

---

# Against Comparativism about Mass in Newtonian Gravity – a Case Study in the Metaphysics of Scale

---

*Author:*

Niels C.M. MARTENS

*Supervisors:*

Dr. Oliver POOLEY  
Prof. David WALLACE  
Dr. Adam CAULTON



MAGDALEN COLLEGE – UNIVERSITY OF OXFORD

Submitted in partial fulfillment of the  
requirements for the degree of Doctor of Philosophy

Michaelmas Term, 2016



# Against Comparativism about Mass in Newtonian Gravity – a Case Study in the Metaphysics of Scale

Niels C.M. Martens

Magdalen College, Oxford – Michaelmas Term 2016

Submitted in partial fulfillment of the  
requirements for the degree of Doctor of Philosophy

[Abstract]

---

This thesis concerns the metaphysics of scale. It investigates the implications of a physical determinable being dimensionful. In particular, it considers the case study of mass, as it features within Newtonian Gravity. Nevertheless, most of the terminology, methodology and arguments developed should be relatively straightforwardly applicable to other determinables and theories.

Weak Absolutism about mass holds that mass ratios obtain in virtue of absolute masses. Weak Comparativism denies this. In the first five chapters I argue in favour of Weak Absolutism over Weak Comparativism. The sixth chapter argues against reducing mass to other non-mass facts. The overall conclusion is Strong Absolutism about mass within Newtonian Gravity: mass ratios obtain in virtue of absolute masses, which themselves are fundamental (i.e. they do not require anything further in order to obtain).

Comparativism promises to recover all the virtues of absolutism, in particular its empirical adequacy, but at a lower ‘metaphysical cost’. Special attention is given to Dasgupta’s recent comparativist proposal. Dasgupta interprets the requirement of empirical adequacy in terms of the undetectability of the absolute mass scale. I argue that undetectability is an unsuitable way of understanding empirical adequacy and that we would do better to understand it in terms of a theory’s ability to correctly generate the set of empirically possible worlds (or at least the actual world). I refute Dasgupta’s comparativism both on my terms and on his own terms. I subsequently develop and strongly criticise alternative versions of comparativism. Chapter five sheds doubt on the supposed ‘metaphysical parsimony’ of comparativism.

This debate should be of particular interest to readers who engage with the substantialism–relationalism debate. These debates are much more entwined than previously acknowledged, which provides a significant source of mutual inspiration, although I do also draw out some important disanalogies.



*To P.*



# Document Statistics

---

Words in Abstract:	333
Words in Body:	55,611
Words in Captions & Footnotes:	7,000
Words in Bibliography:	2,329
Maths (In-Line & Displayed):	644
Figures & Tables:	15
<b>Total (excl. Abstract, Figures &amp; Tables):</b>	<b>65,584</b>



# Acknowledgements

---

I would like to thank the Arts & Humanities Research Council and the Scatcherd European Scholarship Foundation for three years of financial support. A research visit to collaborate with Shamik Dasgupta at Princeton University would not have been possible without the AHRC Research Training Support Scheme and a Santander Academic Travel Award. Magdalen College has my gratitude for the hospitality it has extended to me as a graduate student—many of the ideas advocated in this thesis have blossomed in Magdalen’s gardens. I also thank the alumni of Magdalen College and the Philosophy Department of the University of Oxford for financial support towards attending several conferences and summer schools.

My interest in the topics of this thesis extends back at least to Jan-Willem Romeijn’s lectures on the philosophy of the natural sciences at the University of Groningen. When asking him whether it would be possible to write a BA thesis in philosophy that combined my favourite part of his lectures, the substantivalism–relationalism debate, with the topic of my BSc thesis, parity violation, I expected a negative answer. It turned out that these topics were much more related than I had imagined. My DPhil thesis completes this triangle of analogous absolute-vs-relational flavoured debates: space, handedness and mass. Thinking originally that this BA thesis would be the end of my dabbling in philosophy, which till that point was merely a means towards becoming a better physicist, this exhilarating project under Jan-Willem’s guidance instead drove me to apply to the MSt Philosophy of Physics course at the University of Oxford, which ultimately led to the DPhil course.

Besides Jan-Willem, my greatest intellectual debts are to my DPhil thesis supervisors: Oliver Pooley, David Wallace and Adam Caulton. After having applied to the MSt programme mainly to work with Oliver, I only found out at the start of term that he was on sabbatical for the full duration of the course. Luckily, I was allowed to stay on for the DPhil and have him be my primary supervisor for more than three years. Every time I thought I had finally managed to write a decent piece of work, Oliver would proceed to cover the first page in red and have us discuss it for two hours or more. In the typical Socratic Oxford style, I would be forced to approach the topic from ten different angles I had not even considered before. This has taught me the tough lesson that philosophy of physics is never done—an exciting prospect for the philosopher in me, a scary prospect for the physicist in me. David was my Transfer of Status examiner (with Simon Saunders), and also my Confirmation of Status examiner, making him the ideal candidate to join our team after two years, providing guidance that was

complementary to Oliver's. Adam only joined the team at the very end, after David left Oxford. This timing was unfortunate, in that my conversations with Adam suggested many possible further avenues for research, reiterating the lesson that a philosophy project is never really done. Oliver, David and Adam have shown me how I can, should and want to do philosophy of physics. I could not have asked for a better role model than this amalgam of philosophers of physics.

Two other philosophers have played a crucial role: Shamik Dasgupta and David Baker. Shamik allowed me to visit him at Princeton as a Visiting Student Research Collaborator. He was extremely generous with his time, sometimes at Princeton but mostly in cafés in Brooklyn. Although I have not been able to turn him into an absolutist, nor he me into a comparativist, I believe we now understand much better where the other comes from. Dave has been so kind to share and discuss two manuscripts with me that played an important role in the development of this thesis.

Many other philosophers of physics deserve my thanks for comments on, or discussion of subjects relating to, or sharing their manuscripts related to material in this thesis. These include Harjit Bhogal, Harvey Brown, Eddy Keming Chen, Erik Curiel, Marco Dees, Neil Dewar, Patrick Dürr, Michael Esfeld, Sam Fletcher, Mike Hicks, Dennis Lehmkuhl, Niels Linnemann, Chiara Marletto, Casey McCoy, Tushar Menon, Tom Møller-Nielsen, Zee Perry, Carina Prunkl, John Roberts, Simon Saunders, Syman Stevens (also for his  $\text{\LaTeX}$ -craft), Teru Thomas, Chris Timpson, and several other philosophers mentioned in the footnotes. Chris Timpson and Alastair Wilson provided a refreshingly new set of comments during the *viva voce* of this thesis.

Three friends deserve special mention. Reinier van Straten has helped me through the long Oxford summers with our silly 'who has written most words today' competitions, and our hundreds of punting sessions through the above mentioned Magdalen gardens<sup>1</sup>. Tushar Menon and Carina Prunkl I have come to consider to be family. Tushar has been a part of this journey in so many ways, not in the least by never failing to remind me of the importance of mathematics and of Newtonian Gravity being a false theory<sup>2</sup>. Carina deserves my unending gratitude simply for being Carina. Finally, I do not think I would have stayed sane during that last summer of writing up if it had not been for underwater hockey and Sergio & Katja's bachata.

Needless to say, my family deserves my thanks above all. They have raised me to be one of those annoying millennials who believes they could achieve anything as long as they work hard enough for it. Bedankt voor alle kansen die jullie me hebben gegeven!

NCMM, 21 November 2016 [Revised: 2 May 2017]

---

<sup>1</sup>And by showing me that Luftrausers is way more effective than a stress ball.

<sup>2</sup>But aren't *all* our current theories false, Tushar?

# Nomenclature

- $G \cdot m$  problem, 57–58
- 4D-fundamentalism, 94
- Absolute masses
  - Definition, 9, 15
  - Realism, 51
- Absolutism
  - Strong, Metaphysical, 11, 94
  - Weak, Metaphysical, 8
- At-at theory, 78, 140
- Comparativism
  - Alpha Mass, 89
  - Chain, 120, 122
  - Dasgupta's, 33–34
  - Doubly Machian, 92, 97, 111, 147
  - Dynamic, 21
  - Kinematic, 17
  - Machian, 90, 97, 147
  - Mixed, 80
  - Piggy-back, 109
  - Regularity, 76, 92, 96–97
  - Strong, Metaphysical, 11
  - Weak, Metaphysical, 9
  - Web, 120
- Comparativist's Bucket, 49, 72–73
- Completeness, 65
- Dasgupta's Razor, 31
- Determinate Quidditism, 16
- Determinism
  - Bakerian, 78
  - Laplacean, 64, 71
- Empirical Equivalence
  - Models, 71
  - Theories, 63
- General Recipe for  $\phi$ , 69
- Grounding, 8
- Initial Variables & P'meters Problem
  - Comparativist, 70
  - General, 69
  - Reductionist, 79, 123–142
- Laplace's problem, 1, 60, 71, 123, 143
- Leibniz Mass Scaling
  - Active, 25
  - Passive, 27
- Magnitudes, 14
- Mill-Ramsey-Lewis Best Systems, 95
- Ozma Games, 18, 50, 56
- Primitivism, 11, 94
- Quantities, 14
- Reductionism, 11, 79, 123–142
- Regularity Eliminativism, 105
- Relationalism
  - Enriched, 75
  - Kinematic, 17
  - Regularity/ Have-it-all, 76, 95–96
- Separability
  - Generalised, 102
  - Standard, 94, 101–104, 115–116
- Soundness, 68
- Supervenience, 94

Three approaches to empirical adequacy

Possibility checking, [24](#), [26](#), [60–71](#)

Symmetry, [24](#), [25](#), [146](#)

Undetectability, [24](#), [26](#), [30–59](#)

# Contents

<b>Introduction</b>	<b>1</b>
<b>1 Absolutism and Comparativism about Mass</b>	<b>6</b>
1.1 Defining Absolutism and Comparativism . . . . .	6
1.1.1 How do the determinates relate, mathematically? . . . . .	6
1.1.2 How do the determinates relate, metaphysically? . . . . .	8
1.1.3 Representing mass, or ‘What absolute mass is and is not’ . . . . .	11
1.1.4 Kinematic comparativism . . . . .	17
1.1.5 Operationalising Mass: Ozma Games . . . . .	18
1.2 Arguments for Comparativism . . . . .	20
1.2.1 Naive Arguments & Dynamic Comparativism . . . . .	20
1.2.2 The General Schema & Motivation for the Comparativist Argument . . . . .	21
1.2.3 Modified General Schema . . . . .	23
1.2.4 Three other approaches to empirical adequacy . . . . .	24
1.3 Summary so far & Thesis Outline . . . . .	28
<b>2 Detecting Newtonian Absolute Masses</b>	<b>30</b>
2.1 Dasgupta’s Razor . . . . .	30
2.2 Dasgupta’s Comparativism . . . . .	32
2.3 Dasgupta: absolute masses are undetectable . . . . .	34
2.3.1 Spatial Intra-world Leibniz Scaling . . . . .	35
2.3.2 Temporal Intra-world Leibniz Scaling . . . . .	37
2.3.3 Inter-world Leibniz Scaling . . . . .	41
2.4 Response to Dasgupta . . . . .	44
2.4.1 Detectability: the dependence of trajectories on absolute masses . . . . .	46
2.4.2 Adding gravitation: the comparativist’s bucket . . . . .	48
2.5 Solving the Ozma Mass Problem . . . . .	50
2.6 Evaluating the undetectability approach . . . . .	51
2.6.1 Ozma Games, one last time . . . . .	56

2.7	Further underdetermination of absolute masses . . . . .	56
2.8	Conclusion . . . . .	58
<b>3</b>	<b>The Possibility Checking Approach</b>	<b>60</b>
3.1	Why mass? . . . . .	60
3.2	Empirical Equivalency via possibility checking . . . . .	62
3.3	A challenge for the comparativist: the comparativist's bucket again	72
<b>4</b>	<b>Comparativist Responses</b>	<b>74</b>
4.1	A taxonomy of responses . . . . .	74
4.2	Facing the challenge head-on . . . . .	76
4.2.1	Baker's response . . . . .	77
4.2.2	Alternative Ideologies . . . . .	79
4.2.3	Varying Newton's Constant . . . . .	85
4.3	If not the spirit, then at least the letter: regularity comparativism	92
4.3.1	Empiricism about laws of nature . . . . .	93
4.3.2	Regularity Relationalism . . . . .	95
4.3.3	Regularity Comparativism . . . . .	96
4.3.4	Responses . . . . .	97
4.3.5	Conclusion . . . . .	106
4.4	Avoiding the challenge . . . . .	107
4.4.1	No Initial Condition Formulation . . . . .	107
4.4.2	Giving up soundness and/or completeness . . . . .	111
4.5	Conclusion . . . . .	113
<b>5</b>	<b>Theoretical Virtues</b>	<b>115</b>
5.1	Non-locality . . . . .	115
5.2	Metaphysical Parsimony & Explanatory Power . . . . .	116
5.2.1	Quantitative & Qualitative Metaphysical Parsimony . . . . .	117
5.2.2	The conspiracy of mass relations . . . . .	118
5.2.3	Chain Comparativism . . . . .	120
5.2.4	Metaphysical parsimony at the level of worlds . . . . .	121
5.2.5	Conclusion . . . . .	122
<b>6</b>	<b>Reductionism</b>	<b>123</b>
6.1	The Project . . . . .	124
6.2	A bad argument against reductionism . . . . .	126
6.3	Mach . . . . .	127
6.3.1	Mach & Comparativism . . . . .	129
6.4	Beyond Mach . . . . .	130
6.4.1	Generalising to more particles . . . . .	130
6.4.2	Including other spatiotemporal magnitudes . . . . .	131

6.4.3	The main argument . . . . .	132
6.4.4	Unpacking the argument . . . . .	133
6.5	Bits and bobs . . . . .	139
6.6	An additional argument against reductionism . . . . .	141
6.7	Conclusion . . . . .	142
<b>Conclusion &amp; Outlook</b>		<b>143</b>



# Introduction

Pierre Laplace had a problem. A big one. He wanted to know what information would be sufficient to predict the future. This question was asked in a much more precise way by Poincaré [1,2]. What is the minimal choice of initial variables and parameters that corresponds to a well-posed initial value problem—that is, for any combination of the associated initial determinate values, the relevant laws will determine a unique evolution [3]?

Poincaré was the first to explicitly link this problem to the substantivalism–relationalism debate about space and motion [1,4]. In retrospect, in the context of theories of gravity—in this thesis we will focus on Newtonian Gravity (NG)—we have now been focusing for more than three centuries on the question of whether the initial data need include absolute or merely relational determinables. That is, do we need positions, velocities and accelerations relative to absolute space, or is it sufficient to consider distances, velocities and accelerations relative to other material bodies?

Given the age and the importance of this debate, it should be surprising that only very recently we have started considering whether we need the absolute scales of the dimensionful determinables, or only the ratios between the associated determinates. Do we require an absolute distance scale, or only ratios between distances? Do we require an absolute mass scale, or only ratios between masses (or not even those)? It is mass that will be the central case study of this thesis, although much of the terminology, methodology and arguments developed here should be relatively straightforwardly applicable to other determinables and theories.

The standard formulation of NG seems to explicitly model the *intrinsic* mass of each particle, by a numerical value associated with that particle (where that value depends on the unit chosen, say kilograms), rather than modeling only mass ratios between particles. This particle is 1kg, that particle is 4kg, etc. Naive literalism about our physical theories seems to suggest a metaphysical commitment to that absolute mass scale, and even a metaphysical priority of those absolute masses over the mass ratios. We may call this (weak) absolutism about mass. Especially Lewisian Humean Supervenience has taken this view on board by positing

a spatiotemporal mosaic with intrinsic masses (and charges) sprinkled onto it.

On the other hand, Mach's operational definition of mass has been highly influential (among physicists) [5]. He defined mass relations, namely mass *ratios*, (via acceleration ratios) and considered his work done; no need for a further operational definition of absolute masses, let alone granting them metaphysical priority over the mass ratios. We may call this (weak) comparativism about mass. Similarly, Field's nominalist research programme, as presented in *Science without Numbers* [6], was and is highly influential (among philosophers), and has comparativism as a by-product: its fundamental nominalist predicates are a betweenness and a congruence relation. Why would anyone place (fundamental) importance only on mass ratios? For one thing, the numerical values used to model masses in the standard formulation of NG are conventional. That is because this modeling comprises not only a numerical value but also an arbitrary unit. A unit is nothing but a relation in disguise. Often we choose the unit 'kg', which uses the International Prototype of the Kilogram in Paris as the standard reference object, but any other reference object would do, in principle. A different choice of unit means that the masses of each object are represented by a different numerical value. This arbitrariness should perhaps not be surprising. Is there anything '4-ish' about (the mass of) a square object of 4kg in the same sense as there is something '4-ish' about its shape (i.e. the number of its corners)? No. If we were to send such an object to aliens on another planet and enquire after its properties via morse code, they would unambiguously communicate its number of corners, but would express its mass, say, as 10,000 zorbs. After further enquiry we would find out that the object we sent them just happens to stand in a mass ratio of 10,000:1 to the standard of 1 zorb that they have stored in their capital city. It seems that there is no way of doing better at communicating the mass property than by expressing it comparatively. That just seems to be what it is for a physical determinable (such as mass) to be dimensionful.

The debate between weak absolutism and comparativism thus concerns the relative metaphysical priority (i.e. fundamentality) of absolute masses and mass ratios. Weak absolutism holds that mass ratios are true in virtue of absolute masses (which are thus metaphysically prior, that is more fundamental), whereas weak comparativism either denies the reality of absolute masses, or at the very least their metaphysical priority to mass ratios. Strong absolutism and comparativism add to this the claim that the most fundamental level of mass (i.e. absolute masses for the absolutist and mass ratios for the comparativist) is not itself further reducible to non-mass facts or properties (i.e. spatiotemporal determinables). This thesis argues for strong absolutism about mass within Newtonian Gravity.

The positions of absolutism and comparativism will be developed in detail in [Chapter 1](#). I will introduce the general schema for what I take to be the main argument in favour of (weak) comparativism. Comparativism promises to recover

all the virtues—both empirical and theoretical—of absolutism, but with a sparser metaphysics. Hence, the argument goes, we should favour comparativism. In the following three chapters I argue against weak comparativism by refuting its empirical adequacy, that is by showing that an initial state lacking absolute masses fails to solve Laplace’s problem. [Chapter 5](#) sheds doubt on the supposed ‘metaphysical sparseness’ of comparativism, as well as its explanatory power. [Chapter 6](#) argues against reducing Newtonian mass to other non-mass primitives within Newtonian Gravity. The overall conclusion is that strong absolutism about mass is true in Newtonian Gravity: mass ratios obtain in virtue of absolute masses, which themselves are fundamental (within NG)—that is, they do not obtain in virtue of any of the other non-mass (i.e. spatiotemporal) determinables featuring in Newtonian Gravity.

In slightly more detail, [Chapter 2](#) focuses on Dasgupta’s recent comparativist proposal. He interprets the requirement of empirical adequacy in terms of the undetectability of absolute masses. Although I refute Dasgupta’s theory on his own terms, my main intent there is to argue that undetectability is an unsuitable way of understanding empirical adequacy, and that we would do better to understand it in terms of ‘possibility checking’: the theory’s ability to correctly generate the set of empirically possible worlds (or at least the actual world). This approach is developed in detail in [Chapter 3](#). On those terms Dasgupta’s theory still fails. [Chapter 4](#) develops alternative versions of comparativism—strongly inspired by the analogous substantivalism–relationalism debate about space and motion—and strongly criticises them.

On that last note, the absolutism–comparativism debate should be of particular interest to readers who engage with the substantivalism–relationalism debate. These debates are much more entwined than previously acknowledged, which provides a significant source of mutual inspiration, although I will also draw out some important disanalogies.

Finally, it is important that I be clear on the interpretation and scope of the main claim, which is that strong absolutism about mass is *true in Newtonian Gravity*. One might be tempted to understand this conclusion according to what Williams calls the standard account of theory interpretation, which holds that to give an interpretation of a physical theory is to answer the question “if this theory provided a true description of the [actual] world in all respects, what would the world be like?” [7, p.2]. But Newtonian Gravity—and with it most if not all of our current, best theories—is only approximately true of our actual world, which strictly speaking means that it is false. Now, if it were the case that ‘if absolutism/ comparativism is true, then it is necessarily true’, we could just consider any theory we wanted to and extend our conclusions to the future, final, true theory whatever it may turn out to be. However, it is an important theme of this thesis that the absolutism–comparativism debate is an empirical issue, and

depends on the specific theory under consideration. One modest way to proceed then would be to view the current discussion of the absolutism–comparativism debate in the context of NG mainly and merely as an exercise in developing the methodology relevant to deciding on matters of absolutism vs. comparativism and to chart the potential pitfalls—without necessarily any ramifications for the metaphysics of the actual world. That is, to prepare our metaphysical toolkit for the moment when we have found our final theory. Although I in fact do consider this to be a strong motivation for the current project, and one of the main contributions of this thesis, this thesis also goes beyond this goal, and has to do so. As repeatedly mentioned, a crucial bone of contention in this debate is the empirical adequacy of comparativism about mass in NG. This issue and thereby the whole debate is moot without having a well-defined data-set—which both sides agree on—that the theory is trying to account for. And that data-set of course has to pertain to the actual world. I therefore intend the main claim of this thesis to be understood within William’s alternative to the standard framework of theory interpretation, according to which interpreting a physical theory is to answer the question “given that this theory provides an approximately true description of our world, what is our world approximately like?” [7, p.2]. The merely approximate truth of the theory of NG is cashed out by focusing only on a proper subset of the empirical data that needs to be accounted for. This Newtonian subset ignores data at small length scales (quantum effects), at speeds comparable to the speed of light (relativistic effects), at small accelerations (MONDian effects), and in strong gravitational fields (generally relativistic effects). That the actual world may only approximately, that is only emergently contain absolute Newtonian masses is acknowledged by acknowledging that Newtonian Gravity is not a fundamental theory. Nevertheless, it is consistent to claim that the strong form of absolutism is true of mass *as it features in Newtonian Gravity*. By this I mean that absolute masses cannot be reduced to, or are not grounded in other non-mass (i.e. spatiotemporal) determinables that feature *at that same level of description*, that is within NG. At the same time, the final theory—or maybe theories—will (approximately) ground NG as a whole. It will then be our task to look for a counterpart of Newtonian absolute mass within that more fundamental theory—how to do this, and the related issues of inter-theoretic reduction and essentialism about mass are outside the scope of this thesis. Perhaps that counterpart will not be fundamentally mass-like (cf. the Higgs mechanism in the case of inertial mass, or the curvature of spacetime in the case of gravitational mass), or perhaps that counterpart will even be a comparative notion of mass (in combination with, perhaps, other properties). None of that will change the conclusion that strong absolutism holds of mass as it features in Newtonian Gravity, and that Newtonian Gravity provides an approximately true, emergent description of the world with Newtonian mass being strongly absolute at the emergent, Newtonian level.

If it had turned out that tigers were not composed of atoms, but of tiny little elephants, this would not have made us stop believing in tigers, nor in the claim that at the macroscopic level to which zoology applies tigers are not reducible to (macroscopic) elephants (which are other primitives within that same theory, zoology).

In summary, the claim that absolutism about mass is true in Newtonian Gravity is to be interpreted as follows. Absolutism about mass is the metaphysical picture that fits better with the theory of Newtonian Gravity than the comparativist picture. The theory of Newtonian Gravity provides an approximately true description of the actual world to the extent that it can account for the Newtonian subset of empirical data—as defined above—but not the full set of empirical data. At the emergent, approximate, Newtonian level, e.g. the level of planetary motion, the world thus contains absolute masses, even if these notions may not turn out to appear at the fundamental level. Moreover, the further claim that the *strong* version of absolutism is true about mass in Newtonian Gravity then denies that at this emergent level the world equally well or even better fits a metaphysical picture that substitutes those Newtonian absolute masses for other non-mass notions that feature at that same level, e.g. planetary accelerations—even if such non-mass notions may turn out to be preferable at the fundamental level of reality that a final theory might describe exactly.

# Chapter 1

## Absolutism and Comparativism about Mass

### 1.1 Defining Absolutism and Comparativism

Dasgupta notices the following interesting aspect of the property of having mass:

“The property of having mass is a determinate that appears to have two kinds of determinates. On the one hand, we naturally think that something with mass has a determinate *intrinsic* property, a property it has independently of its relationships with other material bodies. But we also think that things with mass stand in various determinate *mass relationships* with one another, such as  $x$  being more massive than  $y$  or  $x$  being twice as massive as  $y$ .” [8, p.105]

It is the central aim of this thesis to determine how these two determinates, mass relations and absolute masses, relate. This question can be asked both on the formal or mathematical level, and the metaphysical level.

#### 1.1.1 How do the determinates relate, mathematically?

We first ask the question of how these two determinates are related, *mathematically speaking*. Here are some obvious ways in which they might relate: a description in terms of one determinate underdetermines the facts associated with a description in terms of the other determinate (which does settle all the facts about mass); both descriptions are simultaneously required and together completely and uniquely settle the facts about mass; or, the facts associated with one description are inconsistent with any set of facts allowed by the other description. We will as-

sume in this thesis that the relevant mass relationships<sup>3</sup> are the scale-independent ones (see Baker for a justification [9]), which under suitable constraints (detailed below) can be interpreted as mass ratios<sup>4</sup>. We will consider these determinates as they feature in Newtonian Gravity. For simplicity, we will assume standard NG to comprise an ontology of  $n$  particles in a flat (*pace* Knox [10]) substantial Newtonian space and time, with an ideology<sup>5</sup> of absolute distances, velocities and finite, positive, non-zero masses, governed by Newton’s three laws and the Law of Universal Gravitation, and a principle of equivalence between inertial and gravitational mass<sup>6</sup>. It is then easy to see that in such a theory the mass relations are fully and uniquely determined by the intrinsic masses. The mass relation between each ordered pair of particles is given or represented by the ratio of the intrinsic masses of the relata, which is always well-defined under the assumption of finite non-zero masses. Thus, in a world with 3 particles, with masses of 4 kg, 2 kg and 1 kg, the mass relations between object 1 and 2 and between 2 and 3 are in both cases ‘twice as massive as’, and the mass relation between object 1 and 3 is ‘four times as massive as’.

Does each complete set of mass relations between all  $n$  particles also fully and uniquely determine the absolute masses of all  $n$  particles? No, as the following counterexample shows. Consider again a universe consisting of 3 particles. The mass relation between object 1 and 2 is (tentatively) ‘twice as massive as’, the relation between object 2 and 3 is (tentatively) ‘twice as massive as’, and the relation between 1 and 3 is (tentatively) ‘ten times as massive as’. It is obvious that there is no distribution of absolute masses that satisfies this structure of mass relations—hence we were not even justified in interpreting them as mass ratios in the first place.

If the set of mass relations is sufficiently constrained—that is if every closed loop of mass relations combines (i.e. intuitively understood as ‘multiplies’) to a ratio of 1, or, in nominalistic terms, to the relation ‘is as massive as (itself)’—a

---

<sup>3</sup> Strictly speaking, in first order logic, there is just one mass relation, and several instantiations of that single relation. In this thesis, as in most of the literature, when we talk about mass relationships, this is short for several instantiations of this single mass relation. Why this makes sense will become more clear in [Subsection 1.1.3](#), where I will advocate defining absolute mass(es) and the mass ‘relation(s)’ using group theory.

<sup>4</sup>By this I do not mean to rule out nominalistic versions of comparativism—such as Field’s approach which uses primitive congruence predicates as mass relations [6]—in favour of quantitative approaches—such as the numerical mass ratios used by, for instance, Dasgupta. I merely intend to exclude relations such as, in quantitative terms, ‘x is 2kg more massive than y’.

<sup>5</sup>Here I am using ‘ontology’ and ‘ideology’ in the Quinean sense [11]. Roughly, ontology refers to the (primitive) objects, and ideology to their (primitive) properties.

<sup>6</sup>Given their distinct conceptual role it would be interesting to consider whether one could be an absolutist—defined below—about inertial mass but a comparativist—defined below—about gravitational mass, and vice versa. In this thesis we will however follow the current literature in bracketing out such a possibility, by considering only a single mass determinable.

*representation theorem* can be proven [9]. This means that every structure of relations so constrained can be instantiated by a distribution of intrinsic masses. Only then can they properly be understood as mass ratios! It is important to note that a distribution of intrinsic masses corresponding to a structure of mass relations is not unique: multiplying all intrinsic masses of a solution by a scalar provides a new solution. The representation theorem provides a homomorphism—not an isomorphism—from the distribution of absolute masses to the structure of mass relations.

Thus, to answer our initial question: without any constraints on the mass relations they will be inconsistent with the absolute masses for certain situations; with sufficient constraints they are consistent and related in a many-to-one fashion. Starting out with any distribution of absolute masses, on the other hand, *always* provides a *unique* set of mass relations.

### 1.1.2 How do the determinates relate, metaphysically?

We now turn to the metaphysical counterpart of this mathematical question. Absolutism about mass confers metaphysical priority to the determinate absolute masses:

**(Weak, Metaphysical) Absolutism:** The determinate absolute masses ground the mass relations.

I take the notion of ‘to ground’ to be synonymous with ‘to be true or to obtain in virtue of’, ‘to be metaphysically prior to’ and ‘to explain’. In other words, according to absolutism, the mass relations are explained by the intrinsic masses. ‘Explain’ should here not be understood in the causal sense, but in the metaphysical sense: P holds in virtue of the metaphysical facts that make it count as P. Thus, a causal explanation of ‘Niels played underwater hockey last Sunday evening’ would be that I had received an invite, that I saw on YouTube how amazing this sport is<sup>7</sup>, that I biked to the swimming pool, etc. A metaphysical explanation, on the other hand, should mention that what makes it count that Niels was playing underwater hockey last Sunday evening. In this case that might be the combination of facts that I was underwater, wearing a mask, in a team of six people, shooting pucks in the opponents goal, etc. I believe that both the debate between absolutism and comparativism (defined below) and the analogous debate between relationalism and substantivalism about space are most correctly, most concisely and most clearly explicated in terms of this notion of ground. If one prefers to explicate the latter debate in a different manner, for instance in terms merely of possible worlds, feel free to substitute that approach whenever

---

<sup>7</sup>It is! You should check it out online.

I use the notion of ground. Not too much should hinge on it, except perhaps the issue of equivocating between ‘fundamentality’ and ‘reality’ that is brought to light in [Chapter 2.6](#). However, as I will argue there, framing the debate in terms of whether or not absolute masses are detectable is not to be preferred.

The opposing view denies such metaphysical priority of absolute masses:

**(Weak, Metaphysical) Comparativism:** The determinate mass relations are not grounded in determinate absolute masses.

Comparativism is thus silent on the status of the absolute masses. It is often held in conjunction with either the view that facts about (the quantities used to represent) absolute masses hold in virtue of the mass relationships together with a convention, such as deciding to refer to the mass of International Prototype of the Kilogram in Paris as 1 kg, or a view that simply denies the existence or meaningfulness of absolute masses. In [Section 4.3](#) I will however discuss grounding the absolute masses in the mass relations (plus other non-conventional non-mass facts, namely a complete four-dimensional mosaic of spatiotemporal relations).

This latter option brings us to the question of what is exactly meant by ‘absolute masses’. Are we to define ‘absolute’ as ‘intrinsic’, or ‘monadic’, or a conjunction of both? Dasgupta, in the quote at the start of this chapter, refers to them as intrinsic masses. For the absolutist, the absolute masses are indeed intrinsic. However, grounding absolute masses in a complete four-dimensional mosaic of spatiotemporal relations makes them as extrinsic as they could possibly be, even though they are still monadic<sup>8</sup>. I—and Roberts [13]—thus take the important distinction between absolute masses and mass relations to be that the former are monadic properties, and the latter dyadic properties (i.e. binary relations)<sup>9</sup>. That is how I will be interpreting the essence of the qualifier ‘absolute’, although I will define the structure of the absolute masses in much more detail in [Subsection 1.1.3](#).

Perhaps it is fair to say that absolutism is the received view. Roberts claims that it surely is the commonsense position [13]. Views that count as absolutism have been defended by Armstrong [14, 15], Mundy [16] and Lewis [17]. This is not to say that comparativism is without any impressive history of (implicit) advocates. Russell discussed it in his *The Principles of Mathematics* [18]. Ellis defended it in 1966 [19]. Field’s nominalist research programme, as presented in *Science without Numbers* [6], has comparativism as a by-product (see also his 1985 paper [20]). It was central to Bigelow, Pargetter & Armstrong’s 1988

---

<sup>8</sup>The Higgs mechanism for generating inertial masses in the Standard Model of Particle Physics seems to similarly generate monadic masses that are nevertheless extrinsic, as the mass emerges from an interaction with, or immersion in the Higgs field [12].

<sup>9</sup>In the context of absolutism and comparativism about other determinables, say distance, we have to adjust this: absolutism about distance will concern binary relations, and comparativism about distance will concern 4-place relations.

paper [21] (see also Bigelow & Pargetter’s book [22]). Arntzenius briefly touches upon the topic in his *Space, Time, and Stuff* [23] in 2012, but it is not until 2013 that the terms ‘absolutism’ and ‘comparativism’ are coined by Dasgupta [8]. Eddon’s paper [24] in that same year cites Dasgupta’s paper but does not yet adopt the terminology. It is Dasgupta’s paper, and two of Baker’s manuscripts in response [9, 25], especially his *Some Consequences of Physics for the Comparative Metaphysics of Quantity*, that I will engage most with in this thesis.

Do we again need to postulate a constraint on the mass relations, this time a metaphysical constraint, to ensure that they are homomorphic to a distribution of absolute masses? If the models of our physical theories were to consist of mass relations that do not satisfy this constraint, no absolutist version of these physical theories could be formulated, and comparativism would trivially be true. In fact, our physical theories—definitely Newtonian Gravity—are standardly formulated in terms of absolute masses, the tools of the absolutist. The constraint on the mass relations is indeed needed. But now that the constraint is metaphysical we may ask why the world, in the comparativist framework, would be so conspiratorial as to consist of a complicated structure that allows exactly for an instantiation by absolute masses. We will return to this mystery in [Subsection 5.2.2](#), and assume for now that the constraint is satisfied.

If defined as above, weak comparativism is the denial of weak absolutism, which makes this pair of views mutually exhaustive<sup>10</sup>. However, some of the literature tends to focus on stronger versions, either implicitly or explicitly<sup>11</sup>. They do not only concern the relative metaphysical priority of the two types of

---

<sup>10</sup> That is if we assume the existence of mass ratios. It is an interesting question whether we truly need mass ratios (within NG), or whether they are merely an artefact of our representation in terms of quantities. As far as I am aware all of the comparativism literature assumes the existence of mass ratios, so we will do so as well (for now).

<sup>11</sup>Dasgupta defines absolutism as “the view that the most fundamental facts about material bodies vis-à-vis their mass include facts about which intrinsic mass they possess” [8, p.105], and comparativism as “the view that the most fundamental facts about material bodies vis-à-vis their mass just concern how they are related in mass, and all other facts about their mass hold in virtue of those relationships” [8, p.105-6]. (As explained below, I do not agree with this last part of the definition of comparativism, since it is violated by the view called regularity comparativism, which I take to be correctly categorised as comparativism.) As they stand, these statements could be interpreted either in the weak sense or in, what I call below, the strong sense, depending on whether ‘most fundamental’ is taken to be a statement about absolute fundamentality, or whether ‘most fundamental vis-à-vis their mass’ is read as being about relative fundamentality. Since Dasgupta also claims that the absolutist believes that “intrinsic masses are fundamental” [8, p.105], he is best interpreted as explicitly discussing the strong versions of absolutism and comparativism. However, nothing in the rest of his paper turns out to hinge on considering the strong rather than the weak versions. In particular, none of the arguments gives any reason to go beyond the weak versions. Hence, as explained in the text, I believe it best in general to focus on the weak versions, especially since only they are mutually exhaustive. The strong versions will be discussed separately, in [Chapter 6](#).

determinates, but also claim that the metaphysically prior determinate is itself ungrounded.

**Strong (Metaphysical) Absolutism:**

1. Weak (Metaphysical) Absolutism
2. **Primitivism**<sup>12</sup>: Mass is fundamental. That is, the determinate absolute masses are not themselves grounded in anything further.

**Strong (Metaphysical) Comparativism:**

1. Weak (Metaphysical) Comparativism
2. **Primitivism**: Mass is fundamental. That is, the determinate mass relationships are not themselves grounded in anything further.

These two stronger definitions are not mutually exhaustive: both are inconsistent with a denial of the second part, namely with the view that mass (both absolute masses and mass relationships) is not fundamental. This third position within the resulting trilemma may be called reductionism about mass<sup>13</sup>. [Chapter 6](#) will argue against this form of reductionism (and [Section 4.3](#) will argue against a different type of reductionism, namely that of absolute masses to mass ratios and other non-mass facts). When mentioning absolutism/comparativism in this thesis without any further specification, the distinction between the weak and the strong metaphysical versions will either be irrelevant, or it will be clear from the context that the weak metaphysical version is intended.

### 1.1.3 Representing mass, or ‘What absolute mass is and is not’

The standard formulation of NG seems to explicitly model the intrinsic mass of each particle, by a numerical value associated with that particle (where that value depends on the unit chosen, often kilograms). A literal metaphysical interpretation of this standard formulation seems to commit one to intrinsic masses. But what if such standard formulations are merely pragmatic and conventional shortcuts for physically or empirically equivalent formulations that include mass

---

<sup>12</sup>Dees calls this Quantity Primitivism, but in this thesis ‘quantity’ is a highly technical notion ([Subsection 1.1.3](#)), distinct from my equally technical notion of ‘magnitude’, which is presumably what Dees has in mind [26]. I am choosing for Primitivism rather than Magnitude Primitivism though, because in the case of strong comparativism the primitivism refers not to the magnitudes but the ratios.

<sup>13</sup>An analogous distinction in the space debate is made in Martens [27].

relations only? Reflecting on the following question suggests that this may indeed be the case.

When a particle in Newtonian Gravity is characterised as having a mass of, say, 5 kg, is there something intrinsically ‘5-ish’ about that particle (or its mass)? That the answer to this question is ‘no’, becomes clear when we contrast the property of being massive with the property of having a certain shape. ‘Being a polygon with 5 corners’ refers to something intrinsically ‘5-ish’ about an object with that property, which ‘having a mass of 5 kg’ does not. This difference is explained by the number of corners of an object being a 1) discrete and 2) dimensionless quantity; this allows us to simply count the corners. Mass, at least in (the standard formulation of) NG, is a continuous and dimensionful notion, hence the absolute mass of an object cannot simply be counted in the same way. Would a property that is only discrete but not dimensionless suffice to provide said contrast with Newtonian mass? That depends. Take for instance electric charge, which we believe to be quantised in quanta of  $\frac{1}{3}e$ . The existence of this smallest quantum removes the arbitrariness that a conventional unit such as kg has—although we are of course still free to choose any unit we want, however ‘unnatural’—but does not imply that we can count the electric charge of an individual object in the same way that we can count its corners. This would be the case if all electrically charged particles ‘featured’ a discrete number of quanta of  $\frac{1}{3}e$ , in the same way as a polygon features a discrete number of corners. Current theories tell us that this is not the case: the up quark, for instance, has a charge of  $\frac{2}{3}e$ , but this is not taken to mean that it carries 2 packages of electrical charge, just that it carries 1 package with a numerical (dimensionful) value that is twice as large as the minimum possible value<sup>14</sup>. Similarly, a property that resembles mass only by being continuous but not by being dimensionful would also fail to provide said contrast. An example would be (solid) angle. It is true that representing an angle as a fraction of the maximally possible angle (in that space) would make it somewhat objective (i.e. less conventional than the unit of mass), but an angle of 5% of the maximal angle is still not ‘5-ish’ in the way that a pentagon is. It is thus indeed both the discreteness and the dimensionlessness of the number of corners of a pentagon that makes it intrinsically ‘5-ish’ in a way that (particles with) properties that lack either of these features—such as absolute mass which lacks both—are not.

In summary, there is nothing intrinsically ‘5-ish’ about a particle in NG with a mass of 5 kg. Perhaps this should have been obvious, even without contrasting mass with shape, since a mass of 5 kg is equivalent to a mass of 11 pounds (whereas

---

<sup>14</sup>Perhaps distance (or time) as it features in theories of discrete spacetime such as causal set theory [28] would be a more interesting example. In this case, spatial (or temporal) extension boils down to counting the spacetime atoms. However, in such a case it is questionable whether distance (or time) is still truly a dimensionful determinable.

the property of having 5 corners is not equivalent to having 11 ‘anything’s). This points to three related but distinct worries for the absolutist: formal, metaphysical and epistemological. Together they seem to suggest that what matters is not the intrinsic masses, but the mass relations. Each of these three worries will now be discussed in turn. I will subsequently argue that all three worries dissolve once we understand correctly what an absolute mass is.

### Three worries about absolute mass values

The **formal worry** is that the numerical values for the absolute masses are not unique, but depend on the unit, which is conventional. And if absolute masses are to be metaphysically real let alone fundamental, it is a necessary condition for their values to be well-defined (i.e. unique).

Even if one could overcome this worry by somehow arguing for a privileged unit of mass, there is a **metaphysical worry** about the numerical values of the mass in that fundamental unit. Field [6] points out that the absolutist numerical values presuppose a platonic space where these numbers live<sup>15</sup>. What I find perhaps even stranger than that is the required connection between all massive objects and the platonic numbers in that space. Moreover, Field continues, even on that platonistic assumption, physical theories should provide an explanation without appeal to extraneous, causally irrelevant entities<sup>16</sup>. Numbers are such causally inert entities.

If, as at first blush appears to be the case, we need to invoke some real numbers like  $6.67 \times 10^{-11}$  (the gravitational constant in  $m^3/kg^{-1}/s^{-2}$ ) in our explanation of why the moon follows the path that it does, it isn’t because we think that that real number plays a role as a *cause* of the moon’s moving that way; it plays a very different role in the explanation than electrons play in the explanation of the working of electric devices. The role it plays is as an entity *extrinsic to the process to be explained*, an entity related to the process to be explained only by a function (a rather arbitrarily chosen function at that). Surely then it would be illuminating if we could show that a purely intrinsic explanation of the process was possible, an explanation that did not invoke functions to extrinsic and causally irrelevant entities. [6, p.43]

---

<sup>15</sup>One might argue that the comparativist faces that same objection: a relation such as ‘x is twice as massive as y’ refers indirectly to the platonic number two. This is true for the quantitative approach to comparativism of, for instance, Bigelow & Pargetter [21] and Dasgupta [8], but not for congruence-based comparativism, as advocated famously by Field himself [6].

<sup>16</sup>Arntzenius and Dorr echo this thought [23, Ch.8].

Even if we buy into an implausible metaphysics of platonistic numbers plus a (causally efficacious) mapping from that space to the massive objects, it would neither give us direct access to, nor acquaintance with, those ‘absolute mass values’, whereas we do have relatively direct access to shape via touch or almost directly via sight. What would it mean to ‘see a mass’? It is not as if a number appears on our visual image whenever we look at a massive object. In other words, ‘absolute mass values’ are epistemically inaccessible: this is the **epistemological worry**. The best we could do is choose a conventional standard, such as an object in Paris which we define to have a mass of 1 kg, and *compare* all other masses to that mass (and even that would have to go indirectly via weight).

### Magnitudes vs. Quantities

I will now argue that all these worries derive from failing to distinguish between *physical magnitudes* and the *numerical quantities* used to represent them<sup>17</sup>. This failure will keep recurring in the remainder of this thesis, as I believe it to be one of the crucial confusions that misdirects many authors towards comparativism.

Before making this distinction, and arguing that it dissolves all three worries, I will briefly point out one response that is specific to the formal worry—the worry that a fundamental entity must be well-defined, whereas ‘absolute mass values’ seem not to be. The fact that there are several formal modes of representation has no immediate consequences for the metaphysical thesis that absolutism is, as long as only one of those modes represents the actual world, and the other modes are merely mathematical. Of course we then wonder how we can know which of these modes of representation is the real one, but this is part of the epistemological worry. The abundance of these mathematical modes of representation stems from the mere fact that multiplying  $\mathbb{R}^+$  by an overall scalar is a non-trivial automorphism. In fact, all of these different representations represent the exact same metaphysics, which is the core idea behind the following story.

All three objections dissolve when we realise that we have been wrongly equating the absolutist’s commitment to absolute masses with a commitment to absolute numerical (mass) values—let us call those (numerical) **quantities**. Compare this again with shape: that a certain function from the corners of a polygon to  $\mathbb{R}^+$  gives us, say, the number 5, is a mathematical property of that object (plus that function) but does not imply that there is a metaphysical/Platonic numerical value (quantity) or even magnitude (as defined below) attached to that object. Absolutism merely claims that there are brute absolute mass **magnitudes** (which ground the mass relations), where I define physical magnitudes as (meta)physical

---

<sup>17</sup>Perhaps this failure to distinguish magnitudes and quantities is sufficient to undermine what Dasgupta calls ‘The Objection from Kilograms’ [8, §5] (which is, in contrast to the three worries discussed in this section of this thesis, seen as an objection against comparativism).

properties which mirror some of the structure of  $\mathbb{R}^+$  without in fact being a Platonistic version of the numerals in  $\mathbb{R}^+$  or a mapping to those numerals, that is without in fact being quantities. Magnitudes are thus the continuous version of the number of corners of a polygon-shaped object.

More precisely, I define physical magnitudes as follows. The determinate physical magnitudes falling under one physical determinable comprise a set of monadic properties with cardinality  $\aleph_1$ . On the elements of this set we place two<sup>18</sup> structures: 1) a total order, and 2) an associative concatenation structure which is to be interpreted as addition—it fixes where the sum of several magnitudes ‘fits into the total order’<sup>19</sup>. In the case of mass, the first structure is associated (operationally speaking) with comparing the masses of two objects by putting one massive object on each scale of a balance. Adding the second structure corresponds to adding several massive objects on each scale of the balance. In the case of electric charge these structures generate a totally ordered group, since electric charge can be neutral or negative, thereby providing an identity element and an inverse for every element in the set. In the case of mass this generates a totally ordered semi-group. It does not form a totally ordered group since there is no inverse (no negative masses) and no identity element (no zero mass, at least not in NG).

As it stands, these structures render all absolute masses qualitatively identical<sup>20</sup>. Then, in order for the laws to know in each possible world which determinate magnitude they are ‘latching onto’, the determinate magnitudes need to be endowed with non-qualitative, transworld identities (i.e. these identities are required for the forces to be well-defined, in the sense of uniquely matching up instances of initial conditions, including masses, with, say, accelerations)—more on this in [Section 2.6](#). In the case of strong absolutism these identities will be primitive; in the case of weak absolutism these transworld identities will presumably be reducible to the transworld identities of the properties to which the absolute masses reduce. (Primitive) non-qualitative, transworld identity of properties is usually

---

<sup>18</sup>One may wonder whether we also need multiplicative structure, as the gravitational force depends on the product of two masses (and in order to define the Active Leibniz Mass Scaling defined below). One response would be to argue that this structure is part of the laws and not the masses, although it could be retorted that we could make the same move for all the structure that we do attribute to masses here (cf. Dees [26]). Fortunately, in standard NG we need not worry about products of masses, since we have assumed an equivalence principle between empirical and inertial mass, such that the final equation that governs the dynamics,  $a = Gm/r^2$ , does not contain any product of masses.

<sup>19</sup>The abovementioned constraints on mass relations required to prove the representation theorem are exactly the constraints that ensure that the mass relations can be represented by intrinsic properties with these two structures.

<sup>20</sup>It is for this reason that kinematic comparativism, as defined in the next subsection, is true.

referred to as quidditism, but it is often ambiguous<sup>21</sup> whether quidditism is supposed to refer to transworld identities of the determinates or the determinables, so we will specify the current view as determinate quidditism or **quidditism about absolute mass determinates**<sup>22</sup>.

In order to be a realist about absolute masses in Newtonian Gravity we should thus be aware that a mathematical theory representing magnitudes necessarily has to choose one of the conventional homomorphisms of  $\mathbb{R}_+$ , the set of quantities, onto those magnitudes, but it is the magnitude and not the quantity that one should be realist about. The metaphysical and epistemological worries were phrased in terms of quantities, and equally disappear when we realise the absolutist is only committed to (the metaphysical priority of) magnitudes.

