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ABSTRACT

We consider a Hicksian, two-period economy with an exhaustible resource. There is no forward resource market in the first period; traders form expectations about the second period resource price. Current supply plans are defined implicitly by an asset adjustment mechanism. The current, spot resource price adjusts in proportion to the current, ex-ante excess demand for the resource. We are concerned with the stability of the first-period (short-run or temporary) equilibrium which is defined as the point where asset and spot markets clear and no revision of expectations occurs.

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INTRODUCTION

Our purpose in writing this paper is to relax two assumptions which dominate the theory of exhaustible resources and examine the implications for the stability of equilibrium. The first assumption postulates that a complete set of forward markets exists. The second postulates that asset and spot commodity markets clear instantaneously.

Neither of these assumptions hold in the real world. Forward markets are incomplete and cover only some months ahead. This is particularly the case for the most important exhaustible resource, crude oil. Under these circumstances, traders' expectations about future spot prices become important. Since a unit of an exhaustible resource is like an asset (it can either be sold now in the spot market or retained in order to be sold in the future) current supply decisions are guided by the expected resource price. However, the rule governing traders' expectations is not something that can be modelled easily. At best one can hope that traders form expectations taking into account the state of the resource market in the future.

The assumption of instantaneous market clearing provides analytical convenience but cannot be justified on any other grounds.

In a decentralised environment, decisions about demand and supply are taken by different agents. Instantaneous spot market clearing bypasses a very important question: how spot prices change outside equilibrium, ie when individual plans are not consistent in the aggregate.

A different argument applies to asset markets. In economies with incomplete forward markets, the resource own rate of return depends on the expected resource price. Hence, instantaneous asset market clearing imposes certain a priori restrictions on the formation of expectations. This is clearly an undesirable feature of a model which aims to explain how expectations are formed and revised outside equilibrium.

It seems more natural to describe a decentralised economy by a set of adjustment processes rather than a set of market clearing conditions. If such an approach is adopted stability becomes an important issue. This will particularly be the case if the equilibrium is characterised by some efficiency properties. Stability will then imply that the economy will operate efficiently for most of the time.

The distinguishing characteristic of this paper is that it studies the stability properties of an economy in which there are no forward markets and spot and asset markets adjust simultaneously. The absence of a complete set of forward markets and its implications for stability has been discussed in the context of the theory of exhaustible resources (see, among others, Stiglitz, 1974; Dasgupta and Heal, 1979) under the assumption of instantaneous spot and asset market clearing. It has been shown that a sequence of temporary equilibria along which all markets clear will not bring the economy to long-run competitive equilibrium if the initial resource price is too high or too low. In this paper we are concerned with a different question; in effect, we want to know how each point on the sequence of temporary equilibria is achieved. Suppose that the economy starts from a position where spot and asset markets are outside equilibrium. In addition, let us postulate that adjustment occurs according to a certain process. Will then the economy converge to a point which belongs to the sequence of temporary equilibria? In other words, we are concerned with short-term stability and not with the convergence of the entire sequence of temporary equilibria. We believe that this concept is more relevant to short-term issues such as resource market instability and stabilisation policy. However, we will not neglect "long-term" considerations in this paper. On the contrary we will show that expectations based on future scarcity have strong implications for the stability of the temporary equilibrium.

The only systematic effort to introduce disequilibrium elements into the theory of exhaustible resources is that of Heal (1975), (1981). Although the possibility of simultaneous adjustment in spot and asset markets is considered, the main thrust of his analysis assumes instantaneous spot market clearing. In effect, he is only concerned with asset market adjustment. A rather unsatisfactory feature of Heal's model is that the underlying equilibrium concept is not clear. In other words, is he concerned with the short-run or long-run equilibrium? In addition, the way the model is specified does not allow for a rational expectations equilibrium. Despite all these points, our model has many common characteristics to that of Heal and ought to be considered as complementary to it.

The rest of the paper is organised as follows. The first part sets out the assumptions and specifies a competitive model of adjustment. Local stability results are derived under two alternative assumptions on the formation of expectations. The first postulates that expectations are adaptive while the second postulates that expectations are consistent with the intertemporal structure of the model. In the second case stability implies that the economy converges to the rational expectations equilibrium. The second part contains two sections. In the first, local stability results are derived when the basic model is extended to include traders with market power. The under-lying short-term equilibrium is the Nash-Cournot rational expectations, temporary equilibrium with a dominant producer and a competitive fringe. We think that this section's model of disequilibrium adjustment provides a theoretical explanation of the fact that "small" oil producers become price makers when the market is in disequilibrium despite the presence of the dominant producer. Finally, in the last section we examine the conditions under which the competitive, temporary equilibrium is locally stable when the possibility of new resource discoveries is introduced. The purpose of this section is to show the fallacy of an argument that prevailed in the oil industry and was used as an explanation for the behaviour of the oil price in the pre-1970s period.

In what follows we will refer to a unit of an exhaustible resource as a barrel of oil. However, most of the arguments apply to any exhaustible resource. Some parts of the model are also relevant to the analysis of inventory behaviour for any non-depreciating commodity. In this context, it is only through the formation of expectations that an exhaustible good differs from a non-exhaustible one.

1. A COMPETITIVE MODEL OF ADJUSTMENT

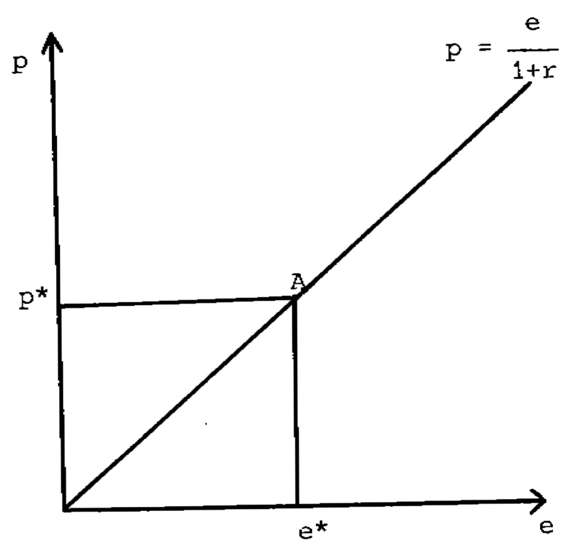
We will consider a two-period economy with no forward markets. Traders observe current prices and form expectations about prices in the next period ("future"). The current price of an oil barrel in terms of a numeraire asset will be denoted by p and its expected price by e. All traders in the oil market expect with confidence that the future oil price will be e. In the background of the model the numeraire asset market will clear and offer a one period rate of return r. Participants in the oil market take r as exogenously given.

