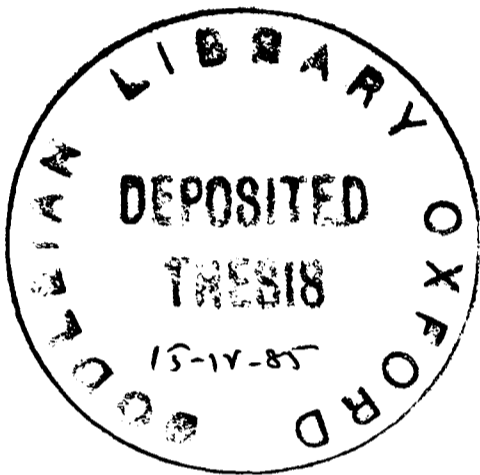


MANAGERIAL INCENTIVES AND CONTROL
IN PUBLIC ENTERPRISES



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ABSTRACT

MANAGERIAL INCENTIVES AND CONTROL IN PUBLIC ENTERPRISES

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The subject matter of the thesis is the study of managerial incentive schemes for public enterprises. The problem of incentives and control is characterized in Chapter I stressing asymmetric information and preferences between principal (Minister) and agent (Manager). Chapter II reconsiders the findings of some previous works on the use of simple mechanisms under certainty and it shows that incentive problems may be solved with a pure-rent type contract. One of the parameters of the contract (profit-sharing ratio) is central to the enforcement of 'managerial' efficiency while the relative weights given to profit and (an approximation to) consumer surplus will influence optimal pricing decisions. This feature is maintained in a risky environment (Chapter III) although reinterpreted in a second-best fashion due to the trade-off between incentives and risk-sharing. The weights given to each side of the contract depend on the impact of price changes upon the degree of profit-uncertainty and the managerially self-selected level of effort. In addition it is shown that, when providing incentives, the Minister will depart from the pricing rule derived under full information. Chapter IV shows that these results are valid when capacity choice and non-price rationing issues become relevant. Chapter V attempts to integrate the issues of performance indicators and efficiency audits into the previous framework. It is shown that there must exist an upper limit to the admissible number of performance indicators and that efficiency audits can be designed as conditional investigation procedures and used according to an expected cost-benefit characterization. Some central underlying factors affecting the form of the optimal investigation strategy are identified. Finally, Chapter VI attempts to consider the previous results and their implications within a brief discussion of U.K. policy for public enterprises in the last decades.

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CHAPTER I
INTRODUCTION

1.1. Introduction

The problem of incentives and control in public enterprises has recently drawn the attention of a number of economists who felt that the traditional normative analysis of the public firm was incomplete, since the optimal rules on which it is based have to be taken by organizations and individuals with goals and objectives of their own, and have to be controlled by poorly informed government departments. In the area of public pricing for example there is no certainty that decision makers will be motivated to follow the appropriate directives or rules if they pursue their own goals. In addition, those government departments in charge of the enterprise cannot directly enforce the rules given their lack of knowledge about demand and, in particular, technical conditions faced by the firm. Moreover, not only allocative but also managerial efficiency defined in a broad sense cannot be directly enforced either: even if we are sure that management follows the optimal allocative rules, there is no guarantee that costs are minimized given the absence or mitigation of automatic, market-related mechanisms that can discipline managerial teams. In fact, to many observers and critics of nationalization as a regulatory mechanism this lack of production or technical efficiency rather than any dead-weight loss triangle due to allocative inefficiency is the central issue that has never been solved.

In this context, and against the background of a recent and flourishing literature on related topics, a study of managerial incentive schemes for public enterprises hardly needs a major justification effort. The orientation and spirit of the present thesis can be seen within the framework of many recent attempts to characterize and design a workable regulatory process of managerial decisions in public enterprises. To this end, this chapter will

provide an overview of some central issues and it will delimit the scope of the thesis. Given that much of the relevant literature is critically surveyed in each one of the following chapters, along with our own treatment of the problems, we will not embark here on a major or extensive review.

1.2. Managerial Discretion and Property Rights in the Public Sector Firm¹

In the traditional textbook analysis of public enterprise economics, the emphasis is put on optimal pricing rules in a partial equilibrium context². Undoubtedly, we have witnessed major developments within this approach that have enriched both theory and practice in the sector. However, there have been no concomitant attempts to provide a coherent regulatory framework in which the derived optimal policies could be enforced. Despite the fact the rules or policies make sense chiefly in a decentralized process of decision making, we did not have until recently even simple theoretical works that could analyze the achievement of those rules within a decentralized process. In the absence of any specification of the likely behaviour of decision makers inside the public firm any elaboration on such a regulatory process seems a very difficult task.

As soon as we begin to see the public enterprise as an organization with its own goals and objectives, we realize that there is ground for using the modern theories of the firm to give substance to some of the above questions³. In particular, stressing the entrepreneurial nature and market-orientated activities of public enterprises makes the use of those received theories even more relevant.

¹For a more detailed discussion see Navajas (1983).

²See for example Bös (1981), Crew and Kleindorfer (1979) and Rees (1984b).

³Baumol (1959), Penrose (1980), Williamson (1964) and Marris (1964) are the most representative works. Another line was associated with H. Simon, Cyert and March. See Marris and Muller (1980) for a review.

A major theme in the literature has been the effects of the separation of ownership and control on the behaviour of managerial teams. Theories of managerial firms emerged in the late fifties and early sixties stressing revenue, growth and emoluments or expenses as alternative objectives followed by Managers in modern corporations. Looking at the dramatic form of separation between "owners" and controllers present in public firms; the non-marketed nature of the public firm's assets which hinders any discipline coming from capital markets; the absence of a clear measure of performance or success in a multi-objective environment and thus the absence of performance-related managerial remuneration; and finally the 'sluggishness' of public sector labour markets; it seems clear that public enterprise Managers perform their activities in a regime fertile to the proliferation of managerial discretion. The difficult task for the analyst however is to find an appropriate abstraction or description of such a regime. The most recent and serious attempt to tackle the problem is the work by Rees (1982) reviewed below in sections 1.3. and 1.4.

Almost two decades ago, in one of the first applications of another major theory of the firm - the property rights approach -, Armen Alchian (1965) arrived at a novel characterization of the problem of incentives and efficiency in public enterprises. For Alchian the crucial point resided in the non-marketed nature of the ownership rights of the public firm, which gave rise to a particular cost-reward structure facing both owners (i.e. citizens) and Managers of the firm. On the one hand, the very large and unrestricted number of 'public owners' diluted the gains that any of them could have to enforce a cost-reducing policy. On the other hand, self-interested Managers derive utility from (but do not bear the cost of) on-the-job consumption and related forms of non-pecuniary income. Thus, two basic ingredients of the property rights theory, namely (a) utility maximizer self-interested agents, and (b) the structure of property rights shaping the constraints faced by each of them,

were first used in this analysis to arrive at a characterization of the poor incentives for efficiency present in public enterprises.

Later elaborations of the general theory itself (e.g. Alchain and Demsetz, 1972) and applications to the public firm¹, clarified concepts and in particular incorporated specific and testable hypothesis related to investment decisions, pricing, productive efficiency, managerial activities and the like. Followers of the property rights approach however have been more interested in questions of comparative efficiency vis-a-vis the private sector² and in aspects of comparative institutional choice (i.e. different property rights arrangements) rather than in problems directly concerned with the design of a regulatory system of decisions within public firms. For a great number of these economists efficiency is best discussed and achieved if we address ourselves to alternative institutional choices in which privatization is a major option. Some have openly rejected public ownership as a workable institutional mechanism to enforce efficiency (e.g. Wiseman, 1978 and Littlechild, 1979).

A similar line of analysis to the property rights theory is represented by the so-called public choice approach, which instead stresses the lack of competition within the government and tries to make more explicit the role of bureaucrats and institutions within a certain political process³. The theory of bureaucracy presented by Niskanen (1971) is the seminal contribution in the area. His model of bureaucratic budget maximizing behaviour resembles the managerial theories of Baumol and Williamson insofar as the specification of managerial preferences is concerned. It is clear however that these works are

¹See for example De Alessi (1969, 1974, 1975, 1980), Wiseman (1978), Rowley and Yarrow (1981), Littlechild (1979).

²See for example the vast literature surveyed by Borcharding et al. (1982).

³See for example Mueller (1979) for a survey of the approach, Borcharding et al. (1982) for a survey of empirical results related to public firms and Lindsay (1976) for a development.

perhaps more useful for explaining the behaviour of government agencies and institutions less entrepreneurial and market-orientated in nature than the public enterprises we are interested in. On the other hand, the public choice approach can be very helpful if we want to explain the behaviour of government officials and politicians toward the objectives and control of public firms, that is to fill the gap between 'public owners' and administrators of public firms in Alchian's characterization given before¹.

To sum up, managerial discretion in public enterprises would be the result of the absence or mitigation of market-related mechanisms of control that in principle could not discipline managerial teams as much as they can in the private sector. The particular configuration of property rights and the derived cost-reward structure facing Managers imply that there may exist substantial barriers to the enforcement of efficiency in the sector. In the next section we make more precise the issues behind the problem of control and incentives in public enterprises using the principal-agent paradigm.

1.3. The Control of Public Enterprises from a Principal-Agent Perspective

A problem of control and incentives is said to exist in an organization when agents have different preferences and information from those possessed by the principals. Asymmetries of information and preferences are thus two major preconditions, otherwise the problem becomes trivially easy to solve. If agents had the same preferences as the principals they would not depart from the actions or decisions the principals themselves would make, while if the principals had the same knowledge and information available to the agents they could directly enforce their best 'centralized' decisions. Modern theoretical treatments of incentives and control in organizations² regard

¹The work of Foster (1971) is a clear example.

²See Holmstrom (1978) Chapter I. Reviews of the area are given by Laffont and Maskin (1982) and Hess (1983).

agents as supplying two basic inputs: (a) information and knowledge for decision-making, and (b) productive inputs. The first are inputs associated with information required to implement the principal's best choice while the second are derived from the fact that agents are central participants in a production process. In practice, there may not be a clear line between these two classes of inputs. However, for theoretical purposes the division has proved fruitful and entire areas in the literature are specialized in the treatment of either pure decision-making or pure production incentives.

The principal-agent relationship has emerged in recent years as a convenient paradigm example in which to discuss problems of control and incentives. In the theory of the firm for example, Jensen and Meckling (1976) have used it to provide a new look at the effects of ownership structure on the activities of Managers and the resulting financial structure. (See also Fama (1980) and Marcus (1982)). At a more general and abstract level, the literature has explored optimal incentive contracts that either emphasize the trade-off between risk and incentives in a pure moral hazard context or, more recently, also look at asymmetric information problems. In the first group, the early contributions of Wilson and Ross on pure risk incentives provided the basis for the analysis of pure production incentives or moral hazard problems¹.

In these models a risk averse or neutral principal employs a risk (and effort) averse agent to perform a given task. The principal derives utility from the value of an outcome or product, net of the agent's remuneration, which is affected by a random variable and depends positively on the agent's supply of an unobservable productive input; while the agent derives utility from income and disutility from his work or effort. The central instrument

¹Wilson (1968, 1969), Ross (1973, 1974), Spence and Zeckhauser (1971) Stiglitz (1974, 1975), Mirrlees (1976), Harris and Raviv (1978, 1979), Holmstrom (1979), Shavell (1979) and Grossman and Hart (1983).

of control is a sharing rule which pays the agent according to the value of the outcome. The literature then has been mainly concerned with the derivation of optimal sharing rules that balance gains and losses from incentives and risk sharing. The pre-contractual information structure in these models is completely symmetrical except for the effort decision (thus the classification as pure production incentives models), that is, both parties know every relevant functional relationship although the principal cannot observe the actual level of effort chosen by the agent, and both are unable to observe the actual state of nature. In Harris and Raviv (1978, 1979, model 2) and Holmstrom (1979, section 3.5) we find treatments of post-contractual asymmetric information where the agent observes the actual realization of the state of nature after he has signed the contract with the principal but before he selects his effort. However, the principal uses no communication channel to take opportunity of the increased information obtained by the agent in the process of decision making. Christiansen (1981) included such a communication structure using developments from the next class of models to be discussed.

More recently, a second group of principal-agent models have considered as a central issue aspects of asymmetric information (Harris and Townsend, 1981; Sappington, 1980). This asymmetry is pre-contractual in nature, that is, the agent knows privately not only his level of effort but also some random parameters of the structure of the problem before the contract is signed, while the principal knows every relevant functional relationship and has prior subjective beliefs on the private information possessed by the agent. Moral hazard due to unobservable effort is still present in some of these models but the emphasis is mainly on the construction of revelation mechanisms instead of on the incentive-risk problems of the previous literature. For this reason, these models are directly related to the vast literature on the implementation of social choice rules and the provision of public goods (see Dasgupta et al., 1979 and Laffont and Maskin, 1982) which concentrates on pure decision-

making incentives. A major result in the area is the so-called "revelation principle" whereby the principal, in his search for incentive compatible mechanisms that are parameter contingent (i.e. depend on the private information possessed by the agent), can restrict himself to simple direct mechanisms¹ that have truthful revelation of the relevant parameters as an optimal equilibrium strategy for the agent.

Moving from the abstract models described so far to the context in which public enterprises are controlled we find many particular features that deserve special comment.

First, it has been argued (e.g. Aharoni, 1982) that in practice it may be difficult to identify a principal with clear and unambiguous objectives as it is described in the theory. On the one hand, ministerial control of public enterprises is a political process in which many principals and interest groups are immersed. It is not so much that there exist many principals - a fact present also in many other institutional settings - but for the particular configuration that this multi-principal situation adopts in the public sector and its impact on the required objectives and goals of the public enterprise. In standard textbook treatments, public enterprise economics provides us with a clear distinction of the relevant objectives that should be followed by a public firm. Allocative efficiency, distributional effects and profitability can be treated within a normative framework. At a more positive level of analysis however, ministerial objectives might be difficult to clarify. In practice for example it is often argued that a central problem in the control of public firms is that ministerial intervention is so frequent, uncoordinated and contradictory that it makes impossible to measure performance. Despite these shortcomings, we maintain in this thesis that as an extension of the mainstream public enterprise economics it is perfectly valid to take the

¹A direct mechanism is one in which the agent is asked to report directly his private information.

organizing principles of the discipline as the objectives that an idealized, altruistic minister would follow. This would amount to an omission of public choice problems at the government level, or to put it in more fashionable terms, to a mis-specification of the positive aspects of choice within the public sector. However, we would defend the present line of inquiry on the grounds that it can still shed light on some central aspects of the problem of control. In addition, we do not believe that the traditional approach is in a bankruptcy situation in the sense of being unworkable. We agree with Rees (1984b) in that ... "to cease to insist on the welfare-maximization approach is to let the case go by default and to leave the field clear to the obfuscations and fudges of accountants and the special pleading of particular interest groups."

Second, the agent can in principle be interpreted as a simplification concept representing the managerial team in charge of the enterprise and with preferences defined over income and effort as in the paradigm. Gravelle (1982a) for example has followed such a specification in his evaluation of different mechanisms of incentives and control. It will also be the one followed in this work since we believe it to be the most convenient and appropriate one for the analysis of simple incentive problems. The term managerial effort or action, however, is a sort of "catch-all" concept representing both a productive managerial input and the inverse of different forms of non-pecuniary income. In addition, as a positive specification this may be less than satisfactory. In actual regimes of control of public firms, where income is fixed (and no profit or performance-related sharing rule is used) it seems certain that management will develop preferences toward some particular activities, the concrete specification of which might prove fruitful for the analysis. Rees (1982), for example, has taken the size of the firm - measured by physical output - as a relevant proxy to consider for the specification of managerial utility. Moving towards a regime of control where bonus payments

are introduced, income should now matter for the Manager. However, the income-effort characterization seems to imply the rather extreme result that in the process any other preferences such as size are eliminated rather than mitigated. Moreover, as Rees (1982) and Gravelle (1982b) in different analyses have suggested, the special power of trade unions in public enterprises might imply that the "objective" function of the firm results from an equilibrium bargaining in which unionized labour may 'impose' preferences for employment and wages. Throughout this thesis we shall ignore these aspects to concentrate on the traditional income-effort specification of managerial preferences.

Finally, the information structure assumed to be possessed by the principal in the paradigm is far too huge and complex compared with the information that in practice is available to ministers in charge of public firms. Rees (1980) made this point in the first characterization of the control problem of public firms made using the principal-agent paradigm. He noticed that on the one hand the problem of information was different since the most relevant 'actions' taken by Managers were observable, such as prices, quantities, capital, labour, etc. On the other hand, the information structure represented by the form of the relevant functional relationships in the problem, (such as demand, costs, managerial preferences, etc.) was completely non-available to Ministers. Given these distinguishing facts, Rees maintained that (in the absence of performance-related remuneration or sharing rules) the principal-agent problem should be reformulated for public firms, in a way that questions of asymmetric information, participative planning or management-by-participation played the leading role, within a context where control constraints such as profit targets and capital rations are adjusted reflecting exchanges in information¹. It follows that the emphasis should be then put on asymmetric information versions of the paradigm rather than in pure moral hazard ones, although

¹This is developed further in Rees (1984a).

we have noticed before that both versions require the principal to know basically the same functional relationships.

Nevertheless, the fact that pricing and other decisions are observable should not diminish the relevance of production incentive problems associated with the provision of some managerial input. We have mentioned before that the problem of managerial or X-inefficiency is often cited as central in public debates. Moreover, the fact that managerial income is fixed in practice and no sharing rule is used should not mean that we cannot study performance-related remuneration systems equivalent to those used in the private sector. If these points are accepted, it follows that we can still have something to learn from moral hazard or pure production incentive formulations.

This however lead us straight to questions about the feasibility of measuring performance in a relatively costless fashion. Take for instance the simple partial equilibrium analysis with no distributional or financial considerations where performance is given by the sum of consumer surplus and profits. The central problem here rests in the measurability of the former. On the one hand, we may think of a situation where demand curves can be made available and a measure of consumer gains approximated. There is a long debate in welfare economics about whether we can even approximate that measure. McKenzie (1983) summarizes this polemical area and perhaps more importantly, shows that an accurate money metric of consumer gains can be built from observable demand functions. Therefore, following his method one could in principle arrive at a measure of social benefits. On the other hand, recent work by Finsinger and Vogelsang (1982) and others have addressed the problem of the lack of demand functions and discussed ways in which consumer surplus can be roughly approximated by a price index.

To sum up, problems of control and incentives result from asymmetries of preferences and information between principal and agents. The principal-agent paradigm can give an organizing framework in which to discuss incentives

and control in public enterprises, even though the case has its own characteristics. Taking social welfare as the objective function of the principal or Minister and income-effort as the arguments of the utility function of the Manager we can explore the use of bonus schemes in a similar fashion to that of the general paradigm.

1.4. Previous Treatments in the Literature

The study of specific schemes to deal with incentives and control in public enterprises has been enriched by many recent contributions. Finsinger and Vogelsang (FV) (1982) developed a simple mechanism to solve information problems when demand is unobservable by adapting a previous mechanism proposed for the regulation of private monopolies (FV, 1979). The scheme attempts to cope with pricing incentives under conditions of (almost extreme) asymmetric information, assuming certainty and stationarity in demand and cost curves. Some work by Scott (1952, 1978) had previously proposed a similar scheme although with less analytical elaboration. In addition there exist striking similarities with the problems and schemes studied by Domar (1974) and in particular Tam (1981) to enforce efficient pricing in socialist firms. Finally, FV (1981) followed a comparative-institutional-choice approach in comparing variations of their scheme for different environments.

Gravelle (1982a) studied production and pricing incentives under certainty, stationarity and assuming the same degree of asymmetric information as FV (1982). His work is also a comprehensive and critical evaluation of proposals put forward implicitly or explicitly in previous works: Rees (1968), Bös (1978), Crew, Kleindorfer and Sudit (1979), Scott (1978) and FV (1982), as well as the traditional profit target constraints described in official papers (White Papers 1967, 1978) and studied by Gravelle and Katz (1976). Gravelle showed that in a simple environment none of the examined mechanisms is totally successful in solving both incentive problems.

As commented before, Rees (1980) provided the first characterization of the control problem of public firms using the principal-agent paradigm. Rees (1982) gives a positive analysis of public enterprise decision making, in which some equilibrium predictions are obtained - concerning the use of labour and capital inputs, pricing, etc. - and accord well with empirical observation of the sector. In Rees (1984a) the analysis is extended to characterize the actual process of control and target setting. Again, the emphasis is on actual rather than optimal (i.e. from the viewpoint of a social welfare maximizer Minister) mechanisms. In the same line of positive analysis, Gravelle (1982b) has studied bargaining problems between Managers and unions, their implications for efficiency and a comparison with results obtained for an unregulated monopoly.

Finally, the paper by Bergson (1976) is a different contribution, concerned mainly with pure risk incentives without modelling any particular decision (although he has in mind investment decisions). Using Ross (1973, 1974) framework as a reference he studies the bonus required to achieve 'similarity' between Manager and Minister with emphasis on the size of the resulting rewards¹.

1.5. Scope and Outline of the Dissertation

Chapter II provides an analysis of linear bonus schemes for Managers in a context of certainty. It can be seen as reconsideration of the results obtained by Gravelle (1982a) and Finsinger and Vogelsang (1982).

¹After completing this thesis a paper by Guesnerie and Laffont (1984) has called my attention, in which they study incentive schemes in public firms under asymmetric information, following the methodology of the second class of principal-agent models described above. It is worth noting that the authors share the same view on the nature of the problem of control in public firms outlined in the previous section. Their main conclusions are that "the design of adequate incentive schemes is likely to improve very much upon the non-informed optimal policies" and that "optimal incentive schemes are sensitive to the nature and the objectives of the firms as well as to the objectives of the Center".

We find that in this environment it may be possible to solve incentive problems for pricing and production even when demand is not observable.

In Chapter III we extend the analysis allowing for demand and cost uncertainty and again restricting the analysis to linear bonus schemes. We study efficient incentive contracts stressing production incentives and assuming a well informed Minister. With uncertainty the information required to enforce managerial and allocative efficiency is high. However, we ask what can we learn from the analysis that may be relevant for the design of actual incentive schemes.

Chapter IV is an extension of the model of the previous chapter to consider aspects of capacity choice and rationing neglected before.

In Chapter V we make a more explicit analysis of the monitoring system and use some treatments in the literature to discuss the role of performance indicators and efficiency audits in the process of control.

Finally Chapter VI concludes the thesis. Instead of being a straightforward summary of conclusions however, this chapter attempts to consider the conclusions of the work within a discussion of some aspects of the actual process of control in the United Kingdom. This would allow us to evaluate the policy implications of the previous results and also to suggest ways in which the theoretical treatment can be improved insofar as policy questions are concerned.

At the outset, we notice ^{four}~~three~~ major aspects that delimit the scope of the thesis:

- (a) We abstract from problems related to the definition of objectives in a control department or Ministry and within a political process, i.e. public choice problems are neglected.
- (b) We assume a well defined, two-person situation with well established objectives for principal and agent; the former being a socially altruistic welfare maximizer and the latter a self-interested utility maximizer in income and effort.

(c) The emphasis is on outcome-based incentive contracts that take the form of sharing rules. In addition, the analysis concentrates more on production (moral hazard) than on decision-making (asymmetric information) incentives.

(d) In the models studied below, the social welfare function of the Minister is represented by the sum of consumer and producer surpluses less managerial income (subject to a given utility level of the manager which reflects employment opportunities elsewhere in the economy). This formulation is in conformity with the traditional principal-agent formulation although it is different from the one adopted in the works of Finsinger-Vogelsang (1982) and Gravelle (1982a) where managerial income is not deducted.

CHAPTER II

LINEAR MANAGERIAL BONUS SCHEMES: SOME RESULTS UNDER CERTAINTY

Introduction

In the previous chapter we have briefly mentioned some previous attempts to study the problem of incentives for pricing and managerial efficiency in a context characterized by asymmetric information and certainty. The basic idea behind these proposals has been the construction of simple mechanisms that are informationally feasible in terms of the knowledge that the regulators must possess.

This chapter attempts to reconsider some of the findings of the received literature. We shall first study the use of managerial bonus schemes in a context where the Minister in charge of the public enterprise can measure the demand function of the firm and obtain information about cost (but not the cost function), profit, prices and quantities. Within this context, we shall show that the Minister is capable of solving the above mentioned incentive problems by offering the Manager an incentive contract that adopts a pure-rent form, that is it pays the sum of consumer and producer surplus less a fee (or equivalently above a given target). This result contradicts a previous claim by Gravelle (1982a) on the unfeasibility of such a scheme and explains why Gravelle's result is incomplete.

Although interesting as an illustration of how to provide optimal incentives, the previous scheme does not satisfy the informative constraint on demand imposed by most of the works. In order to overcome the lack of observability of consumer benefits, Finsinger and Vogelsang (1982) proposed an adjusting mechanism based on a performance index linked to managerial rewards. They first claimed to have solved both pricing and production incentive problems, but this was later re-examined by Gravelle (1982a, 1983) who showed that the problem of managerial incentives - not explicitly considered by Finsinger

and Vogelsang - could only be solved if the scheme were redesigned in such a way that the Manager should appropriate all the welfare gains. Therefore, in the end, the failure of the Finsinger-Vogelsang mechanisms rested on a problem of appropriation. However, showing that Gravelle's conclusions are based on a sub-optimal bonus function we reformulate the problem and then extend the previous result under perfect observability of demand to the present context.

The structure of the chapter is the following. In section 2.1 we study an optimal linear bonus scheme under marginal cost pricing (section 2.1.1) and Ramsey pricing (section 2.1.2) and when demand is perfectly observable by the Minister. Section 2.2 briefly comments on some financing aspects of the scheme. Section 2.3 deals with the problem of the lack of information about demand. Finally in section 2.4 we comment on the adaptation of the proposed scheme to the introduction of distributional aspects, and its likely failure to attend quality regulation and to account for the case of managerial preferences for size (in addition to income and effort). Section 2.5 summarizes our conclusions.

2.1. An Optimal Incentive Scheme when Demand is Observable

We shall formulate our analysis in the context of a partial equilibrium situation, where only the prices of the public firm (monopolist) change and do not affect the equilibrium of an otherwise first best economy. Restrictions to the use of lump sum taxation to finance any deficit of the enterprise are considered in the next section.

The firm is run by a Manager who derives utility from his income and disutility from effort. Effort is in turn seen as a hidden productive input which affects the profit level obtained by the firm. Control of the enterprise is in the hands of a socially altruistic Minister whose objective is the maximization of the sum of producer and consumer surplus. He would like to see the Manager supplying a high amount of effort but the unobservability of managerial behavior plus effort-aversion by the Manager prevent this. Also, the Minister would like to see the traditional marginal cost pricing rule implemented by the Manager but he cannot be sure about the Manager following his directives given that he cannot obtain information about marginal cost. In order to encourage the Manager to supply effort, an incentive scheme which remunerates the Manager according to the performance of the firm is introduced. The ability of the Minister to measure performance is crucial: given that he can observe the demand function of all the products of the firm, and due to the absence of income effects, he measures performance by the sum of consumer surplus and reported profits. The problem for the Minister is then to derive the optimal parameters of a linear incentive contract, where linearity is given by administrative and informational restrictions which make non-linear schemes unfeasible.

Managerial utility is given by the function

$$U = U(y, a) \tag{2.1}$$

with $U(\cdot)$ continuously differentiable with partial derivatives $U_y > 0$, $U_a < 0$, $U_{yy} < 0$, $U_{aa} > 0$, where y is income and a denotes effort. We shall neglect

the impact on managerial behavior of the fact that the Manager is a taxpayer and a consumer of the products of his firm (see Gravelle 1982a, for an inclusion of these aspects).

There exist n products or services provided by the enterprise, with demand functions represented by the vector $x(p)$ where p is the vector of n prices. Consumer surplus is defined by

$$S(p) = \int_p^{\hat{p}} \sum_{i=1}^n x_i(p) dp_i \quad (2.2)$$

where the absence of income effects guarantees the so-called "integrability conditions"¹, and $\hat{p} = (\hat{p}_1, \dots, \hat{p}_i, \dots, \hat{p}_n)$ such that $x_i = 0$ for $p_i \geq \hat{p}_i$ $i = 1, \dots, n$.

The cost function of the firm is written as

$$C = C[x(p), a] \quad (2.3)$$

where $C[\cdot]$ is continuously differentiable with its partial derivative with respect to effort $C_a < 0$, $C_{aa} > 0$. Profits are therefore defined by

$$\pi(p, a) = p \cdot x(p) - C[x(p), a] \quad (2.4)$$

Both cost and profits in (2.3) and (2.4) are computed before managerial rewards of any type. These are condensed in the function

$$y = \alpha_1 \cdot S(p) + \alpha_2 \cdot \pi(p, a) + c \quad (2.5)$$

with parameters α_1, α_2 and c to be determined by the Minister. Both the Manager and Minister can observe demand functions and therefore computed consumer surplus is observable by both parties. The Minister cannot observe the cost function (2.3), the level of effort chosen by the Manager, although he obtains information from bookkeeping data concerning the level of profits, and presumably cost.

The objective of the Minister is to maximize the sum of (2.2) and (2.4) net of managerial income and subject to a given level of managerial utility.

¹New methods (e.g. McKenzie, 1983) have been proposed such that a money metric of consumer benefits is derived without the above restrictions. Traditional treatments of public pricing in partial equilibrium implicitly or explicitly follow this assumption (e.g. Crew and Kleindorfer 1979, Rees 1984b).

However, we do not assume that the Minister knows the utility function of the Manager. Nevertheless, employment opportunities elsewhere put a lower bound to the utility attained by the Manager. In a similar fashion, we shall argue below that the bargaining power of the Minister sets a lower bound for his attainable utility represented by social welfare net of managerial income.

Our main concern will be to see whether the Minister can enforce optimal pricing and managerial efficiency in this context. Notice that managerial effort, which is the variable that determines managerial efficiency appears only in the profit function, while prices are present in both sides of the welfare function. This suggests that the "profit's part" of the incentive scheme (2.5), namely the profit sharing ratio¹, will play the central role in inducing managerial efficiency, while optimal pricing will depend on the relative weights given to both sides of the contract. This is illustrated below. The first result is obtained in a first best context where lump-sum taxation is feasible and marginal cost pricing is therefore the optimal allocative rule.

2.1.1. Marginal Cost Pricing and Managerial Efficiency

Let us first illustrate the optimal conditions for allocative and managerial efficiency in a full information context. Here the Minister knows all the relevant functions and he can observe the level of effort supplied by the Manager; so his problem is represented by

$$\text{Max}_{p,a,y} \quad S(p) + \pi(p,a) - y + \lambda \cdot U(y,a) \quad (2.6)$$

First order conditions of this problem², assuming interior solution are given by

$$\sum_{i=1}^n (p_i - C'_i) \cdot \frac{\partial x_i}{\partial p_k} = 0 \quad \text{for all } i,k = 1, \dots, n \quad (2.7)$$

¹Throughout this and the next chapters we shall be referring to α_2 as the profit sharing ratio although the term "marginal profit sharing ratio" would be more correct.

²Second order conditions are guaranteed by assuming that (2.6) is locally concave in p,a and y at the optimal solution.

$$- C_a + \lambda \cdot U_a = 0 \quad (2.8)$$

$$- 1 + \lambda \cdot U_y = 0 \quad (2.9)$$

where $i, k = 1, \dots, n$ represent indices for goods, $C'_i = \partial C / \partial x_i$ and λ is the multiplier associated with the managerial utility constraint. Condition (2.7) states marginal cost pricing, while from (2.8) and (2.9) we obtain the condition for managerial efficiency, namely that the subjective marginal rate of substitution between managerial income and effort (which represents the social cost of additional effort) must be equal to the marginal social gain from effort (given by the reduction in cost), that is

$$\frac{U_a}{U_y} = C_a \quad (2.10)$$

Condition (2.9) shows that, in equilibrium, the value of the multiplier λ is given by the inverse of the marginal utility of managerial income, or the marginal social value, in f , of managerial utility, and can be seen as a result derived from welfare maximization with no distributional consideration (e.g. Varian, 1978, p. 154; Gravelle, 1982a, p. 85). Alternatively, within a principal-agent, partial equilibrium context, condition (2.9) has been seen as a condition for optimal risk sharing, namely that the ratio of the marginal utilities of principal and agent, here $1/U_y$, equals a constant¹. Of course, we are neglecting risk sharing issues until the next Chapter. In addition, and again in the spirit of partial equilibrium analysis, the value of λ is affected by the minimum acceptable utility level for the Manager, \hat{U} , which is taken as a constraint by the Minister.

It is interesting and important to notice why the rules derived from the previous problem are not implementable under asymmetric information. Of course, the fact that the Minister does not possess information about cost prevents him from choosing marginal cost pricing for a given level of effort.

¹See for example Raiffa (1968), Holmstrom (1979) and the discussion in the next Chapter.

However, suppose we are in a context where the issue of managerial efficiency is irrelevant, either because the productivity of effort is zero, i.e. $C_a = 0$, or because the Manager does not derive dis-utility from additional effort and he decides to supply the maximum effort possible. In this situation the only remaining issue is the decision to price according to marginal cost, and there is no indication whatsoever that the Manager will follow this rule. Nevertheless, with his income fixed and without effort-aversion, there is no indication that he will gain from not revealing the true marginal cost either. In this case we cannot prove that the marginal cost pricing rule fails to be implementable due to incentive-incompatibility problems. When production incentive problems are reintroduced, if managerial income remains fixed the Manager will then supply a minimum amount of effort; let us suppose that this minimum is zero. In this case he has still no incentive to lie about the actual level of marginal cost, but since he is reducing his effort to zero, pricing is determined with respect to the "wrong" marginal cost curve. This could lead us to affirm that the problem lies in the absence of appropriate incentives for the supply of effort and that profit sharing as in the private sector will solve the problem. This however is clearly incorrect since whenever managerial income becomes dependent on profits the Manager will then have incentives to lie about the level of marginal cost: he will claim that marginal costs are given by the level of profit maximizing prices in order to increase his income and utility for any given level of effort he chooses. It is in this context that the contract given by (2.5) is introduced in order to provide incentives for production and to correct the incentive-incompatibility problems (with respect to pricing) derived from profit sharing.

The Manager's problem is to maximize his utility

$$\begin{array}{ll} \text{Max} & U(y, a) \\ & p, a \end{array} \quad (2.11)$$

where y is given by (2.5). First order conditions for this problem, assuming

interior solutions, are given by

$$U_y \cdot \left\{ -\alpha_1 \cdot x_k + \alpha_2 \cdot \left[x_k + \sum_i (p_i - C'_i) \cdot \frac{\partial x_i}{\partial p_k} \right] \right\} = 0 \quad (2.12)$$

for all $i, k = 1, \dots, n$

$$-U_y \cdot \alpha_2 \cdot C_a + U_a = 0 \quad (2.13)$$

These conditions can be rewritten as

$$\sum_i (p_i - C'_i) \cdot \frac{\partial x_i}{\partial p_k} = -x_k \left(1 - \frac{\alpha_1}{\alpha_2} \right) \quad (2.12')$$

$i, k = 1, \dots, n$

$$\frac{U_a}{U_y} = \alpha_2 \cdot C_a \quad (2.13')$$

These expressions make clear what values of the parameters α_1 and α_2 are needed to enforce optimal conditions. As suggested before managerial efficiency crucially depends on the value assigned to the profit sharing ratio α_2 ; setting $\alpha_2 = 1$ (2.13') is equal to (2.10). On the other hand, the relative values of α_1 and α_2 are relevant for allocative efficiency. Notice that marginal cost pricing as a rule is guaranteed by choosing α_1/α_2 equal to one, regardless of the absolute values given to these parameters. Of course, price is set equal to a "too high" marginal cost unless α_2 is unity, but the point is that the Manager will price efficiently for any given level of effort. When $0 = \alpha_1 < \alpha_2$, the Manager will adopt a monopolistic price structure.

A central condition for the satisfaction of the previous rules is that the Minister can set the parameter c as a negative fee, such that the Manager and Minister achieve their respective bargaining levels of utility. The result can be stated as

Proposition 2.1. : Given the previous assumptions, marginal cost pricing and managerial efficiency can be achieved with the incentive contract (2.6) where $\alpha_1 = \alpha_2 = 1$ and $c < 0$. The Manager is given all the increase in consumer and producer surplus above a given target.

Remark 2.1. : The solutions derived from problem (2.11) with $\alpha_1 = \alpha_2 = 1$ are identical to those obtained in problem (2.6), if and only if, the fee c can be chosen such that the utility level obtained by the Manager is identical in both solutions.

The relevance of the last remark is due to the solution of the implicit bargaining problem between Minister and Manager under symmetric and asymmetric information. To claim as in Proposition 2.1. that optimality follows choosing $c < 0$ is not enough unless we are sure that c is reduced until the Manager attains the same utility level as in the full information case. Otherwise, there will be rents going to the Manager in the form of a higher c , as a result of his increased bargaining power under asymmetric information. Nevertheless, even if this is the case, the form of the optimal contract will remain as that characterized by proposition 2.1. and optimality will be redefined in a second-best sense, with respect to the minimum utility level accepted by the Manager under asymmetric information.

It is difficult to see the likely result of the implicit bargaining in the present context where we have avoided considering such a process. On the one hand, the most important are market constraints in the form of competition from potential Managers the most likely is the possibility that both solutions will coincide. On the other hand, if the relation is that of a strict bilateral monopoly we should expect the solutions to differ reflecting the information possessed by each party.

Efficient contracts that adopt a "pure rent" form as the one derived above are well known special cases in the Principal-Agent literature (e.g. Stiglitz, 1974, Shavell, 1979, Harris and Raviv, 1978, 1979). They result when the

agent is risk neutral since in this case both optimal risk-sharing and the provision of incentives coincide. They also follow when the environment becomes riskless (see next Chapter) as in the present case. However, we have not invoked such results since in those models the principal knows every relevant functional relationship. Otherwise, the present result would follow as a trivial extension of those models.

In the next section we introduce profitability or revenue constraints into the pricing problem.

2.1.2. Ramsey Pricing and Managerial Efficiency

Suppose lump-sum taxation becomes unfeasible and the distortionary aspects of other form of taxation requires the treatment of public pricing within the optimal taxation problem (e.g. as in Baumol and Bradford, 1970). The optimal, full information conditions have then to be changed by introducing a profit constraint $\pi \geq \pi_0$ into the problem stated in (2.6). The first order conditions of this reformulated problem can be written as

$$\sum_i (p_i - C'_i) \cdot \frac{\partial x_i}{\partial p_k} = - \frac{\mu}{1 + \mu} \cdot x_k \quad \text{for all } i, k = 1, \dots, n \quad (2.14)$$

$$\frac{U_a}{U_y} = (1 + \mu) \cdot C_a \quad (2.15)$$

where $\mu > 0$ is the lagrange multiplier associated with the (assumed binding) profit constraint.

Expression (2.14) states the classical condition for Ramsey pricing, according to which all demand elasticities have to be inflated by a constant factor and the enterprise should afterwards behave as a monopolist. Expression (2.15) restates the condition for managerial efficiency showing that the social marginal return from effort has increased after the higher valuation given to profit welfare.

Inspection of conditions (2.12') and (2.13') of the Manager's problem

(2.11), reveals that the above conditions are achieved setting $\alpha_1 = 1$ and $\alpha_2 = 1 + \mu$. The simple reason for this choice is that the Minister values profits higher than before and would adjust the incentive contract accordingly. Notice that the required departure of prices from marginal cost must be induced through a reduction in the ratio α_1/α_2 . This can in principle be obtained increasing α_2 , reducing α_1 or moving both coefficients. However only the adjustment in α_2 is appropriate since the need for an increase in the perceived return from effort must also be attended. A reduction in α_1 might affect the equilibrium, self-selected level of effort in an opposite direction to the one required, since it will induce a reduction in output which might in turn reduce the marginal return from effort if $C_{ax} = C'_a < 0$, that is if marginal costs are reduced by increases in managerial effort¹.

Nevertheless, there are two problems that prevent the previous adjustment. First, there exists an informational constraint given by the fact that the Minister does not know the value of the multiplier μ , which depends on the magnitude of the target π_0 in relation to the maximum level of profits. Second, the multiplier μ can adopt very high positive values (it has $+\infty$ as an upper bound) for tight profits constraints and so will the required profit-sharing ratio. However, highly positive profit sharing ratios are unlikely to be institutionally feasible, even if the fee c is adjusted downwards accordingly.

For these reasons, we shall study a modified incentive contract where the Minister selects the fee c such that it will induce the Manager to satisfy the profit target without moving α_2 upwards. We define,

$$\begin{aligned} y &= \alpha_1 \cdot S(p) + \alpha_2 \cdot \pi(p,a) + c_1 & \text{if } \pi \geq \pi_0 \\ &= \alpha_1 \cdot S(p) + \alpha_2 \cdot \pi(p,a) + c_2 & \text{if } \pi < \pi_0 \end{aligned} \quad (2.16)$$

¹See discussion in the next Chapter for more details on this effect.

The assumption that $C_{xa} \leq 0$ is also implicitly used in Lemma 2.1

where $c_1 > c_2$. The fee c_2 is selected such that the Manager will not be able to attain his minimum negotiated utility level if the profit target is not satisfied¹.

Solving the equivalent of problem (2.11), where now y is given by (2.16) and the Manager maximizes his utility subject to $\pi \geq \pi_0$, we obtain as first order conditions with respect to p

$$- U_y \cdot \alpha_1 \cdot x_k + (U_y \cdot \alpha_2 + \psi) \left[x_k + \sum_i (p_i - C_i') \cdot \frac{\partial x_i}{\partial p_k} \right] = 0 \quad (2.17)$$

$$i, k = 1, \dots, n$$

which after some manipulation can be written as:

$$\sum_i (p_i - C_i') \cdot \frac{\partial x_i}{\partial p_k} = - x_k \left[\frac{(\alpha_2 + \psi/U_y) - \alpha_1}{\alpha_2 + \psi/U_y} \right] \quad (2.17')$$

$$i, k = 1, \dots, n,$$

and with respect to effort:

$$\frac{U_a}{U_y} = \left(\alpha_2 + \frac{\psi}{U_y} \right) \cdot C_a \quad (2.18)$$

where ψ is the lagrange multiplier associated with the profit constraint.

The conformity of (2.17') and (2.18) with conditions (2.14) and (2.15) respectively, looks difficult to hold, since it depends not only on the values of α_1 and α_2 as in the previous section, but also on the value of the lagrange multipliers μ and ψ . However, the following Lemma shows that both multipliers are closely related under specific conditions

Lemma 2.1. : When $\alpha_1 = \alpha_2 = 1$, we have in equilibrium that the lagrange multiplier of the Manager's problem is equal to that of the Minister's problem multiplied by the marginal utility of managerial income, that is

$$\psi^* = U_y \cdot \mu^* \quad (2.19)$$

Proof : From the first order conditions under full information, i.e. expressions (2.14) and (2.15) we can obtain:

¹A penalty in the lower part of (2.16) would be enough. See Figure 2.1.

$$\begin{aligned} \mu^* &= - \frac{\sum_i [p_i^* - C_i'(p^*, a^*)] \cdot \partial x_i(p^*) / \partial p_k}{x_k(p^*) + \sum_i [p_i^* - C_i'(p^*, a^*)] \cdot \partial x_i(p^*) / \partial p_k} \\ &= \frac{U_a(y^*, a^*)}{U_y(y^*, a^*)} \cdot \frac{1}{C_a(p^*, a^*)} - 1 \end{aligned} \quad (2.20)$$

where the asteriks denote solutions. Similarly, from the first order conditions of the Manager's problem, (2.17') and (2.18), setting $\alpha_1 = \alpha_2 = 1$ we can obtain:

$$\begin{aligned} \psi^* &= - \frac{U_y(\bar{y}, \bar{a}) \cdot \sum_i [\bar{p}_i - C_i'(\bar{p}, \bar{a})] \cdot \partial x_i(\bar{p}) / \partial p_k}{x_k(\bar{p}) + \sum_i [\bar{p}_i - C_i'(\bar{p}, \bar{a})] \cdot \partial x_i(\bar{p}) / \partial p_k} \\ &= U_y(\bar{y}, \bar{a}) \cdot \left[\frac{U_a(\bar{y}, \bar{a})}{U_y(\bar{y}, \bar{a})} \cdot \frac{1}{C_a(\bar{p}, \bar{a})} - 1 \right] \end{aligned} \quad (2.21)$$

where \bar{p}, \bar{a} and \bar{y} denote the corresponding solutions to the Manager's problem.

