Supersymmetric Five-Point Gluon Amplitudes in AdS Space

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We present the tree-level five-point amplitude of the lowest Kaluza-Klein mode of super-Yang-Mills theory on $\mathrm{AdS}_5 \times \mathrm{S}^3$, dual to the correlator of the flavor current multiplet in the dual $4d$ $\mathcal{N} = 2$ superconformal field theory. Its color and kinematical structure is particularly simple and resembles that of the flat-space gluon amplitude.

Introduction.—Via the AdS/CFT correspondence, correlation functions in conformal field theories (CFTs) are identified with on-shell scattering amplitudes in anti–de Sitter (AdS) space. On the one hand, from these holographic correlators we can extract useful data of the strongly coupled CFTs. On the other, we can use them to explore generalizations of various properties of flat-space amplitudes in curved backgrounds, which might hopefully shed light on the yet-elusive underlying principles.

Recent explorations have led to encouraging results in the search for AdS avatars of flat-space properties. For example, there has been much evidence for AdS realizations of the color-kinematic duality [1–9] and a double copy relation has been identified in AdS_5 [10]. Both properties play a pivotal role in the modern flat-space amplitude program and have numerous applications [11]. However, it should be noted that most of the progress in AdS has been limited to correlators with $n \leq 4$ points. For a systematic understanding of these properties it is imperative to study higher-point functions.

In this Letter, we present the first five-point amplitude of AdS supergluons. In particular, we consider the lowest Kaluza-Klein (KK) modes of super-Yang-Mills (SYM) theory on $\mathrm{AdS}_5 \times \mathrm{S}^3$, which arises as a decoupling sector in the holographic dual of several important $4d$ $\mathcal{N} = 2$ SCFTs [12–14]. Compared to supergravity, the kinematics of supergluons is much simpler, making them the ideal

arena for higher-point explorations. On the other hand, an eventual full-fledge AdS double copy relation would also make gluon amplitudes more fundamental. Unfortunately, computing holographic correlators by traditional methods is in general very difficult. This is partially due to the complicated effective Lagrangian, and also because of the many curved-space diagrams needed to be summed up. Instead, we will apply the modern techniques of [15–19], and in particular [20], to bootstrap the correlator. We show that the five-point function can be fixed by using only symmetries and consistency conditions, without inputing precise details of the effective Lagrangian. Moreover, we find a remarkably simple expression for the supergluon Mellin amplitude

$$\mathcal{M}_5 = \sum_{i=1}^{15} \frac{g_i n_i}{D_i},$$

which has the same form as the gluon five-point amplitude in flat space. Here, g_i are color structures given by cubic graphs and D_i are the associated scalar propagators. The kinematic numerators n_i turn out to satisfy the same relations as the color factors g_i, therefore giving rise to a color-kinematic duality. While the color-ordered supergluon amplitudes satisfy Kleiss-Kuijf relations [21] and the photon-decoupling identity as their flat-space counterpart, we show that interestingly they do not obey Bern-Carrasco-Johansson (BCJ) relations [22]. Furthermore, we find that the correction terms are particularly simple and hint at a pattern that may generalize to higher points.

Kinematics.—AdS supergluons are dual to scalar operators of the form $\mathcal{O}_k^{I;\alpha_1 \ldots \alpha_k}$, labeled by an integer $k = 2, 3, \ldots$.

They are the superconformal primaries of $\frac{1}{2}$-Bogomol'nyi-Prasad-Sommerfield multiplets of the $4d$ $\mathcal{N}=2$ superconformal algebra, with protected conformal dimension $\Delta = k$. These operators transform in the adjoint representation of a color group G_F [23] and in the spin-$k/2$ representation of the $\mathrm{SU}(2)_R$ R-symmetry group. In top-down holographic models [12–14], they arise as the S^3 KK modes of an $\mathcal{N}=1$ vector multiplet on an $\mathrm{AdS}_5 \times S^3$ subspace. In the large central charge limit, their couplings with the gravitational degrees of freedom in the $10d$ ambient space are parametrically suppressed. Therefore, at leading order it is consistent to consider the $\mathrm{AdS}_5 \times S^3$ theory by itself. In this Letter, we will focus on operators with the $k = 2$ KK level.

