Eigenstrain Reconstruction of Residual Strains in an Additively Manufactured and Shot Peened Nickel Superalloy Compressor Blade

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Contents

Abstract

Numerical modelling of the residual stresses and strains within mechanical components is of great importance for improving the quality and reliability of design for structural integrity. A particularly versatile and powerful approach is offered by direct and inverse eigenstrain modelling. The nature of the eigenstrain modelling approach is it not only generates an efficient parametric representation of the residual stress field, but also ensures consistency by enforcing stress equilibrium and strain compatibility. In the present study we propose a particular way of prescribing the eigenstrain field due to surface treatment such as shot peening. Eigenstrain variation is described by a continuous function of the distance from the boundary of the object in a two-dimensional model of its cross-section. The procedure is compatible with the use of commercial numerical simulation software, and allows correct assignment of all eigenstrain components. We apply the technique to the evaluation of residual strain within an additively manufactured nickel superalloy compressor blade that was subsequently subjected to shot peening treatment. Two experimental techniques are used to validate the model, namely, Focused Ion Beam ring core milling (FIB-DIC) and synchrotron X-ray Powder Diffraction (SXRD). Consistency between model prediction and experimental measurements provides verification of the suitability of eigenstrain modelling as consistent basis for the incorporation of residual stress effects on the deformation behaviour of manufactured components.

1. Introduction

The knowledge of residual stresses plays a significant role in the correct assessment of structural integrity and performance reliability [1, 2]. The simplest approach involves considering residual stress as a source of uncertainty that must be accounted for by the introduction of a safety factor to ensure the integrity of mechanical components and assemblies in service. A more refined methodology takes into account the specific spatially varying nature of the residual stress within components and assemblies, by incorporating residual stress effects in the structural integrity predictions. Implementing this approach requires sufficiently detailed knowledge of the spatially resolved residual stress state based on the combination of modelling and experimental evaluation.

Residual stresses arise as a consequence of nearly all manufacturing and thermo-mechanical treatment processes. Depending on the length scale at which residual stresses are present, they give rise to vastly different effects (e.g. shape distortion, increase in the dislocation density, change in hardness, ductility, the resistance to fatigue and/or fretting, etc.) [3-5]. Residual stress may have either beneficial or detrimental effect on the performance depending on their nature and sign. Near surface compressive residual stress may enhance component life by reducing the propensity of cracks to nucleate and propagate. Since it is not always possible to control fully the nature and magnitude of residual stresses that arise due to the primary manufacturing process, additional secondary processing steps can be employed to achieve the reduction of tensile residual stress and/or the introduction of compressive residual stress [6]. Shot peening (SP) is one of the most popular and widespread among these techniques [7, 8]. Localised plastic deformation is induced by the impact of small rounded shot on the surface of a part made from a ductile metal, leading to the formation of a shallow near-surface layer of compressive residual stress within the component. Shot peening is often used for the reduction or sign reversal of tensile residual stresses, such as e.g. in the case of weldments [9].

To enable component design, the simulation of the stress state that arises within it under simple or complex loading conditions is carried out using numerical methods such as Finite Element Analysis (FEA). At this stage the residual stress state is often ignored, introducing a considerable amount of uncertainty. To take into account the residual stress, it must be prescribed within the model in a way that satisfies the requirement of strain compatibility, together with stress equilibrium and traction boundary conditions. This can be achieved particularly well using the concept of permanent inelastic strain, also known as *eigenstrain* [10], that was originally introduced to enable analytical formulation and solution of residual stress problems [11]. The premise of this technique is to consider the consequences of the introduction of eigenstrain in terms of elastic strains and the associated stresses (residual stresses) that must simultaneously ensure continuity and stress balance, respectively. Thus, the residual stress field arises as a one-step (non-iterative) solution of the linear elasticity problem perturbed by eigenstrain. This formulation lends itself to the solution of both direct and inverse problems, be it analytically or numerically (e.g. by least squares) [12]. The inverse eigenstrain approach allows finding the most likely residual stress state that is consistent with *both* the experimental data and the laws of continuum mechanics.

In the past, eigenstrain modelling has been successfully used for the reconstruction of arbitrary residual stress fields in both two- or three-dimensional numerical models [13, 14]. The eigenstrain distribution that induces a typical residual stress profile found after shot peening can be described by a continuous function that depends on the argument given by the distance from component surface that has been subjected to the treatment [15]. This assumption is based on the view that shot peening under given treatment conditions leads to the introduction of a well-defined eigenstrain distribution that, in the absence of particularly sharp tips, corners or notches, does not depend on the sample geometry [16]. When complex component geometries need to be analysed, the boundary shape description may not be possible by global analytical function(s). In such cases, the introduction of eigenstrain into the numerical model must be accomplished by combining the local functional description with the global numerical assignment of values to elements. The present study is devoted to the implementation of this approach for a particular case of aerofoil cross-sectional shape of compressor blade. The required prescription of in-plane eigenstrain tensor as a function of distance from the free surface was performed using a pre-processing routine written in Matlab for the evaluation of distances from boundary outline contour (shortest radius) at each Integration Point (IP) of the FE model. In addition, the angles formed by the shortest radius with global Cartesian axes are used to ensure the correct prescription of the local eigenstrain components, namely, compressive inelastic strain in the direction of surface normal, and expansion eigenstrains of opposite sign and 50% magnitude in the perpendicular plane.

In the present paper we report the evaluation of residual strains within the cross-section of an additively manufactured compressor blade that was subsequently treated by shot peening (SP). Residual strain measurement was conducted by means of two experimental techniques, namely, FIB-DIC micro- ring-core [17-19] and synchrotron X-ray powder diffraction (SXRPD) [20-23] in transmission mode. These techniques perform residual strain evaluation based on two different principles and in different gauge volumes. As well as other variants of the FIB-based milling technique [24-29], the FIB-DIC micro-ring-core technique relies on the principle of the strain relief after the introduction of a cut that creates new traction-free surfaces, whereas as far as SXRPD is concerned, strain is evaluated by monitoring the lattice parameter change for crystalline material. FIB-DIC micro-ring-core method is able to measure local residual strain at or near free surface of material regardless of whether it has crystalline or amorphous structure. In contrast, SXRPD technique is applicable to polycrystalline materials and, due to high penetrating ability of hard X-rays, produces a gauge volume that spans the entire sample depth, while micron scale lateral resolution can be readily achieved. These distinctions between the two techniques are carefully traced through the discussion, and are used in combination for the purpose of cross-validation.