Clearly, the form of (2.20) and (2.21) coincide as it is claimed in expression (2.19), but the result does not follow unless the solutions of p, a and y are identical. Conversely, if (2.19) holds then the solutions must be identical. Also notice that if the solutions of income and effort are identical, then the levels of managerial utility must be identical. (Here is where the assumption about the result of the bargaining between Minister and Manager discussed in Remark 2.1. is implicitly used.)

Thus, we shall prove (2.19) assuming first that it does not hold, then deriving possible solutions for p and a and finally obtaining a contradiction to the original assumption. (Throughout the analysis we shall of course assume the same target π_0 imposed on both cases).

$$\text{Case I : Assume that } \frac{\psi^*}{U_y(\bar{y}, \bar{a})} > \mu^* > 0 \quad (2.22)$$

Using the first order condition (2.17'), with $\alpha_1 = \alpha_2 = 1$, it follows that for any given level of effort for both problems, we must observe $\bar{p}_i > p_i^*$ for all $i = 1, \dots, n$. (If $\bar{a} < a^*$ then from (2.17') \bar{p}_i must be even greater than p_i^* .) This is so because the cost structure in the two problems is the same for any

given level of effort. For this same reason, the above inequality implies the following relationships

$$\text{if } \pi(\bar{p}, \bar{a}) = \pi_0 \quad \Rightarrow \quad \bar{a} < a^* \quad (2.23)$$

$$\text{if } \bar{a} \geq a^* \quad \Rightarrow \quad \pi(\bar{p}, \bar{a}) > \pi_0 \quad (2.24)$$

(2.23) says that if we require that the target π_0 is just achieved with \bar{p} , \bar{a} , then the level of effort \bar{a} must be lower than the level of effort in the Minister's problem. On the other hand, (2.24) says that if the level of effort in the Manager's problem is equal or higher than in the Minister's problem, then it must follow that the profit target π_0 is not binding in the Manager's problem.

A similar analysis, using the first order condition (2.18) indicates that, using (2.22), and for any given prices for both problems, we must observe, if utility is kept at the indifference curve \hat{U} (the bargaining level), that $\bar{a} > a^*$. (If $\bar{p}_i < p_i^*$ then from (2.18) \bar{a} must be even greater than a^* .) This in turn implies the following relationships

$$\text{if } \pi(\bar{p}, \bar{a}) = \pi_0 \quad \Rightarrow \quad \bar{p}_i < p_i^* \quad (2.25)$$

(at least for one i)

$$\text{if } \bar{p}_i \geq p_i^* \text{ for all } i \quad \Rightarrow \quad \pi(\bar{p}, \bar{a}) > \pi_0 \quad (2.26)$$

Now (2.25) contradicts (2.23) since the inequalities go in opposite directions, while (2.26) does not contradict (2.24). Taken this last alternative it must follow since the constraint π_0 is not binding that $\psi^* = 0 < \mu^*$ contradicting the original assumption in (2.22).

$$\text{Case II : Assume that } 0 < \frac{\psi^*}{U_y(\bar{y}, \bar{a})} < \mu^* \quad (2.27)$$

The proof follows the same steps as for Case I. Using the first order condition (2.17') with $\alpha_1 = \alpha_2 = 1$, it follows that for any given level of effort for both problems, $\bar{p}_i < p_i^*$ for all $i = 1, \dots, n$. (If $\bar{a} > a^*$ then from (2.17') \bar{p}_i must be even lower than p_i^* .) This in turn implies the following relationships

$$\text{if } \pi(\bar{p}, \bar{a}) = \pi_0 \quad \Rightarrow \quad \bar{a} > a^* \quad (2.28)$$

$$\text{if } \bar{a} \leq a^* \quad \Rightarrow \quad \pi(\bar{p}, \bar{a}) < \pi_0 \quad (2.29)$$

A similar analysis, using the first order condition (2.18) indicates that for any given prices for both problems, and if utility is kept at the bargaining level \hat{U} in both cases, we must observe $\bar{a} < a^*$ (if $\bar{p}_i > p_i^*$ then \bar{a} must be even lower than a^*). This in turn implies the following relationships

$$\text{if } \pi(\bar{p}, \bar{a}) = \pi_0 \quad \Rightarrow \quad \bar{p}_i > p_i^* \quad (2.30)$$

(at least for one i)

$$\text{if } \bar{p}_i \leq p_i^* \text{ for all } i \quad \Rightarrow \quad \pi(\bar{p}, \bar{a}) < \pi_0 \quad (2.31)$$

(2.30) contradicts (2.28) since the inequalities go in opposite directions.

(2.31) does not contradict (2.29). Taking this last alternative however it follows that the profit constraints cannot be achieved therefore implying that $\psi^* \approx +\infty > \mu^*$ contradicting the original assumption in (2.27).

Q.E.D.

The economic interpretation of result (2.19) is simple. From the standard theory of classical optimisation we know that the equilibrium value of any Lagrange multiplier measures the change of the objective function with respect to the relevant constraint-constant, evaluated at the equilibrium value of the variables of the problem (see for example, Intriligator, 1971, pp. 36-38). In the present problem μ^* and ψ^* measure the value to society (or the Minister) and the Manager respectively of a marginal increase in government revenue from other sources (i.e. a marginal decrease in π_0). Under an incentive contract with $\alpha_1 = \alpha_2 = 1$, marginal changes in social welfare coincide with marginal changes in managerial income. Thus this value is multiplied by the marginal utility of income to the Manager to obtain his valuation of a relaxation of π_0 .

Notice that adapting the proof of Case I above it can be shown that $\mu^* = 0 \Rightarrow \psi^* = 0$. This confirms Proposition 2.1. since this can be seen as a special case of the present one.

Directly from Lemma 2.1. we obtain the following result.

Proposition 2.2. : Given the previous assumptions, Ramsey pricing and managerial efficiency can be achieved with the incentive contract (2.16) choosing $\alpha_1 = \alpha_2 = 1$ and $c_1 < 0$.

The same qualification about the bargaining level of managerial utility made before in Remark 2.1. applies here as well.

An illustration of the equilibrium choice of the Manager under the present scheme is made in the next figure. For simplicity we assume $\pi_a = 1$, or equivalently that effort enters as a fixed cost with unitary coefficient¹, implying that the frontier of the opportunity set of the Manager is linear.

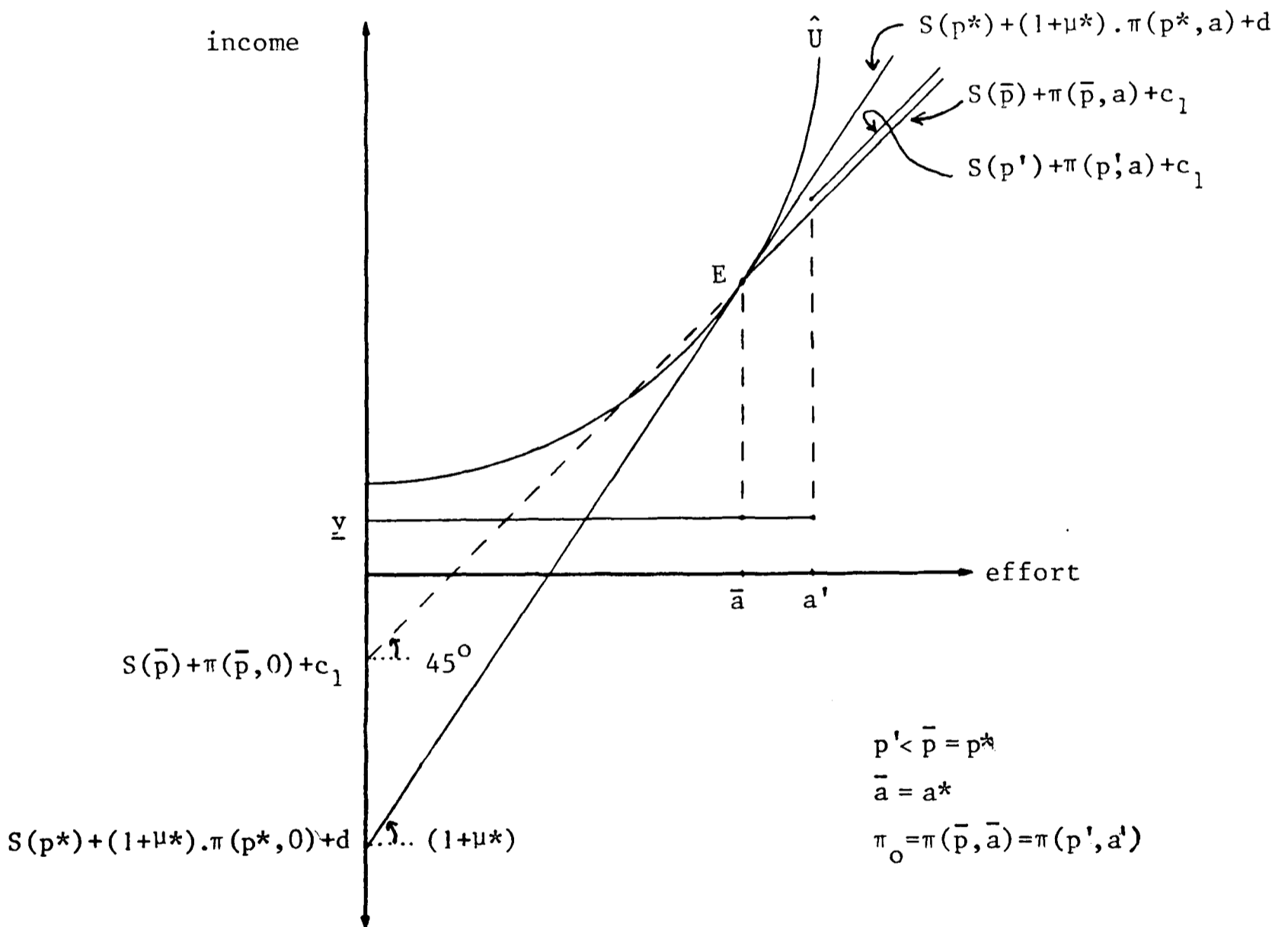


Figure 2.1.

The solution to the Minister's best centralized choice under full information is given by point E. This solution could be achieved by setting $\alpha_1 = 1$

¹The slope of the opportunity set is given by $\alpha_2 \cdot \pi_a$. Notice also that $\pi_a = 1$ implies that marginal cost is not affected by a ; nor the slope of the opportunity set by changes in prices, if α_2 is fixed.

and $\alpha_2 = 1 + \mu^*$, as we discussed before, and charging a fee or a target equal to $d < 0$. Thus, at E the frontier of the opportunity set (with prices p^*) $S(p^*) + (1 + \mu^*)\pi(p^*, a) + d$ is tangent to the indifference curve \hat{U} . Under the proposed scheme, the frontier of the opportunity set (with prices \bar{p}) is given by $S(\bar{p}) + \pi(\bar{p}, a) + c_1$, if the target π_0 is achieved; otherwise we assume that the Manager receives a low salary given by the minimum institutionally possible wage, \underline{y} . The discontinuity of the frontier of the choice set occurs at $a = \bar{a}$ where the profit target is achieved, at this point the Manager receives a reward of $S(\bar{p}) + \pi(\bar{p}, \bar{a}) + c_1$. In the figure we have chosen $\bar{a} = a^*$ and $\bar{p} = p^*$ illustrating the equilibrium. Suppose the Manager reduces prices below \bar{p} . In this case the curve $S(p) + (1 + \mu)\pi(p, a) + d$ will move downwards or to the right since p^* is maximizing this sum, and the Manager will fail to achieve \hat{U} . Under the proposed scheme however, the curve $S(p) + \pi(p, a) + c_1$ should move upwards, since the reduction in prices move us closer to the unconstrained welfare maximization. However, in this case the Manager will fail to achieve the target π_0 unless he increases effort to a' . Thus, in the new situation his maximum income for each effort is given by the wage \underline{y} for all $a < a'$ and by $S(p') + \pi(p', a) + c_1$ for all $a \geq a'$, showing that he will fail to achieve his utility \hat{U} .

2.2. The Finance of Managerial Rewards and the Cost of the Incentive Scheme

One of the features that differentiates the principal-agent problem of the public sector firm from an equivalent analysis of the private firm is that the Minister derives utility from something he cannot freely observe or appropriate, namely consumer surplus. The question of appropriability seems important since we are talking about a sharing rule and it may sound awkward to present the Minister as sharing something he does not have. This however would not create any serious problem if it can be shown that managerial rewards are financed from profits (cf. Finsinger and Vogelsang, 1982). Nevertheless, suppose the equilibrium level of profits turn out to be

zero or negative; should we have a problem in financing managerial rewards then?; and if we are not in a first best economy, is it the case that the finance of these rewards implies a higher departure from marginal cost pricing rules and therefore introduces an explicit extra cost? In this section we briefly show that the answer to both questions is no.

Suppose first that we are in the first best context of section 2.1.1., and that marginal costs are constant. In this case $\pi(p,a) = 0^1$ and in problem (2.6) managerial income is paid out of general revenues. In this case it is treated as any other overhead cost. A similar result follows under increasing returns with $\pi(p,a) < 0$.

The same result follows in the case of problems (2.11) with $\alpha_1 = \alpha_2 = 1$ and $c < 0$. If marginal costs are constant and $\pi(p,a) = 0$ then in equilibrium

$$y = S(p) + c > 0$$

Again, this amount is paid out of general revenues by the Minister as in the case of any overhead cost. Under increasing returns the result is similar. Notice that, given the incentive scheme, the Manager will himself choose to produce with a loss and the Minister will be willing to finance this loss. Unlike in the "marginal cost controversy" of the forties and fifties² where marginal cost pricing was seen as inducing managerial inefficiency, under increasing returns, in the present case the two problems are separated since the incentive scheme guarantees managerial efficiency, even in loss making situations which would only reflect efficient pricing.

Suppose now that the public firm is asked to break-even. In the full information context this would mean that $\pi(p,a) \geq y$, leading to conditions (2.14) and (2.15). Prices are raised to cover managerial rewards and overhead costs.

A similar result follows under the incentive scheme proposed in (2.16).

¹Assuming no fixed cost.

²Coase (1946).

Here $\pi_0 = y$ and in equilibrium we should observe

$$\bar{y} = \pi(\bar{p}, \bar{a})$$

$$\text{with } c_1^* = -S(\bar{p})$$

that is the fee is chosen such that in equilibrium the Manager receives all the profits which is just enough to obtain his compensation. Notice that if the Manager raises prices above \bar{p} which are those given by condition (2.17') with $\alpha_1 = \alpha_2 = 1$ then his income is reduced since now $c_1^* > -S(p')$ with $p' > \bar{p}$. In Lemma 2.1. and Proposition 2.2., with managerial utility given at $U = \hat{U}$, prices, effort and managerial income are the same as in the best centralized decision, showing that there is no "extra cost" associated with the present scheme in relation to the full information context.

2.3. An Optimal Incentive Scheme under Imperfect Observability of Consumer Benefits

In the previous section we studied an optimal incentive scheme in which the Manager receives, at the margin, the full increase in social welfare. Given the simplicity of the situation studied it may seem strange how this result has not been formalized before in the literature. In his extensive and insightful analysis of managerial incentive mechanisms in public firms, Gravelle (1982a) seems to reject the possibility of such a scheme on the basis that it would imply that the Manager should appropriate all the level of social welfare. Since he did not consider explicitly the form of an optimal incentive contract, however, it seems that he failed to recognize that the role of the fee c in such a contract is precisely to reconcile the optimal marginal conditions for pricing and managerial efficiency, along with the optimal share to be taken by both parties (within a process of negotiation of their respective utilities). However, the previous results would not challenge too much Gravelle's conclusions since he is dealing, unlike us, with a situation where demand is not observable. Studying an incentive payment function $y(L, \pi)$ with

L defined as a traditional Laspeyres price index $p \cdot x_o / p_o \cdot x_o$ ¹, Gravelle showed (pp 95-6, expressions (33) and (34) op.cit.) that in order to achieve efficient pricing and managerial efficiency the Minister should set

$$y_{\pi} = 1 \quad (2.32)$$

$$y_L \cdot L'(p) = -x(p) \quad (2.33)$$

where $y_{\pi} > 0$ and $y_L < 0$ are the derivatives of the incentive contract and $L' = dL/dp$. Differentiating $y(L, \pi)$ with respect to price p and for a given level of effort a , Gravelle obtained (see his expression (35); p. 96) using (2.32) and (2.33):

$$\frac{dy}{dp} = (p - C_x) \cdot x'(p) \quad (2.34)$$

where C_x is marginal cost and $x'(p) = dx/dp$. From condition (2.34) Gravelle concluded that "... integrating, we see that for price and x-efficiency y must be the sum of profit and consumer surplus, or total willingness to pay less cost. In other words, the Manager must appropriate all of the social benefits from the output of the good" (op.cit., p. 96).

This result is valid however only if we ignore the constant of integration resulting in the L.H.S. of (2.34). Suppose p^* is the equilibrium, self-selected level of price and the Manager could technically increase price until \hat{p} where $x(\hat{p}) = 0$. From (2.34) we obtain

$$- \int_{p^*}^{\hat{p}} \frac{dy}{dp} dp = - \int_{p^*}^{\hat{p}} (p - C_x) \cdot x'(p) dp$$

or

$$y^* - y[L(\hat{p}), \pi(\hat{p}, a)] = \int_0^{x^*} (p - C_x) \cdot dx$$

(changing the limits of integration appropriately in the R.H.S.). Finally,

$$y^* = S(p^*) + \pi(p^*, a^*) + y[L(\hat{p}), \pi(\hat{p}, a^*)] \quad (2.35)$$

¹He was extending an earlier analysis by Bös (1978) on the use of cost of living indices for public pricing purposes.

where y^* is the equilibrium level of income. Clearly, we would expect from an optimal incentive contract to have

$$y[L(\hat{p}), \pi(\hat{p}, a^*)] < 0.$$

Thus, (2.35) shows that there is no need to pay the Manager social welfare in full, and that expression (2.34) establishes a marginal condition.

Finally, observing the informational requirements to enforce condition (2.33) Gravelle concluded (op.cit. p. 96):

"Unfortunately even if the political authorities are willing to accept the distributional consequences of this result, it does not help them very much in achieving efficiency. To implement the bonus function it is necessary to measure consumer surplus or willingness to pay and this implies knowledge of the demand function."

In the present section we shall investigate the use of an optimal incentive scheme in situations where demand is not observed by the Minister.

Recent works by Finsinger and Vogelsang (FV) and others¹ have tried to provide a solution to the lack of observability of part of the Minister's maximand in the context of a classical model. They have recognized that although consumer surplus cannot be measured in a situation where demand functions are unknown, the change in consumer surplus can be approximated by the change of a price index. This, along with the change in profits, can be used to build a performance indicator that once linked with managerial income can enforce optimal pricing decisions.

The assumptions of FV's (1982) model are:

(a) A partial equilibrium analysis where the Minister's objective is given by the maximization of social benefits defined as before by the sum of consumer and producer surplus,

$$W(p) = S(p) + \pi(p) \tag{2.36}$$

¹See Finsinger and Vogelsang (1979, 1981, 1982). Gravelle (1982a) pointed out the similarity of this method with an earlier contribution by Scott (1952, 1978). Also there exist striking similarities with the problems addressed by Domar (1974) and Tam (1981).

(b) Income effects are assumed away, continuity and differentiability of $S(p)$ is assumed and also the price vector p is taken as belonging to a compact subset of the n -space of positive vectors R_+^n .

(c) The profit function is the difference between revenues and a minimum cost function,

$$\pi(p) = p \cdot x(p) - C[x(p)] \quad (2.37)$$

and is also assumed continuously differentiable. Notice that no managerial effort affects cost. There exists stationarity in both demand and cost functions through time.

(d) Finally the management incentive structure is assumed to be embedded into the payment function:

$$Y_t = A + I_t \quad (2.38)$$

At the end of period t the Manager receives a fixed payment A , plus the value of a performance index I_t defined as:

$$I_t = x_{t-1}(p_{t-1} - p_t) + \pi_t - \pi_{t-1} \quad (2.39)$$

It can be easily seen that the first term of the RHS of (2.39) approximates the change in consumer surplus (under the assumption of no income effects and stationarity in demand). The important point is that given the convexity of $S(p)$, (2.39) always underestimates the actual change in social welfare (see FV, 1979, 1982). This makes sure that every increase in managerial income should also lead to a bigger increase in social welfare.

Finsinger and Vogelsang then show that in his maximization of (2.38) the Manager changes prices generating a sequence such that it gives the maximum present value of $W(p)$ with prices equal to marginal costs.

There are however a number of unsettled issues in their analysis. We shall mention three main criticisms to this work.

First, although allocative efficiency in a first best context is enforced by the use of the performance index (2.39) the question of managerial effici-

ency is not addressed. Costs are rather assumed to be at a minimum and FV claim that the Manager will seek to minimize costs. This is so only under the assumption that it does not cost anything to him in terms of utility. As Gravelle (1982a, 1983) has shown, when managerial effort is explicitly considered, the Manager will under-supply effort in relation to the optimal level in all but his last period in office. The reason for this is simply that the performance index I_t depends on changes in the cost of production rather than on the level. Thus an increase in effort in period t will increase I_t , but ceteris paribus will reduce I_{t+1} . The Manager is rewarded in period t when the cost reduction takes place but penalized in period $t+1$. Only in his last period in office there will be no such reductions in the return from effort. Thus Gravelle (1982a, 1983) considered a solution to this problem by modifying the bonus scheme such that now changes in managerial income could be derived from changes in welfare, namely,

$$\begin{aligned}
 Y_t &= I_t + Y_{t-1} = \sum_{j=1}^t I_j \\
 &= \sum_{j=1}^t (p_{j-1} - p_j) \cdot x_{j-1} + \pi_t - \pi_0
 \end{aligned}
 \tag{2.40}$$

Under this scheme, Gravelle (1982a, p. 100) showed then that efficient pricing and managerial efficiency are achieved. However, again the problem is that the Manager should appropriate all the value of social welfare. As Gravelle puts it, "... the incentive mechanisms "works" because they enable the Manager to reap the rewards from acting as an (almost) perfectly discriminating monopolist." However, from our discussion above, this conclusion neglects the fact that the incentive scheme (2.40) is surely suboptimal from the point of view of the Minister's problem and a consideration of the optimal enforceable incentive scheme should not necessarily give the Manager all the level of welfare.

Second, FV did not take the incentive function (2.38) from the solution

of an optimal incentive contract but rather assumed it. Presumably the unitary coefficient attached to I_t in that function is optimal but this is not discussed. Also the sign of the fixed payment A is not determined but assumed, and the level of utility that the Manager is supposed to achieve in the process is ignored. In one of their presentations (1981, p. 402) they maintain that "... incentive mechanisms are solutions to simple principal-agents problems." However, one distinguishing factor of these problems is the endogenous treatment of the incentive function or the so-called sharing rule.

Third, FV's model can be seen as a class of model in which (extreme) asymmetric information is the central feature and there exist decision-making incentive problems. However, are we absolutely sure that these information revealing problems will inevitably exist in the present model? The way this question is addressed in the modern literature on incentives under asymmetric information is by showing first that in the absence of the proposed scheme there exist, unambiguously, a problem of incentive incompatibility by which it is meant that there would be incentives for the Manager not to reveal his private information truthfully (in the present case, the level of marginal cost in order to set prices)¹. However, we cannot prove this condition in the FV model. With his income fixed, and asked to reveal the level of marginal cost the Manager would have no incentive not to report it truthfully. Without any clue about managerial preferences in the absence of the incentive scheme, we have no clear indication why there should be a departure from the best centralized decision under full information. In this sense, the introduction of managerial effort as an input and providing disutility is important not only to give substance to the discussion about cost minimization but also to justify the introduction of the mechanism as one of information transmission at all. Under effort-aversion, the Manager would be prepared to reveal a lower level of marginal cost in order to reduce his level of effort (cf. dis-

¹See for example, the modern approaches to the so-called transfer pricing problem, Groves and Loeb (1979) and Harris, Kriebel and Raviv (1982).

cussion in section 2.1.1.).

To sum up, although FV provided some justification for the use of managerial incentive schemes in public enterprises, their analysis is still incomplete. Their main contribution however is to show a way of solving the problem created by the absence of demand estimates. We shall now discuss an alternative formulation of the problem under the same set of assumptions about the information available to the Minister.

All the assumptions of section 2.1.2. are maintained except the one concerning demand information: it is now assumed that only the Manager can observe demand functions, while the Minister limits his information to prices and quantities.

For the purpose of measuring performance to compute managerial rewards, the Minister is assumed to construct the following indicator:

$$L(p) + \pi(p,a) - \pi(p_o, a_o) \quad (2.41)$$

where

$$L(p) = x_o \cdot (p_o - p) \quad (2.42)$$

where subindices o denote pre-contractual values, or time zero. The model however is, as the one analyzed before, a static, one-shot equilibrium model and thus no temporal aspects are addressed. Everything occurs within one period, and the variables p and a can be seen as end of period values.

For the purpose of illustrating more clearly the result to be obtained below let us define "base-volume profits" as that level of profits calculated using the end of the period or equilibrium quantities and the beginning of the period or pre-contractual prices, that is¹

$$p_o \cdot x(p) - C[x(p), a] \quad (2.43)$$

adding and subtracting $p \cdot x(p)$ we can write (2.43) as

$$x(p) \cdot (p_o - p) + \pi(p, a) \quad (2.43')$$

The incentive contract proposed is similar to that considered before, when

¹This definition is inspired in the concept of "volume profits" introduced by Scott (1952, 1978) where prices of period t+1 are used to calculate profits of period t.

a profit target is present:

$$\begin{aligned} y &= \underline{\alpha}_1 \cdot L(p) + \alpha_2 \cdot \pi(p,a) + c_1 & \text{if } \pi \geq \pi_0 \\ &= \underline{\alpha}_1 \cdot L(p) + \alpha_2 \cdot \pi(p,a) + c_2 & \text{if } \pi < \pi_0 \end{aligned} \quad (2.44)$$

where π_0 is the profit target (it must not be confused with $\pi(p_0, a_0)$), $c_1 > c_2$, α_2 is the unique profit sharing ratio and $\underline{\alpha}_1$ indicates that there is one coefficient α_1 associated with each good in the index $L(p)$. That is:

$$\underline{\alpha}_1 \cdot L(p) = \sum_{k=1}^n \alpha_{1,k} \cdot x_{o,k} \cdot (p_{o,k} - p_k) \quad (2.45)$$

The reason for the use of a unique profit sharing ratio and n consumer surplus sharing ratios will become evident below.

Notice that the initial or precontractual level of profits $\pi(p_0, a_0)$ is not explicitly considered in (2.44), although it might be seen as embedded into the decision to set the fee c_1 .

We recall that the Minister best centralized decisions under full information are those illustrated in expressions (2.14) and (2.15) at the beginning of section 2.1.2. The Manager's problem is to solve

$$\text{Max}_{p,a} \quad \mathcal{L} = U(y,a) + \psi \cdot [\pi(p,a) - \pi_0] \quad (2.46)$$

First order conditions with respect to prices p_k are (assuming interior solutions)

$$- U_y \cdot \alpha_{1,k} \cdot x_{o,k} + (U_y \cdot \alpha_2 + \psi) \cdot [x_k + \sum_{i=1}^n (p_i - C'_i) (\partial x_i / \partial p_k)] = 0 \quad (2.47)$$

for all $i, k = 1, \dots, n$. As in (2.45) $x_{o,k}$ denotes the k^{th} element in the vector of precontractual quantities x_o .

After some manipulation, we can express (2.47) as

$$\sum_{i=1}^n (p_i - C'_i) \cdot \frac{\partial x_i}{\partial p_k} = - x_k \left[\frac{(\alpha_2 + \psi/U_y) - \alpha_{1,k} \cdot x_{o,k}/x_k}{\alpha_2 + \psi/U_y} \right] \quad (2.47')$$

the first order condition of (2.46) with respect to effort gives:

$$\frac{U_a}{U_y} = \left(\alpha_2 + \frac{\psi}{U_y} \right) \cdot C_a \quad (2.48)$$

which is identical to condition (2.18).

We can see that the solution to the optimal incentive scheme is very simi-

lar to that discussed before in section 2.1.2. Notice that if it were the case that $\alpha_2 = 1$ and $\alpha_{1,k} = \bar{x}_k/x_{o,k}$, where \bar{x}_k denotes the self-selected, equilibrium quantity associated with the above first order conditions, (2.17') and (2.47') become identical. Then we could invoke the result stated in Lemma 2.1. about the equilibrium value of the multipliers, and optimality would result as in that problem.

There is however one fundamental difference with respect to the model of section 2.1.2., and this is that in order to obtain optimality the determination of the n consumer surplus sharing ratios $\alpha_{1,k}$ $k = 1, \dots, n$ must be done in a way that depends on the equilibrium chosen by the Manager. In other terms the scheme can be seen as delegating authority to the Manager to pick one, among many remuneration schedules offered to him. The central question then is to determine whether or not he will have incentives to choose prices effort and the remuneration schedule such that conditions (2.14) and (2.15) are obtained.

Suppose we are already in equilibrium with $\bar{a} = a^*$, $\bar{p}_k = p_k^*$ (and therefore $\bar{x}_k = x_k^*$) for all $k = 1, \dots, n$ and therefore the result (2.19) of Lemma 2.1. Clearly, the Manager will not find it convenient to move from this situation if he loses in the process of doing so, even if he can affect the value of the sharing ratios $\alpha_{1,k}$. This however is guaranteed by the fact that for all $\bar{x}_k < x_k^*$ the Manager will be reducing his income. and for all $\bar{x}_k > x_k^*$ although $\alpha_{1,k}$ increases, the profit target is not achieved and again the Manager is penalized by a reduction in his income.

Therefore, we can summarize the discussion by stating:

Proposition 2.3. : When demand is not observed, Ramsey pricing and managerial efficiency can be achieved with the incentive contract (2.44) setting

$\alpha_{1,k} = x_k/x_{o,k}$, $\alpha_2 = 1$ and $c_1 < 0$. The Manager is given the value of base-volume profits less a fee.

The remarks made in sections 2.1.1. and 2.1.2. about the selection of the fees c_1 and c_2 and the equilibrium bargaining level of managerial utility also applies here. Moreover we have

Remark 2.2. : The mechanism cannot enforce marginal cost pricing, except under constant returns to scale.

This result can be seen by first ignoring the profit constraint. Then from expression (2.47') with $\psi = 0$ we obtain again $\alpha_2 = 1$ and $\alpha_{1,k} = x_k/x_{0,k}$ and again the Manager is given all base-volume profits less a fee. (The contract is no longer dichotomous as (2.44) since the profit constraint is ignored.) However, base-volume profits for the Manager means that his "effective" marginal revenue as a base-volume profit maximizer is fixed at p_0 according to expression (2.43). Thus he will increase output until marginal costs are equal to marginal revenue p_0 . If marginal costs are constant or slightly decreasing the Manager will then expand output until x_k becomes ^{infinite} ~~zero~~. This is illustrated in the next figure, for the one good case

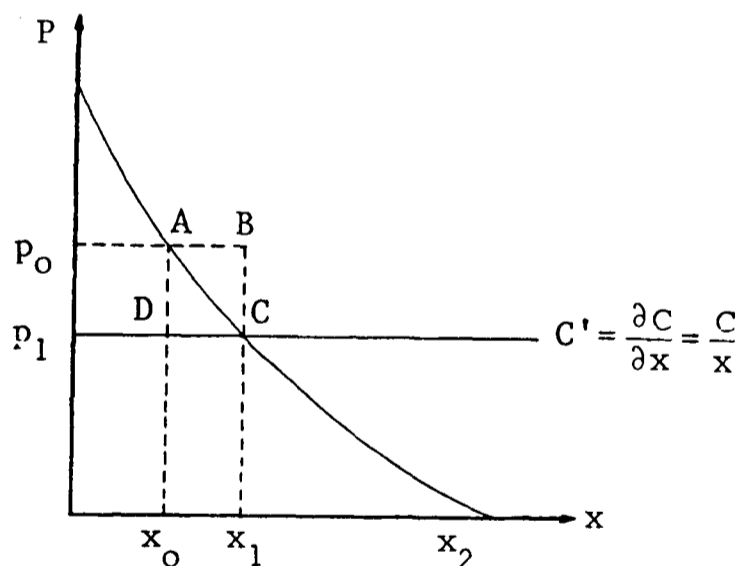


Figure 2.2.

For a given level of effort, a reduction of price from p_0 to p_1 increases base-volume profits by the area ABCD. Thus the Manager would expand output until x_2 . However, since in this particular case (with no fixed cost) marginal cost pricing leads to break-even, the Minister can employ the incentive scheme (2.44) with $\pi_0 = 0$.

The result stated in Proposition 2.3. indicates that there is a way of

giving correct incentives to public enterprise Managers in the context assumed. Of course, we could have arrived at the same conclusion by initially writing the performance index $L(p) = x(p_0 - p)$ and then the analysis would have led to the result that $\alpha_{1,k} = \alpha_2 = 1$. However, we have chosen the previous characterization in order to bring out some important aspects discussed before. First, the price index $L(p)$ in (2.42) is a Laspeyres-type of index which is the same adopted in the work of Finsinger and Vogelsang. Our analysis of the optimal form of the managerial bonus scheme can then be seen as a reconsideration of the appropriate index. Our conclusion is that a Paasche-type index is the appropriate one since it results from the solution of an optimal incentive contract. The reason why FV selected a Laspeyres-type of index is however perfectly clear in the context of their analysis. They were dealing with changes in welfare and wanted to use some index which underestimates the changes in consumer surplus. Our Paasche-type index overestimates this change, as it can be seen in Figure 2.2. However, within the problem of the derivation of an optimal incentive scheme this aspect should not matter since the Manager will not necessarily receive the full amount of that change. Second, the presentation followed also makes clear why the bonus function considered by Gravelle (1982a, 1983) and written as expression (2.40) above, is a suboptimal incentive scheme. From (2.40) and assuming that the Manager goes straight away to choose his equilibrium prices and effort, denoted by t , we can see that the implicit value of the consumer surplus sharing ratios $\alpha_{1,k}$ is taken to be unity instead of the ratio given in Proposition 2.3. In addition there is the neglected issue of the use of a fixed fee in the incentive contract mentioned before.

2.4. Some Possible Extensions

In this section we shall briefly mention how the results obtained before are affected by the consideration of distributional aspects in pricing,

quality regulation and managerial preferences towards the size of the firm.

In a study of the use of cost of living indices in public pricing, Bös (1978) has shown that one of the advantages of using a profit-constrained price-index minimization is the possibility of allowing for adjustments due to the distributional characteristics of each good in a similar fashion to the proposal of Feldstein (1972). Since the scheme obtained in Proposition 2.3. also uses a price index, written in differences, Bös conclusion also carries over to the present model. Therefore, the scheme seems to be flexible enough to take account of distributional aspects.

The consideration of quality aspects in the goods or services offered by the firm seems however to require an additional regulatory apparatus. Suppose "quality" is understood as a simple parameter entering the demand function, in the way analyzed by Spence (1975) and Sheshinski (1976). Then, as it has been shown in these models, the optimal quality decision becomes difficult to enforce unless the Minister has information about the impact of quality on the whole demand schedule, not just the marginal consumer. Thus quality regulation is unlikely to be efficient even in the most informationally favorable environment assumed in section 2.1. Therefore, quality decisions will have to be controlled using an additional framework, perhaps in the form of minimum constraints and involving the use of consumer councils. Finally, notice that one could also question the appropriability of the Spence-Sheshinski framework for the case of public enterprises. First, the sort of departures from the optimum they have characterized are perhaps of second order of magnitude. That is, the change in quality is taken into account by looking at market prices but what is not is the difference between the valuation of quality changes made by marginal and inframarginal consumers. Second, the single dimensionality of the quality parameter does not capture other important quality aspects present in public enterprises providing services, such as reliability.

Managerial preferences in the present model have been based on income and effort, as in some of the works surveyed and as in the classical principal-agent characterization. However, in the system of control of public enterprises that currently operates in most countries, where no bonus schemes are used and managerial income is fixed, it seems very likely that Managers will develop preferences toward certain forms of non-pecuniary income. Rees (1982) for example has explored the equilibrium-choice aspects of a positive model in which Managers derive utility from "size" measured by the output of the firm (or the sum of outputs under multiproduction), and the presence of unionized labour exerts pressure on them to pay attention to employment and wages.

Suppose we neglect the effects of the union, and we move from Rees' formulation towards one in which incentive bonus schemes play a major role in the control process. Implicitly, our formulation in the previous models is saying that in the process of change after the introduction of bonus schemes, managerial preferences toward size vanish since now the Manager is fully aware of the consequences of over-expansion on his income and utility. It seems unlikely however, that such a change will occur so rapidly and up to the extreme of eliminating size preferences. More likely, there will be just a mitigation of size preferences at the time the incentive scheme is introduced.

If preferences towards size, in the form of output, are introduced into our previous model - along with income and effort - then the optimal choice of the ratios α_1 and α_2 will have to be modified in a way that informational problems to enforce efficient pricing may reappear. When the Manager derives utility from output expansion he would tend to underprice the products of the firm (as far as other constraints, e.g. profit target, may allow him to do so). Since this effect goes in the same direction as the maximization of consumer surplus, it becomes clear that the Minister will reduce α_1 giving less weight to consumer surplus in the contract. However, the optimal value of the ratio α_1/α_2 will now depend, among other things, on the subjective managerial

rate of substitution between output and income and this requires information beyond what we have assumed before. On the other hand, the condition for managerial efficiency - for a given level of output - is not altered and therefore the optimal value of the profit sharing ratio would remain at unity as before. It thus remains open the question of what mechanisms or modification of the previous sharing rules can enforce efficient pricing in the presence of this and other managerial discretionary behavior and still maintain minimum informational requirements.

2.5. Conclusions

After examining the major proposals made so far to solve incentive problems in public firms Gravelle (1982a) concluded that "... none of the mechanisms suggested will ensure that Managers of public firms will set allocatively efficient prices and produce x-efficiently." Those mechanisms had in common that they were attempts to solve the problem in a simple environment. Simple mechanisms obtained in simple environments are perhaps unlikely to provide a major breakthrough in the system of control anyway. However, the importance of Gravelle's criticism resides in the fact that if the mechanisms fail in simple contexts they are even less reliable as policy prescription in real-world problems characterized by non-stationarity and uncertainty in demand and cost curves. Given this background, the contribution of this chapter is to show a simple incentive mechanism that may work in the context studied above.

The resulting optimal bonus scheme (among the class of linear schemes) gives the Manager the sum of an approximation to consumer surplus and profits, less a fee. A unitary profit sharing ratio has as a main objective to provide optimal incentives for managerial efficiency, while the relation between both weights in the incentive contract depends on the efficient pricing objective. In more complex environments (e.g. next Chapters) the optimal

values of α_1 and α_2 would generally have to be set using substantial information, but the advantage of the present context is that they are optimal at unity and so the Minister may use this result.

A central assumption however that underlines the achievement of the full information solution is that the Manager gets the same bargaining level of utility under symmetric and asymmetric information. However, if this were not the case, the efficiency of the scheme proposed should be redefined in a second best fashion, in relation to the utility achieved under asymmetric information.

We have shown that Gravelle's criticism of managerial bonus schemes rested on a mispecification of an optimal equilibrium contract for the Manager. Another look at this problem can be given by noting that with $\alpha_1 = \alpha_2 = 1$ and $c = 0$ in the model of section 2.1., the Minister's utility becomes zero. But surely if a bargaining is going on between Minister and Manager the former can convince the latter to accept a contract with $c < 0$ that is preferable to the status quo (of no contract at all). Recognizing that there exist a lower positive bound for the Minister's utility which is better for the Manager than the status quo we can no longer follow Gravelle's conclusion.

Nevertheless, pure-rent or lump-sum contracts such as those derived here are unlikely to be recommendable when real life aspects of policy are considered. Of course they are no longer optimal, and surely no longer viable, in contexts under uncertainty. The point of analyses like the one provided in this Chapter is to enrich our knowledge of how to solve the problem of incentives in a simple environment and to learn something about the important features of the problem.

CHAPTER III

LINEAR MANAGERIAL BONUS SCHEMES UNDER COST AND DEMAND UNCERTAINTY

3.1. Introduction

In this chapter we shall extend the previous analysis to a risky situation. Uncertainty is of course a fundamental ingredient in the study of incentive contracts and if its absence from the previous analysis did not make it completely trivial, it was because we assumed complete lack of knowledge by the Minister of the technical conditions of the firm.

The model considered here makes the incentive or bonus scheme a linear function of consumer surplus plus profits as in the previous chapter, although the existence of uncertainty about demand and cost functions makes the outcome stochastic. We shall study the characteristics of linear efficient contracts concentrating on the determinants of the 'weights' given to each part of the bonus scheme. It will also be shown that the results obtained in the previous chapter are a special, limit case of the present results.

The structure of the chapter is the following: Sections 3.2. and 3.3. introduce the model and characterize the so-called full information solution. Section 3.4. studies an intermediate version of the model in which the Minister retains control of the pricing decision and in addition introduces a profit sharing scheme to induce an efficient level of managerial activities. In Section 3.5. we study decentralized pricing and productive decisions under a "total surplus" sharing rule. Section 3.6. completes the analysis of this case with an examination of the optimal "weights" that characterize an efficient contract. Section 3.7. considers an extension to the case of pricing under a financial constraint. Finally Section 3.8. provides an illustrative example and Section 3.9. summarizes the main conclusions of the chapter.

3.2. General Framework and Basic Assumptions

We begin by assuming that the public firm is controlled by a Von Neumann-Morgestern utility maximizer Manager, with his utility given by the additive function

$$U(y,a) = H(y) - V(a) \quad (3.1)$$

where y is managerial income and a is taken as 'effort' or managerial input. Function (3.1) is assumed to be continuously differentiable with partial derivatives $H' > 0$, $H'' < 0$, $V' > 0$, $V'' > 0$. Thus the Manager exhibits both risk and work aversion.

The principal or Minister is assumed to pursue the maximization of expected social benefits \bar{W} defined in a partial equilibrium context. This objective is measured by the sum of expected consumer (\bar{S}) and producer ($\bar{\pi}$) surpluses,

$$\bar{W}(p,a) = \bar{S}(p) + \bar{\pi}(p,a) \quad (3.2)$$

where p denotes the price of the unique good. Expression (3.2) is a straightforward extension of the same measure under certainty and therefore it implies usual assumptions such as the absence of income effects and distributional considerations. However, the representation of consumers' valuation under uncertainty through expected consumer surplus carries the additional assumption that consumers are risk neutral¹.

Basically, risk can be introduced into the model through oscillating demand and cost functions. Notice however that there are some different consequences of bringing each of these types of uncertainty into the model. On the one hand, cost uncertainty would create uncertainty in the profit's side of the outcome and in the profit's side of the linear contract analyzed in the previous chapter. This would lead to a somewhat straightforward principal-agent formulation with a profit sharing rule showing the well known trade-off

¹For a discussion of this formulation see Schmalensee (1972) and Rees (1982, 1984b, Appendix to Chapter 10). It is worth noticing that it has been employed by most of the literature on regulation under asymmetric information (see, as examples, Baron and Myerson (1982), and Riordan (1984)).

between risk sharing and incentives, and the consumer surplus side being adjusted accordingly to achieve the objective of efficient pricing.

On the other hand, the introduction of demand uncertainty makes the analysis more relevant (and in a position that can be compared with the extensive literature in the area) but much more difficult to handle both conceptually and in terms of the results obtained. First, in a price-setting (public) monopoly model like the one studied here, demand uncertainty affects both demand and cost conditions producing ambiguous results in pricing as Leland (1972) and others have shown. Second, the definition of the Minister's objective function in terms of expected consumer surplus introduces additional restrictions in the set of assumptions to be made. Third, public enterprise pricing under demand uncertainty and a given plant capacity implies that demand will have to be rationed in certain states of nature (as Brown and Johnson (1969) first showed) if a uniform pricing rule is adopted. Thus non-price rationing schemes and their costs have to be considered explicitly. In the present chapter these problems are assumed away (see assumptions below).