Perhaps surprisingly, the mass ratios of the comparativist have a similar structure. They also consist in a set of determinate properties, again with cardinality  $\aleph_1$ . These properties are however binary relations, rather than monadic properties. These determinate properties are again 1) totally ordered, and 2) obey an associative concatenation structure, but this concatenation structure is to be interpreted as multiplication<sup>23</sup>. It includes the relation ‘... is as massive as ...’—or in quantitative terms ‘... stands in a 1:1 mass ratio to ...’—which forms the identity element, and also includes an inverse for every mass ratio. Hence, the mass ratios form a totally ordered group. One might again expect me to endow these determinates with non-qualitative, transworld identities, for the laws to ‘latch onto’. However, the existence of a unique privileged element, the ‘multiplicative’<sup>24</sup> identity element, is sufficient for the laws to know which mass ratio they are ‘latching onto’.

In summary, the absolutism-comparativism debate concerns magnitudes and relations (which are related in a many-to-one fashion), and not quantities and relations. Unfortunately magnitudes need to be represented by quantities, which are related to magnitudes in a many-to-one fashion. I believe, as we will see, that it is this double many-to-one relation that wrongly guides some authors towards

---

<sup>21</sup>Interestingly, this ambiguity raises problems for ‘swapping arguments’ (also called humility arguments [29]) against quidditism. Elsewhere [30] I argue that this ambiguity in fact fatally undermines such arguments.

<sup>22</sup>Insofar as determinables are grounded in determinates, it would only be a small leap to argue from determinate quidditism to transworld identities of determinables.

<sup>23</sup>These structures correspond nicely to Weyl’s operational definition of (inertial) mass [31,32].

<sup>24</sup>The existence of an identity element by itself is not sufficient; it needs to be a ‘multiplicative’ identity element. Only in this case can one prove a unique representation theorem between the mass relations and  $\mathbb{R}_+$ . Take for instance the totally ordered group of absolute electric charges. This group features an ‘additive’ identity element, but this does not ensure a unique representation theorem—this is basically the content of kinematic comparativism, as defined below.

comparativism about mass<sup>25</sup>.

### 1.1.4 Kinematic comparativism

The *representational* redundancy—of quantities when representing magnitudes—that was exposed in the previous subsection stems from some *epistemological* cousin of (metaphysical) comparativism being trivially true.

Consider first the analogous substantivalism–relationalism debate about the metaphysics of space(time). Leibnizian relationalists often point out that distances and velocities can only be *described* or *expressed*, non-dynamically, by referring to some material reference body. Being told the position of your car with respect to the ‘centre of the universe’ or the spatial point labelled ‘Tushar’ is not going to help you find it (and even if it would, it is in some sense still a relational expression as it refers to the distance between one (material) object and another (non-material) object: space). Let us call this *kinematic relationalism*<sup>26</sup>. Metaphysical substantivalists about space have no choice but to agree with this, although they will argue (via Newton’s bucket and/or globes<sup>27</sup>) that this does not imply metaphysical relationalism<sup>28</sup>. We will return to this argument later.

A similar story applies to our topic. Consider the following conversation:

*Teacher*: “Homework question: what is the mass of a proton?”

*Pupil*: “ $1.67 \cdot 10^{-27}$ ”

*Teacher*: “ $1.67 \cdot 10^{-27}$  what? I’m afraid I’ll have to give you 0 marks for this answer.”

*Pupil*: “That’s not fair; it was obvious that I meant kg.

Give me at least half the marks, I got most of it right.”

*Teacher*: “Without a unit that number is meaningless. Adding any arbitrary unit could turn it into any different answer.”

The pupil’s answer was as useless as the report that your car was close to the centre of the (infinitely large) universe. Mass is a *dimensionful* determinable. As such, the following epistemological cousin of comparativism is true of it:

**Kinematic Comparativism:** For any dimensionful determinable, such as mass, the magnitude predicated of any particle can only be *reported* or *expressed*,

---

<sup>25</sup>See, for instance, Roberts [13, §5].

<sup>26</sup>I adapt this terminology from Huggett [33], who uses the term ‘kinematic relativity’. However, 1) the notion of relativity is easily confused—in the context of the philosophy of space—with Einstein’s notion of the relativity of simultaneity, and 2) kinematic relativity is the epistemological or kinematic version of Leibnizian (metaphysical) relationalism, making ‘kinematic relationalism’ the more obvious choice.

<sup>27</sup>In Newton’s Scholium to the Definitions in his *Principia* [34].

<sup>28</sup>Zanstra [35] suggests that Berkeley, Aristotle and Copernicus seem to make the invalid argument of inferring (metaphysical) relationalism from kinematic relationalism. At the very least they are confusing the two notions.

non-dynamically, in terms of how this magnitude relates to the magnitude of another particle having the same determinable property.

In fact, we should take it not merely as a feature but as a *definition* of the dimensionfulness of a determinable that it is kinematically comparative. Only dimensionless determinables, such as the number of corners of an object, can be expressed in a more direct way. We might call them *kinematically absolute*.

It is because at the kinematic level mass is comparative that on the representational level it must be represented by a quantity times a unit. A unit is thus merely a relation—to a standard reference body (*pace* Dasgupta [36])—in disguise. This in turn necessitates a conventional choice of the specific unit (i.e. an arbitrarily designated reference body that is represented by the quantity 1), which introduces representational redundancy. But does this redundancy at the representational level have any implications for the redundancy of absolute masses at the metaphysical level (or at least for their metaphysical priority over mass ratios)? In [Chapter 1.2.1](#) I argue that it does not.

### 1.1.5 Operationalising Mass: Ozma Games

In this brief intermezzo I will illustrate some of the distinctions made so far by introducing another way of thinking about them that will be useful later on. This framework is obtained by generalising Gardner’s Ozma Problem for handedness [37]. Gardner asks whether we could convey the meaning of ‘left(-handedness)’ to aliens, solely by sending them a signal in morse code, without being allowed to ostensibly refer to handed structures on the night sky. This problem can easily be generalised to properties such as shape and mass. Equivalently, we could send the aliens a 5-kg left-handed glove (with 5 fingers) and enquire after its properties.

If we ask the aliens for the shape of the object, that is the number of fingers, no problems arise. No comparative position on (this aspect of) shape could even be formulated, and once a numerical system, say the decimal system, is agreed upon there is also no ambiguity arising from the way the number of fingers is represented. The answer is simply ‘5’. This property is kinematically absolute.

It was long thought that there was no answer to the original handedness version of the Ozma Problem, since many physical theories are mirror-symmetric. That is, for every handed solution its mirror image is also a solution. Hence, it seems the aliens can not do any experiment that will tell them unambiguously what (the thing that we call) left is. Invoking the fact that human hearts are on the left side of the body is of no help either, since this is contingent, and for all we know the alien hearts are in fact on the right side—the laws of nature allow it. The same applies to the right-handed twist of our DNA helices, etc. It seems that handedness is a comparative notion: left is nothing over and above the opposite

of right, and vice versa. However, we now know that the weak interactions do not exhibit mirror symmetry [38]. Decays governed by the weak interaction will predominantly happen in what we call a left-oriented fashion. Thus, the aliens could, say, grab a cobalt-60 nucleus and observe the handedness of its decay (bracketing the stochastic nature of the decay), and compare that to the glove. They could report that comparison to us. In that sense<sup>29</sup>, handedness is absolute. The aliens could of course also report their findings as the glove being, say, ‘right-handed’, if they have chosen to call the orientation of the cobalt-60 decay ‘right-handed’. Nothing in our experiment forces the aliens to label the orientation of the cobalt-60 decay ‘left’. Thus, although in this case we have an experiment that allows us to agree with the aliens on what it is that we *refer* to when we say ‘left’, at the *representational* level—in this case linguistic representation—we may still disagree because of a differing convention. Things would be similar if we would have chosen a mathematical representation. Say we denote a left-handed coordinate system by ‘1’, and a right-handed coordinate system by ‘-1’, to represent the opposite sign of the coordinates of, say, the x-axis. Although we might *a posteriori* come to an agreement with the aliens that ‘1’ is the natural convention for the orientation of weak processes, no such convention is *a priori* privileged.

Since kinematic comparativism about mass is true we should not expect the aliens to give the ‘correct’ answer when we inquire after the mass of the object. We should expect something like ‘10,000 zorb’, where, so they explain after we communicate our non-understanding of this answer, 1 zorb is the mass of their reference mass in their capital city. Upon reflection we realise that, although it is correct that we might expect such an answer, this is equally so if absolutism is true. In the case of absolutism the ambiguous answer will however arise from the ambiguity at the representational level, rather than at the metaphysical level. Whether the aliens could do an experiment to determine the absolute mass or not is the main topic of this thesis<sup>30</sup>. I will provide such an experiment, and the

---

<sup>29</sup>I do not literally believe that a coherent absolutist position of handedness could be formulated, let alone be true. One reason is that, if handedness was an intrinsic property of an object (or process), that determinate property would change under a rigid rotation of the object in the fourth dimension (or if our space is non-oriented, under a rigid translation along certain closed trajectories) [39, 40]. Intrinsic properties can not depend on rigid movements. Handedness is thus ‘kinematically comparative’, or even comparative in a sense stronger than that. Therefore, in order to explain this miraculous consistency of handed decays as governed by the weak force, one requires either space itself to be handed [41] or a global weak quantum field [42], that serves as a global comparison for the decays on both our planet and the aliens’ planet. Since we nor the aliens could see either of these handed reference entities, at the epistemological level it seems that handedness is absolute in the Ozma scenario.

<sup>30</sup>Of course we could just describe a specific elementary particle to them and use its mass as the unit of mass, but here we consider NG only.

natural convention for a mass unit that follows from it: we obtain a dynamically privileged unit. This does however not remove the freedom to use any other unit, that is any other representation. After all, we ourselves have chosen to use an arbitrary mass in Paris as the reference mass, rather than the natural unit provided by the experiment described below. Once again, I believe it is this ambiguity at the representational level that leads to mistaken belief in comparativism, by disguising what is happening at the metaphysical level. In terms of my natural unit the aliens have no problem expressing the mass of the object correctly, even if that means they are still expressing it in terms of a relationship in disguise.

## 1.2 Arguments for Comparativism

### 1.2.1 Naive Arguments & Dynamic Comparativism

We have already seen one naive argument in favour of comparativism<sup>31</sup>. In response to naive absolutism—that is, since the standard formulation of NG explicitly models the *intrinsic* mass of each particle, by a numerical quantity associated with that particle, naive literalism suggests a metaphysical commitment to the absolute masses (and even a metaphysical priority over mass ratios)—it was pointed out that this numerical value depends on a conventional choice of unit (i.e. the formal worry). The value, say 5kg, is not unique, and represents nothing ‘5-ish’ about the particle with that mass; it is merely a redundancy and thus<sup>32</sup> does not represent anything fundamental or even real. We have seen that this argument confuses the absolutist’s commitment to mass magnitudes with a commitment to mass quantities. There is indeed a redundancy at the level of representation, but that does not affect the metaphysical thesis that absolutism is.

A follow-up argument<sup>33</sup> immediately presents itself, by repeating the previous argument one level lower, by invoking kinematical comparativism (which is responsible for the representational redundancy but is not itself a notion that applies to the representational level) directly. If we can only *express* mass ratios and not (non-conventional) absolute masses, then that must be because there simply *are* only mass ratios. In [Chapter 6](#) I will make plausible that this is what Mach had in mind when he attempted to operationally define mass.

That kinematical comparativism does not imply metaphysical comparativism is most easily seen by returning to our analogy with the spatiotemporal debate. Substantivalists have to agree with kinematic relationalism: when *reporting* the location of one’s car one needs to relate it to other material objects, as it cannot

---

<sup>31</sup>Maudlin [43, p.29] seems to be giving this argument for comparativism about distance.

<sup>32</sup>This extra step requires some Occamist norm, several of which will be introduced below.

<sup>33</sup>At various places, it seems to be the case that Roberts is invoking something like either or both of these two naive arguments [13, p.4,12].

be meaningfully reported by relating it to the centre of substantival space. But they insist that it does not follow<sup>34</sup> that physics, that is the evolution of these relational notions, *depends* only on these relational distances (and higher-order time-derivatives) (i.e. dynamic relationalism)—*a fortiori* it does not follow that metaphysical relationalism is true. An independent argument is needed for dynamic relationalism, and (the mere (*prima facie*) possibility of) Newton’s bucket and globes reveal(s) exactly this logical and physical gap between kinematic and dynamic relationalism. Even though we can meaningfully express the location (and acceleration) of the water only with respect to the bucket (or other material objects, but not with respect to absolute space), its subsequent behaviour could be construed as nevertheless depending on its rotation with respect to substantival space (*pace* Barbour [2]).

Define the analogue of dynamic relationalism as follows:

**Dynamic Comparativism:** Physics depends only on the mass ratios, not on further absolute masses grounding those ratios. In other words, metaphysical comparativism is empirically adequate.

What we mean by ‘empirically adequate’ will have to be made more precise in the remainder of this thesis, but an intuitive understanding suffices to see that the mere fact that masses need to be expressed via comparisons does not mean that physics does not depend on the absolute masses over and above the mass comparisons. Kinematic comparativism does not imply dynamic comparativism (and therefore *a fortiori* does not imply metaphysical comparativism), just as kinematic relationalism does not imply dynamic relationalism. An independent argument would be needed for dynamic comparativism, and it is the main aim of this dissertation to show that that argument does not exist.

### 1.2.2 The General Schema & Motivation for the Comparativist Argument

An argument for the empirical adequacy of metaphysical comparativism is what the comparativist needs. But even if this can be found, metaphysical absolutism would still be consistent with the empirical adequacy of comparativism. After all, the absolutist does not deny the reality of mass ratios, only their (relative) fundamentality. It would be true though that in that case the absolute masses would be metaphysically redundant (over and above the mass ratios). A comparativist who is motivated by metaphysical parsimony may then invoke some Occamist norm that urges us to get rid of such redundant properties (i.e. Occam’s Razor). The comparativist’s problem with their opponent is thus, in general, not that they

---

<sup>34</sup>Against the authors mentioned in [fn. 28](#).

believe that the absolutist position is incoherent<sup>35</sup> or inconsistent<sup>36</sup>, but just that their commitment to absolute masses grounding mass ratios is unnecessary and on top of that unnecessarily ‘expensive’ (metaphysically speaking). In analogy with the Occamist arguments that are usually invoked against absolute velocities in NG, the general schema for the comparativist argument that seems prevalent in the literature is then:

$P_{\text{dyn}}$	<b>Dynamic Comparativism:</b> (Metaphysical) comparativism is empirically adequate.
$P_{\text{occ-par}}$	<b>Occamist norm:</b> All other things being equal (i.e. $P_{\text{dyn}}$ —assuming the empirical adequacy of absolutism <sup>37</sup> ), we should favour theories that are metaphysically more parsimonious.
$P_{\text{par}}$	(Metaphysical) comparativism about mass is metaphysically more parsimonious than (metaphysical) absolutism.
$C$	(Metaphysical) comparativism about mass is favoured over (metaphysical) absolutism.

The naive comparativist was motivated by a desire to get rid of absolute masses because they believed absolute masses are redundant when it comes to *describing* masses (cf. kinematic comparativism). Hence, there is no incentive to ‘add’ them to our fundamental metaphysics. This criterion was too strict—or too loose, depending on how you look at it—albeit on the right track. The comparativist in this subsection is motivated by a desire to get rid of absolute masses because they believe those are redundant when it comes to (describing masses and) ensuring the empirical adequacy of Newtonian Gravity, so there is no incentive to pay the extra ‘metaphysical cost’ of adding them to our ideology. (As before, I am using ‘ideology’ in the Quinean sense [11]. I will keep doing so in the remainder of this thesis. Roughly, ontology refers to the (primitive) objects, and ideology to their (primitive) properties. I am using the broader term ‘metaphysical parsimony’ rather than the more common ‘ontological parsimony’, to include parsimony of objects, properties and any other ‘types of things’ we might include in our fundamental metaphysics such as laws of nature.)

<sup>35</sup>Unless one combines comparativism—specifically a version that is anti-realist about absolute masses (see Section 2.6)—with a verification theory of meaning.

<sup>36</sup>There is one interesting, specific scenario that would be inconsistent though. If in the context of regularity comparativism, defined in Section 4.3, one would randomly add fundamental intrinsic masses to a otherwise comparativist Humean mosaic consisting in fundamental mass ratios, then those might well be inconsistent with the emergent absolute masses obtained via the regularity protocol.

<sup>37</sup>In the domain of applicability of the theory.

### 1.2.3 Modified General Schema

I believe that the criterion underlying this comparativist motivation is still too strict (or too loose, depending on how you look at it). The previous argument is valid only if the ‘all other things being equal’ clause in the Occamist norm can indeed be equated to empirical adequacy, but theoretical terms such as absolute masses can in principle do more work than that. *Prima facie*, they could provide explanatory power<sup>38</sup> and the modal consequences of the theory (and, as discussed in [Section 5.1](#), they ensure the locality, i.e. the separability, of the theory). The general schema therefore needs to minimally be extended in the following way:

$P_{\text{emp}}$	<b>Empirical Adequacy:</b> (Metaphysical) comparativism is at least as empirically adequate as (metaphysical) absolutism.
$P_{\text{exp}}$	<b>Explanatory Adequacy:</b> (Metaphysical) comparativism is at least as explanatorily adequate as (metaphysical) absolutism.
$P_{\text{mod}}$	<b>Modal Adequacy:</b> (Metaphysical) comparativism has the same modal consequences as (metaphysical) absolutism.
$P_{\text{occ-par}}$	<b>Occamist norm:</b> All other things being equal (i.e. $P_{\text{emp}} \wedge P_{\text{exp}} \wedge P_{\text{mod}}$ ), we should favour theories that are metaphysically more parsimonious.
$P_{\text{par}}$	(Metaphysical) comparativism about mass is metaphysically more parsimonious than (metaphysical) absolutism.
<hr/>	
<b>C</b>	(Metaphysical) comparativism about mass should be favoured over (metaphysical) absolutism.

It will be discussed in [Subsection 4.4.2](#) whether the comparativist can fruitfully deny the requirement that their theory be modally equivalent to absolutism.

Explanation, prediction (which is related to empirical adequacy) and modality are three interrelated notions within the philosophy of science which are very important but controversial. Although I will provide a sketch of their interpretation and importance in the context of the current debate in [Chapter 3](#), a full-fledged positive account of each of these notions will not be necessary for our purposes. Instead, I will point out along the way clear-cut cases of comparativism losing out

<sup>38</sup>Although Roberts ends up arguing that absolute masses have no explanatory power, he does agree that if they did have such power this would provide justification for absolutism even if comparativism is empirically adequate [13]. In other words, he agrees that the ‘all other things being equal’ clause in the Occamist norm should at least quantify over explanatory power.

to absolutism, especially when it comes to explanatory power. Thus, although the extended general schema is a valid argument, it is the main aim of this thesis to show that it is not sound. I believe that all premises besides the Occamist norm are at least questionable, but the main part of this thesis will argue that comparativism is not empirically adequate, with some losses of explanatory power pointed out along the way, and a separate chapter questioning the supposed metaphysical parsimony of comparativism ([Chapter 5](#)).

### 1.2.4 Three other approaches to empirical adequacy

It will once again prove fruitful to model our discussion of the empirical adequacy of comparativism on the template provided by the analogous debate about the metaphysics of space. Besides the mistaken attempt to prove the empirical adequacy directly from kinematic relationalism/comparativism, three other popular approaches to interpreting arguments in favour or against empirical adequacy dominate the literature. It is generally accepted that Static and Kinematic Leibniz Shifts, if doing anything at all, provide a challenge for the substantialist, whereas Rotational Dynamic Leibniz Shifts (i.e. Newton’s bucket experiment and globes thought experiment) threaten the empirical adequacy of relationalism.

On the first approach, the **symmetry** approach, the former two Leibniz shifts are interpreted as symmetries of Newtonian Gravity, whereas the latter is not. The Occamist norm is then explicated as a direct symmetry-to-(un)reality inference, for instance via Saunders’ Invariance Principle: “only quantities<sup>39</sup> invariant under exact symmetries are real” [42, p.1-2].

On the second approach, the **undetectability approach**, the first two types of shifts are interpreted as exposing the undetectability of absolute positions and velocities, whereas the Rotational Dynamic Leibniz Shift reveals the detectability of absolute rotational accelerations. The Occamist norm should then be interpreted as a razor for undetectable notions.

The third approach, the **possibility checking approach** [44–46], checks whether the theory generates the correct set of empirically possible worlds (or at the very least includes a possible world that is empirically equivalent to the actual world). From each relationalist world, uncountably many metaphysically distinct substantial worlds can be generated via the three types of Leibniz Shifts. Ideally, there should be at least one metaphysically distinct possible world associated with each empirically distinct possible world allowed by (substantialist) Newtonian Gravity. If relationalism satisfies this criterion—which seems to be the case for worlds related by the first two types of shift, but not the latter type of shift—but substantialism generates ‘too many’ metaphysical possibilities, a

---

<sup>39</sup>‘Quantities’ is used more colloquially here than the technical definition in this thesis.

Leibnizian Principle of the Identity of Indiscernibles is then often invoked to argue against substantivalism. This is a specific version of the Occamist norm, which equates metaphysical parsimony with minimalisation of the number of metaphysically distinct possible worlds per associated empirically distinct possible world (see [Subsection 5.2.4](#)).

The thought experiment central to the current debate is not whether it would make a difference if God had placed all the matter in the universe 1 meter to the left, or boosted it with 1 m/s, etc., but whether it would have made a difference if God had decided to, say, double the absolute masses of all the particles in the universe. In other words, the relevant transformation that threatens the empirical adequacy of comparativism is what we may call a Leibniz *scaling*:

**(Active) Leibniz (Mass) Scaling:** A uniform scalar multiplication<sup>40</sup> of each of the absolute mass magnitudes of all the particles, *ceteris paribus* (i.e. whilst keeping everything else unchanged).

(What makes this transformation ‘active’ will be discussed below.) The threat to comparativism that is the focus of this thesis concerns the question of whether a Leibniz Scaling, which changes only the absolute masses but leaves the mass relations and everything else invariant, produces an empirical difference. If it does, and I will argue that it does, this suggests that comparativism may not in fact be empirically adequate. If it does not, this would be a strong reason in favour of comparativism (although we would still have to check its explanatory and modal adequacy—or so I claim).

As in the space debate, there are three approaches to interpreting and responding to the Leibniz Scaling argument against (or in favour of) the empirical adequacy of comparativism.

A comparativist using the **symmetry approach** will claim that Leibniz Scaling is a symmetry of Newtonian Gravity, and therefore, by the Invariance Principle, absolute masses are not (objectively) real (and *a fortiori* not more fundamental than mass ratios). An absolutist will deny that it is a symmetry.

---

<sup>40</sup>Does it make sense to multiply magnitudes (rather than the quantities that represent them) by a scalar (Cian Dorr, personal communication)? We could perhaps respond to this question by adding extra (multiplicative) structure to the absolute masses, but it seems that that structure is not needed within Newtonian Gravity (cf. [fn. 18](#)), and it would reduce the metaphysical parsimony. Alternatively, we could piggy-back on the structure of the quantities: apply the scalar multiplication at the representational level and find out which new magnitudes are now represented by the quantity that represented the old magnitudes. Dasgupta develops a mass-counterpart theory that might be used for this purpose [8] (although this basically boils down to a passive Leibniz Scaling, as defined below). If one feels that either of these options would be cheating, it also suffices for the purposes of this thesis to consider any automorphism that is not the identity.

The **undetectability approach** is a return to a similar epistemic worry (about absolute masses) as before ([Subsection 1.1.3](#)), but this time the worry is not about being unable to detect the absolute numerical values of these masses (which was not a worry since absolute masses are taken to be magnitudes, not the quantities that represent them), but that these absolute mass magnitudes themselves are epistemically redundant, in the sense of being undetectable over and above the mass relations.

The **possibility checking approach** explicates a sufficient condition for empirical adequacy as comparativism’s ability to generate the correct set of empirically possible worlds. From the previous section we now realise that the representation theorem between absolute masses and mass relations is to be understood as being between magnitudes and relations, not quantities and relations. To every metaphysically possible world corresponding to a structure of mass relations, one would expect<sup>41</sup> there to correspond (via active Leibniz scalings) uncountably many possible ‘absolutist worlds’ with different absolute masses (understood as magnitudes) that are all nevertheless compatible with the mass structure in the corresponding ‘comparativist world’. (In turn, to each of these absolutist worlds there correspond (via passive Leibniz scalings, see below) uncountably many representationally distinct possible worlds (also known as models), all differing in the quantities assigned to the absolute masses, but we are not interested in those sets of models.) Comparativism should provide at least one metaphysically distinct (and dynamically allowed) possible world for each empirically distinct possible world allowed by absolutism—how to differentiate empirically distinct worlds is an issue that will be scrutinised in [Chapter 2](#) and [Chapter 3](#). If the metaphysically distinct worlds that comparativism acknowledges fail to differentiate between those distinct empirical possibilities, then comparativism is wrong. Arguably, in General Relativity, for certain absolute masses, objects are not black holes, and for larger absolute masses they do become black holes—although in this general relativistic scenario the distinction between inertial and gravitational mass does become important, as the latter does not exist, making it less obvious if and how the absolutism–comparativism distinction could be applied. Ignoring these issues for now, the absolute masses of objects in a universe can be changed without the mass relations changing, which changes certain objects into black holes. If we take being a black hole or not as empirically distinct (or generating empirically distinct effects)<sup>42</sup>, then comparativism seems to be unable to acknowledge this distinction in some cases. (Here we are of course interested in NG, rather than General Relativity.) If, on the other hand, the set of all the metaphysically distinct possible worlds acknowledged and dynamically allowed by comparativism contains all the

---

<sup>41</sup>Except, in a sense, in the case of regularity comparativism ([Subsection 4.3.4](#)).

<sup>42</sup>In the conclusion to this thesis I will point out that that is not as straightforward as it may seem.

empirically distinct possible worlds (that are dynamically allowed by absolutism), then we may opt for comparativism over absolutism based on an Occamist norm.

Four points are worth mentioning at this stage. Firstly, whereas in the substantivalism–relationalism debate it is generally accepted<sup>43</sup> that the Static and Kinematic Shift provide, if anything, a challenge for the substantivalist, and the Rotational Dynamic Shift a challenge for the relationalist, in the mass debate this is not at all the case. Comparativists believe that a Leibniz scaling has no empirical effect (i.e. Leibniz Scalings are a symmetry/absolute masses are undetectable/Leibniz Scalings relate empirically indistinct worlds) and thereby proves their view correct, whereas absolutists claim that it does have empirical consequences and thus proves their view. In fact, we have already seen some naive, mistaken motivations for the comparativist believing this is the case, which brings us to our second point.

Secondly, then, it is important to note that all three approaches centre around an *active* version of the Leibniz scaling transformation. That is, we ask what happens when we change something *out there*, that is, what happens when we change the magnitudes, which are part of the metaphysics. Thus, we are not asking what happens if we instead apply the following transformation at the level of representation:

**Passive Leibniz (Mass) Scaling:** <sup>44</sup> A uniform scalar multiplication of each of the quantities representing the absolute mass magnitudes of all the particles, *ceteris paribus* (i.e. whilst keeping everything that is represented, including those magnitudes, unchanged).

This is just a change of unit. It is obvious that a change of unit, like a change of coordinates in the space case, does not change anything in the real world (assuming we consistently change all the other units, say of Newton’s constant, as well). It is just a human choice of applying different labels. Especially the first naive argument boiled down to focusing on this passive scaling, rather than the active counterpart.

It should be admitted though that the mentioning of ‘scalar multiplication’ in the definition of an active Leibniz scaling may have facilitated confusing this active scaling with its passive counterpart. In this context, see [fn. 40](#).

Thirdly, this debate concerns a transformation with a multiplicative nature, which makes it clearly distinct from the previous three Leibniz shifts, with their additive nature. In other words, the name ‘comparativism about mass’ rather than ‘relationalism about mass’ has been chosen on purpose, because they would refer to distinct views. Although relationalism about mass might not make sense,

---

<sup>43</sup>Despite, ironically, Clarke having been the one to introduce the Static Shift [47,48].

<sup>44</sup>This is closely related to Dasgupta’s mass-counterpart theory [8].

comparativism about spatial (or temporal) distance does, and this would be distinct from the relationalism (about distance) that we have been mentioning so often in this thesis. Comparativism about distance claims that the single ratio between the two distances between particle 1 and 2 and between 3 and 4 is not grounded in the two distances between particles 1 and 2 and between 3 and 4 [8]. Arguments concerning its empirical adequacy would employ a Leibniz Distance Scaling, rather than any of the Leibniz Shifts.

Fourthly, we may wonder whether these three approaches should be considered as equivalent approaches to the active Leibniz Scaling argument against/in favour of the empirical adequacy of comparativism, or whether they constitute three separate arguments, potentially even with mutually inconsistent conclusions. In the space debate the three approaches seem to be used interchangeably—except perhaps in the context of the Static Shift.

The notion of symmetries of physical theories and a justification of their associated symmetry-to-(un)reality inference are notorious topics in the recent philosophy of physics literature, see for instance Dasgupta [49] and references therein. This is the most important reason why the focus of this thesis will be on the other two approaches. Methodologically, common sense suggests to invest first in the easiest and least controversial options within a set of *prima facie* equivalent approaches. If the symmetry approach is indeed equivalent to the other approaches—or at least to the possibility checking approach which I will champion in this thesis—a fan of the symmetry approach may of course want to use successful defences of comparativism via the other approaches to reverse engineer the best understanding of (mass scaling) symmetries and a justification for the associated symmetry-to-reality inference. However, as this thesis will argue for absolutism—in particular, against the empirical adequacy of comparativism—the Leibniz mass scaling should presumably *not* come out as a symmetry of NG.

The undetectability approach, including the corresponding version of the Occamist norm, will be developed and criticised in [Chapter 2](#). I argue that, to the extent that it is distinct from the possibility checking approach, it is wrong. To the extent that it is equivalent to the possibility checking approach, I suggest that we simply move to that more perspicuous approach, which I will develop in [Chapter 3](#).

### 1.3 Summary so far & Thesis Outline

We have seen that the debate between absolutism and comparativism about mass (in NG) concerns the relative fundamentality of the two determinates associated with the single determinable mass. Are the mass relations grounded in absolute masses, as the weak absolutist contends, or are those mass relations not grounded

in absolute masses, as asserted by the weak comparativist? We could go beyond mere relative fundamentality and consider whether the most fundamental mass (i.e. absolute masses for the absolutist and mass ratios for the comparativist) is itself fundamental or grounded in other non-mass determinables or facts. We will not turn to this latter issue until [Chapter 6](#).

It is important to realise that the absolutist commits herself to absolute mass magnitudes—defined as a set of monadic, quidditistic determinates forming a totally ordered semi-group under a combination rule to be interpreted as addition, as to mirror part of the structure of numerical quantities that can therefore be used to represent them—rather than those representing quantities themselves. It is the double many-to-one relationship between magnitudes and mass relations, and quantities and magnitudes, that leads to much of the confusion misdirecting some authors towards comparativism.

The idea behind the general schema for the argument in favour of (weak) comparativism centers around the promise that (weak) comparativism can recover all the virtues of absolutism—in particular empirical adequacy and explanatory power—but in a metaphysically more parsimonious way. The metaphysical parsimony and explanatory adequacy of comparativism will be discussed in [Chapter 5](#). The next three chapters will argue for the empirical inadequacy of comparativism (i.e. against dynamic comparativism).

I have admitted the truth of kinematic comparativism—absolute magnitudes can only be *expressed* via comparisons, that is just what it is to fall under a dimensionful determinable—but argued that it is invalid to infer dynamic comparativism from this.

Instead, the issue of empirical adequacy could be approached either in terms of symmetry, in terms of the undetectability of absolute masses, or in terms of possibility checking. I will focus on the latter two. The next chapter introduces Dasgupta’s comparativist proposal, and his preferred undetectability approach. Although I will argue that even on this approach Dasgupta’s comparativism fails, the take-home message will be to illustrate the unsuitability of that approach, to the extent that it differs from the possibility checking approach. [Chapter 3](#) develops this possibility checking approach, and reconfirms the unfeasibility of Dasgupta’s comparativism. [Chapter 4](#) considers alternative comparativist proposals and tactics, and evaluates them, negatively, in terms of the possibility checking approach.

## Chapter 2

# Detecting Newtonian Absolute Masses

### 2.1 Dasgupta's Razor

Dasgupta, in *Symmetry as an Epistemic Notion (Twice Over)* [50], considers several reasons one might have for believing that features such as absolute masses are redundant (over and above mass ratios), and ultimately settles on an epistemic notion of redundancy, that is undetectability. After first discussing non-epistemic inferences to the unreality of a feature based on that feature either being subjective, or being redundant in the sense of making no difference, or being arbitrary, or being explanatorily dispensable, etc., he concludes that none of these inferences works. Dasgupta points out that we should never have expected these non-epistemic approaches to work in the first place:

If we could see or detect [this feature, eg. absolute mass], we should believe that it is real regardless of whether it is redundant in *any* of the above senses. So a necessary condition on our reasonably believing that it is unreal is *that we do not believe that it is detectable*. And from there one might argue that a necessary condition on our reasonably believing that it is unreal is *that we believe that it is undetectable*. If that is right then it is hard to see how any non-epistemic reconstruction of the inference in terms of redundancy could work. [50, p.14, italics in the original]

In *Absolutism vs Comparativism about Quantity* [8], Dasgupta applies this reasoning to the absolutism–comparativism debate. From both papers, we may reconstruct his undetectability approach—to which he explicitly commits, despite often sliding between talk about undetectability and about possibility checking—

to the general schema for the comparativist argument, **Dasgupta's Razor**:

$\mathbf{P}_{\text{det}}$	<b>Empirical adequacy:</b> Absolute masses are undetectable (over and above the mass ratios).
$\mathbf{P}_{\text{occ-det}}$	<b>Occamist norm:</b> All other things being equal, we should favour theories that do not posit undetectable structure.
<hr/>	
$\mathbf{C}_c$	The theory without undetectable absolute masses (i.e. metaphysical comparativism) is favoured over the theory that does posit them (i.e. metaphysical absolutism).

Dasgupta's Razor cannot be the whole story. For one thing, as already argued in [Chapter 1.2.3](#), the 'all other things being equal' clause comprises much more than empirical adequacy (eg. explanatory and modal power). In the quote above, Dasgupta has argued that undetectability of a feature such as absolute mass is a necessary condition for inference to the nonexistence of that feature, and I agree, but it does not follow that undetectability is a sufficient condition. Secondly, Dasgupta's undetectability approach skips the link with metaphysical parsimony, and homes in directly on undetectability as an epistemic vice. This would circumvent the criticisms against the metaphysical parsimony of comparativism brought forward in [Chapter 5](#). But it is not directly clear that a theory that posits a feature that, despite being undetectable, provides an enormous overall gain in metaphysical parsimony or simplicity, should be disfavoured. Why should the 'all other things being equal' clause not also range over metaphysical parsimony and simplicity? In other words, if the Occamist norm is explicated directly in terms of undetectability rather than metaphysical parsimony, we would expect an additional 'metaphysical parsimony adequacy' premise. In fact, Dasgupta acknowledges<sup>45</sup> much that is mentioned in these two points:

It remains possible that dispensing with the feature yields a theory that has too many other vices to warrant belief, such as being too inelegant or complex. In that case we would have empirical evidence of sorts that the feature is real, in the sense that our all-things-considered best empirically confirmed theory implies that it is real. But that is a situation in which all else is not equal. [[50](#), p.18]

[W]e can only draw the conclusion ... when we have the alternative theory in hand and have shown that all else is equal. This explains why it was rational for Newton to believe in absolute velocity even though he knew that it was ... undetectable [in NG]. The reason this

---

<sup>45</sup>See also [[8](#), p.133-4].

was rational for him was that he had no good alternative theory to hand [such as Neo-Newtonian spacetime]. [50, p.18]

I will not dwell on the validity of this argument though, since 1) one aim of this chapter is to argue that the first premise is, in some sense, false, 2) another aim is to point out that it is wrong to equate comparativism with the view that necessarily denies that absolute masses are detectable in that sense, and 3) the most important message of this chapter is that the notion of undetectability is ill-suited in the first place.

## 2.2 Dasgupta’s Comparativism

Since we are evaluating the truth of *dynamic* comparativism, we need to first say something more about the dynamics, that is Dasgupta’s comparativist interpretation of Newton’s three laws and the Law of Universal Gravitation. Newton’s first and third laws are the same in the absolutist and comparativist framework. The second law is standardly written in the following symbolic notation:

$$f_q = m_q \cdot a_q, \tag{L0}$$

where I have added the subscripts to indicate that this formula is standardly interpreted as relating the *quantities*  $f$  (force),  $m$  (mass) and  $a$  (acceleration). The Gravitational Force between two particles, in the same symbolic notation, is:

$$f_q = G_q \frac{m_{1,q} m_{2,q}}{r_q^2}, \tag{G0}$$

where  $G$  is the numerical value of Newton’s Constant in some arbitrarily chosen units.

How to interpret these two laws? On one interpretation these laws govern absolute *magnitudes* of mass, acceleration<sup>46</sup> and force. Thus, for Newton’s Second Law:

- (L1) For any material thing  $x$ ,  
 (a) For any reals  $r_1$  and  $r_2$ , if  $x$  has mass  $r_1 M$ <sup>[47]</sup> and acceleration

---

<sup>46</sup>Although I have taken the qualifier ‘absolute’ in ‘absolute masses’ to mean ‘monadic’, for an absolutist about mass these masses are also intrinsic, even though they will not necessarily be for a comparativist who is a realist about absolute masses. In the case of acceleration magnitudes, however, I take it that even an absolutist about those magnitudes will not (necessarily) hold that they are intrinsic, since, according to the “at-at” theory, acceleration is reducible to the limit of velocity differences (which in turn are reducible to a limit of differences in position), cf. [Subsection 4.2.1](#).

<sup>47</sup>See [fn. 40](#)

$r_2A$ , then  $x$  has force  $r_1r_2F$  acting on it.

(b) For any real  $r_3$ , if  $x$  has force  $r_3F$  acting on it, then there are reals  $r_4$  and  $r_5$  whose product is  $r_3$ , such that  $x$  has mass  $r_4M$  and acceleration  $r_5A$ . [8, p.130]

Dasgupta argues that if L1 is the correct interpretation of the law, then under a Leibniz scaling—leaving the forces and the accelerations unchanged—the law will not obtain anymore [8]. This should be surprising to the comparativist, since such a Leibniz scaling is equivalent to the identity operation within their framework, and should thus not change whether the law obtains—that is, it should not turn a solution into a non-solution. Under this interpretation, the truth of this law then depends on absolute masses over and above mass ratios. Dasgupta therefore argues that L0 should be interpreted instead as governing only *magnitude relations*:

(L2) For any material things  $x$  and  $y$  [in the same world],

(a) For any reals  $r_1$  and  $r_2$ , if  $x$  is  $r_1$  times as massive as  $y$  and is accelerating at  $r_2$  times the rate of  $y$ , then  $x$  has  $r_1r_2$  times as much force acting on it as  $y$ .

(b) For any real  $r_3$ , if  $x$  has  $r_3$  times as much force acting on it than  $y$ , then there are reals  $r_4$  and  $r_5$  whose product is  $r_3$ , and such that  $x$  is  $r_4$  times as massive as  $y$  and is accelerating  $r_5$  times the rate of  $y$ . [8, p.130-1]

If this law obtains in one world, then a Leibniz scaling creates a world in which this law also obtains.

Dasgupta justifies this move by claiming that there is no empirical evidence that would confirm L1 over his interpretation of the law L2. This justification is of no use, since it is circular. We are looking for a formulation of the laws of NG such that they render the absolute masses empirically redundant, that is undetectable. We thus can not assume from the start that choosing this law would be empirically unproblematic. Insofar as this begs the question against the absolutist (s)he will never agree on this formulation of the law for this reason.

What seems to me to be the implicit but incorrect justification behind the alternative interpretation of the law, luring authors such as Dasgupta mistakenly towards comparativism, is kinematic comparativism and the resulting many-to-one relation between quantities and the magnitudes they represent. It is one of the aims of this thesis to argue that this justification is indeed mistaken (see for instance [Section 2.7](#)). We will, for now, acquiesce and discuss Dasgupta's analysis of the threat from Leibniz scaling via his interpretation of the law.

Notice that although L2 is technically compatible with absolutism about the forces and accelerations, it makes absolute forces and accelerations redundant—again the presumed motivation here is kinematic comparativism about forces and

accelerations. Although it seems very much in the spirit of comparativism about any determinable to also be a comparativist about all other determinables, these views are logically independent. It is consistent to be, say, a comparativist about mass and an absolutist about distance. In fact, in [Subsection 4.2.3](#) I will develop such a theory.

Surprisingly, Dasgupta does not discuss the actual force law. Presumably force laws are interpreted analogously. Rather than interpreting G0 as

- (G1) For any material things  $x$  and  $y$  in the same world,
- (a) For any reals  $r_1, r_2$  and  $r_3$ , if  $x$  has mass  $r_1M$  and  $y$  mass  $r_2M$ , and if the distance between them is  $r_3R$ , then  $x$  and  $y$  have an equal but opposite force  $\frac{r_1 \cdot r_2}{r_3^2} F$  (due to the other particle, respectively) acting upon them (in the direction of the other particle).
  - (b) For any real  $r_4$ , if  $x$  and  $y$  have an equal but opposite force  $r_4 F$  (due to the other particle, respectively) acting upon them (in the direction of the other particle, respectively), then there are reals  $r_5, r_6$  and  $r_7$  for which  $\frac{r_5 \cdot r_6}{r_7^2} = r_4$ , such that  $x$  has mass  $r_5M$ ,  $y$  has mass  $r_6M$  and the distance between them is  $r_7R$ .

I take it that he would interpret it as

- (G2) For any material things  $w, x, y$  and  $z$  in the same world,
- (a) For any reals  $r_1, r_2$  and  $r_3$ , if  $w$  is  $r_1$  times as massive as  $y$  and  $x$  is  $r_2$  times as massive as  $z$ , and if the distance between  $w$  and  $x$  is  $r_3$  times as large as the distance between  $y$  and  $z$ , then the equal but opposite force between  $w$  and  $x$  is  $\frac{r_1 \cdot r_2}{r_3^2}$  as strong as the equal but opposite force between  $y$  and  $z$ .
  - (b) For any real  $r_4$ , if the force between  $w$  and  $x$  is  $r_4$  times as strong as between  $y$  and  $z$ , then there are reals  $r_5, r_6$  and  $r_7$  for which  $\frac{r_5 \cdot r_6}{r_7^2} = r_4$ , such that  $w$  is  $r_5$  times as massive as  $y$  and  $x$  is  $r_6$  times as massive as  $z$  and the distance between  $w$  and  $x$  is  $r_7$  times as large as the distance between  $y$  and  $z$ .

Once again, under this interpretation of the law it still obtains after a Leibniz scaling.

## 2.3 Dasgupta: absolute masses are undetectable

In this section I present (my interpretation of) Dasgupta's set-up and analysis of several ways in which a Leibniz Mass Scaling may threaten the empirical adequacy of comparativism. Apart from some critical comments along the way, my own analysis will not be presented until the subsequent section, where I argue that both Dasgupta's set-up and his analysis of the scenarios are mistaken.

### 2.3.1 Spatial Intra-world Leibniz Scaling

Consider a Newtonian world  $W$  with two spatially separated subsystems  $S$  and  $S^*$  which differ by a Leibniz Mass Scaling. As Dasgupta interprets this,  $S$  and  $S^*$  are copies of each other insofar as the interparticle distances and the velocities of and forces on each particle in  $S$  are the same as their counterpart in  $S^*$ , but all the masses of the particles in  $S^*$  are, say, twice that of their counterparts in  $S$ . An example is shown at the top of [Figure 2.1](#). Applying L1 to each of the subsystems separately and then comparing the results tells us that the particles in  $S^*$  will accelerate at half the rate of their counterparts in  $S$ .

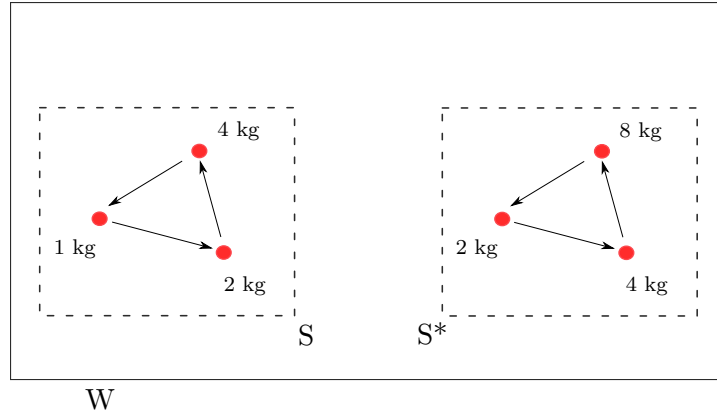
To the comparativist about mass this scenario looks different. Not only are the force ratios and distance ratios in  $S$  identical to their respective counterparts in  $S^*$ , but also the mass relations. This is depicted in subfigure (b) of [Figure 2.1](#). Can the comparativist account for the differences in accelerations? Applying L2 to each subsystem separately seems to fail to provide those different accelerations since everything that is governed by L2 (i.e. force relations, mass relations and acceleration relations) is identical between both systems.

One may respond that it is no problem that the acceleration relations within  $S^*$  are the same as their counterpart relations in  $S$ . In fact, the absolutist picture tells us they should be: the absolute<sup>48</sup> acceleration of each particle in  $S^*$  is half that of the acceleration of its counterpart in  $S$ , which leaves all the intra-system acceleration relations invariant. However, the issue was never that the intra-system relations cannot be accounted for, but the inter-system acceleration relations. In other words, the comparativist is not required to account for *different* accelerations nor acceleration relations *within* each system, but a *difference in* accelerations *between* both systems, or so Dasgupta claims.

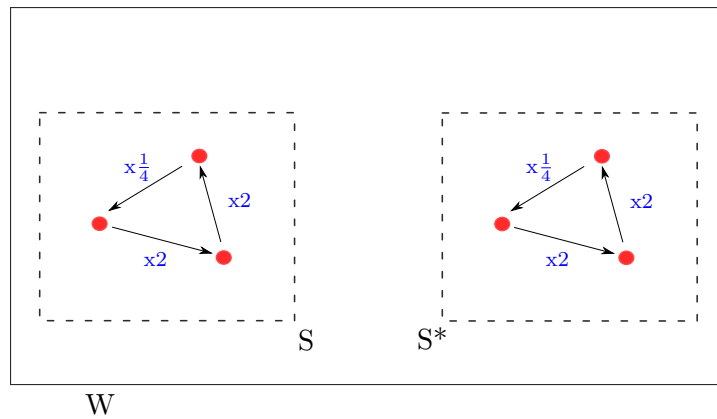
This may seem to indicate that absolute masses are not redundant: they are necessary to account for the empirical data, in this case inter-system acceleration relations. However, the analysis above makes the mistake of only applying L2 to each subsystem separately. We are tempted to do so since in the absolutist framework all the relevant information is stored locally within the subsystem, namely on each particle. Hence, applying the relevant law (i.e. L1) to each subsystem separately gives us all the relevant absolute magnitudes (i.e. accelerations) for each separate subsystem, and *a fortiori* all the relevant magnitude ratios between the two systems. In the comparativist framework not all the information is stored locally: the magnitude ratios between particles in  $S$  and the magnitude ratios between particles in  $S^*$  do not merely fail to determine the inter-system relations, they do not constrain them at all. Only when we take into account the inter-system mass relations do we have all the relevant information. Subfigure (c) in [Figure 2.1](#) depicts the inter-system relations corresponding to the absolutist

---

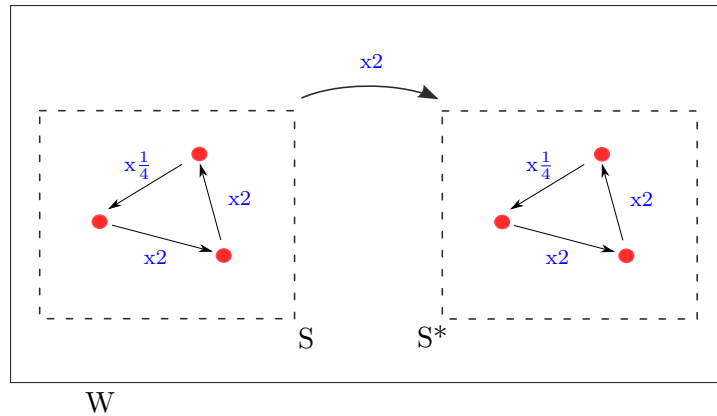
<sup>48</sup>As opposed to the comparative acceleration, rather than the relative acceleration.



