar Coulomb phase

shift: this was first pointed out by Dalitz [50]*. Each

Figure 5.10 the T matrix elements can be written :



pair of initial and final state charges contributes a Coulomb phase factor. Rather than cancelling, these factors change the phase of the wave functions by an amount which approaches infinity as the photon mass $\lambda \rightarrow 0$ (corresponding to the $\ln 2kr$ phase in the Coulomb wave functions [56]). In our case, the divergence in the two-photon-exchange amplitude is merely the second term in an expansion of the Coulomb phase factor, which arises because of the infinite range of the Coulomb field.

It is clear therefore that the argument of Barut and Fronsda is inadequate. It is because the divergence of the two-photon-exchange amplitude appears as the Coulomb phase factor that there is no divergence in the polarization.

5.4 Electron-Proton Scattering with Arbitrary Form Factors

1 Formalism

Using the same notation, we shall now generalise the

* Kacser [55] has given a more complete treatment of the infra-red factor of the third Born approximation.

results of Section 5.2.1. For spin $\frac{1}{2}$ - spin $\frac{1}{2}$ elastic scattering the T matrix element can be written :

The transition probability of the recoil proton is given by

$$T_{fi} = \bar{u}_e(k') \bar{u}_p(p') \mathcal{M} u_e(k) u_p(p) \quad (5.31)$$

where \mathcal{M} is now a matrix in the combined spin-space of the electron and proton. With the assumption of time-reversal-invariance, \mathcal{M} can be written in terms of six scalar amplitudes [57]

$$\mathcal{M} = A_S S + A_V V + A_T T + A_A A + A_P P + A_U U \quad (5.32)$$

where the second term for example, represents

$$A_V V \sim A_V(s,t) \gamma_{(e)}^\mu \cdot \gamma_{(p)}^\mu$$

and S, T, A and P are the other four Fermi invariants.

We have some freedom in the choice of the sixth invariant U , and we can take for example [58]

An expansion of \mathcal{M} in powers of a given spin $M_1 = M_2$.

Applying the trace $U \sim p'_{(e)} \cdot 1_{(p)}$, it is seen that the lowest order contribution to the yield is

We note that \mathcal{M} has the property that

$$\mathcal{M}_R = \mathcal{M} \quad (5.33)$$

where the subscript R denotes all γ -matrices in both the electron and proton spin spaces are written in reverse order.

For example for the tensor term

$$\text{for } e-p \text{ and } \left[\sigma_{(e)}^{\mu\nu} \cdot \sigma_{\mu\nu}^{(p)} \right]_R = \sigma_{(e)}^{\mu\nu} \cdot \sigma_{\mu\nu}^{(p)}$$

The transverse polarization of the recoil proton is given by

$$I_{OP} = \text{Tr}[\gamma_5 \not{s} \Lambda(p') \Lambda(k') \not{m} \Lambda(p) \Lambda(k) \bar{m}] \quad (5.34)$$

with

$$\Gamma I_{O\Delta^2} = -\text{Tr}[\Lambda(p') \Lambda(k') \not{m} \Lambda(p) \Lambda(k) \bar{m}]$$

and where $\gamma_5 \not{s}$ acts only in the proton spin-space.

The elastic unitarity condition now reads

$$\Lambda(k') \Lambda(p') [\not{m} - \bar{m}] \Lambda(k) \Lambda(p)$$

$$= (+1) \int \frac{d^3 k_n}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} (2\pi)^4 \delta(Q - k_n - p_n) N_n^2$$

$$\Lambda(k') \Lambda(p') \not{m} \Lambda(k_n) \Lambda(p_n) \bar{m} \Lambda(k) \Lambda(p) \quad (5.35)$$

An expansion of \not{m} in powers of a gives again $\not{m}_1 = \bar{m}_1$.

Applying the trace theorem 6 as before, it is easy to prove that the lowest order contribution to the polarization must result from an interference between \not{m}_1 and \not{m}_2 :

$$I_{OP} = \text{Tr}[\gamma_5 \not{s} \Lambda(p') \Lambda(k') (\not{m}_2 - \bar{m}_2) \Lambda(p) \Lambda(k) \not{m}_1] \quad (5.36)$$

$\not{m}_2 - \bar{m}_2$ is given in terms of first order amplitudes in Eq. (5.35). \not{m}_1 is just the one-photon-exchange amplitude

for e-p scattering and is given by the Feynman rules [49] :

$$M_1(\Delta^2) = \gamma_{\mu}^{(e)} \cdot \frac{e e_p}{\Delta^2} \cdot \Gamma_{\mu}^{(p)}(\Delta^2) \quad (5.37)$$

This is the basic expression for all our calculations. The standard form for the proton current $\Gamma_{\mu}(\Delta^2)$ is (see the literature (references [49], [50] and the preceding chapters)

$$\Gamma_{\mu}(\Delta^2) = G_M(\Delta^2) \gamma_{\mu} - \frac{[G_M(\Delta^2) - G_E(\Delta^2)]}{2M(1+\tau)} \mathbb{P}_{\mu}$$

Using the shorthand notation $\tau = -\Delta^2/4M^2$ and so on, $\equiv A(\Delta^2) \gamma_{\mu} + B(\Delta^2) \mathbb{P}_{\mu}$ (5.38)

where $\tau = -\Delta^2/4M^2$ and \mathbb{P} is defined in Eq. (5.1).

Similarly we define:

$$\mathbb{P}_1 = p + p_n \quad \mathbb{P}_2 = p' + p_n \quad (5.39)$$

The trace factorizes into traces in the electron and proton spin-spaces and the polarization is now given by

$$I_{OP} = (+1) \int \frac{d^3 k_n}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} (2\pi)^4 \delta^4(Q - k_n - p_n) N_n^2$$

$$\times \frac{(e e_p)^3}{\Delta^2 \Delta_1^2 \Delta_2^2} \cdot \frac{T_e^{\mu\nu\rho} T_{p\mu\nu\rho}}{(2\mu)^3 (2M)^3} \quad (5.40)$$

where

$$T_e^{\mu\nu\rho} = \text{Tr}[(\not{K} + \mu) \gamma^{\mu} (\not{K}_n + \mu) \gamma^{\nu} (\not{K} + \mu) \gamma^{\rho}]$$

(5.16) and (5.17) then become

and

$$T_{p\mu\nu\rho} = \text{Tr}[\gamma_5 \not{\epsilon} (\not{p}' + M) \Gamma_\mu(\Delta_2^2) (\not{p}_n + M) \Gamma_\nu(\Delta_1^2) (\not{p} + M) \Gamma_\rho(\Delta^2)]$$

This is the basic expression for all our calculations. In the literature (references [46], [42] and [47]) a less obviously covariant formalism has been used.

2 Trace Calculations

in the following form:

Using the shorthand notation $A \equiv A(\Delta^2)$, $A_1 \equiv A(\Delta_1^2)$ and so on, there are eight separate contributions to the proton trace - arising from the eight combinations AA_1A_2 , BA_1A_2 , etc.. We write

$$T_e T_p \equiv T_e^{\mu\nu\rho} T_{p\mu\nu\rho} \quad (5.41a)$$

and the trace products T_I to T_{VIII} are defined by

$$T_e T_p = AA_1A_2 T_I + A_2A_1B T_{II} + A_2B_1A T_{III} + B_2A_1A T_{IV} \\ + A_2B_1B T_V + B_2A_1B T_{VI} \\ + B_2B_1A T_{VII} + B_2B_1B T_{VIII} \quad (5.41b)$$

The evaluation of these traces is perhaps best described as 'straightforward but extremely tedious'. To simplify the calculation, terms proportional to the square of the electron mass μ^2 , have been neglected. Our trace identities (Eqs. (5.16) and (5.17)) then become

$$K^2(\text{skpk}_n) = \Delta_1^2(\text{skk}'p) + (2k \cdot p)(\text{skk}'k_n) \quad (5.16')$$

$$K^2(\text{spk}'k_n) = \Delta_2^2(\text{skk}'p) + (2k \cdot p + \Delta^2)(\text{skk}'k_n) \quad (5.17')$$

and $K^2 = -\Delta^2$.

For ease of integration, it is convenient to write the results in terms of $(\text{skk}'k_n)$ and $(\text{skk}'p)$ - Eq. (5.18) gives us some freedom in the actual form of these results.

The results of the trace calculations may be written in the following form:

$$\Delta^2 T_N = a_N(\text{skk}'k_n) + b_N(\text{skk}'p) \quad (5.42)$$

$$T_I \quad a_I = 8M [(k \cdot p) [\Delta_1^2 + \Delta_2^2] - \Delta^2 \Delta_1^2]$$

$$b_I = 16M \Delta_1^2 \Delta_2^2$$

$$T_{II} \quad a_{II} = 2 \left\{ (\Delta_1^2 + \Delta_2^2) [4M^2(4k \cdot p) - \Delta^2] - \Delta^2(8k \cdot p) \right. \\ \left. - (4k \cdot p + \Delta^2) 4\Delta^2 Q^2 \right\}$$

$$b_{II} = 2 \left\{ 4\Delta_1^2 \Delta_2^2 [4M^2 - \Delta^2] \right. \\ \left. + \Delta^2(8k \cdot p)(4k \cdot p + \Delta^2) \right\}$$

$$T_{III} \quad a_{III} = 2(2k \cdot p) \left\{ 4Q^2 \Delta_1^2 + 4M^2 \Delta_2^2 - 4\Delta^2 Q^2 \right. \\ \left. - (8k \cdot p) [4k \cdot p + \Delta^2] \right\}$$

$$b_{III} = 2 \left\{ \Delta_1^2 \Delta_2^2 (8M^2 + 8k \cdot p) - \Delta_1^2 (8k \cdot p) (4k \cdot p + \Delta^2) \right\}$$

$$a_{IV} = 2 \left\{ (4k \cdot p + \Delta^2) [4\Delta^2 Q^2 + (4k \cdot p)^2] \Delta^2 \right. \\ \left. + 4\Delta^2 Q^2 \cdot (2k \cdot p) + \Delta_1^2 \cdot 4M^2 [2k \cdot p - \Delta^2] \right. \\ \left. + \Delta_2^2 \cdot 4Q^2 [2k \cdot p + \Delta^2] \right\}$$

T_{IV}

$$b_{IV} = 2 \left\{ \Delta_1^2 \Delta_2^2 (8k \cdot p + 8M^2 + 4\Delta^2) \right. \\ \left. + \Delta_1^2 (8k \cdot p) (4k \cdot p + \Delta^2) \right. \\ \left. - \Delta^2 (8k \cdot p) (4k \cdot p + \Delta^2) \right\}$$

$$a_V = M \left\{ 2(4k \cdot p + \Delta^2) (8k \cdot p [4k \cdot p + \Delta^2] + 4M^2 \Delta^2) \right.$$

have a different $+ \Delta_1^2 \cdot (4k \cdot p + \Delta^2) \cdot 8Q^2$

terms must be integrated separately. In the case of Δ_1 and Δ_2 were taken to be $\Delta_1^2 (8k \cdot p [4M^2 - 2\Delta^2] - 8M^2 \Delta^2)$

"V₃" terms, and the three "AB" terms must be added

together before integration. The result is

behaviour of Δ_1 and Δ_2 is the same as in the case of Δ_1 and Δ_2 have the same Δ -dependence.

As a check on our calculations,

$$[47] \text{ was verified } a_{VI} = M \left\{ 2(4k \cdot p + \Delta^2) (8k \cdot p [4k \cdot p + \Delta^2] + 4M^2 \Delta^2) \right. \\ \left. + \Delta_1^2 (8k \cdot p [4M^2 - 2\Delta^2] - 8M^2 \Delta^2) \right. \\ \left. + \Delta_2^2 (4k \cdot p + \Delta^2) \cdot 8Q^2 \right\}$$

T_{VI} Note that the integral in (47) can be simplified by using the result of the calculation of the polarizability of the magnetic moments. This result can also be used to calculate the electric mass terms in (47) with the same result. The expression obtained for the polarizability

$$b_{VI} = M \left\{ 4\Delta_1^2 \Delta_2^2 (8k \cdot p + 8M^2) \right. \\ \left. + 4\Delta_1^2 (8k \cdot p) (4k \cdot p + \Delta^2) \right\}$$

$$a_{VII} = M \left\{ \Delta^2 \cdot 8Q^2 \cdot ([8k \cdot p + \Delta^2] + [\Delta_1^2 + \Delta_2^2]) \right\}$$

 T_{VII}

$$b_{VII} = M \left\{ -2 \Delta^2 \cdot (8k \cdot p) ([8k \cdot p + \Delta^2] + [\Delta_1^2 + \Delta_2^2]) \right\}$$

The integrals are performed in

section 5.2.3.

$$a_{VIII} = \Delta^2 (4k \cdot p + 4M^2) \left\{ (2(4k \cdot p)^2 + 4Q^2 \Delta^2) + 4Q^2 (\Delta_1^2 + \Delta_2^2) \right\}$$

 T_{VIII}

$$b_{VIII} = 0$$

In general, the A and B terms of the proton current have a different Δ^2 -dependence, and consequently all eight terms must be integrated separately. In reference [47], A and B were taken to have the same Δ^2 -dependence: the three "A²B" terms, and the three "AB²" terms could then be added together before integration. The reported experimental behaviour of G_E and G_M [12] does not suggest that A and B have the same Δ^2 -dependence.

As a check on our calculations, the result of reference [47] was verified by adding the three "A²B" terms, and the three "AB²" terms*. This result is a special case of our calculation** which applies for any Δ^2 -dependence of G_E and G_M .

* Note that the symmetry of the integral has been used to simplify one term in the expression quoted in ref. [47] as the result of the trace calculation.

** The term proportional to AA_1A_2 is the trace required for the polarization of two Dirac particles with no anomalous magnetic moments. This term was also calculated keeping the electron mass terms to compare with the result of ref. [46]. The expression obtained for the polarization in this case is

I thank Dr. Flückiger for a private communication concerning these points, and am informed that the term omitted in [47] was not omitted in their calculations.

The denominator trace (I_0) is just that involved in the Rosenbluth cross section.

collation of the divergences

3. Integrals and the Infra-red Divergence

The integrals are performed in the overall CMS as in section 5.2.3. We require integrals of the form

$$I \sim \int \frac{F(\Delta_1^2, \Delta_2^2)}{\Delta_1^2 \Delta_2^2} d\Omega_n \quad (5.43)$$

where F is some known function of Δ_1^2 and Δ_2^2 . We must now verify that there is no infra-red divergence. This is easy to show if A and B are constant: the traces can then be added and reduce to the result of reference [47]. This result may be written in the form

$$T_e T_p = (s k k' k_n) H + \Delta_1^2 \Delta_2^2 (s k k' p) J \quad (5.44)$$

where H and J are polynomials in $\Delta_1^2, \Delta_2^2, k \cdot p$, etc.. In the unitarity integral Eq. (5.43), the J term clearly gives no divergence. For the H term we have seen in Section 5.2.3, that the $(s k k' k_n)$ factor automatically guarantees the cancellation of the divergence: thus the total result is finite as we expect*. However, for our case, where we allow a

identical with the result of Barut and Fronsdal except for the overall sign. We believe the sign given here to be correct.

* A similar argument has been applied for the inelastic intermediate states when, in the limit of zero electron mass, current conservation restricts the trace to be of the form of Eq. (5.44). This is shown in the paper of Guerin and Piketty [42] but there is a misprint in Eq. 12 of their paper, where a term like the J term in Eq. 5.44 above, has been omitted. I thank Dr. Piketty for a private communication concerning these points, and am informed that the term omitted in Eq. 12 was not omitted in their calculations.

different Δ^2 -dependence for A and B, the trace cannot be written in the form of Eq. (5.44). To check for the cancellation of the divergence therefore, we need only consider the terms not of the 'J' or 'H' type. For the "A²B" terms, these result in the integral

$$\begin{aligned} \Pi_{AB^2} \sim \int \left\{ B \Delta^2 \cdot \frac{A_1 A_2}{\Delta_1^2 \Delta_2^2} - A \cdot \frac{B_1 A_2}{\Delta_2^2} \right. \\ \left. + A \cdot \frac{A_1 B_2}{\Delta_2^2} - A \Delta^2 \cdot \frac{A_1 B_2}{\Delta_1^2 \Delta_2^2} \right\} d\Omega_n \quad (5.45) \end{aligned}$$

Consider the limit $\Delta_1^2 \rightarrow 0$. Then $\Delta_2^2 \rightarrow \Delta^2$; $B_2 \rightarrow B$; and $A_2 \rightarrow A$. Thus the divergent terms cancel since

$$\lim_{\Delta_1^2 \rightarrow 0} [B A_1 A_2 - A A_1 B_2] \rightarrow 0$$

A similar cancellation takes place as $\Delta_2^2 \rightarrow 0$. The same is true for the remaining "B²A" terms

$$\Pi_{AB^2} \sim \int \left\{ B \cdot \frac{B_1 A_2}{\Delta_1^2} + B \cdot \frac{A_1 B_2}{\Delta_2^2} - A \Delta^2 \cdot \frac{B_1 B_2}{\Delta_1^2 \Delta_2^2} \right\} d\Omega_n \quad (5.46)$$

Thus, as we expect, the cancellation is independent of the explicit Δ^2 -dependences of A and B.

For our numerical results (see next section) we take the specific case of a dipole Δ^2 -dependence for G_E and G_M .

This is the so-called 'Coulomb-like' and 'Coulomb-like' form
Then

$$A(\Delta^2) \sim \frac{1}{(a-z)^2} \quad a, b > 1 \quad (5.47)$$

$$B(\Delta^2) \sim \frac{1}{(b-z)} \frac{1}{(a-z)^2}$$

Using partial fractions, the integrals can be reduced to simpler ones, which all derive from the 'basic' unitarity integral [59]

$$I(z) = \int \frac{d\Omega_n}{(T_1 - z_1)(T_2 - z_2)} \quad T_1, T_2 > 1 \quad (5.48)$$

After inserting a small photon mass λ into the propagators as before, the integrals may be performed analytically. Taking the limit as $\lambda \rightarrow 0$, (i.e., T_1 and/or $T_2 \rightarrow 1$), the infra-red divergent terms ($\sim \ln \lambda^2$), explicitly cancel and we obtain finite expressions. (See for example Section 5.2.3.) More details of the integrals are given in Appendix 3.

4 Numerical Results

The numerical calculations were carried out using the usual experimental parameterization of the two form factors

$$G_M(\Delta^2) = \mu_p G_E(\Delta^2) = \mu_p G_D(\Delta^2)$$

with

$$G_D(\Delta^2) = \frac{1}{(1 - \Delta^2/\beta)^2}$$

This is the so-called 'Dipole Fit' and 'Scaling Law' for G_E and G_M which appears to be a fairly good representation of the experimental data [12]. Experimentally $\beta = 0.71$ (GeV/c)². The integrals were carried out analytically and the infra-red divergent terms cancelled as explained in the previous section. The explicit expressions are too long to be given here: they were programmed and the polarizations calculated using a KDF9 computer.