Thus, we assume that there exist two continuous random variables \tilde{s}_1 and \tilde{s}_2 that affect demand and cost in the way described below. These two variables are jointly distributed with a distribution function $F(\tilde{s}_1, \tilde{s}_2)$ continuously differentiable.

The demand function that the firm faces is written as:

$$x = x(p, \tilde{s}_1) \tag{3.3}$$

with partial derivatives $x_p < 0$, $x_{s_1} > 0$.

Given (3.3) we can define actual and expected consumer surplus:

$$S(p, \tilde{s}_1) = \int_p^{p^0} x(v, \tilde{s}_1) dv \tag{3.4}$$

and

$$\bar{S}(p) = \int_{s_1} S(p, \tilde{s}_1) dF_1(\tilde{s}_1) \tag{3.5}$$

where p^0 is a price level at which demand vanishes, and $F_1(\tilde{s}_1) = F(\tilde{s}_1, \infty)$ is the marginal distribution of s_1 .

The cost function¹ is defined as:

$$C = C[x(p, \tilde{s}_1), a, \tilde{s}_2] \quad (3.6)$$

with partial derivatives $C_x > 0$, $C_a < 0$, $C_{s_2} > 0$, $C_{x, s_2} > 0$, $C_{xa} < 0$, $C_{aa} > 0$. The random variable \tilde{s}_2 introduces a direct shock into the cost function. An increase in \tilde{s}_2 increases both total and marginal costs. Managerial effort reduces total cost (at a decreasing rate) and marginal cost.

Actual and expected profits are thus defined as:

$$\pi(p, a, \tilde{s}_1, \tilde{s}_2) = p \cdot x(p, \tilde{s}_1) - C[x(p, \tilde{s}_1), a, \tilde{s}_2] \quad (3.7)$$

$$\bar{\pi}(p, a) = \int \int_{s_1 s_2} \pi(p, a, \tilde{s}_1, \tilde{s}_2) f(\tilde{s}_1, \tilde{s}_2) d\tilde{s}_1 d\tilde{s}_2 \quad (3.8)$$

where $f(\cdot)$ is the joint density function of \tilde{s}_1 and \tilde{s}_2 .

Notice that while consumer surplus is affected by \tilde{s}_1 , profit is affected by both \tilde{s}_1 and \tilde{s}_2 . But perhaps a more important difference between profits and consumer surplus is that while the actual value of the former can be observed by the Minister (even when he does not compute equation (3.7)), the same does not occur with the latter. This distinction has an important implication for the definition of the "outcome" in the present model. The fact is that consumer surplus is not an outcome that the Minister can readily and costlessly observe but rather a magnitude that he has to compute from available information about demand. Making the reasonable assumption that the Minister cannot observe the demand schedule at each state of nature² he is left with an average or expected demand given, say, by some econometric estimation. Furthermore, even if he had the possibility to compute $S(p, \tilde{s}_1)$ for for each realization of \tilde{s}_1 , the Minister would prefer to take a fix value of

¹The cost function does not include any managerial salary or bonus.

²This is also in conformity with the literature on the principal-agent relationship where the principal cannot observe states of nature, otherwise the problem becomes trivial, e.g. Harris and Raviv (1978).

S, to incorporate into the bonus scheme, rather than to take it random, i.e. given by its actual value. The reason for this is given by an argument concerning the class of efficient incentive or bonus schemes: Given the differences in attitude toward risk, optimal risk sharing would indicate that the Manager receives a fixed income. Nevertheless the Minister will have to deviate from this in a situation where moral hazard problems are present. However, notice that given $S(p, \tilde{s}_1)$ and $\pi(p, a, \tilde{s}_1, \tilde{s}_2)$ moral hazard is present only in profits. Actually, to make managerial income random due to $S(p, \tilde{s}_1)$ will increase risks without providing the Manager any incentives for increasing his effort. In other words, and using the terminology of Holmstrom (1979) and explained in Chapter V, $S(p, \tilde{s}_1)$ as a signal is "uninformative" about effort. Its inclusion into the contract is justified to induce optimal pricing, but for this it will be enough to take it as fixed at its expected value.

Given these considerations we restrict our study of incentive schemes to the following class:

$$y = \alpha_1 \cdot \bar{S}(p) + \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + c \quad (3.9)$$

Thus the "outcome" by which the Manager is rewarded or penalized in this context is given by the sum of expected consumer surplus and actual profits. Here the price decision taken by the Manager will affect his income through changes in $\bar{S}(\cdot)$ and $\pi(\cdot)$; while effort decisions will affect income through profits only. Uncertainty is present only in the profit's side of the contract.

To complete the presentation we further assume explicitly:

- A.1. Symmetric information between Minister and Manager concerning the distribution of $(\tilde{s}_1, \tilde{s}_2)$ and the structure of demand, technology and preferences, except of course in relation to managerial effort.
- A.2. No administrative costs to measure \bar{S} , π and to write down and administer the contract. Monitoring activities are not considered (until Chapter V).

A.3. The sharing rule (3.9) will be bounded below and (perhaps) above by some institutional limits. However, we assume that the equilibrium solutions imply that these bounds are not reached. (This "interior solution" assumption also rules out the possibility that the sharing rule does not exist and that the bounds are reached for this reason rather than because of institutional constraints, see discussion in Chapter V.)

A.4. As noted before we assume a single good in a first best economy, (until section 3.7).

A.5. The price level, either under centralization or under decentralization, operates as an ex-ante control (using Leland's (1972) terminology) and demand must be satisfied in each state of nature. We assume that there are no capacity limits and that demand can be met instantaneously by changing the production level. That is, the issues of capacity choice and non-price rationing are neglected (until the next chapter).

3.3. The Full Information Solution

The best centralized decisions in a full information context (i.e. where the Minister observes managerial effort) are derived from the following problem

$$\text{Max}_{p,a,y} \quad \bar{S}(p) + \bar{\pi}(p,a) - y + \lambda \cdot [H(y) - V(a)] \quad (3.10)$$

First order conditions with respect to p, a and y are given by¹:

$$E\{ [p^* - C_x(x(p^*, \tilde{s}_1), a^*, \tilde{s}_2)] \cdot x_p(p^*, \tilde{s}_1) \} = 0 \quad (3.11)$$

$$E\{ -C_a(x(p^*, \tilde{s}_1), a^*, \tilde{s}_2) \} - \lambda \cdot V'(a^*) = 0 \quad (3.12)$$

$$\frac{1}{H'(y^*)} = \lambda \quad (3.13)$$

where $E\{\cdot\}$ denotes the expectation operator defined over \tilde{s}_1 and \tilde{s}_2 . In order to simplify the analysis let us further assume

¹Assumptions are made such that the maximand in (3.10) is locally concave in p, a and y implying that these conditions are also sufficient. Also, interior solutions are assumed.

A.6. x_p is state independent, i.e. $\frac{\partial x_p}{\partial s_1} = 0$

This is equivalent to assuming additive demand uncertainty¹, that is

$$x(p, \tilde{s}_1) = x(p) + \tilde{s}_1 \quad (3.14)$$

Suppose, as an illustration, that A.6. is not satisfied, then we would obtain from (3.11)

$$p^* = E \left\{ \frac{x_p}{E\{x_p\}} \cdot C_x \right\} \begin{matrix} \geq \\ \leq \end{matrix} E\{C_x\}$$

$$\text{if } x_{p,s_1} \text{ and } C_{xx} \begin{matrix} \geq \\ < \end{matrix} 0$$

thus, the departure of p^* from expected marginal cost depends on the slope of the demand function across states and on whether marginal costs are increasing or decreasing. In the case of multiplicative uncertainty we have $x_{p,s_1} > 0$ and therefore this departure depends only on the sign of C_{xx} .

In the present case we can write from (3.11)

$$x_p \cdot [p^* - E\{C_x\}] = 0 \quad (3.15)$$

stating that optimal pricing is given by setting price equal to expected marginal cost. From (3.12) and (3.13) we can write

$$\bar{\pi}_a = \frac{V'(a^*)}{H'(y^*)} \quad (3.16)$$

where $\bar{\pi}_a = -E\{C_a\}$, stating that (social) marginal benefits and costs of effort must be equalized. The weight λ in (3.10) can be seen as the social marginal value, in £, of managerial utility. Condition (3.13) also states the well known result that optimal risk sharing requires that the ratio of marginal utilities be equal to a constant (see Raiffa, 1968, Chapter VII). In the present context this means that managerial marginal utility of income must be constant, in other words that the Manager receives a fixed income.

¹Under this assumption, p^0 in expression (3.4) becomes equal to $x^{-1}(-\tilde{s}_1)$ thus it will depend on \tilde{s}_1 . Additionally, since demand cannot be negative we need to specify a lower bound for \tilde{s}_1 . In the literature either this assumption or simply the assumption that $x(p^*) + \tilde{s}_1 \geq 0$ have been employed. (This latter to avoid complications from having \tilde{s}_1 with a truncated distribution, see Sherman and Visscher (1978)).

Finally, the level of managerial utility U^* obtained from the allocation (a^*, y^*) can also be seen as representing an opportunity cost for the Manager of employment opportunities elsewhere.

3.4. Profit Sharing with Centralized Pricing

Suppose the Minister is no longer able to observe or monitor managerial activities at any cost, but he still retains command of the pricing decision. In this case, since managerial effort affects profit only, he may introduce some profit related bonus scheme to alleviate the moral hazard problem but exposing the Manager to some uncertainty. The fact that he has to provide incentives for the supply of effort (and that the first best solution of the previous section is no longer attainable) implies that the Minister will also seek to deviate from the pricing condition given by expression (3.15), as it will be shown below. We define the incentive contract as

$$y = \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + c \quad (3.17)$$

where the notation adopted in (3.9), for the profit sharing ratio α_2 and the fixed fee c , is maintained only for notational convenience.

Under (3.17) the Manager will select his level of effort to maximize expected utility. The first order condition of this maximization problem is given by¹:

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a\} = V'(a) \quad (3.18)$$

This expression is the equivalent, under risk, of the condition used in the previous chapter although we will show that the first best solution is not attainable due to the well known trade off between incentives and risk.

The Minister's problem is to select the parameters α_2 and c , and a price level p such that he maximizes expected social benefits net of managerial

¹The assumptions already made about the concavity of the managerial utility function guarantee sufficiency along with a unique interior solution. The uniqueness of the solution to this problem is a crucial assumption as the works of Mirrlees (1975) and more recently Grossmann and Hart (1983) illustrate.

rewards, subject to a bargaining level of managerial utility \hat{U} and to the choice of effort given by (3.18), which is defined as an incentive-compatibility or self-selection constraint.

$$\text{Max}_{\alpha_2, c, p} \quad \bar{S}(p) + (1 - \alpha_2)\bar{\pi}(p, a) - c \quad (3.19)$$

subject to

$$E\{H(y)\} - V(a) \geq \hat{U} \quad (3.20)$$

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a\} = V'(a) \quad (3.21)$$

The assumptions made so far guarantee that this problem has unique interior solutions. The procedure to solve it in order to arrive at a characterization of the optimal profit sharing ratio is similar to that of Weitzman (1980). Taking (3.20) as an equality and (3.21) we can solve for the (unique) values of a and c as functions of α_2 and p :

$$a^* = a^*(\alpha_2, p) \quad (3.22)$$

$$c^* = c^*(\alpha_2, p) \quad (3.23)$$

Substituting these solutions into (3.20) we write

$$E\{H(\alpha_2 \cdot \pi(p, a^*(\alpha_2, p), \tilde{s}_1, \tilde{s}_2) + c^*(\alpha_2, p))\} - V(a^*(\alpha_2, p)) = \hat{U} \quad (3.24)$$

Differentiating this expression with respect to α_2 we get:

$$E\{H'(y) \cdot (\pi + \alpha_2 \cdot \pi_a \cdot \frac{\partial a^*}{\partial \alpha_2} - \frac{\partial c^*}{\partial \alpha_2})\} - V' \cdot \frac{\partial a^*}{\partial \alpha_2} = 0$$

using (3.21) and since $\partial c^*/\partial \alpha_2$ and $\partial a^*/\partial \alpha_2$ are state independent we obtain

$$\frac{\partial c^*}{\partial \alpha_2} = - E\left\{\frac{H'}{E\{H'\}} \cdot \pi\right\} \quad (3.25)$$

Differentiating (3.24) with respect to p :

$$E\{H'(y) \cdot (\alpha_2 \cdot \pi_p + \alpha_2 \cdot \pi_a \cdot \frac{\partial a^*}{\partial p} - \frac{\partial c^*}{\partial p})\} - V' \cdot \frac{\partial a^*}{\partial p} = 0$$

using (3.21) again and since $\partial a^*/\partial p$ and $\partial c^*/\partial p$ are state independent we have:

$$\frac{\partial c^*}{\partial p} = - \alpha_2 \cdot E\left\{\frac{H'}{E\{H'\}} \cdot \pi_p\right\} \quad (3.26)$$

Subscripts in π will denote partial derivatives. Expressions (3.25) and (3.26) have a simple interpretation: They show how the fee c has to be changed after

a change in one of the other two instruments, to keep expected managerial utility at its negotiated level \hat{U} .

Substituting expressions (3.22) and (3.23) into (3.19) we can write the Minister's problem as an unconstrained maximization

$$\text{Max}_{\alpha_2, p} \quad \bar{S}(p) + (1 - \alpha_2) \bar{\pi}(p, a^*(\alpha_2, p)) - c^*(\alpha_2, p) \quad (3.27)$$

First order conditions for this problem are:

$$-\bar{\pi} + (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial \alpha_2} - \frac{\partial c^*}{\partial \alpha_2} = 0 \quad (3.28)$$

$$\bar{S}_p + (1 - \alpha_2) \cdot (\bar{\pi}_p + \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p}) - \frac{\partial c^*}{\partial p} = 0 \quad (3.29)$$

Introducing the following definitions:

$$\bar{\pi}_H = E \left\{ \frac{H'}{E\{H'\}} \cdot \pi \right\} \quad (3.30)$$

$$\bar{\pi}_{Hp} = E \left\{ \frac{H'}{E\{H'\}} \cdot \pi_p \right\} \quad (3.31)$$

$$\bar{\pi}_{\alpha_2} = \bar{\pi}_a \cdot \frac{\partial a^*}{\partial \alpha_2} \quad (3.32)$$

$$\epsilon = \alpha_2 \cdot \frac{\bar{\pi}_{\alpha_2}}{\bar{\pi}} \quad (3.33)$$

Expressions (3.30) and (3.31) were discussed above. They represent a weighted expectation on profits and on the change in profits with respect to price, with the weights provided by the relative (to the mean) magnitude of the marginal utility of managerial income in each state of nature. Expression (3.32) measures the effect of a change in α_2 on expected profits, through its impact on the equilibrium level of effort. Finally, expression (3.33) is an elasticity-like measure. Using expression (3.25) and definitions (3.30) to (3.33) we can solve expression (3.28) for α_2 , after some algebraic manipulation:

$$\alpha_2^* = \frac{\epsilon}{\epsilon + 1 - \frac{\bar{\pi}_H}{\bar{\pi}}} \quad (3.34)$$

This is the formula obtained by Weitzman (1980), adapted to our case. It

illustrates the trade-off between the incentive effect of an increase in α_2 given by elasticity ϵ and the risk sharing effect given by the ratio of weighted to unweighted expected profits. If the impact of an increase in managerial effort is huge (i.e. $\epsilon \rightarrow \infty$) or the Manager becomes risk neutral ($\bar{\pi}_H = \bar{\pi}$) we shall obtain $\alpha_2^* = 1$ and therefore the full information conditions (3.15) and (3.16). Conversely, if the productivity of effort is zero ($\epsilon = 0$) or managerial utility losses from risk bearing are huge ($\bar{\pi}_H \rightarrow -\infty$) we shall obtain $\alpha_2^* \approx 0$. In general though, the solution of α_2^* will be between these two extreme cases.

Nevertheless, although useful to interpret the underlying effects, (3.34) is not a closed-form solution since α_2 affects also the right hand side of the expression. Thus, it does not allow us for instance to study some comparative statics properties or to compute its value. Weitzman, however, shows that introducing further simplifications (i.e. restricting the analysis to two states of nature) we can easily employ (3.34) for these purposes.

In addition, notice that unlike what occurred in the certainty case, the Minister needs a great deal of information to compute the optimal value of α_2 ; in particular information about cost conditions (to obtain ϵ) and about the Manager's utility function. Of course, these information requirements are too strong looking at the practical experience of public enterprise regulation. Nevertheless one could take the previous analysis as a characterization of Pareto Efficient contracts that allow us to look at the determinants of the optimal solution. From a practical viewpoint, perhaps it would be possible for the Minister to get some rough estimates of ϵ and $\bar{\pi}_H/\bar{\pi}$ which would be used to determine the profit sharing ratio (see, for instance, Weitzman's numerical exercise based on some estimation of these parameters for the area of defense contracting).

Next, we shall discuss the optimal pricing rule that the Minister will derive from the present context. Using (3.26) we can write expression (3.29) as:

$$\bar{S}_p + \bar{\pi}_p + (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p} + \alpha_2 \cdot (\bar{\pi}_{Hp} - \bar{\pi}_p) = 0 \quad (3.35)$$

Then we can state the following result:

Proposition 3.1. : When incentive problems are present and the Minister has to introduce an incentive contract or bonus scheme to induce the Manager to supply effort, his optimal pricing rule will deviate in general from the rule under full information.

Proof: Notice that (3.15) is obtained with $\bar{S}_p + \bar{\pi}_p = 0$.

According to (3.34) α_2 will in general oscillate in the interval $[0,1]$. $\bar{\pi}_a$ is positive while the partial derivative $\partial a^*/\partial p$ is likely to be negative, as it can be shown by differentiating the first order condition (3.18) and abstracting from income effects, that is keeping utility at \hat{U} . Finally the last term in (3.35) will be different from zero since the Manager is risk averse. Q.E.D.

The relationship between the optimal price level obtained from (3.35) and the level of expected marginal cost will depend on the sign and magnitude of two effects.

First, the effect given by $(1 - \alpha_2) \cdot \bar{\pi}_a \cdot \partial a^*/\partial p$, which is likely to be negative, increasing the gains from a price reduction above the traditional gains in consumer benefits. The reason for this is that any increase (decrease) in the price level reduces (increases) the equilibrium level of effort chosen by the Manager and therefore reduces (increases) the level of expected benefits, net of managerial rewards, by the term under discussion.

Second, the effect given by $\alpha_2 \cdot (\bar{\pi}_{Hp} - \bar{\pi}_p)$ is related to the lack of similarity¹ between the Minister's and Manager's utility functions and to the increasing-risk effect of a change in the price level. When he decides to

¹In Ross (1974), similarity is obtained when the Minister's utility can be expressed as an affine transformation of the Manager's utility implying that both individuals have the same assessments of wealth and will make the same decisions in all risky situations.

change the price level, the Minister affects expected profits by $\bar{\pi}_p$ and thus the share going to the Manager by $\alpha_2 \cdot \bar{\pi}_p$. However, since the Manager is risk averse, changes in "weighted" expected profits are relevant for him and therefore the compensation required to keep him at the negotiated utility level is given by $\alpha_2 \cdot \bar{\pi}_{Hp}$ (see expression (3.26)). Thus the term discussed here represents an extra payment that the Minister will have to consider when changing the price level. Its relation to increasing-risk effects is illustrated by the next result¹: If the distribution of $\pi(p, a, \tilde{s}_1, \tilde{s}_2)$ becomes less risky (riskier) after an increase in price p , then $\bar{\pi}_{Hp} - \bar{\pi}_p$ is positive (negative). To show this let us define $\xi = \bar{\pi}_p \cdot \delta p$ as the change in expected profits after a small increase ($\delta p > 0$) in price. Taking $\bar{\pi}_p$ as positive we have $\xi > 0$. A well known result in the literature on uncertainty (Rothschild and Stiglitz, 1970; 1971) defines (among other definitions) the distribution of $\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2)$ as less risky than the distribution of $\pi(p, a, \tilde{s}_1, \tilde{s}_2)$, if any risk averse agent prefers the distribution of $\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2) - \xi$ to the distribution of $\pi(p, a, \tilde{s}_1, \tilde{s}_2)$; where ξ has been deducted from $\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2)$ in each state of nature $(\tilde{s}_1, \tilde{s}_2)$ giving rise to a distribution with the same mean as in $\pi(p, a, \tilde{s}_1, \tilde{s}_2)$. That is

$$E\{H[\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2) - \xi]\} - E\{H[\pi(p, a, \tilde{s}_1, \tilde{s}_2)]\} > 0$$

for any utility function with $H'' < 0$. Expanding $H[\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2) - \xi]$ about $H[\pi(p, a, \tilde{s}_1, \tilde{s}_2)]$, we have for small δp (and therefore ξ) and neglecting terms of order δp^2 and higher:

$$\begin{aligned} H[\pi(p + \delta p, a, \tilde{s}_1, \tilde{s}_2) - \xi] &= H[\pi(p, a, \tilde{s}_1, \tilde{s}_2)] \\ &+ H'[\pi(p, a, \tilde{s}_1, \tilde{s}_2)] \cdot (\pi_p(p, a, \tilde{s}_1, \tilde{s}_2) \cdot \delta p - \xi) \end{aligned}$$

Taking expectations, arranging terms and using the previous definitions, we can obtain

$$E\{H'[\pi(p, a, \tilde{s}_1, \tilde{s}_2)] \cdot (\pi_p(p, a, \tilde{s}_1, \tilde{s}_2) - \bar{\pi}_p(p, a))\} > 0$$

¹The next discussion adapts a similar result obtained by Leland (1972, appendix) in his treatment of monopolist pricing under demand uncertainty.

which can finally be rearranged using definition (3.31) to give the desired result. In short, this result implies that while making changes in the price level the Minister must consider how this affects the degree of uncertainty in the environment and its impact on the welfare of the agency.

Finally, it could be argued that Proposition 3.1. may be qualified to a great extent if we consider a more general definition of expected marginal cost. Notice that the cost function defined in (3.6) and used throughout the chapter does not include managerial rewards or any other managerial payment. This seems a natural formulation since we are dealing with the question of managerial rewards separately and thus we need to consider it explicitly. It follows also that marginal cost does not include managerial income. In the full information case this inclusion becomes irrelevant since income is fixed. In the present case however managerial income is affected by changes in output as it is the equilibrium level of effort. Thus a redefinition of marginal cost, including the monetary value of the last two terms in (3.35) would suffice to keep the marginal cost pricing rule valid. In this case, what Proposition 3.1. establishes is that the Minister has to recompute marginal cost taking account of the additional effects discussed above and, of course, that the price level will be different from that obtained under full information.

3.5. Profit and Consumer Surplus Sharing with Decentralized Pricing

In this section we begin the study of the main topic of the chapter, namely the implementation of an efficient incentive scheme designed in the way described by expression (3.9) and when, of course, the pricing decision has been delegated to the Manager.

The Minister's problem is now to select an incentive contract characterized by the triple (α_1, α_2, c) such that he maximizes social welfare net of managerial rewards, subject to the negotiated utility level \hat{U} and to the

self-selected levels of effort and price decided by the Manager. In notation

$$\begin{aligned} \text{Max}_{\alpha_1, \alpha_2, c} \quad & (1 - \alpha_1)\bar{S}(p) + (1 - \alpha_2)\bar{\pi}(p, a) - c & (3.36) \\ \text{subject to} \quad & \end{aligned}$$

$$E\{H(y)\} - V(a) \geq \hat{U} \quad (3.37)$$

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a(p, a, \tilde{s}_1, \tilde{s}_2)\} = V'(a) \quad (3.38)$$

$$E\{H'(y) [\alpha_1 \cdot \bar{S}_p(p) + \alpha_2 \pi_p(p, a, \tilde{s}_1, \tilde{s}_2)]\} = 0 \quad (3.39)$$

This problem characterizes Pareto efficient contracts for the present case.

Notice that in relation to the previous section, the Minister substitutes α_1 for p as an instrument. In fact (as it will be made clear below) α_1 main role is to help to enforce optimal pricing and thus operates as a subrogate for direct pricing. In this section we show that the formula for the optimal profit sharing ratio is slightly modified, and we discuss the relative values of α_1 and α_2 required to induce optimal pricing. In the next section we explore this latter subject further, making more explicit the determinants of those values.

Before we proceed, a final comment about the structure of the present model seems necessary to avoid misinterpretations. This relates to the amount of information possessed by the Minister, its relation to the decentralization of the pricing decision and the fact that this is in reality observable, unlike the level of effort.

On the one hand, as in most principal-agent models the problem given by (3.36)-(3.39) assumes that the Minister has perfect knowledge about demand and cost conditions (including the impact of managerial effort on costs) and the utility of the Manager. He is only unable to know the actual value of effort and of the state of nature. However, if this were the case, the introduction of the pricing decision as decentralized would be inappropriate: since the Minister can select the optimal price for a given level of effort, he would not decentralize this decision but instead would write a profit sharing contract and choose the price level as we have indicated in the previous section.

On the other hand, if we assume that the Minister does not possess information about costs (demand is still necessary for him to be able to calculate $S(p)$) then the decentralization of pricing does make sense to allow for the superior information of the Manager to be used, but then we are left with the problem that the Minister will not be entirely able to solve (3.36)-(3.39) due to his lack of information.

Hence, the intention of the approach followed here has to be interpreted taking into account the foregoing discussion. First, we want to characterize the properties of an efficient incentive contract of the class given by (3.9) and for this we assume that the Minister solves (3.36)-(3.39) as if he were completely informed about the enterprise (even though the decentralization of pricing is not totally clear in this context). Second, we look at the properties of these efficient contracts and ask what recommendations, if any, they can provide to the less informed Minister (that is likely to exist in practice) to introduce managerial rewards and incentive schemes in public enterprises.

The method followed to solve (3.36)-(3.39) is the same as that used in the previous section. First, using (3.37) as an equality, (3.38) and (3.39) we solve for the (unique) values of a , p and c in terms of α_1 and α_2 :

$$a^* = a^*(\alpha_1, \alpha_2) \quad (3.40)$$

$$p^* = p^*(\alpha_1, \alpha_2) \quad (3.41)$$

$$c^* = c^*(\alpha_1, \alpha_2) \quad (3.42)$$

Second, substituting these functions into (3.37) and differentiating with respect to α_1 and α_2 we obtain after some simplifications allowed by (3.38) and (3.39)

$$\frac{\partial c^*}{\partial \alpha_1} = -\bar{S} \quad (3.43)$$

$$\frac{\partial c^*}{\partial \alpha_2} = -E\left\{\frac{H'}{E\{H'\}} \cdot \pi\right\} \quad (3.43')$$

Notice again the interpretation given to these conditions as the change required in the fee c after a change in any of the sharing ratios to maintain expected utility at the level \hat{U} . The final step is to rewrite the Minister's problem, using (3.40)-(3.42) as an unconstrained maximization problem with α_1 and α_2 as instruments, that is

$$\text{Max}_{\alpha_1, \alpha_2} (1 - \alpha_1) \cdot \bar{S}(p^*(\alpha_1, \alpha_2)) + (1 - \alpha_2) \cdot \bar{\pi}(p^*(\alpha_1, \alpha_2), a^*(\alpha_1, \alpha_2)) - c^*(\alpha_1, \alpha_2)$$

The first order conditions of this problem can be written, with the help of (3.43) and (3.43'), after some manipulation as:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} (1 - \alpha_1^*) \\ (1 - \alpha_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{\pi} - \bar{\pi}_H \end{bmatrix} \quad (3.44)$$

where

$$\begin{aligned} A &= \bar{S}_p \cdot (\partial p^* / \partial \alpha_1) \\ B &= \bar{\pi}_p \cdot (\partial p^* / \partial \alpha_1) + \bar{\pi}_a \cdot (\partial a^* / \partial \alpha_1) \\ C &= \bar{S}_p \cdot (\partial p^* / \partial \alpha_2) \\ D &= \bar{\pi}_p \cdot (\partial p^* / \partial \alpha_2) + \bar{\pi}_a \cdot (\partial a^* / \partial \alpha_2) \end{aligned}$$

By assumptions already made, $\bar{S}_p < 0$, $\bar{\pi}_a > 0$. We shall take $\bar{\pi}_p < 0$, that is, the price level is set below the expected profit maximizing level. In addition, for the next discussion we take the signs of the partial derivatives¹

$$\partial p^* / \partial \alpha_1 < 0, \partial p^* / \partial \alpha_2 > 0, \partial a^* / \partial \alpha_1 > 0, \partial a^* / \partial \alpha_2 > 0 \quad (3.45)$$

In Appendix 3.A. we show that these signs are very likely to occur under normal conditions. These signs imply that $A > 0$, $B \geq 0$, $C < 0$, $D > 0$. Further we take the determinant

$$\Delta = A \cdot D - B \cdot C > 0 \quad (3.46)$$

(see Appendix 3.A, expression (3.A.12)).

This guarantees that unique solutions exist for the system (3.44). Notice however that these do not represent explicit solutions for α_1 and α_2 since they are arguments of the functions forming Δ . Instead, we solve (3.44) to arrive at a characterization of the effects determining α_1 and α_2 as well as the most likely range of values. Solving for $1 - \alpha_2$, we obtain

$$1 - \alpha_2^* = \frac{A}{\Delta} (\bar{\pi} - \bar{\pi}_H) \quad (3.47)$$

This implies in turn that $\alpha_2^* < 1$, since $\bar{\pi} > \bar{\pi}_H$, $A > 0$ and $\Delta > 0$. Furthermore, from Appendix 3.A (see expression A.3.11) we obtain that the ratio Δ/A can be

¹The derivatives in (3.45) represent changes in the equilibrium (expected utility constant) levels of effort and price after changes in α_1 and α_2 . They are obtained differentiating condition (3.38) and (3.39), see Appendix 3.A.

written as:

$$\frac{\Delta}{A} = \bar{\pi}_a \cdot \frac{\partial a^*}{\partial \alpha_2} - \bar{\pi}_a \cdot \frac{\partial a^*}{\partial \alpha_1} \cdot \frac{\partial p^*/\partial \alpha_2}{\partial p^*/\partial \alpha_1} \quad (3.48)$$

This ratio denotes the total effect of a change in α_2 on expected profits and it will be denoted by $\bar{\pi}_{\alpha_2}$. To the direct effect given by the first term in (3.48), we add a second effect which comes via price and α_1 . Thus, using this definition we can write (3.47) as

$$1 - \alpha_2^* = \frac{\bar{\pi} - \bar{\pi}_H}{\bar{\pi}_{\alpha_2}}$$

introducing again an elasticity-like measure defined by

$$\varepsilon' = \alpha_2 \cdot \frac{\bar{\pi}_{\alpha_2}}{\bar{\pi}}$$

we can obtain, after some manipulation,

$$\alpha_2^* = \frac{\varepsilon'}{\varepsilon' + 1 - \frac{\bar{\pi}_H}{\bar{\pi}}} \quad (3.49)$$

This is the same formula as that obtained in the previous section, except for the inclusion of an indirect effect of α_2 on expected profits (thus the distinction of ε' from ε) which in the previous model was of course unnecessary. The same properties and limit values discussed before apply here as well.

A central aspect of the model is given by the role of the profit sharing ratio in the solution of the incentive-risk trade off. As we have noticed in the previous chapter, working in a riskless environment, given that managerial effort affects profits only, α_2 plays a central role in providing incentives for the supply of effort. This is also observed in the present context since α_2 is mainly determined by effects operating in the profit's part of welfare. In addition, given that consumer surplus was taken as a risk-free measure (but as an outcome depending on the price level selected), α_2 also determines the relative risks allocated to each party. In consequence, the model shows that α_2 is the central policy parameter so far as the incentive-risk trade off is concerned, while α_1 will have to be adjusted in relation to α_2 to take account

of the pricing objective.

The optimal value of α_1 in relation to α_2 can be seen dividing both solutions from (3.44):

$$\frac{1 - \alpha_1^*}{1 - \alpha_2^*} = - \frac{B}{A} \quad (3.50)$$

Since $A > 0$ and $\alpha_2^* < 1$ we first notice that

$$\alpha_1^* \begin{cases} > \\ = \\ < \end{cases} 1 \quad \text{iff} \quad B \begin{cases} > \\ = \\ < \end{cases} 0 \quad \text{that is} \quad \text{iff} \quad \bar{\pi}_a \cdot \partial a^* / \partial \alpha_1 \begin{cases} > \\ = \\ < \end{cases} - \bar{\pi}_p \cdot \partial p^* / \partial \alpha_1 \quad (3.51)$$

Suppose that $\partial a^* / \partial \alpha_1$ is positive (otherwise B is always negative). Then the above condition says that α_1 will be set above (below) unity if the impact of a change in α_1 on the level of expected profits, through a change in the equilibrium level of effort, is higher than the similar effect operating through the equilibrium price level. This is in conformity with our previous discussion of expression (3.35) in the previous section: there we mentioned that a reduction in price would increase in general the equilibrium level of effort and thus the gains of a price reduction. Here we can write the inequality on the right of (3.51) as

$$- \bar{\pi}_a \frac{\partial a^* / \partial \alpha_1}{\partial p^* / \partial \alpha_1} \begin{cases} > \\ = \\ < \end{cases} \bar{\pi}_p \quad (\text{since } \partial p^* / \partial \alpha_1 < 0) \quad \text{and the L.H.S. can be seen as}$$

the (managerial) rate of change between equilibrium effort and price times the impact of effort on expected profits. If this effect is relatively high, α_1 will be selected above unity to induce a lower price and exploit this effect.

Nevertheless, as we show in Appendix 3.A (expression 3.A.14) the sign of B is very likely to be negative. Indeed, it must be negative if the increasing-risk effects of a rise in α_2 are of relative small magnitude and we assume that $\partial p^* / \partial \alpha_2 > 0$ (an increase in the profit sharing ratio raises the equilibrium level of price) which is a most likely result under normal conditions. It follows that except for an extremely risky environment or risk averse Manager we should observe $\alpha_1^* < 1$.

What will be the relative magnitudes of α_1 and α_2 if both are below unity?

Suppose that the Manager chooses (effort and) price such that $\bar{S}_p + \bar{\pi}_p = 0$ at the self-selected equilibrium. It follows that

$$A + B = \bar{\pi}_a \cdot \frac{\partial a^*}{\partial \alpha_1} \begin{cases} > \\ = \\ < \end{cases} 0 \quad \text{iff} \quad \frac{\partial a^*}{\partial \alpha_1} \begin{cases} > \\ = \\ < \end{cases} 0$$

Since we take $\frac{\partial a^*}{\partial \alpha_1} > 0$ as the most likely case, we then have $A + B > 0 \Rightarrow A > -B \Rightarrow \alpha_1^* > \alpha_2^*$. However, due to the lack of similarity, the Manager will choose the price level such that $\bar{S}_p + \bar{\pi}_p \neq 0$ when the Minister chooses $\alpha_1 = \alpha_2$, as it is shown in the next section. Therefore it follows that the decision to select $\alpha_1^* > \alpha_2^*$ will be guaranteed whenever the Manager chooses $\bar{S}_p + \bar{\pi}_p > 0$ under $\alpha_1 = \alpha_2$. The next section explores in detail the effects underlying the selection of the relative magnitudes of α_1 and α_2 .

To conclude this section we shall make two final comments on two aspects of the solution of the present model. First, if risk is eliminated we shall obtain $\alpha_1^* = \alpha_2^* = 1$ and the full information results given by the results of the previous chapter. It would follow that the models discussed in that chapter are a special, limit case of the present one. However, this conclusion should be qualified in view of the different set of assumptions concerning the information possessed by the Minister employed in the two chapters. In the previous chapter we assumed that the Minister was totally uninformed on the technological conditions facing the firm and two models were studied according to the knowledge of demand conditions assumed. In the present chapter however the Minister is fully informed about these conditions. Moreover, if the environment becomes riskless he can achieve the first best solution either by using a linear scheme with $\alpha_1^* = \alpha_2^* = 1$ or more simply using a dichotomous contract¹ and ordering the Manager to select the best levels of effort and price. This latter alternative however is not available for the models of the previous chapter, and even the linear contract would have to be modified if we are in a

¹A dichotomous contract is given by $y = y^* \cdot I(a \geq a^*, p = p^*) + c$, where y^*, a^*, p^* are first best levels and $I(\cdot)$ is the indicator function, being unity at $a \geq a^*, p = p^*$ and zero otherwise. c is a penalty.

situation in which demand is known imperfectly. Therefore, it is incorrect to argue that the models of the previous chapter (and indeed the works of Fisinger-Vogelsang and Gravelle) are trivial, limit cases of a principal-agent model. They are not, since they assume an almost complete misinformation on the side of the Minister.

Second, the fact that we have provided a characterization rather than a full, explicit solution of the model has obscured the result that effort, price and the profit sharing ratio obtained from (3.36)-(3.39) are the same as those obtained in the previous section. That is, the delegation of the pricing decision simply involves the substitution of α_1 for p as an instrument; the selection of α_1 to achieve the pricing condition obtained in (3.35) and the adjustment of c to keep managerial utility at \hat{U}^1 . The reason why the profit sharing ratio, effort and price should be the same is that none of the aspects of the structure of the model of profit sharing with centralized pricing has been changed (in particular the productivity of effort, the structure of risk or the preferences of the Manager)². Thus, the pricing decision implicit in this model should agree with expression (3.35). Notice that from expression (3.50) (or equivalently from the first equation in (3.44)) we can write

$$(1 - \alpha_1^*) A + (1 - \alpha_2^*) B = 0$$

using the definitions of A and B we can write after some changes,

$$\bar{S}_p + \bar{\pi}_p - \alpha_1^* \bar{S}_p - \alpha_2^* \bar{\pi}_p + (1 - \alpha_2^*) \cdot \bar{\pi}_a \cdot \frac{\partial a^* / \partial \alpha_1}{\partial p^* / \partial \alpha_1} = 0$$

From the self-selection constraint (3.39) we have, after some algebraic manipulation and using definition (3.31),

$$\bar{S}_p = - \frac{\alpha_2}{\alpha_1} \bar{\pi}_{Hp}$$

¹The difference between the fee c in (3.36) and the equivalent fee in (3.27) is given by the term $\alpha_1^* \bar{S}(p)$.

²Thus, there would not be differences between ε and ε' in the characterizations of the optimal profit sharing ratios.

Thus, substituting this condition above and arranging we obtain a condition identical to (3.35).

This final observation would imply that although there are no gains from delegating the pricing decision in this context, there are no losses either, if the parameter α_1 is appropriately chosen.

3.6. Optimal Pricing and the Relative Value of the 'Surplus' and 'Profit' Sharing Ratios

In this section we study the effects leading to a choice of the relative values of α_1 and α_2 required for optimal pricing, where this is understood as the price level implied by condition (3.35) of section 3.4. We shall work using the self-selection constraint (3.39) and study the values of α_1 and α_2 that according to this constraint induce the Manager to choose the optimal price level.

Since $\bar{S}_p = -\bar{x} = -E\{x\}$ and $\pi_p = x + x_p \cdot (p - C_x)$ we can write (3.39) as

$$E\{H'(y) \cdot [-\alpha_1 \cdot \bar{x} + \alpha_2 x + \alpha_2 x_p \cdot (p - C_x)]\} = 0$$

or, distributing terms,

$$-\alpha_1 \cdot \bar{x} \cdot E\{H'\} + \alpha_2 \cdot E\{H'x\} + \alpha_2 \cdot x_p \cdot p \cdot E\{H'\} - \alpha_2 \cdot x_p \cdot E\{H'C_x\} = 0 \quad (3.52)$$

Using the definition of covariance we can write $E\{H'x\}$ and $E\{H'C_x\}$ as:

$$E\{H'x\} = \text{cov}(H', x) + E\{H'\} \cdot \bar{x} \quad (3.53)$$

$$E\{H'C_x\} = \text{cov}(H', C_x) + E\{H'\} \cdot E\{C_x\} \quad (3.54)$$

Substituting (3.53) and (3.54) into (3.52), dividing by $\alpha_2 \cdot E\{H'\}$ and arranging we can obtain the following expression

$$p - E\{C_x\} = \frac{\text{cov}(H', C_x)}{E\{H'\}} - \frac{\text{cov}(H', x)}{x_p \cdot E\{H'\}} + \left(\frac{\alpha_1}{\alpha_2} - 1\right) \cdot \frac{\bar{x}}{x_p} \quad (3.55)$$

This expression shows that in order to encourage the Manager to choose price, say, equal to expected marginal cost $E\{C_x\}$, the relative values of α_1 and α_2 will have to be chosen such that the right hand side of (3.55) becomes zero. The ratio α_1/α_2 will therefore depend primarily on the values of the covariances.

However, $p = E\{C_x\}$ is not the optimal pricing rule in the present context, according to Proposition 3.1. Instead, we must derive it from expression (3.35). Rewriting this expression in terms of covariances (this is a straightforward translation of the steps followed above), since

$$\bar{\pi}_{Hp} - \bar{\pi}_p = \frac{\text{cov}(H', x)}{E\{H'\}} - x_p \cdot \frac{\text{cov}(H', C_x)}{E\{H'\}} \quad (3.56)$$

we can write from (3.35):

$$p - E\{C_x\} = -\alpha_2 \cdot \frac{\text{cov}(H', x)}{x_p \cdot E\{H'\}} + \alpha_2 \cdot \frac{\text{cov}(H', C_x)}{E\{H'\}} - (1 - \alpha_2) \cdot \frac{\bar{\pi}_a}{x_p} \cdot \frac{\partial a^*}{\partial p^*} \quad (3.57)$$

Thus, to agree with optimal pricing the L.H.S. of expression (3.55) must satisfy expression (3.57). Substituting this into (3.55) we can write after some simple arrangements the expression:

$$(1 - \alpha_2) \left[x_p \cdot \frac{\text{cov}(H', C_x)}{E\{H'\}} - \frac{\text{cov}(H', x)}{E\{H'\}} \right] + (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p^*} + \left(\frac{\alpha_1}{\alpha_2} - 1 \right) \cdot \bar{x} = 0 \quad (3.58)$$

This expression shows that the optimal ratio α_1^*/α_2^* will depend on two kinds of effects. First, the increasing-risk effect of a change in the price level which determines part of the compensation the Minister will have to pay to the Manager to induce him to choose a given price. This effect is given by the sign and magnitude of the term containing the covariances which is equal to the difference between $\bar{\pi}_p$ and $\bar{\pi}_{Hp}$, discussed at the end of section 3.4. Since he is risk averse, the Manager will be choosing price so that he can also reduce the level of risk in the environment (i.e. as a hedge against uncertainty). The Minister in turn will adjust the relative weights given to the incentive contract to affect that decision, in other words he will have to compensate (charge) the Manager for the increase (reduction) in risk associated with the optimal price level. Second, the impact of a change in price upon the equilibrium level of effort represented by the second term of (3.58)

and also discussed before. Since $\bar{\pi}_a > 0$, $x_p < 0$ and $\partial a^*/\partial p^*$ is most likely to be negative, we have that this effect is unambiguous insofar as it will favour an increase in α_1^*/α_2^* over unity. The first effect however will depend on the sign and magnitude of the covariances, which we will study in this section. Notice that the discussion and results of this section will help us to determine at the same time how the increasing-risk effect affects (a) the optimal price level implied by expression (3.35); (b) the price level chosen by the Manager under different values of the ratio α_1/α_2 ; and finally (c) the optimal ratio α_1/α_2 that will be selected by the Minister to make (a) and (b) similar.