(a)



(b)



(c)

Figure 2.1: Example of the spatial version of an intra-world Leibniz scaling scenario. An arrow with for instance the label 'x2' represents the following mass ratio: the object at which the arrow points is twice as massive as the object at which the arrow starts. The force on each particle in system S is, *ex hypothesi*, identical to the force on its counterpart **36** S\* ; they are omitted from this picture.

scenario in subfigure (a). Once we take into account that the inter-system mass relations are such that each particle in  $S^*$  is twice as massive as its counterpart in  $S$ , that is by applying L2 to the whole world, we have no problem recovering the inter-system acceleration relations. And that is all the comparativist about mass needs to explain, or so Dasgupta claims; no further absolute accelerations (or anything else) are to be accounted for. Getting the acceleration ratios right implies that absolute masses are undetectable. It is not directly clear why it would be true though that no further absolute accelerations (or anything else) are to be accounted for. Comparativism about mass does not by itself imply comparativism about accelerations. Presumably Dasgupta is implicitly invoking kinematic comparativism about accelerations. Since absolute accelerations cannot be expressed directly, but only via comparisons, acceleration ratios are the only empirical data that is to be accounted for. (As mentioned above, these observed acceleration ratios (and L2) are nevertheless consistent with underlying absolute acceleration magnitudes, and thereby in principle with metaphysical absolutism about accelerations.)

The above analysis makes salient that comparativism violates a specific kind of locality principle, namely (spatial) separability. A quantity or magnitude such as mass is separable if all the facts about mass in a region supervene on all the facts about mass in the subregions that make up that region. We have seen that the mass ratios of  $W$  do not supervene on the mass ratios within  $S$  and within  $S^*$ . Comparativistic mass is non-separable. This non-separability seems undesirable or at least surprising for at least two reasons. First of all, one of the core assumptions in every scientific experiment is that you can isolate subsystems, i.e. a lab, and ignore the rest of the world for all practical purposes. To this one might respond that all the intra-system relations are fully determined by intra-system mass relations; it is only inter-system relations that require inter-system mass relations. Nevertheless some awkwardness remains: the comparativist needs more ‘stuff’ than the absolutist to explain everything there is to explain, even if we would end up concluding that both positions can explain everything there is to explain. This puts pressure on the metaphysical parsimony of comparativism. We will return to the issue of non-separability and its consequences in [Chapter 5](#).

### 2.3.2 Temporal Intra-world Leibniz Scaling

In this scenario the two sub-systems are temporally separated, rather than spatially. Consider a possible world  $W$  in which a uniform mass doubling of all particles in  $W$  occurs at midnight, again keeping all the distances, velocities and forces the same [8].

Although Dasgupta considers this scenario only in a different context, we can once again ask whether his comparativism can get the empirical data right. I

take it he would interpret this as being able to tell the correct story about the inter-system acceleration relations (i.e. the comparative change). In this case the subsystems are separated in time, and thus the trans-temporal differences in acceleration need to be explained: every particle in today's world ( $= S$ ) bears the relation 'is accelerating at half the rate of' to its counterpart in yesterday's world ( $= S^*$ ). By analogy with the previous, spatial scenario, we may employ inter-system mass relations, that is trans-temporal mass relations—fundamental facts about the way the mass of a particle at one time is related in mass to that particle (or another particle) at another time—in order to explain the trans-temporal acceleration relations. This is what I assume Dasgupta would suggest—he in fact uses this solution in the context in which he himself discusses the temporal intra-world Leibniz scaling [8, §3]. The change that does require an explanation, namely the comparative change in acceleration relations, is then explained by invariant force relations over time combined with mass relations that are doubled over time.

An important consequence of this solution is that the comparativist is now not only committed to spatial non-separability, but also temporal non-separability. The facts about mass relations of a set of particles, across space and time, do not supervene on the set of instantaneous mass relations of those particles. Additional fundamental transtemporal relations are required, reducing the metaphysical parsimony of comparativism—more on this in [Chapter 5](#).

Are these additional relations really required though? Is there truly any relevant comparative change between the subsystems that requires an explanation? Why did we think so for the spatial case? The way that scenario has been set up it seems we have been assuming substantival space and time, and have been acting as an omniscient God with direct access to the metaphysical facts, and who therefore recognises a comparative difference in accelerations in terms of such substantival spatial and temporal measures. This would indeed give us, in both the spatial and temporal case, a comparative change that requires explanation.

But we are not omniscient beings who know all the metaphysical facts. We have only limited epistemological access to those metaphysical facts. In the spatial version of the objection, if we want to go about and check if there is an inter-system difference in acceleration it seems like we need to get our hands dirty and use rods and clocks. We first go to  $S$  and measure, say, that the acceleration is one rod-length per one period of our clock squared, and find for that particle's counterpart in  $S^*$ , once we have moved over there, an acceleration of only half a rod-length per one period squared. This is then why we think that there is a comparative difference in accelerations that requires an explanation.

Two worries concerning this method come to mind. First of all, rods and clocks are not primitive ideal things that are just there for us to use in any possible circumstance and without requiring any further thought as to their constitution. In other words, we require a theory of rods and clocks, including a justification

for their being suited as waywisers of space and time [51, 52]. The second worry derives from the first. Assume that we have found rods and clocks which function as ideal waywisers when at rest in some inertial frame. According to the above method, we use rods and clocks to measure accelerations in one system, and then accelerate those tools towards the other system followed by deceleration in order to use them in the other system. If rods and clocks are not primitive, how can we be sure that this acceleration does not affect them, and influences their ability to be waywisers of space and time [51, 52]? I will refer to this as the *dynamical rod hypothesis* and the *dynamical clock hypothesis*<sup>49</sup>: the hypotheses that accelerations do not affect the abilities of rods and clocks to function as perfect waywisers of space and time, respectively. Whether this hypothesis is correct for specific (or even any) rods and clocks is far from obvious. For instance, we do not expect a cuckoo clock to survive just any series of accelerations (i.e. forces ‘hitting’ the clock), nor, *a fortiori*, to retain its ability to measure time after such beating [53].

I take these to be serious worries, but in the spatial version of the objection we can largely evade them. If we assume that there is at least one type of rod, and one type of clock, each of which function as a waywiser, then we can use them without having a theory of rods and clocks. We can be instrumentalists about our instruments. Why are we justified in viewing our instruments as black boxes, at least when it comes to the spatial version of the objection? Because we are not using separate rods and separate clocks for the separate subsystems. If we would for instance use a wooden rod for S and a metal rod for S\*, we would indeed have to justify that they both measure the same length. However, we only need one rod and one clock. Having assumed that these work properly in S, we can subsequently use those same instruments in S\*. Since re-using them does not change their constitution, they will work equally well in S\*.

This is of course only the case if the acceleration during transport between the systems does not affect our single rod and our single clock. But in principle we can transport them without acceleration. If they are both on a constant velocity trajectory intersecting both subsystems, they can measure accelerations in S when passing through S, and accelerations in S\* when passing through S\*. If we want we could even ‘go back’ to S if we have a second rod and clock (of the same constitution) which move at constant speed in the other direction. This does not make our previous worry resurface, because in S\* we can check if both the old and the new instruments function equivalently, before the new tools are sent to S. This local synchronisation of the tools removes the need for a detailed knowledge of a theory of those instruments.

With respect to the temporal version of the objection, everything seems to change. In order to empirically recognise an inter-system comparative difference

---

<sup>49</sup>In Brown’s terminology this is the Clock Hypothesis [51, 52].

that would then require an explanation, it seems we would need to have rods and clocks that go between the two systems in order to compare the accelerations in the two systems (since we are still no omniscient Gods). The fact that we cannot time-travel then suggests that there is no explanandum in the temporal version of the objection<sup>50</sup>.

One might respond that we can in fact ‘move’ the instruments between the subsystems. We just write down what the instruments measure yesterday, wait till ‘time moves the instruments to tomorrow’, and then write down which accelerations are measured then. Compare the two notes, et voilà!

But now our previous worries resurface as one combined worry, which might be called the *modal clock hypothesis* and *modal rod hypothesis*. Does this modal change, the uniform mass doubling, affect the waywising ability of the rods and clocks? In order for a rod to be stable, we presumably need to go beyond Newtonian Gravity and include a repulsive force that counterbalances the attractive gravitational force and creates a stable rod<sup>51</sup>. Our theory of rods now becomes important, since a uniform mass doubling will quadruple the gravitational interaction between the constituents of the rod and presumably either break it or make it shrink<sup>52</sup>. Similarly, if we use as a clock a gravity pendulum, which is basically a mass attached to a rod, its period not only depends on the length of the rod, but also on the local acceleration of gravity, which depends (among other things) on the mass of the Earth. It seems unlikely that either of the modal hypotheses holds. Our theory of rods was unimportant in the spatial version of the objection since we used the same rod for each subsystem. Although in the temporal version the rod used today is the same as the rod used yesterday in the sense that the two are connected through their worldline, they are not the same insofar as they are made up out of constituents with different (mass) properties. It is unclear whether any explanandum exists in the temporal version of the objection, and if so what that explanandum would be.

Let us take a step back. It seems like we have been cheating, in the sense that the temporal version of the intra-world Leibniz scaling objection is not exactly the temporal counterpart of the spatial version. Within the spatial version, the rod(s) and clock(s) effectively form a third subsystem. This third subsystem is then used

---

<sup>50</sup>It may be argued that exactly at midnight we might be able to recognise that something changes, even as comparativists. Since at midnight highly artificial discontinuities occur I will not further pursue this route.

<sup>51</sup>Even then we might still not have stable rods until we move to quantum mechanics.

<sup>52</sup>After midnight the mass of the platinum cube in Paris that serves as a reference standard for mass is doubled as well, which might give the impression that the mass of the constituents of the rod has not changed, but it is only the quantities that remain unchanged, not the magnitudes they represent. (Incidentally this is another way in which quantities are a suboptimal representational tool.) We should be careful not to let this misinform our theory of the rod and not to let it disguise that something relevant has really changed (if absolutism is indeed correct).

to determine (spatial and temporal) congruence with  $S$ , and then moved to  $S^*$ —without suffering uniform mass doubling!—in order to determine congruence with  $S^*$ . In the temporal version, the instruments are part of both subsystems, and as such undergo mass doubling when ‘moved’ (across time) between the subsystems. A fair comparison would then thus be to consider the instruments in the temporal version of the objection also as a proper subsystem, by not having their masses doubled at midnight (or to have them be part of the two subsystems in both versions of the objection, which would eliminate the explanandum in each case). In such a way the inter-system comparative change can be measured, and this explanandum then requires trans-temporal mass relations as an explanans.

However, within the scenario of the spatial version of the objection, we can in principle cut out the middle-man: we do not actually need the instruments to be able to formulate the explanandum. The two subsystems could be made to overlap<sup>53</sup> (either right from the start, or by having them approach each other at constant speed). The inter-system comparative accelerations can then be determined by local (spatial and temporal) congruence. This is of course not an option with the temporal version of the objection: we cannot make yesterday’s and today’s subsystem overlap. Instruments are truly required to formulate the explanandum, but if these undergo mass doubling themselves it is unclear how the explanandum should be formulated exactly.

A final response is then to strip down our measurement apparatus to their bare essentials. Instead of a rod, we can do with just the ‘endpoints’. That is, if we use two force-free particles with non-zero relative velocity, this can serve as a distance measure that is constant under midnight mass doubling. With this measure of length, a third force-free particle moving away at a constant speed then provides a measure of time: one unit of time is defined as the ‘time it takes’ for the particle to move one unit of distance, as determined by the first two particles. Since these ‘rods’ and ‘clocks’ have no forces on them, a midnight mass doubling will not affect their functioning: the modal hypotheses are correct.

This then finally provides us with an explanandum analogous to the explanandum in the spatial version of the objection. Dasgupta’s trans-temporal mass relations are indeed required to explain it (within the framework of Dasgupta’s Comparativism).

### 2.3.3 Inter-world Leibniz Scaling

The final scenario is a different modification of the first (i.e. the spatial version of the intra-world Leibniz Scaling objection). Here the two systems under considera-

---

<sup>53</sup>Here I am assuming, as in the rest of this section, that the systems remain dynamically isolated once brought together. Once we take gravity into account in [Section 2.4.2](#) we will see that this is not in fact the case.

tion are not part of the same world  $W$ , but are each complete and distinct possible worlds:  $S \equiv W$ ,  $S^* \equiv W^*$ . They are related via an inter-world Leibniz Mass Scaling. In Dasgupta’s words, the objection asks us to consider a possible world  $W^*$  in which all absolute masses are uniformly doubled compared to the first world  $W$ , whilst the forces, distances and velocities remain the same. Subfigure (a) of [Figure 2.2](#) is an attempt to depict this scenario, but it is important to realise that certain difficulties arise when representing such a scenario due to the one-to-many relation between magnitudes and the quantities used to represent those magnitudes. The particle that is labelled ‘1kg’ in  $W$  has a counterpart labelled ‘2kg’ in  $W^*$ . This is to represent that the mass magnitude of the counterpart in  $W^*$  is doubled compared to the original particle in  $W$ . However, ‘kg’ usually obtains its meaning—within a world!—by choosing one privileged particle—within that world—that is represented by the quantity 1, or 1kg. The masses of the other particles are then represented comparatively, by expressing them in multitudes of this conventional kg. In the actual world that privileged object is a platinum alloy cube in Paris. The 1kg particle  $W$  is the analogue of the cube in Paris. In  $W^*$  the magnitude of the counterpart of the ‘cube in Paris’ is doubled compared to the original. However, if ‘kg’ is supposed to label the standard mass in Paris, then the counterpart in  $W^*$  should, by definition, also be labelled ‘1kg’, even though it would refer to a magnitude twice as large as the magnitude referred to by ‘1kg’ in  $W$ . This would of course completely hide exactly what the figure is supposed to convey. Hence, usage of ‘x kg’ in  $W^*$  is to be understood as representing the magnitude that would be represented by that label in  $W$ .

Once again, we consider whether Dasgupta’s comparativism gets the empirical data right, in accordance with the first premise of Dasgupta’s razor. As Dasgupta interprets the explanandum raised by this objection: in  $W^*$  the masses are uniformly doubled compared to  $W$ , so the accelerations should be uniformly halved. At this point one might worry (or rejoice) that this scenario indeed seems to be a much stronger threat to the empirical adequacy than the first two. The analogous explanandum of the first two objections was explained by an inter-system application of L2, that is via inter-system mass relations. Here the systems are worlds, rendering the analogue of the explanans fundamental inter-world mass relations, something Dasgupta is reluctant to commit to [[8](#), p.115]. There is no arrow in [Figure 2.2](#) that would serve as the analogue to the inter-system arrow that appears in subfigure (c) of [Figure 2.1](#). The other side of this coin is however that there is in fact also no inter-world explanandum (in Dasgupta’s sense). There are no fundamental inter-world acceleration relations, nor are there any emergent or epistemically accessible inter-world relations that need to be explained. We have no means of magically travelling to other possible worlds with our rod and clock to see how the accelerations there relate to our actual world. Applying L2 to the mass relations in  $W$  gives the correct acceleration relations in  $W$ ; applying

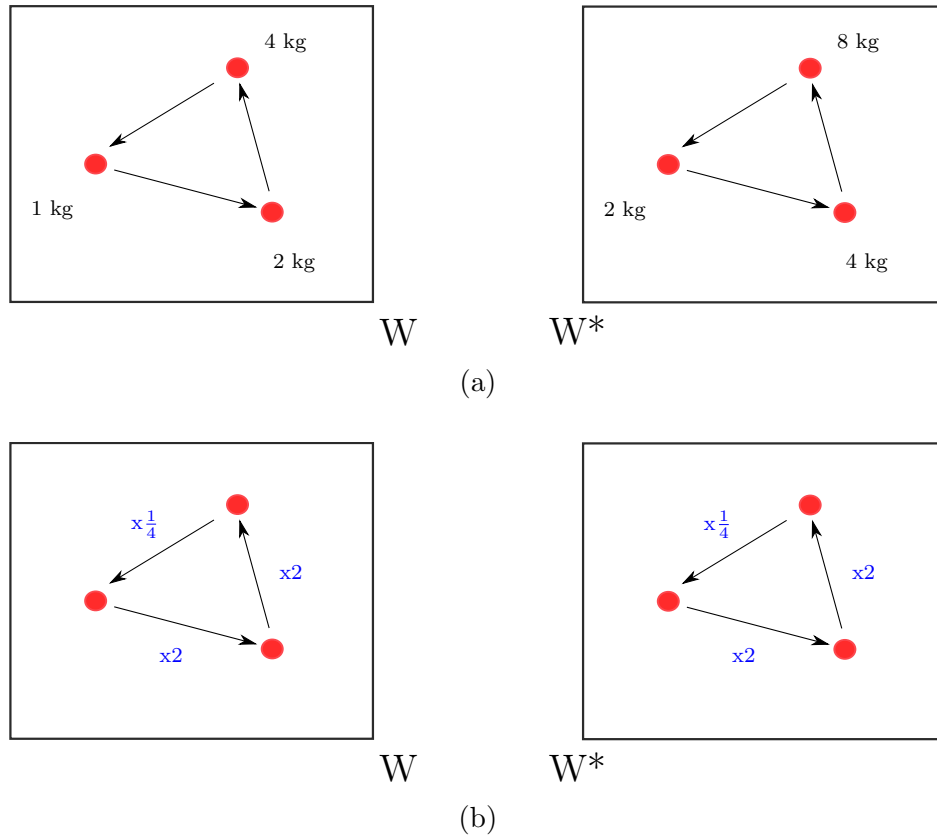


Figure 2.2: Example of an inter-world Leibniz scaling scenario. Subfigure (a) represents the scenario in absolutist terms. For the usage of the label 'kg' in  $W^*$ , see the comments in the text. Subfigure (b) represents the information about mass available to the comparativist about mass.

L2 to the mass relations in  $W^*$  gives the correct accelerations in  $W^*$  (namely the same as in  $W$ ). And this is all that needs explaining. Or so it seems when the explanandum is explicated in the way Dasgupta does.

## 2.4 Response to Dasgupta

The previous three scenarios purported to show that three different versions of Leibniz Scalings all lead to detectable differences, which according to Dasgupta is to be equated with the detectability of absolute masses, which in turn is to be equated with the empirical inadequacy of comparativism. In the two intra-world scenarios Dasgupta explicates this explanandum of ‘getting the empirical data right’ as explaining the acceleration relations, specifically the inter-system accelerations relations. In these scenarios he applies L2 to both systems as a whole—in particular the inter-system mass ratios—to explain this explanandum; in the inter-world scenario there is in fact no such explanandum. It seems that Dasgupta’s comparativist can account for all the relevant empirical data, which supposedly renders the absolute masses undetectable.

It is in formulating the explananda that the first of two crucial flaws appears in Dasgupta’s analysis of and response to the three scenarios. Observables in Newtonian Gravity are the trajectories of particles and the relations between them. Those follow, in the case of absolutism (L1 and G1), from initial conditions plus the laws of the theory. All Dasgupta has done is check whether (his interpretation of) the ‘law’ (L2) obtains in all scenarios. L2 governs mass relations, force relations and acceleration relations. (We will get to the forces in a second.) It is indeed the case that the acceleration relations in each system, and between each system, are correctly predicted by L2 for each scenario. Checking only whether the correct acceleration relations obtain in no way ensures that uniform mass doubling does not change the particle trajectories and the relations between those trajectories. These might depend on more than merely acceleration relations, and as I will show below they in fact do. If L2, perhaps with G2, fails to provide those trajectories, we should question whether they even deserve to be called laws, or are merely regularity statements that happen to be true of all the three scenarios.

The second flaw sneaks in right at the beginning, when the scenarios are described. We are asked to imagine that only the masses are doubled, and that everything else remains the same, in particular the forces. The *ceteris paribus* clause in the Leibniz Mass Scaling is misinterpreted to include forces<sup>54</sup>. In other words, the force magnitude (or magnitude relation) mentioned in Newton’s Second Law is viewed as an independent variable, whereas in fact it represents a ‘slot’

---

<sup>54</sup>I believe Baker’s Friction World and Shell World arguments to suffer from the same mistake [9, 25].

which needs to be filled by another specific force law. Forces do not ‘float free’, but have sources, and in Newtonian Gravity the source of the only force available is mass, exactly the thing that we are asked to vary in these scenarios. Dasgupta does acknowledge that he is ignoring this issue (footnotes 27 and 30), but it is wholly unclear why this would be justified. Indeed, on reflection it might strike one as rather strange that the gravitational force has not been mentioned a single time in our lengthy discussion of the detectability of mass. Of course we could introduce, for instance, electromagnetic forces, which would indeed remain invariant under uniform mass doubling. However, this does not mean that the gravitational force suddenly disappears. Although for some choices of parameters the gravitational force becomes negligible and the forces involved seem independent of mass for all practical purposes, this would not hold anymore once we start scaling the masses uniformly by large scalars. Moreover, including an electromagnetic force means extending Newtonian Gravity, whereas here we are concerned with standard NG. (Admittedly, Dasgupta seems to be interested in Newtonian Mechanics more broadly, but 1) no realistic approximation of the actual world could leave out the gravitational force<sup>55</sup>, and 2) considering Newtonian Mechanics more broadly does not mean considering only Newton’s three laws. Newton’s three laws are by themselves not a theory: the F-slot needs to be filled by *something* in order for it to be a well-defined theory. Forces require a source<sup>56</sup>).

Not only does Dasgupta analyse the objections incorrectly by explicating the explanandum incorrectly, but the scenarios he proposes are inconsistent in the first place—he misinterprets the *ceteris paribus* clause in the Leibniz Scaling transformation. In the following section I will first provide the correct analysis within the inconsistent scenario. Even then the absolute masses turn out to be empirically meaningful (a corollary of which is that they would be empirically meaningful as well in an ‘NG plus dominant electromagnetic force’ theory). Perhaps two wrongs make a right: perhaps the dependence of the force on the mass can compensate for the empirical difference arising directly from the mass doubling. In the subsequent section I give the full-fledged analysis including gravitation. Only a force law that is  $\propto m^1$  would right the wrong; with a gravitational force law that varies as  $\propto m^2$ , a uniform mass doubling, properly interpreted, leads once again to a detectable difference.

---

<sup>55</sup>Unless it is being reduced to, say, the curvature of spacetime, of course.

<sup>56</sup>I believe that both these problems also (fatally) plague Baker’s Friction World argument [9, 25].

### 2.4.1 Detectability: the dependence of trajectories on absolute masses

In this section I show an example of a set of trajectories of massive particles that is detectably different when we uniformly vary the masses of those particles (while keeping the forces constant). (And if the two systems under consideration are two distinct possible worlds, there are no inter-system mass ratios that could be used in any way as an explanans.)

Trajectories are collections of spatial points labelled with a time parameter. As such, a substantialist framework might seem best to formulate the following example. However, 1) I want the example to convince relationalists as well, and 2) there being substantial space and time does not by itself give us epistemic access to length and duration. We could use some extra particles to serve as a rod and a clock. Imagine a world with two force-free particles, subfigure (a) of [Figure 2.3](#). Without loss of generality I will assume that, within a substantialist framework, they are both at rest. The distance between these two particles, ‘the rod’, serves as the standard for the length unit  $d$ . This allows us to introduce a third force-free particle which, in substantialist terms, is moving away from the rod with a non-zero constant velocity (subfigure (c)), and thereby serves as a clock. The unit of time  $t$  is defined as the time it takes the third particle to move a distance  $d$ . We now add a fourth particle, to which a force is applied (subfigure (d)). Applying L1<sup>57</sup> gives us its trajectory. If the masses would be uniformly doubled whilst keeping the distances, velocities and forces the same (subfigure (e))—that is, we are considering a specific instance of the inter-world Leibniz Mass scaling scenario—L2 would not recognise the difference; the force relations and mass relations do not change, and therefore neither do the acceleration relations. However, the absolute acceleration—as opposed to comparative rather than relative acceleration—does change. Given that the initial velocities of the particles do not change, the trajectories of the first three particles do not change since they are force-free, but the trajectory of the fourth particle changes due to its change in *absolute* acceleration. Using our rod and clocks as standards for length and time, we measure a difference in (relative) trajectories that even the comparativist must acknowledge. Absolute masses are empirically meaningful!

Dasgupta’s analysis only works in situations with for instance two particles starting at the same location whilst being at rest. In that case  $\frac{s_1}{s_2} \propto \frac{a_1}{a_2} \propto \frac{m_1}{m_2}$ , where  $s$  is the distance traversed. Transformations that leave the mass relations invariant are then not detectable. In general however we need to take into account

---

<sup>57</sup>Technically L2 could not even be applied here, since there are no non-zero force relations. This could be remedied by introducing another particle with a force acting on it.

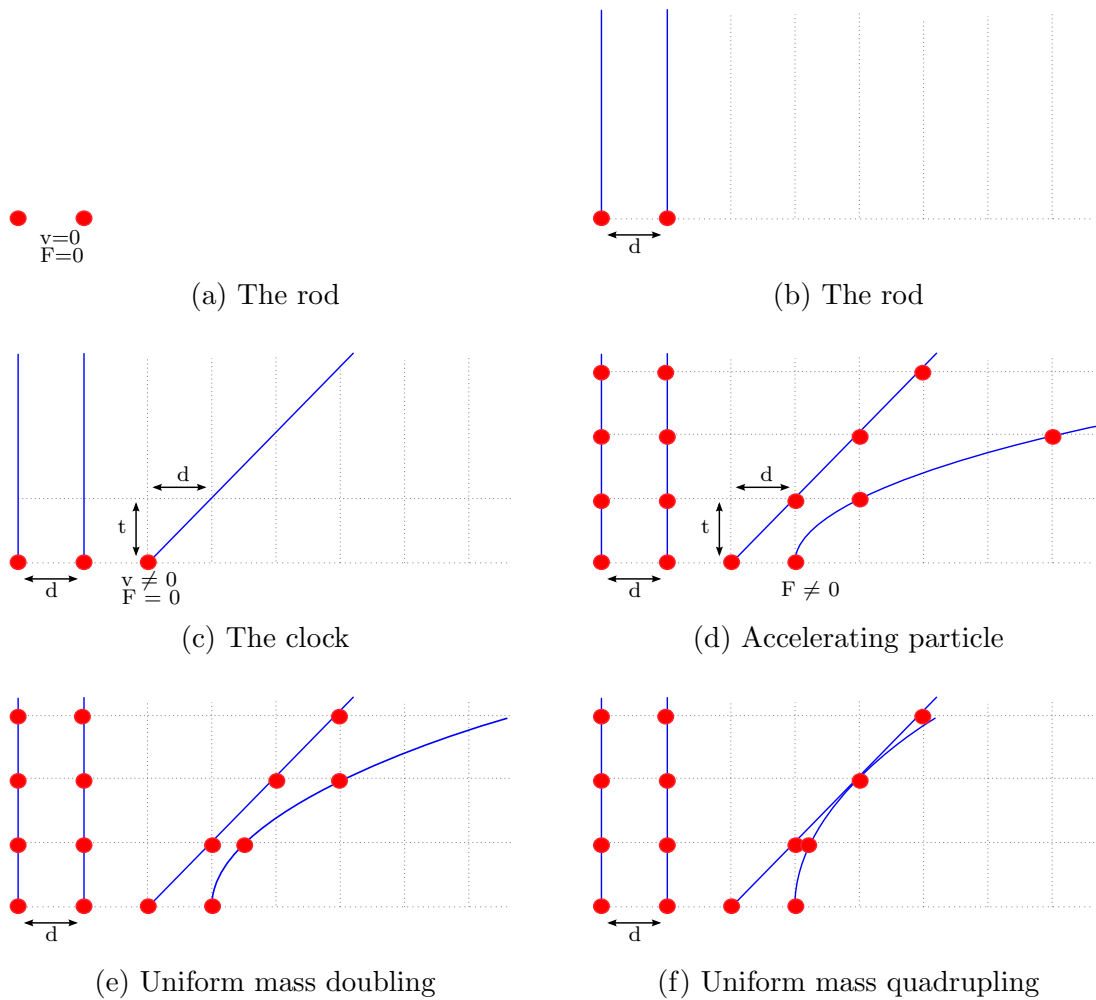


Figure 2.3: Example of particle trajectories detectably changing under uniform mass doubling. The red dots are massive particles; the blue lines their world lines. The two force-free particles in (a) and (b) serve as a rod; with this standard for length a third force-free particle, (c), can be used to function as a clock.

the initial locations and velocity:

$$\frac{s_1}{s_2} = \frac{x_1 + v_1 t + \frac{1}{2} a_1 t^2}{x_2 + v_2 t + \frac{1}{2} a_2 t^2} = \frac{x_1 + v_1 t + \frac{1}{2} \frac{F_1}{m_1} t^2}{x_2 + v_2 t + \frac{1}{2} \frac{F_2}{m_2} t^2} \not\propto \frac{m_1}{m_2} \quad (2.1)$$

Mass only appears in front of one term in each of the numerator and denominator; a Leibniz Scaling will not leave the relative trajectories invariant.

I introduced a rod and clock to be able to operationally differentiate the different scenarios resulting from a Leibniz Scaling. If we had a universe with only, say, the third and the fourth particle, epistemologically speaking the bets would be off. In such a universe any coordinate system could be chosen that has the same ordering of coordinates as the (imagined) substantial coordinate system—that is any coordinate system in which the particles do not intersect. We might for instance choose a coordinate system in which both trajectories are rectilinear [54]. However, if we would uniformly quadruple the masses in our previous example (subfigure (f)), two of the trajectories coincide at some point in time. Multiplying the masses even further would result in the trajectory of the fourth particle intersecting the trajectory of the third to ‘reach its other side’, followed by a later intersection to ‘return to its own side’. Such intersections of trajectories are recognisable for both the absolutist and comparativist, without even the need for rods and clocks. Coincidence or no coincidence is the crucial empirical difference. This is fortunate since we will not be able to construct rods and clocks from force-free particles once we take gravitation into account, as every massive particle will feel a force from the other massive particles.

### 2.4.2 Adding gravitation: the comparativist’s bucket

Let us finally use the full power of Newtonian Gravity by adding gravitation. We will consider yet another, even simpler instance of the inter-world Leibniz Mass Scaling scenario—a simplification of Baker’s Earth-Pandora scenario [9]—so that there are no inter-system mass ratios available anyway that could potentially be used as a comparativist explanans. Consider a world, governed by the laws of Newtonian Gravity, with two equally massive particles a distance  $r$  apart, each initially moving away from the other with velocity  $v$  (such that there is zero angular momentum). What will happen? That depends on whether the gravitational force provides enough of a ‘brake on the velocity’. If not, the particles will escape each other; otherwise they will coincide. Using L1 and G1 we can calculate the escape velocity:

$$v_{e,1} = \sqrt{\frac{2Gm_2}{r}}. \quad (2.2)$$

If the initial velocity is larger than the escape velocity the particles will escape, otherwise they will coincide. It is crucial to notice that the escape velocity depends only on the absolute mass of each of the particles, not the mass ratio between the two particles! Hence, if the absolute masses of the particles in our scenario are such that the particles barely escape each other, they will ‘collide’ in another world which differs only by uniform mass doubling (properly interpreted). The comparativist cannot distinguish these two worlds as far as the initial conditions are concerned, as these—the mass relations, distance relations, velocity relations and the force relations (and therefore also the acceleration relations that follow dynamically from these initial conditions, which are the only things Dasgupta cared about)—are identical. In the comparativist framework this scenario is indeterministic: the initial comparativistic conditions do not predict whether the particle trajectories will intersect or not, an observable difference that even the comparativist recognises. The absolutist has no problem predicting what happens. That we do not have a clock or rod available is no problem, since in one world the particles will never collide, even if one waits infinitely long, while in the other world the particles will collide within finite time. Absolute masses are empirically meaningful!<sup>58</sup>

Newton’s bucket similarly shows that the relationalist initial conditions, in particular relative accelerations, are insufficient for an empirically deterministic evolution of the system (i.e. they fail to solve Laplace’s problem). Absolute (rotational) accelerations are empirically meaningful. By analogy<sup>59</sup>, we will call this specific two-particle instance of the inter-world Leibniz Mass Scaling the **comparativist’s bucket**—although it is important to realise that the same issue arises for any other pair of worlds with any number of particles<sup>60</sup>; it is just that the two-particle scenario is the simplest scenario for my purpose as it allows for analytic solutions. Absolute masses are empirically meaningful. (In terms of the symmetry approach: a Leibniz Mass Scaling, interpreted as keeping the distances and velocities the same, is not a symmetry of the theory.)

It is interesting to note that it is not the case that any dependence of the

---

<sup>58</sup>In fact, if we place particle detectors infinitely far to the ‘left’ and ‘right’, we have in effect constructed the absolute mass detector of which Dasgupta argued it could not exist [8, §8.3].

<sup>59</sup> An even closer analogy, using particles rather than a bucket, would be Skow’s two-particle scenarios [3] (discussed by Pooley [55]) or Barbour’s three-particle scenarios [2, Fig.13]. As all these scenarios are used to make the same point, and Newton’s bucket is the most famous of the three, I have chosen to adopt the name ‘comparativist’s bucket’.

<sup>60</sup>However, in the general multi-dimensional, multi-particle case, presumably only a subset of the space of solutions of measure zero will have colliding particles at all. Perhaps this is even a reason to believe that we can and should ignore that subset. In the general scenario it will then be necessary to make our criterion of empirical equivalence of worlds more strict by considering not only coincidence of trajectories but also angles (i.e. shapes) or ratios of distance over time. The absolutist and comparativist should still be able to agree on this criterion.

force on mass would produce this problem for the comparativist. If the force would for instance be  $F_{g'} = \frac{G'm_2}{r^2}$  or  $F_{g''} = \frac{G''m_1}{r^2}$ , the escape velocities would be  $v_{e',1} = \sqrt{\frac{G'm_2}{rm_1}}$  and  $v_{e'',1} = \sqrt{\frac{G''}{r}}$  respectively. These escape velocities are invariant under uniform mass scaling. These forces would thus cancel out exactly the effect of the mass doubling on the trajectories discussed in the previous section. This result is in sharp contrast with the argument considered in the previous chapter—that dynamic comparativism is logically implied by kinematic comparativism—which would have proven comparativism true *a priori*. Such a result would both be very surprising, and at the same time make the debate much less interesting. We have now seen that the empirical meaningfulness of absolute masses is truly a non-trivial empirical (i.e. *a posteriori*) question, although I hope that that was not too large of a surprise.

## 2.5 Solving the Ozma Mass Problem

Solving the Ozma Mass Problem seemed impossible, since the aliens could only directly (i.e. non-dynamically) express the mass of an object by comparing it to a standard reference mass in their capital (i.e. kinematic comparativism). On the other hand, the Ozma Handedness Problem can be solved by asking the aliens to perform a certain experiment (namely any decay that is governed by the weak interactions) to determine unequivocally what (the orientation that we call) left is. It now seems we have a similar experiment available when it comes to having the aliens pick out a specific absolute mass of an object. Ask the aliens to set up two particles of equal mass according to the scenario above and let them observe whether they escape or not. The mass of these particles can then be expressed in a ‘natural’ mass unit—that is a dynamically privileged mass unit—which is such that 1 unit of mass would just about escape. (Of course, kinematic comparativism tells us that this natural unit cannot be forced upon us. The aliens are still free to use another unit. But what the existence of this dynamically privileged natural unit does allow is an unambiguous way of communicating and comparing units.)

This solution is however not as neat as the solution to the Ozma Handedness Problem. For that experiment no other information (other than the type of particle), that is no initial condition, is relevant. In the mass case the answer will depend on the initial distance and initial velocity of the two particles in the experiment. In other words, the mass unit depends on the length and time units. We have nevertheless made some progress. We have solved the Simple Ozma Mass Problem, where we are allowed to send the aliens a rod and clock. In other words, we have reduced the Ozma Mass Problem to the Ozma Length Problem and Ozma Time Problem. The claim that we have defined a natural mass unit is thus to be interpreted as there being a dynamically privileged unit for mass, but

only relative to a particular pair of rod and clock.

## 2.6 Evaluating the undetectability approach

Now we have realised that the real explanandum is the (relative) particle trajectories (*pace* Barbour [2]) and not the acceleration ratios, and that those trajectories detectably differ depending on the absolute masses, it seems that we should conclude from the discussion so far that the undetectability approach to the argument for comparativism actually backfires and proves absolutism correct instead. In this section we will take a U-turn, arguing that the undetectability approach is a bad way of explicating the issue of the empirical adequacy of comparativism in the first place. Fortunately our efforts so far in this chapter will not have been for nothing, as we will use many of the lessons learnt when applying the possibility checking approach next.

There are three reasons why the empirical meaningfulness/meaninglessness of absolute masses comes apart from their (un)detectability and from the empirical (in)adequacy of comparativism. Firstly, the empirical meaningfulness of absolute masses comes apart from the empirical adequacy of comparativism once we realise that the discussion so far has **confused fundamentality and reality**. This is not uncommon in the analogous debate about space: relationalism is sometimes portrayed as denying the reality of spacetime<sup>61</sup>, which is false for most relationalists (except perhaps Leibniz himself). The issue at the heart of that debate is whether the spatiotemporal relations between material objects are fundamental, or grounded in the spatiotemporal relations between the spacetime points they occupy. In the former case, spacetime is still a real feature that emerges<sup>62</sup> from the network of fundamental spatiotemporal relations between matter. Similarly, the comparativist does strictly speaking not deny what we may call realism about absolute masses<sup>63</sup>:

### Realism about Absolute Masses:

Absolute masses are empirically meaningful.

---

<sup>61</sup>One may, for instance, have expected this mistake from the title of Pooley's forthcoming book on the substantivalism–relationalism debate, *The Reality of Spacetime*, although this does not in fact occur in the book [55].

<sup>62</sup>For instance, but not necessarily, as a 'real pattern', a phrase coined by Dennett [56].

<sup>63</sup>In fact, Dasgupta seems to have some sympathy with realism about absolute masses, insofar as he finds it important that comparativism must be able to explain what he calls the kilogram facts (e.g. the fact that my laptop is 2 kgs), which seem closely related to absolute masses. He even develops a novel, plural notion of grounding [8, 36] in order to be able to explain these facts. It is not directly clear to me how he takes this sympathy with realism about absolute masses to be compatible with denying the detectability of absolute masses.

Comparativists only deny that absolute masses are more fundamental than mass ratios. Not being fundamental does not imply not being real. A chair is not fundamental, but it is real. An absolutist does not deny the reality of mass ratios, but merely their fundamentality. Similarly, absolute masses could in principle be grounded in mass ratios (and perhaps other non-mass properties). This is admittedly a counter-intuitive position, at least *prima facie*, and anti-realism about absolute masses is certainly very much in the spirit of comparativism which is probably the reason why to my knowledge nobody so far has adopted such a realist comparativist position. Nevertheless, if one agrees with my argument for the empirical meaningfulness of mass in [Subsection 2.4.2](#), absolute masses are clearly real. This is by itself an important result, even if it does not immediately prove absolutism. In fact, many may find the issue of the realism of absolute masses more interesting than the actual absolutism–comparativism debate as it has been defined in this thesis. They may even want to go as far as to redefine absolutism (comparativism) as realism (anti-realism) about absolute masses<sup>64</sup>. This is however mere semantics, so I have chosen to stick to the standard definitions as they occur in most of the literature on this topic. In light of this argument for realism about absolute masses (i.e. the comparativist’s bucket), an obvious loophole<sup>65</sup> the comparativist might want to exploit is to remain insistent on the non-fundamentality of absolute masses—the letter of comparativism—whilst at the same time accepting their reality—giving up the spirit of comparativism. We will discuss one way of pursuing this option, called regularity comparativism, in [Chapter 4.3](#). Although I will conclude against that version of comparativism, it remains a coherent position that forms a part of the logical space of possible positions. Comparativism thus clearly is consistent with absolute masses being empirically meaningful.

The other two reasons are two sides of the same coin. They relate to equating the empirical meaningfulness/meaninglessness of absolute masses with their (un)detectability. In [Section 2.4](#) we showed that absolute masses are empirically meaningful, because they are correlated with trajectories in an observable way. In other words, they are empirical-difference-makers. Does that mean we have detected/determined/gained knowledge of *which* absolute mass *this* object has [imagine me pointing at any of the objects in an instance of the scenarios described in the previous sections]? Can you express to me what the absolute

---

<sup>64</sup>Arguably, Roberts’ rough definition of comparativism could be interpreted as such: “A *comparativist* about a quantity-type – about mass, for instance – says that what has significance is relations among the values of quantities of that type, rather than values of particular quantities of that type” (italics in original) [[13](#), p.3] (see also his fn. 3).

<sup>65</sup>If one had chosen to redefine absolutism (comparativism) as realism (anti-realism) about absolute masses, this loophole would not be an option. Regularity comparativism—or in fact the whole second category of the comparativist research program ([Chapter 4.3](#))—would be off the table, *ab initio*.

mass—the quiddity—is of this object? No. Conversely, if absolute masses had not been empirically meaningful, what would it have been exactly that we would have failed to detect?

Consider again sending objects to aliens; this time let us make it two 4kg square tiles. When we ask after their mass ratio and shape, the aliens have no problem determining and then reporting those to us: 1 and 4 (corners), respectively. But when we ask what the absolute mass is, they can only determine that the existence of the absolute masses (whichever specific absolute masses they are) of the tiles that ground their mass ratios influences the trajectories of those tiles. They cannot express the specific absolute masses (more) directly, that is they cannot express them beyond that—i.e. beyond the indirect statements ‘this is the mass that corresponds to this/ that trajectory’ or ‘this mass is the same as/ different from that mass’—in contrast to the way in which they can (more) directly state the number of corners. Conversely, if those absolute masses had not made a difference to the trajectories, there was no well-defined question we could ask the alien (about absolute masses) that they would not have the answer to. Asking them what the absolute mass is was not a well-defined question, since it is not clear what could ever count as an answer (other than, eg. ‘the same absolute mass as that other object’ or ‘the mass that corresponds to that trajectory’). We could only ask them if the mass of the tiles is twice as large as it actually is, to which the trivial answer is no, or how the masses compare to those of other objects.

A parallel to this conundrum is generated by the (supposed) undetectability of absolute positions in NG [29, 48, 57]. Maudlin contrasts the inexpressibility of our supposed ignorance of absolute positions (which is therefore no true ignorance, since there is no epistemic vice that an Occamist norm ( $P_{occ-det}$ ) can latch onto) with the expressibility of our actual ignorance about (the direction of) absolute velocities.

The essential difference between the static and kinematic cases lies in the semantical role that indexicals or demonstratives play for terms denoting *places* as opposed to the roles they play in terms denoting *velocities*. Since all absolute places are qualitatively identical, the only way we can possibly refer to them is either by direct ostension or by using a definite description which makes reference to some material object [i.e. kinematic relationalism]... We can then formulate meaningful counterfactuals about worlds where everything would be displaced from its actual location, but we can also be assured that they *are* counterfactuals, that they do not describe the world as it is.

...

We can ... sensibly ask about the Earth’s absolute velocity without having, even in principle, a means to determine what it is. But we

can only sensibly ask about the position of the Earth by asking for its position relative to some determinate set of coordinates [i.e. kinematic relationalism], and the linguistic wherewithal needed to establish the coordinates also provides us the means of answering the question. In sum, the *only* way that the static shift can be formulated is something like, “what if God had created the material universe oppositely oriented *to the way it is oriented now?*”, and this is clearly a counterfactual situation. But we can ask “what if God created the material universe at absolute rest?”, not knowing whether we describe a counterfactual situation or not. [48, p.190-1, italics in original]

In our case, if absolute masses had turned out to be empirically meaningless, I could not detect/determine and get to know *how* (absolutely) massive a specific object was, but I could not express that ignorance by stating *which specific* absolute mass I do not know the object has. For example, ‘I cannot know whether the absolute mass is twice the actual mass’ is clearly false, as is ‘I cannot know, in principle, whether the absolute mass is the same as the absolute mass of that other object’. This failure of expressibility should not surprise us: this is the content of kinematic comparativism, this is what it means, by definition, to be a dimensionful magnitude—in fact, as Table 2.1 shows, typical examples of expressible ignorance and knowledge are all dimensionless notions, and typical examples of inexpressible ignorance and knowledge are all dimensionful notions. This is why the Leibniz Scaling is (necessarily) formulated ‘comparatively’, as a multiplication mapping from the absolute masses in one possible world to the absolute masses in another possible world. It is true that the Kinematic Shift is standardly formulated in the same way, by adding velocities to the absolute velocities in one world to obtain the velocities in a different world, but, as Maudlin points out in the quote, in principle the Kinematic Shift Argument could have been reformulated to ‘absolutely’ or ‘intrinsically’ pick out the world that is at absolute rest, without ‘comparing’ it to other possible worlds. There is no such reformulation of the Leibniz Scaling.

In the case of absolute masses in fact being empirically meaningful, as I have argued for, we face the problem that we cannot express that knowledge by non-trivially expressing *which* absolute mass this object has. We can state that it has or does not have the same mass as another object, and we can state that we know that it does not have twice the absolute mass that it actually has, but that is as good as it gets.

This latter case is similar to a Newtonian possible world with a force law that does depend on position. Since this is not true in standard NG, substantivalists often turn to a sophisticated version, which denies primitive identities of space-time points, but if the force law(s) had in fact depended on position these prim-

	<b>inexpressible</b>	<b>expressible</b>
<b>ignorance</b>	<ul style="list-style-type: none"> <li>▪ absolute position</li> <li>▪ absolute &amp; relative speed (against Maudlin [48])</li> <li>▪ absolute mass, if it would have been empirically meaningless</li> </ul>	<ul style="list-style-type: none"> <li>▪ direction of absolute velocity</li> <li>▪ being at absolute rest or not</li> </ul>
<b>knowledge</b>	<ul style="list-style-type: none"> <li>▪ absolute mass</li> <li>▪ absolute position, if the force law had depended on position</li> <li>▪ absolute rotation</li> </ul>	<ul style="list-style-type: none"> <li>▪ number of corners</li> <li>▪ mass ratios</li> </ul>

Table 2.1: Examples of expressible and inexpressible knowledge and ignorance.

itive identities would have been needed for the force(s) to latch onto. Similarly, since absolute masses are empirically meaningful, the magnitudes require non-qualitative identities to which the forces latch on (i.e. these identities are required for the forces to be well-defined/ referring, in the sense of uniquely matching up instances of initial conditions, including masses, with, say, accelerations).

In summary, to require expressible knowledge of absolute masses as criterion for their detectability, or expressible ignorance as criterion for their undetectability, would be incorrect. This would make absolutism in some sense *a priori* false, which would be strange since detectability it supposed to be something that is to be determined *a posteriori*. This is basically the naive argument in another guise. Similarly, comparativism is in some sense also *a priori* false, since there is nothing that one could ever have been ignorant of in the first place. The obvious response, on behalf of the undetectability approach, is that we we were never interested in this narrow sense of (un)detectability (“detecting-which”), which requires one to be able to express the specific absolute masses that we are either ignorant of or have knowledge of. The notion of detectability is supposed to line up with the empirical meaningfulness of absolute masses, not to be divorced from it. *That* changing the absolute masses results in a detectable difference is sufficient for the masses to be detectable (“detecting-that”), even if we cannot detect *which specific* mass obtains. Similarly, if we know that absolute masses make no detectable difference, then it does not matter that we cannot further specify which absolute mass it is of which we do not know that it obtains. I completely agree. But once we accept this broader notion of detectability, that uniformly changing the masses leads to a detectable difference, that is a detectably different possible world, the undetectability approach has collapsed onto the possibility checking approach—it is not merely equivalent but literally becomes the same approach. Detectability is now an explicitly modal notion, distinct from the non-modal manner in which we detected the number of corners of an object. We are now in the business of

comparing the detectable features of possible worlds that differ in their distribution of absolute masses (but not mass ratios)<sup>66</sup>. We are thus naturally pushed towards the possibility checking approach. This approach focuses merely on checking whether the correct empirically distinct possible worlds are included, not on ‘labeling’ or ‘matching up’ those possible worlds with expressible absolute masses.

### 2.6.1 Ozma Games, one last time

It is important to realise the implications of the discussion in this section for the evaluation of different variants of the Ozma Mass Problem. The Ozma Mass Problem concerns the question of whether we can convey the meaning of, say, ‘1kg’ to the aliens (ideally in the same sense as we can convey dimensionless determinates). So far we have equivocated between three interpretations of the problem. 1) Can we express a quidditistic absolute mass (say the one that we have labelled ‘1kg’) to the aliens, in the same direct (i.e. kinematic) sense as we can convey dimensionless determinates? 2) Can the aliens discover (i.e. detect) that absolute masses are empirically relevant? 3) Can we ensure that the aliens pick out a specific object with an absolute mass that is the same as the mass that we have labelled, say, ‘1kg’?

