Children’s Mathematical Difficulties: Some Contributory Factors and Interventions

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Abstract

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Mathematical difficulties are a widespread problem for many children all over the world. The chapter discusses factors that predict arithmetical difficulties and interventions that are used to foster children’s mathematical skills in primary school. As factors that contribute to mathematical difficulties, national and cultural aspects, socio-economic differences, attitudes and emotions are examined. In this regard, the chapter highlights the influences of teaching methods, poverty and mathematical anxiety for math abilities and partially mathematical learning problems. Interventions in numeracy are classified in varying degrees of intensiveness (‘three waves’). The chapter analyses advantages and disadvantages of all three ‘waves’ (whole-class approach, light-touch individualized and small-group interventions, highly intensive approach) so that factors can be identified, which make an intervention effective.

Keywords

Math difficulties
Predictors
Arithmetical achievement
Children
Interventions

Difficulty with arithmetic is a common problem (Butterworth, Sashank, & Laurillard, 2011). For example, about 22% of the adult population in the UK have severe numeracy difficulties that have a serious practical and social impact on their daily lives, whereas only about 5% have similar levels of difficulty in literacy assessed by the same criteria (Bynner & Parsons, 1997; Parsons & Bynner, 2005).

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This chapter will first discuss some of the factors that contribute to arithmetical difficulties. There are genetic and other brain-based developmental factors that contribute to mathematical difficulties, which are discussed elsewhere in this book. This chapter will focus on some

environmental and motivational factors
. It will then proceed to discuss some interventions that have been used for mathematical difficulties, especially at primary school level.

National and Cultural Factors: What Do We Learn from International Comparisons?

International comparisons
, such as those of TIMSS (Mullis, Martin, Foy, & Hooper, 2016; Mullis, Martin, & Loveless, 2016) and PISA (OECD, 2013), report considerably better performance in mathematics by children in some countries than in others. In particular children from countries in the Far East, such as Japan, Korea, Singapore and China, tend to perform better in arithmetic than do children in most parts of Europe and America.
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It is important to be cautious in interpreting the results of such international comparisons (Jerrim, 2011; Sturman, 2015). For example, there may be issues with sampling. Subtle differences in the ways in which pupils or schools are selected for testing may significantly affect the results. For example, the implications of selecting a class of pupils of a particular age may differ according to whether promotion by age is automatic or is dependent on school achievement and on whether the schools divide pupils into ability groups by streaming or setting. Moreover, differences are typically studied at a whole-country level and may fail to take into account differences between regions or school types within a country. For example, in the TIMSS 2011 study, Massachusetts obtained mathematics scores similar to those in the highest-achieving countries, while Alabama obtained mathematics scores below the international median. China usually comes close to the top in international comparisons; but it must be remembered that this is an extremely large and diverse country and that it can be difficult to be sure that the sampling is representative. Children in the larger, more readily accessible cities are more likely to be sampled than those in remote rural schools. For example, the PISA 2012 study included only schools in Shanghai and Hong Kong (OECD, 2014). There is a lot of evidence that rural Chinese children are relatively economically and educationally disadvantaged compared to urban children and their omission from most international comparisons may lead to somewhat misleading results. Nevertheless, despite such cautions, there are some fairly consistent findings of relatively high performance by the Pacific Rim countries

and by Finland (Mullis, Martin, & Loveless, 2016). Educators, researchers and policymakers have made several attempts to examine the reasons.

Might International Differences in Teaching Methods Affect Performance?

For example, people have attempted to investigate whether there are particular
teaching methods
or school characteristics in higher-achieving countries that might be transferable to other countries and lead to improvement.

