Asset Pricing in Dynamic Macroeconomics



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A thesis submitted in partial fulfilment for the degree of

Doctor of Philosophy

Hilary 2017

This thesis is dedicated to my Papa, you taught me to have the confidence to follow my dreams. I am the man I am today because of you. I miss you...

Statement of Originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes.

I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

Rahul Raagvindra Babu Nath

Acknowledgements

First and foremost I would like to thank my supervisor Professor David Vines, without whom this thesis would not have been possible. Your constant support and encouragement was more than I could have asked for. I also enjoyed our occasional differences of opinion and the frank discussions these prompted - this thesis and the argument in it are the better for our discussions. You have forced me to really think about the economics in this thesis and how to express my opinions much better. Your intuitive grasp of so much of economics continues to inspire me to be a better economist. Thank you for teaching me so much, especially about how to think about economic problems and economics in general.

I would like to thank the Department of Economics at the University of Oxford for their support during what was at times have been very difficult times. I would also like to thank all of the staff at Balliol College for their constant support during my studies. In particular I would like to thank Dr. Nicola Trott, Rev. Bruce Kinsey and Prof. Thomas Melham for all of their support on behalf of the Balliol, the three of you have provided unwavering support, advice and friendship for which I will be eternally grateful.

I am grateful to Balliol MCR for their support and the great friendships that I have made there. You have all been my family and made Balliol MCR my home away from home. I would particularly like to thank

my friends, both at Balliol and further afield, you all know who you are! While there are too many to name individually, there are some who deserve special mention - in particular I would like to thank Phoebe, Sara, Elisabetta, Calum, Pierre, Fabio, Helen, Megan, Lucy, AK, Vassili, Chris K, Guy, Robin and Leea. Your unwavering support, holiday ideas, dinners, conversations over drinks and all of the other crazy things we have done over the past five years have made this a time in my life that I will always cherish and look back on with the fondest of memories. I am a better person for having met each and every one of you. At this time I would also like to pay tribute to the countless bottles of wine, whiskey and beer that made the ultimate sacrifice in the completion of this thesis; I only hope it was not too detrimental to the quality of the argument presented here. A lot of coffee went into this thesis so I would also like to thank Zappi's Cafe for keeping me caffeinated and for introducing me to my Zappi's family - thanks for keeping me sane!

Finally I would like to thank my family for without their love and support I would not have been able to achieve all that I have.To my brother, Prashaant - your support from the other side of the world has meant a lot and helped me in the darkest times. In particular I would like to thank my Dad for everything, I would not be here today without your love, unwavering support and confidence, and all of your sacrifices. I am proud that I was able to fulfil your dream for me to complete this thesis. My only regret is that you could not be here to share this moment with me.

This work was completed through the generous support of the University of Oxford Clarendon Fund Scholarship.

Abstract

How does monetary policy affect financial markets? Coming out of the financial crisis much ink, both academic and non-academic, has been spilled trying to understand how policy makers should react to mitigate such a financial crisis. While there has been significant focus, justifiably, on crises, policy design and financial market transmission, there is very little theoretical work on how the effects of central bank policy transmits to asset prices. This thesis contends that the reasons for this dearth of theoretical research are two-fold - the dominance of log-linearisation in the New Keynesian tradition and a move to marginalise labour market channels in the asset pricing literature. The first means that these models fail to satisfactorily explain asset prices, while the second is a response to the perverse movements of employment in these models. Furthermore, attempts to fix the second problem make the first problem even worse. I begin this thesis by addressing these two concerns. I show that by addressing them one can better explain reality both in terms of the business cycle and asset prices within this models. I proceed to develop a non-linear New Keynesian model and use it to construct a non-linear asset pricing relationship. Finally, I use this non-linear model, as a laboratory to analyse the transmission to financial markets of central banking policy following various simple Taylor rules. This work allows one to move from a situation where the impact of policy is well understood to one where the understanding is as yet incomplete. The insights gleaned from this research can be used to inform policy design with some understanding of the expected impact on asset markets.

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CHAPTER 1

Introduction

> *Clearly finance has a lot to say about macroeconomics, and it says that something is desperately wrong with most macroeconomic models.*
>
> — **John Cochrane**
>
> Cochrane (2005)

> *All models are wrong but some are useful*
>
> — **George Box**

This thesis is comprised of four chapters, in three distinct parts. In this thesis I contend that the reasons for the dearth of theoretical research into asset pricing in New Keynesian Models is two-fold - the dominance of log-linearisation in the New Keynesian tradition and a marginalisation of the labour market channel in the equilibrium asset pricing literature. I begin the thesis by addressing these two concerns before proceeding to show that one can price assets in the New Keynesian model if one

is willing to proceed with a non-linear analysis. Part I deals with these themes. Part II presents the New Keynesian extension of Part I. While Part III uses the model of Part II as a policy laboratory to analyse how monetary policy characterised by simple Taylor Rules affect asset prices. A brief overview of each of these main parts of the thesis are presented below as a precursor to the broader motivation for this thesis.

Part I

The first part of the thesis is made up of two chapters - 'A Countercyclical Labour Problem' and 'Asset Pricing with GHH Preferences'. The former addresses the reasons behind the dearth of asset pricing in New Keynesian models and demonstrates that these are not insurmountable. The latter builds on the findings of the former to propose a preference based solution to the excessive countercyclical labour problem outlined above. The models of Part I are all Real Business Cycle models and abstract from any New Keynesian features, e.g. nominal rigidities. The Real Business Cycle framework provides the core model of New Keynesian models; I use this model to begin at the beginning so to speak. By stripping away all Keynesian features and address the concerns in a purely real model, one can better understand what the Keynesian features add to the explanation in latter chapters.

'A Countercyclical Labour Problem', the first chapter of the thesis, serves a dual purpose - to explain the excessive countercyclical movement of labour and to present the non-linear solution method used throughout the thesis. First, it provides a clear diagnosis of the excessive countercyclical movement of labour that has been lacking in real models purporting to explain asset prices. I show that the reason for this excessive countercyclical movement is a consequence of the income effect dominating the substitution effect, and that this is endemic to CRRA preferences. Second, it presents the Parametrised Expectations Algorithm, the non-linear solution method

used throughout the thesis. I show that there is a better joint replication of asset pricing and business cycle facts under the non-linear methods than under standard log-linear methods. This improvement is significant and hints at the gains that can be made in models with greater non-linearities, like the New Keynesian models studied in later chapters.

That CRRA preferences are not well suited to address the excessive countercyclical labour problem using a pure preference based approach is established as one of the key conclusions of 'A Countercyclical Labour Problem'. However, I note that literature seems to prefer CRRA preferences leading to the common solution being to introduce frictions into the model that restrict the contemporaneous response of labour as discussed previously. 'Asset Pricing with GHH Preferences', the second chapter of this thesis, shows that a preference based solution is possible if one is willing to look beyond CRRA preferences to the GHH preferences of Greenwood et al. (1988). GHH preferences have all of the features that recommend Epstein-Zin preferences, the "go to" alternative to standard CRRA preferences in asset pricing. GHH preferences are considered instead of Epstein-Zin since standard GHH preferences have no income effect; so it is a natural candidate to remedy the labour market given the diagnosis of the first chapter. The key finding of the second chapter is that once internal habits are introduced into standard GHH preferences the income effect re-emerges but one may control it using the habit intensity parameter. Thus one may control the cyclical movement of labour via the habit intensity parameter and this can be calibrated to induce an appropriate cyclical movement in labour.

There is a long tradition in the equilibrium asset pricing literature of altering the preference structure to better replicate asset pricing facts. This tradition was started by Epstein and Zin (1989) andConstantinides (1990) in their attempts to 'solve' the

Equity Premium Puzzle and has persisted into the present, a recent example being the preferences introduced in Jaimovich (2008) and Jaimovich and Rebelo (2009). The second chapter of Part I differs from this approach in one fundamental respect. GHH preferences are introduced to remedy the labour market consequences of frictions rather than deal directly with the asset pricing results.

The study of labour in Part I serves as the necessary first step in this thesis since the labour market plays a central role in New Keynesian models. It is via the labour market that nominal rigidities are generally justified, indeed rigidities in wage setting is the clearest and most visible example of nominal rigidities. The impact of nominal rigidities is also most clearly seen in the labour market. Thus an understanding of the labour market behaviour in the absence of nominal rigidities allows one to differentiate between labour market mechanisms that arise due to asset pricing features (i.e. consumption habits and capital adjustment costs) in RBC models and those that are the result of introducing the New Keynesian features (i.e. monopolistic competition and nominal rigidities).

Part II

Part II consists of a single chapter - 'Asset Pricing in a New Keynesian Model with Nominal Price and Wage Rigidities'. It extends the analysis of Part I to a New Keynesian model with capital and a simple financial sector consisting of an equity market and a market for risk free bonds. The New Keynesian framework leads to the non-neutrality of monetary policy due to the presence of nominal rigidities that prevent the economy from reaching the first best allocation. Hence it is the natural class of models for an analysis of central banking policy and asset prices as it gives central bank policy a role. This chapter therefore provides the necessary intermediate step to achieve the aim of this thesis in studying the asset pricing implications of

macroeconomic policy.

The key result of this chapter is the derivation of a relationship linking the return on equity to real economic variables (i.e. consumption, investment, labour and capital). The approach uses purely non-linear methods and a general definition of the financial structure of the firm. Such a relationship has not been derived before. I show that it can be used to understand the behaviour of asset prices in New Keynesian models. It establishes the fundamental link between the real economy and the financial economy. How policy affects real variables is well studied and understood - the relationship derived here provides the link that allows one to understand how that policy impact flows onto affect asset prices via the real economy.

In addition to this theoretical asset pricing result, the chapter sheds further light on the dynamics of the labour market in this class of models. The quantitative results suggest that the New Keynesian approach fares no better at the joint replication of asset pricing and business cycle results of the models presented in Chapter 3. Further, I find that the countercyclical movement of labour in Part I re-emerges.[1] This requires more subtle diagnosis of the income and substitution effects in the non-linear New Keynesian framework. Such an analysis of the labour market has not been, to my knowledge, provided in the literature before.

Part III

Part III also consists of a single chapter - 'Monetary Policy Impact on Asset Prices' - the fourth, and final, substantial chapter of the thesis. This chapter takes the model developed in Part II and treats it as a laboratory in which to run monetary policy experiments.

[1]This is a priori expected in the New Keynesian model and discussed extensively throughout the thesis.

In the presence of nominal rigidities in prices and wages, i.e. when there is a degree of stickiness in these variables, a central bank fully stabilising price inflation is no longer the optimal policy response. Thus there does not exist an optimal policy and the choice of policy regime forces the central bank into a trade-off. This is simply due to the central bank having a single instrument, the nominal interest rate, to achieve two objectives - price and wage stability. In particular I consider two popular characterisations of central banking policy regimes - simple non-linear Taylor rules with and without interest rate smoothing.

I analyse the asset market impact of these policy rules and find that the degree of smoothing affects asset markets via the volatility of returns with little impact on mean returns. This suggests that central banks following a smoothing regime affect asset markets via an expectations channel not present in simple rules without smoothing. This has implications for the design of policy - namely that by following a smoothing regime the central bank can generate lower asset market volatility (i.e. financial stability) and achieve macroeconomic stability. While the asset pricing analysis is the primary focus of this chapter, I also show that there is a non-trivial dynamic response of labour to the degree of smoothing. I show that this response is central to understanding all other dynamic responses of the economy and is the consequence of the non-linear nature of the labour market.

Finally, this chapter provides more than insight into the design of monetary policy regimes when both macroeconomic stability and financial market stability are objectives; it also acts as an objective test of the model developed in the thesis. The data suggest that the nominal interest rate exhibits significant inertia, i.e. a smoothing regime is a better depiction of reality than non-smoothing regimes. When faced with

this test the model continues to perform no worse with respect to mean asset returns but moves significantly away from the data with respect to asset market volatility. This highlights that there is still work to do. In particular one needs more than the two shocks in the current model. These avenues for future research are discussed at some length in the Conclusions chapter.

1.1 Motivation: Why is this important?

The past decade has witnessed significant upheaval in the global economy. One has seen the bursting of the housing bubble in the United States precipitate a global financial crisis which ultimately led to a global sovereign debt crisis. This period of upheaval, not yet over, has witnessed many an economy teeter on the edge. Central banks, given their broad mandate of ensuring macroeconomic stability, have, for better or worse, found themselves one of the key protagonists in all of these crises and central to any subsequent narratives of this period. Central banks have been, and are, charged with seeing into the abyss and finding a way of ensuring the economy is brought back from the brink. But how did central bank policy response to the bursting of the housing bubble affect financial markets? Did the actions of central banks during the global financial crisis, aiming to generate some form of macroeconomic stability, actually lead to a more severe crisis? These are some of the questions that motivate this work and inform its agenda; but this is not a thesis about crises, central banking policy and financial markets, it is about the more mundane non-crisis periods. Before one can meaningfully understand how central banks affect asset markets in a crisis one must get a grip on how central bank policy affects asset markets during non-crisis periods.

How does monetary policy affect financial markets? Coming out of the financial

crisis much ink has been spilled, both academic and non-academic, in answering this question. There has been an apparently single-minded focus in trying to understand how policy makers should react to mitigate such crises. In the context of a prolonged period of crises, as in the past decade, this is understandable. But, how are assets fundamentally priced in the models that frame our current understanding of the macroeconomy - the New Keynesian Model? This question has received much less attention. Attempting to answer the former at the expense of the latter is a little like putting the cart before the horse. This thesis begins to fill this gap and addresses the latter question as a way to better understand the former. That is, this thesis is predominantly an exploration of asset pricing questions in macroeconomics with a view to provide a framework that permits one to understand how macroeconomic policy, in particular central bank policy, affects financial markets.

There is a limited literature looking at how asset prices behave in New Keynesian models and all of this work has only been published very recently; Wei (2009), De Paoli et al. (2010),Challe and Giannitsarou (2014) and Swanson (2015) being the most relevant literature for the central aims of the thesis. De Paoli et al. (2010) stress that the importance of New Keynesian models in policy analysis requires one to at least understand the asset pricing implications of these models. Furthermore the recent financial crisis has put into the spotlight the importance of understanding how central banks affect asset prices in both crises and non-crisis periods.

The Dynamic Stochastic General Equilibrium (DSGE) modelling paradigm, first through Real Business Cycle (RBC) models and more recently through New Keynesian models, has become the dominant modelling paradigm in modern macroeconomics. Despite its dominance in modern macroeconomics, acceptance of the DSGE paradigm is not universal among macroeconomists. A good overview of both sides

of the debate may be found in the September 2011 issue of The Manchester School arising from discussions at the Money, Macroeconomics and Finance Research Group meeting of 2011. (Driffill, 2011; Pesaran and Smith, 2011; Ascari, 2011; Miller, 2011; Werner, 2011; Driffil, 2011) Despite the debate surrounding the DSGE paradigm, Blanchard (2016), in a pragmatic defence of DSGE models concludes that "...DSGE models are flawed. But DSGE models can fulfil an important need in macroeconomics, that of offering a core structure around which to build and organise discussions". Indeed, even the most vehement detractors cede that DSGE models are still useful and should not be scrapped in their entirety. In this thesis I follow the Blanchard (2016) approach to the use of DSGE models; namely that they are a useful tool that allows one to answer questions about the fundamental links between the real economy and financial markets. There may - or may not - be others, this a question which, deliberately, I do not examine.

The success of the DSGE paradigm can be attributed to a certain degree of success they have achieved in providing a quantitative description of business cycle fluctuations of macroeconomic quantities (such as GDP, investment and consumption). These same models, however, have had limited success when faced with the quantitative replication of asset pricing facts. The most famous examples being the "Equity Premium Puzzle"[2] of Mehra and Prescott (1985) and the "Risk Free Rate Puzzle"[3] of Weil (1989). That is, models that can explain quantities cannot explain prices. But, the DSGE paradigm assumes a general equilibrium framework where one must necessarily be able to move with ease between quantities and prices without loss of fi-

[2]The Equity Premium Puzzle is a quantitative puzzle in macroeconomics. Mehra and Prescott (1985) showed that one cannot quantitatively replicate the empirical equity premium in the U.S. in a simple RBC model without resorting to an implausibly high degree of risk aversion on the part of agents or empirically unacceptable consumption volatility.

[3]The Risk Free Rate Puzzle is a counterpart of the Equity Premium Puzzle. Weil (1989) shows that solution of the Equity Premium Puzzle by resorting to high degree of risk aversion leads to risk free rates that are too high. Thus solutions of the Equity Premium Puzzle lead to a deterioration of risk free rates.

delity. That one cannot do so highlights a major shortcoming of the DSGE paradigm, indeed one that many of its detractors have latched onto as a means to discredit the paradigm.

Modern dynamic macroeconomic methodology, i.e. the DSGE paradigm, assumes that there exist optimising agents who allocate quantities over time to maximise lifetime utility. Such optimal allocation decisions are the result of equating marginal utilities and marginal rates of transformation both intra- and intertemporally. Cochrane (2005) highlights the fact that asset markets are inextricably linked to the real economy as they provide the mechanism that "does all this equating". Nevertheless, the inability of modern macroeconomic models to adequately explain asset pricing results is a fundamental shortcoming that cannot be dismissed lightly as it forms the basis of much of the intuition about macroeconomic fluctuations. (Mehra, 2003) In effect RBC models can describe the characteristics of the business cycle but not one of the core mechanisms which bring about such facts. Furthermore, Mehra (2003) contends that this inability to account for asset pricing facts undermines the viability of this entire class of models as a tool for policy analysis. Asset markets are therefore the truest test of the usefulness of this class of models and their ability to fulfil the challenge of Lucas (1980) to the DSGE modelling paradigm, then in it's infancy:

> *"One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies that would be prohibitively expensive to experiment with in actual economies can be tested out at much lower cost...Any model that is well enough articulated to give clear answers to the questions we put to it will necessarily be artificial, abstract, patently 'unreal'...The more dimensions on which the model mimics the answers actual economies give to simple questions the more we trust its answers to harder questions. This is the*

sense in which more 'realism' is clearly preferred to less."

The emergence of the equilibrium asset pricing literature in response to asset pricing concerns, i.e. the behaviour of prices, within DSGE models may be viewed as a response to this challenge. This thesis follows the equilibrium asset pricing research agenda; it considers models that can jointly replicate the salient business cycle and asset pricing facts. Unlike the vast majority of the equilibrium asset pricing literature, the driving force behind this thesis is not to solve the famous asset pricing puzzles. It is a priori known that the asset pricing results will not be close to the data for two main reasons. First there are not enough shocks to adequately replicate the data. In my model I only consider two types of exogenous shocks, a far cry from the seven such shocks used in Smets and Wouters (2007), the accepted benchmark model for medium scale New Keynesian DSGE analysis. Secondly I focus on fundamental asset pricing, i.e. that implied by the fundamentals of the real economy, abstracting from insights from behavioural economics which, I believe, will ultimately prove essential to any meaningful attempt at quantitative replication of asset prices. Rather, it is the aim of this thesis to give a clearer understanding of how assets behave in DSGE models, in particular the New Keynesian models which have become the dominant theoretical model in macroeconomic policy analysis. I am interested in how the various assumptions and frictions of these models interact to fundamentally impact on asset prices and ultimately to study how the conduct of monetary policy affects these interactions in asset markets. This is in keeping with the 'neoclassical' tradition espoused in Chari et al. (2009) who state:

> *"The tradition favoured by many neoclassicals (including us) is to keep a macromodel simple, keep the number of its parameters small and well motivated by the microfacts, and put up with the reality that no model can, or should, fit most aspects of the data."*

To bring all of this back to the motivation for this thesis stemming from the crisis - a final quote from Ascari (2011),

> *"What one can conclude, certainly, is that current macroeconomic models lack an important part: the linkage between finance and macroeconomics. This part is very stylised in current DSGE macromodels, and the crisis made it clear that this link is much more important than we once thought."*

This thesis will look at how asset prices behave in the 'stylised DSGE macromodels' referred to above and what, if anything, one can say about the effect of macroeconomic policy on asset prices within these models. However, before we proceed on that journey, I briefly introduce the literature on asset pricing in DSGE models and put forward two hypotheses as to why the study carried out in this thesis has not yet been carried out in New Keynesian models.

1.2 Asset Pricing in DSGE Models - A Critical Introduction

The inability of modern macroeconomic models to replicate asset pricing facts is well documented (Mehra and Prescott, 1985; Weil, 1989; Cochrane, 2005). This has led to the emergence of the equilibrium asset pricing literature, the aim of which is to find a model that can jointly replicate asset pricing and business cycle facts. The idea of joint replication, first made explicit in Rouwenhorst (1995), provides an objective criterion by which to judge equilibrium asset pricing models as one does not want to improve asset pricing results at the expense of business cycle results. The models in this literature look to find explanations of the equity premium, and routes to increase the quantitative equity premium, that do not simultaneously lead to significant deterioration of the business cycle characteristics in these models. That is, the literature

is an attempt to understand asset prices (most commonly the equity premium) within the constraints of well founded macroeconomic models.

The equilibrium asset pricing literature is driven by a philosophy that the frustrating of agent desires through the introduction of frictions is essential to improve asset pricing results. Boldrin et al. (1995) suggest that two ingredients are necessary, "(i) households must have a strong incentive to buy (sell) assets when the marginal utility of consumption is low (high), and (ii) a technology which frustrates these desires." (Boldrin et al., 1995) This broad approach has led the literature to a consensus that capital adjustment costs and habits in consumption are both essential ingredients in any successful attempt at joint replication. Indeed Jermann (1998), one of the earliest attempts at joint replication, has a measure of success in a model with capital adjustment costs, consumption habits and inelastic labour supply. The presence of capital adjustment costs mutes the contemporaneous response of investment to shocks thus forcing consumption to respond more vigorously than otherwise. However, habits in consumption mute the contemporaneous response of consumption. Together they force agents to both use the costly inter-temporal investment channel in order to achieve smoother consumption and also have to suffer a greater degree of consumption volatility. Thus they capture the idea that agents "have to both care [about smooth consumption] and be prevented from doing anything about it [due to capital adjustment costs]." (Jermann, 1998)

Consumption habits, introduced by Constantinides (1990) relax the non-separability of consumption across time in agent preferences. This captures the idea that past consumption decisions affect current marginal utility. Such past decisions may be captured by the agent's own consumption (internal habits) or the consumption of the population (external habits). In either context, the presence of consumption habits

introduces inertia into the consumption decisions of agents. This inertia accentuates the desire for smooth consumption leading to a greater movement in investment if labour is inelastic. However, as Jermann (1998) recognises, this is not enough. Having forced agents to use investment channel more intensively through the introduction of internal consumption habits one needs to make it costly to use the investment channel in order to generate a sizeable equity premium.