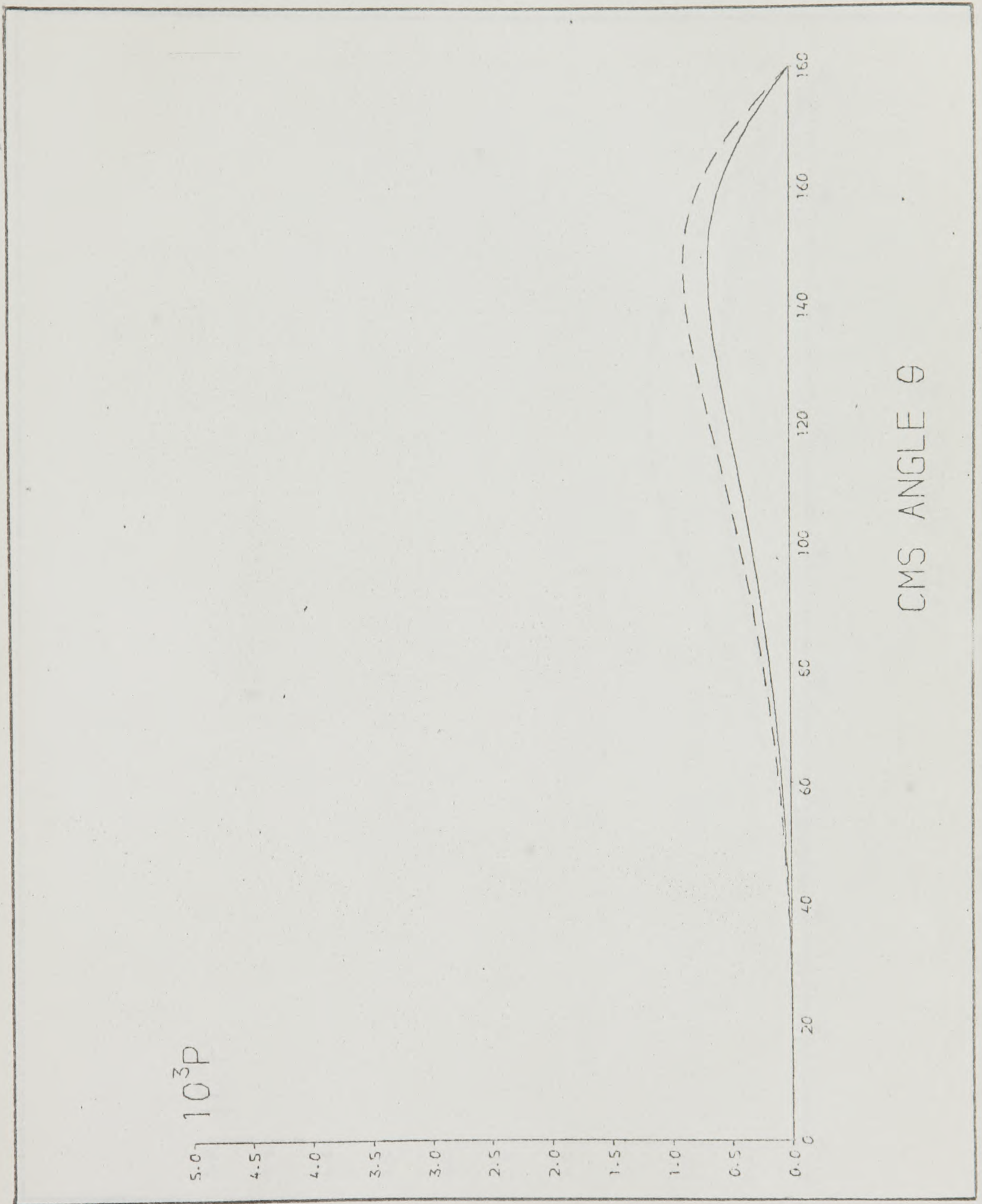
In the figures, the full line curves represent the polarization calculated using the experimental form factors, i.e., $\beta = 0.71$. The broken-line curves represent the polarization obtained for $G_D(\Delta^2) = 1$ ($\beta = \infty$). This can be considered as some sort of 'point-like' limit for the proton, although such a limit is rather arbitrary. We note here that if we take as our 'point-like' form factors the definitions $F_1(\Delta^2) = F_2(\Delta^2) = 1$, then we obtain the zero in the polarization noted in reference [47]. Their approximation is equivalent to parameterizing the form factors F_1 and F_2 by a dipole formula:

$$F_1(\Delta^2) = F_2(\Delta^2) = G_D(\Delta^2)$$

The graphs in Figs. 5.11 show the behaviour of this elastic effect with respect to the CMS scattering angle for various electron LAB. energies. Figure 5.12 shows the behaviour with respect to electron LAB. energy for a fixed scattering angle: this angle is chosen so that the effect at 10 GeV is near its maximum (see Fig. 5.11d). Note that this angle is different for the two values $\beta = 0.71$ and $\beta = \infty$.

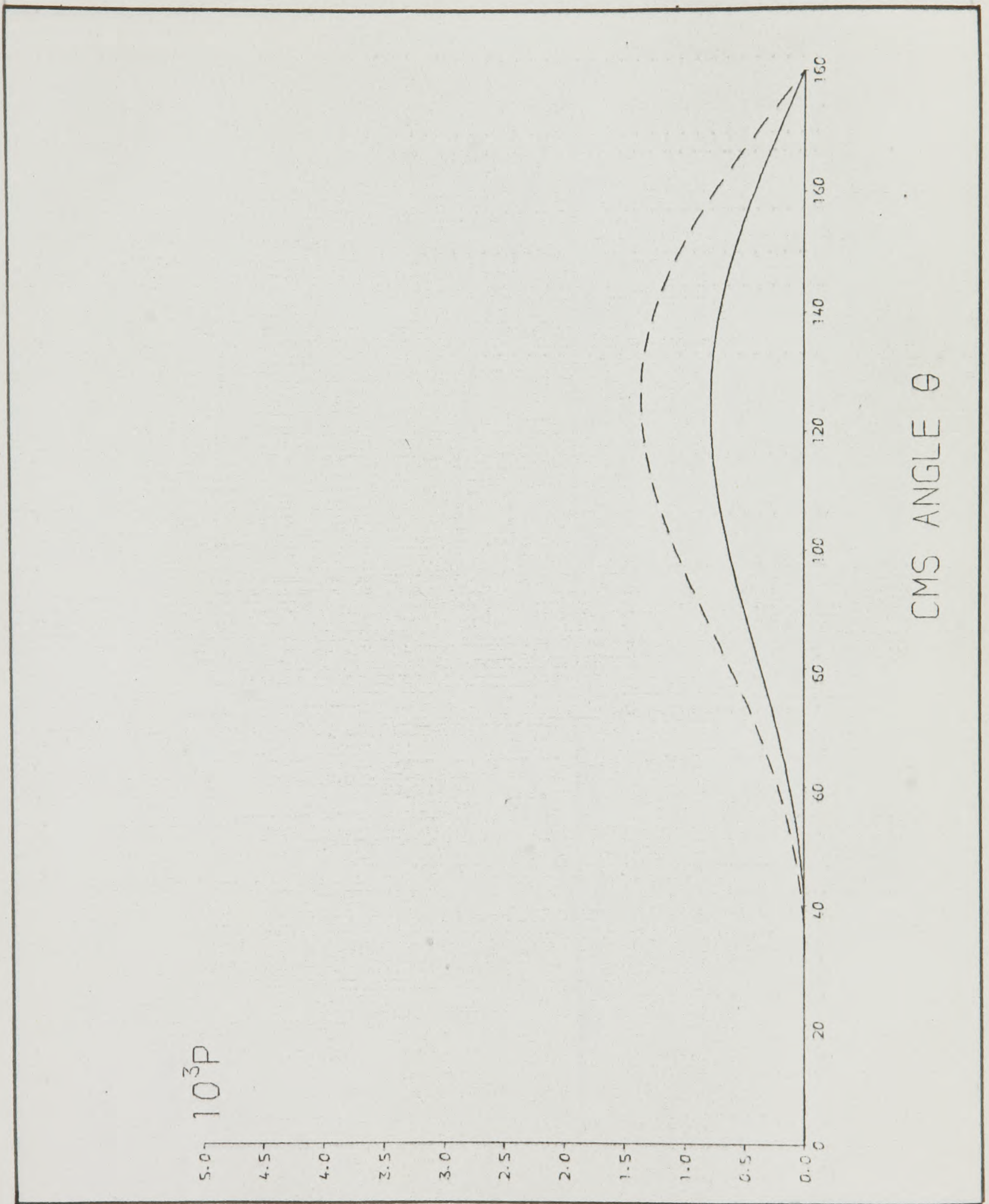
(w₁ is also shown in the figure)

Figure 5.11a



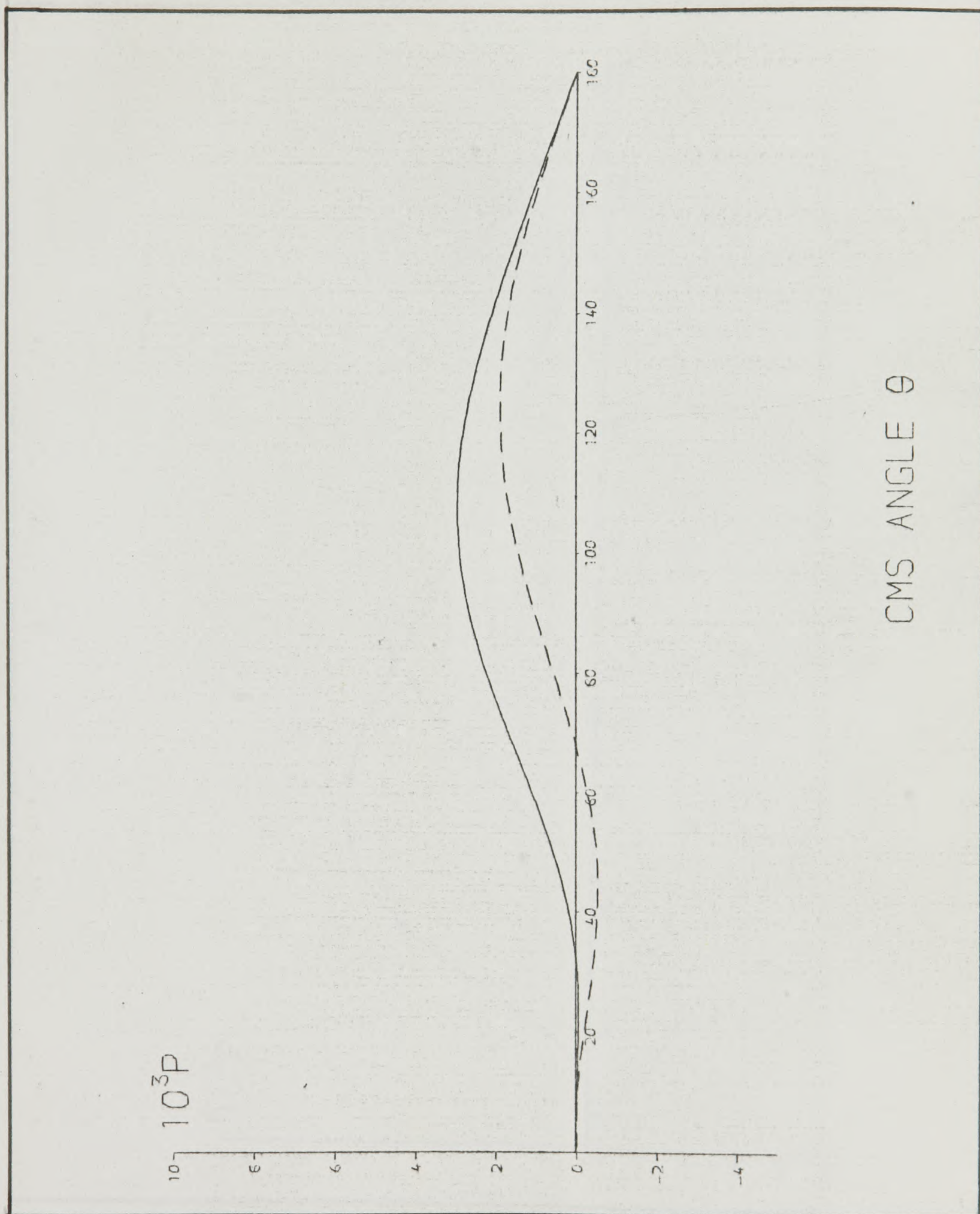
10^3 x Polarization vs. CMS scattering angle θ
for $\omega_L = 100$ MeV.

(ω_L is electron LAB energy)

Figure 5.11b

10^3 x Polarization vs. CMS scattering angle θ for
 $\omega_L = 400$ MeV
 (ω_L is electron LAB energy)

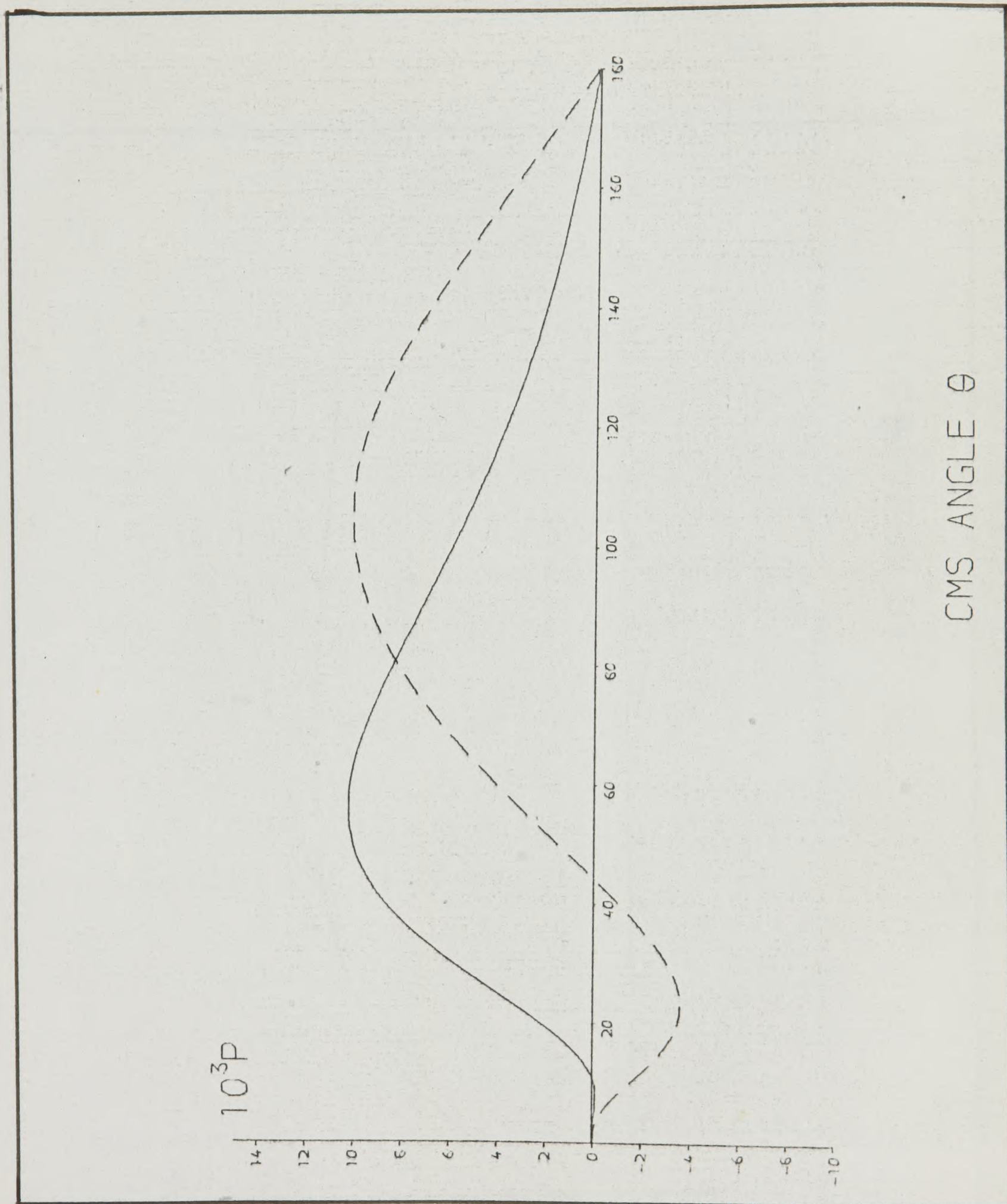
Figure 5.11c



10^3 x Polarization vs. CMS scattering angle θ for
 $\omega_L = 1$ GeV

(ω_L is electron Lab. energy)

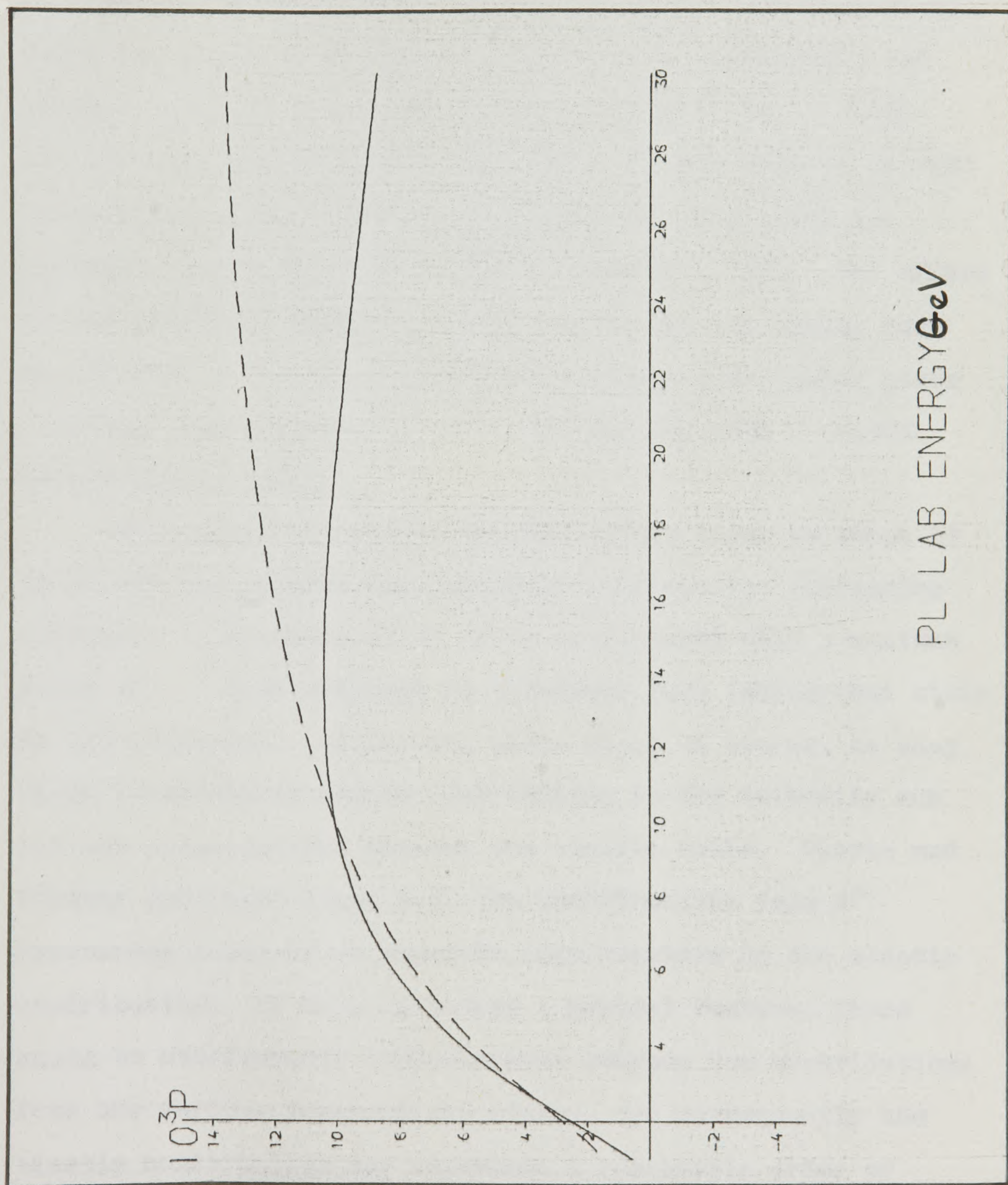
Figure 5.11d



10^3 x Polarization vs. CMS scattering angle θ
for $\omega_L = 10$ GeV

(ω_L is electron LAB energy)

Figure 5.12



10^3 x Polarization vs. electron LAB. energy in GeV,
for fixed CMS scattering angle.

Full Curve ($\beta = 0.71$) :: $\theta = 50^\circ$
Broken Curve ($\beta = 8$) :: $\theta = 100^\circ$.

5.5 Conclusions

It is to be emphasized that these calculations are expected to correspond closely to the physical situation up to electron energies where pion production becomes important. Using the dipole form factors suggested by experiment, the maximum value of the polarization is found to be $\sim 0.08\%$ for electron energies below 400 MeV. We note that at 105 MeV the maximum value is about 0.07% compared with 0.06% in reference [47] and 0.13% quoted in reference [42]. The effect of the anomalous magnetic moment can be seen by noting that at 100 MeV, a 'Dirac' proton with a dipole form factor gives a maximum polarization of $\sim 0.02\%$ compared with $\sim 0.07\%$ as calculated above.

The calculated maximum elastic effect rises to about 1% at 10 GeV and remains approximately constant for increasing energies. This value of 1% is to be compared with a maximum value of $\sim 0.6\%$ obtained in reference [47] (which they state is not reliable). At 10 GeV, there will, of course, be many other intermediate states contributing in the unitarity sum for the polarization, besides the elastic state. Guerin and Piketty [42] have found that the contributions from N^* resonances could be of opposite sign relative to the elastic contribution. If this is indeed a general feature, there could be considerable cancellations between the contributions from the various intermediate states, and consequently the elastic contribution may represent a reasonable order of magnitude estimate even at high energies.