For notational convenience, let us write H' and C_x directly as functions of the random variables \tilde{s}_1 and \tilde{s}_2 :

$$H' [y(\bar{S}, \pi(p, a, \tilde{s}_1, \tilde{s}_2))] \equiv H'(\tilde{s}_1, \tilde{s}_2) \quad (3.59)$$

$$C_x(x(p, \tilde{s}_1), a, \tilde{s}_2) \equiv C_x(\tilde{s}_1, \tilde{s}_2) \quad (3.60)$$

Without loss of generality, let us further assume that \tilde{s}_1 and \tilde{s}_2 are distributed with zero mean, i.e. $E\{\tilde{s}_1\} = E\{\tilde{s}_2\} = 0$. Then expanding (3.59), (3.60) and $x(p, \tilde{s}_1)$ in Taylor series about (0,0) we can write:

$$\begin{aligned} H'(\tilde{s}_1, \tilde{s}_2) &= H'(0,0) + H'_{s_1}(0,0) \cdot \tilde{s}_1 + H'_{s_2}(0,0) \cdot \tilde{s}_2 \\ &\quad + \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 H'_{s_i s_j}(0,0) \cdot \tilde{s}_i \cdot \tilde{s}_j + O_H \end{aligned} \quad (3.61)$$

$$\begin{aligned} C_x(\tilde{s}_1, \tilde{s}_2) &= C_x(0,0) + C_{xs_1}(0,0) \cdot \tilde{s}_1 + C_{xs_2}(0,0) \cdot \tilde{s}_2 \\ &\quad + \frac{1}{2} \sum_{i=1}^2 \sum_{j=1}^2 C_{xs_i s_j}(0,0) \cdot \tilde{s}_i \cdot \tilde{s}_j + O_c \end{aligned} \quad (3.62)$$

$$x(p, \tilde{s}_1) = x(p,0) + x_{s_1}(p,0) \cdot \tilde{s}_1 + x_{s_1 s_1}(p,0) \cdot \tilde{s}_1^2 + O_x \quad (3.63)$$

where O_H , O_c and O_x are formed by terms of order at least $\tilde{s}_i \cdot \tilde{s}_j \cdot \tilde{s}_h$ ($i, j, h = 1, 2, 3$). Subscripts in H' and C_x denote partial derivatives. In the following analysis we shall neglect higher order terms in O_H , O_c and O_x (as well as in some of the $H'_{s_i s_j}$ and $C_{xs_i s_j}$) by assuming that risks are small such

that the utility of income and cost functions can be approximated by quadratic expressions¹. Taking this into account we will write: (notice also that given additive demand uncertainty $x_{s_1} = 1, x_{s_1 s_1} = 0$)

$$H'_{s_1}(0,0) = H''(0,0) \cdot \alpha_2 [p - C_x(x(p,0), a, 0)] = 0 \quad (3.64)$$

$$H'_{s_2}(0,0) = -H''(0,0) \cdot \alpha_2 \cdot C_{s_2}(x(p,0), a, 0) > 0 \quad (3.65)$$

$$H'_{s_1 s_1}(0,0) = -H''(0,0) \cdot \alpha_2 \cdot C_{xx}(x(p,0), a, 0) \quad (3.66)$$

$$H'_{s_1 s_2}(0,0) = -H''(0,0) \cdot \alpha_2 \cdot C_{xs_2}(x(p,0), a, 0) \quad (3.67)$$

$$C_{xs_1}(0,0) = C_{xx}(x(p,0), a, 0) \quad (3.68)$$

$$C_{xs_2}(0,0) = C_{xs_2}(x(p,0), a, 0) > 0 \quad (3.69)$$

$$H'_{s_2 s_2}(0,0) = C_{xs_1 s_1}(0,0) = C_{xs_1 s_2}(0,0) = C_{xs_2 s_2}(0,0) = 0 \quad (3.70)$$

Expression (3.64) becomes zero taking $p = C_x(x(p,0), a, 0)$, that is, starting from a riskless situation and then introducing small changes in \tilde{s}_1 does not affect profits, managerial income and thus its marginal utility². On the other hand an increase in \tilde{s}_2 increases marginal and total costs everywhere and, ceteris paribus, reduces managerial income, giving the sign of (3.65) due to the concavity of $H(y)$. Expressions (3.66) and (3.67) are straightforward second order effects, while condition (3.70) is guaranteed by the quadratic form of the relevant functions and small changes in \tilde{s}_1 and \tilde{s}_2 .

Taking expectations in (3.61)-(3.63) and using (3.64)-(3.70) we can write

$$E\{H'(\tilde{s}_1, \tilde{s}_2)\} = H'(0,0) + \frac{1}{2} H'_{s_1 s_1}(0,0) \cdot \sigma_1^2 + \frac{1}{2} H'_{s_1 s_2}(0,0) \cdot \text{cov}(\tilde{s}_1, \tilde{s}_2) \quad (3.71)$$

$$E\{C_x(\tilde{s}_1, \tilde{s}_2)\} = C_x(0,0) \quad (3.72)$$

¹See Weitzman (1974) for an application and Samuelson (1970) for a theoretical justification of this approximation result.

²Some graphical analysis of finite changes would apparently indicate a reduction (increase) in profits for $\tilde{s}_1 > (<) 0$ if marginal cost are increasing in output. This however is not captured in (3.64) but in (3.66) since these are second order effects. Nevertheless these effects will disappear later in the analysis if we assume symmetric distribution of \tilde{s}_1 . In any case these effects would not affect the result stated in Proposition 3.2. in any dramatic way.

$$E\{x(p, \tilde{s}_1)\} = x(p, 0) \quad (3.73)$$

where $\sigma_i^2 = \text{var}(\tilde{s}_i)$ $i = 1, 2$.

Multiplying expression (3.61) by (3.62), taking expectations and making use of conditions (3.64) and (3.70), using the definition of $\text{cov}(H', C_x)$ from (3.54), and expressions (3.71) and (3.72) we can obtain after some simple arrangements:

$$\begin{aligned} \text{cov}(H', C_x) &= H'_{s_2}(0, 0) \cdot C_{xs_1}(0, 0) \cdot \text{cov}(\tilde{s}_1, \tilde{s}_2) + H'_{s_2}(0, 0) \cdot C_{xs_2}(0, 0) \cdot \sigma_2^2 \\ &+ \frac{1}{2} H'_{s_1 s_1}(0, 0) \cdot C_{xs_1}(0, 0) \cdot E\{\tilde{s}_1^3\} + \frac{1}{2} H'_{s_1 s_2}(0, 0) \cdot C_{xs_1}(0, 0) \cdot E\{\tilde{s}_1^2 \cdot \tilde{s}_2\} \\ &+ \frac{1}{2} H'_{s_1 s_2}(0, 0) \cdot C_{xs_2}(0, 0) \cdot E\{\tilde{s}_1 \cdot \tilde{s}_2^2\} \end{aligned} \quad (3.74)$$

In like fashion, multiplying (3.61) by (3.63), taking expectations and using (3.64) and (3.70), the definition of $\text{cov}(H', x)$ from (3.53) and expressions (3.71) and (3.73) we can write:

$$\begin{aligned} \text{cov}(H', x) &= H'_{s_2}(0, 0) \cdot \text{cov}(\tilde{s}_1, \tilde{s}_2) + \frac{1}{2} H'_{s_1 s_1}(0, 0) \cdot E\{\tilde{s}_1^3\} \\ &+ \frac{1}{2} H'_{s_1 s_2}(0, 0) \cdot E\{\tilde{s}_1^2 \cdot \tilde{s}_2\} \end{aligned} \quad (3.75)$$

Finally assuming that \tilde{s}_1 is symmetrically distributed¹ we have $E\{\tilde{s}_1^3\} = 0$; and we will further ignore higher order terms by taking $E\{\tilde{s}_1^2 \cdot \tilde{s}_2\} = E\{\tilde{s}_1 \cdot \tilde{s}_2^2\} \approx 0$.

This allows us to write, from (3.74) and (3.75):

$$\begin{aligned} \text{cov}(H', C_x) \cdot x_p - \text{cov}(H', x) &= x_p \cdot H'_{s_2}(0, 0) \cdot C_{xs_2}(0, 0) \cdot \sigma_2^2 \\ &+ H'_{s_2}(0, 0) \cdot \text{cov}(\tilde{s}_1, \tilde{s}_2) \cdot [C_{xs_1}(0, 0) \cdot x_p - 1] \end{aligned} \quad (3.76)$$

Considering the foregoing discussion we can state the following result:

Proposition 3.2. : Given the previous assumptions, a sufficient condition for the optimal ratio α_1^*/α_2^* to be greater than unity is that $\text{cov}(\tilde{s}_1, \tilde{s}_2) \geq 0$.

¹If skewness were to be considered it might be argued that a relevant assumption would be to take the distribution of demand skewed to the right, since in most public utilities one would expect to see states of very high demand occurring with some low probability while the same is not verified for extremely low demand states. In this case taking $E\{s_1^3\} > 0$ would not affect the results.

Proof : The first term in (3.76) is unambiguously negative given the signs of (3.65), (3.69), $x_p < 0$ and σ_2^2 . We notice that the first and second order conditions for a local maximum of the welfare function at $\tilde{s}_1 = 0, \tilde{s}_2 = 0$ are:

$$S_p + \pi_p = [p^* - C_x(0,0)] \cdot x_p = 0 \quad (3.77)$$

$$(S_{pp} + \pi_{pp}) \Big|_{p^* = C_x(0,0)} = x_p - C_{xx}(0,0) \cdot x_p^2 < 0 \quad (3.78)$$

This last condition guarantees, using (3.68), that the term within square brackets in (3.76) is negative. Finally, using (3.65) and referring to (3.58) we obtain the desired result.

Q.E.D.

From this proposition we can derive results for more specific cases:

Corollary 3.2.1. : Either under cost uncertainty only or when demand and cost shocks are uncorrelated, the optimal ratio α_1^*/α_2^* is unambiguously greater than one.

Proof : evident from (3.76) taking $\text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$. \square

Corollary 3.2.2. : Under demand uncertainty only, we obtain $\alpha_1^* = \alpha_2^* = 1$, that is the first best solution is attainable.

Proof : Taking $\sigma_2^2 = \text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$ we obtain (3.76) equal to zero as well. The central point here is that the marginal utility of income evaluated at $\tilde{s}_1 = 0, p = C_x(x(p,0), a)$, does not change with small changes in \tilde{s}_1 , by condition (3.64). This implies that we can take $\bar{\pi} = \bar{\pi}_H$ in expression (3.49) of the last section (and in (3.34) of section 3.4 as well) which gives $\alpha_2^* = 1$. Finally, since (3.76) is zero and $\alpha_2^* = 1$ we obtain $\alpha_1^* = \alpha_2^*$ from (3.58). \square

These results summarize sufficient conditions under which the Minister will give more weight to the consumer-surplus side of the contract. The crucial requirement is that the correlation between cost and demand shocks is non-negative, provided we take for granted the concavity of the social welfare function as in the standard theory of public monopoly pricing. The result stated in Corollary 3.2.2. is perhaps surprising since one would have thought a priori that even a small uncertainty in demand would suffice to bring about

a second best result. This however does not mean that demand uncertainty should be neglected since when both cost and demand uncertainty are present their relationship become important for the result given in Proposition 3.2. The impact of high costs on profits and managerial income is amplified (reduced) if at the same time we have positive (negative) demand shocks, thus the requirement that $\text{cov}(\tilde{s}_1, \tilde{s}_2) \geq 0$.

3.7. Extension to Pricing Under a Financial Constraint

In the previous sections we have assumed that the Minister can finance any deficit of the public enterprise by resorting to lump-sum forms of taxation. Notice that, for illustrative purposes, we can speak of two types of deficit: First, that associated with pricing under increasing returns for given, say 'riskless' or 'average' cost and demand schedules. Second, that associated with cost and demand shocks for a given choice of price and managerial effort. The Manager will himself choose to produce under losses of the first class since it is in his interest to do so given the form of the bonus scheme (cf. Chapter II). The Minister on the other hand will provide the subsidy to cover the losses since he is sure that they result under conditions of efficient pricing and choice of effort. (Of course, the bonus scheme is based on profits or deficits before subsidies). On the other hand the second type of deficits are not welcomed by the Manager insofar as they are partially financed by reductions in managerial bonuses due to incentive problems. The Minister will be willing to cover these deficits, if he can resort to lump-sum taxation, since they are due to environmental factors and not to any managerial decision. (He nevertheless must charge part of the loss to the Manager to provide incentives).

When we abandon the assumption of perfect financeability of deficits by non-distortionary means, Ramsey prices become the optimal rule (restricting the analysis to the class of uniform pricing rules) and the incentive scheme

must be modified accordingly, in order to induce the choice of the new optimal values of price and managerial effort. Given that expected profits enter into the social benefit function with a weight greater than one, we should expect a modification of the sharing ratios α_1 and α_2 so that the required level of expected profits is achieved. This could in principle be achieved by a reduction in α_1 or an increase in α_2 . Nevertheless, it is the latter form which seems the appropriate one: to say that the social valuation of expected profits is higher does not only imply that the price level must rise in relation to the first best outcome, but also that the social value of effort will be higher implying that more effort will be demanded from the Manager. A reduction in α_1 will induce the first effect but not the second; in fact it is likely that effort will be reduced after a reduction in α_1 (see Appendix 3.A., expression 3.A.4). This latter effect would also rule out any combination of increases in α_2 along with decreases in α_1 , since decreases in α_1 must be compensated with further increases in α_2 . Therefore, it follows that the profit sharing ratio seems to be the most appropriate instrument to change to induce an optimal choice in the new constrained situation. Nevertheless, we must notice that an increase in α_2 has also its cost: it increases the degree of risk that must be borne by the Manager and therefore the losses given by a further departure from the efficient risk sharing arrangement. Thus any increase in α_2 resulting from the need to have higher prices and effort must be balanced against the losses due to more inefficient risk sharing.

In order to study these effects in more detail, let us reformulate the maximization problem given by expressions (3.36) to (3.39) by introducing the additional constraint $\bar{\pi} \geq \pi_0$. Thus now the Minister will select α_1 , α_2 and c taking into account this constraint as well as those given by expressions (3.37)-(3.39).

The solution to this modified programme goes through similar steps as those described in section 3.5. It can then be shown that the first order

conditions of the unconstrained maximization of the Minister's objective function with respect to α_1 and α_2 can be written as (cf. expression (3.44)):

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 - \alpha_1^* \\ 1 - \alpha_2^* \end{bmatrix} = \begin{bmatrix} -\mu \cdot B \\ \bar{\pi} - \bar{\pi}_H - \mu \cdot D \end{bmatrix} \quad (3.79)$$

where A, B, C and D are the same expressions as in section 3.5, and μ is the Lagrange multiplier associated with the profit constraint. Solving for $1 - \alpha_1^*$ and $1 - \alpha_2^*$ we obtain:

$$1 - \alpha_1^* = -\frac{B}{\Delta} (\bar{\pi} - \bar{\pi}_H) \quad (3.80)$$

$$1 - \alpha_2^* = \frac{A}{\Delta} (\bar{\pi} - \bar{\pi}_H) - \mu \quad (3.81)$$

where $\Delta = A \cdot D - B \cdot C > 0$. Expression (3.80) gives the same characterization of α_1^* as that obtained in section 3.5. From expression (3.81) and following the same steps described in section (3.5) (cf. expression (3.48), etc.) we obtain

$$\alpha_2^* = \frac{(1 + \mu) \cdot \epsilon'}{\epsilon' + 1 - \frac{\bar{\pi}_H}{\bar{\pi}}} \quad (3.82)$$

Thus the formula characterizing the optimal profit sharing ratio is increased by the factor $1 + \mu$. The lower bound of this formula continues to be zero (when $\bar{\pi}_H = \bar{\pi}$, or $\epsilon' \rightarrow 0$). The upper bound however (when $\bar{\pi}_H \rightarrow -\infty$, or $\epsilon' \rightarrow \infty$) is increased to $1 + \mu > 1$. From (3.80) and (3.82) we might be tempted to conclude that the introduction of a profit constraint would not affect α_1^* and would raise α_2^* by $1 + \mu$ from those values obtained without the constraint. The conclusion is unjustified however, since both in section 3.5 and here we are providing a characterization, not a solution. The point is that the values of ϵ' and in particular $\bar{\pi}_H/\bar{\pi}$ will not be the same here as in section 3.5. An increase in α_2 will affect the elasticity ϵ' and in particular it will reduce $\bar{\pi}_H/\bar{\pi}$ due to the increasing risk borne by the Manager. For the same reason, we shall not see in general $\alpha_2^* > 1$ except if the Manager can bear high risks (i.e. he is almost risk neutral). This will be so even if μ adopts large values, for

even though there will be gains from increasing α_2 , the losses due to increasing risk will eventually dominate. Nevertheless we shall observe an increase in α_2^* above the level obtained under no profit constraints, and also a reduction in the ratio α_1^*/α_2^* . This would make the condition obtained in Proposition 3.2. no longer sufficient for having α_1^*/α_2^* greater than unity (see below).

The expression associated with the pricing condition (cf. expression (3.35) and the equivalent obtained in section 3.5) is:

$$\bar{S}_p + (1 + \mu) \cdot \bar{\pi}_p + (1 + \mu - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p^*} + \alpha_2 (\bar{\pi}_{Hp} - \bar{\pi}_p) = 0 \quad (3.83)$$

This has a similar interpretation to that given to expression (3.35). The introduction of a binding profit constraint increases the value of changes in profits by μ . The first two terms characterize the Ramsey-pricing condition

$$\frac{p - E\{C_x\}}{p} = \frac{\mu}{1 + \mu} \cdot \frac{1}{\bar{\eta}}$$

where $\bar{\eta} = x_p \cdot \frac{p}{x}$, is the expected elasticity of demand.

The last two terms in (3.83) again represent the effect of changes in price upon the equilibrium level of effort and then profits (and therefore its valuation is increased by μ), and the increasing risk effect of a change in the price level, respectively. Therefore the point illustrated by Proposition 3.1. also applies to the present situation.

The analysis performed through section 3.6 can also be applied here to show that the conditions stated in Proposition 3.2. are no longer sufficient for $\alpha_1^*/\alpha_2^* > 1$. The equivalent to expression (3.58) for the present case is¹:

$$\frac{(1 + \mu - \alpha_2)}{1 + \mu} \left[x_p \frac{\text{cov}(H', C_x)}{E\{H'\}} - \frac{\text{cov}(H', x)}{E\{H'\}} + \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p^*} \right] + \frac{\mu}{1 + \mu} \cdot \bar{x} + \left(\frac{\alpha_1}{\alpha_2} - 1 \right) \cdot \bar{x} = 0 \quad (3.84)$$

We have a new term which is negative, thus $\text{cov}(\tilde{s}_1, \tilde{s}_2) \geq 0$ is no longer suffi-

¹Obtained, as in that case, from the first order condition of the Manager's problem w.r.t. price and using expression (3.83) and definitions (3.53), (3.54) and (3.56).

cient for having α_1^*/α_2^* greater than one. The size of the profit constraint will now play a role in the determination of that ratio. Nevertheless, although reduced, the possibility of having $\alpha_1/\alpha_2 > 1$ as an optimal result is still open and it will be more likely the larger the size of the covariances, if they are positive, and the larger the impact of changes in prices on expected profits through changes in the equilibrium value of effort.

A possible way out of the indeterminacy of the sign of $(\frac{\alpha_1}{\alpha_2} - 1)$ in expression (3.84) would be to redefine the incentive scheme such that the profit constraint is introduced directly into the bonus scheme. Define

$$\begin{aligned} y &= \alpha_1 \cdot \bar{S}(p) + \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + d && \text{if } \bar{\pi} \geq \pi_0 \\ &= \alpha_1 \cdot \bar{S}(p) + \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + c && \text{if } \bar{\pi} < \pi_0 \end{aligned} \quad (3.85)$$

where $d > c$. Here the Minister chooses the fees d and c , such that he induces the Manager to choose p and a such that the profit constraint is achieved; for example, selecting c in a way that the Manager will fail to achieve his minimum acceptable utility level \hat{U} .

The constraint $\bar{\pi} - \pi_0 \geq 0$ is based as before on the expected value of profits since this is the relevant performance measure for the Minister (cf. Sherman and Visscher (1978)). The problem however, is how expected profits are calculated, since although it is perfectly valid in this context to speak of a subjective distribution of profits for the Minister, this is parameterized in the level of effort. In addition, how this constraint can be made effective after its setting is not clear either, since actual profit can take different values and the Minister has no means of knowing if the actual level belongs to the distributions with means at least equal to π_0 . One way round the problem, would be to announce the target and to ask the Manager to satisfy it taking some years together, in order to average 'good' and 'bad' years. This has an obvious counterpart in the actual practice of setting targets for periods between three and five years (see White Paper (1978)). Thus, insofar

as the Manager perceives it as an "average" target one could in principle take expected profits as the target value¹ (see Rees (1982) for a similar specification, although without the problems mentioned above for the present case).

Assuming that a contract like (3.85) can be enforced, the Manager's problem is as before to maximize his expected utility with his income given by (3.85), that is

$$\begin{aligned} \text{Max}_{a,p} \quad & E\{H(y)\} - V(a) \\ \text{subject to} \quad & \bar{\pi} \geq \pi_0 \end{aligned}$$

where $y = \alpha_1 \cdot \bar{S}(p) + \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + d$

First order conditions are given by

$$E\{-H'(y) \cdot \alpha_2 \cdot \pi_a\} + \gamma \cdot E\{\pi_a\} = V'(a) \quad (3.86)$$

$$E\{H'(y) \cdot (\alpha_1 \cdot \bar{S}_p + \alpha_2 \cdot \pi_p)\} + \gamma \cdot E\{\pi_p\} = 0 \quad (3.87)$$

where γ is the lagrange multiplier associated with the profit constraint.

The Minister's problem is then given by the maximization of $\bar{S} + \bar{\pi} - \bar{y}$ subject to a minimum utility for the Manager and to the self-selection constraints (3.86) and (3.87). The Minister does not take the profit constraint as a direct constraint since this works through the Manager's problem.

From (3.86) we observe an increase in the return from effort, an effect which is required to induce more managerial effort. From (3.87) we can arrange terms as we did in section 3.6 to obtain

$$\begin{aligned} p - E\{C_x\} &= -\frac{\theta}{1+\theta} \cdot \frac{\bar{x}}{x_p} - \frac{\text{cov}(H', x)}{(1+\theta) \cdot x_p \cdot E\{H'\}} + \frac{\text{cov}(H', C_x)}{(1+\theta) \cdot E\{H'\}} \\ &\quad - \left(1 - \frac{\alpha_1}{\alpha_2}\right) \cdot \frac{\bar{x}}{x_p} \cdot \frac{1}{(1+\theta)} \end{aligned} \quad (3.88)$$

where $\theta = \frac{\gamma}{\alpha_2 \cdot E\{H'\}}$

¹There is no reason however to believe that a few years, i.e. a small sample of states of nature, will be representative enough. More importantly, speaking in terms of targets and rewards based on a number of years we are moving into a multiperiod situation while on the other hand (3.85) is a supposedly one period contract.

This expression shows for example that if the Minister selects α_1/α_2 in (3.88) such that the last three terms disappear, the Manager will choose a Ramsey-like price. However, we know that the optimal pricing rule is embedded into expression (3.83) above.

Thus, using (3.83) and (3.88) it can be shown that, provided that in equilibrium $\mu^* = \theta^*$, we obtain an expression like (3.84) above but without the term $[\mu/(1+\mu)]\bar{x}$. Therefore, the indeterminacy of the sign of $(\alpha_1^*/\alpha_2^* - 1)$, disappears. The Minister chooses the relative values of α_1 and α_2 on the basis of the same effects discussed in section 3.6 (cf. expression (3.58)). The increase in the value of profits does not affect α_1 or α_2 since it works through the choice of the fee d , inducing the Manager to select price and effort such that $\bar{\pi} \geq \pi_0$ is satisfied.

The similarity between μ and θ is essential for this result. The multiplier μ represents the marginal loss (gain) to society - given by the change in $S + \pi$ - of an increase (relaxation) of the constraint π_0 . Similarly, the multiplier γ represents the equivalent loss (or gain) for the Manager and it is expressed in utility units (while μ is expressed in monetary terms). To obtain θ , this multiplier is deflated by $\alpha_2 \cdot E\{H'\}$ which can be interpreted as the expected marginal utility of 1 £ of "profit income" for the Manager. Using an argument similar to that used in the previous chapter, we have that if the Ministry chooses α_2 and α_1 in an optimal fashion, described in the previous section, then in equilibrium we obtain the similarity between μ and θ .

3.8. An Example

In this section we provide an illustration of the issues discussed in the model of sections 3.5 and 3.6, in particular the factors behind the determination of the optimal profit sharing ratio α_2 and the adjustment required in the consumer surplus sharing ratio α_1 to induce the selection of the appropriate pricing rule. The example chosen is a simplification in which we assume away

some of the complexities of the model in order to arrive at a characterization of those issues with a minimum possible formulation.

3.8.1. Functions

Let us assume that the managerial utility function is given by

$$\begin{aligned} U &= H(y) - V(a) \\ &= m - n \cdot \exp(-r \cdot y) - a^2 \end{aligned} \tag{3.E.1}$$

where m , n and r are positive parameters. The form of U adopted is a separable function with a negative exponential form for $H(\cdot)$ and quadratic for $V(\cdot)$. Both $H(\cdot)$ and $-V(\cdot)$ are concave; r is the (constant) coefficient of absolute risk aversion, i.e.

$$r = -H''/H' > 0.$$

The demand facing the firm is assumed to be linear, with additive uncertainty

$$x(p, \tilde{s}_1) = b \cdot (\hat{p} - p) + \tilde{s}_1 \tag{3.E.2}$$

It is assumed that there exists a price level $p' = \hat{p} + \tilde{s}_1/b$ at which demand becomes zero. We assume also that \tilde{s}_1 is normally distributed with mean zero and variance σ_1^2 ¹.

Consumer surplus is defined as

$$S(p, \tilde{s}_1) = \int_p^{p'} (b \cdot (p - v) + \tilde{s}_1) dv \tag{3.E.3}$$

and expected consumer surplus is

$$\begin{aligned} \bar{S}(p) &= S(p, 0) = \int_p^{\hat{p}} b (\hat{p} - v) dv \\ &= (\hat{p} - p)^2 \cdot b/2 \end{aligned} \tag{3.E.4}$$

The cost function of the firm is given by

$$C = (k + \tilde{s}_2) \cdot x(p, \tilde{s}_1) + t_0 + \tilde{s}_2 - t_1 \cdot a \tag{3.E.5}$$

¹We also need an assumption ruling out the possibility of negative quantities (and costs). Since $\tilde{s}_1 \in (-\infty, \infty)$ we assume that the density of \tilde{s}_1 becomes virtually zero at large values of \tilde{s}_1 so that the probability of negative quantities is ignored.

where k , t_0 and t_1 are positive parameters. Notice that costs are linear in output, effort affects fixed costs only and there exists "direct" cost uncertainty in additive form given by \tilde{s}_2 . We also assume that \tilde{s}_2 is normally distributed, independently from \tilde{s}_1 , with mean zero and variance σ_2^2 . (Thus $\text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$). Profits can be written as:

$$\pi(p, a, \tilde{s}_1, \tilde{s}_2) = [p - (k + \tilde{s}_2)] \cdot [b \cdot (\hat{p} - p) + \tilde{s}_1] - t_0 - \tilde{s}_2 + t_1 \cdot a \quad (3.E.6)$$

k is expected marginal cost and $t_0 - t_1 \cdot a$ is expected fixed cost. The incentive contract is characterized by

$$y = \alpha_1 \cdot \bar{S}(p) + \alpha_2 \cdot \pi(p, a, \tilde{s}_1, \tilde{s}_2) + c \quad (3.E.7)$$

with mean

$$\bar{y} = \alpha_1 \cdot (\hat{p} - p)^2 \cdot (b/2) + \alpha_2 [(p - k) \cdot b(\hat{p} - p) - t_0 + t_1 \cdot a] + c \quad (3.E.8)$$

and variance

$$\text{var}(y) = \alpha_2^2 \cdot \sigma_\pi^2 \quad (3.E.9)$$

where σ_π^2 is the variance of profits and it can be shown to be

$$\sigma_\pi^2 = (p - k)^2 \cdot \sigma_1^2 + (b \cdot (\hat{p} - p) + 1)^2 \cdot \sigma_2^2 + \sigma_1^2 \cdot \sigma_2^2 \quad (3.E.10)$$

3.8.2. Solutions: The Manager's Problem

The Manager's problem is to maximize his expected utility with y given by the incentive contract. Since y is normally distributed, i.e. $y \sim N(\bar{y}, \alpha_2^2 \sigma_\pi^2)$ the expected utility of the Manager can be written as¹:

$$E\{H(y)\} - V(a) = m - n \cdot \exp\left[-r\left(\bar{y} - \frac{r}{2} \cdot \alpha_2^2 \cdot \sigma_\pi^2\right)\right] - a^2 \quad (3.E.11)$$

Maximization of (3.E.11) with respect to p and a give the first order conditions (using (3.E.8) and (3.E.10)):

$$-n \cdot \exp[-r \cdot g(\bar{y})] \cdot [-r \cdot (\partial \bar{y} / \partial p) + r^2 \cdot \alpha_2^2 \cdot (\partial \sigma_\pi^2 / \partial p)] = 0 \quad (3.E.12)$$

$$-n \cdot \exp[-r \cdot g(\bar{y})] \cdot [-r \cdot (\partial \bar{y} / \partial a)] - 2 \cdot a = 0 \quad (3.E.13)$$

¹Since the expected value of $\exp(-r \cdot y)$ gives the moment generating function of y with parameter r , and when y is normally distributed this takes the form given in (3.E.11).

where

$$g(\bar{y}) = \bar{y} - \frac{r}{2} \cdot \alpha_2^2 \cdot \sigma_\pi^2 \quad (3.E.14)$$

$$\partial \bar{y} / \partial p = -\alpha_1 (\hat{p} - p) \cdot b + \alpha_2 \cdot b [(\hat{p} - p) - (p - k)] \quad (3.E.15)$$

$$\partial \sigma_\pi^2 / \partial p = 2(p - k) \cdot \sigma_1^2 - 2 \cdot b [b(\hat{p} - p) + 1] \cdot \sigma_2^2 \quad (3.E.16)$$

$$\partial \bar{y} / \partial a = \alpha_2 \cdot t_1 \quad (3.E.17)$$

From expression (3.E.12) we can obtain after some arrangements, ignoring the exponential term and using (3.E.15) and (3.E.16):

$$\begin{aligned} (1 - \alpha_1 / \alpha_2) \cdot b \cdot (\hat{p} - p) + b \cdot r \cdot \alpha_2 [b(\hat{p} - p) + 1] \sigma_2^2 \\ = (b + r \cdot \alpha_2 \cdot \sigma_1^2) (p - k) \end{aligned} \quad (3.E.18)$$

This expression shows that if the Minister sets $\alpha_1 = \alpha_2$ the Manager will price above expected marginal cost. Suppose the Minister wants price equal to expected marginal cost, then since $(b + r \cdot \alpha_2 \cdot \sigma_1^2) > 0$ we need the L.H.S. of (3.E.18) to be zero or, after arrangements,

$$\alpha_1 / \alpha_2 = 1 + r \cdot \alpha_2 \cdot \sigma_2^2 [b(\hat{p} - p) + 1] / (\hat{p} - p) \quad (3.E.19)$$

Notice that in (3.E.19) demand uncertainty does not affect the ratio α_1 / α_2 . Of course, in the present case the Minister will like to set prices different from expected marginal cost. Nevertheless, we know by Corollary 3.2.2., since in the present case $\text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$ that demand uncertainty will not affect the optimal ratio α_1^* / α_2^* .

From the first order condition (3.E.12) which is rewritten as (3.E.18), we can obtain the self selected price level which is independent of effort a . From the first order condition (3.E.13) we can determine the self-selected effort for a given level of expected managerial income \bar{y} (which does depend on the price level). Introducing a third equation representing the negotiated utility level, we close the solution restricting the possible pairs (\bar{y}, a) and thus finding the solution of the self-selected level of effort as independent of the price level. That is

$$\hat{U} = m - n \cdot \exp[-r \cdot g(\bar{y})] - a^2 \quad (3.E.20)$$

Define

$$- \hat{h} = \hat{U} - m, \quad \text{then from (3.E.20) we get}$$

$$\exp[-r \cdot g(\bar{y})] = (\hat{h} - a^2)/n \quad (3.E.21)$$

substituting this expression into (3.E.13), using (3.E.17), we obtain after some manipulation

$$a^2 + 2a/r \cdot \alpha_2 \cdot t_1 - \hat{h} = 0 \quad (3.E.22)$$

The positive solution to this quadratic equation is given by

$$a^*(\alpha_2) = -\frac{1}{t_1 \cdot r \cdot \alpha_2} + \frac{1}{2} \left[\frac{4}{r^2 \cdot \alpha_2^2 \cdot t_1^2} + 4 \hat{h} \right]^{1/2} \quad (3.E.23)$$

According to this expression, the self-selected level of effort a^* is increasing in α_2 with limits $h^{1/2}$ as α_2 tends to infinity and zero as α_2 tends to zero. It can also be shown that it is a concave function. Finally, notice that the independence of a^* from p and then α_1 comes from the simplifying assumption that effort affects fixed costs only, or that $C_{xa} = 0$.

From (3.E.21), and using expression (3.E.14) we can write

$$\bar{y}^*(\alpha_1, \alpha_2) = -(1/r) \cdot \log[(\hat{h} - a^*)^2/n] + (r/2) \cdot \alpha_2^2 \cdot \sigma_\pi^2 \quad (3.E.24)$$

\bar{y}^* depends on α_1 indirectly, through σ_π^2 and price (see (3.E.10)).

3.8.3. Solutions: The Minister's Problem

The Minister's problem is given by:

$$\begin{aligned} \text{Max}_{\alpha_1, \alpha_2, c} \quad \bar{S} + \bar{\pi} - \bar{y} &= (1 - \alpha_1) \bar{S}(p) + (1 - \alpha_2) \bar{\pi}(p, a) - c & (3.E.25) \\ \text{subject to} \quad E\{U\} &\geq \hat{U} \end{aligned}$$

p and a satisfy (3.E.12) and (3.E.13)

One method of characterizing (rather than solving explicitly) the optimal ratios α_1^* and α_2^* was described in section 3.5. There we have indicated an indirect method, solving first the Manager's problem for a given level of utility \hat{U} , obtaining solutions for a^* , p^* and c^* (or equivalently \bar{y}^*) as functions of α_1 and α_2 . Then substituting these into the Minister's maximand we have an unconstrained optimization problem in terms of α_1 and α_2 . This

method however is somewhat difficult to work out explicitly in this example, since the maximand of the Minister becomes a very large and complicated expression in α_1 and α_2 (notice for example that a^* and p^* are nonlinear in α_2), and of course the first order conditions are even more complicated to work with.

Here instead we shall follow a short cut method that will allow us to characterize the solution of the optimal profit sharing ratio α_2 . Suppose we want to write the problem (3.E.25) as a maximization with respect to α_2 only. We can do this by assuming that the price level is fixed at the optimal level say p^* . This will give σ_π^2 independent of price (see (3.E.10)). Then substituting a^* from (3.E.23) and \bar{y}^* from (3.E.24) into (3.E.25) we can solve with respect to α_2 . The next stage however is to make the value of α_2 obtained in this form compatible with p^* . However, we need not worry about this since for that purpose α_1 can be used. Therefore α_1 is chosen as a residual, after α_2 has been selected to make this selection compatible with p^* . The value of α_1 obtained in this way should not affect a^* given by (3.E.23), nor \bar{y}^* since the price level remains fixed. To put it in another way, the assumptions made in the present example allow us to "decompose" the problem such that α_1 and α_2 are chosen separately, the latter solving the incentive-risk problem and the former solving the optimal pricing problem for a given choice of α_2 .

Thus assuming $p = p^*$, $\bar{S}(p)$ is fixed at $\bar{S}(p^*)$ and the maximization in (3.E.25) is equivalent to maximize $\bar{\pi}(p^*, a) - \bar{y}$. Or equivalently (substituting from (3.E.6), (3.E.23) and (3.E.24)) :

$$\text{Max}_{\alpha_2} \quad t_1 \cdot a^*(\alpha_2) - \bar{y}^*(a^*(\alpha_2), \alpha_2) \quad (3.E.26)$$

Substituting explicitly (24) into (26) and defining

$$z = \hat{h} - a^{*2} \quad (3.E.27)$$

we have

$$\text{Max}_{\alpha_2} \quad t_1 \cdot a^*(\alpha_2) + \frac{1}{r} \log z(a^*(\alpha_2)) - \frac{r}{2} \cdot \alpha_2^2 \cdot \sigma_\pi^2 - \frac{1}{r} \cdot \log n \quad (3.E.28)$$

The first order condition of this problem (second order condition is guaranteed by the concavity of $a^*(\alpha_2)$ and of the third term as well) is:

$$t_1 \cdot \frac{da^*}{d\alpha_2} + \frac{1}{r} \cdot \frac{1}{z} \cdot \frac{dz}{da^*} \cdot \frac{da^*}{d\alpha_2} - r \cdot \alpha_2^* \cdot \sigma_\pi^2 = 0 \quad (3.E.29)$$

which can be written as (using (3.E.27)):

$$\frac{da^*}{d\alpha_2} \left(t_1 - \frac{2 \cdot a^*}{r \cdot z} \right) - r \cdot \alpha_2^* \cdot \sigma_\pi^2 = 0 \quad (3.E.30)$$

In order to get an expression for $da^*/d\alpha_2$ notice that expression (3.E.23) can be written as

$$\left[a^* + \frac{1}{t_1 \cdot r \cdot \alpha_2^*} \right]^2 = \frac{1}{t_1^2 \cdot \alpha_2^{*2} \cdot r^2} + \hat{h}$$

expanding the square on the L.H.S. we can obtain after eliminating one term and using (3.E.27):

$$a^* = \alpha_2^* \cdot r \cdot t_1 \cdot z / 2 \quad (3.E.31)$$

then

$$\begin{aligned} \frac{da^*}{d\alpha_2} &= \frac{1}{2} \alpha_2^* \cdot r \cdot t_1 \cdot \frac{dz}{da^*} \cdot \frac{da^*}{d\alpha_2} + \frac{1}{2} r \cdot t_1 \cdot z \\ &= \frac{1}{2} \frac{r \cdot t_1 \cdot z}{(1 + r \cdot \alpha_2^* \cdot t_1 \cdot a^*)} \end{aligned} \quad (3.E.32)$$

Substituting (3.E.32) into (3.E.30) we can write after some arrangements

$$\sigma_\pi^2 = \frac{1}{2} \frac{t_1 \cdot z}{\alpha_2^* (1 + r \cdot \alpha_2^* \cdot t_1 \cdot a^*)} \left[t_1 - \frac{2a^*}{r \cdot z} \right] \quad (3.E.33)$$

This is our key relationship between a^* , α_2 and σ_π^2 and the main result of this section. Notice some of the following properties of expression (3.E.33):

$$\text{as } a^* \rightarrow 0 \quad (\text{i.e. as } \alpha_2 \rightarrow 0 \text{ and } z \rightarrow \hat{h}) , \quad \sigma_\pi^2 \rightarrow \infty$$

$$\text{as } a^* \rightarrow h^{1/2} \quad (\text{i.e. as } \alpha_2 \rightarrow \infty \text{ and } z \rightarrow 0) , \quad \sigma_\pi^2 \rightarrow 0$$

$$\text{as } a^* \rightarrow a_{\text{FB}} \quad (\text{i.e. as } \alpha_2 \rightarrow 1 \text{ and } z \rightarrow \frac{2a^*}{r \cdot t_1}) , \quad \sigma_\pi^2 \rightarrow 0$$

where a_{FB} denotes the first best level of effort. The next figure illustrates these relationships (and also it indicates the range of values of α_1/α_2).

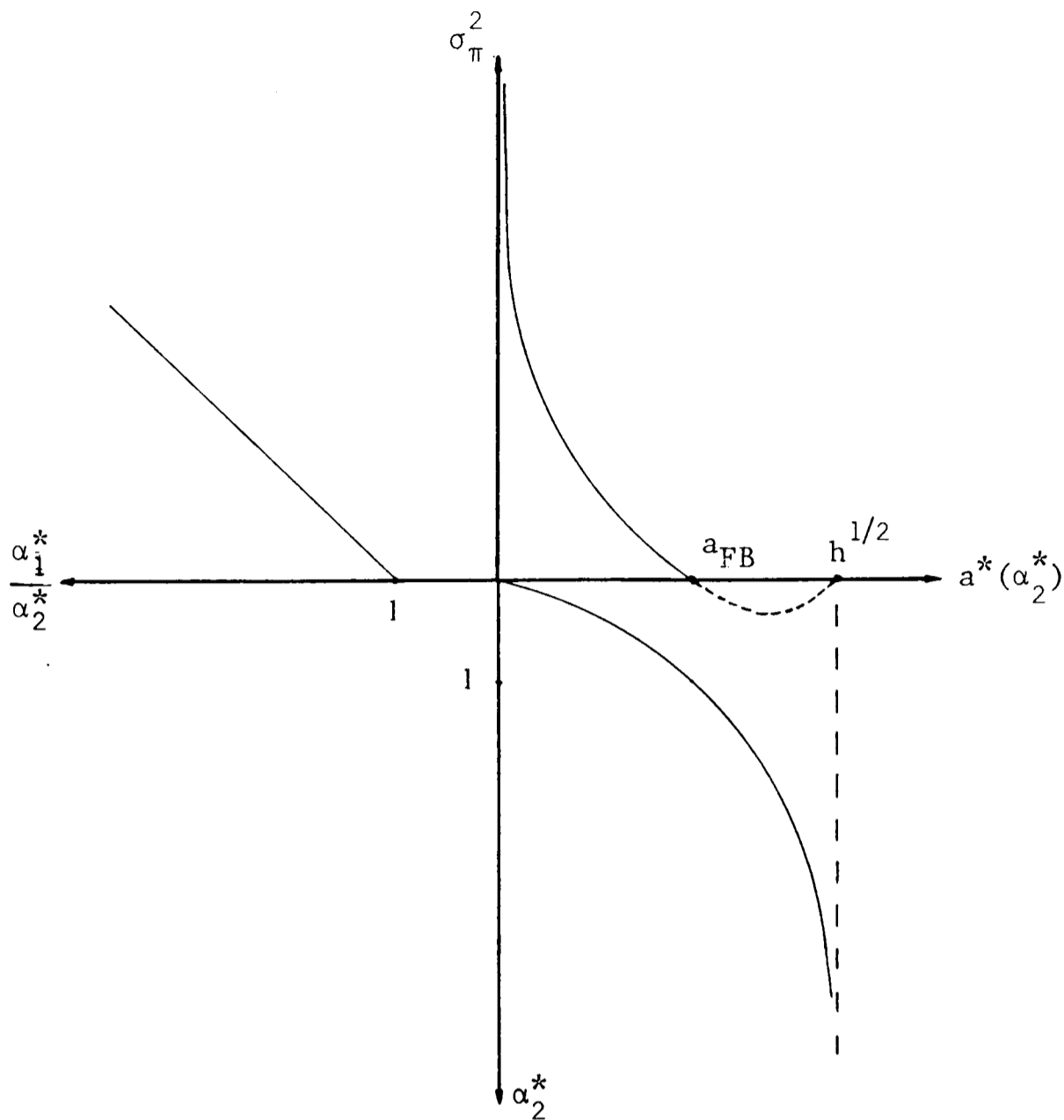


Figure 3.1.

As the variance of profits tends to zero we approach the first best solution in which there is no risk and α_2 is unity (α_1 is also unity and the price level equal to marginal cost, see (3.E.18)). $a_{FB} = a^*(1) = r \cdot t_1 \cdot z/2$. For values of $a > a_{FB}$ expression (3.E.33) gives $\sigma_{\pi}^2 < 0$ which is not possible by definition, leaving the relevant intervals for a^* as $[0, a_{FB}]$ and for $\alpha_2^* \in [0, 1]$. The value of α_1/α_2 is given by (3.E.18) at $p = p^*$ and is increasing in σ_{π}^2 .