Capital adjustment costs slows down the capital accumulation process.[4] This captures the idea that investment is not costless and thereby introduces inertia into the capital accumulation process. Capital adjustment costs allow Tobin's Q to vary over the business cycle, a feature not present in the prototypical Real Business Cycle model. This variation of Tobin's Q has been found to be useful when attempting to replicate asset pricing facts (e.g. Jermann (1998); Boldrin et al. (2001)).

A model with capital adjustment costs, internal consumption habits and inelastic habits can only get one so far in the quantitative replication of asset prices, especially if one is also simultaneously concerned about reasonable quantitative business cycle results within the model, i.e. the equity premium is still either too small or if there is a significant equity premium then there are features of the business cycle that are at odds with the data (e.g. cycle length and the movement of labour). In his review of the equity premium puzzle Mehra (2003) highlights the following subsequent development of the equilibrium asset pricing literature. The most fruitful avenues have focused on alternate preference structures (e.g.Epstein and Zin (1989); Constantinides (1990)), issues of idiosyncratic and uninsurable risks that lead to market incompleteness (e.g. Heaton and Lucas (1996)), disaster states where small probability events lead to large consumption falls (e.g. Rietz (1988)) and borrowing constraints and transaction costs

[4]The standard capital accumulation function is a linear function of capital, investment and depreciation.

(e.g. Constantinides and Duffie (1996); Constantinides et al. (2002)). While these features have been important in the development of the understanding of the equity premium, and prices in DSGE models, I abstract from all except consumption habits in this thesis. Such features are necessary in any meaningful quantitative attempt to replicate asst pricing results but are not considered here so as to focus on simple DSGE models and isolate the impact of various modifications introduced throughout this thesis. Indeed the model in this thesis only takes the core modifications of the DSGE model by this literature, namely internal habits in consumption and capital adjustment costs.

1.3 The Dearth of Asset Pricing in New Keynesian Models

In this thesis I contend that the reasons for the dearth of theoretical research into asset pricing in New Keynesian Models is two-fold - the dominance of log-linearisation in the New Keynesian tradition and a marginalisation of the labour market channel in the equilibrium asset pricing literature. These hypotheses, presented below, constitute two of the central themes that run throughout this thesis.

1.3.1 Log-Linearisation: A Folly for Asset Pricing

A central methodological theme maintained throughout this thesis, and indeed my research, is the eschewing of log-linear/log-normal solution techniques in favour of purely non-linear techniques to solve the inherently non-linear systems that characterise the economy in DSGE models. The modus operandi of the New Keynesian tradition is to work with linearised models. By using non-linear methods I show that there are significant gains in the quantitative replication of asset pricing results. The reason lying behind the finding is actually very straightforward. The explanation of

the equity risk premium - and the response of this premium to macroeconomic shocks - depends fundamentally on the non-linearity which comes from the risk aversion of these who hold financial assets. This means that linearising a model prevents one from getting a proper grip on the determination of the equity premium, and asset prices more generally. I show this using a very simple example of the equity risk premium in what follows.

The Euler equation is the key to pricing assets in dynamic macroeconomic models and takes general form,

$$1 = E_t \left[m_{t,t+1} R_{t+1} \right]. \tag{1.3.1}$$

The stochastic discount factor, $m_{t,t+1} = \beta \frac{U_{c,t+1}}{U_{c,t}}$, is the ratio of marginal utilities of consumption in adjacent periods. It therefore tells one how agents within the model value future consumption in terms of current consumption. The return of the asset in question is given by R_{t+1}. So the Euler Equation gives the return, in equilibrium, that makes agents indifferent between deferring consumption by investing in an asset and contemporaneous consumption.

The "folly of log-linearisation" for asset pricing in macroeconomics lies in how it deals with the Euler Equation. The method for dealing with the Euler Equation rests on the assumption that,

$$\mathcal{G}\left(E\left[X\right]\right) = E\left[\mathcal{G}\left(X\right)\right]. \tag{1.3.2}$$

This only holds when $\mathcal{G}\left(\cdot\right)$ is a linear function. There is inherent folly in assuming this continues to hold when $\mathcal{G}\left(\cdot\right)$ takes the logarithmic functional form. For in doing so one has assumed that the log of the sum is the sum of the logs - the expectations operator, $E\left[\cdot\right]$, is just a probability weighted sum. The mathematically correct way to deal with the logarithmic functional form is to use Jensen's Inequality, but this

renders the solution method unusable. In fact most approaches pay lip service to Jensen's Inequality but argue that the term is small since one is only considering small deviations around the steady state. While this is a valid argument when dealing with real economic variables, it makes log-linearisation fundamentally unsuited to the study of crises and asset pricing where the Jensen's Inequality term is crucial to our understanding. In crises the Jensen's Inequality term is important because one is by definition far from the steady state. In the case of asset prices the Jensen's Inequality term is important as it captures higher order moments essential to understanding asset prices. More generally, ignoring Jensen's Inequality requires that one only consider small shocks that don't move the economy "too much" from the steady state. But this presupposes that only certain small shocks may be permitted to hit the economy and that they may not combine to move the economy by "too much" away from the steady state. In the highly non-linear systems log-linearisation seeks to approximate such restrictions on the shocks is unlikely to hold. Indeed one may reasonably expect that due to the highly non-linear form, even small shocks may lead to points far from the steady state.

To see the damage done by log-linearisation techniques for asset pricing let us consider two types of asset - a risk free bond whose return is known at t $\left(R^f_{t+1}\right)$ and a share of equity whose return is a random variable at t $\left(R^e_{t+1}\right)$. Using (1.3.1) one gets,

$$R^f_{t+1} = \frac{1}{E_t\left[m_{t,t+1}\right]}, \tag{1.3.3}$$

since the is risk free return is known. The risky asset is a little more complicated since it is a random variable and must be written using a covariance form[5] which,

[5]The covariance between two variables is given by $\operatorname{cov}(X,Y) = E[XY] - E[X]E[Y]$, which holds both conditionally and unconditionally.

after some algebra yields,

$$E_t\left[R^e_{t+1}\right] = R^f_{t+1} - R^f_{t+1}\text{cov}_t\left(m_{t,t+1}, R^e_{t+1}\right). \tag{1.3.4}$$

Thus the expected return on equity depends on how it moves with the stochastic discount factor and it is priced relative to the risk free return. If the covariance is negative then equity destabilises consumption and therefore it must have a return higher than the risk free rate to induce agents to hold it. Conversely if the covariance is positive then equity is insurance as it smooths consumption. This is a standard result in asset pricing that illustrates that higher order moments, captured by the covariance, are essential to the pricing of risky assets since it is that term that captures the risk profile of the asset.

Under the log-linearisation assumption (1.3.2) one gets, to a first order Taylor approximation,

$$R^{f,LL}_{t+1} = -E_t\left[ln\left(m_{t,t+1}\right)\right] \tag{1.3.5}$$

$$E_t\left[R^{e,LL}_{t+1}\right] = -E_t\left[ln\left(m_{t,t+1}\right)\right] \tag{1.3.6}$$

so that there is no difference between the equity and the risk free return. This makes sense - simply put the use of the logarithmic function leads to a breaking of the covariance structure thereby removing the impact of higher order moments.

Log-linearisation is therefore not useful for asset pricing as it removes the higher order moments which are central to asset pricing results. Log-linearisation also implies linearity between the certainty equivalents of assets (Jermann, 1998), which is not a priori a desirable feature.

The response of the literature has been to modify this basic approach to recover the higher order moments. This is commonly done by either (1) recognising that the expectations operator is a sum and taking ever higher order Taylor approximations before imposing the log-linear assumptions or (2) imposing a distributional assumption on the terms within the Euler equation. If one follows the first approach then, in order to have any reasonable asset market features one needs to take at least third or fourth order Taylor approximations[6] which is both cumbersome and fraught with computational errors. Following the second approach, an assumption of joint log-normality of terms within the Euler Equation is commonly used as a means of using the characteristics of an entire distribution. But this begs the question - what distributional assumption is reasonable? There are many distributions that look like the log-normal and one suspects that the use of the log-normal is rooted deeply in the questionable idea that asset returns can be completely characterised by mean and variance. Furthermore, imposing a distributional assumption adds extra a priori structure on the model which prevents the model from "speaking".

1.3.2 The Importance of the Labour Market Channel - What's Missing in Asset Pricing Models

A central tenet of macroeconomics is the idea that economic agents desire smooth consumption profiles. The desire for smooth consumption leads agents to exploit any opportunity to achieve this goal. Asset markets allow households to satisfy their preference for smooth, stable consumption. As discussed previously, in order to generate plausible asset pricing results one must introduce frictions which frustrate this desire for smooth consumption. The asset pricing literature has settled on the combination of strong internal habits in consumption and strong capital adjustment costs as necessary for any viable quantitative replication of asset pricing and business cycle results.

[6]See Swanson (2015)

Indeed both Jermann (1998) and Boldrin et al. (2001) have a degree of success jointly replicating quantitative asset pricing and business cycle results, however this is at the expense of the labour market.

The need to marginalise the labour market channel is pervasive in all successful explanations of asset prices in macroeconomic models. Jermann (1998) and Boldrin et al. (2001) provide two illustrative examples. In his seminal work on asset pricing in macroeconomic models Jermann (1998) assumes completely inelastic labour supply. Boldrin et al. (2001), while allowing for an endogenous labour market, can achieve superior asset pricing results in their preferred two sector model only by assuming limited contemporaneous inter-sectoral mobility of labour. The reasoning behind this response of the literature is straightforward - by marginalising the labour channel one forces agents in the economy to use the costly inter-temporal channel, i.e. investment, to smooth consumption. This channel is costly due to the presence of capital adjustment costs.

When labour is endogenous, households, rubbing up against constraints on consumption and investment, mercilessly exploit the labour channel as a means to smooth income. Households, by smoothing income, thereby achieve their consumption smoothing goal. The reason is obvious - endogenous labour provides an intra-temporal channel that gives agents a way out from the constraints imposed on the inter-temporal channel. However, an endogenous labour market is not compatible with realistic asset pricing results, even in the face of strong consumption habits and strong capital adjustment costs. By allowing for fully endogenous labour response without labour frictions, i.e. by making the labour channel costly, one gets a deterioration of asset pricing results. This finding is not new, indeed it frames the basis of the literature's response to the labour market - marginalise the labour channel, i.e. the

intra-temporal channel, through various ad-hoc frictions to force agents to use the costly inter-temporal channel to achieve consumption smoothing.

If one can find a way to relax this trade-off between fully endogenous labour and asset prices then one has a framework that provides sharper insights to policy and theoretical questions. I show that one can resolve this trade-off without need to resort to assumptions that marginalise the labour market or the nominal rigidities necessary for New Keynesian models. I show that a simple preference based approach, using so-called GHH preferences, enables one to fix this problem in a non-arbitrary manner. This approach is deemed 'cleaner' than those hitherto proposed as it doesn't force one to rely on a sophisticated edifice of evermore subtle assumptions introduced to 'solve' the labour market. Furthermore, I hold that the current form of model specific solutions in RBC models are difficult to generalise to the New Keynesian framework. This makes all such solutions fit for purpose and severely limits the questions one can answer with this approach.

There is fierce debate surrounding the response of labour to a productivity shock - does labour rise or fall when the economy is hit by a productivity shock? In the Real Business Cycle paradigm a productivity shock is expansionary, while in the New Keynesian paradigm the response is contractionary. Indeed, Cantore et al. (2014) contends that this dichotomy has driven the majority of the literature, with respect to the cyclical response of labour, since Gali (1999).[7] Cantore et al. (2014) notes that in a model with fully flexible prices, like the RBC model, one can only expect a positive response of labour if one is to assume agents behave as expected - an increase in productivity leads to an increase in output which is fully met by firms who

[7]Smets and Wouters (2007) and Cantore et al. (2014) provide a good overview of this debate surrounding the response of labour to a productivity shock. For a detailed review of this debate the reader is referred to these two papers.

can adjust prices freely. In contrast, in a model with nominal rigidities, like the New Keynesian model, firms cannot freely adjust prices to meet excess output demand and so must reduce labour employed. I take an agnostic stance on this debate. However, the presence of such a debate underscores the importance of the labour channel in macroeconomic analysis. In marginalising the labour channel, I contend that the literature seeking to explain asset prices in macroeconomic models is counterproductive for macroeconomic models that presume to provide insights for policy advice. As such the models in this thesis all contain fully endogenous labour markets. I embrace the view that the labour channel is essential to the understanding of the dynamics of the economy and models that ignore the labour channel ignore a crucial part of the story.

1.4 Non-Linear Methods, Macroeconomics and Asset Pricing: A Short Literature Survey

The need for non-linear/global solution methods constitutes a significant motivation in the arguments presented above. In this section I provide a brief survey of the literature dealing with global solution methods and their application in various macroeconomic contexts, ending with a survey of the application of global solution methods for asset pricing.

The proposal and use of global solution methods is not new in macroeconomics. It has roots that go back at least to the work of den Haan and Marcet (1990) (and related earlier work) on the Parameterised Expectations Algorithm and the work of Judd (1992) in the development of general projection methods.[8] Despite this long tradition in the macroeconomics literature, global methods have generally been shunned

[8]The Parameterised Expectations Algorithm is a specific example of the projection method.

in favour of the much faster perturbation methods. Such perturbation methods use Taylor approximations around the non-stochastic steady state to solve the model; the first order approximation using a log transform gives the log-linear approach discussed above.

The emergence of the asset pricing concerns, and the creation of larger, more complex macroeconomic models that incorporate various non-linearities and many state variables has led to a burgeoning literature studying the numerical characteristics of different solution techniques. Examples of this literature include Aruoba et al. (2006) and Caldara et al. (2012) for closed economy DSGE models and Kollmann et al. (2011) for open economy DSGE models. Maliar and Maliar (2014) provide a thorough overview of this literature. The key findings of this literature highlight that projection methods (i.e. global methods) provide a more accurate solution than perturbation methods (i.e. local approximation techniques), but at the expense of computation time as projection methods suffer from the "curse of dimensionality". Perturbation methods, being inherently local methods, suffer from concerns about higher order dynamics driving the economy far from the local approximation point; a concern that has become pressing as macroeconomic models become evermore non-linear as their size increases. A related concern about higher order perturbation methods is the order of approximation to use in the Taylor approximation so that the researcher is satisfied that there are no higher order dynamics at work that are not captured.(Levintal, 2017) Recent work on 'Taylor projections', a hybrid projection-perturbation method, proposed in Levintal (2017) and Fernández-Villaverde and Levintal (2016) shows promise in achieving the accuracy of projection methods while benefiting from the speed of perturbation methods.

The application of global methods with reference to asset prices covers models with

recursive preferences (e.g. Caldara et al. (2012); Aldrich and Kung (2010); Pohl et al. (2014); Branger et al. (2016)); financial frictions (e.g. Gourio (2013)); rare disasters (e.g. Gourio (2012); Fernández-Villaverde and Levintal (2016)); stochastic volatility(e.g. Caldara et al. (2012); de Groot (2015)); heterogeneous agents (e.g. Krueger and Kubler (2004)); and heterogeneous capital (e.g. Branger et al. (2016)).

If one is determined to use global solution methods then the curse of dimensionality forces one to work with small scale models which have a relatively small number of state variables. The models dealt with in this thesis are sufficiently simple that the projection method used in this thesis, the Parameterised Expectations Algorithm, can be implemented without significant computational burden. Future work looking at more complicated models with more state variables would need to consider hybrid approaches like that of Fernández-Villaverde and Levintal (2016).

1.5 Some Stylised Facts

The idea of joint replication plays an important role in this thesis. One is interested in models with the ability to broadly replicate both quantitative asset pricing and business cycle results. The stylised facts presented below play a central role in the evaluation of the models and their performance at satisfying the joint replication criterion.

The business cycle stylised facts are well known and relatively standard in the literature. The stylised facts of interest are the second order moments of the data since the models are calibrated to match long run means. The business cycle stylised facts are taken from Kehoe and Perri (2002). Kehoe and Perri (2002) generate these stylised facts from a quarterly time series for the U.S. over the period 1970-1998. The volatil-

ity of output is measured in percentage standard deviation terms and values for all other variables are stated relative to output.

Table 1.1: Business Cycle Stylised Facts

Volatility of Output	1.72	
	Volatility	Contemporaneous Correlation
Consumption	0.79	0.87
Hours Worked	0.63	0.93
Investment	3.24	0.86
Productivity	0.54	0.78

Kehoe and Perri (2002)

In this thesis I am interested in understanding how one can improve the quantitative results with respect to hours worked when one allows labour to be fully endogenous in the model. In addition I look at how the change in the quantitative results for hours worked affects the quantitative results for other variables, i.e. is labour quantitatively important for the quantitative replication of other variables. The asset pricing stylised facts tend to be more variable than those for the business cycle depending on the time period sampled. Despite this variability across sample periods the same trends are always evident. Returns on 1-month Treasury Bills (the risk free proxy asset) are low and relatively stable, while the return on equity is substantially larger and more volatile, and an equity premium of $5 - 7\% p.a.$ is exists.[9] The asset pricing stylised facts are taken from In and Yoon (2007). In and Yoon (2007) generate these stylised facts from asset market data taken from the "Stocks, Bonds, Bills and

[9]The equivalent quarterly returns are in the range $1.2 - 1.7\%$ per quarter

Inflation Year Book" published annually by Ibbotson and Associates. So as to be comparable to the business cycle results, I truncate the sample of In and Yoon (2007) to be equal to that for the business cycle results, i.e. the sample used is from 1970, Q1 to 1998, Q4. The Treasury bills considered are 1-month U.S. Treasury Bills and the equity return is measured on the Standard & Poor's 500 stock index. All values are expressed as annualised percentages with quarterly values in parentheses.

Table 1.2: Asset Pricing Stylised Facts

	Mean Return	Standard Deviation
T-Bills	1.49	2.62
	(0.37)	(1.31)
Equity Return	7.87	16.72
	(1.96)	(8.36)
Equity Premium	6.52	15.69
	(1.59)	(7.84)

In and Yoon (2007); Ibbotson and Associates (2004)

In this thesis I am interested in understanding how the mean asset pricing results behave in the various models studied. There is less interest placed on the volatilities since it is a priori known that these results will be far from the data since my models only focus on two exogenous shocks.[10]

[10]See above, in the general introduction, for a more complete discussion

Part I

CHAPTER 2

A Countercyclical Labour Problem

> *Labour was the first price, the original purchase - money that was paid for all things. It was not by gold or by silver, but by labour, that all wealth of the world was originally purchased.*
>
> — **Adam Smith**

2.1 Introduction

The equilibrium asset pricing literature highlights that any viable DSGE model seeking to successfully replicate both quantitative asset price and business cycle facts must at a minimum have both capital adjustment costs and internal habits in consumption.[1] Strong capital adjustment costs are necessary for the replication of quantitative asset

[1] See Thesis Introduction

pricing results. Ideally, for any given habit intensity, one could increase the strength of adjustment costs sufficiently until results match the data. As highlighted in Boldrin et al. (2001), the strength of adjustment costs required to replicate asset pricing facts wreaks havoc on the business cycle; the most concerning (for RBC variants) being the excessive countercyclical movement of labour. This excessive countercyclical movement of labour is the result of the income effect significantly dominating the substitution effect in the presence of strong, empirically plausible capital adjustment costs. Strengthening the adjustment cost also has deleterious implications for the period of the cycle; as the adjustment cost is strengthened the period of the cycle becomes progressively longer.

The countercyclical movement of labour does not make theoretical sense in a RBC model as there are no features that cause agents to be demand constrained and therefore create the need for a reduction labour. While the response of labour to a positive technology shock is debated in the empirical literature, from a theoretical standpoint there is no doubt as to the direction of movement. Indeed, Cantore et al. (2014) notes that in a model with fully flexible prices, like the RBC model, one can only expect a positive response of labour to a technology shock if one is to assume agents behave as expected - an increase in productivity leads to an increase in output which is fully met by firms who can adjust prices freely. Thus, features in a RBC model which lead to the countercyclical movement of labour must be doing so via dubious mechanisms[2] within the model.[3] I study this mechanism in the face of the minimum features needed for reasonable quantitative asset pricing results - i.e. capital adjustment costs and habits in consumption.

[2]Gali (1999), despite favouring the view that labour does indeed move countercyclically, does not believe it is the result of the income effect dominating the substitution effect.

[3]In later chapters I consider New Keynesian models with various nominal rigidities. In these models the countercyclical movement of labour is may be rationalised by the presence of nominal rigidities in both prices and wages, and the presence of both demand and supply side shocks.

There are three instruments available to agents in the prototypical RBC model when responding to shocks in order achieve their consumption smoothing goals - consumption, investment and labour. Habits in consumption and capital adjustment costs act to marginalise the consumption and investment channels respectively. The former by inducing unacceptably high psychological costs to changes in consumption while the latter leads to unacceptably high physical costs. As a consequence only the labour channel is unfettered in this environment and agents exploit this by using labour to smooth income. Thus agents smooth consumption indirectly by smoothing income, which requires the countercyclical response of labour. This mechanism does not operate in Jermann (1998) as labour is supplied inelastically. The mechanism is significantly muted in Boldrin et al. (2001) by their assumption of limited intersectoral mobility of labour which essentially makes labour supply in each of their two sectors inelastic in the period of the shock[4]. It is these assumptions on the labour market that allow both Jermann (1998) and Boldrin et al. (2001) to generate their superior joint replication results. The basic idea, as explained in the Introduction, is to force agents to the costly inter-temporal channel by muting the costless intra-temporal labour channel.

This chapter serves to provide a clear diagnosis of the excessive countercyclical labour problem in a RBC model with capital adjustment costs, internal habits in consumption and endogenous labour supply. It serves to understand how the labour market operates when features are introduced into RBC models to improve their asset pricing results. I show that while labour moves in the way the empirical literature suggests in such models it does so via a dubious mechanism and by much more than is empirically plausible. In diagnosing the excessive countercyclical labour problem this

[4]Labour can, however, be reallocated between periods in the preferred model of Boldrin et al. (2001)

chapter also develops the formal benchmark model that will serve as a point of reference when assessing the impact of the additions introduced in later chapters. Finally, it also formerly presents the non-linear solution method used throughout this thesis and an analysis of the asset pricing implications of the benchmark model.

2.2 Model Set Up

The benchmark model is a decentralised RBC model. It consists of infinitely lived, homogeneous households and identical perfectly competitive firms. Households own all of the factors of production and allocate resources over time to maximise lifetime utility. Firms use the available resources and production technology to produce output. Finally the economy is subject to aggregate uncertainty via random shocks to the productivity of technology.

2.2.1 Households

Households seek to maximise lifetime utility,

$$U_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left(c_{t+j} - \nu c_{t+j-1}, 1 - n^s_{t+j} \right) \right\} \tag{2.2.1}$$

where β is the time discount factor.