For the purposes of analysis, the Leading Edge (LE), Trailing Edge (TE) and Middle position (M) on the blade were analysed using the techniques described above along several lines. Once the reconstruction of eigenstrain distribution was achieved, it was employed to visualise residual stress and strain fields everywhere within the blade and in specific regions of interest.

2. Eigenstrain residual stress description

2.1. Eigenstrain theory

Eigenstrain theory is based on the description of the inelastic strain distribution that gives rise to residual stress. Permanent ('frozen-in') strain that is termed *eigenstrain* describes shear, shrinkage or dilatation induced by an arbitrary inelastic process that may be associated with phase transformation, temperature change, plastic deformation, etc.

The total strain within a solid can always be decomposed additively into the irreversible and reversible parts. The former part is the permanent inelastic strain (eigenstrain ε^*), whilst the latter part corresponds to the elastic strain, ε_{el}:

$$\overline{\overline{\varepsilon_{tot}}} = \overline{\overline{\varepsilon}}^* + \overline{\overline{\varepsilon_{el}}}. \qquad (1)$$

In most cases, the presence of an eigenstrain distribution within a body means that an elastic strain field is required to maintain strain compatibility. It is important to note that this elastic strain field is subject to further mathematical requirements: the generalised Hooke's law that imposes the linear relationship between the elastic strain and stress within the body means that the elastic strain must be *statically admissible*, i.e. correspond to an equilibrated stress field. If no external tractions are applied to the body, then the aforementioned tensor fields are referred to as *residual elastic strain* (r.e.s.) and *residual stress*, respectively.

The eigenstrain method has already been adopted widely for modelling residual stress at various scales and fields of application. An important field of eigenstrain analysis concerns inclusions in elastic solids for which a whole range of fundamental solutions are available [30-32], notably for Eshelby ellipsoidal inclusions, cuboidal inclusions, etc. Eigenstrain-based analytical solutions have also been presented for residual stress distributions within a plate subjected to shot peening [11, 33-37] and a hollow tube subjected to autofrettage [38].

In cases where the geometry of the problem cannot be described using an analytical formulation, numerical methods can be utilised. Since real mechanical components are usually geometrically complex, the Finite Element Method (FEM) is widely employed. Within the framework of FEM it is convenient to introduce arbitrary eigenstrain fields into the model using pseudo-thermal strain expressed in terms of a temperature change ΔT and a tensor of thermal expansion coefficients $\bar{\bar{\alpha}}$. Then:

$$\overline{\overline{\varepsilon^*}} = \bar{\bar{\alpha}}\,\Delta T \qquad (2)$$

The thermal expansion coefficient tensor $\bar{\bar{\alpha}}$ contains six independent components and allows the description of all possible eigenstrain states. The prescription of the thermal expansion coefficient tensor at the totality of the Integration Points (IP) of the FEM model, in combination with the application of a uniform temperature change, furnishes an entirely flexible description of the eigenstrain field with the help of a user subroutine to define $\bar{\bar{\alpha}}$. Once the known eigenstrain distribution is prescribed, the one step solution of the thermo-elastic problem follows which returns the residual elastic strain distribution, and with it the associated residual stress distribution that is self-equilibrated by construction.

The evaluation of the elastic strain field when the imposed eigenstrain is known is called the *direct problem*. An example of a direct problem is the Eshelby treatment of an ellipsoidal inclusion [39], whereby uniform eigenstrain distribution is imposed within an ellipsoidal domain (inclusion), and the inner and outer elastic strain and stress fields are determined.

It is worth mentioning that the inelastic deformation (eigenstrain), produced by a mechanical treatment, if known because evaluated through a simple case, this can be transferred to more complex geometries. This was firstly introduced by Korsunsky [11] and named *Principle of Transferability of Eigenstrain*.

On the other hand, if the elastic strain is known and the eigenstrain the generated it is sought, this is called *inverse problem*. This is the case where experimental evidence provides information regarding the elastic strain field and thus the reconstruction of the eigenstrain that induced that elastic strain change is evaluated. In the past years, this procedure has been widely performed to several relevant engineering case such as: shot peening [40], including in the context of Crystal Plasticity [34], friction stir welding [41, 42], phase transformation [43], etc.

2.2. An approach to eigenstrain analysis for arbitrary 2D geometry

The distribution of residual stress/strain generated by a surface treatment process can be reproduced by the eigenstrain approach. The introduction of an eigenstrain distribution within the model by means of a continuous function can capture the intrinsic strain distribution generated by the underlying process operation by applying eigenstrain to simulate the inelastic deformation arising as a consequence of a treatment such as surface processing.

Generally, a two-dimensional (2D) solid mechanics problem can be considered in the plane-stress or plane-strain approximation. The choice of the most appropriate approximation is determined by the constraint experienced by the material in the out-of-plane direction. Depending on the nature of the problem, the eigenstrain distribution in a 2D domain can be specified. In particular it is interesting to consider a situation when eigenstrain components are also confined to the plane of consideration, so that the problem arising can be referred to as the plane eigenstrain problem.

Schematic diagram in the Fig.1 shows how at each arbitrary point P lying within the model, the x-axis orientation of a Local Cartesian coordinate System (LCS) can be identified as the direction of vector that connects to the nearest contour point C, $\overline{PC}$. Consequently, the angle α denotes the rotation angle of the LCS with respect to the Global Cartesian coordinate System (GCS). According to this schematisation, the components of eigenstrain in the LCS ($\varepsilon^*_{x'}$ and $\varepsilon^*_{y'}$) can be prescribed by using ε^*_x and ε^*_y in the GCS upon the knowledge of the rotation angle α.

<< FIGURE 1 HERE >>

Prescribing eigenstrain distribution using a continuous function requires the computation of the minimum distance for arbitrary point P from the nearest point on the sample contour, C. In order to impose correct eigenstrain components along the directions normal and parallel to the boundary contour, the angle α must be found that is formed by the surface normal at point C and the abscissa axis of the GCS.