The content of kinematic comparativism boils down to a negative answer to the first question. Nevertheless, we now know which experimental instructions we need to give the aliens to demonstrate to them that absolute masses are empirically relevant—they can “detect-that” (even if they cannot “detect-which”). This experiment, the comparativist’s bucket, provides a dynamically privileged unit. Although kinematic comparativism implies that we cannot force the aliens to use this natural unit rather than any other unit, we can use it to express to them our privileged unit, the kg, by telling them the conversion ratio between kilograms and the natural unit. In this way, then, we can ensure (given a particular pair of rod and clock) that we can have the aliens pick out a specific object that has the same quidditistic mass as our standard reference object in Paris—even though they cannot “detect-which”.

## 2.7 Further underdetermination of absolute masses

In the context of the undetectability approach, one final, interesting, additional but related problem for absolutism should be pointed out.

---

<sup>66</sup>§8.3 of Dasgupta’s 2013 paper [8] suggests that Dasgupta is sympathetic to this way of thinking. §8 of his 2015 paper [58] suggests otherwise.

We have seen that the quantities—at the representational level—both underdetermine and are underdetermined by the magnitudes—at the metaphysical level—because of the freedom of a choice of unit. This two-way underdetermination arose from kinematical comparativism, that is mass being dimensionful and hence requiring an arbitrary unit to be represented. We have seen that in this narrow sense absolute masses are undetectable/undeterminable: it is inexpressible which specific mass obtains. However, this did not change the argument that a uniform change of absolute masses leads to a detectable difference, thereby rendering the existence of absolute masses detectable, if not which specific absolute mass obtains.

There is however another sense in which absolute masses are underdetermined, which goes beyond even their inexpressibility: even the inexpressible quiddities of the determinate masses themselves are underdetermined by the observable trajectories. So far we have taken the (absolutist) laws (L1 and G1) as given, but we in fact only have empirical access to the (strength of the) gravitational law and the (mass) magnitudes as a package deal<sup>67</sup>. To illustrate this, consider for simplicity a world with two particles, a certain distance apart. If the data—that is the trajectories, or more specifically the accelerations  $a_1$ ,  $a_2$ —are consistent with one instance of L1 and G1 and a distribution of absolute masses  $m_1$  and  $m_2$  which maps those masses (given the distance) to those accelerations, then it will be equally consistent with a distribution of masses  $\beta m_1$  and  $\beta m_2$ , where  $\beta$  is a scalar, provided that we choose different versions of L1 and G1 which now map this alternative set of mass magnitudes to  $a_1$  and  $a_2$ . (At the representational level this ‘active’ underdetermination is mirrored by a ‘passive’ counterpart: the freedom of, given a choice of units, uniformly multiplying all the mass quantities as long as one compensates by changing the value of Newton’s constant.) Thus, absolute masses are supposedly undetectable/undeterminable/underdetermined in a sense that goes even beyond their inexpressibility<sup>68</sup>.

One response might be that, insofar as this underdetermination issue would be a problem for absolutism about mass in NG, it would be a problem for any physical property and any physical theory, including mass ratios in NG. One can always add anything to physical magnitudes or ratios or modify them in any other way, and modify the laws accordingly to screen-off this modification in the physical

---

<sup>67</sup>Dasgupta discusses this same problem for absolute velocities [58, p.610] and absolute accelerations (his so-called ‘curvature’ argument) [58, p.615]. In his paper on comparativism [8], Section 8.4 might be interpreted as hinting at this problem for the case of mass.

<sup>68</sup>One might consider using extra-theoretical virtues, such as metaphysical parsimony or explanatory power, to remove the underdetermination. For instance, Maudlin uses (a theological version of) the Principle of Sufficient Reason to argue that one absolute velocity, namely absolute rest, is preferred over any other, non-zero absolute velocity in an arbitrary (i.e. not-reasoned-for) direction [48] (*pace* Dasgupta [58, p.607-8, 610]). However, in our scenario all options seem to be on a par.

properties. We have thus entered the murky waters of a much more general debate in the philosophy of science, rather than a problem that is specific to absolutism only.

However, the most obvious and much stronger response is exactly the same as the response in the previous section. After all, this new underdetermination basically boils down to the inexpressibility of the ‘strength’ of the force as represented by the value of Newton’s constant—a dimensionful ‘quantity’!—which parameterises the family of laws that the data underdetermines. Just as we cannot express the absolute masses, we cannot express which (version of the) law obtains. But the argument in [Chapter 2.4.2](#) still shows that either changing the uniform masses whilst keeping the law the same or changing the law whilst keeping the masses the same leads to a detectable difference. In the same way that the inexpressibility of absolute masses did not imply that we could not detect their existence, the inexpressibility of the specific version of the law does not imply that we cannot detect that an instance of an L1-type law obtains. Nor does the empirical accessibility of the law and the magnitudes only as a package deal imply that we cannot determine that a combination of absolute masses and L1-type and G1-type laws obtains—the different trajectories in [Chapter 2.4.2](#) still need to be explained. In the same way that kinematic comparativism provided a naive and misguided motivation for comparativism, the inexpressibility of L1 and G1 provided a bad motivation for adopting L2 and G2. (We will return to the failures of L2 and G2 below.)

None of this is to say that the empirical accessibility of the laws and the magnitudes as a package deal only does not present a severe epistemological constraint. In fact, we will discuss in [Subsection 4.3.4](#) that it is exactly this issue that seems to undermine the route of saving comparativism via the regularity approach (although it in fact does end up temporarily saving it).

## 2.8 Conclusion

This chapter focused on Dasgupta’s comparativism, as encapsulated in L2 ([p.33](#)), and presumably G2 ([p.34](#)). His preferred way of explicating the requirement of comparativism being empirically adequate in terms of the undetectability of absolute masses leads to a specific instance of the general schema for the comparativist argument: Dasgupta’s Razor.

I have attempted to bring to light two mistakes in Dasgupta’s take on the threat to comparativism by Leibniz Mass Scaling scenarios. He misinterprets the *ceteris paribus* clause in the Leibniz Scaling so as to include forces, whereas the gravitational force of course depends on mass and thereby potentially changes when the masses change uniformly. Additionally, he takes acceleration ratios to

be the only detectable data that need to be accounted for, whereas those facts underdetermine what is really observable: (relative) trajectories. Once we correct these mistakes, inter-world Leibniz Mass Scaling scenarios, such as Baker’s simple two-particle instance, indicate that absolute masses are empirically meaningful, since varying them uniformly generates detectably different (relative) trajectories.

Nevertheless, I disagree with Dasgupta equating the empirical adequacy of comparativism, (a narrow definition of) the undetectability of absolute masses, and the empirical meaninglessness of absolute masses (i.e. anti-realism about absolute masses). The empirical meaninglessness of absolute masses comes apart from the empirical adequacy of comparativism, since comparativism is a claim only about the relative fundamentality of absolute masses, and is thus in principle consistent with their reality. Moreover, if detectability requires one to express which specific quidditistic mass has been detected, and undetectability requires one to express which mass has been failed to detect, then neither absolutism nor comparativism can be proven correct since kinematic comparativism precludes such “detecting-which”. If we adopt a broader notion of detectability, that an inter-world Leibniz Mass Scaling—that is an expressible difference in the sets of absolute masses between two possible worlds, even though those masses are not expressible themselves—leads to empirically distinct (relative) trajectories, then detectability has explicitly become a modal notion and collapses into the possibility checking approach. Thus, to the extent that the undetectability approach is distinct from the possibility checking approach it is problematic. To the extent that it is (not merely equivalent but literally) the same, let us simply develop the more perspicuous possibility checking approach in detail.

Additionally, it was pointed out that epistemic access to quidditistic masses is constrained even beyond their inexpressibility: we only have empirical access to (that which is represented by) ‘Newton’s Constant times the absolute mass scale’. Nevertheless, as before, this does not make it meaningless to consider a pair of worlds that agree on the inexpressible strength of the gravitational law but differ uniformly in the inexpressible masses (or vice versa).

## Chapter 3

# The Possibility Checking Approach

In this chapter I will develop the condition of empirical adequacy according to the possibility checking approach. A sufficient criterion for empirical adequacy is to correctly generate the set of empirically possible worlds. I claim that this approach presents the most direct and perspicuous way of expounding Laplace's problem, simultaneously incorporating explanatory and modal adequacy (cf. [Subsection 1.2.3](#)). It is explicit about a choice of ideology, and thereby specifies which things are to be kept invariant when uniformly doubling the mass, and explicit about the trajectories being that what is observable. Moreover, it side-steps the issues that plagued the undetectability approach. On top of all this, it suggests a clear research programme for the comparativist in response to the comparativist's bucket argument, which will be developed and evaluated in the subsequent chapter.

### 3.1 Why mass?

Why did we ever introduce mass in the first place? We cannot observe (a particle having the property of having) mass as directly as, for instance, the location of a massive object (relative to us). What we do observe however—*pace* Barbour [2]—is that some (relative) trajectories (i.e. relative locations over time) do occur in nature, say two celestial bodies approaching each other at ever increasing speed, and other trajectories never occur, say two celestial bodies executing the Argen-

tinian tango<sup>69,70</sup>. The (strong) absolutist posits the notion of primitive absolute mass because it explains why certain patterns are allowed by nature and other patterns are not. Mass therefore becomes indirectly observable, i.e. its effects become observable. More specifically, if we include the absolute mass magnitudes in the initial state of our models of NG, and postulate laws that refer to this notion of mass, it turns out that we can find *unique* solutions to the corresponding initial value problems and thereby fix the evolution of the system up to infinity (i.e. solving Laplace’s problem, unless a smaller set of initial data can provide the same results). Including mass in our theory thus allows us to 1) **predict** future states of the world based on past data. But not only that. It turns out that in addition we can 2) **explain** the observed particle trajectories<sup>71</sup>. Why this is the case is most easily illustrated by showing why the comparativist (or the reductionist, as discussed in [Chapter 6](#)) theory is lacking in this respect, as will be done in [Chapter 5](#). In a nutshell, in the absolutist theory the initial variables and parameters can take on all<sup>72</sup> possible values, whereas the comparativist theory exhibits brute (i.e. *unexplained!*) constraints on the initial values (against  $P_{\text{exp}}$ , [p.23](#)). Finally, if we range over all the values the initial variables and parameters could take, and solve the initial value problems for each of these cases, we obtain the supposedly correct<sup>73</sup>—but see [Subsection 4.4.2](#)—set of empirically possible worlds. In other words, including mass gives us the correct 3) counterfactuals and other **modal claims** of the Newtonian Theory of Gravity. What more could we want from a physical theory?

The three virtues above provide a rough characterisation of the ‘work’ that the notion of absolute mass seems to do within our theory despite not being directly observable (or more precisely, not as directly observable as relative distances). If

---

<sup>69</sup>One might respond that we could simply hold a massive object in each hand, and feel the potentially different weights, despite there being no spatiotemporal difference. However, the experience of the strength of that force in one’s brain is correlated with and reducible to the depth to which the massive object protrudes into our skin, that is our somatosensory nervous system. Even weighing an object in one’s hand is consistent with the claim that all observation is spatiotemporal.

<sup>70</sup>Similar sentiments are echoed by Bell (“[I]n physics the only observations we must consider are position observations...” [59, p.166]), Holland (“[A]ll experiments in the real world ultimately reduce to the determination of position. As far as we are aware this [assumption] is completely in accord with actual laboratory practice” [60, p.350]) and Jammer (“[I]n the last analysis all measurements in physics are kinematic [i.e. spatiotemporal] in nature...” [32, p.6]).

<sup>71</sup>Assuming that the masses are not themselves explained by the particle trajectories, as is the case for regularity comparativism, as will be pointed out in [Subsection 4.3.4](#).

<sup>72</sup>Except perhaps for negative mass values, but these can be made irrelevant if we take it as a fundamental feature of the gravitational law that it is attractive and thus only cares about the magnitudes of the masses. (See Jammer [32, Ch.4] for a historical overview of the search for negative (gravitational) masses.)

<sup>73</sup>In the regime of applicability of the theory.

we can show that mass ratios could do that work all by themselves, then it was never really (i.e. fundamentally) the notion of absolute mass which did all that work—Laplace’s problem can still be solved with a smaller set of initial variables and parameters—and we could and should get rid of a notion of absolute mass (grounding those mass ratios) for reasons of metaphysical parsimony. More specifically, for the comparativist project to succeed, we would need to purge absolute mass from our initial state, and have our laws refer only to those comparativist determinates and other non-mass determinates. If the corresponding initial value problems give the correct, unique solutions, we can 1) *predict* future data using past data that is purely comparativistic. Provided there are no unexplained constraints on the initial data and (part of the) initial data is not itself explained by the particle trajectories—this provision is not satisfied in the case of regularity comparativism, as pointed out in [Subsection 4.3.4](#)—we can also 2) *explain* the observed or allowed particle trajectories. Finally, if we then range over all the associated possible initial states, and again get the correct set of empirically possible worlds, the 3) counterfactuals and other *modal claims* are also accounted for without having invoked the notion of absolute masses (grounding the mass ratios).

## 3.2 Empirical Equivalency via possibility checking

It is the aim of this section to give a *sufficient* condition for the empirical adequacy of comparativism. Given that absolutism is empirically adequate (within the Newtonian regime)<sup>74</sup>, we can do so by requiring comparativism to be *empirically equivalent* to the absolutist theory (which will simultaneously ensure modal adequacy). After all, we are *in the first instance* interested in the question of whether the *standard* formalism of Newtonian Gravity is compatible with a comparativist ideology. If this turns out to fail, the comparativist could subsequently attempt to design a comparativist *alternative* to Newtonian Gravity that corresponds to a set of empirically possible worlds that may differ from that of standard NG as long as it includes a world that is consistent with the empirical data of the actual world (within the Newtonian regime)—that is, give up on modal adequacy. We will return to this revisionist move in [Subsection 4.4.2](#). For now, I will aim to incorporate all of the desiderata highlighted in the previous section.

Consider the sets of kinematically possible worlds (KPWs) of the absolutist and comparativist theories, represented in [Figure 3.1](#) as  $K_1$  and  $K_2$  respectively.

---

<sup>74</sup>In order to avoid circularity, this regime could be defined as the regime of medium-sized goods at speeds small compared to the speed of light, at accelerations above the MOND-scale, and in weak gravitational fields.

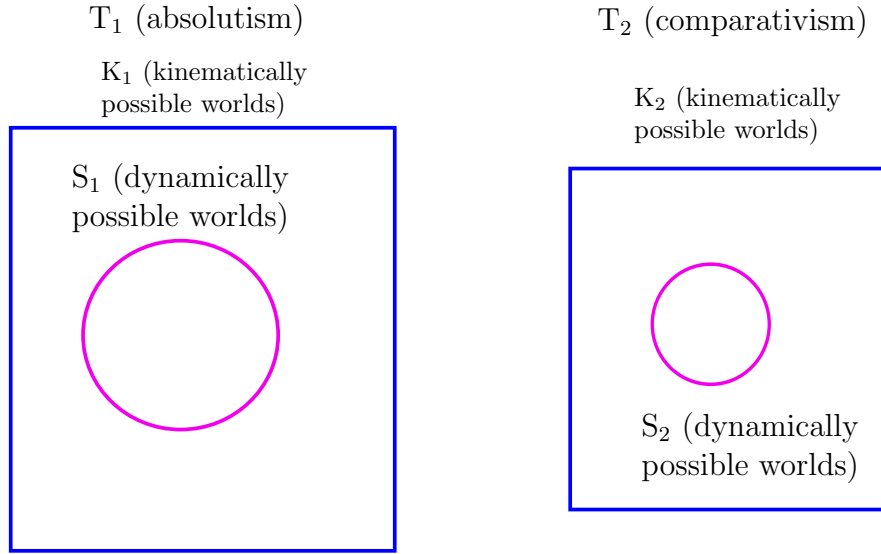


Figure 3.1: The sets of kinematically possible worlds for the absolutist and comparativist, and their subsets of dynamically possible worlds picked out by the respective laws.

The KPWs can be thought of as representing the range of metaphysical possibilities consistent with the theory’s basic ontological and ideological assumptions [55,61]. Given a particle ontology and the standard ideology (but see [Subsection 4.2.2](#)), these worlds consist of the specifications of the distances and masses—absolute masses for the absolutist and mass ratios for the comparativist—of all the particles at all times, and the non-instantaneous distances. The laws of each theory can be viewed as restricting the space of kinematically possible worlds to the subset that is physically allowed to occur, which we call the set of dynamically possible worlds (DPWs;  $S_1$  and  $S_2$ ). Empirical equivalence can then be defined as follows:

**Empirical Equivalence of Theories:** Theory  $T_2$  (comparativism) is empirically equivalent to theory  $T_1$  (absolutism) if and only if the laws of nature of theory  $T_2$  pick out a set of dynamically possible worlds  $S_2$  from the set of kinematically possible worlds  $K_2$  such that 1) for *each* of the empirically distinct worlds picked out by theory  $T_1$  as a dynamically possible world ( $S_1$ )  $S_2$  includes a world empirically equivalent to that world (i.e. *completeness*), and 2) all worlds that are empirically distinct from each of the absolutist solutions in  $S_1$  are excluded from  $S_2$  (i.e. *soundness*).

This definition of empirical equivalence of *theories* presupposes a definition of empirical equivalence (and its counterpart, empirical distinctness) of *worlds*, both of worlds within the same  $T_i$  and of worlds across  $T_1$  and  $T_2$ . I will assume

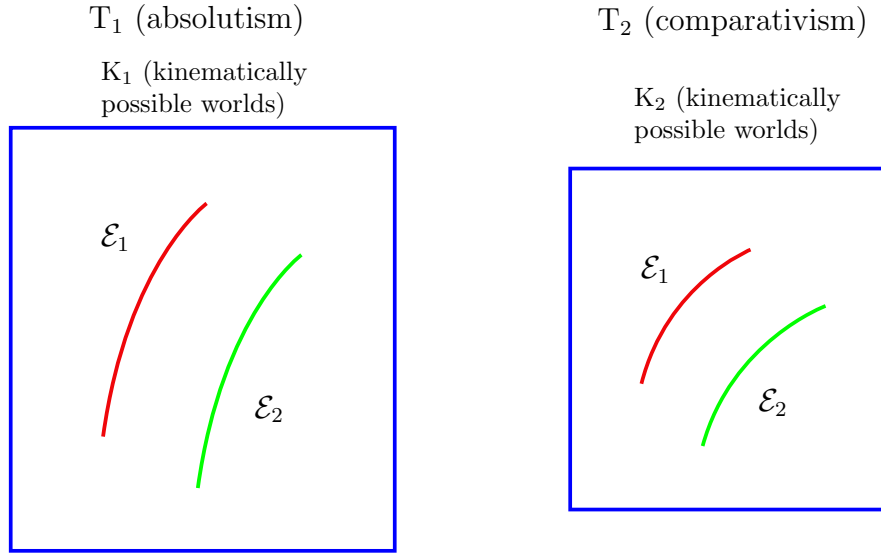


Figure 3.2: Coloured lines represent empirical equivalence classes  $\mathcal{E}_i$ .

for now that there is such a relation  $\mathcal{E}$  that groups all the worlds into empirical equivalence classes  $\mathcal{E}_i$  (see Figure 3.2), and will discuss this notion in more detail at the end of this section.

The set of dynamically possible worlds of the absolutist interpretation of NG exhibits a particularly nice feature: it makes NG a deterministic theory (modulo some well-known exotic counter-examples [62, 63]). A popular way of explicating determinism is what Earman dubs Laplacean determinism [62]:

**Laplacean Determinism (LD):** A world  $W$  is deterministic iff, for any time  $t$ , there is only one empirically possible world whose state at  $t$  is identical to  $W$ 's.

A theory is then deterministic if all its DPWs are deterministic. The state of  $W$  at  $t$  is usually called the set of initial conditions of  $W$ . Assuming that the laws of a theory have an initial value formulation, which is the case for absolutist NG, LD is equivalent to there being a unique solution to the corresponding initial value problem (cf. Laplace's problem).

In the spirit of the 'all other things being equal' clause in the argument for comparativism, it may seem reasonable to expect a comparativist interpretation of NG to be deterministic as well. Could comparativism be stochastically indeterministic—predicting only that *either* this evolution, *or* that evolution will happen, each with a specific likelihood? Or even more extreme, should we allow the possibility of inscrutably indeterministic laws—laws that are disjunctions of several possible evolutions, without providing any probability distribution over the disjuncts whatsoever? For one thing, this would lead to a slippery slope. If

we allow such laws to pick out the set of DPWs, why not a law that is simply a disjunction of all the DPWs, expressed purely in spatiotemporal terms and thereby eliminating the need for a concept of mass altogether? Secondly, indeterminism would correspond to a loss in predictive power compared to the deterministic absolutist theory.

We will require for now that comparativism has an initial value formulation and expect it to exhibit the feature of determinism—both these issues will be questioned in [Section 4.4](#). This will allow us in the remainder of this section to develop a framework for determining whether comparativism satisfies our definition of empirical equivalence (with absolutism). This will ultimately lead to the formulation of a problem for the comparativist, the Comparativist Initial Variables & Parameters Problem. If the comparativist is able to solve this problem, it follows that comparativism is empirically equivalent to absolutism, and *a fortiori* empirically adequate.

In order to determine whether comparativism satisfies our definition of empirical equivalence to absolutism, we first introduce a surjective mapping  $\phi : K_1 \rightarrow K_2$ , satisfying three conditions.

The first condition on  $\phi$  requires it to map worlds  $W_i$  in  $K_1$  to empirically equivalent worlds in  $K_2$ :

**Preservation of Empirical Equivalence Classes:**  $\forall W_i \mathcal{E}(W_i, \phi(W_i))$ .

[Figure 3.3](#) depicts a  $\phi$  that satisfies this condition. The mapping in [Figure 3.4](#) violates this condition by mapping empirically distinct worlds to empirically equivalent worlds. The mapping in [Figure 3.5](#) violates this condition by mapping empirically equivalent worlds to empirically distinct worlds.

The second condition on  $\phi$  is required to make comparativism complete:

**Completeness:**  $\forall \mathcal{E}_i (\mathcal{E}_i \cap S_1 \neq \emptyset \rightarrow (\exists W \in \mathcal{E}_i : \phi(W) \in S_2))$ .

Each empirical equivalence class of worlds in  $K_1$  that contains at least<sup>75</sup> one world that is dynamically possible according to  $T_1$  must contain one world that gets mapped under  $\phi$  to the set of dynamically possible worlds of  $T_2$  ([Figure 3.6](#)).

---

<sup>75</sup>As indicated in the figure, it is fine for an empirical equivalence class to contain both worlds that are dynamically possible and worlds that are not. For instance, given an absolutist theory  $T_1$  with a specific choice for the strength of the gravitational force, as represented by Newton’s constant, we can consider a world  $W_2$  that is related to a dynamically possible world  $W_1$  of that theory by having exactly the same spatiotemporal history but a set of absolute masses that is uniformly scaled.  $W_2$  will be empirically equivalent to  $W_1$ , but will not be dynamically possible, although it would be dynamically possible according to another absolutist theory  $T'_1$  which differs from  $T_1$  merely by a different strength of the gravitational force (cf. [Section 2.7](#)). Another example would be a uniform ‘acceleration boost’. As long as this boost is not accompanied by a change in the gravitational field, it will not be a symmetry of the theory, that is the boost will turn a dynamically possible world into a world that is not dynamically possible, but nevertheless empirically equivalent to the original world.

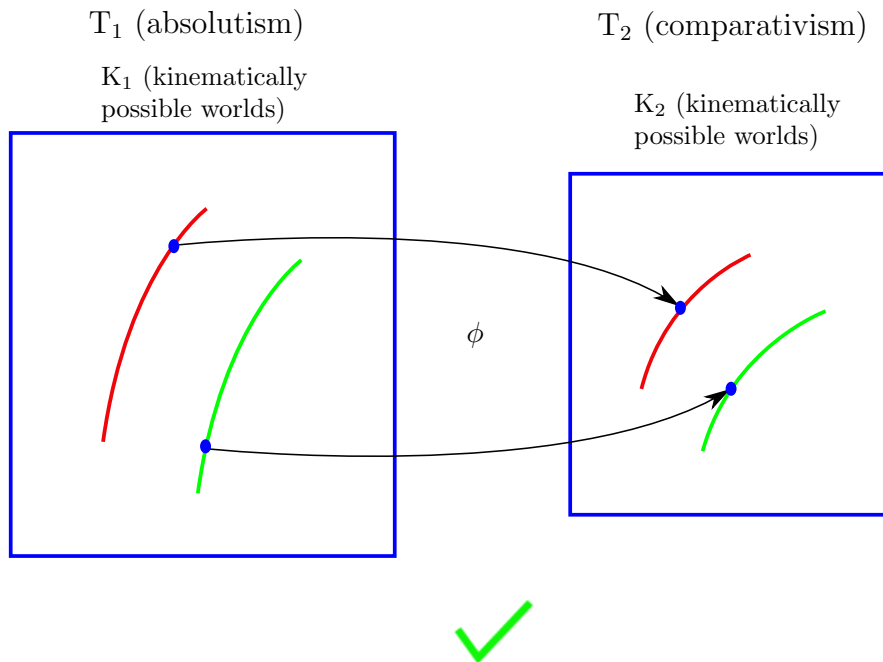


Figure 3.3:  $\phi$  maps worlds in  $K_1$  to empirically equivalent worlds in  $K_2$ .

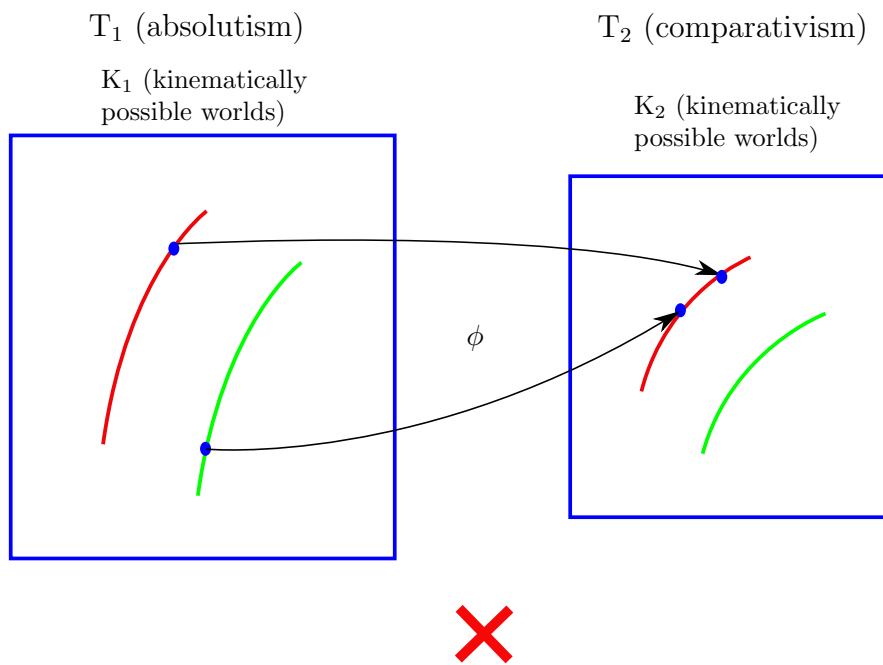


Figure 3.4:  $\phi$  may not map a pair of empirically distinct worlds in  $K_1$  to a pair of empirically equivalent worlds in  $K_2$ .

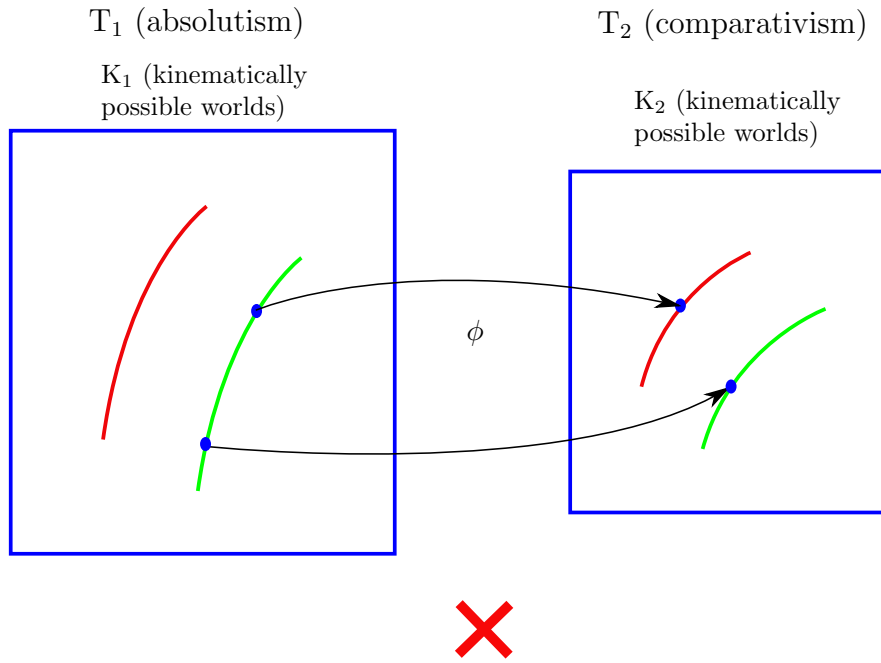


Figure 3.5:  $\phi$  may not map a pair of empirically equivalent worlds in  $K_1$  to a pair of empirically distinct worlds in  $K_2$ .

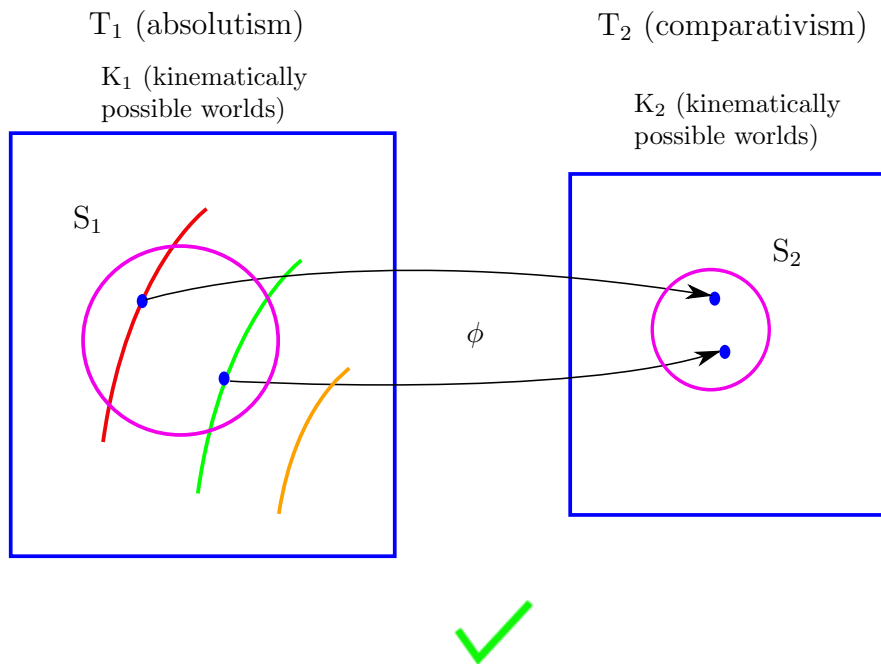


Figure 3.6: Each empirical equivalence class of worlds in  $K_1$  that contains at least one world that is dynamically possible according to  $K_1$  must contain one world that gets mapped under  $\phi$  to the set of dynamically possible worlds of  $T_2$

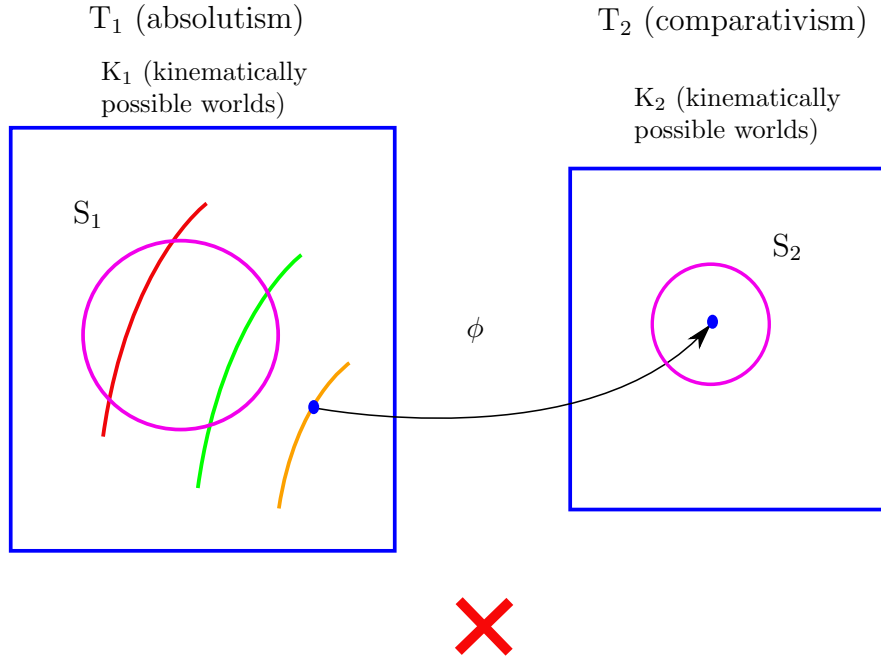


Figure 3.7: Worlds that are neither dynamically possible according to  $T_1$  nor empirically equivalent to a dynamically possible world of  $T_1$  must not be mapped under  $\phi$  to the set of dynamically possible worlds picked out by the laws of  $T_2$ .

The final condition ensures that comparativism is sound:

**Soundness:**  $\forall \mathcal{E}_i (\mathcal{E}_i \cap S_1 = \emptyset \rightarrow (\neg \exists W \in \mathcal{E}_i : \phi(W) \in S_2))$ .

Worlds that are neither dynamically possible according to  $T_1$  nor empirically equivalent to a dynamically possible world of  $T_1$  must not be mapped under  $\phi$  to the set of dynamically possible worlds picked out by the laws of  $T_2$  (Figure 3.7).

We need a positive recipe for finding a  $\phi$  that satisfies these conditions. The most obvious choice, albeit not necessarily the only option, is as follows. Worlds in  $K_1$  can be labelled by the (inexpressible) determinates of a chosen minimal<sup>76</sup> set of initial variables and parameters  $\alpha = \{\alpha_i, \dots, \alpha_n\}$ —where  $\alpha_1 = m$  (= absolute mass) for the absolutist—since the world is completely determined by those initial determinates and their evolution according to the absolutist laws:  $W_{\{m_i=c_{1,i}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\}, abs}$ , where the  $c$ 's are the initial determinates and  $i$  ranges over all the particles<sup>77</sup>.  $\phi$  can then be chosen to map this world to the comparativist world that is obtained from the same determinates for the same initial

<sup>76</sup>That is, a set of independent variables and parameters.

<sup>77</sup>This will not work outside of the sets of DPWs, but this is not necessarily problematic [55, p.153].

conditions, except for the masses being replaced with mass relations, evolved by the comparativist laws:

**General Recipe for  $\phi$ :** <sup>78</sup>

$$W_{\{m_i=c_{1,i}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},abs} \mapsto W_{\{\frac{m_i}{m_j}=\frac{c_{1,i}}{c_{1,j}}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},comp}$$

Thus, for the standard choice of initial variables and parameters as inter-particle distances, relative velocities and masses, we would obtain:

$$W_{\{m_i=c_{1,i}, r_{ij}=c_{2,ij}, v_{ij}=c_{3,ij}\},abs} \mapsto W_{\{\frac{m_i}{m_j}=\frac{c_{1,i}}{c_{1,j}}, r_{ij}=c_{2,ij}, v_{ij}=c_{3,ij}\},comp}$$

Note that for the standard choice of initial variables and parameters this mapping will be many-to-one: for every distribution of mass ratios there is a continuum infinity of choices of distributions of absolute masses<sup>79</sup>. For this reason  $K_1$  is symbolically depicted as being larger than  $K_2$  in all the figures. According to the *Preservation of Empirical Equivalence Classes Condition*, each of these many absolutist worlds—all related by a Leibniz Mass Scaling, where the *ceteris paribus* clause is explicated via the choice of  $\alpha$ —is empirically equivalent to the single comparativist world. After all, according to the comparativist the addition of independent absolute masses to a comparativist ideology  $\alpha$  is (empirically) redundant.

Finally, we can put all of this together and turn the criterion of empirical equivalence with  $T_1$  (absolutism) into a task that needs to be solved by the comparativist. First let us define this task—which we might call Laplace’s challenge—for a general theory  $T_2$ :

**General Initial Variables & Parameters Problem (GIVPP):** <sup>80</sup>

Finding a set of independent initial variables and parameters  $\alpha$  such that when specifying the initial conditions of a theory  $T_2$  in terms of the values (i.e. determinates) of that set of variables and parameters, the corresponding Initial *Value* Problems produce unique solutions, and there exists a surjective mapping  $\phi$  from the space of kinematically possible worlds  $K_1$  of  $T_1$  to the

---

<sup>78</sup>Here, and below, I have for simplicity’s sake omitted the possibility of the comparativist’s world’s label also including absolute masses (that do not ground the mass ratios). I will return to this issue, briefly, in [Subsection 4.3.4](#).

<sup>79</sup>This may seem inconsistent with the logical possibility, discussed in [Chapter 4.3](#), that the mass ratios partially ground the absolute masses. However, in that scenario the absolute masses are not grounded in the mass ratios by themselves but in combination with the full history of particle trajectories. It is unclear how the determinate ‘values’ of  $\alpha$  at the initial time could ever fix the absolute masses by themselves.

<sup>80</sup>I am using ‘variables and parameters’ rather than ‘determinables’ to distinguish between different types of determinates that fall under one determinable, such as mass ratios and absolute masses.

space  $K_2$  of  $T_2$  which preserves empirical equivalence classes, is sound, and complete.

The Comparativist Initial Variables & Parameters Problem then adds as extra constraints that  $\alpha$  needs to be a comparativist ideology, and that  $\phi$  maps absolutist worlds to comparativist worlds labelled by the same determinate ‘values’ (other than the absolute mass determinates):

**Comparativist Initial Variables & Parameters Problem (CIVPP):**

Finding a set of independent initial variables and parameters  $\alpha = \{\alpha_1 = \frac{m_i}{m_j}, \alpha_2, \dots, \alpha_n\}$  ( $\alpha_2, \dots, \alpha_n$  may not be the absolute masses—unless they do not ground the mass ratios—nor be reducible to them over and beyond their mass ratios) such that when specifying the initial conditions of a theory in terms of the values (i.e. determinates) of that set of variables and parameters ( $\frac{m_i}{m_j} = \frac{c_{1,i}}{c_{1,j}}, \dots, \alpha_{n,i} = c_{n,i}$ ), the corresponding Absolutist and Comparativist Initial *Value* Problems produce unique solutions  $W_{\{m_i=c_{1,i}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},abs}$  and  $W_{\{\frac{m_i}{m_j} = \frac{c_{1,i}}{c_{1,j}}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},comp}$ , such that the surjective mapping  $\phi$  from  $W_{\{m_i=c_{1,i}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},abs}$  to  $W_{\{\frac{m_i}{m_j} = \frac{c_{1,i}}{c_{1,j}}, \alpha_{2,i}=c_{2,i}, \dots, \alpha_{n,i}=c_{n,i}\},comp}$  preserves empirical equivalence classes, is sound, and complete.

In other words, the comparativist needs to successfully complete the following iterative four-step programme:

- Step 1** Choose a set of absolutist independent initial variables and parameters  $\alpha_{abs} = \{m_i, \alpha_{2,i}, \dots, \alpha_{n,i}\}$ , thereby fixing the corresponding set for the comparativist  $\alpha_{comp} = \{\frac{m_i}{m_j}, \alpha_{2,i}, \dots, \alpha_{n,i}\}$ .
- Step 2** Choose determinate ‘values’ for each variable and parameter, and find a unique solution to both of the corresponding Initial *Value* Problems.
- Step 3** Check whether the absolutist and comparativist solution are empirically equivalent.
- Step 4** Repeat step 2 & 3 for all possible determinate values and check the whole set of solutions for soundness and completeness.

The choice of  $\alpha$  corresponds to fixing the ideology of the theory. It is a metaphysical choice that removes the underdetermination between the many mathematical ways in which the *ceteris paribus* clause in the Leibniz Mass Scaling, that is the things that are to be kept fixed when we vary the masses, could be interpreted.

Step 2 is where the dynamics becomes relevant. The ability to carry out step 2 presupposes an initial value formulation. If step 2 succeeds for all allowed choices of determinate ‘values’, the theory exhibits Laplacean Determinism. If so, this step contributes to the predictive power of the theory: we have solved Laplace’s Problem<sup>81</sup>. If there are no restrictions on the range of determinate values, or at least no unexplained constraints—as there are not in absolutism, but there are for comparativism (Subsection 5.2.2) and reductionism (Chapter 6)—this will furthermore ensure explanatory adequacy. Step 3 takes into account our first restriction on  $\phi$ —shortly we will see that our notion of empirical equivalence of worlds focuses on trajectories rather than mere acceleration ratios, as promised. Step 4 takes into account the remaining two restrictions on  $\phi$ , thereby checking for modal adequacy. Everything that was problematic in the previous chapter has now been resolved.

Before we confront the comparativist with the CIVPP, I need to make good on one promise: discussing the notion of empirical equivalence between two worlds,  $\mathcal{E}$ , which was presupposed in the definition of empirical equivalence of theories above. One might now expect a story including rods and clocks. However, the r-word and the c-word are far from unproblematic, as they themselves are complex physical systems, and in particular physical systems with mass. To treat them as primitive systems and use them to compare kinematical notions such as length and time across worlds would be to commit ‘Einstein’s sin’ [51]. Fortunately, for our purposes we merely need a *necessary* condition for the empirical *equivalence* of worlds. Worlds, both within a  $T_i$  and across  $T_i$ ’s, need to agree on whether particles coincide or not<sup>82,83</sup> (i.e. whether their trajectories intersect)—cf. Subsection 2.4.2 and the following section. This corresponds to a *sufficient* condition for the empirical *distinctness* of worlds, and that is all we need to argue in the next section that worlds with identical comparativist initial conditions evolve in empirically inequivalent ways. This topological criterion<sup>84</sup> is an empirical feature that both the absolutist and the comparativist can agree on without having to invoke rods and clocks.

---

<sup>81</sup>Assuming we could not still have achieved the same thing after purging  $\alpha$  even further.

<sup>82</sup>Any escape solution could always be turned into a collision solution by evolving the ‘initial conditions’ in the opposite temporal direction. In this thesis we will only consider worlds from the initial time ‘onwards’, not from infinitely far into the ‘past’.

<sup>83</sup>When two particles collide, this results in a singularity and the theory technically breaks down. At that point we would have to resort to a different branch of mechanics, which McKinsey, Sugar and Suppes refer to as ‘impact mechanics’ [64].

<sup>84</sup>See also Schlick’s ‘method of coincidences’ [65–68].

### 3.3 A challenge for the comparativist: the comparativist’s bucket again

In this section we will briefly review Baker’s two-particle instance of the inter-world Leibniz Mass Scaling scenarios in [Subsection 2.4.2](#), and use it to argue that the comparativist equivalent of the standard choice of ideology ( $\alpha_s$ ), which I take to be inter-particle distances, relative velocities and mass relations, fails to solve the Comparativist Initial Variables & Parameters Problem (at least if the dynamics are Dasgupta’s L2 and G2). *Prima facie*, this seems to suggest that a comparativist version of Newtonian Gravity is empirically inadequate, rendering it untenable.

Consider a simple world, governed by the laws of Newtonian Gravity, with two equally massive particles a distance  $r$  apart, with a relative positive initial velocity  $v$  (such that there is zero angular momentum). How will this world evolve?

Whereas this description corresponds to a unique choice of initial variables and parameters for the comparativist, the absolutist will demand that more information is needed: this description is compatible with continuum infinitely many absolute masses (i.e. absolutist worlds differing by a Leibniz Mass Scaling). And, she claims, this choice is important, because for some choices of absolute masses the particles will escape each other and for other choices they will collide, depending on whether the following inequality—derivable from L1 and G1—is satisfied:

$$v > v_e = \sqrt{\frac{2Gm}{r}}. \quad (3.1)$$

It is clear from this inequality that the evolution depends, deterministically, on the initial absolute masses of the particles, over and above their mass ratios. (Remember our earlier discussion of empirical equivalence of two worlds: no rods and clocks are required to distinguish collision and escape; these two results are easily distinguished by both the absolutist and comparativist.) Once the initial masses are fixed, the corresponding absolutist initial *value* problem has a unique solution.

How does this pan out in the comparativist framework? G2 and L2 are not even in the form of a differential equation. They are not suitable for solving initial value problems. Hence, it is not even clear that they deserve the status of lawhood. (Further issues with this interpretation of the ‘laws’ will be offered in [Subsection 4.4.1](#).) Different comparativist laws are needed. However, regardless of the specific form of the comparativist laws, they will either fail to evolve this single set of comparativist initial determinates into a unique solution but allow instead for both escape and collision—in which case determinism is violated (Step 2)—or they will provide a unique solution to the initial *value* problem, say collision,

and thereby fail to generate the complete set of solutions (Step 4)<sup>85</sup> (or so it seems, but see [Subsection 4.2.3](#)). Either way, the Comparativist Initial Variables & Parameters Problem is not solved (by this choice of ideology). This seems to threaten the empirical adequacy of comparativism.

To compare all this with the undetectability approach, notice that it made sense to distinguish worlds agreeing on the network of mass ratios and the strength of the gravitational law but disagreeing on the absolute masses, even though I have not in any way expressed to you the absolute masses—nor the strength of the gravitational law—within each individual world, other than via their dynamical effects. Besides, the acceleration ratios—Dasgupta’s explanandum—are the same in all the above worlds, and they thus underdetermine all that is observable: the (relative) trajectories.

To sum up: a uniform variation of the allegedly redundant absolute masses—that is, they are not supposed to be an independent variable over and beyond the mass ratios and other non-mass variables—is, *ceteris paribus*, not supposed to lead to any empirical difference, but in this case it results in a dramatic empirical difference: particles escape each other instead of colliding. From the comparativist point of view, uniformly varying the redundant initial masses is supposed to amount to nothing more than the identity operation, but instead the evolution changes in an observable manner.

---

<sup>85</sup>In this case step 3 is also violated for some absolutist worlds, namely the worlds where the particles escape, as they get mapped by  $\phi$  to a comparativist world in which the particles collide.

# Chapter 4

## Comparativist Responses

### 4.1 A taxonomy of responses

In the previous chapter we considered possible worlds of which the initial conditions differed by a Leibniz scaling, which turned out to result in empirically distinct evolutions. If comparativism is true, this scaling should have been equivalent to the identity operation, thus suggesting that comparativism is empirically indeterministic and hence empirically inadequate. I have formulated this problem in the form of a challenge: can the comparativist solve the Comparativist Initial Variables & Parameters Problem? This predicament is clearly analogous to the Rotational Dynamic Leibniz Shift argument (aka Newton's bucket and globes<sup>86</sup>) against relationalism about space. Unsurprisingly then, the relationalist responses inspire many of the comparativist responses discussed below, albeit not all of them. Hence, in categorising the comparativist responses to their challenge it will be tempting to follow Huggett [69] and Pooley's [55, 70] three categories of relationalist responses (introduced below). In the space case these happen to coincide with three broader categories, but some of the responses in the mass case only fit into the broader categories, which is why we will ultimately be using those broader categories here. I will start with the most ambitious category, which accepts the challenge head-on, continue with the more compromising second category which saves the letter of comparativism (i.e. mass ratios are not grounded in absolute masses) but gives up its spirit (i.e. anti-realism about objective, non-conventional absolute masses) and end with the most defeatist and revisionary category, which criticises several assumptions made in the previous chapter in order to avoid having to face the challenge in the first place.