The period utility of the household depends on consumption, c, and labour supply n^s. It exhibits internal habits in consumption, the strength/intensity of which is governed by the parameter ν. Habits in consumption relax the time separability of consumption by introducing complementarity in the consumption decisions in adjacent periods. (Constantinides, 1990) The more a household consumes in the current period the more it will have to consume in subsequent periods to maintain the

same level of utility due to the accumulation of habit stock. So if consumption rises unexpectedly this will lead to a fall in the contemporaneous marginal utility of consumption[5] and an increase in the marginal utility of consumption in the subsequent period.[6]

The period utility function is of the CRRA form. This is a standard utility function that has been studied extensively in the literature and its properties are well documented. Specifically, the functional form is

$$u(c_t - \nu c_{t-1}, 1 - n_t^s) = \frac{\left[(c_t - \nu c_{t-1})^{\gamma} (1 - n_t^s)^{1-\gamma}\right]^{1-\sigma} - 1}{1 - \sigma}$$

where γ is the relative share of consumption in the composite good[7] and σ is the curvature parameter. The curvature parameter, σ, measures the degree of risk aversion, with an increase in σ being understood as an increase in agent risk aversion.

The period budget constraint for the household is,

$$c_t + i_t \leq w_t n_t^s + r_t k_t \tag{2.2.2}$$

Household income is derived from rental of the factors of production to firms via perfectly competitive factor markets. This income is used to purchase the consumption good, c_t and invest in capital, i_t. They supply labour at the competitive wage rate w_t and rent capital to firms at the competitive rental rate r_t. The preferences of the household exhibit local non-satiation so all income is either consumed or invested, hence the budget constraint holds with equality. It is further assumed that households have no productive activity at home so that all labour and capital are supplied

[5] An increase in c_t will lead to an increase in $c_t - \nu c_{t-1}$ causing the marginal utility to fall.
[6] An increase in c_t will lead to a fall in $c_{t+1} - \nu c_t$ causing the marginal utility to rise.
[7] The composite good is a combination of actual consumption and leisure

to firms, i.e. the only decision is the quantum of each factor to supply not whether to supply the factors.[8]

As owners of capital, households face the capital accumulation process,

$$k_{t+1} = (1-\delta)k_t + \phi\left(\frac{i_t}{k_t}\right)k_t \tag{2.2.3}$$

where δ is the fixed depreciation rate. Capital accumulation consists of two components; undepreciated capital and investment. Investment is subject to capital adjustment costs, governed by the $\phi(\cdot)$, which assumes that resources are required to transform investment into capital. This introduces inertia into the accumulation of capital since it takes several periods to achieve the desired levels of capital. The economy can no longer achieve any feasible level of capital both costlessly and immediately, as is the case when adjustment costs are absent (Abel, 1981). Non-linearity of the function $\phi(\cdot)$ leads to non-linearity in the accumulation of capital. In particular there is a non-linearity in how the relative proportion of investment, captured by the investment-capital ratio, translates into additional capital.[9] The function $\phi(\cdot)$ is assumed to have the following properties,

$$\phi'(\cdot) > 0, \quad \phi''(\cdot) < 0, \tag{2.2.4}$$

$$\phi(0) = 0, \quad \phi(\delta) = \delta. \tag{2.2.5}$$

[8]The assumption that households have no productive activity at home will be maintained throughout this thesis.

[9]Since $\frac{i_t}{k_t} \in [0,1]$ this means $\phi\left(\frac{i_t}{k_t}\right) \in [0,1]$ so that not all investment is converted into capital hence introducing inertia into capital accumulation.

Specifically, the functional form for the adjustment cost function is given by the following split function,

$$\phi\left(\frac{i}{k}\right) = \begin{cases} \frac{b_1}{1-\frac{1}{\xi}}\left(\frac{i}{k}\right)^{1-\frac{1}{\xi}} + b_2 & \text{if } i > 0 \\ 0 & \text{if } i = 0 \end{cases} \tag{2.2.6}$$

This general functional form is used in both Jermann (1998) and Boldrin et al. (2001), although both only report the upper branch of the split function.[10] The above function separates the conversion of investment into a fixed component and a variable component. The parameter b_2 governs the fixed amount of investment that flows through to capital, it captures the idea that there is a certain minimum proportion of all investment that will be converted into capital. The variable component is governed by the parameters b_1 and ξ, where b_1 is the additional investment converted into capital and ξ governs the curvature of the adjustment cost function, hence the penalty imposed on investment. As ξ fall there is greater curvature and hence stronger adjustment costs. Finally, as $\xi \to \infty$ the functional form collapses to a one without capital adjustment costs.[11]

Households optimisation of lifetime utility subject to the budget constraint and cap-

[10]The lower branch of this split function ensures that if there is no investment then there are no adjustment costs, so that it satisfies the regulatory conditions in (2.2.5). In the DSGE models considered in this thesis consumption smoothing arguments ensure that there will always be positive investment. As such one can safely ignore the lower branch as it will never be invoked. Simulations were conducted with the full split function, i.e. with the possibility of zero investment, and I find that the lower branch is never invoked, i.e. investment is always positive.

[11]Steady state equations imply that $b_1 = \delta^{\frac{1}{\xi}}$ and $b_2 = \frac{\delta}{1-\xi}$. So as $\xi \to \infty$ one gets the following,

$$\begin{aligned} b_1 = \delta^{\frac{1}{\xi}} &\to 1, \\ b_2 = \frac{\delta}{1-\xi} &\to 0, \\ \frac{1}{1-\frac{1}{\xi}}\left(\frac{i}{k}\right)^{1-\frac{1}{\xi}} &\to \frac{i}{k}, \end{aligned}$$

so that the adjustment cost function approaches a linear function. Thus the capital accumulation function approaches one that does not exhibit capital adjustment costs.

ital accumulation process yields,

$$w_t = -\frac{u_{n,t}}{u_{c,t} - \beta\nu E_t\left[u_{c,t+1}\right]} \tag{2.2.7}$$

$$\begin{aligned} 1 &= E_t\left\{m_{t+1}R_{t+1}\right\} \\ R_{t+1} &= \frac{\phi'(g_t)}{\phi'(g_{t+1})}\left[z_{t+1}f_{k,t+1}\phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right] \end{aligned} \tag{2.2.8}$$

where $u_{c,t}$ and $u_{n,t}$ are the marginal products of consumption and labour respectively, $m_{t+1} = \beta\left(\frac{u_{c,t+1}}{u_{c,t}}\right)$ is the stochastic discount factor and $g_t = \frac{i_t}{k_t}$ is the investment capital ratio. The condition (2.2.7) is the intratemporal labour supply decision, while (2.2.8) is the intertemporal Euler equation governing capital investment in the presence of capital adjustment costs. Both conditions have been studied extensively in the literature.

2.2.2 Firms

Firms seek to maximise lifetime profit,

$$\Theta_t = E_t\left\{\sum_{j=0}^{\infty} m_{t,t+j}\left[y_{t+j} - r_{t+j}k_{t+j} + w_{t+j}n^d_{t+j}\right]\right\} \tag{2.2.9}$$

where y is output and n^d is labour demand. The firm must pay wages, w, for each unit of labour employed, n^d, and rental rate, r, on each unit of capital employed, k^d. These factor prices are determined in competitive markets so firms have no control over them. Firms are owned by households so discount future profits at the households stochastic discount factor.

The only restriction on the firm is that production be feasible given the production technology available to the firm, characterised by the neo-classical production

function $f(\cdot)$. The production function takes the Cobb-Douglas form,

$$y_t = z_t f(k_t, n_t) = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{2.2.10}$$

where $\alpha \in (0, 1)$ is the capital share of production. This technology is assumed to be subject to random shocks to total factor productivity, z_t, which follows an auto-regressive process, i.e.

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_t \tag{2.2.11}$$

where ρ measures the persistence of the shock and ε_t is a random shock. Households, as owners of capital, face the intertemporal decisions in the economy. Thus firms, freed of all intertemporal decisions, maximise lifetime profit, (2.2.9), by maximising the period profit subject to output being feasible. This yields the standard first order conditions that the factor prices are equal to their marginal products. Hence the firm makes zero profit in all time periods as perfectly competitive markets ensure that the value of output is equal to the cost of the inputs.

2.2.3 Equilibrium

Equilibrium in the economy is a set of prices $\{r_t, w_t\}_{t=0}^{\infty}$, allocations $\left\{\hat{c_t}, \hat{n_t^s}, \hat{i_t}, \hat{k_{t+1}}\right\}_{t=0}^{\infty}$ for households and $\left\{\hat{n_t^d}, \hat{k_t}\right\}_{t=0}^{\infty}$ for firms such that, given prices $\{r_t, w_t\}_{t=0}^{\infty}$:

1. The allocation $\left\{\hat{c_t}, \hat{n_t^s}, \hat{i_t}, \hat{k_{t+1}}\right\}_{t=0}^{\infty}$ solves the households problem.
2. The allocation $\left\{\hat{n_t^d}, \hat{k_t}\right\}_{t=0}^{\infty}$ solves the firms problem.
3. Markets clear,

$$y_t = c_t + i_t,$$

$$n_t^s = n_t^d = n_t$$

2.2.4 Full Macroeconomic System

Deriving the first order conditions for the household's problem and the firm's problem, and under equilibrium we arrive at the following system for the benchmark economy.

$$z_t f_{n,t} = -\frac{u_{n,t}}{u_{c,t} - \beta\nu E_t\left[u_{c,t+1}\right]} \tag{2.2.12}$$

$$1 = E_t\left\{m_{t+1} R_{t+1}\right\} \tag{2.2.13}$$

$$R_{t+1} = \frac{\phi'(g_t)}{\phi'(g_{t+1})}\left[z_{t+1} f_{k,t+1}\phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right] \tag{2.2.14}$$

$$g_t = \frac{i_t}{k_t} \tag{2.2.15}$$

$$c_t + i_t = z_t f\left(k_t, n_t\right) \tag{2.2.16}$$

$$k_{t+1} = (1-\delta)k_t + \phi\left(\frac{i_t}{k_t}\right)k_t \tag{2.2.17}$$

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_t \tag{2.2.18}$$

where (2.2.12) is the equilibrium labour market condition, (2.2.13) is the equilibrium Euler equation, (2.2.14) is the equilibrium return on capital, (2.2.16) is the equilibrium budget constraint, (2.2.17) defines the capital accumulation process and (2.2.18) defines the process governing the perpetuation of productivity shocks.

2.2.5 Calibration

Kydland and Prescott (1982) stress that RBC models should be consistent with long run growth facts. The purpose of calibration is to ensure that models are rooted in the real world. This is achieved by a restriction on the model parameters so that the long run characteristics of the model match certain long run characteristics of the economy. Furthermore calibration allows one to focus on the cyclical properties of the model without having to analyse the long-run growth implications of the model.

Hence it adds a degree of objectivity by ensuring that the models are consistent in the long run, even if the short run dynamics are markedly different.

The long run characteristics of the benchmark model are captured in the steady state, characterised by a situation when all variables have settled to their long run means. The benchmark model is calibrated so that the steady state matches the long run characteristics of the U.S. economy.

Technology Parameters

The technology parameters govern the production function, the capital accumulation process and the total factor productivity shock process. The numerical values for these parameters are relatively standard across the literature. The calibrated parameter values are taken from Kehoe and Perri (2002). A similar parametrisation is used in Boldrin et al. (2001) and Christiano and Eichenbaum (1992), except the standard deviation of innovations differs as they assume labour augmenting technology shocks. The standard deviation of technology shocks is the same as that assumed in Cooley et al. (1995). The depreciation parameter is set to 0.025 so that the annual depreciation rate is 0.1; again a common assumption in the literature. The values are in percentages and the calibration time period is three months (i.e. quarterly).

Table 2.1: Calibrated Technology Parameters for U.S.

Capital Share	$\alpha = 0.36$
Depreciation Rate	$\delta = 0.025$
Autocorrelation Coefficient	$\rho = 0.95$
Standard Deviation of Innovations	$\varepsilon = 0.007$

Kehoe and Perri (2002)

The functional form for the adjustment cost function is calibrated so that the model is invariant to ξ in the long run. This captures the idea that adjustment costs are a short term phenomenon. The steady state equations imply that the adjustment cost function parameters satisfy,

$$b_1 = \delta^{\frac{1}{\xi}},$$
$$b_2 = \frac{\delta}{1-\xi}.$$

The parameter ξ is a free parameter and cannot be derived from the deep parameters of the model. The baseline parametrisation of ξ is 0.23, which is used in both Jermann (1998) and Boldrin et al. (2001) facilitating comparison of results. Boldrin et al. (2001) cite that this value for ξ is near the lower end of possible parametrisations. Thus the calibration of the capital adjustment cost parameter implies very strong adjustment costs.

Preference Parameters

The preference parameters relate to the rate of time preference and the two parameters in the utility function. The rate of time preference, β, across the literature is calibrated to match the long run risk free rate. (Kehoe and Perri, 2002; Boldrin et al., 2001) In line with the literature I choose $\beta = \frac{1}{1.0037}$ to match the average quarterly return on U.S. T-Bills from the stylised facts presented in the Introduction.

The parameter γ is the share of consumption in the composite consumption good defined in the utility function. It is calibrated from the steady state labour supply equation as,

$$\gamma = \frac{\left(\frac{1-\nu}{1-\beta\nu}\right)\left(1-\frac{\bar{i}}{\bar{y}}\right)\frac{\overline{n}}{1-\overline{n}}}{(1-\alpha)+\left(1-\frac{\bar{i}}{\bar{y}}\right)\left(\frac{\overline{n}}{1-\overline{n}}\right)\left(\frac{1-\nu}{1-\beta\nu}\right)}, \tag{2.2.19}$$

where $\frac{\bar{i}}{\bar{y}}$ is the steady state investment-output ratio and $\frac{\overline{n}}{1-\overline{n}}$ is the steady state labour-leisure ratio. The investment-output ratio for the U.S. is 0.25 (Cooley et al., 1995). Finally it is assumed that agents work one third of the time, i.e. a standard 8 hour work day, in steady state.

The parameter σ controls the curvature of the utility function and is associated with the level of risk aversion. For the CRRA utility function assumed there is a relatively simple relationship between the intertemporal elasticity of substitution (IES) and the parameters of the utility function,

$$IES = \frac{1}{1-\gamma(1-\sigma)}. \tag{2.2.20}$$

Once γ has been calibrated, the calibrated value for σ satisfies,

$$\sigma = 1 - \frac{\zeta - 1}{\gamma\zeta}, \tag{2.2.21}$$

where ζ is the value of the IES. The permissible values for IES range between zero and one. (Harashima, 2005) The common assumption in the literature, used here, is that $IES = \frac{1}{2}$.

The parameter governing habit intensity, ν, is the only free parameter in the model for which there is no clear calibration scheme. The calibration scheme adopted in the macro-finance literature is to select ν to maximise the ability of the model to account for the mean equity premium and the mean risk free rate.[12] The resulting search algorithm over the range $\nu \in [0, 0.9]$ often forces a corner solution with $\nu = 0.9$. (Boldrin et al., 2001) I use the corner solution parametrisation of $\nu = 0.9$ for the benchmark model.

[12] e.g. see Jermann (1998); Boldrin et al. (2001)

Table 2.2: Calibrated CRRA Preference Parameters for U.S.

Time Preference	$\beta = 0.9963$
Proportion of Consumption in Composite Good	$\gamma = 0.2638$
Curvature Parameter	$\sigma = 4.5628$
Habit Parameter	$\nu = 0.9$

2.3 Solution Technique

The simulation results are obtained using the Parameterised Expectations Algorithm (PEA). PEA is a standard non-linear method for solving the system of equations that define the economy. Theoretical validation of PEA can be found in den Haan and Marcet (1990) while its wide applicability to economic models has been studied in Marcet and Marshall (1994) and Marcet and Lorenzoni (1998)

A non-linear approach has been used rather than the standard log-linear solution techniques because I am ultimately interested in the behaviour of asset prices. Log-linearisation involves converting complex non-linear systems into approximately linear systems in terms of log deviations from the steady state. While this provides fast and generally accurate results for most purposes there are two cases when log-linearisation is not advisable - (i) asset pricing and (ii) analysis of economy's far from the steady state, e.g. crisis economics. In both of these situations the non-linearity of the system plays a crucial role in the behaviour of the economy and assuming it away via log-linearisation marginalises mechanisms that are important components of these models.[13] Asset pricing is particularly unsuited to log-linearisation as it is the Euler

[13]In Part II, I will show how non-linear methods applied to New Keynesian models provides deeper insight into labour market responses that are ignored (or treated in the wrong way) when one uses log-linear solution methods.

Equation, an inherently non-linear object, which is central to pricing assets in these models. The log-linear approach imposes a linear relationship between the certainty equivalents of different assets (Jermann, 1998), this is a priori neither sensible nor advisable. If one assumes that some portion of agent wealth is tied up in financial assets then the linearity imposed on the system under log-linearisation implies that agents face a symmetric response of marginal utility when asset prices change. The concavity of the utility function due to risk aversion means that this is not the case for a fall in asset prices (and hence wealth) as for an equal rise in asset prices.[14] Agents must be compensated for this asymmetric response of utility via a higher return leading to the existence of risk premia. The use of log-linearisation removes such premia by assuming risk averse agents are in fact risk neutral across gambles within the period. This makes the log-linear approach, and its variants, methodologically unsuited to the study of asset prices within New Keynesian models, and indeed DSGE models in general.

The accepted recourse of the macro-finance literature out of this is the log-linear/log-normal approach which makes a distributional assumption about the terms within the Euler Equation and proceeds to derive asset pricing results. The rationale behind the use of non-linear methods in this thesis is to derive more general results which do not rest on distributional assumptions or the undesirable consequences of log-linearisation.

The conditional expectation in the system is the cause of most concern when solving the system. Not only is it inherently non-linear but it is not even possible to derive formulae for these conditional expectations in the models considered. (Marcet and Lorenzoni, 1998) The Parameterised Expectations Algorithm uses functional approximation techniques and exploits the recursive nature of the system in order to obtain

[14]The fall in marginal utility is much larger than the rise in marginal utility for opposite (but otherwise identical) movements in the asset price.

a solution. The central assumption is that the conditional expectation $E_t(\cdot)$ can be approximated by some unknown, time invariant function $\Psi(\cdot)$, of the state variables. Marcet and Lorenzoni (1998) state that under rational expectations, if agents use $\Psi(\cdot)$ to form decisions then "the series generated is such that $\Psi(\cdot)$ is the best predictor of future variables inside the conditional expectation." The algorithm solves the model by finding a fixed point for the parameters of $\Psi(\cdot)$ rather than looking for the policy function.

Intuitively the algorithm exploits the recursive nature of the system and the rational expectations equilibrium to solve the model. Together these features ensure that the conditional expectation is a time invariant function of the state variables. (Marcet and Marshall, 1994) PEA approximates the true form of the conditional expectation with a "simple function", $\Psi(\cdot)$, of the state variables. The aim of PEA is to parametrise this function so that it may be considered a good approximation of the true form used by agents to make decisions. The model is simulated on the assumption that this function is correctly parametrised. The parameters of the function are then re-estimated using non-linear least squares estimation on the simulated data. The updated parameter set is checked to determine if they are "close enough" to the original parameters, i.e. if a fixed point of $\Psi(\cdot)$ has been found. If not then the procedure is repeated with a suitably updated parametrisation for the "simple function". The algorithm is considered to have solved the model when it either finds the fixed point of $\Psi(\cdot)$, or is arbitrarily close to it. The algorithm is formalised below and the discussion follows that of Marcet and Marshall (1994).

PEA assumes that the economy can be described by a system of the form,

$$g\left(E_t\left[\kappa_t\left(z_{t+1}, z_t\right)\right], z_t, z_{t-1}, \varepsilon_t\right) = 0, \tag{2.3.1}$$

where z_t is a vector of all endogenous and exogenous variables contained within the conditional expectation, and ε_t is a vector of the exogenous shock process. The functions $g(\cdot)$ and $\kappa(\cdot)$ are known from the form of the system and the conditional expectation. The following relationship is assumed to hold for the conditional expectation,

$$E_t[\kappa_t(z_{t+1}, z_t)] = E[\kappa_t(z_{t+1}, z_t)|x_t], \tag{2.3.2}$$

where x_t is a vector of the state variables. State variables are defined as those that satisfy,

$$x_t = b(z_{t-1}, \varepsilon_t) \tag{2.3.3}$$

for a known function b, i.e. they are completely predetermined within the period or are outside the control of the decision maker. Thus the information set relevant for the problem can be restricted to the information contained in x_t. The central assumption of PEA is that the above conditional expectation may be represented by a time invariant function of the state variables, i.e.

$$\Psi(x_t) = E[\kappa_t(z_{t+1}, z_t)|x_t]. \tag{2.3.4}$$

This assumption is justified by the recursive nature of the system and the characterisation of the system as a rational expectations equilibrium. Taken together these imply that the conditional expectation must be time invariant.[15] Critically the function $\Psi(\cdot)$ is unknown until the system has been solved, resulting in circular reasoning whereby the system cannot be solved until $\Psi(\cdot)$ is known and $\Psi(\cdot)$ is unknown until the system has been solved. PEA breaks this cycle by assuming that the function $\Psi(\cdot)$ can be approximated by a simple function

$$\psi(\cdot) = \psi(\theta, x(\theta)) \tag{2.3.5}$$

[15] A proof can be found in Heer and Maussner (2005, Ch 3.1)

defined over a parameter set, θ, and the state variables. Note that the state variables, indeed all variables, are now a function of the parameter set. Marcet and Marshall (1994) show that $\psi(\theta, x(\theta))$ will become arbitrarily close to $\Psi(\cdot)$ within a finite number of iterations. PEA does not assume that the agents in the model do not know the true form $\Psi(\cdot)$, it is the true form that agents use to make decisions. Rather it attempts to approximate this true form through successive iterations of $\psi(\theta, x(\theta))$.

The PEA procedure is to find a parameter set $\theta \in \Theta$ such that it is a fixed point of $\Psi(\cdot)$. Define the function

$$G(\theta) = \underset{\theta \in \Theta}{argmin} E\left[\|\kappa_t(z_{t+1}(\theta), z_t(\theta)) - \psi(x(\theta), \theta)\|^2\right]. \tag{2.3.6}$$

so that $G(\theta)$ is the mean square error. PEA looks for a fixed point, i.e. $\theta = G(\theta)$. The practical implementation of the algorithm estimates $\hat{\theta}$ from the non-linear least squares regression,

$$\hat{\theta} = \underset{\hat{\theta} \in \Theta}{argmin} \frac{1}{T-1} \sum_{t=1}^{T-1} [\kappa_t(z_{t+1}(\theta), z_t(\theta)) - \psi(x_t(\theta), \theta)]^2, \tag{2.3.7}$$

where the 'explanatory variable' is the approximating function, $\psi(\theta, x(\theta))$, and the dependent variable is the function inside the conditional expectation, $\kappa(z_{t+1}, z_t)$. This procedure is repeated iteratively until the parameter vector converges, given a critical value for convergence. The updating procedure for the parameter vector is,

$$\theta^{i+1} = \eta\hat{\theta} + (1-\eta)\theta^i, \tag{2.3.8}$$

where η is a smoothing parameter and θ^i the initial guess. This updating procedure can be viewed as a learning process where new information is incorporated into the forecasts in a smooth fashion using least squares learning. (Marcet and Marshall,

1994; Marcet and Lorenzoni, 1998) Furthermore, agents using least squares learning form rational expectations (Marcet and Marshall, 1994; Marcet and Lorenzoni, 1998), so the algorithm has economic intuition.