The practical implementation of such algorithm within the Abaqus environment represents a challenge, given the difficulty of defining the boundary contour from element node data. An alternative approach is to evaluate the required value of distance and angle prior to the FE computation using a Matlab-based routine. The procedure involves the use of a binary mask representing the sample geometry, with the values set to 1 within the sample, and 0 outside. For each pixel within the mask that has the value of unity, the distance to the nearest null value pixel can be evaluated readily. The output from this pre-processing step is incorporated into the Abaqus model by extracting integration point (IP) coordinates and computing the distances and angles by interpolation of the values from Matlab image analysis. Abaqus subroutines are then used to set the distance and angle data for each IP as internal variables, and to compute the correct eigenstrain components accordingly.

The detailed list of the steps needed for the implementation of the above procedure, along with comprehensive description, is reported in the Appendix. With the aim of checking the correctness of the imposed eigenstrain, in particular the right component discrimination, the principal directions can be visualised during the post-processing. Figure 2(b) below illustrates a quiver plot of principal directions. In this specific case, two principal components of eigenstrain imposed were perpendicular and parallel to the model edges (blue line). The vectors indicate the principal directions are seen to be consistent with the contour orientation, particularly as the boundary is approached. It is possible to observe in Figure 2(b) that the red arrows (directions of maximum principal stress/strain) are actually locally perpendicular to the contour, whilst the other direction (indicated by the black arrows) is locally tangent to the contour.

<< FIGURE 2 HERE >>

3. Residual stress reconstruction in ALM shot peened blade

3.1. Additive Layer Manufacturing

Additive Layer Manufacturing (ALM) is an emerging technique for the production of complex three-dimensional mechanical components. If the material involved is metal or alloy, the fundamental idea of this technique is the melting of a stationary bed or a jet of compound powder, usually by means of rastering a high energy focused laser beam or, in some cases, using conventional weld arc. Therefore, the final product is built up by adding one cross-sectional layer of material at a time [44]. This process is usually interfaced with 3D CAD models, so that near-net-shaped components can be formed by reproducing the sequence of cross section contours that compose the model. The introduction of this technique has allowed the designers to optimise their objects, particularly to improve the performance and reduce weight. The application of ALM is not limited only to the macroscopic scale, but also applied to micro-scale and nano-structures [45]. The technique delivers outstanding performance in the welding of dissimilar materials, reducing various unwanted side effects that arise with the use of other methods [46].

In recent years ALM has begun to be increasingly utilised in aerospace applications, particularly when high temperature performance is sought [47]. Compressor blades are a prominent example of a mechanically loaded component that is simultaneously subjected to high temperature. Gas turbine engines contain numerous blades that are exposed to cyclic loading and whose failure leads to the loss of efficiency or even destruction of the entire system. Thus, periodic monitoring and replacement of turbine blades is necessary. In some cases the repair of damaged parts using ALM (instead of replacement) is a feasible solution that is also significantly less costly [48, 49].

Despite its supreme versatility, ALM may give rise to some undesired effects in the component manufacturing. The most obvious is porosity: since the technique does not involve the application confining pressure during solidification from melt, gradients in material density may arise. The combination of this effect with the high gradients in temperature may lead to the generation of defects [50]. Furthermore, the rapid changes in the local temperature and thus steep temperature gradients within the component may give rise to thermal stresses. If this stresses exceed the material strength, voiding or cracking may appear [51]. Finally, during cooling from the processing temperature, uneven material shrinkage may give rise to residual stress and distortion [52].

3.2. Methods for experimental residual strain measurement

Residual stress measurement can be accomplished using several techniques, but only a few of them are able to provide resolution down to the finest scale of e.g. microns. In the present study two experimental techniques were employed for residual stress evaluation, FIB-DIC micro-ring-core milling [18, 19, 53] and synchrotron XRD [54]. FIB-DIC methodology is based on the strain relief phenomena occurring within the surface layer of the when the constraint is modified by making a cut. The flexible control of Focused Ion Beam (FIB) scanning means that the cut can be performed in any shape, e.g. that of a circular trench. The isotropy of this arrangement allows obtaining information about the strain relief in all in-plane directions. The central core of material remaining after the trench milling procedure is similar to micro-pillars used for micro-scale mechanical testing. The monitoring of the displacement and therefore of the apparent strain evolution at the top surface of the pillar using Digital Image Correlation (DIC) provides information regarding the residual stress that was locked in the material prior to milling. The fitting of the apparent strain evolution along a certain direction with a master curve allows quantifying the average residual stress within the gauge volume [15]. The fitting function is derived from numerical simulations that assume uniform residual strain within the host material along the depth [17]. Of the other candidates for residual stress measurement at the micron scale the one suitable for crystalline materials is the X-ray diffraction-based technique. Particularly, if bulk stresses need to be studied and high spatial resolution is sought, the required high flux and accurate beam collimation can be achieved using Synchrotron X-Ray Powder Diffraction (SXRPD) [20-23].

3.3. Compressor blade description and sample preparation

Additive manufacturing techniques were used to create an IN718 compressor aerofoil as shown in Fig 2(a). Particularly, the technique adopted was the Laser Metal Deposition (LMD) using a laser and blown powder. Shot peening was applied along the entire contour of the component in order to induce compressive stresses close to the surface regions. Based on the results of prior vibrational testing, a cross section of the aerofoil was selected as the region of interest for this study. This fatigue test enabled to pinpoint the critical region of this component (i.e. crack nucleation location).This was at 17mm from the blade root, as indicated in Fig.2(a).

In order to implement the semi-destructive FIB-DIC micro ring-core and SXRPD analysis techniques for the residual strain measurement, free surfaces at the region of interest and a near-constant stress distribution along the blade extension were sought. For this reason, sectioning of the aerofoil was performed in the circumferential direction parallel to the outer edge of the aerofoil using a diamond cutting wheel. As result, a 2mm thick cross-section across the entire blade was extracted.

Surface material removal of the layers of material affected by cutting was performed using Grade 320 grinding paper. Following this, increasingly fine grit sizes (up to Grade 4000) were used to improve the surface finish. Polishing of the surfaces was then performed using 0.1 µm diamond suspension. In order to minimise induced surface stress and increase the surface contrast of the sample, etching was then performed on the surface using a weak acetic acid solution. In fact, controlled chemical etching enabled the removal of the shallowest layer impaired by sample preparation.

Three milling regions on the cross section were selected for FIB-DIC and SXRPD residual strain analysis. A montage cross section showing regions of stress analysis and approximate mapping positions is reported in Fig.3.