There has, for instance, been considerable emphasis recently on the 'mastery' approaches of East Asian countries, resulting in some attempts to introduce methods based on this approach into schools in the UK and the USA (see discussion of whole-class interventions at the beginning of this chapter). One difficulty in doing so is deciding which of the numerous aspects of the 'mastery' approaches are particularly important. For example, such approaches involve (1) breaking down different parts of the curriculum into units and (2) clearly defining the goals, and the aim is to ensure that all pupils have mastered each unit before going on to another one. While the approach is sometimes misinterpreted as involving whole-class teaching without any adaptations to weaker pupils, in fact teachers are expected to look at the children's work, and to intervene immediately with individuals' misconceptions before moving on. Thus, at least according to the ideal, the same person combines, within a short space of time, the role of class teacher and deliverer of interventions. This of course places high demands on the teacher.

Finland, another high-achieving country, differs from the Pacific Rim countries in some key ways. Whereas children in the Pacific Rim countries start school quite early and experience repeated high-stake testing, Finnish children do not begin formal instruction until 7 and experience little high-stake testing.

One thing that Finland and the Pacific Rim countries appear to have in common is the high status of the teaching profession, with high levels of selection for education courses, extended courses, and opportunities for continuous professional development
. It may be that this is at least as important as any details of the curriculum.

Socio-economic Differences

Parental

social class
is an important
predictor
of children’s academic performance in all subjects, including mathematics (Perry & Francis, 2010). British adults with severe persisting numeracy difficulties are far more likely than those without such difficulties to have come from ‘working-class’ backgrounds and to have been poor (Bynner & Parsons, 1997, 2006). In countries with greater social inequalities, such as Brazil, social class effects on academic performance are even greater (Nunes, Schliemann, & Carraher, 1993). For example, Davie, Butler, and Goldstein (1972) found that even at the age of 4, there was a year’s difference in British children’s performance on intellectual tasks, including numeracy tasks, between working-class children living in a deprived area and middle-class children living in a comfortable area.

There are many possible ways in which
social class
could influence academic performance, including mathematical performance. Better-off parents can afford more books and other academic materials for their children. On average, they may have more time to talk to and interact with their children (though this is not always the case). The children are likely to attend schools with better resources. Moreover, better-off parents have usually had more formal education themselves and are therefore likely to be more able to help their children to learn academic subjects. Also, the culture of the school is likely to be less alien to a child from a family with a high level of formal education than one from a family with lower formal education (Biddle, 2001; Brooker, 2002).

Evidence suggests that early influences of
social class
are much greater on verbal than nonverbal aspects of mathematics. For instance, Jordan, Huttenlocher, and Levine (1992, 1994) found no social class differences in pre-schoolers’ ability to do nonverbal addition and subtraction problems; but middle-class children were better than working-class children at verbal arithmetic. Presumably, middle-class children have more experience than working-class children with conventional mathematical language, which gives them an advantage in verbal arithmetic, but not necessarily in nonverbal arithmetic.

There are several preschool intervention programmes for children at risk, usually children living in poverty. They appear to be commonest in the USA and include, for example, the mathematical components of the Head

Start programme (Arnold, Fisher, Doctoroff, & Dobbs, 2002), the Pre-K Mathematics curriculum (Klein, Starkey, Clements, Sarama, & Iyer, 2008), the Big Math for Little Kids programme of Ginsburg and his colleagues (Greenes, Ginsburg, & Balfanz, 2004) and the Building Blocks programme of Clements and Sarama (2011).

Ramani and Siegler (2008, 2011) proposed that SES differences in early numeracy may reflect differing prior experience with informal numerical activities, such as numerical board games. They found that the numerical magnitude knowledge of pre-schoolers from low-income families lagged behind that of peers from better-off backgrounds. But playing a simple numerical board game for four 15-min sessions eliminated the differences in numerical estimation proficiency. Playing games that substituted colours for numbers did not have this effect. These findings have been replicated with groups of children in the UK (Whyte & Bull, 2008) and Sweden (Elofsson, Gustafson, Samuelsson, & Traff, 2016).