Assuming perfect competition and no extraction and storage costs, the owner of an oil barrel can either sell it now and get a return equal to p or wait and sell it in the future at an expected price e. He will be indifferent between the two if their present value is the same.

$$p = \frac{e}{1+r} \qquad (1)$$

Let us assume that the price which clears the oil market in the first period (that is, the price consistent with spot market equilibrium) is p^*. The expected price consistent with condition (1) is $e^* = p^*(1+r)$. The pair (p^*,e^*) is point A in Figure 1.

Figure 1



(p*, e*) will be a rational expectations equilibrium if it is consistent with market clearing over both periods; in this case the position of (p*, e*) will be given by the condition that oil reserves will be exhausted by the end of the second period (not depicted in Figure 1).

We will be interested in the local stability of point A; ideally, the equilibrium price path consists of a sequence of temporary equilibria similar to A, and we would like to know if the economy will return to this sequence after a "shock" has moved it away from it.

First we will specify the behaviour of the economy outside equilibrium. For this purpose it would be helpful to imagine each period as being a finite time interval, within which price and quantity adjustments are taking place. In a decentralised environment, similar to that prevailing in the current oil market, one would not expect ex ante (planned) demand to match ex ante supply instantaneously. To capture the process that characterises decentralised markets we assume that, within the first period, the current (spot) price adjusts according to the following rule[1]

$$\dot{p} = aE \tag{2}$$

where a is a positive constant and E is the ex-ante current excess demand function. It is defined as

$$E = C(p) - S \tag{3}$$

where C(p) is total ex-ante oil demand and S is total ex-ante oil supply. S will be such as to satisfy the following asset adjustment rule:

$$\dot{S}[(1+r)\ p-e] > 0,\ p \neq \frac{e}{1+r}$$

$$\dot{S} = 0,\ p = \frac{e}{1+r} \tag{4}$$

(4) says how, within the first period, resource owners revise their plans concerning the optimal level of future reserves. If the current price exceeds the present value of the price expected to rule in the future, they revise their plans downwards (ie they would like to increase their current supply) and vice-versa. If the present value of these two prices is the same, they do not alter their plans.[2]

It might be useful to make some remarks about rule (2). We do not intend to present it as a tâtonnement process but as a rule mimicking certain elements of an actual adjustment mechanism. A possible interpretation of it (though not necessarily the only one and certainly not a rigorous one) might be as follows. Assume that, at a certain point in time, ex-ante oil supply exceeds ex-ante demand. If the spot price remains unchanged the spot market will clear by quantity rationing. Some resource owners will be rationed to sell (deplete at a rate) less than they had planned or, what amounts to the same thing, to carry forward an amount of reserves which, in general, differs from that derived by the stock adjustment rule (4). Under these circumstances, a competitive resource owner who has no ex-ante market power will reduce the price, perhaps by a very small amount, in order to avoid rationing.[3] This is exactly Arrow's (1959) well known argument that competitive producers become price makers when markets are outside equilibrium.

The actual rate of depletion will be given by the "short" side of the market[4] and will not appear as an explicit variable of this model. However, if traders form consistent (rational) expectations based on future scarcity they will take it into account by forming an estimate about the oil demand (consumption) function. We will formalise this idea more precisely later in the paper.

We now turn to the formation of expectations e. Traders' expectations are not something which one can model with any degree of accuracy; here we will examine the implications of two types of assumptions that are widely used in temporary equilibrium theory. The first type postulates that traders form expectations by extrapolating current market signals. The second postulates that expectations are consistent with the intertemporal structure of the model.

Adaptive Expectations

We assume that, within the first period, expectations are formed according to the following rule:

$$\dot{e} = c\ (p - \frac{e}{1+r}),\ c > 0 \qquad (5)$$

We may interpret the above equation as follows. Since $(1+r)\ p-e>0(<0)$ is equivalent to $r-[(e/p)-1]>o(<o)$, and $(e/p)-1$ is the expected rate of return from holding a barrel of oil for one period, (5) postulates that the expected price adjusts in such a way as to equalise the rates of return on the two alternative assets.[5] It is an adaptive expectations rule since an increase (decrease) in p implies an increase (decrease) in e.

It is not easy to derive global stability results for the system of equations (2), (4) and (5). However, limiting the analysis to local stability only, things become much easier. First we can eliminate $\dot{S}$. We will consider S as a function of p and e and use equation (4) to find the (sign of the) partial derivatives S_p, S_e (and, through (3), E_p, E_e) at the temporary equilibrium. Denoting partial derivatives by subscripts, equation (3) yields $E_p=C_p-S_p$. Assuming a well-behaved oil consumption function (eg excluding increasing returns to scale in refineries), we have $C_p<0$. Evaluating partial derivatives at the temporary equilibrium p^*,e^* (that is, at the point where $\dot{S}=0$) it follows from (4) that $S_p>0$. This is obvious since an increase in the current price p keeping the expected price e equal to $(1+r)p^*$ implies that the present value of the expected price is less than the current price $(p^*<p^*+dp)$; as a consequence,

resource owners wish to reduce their stocks (ie to increase their current supply). Hence $E_p<0$. Using the same argument, it is also straightforward that $E_e>0$. We have thus established that, at a neighbourhood of the temporary equilibrium

$$E_p < 0, \; E_e > 0 \qquad (6)$$

Now we will employ the phase diagram technique to obtain stability results for the system of equations (2) and (5) using the results in (6).