<< FIGURE 3 HERE >>

3.4. FIB-DIC measurement

Residual stress distributions were obtained by repeatedly performing the strain analysis procedure with spacing ranging from 100 to 50 microns between each FIB-DIC measurement. Following this testing procedure, the final FIB milling parameters were chosen to be 30keV and 0.17nA. The optimal SEM parameters were found to be 5keV and 260nA. The incremental FIB milling of the surface was performed in synergy with the SEM image acquisition. A SEM image was collected, at the resolution of 2001x2001 pixels, after each milling process and 50 images were recorded for each measurement. Fig.4 shows an example of images taken at different stages as the ring core milling advanced.

<< FIGURE 4 HERE >>

The introduction of DIC [55] at this stage allowed the reconstruction of the strain relief at the stub top surface. For the sake of convenience the average strains were measured in the two perpendicular directions (*x* and *y* in Fig.4). These directions were accurately chosen to match the sample contour directions that were thought to be coincident with the principal strain/stress direction. At each milling point therefore, a pair of relief curves were obtained. The interpretation process of the relief curves leading to the abstraction of residual strains present within the material prior to milling was conducted by master curve fitting [15]. The uncertainties arising from this process were assessed and reported with the final residual strain values as error bars, as will be shown in the subsequent sections of the paper.

3.5. SXRPD measurements

SXRPD analysis was performed at beamline I15 at Diamond Light Source, Harwell. A monochromatic beam with the photon energy of 76 keV was focused down to less than 70 × 70 μm^2 in size by two 1.2m long Kirkpatrick-Baez mirrors, and then cleaned up by a collimator of 70 µm diameter positioned close to the sample. The sample was placed into a specially manufactured mount and an optical system was used to align the beam with the FIB-DIC marker locations at micron-scale precision. A raster scan was then used to collect diffraction patterns in steps of 50 µm from the edge of the sample. A PerkinElmer flat panel 1621-EN detector (2048x2048 pixels, pixel size $0.2x0.2mm^2$) was used to record the resulting diffraction patterns.

A general observation can be made regarding the diffraction patterns collected. As seen from Fig.5, the relatively large grain size present within the sample induced pattern “graininess”. In particular, the larger grain sizes at the two extremes of the blade (TE and LE) produced poor powder diffraction pattern quality compared to the pattern obtained at the middle position (M). Due to these limitations it was only possible to obtain SXRPD patterns that lend themselves to reliable interpretation only from the middle position of the blade, and also for the *yy* component of strain at the blade Trailing Edge (TE).

<< FIGURE 5 HERE >>

For the reasons given above, 60° azimuthal integration was used to improve the grain sampling statistics of the resulting 1D spectra. A critical examination of the Debye–Scherrer rings revealed that sample graininess had least impact on the ϒ phase <200> peak. Lattice parameter quantification was performed using this peak for the scattering vectors parallel and perpendicular to the sample contour. Furthermore, in order to improve the sampling statistics, integration of the patterns for the relevant strain direction was performed for both diametrically opposing segments spanning 60° angular range. Final interpretation was performed using least squares minimisation to reduce noise further and minimise the error due to small error in the ring centre determination.

For a given Miller index hkl, the conversion between the XRPD lattice parameter variation (d_{hkl}) and the estimate of lattice strain (ε_{hkl}) is given by:

$$\varepsilon_{hkl} = \frac{d_{hkl} - d_{hkl}^{0}}{d_{hkl}^{0}} \qquad (1)$$

where d_{hkl}^{0} is the unstrained lattice parameter. In order to provide reliable measure of the absolute residual strain variation, accurate quantification of d_{hkl}^{0} is essential. In this study, the direct comparison between the absolute residual strain values obtained by FIB-DIC, at the middle position of the blade (MP), and relative values obtained by XRPD was used. This process enabled the determination of the optimal value for the unstrained lattice parameter of face-centred cubic γ phase of IN718, which was determined to be a_{γ}^{0}=3.59756 Å.

3.6. Residual strain reconstruction and comparison with experiments

Due to the nature of the studied sample, the residual stress problem can be treated in the plane stress approximation, with the out-of-plane components (longitudinal direction of the blade indicated by the z axis in Fig.2(b)) of stress being assumed null. This is also true for the case of FIB-DIC measurements. Preliminary numerical simulations showed that the imposition of non-null eigenstrain distribution of any shape along the direction perpendicular to the local contour line did not give rise to noticeable residual elastic strain and residual stress. This can be understood by considering the fact that material expansion along these directions is not constrained, so that the material deforms freely and no permanent misfit arises due to the eigenstrain introduction.

Consequently, the present problem was reduced to the sole analysis of the eigenstrain component in the tangent direction to the model contour.

Eigenstrain distribution was imposed in a point-wise fashion at each IP of the FE mesh. For the purposes of algorithmic implementation, eigenstrain was defined by means of a continuous function of the shortest distance of each IP from the component contour.

Additive Layer Manufacturing process intrinsically introduces a residual stress field within the mechanical part, on which the additional eigenstrain generated by Shot Peening (SP) is superimposed. The outcome of SP can be described by a tensile tangential eigenstrain profile which has non-zero value at the surface, increases in magnitude with depth, and vanishes at the depth up to a few hundred microns [9]. The residual stress profiles generated by SP can be described using a shifted cosine function superimposed on a linear background, as introduced in the paper by Watanabe et al. [56]. A further extended analysis of the residual state in a shot-peened plate was presented in the paper by Korsunsky [11], which demonstrates that the residual stress profile arises from the superposition of linear bending on the inverted eigenstrain profile. In other words, in the Watanabe et al. formulation the shifted cosine term (truncated at the depth where it reaches zero value) corresponds to the eigenstrain introduced by shot peening.

It is clear that this choice of eigenstrain representation is made on the basis of matching experimental observation, and is not unique. As an alternative, in the present study we consider the eigenstrain profile to be represented by a shifted Gaussian function. In practice, the difference in the profile representation between shifted Gaussian and shifted cosine is minimal, as illustrated in Fig.6a. However, the choice of Gaussian function obviates the need for truncation, and also paves the way for the representation of more complex distributions by linear combination of multiple Gaussian peaks, as illustrated below.