3.8.4. Comparative Statics

We can use a modified version of expression (3.E.33) to perform some comparative statics analysis. From (3.E.31) we can write the profit sharing ratio as

$$\alpha_2^* = \frac{2a^*}{r \cdot t_1 \cdot z}$$

Substituting this into the term within parentheses in the denominator of expression (3.E.33) we can write:

$$\sigma_{\pi}^2 = \frac{1}{2} \frac{z}{\left(1 + 2 \cdot \frac{a^{*2}}{z}\right)} \cdot \left(\frac{t_1}{\alpha_2^*} - \frac{2a^*}{\alpha_2^* \cdot r \cdot z}\right)$$

Finally, using (3.E.31) and the definition of z we obtain

$$\sigma_{\pi}^2 = \frac{1}{2} \frac{z^2 \cdot t_1}{(\hat{h} + a^{*2})} \cdot \left(\frac{z \cdot r \cdot t_1}{2a^*} - 1\right) \quad (3.E.34)$$

The advantage of (3.E.34) over (3.E.33) is that it does not depend explicitly (but through a^*) on α_2 , thus we can obtain

$$\frac{d\alpha_2^*}{d\sigma_{\pi}^2} = \frac{d\alpha_2^*}{da^*} \cdot \frac{da^*}{d\sigma_{\pi}^2} \quad (3.E.35)$$

That is, we use (3.E.34) to get $da^*/d\sigma_{\pi}^2$ and expression (3.E.32) to get $d\alpha_2^*/da^*$.

We can already see that (3.E.35) will have a negative sign (as it is also shown in the figure). Differentiating (3.E.34) with respect to a^* , we can obtain after simple manipulation, evaluating the result at $a^* = a_{FB}$

$$\left. \frac{d\sigma_{\pi}^2}{da^*} \right|_{a^*=a_{FB}} = \frac{1}{2} \frac{z^2 \cdot t_1^2 \cdot r}{(\hat{h} + a_{FB}^2)} \cdot \left[-\frac{(2a_{FB}^2 + z)}{2a_{FB}^2} \right]$$

since by the definition of z , $z + 2a^{*2} = \hat{h} + a^{*2}$ we further simplify

$$\left. \frac{d\sigma_{\pi}^2}{da^*} \right|_{a^*=a_{FB}} = -\frac{1}{4} \frac{r \cdot t_1^2 \cdot z^2}{a_{FB}^2} = -\frac{1}{r} \quad (3.E.36)$$

since at $a^* = a_{FB}$, $z = 2a_{FB}/r \cdot t_1$.

To complete the evaluation of (3.E.35), we invert expression (3.E.32) evaluated at $a^* = a_{FB}$; since $\alpha_2^* = 1$ and $r \cdot t_1 \cdot z = 2a_{FB}$.

$$\left. \frac{d\alpha_2^*}{da^*} \right|_{a^*=a_{FB}} = \frac{1}{a_{FB}} + r \cdot t_1 \quad (3.E.37)$$

Thus using (3.E.36) and (3.E.37) we finally obtain

$$\left. \frac{d\alpha_2^*}{d\sigma_{\pi}^2} \right|_{a^*=a_{FB}} = -r^2 \cdot t_1 - r \cdot a_{FB}^{-1} < 0 \quad (3.E.38)$$

Notice that the absolute value of this derivative increases (at an increasing rate) with the size of the coefficient of absolute risk aversion and (linearly) with the size of the coefficient of effort in the cost function.

The effect of a change in σ_{π}^2 on the outcome value is relatively simple to evaluate. We have to obtain

$$\frac{\partial X}{\partial \alpha_2^*} \cdot \frac{d\alpha_2^*}{d\sigma_{\pi}^2} + \frac{\partial X}{\partial \sigma_{\pi}^2}$$

where $X = \bar{S} + \bar{\pi} - \bar{y}$. However, since at the optimum we observe $\partial X / \partial \alpha_2^* = 0$, we have (from (3.E.28)),

$$\frac{dX}{d\sigma_{\pi}^2} = -\frac{r}{2} \cdot \alpha_2^{*2} < 0$$

Evaluated at $a^* = a_{FB}$, or $\alpha_2^* = 1$ (first best) this derivative becomes

$$\left. \frac{dX}{d\sigma_{\pi}^2} \right|_{a^*=a_{FB}} = -\frac{r}{2} \tag{3.E.39}$$

An infinitesimal increase in risk, starting from the first best (riskless) situation, reduces the value of the outcome by minus the risk premium paid to the Manager per unit of risk, i.e. minus half the Arrow-Pratt coefficient of absolute risk aversion.

3.9. Conclusions

In this chapter we have addressed the issue of managerial incentive schemes in public enterprises facing uncertainty in demand and cost functions. The incentive contract studied was a linear scheme relating managerial income with expected consumer surplus and actual profits. Our main results can be summarized as follows:

1. The presence of incentive-risk sharing problems implies that, when providing an incentive scheme to solve them in an optimal way, the Minister will

generally depart from the full information pricing rule (Proposition 3.1.). This departure results when the Minister takes into account the impact of pricing upon the supply of the unobservable productive input by the Manager as well as upon the degree of uncertainty in the outcome.

2. The optimal profit-sharing ratio α_2^* will reflect the trade-off between incentives and risk-sharing in profits. Given the form of the incentive contract studied here this trade-off has to be solved mainly by the choice of α_2 (expression (3.49)).

3. To achieve the objective of efficient pricing, the relevant policy parameter is the ratio between the consumer-surplus and profit sharing ratios, that is α_1/α_2 . The optimal value of this ratio will crucially depend on the correlation between cost and demand shocks. Under the simplifying assumptions of the model studied, we could identify as a sufficient condition for α_1^*/α_2^* greater than one that the covariance between cost and demand shocks must be non-negative (Proposition 3.2.). Under cost uncertainty only, α_1^*/α_2^* will be always greater than one (Corollary 3.2.1.) while under demand uncertainty only, the first best outcome can be attained (Corollary 3.2.2.).

4. When pricing under a financial constraint is considered we should expect a higher profit sharing ratio α_2^* . This makes the condition for having α_1^*/α_2^* greater than unity (i.e. $\text{cov}(\tilde{s}_1, \tilde{s}_2) \geq 0$) no longer sufficient. However if the incentive scheme can be designed such that it takes account of the profit target, expressed in expected units, we have suggested that the previous result is still valid.

5. Finally it is worth noting that in order to implement the optimal parameter values α_1^* and α_2^* the Minister needs to have information about cost and demand conditions as well as about managerial preferences.

This last condition is particularly worrying if we think in terms of the potential applicability of a scheme like the one studied here. Taken too seriously it would invalidate any further consideration of the scheme, in

particular it would imply to reject any further study of its empirical properties.

There is reason to believe however that this is a too pessimistic view. On the one hand, there exists evidence provided from some areas of economic activities (e.g. defense contracting, managerial profit schemes in the private sector, etc.) where these sort of schemes are used though one cannot be sure that they entirely satisfy Pareto efficient conditions. At least we know they are used because they improve the situation, perhaps considerably, with respect to other alternative arrangements. On the other hand, the argument that if enough knowledge to implement an optimal solution is not available then we should forget about the whole scheme cannot be accepted in those circumstances where we have enough information to know that changes in certain directions will improve the outcome. In some of the special cases discussed before, appropriate increases in α_2 and setting α_1/α_2 above unity might considerably improve managerial efficiency and the search for efficient pricing in relation to the actual framework of incentives in public enterprises.

Appendix 3.A.

The Self-Selected Levels of Effort and Price, and some Comparative Statics Results

Solving for effort and price in the first order conditions (3.38) and (3.39) in the text, and substituting the solutions into these conditions we can write them as identities:

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a(p^*(\alpha_1, \alpha_2), a^*(\alpha_1, \alpha_2), \tilde{s}_1, \tilde{s}_2)\} - V'(a^*(\alpha_1, \alpha_2)) = 0 \quad (3.A.1)$$

$$E\{H'(y) \cdot [\alpha_1 \cdot \bar{S}_p(p^*(\alpha_1, \alpha_2)) + \alpha_2 \cdot \pi_p(p^*(\alpha_1, \alpha_2), a^*(\alpha_1, \alpha_2), \tilde{s}_1, \tilde{s}_2)]\} = 0 \quad (3.A.2)$$

Differentiating these expressions with respect to α_1 keeping expected utility at \hat{U} , that is considering the change in $c^*(\alpha_1, \alpha_2)$ needed to achieve this, we obtain:

$$\begin{bmatrix} L_{aa} & L_{ap} \\ L_{pa} & L_{pp} \end{bmatrix} \begin{bmatrix} \partial a^*/\partial \alpha_1 \\ \partial p^*/\partial \alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -E\{H'(y) \cdot \bar{S}_p\} \end{bmatrix} \quad (3.A.3)$$

where $L_{aa} < 0$, $L_{pp} < 0$ (are the second direct derivatives of (3.A.1) and (3.A.2)) and $J = L_{aa} \cdot L_{pp} - (L_{ap})^2 > 0$ are given by assuming that the second order conditions are satisfied. Thus solving (3.A.3) by Cramer's rule we obtain

$$\partial a^*/\partial \alpha_1 = E\{H'(y) \cdot \bar{S}_p\} \cdot L_{ap} / J \quad (3.A.4)$$

$$\partial p^*/\partial \alpha_1 = E\{H'(y) \cdot \bar{S}_p\} \cdot L_{aa} / J \quad (3.A.5)$$

Expression (3.A.5) is unambiguously negative while from expression (3.A.4) we have $\text{sign } \partial a^*/\partial \alpha_1 = -\text{sign } L_{ap}$. It can be shown that $L_{ap} = E\{H''(y) \cdot \alpha_2 \cdot \pi_a \cdot (\alpha_1 \cdot \bar{S}_p + \alpha_2 \cdot \pi_p) + H'(y) \cdot \alpha_2 \cdot \pi_{ap}\}$; this expression can be positive or negative. The second term in L_{ap} is negative since $\pi_{ap} = -C_{xa} \cdot x_p < 0$ by assumptions made before. The first term cannot be signed and it also depends on the chosen values of α_1 and α_2 . Assuming that the second term dominates in case that the first is positive, we have $\partial a^*/\partial \alpha_1 > 0$. An increase in α_1 reduces the price level (increases output) and then it would tend to increase the productivity of effort according to π_{ap} . This in turn will induce an increase in the equilibrium level of effort.

Differentiating (3.A.1) and (3.A.2) with respect to α_2 and again keeping expected utility at \hat{U} , we can obtain

$$\begin{bmatrix} L_{aa} & L_{ap} \\ L_{pa} & L_{pp} \end{bmatrix} \begin{bmatrix} \partial a^*/\partial \alpha_2 \\ \partial p^*/\partial \alpha_2 \end{bmatrix} = \begin{bmatrix} -E\{H'(y) \cdot \pi_a\} - E\{H''(y) \cdot (\pi + \partial c^*/\partial \alpha_2) \cdot \alpha_2 \pi_a\} \\ -E\{H'(y) \cdot \pi_p\} - E\{H''(y) \cdot (\pi + \partial c^*/\partial \alpha_2) \cdot (\alpha_1 \bar{S}_p + \alpha_2 \pi_p)\} \end{bmatrix} \quad (3.A.6)$$

Each element of the R.H.S. vector of expression (3.A.6) is formed by two terms, the first associated with a pure substitution effect and the second associated with an increasing-risk effect. These latter terms cannot generally be signed. Further, assuming that they have a small magnitude we can write

$$\text{sign } \partial a^*/\partial \alpha_2 = \text{sign}[E\{H'(y) \cdot (\pi_p \cdot L_{ap} - \pi_a \cdot L_{pp})\}/J] \quad (3.A.7)$$

$$\text{sign } \partial p^*/\partial \alpha_2 = \text{sign}[E\{H'(y) \cdot (\pi_a \cdot L_{pa} - \pi_p \cdot L_{aa})\}/J] \quad (3.A.8)$$

Necessary and sufficient conditions for these to be positive are given, respectively, by

$$\pi_p \cdot L_{ap} - \pi_a \cdot L_{pp} > 0 \quad (3.A.9)$$

$$\pi_a \cdot L_{pa} - \pi_p \cdot L_{aa} > 0 \quad (3.A.10)$$

However, these conditions, taken together, imply and are implied by $J > 0$.

To demonstrate the latter result suppose that (3.A.9) and (3.A.10) do not hold, thus $\pi_a \cdot L_{pp} > \pi_p \cdot L_{ap}$ and $\pi_p \cdot L_{aa} > \pi_a \cdot L_{pa}$. Multiplying these inequalities by -1 to obtain positive values in each side (we are taking $L_{ap} = L_{pa} < 0$ otherwise (3.A.9) and (3.A.10) follow trivially) and then multiplying each corresponding side of the resulting inequalities we obtain

$$\pi_a \cdot \pi_p \cdot L_{pp} \cdot L_{aa} < \pi_a \cdot \pi_p \cdot (L_{ap})^2, \text{ which implies that } J < 0. \text{ Therefore we take (3.A.7) and (3.A.8) as positive.}$$

Next we show that the determinant Δ given by expression (3.46) in the text is positive if the increasing-risk effects associated with changes in α_2 are relatively small. First, it is easy to show that Δ can be written after some manipulation as

$$\Delta = \bar{S}_p \cdot \bar{\pi}_a [(\partial p^*/\partial \alpha_1) \cdot (\partial a^*/\partial \alpha_2) - (\partial p^*/\partial \alpha_2) \cdot (\partial a^*/\partial \alpha_1)] \quad (3.A.11)$$

Second, using expressions (3.A.4), (3.A.5), (3.A.7) and (3.A.8), we can finally write (3.A.11) after some cancellations of terms

$$\Delta = \bar{S}_p \cdot \bar{\pi}_a \cdot E\{H'(y) \cdot \pi_a\} \cdot E\{H'(y) \cdot \bar{S}_p\} / J > 0 \quad (3.A.12)$$

Finally the term B in the text can be written using (3.A.4) and (3.A.5) as:

$$B = -E\{H'(y) \cdot \bar{S}_p\} (\bar{\pi}_p \cdot L_{aa} - \bar{\pi}_a \cdot L_{ap}) / J \quad (3.A.13)$$

since $\bar{S}_p < 0$, we have $\text{sign } B = \text{sign} (\bar{\pi}_p \cdot L_{aa} - \bar{\pi}_a \cdot L_{ap})$. However since L_{aa} and L_{ap} are state independent (they are expected values) we can write

$$\bar{\pi}_p \cdot L_{aa} - \bar{\pi}_a \cdot L_{ap} = E\{\pi_p \cdot L_{aa} - \pi_a \cdot L_{ap}\} \text{ and therefore}$$

$$\text{sign } B = \text{sign} (\pi_p \cdot L_{aa} - \pi_a \cdot L_{ap}) \quad (3.A.14)$$

Finally using (3.A.10) we have that (3.A.14) is negative.

CHAPTER IV

LINEAR MANAGERIAL BONUS SCHEMES: ASPECTS OF CAPACITY CHOICE

4.1. Introduction: Some Issues in Incentives for Investment Decisions

The control of investment decisions is one of the central aspects of the general system of control in public enterprises. Its importance results from the heavy investments that the industries usually undertake and therefore from the corresponding amount of public money that must be allocated. In this chapter we attempt to extend the previous analysis of managerial incentive schemes to take account of investment decisions. The issue of the control of investment under decentralization however is not an easy one. First, optimal public investment decisions are a difficult subject even in a world without incentive problems. One example is given by the past and current debate on the appropriate social discount rate to evaluate investment programmes. Second, within the literature on incentives and control in organizations there is still no completely satisfactory analysis that can cope with the complexities of the subject. In this section we shall briefly discuss different aspects that can be considered for potential modelling, while reviewing some relevant literature.

4.1.a. The most straightforward extension of our previous analysis would be to relax the assumption of flexible capacity in order to introduce capacity-choice and non-price rationing issues. With capacity fixed at a given level, demand uncertainty gives rise to excess demand in some states of nature. The costs associated with this excess demand are given by the costs of rationing through a non-price mechanism due to the restriction imposed by uniform pricing across all states of nature. In this context capacity output is optimally decided in an ex-ante fashion taking into account costs and benefits, as in the case of the pricing decision. As in the previous chapter we shall benefit from studying two different situations: First, one in which pricing-capacity

decisions are centralized (i.e. taken by the Minister) and a profit sharing contract is introduced to provide incentives for the supply of managerial effort. Second, one in which all decisions are decentralized and an incentive contract based on an approximation to total welfare is introduced to provide production as well as decision making (pricing-capacity) incentives. The study of this model will be the central subject of this chapter. The purpose of the proposed extension is twofold: First we would like to know (in the spirit of Proposition 3.1.) how the presence of managerial incentive problems affect the optimal capacity and rationing decisions in relation to the existing literature where the issue is ignored¹. Second and more important we would like to know whether or not the result obtained before concerning the most likely range of values of the parameters of the bonus scheme are maintained when capacity-choice and non-price rationing problems are introduced. This extension would give more relevance to the previous result since the problems addressed in this chapter are pervasive in some nationalised industries such as electricity and gas.

4.1.b. As we have noted in Chapter I, the standard principal-agent model fails to tackle one central problem of decentralized decisions in public firms: the fact that many 'actions' taken by the Manager are observable but the structure (i.e. technology, etc.) of the problem is unknown to the Minister. That is, models of asymmetric information are perhaps a better characterization of the investment decisions involved. One way of modelling such a situation would be to eliminate production incentives problems from the analysis and instead concentrate on the case where the Manager possesses superior information concerning some central parameters of the problem. Due to the particular nature of his preferences (which in general may differ from the income-effort characterization used before) the Manager has incentives to misinform the Minister about the actual value of the unknown parameters. The Minister then has to

¹See for example Brown and Johnson (1969), Meyer (1975), Crew and Kleindorfer (1978), Sherman and Visscher (1978) and Rees (1982, 1984b).

consider the introduction of a truth-telling mechanism that will induce the Manager to reveal his private information. One potential avenue for research would be for example to extend a model due to Rees (1984a) introducing techniques associated with the so-called 'revelation principle'¹.

4.1.c. Since we are interested in the issue of investment decisions it becomes relevant to ask how to relate the managerial 'action' of the agency model to those decisions. Interpreted in terms of managerial effort this action may be seen for example affecting the marginal cost of capacity output, i.e. the cost of capital. This may in turn result from managerial activities directed towards the maintenance of existing equipments, stretching the life of some machinery and other activities that reduce the depreciation component of that cost. Nevertheless, other types of actions not necessarily interpreted as 'effort' may be more relevant for the analysis. Hughes (1982) for example has investigated the interesting case in which the managerial action affects the distribution of the 'outcome' not in a first-order stochastic dominance (FOSD) sense as assumed in the standard agency model but rather in a second or third order². Second-order stochastic dominance (SOSD) for example is important for investment decisions because it amounts to a reduction in risk equivalent to (or identical if the mean of the distribution is invariant) that studied by Rothschild and Stiglitz (1970, 1971) and Stiglitz and Diamond (1974). Thus, from this setting, one might infer that an increase in the 'action' taken by the Manager, through a reduction of risky prospects, would be beneficial to the agency. Unfortunately, Hughes results contradict this assertion for the most likely class of agency situations: he obtained that the principal will assign a positive value to this (SOSD-inducing) action if he is marginally more risk

¹See Myerson (1979) and Harris and Townsend (1981) for the theoretical aspects and Baron and Myerson (1982) and Sappington (1983) for applications to a regulated monopolist case.

²See the next chapter for the discussion of this FOSD effect which allows us to term the managerial action as a productive input or 'effort'.

averse than the agent¹. Moreover, if we are in a context in which the principal is risk neutral and the Manager risk averse this valuation is equal to zero regardless of the type of utility functions assumed. Quite intuitively, reductions in risk are irrelevant for the welfare of a risk neutral principal while they are costly for an 'action-averse' Manager. Nevertheless, it would be interesting to see whether Hughes results are maintained in a context where we have two types of action, one interpreted as effort in the standard sense of the theory and the other as an SOSD-inducing action. In this case perhaps a reduction in risk may turn out to be beneficial to the agency because it improves the trade off between risk and incentives for the first type of action.

4.1.d. There have been some attempts to deal with private firms investment decisions under agency problems (examples are Heckermann, 1975 and Marcus, 1982). These attempts belong to a growing body of literature on managerial models of financial decisions, in which the earlier paper by Jensen and Meckling (1976) was a seminal contribution. These models however are of limited value for our present analysis because of two main reasons: First, they are conceived in the context of a capital market, making explicit use of the capital assets pricing model (CAPM). This is clearly irrelevant for the public enterprise whose assets are of course "non-marketed". Second, they stress the nature of the investment decision within the capital market equilibrium but they are much less explicit in treating the choice of effort or action by the Manager and the sharing rule as understood in the standard agency model. Marcus (1982) for example concentrates his analysis on the investment decisions made by the Manager while he is the only trader in a CAPM who is constrained in the sales of his assets of the firm he works. This 'no-sale' constraint is justified as part of the compensation package to which both the Manager and the

¹The study is restricted to the case of HARA utility functions but since this class encompasses most of the functions studied in the literature the analysis suffers no great loss of generality.

firm are supposed to agree.

In relation to the first point, - the non-marketed nature of the public enterprise assets -, someone could argue along with some finance-orientated literature (see for example Lintner, 1981 and Gillis, Jenkins and Lessard, 1982) that still the CAPM is of value to make investment decisions in public firms, in particular in the evaluation of risks that lead to the social discount rate or cost of capital. However, this is an area in which there exist considerable debate and the different results are not widely accepted. Moreover they concern to an aspect which is not central to the issues addressed in this work.

4.1.e. Finally, we must recognize that models of capacity choice under oscillating demand are one-period equilibrium analysis and therefore have limited power in capturing the complexity of long term investment decisions. In particular the low time horizon of managerial tenure may imply that a Manager can face no incentive constraint on his decision to invest in programmes that have repercussions beyond his tenure period. There are at least two relevant and related issues here: First, is the problem of how to limit managerial discretion through remuneration packages in the context described above. Second, is the consideration of multiperiod effects into the agency relationship. The first issue has been considered by Eaton and Rosen (1983) in a simplified setting, while the second is currently being expanded (see Radner, 1981; Rubinstein and Yaari, 1983 and in particular Lambert, 1983). Both topics can be of central interest in the study of the control of investment decisions in a multiperiod setting.

The structure of the present chapter is the following: In section 4.2. we reformulate the model of the previous chapter adapted to the present case. Section 4.3. describes the full information solution. Section 4.4. deals with profit sharing under centralized pricing and capacity choice while in section 4.5. we study the decentralization of pricing and capacity under a 'total surplus' bonus scheme.

4.2. Formulation of the Model

As before we begin assuming that the Manager maximizes a Von Neumann-Morgenstern utility function, separable in income (y) and effort (a):

$$U(y,a) = H(y) - V(a) \quad (4.1)$$

with partial derivatives $H' > 0$, $V' > 0$, $H'' < 0$ and $V'' > 0$.

The Minister is interested in the maximization of expected social welfare in a partial equilibrium context

$$\bar{W}(p,q,a) = \bar{S}(p,q) + \bar{\pi}(p,q,a) \quad (4.2)$$

where p is price (only one good is assumed), $\bar{S}(\cdot)$ is expected consumer surplus, $\bar{\pi}(\cdot)$ are expected profits, and q is capacity output (maximum capacity plant in output units). The arguments of \bar{S} and $\bar{\pi}$ are clarified below.

The structure of the model will maintain the features of the formulation adopted in the previous chapter, except of course for the necessary modifications to take account of capacity decisions and the possibility of non-price rationing. To allow for these aspects we shall follow a formulation by Rees (1984b) which is adapted to the present case¹.

To arrive at an expression for $\bar{S}(\cdot)$ let us first define the demand of the i^{th} consumer of the public enterprise good as:

$$x_i = x_i(p, \tilde{s}_1) \quad i = 1, \dots, N \quad (4.3)$$

where $\tilde{s}_1 \in [s_1^0, s_1^1]$ is a continuous random variable with density $f_1(s_1)$. The consumer surplus of the i^{th} individual, in the case he suffers no rationing is therefore

$$S_i(p, \tilde{s}_1) = \int_p^{p^0} x_i(p, \tilde{s}_1) dp \quad (4.4)$$

where p^0 is such that $x_i(p^0, \tilde{s}_1) = 0$.

There exists a chance, however, that the individual i will be rationed in some states of nature. Let us write a partition of the state space such that

¹See Rees (1984b, Appendix to Chapter X). The present formulation is in a "continuous form".

$$S_u = \{s_1 \mid x < q\} \quad S_r = \{s_1 \mid x > q\} \quad (4.5)$$

where $x = \sum_{i=1}^N x_i$. If $s_1 \in S_r$ then there is a probability that i will be rationed. Let us define the probability that the i^{th} consumer will not realize his demand:

$$\theta_i = \theta_i(x(p, \tilde{s}_1), q) \quad (4.6)$$

a function of total demand and capacity. Expression (4.6) defines what has been termed in the literature a random rationing scheme. We shall not specify the functional form of (4.6) but rather assume that its form is jointly decided by Minister and Manager and observed by both¹. The partial derivatives of (4.6) are $\partial\theta_i/\partial x > 0$ and $\partial\theta_i/\partial q < 0$ which make obvious sense.

The expected consumer surplus of the i^{th} individual is then defined:

$$\bar{S}_i(p, q) = \int_{S_1} (1 - \theta_i) S_i(p, \tilde{s}_1) \cdot f_1(s_1) ds_1 \quad (4.7)$$

The probability of being rationed, makes this measure a function of capacity, unlike in the previous formulation. Using (4.6) we can write (4.7) as:

$$\bar{S}_i(p, q) = \int_{s_1 \in S_u} S_i(p, \tilde{s}_1) \cdot f_1(s_1) ds_1 + \int_{s_1 \in S_r} (1 - \theta_i) \cdot S_i(p, \tilde{s}_1) \cdot f_1(s_1) ds_1 \quad (4.7')$$

Notice that θ_i is a conditional (on \tilde{s}_1) probability and by definition $\theta_i = 0$ for all $\tilde{s}_1 \in S_u$.

Total expected consumer surplus is simply

$$\bar{S}(p, q) = \sum_{i=1}^N \bar{S}_i(p, q) \quad (4.8)$$

In order to define expected profits, let us first write the profit function:

$$\pi(p, a, q, \tilde{s}_1, \tilde{s}_2) = p \cdot x(p, \tilde{s}_1) - C[x(p, \tilde{s}_1), a, q, \tilde{s}_2] \quad (4.9)$$

Let us adopt, to keep the model simple and to compare it with the literature, the traditional formulation of constant marginal cost of both output and

¹We are therefore abstracting the analysis from the problem of selecting an optimal rationing mechanism.

capacity. Thus the cost function becomes:

$$C[x(p, \tilde{s}_1), a, q, \tilde{s}_2] = v(a, \tilde{s}_2) \cdot x(p, \tilde{s}_1) + \beta(a, \tilde{s}_2) \cdot q \quad (4.10)$$

with partial derivatives $v_a < 0$, $v_{s_2} > 0$, $\beta_a < 0$, $\beta_{s_2} > 0$, $v_{aa} > 0$ and $\beta_{aa} > 0$ ¹.

The random variable \tilde{s}_2 is assumed continuously distributed (and independent from \tilde{s}_1) with density $f_2(s_2)$. Thus expected profits are:

$$\bar{\pi}(p, a, q) = \int_{s_1} \int_{s_2} \{ [p - v(a, \tilde{s}_2)] \cdot x(p, \tilde{s}_1) - \beta(a, \tilde{s}_2) \cdot q \} f_1(s_1) \cdot f_2(s_2) ds_1 ds_2 \quad (4.11)$$

Introducing the constraint that $x \leq q$ and using (4.6) we can write (4.11):

$$\begin{aligned} \bar{\pi}(p, a, q) &= \int_{s_1 \in S_u} \int_{s_2} [p - v(a, \tilde{s}_2)] \cdot x(p, \tilde{s}_1) \cdot f_1(s_1) \cdot f_2(s_2) ds_1 ds_2 \\ &+ \int_{s_1 \in S_r} \int_{s_2} [p - v(a, \tilde{s}_2)] \cdot q \cdot f_1(s_1) \cdot f_2(s_2) ds_1 ds_2 \\ &- \int_{s_2} \beta(a, \tilde{s}_2) \cdot q \cdot f_2(s_2) ds_2 \end{aligned} \quad (4.11')$$

Thus, we can now write (2) more explicitly, using (7') and (10'):

$$\begin{aligned} \bar{W}(p, a, q) &= \int_{s_1 \in S_u} \int_{s_2} \{ \sum_i S_i(p, \tilde{s}_1) + [p - v(a, \tilde{s}_2)] \cdot x(p, \tilde{s}_1) \} f_1(s_1) \cdot f_2(s_2) ds_1 ds_2 \\ &+ \int_{s_1 \in S_r} \int_{s_2} \{ \sum_i [1 - \theta_i(x(p, \tilde{s}_1), q)] \cdot S_i(p, \tilde{s}_1) + [p - v(a, \tilde{s}_2)] \cdot q \} \\ &\cdot f_1(s_1) \cdot f_2(s_2) ds_1 ds_2 - \int_{s_2} \beta(a, \tilde{s}_2) \cdot q \cdot f_2(s_2) ds_2 \end{aligned} \quad (4.12)$$

4.3. The Full Information Solution

In this case the Minister observes a and will choose p, q and a in order

¹Notice that v and β are both affected by the same managerial effort and by the same random variable.

to maximize (4.12), net of managerial income, subject to the utility achieved by the Manager, that is:

$$\text{Max}_{p,a,q} \quad \bar{S}(p,q) + \bar{\pi}(p,a,q) - y + \lambda \cdot [H(y) - V(a)] \quad (4.13)$$

where λ is interpreted (as before) as the marginal social benefit of managerial utility.

The first order condition (F.O.C.) of (4.13) with respect to p is¹:

$$\int_{s_1 \in S_u} \int_{s_2} \{x_p(p, \tilde{s}_1) \cdot [p - v(a, \tilde{s}_2)]\} f_1(s_1) \cdot f_2(s_2) ds_1 ds_2$$

$$+ \int_{s_1 \in S_r} \int_{s_2} \{-\sum_i [1 - \theta_i] \cdot x_i(p, \tilde{s}_1) + q - \sum_i S_i(p, \tilde{s}_1) \cdot \frac{\partial \theta_i}{\partial x} \cdot x_p(p, \tilde{s}_1)\} f_1(s_1) f_2(s_2) ds_1 ds_2 = 0$$

Before we proceed, we can show that

$$q - \sum_i [1 - \theta_i(\cdot)] \cdot x_i(p, \tilde{s}_1) = 0 \quad \text{for all } \tilde{s}_1 \in S_r \quad (4.14)$$

Notice that we can write the L.H.S. of (4.14) as:

$$q - x(p, \tilde{s}_1) + \sum_i \theta_i(\cdot) \cdot x_i(p, \tilde{s}_1) = \sum_i \theta_i(\cdot) \cdot x_i(p, \tilde{s}_1)$$

For all $\tilde{s}_1 \in S_r$, since the capacity constraint will be binding for all these states of demand. Now we can show that θ_i and x_i have a complementary property in the sense that each one is zero when the other is different from zero. Given an equilibrium capacity and price, q and p respectively, θ_i and x_i depend only on the value of \tilde{s}_1 . If \tilde{s}_1 is such that the i^{th} consumer is not rationed, then $x_i > 0$ and $\theta_i = 0$. If, conversely, the i^{th} consumer is rationed, then $x_i = 0$ and $\theta_i = 1$. Using (4.14), and introducing the assumption that x_p is independent of \tilde{s}_1 , (i.e. additive uncertainty) we can write the F.O.C. as:

$$\left[p - \int_{s_2} v(a, \tilde{s}_2) \cdot f_2(s_2) ds_2 \right] \cdot F_{s_u} = \int_{s_1 \in S_r} \sum_i S_i(p, \tilde{s}_1) \cdot \frac{\partial \theta_i}{\partial x} f_1(s_1) ds_1 \quad (4.15)$$

$$\text{where } F_{s_u} = \int_{s_1 \in S_u} f_1(s_1) ds_1$$

¹The assumptions made so far guarantee the achievement of second order conditions.

The left hand side of (4.15) is the difference between price and expected marginal cost (for a given q) for unrationed states of demand. Notice that in our previous formulation, where there was no rationing problems (i.e. $\theta_i = 0$ for all i , \tilde{s}_1) we obtained the condition that price would be set equal to expected marginal costs. In the present model since $\partial\theta_i/\partial x > 0$ (i.e. the probability that i will be rationed increases with total demand), optimal price p will be set above the expected marginal costs for a value reflecting the marginal reduction in the expected cost of rationing. The argument is well known (see Rees 1984b): The expected cost of rationing for the i^{th} individual is measured here by his expected willingness to pay for having the good at price p rather than being prevented from consumption, and this is given by the integral of S_i times θ_i over the set S_r . An increase in price will reduce total demand, thus the probability θ_i and therefore the expected cost of rationing. This reduction (gain) has to be balanced against the expected loss in welfare given by the increase in p above expected marginal costs and which is relevant only for unrationed states.

The first order condition with respect to a can be written after some manipulation as:

$$\begin{aligned}
 & - \left[\int_{s_2} v_a(a, \tilde{s}_2) \cdot f_2(s_2) ds_2 \right] \cdot \left[\int_{s_1 \in S_u} x(p, \tilde{s}_1) \cdot f_1(s_1) ds_1 \right] \\
 & - \int_{s_2} [v_a(a, \tilde{s}_2) + \beta_a(a, \tilde{s}_2)] \cdot q \cdot f_2(s_2) ds_2 \cdot \int_{s_1 \in S_r} f_1(s_1) ds_1 = \frac{V'(a)}{H'(y)}
 \end{aligned} \tag{4.16}$$

Expression (4.16) is the well known condition that (social) marginal returns from effort must equal (social) marginal costs. The left hand side of (4.16) measures the returns in unrationed and rationed states respectively. In unrationed states, this return is measured by the reduction in expected marginal costs (given q) times the expected demand for those states. In rationed states, demand is irrelevant since we meet capacity constraints, and we measure the

reduction in expected total marginal costs, times the probability that $\tilde{s}_1 \in S_r$.

The F.O.C. w.r.t. q can be written after some manipulation as:

$$\int_{s_2} [p - v(a, \tilde{s}_2)] \cdot f_2(s_2) ds_2 \cdot F_{s_r} - \int_{s_1 \in S_r} \sum_i S_i(p, \tilde{s}_1) \frac{\partial \theta_i}{\partial q} \cdot f_1(s_1) ds_1 \quad (4.17)$$

$$= \int_{s_2} \beta(a, \tilde{s}_2) f_2(s_2) ds_2$$

where $F_{s_r} = 1 - F_{s_u}$.

This is a benefit-cost characterization. An expansion in capacity will increase expected operating profits in rationed states by the difference between price and operating marginal costs. In addition, since $\partial \theta_i / \partial q < 0$ (i.e. an increase in capacity reduces the probability that i will be rationed) the second term of the L.H.S. represents a marginal decrease in expected rationing costs. Therefore, these gains must be balanced against the expected marginal cost of capacity output.

Conditions (4.15), (4.16) and (4.17) characterize the first best, full information solution. They are well known results in the literature. In the following section we shall see how they are affected when managerial effort is unobservable and a profit sharing rule has to be introduced to provide incentives.

4.4. Profit Sharing with Centralized Pricing and Capacity Choice

In this section we investigate the case in which the Minister cannot observe the action a , but instead retains full control of the pricing and capacity decisions. Thus it should be seen as an extension of section 3.4 of the previous chapter. Our purpose here is to show that those results are unmodified except for the consideration of the choice of capacity. We again assume that managerial income is determined in this setting by the linear

contract:

$$y = \alpha_2 \cdot \pi(p, a, q, \tilde{s}_1, \tilde{s}_2) + c \quad (4.18)$$

The Manager's problem is to maximize his expected utility

$$\text{Max}_a \quad E\{H(y)\} - V(a)$$

where y is given by (4.18). The first order condition for this problem is identical to (3.18) that is

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a\} = V'(a) \quad (4.19)$$

The Minister's problem is to select an incentive contract characterized by the pair (α_2, c) and price p and capacity q such that he maximizes expected social benefits net of managerial rewards subject to a bargaining level of managerial utility \hat{U} and to the choice of a given by (4.19) (which we assume to be unique) i.e. the incentive-compatibility or self-selection constraint.

$$\text{Max}_{\alpha_2, c, p, q} \quad \bar{S}(p, q) + (1 - \alpha_2) \cdot \bar{\pi}(p, a, q) - c \quad (4.20)$$

$$\text{subject to} \quad E\{H(y)\} - V(a) \geq \hat{U} \quad (4.21)$$

$$\text{and } a \text{ satisfies eq. (4.19)} \quad (4.22)$$

Following the same method used in section 3.4 of the previous chapter it becomes straightforward to show that the optimal profit sharing ratio can be expressed again as in (3.34):

$$\alpha_2^* = \frac{\epsilon}{\epsilon + 1 - \frac{\bar{\pi}_H}{\bar{\pi}}} \quad (4.23)$$

where $\bar{\pi}_H = E\{H' \cdot \pi\} / E\{H'\}$; $\bar{\pi} = E\{\pi\}$; $\pi_a = E\{\pi_a\}$; $\epsilon = \alpha_2 \cdot \bar{\pi}_{\alpha_2} / \bar{\pi}$;

$$\bar{\pi}_{\alpha_2} = \bar{\pi}_a \cdot \partial a^* / \partial \alpha_2.$$

In addition we obtain that the first order conditions for optimal pricing and capacity choice are:

$$\bar{S}_p + \bar{\pi}_p + (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p} + \alpha_2 \cdot (\bar{\pi}_{Hp} - \bar{\pi}_p) = 0 \quad (4.24)$$

$$\bar{S}_q + \bar{\pi}_q + (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial q} + \alpha_2 \cdot (\bar{\pi}_{Hq} - \bar{\pi}_q) = 0 \quad (4.25)$$

where $\bar{\pi}_p = E\{\pi_p\}$; $\bar{\pi}_{Hp} = E\{H'\pi_p\}/E\{H'\}$; $\bar{\pi}_q = E\{\pi_q\}$ and $\bar{\pi}_{Hq} = E\{H'\pi_q\}/E\{H'\}$, and the subscripts in \bar{S} and $\bar{\pi}$ indicate partial derivatives.

These results indicate that the introduction of capacity choice considerations does not affect the formula for the optimal profit sharing ratio obtained before nor the condition for optimal pricing. Expression (4.25) on the other hand is new and shows a modification of the optimal capacity decision written as expression (4.17) (which can be denoted by $\bar{S}_q + \bar{\pi}_q = 0$) in the same way as the pricing decision has to deviate from $\bar{S}_p + \bar{\pi}_p = 0$ represented by expression (4.15). Thus we could have an extension of Proposition 3.1. taking into account the change in the capacity decision. Furthermore the effects that determine the departure from (4.17) are those discussed in section 3.4 of the previous chapter. First, the impact of changes in the capacity level selected upon the equilibrium, self-selected level of managerial effort. Second, the increasing-risk effect of a change in the capacity level which is represented by changes in the degree of profit-uncertainty and therefore in the corresponding compensation that the Manager should receive to take those risks (maintaining his expected utility at \hat{U}). The term representing the first effect is likely to be positive (since $\partial a^*/\partial q$ is likely to be positive) implying that an increase in the selected capacity level will increase the level of effort and therefore expected net welfare by the term in question. On the other hand the term representing the second effect will be negative if it can be shown that a reduction in capacity makes the distribution of profit less risky. The nature of this effect is further considered in the next section.

4.5. Profit and Consumer Surplus Sharing with Decentralized Pricing and Capacity Choice

In this section, we would like to extend the present formulation to the case in which pricing and capacity decisions are decentralized. In particular, we shall be interested in finding out whether the introduction of capacity decisions and non-price rationing costs change the sort of qualitative results

obtained earlier in section 3.6 of the previous chapter, namely that in a linear incentive scheme based on expected consumer surplus and actual profits, the former would receive a higher 'weight' than the latter under some conditions¹. The incentive contract is represented by the function:

$$y = \alpha_1 \cdot \bar{S}(p,q) + \alpha_2 \cdot \pi(p,a,q,\tilde{s}_1,\tilde{s}_2) + c \quad (4.26)$$

we take the consumer surplus side of the contract fixed at its expected value for the same reason as in the previous chapter, namely that in general the Minister will not be able to calculate S for each state of nature due to his inability to know the state of nature that has actually occurred. Moreover, even if he were able to calculate such a surplus measure in each state, he would prefer to take it fixed rather than state dependent since this latter form would involve a loss in welfare to the agency due to increased risk, without improving the incentives for the supply of effort.

The Minister's problem again is to select an incentive function characterized by the triple (α_1, α_2, c) such that he maximizes expected social benefits net of managerial rewards, subject to a bargaining level of utility for the Manager and to the self-selection constraints representing the decentralized choice of a , p and q . Written in full,

$$\begin{aligned} \text{Max}_{\alpha_1, \alpha_2, c} & (1 - \alpha_1) \cdot \bar{S}(p,q) + (1 - \alpha_2) \cdot \pi(p,a,q,\tilde{s}_1,\tilde{s}_2) - c & (4.27) \\ \text{subject to} & E\{H(y)\} - V(a) \geq \hat{U} & (4.28) \end{aligned}$$

$$E\{H'(y) \cdot \alpha_2 \cdot \pi_a(p,a,q,\tilde{s}_1,\tilde{s}_2)\} = V'(a) \quad (4.29)$$

$$E\{H'(y) \cdot (\alpha_1 \cdot \bar{S}_p(p,q) + \alpha_2 \cdot \pi_p(p,a,q,\tilde{s}_1,\tilde{s}_2))\} = 0 \quad (4.30)$$

$$E\{H'(y) \cdot (\alpha_1 \cdot \bar{S}_q(p,q) + \alpha_2 \cdot \pi_q(p,a,q,\tilde{s}_1,\tilde{s}_2))\} = 0 \quad (4.31)$$

From this programme it is easy to show that we can obtain results similar to those reported in section 3.5 of the previous chapter, in particular we can obtain expressions like (3.47), (3.49) and (3.50). However, we shall concentrate instead on an analysis similar to that of section 3.6. The steps of the

¹Of course, the shortcomings of the analysis discussed briefly at the beginning of section 3.5 are also valid here.

analysis are the following: First, we shall write the first order conditions of the Manager's problem with respect to price and capacity (given by expressions (4.30) and (4.31) respectively) explicitly. Second, through some algebraic manipulations we shall ask what values of α_1 and α_2 make these conditions to satisfy the choice of p and q that the Minister would prefer (which are embedded in conditions (4.24) and (4.25) respectively).

We shall first work with the first order condition with respect to price. Writing expression (4.30) more explicitly and taking unconditional expectations¹ over the appropriate subsets of random variables we can write:

$$\begin{aligned}
& E_{s_u, s_2} \{ H'(y) \cdot [-\alpha_1 \cdot E_{s_u} \{x\} + \alpha_2 \cdot x(p, \tilde{s}_1) + \alpha_2 \cdot x_p [p - v(a, \tilde{s}_2)]] \} \\
& + E_{s_r, s_2} \{ H'(y) \cdot [-\alpha_1 (E_{s_r} \{ \sum_i (1 - \theta_i) \cdot x_i(p, \tilde{s}_1) \}) + E_{s_r} \{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial x} \cdot x_p \}] \\
& + \alpha_2 \cdot q \} = 0
\end{aligned} \tag{4.32}$$

where $E_{s_u, s_2} \{ \cdot \} = \int_{s_1 \in S_u} \int_{s_2} (\cdot) f_1(s_1) \cdot f_2(s_2) ds_1 ds_2$

and $E_{s_r, s_2} \{ \cdot \} = \int_{s_1 \in S_r} \int_{s_2} (\cdot) f_1(s_1) \cdot f_2(s_2) ds_1 ds_2$

indicates expectations taken over $\tilde{s}_1 \in S_u$ (and $\tilde{s}_1 \in S_r$ respectively) and all \tilde{s}_2 . Expression (4.32) was written making direct use of the definition of expected consumer surplus \bar{S} and profits π given in section 4.2. It shows the form of condition (4.30) taking into account unrationed and rationed states of nature. For example, in unrationed states \bar{S}_p is given by $-E_{s_u} \{x\}$ while the corresponding value for rationed states is $E_{s_r} \{ -\sum_i (1 - \theta_i) \cdot x_i - \sum_i S_i \cdot \partial \theta_i / \partial x \cdot x_p \}$. Also, π_p for unrationed states is given by $x + x_p \cdot (p - v)$ while for rationed ones is given by q . Separating terms in (4.32) we obtain:

¹See Appendix 4.A. for a clarification of the notation and definitions used.

$$\begin{aligned}
& -\alpha_1 \cdot E_{S_u} \{x\} \cdot E_{S_u, S_2} \{H'\} + \alpha_2 \cdot E_{S_u, S_2} \{H' \cdot x\} + \alpha_2 \cdot x_p \cdot p \cdot E_{S_u, S_2} \{H'\} \\
& -\alpha_2 \cdot x_p \cdot E_{S_u, S_2} \{H' \cdot v\} - \alpha_1 \cdot E_{S_r} \left\{ \sum_i (1 - \theta_i) \cdot x_i \right\} \cdot E_{S_r, S_2} \{H'\} \\
& -\alpha_1 \cdot E_{S_r} \left\{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial x} \right\} \cdot x_p \cdot E_{S_r, S_2} \{H'\} + \alpha_2 \cdot q \cdot E_{S_r, S_2} \{H'\} = 0 \quad (4.33)
\end{aligned}$$

We know, from expression (4.14), that $q - \sum_i (1 - \theta_i) \cdot x_i = 0$ for all $\tilde{s}_i \in S_r$.