A polarization of the order of 1% is probably too small

to be measured with present experimental errors. To date, experimental data [18] exists for electron Lab. energies up to 18 GeV and for momentum transfers up to about 2 (GeV/c)^2 . The measurements appear to be consistent with $P = 0$ (see Chapter 2). It is to be noted however, that the cross sections for inelastic e-p scattering in the so-called deep inelastic region [60] appear to show that resonance contributions are not the most important: thus the possibility exists that in some kinematic region the polarization may be much larger than the elastic effect.

In the previous * section, calculations of the polarization to the polarization via the exchange of a photon. As energy increases, more inelastic interactions become important and there is the possibility of an enhancement of the polarization importance of the two-photon-exchange term due to the interaction effects. As exact calculations of the polarization of the inelastic multi-particle systems and as theory is yet able to predict these. However, there have been several attempts to estimate the importance of the two-photon-exchange term and we shall now briefly review these calculations.

The earliest attempt to estimate the contribution of two-photon-exchange effects to the polarization of e-p scattering by Drell and Suderman is [19]. This work led to the first of which involved a calculation of the polarization of virtual Compton scattering and the second of which was an approximation with a two-photon-exchange term.

*

A recent preprint by Bloom and Gilman [61] appears to cast some doubt on this conclusion.

Two-photon corrections to the polarization of e-p scattering are discussed in [62].

were negligible for CHAPTER 6

and Fubini [36] used a more realistic wave

function MULTIPARTICLE INELASTIC INTERMEDIATE STATES AND A

and gave RIGOROUS UPPER BOUND FOR THE POLARIZATION

extended the WKB approach to the

6.1 Introduction

virtual photon-exchange processes. In the previous chapter, calculations of the elastic two-photon-effects would suggest a contribution to the polarization of the recoil proton suggested scattering at all energies. At high energies, there a maximum elastic effect of about 1%. At high energies, there will be many other intermediate states which can contribute and Drell and Sullivan [37] have already shown that to the polarization via the unitarity sum. As the energy necessarily involves higher-order terms, more inelastic intermediate states are allowed and quantitative predictions are not possible. There is the possibility of an enhancement of the relative of Guerin and Piquet [32] have already shown that importance of the two-photon-exchange term due to strong interaction effects. An exact calculation would require knowledge of the inelastic multiparticle amplitudes and no theory is yet able to predict these. However, there have been calculation of the effect of exchange resonances in that several attempts to estimate the importance of the two-photon-exchange terms and we shall now briefly review these by neglecting the contributions of longitudinal modes and calculations.

considering only the $\Delta(1232)$ resonance. The earliest attempt to estimate the contribution of the ratio of the σ_{ep} to σ_{ep} due to two-photon-exchange effects in elastic e-p scattering was made by Drell and Ruderman in 1957 [38]. They made two estimates; [39], and the more detailed results of [40] the first of which involved a very crude approximation of the effect of the resonance contribution to the virtual Compton scattering amplitude, and the second, used a few per cent and moreover, the elastic scattering contributions tend to cancel each other out for energies greater for the virtual photon flux. Both estimates indicated that than 500 MeV. His calculations are only valid for energies two-photon corrections to the one-photon-exchange cross sections up to about 1 GeV.

were negligible for electron energies below 500 MeV. Drell and Fubini [39] used a more realistic model for the contribution of the $\Delta(1236)$ to the virtual Compton amplitude and reached the same conclusion. Werthamer and Ruderman [40] extended the WKB approach using the Weiszäcker-Williams approximation, and, after some assumptions concerning the virtual photon-meson production amplitude, concluded that two-photon-effects would remain negligible for small-angle scattering at all energies. Flamm and Kummer [41] have considered the effect of $J^{PC} = 2^{++}$ meson exchange in the s-channel, and Drell and Sullivan [43], 1^{++} meson exchange. Such models necessarily involve unknown coupling constants and precise quantitative predictions are not possible. The calculations of Guerin and Piketty [42] have already been mentioned: they indicate that the resonance contributions are of the same order of magnitude as the elastic contribution, and can be of opposite sign relative to the elastic effect. The most recent calculation of the effect of s-channel resonances is that of Greenhut [44]. The virtual Compton amplitude is approximated by neglecting the contributions of longitudinal photons and considering only the $\Delta(1236)$ resonance. His results for the ratio of the e^+p to e^-p differential cross sections compare well with the qualitative results of Drell and Fubini [39], and the more detailed results of Campbell [63]. The effect of the resonance contribution is found to be less than a few per cent and moreover, the elastic and resonance contributions tend to cancel each other for energies greater than 500 MeV. His calculations are only valid for energies up to about 1 GeV.

We see therefore, that calculations of the multiparticle intermediate states in the two-photon-exchange graphs, have been mainly* restricted to resonance approximations valid up to about 1 GeV. The general conclusion of these calculations is that two-photon effects are unimportant**. However, at high energies many inelastic states are possible and the resonance contributions may be relatively unimportant. The inelastic e-p cross sections, with observation of only the final electron, have now been measured [65] over a large range of q^2 and W^2 - the invariant mass squared of the hadronic final state. In the resonance region, which we define roughly as $W \lesssim 2$ GeV, the inelastic cross sections exhibit a q^2 behaviour qualitatively similar to the elastic cross section above $|q^2| = 1$ (GeV/c)². For larger invariant masses W , the inelastic cross sections have a very different q^2 -dependence. For example [65], at $W = 3.5$ GeV, the quantity $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$ changes by less than a factor of 2 over the q^2 range, while the analogous quantity for elastic scattering changes by some 3 orders of magnitude. (σ_{Mott} is the Mott differential cross section for the scattering of an electron by a point charge with no recoil.) The observation of this weak q^2 behaviour is a fundamental result of the 'deep inelastic' experiments. There are many other interesting features of the deep inelastic data [60] but here we stress only the marked

* There have been several attempts to 'Reggeize' the photon [64], [58]; however, the existing experimental data for e-p scattering is well described by the one-photon-exchange approximation and no specifically Regge-pole effects have been observed.

** There is also a calculation by Harte [45] using a specific 'Bootstrap' model which indicates that two-photon effects may be important at large momentum-transfers.

difference in behaviour of the deep inelastic region from that of the resonance region. Because of this, it may be an unjustifiable extrapolation from the conclusions of resonance calculations, to presume that two-photon effects are negligible at all values of s and t .

In this chapter, we derive a rigorous upper bound for the contribution to the polarization of all the inelastic intermediate states, in terms of measurable cross sections. A numerical estimate of this bound based on existing data, is then given and the results discussed. The next section contains some comments on two types of models for the intermediate state contributions.

6.2 Upper Bound for the Inelastic Contribution

1 Derivation of the Bound

In Chapter 5, the general expression for the polarization in elastic e - p scattering to order α^3 was obtained

$$I_{0P} = \text{Tr}[\gamma_5 \not{\epsilon} \Lambda(k') \Lambda(p') [\mathfrak{m}_2 - \bar{\mathfrak{m}}_2] \Lambda(p) \Lambda(k) \mathfrak{m}_1] \quad (6.1)$$

This can be disentangled from the trace form back to summations over spins

$$I_{0P} = \sum_{\substack{\lambda_f \lambda_1 \\ \mu_f \mu_1}} \left\{ \bar{u}_{\lambda_f}(p') \bar{u}_{\mu_f}(k') [\mathfrak{m}_2 - \bar{\mathfrak{m}}_2] u_{\lambda_1}(p) u_{\mu_1}(k) \right\} \\ \times \left\{ \bar{u}_{\lambda_1}(p) \bar{u}_{\mu_1}(k) \mathfrak{m}_1 \gamma_5 \not{\epsilon} u_{\lambda_f}(p') u_{\mu_f}(k') \right\} \quad (6.2)$$

As before, we may express $\mathfrak{m}_2 - \bar{\mathfrak{m}}_2$ via the unitarity

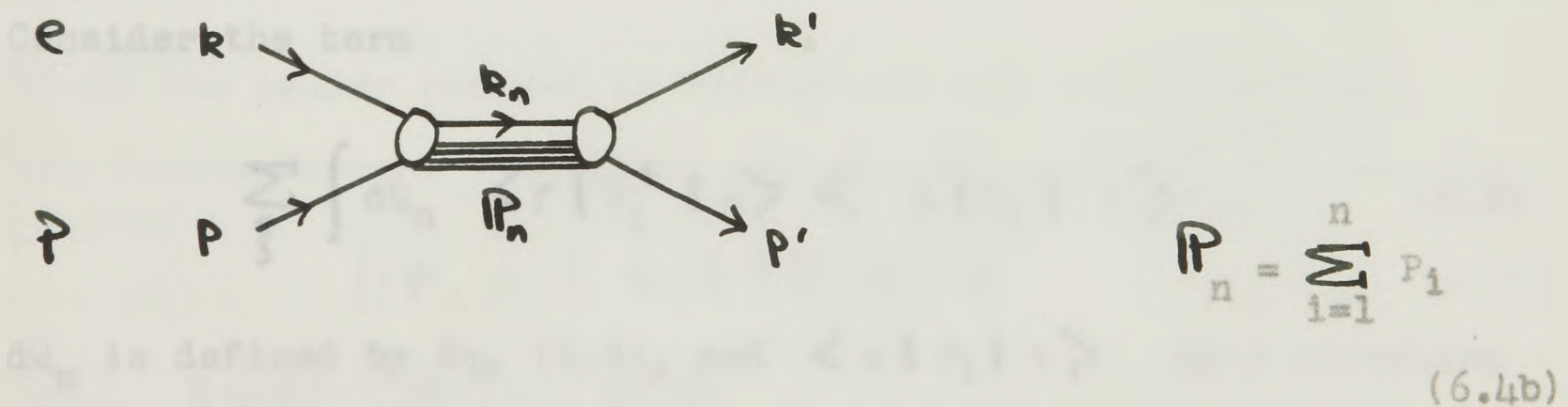
relation:

$$\bar{u}_{\lambda_f} \bar{u}_{\mu_f} [m_2 - \bar{m}_2] u_{\lambda_1} u_{\mu_1} = (+1) \sum_n (2\pi)^4 \delta^4(P_1 - P_n) N_n^2 \langle f | T_1^+ | n \rangle \langle n | T_1 | i \rangle \quad (6.3)$$

The summation over n runs over all possible intermediate states: in our previous calculation only the elastic state $e p$ was retained. We now wish to bound the contribution from all the inelastic states that can contribute when $s \gg (M + m_\pi)^2$ ($m_e \sim 0$). Each intermediate state contains an electron and a certain number n hadrons - as shown in Fig. 6.1.

$$P_n = k_n + \underbrace{p_1 + p_2 + \dots + p_n}_{n \text{ hadrons}} \quad (6.4a)$$

Figure 6.1



Thus

$$\sum N_n^2 \rightarrow \sum \frac{\mu}{(2\pi)^3} \int \frac{d^3 k_n}{\omega_n} \prod_{j=1}^n \frac{d^3 p_j}{p_j^0} \frac{(R_j)}{(2\pi)^{3n}} \quad (6.5)$$

k_n and P_n are held constant.

(ξ represents the different intermediate states and the spin variables).

The hadronic integration may be written

$$\int \prod_{j=1}^n \frac{d^3 p_j}{p_j^0} \cdot \frac{R_j}{(2\pi)^{3n}} = \int d^4 P_n \prod_{j=1}^n \frac{d^3 p_j}{p_j^0} \delta^4(P_n - \sum_j p_j)$$

$$\times \frac{R_j}{(2\pi)^{3n}}$$

In a similar notation

$$\equiv \int d^4 P_n dQ_n \quad (6.6)$$

Therefore : k_n^2 .

$$I_n(f_i) = \sum_n (2\pi)^4 \delta^4(P_1 - P_n - k_n) N_n^2 \langle f | T_1^+ | n \rangle \langle n | T_1 | i \rangle$$

Ψ and ϕ :

$$= (2\pi) \mu \sum_{\xi} \int \frac{d^3 k_n}{\omega_n} dQ_n \langle f | T_1^+ | n \rangle \langle n | T_1 | i \rangle \quad (6.7)$$

Consider the term

Since the scalar product is defined in this function space,

$$\sum_{\xi} \int dQ_n \langle f | T_1^+ | n \rangle \langle n | T_1 | i \rangle \quad (6.8)$$

dQ_n is defined by Eq. (6.6), and $\langle n | T_1 | i \rangle$ is a shorthand for the amplitude:

$$\langle k_n; p_1 \dots p_n; \xi | T_1 | k, p; \lambda_1 \mu_1 \rangle$$

k, p, λ_1, μ_1 are fixed, and for the integration over dQ_n ,

k_n and P_n are held constant. Thus $\langle n | T_1 | i \rangle$ is a

complex amplitude which is a function only of the variables ξ , and the $3n-4$ momentum variables. We write symbolically

$$\langle n | T_1 | i \rangle \sim \phi(p_1 \dots p_n; \xi) \quad (6.9a)$$

which is to be understood as representing $\langle n | T_1 | i \rangle$ at fixed k, p, k_n and P_n ; for initial spin variables λ_1, μ_1 . We define also $P_n^2 = W^2$ and $t_1 = (k - k_n)^2$.

In a similar notation

$$\langle f | T_1^+ | n \rangle \sim \psi^*(p_1 \dots p_n; \xi) \quad (6.9b)$$

at fixed k', p', k_n, P_n for λ_f, μ_f . Likewise we define $t_2 = (k' - k_n)^2$.

In this notation it is easy to see that Eq. (6.8) defines the invariant scalar product of the two functions ψ and ϕ :

$$\sum_{\xi} \int dQ_n \psi^*(p_1 \dots p_n; \xi) \phi(p_1 \dots p_n; \xi) \equiv (\psi, \phi) \quad (6.10)$$

Since the scalar product is defined in this function space, the Cauchy-Schwarz inequality may be used to bound this product

$$|(\psi, \phi)| \leq \|\psi\| \cdot \|\phi\| \quad (6.11)$$

where $\|\psi\|^2 = \sum_{\xi} \int dQ_n \psi^* \psi$

and similarly for $\|\phi\|^2$.

Now $\|\phi\|^2$ and $\|\psi\|^2$ can easily be related to the differential cross section for inelastic e-p scattering, in which only the final electron is detected:

is obtained from (6.2) and (6.3)

$$I_n \leq \sum_{\lambda, \lambda_1} | \dots |$$

For given initial helicities $\lambda_i \mu_i$, the differential cross section is given, in one-photon-exchange approximation, by

$$d\sigma_{\lambda_i \mu_i} = \frac{2\pi}{[(k \cdot p)^2 - \mu^2 M^2]^{\frac{1}{2}}} \cdot \frac{d^3 k_n}{\omega_n} \cdot \sum_{\{j=1\}^n} \prod_{j=1}^n \frac{d^3 p_j}{p_j^0} \times \frac{\delta^4(p+q-n)}{(2\pi)^{3n}} \quad (6.12)$$

Consequently, where $\gamma = \mu^2 M \prod_{j=1}^n R_j |\langle n | T_1 | i \rangle|^2$.

$$\frac{d\sigma_{\lambda_i \mu_i}}{d^3 k_n} = \frac{2\pi \mu^2 M / \omega_n}{[(k \cdot p)^2 - \mu^2 M^2]^{\frac{1}{2}}} \cdot \|\phi\|^2 \quad (6.13)$$

since useful

$$I_n(f_i) = (2\pi)\mu \int \frac{d^3 k_n}{\omega_n} (\psi, \phi)$$

We obtain :

$$|I_n(f_i)| \leq \frac{[(k \cdot p)^2 - \mu^2 M^2]^{\frac{1}{2}}}{\mu M} \int d^3 k_n \left\{ \frac{d\sigma'_{\lambda_f \mu_f}}{d^3 k_n} \cdot \frac{d\sigma_{\lambda_i \mu_i}}{d^3 k_n} \right\}^{\frac{1}{2}} \quad (6.14)$$

(1) Integral over inelastic states $I_n(f_i)$ where $d\sigma$ is evaluated at k, p, k_n fixed and $d\sigma'$ at k', p', k_n fixed. This is a rigorous upper bound for the contribution of the inelastic intermediate states in terms of measurable inelastic cross sections. The upper bound for the polarization is obtained from (6.2) and (6.3) :

$$I_0^P \ll \sum_{\substack{\lambda_f \lambda_i \\ \mu_f \mu_i}} |I_n(f_i)| \cdot \left| \left\{ \bar{u}_{\lambda_i \mu_i} m_1 \gamma_5^{\lambda_f} u_{\lambda_f \mu_f} \right\} \right| \quad (6.15)$$

Thus, in the absence of polarized cross sections, only an inequality may be obtained. The one-photon-exchange amplitude in Eq. (6.15) can be evaluated in terms of the form factors G_E and G_M .

Before we proceed to make a rough estimate of the bound, we notice the following points. I_0 is proportional to the elastic cross section and falls off very rapidly as q^2 increases. The one-photon amplitude in the sum falls off roughly as $(I_0)^{\frac{1}{2}}$. Consequently if $|I_n(f_i)|$ - the integral over the square-root of the inelastic cross sections - falls off slowly with q^2 , then the bound can become very large. In fact, since the form in which the bound is written has lost the property of the polarization as being a difference of cross sections, we have the possibility that at high q^2 the bound may be greater than unity. In this case, the bound is no longer useful!

2 Numerical Estimate of the Bound

In this section we shall make an order of magnitude estimate of the bound.

(1) Integral over Inelastic Cross Sections : $I_n(f_i)$

At present, only unpolarized cross sections have been measured :

$$\frac{d\sigma_0}{d^3k_n} = \frac{1}{4} \sum_{\lambda \mu} \frac{d\sigma_{\lambda \mu}}{d^3k_n}$$

Parity gives the relation

$$\frac{d\sigma_{-\lambda-\mu}}{d^3k_n} = \frac{d\sigma_{\lambda\mu}}{d^3k_n}$$

Thus, in the absence of polarized cross sections, only an inequality may be obtained

$$\frac{d\sigma_{\lambda\mu}}{d^3k_n} \leq 2 \frac{d\sigma_0}{d^3k_n}$$

However, for an order of magnitude estimate we shall assume

$$\frac{d\sigma_{\lambda\mu}}{d^3k_n} \sim \frac{d\sigma_0}{d^3k_n} \quad (6.16)$$

The integrals will be evaluated in the overall CMS as in Chapter 5, except that for an inelastic intermediate state : $|\underline{k}| \neq |\underline{k}_n|$. We have (see Fig. C.1)

$$\begin{aligned} t_1 &= -2 k k_n (1 - \cos\theta_1) \\ t_2 &= -2 k k_n (1 - \cos\theta_2) \end{aligned}$$

with

$$\cos\theta_2 = \cos\theta \cos\theta_1 + \sin\theta \sin\theta_1 \cos\phi$$

$$W^2 = E^2 - 2 E k_n$$

where E is the total CMS energy. (The electron mass is neglected.)

Since for unpolarized cross sections, $d\phi \rightarrow 2\pi$, we can rewrite $I_n(\text{fi})$ by making the change of variable:

virtual photon $\frac{d^3 k_n}{\omega_n} = \frac{1}{4k \mathcal{E}} dW^2 dt_1 d\phi$ (6.17)

We obtain

The behaviour of $I_n(f_i) \sim I_n(0) = \frac{k \mathcal{E}}{2\pi\mu M} \int dW^2 dt_1 d\phi \left\{ \frac{d\sigma_0(t_2)}{dt dW^2} \frac{d\sigma_0(t_1)}{dt dW^2} \right\}^{\frac{1}{2}}$ (6.18)

The limits of integration are

with $\int_{W^2=(M+m_\pi)^2}^s dW^2 \int_{-4kk_n}^0 dt_1 \int_0^{2\pi} d\phi$

and where the factor $4kk_n = (s-M^2)(s-W^2)/s$ is the photoproduction value.

We must now parameterize the inelastic cross section: this may be written in invariant form as

For the integral (6.18) we now make the following ansatz $\frac{d^2\sigma}{dt dW^2} = H(t, W^2) \sigma_{\text{exp}}(t, W^2)$ (6.19)

(i) The lower limit for the W^2 integration is taken to be where

$H(t, W^2) = \frac{a}{4\pi} \cdot \frac{K}{t} \cdot \frac{4M}{(s-M^2)^2} \cdot \frac{1}{(1-\mathcal{E})}$

to the polar angle $K = (W^2 - M^2)/2M$ (The Hand Flux factor)

from Fig. 6.2.