The effect of severe plastic deformation that occurs in the region affected by SP tends to dominate over any pre-existing residual stress state inherited from prior processing. In fact, the SP effect overcomes and removes the residual elastic strain caused by the ALM process [57-60]. Therefore, in the vicinity of the component contour it is correct to impose solely the SP residual elastic strain (eigenstrain) profile. On the other hand, within the bulk of the component away from the contour the consequences of the ALM process are expected to be present, and needs to be captured by the introduction of an appropriate eigenstrain profile.

In our experiments the residual elastic strain was evaluated with greatest accuracy at the Middle position of the blade, compared to the other positions. Therefore, this region was chosen for calibrating the eigenstrain distribution. Given the predominance of SP eigenstrain in the near contour region, we first use error minimisation to calibrate the parameters of the Gaussian profile that describes the eigenstrain in the form:

$$\varepsilon_{SP}^{*} = A_{SP}\, e^{-\frac{(d-b_{SP})^2}{2c_{SP}^2}} \tag{2}$$

In the expression in Eq.2, A_{SP} is the Gaussian amplitude, b_{SP} is the coordinate shift, and c_{SP} is the Gaussian width. Argument d is the distance of the IP from the model contour. FEM calibration were performed by varying the three parameters and minimising the deviation from the experimental data. Once satisfactory agreement was found, the background eigenstrain that represents the ALM process in the inner region of the blade was sought in the same way. Again, a Gaussian function was adopted (3) for the FEM optimisation process. Subscript BG indicates the reference to the background profile:

$$\varepsilon_{BG}^{*} = A_{BG}\, e^{-\frac{(d-b_{BG})^2}{2c_{BG}^2}} \tag{3}$$

Table 1 summarises the coefficients found by performing FEM calibration that were adopted for residual elastic strain prediction.

<< TABLE 1 HERE>>

The complete eigenstrain profile was constructed by summing the two contributions:

$$\varepsilon^{*} = \varepsilon_{SP}^{*} + \varepsilon_{BG}^{*} \tag{4}$$

The plot in Fig.6b illustrates the two contributions (SP and background), shown in solid and dashed lines. In addition, the total eigenstrain imposed to the model is shown by the thick black line.

<< FIGURE 6 HERE >>

The blade geometry was modelled by contouring the SEM blade image acquired within a 2D modelling software. Following the steps described in the Appendix, the eigenstrain distribution was implemented in the FEM model, and the solution obtained for the three regions analysed. Consistent eigenstrain profile of Fig.6b was imposed in the subsequent analysis of other regions of the blade, in order to obtain validation of the correctness of the calibration approach adopted in the present study.

The results provided by FEM post-processing can be visualised using coloured contour maps and by plotting 2D strain profiles of each component along selected paths. The residual elastic strain variation was compared with experimental measurement lines.

The experimental results at middle position of the blade (MP) were collected across the entire blade thickness using SXRPD and across a reduced length close to one edge using FIB-DIC. Fig.7 shows experimentally measured residual strain components along with the residual elastic strain reconstructed by eigenstrain. Contour maps in the following Figures show the maximum principal residual elastic strain.

<< FIGURE 7 HERE >>

<< FIGURE 8 HERE >>

<< FIGURE 9 HERE >>

<< FIGURE 10 HERE >>

At the blade Leading Edge (LE), the residual strain was measured along two perpendicular lines. The first line originated at the tip of the blade in the LE position and ran along the bisector of the LE tip contour line. The results are shown in Fig.8. For this location no reliable SXRPD measurements can be reported due to the excessive graininess of diffraction patterns.

The second measurement line at the blade Leading Edge (LE) ran across the blade thickness at the location where it equals 1mm. The results are shown in Fig.9. Once again, no SXRPD measurements are reported for the same reason stated above.

The final set of measurements was collected at the blade Trailing Edge (TE). At this location the residual strain distribution was spatially resolved along a line perpendicular to the blade tip as shown in Fig.10(a). The same figure also shows the residual strain plots. Here the SXRPD are reported solely for *yy* strain component, for which the values could be seen as useful even in the presence of large errors indicated by the error bars.

However, in order to quantitatively describe the dispersion of experimental data about the FEM strain field, the root-mean-square deviation was evaluated for each set of measurements. The calculated values are reported in Table 2.

<<TABLE 2 HERE>>

4. Discussion

A procedure for efficient description of eigenstrain distribution due to surface treatment in two-dimensional models has been successfully developed and presented. The correctness of the implementation was verified by visualising principal strain directions over the model geometry.

The problem being studied concerned shot peening (SP) process applied to compressor blade. Residual stresses arising in this situation represent the superposition and perhaps interaction between the two manufacturing processes (ALM and SP). The assessment of the residual strain profile was conducted experimentally by means of FIB-DIC and SXRPD. The eigenstrain profile induced by SP over the pre-existing ALM background was assessed, and the predicted residual elastic strain distribution compared to experimental measurements. Acceptable overall agreement was found, particularly in view of the large uncertainty associated with the ALM outcomes. Specifically, at the blade Middle position the *xx* component of strain (ε_{xx}) showed good agreement between experimental and numerical solutions (Fig.7b). Less accurate agreement between reconstruction and measurements was found for the perpendicular component of strain, ε_{yy}. In fact, this reconstructed quantity failed to match the magnitude in the central part of blade thickness, although the overall trend match is satisfactory (Fig.7c). It is worth noting that the mismatch was primarily observed at the central region of the profile, where the background residual stress was imposed. This disagreement may be imputed to the complex residual stress distribution produced by the manufacturing process that evidently may slightly diverge from the nominal eigenstrain distribution imposed as background.

Regarding the other two regions of the blade, it is worth reminding that the FEM simulation are the outcome of the calibration against experiments conducted at the MP. Therefore, a limited discrepancy from the experimental outcomes was expected. At the blade Leading Edge, the measurements were affected by large errors, as is evident in Fig.8b-c. Nevertheless, the majority of experimental points are seen to lie close to the predicted eigenstrain curve. Nevertheless, the trend highlighted by the measurements was not captured satisfactorily. This divergence is thought to be due changes in the operation of both ALM and SP processes at blade tips. It is reasonable to anticipate that at the tip the effectiveness of SP may deviate significantly from parallel surface regions. Similarly, the conditions of material deposition by ALM are likely to be modified by the proximity of the sample boundary. Furthermore, local deviation between FEM prediction and FIB-DIC measurements can be caused by the presence of pores in the material. In fact, it is well known that one of the most prominent disadvantages of this manufacturing technique is the generation of undesired porosity [61].On the other hand, good agreement was found along the line across the blade thickness at this location (Fig.9b-c).

