

Simple scaling law predicts peak efficiency in oscillatory propulsion

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Oscillatory propulsion is ubiquitous among swimming and flying animals, and may some day be practical as a replacement for rotary propulsion in watercraft and small air vehicles. The strength and efficiency of flapping thrust production closely depends on a dimensionless parameter called the Strouhal number (St), representing the ratio of the transverse oscillation speed to forward speed of the propulsor ($St = 2fA/U$, where f is the oscillation frequency, A its amplitude at the trailing edge, and U the forward speed; Fig. 1a). Crucially, the propulsive efficiency, defined as the ratio of propulsive power output to mechanical power input, usually peaks at a Strouhal number intermediate between the value associated with the onset of thrust production and the value associated with maximal thrust production. Whilst the exact Strouhal number giving peak propulsive efficiency depends on the relative amplitude and phase of the rotational pitch and translational heave components of flapping, its optimum typically falls in the range $0.2 < St < 0.4$ for most natural combinations of pitch and heave. Remarkably, nature's swimmers and fliers have been found to operate in the same narrow range of Strouhal number when cruising (1,2). What determines the Strouhal number giving peak propulsive efficiency? Writing in PNAS, Floryan et al. (3) think they have the answer.

The fluid dynamics of a flapping wing, fin, flipper, or tail-fluke are fiendishly complicated, but have been captured in exhaustive detail over the past three decades, through a combination of experimental measurements and numerical modelling. These studies confirm the old intuition that the undulating wake behind an oscillating wing will form a staggered array of vortices of alternate sense (4), the precise details of which depend on the energy of the motion (5). When the propulsor's transverse oscillations are very slow compared to its forward motion (i.e. at very low St), the work done may be insufficient to overcome the drag of its own profile. In this case, the wake will resemble the Kármán vortex street that typically forms behind drag-producing bodies (Fig. 1b). Net thrust production becomes possible at higher transverse oscillation speeds (i.e. at higher St), and is associated with the formation of a jet wake in reaction to the pressure forces on the propulsor. In this case, the wake will typically form a superficially similar flow structure called a reverse Kármán vortex street, in which the sense of the paired vortices is reversed (Fig. 1c). Other wake structures are possible (5), especially at higher oscillation speeds, but most empirical studies agree that maximal propulsive efficiency is associated with maximal development of the paired vortices in the wake (1,6), in a phenomenon called wake resonance (7).

The flow about an oscillating propulsor is closely coupled to the flow in its wake, but an explanation of propulsive efficiency given only in terms of the wake feels a little incomplete. Indeed, it has been suggested that efficient development of the wake may be a consequence, rather than a cause, of efficient propulsion (8). At any rate, there ought at least to be some complementary explanation involving the fluid dynamic forces that produce the wake. Recent analytical and experimental work by the same team of researchers (9,10) has analysed the scaling of thrust production in pitching and heaving motions in great depth, but the detail of those analyses obscures the simplicity of the key underlying principle. Enter the neat scaling argument now advanced in PNAS by Floryan et al. (3). The fluid dynamic forces acting on the surface of a propulsor unavoidably produce a small amount of rearward drag, which the pressure force arising from its oscillation must overcome to generate a net forward thrust. The forward component of this pressure force depends quadratically on the transverse oscillation speed ($\sim fA$), whereas the rearward

component of the drag depends approximately quadratically on the forward airspeed (U). This tension between the numerator and denominator of the Strouhal number ($St=2fA/U$) is key, because the drag will dominate the thrust at very low St , resulting in a negative propulsive efficiency. Propulsive efficiency quickly becomes positive with increasing St as thrust begins to dominate drag, but it drops off subsequently because the power required to generate thrust increases faster with St than the thrust itself. The overall result is that propulsive efficiency has a well-defined optimum that critically depends on the drag of the propulsor (3). Absent this drag, a net thrust can always be produced, and the propulsive efficiency increases without limit as St approaches zero. Thus, the optimal Strouhal number is determined by a trade-off between the diminishing importance of drag and the diminishing return on energy expenditure, as thrust increases with St .

There is more to Floryan et al.'s analysis (3) than this. The simplified scaling relationships that they develop from unsteady airfoil theory and new experimental data (10) hold for a quite general set of motions, and their results confirm the importance of getting the amplitude and phase of the rotational pitch and translational heave components just right. For biologically relevant motions, in which heaving leads pitching by a quarter of a cycle (Fig. 1a), their model predicts that heave and pitch should ideally contribute equally to the total motion (3). Still, the end result of this elegant scaling argument is a simple efficiency curve shaped like the example in Fig. 1d, with a well-defined peak at some optimal value of St that scales as the square root of the drag on the propulsor. The resulting family of curves fits the accompanying validation data very closely indeed (3).