Consider the (p,e) locus along which E(p,e)=0. Taking the total derivative of both sides of this equation and using (6) we obtain

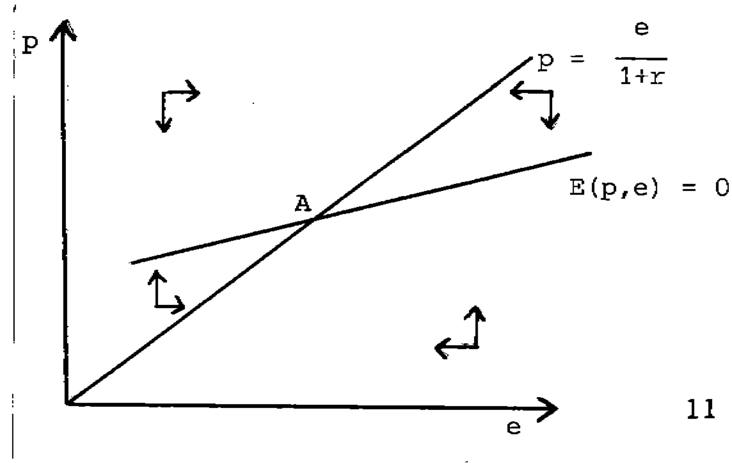
$$\left[\frac{dp}{de}\right]_{E=0} = -\frac{E_e}{E_p} > 0 \qquad (7)$$

Hence, near the temporary equilibrium, the E (p,e)=0 locus has a positive slope. Its position relative to the (1+r) p=e locus depends on the sign of $|E_p| - (1+r) E_e$. We first assume

$$|E_p| > E_e\,(1+r) \qquad (8)$$

Under assumption (8), the phase diagram of the system of differential equations (2) and (5) is as in Figure 2; it follows that A is a (locally) stable equilibrium.[6]

Figure 2

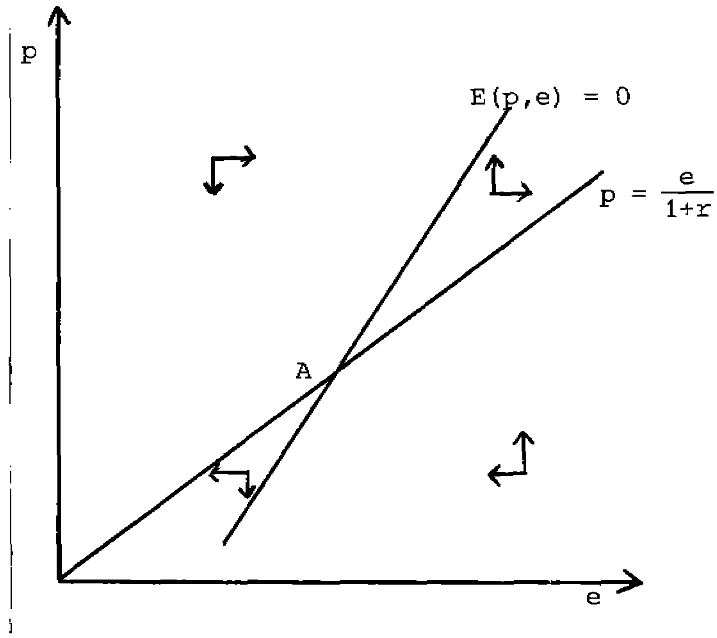


If, instead of (8), we assume

$$|E_p| < E_e\ (1+r) \tag{9}$$

the phase diagram will be as in Figure 3 and A will be an unstable temporary equilibrium (In fact, A will be a saddle point[7]).

Figure 3



These results may be interpreted as follows. If a barrel of oil now is considered to be a different commodity to a barrel of oil in the future, assumption (8) postulates that the own price effect "dominates" the cross price effect; in fact (8) looks like the well-known "Diagonal Dominance" condition (Arrow and Hahn, 1971). Hence it is not surprising that the equilibrium is stable. Adaptive expectations imply inelastic expectations in the Hicksian sense and, combined with Diagonal Dominance in the commodity market, the result follows[8] (Arrow and Hahn, op cit).

One might ask which of the two conditions (8) or (9) is the most plausible. For well-behaved oil consumption functions and a constant speed of stock adjustment (that is, when (4) takes the special form $\dot{S} = d[(1+r)\ p - e]$, $d > o$) it is easy to show that, close to the temporary equilibrium, condition (8) is satisfied. However, non-linear stock adjustment rules cannot be excluded and, in this case, condition (8) might or might not be satisfied.[9]

Even if A (in Figure 2) is a stable temporary equilibrium, one should be aware that it is a myopic one. Expectations about future prices are independent of the state of the market in the future; that is, future resource scarcity has not been taken into account. In spite of the fact that the economy is competitive, a sequence of temporary equilibria like A might lead to inefficiency in resource use. The problem is familiar from the literature on exhaustible resources. In the absence of a complete set of forward markets, the initial resource price might be too high or too low. However, it has been suggested that many important phenomena and episodes in the oil market can be explained by the fact that market participants had expected current price trends to prevail in the future (Dasgupta and Heal, op cit, ch 15).

A natural substitute for the missing forward markets is to assume that traders expect future prices to be consistent with the state of future excess demand. In the context of exhaustible resources the difficulty with this assumption is that information about relevant parameters (eg the level of stocks) is far from perfect and, in most cases, not even public. It is well-known that in such circumstances a market equilibrium might not exist so that the system might be inherently unstable. (For a neat exposition of the arguments see Dasgupta and Heal, op cit, ch 14). But even when information is freely available, it is not clear whether "consistent" expectations (a precise definition will be given below) produce a stable temporary equilibrium. In the next section we will examine this question in detail.

Consistent Expectations

We assume that, in the first period, traders form an estimate of the consumption function, which we denote by $\hat{C}(\cdot)$. Hence, if $\bar{R}$ is the initial oil stock and $\hat{C}(p)$ is estimated oil consumption in the first period, the future excess demand is expected to be

$$\hat{E}(p,e) = \hat{C}(e) - [\bar{R}-\hat{C}(p)] \tag{10}$$

We now assume that expectations about the future price e are formed according to the following rule

$$\dot{e} = b\hat{E}(p,e) \ , \ b>0. \tag{11}$$

Rule (11) is an algorithm about the expected price e. Traders estimate future excess demand, and adjust their estimate of the future price accordingly. It might be helpful to trace the precursors of this assumption. The most obvious one is the concept of rational expectations, according to which traders expect the future price to clear the market in the second period. Rule (11) is a more realistic version of this principle since it does not require that traders know the rational expectations equilibrium vector $(\tilde{p}, \tilde{e})$. Our decentralised economy will converge to it provided that the system of differential equations (2) and (11) is stable. Also, certain authors who have contributed to the theory of exhaustible resources have suggested that, in the absence of forward markets, expectations about future prices might be based on price as well as on quantity signals. For instance, Dasgupta and Heal (op cit) have made the expected rate of return from holding oil stocks a function of the resource-utilisation ratio; Heal (1975), (1981) has assumed that it is a combination of past rates of return and the rate of resource depletion. The main difference between those assumptions and (11) is in the underlying equilibrium concept itself. Equilibrium in those models occurs at the point of exhaustion only, while our attention is on short-run (temporary) equilibria.

