THE INDUSTRIAL ORGANIZATION OF
INPUT MARKETS

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Declaration

I declare that (with the exception of Chapter 2, which is joint work with Dr. John Thanassoulis) this thesis represents my own work, and that (with the exception of Chapter 5, which is based on my M.Phil. thesis) none of it has been accepted, or is currently being submitted, for any degree or diploma or certificate or other qualifications in this University or elsewhere.

Signature............................................. Date...................................

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Abstract

This thesis consists of three closely connected pieces of work and an enhanced version of my M.Phil. thesis. The first three substantive chapters analyse vertical contracting in input markets under the exercise of differential buyer power. Chapters 2 and 3 consider the case of a supplier selling its output via a supermarket that offers captive demand (due to customers who anyway make a trip for their weekly shopping), which its rival, a local store is not able to offer. It is shown that the supermarket can negotiate an input price lower than the local store’s only if its advantage translates into sufficient bargaining strength in setting contracts. The existence of a waterbed effect, the implications of a partially covered market, a nonlinear pricing structure and welfare implications of a ban in discrimination are also explored. Chapter 4 modifies the standard model where size determines buyer power to show that if quantities need to be decided in advance, an increase in a retailer’s size is always welfare improving. For the presence of waterbed effects, we propose a novel insight that runs across different classes of models: following a discount to one retailer, the supplier faces two competing incentives - it wants to extract profits from the rival retailer but it also wants to transfer sales towards it. The waterbed effect is shown to be present only if the discount to the retailer is small, so incentives for profit extraction outweigh those for transferring business.

Finally chapter 5 studies a firm’s strategic incentive to outsource when its product displays network effects. It shows that a firm would choose to increase its observable marginal cost to make its competitor less aggressive and thereby increase its own probability of winning competition for the market. This is robust to small levels of uncertainty.
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Chapter 1

Introduction

The main theme of chapters 2, 3 and 4 is pricing in the wholesale market and the exercise of differential buyer power. The final chapter (an enhanced version of my M.Phil. thesis) explores vertical contracting in a market with network effects. Chapter 2 is joint work with Dr. John Thanassoulis.

Chapters 2, 3, and 4 look at the application of differential buyer power and ask what prices retailers would pay to a monopolist supplier if they were in some ways, asymmetric. In chapters 2 and 3 we have a different motivation for this asymmetry than what has, to our knowledge, been discussed in the literature. The retailers are conceptualised as a supermarket and a local store, with the supermarket having some potentially ‘captive demand’ due to the customers who travel to it anyway to do their weekly shopping. In chapter 4 we follow the standard notion of discounts based on size.

In these chapters we ask two fundamental questions. First, who pays a higher price? The results depend on whether the advantage possessed by the retailer translates into a better bargaining position. Second, we ask if there are any waterbed effects - i.e. is a reduction in input price to one retailer harmful for
CHAPTER 1.

its rivals? The results depend on the interplay between two competing forces: a discount to one retailer makes the supplier more able to extract profits from the rival retailer; but a discount to one retailer also makes the supplier want to transfer sales away from it. We argue that if the discount is small, the first effect dominates and we have a waterbed effect. However if the size of the discount is large, then the second effect dominates. Then a discount to a retailer is in fact, beneficial for its rivals. We show that this interplay of forces holds across a class of models and is central to the idea of this thesis.

Having shown that a waterbed effect exists under certain conditions, Chapter 4 asks whether the exercise of differential buyer power is harmful for customers. The answer depends on the size of the waterbed effect, which is directly related to the supplier’s incentives to extract profits from the smaller retailer. We show that these incentives are weaker if retailers compete in quantities. Consequently, while a retailer becoming bigger in markets where firms set prices might be a cause for concern, for goods where quantities need to be decided in advance, there is no such concern.

Chapter 5 considers a market with network effects and asks whether a firm has any strategic incentives to alter its observable marginal costs. We show that it is in a firm’s interest to increase its marginal cost because this makes its rival less aggressive. Outsourcing is relevant as it is a means by which the firm can achieve this end. The chapter also presents an extension of this framework to allow for uncertainty in the knowledge of these costs.

We now provide a summary of the literature in these related fields and then an introduction to the chapters in more detail.
1.1 Literature review of the field

Most of this thesis is concerned with the impact of differential buyer power on the level of input prices paid by downstream firms, on the relationship between the firms’ input prices, and on the welfare consequences of it. We now discuss the general literature around these issues.¹

Input price levels (linear contracts)

The first question of interest this thesis explores is: how does a monopolist supplier² set discriminatory input prices for retailers that are in some ways asymmetric? This question is particularly relevant because of its implications on antitrust law. It is generally believed that larger, more powerful chain stores can negotiate better deals with their suppliers, and that this harms their weaker competitors and competition in general. The most cited of these laws is the Robinson-Patman Act in the US which prevents suppliers from discriminating between buyers. Under EC law there is no direct equivalent but price discrimination issues do arise in the context of abuse by a dominant undertaking. For example, article 82C prevents ‘dissimilar conditions to equivalent transactions’.³

This is supported in final good markets where price discrimination has been shown to be welfare reducing. Pigou (1929) and Robinson (1969) showed that as long as all markets are served, with linear demand curves and constant marginal cost, total welfare is lower under discriminatory pricing regimes. This is because total output produced is unchanged. Schmalensee (1981) and Varian (1985) have

¹We will not address the literature relating to outsourcing and network effects here because there is a detailed discussion in Chapter 5.
²In what follows, and in the literature that assumes a monopolistic supplier, we typically think of the supplier as selling a brand rather than a product. This makes the assumption of a monopolist more convincing.
³See Whelan and Marsden (2006) for a comparative overview of the legal stance on price discrimination.
shown that a necessary condition for discrimination to raise welfare is that total output increases. Cowan (2007) considers markets where demand functions are additively shifted versions of each other and shows that in all commonly used in models of imperfect competition, social welfare is lower under discrimination.\footnote{However Cowan (2012) argues that discrimination can in fact be beneficial for customers under reasonable conditions.}

However, price discrimination in input markets works differently than in final markets. This is true for two reasons - first, the retailers’ demands are not independent of each other as in final markets and second, retailers have the ability to integrate backwards into the production of the good itself. Hence the results from final good markets cannot be automatically extended to input markets. Consequently, the academic literature from this area of research comes to rather different conclusions.

The main insight from the literature is a counterintuitive one - larger, more efficient retailers are charged higher input prices. If this is true then a ban on price discrimination would in fact be beneficial for these larger stores. DeGraba (1990) considers a framework where a monopolist upstream supplier $U$ sells an input to downstream Cournot competitors $D_i$ and $D_j$. They use this input along with others to produce a homogeneous output. The downstream firms are asymmetric in the sense that they have different constant marginal costs of production, in particular $MC_i < MC_j$. The supplier will then charge the low cost firm $D_i$ a higher input price than a high cost firm $D_j$, thereby partially offsetting the cost advantage. This is because $D_i$ has a less elastic derived demand for the input. Given the same input price, $D_i$ will sell a higher quantity than $D_j$. When the input price is increased, both would see a decrease in quantity, but for $D_i$, this represents a small percentage decrease. Hence $D_i$ is willing to pay more for the input than $D_j$. This can be exploited by $U$ who will charge the more efficient
firm a higher price. The optimal discriminatory contracts then serve to dampen the downstream asymmetry. In the long run this also has an adverse affect on firms’ incentives to invest in cost reducing strategies.\footnote{See Haucap and Wey (2004) for an application to the labour market - ‘equal pay for equal work’ rules may be beneficial as they can encourage innovation.}

Yoshida (2000) extends this framework by allowing the downstream firms to differ not just in efficiency in the use of other inputs ($\beta$ efficiency) but also in the use of the input bought from $U$ ($\alpha$ efficiency). This has the impact of changing the total quantity produced under discriminatory and uniform pricing regimes. He shows that similar to DeGraba’s case (where $\alpha_i = \alpha_j$), more efficient firms are handicapped in the discriminatory pricing regime. The more efficient firm still has the less elastic derived demand for the input and will be charged a higher price by the supplier.\footnote{The two however predict different welfare consequences - In DeGraba total quantity is constant but $D_i$ supplies less quantity and $D_j$ supplies more quantity than they would under uniform pricing. Hence under discrimination total quantity is produced inefficiently and this reduces welfare. Yoshida however shows that an contrary to the accepted results in final markets, in input markets an increase in total quantity is a sufficient condition for welfare reduction.}

However, the reason why these models provide such counterintuitive results about discriminatory pricing is because while they account for ‘willingness to pay’, they don’t account for ‘need to buy’ - i.e. there is no measure of buyer power. It must be the case that more efficient retailers have better outside options. In that case, $U$ is no longer an unconstrained monopolist and we can reverse the earlier results. The larger, more efficient store now pays a lower price.\footnote{Raskovich (2003) however shows that being ‘pivotal’ for a supplier’s decision to produce is bad for a large retailer because it has to cover the shortfall of other retailers’ payments.}

Inderst and Valletti (2009) model buyer power as the ability to source from a rival supplier. $D_i, D_j$ can continue buying from $U$ at the price it sets or they can reject the offer and incur a fixed cost $F$ to source from another rival supplier. A lower marginal cost means that the value of profits increases, both if the retailer chooses to source from the given supplier or if it chooses to exercise its outside
option. However, the profits from exercising the outside option increase more. Hence the more efficient retailer is more likely to reject $U$’s offer and therefore will need to be given a higher surplus in the form of a lower input price to ensure its participation. The supplier is hence constrained by this demand side substitution and will charge the more efficient firm a lower price.

Katz (1987) models buyer power as the ability to integrate backwards into the production of the input. He considers a setup where the downstream firms are equally efficient but differ in size. $D_i$ is a chain store and $D_j$ is a local store. He argues that the chain store has a viable threat to integrate backwards due to its larger demand and the economies of scale in production of the input. This allows $D_i$ to get a lower input price from $U$.\footnote{This assumes that as the input price offered to $D_j$ reduces, the incentives of $D_i$ to integrate also reduce since a low price signals that the supplier is a low cost one and will set low prices in the post integration regime. This implies that the integration frontier is downward sloping. If on the other hand, $D_i$ has complete information about the supplier, a low input price to the rival doesn’t have a signalling role and only serves to reduce the non-integration profits. It then increases incentives to integrate and the frontier is upward sloping. In this case $D_i$ is not able to get a lower price than $D_j$.}

Additionally, there might be other reasons why large buyers are able to secure lower prices. DeGraba (2003) considers the impact of incomplete information. $U$ can distinguish larger retailers from smaller ones but cannot observe retailers’ valuations. If $U$ is risk averse then a single buyer that demands a specific quantity represents a riskier profit source than multiple smaller buyers. The larger retailer will hence be charged lower prices. Smith and Thanassoulis (2006) consider the case of upstream competition with supplier-level volume uncertainty. They show that increasing marginal costs imply that large buyers will receive lower prices. Snyder (1996) shows that large buyers receive lower prices because they destabilise collusion. On the other hand, Chipty and Snyder (1999) and Inderst and Wey (2007) show that with increasing marginal costs, if volumes are uncertain, the supplier penalises the larger buyer.
1.1. LITERATURE REVIEW

Input price levels (nonlinear contracts)

While the previous literature assumed that the contract structure between the upstream and downstream firms was of a linear form, there is sufficient evidence that large retailers get offered volume discounts, fixed payments and rebates. The investigation into the Supply of groceries in the UK, carried out by the Competition Commission, conducted a survey where 70% of the respondents said that suppliers frequently make marketing contributions and promotional investments and 43% claimed that suppliers gave ‘other’ non unit price rebates. In fact, respondents said they prefer to get fewer discounts on the unit price and would rather differentiate on other aspects. 9

In this context, Inderst and Shaffer (2009) consider $U$ setting nonlinear contracts for the downstream firms. The supplier can then use the per unit price to increase the size of the total surplus and the fixed payment to maximise its own share of it. They show that $U$ will charge a wholesale price that is increasing in the joint profit margin of the rival and itself and in the diversion ratio between the retailers. Therefore the firms are charged equal prices if they are equally efficient (in which case they have the same joint profit margin with the supplier) or if they are in separate markets (in which case the diversion ratio is zero). If the demand is linear then the diversion ratio is constant, and the more efficient firm will be charged the lower per unit price. This implies that allocative efficiency is higher since more productive firms are favoured. Therefore a ban on price discrimination reduces allocative efficiency and leads to lower consumer surplus.

The crucial assumption however is that all contracts are observable. If they are not O’Brien and Shaffer (1992, 1994); McAfee and Schwartz (1994); Rey and Verge (2004) show that $U$ chooses to maximise bilateral profits instead of joint profit. $U$ is then forced to set wholesale prices equal to the marginal cost of

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production. A ban on price discrimination makes contracts observable and leads to an increase in all wholesale prices.

Expression of buyer power

The papers discussed previously (Katz, 1987; Inderst and Valletti, 2009) have accounted for buyer power by arguing that the stronger retailer has a better outside option. However, buyer power can also be modelled directly. O’Brien and Shaffer (1992, 1994) consider $U$ as bargaining simultaneously and independently with $D_i, D_j$. Dobson and Waterson (2007) and Björnerstedt and Stennek (2007) consider a bilateral oligopoly framework with multiple buyers and sellers where prices are determined through bargaining. Smith and Thanassoulis (2006) also consider multiple buyers and suppliers who choose each other randomly and engage in bilateral negotiations over a per unit price.

While these capture buyer power, they are not able to capture differential buyer power. Gans and King (2002); Mathewson and Winter (1997); Majumdar (2005) all make the claim that a buyer group with a first mover advantage will benefit over other smaller buyers. Chen (2003) also considers a sequential setup that allows for the larger store to have a stronger bargaining position. Following McAfee and Schwartz (1994) this paper says that the supplier has an incentive to make upfront contracts with all the fringe buyers. However, with the large buyer which possesses countervailing power, such contracts can be renegotiated. Therefore after $U$ decides input prices for the fringe, the large retailer can renegotiate with $U$ to determine a separate contract. This is the primary motivation for our bargaining model in chapters 2 and 3. However the fringe in this model is passive, while ours allows the smaller retailer to act strategically.
Consequences of buyer power (welfare increase)

The papers discussed above have highlighted how buyer power has been modelled in the literature. However we are principally interested in the consequences of these expressions of buyer power, both on their rivals, and on the customers. One of the early works done in this area is due to Galbraith (1952) who claims that the presence of large, powerful retailers is good for customers. The argument is the following - large retailers possess countervailing power which neutralises the monopoly power possessed by large suppliers. They use this countervailing power, ‘by proxy’, on behalf of the individual customer, since they pass on some of their cost savings in the form of lower prices. Buyer power is hence a desirable feature of markets. However, Stigler (1954) argues that Galbraith provides no rational basis to this ‘dogma’ since there is no reason why buyers would ‘lose sight of long run advantages of monopolistic behaviour in a welter of irrational competitive moves’. Hunter (1958) examines the countervailing function of the British Co-operative movement. He argues that the restricted commodity base of the Co-operative meant it could not harness such countervailing power. Therefore the existence of large buyers does not ensure the favourable outcome Galbraith suggested. This is particularly relevant in the case where suppliers negotiate non-linear tariffs with the retailers. If the retailer negotiates a discount on the fixed component only, then there are no incentives for these savings to be passed on to the customers.

Chen (2003) shows how countervailing power in the presence of nonlinear tariffs is still beneficial for customers. The model consists of a supplier facing a dominant retailer and a competitive fringe. As mentioned before, $U$ sets linear input prices for the fringe, but the dominant retailer can negotiate a separate nonlinear deal. Chen shows that a rise in countervailing power leads to lower retail prices for the customers, but this is not for the reason Galbraith suggested.
A rise in countervailing power reduces the supplier’s share of the profits coming from the dominant retailer. It is then in the supplier’s interest to make up for the lost profits by boosting sales to the fringe. It hence reduces their input price which shifts their supply curve to the right and decreases retail prices. This captures the idea that once the share of the joint profits the supplier can keep reduces, it is less costly for it to decrease prices to the fringe. Therefore an increase in countervailing power is beneficial for the rivals. Customers are then better off not because of a proxy minded retailer who passes on its cost savings, but because the supplier, in an attempt to counter the reduction in its profits, creates better terms of trade for the rival retailers.

Consequences of buyer power (welfare decrease)

Galbraith (1952) and Chen (2003) both show that an increase in buyer power is beneficial for the customers, either by the proxy minded retailer who passes on some of the cost savings to the customers, or by the supplier’s reaction to boost supply terms for the rival retailers. However Adams (1953) highlights that strong buyers, rather than having countervailing power, might develop ‘coalescing power’. He cites the example of wage increase and inflation in the labour market to argue that the upstream and downstream firms can enhance each other’s powers in a way that is detrimental to the consumers. Dobson (2006) also argues that dominant positions by different parties, rather than checking market power to the public benefit, might exacerbate detrimental economic effects since they coalesce in a manner to mutually protect incumbent positions.

The argument by Galbraith and Chen seems counterintuitive for another reason. Their implicit finding is that an increase in buyer power possessed by one retailer is weakly beneficial for its rivals. But surely, an increase in a retailer’s
buyer power enhances its bargaining position \textit{vis-a-vis} the rival retailers. It should then impose a negative externality on the fringe?

This is the motivation for what has come to be known as the ‘waterbed effect’ whereby an improvement in terms of trade offered to one retailer creates worse terms of trade for the other retailers. The early informal argument for the waterbed effect was that if a supplier gives a discount to one retailer, it must increase prices to other retailers otherwise it would be forced to shut down. This however begs the question - if the supplier could increase prices to the rival retailers, why did it not do so before? Why wait for the terms of trade for one retailer to improve? This is additionally not convincing in the context of a supplier with market power.

Inderst and Valletti (2011) have proposed the most widely accepted model for the presence of waterbed effects. Their argument relies on the fact that discounts are based on size. The supplier sells via two rival retailers in several separate markets. The retailers can buy the good at the input price set by the supplier, or they can reject the offer and incur a fixed cost $F$ to source from a rival retailer who will sell the input to them at cost of production. If one retailer then starts operating in more than one market, it is able to spread this fixed cost over a larger number of units. It is then more likely to reject $U$’s offer. This gives it buyer power. If the supplier wishes not to lose the business from this retailer, it will have to offer it a lower input price.

Crucially however, this imposes a \textit{negative externality} on the smaller rival. A decrease in the larger retailer’s input price decreases the rival’s profits both from sourcing from the given supplier and from exercising its outside option. However the profits from exercising the outside option decrease more. Hence, ceteris paribus, the smaller rival is less likely to exit the game. This means that it is now \textit{feasible} for the supplier to increase its input price, while maintaining its
participation. A decrease in the larger buyer’s price then creates an opportunity for an increase in the smaller rival’s price that was not present before.

As a result of paying lower input prices, the larger buyer will pass on some of these cost savings to the final customers in the form of a lower retail price. As a result of the waterbed effect however, the smaller buyer might be forced to pass on some of these cost increases to the final customers in the form of a higher retail price. Inderst & Valletti show that the net effect on welfare is negative if the firms were already quite different in size, or if the fixed cost of searching for the rival retailer was high. Therefore the exercise of differential buyer power might not be beneficial for the customers.\textsuperscript{10}

Dobson and Inderst (2008) argue that there might be other reasons for welfare reduction. Even if retail prices don’t increase in the short run, the smaller rival’s margins will be squeezed. This causes long run exit and monopolisation of the retail industry which is not in the customers’ interest. Moreover, there is inefficiency created because retailers substitute variety for quantity in order to increase their buyer power. Allocative inefficiency also might arise as customers are forced to travel to distant large retailers instead of the local off-licence which is now less competitive. Additionally, since larger buyers continue to get discounts anyway, they have fewer incentives to engage in cost reducing investments.

There are caveats however. With non-linear pricing structures there is no reason for the large buyer to pass on part of the discount to the customers so the proposed mechanism breaks down. The argument from Chen (2003) that a discount to one retailer makes a discount to the other retailers less costly is still valid.\textsuperscript{10}Smith and Thanassoulis (2006) analyse a model of supplier level uncertainty and the effect of market structure changes on seller cost expectations. They show that decreasing returns to scale in upstream technology give rise to a waterbed effect but upstream economies of scale lead to an ‘inverse’ waterbed effect.
valid - if the contracts are perfectly observable and buyers are roughly symmetric, this undermines the supplier’s bargaining position vis-a-vis other retailers who can now ask for a discount. Additionally, it might be in the supplier’s interest to reduce the rival retailer’s prices so as to keep alternative channels of supply open in case negotiations break down with the large retailer. Moreover, if the upstream industry is oligopolistic, then buyer power reduces the probability and efficacy of supplier collusion which implies that all retailers pay lower prices. The presence of upstream competition also implies that suppliers must coordinate the price increase and it is not clear how they would do so. Small retailers also buy from buyer groups which means they should be able to get part of the size related discounts. Finally, the usual models for waterbed effects also perhaps overstate the size because the large retailer faces competition only from the smaller rivals, not from other large retailers. In the presence of those, final customer detriment is likely to be lower.

In light of this review of the literature, we move on to discuss the contributions of each chapter in more detail and show where our research fits in.

1.2 Chapter 2 - Input price discrimination with different routes to market

Chapter 2 looks at equilibrium pricing in input markets where downstream retailers are asymmetric. Although this question has been addressed in the literature, the source of the retailers’ asymmetry has been recognised only as efficiency (DeGraba, 1990; Yoshida, 2000; Inderst and Valletti, 2009; Inderst and Shaffer, 11 The next three chapters of this thesis will argue that there is another reason why the waterbed effect might not exist: following a discount to one retailer, the supplier earns a smaller markup on those sales. It hence wants to ‘transfer business’ to the rival retailers.
and as size (Katz, 1987; Inderst and Valletti, 2011). However, retailers differ in many other aspects. The key difference in our model from the rest of the literature is that we consider a new source of asymmetry - the basket of goods provided at each retailer. Chapter 2 and Chapter 3 consider the case of a supplier selling its output via a local store and a supermarket. The supermarket is not modelled as selling higher volumes or being more efficient than local store, but rather as the store that offers a larger variety of goods. This captures the idea that customers visit supermarkets to do their weekly shopping. The model is set up as a Hotelling framework with two types of customers - ‘one off shoppers’ who want to buy only the good in question and hence pay symmetric transport costs to go to either retailer, and ‘supermarket shoppers’ who want to purchase a basket of goods large enough that they pay zero cost to travel to the supermarket. The supermarket is then a cheaper route to access the market because it has this potentially ‘captive demand’.

Given this new source of asymmetry we ask the standard question - does the supermarket pay a lower price than the local store? We find, similar to DeGraba (1990); Yoshida (2000), that if the supplier has all the bargaining power, the supermarket is charged a higher price than the local store. This is because the captive demand possessed by the supermarket means that it is willing to pay more for the input than the local store. If the supplier has all the bargaining power, it will exploit this low elasticity of derived demand and charge the supermarket a higher price. This is our benchmark case.

Of course the reason for this counterintuitive result is that we have not accounted for buyer power. We incorporate the notion of buyer power in our model but we do so differently from Katz (1987); Inderst and Valletti (2009). While supermarkets do have better outside options than their smaller counterparts, they are also more likely to have a direct role in the negotiations. This is our motiva-
tion for using bargaining strength as the expression of buyer power. However, we do not model this as the supplier bargaining simultaneously and independently with all the retailers as in Dobson and Waterson (2005); Smith and Thanassoulis (2006); Björnerstedt and Stennek (2007), since the assumption of simultaneous independent negotiation ignores downstream strategic interaction. We instead propose a simple sequential game where the supermarket has the bargaining power to make preemptive offers to the supplier. The supplier can accept or reject the supermarket’s offer and then makes an offer to the local store.\textsuperscript{12}

Accounting for buyer power in this way, the analysis shows that the supermarket is for sure able to lower its input price. The supplier could still punish the supermarket by lowering the input price to the local store. However, this is now sub-optimal. When the supplier had bargaining power, it offered a high price to the supermarket to extract rents from the captive demand. When the supermarket has bargaining power the supplier cannot access the extra surplus the supermarket secures. However, the supplier can seek to capture more of the customers’ surplus via the local store which has little bargaining power. Thus it raises the price offered to the local store.

The analysis also shows that while the local store pays a higher price than the supermarket, it also might pay a higher price than it did in the benchmark case; i.e. there might be a waterbed effect. Similar to Inderst and Valletti (2011), we find that a discount to the supermarket makes an increase the the local store’s price feasible. In Inderst and Valletti (2011) this was because a discount to the large retailer made the smaller retailer less likely to exercise its outside option. In our model this is because we make the standard assumption of full market cover. This ensures the supplier’s pricing strategy in the benchmark case is to capture all the valuation from the indifferent customer. When the supermarket

\textsuperscript{12}Chapter 3 will show that this order of moves is robust.
has bargaining power, it makes a smaller price offer. This means that its retail price is now lower. Due to strategic complementarity the retail price of the local store is lower too. Therefore all customers are still buying the good but paying lesser for it. Hence it is now feasible to increase the local store’s input price while maintaining full cover.

It is however, not always optimal to do so. Our analysis shows that there are two competing forces at work. Following a discount to the supermarket, the supplier would like to make use of its enhanced ability to extract profits from the local store. This requires it to raise the local store’s price. However, it is also earning a lower mark-up on the supermarket’s sales. It would hence like to shift sales away from it towards the local store where it earns a higher mark-up. This requires it to lower the local store’s price. We show that if the supermarket receives a small discount then the supplier’s incentives to extract profits are greater because the large market share of the local store makes transferring business a costly strategy. This gives us the waterbed effect. However, if the supermarket receives a large discount then the low market share of the local store increases the supplier’s incentives to transfer business. In this case a discount to the supermarket is beneficial for its rivals. Therefore, while small discounts are harmful for the rivals, large discounts are in fact, beneficial.

The main contribution of this chapter then is to highlight a different source of buyer power - variety; to explore a different mechanism for its expression - the bargained setup, and a different intuition for the understanding of waterbed effects.
1.3 Chapter 3 - Partial cover and robustness checks

Chapter 3 considers some extensions and robustness checks on the framework presented in Chapter 2. Its first contribution is to recognise that although full cover of the market is the standard assumption, it is not entirely an appropriate one. Full cover in our model is optimal for the supplier only if the value of the good is very large compared to the transport cost. These would be ‘emergency goods’ for which it is too costly for customers to wait for a supermarket shop - such as lightbulbs, or milk for breakfast. For all other types of goods, the supplier would prefer to keep prices high and create scarcity among the customers. Here a reduction in retail prices will affect whether customers decide to buy. Therefore we need a model that captures these volume effects.

However, to capture them we do not need to change our framework to a Cournot setting because the presence of two types of customers in our model allows the retailers to compete under partial cover as well; they can become monopolists over the one off shoppers and compete over the supermarket shoppers. In this context, our analysis shows that if the supplier has all the bargaining power, it still charges the supermarket a higher price than the local retailer. This is because there is no additional asymmetry created between the retailers due to partial cover. The supermarket still has a less elastic demand for the input due to the captive demand and thus is exploited by the supplier.

However, when we account for buyer power in a similar way to Chapter 2, it is now no longer the case that the supermarket is always able to negotiate a better deal than its rival. This might seem counterintuitive since the supermarket is now in a stronger bargaining position. However this is so because the supplier might
want to attract new customers by keeping the input price of the local store below
the supermarket’s. If the local store is able to give the supplier a high level of
profits exclusively then the supermarket is forced to make a high offer otherwise
it will be rejected. This raises its retail price and reduces the volumes it sells.
The supplier would then prefer to decrease the local store’s input price below
the supermarket’s so it can access more customers. Under full cover, this was
not an option. The only incentives then were to exploit the local store’s weak
bargaining position. Therefore under partial cover, despite having bargaining
power, the supermarket will not always be able to negotiate a better deal than
its rival.

Additionally, our analysis shows that there is never a waterbed effect. This
follows directly from the observation that the supplier is now unconstrained in the
benchmark equilibrium. Unlike under full cover, a discount to the supermarket
does not create any additional opportunity for profit extraction. The only force
at play then is that of transferring business. A decrease in the supermarket’s
price always serves to decrease the local store’s market share. This increases the
supplier’s incentives to transfer business. As a result, a discount to the super-
market is always followed by a discount to the local store.

Given the two regimes of full and partial cover presented in Chapters 2 and 3,
we also ask what the supplier would prefer to do. Since the supplier is a monop-
olist, given its choice of input prices, it can implicitly decide whether it wants to
supply the entire market or only a part of it. We show that over the parameter
range where the supplier makes non zero profits under both full and partial cover,
it will always find it optimal to supply only part of the market. This is because
of the standard monopoly outcome - it can raise profits by increasing prices and
decreasing volumes until the demand elasticity is unitary.
In the second part of Chapter 3 we perform a robustness check on the bargaining game presented in Chapter 2. The bargaining game allowed the supermarket to make preemptive offers to the supplier. It therefore explicitly gave it a first mover advantage. One could argue that the results are artificial since we had not shown why it was reasonable to assume so. Chapter 3 demonstrates that this assumption is robust by adding another stage before the actual game where the retailers bid for the right to be the first mover. The supermarket is indeed willing to bid more in equilibrium. This is because it is punished disproportionately by the supplier if it moves second. Hence it has a lot more to lose by not going first and will be willing to bid more for that right. The bargaining order is indeed as we have modelled in Chapter 2.

The third part of this chapter recognises that while we have so far only allowed the supplier to charge linear tariffs, empirically a lot of contracts are nonlinear (in the form of rebates, fixed payments and volume discounts). Our analysis shows that with nonlinear contracts, the incentives of the supplier to discriminate are reversed. Similar to Inderst and Shaffer (2009), if the supplier has all the bargaining power and can set nonlinear tariffs then it will maximise industry surplus and charge the supermarket a lower per unit price than the local store. If on the other hand, the supermarket has bargaining power then it will, as with linear pricing, always be able to secure a lower per unit price than the local store. However, the difference between the equilibrium input prices is smaller with nonlinear pricing. This is because the supplier can extract the local store’s profits via the fixed fee and therefore is less willing to punish it.

Finally we return to the linear pricing model of Chapter 2 and conduct welfare analysis. We show that welfare of consumers is always higher if the supplier is not allowed to discriminate. This is similar to DeGraba (1990). In his setup the
removal of discrimination means the low cost firm does not subsidise the high cost firm so total output is produced at a lower average cost. This increases welfare. In our model, the supplier discriminates by extracting surplus from the supermarket shoppers while keeping the one off shoppers better off. When this is no longer allowed, the supermarket shoppers benefit and the one off customers are worse off. However total welfare always increases because as the proportion of supermarket shoppers decreases, the damage done to the one off customers also decreases. A ban on discrimination then is always welfare improving.

1.4 Chapter 4 - Waterbed effect and welfare

Chapter 4 addresses the question of waterbed effects and welfare directly in the standard setting of Inderst and Valletti (2011). As outlined previously, their model consists of a supplier selling its output via retailers that are asymmetric in size. The retailers can either accept the ‘take-it-or-leave-it’ offer made by the supplier, or they can reject it and incur a fixed cost to find another supplier which will sell the good at cost. The larger retailer can spread these fixed costs over a larger number of units and hence needs to be given a higher amount of surplus in the form of a lower input price to ensure its participation. More importantly, the larger retailer also imposes an externality on its smaller rival who is now less likely to exercise its outside option. The supplier can exploit this and charge it a higher price. This gives the waterbed effect. Inderst and Valletti (2011) highlight that if the fixed cost of going to a rival supplier is high, and if the retailers are already asymmetric, this waterbed effect can be welfare reducing.

Crucially, the setup used for welfare analysis is a fully covered one. The UK grocery market investigation conducted by the Competition Competition-Commission (2008) uses this model to analyse the presence of a waterbed effect. They argue that since the Hotelling model uses a fully covered market, it over-
states the size of the waterbed effect.\textsuperscript{13} With a market that is partially covered an increase in the large retailer’s market share is not entirely at the cost of the small retailer’s market share. Hence consumer detriment might be overstated. We take account of this criticism and extend the Inderst and Valletti (2011) framework to allow for a partially covered market. We keep the notion for discounts unchanged but allow the firms to compete in quantities. This has the effect of keeping the motivation for the discounts unchanged but allows us to conduct a more comprehensive welfare analysis. Additionally, quantity competition is a better representation of the type of competition in case of goods where shelf space and perishability impose significant capacity constraints. An increase in one retailer’s size in this setup continues to imply that it can spread the fixed costs of going elsewhere over a larger number of units. Hence the larger retailer is still able to get a lower price. Everything else being equal, it will continue to pass on some of these cost savings to the customers and steal some customers from its rival. Hence the waterbed effect continues to exist. This is in line with Inderst & Valletti’s initial general setup.

Inderst and Valletti (2011) however consider a restricted parameter space where the fixed cost of going to the rival supplier is small. Equivalently, this requires that the increase in size of the retailer also be small. This ensures both participation constraints bind. We expand the parameter space under consideration to account for large increases in size too. Similar to the themes in the previous chapters, we find there are again two forces at work. For small increases in size, the supplier prefers to extract profits from the smaller retailer who is now less likely to reject its offer. However if a retailer becomes very large then the supplier’s incentives to transfer business away from it to the smaller retailer

\textsuperscript{13}See Competition-Commission (2008) Para 5.30
take over and it, in fact, reduces the price offered to the smaller retailer. Therefore, for large increases in size, there might not be a waterbed effect. This is not specific to quantity competition. However, the incentives to transfer business are indeed higher under quantity competition. This is because strategic substitutability ensures than an increase in a rival’s competitiveness is more damaging to own market share compared to strategic substitutability associated with price competition. A small market share reduces the costs of transferring business, and the supplier is less likely to continue extracting profits.

This leads to our main result. An increase in size under quantity competition is always welfare improving. Consequently, while the increase in size of one retailer might be detrimental to its rivals, it is always beneficial for customers because the damage to the smaller rival is not disproportionate. This means that the smaller rival might face higher costs and decrease the quantity it supplies, but it will not be enough to overpower the increase in quantity provided by the larger retailer. Under price competition an increase in size might be welfare reducing because the supplier is willing to allow a disproportionate waterbed effect. With quantity competition, the incentives to transfer profits take over much sooner and the supplier is not willing to allow it.