Floryan et al. validate their scaling relationship (3) against published data from a symmetric airfoil undergoing one particular combination of pitch and heave (11), so it remains to be seen how well it will fit empirical data collected from a wider range of airfoil sections and kinematics. It would be particularly informative to test whether the value of St maximising propulsive efficiency does indeed scale with the airfoil drag coefficient as predicted, and to test the model against data collected for wing sections resembling the thin and highly-cambered airfoils of swimming and flying animals. Certainly, the model's assumptions mean that it will not capture the higher-order nonlinearities that result from flow separation at high angles of attack, and its simplicity is such that the propulsive efficiency is always predicted to peak at a Strouhal number $\sqrt{3}$ times that at which the propulsor transitions from producing a net drag to a net thrust. Nevertheless, the beauty of any good scaling argument lies in its combination of simplicity and generality, and in light of the self-similarity observed in oscillatory propulsion across many different scales in nature, it is reasonable to hope that their relationship should apply to a wide range of wing morphology and kinematics. Floryan et al.'s argument (3) certainly has the principled generality that we should look for, and it seems likely to complement rather than conflict with the established explanation in terms of wake resonance.

This is not the first time that a balance between thrust and drag has been invoked to explain the significance of the Strouhal number in oscillatory propulsion (8,12,13). The key difference here rests in Floryan et al.'s emphasis on the intrinsic drag of the propulsor, which they define for the limiting case of a wing undergoing the same motion at vanishingly small frequency (3). As is the case for the many empirical studies measuring the propulsive efficiency of isolated wings, this approach allows the efficiency of the propulsor to be defined independently of the drag on the body. In contrast, other recent scaling studies (8,12,13) have only explicitly considered the drag on the body, which is necessarily balanced by the net thrust of the propulsor at equilibrium. Those studies have concluded, perhaps unsurprisingly, that the value of St attained in cruising is set by this balance between thrust and drag, in conjunction with some other constraint upon amplitude. This is reasonable, because forward speed (U) can only be varied by controlling oscillation

frequency (f) or amplitude (A). It follows that the Strouhal number ($St=2fA/U$) cannot be set independent of the forward speed, except insofar as different combinations of frequency and amplitude giving different transverse oscillation speeds ($\sim fA$) can yield the same amount of thrust at the same cruise speed (U). Whilst this wider scaling argument is undoubtedly correct – for neutrally buoyant swimmers, at least – it does not provide the clear mechanistic explanation of how and why propulsive efficiency varies with Strouhal number that Floryan et al.’s new analysis supplies (3).

What significance does all this have for the observation (1,2) that a wide range of swimming and flying animals cruise in the range $0.2 < St < 0.4$ associated with efficient propulsion? The simplest plausible answer is that because peak propulsive efficiency is reached at a Strouhal number only a little higher than that at which net thrust is first produced (Fig. 1d), any reasonably streamlined animal with a small amount of body drag and a large enough propulsor is bound to operate at a near-optimal value of St . This argument, which is consistent with the conclusions of earlier scaling studies (8,12,13), would reduce the role of natural selection in determining St to one of minimising drag on the body. Given that the concept of streamlining was arrived at through observation of nature’s swimmers and fliers (14), there can be little doubt that this forms a part of the story. Even so, this perspective almost certainly underplays the role of natural selection in tuning St . Recall that thrust production is an increasing function of St , at least initially, and continues to be so well beyond the point at which propulsive efficiency is maximised. It follows that even an undersized propulsor may be able to drive an animal forward at a given speed, but only by operating at higher-than-optimal St . Conversely, an oversized propulsor may already produce enough thrust to balance the drag on the body at lower-than-optimal St . Natural selection can therefore be expected to match the morphology of any propulsor to the body it has evolved to propel, such that the value of St at which its net thrust balances body drag when cruising should ideally be the value of St at which its efficiency is maximised.

In flying animals, this optimisation is subject to the further constraint that lift must balance body weight at equilibrium, which may go some way to explaining the more complex kinematics of wings over fins, flukes, and flippers. It is tempting to wonder whether the challenge of dealing with this additional constraint whilst flying in an energetically efficient manner might also explain the prevalence of intermittent, non-equilibrium flight modes in smaller birds. Whether or not this proves to be the case, it is clear that simplified scaling arguments will continue to play a leading role in teasing apart the interwoven influences of natural selection and physical constraint that shape the emergent patterns of life (15). The paper by Floryan et al. is an important step on this road.