The approximating function, $\psi(\cdot)$, is assumed to take the exponential form,

$$\psi(\theta, x(\theta)_t) = \exp[\theta \cdot \ln(x(\theta))] = \exp\left[\sum_{j \in \mathcal{J}} \theta_j \ln(x(\theta)_j)\right], \tag{2.3.9}$$

where $\mathcal{J}$ is the set of state variables. Marcet and Lorenzoni (1998) suggest that this is the appropriate functional form for models of the type considered in this thesis as it ensures that the term being approximated is always positive, which is appropriate as the terms inside the expectation are always positive. Marcet and Lorenzoni (1998) further state that using this functional form will lead to a better fit with fewer parameters.

2.3.1 Homotopy

PEA is a numerical procedure and so requires an initial guess, θ^0, to start the procedure. There is a valid concern that the PEA solution is dependent on this initial guess. PEA is locally stable if and only if the initial guess of θ is "close enough" to the true estimate (Marcet and Marshall, 1994; Marcet and Lorenzoni, 1998).

To ensure that the initial guess for the benchmark model is "close enough" it is estimated from the solution provided by the log-linearised approach solved using Dynare.[16] Marcet and Lorenzoni (1998) describe the concept of homotopy in PEA as starting "with a version of the model that is easy to solve, then [modifying] the parameters slowly to get the desired solution". This will lead to a stable solution so

[16] I use a second order log-linear solution from Dynare as the initial guess for all models in this thesis.

long as the parameters change in a stable fashion. A number of the models can be considered "close enough" to each other that the solution in one model can be used as the initial guess in the other.[17] Homotopy also guarantees that we move closer to a solution as the parameter set is updated, as long as the updating is in a smooth fashion.

2.3.2 Practical Implementation

The practical implementation of the Parameterised Expectations Algorithm takes the following steps.

1. Choose stopping criterion $\upsilon > 0$ and a smoothing parameter $\eta \in (0, 1)$. The smoothing parameter defines the rate at which the initial guess is updated, i.e. how much new information is incorporated into the updated guess.

2. Choose an approximating function, $\psi(\theta, x_t(\theta))$, and a reasonable initial guess for θ, the initial guess is θ^i. To start the process, the first initial guess is found via a solution from Uhlig's Toolkit to run the initial non-linear least squares regression.

3. Generate a series of shocks $\{\epsilon_t\}_{t=1}^{T+1}$

4. Given an initial amount of capital, k_0, and $\{\epsilon_t\}_{t=1}^{T+1}$, recursively generate the series for each variable in the system from the system of first order conditions.

5. Find the parameter $\hat{\theta}$ by solving the non-linear least squares regression,

$$\hat{\theta} = \underset{\hat{\theta} \in \Theta}{argmin} \frac{1}{T-1} \sum_{t=1}^{T-1} [\kappa_t(z_{t+1}(\theta), z_t(\theta)) - \psi(x_t(\theta), \theta)]^2, \tag{2.3.10}$$

[17] e.g. One can use the solution to the model without capital adjustment costs as the initial guess to a model with very weak (almost negligible) capital adjustment costs. The adjustment costs can then be strengthened by moving from a model with weak capital adjustment costs to one with much stronger adjustment costs in a smooth fashion.

6. Update the initial guess parameter vector as $\theta^{i+1} = \eta\hat{\theta} + (1-\eta)\theta^i$

7. If $\left|\hat{\theta} - \theta^i\right| > \upsilon$ then the new parameter vector is not close enough to the initial guess, so convergence has not occurred. In this case repeat from Step 2 using the updated parameter value θ^{i+1}. If $\left|\hat{\theta} - \theta^i\right| \leq \upsilon$ then stop as the algorithm has converged, i.e. the optimal solution has been found.

2.4 Business Cycle Results

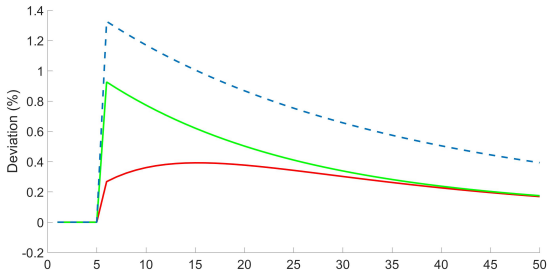
The business cycle results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique.

2.4.1 Impulse Response Analysis

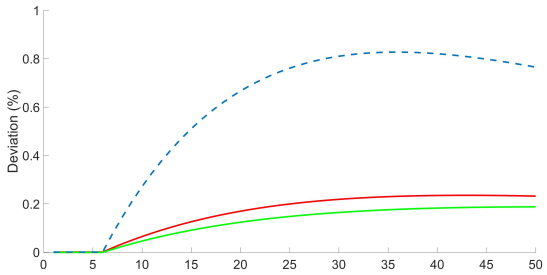
Impulse response functions illustrate how variables respond to a single perturbation of the dynamic system. In order to derive the impulse responses the system is first placed in steady state by setting the innovations of the system to zero. This steady state system is then perturbed by a single shock in period τ with all innovations to the process after τ once again set to zero. This allows the propagation of the single shock at τ to be followed through the system without any conflating shocks. I

The impulse response functions presented are percentage deviations from steady state for a one percentage point shock to technology[18]. Percentage deviations from steady state remove any scale effects thus focusing only on the dynamics of the system. The impulse responses of a pure RBC model, i.e. one without capital adjustment costs and consumption habits, and a model without habits (i.e. $\nu = 0$) are presented as

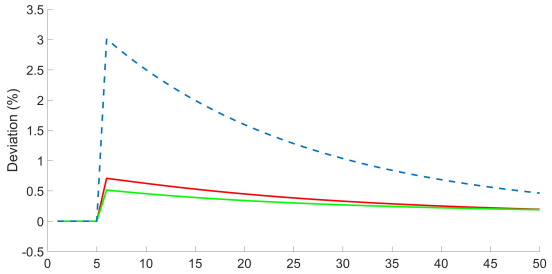
[18]This is slightly higher than a one standard deviation shock but is used to make the analysis clearer.



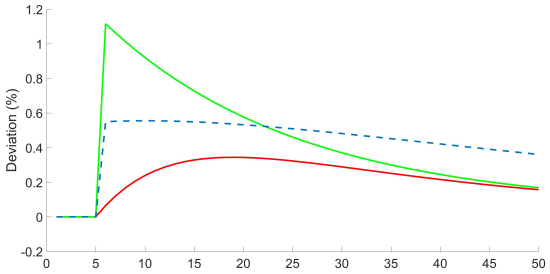
(a) Output



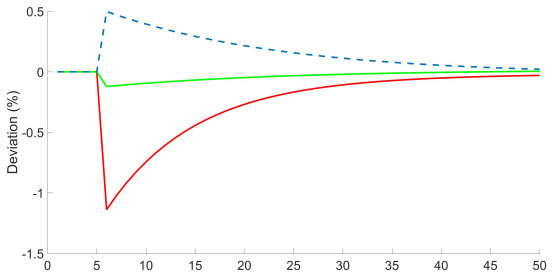
(b) Capital



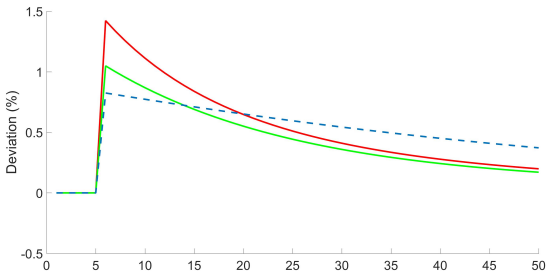
(c) Investment



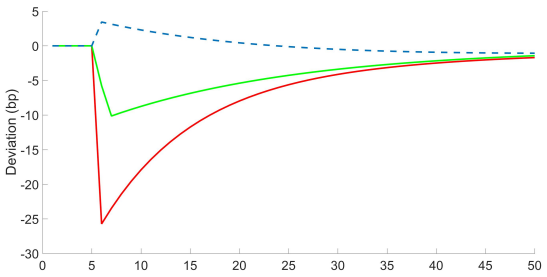
(d) Consumption



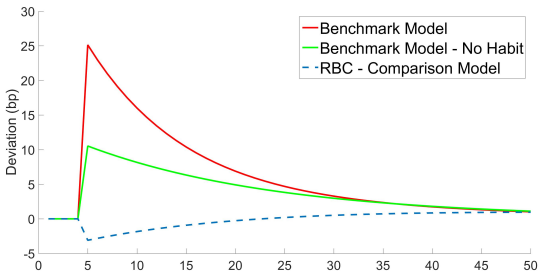
(e) Labour



(f) Real Wage



(g) Interest Rate (Basis Point)



(h) Stochastic Discount Factor (Basis Point)

Figure 2.1: Impulse Responses

baseline comparison models.

At impulse only investment, consumption or labour may respond contemporaneously. Capital adjustment costs significantly mute the response of investment as costs are incurred from changing investment quickly. The muted investment response leads to an expected slower capital accumulation. Since the response of investment has been restricted by the introduction of capital adjustment costs, consumption and labour need to adjust to pick up the slack.

Productivity shocks increase household income by increasing output owing to the fact that

$$z_t f\left(k_t, n_t\right) = z_t f_{k,t} k_t + z_t f_{n,t} n_t,$$

since the neo-classical production functions are homogeneous of degree zero. Any contemporaneous increase in income must be either consumed or invested. Since the response of investment has been restricted by capital adjustment costs, consumption must take up the slack as income increases. However, in the benchmark model households are averse to large changes in consumption due to the presence of internal habits in consumption. Alternatively the factors of production, i.e. labour and capital, may move so as to mute the contemporaneous increase in income. Since capital is contemporaneously fixed, labour must be used to smooth income. This is the case for the benchmark model as evidenced by the muted response of output and the large fall in labour, I study this income smoothing argument in more detail below.[19]

The pronounced hump shaped response for consumption in the benchmark model is characteristic of the presence of habits in consumption inducing inertia into the

[19]This is also the case for the benchmark model without consumption habits but to a much smaller degree

consumption process. The consumption response drives the response in the risk free interest rate; the rate at which households transfer consumption across periods. In the RBC model, households require a higher return to transfer consumption as extra consumption is more valued in the current period than in the future periods. In the pure RBC case the the interest rate must increase to make agents want to defer consumption, simultaneously inducing increased investment. The market clearing condition requires that all output not consumed be invested, so the interest rate determines the relative desire to invest over consuming output. The introduction of investment adjustment costs leads to a very large increase in contemporaneous consumption induced by a fall in interest rates which facilitate the desire for consumption over investment in the face of the capital adjustment cost. In the benchmark model habits induce consumption inertia by forcing households to care about not just contemporaneous consumption but also future consumption.[20] Habits cause households to prefer excessively smooth consumption so they are willing to transfer their consumption into future periods at a significantly lower return. The large initial fall in the risk free rate in the benchmark model is a consequence of this extreme desire to smooth consumption resulting from the presence of inter-temporal factors. The risk free rate approaches steady state from below as agents progressively value contemporaneous consumption more once consumption begins to fall.

[20]The risk free rate has general form,

$$1 = E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \right] R^f_{t+1}$$

In models without habits Λ_t takes the form,

$$\Lambda_t = u_{c,t},$$

i.e. it is the contemporaneous marginal utility of consumption. However in models with habits there appears an inter-temporal term as Λ_t takes form,

$$\Lambda_t = u_{c,t} - \beta \nu E_t \left[u_{c,t+1} \right],$$

i.e. it now incorporates both the contemporaneous marginal utility of consumption and also the marginal utility of consumption in the subsequent period.

2.4.2 Income and Substitution Effects

The labour market is described by the labour supply and labour demand curves. The equation describing labour demand is given by the firm's first order condition for labour,

$$w_t = z_t f_n (k_t, n_t) . \tag{2.4.1}$$

The supply side of the labour market is driven by the household's labour supply condition[21],

$$w_t = -\frac{u_n (c_t - \nu c_{t-1}, 1 - n_t)}{u_c (c_t - \nu c_{t-1}, 1 - n_t) - \beta \nu E_t \left[u_{c_{t+1}} (c_{t+1} - b c_t, 1 - n_{t+1}) \right]} \tag{2.4.2}$$

The response of labour may be decomposed into two offsetting effects - the income and substitution effects. I make use of the stylised labour market diagram[22] below, and a partial equilibrium analysis, to aid in defining these effects in the present model. This diagram illustrates the response of labour demand and labour supply to a positive productivity shock. While both curves move simultaneously, I decompose the responses into those that are a direct consequence of the shock and those that are indirectly related to the shock. The labour market begins in equilibrium at point A. The positive productivity shock has a direct impact on labour demand, (2.4.1), and shifts it to the right. This increase in demand for labour leads to an increase in both the real wage and the level of labour employed as the economy moves to point B. There is no direct change in the labour supply resulting from the positive productivity shock. I refer to this shift in labour demand as the substitution effect.

[21] The benchmark model without habits is a special case where $\nu = 0$ so that one gets the standard labour supply condition,

$$w_t = -\frac{u_n (c_t, 1 - n_t)}{u_c (c_t, 1 - n_t)}$$

[22] All curves have been drawn as straight lines for the sake of clarity.

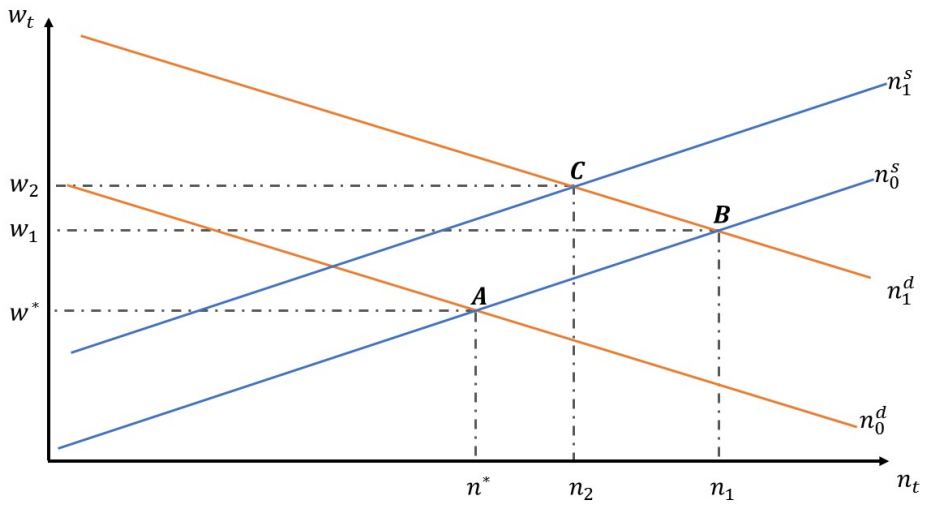


Figure 2.2: Stylised Labour Market

The labour demand curve may also shift due to a change in the capital stock. However, since capital does not respond contemporaneously to the productivity shock it will not cause a contemporaneous shift in the labour demand. Such capital induced shifts will only affect the labour demand curve with a lag.

The labour supply curve, (2.4.2), may shift as a result of changes in any of its non-labour components. I refer to the shift of the labour supply curve resulting from changes in non-labour components as the income effect. For example, the positive productivity shock also has an indirect impact on consumption. In particular it leads to an increase in both consumption and investment as household income increases due to the fact that they are working more and at higher wages. The increase in consumption leads to a shift leftwards of the labour supply curve as households desire to consume more of all goods, including leisure. Thus there is a decline in the amount of labour supplied and wages rise further in an effort to induce households to supply labour. Graphically this is a movement from B to C, with C being the new equilibrium in the labour market. This shift is also affected by the changes in the

expected value of the future marginal utility of consumption in models with internal consumption habits, which will also change contemporaneously as a consequence of the productivity shock.

One sees that the income effect acts to decrease labour supply as household income increases while the substitution effect acts to increase labour supply as the real wage increases. Thus these effects, as defined above, continue to have their usual economic interpretations.

2.4.3 The Response of Labour

The inertia in both consumption and investment forces the countercyclical response of labour as this is the only variable remaining which can respond contemporaneously. Agents therefore reduce labour in order to smooth income, thereby smoothing consumption. It may alternatively be thought of as the income effect dominating the substitution effect as they have been defined above. Using the stylised labour market diagram; this is a case of C lying to the left of A, i.e. the shift in the labour supply curve greatly exceeds the shift in the labour demand curve.

The presence of capital adjustment costs restricts the use of investment as an instrument to smooth consumption, while habits in consumption induce agents to prefer very smooth consumption profiles. Thus labour is the only unrestricted instrument available to households that allows them to achieve their consumption smoothing objective. Habits therefore induce a very strong income effect so as to reduce the equilibrium supply of labour, reducing household income and thereby smoothing consumption. The figure below shows how labour responds at impulse for a range of values for the habit parameter, ν.

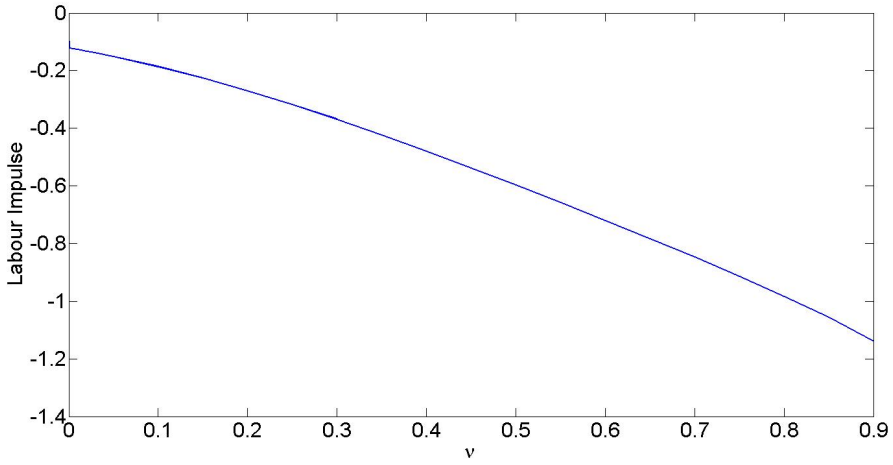


Figure 2.3: Labour Impulse Sensitivity to Habit Parameter

Even without habits the income effect dominates the substitution effect in the benchmark model. The negative response of labour is only magnified as the habit parameter is strengthened. This may be attributed to a strengthening of the income effect through the introduction of inter-temporal concerns by agents. There is no change in the substitution effect as the shock remains the same. As the only difference between the models is the strength of the habit parameter we can therefore infer that it is the habit parameter that is leading to a strengthening of the income effect. This is because agents become increasingly more concerned about inter-temporal consumption in addition to contemporaneous consumption as habits are increased.

The large income effect leads to a fall in labour despite the significant increase in the real wage. The real wage increases due to the positive productivity shock shock but is moderated in the comparison model by an increase in the labour supply. In the presence of capital adjustment costs the fall in labour, resulting from the use of labour to smooth consumption, magnifies the wage increase resulting from the shock. This is not optimal since households income is only increased further as a result of the magnified wage. I consider this a shortcoming of the model not because it doesn't

adequately replicate the data (see discussion below) but rather the mechanism bringing it about - an income effect so strongly dominating the substitution effect - is not plausible. Indeed in a model with a single supply side shock (i.e. the technology shock) and fully flexible prices one expects labour to move cyclically with the supply shock, i.e. for the substitution effect to dominate the income effect.

Gali (1999) contends that empirically labour has a near zero and even slightly negative correlation with productivity, and hence output.[23] Gali (1999) uses this result as evidence of the unsuitability of the RBC framework and the need to move to New Keynesian models, i.e. ones with nominal rigidities. This would support the argument that labour is in fact countercyclical and doom any attempt to resolve this countercyclicality in the RBC framework is misguided. I do not refute the results in Gali (1999), indeed I agree that labour may not move significantly with productivity and if it does then the most likely response is countercyclical. However, I contend that this response of labour is not via the mechanism described above.[24] As discussed in the introduction, the response of labour should not be countercyclical and so highlights a mechanism that is misunderstood and misbehaving within the benchmark model. It is on these grounds that I refute the second part of the argument - the countercyclical response of labour is a shortcoming of the benchmark model and worthy of study. Indeed, by studying the operation of this labour market mechanism and understanding it properly in the RBC framework (as this chapter has done) one can gain deeper insights into the operation of the labour market in models with nominal rigidities.

Table 2.3: Business Cycle Results

	Data	Pure RBC	Benchmark
Standard Deviation (%)			
Output	1.72	1.24	0.31
Standard Deviation Relative to Output			
Consumption	0.79	0.42	0.66
Hours Worked	0.63	0.38	3.40
Investment	3.24	2.28	2.20
Productivity	0.54	0.76	3.05
Contemporaneous Correlation with Output			
Consumption	0.87	0.99	0.80
Hours Worked	0.93	0.99	-0.81
Investment	0.86	0.99	0.92
Productivity	0.78	0.99	0.90

2.4.4 Quantitative Business Cycle Results

The quantitative business cycle results are obtained from a simulation of the Parameterised Expectations Algorithm solution for each model with randomly drawn shocks. Impulse responses provide some insight into the quantitative simulation results, but for the quantitative results the scale must also be considered.[25] The results presented in Table 2.5 are the sample statistics for the second order moments for the relevant series obtained. One sees a deterioration in some of the quantitative business cycle results, the most striking of which is the countercyclical movement of labour.

2.5 Asset Pricing Implications

An asset is a means of transferring consumption between periods. The nature of an asset is completely determined by nature of the return it offers, i.e. the states of the world in which it makes payments and when payoffs are realised. This re-

[23]Gali (1999) reports that the unconditional correlation between labour and productivity has empirically range -0.28 and 0.29 with mean of -0.14.

[24]See the Introduction for a discussion.

[25]An appreciation of the scale will be important in later chapters.

turn can be either fixed or random depending on the asset. A risk free asset is one which guarantees a certain return in the next period, regardless of the realisation of uncertainty. A risky asset is one where the return next period is contingent on the particular realisation of uncertainty, making it a random variable. The asset pricing statistic of concern is the rate of return as this fully characterises the asset in question.