To conveniently keep track of the $\mathrm{SU}(2)_R$ indices, we contract them with two-component polarization spinors $\mathcal{O}_2^I(x; v) \equiv \mathcal{O}_2^{I;\alpha_1,\alpha_2}(x) v_{\alpha_1} v_{\alpha_2}$. The five-point function

$$G_5(x_i; v_i) = \langle \mathcal{O}_2^{I_1}(x_1; v_1) \cdots \mathcal{O}_2^{I_5}(x_5; v_5) \rangle \quad (1)$$

becomes a function of both the spacetime and the internal coordinates. Here, we suppressed the color indices on the left-hand side to avoid cluttering the notation. Clearly, G_5 is a polynomial of $(v_i, v_j) \equiv \epsilon^{\alpha\beta} v_{i,\alpha} v_{j,\beta}$ and every v_i should appear in each monomial precisely twice. Covariance under conformal symmetry and R symmetry allows us to write G_5 in terms of cross ratios. Moreover, fermionic generators in the superconformal group impose additional constraints. In particular, the chiral algebra construction [24] constrains the form of correlators when all operators are inserted on a $2d$ plane. Let us parametrize the plane by complex coordinates $(z, \bar{z})$. For the special coordinate-dependent polarizations $v_i^\alpha = (1, \bar{z}_i)$, the resulting correlator is independent of the antiholomorphic coordinates [24]

$$\partial_{\bar{z}_j} G_5(z_i, \bar{z}_i; v_i^\alpha = (1, \bar{z}_i)) = 0. \quad (2)$$

The five-point function has multiple color structures. In this Letter, we will focus on correlators which correspond to tree-level scattering in AdS. Moreover, all the particles are in the adjoint representation. Therefore, as in flat space, the color factors are linear combinations of the cubic tree diagrams $\{g_1, \ldots, g_{15}\}$ enumerated in Fig. 1. Each diagram represents a contraction of the color group structure constants f^{IJK}, e.g.,

$$g_1 = f^{I_1 I_2 J} f^{J I_3 K} f^{K I_4 I_5}. \quad (3)$$

These cubic diagrams are not all independent but are related by the Jacobi identity. For instance,

$$g_1 - g_2 + g_8 = 0, \qquad g_1 - g_6 + g_7 = 0, \quad (4)$$

with the complete set of relations given in (1) of Supplemental Material in Ref. [25]. These relations reduce the number of independent g_i to six.
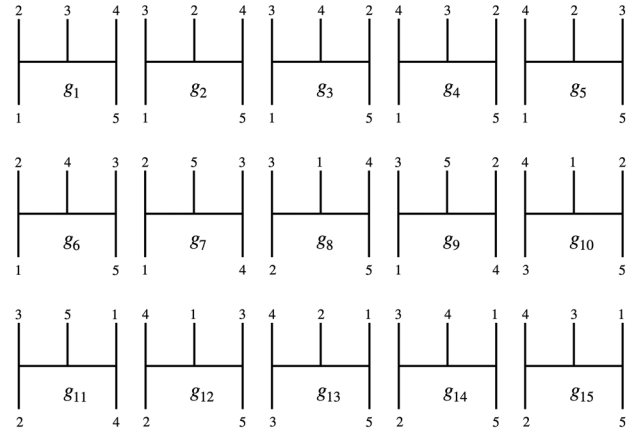

FIG. 1.   All possible cubic tree diagrams.

Finally, a convenient way to represent holographic correlators is the Mellin representation formalism [26,27] where the analytic structure is manifest. An n-point function can be written as

$$G_n(x_i) = \int [ds_{ij}] \left(\prod_{i<j} (x_{ij}^2)^{-s_{ij}} \Gamma[s_{ij}] \right) \mathcal{M}_n(s_{ij}), \quad (5)$$

where $\mathcal{M}(s_{ij})$ is the *Mellin amplitude*. The Mellin-Mandelstam variables s_{ij} satisfy relations identical to those of the flat-space Mandelstam variables $s_{ij}^\flat$

$$s_{ij} = s_{ji}, \qquad s_{ii} = -\Delta_i, \qquad \sum_{j=1}^{n} s_{ij} = 0, \quad (6)$$

except that the squared masses are replaced by the conformal dimensions. One can also recover the flat-space amplitude by taking the high energy limit [27]

$$T_n^\flat(s_{ij}^\flat) \propto \lim_{\beta \to \infty} \beta^a \mathcal{M}_n(\beta s_{ij}^\flat), \quad (7)$$

where a is some appropriate power. But we note that the limit can be zero because the polarizations in $T_n^\flat(s_{ij}^\flat)$ are restricted to special configurations [19].