The main contribution of this chapter is then to highlight that while small increases in a retailer’s advantage are detrimental to its rivals, large increases might be beneficial; and that contrary to price competition, with quantity competition an increase in a retailer’s size is always welfare improving.
1.5 Chapter 5 - Outsourcing in a market with network effects

Chapter 5 is an enhanced version of my M.Phil. thesis. It falls within the scope of vertical contracting but is not directly related to the earlier theme of price discrimination.

This chapter considers a market with network effects. The idea of network effects is an increasingly important one\textsuperscript{14}, however very little has been written about the nature of vertical contracting in such a setting. These markets are characterised by high switching costs, hence firms use a ‘bargain followed by rip off’ strategy to ensure consumer lock in. The incentives to delegate in a market with network effects are then starkly different from those in regular markets because firms compete \textit{for} the market.

In markets without network effects, it is known that delegation is in the individual interest of the firm (Bonanno and Vickers, 1988; Vickers, 1985). We extend the setup in these models to account for network effects and conduct our analysis. These are modelled in an all or nothing way; firms compete in a two stage game: in stage 1 firms compete in quantities; and in stage 2, the firm that sold a higher quantity in stage 1 wins the whole market and can make monopoly profits. This captures the idea of network effects through technology adoption and switching costs. To ensure that the firm that wins the market has a conclusive and unambiguous win, we also assume the firms compete in predetermined discrete quantities. Additionally, we only consider the mixed strategy equilibrium since the others can be discounted by forward induction arguments.

\textsuperscript{14}See for example the discussion in Farrell and Klemperer (2006) about examples of network effects in computing (operating systems and applications), telecommunications, video recordings (betamax versus VHS, DVD), credit cards, keyboards (QWERTY versus DSK) and securities markets and exchanges markets.
Our analysis shows that it is in a firm’s strategic interest to \textit{increase} its observable marginal cost in order to win competition for the market. This is because an increase in own costs makes the rival less aggressive. The mechanism is as follows: an increase in own costs reduces own profits irrespective of action, but it reduces them asymmetrically. Profits from choosing a high quantity fall relatively more than from those from choosing low quantity. Indifference and subsequent equilibrium requires its rival to make choosing the low quantity less attractive for firm $i$ - which is achieved by the rival itself choosing the low quantity more often. This asymmetry changes the probability distribution of the rival’s strategies to a more ‘friendly’ one. Outsourcing is then useful as it allows the firm to achieve this increase in costs by charging a wholesale price above the cost of production. Subsequently, ‘vertically integrated’ profits can be extracted through a fixed franchise fee.

The chapter then considers the impact of uncertainty on the part of the opponent. It can be shown that this increases aggression. If a firm cannot credibly communicate its change in marginal cost, then aggression due to uncertainty could overpower the friendly behaviour induced by increasing costs. As a result, a firm may not always want to increase its marginal cost. Our previous result is hence robust to only small levels of uncertainty.

The main contribution of this chapter is to show that in markets with network effects, a firm that wants to increase its probability of winning the market will choose to outsource its product and charge a wholesale price higher than the cost of production. However, it will do so only if it can communicate this increase well enough to its rival.
Chapter 2

Input price discrimination with different routes to market †

Abstract: This chapter considers how a supplier would choose to price discriminate when the retailers offer different routes to access the market. We study a supplier selling its good via either a supermarket, a local store, or both; but the supermarket offers a cheaper route to access the market. We demonstrate that if the supplier has bargaining power it raises the input price of the supermarket above that of the local store. This allows it to extract rents from those customers who travel to the supermarket anyway to buy other goods. However, if the supermarket has bargaining power, then the supplier is forced to try and harness rents via the local store and the ordering of the input prices is reversed. We show that small discounts to the supermarket create a waterbed effect because the supplier extracts profits by raising the local store’s input price. However, large discounts to the supermarket are beneficial for the local store because the supplier lowers its input price to transfer business.

†Joint work with Dr. John Thanassoulis

†The word ‘discrimination’ here might be a misnomer because during the course of this chapter, we will directly endow the downstream firms with bargaining power.
2.1 Introduction

This chapter assesses third degree price discrimination in markets where a monopolist supplier sells its product to the final customers via retailers. In particular it considers a setup where one retailer might offer a more privileged route to access the market than its rivals. In this context, the chapter asks the following two questions - first, is offering a better route to market advantageous or detrimental to a retailer? and second, do lower prices for one retailer result in higher prices for the other, i.e. is there a waterbed effect?

The question of retailers being charged different prices has attracted interest due to its implications on competition law. While intuition suggests that larger, more efficient retailers should pay lower prices, a lot of the literature makes the somewhat counter-intuitive argument that larger, more efficient firms should face higher input prices. The main proponent of this idea is DeGraba (1990) who considers a setup where downstream firms are asymmetric in efficiency. He argues that a price discriminating upstream monopolist will disadvantage the low cost downstream retailer by charging it a higher price than the high cost retailer. This is because of the following argument: the more efficient retailer is, everything else being equal, more competitive in the downstream market. Given a certain input price, it will sell larger quantities than its less efficient rivals. This makes its derived demand for the input less elastic. The profit maximising supplier has the power to exploit this and will do so by charging it a higher price. Offering a better route to market is therefore disadvantageous for a downstream retailer.

This result is robust to retailers being asymmetric in different forms of efficiency. Yoshida (2000) considers a more general case where the downstream firms differ not only in the efficiency in the use of other inputs ($\beta$-efficiency), but also in the use of the input bought from $U$ ($\alpha$-efficiency). However, it is still the case
that the more efficient retailer has a less elastic demand for the input and the above result stands. Hence a retailer offering a better route to market is still handicapped in the discriminatory pricing regime.

This counter-intuitive and largely empirically unsupported\(^2\) result holds because these models do not account for buyer power. One would expect bigger, more efficient firms to have better outside options and hence be offered more favourable contracts. Katz (1987) tries to solve this problem by modelling buyer power as the ability to integrate backwards into the production of the input. The argument is that because large stores can integrate backwards, it allows them to get a lower price compared to smaller firms without this ability. His model differs from DeGraba’s in the sense that the downstream firms, while being equally efficient, consist of a chain store and a local store. The chain has a viable threat to integrate backwards due to its larger demand and the economies of scale in production of the input. The supplier then is no longer an unconstrained monopolist, and offers the chain store a lower price than the local store.

Alternatively, Inderst and Valletti (2009) solve this problem by introducing buyer power via the threat of demand side substitution. They propose that the more efficient retailer has a tighter participation constraint. As a result of the threat of demand side substitution, the upstream supplier is again no longer an unconstrained monopolist and the more efficient retailer receives a discount from the discriminating upstream firm. Hence once we account for buyer power, offering a better route to market is beneficial for the retailer.

We are also interested in whether being a better route to market is beneficial or harmful for a retailer. However, we propose a different motivation for buyer power, and a different expression of it. For the expression of buyer power, we argue that larger retailers are more likely to have a direct say in the negotiation

\(^2\)See Competition-Commission (2008) para 5.22 - The four largest grocery retailers (Asda, Morrisons, Sainsburys and Tesco), when analysed together, pay on average, between 4 and 6 per cent less than the mean.
process; that is, they have the ability to preemptive offers to the supplier.\textsuperscript{3} As a result, this chapter models buyer power as bargaining strength in setting contracts. This has the effect of giving the stronger buyer a first mover advantage. We show that offering a better route to market is advantageous for the retailer once we express buyer power in this way.

While the papers cited above ask which retailer has the better ‘absolute’ price, the literature on waterbed effects asks how a change in the price offered to one retailer affects the prices charged to its rivals. The waterbed effect suggests that a discount to large retailers causes an increase in input prices offered to the smaller retailers. Inderst and Valletti (2011) show how the increase in size of one retailer makes increasing prices to the other retailer both feasible and optimal. Their argument for the presence of waterbed effects relies on the assumption that discounts are based on size. If a large retailer manages to secure an additional discount due to some exogenous factors then, given everything else, it will pass on some of these savings to its customers. This means that, everything else being equal, it will steal some customers from its rivals and become an even larger retailer. Its rivals will then become even smaller, and as long as discounts are based on size, they will no longer be able to secure the low price they had before. Hence lower prices for the larger retailer result in higher prices for its rivals.\textsuperscript{4}

The Competition Commission, whist conducting its grocery markets investigation in 2008, found no evidence of the waterbed effect in the UK grocery market

\textsuperscript{3}O’Brien and Shaffer (1994), Dobson and Waterson (2005), Smith and Thanassoulis (2006) and Björnerstedt and Stennek (2007) also consider a bargained upstream-downstream interface but Chen (2003) is the main modelling motivation for this part.

\textsuperscript{4}There is also an informal long run argument for the presence of waterbed effects that suggests that discounts to large stores mean upstream margins are being squeezed. This causes exit and consolidation in the upstream industry, which implies that all downstream retailers get a worse deal. If this is not disproportionate then it might be that large retailers get a discount while small retailers are forced to pay more.
because the testable predictions of the model presented did not hold through. In particular, it questioned the nature of discounts based on size. It found the relationship between size and discounts to be first, highly non-linear and second, insignificant for primary brands.\textsuperscript{5}

Given this, we bring in a different motivation for the source of buyer power. While size related discounts are plausible, it is more likely that size gets retailers a place on the negotiating table. Hence this relationship should be strong for medium sized retailers. For very small retailers, this strategy is unlikely to be cost effective for the supplier. Similarly for very large stores, there might be ceiling effects that constrain the discounts.\textsuperscript{6} Our motivation for a different source of discounts comes from the observation that large supermarkets are not just big local stores - they are fundamentally different because they offer not just higher quantity, but \textit{variety}. Customers go to a supermarket because they want to do their weekly shopping and purchase a basket of goods rather than to buy more of the same good. This gives the supermarket a potentially ‘captive demand’ which is the source of its buyer power.

We bring in this idea by locating the two retailers at opposite ends of a Hotelling line. The supermarket offers a population of customers who will visit it anyway to buy other goods, while customers purchasing from the local store must make a specific trip. This captures the idea that the supermarket offers a better route to market than a local store because it has this potentially captive demand. We make the standard assumption of full market cover.

We first consider a benchmark case where the upstream supplier has all the

\textsuperscript{5}See Competition-Commission (2008) para 5.32

\textsuperscript{6}Competition-Commission (2008) - despite Tescos increase in size and national sales share since 2003, its advantage over other grocery retailers in terms of the prices it pays to suppliers has not increased.
bargaining power, similar to DeGraba (1990); Katz (1987); Inderst and Valletti (2009). We analyse the supplier’s pricing decision in the input market. One might think that the supermarket should get a lower price than its rival because the supplier would be able to access more of the market through this superior route. However, similar to the rest of the literature, we find that the supplier would raise the input price of the supermarket above that of the local store. The captive demand and subsequently the higher volumes sold by the supermarket, make its demand for the input less elastic than that of the local store. The price discriminating monopolist will then choose to exploit this and charge the supermarket a higher input price. This allows it to capture rents from the supermarket shoppers. Consequently, an increase in downstream asymmetry increases the elasticity differential and magnifies the differences between the equilibrium input prices. This result adds to the current literature by showing that the supplier continues to punish the better route to market even if we account for this new source of asymmetry between the downstream firms.

We then incorporate buyer power into the modelling setup. Similar to Chen (2003), we propose a two stage model to capture the strategic interaction between the firms. The supermarket has the ability to make an input price offer to the supplier, who then makes an offer to the local store. The supermarket can now for sure negotiate a lower price. The supplier could in turn lower the input price to the local store but that would be sub-optimal. Earlier the supplier could extract rents from the captive demand of the supermarket. Now it can not do that anymore, so it will choose to extract rents from the weaker retailer - the local store. This necessitates raising input prices to the local store. Interestingly, as the asymmetry between the downstream firms increases, the full cover assumption implies that the input price offer made by the supermarket increases. Hence, an increase in downstream asymmetry will dampen the differences between the
2.1. INTRODUCTION

Consequently, the only gains for the supermarket from being a offering route to market come via the ability to propose prices to the supplier. In all other respects, offering a better route to market is still a disadvantage. Hence the chapter’s first major contribution is to make the claim that being a offering route to market is harmful for a retailer unless it translates into bargaining power.

The second major contribution of the chapter examines whether, in response to the supermarket getting a discount, the local store is forced to pay a higher price than it did in the benchmark game - i.e. is there a waterbed effect? For this we assess the impact of a discount to the supermarket on the supplier’s reaction function. A discount to the supermarket reduces all retail prices and weakens the full cover constraint. This makes it feasible for the supplier to increase prices and extract profits from the local store. The supplier’s enhanced ability to extract profits gives us the waterbed effect.

However, a discount to the supermarket also creates the opposite incentives for the supplier because it reduces the its markup on the supermarket’s sales. Given that the local store pays a higher level of input price, the supplier wants to transfer sales away from the supermarket towards the local store. This makes it want to reduce the input price to the local store. If this effect dominates, the incentive to extract profits then there is in fact, no waterbed effect. We can show that the waterbed effect is more likely if the cost of transferring business - which is the loss in profits felt on the number of infra marginal customers at the local store - is high. In other words, the waterbed effect is more likely if the supermarket gets only a ‘small’ discount. A small discount means the local store is still able to compete with the supermarket and has a sizeable market share. The costs of transferring business are too high and the supplier raises the local store’s price. This is comparable to Inderst and Valletti (2011). If on the other hand the
supermarket gets a ‘large’ discount then the local store will not be competitive and will only have a small market share. Now transferring business is less costly and the supplier will prefer to reduce the local store’s price. Therefore a large discount to the supermarket is beneficial for its rivals.

Plan of the chapter: The model is presented in Section 2.2. Section 2.3 analyses the case where the upstream supplier has all the bargaining power. Section 2.4 introduces a bargained supplier-retailer interface and analyses the waterbed effect. Section 2.5 concludes and proofs are contained in the Appendix.

2.2 The Model

There is a monopolist upstream supplier $U$ who has two routes to market which are the downstream retailers, $D_L$ and $D_S$. $U$ agrees a per unit price with the two retailers. The retailers then set retail prices upon which they compete. We assume the retailers are equally efficient and normalise all marginal costs of production to zero.

The two downstream stores provide different routes to market. In particular, $D_L$ is a local store and $D_S$ is a supermarket. To model this we suppose that $D_L$ and $D_S$ sit at opposite ends of a Hotelling line of length one. We introduce two types of customers: a proportion $1 - A$ of total customers (referred to as type $B$ customers) are ‘one off shoppers’. They make a shopping trip purely for the good under consideration. These customers incur a transport cost of $t$ times the distance travelled to their preferred store. All other customers, (a proportion $A$ of the total customers, referred to as type $A$ customers) are ‘supermarket shoppers’. They will, for other purposes, travel to the supermarket. These customers incur no additional transport cost if they purchase the good in question from the
supermarket. They will however incur a transport cost if they decide to purchase from \( D_L \) (equal to \( t \) times the distance travelled to the local store). Both A and B customers are distributed uniformly between \( D_L \) and \( D_S \). A graphical representation of transport costs for a customer located at \( \theta \) is provided in Figure 2.1.

Figure 2.1: Type B customers pay a transport cost to go to either store; Type A pay the transport cost only to go to the local store.

This model cleanly captures that the supermarket, \( D_S \) offers a superior route to market, than the local store, \( D_L \). By selling through \( D_S \), the supplier receives cheaper access to the type A customers who are potentially already there. It is still possible for them to reach \( D_L \) but they do so at a cost. The first research question now is: under what circumstances does this advantageous route to market translate into beneficial input prices for the supermarket?

To complete the model we assume that all customers have a value \( V \) associated with the good. This could represent the utility received from purchasing the good from a third firm. Consequently, \( V \) is the maximum that any customer is willing to pay.
2.3  The Standard Approach: Single supplier has the bargaining power

We first consider the supplier $U$ having all the bargaining power. This is the standard bargaining power distribution which the literature on input price discrimination has considered. In particular DeGraba (1990), Katz (1987) and Inderst and Valletti (2009) all assume that the supplier $U$ is in a position to make ‘take-it-or-leave it’ offers. We will show that this assumption has the effect of rendering $D_S$ weak, notwithstanding the superior route to market which it offers.

Denote the equilibrium input prices by $\{c^*_L, c^*_S\}$. Since marginal costs of production have been normalised to zero, these offers form the composite marginal costs for the two retailers. We make the standard assumption of full market cover so all the customers of both types buy the good.\footnote{We relax this assumption in Chapter 3. Our first result (Proposition 2.1) is unchanged.} This would be the case if the transport cost is small enough compared to the value of the good, so the customers towards the middle of the Hotelling line will find it optimal to buy the good. We can think of these as goods for which customers find ‘waiting for a supermarket trip’ too costly, for example ‘emergency goods’ such as lightbulbs, or milk for breakfast.

We begin by noting that the marginal type $B$ customer will be located at the point $\hat{\theta}_B$ where:

\[ V - (P_L + t\hat{\theta}_B) = V - (P_S + t(1 - \hat{\theta}_B)) \quad \Rightarrow \quad \hat{\theta}_B = \frac{P_S - P_L}{2t} + \frac{1}{2} \]

where $P_L$ and $P_S$ are the retail prices set by the local store and the supermarket.
respectively. The marginal type \( A \) customer will instead be located at \( \hat{\theta}_A \) where

\[
V - (P_L + t\hat{\theta}_A) = V - P_S \quad \Rightarrow \quad \hat{\theta}_A = \frac{P_S - P_L}{t}
\]
as long as these locations lie between 0 and 1. It should be noted that the location of type \( A \) customers is more responsive to price changes than the type \( B \) customers. In preparation for the main result, we develop the following lemmas:

**Lemma 2.1.** The local store \( D_L \), serves a positive number of type \( B \) customers, so \( \hat{\theta}_B > 0 \).

The intuition is as follows. Suppose for a contradiction that \( \hat{\theta}_B = 0 \). Then it must be the case that \( c_L \) is so high that \( P_L > V \) and the local store, \( D_L \), is not selling to any type \( B \) customers. If type \( B \) customers do not shop at the local store, neither do type \( A \) customers who can have other reasons to go to the supermarket. Hence in this case the local store does not serve any customers. Then we could have only two potential scenarios: one where the supermarket was supplying to all the type \( B \) customers, and another where it was supplying to only some of them. In either case it would be better for the supplier to reduce \( c_L \) such that the local store, \( D_L \), serves some type \( B \) customers. This can be done without altering the preference of all \( A \) customers who go to the supermarket, and yet it raises the supplier’s profits. This is a contradiction to optimality.

**Lemma 2.2.** The local store \( D_L \), serves a non negative number of type \( A \) customers, so \( P_S \geq P_L \) implying that \( \hat{\theta}_A \geq 0 \).

Suppose again for a contradiction that Lemma 2.2 did not hold and instead that the local store was more expensive than the supermarket. All \( A \) customers would buy from the supermarket, and the local store would compete only for the \( B \) customers. The equilibrium retail prices can then be found for any input prices \( \{c_L, c_S\} \). If it were the case that \( P_S < P_L \), then \( \hat{\theta}_B < \frac{1}{2} \) since the transport costs
are symmetric to both locations. It then follows that the input prices must be such that $c_S < c_L$. Now consider $U$ deviating by marginally raising input prices to the supermarket and lowering them to the local store so as to marginally increase the share of $B$ type shoppers buying from the local store. Such a deviation raises $U$’s profits on sales to all $A$ customers who continue to go to the supermarket. The $B$ customers who swap from the supermarket to the local store contribute more as $c_S < c_L$. Finally, profits are lost on the former local store customers who remain with the local store, and gained on the $B$ customers who remain with the supermarket. As $\hat{\theta}_B < \frac{1}{2}$, the gains with those remaining in the supermarket are larger. Hence, this is a profitable deviation and so the result is proved.

**Lemma 2.3.** The supermarket $D_S$, serves a positive number of both type $A$ and type $B$ customers. So $\hat{\theta}_A < 1$ and $\hat{\theta}_B < 1$.

First note that the type $A$ customers at location 1 are identical to the type $B$ customers at location 1 because they are already at the supermarket and so do not pay any extra transport costs. Hence the supermarket serves a positive number of type $A$ customers if and only if it serves a positive number of type $B$ customers. The intuition for Lemma 2.3 is as follows. Suppose for a contradiction that $\hat{\theta}_A$ and $\hat{\theta}_B$ are both equal to 1. Then it must be the case that $c_S$ is so high that $P_S \geq P_L + t$ and $D_S$ is not selling to any customers at all. Then we could have only two potential scenarios: one where the local store was supplying to all customers, and another where it was supplying to only some of them. In either case it would be better for the supplier to reduce $c_S$ such that the supermarket, $D_S$, serves some customers. In the first case the supplier gains profits on additional customers without affecting those who buy from the local store, and in the second case the supplier gains profits on those customers that switch from the local store to the supermarket because the supermarket pays a higher input price. This is a contradiction to optimality.
We have proved that the two firms weakly share the type $A$ and $B$ customers in equilibrium. We now wish to show that the supplier $U$ will always seek to extract a higher price from the supermarket.

**Proposition 2.1.** For low levels of transport cost, there exists an equilibrium where the supplier serves the entire market and charges the supermarket a higher price than the local store: $c^*_S > c^*_L$.

When there are no supermarket shoppers ($A = 0$), the downstream firms are symmetric and the supplier charges them the same price. As the proportion of supermarket shoppers increases, the downstream industry begins to offer increasingly different routes to market. Further, the larger volumes served by the supermarket make its derived demand for the input less elastic. If the supermarket were to raise its retail price then it would lose the same number of customers as the local store would gain, but proportionately this would be a small part of the total served. Thus the supermarket will, other things equal, seek to raise price higher than the local store would. The supplier sees this opportunity for a higher margin at the supermarket and wishes to profit itself. Thus the supplier raises the price to the supermarket beyond that to the local store. This allows it to profit directly from the larger market share of the supermarket.

To calculate the level of these input prices we determine that the supplier’s profits are increasing in both input prices. It will hence charge as high input prices as it can subject to the constraint that full cover is indeed maintained. In such a way, it will exhaust all the value from the indifferent type $B$ customer.

The intuition for $c^*_S > c^*_L$ parallels that in DeGraba (1990). Here however the supermarket has no cost advantage. Rather the supermarket is a better route to market as customers can be thought to be already present in the store. It is the
higher volume gained due to these customers that translates into the supermarket having a lower elasticity of derived demand. This in turn yields the result of Proposition 2.1. The difference is that in DeGraba (1990), while an increase in efficiency of one retailer is damaging for itself, it is not beneficial for its less efficient rival. In our model however, an increase in the proportion of $A$ customers is beneficial for the local store.

We conclude this section by studying how retail prices vary as the proportion of supermarket shoppers increases:

**Corollary 2.1.** As the proportion of supermarket shoppers ($A$) increases, the retail price set by the local store decreases while that set by the supermarket increases.

As the proportion of supermarket shoppers grows, the higher is the overall elasticity of the customers. This is because supermarket shoppers are more price sensitive than the one-off shoppers. This drives down the retail price from the supermarket because given the customers have a higher overall elasticity they will now accept a lower price rise before a given proportion of them is lost. Hence the supermarket lowers price. However, an increase in the proportion of supermarket shoppers means that the number of customers served by the supermarket increases at every level of retail price. The supplier seeks a share of this increased volume and so drives up the input price of the supermarket. This makes the supermarket raise retail prices. The second effect dominates and the supermarket always wants to increase retail prices in response to an increase in the proportion of supermarket shoppers. Having raised the input price to the supermarket, the supplier additionally seeks to lower the input price to the local store. This allows full market cover to continue. This reduction in input prices and in markup (due to higher elasticity) overrides the strategic complementarity and allows the local
store to lower its retail price.

2.4 A Bargained Supplier-Retailer Interface

The work in section 2.3 has given the supplier all the bargaining power. This approach parallels that in the existing literature. However the results are somewhat counterintuitive. The supermarket provides a better route to market as it can provide access to a large proportion of potentially captive customers. And yet this benefit translates into a higher input price than the alternative less good routes to market. We now explore how allowing the supermarket some measure of bargaining power alters this result.

There is no clear consensus in the literature as to how bargaining between multiple retailers and a supplier should be modelled. Some authors have decided to model this as $U$ bargaining simultaneously and independently with all the retailers eg. Dobson and Waterson (2005); Smith and Thanassoulis (2006); Bjornerstedt and Stennek (2007). However this has attracted criticism as the downstream strategic interaction is ignored by the assumption of simultaneous independent negotiation. We instead propose a simple game which yields an equilibrium in pure strategies. This allows our formulation to be readily understood.

We propose that the supermarket has the bargaining power to make a price offer to the supplier $U$, while $U$ has the power to make a price offer to the local store. This balance of bargaining power is a natural situation to compare to $U$ having all the bargaining power. Gans and King (2002); Mathewson and Winter (1997); Majumdar (2005) all

---

8This has some behavioural justifications too as we expect supermarkets to be able to update their contracts more frequently than local stores. Large supermarkets are then more likely to make preemptive offers.

9Chapter 3 endogenises the bargaining order by adding in a stage 0 before the actual game allowing the local store and supermarket to bid for the right to make the offer to the supplier. It demonstrates that the supermarket would be willing to bid more and so the order of moves is as we have modelled it.
make the claim that a buyer group with a first mover advantage will benefit over other smaller buyers. Chen (2003) also proposes a sequential structure of bargaining power. There the supplier makes a ‘take-it-or-leave-it’ offer to the fringe retailers. The dominant retailer possesses countervailing power and can ex-post renegotiate a separate deal with the supplier. However, the fringe in Chen’s model is a price taker while our model allows the local store to act strategically. We have the following game:

**Stage 1:** \( D_S \) makes a take-it-or-leave-it offer \( c^*_S \) to the supplier \( U \).

**Stage 2:** \( U \) decides whether to accept / reject the offer. It then makes a take-it-or-leave-it offer of \( c^*_L \) to \( D_L \).

Allowing the supermarket to have some measure of bargaining power will for sure lower its input price. It is unclear however whether the supermarket will be in a position to secure a more advantageous deal than the local store. The issue is that the supplier has an outside option. Thus if the supermarket is too aggressive in its demands then the supplier will refuse to use its route to market and supply exclusively through the local store. We present the following result.

**Proposition 2.2.** If the supermarket has bargaining power, then it secures a lower price than the local store: \( c^*_S < c^*_L \).  

The supermarket will always ensure that the offer it makes is acceptable because if it did not then it would secure zero profit from this product. Given that the supermarket is able to set its input price the supplier must now decide what price to offer the local store. We now have an entirely new intuition. The supplier could still punish the supermarket by lowering the input price to the local store and preserve the same input price ordering as in the case when \( U \) had all the bargaining power. However, this is now sub-optimal. \( U \) would rather raise

\[ \text{10} \]

\[ \text{It should be noted however, that in section 2.3 the supplier is at a boundary optimum. If the supermarket makes an offer very close to the benchmark level then we could have } c_S > c_L \text{ since full cover will not allow } c_S < c_L \text{ but this is an artificial result. Hence we do not consider these very high values of } c_S. \]
the input price offered to the local store above the supermarket’s level. When $U$ had bargaining power it offered a high price to the supermarket to extract rents from the captive demand. When the supermarket has bargaining power $U$ cannot access the extra surplus the supermarket secures. However, it can try to capture more of the customers’ surplus via the local store who has little bargaining power. Thus $U$ raises the price offered to the local store. This allows $U$ to extract more of the surplus from the customers who continue purchasing from the local store. Additionally, we determine that due to full cover, $U$ charges the local store a constant mark up over the supermarket’s offer.

Having developed a model for analysing bargaining within the supply chain, we use it study the impact of a change in the proportion of supermarket shoppers on prices. We determine that the supermarket’s profit only depends on the difference in the input prices between itself and the local store. This again follows because we are assuming full market cover. Hence there are multiple equilibria. We select from amongst these possible equilibria the one in which the supermarket demands the lowest acceptable input price. This equilibrium would be selected if there were any marginal customers who would grow the total industry demand in the event that industry prices were reduced. Under this convention we have:

**Proposition 2.3.** If the proportion of supermarket shoppers (type A customers) grows then:

1. The equilibrium input prices secured by the supermarket rise, while those secured by the local store decline, and
2. The retail prices at both the supermarket and the local store rise.

If the supermarket is too aggressive in its demands then the supplier will reject it and use the local store as its only route to market. All of the customers can, in
principle, be supplied by the local store as even supermarket shoppers can pay the travel cost and visit it. The profit available to the supplier in the event of using only the local store is therefore independent of the proportion of supermarket shoppers in the population. Suppose now that the supermarket did not alter its offer as the proportion of supermarket shoppers in the population grew. The local store will now be at a disadvantage, and so the profits the supplier secures via the local store decline. This reduction in profits for $U$ would mean $U$ would rather decline the supermarket’s offer. To prevent this, the supermarket must make some more surplus available, and so it agrees to increase the input price it offers. This will in turn drive up the retail prices of the supermarket. The supplier is less able to extract rents via the local store as the local store is now a weaker route to market. Hence the optimal response is for the supplier to lower the input prices to the local store (while holding them above the supermarket’s input prices (Proposition 2.2)). As prices are strategic complements the higher supermarket prices allow the local store to raise its prices also.

Note that Proposition 2.3 highlights a slightly counterintuitive part of our model. As the proportion of supermarket shoppers increases, the upstream chooses to dampen the differences between the downstream input prices.\(^\text{11}\)

### 2.4.1 Waterbed Effect

We have shown in Proposition 2.2 that once we account for the supermarket’s buyer power, it is able to negotiate a lower price than the local store. Now we would like to see whether this reduction in the supermarket’s input price has increased the level of price paid by the local store. In other words, does a discount

\(^\text{11}\)As a consequence of this, if the proportion of type A customers is small, then the downstream input prices and retail prices are quite dissimilar. It might then be the case that all the type A customers want to buy from the supermarket. This kind of an equilibrium will not be a correct comparison to our benchmark model and hence is not discussed. For the feasibility of the equilibrium where both types of customers are shared, we require then, for the proportion of type A customers in the population to be large.
to the supermarket make necessary, an increase in the local store’s input price?\(^{12}\)

A decrease in the supermarket’s input price does two things - first it decreases the supermarket’s retail price. Due to strategic complementarity the local store’s retail price also decreases, but by lesser. This means average retail prices are lower. Since the market is fully covered, everyone who was buying before is still buying, but paying lesser for it. This implies that the supplier can now increase the local store’s input price without losing full cover of the market. This gives it an enhanced ability to ‘extract profits’ from the local store. However, a second impact of the decrease in the supermarket’s price is that its market share increases. The supplier is now earning a lower markup on a larger number of units. This creates incentives for the supplier to ‘transfer business’ from the supermarket to the local store where it earns a higher markup (from Proposition 2.2). The only way for the supplier to do so is by reducing the local store’s input price.

These produce two competing effects on the supplier’s pricing strategy - (i) it wants to keep the local store’s input high because it can extract profits from it, but (ii) it wants to also reduce its price so it can transfer business to it. The effect that dominates depends on the level of the offer made by the supermarket:

Proposition 2.4. The waterbed effect exists only if the input price offer made by the supermarket is large.

Proposition 2.4 might seem a little counterintuitive. How can small discounts attained by a supermarket be harmful for the local store and large discounts be beneficial?

\(^{12}\)There might be a concern here that the benchmark model and the bargained setup are not the right points of comparison for the analysis of waterbed effects since we are not comparing ‘the case of buyer power’ with ‘the case of differential buyer power’ as in other formal models. We are in fact, comparing ‘the case of no buyer power’ with ‘the case of differential buyer power’. However, our setup extends to the standard comparison and we maintain that this constitutes a valid comparative static.
The result can be understood by recognising that for the supplier, the ‘cost’ of transferring business is measured by the loss in profits on the infra marginal customers at the local store. If the supermarket makes a high offer, its retail price is high and it has a small market share. Consequently the local store has a high market share. In this case the number of infra marginal customers at the local store is too large to make transferring business an optimal strategy. The supplier instead chooses to extract profits from the local store and increases its input price. Effect (i) dominates and we see a waterbed effect.

If however the supermarket makes a low offer then it has a low retail price and a high market share. The local store consequently has a small market share. Now the set of infra marginal customers at the local store is small enough that effect (ii) dominates. The supplier chooses to decrease the local store’s input price to transfer business away from the supermarket. Here there is no waterbed effect.\(^{13}\)

\[c_L < c_L^{Bench}\]

\[c_L > c_L^{Bench}\]

---

\[\text{Figure 2.2: The waterbed effect exists if } c_S \text{ is high enough}\]

Figure 2.2 represents this graphically. The market is under full cover if the retail prices and consequently the input prices are low enough - as shown by the

---

\(^{13}\)Rather we see the opposite of the waterbed effect, perhaps an ‘anti-waterbed effect’ where a discount for one firm is beneficial for its rivals.
2.4. BARGAINING

set of points under downward sloping line. The isoprofit curves of the upstream are convex and the benchmark equilibrium is shown by the point $O^{Bench}$ where the highest isoprofit curve is tangent to the full cover constraint. From Proposition 2.1, the supermarket pays a higher input price than the local store. The supplier’s reaction function in the bargaining game $c_L(c_S)$, is given by identifying the highest isoprofit curve given any level of $c_S$ offered by the supermarket - in other words, the set of points where the isoprofit curves have infinite slope. This lies entirely above the 45 degree line as argued by Proposition 2.2. For higher levels of the offer, the full cover constraint binds and the composite reaction function is highlighted in bold. The isoprofit curve representing the profits made if the supplier sells though the local store exclusively is the supplier’s participation constraint. Any $c_S$ that gives the supplier profits above this level will be accepted. In Figure 2.2 these are the set of input prices from $A$ to $O^{Bench}$.