Thus we must observe that

$$E_{S_r} \left\{ \sum_i (1 - \theta_i) \cdot x_i \right\} = E_{S_r} \{q\} = q \cdot F_{S_r} \quad (4.34)$$

where $F_{S_r} = 1 - F_{S_u} = \int_{s_1 \in S_r} f_1(s_1) ds_1$. Substituting (4.34) into (4.33) we can

write this expression as:

$$\begin{aligned}
& -\alpha_1 \cdot E_{S_u} \{x\} \cdot E_{S_u, S_2} \{H'\} + \alpha_2 \cdot E_{S_u, S_2} \{H' \cdot x\} + \alpha_2 \cdot x_p \cdot p \cdot E_{S_u, S_2} \{H'\} \\
& -\alpha_2 \cdot x_p \cdot E_{S_u, S_2} \{H' \cdot v\} + (\alpha_2 - \alpha_1) \cdot q \cdot E_{S_r, S_2} \{H'\} + \alpha_1 \cdot q \cdot E_{S_r, S_2} \{H'\} \cdot F_{S_u} \\
& -\alpha_1 \cdot E_{S_r} \left\{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial x} \right\} \cdot x_p \cdot E_{S_r, S_2} \{H'\} = 0 \quad (4.35)
\end{aligned}$$

Before we proceed we shall introduce a definition of covariance over the subsets $S_u \cup S_2$ and $S_r \cup S_2$. Let us define (see Appendix 4.A.):

$$\begin{aligned}
\text{cov}_{S_u, S_2} (H', x) &= E_{S_u, S_2} \left\{ \left[H' - \frac{E_{S_u, S_2} \{H'\}}{F_{S_u}} \right] \left[x - \frac{E_{S_u, S_2} \{x\}}{F_{S_u}} \right] \right\} \\
&= E_{S_u, S_2} \{H' \cdot x\} - \frac{E_{S_u, S_2} \{H'\} \cdot E_{S_u, S_2} \{x\}}{F_{S_u}} \quad (4.36)
\end{aligned}$$

similarly we have

$$\text{cov}_{S_u, S_2} (H', v) = E_{S_u, S_2} \{H' \cdot v\} - \frac{E_{S_u, S_2} \{H'\} \cdot E_{S_u, S_2} \{v\}}{F_{S_u}} \quad (4.37)$$

In addition let us prove the following result which will be needed for the subsequent discussion.

Lemma 4.1. : For small risks and taking $E\{\tilde{s}_1\} = E\{\tilde{s}_2\} = 0$, if $E_{s_u, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = E_{s_r, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = E_{s_1, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = 0$ (this last equality is given by the assumption of independence between \tilde{s}_1 and \tilde{s}_2) then we can write

$$E_{s_u, s_2}\{H'(y)\} = E_{s_1, s_2}\{H'(y)\} \cdot F_{s_u} \quad (4.38)$$

$$E_{s_r, s_2}\{H'(y)\} = E_{s_1, s_2}\{H'(y)\} \cdot F_{s_r} \quad (4.39)$$

where $E_{s_1, s_2}\{\cdot\}$ is the expectation taken over all \tilde{s}_1, \tilde{s}_2 .

Proof : Defining $H'(y)$ as a function of \tilde{s}_1, \tilde{s}_2 , as in expression (3.59) and expanding it in Taylor series about (0,0) we obtain expression (3.61). Taking expectations, first over all \tilde{s}_2 and then over $\tilde{s}_1 \in S_u$, and using conditions (3.64)-(3.70) we get $E_{s_u, s_2}\{H'(\tilde{s}_1, \tilde{s}_2)\} = H'(0,0) \cdot F_{s_u}$ since $H_{s_1, s_1}(0,0) = 0$ (given that $C_{xx} = 0$, see (3.66)) and using the assumption that $E_{s_u, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = 0$. In addition, since $\text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$ by assumption we have (see (3.71)) that $E_{s_1, s_2}\{H'\} = H'(0,0)$ which proves the result stated in (4.38). The result given in (4.39) can be proved in a similar way. Q.E.D.

This Lemma is a re-statement of the result that the marginal utility of managerial income does not change across different states of demand. It follows that $\text{cov}_{s_u, s_2}(H', x) = 0$.

Now dividing expression (4.35) by $\alpha_2 \cdot E_{s_1, s_2}\{H'\}$, using the result in Lemma 4.1. and definitions (4.36) and (4.37) we can write after some manipulation:

$$\begin{aligned} & x_p \cdot (p - E_{s_2}\{v\}) \cdot F_{s_u} - x_p \cdot E_{s_r}\left\{\sum_i S_i \cdot \frac{\partial \theta_i}{\partial x_i}\right\} \\ & - x_p \cdot \frac{\text{cov}_{s_u, s_2}(H', v)}{E_{s_u, s_2}\{H'\}} \cdot F_{s_u} + \left(1 - \frac{\alpha_1}{\alpha_2}\right) \cdot E_{s_u}\{x\} \cdot F_{s_u} \\ & + \left(1 - \frac{\alpha_1}{\alpha_2} \cdot F_{s_r}\right) \left[q \cdot F_{s_r} + E_{s_r}\left\{\sum_i S_i \frac{\partial \theta_i}{\partial x_i}\right\} x_p\right] = 0 \end{aligned} \quad (4.40)$$

The first two terms in this expression represent $\bar{S}_p + \bar{\pi}_p$ (see expression (4.15)).

The third term summarizes the increasing-risk effect of a change in price (see

below) and it can be shown to be positive (see Appendix 4.A.). Under certainty the covariance is zero, $F_{s_r} = 0$ (as θ_i and its derivative with respect to x), and since $\bar{S}_p + \bar{\pi}_p = 0$ embodies the optimal pricing rule we have that $\alpha_1^* = \alpha_2^* = 1$. Under cost uncertainty only, the last term also vanishes since we cannot have demand rationing.

As we have noted in section 4.4, the optimal condition for pricing is given by expression (4.24). From this we notice that the first two terms in expression (4.40) above must be equal to

$$- (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \partial a^* / \partial p - \alpha_2 \cdot (\bar{\pi}_{Hp} - \bar{\pi}_p) \quad (4.41)$$

Using the result in Lemma 4.1. it can be shown (see Appendix 4.A.1.) that

$$\bar{\pi}_{Hp} - \bar{\pi}_p = - x_p \cdot \frac{\text{cov}_{s_u, s_2}(H', v)}{E_{s_u, s_2}\{H'\}} \cdot F_{s_u} \quad (4.42)$$

Thus substituting this result into (4.41) and this into (4.40) we can write the condition for the optimal choice of α_1 and α_2 needed to satisfy the pricing condition (4.24):

$$\begin{aligned} & - (1 - \alpha_2^*) \cdot x_p \cdot \frac{\text{cov}_{s_u, s_2}(H', v)}{E_{s_u, s_2}\{H'\}} \cdot F_{s_u} - (1 - \alpha_2^*) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial p} \\ & + (1 - \frac{\alpha_1^*}{\alpha_2^*}) E_{s_u}\{x\} \cdot F_{s_u} + (1 - \frac{\alpha_1^*}{\alpha_2^*} \cdot F_{s_r}) [q \cdot F_{s_r} + E_{s_r}\{\sum_i S_i \cdot \frac{\partial \theta_i}{\partial x}\} x_p] = 0 \quad (4.43) \end{aligned}$$

Since $\alpha_2^* < 1$, $\partial a^* / \partial p$ is generally negative and the expression containing the covariance can be shown to be positive (see Appendix 4.A) we obtain that $\alpha_1^* > \alpha_2^*$ unless the term in square brackets in the last term is highly negative. On the other hand if this term is positive and relatively large we need to have $\alpha_1^* / \alpha_2^* > 1 / F_{s_r} > 1$. It is most likely that this term will be positive, while its magnitude may be high and it will depend upon the probability of having demand rationing. The next example illustrate the point for a simple case

Example 4.1. : Let us assume that all consumers are identical and thus S_i is the same for all i ; that demand is linear and that θ_i is also the same for

all i and given by a simple random rationing scheme with $\theta_i = 1 - q/x$ for $x \geq q$ and zero otherwise. The next figure illustrates the case.

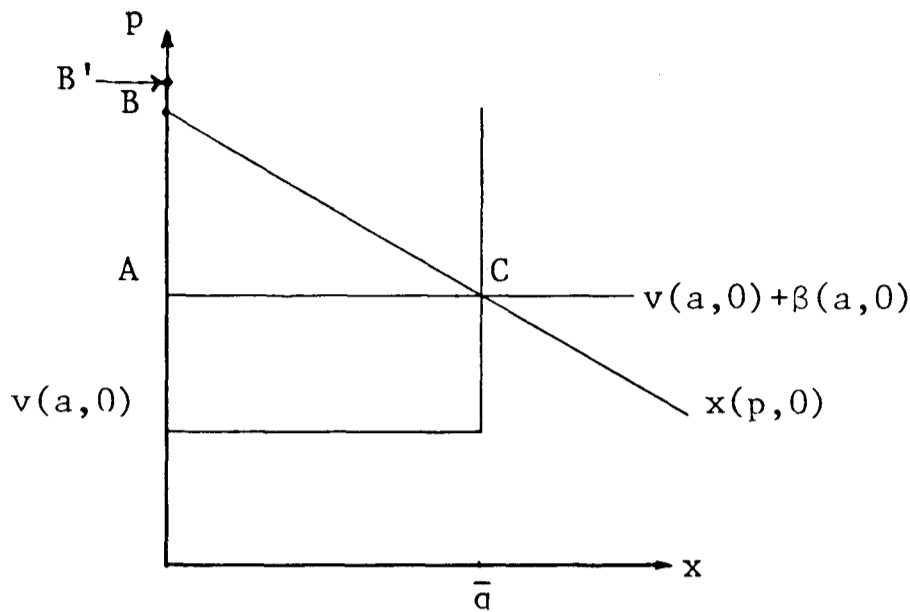


Figure 4.1.

Following the figure, starting from a situation of no risk, $\tilde{s}_1 = 0$ and $x = \bar{q}$.

Thus aggregate consumer surplus is

$$\sum_i S_i(p,0) = \triangle ABC = \frac{1}{2} \cdot \overline{AB} \cdot \bar{q}$$

Since we have additive demand uncertainty then

$$x_p(p,0) = -\bar{q}/\overline{AB} = x_p(p,\tilde{s}_1) = -(\bar{q} + \tilde{s}_1)/\overline{AB'}$$

where $\overline{AB'} > \overline{AB}$.

For all $\tilde{s}_1 \in S_r$, $\tilde{s}_1 > 0$ and $x = \bar{q} + \tilde{s}_1$ then

$$\sum_i S_i(p,\tilde{s}_1) = \frac{1}{2} \overline{AB'} \cdot (\bar{q} + \tilde{s}_1) > \sum_i S_i(p,0)$$

$$\frac{\partial \theta}{\partial x}(p,\tilde{s}_1) = \frac{\bar{q}}{(\bar{q} + \tilde{s}_1)^2} < \frac{\partial \theta}{\partial x}(p,0) = \frac{\bar{q}}{x^2} = \frac{1}{\bar{q}}$$

The term under discussion is

$$q \cdot F_{S_r} + E_{S_r} \left\{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial x} \right\} x_p = E_{S_r} \left\{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial x} \cdot x_p + q \right\}$$

the term within curly brackets in this example becomes approximately equal to

$$\approx -\frac{1}{2} \overline{AB'} (\bar{q} + \tilde{s}_1) \cdot \frac{\bar{q}}{(\bar{q} + \tilde{s}_1)^2} \cdot \frac{(\bar{q} + \tilde{s}_1)}{\overline{AB'}} + \bar{q} = \frac{\bar{q}}{2}$$

thus it becomes approximately of a magnitude (in quantity units) of half of installed capacity. To obtain its expected value over S_r we should then multiply it by F_{S_r} . \square

The analysis of the selection of α_1/α_2 required to induce the optimal capacity decision follows similar steps to those just studied for pricing. First, writing condition (4.31) more explicitly:

$$\begin{aligned} E_{S_r, S_2} \{ H'(y) \cdot [-\alpha_1 \cdot E_{S_r} \{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial q} \} + \alpha_2 (p - v)] \} \\ - E_{S_1, S_2} \{ H'(y) \cdot \alpha_2 \cdot \beta \} = 0 \end{aligned} \quad (4.44)$$

In order to arrive at an expression equivalent to (4.43) let us first define:

$$\text{cov}_{S_1, S_2} (H', \beta) = E_{S_1, S_2} \{ H' \cdot \beta \} - E_{S_1, S_2} \{ H' \} \cdot E_{S_1, S_2} \{ \beta \} \quad (4.45)$$

$$\text{cov}_{S_r, S_2} (H', v) = E_{S_r, S_2} \{ H' \cdot v \} - \frac{E_{S_r, S_2} \{ H' \} \cdot E_{S_r, S_2} \{ v \}}{F_{S_r}} \quad (4.46)$$

Separating terms in (4.44); using definitions (4.45) and (4.46); dividing by $\alpha_2 \cdot E_{S_1, S_2} \{ H' \}$; using Lemma 4.1. and finally adding and subtracting the term $E_{S_r} \{ \sum_i S_i (\partial \theta_i / \partial q) \}$ we can arrive at:

$$\begin{aligned} [p - E_{S_2} \{ v \}] \cdot F_{S_r} - E_{S_r} \{ \sum_i S_i \frac{\partial \theta_i}{\partial q} \} - E_{S_2} \{ \beta \} \\ - \frac{\text{cov}_{S_r, S_2} (H', v)}{E_{S_r, S_2} \{ H' \}} \cdot F_{S_r} - \frac{\text{cov}_{S_1, S_2} (H', \beta)}{E_{S_1, S_2} \{ H' \}} \\ + (1 - \frac{\alpha_1}{\alpha_2} F_{S_r}) E_{S_r} \{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial q} \} = 0 \end{aligned} \quad (4.47)$$

The first two terms in this expression represent $\bar{S}_q + \bar{\pi}_q$ (see condition (4.17)).

In order to satisfy the condition for optimal capacity choice given by (4.25) they must be equal to

$$- (1 - \alpha_2) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial q} - \alpha_2 \cdot (\bar{\pi}_{Hq} - \bar{\pi}_q) \quad (4.48)$$

It can be shown (see Appendix 4.A.) that

$$\bar{\pi}_{Hq} - \bar{\pi}_q = - \frac{\text{cov}_{s_r, s_2}(H', v)}{E_{s_r, s_2}\{H'\}} \cdot F_{s_r} - \frac{\text{cov}_{s_1, s_2}(H', \beta)}{E_{s_1, s_2}\{H'\}} \quad (4.49)$$

Substituting (4.49) into (4.48) and this into (4.47) we obtain

$$\begin{aligned} & - (1 - \alpha_2^*) \left[\frac{\text{cov}_{s_r, s_2}(H', v)}{E_{s_r, s_2}\{H'\}} \cdot F_{s_r} + \frac{\text{cov}_{s_1, s_2}(H', \beta)}{E_{s_1, s_2}\{H'\}} \right] \\ & - (1 - \alpha_2^*) \cdot \bar{\pi}_a \cdot \frac{\partial a^*}{\partial q} + \left(1 - \frac{\alpha_1^*}{\alpha_2^*} \cdot F_{s_r}\right) \cdot E_{s_r} \left\{ \sum_i S_i \cdot \frac{\partial \theta_i}{\partial q} \right\} = 0 \end{aligned} \quad (4.50)$$

Since $\bar{\pi}_a > 0$, $\alpha_2^* < 1$, $\partial a^*/\partial q$ is most likely to be positive, $\partial \theta_i/\partial q < 0$ and the covariances are positive (see Appendix 4.A.) we obtain

$$\frac{\alpha_1^*}{\alpha_2^*} > \frac{1}{F_{s_r}} > 1 \quad (4.51)$$

This result would not contradict the one obtained while discussing the pricing condition since as we have seen in expression (4.43) the ratio α_1^*/α_2^* can in fact satisfy condition (4.51) to induce optimal pricing. The result would also indicate that the optimal ratio α_1^*/α_2^* is higher in the case of fixed capacity and non-price rationing than in the case of flexible capacity.

We must be careful in finding an explanation for this last result since there seems to be more than one effect involved. Before performing the analysis of this section one would have suggested that since rationing costs are important in the present case, this would lead the Minister to seek a higher price than in the flexible capacity case and therefore a lower ratio α_1^*/α_2^* . This type of reasoning however is incorrect on two accounts. First, we notice that capacity is also an instrument in this context whereby rationing costs can be reduced, and in order to induce a higher choice of capacity the ratio α_1/α_2 should be increased. Second and more fundamentally, - as we have illustrated in the previous chapter as well - there exists two reasons for departing from an 'equal-weights' contract with $\alpha_1 = \alpha_2$, namely an increasing-risk effect and an effect through changes in the equilibrium level of effort. The cost of rationing in itself does not give rise to a further departure since

it is already embodied in the sharing rule and evaluated in a similar way by Minister and Manager.

Therefore the selection of α_1^* / α_2^* above unity must depend only on the two above mentioned effects, and further inspection of the analysis leading to conditions (4.43) and (4.50) suggests that the increasing-risk effect of changes in price and capacity plays the major role in the results. Since an increase in price reduces the degree of profit uncertainty the Manager will set the price level 'too high' and for this reason the Minister will adjust α_1 / α_2 upwards. This adjustment is reinforced when the capacity decision is considered, since in the present context a reduction in capacity reduces the degree of profit uncertainty and the Manager will tend to select a lower than optimal capacity. Thus in order to induce the optimal choice of capacity and price the ratio α_1 / α_2 is selected according to (4.51).

4.6. Conclusions

In this chapter we have addressed the issues of capacity choice and non-price rationing within our previous treatment of managerial bonus schemes in public enterprises. The extension seems justified due to the empirical importance of these issues for many public utilities and as an integration of our analysis with the existing literature on the subject. We have simplified the setting in relation to the previous chapter by assuming constant marginal cost of output and capacity, and uncorrelated shifts in demand and costs.

The first result is a re-statement of a result obtained before, namely that when he has to provide incentives for the supply of effort the Minister will generally depart from the rules adopted when there are no incentive problems. Two effects will influence the optimal level of capacity: First, the effect of a change in capacity level upon the self-selected level of managerial effort; and second an increasing-risk effect of a change in the capacity level which is represented by changes in the degree of profit-uncertainty and thus

by the corresponding compensation that the Manager should receive to take those risks. The first effect would tend to make the optimal capacity level higher, *ceteris paribus*, than in the case without incentive problems; while the second effect goes in the opposite direction.

When pricing-capacity decisions are decentralized and a linear scheme based on expected consumer surplus and actual profits is used to provide incentives, the qualitative result obtained before about the relative weights given to each part of the incentive contract is maintained. That is, the expected consumer surplus side of the contract receives a relatively higher weight; while the profit sharing ratio oscillates between zero and one. Thus the result stated in corollary 3.2.1. of the previous chapter, namely that $\alpha_1^*/\alpha_2^* > 1$ whenever $\text{cov}(\tilde{s}_1, \tilde{s}_2) = 0$ is unaffected by considerations of capacity choice. In addition, the analysis made in section 4.5. indicates that the optimal ratio between α_1 and α_2 is higher in the present context than in the flexible capacity case in order to induce the required adjustment of capacity output by the Manager.

Appendix 4.A.

Let us define the expected value of demand $x(p, \tilde{s}_1)$ conditional on $\tilde{s}_1 \in S_u$ and $\tilde{s}_1 \in S_r$ respectively:

$$E\{x \mid \tilde{s}_1 \in S_u\} = \int_{s_1 \in S_u} x(p, \tilde{s}_1) \cdot \frac{f_1(s_1)}{F_{s_u}} \cdot ds_1 \quad (4.A.1)$$

$$E\{x \mid \tilde{s}_1 \in S_r\} = \int_{s_1 \in S_r} x(p, \tilde{s}_1) \cdot \frac{f_1(s_1)}{F_{s_r}} \cdot ds_1 \quad (4.A.2)$$

These can be seen as conditional expected values of demand over unrationed and rationed states respectively. The notation used throughout section 4.5 for x (and any other variable) is:

$$E_{s_u, s_2}\{x\} = E\{x \mid \tilde{s}_1 \in S_u\} \cdot F_{s_u} \quad (4.A.3)$$

$$E_{s_r, s_2}\{x\} = E\{x \mid \tilde{s}_1 \in S_r\} \cdot F_{s_r} \quad (4.A.4)$$

where $F_{s_u} = \int_{s_1 \in S_u} f_1(s_1) ds_1$ and $F_{s_r} = 1 - F_{s_u} = \int_{s_1 \in S_r} f_1(s_1) ds_1$; and where the

indication that the expected value over all \tilde{s}_2 is considered does not alter the result. Similarly, let us define the covariances between $H'(y)$ and $x(p, \tilde{s}_1)$ conditional on $\tilde{s}_1 \in S_u$ and $\tilde{s}_1 \in S_r$

$$\text{cov}(H', x \mid \tilde{s}_1 \in S_u) = \int_{s_1 \in S_u} \int_{s_2} \left[H' - \frac{E_{s_u, s_2}\{H'\}}{F_{s_u}} \right] \cdot \left[x - \frac{E_{s_u, s_2}\{x\}}{F_{s_u}} \right] \frac{f_1(s_1)}{F_{s_u}} \cdot f_2(s_2) ds_1 ds_2 \quad (4.A.5)$$

$$\text{cov}(H', x \mid \tilde{s}_1 \in S_r) = \int_{s_1 \in S_r} \int_{s_2} \left[H' - \frac{E_{s_r, s_2}\{H'\}}{F_{s_r}} \right] \cdot \left[x - \frac{E_{s_r, s_2}\{x\}}{F_{s_r}} \right] \frac{f_1(s_1)}{F_{s_r}} \cdot f_2(s_2) ds_1 ds_2 \quad (4.A.6)$$

The definition used throughout section 4.5 is:

$$\text{cov}_{s_u, s_2}(H', x) = \text{cov}(H', x \mid \tilde{s}_1 \in S_u) \cdot F_{s_u} \quad (4.A.7)$$

$$\text{cov}_{s_r, s_2}(H', x) = \text{cov}(H', x \mid \tilde{s}_1 \in S_r) \cdot F_{s_r} \quad (4.A.8)$$

The result obtained in section (4.5), summarized by expression (4.51) is not altered if we use any of the two definitions shown above, that is $E\{x \mid \tilde{s}_1 \in S_u\}$ instead of $E_{s_u, s_2}\{x\}$, $\text{cov}(H', x \mid \tilde{s}_1 \in S_r)$ instead of $\text{cov}_{s_r, s_2}(H', x)$, etcetera; for all expectations and covariances taken. Notice also that the result in Lemma 4.1. can be written as $E\{H'(y) \mid \tilde{s}_1 \in S_u\} = E\{H'(y)\} = E\{H'(y) \mid \tilde{s}_1 \in S_r\}$ which makes obvious sense.

From the definition of expected profits (see expression (4.11') in the text, and using the above definitions we can write:

$$\bar{\pi}_p = E_{s_u, s_2}\{x + x_p \cdot (p - v)\} + E_{s_r, s_2}\{q\} \quad (4.A.9)$$

Similarly, we define

$$\bar{\pi}_{Hp} = E_{s_u, s_2}\{H' \cdot (x + x_p(p - v))\} + E_{s_r, s_2}\{H' \cdot q\} / E_{s_1, s_2}\{H'\} \quad (4.A.10)$$

Separating terms, and subtracting (4.A.9) from (4.A.10), using Lemma 4.1. some terms will disappear. Finally, using the definition of covariance given in (4.36) and (4.37) in the text, and since $\text{cov}_{s_u, s_2}(H', x) = 0$ we get expression (4.42).

Similarly we can write,

$$\bar{\pi}_q = E_{s_r, s_2}\{p - v\} - E_{s_1, s_2}\{\beta\} \quad (4.A.11)$$

$$\bar{\pi}_{Hq} = E_{s_r, s_2}\{H' \cdot (p - v)\} - E_{s_1, s_2}\{H' \cdot \beta\} / E_{s_1, s_2}\{H'\} \quad (4.A.12)$$

Exactly the same steps described above lead to expression (4.49).

Finally, to prove the sign of the covariances in (4.42) and (4.49) we can follow the same approach as that of the previous chapter. We want to show that

$$\text{cov}_{s_u, s_2}(H', v) > 0 \quad (4.A.13)$$

$$\text{cov}_{s_r, s_2}(H', v) > 0 \quad (4.A.14)$$

$$\text{cov}_{s_1, s_2}(H', \beta) > 0 \quad (4.A.15)$$

$$\text{cov}_{s_u, s_2}(H', x) = 0 \quad (4.A.16)$$

First, writing all functions directly in terms of the random variables we have $H'(\tilde{s}_1, \tilde{s}_2)$; $v(a, \tilde{s}_2)$; $\beta(a, \tilde{s}_2)$; $x(p, \tilde{s}_1)$. (We assume also $E\{\tilde{s}_1\} = E\{\tilde{s}_2\} = 0$ and $E_{s_1, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = E_{s_u, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = 0$.) Expanding these functions in Taylor series about $(0,0)$ and ignoring the terms that vanish, such as $H'_{s_1}(0,0)$; $H'_{s_1, s_2}(0,0)$; $v_{s_2, s_2}(a,0)$; $\beta_{s_2, s_2}(a,0)$ and higher order terms we can write:

$$H'(\tilde{s}_1, \tilde{s}_2) = H'(0,0) + H'_{s_2}(0,0) \cdot \tilde{s}_2 + H'_{s_1, s_2}(0,0) \cdot \tilde{s}_1 \cdot \tilde{s}_2 \quad (4.A.17)$$

$$v(a, \tilde{s}_2) = v(a,0) + v_{s_2}(a,0) \cdot \tilde{s}_2 \quad (4.A.18)$$

$$\beta(a, \tilde{s}_2) = \beta(a,0) + \beta_{s_2}(a,0) \cdot \tilde{s}_2 \quad (4.A.19)$$

$$x(p, \tilde{s}_1) = x(p,0) + x_{s_1}(p,0) \cdot \tilde{s}_1 \quad (4.A.20)$$

In order to prove (4.A.13), let us multiply (4.A.17) by (4.A.18) and take expectations over $\tilde{s}_1 \in S_u$ and all \tilde{s}_2 , to obtain

$$E_{s_u, s_2}\{H'(\tilde{s}_1, \tilde{s}_2) \cdot v(a, \tilde{s}_2)\} = H'(0,0) \cdot v(a,0) \cdot F_{s_u} + H'_{s_2}(0,0) \cdot v_{s_2}(a,0) \cdot \sigma_2^2 \cdot F_{s_u} \quad (4.A.21)$$

(since $E_{s_1, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = E_{s_u, s_2}\{\tilde{s}_1 \cdot \tilde{s}_2\} = 0$).

Moving the first term of the R.H.S. of (4.A.21) to the L.H.S. we have on the left of that expression:

$$E_{s_u, s_2}\{H' \cdot v\} - H'(0,0) \cdot v(a,0) \cdot F_{s_u} = E_{s_u, s_2}\{H' \cdot v\} - E_{s_1, s_2}\{H'\} \cdot E_{s_u, s_2}\{v\}$$

using Lemma 4.1. we have $= E_{s_u, s_2}\{H' \cdot v\} - \frac{E_{s_u, s_2}\{H'\} \cdot E_{s_u, s_2}\{v\}}{F_{s_u}}$

and finally using definition (4.37) $= \text{cov}_{s_u, s_2}(H', v)$

Thus from (4.A.21) we can write:

$$\text{cov}_{s_u, s_2}(H', v) = H'_{s_2}(0,0) \cdot v_{s_2}(a,0) \cdot \sigma_2^2 \cdot F_{s_u} > 0 \quad (4.A.22)$$

since all the terms are positive.

The proof of (4.A.14) is quite similar and it will be omitted. To prove (4.A.15) we multiply (4.A.17) by (4.A.19); take expectations over \tilde{s}_1, \tilde{s}_2 and arranging terms we obtain

$$\text{cov}_{s_1, s_2}(H', \beta) = H'(0,0) \cdot \beta_{s_2}(a,0) \cdot \sigma_2^2 > 0 \quad (4.A.23)$$

Finally to prove (4.A.16) we notice first that

$$E_{s_u, s_2} \{H'(\tilde{s}_1, \tilde{s}_2)\} = H'(0,0) \cdot F_{s_u} \quad (4.A.24)$$

$$E_{s_u, s_2} \{x(p, \tilde{s}_1)\} = x(p,0) \cdot F_{s_u} + E_{s_u} \{\tilde{s}_1\} \quad (4.A.25)$$

Multiplying (4.A.17) by (4.A.20) and taking expectations over $\tilde{s}_1 \in S_u$ and all \tilde{s}_2 , we obtain

$$E_{s_u, s_2} \{H' \cdot x\} = H'(0,0) \cdot x(p,0) \cdot F_{s_u} + H'(0,0) \cdot E_{s_u} \{\tilde{s}_1\}$$

using (4.A.24) and (4.A.25) we obtain

$$\text{cov}_{s_u, s_2} (H', x) = E_{s_u, s_2} \{H' \cdot x\} - \frac{E_{s_u, s_2} \{H'\} \cdot E_{s_u, s_2} \{x\}}{F_{s_u}} = 0 \quad (4.A.26)$$

CHAPTER V

ON MONITORING SYSTEMS, PERFORMANCE INDICATORS AND EFFICIENCY AUDITS

5.1. Introduction

In previous chapters we have studied a standard model of agency for a public enterprise assuming that the Manager's action or effort was completely unobservable and thus ruling out any form of monitoring. In practice however the monitoring technology will not be so underdeveloped and there will exist some attempts (although imperfect and costly) to monitor managerial activities. At a theoretical level the literature has dealt explicitly with the issue in particular since the contributions by Holmstrom (1979) and Shavell (1979). In this chapter we try to relate these and other treatments to two topics on the control of public enterprises, namely performance indicators and efficiency audits. These topics have been recently put by many writers and official statements in the U.K. at the forefront of the policies directed to achieve managerial efficiency in public firms. They are also part of the more general (and broadly defined) issue of the design of information systems used in the control of public firms. Here we shall attempt to discuss the rationale for the use of these mechanisms in the context of a well defined incentive-scheme system like the one treated in previous chapters.

One can identify in the literature at least three types of models or formulations concerned with monitoring in agencies. The first is best represented by the works of Harris and Raviv (1979), Holmstrom (1979) and Shavell (1979) in which the question addressed is when it will be valuable to introduce some costlessly available information on managerial activities, into an efficient (second best) incentive contract. In these models, the decision whether or not to include such available signal or monitor becomes unconditional, in the sense that its inclusion will not depend on any other event (e.g. the realization of a particular value of the outcome) but only on its overall valuation. Perhaps the most clear and useful characterization of this decision

is shown in Holmstrom's (1979) informativeness condition. Roughly speaking, a new or additional signal will be informative if it can tell the principal something more about the possible action chosen by the agent. Thus the important result of these models is that the demand for additional information (on the agent's activities) in an agency is non-trivial, that is, it must have positive value or be informative.

In section 5.3 below, we try to relate the informativeness condition to the general principles that must guide the selection of performance indicators once a well defined incentive scheme has been implemented. In other words, we ask: Is it valuable to introduce performance indicators into a given incentive contract in order to evaluate and compensate the Manager?

The previous models do not tell us how valuable a given monitor will be nor do they consider the cost of obtaining such an information. Thus, a second type of works have recognized explicitly the costs involved in the use of the monitoring technology and therefore suggested ways in which resources can be saved using it occasionally or rather 'conditionally'. Demski and Feltham (1978) and Baiman and Demski (1980a,b) are representatives of this line of inquiry. In their framework, evaluation systems become conditional upon a certain (extreme) realisation of performance by the firm where the expected benefits from calling for an investigation outweigh the costs.

Interpreting efficiency audits as costly investigation procedures to monitor managerial efficiency, it seems that the previous analysis can help us in the discussion of the general principles involving the design and operation of such audits. Thus section 5.4 below attempts to provide an introduction to such discussion.

It must be clear however, when we refer to efficiency audits, that we are talking about evaluation mechanisms that try to assess the level of managerial efficiency in the context of a perfect measurement of the outcome or performance. The distinction is relevant since this type of activity is usually known

in the literature as monitoring rather than auditing. The usual interpretation of the latter covers not only the evaluation of managerial efficiency but also and more fundamentally an assessment of the level of performance actually achieved. This comes as a result of imperfect observation of performance (e.g. the traditional distinction between reported and actual profits or in our context the inability to observe social benefits) and is the concern of the third class of models discussed. These models (examples are Evans (1980) and Balachandran and Ramakrishnan (1980)) recognize that auditing is made to assess the actual value of the outcome as well as the level of managerial efficiency, with both seen as joint products of the evaluation system. In this context, the efficiency of any incentive mechanism is obviously lower than in the case of perfect observation of the outcome and it approaches this case as the costs of auditing tend to zero.

In the analysis of section 5.4 however we shall interpret the term efficiency audit as one concerned with the investigation of managerial efficiency in a perfectly-measured-outcome context¹.

Before we deal with the questions of informativeness and performance indicators in section 5.3 and conditional information systems and efficiency audits in section 5.4 we shall discuss a reformulation of the treatment of incentive contracts, that we shall employ in the present chapter.

5.2. A Reformulation of the Problem: The 'Distribution Function' Approach

The formulation adopted in the two previous chapters followed what has been termed the 'state-space approach', in the sense that the random variables

¹The evidence available in the U.K. suggests that the meaning of efficiency audits has not been totally clarified in the sense that it involves attempts to evaluate managerial efficiency and to assess performance. This of course comes as a result of the well known difficulties of dealing with performance measurement in public firms and also as a result of more general informational asymmetries between Managers and Ministers. In recent years however the interpretations of efficiency audits have attached much of monitoring (even in the sense of regular monitoring) to the concept, see for example White Paper (1978).

representing uncertainty in the outcome function are treated explicitly, and all distribution functions are unconditional distributions of such random variables. This has been the approach followed in the early literature on agency (e.g. Spence and Zeckhauser, 1971; Ross, 1973; Stiglitz, 1974). In these models the relationship between outcome, the agent's action and the state of nature is considered explicitly, the sharing rule is assumed differentiable and the solution to the optimal non-linear sharing rule is obtained applying the calculus of variations.

This approach has been critically examined by Holmstrom (1978) who noticed that the assumption of the differentiability of the sharing rule employed by the approach may not be valid in a more general context. First, he extended the analysis of a point first made by Mirrlees (1974, p. 248) that there may exist no optimal solution for the class of unbounded sharing rules. For this reason one has to restrict the available sharing rules to a finite interval and in these conditions such optimal (restricted) sharing rule may become non-differentiable, invalidating the procedure used by the previous authors. Second, even if the optimal solution exists for unbounded sharing rules, this may become non-differentiable anyway, as an example by Gjesdal (1976) first showed¹.

Furthermore, and perhaps more relevant for our present analysis, while examining the characterization of the optimal non-linear sharing rule made by Ross (1973), Holmstrom recognized the weakness of the state-space approach in (explicitly) considering the impact of the form of the distribution of the random variable upon the optimal sharing rule. This criticism is very important because of the following fact. The distribution of the random variable can be seen as inducing another distribution in the outcome the principal is

¹In addition, we notice that the problem of uniqueness of the solution to the agent's problem first raised by Mirrlees (1975) and recently examined by Grossman and Hart (1983) is present in any of the formulations here discussed.

interested in. Since this distribution is parameterized in the action chosen by the agent, one can interpret the realisation of the outcome as giving some indication, in the traditional bayesian sense, of the actual level of effort chosen by the agent¹. In this sense, one would expect that the form of the distribution of the random variable would play a central role in the characterization of the optimal sharing rule, but the state-space formulation fails to make this distinction clear.

To overcome these difficulties, another approach originated in the works of Mirrlees (1974, 1976) and extended by Holmstrom has suppressed the random variable from the formulation of the problem and replaced its distribution by the distribution of the outcome parameterized in the agent's action. This method, which may be called the distribution function approach, has proved to be very robust in dealing with the statistical-decision like aspects commented above (see also Milgrom, 1981).

We notice however that our previous treatment can still be justified on two different grounds. First, as it has been recognized by Holmstrom (1978) there exist circumstances, particularly those in which the class of available sharing rules is restricted a priori (say for administrative reasons) to for example the linear class, where the state-space approach is the appropriate formulation to follow. Second, our treatment of the problem of efficient pricing made clear the need to take explicit account of the role of random variables associated with demand and cost shocks. This, obviously, would have become obscured suppressing these random variables as in the distribution function approach.

Nevertheless, in dealing with aspects of evaluating alternative information

¹This is an interpretation rather than a straightforward application of statistical decision theory, since the agent's action is not a random variable but a variable strategically chosen by an economic agent. Furthermore, given his knowledge of the structure of the problem, the Minister knows exactly what effort the agent will supply for a given sharing rule, see Holmstrom (1980).

and monitoring systems, the robustness of the distribution function approach suggests that we can benefit from translating our problem into this formulation. In particular this will allow us to make direct use of some central propositions derived under this framework. Also, in order to take full advantage of the methodology we shall work at a general level, studying nonlinear sharing rules, although we shall make references to the linear case whenever possible. Finally, since our prime interest is the study of the monitoring of managerial efficiency we shall not consider the decentralization of pricing or capacity decisions¹, leaving the treatment as one of pure production incentives. The resulting model is thus equivalent to the case of profit sharing with centralized pricing studied in previous chapters.

Let us define the outcome as social welfare $W = S(p) + \pi(p, a, \tilde{s})$. As before S is consumer surplus, π profits, p price of the unique good, a managerial effort and \tilde{s} the state of nature (which can be interpreted as cost uncertainty). The incentive contract offered to the Manager is summarized by the sharing rule $y(\pi, m)$ where m is a monitor or information signal that can be introduced into the contract if certain conditions described below are satisfied. We also assume that the sharing rule is bounded above and below $\underline{y} \leq y(\cdot) \leq \bar{y}$.

Define $F(\pi, m; a, p)$ as the joint distribution function of profits and the signal m , parameterized in managerial effort and the price level. (We shall suppress the random variable \tilde{s} and the functional relationship between π , a and p). We further assume as before that the derivative $\pi_a \geq 0$ implying that $F_a = \int \int f_a(\pi, m; a, p) d\pi dm \leq 0$. An increase in effort produces a shift to the right of the distribution function in the sense of first order stochastic dominance. Also we assume that f_a , f_{aa} , f_p and f_{ap} , the partial derivatives

¹The decentralization of pricing studied before however was rather artificial and clearly it does not make sense to monitor the price level since it is costlessly available.

of the joint p.d.f. of π and m , all exist and are well defined for every (π, m, a, p) . As before, the Minister is assumed risk neutral and thus maximizing the expected value of $W - y(\pi, m)$, i.e. welfare less managerial remuneration. The Manager is assumed risk averse with a separable utility function $U(y, a) = H(y) - V(a)$, with partial derivatives $H' > 0$, $H'' < 0$, $V' > 0$, $V'' > 0$. Finally, we assume that both Minister and Manager have homogeneous pre-contractual beliefs concerning the distribution of the state of nature \tilde{s} .

5.3. Informativeness and the Choice of Performance Indicators

Given the previous notation and assumptions we can write the Minister's problem as:

$$\begin{array}{l} \text{Max} \\ y(\pi, m), a, p \\ \underline{y} \leq y(\cdot) \leq \bar{y} \end{array} \quad S(p) + \iint [\pi - y(\pi, m)] \cdot f(\pi, m; a, p) d\pi dm \quad (5.1)$$

$$\text{subject to} \quad \iint H[y(\pi, m)] \cdot f(\pi, m; a, p) d\pi dm - V(a) \geq \hat{U} \quad (5.2)$$

$$\iint H[y(\pi, m)] \cdot f_a(\pi, m; a, p) d\pi dm = V'(a) \quad (5.3)$$

Expression (5.1) simplifies the Minister's objective function to the sum of (deterministic) consumer surplus and expected 'net' profits. Expression (5.2) reflects the constraint imposed on expected managerial utility given by employment opportunities elsewhere; this constraint is often called the 'rationality' constraint, showing that the Manager will not agree on the incentive contract unless a minimum (autarky) level of utility is guaranteed. Expression (5.3) is the first order condition of the Manager's choice of a , and it is usually called the self-selection, or incentive-compatibility constraint. We further assume that there exist restrictions in the form of the managerial utility function and the distribution function $F(\cdot)$ such that the agent's expected utility is concave in effort; thus validating the use of the first order condition approach to model the agent's choice of effort¹. The

¹Grossman and Hart (1983) provide a general treatment of the issue and specify these required restrictions.

Lagrangean function associated with the above problem can be written as

$$\begin{aligned}
 L = & S(p) + \iint \{ [\pi - y(\pi, m)] + \lambda \cdot H[y(\pi, m)] \\
 & + \mu \cdot H[y(\pi, m)] \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)} \} f(\pi, m; a, p) d\pi dm \\
 & - \lambda \cdot V(a) + \mu \cdot V'(a)
 \end{aligned} \tag{5.4}$$

Pointwise optimization of L with respect to $y(\pi, m)$ gives after some arrangements

$$\frac{1}{H'[y(\pi, m)]} = \lambda + \mu \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)} \quad \text{for all } \pi, m. \tag{5.5}$$

In addition we obtain two adjoint equations, for a and p respectively which can be written as

$$\begin{aligned}
 & \iint [\pi - y(\pi, m)] \cdot f_a(\pi, m; a, p) d\pi dm \\
 & + \mu \cdot \{ \iint H[y(\pi, m)] \cdot f_{aa}(\pi, m; a, p) d\pi dm - V''(a) \} = 0
 \end{aligned} \tag{5.6}$$

and

$$\begin{aligned}
 S_p(p) + & \iint \pi \cdot f_p(\pi, m; a, p) d\pi dm \\
 & + \lambda \cdot \{ \iint [H[y(\pi, m)] - \frac{y(\pi, m)}{\lambda}] \cdot f_p(\pi, m; a, p) d\pi dm \} \\
 & + \mu \cdot \iint H[y(\pi, m)] \cdot f_{ap}(\pi, m; a, p) d\pi dm = 0
 \end{aligned} \tag{5.7}$$

The characterization given by (5.5)-(5.7) is identical to the one shown by Holmstrom (1979) except for the addition of expression (5.7) which shows the pricing decision taken by the principal. Expression (5.5) shows the departure from optimal risk sharing¹ that has to be made to attend incentive problems. This departure is proportional to the ratio f_a/f provided that the multiplier μ is positive. This last result follows from Holmstrom's (1979) proposition 1, and it implies that in the second-best equilibrium the Minister would like to see the Manager supplying more effort than the one observed. The ratio f_a/f can be seen as the derivative of the maximum likelihood function $\log f$ with a

¹Optimal risk sharing is satisfied by the condition that the ratio of marginal utilities of income of principal and agent are equal to a constant, i.e. $1/H'[y(\pi, m)] = \lambda$, see Raiffa (1968).

as an unknown parameter (see, however, note 1 in p. 128 above). Taking this derivative as monotonically increasing in profits π we assume what has been termed the monotone likelihood ratio property (MLRP), (see Milgrom, 1981)¹. Under this interpretation the Minister infers the level of effort put by the Manager from the realization of the outcome π (see next section). In addition, and for a given level of profits, but under different contingencies signalled by m , the ratio f_a/f will be affected implying different remunerations for the Manager. For example, if for one value of m it is possible to infer less about a through the outcome π (i.e. f_a is smaller) than in the absence of the information signalled by m , then the deviation from optimal risk sharing must be correspondingly smaller.