(ii) $\mathcal{E} = \left\{ 1 + \frac{2[(p \cdot q)^2 - M^2 t]}{[(2k \cdot p)(2k' \cdot p) + M^2 t]} \right\}^{-1}$ (The electron type) $(t \equiv q^2)$

* In fact, since $\mathcal{E} = \sigma/\sigma_0$ appears to be unambiguously

$\sigma_{\text{exp}}(t, W^2)$ is a combination of the transverse and scalar

virtual photon cross sections *

$$\sigma_{\text{exp}}(t, W^2) = \sigma_{\text{T}}(t, W^2) + \epsilon \sigma_{\text{S}}(t, W^2)$$

The behaviour of σ_{exp} with W at constant t is displayed in Fig. 6.2. For muon-proton inelastic scattering [66], Perl et al. [67] have given some simple parameterizations for σ_{exp} for $W > 2$ GeV. Their best fit is

$$\sigma_{\text{exp}}^{(\mu)}(t, W^2) = \frac{S_{W^2}}{(1-Rt)} \quad (6.20)$$

with

$$R = 1.38 \pm 0.22 \quad (\text{GeV}/c)^2$$

and where the fitted values of S_{W^2} are consistent with the photoproduction values.

The muon data seems to be compatible with the electron data [66].

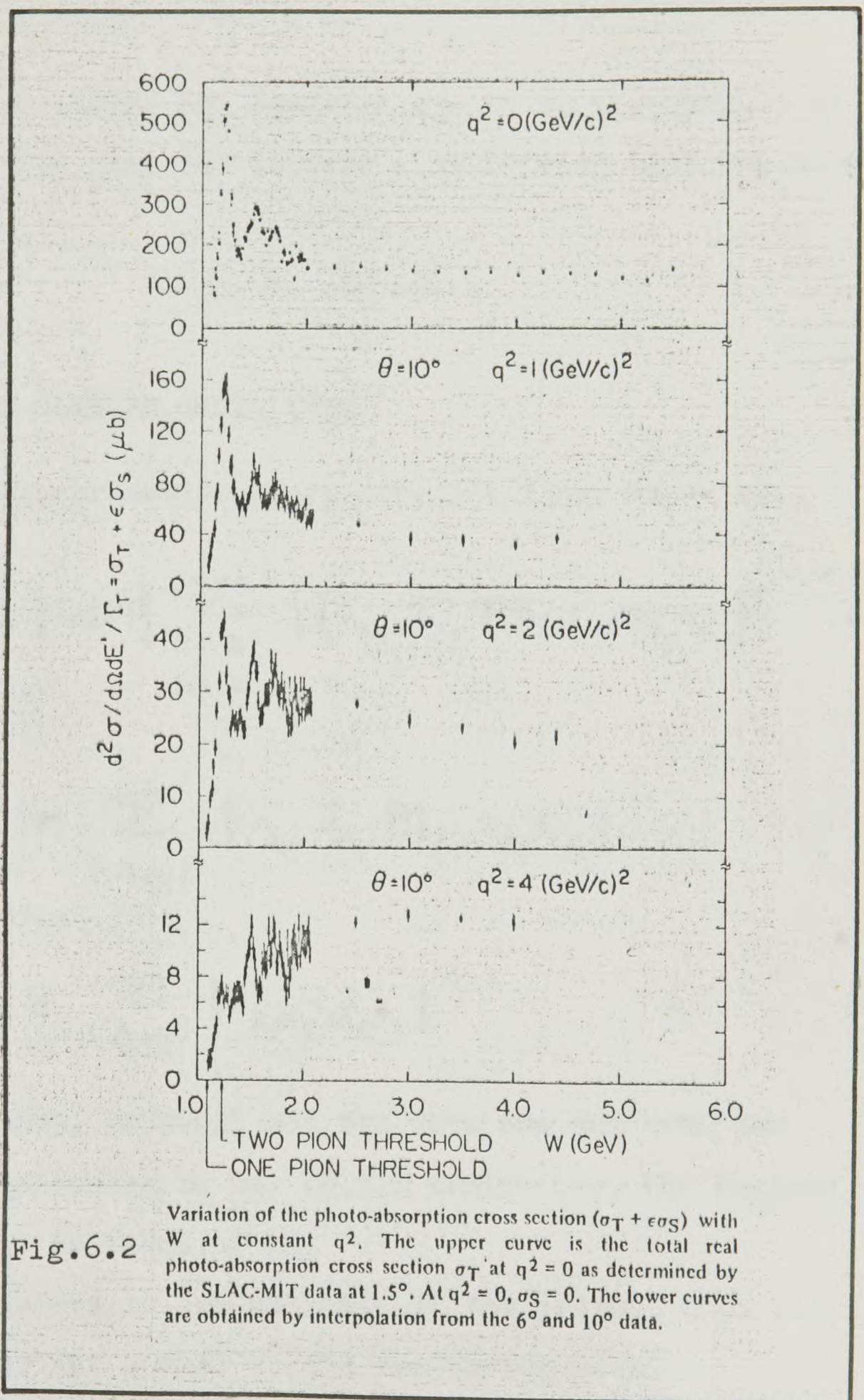
For the integral (Eq. (6.18)) we now make the following crude approximations :-

(i) The lower limit for the W^2 integration is taken to be $W^2 = 4$. Below this value of W^2 , resonance production is important and we assume that this region gives no sizable contribution to the polarization. This cut-off can be seen to be plausible from Fig. 6.2.

(ii) The fit of Eq. (6.20) is assumed for the electron data.

* In fact, since $R = \sigma_{\text{S}}/\sigma_{\text{T}}$ appears to be considerably less than 1 [65], σ_{exp} is approximately σ_{T} .

Figure 6.2



$\sigma_{\text{exp}}(q^2, W^2)$ taken from reference [65]
 (Note that q^2 in figure is $|q^2|$ in our metric.)

Since this fit as assumed for the whole range of integration over t , at high s this involves a considerable extrapolation of the data.

(iii) We also assume the function S_{W2} to be a constant: as can be seen from Fig. 6.2 this is a reasonable approximation.

We take

$$S_{W2} = 120 \mu\text{b.}$$

(2) Sum over Elastic Amplitudes

In the approximation above, the bound may be written

$$I_{OP} \leq |I_n(0)| \sum_{\substack{\lambda_f \lambda_i \\ \mu_f \mu_i}} | \{ \bar{u}_{\lambda_i} \bar{u}_{\mu_i} m_1 \gamma_5 u_{\lambda_f} u_{\mu_f} \} | \quad (6.21)$$

where

$$\begin{aligned} I_0 &= \sum_{(\lambda, \mu)} | \bar{u}_{\lambda_f} \bar{u}_{\mu_f} m_1 u_{\lambda_i} u_{\mu_i} |^2 \\ &\equiv \sum_{(\lambda, \mu)} | T_{\lambda_f \mu_f : \lambda_i \mu_i} |^2 \end{aligned} \quad (6.22)$$

For our estimate, we shall not evaluate the sum over the one-photon amplitudes in Eq. (6.21) explicitly, but instead approximate it in terms of I_0 .

If λ refers to the helicity of the proton, then since s is the covariant normal to the scattering plane

$$\gamma_5 \not{s} u_{\lambda} \sim u_{-\lambda}$$

Thus the sum in Eq. (6.21) may be written

$$\sum_{(\lambda, \mu)} |T_{\lambda_f \mu_f: \lambda_i \mu_i}|$$

Using the approximations described in the previous section, since, in the limit of zero electron mass, the electron helicity is conserved, this involves a sum over 8 CMS amplitudes. For our rough estimate we assume these are all of the same order of magnitude, and therefore

$$I_0 = \sum_{(\lambda, \mu)} |T|^2 \sim 8 |\bar{T}|^2$$

Then:

$$\sum_{(\lambda, \mu)} |T| \sim 8 |\bar{T}| = [8 I_0]^{1/2}$$

The Rosenbluth cross section can be written in terms of I_0 :

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} \cdot \left(\frac{\mu M}{k E}\right)^2 \cdot \frac{1}{4} I_0 \tag{6.23}$$

a less crude parameterization of the inelastic cross section* and the polarization bound becomes with these approximations:

$$P \ll |In(0)| \cdot \left(\frac{8}{I_0}\right)^{1/2}$$

$$\sim \int d\omega^2 dt_1 d\phi \left\{ \frac{d^2\sigma_0}{dt d\omega^2}(t_2) \frac{d^2\sigma_0}{dt d\omega^2}(t_1) \right\}^{1/2}$$

$$2\pi \left[2\pi \frac{d\sigma}{dt}(t) \right]^{1/2} \tag{6.24}$$

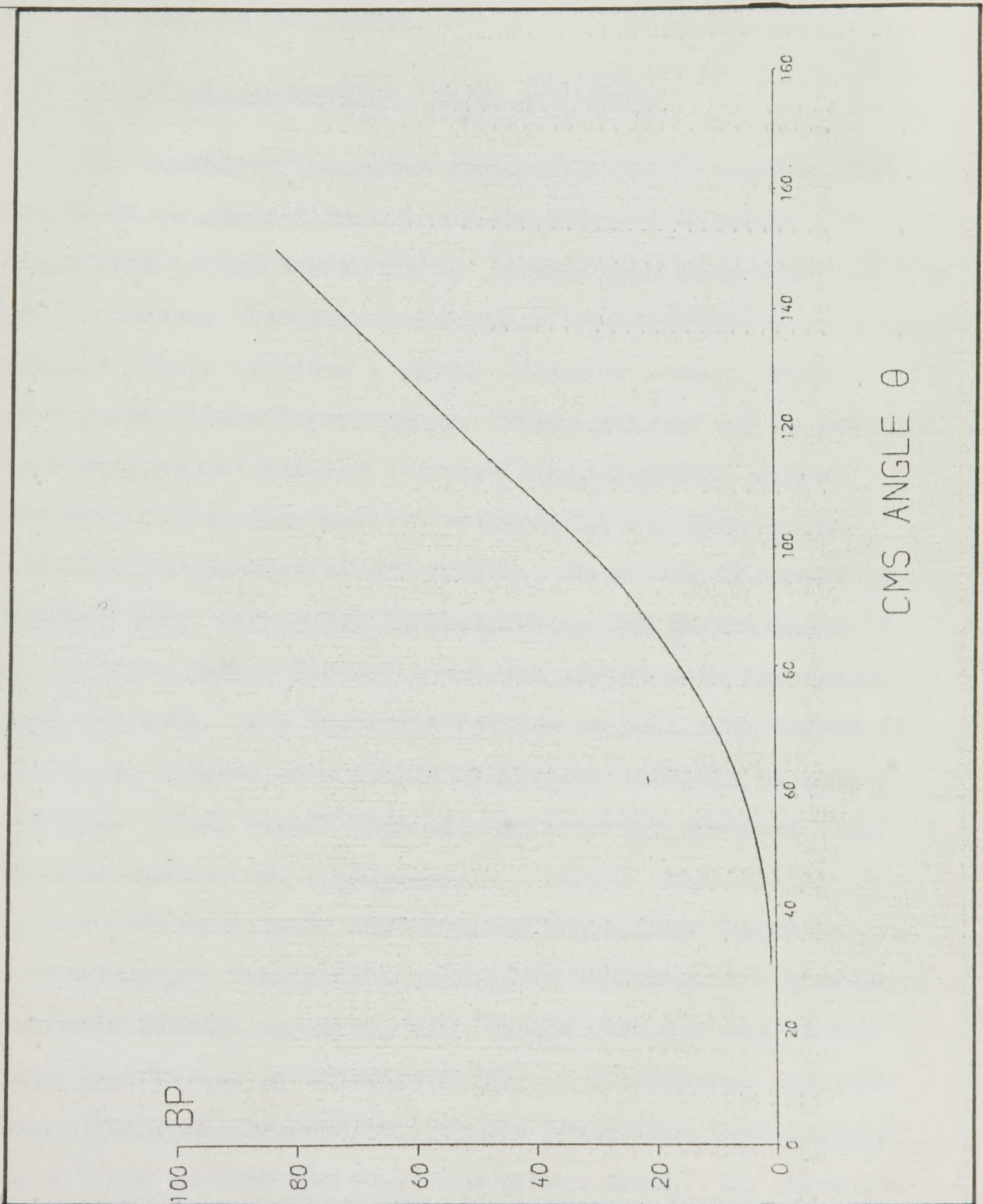
The numerical results obtained with these approximations are discussed in the next section.

* In this form, which involves the above approximation, the bound is very similar to one quoted by De Rujula and de Rafael for neutrino reactions [68].

3 Results and Discussion

Using the approximations described in the previous section, the bound was evaluated using a KDF9 computer: the integrals were evaluated numerically using Simpson's Rule. The integral is convergent but care must be taken with the singularities of the integrand. Figure 6.3 shows the result of the calculation for the angular distribution of the bound, at an incident electron LAB. energy of 3 GeV ($s \sim 7 \text{ (GeV)}^2$). As discussed at the end of Section 6.2.1, the magnitude of the bound increases rapidly with θ ($|t|$). For higher energies, the general shape is similar but the curve rises more steeply. For example at 5 GeV, the bound rises from $\sim 4\%$ at $\theta \sim 30^\circ$, to 30% at $\theta \sim 60^\circ$ and exceeds unity at $\theta \sim 90^\circ$. It seems probable that a more accurate evaluation of the bound, using a less crude parameterization of the inelastic cross sections, and explicitly evaluating the sum over the one-photon-exchange elastic amplitudes, would lead to similar results.

It is clear therefore, that this bound is of very limited use. Even at 3 GeV, for $t \sim -1.2 \text{ (GeV/c)}^2$, the value of the bound is $\sim 6\%$; while the measured polarization at this s and t , is compatible with zero, with an estimated error of $\pm 3\%$ (Kirkman et al. reference [18]). The remaining question is therefore whether it is possible to obtain a better bound. As we showed in Chapter 2, the polarization is constrained to vanish at $\theta = 0^\circ$ and $\theta = 180^\circ$ by angular momentum conservation. It would therefore be desirable to extract this 'sin θ ' factor and thereby obtain a more realistic bound. We have been unable to obtain such a bound, which

Figure 6.3

Polarization Bound BP (in percent) vs. CMS scattering angle θ for incident electron LAB energy $\omega_L = 3$ GeV.

can still be simply related to measured quantities.

6.3 Comments on Two Models

1 An Infinite-Component Theory of Hadrons

The so-called dynamical group theories of hadrons [69] are based on generalizations of the group theoretical treatments of the H-atom [70]. It has been shown that the use of unitary, infinite-dimensional representations of non-compact groups provides a useful framework within which to re-fashion Schroedinger theory. These methods can be extended to relativistic systems: a chosen group together with a suitable representation then contains information on the dynamical properties of the system. In an $O(4,2)$ theory of hadrons [69], the proton is regarded as the ground state of an infinite baryon 'tower' - excited states with increasing mass and spin. The important feature is that each hadron is no longer treated as a single elementary particle of mass M and spin j , but rather that all hadrons with the same internal quantum numbers are considered as a single relativistic system described by an infinite-component wave function. In such an $O(4,2)$ theory with a suitable choice of the electromagnetic current operator, the 'dipole fit' for the proton form factors can be reproduced [69]. Furthermore, explicit expressions may be obtained for the transition form factors to all the excited states of the proton tower. In principle therefore, the contributions from all these excited states may be calculated in a similar fashion to the calculation for

the elastic intermediate state in Chapter 5. For a consistent calculation it would be necessary to use the mass M_j of the intermediate state as predicted by the theory and also the appropriate spin j propagator: a straightforward calculation including more than a few intermediate states appears to involve a prohibitive amount of labour. Moreover, if we wish to use a more realistic model including transitions to baryon states with different internal quantum numbers from the proton - such as the $\Delta(1236)$ - the theory becomes much more involved [71].

This model is entirely a 'resonance type' model and qualitatively, we expect the form factors of the excited states to be of the form of the product of a spin-dependent 'threshold' factor and a factor similar to the elastic form factor [72]. Since the major contributions to the unitarity integrals arise from the regions where one of the denominators is small ($\Delta_1^2 \rightarrow 0$ or $\Delta_2^2 \rightarrow 0$), we expect the threshold factor to damp the contribution of the resonances in these regions. It is therefore plausible that the contributions from resonances are never large*.

In view of these practical considerations, apart from any possible theoretical objections to the use of an infinite component theory, this approach was not pursued any further.

2 An 'Overlap Function' Model

A possible approach for an estimate of the effects of the

* Guerin and Piketty reach a similar conclusion in reference [42].

inelastic states in the unitarity sum, is to approximate their effect by an 'overlap function'. The overlap function arises in the theory of high energy elastic collisions of two hadrons in the following way [73]. At high energy and for small momentum transfers, it is probable that the elastic scattering amplitude, $\langle a'b' | T | a b \rangle = T_{el}$, for the collision of two hadrons a and b , becomes very nearly purely imaginary. The differential cross section also becomes very small except at small values of t , and one concludes that the elastic scattering is essentially shadow scattering caused by the existence of inelastic processes. The overlap function represents an attempt to relate the bulk properties of the inelastic processes to the elastic scattering amplitude: we sketch the derivation below [73].

Unitarity of the S-matrix

$$T - T^\dagger = i T^\dagger T$$

relates the imaginary part of the elastic amplitude to a sum over both elastic and inelastic intermediate states. If we define :

$$|\Psi(a,b)\rangle = \sum_{n'} |n'\rangle \langle n' | T | a b \rangle$$

where n' denotes only inelastic states to be included in the sum, then the wave function $|\Psi(a,b)\rangle$ represents the inelastic states derived from the state $|a b\rangle$. Thus the inelastic unitarity sum may be written

$$\langle \Psi(a'b') | \Psi(a,b) \rangle \sim \delta^4(p_a + p_b - p_{a'} - p_{b'}) F(s,t)$$

The relativistically invariant function $F(s,t)$ measures the overlap between the two inelastic wavefunctions. With the assumption that T_{el} is pure imaginary, we can obtain the approximate relation

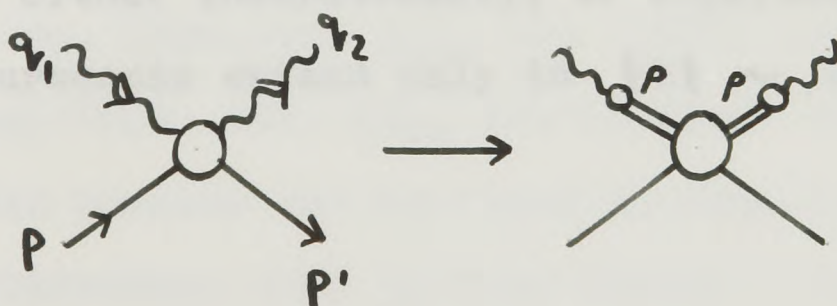
$$T_{el}(s,t) \sim \frac{i}{2} F(s,t)$$

The t -dependence of $F(s,t)$ can therefore be determined from the elastic differential cross section. An approximate parameterization for $F(s,t)$ is

$$F(s,t) = F(s,0) \exp(bt)$$

To apply such a model to virtual Compton scattering, we must make the assumption that the virtual photons behave like hadrons. If we use the ideas of vector-dominance [74] the virtual Compton amplitude may be related to the elastic ρ - P amplitude (ignoring contributions from ω and ϕ): this is represented symbolically in Fig. 6.4.

Figure 6.4



However for photon scattering, we must apply the constraints imposed by current conservation and ascribe tensor indices to the overlap function. Consequently a complete calculation would require knowledge of the invariant amplitudes for

virtual Compton scattering. Nevertheless the idea of approximating all the inelastic effects by an overlap function is a very attractive one, and seems likely to lead to fairly definite predictions. Unfortunately time did not permit a detailed investigation of this type of model.

6.4 Concluding Remarks

[2] The contributions of resonant intermediate states to the recoil proton polarization, are likely to be of the same order of magnitude as that of the elastic intermediate state. Calculations suggest that these resonance effects may be of arbitrary sign with respect to the elastic effect. A rigorous bound for the contribution of all the inelastic intermediate states, which uses the measured inelastic cross sections, yields an unrealistically high bound at large $|t|$. Although it is perhaps unlikely that at large s and $|t|$, the effects of multiparticle intermediate states could conspire to produce an appreciable polarization, such a possibility cannot yet be ruled out, either theoretically, or experimentally - since existing measurements extend only to $|t| \sim 2.0 \text{ (GeV/c)}^2$ [18].

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helicity states. Instead, we shall give a detailed discussion of the phase convention that we use, and of our treatment of the density matrix formalism.

1. One-Particle Helicity States

We consider a system of one free particle of mass m , and spin s . All the possible state vectors of this system form a complex Hilbert space $\mathcal{H}(ms)$, which is the carrier space of the irreducible unitary representation $[ms]$ of the proper Poincaré group. A convenient basis for the space $\mathcal{H}(ms)$ is given by the set of common eigenstates $|p, \lambda, [ms]\rangle$ of the momentum p , helicity λ , and of the two invariants $p_\mu p^\mu = m^2$ and $p_\mu p^\mu = -s^2$ (the internal invariants).