Now, to assess whether there is a waterbed effect in our model we need to determine the size of the supermarket’s offer. For this we assess the decision making process more closely. As we have argued previously, the supermarket’s profits are only a function of the difference between the input prices. Hence it is indifferent between all offers from $A$ to $B$. Under the previous convention where the supermarket demands the lowest acceptable input price (point $A$) it is the case that the supermarket’s offer is low enough such that there is in fact no waterbed effect. The supermarket’s offer is low enough that the costs of transferring business are outweighed by the benefits and the price offered to the local store is lower than in the benchmark case. However, if we consider an equilibrium where the supermarket and the supplier engage in Nash bargaining in stage 1 then the equilibrium selected is somewhere between $B$ and $O^{Bench}$.

\footnote{This is because the supermarket’s profits are independent of its input price offer between $A$ and $B$ and decreasing in them beyond $B$. The supplier’s profits on the other hand are always increasing with the offer. The Nash product is then maximised somewhere between $B$ and $O^{Bench}$.}
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is a waterbed effect. The offer made by the supermarket is high enough to ensure that the supplier’s incentives to extract profits from the local store dominate its incentives to transfer business.

The intuition presented is a little different from the prevailing explanation of waterbed effects given by Inderst and Valletti (2011). In their model, the source of discounts is the size of the retailer. As one retailer becomes bigger, it is able to spread the fixed costs of switching suppliers over a larger number of units. Therefore it can negotiate a lower price. Additionally, it imposes an externality on its rival because it makes it less inclined to source from a rival supplier. This makes it feasible for the supplier to increase the small retailer’s price. This argument is equivalent to the relaxation of the full cover constraint in our model. However in their model, there is always a waterbed effect because the parameter values they consider (i.e. a small increase in size) imply that extraction of profits is the optimal strategy.

Chapter 4 will argue, that for other parameter values (i.e. a large increase in size) their model, like ours, also predicts a ‘transferring business’ effect. If the larger retailer increases a lot in size, the supplier would prefer to transfer business to the smaller retailer and reduce its input price below the benchmark level.

2.5 Conclusions

We have considered a monopolist supplier selling an input through two different routes to market - a local store and a supermarket. The supplier’s aim is to extract surplus from the customers via its retailers. Those retailers which have a less elastic derived demand for the input are where the supplier would seek to extract more rents. This is accomplished by raising input prices. In our model the supermarket sold to more customers since some of them could buy
without incurring transport costs. This gave it a less elastic demand. In previous literature one retailer was exogenously more efficient (such as DeGraba (1990); Yoshida (2000)) and this led to lower elasticity. Input prices for other retailers fall as lower volumes and hence higher elasticity means less extraction of rents is possible. An increase in these supermarket shoppers hence magnifies the elasticity differential and differences between the retailers’ equilibrium input prices.

However these retailers offering advantageous routes to market are also more likely to be the ones with higher bargaining power. When retailers have bargaining power they do not allow the surplus harnessed from their customers to be transferred to the supplier. They also lower their retail prices by securing lower input prices. The supplier in response is forced to seek surplus via those retailers with weak bargaining power rather than those with low elasticity of demand. Now the retailers with weak bargaining power find their input prices forced up as the supplier cannot harness rents otherwise. Hence better routes to market are indeed able to extract superior input prices via bargaining. An increase in the proportion of supermarket shoppers now produces different results as it dampens the differences between the retailers’ equilibrium input prices. However, the differences in retail prices are magnified.

There are also two forces at work following a reduction in the supermarket’s input price. A discount to the supermarket makes the supplier more able to extract profits from the local store. This can be achieved by raising its input price. However the discount also increases the supplier’s incentives to transfer business towards the local store. This can be achieved by reducing its input price. We have argued that it is in fact, small discounts to the supermarket which are harmful for the local store while large discounts are beneficial.

Aside from the size of the discount, we can also think about the mechanics
2.5. CONCLUSIONS

in terms of the type of supplier. Moving outside the model a little, the waterbed effect is more likely if the supplier is a primary brand like Coca Cola or Nestle. Here the supermarket has little bargaining power vis-a-vis the supplier and will be forced to make a high offer. If on the other hand, the supplier is not a primary brand then the supermarket will have a lot of bargaining power against it. It will then only make it a small offer. In this scenario a discount to the supermarket is beneficial for its rivals. Our model can therefore be used to narrow down the type of suppliers that might allow a waterbed effect.  

To conclude, the insights from this chapter are that a better route to market receives a discounted price compared to worse routes to market only if its advantage translates into bargaining power. However, this is damaging for its rival only if the size of the discount is small.

\textsuperscript{15}There are caveats however. Our model assumes that the market is of a fixed size. This is likely to overestimate the size of the waterbed effect since the supermarket’s gain comes entirely at the local store’s loss. In Chapter 3 we extend this framework to allow for quantity effects and we show that this has significant implications for the presence of the waterbed effect. Additionally, our model can not capture the impact of buyer groups that small retailers source from. When discounts are based on volumes, the presence of these buyer groups allows small stores to make use of size related discounts as well. However with our model, the source of buyer power is variety, which the buyer groups would not be able to take advantage of. Having said that, variety and quantity are highly correlated in practice so it is not clear how the testable predictions of this model might be different.
2.6 Appendix

Proof of Lemma 2.1. Suppose, for a contradiction, that $D_L$ serves no type $B$ customers, thus $\hat{\theta}_B \leq 0$. Therefore $P_S - P_L \leq -t$, hence $\hat{\theta}_A < 0$. Therefore $D_L$ serves no customers. First suppose that $P_S > V - t$. In this case not all $B$ customers buy from $D_S$. But then $U$ could set $c_L = V - \varepsilon$ and the local store would then sell to some of the unserved $B$ customers. $A$ customers would be unaffected as $\hat{\theta}_A$ would remain negative. $U$’s profits would increase. This is a contradiction to optimality. Suppose instead that $P_S \leq V - t$ so that all customers buy from $D_S$. Now $U$ could set $c_L = P_S + t - \varepsilon$. This would cause some type $B$ customers to swap from $D_S$ to $D_L$. However $c_L > P_S > c_S$ and hence this will increase $U$’s profits. This is again a contradiction to optimality. Thus we must have $\hat{\theta}_B > 0$ in equilibrium. \hfill $\blacksquare$

Proof of Lemma 2.2. Suppose, for a contradiction, that $P_S < P_L$. Hence $\hat{\theta}_A = 0$, $\hat{\theta}_B < \frac{1}{2}$. We have $\hat{\theta}_B > 0$ by Lemma 2.1. We now determine the retail prices. The local store, $D_L$ maximises $\hat{\theta}_B (P_L - c_L)$ which yields the FOC: $t + P_S - 2P_L + c_L = 0$. The supermarket $D_S$ serves a population of $A + (1 - A) \left(1 - \hat{\theta}_B\right)$ and so maximises $[1 - (1 - A)\hat{\theta}_B](P_S - c_S)$ which yields the FOC: $1 - (\frac{1 - A}{2t})(2P_S - P_L - c_S + t) = 0$. These FOCs can be combined into the following system:

$$
\begin{pmatrix}
2 & -1 \\
-(1 - A) & 2(1 - A)
\end{pmatrix}
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \begin{pmatrix}
t + c_L \\
(1 + A) t + (1 - A)c_S
\end{pmatrix}
$$

$$
\Rightarrow
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \frac{1}{3(1 - A)}
\begin{pmatrix}
2(1 - A) & 1 \\
(1 - A) & 2
\end{pmatrix}
\begin{pmatrix}
t + c_L \\
(1 + A) t + (1 - A)c_S
\end{pmatrix}
$$

(2.1)
We now determine that the marginal type B customer is at

\[
\hat{\theta}_B = \frac{1}{2} + \frac{1}{2t} \frac{1}{3(1-A)} \left[ (1-A)t + (1-A)c_L + 2(1+A)t + 2(1-A)c_S \right] - 2(1-A)t - 2(1-A)c_L - (1+A)t - (1-A)c_S
\]

\[
= \frac{1}{2} + \frac{1}{2t} \frac{1}{3(1-A)} [(1-A)(c_S - c_L) + 2At] \tag{2.2}
\]

By assumption \( P_S < P_L \), and so (2.2) implies that \( c_S < c_L \).

Now suppose that \( U \) raises \( c_S \) and lowers \( c_L \) slightly, so as to move \( \hat{\theta}_B \) to \( \hat{\theta}_B + \varepsilon \). \( U \) clearly gains profit on all A customers who now pay more. \( U \) will set prices which allow it to extract all of the surplus of the marginal type B customer. Hence for \( D_L \) its price changes from \( P_L = V - t\hat{\theta}_B \) to \( V - t \left( \hat{\theta}_B + \varepsilon \right) \), a reduction of \( \varepsilon t \). Likewise \( P_S = V - t \left( 1 - \hat{\theta}_B \right) \) and so its price must grow by \( \varepsilon t \). Using (2.1) we can determine the change in prices which deliver these price changes:

\[
\left( \frac{2(1-A) \cdot \delta c_L + (1-A) \cdot \delta c_S}{3(1-A)} \right) = -\varepsilon t \quad \text{and} \quad \left( \frac{(1-A) \cdot \delta c_L + 2(1-A) \cdot \delta c_S}{3(1-A)} \right) = \varepsilon t
\]

\[
\Rightarrow \delta c_S = -\delta c_L = 3\varepsilon t
\]

This deviation in prices causes the following change in \( U \)'s profit arising from type B customers:

\[
\delta \Pi_U|_B = \varepsilon (c_L - c_S) - 3\varepsilon t\hat{\theta}_B + 3\varepsilon t \left( 1 - \hat{\theta}_B \right) > 0
\]

the inequality follows as \( \hat{\theta}_B < 1/2 \) and \( c_L > c_S \). In addition, \( U \)'s profit arising from type A customers has increased as given \( P_S < P_L \), they are all still being served by \( D_S \) and \( \delta c_S > 0 \). Hence \( U \)'s total profits have increased. This is a contradiction to \( U \)'s optimal behaviour. Hence we must have \( P_S \geq P_L \) after all. \( \square \)
2.6. APPENDIX

Proof of Lemma 2.3. First note that the type A customer at location 1 is like a type B customer at location 1 - as type B customers at the supermarket pay no transport costs to travel to it. Hence \( \hat{\theta}_A < 1 \iff \hat{\theta}_B < 1 \). Now, suppose for a contradiction that \( c_s \) was so high that \( P_S \geq P_L + t \). In this case \( \hat{\theta}_B \geq 1 \) and \( \hat{\theta}_A \geq 1 \), and the supermarket is not serving any customers. First suppose that \( P_L + t > V \). In this case not all customers are being served by the local store. But then \( U \) could set \( c_s = V - \varepsilon \) and the supermarket could serve to some additional customers without affecting those already being served by the local store. \( U \)'s profits would increase. This is a contradiction to optimality. Suppose instead that \( P_L + t < V \) so the local store is serving all the customers. Now \( U \) could set \( c_s = P_L + t - \varepsilon \). This would mean that some A and some B customers swap from the local store to the supermarket. However, \( c_s > P_L > c_L \) and hence this will increase \( U \)'s profits. This is again a contradiction to optimality. Thus we must have \( P_S < P_L + t \) and hence \( \hat{\theta}_A < 1 \) and \( \hat{\theta}_B < 1 \). \( \square \)

Proof of Proposition 2.1. Suppose, for a contradiction, that \( U \) proposes \( c_s \leq c_L \). By Lemma 2.2, \( P_S \geq P_L \), so \( \hat{\theta}_B > \frac{1}{2} \). By Lemmas 2.1 and 2.2 \( 0 \leq \hat{\theta}_A < \hat{\theta}_B < 1 \). The retail prices in this case can be found as follows. The local store, \( D_L \) maximises \((P_L - c_L)(A\hat{\theta}_A + (1 - A)\hat{\theta}_B)\) which yields the FOC: \(-(1 + A)(P_L - c_L) + (1 + A)P_S - (1 + A)P_L + (1 - A)t = 0\). The supermarket \( D_S \) maximises \((P_S - c_S)(A(1 - \hat{\theta}_A) + (1 - A)(1 - \hat{\theta}_B))\) which yields the FOC: \( t - 2P_S + P_L + c_S = 0 \).
These FOCs can be combined into the following system:

\[
\begin{pmatrix}
2(1 + A) & -(1 + A) \\
-1 & 2
\end{pmatrix}
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \begin{pmatrix}
(1 + A)c_L + (1 - A)t \\
\quad c_S + t
\end{pmatrix}
\]

\[
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \frac{1}{3(1 + A)} \begin{pmatrix}
2 & (1 + A) \\
1 & 2(1 + A)
\end{pmatrix}
\begin{pmatrix}
(1 + A)c_L + (1 - A)t \\
\quad c_S + t
\end{pmatrix}
\]

\[
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \frac{1}{3} \left(2c_L + c_S\right) + \frac{1}{3(1 + A)} \begin{pmatrix}
3 - A \\
3 + A
\end{pmatrix} t
\]

(2.3)

Now suppose that \( U \) raises \( c_S \) and lowers \( c_L \) slightly so as to move \( \hat{\theta}_B \) to \( \hat{\theta}_B + \varepsilon \). The supplier \( U \) will set the input price so as to extract all the surplus of the marginal type \( B \) customer as this customer is the one who derives the lowest utility from its purchase and so is most tempted by the outside option: \( V \). Hence for \( D_L \) its price changes from \( P_L = V - t\hat{\theta}_B \) to \( V - t \left( \hat{\theta}_B + \varepsilon \right) \), a reduction of \( \varepsilon t \). Likewise \( P_S = V - t \left( 1 - \hat{\theta}_B \right) \) and so its price must grow by \( \varepsilon t \). Using (2.3) we require:

\[
\begin{align*}
\frac{1}{3} (2 \cdot \delta c_L + \delta c_S) &= -\varepsilon t \\
\frac{1}{3} (\delta c_L + 2 \cdot \delta c_S) &= \varepsilon t
\end{align*}
\]

\[
\Rightarrow \delta c_S = -\delta c_L = 3\varepsilon t
\]

Therefore this deviation in prices causes the following change in \( U \)'s profit arising from type \( B \) customers:

\[
\delta \Pi_U|_{\theta_B} = \varepsilon (c_L - c_S) - 3\varepsilon t\hat{\theta}_B + 3\varepsilon t \left( 1 - \hat{\theta}_B \right)
\]

The deviation in prices also causes a change in \( U \)'s profits arising from type \( A \) customers. We can calculate \( \delta \hat{\theta}_A = \frac{1}{t} \cdot \delta (P_S - P_L) = 2\varepsilon \). Therefore, the change

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in $U$’s profits arising from type $A$ customers is:

$$\delta \Pi|_A = 2\varepsilon (c_L - c_S) - 3\varepsilon t\hat{\theta}_A + 3\varepsilon t \left(1 - \hat{\theta}_A\right)$$

The total change in $U$’s profits is:

$$\delta \Pi_U = A \cdot \delta \Pi|_A + (1 - A) \cdot \delta \Pi|_B$$

$$= (1 + A)\varepsilon (c_L - c_S) + 3\varepsilon [At - (1 + A)(P_S - P_L)]$$

$$= 2\varepsilon \cdot (1 + A)(c_L - c_S) + A\varepsilon t$$

Where we have substituted for $3(1 + A)(P_S - P_L) = 2At - (1 + A)(c_L - c_S)$ from (2.3). It is clear that $\delta \Pi_U > 0$ as we have assumed $c_L \geq c_S$. This shows a contradiction to $U$’s optimal behaviour. Hence we must have $c_S > c_L$ after all.

To find explicitly the optimum values of input prices, we allow $U$ to optimise over them, given retail prices (2.3). The supplier will increase $\{c_L, c_S\}$ to the point where all the value is extracted from the indifferent type $B$ customer.

$$V = P_L + t \cdot \hat{\theta}_B \Rightarrow V = \frac{1}{2}[P_S + P_L + t]$$

using (2.3): $V = \frac{1}{2}[c_L + c_S + \frac{2t}{1 + A} + t]$

rearranging $c_L = 2V - c_S - \frac{3 + A}{1 + A} t$ (2.4)

The supplier then maximises:

$$\Pi(U) = c_L \left[A \cdot \hat{\theta}_A + (1 - A) \cdot \hat{\theta}_B\right] + c_S \left[A \cdot (1 - \hat{\theta}_A) + (1 - A) \cdot (1 - \hat{\theta}_B)\right]$$

$$= c_S + (c_L - c_S) \left[A \cdot \hat{\theta}_A + (1 - A) \cdot \hat{\theta}_B\right]$$

$$= c_S + (c_L - c_S) \left[\hat{\theta}_A (1 + A) / 2 + (1 - A) / 2\right]$$ (2.5)
From (2.3) we have
\[ \hat{\theta}_A = \frac{1}{3t} \left\{ c_S - c_L + \frac{2tA}{1 + A} \right\} \tag{2.6} \]
Substituting from (2.4) for \( c_L \) we can write the optimisation solely in terms of \( c_S \):

\[
\Pi(U) = c_S + \left[ 2V - 2c_S - \frac{3 + A}{1 + A} t \right] \cdot \frac{1}{6t} [(c_S - c_L) (1 + A) + 2tA + 3t (1 - A)]
\]

which leads us to the first order condition for \( c_S \) of: \( 4(1+A)(V - c_S) - 6t - At = 0 \).

Hence:
\[
c_L^* = V - \frac{6 + 3A}{4(1 + A)} t, \quad \text{and} \quad c_S^* = V - \frac{6 + A}{4(1 + A)} t \tag{2.7}
\]

Note however that for the full market equilibrium to be feasible, the following must be true:

- \( \hat{\theta}_A > 0 \Rightarrow P_S - P_L > 0 \). This can be written as a condition on the transport cost being big enough. We require:

\[
t > \frac{(1 + A)(c_L - c_S)}{2A} \tag{2.8}
\]

Note that this is automatically satisfied if \( c_S \geq c_L \).

- \( \hat{\theta}_A < 1 \Rightarrow P_S - P_L < t \). This can be written in terms of the transport cost being big enough:

\[
t > \frac{(1 + A)(c_S - c_L)}{3 + A} \tag{2.9}
\]

Finally, note that \( 0 < \hat{\theta}_A < 1 \) is satisfied if (2.8) and (2.9) hold. Since \( c_L^* < c_S^* \) from Proposition 2.1 we have that (2.9) binds. Substituting for the optimal values of of input prices from (2.3), \( \{c_L^*, c_S^*\} \), we find that this is
satisfied $\forall A > 0$.

- $0 < \hat{\theta}_B < 1$. This is satisfied if $0 < \hat{\theta}_A < 1$ because:
  
  \[
  \hat{\theta}_B = \frac{1}{2} + \frac{\hat{\theta}_A}{2} \quad \text{so} \quad 0 < \hat{\theta}_A < 1 \implies 0 < \hat{\theta}_B < 1
  \]

  Note again, that $0 < \hat{\theta}_B < 1$ is satisfied at the optimal values of input prices, $\{c_L^*, c_S^*\}$ without any further restrictions.

- $P_L + t\hat{\theta}_B \leq V$. This implies that the indifferent customer is still being served (and hence, all customers are being served). Substituting for prices and market shares from the proof of Proposition 1 in Chapter 1, this condition reduces to:
  
  \[
  c_L + c_S \leq 2V - \frac{3 + A}{1 + A} t \quad (2.10)
  \]

  Substituting for the optimal input prices $\{c_L^*, c_S^*\}$ reduces this to an equality.

- $c_L^* \geq 0$ and $c_S^* \geq 0$. We have shown that $c_L^* < c_S^*$. This means that $c_L^* \geq 0$ is binding. From equation 2.7 below, this requires:
  
  \[
  c_L^* = V - \frac{6 + 3A}{4(1 + A)} t \quad \geq 0 \implies \frac{t}{V} \leq \frac{6 + 3A}{4(1 + A)} \quad (2.11)
  \]

  Hence, the equilibrium where the entire market is served is feasible is where the transport cost is not too big compared to the value. Now the range of effective transport cost that supports the full market equilibrium is given by (2.11) as $\frac{t}{V} \leq \frac{6 + 3A}{4(1 + A)}$. Now, $\frac{d(t_F)}{dA} =_{sign} [(6+3A)-(1+A)3] = 3 > 0$. Hence as $A$ increases, the range of relative transport cost that supports the full market equilibrium widens.

Proof of Corollary 2.1. Substituting the equilibrium values form (2.7) into (2.3),
we get
\[ P_L^* = V - \frac{6 + 11A}{12(1 + A)} t, \quad P_S^* = V - \frac{6 + A}{12(1 + A)} t \] (2.12)

It is easy to show that
\[ \frac{dP_L}{dA} = \frac{-5t}{12(1 + A)^2} < 0 \quad \text{and} \quad \frac{dP_S}{dA} = \frac{5t}{12(1 + A)^2} > 0 \]

Proof of Proposition 2.2. First suppose that \( U \) were to reject \( D_S \)'s offer. In this case \( U \) only supplies the market via the local store. Both types of customers pay transport costs to secure the good. Given \( c_L \), \( D_L \) can choose whether it wants to serve all the customers or only a part of them. The local store would rather serve the whole market if profits shrink if \( D_L \) raises prices to lose some customers. If serving only part of the market, \( D_L \) would serve all the customers up to location \( \theta \) where \( V - \theta t - P_L = 0 \). Hence a price of \( P_L \) would secure a market share of \( (V - P_L)/t \). The local store’s profits would be \( \Pi_{D_L}^{X}(c_L) = (P_L - c_L)(V - P_L)/t \) and the optimal price would be \( P_L = (V + c_L)/2 \). Thus the whole market is served if the optimal market share, \( (V - c_L)/2t \), would be greater than 1. Hence the whole market is served if \( c_L \leq V - 2t \). The supplier’s profits are therefore

\[ \Pi_{U}^{X_{D_L}}(c_L) = \left\{ \begin{array}{ll}
    c_L & \text{if } c_L \leq V - 2t \\
    \frac{(V - c_L)}{2t} c_L & \text{if } c_L > V - 2t
\end{array} \right. \] (2.13)

\( \Pi_{U}^{X_{D_L}} \) is maximised at \( c_L = V - 2t \) if we assume \( V \geq 4t \). We continue to assume that differentiation \( t \) is small and so restrict attention to the full coverage case. Hence if \( D_S \)'s offer is rejected then \( U \) will demand \( c_L = V - 2t \) and make a profit of \( V - 2t \).
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The supermarket must make an offer of \( c_S \) such that

\[
\Pi^D_{S,D_S}(c_S, c_L) \geq \Pi^D_{U,U^*}(c_L^*)
\]  

(2.14)

Only offers of \( c_S \) ensuring the participation constraint of \( U \) is met will be accepted.

Suppose now that \( D_S \)'s offer of \( c_S \) is accepted. Then \( U \) sells through both \( D_L \) and \( D_S \) at prices \((c_L, c_S)\). By assumption on \( t \) all customers will be served. The marginal type \( A \) customer is at \( \hat{\theta}_A \) and the marginal type \( B \) customer is at \( \hat{\theta}_B \). The prices and market shares are given by (2.3) and (2.6). The supplier firm would derive a profit in this case equal to (2.5). Simplifying we have:

\[
\Pi_U(c_S, c_L) = c_S + (c_L - c_S) \left[ \frac{1}{3t} \left\{ c_S - c_L + \frac{2tA}{1 + A} \right\} \left( \frac{1 + A}{2} \right) + \left( \frac{1 - A}{2} \right) \right]
\]

\[
= c_S + (c_L - c_S) \frac{1}{3t} \left[ (c_S - c_L) \left( \frac{1 + A}{2} \right) + tA + 3t \left( \frac{1 - A}{2} \right) \right]
\]

\[
= c_S + \frac{1}{2} (c_L - c_S) \left( \frac{1 + A}{3t} \right) \left\{ c_S - c_L + \frac{3 - A}{1 + A} t \right\}
\]

(2.15)

The optimal \( c_L \) can then be found directly as a function of \( c_S \):

\[
c_L \in \arg\max_{c_L} \Pi_U(c_S, c_L) = c_S + \frac{1}{2} \frac{3 - A}{1 + A} t > c_S
\]

(2.16)

Thus we have the desired result. Note however that this holds as long as full cover is maintained. So the final reaction function of the supplier can be written using (2.10) and (2.16) as:

\[
c_L(c_S) = \begin{cases} 
  c_S + \frac{3 - A}{2(1 + A)} t & \text{if } c_S \leq V - \frac{3 + A}{4(1 + A)} t \\
  2V - c_S + \frac{3 + A}{1 + A} t & \text{if } c_S \geq V - \frac{3 + A}{4(1 + A)} t
\end{cases}
\]

(2.17)

Note that for values of \( c_S \) very close to the benchmark equilibrium \((i.e. c_S > V - \frac{3 + A}{2(1 + A)} t)\),
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we do not have \( c_S < c_L \) in equilibrium but this is a boundary case and hence we do not consider it.

\[ \square \]

Proof of Proposition 2.3. Given the supermarket’s offer of \( c_S \), the supplier, if it accepts, will demand input prices of \( c_S \) as given by (2.16). The supermarket’s profit will therefore be

\[
\Pi_{DS} (c_L, c_S) = (P_S - c_S) [A \cdot (1 - \hat{\theta}_A) + (1 - A) \cdot (1 - \hat{\theta}_B)]
\]

\[
= (P_S - c_S) \left[ 1 - A\hat{\theta}_A - \left( \frac{1 - A}{2} \right) \left( \hat{\theta}_A + 1 \right) \right]
\]

Using (2.3) and (2.6) we can simplify this to

\[
\Pi_{DS} (c_L, c_S) = \left( \frac{1}{3} (c_L - c_S) + \frac{3 + A}{3(1 + A)} t \right) \left( \frac{1 + A}{2} \right) \left[ 1 - \frac{1}{3t} \left( c_S - c_L + \frac{2tA}{1 + A} \right) \right]
\]

\[
= \left( \frac{1 + A}{2t} \right) \left( \frac{1}{3} (c_L - c_S) + \frac{3 + A}{3(1 + A)} t \right)^2
\]

(2.18)

Now \( c_L - c_S \) is chosen by \( U \) to have a constant value (2.16). Hence the supermarket is indifferent between different price offers as noted in the main text. We select the equilibrium with the lowest acceptable price. To determine this the supplier makes a final profit found by substituting (2.16) into (2.15) of:

\[
\Pi_U = c_S + \frac{1}{2} \left( \frac{1}{2} \frac{3 - A}{1 + A} t \right) \left( \frac{1 + A}{3t} \right) \left\{ -\frac{1}{2} \frac{3 - A}{1 + A} t + \frac{3 - A}{1 + A} t \right\}
\]

\[
= c_S + \frac{1}{24} \left( \frac{3 - A}{1 + A} \right)^2 t
\]

If \( U \) rejects the offer its profits are given by maximising (2.17) as \( V - 2t \). Hence the lowest input price offer \( U \) will accept from the supermarket, \( D_S \) is \( c_S \) given by:

\[
c_S + \frac{1}{24} \left( \frac{3 - A}{1 + A} \right)^2 t \geq V - 2t \Rightarrow c_S = V - 2t - \frac{1}{24} \left( \frac{3 - A}{1 + A} \right)^2 t
\]
We can now determine the comparative statics results:

\[
\frac{dc_S}{dA} = \text{sign} \left[ - (1 + A) \left( 2 (3 - A) - (3 - A)^2 \right) \right] = \text{sign} \ 5 + A > 0
\]

\[
\frac{dc_L (c_S)}{dA} = \frac{d}{dA} \left[ V - 2t - \frac{1}{24} \frac{(3 - A)^2}{1 + A} t + \frac{13 - A}{21 + A} t \right]
\]

\[
= \text{sign} \frac{d}{dA} \left[ \frac{(9 + A) (3 - A)}{1 + A} \right] = \text{sign} \ [(1 + A) (3 - A - 9 - A) - (9 + A) (3 - A)]
\]

\[
= \left[ - (6 + 8A + 2A^2) - (27 - 6A - A^2) \right] = -A^2 - 2A - 33 < 0
\]

The retail prices are provided by (2.3). Inputing the prices from (2.16) we have

\[
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix} = \frac{1}{3} \left( 3c_S + \frac{3-A}{1+A} t \right) + \frac{1}{3(1+A)} \left( 3 - A \right) t
\]

Hence, using the equilibrium price \(c_S\) we have

\[
P_S = \ c_S + \frac{13 - A}{6 (1 + A)} t + \frac{3 + A}{3(1 + A)} t = V - 2t - \frac{1}{24} \frac{(3 - A)^2}{1 + A} t + \frac{36 + 4A}{1 + A} t
\]

\[
\Rightarrow \frac{dP_S}{dA} = \text{sign} \left[ (1 + A) (-2 + 2A) - (-2A + A^2 + 45) \right] > 0
\]

while

\[
P_L = \ c_S + 2 \frac{3 - A}{3(1 + A)} t = V - 2t - \frac{1}{24} \frac{(3 - A)^2 + 16 (3 - A)}{1 + A} t
\]

\[
\Rightarrow \frac{dP_L}{dA} = \text{sign} \left[ (1 + A) (-2 (3 - A) - 16) - (3 - A)^2 - 16 (3 - A) \right]
\]

\[
= \text{sign} \left[ (1 + A) (-22 + 2A) - (9 + A^2 - 6A) - 48 + 16A \right] > 0
\]

Yielding the results.

Consider now the feasibility of this equilibrium. From Chapter 2, we see that necessary conditions for \(0 < \hat{\theta}_A < 1\) are \(t > \frac{(1 + A)(c_L - c_S)}{2A}\) and \(t > \frac{(1 + A)(c_S - c_L)}{3 + A}\). As here \(c_L > c_S\), the second condition binds. We substitute
for $\tilde{c}_L, \tilde{c}_S$ and find that this holds when

$$t > \frac{1 + A}{2A} \cdot \frac{3 - A}{2(1 + A)} t \quad \Rightarrow \quad A > \frac{3}{5} \quad (2.19)$$

\[\Box\]

**Proof of Proposition 2.4.** To show that a waterbed effect exists we need to show that $c_L$ as defined in (2.16) is bigger than $c^*_L$ of the benchmark game as defined in (2.7). Call the lowest acceptable offer by $\tilde{c}_S$. This must be such that $\Pi_{U}^{D_S,D_L} = \Pi_{U}^{X_{DL^*}}$. Substituting for these and assuming $V > 4t$ we get

$$\tilde{c}_S + \frac{1}{2}(c_L - \tilde{c}_S) \left( \frac{1 + A}{3t} \right) \left\{ \tilde{c}_S - c_L + \frac{3 - A}{1 + A} t \right\} = V - 2t$$

Now substituting for the supplier’s optimal response of $c_L$ from (2.16) we get

$$\tilde{c}_S = V - 2t - \frac{(3 - A)^2}{24(1 + A)} t \quad (2.20)$$

Substituting (2.20) into (2.16) we get

$$\tilde{c}_L = V - 2t + \frac{(3 - A)(9 + A)}{24(1 + A)} t \quad (2.21)$$

To prove that there is no waterbed effect in this equilibrium it is sufficient to show that $\tilde{c}_L < c^*_L$ of the benchmark game. Using (2.7) and (2.21), this is true if

$$V - 2t + \frac{(3 - A)(9 + A)}{24(1 + A)} t < V - \frac{6 + 3A}{4(1 + A)} t \quad \Rightarrow \quad \frac{1}{24(1 + A)} (A^2 + 36A - 15) > 0 \quad (2.22)$$

Which is satisfied for all values given by (2.19). Hence for the values of $A$ that make this equilibrium feasible, (2.22) is always true. Hence $\tilde{c}_L < c^*_L$. There is no waterbed effect. \[\Box\]
Abstract: This chapter considers four extensions of chapter 2. First, we analyse the non-standard Hotelling case with partial cover. We show that if the supplier has all the bargaining power, it continues to charge the supermarket a higher price than the local store. However, when the supermarket has bargaining power, it might not be able to negotiate a better deal than its rival since the supplier can gain new customers by helping the local store. Moreover, in response to giving a discount to the supermarket, the supplier only faces incentives to transfer business. Hence there is no waterbed effect. Second, we show that the order of moves in the bargaining model is robust because the supermarket would be willing to bid more for the right to move first. Third, we allow for nonlinear pricing and show that the supplier’s incentives to discriminate are reversed. Additionally, we argue that the size of the supermarket’s advantage in the bargaining game is smaller. Finally, we show that a ban on discrimination is always welfare improving.
3.1 Introduction

This rather large chapter addresses four different yet interconnected issues. Using the same parent model presented in Chapter 2, we sequentially relax a few assumptions to assess the impact on our results.

First we address the issue of full cover. While the full cover assumption is standard in the literature, it does not have strong empirical backing. One would expect that buying decisions of customers change when price levels are altered. This is likely to be an important factor in determining our results. For the analysis of waterbed effects especially, the assumption of full cover is likely to overstate the size of the effect since the gain of one firm’s market share comes entirely at the cost of its rivals. If new customers can enter the market when prices are reduced, the size of the externality one retailer imposes on another is reduced.\(^1\)

Therefore for the first part of this chapter we expand the framework presented in Chapter 2 to allow for partial cover. A natural extension for this purpose would have been to construct a Cournot setting because Hotelling models usually cannot capture competition with partial cover. However, given the two types of customers in our setup, it is still possible for the firms to compete over the supermarket shoppers while being monopolists over the one off customers. This allows us to continue with our existing framework and isolate the key differences.

In a Hotelling setup partial cover would be chosen if the transport costs are high enough so it is too costly for the supplier to serve the customers far away from the retail outlets. It will prefer to charge high input prices and extract surplus from customers closer to the retail stores rather than lower prices so as to serve everyone. Having allowed for partial cover, the question we want to answer is - will serving part of the market make a difference to the pricing strategy of the

\(^1\)See Competition-Commission (2008) Para 5.3
supplier? In particular, will it continue to punish the better route to market? We find that it does. The better route to market (supermarket) continues to have a less elastic demand for the input. The price discriminating supplier will choose to exploit that by setting it a high input price. Serving part of the market does not change anything fundamentally in our model because it does not create any additional asymmetry. The supplier now charges a higher level of input prices to both the retailers, but does not vary the difference.