The Role of Attitudes and Emotions

Mathematical development
 and performance depend not only on our learning and intellectual abilities, and the teaching that we have received, but also on emotions and attitudes. There is a wide spectrum of attitudes that people have with regard to mathematics, ranging from the extremely positive to the extremely negative. Unfortunately, the latter are all too common, and while they sometimes involve mere dislike or boredom with mathematics, many people suffer from severe anxiety, even fear, with regard to mathematics (Ashcraft, 2002; Maloney & Beilock, 2012).

Mathematics anxiety
 has been defined as ‘a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in …ordinary life and academic situations’ (Richardson & Suinn, 1972). People who fear mathematics are seriously restricted in their choice of occupation (Brown, Brown, & Bibby, 2008; Chipman, Krantz, & Silver, 1992). They may experience difficulty and anxiety in managing their finances, and, if the fear is severe, even in such activities as reading train and bus timetables.

Estimates as to the frequency of mathematics anxiety are varied and depend both on the ways in which mathematics anxiety
 is defined and assessed and on the nature of the sample. Given that people with extreme mathematics anxiety are probably less likely to attend university, university student samples may be biased toward comparatively low levels of such anxiety: nevertheless, mathematics anxiety is common even in such samples. Richardson and Suinn (1972)

estimated that 11% of university students show high enough levels of mathematics anxiety to be in need of counselling. Johnston-Wilder, Brindley, and Dent (2014) found a higher figure in a group of apprentices showed high mathematics anxiety, with about 30% showing high mathematics anxiety and a further 18% affected to a lesser degree. Ashcraft and Moore (2009) estimated that 17% of the population have high levels of mathematics anxiety.

Mathematics anxiety
 is an important problem, not only because it is an unpleasant and stressful emotion but because many studies in different countries show that attitudes to mathematics are correlated with actual mathematical performance and in particular that mathematics anxiety is negatively correlated with performance (Baloğlu & Koçak, 2006; Dulaney et al., 2017; Hembree, 1990; Ho et al., 2000; Ma & Kishor, 1997; Miller & Bichsel, 2004). What is less clear is the direction of causation. On the one hand, mathematics anxiety may cause poorer performance by reducing motivation and leading to reduced practice, or to active avoidance (Chinn, 2009), or by overloading working memory (Ashcraft, 2002). On the other hand, mathematical difficulties may lead to mathematics anxiety, by causing experiences of failure, confusion and embarrassment associated with mathematics.

Maloney and Beilock (2012) proposed that mathematics anxiety is likely to be due both to pre-existing difficulties in mathematical cognition and to social factors, e.g. exposure to teachers who themselves suffer from mathematics anxiety. They suggest that children with initial mathematical difficulties are also likely to be more vulnerable to the negative social influences and that this may create a vicious circle.

Most studies of attitudes of mathematics anxiety
 have dealt with the problem of negative attitudes. But positive attitudes such as enjoyment of mathematics and self-confidence in mathematics are also important topics to study, and cannot be reduced to the simple absence of anxiety. For example, Villavicencio and Bernardo (2016) found that positive emotions toward mathematics predicted achievement in a Filipino adolescent group, even after controlling for anxiety. It is possible that positive attitudes to mathematics could act as a protective factor in pupils with risk factors for low mathematical attainment.

The importance of attitudes to mathematics makes it important to find ways of intervening to improve attitudes and in particular to treat or prevent mathematics anxiety. Treatments include desensitization and cognitive behaviour therapy and related treatments that are used for many forms of anxiety (e.g. Hembree, 1990). Beilock and colleagues have found that ‘writing out’ one’s anxieties can reduce anxiety and improve performance, both in mathematics anxiety and other forms of academic performance anxiety (Park, Ramirez, & Beilock, 2014; Ramirez & Beilock, 2011). Such treatments are of course only feasible with those who are old enough to have reasonable facility with writing.