(In what follows the labels $[ms]$ will be omitted.) The relative phases of the basis vectors are uniquely specified

APPENDIX I

The Helicity Formalism and Covariant Formulation
of the Spin Density Matrix

Most of the material in this appendix is quite well-known and can be found in the original paper of Jacob and Wick [13] (JW), or for example, in the books by Werle [19], and Martin and Spearman, [16], where the helicity formalism is discussed in great detail. Consequently this appendix will only briefly mention if at all, many of the important properties of the helicity states. Instead, we shall give a detailed discussion of the phase convention that we use, and of our covariant density matrix formalism.

1. One-Particle Helicity States

We consider a system of one free particle of mass w , and spin s . All the possible state vectors of this system form a complex Hilbert space $\mathfrak{h}(ws)$, which is the carrier space of the irreducible unitary representation $[ws]$ of the proper Poincare group. A convenient basis for the space

$\mathfrak{h}(ws)$ is given by the set of common eigenvectors $|p, \lambda, [ws]\rangle$ of the momentum \underline{p} , helicity $\underline{J} \cdot \underline{p}/p$ and of the two invariants:

$$p_\mu p^\mu = w^2.$$

$$\text{and } p_\mu p^\mu = -w^2 s(s+1) \quad (\text{the Lubanski invariant}).$$

(In what follows the labels $[ws]$ will be omitted.) The relative phases of the basis vectors are uniquely specified

by requiring the vectors $|\underline{p} \lambda\rangle$ to be suitable Lorentz transforms of one fixed basis vector $|000 \lambda_0\rangle$. Since this vector $|000 \lambda_0\rangle$ describes a particle at rest the angular momentum operators \underline{j} coincide with the spin operators \underline{s} . By applying s_{\pm} to our standard vector, we obtain the remaining linearly independent vectors $|000 \lambda\rangle$ with $\lambda = -s, \dots, s$.

We then define the vectors $|\underline{p} \lambda\rangle$ for $0 < \theta < \pi$ and $-\pi < \phi \leq \pi$ by :

$$|\underline{p} \lambda\rangle = |\phi \theta \underline{p} \lambda\rangle = R(\phi, \theta, 0) L_z(v) |000 \lambda\rangle \quad (\text{A.1})$$

where ϕ, θ are the polar angles of \underline{p} . R and L are the appropriate rotation and 'boost' operators respectively. For $\theta = 0$ and $\theta = \pi$, and for $\frac{1}{2}$ integer spin s also along the meridian $\phi = -\pi$, the definition (A.1) is not unique. In fact for helicity states, it is impossible to define the phase of the vector $|\phi \theta \underline{p} \lambda\rangle$ so that it is continuous over the whole unit sphere [20]. We note here that our convention is not identical to that of (JW) where they attempted to minimise these phase discontinuities. However, as we shall show later, for discussion of polarizations, it is more convenient to use the above convention. (In fact, (JW) in their discussion of polarization formulae resort to this convention.)

We must therefore define for \underline{p} parallel or anti-parallel to the z-axis, the corresponding helicity states. We set :

$$|00 \underline{p} \lambda\rangle = L_z(v) |000 \lambda\rangle \quad (\text{A.2})$$

$$\begin{aligned}
 |0 \pi p \lambda\rangle &= |0 0 -p \lambda\rangle = L_{-z}(v) |0 0 0 -\lambda\rangle \\
 &= e^{-i\pi S} R(\pi, \pi, 0) L_z(v) |0 0 0 \lambda\rangle \\
 &= (-1)^{S-\lambda} R(0, \pi, 0) L_z(v) |0 0 0 \lambda\rangle
 \end{aligned} \tag{A.3}$$

with $v = p/p_0$.

These definitions are completely equivalent to (JW) and no modifications of the symmetry properties are needed. The vectors $|p \lambda\rangle$ are assumed to be normalized in a Pioncare invariant manner :

$$\langle p' \lambda' | p \lambda \rangle = \varepsilon \delta^3(p' - p) \delta_{\lambda', \lambda} \tag{A.4}$$

with $\varepsilon = (p^2 + w^2)^{\frac{1}{2}}$.

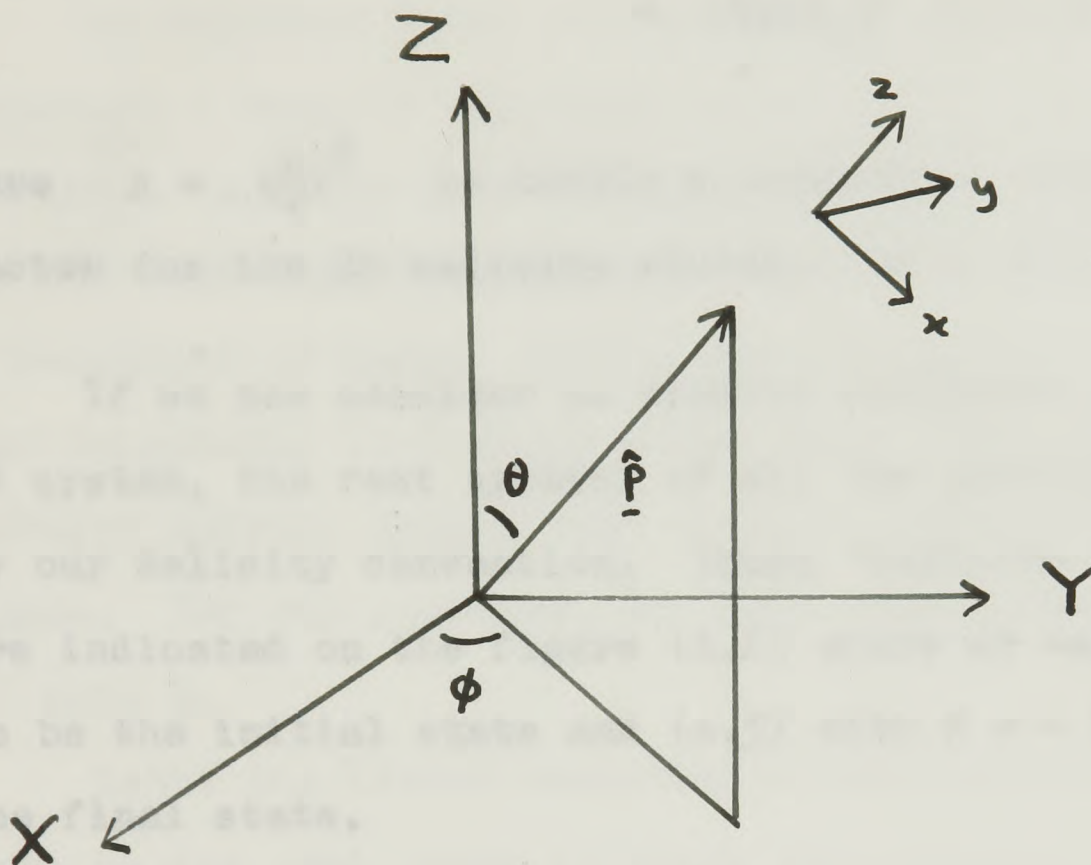
The derivation of the symmetry properties and the construction of the $|j m p \lambda\rangle$ basis is straightforward. We notice here that (A.1) so far has been regarded in the 'Active' sense as a prescription for generating consistently all possible states of a particle with respect to a fixed reference frame. However, it can also be regarded in the 'passive' sense as representing the same state viewed in two different reference systems

$$|0 \theta p \lambda\rangle = R(0, \theta, 0) L_z(v) |0 0 0 \lambda\rangle \tag{A.1}$$

This 'Helicity convention' therefore amounts to a definition of the orientation of the rest frame axes for the particle. If the particle has momentum $(0 \theta p)$ in the frame XYZ then

the rest frame axes xyz are obtained by a rotation $R(\theta \ 0 \ 0)$ from the axes XYZ (see Fig. A.1).

Figure A.1



For a particle travelling in the negative Z -direction our convention

$$|0 \ 0 \ -p \ \lambda\rangle = e^{-i\pi s} R(\pi, \pi, 0) |0 \ 0 \ p \ \lambda\rangle \quad (\text{A.3})$$

amounts to an additional rotation by π about the Z direction for the rest frame axes.

2 Two Particle States

In the CM system, we construct Helicity states for two particles by the following direct product of one particle states:

$$|\underline{P} = 0; 00p; \lambda_1 \lambda_2\rangle = A |0 \ 0 \ p \ \lambda_1\rangle |0 \ 0 \ -p \ \lambda_2\rangle \quad (\text{A.4})$$

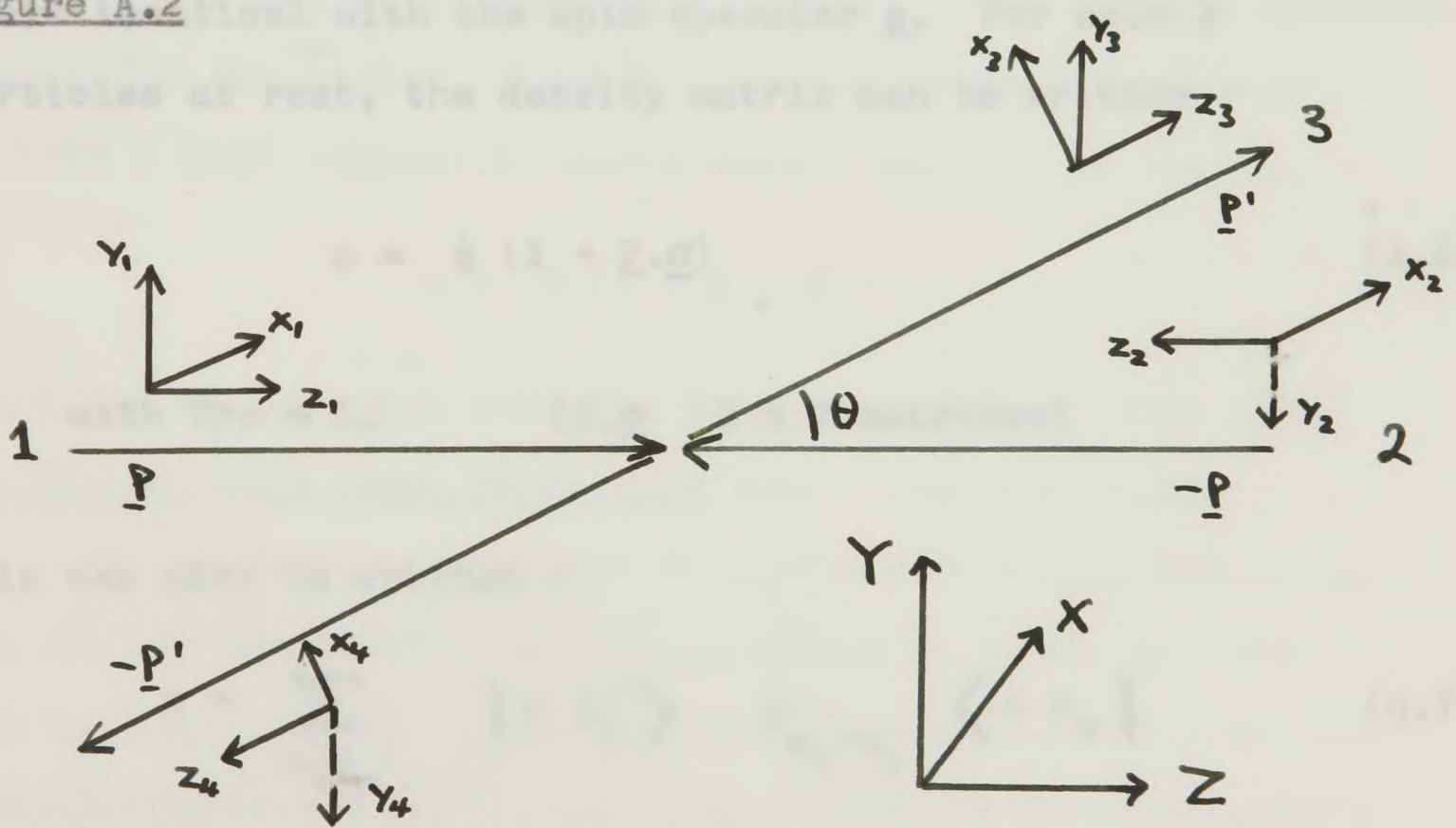
and

$$\begin{aligned}
 | \underline{P} = 0; \theta \neq 0; \lambda_1 \lambda_2 \rangle &= R(\theta, \theta, 0) | \underline{P} = 0; 0 \neq 0; \lambda_1 \lambda_2 \rangle \\
 &= A | \underline{P} \lambda_1 \rangle | -\underline{P} \lambda_2 \rangle
 \end{aligned}
 \tag{A.5}$$

Here $A = \left(\frac{P}{W}\right)^{\frac{1}{2}}$ is merely a convenient normalization factor for the CM helicity states. ($W = P^0$).

If we now consider an elastic collision process in the CM system, the rest systems of all the particles, are defined by our Helicity convention. These 'Helicity Rest Frames' are indicated on the figure (A.2) where we have taken (A.4) to be the initial state and (A.5) with $\theta = 0$ for simplicity, the final state.

Figure A.2



where the $\underline{p}, \underline{p}'$ plane is XZ plane and $\underline{n} = \frac{\underline{p} \wedge \underline{p}'}{|\underline{p} \wedge \underline{p}'|}$ defines the Y axis.

We note that Y_1 and Y_3 are parallel to Y and that Y_2 and Y_4

are antiparallel.

3 Covariant Formulation of the Density Matrix

We consider first the density matrix for one particle. Consider a beam of particles whose linear momentum is well defined, (and the same for each particle in the beam), which in the rest frame possesses a non-zero expectation value for a component of spin. Such a beam is said to be polarized and in general is a mixed state which must be described by a density matrix, rather than a state vector of the Hilbert space. For our purposes, we shall only consider particles of spin $\frac{1}{2}$.

Since the linear momentum is well-defined we can transform to the rest frame in which the angular momentum operators \underline{j} are identical with the spin operator \underline{s} . For spin $\frac{1}{2}$ particles at rest, the density matrix can be written :

$$\rho = \frac{1}{2} (1 + \underline{P} \cdot \underline{\sigma}) \quad (\text{A.6})$$

with $\text{Tr} \rho = 1$. ($1, \underline{\sigma}$ 2 x 2 matrices)

This can also be written :

$$\rho = \sum_{m_s, m_s'} |s, m_s'\rangle \rho_{m_s' m_s} \langle s, m_s| \quad (\text{A.7})$$

where m_s is z-cpt. of spin in the rest frame.

Consider now a frame in which the particles in the beam are travelling with momentum p in the Z-direction. Clearly

we need to boost the state vectors $|0\ 0\ 0\ m_s\rangle$ with

$L_z(v)$:

$$L_z(v) \rho L_z^{-1}(v) = \sum_{m_s' m_s} |0\ 0\ p\ m_s'\rangle \rho_{m_s' m_s} \langle 0\ 0\ p\ m_s|$$

The elements $\rho_{m_s' m_s}$ are clearly the same in the frame XYZ

as they were in the rest frame xyz, and furthermore, these two sets of axes are parallel to each other.

Thus we can write ρ for a beam of particles moving in the z-direction as before (A.6), provided we remember that the spin operators, and polarization \underline{P} refer to the rest frame axes xyz, which in this case are parallel to XYZ. However, for a beam of particles with momentum $\underline{p} = (\emptyset\ \emptyset\ p)$ in the frame XYZ, the helicity formalism implies an additional rotation $R(\emptyset, \emptyset, 0)$ of the rest frame axes. Thus, if we use the helicity basis for the states $|\underline{p}\ \lambda\rangle$, we can write the density matrix ρ with respect to these states exactly as before

$$\rho = \frac{1}{2} (1 + \underline{P} \cdot \underline{\sigma}) \quad (\text{A.6})$$

provided we remember that the spin operators σ_x etc. refer to the helicity rest frame axes xyz. (See Fig. A.1).

In conclusion, the density matrix for moving particles can be written exactly as for particles at rest, provided the moving particle states are specified by Helicity States, and the operators σ_i, P_i are interpreted in the appropriate helicity rest frame moving with the particle.

This procedure is fully relativistic even though the are the expectation values of definite axis polarizations.

result is not manifestly covariant.

The density matrix for two particles can be constructed by a straightforward generalization of this procedure. It can be written exactly as for particles at rest provided we interpret the operators etc. as referred to the appropriate rest frames.

We now specialise to two particles, a and b, both of spin $\frac{1}{2}$. A complete set of operators in the combined spin space is :

$$I = I_a \otimes I_b, \quad I_a \otimes \sigma_n^k, \quad \sigma_a^i \otimes I_b, \quad \sigma_a^i \otimes \sigma_b^k \quad (\text{A.7})$$

(We often abbreviate this notation and write for example $\sigma_a^i \sigma_b^k$ bearing in mind that $\underline{\sigma}_a$ and $\underline{\sigma}_b$ refer to different subspaces.) The density matrix for two uncorrelated particles can be written as a direct product of ρ_a and ρ_b , the density matrices corresponding to the individual particles:

$$\rho = \rho_a \otimes \rho_b = \frac{1}{4} (I_a + \underline{P}_a \cdot \underline{\sigma}_a) (I_b + \underline{P}_b \cdot \underline{\sigma}_b) \quad (\text{A.8})$$

In the general case we have :

$$\rho = \frac{1}{4} (I + \underline{P}_a \cdot \underline{\sigma}_a + \underline{P}_b \cdot \underline{\sigma}_b + \sum_{i,k} C_{ik} \sigma_a^i \sigma_b^k) \quad (\text{A.9})$$

where the real coefficients C_{ik} have the physical meaning of measuring simultaneously the i^{th} cpt of σ_a for particle a, and the k^{th} cpt of σ_b for particle b. The C_{ik} therefore are the expectation values of definite spin correlations.

In the relativistic case, we must remember that $\underline{\sigma}_a^P$ and σ_b^P refer to different inertial frames.

To summarize, the density matrix will be specified in the helicity basis :

$$\rho_{\lambda'_a \lambda'_b; \lambda_a \lambda_b} \quad (A.10)$$

where λ_a, λ'_a and λ_b, λ'_b refer to the z-cpt of spins in the helicity rest frames of particles a and b respectively.

The use of different coordinate systems for each particle is characteristic of the use of helicity states. It provides an easy way to circumvent the technical problems associated with the polarization of relativistic particles.

where $g^{\mu\nu}$ is the metric tensor

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Unless otherwise stated, we use above notation, a representation with γ^0 hermitian and the γ_i antihermitian with γ_2 imaginary:

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \gamma_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix} \quad (A.11)$$

with the usual representation of the 2×2 Pauli matrices

APPENDIX 2

METRIC, DIRAC MATRICES AND TRACE THEOREMS

Our notation is that used by Bjorken and Drell [49] and is explained in detail in their Appendix A. Here we present a summary and some further results that we need in our calculations. The scalar product of two four vectors a^μ and b^μ is denoted by

$$a^\mu b_\mu = a \cdot b = a^0 b^0 - \underline{a} \cdot \underline{b} \quad (\text{B.1})$$

The γ -matrices in the Dirac equation satisfy the anti-commutation relations

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (\text{B.2})$$

where $g^{\mu\nu}$ is the metric tensor with the properties:

$$g^{\mu\nu} = \begin{cases} +1 & \text{for an even permutation of } 0, 1, 2, 3 \\ -1 & \text{for an odd permutation of } 0, 1, 2, 3 \\ 0 & \text{if two indices are the same} \end{cases}$$

then γ_5 may be written:

Unless otherwise stated, we use where necessary, a representation with γ^0 hermitian and the γ_i antihermitian with γ_2 imaginary:

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \underline{\gamma} = \begin{bmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{bmatrix} \quad (\text{B.3})$$

Further, using

with the usual representation of the 2 x 2 Pauli Matrices

$\sigma^\mu = (1, \underline{\sigma})$. For the inner product of a γ matrix with a four vector the Feynman 'slash' notation is used:

$$\gamma_\mu A^\mu = \not{A} = \gamma^0 A^0 - \underline{\gamma} \cdot \underline{A} \quad (\text{B.4})$$

It is useful to define the combinations

$$\sigma^{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu] \quad (\text{B.5})$$

$$\text{and } \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \gamma_5 \quad (\text{B.6})$$

In the above representation

$$\gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

If we define the antisymmetric tensor of rank 4, $\epsilon_{\alpha\beta\gamma\delta}$ with the properties:

$$\begin{aligned} \epsilon_{\alpha\beta\gamma\delta} &= +1 \quad \text{for an even permutation of } 0, 1, 2, 3 \\ &= -1 \quad \text{for an odd permutation of } 0, 1, 2, 3 \\ &= 0 \quad \text{if two indices are the same} \end{aligned}$$

then γ_5 may be written:

$$\gamma_5 = \frac{i}{4!} \epsilon_{\alpha\beta\gamma\delta} \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \quad (\text{B.7})$$

With this form it is easy to show

$$\gamma_5 \gamma^\sigma = \frac{i}{3!} \epsilon_{\alpha\beta\gamma\sigma} \gamma^\alpha \gamma^\beta \gamma^\gamma \quad (\text{B.8})$$

Further, using

$$\epsilon^{\sigma\mu\nu\rho} \epsilon_{\sigma\alpha\beta\gamma} = - \begin{vmatrix} \delta_{\mu\alpha} & \delta_{\mu\beta} & \delta_{\mu\gamma} \\ \delta_{\nu\alpha} & \delta_{\nu\beta} & \delta_{\nu\gamma} \\ \delta_{\rho\alpha} & \delta_{\rho\beta} & \delta_{\rho\gamma} \end{vmatrix}$$

(remembering that $\epsilon^{0123} = -1$) we find

$$\gamma^\mu \gamma^\nu \gamma^\rho = g^{\mu\nu} \gamma^\rho - g^{\mu\rho} \gamma^\nu + g^{\nu\rho} \gamma^\mu + i\gamma_5 \epsilon^{\mu\nu\rho\sigma} \gamma_\sigma \quad (\text{B.9})$$

Finally, we collect together a list of some useful trace theorems. Proofs when they are not indicated, can be found for example in reference [49].