Expression (5.7) must look familiar since it is the equivalent - under the present formulation - to expression (3.35) and describes the same effects commented in Proposition 3.1. The first two terms in (5.7) are respectively the derivatives of consumer surplus and expected profits, with respect to price. The second term is associated with the extra compensation that the Manager must receive after a change in price, to stay at the bargaining utility level. Finally, the third term reflects the effect of a change in price upon the equilibrium, self-selected effort.

Turning to the central issues of this section, we shall follow Holmstrom (1979) in defining a signal or monitor m as valuable, if when incorporated into a contract, i.e. $y(\pi, m)$, both Minister and Manager can be made better off than without the signal, i.e. relying on the contract $y(\pi)$. Thus, a signal m is said to be informative about the level of managerial effort if it is false that

$$f(\pi, m; a, p) = g(\pi, m) \cdot h(\pi; a, p) \quad (5.8)$$

This condition has a straightforward interpretation in terms of statistical decision theory since it is the condition that π is a sufficient statistic for the pair (π, m) with respect to managerial effort when this is seen as a random

¹The result that $\mu > 0$ however is obtained without relying on this condition.

variable.

The result obtained by Holmstrom (1979, Proposition 3) is that informativeness is a necessary and sufficient condition for a signal or monitor m to be valuable. The purpose of this section is to use this general principle to discuss the selection (in terms of value of information) of performance indicators - interpreted as monitors - for public firms once an incentive contract has been implemented. There is however a technical aspect of Holmstrom's result that we should mention. The 'necessary' part of the result follows for all types of contracts. However, restricting the class of available contracts beforehand for administrative reasons to for example linear ones the 'sufficiency' part of the result may not be valid in some circumstances. Since this point is rather technical and tangential to the purpose of this section we shall discuss it briefly in Appendix 5.A. For the rest of the section we shall assume that, if informative, a signal m will be valuable even under linear contracts¹. Notice however that in the discussion below we are more interested in detecting signals that fail to be informative and therefore we are relying on the 'necessary' part of the result.

In this context we have profits as the only outcome in which the Minister can rely to infer the level of managerial effort² and we consider additional signals in the form of performance indicators. Those indicators normally mentioned are measures of physical productivity, overhead costs, manning levels etc., thus we can take m as any of these measures. Nevertheless the standard model of agency used here is too abstract in one fundamental aspect: the variable managerial action or effort is a 'catch-all' concept representing probably numerous activities. Single-valuedness of a is normally assumed for

¹For this we implicitly assume that the magnitude of the information provided by m makes this possible.

²Consumer surplus is of course part of the 'outcome' insofar as the pricing decision is concerned, however for the purpose of enforcing managerial efficiency it does not help.

the sake of tractability. However, when it comes to the practical discussion of the usefulness of different specific indicators or signals, it really matters what is the actual form taken by those managerial activities we are trying to monitor. Thus we cannot properly evaluate specific indicators unless we specify in more detail the structure of the problem and in particular the form and nature of the managerial action. For this reason, we prefer to restrict the discussion to two specific examples. In the first, we illustrate a case in which a performance indicator takes the form of a simple measure of input productivity which is affected by a managerial input.

Example 5.1. Suppose profits take the following simple form

$$\pi(a, p, \tilde{s}) = p(x) \cdot x(a, \tilde{s}, L) - w \cdot L \quad (5.9)$$

where

$$x(a, \tilde{s}, L) = (r + a + \tilde{s}) \cdot L^\alpha \quad (5.10)$$

$x(\cdot)$ is the production function of the firm, L is some input and w the input price. $p(x)$ is the inverse demand function but we shall take p as given or already determined in the next discussion. r is a constant parameter and \tilde{s} is a random variable which for illustrative purposes is assumed to be normal with mean zero and variance σ_s^2 . Then profits are distributed as

$$\pi \sim N(p \cdot (r + a) \cdot L^\alpha - w \cdot L, p^2 \cdot L^{2\alpha} \cdot \sigma_s^2)$$

From (5.9) we can write, dividing by L , and arranging

$$m = \gamma \cdot \pi + \beta \quad (5.11)$$

where $m = (r + a + \tilde{s}) \cdot L^{\alpha-1}$ is the average productivity of L and $\gamma = 1/p \cdot L$,

$\beta = w/p$. m is taken as the monitor or performance indicator, and from (5.11)

$$E\{m\} = \gamma \cdot E\{\pi\} + \beta \quad (5.12)$$

$$\text{Var}\{m\} = \sigma_m^2 = \gamma^2 \cdot \sigma_\pi^2 \quad (5.13)$$

For notational convenience we shall denote π for (the variable) profits and $\hat{\pi}$ for the number 3.1416. Thus we can write the p.d.f. of π

$$h(\pi; a, p) = \frac{1}{\sigma_\pi \cdot (2\hat{\pi})^{1/2}} \cdot \exp \left\{ -[\pi - E\{\pi\}] / 2 \cdot \sigma_\pi^2 \right\} \quad (5.14)$$

using (5.11) we can write the p.d.f. of m as

$$k(m,a) = \frac{1}{\gamma \cdot \sigma_{\pi} \cdot (2\hat{\pi})^{1/2}} \cdot \exp \{ - [\pi - E\{\pi\}] / 2 \cdot \gamma \cdot \sigma_{\pi}^2 \} \quad (5.15)$$

or equivalently

$$k(m,a) = g(\pi,m) \cdot h(\pi; a,p) \quad (5.16)$$

where $g(\pi,m) = 1/\gamma = \pi/(m - \beta)$.

Since π and m are not two separate random variables we cannot define a joint density for (π,m) . Therefore, condition (5.8) cannot be strictly discussed. Instead, we have a deterministic relationship between π and m in the simple linear form (5.11). The densities of π and m , both induced by the same random variable \tilde{s} , are identical up to a monotonic transform given by γ . In this context m does not add any new information about a not already captured by π and one can loosely say that in this sense π is sufficient for (π,m) with respect to a . \square

This example illustrates a perhaps trivial but nevertheless simple case which I believe may occur in relation to some indicators: they come from the same structure (both stochastic and deterministic) that determines an already used measure of performance, such as profits. The example above assumes the most simple one input-one product case but the issue discussed may suffer no loss of generality. In multiproduct-multifactor contexts the usually derived Divisia indices are also closely related to profits (see Crew et al., 1979) and therefore one would expect a similar result. The next example illustrates a direct monitoring system.

Example 5.2. Suppose that profits take the form:

$$\pi(a,p,\tilde{s}) = (p - c) \cdot x(p) - q + a + \tilde{s} \quad (5.17)$$

c is marginal and average variable cost; managerial effort a is seen as activities that reduce the level of fixed costs below q . p is price, assumed given and $x(p)$ is demand. Suppose that the Minister can monitor the level of overhead costs and detect some overspending through a noisy observation of a , that is

$$m = a \cdot \tilde{r} \quad (5.18)$$

Further, assume that the random variables \tilde{s} and \tilde{r} are independent and normally distributed with mean 0 and 1 and variance σ_s^2 , σ_r^2 respectively. Then we can show that

$$f(\pi, m; a, p) = k(m, a) \cdot h(\pi, a, p) \quad (5.19)$$

In this context m is informative; we further have $\frac{f_a}{f} = \frac{k_a}{k} + \frac{h_a}{h}$ showing that m is used to infer the level of a in the same way the outcome is. High levels of overhead costs for example will indicate (independently of the level of profits) that overspending is high, that is, a is low. Notice that $k(m, a)$ is monotonically increasing in m and h is monotonically increasing in π since the normal distribution satisfies the MLRP. Furthermore suppose the Minister can separate the information provided by m into two disjoint sets M and M^c such that

$$k_a(M, a) / k(M, a) > k_a(M^c, a) / k(M^c, a) \quad \text{where}$$

$k(M, a) = \int_M k(m, a) dm$, $k_a = \int_M k_a(m, a) dm$ and the same corresponding expressions for $k(M^c, a)$ and $k_a(M^c, a)$. In this case the Minister can offer to the Manager a dichotomous contract whereby $y(\pi, M) > y(\pi, M^c)$ for all π , and he uses the information about m only qualitatively, that is just to know whether $m \in M$ or $m \in M^c$. (See also Appendix 5.A. for a reference to a linear contract.) In other words, despite being remunerated according to profits, the Manager gets an additional prize (penalty) if the level of overhead costs is below (above) a certain amount. \square

These two examples were not selected in order to exhaust the list of possible cases but to try to illustrate simple and general principles in the discussion of performance indicators. It seems clear that the informativeness condition obtained in the standard principal-agent model has some limitations when one wants to be more specific about a given problem of monitoring. First, the condition does not tell us how much valuable any given indicator will be nor

does it consider the cost of collecting the information. Second, it may not be sufficient even to obtain a positive valuation of a monitor in a context of a priori restricted contracts (cf. Appendix 5.A.). In addition the generality of the concept managerial action needs to be substantiated to give an indication of what sort of anomalies we are trying to monitor. Nevertheless, the discussion of this section can provide two teachings of a general character: First, as a prerequisite (necessity), informativeness puts an upper bound to the number of admissible indicators that should be considered for the purpose of monitoring. In terms of the actual decision of selecting these indicators this result may cast doubts (shown in example 5.1) on the efficacy of using a myriad of indices of physical productivity, most of them closely related to each other and to existing measures of performance such as profits. Second, it might be suggested that those indicators which concentrate on specific decisions such as overhead costs, manning levels, etc. can be more useful than general indices, in particular if there is a presumption that unobservable managerial activities are closely linked to those decisions

5.4. Conditional Information Systems and Efficiency Audits

Suppose the monitoring technology is such that the Minister can have access to a signal m about the Manager's action only at some positive cost K . This signal can be seen as the product of an investigation or audit mechanism that is costly. If the Minister takes the decision to launch the investigation, m is obtained and K is subtracted from the value of the outcome obtained. The gains are of course the information provided by m , in a sense similar to that discussed in the previous section. If, on the other hand, the Minister decides not to launch the investigation, a standard incentive scheme $y(\pi)$, i.e. based on the outcome only, will prevail and the Manager will be remunerated accordingly. How can the Minister design the monitoring mechanism represented by this investigation procedure? In particular, when will he take the

decision to investigate, what factors will influence such a decision and to what extent can this decision be used as a threat to influence managerial decisions? The general principles behind the answers to these questions can be of great help in the design and use of efficiency audits in public firms, interpreted as conditional monitoring systems to enforce managerial efficiency. Although the analysis adopted in this section will be restricted to the case where an optimal incentive scheme is implemented, it nevertheless may provide useful insights into questions that have remained unexplored at an analytical level.

Why informativeness is not enough in the present context? The informativeness condition tell us simply that whenever the ratio f_a/f is affected by the signal m , there exists a demand for such signal or information. It does not matter how much noisy m is since this can be eliminated making the sharing rule to depend on m marginally. Thus we eliminate risk effects associated with m and we get only the incentive effects, improving the welfare of both parties. Nevertheless, the presence of positive costs to obtain the signal makes this contract design insufficient to guarantee positive net benefits, since the costs may be higher than the marginal benefits obtained. Therefore, in the context of costly investigation procedures, we need to take into account in a more clear way the magnitude of the benefits associated with the use of m .

To begin with, notice that the characterization of the second best sharing rule when m is not used is given by

$$\frac{1}{H'[y(\pi)]} = \lambda + \mu \cdot \frac{f_a(\pi; a, p)}{f(\pi; a, p)} \quad (5.20)$$

We know that since the ratio f_a/f is generally different from zero for different realizations of π and since μ can be shown to be positive there are gains for departing from the first best (optimal risk sharing) situation. However, notice that since f_a/f is assumed increasing in the outcome π , these gains are

greater in the tails of the distribution of π . In other words, starting from the first best situation, characterized by $f_a = 0$, the losses associated with the failure to observe the Manager's action are greater for extreme realizations of π , making the value or demand for information about a higher at these points. Holmstrom (1979, p. 79) gave a clear benefit-cost interpretation to the ratio f_a/f :

"The characterization in [(5.20)] has an intuitive interpretation in terms of deviation from optimal risk sharing to provide incentives for increased effort on the part of the agent. This is accomplished by taking $[y(\pi) > y_\lambda]$ ¹ when the marginal return from effort is positive to the agent², and $[y(\pi) < y_\lambda]$ when it is negative... The incentive effect of deviating from optimal risk sharing is stronger the larger is $|f_a|$, and it is more costly (in terms of lost risk-sharing benefits) the greater is f . Thus $|f_a|/f$ may be interpreted as a benefit-cost ratio for deviation from optimal risk sharing and [(5.20)] states that such deviations should be made in proportion to this ratio, with individual risk aversion taken into account."

The next figure helps to illustrate the point and it is self-explanatory.

The foregoing discussion thus suggests that the benefits of investigating managerial activities would be higher for extreme realizations of the outcome π , since it is at these values where the departure from the first best situation is the largest one (see also Holmstrom, 1980). In other words, facing non-trivial costs of launching the investigation, the Minister would call for efficiency audits at very low or very high realizations of profits. A striking characteristic of this interpretation however is that it suggests a pattern (i.e. two-tail investigations) which is different from that most commonly thought in practice, namely that investigations are called when an extremely low (but not high) profit outcome is observed³. This however has been one of

¹ y_λ denotes the first best sharing rule, where $1/H'[y_\lambda] = \lambda$. The difference between y_λ and $y(\pi)$ mentioned in the text can be proved as a corollary of the proposition that $\mu > 0$, see Holmstrom (1979).

²This effect can be seen looking at the first order condition of the Manager's choice of a , such as expression (5.3) where the LHS is the marginal return from effort to the Manager and the RHS the marginal cost.

³In a similar fashion, sharing rules in the form of profit bonus schemes are commonly bounded from below but it does not appear that they are equally bounded from above.

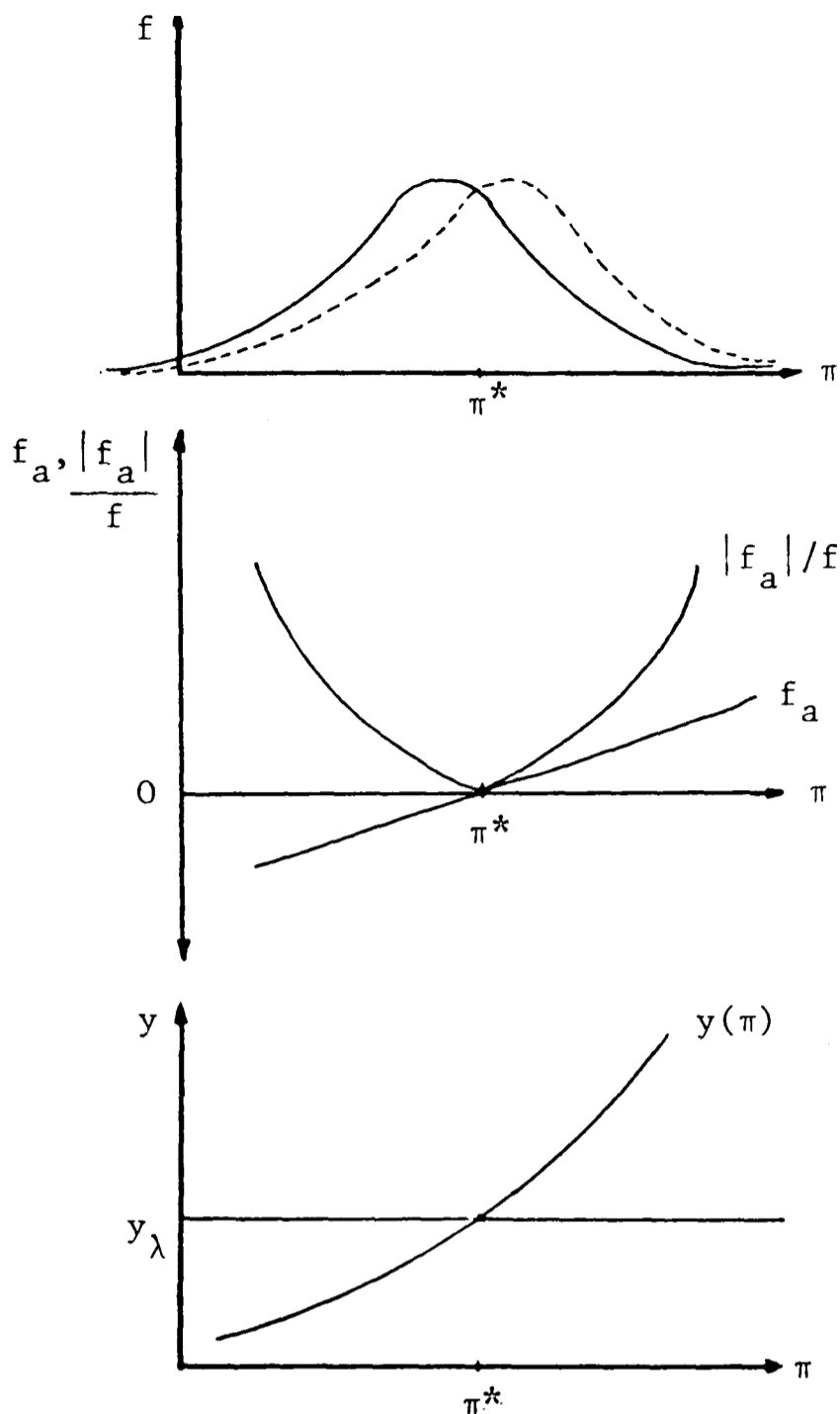
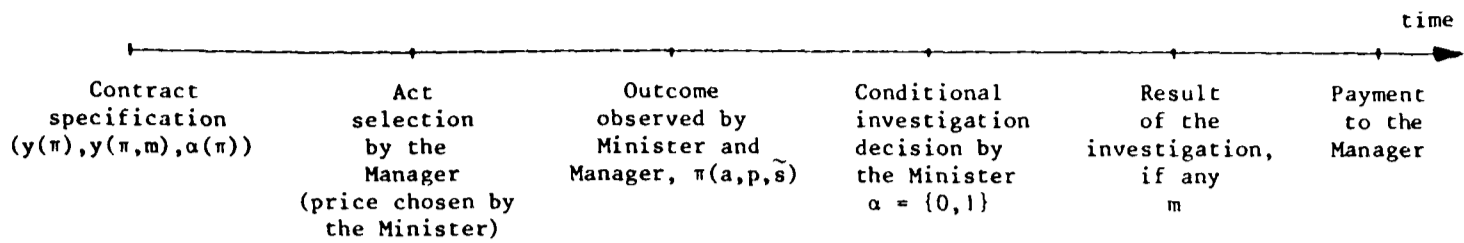


Figure 5.1.

the main issues studied in the literature, that is, under what circumstances an optimal investigation strategy will adopt a "lower-tail" (or upper-tail) form.

In order to study the form of optimal conditional investigation strategies we shall follow a formulation due to Baiman and Demski (1980a, b). We maintain all the features described at the end of section 5.2. except for the incentive contract which is now characterized by the triple $(y(\pi), y(\pi, m), \alpha(\pi))$. The decision to investigate is conditional on the value of profits observed; $y(\pi): \pi \rightarrow [\underline{y}, \bar{y}]$ gives the Manager's remuneration when no investigation is performed, $y(\pi, m): \pi \times M \rightarrow [\underline{y}, \bar{y}]$ his remuneration if the investigation is carried out and a signal m obtained. Finally $\alpha(\pi): \pi \rightarrow [0, 1]$ can be seen as the conditional probability of investigation given the observation of the outcome. The structure of the decision process associated with the administration of the incentive scheme can be depicted as follows:



The Minister's problem is now given by the following programme

$$\begin{aligned} \text{Max} \\ y(\pi), y(\pi, m) \in [y, \bar{y}] \\ \alpha(\pi) \in [0, 1] \\ a, p \end{aligned} \quad \iint \{ \alpha(\pi) \cdot [\pi - y(\pi, m) - K] \\ + [1 - \alpha(\pi)] \cdot [\pi - y(\pi)] \} f(\pi, m; a, p) d\pi dm \quad (5.21)$$

subject to

$$\iint \{ \alpha(\pi) \cdot H[y(\pi, m)] + (1 - \alpha(\pi)) \cdot H[y(\pi)] \} f(\pi, m; a, p) d\pi dm - V(a) \geq \hat{U} \quad (5.22)$$

$$\iint \{ \alpha(\pi) \cdot H[y(\pi, m)] + (1 - \alpha(\pi)) \cdot H[y(\pi)] \} f_a(\pi, m; a, p) d\pi dm = V'(a) \quad (5.23)$$

Since the resulting Lagrange function formed by (5.21)-(5.23) is linear in $\alpha(\pi)$, then this probability will be either zero or one (Baiman and Demski, 1980a, Proposition 3). The first order conditions with respect to $y(\pi)$, $y(\pi, m)$ and $\alpha(\pi)$ are, respectively (where λ and μ are the Lagrange multipliers associated with (5.22) and (5.23) respectively):

$$(1 - \alpha(\pi)) \cdot [-1 + \lambda \cdot H'[y(\pi)] + \mu \cdot H'[y(\pi)] \cdot \frac{\pi f_a(\pi; a, p)}{\pi f(\pi; a, p)}] = 0 \quad (5.24)$$

for all π

$$\alpha(\pi) [-1 + \lambda \cdot H'[y(\pi, m)] + \mu \cdot H'[y(\pi, m)] \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)}] = 0 \quad (5.25)$$

for all π, m

$$\int \{ y(\pi) - y(\pi, m) - K + \lambda \cdot H[y(\pi, m)] - \lambda \cdot H[y(\pi)] \} f(\pi, m; a, p) dm = 0 \quad (5.26)$$

$$+ \mu \cdot H[y(\pi, m)] \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)} - \mu \cdot H[y(\pi)] \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)} \} f(\pi, m; a, p) dm = 0$$

for all π

Expressions (5.24) and (5.26) result from pointwise optimization over π since the respective instruments are functions of π only; while in expression (5.25)

it is over (π, m) . Notice also that in (5.24) we have integrated over m since the resulting expression does not depend on the signal and therefore the condition is written defining the marginal densities ${}_{\pi}f(\pi; a, p) = \int f(\pi, m; a, p) dm$ and ${}_{\pi}f_a(\pi; a, p) = \int f_a(\pi, m; a, p) dm$.

If the investigation is not launched, i.e. $\alpha(\pi) = 0$, expression (5.25) vanishes and from expression (5.24) we obtain the characterization given in expression (5.20) above. On the other hand, if the Minister launches the investigation, i.e. $\alpha(\pi) = 1$, (5.24) vanishes and from (5.25) we obtain the characterization given in expression (5.5) of section 5.3. Thus, the central novelty of the model is represented by the decision of whether or not to call for an investigation, summarized in condition (5.26). From this, Baiman and Demski (1980b) derived the following benefit-cost characterization: $\alpha(\pi) = 1$ if, and only if,

$$B(\pi) = \int \left\{ y(\pi) - y(\pi, m) + \left[\lambda + \mu \cdot \frac{f_a(\pi, m; a, p)}{f(\pi, m; a, p)} \right] \cdot [H[y(\pi, m)] - H[y(\pi)]] \right\} f(\pi, m; a, p) dm \quad (5.27)$$

$$\geq K \cdot {}_{\pi}f(\pi; a, p) = C(\pi)$$

Where $B(\pi)$ and $C(\pi)$ are benefit and cost respectively of launching the investigation after a given value of π has occurred. On the other hand, $\alpha(\pi) = 0$ if, and only if, $B(\pi) < C(\pi)$. Finally, notice that substituting expression (5.5) into (5.27) allow us to write

$$B(\pi) = \int \left\{ y(\pi) - y(\pi, m) + \frac{1}{H'[y(\pi, m)]} \cdot [H[y(\pi, m)] - H[y(\pi)]] \right\} \cdot \frac{f(\pi, m; a, p)}{{}_{\pi}f(\pi; a, p)} dm \geq K \quad (5.28)$$

Baiman and Demski did not provide a detailed discussion of the terms forming $B(\pi)$ but we can see from expression (5.28) that these benefits are given by the expected value, in terms of m^1 , of the sum of the change in the Minister's

¹Notice that in (5.28) the ratio $f(\pi, m; a, p)/{}_{\pi}f(\pi; a, p)$ is the conditional probability of m for a given π . Since (5.27) can also be written in this way, the notation used by Baiman and Demski, $C(\pi)$ is not totally correct since the costs of the investigation do not depend on π but they are fixed at K .

utility (i.e. $y(\pi) - y(\pi, m)$, since π is given) and a term, in monetary units, reflecting the change in managerial utility, when $y(\pi)$ is substituted by $y(\pi, m)$ as a result of the investigation. Since the outcome of the investigation, i.e. the signal m , is informative about π there will be potential gains from using it to change managerial remuneration, but since m is also noisy there will be a loss due to a further departure from optimal risk sharing. It follows that the main factors affecting the magnitude of $B(\pi)$ will be the degree of accuracy (or noise) assumed for m ; the degree of risk aversion displayed by the Manager and finally the shape of the distribution $F(\pi, m; a, p)$ since this will affect the ratio f_a/f which in turn affects the weight $1/H'[y(\pi, m)]$ in (5.28). Baiman and Demski then move a step further by assuming independence between π and m and thus obtaining a characterization of the optimal investigation policy according to the degree of risk aversion assumed for the Manager. For instance, a 'lower-tail only' investigation strategy follows if the Manager is highly risk averse, since in this case - according to Baiman and Demski - a noisy investigation mechanism acts as a punishment and therefore the Manager will try to avoid low realizations of π by increasing his effort (see however Holmstrom's comments, 1980).

Since according to this discussion there exist more than one effect involved in the decision to call for an investigation we will find it convenient to explore a more simple and extreme situation in which the Minister can have access, at a cost K , to a perfect monitoring technology. This analysis can give a complementary perspective to the one provided by the previous authors since they seem to overemphasize the noise rather than the accuracy present in the investigation mechanism.

5.4.1. The Case of a Perfect Auditing Mechanism

By a perfect auditing mechanism we shall mean an investigation procedure that will give $m = a$, that is, it will signal the actual level of effort chosen

by the Manager. What are the costs and benefits of launching the investigation in this particular case?

Whenever the Minister can have access to the signal $m = a$ we know that the first best situation is attainable with a dichotomous contract of the following form

$$\begin{aligned} y &= y_\lambda && \text{if } a \geq a^* \\ &= \phi && \text{if } a < a^* \end{aligned} \quad (5.29)$$

where ϕ is a penalty and a^* is the first best level of effort. Since the ratio f_a/f in (5.5) becomes zero, since effort is observable, we can compare the marginal utilities $H'[y(\pi)]$ and $H'[y(\pi, m)]$ on the basis of the following equilibrium conditions

$$\frac{1}{H'[y(\pi)]} = \lambda + \mu \cdot \frac{f_a(\pi; a, p)}{f(\pi; a, p)} \quad (5.30)$$

$$\frac{1}{H'[y(\pi, m)]} = \begin{cases} \frac{1}{H'[y_\lambda]} = \lambda & \text{if } a \geq a^* \\ \frac{1}{H'[\phi]} = \psi & \text{if } a < a^* \end{cases} \quad (5.31)$$

where $\lambda > \psi$ and $\psi \rightarrow 0$ as a large penalty is imposed on the Manager. Since $m = a$ we can write $B(\pi)$ for the present case as:

$$\begin{aligned} B(\pi) &= \int_{\underline{a}}^{a^*} \{y(\pi) - \phi + \psi [H[\phi] - H[y(\pi)]]\} \frac{f(\pi; a, p)}{\pi f(\pi; p)} da \\ &+ \int_{a^*}^{\bar{a}} \{y(\pi) - y_\lambda + \lambda [H[y_\lambda] - H[y(\pi)]]\} \frac{f(\pi; a, p)}{\pi f(\pi; p)} da \end{aligned} \quad (5.32)$$

where \underline{a}, \bar{a} are the lower and upper values of the set of managerial actions, and we define $\pi f(\pi; p) = \int_{\underline{a}}^{\bar{a}} f(\pi; a, p) da$.

The characterization given by (5.32) has the following interpretation.

First, notice that $\gamma(\pi) = \int_{\underline{a}}^{a^*} \frac{f(\pi; a, p)}{\pi f(\pi; p)} da$ and $1 - \gamma(\pi) = \int_{a^*}^{\bar{a}} \frac{f(\pi; a, p)}{\pi f(\pi; p)} da$

can be seen as the subjective conditional probabilities (from the Minister's viewpoint) of $a < a^*$ and $a \geq a^*$ respectively, for a given realization of π .

Since, by assumption the density $f(\pi; a, p)$ satisfies the monotone likelihood ratio property (MLRP), these conditional probabilities are strongly affected by the particular realisation of π : the higher (lower) is π the lower (higher) is the conditional probability $\gamma(\pi)$.

Thus $B(\pi)$ in (5.32) can be seen again as the expected value of the changes in utility for both parties. Each term within curly brackets shows these changes in the two different sets of values of a ($a \leq a^*$ and $a \geq a^*$) and it is weighted by the subjective probability of that set of values. Notice that the change in managerial utility is weighted differently depending on the value of a in relation to the first best a^* .

In order to give a more precise discussion of the factors affecting the decision to call for an investigation and the form of the optimal investigation policy we can identify three factors affecting the magnitude of $B(\pi)$: 1) The size of the penalty ϕ , 2) the degree of risk aversion displayed by the Manager, and 3) the shape of the probability distribution $F(\pi; a, p)$. We will now discuss each one of these factors in turn.

An increase in the penalty associated with finding the Manager responsible for a low realisation of profits, that is a reduction in the algebraic value of ϕ , will increase the benefits $B(\pi)$, that is

$$\frac{\partial B(\pi)}{\partial \phi} = \{-1 + \psi \cdot H'[\phi] + \frac{\partial \psi}{\partial \phi} \cdot [H[\phi] - H[y(\pi)]]\} \cdot \gamma(\pi) > 0 \quad (5.33)$$

since $\psi \cdot H'[\phi] = 1$ according to (5.31) and $\frac{\partial \psi}{\partial \phi} = -H''[\phi]/H'[\phi]^2 > 0$ and $H[\phi] - H[y(\pi)] < 0$. Notice that since the probability $\gamma(\pi)$ is decreasing in π , the derivative in (5.33) will be higher for lower rather than upper realisations of π . In addition the relative value of ϕ with respect to the values adopted by $y(\pi)$ over the subset $\pi^- = \{\pi: f_a/f < 0\}$, (see Figure 5.1), is important for the verification of a lower-tail investigation policy. Suppose, as it is likely to be the case in practice, that for extremely low realisations of π , the lower bound \underline{y} is reached (although one can construct cases in which this

will never occur, e.g. see the example in Holmstrom, 1979). In this case, the difference between y and ϕ becomes important for the verification of a lower tail strategy as the next result illustrates.

Proposition 5.1. : Suppose there exists a value $\pi_1 \in \pi^-$ for which the lower bound \underline{y} is reached. If $\phi = \underline{y}$ then we shall not observe a lower tail investigation strategy being followed.

Proof: By the definition of lower tail investigation we have that there must exist an outcome π_0 such that whenever $\pi \leq \pi_0$ then $\alpha(\pi) = 1$. By assumption, there exists an outcome $\pi_1 \in \pi^-$ such that

$$\frac{1}{H'[\underline{y}]} = \frac{1}{H'[y(\pi_1)]} = \lambda + \mu \cdot \frac{f_a(\pi_1; a, p)}{f(\pi_1; a, p)}$$

and \underline{y} is the managerial payment for all $\pi \leq \pi_1$, under the incentive scheme $y(\pi)$. From (5.32) and since by assumption $\phi = \underline{y}$ we have that the first term disappears for all $\pi \leq \pi_1$ and

$$B(\pi_1) = \{\underline{y} - y_\lambda + \lambda \cdot [H[y_\lambda] - H[\underline{y}]]\} \cdot [1 - \gamma(\pi_1)]$$

The term within curly brackets is independent of π and if it is zero or negative the result follows trivially. If it is positive we notice that $1 - \gamma(\pi)$ is increasing in π , so $B(\pi) < B(\pi_1)$ for all $\pi < \pi_1$. Since $B(\pi)$ decreases with π for $\pi \leq \pi_1$ we cannot find an outcome $\pi_0 \leq \pi_1$ such that $B(\pi_0) = K$ and $B(\pi) > K$ for all $\pi < \pi_0$.

Q.E.D.

The importance of this result will of course depend on the verification that the optimal sharing rule $y(\pi)$ is bounded below. If this is the case, Proposition 5.1. has a clear implication insofar as the required size of the penalty ϕ is concerned, that is we need a penalty which can be set below the minimum salary of the Manager under normal conditions. In some models, \underline{y} is taken as minus the wealth of the Manager and therefore it seems difficult in this context to justify that ϕ can be lower than this amount. In practice however, \underline{y} will be given probably by some institutional constraint (in the

form of minimum wage laws, etc.) and in this context the penalty for finding the Manager cheating can be higher than the penalty represented by \underline{y} . It might be possible that ϕ is substantially lower than \underline{y} , in terms of loss of job, reputation and employment opportunities elsewhere¹.

In order to discuss the role of managerial risk aversion, let us differentiate (5.30) with respect to π , to obtain after some arrangements

$$y'(\pi) = \frac{\mu}{R_A} \cdot H'[y(\pi)] \cdot \frac{\partial}{\partial \pi} (f_a/f) \quad (5.34)$$

where $R_A = -H''[y(\pi)]/H'[y(\pi)]$ is the Arrow-Pratt measure of absolute risk aversion. Expression (5.34) shows that the slope of the optimal sharing rule $y(\pi)$ will depend on the change of f_a/f with respect to π (which is positive due to the MLRP) and on the degree of absolute risk aversion, for given $H'(\cdot)$ and μ . It also shows that $y'(\pi)$ will depend on π , so linear sharing rules are clearly suboptimal (except for very special cases).

The greater R_A is the lower will be the slope of $y(\pi)$ for a given π and therefore the smaller its departure from the first best schedule given by y_λ . This implies, intuitively, that for any outcome $\pi \in \pi^- = \{\pi: f_a/f < 0\}$ the lower will be the increase in income that the Minister will have to grant to the Manager in the event that $m = a \geq a^*$ is shown by the investigation. In addition, the difference between $y(\pi)$ and the penalty ϕ widens, increasing also the potential gains the Minister has in finding the Manager cheating. Thus these effects will make, ceteris paribus, $B(\pi)$ larger for all $\pi \in \pi^-$. A similar argument can show that for all $\pi \in \pi^+ = \{\pi: f_a/f > 0\}$, $B(\pi)$ will be reduced. These effects however ignore the changes in managerial utility taking place which go in the opposite direction (see expression (5.32)). However,

¹There is a well known result in the literature, due to Mirrlees (1974), which establishes that if huge penalties are available, the first best situation can always be attained. This result is also present in this model, since if we can make $\phi \rightarrow -\infty$ then, for non prohibitive costs K , the investigation will always be called.

if the Manager is risk averse (and as he becomes more risk averse) changes in managerial utility of income will be lower for those $\pi \in \pi^+$, where income is relatively high, than for those $\pi \in \pi^-$ where it is relatively low. This reasoning implies that we should see a reduction in $B(\pi)$ for $\pi \in \pi^+$ as the Manager becomes more risk averse, but it is less clear about the effect for the subset π^- . The next proposition however makes the analysis neater since it also considers the role of the probabilities $\gamma(\pi)$ and $1 - \gamma(\pi)$ neglected in the above discussion.

Proposition 5.2. : The value of the benefits $B(\pi)$ becomes lower over π^+ and higher (at least for extreme realisations of π) over π^- as the Manager becomes more risk averse.

Proof: Let $H[y(\pi)]$ denote the initial utility (of income) function of the Manager. Let us define a concave transform g of $H[y(\pi)]$, $g: H[y(\pi)] \rightarrow H_T[y_T(\pi)]$. Associated with $H_T(\cdot)$ there will be another optimal sharing rule denoted by $y_T(\pi)$. g is defined so that it has the following properties:

$$H[y(\pi^*)] = H_T[y_T(\pi^*)] \tag{5.35}$$

$$\text{and } H[y(\pi)] > H_T[y_T(\pi)] \quad \text{For all } \pi \neq \pi^*$$

where π^* is the point where $f_a/f = 0$ (see Figure 5.1). The next figure illustrates the relationship between $H(\cdot)$ and $H_T(\cdot)$.

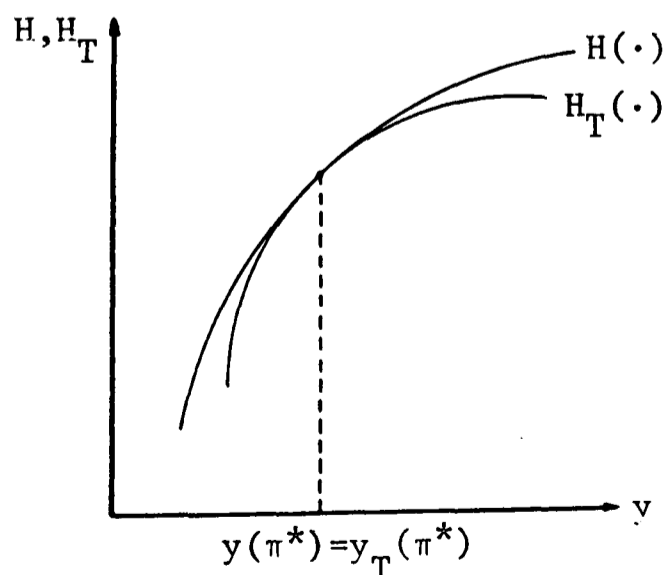


Figure 5.2.

Thus, it follows that

$$H' [y(\pi)] \begin{cases} \geq \\ = \\ < \end{cases} H'_T [y_T(\pi)] \quad \text{as} \quad \pi \begin{cases} \leq \\ = \\ > \end{cases} \pi^* \quad (5.36)$$

Using the equilibrium condition in (5.30) it can be shown that (since λ , μ , and f_a/f are unaltered)

$$y(\pi) \begin{cases} > \\ = \\ < \end{cases} y_T(\pi) \quad \text{as} \quad \pi \begin{cases} > \\ = \\ < \end{cases} \pi^* \quad (5.37)$$

Thus we have that after the transformation g , $y(\pi)$ increases for all $\pi \in \pi^-$ and decreases for all $\pi \in \pi^+$. To see the impact of this change on $B(\pi)$ let us define, from (5.32),

$$X(\pi) = y(\pi) - \phi + \psi \cdot [H[\phi] - H[y(\pi)]] \quad (5.38)$$

$$Z(\pi) = y(\pi) - y_\lambda + \lambda \cdot [H[y_\lambda] - H[y(\pi)]] \quad (5.39)$$

thus we can write

$$B(\pi) = X(\pi) \cdot \gamma(\pi) + Z(\pi) \cdot (1 - \gamma(\pi)) \quad (5.40)$$

Since the densities $f(\pi; a, p)$ and ${}_\pi f(\pi; p)$ are unaltered after g , so are the probabilities γ and $1 - \gamma$. Notice that when $\pi \in \pi^+$ we have $f_a/f > 0$ and using (5.30) and (5.31) we can sign

$$\frac{\partial X(\pi)}{\partial y(\pi)} = 1 - \psi \cdot H' [y(\pi)] > 0 \quad (5.41)$$

$$\frac{\partial Z(\pi)}{\partial y(\pi)} = 1 - \lambda \cdot H' [y(\pi)] > 0 \quad (5.42)$$

Therefore the reduction from $y(\pi)$ to $y_T(\pi)$ reduces both $X(\pi)$ and $Z(\pi)$, and therefore $B(\pi)$, for all $\pi \in \pi^+$.

When $\pi \in \pi^-$ we have that (5.41) is still valid but (5.42) has the opposite sign, i.e. $\partial Z(\pi)/\partial y(\pi) < 0$. Thus the increase in $y(\cdot)$ from $y(\pi)$ to $y_T(\pi)$ increases $X(\pi)$ but reduces $Z(\pi)$. However, notice that from (5.40) we can write

$$\begin{aligned} \frac{\partial B(\pi)}{\partial y(\pi)} &= \left[\frac{\partial X(\pi)}{\partial y(\pi)} - \frac{\partial Z(\pi)}{\partial y(\pi)} \right] \cdot \gamma(\pi) + \frac{\partial Z(\pi)}{\partial y(\pi)} \\ &= (\lambda - \psi) \cdot H' [y(\pi)] \cdot \gamma(\pi) + 1 - \lambda \cdot H' [y(\pi)] \\ &= 1 - \eta \cdot H' [y(\pi)] \end{aligned} \quad (5.43)$$

where $\eta = \gamma(\pi) \cdot \psi + (1 - \gamma(\pi)) \cdot \lambda$. Since $\psi < \frac{1}{H'[y(\pi)]} < \lambda$ whenever $\pi \in \pi^-$ (i.e. $f_a/f < 0$, and using expressions (5.30) and (5.31)); and since $\gamma(\pi)$ will be high ((1 - $\gamma(\pi)$) low) when $\pi \in \pi^-$, we shall have, at least for extreme realisations of π , $\eta < 1/H'[y(\pi)]$, implying that (5.43) is positive.

Q.E.D.

As an extreme but interesting case, suppose the Manager becomes hugely risk averse, implying that no bonus scheme can be used and thus $y_T(\pi) = y_\lambda$. Then, from (5.32), we obtain

$$B(\pi) = \{y_\lambda - \phi + \psi[H[\phi] - H[y_\lambda]]\} \cdot \gamma(\pi) \quad (5.44)$$

since the term within curly brackets does not depend on π , and $\gamma(\pi)$ is decreasing in π , so is $B(\pi)$. This implies that if there is going to be an investigation¹, this must necessarily take a lower-tail form.

More generally, the result stated in Proposition 5.2. provides a useful description of the effect of managerial risk aversion on the form of the investigation policy. It shows that if managerial bonuses have to be a small percentage of managerial income due to the inability of the Manager to take risks, then the chance that the investigation strategy will be lower-tail increases. A similar effect was obtained by Baiman and Demski (1980b) in the context of a noisy investigation mechanism.

Finally, it is clear from expressions (5.30) and (5.32) that the actual shape of the probability distribution $F(\pi; a, p)$ will also play a role in determining the value of $B(\pi)$ over π^- and π^+ . This effect will probably depend on the particular form that this distribution will adopt and we have not been able to obtain results of the same level of generality as the previous ones. We notice however two effects of a change in the shape of the density $f(\pi; a, p)$ on the value of $B(\pi)$. First, from expressions (5.30) and (5.34)

¹For this to be verified however we first need $B(\pi) > 0$ and this will occur, in the present example, only if $\psi \approx 0$ that is only if a very large penalty ϕ is available. Thus, with very large penalties available and if the Manager is extremely risk averse the optimal investigation adopts a lower tail form.

there will be a change in the ratio f_a/f that will affect the form of the sharing rule $y(\pi)$. Second, there will be a change in the conditional probabilities $\gamma(\pi)$ and $1-\gamma(\pi)$. The final effect will thus depend on the sign and magnitude of these two effects.

5.5. Conclusions

In this chapter we have attempted to integrate the issues of the selection of performance indicators and the design of efficiency audits into the general framework of managerial incentive schemes discussed previously. The analysis has been made in a highly stylized context and it should be seen only as an introductory step towards a more well-structured and policy-relevant modelling of the issues. However, we believe that some interesting insights, that can guide general principles used in the design of monitoring mechanisms, have been obtained in the course of the analysis.