The chapter further argues that the results are quite different if we allow the downstream firms to have some bargaining power. With full cover, allowing for buyer power meant that the supermarket was always able to negotiate a better price. This does not hold if only part of the market is being served. Now that seems counterintuitive. The supermarket was able to secure a lower price than its rival under full cover. So surely, when it additionally becomes a monopolist over some customers, it should still be able to secure a lower price? However since both the local store and the supermarket become monopolists over a range of customers, this does not create any additional advantage for the supermarket. The only implication of partial cover then is that the supplier has an option to increase its customer base by lowering input prices. We show that it will choose to do so if supermarket doesn’t buy enough volume. So when the supermarket makes a high offer, the supplier reduces the local store’s price below the supermarket’s to increase the customer base. But when the supermarket makes a low offer, the supplier continues to extract profits from the local store by charging it a higher price. Thus, even if we allow the downstream industry to have some measure of bargaining power, the supermarket still might end up paying a higher price than its rivals.

The paper then addresses the issue of waterbed effects. With full cover, a discount to the supermarket created a new opportunity for the supplier to increase
prices and extract profits from the local store. It also created incentives for the supplier to transfer business towards the local store. If the size of the discount was small, the first effect dominated and we saw a waterbed effect. However, in this chapter with partial cover the supplier is unconstrained in the benchmark equilibrium. Here a discount to the supermarket does not alter the supplier’s ability to extract profits from the local store. The only incentive the supplier faces then is to transfer business. Therefore, a reduction in the supermarket’s price is always followed by a reduction in the local store’s price.

Given that the supplier’s behaviour is different under full and partial cover, the paper also asks which equilibrium it would prefer. It is shown that over the parameter range that the supplier makes positive profits under both full and partial cover, it is always optimal for it to make the good scarce and supply part of the market. This is because the supplier can increase profits by raising prices and reducing quantity until the demand elasticity is unitary.

The second part of this chapter returns to the framework of full cover and performs a robustness check on the bargaining game. Chapter 2 argued that if we allow the downstream industry to have some bargaining power, the input prices would be reversed. This was demonstrated by a sequential game where the supermarket had the ability to propose a price to the supplier, who then proposed a price to the local store. However, bargaining power was still exogenously given. While there might be behavioural justifications, our model didn’t show why it was reasonable to assume that the supermarket is able to make preemptive offers. We endogenise the bargaining order by adding a stage 0 before the actual game where the local store and the supermarket bid for the right to be the first mover. We show that the supermarket is indeed willing to bid more. This is because it is punished disproportionately by the supplier if it moves second - the supplier takes advantage of the captive demand and extracts surplus from it by charging
it a very high price. The supermarket hence has more to lose by going second and will bid more in stage 0. Therefore the order of moves is indeed as we have modelled in Chapter 2.

The third part of this chapter considers the impact of changing the structure of the contracts. So far we have assumed linear tariffs. Although there is evidence that larger retailers do benefit at the margin\(^2\), not all the gains for the larger retailer can be measured at the margin. These include fixed payments, volume discounts and rebates. Hence we modify the full cover framework of Chapter 2 to include non linear tariffs. Similar to Inderst and Shaffer (2009) we show that optimal discriminatory contracts amplify the differences in competitiveness. This is because using a fixed fee to extract the downstream surplus, the supplier is now interested in maximising joint profits. It will achieve that by charging the supermarket a lower price than the local store. We then consider the bargaining scenario and find that similar to the case with linear tariffs, the supermarket is able to negotiate a lower price than the local store. This result is not as significant because it was already paying a lower price without accounting for bargaining power. However when it has bargaining power, it is able to keep some surplus for itself. More importantly, the degree to which the supermarket is better off in the bargaining game (at the margin) is lower in this case. This is because the supplier now cares about the local store’s profits since it can extract them via the fixed fee. Hence it is less inclined to punish it.

Finally the chapter considers the case where price discrimination might be banned and conducts welfare analysis. We show that the optimal linear uniform price lies in between the optimal discriminatory prices. Since discriminatory

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\(^2\)See Competition-Commission (2008) and the Working Paper on Supplier Pricing (2007). They find that after excluding the fixed parts of contracts, the relative discounts enjoyed by the bigger retailers were even higher.
Pricing meant type A customers were subsidising the type B customers, a ban on discrimination makes type A customers better off but type B customers worse off. It can be shown however that total welfare of customers always increases.

Plan of this paper: Section 3.2 considers the case where only part of the market is supplied. It analyses the impact on input prices both in the benchmark and bargaining case and asks whether any waterbed effects exist. It then moves on to equilibrium selection. Section 3.3 endogenises the notion of bargaining power presented in Chapter 2. Section 3.4 analyses the case of non linear tariffs and Section 3.5 conducts welfare analysis. Finally Section 3.6 concludes and the proofs are contained in the Appendix.

3.2 Partial Cover

Most of the Hotelling literature makes the assumption that transport costs are low enough that there is full market cover. This holds for goods for which customers have a high valuation and a low cost of travelling. But for other non emergency goods for which ‘waiting’ is not costly, we would not expect full cover to hold - for example cans of tuna or large bottles of water to stock at home. A decrease in retail prices in this case increases the size of the market as customers decide whether they want to buy. Hence we need to assess the issue of partial cover. Incentives to discriminate might be different in this setup as the supplier is now not concerned with keeping prices low to ensure full cover.

3.2.1 U has all the bargaining power

This section assesses the impact of serving part of the market on the input prices offered to the downstream retailers when the supplier has all the bargaining power. Let us begin by noting that the marginal type B customer for the local
store is at a point $\hat{\theta}_{BL}$ where:

$$P_L + t\hat{\theta}_{BL} = V \Rightarrow \hat{\theta}_{BL} = \frac{V - P_L}{t}$$

and the marginal type $B$ customer for the supermarket is at a distance $\hat{\theta}_{BS}$ where:

$$P_S + t\hat{\theta}_{BS} = V \Rightarrow \hat{\theta}_{BS} = \frac{V - P_S}{t}$$

Note that while $\hat{\theta}_{BL}$ represents the location of the marginal $B$ customer for the local store, $\hat{\theta}_{BS}$ is not a representation of the location but the distance of of the marginal customer from the supermarket. This non-traditional notation is used because not everyone is served. Before determining the location of the type $A$ customers, we present a lemma:

**Lemma 3.1.** If $U$ is making tioli linear price offers and serving part of the market, then in equilibrium all type $A$ customers are served.

Let us suppose for a contradiction that not all type $A$ customers are being served. Since they pay no transport cost to go to the supermarket, this means that no type $A$ are being served by the supermarket. This implies that $P_S > V$ and $P_L + t > V$. Thus some type $A$ and type $B$ customers are being served by the local store, while the supermarket serves no one. Consider the upstream lowering $c_S$ such that $P_S$ is just under $V$. This means all $U$ gains profits from all originally unserved type $A$ customers who now go to the supermarket, while it doesn’t lose any profit from the local store. This is a valid deviation. Hence it must be the case that all type $A$ customers are being served.

Given that all $A$ customers continue to be served, let us also define the location of the marginal type $A$ customer as $\hat{\theta}_A$ where:

$$P_L + t\hat{\theta}_A = P_S \Rightarrow \hat{\theta}_A = \frac{P_S - P_L}{t}$$
Note that unlike under full cover, now both $A$ and $B$ customers are equally price sensitive. We now wish to show that $U$ will again seek to extract a higher price from the supermarket.

**Proposition 3.1.** *For intermediate values of the transport cost, there exists an equilibrium where the supplier serves part of the market and charges the supermarket a higher price than the local store. In addition, $c^*_S$ is chosen as the same constant mark up over $c^*_L$ as under full cover.*

The result here is almost identical to Proposition 2.1, and a similar intuition applies. Whether the firms are serving the market in part or in full, the supermarket remains a better route to market since all type $A$ customers continue to buy. This makes its derived demand for the input less elastic, which gets translated into a higher input price from the supplier.

In addition, it is also the case that the upstream still exploits the supermarket to the same degree. Serving part of the market does not change anything fundamentally in our model as it does not create any additional asymmetry. Downstream demand is now more price sensitive because the $B$ customers are as price sensitive as the $A$ customers. However, this happens for both downstream stores to the same degree. Hence there is no additional incentive for the upstream to price discriminate. Therefore both stores are charged a higher level of prices but the difference between them is held the same. Further, the comparative statistics remain unchanged - as the proportion of supermarket shoppers increases, the downstream asymmetry increases and the supplier charges the supermarket an even higher and the local store an even lower price.

Note however that the supplier will choose this equilibrium as long as the transport cost is intermediate compared to the value. If the transport cost was too small compared to the value, then customers towards the middle of the Hotelling line would always want to buy. If on the other hand the transport cost was
too large, then the retail prices could be too high for anyone to buy. This is because a high transport cost means the customers’ demands are less elastic so the downstream stores increase their markup. We also have the following comparative statics on retail prices:

**Corollary 3.1.** Suppose only part of the market is being served. The equilibrium retail price set by the supermarket is always increasing in the proportion of supermarket shoppers \((A)\), while the retail price set by the local store is increasing in \((A)\) only if it is high.

With an increase in the proportion of supermarket shoppers, from Proposition 3.1, the supplier increases the input price to the supermarket. This causes an increase in the supermarket’s retail price. The supplier also decreases the input price it charges the local store. This direct effect makes the local store’s retail price fall. However, strategic complementarity makes it’s retail price rise. The strength of this complementarity is measured by the number of customers the two downstream stores compete over - which are the supermarket shoppers. Thus, if there are a lot of these shoppers then the strategic effect is enough to over-ride the direct effect and the retail price of the local store also rises.

### 3.2.2 Implications of bargaining power

We would now like to examine how the the supplier’s pricing incentives change when the downstream industry has bargaining power. The game is identical to that presented in Chapter 2:

**Stage 1:** \(D_S\) makes a take-it-or-leave-it offer \(c^*_S\) to the supplier \(U\).

**Stage 2:** \(U\) decides whether to accept / reject the offer. It then makes a take-it-or-leave-it offer of \(c^*_L\) to \(D_L\).

In Proposition 2.2 of Chapter 2 we had shown that under full cover, the supermarket is always able to negotiate a better price than the local store if it
has bargaining power. Since the supplier can no longer extract rents form the better route to market, it will have to extract rents from the weaker retailer. Under full cover this amounts to raising the input price of the local store since all customers continue to buy. However, under partial cover, increasing the local store’s input price leads to some customers falling out of the market entirely. Hence it might not always be in the supplier’s interest to tilt the playing field against the local store. We have the result:

**Proposition 3.2.** Suppose only part of the market is being served. Then despite having bargaining power, the supermarket is able to negotiate a better deal than the local store only if it makes an offer that is low enough.

Suppose for a contradiction that \( c^*_L \leq c^*_S \) always. To see if there are any valid deviations let us consider the impact of increasing \( c^*_L \) on the profits of the upstream. Increasing the input price of the local store will increase its retail price. Due to strategic complementarily it will also increase the retail price of the supermarket but by a smaller amount. Hence some customers will switch to the supermarket and some other customers will fall out of the market. The upstream will gain profits on all the infra marginal customers who continue to buy form the local store and on those marginal customers who switch to the supermarket (since \( c^*_L \leq c^*_S \)) but will lose profits on all the marginal customers who stop buying.

Crucially, the size of the profits lost by the supplier is increasing in the level of the supermarket’s offer. For any level of offer made by the supermarket, an increase in the local store’s input price will increase both retail prices. Consequently, some type B supermarket customers will stop buying the good. If the level of the offer made by the supermarket is large, this represents a larger loss in the supplier’s profits. Therefore, the supplier has a higher incentive to increase the local store’s price if the offer made by the supermarket is low. For high offers then, volume effects mean that it is too damaging to increase the local store’s
price. Thus we have \( c^*_L > c^*_S \) only for low levels of \( c^*_S \).\(^3\)

Another way asses this question is to consider the participation constraint of the upstream. Since there is partial cover, the supermarket’s profits are decreasing in the offer it makes. It will then make the lowest acceptable offer, and the level of that offer is determined by the amount of profits the local store can provide the upstream if it sells through it exclusively. If the local store can give the upstream a lot of exclusive profits then the supermarket is forced to make a high offer and consequently the local store can pay a lower price than the supermarket. However, if the local store is unable to provide the upstream with high profits then it will have to pay a higher input price in equilibrium.

\[ c^*_L = c^*_S \]

\[ c^*_L < c^*_L \]

Figure 3.1: \( c^*_S < c^*_L \) if \( c^*_S \) is small, No waterbed effect

\(^3\)Note that similar to Chapter 2, a higher offer made by the supermarket increases the number of infra marginal customers of the local store and this makes the supplier want to increase the local store’s input price. However the damage done to the supplier because of the customers who exit is too large if the offer made by the supermarket is high. This overpowers the first effect and we have that the supplier in fact wants to increase the local store’s price only when the offer made by the supermarket is low enough.
3.2. PARTIAL COVER

3.2.3 Waterbed effect

We now address the question of waterbed effects. We have shown in Proposition 3.2 that the supermarket may or may not be able to negotiate a better deal than the local store. We now ask, what the implications are on the level of input price paid by the local store in comparison to the benchmark case. We have the result:

Proposition 3.3. Suppose only part of the market is being served. Then there is no waterbed effect - a fall in the input price paid by the supermarket always leads to a fall in the input price paid by the local store.

Notice first that if the offer made by the supermarket is extremely low, then we might have a waterbed effect. This is because the upstream must set the input price of the local store high enough to maintain partial cover. We are not interested in this boundary case.

Proposition 3.3 here is different from Proposition 2.4. Under full cover, the waterbed effect existed if the offer made by the supermarket was high enough. This was because a high offer by the supermarket meant that the local store had a large market share. This increased the costs associated with transferring business to it and made the supplier more likely to instead extract profits by increasing the local store’s price. Crucially, the supplier in the benchmark game was constrained by full cover. A decrease in the supermarket’s price weakened that constraint and made the supplier more able to extract profits from the local store. However, under partial cover, the supplier is not constrained in the benchmark case. Therefore it does not see an increase in its ability to extract profits from the local store following a decrease in the supermarket’s price. The only impact then of a discount to the supermarket is an increase in the incentives to transfer business away from it. Therefore in response to a discount to the
supermarket, the supplier always charges the local store a lower price. 4

This can be seen in Figure 3.1. The isoprofit curves of the upstream are ellipses around the bliss point at the benchmark equilibrium.5 The participation constraint of the upstream is also indicated by one of these curves. The market is under partial cover if the input prices are not too low, denoted by the input price combinations above the solid downward sloping line.6 The reaction function of the upstream to any offer made by the supermarket is located at the tangency of the isoprofit curve with the vertical line at that level. The locus of these points is highlighted in bold. If the local store is able to give the upstream only a low level of exclusive profit then the supermarket makes a low offer and we have $c_L^* > c_S^*$. If however the local store can give the upstream a high level of exclusive profits then the supermarket is forced to make a high offer and then $c_L^* < c_S^*$. As the supermarket makes an offer lower than the benchmark case, the optimal input price offered to the local store is also below the benchmark level. Hence there is no waterbed effect. Our results here are similar to Chen (2003). The intuition is a little different. In Chen (2003), a rise in countervailing power possessed by a large retailer doesn’t change the per unit price it pays but leads to a decrease in the supplier’s profits accruing from it. The supplier then tries to offset the decrease in profits by boosting sales to the fringe. In our model, an increase in

4Note that this lack of a waterbed effect is not due to the argument that with partial cover the externality one retailer imposes on another is smaller. Rather this is because partial cover in our model removes the constraint on the supplier. This means all extraction of profits has already happened, and it is not optimal to disadvantage the local store following an increase in the supermarket’s competitiveness.

5Note that the supplier keeps the difference between the input prices constant under both full and partial cover (from Proposition 3.1).

6It is interesting to note however that the level of retail prices given a $\{c_L, c_S\}$ is lower under partial cover. This seems counterintuitive since the firms are monopolists over the one off shoppers and we would expect them to increase their mark-up. However, the removal of strategic complementarity serves to reduce these incentives as a given increase in price now means that the retailer loses more customers than under full cover. Therefore there is a set of input prices where full cover is not feasible (since retail prices are too high) but also partial cover is not feasible (since retail prices are too low).
buyer power reduces the supermarket’s price. It hence creates incentives for the supplier to transfer sales away from the supermarket towards the local store. In both models, a discount to the larger retailer makes a discount to the smaller retailer less costly for the supplier.

### 3.2.4 Equilibrium Selection

We have shown the equilibrium under full and partial cover. However note that the supplier, via its choice of \( \{c_L, c_S\} \) is implicitly deciding whether it wants to serve the entire market. This section then asks the question of optimality. Given that it is feasible to supply both a part of and the entire market (so \( U \) makes nonzero profits in both), we would like to know which regime the supplier would find more profitable. The ex-ante intuition might be that if there are a lot of supermarket shoppers, it is probably not in the supplier’s interest to serve the entire market because it would have to lower input prices to ensure that retail prices are low enough to maintain participation. On the other hand, with a low proportion of supermarket shoppers, it might be worth keeping input prices low to capture rents from all the one-off shoppers who are now more numerous. However, we find that the supplier always prefers to serve only part of the market.

**Proposition 3.4.** Suppose the transport cost is such that \( U \) makes nonzero profits under both full and partial cover. Then the supplier will always prefer to supply only part of the market.

Let us first consider the case where \( A = 0 \). The upstream will set symmetric input prices and the downstream firms will serve equal number of customers. It can be shown that even in this case, the upstream is better off supplying only part of the market. The analogy is similar to any monopoly’s pricing decision - if \( U \) supplies the entire market, it is not maximising its profits because the elasticity of demand is not unitary. It would gain from increasing input prices and reducing
quantity until a point where the revenue is maximised.

If $A > 0$, then from Lemma 3.1 all supermarket shoppers will continue to be served. The supplier then for sure gains higher profits on these customers because it is charging higher level of input prices under partial cover. Therefore its profits are higher for both types of customers in serving part of the market. Hence, it is always in the supplier’s interest to make the good scarce.

3.3 Endogenising the Bargaining Order

Let us return to the framework of Chapter 2. We have modelled bargaining power as the ability to propose prices to the supplier. A criticism of the results in Chapter 2, in particular Proposition 2, might then be that it is so because we have artificially given the supermarket a first mover advantage, and hence the result is true by construct. We haven’t explicitly shown why it is reasonable to assume that the supermarket has this first mover advantage. To show the robustness of our assumption, we now introduce a stage 0 before the actual game where $D_L$ and $D_S$ bid for the right to be the first mover. Endogenising the bargaining power in this way provides a clear basis for our modelling choice in Chapter 2. The game is as follows:

**Stage 0:** $D_L$ and $D_S$ bid for the right to be the first mover.

**Stage 1:** $D_i$ that wins in stage 0 makes a ‘take-it-or-leave-it’ offer $c^*_i$ to $U$.

**Stage 2:** $U$ decides whether to accept / reject the offer, and makes a ‘take-it-or-leave-it’ offer of $c^*_j$ to $D_j$.

This game cleanly captures whether the supermarket indeed has the ability to propose prices. Our modelling choice is confirmed by the result:

**Proposition 3.5.** If the downstream firms are allowed to bid for the right to be the first mover, the supermarket is willing to bid a higher amount.
3.4 **NONLINEAR PRICING**

Both players would prefer to move first rather than second. If the supermarket moves first (as in Chapter 2) then it will be able to command a low price. The supplier will try to recapture it’s rents via the weaker local store and in turn set it a high price. Consider now the scenario where the local store moves first. The first mover will still be able to command a low price, but now the weaker retailer also has a more inelastic demand. The supplier will now set the supermarket an extremely high price. This is for two reasons - because the supermarket has a weaker bargaining position, but also because the supermarket is a better route to market. Hence the supermarket is punished disproportionately by the supplier if it moves second. It then has a lot more to lose by going second and will therefore be willing to bid a higher amount in stage 0 to win the right to go first.

**3.4 Nonlinear Pricing**

Our analysis so far, both in the full and partial cover cases has still consisted of a linear contract. This is on the same lines as Katz (1987); DeGraba (1990); Yoshida (2000); Inderst and Valletti (2009, 2011). However there is significant empirical motivation for use of nonlinear tariffs. It is well known in the academic literature that contracts between suppliers and retailers rarely consist of simple linear pricing and involve volume discounts, fixed payments and rebates.\(^7\) Theoretically too, the presence of nonlinear tariffs means the supplier can disentangle the objectives of maximising the size of the surplus and its own share of it.\(^8\) Therefore if nonlinear tariffs were allowed, one would expect that the supplier

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\(^7\)The Supply of groceries in the UK investigation carried out by the competition commission conducted a survey where 70% of the respondents said that suppliers frequently make marketing contributions and promotional investments. 43% claimed that suppliers gave ‘other’ non unit price rebates. In fact, retailers prefer to get fewer discounts on the unit price and differentiate their terms of trade more. See Competition-Commission (2008) Appendix 5.4 para 40. See also Chen (2003) and Bloom et al. (2000) - slotting fees account for up to 9 billion in annual promotional expenditures, or approximately 16% of all new product introduction costs, and per-item store costs approximate 5,000 to 25,000.

\(^8\)See Inderst and Shaffer (2009)
should choose the allocatively efficient outcome and supply largely through the better route to market. This should give us a more intuitive result even in the benchmark case.

3.4.1 The Standard Approach: Single supplier has the bargaining power

We return to the benchmark game (of Chapter 2) and see how the results vary as we change the pricing structure from linear to two part tariffs. The supplier now sets a fixed fee \( F_i \) as well as a per unit price \( c_i \) for both firms \( i \in \{L, S\} \). The game is otherwise identical to that presented in Chapter 2. In stage 1, \( U \) makes a ‘take-it-or-leave-it’ offer \( \{c^*_L, F^*_L\} \) to \( D_L \) and \( \{c^*_S, F^*_S\} \) to \( D_S \), and in stage 2, \( D_L, D_S \) decide whether to accept / reject the offers, compete and realise profits. We present the following result:

**Proposition 3.6.** If the upstream is allowed to set two part tariffs then the supermarket pays a lower per unit price: \( c^*_S < c^*_L \).  

Similar to Inderst and Shaffer (2009) we find that the supplier will charge the better route to market a lower price at the margin. Optimal discriminatory contracts then amplify differences in competitiveness. So allocative efficiency is gained if the supplier is allowed to discriminate. With linear tariffs, \( U \) is more able to extract rents form the supermarket so it charges it a higher price. However with nonlinear tariffs, \( U \) is equally able to extract rents from both the downstream retailers. It then maximises joint industry surplus which requires charging the supermarket a lower per unit price.  

\[ F^*_S > F^*_L. \]

\[ \text{Inderst and Shaffer (2009) argue that this is because ‘discrimination with respect to the quantities that each downstream firm purchases’ is now feasible.} \]
3.4.2 A Bargained Supplier-Retailer Interface

Given that the supermarket already pays a lower price than its rival in the benchmark case, it would seem that we do not really need to bring in the notion of buyer power to make our results empirically sound. However, the supplier’s incentives to punish the local store are different when it can use nonlinear pricing. Intuitively it would seem that since the supplier can extract all the surplus from the local store, it would be less inclined to punish it vis-a-vis the supermarket. For this purpose we consider the bargaining game of Chapter 2 with nonlinear pricing. In stage 1, $D_S$ makes a tioli offer of \( \{c^*_S, F^*_S\} \) to $U$ and in stage 2, $U$ accepts/rejects and makes a tioli offer \( \{c^*_L, F^*_L\} \) to $D_L$. We present the following result.

**Proposition 3.7.** If the supermarket has bargaining power then it is able to negotiate a lower price than the local store so $c^*_S < c^*_L$. However the size of its advantage is smaller under nonlinear pricing.

We continue to get the result that the supermarket should pay a lower input price, however, the results are quantitatively different from those presented in Chapter 2. In particular, the upstream’s reaction function still sets the local store’s price as a fixed markup over the supermarket’s offer, but this markup is lower if the pricing structure is nonlinear.

The intuition is as follows. Given any acceptable offer made by the supermarket, the supplier will set $F^*_L$ in such a way to give the local store zero profit. It will choose the input prices to maximise the sum of its own profits and the local store’s profits, using the variable component of the price to maximise the size of the surplus and the fixed component to extract the surplus for itself. Since the local store’s profits decrease with the input price, the costs to the supplier of increasing it are higher. $U$ is then less inclined to increase the local store’s input price. Thus the presence of a two part structure in the pricing weakens the
supermarket’s position a bit in terms of the variable priced component. However, the supermarket will use the fixed component to extract the surplus from the supplier.

3.5 Welfare

We now return to the linear pricing case with full cover of Chapter 2. We showed in Proposition 2.1 that, if allowed to discriminate, the supplier would set the local store’s input price below that of the supermarket. We would now like to know whether this is damaging for the customers. This is particularly relevant for the discussion of the Robinson-Patman Act and article 82C issues in competition law. We examine what would happen if there were regulatory barriers such that $U$ were forced to set the same input price for both downstream firms, and its implication on consumer welfare.

**Proposition 3.8.** If the upstream firm is not allowed to discriminate, it will set a uniform input price $\hat{c}^\ast$, such that $c^\ast_L < \hat{c}^\ast < c^\ast_S$, which is welfare improving.

The intuition for the input price levels is the following - when the upstream firm chooses to discriminate, it does so by letting the supermarket subsidise the local store. When it is forced to set the same input price, the upstream is unable to extract the extra surplus from the $A$ consumers by tilting the playing field against the supermarket. Hence rather than helping the smaller (and perhaps the weaker) route to market, a regulation banning price discrimination will actually help the supermarket.

For the welfare of the customers - if the supplier is allowed to discriminate, it makes the type $B$ customers better off at the cost of the type $A$ customers. When it can no longer discriminate, type $A$ customers no longer subsidise the type $B$
3.6. CONCLUSIONS

ones, and their welfare increases. Conversely, the welfare of type $B$ customers decreases. One would then think that if there are a lot of type $B$ customers then total welfare must decrease. However the size of the welfare change following a ban on discrimination depends on how advantaged these $B$ customers originally were. If there are a lot of $B$ customers then the discriminatory prices weren’t too different to begin with. Hence when the upstream is not allowed to discriminate the welfare of type $B$ customers doesn’t fall by much. Total welfare then always increases.

Our result here is similar to the findings of DeGraba (1990). In that setup the firms are asymmetric in cost. Similar to Proposition 2.1, DeGraba shows that the supplier charges the high cost firm a lower input price than the low cost firm. The removal of discrimination then means that the supplier does not subsidise the high cost firm at the expense of the low cost firm. Given that industry output remains unchanged under the two regimes, total output then is produced at a lower average cost when discrimination is disallowed. This leads to an increase in welfare.

3.6 Conclusions

This chapter has considered some extensions and some robustness checks of the results presented in Chapter 2. The basic model remains the same - there a monopolist supplier selling its product to final customers via retailers that offer differing routes to market - a local store and a supermarket. The supermarket offers a better route to market because of the potentially captive demand it enjoys.

We have shown in chapter 2 that under full cover, the supplier would will choose to punish the better route to market because it is willing to pay more. In this chapter we argue that this result is robust even when we account for partial cover and volume effects. In addition, the supplier punishes the better route to
market to the same degree. This is because partial cover does create any additional asymmetry between the retailers. The supermarket still has a more inelastic demand for the input and hence pays a higher input price. The only consequence of serving part of the market then is that the supplier charges a higher level of input prices. The difference between the two input prices is, however, unchanged.

We have also shown that the other results of Chapter 2 do not hold through when we allow for quantity effects. In particular, if the downstream industry has bargaining power, it is no longer the case that the supermarket is always able to negotiate a lower price than its rival. If the local store is able to give the supplier a high level of profits exclusively then the supermarket is forced to make a high offer. This means that the supermarket does not buy enough volume. Therefore the supplier reduces the input price to the local store to increase its customer base. Hence we find that the despite having bargaining power, the supermarket will not always be able to negotiate a better deal than its rival.

There is also no waterbed effect. This is because in the supplier is not constrained in the benchmark case of this model. A discount to the supermarket does not create any additional means by which the supermarket can extract profits from the local store. Then the only incentives it has are to transfer sales away from the supermarket towards the local store. This is achieved by reducing the local store’s price and hence we see no waterbed effect.

Given the two scenarios presented in Chapter 2 and 3, we also ask the question of optimality. We show that the supplier will always find it optimal to supply only part of the market. This is because it can raise profits by increasing prices and decreasing volumes until the demand elasticity is unitary. Since all the type A supermarket shoppers continue to be served even in the part market regime, the supplier achieves this by creating scarcity of the good among the one-off type B shoppers. Therefore while we expect results of chapter 2 to hold for ‘emergency
In the second part of this chapter we consider the robustness of the model presented earlier. The bargaining game in Chapter 2 gave the supermarket a first mover advantage. This chapter has demonstrated that this assumption is correct because the supermarket is punished disproportionately by the supplier if it moves second. Hence it will be willing to bid more for the right to be the first mover and the order of moves is indeed as we have modelled in Chapter 2.

The third part of the chapter shows that with nonlinear tariffs, the incentives of the supplier to discriminate are reversed. We show that if the supplier has all the bargaining power, it will maximise industry surplus and charge the supermarket a lower per unit price than the local store. If the supermarket has bargaining power then it will, as with linear pricing, always be able to secure a lower per unit price than the local store but the size of its advantage is smaller. This is because the supplier can extract the local store’s profits via the fixed fee and is therefore less willing to punish it.

Finally we have shown that welfare of consumers is always higher if the supplier is not allowed to discriminate. The supplier discriminates by extracting surplus from the type $A$ customers while keeping the type $B$ customers better off. When this is no longer allowed, the $A$ customers benefit and the $B$ customers are worse off. However, as the proportion of $B$ customers increases, the damage done to them also decreases. Therefore total welfare increases.


## 3.7 Appendix

**Proof of Lemma 3.1.** Let us suppose for a contradiction that not all type A customers are being served. This means that no type A are being served by the supermarket. So we must have \( P_S > V \) and \( P_L + t > V \). Consequently, \( \hat{\theta}_{BS} = 0 \).

Thus some type A and type B customers are being served by the local store, while the supermarket serves no one. Consider the upstream marginally lowering \( c_S \) such that \( P_S < V \). This means all U gains profits from all originally unserved type A customers who now go to the supermarket, while it doesn’t lose any profit from the local store. This is a valid deviation. Hence it must be the case that all type A customers are being served.

**Proof of Proposition 3.1.** Given a \( \{c_L,c_S\} \), we know \( \hat{\theta}_A = \frac{1}{t}(P_S - P_L) \), \( \hat{\theta}_{BL} = \frac{1}{t}(V - P_L) \) and \( \hat{\theta}_{BS} = \frac{1}{t}(V - P_S) \). We can calculate

\[
\Pi_{DL} = \frac{1}{t}(P_L - c_L) [A(P_S - P_L) + (1 - A)(V - P_L)]
\]

which gives FOC : \( 2P_L - AP_S - (1 - A)V - c_L = 0 \). We can also calculate

\[
\Pi_{DS} = \frac{1}{t}(P_S - c_S) [A(t - P_S + P_L) + (1 - A)(V - P_S)]
\]

which gives FOC : \( 2P_S - AP_L - (1 - A)V - c_S - At = 0 \). The FOCs can be combined into the following system:

\[
\begin{pmatrix}
2 & -A \\
-A & 2
\end{pmatrix}
\begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \begin{pmatrix}
(1 - A)V + c_L \\
(1 - A)V + c_S + At
\end{pmatrix}
\]

\[
\Rightarrow \begin{pmatrix}
P_L \\
P_S
\end{pmatrix}
= \frac{1}{4 - A^2} \begin{pmatrix}
2 & A \\
A & 2
\end{pmatrix}
\begin{pmatrix}
(1 - A)V + c_L \\
(1 - A)V + c_S + At
\end{pmatrix}
\]

(3.1)

The supplier’s profit can be found to be

\[
\Pi_U = c_L [A\hat{\theta}_A + (1 - A)\hat{\theta}_{BL}] + c_S [A(1 - \hat{\theta}_A) + (1 - A)\hat{\theta}_{BS}]
\]

(3.2)
Using prices from (3.1) we can calculate:

\[ \hat{\theta}_A = \frac{c_S - c_L + At}{(2 + A)t} \quad \hat{\theta}_{BL} = \frac{(2 + A)V - 2c_L - Ac_S - A^2t}{(4 - A^2)t} \quad \hat{\theta}_{BS} = \frac{(2 + A)V - Ac_L - 2c_S - 2At}{(4 - A^2)t} \]

Substituting these into (3.2) we get:

\[ \Pi_U = \frac{1}{(4 - A^2)t} \left[ (2 + A)(1 - A)(c_L + c_S)V + 2Ac_Lc_S - (2 - A^2)(c_L^2 + c_S^2) + At(Ac_L + 2c_S) \right] \tag{3.3} \]

Maximising over \( \{c_L, c_S\} \) gives the system of FOCs:

\[
\begin{pmatrix}
(2 - A^2) & -A \\
-A & (2 - A^2)
\end{pmatrix}
\begin{pmatrix}
c_L^* \\
c_S^*
\end{pmatrix}
= \frac{1}{2} (2 + A)(1 - A)V + At \begin{pmatrix} A/2 \\ 1 \end{pmatrix}

\Rightarrow \begin{pmatrix}
c_L^* \\
c_S^*
\end{pmatrix} = \frac{1}{2}V + \frac{At}{2(1 - A^2)} \begin{pmatrix} A \\ 1 \end{pmatrix} \tag{3.4}
\]

From (3.4) it is immediate that \( c_L^* < c_S^* \) \( \forall A > 0 \). The equilibrium where part of the market is served is feasible if:

- \( \hat{\theta}_A > 0 \implies P_S > P_L \). This can be written in terms of the transport cost being high enough: \( t > \frac{c_L - c_S}{A} \). This is automatically satisfied under Proposition 3.1 as \( c_L^* < c_S^* \).