(1) Trace of odd number of γ 's vanishes

(2) $\text{Tr } 1 = 4$; $\text{Tr } \not{a} \not{b} = 4 a \cdot b$

(3) $\text{Tr}[\not{a}_1 \dots \not{a}_n] = (a_1 \cdot a_2) \text{Tr}[\not{a}_3 \dots \not{a}_n] - (a_1 \cdot a_3) \text{Tr}[\not{a}_2 \not{a}_4 \dots \not{a}_n]$
 $+ \dots + (a_1 \cdot a_n) \text{Tr}[\not{a}_2 \dots \not{a}_{n-1}]$

(4) $\text{Tr}[\gamma_5] = 0$

$\text{Tr}[\gamma_5 \not{a} \not{b}] = 0$

$\text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d}] = 4i \epsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta$

For this last trace product, we often use the shorthand ^{*}:

$$\text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d}] \equiv (a b c d)$$

* Note this notation differs by factors of 4i etc. from that used by other authors, e.g. reference [42].

$$(5) \quad \text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d} \not{e} \not{f}]$$

Using $\not{a}_1 \not{a}_2 = -\not{a}_2 \not{a}_1 + 2a \cdot b$ leads to identities of the form:

$$\begin{aligned} 0 &= (2a \cdot b) (cdef) - (2a \cdot c) (bdef) \\ &\quad + (2a \cdot d) (bcef) - (2a \cdot e) (bcdf) \\ &\quad + (2a \cdot f) (bcde) \end{aligned}$$

Use of the γ -matrix identity (B.9), leads to a trace theorem which to our knowledge has not been given in the literature. We obtain for example

$$\begin{aligned} \text{Tr}[\gamma_5 \not{a} \not{b} \not{c} \not{d} \not{e} \not{f}] &= (a \cdot b) (cdef) + (b \cdot c) (adef) \\ &\quad - (a \cdot c) (bdef) + (d \cdot e) (abcf) \\ &\quad - (d \cdot f) (abcf) + (e \cdot f) (abcd) \end{aligned}$$

$$(6) \quad \text{Tr}[\not{a}_1 \not{a}_2 \dots \not{a}_{2n}] = \text{Tr}[\not{a}_{2n} \dots \not{a}_2 \not{a}_1]$$

The proof of this theorem rests on the fact that there exists a matrix C such that

$$C \gamma_\mu C^{-1} = -\gamma_\mu^T$$

(The superscript T indicates transpose).

This theorem is very useful in polarization calculations.

(7) Finally, for the sake of completeness although the result is not needed in this thesis, we give the following interesting trace theorem :

$$\text{Tr}[\gamma_\mu P] \neq \text{Tr}[\gamma^\mu P']$$

$$= 2 \text{Tr}[(P + P_R) P']$$

$$= 2 \text{Tr}[P (P' + P'_R)]$$

where P and P' are products of odd numbers of γ -matrices and any number of γ_5 matrices. P_R and P'_R are the same products with all the γ -matrices written in reverse order.

A proof of this theorem may be found in the book by Eisele [76].



$$\begin{aligned} \epsilon &= \text{small} \\ R &= \text{large} \\ \epsilon &= \text{small} \end{aligned}$$

For $\epsilon < a < b < R$, the denominator of the integrand is never zero (for a in the physical region $|a| < 1$), and the result is real and well-defined. The integral may be evaluated using Feynman's method [63] based on the identity

$$\frac{1}{a^2} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2}$$

INTEGRALS FOR THE UNITARITY CALCULATIONS

The basic unitarity integral is of the form

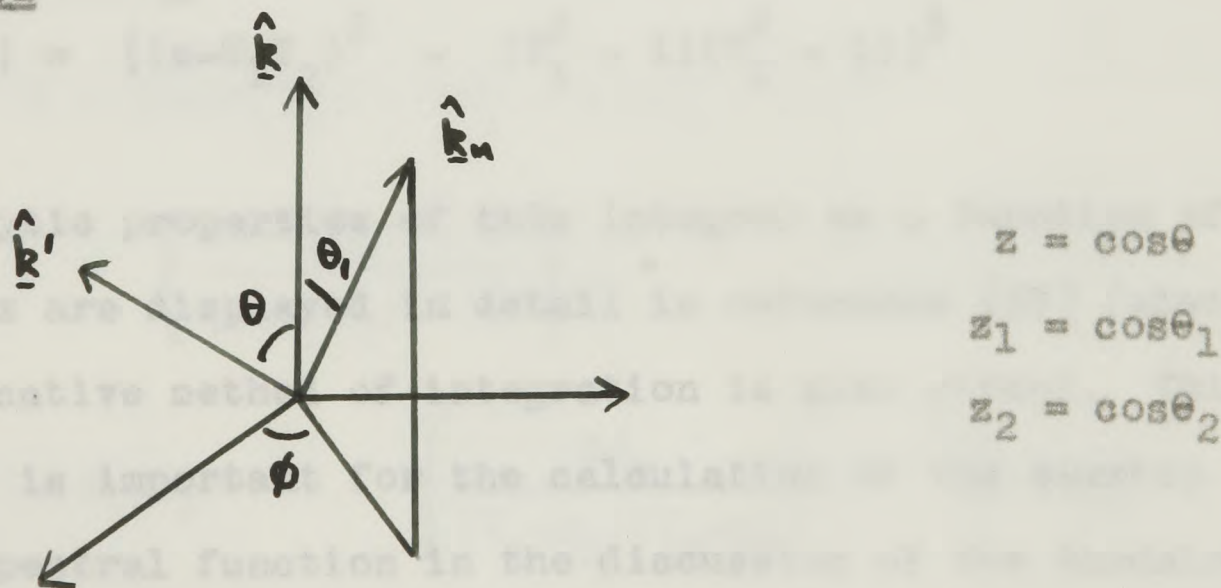
$$I(z) = \int \frac{d\Omega_n}{(T_1 - z_1)(T_2 - z_2)} \quad T_1, T_2 > 1 \quad (C.1)$$

where $z_1 = \hat{\underline{k}} \cdot \hat{\underline{k}}_n$ and $z_2 = \hat{\underline{k}}' \cdot \hat{\underline{k}}_n$: $\hat{\underline{k}}$ and $\hat{\underline{k}}'$ are two fixed directions and the integration is over the direction of $\hat{\underline{k}}_n$. z_1 and z_2 are related by

$$z_2 = z z_1 + \sqrt{1 - z^2} \sqrt{1 - z_1^2} \cos \phi \quad (C.2)$$

as is evident from Fig. C.1.

Figure C.1



For $T_1, T_2 > 1$, the denominator of the integrand is never zero (for z in the physical region $|z| < 1$), and the result is real and well-defined. The integral may be evaluated using Feynman's method [62] based on the identity

$$\frac{1}{ab} = \int_0^1 da \frac{1}{[aa \pm b(1-a)]^2} \quad (C.3)$$

I reduces to the form

$$I(z) = \int_0^1 da \int \frac{d\Omega_n}{[k_n \cdot a + b]^2}$$

where \underline{a} and b are easily determined.

Thus:

$$I(z) = 4\pi \int_0^1 da \frac{1}{[b^2 - a^2]}$$

After some manipulation the result of this integration may be written

$$I(z) = \frac{2\pi}{K(z)} e_n \left\{ \frac{z - T_1 T_2 - K(z)}{z - T_1 T_2 + K(z)} \right\} \quad (C.4)$$

where

$$K(z) = [(z - T_1 T_2)^2 - (T_1^2 - 1)(T_2^2 - 1)]^{\frac{1}{2}} \quad (C.5)$$

The analytic properties of this integral as a function of complex z are displayed in detail in reference [59] (where an alternative method of integration is also given). This integral is important for the calculation of the elastic double spectral function in the discussion of the Mandelstam representation (see for example the books of references [16] or [59]). However, for our purposes we remain in the physical region ($|z| < 1$) and need only the limits as T_1 and/or T_2 tend to 1. We write

$$T = 1 + \varepsilon \quad ; \quad \varepsilon = \lambda^2 / 2k^2$$

where ϵ is the small photon mass. We then find:

$$\lim_{\epsilon \rightarrow 0} \int \frac{d\Omega_n}{(T-z_1)(T-z_2)} = \frac{4\pi}{(1-z)} \ln \left(\frac{1-z}{\epsilon} \right)$$

For the calculation involving dipole form factors further integrals are needed. For example

$$\int \frac{d\Omega_n}{(T_1-z_1)^2(T_2-z_2)}$$

These integrals may be obtained from $I(z)$ by differentiation with respect to the parameters T_1 and T_2 . The relevant integrals are as follows.

I. $T_1, T_2 > 1$

$$K = [(z-T_1T_2) - (T_1^2 - 1)(T_2^2 - 1)]^{\frac{1}{2}}$$

$$\xi = \left\{ \frac{(z-T_1T_2) - K}{(z-T_1T_2) + K} \right\}$$

$$(1) \quad VI1(z, T_1, T_2) = \int \frac{d\Omega_n}{(T_1-z_1)(T_2-z_2)} = \frac{2\pi}{K} \ln \xi$$

$$(2) \quad VI2(z, T_1, T_2) = \int \frac{d\Omega_n}{(T_1-z_1)^2(T_2-z_2)}$$

$$= \frac{2\pi}{K^2} \left\{ \frac{(T_1-T_2z)}{K} \ln \xi + 2 \frac{(T_2-T_1z)}{(T_1^2-1)} \right\}$$

$$\begin{aligned}
 (3) \quad \text{VI3}(z, T_1, T_2) &= \int \frac{d\Omega_n}{(T_1 - z_1)^2 (T_2 - z_2)^2} \\
 &= \frac{2\pi}{K^5} \ln \left\{ -2z (T_1^2 + T_2^2) + T_1 T_2 (3 + z^2) + z(z^2 - 1) \right\} \\
 &+ \frac{4\pi}{K^4} \cdot \frac{1}{(T_1^2 - 1)(T_2^2 - 1)} \times \left\{ T_1^4 + T_2^4 + T_1^2 T_2^2 (3z^2 - 1) \right. \\
 &\quad \left. + (z^2 - 1) \right. \\
 &\quad \left. - 2 T_1 T_2 z [T_1^2 + T_2^2 - 2] \right. \\
 &\quad \left. - 2z^2 (T_1^2 + T_2^2) \right\}
 \end{aligned}$$

All these integrals are finite, since T_1 and $T_2 > 1$.

The following limits as $T \rightarrow 1$ are also required:

II Limit $\epsilon \rightarrow 0$; $B > 1$

$$\begin{aligned}
 (1) \quad \text{WI1}(z, B) &= \int \frac{d\Omega_n}{(T - z_1)(B - z_2)} = \int \frac{d\Omega_n}{(B - z_1)(T - z_2)} \\
 &= \frac{2\pi}{(B - z)} \cdot \ln \left[\frac{2(B - z)^2}{(B^2 - 1)} \right] - \frac{2\pi}{(B - z)} \ln \epsilon
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \text{WI2}(z, B) &= \int \frac{d\Omega_n}{(T - z_1)(B - z_2)^2} = \int \frac{d\Omega_n}{(B - z_1)^2 (T - z_2)} \\
 &= \frac{2\pi}{(B - z)^2} \cdot \ln \left[\frac{2(B - z)^2}{(B^2 - 1)} \right] - \frac{4\pi}{(B^2 - 1)} \cdot \frac{(Bz - 1)}{(B - z)^2} \\
 &\quad - \frac{2\pi}{(B - z)^2} \ln \epsilon
 \end{aligned}$$

In computations, all the $Q_n \varepsilon$ terms may be dropped since they cancel explicitly.

With the liberal use of partial fractions, all the integrations of Chapter 5 may be reduced to these above, or to simpler integrals. Thus an explicit finite expression may be obtained for the polarization.

ELASTIC CONTRIBUTION TO THE POLARIZATION

(CHAPTER 5)

PROGRAM LISTING

Only part of the main program is listed here. Besides containing the plotting routines, this calculates the CM momentum PC etc. It also sets a flag NF for the calculation with the two types of form factors:

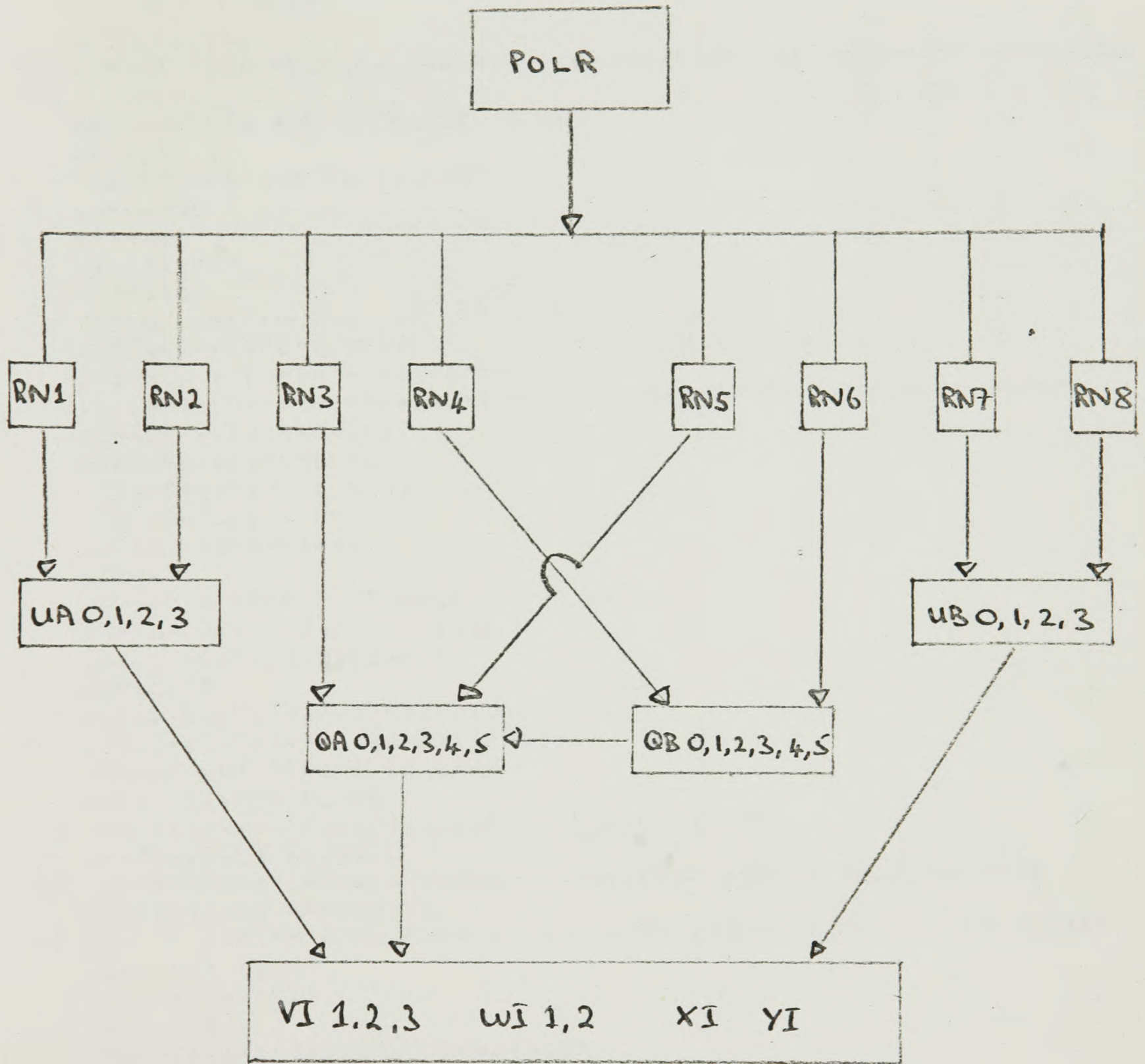
$NF = 0$ 'Point-like' form factor $G_D = 1$ ($\beta = \infty$)

$NF = 1$ Dipole form factor ($\beta = 0.710$)

The function $POLR(TD)$ calculates the polarization as a function of the CMS scattering angle. A block diagram shows a calling breakdown for the other function sub-routines (shown for $NF = 1$).

The functions VI 1,2,3 and WI 1,2 are given in Appendix 3 (omitting the $\ln \epsilon$ term).