Bootstrap.—In principle, G_5 can be computed by standard diagrammatic techniques. However, this would require explicit details of the off-shell effective Lagrangian whose complexity obscures the simplicity of the final "on-shell" correlator. A more efficient strategy is to bootstrap the answer using supersymmetry, as pioneered in [15,16]. Here, we will use the method of [20] which is specifically tailored for five-point functions.

We start with an ansatz which is a linear combination of all possible Witten diagrams. The only possible exchanged fields are the scalar supergluon field itself and a spin-one gluon field. The latter vector field is a superconformal descendant and therefore also carries the adjoint
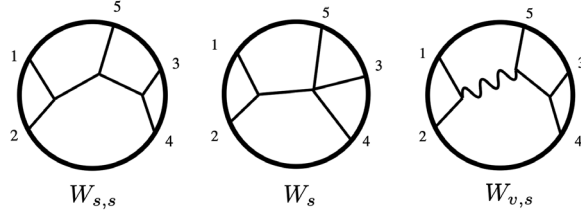
FIG. 2. Tree-level Witten diagrams. Straight lines represent scalar supergluons (s) and wavy lines are spin-one gluons (v).

representation of the color group. It has conformal dimension $\Delta = 3$ and is neutral under $SU(2)_R$. The ansatz is a linear combination of the exchange diagrams listed in Fig. 2

$$G_5^{\text{ansatz}} = Y_{s,s}(g,v)W_{s,s} + Y_s(g,v)W_s$$
$$+ Y_{v,s}(g,v)W_{v,s} + (\text{permutations}). \quad (8)$$

Here, $Y_X(g,v)$ are color and R-symmetry structures compatible with the diagram, parametrized by unknown coefficients. For example, there is a unique color and R-symmetry structure associated with the first diagram

$$Y_{s,s}(g,v) = \lambda_{s,s}g_7(V_{13245} + V_{13254} + V_{13524} - V_{14235}), \quad (9)$$

where $\lambda_{s,s}$ is an unknown parameter and V_{abcde} are pentagon-shaped contractions of the polarization spinors

$$V_{abcde} = (v_a, v_b)(v_b, v_c)(v_c, v_d)(v_d, v_e)(v_e, v_a). \quad (10)$$

The Witten diagrams $W_{s,s}$, W_s, and $W_{v,s}$ were evaluated in [20] as finite linear combinations of D functions $D_{\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5}$. For example,

$$W_{s,s} = \frac{1}{16x_{12}^2 x_{34}^2} D_{1,1,1,1,2}. \quad (11)$$

More details about R-symmetry structures and Witten diagrams can be found in Supplemental Material in Ref. [25]. In arriving at the ansatz, we have also used symmetry and compatibility with the flat-space limit. R symmetry prohibits double-exchange diagrams where both internal lines are spin-one gluons. On the other hand, the polarizations in the flat-space limit $\mathcal{T}_5^\flat(s_{ij}^\flat)$ are orthogonal to all momenta [19]. This forces the flat-space limit to vanish [28], forbidding vector single-exchange diagrams and five-point contact diagrams in the ansatz [29]. In total, there are eight unknown coefficients in the ansatz (8). Crossing symmetry reduces this number down to three.

Finally, as was noted in [20] all D functions in the ansatz are derivatives of $D_{1,1,1,1,2}$ (and its permutations) with respect to x_{ij}^2. On the other hand, the basic D function $D_{1,1,1,1,2}$ was studied in the amplitude literature [30,31] and can be expressed in terms of one-loop box diagrams [32]. Using these properties we can explicitly evaluate the x_{ij}^2

dependence of the ansatz, and implementing the chiral algebra condition (2) becomes straightforward. This condition is highly nontrivial and translates to an overdetermined set of linear equations for the unknown parameters. We find that these equations fix the ansatz completely up to an overall constant.

Five-point function.—We now present the five-point function. The Mellin amplitude is remarkably simple and can be written in a form resembling the flat-space gluon amplitude

$$\mathcal{M}_5 = \sum_{i=1}^{15} \frac{g_i n_i}{D_i}. \quad (12)$$

Here, D_i are Mellin scalar propagators associated with the ith diagram. More precisely, if the external labels of g_i are clockwise ($abcde$) [e.g., g_1 reads (12345)], then

$$D_i = (s_{ab} - 1)(s_{de} - 1). \quad (13)$$

The kinematic factors n_i are given by [33]

$$n_i = s_{ad}V_{acdeb} - s_{ae}V_{acedb} + s_{bd}V_{aedcb} - s_{be}V_{adecb}. \quad (14)$$

Remarkably, the kinematic factors satisfy the same algebraic relations as the color factors g_i, e.g.,

$$n_1 - n_2 + n_8 = 0, \qquad n_1 - n_6 + n_7 = 0. \quad (15)$$

This gives rise to an AdS version of the *color-kinematic duality* [22] at the level of five-point functions, and generalizes the previous observation at four points [5]. As a consistency check, (12) also factorizes correctly into lower-point amplitudes at its poles [34].