First we have shown that there must exist, from the viewpoint of the value of information, an upper bound in the number of admissible performance indicators and that some broadly used or advocated measures such as productivity indices may be redundant in some circumstances. In addition, those indicators that concentrate on specific decisions which are presumed to be closely related to unobservable managerial activities can be more useful from an informational viewpoint.

Second, a central principle behind conditional information mechanisms is that since investigations are likely to be costly in practice we must save resources using them only in specific circumstances. These circumstances are determined according to expected benefits and costs. Thus, following this result we should not observe excessive, day-to-day intervention into the 'affairs' of the public firm, but deep investigations only when an extreme outcome occurs. Notice that the justification for this type of policy is not based on the "excessive-intervention-is-bad-for-management" argument used in

the debates reflected by official papers and previous specialized writings in the United Kingdom. Instead, it results from a clear expected cost-benefit approach on the use of an information system.

In section 5.4.1. we studied a perfect auditing mechanism, concentrating on the factors that affect the optimal form of such a mechanism. We have shown (Proposition 5.1.) that if the Manager is 'safe' from potential penalties that cannot be made effective due to minimum wages or other limited liability institutions, then it will not be beneficial to adopt a lower-tail investigation strategy. The implication of this result for the design of an investigation mechanism is that investigations must necessarily carry an implementable punishment in case the Manager is found responsible for low profits. In addition, we have shown (Proposition 5.2.) that as the Manager is less able to take risks and thus to accept contingent payment systems the optimal investigation tends towards the lower-tail form. Thus, designing lower-tail investigation procedures can be an optimal policy under these circumstances.

Appendix 5.A.

Here we shall briefly discuss why informativeness may not be sufficient for getting a positive value of a monitor when the sharing rule is restricted to the linear class. The strategy followed by Holmstrom (1979) to prove the result under a general sharing rule is (a) to assume that m is informative and to introduce a partition in the set of values of m , M and M^C , such that the ratio f_a/f is higher, on aggregate, in M than in M^C ; (b) to choose an additive variation $\delta y(\pi, m)$ in the sharing rule $y(\pi)$, such that $\delta y(\pi, m)$ is constant for all $m \in M$ and for all $m \in M^C$ and satisfies the condition that it equals zero integrated over m , for each given π, a, p ; and finally (c) to show that the change in the expected utility of the agent, induced by $\delta y(\pi, m)$, is zero while the corresponding change for the principal is positive.

The problem with linear sharing rules is that the step (b) above may not be satisfied since it may be impossible to find an additive variation satisfying that condition. In notation, this condition (expression (21) in Holmstrom, 1979) is

$$\delta y(\pi, M) \cdot f(\pi, M; a, p) + \delta y(\pi, M^C) \cdot f(\pi, M^C; a, p) = 0 \quad \text{For all } \pi, a, p \quad (5.A.1)$$

where

$$f(\pi, M; a, p) = \int_{m \in M} f(\pi, m; a, p) dm$$

$$f(\pi, M^C; a, p) = \int_{m \in M^C} f(\pi, m; a, p) dm$$

Let us define the marginal p.d.f. of π ,

$$\int_m f(\pi, m; a, p) dm = \int_{\pi} f(\pi; a, p) = \int_{\pi}^M f(\pi; a, p) + \int_{\pi}^{M^C} f(\pi; a, p)$$

thus (5.A.1) can be written as

$$\delta y(\pi, M) \cdot \int_{\pi}^M f(\pi; a, p) + \delta y(\pi, M^C) \cdot \int_{\pi}^{M^C} f(\pi; a, p) = 0 \quad \text{For all } \pi, a, p \quad (5.A.2)$$

By construction $\delta y(\pi, M)$ is constant over M and $\delta y(\pi, M^C)$ constant over M^C . By

linearity the admissible $\delta y(\pi, M) = \delta \alpha_1(M) \cdot \pi + \delta c(M)$ and $\delta y(\pi, M^c) = \delta \alpha_1(M^c) \cdot \pi + \delta c(M^c)$ must be linear in π . In other words $\delta \alpha_1(M)$, $\delta \alpha_1(M^c)$, $\delta c(M)$ and $\delta c(M^c)$ must be constant over π . For this reason it may be difficult to find an additive variation in a linear contract such that (5.A.2) holds for all π . Under non-linear sharing rules we work as if we have a different α_1 and c for each π and thus things are easy. Therefore, if (5.A.2) is not achievable the expected value of the additive variation will be non-zero and there will be non-trivial effects on the utility of the Manager due to risk aversion.

CHAPTER VI

CONCLUSIONS

6.1. Introduction

The design of a system of incentives and control in any big organization is a complex task involving many aspects: the definition of objectives and the agreed measure of performance; the determination of hierarchical structures and the optimal degree of delegation; the design of the underlying information or monitoring system and finally of the rewards and penalties embedded into the labour contracts offered to decision takers. By making simplifying assumptions we were able to start this thesis concentrating the analysis on the last aspect, i.e. the design of managerial incentive schemes for public enterprises. Although the conclusions of the previous chapters should be taken within the limited scope of the work, we believe that the results will be more clearly interpreted and evaluated if we can frame them in relation to the more general question of control in public enterprises. For this purpose, throughout the sections of this chapter we shall make reference to some available evidence on the system of control of public firms in the United Kingdom. This will help to put into perspective the conclusions drawn from the previous chapters vis-à-vis general aspects of the policy of control and regulation of decisions in public enterprises.

At the outset, however, we do not claim that the results obtained in the course of this investigation have a direct application to the existing framework of control operating in the U.K. Firstly, the emphasis put in the previous chapters has been theoretical rather than empirical since we have not looked for results of immediate practical applicability and the empirical properties of the schemes studied were not considered. Secondly, as the discussion of the next section will illustrate, there are in practice other more central aspects of policy that must be first clarified - in order to obtain a coherent system

of control - before we can consider the applicability of incentive schemes for Managers. Nevertheless, it is within the discussion of the design of the system of managerial incentives and control, and the institutional changes required, that the results of the thesis can be valuable for future policy debates as well as for further research into the area.

Section 6.2. will largely provide the referring policy framework. Discussing the different views on the system of control in the U.K. we observe some failure of the official reports to take into account the asymmetries of information and preferences that are at the heart of the problem of control, and therefore to make explicit the regulatory aspects of public enterprise decisions. Section 6.3. comments on the empirical evidence that suggests that managerial labour contracts in public firms are poorly designed and then goes on to summarize the results of Chapters II, III and IV on the design of linear managerial bonus schemes and their implications. Section 6.4. considers the issue of control and regulation through performance indicators and efficiency audits criticising some recent institutionally orientated literature and summarizing the results obtained in Chapter V. Finally section 6.5. looks at some directions for further research into the area.

6.2. Some Views on the Design of the System of Control of Public Enterprises in the United Kingdom

The lack of clarity of the statutes of nationalisation in the U.K. concerning the objectives of the public corporations is often quoted as the starting point of an evolving process of reforms in the control system, characterized by the absence of a well defined set of objectives. There exist, however, at least two aspects of the original conception of the public corporation that are important for our present discussion. The first is the idea of the creation of an organization operating outside the central government administration. The second is that elements of economic efficiency (although vaguely defined) should occupy a primary place in the objectives of the corpora-

tions. The fact that these organizations are productive units orientated to the market (at least in the sense of the commercialization of products and services) was seen as a predominant element in the determination of organizational structure, objectives and control system.

The absence of such a well defined system of control from the beginning obviously led to a period in which interpretations of the statutes was made freely by Ministers, sponsoring departments and corporations, creating a fairly centralized and interventionist system of control without clear policy objectives. This situation was then modified with the introduction by the Government of three White Papers (1961, 1967 and 1978) trying to establish a clear set of economic and financial guidelines that should be followed by the industries. Along with these documents there has been two other major reports by the Select Committee of Nationalised Industries (SCNI, 1967/68) and the National Economic Development Office (NEDO, 1976) that dealt mainly with the design of the institutional aspects of the system of control¹.

One could identify, reading the evidence presented by these documents, at least three views on the institutional framework that should guide the control of public enterprises (Rees, 1979a). The first, which has been the official framework for the last decades (and has been confirmed by the last White Paper) makes the public enterprise accountable to a sponsoring department or Ministry which in turn is accountable to Parliament. In addition to this relationship there is a strong element of control coming from the role of the Treasury as a financial controller as well as proposer of economic guidelines. Moreover, other organisms such as the (now dissolved) SCNI, special parliamentary subcommittees, the Monopolies and Mergers Commission, etc., can participate in the control mechanism.

¹This separation is not exact since these reports also dealt explicitly with economic and financial guidelines while the White Papers - particularly the 1978 one, in reply to NEDO - also touched aspects of institutional design.

The criticisms to this framework have centered traditionally on the problems of excessive ministerial intervention, confusion of objectives and responsibilities, etc. The main point however is that under this system there seems to be many principals acting at the same time. Disagreements and different personal interpretations of policies have in this context reduced the accountability of the enterprises. Thus the issue is not only that there are too many objectives but rather that there exist many principals. In addition, the regulatory aspects of the economic guidelines for the sector have never been properly addressed, although we notice below some changes in the rules over the years that are related to their ability to control decisions. To some critics, the economic rules introduced by the 1967 White Paper are unlikely to be attended seriously under this regime where Ministers exercise discretion over a multiple objective policy.

The second view, which originated as a criticism to the previous one, was put forward by the SCNI (1967/68). According to this report, there was a need for a radical institutional change that would solve the problems derived from the multiplicity of principals and objectives. The recommendation of the SCNI (i.e. the creation of a Ministry for Nationalised Industries) can be seen therefore as a serious attempt to define a principal with clear objectives who would take over the control of public enterprises in the U.K.. In addition, the effort put by the SCNI to organize the objectives of the public enterprises giving priority to economic efficiency made it a very coherent document.

Nevertheless although capable of understanding the problems of having too many principals and too many objectives, the SCNI failed to notice some fundamental aspects of the process of control. They overstressed the view that public enterprises would operate in a "largely self-regulating" system if proper decentralization of decisions under the proposed guidelines could take place (e.g. Chapter V, para 200 op.cit.). They did not consider the regulatory

aspects of those guidelines and did not deal with the question of incentives for decision takers. In sum, the SCNI's report made a very good contribution to the debate about the appropriate control system but its main deficiency lies in the misunderstanding of the asymmetries of preferences and information that undermine the sort of delegated choice system proposed¹.

The third view, put forward by NEDO's extensive study of the sector, was again conceived as a criticism of the traditional framework but it was based on a different interpretation of the problems and of the solutions needed. Although the diagnosis of the problem also stressed excessive ministerial intervention, NEDO presented as an alternative the idea of a 'policy council' at each enterprise level, that would unificate the views of the different interest groups involved in the sector. This council would therefore obtain a 'consensus' for an agreed strategy towards public enterprise policy acting as a buffer to avoid direct ministerial interference into the management of the enterprises.

However there are reasons to criticize this proposal on the basis that far from solving the lack of agreed policies its implementation could complicate the situation still further². In fact, it seems as if NEDO moved in the opposite direction to that followed by the SCNI, introducing even more principals into the control process. In addition, and unlike the SCNI, NEDO tended to relegate to a second plane argument for economic efficiency and replaced them by indicators of performance of diverse character (see also section 6.4. below). In short, NEDO's proposals are based on the view that administrative failure is the basis of the control problem in public firms, stressing therefore administrative procedures and neglecting central principles of economic

¹Foster (1971) has provided a different but nonetheless robust critique based on the political and bureaucratic process of decision making within the ministerial and parliamentary system.

²This view was apparently shared by the White Paper (1978) who rejected the main proposal.

regulation.

The definition of objectives and the corresponding agreement on and measurement of performance is clearly a key or primary element in any system of control. One could say that any other refinement of the process of control let it be improving monitoring devices, compensation systems, etc., all suffer from an institutional-second-best type of failure, in the sense that unless minimum conditions in terms of agreed objectives and measurement of performance are satisfied first, they can lose most of their significance. This is a central issue in the control of public firms in practice and it explains in part why it has been so difficult to develop well structured devices and mechanisms of control. The objectives implicitly recognized in the U.K. system of control are (Rees, 1984b): (i) Economic (encompassing technical and managerial) efficiency; (ii) Profitability, in the sense of contribution to the Exchequer; (iii) Distributional effects; and (iv) Macroeconomic effects; with the weights allocated to each one changing through time. We share the view that for reasons related to the evaluation of performance¹ there is a strong case for organizing the system of control around the objectives of economic efficiency and profitability, with some relatively minor adjustments due to distributional effects and practically no attention to macroeconomic aspects². We are aware however that this would imply the need for some institutional change that - as in the case of the SCNI's report - governments are not always prepared to accept.

Turning into the economic guidelines proposed since 1967, we notice some changes in the definition and design of the rules over the years and their association with attempts to control decisions. Although they represented a

¹Which is in turn related to the problem of coping with multidimensionality, conflicts among objectives and stability through time.

²Aspects of macroeconomic policy are of course related to the objective of profitability, particularly in recent years with the attention paid to the size of the Public Sector Borrowing Requirements. The point however is against using public enterprises as an active instrument of macroeconomic policy.

substantial improvement in relation to previous practices, the economic rules introduced by the 1967 White Paper were not conceived in terms of a workable regulatory system but as simple guidelines. Thus, within a system of control with the characteristics described previously, the implementation of such rules became very difficult to achieve. This fact would explain, although partially, later attempts to introduce more imperfect but 'controllable' alternatives and also the re-emergence of financial instruments playing a central role in the regulation of decisions.

Most of the debate in recent years has concentrated on the analysis of the internal consistency of the rules proposed by the different White Papers and particularly on the potential conflicts between different rules (e.g. pricing - financial target - cost of capital). Given the previous interpretation, however, the attacks to the 1978 White Paper put forward by Heald (1980) and Webb (1980) could be regarded as partial or incomplete since they do not stress the regulatory aspects of the system and rules discussed which are also important to understand the change of emphasis observed¹. Three examples can be given in relation to this point.

The first is the substitution of the Required Rate of Return (RRR) approach for the Test Discount Rate (TDR) methodology for investment appraisal (see White Paper, 1978 and Treasury, 1979). Conceived as a shadow price of capital input, the TDR failed however to regulate investment decisions. Large (up to 80 percent in some reported cases) portions of annual investment were left out of the calculation for reasons such as 'inescapable' investment, that raise doubts on whether Managers took such control as a constraint. In addition the TDR methodology, when used, was applied to the determination of the choice of technique for a given investment volume rather than to the whole

¹This point does not imply that the 1978 proposals were an acceptable compromise between enunciation and regulation. Rather the idea is that they have tried to be more effective in the control of investment and pricing decisions though sacrificing some theoretical consistency.

investment programme. Redefining the control process by taking the investment made annually as a whole (without attention to individual projects) the RRR approach attempted to control total investment in a given year at the cost of violating marginality conditions.

The second example concerns the determination of prices according to marginal cost. A central property of the measure of cost of capital is the determination of the rent attributed to the capital input that is needed to obtain marginal costs. The TDR system did not consider this relationship explicitly. On the other hand, the RRR approach related the investment and the additional output associated with it through what might be termed 'average-incremental-cost' pricing (Rees, 1979b). This again sacrificed pure marginal conditions for the sake of implementation.

The final example is given by the proliferation of financial controls. The role of financial targets was strengthened after the 1978 White Paper which also tried to make explicit the interrelationship between these targets and the RRR approach. Nevertheless, the criticisms (e.g. Heald, 1980; Webb, 1980) that the financial targets will be a determining (rather than determined) factor of pricing and investment policies seem to be correct in view of the leading role that the White Paper gives to the targets. Notice however, that these targets operate over a period of three to five years and thus they are not an immediate constraint on annual investment. To play this role, the Government re-emphasized the use of short term cash limits called External Financial Limits (EFL), which can be seen as a mechanism of capital rationing. There are two possible interpretations of the new emphasis put on EFL. The first is simply as a rationing device that re-emerged as a consequence of the importance attached by macroeconomic policies to the control of the Public Sector Borrowing Requirements. The second can be derived from the fact that under asymmetric information and managerial strategic behaviour to influence the acceptance of high investment levels (a policy which would favour

management own goals) the control system cannot rely entirely on a shadow price. The evidence suggests however that it is the first reason the one that has motivated the renewed emphasis on EFL (Heald and Steel, 1981).

6.3. Incentives and Labour Contracts for Top Management

Over the past two or three decades, different reports such as the Herbert Committee (1957), the SCNI (1967/68) and NEDO (1976) have stressed the problems observed in the design of top managerial labour contracts in the U.K., in particular concerning problems of appointment, promotion and the low (relative to the private sector) pay received by members of the different Boards. The study by Elliott and Fallick (1981) also endorsed these criticisms. The quantitative evidence presented in all these studies is however small, not very well organized, and the issue of incentives not addressed. More recently Jones and O'Brien (1982) performed a comparative study of several aspects of the Boards in public (the main nationalized industries) and a group of large private corporations. Their main findings were that: (a) In terms of size the public sector Boards were smaller on average than their private sector counterparts; (b) public enterprises have a significantly higher ratio of part-time to full-time members in their Boards; (c) public enterprise Board members come from a wider spectrum of society, representing different interest groups; (d) in terms of length of appointment, public enterprise directors appear to be at a disadvantage in that they tend to have much shorter tenure periods¹; (e) in terms of pay, the study merely confirms the existing knowledge that public sector directors are paid substantially less than private sector ones. It follows from this study that the problems first raised by the Herbert Committee are still relevant.

¹This particular result seems to contradict the evidence provided by an often quoted paper of De Alessi (1974) on managerial tenure in public firms, using evidence from the U.S. electricity sector.

The main criticism emerging from these reports and studies is that managerial labour contracts in public firms are poorly designed, maintaining features present in the central government and civil service and therefore unsuitable for modern entrepreneurial organizations. It seems that the idea that the public corporation should be distinguished from central government agencies has not been fully translated into the design of top managerial contracts. The relevant point for our discussion however is that all the reports and studies mentioned above call, implicitly or explicitly, for a major review of the design of these contracts. It is within this reconsideration, that the issues addressed in this thesis can gain relevance and the result obtained provide some general principles that should be taken into account. Thus we can summarize the main findings of the study of linear managerial bonus schemes performed in Chapters II, III and IV.

Chapter II provided a reconsideration of some previous literature in the area which studied the properties of simple incentive mechanisms in correspondingly simple (i.e. under certainty and stationarity) environments. We found that managerial incentive problems can be solved writing a contract which pays the Manager the sum of profits and some approximation to consumer surplus, less a fee. The profit sharing ratio is seen as the appropriate instrument for the achievement of managerial efficiency while the relative weights given to each part of the contract will influence the pricing decision. Simple as it looks, this result can serve as a basis for the search of efficient managerial bonus schemes in practice. The central question however is whether a reliable and stable approximation to consumer benefits can be built in practice, but we have left this topic as an empirical issue. In addition, we have mentioned that - in the manner suggested by some previous works - distributional aspects can be incorporated into the proposed scheme; although this is not the case of aspects of quality regulation. Also if managerial preferences for certain types of non-pecuniary income are relevant, the informational requirements associated with the scheme will increase.

It is interesting to see that this theme is in a way repeated in Chapter III although reinterpreted appropriately to take account of uncertainty. The linear contract studied throughout this chapter pays the Manager according to actual profits and an average or expected value of consumer benefits. Given the form of the contract studied, the profit sharing ratio will now reflect the trade off between incentive and risk sharing in profits while the weights given to each part of the contract are again determined to influence optimal pricing. The optimal value of the ratio between these weights will crucially depend on the correlation between cost and demand shocks. Under the simplifying assumptions of the model we could identify as a sufficient condition for the optimal ratio between the 'consumer-surplus' and profit sharing ratios to be greater than one, that the covariance between cost and demand shocks must be non-negative. The determining factors behind this result are two: First, the increasing-risk (in profits) effect of a change in price and the corresponding compensation that the Manager must receive before taking these risks, given his degree of risk aversion. Second, the impact of changes in price upon the self-selected level of managerial effort also gives a reason for an adjustment of the relative weights. These two effects also give rise to another result obtained in the same chapter: the presence of incentive-risk sharing problems imply that, when providing an incentive scheme to solve them in an optimal way, we should depart from the pricing rule followed under full information. The point illustrated by this result is that the derivation of the rules must be made taking into account the fact that they are now regulated in the context of managerial incentive problems. The empirical importance of the two underlying effects however is an open question.

The purpose of Chapter IV was to further extend the model to a context where aspects of capacity choice and non-price rationing become relevant. The results merely constitute a restatement of the ones previously obtained, showing that they are robust enough to take account of these issues.

6.4. Control through Performance Indicators and Efficiency Audits

The idea that the internal efficiency of public enterprises could be controlled through efficiency audits was first put forward a long time ago in the U.K. (see Robson, 1962 p. 203). More recently the topic has been discussed by most of the reports reviewed before. NEDO (1976) also emphasized the need for the publication by the Boards of a wide range of indicators of performance that was later accepted by the 1978 White Paper. However, the distinction between indicator and 'target' has not been made clear, nor the control aspects of the introduction of such measures.

Since the 1978 White Paper, a growing body of literature on the sector¹ has stressed the role of performance indicators but included within a framework of comprehensive auditing of public enterprise decisions. The basic idea behind these works is that by making management information systems more transparent to Ministers, external bodies and interest groups (such as consumer councils) the process of control of public enterprises will be greatly improved.

Previous criticism to the ability of audits to solve the managerial control problem have focused on the intrinsic difficulties of making objective and unambiguous recommendations and in the power to enforce such changes once they are suggested (see Schmalensee, 1979 pp. 131-2). The SCNI's report (1967/68, Chapter XV paras 777-789) also reflected the criticism - shared by many of the industries at that time - that a system of efficiency audits would interfere excessively with the management of the enterprises. In this section, however, we can criticize the above mentioned literature on two different grounds: First, is that they invariably relegate economic efficiency rules to a secondary plane concentrating instead on a huge number of ambiguous indicators. That is, they have started with the idea that financial indicators are incomplete measures of performance and then proposed (in addition) the use of

¹Examples are Reedwood and Hatch (1982); Likierman (1983) and Aharoni (1983).

technical measures, neglecting central principles of economic regulation. Second, and equally important, even though they stress questions of management information systems, they also neglect aspects central to the problem of incentives and control - i.e. the asymmetries of preferences and information underlined before - in such a way that the effectiveness of the proposed control mechanisms seems dubious.

Contrasting with the view of the role of auditing and management information systems held by the previous works, there has been in recent years an explosion in the theoretical research on managerial accounting which has increasingly used developments from the economics of information and thus changed the emphasis from the "decision-facilitating" to the "decision-influencing" aspects of managerial accounting systems (see Demski and Kreps, 1982 for a review of the area). The emphasis put on Managers as agents with private information and providing productive inputs explains why the agency paradigm has become a natural instrument of analysis in this area.

Chapter V has followed this line of inquiry and thus is an attempt to integrate the issues of the selection of performance indicators and the design of efficiency audits into the general framework of managerial incentive schemes discussed before. The analysis can be seen only as an introductory attempt but some interesting insights have nevertheless been obtained. First, we have maintained that there must exist, from the viewpoint of the value of information, a limit in the number of admissible performance indicators and that some broadly used or advocated measures such as productivity indices may be redundant if a well defined (outcome-based) bonus incentive scheme is already in use. Second, a central principle behind efficiency audits seen as conditional information mechanisms is that since investigations are likely to be costly in practice we must save resources using them only in specific circumstances. These circumstances depend on extreme realisations of the outcome which is used to evaluate managerial performance, and are determined according

to expected benefits and costs. Thus according to this result we should not observe excessive day-to-day or continuous intervention into the activities of the firm but only deep investigations that are triggered by extreme outcomes. Notice that this type of policy would avoid meeting the criticism to efficiency audits put forward by the SCNI's report. Rather, it is justified from the viewpoint of an expected cost-benefit approach to the use of an information system.

The case of a perfect auditing mechanism studied also in Chapter V has helped to illustrate the factors that affect the optimal form and design of such a mechanism. We have shown that if the Manager is 'safe' from potential penalties that cannot be made effective due to minimum wages or other limited liability institutions then it will not be optimal to adopt a lower-tail investigation strategy (i.e. investigate only extremely bad outcomes). The obvious implication of this result for the design of an investigation mechanism is that investigations must necessarily carry an implementable punishment in case Managers are found responsible for low profits (or high deficits). In addition we have shown that as the Manager is less able to take risks, and thus to accept contingent payment systems, the optimal investigation tends towards the lower-tail form. Thus designing lower-tail investigation procedures can be an optimal policy under these circumstances.

6.5. Directions for further Research

The list of topics on the control and regulation of decisions in public firms that could be potentially interesting to explore is very long. Following the type of analysis performed in this thesis, some directions for further research call our attention and thus they are worth mentioning:

1. A more serious treatment of the problem of asymmetric information and decision-making incentives can give a better understanding of the problems to regulate pricing and investment decisions. Attention to the generation of

information and the design of transmission channels between Managers and Ministers can enhance our understanding of the process of participative planning or management-by-participation that is used in practice (cf. Rees, 1984a).

2. The topics investigated in Chapter V are very interesting and relevant and thus would require further attention. Again, the model can be enriched introducing pre or post-contractual asymmetric information in order to study optimal auditing of pricing and investment decisions.

3. The type of schemes studied before are based on the value of some outcome and therefore depend on the availability of a correct measure of performance. If it were unfeasible (for institutional or informational reasons) to obtain such a measure, the study of the properties of specific goal-based schemes (cf. Weitzman, 1976) under moral hazard and asymmetric information can provide useful insights for the design of a system of control based on management-by-objectives.

REFERENCES

- Aharoni, Y. (1982), "State-Owned Enterprise: An Agent without a Principal", in L.P. Jones (ed.), pp. 67-76.
- (1983), "Comprehensive Audit of Management Performance in U.S. State-Owned Enterprises", Annals of Public and Co-operative Economy, 54, 1, pp.
- Alchian, A. (1965), "Some Economics of Property Rights", Il Politico, 30, pp. 816-29.
- and H. Demsetz (1972), "Production, Information Costs, and Economic Organization", American Economic Review, 62, pp. 777-95.
- Baiman, S., and J. Demski (1980a), "Variance Analysis Procedures as Motivational Devices", Management Science, 26, 8, pp. 840-48.
- (1980b), "Economically Optimal Performance Evaluation and Control Systems", Journal of Accounting Research, 18, Supplement, pp. 184-220.
- Balachandran, B.V., and R.T. Ramakrishnan (1980), "Internal Control and External Auditing for Incentive Compensation Schedules", Journal of Accounting Research, 18, Supplement.
- Baron, D., and R. Myerson (1982), "Regulating a Monopolist with unknown Cost", Econometrica, 50, 4, pp. 911-30.
- Baumol, W.J. (1959), Business Behavior, Value and Growth, New York: MacMillan.
- , and D.F. Bradford (1970), "Optimal Departures from Marginal Cost Pricing", American Economic Review, 60, pp. 265-83.
- Bergson, A. (1978), "Managerial Risks and Rewards in Public Enterprise", Journal of Comparative Economics, 2, pp. 211-25.
- Borcherding, T.E., W.W. Pommerehne, and F. Schneider (1982), "Comparing the Efficiency of Private and Public Production: The Evidence from Five Countries", Zeitschrift für Nationalökonomie, supp.2, pp. 127-56.
- Bös, D. (1978), "Cost of Living Indices and Public Pricing", Economica, 45, pp. 59-69.
- (1981), Economic Theory of Public Enterprise, New York: Springer-Verlag.
- Brown, G., and M. Johnson (1969), "Public Utility Pricing and Output under Risk", American Economic Review, 59, 1, pp. 119-28.
- Christensen, J. (1981), "Communication in Agencies", Bell Journal of Economics, 12, Autumn.
- Coase, R.H. (1946), "The Marginal Cost Controversy", Economica, 13, pp. 169-82.
- Crew, M.A., and P. Kleindorfer (1978), "Reliability and Public Utility Pricing", American Economic Review, 68, 1, pp. 31-40.
- (1979), Public Utility Economics, London: MacMillan.
- , and E.F. Sudit (1979), "Incentives for Efficiency in the Nationalised Industries: Beyond the 1978 White Paper", Journal of Industrial Affairs, 7, Autumn, pp. 11-15.
- Dasgupta, P., P. Hammond, and E. Maskin (1979), "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility", Review of Economic Studies, 46, pp. 185-216.
- De Alessi, L. (1969), "Implications of Property Rights for Government Investment Choices", American Economic Review, 59, pp. 13-24.
- (1974), "Managerial Tenure under Private and Government Ownership in the Electricity Power Industry", Journal of Political Economy, 82, pp. 645-53.
- (1975), "Some Effects of Ownership on the Wholesale Prices of Electric Power", Economic Inquiry,

- (1980), "The Economics of Property Rights: A Review of the Evidence", Research in Law and Economics, 2, pp. 1-47.
- Demski, J., and G. Feltham (1978), "Economic Incentives in Budgetary Control Systems", The Accounting Review, 53, 2, pp. 336-59.
- , and D.M. Kreps (1982), "Models in Managerial Accounting", Journal of Accounting Research, 20, Supplement, pp. 117-48.
- Diamond, P.A., and J. Stiglitz (1974), "Increases in Risk and in Risk Aversion", Journal of Economic Theory, 8, pp. 337-60.
- Domar, E.D. (1974), "Optimal Compensation of a Socialist Manager", Quarterly Journal of Economics, 88, pp. 1-18.
- Eaton, J., and H. Rosen (1983), "Agency, Delayed Compensation and the Structure of Executive Remuneration", Journal of Finance, 38, 5, pp. 1489-1505.
- Elliott, R.F., and J.L. Fallick (1981), Pay in the Public Sector, London: Mac-Millan.
- Evans, J.H. III (1980), "Optimal Contracts with Costly Conditional Auditing", Journal of Accounting Research, 18, Supp, pp. 108-28.
- Fama, E. (1980), "Agency Problems and the Theory of the Firm", Journal of Political Economy, 88, pp. 288-307.
- Feldstein, M.S. (1972), "Distributional Equity and the Optimal Structure of Public Prices", American Economic Review, 62, pp. 32-36.
- Finsinger, J., and I. Vogelsang (1979), "A Regulatory Adjustment Process for Optimal Pricing by Multiproduct Monopoly Firms", Bell Journal of Economics, 10, pp. 157-71.
- (1981), "Alternative Institutional Frameworks for Price Incentive Mechanisms", Kyklos, 34, pp. 388-404.
- (1982), "Performance Indices for Public Enterprises", in L.P. Jones (ed.), pp. 281-96.
- Foster, C.D. (1971), Politics, Finance and the Role of Economics, London: George Allen and Unwin.
- Gillis, M., G.P. Jenkins, and D.R. Lessard (1982), "Public Enterprise Finance: Toward a Synthesis", in L.P. Jones (ed.), pp. 257-77.
- Gjesdal, F. (1976), "Accounting in Agencies", Mimeo, Graduate School of Business, Stanford University.
- Gravelle, H.S.E. (1982a), "Incentives, Efficiency and Control in Public Firms", Zeitschrift für Nationalökonomie, supp 2, pp. 79-104.
- (1982b), "Bargaining and Efficiency in Public and Private Sector Firms", Mimeo, Dept. of Economics, Queen Mary College, London, August (Published in M. Marchand et.al. (eds), pp. 193-220).
- (1983), "Alternative Institutional Frameworks for Price Incentive Mechanisms: Some Comments", Kyklos, 36, 1, pp. 115-20.
- , and E. Katz (1976), "Financial Targets and X-efficiency in Public Enterprises", Public Finance, 31, pp. 218-34.
- Grossman, S., and O. Hart (1983), "An Analysis of the Principal-Agent Problem", Econometrica, 51, 1, pp. 7-45.
- Groves, T., and M. Loeb (1979), "Incentives in a Divisionalized Firm", Management Science, 25, pp. 221-30.
- Guesnerie, R., and J.J. Laffont (1984), "The Government Control of Public Firms and the Economics of Incomplete Information: An Introduction", in M. Marchand et.al. (eds), pp. 159-178.
- Harris, C., C. Kriebel and A. Raviv (1982), "Asymmetric Information, Incentives and Intrafirm Resource Allocation", Management Science, 28, pp. 604-20.
- , and A. Raviv (1978), "Some Results on Incentive Contracts with Application to Education and Employment, Health Insurance and Law Enforcement", American Economic Review, 68, pp. 20-30.

- (1979), "Optimal Incentive Contract with Imperfect Information", Journal of Economic Theory, 20, pp. 231-59.
- , and R. Townsend (1981), "Resource Allocation Under Asymmetric Information", Econometrica, 49, pp. 33-67.
- Heald, D. (1980), "The Economic and Financial Control of U.K. Nationalised Industries", Economic Journal, 90, pp. 243-65.
- , and D. Steel (1981), "Nationalised Industries: A Search for Control", Public Money, June, pp. 13-19.
- Heckermann, D. (1975), "Motivating Managers to Make Investment Decisions", Journal of Financial Economics, 2, pp. 273-92.
- (Herbert Committee) (1956), Report of the Committee of Enquiry into the Electricity Supply Industry, Cmnd 9672, London: H.M.S.O.
- Hess, J.D. (1983), The Economics of Organization, Amsterdam: North Holland.
- Holmstrom, B.R. (1978), On Incentives and Control in Organizations, unpublished Ph.D. Thesis, Graduate School of Business, Stanford University.
- (1979), "Moral Hazard and Observability", Bell Journal of Economics, 10, Spring, pp. 74-91.
- (1980), "Discussion of Economically Optimal Performance Evaluation and Control Systems", Journal of Accounting Research, supp., pp. 221-26.
- Hughes, J. (1982), "Agency Theory and Stochastic Dominance", Journal of Financial and Quantitative Analysis, 27, 3, pp. 341-61.
- Intriligator, M.D. (1971), Mathematical Optimization and Economic Theory, Englewood Cliffs: Prentice-Hall.
- Jensen, M., and W. Meckling (1976), "Theory of the Firm: Managerial Behavior, Agency Cost and Ownership Structure", Journal of Financial Economics, 3, p. 305-60.
- Jones, L.P. (ed.) (1982), Public Enterprise in Less-Developed Countries, Cambridge: Cambridge University Press.
- Jones, T.T., and P.M. O'Brien (1982): "The Composition, Background and Salaries of Board Members of Nationalised Industries with Some Private Sector Comparisons", Occ. Paper 8211, Dept. of Management Sciences, University of Manchester, September.
- Laffont, J.J., and E. Maskin (1982), "The Theory of Incentives: An Overview", in W. Hildenbrand (ed.) Advances in Economic Theory, Cambridge; Cambridge University Press.
- Lambert, R. (1983), "Long Term Contracts and Moral Hazard", Bell Journal of Economics, 14, Autumn, pp. 441-52.
- Leland, H. (1972), "Theory of the Firm Facing Uncertain Demand", American Economic Review, 62, 3, pp. 278-91.
- Likierman, J.A. (1983), "Nationalised Industries", in Henley et.al. (eds), Public Sector Accounting and Financial Control, London: Van Nostrand Reinhold.
- Lindsay, C.M. (1976), "A Theory of Government Enterprise", Journal of Political Economy, 84, pp. 1061-77.
- Lintner, J. (1981), "Economic Theory and Financial Management", in Y. Aharoni and R. Vernon (eds), State-Owned Enterprises in the Western Economies, New York: Crom Helm.
- Littlechild, S.C. (1979), "Controlling the Nationalised Industries: Quis Custodiet Ipsos Custodes", Disc. Paper 56, Series B, Faculty of Commerce and Social Sciences, University of Birmingham.
- Marchand, M., P. Pestieau, and H. Tulkens (eds) (1984), Public Enterprise Performance: Theory and Measurement, Amsterdam: North-Holland.
- Marcus, A.J. (1982), "Risk Sharing and the Theory of the Firm", Bell Journal of Economics, 13, Autumn, pp. 369-78.
- Marris, R. (1964), The Economic Theory of Managerial Capitalism, London: MacMillan.

- , and D.C. Mueller (1980), "The Corporation, Competition and the Invisible Hand", Journal of Economic Literature, 18, pp. 32-63.
- McKenzie, G.W. (1983), Measuring Economic Welfare: New Methods, Cambridge: Cambridge University Press.
- Meyer, R.A. (1975), "Monopoly Pricing and Capacity Choice under Uncertainty", American Economic Review, 65, 3, pp. 326-37.
- Milgrom, P. (1981), "Good News and Bad News: Some Representation Theorems and Applications", Bell Journal of Economics, 12, pp. 380-91.
- Mirrlees, J.A. (1974), "Notes on Welfare Economics, Information and Uncertainty" in Balch et.al. (eds), Essays on Economic Behaviour under Uncertainty, Amsterdam: North Holland.
- (1975), "The Theory of Moral Hazard and Unobservable Behaviour: Part I" Mimeo, Nuffield College, Oxford, October.
- (1976), "The Optimal Structure of Incentives and Authority within an Organization", Bell Journal of Economics, 7, Spring, pp. 105-131.
- Mueller, D.C. (1979), Public Choice, Cambridge: Cambridge University Press.
- Myerson, R.B. (1979), "Incentive Compatibility and the Bargaining Problem", Econometrica, 47, pp. 61-74.
- National Economic Development Office (1976), A Study of U.K. Nationalised Industries, London.
- Navajas, F.H. (1983), Managerial Discretion, Incentives and the Control of the Public Sector Firm, M.Phil Thesis, Oxford.
- Niskanen, W.A. (1971), Bureaucracy and Representative Government, Chicago: Aldine-Atherton.
- Penrose, E. (1980), The Theory of the Growth of the Firm, 2nd Ed., Oxford: Basil Blackwell.
- Radner, R. (1981), "Monitoring Cooperative Agreements in a Repeated Principal-Agent Relationship", Econometrica, 49, pp. 1121-48.
- Raiffa, H. (1968), Decision Analysis: Introductory Lectures of Choices Under Uncertainty, Reading, Mass: Addison-Wesley.
- Redwood, J., and J. Hatch (1982), Controlling Public Industries, Oxford: Basil Blackwell.
- Rees, R. (1968), "Second Best Rules for Public Enterprise Pricing", Economica, 35, pp. 260-73.
- (1979a), The Control of Nationalised Industries, M. Keynes: Open University Press.
- (1979b), "The Test Discount Rate - Comment", in Treasury (1979), pp. 41-7.
- (1980), "The Principal-Agent Relationship and Control of Public Enterprise", paper presented at a Seminar on Regulation, Centre for Socio Legal Studies, Wolfson College, Oxford, March.
- (1982), "A Positive Theory of the Public Enterprise", Mimeo, Economics Dept., University College, Cardiff, September. (Published in Marchand et.al. (eds), pp. 179-91).
- (1982b), "Some Problems in Optimal Pricing under Uncertainty", Zeitschrift für Nationalökonomie, Supp. 2, pp. 63-78.
- (1984a), "The Public Enterprise Game", Economic Journal, Supp, March.
- (1984b), Public Enterprise Economics, 2nd Edition, London: Weidenfeld and Nicolson.
- Riordan, M. (1984), "On Delegating Price Authority to a Regulated Firm", Rand Journal of Economics, Spring, pp. 108-15.
- Robson, W.A. (1962), Nationalised Industry and Public Ownership, 2nd Edition, London: George Allen and Unwin.
- Ross, S. (1973), "The Economic Theory of Agency: The Principal's Problem", American Economic Review, Papers and Proceedings, May, pp. 134-39.

- (1974), "On the Economic Theory of Agency and The Principle of Similarity", in M. Balch et.al. (eds) Essays on Economic Behavior Under Uncertainty, Amsterdam: North Holland.
- Rothschild, M., and J. Stiglitz (1970), "Increasing Risk I: A Definition", Journal of Economic Theory, 2, pp. 225-43.
- (1971), "Increasing Risk II: Its Consequences", Journal of Economic Theory, 3, pp. 66-84.
- Rowley, C.K., and G.K. Yarrow (1981), "Property Rights, Regulation and Public Enterprise: the Case of the British Steel Industry 1957-75", International Review of Law and Economics, 1, pp. 63-96.
- Rubinstein, A., and M. Yaari (1983), "Repeated Insurance Contracts and Moral Hazard", Journal of Economic Theory, 17.
- Samuelson, P.A. (1970), "The Fundamental Approximation Theorem in terms of Means, Variances and Higher Moments", Review of Economic Studies, 37, pp. 50-65.
- Sappington, D. (1980), "Precontractual Information Asymmetry between Principal and Agent: the Continuous Case", Economics Letters, 5, pp. 371-75.
- (1983), "Optimal Regulation of a Multiproduct Monopoly with unknown Technological Capabilities", Bell Journal of Economics, 14, pp. 453-63.
- Schmalensee, R. (1972), "Option Demand and Consumer's Surplus: Valuing Price Changes under Uncertainty", American Economic Review, 62, pp. 813-24.
- (1979), The Control of Natural Monopolies, Lexington, Mass: D.C. Heath.
- Scott, M.F.G. (1952), Criteria of Efficiency for Nationalised Industries, unpublished B.Litt. Thesis, University of Oxford.
- (1978), "A Profit Test for Public Monopolies", in D. Butler and A.H. Halsey (eds.), Policy and Politics, Essays in Honour of Norman Chester, London, pp. 152-68.
- Select Committee on Nationalised Industries (1967/68), Ministerial Control of the Nationalised Industries, HC371 Vols. I, II, and III, London: H.M.S.O.
- Shavell, S. (1979), "Risk Sharing and Incentives in the Principal and Agent Relationship", Bell Journal of Economics, 10, pp. 55-73.
- Sherman, R., and M. Visscher (1978), "Second-best Pricing with Stochastic Demand", American Economic Review, 68, 1, pp. 41-53.
- Sheshinski, E. (1976), "Price, Quality and Quantity Regulation in Monopoly Situations", Economica, 43, pp. 127-37.
- Spence, M. (1975), "Monopoly, Quality and Regulation", Bell Journal of Economics, 6, pp. 417-29.
- , and R. Zeckhauser (1971), "Insurance, Information and Individual Action", American Economic Review, 61, pp. 380-87.
- Stiglitz, J. (1974), "Risk Sharing and Incentives in Sharecropping", Review of Economic Studies, 61, pp. 219-56.
- (1975), "Incentives, Risk and Information: Notes Toward a Theory of Hierarchy", Bell Journal of Economics, 6, pp. 552-79.
- Tam, M.S. (1981), "Reward Structures in a Planned Economy: The Problem of Incentives and Efficient Allocation of Resources", Quarterly Journal of Economics, 96, pp. 111-28.
- Treasury (1979), "The Test Discount Rate and the Required Rate of Return", Government Economic Service Working Paper 22, London.
- Varian, H.R. (1978), Microeconomic Analysis, New York: Norton.
- Webb, M.G. (1980), "A Critical Appraisal of United Kingdom Policy for the Nationalised Industries", in B. Mitchell and P. Kleindorfer (eds.), Regulated Industries and Public Enterprise, Lexington Mass: D.C. Heath.
- Weitzman, M.L. (1974), "Prices vs. Quantities", Review of Economic Studies, 41, pp. 477-91.

- (1976), "The New Soviet Incentive Scheme", Bell Journal of Economics, 7, pp. 251-57.
- (1980), "Efficient Incentive Contracts", Quarterly Journal of Economics, 95, pp. 719-30.
- (White Paper) (1961), The Financial and Economic Obligations of Nationalised Industries, Cmnd 1337, London: H.M.S.O.
- (1967), Nationalised Industries: A Review of Economic and Financial Objectives, Cmnd 3437, London: H.M.S.O.
- (1978), The Nationalised Industries, Cmnd 7131, London: H.M.S.O.
- Williamson, O.E. (1963), "Managerial Discretion and Business Behavior", American Economic Review, 53, pp. 1032-57.
- Wilson, R. (1968), "On the Theory of Syndicates", Econometrica, 36, pp. 119-32.
- (1969), "The Structure of Incentives for Decentralization under Uncertainty", La Decision, Paris.
- Wiseman, J. (1978), "The Political Economy of Nationalised Industry" in J.M. Buchanan et.al. (eds.), The Economics of Politics, I.E.A. Readings 17, London, pp. 73-92.