Finally, the analysis at the blade Trailing Edge (TE) did not reveal high magnitudes of residual strain (Fig.10). This was confirmed by the eigenstrain reconstruction that predicted this residual elastic strain state. Low residual strain magnitudes are the results of large plastic deformation occurring almost throughout the entire blade thickness. Therefore, only a very limited amount of residual elastic strain may arise to restore the equilibrium disturbed by eigenstrain introduction.

One remark needs to be made regarding the effectiveness of SP process. It has been revealed in the present work that in the presence of tight contour radii, the effectiveness of SP process may be reduced. This implies that the induced residual strain may diverge from the expectation based on the ideal case where the local curvature radius is thought to be large. Same findings were earlier experienced by DeWald and Hill [14].

Further efforts can be addressed at implementing the presented method for the simulation of three-dimensional geometries. Full reconstruction of the residual stress contained in a *three-dimensional* object requires further measurements in order to capture evolution of the out-of-plane component of residual stress arising as background (arising from AL manufacturing). However, it is worth noting that shot peening is thought to introduce isotropic deformations along all the directions perpendicular to the local shot trajectory. Therefore, in principle the 3D residual stress field given by the sole shot peening could be simulated by imposing the same eigenstrain distribution along this third direction.

5. Conclusion

The proposed eigenstrain method for the in-plane strain reconstruction was successfully applied to a real case study of ALM and SP compressor blade section. The computed strain maps showed good agreement with the experimental outcomes. The application of the eigenstrain distribution to the entire geometry allowed the prediction of the strain field not only at positions where experimental evidence was available, but *everywhere within the cross section*. Furthermore, numerical simulation confirmed the consistency of the experiments in terms of stress equilibrium, strengthening the confidence in their validity. For these reasons, the proposed method can be considered as complementary to the experimental evaluation of residual elastic strains. Once the intrinsic inelastic deformation (i.e. eigenstrain) of a specific process is known (e.g., SP), the great advantage of the approach lies in the applicability of this description to 2D geometries of arbitrary shape. Therefore, the simulation of residual elastic strain arising from SP can be performed for any 2D model.

Appendix

The relevant steps involved in the model building process are listed below in a sequential order, that is also schematically represented in Fig.11 below.

i. 2D modelling
   - The geometry shape is obtained by pre-existing model, e.g. CAD, or by contouring a real mechanical component shape (e.g. importing SEM image)
   - A model containing the geometric contour is exported for Abaqus meshing, and a second model is exported as a binary mask by filling the solid part of the model with value 1, and outside with 0.

ii. Abaqus mesh generation
   - The geometry is imported and meshed by refining the elements where high strain gradient is expected.

iii. Abaqus IPs extraction
   - A dummy solution is run with the purpose of extracting the IPs coordinates of the meshed model.

iv. Matlab distances and angles computation
   - An image format version of the model (e.g. TIFF, JPEG) is imported into Matlab and treated as matrix.

- At each position of the matrix (equivalent pixel), two new matrices are generated where the distances from all the surrounding pixels are computed in terms of the x and y differences.
- The element-wise multiplication by the binary mask representing the model geometry allows the values at positions outside the sample to be set to 0.
- A new matrix containing the distances is populated by seeking the minimum value of distance (both x and y components), and calculating the resulting vector distance.
- A matrix containing the angles is populated by calculating the angles made by the distance vector with the x axis at each pixel position.

v. Interpolation
   - Matrices containing IPs coordinates are imported in Matlab
   - The interpolation between the grid generated by the imported mesh (IPs) and the image pixels positions is performed. As outcome, at each IP, the distances and angles are evaluated.
   - The arrays containing IPs number, distances and angles are exported

vi. Eigenstrain implementation
   - The UEXTERNALDB subroutine is used for the importation of the computed values in an Abaqus script.
   - The UEXPAN subroutine is then used for the assignment of the thermal expansion coefficient at each IP. This subroutine at each loop computes the eigenstrain assignment by recalling the value of the distance and angle assigned to the datum IP.

vii. Solution
   - The Finite Element computation is run and the solution can be visualised in the post-processing environment of Abaqus.

<< FIGURE 11 HERE >>

References

[1] P.J. Withers, H.K.D.H. Bhadeshia, Residual stress. Part 2 – Nature and origins, Materials Science and Technology 17(4) (2001) 366-375.

[2] M.N. James, Residual stress influences on structural reliability, Engineering Failure Analysis 18(8) (2011) 1909-1920.

[3] C. Garcia, T. Lotz, M. Martinez, A. Artemev, R. Alderliesten, R. Benedictus, Fatigue crack growth in residual stress fields, International Journal of Fatigue 87 (2016) 326-338.

[4] K. Lin, S.D. Pang, The influence of thermal residual stresses and thermal generated dislocation on the mechanical response of particulate-reinforced metal matrix nanocomposites, Composites Part B: Engineering 83 (2015) 105-116.

[5] G. Fu, M.I. Lourenço, M. Duan, S.F. Estefen, Influence of the welding sequence on residual stress and distortion of fillet welded structures, Marine Structures 46 (2016) 30-55.

[6] K.J. Kirkhope, R. Bell, L. Caron, R.I. Basu, K.T. Ma, Weld detail fatigue life improvement techniques. Part 1: Review, Marine Structures 12(6) (1999) 447-474.

[7] A.H. Mahmoudi, A. Ghasemi, G.H. Farrahi, K. Sherafatnia, A comprehensive experimental and numerical study on redistribution of residual stresses by shot peening, Materials and Design 90 (2016) 478-487.

[8] A.A. Ahmed, M. Mhaede, M. Wollmann, L. Wagner, Effect of micro shot peening on the mechanical properties and corrosion behavior of two microstructure Ti-6Al-4V alloy, Applied Surface Science 363 (2016) 50-58.

[9] L. Zhiming, S. Laimin, Z. Shenjin, T. Zhidong, J. Yazhou, Effect of high energy shot peening pressure on the stress corrosion cracking of the weld joint of 304 austenitic stainless steel, Materials Science and Engineering A 637 (2015) 170-174.