The Mellin expression (12) also leads to a very compact representation of the five-point function in position space

$$G_5 = -\frac{32}{\pi^2} \sum_{i=1}^{15} g_i \mathbb{D}_i W_i. \quad (16)$$

Here, W_i is the scalar double-exchange Witten diagram associated with g_i [$W_{s,s}$ in (11) up to permutations], and $\mathbb{D}_i$ are differential operators defined by

$$\mathbb{D}_i = \theta_{ad}V_{acdeb} - \theta_{ae}V_{acedb} + \theta_{bd}V_{aedcb} - \theta_{be}V_{adecb}, \quad (17)$$

with $\theta_{ab} \equiv x_{ab}^2(\partial/\partial x_{ab}^2)$. We checked that (16) is equivalent to the position space expression directly obtained from (8), upon using D-function identities. Setting $v_i^\alpha = (1, \bar{z}_i)$ in the coplane configuration, we find a holomorphic function as required by the chiral algebra construction

$$G_5(z_i, \bar{z}_i; v_i^\alpha = (1, \bar{z}_i)) = \frac{4}{5} \sum_{i=1}^{15} Z_i g_i, \quad (18)$$

where

$$Z_i = [abcde] + [baced] - [bacde] - [abced], \quad (19)$$

with $[abcde] = (z_{ab}z_{bc}z_{cd}z_{de}z_{ea})^{-1}$ and $z_{ij} = z_i - z_j$.

Amplitude relations.—The resemblance between flat-space and AdS amplitudes is clear from (12). To further explore the analogy, we will study the partial amplitudes

$$\mathcal{M}_5 = \sum_{\sigma \in S_4} A_5(1, \sigma(2), ..., \sigma(5)) \mathrm{Tr}(T^{I_1} T^{I_{\sigma(2)}}, ..., T^{I_{\sigma(5)}}).$$

Since all particles are in the adjoint color representation, the Kleiss-Kuijf relations and the photon-decoupling identity are automatically satisfied. On the other hand, we will show that the color-ordered amplitudes do *not* obey the flat-space BCJ relations, even though the Mellin amplitude satisfies color-kinematic duality. Interestingly, the correction terms exhibit a very simple structure.

Concretely, let us consider the following partial amplitudes in the Del Duca-Dixon-Maltoni basis $\{g_1, ..., g_6\}$ [35]

$$A_5(1,2,3,4,5) = \frac{n_1}{D_1} - \frac{n_7}{D_7} - \frac{n_8}{D_8} + \frac{n_{13}}{D_{13}} - \frac{n_{14}}{D_{14}},$$

$$A_5(1,3,2,4,5) = \frac{n_2}{D_2} + \frac{n_8}{D_8} - \frac{n_9}{D_9} + \frac{n_{14}}{D_{14}} + \frac{n_{15}}{D_{15}},$$

$$A_5(1,3,4,2,5) = \frac{n_3}{D_3} + \frac{n_9}{D_9} - \frac{n_{10}}{D_{10}} - \frac{n_{13}}{D_{13}} - \frac{n_{15}}{D_{15}},$$

$$A_5(1,4,3,2,5) = \frac{n_4}{D_4} + \frac{n_{10}}{D_{10}} - \frac{n_{11}}{D_{11}} + \frac{n_{13}}{D_{13}} - \frac{n_{14}}{D_{14}},$$

$$A_5(1,4,2,3,5) = \frac{n_5}{D_5} + \frac{n_{11}}{D_{11}} + \frac{n_{12}}{D_{12}} + \frac{n_{14}}{D_{14}} + \frac{n_{15}}{D_{15}},$$

$$A_5(1,2,4,3,5) = \frac{n_6}{D_6} + \frac{n_7}{D_7} - \frac{n_{12}}{D_{12}} - \frac{n_{13}}{D_{13}} - \frac{n_{15}}{D_{15}}.$$