There is evidence that interventions that improve mathematical performance may also improve attitudes and reduce anxiety. Levitt and Hutton (1983) found that training in basic arithmetical skills and in relevant study skills such as note taking can reduce mathematics anxiety . Supekar, Iuculano, Chen, and Menon (2015) used an intensive 8-week programme to improve the mathematical skills of children in Grade 3 with high and low mathematics anxiety. The programme was based on MathWise (Fuchs et al., 2008, 2013) and included both games and more formal activities involving practice with addition and subtraction, training in efficient counting strategies for arithmetic and exposure to concepts such as the commutativity of addition and that adding zero to a number does not change it. Children in general improved in mathematical problem-solving, and those who started with high mathematics anxiety showed a significant reduction in mathematics anxiety on a questionnaire measure. Moreover, fMRI scanning showed that before training, children with high mathematics anxiety showed brain activation differences from those with low mathematics anxiety, and in particular higher amygdala; but after the training these group differences disappeared.

Interventions for Mathematical Difficulties

As indicated in the above paragraph, interventions can be effective in ameliorating mathematical difficulties and possibly improve attitudes as well. How best can we intervene?

There have been some intervention programmes for children with mathematical difficulties since at least the first half of the twentieth century, especially in the USA (Brownell, 1929; Tilton, 1947; Williams & Whitaker, 1937). However, there are relatively few numeracy interventions available until recently; and mostly such interventions have not been used on a large scale.

However, in the twenty-first century, there has been increasing interest in developing interventions for children with numeracy difficulties (Butterworth et al., 2011; Chodura, Kuhn, & Holling, 2015; Clements & Sarama, 2011; Cohen Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013; Dowker, 2017; Dowker & Sigley, 2010; Gersten et al., 2009; Kucian et al., 2011; Rasanen, Salminen, Wilson, Aunio, &

Dehaene, 2009). Only a few of these interventions will be discussed in detail here: for a more comprehensive account, see Dowker (2017). Interventions in numeracy (as well literacy) have sometimes been classified into three categories of varying degrees of intensiveness termed 'waves' in the UK and 'tiers' in the USA. Wave 1 involves whole-class teaching designed to be suitable for children of a variety of attainment levels. Wave 2 involves lighter-touch, less-intensive interventions in small groups (or sometimes limited-time one-to-one interventions) with children who are experiencing mild or moderate difficulties in the subject. Wave 3 involves more intensive, usually individualized interventions for children with more significant problems.

Whole-Class Approaches

While there has been interest in developing and improving mathematics curricula and educational techniques for quite a long time, there has been increasing recent interest in investigating the possible role of certain new whole-class programmes in improving overall performance and reducing the incidence of numeracy difficulties. One programme which is attracting current interest from this point of view, especially in the UK, is Mathematics Mastery, a programme inspired by some aspects of Singapore mathematics education. This is supported by NCETM (https://www.ncetm.org.uk). Compared to traditional curricula, fewer topics are covered in more depth, and greater emphasis is placed on problem-solving and on encouraging mathematical thinking. A current evaluation by the Education Endowment Foundation has so far indicated that the use of the programme in Year 1 results in an average increased gain in mathematics age of 2 months in the first year, and the use of the programme in Year 7 results in an average increased gain in mathematics age of 1 month. Further investigation is desirable to see whether these gains are maintained or extended over time.

Adapting classroom instruction to take account of individual needs may also involve allowing for independent individualized or small-group work within a class (El-Naggar, 1996). This may involve progressing through a textbook at one's own pace, the use of individualized worksheets and/or the individualized use of educational computer software.

Such approaches are potentially more flexible and have more potential for taking account of the componential nature of arithmetical ability, than giving all children the same instruction, or streaming or setting. Unlike streaming and setting, which have usually been found to have negative effects on low attainers' performance, Lou et al. (1996) indicated that within-class grouping had a positive effect on the performance of low achievers, but only if it was accompanied by provision of appropriate materials and activities. The potential disadvantages of such approaches include the risks that even within one class, some pupils may be labelled as ‘low attainers’ and live down to expectations and that work will become so individualized that pupils will not benefit from mutual discussion and exploration of ideas.