BLOCK DIAGRAM



MAIN PROGRAM

FORTRAN COMPILER

MARK NO. 302

DATE

15/06/70

TIME

COMMON BH,AD,B,AMP,PHI,C

COMMON FC,WC,EC,PI

COMMON IF

EXTERNAL PR3

ROSENBLUTH PROTON

WRITE(1,10)

10 FORMAT (54H POLM CALCULATES POLARIZATION FOR ROSENBLUTH PROTONS

1 ,//)

ALL MOMENTA AND ENERGIES IN GEV

SECTION 1

CONSTANTS AND VARIABLES

✓ BM=0.938

✓ CM=0.0

✓ PI = 3.1416

✓ PL=0.100

CALCULATE CM MOM PC

WL=SQRT(PL*PL+CM*CM)

S=CM*CM + BM*BM + 2.0*BM*WL

XL=(S*S+CM**4+BM**4-2.0*S*CM*CM-2.0*S*BM*BM-2.0*CM*CM*BM*BM)

PC=SQRT(XL/(4.0*S))

LC=SQRT(PC*PC+BM*BM)

WC=SQRT(PC*PC+CM*CM)

ALL VPLES SET UP

BD IN (GEV/C)**2

✓ BD=0.71

-- BD = BETA IN DIPOLE FIT -----

AD=BD*BD/((2.0*PC*PC)**2)

B=1.0+(BD/(2.0*PC*PC))

AMP=2.79

PHI=2.0*BM*(AMP-1.0)/(2.0*PC*PC)

C=1.0+(4.0*BM*BM/(2.0*PC*PC))

PRINT OUT BEFORE DO LOOP

WRITE(1,20) PL,PC

20 FORMAT(5X,4H PL=,F8.4,10X,4H PC=,F8.4,//)

WRITE(1,30) BD,AD,B

30 FORMAT(5X,4H BD=,F8.4,10X,4H AD=,E12.4,5X,3H B=,E12.4,//)

WRITE(1,40) AMP,PHI,C

40 FORMAT(4X,5H AMP=,F8.4,10X,5H PHI=,E12.4,5X,3H C=,E12.4,///)

NOW PLOT CARDS

CALL INCPLOT(1)

CALL CHS

CALL LIMITS(0.0,20.0,0.0,15.0)

CALL REGION(1.0,19.0,3.0,13.0)

NOW LABEL AXES

CALL CRSET(1)

CALL CRSIZE(0.6)

CALL PLOTCL(10.0,1.0,10HCMS ANGLE,10)

CALL CRSET(4)

CALL TYPECS(3H20,,1)

CALL CRSET(1)

CALL PLOTCL(1.5,13.0,2H10,2)

CALL SUFFIX

CALL TYPECL(1H3,1)

CALL NORMAL

CALL TYPECL(1HP,1)

FUNCTION POLR(TD)

COMMON BR,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

COMMON DF

DIMENSION DR(3)

A=1.0/137.0

IF(TD)10,10,20

10 P=0.0

GO TO 30

20 CONTINUE

TR=TD*PI/180.0

TR IS ANGLE IN RADIAHS

Z=COS(TR)

SZ=SIN(TR)

CALCULATE STANDARD KINEMATIC QUANTITIES

Q2=(EC+WC)**2

DL2=-2.0*PC+PC*(1.0-Z)

QK=WC*(EC+WC)

IF(NF-1) 100,200,200

100 AF=AMP

BF=-PHI/(C-Z)

GO TO 300

! FORN FACTORS A AND B

200 CONTINUE

AF=AMP*AD/((B-Z)**2)

BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))

300 CONTINUE

; END OF SECTION 1

; SECTION 2

FRS=-A*(PC**3)*SZ/PI

; CALCULATE FRS

; CALCULATE DO ROSENBLUTH

; THIS IS WITH DIPOLE FORM FACTORS

DR(1)=8.0*AF*AF*(DL2*DL2+2.0*Q2*DL2+2.0*((Q2-BM*BM)**2))

DR(2)=2.0*AF*BM*BF+(BF*BF*(4.0*BM*BM-DL2)/2.0)

DR(3)=32.0*(Q2*DL2+((Q2-BM*BM)**2))

DO=DR(1)+DR(2)*DR(3)

C END OF SECTION 2

C SECTION 3

C CALCULATE POLARIZATION

RJ=RN1(Z)+RN2(Z)+RN3(Z)+RN4(Z)+RN5(Z)+RN6(Z)+RN7(Z)+RN8(Z)

P=FRS*RJ/DO

C END OF SECTION 3

30 CONTINUE

POLR=P

RETURN

END

REFERENCE TABLES

0	00000/0	10	00003/1
200	00031/0	300	00043/2

20 00004/4

ROUTINE COMPILED

TIME LESS THAN 4 SECS

NUMBER OF INSTRUCTION WORDS 87

* * * * *

```

FUNCTION RN2(Z)
DIMENSION GA(3),GB(4),GM(4),GR(3)
COMMON BN,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON NF

```

```

C CALCULATE STANDARD KINEMATIC QUANTITIES

```

```

Q2=(EC+WC)**2
DL2=-2.0*PC*PC*(1.0-Z)
QK=WC*(EC+WC)
IF(NF-1) 10,20,20

```

```

10 AF=AMP
BF=-PHI/(C-Z)
GO TO 30

```

```

20 CONTINUE

```

```

C FORM FACTORS A AND B
AF=AMP*AD/((B-Z)**2)
BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))

```

```

30 CONTINUE
R=QK/(DL2*Q2+4.0*QK*QK)

```

```

C END OF THIS SECTION

```

```

C ARRAY GA FOR RN2
GA(1)=-8.0*DL2*Q2*(4.0*QK+DL2)
GA(2)=2.0*(4.0*BM*BM*(4.0*QK-DL2)-DL2*8.0*QK)
GA(3)=GA(2)

```

```

C NOW STANDARD GB FOR RN 1,2,7,8
GB(1)=(GA(2)+GA(3))
GB(2)=-((2.0*GA(1)-DL2*(GA(2)+GA(3)))/(2.0*PC*PC)
GB(3)=-GA(1)*DL2/((2.0*PC*PC)**2)
GB(4)=GA(2)+GA(3)

```

```

C ARRAY GH FOR RN2
GH(1)=2.0*DL2*8.0*QK*(4.0*QK+DL2)
GH(2)=0.0
GH(3)=0.0
GH(4)=8.0*(1.0*BM*BM-DL2)

```

```

C ARRAY GR SAME FOR RN 1,2,7,8
GR(1)=GH(4)
GR(2)=(GH(2)+GH(3))/(2.0*PC*PC)
GR(3)=GH(1)/((2.0*PC*PC)**2)

```

```

C NOW CALCULATE RESULT
W1=(R*GB(1)+GR(1))*UA0(Z)
W2=(R*GB(2)+GR(2))*UA1(Z)
W3=(R*GB(3)+GR(3))*UA2(Z)
W4=R*GB(4)*UA3(Z)
V=BF*(W1+W2+W3+W4)
RN2=V
RETURN
END

```

REFERENCE TABLES

0 00000/0

10 00013/3

20 00017/3

ROUTINE COMPILED

TIME LESS THAN 5 SECS

NUMBER OF INSTRUCTION WORDS 98

* * * * *

```

FUNCTION RN3(Z)
DIMENSION GA(3),GG(6),GH(4),GN(4)
COMMON BN,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON HF
CALCULATE STANDARD KINEMATIC QUANTITIES
Q2=(EC+PC)**2
DL2=-2.0*PC*PC*(1.0-Z)
QK=PC*(EC+WC)
IF (HF-1) 10,20,20
10 AF=AMP
   BF=-PHI/(C-Z)
   GO TO 30
20 CONTINUE
   FORM FACTORS A AND B
   AF=AMP*AD/((B-Z)**2)
   BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))
30 CONTINUE
   R=QK/(DL2*Q2+4.0*QK*QK)
   END OF THIS SECTION
   ARRAY GA FOR RN3
   GA(1)=4.0*QK*(-DL2*4.0*Q2-8.0*QK*(4.0*QK+DL2))
   GA(2)=16.0*QK*Q2
   GA(3)=16.0*QK*BN*BN
   ARRAY GG STANDARD FOR RN3,4,5,6
   GG(1)=GA(2)+GA(3)
   GG(2)=-((GA(1)-GA(3)*DL2)/(2.0*PC*PC))
   GG(3)=-((GA(1)-GA(2)*DL2)/(2.0*PC*PC))
   GG(4)=-((GA(1)*DL2)/((2.0*PC*PC)**2))
   GG(5)=GA(3)
   GG(6)=GA(2)
   ARRAY GH FOR RN3
   GH(1)=0.0
   GH(2)=-16.0*QK*(4.0*QK+DL2)
   GH(3)=0.0
   GH(4)=2.0*(8.0*BN*BN+8.0*QK)
   ARRAY GN STANDARD FOR RN3,4,5,6
   GN(1)=GH(4)
   GN(2)=-GH(3)/(2.0*PC*PC)
   GN(3)=-GN(2)/(2.0*PC*PC)
   GN(4)=+GH(1)/((2.0*PC*PC)**2)
   W1=(R*GG(1)+GN(1))*QA0(Z)
   W2=(R*GG(2)+GN(2))*QA1(Z)
   W3=(R*GG(3)+GN(3))*QA2(Z)
   W4=(R*GG(4)+GN(4))*QA3(Z)
   W5=R*GG(5)*QA4(Z)
   W6=R*GG(6)*QA5(Z)
   W=AF*(W1+W2+W3+W4+W5+W6)
   RN3=W
RETURN
END

```

```

FUNCTION PH1(Z)
  DIMENSION GA(3),GG(6),GN(4),GN(4)
  COMMON BM,AD,B,AMP,PHI,C
  COMMON PC,WC,EC,PI
  COMMON DE
  CALCULATE STANDARD KINEMATIC QUANTITIES
  Q2=(EC+WC)**2
  DL2=-2.0*PC*PC*(1.0-Z)
  QK=WC*(EC+WC)
  IF(DE-1) 10,20,20
10 AF=AMP
  BF=-PHI/(C-Z)
  GO TO 30
20 CONTINUE
  FORM FACTORS A AND B
  AF=AMP*AD/((B-Z)**2)
  BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))
30 CONTINUE
  R=-QK/(DL2*Q2+4.0*QK*QK)
  END OF THIS SECTION
  ARRAY GA FOR RN4
  GA(1)=2.0*((4.0*QK+DL2)*(DL2*4.0*Q2+16.0*QK*QK)+8.0*QK*DL2*Q2)
  GA(2)=8.0*BM*BM*(2.0*QK-DL2)
  GA(3)=8.0*Q2*(2.0*QK+DL2)
  ARRAY GG STANDARD FOR RN3,4,5,6
  GG(1)=GA(2)+GA(3)
  GG(2)=-GA(1)-GA(3)*DL2/(2.0*PC*PC)
  GG(3)=-GA(1)-GA(2)*DL2/(2.0*PC*PC)
  GG(4)=-GA(1)*DL2/((2.0*PC*PC)**2)
  GG(5)=GA(2)
  GG(6)=GA(3)
  ARRAY GN FOR RN4
  GN(1)=16.0*QK*DL2*(4.0*QK+DL2)
  GN(2)=16.0*QK*(4.0*QK+DL2)
  GN(3)=0.0
  GN(4)=2.0*(9.0*QK+8.0*BM*BM+4.0*DL2)
  ARRAY GN STANDARD FOR RN3,4,5,6
  GN(1)=GN(4)
  GN(2)=-GN(3)/(2.0*PC*PC)
  GN(3)=-GN(2)/(2.0*PC*PC)
  GN(4)=+GN(1)/((2.0*PC*PC)**2)
  W1=(R*GG(1)+GN(1))*QB0(Z)
  W2=(R*GG(2)+GN(2))*QB1(Z)
  W3=(R*GG(3)+GN(3))*QB2(Z)
  W4=(R*GG(4)+GN(4))*QB3(Z)
  W5=R*GG(5)*QB4(Z)
  W6=R*GG(6)*QB5(Z)
  W=AF*(W1+W2+W3+W4+W5+W6)
  RN4=W
  RETURN
END

```

```

FUNCTION RN5(Z)
DIMENSION GA(3),GG(6),GN(4),GN(4)
COMMON /1/ AD,B,AMP,PHI,C
COMMON PC,WC,EC,PJ
COMMON BF
CALCULATE STANDARD KINEMATIC QUANTITIES
G2=(EC+WC)**2
DL2=-2.0*PC+PC*(1.0-Z)
QK=WC*(EC+WC)
IF(NF-1) 10,20,20
10 AF=AMP
BF=-PHI/(C-Z)
GO TO 30
20 CONTINUE
FORM FACTORS A AND B
AF=AMP*AD/((B-Z)**2)
BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))
30 CONTINUE
R=QK/(DL2*Q2+4.0*QK*QK)
END OF THIS SECTION
ARRAY GA FOR RN5
GA(1)=2.0*BM*(4.0*QK+DL2)*(8.0*QK*(4.0*QK+DL2)+4.0*BM*BM*DL2)
GA(2)=8.0*BM*Q2*(4.0*QK+DL2)
GA(3)=BM*(8.0*QK*(4.0*BM*BM-2.0*DL2)-8.0*BM*BM*DL2)
ARRAY GG STANDARD FOR RN3,4,5,6
GG(1)=GA(2)+GA(3)
GG(2)=-((GA(1)-GA(3)*DL2)/(2.0*PC*PC)
GG(3)=-((GA(1)-GA(2)*DL2)/(2.0*PC*PC)
GG(4)=-((GA(1)*DL2)/((2.0*PC*PC)**2)
GG(5)=GA(3)
GG(6)=GA(2)
ARRAY GN FOR RN5
GN(1)=0.0
GN(2)=0.0
GN(3)=8.0*BM*4.0*QK*(4.0*QK+DL2)
GN(4)=4.0*BM*(8.0*QK+8.0*BM*BM)
ARRAY GN STANDARD FOR RN3,4,5,6
GN(1)=GN(4)
GN(2)=-GN(3)/(2.0*PC*PC)
GN(3)=-GN(2)/(2.0*PC*PC)
GN(4)=+GN(1)/((2.0*PC*PC)**2)
W1=(R*GG(1)+GN(1))*QA6(Z)
W2=(R*GG(2)+GN(2))*QA1(Z)
W3=(R*GG(3)+GN(3))*QA2(Z)
W4=(R*GG(4)+GN(4))*QA3(Z)
W5=P*GG(5)*QA4(Z)
W6=P*GG(6)*QA5(Z)
W=BF*(W1+W2+W3+W4+W5+W6)
RN5=W
RETURN
END
    
```

```

FUNCTION RN6(Z)
DIMENSION GA(3),GG(6),GH(4),GN(4)
COMMON PH,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON IF
CALCULATE STANDARD KINEMATIC QUANTITIES
Q2=(EC+WC)**2
DL2=-2.0*PC*PC*(1.0-Z)
QK=WC*(EC+WC)
IF(IF-1) 10,20,20
10 AF=AMP
BF=-PHI/(C-Z)
GO TO 30
20 CONTINUE
FORM FACTORS A AND B
AF=AMP*AD/((B-Z)**2)
BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))
30 CONTINUE
R=(QK/(DL2*Q2+4.0*QK*QK))
END OF THIS SECTION
ARRAY GA FOR RN6
GA(1)=2.0*BM*(4.0*QK+DL2)*(8.0*QK*(4.0*QK+DL2)+4.0*BM*BM*DL2)
GA(2)=BF*(8.0*QK*(4.0*BM*BM-2.0*DL2)-8.0*BM*BM*DL2)
GA(3)=8.0*BM*Q2*(4.0*QK+DL2)
ARRAY GG STANDARD FOR RN3,4,5,6
GG(1)=GA(2)+GA(3)
GG(2)=-GA(1)-GA(3)*DL2/(2.0*PC*PC)
GG(3)=-GA(1)-GA(2)*DL2/(2.0*PC*PC)
GG(4)=-GA(1)*DL2/((2.0*PC*PC)**2)
GG(5)=GA(3)
GG(6)=GA(2)
ARRAY GH FOR RN6
GH(1)=0.0
GH(2)=BM*32.0*QK*(4.0*QK+DL2)
GH(3)=0.0
GH(4)=4.0*BM*(8.0*QK+8.0*BM*BM)
ARRAY GN STANDARD FOR RN3,4,5,6
GN(1)=GH(4)
GN(2)=-GH(3)/(2.0*PC*PC)
GN(3)=-GH(2)/(2.0*PC*PC)
GN(4)=+GH(1)/((2.0*PC*PC)**2)
W1=(R*GG(1)+GN(1))*QB0(Z)
W2=(R*GG(2)+GN(2))*QB1(Z)
W3=(R*GG(3)+GN(3))*QB2(Z)
W4=(R*GG(4)+GN(4))*QB3(Z)
W5=R*GG(5)*QB4(Z)
W6=R*GG(6)*QB5(Z)
W=BF*(W1+W2+W3+W4+W5+W6)
RH6=W
RETURN
END

```

FUNCTION RN7(Z)
 DIMENSION GA(3),GB(4),GM(4),GR(3)
 COMMON BN,AD,B,AMP,PHI,C
 COMMON PC,WC,EC,PI
 COMMON NF

C CALCULATE STANDARD KINEMATIC QUANTITIES

Q2=(EC+WC)**2
 DL2=-2.0*PC*PC*(1.0-Z)
 QK=WC*(EC+WC)
 IF(NF-1) 10,20,20

10 AF=AMP

BF=-PHI/(C-Z)
 GO TO 30

20 CONTINUE

C FORM FACTORS A AND B

AF=AMP*AD/((B-Z)**2)
 BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))

30 CONTINUE

R=-QK/(DL2*Q2+4.0*QK*QK)

C END OF THIS SECTION

C ARRAY GA FOR RN7

GA(1)=BN*2.0*DL2*Q2*(8.0*QK+DL2)
 GA(2)=BN*2.0*DL2*Q2
 GA(3)=GA(2)

C NOW STANDARD GB FOR RN 1,2,7,8

GB(1)=(GA(2)+GA(3))
 GB(2)=-((2.0*GA(1)-DL2*(GA(2)+GA(3)))/(2.0*PC*PC))
 GB(3)=-GA(1)*DL2/((2.0*PC*PC)**2)
 GB(4)=GA(2)+GA(3)

C ARRAY GM FOR RN7

GM(1)=-BN*16.0*QK*DL2*(8.0*QK+DL2)
 GM(2)=-BN*16.0*QK*DL2
 GM(3)=GM(2)
 GM(4)=0.0

C ARRAY GR SAME FOR RN 1,2,7,8

GR(1)=GM(4)
 GR(2)=-((GM(2)+GM(3))/(2.0*PC*PC))
 GR(3)=GM(1)/((2.0*PC*PC)**2)

W1=(R*GR(1)+GR(1))*UB0(Z)

W2=(R*GR(2)+GR(2))*UB1(Z)

W3=(R*GR(3)+GR(3))*UB2(Z)

W4=R*GB(4)*UB3(Z)

W=AF*(W1+W2+W3+W4)

RN7=W

RETURN

END

0 00000/0

10 00013/3

REFERENCE TABLES
 20 00017/3

ROUTINE COMPILED

TIME LESS THAN 5 SECS

NUMBER OF INSTRUCTION WORDS 94

* * * * *

#PUNCH

#IDENTIFIERRN3

FUNCTION RN(Z)

DIMENSION GA(3),GB(4),GN(4),GR(3)

COMMON BN,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

COMMON HF

C CALCULATE STANDARD KINEMATIC QUANTITIES

QZ=(EC+WC)**2

DL2=-2.0*PC*PC*(1.0-Z)

QK=VC*(FC+WC)

IF(HF-1) 10,20,20

10 AF=AMP

BF=-PHI/(C-Z)

GO TO 30

20 CONTINUE

C FORM FACTORS A AND B

AF=AMP*AD/((B-Z)**2)

BF=-PHI*AD/((C-Z)*(B-Z)*(B-Z))

30 CONTINUE

R=-QK/(DL2*QZ+4.0*QK*QK)

C END OF THIS SECTION

C ARRAY GA FOR RN

GA(1)=DL2*(4.0*QK+4.0*BM*BM)*(2.0*((4.0*QK)**2)+4.0*QZ*DL2)

GA(2)=4.0*DL2*QZ*(4.0*QK+4.0*BM*BM)

GA(3)=GA(2)

C NOW STANDARD GB FOR RN 1,2,7,8

GB(1)=(GA(2)+GA(3))

GB(2)=-((2.0*GA(1)-DL2*(GA(2)+GA(3)))/(2.0*PC*PC)

GB(3)=GA(1)*DL2/((2.0*PC*PC)**2)

GB(4)=GA(2)+GA(3)

C ARRAY GN FOR RN

GN(1)=0.0

GN(2)=0.0

GN(3)=0.0

GN(4)=0.0

C ARRAY GR SAME FOR RN 1,2,7,8

GR(1)=GN(4)

GR(2)=-((GN(2)+GN(3)))/(2.0*PC*PC)

GR(3)=GN(1)/((2.0*PC*PC)**2)

W1=(R*GR(1)+GR(1))*UB0(Z)

W2=(R*GB(2)+GR(2))*UB1(Z)

W3=(R*GB(3)+GR(3))*UB2(Z)

W4=R*GB(4)*UB3(Z)

W=BF*(W1+W2+W3+W4)

RWB=W

RETURN

END

REFERENCE TABLES

20 00017/3

0 00000/0

10 00013/3

ROUTINE COMPILED

TIME LESS THAN 5 SECS

NUMBER OF INSTRUCTION WORDS 95

* * * * *

RAP COMPILER

MARK NO. 302

DATE

15/06/70

TIME

FUNCTION UAG(Z)

COMMON BR,AD,B,ANF,PHI,C

COMMON FC,WC,EC,PI

COMMON NF

IF(NF-1)100,200,200

100 NF=UAG(Z)

UAG=WNF

RETURN

200 CONTINUE

ATA> TERMS ROSENBLUTH PROTON

ADR=ALP*AD

W=ADR*ADR*VIG(Z,B,B)

UAG=W

RETURN

END

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 2 SECS

NUMBER OF INSTRUCTION WORDS 14

* * * * *

INCH

IDENTIFIERUA1

ORTRAN

FUNCTION UAI(Z)
COMMON BR,AD,B,AMP,PI,C
COMMON DC,W,C,EC,PI
COMMON MF

IF (MF-1) 100, 200, 200

100 WMF=UAI(Z)

UAI=WMF

RETURN

200 CONTINUE

ALAZ TERMS ROSENBLUTH PROTON

W1=W12(Z,B)

W2=-V12(Z,B,B)

W3=- (B-1.0)*V13(Z,B,B)

ADR=AMP*AD

W=ADR*ADR*(W1+W2+W3)/((B-1.0)*(B-1.0))