- \( \hat{\theta}_A < 1 \implies P_S - P_L < t \). This can again be written in terms of the transport cost being high enough: \( t > \frac{c_S - c_L}{2} \). Substituting for optimal input prices from (3.4) this condition reduces to \( t > \frac{A}{4(1 + A)}t \) which is true for all \( A > 0 \). Hence \( 0 < \hat{\theta}_A < 1 \) is satisfied under the optimal input price choices without any further restrictions.

- \( \hat{\theta}_{BL} > 0 \implies P_L < V \). This can also be written in terms of \( t \) as: \( t < \frac{A}{4(1 + A)}t \).
\[(2 + A)V - 2c_L - Ac_S \]

\[\frac{A^2}{A^2}.\]

- \(\hat{\theta}_{BS} > 0 \Rightarrow P_S < V\). This can also be written in terms of \(t\) as: \(t < \frac{(2 + A)V - Ac_L - 2c_S}{2A}\). Note that \(\hat{\theta}_{BS} > 0\) binds. This is because the transport costs affect \(P_S\) more than \(P_L\) due to the presence of \(A\) customers.

Substituting for \(\{c^*_L, c^*_S\}\) from (3.4), this condition reduces to:

\[
\frac{t}{V} < \frac{(1 - A^2)(2 + A)}{(6 - 3A^2)A}
\]

(3.5)

Thus, the transport costs must be low enough so that the supermarket is active.

- \(\hat{\theta}_{BL} + \hat{\theta}_{BS} < 1 \Rightarrow P_S - P_L > 2(V - t)\). This reduces to

\[
c_L + c_S > V - 2t
\]

(3.6)

This condition can be written in terms of \(t\) as: \(t > V - \frac{c_L + c_S}{2}\). Substituting for \(\{c^*_L, c^*_S\}\), this reduces to:

\[
\frac{t}{V} > \frac{2(1 - A)}{4 - 3A}
\]

(3.7)

Thus, the transport cost must be high enough so that the customers in the middle find it too expensive to buy.

- \(c^*_L \geq 0\) and \(c^*_S \geq 0\). We have shown that \(c^*_L < c^*_S\). This means \(c^*_L \geq 0\) is binding: \(c^*_L = \frac{V}{2} + \frac{A^2}{2(1 - A^2)}t\) which is true \(\forall V, t \geq 0\) & \(1 \geq A \geq 0\).

Hence there are no further constraints imposed.

We can summarise the feasibility constraints from (3.5) and (3.7) under:

\[
\frac{2(1 - A)}{4 - 3A} < \frac{t}{V} < \frac{(1 - A^2)(2 + A)}{(6 - 3A^2)A}
\]

(3.8)
Hence the equilibrium where part of the market is being served is feasible if the transport cost is low enough compared to the value to ensure retail prices are such that someone is buying, but they must also be high enough to ensure that not everyone is buying. Now the range of effective transport cost that supports the part market equilibrium is given by (3.8) as

\[
\frac{t^P}{V} = \frac{(1 - A^2)(2 + A)}{(6 - 3A^2)A} - \frac{2(1 - A)}{4 - 3A} = \frac{(1 - A)^2(2 - A)(4 + 3A)}{3A(2 - A^2)(4 - 3A)}
\]

Now,

\[
\frac{d(t^P)}{dA} = \text{sign} \ A(2 - A^2)(4 - 3A)[2(1 - A)(6A^2 - 6A - 7)] - (1 - A)^2(2 - A)(4 + 3A)[4(3A^3 - 3A^2 - 3A + 2)]
\]

\[
= \text{sign} \ 2(1 - A)(6A^5 - 9A^4 + 26A^2 + 16A - 32) < 0
\]

Thus as \( A \) increases, the range of transport cost supporting the part market equilibrium, becomes narrower.

**Proof of Corollary 3.1.** Using (3.1) and (3.4) we get:

\[
P^*_L = \frac{(3 - 2A)V}{2(2 + A)} + \frac{(5 - 2A^2)A^2t}{2(4 - A^2)(1 - A^2)} \quad , \quad P^*_S = \frac{(3 - 2A)V}{2(2 + A)} + \frac{3(2 - A^2)At}{2(4 - A^2)(1 - A^2)}
\]

It can be shown numerically that \( \frac{dP^*_L}{dA} > 0 \) for \( A > \hat{A} \), and \( \frac{dP^*_S}{dA} > 0 \) for \( A > 0 \).

To see why, let us consider the profits of first the local store:

\[
\Pi_{DL} = (P_L - c_L)[A\hat{\theta}_A + (1 - A)\hat{\theta}_{BL}]
\]

\[
\frac{d\Pi_{DL}}{dP_L} = (A\hat{\theta}_A + (1 - A)\hat{\theta}_{BL}) + (P_L - c_L) \left[ A \left( \frac{dP^*_S}{dP_L} - \frac{1}{t} \right) + (1 - A) \left( \frac{-1}{t} \right) \right]
\]

\[
= \underbrace{N_L}_{\text{Profit gain on existing customers}} + \underbrace{(P_L - c_L) \left[ A \left( \frac{dP^*_S}{dP_L} - \frac{1}{t} \right) \right]}_{\text{Profit lost of customers that switch to } D_S}
\]
The optimal price is chosen to set the gain in profits from increasing $P_L$ equal to the loss in profits. Let us now consider the behaviour of these gains and losses as $A$ changes. The gain can be represented by:

$$G_L = N_L = A\hat{\theta} + (1-A)\hat{\theta}_BL = \frac{1}{t}(AP_S + (1-A)V - P_L) \quad \Rightarrow \quad \frac{dG_L}{dA} = -(V - P_S) \frac{1}{t} < 0$$

If $A = 0$, the firms are symmetric and both serve half the market. As $A$ increases, the local store serves fewer and fewer customers and hence the gain from increasing prices decreases with $A$. The losses can be represented by:

$$|L_L| = (P_L - c_L) \left[ 1 - A\frac{dP_L}{dP_L} \right]$$

$$\Rightarrow \quad \frac{d|L_L|}{dA} = (P_L - c_L) \left[ -\frac{dP_L}{dP_L} \right] < 0$$

When $A$ is small, the downstream firms share fewer customers and there is lesser complementarity between the prices, thus given an increase in $P_L$, $P_S$ doesn’t increase by as much. Hence more customers switch to the supermarket and it is not optimal for $D_L$ to increase prices. Hence, $\frac{dP_S}{dA} < 0$ if $A$ is small. Conversely, when $A$ is big, there is more complementarity between the prices, and given an increase in $P_L$, $P_S$ increases by a lot more. Hence fewer customers switch to the supermarket and it is in fact optimal for $D_L$ to increase prices. Hence, $\frac{dP_S}{dA} > 0$ if $A$ is large. Consider now the profits of the supermarket:

$$\Pi_{DS} = (P_S - c_S)[A(1 - \hat{\theta}A) + (1 - A)\hat{\theta}_BS]$$

$$\frac{d\Pi_{DS}}{dP_S} = A(1 - \hat{\theta}A) + (1 - A)\hat{\theta}_BS + (P_S - c_S) \left[ A \left( \frac{dP_L}{dP_L} - \frac{1}{t} \right) + (1 - A) \left( \frac{-1}{t} \right) \right]$$

$$= \underbrace{N_S}_{\text{Profit gain on existing customers}} + \underbrace{(P_S - c_S) \left[ \frac{dP_L}{dP_L} - \frac{1}{t} \right]}_{\text{Profit lost of customers that switch to } D_L}$$
The optimal price is chosen to set the gain in profits from increasing $P_S$ equal to the loss in profits. Let us now consider the behaviour of these gains and losses as $A$ changes.

$$G_S = N_S = A(1 - \hat{\theta}_A) + (1 - A)\hat{\theta}_{BS} = \frac{1}{t}[(1 - A)V + At + AP_L - P_S]$$

$$\Rightarrow \quad \frac{dG_S}{dA} = \frac{P_L + t - V}{t} > 0$$

If $A = 0$, the firms are symmetric and both serve half the market. As $A$ increases, the supermarket serves more and more customers and hence the gain from increasing prices increases with $A$.

$$|L_S| = (P_S - c_S) \left[1 - A\frac{dP_L}{dP_S}\right] = (P_S - c_S) \left[-\frac{dP_L}{dP_S}\right] < 0$$

Again, as $A$ increases, $P_L$ increases, and fewer customers move to the local store for a given increase in $P_S$. Thus the loss of increasing $P_S$ is decreasing in $A$. Therefore, as $A$ increases, $D_S$ will find it optimal to increase $P_L$. Hence we must have $\frac{dP_S}{dA} > 0$ for all $A$.

**Proof of Proposition 3.2.** First suppose that $D_S$’s offer is rejected. $U$ can now make an offer to $D_L$. Given $c_L$, $D_L$ can choose whether it wants to serve all the customers or only a part of them. Both types of customers face transport costs going to the local store. Thus, if the market is only partially served then $D_L$ would serve all the consumers up to location $\theta_L$ where $V - \theta_L t - P_j = 0$. Hence a price of $P_L$ would secure a market share of $(V - P_L)/t$. Hence the profits would be $\Pi_{D_L}^U(c_L) = (P_L - c_L)(V - P_L)/t$ and the optimal price would be $P_L = \frac{1}{2} [V + c_L]$ yielding a profit of $\Pi_{D_L}^U(c_L) = \frac{1}{4t} (V - c_L)^2$. The upstream firm would make a profit of $\Pi_{D_L}^U(c_L) = \left(\frac{V - c_L}{2t}\right) c_L$. Serving only part of the market holds if the calculated market share of consumers $(V - P_L)/t < 1$ iff $P_L > V - t$ and this
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holds if \( c_L > V - 2t \). Otherwise, if \( c_L \leq V - 2t \) the local store would serve the whole market and price to make the consumer at location 1 indifferent. We would then have \( P_L = V - t \) and therefore \( \Pi_{DL}^X(c_L) = V - t - c_L \) and \( \Pi_{UL}^{XD}(c_L) = c_L \).

This gives us:

\[
\Pi_{UL}^{XD}(c_L) = \begin{cases} 
  c_L & c_L \leq V - 2t \\
  \frac{(1}{2t})(V - c_L)c_L & c_L > V - 2t 
\end{cases}
\]

\( \Pi_{UL}^{XD} \) is maximised at \( c_L = V - 2t \) as long as we assume \( V \geq 4t \). Hence if \( D_S \)'s offer is rejected then \( U \) will demand \( c_L = V - 2t \) and make a profit of \( V - 2t \).

If however, \( V < 4t \), then we have that \( \Pi_{UL}^{XD} \) is maximised at \( c_L = V/2 \), and \( U \) makes a profit of \( V^2/8t \). Essentially, we have:

\[
\Pi_{UL}^{XD*} = \begin{cases} 
  V - 2t & V \geq 4t \\
  \frac{V^2}{8t} & V < 4t 
\end{cases}
\]

We consider only the case where \( V < 4t \) as this is where the benchmark equilibrium is valid. Hence \( \Pi_{UL}^{XD*} = \frac{V^2}{8t} \). The rival retailer then just needs to make an offer of \( c_S \) such that

\[
\Pi_{UL}^{D_D}(c_L, c_S) \geq \Pi_{UL}^{XD*} \tag{3.9}
\]

Only offers of \( c_S \) ensuring the participation constraint of \( U \) is met will be accepted. This provides a constraint on the offers made by the supermarket.

Suppose instead that \( D_S \)'s offer of \( c_S \) is accepted. Then \( U \) sells through both \( D_S \) and \( D_L \) and makes profits as shown in (3.3). The optimal value of \( c_L \) given a \( c_S \) can then be found to be

\[
c_L \in \arg\max_{c_L} \Pi_U(c_S, c_L) = \frac{(2 + A)(1 - A)V + A^2t}{2(2 - A^2)} + \frac{A}{2 - A^2}c_S \tag{3.10}
\]
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It can be shown that we only have $c_L > c_S$ for low values of $c_S$.

$$c_L(c_S) > c_S \quad \text{if} \quad c_S < \hat{c}_S \quad \text{where} \quad \hat{c}_S = \frac{(2 + A)(1 - A)V + A^2 t}{2(2 + A)(1 - A)}$$

Of course this is true as long as the input prices are high enough to maintain partial cover. For too low values of $c_S$ it might be that the prices are given instead by (3.6).

\textit{Proof of Proposition 3.3.} Notice from above that $\frac{d \hat{c}_L}{dc_S} > 0$ and that $\hat{c}_L(c_S = c^*_S) = c^*_L$. Hence the reaction function of the upstream is upward sloping and passes through the discriminatory equilibrium. Consider now the profits of the supermarket:

$$\Pi_{DS} = (P_S - c_S) \left[ A \left( 1 - \frac{P_S - P_L}{t} \right) + (1 - A) \left( \frac{V - P_S}{t} \right) \right]$$

Substituting from (3.1) and (3.10) we get:

$$\Pi_{DS} = \frac{1}{2[2 - A^2)(4 - A^2)]^2 t} \left[ (2 + A)(1 - A)(-2A^2 + A + 4)V - 2(4 - A^2)(1 - A^2)c_S + (8 - 3A^2)At \right]^2$$

This is decreasing in $c_S$. Hence the supermarket will make the lowest offer acceptable. Given now that the supermarket will make an input price offer lower than $c^*_S$, and given the conditions above, all input price offers of $\hat{c}_L$ made by the supplier will lie below the benchmark value of $c^*_L$.

\textit{Proof of Proposition 3.4.} First note that both the input prices in the part market regime are higher than the input prices in the full market regime. $c^F_L = V - \frac{3(2 + A)}{4(1 + A)}t < c^p_L = \frac{V}{2} + \frac{A^2 t}{2(1 - A^2)}$ and $c^F_S = V - \frac{6 + A}{4(1 + A)}t < c^p_S = \frac{V}{2} + \frac{At}{2(1 - A^2)}$. From Lemma 3.1 it is clear that all $A$ customers will continue to be served in the part market regime. Hence the supplier’s profits from these customers must be higher in the part market regime. The profit from type $B$ customers might be
higher or lower because the supplier gets more rent per customer, but now fewer of them are being served. We need to assess if the potential decrease in profits from $B$ customers is outweighed by the increase in profit from $A$ customers. Below we will show that the upstream supplier increases profits from even the $B$ customers if it doesn’t serve everyone. This is true even for $A = 0$. For $A > 0$, it gains further profits from the $A$ customers.

Consider now the case if $A = 0$. If the supplier decides to serve everyone then $c_F^L = c_F^S = V - \frac{3t}{2}$. Both the retailers now serve half the market each and the upstream supplier makes a profit of

$$\Pi_U = \left(V - \frac{3t}{2}\right)\left(\frac{1}{2} + \frac{1}{2}\right) = V - \frac{3t}{2}$$

If it decides not to serve everyone then $c_P^L = c_P^S = \frac{V}{2}$. Both retailers serve equal number of customers and the upstream supplier makes a profit of

$$\Pi_U = \frac{V}{2} \left(\frac{V}{4t} + \frac{V}{4t}\right) = \frac{V^2}{4t}$$

Note that $\frac{V^2}{4t} > V - \frac{3t}{2}$ if $V^2 - 4Vt + 6t^2 > 0$ which is true for all $V, t > 0$. So when $A = 0$, $U$ is better off supplying only part of the market. Given that, when $A > 0$ $U$ will always be better off serving part of the market. \(\square\)

**Proof of Proposition 3.5.** The winner in stage 0 will be the retailer that has more to gain by moving first. Consider the maximum the local store would be willing to pay to win the right to be the first mover. It’s profits are given by $\Pi_{DL} = (P_L - c_L)(A\hat{\theta}_A + (1 - A)\hat{\theta}_B)$. Reiterating from the proof of Proposition 2.1 in Chapter 2, the retail prices are given by:

$$\begin{pmatrix} P_L \\ P_S \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2c_L + c_S \\ c_L + 2c_S \end{pmatrix} + \frac{1}{3(1 + A)} \begin{pmatrix} 3 - A \\ 3 + A \end{pmatrix} t$$  \hspace{1cm} (3.11)
Substituting for (3.11) into the local store’s profit we get:

$$\Pi_{DL} = \left(\frac{2c_L + c_S}{3} + \frac{(3 - A)t}{3(1 + A)} - c_L\right) \left[\frac{1 + A}{6t} \left(c_S - c_L + \frac{3 - A}{1 + A}t\right)\right]$$

(3.12)

If $D_L$ does not win in stage 0, it will have to take the price set by $U$ in stage 2. Again, reiterating from the proof of Proposition 2 in Chapter 2, the input price set to the local store following an acceptable offer of $c_S$ from the supermarket is given by:

$$c_L(c_S) = c_S + \frac{1}{2} \frac{3 - A}{1 + A}t > c_S$$

(3.13)

Substituting from (3.13) into (3.12) we can calculate a profit of $\Pi_{DL}^{2nd} = \frac{(3 - A)t}{72(1 + A)}$.

Now if $D_L$ wins in stage 0, it will be in a position to propose a price to $U$. $U$’s profits are given by

$$\Pi_U(c_S, c_L) = c_S + (c_L - c_S) \left[\frac{1}{3t} \left\{c_S - c_L + \frac{2tA}{1 + A}\right\} \left(\frac{1 + A}{2}\right) + \left(\frac{1 - A}{2}\right)\right]$$

$$= c_S + (c_L - c_S) \frac{1}{3t} \left[c_S - c_L\right] \left(\frac{1 + A}{2}\right) + tA + 3t \left(\frac{1 - A}{2}\right)\right]$$

$$= c_S + \frac{1}{2}(c_L - c_S) \left(\frac{1 + A}{3t}\right) \left\{c_S - c_L + \frac{3 - A}{1 + A}t\right\}$$

(3.14)

The optimum $c_S$ can then be found as a function of $c_L$ as:

$$c_S^*(c_L) = \operatorname{argmax}_{c_S} \frac{1}{2}(c_L - c_S) \left(\frac{1 + A}{3t}\right) \left\{c_S - c_L + \frac{3 - A}{3 + A}t\right\} + c_S = c_L + \frac{3 + A}{2(1 + A)}t$$

(3.15)

Substituting (3.15) into (3.12) we calculate the profits the local store makes if it moves first $\Pi_{DL}^{1st} = \frac{1 + A}{2t} \left[\frac{1}{3} \left(\frac{1}{2} \frac{3 + A}{1 + A}t + \frac{3 - A}{1 + A}t\right)\right]\right] = \frac{1}{2t} \left(\frac{1}{2} \frac{3 + A}{1 + A}t + \frac{3 - A}{1 + A}t\right) = \frac{(9 - A)t}{12(1 + A)}$ Hence the maximum that $D_L$ would be willing to bid for the right to be the first mover would be

$$B_{DL} = \Pi_{DL}^{1st} - \Pi_{DL}^{2nd} = \frac{(6 - A)t}{6(1 + A)}$$

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Similarly, consider the maximum the supermarket would be willing to pay to win the right to be the first mover. The supermarket’s profits are given by

\[ \Pi_{DS} = (P_S - c_S)(A(1 - \hat{\theta}_A) + (1 - A)(1 - \hat{\theta}_B)). \]

Substituting from prices in (3.11) we get:

\[ \Pi_{DS}(c_L, c_S) = \left( \frac{1 + A}{2t} \right) \left( \frac{1}{3} (c_L - c_S) + \frac{3 + A}{3(1 + A)} t \right)^2 \]

If \( DS \) wins in stage 0, it will propose a price to \( U \) in stage 1. From (3.13) and (3.16) we get:

\[ \Pi_{1st}^{DS} = 1 + A \left[ \frac{1}{3} \left( \frac{13 - A}{2(1 + A)} + \frac{3 + A}{1 + A} t \right) \right]^2 = \frac{(9 + A)^2 t}{72(1 + A)}. \]

If \( DS \) does not win in stage 0, it will have to take the price set by \( U \) in stage 2. From (3.15) and (3.16) we can calculate it’s profits as

\[ \Pi_{2nd}^{DS} = \frac{(3 + A)^2 t}{72(1 + A)}. \]

Hence the maximum that \( DS \) would be willing to bid for the right to be the first mover would be

\[ B_{DS} = \Pi_{1st}^{DS} - \Pi_{2nd}^{DS} = \frac{(6 + A)t}{6(1 + A)}. \]

It is immediate that \( B_{DS} > B_{DL} \) for all \( A > 0 \). \( \square \)

Proof. of Proposition 3.6: From earlier calculations, we can show that given a \( \{c_L, c_S\} \),

\[ \Pi_{DL} = \left( \frac{1 + A}{18t} \right) \left( c_S - c_L + \frac{3 - A}{1 + A} t \right)^2 - F_L \]

\[ \Pi_{DS} = \left( \frac{1 + A}{18t} \right) \left( c_L - c_S + \frac{3 + A}{1 + A} t \right)^2 - F_S \]

\[ \Pi_U = \left( \frac{1 + A}{6t} \right) (c_L - c_S) \left( c_S - c_L + \frac{3 - A}{1 + A} t \right) + c_S + F_L + F_S \]

Again, \( F_L, F_S \) are chosen to set \( \Pi_{DL} = \Pi_{DS} = 0 \), so we get:

\[ \Pi_U = \left( \frac{1 + A}{6t} \right) (c_L - c_S) \left( c_S - c_L + \frac{3 - A}{1 + A} t \right) + c_S 
+ \left( \frac{1 + A}{18t} \right) \left( c_S - c_L + \frac{3 - A}{1 + A} t \right)^2 
+ \left( \frac{1 + A}{18t} \right) \left( c_L - c_S + \frac{3 + A}{1 + A} t \right)^2 \]

(3.17)
Again, this is increasing in \( \{c_L, c_S\} \). Thus the optimal input prices would be the highest that can be sustained given the entire market is served. Thus \( U \) maximises \( \Pi_U \) subject to the constraint that \( P_L + t\theta_B \leq V \), which finally gives us the constraint:

\[
c_L \leq 2V - \frac{3 + A}{1 + A}t - c_S
\] (3.18)

Maximising (3.17) subject to (3.18) gives us

\[
\{c^*_L, c^*_S\} = \{V - \frac{6 + A}{4(1 + A)}t, \ V - \frac{6 + 3A}{4(1 + A)}t\}
\]

These are mirror images of the input prices set if the upstream is setting linear prices. Note that we now have \( c^*_L > c^*_S \) for all \( A > 0 \). We can also calculate

\[
\{F^*_L, F^*_S\} = \left\{ \frac{(6 - 3A)^2}{72(1 + A)}t, \ rac{(6 + 3A)^2}{72(1 + A)}t \right\}
\]

The levels of final profits can be calculated to be:

\[
\Pi^*_D L = \Pi^*_D S = 0, \quad \Pi^*_U = V - \frac{(9A^2 - 36A - 36)}{72(1 + A)}t
\]

\[\square\]

**Proof.** of Proposition 3.7: Consider the case where \( D_S \)’s offer is rejected. In this case the upstream is selling exclusively through \( D_L \). Assume it offers \( \{c_L, F_L\} \).

\[
\Pi_{DL} = (P_L - c_L)\theta - F_L = (P_L - c_L)\left(\frac{V - P_L}{t}\right) - F_L
\]

\[FOC \Rightarrow P^*_L = \frac{V + c_L}{2}, \quad \Pi^*_D L = \frac{(V - c_L)^2}{4t} - F_L, \quad \Pi^*_U^X D_L = \frac{c_L(V - c_L)}{2t} + F_L\]
Now, the upstream firm chooses $F^*_L$ such that $\Pi^*_D = 0$, hence

$$F^*_L = \frac{(V - c_L)^2}{4t}$$

$$c^*_L \in \arg \max_{c_L} \Pi^{XD}_U = \frac{c_L(V - c_L)}{2t} + \frac{(V - c_L)^2}{4t} = \frac{V^2 - c_L^2}{4t}$$

$$FOC \Rightarrow c^*_L = 0$$

Hence the upstream will set the linear price to zero to maximise the size of the pie and set the fixed component to extract all the surplus from the downstream. We have finally $\{c^*_L, F^*_L\} = \{0, \frac{V^2}{4t}\}$.

However, this holds only if $\theta = \frac{V - c_L}{t} < 1$, or under the condition that $c_L > V - t$. We know that $c^*_L = 0$ so the condition collapses to $V < 2t$. If on the other hand we had $V > 2t$ then we would have $\theta = 1$ and hence $P_L = V - t$. This finally gives us $\Pi^*_D = V - t - c_L - F_L$ and $\Pi^{XD}_U = c_L(1) + F_L = V - t$. Hence any $\{c^*_L, F^*_L\} = \{c^*_L, V - t - c_L\}$ will be an equilibrium. So we have

$$\{c^*_L, F^*_L\} = \begin{cases} 
\{0, V^2/4t\} & V < 2t \\
\{c^*_L, V - t - c_L\} & V \geq 2t 
\end{cases}$$

$$\Pi^{XD}_U = \begin{cases} 
V^2/4t & V < 2t \\
V - t & V \geq 2t 
\end{cases}$$

(3.19)

This represents the participation constraint of the upstream firm that the offer from $D_S$ must meet for it to be accepted.

If $D_S$’s offer is accepted

$$\Pi^{DL,DS}_U = c_S + \frac{1 + A}{2t}(c_L - c_S) \left( \frac{c_S - c_L}{3} + \frac{3 - A}{3(1 + A)} t \right) + F_S + F_L$$
\[ \Pi_{DL} = \frac{1 + A}{2t} \left( \frac{c_S - c_L}{3} + \frac{3 - A}{3(1 + A)} t \right)^2 - F_L = \frac{1 + A}{18t} \left( c_S - c_L + \frac{3 - A}{1 + A} t \right)^2 - F_L \]

Given \( \{c_S, F_S\} \), the upstream will choose

\[ F_L^* = \frac{1 + A}{18t} \left( c_S - c_L + \frac{3 - A}{1 + A} t \right)^2 \]

to set \( \Pi_{DL} = 0 \). This gives us:

\[ \Pi_{U,U-DS}^{DL,DS} = c_S + \frac{1 + A}{18t} (c_L - c_S) \left( c_S - c_L + \frac{3 - A}{1 + A} t \right) + F_S + \frac{1 + A}{18t} \left( c_S - c_L + \frac{3 - A}{1 + A} t \right)^2 \]

\[ c_L^* \in \arg\max_{c_L} \Pi_{U,U-DS}^{DL,DS} = c_S + \frac{3 - A}{4(1 + A)} t \quad (3.20) \]

Note that this implies \( c_S^* < c_L^* \). Using this we can also calculate

\[ F_L^* = \frac{(3 - A)^2}{32(1 + A)} t \]

Now \( D_S \)'s profit can be calculated to be:

\[ \Pi_{DS} (c_S, c_L^*) = \frac{1 + A}{2t} \left( \frac{c_L^* - c_S}{3} + \frac{3 + A}{3(1 + A)} t \right)^2 - F_S \]

\[ \Pi_{DS} \left( c_S, c_S + \frac{3 - A}{4(1 + A)} t \right) = \frac{(5 + A)^2}{32(1 + A)} t - F_S \]

This is independent of \( c_S \) and depends only on \( F_S \). Hence any \( c_S \) that ensures that the full market is served (i.e. satisfies (3.18)) will be an equilibrium. Moreover, \( D_S \) will choose \( \{c_S, F_S\} \) in such a way that \( F_S \) is as small as possible. Further,
we have

\[
\Pi_{U,D^L,D^S} = \frac{1 + A}{2t} (c_L - c_S) \left( \frac{c_S - c_L}{3} + \frac{3 - A}{3(1 + A)} t \right) + F_S + \frac{1 + A}{2t} \left( \frac{c_S - c_L}{3} + \frac{3 - A}{3(1 + A)} t \right)^2
\]

\[
= c_S + \frac{(3 - A)^2}{16(1 + A)} t + F_S
\]

From (3.19) \( D^S \) needs to pick \{\( c_S, F_S \)\} such that :

\[
c_S + \frac{(3 - A)^2}{16(1 + A)} t + F_S \geq \begin{cases} 
\frac{V^2}{4t} & \text{if } V < 2t \\
V - t & \text{if } V \geq 2t
\end{cases}
\]

\[
\Rightarrow F_S \geq \begin{cases} 
\frac{V^2}{4t} - \frac{(3 - A)^2}{16(1 + A)} t - c_S & \text{if } V < 2t \\
V - t - \frac{(3 - A)^2}{16(1 + A)} t - c_S & \text{if } V \geq 2t
\end{cases}
\]

(3.22)

This shows that to choose as small a fixed component as possible, \( D^S \) must choose as large a per unit price as possible. This is constrained by the full market being served. So from (3.18) we have:

\[
c^*_L + c^*_S \leq 2V - \frac{3 + A}{1 + A} t \quad \Rightarrow \quad c^*_S \leq V - \frac{3(5 + A)}{8(1 + A)} t
\]

(3.23)

Hence, \( D^S \) will choose

\[
c^*_S = V - \frac{3(5 + A)}{8(1 + A)} t
\]

\[
\Rightarrow F^*_S = \begin{cases} 
\frac{V^2}{4t} - V + \frac{(-A^2 + 12A + 21)}{16(1 + A)} t & \text{if } V < 2t \\
\frac{(-A^2 - 4A + 5)}{16(1 + A)} t & \text{if } V \geq 2t
\end{cases}
\]

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Proof. of Proposition 3.8: Note that in such a case, the local store, $D_L$ maximises $(P_L - \tilde{c})(A\tilde{\theta}_A + (1 - A)\tilde{\theta}_B)$ which yields the FOC: $(P_L - \tilde{c})(-(1 + A)) + (1 + A)P_S - (1 + A)P_L + (1 - A)t = 0$. The supermarket $D_S$ maximises $(P_S - \tilde{c})(A(1 - \tilde{\theta}_A) + (1 - A)(1 - \tilde{\theta}_B))$ which yields the FOC: $(-1)(P_S - \tilde{c}) + (t + P_L - P_S) = 0$. These FOCs can be combined into the following system:

$$\begin{pmatrix} 2(1 + A) & -(1 + A) \\ -1 & 2 \end{pmatrix} \begin{pmatrix} P_L \\ P_S \end{pmatrix} = \begin{pmatrix} (1 + A)\tilde{c} + (1 - A)t \\ \tilde{c} + t \end{pmatrix}$$

$$\begin{pmatrix} P_L \\ P_S \end{pmatrix} = \tilde{c} + \frac{t}{3(1 + A)} \begin{pmatrix} 3 - A \\ 3 + A \end{pmatrix}$$

(3.24)

The upstream firm then maximizes:

$$\Pi(U) = \tilde{c}[A\tilde{\theta}_A + (1 - A)\tilde{\theta}_B + A(1 - \tilde{\theta}_A) + (1 - A)(1 - \tilde{\theta}_B)] = \tilde{c}$$

This makes sense. If the upstream firm is supplying the whole market, then it is charging all the consumers the same input price $\tilde{c}$. Thus with a market size of 1, the profits made by the upstream are just equal to the input price it sets. Now, to find this highest value of $\tilde{c}$ that the upstream can set conditional on serving the full market, it again needs to ensure that the price faced by the indifferent
type $B$ consumer is equal to the outside option. In particular, set $P^*_{L} = V$:

$$V = P_L + t \cdot \hat{\theta}_B = P_L + t \left[ \frac{1}{2} + \frac{1}{2t} (P_S - P_L) \right]$$

$$\Rightarrow \quad V = \frac{1}{2} [P_S + P_L + t]$$

$$V = \hat{c} + \frac{1}{2} \left[ \frac{2t}{1 + A} + t \right] \quad \text{Using (3.24)}$$

or $$\hat{c}^* = V - \frac{3 + A}{2(1 + A)} t$$ \hspace{1cm} (3.25)

Comparing with previous results, we can see that $c^*_L < \hat{c}^* < c^*_S$. In fact, due to the linear demand curves and no ‘output effect’, $\hat{c}^* = \frac{1}{2} [c^*_L + c^*_L]$.

Substituting from (3.25) above, we get:

$$\Pi(U) = V - \frac{3 + A}{2(1 + A)} t$$

Using (3.24) we can solve to get the final good prices:

$$\hat{P}^*_L = V - \frac{(3 + 5A)}{6(1 + A)} t, \quad \hat{P}^*_S = V - \frac{(3 + A)}{6(1 + A)} t$$ \hspace{1cm} (3.26)

Note that $\hat{P}^*_L > P^*_L$ and $\hat{P}^*_S < P^*_S$ for all $A > 0$.

We have shown that if the upstream firm was not allowed to discriminate, it would set an input price lower than $c_S$ but higher than $c_L$. This change would reduce the proportion of Type $B$ customers served by the local store. Let this new proportion, $\hat{\theta}_B = \hat{\theta}_B - \kappa$, so that $\delta \hat{\theta}_B = -\kappa$. The price offered by the local store changes from $P_L = V - \hat{\theta}_B$ to $\hat{P}_L = V - \hat{\theta}_B + t\kappa$, so we have that $\delta P_L = t\kappa$.

Similarly, $\delta P_S = -t\kappa$, and $\delta \hat{\theta}_A = -2\kappa$.

We can show that If the upstream firm cannot discriminate, the welfare of Type $B$ customers falls. Consider the welfare of Type $B$ customers: We have that
3.7. APPENDIX

\[ CS_B = \frac{1}{2}(V - P_L)\hat{\theta}_B + \frac{1}{2}(V - P_S)(1 - \hat{\theta}_B) \]. The change in consumer surplus can be measured as:

\[ \delta CS_B = \frac{1}{2} \kappa (P_L - P_S) + t\kappa \left( \frac{1}{2} - \hat{\theta}_B \right) = -\kappa t \hat{\theta}_A < 0 \]

Where we have substituted for \( \hat{\theta}_B = \frac{\hat{\theta}_A}{2} + \frac{1}{2} \). So, the welfare of Type B customers always decreases when the upstream cannot discriminate. Note that if \( A = 0 \), or when there are only Type B customers, the optimal strategy of the upstream is to set \( c_L = c_S \), which leads to \( P_L = P_S \) and consequently there is no change in the equilibrium and in the consumer surplus when the upstream is not allowed to discriminate.