The central equation for pricing assets is the Euler Equation which has general form,

$$1 = E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda t} R_{t+1} \right], \tag{2.5.1}$$

where β is the rate of time preference, Λ_t is the marginal utility of consumption in period t and R_{t+1} is the return profile of the asset between t and $t+1$. The Euler Equation is central to understanding asset prices as it provides a link between real macroeconomic variables and the return on financial assets.

The stochastic discount factor, $\beta \frac{\Lambda_{t+1}}{\Lambda t} = m_{t+1}$, is a generalised discount factor that captures both time discounting and the preferences of the agent. It values the marginal unit of consumption in the next period in terms of the marginal utility of consumption in the current period. So if m_{t+1} is small, the value attached to the marginal unit of consumption in the next period is small compared to the marginal unit of consumption in the current period. According to (2.5.1) assets are priced such that the product of the stochastic discount factor and the return equal one in expectation at time t. Thus the agent is indifferent between an additional unit of consumption in the current period and deferring this additional consumption into the next period by investing in the asset.

The two assets of interest in this chapter are a risk free bond and equity, both are

single period assets. All asset contracts in the economy are defined so that investment is made prior to the realisation of the shock, i.e. investment is made in the face of uncertainty. The characteristics of these assets will remain the same throughout this thesis.

2.5.1 Risk Free Rate

The risk free asset contract guarantees to pay the market clearing rate implied by the Euler equation before the shock is realised. This return is then paid once the shock has been realised. Thus the risk free bond pays a fixed amount regardless of the realisation of uncertainty. The bond is risk free as the return earned over the period is known at the start of the period and not subject to any random fluctuations. The risk free rate is a market clearing rate since it is the rate at which agents are indifferent between consumption now and deferring consumption for a single period. Thus it is the rate that clears the consumption market, i.e. the rate at which agents do not have any surplus consumption requirements or wish to consume more.

Let R^f_{t+1} be the gross rate of return on the risk free bond, i.e. $R^f_{t+1} = 1 + r^f_{t+1}$. The return on the risk free bond is known in period t so (2.5.1) becomes,

$$1 = E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda t}\right]R^f_{t+1} \Rightarrow r^f_{t+1} = \frac{1}{E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda t}\right]} - 1. \tag{2.5.2}$$

The risk free rate is that return the agent is willing to accept to give up a unit of consumption before uncertainty has been realised. This deferred consumption must be invested since agents can either consume or invest in this economy. The risk free rate may therefore also be considered the certainty equivalent of the ex-ante expected return on equity since it is the certain return at which agents are indifferent between consuming in the current period and investing (or alternatively between investing in

equity or not).

2.5.2 Return on Equity

The equity contract does not guarantee a specific return ex-ante but is determined once the shock has been realised. As with the risk free bond, investment is made before the shock is known and the asset makes payment only after the shock has been realised. The pricing of the equity contract within the model requires a relationship between the ex-post return on equity and the real variables in the model.

Equity is defined as a claim to all or part of the output of a productive unit of the economy. (Lucas, 1978) As none of the Welfare Theorems are violated in the economy set-up the problem can be re-cast as a social planners problem. In such a case it has been shown that there is a clear link between the return on equity and the return on capital, specifically that they are equivalent. Altug and Labadie (2008, Ch. 10) provide a proof of this equivalence in a simple RBC model which can be trivially extended to the economy set up considered in this chapter, i.e. a model with capital adjustment costs. This equivalence result has a certain intuitive appeal since capital is the only productive asset in the economy set-up. If we assume that the firms sole objective is to maximise the pay-off to shareholders, then an investment into shares of the firm must be invested by the firm into capital stock as this is the only productive use of the funds. The ex-post gross return on equity is therefore the gross ex-post return on capital over the period from t to $t+1$.

Let R^e_{t+1} denote the gross return on equity. This return is random at the time of investment since the return is only known once the shock has been realised. The

return on equity is,

$$R^e_{t+1} = \frac{\phi'(g_t)}{\phi'(g_{t+1})}\left[z_{t+1}f_{k,t+1}\phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right],$$

which has the same form across all models with capital adjustment costs. Since this return depends on the realisation of the shock it is unknown at the time of investment. This feature makes equity a "risky" asset. This from also provides an explicit link between the financial return and the real economy thereby permitting an analysis of how equity returns are affected by movements in real variables. Boldrin et al. (2001) (BCF) decompose this return as,

$$R^e_{t+1} = \frac{\mathcal{K}_{t+1} + P_{k,t+1}}{P_{k',t}}, \tag{2.5.3}$$

where $P_{k',t}$ is the consumption value of newly installed capital that will be used for production at the beginning of period $t+1$ (i.e. it is the value of capital at the beginning of the period), $P_{k,t+1}$ is the value of this capital at the end of the period $t+1$ and $\mathcal{K}_{t+1}$ is the marginal product of capital. The ratio $\frac{\mathcal{K}_{t+1}}{P_{k',t}}$ may be considered a measure of the productivity of investment (or alternatively the benefit of investment) as the numerator is the marginal return from investing in capital while the denominator measures the price of investment. The ratio $\frac{P_{k,t+1}}{P_{k',t}}$ measures the capital gain from investment as it measures the value of capital at the end of the period as a ratio of the price of investment. The capital gain will generally be less than unity since it takes into account the depreciation of the capital stock investment. This decomposition will play an important role in the study of asset prices throughout this thesis, and will henceforth be called the BCF decomposition.

The BCF decomposition for models with capital adjustment costs is,

$$\begin{aligned}
\mathcal{K}_{t+1} =& z_{t+1} f_{k,t+1} \\
P_{k,t+1} =& \left[1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right] P_{k',t+1} \\
P_{k',t} =& \frac{1}{\phi'(g_t)}
\end{aligned}$$

Tobin's Q is defined as the ratio of the value of capital within the firm to its replacement cost (or value outside the firm),

$$Q = \frac{\text{Value of New Capital}}{\text{Replacement Cost of Capital}}.$$

Under (2.5.3) Tobin's Q is given by $P_{k',t}$, i.e.

$$Q_t = P_{k',t} = \frac{1}{\phi'\left(\frac{i_t}{k_t}\right)}. \tag{2.5.4}$$

The cost of investment is assumed to be unity, i.e. one unit of investment is always assumed to cost one unit of consumption, so that investment and consumption are perfect substitutes.[26] Under this assumption Tobin's Q is the same as the value of new capital and so may be considered the price of capital. Note that the denominator measures how $\phi(g_t)$, the proportion of investment converted into capital, varies as the level of investment changes as capital is fixed in the short run. If $\phi'(g_t)$ is small then each additional unit of investment increases capital by very little. This must be because the price of capital is high at the current level of g_t so that more investment is needed for any given increase in capital. The term $\phi'(g_t)$ therefore measures the marginal benefit of investment.

[26] This implies that the marginal cost of investment is unity

The behaviour of $\phi(g_t)$ defines the behaviour of Tobin's Q and has a large impact on the behaviour of the return on equity. We assume that $\phi'(\delta) = 1$ and that $\phi''(\cdot) < 0$ which implies the following,

$$\phi'(g_t) > 1 \quad \text{for } g_t < \delta \Longrightarrow P_{k',t} < 1 \quad \text{for } g_t < \delta$$
$$\phi'(g_t) \leq 1 \quad \text{for } g_t \geq \delta \Longrightarrow P_{k',t} \geq 1 \quad \text{for } g_t \geq \delta$$

Under the Q-theory of investment, firms will invest in capital so long as $Q \geq 1$, i.e. the value of new capital, $P_{k',t}$, exceeds it's cost. Thus there is an incentive for investment, according to the Q-theory, only when the level of investment is greater than the replacement level, i.e. depreciation. At such a level firms are investing in capital and so the capital stock is growing. Since the capital stock is growing this dampens the return on capital, i.e. the equity return. This occurs since $P_{k',t}$ appears in the denominator of the ex-post return, (2.5.3), and so if it exceeds unity then it acts to dampen the return. This dampening of the equity return as capital grows plays an important role in ensuring investment returns to long run steady state. This is because those providing the equity capital will begin to reduce investment as the return falls. If the level of investment is below the replacement level then firms are allowing the capital stock to decline which magnifies the return on capital. This magnification of the equity return generates an incentive for households to invest in equity. If investment spending does not grow at the same rate as capital then g_t will necessarily vary, leading to the dynamics discussed above. The effect of a variable Tobin's Q is to move capital levels back to the steady state by magnifying or dampening the equity return.

Finally, I define the equity premium, Ξ, as the difference between the return on

equity and the risk free rate, i.e.

$$\Xi_{t+1} = R^e_{t+1} - R^f_{t+1}. \tag{2.5.5}$$

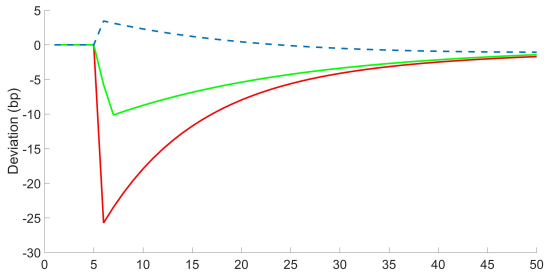
2.5.3 Asset Pricing Impulse Response Analysis

The response of the risk free rate is driven by the dynamics of consumption. The dynamics of consumption have been discussed in detail in the analysis of the business cycle dynamics of the various models.

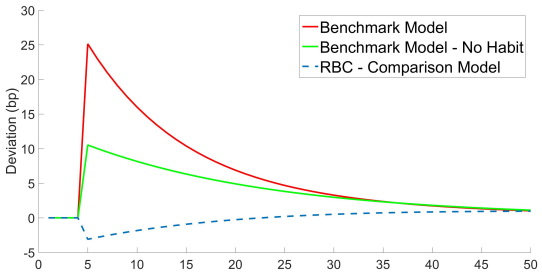
In order to understand the drivers of the return on equity I also look at the impulse responses of the different components of the BCF decomposition, (2.5.3), of the equity return.

Let us define $R^e_{t,t+1}$ as the return at $t+1$ on investment at t and let τ denote the period when the shock occurs. When analysing the impulse responses one is particularly interested in $R^e_{\tau-1,\tau}$ and $R^e_{\tau,\tau+1}$. $R^e_{\tau-1,\tau}$ is the return on investment made at $\tau-1$ and realised at τ so captures the impact on the return at impulse, i.e. how the unexpected shock affects the return. $R^e_{\tau,\tau+1}$ is the return realised at $\tau+1$ from investment at τ so captures the response of the return to the shock. From the impulse response it is clear that there is a large increase in the return at impulse followed by a negative return in the subsequent period causing the ex-post return on equity to approaches the steady state from below.

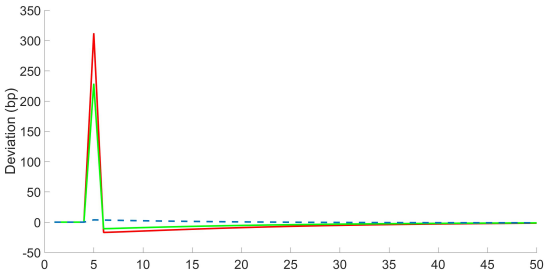
In order to understand this impulse profile one must look at how agents value consumption, i.e. the stochastic discount factor. Agents have a desire to smooth consumption so that when consumption is high (low) they seek to buy (sell) assets in order to build up (deplete) their habit stock and reduce variation in consumption si-



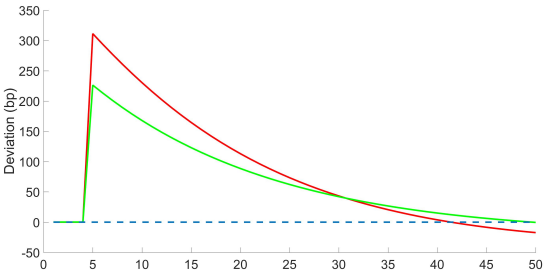
(a) Risk Free Rate



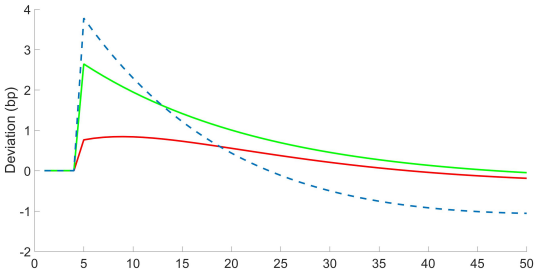
(b) Stochastic Discount Factor



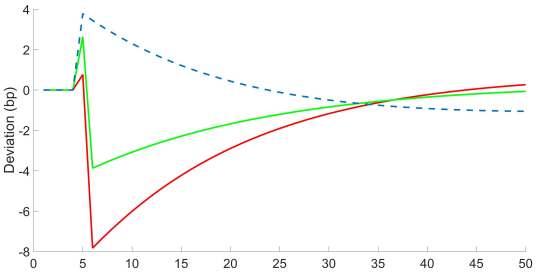
(c) Ex-Post Equity



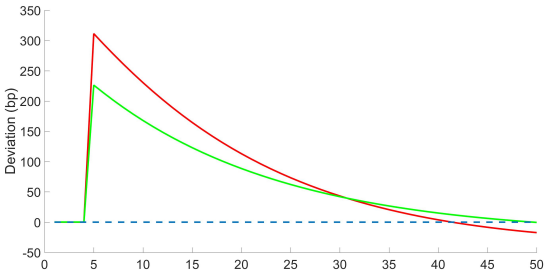
(d) Tobin's Q: $P_{k',t}$



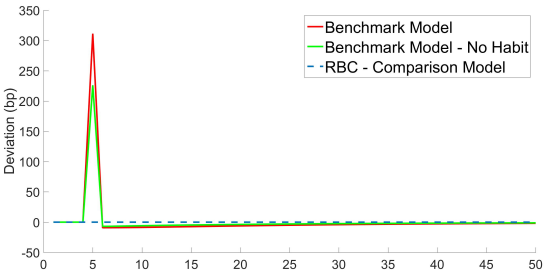
(e) Marginal Product of Capital: $\mathcal{K}_{t+1}$



(f) $\frac{\mathcal{K}_{t+1}}{P_{k',t}}$



(g) $P_{k,t+1}$



(h) Capital Gain: $\frac{P_{k,t+1}}{P_{k',t}}$

Figure 2.4: Ex-Post Equity Impulse Responses

multaneously. (Boldrin et al., 1995) Capital is contemporaneously fixed so any change in demand for assets will lead to large fluctuations in the price of capital, but not the contemporaneous return as this depends on the price of capital at investment. This leads to an excess demand which drives up prices and therefore reduces the return in subsequent periods. This channel does not exist in the RBC model as evidenced by the constant Tobin's Q.

The marginal product of capital does not vary sufficiently at impulse to account for the response of the ex-post return on equity. The small ex-post equity return in RBC models is due to the operation of this channel only and not also the capital gains channel which results from a variable Tobin's Q.

It is clear from the impulse responses that the dynamics of the equity return is driven by the capital gains channel in the benchmark model since it is the only only component of the return that responds to the contemporaneous price of capital. The capital gain at impulse and after is given by,

$$\frac{P_{k,t+1}}{P_{k',t}} = \begin{cases} [1-\delta+\phi(g_\tau)-g_\tau\phi'(g_\tau)]\,\frac{P_{k',\tau}}{P_{k',\tau-1}} & \text{for } t=\tau \\ [1-\delta+\phi(g_{\tau+1})-g_{\tau+1}\phi'(g_{\tau+1})]\,\frac{P_{k',\tau+1}}{P_{k',\tau}} & \text{for } t=\tau+1 \\ [1-\delta+\phi(g_t)-g_t\phi'(g_t)]\,\frac{P_{k',t}}{P_{k',t-1}} & \text{for } t>\tau \end{cases} \tag{2.5.6}$$

The contemporaneous response of the capital gain is driven by the fact that at impulse the value of the capital at the end of the period is valued at the contemporaneous price of capital but the return is determined by the price of capital in the period before impulse. The precipitous decline results from returns post-impulse (i.e. $t > \tau$) using the higher price of capital. Hence the ratio $\frac{P_{k',t+1}}{P_{k',t}}$ is less than unity for $t > \tau$ from the impulse response of Tobin's Q. Thus it is the movement in the price of capital that

Table 2.4: Asset Pricing Results

	Data	Pure RBC	Benchmark
Mean (% p.a)			
Risk Free	1.49	1.036	1.37
Equity	7.87	1.042	1.88
Equity Premium	6.52	0.005	0.51
Standard Deviations (% p.a)			
Risk Free	2.62	0.196	0.95
Equity	16.72	0.206	4.52
Equity Premium	15.696	0.064	4.45

drives the ex-post equity return and consequently the ex-post equity premium.

2.5.4 Quantitative Asset Pricing Results

The quantitative asset pricing results show a dramatic improvement of the asset pricing statistics of the benchmark model over the pure RBC model. There is a improvement in all asset pricing statistics compared to the the pure RBC model providing a reason for the capital adjustment cost, internal consumption habit combination becoming so dominant in the literature concerned with joint replication of asset pricing and business cycle facts. This improvement in asset pricing results is offset by a deterioration in the quantitative business cycle results, the most significant of which is the excessive countercyclical response of labour.

2.6 Where Do Non-Linear Methods Get Us?

This chapter has advocated the use of non-linear methods as the only theoretically sound solution methodology when one is interested in jointly explaining asset pricing and business cycle results. I present here the business cycle and asset pricing results for the Benchmark model obtained from both the Parameterised Expectations Algorithm and those obtained from first, second, and third order log-linear solution

Table 2.5: Business Cycle Results Comparison for Benchmark Model

	Data	PEA	Log-Linear Approximations		
			First Order	Second Order	Third Order
Standard Deviation (%)					
Output	1.72	0.31	0.30	0.37	0.47
Standard Deviation Relative to Output					
Consumption	0.79	0.66	0.63	0.57	0.41
Hours Worked	0.63	3.40	3.35	2.40	1.57
Investment	3.24	2.20	2.32	1.75	1.88
Productivity	0.54	3.05	3.05	2.49	1.98
Contemporaneous Correlation with Output					
Consumption	0.87	0.80	0.75	0.68	0.59
Hours Worked	0.93	-0.82	-0.86	-0.92	-0.95
Investment	0.86	0.92	0.92	0.96	0.98
Productivity	0.78	0.99	0.93	0.96	0.98

Table 2.6: Asset Pricing Results Comparison for Benchmark Model

	Data	PEA	Log-Linear Approximations		
			First Order	Second Order	Third Order
Mean (% p.a)					
Risk Free	1.41	1.29	1.00	0.98	0.98
Equity	7.40	1.84	1.00	0.99	0.99
Equity Premium	5.93	0.55	0	0.01	0.01
Standard Deviations (% p.a)					
Risk Free	1.34	0.95	0.24	0.19	0.25
Equity	16.12	4.52	0.24	0.19	0.25
Equity Premium	16.16	4.45	0	0.19	0.25

methods using Dynare.[27]

One notes that the non-linear approach does not differ markedly from the log-linear counterparts for the business cycle results. Given it's speed at finding a solution, and the small differences between the log-linear and non-linear solutions, one can understand why the literature has been dominated by log-linear solution methods.

However, the asset pricing results present a very different story. Here there is a signif-

[27]Dynare only allows one to solve models up to a third order approximation. I present results up to a third order approximation for this reason.

icant difference between the two solution methods with non-linear methods providing the more realistic results. An analytic derivation of this result, and some intuition, has already been provided in the introduction and will not be repeated here. Suffice it to say that the unsuitability of log-linear methods for asset prices results from the importance of non-linearities which are central to asset pricing results. The results here show that higher order approximations fail to provide a solution to this problem, indeed one suspects that at the very least one would need something higher than a fifth order approximation to begin capturing the behaviour of moments deemed important for asset pricing.[28]

These results highlight the unsuitability of log-linear approximation methods for any realistic attempt at the joint replication of quantitative asset pricing and business cycle results. Furthermore, it is worth noting how abjectly the log-linear approach fails in the simple model of this chapter, which is the least non-linear of the models studied in this thesis. The features introduced throughout this thesis will only lead to a greater degree of non-linearity in the models. This supports the approach maintained in this chapter, and this thesis, that non-linear methods are necessary for the models to have any realistic asset pricing results.

2.6.1 Business Cycle Results - What is going on?

The failure of the log-linear approach for quantitative asset pricing results is a priori expected. However, one notes that as the order of approximation increases the quantitative business cycle results are moving further away from the PEA solution. This deserves some explanation as one would expect the log-linear solution to approach the PEA solution as the order of approximation increases. In order to gain some insight

[28] A first order approximation permits no premium. A second order approximation has a premium that is proportional to a constant volatility. A third order allows this volatility to vary, while a fourth order allows the variation to be asymmetric. Thus before one even considers higher order moments one is forced to at least a fourth order approximation. See Swanson (2015)

Table 2.7: Euler Equation Errors

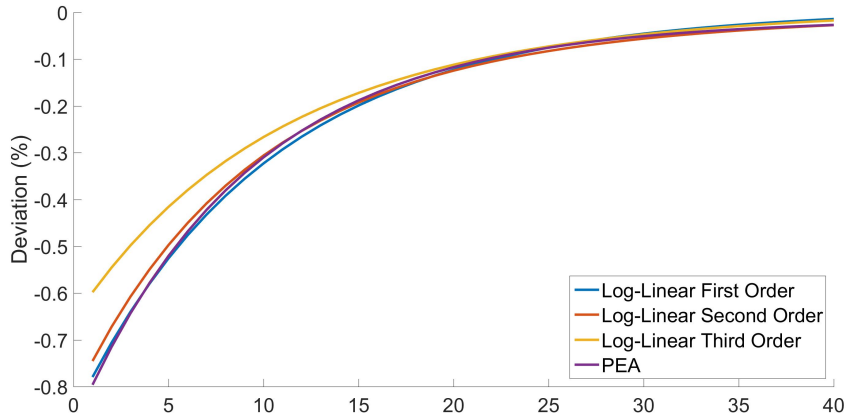
	PEA	Log-Linear Approximations		
		First Order	Second Order	Third Order
Full Benchmark Model	-3.6455x10^{-5}	-0.025	-0.0176	-0.0059
Benchmark Model without Habits	-2.1367x10^{-4}	-2.4512x10^{-4}	-2.2982x10^{-4}	-2.2980x10^{-4}

into this I present the Euler Equation errors for each of the approximations and the PEA solution, and I also study the evolution of the impulse response functions as the order of approximation increases.