[10] R. Furuhashi, T. Mura, On the equivalent inclusion method and impotent eigenstrains, Journal of Elasticity 9(3) (1979) 263-270.

[11] A.M. Korsunsky, On the modelling of residual stresses due to surface peening using eigenstrain distributions, Journal of Strain Analysis for Engineering Design 40(8) (2005) 817-824.

[12] A.M. Korsunsky, Variational eigenstrain analysis of synchrotron diffraction measurements of residual elastic strain in a bent titanium alloy bar, Journal of Mechanics of Materials and Structures 1(2) (2006) 18.

[13] T.S. Jun, A.M. Korsunsky, Evaluation of residual stresses and strains using the Eigenstrain Reconstruction Method, International Journal of Solids and Structures 47(13) (2010) 1678-1686.

[14] A.T. DeWald, M.R. Hill, Eigenstrain-based model for prediction of laser peening residual stresses in arbitrary three-dimensional bodies Part 2: Model verification, The Journal of Strain Analysis for Engineering Design 44(1) (2009) 13-27.

[15] E. Salvati, M. Benedetti, T. Sui, A.M. Korsunsky, Residual stress measurement on shot peened samples using FIB-DIC, Conference Proceedings of the Society for Experimental Mechanics Series, 2016, pp. 275-283.

[16] T.S. Jun, A.M. Venter, A.M. Korsunsky, Inverse Eigenstrain Analysis of the Effect of Non-uniform Sample Shape on the Residual Stress Due to Shot Peening, Experimental Mechanics 51(2) (2011) 165-174.

[17] X. Song, K.B. Yeap, J. Zhu, J. Belnoue, M. Sebastiani, E. Bemporad, K.Y. Zeng, A.M. Korsunsky, Residual stress measurement in thin films using the semi-destructive ring-core drilling method using Focused Ion Beam, Procedia Engineering, 2011, pp. 2190-2195.

[18] A.M. Korsunsky, M. Sebastiani, E. Bemporad, Focused ion beam ring drilling for residual stress evaluation, Materials Letters 63(22) (2009) 1961-1963.

[19] E. Bemporad, M. Brisotto, L.E. Depero, M. Gelfi, A.M. Korsunsky, A.J.G. Lunt, M. Sebastiani, A critical comparison between XRD and FIB residual stress measurement techniques in thin films, Thin Solid Films 572 (2014) 224-231.
[20] E. Salvati, H. Zhang, K.S. Fong, X. Song, A.M. Korsunsky, Separating plasticity-induced closure and residual stress contributions to fatigue crack retardation following an overload, Journal of the Mechanics and Physics of Solids 98 (2017) 222-235.
[21] E. Salvati, T. Sui, H. Zhang, A.J.G. Lunt, K.S. Fong, X. Song, A.M. Korsunsky, Elucidating the Mechanism of Fatigue Crack Acceleration Following the Occurrence of an Underload Advanced Engineering Material, (2016).
[22] B. Lin, S. Zabeen, J. Tong, M. Preuss, P.J. Withers, Residual stresses due to foreign object damage in laser-shock peened aerofoils: Simulation and measurement, Mechanics of Materials 82 (2015) 78-90.
[23] M.K. Khan, M.E. Fitzpatrick, S.V. Hainsworth, A.D. Evans, L. Edwards, Application of synchrotron X-ray diffraction and nanoindentation for the determination of residual stress fields around scratches, Acta Materialia 59(20) (2011) 7508-7520.
[24] R. Zhu, H. Xie, X. Dai, J. Zhu, A. Jin, Residual stress measurement in thin films using a slitting method with geometric phase analysis under a dual beam (FIB/SEM) system, Measurement Science and Technology 25(9) (2014).
[25] B. Winiarski, P.J. Withers, Mapping residual stress profiles at the micron scale using FIB micro-hole drilling, Applied Mechanics and Materials, 2010, pp. 267-272.
[26] B. Winiarski, M. Benedetti, V. Fontanari, M. Allahkarami, J.C. Hanan, P.J. Withers, High Spatial Resolution Evaluation of Residual Stresses in Shot Peened Specimens Containing Sharp and Blunt Notches by Micro-hole Drilling, Micro-slot Cutting and Micro-X-ray Diffraction Methods, Experimental Mechanics 56(8) (2016) 1449-1463.
[27] B. Winiarski, M. Benedetti, V. Fontanari, M. Allahkarami, J.C. Hanan, G.S. Schajer, P.J. Withers, Comparative analysis of shot-peened residual stresses using micro-hole drilling, micro-slot cutting, X-ray diffraction methods and finite-element modelling, Conference Proceedings of the Society for Experimental Mechanics Series, 2016, pp. 215-223.
[28] Q. Wang, H. Xie, Z. Liu, X. Lou, J. Wang, K. Xu, Z. Zhang, J. Liao, C. Gu, Residual stress assessment of interconnects by slot milling with FIB and geometric phase analysis, Optics and Lasers in Engineering 48(11) (2010) 1113-1118.
[29] M. Krottenthaler, L. Benker, M.Z. Mughal, M. Sebastiani, K. Durst, M. Göken, Effect of elastic anisotropy on strain relief and residual stress determination in cubic systems by FIB-DIC experiments, Materials and Design 112 (2016) 505-511.
[30] T. Mura, Micromechanics of Defects in Solids, (1987).
[31] L. Ma, A.M. Korsunsky, The principle of equivalent eigenstrain for inhomogeneous inclusion problems, International Journal of Solids and Structures 51(25–26) (2014) 4477-4484.
[32] S.A. Faghidian, A smoothed inverse eigenstrain method for reconstruction of the regularized residual fields, International Journal of Solids and Structures 51(25–26) (2014) 4427-4434.
[33] A.M. Korsunsky, Residual elastic strain due to laser shock peening: Modelling by eigenstrain distribution, The Journal of Strain Analysis for Engineering Design 41(3) (2006) 195-204.
[34] W.D. Musinski, D.L. McDowell, On the eigenstrain application of shot-peened residual stresses within a crystal plasticity framework: Application to Ni-base superalloy specimens, International Journal of Mechanical Sciences 100 (2015) 195-208.
[35] S.A. Faghidian, A smoothed inverse eigenstrain method for reconstruction of the regularized residual fields, International Journal of Solids and Structures 51(25-26) (2014) 4427-4434.
[36] M. Achintha, D. Nowell, K. Shapiro, P.J. Withers, Eigenstrain modelling of residual stress generated by arrays of laser shock peening shots and determination of the complete stress field using limited strain measurements, Surface and Coatings Technology 216 (2013) 68-77.
[37] M. Achintha, D. Nowell, Eigenstrain modelling of residual stresses generated by arrays of LSP shots, Procedia Engineering, 2011, pp. 1327-1332.