Light-Touch Individualized and Small-Group Interventions

There are clearly many children who have significant need of more targeted intervention, but do not need extremely intensive intervention. Relatively light-touch interventions are needed for such individuals. Such interventions are usually delivered by teachers or teaching assistants within the school. They are often delivered in small groups, or sometimes individually but on a relatively infrequent basis.

For example, in the UK, Askew, Bibby, and Brown (2001) developed a small-group intervention technique that involved the use of derived fact strategies. Teachers worked with small groups (four per group) of 7- to 8-year-old children, who had performed below average in school achievement at age 7. The children underwent intervention once a week for 20 weeks. These children improved significantly more than the controls, both in accuracy and in their use of known and derived facts rather than needing to resort to counting strategies.

In an American study, Bryant, Bryant, Gersten, Scammacca, and Chavez (2008) delivered 15-min small-group interventions 3–4 days a week for 18 weeks. The interventions dealt with counting, quantity representation, basic facts and place value concepts. The 26 pupils in the intervention did not differ significantly from controls on a standardized test at the end of the intervention period. However, intervention programmes lasting longer than 15 min at a time and/or continuing over a longer period of time have given positive results. Bryant et al. (2008) used similar interventions to the above, but lasting 20 min at a time, 4 days a week, for 23 weeks. The 42 children in this study did perform significantly better than controls on a standardized test.

Thus, relatively small-group interventions can have a significant impact on the progress of children with mathematical difficulties. The ways in which such interventions are delivered seem to affect the level and nature of their effectiveness.

There are also programmes which, though administered individually, involve relatively small amounts of time and may often be delivered by teaching assistants rather than specialist teachers.

An example is Catch Up® Numeracy, which was developed through a collaboration between myself and Catch Up®, a not-for-profit charity (Dowker, 2017; Dowker & Sigley, 2010; Holmes & Dowker, 2013). The main target pupils have been pupils aged from 6 to 11 years, who have moderate difficulties with mathematics. It has recently been extended for use with 11 to 14 year olds. It consists of two 15-min sessions per week for approximately 30 weeks.

Catch Up Numeracy focuses on assessing and targeting specific strengths and weaknesses. The intervention begins by assessing the children on the ten components of numeracy. Each child is assessed individually by a trained teacher or more usually a teaching assistant. This assessment is used to construct a 'Catch Up Numeracy' learner profile, which determines the entry level for each of the ten Catch Up Numeracy components. Children are provided with mathematical games and activities targeted to their specific levels in specific activities.

The ten components include (1) counting orally; (2) counting objects; (3) reading and writing numbers; (4) comparing, adding and subtracting tens and units; (5) ordinal numbers; (6) word problems; (7) translation between different formats (numerals, number words and sets of objects); (8) derived fact strategies (the use of known facts, combined with arithmetical principles such as commutativity, to derive new facts; e.g. if 8 + 6 = 14, then 6 + 8 must also be 14); (9) estimation of quantities and of answers to arithmetic problems; and (10) remembered number facts.

Studies where children were pretested and post-tested on the Hodder Basic Number Screening Test (Gillham & Hesse, 2012) have shown that children make about twice as much progress as would be expected from the passage of time alone and that they make significantly more progress than business-as-usual controls (Dowker, 2017; Holmes & Dowker, 2013). A randomized controlled study is currently underway to investigate whether the gains are significantly greater than those of children receiving equivalent-time mathematics intervention.