The dynamic adjustment of the economy is now described by

$$\begin{aligned} \dot{p} &= aE \\ \dot{e} &= b\hat{E} \end{aligned} \tag{12}$$

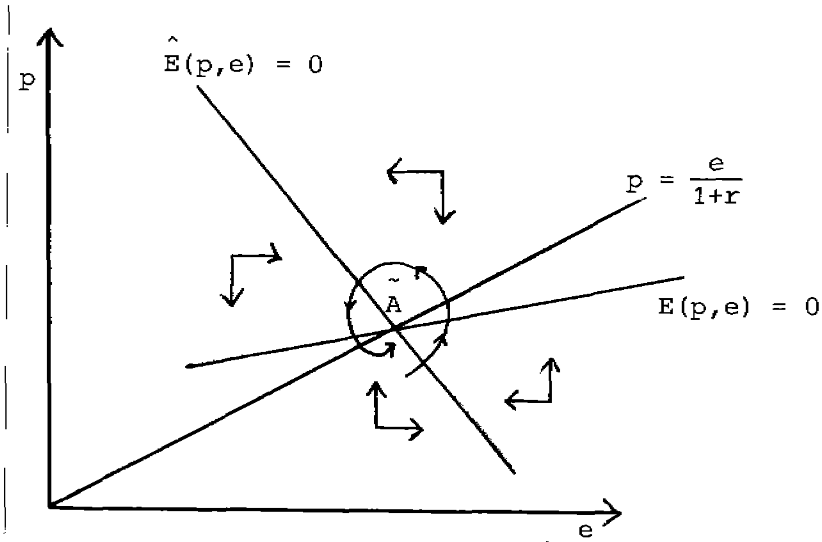
where E has been defined in (3). To construct the phase diagram of (12) we need to determine the slope of the $\hat{E}$=o locus. Since $\hat{E}_p = \hat{C}_p < o$, $\hat{E}_e = \hat{C}_e < o$ (see (10)), it follows that

$$\left[\frac{dp}{d_e}\right]_{\hat{E}=o} = - \frac{\hat{E}_e}{\hat{E}_p} < 0 \tag{13}$$

The intuition behind (13) is clear. A high current oil price implies low current consumption, high future stocks and, therefore, a low future oil price in order to raise future consumption to exhaust the stocks.

The E=o locus has a positive slope (a result established earlier) and its position relative to the asset market equilibrium locus $(1+r)\,p = e$ depends on the sign of $|E_p| - (1+r)\,E_e$. If $|E_p| > (1+r)\,E_e$, the phase diagram of the system of differential equations (12) will be as in Figure 4. It follows that the equilibrium (point A) is stable.

Figure 4



By constructing the phase diagram under the condition $|E_p| < (1+r)\,E_e$ it is straight forward to show that A is again a stable equilibrium. This does not seem surprising. Since traders have consistent expectations about future prices they do not drive the current price away from its equilibrium level; a high current price implies future excess supply and a lower future price; that induces traders to sell now, driving the current price down.[10]

In effect we have shown that the rational expectations, temporary equilibrium is locally stable. However, this does not say anything about the pattern or the speed of convergence. It is clear (see Figure 4) that convergence might occur through dampening oscillations, or that the price might over-react to a supply or demand shock; it is only at the limit that the system converges to equilibrium.

It might be argued that given the simplicity of the present partial equilibrium model, there is no reason why the market clearing future price should not be computed instantaneously for any observed, current price p. In fact, adopting this (stronger) version of consistent expectations will enable us to consider current supply plans as an explicit state variable of the model, instead of treating it as a function of p and e using the equilibrium (local) approximations derived before (see (6)).

If the market clearing future price is computed instantaneously (that is, if traders solve for the rational expectations price e instantaneously) the dynamic adjustment of the economy will be described by the following equations

$$\dot{p} = a[C(p) - S], \tag{14}$$

$$\dot{S} = d[(1+r)\, p - e], \quad d > 0, \tag{15}$$

$$\hat{E} = \hat{C}(e) + \hat{C}(p) - \bar{R} = 0 \tag{16}$$

Equation (14) is similar to equation (2). Equation (15) is similar to (4) but with a constant speed of adjustment. In fact, none of the results will change if we adopt equation (4) instead. Finally, equation (16) defines the rational expectations price e given p, $\bar{R}$ and an estimate of the demand function $\hat{C}(.)$.

From (16), e=e(p) with $e_p = (-\hat{E}_p/\hat{E}_e) < 0$ (see also equation (13)). Substituting e(p) for e in (15) we obtain a dynamic system consisting of equations (14) and (15) and the state variables p and S. Formally, this system looks similar to that of Heal (1981, p204). It differs though in one very important respect; it considers the expected price e as endogenous to the model rather than as an exogenous constant.

Let us denote by p^* the solution of the equation $p(1+r) = e(p)$. Since $e_p < 0$, $p > p^*$ implies $(1+r)p>e(p)$ and, from equation (15), $\dot{S} > 0$. Similarly, $p < p^*$ implies $\dot{S} < 0$. Hence, the phase diagram of the system of equations (14) and (15) will be as follows

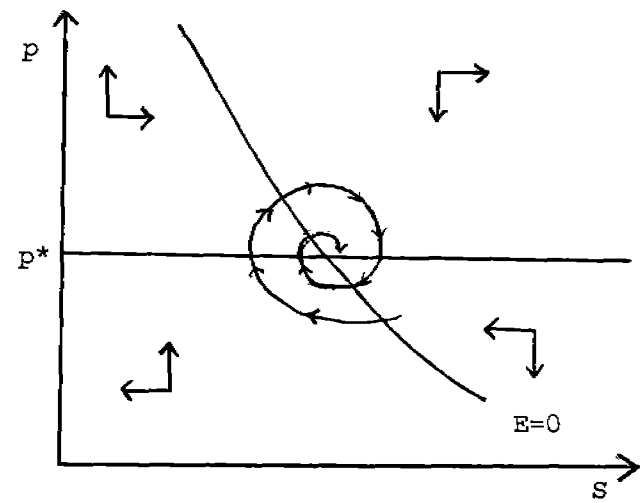


Figure 5

The equilibrium will be stable although convergence occurs via dampening cycles.[11]

In this section the temporary, competitive equilibrium is characterised by the following properties. First, the returns from holding the resource and the exogenous asset are equal. Second, expectations are based on future scarcity. Therefore, showing that the equilibrium is stable implies that, for most of the time, the market will operate at a point where resources are allocated efficiently. We should notice, however, that we have ignored subjective uncertainty. For instance, if we had incorporated risk-averse traders into the model (to reflect missing insurance markets or imperfect capital markets) this would not have been a general result. In particular, the possibility of oscillatory behaviour suggests that some kind of insurance (eg a buffer stock) is needed against market fluctuations.