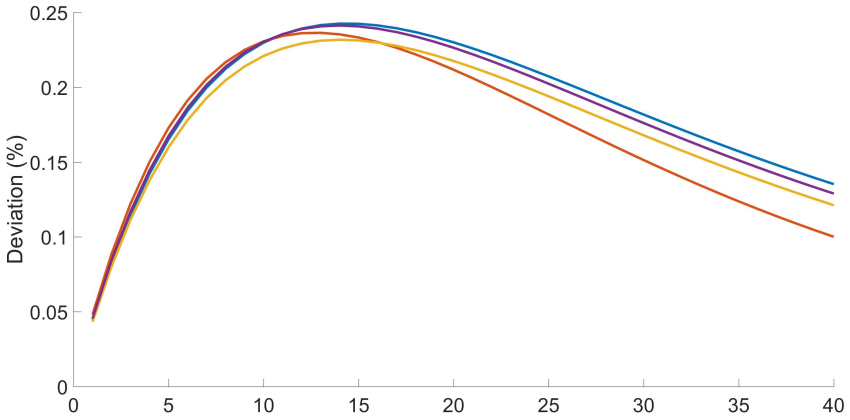
Euler Equation errors provide a means of assessing the accuracy of the solution. The Euler Equation error is defined as,

$$Error = E_t\left[\widehat{m_{t,t+1}R_{t+1}}\right] - 1, \tag{2.6.1}$$

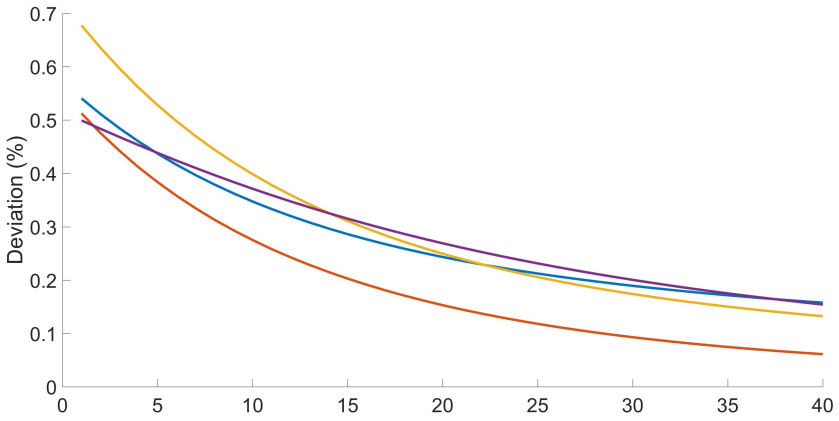
where $E_t\left[\widehat{m_{t,t+1}R_{t+1}}\right]$ is the estimated value of the expectation under the solution. The true solution would have an error of zero, so the closer the error is to zero the greater the accuracy of the solution method. The Euler Equation errors for the benchmark model under each approximation are presented in Table: 2.7, also presented are errors for the model without habits. One notes that the error falls as the order of approximation increases and that the PEA error is several orders of magnitude superior to the log-linear approximations. The fall in the error suggests that the higher order approximations are getting closer to the true solution, but also that one requires even higher approximations to achieve errors within the same magnitude as PEA.



(a) Labour



(b) Consumption



(c) Investment

Figure 2.5: Impulse Response Evolution to Order of Approximation

The fall in the Euler Equation error further suggests that the benchmark model is sufficiently non-linear that higher order approximations are capturing something important. Indeed, for the model without habits the different approximations have an error that is in the same order of magnitude as the PEA solution. This suggests that a model with only capital adjustment costs is sufficiently linear that higher order approximations are not capturing much of importance. To delve into this further I consider the evolution of the impulse responses for investment, consumption and labour over the various approximations for the benchmark model. I choose these variables as it is in these markets that the introduction of capital adjustment costs and consumption habits have the largest impact.[29] Internal consumption habits cause both the consumption process and labour supply to become non-linear, while capital adjustment costs explicitly introduce non-linearities into the accumulation of capital.[30]

The impulse response evolution shows that increasing the order of approximation has significant impact on the dynamics of the benchmark model. This appears to be due to the presence of both frictions as the impulse responses for benchmark model without habits (see Appendix) are unchanged, and very close, as the order of approximation increases. The clearest example is in the dynamics of investment where one sees marked differences in the various impulse responses. It appears that the impulse responses are approaching the PEA response, the closest approximation of the 'true' response, albeit in a 'jumpy' fashion.

The fall in the Euler Equation error coupled with the differences in the impulse responses suggests that there are higher order general equilibrium interactions between

[29]The impulse responses for the model without habits are presented in the Appendix to this chapter. All of the responses for this model are essentially identical which is to be expected given the proximity of the Euler Equation errors for this specification.

[30]The impact of these non-linearities have been covered in earlier parts of this chapter.

the two frictions that are being captured as the order of approximation increases. The interaction between capital adjustment costs and habits in consumption on consumption, investment and labour has been discussed in detail in earlier parts of this chapter. It is the successive approximation of these general equilibrium interactions that appear to have such a large impact on the quantitative results.

The analysis of this section is an example of more general concerns surrounding the use of higher order perturbation solution methods. Namely that ignoring higher order approximations ignores higher order dynamics that are important for the economy that may have consequences for quantitative results.

2.7 Conclusion

This chapter has provided a clear diagnosis of the excessive countercyclical labour problem arising in macro-finance models with capital adjustment costs, internal habits in consumption and endogenous labour supply. Such a diagnosis of the problem has, as yet, been lacking in the literature. This does not mean that the results presented here are unknown to the literature, but rather there has been no clear exposition of the problem and why it is a problem. As such this chapter serves as more than just a literature survey, and introduction to the benchmark model and solution method used in this thesis.

I find that when labour is endogenous agents exploit the unrestricted labour channel to smooth their income and hence achieve their consumption smoothing objective. The appeal of models with both capital adjustment costs and internal habits in consumption in the joint replication literature was further highlighted by showing its significant improvement of the asset pricing results at only a minor deterioration of

the business cycle results. However this ignores the major deterioration of the results with respect to the correct cyclical movement of labour; a fact most of the literature appears to ignore or discount. By providing a clearer exposition of the problem one can undertake a more effective search for a solution. The diagnosis presented here highlights that the root cause of the observed movement of labour in the presence of internal habits and capital adjustment costs is a malfunctioning of the labour market mechanism that brings about equilibrium in that market. Such a malfunctioning of the labour market mechanism is at odds with the theoretical response of labour in RBC models and brings into question the functioning of other mechanisms within RBC models.

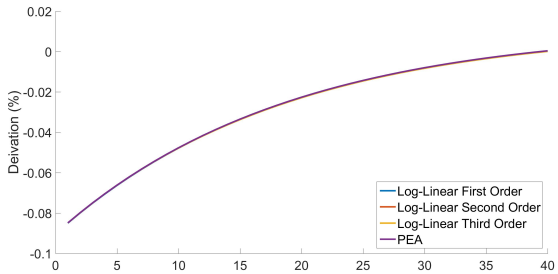
The attempts in the literature to resolve the excessive countercyclical labour problem focus on the introduction of frictions aimed at making labour inelastic in the period of the shock. These approaches prevent labour from reacting contemporaneously to the shock and thereby restrict the only free variable remaining in the system, e.g. inelastic labour supply (Jermann, 1998) or multi-sector models with limited inter-sectoral mobility (Boldrin et al., 2001). The use of frictions to make labour inelastic, whether overall or in the period of the shock, is unsatisfying for two reasons. First it does not allow one to study the impact the cyclical nature of labour has on the results of the model, i.e. the models are not flexible enough to model a variety of labour responses. Thus a view is taken on the cyclical response of labour and built into the model. Secondly such frictions make the models built for purpose, i.e. using these models to answer other questions may be difficult. In particular the extension of models into one with nominal rigidities is not trivial with frictions of the type introduced in Boldrin et al. (2001).

The difficulties faced by the proposed solutions in the literature could be overcome

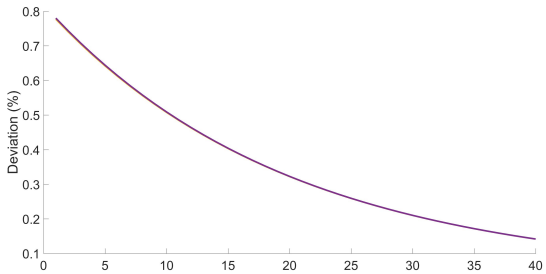
were one to find a preference structure that permitted the correct cyclical response of labour yet with the flexibility to model various responses of labour. The extension of the models into a world of nominal rigidities would be much easier as such extensions do not place any assumptions on the preference structure. The analysis of this chapter suggests that the simplest route to such a solution is to use a preference structure that removes the income effect (or significantly weakens the income effect). I deem this the simplest route as it attacks the problem of the income effect dominating the substitution effect directly, rather than indirectly via additional frictions or the introduction of additional types of shocks to correct the problem. Such a route to solving the countercyclical labour problem has, as yet not been proposed in the literature and is the focus of the next chapter.

Appendix

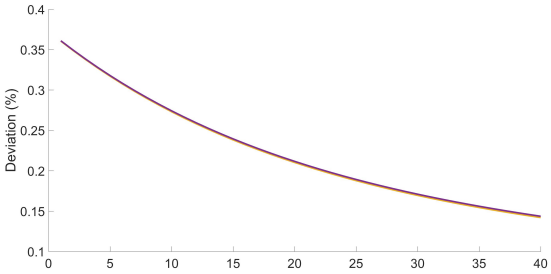
2.A Benchmark Model without Habit IRF Evolution



(a) Labour



(b) Consumption



(c) Investment

Figure 2.A.1: Impulse Response Evolution to Order of Approximation for Benchmark Model without Habits

CHAPTER 3

Asset Pricing with GHH Preferences

3.1 Introduction

The excessive countercyclical movement of labour serves as a major hurdle to the joint replication of business cycle and asset pricing results. The correct cyclical movement of labour will not only improve the business cycle results but it should also lead to better asset pricing results. Thus correcting the cyclical movement of labour serves as a costless way to improve both business cycle and asset pricing results. The solution proposed in this chapter corrects the malfunctioning labour market mechanism highlighted in the previous chapter in an attempt to provide greater faith in the model results elsewhere.

Once labour is prevented from being excessively countercyclical - there is much debate on whether it is procyclical or acyclical- the income of agents cannot decrease response to a positive productivity shock. This income will either have to be consumed or invested, both of which agents dislike due to consumption habits and capital adjustment costs, forcing agents to once again face a real tradeoff. This will therefore

affect asset prices either through the risk free rate or via Tobin's Q.

Given the possibility of a "free lunch" via the simultaneous improvement of both asset pricing and business cycle results there have been various responses to excessive countercyclical labour in the literature. All such attempts to resolve the problem, however, have focused on assumptions or frictions which make labour inelastic in the period of the shock. These approaches prevent labour from reacting contemporaneously to the shock and thereby restrict the only free variable remaining in the system. Jermann (1998) showed that in a model with inelastic labour, habits in consumption and capital adjustment costs joint replication results were superior to those without all three features. While this may allow for replication of the quantitative results the assumption is unsatisfying given the absence of endogenous labour supply. An alternative approach proposed by Boldrin et al. (2001) is a multi-sector model with limited inter-sectoral mobility. In such a model factors of production may be reallocated between periods in response to the shock but cannot be reallocated in the period of the shock. The assumption of limited inter-sectoral mobility therefore makes labour supply in each of their two sectors inelastic in the period of the shock. While this is much closer to reality it continues to rest on inelastic labour in the period of the shock to garner the correct cyclical movement of labour.

There are two routes to remedying the response of labour in the benchmark model without resort to inelastic labour (Jermann, 1998) or multisector production with limited factor mobility (Boldrin et al., 2001). The first is to remove the income effect in the labour supply decision by an alteration of the utility function. By doing so the labour supply will respond solely to the higher wage and not to the higher level of consumption. The natural choice of utility function for this purpose is that proposed by Greenwood et al. (1988), the so called GHH preferences. While GHH preferences

have become popular in various strands of the macroeconomic literature, the use of this preference structure is justified on purely pragmatic grounds - i.e. it permits one to marginalise the income effect. I do not contend that this is the preference structure that describes the world, but rather seek to understand how much improvement one can achieve if the cyclical movement of labour is corrected.[1] The second route is to make the wage less sensitive to the contemporaneous labour supply decision. This requires households to be forward looking in their labour supply decisions rather than making period-by-period decisions. The obvious solution following this route is to move to a New Keynesian model and introduce sticky wages, which allows households to be off their labour supply curve in the short run. Sticky wages will also break the strict relationship between the real wage and the contemporaneous marginal product of labour. In this chapter we will focus on the former, while the introduction of nominal rigidities will be the subject of Part II. This chapter shows that one can indeed induce the correct cyclical response of labour by simple alteration of the utility function from the CRRA form to the GHH form.

A solution using the utility function is the preferred approach as it permits extensions into models with nominal rigidities with relative ease as such extensions do not rely on the specific form of the utility function. It also removes the need for onerous assumptions about frictions in the labour market so providing a resolution to the problem that I deem to be "cleaner".

One could argue that introducing nominal rigidities immediately would be the obvious route to a solution of the excessive countercyclical labour problem. The ultimate aim of this thesis is to study asset pricing responses to central bank policy rules. Such a study necessarily requires models with nominal rigidities, thus one could have

[1] GHH preferences are the simplest and most widely studied preference structure that allows one to study this.

moved directly to New Keynesian models. Indeed the same concerns surrounding the countercyclical movement of labour are not theoretically well founded in a model with nominal rigidities, as pointed out by Cantore et al. (2014). I feel that moving to such a model misses the point as we have given up the opportunity to further diagnose the causes of the problem since the labour market plays such a central role in New Keynesian models. Thus by studying the labour market without New Keynesian features allows one to disentangle the causes due to the asset pricing features and those due to New Keynesian features. Furthermore a utility function approach is sufficiently flexible that it can be applied to both models with and without nominal rigidities and so provides a general solution to the problem. Such a utility function approach to solving the problem does not appear to have been studied in the literature.

This chapter explores the impact of replacing the standard constant relative risk aversion utility function with GHH preferences with a view to resolving the excessive countercyclical movement of labour. In the presence of internal habits in consumption the income effect cannot be removed by introduction of GHH preferences. However, I find that the income effect becomes habit induced with the introduction of internal habits and that consequently the habit intensity controls the strength of the resulting income effect. This allows one to control the cyclical behaviour of labour with the habit intensity parameter. I show that a direct consequence of this property is that the habit strength parameter can be used to define an alternative calibration approach for the previously free habit intensity parameter. This calibration of the habit parameter relies on setting the habit intensity so as to induce the correct cyclical movement in labour. This calibration scheme has not been proposed before, mainly as a result of very little study having been carried out on GHH preferences with internal habits. A similar approach is proposed in Jaimovich and Rebelo (2009) in their proposed preference structure, while Jaimovich (2008) studies how their equivalent of a habit

parameter in the specification of Jaimovich and Rebelo (2009) can be used to vary the income effect, however both stop short of suggesting the alternate calibration scheme proposed here.

I find that using this alternate calibration scheme for the habit parameter does allow for a resolution of the countercyclical labour problem that results from capital adjustment costs. Thus a utility function approach to the solution of the problem is viable. However this is at the cost of a weaker habit intensity parameter. I find that once the correct cyclical behaviour is induced in labour, there is a significant improvement in the business cycle dynamics and quantitative business cycle results. We further find that the quantitative asset pricing results are significantly improved when GHH preferences are introduced. This improvement in asset pricing results is despite the weakening of the habit intensity which is a priori unexpected.

3.2 An Alternate Preference Structure: GHH Preferences

Countercyclical labour is the consequence of the income effect dominating the substitution effect. The solution proposed here is to introduce preferences of the form,

$$u(c,n) = U(c - \mathcal{G}(n)) \tag{3.2.1}$$

where $U'(\cdot) > 0$, $U''(\cdot) < 0$, $\mathcal{G}'(\cdot) > 0$ and $\mathcal{G}''(\cdot) > 0$. This form was first introduced by Greenwood et al. (1988) and are referred to as GHH preferences.

The equilibrium labour market condition requires that,

$$w_t = z_t f_{n,t} = -\frac{u_{n,t}}{u_{c,t}}. \tag{3.2.2}$$

Under the general GHH specification this becomes,

$$w_t = z_t f_{n,t} = \mathcal{G}'(n). \tag{3.2.3}$$

The advantage of GHH preferences for the current purpose is that there is no income effect in the labour supply equation since the marginal rate of substitution between labour and consumption is independent of consumption. Thus the labour supply decision depends only on the real wage. As a more concrete example, let us consider the following functional forms,

$$u(c_t, n_t) = \begin{cases} \dfrac{\left[c_t - \chi \frac{n_t^{1+\varphi}}{1+\varphi}\right]^{1-\sigma} - 1}{1-\sigma} & \text{GHH} \\ \\ \dfrac{[c_t^{\gamma}(1-n)^{1-\gamma}]^{1-\sigma} - 1}{1-\sigma} & \text{CRRA} \end{cases}$$

where σ measures the degree of risk aversion, χ is the utility weight of the disutility of labour and φ is the inverse of the Frisch elasticity of labour supply for the GHH utility function and the CRRA parameters are as defined previously. Under these two functional forms we get the following equilibrium labour conditions,

$$w_t = \begin{cases} \chi n_t^{\varphi} & \text{GHH} \\ \\ \dfrac{1-\gamma}{\gamma}\dfrac{1-n_t}{c_t} & \text{CRRA} \end{cases}$$

Under CRRA preferences the labour supply decision depends on both the wage and consumption. While the labour supply decision only depends on wages under GHH preferences. Hence there is an income effect, as defined in the previous chapter, present in standard CRRA preferences but not in standard GHH preferences.

Standard GHH preferences present a simple way in which to remove the income effect without assuming risk neutral agents. Let us consider the separable utility function,

$$u(c, n) = g(c) - h(n),$$

where $g(\cdot)$ and $h(\cdot)$ are both twice differentiable functions. Under this separable utility function the labour supply decision is,

$$w_t = \frac{h'(n)}{g'(c)}.$$

The only way in which the separable utility function does not result in an income effect is if $g'(\cdot) = 1$, i.e. if consumption enters linearly into the utility function. However such linearity implies that agents are risk neutral over consumption gambles. Thus the linearity of consumption in the utility function, which is central to the removal of the income effect, requires the assumption of risk neutral agents. Standard GHH preferences provide a simple way in which to remove the income effect via consumption entering linearly and yet retain agents that are risk averse over consumption gambles via the concave function $U(\cdot)$.

3.2.1 GHH with Habits: Re-emergence of the Income Effect

Internal habits in consumption are a key to the benchmark model; and macro-finance models with capital adjustment costs in general. The presence of habits in consump-

tion leads to the denominator of the labour supply equation, (3.2.2), becoming,

$$\Lambda_t = u_{c,t} - \beta\nu E_t \left[u_{c,t+1}\right] \tag{3.2.4}$$

where $u_{c,t}$ is the marginal utility of consumption in period t. The strengthening of the already present income effect as internal habits are introduced and strengthened has already been discussed for CRRA preferences. However, one notes that even under GHH preferences the introduction of internal habits in consumption leads to the emergence of an income effect not present in the absence of habits. (Monacelli and Perotti, 2008)

We consider GHH preferences with internal habits in consumption of the following functional form,

$$u\left(c_t, c_{t-1}, n_t\right) = \frac{1}{1-\sigma}\left[\left(c_t - \nu c_{t-1}\right) - \chi\frac{n_t^{1+\varphi}}{1+\varphi}\right]^{1-\sigma} \tag{3.2.5}$$

where ν is the strength of the habit and all other parameters are as defined for the simple GHH utility function. The equilibrium labour condition, (3.2.2), becomes,

$$w_t = \frac{\chi n_t^{\varphi}\left[\left(c_t - \nu c_{t-1}\right) - \chi\frac{n_t^{1+\varphi}}{1+\varphi}\right]^{-\sigma}}{\left[\left(c_t - \nu c_{t-1}\right) - \chi\frac{n_t^{1+\varphi}}{1+\varphi}\right]^{-\sigma} - \beta\nu E_t\left\{\left[\left(c_{t+1} - \nu c_t\right) - \chi\frac{n_{t+1}^{1+\varphi}}{1+\varphi}\right]^{-\sigma}\right\}}. \tag{3.2.6}$$

In contrast to the no habit case, equilibrium labour is no longer independent of consumption and so an income effect exists. The emergence of an income effect is a consequence of the complementary adjacency in consumption induced by the introduction of internal habits. The income effect only emerges for GHH preferences in the presence of internal habits. Were the habits external then the equilibrium labour condition, for both GHH and CRRA, would be unchanged from the no habit

case. External habit models were considered but have not been reported as they induce an unacceptably high level of volatility in the risk free rate. Furthermore, preferences with external habits are not able to jointly account for moderate levels of risk aversion and moderate levels of risk aversion in wealth. (Boldrin et al., 1997)

3.2.2 Calibration

The introduction of GHH preferences does not affect the technology parameters so that the calibration of the technology parameters remains unchanged from the benchmark model. The presence of GHH preferences requires calibration of σ, ν, χ and φ. The calibration of ν is unchanged.

The Frisch elasticity of labour supply is defined as the elasticity of labour supply with respect to the real wage, holding constant the marginal utility of consumption. (Monacelli and Perotti, 2008) Monacelli and Perotti (2008) provides a mapping of the Frisch elasticity of substitution into a GHH setting from a number of authors. Mapping these values across to our functional form implies that $\varphi = \zeta - 1$, where ζ is the inverse of the Frisch elasticity in Monacelli and Perotti (2008). Under this transform the parametrisations of φ range from $\varphi = \frac{1}{1.85}$ (Smets and Wouters, 2007) to $\varphi = \frac{1}{0.4}$ (Schmitt-Grohe and Uribe, 2008; Jaimovich and Rebelo, 2009), while Monacelli and Perotti (2008) use an intermediate value of $\varphi = \frac{1}{0.8}$. I employ a calibration of $\varphi = \frac{1}{0.75}$. This calibration is supported by Chetty et al. (2011) who suggest that Frisch elasticities of aggregate labour supply above 1 are inconsistent with micro evidence.

A key characteristic of CRRA preferences is that there is a strict relationship between the inter-temporal elasticity of substitution (IES) and the coefficient of risk aversion. IES is a measure of the aversion to variation in consumption across time periods, while the coefficient of risk aversion measures the aversion to variation of

consumption over different states within the period. CRRA preferences require that agents who dislike variation of consumption over states will also dislike variation in consumption over time (Mehra, 2003). While this relationship is useful in the calibration of CRRA utility functions, there is no a priori reason why this should be the case (Mehra, 2003). GHH preferences lead to a decoupling of IES from the coefficient of risk aversion.[2] (Jahan-Parvar et al., 2013) Since GHH preferences decouple IES and risk aversion we employ a calibration of $\sigma = 2$.