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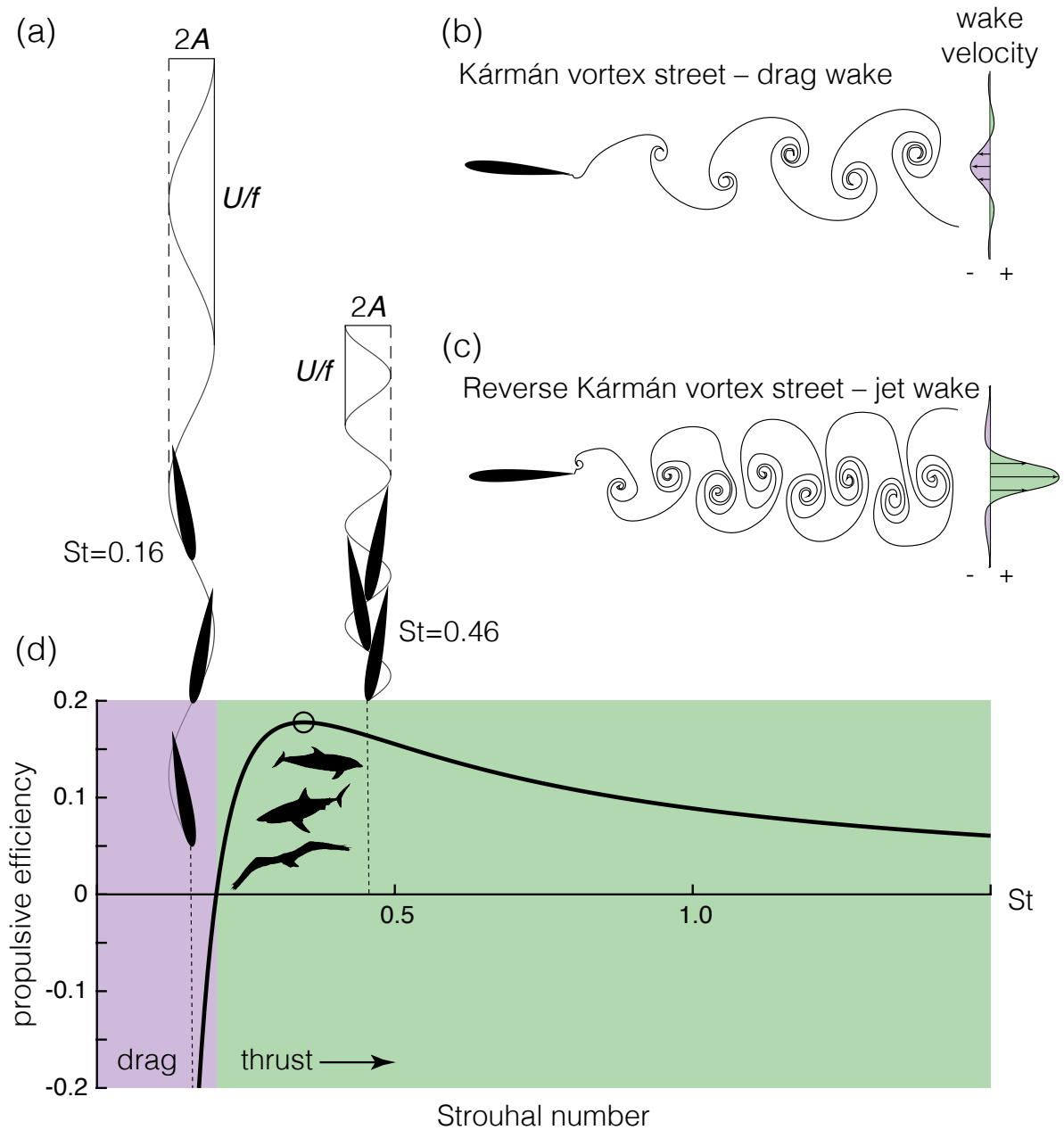


Figure 1. Significance of Strouhal number (St) in oscillatory propulsion. (a) $St=2fA/U$ represents the ratio of transverse oscillation speed to forward speed of the propulsor, given by the ratio of trailing-edge excursion ($2A$) to distance travelled per oscillation (U/f), where f is frequency, A is amplitude, and U is speed. (b) At low St, the wing produces a net drag and typically forms a wake structure called a Kármán vortex street. (c) At higher St, the wing generates a net thrust and the jet wake typically forms a reverse Kármán vortex street. Wake structures in (c,d) drawn from experimental data (5) at $St=0.12$ and $St=0.18$. (d) Propulsive efficiency predicted by Floryan et al.'s scaling (3) as a function of St, with parameters fitted to validation data (11) for an airfoil with heave leading pitch by 90° . Propulsive efficiency is negative in the purple region, where a net drag is produced, and is positive in the green region where net thrust is produced. Efficiency peak circled. Dotted lines show relate kinematics in (a) to efficiency curve in (d). Nature's swimmers and fliers typically cruise in the range $0.2 < St < 0.4$.