We can also show that if the upstream firm cannot discriminate, the welfare of Type A customers increases. Consider the welfare of Type A customers:

We have that \( CS_A = (V - P_S) + \frac{1}{2}(P_S - P_L)\hat{\theta}_A \). The change in consumer surplus can be measured as:

\[ \delta CS_A = \kappa (t - t\hat{\theta}_A - (P_S - P_L)) = \kappa t (1 - 2\hat{\theta}_A) > 0 \]

Which is positive as \( \hat{\theta}_A < \frac{1}{2} \) from the lemma before. So, the welfare of Type A customers always increases when the upstream cannot discriminate.

Now looking at the total consumer surplus, we have that \( CS_T = ACS_A + (1 - A)CS_B \). The change in consumer surplus can be measured as:

\[ \delta CS_T = -A \cdot \kappa t (1 - 2\hat{\theta}_A) - (1 - A) \cdot \kappa t \hat{\theta}_A \]

\[ = \kappa t [A(1 - \hat{\theta}_A) - \hat{\theta}_A] \]
Which is positive if $A > \frac{\theta_A}{1-\theta_A}$. Substituting for $\hat{\theta}_A = \frac{5A}{6(1+4)}$, we require:

$$A > \frac{5A}{6 + A} \quad \Rightarrow \quad A^2 + A > 0$$

Which is always true. Hence it must always be the case that consumers’ welfare increases when the upstream is not allowed to discriminate. \qed
Chapter 4

Waterbed effect and welfare

Abstract: This chapter studies the welfare implications of an increase in retailer size. We modify the framework of Inderst and Valletti (2011) to allow for volume effects via quantity competition. Retailers that differ in size have the option of either buying from the given supplier at the price it sets, or rejecting that offer and incurring a fixed cost to search for a rival supplier who will charge them lesser. We show that similar to their model, for small increases in size there is a waterbed effect since a discount to the larger retailer makes the smaller retailer less likely to reject the supplier’s offer. This allows the supplier to extract profits from the smaller retailer by increasing its price. For large increases in size however, the supplier prefers to transfer business away from the larger retailer and there is no waterbed effect. Crucially, quantity competition and the resultant strategic substitutability ensure that the incentives to transfer business are higher. Consequently, an increase in retailer size is always welfare improving because while the smaller retailer is punished, it is not punished disproportionately so.
4.1 Introduction

Is the presence of large, powerful retailers good for customers?

The main argument in favour of this idea comes from Galbraith (1952) who argues that large retailers act as customers’ champions because they possess countervailing power. These large retailers can negotiate favourable terms of supply from their upstream suppliers and then pass on some of these cost savings to the customers. Galbraith’s argument however breaks down when retailers negotiate discounts on nonlinear tariffs since there are no incentives for these to be passed on. Another proponent is Chen (2003) who shows that even if contracts are nonlinear, an improvement in terms of trade for the larger retailer makes an improvement in terms of trade for the smaller retailer less costly. Subsequently better terms of trade for both retailers ensure that customer welfare improves.

The ‘waterbed effect’ warns against this line of reasoning. It argues that while an increase in size creates favourable terms of supply for the large retailers, it does so at the cost of the smaller rivals. Hence a discount offered to large buyers imposes a negative externality on their rivals, leading to worsening of terms of trade for them. Therefore while larger retailers pass on their cost savings to the final customers, smaller retailers pass on their cost increases. It is then not obvious that welfare will increase. This has attracted considerable amount of attention recently form academics as well as policy makers.\footnote{In 2008 the Competition Commission (CC) conducted an enquiry into the U.K. groceries market and analysed the possibility of a waterbed effect. The CC also acknowledged the effect in Safeway/Asda (2003), and this concept was discussed in the OECD round table on Buyer power (2008), and by the German Antitrust Authority (2008). The term has also been used to address rebalancing issues in tariffs in the telecommunications industry. See Genakos and Valletti (2011).}

The main formalisation of this idea comes from Inderst and Valletti (2011).
Their model shows why a reduction in one retailer’s price, makes feasible, an increase in the rival retailer’s price. There is a monopolist supplier selling through two retailers in several independent markets. The retailers can choose to buy the good from the supplier at the ‘take-it-or-leave-it’ input price it charges, or they can reject that offer and incur a fixed cost to search for another rival supplier who sells it to them at the cost of production. The supplier’s pricing decision is then constrained by this outside option. If retailers are symmetric (i.e. each retailer is active only in one market) then the supplier charges them the same input price. However, if one retailer becomes bigger by purchasing retail outlets in the other markets then it can spread the fixed cost of going elsewhere over a larger number of units. It then, everything else being equal, is more likely to exercise its outside option. Hence the supplier needs to provide it a larger amount of surplus to it in the form of a lower input price. This shows why the larger retailer is charged a lower price.

More importantly, a decrease in the larger retailer’s price decreases the smaller retailer’s profits both if it continues to buy from the supplier and if it exercises its outside option. However, the profits from exercising the outside option decrease by more. Hence, everything else being equal, the smaller rival is less likely to reject the supplier’s offer. This means that it is now feasible for the supplier to increase the input price offered to the smaller rival without losing its business. The supplier can take away some surplus from the smaller retailer in the form of a higher input price. It is additionally optimal\footnote{Inderst and Valletti (2011) restrict the parameter space such that the supplier finds it optimal to let both participation constraints bind. We will argue later that for other parameter values, the supplier prefers to keep the small store’s participation constraint slack.} for the supplier because it’s profits accruing from the retailers are increasing in the input price they pay. The increase in size of a retailer and the subsequent discount it achieves, hence
imposes a negative externality on its rivals who are now forced to pay more.\textsuperscript{3}

Policymakers are interested in this phenomenon because of its welfare implications. If the waterbed effect exists then while the larger store will reduce its retail price, the smaller store might increase its retail price. If the average price increases, this results in short run welfare loss for the customers.\textsuperscript{4} Inderst and Valletti (2011) show that with a fully covered market, the waterbed effect can lead to an increase in average retail prices if the fixed cost of going to the rival supplier is high or if the retailers are already very asymmetric.

There are however some concerns with their model. The Competition Commission of the UK conducted an enquiry into the supply of groceries. In its analysis of the waterbed effect it questioned the assumption of a fully covered market. The worry was that full cover might overstate the waterbed effect results. If we instead assumed only partial cover, an increase in the large retailer’s market share would not be entirely at the cost of the smaller retailer because new customers start buying the good. The supplier then has less of a reason to increase prices to smaller retailers. Consequently, customer detriment should also be lower. The Commission in it’s grocery enquiry stated:\textsuperscript{5}

\textit{In effect, the model assumes that the market is of a fixed size and that, as a result, one retailer’s gain is another retailer’s loss. By ignoring the possibility that the total market size might expand in response to}

\textsuperscript{3}There is also an informal long run argument for the presence of waterbed effects which is of the following form: if suppliers keep giving better deals to large retailers then supplier margin is squeezed. This will lead to exit and consolidation of the upstream industry. As a result all retailers are charged a higher price. If this increase is not disproportionate then it could be the case that large retailers are charged lower prices and small retailers are charged higher prices. This paper will focus however on the possibility of welfare effects in the short run only.

\textsuperscript{4}Additionally, even if the average price doesn’t increase, while there might be benefit for customers in the short run, in the long run we would see exit and monopolisation of the retail industry which is bad for the customers. Moreover, the waterbed effect will cause an inefficient distribution of customers, promote too little variety when retailers substitute variety for quantity and reduce incentives to invest by larger retailers since they already face very favourable terms of trade. For a detailed discussion, see Dobson and Inderst (2008).

\textsuperscript{5}See Competition-Commission (2008) Para 5.30
lower retail prices (a market expansion effect), the ACS model overemphasises the loss of customers by convenience stores, and their consequent loss of bargaining power with suppliers. Further, by omitting a market expansion effect, the model overstates the consumer detriment arising from the waterbed effect by not taking into account the new, additional customers at larger grocery stores that benefit from the lower prices at these stores.

We therefore propose a model to incorporate this very idea. We extend the Inderst and Valletti (2011) framework to a Cournot setting so we can establish the impact of quantity effects. Additionally, a quantity setting model might be more appropriate for the retail industry because while it is more intuitive to think of retailers as competing in prices, their choices are heavily limited by capacity and time constraints. In the face of these constraints, a price setting model is akin to a quantity setting one.\(^6\) Additionally, there is informal evidence that supermarkets do indeed negotiate only quantity and quality with farmers at the time of negotiations but the price is not decided until the produce reaches distribution centres.\(^7\)

Our first contribution is to show that forces similar to those highlighted in Chapters 2 and 3 also arise in this set up. An increase in size does ensure the larger retailer pays a lower price than before, but it again creates two competing forces for the supplier. Following a reduction in the larger retailer’s price, the smaller retailer is less likely to exit. This means the supplier now has an opportunity to extract profits from the smaller retailer that was not available before. However since the larger retailer is now paying the supplier a lower price, the supplier also wants to transfer some business away form it towards the smaller retailer. The first effect dominates if the increase in size is small (which is the parameter space considered by Inderst and Valletti (2011)). Here we have a waterbed effect. If however there is a large increase in size then the supplier would

\(^6\)This is following the argument by Kreps and Scheinkman (1983).
\(^7\)See for example the South African export fruit sector - http://www.eldis.org/id21ext/s7csb1g1.html
rather reduce the price offered to the smaller retailer. In this case, the larger retailer imposes a positive externality on its rival.

On the question of welfare, our Cournot model implies that following a reduction in own input price, the large retailer produces a greater quantity. The waterbed effect and the associated increase in input prices to the small retailer will lead it to produce a smaller quantity. The research question now is - are there any conditions under which the total quantity produced by the market falls? This will mean that there is a clear reduction in welfare even in the short run. Our final contribution is to show that this is never the case. This stems from the observation that the incentives to transfer business are stronger under quantity setting than under price setting. Strategic substitutability implies that an increase in a rival's competitiveness is more damaging to own market share under Cournot competition. If firms were setting prices, strategic complementarity would ensure that the smaller retailer's market share did not fall by as much. This means that the 'cost' of transferring business - as measured by the loss in profits felt on the infra marginal customers - is also lower under Cournot competition. Therefore, the incentives to transfer business override the incentives to extract profits more easily in our model. Hence while there might be a waterbed effect, it is never damaging enough to be welfare reducing. The main contribution of this paper is thus to highlight that in markets where quantities need to be decided in advance, an increase in retailer size is not a cause for concern.

The plan of the paper is as follows: Section 4.2 presents the model, Section 4.3 analyses the impact of buyer power on input prices, Section 4.4 analyses the impact of differential buyer power on input prices, and Section 4.5 presents the welfare analysis. Section 4.6 concludes and proofs are contained in the Appendix. Finally, Section 4.7 provides a cross chapter comparison of the various models.
4.2 The Model

There is a monopolist upstream supplier $U$, selling its good in $N$ symmetric markets via downstream retailers. Each market has two downstream firms $D_i$ and $D_j$. The supplier makes ‘take-it-or-leave-it’ input price offers of $\{w_i, w_j\}$ to these firms in each market. We normalise all marginal costs of production to zero\(^8\) and hence $\{w_i, w_j\}$ represent the composite marginal costs for each of these downstream firms. If the firms do not accept the supplier’s offer, they can source the input at zero cost from a rival supplier but they will incur a fixed cost $F^9$ to do so. This represents their outside option. $D_i, D_j$ then compete in quantities in the downstream market. Suppose the demand curve is linear and given by

$$P = A - \frac{1}{B}Q$$

where $Q = q_i + q_j$, and $A$ determines customers’ valuation and $B$ determines the size of the market.

4.3 Symmetric Case

We first analyse the impact of symmetric buyer power. Chapters 2 and 3 have shown that if we don’t account for buyer power, the larger firm is charged a higher price. However, Inderst and Valletti (2009); Katz (1987) have shown that when we do, the larger firm is charged a lower price. We analyse this in our model by assuming first that each firm is active only in one market, so the firms are symmetric in size. The firms will buy from the supplier only if they make at least as much profit as they do compared to rejecting the supplier’s offer and

---

\(^8\)Here we deviate from Inderst and Valletti (2011) since we assume that the supplier’s marginal cost of production is the same as the retailers’. However, we maintain that this does not alter the direction of our results.

\(^9\)This fixed cost can also be thought of the cost to the retailers of producing the good themselves. These would take the form of private label products.
buying from the rival supplier. Their participation constraints can then be given by:

\[
PC_i : \Pi_i(w_i, w_j) \geq \Pi_i(0, w_j) - F
\]
\[
PC_j : \Pi_j(w_i, w_j) \geq \Pi_j(w_i, 0) - F
\]

where \( \Pi_i(w_i, w_j) \) represents the profit if \( D_i \) sources from \( U \) and \( \Pi_i(0, w_j) - F \) represents the profit if it sources from a rival supplier. Note first that if the fixed cost of going to the rival supplier is too high, then these participation constraints are not binding, and there is really no ‘buyer power’. Therefore we assume that \( F \) is low enough such that the supplier is constrained. Then we have the following result.

**Proposition 4.1.** Consider the benchmark case where both firms are symmetric. Then for low costs of going to the rival supplier, there exists a unique equilibrium where the supplier offers each firm the same input price \( w_1 \), which is increasing in the cost of going to the rival supplier \( F \), and decreasing in the size of the market \( B \), and in the valuation of the customers \( A \).

Since the firms are symmetric, there is no reason for the supplier to charge them different prices. Over the range where \( F \) is small enough that the outside option is binding, the supplier’s profits are increasing in the input price it offers. Therefore the supplier will charge the highest input price it can. This input price is also increasing in the cost of going to the rival supplier because the lower this cost is, the more willing the downstream firms are to exit. To make them participate the supplier has to make some surplus available to them in the form of a lower input price. Similarly an increase in the size and the valuation of the market has the effect of reducing the effective cost of going to the rival supplier because the cost can be spread over more units or be recovered by a high retail
price. Hence as these increase, the effective cost of going to the rival supplier decreases and the firms are offered a lower input price.

4.4 Asymmetric Case

Having analysed the impact of symmetric buyer power, we now want to address the issue of differential buyer power. Suppose that a downstream firm, via acquisitions is now active in $n > 1$ markets. The participation constraint of the smaller retailer (now labelled as $j = S$) is unchanged but the participation constraint of the larger retailer (now labelled as $i = L$)\textsuperscript{10} becomes harder to satisfy since it is able to spread the fixed cost of going to the rival supplier over a larger number of units. These constraints can be given by:

\[
\begin{align*}
PC_L & : \Pi_L(w_L, w_S) \geq \Pi_L(0, w_S) - \frac{F}{n} \\
PC_S & : \Pi_S(w_L, w_S) \geq \Pi_S(w_L, 0) - F
\end{align*}
\]

and we have the result:

\textbf{Proposition 4.2.} If the retailers are not too different in size and if the fixed cost of going to the rival supplier $F$ is small, the larger retailer’s input price, $w_L$, is strictly smaller than the (benchmark) input price in case of symmetric retailers, $w_I$, while the input price of competing smaller firms, $w_S$, is strictly larger than $w_I$.

The larger retailer is able to obtain a lower input price because an increase in size makes its outside option more attractive. Therefore to ensure it’s participation, the supplier needs to provide it a higher surplus in the form of a lower

\textsuperscript{10}Note that the indicators \{L,S\} in Chapter 2 and 3 have referred to the local store and the supermarket. Here however they refer to the larger and smaller retailers. This perhaps confusing notation is used to make our results directly comparable with Inderst and Valletti (2011).
input price.

This also imposes an externality on the participation decision of the smaller rival. Given that the large retailer is paying a lower input price, the smaller retailer’s profits reduce, both from staying in the game and from exercising its outside option. However, the profits from exercising its outside option reduce more. The intuition for this is the following - a decrease in the rival’s price means that the rival’s quantity will increase. Due to strategic substitutability, own quantity will decrease. Since own quantity will not decrease one for one, this means that the final price will be lower. Now in the case where the smaller rival chooses to exercise its outside option, it feels this reduction in price on a larger number of units because it is more competitive.\footnote{Note additionally that the smaller rival suffers a greater reduction in profits from the marginal customers too. In the case where it exercises its outside option, its mark-up is higher. Hence it loses a larger amount for the same per unit quantity reduction.} This makes the smaller retailer less likely to reject the supplier’s offer and its participation constraint becomes slack. This means there is now a new opportunity for the supplier to extract profits from the smaller retailer without losing its business. This gives us the waterbed effect.

Having established the potential presence of the waterbed effect in our model, we would like to do some comparative statics. We are interested in how the size of this waterbed effect changes with the parameters of the model.

\textbf{Corollary 4.1.} \textit{The size of the waterbed effect is increasing in the cost of going to the rival supplier $F$, and decreasing in the size of the market $B$ and valuation of the customers, $A$.}

A larger fixed cost of going to the rival supplier and a smaller size of the market both mean that the effective per unit cost a retailer faces to source from a rival supplier is higher. This means that an increase in size is more valuable for the larger retailer. This is reflected in greater asymmetry between the offered...
input prices. Additionally, similar to the full cover model, the convexity of the participation constraints implies that the waterbed effect is larger if the firms are already asymmetric.

Note that similar to Chapters 2 and 3, the supplier still has some incentives to transfer business from the larger retailer to the smaller one. However, this strategy is too costly if the increase in size is small. A small increase in size means the larger retailer can negotiate only a small discount and hence the smaller retailer has a significant market share. This means the supplier faces the loss in profits over a large number of infra marginal customers. Therefore the incentives to extract profits are higher than those of transferring business and we see the waterbed effect. However a large increase in size means the larger retailer can negotiate a sizeable discount and the smaller retailer has only a small market share. Then the incentives to transfer business away from the larger store are higher since the supplier faces the loss in profits over fewer customers. If this effect is stronger than the first effect then an increase in retailer size is in fact beneficial for its rivals.

Consider figure 4.1 for the graphical representation in \( \{w_L, w_S\} \) space. The participation constraints are both downward sloping. This represents that given a decrease in a rival’s costs, own profits decrease - both in the game and outside the game. However the profits from going elsewhere decrease more. The isoprofit curves of the upstream are ellipses around the bliss point of \( \left\{ \frac{A}{2}, \frac{A}{2} \right\} \). If the firms are symmetric, the equilibrium is given by the point \( O' \) where both participation constraints bind and each firm pays the same input price \( w_I \). When one firm becomes a little bigger due to acquisitions, its participation constraint shifts to the left. The larger retailer is able to negotiate a lower price. Given that \( PC_S \) is downward sloping, this necessarily means that the input price offered to the
small retailer increases. This gives us the waterbed effect as shown at $O^s_{\text{small}}$. This is indeed the correct equilibrium as long as the upstream has no incentive to deviate - which is when the isoprofit curve is steeper than $PC_L$ at $O^s_{\text{small}}$.

\[
PCL(n=1) \quad PCS
\]

Figure 4.1: Waterbed effect in a Cournot model

Suppose the increase in size is substantial. This will shift $PC_L$ considerably to the left. If the supplier decides to, as before, make both downstream firms just participate, then this will lead to a very small $w_L$ and a very large $w_S$ at $O^s_{\text{large}}$. This leads to weak competition in the downstream market and is not in the supplier’s interest. Now the upstream’s incentives to transfer business are also higher because the smaller retailer has only a small market share. The supplier then gives the smaller retailer more than the bare minimum profit and chooses the input price combination such that its isoprofit curve is tangent to the large store’s participation constraint. This way, it leaves $PC_S$ slack while letting $PC_L$ bind. The reaction function of the supplier is highlighted in bold. For small in-
creases in size it is just represented by $PC_S$ but for large increases it is the locus of points where $PC_L$ is tangent to the isoprofit curve. If the transferring business effect is strong enough then we would see the opposite of a waterbed effect at $O_{NWB}^n$.

Note that this potential lack of the waterbed effect if retailers are very different in size is not because we have a Cournot model. This effect can be seen in Inderst and Valletti (2011)’s Hotelling model too if we extend the parameter space considered. If there is too uneven a distribution of input prices downstream then customers will end up travelling a lot to go to the cheaper retailer. This means that a lot of the customer valuation, which could be the supplier’s surplus is wasted in travelling. Hence the supplier still wants to transfer business to the smaller retailer. However, the difference between the models is that the supplier is willing to tolerate a much wider variation in competitiveness under Hotelling than under Cournot. This can be understood by assessing the cost of transferring business in the two models. The strategy of transferring business is costly because the supplier faces a loss in profits on the infra marginal customers at the smaller retailer. The market share of the smaller retailer then inversely corresponds to the incentives for transferring business. Under Hotelling, a decrease in a retailer’s input price reduces its retail price. Due to strategic complementarity it also decreases the rival’s retail price. Therefore the market share of the rival doesn’t suffer as much. Under Cournot, strategic substitutability ensures that the rival’s market share suffers a lot. Therefore given the same reduction in input price, the rival has fewer infra marginal customers in the Cournot model. Hence the costs of transferring business to it are also lower.

Note additionally that while the parameters highlighted in Corollary 4.1 increase the size of the waterbed effect, they also decrease the probability that it
exists. A high fixed cost of going to a rival supplier might mean that the supplier could increase the smaller retailer’s price by a lot, but it also means that it is less likely to do so since its incentives to transfer business take over. Similarly, if the firms are already asymmetric, there is more scope for the supplier to punish the smaller retailer but it is less likely to do so since it already faces some incentives to transfer business. Therefore care needs to be taken when we make predictions for the industry. This is not relevant, of course if we are only considering marginal increases in size.\(^{12}\)

### 4.5 Welfare

We have shown that if a retailer increases in size it is able to negotiate a better price, but it potentially imposes a negative externality on its rival who is forced to pay a higher price. The waterbed effect is *per se* only a problem when it is welfare decreasing. We would now like to examine the conditions under which this might be the case. The larger retailer faces a lower marginal cost and will increase the quantity it supplies. The smaller retailer faces a higher marginal cost and will decrease the quantity it supplies. Welfare of customers is then lower if total quantity supplied in the market falls. We have the result:

**Proposition 4.3.** *The increase in size of a retailer is always welfare improving despite the presence of a waterbed effect.*

Given the linearity of the model, the waterbed effect is welfare reducing only when it is disproportionate - i.e. the increase in input price faced by the smaller retailer is larger than the decrease in price faced by the larger retailer. A necessary condition for welfare reduction then is that the slope of the smaller retailer’s

\(^{12}\)Note that if the retailers were already very asymmetric then even marginal increases in size need not result in a waterbed effect.
participation constraint be bigger than unity.

In such a case, the supplier will prefer to decrease the price to the smaller retailer to transfer sales towards it. The insight from this is that it is not in the supplier’s interest to allow the input price to the smaller retailer to become so high that total quantity in the market decreases. An increase in size then is always welfare improving - the supplier will decrease the price offered to the larger retailer and increase the price offered to the smaller retailer, but lesser than one for one. Consequently welfare will always increase.\textsuperscript{13}

This is not the case in the full cover Hotelling model. Suppose following a decrease in the large retailer’s price, the supplier increases the small retailer’s price disproportionately. Then while average retail prices are higher, total quantity served is unchanged (due to full cover assumption). Thus all the customers who were buying before are still buying, but paying higher prices for it. The supplier gains profits on the infra marginal customers of the smaller retailer but loses profits on those customers that switch to the larger retailer. While this might be profit increasing for the supplier under full cover, it is never so under partial cover. A disproportionate increase in the smaller retailer’s price here will increase the retail price only by reducing total quantity served. The supplier now still gains profits on the infra marginal customers but loses a lot more on the marginal customers since they stop buying.

\textsuperscript{13}This is not however a very general result because we consider only the linear case. With a linear demand, welfare reduction requires the waterbed effect to be disproportionate. With constant elasticity demand curves, we could probably come up with weaker conditions and it might be the case that welfare decreases.
4.6 Conclusions

This chapter has modified the full cover Hotelling model of Inderst and Valletti (2011) to allow for volume effects via downstream Cournot competition. A single supplier sells its output in several markets via retailers who can either accept the input prices offered by the supplier or can incur a fixed cost to find another supplier who will sell it to them at zero cost.\footnote{There are certain downsides of using this approach - an increase in size always increases a retailer’s outside option. Hence, in some sense we are assuming ‘unlimited’ economies of scale. Recent empirical evidence suggests however that this relationship between size and discount is nonlinear at best. See Competition-Commission (2008) Appendix 5.4 para 23.} In this way the supplier is a constrained monopolist.

We have shown that similar to their model, if the firms are symmetric, they are charged equal input prices which are increasing in the cost of going to a rival supplier. They are also strictly decreasing in the size of the market and valuation of the customers. This is because a higher fixed cost and a smaller size and value of the market all mean that the effective per unit cost of searching for a rival supplier are higher. The firms are less likely to exit and the supplier can extract that in the form of a higher input price.

We have also shown, again similar to their model, that if a retailer becomes larger via acquisitions in different markets then it is able to spread these fixed cost over more units, thereby reducing the effective per unit cost. It will therefore need to be given a higher surplus in the form of a lower per unit price to ensure its participation. As a result of the larger retailer being offered a lower price, the smaller retailer’s profits reduce, both from staying in the game and from exiting. However since it has a higher market share if it exits, the profits from exiting reduce more. This makes it less likely to reject the supplier’s offer. The supplier now has a new opportunity to extract the smaller retailer’s profits and will do so...
by charging it a higher price. This creates a waterbed effect.

However, if a retailer increases a lot in size, then the smaller rival will have to be charged a very high price in order to extract all the surplus. Consequently, the downstream firms will sell very different quantities. This is not in the supplier’s interest and it will want to transfer sales from the larger retailer to the smaller retailer where it gets a higher markup. This is achieved by reducing the smaller retailer’s price and giving it more surplus than necessary to create a more even competition downstream. Therefore, small increases in size of a large retailer are detrimental for its smaller rivals but large increases in size are beneficial for them.

Finally, we show that under quantity competition, an increase in a retailer’s size is always welfare improving. This is because the supplier’s incentives to transfer business are higher. The reason for that is strategic substitutability associated with quantity competition increases the damage done due to an improvement in a rival’s competitiveness to own market shares. It therefore reduces the costs associated with the transferring business and the supplier is less likely to accept large differences in input prices. Therefore while the increase in size of one retailer still imposes a negative externality on the smaller retailer, the size of this externality in the Cournot model is always small enough that customer welfare actually increases.\(^{15}\)

### 4.7 Comparing across chapters

We have presented three models for the presence of waterbed effects. There are two primary differences between the models - the source of buyer power and

\(^{15}\)Note that the larger retailer in our model faces competition only from the smaller retailer, not from other large retailers. In the presence of those it is likely that welfare improvement would be an even stronger case. The presence of buyer groups who supply to smaller retailers would also strengthen this case because they can take advantage of large volumes.
the extent of market cover. Chapters 2 and 3 determined variety present at the supermarket as the source of its buyer power. Variety gave it a potentially captive demand which gave it buyer power because it increased its probability of making preemptive offers to the supplier. In the current chapter we have used the standard notion of size determining buyer power. A larger size increases the value of the outside option since the fixed cost of finding a rival supplier can be spread over more units. This means, to ensure a larger retailer’s participation, the supplier has to give it a higher surplus in the form of a lower input price.

Figure 4.2 shows a comparison between the different models. The mechanism at play is similar. Following a discount to the local store the supplier faces two opposite incentives. In chapters 2 and 4 and in Inderst & Valletti’s model, the supplier is constrained in the benchmark equilibrium. In chapter 2 it is constrained by the full cover assumption and in the other two it is constrained by the retailers’ outside options. A discount to the larger retailer weakens this constraint. In chapter 2 this lowers average retail prices and in the other two it makes the rival retailer less likely to source from another supplier. Both imply that the supplier now has an opportunity to increase the smaller retailer’s price that was not available before. This allows it to extract profits from the smaller retailer. In this way, a discount to one retailer is harmful for its rivals.

However, the supplier will find it optimal to do so only as long as the level of input prices paid by the two downstream retailers is not too different. When a retailer increases a lot in size (in chapter 4) or when supplier is not a primary brand so the supermarket negotiates a large discount (in chapter 2), competition in the downstream industry is too asymmetric. The supplier will then prefer to instead shift sales to the smaller retailer that pays it a higher input price. The only means by which it can do so is by reducing its price. Therefore if the ad-
4.7. COMPARING ACROSS CHAPTERS

Chapter 2: Hotelling

Chapter 3: Hotelling

IV(2011): Hotelling

Chapter 4: Cournot

Figure 4.2: Comparing across models
vantage possessed by the large retailer is substantial, we expect that there would in fact be no waterbed effect. This is always the case in chapter 3 because the supplier is unconstrained in the benchmark equilibrium. A discount to the supermarket then does not create any additional avenues to extract profits from the local store. The only force at play is the supplier’s desire to transfer sales from the supermarket to the local store. Therefore we see the opposite of a waterbed effect.

Moreover, we have shown that even if the increase in size of a retailer is detrimental to its rivals, it is never detrimental to the customers if quantities need to be set in advance. The supplier’s incentives to transfer sales to the smaller retailer are stronger under quantity setting since it feels the associated loss in profits on fewer units. Therefore while it increases the smaller rival’s price, it never increases it disproportionately. Given the linearity of the model, this always implies a welfare increase.\footnote{The grey dotted line in Figure 4.2 shows the locus of points where the slope of the participation constraint of the smaller retailer is unity. So at all input price combinations above this line, we have a reduction in welfare but at all points below, welfare increases. As can be seen, under the Hotelling model a welfare reduction can be allowed by the supplier but under Cournot it will never be.}

There are additionally, certain implications regarding the type of goods and market characteristics. For goods that are sold in local stores, but for which waiting is cheap, we might not expect the waterbed effect to arise. For goods that are too expensive to wait for, we do expect the waterbed effect to arise. Similarly, small increases in a retailer’s advantage (due to fewer acquisitions or as a consequence of dealing with primary brands) might be harmful for its rivals, but large increases should be beneficial. Moreover, the probability of seeing a waterbed effect given some increase in size is lower if the fixed cost of going to a rival retailer is high. But if we do see it, it is likely to be substantial. Finally, while this waterbed effect might be damaging for customer welfare if firms compete in
prices, for the type of goods where capacity and perishability constraints are significant, an increase in a retailer’s size is always welfare improving.
4.8 Appendix

Proof. of Proposition 4.1: Given a \( w_i \), firm \( i \) maximises \( \Pi_i = (A - \frac{q_i + q_j}{B} - w_i) \) leading to the FOC: \( AB - 2q_i - q_j - Bw_i = 0 \). Solving these for the two firms \( i, j \) gives

\[
q_i = \frac{B}{3} (A - 2w_i + w_j)
\]

and we can calculate profits to be \( \Pi_i = \frac{B}{9} (A - 2w_i + w_j)^2 \). The participation constraints are:

\[
PC_i : \quad \frac{B}{9} (A - 2w_i + w_j)^2 \geq \frac{B}{9} (A + w_j)^2 - F \quad \Rightarrow \quad w_i (A - w_i + w_j) \leq \frac{9F}{4B}
\]

\[
PC_j : \quad \frac{B}{9} (A - 2w_j + w_i)^2 \geq \frac{B}{9} (A + w_i)^2 - F \quad \Rightarrow \quad w_j (A - w_j + w_i) \leq \frac{9F}{4B}
\]

The supplier’s profits can be given by \( \Pi = \frac{B}{3} [w_i (A - 2w_i + w_j) + w_j (A - 2w_j + w_i)] \) Defining \( C_i = w_i (A - w_i + w_j) - \frac{9F}{4B} \) and \( C_j = w_j (A - w_j + w_i) - \frac{9F}{4B} \), the Lagrangian problem is

\[
\max_{w_i, w_j, \lambda_i, \lambda_j} L = \Pi - \lambda_i C_i - \lambda_j C_j
\]

\[
\lambda_i \geq 0, \lambda_i C_i = 0
\]

\[
\lambda_j \geq 0, \lambda_j C_j = 0
\]

are the complementary slackness conditions. The problem can be solved by obtaining

\[
\lambda_j > 0 \quad \Rightarrow \quad \frac{A - 2w_i + w_j}{w_j} > \frac{A - 4w_i + 2w_j}{A - 4w_j + 2w_i} \tag{4.1}
\]

\[
\lambda_i > 0 \quad \Rightarrow \quad \frac{A - 4w_j + 2w_i}{w_i} > \frac{A - 4w_j + 2w_i}{A - 2w_i + w_j} \tag{4.2}
\]

These represent that the slope of the isorprofit curve is flatter than the participation constraints. Note that \( C_i = C_j = 0 \) imply \( w_i = w_j = w_I = \frac{9F}{4AB} \).\(^{17}\) Under this solution (4.1) and (4.2) are satisfied if \( w_I < \frac{A}{2} \). This requires that the fixed

\(^{17}\) Another solution of the quadratic is \( w_i = A - w_j \), but the resultant solution does not ensure positive quantity sold.
cost of going elsewhere be small: \( F < \frac{2A^2B}{9} \). Note also that since the profit function is concave and since the constraints are monotonic and therefore quasi convex, our solution is indeed a maximum.

It is also straightforward to note that \( \frac{dw_I}{dF} > 0, \frac{dw_I}{dA} < 0, \frac{dw_I}{dB} < 0 \).