As we will soon see, Huggett and Pooley's categorisation depends on whether the comparativist response modifies the original theory of NG, or does not. They

---

<sup>86</sup>See [fn. 59](#).

interpret this distinction not according to the syntactic view of theories—which equates a theory to an axiomatized set of sentences of some formal or metamathematical language [71]—but according to the semantic view of theories, which equates a theory with a set of empirically allowed models [72, 73]. These models—in terms of quantities—then represent the empirically possible worlds—consisting of magnitudes. In light of [Subsection 1.1.3](#) on the confusion between quantities and magnitudes, I propose to cut out the middleman, and equate a theory directly with its set of empirically possible worlds. Having said that, in most cases this distinction between the standard and modified semantic views will be an unimportant<sup>87</sup> technicality, and I will refer simply to the semantic view, even though I will mostly consider worlds rather than models. Finally, it is important to point out that I do not intend to be taking a strong stance within the semantic–syntactic debate; adopting the (standard or modified) semantic view of theories is merely a convenient way of categorising the comparativist responses, which is both in line with the literature (Huggett & Pooley) and (in the case of the modified semantic view) in the spirit of the possibility checking approach.

Huggett and Pooley’s first category, then, does not change the theory<sup>88</sup>—semantically speaking, that is it adheres to the requirement of soundness and completeness with respect to the set of empirically allowed possible worlds of standard NG—but does adopt an *alternative* ideology.

In the case of relationalism, this means *enriching* the ideology in a relationally acceptable way as to remove the indeterminism. Sklar suggests adding primitive monadic (i.e. instantaneous, albeit time-varying) accelerations [74], whereas other versions of Enriched Relationalism add transtemporal relations: Poincaré considers accelerations albeit not necessarily Sklar’s instantaneous accelerations [1]; Maudlin considers adding all the spatiotemporal metrical relations between particles at different times [48]; Maudlin and Binkoski a collinearity relation [48, 75]; Pooley a three-place relation that measures the departure from collinearity [55]; Saunders a notion of parallelism of spatial directions at different times [76].

In the case of comparativism, enriching the ideology with transtemporal relations is useless. The evolution of mass ratios (and absolute masses for that matter) is trivial: they are constant. Rather than enriching the ideology, the comparativist may instead seek to replace the standard ideology with a different (transtemporal or instantaneous) one. Various options for doing so are discussed in [Chapter 4.2](#), besides an option which faces the challenge head-on not by modifying the ideology but by modifying the laws.

---

<sup>87</sup>One potential novelty of the modified semantic view, however, is the option of including (dispositionalist or primitivist) laws as part of the possible worlds, whereas it may not make sense to attribute or attach them to models (in the same sense). Cf. [Subsection 4.2.3](#).

<sup>88</sup>Cf. [fn. 97](#).

Huggett and Pooley’s second category—dubbed ‘have-it-all relationalism’<sup>89</sup>—attempts to solve (the space analogue of) the CIVPP without any change in ideology. In a sense this approach thus seems more ambitious than the first category, but the only view that falls in this category in fact compromises by admitting that the spirit of absolutism/substantivalism is correct, just not the letter—in that sense they do thus not ‘have it all’. Remember from [Section 2.6](#) that it is important not to confuse the debate between absolutism and comparativism, which is a debate about metaphysical priority (that is, grounding or relative fundamentality), with a debate about the reality (i.e. empirical meaningfulness) of absolute masses. The reality of absolute masses (inertial frames) is in principle compatible with their non-fundamentality, even though it seems that anti-realism about absolute masses (inertial frames) is very much in the spirit of comparativism (relationalism). Huggett uses the Mill-Ramsey-Lewis Best Systems account of Newton’s laws to ground the inertial frames in a fully relational Humean mosaic [77]—these terms will be explained below. The comparativist who accepts the empirical meaningfulness of absolute masses may nevertheless use Huggett’s approach to attempt to get those masses ‘for free’ by grounding them in mass ratios (and other non-mass facts). [Chapter 4.3](#) discusses whether the comparativist can indeed successfully make use of this wiggle room.

After the ‘same theory, different ideology’ and ‘same theory, same ideology’ categories, Huggett and Pooley’s third category sticks to the ideology but changes the theory (semantically speaking). Defining the criterion of empirical adequacy for comparativism by complete empirical equivalence with the absolutist theory, as explicated via the CIVPP, may have been too ambitious. Although [Section 3.2](#) did provide a sufficient criterion for empirical adequacy, this is not a necessary criterion. In the space case, Mach and Barbour give up on the requirement of completeness. As long as the actual world is included in  $S_2$ , the set of DPWs, this should be sufficient. In [Chapter 4.4](#) we will discuss the analogous response for the mass case, but also generalise this approach by considering several other bits and pieces of the previous chapter that could be given up in order to have to avoid facing the (complete) challenge.

## 4.2 Facing the challenge head-on

The most obvious response to the argument posed in the previous chapter is that it only shows, if anything, that the standard choice for the ideology  $\alpha$ —distance, velocity and mass ratios—does not solve the CIVPP, not that it is *unsolvable!* In other words, the *ceteris paribus* clause in the Leibniz Mass Scaling transformation

---

<sup>89</sup>This category is the only category that completely coincides with my broader categories: in both the space and mass case this second category only contains a single response.

(p.25) is ambiguous. It tells you to change the masses and leave the remainder of the ideology unchanged, but this obviously depends on the choice of ideology. A simple example: if you change the masses and keep the velocities the same this will change the momenta, but if you instead keep the momenta constant then this will change the velocities. When keeping the standard ideology the same, a uniform change in masses leads to an empirically distinct evolution. If we choose to keep other things the same, this might change the standard ideology in exactly the right way to compensate for the change in masses by producing no empirically distinct evolution after all. Recall that Dasgupta interpreted the *ceteris paribus* in an inadmissible way (Section 2.4); perhaps we can do better.

### 4.2.1 Baker's response

We begin by discussing Baker's own response to the two-particle scenario, since, as I will argue, it ultimately boils down to an alternative (but unsuccessful) choice of the ideology  $\alpha$  [9]. Using the fact that  $a = \frac{GM}{r^2}$ , Baker substitutes the corresponding equation for mass in the escape velocity inequality (Equation 2.2) to obtain  $v > v_{e,1} = \sqrt{2ar}$ . He then concludes that, since no mass appears in this equation, uniform mass doubling does not change the trajectories of the particles. This incorrect conclusion would imply that, if the two particles escape/ collide in one world they must do so in all dynamically possible worlds (where the particles are the same initial distance apart and have the same initial velocities)—a violation of *completeness* looms. The mistake made is that  $a$  is interpreted as an independent variable. Finding a new symbol and substituting this into an equation does of course not mean that we should forget what this symbol meant, and in particular that it depends on  $m$ . Baker's formulation of the escape velocity only serves to *disguise* the crucial dependence on mass, not to prove it is not there:  $v > \sqrt{2ra(m)}$ .

One might be tempted to respond by promoting acceleration to a fundamental concept, but in order to do so one has to demote another variable (or at least remove it from the set of initial variables  $\alpha$ ), on pain of overdetermining the dynamics, which may lead to inconsistencies. Here we might anticipate that it is possibly the concept of mass that gets thrown overboard altogether, which may not be in the best interest of the comparativist. We will return to this below.

Although it is unclear why one would do so at this point in the dialectic, Baker then turns to an analysis of the higher-order spatiotemporal notions, velocity and acceleration. Do instantaneous velocities and accelerations (at the initial time) make sense? An object's velocity at time  $t$  is ordinarily defined as a derivative, that is as a limit:

$$v(t) = \frac{dr}{dt}(t) = \lim_{\Delta t \rightarrow 0} \frac{r(t + \Delta t) - r(t)}{\Delta t} = \frac{r(t + dt) - r(t)}{dt}. \quad (4.1)$$

On this view, sometimes called the “at-at” theory<sup>90</sup>, only the mass and distances remain as independent variables; the velocity is defined reductively via the change in the distance across an infinitesimal period of time. Acceleration can be reduced analogously. We can now again rewrite the escape velocity inequality:

$$\left(\frac{dr}{dt}\right)^2 > 2r\frac{d^2r}{dt^2}. \quad (4.2)$$

Baker concludes from this equation that not only a uniform scaling of mass does not change whether the inequality is satisfied, but neither does a uniform scaling of length: multiplying  $r$  by a constant  $c$  multiplies each side of the equation by  $c^2$ .

But this means that something must have gone horribly wrong. Besides length (and time), there are no variables left in the equation which might influence whether the inequality is satisfied or not. Thus, if the particles escape (collide) in one possible world and hence satisfy (violate) the escape inequality, then the inequality will hold in all<sup>91</sup> (none of the) dynamically possible worlds: in no dynamically possible worlds do the two particles collide (escape). To the extent that we believe that such collision (escape) is empirically possible, we have a violation of the completeness condition.

What is going on here? Before answering this, let us briefly get out of the way one other feature of Baker’s framework that is also novel, but not necessarily problematic. Under Laplacean Determinism, the initial state was explicated as one instant of time  $t$ . This stops being tenable once velocity and/or acceleration are chosen as initial variables, and they are defined reductively. On this view, at the initial  $t$ , the fate of the world is still up for grabs; only after one or two infinitesimally small periods  $dt$  does the whole future become fixed—metaphorically, God was not done once he fixed the laws and the initial instant, he had to labor one or two infinitesimal periods longer. In effect, Baker has replaced Laplacean Determinism with, say, the following:

**Bakerian Determinism (BD):** A world  $W$  is deterministic iff, for any initial infinitesimal period of time  $[t, t + 2dt]$ , there is only one empirically possible world whose state at  $[t, t + 2dt]$  is identical to  $W$ ’s.

---

<sup>90</sup>This idea originates with Aristotle and was developed in the Middle Ages [78–80]. In more modern times it is associated, amongst others, with Russell [18].

<sup>91</sup>Strictly speaking this is only true for possible worlds with only two particles, call these the scarce worlds. If there are more particles, for simplicity say several (approximately) isolated two-particle systems, these systems might stand in the correct mass relation to the alpha-two-particle system that is the counterpart of the system in the scarce worlds to allow for a different evolution. The question then remains however how to determine which of the two-particle systems is the alpha-system. We will return to related issues in [Subsection 4.2.3](#).

I will not dwell on whether this modification is justified; I merely intended to point out that Baker’s view is implicitly committed to defining determinism in this alternative way.

Let us return to the violation of the completeness condition. It seems that Baker in effect has chosen  $\alpha = \{r_{ij}\}$  in his attempt to solve the CIVPP. It is then not surprising that all possible worlds have the same evolution: in order to obtain more than one evolution, we need at least two dimensionful magnitudes that can be compared to each other. What has gone wrong in the analysis of Equation 4.2 becomes clear when we return to the reductive definition of velocity (Equation 4.1) and acceleration:

$$a(t) = \frac{r(t + 2dt) - 2r(t + dt) + r(t)}{dt^2}. \quad (4.3)$$

We see that the notion of ‘uniformly scaling  $r$ ’ is ill-defined. Which  $r$ ?  $r(t)$ , or  $r(t + dt)$  or  $r(t + 2dt)$ ? Indeed, if all three are simultaneously scaled uniformly by the same factor, Baker’s analysis goes through, but this means that a change in position is coupled to a change in velocity and in acceleration, and these are supposed to be independent. It is unclear why a change in one would have to be accompanied by a change in the other. It seems that Baker’s analysis is better interpreted as an attempt to solve the GIVPP with  $\alpha = \{r_{ij}, v_{ij}, a_{ij}\} \equiv \{r(t)_{ij}, r(t + dt)_{ij}, r(t + 2dt)_{ij}\}$ .

But then we see, as anticipated, that this approach is a reductionist approach: it attempts to reduce mass to spatiotemporal notions. Whether this Reductionist Initial Variables & Parameters Problem can be solved will be the topic of Chapter 6. But even if it is solvable, this reductionist approach throws out the massive baby with the bathwater—it throws away fundamental absolute masses not in exchange for fundamental mass ratios but fundamental non-mass magnitudes. This reductionism, if it works, rules out both strong absolutism and comparativism, but is compatible with either weak absolutism or comparativism. It goes no way towards defending weak comparativism over weak absolutism though: the argument in the previous chapter suggests that it should be supplemented with weak *absolutism*. The mass ratios are grounded in absolute masses which in turn are grounded in spatiotemporal (i.e. non-mass) magnitudes.

## 4.2.2 Alternative Ideologies

**An example:**  $\varphi \equiv \frac{v}{\sqrt{m}}$

Let us remind ourselves of the task at hand. For comparativism about mass to be empirically adequate, changing all the masses uniformly whilst keeping everything else the same should not make any empirical difference. What does it mean for

all other things to be the same? We have seen that if one chooses as other initial variables the inter-particle distances and their velocities, empirical differences do arise. What other set of initial variables and parameters (i.e. the ideology) could we choose in order to solve the Initial Variables & Parameters Problem? Consider a collision solution. Doubling all the masses leads to an increased gravitational pull. If nothing empirical is to change, the comparativist needs a kind of ‘mechanism’ to ‘simultaneously’ increase the velocities in the right way to compensate for the enhanced gravitational pull. Incorporating momenta in the (fundamental) ideology—as will be embraced by proponents of the primacy of the Hamiltonian formalism and its metaphysics—might seem to do the trick: doubling the masses whilst keeping the momenta the same will change the velocities. However, not only is this compensation too large—the escape velocity varies with the square root of the mass rather than being proportional to it—it also compensates in the wrong direction: the initial velocity decreases whereas the gravitational pull it is supposed to overcome increases. Nevertheless, we could construct a generalised notion of momentum, in this case  $\varphi = \frac{v}{\sqrt{m}}$ , which does serve the intended purpose. The escape ‘velocity’ inequality becomes:

$$\varphi > \sqrt{\frac{2G}{r}} \tag{4.4}$$

Satisfaction of this equality will not change when the initial masses are changed uniformly, *ceteris paribus*. If we compare two different worlds with the same initial conditions, now including  $\varphi$ , except for a universal doubling of mass, pairs of particles that collide or escape in one world will also do so in the other world. This choice of ideology provides empirical adequacy.

Before arguing against this response, we need to reject some confusing terminology that might be incorrectly applied to it. Baker introduces the terms mixed relations and mixed comparativism [25]. The relations of standard comparativism compared the determinate mass of one particle to the determinate value of the same determinable, mass, of another particle. A mixed relation instead compares magnitudes of two different determinables. Mixed comparativism then focuses on mixed relations, and claims that these mixed relations are not grounded in the associated determinates (of the distinct determinables). Since  $\varphi$  is defined as the ratio of the determinate of one determinable, velocity, to that of (the square root of) a determinate of a different determinable, mass, one might be tempted to classify this approach as a version of mixed comparativism.

This should be resisted for two reasons. Firstly, this is in fact a form of absolutism. The forms and examples of pure (i.e. non-mixed) comparativism that we have been using so far happened to refer to relations that were two-place both at the first-order and second-order level. That is, these mass properties had as their two relata two distinct objects, and these properties themselves were

explicitly formed from two other properties (for instance from  $m_1$  and  $m_2$ )—this latter fact is particularly the case since we have been using examples of relations that were quantitative rather than congruence-based. Similarly, we have only encountered forms of absolutism where the absolute mass was a 1-place property at both the first-order and second-order level: the absolute masses were properties that had one relatum each (i.e. they were monadic), and they were not formed or defined, at least not explicitly, out of two or more properties. Since  $\varphi$  is above defined explicitly as a second-order two-place property—it is formed from the properties  $m$  and  $v$ —it is tempting to consider the approach above to be a form of comparativism. However, it was implicit that it is a one-place property at the first-order level, that is a monadic property:  $\varphi'_i = \frac{v_i}{\sqrt{m_i}}$ <sup>92</sup> contains a single index and thus refers to a single particle. Does this mean we have some hybrid version of absolutism and comparativism? No. The essence of comparativism (absolutism) is that it is a two-place (one-place) property at the first-order level! It is merely a coincidence that the forms of comparativism (absolutism) that we have encountered so far were also two-place (one-place) at the second-order level. The approach in this section is thus a form of absolutism about  $\varphi$ !

There is one obvious way though of modifying this into a form of comparativism: using  $\varphi'_{ij} = \frac{v_i}{\sqrt{m_j}}$  would also leave the corresponding escape velocity inequality invariant under the associated Leibniz Scaling<sup>93</sup>, but is a two-place relation at the first-order level and thus corresponds to a form of comparativism. However, at this point any interesting difference between absolutism and comparativism has evaporated: both forms allow for the same possible worlds, both are equally metaphysically parsimonious, and they are the same in all other respects except for the comparativist version being non-separable.

The second problem with this categorisation as mixed comparativism about  $\varphi$  takes issue not with the label ‘comparativism’ but with the label ‘mixed’. The distinction between mixed and non-mixed relations is ill-defined. For example, instead of expressing  $\varphi$  as a ratio of functions of  $m$  and  $v$ , we might equally express  $m$  as a mixed relation between  $v$  and  $\varphi$  (and/or  $v$  as a mixed relation between  $m$  and  $\varphi$ ). Not only is this mathematically always possible (for non-zero velocities and momenta), if  $\varphi$  is not grounded in  $m$  and  $v$  these alternative mixed definitions even reflect the underlying metaphysics more perspicuously. (In fact,  $\varphi_{ij}$ , say, could not be represented as a ratio at all if it were not by the grace of a condition similar to the one in [Section 1.1](#).) Under this simple algebraic manipulation the original, pure forms of absolutism and comparativism about mass suddenly become mixed forms. Perhaps Baker implicitly holds that some

---

<sup>92</sup>I am assuming absolute velocities here, for simplicity’s sake.

<sup>93</sup>Using instead a form of comparativism that takes ratios of the type  $\frac{\varphi_i}{\varphi_j}$  to not be grounded in  $\varphi_k$ ’s would not do so.

determinables should be privileged, say those of the standard ideology. Only those count as pure. I will develop this intuition further below.

In summary, this approach should be classified as absolutism about  $\varphi$ . Whether it is mixed or pure is moot. But this thesis concerns absolutism and comparativism about mass. Where does absolutism about  $\varphi$  leave us in that regard?

Before turning to this important question, let us briefly evaluate the viability of absolutism about  $\varphi$  in more detail. Taking  $\varphi$  to be an independent member of the ideology probably seems strange, unnatural and counter-intuitive to many. As Baker reminds us, such pre-theoretic intuitions are not an argument against magnitudes like  $\varphi$  per se [25]. Let us then try to unpack what is so strange about magnitudes like  $\varphi$ . The magnitudes of the standard ideology, that is distance, velocity and mass, are each either purely spatiotemporal or purely dynamical.  $\varphi$  mixes this up. Perhaps it was even this distinction Baker had in mind when distinguishing between mixed and pure relations. One reason such mixing of spatiotemporal ( $v$ ) and dynamical ( $\sqrt{m}$ ) notions might worry us is that it blurs the boundary between explanandum and explanans. Recall from Section 3.1 that dynamical, (directly) unobservable notions such as mass are introduced to dynamically explain the subset of kinematically possible worlds that is physically allowed, that is the possible ways in which the spatiotemporal—that is observable—notions could evolve. Explanans and explanandum should be clearly distinguishable. That  $\varphi$  is defined using the observable notion of velocity brings us to two further related worries. Firstly, when  $\varphi$  is part of our ideology, measuring  $v$  (or more correctly,  $dx/dt$ ) would thereby fix  $m$  as well. This suggests that absolutism about  $\varphi$  may go hand in hand with absolutism about mass. The second worry is similar to an argument against opponents of “at-at” theorists who take  $v$  to be an instantaneous velocity that is not reducible to  $dx/dt$ , an observable notion [81, 82]. This view implies the metaphysical possibility of a world where  $v$  does not equal the associated limit  $dx/dt$ . In our case, taking  $\varphi$  to be independent from  $m$  and  $v$  implies the metaphysical possibility of worlds where  $dx/dt$  does not equal  $\varphi\sqrt{m}$ . This is problematic: if the velocity (or  $\varphi$ ) is ambiguous, the trajectories of the particles are ill-defined. Perhaps in such worlds the trajectories of the particles are determined from  $dx/dt$ , and  $v$  is simply redundant. But if so, why would they not be redundant even in worlds where they equal  $dx/dt$ ? Alternatively, such deviant possible worlds could be ruled out by imposing, as a matter of additional law, that the relevant equalities do hold<sup>94</sup>. Adding such a law would most strongly reduce the metaphysical<sup>95</sup> parsimony however, the only thing that comparativism had going for it.

We finally come to the issue of where including (absolute)  $\varphi$ s as independent

---

<sup>94</sup>Pooley discusses the analogous problem and solution in the context of Sklarations [70].

<sup>95</sup>It will depend on one’s views about laws whether the laws are part of the ideology or some other metaphysical category, so I have here chosen the broadest possible category of parsimony.

members of the ideology leaves us with respect to absolutism and comparativism about mass. Merely adding the  $\varphi$ s will overdetermine the system; they need to be substituted for something else. In other words, what are the notions that need to be ‘kept equal’ when applying the Leibniz scaling? Is the ideology  $\{r_{ij}, \varphi_i\}$ ? Or  $\{r_{ij}, v_{ij}, \varphi_i\}$ ?  $\{r_{ij}, m_i, \varphi_i\}$ ?  $\{r_{ij}, \frac{m_i}{m_j}, \varphi_i\}$ ? Or something else?

$\{r_{ij}, \frac{m_i}{m_j}, \varphi_i\}$  may seem most desirable for the comparativist about mass. This would not be a very metaphysically parsimonious ideology though, compared to the standard absolutist ideology: besides the spatiotemporal ideology, we have to add two types of directly unobservable, dynamical notions: absolutist  $\varphi$ s and comparativist masses. (To be fair, we seem to compensate for this by having left out the velocity, but if we define the velocity reductively according to the “at-at” theory it adds no actual ‘metaphysical cost’ to that of the standard ideology.) Metaphysical parsimony was the only thing that comparativism had going for it: avoiding the single extra degree of freedom of the absolute mass scale at the cost of many  $\varphi$ s is throwing away the baby with the bathwater.

Consider  $\{r_{ij}, v_{ij}, \varphi_i\}$ . This ideology precludes Leibniz Mass Scalings altogether. The  $v$ s and  $\varphi$ s together fix the masses (via the additional law which ensures that the definition of  $\varphi$  is satisfied), so if they are kept constant one cannot vary the masses (uniformly), since it is not an independent parameter. Moreover, since they ground the absolute masses (which in turn ground the mass ratios), this would constitute weak absolutism about mass, not a form of comparativism.

The correct choice of ideology corresponding to this approach is then  $\{r_{ij}, m_i, \varphi_i\}$ , where (the extra law that ensures) the definitional relation between the  $m$ s and  $\varphi$ s fixes the magnitude of the initial velocity. A Leibniz Mass Scaling then corresponds to keeping the  $r$ s and  $\varphi$ s constant, and thus changing the  $v$ s in order to ‘compensate’ for the altered gravitational pull. If both  $m$  and  $\varphi$  are taken to be fundamental, that is strong absolutism about  $m$  and  $\varphi$ , this ideology is definitely metaphysically less parsimonious than strong absolutism about mass by itself, and also than strong comparativism about mass. Most importantly, this of course means that comparativism about mass is false. If we only assume strong absolutism about  $\varphi$ , the  $\varphi$ s and  $v$ s together still fix and thereby ground an absolute notion of mass (which in turn grounds the mass ratios), rather than a comparative notion of mass. This approach thus corresponds to weak absolutism about mass. Changing the ideology in this way does nothing to defend (any form of) comparativism over absolutism.

### More generally

We have seen in the previous chapter that the following mapping of the initial conditions leads to an empirically distinct evolution (assuming L1 and G1):

$$\begin{aligned} m &\mapsto \beta m \\ r &\mapsto r \\ v &\mapsto v \end{aligned} \tag{4.5}$$

where  $\beta$  is a scalar. If we instead use a ‘combined’ or ‘mixed scaling’, which simultaneously changes the velocity,

$$\begin{aligned} m &\mapsto \beta m \\ r &\mapsto r \\ v &\mapsto \sqrt{\beta} \cdot v \end{aligned} \tag{4.6}$$

we again get the original evolution. (On the symmetry approach, we would say that even though [transformation 4.5](#) is not a symmetry of NG, [transformations 4.6](#) and [4.7](#) are.). This scaling is of course only a ‘mixed’ scaling with respect to the standard ideology (cf. [Eq. 4.5](#)), since we could just as well have written it as

$$\begin{aligned} m &\mapsto \beta m \\ r &\mapsto r \\ \wp &\mapsto \wp \end{aligned} \tag{4.7}$$

(which enforces  $v \mapsto \sqrt{\beta} \cdot v$ ). Choosing then an alternative ideology including a variable that remains constant under this mapping,  $\alpha = \{r_{ij}, v_{ij}, \wp_i\}$ , ensures (i.e. it ‘provides a mechanism’) that a Leibniz Mass Scaling corresponds to the [transformation 4.6](#) (= [4.7](#)). Alas, if only this ideology did not correspond to (weak) absolutism about mass, rather than comparativism. It does solve the IVPP, but not in a comparativism-friendly manner.

The comparativist then obviously wants to continue searching for other ‘combined scalings’ (i.e. symmetries) which do correspond to a comparativist ideology. For instance, they may consider the mapping:

$$\begin{aligned} m &\mapsto \beta m \\ r &\mapsto \beta r \\ v &\mapsto v \end{aligned} \tag{4.8}$$

or non-trivial mappings of both  $r$  and  $t$ ; or even playing around with forces as part of the ideology. It should be clear from the discussion of  $\wp$  though that similar problems are to be expected. Hence I will not dwell on listing and discussing every possible ‘combined scaling’, but trust that those have been rendered sufficiently implausible by now—except for one alternative which is sufficiently distinct and interesting to deserve its own subsection.

### 4.2.3 Varying Newton’s Constant

Perhaps the most popular response goes as follows<sup>96</sup>: the Leibniz Mass Scaling is ill-defined until we are told what happens to Newton’s constant when the masses change. What if Newton’s constant is not actually a constant—across possible worlds that is, not across time and/or space? If  $G$  is the right function of mass, a uniform scaling of the masses will not produce any empirical difference after all. If  $G = G(W) = \gamma/m_W$  where  $\gamma$  is a true constant (across possible worlds) with mass dimension zero, the escape velocity inequality becomes

$$v > \sqrt{\frac{\gamma}{r} \frac{m_i}{m_W}}. \quad (4.9)$$

A uniform mass scaling will not affect whether this inequality is satisfied or not, because it will not affect the ratio  $\frac{m_i}{m_W}$ . One may even be tempted to say that this boils down to the following choice for the ideology:  $\alpha = \{r_{ij}, v_{ij}, \frac{m_i}{m_W}, \gamma\}$ , thereby justifying a categorisation of this response as ‘same worlds, different ideology’.

One line of thought that may seem to motivate this response is as follows. When we change units (and quantities)—a passive transformation—say we change  $1kg$  into  $2.2lb$ , why does the physics not change? Because we are used to concurrently changing the value (and unit) of Newton’s constant. Why not do something similar when actively transforming the masses—that is when transforming the magnitudes rather than merely the quantities that represent them?

(This motivation is an extended version of the idea behind §5 of Roberts [13]. In fact, Roberts only seems to consider passive Leibniz Scalings, and does not even get to active scalings. He points out that consistent passive Mass Scalings should also scale Newton’s Constant, such that the escape velocity does not change. This is true, but completely misses the point of the comparativist’s bucket, which crucially depends on the Leibniz Scaling being active, as pointed out in [Subsection 1.2.4](#).)

A second line of thought that may seem to motivate this response, in particular the idea that we are either 1) completely free to change  $G$ , or 2) perhaps even required to change  $G$  precisely to the extent as to compensate for changes in  $m$ , is once again the empirical accessibility of  $G$  and  $m$  only as a product ( $G \cdot m$ ), and the inexpressibility of  $G$  ([Section 2.7](#)).

It is however plain wrong that the Active Leibniz Mass Scaling is ill-defined until we tell a story about  $G$ . We want to know if changing the masses leads to an empirical difference, *ceteris paribus*. Clearly that means keeping the laws the

---

<sup>96</sup>Although I have never seen this in print—apart perhaps by Roberts, who could be argued to subscribe to it, as discussed below—it comes up at every single conference. In particular, I would like to thank Erik Curiel for insisting that this response is more worthwhile than I originally maintained.

same. It is not at all surprising that if one were allowed to change the laws at will for each possible world one could get any (or at least many) of the evolutions one may have wanted. That is simply not an option within the rules of the game we are playing. Symbolically including  $G$  or  $\gamma$  in  $\alpha$ , as if it were on a par with the magnitudes, may confuse one into believing that this is an option. Newton's constant is a quantity that parametrises a family of laws; it is not itself a magnitude, nor part of the ideology.

The first motivation was misguided for the following reason. When in common parlance we say 'changing units', we in fact mean changing the [quantity · unit] combination, and we do so in such a way (say from  $1kg$  to  $2.21lb$ ) that the referent (i.e. the mass magnitude, or the specific G1-type law in the case of Newton's constant) does not change. In principle we would not even have to simultaneously change the [quantity · unit] combination of Newton's constant, but it provides calculational simplicity if the same units are chosen to match up across an equation. This is the only reason that a change in the unit (and the value) of the mass quantities is in practice always combined with a change in the unit (and the value) of Newton's constant. It would be a perfectly viable question to ask what would happen if we (uniformly) changed only the unit or only the values of the masses, whilst leaving both the referents of all units the same as well as the value and unit of Newton's constant (or at least the [quantity · unit] combination such that the G1-type law that is referred to remains unchanged). This would of course change the referents of the mass [quantity · unit] combinations, namely the mass magnitudes, and would thus correspond to an active scaling. And at that point we have no obvious reason to believe that either 1) the referent of (the [quantity · unit] combination of) Newton's constant must be changed as well, or 2) that other things that depend on the mass magnitudes, such as the trajectories, do not change. In summary, a passive Leibniz Scaling is *by intention* merely a change of representation—of one and the same state of affairs. Hence, it is on purpose that nothing changes 'out there'. It is thereby simply convenient to also change the units of Newton's constant. None of this has any implications for the well-definedness and relevance of active Leibniz Mass Scalings which do leave the referent of Newton's constant unchanged. And the comparativist's bucket applies exactly those active rather than passive Leibniz Scalings.

The second motivation was equally misplaced. It is true that in both the actual and another possible world we will not be able to express which of a set of G1-like laws, each represented by a different value of the quantity  $G$  (given a fixed unit and a fixed referent of that unit), obtains. But that is a whole different issue, of an epistemological kind. It is irrelevant to a coherent definition of the operation of keeping whatever law we may have in one world the same across other possible worlds which we are comparing that first possible world to. And it is clearly this perfectly well-defined operation that we are considering when we

are determining the empirical effect of a uniform change of masses, *ceteris paribus*. In other words, even if advocates of this response would have shown that we *may* change Newton’s constant and would avoid an empirical difference resulting from a uniform scaling of the masses by doing so, it has thereby neither shown that we *must* change Newton’s constant so, nor that no empirical difference would obtain when we decide not to change Newton’s constant so even if we may. The latter claim is the only interesting one: it is not sufficient for the comparativist to show that it is *possible in some technical way* to avoid an empirical difference; instead it has to be shown that it is *impossible in any way* to observe a difference resulting from a uniform mass scaling.

Either motivation, if it had been justified, would have proven comparativism to be true *a priori*. As pointed out repeatedly, eg. in Subsections 1.2.1 and 2.4.2, the absolutism–comparativism debate is an *empirical* debate. There can not be any *a priori* arguments. After all, it is certainly logically possible that absolute masses generate empirical effects. The specific dynamics matters. This brings us to the following point.

Although I have argued that the above response fails, it does contain the seeds for a more serious response, albeit a more revisionary one. What if we modify the laws of nature, in particular the gravitational law, such that in the actual world the absolutist law is an effective form of this more general, comparativist law? In particular, we may create this new law from the old absolutist law G1 by replacing  $G$ , as above, with  $\gamma/m_W$ :

$$F_{grav,ij}(W) = \gamma \frac{m_i m_j}{r^2 m_W}. \tag{G3W}$$

Recall the earlier claim in Subsection 2.4.2 that a Leibniz Scaling would have had no effect if the force laws had depended on a combination of masses with a dimension of 1 rather than 2. This is such a law. To be clear, we are not violating the *ceteris paribus* clause, nor are we making use of the underdetermination of  $G$ . We are (syntactically) revising the theory by formulating a new law. We then apply the Leibniz Mass Scaling to this new theory, by applying the *ceteris paribus* clause to the ideology and the laws of this new theory—this new law truly is kept constant across possible worlds, not adjusted at will in each possible world to compensate for any unwanted empirically distinct evolution. The justification for this revisionary move is not the underdetermination of  $G$ , but rather the hope that this revised theory will nevertheless be able to solve the CIVPP; the proof of the pudding, if any, will be in the eating.

Note that this only counts as a distinct theory under the syntactic view of theories, since we have adopted new equations to represent new laws. If this supposedly new theory indeed solves the CIVPP, that is it correctly generates the empirically distinct set of worlds that should be dynamically allowed by Newtonian Gravity, the (modified) semantic view of theories will in fact consider this

‘new theory’ as identical<sup>97</sup> to the original one, since all there is to a theory on this view is that set of possible worlds. Thus, if the labels ‘same/different theory’ used by Huggett and Pooley for their categorisations would have been interpreted syntactically, this response would be categorised as part of Huggett and Pooley’s third category: ‘different theory, same ideology’. However, since we have so far interpreted the labels ‘same/different theory’ semantically, this response would belong to their first or second category, since it aims for ‘the same worlds’. Since, as mentioned, it is wrong to treat Newton’s constant on a par with other magnitudes and include it in the ideology, this would suggest that this response belongs to Huggett and Pooley’s second category: ‘same theory, same ideology’ or ‘have-it-all relationalism (comparativism)’. In fact, this response, if successful, seems to deserve the name have-it-all comparativism even more than the only other response in that category, regularity relationalism (comparativism), since it does not give up on the spirit of comparativism but does insist on denying the reality/ empirical meaningfulness of absolute masses. This greater ambition would however place this response in my first rather than my second category. Another reason why this response sits uneasily in Huggett and Pooley’s second category is that whilst the (non-mass sector of the) ideology is indeed the same, and the theory may be the same as well according to the standard semantic view, there is something that is different: the laws. Huggett and Pooley’s focus merely on ontology, ideology and the standard semantic account of theories fails to acknowledge this difference which is not altogether uninteresting—by showing that the same set of dynamically possible models can be picked out via different (syntactic) laws which have different consequences for the mass sector of the ideology, it suggests that the laws should have some significance over and above the dynamically possible models even on the standard semantic view of theories. Perhaps this difference is more easily accommodated in my modified semantic account of theories, which equates theories directly with a set of empirically possible *worlds*, which may include (eg. dispositionalist or primitivist) laws over and above the ontology and the ideology.

Rather than dwelling on the relatively unimportant categorisation of this response, and the best definition of the semantic view of theories, let us evaluate the success of this response. In particular, we will do so along the way of filling in the notion that is at the heart of this response but has been left undefined so far: what is this  $m_W$  in  $G = \gamma/m_W$  actually supposed to refer to?

---

<sup>97</sup>Insofar as we pairwise compare the members of the sets of DPWs only with respect to their empirical equivalence. In general, an absolutist world that is empirically equivalent to a given comparativist world will not be metaphysically equivalent, even with respect to determinables other than the mass determinable. Cf. our discussion of the completeness of  $G3_W$  below.

## The Alpha Mass

A first suggestion is that  $m_W$  is some privileged mass, a primitive ur-mass that serves as a standard across all possible worlds with respect to which all other intra-world mass comparisons can be made<sup>98,99</sup>. We may also call it the Alpha Mass, by analogy<sup>100</sup> with Neumann’s Body Alpha:  $m_W \equiv m_\alpha$ . Neumann’s response to the argument from inertial motion (i.e. Newton’s bucket) was to privilege one body as moving inertially [84]. All and only those other bodies in uniform rectilinear motion with respect to that Body Alpha are also moving inertially. However, as Newton [85] pointed out in response to Descartes [86]—who had in effect been considering the ‘fixed’ stars to be Bodies Alpha 226 years before Neumann—in a world governed by Newtonian Gravity there are no inertially moving bodies, since gravity is an attractive, long-range force which can thus never be shielded-off. There cannot be a body that is the Body Alpha. The mass case is more promising though. Whereas the Body Alpha had to be inertially moving, it does not matter what the mass of Body Alpha is. All that matters is that any one mass is privileged; it does not matter which one. Thus, it may be the platinum alloy cube in Paris, but it need not be.

Four other problems do still arise. Firstly, it may seem that this response does not in fact succeed in generating the full set of empirically distinct worlds that are dynamically allowed by NG, but violates the completeness criterion. If we have one solution for a pair of two equally massive particles, say an escape (collision) solution, then varying the absolute masses *ceteris paribus* can not turn that solution into a collision (escape) solution. It seems that this theory rules out the possibility that two equally massive particles will collide (escape). However, an escape (collision) solution can still be turned into a collision (escape) solution by varying either the initial velocities or distances—although this of course means committing to absolutism about velocity or distance. The task for the comparativist was obviously not to generate the whole set of dynamically allowed solutions merely from applying Leibniz Mass Scalings to a first solution. Rather it demands us to range over all the possible initial values of the chosen ideology, and check whether that provides the desired set of worlds. In this case it does. Nevertheless,

---

<sup>98</sup>Baker considers this option, but ends up rejecting it [25].

<sup>99</sup>In MODified Newtonian Dynamics (MOND) [83], the new constant  $a_0$ , with dimensions of acceleration, seems to be similarly used to serve as a standard for acceleration comparisons. However, it would be incorrect to conclude from this that comparativism about acceleration is true of MOND.  $a_0$  is not the acceleration of some privileged body, and thus not an “Alpha Acceleration” in the sense of Neumann’s Body Alpha (see text), but a new constant of nature—a consequence of which is that MOND avoids the second and third objection discussed in the text. A Leibniz Acceleration Scaling would therefore change only the absolute accelerations  $a_i$  ( $i = 1, \dots, n$ ) but not  $a_0$ , which implies a change in the ratios  $\frac{a_0}{a_i}$  and thereby an empirical difference. I believe absolutism about acceleration to be true of MOND.

<sup>100</sup>I would like to thank John Norton for reminding me how interesting this analogy is.

in step 2 of the CIVPP and ‘The General Recipe for  $\phi$ ’ (p.69) it seemed very natural to compare absolutist and comparativist worlds that agreed on all the determinate values of the non-mass determinables. Perhaps that requirement was slightly too strict: kinematic comparativism about, say,  $r$  might allow  $\phi$  to instead map absolutist worlds labelled by  $r_{ij} = c_{2,ij}$  to comparativist worlds labelled by  $r_{ij} = \beta c_{2,ij}$ , where  $\beta$  is a scalar, as long as we do not have independent empirical access to the absolute value of  $r$ .

Secondly, even though there are some dynamically possible worlds containing  $m_\alpha$  whereas there are none containing Body Alpha, there will still be some that do not contain  $m_\alpha$  either. In those worlds the law fails to refer. As Baker puts it: the privileged massive object would have to exist with physical necessity, but that is an extremely implausible posit for a scientific theory to make [25].

Thirdly, having the law refer to one particular, privileged body or property violates a generally accepted view about laws of nature called ‘generalism’. It claims that the world is fundamentally purely qualitative: a language or theory devoid of reference to particular individuals (such as  $m_\alpha$ ) suffices to give a complete and perspicuous description of fundamental reality [87]. I will not discuss the merits of this view here—see Saunders [88], Dasgupta [89], Pooley [55] and Møller-Nielsen [87] for convincing arguments—but rest content with having pointed out this consequence of the  $m_\alpha$  solution.

Fourthly, it is not clear how this positively and significantly differs from absolutism. The debate between comparativism and absolutism was only ever about a single degree of freedom: the absolute mass scale. Adding a primitive  $m_\alpha$  seems very much like adding something like an absolute mass scale. Even if it is supposed to be a different type of beast, its loss in metaphysical parsimony seems to be enough to counteract the gain the comparativist was supposed to obtain from dismissing an absolute mass scale. In fact, this approach boils down to imposing determinate quidditism, but then only for a single mass rather than all absolute masses. If this would have succeeded, we could equally well have modified our definition of absolute masses to include determinate quidditism about a single element of the set of absolute masses only. By imposing determinate quidditism about all absolute masses the absolutist avoids the previous two problems.

### **Summing over all the masses: ‘Machian’ comparativism**

In the substantivalism–relationalism debate about space, Mach’s approach may be seen as an attempt at an improvement upon the Body Alpha response, by avoiding the requirement of there being one privileged body (or mass) in all possible worlds (on pain of failure of reference of the law) and the associated violation of generalism. Rather than choosing one specific body as a standard of inertial motion, inertial motion is abstracted from the bulk matter in the whole universe.

By analogy, one might wonder whether  $m_W$  should refer to the sum over all the masses in the universe. The corresponding **Machian gravitational law** is:

$$F_{grav,ij} = \gamma \frac{m_i m_j}{r^2 \sum_k m_k}. \quad (\text{G3})$$

This would result in the following escape velocity inequality:

$$v > \sqrt{\frac{\gamma}{r} \frac{m_j}{\sum_k m_k}}. \quad (4.10)$$

It is important to not take from this equation that the sum of all the absolute masses—which is an absolutist notion, unavailable to the comparativist<sup>101</sup>—plays a crucial role all by itself. Rewriting the escape velocity inequality as follows will make explicit that it really is the sum of all mass ratios that matters:

$$v > \sqrt{\frac{\gamma}{r} \sum_k \frac{m_k}{m_j}}. \quad (4.11)$$

Uniformly scaling the masses does not affect whether this inequality is satisfied.

This view has the following counter-intuitive consequences. If God were to suddenly add a massive particle or increase the mass of a massive particle, however far away from our two-particle system of interest, this would nevertheless reduce the force between these two particles. Comparativism has become even more holistic. Similarly, the gravitational force between each pair of particles is weaker in a world  $W_2$  that differs from a world  $W_1$  solely by the addition of some massive particles somewhere. Beyond perhaps running counter to one’s intuition, these two scenarios do not provide any empirical problem though. God has never suddenly increased the number of particles or their masses; if he were to, it might indeed reduce the gravitational force, for all we know. Similarly, in the second case the ‘strength of the law’, represented by Newton’s Constant, would still be constant within both worlds. Its constancy within the actual world is all the empirical data we have.

Although no comparativist has proposed this theory—as far as I am aware—I believe it to be the most viable version of comparativism. However, besides the extreme holism, another issue requires more serious attention. In [Chapter 1](#) I defined absolute masses as a totally ordered semi-group, with the concatenation rule to be interpreted as addition. Under this structure, the sum over all the masses (or mass ratios) is well-defined. Comparativist masses, on the other hand,

---

<sup>101</sup>Unless they are a realist about absolute masses. This option will be discussed in the next section.

were defined as a totally ordered group, where the concatenation rule was to be interpreted as multiplication. Under this structure, sums over mass ratios are not yet defined. The current version of comparativism would thus require additional structure, namely an associative concatenation rule to be interpreted as addition (although no inverse or identity elements are required<sup>102</sup>, since we do not have 0 mass ratios). But the whole point of comparativism was to do with less structure than the absolutist, that is to be more ideologically parsimonious. It is unclear where the current response stands in that regard. The mass ratios now require a total order and two concatenation structures, whereas the absolute masses required one concatenation rule less—although the absolute masses also require transworld identities. That some of these rules lead to semi-groups rather than groups is irrelevant, since the existence of identity and inverse elements is merely a fact that is true or false of a set plus a concatenation rule, not some extra structure that needs to be imposed. In sum, it is unclear where this leaves this response. To make things worse, additional metaphysically unparsimonious aspects of (any form of) comparativism will be discussed in [Chapter 5](#).

Moreover, it is worth repeating what was discussed in the context of the Alpha Mass: to ensure completeness, one needs to commit to absolutism about  $r$  (or  $v$ )<sup>103</sup>. Replacing  $G$  not with  $\gamma / \sum_k m_k$  but with  $\gamma \sum_i r_i / \sum_k m_k$  (**Doubly Machian Comparativism**) would violate completeness. Although it seems very much in the spirit of comparativism to be a comparativist about all determinables, comparativism about mass is logically consistent with absolutism about distance, as pointed out in [Section 2.2](#). There remains an arbitrary choice though between opting for the comparativism about mass and absolutism about distance combination, rather than the other way round.

### 4.3 If not the spirit, then at least the letter: regularity comparativism<sup>104</sup>

In response to Newton’s bucket, Van Fraassen [90] has pointed out that it is not the structure of absolute space that is required to privilege certain frames, but merely the truth of Newton’s laws in those frames. Huggett [77] uses this

<sup>102</sup>If an identity and inverse did exist, the structure would be a ring (apart from the requirement that the multiplicative rule be distributive over the addition rule).

<sup>103</sup>This is presumably a problem for Maudlin [43, p.29], who is motivated by the (invalid—see [Subsection 1.2.1](#)—)argument from kinematic comparativism to adopt metaphysical comparativism about distance, and thus plausibly for the same reasons also metaphysical comparativism about mass.

<sup>104</sup>An edited version of this section is forthcoming in *Philosophy of Science* 84(5) under the title “Regularity Comparativism about Mass in Newtonian Gravity”.

insight to formulate a version of relationalism that ‘gets inertial frames for free’. Regularity Relationalism employs the framework of Humean Supervenience—in particular the Mill-Ramsey-Lewis Best System Account—to have both the inertial frames and the (absolutist) laws (L1 and G1) supervene as a package deal from a relational Humean mosaic.

Since the comparativist’s bucket is analogous to Newton’s bucket—the former is an argument for realism about the absolute masses and the latter an argument for realism about inertial frames—an obvious comparativist response seems to be a regularity version of comparativism, that attempts to get both the absolute mass scale and the laws of Newtonian Gravity as a package deal from a comparativist Humean mosaic. Regularity Comparativism thus accepts defeat with respect to the comparativist’s bucket proving realism about absolute masses—what one may consider to be against the spirit of comparativism—but insists on the letter of comparativism: there is, *prima facie*, some wiggle room to get absolute masses ‘for free’ from more fundamental mass ratios. In this comparativist framework absolute masses are thus as detectable as they could possibly be, contrary to Dasgupta’s preferred undetectability approach.

After outlining the framework of Humean Supervenience and in particular the Mill-Ramsey-Lewis Best Systems Account in [Subsection 4.3.1](#), I discuss regularity relationalism ([Subsection 4.3.2](#)) and regularity comparativism ([Subsection 4.3.3](#)). The final subsection discusses four arguments against regularity comparativism, leading to the conclusion that this view is untenable.

### 4.3.1 Empiricism about laws of nature

Perhaps the most popular incarnation of empiricism about laws of nature goes under the name of **Humean Supervenience**. Earman already considered whether “laws are parasitic on occurrent facts” [91] in 1984, but the most well-known formulation of the view stems from Lewis in 1986:

Humean supervenience is named in honor of the greater [sic] denier of necessary connections. It is the doctrine that all there is to the world is a vast mosaic of local matters of fact, just one little thing and then another. [...] We have geometry: a system of external relations of spatio-temporal distances between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which require nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. All else supervenes on that. [17, p.ix-x]

The name ‘Humean supervenience’ suggests that this view in fact goes much further back than 1986, but we will question below whether Lewis’ motivations and justifications were indeed the same as Hume’s.

As Maudlin points out, Lewis’ thesis of Humean supervenience in fact comprises two logically independent theses [92].

The first states that:

**Separability:** “The complete physical state of the world ... supervenes on ... the intrinsic physical state of each spacetime point (or each pointlike object) and the spatio-temporal relations between those points.” [92, p.51]

Or, as Maudlin informally glosses it, all fundamental properties are intrinsic properties, except for spatio-temporal relations, which are the only fundamental external properties (i.e. relations).

In the context of this thesis it will prove useful to note that Separability—at least according to the informal gloss—seems to comprise two additional theses. Firstly, it is committed to **Strong Absolutism** about all non-spatiotemporal determinables such as mass and electric charge, i.e. both **Weak Absolutism** and **Primitivism**. The latter is the view that these magnitudes are fundamental (cf. p.11). Secondly, it is committed to **4D-fundamentalism**: the view that the four-dimensional spatio-temporal relations are fundamental (as opposed to for instance a 3N-dimensional configuration space)<sup>105</sup>.

It is not directly clear that the formal definition of Separability does indeed commit to either Primitivism or 4D-fundamentalism, because both theses only follow from twice sneaking in the notion of fundamentality into the informal gloss. One could perhaps argue though that Lewis’ original quote is most naturally interpreted by adding those ‘fundamentality’ qualifiers.