The parameter χ measures the utility weight of the disutility of from labour and is parametrised so that one-third of time is spent working in the deterministic steady state.

Table 3.1: Calibrated GHH Preference Parameters for U.S.

Time Preference	$\beta = 0.9963$
Frisch Elasticity of Labour Supply	$\frac{1}{\varphi} = 0.75$
Curvature Parameter	$\sigma = 2$
Habit Parameter	$\nu = 0.9$

3.3 Business Cycle Results

The business cycle results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique.[3] In the results below the "Benchmark Model" refers to the model presented in the previous chapter where preferences are defined by a CRRA utility function rather than the GHH utility function introduced in this chapter.

[2] A proof is provided in Appendix A of this chapter

[3] See Chapter 1 for a detailed discussion of the Parameterised Algorithm.

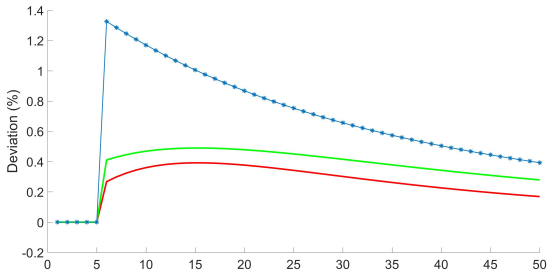
3.3.1 Business Cycle Impulse Response Analysis

The impulse responses under GHH preferences are presented in Figure 3.1. The introduction of GHH preferences does not change the dynamics of the economy, with responses remaining similar to those in the benchmark model. In particular the response of labour remains countercyclical. While the response of labour remains negative it exhibits a smaller response than the benchmark. The habit induced income effect continues to dominate the substitution effect, but the smaller response under GHH suggests significant weakening of the income effect compared to the benchmark model. This smaller response may be attributed to a weakening of the income effect.

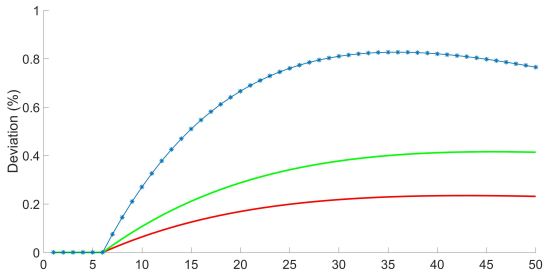
The sensitivity of the labour impulse response to the strength of the habit parameter is presented Figure 3.2. There are two points to note from this figure. The first is that the income effect is significantly weakened by the introduction of GHH preferences and that it is weaker as the habit intensity parameter falls. This is intuitive - under GHH preferences the income effect is habit induced and so would be expected to vary in strength with the habit parameter. The second, and more important, point to note is that the introduction of GHH preferences permits a region, $\nu \in [0, 0.48]$, where the labour response is procyclical. Within this region the income effect is present but sufficiently weak that it cannot dominate the substitution effect resulting in the correct cyclical movement of labour. Thus the labour market mechanism is operating as one would expect in a DSGE model with flexible prices.

3.3.2 An Alternative Calibration Scheme for Habit Intensity

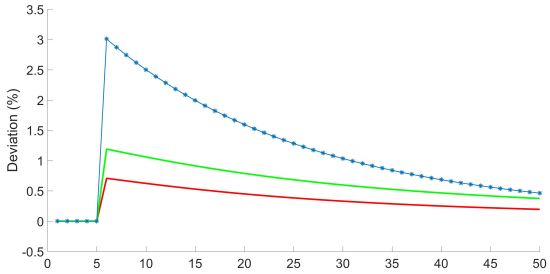
The income effect under GHH preferences, as noted previously, is habit induced rather than inherent. This allows for an alternative calibration scheme for the habit parameter under GHH preferences. Under the alternate calibration scheme the habit



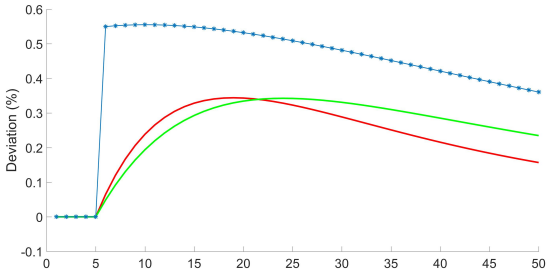
(a) Output



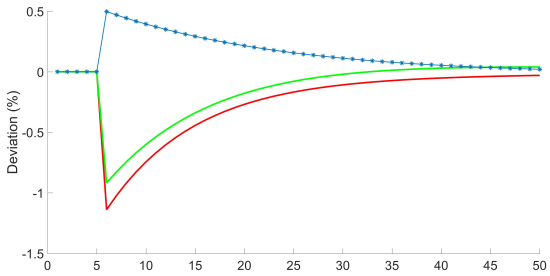
(b) Capital



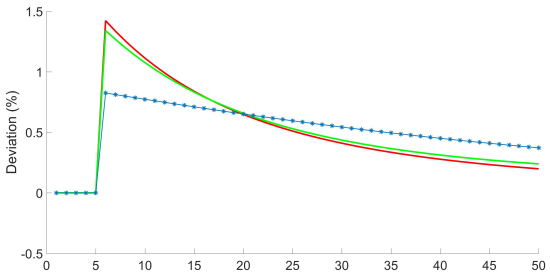
(c) Investment



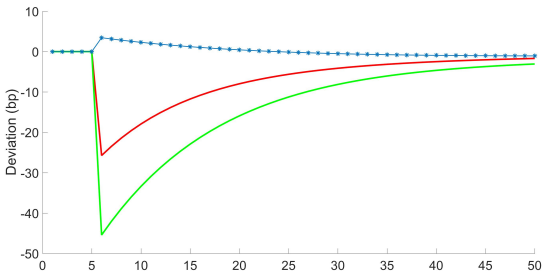
(d) Consumption



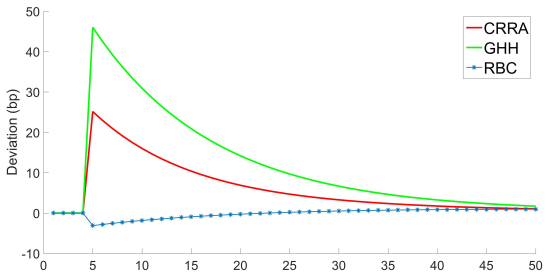
(e) Labour



(f) Real Wage



(g) Interest Rate (Basis Point)



(h) Stochastic Discount Factor (Basis Point)

Figure 3.1: Impulse Responses

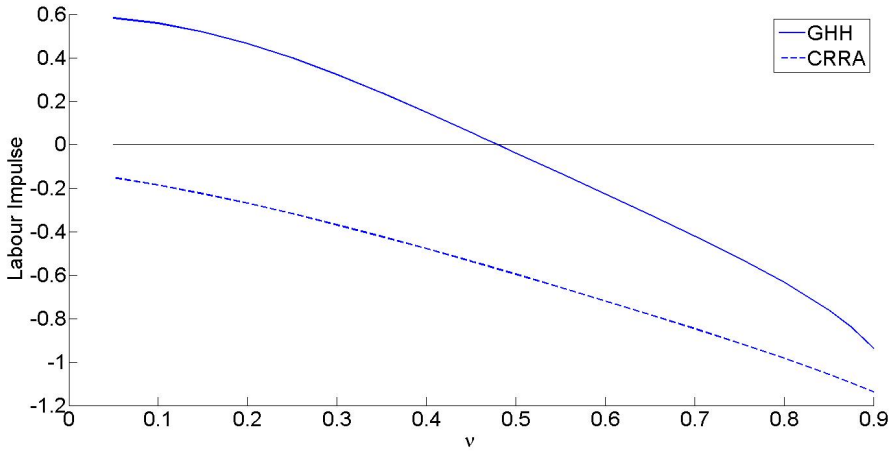


Figure 3.2: Labour Impulse Sensitivity to Habit Parameter

parameter is calibrated to induce the appropriate cyclical movement in labour for the DSGE model of concern - i.e. strictly procyclical in RBC models or possibly countercyclical in New Keynesian models with sticky prices. Thus for GHH preferences we can use the habit intensity to control the strength of the income effect and hence the cyclical behaviour of labour. Jaimovich and Rebelo (2009) introduce a preference specification whereby they can control the size of the income effect using a preference parameter that is similar to the habit intensity parameter, ν, in the model with GHH preferences. In fact both Jaimovich and Rebelo (2009) and Jaimovich (2008) use a similar calibration scheme as suggested here to control the size of the income effect. The approach proposed here is preferred to that of Jaimovich and Rebelo (2009) as I achieve their result without need to assume their specific utility function. Rather the utility function used here has been studied, and indeed vetted, extensively in the literature.

There has been a large debate surrounding the effect of a productivity shock on labour (e.g. see discussion in Gali (1999) and Smets and Wouters (2007)). The advantage of using this alternative calibration scheme is that one can take a relatively

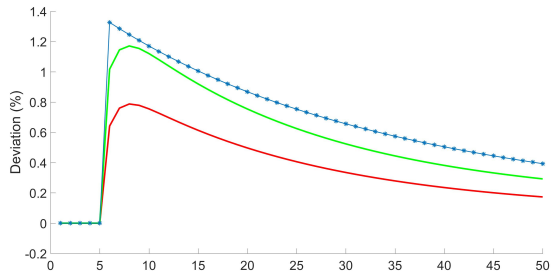
neutral approach to the impact of a productivity shock on labour. Furthermore given the flexibility of the response of labour one can also study the effect of varying responses rather than being constrained to that implied by the specific model. Under this scheme one could take the labour response to a technology shock from an empirical impulse response function and vary ν until the initial response matches the empirical one.[4]

The alternative calibration scheme has the added advantage of being able to study asset pricing results without calibrating parameters to maximise the ability to match asset pricing facts, i.e. without a priori biasing asset pricing results. This is in contrast to the calibration methodology adopted in the literature where the habit parameter is calibrated to maximise the ability of the model to match asset pricing facts.

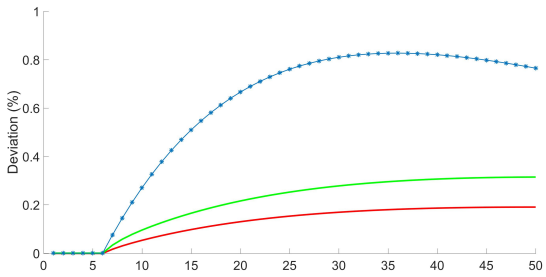
I employ an alternative calibration of $\nu = 0.47$ under this scheme. This value has been chosen as the largest possible value of the habit intensity parameter which induces a positive response in labour, i.e. where the substitution effect just dominates the income effect.

The impulse responses under this alternative calibration are presented in Figure 3.3. The ability to costlessly exploit the labour channel to smooth consumption was the main reason for the countercyclical movement of labour in the benchmark model. Under the alternate calibration scheme the response of labour is restricted to be not countercyclical thereby restricting the ability of agents to exploit the labour channel to smooth income and ultimately consumption. Agents are in an environment where they face a trade-off and consequently cannot achieve their goal of smooth consumption in a costless manner, i.e. by reducing labour supply.

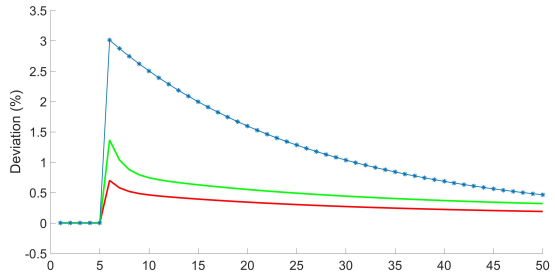
[4]I do not take this approach in this thesis, but it is left for future work.



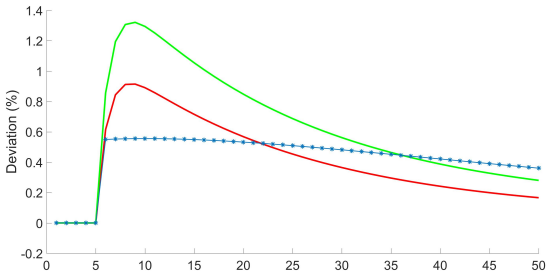
(a) Output



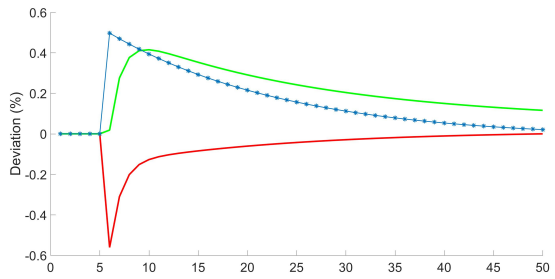
(b) Capital



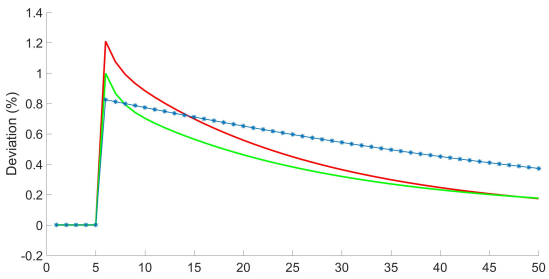
(c) Investment



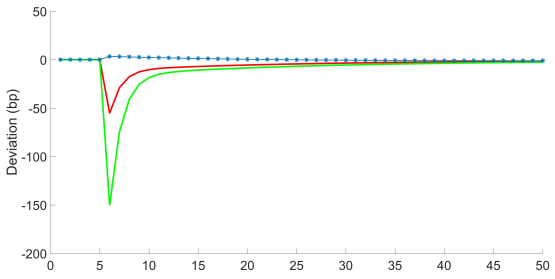
(d) Consumption



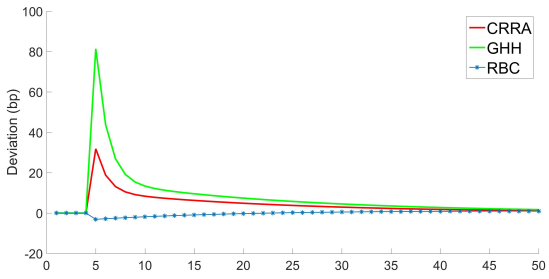
(e) Labour



(f) Real Wage



(g) Interest Rate (Basis Point)



(h) Stochastic Discount Factor (Basis Point)

Figure 3.3: Impulse Responses under Alternate Habit Calibration Scheme ($\nu = 0.47$)

The constraint that labour not be countercyclical leads to amplified responses in all variables when compared to the benchmark. Countercyclical labour mutes the potential increase in output from a positive shock thereby muting the increase in income available to households. The increase in output resulting from non-countercyclical labour leads to an increase in income thereby leading to amplified responses in the other variables. The power of non-countercyclical labour may be seen by comparing the benchmark, where labour remains countercyclical, with that of the GHH model, where labour is restricted to being non-countercyclical. The difference between the benchmark and GHH models are even larger once the response of labour has been corrected when compared to Figure 3.1.

The trade-off agents face is between smooth consumption, which requires high capital adjustment costs as investment must respond significantly, or volatile consumption, to minimise adjustment costs. Agents continue to prefer smooth consumption but weaker habits under the alternate calibration mean they are willing to accept a larger movement in consumption compared to models where the habit is much stronger. Combined with the increase in income and continued strong capital adjustment costs, which restrict the movement of investment, consumption rises significantly in order to satisfy the budget constraint. This large response of consumption leads to the large movement in the stochastic discount factor, which in turn leads to the large movement in the risk free rate.

In the presence of habits there is a positive feedback between consumption and investment. Agents have a strong desire to smooth consumption so that when consumption is high (low) they seek to increase (decrease) investment. This simultaneously builds up (depletes) their habit stock and reduces variation in consumption. (Boldrin et al.,

1995) Procyclical labour, which amplifies the consumption response, together with this positive feedback allows GHH preferences to weaken the the impact of the adjustment cost, regardless of the calibration, as evidenced by the larger response of investment in both Figure 3.1 and Figure 3.3. This is not possible in the benchmark model due to the strong income effect generated by the CRRA preferences which mutes the consumption response by muting the increase in output and income.

3.3.3 A General Equilibrium Decomposition of the Income and Substitution Effects

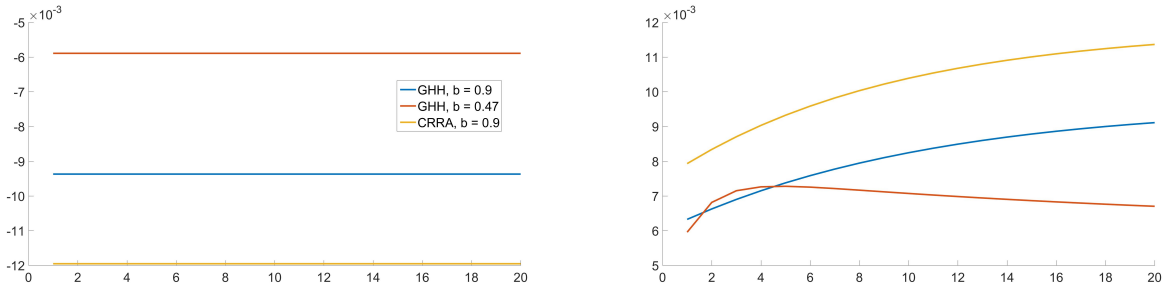
The income and substitution effects, as defined so far, have relied on a partial equilibrium analysis. While this has been sufficient for the analysis thus far, a comparison across different utility specification requires a general equilibrium analysis of the income and substitution effects. A general equilibrium analysis of the income and substitution effects may be carried out using the decomposition proposed in King (1991). This decomposition considers the impulse response of labour under two scenarios:

1. All variables may respond optimally to the shock, i.e. standard impulse response analysis

2. The agent faces no change in income

Under the second scenario the household also receives a one off transfer at impulse to compensate for the change in lifetime utility that results from the shock.[5] This decomposition allows one to isolate the income effect and thereby also the substitution effect. Figure 3.4 presents this decomposition for the CRRA model with $b = 0.9$ and the GHH model with $b = 0.9$ and $b = 0.47$. The decomposition highlights that the income effect on labour is much larger under CRRA preferences, compare the yellow

[5]Let U be the lifetime utility in the non-stochastic steady state, and let U^* be the lifetime utility in response to the shock. Then the one off transfer made to the agent will be the difference between U and U^*.



(a) Income Effect on Labour

(b) Substitution Effect on Labour

Figure 3.4: King (1991) Decomposition of Income and Substitution Effects

and blue lines. Secondly it highlights that the income effect is significantly weaker as the habit intensity is lowered. Finally it shows that under the alternate calibration scheme the income effect is dominated by the substitution effect.

3.3.4 Quantitative Business Cycle Results

The quantitative business cycle results are obtained from a simulation of the Parameterised Expectations Algorithm. The results are presented for both habit parameter calibrations for the Benchmark and GHH models. The Benchmark model with $\nu = 0.47$ is reported for the benefit of comparison. The significant improvement in output volatility for the GHH model with $\nu = 0.47$ is a result of the calibration inducing procyclical movement in labour.

The volatility of labour relative to output falls as habits are weakened and GHH preferences are introduced. The largest share of this reduction is driven by the weakening of habits. The weakening of the habit has been shown to lead to a weakening of the income effect. As habit intensity is reduced there is a progressively smaller contemporaneous labour response to the shock. This muted response causes labour to be less volatile as there is a reversion to zero as habits are weakened before the substitution effect begins to dominate. The introduction of GHH preferences reduces

Table 3.1: Business Cycle Results

		Benchmark		GHH Preferences	
	Data	$\nu = 0.9$	$\nu = 0.47$	$\nu = 0.9$	$\nu = 0.47$
Standard Deviation (%)					
Output	1.72	0.31	0.86	0.42	1.13
Standard Deviation Relative to Output					
Consumption	0.79	0.66	1.14	0.40	1.09
Hours Worked	0.63	3.40	0.53	2.07	0.34
Investment	3.24	2.20	0.76	2.74	0.94
Productivity	0.54	3.05	1.25	2.25	0.84
Real Wage	0.47	4.25	1.41	2.90	0.76
Contemporaneous Correlation with Output					
Consumption	0.87	0.80	0.99	0.61	0.99
Hours Worked	0.93	-0.81	-0.66	-0.85	0.80
Investment	0.86	0.92	0.93	0.97	0.90
Productivity	0.78	0.90	0.98	0.94	0.98
Real Wage	0.21	0.88	0.96	0.94	0.96
Euler Equation Errors		-3.6455×10^{-5}	-1.8994×10^{-5}	-7.0131×10^{-5}	-1.4264×10^{-5}

the labour volatility as it too leads to a weaker income effect for all values of ν.

The quantitative results indicate that the improvement along the labour dimension leads to deterioration along both the consumption and investment dimensions. In the discussion that follows, it is clear that GHH preferences are driving this deterioration in the quantitative results along the consumption and investment dimensions. However, it must also be noted that in both cases the GHH results are closer to the data than their CRRA counterparts.[6]

The reversion of labour to zero sees relative consumption volatility fall as one moves from CRRA preferences to GHH preferences. The weakening of the income effect under GHH preferences leads to smoother household income which, ceteris paribus, allows for smoother consumption under GHH preferences. This result has been found

[6]That is, comparing Columns (2) and (4) for high habit intensity and Columns (3) and (5) for low habit intensity.

in previous studies using GHH preferences, (e.g. Jahan-Parvar et al. (2013); Guvenen (2009)).

Investment volatility increases with the introduction of GHH preferences. This increase is both relative to output and in absolute terms despite the inertia induced by capital adjustment costs. The introduction of GHH preferences has been shown to lead to a weaker income effect at all habit levels thereby weakening their ability to exploit the labour channel to smooth consumption. Agents actually receive higher income under GHH preferences as a result. This higher income must be consumed or invested. Since agents are receiving higher income they can consume the same/more under GHH preferences by lowering the proportion of income spent on consumption. The presence of strong habits also causes agents to dislike very volatile consumption. This leaves more income to be invested and therefore leads to a more volatile investment series. Thus the smoother consumption series places higher pressure on investment to respond to the shock despite the presence of capital adjustment costs. The mechanism at work here has been discussed in the previous section where it was noted that under the influence of habits there is a positive feedback between consumption and investment. The fall in volatility as the habit parameter is weakened is expected as consumption takes on more of the contemporaneous response to shocks as this channel is opened up. The opening up of the consumption channel is in the sense that it becomes less costly to alter consumption as habit intensity is weakened.

3.4 Asset Pricing Implications

The assets considered in this section are one period risk free bonds and one period equity. These assets are identical to those introduced and defined in the previous

chapter. In particular there is no change in the form of the gross equity return,

$$R^e_{t+1} = \frac{\phi'(g_t)}{\phi'(g_{t+1})}\left[z_t f_{k,t}\phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right],$$

where all terms continue to be defined as in the previous chapter.