2. EXTENSIONS OF THE BASIC MODEL
Adjustment in a mixed market structure

In the previous section, the equilibrium structure of the economy was that of a competitive market. This was a simplifying assumption imposed to help us concentrate on the specification of the adjustment process. However, markets are not competitive (especially in the short-run) and it is reasonable to ask how this process applies to a market characterised by imperfect competition. This question is not easy to answer. First, one needs to choose among the numerous equilibrium theories on imperfect competition in order to select the appropriate equilibrium concept. Second, one has to specify how the economy behaves outside equilibrium.

In the context of the theory of exhaustible resources it is common to use the Nash-Cournot equilibrium concept in order to derive the intertemporal equilibrium price trajectory for an oligopolistic resource market. The assumption that is usually made (see, among others, Salant, 1976, 1982; Dasgupta and Heal, 1979; Ulph and Folie, 1980; Newbery, 1981) is that the supply side consists of a group of colluding producers (cartel) and a group of small, competitive producers (the competitive fringe). It has been suggested that this assumption is a reasonable approximation to the structure of the oil market.

According to this model, the competitive fringe maximises profits taking prices as given, while the cartel maximises profits taking into account the fringe's supply and the market response. The model and its equilibrium properties are well known and we will not reproduce them here. Our purpose is to examine how the model of adjustment developed in the previous part ought to be modified when the market structure is as described above.

We will retain our assumption (2) to describe how the current price adjusts outside equilibrium. The presence of a dominant producer (cartel) does not affect the nature of the adjustment process described in the previous section. The dominant producer's current supply plans will be implicitly defined by an adjustment process similar to (4). The only difference is that he will arbitrage his perceived marginal revenue over the two periods rather than prices. The fringe's supply plans will be defined exactly as before. Given that the cartel's current supply plans are based on the perceived total demand $\hat{C}(p)$ (which might differ from the actual demand) and its conjecture about the fringe's supply (which might also differ from the actual one) the spot market might not clear ex-ante. Suppose that (ex-ante) excess supply prevails. If the spot price remains unchanged, the spot market will actually clear by quantity rationing. The fringe or the cartel (or both) will be rationed to sell less than they had planned. Without loss of generality we might assume that rationing is proportional to the equilibrium shares of the cartel and the fringe. Since each member of the fringe is "small" it faces an "almost" horizontal demand curve; that is, its perceived elasticity of demand is very large. Therefore it will find it profitable to cut the price until it eliminates all quantity rationing. This is exactly Arrow's (op cit) argument mentioned above that small firms become price makers if markets are outside equilibrium. Notice that the cartel will also find it profitable to reduce the price if its (perceived) own demand elasticity exceeds unity.

It is this kind of adjustment that assumption (2) is supposed to capture. In fact, it seems that the above argument does fit actual events in the oil market during the post-1973 era. As has been explained elsewhere (Mabro, 1981; Roberts, 1984) small producers were the first to change the price of oil when the market was in disequilibrium.

We will now specify the adjustment process in detail. The current price p will adjust according to the rule

$$\dot{p} = aE \qquad (17)$$

E is now defined as

$$E = C(p) - S^f - S^m, \qquad (18)$$

where S^f and S^m are the ex-ante supply plans of the competitive fringe and the cartel respectively.

S^f will be implicitly defined by an asset adjustment rule similar to (4). To simplify matters we will now assume that the speed of adjustment is a constant, h_1:

$$\dot{S}^f = h_1 \left(p - \frac{e}{1+r}\right), \; h_1 > 0, \qquad (19)$$

S^m will also be given by an asset adjustment rule. Since the cartel has market power it will arbitrage its marginal revenue over the two periods; for instance, it will prefer to carry the marginal unit forward rather than sell it now if the discounted expected marginal revenue in the second period is higher than its perceived, current marginal revenue. Hence we postulate

$$\dot{S}^m = h_2 \left[p\left(1 - \frac{\sigma}{\varepsilon}\right) - \frac{e}{1+r}\left(1 - \frac{\hat{\sigma}}{\hat{\varepsilon}}\right)\right], \quad h_2 > 0 \tag{20}$$

where σ is the cartel's conjectural market share (it depends on the cartel's conjecture about the fringe's supply) and is the (negative of the) elasticity of the perceived demand for oil $\hat{C}(p)$; $\hat{\sigma}$ and $\hat{\varepsilon}$ are the expected values for the same variables in the second period. σ, ε, $\hat{\sigma}$ and $\hat{\varepsilon}$ depend on the state variables of the economy. We will later need the first partial derivatives of σ, $\hat{\sigma}$, ε and $\hat{\varepsilon}$ with respect to p and e; as we will see, these can be estimated without much difficulty given the Nash-Cournot assumption about the market structure.

All agents in the economy expect a common future price e. By assumption they all use the same forecasting rule (algorithm) according to which the future oil price adjusts according to the expected future excess demand:

$$\dot{e} = \hat{E}(p,e) = \hat{C}(p) + \hat{C}(e) - \bar{R}^f - \bar{R}^m, \tag{21}$$

where $\bar{R}^f$, $\bar{R}^m$ are the initial reserves belonging to the fringe and the cartel.