What is clear is that the formal definition commits one to some form of absolutism about all non-spatiotemporal determinables. As it stands, Humean Supervenience is thus clearly not usable by comparativists. Below we will discuss the option of liberalising Humean Supervenience, in order to make it suitable for comparativists.

Besides Separability, Humean Supervenience comprises Supervenience<sup>106</sup>:

**Supervenience:** “All facts about a world, including modal and nomological facts, are determined by its [complete] physical state.” [92, p.51]

Separability of course requires the physical state referred to in Supervenience to be separable, but Supervenience by itself does not do so as it is a logically independent thesis. How each thesis is motivated, and whether they can be motivated independently is an issue that we will return to below.

---

<sup>105</sup>Here I use terminology that is similar to Chen’s [93].

<sup>106</sup>‘Physical Statism’ in Maudlin’s terminology.

Within this broad framework of Humean Supervenience, the exact manner in which the nomological facts supervene on the separable physical state (i.e. the four-dimensional Humean mosaic) is usually explicated via the Mill-Ramsey-Lewis Best Systems Account (MRL) [91,94]. According to this approach, laws of nature are generalisations which are axioms or theorems of the ‘best’ axiomatisations of the Humean mosaic. What makes an axiomatisation the best axiomatisation is an optimisation of two virtues that pull in opposite directions: simplicity and strength. The desideratum of strength, or informativeness, is often quantified by the ‘amount’ of possible worlds it rules out; simplicity is often explicated syntactically.

### 4.3.2 Regularity Relationalism

Huggett defends a version of relationalism about space, regularity relationalism, which makes crucial use of the MRL approach [77]. This version is supposed to deal with *inter alia* the problem posed by Newton’s bucket, i.e. inertial effects. The substantivalist posits absolute space to provide the inertial frames in which Newton’s laws hold and with respect to which the absolute acceleration is determined that underlies Newton’s bucket. Huggett however builds upon the key insight by Van Fraassen [90] that it is not the structure of absolute space that privileges certain frames, but merely the truth of Newton’s laws in those frames. Thus, if we consider all the possible reference frames that are naturally adapted—to be specified in more detail below—to the spatiotemporal relations of the Humean mosaic, for some of these frames the laws that are the best axiomatisations will be Newton’s laws, whereas the other frames will have best axiomatisations comprising laws that include additional correction terms. The plausible claim is then that the former set of reference frames provide the simplest (and strongest) axiomatisation overall. It is only those frames in which Newton’s laws are true; it is merely this fact that privileges those frames, no further metaphysical implications can be derived from their privileged status. Thus, both the inertial frames and Newton’s laws supervene as a package deal; the inertial frames come for free.

Now in slightly more detail. Huggett’s sparse mosaic consists merely of the Leibnizian spatial relations of the particles over time, and their fundamental intrinsic properties such as mass and charge. Call this the relational state, or the Leibnizian-Humean mosaic. It does not include other geometric relations, and in particular not an affine structure.

We then consider the set of what we will call *ontological coordinate frames*. As a first pass, we might choose them to be the set of all adapted frames of all the bodies in the world. A frame is adapted to a reference body if that body is at rest at the origin of that frame. But, since in a world governed by Newtonian Gravity we would generically not expect there to be any inertial bodies (cf. our

Body Alpha discussion in [Subsection 4.2.3](#)), this set of adapted frames would not include any inertial adapted frames. Instead we choose as our set of ontological frames the set of adapted frames plus all frames related to those adapted frames by arbitrary continuous spatially rigid transformations<sup>107</sup>.

In the final step we then axiomatise the Leibnizian-Humean mosaic separately for each of these ontological frames. This will, Huggett claims, result in a privileged set of pairs of Best System Frames and laws written in terms of those coordinates which are the best laws overall. These laws are Newton’s laws, and these frames are the inertial frames.

### 4.3.3 Regularity Comparativism

Regularity relationalism was *inter alia* a response to Newton’s bucket, which showed the empirical meaningfulness of a standard of inertia. The main argument in this thesis is the analogue of Newton’s bucket for the comparativist, as it makes clear the empirical meaningfulness of an absolute mass scale. An obvious move for the comparativist is to use the regularity approach: getting the absolute mass scale for free by having it supervene on a mosaic of fundamental mass relations (as a package deal together with the laws of Newtonian Gravity).

Huggett’s insight was that if we quantify over the best axiomatisations of the Leibnizian-Humean mosaic as coordinatised by different ontological frames, the inertial subset of those frames will drop out, since only in those frames do the laws come out in their simple Newtonian form (and those laws are the best overall). This easily translates to the case of comparativism. That one absolute mass scale is privileged does not imply that we should attribute it fundamental metaphysical status (relative to the mass ratios); it only means that when we quantify over the best axiomatisations of the Humean mosaic as ‘coordinatised’ by different absolute mass scales, the simple Newtonian laws will be true for only one of those choices of scale.

Again in slightly more detail. Our Comparativist Humean mosaic consists of the relevant geometric structure (either merely the Leibnizian relations, or more structure) of all the bodies over time, plus fundamental mass ratios between all<sup>108</sup> bodies, plus their fundamental intrinsic properties such as charge.

Clearly this constitutes a further liberalisation of the standard form of Humean Supervenience, since both the formal and informal definition of Separability presupposed absolutism about all non-spatiotemporal determinables. Moreover, the informal definition comprised primitivism about absolute masses, whereas, although absolute masses still ‘exist’ in regularity comparativism, they are not

---

<sup>107</sup>Pooley chooses these frames *ab initio* [70, §3.1].

<sup>108</sup>In the case of Chain Comparativism, see [Subsection 5.2.3](#), there will only be a single chain of fundamental relations, not a complete graph/web.

fundamental. We will discuss below whether this is a problem, and if so how it might be solved. For now we will just go with it.

The analogue of the sets of ontological coordinates is the sets of distributions of absolute mass magnitudes that are compatible with the fundamental mass ratios. Since a choice of absolute mass for one particle fixes all the other absolute masses via the mass ratios, this set of ontological mass coordinates can be parameterised by the (quantity representing the) absolute mass magnitude of one specific body.

For each ‘value’ of the absolute mass scale parameter we consider the best axiomatisation of the mosaic so ‘coordinatised’. The claim is then that only for the single correct ‘value’ of the parameter the best axiomatisation comprises the laws of Newtonian Gravity, and that these are the best laws overall<sup>109</sup>.

### 4.3.4 Responses

In this subsection I discuss four critical responses to regularity comparativism. The first one is mistaken, but the latter three pose a more serious threat. The fourth argument is especially fatal.

#### A disanalogy with regularity relationalism?

One may concede that the regularity protocol provides the inertial frames, but nevertheless fails to provide the absolute mass scale, by arguing that the mass and space cases are disanalogous.

The regularity approach claims that only for a subset of the ontological ‘coordinates’ the laws are best, and those best laws are the laws of Newtonian Gravity. In other words:  $L(c_o) = L_{NG}(c_o) + L_{corr}(c_o)$ , that is in general there will be correction terms (i.e. fictional forces in the relationalism case). Only for the Best System Coordinates (i.e. the inertial frames in the relationalism case) will the correction terms go to zero:  $L(c_{bs}) = L_{NG}(c_{bs})$ . Finally, and crucially, there is no alternative way of writing  $L_{NG}(c_o) + L_{corr}(c_o)$  (for coordinates that are not part of the best system) that is as simple as  $L_{NG}(c_{bs})$ .

Now, this final claim seems plausible in the case of regularity relationalism. But, the argument goes, it is much less plausible in the case of comparativism. It is true that if we do not change the units of mass, position, Newton’s constant and the accelerations involved, nor the things they represent, choosing the wrong quantity representing the absolute mass scale will mean that  $L_{NG}(c_o)$  maps those masses (given the distances) to the wrong numerical values of the accelerations (i.e. a crucial ingredient of the data that we are trying to account for). This

---

<sup>109</sup>As pointed out in the conclusion of this thesis, it would be interesting to consider how the gravitational law of absolutism (G1) compares in simplicity and strength to the gravitational laws of Machian Comparativism and Doubly Machian Comparativism (Subsection 4.2.3).

would imply that a correctional term is required. But, the argument goes, what if, whenever we choose the ‘wrong’ absolute mass scale, we just adjust the numerical value of Newton’s constant accordingly.  $L_{NG}(c_o(G)) + L_{corr}(c_o)$  can always be rewritten as  $L_{NG}(c_o(G'))$ . Thus, for each choice of mass scale we could get an equally simple law; the only difference is the numerical value of  $G$  in each of these laws. In other words, the regularity approach fails in its main aim: picking out a unique absolute mass scale.

One might respond by pointing out that this is a trick at the representational level—the level of quantities—only; and in that respect it is actually not disanalogous to the relationalism case. In that case it is also not just one ‘coordinatisation’ that corresponds to the best axiomatisation, but several. Each of them related by a symmetry of the theory. When we decide to use a coordinate system with a different origin (i.e. a passive Static Leibniz Shift), we know that this origin has no physical meaning. It is just representational redundancy. Similarly, in the case of comparativism, what a law is at the *metaphysical* level is a mapping from initial positions and mass magnitudes to acceleration magnitudes. One can choose to *represent* that single law in different ways by choosing different units for acceleration and mass, which will in turn fix the unit of Newton’s constant (i.e. a Passive Leibniz Scaling). But that is just a matter of representation, not of metaphysics: each of these different representations represents one and the same situation.

However, the original criticism can also be applied at the metaphysical level, the level of magnitudes. When we are axiomatising the mosaic for a wrong choice of absolute mass magnitudes, we still get equally simple (and strong) laws. The reason for this is the empirical accessibility of (that what is represented by)  $G$  and  $m$  only via the product  $G \cdot m$  (Section 2.7). Scaling the mass magnitudes uniformly can be compensated for by scaling the strength of the law (as represented by  $G$ ) accordingly<sup>110</sup>. Each of these axiomatisations provides a different law—if we choose to call two laws differing only by a different value of  $G$  distinct laws—but these are nevertheless equally simple (and strong), so the regularity approach will pick out all these pairs of mass scales and corresponding laws, rather than just one.

We should respond to this by claiming that requiring the regularity protocol to pick out one unique mass scale is too strict a demand<sup>111</sup>. It suffices to pick out *at least* one pair of mass scale and law that reproduces the data, and it is

---

<sup>110</sup>In the symmetry approach, this would be an interesting ‘mixed’ symmetry, which not only transforms the magnitudes but simultaneously ‘transforms’ the (strength of the) law, such that  $G \cdot m$  remains invariant. (It should be obvious however that it does not follow from this symmetry alone that a transformation of just the mass magnitudes (i.e. a Leibniz Mass Scaling) is also a symmetry.)

<sup>111</sup>I would like to thank Chris Wüthrich for insisting that this response has more to it than I originally thought.

fine if several other pairs are picked out as well as long as they also reproduce the data. Due to the empirical accessibility of (that what is represented by)  $G$  and  $m$  only via the product  $G \cdot m$ , all these ‘winning pairs’ satisfy this weaker condition. As mentioned above, this is not that dissimilar from the regularity relationalism case. In that case the regularity protocol also fails to provide a unique answer. It is true that if we start with a relational Humean mosaic all those answers will represent the same scenario. However, if we would have started out with a substantivalist mosaic we would have arrived at a set of equally ‘best’ inertial frames, all differing by an active Leibniz (Static or Kinematic) Shift, which makes them metaphysically distinct.

In summary, the regularity approach indeed fails to pick out an absolute mass scale, but it does pick out a set of pairs of mass scales and laws, each of which by themselves solves the CIVPP. Is this not all that can be asked of comparativism? Yes. Besides sufficient explanatory power, as discussed below.

### Explanatory Inadequacy

Having conceded that the regularity approach succeeds in its aim (once that has been correctly interpreted), the remaining criticisms assert that it only manages to do so by cheating. In this and the next two sections we will discuss several of the ways in which the regularity approach is supposed to be doing so.

Should we be surprised that the regularity approach to comparativism retrieves the correct form of the laws? No, not at all. The mosaic contains (approximations of) the two-particle experiments (discussed above) and Mach’s experiments (discussed below). It contains all the trajectories of all the particles, infinitely far into the past and future. Everything that operationalises the mass notions we are after is included in the mosaic. Thus, if we were ever going to be able to find and apply Newton’s laws and the law of Universal Gravitation—and we have indeed done successfully done so for three centuries now, using only part of the mosaic—this better be enough. (More on this in [Subsection 6.4.2](#).)

But this never was the problem. The Humean empiricists and the opposing camp agreed on the epistemology of finding the specific forms of the laws of nature. What they disagreed on was the metaphysics. For the Humean Empiricists this operationalisation, this epistemological procedure of retrieving the specific form of the laws by axiomatising the observable data in the way that best optimises both strength and simplicity, is all there is to being a law. For the opposing camp this is merely the method of finding out the specific form of the law, but this is not what it is to be a law. Laws have some metaphysical status, and they are used to *explain* how the mosaic of trajectories can be *generated* from initial conditions. In other words, the regularity comparativist puts the cart before the horse. The task at hand was the Comparativist Initial Variables & Parameters

Problem: using the comparativist initial conditions to explain the behaviour of the world, that is to generate the mosaic<sup>112</sup>. The regularity approach has helped itself to the mosaic—that is the data, the explanandum—and has told us how we are supposed to retrieve the laws from this. Even if that descriptive job has been done well, it is not the explanatory job we were after in the absolutism–comparativism debate.

Two problems regarding explanation are to be distinguished<sup>113</sup>. First, folk philosophy/science/metaphysics tells us that it is gravity (together with its source, gravitational mass) which makes things attract each other. Regularity comparativism gets the arrow of explanation wrong: it claims that gravity and mass are true in virtue of attractive trajectories of particles, rather than gravity and mass explaining the universality of the attractivity of gravity<sup>114</sup>. Secondly, when explaining a specific part of a trajectory, regularity comparativists will do so in the same way as absolutists: they start from initial conditions and evolve them forwards using the laws. However, for the regularity comparativist, those laws and the absolute masses in the initial data are obtained from the whole mosaic. So the whole mosaic—in effect a conjunction of all the localised, particular matters of fact—is used as explanans to explain one specific part of the mosaic, one particular matter of fact. This is explanatorily (and logically) circular. Both these issues suggest that the explanatory adequacy premise  $P_{\text{exp}}$  in the argument for comparativism is false.

As these issues are specific instances of the explanatory inadequacy objections against Humeanism [95, Ch.6] [96–98], I will not further engage with this general debate here, but see Loewer [99], Lange [100], Miller [101], Hicks & Van Elswyk [102], and Bhogal [103].

This is however a good place to say a bit more about the sense in which regularity comparativism solves the CIVPP. In [Section 3.2](#) I have allowed in principle that  $\alpha$  contains absolute masses, as long as they do not ground the mass ratios. This was in order not to commit Dasgupta’s mistake of equivocating between the concepts of reality and fundamentality. However, it is of course very much in the spirit of comparativism to include no absolute masses. Similarly, the expectation going along with the general recipe for  $\phi$  was that it would be a many-to-one mapping, and that the comparativist worlds would be uniquely labelled by the mass ratios and other initial non-mass determinates. All this is false for regularity

---

<sup>112</sup>By this I do not mean an explanation of the existence of a whole (mosaic) as such, but rather an explanation of any or all of the specific matters of fact that make up the whole—as any mosaic is in effect a conjunction of particular matters of fact. I would like to thank Christopher Timpson for pointing out this distinction.

<sup>113</sup>A third problem is the conspiracy of mass relations, as introduced in [Subsection 5.2.2](#).

<sup>114</sup>As mentioned before, this is not to be confused with us finding out about the truth of gravity and mass by observing that particles exhibit attractive trajectories. This epistemological truth is independent of the question of the arrow of explanation.

comparativism. Absolutist worlds are mapped in a one-to-one fashion to comparativist worlds, labelled by exactly the same (absolute) determinates. The only difference is that the mass determinates labeling the comparativist worlds are extrinsic: they are grounded in the mass ratios plus the full history. Including those non-fundamental masses in the ideology gives unique solutions to the initial value problems, but those initial values for the absolute masses cannot be freely chosen but are to be obtained exactly from those solutions. The circularity is obvious. Leibniz Mass Scalings have a similarly contrived interpretation in the regularity framework. The *ceteris paribus* clause must here be interpreted to *not* include the trajectories, something that the regularity comparativist takes to be fundamental. If the trajectories and the mass ratios are kept invariant, it is impossible to scale the absolute masses, since those are fixed by the former. A Leibniz Scaling thus means that one keeps the mass ratios the same, but moves to a completely different mosaic, as to obtain scaled emergent, absolute (i.e. monadic, not intrinsic!) masses.

## Separability

As pointed out earlier, regularity comparativism is blatantly contradicting one of the two theses comprising Humean Supervenience, the thesis of Separability, which presupposes absolutism about all non-spatiotemporal magnitudes. Two options immediately spring to mind: either **liberalising or generalising** the definition of Separability such that (regularity) comparativism does in fact satisfy it, or **giving up Separability altogether** and arguing that Supervenience can be used by itself to carry out the regularity protocol.

The latter option might seem best. The crux of the regularity approach is to have the inertial frames or the absolute mass scale *supervene* on some appropriate mosaic. The separability of that mosaic does not seem to play any role. Nevertheless, the Humeans have had to deal with one notorious charge of non-separability before, and in that case they were very reluctant to stray too far from the Separability condition. This is the case of entanglement in quantum mechanics. The details of this case are neatly rehearsed in, for instance, Maudlin [95], and need not detain us here. What is relevant here is that the Humeans accept that entangled quantum states are non-separable in the strict sense, but insist on defending some generalised version of Separability rather than just giving up any sort of separability condition altogether<sup>115</sup>. Albert [106] notes that the wave function lives in  $3N$ -dimensional configuration space (with  $N$  the number of particles in the universe), and the wavefunction does specify intrinsic values (namely two, the amplitude and the phase) for each point of that higher-dimensional space. It is

---

<sup>115</sup>Alternatively, Darby suggests that we add the quantum entanglement relations to the mosaic [104], but, as Dewar correctly points out [105], this is simply to relinquish Separability.

thus suggested that we should give up on 4D-fundamentalism and take configuration space to be the fundamental arena of physics. (Of course one then needs to tell a story of how this is consistent with our manifest image of the world around us being four dimensional. Albert tells such a story, but we need not dwell on it here.) We could then define an analogue of Separability by quantifying instead over the points of that configuration space, and we may naturally want to call this new condition **Generalised Separability**.

Enter Shakespeare. “What’s in a name?” Why should we care whether this new condition is satisfied or not? Simply because its name is similar to a previous condition we cared about? In fact, why did we ever care about Separability? Albert’s implicit motivation seems to be that the **classical world**, that is “ ‘familiar macroscopic objects’ under so-called ‘familiar macroscopic circumstances’ ” [106, p.282], is manifestly separable (assuming absolutism of course). Maudlin finds similar motivations in Lewis<sup>116</sup> and Einstein. But the argument that new physics should be separable because physics so far has been separable is an inductive argument. We would expect Humeans, of all people, to be the last to endorse such an invalid argument. Even if this motivation were valid, it is not clear why this would in any way motivate Generalised Separability. This motivation is clearly about the 4D manifestation of the classical world. Moreover, even if a separate motivation for General Separability were to be provided (for instance realism about quantum mechanics, as Albert suggests), General Separability never did *replace* Separability. As hinted at earlier, the formal definition of Separability is consistent with quantification over emergent (i.e. non-fundamental) spacetime points. In other words, even if the wave function is separable in the generalised sense in configuration space, it would still be non-separable at the level of the 4D spacetime that has to emerge somehow from the configuration space.

We have arrived back at the suggestion of just leaving Separability behind (but see [Section 5.1](#)), and sticking only with Supervenience (as Dasgupta does [8]). From the standpoint of the regularity approach, this is all that we need. However, even though Supervenience is logically independent from Separability, we need to make sure that we can still motivate Supervenience once Separability is given up. We need to make sure that they did not come as a package deal. Why might we be motivated to hold Supervenience?

---

<sup>116</sup>Arguably, Lewis’ main theoretical motivation for Separability is that it underwrites his ‘principle of recombination’ as a source for the set of possible worlds [107]. Perhaps this principle can still produce all required possible worlds even if the supervenience base does include dyadic relations (such as mass relations), as Darby seems to be suggesting [104, §4]. If Darby is correct, then we can safely ignore the requirement of Separability, as far as Lewis’ theoretical motivation is concerned. (See Dasgupta [8, §6] for another sense in which Lewis does not in fact require Separability.) If not, then one would have to give up either regularity comparativism or the principle of recombination. Here I will not dwell on the large literature on the principle of recombination.

Let us recall the first sentence of the quote from Lewis in [Subsection 4.3.1](#): “Humean supervenience is named in honor of the greater [sic] denier of necessary connections” [17, p.ix]. Hume is famous for arguing that **necessary connections** between two events (the first of which we call the cause and the second of which we call the effect) are merely ideas of the mind, but not ‘things’ that can be observed [108]. All that can be observed is constant conjugation of pairs of events. **Humean empiricists** therefore urge us to purge necessary connections (between what we call causes and effects), that is causation, from our metaphysics. Insofar as the notions of causation and laws are related, this form of empiricism also gets rid of laws as fundamental concepts.

But if it is an empiricist dislike for necessary connections that motivates Supervenience, then Humean Supervenience comes as a package deal. This type of empiricist will also be committed to a separable mosaic (at least insofar as mass is concerned). As Dewar puts it: “Prima facie, the kind of world that violates Separability is one in which there are necessary connections between distinct existents: that is, in which there are fundamental and irreducible *relations* between pointlike things” [105, p.15] (*italics in the original*). Entangled states are such that, say, if a measurement on one of the entangled particles results in ‘up’, the measurement of the other one will necessarily result in ‘down’. Note that ‘necessarily’ is here not to be read (just) as an inter-world notion, but rather an intra-world notion in the same sense as when Hume argues against one event (the cause) necessitating another later event in the same world (the effect). If there are two distinct physical systems consisting of massive particles, the behaviour of one will necessitate specific behaviour of the second given the inter-system mass ratios. This does not detract from the mass ratios being totally contingent in the sense that they can differ between worlds/mosaics. The Humean might retort that on their framework the mass ratios and the trajectories are completely independently specifiable. In particular, we are free to have the second system behave differently from the way one would expect given the inter-system mass ratios. However, not only would this result in a world that is nomologically impossible according to Newtonian Gravity, it is also to admit that mass is in no way related to anything that is observable and it is therefore redundant. As will be discussed below, purging any notion of mass from the mosaic (since the trajectories do all the work by themselves) leads to what I believe to be the most defensible Humean position—regularity eliminativism—but is fatal for regularity comparativism. To sum up, if this dislike for necessary connections is the only motivation for Humean Supervenience, then this motivates both Supervenience and Separability (about mass). Separability cannot be given up without the whole thesis collapsing. A combination of comparativism and Humeanism is doomed from the start.

We have yet one other motivation to consider though. Huggett himself mentions in a footnote that **metaphysical parsimony** is the best justification for

Humeanism: “The best argument for MRL is that it is metaphysically parsimonious and faces no knock-down objection” [77, p.43]. Especially comparativists, motivated similarly by metaphysical parsimony, must find this attractive. Whether a Separable physical state is indeed more metaphysically parsimonious than a non-separable state will be discussed in [Chapter 5](#). Here this is not immensely relevant. In contrast to the motivation from Humean Empiricism, where the dislike for necessary connections was a black-and-white matter which implied both Separability and Supervenience, metaphysical parsimony is a matter of degree. If Supervenience produces any ‘amount’ of gain in metaphysical parsimony, this will speak in favour of Supervenience. Of course it would be ideal if any loss in metaphysical parsimony due to the non-separability of comparativism would not outweigh the gain due to Supervenience, but when comparing regularity comparativism to comparativism *simpliciter* any degree of gain in metaphysical parsimony from Supervenience would provide a justification for Supervenience by itself.

Why then would Supervenience be more metaphysically parsimonious than governing law views such as primitivism which take laws to be irreducible nomic facts over and above the occurrent facts [95, Ch.1]? Presumably because the Humean helps herself *only* to the mosaic, and does not require any *additional* nomic facts over and above the occurrent facts. But that is simply the wrong comparison. The governing law camp helps themselves not to the full 4D mosaic, but merely to some initial conditions (i.e. a 3D mosaic)<sup>117</sup> plus fundamental laws (or dispositions). The remaining dimension of the Humean mosaic, eternally far towards the past and into the future, supervenes on those initial conditions and the laws. Moreover, the governing law camp can help themselves to a small set of ‘timeless’ mass ratios—they hold at the initial time, and since the law does not evolve them, they hold at all times—whereas the Humean needs to ‘buy into’ instantaneous mass ratios for every instant of the 4D mosaic (and these mass ratios need to always be the same, without any explanation given).

Where does this leave us? Now it is clear that the Humean’s fundamental metaphysics is not simply a proper subset of the fundamental metaphysics of the opposing camp, but instead a different set of ‘things’, it becomes difficult to say much about their relative metaphysical parsimony. I do not intend to settle this debate here. Instead I will end this part of the discussion by pointing out that, even if the criterion of metaphysical parsimony would favour a Humean account of laws and regularity comparativism, in the next part of the discussion we will see that such a principle brings with it a whole other threat to comparativism: eliminativism. Regularity comparativism finds itself in dangerous waters between the non-separable Scylla of Humean Empiricism (i.e. the dislike of necessary connections) and the eliminativist Charybdis of metaphysical parsimony.

---

<sup>117</sup>The issue remains of deciding which time is supposed to be the fundamental initial time (Casey McCoy, personal communication).

## Stopping criterion & Eliminativism

Another way in which the regularity approach might seem to cheat, is by being “too easy” [77, p.70], but in a different sense from the explanatory circularity cheat of helping itself to the explanandum. What is meant here is that, if the regularity approach succeeds in reducing inertial frames and absolute masses, why not use it to reduce any other notion as well [23]? Pooley, for instance, worries that the approach could be used to reduce even the temporal metric and the Leibnizian relations [70, p.567]? It seems we need a non-ad-hoc criterion for determining how much stuff we can heave over from the supervenience base to the supervenient level.

But in the case of regularity relationalism it is not really clear why this is worrisome. As long as purging the supervenience base still produces the correct laws, is that not a good thing? Especially Humean Empiricists should be happy to give up as many metaphysically fundamental notions as possible, under the banner of metaphysical parsimony.

In fact, Stevens successfully outlines how one might reduce metrical notions—as Pooley mentioned—in the Special Relativistic case, by reducing them to a mosaic consisting only of topological relations [68, 109, 110]. Pooley even discusses such a project a bit later in the same paper in which he expressed his worry [70, 6.3.2]. He does correctly warn us that we might not be able to throw away everything from the supervenience base, because at some point we would not expect the regularity approach to generate the Newtonian laws anymore (but presumably some ‘simpler’ laws) [70, fn. 88]. But from this it does not follow that we need some independent criterion to ensure that we stop throwing away stuff from the supervenience base at exactly that point; it is the regularity approach itself that tells us that it will break down at that point and we should thus not purge the supervenience base any further.

The case of regularity comparativism is importantly different. Empiricism, or more generally the desideratum of metaphysical parsimony, should welcome the opportunity of not only reducing absolute masses, but also mass ratios. In other words, **regularity eliminativism**—where the mosaic is purged of any fundamental mass determinates—if successful<sup>118</sup>, should be preferable over regularity comparativism. (In fact, this view was suggested for independent reasons by

---

<sup>118</sup>As mentioned in [Section 1.1](#), the mass relations require a certain constraint to be interpretable as mass ratios. As will be discussed in [Subsection 5.2.2](#), one might impose this constraint via an additional law. Requiring such an extra law would make any axiomatic system less ‘simple’ and thereby less ‘good’, and might thus suggest that the regularity protocol naturally breaks down. However, 1) this seems to be as much a problem for regularity comparativism as for regularity eliminativism, and 2) this only means that our overall best system—either in the comparativism or eliminativism case—is less good than we thought, not that it is not still the best axiomatisation of all the possible axiomatisations.

Hall [111], and praised for its virtues by Esfeld *et al.*<sup>119</sup>) But this would imply that, by adopting the regularity approach, the comparativist has once again thrown away the massive baby with the bath water. Given that the regularity comparativist had already recognised the empirical meaningfulness of absolute masses, as encoded in the different particle trajectories in the mosaic, regularity eliminativism would combine with weak absolutism—the corresponding picture of the world would be a purely spatiotemporal mosaic which grounds the masses which in turn ground the mass ratios. Comparativism has overshot its goal.

For the comparativist it then does become important to provide an independent criterion for preventing the reduction of mass beyond the absolute masses. In Chapter 6 I do in fact provide an argument against eliminativism *simpliciter* (which I there call ‘reductionism’). However, as will become clear in that chapter, it is exactly the regularity approach (to comparativism) which cannot help itself to that argument. I consider whether initial conditions and laws which are purged from any notion of mass by replacing it with spatiotemporal magnitudes could still solve the corresponding Initial Variables & Parameters Problem. The answer is negative. But along the way I discuss a less restricted attempt at reducing mass to spatiotemporal magnitudes by Narlikar (Subsection 6.4.2). Narlikar argues that, if we help ourselves to spatiotemporal magnitudes at a certain number of distinct instants—something we were not allowed to help ourselves to in the previous setting—we can reduce the mass ratios. Admittedly 1) there are some unsolved issues that are mentioned in a footnote, and 2) it is also not clear how the absolute mass scale can be reduced beyond the mass ratios. But the latter is as much a problem for regularity comparativism, as it is for regularity eliminativism. Regarding the former, we only ever successfully discovered Newton’s laws and the law of universal gravitation from a set of data consisting of a subset of the eliminativist mosaic; the eliminativist project in Narlikar’s sense must be true somehow.

In other words, it is unclear why any Humean Empiricist was committed to Primitivism—either about mass magnitudes or mass ratios—in the first place. Regularity eliminativism seems the most harmonious version of the regularity approach anyway. So much the worse for comparativism.

### 4.3.5 Conclusion

In this section we considered whether we could accept giving up the spirit of comparativism—anti-realism about absolute masses—whilst nevertheless retaining the letter—having those absolute masses not ground the mass ratios but being (partially) grounded in them. We used the the regularity approach, that is the

---

<sup>119</sup>See Loewer [112], Esfeld *et al.* [113–117] and Hubert [118].

Mill-Ramsey-Lewis Best Systems Account within the framework of (Humean) Supervenience, in an attempt to have the absolute masses supervene on a mosaic with fundamental mass ratios: regularity comparativism. Besides explanatory inadequacy, and a questionable consistency with the separability feature of the Humean framework, the fatal blow to regularity comparativism is that to the extent that it works, it works too well: there is no way of stopping at moving not only absolute masses but also the mass ratios to the supervenient level. This approach throws out the massive babies with the bathwater. It ultimately leads to weak absolutism, rather than a defence of (strong) comparativism over (strong) absolutism.

## 4.4 Avoiding the challenge

Hitherto we have considered comparativist attempts to face the challenge posed in [Chapter 3](#)—either head-on by denying the reality of absolute masses, or merely in letter—but to no avail. The only option that remains is to deny that comparativism has to face that challenge, or at least in its current formulation. A first route suggests that we can obtain full empirical equivalence with NG without solving the CIVPP challenge. In particular, it denies that theories require an initial value formulation, and they thereby perhaps even lack determinism. If we have some method to pick out the correct set of possible worlds as a whole, rather than by matching up (inexpressible) initial conditions with possible worlds, this should be sufficient. A second route, the most radical of all, points out that whilst empirical equivalence (obtained either via solving the CIVPP or via the previous route) is sufficient for empirical adequacy, it is not necessary. As long as we have a comparativist theory that includes a world that is consistent with the empirical data of the actual world (in the Newtonian regime), it is irrelevant whether it agrees with standard NG on the other dynamically allowed worlds. This revisionary route gives up on the criteria of soundness and/or completeness. In other words, it suggests creating a new theory that is different (at least semantically) from standard NG.

### 4.4.1 No Initial Condition Formulation

One assumption in [Chapter 3](#) that could be rejected is the demand for an initial condition formulation of the laws. If comparativist laws, or some other comparativist protocols, correctly pick out the set of DPWs as a whole, without generating each of those individual worlds from the initial conditions as per the CIVPP 4-step protocol, then there is nothing more that could be asked of the comparativist (contrary to what Skow [\[3\]](#) and Poincaré [\[1\]](#) claim).

Whereas the absolutist law G1 (p.34) and the ‘Machian’ comparativist law G3 (p.91) are formulated by reference to initial conditions—inserting them into Newton’s second law gives a differential equation—Dasgpta’s G2 (p.34) is not (as mentioned in Chapter 3.3). That is, given the initial conditions, G2 does not suffice to pick out a unique trajectory. As argued in Chapter 2, all Dasgupta did was check whether G2 was a statement that is true of all the scenarios described in that chapter. This is a much weaker requirement than picking out a unique trajectory given initial conditions. In fact, we should even question why L2 and G2 would deserve to be called laws. Nevertheless, we may ask whether G2 might serve as a constraint to pick out the set of DPWs as a whole (and thereby potentially earn the status of lawhood). That is, if we consider the subset of KPWs of which G2 is true at any instant of time, would this pick out all and only those worlds that are part of the desired set of DPWs?

It does not. It picks out too many worlds, thereby violating the soundness requirement. Start with a known solution (to absolutist NG),  $W1$ . Any world differing from  $W1$  by a uniform multiplication of the accelerations of all the particles by any function of time is also an allowed world, since this does not alter the acceleration ratios and G2 only constrains those ratios, not the absolute accelerations. This holds in particular for any discontinuous function of time, which would generate undesired discontinuous trajectories (or no well-defined trajectories at all)—a feature which is ruled out automatically if the law has the form of a differential equation. Even if we were to add some postulate that rules out such discontinuous absolute accelerations—although it seems impossible for the comparativist to formulate such a postulate without referring to the absolutist notion of absolute accelerations—other problematic functions would remain. For instance, in the two-particle scenario, having the two particles oscillate back and forth, with the acceleration relations being equal to 1, never resulting in escape nor collision, will also be a solution, that is it will also satisfy G2<sup>120</sup>. Using G2 as a criterion to pick out the whole set of DPWs violates the soundness condition. This condition is violated so extremely that determinism is violated in the most extreme way possible: not only are the worlds indeterministic at the initial point in time (against Laplacean Determinism), they are indeterministic at every time (against Bakerian Determinism). There is basically no predictive power as far as the trajectories are concerned, besides an extremely weak constraint on the evolution of the ratios of the magnitudes involved.

Perhaps we should consider protocols that operate directly on either the con-

---

<sup>120</sup>Using the conservative notion of empirical equivalence of worlds used in this thesis, which cares only about coincidence, this solution will technically be classified as ‘escape’. However, it seems plausible that more problematic examples arise once we move to systems with more particles and/or widen our notion of empirical equivalence of worlds to be sensitive to the variation of either ratios of distances or angles (i.e. shapes) over time.

figuration space or even the space  $K$  of KPWs. As Pooley points out, “[i]n some formulations of dynamics, the DP[W]s are singled out in terms of their relations to other KP[W]s” [70, p.558]. One such approach, in the space debate, is Barbour and Bertotti’s Relationalism [2, 55, 119]. They construct a metric (which is a function of the absolute masses) on shape space—a relative configuration space which treats only the ratios of distances within a configuration as physically meaningful—and pick out the DPWs via an action or geodesic principle, that is by extremising paths in shape space relative to that metric. This process is referred to as ‘best matching’. Since we only care about extremising the paths—that is we are *comparing* which one is shortest or longest—it might seem that the absolute mass scale is not relevant after all. However, each path in shape space corresponds to a different world, and we are thus in some sense comparing absolute notions between worlds, something that goes beyond the comparativist fundamental physics. It is nevertheless tempting to consider whether this protocol can still be successful with some metric which depends only on mass ratios, but such endeavour is beyond the scope of this thesis. If one decides to pursue this route, it is important to be aware that Barbour and Bertotti explicitly commit to comparativism about distance. Although this is *a priori* unrelated to comparativism about mass, we have seen in the case of ‘Machian’ comparativism that sometimes committing to comparativism about mass does have consequences for one’s views on distance, and vice versa. It should furthermore be noted that their approach is intended to pick out only a subset of the absolutist DPWs, and violates completeness. In that sense this approach belongs to the category of approaches discussed in the next subsection, where we will discuss this violation.

Instead of Barbour & Bertotti’s Relationalism, we will consider two simpler protocols. The first one starts from the absolutist set of DPWs and specifies a suitable mapping from that set to the set of comparativist KPWs. If that mapping is such that it maps absolutist DPWs to comparativist KPWs with exactly the same trajectories of particles and with mass ratios equal to those grounded by the absolute masses, this mapping will pick out the correct set of comparativist DPWs. Thus, rather than 1) creating absolutist models by evolving absolutist initial states via absolutist laws, 2) creating comparativist models by evolving comparativist initial states via comparativist law, and 3) checking for empirical equivalence, soundness and completeness by finding a suitable mapping  $\phi$  that maps models to each other based on their initial states (cf. the General Recipe for  $\phi$ , p.69), we skip step 2 and find the set of comparativist DPWs via a  $\phi$  that compares full histories. The second protocol also starts from the absolutist set of DPWs but simply strips them from their absolute ideology (i.e. the absolute masses, but not the mass ratios) to create the set of comparativist DPWs.

These two protocols, together with the regularity approach, form a category that I will call **piggy-back comparativism**, by analogy with what Arntzenius

calls piggy-back relation[al]ism [23]—hence they face some of the same problems as regularity comparativism. Arntzenius explains the basic idea behind piggy-back relationalism as follows. Start from the relationalist ideology. Decline the project of finding laws formulated directly in terms of this ideology. Instead, pick one’s favourite substantival theory, and state constraints on how histories of the relational ideology are allowed to be embedded in substantival histories. Piggy-back comparativism adopts this same protocol but with an ideology that includes mass ratios but no underlying absolute masses.

Earman’s response to a version of piggy-back relationalism makes salient the uneasy feeling that the piggy-back relationalist is somehow cheating:

This ... leaves the relation[al]ist in the position of claiming that the substantivalist tells a fairy tale using all sorts of fictive entities and at the same time admits that he cannot distinguish between the physically possible and the physically impossible without resort to those fictive entities. This is not a position I would feel comfortable occupying. [120, p.172]

Arntzenius himself expresses a similar sentiment:

[The piggy-back approach] would make life too easy for people who want to get rid of objects and properties that they do not like for some whimsical reason. For instance, suppose I were to claim that the world is pretty much as you think it is, except that rocks do not exist. I then go on to claim that the true theory of the world is that there are no rocks, and that the true theory of the world merely says that the things and properties and relations that there are (namely, everything other than rocks and their properties) are embeddable in a non-existing make-belief world which includes rocks and in which your favourite make-belief laws hold. It seems obvious to me that ... one should not believe that it is true. Theft is theft. Honest toil is honest toil. [23, p.170]

I take it that these feelings that the relationalist or comparativist is cheating should be fleshed out as the following two concrete worries—distinct but related—familiar from the criticism against regularity comparativism. Why cannot the comparativist make use of the absolutist worlds as a tool, without reifying them? Firstly, without reifying the absolutist worlds that are used to constrain the comparativist DPWs, this mysterious, conspiratorial constraint is completely unexplained<sup>121</sup>. Not only is this view faced with a huge explanatory loss—with explanatory power being one of the main reasons of introducing any notion of mass

---

<sup>121</sup>See Pooley [55, Ch.5] for a discussion of this worry in the case of relationalism.

in the first place (cf. [Section 3.1](#))—but there is an obvious cry for inference to the best (i.e. only!) explanation: the comparativist worlds are constrained as if there were fictitious absolute masses exactly because there are absolute masses! Secondly, this cheat creates a slippery slope, as Arntzenius’ quote emphasises: if we can conjure away absolute masses at a whim, why not mass ratios as well? The comparativist has been throwing away a jacuzzi full of massive babies with the bath water by now.

The only potential option left<sup>122</sup> to a comparativist who wants to generate the standard set of DPWs is a recently developed, novel mode of explanation called Constructor Theory, which explicitly rejects the “prevailing conception of fundamental physics” in terms of initial conditions and laws evolving those initial conditions [[121–123](#)]. Instead it proposes a reformulation of all of physics in terms of fundamental statements specifying which tasks are possible, which are impossible, and why. As constructor theory has not yet been applied to Newtonian Gravity, I will leave this approach to future work.

#### 4.4.2 Giving up soundness and/or completeness

G3 is analogous to Mach’s solution to the arguments from inertial effects—Newton’s bucket and globes—in one respect, but interestingly not in another more radical aspect. Both approaches crucially rely on all the matter in the universe, but G3 reproduces all the DPWs whereas Mach’s proposal violates the completeness condition<sup>123</sup>. Since our belief in the full set of DPWs of any theory has been arrived at only indirectly via our belief in a theory whose set of DPWs includes a world consistent with the empirical data of the actual world (at least in the relevant (eg. Newtonian) regime), we have no reason, *ceteris paribus*, to favour it over another theory with a different set of DPWs as long as that set also includes (a world consistent with the empirical data of) the actual world. After all, we perform experiments in the actual world, not in counterfactual worlds. In this final subsection we consider achieving empirical adequacy without requiring empirical equivalence to NG, that is by allowing violations of either the soundness or completeness condition, or both. In other words, we are searching for a revision (semantically speaking) of standard NG, that is a theory with different modal consequences, which is nevertheless both comparativist and empirically adequate. That the requirement of modal adequacy ([Subsection 1.2.3](#)) is violated is supposed

---

<sup>122</sup>Except perhaps for Hawking’s research programme, as discussed by Barbour [[2](#), Ch.22].

<sup>123</sup>Moving from Machian Comparativism to Doubly Machian Comparativism ([p.92](#)) by replacing  $r$  in G3 by  $r/\sum_j r_j$  would make it analogous to Mach’s response in both ways. This would mean the theory does violate completeness. On the one hand this might seem to make it stronger—in the Humean sense of ruling out more possible worlds, but see [p.147](#)—but it would also be more revisionary.

to be a feature rather than a bug: we revise the original theory by committing to different counterfactuals than the ones we originally thought to be true.

Let us briefly recall Mach's proposal in more detail. Mach insisted that it does not follow from Newton's bucket that relative distances and velocities do not suffice for a deterministic evolution [5]. Indeed, the relative velocity of the water and the bucket fail to do so, but other velocities—most notably those with of the bucket/water with respect to the fixed stars, or more precisely the bulk matter of the universe—may solve the problem. The only interesting scenario is an absolute rotation of the bulk matter—either in a crowded universe such as ours or in an empty universe containing only the globes or bucket experiment. In other words, the relationalist has no problem accounting for effects arising from relative local angular momentum, but only from absolute global angular momentum. There is thus a neat way of dividing the possible evolutions in two categories: those corresponding to a zero absolute global angular momentum, and those corresponding to a non-zero absolute global angular momentum. 'Machian relationalism' denies that the latter category is meaningful, and *a fortiori* that it is dynamically possible. Since measurements of the global angular momentum of our universe are consistent with a zero value, the set of DPWs of 'Machian relationalism' does include that of the actual universe, despite violating completeness by being a proper subset of the set of DPWs of standard NG.

It is important to realise though that one cannot simply deny that the inconvenient worlds are to be accounted for, without any background theory that indeed systematically excludes these worlds. To do so would be *ad hoc* (and it would be very unclear how to translate this to the symmetry and undetectability approaches). In fact, Mach himself failed to provide such a background theory, which is why I have been using quotation marks around 'Machian relationalism'. The above mentioned version of relationalism by Barbour and Bertotti is in fact an attempt at providing such a theory. Their best matching procedure picks out only possible worlds that, in substantivalist terms, feature a zero global angular momentum. If we were to apply the best matching process using a metric on shape space that depends only on mass ratios, perhaps we will obtain a further violation of completeness (by leaving out certain absolutist DPWs) whilst nevertheless being empirically adequate. However, although in the relationalism case it was clear which category of worlds we expect the theory to predict to be impossible—the ones that correspond to a non-zero global angular momentum, since the relationalist does not recognise that notion as meaningful—this is much less clear in the case of comparativism. The analogous global parameter might be the total energy. In the two particle case, the sign of the total energy, that is the comparison of gravitational and kinetic energy, determines whether the particles escape and collide. In multi-particle worlds, the total energy will play a similar role. However, which constraint on the total energy might the comparativist pro-

vide? Choosing any non-zero value is arbitrary, leaving the option of setting the total energy to zero, as if often believed to be the case in our universe. What would however be a justification for such a constraint, and in particular why would that justification be specific to comparativism—how would it follow naturally from a comparativist framework? Moreover, the absolutist story has no need for energy. Why would energy play an indispensable role in the comparativist framework? Not only does replacing the need for a single extra degree of freedom—the absolute mass scale—by a constraint on a new ‘entity/property/magnitude’, energy, sound metaphysically unparsimonious, fixing the total energy is mathematically equivalent to fixing the absolute mass scale, suggesting that this might just be absolutism in disguise.

None of this is to say that it is strictly impossible to come up with an empirically adequate comparativist theory that is not empirically equivalent to standard NG—G2 in fact includes the actual world in its set of DPWs but does so by violating soundness so egregiously that it is hardly better than a theory which just takes the full space of KPWs to be the set of DPWs and therefore has no predictive power whatsoever. The main point is that there is no guide at all as to what a sensible set of DPWs of such a theory would look like. If the comparativist manages to come up with a suitable (i.e., among other things, a theory that does not violate soundness to the extent that all or most of the determinism and predictive power vanishes), non-ad-hoc theory that is empirically adequate, it will be forgiven if that theory is not empirically equivalent to standard NG, but at this point the onus is on the comparativist to come up with any such theory in the first place. I have small hope this will occur.

## 4.5 Conclusion

I have considered several options available to the comparativist in response to the challenge posed in the previous chapter, the Comparativism Initial Variables & Parameters Problem. The most ambitious category of responses faces the challenge head-on. I have argued that Baker’s response boils down to reductionism about mass (i.e. solving the IVPP with an ideology purged from any mass properties). The viability of reductionism will be discussed in [Chapter 6](#). Baker thus moves away from both strong absolutism and comparativism, but in no way argues for (weak) comparativism over (weak) absolutism. Other responses consider alternatives to the standard ideology. They similarly boil down to weak absolutism, and/or face issues of metaphysical parsimony. I propose a Machian version of comparativism which replaces Newton’s constant in the absolutist law with a variable across possible worlds (since it depends on the sum of the mass ratios in a world). I believe this novel version of comparativism to be the most viable. Its metaphys-

ical parsimony compared to absolutism is still not obvious though. Moreover, it manages to satisfy the completeness criterion only in a slightly unnatural manner.

The second category contains only a single response: regularity comparativism. This response accepts that the spirit of comparativism—anti-realism about absolute masses—has to be given up, but insists that the letter can be saved: the absolute masses scale can be obtained for free at the supervenient level, from a comparativist Humean mosaic. This approach faces severe problems. It is explanatorily inadequate, the non-separability of comparativism clashes with the desideratum of a separable mosaic, and most fatally the approach works too well since it can and should be used to get rid not only of fundamental masses but also fundamental mass ratios. The regularity approach fits best with weak absolutism.

The responses in the final, least ambitious category deny having to respond to the challenge, by disagreeing with various ingredients that went into the set-up of the CIVVP. In particular, I considered giving up the requirement of 1) picking out the set of DPWs via laws and initial conditions, in favour of picking out the whole set of DPWs via more holistic constraints; and 2) completeness and/or soundness. Several simple options were dismissed. Barbour and Bertotti's best matching approach and Constructor Theory might provide interesting avenues for further research.

# Chapter 5

## Theoretical Virtues

Up to this point the focus has been solely on the empirical adequacy premise in the argument for comparativism. In this chapter I will question some of the supposed theoretical virtues of comparativism, which feature in the other premises. I will begin with a brief discussion of the non-locality of comparativism before moving on to the main focus of this chapter: metaphysical parsimony and explanatory power.

### 5.1 Non-locality

Einstein, amongst many others, pointed out that one of our main desiderata for physical theories is for them to satisfy some principle of locality [124] (cf. [Subsection 1.2.3](#)). As Dasgupta admits, comparativism is non-local. We may further distinguish *dynamical locality* (or a principle of local action) and *structural locality* (or a principle of separability [125]). Dynamical non-locality comes in two forms, although they often appear together: instantaneous action-at-a-distance and unmediated action-at-a-distance. Non-separable properties are such that the properties of a region (in space and/or time) do not supervene on the corresponding properties of the subregions that make up the larger region ([Subsection 4.3.1](#)). As mentioned above, and is particularly clear from [Figure 2.1](#), comparativism about mass is non-local in the sense that it is non-separable—especially the version expounded in [Subsection 4.2.3](#).