UAI = W

RETURN

END

REFERENCE TABLES

0 00000/0 100 00002/1 200 00005/4

ROUTINE COMPILED

TIME LESS THAN 2 SECS

NUMBER OF INSTRUCTION WORDS 25

* * * * *

JNCH

JENTIFIRUA2

JRTRAH

```

FUNCTION UA2(Z)
COMMON DM,AD,B,AMP,PI,C
COMMON FC,WC,EC,PI
COMMON WF

```

```

100 IF(WF-1)100,200,200
    WF=UA2(Z)
    UA2=WF
    RETURN

```

```

200 CONTINUE
    A1A2 TERMS ROSENBLUTH PROTON
    R=1.0-Z
    U12=(4.0*PI*ALOG(R))/R
    W1=U12
    W2=-2.0*W1(Z,B)
    W3=+V11(Z,B,B)
    W4=-2.0*(B-1.0)*W12(Z,B)
    W5=+2.0*(B-1.0)*V12(Z,B,B)
    W6=+(B-1.0)*(B-1.0)*V13(Z,B,B)
    ADR=AMP*AD
    W=(ADR*ADR/((B-1.0)**4))*(W1+W2+W3+W4+W5+W6)
    UA2=W
    RETURN
END

```

REFERENCE TABLES
 200 00005/4

0 00000/0 100 00002/1

ROUTINE COMPILED
 TIME LESS THAN 3 SECS
 NUMBER OF INSTRUCTION WORDS 45
 * * * * *

UNCIH
 IDENTIFIERUAD
 ORTRAN

TRAN COMPILER

PAGE NO. 302

DATE

15/06/70

TIME

FUNCTION UAB(Z)

COMMON FB,AD,B,AMP,PHI,C

COMMON FL,WC,EC,PI

COMMON FF

IF (NF-1) 100,200,200

100 WNF=UAB(Z)

UAB=WNF

RETURN

200 CONTINUE

A1A2 TERMS ROSENFLOTH PROTON

W1=+W11(Z,B)

W2=-(B-1.0)*W12(Z,B)

W3=-W11(Z,B,B)

W4=+(B-1.0)+(B=1.0)*W13(Z,B,B)

ADR=AMP*AD

W=(ADR*ADR/((B=1.0)*(B-1.0)))*(W1+W2+W3+W4)

UAB=W

RETURN

END

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 2 SECS

NUMBER OF INSTRUCTION WORDS 31

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UNCH

IDENTIFIERUBO

ORTPAN

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ITRAH COMPILER

MARK NO. 302

DATE

15/06/70

TIME

```

FUNCTION UBO(Z)
COMMON BM,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON DF
IF (DF-1)100,200,200
100 WDF=UBMO(Z)
    UBO=WDF
    RETURN
200 CONTINUE
    BIB2 TERMS ROSENBLUTH PROTON
    B11=V11(Z,C,C)
    B12=V11(Z,B,B)
    B13=-V11(Z,C,B)
    B14=-V11(Z,B,C)
    W1 = B11+B12+B13+B14
    B15=-V12(Z,B,C)
    B16=B15
    B17=+V12(Z,B,B)
    B18=B17
    W2=(B-C)*(B15+B16+B17+B18)
    B19=+V13(Z,B,B)
    W3=(B-C)*(B-C)*B19
    W4=(PHI*AD/((B=C)*(B-C)))*W2
    W=W4*(W1+W2+W3)
    UBO=W
    RETURN
END

```

0 00000/0 100 00002/1

REFERENCE TABLES
200 00005/4

ROUTINE COMPILED
TIME LESS THAN 3 SECS
NUMBER OF INSTRUCTION WORDS 48
* * * * *

PUNCH
IDENTIFIRUB1
FORTRAN

```

FUNCTION UB1(Z)
COMMON BA,AD,B,AMP,PHI,C
COMMON BC,WC,EC,PI
COMMON WF
IF (BF-1) 100,200,200
100 WNF=UBN1(Z)
    UB1=WNF
    RETURN
200 CONTINUE
    BIN2 TERMS ROSEBLUTH PROTON
    B11=+(W11(Z,C)=V11(Z,C,C))/(C-1.0)
    B12=+(W11(Z,B)=V11(Z,B,B))/(B-1.0)
    B13=+(W11(Z,B)=V11(Z,C,B))/(C-1.0)
    B14=+(W11(Z,C)=V11(Z,B,C))/(B-1.0)
    W1=B11+B12+B13+B14
    B15=+(W12(Z,B)=V12(Z,B,C))/(C-1.0)
    B16=+(W11(Z,C)=V11(Z,B,C)-(B-1.0)*V12(Z,B,C))/((B-1.0)*(B-1.0))
    B17=+(W12(Z,B)=V12(Z,B,B))/(B-1.0)
    B18=+(W11(Z,B)=V11(Z,B,B)-(B-1.0)*V12(Z,B,B))/((B-1.0)*(B-1.0))
    W2=(B-C)*(B15+B16+B17+B18)
    B19=+(W12(Z,B)=V12(Z,B,B)-(B-1.0)*V13(Z,B,B))/((B-1.0)*(B-1.0))
    W3=B19*(B-C)*(B-C)
    W4=(PHI+AD)/((B=C)*(B-C))**2
    W=W4*(W1+W2+W3)
    UB1=W
    RETURN
END

```

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 4 SECS

NUMBER OF INSTRUCTION WORDS 106

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*PUNCH

*IDENTIFIERUB2

*FORTRAN

EGTRAM COMPILER

MARK NO. 302

DATE 15/06/70

TIME

FUNCTION RUB2(Z)
COMMON DA,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON DF

IF(DF-1)100,200,200

100 WNF=UBN2(Z)

UB2=WNF

RETURN

200 CONTINUE

BIB2 TERMS ROSENBLUTH PROTON

R=(1.0-Z)

UI2=(4.0*PI*ALOG(R))/R

B11=(UI2+VI1(Z,C,C)-2.0*W11(Z,C))/((C-1.0)*(C-1.0))

B12=(UI2+VI1(Z,B,B)-2.0*W11(Z,B))/((B-1.0)*(B-1.0))

B13=(UI2+VI1(Z,C,B)-W11(Z,B)-W11(Z,C))/((C-1.0)*(B-1.0))

B14=B13

W1=B11+B12+B13+B14

B15=(UI2+VI1(Z,B,C)-W11(Z,B)-W11(Z,C)-(B-1.0)*W12(Z,B)

1 +(B-1.0)*VI2(Z,B,C))/((C-1.0)*(B-1.0)*(B-1.0))

B16=B15

B17=(UI2+VI1(Z,B,B)-2.0*W11(Z,B)-(B-1.0)*W12(Z,B)

2 +(B-1.0)*VI2(Z,B,B))/((B-1.0)**3)

B18=B17

W2=(B-C)*(B15+B16+B17+B18)

B19=(UI2+VI1(Z,B,B)-2.0*W11(Z,B)-2.0*(B-1.0)*W12(Z,B)

3 + 2.0*(B-1.0)*VI2(Z,B,B)+(B-1.0)*(B-1.0)*VI3(Z,B,B))/((B-1.0)

4 **4)

W3=(B-C)+(B-C)*B19

W4=(PHI*AD/((B=C)*(B-C)))**2

W=W4*(W1+W2+W3)

UB2=W

RETURN

END

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 5 SECS

NUMBER OF INSTRUCTION WORDS 131

* * * * *

*PUNCH

*IDENTIFIERUB3

*FORTRAN

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EGTRAM COMPILER

MARK NO. 302

DATE

15/06/70

TIME

FUNCTION GAO(Z)

COMMON BH,AD,B,AMP,PHI,C

COMMON FC,WC,EC,PI

COMMON IF

IF(NF-1)100,200,200

100 WNF=GAO(Z)

GAO=WNF

RETURN

200 CONTINUE

C DIA2 TERMS ROSENBLUTH PROTON DIPOLE F F

BA11=+V12(Z,B,C)

BA12=-V12(Z,B,B)

BA13=-V13(Z,B,B)

W1=BA11+BA12+(B-C)*BA13

W2=- (AMP*PHI*AD*AD/((B-C)*(B-C)))

W=W2*W1

GAO=W

RETURN

END

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 2 SECS

NUMBER OF INSTRUCTION WORDS 27

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*PUNCH

*IDENTIFIERCAI

*FORTRAN

FUNCTION UB3(Z)
COMMON PH,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
COMMON HF
IF (HF-1) 100,200,200

```
100 WNF=UBW3(Z)
    UB3=WNF
    RETURN
```

```
200 CONTINUE
```

C B1B2 TERMS ROSENBLUTH PROTON

```
UI1=2.0*PI*ALOG(2.0)
B11=+(UI1-XI(C)-(C-1.0)*WI1(Z,C)+(C-1.0)*VI1(Z,C,C))/(C-1.0)
B12=+(UI1-XI(B)-(B-1.0)*WI1(Z,B)+(B-1.0)*VI1(Z,B,B))/(B-1.0)
B13=-(UI1-XI(C)-(B-1.0)*WI1(Z,B)+(B-1.0)*VI1(Z,C,B))/(C-1.0)
B14=-(UI1-XI(B)-(C-1.0)*WI1(Z,C)+(C-1.0)*VI1(Z,B,C))/(B-1.0)
W1=B11+B12+B13+B14
B15=-(WI1(Z,B)-VI1(Z,C,B)-(B-1.0)*WI2(Z,B)+(B-1.0)*VI2(Z,B,C))
1 / (C-1.0)
B16=-(UI1-XI(B)-(B-1.0)*YI(B)-(C-1.0)*WI1(Z,C)+(C-1.0)*VI1(Z,C,B)
2 +(C-1.0)*(B-1.0)*VI2(Z,B,C)) / ((B-1.0)*(B-1.0))
B17=+(VI1(Z,B)-VI1(Z,B,B)-(B-1.0)*WI2(Z,B)+(B-1.0)*VI2(Z,B,B)
3 ) / (B-1.0)
B18=+(UI1-XI(B)-(B-1.0)*YI(B)-(B-1.0)*WI1(Z,B)+(B-1.0)*VI1(Z,B,B)
4 +(B-1.0)*(B-1.0)*VI2(Z,B,B)) / ((B-1.0)*(B-1.0))
W2=(B-C)*(B15+B16+B17+B18)
B19=+(WI1(Z,B)-(B-1.0)*WI2(Z,B)-VI1(Z,B,B)+(B-1.0)*(B-1.0)*
5 VI2(Z,B,B)) / ((B-1.0)*(B-1.0))
W3=(B-C)*(B-C)*B19
W4=(PHI*AD/((B-C)*(B-C)))**2
W=W4*(W1+W2+W3)
UB3=W
RETURN
END
```

REFERENCE TABLES

0 00000/0 100 00002/1 200 00005/4

ROUTINE COMPILED
TIME LESS THAN 6 SECS
NUMBER OF INSTRUCTION WORDS 174
* * * * *

*PUNCH
*IDENTIFIERQAD
*FORTRAN

EGTRAH COMPILER

MARK NO. 302

DATE

15/06/70

TIME

FUNCTION GA1(Z)

COMMON RB,AD,B,AMP,PHI,C

COMMON PC,WC,EC,P1

COMMON IF

IF(IF-1)100,200,200

100 WPF=GAN1(Z)

GA1=WPF

RETURN

200 CONTINUE

C DIA2 FERUS ROSENBLUTH PROTON DIPOLE F F

GA11=(V12(Z,B)-V12(Z,B,C))/(C-1.0)

GA12=-(V12(Z,B)-V12(Z,B,B))/(B-1.0)

GA13=-(V12(Z,B)-V12(Z,B,B)-(B-1.0)*V13(Z,B,B))/((B-1.0)*(B-1.0))

W1=GA11+GA12+(B-C)*GA13

W2=-(AMP*PHI*AD*AD/((B-C)*(B-C)))

W=W2*W1

GA1=W

RETURN

END

0 00000/0

100

00002/1

REFERENCE TABLES

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 2 SECS

NUMBER OF INSTRUCTION WORDS 46

* * * * *

*PUNCH

*IDENTIFIER0A2

*FORTRAN

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FUNCTION GA2(Z)
COMMON BN,AD,B,AMP,PHI,C
COMMON FC,WC,EC,PI
COMMON IF

```

```

100 WNF=GA2(Z)
    GA2=WNF
    RETURN

```

```

200 CONTINUE
C  DIA2 TERMS ROSENBLUTH PROTON DIPOLE F F
   BA11=+(V11(Z,C)-V11(Z,C,B)-(B-1.0)*V12(Z,B,C))/((B-1.0)*(B-1.0))
   BA12=-(V11(Z,B)-V11(Z,B,B)-(B-1.0)*V12(Z,B,B))/((B-1.0)*(B-1.0))
   BA13=-(V12(Z,B)-V12(Z,B,B)-(B-1.0)*V13(Z,B,B))/((B-1.0)*(B-1.0))
   W1=BA11+BA12+(B-C)*BA13
   W2=-(AMP*PHI*AD*AD/((B-C)*(B-C)))
   W=W2*W1
   GA2=W
   RETURN
END

```

REFERENCE TABLES

0	00000/0	100	00002/1	200	00005/4
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ROUTINE COMPILED
 TIME LESS THAN 3 SECS
 NUMBER OF INSTRUCTION WORDS 57
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- *PUNCH
- *IDENTIFIERQAB
- *FORTRAN

FUNCTION GA(Z)

COMMON BA,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

COMMON IF

IF(NE=1)100,200,200

100 WNF=GA(Z)

WAB=WNF

RETURN

200 CONTINUE

C B1A2 TERMS ROSENBLUTH PROTON DIPOLE F F

R=1-C-Z

UI2=(4.0*PI*ALOG(R))/R

BAI1=+(UI2+VII(Z,C,B)-WII(Z,B)-WII(Z,C))/(B-1.0)*WI2(Z,B)

1 +(B-1.0)*VI2(Z,B,C))/((C-1.0)*(B-1.0)*(B-1.0))

BAI2=-(UI2+VII(Z,B,B)-2.0*WII(Z,B)-(B-1.0)*WI2(Z,B)

2 +(B-1.0)*VI2(Z,B,B))/((B-1.0)**3)

BAI3=-(UI2-2.0*VII(Z,B)+VII(Z,B,B)-2.0*(B-1.0)*WI2(Z,B)

3 +2.0*(B-1.0)*VI2(Z,B,B)+(B-1.0)*(B-1.0)*VI3(Z,B,B))/((B-1.0)**4)

W1=BAI1+BAI2+(B-C)*BAI3

W2=-(AMP*PHI*AD*AD/((B-C)*(B-C)))

W=W2*W1

WAB=W

RETURN

END

REFERENCE TABLES

0 00000/0

100

00002/1

200

00005/4

ROUTINE COMPILED

TIME LESS THAN 4 SECS

NUMBER OF INSTRUCTION WORDS 89

* * * * *

*PUNCH

*IDENTIFIERQA4

*FORTRAN

EGTRAM COMPILER

MARK NO. 302

DATE 15/06/70

TIME

FUNCTION GA4(Z)
COMMON BH,AD,B,AMF,PHI,C
COMMON PC,WC,EC,PI
COMMON BF

IF (BF-1) 100,200,200
100 WNF=GA4(Z)

GA4=WNF
RETURN

200 CONTINUE

C

B1A2 TERMS ROSENBLUTH PROTON DIPOLE F F

BA11=+(V11(Z,B)-V11(Z,C,B)-(B-1.0)*W12(Z,B)+(B-1.0)*V12(Z,B,C))
1 / (C-1.0)

BA12=-(W11(Z,B)-V11(Z,B,B)-(B-1.0)*W12(Z,B)+ B-1.0)*V12(Z,B,B))
2 / (B-1.0)

BA13=(V11(Z,B)-(B-1.0)*W12(Z,B)-V11(Z,B,B)
3 +(B-1.0)*(B-1.0)*V13(Z,B,B))/((B-1.0)*(B-1.0))

W1=BA11+BA12+(B-C)*BA13

W2=-(AMP*PHI*AD*AD/((B-C)*(B-C)))

W=W2*W1

GA4=W

RETURN

END

0 00000/0

100 00002/1

REFERENCE TABLES
200 00005/4

ROUTINE COMPILED

TIME LESS THAN 3 SECS

NUMBER OF INSTRUCTION WORDS 68

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*PUNCH

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*FORTRAN

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FUNCTION GA5(Z)
COMMON BA,AD,B,AMP,PHI,C
COMMON BC,WC,EC,PI
COMMON IF
IF(NE-1)100,200,200
100 WNF=GA5(Z)
    GA5=WNF
    RETURN
200 CONTINUE
C      DIAZ TERMS ROSENBLUTH PROTON DIPOLE F F
    U11=2.0*PI*ALOG(2.0)
    BA11=+(U11-XI(B)-(B-1.0)*YI(B)-(C-1.0)*WI1(Z,C)+(C-1.0)*VI1(Z,C,B)
1    +(C-1.0)*(B-1.0)*VI2(Z,B,C))/((B-1.0)*(B-1.0))
    BA12=-+(U11-XI(B)-(B-1.0)*YI(B)-(B-1.0)*WI1(Z,B)+(B-1.0)*VI1(Z,B,B)
2    +(B-1.0)*(B-1.0)*VI2(Z,B,B))/((B-1.0)*(B-1.0))
    BA13=-+(VI1(Z,B)-(B-1.0)*WI2(Z,B)-VI1(Z,B,B)+(B-1.0)*(B-1.0)*
3    VI3(Z,B,B))/((B-1.0)*(B-1.0))
    W1=BA11+BA12+(B-C)*BA13
    W2=-+(AMP*PHI*AD*AD/((B=C)*(B-C)))
    W=W2*W1
    GA5=W
    RETURN
END

```

REFERENCE TABLES

0 00000/0

100 00002/1

200 00005/4

ROUTINE COMPILED

TIME LESS THAN 4 SECS

NUMBER OF INSTRUCTION WORDS 86

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*RLB
*RLR
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*IDENTIFIERUADQ
*FORTRAN

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EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

```
FUNCTION QBO(Z)
COMMON BM,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI
C A1B2 TERMS ROSENBLUTH PROTON
W=QAO(Z)
QBO=W
RETURN
END
```

REFERENCE TABLES

0 00000/0

```
ROUTINE COMPILED
TIME LESS THAN 1 SEC
NUMBER OF INSTRUCTION WORDS 5
* * * * *
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*PUNCH
*IDENTIFIERQB1
*FORTRAN
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EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION QB1(Z)

COMMON BM,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C A1B2 TERMS ROSENBLUTH PROTON

W=QA2(Z)

QB1=W

RETURN

END

0 00000/0

REFERENCE TABLES

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 5

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*PUNCH

*IDENTIFIERQB2

*FORTRAN

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EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION QB2(Z)

COMMON BM,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C

AIB2 TERMS ROSENBLUTH PROTON

W=QA1(Z)

QB2=W

RETURN

END

0 00000/0

REFERENCE TABLES

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 5

* * * * *

*PUNCH

*IDENTIFIERQB3

*FORTRAN

EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION QB3(Z)

COMMON BM,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C

A182 TERMS ROSENBLUTH PROTON

W=QA3(Z)

QB3=W

RETURN

END

REFERENCE TABLES

0 00000/0

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 5

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*PUNCH

*IDENTIFIERGB4

*FORTRAN

EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION QB4(Z)

COMMON BM,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C

AIBZ TERMS ROSENBLUTH PROTON

"=QB5(Z)

QB4=W

RETURN

END

REFERENCE TABLES

0 00000/0

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 5

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*PUNCH

*IDENTIFIERQB5

*FORTRAN

EGTRAN COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION QB5(Z)

COMMON BM,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C A1B2 TERMS ROSENBLUTH PROTON

H=QA4(Z)

QB5=W

RETURN

END

REFERENCE TABLES

0 00000/0

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 5

* * * * *

*PUNCH

*IDENTIFIERXI

*FORTRAN

FUNCTION XI(T1)

COMMON BH,AD,B,AMP,PHI,C

COMMON PC,WC,EC,PI

C DENOM (T1-Z1) T1 GREATER THAN 1

XY=(T1+1.0)/(T1-1.0)

W= 2.0*PI*ALOG(XY)

XI=W

RETURN

END

REFERENCE TABLES

0 00000/0

ROUTINE COMPILED

TIME LESS THAN 1 SEC

NUMBER OF INSTRUCTION WORDS 9

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*PUNCH

*IDENTIFIERYI

*FORTRAN

EGTRAM COMPILER

MARK NO. 302

DATE

10/06/70

FUNCTION Y1(T1)
COMMON BM,AD,B,AMP,PHI,C
COMMON PC,WC,EC,PI

C DENOM (T1-Z1)**2 T1 GREATER THAN 1
W=4.0*PI/(T1*T1-1.0)
Y1=W
RETURN
END

0 00000/0

REFERENCE TABLES

ROUTINE COMPILED
TIME LESS THAN 1 SEC
NUMBER OF INSTRUCTION WORDS 7

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JOB ORGANISED 11.55.27

10/06/70