Highly Intensive Interventions

There are some children whose difficulties are so severe and/or resistant to intervention that light-touch interventions will not prove sufficient. Much more intensive interventions, perhaps involving daily individualized sessions with a teacher highly trained in intervention techniques, may be necessary. A well-known example of such an intensive intervention is Mathematics Recovery. The

Mathematics Recovery programme

was designed in Australia by Wright and his colleagues (Wright & Ellemor-Collins, 2018; Wright, Martland, & Stafford, 2006). In this programme, teachers provide intensive individualized intervention to low-attaining 6- and 7-year-olds. Children in the programme undergo 30 min of individualized instruction per day over a period of 12–14 weeks. The choice of topics within the programme is based on the Learning Framework in Number, originally devised by Steffe (1992). This divides the learning of arithmetic into five broad stages. These stages are (1) emergent (some simple counting, but few numerical skills), (2) perceptual (can count objects and sometimes add small sets of objects that are present), (3) figurative (can count well and use 'counting all' strategies to add), (4) counting on (can add by 'counting on from the larger number' and subtract by counting down, can read numerals up to 100 but have little understanding of place value) and (5) facile (know some number facts, are able to use some derived fact strategies, can multiply and divide by strategies based on repeated addition, may have difficulty with carrying and borrowing). Children are assessed, before and after intervention, in a number of key topics. They undergo interventions based on their initial performance in each of the key topics. The key topics that are selected vary with the child's overall stage. For example, the key topics at the emergent stage are (i) number word sequences from 1 to 20, (ii) numerals from 1 to 10, (iii) counting visible items (objects), (iv) spatial patterns (e.g. counting and recognizing dots arranged in domino patterns and in random arrays), (v) finger patterns (recognizing and demonstrating quantities up to 5 shown by number of fingers) and (vi) temporal patterns (counting sounds or movements that take place in a sequence). The key topics at the next perceptual stage are (i) number word sequences from 1 to 30, (ii) numerals from 1 to 20, (iii) figurative counting (counting on and counting back, where some objects are visible but others are screened), (iv) spatial patterns (more sophisticated use of domino patterns; grouping sets of dots into 'lots of 2', 'lots of 4', etc.), (v) finger patterns (recognizing, demonstrating and manipulating patterns up to 10 shown by numbers of fingers) and (vi) equal groups and sharing (identifying equal groups and partitioning sets into equal groups). The key topics at later stages place greater emphasis on arithmetic and less on counting. Despite the overall division into stages, the programme acknowledges and adapts to the fact that some children can be at a later stage for some topics than for others.

Smith, Cobb, Farran, Cordray, and Munter (2013) found significant improvement in both standardized tests and researcher-derived tests in a randomized field trial of first-grade pupils assigned to Mathematics Recovery versus a waiting list control group. Effect sizes ranged from 0.21 to 0.28 for standardized tests and were higher for the researcher-derived tests. Future studies should assess the longer-term impact.

Numbers Count

In order to set up an intensive intervention for children with serious numeracy difficulties in the UK, and to test its effectiveness, Every Child Counts was set up as a partnership initiative between the Every Child a Chance charity (a coalition of business partners and charitable trusts) and government. The aim was to enable the lowest-attaining children to make greater progress toward expected levels of attainment in mathematics, catching up with their peers and performing at least at average levels on school assessment tests, wherever possible, by the end of the second year of primary school. The original intention was to provide intensive support in mathematics to 30,000 Year 2 children annually, though this has been significantly reduced due to the financial crisis of 2008 and subsequent government spending cuts.

Dunn, Matthews, and Dowrick (2010) developed the Numbers Count programme, which draws on aspects of three existing interventions: Multi-Sensory Mathematics (developed in Leeds using Numicon materials), Numeracy Recovery (developed in Hackney) and Mathematics Recovery (Wright et al., 2006). This programme involved careful assessment of individual children’s strengths and weaknesses, followed by intervention targeted to addressing specific weaknesses, and emphasizes the development of number concepts through multisensory teaching. It included a wide variety of components of arithmetic but places particular emphasis on methods of counting and number representation. Children received a half an hour of individualized or sometimes very small-group (two or three children to a teacher) intervention per day. It was delivered by teachers who have received Masters level training. In the initial stages of the project, 2621 Year 2 children, across 27 English local authorities, took part in Numbers Count. They received an average of 40 half-hour individualized Numbers Count lessons in a term, delivered by teachers who had received Masters level training. The participating children were given the Sandwell Test, a standardized arithmetic test, before and after entering the programme, and were retested 3 months and 6 months later.