**Proof.** of Proposition 4.2: Now the retailers are active in different number of markets. Their participation constraints are:

\[
\begin{align*}
PC_S : & \quad \frac{B}{9}(A - 2w_S + w_L)^2 \geq \frac{B}{9}(A + w_L)^2 - F \Rightarrow w_S(A - w_S + w_L) \leq \frac{9F}{4B} \\
PC_L : & \quad \frac{nB}{9}(A - 2w_L + w_S)^2 \geq \frac{nB}{9}(A + w_S)^2 - F \Rightarrow w_L(A - w_L + w_S) \leq \frac{9F}{4Bn}
\end{align*}
\]

Defining \( C_S = w_S(A - w_S + w_L) - \frac{9F}{4B} \) and \( C_L = w_L(A - w_L + w_S) - \frac{9F}{4Bn} \), the Lagrangian problem is

\[
\max_{w_S, w_L, \lambda_S, \lambda_L} L = \Pi - \lambda_S C_S - \lambda_L C_L
\]

where \( \lambda_S \geq 0, \lambda_S C_S = 0 \)

\( \lambda_L \geq 0, \lambda_L C_L = 0 \)

are the complementary slackness conditions. The problem can be solved by obtaining

\[
\begin{align*}
\lambda_S > 0 & \Rightarrow \frac{A - 2w_L + w_S}{w_L} > \frac{A - 4w_L + 2w_S}{A - 4w_S + 2w_L} \\
& \Rightarrow A^2 - 4w_L^2 - 3Aw_L - 8w_sw_L > 0 \quad (4.3)
\end{align*}
\]

\[
\begin{align*}
\lambda_L > 0 & \Rightarrow \frac{A - 2w_S + w_L}{w_S} > \frac{A - 4w_S + 2w_L}{A - 4w_L + 2w_S} \\
& \Rightarrow A^2 - 4w_L^2 - 3Aw_L - 8w_sw_L > 0 \quad (4.4)
\end{align*}
\]
Note also that

\[
C_S = 0 \implies w_S = \frac{1}{2} \left\{ (A + w_L) - \sqrt{(A + w_L)^2 - \frac{9F}{B}} \right\} \\
C_L = 0 \implies w_L = \frac{1}{2} \left\{ (A + w_S) - \sqrt{(A + w_S)^2 - \frac{9F}{B}} \right\}
\]

(4.5) \hspace{1cm} (4.6)

See from (4.6) that an increase in size will ceteris paribus reduce \( w_L \). A reduction in \( w_L \) will decrease the left hand side of (4.3). If the reduction in size is small (and consequently the reduction in \( w_L \) is also small) then \( \lambda_S > 0 \) and both participation constraints are binding.

To do comparative statics of marginal increases in size in this region (where both constraints bind), we will define \( \hat{F} = \frac{9F}{4B} \) and \( \hat{F}_L = \frac{9F}{4Bn} \). Taking total derivatives of the binding participation constraints we get the system:

\[
\begin{pmatrix}
A - 2w_S + w_L & w_S \\
w_L & A - 2w_L + w_S
\end{pmatrix}
\begin{pmatrix}
dw_S \\
dw_L
\end{pmatrix}
= \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

\( d\hat{F}_L \)

By Cramer’s rule we get

\[
\frac{dw_S}{d\hat{F}_L} = -\frac{w_S}{D_C}, \quad \frac{dw_L}{d\hat{F}_L} = \frac{(A - 2w_S + w_L)}{D_C}
\]

where \( D_C \) is the discriminant and is positive as long as \( q_L, q_S > 0 \) and \( w_L, w_S < \frac{A}{2} \).

Substituting for \( \hat{F}_L = \frac{9F}{4Bn} \) and hence \( d\hat{F} = -\frac{9F}{4Bn^2}dn \) we get:

\[
\frac{dw_S}{dn} = \frac{9Fw_S}{4Bn^2D_C} > 0, \quad \frac{dw_L}{dn} = -\frac{(A - 2w_S + w_L)9F}{4Bn^2D_C} < 0
\]

This gives is the waterbed effect.

\hspace{1cm} ^{18}\text{Since } w_S \geq \frac{A}{4} \text{ in the positive quadrant.}
However if the increase in size is large (and consequently the reduction in \( w_L \) is also large), (4.3) is no longer satisfied. Now the participation constraint of the small retailer stops binding and we set \( \lambda_S = 0 \). In such a case, the optimum is along the path described by (4.6) and the equality of (4.3). It can be shown that 

\[
\frac{dw_S}{dw_L} > 0^{19} \quad \text{this shows the supplier’s incentives to transfer business.}
\]

To show the possibility of no waterbed effect it is enough to show that some part of this curve lies below \( w_I \). Simple algebra shows that if \( w_L = 0 \), then \( w_S = \frac{A}{4} \). Now a necessary condition for the absence of the waterbed effect would be that there is some equilibrium \( w_S \) that lies below \( w_I = \frac{9F}{4AB} \). We require:

\[
\frac{A}{4} < \frac{9F}{4AB} \implies F > \frac{A^2B}{9}
\]

Note from the analysis of the symmetric equilibrium that \( F < \frac{2A^2B}{9} \). Therefore the range of \( F \) that allows the absence of the waterbed effect is \( \frac{A^2B}{9} < F < \frac{2A^2B}{9} \) which is nonempty. Therefore for \( n \) big enough and \( F \) big enough there is no waterbed effect. To conclude:

\[
\begin{align*}
\text{If } & F < \frac{A^2B}{9} \quad \text{there is a waterbed effect for all increases in size} \\
\text{If } & \frac{2A^2B}{9} > F > \frac{A^2B}{9} \quad \begin{cases} \\
\text{there is a waterbed effect for small increases in size} \\
\text{there is no waterbed effect for large increases in size} \\
\end{cases}
\end{align*}
\]

Proof. of Corollary 4.1: The size of the waterbed effect in our model is given by 

\[
\left| \frac{dw_S}{dn} \right| = \frac{9Fw_S}{4Bn^2D_C}.
\]

If we start from the symmetric equilibrium, \( w_S = w_I = \frac{9F}{4AB} \).
and \( D_C = (A - w_L)^2 - w_L^2 = A(A - 2w_L) = A \left( A - \frac{9F}{2AB} \right) \). Substituting into the size of the waterbed effect we find that

\[
\left| \frac{dw_S}{dn} \right|_{w_L}^C = \frac{81F^2}{8Bn^2A(2A^2B - 9F)}
\]

Note that

\[
\left| \frac{dw_S}{dn} \right|_{w_L}^C = \frac{-81F^2(6A^2B - 9F)}{8Bn^2[A(2A^2B - 9F)]^2} < 0, \quad \left| \frac{dw_S}{dn} \right|_{w_I}^C = \frac{-81F^2(4A^2B - 9F)}{8n^2A(2A^2B^2 - 9FB)^2} < 0
\]

\[
\left| \frac{dw_S}{dn} \right|_{w_L}^C = \frac{81}{8Bn^2A} \cdot \frac{(2A^2B - 9F)2F + 9F^2}{(2A^2B - 9F)^2} > 0
\]

So the waterbed effect is decreasing in \( A, B \) and increasing in \( F \).

**Proof.** of Proposition 4.3: If we assume a standard linear demand function \( P = A - \frac{Q}{B} \), where \( Q = q_L + q_S \), then from solving the first order conditions we can find the equilibrium quantities to be \( q_L = \frac{B}{3}(A - 2w_L + w_S) \) and \( q_S = \frac{B}{3}(A - 2w_S + w_L) \).

We can also find the final market price to be \( P = A - \frac{Q}{B} = A - \frac{[q_L + q_S]}{B} = \frac{B}{3}(A + w_L + w_S) \). Now following a change in input prices, the change in retail price in the market can be represented by \( \Delta P = \frac{B}{3} \Delta w_L + \Delta w_S \) so \( \Delta P > 0 \) if \( |\Delta w_L| > |\Delta w_S| \) In other words, the retail price increases if the waterbed effect is disproportionate.

Now we proceed to check if it is possible to have a disproportionate waterbed effect in our model. For that we require the slope of the small store’s participation constraint to be steeper than unity. The slope is given by \( \frac{dw_S}{dw_L} = \frac{-w_S}{A - 2w_S + w_L} \) and the absolute value of this is weakly bigger than unity if \( w_S \geq \frac{A + w_L}{3} \).

Substituting \( w_S = \frac{A + w_L}{3} \) into (4.3) and solving for \( w_S \):

\[
\frac{A - 2w_L - \left( \frac{A + w_L}{3} \right)}{w_L} = \frac{A - 4w_L - 2 \left( \frac{A + w_L}{3} \right)}{A - 4 \left( \frac{A + w_L}{3} \right) + 2w_L} \Rightarrow w_S = \frac{A}{2}
\]
Therefore as seen in Figure 4.3 below in \( \{w_L, w_S\} \) space, this never intersects the region defined by (4.3) and positive quantity constraints among the values of interest.

Figure 4.3: Welfare increase under Cournot
Chapter 5

Outsourcing in a market with network effects

Abstract: This is a study of a firm’s strategic incentive to outsource when its product displays network effects. A two stage model of Cournot duopoly is used to show that when plant sizes are fixed, a firm would choose to increase its observable marginal cost to make its competitor less aggressive and thereby increase its own probability of winning competition for the market. We show that delegation is a credible way to achieve this and propose a model of strategic outsourcing when firms compete for the market. Our results are robust to small levels of uncertainty.
5.1 Introduction

*If you want something done right, go do it yourself.*

The quote above seems logical. However, a range of literature on delegation disagrees. In markets where firms are interdependent, it may be the case that maximum profits are earned by firms with objectives other than profit maximization.\(^1\) The implications of this on the theory of the firm and vertical contracting have been established by Vickers (1985) and Bonanno and Vickers (1988) who show that vertical separation or delegation is profitable for the firm trying to win competition *in* the market. The aim of this paper is to consider the incentives to delegate in slightly different markets - those with network effects. We question whether delegation is profitable for a firm trying to win competition *for* the market.

(Direct) Network effects\(^2\) are said to exist when each user’s utility of consumption of a good and/or his incentive to consume it, is increasing in the number of existing users of that good. In other words, the number of people consuming a good is a representation of its quality.\(^3\) These effects can be easily seen in several industries such as telecommunications, operating systems, credit cards and even languages. It makes sense for me to buy a telephone or a fax machine only when people I want to communicate with have these too. I would like to use the operating system whose files are compatible with what most of my colleagues

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\(^1\)See Vickers (1985) - say a firm A is deciding whether to enter a market currently monopolised by firm B. B has to decide whether it will fight entry or accommodate. Assuming once firm A enters it is profitable for B to accommodate, backward induction implies that entry and accommodation will happen. However if firm B had an objective of maintaining market share rather than profit maximisation it would choose to fight entry and firm A would not enter. Thus profits can be higher when managers are not trying to maximise profits.

\(^2\)See Klemperer (2006) - Indirect network effects arise if adoption by different users is complementary because of its affect on a related market. Eg. consumers of hardware products may gain when other users consume them too, not because of any direct benefit, but because of the provision of more softwares.

\(^3\)We mean ‘quality’ here in a subjective sense. See Bental and Spiegel (1995).
use, and it is not in my interest to learn a language that nobody who I need to communicate with understands. Essentially these effects represent economies of scale - the ‘per buyer surplus of a coalition of buyers increases with the size of the coalition’.

A critical characteristic of such markets is that once enough consumers have adopted the good, it is in the interest of the rest of the market to adopt it too. The resulting equilibrium is hence history dependent - long term choices depend on early preferences. Most of the literature agrees that firms which offer incompatible products in such an industry compete for early adoption. In contrast to competition in the market, here we have Schumpeterian competition for the market where firms struggle for dominance. Firms typically have a ‘bargain followed by a rip off’ strategy. They charge low prices to attract early users and once they are ‘locked in’, increase prices to create ex-post rents. In equilibrium an ex-post monopoly is created, for which firms compete ex-ante.4

Intuitively, why would delegation help a firm trying to win competition for the market? Most early work on the theory of the firm argued that integration ‘harmonises interests’. Manufacturers integrate with retailers to avoid transaction costs.5 After a point however, there are increasing distortions associated with transactions as the firm becomes more complex - leading to an upper limit on integration. It has been argued however, by Schelling and later Vickers that there may be a strategic reason for non-integration. Delegation creates a ‘suitable divergence of interests, rather than a harmony of interests which may be essential for the credibility of some threats, promises and commitments’.6 Indeed Mathewson and Winter (1984) have argued that vertical restraints are devices that bring distributors’ actions in line with manufacturers’ interests. Hence delegation or

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5This is proposed largely by Williamson (1971).
6See Vickers (1985)
outsourcing (as in our model) is a means of ‘making credible a response pattern that the original source of decision might have been thought to find profitless’.\(^7\) This paper argues that these effects are further strengthened in the face of demand side externalities as the firms are fighting for the whole market.

We first build a benchmark model for the presence of network effects. To capture this idea we present a two stage game. In stage one firms engage in Cournot competition in the market while in stage 2, the firm that wins a larger market share in stage 1, can capture the whole market and become a monopolist. This models directly the technology adoption highlighted by switching cost literature. Additionally, we allow the firms to compete only in predetermined discrete quantities and assess the probability with which firms chooses the ‘high’ quantity. This ensures that the firm that ‘wins’ the market has an unambiguous win. Crucially, we consider only the mixed strategy equilibria. This allows us to conduct comparative statics on the probability with which a firm tries to capture the market.

We show that it is in a firm’s interest to increase its observable marginal cost. This seems counterintuitive. The reason for this is that while an increase in own costs serves to reduce own profits, it also serves to make the rival less aggressive. The latter effect dominates and an increase in own costs is in fact, beneficial for a firm. We show that outsourcing is a means to achieve this by charging a wholesale price above the cost of production. This will however, only be chosen as long as the upstream firm has the bargaining power to extract a large enough surplus from the downstream firm. We then show that uncertainty increases aggression. Consequently, a firm will want to outsource the product only if it is able to communicate that well enough to its rival.

\(^7\)See Schelling (1999)
5.2. A REVIEW OF THE LITERATURE

Plan of the paper: section 5.2 sets out a review of the current literature. Section 5.3 presents the benchmark model when firms have symmetric costs. Section 5.4 then introduces asymmetry in costs to study its effect on the equilibrium strategies. Section 5.5 argues outsourcing is a natural means of achieving this change in marginal cost and highlights conditions under which a firm would choose to do so. Section 5.6 then tries to incorporate asymmetry in information and checks whether our results are robust to this form of uncertainty. Finally Section 5.7 concludes and proofs are contained in the Appendix.

5.2 A Review of the Literature

There are two strands of literature that are relevant for the work here - literature on delegation or theory of the firm, and the literature on network effects. Though there has been a lot of work on those two individually, to our knowledge, there has been surprisingly little on the two together. This paper is an attempt to bridge that gap.

Network effects\(^8\) are well studied. Leibenstein (1950), taking off from Veblen’s notion of ‘conspicuous consumption’ was one of the first to recognise that consumption of an individual is not independent of consumption of another, and that we needed some sort of dynamic theory of demand. He develops a transition from individual to collective demand curves focusing on the complementarity between individuals’ decisions. Most of the literature that followed has applied this to markets such as the telecommunications industry.\(^9\) Rohlfs (1974) presented a model of the telecommunications industry and argued that as firms’ strategic choices do not fully internalize the network effects as single network product tends

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\(^8\)Katz and Shapiro (1985) used the term network externalities, but as Liebowitz and Margolis (1994) argue, network effects may not always be externalities as many indirect network effects are pecuniary. Moreover, network effects can be called externalities only if they haven’t been internalized by pricing or side payments.

\(^9\)See Klemperer (2006)
to be under-adopted at the margin. It shows that there are typically multiple equilibria at any chosen price.

However, these still study monopoly outcomes in markets. With an oligopolistic setting, adopting one network implies not adopting another, and this dilutes the externality that Rohlfs talks about.\textsuperscript{10} The seminal work in oligopolistic setting was done about ten years later by Katz and Shapiro (1985) where they propose a formal model of network competition. Their model shows the possibility of multiple equilibria when the products are incompatible. Thus firms’ reputations play a crucial role in determining which equilibrium is played. They also look at compatibility choices as endogenous and argue that private decisions will depend on whether firms can act unilaterally or whether consensus is required and also on whether side payments can be made. Later work looked at incentives to ‘herd’ whereby we get multiple equilibria in static models and feedback or ‘tipping’ effects in dynamic ones.\textsuperscript{11} For example, David (1985) looked at the QWERTY vs. DSK story from an economic history point of view to highlight the ‘path dependent’ nature of the adoption processes. It showed that the market could reach a Pareto inferior equilibrium.

The consequence of network effects on competition is an important one. Intuitively, once a network has been established, it would not be easy for a firm to enter with an incompatible product. However, Farrell and Shapiro (1988) considering a model of overlapping generations in a duopoly argue that though an incumbent has the ability to restrict entry (via the economies of scale it enjoys), it will choose not to do so when it can not discriminate between the new and old users. In equilibrium, as the market size grows, a firm with attached consumers will specialise in serving them and concede new customers to the rival. This leads to inefficiency in a strange sense - the incumbent encourages too much entry! In

\textsuperscript{10}See Klemperer (2006)
\textsuperscript{11}See Klemperer (2006)
that sense, a firm wanting to remain a fierce competitor, will choose to keep its consumer base small.

Unlike the literature on network effects, the literature on vertical contracting has been around for much longer. Coase (1937) was perhaps the first to point out that firms have a choice between placing a transaction in the market and locating it within the firm.\textsuperscript{12} Perry (1989) argues that vertical integration implies the elimination of contractual and market relations. The literature has developed several schools of thought on why firms choose to integrate vertically. Among them is Williamson (1975) who emphasised the relationship of firms with labour to explain this. Vertical integration involves substitution of purchase of goods, by production of goods that is possible by hiring labour. The transactional economies argument argued that investments entailed a high degree of asset specificity leading to under investment at the margin. Thus vertically integrated firms are superior to separated ones. Grossman and Hart (1986) on the other hand provide an argument for the property rights theory of the firm and say that Williamson’s explanation does not explain what exactly changes with integration. They argue that ownership establishes residual rights - which here is the control variable. The stress is on asset ownership being an instrument for influencing individual incentives.

For our question, Bonanno and Vickers (1988) and Vickers (1985) provide the basic motivation and structure of the model. Bonanno and Vickers (1988) have a model of duopoly where firms compete in prices. They show that vertical separation - i.e. selling a product through a retailer is in the collective as well as in the individual interest of firms. As firms compete in complements, each manufacturer can charge a wholesale price above the cost of production to induce all manufacturers to raise their prices. Thus integration facilities collusion. Vickers

\textsuperscript{12}See Holmstrom (1999)
(1985) considers quantity competition in a similar setting and argues that it is in the individual but not the collective interest of firms. Our model also considers quantity competition but is different in two respects - firms choose pre-determined discrete quantities, and there is a second stage in the model which represents the network effects. Consequently our results are quite different.

5.3 The Benchmark Model

Outsourcing, in this chapter, is relevant insofar as it is a means for a firm to alter its marginal cost. However, prior to analysing the impact of heterogeneous costs, we present a benchmark model with symmetric firms.

There are two identical single product firms $i, j$ producing incompatible goods that display network effects. The firms compete in quantities, and face a linear demand curve of the form $P = 1 - Q$ where $Q = q_i + q_j$. The firms have constant marginal costs that we normalise to be zero, so $MC_i = MC_j = 0$. This is a standard Cournot model with a linear industry demand curve.

To incorporate the idea of network effects we construct a second stage. In stage one the firms simultaneously compete in quantities, and realise profits. This represents short run competition for the market. In stage two, the firm that gained a higher market share in period one subsequently wins the whole market and makes monopoly profits into the future. This captures the presence of network effects through technology adoption and switching costs. Once a firm captures most of the market, incompatibility ensures that consumers gravitate to the larger network in the long run. Firms also have a discount factor $\delta$.

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13If they produce equal quantities then they win the market with equal probability in stage 2.

14Katz and Shapiro (1985) argue that the central feature of the market that determines the scope of the relevant network is whether products can be used together. For example in communication networks, it is whether consumers using one network can contact those on
This represents how much the firms ‘value’ long term profits. It could also be interpreted as the expected future size of the market. We assume firms value the future equally and set $\delta_i = \delta$.

We also assume that in stage 1 the firms choose predetermined discrete quantities, i.e. $q_i, q_j \in \{q_L, q_H\}$. We do this for two reasons: first, the restriction can be interpreted as a realistic technological constraint due to which the firms are forced to choose predetermined capacities or fixed plant sizes. Second, the significant difference in market shares ensures that the firm that eventually wins the market has a conclusive and unambiguous win. This makes the argument that it can make monopoly profits in the future more convincing.

The restriction is however relevant only for the short run when we have competition for the market. In the long run (i.e. stage 2), we argue that it reasonable to assume that the firms can adjust quantity completely. Once a firm has established itself as the winner, it can choose to build plants that produce the monopoly quantity. For our model, note that as a result of this assumption, a change in marginal cost affects only the payoffs of the firms but not their strategy space. A natural extension would be to think about allowing quantities in stage 1 to be continuous as well and a continuous version of this symmetric benchmark case is explored in the appendix, but has not been pursued for the asymmetric model in light of tractability concerns.\textsuperscript{15}

Intuitively, the game presented here must have an equilibrium in mixed strategies. If firm 1 produces a large quantity, then firm 2, knowing that its chances of winning the market are small, would like to produce a low quantity to keep the price high and make profits this period at least. If on the other hand, firm 1 produces low quantity then it is in firm 2’s interest to overproduce and try to win another. If they can, then the appropriate network is the total number of consumers that can contact one another. Thus incompatibility is the central feature of our argument here.\textsuperscript{15}

\textsuperscript{15}See Appendix, Section 5.8.2
the market in stage 2. In our model, we also have two pure strategy equilibria (which involve one firm producing high quantity and the other producing low quantity), but they are not that interesting. Equivalently, we could let the profit in the case the firm lost be a ‘fixed cost of entry’. If a firm knew it was going to lose then it wouldn’t make sense for it to enter. By forward induction, if a firm enters, it must have a strategy that does not rule out winning. The paper will, hence, focus only on the mixed strategy equilibria.\textsuperscript{16}

We can now think about the relevant bounds for the choice of predetermined plant sizes. Note that if we allowed for continuous strategy spaces, and if the firms did not value the future at all ($\delta = 0$), then each firm would choose to maximise profits in the first period and produce the Cournot quantity, $q_C$. If however $\delta > 0$ then they have an incentive only to increase production in this period in the hope of winning competition for the market in the next period. Hence we set $q_L = q_C$. The profit each firm makes is given by

$$\Pi(q_i, q_j) + \text{prob}(q_i > q_j)\delta \Pi_M$$

where $\Pi(q_i, q_j)$ represents the profit in stage 1 and $\Pi_M$ represents the monopoly profit to be made in stage 2. The payoff matrix for the game is hence:

<table>
<thead>
<tr>
<th>$i,j$</th>
<th>$q_C$</th>
<th>$q_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_C$</td>
<td>$\Pi(q_C, q_C) + 1/2\delta \Pi_M$</td>
<td>$\Pi(q_H, q_C) + \delta \Pi_M$</td>
</tr>
<tr>
<td>$q_H$</td>
<td>$\Pi(q_C, q_H)$</td>
<td>$\Pi(q_H, q_H) + 1/2\delta \Pi_M$</td>
</tr>
</tbody>
</table>

Define $\sigma_i$ as the probability that firm $i$ chooses to produce the high quantity $q_H$.

\textsuperscript{16}This assumption also makes sense if we consider the continuous strategy space model as a limiting case of our present model. In the continuous model, we would not expect any pure strategy equilibria to exist. If firm 1 produces $q$ then firm 2 would produce $q + \varepsilon$.  

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For an equilibrium in mixed strategies we must have

\[(1 - \sigma_i) \left[ \Pi(q_c, q_c) + \frac{1}{2} \delta \Pi_M - (\Pi(q_h, q_c) + \delta \Pi_M) \right] \]

\[+ \sigma_i \left[ \Pi(q_c, q_h) - \left( \Pi(q_h, q_h) + \frac{1}{2} \delta \Pi_M \right) \right] = 0\]  

(5.1)

Given the game is symmetric, this solves to give

\[\sigma_i = \sigma_j = \sigma = \frac{\Pi(q_h, q_c) + \frac{1}{2} \delta \Pi_M - \Pi(q_c, q_c)}{\Pi(q_h, q_c) - \Pi(q_c, q_c) - \Pi(q_h, q_h) + \Pi(q_c, q_h)}\]

Substituting for profit values we get

\[\sigma = \frac{\delta}{8(q_h - \frac{1}{3})^2} - 1\]

**Proposition 5.1.** Suppose the firms are symmetric. For intermediate patience values, there exists an equilibrium in mixed strategies where the probability a firm chooses to produce the high quantity is increasing in the discount factor \(\delta\).

First note that that if the firms are completely impatient, or \(\delta = 0\), then it is a dominant strategy to choose the Cournot quantity and we have \(\sigma = 0\). Hence we require the firms to care about the future for them to choose the high quantity with non zero probability. If, on the other hand, the firms are very patient then it is a dominant strategy again to choose the high quantity and we will have \(\sigma = 1\). Hence for intermediate levels of patience, we have an equilibrium in mixed strategies. Given that we have an equilibrium in mixed strategies, it is certainly more likely that firms will choose the high quantity if they are more patient.
5.4 Introducing Asymmetry in Costs

Now we allow the marginal costs of the two firms to be different. Let $MC_j$ continue to be zero but $MC_i = c_i$. Continuing with the previous example, we set $q_L = q_C$.\(^{17}\) Denoting by $\Pi(\cdot; c_i)$ the profits of firm $i$ and $\Pi(\cdot; 0)$ as the profits of firm $j$, we get the payoff matrix:

<table>
<thead>
<tr>
<th>$i,j$</th>
<th>$q_C$</th>
<th>$q_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_C$</td>
<td>$\Pi(q_C, q_C; 0) + \frac{1}{2} \delta \Pi_M(0)$</td>
<td>$\Pi(q_H, q_C; 0) + \delta \Pi_M(0)$</td>
</tr>
<tr>
<td>$q_H$</td>
<td>$\Pi(q_C, q_H, c_i) + \delta \Pi_M(c_i)$</td>
<td>$\Pi(q_H, q_H, 0) + \frac{1}{2} \delta \Pi_M(0)$</td>
</tr>
</tbody>
</table>

Now to solve for $\sigma_j$ we again require (5.1) to hold. This gives us $\sigma_i = \sigma$. The solution for $\sigma_j$ however is different. We require:

$$(1 - \sigma_j) \left[ \Pi(q_C, q_C; c_i) + \frac{1}{2} \delta \Pi_M(c_i) - \left( \Pi(q_H, q_C; c_i) + \delta \Pi_M(c_i) \right) \right] \tag{5.2}$$

$$+ \sigma_j \left[ \Pi(q_C, q_H; c_i) - \left( \Pi(q_H, q_H; c_i) + \frac{1}{2} \delta \Pi_M(c_i) \right) \right] = 0$$

This solves to give

$$\sigma_j = \frac{\Pi(q_H, q_C; c_i) + \frac{1}{2} \delta \Pi_M(c_i) - \Pi(q_C, q_C; c_i)}{\Pi(q_H, q_C; c_i) - \Pi(q_C, q_C; c_i) - \Pi(q_H, q_H; c_i) + \Pi(q_C, q_H; c_i)}$$

Substituting for profit values we get

$$\sigma_j = \frac{\delta (1 - c_i)^2}{8(q_H - \frac{1}{3})^2} - \frac{c_i}{(q_H - \frac{1}{3})} - 1$$

\(^{17}\)Note that given the firms are asymmetric, this is now not the precise Cournot quantity. However, recall our assumption on predetermined plant sizes. Therefore this can be thought of as a firm changing its marginal cost after the plant sizes have been decided. Hence we will continue with the same definition for $q_L$ as before.
which leads us to our primary results:

**Proposition 5.2.** Suppose the firms are asymmetric in efficiency. For intermediate patience levels there exists an equilibrium in mixed strategies where the probability a firm chooses the high quantity is decreasing in the marginal cost of its rival.

A change in marginal costs affects own expected profits from choosing both low quantity and high quantity. However it affects them asymmetrically. Profits from choosing high quantity decrease more than those from choosing low quantity.\(^{18}\)

Firm \(i\) would now prefer to produce low quantity more often. Firm \(j\), knowing this, and choosing its optimal strategy to maintain firm \(i\)’s indifference would then try to make producing low quantity less attractive for firm \(i\), and the only way it can do so is by choosing low quantity less often itself. It is this asymmetry in expected profits makes \(\sigma_j\) decreasing in \(c_i\). Thus an increase in \(c_i\) is beneficial because of the asymmetry it creates in firm \(i\)’s profits.

Essentially, the idea is that a firm can choose cost manipulation as a means to alter its opponent’s reaction function. Note however that this result represents the least intuitive part of all mixed strategy equilibria - own strategies are independent of own payoffs insofar as they don’t change the qualitative aspects of the game - i.e. as long as the preference relation between own payoffs remains unchanged, a relative change in their values has little consequence on the equilibrium strategy of that player. It has consequence only on the strategy of the opponent. Here an increase in own marginal costs only serves to make the rival

\(^{18}\)To see how consider the case where \(q_H = q_M\). Here \(E\Pi_i(q_C) = (1 - \sigma_j)((1 - c_i) + \frac{4(1-c_i)^2}{8} + \sigma_j)\). Also \(E\Pi_i(q_M) = (1 - \sigma_j)((1 - c_i) + \frac{4(1-c_i)^2}{4} + \sigma_j)\). Now, \(\partial E\Pi_i(q_M)/\partial c_i = -(1 - \sigma_j)(\frac{1}{2} - \frac{3}{4}(1 - c_i) + \frac{1}{2} \sigma_j) > 0\) and \(\partial E\Pi_i(q_M)/\partial c_i = -(1 - \sigma_j)(\frac{1}{2} + \frac{1}{2} + \sigma_j(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2})) < 0\).

We can see that, \(|dE\Pi_i(q_M)/dc_i| > |dE\Pi_i(q_C)/dc_i|\)
less aggressive. We can now calculate the expected profits for firm $i$ to be

$$E\Pi_i = (1 - \sigma) [\Pi(q_C, q_C; c_i) + \Pi_M(c_i)] + \sigma\Pi(q_C, q_H; c_i) \quad (5.3)$$

This gives us the result

**Proposition 5.3.** Suppose the firms are not too asymmetric in efficiency and playing the mixed strategy equilibrium. Then the expected payoff of a firm is increasing in its own marginal cost.

This seemingly counter-intuitive result follows from Proposition 5.2. Since a higher marginal cost of firm $i$ makes firm $j$ less aggressive while leaving the strategy of firm $i$ unaffected, it increases the probability with which firm $i$ wins the market in stage 2 where it can make monopoly profits. The strategy of cost manipulation has its own costs and benefits. The ‘benefit’ of this cost manipulation is the reduction in the opponent’s aggression, while the ‘cost’ of this cost manipulation is the reduction in per period profits of the manipulating firm. We find that at all positive levels of the marginal cost, the benefit is greater than the cost, and so the expected profits are in fact increasing with $c_i$. 19 Thus, as the benefits outweighs the costs, it makes sense that expected profits increase with an increase in $c_i$.

---

19Consider the special case where $q_H = q_M$, the monopoly output. Here we can define the benefit of cost manipulation as $B = \sigma - \sigma_j = \frac{1}{2}(18c_0\delta + 36c_i - 9\delta c_{i}^{2})$. The cost is quantified as $C = E\Pi(c_i = 0) - E\Pi(c_i > 0) = (1 - p)\left(\frac{c_i}{3} + \frac{\delta}{8}(-c_i^2 + 2c_i)\right) + p\left(\frac{c_i}{3}\right)$, where $p$ is some probability that that other firm goes high. We need to keep that the same to evaluate just the cost of this cost manipulation. We have that $B > C$ if $c_i > 0$. This is because the condition is strongest when $p = 0$ which requires $\frac{-1}{2}(9\delta c_{i}^{2} - 18c_0\delta - 12c_i) > \left(\frac{c_i}{3} + \frac{\delta}{8}(-c_i^2 + 2c_i)\right)$, which in turn requires $-105\delta c_{i}^{2} + 210c_i\delta + 136c_i > 0$ which is true for all $0 < c_i < 1$. 

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5.4.1 Discussion on parameters

This paper has argued that the mixed equilibrium is the relevant equilibrium to be studied. The range of $c_i$ that allows mixing is however, quite small.\(^{20}\) If the costs of the two firms are very different, it is likely that there is a pure strategy equilibrium. The range of $c_i$ that supports the mixing equilibrium is also decreasing in patience. If the firms are very patient, both will prefer to choose high quantity. It is interesting to see also how the existence of other equilibria change as we move along the $\delta$ space. For low values of $\delta$, the firms do not value next period profits too much, and producing the Cournot quantity is a dominant strategy. Thus the equilibrium that prevails is both firms going choosing the low quantity. For high values of $\delta$, it is a dominant strategy to produce high quantity as the firms value next period profits a lot more. Hence the prevailing equilibrium involves both firms producing high quantity. For intermediate values of $\delta$, there are no dominant strategies. There exists a mixed strategy equilibrium where each firm has a certain probability of choosing high quantity and as argued by Proposition 5.2, this probability depends negatively on the opponent’s costs. There are also two pure strategy equilibria that exist for these intermediate values of $\delta$ (where one firm goes high and the other firm goes low). These however are not plausible because they are inconsistent with forward induction arguments.

Note also that expected profits in the uncoordinated equilibria ($HH$ and $LL$) are increasing in $\delta$. This follows from the observation that all profits are increasing functions of $\delta$: the value of the second period payoffs increases if firms are more patient. For the mixed equilibrium, interestingly, these expected profits decrease with $\delta$. This is because the mixed equilibrium essentially represents the move from one coordinated equilibrium to the other. An increase in $\delta$ makes both firms more likely to produce high quantity. With an increase in quantity the

\(^{20}\)For $q_H = q_M$, the size of the range of $c_i$ that allows mixing is $\sqrt{\frac{1+14\delta}{3\delta}} - \sqrt{\frac{1+14\delta}{3\delta}}$
price falls and the current period profits follow suit. The probability of winning the market in period two remaining the same, the expected profits have to fall. Note also that if $\delta$ is such that mixing is allowed, profits are certainly higher from choosing an optimal $c_i > 0$.

### 5.4.2 Mixed strategy equilibria and counter-intuitive comparative statics

The game presented can essentially be looked at as an entry game. We can define $\{q_L, q_H\} = \{\text{not enter, enter}\}$, where ‘enter’ implies ‘fight for the market’. Our results can then be compared to the standard Dixit-Spence model for entry deterrence. Their model considers investment and a subsequent decrease in marginal costs as a credible mechanism for altering the reaction function of their competitor. An increase in marginal cost in their model would shift the reaction function of the competitor outwards and make it more aggressive. It would hence move the equilibrium such that own quantity deceases and the rival’s quantity increases. Thus here, it is in a firm’s strategic interest to decrease its marginal cost. As Vickers (1985) and Bonanno and Vickers (1988) show, separation is in
the individual interest of the firm and involves a wholesale price lower than the
cost of production, accompanied by a negative fixed fee.