We would like to know if the temporary equilibrium, defined as the stationary point of the system of differential equations (17), (19), (20), (21), is stable. Because of the analytical difficulties that this question involves we will again limit the analysis to local stability. Following exactly the same method as before, we will first eliminate $\dot{S}^f$, $\dot{S}^m$ and consider S^m, S^f as functions of p and e; then we will use equations (19) and (20) to find the sign of the partial derivaties S^f_p, S^f_e, S^m_p, S^m_e (and, through (18), E_p, E_e) at the temporary equilibrium. After this, we will study the local stability of the system of equations (17) and (21) in the two variables p and e.[12]

Asset market equilibrium requires that both $\dot{S}^f$, $\dot{S}^m$ (see equations (19), (20)) are zero. Denoting by an asterisk the temporary equilibrium values of all variables, the conditions for asset market equilibrium are

$$p^* = e^*/1+r \tag{22}$$

and

$$p^* \left[1 - \frac{\sigma^*}{\varepsilon^*}\right] = \frac{e^*}{1+r} \left[1 - \frac{\hat{\sigma}^*}{\hat{\varepsilon}^*}\right] \tag{23}$$

We assume that at the temporary, Nash-Cournot equilibrium the cartel's own demand curve is elastic ($\sigma^* < \varepsilon^*$) so that both the cartel and the competitive fringe are supplying the market. In this case (22) and (23) are both satisfied; they imply that equilibrium is such that the cartel's market power (σ^*/ε^*) is the same in both periods.

We will now evalute E_p, E_e at the temporary equilibrium where (22) and (23) are satisfied (and S^f, S^m are constant). Equation (18) yields

$$E_p = C_p - S^f_p - S^m_p \tag{24}$$

From (19) we have

$$S^f_p = h_1 > 0 \tag{25}$$

From (20) we have

$$S_p^m = h_2 \left[\left(1 - \frac{\sigma^*}{\varepsilon^*}\right) + p\,\frac{\sigma\varepsilon_p - \varepsilon\sigma_p}{\varepsilon^2} + \frac{e\hat{\sigma}_p}{(1+r)\hat{\varepsilon}}\right] \tag{26}$$

The first term $(1 - \sigma^*/\varepsilon^*)$ on the right hand side of (26) is positive, given our previous assumption that at equilibrium the cartel is supplying the market. The term σ_p is negative (so that $-\varepsilon\sigma_p$ is positive) since a higher current price implies a higher current supply from the competitive fringe-see (25)-and, therefore, a lower share for the cartel. The term ε_p depends on the form of the demand function C(p). If C(p) is iso-elastic, $\varepsilon_p = 0$. Although the case $\varepsilon_p < 0$ cannot be excluded, it is common to assume that the (absolute) elasticity is a non-decreasing function of the price. For instance, this will be the case for a linear demand function. We will adopt this assumption here as well; as a consequence the term $\sigma\varepsilon_p$ is non-negative. Finally, the last term, $e\hat{\sigma}_p/\hat{\varepsilon}(1+r)$, is also positive: a higher current price implies a higher current supply from the fringe and, given the fringe's total reserves, a lower future fringe supply; therefore it implies a higher future cartel share.

Hence we have shown that $S_p^m > 0$. Given (25), it follows from (24) that

$$E_p < 0 \tag{27}$$

Using the same arguments we can also show that

$$E_e > 0 \tag{28}$$

The proof of this result is exactly the same as that of (27) and we omit it.

From equation (21) it follows that

$$\hat{E}_p, \hat{E}_e < 0 \qquad (29)$$

Now we are ready to study the (local) stability of the equilibrium for the system of equations (17), (21). In fact, given the results in (27), (28), (29) it is straightforward to show that the phase diagram for this system is similar to that in Figure 4. Hence, the equilibrium is locally stable.

Some remarks may be useful here. First, the assumption that the market structure is given is quite restrictive. Analysts in the oil industry are concerned with the stability of the structure itself. For instance, "Will OPEC act as a monolithic cartel when there is excess supply in the market?" Such questions cannot be answered in the context of the present paper. A number of additional variables ought to be introduced to account for the differences between producers (eg differences in discount rates; see Hnylicza and Pindyck, 1976). Second, since we consider a two-period economy and do not divide the second period into smaller intervals we cannot discuss a number of issues related to entry decisions. However, these have been dealt with in equilibrium models and are rather outside the scope of this paper.

Introducing New Discoveries

We have already noticed that the ability to form consistent (rational) expectations depends on whether information about the level of initial reserves $\bar{R}$ is perfect. As can be seen from equations (10) and (11) e is conditional upon $\bar{R}$. Hence if $\overline{R}$ is not known with certainty the equilibrium e will also be a random variable. In this case the market will be characterised by some "equilibrium" degree of instability.

Information about $\overline{R}$ may not be perfect for various reasons. For instance, in models of imperfect competition owners of natural resources with market power might not be willing to reveal the actual size of their reserves in order to affect market expectations. Another, perhaps more important reason has to do with the fact that initial (proven) reserves can be augmented by further discoveries. The process of exploration and discovery is a costly one and its outcome is subject to (geological) uncertainty. Under these conditions, it is likely that a private economy will not produce the "right" amount of information about reserves. A market failure similar in nature to the distortion associated with the production of public goods might occur. (See, among others, Dasgupta and Heal 1979, ch 14).

Here we are not concerned with this sort of problem. We will assume that all information is public, and all we want to know is how the introduction of new discoveries affects our previous results on the stability of the short-run equilibrium. Although this seems to be a simple problem in comparison to the issues discussed above (on incentives to produce and reveal information) it is in fact related to a fundamental empirical question about the oil market.

It has been suggested by many oil specialists that the falling price of oil in the pre-1970 period might be explained by the fact that for a substantial period the "average" rate of additions to reserves exceeded the rate of consumption; and that (part of) the post-1970 price explosion might be explained by the reversal of this relationship. These arguments suggest that although traders might have had consistent expectations about future prices they had (erroneously) assumed that the past trend of the oil consumption/oil discoveries ratio would prevail in the future; in effect, they had not thought that oil was an exhaustible resource until they observed that the rate of additions to reserves fell below the consumption rate.

Although a two-period model similar to that considered in this paper is hardly suitable to acommodate the concepts of exploration and discovery, some ideas can still be presented. In what follows we will extend the model developed in the first part to take into account the possibility of new discoveries. The first question that we will attempt to answer is how our previous stability results are affected if traders form estimates about new discoveries using the rule of thumb discussed above. Second we will consider a more consistent way to form such estimates and examine its implications for stability.