After using the relations among the kinematic factors, we can express the right-hand side in terms of the six independent kinematic factors $n_1, ..., n_6$. In other words, the six amplitudes are related to $n_1, ..., n_6$ by the multiplication of a matrix which is made of scalar propagators D_i. One could try to invert the matrix and solve for $n_1, ..., n_6$ in terms of the partial amplitudes. In flat space, this is impossible because the matrix contains vectors with zero eigenvalues. Multiplying such a vector from the left gives rise to the BCJ relations [22] which are permutations of

$$s_{12}^\flat A_5^\flat(1,2,3,4,5) + (s_{12}^\flat + s_{23}^\flat) A_5^\flat(1,3,2,4,5)$$
$$+ (s_{12}^\flat + s_{23}^\flat + s_{24}^\flat) A_5^\flat(1,3,4,2,5) = 0, \quad (20)$$

and reduce the number of independent amplitudes from six to two. Crucially, all flat-space particles are massless, i.e., $s_{ii}^\flat = 0$ and all poles are at zero. In AdS space, however, the

matrix becomes *invertible* thanks to the different relations (6) and the massive poles in D_i. Therefore, there are no BCJ relations and all six color-ordered amplitudes are independent. Let us comment here that the absence of BCJ relations should not raise concerns. In flat space, the noninvertibility of the matrix could be argued by noting that the kinematic factors are gauge dependent while the color-ordered amplitudes are gauge independent. Being able to express the former in terms of the latter would violate gauge invariance. This argument rests on the scattering particles being quanta of a spin-one gauge field. Here, instead, we consider the scattering amplitudes of a scalar field. Gauge invariance is trivial and does not change the kinematic factors. Our statement of no BCJ relations also does not contradict the "differential BCJ relations" found in [7]. For their construction, it is important that the external particles are *massless* while in our case they are *massive*.

Let us now examine the analog of (20) in AdS space. Elementary manipulations lead to

$$s_{12}A_5(1,2,3,4,5) + (s_{12} + s_{23})A_5(1,3,2,4,5)$$
$$+ (s_{12} + s_{23} + s_{24})A_5(1,3,4,2,5)$$
$$= \frac{n_1}{D_1} + \frac{n_2}{D_2} + \frac{n_3}{D_3} - \frac{n_7}{D_7} + \frac{n_8}{D_8} + \frac{n_9}{D_9}$$
$$- \frac{n_{10}}{D_{10}} - \frac{n_{13}}{D_{13}} + \frac{n_{14}}{D_{14}} - \frac{n_{15}}{D_{15}}.$$

Note that the right-hand side is remarkably simple and consists of the same building blocks appearing in the color-ordered amplitudes (20) with ± 1 coefficients. It is instructive to compare it with the four-point case computed in [5]

$$s_{12}A_4(1,2,3,4) + (s_{12} + s_{23})A_4(1,3,2,4)$$
$$= \frac{n_s}{s-2} + \frac{n_t}{t-2} - \frac{n_u}{u-2},$$

where we find the same interesting pattern. The color-ordered amplitudes are given by

$$A_4(1,2,3,4) = \frac{n_s}{s-2} - \frac{n_t}{t-2},$$
$$A_4(1,3,2,4) = \frac{n_t}{t-2} - \frac{n_u}{u-2},$$

with $s + t + u = 8$ and $n_s + n_t + n_u = 0$.

Conclusions and outlook.—In this Letter we computed the five-point supergluon amplitude of the lowest KK level by imposing symmetry constraints and consistency conditions. We found extremely simple closed-form expressions both in Mellin space and in position space. We expect that the techniques can be used to compute amplitudes of higher KK modes. It would also be interesting to see if the same strategy allows us to fix higher-point functions. On the other hand, we noted that there is a remarkable resemblance between the Mellin amplitude and

the flat-space gluon amplitude. Therefore, it might be possible to borrow techniques from the flat-space amplitude program and develop more constructive methods (e.g., along the lines of Britto-Cachazo-Feng-Witten recursion relations [36]). Let us also note that although (12) displays color-kinematic duality, naively replacing the color factors with kinematic factors does not reproduce the five-point supergravity amplitude obtained in [20]. In fact, such a prescription already fails at four points. However, the recent work [10] pointed out that such an AdS double copy relation does exist for four-point functions at the level of superconformal *reduced* amplitudes (corresponding to stripping off supercharge delta functions in flat space). It would be interesting to extend the definition of reduced amplitudes to higher points and to check if the double copy structure persists.

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