[38] A.M. Korsunsky, Residual elastic strains in autofrettaged tubes: Elastic-ideally plastic model analysis, Journal of Engineering Materials and Technology, Transactions of the ASME 129(1) (2007) 77-81.

[39] J.D. Eshelby, The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems, Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 241(1226) (1957) 376-396.

[40] X. Song, W.C. Liu, J.P. Belnoue, J. Dong, G.H. Wu, W.J. Ding, S.A.J. Kimber, T. Buslaps, A.J.G. Lunt, A.M. Korsunsky, An eigenstrain-based finite element model and the evolution of shot peening residual stresses during fatigue of GW103 magnesium alloy, International Journal of Fatigue 42 (2012) 284-295.

[41] L. Ma, A.M. Korsunsky, The principle of equivalent eigenstrain for inhomogeneous inclusion problems, International Journal of Solids and Structures 51(25-26) (2014) 4477-4484.

[42] T.S. Jun, K. Dragnevski, A.M. Korsunsky, Microstructure, residual strain, and eigenstrain analysis of dissimilar friction stir welds, Materials & Design 31, Supplement 1 (2010) S121-S125.

[43] T. Hensl, U. Mühlich, M. Budnitzki, M. Kuna, An eigenstrain approach to predict phase transformation and self-accommodation in partially stabilized zirconia, Acta Materialia 86 (2015) 361-373.

[44] L.E. Murr, S.M. Gaytan, D.A. Ramirez, E. Martinez, J. Hernandez, K.N. Amato, P.W. Shindo, F.R. Medina, R.B. Wicker, Metal Fabrication by Additive Manufacturing Using Laser and Electron Beam Melting Technologies, Journal of Materials Science & Technology 28(1) (2012) 1-14.

[45] O. Ivanova, C. Williams, T. Campbell, Additive manufacturing (AM) and nanotechnology: Promises and challenges, Rapid Prototyping Journal 19(5) (2013) 353-364.

[46] A. Hinojos, J. Mireles, A. Reichardt, P. Frigola, P. Hosemann, L.E. Murr, R.B. Wicker, Joining of Inconel 718 and 316 Stainless Steel using electron beam melting additive manufacturing technology, Materials & Design 94 (2016) 17-27.

[47] I. Kelbassa, P. Albus, J. Dietrich, J.I. Wilkes, Manufacture and repair of aero engine components using laser technology, (2008).

[48] J. Jones, P. McNutt, R. Tosi, C. Perry, D. Wimpenny, Remanufacture of turbine blades by laser cladding, machining and in-process scanning in a single machine, 23rd Annual International Solid Freeform Fabrication Symposium - An Additive Manufacturing Conference, SFF 2012, 2012, pp. 821-827.

[49] G. Bi, A. Gasser, Restoration of Nickel-Base Turbine Blade Knife-Edges with Controlled Laser Aided Additive Manufacturing, Physics Procedia 12, Part A (2011) 402-409.

[50] X. Zhou, D. Wang, X. Liu, D. Zhang, S. Qu, J. Ma, G. London, Z. Shen, W. Liu, 3D-imaging of selective laser melting defects in a Co-Cr-Mo alloy by synchrotron radiation micro-CT, Acta Materialia 98 (2015) 1-16.

[51] R.S. Khmyrov, C.E. Protasov, S.N. Grigoriev, A.V. Gusarov, Crack-free selective laser melting of silica glass: single beads and monolayers on the substrate of the same material, International Journal of Advanced Manufacturing Technology 85(5-8) (2016) 1461-1469.

[52] H. Liu, Sparks T., F. Liou, Residual Stress and Deformation Modelling for Metal Additive Manufacturing Processes, Proceedings of the World Congress on Mechanical, Chemical, and Material Engineering (MCM 2015) Barcelona, Spain – July 20 - 21, 2015 (2015) Paper No. 245.

[53] E. Salvati, T. Sui, A.J.G. Lunt, A.M. Korsunsky, The effect of eigenstrain induced by ion beam damage on the apparent strain relief in FIB-DIC residual stress evaluation, Materials & Design 92 (2016) 649-658.

[54] A.M. Korsunsky, K.E. Wells, P.J. Withers, Mapping two-dimensional state of strain using synchroton X-ray diffraction, Scripta Materialia 39(12) (1998) 1705-1712.

[55] M. Senn, Digital Image Correlation and Tracking, [Computer Software] https://uk.mathworks.com/matlabcentral/fileexchange/50994-digital-image-correlation-and-tracking (2015).

[56] Y. Watanabe, Simulation of Residual Stress Distribution on Shot Peening, Journal of the Society of Materials Science, Japan 44(496) (1995) 110-115.
[57] Z. Wang, E. Denlinger, P. Michaleris, A.D. Stoica, D. Ma, A.M. Beese, Residual stress mapping in Inconel 625 fabricated through additive manufacturing: Method for neutron diffraction measurements to validate thermomechanical model predictions, Materials & Design 113 (2017) 169-177.
[58] G. Vastola, G. Zhang, Q.X. Pei, Y.W. Zhang, Controlling of residual stress in additive manufacturing of Ti6Al4V by finite element modeling, Additive Manufacturing 12, Part B (2016) 231-239.
[59] B.A. Szost, S. Terzi, F. Martina, D. Boisselier, A. Prytuliak, T. Pirling, M. Hofmann, D.J. Jarvis, A comparative study of additive manufacturing techniques: Residual stress and microstructural analysis of CLAD and WAAM printed Ti–6Al–4V components, Materials & Design 89 (2016) 559-567.
[60] T. Mukherjee, W. Zhang, T. DebRoy, An improved prediction of residual stresses and distortion in additive manufacturing, Computational Materials Science 126 (2017) 360-372.
[61] G. Tapia, A.H. Elwany, H. Sang, Prediction of porosity in metal-based additive manufacturing using spatial Gaussian process models, Additive Manufacturing 12, Part B (2016) 282-290.