Assume that exploration occurs in the first period. The size of newly discovered reserves will not become known until the beginning of the second period. However, traders have some beliefs about the process of exploration and discovery. We assume that these beliefs are summarised as follows

$$N = N\,[\,\hat{C}(p), e],\ N_{\hat{C}} \geq 0,\ N_e > 0 \qquad (30)$$

Equation (30) is an ad-hoc assumption. It expresses the idea that the higher the current consumption and/or the expected price are, the higher the incentive to explore for new reserves and, therefore, the higher the probability of new discoveries.

The future excess demand function is now estimated as

$$\hat{E}\ (p,e) = \hat{C}\ (e) - [\ \overline{R} + N\ [\hat{C}(p),e] - \hat{C}(p)] \tag{31}$$

We will again assume that the economy is competitive. The dynamic motion of the system will be described by (12) where $\hat{E}$ is now defined in (31). To construct the phase diagram of this system of differential equations we need to find the slope of the $\hat{E}$=0 locus. This is given by

$$\left[\frac{dp}{de}\right]_{\hat{E}=0} = - \frac{\hat{E}_e}{\hat{E}_p} = \frac{N_e - \hat{C}_e}{\hat{C}_p\ (1-N_C)} \tag{32}$$

Since $N_e > 0$; $\hat{C}_p$, $\hat{C}_e < 0$ the sign of (32) is the same as the sign of (N_C-1). If $(N_C-1) < 0$, the $\hat{E}$=0 locus will have a negative slope in the (p,e) space and the phase diagram will be similar to that in Figure 4 (with a steeper $\hat{E}$=0 locus). Hence the equilibrium will be locally stable.

The condition $N_C-1 < 0$ is related to the above argument. In effect, it imposes a restriction on traders' beliefs: although an increase in current oil consumption, dC, might induce new oil discoveries, dN, these are expected to be less than the increase in consumption: dN < dC.

However, the condition dN < dC is only a sufficient condition for stability. If it is violated the equilibrium might still be stable. It can be easily shown that if $N_C>1$ the equilibrium will still be stable provided that the $\hat{E}$=0 locus is steeper than the E=0 locus. But it is rather uninteresting to investigate these conditions further given that the analysis depends on the ad-hoc assumption (30). After all, if oil can be discovered through exploration one should look at the cost of this activity in order to have an idea about scarcity. (For a treatment of the relationship between scarcity and exploration costs see Devarajan and Fisher, 1982). This suggests that the ad-hoc rule (30) should be abandoned in favour of a careful modelling of the exploration/discoveries activity.

Let us imagine that exploration is an activity which produces (discovers) oil. This activity requires inputs which are available at a price. To keep the analysis in the context of the partial equilibrium model considered in this paper we will assume that the only input required to produce (discover) oil is oil.

We will again ignore subjective uncertainty and assume that traders believe with confidence that the production (discoveries) of oil occurs according to the following "discoveries" function

$$N = N\ (C) \qquad (33)$$

where C is oil input. Of course, it may turn out that actual discoveries are more or less than expected but this does not matter since we are only concerned with the first-period equilibrium.

Assuming that exploration is being undertaken by competitive firms, the demand for oil as an input to exploration will be determined by the profit maximising condition

$$p = N_C\ e/1+r \qquad (34)$$

Condition (34) says that the cost of the marginal barrel of oil used as an input to the exploration activity is equal to the discounted expected value of its marginal product.

If exploration activity is believed to be a well-behaved production activity with $N_{\tilde{C}} > 0$, $N_{\tilde{C}\tilde{C}} < 0$ then equation (34) will give a well-behaved input demand function $\tilde{C}(p,e)$ and a well-behaved supply function $N\ [\tilde{C}(p,e)]$. The sign of the partial derivatives of $\tilde{C}\ (p,e)$ and $N\ [\tilde{C}(p,e)]$ with respect of p and e will be as follows:

$$\tilde{C}_p < 0,\ N_p = N_{\tilde{C}}\tilde{C}_p < 0,\ \tilde{C}_e > 0,\ N_e = N_{\tilde{C}}\tilde{C}_e > 0$$

Now we have to include these two functions in the excess demand system. The current, ex-ante excess demand for oil function will now be

$$E\ (p,e) = C(p) + \tilde{C}(p,e) - S(p,e) \tag{35}$$

and the future excess demand is expected to be

$$\hat{E}\ (p,e) = \hat{C}(p) + \tilde{C}(p,e) + \hat{C}(e) - [\bar{R} + N\ [\tilde{C}(p,\ e)] \tag{36}$$

When the partial derivatives of $E(.)$ and $\hat{E}(.)$ are evaluated at the temporary equilibrium, where the asset market equilibrium condition $(1+r)p=e$ is satisfied, it is straightforward to show that the $E(p,e) = 0$ locus has a positive slope in the (p,e) space and the $\hat{E}(p,e) = 0$ locus a negative one. The phase diagram of the system of differential equations $\dot{p} = aE(p,e)$, $\dot{e} = b\hat{E}(p,e)$ will be (qualitively) similar to that in Figure 4 and the temporary equilibrium will be stable.

The crucial assumption in the above simple model is that exploration is expected to be an increasing cost activity.

This is related to the previous discussion about the falling price of oil in the pre-1970 period. In fact it might be suggested that the extrapolation of the past trend of the oil consumption/oil discoveries ratio could have reflected the at that time prevailing opinion that discovery is a decreasing cost activity. However, average exploration costs were increasing during the same period (see Devarajan and Fisher, op cit), which suggests that the above explanation is not very convincing. Other theories dealing with this issue exist, but it is outside the scope of this paper to present them.