Torgerson, Wiggins, Torgerson et al. (2011) carried out an independent evaluation of the programme. About 12 children within each of 44 schools were randomly allocated to either an intervention group or a waiting-list control group. Children in the intervention group received an average of 40 half-hour individualized Numbers Count lessons in a term, delivered by teachers who had received Masters level training. The participating children were given the Sandwell Test, a standardized arithmetic test, as a pretest and were post-tested on the Sandwell Test after 3 months and 6 months and also the Progress in Maths 6 (PIM 6) test after 3 months. Findings showed that the intervention group performed significantly better than the controls on the PIM 6 test (effect size 0.33).

The changes in the Sandwell scores were greater. Before entering the programme, the children's Number Age was on average 11 months below their Chronological Age. On average, they gained 14 months in Number Age in one term, a ratio gain of over 4 (months gained in Mathematical Age divided by mean duration of intervention in months), and were scoring at chronological age level by the time they exited the programme. However, it must be noted that, while the PiM scores were marked by people blind to the children's group assignment, the Sandwell scores were not, so that there could have been unconscious bias with regard to the latter. As always, the question arises of whether the gains will be maintained over the long term. A long-term evaluation is currently being carried out to investigate whether the effects of the intervention persist to the end of primary school and into secondary school.

What Makes Interventions Effective?

As can be seen, programmes vary considerably, both according to the theories and interests of the researchers and according to the target group, e.g. age, nature and extent of mathematical difficulties, etc. However, there are several features that effective programmes share. These include effective assessment of pupils' initial performance level, including, in the case of pupils with mathematical difficulties or low attainment, diagnosis of individual strengths and weaknesses. They also involve taking a developmental approach and applying knowledge of how knowledge and performance typically develop in the age group being studied. They also involve careful planning, taking availability of resources into account, and appropriate use of school staff: the best-designed programme will not work if teaching staff are unavailable, excessively overburdened or not adequately trained to deliver the programme. Another key feature of an effective programme is its ability to motivate pupils, and to prevent or counteract the association of mathematics with boredom, or worse, fear and anxiety. The use of games, for example, is a recurring feature of promising programmes, especially with preschool and primary school children.

Primary school age is where interventions have been most focussed. There are several reasons for this. Mathematical difficulties become noticeable by this stage, whereas they are harder to detect at an earlier stage. Moreover, primary and early secondary pupils are the ones who are most universally doing mathematics at school. Pre-schoolers are by definition not attending formal school and until recently were unlikely to be in an educational setting at all. Until relatively recently secondary pupils who were struggling with mathematics could partially or completely abandon the subject at an early stage. But all primary pupils are spending a significant amount of time on mathematics.

Moreover, it is desirable to intervene at the primary school stage in case of difficulties with numeracy, in order to prevent these problems from having a negative impact on pupils reaching later stages in the curriculum or leading to their developing serious mathematics anxiety.

It is noteworthy that nearly half of the British adult working population are performing only at primary school level in mathematics (BIS, 2011). While part of this is undoubtedly because many pupils struggle with mathematical concepts taught in secondary school, it is also likely that pupils are failing to progress because they have not fully come to grips with primary school material. This makes interventions at the primary school level important: ranging from whole-school interventions to improve overall performance to highly individualized interventions for pupils who are seriously struggling.

There is still much work to be done in developing and evaluating programmes. Ideally, there needs to be long-term follow-up of programmes, to see whether the programmes have an impact on pupils' long-term mathematical performance and on other outcomes. There should also be more comparisons between different programmes, with a particular focus on investigating whether different programmes are more effective with different groups of pupils. Furthermore, there should be more study of whether the size and nature of the group, in which programmes are delivered, affects effectiveness: e.g. whether and under what circumstances individualized interventions are more effective than group interventions, or small-group than large-group interventions.

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