No theory that is non-local in any of the above senses is thereby logically inconsistent. Although all three senses of non-locality may be counter-intuitive, none of them forms a contradiction—unless we move to Special Relativity where instantaneous action-at-a-distance leads to temporal paradoxes (*pace* Maudlin [126]). In fact, the Gravitational Force in Newtonian Gravity is notoriously non-local,

in both of the dynamical senses<sup>124</sup>. Entangled systems in quantum mechanics are, arguably, non-separable. There is nevertheless a difference with this example and comparativism about mass: when explaining entanglement phenomena with a non-separable theory we are explaining something counter-intuitive that could not be explained other than by counter-intuitive building blocks; when explaining phenomena in NG by making reference to non-separable mass relations we are explaining something by positing counter-intuitive building blocks that could have been explained extremely simply by absolute masses instead. Although I find this non-separability both counter-intuitive and unnecessary—absolutism does not require it—I imagine the comparativist will simply bite the bullet. After all, Einstein’s motivation for locality was that without it we could not ever perform any experiment, since there would be no such thing as an isolated laboratory, but 1) this just reduces to the requirement of empirical adequacy—if comparativism were empirically adequate (contrary to this thesis) then we could in fact do localised experiments—and 2) this worry only applies to dynamical locality and not necessarily to non-separability.

## 5.2 Metaphysical Parsimony & Explanatory Power

Dasgupta and Baker take it for granted that comparativism is more metaphysically parsimonious than absolutism. I take the naive intuition behind this assumption to be the thought that absolutists acknowledge both intrinsic masses and mass relations<sup>125</sup> while comparativists only recognise the latter, and therefore the comparativist has a ‘lower metaphysical bill’. But we have already seen that the absolutism–comparativism debate is a debate about grounding, that is about relative fundamentality, not about realism about absolute masses. It is consistent for the comparativist to ‘acknowledge’ absolute masses. But even if we were to focus only on realist absolutists and anti-realist comparativists, this intuition is mistaken. When an absolutist god builds the world (s)he ‘pays’ for intrinsic masses only. The mass relations emerge, that is they are facts that are true of this world in virtue of the fundamental intrinsic masses. They come free of charge. (Similarly, a comparativist who is a realist about absolute masses gets those absolute masses for free.) The comparativist god uses a completely different type of building block: mass relations. It is *prima facie* unclear which building block ‘costs more’, metaphysically speaking. The naive intuition did not require any (detailed) measure of metaphysical parsimony since it took the comparativist

---

<sup>124</sup>Assuming, as we have been doing in this thesis, that we are focusing on a particle theory, rather than a field theory. A field theory need not violate unmediated action-at-a-distance.

<sup>125</sup>In this chapter it is important to recall [fn. 3](#).

to require only a proper subset of the things the absolutist needed. Since they in fact require distinct sets of things a measure of metaphysical parsimony becomes relevant.

### 5.2.1 Quantitative & Qualitative Metaphysical Parsimony

Do we care not about the number of types of ‘things’ our theory postulates, but only about the number of tokens or instantiations? Or do we aim for qualitative parsimony rather than quantitative parsimony: minimalisation of the number of types of things (entities, relations, spaces, laws, etc.), whilst getting an infinite stock of instantiations of each type for free [127–129]? Or should we aim for a mix of both, and if so, how? And do we weigh each type equally?

I am skeptical that there is a rigorous answer to these questions in general, which is why I will not attempt to provide a rigorous measure of parsimony here. Nevertheless, as will be discussed in this subsection, in the specific case of comparativism both the quantitative and qualitative metaphysical parsimony are sufficiently questionable to at least shift the onus of proof to the comparativist. More importantly, in the next subsection I will point out a strange conspiracy implicit in the standard formulation of comparativism. Although I believe that under any plausible measure of (qualitative) metaphysical parsimony the comparativist would lose all its (alleged) parsimonious advantage after having to introduce an additional structure to dissolve this conspiracy, a simple inference to the best explanation argument will also do to argue that the conspiracy defeats the comparativist.

With regards to quantitative parsimony, it seems comparativism obviously loses out (but see [Subsection 5.2.3](#)): for a world consisting of  $n$  particles, its ideology consists of  $n(n - 1)$  (equally) fundamental instantiations (of relations)—or perhaps even  $n^2$  if we include fundamental mass relations between the particle and itself—whereas the absolutists needs only  $n$  (equally) fundamental instantiations (of monadic properties). Things get even worse if we add the fundamental trans-temporal relations that Dasgupta’s Comparativism requires to deal with the temporal intra-world Leibniz scaling scenarios ([Subsection 2.3.2](#))<sup>126</sup>. Comparing the qualitative parsimony is less straightforward. It seems the only way in which we can compare these different types of beasts is via their structure<sup>127</sup>. As mentioned in [Subsection 1.1.3](#), absolute masses are in fact structurally quite similar

---

<sup>126</sup> Although, one could argue that no serious problem arises from temporal intra-world Leibniz scalings, since such scalings are not physically possible within NG (Christopher Timpson, personal communication). No modifications of the ideology are then required.

<sup>127</sup> Although, arguably, mass ratios being closer to an operational understanding of mass, in the sense of kinematic comparativism, may be a reason for considering them to ‘cost less’ than absolute masses (Christopher Timpson, personal communication).

to mass relations. Both types of determinates are structured in two ways: a total order and a concatenation rule. In the case of strong absolutism, the absolutist requires additional primitive transworld identities. Qualitatively speaking the metaphysical parsimony of both views thus seems almost on a par, with strong absolutism perhaps losing out slightly. That is, if we consider standard (say Dasgupta’s) comparativism; the ‘Machian’ form of comparativism expounded in [Subsection 4.2.3](#), for instance, actually requires additional structure. Moreover, in the next subsection I will suggest that even standard comparison needs even more structure. Perhaps a comparison of these structures is not the best way of measuring qualitative parsimony, but at this point the onus of proof lies with the comparativist to suggest an alternative measure.

## 5.2.2 The conspiracy of mass relations

In [Subsection 1.1.1](#) I discussed how the two types of mass determinates relate, mathematically. A set of absolute masses *always* determines a unique set of mass ratios. A set of mass relations on the other hand can only be interpreted as a consistent set of mass ratios if they satisfy a certain *constraint* (i.e. concatenating the relations of any closed loop needs to result in the identity—see also [Eq. 6.2](#) and the surrounding discussion). Not any set of mass relations will do. The mass relations conspire<sup>128</sup> in such a way as to allow for an interpretation as mass ratios, that is to allow for a representation by absolute masses ([Figure 5.1.a](#)) (albeit a non-unique one). To ensure/explain this behaviour it seems the comparativist needs constraining relations between the mass relations: meta-relations. Whereas the absolutist needs ( $n$ ) absolute properties, the comparativist seems to not only need (between  $n(n - 1)$  and  $n^2$ )<sup>129</sup> relations, but in addition a number of meta-relations. This significantly reduces both the qualitative and quantitative parsimony of comparativism. Alternatively one might impose an additional law that ensures this constraint, but this also comprises a cost in (quantitative) parsimony.

This situation is closely related to two other *prima facie* conspiracies, which arise within the framework of relationalism about space. Arntzenius points out that the possible distances between objects are severely constrained [[23](#), Ch. 5]. For instance, in Euclidean space, five particles cannot be equidistant; the distance between particle A and B cannot be shorter than the sum of the distances between A and C and B and C, etc. If distances are fundamental these constraints seem rather conspiratorial—cf. our discussion of piggy-back relationalism and comparativism in [Subsection 4.4.1](#). Meta-relations between the distances are needed to

<sup>128</sup>This conspiracy has recently been acknowledged as well by Roberts [[13](#)], who traces it back to Russell [[18](#)] and Armstrong [[15](#)].

<sup>129</sup>Even more if we include trans-temporal relations ([Subsection 2.3.2](#)).

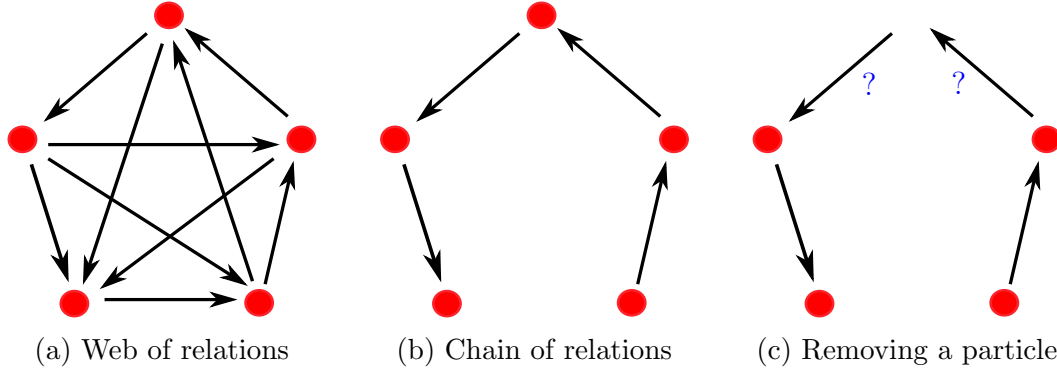


Figure 5.1: Examples of five-particle worlds and their fundamental mass relations, illustrating (a) the overdetermination of mass relations and the corresponding conspiracy that is required for the relations to be interpretable as mass ratios (i.e. all ‘loops’ need to combine to the identity; not illustrated), (b) the Chain Comparativism response, and (c) the counter-response.

ensure that all the constraints hold. What about the substantialist framework? If the metric is interpreted as a mapping from all pairs of points to the real numbers, we obtain a similar overdetermination which requires constraints to avoid inconsistencies<sup>130</sup>. Instead the metric is to be interpreted locally: it maps ‘neighbouring’ spatial points to the real numbers. Distances between distant points or objects are then obtained by integrating over the infinitesimal distances (along the shortest path): no overdetermination, no meta-relations needed to constrain anything. The relationalist about space has no such local metric available, or at least not in theories such as Newtonian Gravity which consist of point-particles rather than a plenum: distances are only defined between (distant) material objects. The comparativist about mass has a similar problem: if the universe was filled with a (relative) density field, mass relations of distant objects could be obtained by multiplying a continuous chain of mass ratios. Here we are however treating Newtonian Gravity as a particle theory, not a field theory.

A second conspiracy arises when we attempt to interpret Newton’s First Law within the relationalist framework about space. “[F]orce-free bodies ... conspire to move in straight lines at uniform speeds while being unable, by *fiat*, to communicate with each other” [51, p.14-15]. This is what Brown dubs the conspiracy of inertia. All force-free particles move as if they are following the affine structure of a single background space. In the relationalist framework particles do not live in such a space, and it is therefore a miracle (cf. the explanatory circularity of regularity relationalism/ comparativism in Subsection 4.3.4) that all the primitive independent (relative) velocities of all the (pairs of) particles allow for a simple co-

<sup>130</sup>See Belot [130, p.25-27] and references therein.

ordinate system—what we call the inertial coordinate system—in which the above mentioned conspiracy of inertia obtains. It seems the relationalist again needs to invoke some meta-structure to constrain these otherwise independent velocities.

Not only do all the meta-structures required to deal with these conspiracies make the comparativist/relationalist theory fatally unparsimonious—against  $P_{\text{occ-par}}$  (p.22)—but, more strikingly, all these conspiracies follow the schema ‘the relations conspire to behave exactly as if the absolutist/substantialist theory were true’. The obvious move is to *infer to the best explanation*: absolutism (substantialism) is correct. These conspiracies can be *no miracle*—this would constitute a substantive loss of explanatory power relative to the absolutist theory, against  $P_{\text{exp}}$  (p.23). If the absolutist type solutions to these problems dissolve these conspiracies trivially, this is sufficient reason to opt for absolutism, regardless of metaphysical parsimony.

### 5.2.3 Chain Comparativism

The comparativist might respond by conceding that once we attribute to them a web of fundamental relations between each pair of objects then the overdetermination of mass relations suggests a weird conspiracy (if inconsistencies are to be avoided when attempting to interpret them as mass ratios), but they never in fact insisted on overdetermining these mass relations<sup>131</sup>. If we have the mass relations form a single chain, as in Figure 5.1.b, by having each particle be the relatum of only *two* mass relations (except for the particles at the beginning and end of the chain which each are the relatum of only one mass relation), the mass relations between all particles in the universe are determined without any overdetermination<sup>132</sup>. Not only does **Chain Comparativism** avoid the conspiracy of mass relations, it also seems quantitatively more parsimonious, and even qualitatively insofar as it does not require meta-relations. I will push back against this supposed gain in parsimony in the next subsection.

I am not aware of any Chain Comparativists. I read authors such as Field [6] and Dasgupta [8] to assume **Web Comparativism**, at least implicitly, and when

---

<sup>131</sup>One might respond that it is simply in the essence of mass that all massive objects stand in fundamental—or equally fundamental, in the case of weak comparativism—mass relations to all other massive objects. In other words, it is in the essence of mass that all forms of comparativism must be web comparativism and cannot be chain comparativism. In the space case this may seem plausible: if points 1 and 2 stand in a distance relation, and points 2 and 3 stand in a distance relation, then points 1 and 3 must also instantiate a distance relation—this is just what it is to be a distance. It is not directly clear that this intuition is as obviously correct in the mass case.

<sup>132</sup>Whether such a move is desirable for the relationalist about space, as a response to the conspiracy pointed out by Arntzenius, is an interesting question which is nevertheless outside of the scope of this thesis.

Dasgupta discusses mass-counterpart theory even quite explicitly [8, §3]. Similarly I am not aware of any Chain Relationalists about space. Perhaps none of these authors has any problems moving to Chain Comparativism/Relationalism. It would imply a certain holism. One of the basic assumptions in experimental physics is the assumption that one can do experiments that are isolated from the rest of the world (for all practical purposes)—cf. [Section 5.1](#). For instance, when we measure the force between two particles in a lab that are interacting gravitationally (for instance the two endpoints of the chain in [Figure 5.1.b](#)), we assume that the force only depends on those two particles. Chain Comparativism would imply that the force between those two endpoints depends on the mass relation determined by the whole chain across the whole universe. It seems however that this would not lead to any different empirical predictions. For all we know the world is just like this—something that we have seen the Machian comparativist already commit to. Similarly, it does not seem elegant that two particles are the relatum of only one mass relation, and the other particles of two mass relations. Again, this might just be the way the world is.

If we however consider the following intuition a valid intuition, Chain Comparativism faces a problem. It seems that if God would remove one massive particle at the far end of the universe, this should (for all practical purposes) not have any empirical effect on Earth. If Chain Comparativism were true, the removal of one such particle would break the chain into two chains ([Figure 5.1.c](#)). The mass relations between any pair of particles on different chains are then not defined. It is not even just the case that Chain Comparativism predicts that such a removal would have an effect on Earth: physics on Earth would become ill-defined.

## 5.2.4 Metaphysical parsimony at the level of worlds

In light of the focus of this thesis on possible worlds, why not adopt an inter-world version of the Occamist norm (of metaphysical parsimony), rather than the intra-world approach above? As suggested in [Subsection 1.2.4](#), a Leibnizian-style Principle of the Identity of Indiscernibles may be applied to whole worlds, by equating metaphysical parsimony with minimalisation of the amount of metaphysically distinct possible worlds per empirical equivalence class (or at least so within the set of dynamically allowed possible worlds). In other words, if  $\phi$  is many-to-one (within  $S_2$ ), this would imply that  $T_2$  (comparativism) is metaphysically more parsimonious.

As pointed out on [p.69](#), a  $\phi$  that maps worlds labelled by the standard absolutist ideology to worlds labelled by the associated comparativist ideology is indeed many-to-one. But the absolutist point was exactly that the comparativist was not acknowledging a metaphysical distinction that does in fact make an empirical difference. It is then an open question for any alternative choice of

ideology, or any completely different version of comparativism, whether it is more parsimonious than absolutism in the sense just described. The choice of ideology  $\{r_{ij}, m_i, \varphi_i\}$  (Subsection 4.2.2) seems to correspond to a one-to-one  $\phi$ , but that is due to this being in fact a (weak) absolutist theory. Regularity comparativism, by accepting realism about absolute masses, also recognises possible worlds that differ in the absolute masses but not in the mass ratios (since the latter only partially ground the former). Only Machian Comparativism seems to correspond to a many-to-one  $\phi$ .

It should be noted that the choice between this inter-world version of the measure of metaphysical parsimony and the intra-world version is not mutually exclusive. In fact, in light of the ‘all other things being equal’ clause in the argument for comparativism, we would require comparativism to come out favourably with respect to each of these measures, or at least with respect to the overall (i.e. both intra- and inter-world) parsimony. It is not obvious that comparativism is the winner here.

In this light, although moving from web comparativism to chain comparativism indeed improves both the quantitative and qualitative intra-world parsimony, it comes at the cost of a reduced inter-world parsimony. There are many ways in which one could generate a ‘chain world’ corresponding to a single ‘web world’. For a chain comparativist, choosing any single chain that connects all the particles in the world is an arbitrary choice. All of these choices are (empirically) equivalent, but nevertheless metaphysically distinct. This leads to a large proliferation of redundant possible worlds, whereas the whole point of comparativism was to do with no metaphysical distinctions without an empirical difference. Chain comparativism does not seem to do much better than absolutism in this regard.

### 5.2.5 Conclusion

Comparativists take it to be obvious that comparativism is metaphysically more parsimonious than absolutism, and therefore do not provide any detailed justification for this assertion. Comparing metaphysical parsimony is a tricky business, if at all possible. The issue is straightforward only in cases where one theory commits to a fundamental metaphysics that is a proper subset of the opposing theory’s fundamental metaphysics. This does not apply to the absolutism–comparativism debate. In this section I have considered the metaphysical parsimony of comparativism along the dimensions of quantitative vs. qualitative parsimony, and intra-world vs. inter-world parsimony. Along all these dimensions the parsimony of comparativism is questionable. In order to make their case, the onus is on the comparativist to tell us a proper story of why their view is so clearly more parsimonious.

# Chapter 6

## Reductionism

The previous chapters formed a single, sustained argument against comparativism, which has now been completed. This chapter, although it makes use of much of the terminology and methodology developed in the previous chapters, is a standalone argument. It negatively evaluates the viability of reducing mass (to spatiotemporal magnitudes within Newtonian Gravity), that is eliminating it from the fundamental ideology of Newtonian Gravity—in the specific sense of solving the General Initial Variables & Parameters Problem without any mass determinates, neither absolute masses nor mass ratios—a *prima facie* possibility that was introduced in [Subsection 1.1.2](#). It nevertheless relates to the overall project in four ways. Firstly, it is yet another aspect of Laplace’s problem, which, as explained in the introduction to this thesis, is the umbrella problem that connects the substantivalism–relationalism debate and the absolutism–comparativism debate. The question that was central to that latter debate—can we solve Laplace’s problem with mass ratios but without absolute masses—is taken one step further: can we solve Laplace’s problem without any fundamental notion of mass whatsoever? A second, specific motivation for evaluating whether the GIVPP could be solved with a purely spatiotemporal ideology is that, as I have argued, that is what Baker’s project boils down to ([Subsection 4.2.1](#)). Thirdly, as part of the historical set-up of this specific type of reduction, we will come across a less restricted type of reduction ([Subsection 6.4.2](#)) that relates to regularity eliminativism ([Subsection 4.3.4](#)). Finally, the previous chapters argued for absolutism over comparativism, without distinguishing between the weak and strong versions. This chapter argues for the strong versions, rendering the overall conclusion that we should favour strong absolutism.

## 6.1 The Project

Recall from [Section 3.1](#) that the notion of (absolute) mass was originally postulated to explain why certain observable trajectories are allowed by nature and other trajectories are not. In particular, postulating this primitive notion provided three virtues. Including mass in our theory allows us to 1) uniquely *predict* future states of the world based on past data (i.e. solve Laplace’s problem). But not only that. Provided there are no unexplained constraints on the range of the initial data, we can also 2) *explain* the observed or allowed particle trajectories. Finally, if we range over all the values the initial variables and parameters could take, and solve the initial value problems for each of these cases, we obtain the correct set of empirically possible worlds. In other words, including mass gives us the correct 3) counterfactuals and other *modal claims* of the Newtonian Theory of Gravity. The requirement for any alternative to (strong) absolutist NG to recover these three virtues was incorporated in the General Initial Variables & Parameters Problem.

In the previous chapters, motivated by a preference for metaphysical parsimony, we considered whether this problem could still be solved after purging the absolute mass scale from the initial conditions. In this chapter we consider whether mass can be reduced to spatiotemporal facts, that is whether the GIVPD can be solved with a purely spatiotemporal initial state—distances, velocities, accelerations and perhaps higher-order time derivatives—and laws referring only to those spatiotemporal magnitudes. One, modest motivation for this intra-theoretical<sup>133</sup> reduction is again the by now familiar desideratum of metaphysical parsimony, which also motivated the (strong) comparativist. This is to be distinguished from the much more radical motivation that drives many empiricists: the urge to get rid of all unobservables, regardless of any other virtues they might have, such as explanatory power. It will become important later on that it is merely the more modest motivation—a methodological principle that I believe everyone should apply, regardless of any sympathies for empiricism—that drives the specific project of this chapter.

I will argue against reductionism about mass, and thereby in favour of the strong versions of absolutism and comparativism. (Together with the earlier argument for absolutism this combines to an argument in favour of strong absolutism.) In particular, it will become clear that the reductionist project lacks explanatory power. Whereas in the ‘mass theory’—that is the standard theory which postulates a primitive notion of mass—the initial variables and parameters

---

<sup>133</sup>I am not considering the option of reducing Newtonian Gravity as a whole to another theory, and identifying the counterpart of mass within that theory. Here I am purely interested in reducing mass to the other primitives within Newtonian Gravity.

can take on all<sup>134</sup> possible values, the reductionist theory exhibits ad-hoc, holistic, brute (i.e. *unexplained!*) constraints on the initial values (which cannot even be formulated without piggy-backing on the mass theory). Moreover, even the initial conditions that are allowed fail to correspond to unique evolutions of the system. The reductionist project also lacks in predictive power.

The most obvious way to proceed with this reductionist project seems to be to find an operational definition of mass in terms of spatiotemporal magnitudes. We can then directly substitute<sup>135</sup> the notion of mass in the initial state and the laws with these spatiotemporal magnitudes, and our work is done. Mach's famous operational definition of mass immediately springs to mind. Indeed, we will shortly start our discussion with Mach. However, it is important to flag at this point once more that Mach's more radical, empiricist project is substantially different from the more specific project just outlined, as will be discussed in more detail below. Thus, although it would be imprudent not to start off our discussion with Mach's famous operational definition, Mach exegesis is not the aim of this chapter; as soon as our project diverges from his, we will leave Mach behind.

Before moving on we need to make more explicit what was already implicit in the previous story: how far do we want to go? One current strand of research considers a moderate version of reducing mass. It considers taking *some* notion of mass to be fundamental, namely the mass determinates—i.e. it is a matter of fact whether two massive particles are equally massive or not—but aims to derive its further quantitative structure—ordering<sup>136</sup>, metric<sup>137</sup> and additive structure<sup>138</sup>—from for instance the dynamics<sup>139</sup>. Both the current and the Machian project are interested in reducing mass altogether<sup>140</sup>, not merely its quantitative structure. Keeping in mind the desideratum of metaphysical parsimony, it is simply not clear what work the fundamental mass determinates are still supposed to do once their quantitative structure has already been reduced away.

In the next section I discuss a bad argument against reductionism by McKinsey, Sugar & Suppes, before proceeding to Mach's operational definition of mass in [Section 6.3](#). [Section 6.4](#) treats historical responses to Mach, culminating in

---

<sup>134</sup>See [fn. 72](#).

<sup>135</sup>Zanstra [[35](#)] adopts a similar 'substitution approach' in the analogous debate on relationalism about space.

<sup>136</sup>Whether a massive particle is less or more massive than another particle with a different mass determinate.

<sup>137</sup>The ratio between the masses of two massive particles.

<sup>138</sup>How the mass of one massive particle compares to the combined mass of two other massive particles.

<sup>139</sup>Dees [[26](#)] advocates a position like this. Roberts' view boils down to the same thing [[13](#), Appendix]. Perry [[131](#)] considers a more varied range of reductionist projects, including the Machian project.

<sup>140</sup>See also Esfeld & Deckert's project [[116](#), [118](#)].

my main argument against reductionism in [Subsection 6.4.3](#). [Section 6.5](#) responds to two loopholes in the argument. The final section teases out a different line of argument that has been looming in the background, resulting once more in the conclusion that Newtonian mass is not reducible to spatiotemporal magnitudes without loss of explanatory and predictive power.

## 6.2 A bad argument against reductionism

Before properly starting our main discussion with Mach in the next section, we will quickly discuss and even more quickly dismiss a famous argument against reducing mass to spatiotemporal magnitudes. McKinsey, Sugar & Suppes [64] famously provide an axiomatization of Newtonian mechanics in terms of the primitive notions of mass, position and force. They point out that for mass to be reducible it would have to be definable in terms of the other notions<sup>141</sup>. Hence, it should be impossible to find two distinct worlds allowed by the theory which differ solely with respect to the primitive mass, but not with respect to the other primitives. They then claim to provide two such worlds: both worlds contain one particle only, which is at rest at all times, and has zero force acting upon it, but the mass values differ between the worlds.

As mentioned, we are here not interested in eliminating mass by reducing it to an alternative unobservable, metaphysical primitive such as force. Nevertheless, if an axiomatization of Newtonian Gravity were to be provided in terms of the primitives mass, position, velocity and acceleration, an analogous argument to the one above might be generated. Presumably such a theory would include two allowed worlds, each with one particle at eternal rest, but with different masses.

Both arguments deserve the same response. Although both worlds are indeed distinct (metaphysical) worlds allowed by the theory, they fall in the same empirical equivalence class—in both cases the particle is simply at rest and its property of being massive is trivially irrelevant, observationally speaking. In so far as we only care about worlds to the extent that they get the observables right, this pair of worlds does not provide an interesting counter-example to reductionism.

*A fortiori*, the reductionist may insist that McKinsey, Sugar & Suppes' argument backfires, since it seems to in fact work against the mass theory that it acknowledges metaphysical distinctions that are empirically indistinguishable. This violates the Principle of the Identity of Indiscernibles (cf. our discussion of the inter-world version of Occam's razor in [Subsection 5.2.4](#)). But this would be an overreaction. It is not at all surprising that a property which is supposed

---

<sup>141</sup>Regularity comparativism seems to be a counterexample to this claim, unless we have such a broad notion of 'definiability' that applying the regularity protocol to obtain emergent masses from the full Humean mosaic counts as defining those masses.

to manifest itself by influencing other particles is empirically idle or dormant in a world where there are no other particles to interact with. What would count against mass is if it were to have no observable consequences in any world allowed by the theory, or at least not in the world that corresponds to the actual world. The pair of worlds discussed in this section does nothing to support this stronger claim.

## 6.3 Mach

Having put aside this argument against reductionism, we turn to the most famous positive attempt to reduce mass. Mach is well-known for providing the first operational definition of inertial mass [5]. He vehemently opposed employing ‘hidden’ metaphysical notions such as mass in physics in order to *explain* observable phenomena. The task of physics is merely the “abstract quantitative expression of facts” [5, p.502] concerning the relations between observable phenomena. He defines inertial mass (relations) in terms of observable accelerations (or more correctly, acceleration relations) only, a feat that so inspired the logical empiricists.

Consider two particles, which are either alone in the universe or approximately dynamically isolated from any other matter. If  $F_{12}$  is the force exerted on particle 1 by particle 2, and  $F_{21}$  the force exerted on particle 2 by particle 1, then Newton’s third law gives  $F_{12} = -F_{21}$ . But Newton’s second law also gives  $F_{12} = m_1 a_{12}$  and  $F_{21} = m_2 a_{21}$ , where  $a_{12}$  denotes the acceleration of particle 1 due to particle 2, and vice versa. Combining this to eliminate the (directly) unobservable notion of force, we obtain<sup>142</sup>:

$$\frac{m_1}{m_2} = -\frac{a_{21}}{a_{12}}. \quad (6.1)$$

We have operationally defined the mass ratio of these two isolated particles, or simply “named” [5, p.266] their acceleration relations as Mach would put it.

---

<sup>142</sup>Note that in general  $\frac{a_{21}}{a_{12}}$  depends on the reference frame, although it will be constant across inertial reference frames. Mach’s definition therefore also depends on the first law [32, p.15], which provides the notion of an inertial frame. However, operationalising the inertial frames brings with it its own problems. Pendse [132] proves that there exists a set of infinitely many special non-inertial frames such that observers at rest with respect to those frames obtain positive and constant values for the mass ratios via Mach’s operational definition, which nevertheless differ from the corresponding values found in the inertial frames. Importantly, those observers will not be able to tell that they are not in an isolated, inertial frame. Note that these problems are irrelevant to the project in this chapter: we take the initial spatiotemporal magnitudes *with respect to some inertial frame* to be given, and use them to attempt to calculate the emergent masses and the evolution of the system. The Machian project—“the heuristic aspect” in Pendse’s terminology [132, p.55]—on the other hand starts out with observed trajectories only, and needs to somehow reconstruct the *inertial* spatiotemporal magnitudes before one may derive the mass (ratios). (Cf. Subsection 4.3.2.)

Of course our actual universe does not consist merely of (subsystems of) two isolated particles. Perhaps we could manually approximately isolate such systems (in turns) on the surface of the Earth where we dwell—or, since we can never perfectly isolate any system from gravity, obtain the mass ratios via the limit of a series of better and better isolated two-particle subsystems—but this would not be an option when considering celestial objects [133]. It definitely will not be an option in the generic messy, crowded worlds that we consider in this thesis<sup>143</sup>. This raises the important question of whether the previous procedure can be consistently generalised to a larger system of interacting particles. Two issues arise. Firstly, the operational definition depends on the component of the acceleration of a particle that is induced by a *single* other particle, whereas we only have empirical access to the *total* acceleration. In [Subsection 6.4.1](#) we will consider whether these individual components can be retrieved from the total acceleration.

The second issue is prior to the first, since it arises even if we could (*per impossibile*, in general) isolate each pair of particles (in turns). Consider a universe with three or more particles. Step 1: take particle 1 and 2 away from the other matter, such that they form an effectively isolated subsystem. Obtain their mass ratio from the acceleration ratio via Mach’s protocol. Step 2: repeat for particle 2 and 3. What can we now expect if we repeat this for particle 1 and 3? Will it satisfy the following consistency check:

$$\left(\frac{m_1}{m_3}\right)_{s_3} = \left(\frac{m_1}{m_2}\right)_{s_1} \cdot \left(\frac{m_2}{m_3}\right)_{s_2}, \quad (6.2)$$

where  $s_i$  indicates the instance of the operational procedure (i.e. step) used to determine that mass ratio? If absolute masses are primitive properties of the particles, this condition is satisfied as a matter of *logical necessity*. In Mach’s framework, this equation is satisfied only if

$$\left(-\frac{a_{31}}{a_{13}}\right)_{s_3} = \left(-\frac{a_{21}}{a_{12}}\right)_{s_1} \cdot \left(-\frac{a_{32}}{a_{23}}\right)_{s_2}. \quad (6.3)$$

But the accelerations which particle 1 and 2 induce in each other and which particle 2 and 3 induce in each other place no (logical) constraints on the accelerations which particle 1 and 3 induce in each other. Mach acknowledges this. For him it is just a *brute empirical fact* that it does not matter which particle we use as a standard to compare every other particle to; any standard will provide the same

---

<sup>143</sup>Here I sympathise with Barbour’s warning not to be “misled by the special circumstances of our existence. ... Take a billion of particles and let them swarm in confusion - that is the reality of ‘home’ almost everywhere in the universe. The stars do seem to swarm... We must master celestial [determination of mass ratios] and not be content with the short cuts that can be taken on the Earth, for they hide the essence of the problem” [2, p.137-8].

mass ratios. But this is just another way of saying that the reductionist assumes a highly mysterious and holistic fact without any explanation whatsoever (cf. the conspiracy of mass ratios in [Subsection 5.2.2](#)). A fact that is trivially explained—as a matter of logical necessity!—if absolute masses are taken to be fundamental. We here encounter the first loss of explanatory power for the reductionist.

If this empirical fact is nevertheless assumed, one can then proceed by *choosing* one of the particles as the standard unit of mass, say 1kg or 1lb, in order to fix all the other masses (or more precisely the quantities representing them) via the consistently determined mass ratios. (Note that knowing the absolute accelerations would not by itself help to fix the absolute masses nor the quantities representing them.) Mass thus seems to have been reduced to acceleration relations.

### 6.3.1 Mach & Comparativism

Before evaluating Mach’s definition *qua* reductionist project, it is worthwhile pointing out that this definition also makes him a weak comparativist about mass. He operationally defines only mass *ratios*, if anything, and arrives at *absolute magnitudes* (or more precisely the quantities that represent them) merely via a convention.

However, one of the main points of this thesis was to expose the empirical meaningfulness of absolute masses over and above mass ratios. There is no justification for Mach stopping at this point and not continuing to provide a further operational definition of the absolute mass scale. Perhaps Mach was similarly misguided by the invalid argument from kinematic comparativism to metaphysical comparativism.

It must be admitted of course that the absolutist should rejoice in Mach’s achievement, as far as it goes. The absolutist is a realist about mass ratios<sup>144</sup>, and has never claimed that masses (either the absolute masses or mass ratios) are ‘directly observable’ (or more correctly, as directly observable as relative distances). If mass were ‘directly observable’ the whole debate between absolutism and comparativism would not exist in the first place—so the absolutists always admitted the need for a method of measuring those mass relations. And this can of course only be done via (more directly) observable, spatiotemporal notions, such as acceleration.

Could we supplement Mach’s project with an operational definition of the absolute mass scale? It seems that attempting to do so would not violate the spirit of the original project. The main thrust of the Machian project was the reduction of mass. Mach incorrectly—or so I contend—interpreted ‘mass’ to refer to mass ratios only. If we manage to additionally reduce the absolute mass scale, this

---

<sup>144</sup>But see [fn. 10](#).

would complete the original project of reducing mass (now correctly understood as both mass ratios and an absolute mass scale).

The obvious candidate for such an operational definition is exactly the escape velocity scenario that was used to prove the empirical relevance of absolute masses in the first place. The escape velocity inequality<sup>145</sup> can be reformulated in terms of spatiotemporal magnitudes only:  $v^2 > v_e^2 = 2ar$ . This suggests that the absolute mass scale could be defined in terms of some ratio of  $r$ ,  $v$  and  $a$ . We may call this a local operational definition, in contrast with the regularity comparativist’s global definition<sup>146</sup>. Although this seems unproblematic for the case of two particles, we will see below (Section 6.5) that this does not in fact generalise to more particles.

## 6.4 Beyond Mach

### 6.4.1 Generalising to more particles

Let us now evaluate Mach’s project *qua* reductionism. Pendse famously points out that Mach’s definition depends crucially on the simple two-particle scenario—which initially seemed like a mere pedagogical simplification—and does not generalise to any number of particles [134]. Mach’s definition requires the *separate contributions* induced by every other particle to the acceleration of a specific particle, whereas we only have empirical access to the *total* acceleration of that particular particle. In systems with too many particles the total acceleration underdetermines the individual contributions. More specifically, Pendse argues that, if we use only acceleration relations at one instant, the mass-ratios are not uniquely determined for systems of more than four particles. Moreover, even if we consider acceleration relations at *any* number of instants, systems with more than seven particles will not give a unique set of mass ratios. I will briefly outline the first argument here [32].

Let  $n$  be the number of particles.  $\mathbf{a}_k$  is the observed, induced total acceleration of the  $k$ th body at  $t_0$ , and  $\hat{\mathbf{u}}_{kj}$  the unit vector in the direction from body  $k$  to body  $j$  at  $t_0$ . Then

$$\mathbf{a}_k = \sum_{j=1}^n a_{kj} \hat{\mathbf{u}}_{kj}, \quad (k = 1, \dots, n) \quad (6.4)$$

where we solve for  $a_{kj}$  ( $a_{kk} = 0$ ), the  $n(n - 1)$  unknown coefficients in  $3n$  linear equations, which represent the induced acceleration on particle  $k$  by particle  $j$  at

<sup>145</sup>This inequality governs the special case where the mass ratio is one, but this could easily be generalised.

<sup>146</sup>Dasgupta provides an example of an alternative global definition. He introduces a notion of plural grounding, and argues that the totality of kilogram facts is plurally grounded in the totality of mass ratios [36].

$t_0$ . It is these coefficients that Mach needs to fix the mass ratios. They are uniquely determined only if their number does not exceed the number of equations,  $n(n - 1) \leq 3n$ , and this is not the case for systems with more than 4 particles. QED. The reader is referred to Pendse’s paper for the proof concerning acceleration data at any number of instants.

## 6.4.2 Including other spatiotemporal magnitudes

Narlikar responds by echoing the thought that underlies the suggested operational definition of the absolute mass scale: accelerations might be insufficient, but we have other spatiotemporal notions at our disposal [135]. In particular, we can measure inter-particle distances as well as accelerations, and insert them into the Gravitational Law<sup>147</sup>. Setting Newton’s constant to one for convenience, we get the following equation for the (arbitrarily chosen<sup>148</sup>) x-component of the acceleration of particle 1 due to the gravitational interaction of all the other particles, at  $t_0$ :

$$a_{1,x}(t = t_0) = \frac{m_2(x_2 - x_1)}{r_{12}^3} + \frac{m_3(x_3 - x_1)}{r_{13}^3} + \dots + \frac{m_n(x_n - x_1)}{r_{1n}^3}, \quad (6.5)$$

where it is understood that the positions and distances are measured at  $t = t_0$  also. These, together with  $a_{1,x}$ , can be observed, resulting in a linear equation of the form

$$A_{12}m_2 + A_{13}m_3 + \dots + A_{1n}m_n = X_1, \quad (6.6)$$

where only the  $m$ ’s are unknown. Repeating this procedure for a total of  $(n - 1)$  different instants, we get  $(n - 1)$  (supposedly)<sup>149</sup> linearly independent equations, allowing us to solve for  $m_2, m_3, \dots, m_n$ . Observing in addition a single acceleration-component of any of the other particles at  $t = t_0$  only is sufficient to determine the remaining  $m_1$ .

---

<sup>147</sup>Pendse [136] objects that we do not have independent empirical access to the Gravitational Law. However, in the context of the project in this chapter we simply take the laws as given. In fact, Mach and Pendse’s own projects take Newton’s Laws as given, so why could Narlikar not add the Gravitational Law to this?

<sup>148</sup>The arbitrariness of this choice will be discussed in Section 6.6.

<sup>149</sup>Although these equations may be linearly independent in general, presumably not all specific instances will be so. What to do with those deviant cases? Perhaps it will turn out that these specific systems are of measure zero in the space of solutions, and that that gives us some reason to ignore them. Or perhaps these cases result in infinitely many solutions which are all empirically equivalent. Or perhaps choosing a different set of instants to measure the distances suffices to restore linear independence. All of this remains to be shown though.

### 6.4.3 The main argument

It is here that we diverge from Mach’s project. Mach’s project was of a re-constructive, *descriptive* and epistemological/empiricist nature. It is the project of humans reconstructing (after the fact!) the masses from the appearance of the four-dimensional mosaic generated by God (cf. regularity comparativism and eliminativism). Therefore, using spatiotemporal data at any number of instants is perfectly acceptable; we are here not in the business of explaining part of the data (the future data) from other parts of the data (the initial state). And this project had better work! We have been applying Newtonian physics successfully for over three centuries now. We have modelled and predicted the behaviour of the planets in our solar system, based on presumed knowledge of the masses of those planets. Thus, there had better be some response to the potential problems with Narlikar’s argument as elaborated upon in [footnote 149](#), unless we want to invoke some error theory<sup>150</sup> about the way we have been doing Newtonian physics for the past three centuries.

In this thesis we are however interested in the much more specific, metaphysical project of *explaining* our actual world by deterministically generating it from the initial conditions. That is, we are ‘playing God’, rather than reconstructing some true, after-the-fact statements about God’s creation. Hence, we are only allowed to use spatiotemporal data at the initial time. The future data is part of the explanandum, not the explanans. Using it would be explanatorily circular. The tools used by Narlikar (and by Pendse when proving his second claim) are not available in the context of this project<sup>151</sup>.

Does this mean that the reductionist project is doomed? No. We can retain Narlikar’s insight—that we have more spatiotemporal data at our disposal than merely accelerations—but restrict ourselves to that additional data *at the initial time only*.

If we could find an operational definition of the masses in terms of the *initial* spatiotemporal notions, then this would guarantee that these initial spatiotemporal notions would suffice (via some law which is obtained by substituting all references to mass by its operational definition) to generate a unique evolution, since this is guaranteed by the initial masses (plus distances positions and relative velocities). As we have seen that initial accelerations are insufficient, we might follow Narlikar’s lead by including distances and inserting them into the gravitational law. We start off with his [Eq. 6.5](#) for the x-component of the acceleration of particle 1 at  $t_0$ , but instead of supplementing it with similar equations at different instants, we consider the analogous equations for the other particles *at the same*

---

<sup>150</sup>Beyond of course the obvious errors in the quantum, relativistic and, potentially, MONDian regimes.

<sup>151</sup>For similar reasons Schmidt’s reduction of mass [[32](#), [137](#)] is disqualified.

*instant.* For instance:

$$a_{2,x}(t = t_0) = \frac{m_1(x_1 - x_2)}{r_{21}^3} + \frac{m_3(x_3 - x_2)}{r_{23}^3} + \frac{m_4(x_4 - x_2)}{r_{24}^3} + \dots + \frac{m_n(x_n - x_2)}{r_{2n}^3}. \quad (6.7)$$

We obtain the matrix equation  $G\mathbf{m} = \mathbf{a}$ , where  $G$  is the following  $n \times n$  matrix:

$$G = \begin{pmatrix} 0 & \alpha_{12} & \cdots & a_{1n} \\ \alpha_{21} & 0 & & \vdots \\ \vdots & & \ddots & \vdots \\ \alpha_{n1} & \cdots & \cdots & 0 \end{pmatrix} \quad (6.8)$$

where  $\alpha_{ij} = \frac{x_j - x_i}{r_{ij}^3}$ . Since  $\alpha_{ij} = -\alpha_{ji}$ <sup>152</sup>,  $G$  is an antisymmetric matrix. But the determinant of an antisymmetric matrix with odd dimensions is singular! Recall that it is a property of the determinant that  $|G| = |G^T|$  and  $|-G| = (-1)^n|G|$ . For an antisymmetric matrix ( $G^T = -G$ ) these properties combine to give  $|G| = |G^T| = |-G| = (-1)^n|G|$ . For odd  $n$  then  $|G| = -|G| = 0$ . Since a unique solution requires a non-zero determinant, this proves that there is no unique solution of masses. QED.

#### 6.4.4 Unpacking the argument

What exactly follows from this? If the determinant had been non-zero, then reductionism would have been straightforwardly successful. It is less straightforward whether the vanishing of the determinant rules out reductionism. A vanishing determinant (for systems with an odd number of particles) proves that *either* there are no solutions *or* there are infinitely many solutions. Given that standard Newtonian Gravity has some solutions, we know that there are at least some sets of initial spatiotemporal magnitudes that fall into the latter category. Are there any sets that fall into the former?

---

<sup>152</sup>This is true only because Newtonian Gravity contains both Newton's third law and the principle of equivalence of gravitational and inertial mass—without for instance the latter Eq. 6.5 would not have been as simple. This seems to suggest that if we were to go beyond Newtonian Gravity by adding other forces which do not obey a similar equivalence principle (such as the Coulomb Force), the argument against reductionism would collapse. This cannot be true however, since this would only introduce more unknowns (i.e. the electric charges) without extra 'knowns' to determine those unknowns (unless perhaps the additional force depended on velocity and we could measure the velocities to aid us). It may be true in Newtonian theories with, say, just such a Coulomb Force and no gravity, but those are not theories that we are considering here.

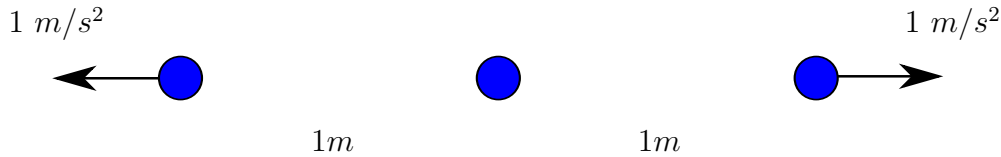


Figure 6.1: First example of a set of accelerations to which no (positive) mass distribution corresponds.

### Horn 1: No solutions

One might think that the following set of initial spatiotemporal magnitudes does not correspond to any (physical) solution. Consider a simple example of a world consisting of three particles. Figure 6.1 depicts three collinear particles, with the middle particle being one meter away from each of the outer particles. The middle particle has zero acceleration, and the outer particles each an acceleration of  $1 m/s^2$  *outwards*. Since gravity is supposed to be attractive, one might think that there are no mass solutions corresponding to this scenario, but there are in fact two categories of (mathematical) solutions: one in which the middle particle has a negative mass and the other two a positive mass, and vice versa. Although there is in fact a solution, this seems ‘unphysical’, since standard NG with mass includes the postulate that masses are always positive<sup>153</sup>. Standard NG thus does not contain these types of solutions. What should we do with such ‘non-physical’ solutions? Perhaps the reductionist could respond by claiming that we can somehow throw away these mathematical solutions since they are non-physical. We discuss such moves below. Instead I will now move on to a more decisive example, where there are not even any mathematical solutions.

In the second example all accelerations are ‘inwards’, which seems *prima facie* compatible with the attractive nature of gravity. In Figure 6.2 the two particles on the left accelerate with  $1 m/s^2$  to the right, and the third particle accelerates in the opposite direction with  $13 m/s^2$ . It is easy to show that there is no solution in terms of masses, not even negative masses.

Could the reductionist just choose to (*a priori*) rule out those deviant sets of initial spatiotemporal magnitudes? Especially Humeans about laws of nature might be tempted by this approach. For instance, in the Mill-Ramsey-Lewis Best Systems approach any true statement that is part of the best system to axiomatise the data counts as a law. Thus, if the statement that rules out these deviant sets

<sup>153</sup>Or, that the gravitational law does not care about the sign of the masses.

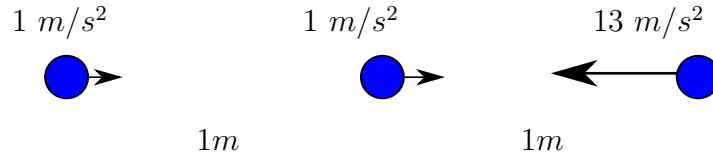


Figure 6.2: Second example of a set of accelerations to which no mass distribution (either positive or negative) corresponds.

of initial spatiotemporal magnitudes is part of the best system, we could just postulate it as a law of our reductionist theory. Compare this to the Humean solution to the problem of the arrow of time: if the Past Hypothesis (i.e. the claim that the initial entropy of the universe was sufficiently low [138]) forms part of the best system, this allows us to promote it to the status of law.

Apart from the standard complaints that such statements are not at all the type of beast that we normally consider as a candidate for law-hood, it is important that any such postulated constraint on the initial conditions is neither ad hoc, nor *unexplained*. Moreover, this constraint should be formulatable without (implicitly) referring to masses, that is without piggy-backing on the theory that takes masses to be primitive<sup>154</sup>. There are several reasons to believe that these conditions are not satisfied.

Whereas the restriction on entropy was straightforward—the initial entropy had to be below a certain value—the restrictions that would rule out the deviant set of initial spatiotemporal magnitudes—or more specifically initial accelerations—that do not correspond to any mass solution are much more complicated. In fact, no value of initial acceleration for any individual particle is ruled out from the start; the constraint takes on a holistic form instead. Only if a particle is located ‘on the outside’, do we all of a sudden require that its acceleration is not directed ‘outwards’<sup>155</sup>. Similarly, once the initial accelerations of all but one particle have been chosen, this can restrict the allowed values of the acceleration of the ‘final’ particle (even if that particle was on the ‘inside’). Leaving out the acceleration of a single particle from the initial conditions is not an option since even when we do include this piece of information the mass solutions are already underdetermined in some cases (see below), nor would this solve the former problem regarding

<sup>154</sup>Pooley [55, Ch.5.4] discusses analogous issues (concerning Sklar’s relationalist manoeuvre of adding primitive accelerations to the initial conditions) in the analogous substantivalism–relationalism debate about space. See also Arntzenius [23, Ch.5.7] on piggy-back relationalism, as discussed in Subsection 4.4.1.