Now, this is clearly orthogonal to what our model predicts. This might be
because mixed strategy equilibria are well known to show counter-intuitive com-
parative statics.\footnote{See Crawford and Smallwood (1984)} We however maintain the mixed strategy equilibrium is the
correct equilibrium to study. It seems safe to conclude though that part of the
results are indeed caused by the mixed strategy equilibrium and returning to the
continuous case may reduce some differences with the literature.

5.5 Outsourcing

This aim of the previous section was essentially to show Proposition 5.3. Ex-
pected profits are increasing in own marginal costs. Thus to maximise its profit,
a firm would like to find a way to increase its marginal cost. A natural way of
doing this would be to delegate the sale of its product to a third party. This sec-
tion explores this point further by addressing the question of whether outsourcing
might allow it to do so.

5.5.1 The Game

Consider the following model with the same basic assumptions: Two identical
firms $i, j$ facing a linear demand curve compete in an industry with network
effects. As before we normalise the marginal costs to zero, and continue to assume
that they value the future equally ($\delta_i = \delta$). We also continue to assume that the
firms choose predetermined discrete quantities $q \in \{q_L, q_H\}$. However, we restrict
our attention to the case where $q_H = q_M$, the monopoly quantity.\footnote{Again, note that this is the monopoly quantity of the benchmark case where there is no
delegation.} This has the
impact of simplifying our calculations, leading to neater results. There is also an additional period in the model where firm $i$ can consider outsourcing the sale of its product to a third party $D_i$. If firm $i$ decides to outsource, it becomes an upstream seller and bargains with its downstream buyer over a two part tariff consisting of a per unit wholesale price $w$ and a fixed fee $F$ that is to be paid up-front. We also assume that the bargained quantities $\{w, F\}$ are fixed for both the periods that follow. Schematically,

\[
\begin{array}{c|c|c}
MC = 0 & \text{(Firm 1) US} & MC = 0 \\
\uparrow & \text{Bargaining over } \{F, w\} & \text{Firm 2} \\
MC = w & DB_1 & MC = 0 \\
MC = w & MC = 0 & MC = 0
\end{array}
\]

Figure 5.2: A schematic

The game now proceeds as follows:

**Stage 0:** Firm $i$ publicly decides whether or not to outsource the sale of its product to a third party. The bargained quantities $\{F, w\}$ are decided.

**Stage 1:** If firm $i$ has outsourced the sale of its product, then the downstream buyer and firm 2 compete with asymmetric costs. If firm 1 has decided not to outsource, then the two firms compete with symmetric costs. Profits are realised.

**Stage 2:** The firm that produced a higher quantity in stage 1 wins the whole market and makes monopoly profits.

To find the Nash equilibrium of this game, we need to compare firm $i$’s payoffs when it chooses to outsource with its payoffs if it chooses not to. If firm $i$ competes with firm $j$ directly, then they have the same marginal costs and we have the model of Section 5.4 with $\sigma_i = \sigma_j = \sigma = \frac{1}{2}(9\delta - 2)$, and $\Pi_i$ given by (5.3). Call this $\Pi_{N.O}$. However, if firm $i$ does choose to outsource to a downstream firm $D_i,$
the cost structure changes. The bargained wholesale input price between firm \( i \) and \( D_i \) becomes the buyer’s marginal cost. Hence while \( MC_{U_i} = 0 \), we have \( MC_{D_i} = w \) and \( MC_j = 0 \). The profits for firm \( D_i \) now include a fixed upfront fee as well. Hence the payoff matrix is given by:

<table>
<thead>
<tr>
<th>( i, j )</th>
<th>( q_C )</th>
<th>( q_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_C )</td>
<td>( \Pi(q_C, q_C; 0) + \frac{1}{2} \delta \Pi_M(0) )</td>
<td>( \Pi(q_M, q_C; 0) + \delta \Pi_M(0) )</td>
</tr>
<tr>
<td>( q_M )</td>
<td>( \Pi(q_M, q_H, 0) )</td>
<td>( \Pi(q_M, q_M, 0) + \frac{1}{2} \delta \Pi_M(0) )</td>
</tr>
</tbody>
</table>

In this case firm \( i \) chooses \( \sigma_i = \sigma \) that satisfies (5.1) and firm \( j \) chooses \( \sigma_j \) that satisfies (5.2). Since \( F \) is only a sunk cost, it does not change our equilibrium is identical to the one in Section 5.4 and still given by \( \{\sigma, \sigma_j\} \) but with the argument \( w \) rather than \( c_i \). We can calculate expected profits from outsourcing:

\[
E\Pi_{Di} = (1 - \sigma_j)[\Pi(q_C, q_C; w) + \delta \Pi_M(w)] + \sigma_j[\Pi(q_C, q_H; w)] - F
\]

\[
E\Pi_{Ui} = \begin{align*}
&= w[\text{Expected qty in period 1}] + \delta w[\text{Prob}(\sigma > \sigma_j)][\text{Expected qty in period 2}] + F \\
&= w \left[ (1 - \sigma)q_C + \sigma q_M + \frac{\delta(1 - w)}{2} \right] + F
\end{align*}
\]

since \( \text{Prob}(\sigma > \sigma_j) = 1 \) for all \( w > 0 \). Note that due to the two part nature of the tariff, the resultant bargaining will be efficient. The firms will use \( w \) to maximise the size of the joint surplus \( E\Pi_{Ui} + E\Pi_{Di} \) and use \( F \) to divide it amongst themselves.\(^{23}\) From Proposition 5.2 this joint surplus is increasing in \( w \), hence the firms will set as high an input price as they can. Given however that we want to continue looking at the mixed strategy equilibrium, this cannot be too high as otherwise firm \( j \) will choose the low quantity for sure. We can show that the

\(^{23}\)See Muthoo (1999)
highest input price set is given by

\[ w^* = 1 - \frac{\sqrt{4 + 14\delta} - 2}{3\delta} \]

and \( F^* \) is chosen depending on the relative bargaining strengths of the two firms.

In this setup we have the following results:

**Proposition 5.4.** If firm \( i \) has enough bargaining power (such that it can command a high upfront fee, \( F \)), then it will always prefer to outsource the sale of its produce to a third party rather than compete with firm \( j \) directly.

This result follows from Proposition 5.2. A firm would like to increase its observable marginal cost in order to make its rival less aggressive, thereby increasing its own probability for winning the market. Outsourcing is a means to do that. However, the firm needs to ensure it can recover some minimum share of the joint upstream-downstream surplus for outsourcing to be profitable. For this, it needs to have enough bargaining power.

**Corollary 5.1.** For the same bargaining power distribution, a firm is more likely to outsource the sale of its produce if it is more patient.

This result is also intuitive. As the benefits of winning the market increase, the incentive to win it also increases. Hence the use of mechanisms that facilitate that win will increase too.

This argument has interesting consequences for analysis of investment decisions. Consider a possible visible investment firm \( i \) can make which costs some fixed fee \( F \) and reduces its marginal cost \( w\% \) below its rival’s. As the plant sizes are fixed, the only consequence of such a move will be to make the rival more aggressive. Firm \( i \)’s strategy remaining the same, it would decrease firm \( i \)’s chances of winning the market. Hence, no such cost reducing investments will be made. This
seems highly counter-intuitive. We would expect that the firm that can lower its marginal cost via investments or technological improvements would increase production and win the market. The driving assumption for our result is again that firms compete in discrete predetermined quantities. Thus, an improvement in efficiency only makes the rival more aggressive and leads to phenomena such as ‘standards wars’. The argument here is similar to the ‘lean and hungry look’ of Fudenberg and Tirole (1984) where a firm wants to be weak to look tough.

5.6 Asymmetric Information

We would now like to examine the impact of asymmetric information on our previous results. Suppose that firm $i$ knows its ex-post marginal cost but firm $j$ does not observe it perfectly. It only has some priors on it. We would like to know whether a firm would have the same incentives to increase its marginal cost.

5.6.1 The impact of uncertainty:

We return to our symmetric model to isolate the effect of uncertainty. Suppose there are two equally efficient vertically integrated firms facing a linear demand curve and competing in quantities. The game is identical to what is described in the earlier sections and we continue to consider the case where $q_H = q_M$. The difference lies in the information structure. We introduce uncertainty in the form of asymmetry of information where firm $j$ believes that firm $i$’s marginal cost is along some distribution with mean 0 (i.e. firm $i$’s marginal cost in expectation is the same as firm $j$’s marginal cost) and some variance $\eta$. Hence,

\[ E(MC_i) = 0, \quad \text{and} \quad Var(MC_i) = \eta, \implies E[MC_i^2] = \eta \]
We can calculate expected profits as $E\Pi(q_i) = E[(1 - Q - C(q_i))q_i]$. Using similar calculations in expectation as (5.1) we get the result

$$\sigma_j^U = \frac{9\delta}{2}(\eta + 1) - 1$$

Note that if we set $\eta = 0$ as in the case of certainty then we are back to the original symmetric game. Now we have the result

**Proposition 5.5.** Uncertainty in the distribution of the opponent’s marginal cost makes a firm more aggressive.

Uncertainty changes the expected value of firm $i$’s profits. More importantly it changes them asymmetrically. Note that monopoly profits are convex in the marginal cost because once a firm becomes a monopoly it is able to set the optimum quantity it desires as it is no longer constrained by the assumption of predetermined plant sizes. Jensen’s inequality implies that the expected value of monopoly profits is then higher with uncertainty. However, the probability of firm $i$ being a monopoly is larger if it chooses the high quantity. Therefore, uncertainty increases expected value of profits from choosing high more than choosing low. Firm $j$ then thinks that, everything else being equal, firm $i$ is more likely to choose high. Indifference and subsequent equilibrium require it to make choosing high less attractive for firm $i$ and the way it can do that is by choosing high more often itself. Therefore, uncertainty makes the rival more aggressive. The implication is that firm $i$ would like to be as informative as possible about its costs.

### 5.6.2 Effect of uncertainty and a change in marginal cost:

Proposition 5.2 and Proposition 5.5 lead us to two competing effects. An increase in uncertainty makes the rival more aggressive, but an increase in marginal
cost makes the rival less aggressive. If a firm was considering increasing its marginal cost (say via outsourcing) but it could not ensure that its rival knew exactly by how much it was being increased, the two effects would come into play and the result would depend on their relative strengths. Hence we have the result:

**Proposition 5.6.** If the firm $j$’s prior on firm $i$’s cost distribution is centred enough, then firm $i$ would like to increase its marginal cost despite the uncertainty.

This result highlights the tradeoff created. An increase in marginal cost is beneficial for firm $i$. However, if it cannot signal this increase precisely enough then it will lose the advantage created as firm $j$ would prefer to be more aggressive. Hence for small levels of uncertainty created, an increase in own marginal cost is still beneficial for firm $i$.

![Figure 5.3: Effect of uncertainty and an increase in marginal cost](image)

Figure 5.3 shows the model in \{c_i, \sigma\} space. From Proposition 5.2, $\sigma$ is decreasing in $c_i$. From Proposition 5.5, $\sigma^U$ lies above $\sigma_2$ at all values of $c_i$. Consider case 2, the increase in $E(c_i)$ is high compared to the level of uncertainty, hence
firm \(i\) would choose to increase its marginal cost. In case 1 on the other hand, a low increase in expected costs coupled with a relatively higher variance will lead to aggressive behaviour from the rival, and hence firm \(i\) would choose not to increase its marginal cost. The results in section 5.4 are thus robust only to low levels of uncertainty.

### 5.7 Conclusions

It is in a firm’s strategic interest to increase its observable marginal cost in order to win competition for the market. An increase in costs reduces its profits irrespective of its action, but it reduces them asymmetrically. Profits from choosing high quantity fall relatively more than from those from choosing low quantity. Indifference and subsequent equilibrium requires its rival to make choosing low quantity less attractive for firm 1 - which is achieved by the rival itself choosing low quantity more often. Hence this asymmetry created changes the probability distribution of the rival’s strategies to a more ‘friendly’ one. Outsourcing is useful as it allows the firm to achieve this increase in costs through an upstream firm charging a wholesale price above the cost of production. Subsequently, ‘vertically integrated’ profits can be extracted through a fixed franchise fee.

Asymmetry in information and uncertainty on the part of the opponent increases aggression. If a firm cannot credibly communicate its change in marginal cost, then aggression due to uncertainty could overpower the friendly behaviour induced by increasing costs. As a result, a firm may not always want to increase its marginal cost. The previous result is therefore robust to only small levels of uncertainty.

There are of course, many issues that the paper does not consider. For starters,
we do not model network effects. We assume they exist and only model their consequences. Perhaps a micro founded theory of how network effects are generated would lead to mode industry specific conclusions. Also, as mentioned before, allowing stage 1 quantities to be continuous would allow for a change in costs to affect the strategy space, and that could give very different results. It should also be noted that we have modelled the bargaining process rather sparsely. By assuming that the fixed fee is paid up front we implicitly assume the firms have the ability to shell out large sums of money (note that $F$ includes the next period profit adjustment which could be substantial). So, if we allow for limited liability then our results may be quite different. Additionally, we have assumed a ‘winner take all policy’. This introduces discontinuity in the expected profits. If we allowed expected profits to be continuous, say by making the probability of winning the market proportional to the market share (by a Poisson or Logistic function), we could possibly generate pure strategy equilibria in the continuous case as well. This has some consequence on which equilibrium we choose to look at and could be an avenue worth exploring.
5.8 Appendix

5.8.1 Technical Proofs

Proof. of Proposition 5.1: From equation (5.1) we have

\[ \sigma_i = \frac{\Pi(q_H, q_C) + \frac{1}{2} \delta \Pi_M - \Pi(q_C, q_C)}{\Pi(q_H, q_C) - \Pi(q_C, q_C) - \Pi(q_H, q_H) + \Pi(q_C, q_H)} \]

Now given \( P = 1 - Q \) with \( Q = q_i + q_j \) and \( MC_i = MC_j = 0 \) we can find

\[ \Pi(q_C, q_C) = \frac{1}{9}, \quad \Pi(q_C, q_H) = \frac{1}{3} \left( \frac{2}{3} - q_H \right), \quad \Pi(q_H, q_C) = q_H \left( \frac{2}{3} - q_H \right) \]
\[ \Pi(q_H, q_H) = q_H (1 - 2q_H), \quad \Pi_M = \frac{1}{4} \]

Substituting for these we get

\[ \sigma = \frac{\delta}{8 \left( q_H - \frac{1}{3} \right)^2} - 1 \]

Note that for \( 0 < \sigma < 1 \) we require \( 8 \left( q_H - \frac{1}{3} \right)^2 < \delta < 16 \left( q_H - \frac{1}{3} \right)^2 \). It is easy to see that \( \frac{d\sigma}{d\delta} > 0 \).

Proof. of Proposition 5.2: From (5.2) we have

\[ \sigma_j = \frac{\Pi(q_H, q_C; c_i) + \frac{1}{2} \delta \Pi_M(c_i) - \Pi(q_C, q_C; c_i)}{\Pi(q_H, q_C; c_i) - \Pi(q_C, q_C; c_i) - \Pi(q_H, q_H; c_i) + \Pi(q_C, q_H; c_i)} \]

Now given \( P = 1 - Q \) with \( Q = q_i + q_j \) and \( MC_i = c_i, \ MC_j = 0 \) we can find

\[ \Pi(q_C, q_C; c_i) = \frac{1}{3} \left( \frac{1}{3} - c_i \right), \quad \Pi(q_C, q_H; c_i) = \frac{1}{3} \left( \frac{2}{3} - q_H - c_i \right) \]
\[ \Pi(q_H, q_C; c_i) = q_H \left( \frac{2}{3} - q_H - c_i \right), \quad \Pi(q_H, q_H; c_i) = q_H (1 - 2q_H - c_i), \quad \Pi_M = \frac{(1 - c_i)^2}{4} \]
Substituting for these we get

$$\sigma_j = \frac{\delta(1 - c_i)^2}{8(q_H - \frac{1}{3})^2} - \frac{c_i}{(q_H - \frac{1}{3})} - 1$$

Note that for $0 < \sigma_j < 1$ we require

$$\frac{8}{(1 - c_i)^2} \left( q_H - \frac{1}{3} \right) \left( q_H - \frac{1}{3} - c_i \right) < \delta < \frac{16}{(1 - c_i)^2} \left( q_H - \frac{1}{3} \right) \left( q_H - \frac{1}{3} - c_i \right)$$

It is easy to see that

$$\frac{d\sigma_j}{dc_i} = -2 \frac{\delta(1 - c_i)}{8(q_H - \frac{1}{3})^2} - \frac{1}{(q_H - \frac{1}{3})} < 0.$$

Proof. of Proposition 5.3: The expected profits of firm $i$ can be calculated as:

$$E\Pi_i = (1 - \sigma_j) \left[ \Pi(q_C, q_C; c_i) + \frac{\delta}{2} \Pi_M(c_i) \right] + \sigma_j \Pi(q_C, q_H; c_i)$$

$$= \left[ \Pi(q_C, q_C; c_i) + \frac{\delta}{2} \Pi_M(c_i) \right] - \sigma_j \left[ \left\{ \Pi(q_C, q_C; c_i) + \frac{\delta}{2} \Pi_M(c_i) \right\} - \Pi(q_C, q_H; c_i) \right]$$

$$= \frac{1}{3} \left( \frac{2}{3} - c_i \right) + \frac{\delta(1 - c_i)^2}{8} - \left\{ \frac{\delta(1 - c_i)^2}{8 \left( q_H - \frac{1}{3} \right)^2} - \frac{c_i}{(q_H - \frac{1}{3})} - 1 \right\}$$

$$\cdot \left\{ \frac{1}{3} \left( \frac{2}{3} - c_i \right) + \frac{\delta(1 - c_i)^2}{8} - \frac{1}{3} \left( 2q_H - c_i \right) \right\}$$

$$\frac{dE\Pi_i}{dc_i} = -\frac{1}{3} - \frac{\delta(1 - c_i)}{4} + \frac{1}{(q_H - \frac{1}{3})^2}$$

$$\cdot \left\{ \frac{\delta(1 - c_i)^2}{8} - \left( q_H - \frac{1}{3} \right)^2 - c_i \left( q_H - \frac{1}{3} \right) \right\} \left\{ \frac{\delta(1 - c_i)}{4} \right\}$$

$$+ \frac{1}{(q_H - \frac{1}{3})^2} \left\{ \frac{1}{3} \left( q_H - \frac{1}{3} \right) + \frac{\delta(1 - c_i)^2}{8} \right\} \cdot \left\{ \frac{\delta(1 - c_i)}{4} + \left( q_H - \frac{1}{3} \right) \right\}$$

Which is positive for small $q_H$ and small $c_i$. The region representing $\frac{dE\Pi_i}{dc_i} > 0$ in $q_H, c_i$ space is given by:

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Proof. of Proposition 5.4: Profits from outsourcing can be given by:

\[ E\Pi_{Ui} = w \left( (1 - \sigma)q_C + \sigma q_M + \frac{\delta(1 - w)}{2} \right) + F \]
\[ E\Pi_{Di} = (1 - \sigma_j)[\Pi(q_C, q_C; w) + \delta \Pi_M(w)] + \sigma_j[\Pi(q_C, q_H; w)] - F \]
\[ TS_i = (1 - \sigma_j)[\Pi(q_C, q_C; 0) + \delta \Pi_M(0)] + \sigma_j[\Pi(q_C, q_H; 0)] \]

Now, at \( q_H \) equal to the monopoly quantity (\( q_M = 1/2 \)), \( \sigma_j \) is given by

\[ \sigma_j = \frac{9\delta(1 - w)^2}{2} - 6w - 1 \]

This is bounded by zero and one as long as

\[ \frac{3\delta + 2 - \sqrt{4 + 16\delta}}{3\delta} < w < \frac{3\delta + 2 - \sqrt{4 + 14\delta}}{3\delta} \]

Hence the firms will set

\[ w^* = \frac{3\delta + 2 - \sqrt{4 + 14\delta}}{3\delta} \]

The precise value of \( F^* \) however will depend on the distribution of bargaining power. If firm \( U_i \) has all the bargaining power, then it will set the franchise fee \( F^* \) to extract all the joint profits and give the downstream firm zero profit in
expectation. This will be the maximum value of $F^\ast$.

\[
E\Pi_{Di} = \frac{2\sqrt{4 + 14\delta} - 4 - \delta}{36\delta} - F_{Max}^\ast = 0 \implies F_{Max}^\ast = \frac{2\sqrt{4 + 14\delta} - 4 - \delta}{36\delta}
\]

If, on the other hand, all the bargaining power lies with the downstream firm then it will set $F^\ast$ to give the upstream firm zero profit. This will be the lowest value of $F^\ast$.

\[
E\Pi_{Ui} = F_{Min}^\ast + \frac{(3\delta + 2 - \sqrt{4 + 14\delta})(9\delta + 2\sqrt{4 + 14\delta} - 2)}{36\delta} = 0
\]

\[\implies F_{Min}^\ast = \frac{(3\delta + 2 - \sqrt{4 + 14\delta})(9\delta + 2\sqrt{4 + 14\delta} - 2)}{36\delta}
\]

It follows that if both the upstream and downstream firms have some bargaining power, then the value of $F$ chosen will be such that $F_{Min}^\ast < F^\ast < F_{Max}^\ast$. Firm $i$ will choose to outsource if

\[
E\Pi_O > E\Pi_{N,O}
\]

\[\implies F^\ast + \frac{(3\delta + 2 - \sqrt{4 + 14\delta})(9\delta + 2\sqrt{4 + 14\delta} - 2)}{36\delta} > \frac{1}{6} - \frac{9\delta^2}{16}
\]

and this holds if $F^\ast > \hat{F}$ where

\[
\hat{F} = \frac{1}{6} - \frac{9\delta^2}{16} - \frac{(3\delta + 2 - \sqrt{4 + 14\delta})(9\delta + 2\sqrt{4 + 14\delta} - 2)}{36\delta}
\]

**Proof.** of Corollary 5.1: From above,

\[
\frac{dF^\ast}{d\delta} = -\frac{9\delta}{8} + \frac{(3\delta + 2 - \sqrt{4 + 14\delta})(9\delta + 2\sqrt{4 + 14\delta} - 2)}{36\delta^2}
\]

\[\quad - \frac{1}{36\delta} \left[ (9 + \frac{14}{\sqrt{4 + 14\delta}})(3\delta + 2 - \sqrt{4 + 14\delta}) \right]
\]

\[\quad + \frac{1}{36\delta} \left[ (3 - \frac{7}{\sqrt{4 + 14\delta}})(9\delta + 2\sqrt{4 + 14\delta} - 2) \right] < 0
\]

**Proof.** of Proposition 5.5: The following analysis of the Bayesian Nash equilib-
rium of the game shows that the equilibrium probability of firm 2 going high increases. Define \( \Pr(q_i = \frac{1}{2}) = \sigma^U_i \). For an equilibrium in mixed strategies, firm \( i \) will choose \( \sigma^U_i \) such that

\[
(1 - \sigma^U_i) E \left[ \left( \frac{1}{9} + \frac{\delta}{8} \right) - \left( \frac{1}{12} + \frac{\delta}{4} \right) \right] + \sigma^U_i E \left[ \frac{1}{18} - \frac{\delta}{8} \right] = 0
\]

\[
\implies \sigma^U_i = \sigma = \frac{1}{2}(9\delta - 2)
\]

Hence the strategy of firm \( i \) is unchanged. Firm \( j \) chooses \( \sigma^U_j \) in such a way that \( \sigma^U_j \) solves:

\[
(1 - \sigma^U_j) E \left[ \left( \frac{1 - 3c}{9} + \frac{\delta(1-c)^2}{8} \right) - \left( \frac{1 - 6c}{12} + \frac{\delta(1-c)^2}{4} \right) \right] + \sigma^U_j E \left[ \frac{1 - 6c}{18} - \frac{\delta(1-c)^2}{8} + \frac{c}{2} \right] = 0
\]

\[
\implies \sigma^U_j = \frac{1}{2}(9\delta \eta + 9\delta - 2)
\]

Note that the extra term \( 9\delta \eta \) which is always positive, implying \( \sigma^U_2 \geq \sigma^S \).

**Proof.** of Proposition 5.6: Assume now that \( E(C(q_i)) = c \), and \( Var(C(q_i)) = \eta \), \( \implies E[(C(q_i))^2] = \eta + c^2 \) and define \( \Pr(q_j = q_M) = \sigma^U_j \). From before, firm \( i \) chooses \( \sigma^U_i = \sigma \). Now firm \( j \) chooses \( \sigma^U_j \) that solves

\[
(1 - \sigma^U_j^*) E \left[ \left( \frac{1 - 3c}{9} + \frac{\delta(1-c)^2}{8} \right) - \left( \frac{1 - 6c}{12} + \frac{\delta(1-c)^2}{4} \right) \right] + \sigma^U_j^* E \left[ \frac{1 - 6c}{18} - \frac{\delta(1-c)^2}{8} + \frac{c}{2} \right] = 0
\]

\[
\implies \sigma^U_j^* = \frac{1}{2}(9c^2\delta - 18c\delta - 12c + 9\delta \eta + 9\delta - 2)
\]

As long as \( \frac{3\delta + 2 - \sqrt{4 + 16\delta - 9\delta^2\eta}}{3\delta} < c < \frac{3\delta + 2 - \sqrt{4 + 14\delta - 9\delta^2\eta}}{3\delta} \), now, firm \( i \) would like to increase its marginal cost despite the uncertainty if \( \sigma^U_j^* \leq \sigma \), which holds if \( \eta \leq \eta^* \). \( ^{24} \)

This gives us the result

\[
\eta^* = \frac{18c\delta + 12c - 9c^2\delta}{9\delta}
\]
5.8.2 A continuous symmetric model

Consider the case where we have continuous quantities. The structure of the game is as in section 5.3. In period one - the firms simultaneously compete in quantities, and realise profits. This is short run competition for the market. In period two - the firm that produced a higher quantity in period one subsequently wins the whole market and makes monopoly profits into the future. In this case it can be shown that there are no pure strategy equilibria. This is because if firm 1 produces $q$, then firm 2 will produce $q + \varepsilon$ to make profits this period and win the market in period two. Thus there will be no pure strategy equilibria. There is only a mixed strategy equilibrium where if one firm goes high, then it is in the interest of the other to go low, and vice versa. The mixed strategy equilibrium will now be a probability distribution over the continuous strategy space. Call the cumulative distribution function $F$ (not to be confused with the fixed bargained fee in section 5.5). Let the firms mix over a range $[q_L, q_H]$. Thus we have,

$$q \sim F[q_L, q_H]$$

Define $T = \int_{q_L}^{q_H} q \ast f(q) \, dq$ as the expectation of each firm’s output. Note that in this symmetric game, $T_i = T_j = T$. The expected profits can be found to be:

$$E\Pi_i(\text{stage 1}) = (1 - q_i - T)q_i$$
$$E\Pi_i(\text{stage 2}) = \delta \ast \Pr(q_i > q_j)(\text{Monopoly profit}) = \delta \ast \Pr(q_i > q_j)\frac{1}{4}$$
$$E\Pi_i(\text{Total}) = (1 - q_i - T)q_i + \delta \ast \Pr(q_i > q_j)(\text{Monopoly profit})$$
$$= (1 - q_i - T)q_i + \delta \ast \Pr(q_i > q_j)\frac{1}{4}$$

Now, we can make an educated guess about the lowest quantity a firm will produce for sure. Suppose firm 1 knew that firm 2 was going to win the market for sure in period 2. Given that firm 2 is winning the market with probability 1 in period 2, the best firm 1 can do, is to just best respond to the expectation of firm 2’s quantity ($= T$). This is the lowest quantity firm 1 will produce. Moreover,
this works because, given that firm 1 is best responding to $T$, it indeed has zero probability of winning the market in stage 2. Firm 1 solves:

$$\max \Pi_1 = q_1(1 - q_1 - T) \quad F.O.C. \implies q_1 = \frac{1 - T}{2}$$

The argument above holds for firm 2 as well. Thus, firm 2 solves:

$$\max \Pi_2 = q_2(1 - q_2 - T) \quad F.O.C. \implies q_2 = \frac{1 - T}{2}$$

Thus we have

$$\min \Pi_1 = \min \Pi_2 = \left(\frac{1 - T}{2}\right)^2$$

Now, for a mixed strategy equilibrium, we require that expected profits of a firm be equal at every choice of the strategy space that the firm is mixing over. Thus

$$E\Pi_i = (1 - q_i - T)q_i + \delta \cdot \Pr(q_i > q_j)\frac{1}{4} = K$$

where $K$ is some constant. Note that $\Pr(q_i > q_j) = F(q_i) = F(q)$. Now all we need to do is get a value for $K$ so we can solve out for $F(q)$. But we already know that $\min \Pi_1 = \min \Pi_2 = \left(\frac{1 - T}{2}\right)^2$ hence this must be equal to $K$. Substituting and solving:

$$(1 - q_i - T)q_i + \delta \cdot F(q)\frac{1}{4} = \left(\frac{1 - T}{2}\right)^2 \implies F(q) = \frac{(1 - T - 2q)^2}{\delta} \implies f(q) = \frac{4(T - 1 + 2q)}{\delta}$$

We know that $F(q_L) = 0$, and $F(q_H) = 1$. So we can calculate: $q_L = \frac{1 - T}{2}$ as
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above and \( q_H = \frac{1 - T + \sqrt{\delta}}{2} \). Using This we can calculate \( T \):

\[
T = \int_{q_L}^{q_H} q \ast f(q) dq = \int_{\frac{1-T}{2}}^{\frac{1-T + \sqrt{\delta}}{2}} q \left( \frac{4(T - 1 + 2q)}{\delta} \right) dq = \frac{4}{\delta} \left[ 2(T - 1)q^2 + \frac{8q^3}{3} \right]_{\frac{1-T}{2}}^{\frac{1-T + \sqrt{\delta}}{2}}
\]

\[
= \frac{4}{\delta} \left[ 2(T - 1) \left( \frac{1-T + \sqrt{\delta}}{2} \right)^2 + \frac{8(1-T + \sqrt{\delta})^3}{3} - (2(T - 1) \left( \frac{1-T}{2} \right)^2 + \frac{8(1-T + \sqrt{\delta})^3}{3} \right]
\]

\[
= \frac{3 + 2\sqrt{\delta}}{9}
\]

and this gives:

\[
q_L = \frac{1 - \left( \frac{3 + 2\sqrt{\delta}}{9} \right)}{2} = \frac{3 - \sqrt{\delta}}{9} \quad \text{and} \quad q_H = \frac{1 - \left( \frac{3 + 2\sqrt{\delta}}{9} \right) + \sqrt{\delta}}{2} = \frac{6 + 7\sqrt{\delta}}{18}
\]

\[
(q_L, q_H) = \left( \frac{3 - \sqrt{\delta}}{9}, \frac{6 + 7\sqrt{\delta}}{18} \right)
\]

Also we can calculate:

\[
F(q) = \frac{1}{\delta} (1 - T - 2q)^2 = \frac{1}{\delta} \left( 1 - \left( \frac{3 + 2\sqrt{\delta}}{9} \right) - 2q \right)^2 = \frac{4}{81\delta} (3 - \sqrt{\delta} - 9q)^2
\]

\[
f(q) = \frac{4}{\delta} (T - 1 + 2q) = \frac{4}{\delta} \left( \left( \frac{3 + 2\sqrt{\delta}}{9} \right) - 1 + 2q \right) = \frac{8}{9\delta} (3 - \sqrt{\delta} - 9q)
\]

and \( f(q) \geq 0 \) when \( q \in (q_L, q_H) \). Note however, for positive price we need \( 1 - 2T > 0 \iff 1 - 2 \left( \frac{3 + 2\sqrt{\delta}}{9} \right) > 0 \iff \delta < \frac{9}{16} \). Thus large values of \( \delta \) do not support the mixed equilibrium. This is different from what we observed in Proposition 5.1 which argued that even too small values of \( \delta \) did not allow the mixed equilibrium to exist. The difference can be attributed to the continuous nature of the strategy space. Here in this model, a firm can just produce \( \epsilon \) more than its rival and win the market. It thus has to sacrifice lesser profits this period for a win in the next period. Hence small \( \delta \) is not such a big problem here. Note however that costs here have been normalized to zero. Thus what we have calculated here is in a sense the profit margin - which in general need not be positive.

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High valuation of the future could just imply that firms are willing to sell at a price lower than their marginal cost in hope that the future profits will make up for the loss.

It can also be shown that the expected quantity of a firm is increasing in $\delta$, and its expected profits are decreasing in $\delta$. Using $T = \frac{3 + 2\sqrt{\delta}}{9} \Rightarrow \frac{dT}{d\delta} = \frac{1}{9\sqrt{\delta}} > 0$. This is intuitive. As firms become value the second period more, they are more willing to increase quantity. Using $E\Pi = \left(\frac{3 - \sqrt{\delta}}{9}\right)^2 \Rightarrow \frac{dE\Pi}{d\delta} = -\frac{(3 - \sqrt{\delta})}{81\sqrt{\delta}} < 0$. Thus as firms become more likely to increase their quantity, their expected profits fall.

The expected profits can be shown to be convex in $\delta$. Using $E\Pi = \left(\frac{3 - \sqrt{\delta}}{9}\right)^2 \Rightarrow \frac{d^2E\Pi}{d\delta^2} = \frac{3}{162\delta\sqrt{\delta}} > 0$. In addition, the range of $q$ over which firms mix is increasing in $\delta$. Since $q_H - q_L = \frac{1}{18}(7\sqrt{\delta} - 2\delta) \Rightarrow \frac{d(q_H - q_L)}{d\delta} = \frac{1}{18}\left(\frac{7}{2\sqrt{\delta}} - 2\right) > 0$. 
Bibliography