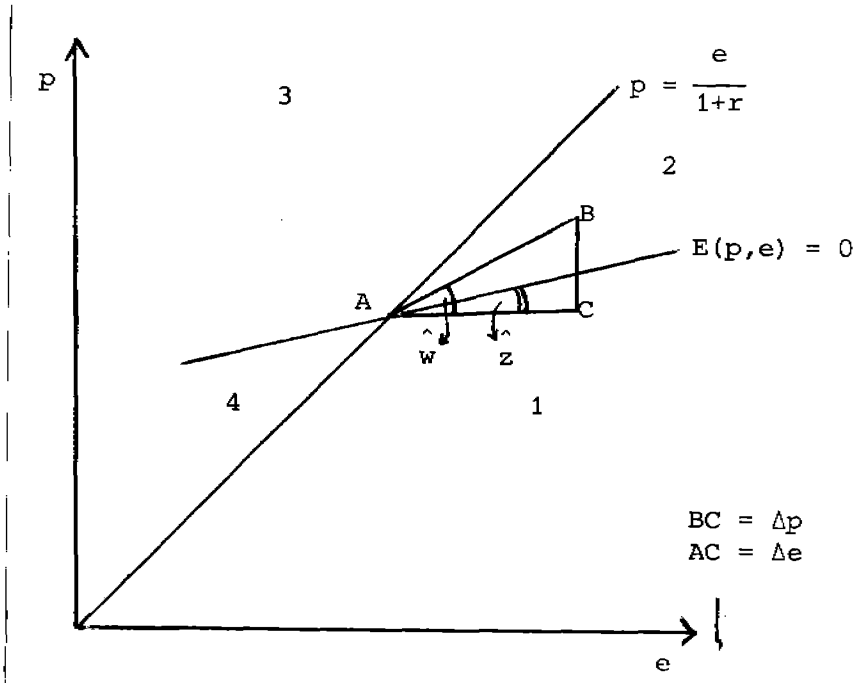
3. CONCLUDING COMMENTS

We have constructed a Hicksian two-period model and studied the (local) stability properties of the short-run equilibrium of an exhaustible resource market. Our motive was to examine whether the economy converges to this equilibrium if it starts from a point where spot and asset markets do not clear. We have derived local stability results under various assumptions about the formation of expectations and the market structure. A general result that has emerged from the paper is that rational expectations (ie expectations based on future scarcity) are sufficient for local stability. As far as the oil market is concerned, the most important sections are perhaps the one with the dominant producer and the competitive fringe, and the one dealing with the introduction of new resource discoveries. In particular, the section on imperfect competition assumes that the temporary, rational expectations equilibrium is the Nash-Cournot equilibrium. In effect, the question that we try to answer is how this equilibrium is achieved if the economy starts from a point where asset and spot markets do not clear.

Many extensions to the model are possible. For instance, one might wish to study the stability of the short-run equilibrium in a model with more than two periods and a limited number of forward markets. Another possibility is to consider a general equilibrium model with more than one financial asset. In a study parallel to the present one (see Stournaras, 1984) we have extended the basic model considered in this paper to a two country world economy trading in oil, a consumption commodity and money.

APPENDIX

Figure 5



In the above diagram we have reproduced Figure 2 and separated the phase space in four regions. It is straightforward to show that in regions 1 and 3 there is no ambiguity about the sign of $\dot{p}$. We will examine the case in region 2; because of symmetry, the results will also apply to region 4.

Take any point in region 2, say B, and consider the change in the ex-ante excess demand E, starting from the equilibrium point A. This is

$$\Delta E = E_p \Delta p + E_e \Delta e = E_p \Delta_p + E_e \frac{\Delta_p}{\tan\hat{w}} =$$

$$= \Delta p \left(E_p + \frac{E_e}{\tan\hat{w}}\right).$$

Since $-\dfrac{E_e}{E_p} = \tan \hat{z} < \tan \hat{w} < \dfrac{1}{1+r}$ (see diagram), it follows that $\Delta E<0$. Hence, there is excess supply at B and therefore, $\dot{p} < 0$.

FOOTNOTES

1. The assumption that the first period is a finite time interval does not contradict the concept of asymptotic stability of the first period equilibrium, since this only requires that the number of spot market iterations within the first period can become very large. (See Arrow and Hahn, 1971, ch 12.9.) In practice, this requires a highly organised spot market.

2. This idea is familiar from the theory of investment in a temporary equilibrium context. (See Bliss, 1975, p313.) In the context of the theory of exhaustible resources a similar rule is used by Heal (1981). Here we do not assume a constant speed of asset adjustment. We will later see that for certain rules on the formation of expectations, the results are sensitive to the form of rule (4).

3. In practice, the presence of positive <u>storage</u> costs can explain even better why this might happen. In our model we have assumed zero storage costs to simplify the analysis. Had we incorporated positive storage costs we would have had to distinguish between oil producers and speculators, while in this model we can aggregate them in the total supply variable S.

4. Since this is a partial equilibrium model we ignore the effects of rationing on other markets (the "spill-over" effects).

5. Rule (5) is an algorithm which says how traders revise their expectations within the first period. By assumption, they all use the same forecasting rule and do not divide the second period into smaller intervals. It differs from the adaptive expectations rules of Dasgupta and Heal (1979 ch.3) and Heal (1981) since these refer to the revision of expectations along the sequence of temporary equilibria. Rule (5) is closer to the informal discusion in Fisher (1981, p56) on the revision of expectations outside equilibrium.

6. The phase diagram has been constructed given the results in (6) and assumption (8). Since there might be some doubt about the direction of movement in the area between the (1+r) p=e locus and the E(p, e)=0 locus to the right of

point A we give a geometric proof of the result in the Appendix. In Footnote 8 we also give the algebraic proof.

7. Saddle point instability is a desirable feature of equilibrium growth models with perfect foresight but not in models like the one considered here.

8. The fact that (8) turned out to be a necessary and sufficient condition for stability is not surprising. Given the results in (6), condition (8) ensures that the Jacobian matrix J of the system of differential equations (2) and (5) satisfies the conditions: trace $J=aE_p-c<0$, determinant $J=-ac[E_p+(1+r)E_e]>0$, which are the necessary and sufficient conditions (the Routh-Hurwitz conditions) for stability. We should also stress that the case $(1+r)E_e=|E_p|$ cannot occur since the excess demand function E is not homogeneous of degree zero in p and e.

9. Consider the piece-wise linear adjustment rule

$$\dot{S} = \begin{cases} d[(1+r)p-e], & (1+r)p>e \\ 0, & (1+r)\,p=e \\ g[(1+r)p-e], & (1+r)p<e \end{cases}$$

At the temporary equilibrium (p^*, e^*) we have $E_p=C_p-d(1+r)$, $E_e=g$. Condition (8) will be violated if $(1+r)\,(d-g)<C_p$.

10. It is straightforward to show that the Jacobian of (12) satisfies the Routh-Hurwitz conditions irrespective of the sign of $|E_p|-(1+r)E_e$.

11. It is easy to show that the Jacobian of (14) and (15) satisfies the Routh-Hurwitz conditions for stability when e is given by (16).

12. Alternatively, we can study the local stability properties of the system of equations (17), (19), (20) and (21) in the four state variables p, S^f, S^m, e.

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