<sup>155</sup>Notice that this also holds for systems with an even number of particles.

outward accelerations. The choices of the initial accelerations of a particle thus depend on the choices for the initial accelerations of the other particles. It is as if the laws determine, after the fact, in a holistic sense, which initial accelerations were allowed in the first place. Namely, exactly those that correspond to initial masses. Inference to the best explanation suggests that that is the case exactly because there are fundamental (initial) masses. There is no non-ad-hoc, reductionist *explanation* for ruling out the deviant sets of initial accelerations, especially not one that does not piggy-back on the concept of mass. In contrast, these constraints are trivially explained (by the attractive nature of gravity) if we do take masses to be primitive.

## Horn 2: Infinitely many solutions

Let us turn to the sets of initial spatiotemporal magnitudes that correspond to a set of infinitely many solutions in terms of (initial) masses. These sets underdetermine the masses, and since different masses correspond, in general, to different (metaphysical) evolutions of the system, an initial state that contains only spatiotemporal data leads to an indeterministic evolution (if there is any well-defined evolution in the first place). Such a reductionist theory will not provide the explanatory and predictive power that NG with primitive masses does.

The first, most obvious line of responses consists of variations on the theme that perhaps each set of infinitely many solutions is similar enough, *in some sense*, to ‘count as one’ and to therefore *effectively* form a single unique solution.

Variation 1: it might be the case that, even though each of these sets of initial spatiotemporal magnitudes corresponds to several distinct possible sets of masses each of which lead to *metaphysically distinct* evolutions, these respective distinct evolutions are in fact all *empirically equivalent*. If this is true for each set of initial spatiotemporal magnitudes that has multiple solutions, this would not only save the reductionist project, but also prove (as was suggested once before, but incorrectly, in [Section 6.2](#)) that the mass theory recognises distinct metaphysically possible worlds that are empirically indistinguishable, which violates the Principle of the Identity of Indiscernibles (i.e. the inter-world Occam’s Razor).

Variation 2: Perhaps each set of infinitely many solutions consists of solutions that differ only with respect to the absolute masses but not with respect to the mass ratios. If so, the reductionist has at least partially succeeded by reducing the mass ratios, if not the absolute masses.

The easiest, most conclusive way to kill both variations with one stone is by providing a single counter-example to both. [Figure 6.3](#) shows two superimposed numerical solutions of the three-body problem in one dimension. The solutions are generated from initial conditions that agree with respect to the spatiotemporal magnitudes, but disagree with respect to their initial masses (which in both cases

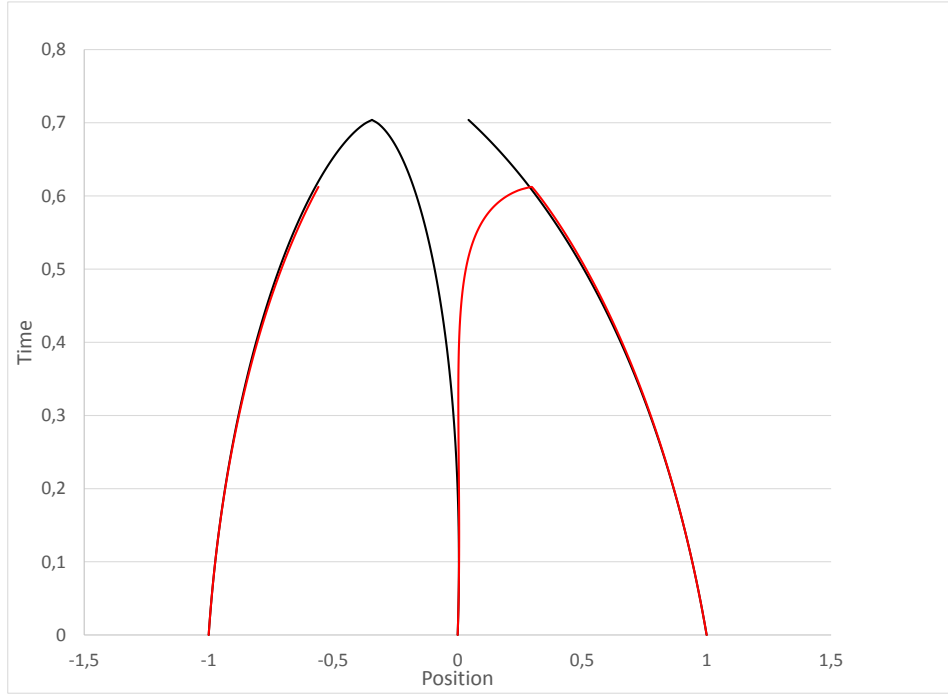


Figure 6.3: **The Smoking Gun:** A numerical solution of the three-body problem in one dimension (black trajectories), superimposed on an alternative solution (red trajectories). Each three-body problem has only been solved until the first collision, as the theory breaks down at that point (see [fn. 83](#)). The initial states of each set of three particles are identical with respect to the spatiotemporal magnitudes ( $G = 1$ ,  $d_{12} = d_{23} = 1$ ,  $v_1 = 0.2$ ,  $v_2 = 0.1$ ,  $v_3 = -0.5$ ,  $a_1 = 1.25$ ,  $a_2 = -1$ ,  $a_3 = -1.5$ ), but they differ in terms of their masses ( $m_1^r = 5.5$ ,  $m_2^r = 0.125$ ,  $m_3^r = 4.5$ ;  $m_1^b = m_2^b = 1.2$ ,  $m_3^b = 0.2$ ). Note that they do not only differ in their absolute masses but also their mass ratios! These different sets of masses generate empirically distinct evolutions! Particle 2 collides first with particle 3 within the red solution, but first with particle 1 within the black solution.

are compatible with the spatiotemporal initial state) and *moreover their mass ratios* (against variation 2). Both solutions clearly generate *empirically distinct* evolutions (against variation 1), since in one case the middle particle collides first with the particle on the left, and in the other case its first collision is with the particle on the right<sup>156</sup>. Moreover, even if the mass ratios had been the same in this example, it would have served to reiterate the point made in [Subsection 6.3.1](#) that absolute masses make an empirical difference. Thus, it would make salient that under variation 2 the need for fundamental intrinsic masses would remain, which anyway provide the mass ratios for free, thereby making such a partial reduction of the mass ratios good for nothing.

Secondly, the reductionist might suggest that including the ‘y’ and ‘z’ components of the acceleration might serve to remove the underdetermination and provide unique mass solutions. We should immediately feel uneasy about this suggestion: when attempting to fix  $n$  mass degrees of freedom one would expect to need  $n$  acceleration degrees of freedom, not an additional  $2n$  more! More on this below (Sections [6.5](#) & [6.6](#)). But even when we do allow ourselves these extra degrees of freedom, this move will not work. The one-dimensional case is still a specific instance of the three-dimensional case. In scenarios where the ‘y’ and ‘z’ components of acceleration are zero, all components of acceleration together still underdetermine the masses and thereby the evolution of the system.

Thirdly, the reductionist might bite the bullet and accept indeterminism (at the initial time only). Perhaps there are alternative, reductionist laws which allow for several possible evolutions of the initial spatiotemporal state—one for each of the evolutions that correspond to the mass solutions compatible with that initial spatiotemporal state—but once a specific evolution has ‘begun’ it follows through, deterministically, until the end. In other words, the laws are indeterministic relative to the initial instantaneous state, but not relative to an initial chunk of the evolution. At this point I can only respond by pointing out that the onus is on the reductionist to provide such indeterministic laws that generate the correct set of empirically possible evolutions<sup>157</sup>. Even if successful, it seems that such an approach would nevertheless weaken the predictive power of the theory.

---

<sup>156</sup>It might be argued that one cannot compare which particle is left or right of the middle between different solutions. One could avoid this by adding an extra particle sufficiently far from these free particles to be dynamically isolated from them, in order to serve as a reference for, say, ‘left’. However, the two solutions are clearly not each other’s mirror image, so if we were to acknowledge distinct histories of distance ratios as a criterion for empirical distinctness of worlds (rather than the more conservative, topological criterion of coincidence), adding the extra reference particle would not even be required.

<sup>157</sup>Dasgupta [[139](#)] is developing an analogue of this project in response to the accusation that relationalism about handedness, space and mass are all indeterministic. That case seems much simpler though than the case considered in this chapter.

## 6.5 Bits and bobs

Have we ruled out that mass can be reduced to spatiotemporal magnitudes? At least two issues need to be dealt with before we can conclude so.

The main argument rests on the substantive premise that the number of particles  $n$  is odd. This may not be true of the actual world. Especially Humeans about laws of nature might jump on this loophole, and just take the statement that  $n$  is even to be part of the best reductionist system, which justifies promoting it to the status of a law, thereby avoiding my main argument. However, first of all, it just seems that such a statement is not at all the kind of statement that is a candidate for being a law—why would it be nomologically necessary that  $n$  is even? Secondly, it could well be false of the actual world that  $n$  is even. Thirdly, even if  $n$  just happens to be even in the actual world, the reductionist still has to prove that  $G\mathbf{m} = \mathbf{a}$  is solvable (where  $G$  is given by Eq. 6.8). The attractive nature of gravity is enough to show that even in those worlds there will be initial spatiotemporal conditions that do not correspond to any set of positive<sup>158</sup> masses, namely those where the particles ‘on the outside’ have an acceleration that points away from all the other matter. Fourthly, it may seem that adding this extra law makes the axiomatic system ‘better’ by making it much stronger at the cost of only a small reduction of simplicity, but given the sheer amount of dynamically possible worlds ( $\gg n^n$  where  $n$  is a large number), merely halving that large number at the cost of moving from four laws to five is actually not that advantageous<sup>159</sup>. Fifthly, assuming a non-revisionary reductionist—as suggested in Section 6.1—who wants to reproduce all the consequences of and the work being done by the standard form of NG (i.e. with primitive masses), the reductionist theory needs to generate all the empirically possible worlds of standard NG. This includes worlds with an odd number of particles, even if none of those represents the actual world. Finally and most importantly, the main argument still goes through for quasi-isolated subsystems of an odd number of particles. Thus, even if our universe consisted of an even number of particles, there will (probably) still be solar systems with an odd number of celestial objects. (It would have been nice for my purposes if that were true of our own solar system—ignoring asteroids etc.—but alas!)

Let us now turn to the last cluster of related issues. The focus in this chapter has mainly been on accelerations. Have we ruled out a reduction of mass to any

---

<sup>158</sup>Although, for e.g. a system with four masses on the vertices of a square, all with accelerations of equal magnitude pointing outwards along the diagonals, there is one unique solution if we were to allow negative masses. (It consists of masses of equal magnitude (the exact value depending on the acceleration magnitude), but the masses on one diagonal have a negative sign, whereas the masses on the other diagonal have a positive sign.)

<sup>159</sup>I would like to thank David Wallace for pointing this out to me.

type or combination of types of spatiotemporal magnitudes, or only a reduction to accelerations (and distances)? We gain some insight into this question when we return to the issue of operationally defining the absolute mass scale (Section 6.3.1; assuming we would have been able to fix the mass ratios). I suggested that we could perhaps use the escape velocity scenario for this purpose. The escape velocity inequality governing that scenario can be rewritten in terms of  $r$ ,  $v$  and  $a$  only, suggesting that we define the absolute mass scale via some ratio of  $r$ ,  $v$  and  $a$ . However, Figure 6.3 has not only proven that mass ratios cannot be reduced to accelerations (and distances), it also proves that the absolute mass scale cannot be defined in terms of  $r$ ,  $v$  and  $a$  once we have more than two particles. For in that figure not only the initial distances and accelerations of the two superimposed solutions agreed, but also the initial velocities. Thus, an initial purely spatiotemporal state containing spatiotemporal magnitudes up till second order fails to solve the reductionist project.

Could we include higher-order spatiotemporal magnitudes? Since these cannot be analytically determined from NG with primitive masses, it is difficult to answer this question<sup>160</sup>, but since this would mean adding even more ‘degrees of freedom’ this does not seem to be a viable option (see also below). We are trying to reduce  $n$  mass degrees of freedom to more than  $n$  spatiotemporal magnitudes. These extra magnitudes cannot be truly degrees of freedom; they cannot be *independent* of the  $n$  degrees of freedom. Either they 1) will lead to inconsistencies in the determination of the masses, or 2) they will always conspire to take on exactly the right values as to avoid inconsistencies. Such a mysterious, conspiratorial constraint—which presumably cannot even be formulated without referring to mass<sup>161</sup>—would be totally *unexplained*, even if imposing this constraint on the initial spatiotemporal state would uniquely fix the evolution.

---

<sup>160</sup>Perhaps the following serves as a plausibility argument for an upper bound on the order  $k$  of initial spatiotemporal data that would guarantee removing the underdetermination of mass (although the overdetermination problem, resulting in conspiratorial (i.e. *unexplained!*) constraints, still remains). On the “at-at” theory of motion (Section 4.2.1), (initial) velocities are not in fact properties of an (initial) instant, but of an infinitesimal (initial) period of time. After all, velocity is usually defined as  $\lim_{dt \rightarrow 0} \frac{r(t+dt) - r(t)}{dt}$ , which is a property intrinsic to  $[t, t + dt]$ .

In general, the initial  $k^{\text{th}}$ -order time derivative of  $r$  is a property of  $[t_0, t_0 + kdt]$ . In a slogan: ‘God was not done when he created the initial configuration and the laws, but he had to also specify the subsequent  $k - 1$  configurations (depending on the order of initial spatiotemporal data that we are considering)’. Now, if  $k = n + 1$ , this initial period (of  $n + 1$  instants) effectively contains  $n - 1$  independent sets of accelerations (and even more sets of distances). Narlikar’s method then guarantees that this initial data fixes the masses.

<sup>161</sup>See fn. 154.

## 6.6 An additional argument against reductionism

As a little bonus, let us bring two earlier strands together, which inspire an additional argument against the reductionist project. Strand 1: Pendse approached the reductionist project as a matter of *counting degrees of freedom*. This aspect returned when we considered using the additional  $2n$  degrees of freedom of the ‘y’ and ‘z’ components of the accelerations to remove the underdetermination of the masses by the ‘x’ components of the accelerations (and the distances) (Subsection 6.4.4). Strand 2: when considering ruling out deviant sets of initial conditions that did not correspond to any mass solutions, we realised the holistic and *conspiratorial* nature of the constraint on the allowed sets of initial spatiotemporal magnitudes (Subsection 6.4.4; see also the end of Section 6.5).

On reflection, it is quite strange that we were trying to reduce the  $n$  degrees of freedom of mass, a scalar, to acceleration, which—as a vector—has  $3n$  degrees of freedom, in the first place. We implicitly tried to avoid this awkwardness by only using one part of the acceleration degrees of freedom, say the ‘x’ components—cf. the *one-dimensional* solutions in Figure 6.3. However, especially in the homogeneous Euclidean space in which Newtonian Gravity lives (*pace* Knox [10]), it is arbitrary to use only one component of this vector magnitude. Even if we were to do so it would be even more arbitrary to determine exactly which component we should use. We seem to have implicitly chosen some preferred axis, in a homogeneous space which has no structure to ground such a notion.

Should we then have used all components of acceleration instead? We already mentioned that this, despite *prima facie* seeming to actually make the reductionist project easier—surely more initial spatiotemporal data will help to further pinpoint the corresponding initial masses and remove the underdetermination—it actually is of no use: one-dimensional examples of underdetermination are just specific cases of three-dimensional examples of underdetermination. In fact, adding these  $2n$  degrees of freedom makes things worse. We expected that, if the reductionist project had worked at all (contrary to the conclusion of this chapter), it would have fixed the  $n$  mass degrees of freedom via some set of  $n$  spatiotemporal degrees of freedom (plus the distances and velocities which were needed additionally in the mass theory as well). If that had worked, the additional  $2n$  overdetermining degrees of freedom would either 1) have lead to inconsistencies, or they would 2) have to always take on exactly the right values to not lead to any inconsistencies. But this latter situation would be extremely conspiratorial and unjustified<sup>162</sup>—as before in Subsection 6.4.4 and Section 6.5. Except of course for the mass theorist, who can trivially explain why the ‘y’ and ‘z’ component of the

---

<sup>162</sup>See again fn. 154.

accelerations always line up in a specific way ‘depending’ on the ‘x’ components.

Summarising, we should have been worried about reducing mass—a scalar—to acceleration—a vector, from the start!

## 6.7 Conclusion

It has been argued that Newtonian mass cannot be reduced to spatiotemporal magnitudes—distance, velocity and acceleration—in the specific sense of a purely spatiotemporal initial state failing to solve the General Initial Variables & Parameters Problem. This reductionism exhibits a lack of explanatory and predictive power with respect to primitivism (i.e. Newtonian Gravity with primitive absolute masses). Thus, the initial state does require primitive masses. The previous chapters argued for the relative metaphysical priority of absolute masses over mass ratios: weak absolutism. This chapter adds to this conclusion the conclusion that absolute masses are primitive within Newtonian Gravity. We end up with strong absolutism about mass in Newtonian Gravity: absolute masses are themselves ungrounded (i.e. fundamental/ primitive) and they ground the mass ratios.

# Conclusion & Outlook

To round off the thesis, I wish to summarize the principal conclusions reached and to make some tentative remarks, in the light of them, about what seem to me the most interesting avenues for further research.

Laplace was interested in the minimal choice of initial variables and parameters that corresponds to a well-posed initial value problem. In the context of Newtonian Gravity, the focus for the past three centuries has been on whether the initial state needs to include absolute spatiotemporal notions—determined with respect to absolute space—or merely relational notions—determined merely with respect to other material bodies. This thesis has used this debate as an inspiration to tackle Laplace’s problem from two novel angles. Firstly, do we need the absolute scales of those variables and parameters, or merely their ratios? In particular, I have considered the case study of mass: does solving Laplace’s problem require absolute masses, or merely mass ratios. Secondly, I have considered whether the initial state requires any notion of fundamental mass whatsoever, or whether we can substitute spatiotemporal notions for it.

In the first five chapters of this thesis I have sought to develop and defend a position that I call Weak Absolutism about mass within Newtonian Gravity—absolute masses ground mass ratios; they are thus more fundamental than mass ratios. This view is the denial of Weak Comparativism. In terms of Laplace’s problem: the initial state requires absolute masses (whether they are absolutely fundamental or not). In the sixth chapter I have argued against reductionism about mass within Newtonian Gravity: the absolute masses in the initial state need to be fundamental masses. Overall this leads me to conclude in favour of what I call Strong Absolutism: Newtonian mass ratios are grounded in absolute masses, which are themselves ungrounded (i.e. they are fundamental, within Newtonian Gravity).

[Chapter 1](#) introduced the main argument in favour of (weak) comparativism, which formalises comparativism’s promise to recover all the virtues of absolutism—in particular empirical adequacy and explanatory power—but in a metaphysically more parsimonious way. Evaluating the empirical adequacy of comparativism took up most of this thesis. It was of particular importance to distinguish between

absolute masses—defined as a totally ordered semi-group of monadic properties with non-qualitative, transworld identities, which I call mass magnitudes—and the numerical quantities used to represent them. Mass is a dimensionful determinable. To be dimensionful is to be kinematically comparative: the magnitude predicated of any particle can only be *reported* or *expressed*, non-dynamically, in terms of how this magnitude relates to the magnitude of another particle having the same determinable property. Because of this reason, magnitudes must be represented by a numerical quantity times a unit, but this unit is conventional: there is a many-to-one relationship between the set of quantities that could be used to represent a set of magnitudes and those magnitudes themselves. It is this many-to-one relationship at the representational level that is often confused with the metaphysical many-to-one relationship between magnitudes and mass relations that is at stake in this debate. In particular, I have argued that it does not follow from kinematic comparativism that physics only depends on the mass ratios. That is, kinematic comparativism does not imply dynamic comparativism, that is the empirical adequacy of comparativism. A proper evaluation of the empirical adequacy of comparativism can be approached in at least three different ways: in terms of symmetries, in terms of the undetectability of absolute masses, and in terms of correctly generating the set of empirically possible worlds. I criticise the second, and develop the third.

Chapter 2 introduced and criticised Dasgupta’s comparativism and his preferred undetectability approach: according to Dasgupta’s comparativist laws the absolute masses are undetectable, and hence comparativism is empirically adequate. It brought forward two flaws in Dasgupta’s analysis. Most importantly, he overlooked the need to account for the trajectories of particles, and focused merely on accounting for their acceleration ratios, which in fact underdetermine the detectable trajectories. This was illustrated by a specific instance of an inter-world Leibniz Mass Scaling, inspired by Baker: uniformly changing the absolute masses leads to detectably distinct trajectories. This scenario, dubbed the comparativist’s bucket by analogy to Newton’s bucket, was posed as the central threat to (the empirical adequacy of) comparativism. Although Dasgupta’s comparativism was rejected on his own terms, the main thrust of this chapter was to argue that the undetectability approach is unsuitable. It mistakes the absolutism–comparativism debate, which is a debate about relative fundamentality, for a debate about realism about absolute masses. Realism about absolute masses is in principle compatible with both absolutism and comparativism. Moreover, whether uniformly varying the masses does or does not lead to a detectably different trajectory, it is still impossible to *express which* absolute mass one has detected or failed to detect. This is the content of kinematic comparativism. Moving to a broader notion of detectability—*detecting that* varying the absolute masses leads to a detectable difference—is just to move to the possibility checking approach. To the extent

that the undetectability approach differs from the possibility checking approach, it is problematic. To the extent that they coincide, I suggest we simply move to that more perspicuous approach.

[Chapter 3](#) develops the possibility checking approach to the empirical adequacy of comparativism in detail. It formulates a task that the comparativist needs to be able to solve. This task is modelled on Laplace’s problem, but is extended to incorporate not only empirical adequacy/ predictive power, but also explanatory and modal adequacy. Dasgupta’s comparativism is again shown to be problematic, as it fails to solve this task.

The possibility checking approach helpfully suggests a research programme of comparativist responses—inspired by the analogous relationalism–substantivalism debate—which is developed and criticised in [Chapter 4](#). The first, most ambitious category of responses faces the task head-on. Baker’s own response falls in this category, but I argue that it boils down to reductionism (which is refuted in the final chapter) and thereby throws away the massive baby with the bathwater. It favours weak absolutism, rather than defending (strong) comparativism over (strong) absolutism. Attempting instead to solve the task with non-standard ideologies similarly leads to weak absolutism, and reduces the amount of metaphysical parsimony that was supposed to be the main reason for favouring comparativism over absolutism. Perhaps the most interesting line of response changes not the ideology but the absolutist laws. I develop a Machian form of comparativism, which is syntactically distinct from absolutism but identical<sup>163</sup> according to the standard semantic view of theories (although the completeness criterion is satisfied in a slightly unnatural way). This view does come with the extra metaphysical cost of an additive structure for the mass ratios.

The second category of responses accepts that the comparativist’s bucket forces us to give up the spirit of comparativism—anti-realism about absolute masses—but insists that we can retain the letter—having those real absolute masses not ground the mass ratios but being (partially) grounded in them. The only response in this category, regularity comparativism, attempts to obtain the absolute mass scale ‘for free’ by having it supervene on a Humean mosaic containing fundamental mass ratios but no fundamental absolute masses. Besides explanatory inadequacy, and a questionable consistency with the separability feature of the Humean framework, the fatal blow to regularity comparativism is that to the extent that it works, it works too well: there is no way of stopping at moving not only absolute masses but also the mass ratios to the supervenient level. As with Baker’s view, this view sketches a picture of reality where the spatiotemporal stuff grounds the absolute masses which in turn ground the mass ratios. This is weak absolutism, not a form of comparativism.

---

<sup>163</sup>Cf. [fn. 97](#).

The responses in the final, least ambitious category deny having to respond to the challenge, by disagreeing with various ingredients that went into the set-up of the comparativist task. In particular, I considered giving up the requirement of 1) picking out the set of dynamically possible worlds via laws and initial conditions, in favour of picking out the whole set of dynamically possible worlds via more holistic constraints; and 2) completeness and/or soundness. Several simple options were dismissed. Barbour and Bertotti's best matching approach and Constructor Theory might provide interesting avenues for further research.

[Chapter 5](#) rejected a naive argument for the supposed relative metaphysical parsimony of comparativism. I have considered the metaphysical parsimony of comparativism along the dimensions of quantitative & qualitative parsimony, and intra-world & inter-world parsimony. Insofar as anything concrete can be said about the fickle notion of parsimony at all, along each of these dimensions the parsimony of comparativism is highly questionable. The onus is on the comparativist to provide a convincing argument here. I furthermore point out a fact about mass ratios which is trivially explained on the absolutist account, but is a highly conspiratorial, unexplained feature on the comparativist framework.

The first five chapters suggest that we should favour weak absolutism over weak comparativism. The final chapter considers reductionism about mass. Can Laplace's problem be solved using a purely spatiotemporal initial state? I argue that not only does reductionism require unexplained, conspiratorial constraints, even with those it still fails to provide unique solutions to the initial value problems. The initial state needs primitive masses. The overall conclusion is then strong absolutism about masses in Newtonian Gravity.

I have already mentioned further research into combining comparativism about mass with Barbour & Bertotti's programme, and Constructor Theory. Other avenues one might want to explore are as follows. Fans of the symmetry approach might want to translate the conclusions of this thesis into the language of their approach, and reverse engineer the best way of explicating the symmetry approach to debates like these. In light of the current conclusions it is tempting to say that a Leibniz Mass Scaling is not a symmetry, but it was a main theme of this thesis that correctly interpreting the *ceteris paribus* clause of the Leibniz Scaling was non-trivial and important. Depending on the interpretation, it may be considered a symmetry (cf. [Subsection 4.2.2](#)). Machian comparativism should provide an interesting playground for the symmetry approach. Although this theory is identical<sup>164</sup> to absolutism on the standard semantic view of theories, the Leibniz Scaling does seem to be a symmetry of the former but not of the latter. How should we interpret symmetries on the semantic view of theories?

In this thesis we considered the case study of comparativism about mass.

---

<sup>164</sup>Cf. [fn. 97](#).

What about comparativism about other determinables, such as distance? It may seem to be in the spirit of comparativism that if one is comparativist about one determinable, it makes sense to also be a comparativist about other determinables, even though this does not logically follow. One of the main themes of this thesis was however that there are no *a priori* arguments for comparativism. We are interested in dynamic comparativism, which obviously depends on the specific dynamics of the theory. As pointed out in [Subsection 2.4.2](#), alternative force laws could have rendered comparativism (about mass) empirically adequate after all. In the case of Machian comparativism it was noted that one either needs to be an absolutist about distance to be a comparativist about mass, or vice versa (or one needs to give up on completeness). Although much of the terminology and many of the type of arguments used in this thesis can be used to consider the case of comparativism about other variables, each case study needs to be considered in the light of the specific dynamics of the relevant theory.

This brings us to comparativism about mass in other theories, such as Newtonian theories with additional forces, but especially more modern theories, such as General Relativity. Again, much of what has been discussed in this thesis applies. Kinematic comparativism holds; the naive arguments fail; the undetectability approach is unsuitable. It may even seem that there is a straightforward relativistic version of the comparativist’s bucket, leading to the same negative conclusions for comparativism about mass. Take two massive bodies. Keep Leibniz Scaling their mass (density). At some point the bodies will form black holes, a difference which supposedly has detectable consequences. However, in the Levi-Civita spacetime, no two bodies will ever ‘escape’ each other, however low their mass density is [[140](#), §10.2]. In this spacetime the escape velocity is infinitely large<sup>165</sup>. It should be interesting to further probe these matters.

Finally, in the context of the regularity approach, it would be worthwhile to determine how the gravitational laws of absolutism (G1), Machian Comparativism (G3) and Doubly Machian Comparativism ( $G = \gamma \sum_i r_i / \sum_k m_k$ ) compare with respect to being a good axiomatisation, given a specific mosaic. Are they equally simple—since none of them contains correction terms—or do the holistic terms in the Machian and Doubly Machian laws make those laws less simple? What about strength? Doubly Machian comparativism violates completeness—it forbids many empirically possible worlds that absolutism does allow—but if we start out with a mosaic with absolute masses and/or absolute distances, these can be used to generate possible worlds/mosaics that are metaphysically distinct

---

<sup>165</sup>It should be interesting to determine how large the class is of spacetimes which have an infinitely large escape velocity. Some candidates, which share the (non-asymptotically flat) asymptotic behaviour of Levi-Civita spacetime, are the Korotkin-Nicolai blackhole solution and certain members of the family of rotating disc solutions due to Meinel and Neugebauer (Gordon Belot, personal communication).

but nevertheless empirically equivalent. Does this reduce the strength of Doubly Machian Comparativism, or is it mainly a suggestion that we should move to sparser mosaics, which are more clearly best axiomatised by a Doubly Machian law?

# Bibliography

- [1] Henri Poincaré. *Science and Hypothesis*. Dover, New York, 1952 [1902], Translated by W. Scott.
- [2] J. Barbour. *The end of time: The next revolution in physics*. Oxford: Oxford University Press, 2000.
- [3] Bradford Skow. Sklar's maneuver. *British Journal for the Philosophy of Science*, 58:777–786, 2007.
- [4] O. Pooley. *The Reality of Spacetime*. Dphil thesis, University of Oxford, 2002.
- [5] E. Mach. *The Science of Mechanics*. The Open Court Publishing Co., translated by T.J. McCormack, 1960 [1893].
- [6] Hartry H. Field. *Science without Numbers: A Defence of Nominalism*. Oxford: Basil Blackwell, 1980.
- [7] Porter Williams. Scientific realism made effective. <http://philsci-archive.pitt.edu/12889/>.
- [8] Shamik Dasgupta. Absolutism vs comparativism about quantity. In Karen Bennett and Dean W. Zimmerman, editors, *Oxford Studies in Metaphysics*, volume 8, pages 105–147. Oxford University Press, 2013.
- [9] David J. Baker. Some consequences of physics for the comparative metaphysics of quantity. Manuscript, May 2013.
- [10] Eleanor Knox. Newtonian spacetime structure in light of the equivalence principle. *British Journal for the Philosophy of Science*, 65(4):863–80, 2014.
- [11] W.V.O. Quine. Ontology and ideology. *Philosophical Studies*, 2:183–204, 1951.
- [12] W.A. Bauer. An argument for the extrinsic grounding of mass. *Erkenntnis*, 74(1):81–99, 2011.

- [13] John T. Roberts. A case for comparativism about physical quantities. Manuscript, accessed on 20 Sept 2016, [https://www.academia.edu/28548115/A\\_Case\\_for\\_Comparativism\\_about\\_Physical\\_Quantities\\_--\\_SMS\\_2016\\_Geneva](https://www.academia.edu/28548115/A_Case_for_Comparativism_about_Physical_Quantities_--_SMS_2016_Geneva).
- [14] D. Armstrong. *A Theory of Universals: Volume 2*. Cambridge: Cambridge University Press, 1978.
- [15] D. Armstrong. Are quantities relations? a reply to bigelow and pargetter. *Philosophical Studies*, 54:305–316, 1988.
- [16] Brent Mundy. The metaphysics of quantity. *Philosophical Studies*, 51(1):29–54, 1987.
- [17] D. Lewis. *Philosophical Papers*, volume ii. Oxford University Press, 1986.
- [18] Bertrand Russell. *The Principles of Mathematics*. W.W. Norton & Company, 1903.
- [19] B. Ellis. *Basic Concepts of Measurement*. Cambridge: Cambridge University Press, 1966.
- [20] H. Field. Can we dispense with space-time? In *PSA 2*, 1985.
- [21] J. Bigelow, R. Pargetter, and D.M. Armstrong. Quantities. *Philosophical Studies*, 54:287–316, 1988.
- [22] J. Bigelow and R. Pargetter. *Science and Necessity*. Cambridge: Cambridge University Press, 1990.
- [23] Frank Arntzenius. *Space, Time, & Stuff*. Oxford University Press, 2012.
- [24] Maya Eddon. Quantitative properties. *Philosophy Compass*, 8(7):633–645, 2013.
- [25] David J. Baker. Comparativism with mixed relations. June 2013 Manuscript.
- [26] M. Dees. Against quantity primitivism. Manuscript.
- [27] N.C.M. Martens. The metaphysics of emergent spacetime theories. Manuscript.
- [28] F. Dowker. Causal sets and the deep structure of spacetime. arXiv:gr-qc/0508109v1, 2008.

- [29] David Lewis. Ramseyan humility. In David Braddon-Mitchell and Robert Nola, editors, *Conceptual Analysis and Philosophical Naturalism*, pages 203–22. Cambridge, MA: MIT Press, 2009.
- [30] N.C.M. Martens. The swapping argument against quidditism: Determinables & determinates. Manuscript.
- [31] H. Weyl. *Philosophy of Mathematics and Natural Science*. Princeton: Princeton University Press, 1949.
- [32] Max Jammer. *Concepts of Mass in Contemporary Physics and Philosophy*. Princeton University Press, 2000.
- [33] Nick Huggett. *Space from Zeno to Einstein: Classic Readings with a Contemporary Commentary*. Cambridge, MA: MIT Press, 1999.
- [34] Isaac Newton. *Philosophiæ Naturalis Principia Mathematica*. London: Joseph Streater, 1686/7. The Scholium to the Definitions has been reprinted in Alexander [47].
- [35] H. Zanstra. A study of relative motion in connection with classical mechanics. *Physical Review*, 23(4):528–545, 1924.
- [36] Shamik Dasgupta. On the plurality of grounds. *Philosophers’ Imprint*, 14(20):1–28, 2014.
- [37] M. Gardner. *The New Ambidextrous Universe: Symmetry and Asymmetry from Mirror Reflections to Superstrings*. Mineola, New York: Dover Publications, Inc, dover 2005 (= third) edition, 1964 [revised and retitled in 1990].
- [38] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes, and R.P. Hudson. Experimental test of parity conservation in beta decay. *Physical Review*, 105:1413–15, 1957.
- [39] G. Nerlich. *The shape of space*. Cambridge: Cambridge University Press, 1976.
- [40] O. Pooley. Handedness, parity violation and the reality of space. In Katherine Brading & Elena Castellani, editor, *Symmetries in Physics: Philosophical reflections*, pages 250–280. Cambridge University Press, 2003. See also [4].
- [41] N.C.M. Martens. Parity violation and the reality of space. In *Proceedings of the VSNU Student Research Conference, Utrecht, the Netherlands, 21 November 2012*, pages 221–224, 2012.

- [42] Simon Saunders. Mirroring as an a priori symmetry. *Philosophy of Science*, 74:452–480, 2007.
- [43] T. Maudlin. *Philosophy of Physics: Space and Time*. Princeton University Press, 2012.
- [44] Gordon Belot. Rehabilitating relationalism. *International Studies in the Philosophy of Science*, 13(1):35–52, 1999.
- [45] Gordon Belot. Geometry and motion. *British Journal for the Philosophy of Science*, 51(4):561–595, 2000.
- [46] Gordon Belot and John Earman. Pre-socratic quantum gravity. In Craig Callendar and Nick Huggett, editors, *Physics meets philosophy at the Planck scale: Contemporary theories in quantum gravity*, chapter 10, pages 213–55. Cambridge University Press, 2001.
- [47] H.G. Alexander, editor. *The Leibniz-Clarke Correspondence*. Manchester: Manchester University Press, 1956 [1717]. Originally written by Leibniz, G.W. & Clarke, S. in 1715-16 and published by Clarke, S. in 1717.
- [48] Tim Maudlin. Buckets of water and waves of space: Why spacetime is probably a substance. *Philosophy of Science*, 60:183–203, 1993.
- [49] Shamik Dasgupta. Symmetry as an epistemic notion (twice over). Forthcoming in *The British Journal for the Philosophy of Science*, page numbers of quotes refer to online version ([www.shamik.net](http://www.shamik.net); 9 Sept 2014).
- [50] Shamik Dasgupta. Symmetry as an epistemic notion (twice over). *The British Journal for the Philosophy of Science*, Advance Access:1–42, 2015.
- [51] Harvey R. Brown. *Physical Relativity: Space-time structure from a dynamical perspective*. Oxford University Press, 2005.
- [52] Harvey R. Brown. The behaviour of rods and clocks in general relativity, and the meaning of the metric field. [arXiv:0911.4440](https://arxiv.org/abs/0911.4440), 2009.
- [53] Arthur Eddington. *Space, time, and gravitation: an outline of the general relativity theory*. Cambridge: Cambridge University Press, 1987 [1920].
- [54] R. DiSalle. Space and time: Inertial frames. *Stanford Encyclopedia of Philosophy*, 2002 [revised in 2009].
- [55] Oliver Pooley. *The Reality of Spacetime*. Manuscript.

- [56] Daniel C. Dennett. Real patterns. *The Journal of Philosophy*, 88(1):27–51, 1991.
- [57] Shamik Dasgupta. Inexpressible ignorance. *Philosophical Review*, 124(4), 2015.
- [58] Shamik Dasgupta. Substantivalism vs relationalism about space in classical physics. *Philosophy Compass*, 10(9):601–624, 2015.
- [59] John S. Bell. *Speakable and unspeakable in quantum mechanics*. Cambridge University Press, 1987.
- [60] Peter R. Holland. *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics*. Cambridge University Press, 1995.
- [61] O. Pooley. *The Reality of Spacetime*. Forthcoming, 2013.
- [62] John Earman. *A Primer on Determinism*. D. Reidel Publishing Company, 1986.
- [63] John D. Norton. Causation as folk science. *Philosophers' Imprint*, 3(4):1–22, 2003.
- [64] J.C.C. McKinsey, A.C. Sugar, and P. Suppes. Axiomatic foundations of classical particle mechanics. *Journal of Rational Mechanics and Analysis*, 2(2), 1953.
- [65] Moritz Schlick. Space and time in contemporary physics: An introduction to the theory of relativity and gravitation. see mulder and van de velde-schlick [141, ch.9, p. 207-269]. 1979 [1922]. Translated by Henry Brose and Peter Heath from *Raum und zeit in der gegenwärtigen physik*, 4th edition, Berlin.
- [66] Moritz Schlick. *General Theory of Knowledge*. Library of Exact Philosophy Series. Springer-Verlag, 1974 [1925]. Translated by Albert Blumberg, from the 2nd German Edition of *Allgemeine Erkenntnislehre*, Berlin.
- [67] Moritz Schlick. *Philosophy of Nature*. Philosophy Library, Inc., 1949. Translated by Amethe von Zeppelin.
- [68] Syman Stevens. *The Dynamical Approach to Relativity as a Form of Regularity Relationalism*. PhD thesis, University of Oxford, 2014.

- [69] Nick Huggett. Why manifold substantivalism is probably not a consequence of classical mechanics. *International Studies in the Philosophy of Science*, 13:17–34, 1999.
- [70] Oliver Pooley. Substantivalist and relationalist approaches to spacetime. In Robert Batterman, editor, *The Oxford Handbook of Philosophy of Physics*. Oxford University Press, 2013.
- [71] Rasmus Grønfeldt Winther. The structure of scientific theories. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.
- [72] P. Suppes. A comparison of the meaning and uses of models in mathematics and the empirical sciences. *Synthese*, 12:287–301, 1960.
- [73] B.C. Van Fraassen. *The Scientific Image*. Oxford: Oxford University Press, 1980.
- [74] Lawrence Sklar. *Space, Time and Spacetime*. Berkeley: University of California Press, 1974.
- [75] James P. Binkoski. On the viability of galilean relationalism. *British Journal for the Philosophy of Science*, 2016. (Forthcoming).
- [76] Simon Saunders. Rethinking newton’s principia. *Philosophy of Science*, 80(1):22–48, 2013.
- [77] Nick Huggett. The regularity account of relational spacetime. *Mind*, 115(457):41–73, 2006.
- [78] F. Arntzenius. Are there really instantaneous velocities? *The Monist*, 83(2):187–208, 2000.
- [79] E. Grant. *Physical Science in the Middle Ages*. Wiley, New York and London, 1981.
- [80] E.J. Dijksterhuis. *The Mechanisation of the World Picture*. Princeton: Princeton University Press, 1986.
- [81] Claus Beisbart. How to fix directions or are assignments of vector characteristics attributions of intrinsic properties? *Dialectica*, 63(4):503–524, 2009.
- [82] Ralf Busse. Humean supervenience, vectorial fields, and the spinning sphere. *Dialectica*, 63(4):449–489, 2009.

- [83] M. Milgrom. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal*, 270:365–370, 1983.
- [84] C. Neumann. *Ueber die Principien der Galilei-Newton’schen Theorie*. Leipzig: B.G. Teubner, 1870.
- [85] Isaac Newton. De gravitatione. In *Unpublished Papers of Isaac Newton: A Selection from the Portsmouth Collection in the university Library, Cambridge*, pages 123–146. Cambridge University Press, 1962 [late 1660’s]. Translated and edited by A.R. Hall and M.B. Hall. Excerpts reprinted in [33].
- [86] René Descartes. Principles of philosophy. In *The Philosophical Writings of Descartes (vol. 1, pp. 207-252)*. Cambridge University Press, 1985 [1644]. Translated and edited by J. Cottingham, R. Stoothoff, and D. Murdoch. Excerpts reprinted in [33].
- [87] Thomas Møller Nielsen. *Symmetry, Indiscernibility, and the Generalist Picture*. PhD thesis, University of Oxford, 2015.
- [88] S. Saunders. Physics and leibniz’s principles. In K. Brading and E. Castellani, editors, *Symmetries in Physics: Philosophical Reflections*. Cambridge: Cambridge University Press, 2003.
- [89] Shamik Dasgupta. Individuals: An essay in revisionary metaphysics. *Philosophical Studies*, 145(1):35–67, 2009.
- [90] B.C. Van Fraassen. *An introduction to the philosophy of time and space*. New York: Columbia University Press, 1970.
- [91] John Earman. Laws of nature: The empiricist challenge. In R. Bogdan D.M. Armstrong, editor, *Profiles*, volume 4. Dordrecht: D. Reidel Publishing Company, 1984.
- [92] T. Maudlin. Why be humean. In [95].
- [93] Eddy Keming Chen. Our fundamental physical space: An essay on the metaphysics of the wave function. Manuscript.
- [94] D. Lewis. *Counterfactuals*. Cambridge: Harvard University Press, 1973.
- [95] T. Maudlin. *The Metaphysics within Physics*. Oxford University Press, 2007.

- [96] D.M. Armstrong. *What is a Law of Nature?* Cambridge: Cambridge University Press, 1983. Cambridge Studies in Philosophy.
- [97] A. Bird. *Nature's Metaphysics*. Oxford: Oxford University Press, 2007.
- [98] A. Hajek. Mises redux-redux: Fifteen arguments against finite frequentism. In *Philosophy of Probability: Contemporary Readings*, pages 209–227. Routledge, 2010.
- [99] B. Loewer. Two accounts of laws and time. *Philosophical Studies*, 160(1):115–137, 2012.
- [100] Marc Lange. Grounding, scientific explanation, and humean laws. *Philosophical Studies*, 164(1):255–261, 2013.
- [101] Elizabeth Miller. Humean scientific explanation. *Philosophical Studies*, 172(5):1311–1332, 2015.
- [102] Michael Townsen Hicks and Peter van Elswyk. Humean laws and circular explanation. *Philosophical Studies*, 172:433–443, 2015.
- [103] Harjit Bhogal. The metaphysical and the scientific: One weird trick for defending humeanism about laws of nature. Manuscript.
- [104] George Darby. Relational holism and humean supervenience. *British Journal for the Philosophy of Science*, 63:773–788, 2012.
- [105] Neil Dewar. What the humean cannot say about entanglement. <http://philsci-archive.pitt.edu/12046/>.
- [106] D.Z. Albert. Elementary quantum metaphysics. In J.T. Cushing *et al.*, editor, *Bohmian Mechanics and Quantum Theory: An Appraisal*, pages 277–84. Kluwer Academic Publishers, 1996.
- [107] David Lewis. *On the Plurality of Worlds*. Blackwell Publishers, 1986.
- [108] D. Hume. *An enquiry concerning the human understanding*. Oxford Clarendon Press, Reprint 1894 [1777].
- [109] Syman Stevens. The dynamical approach as practical geometry. *Philosophy of Science*, 82(5):1152–1162, 2015.
- [110] Syman Stevens. Regularity relationalism and the constructivist project. *British Journal for the Philosophy of Science*, Forthcoming.

- [111] Ned Hall. Humean reductionism about laws of nature. Manuscript, <http://philpapers.org/archive/HALHRA.pdf>.
- [112] Barry Loewer. Humean supervenience. *Philosophical Topics*, 24(1):101–127, 1996.
- [113] Michael Esfeld. Quantum humeanism, or: physicalism without properties. *The Philosophical Quarterly*, 64(256):453–470, 2014.
- [114] Antonio Vassallo, Dirk-André Deckert, and Michael Esfeld. Relationalism about mechanics based on a minimalist ontology of matter. *European Journal for Philosophy of Science*, pages 1–20, 2016.
- [115] Michael Esfeld, Dustin Lazarovici, Vincent Lam, and Mario Hubert. The physics and metaphysics of primitive stuff. *British Journal for the Philosophy of Science*, Forthcoming.
- [116] Michael Esfeld & Dirk-André Deckert. What there is: A minimalist ontology of the natural world. Manuscript, 2016.
- [117] Michael Esfeld, Dirk-André Deckert, and Andrea Oldofredi. What is matter? the fundamental ontology of atomism and structural realism. Manuscript.
- [118] Mario Hubert. Quantity of matter or intrinsic property: Why mass cannot be both. Manuscript, <http://philsci-archive.pitt.edu/11806/>.
- [119] J.B. Barbour and B. Bertotti. Machs principle and the structure of dynamical theories. In *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, volume 382, pages 295–306, 1982.
- [120] J. Earman. *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time*. Cambridge, Massachusetts: MIT, 1989.
- [121] D. Deutsch. Constructor theory. *Synthese*, 190(18), 2013.
- [122] D. Deutsch and C. Marletto. Constructor theory of information. *Proceedings of the Royal Society A*, 471(2174), 2015.
- [123] C. Marletto. Constructor theory of probability. *Proceedings of the Royal Society A*, 472(2192), 2016.
- [124] Albert Einstein. Quantum mechanics and reality. *Dialectica*, 2:320–324, 1948.
- [125] R. Healey. Nonlocality and the aharonov-bohm effect. *Philosophy of Science*, 64:18–41, 1997.

- [126] Tim Maudlin. *Quantum Non-Locality & Relativity: Metaphysical Intimations of Modern Physics*. Wiley-Blackwell, 2011. Third Edition.
- [127] D. Nolan. Quantitative parsimony. *British Journal for the Philosophy of Science*, 48:329–443, 1997.
- [128] A. Baker. Quantitative parsimony and explanatory power. *British Journal for the Philosophy of Science*, 54:245–259, 2003.
- [129] Lina Jansson & Jonathan Tallant. Quantitative parsimony: Probably for the better. *British Journal for the Philosophy of Science*, forthcoming.
- [130] Gordon Belot. *Geometric Possibility*. Oxford University Press, 2011.
- [131] Z.R. Perry. Chapter 4. problems for a dynamic theory of quantity. Part of PhD thesis manuscript, 2016.
- [132] C.G. Pendse. A further note on the definition and determination of mass in newtonian mechanics. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science: Series 7*, 27(180):51–61, 1939.
- [133] Herbert A. Simon. The axioms of newtonian mechanics. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science: Series 7*, 38(287):888–905, 1947.
- [134] C.G. Pendse. A note on the definition and determination of mass in newtonian mechanics. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science: Series 7*, 24(164):1012–1022, 1937.
- [135] V.V. Narlikar. The concept and determination of mass in newtonian mechanics. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science: Series 7*, 27:180:33–36, 1939.
- [136] C.G. Pendse. On mass and force in newtonian mechanics—addendum to “mass i.” and “mass ii.”. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science: Series 7*, 29(196):477–484, 1940.
- [137] H.-J. Schmidt. A definition of mass in newton-lagrange mechanics. *Philosophia Naturalis*, 30:189–207, 1993.
- [138] D.Z. Albert. *Time and Chance*. Cambridge, MA: Harvard University Press, 2000.
- [139] Shamik Dasgupta. Relationalism and the limits of possibility. Manuscript, Oct 2015.

- [140] J.B. Griffiths and J. Podolsky. *Exact Space-Times in Einstein's General Relativity*. Cambridge: Cambridge University Press, 2009.
- [141] Henk Mulder and Barbara van de Velde-Schlick, editors. *Moritz-Schlick: Philosophical Papers*, volume 11-I (1909-1922) of *Vienna Circle Collection Series*. Reidel Publishing Company, 1979.