The risk free rate does have a different form since the marginal rate of substitution will be different under the new preference specification. It continues to be priced according to,

$$R^f_{t+1} = r^f_{t+1} + 1 = \frac{1}{E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda t}\right]},$$

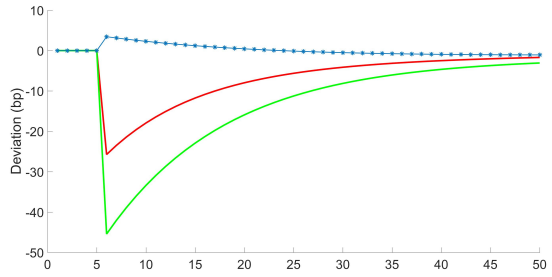
where Λ_t will have a different form than in the previous chapter as the utility function has been altered.

3.4.1 Asset Pricing Impulse Response Analysis

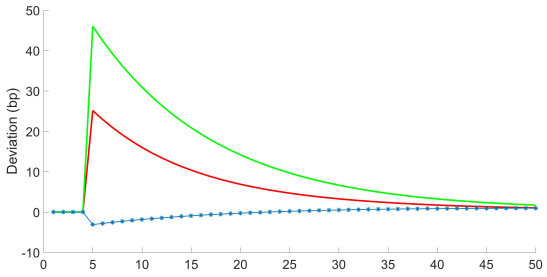
Two sets of impulse response figures are presented. Those presented in Figure 3.1 are for the GHH model with $\nu = 0.9$ so as to permit a study of the impact of the introduction of GHH preferences on asset pricing dynamics. Figure 3.2 compares asset pricing dynamics for the two GHH models, i.e. for $\nu = 0.9$ and $\nu = 0.47$, allowing the study of the impact of correcting the cyclical behaviour of labour on asset pricing dynamics. In order to understand the drivers of the return on equity I also look at the impulse responses of the different components of the Boldrin et al. (2001) decomposition[7] of the equity return which was discussed in the previous chapter.

[7]The Boldrin et al. (2001) decomposition for models with capital adjustment costs are,

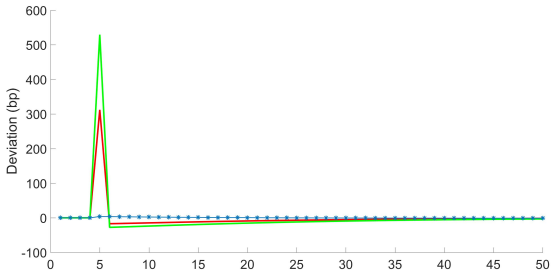
$$\begin{aligned}
\mathcal{K}_{t+1} =& z_{t+1} f_{k,t+1} \\
P_{k,t+1} =& \left[1 - \delta + \phi(g_{t+1}) - g_{t+1}\phi'(g_{t+1})\right] P_{k',t+1} \\
P_{k',t} =& \frac{1}{\phi'(g_t)}
\end{aligned}$$



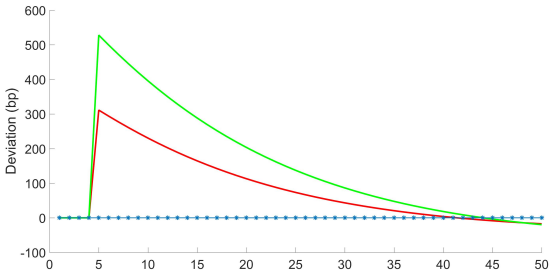
(a) Risk Free Rate



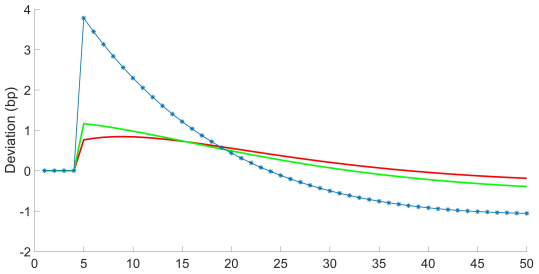
(b) Stochastic Discount Factor



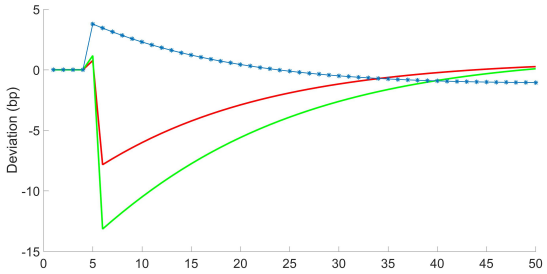
(c) Ex-Post Equity



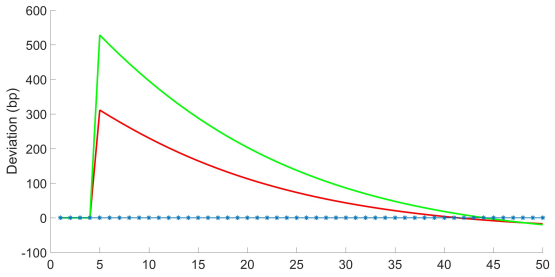
(d) Tobin's Q: $P_{k',t}$



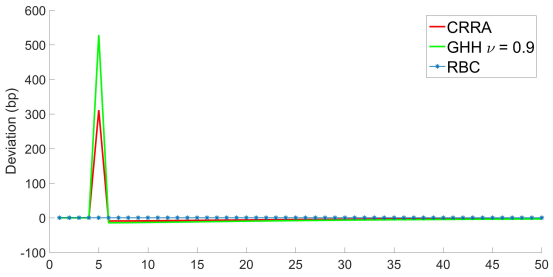
(e) Marginal Product of Capital: $\mathcal{K}_{t+1}$



(f) $\frac{\mathcal{K}_{t+1}}{P_{k',t}}$

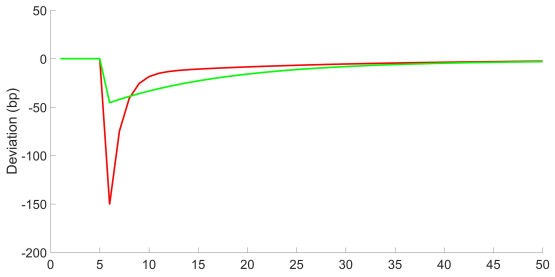


(g) $P_{k,t+1}$

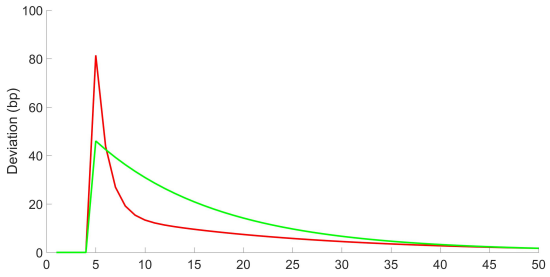


(h) Capital Gain: $\frac{P_{k,t+1}}{P_{k',t}}$

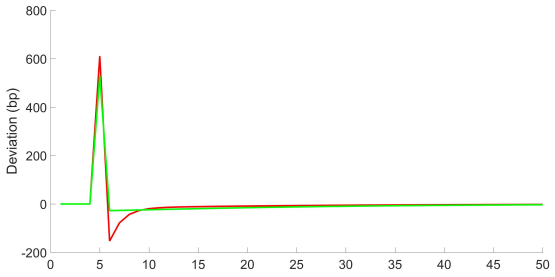
Figure 3.1: Ex-Post Equity Impulse Responses



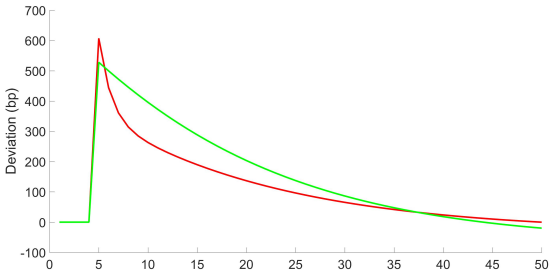
(a) Risk Free Rate



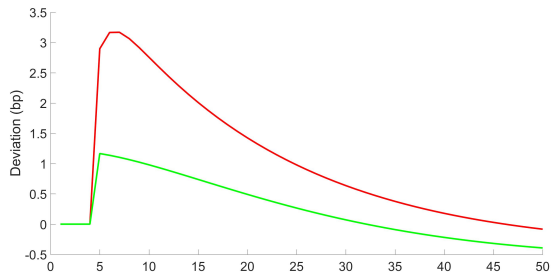
(b) Stochastic Discount Factor



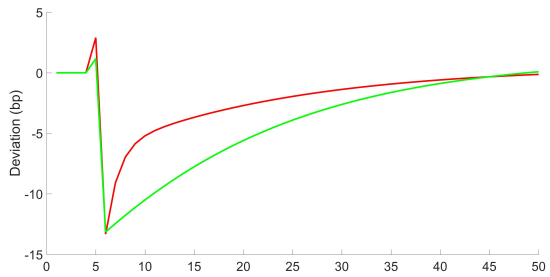
(c) Ex-Post Equity



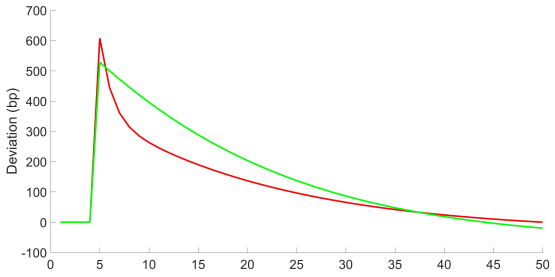
(d) Tobin's Q: $P_{k',t}$



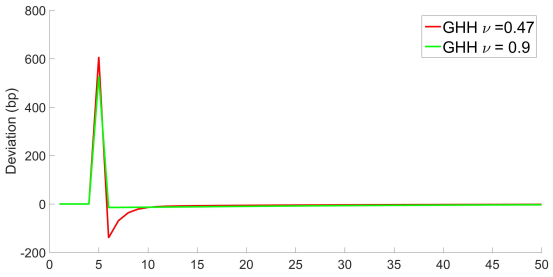
(e) Marginal Product of Capital: $\mathcal{K}_{t+1}$



(f) $\frac{\mathcal{K}_{t+1}}{P_{k',t}}$



(g) $P_{k,t+1}$



(h) Capital Gain: $\frac{P_{k,t+1}}{P_{k',t}}$

Figure 3.2: Ex-Post Equity Impulse Responses

The response of the risk free rate is driven by the response of consumption in the various models. The dynamics of consumption have been discussed in detail when discussing the business cycle dynamics of the various models. Suffice it to say that the introduction of GHH preferences leads to an amplification of the risk free rate response. The weakening of the habit parameter in GHH models causes a magnification of the risk free response as consumption takes on more of the adjustment and a faster return to steady state as consumption inertia is weakened. Both of these results make intuitive sense given the business cycle dynamics.

The response of the return on equity is higher for the GHH models due to the larger response of Tobin's Q in these models. This amplified response in GHH models is the consequence of the greater response of investment at impulse resulting from a weakening of the capital adjustment cost as discussed in previous sections. As noted in the previous chapter, in the presence of capital adjustment costs, it is the capital gain that drives the response of the ex-post equity return. It's characteristic one period spike and subsequent fall is due to the one period delay in the price of capital being used for determining the return.

3.4.2 Quantitative Asset Pricing Results

The quantitative asset pricing results show an improvement in the asset pricing results of the model once GHH preferences are introduced.

There is a deterioration in the mean risk free rate under GHH preferences, which can be understood from the impulse response of consumption in the two models. Both have the same habit intensity but under GHH preferences this induces greater

Table 3.1: Asset Pricing Results

		Benchmark		GHH Preferences	
	Data	$\nu = 0.9$	$\nu = 0.47$	$\nu = 0.9$	$\nu = 0.47$
Mean (% p.a)					
Risk Free	1.49	1.37	1.41	1.05	1.25
Equity	7.87	1.88	1.89	2.63	3.07
Equity Premium	6.52	0.51	0.47	1.59	1.81
Standard Deviations (% p.a)					
Risk Free	2.62	0.95	1.04	1.81	2.61
Equity	16.72	4.52	4.44	7.67	8.84
Equity Premium	15.69	4.45	4.32	7.54	8.45

inertia in consumption, as evidenced by the impulse responses.[8] One sees consumption takes much longer to return towards steady state hence exhibits a greater degree of inertia. Households appear to prefer smoother consumption under GHH preferences so are willing to accept a smaller risk free rate to achieve this goal. The increase in the risk free rate as habits are weakened is consistent with agents accepting less smooth consumption. The increased volatility of the risk free rate is expected given the amplified response to shocks both once GHH preferences are introduced and as habits are weakened.

There is a low return on equity and consequently low equity premium in the benchmark model. While much better than a pure RBC model[9] these results are still very far from the data. Part of the small equity premium may be attributed to the countercyclical movement of labour in the benchmark model. Introducing GHH preferences increases the equity return. This increase is attributed to two effects - the larger income available to households as the income effect on labour is weakened, and the greater consumption inertia induced under GHH preferences. Once labour is restricted to being non-countercyclical under the alternate calibration, an even larger

[8] e.g. see Figure 3.1

[9] Based on the same calibration we get $R^e = 1.40\%$p.a. and an equity premium of 0.005%p.a.

increase in the return results. This is attributed to the procyclical movement of labour and the resulting increase in income leading to higher investment levels, despite the weakening of habit intensity. While the premium is still far from the data it highlights that the procyclical movement of labour, and the re-emergence of a real trade-off by agents it represents, allows asset pricing results to be significantly improved. This is particularly true in the preferred model with GHH preferences and the alternate calibration scheme for habits.

3.4.3 GHH: Decoupling Risk Aversion and IES

The quantitative asset pricing results demonstrate that GHH preferences improve asset pricing results without a significant deterioration in the business cycle results. Thus they come closer to the ideal of joint replication than the Benchmark specification. This can be explained by the ability of GHH preferences to decouple risk aversion from the inter-temporal elasticity of substitution (IES). I refer to GHH preferences decoupling the IES from risk aversion in the sense that knowledge of the preference parameters is not sufficient to pin down the IES.[10]

A key feature of CRRA preferences is that there is a strict relationship between IES and the coefficient of risk aversion. IES is a measure of the aversion to variation in consumption across time periods, while the coefficient of risk aversion measures the aversion to variation of consumption over different states within the period. CRRA preferences require that agents who dislike variation of consumption over states will also dislike variation in consumption over time. (Mehra, 2003) While this relationship is useful in the calibration of CRRA utility functions, there is no a-priori reason why this should be the case. (Mehra, 2003) Often the macro-finance literature looks to Epstein-Zin preferences as the go to alternative to the standard CRRA prefer-

[10]In Appendix A to this chapter I provide the derivation of IES under standard GHH preferences.

ences. This is largely due to a significant body of work which highlights the superior asset pricing results of Epstein-Zin preferences due to their ability to separate the coefficient of risk aversion from the intertemporal elasticity of substitution. GHH preferences are preferred to Epstein-Zin as standard GHH preferences have no income effect. As shown in the previous chapter it is the dominance of the income effect over the substitution effect in the presence of capital adjustment costs which leads to the countercyclical labour problem, hence any preference structure which allows its marginalisation will improve the quantitative results as shown. In addition Jahan-Parvar et al. (2013) show that standard GHH preferences also have the property which commends Epstein-Zin preferences, namely it separates the coefficient of risk aversion from the intertemporal elasticity of substitution.

3.5 Conclusion

In this chapter we have demonstrated that the countercyclical labour problem of the previous chapter can be resolved with a purely preference based approach. Introducing GHH preferences with internal habits I showed that the income effect can be controlled via the habit intensity parameter. This feature of GHH preferences with internal habits led to the proposal of an alternate calibration scheme for the habit parameter which led to overall improved asset pricing and business cycle results. Such a preference based approach and alternate calibration scheme have not been proposed in the literature previously. Finally, the existence of a solution to the countercyclical labour problem via the utility function highlights that one does not need to resort to onerous assumptions on the labour market in order to garner the correct cyclical response of labour in models without nominal rigidities.

3.5.1 The Need for Nominal Rigidities

The contemporaneous correlations for the preferred model with the correct cyclical behaviour of labour are all very close to unity meaning that variables move in lock step with output (or alternatively the productivity shock). This is a well known shortcoming of RBC models which are the result of a weak internal propagation mechanism in these models. (Christiano, 1988; Cogley and Nason, 1995; Burnside and Eichenbaum, 1996) The propagation of shocks in RBC models relies on capital accumulation and intertemporal substitution making output dynamics the same as impulse dynamics (Cogley and Nason, 1995). Cogley and Nason (1995) further argue that capital adjustment cost models, while altering the flow of investment, do not significantly affect the capital stock. Thus even with capital adjustment costs it is impulse dynamics that drive the dynamics of the model. The introduction of nominal rigidities should lead to an improvement in the contemporaneous correlations as they make agents forward looking and thereby less sensitive to the contemporaneous impulse.

Introducing nominal rigidities requires that firms are endued with a degree of market power. This is achieved by assuming monopolistic competition among intermediate goods firms that produce a differentiated product thereby allowing some degree of pricing power. Since intermediate goods firms can set prices and make profit in equilibrium they set prices at a mark-up on factor prices in the absence of price rigidities. Intuition would suggest that wages are lower than otherwise allowing for the possibility of weakening the income effect of a technology shock as not all of the shock will be apportioned to higher wages. In particular it may be that profits respond asymmetrically to a shock, i.e. they rise more when shocks are positive and fall less when shocks are negative. This would lead to wages rising less when shocks are positive due to lower labour demand and falling more when shocks are negative as profits are

protected. So enduing firms with market power should augment the benefits of GHH preferences found in this chapter.

In the presence of nominal rigidities, firms are somehow restricted in their ability to set prices every period. The pricing decision becomes an inter-temporal decision. In particular if we assume price stickiness *à la* Rotemberg (Rotemberg, 1982) then firms face a menu cost when they change their prices across periods. This makes the pricing decision forward looking as firms must consider future period prices in their current pricing decision. Firms set their prices in the given period in response to the realisation of the shock. However, their pricing response will be tempered by the possibility that it may not be optimal to change prices in future periods as the benefit in terms of additional profit is outweighed by the resulting menu cost associated with changing price in the subsequent period. Thus any price rigidity will divorce, to an extent, the effect a contemporaneous shock has on the responsiveness of the price of the aggregate good. This would weaken the substitution effect thus acting in opposition to GHH preferences.

Household demand for the aggregate good depends on aggregate price, so the responsiveness of consumption will be directly affected by the change in the responsiveness of price to the shock. This will alter the contemporaneous correlation between consumption and output. It will also have an impact on labour supply through the income effect, thereby influencing the contemporaneous correlation of labour with output. If households are assumed to have power in the wage setting market in an analogous fashion to firms then any nominal wage rigidities will act to weaken the income effect. This is due to the fact that households too become forward looking. This will act in concert with GHH preferences. Hence we see that nominal rigidities make agents more forward looking and so make them less sensitive to the contempo-

raneous impulse. Nominal rigidities lead to a richer internal propagation mechanism as a result.

The RBC framework does not allow for any interplay between real and nominal variables. This is a well trodden critique of the RBC paradigm that complements the critique above regarding the contemporaneous correlations and the relative simplicity of the propagation mechanism of the RBC framework. For present purposes the move to a world with nominal rigidities in a New Keynesian framework advances a more pragmatic aim - the study of the relationship between central bank policy rules and asset prices necessitates the need for the New Keynesian framework.

This chapter has demonstrated that a utility function approach is a viable route to resolving the excessive countercyclical movement of labour in the RBC model. The utility function approach allows nominal rigidities to be introduced in a relatively straightforward manner into the model developed in this chapter. The advantage of the utility function approach is that it allows for easy transition between models with nominal rigidities and without them as the introduction of nominal rigidities is independent of the utility function specification. In contrast, the introduction of frictions to solve the countercyclical response of labour dos not lend itself to extension to a world of nominal rigidities in as straightforward a manner. This places us in a position to isolate the impact of nominal rigidities and gain deeper insights into how nominal rigidities affect the dynamics and quantitative results of the model developed in this chapter. The impact of nominal rigidities, especially on asset prices, is the focus of the remaining chapters of this thesis.

Appendix

3.A IES under GHH Preferences

This appendix derives the Intertemporal Elasticity of Substitution (IES) under GHH preferences and is taken from Jahan-Parvar et al. (2013).

The IES is defined as the responsiveness of the growth rate of consumption to the real interest rate,

$$IES = \frac{\partial \ln\left(\frac{c_{t+1}}{c_t}\right)}{\partial \ln r_t^f}. \tag{3.A.1}$$

The Euler equation for the risk free rate is given by,

$$u_{c,t} = \beta u_{c,t+1} r_t^f, \tag{3.A.2}$$

which in logs gives,

$$\ln\left(u_{c,t}\right) = \ln\beta + \ln\left(u_{c,t+1}\right) + \ln r_t^f. \tag{3.A.3}$$

Rearranging and taking the partial derivative with respect to the risk free rate gives,

$$\frac{\partial \ln\left(\frac{u_{c,t+1}}{u_{c,t}}\right)}{\partial \ln r_t^f} = -1. \tag{3.A.4}$$

Combining (3.A.1) with (3.A.4) gives the following definition for the IES,

$$IES = -\frac{\partial \ln\left(\frac{c_{t+1}}{c_t}\right)}{\partial \ln\left(\frac{u_{c,t+1}}{u_{c,t}}\right)}. \tag{3.A.5}$$

Jahan-Parvar et al. (2013) proceed to take the first order Taylor approximation of $\ln u_{c,t+1}$ and $\ln u_{c,t}$:

$$\ln u_{c,t+1} = \ln u_c + \left[\frac{\partial \ln u_{c,t+1}}{\partial \ln c_{t+1}} \mid_{c_{t+1}=c}\right](\ln c_{t+1} - c) \tag{3.A.6}$$

$$\ln u_{c,t} = \ln u_c + \left[\frac{\partial \ln u_{c,t}}{\partial \ln c_t} \mid_{c_t=c}\right](\ln c_t - c) \tag{3.A.7}$$

Taking the difference between these equations yields,

$$\ln\left(\frac{u_{c,t+1}}{u_{c,t}}\right) = \frac{\partial \ln u_c}{\partial \ln c} \ln\left(\frac{c_{t+1}}{c_t}\right). \tag{3.A.8}$$

Using the above one can easily see that,

$$IES = -\frac{\partial \ln\left(\frac{c_{t+1}}{c_t}\right)}{\partial \ln\left(\frac{u_{c,t+1}}{u_{c,t}}\right)} \tag{3.A.9}$$

$$= \frac{-1}{\frac{\partial \ln u_c}{\partial \ln c}} \tag{3.A.10}$$

GHH preferences in steady state are given by,

$$u = \frac{\left[c - \chi\frac{n^{1+\varphi}}{1+\varphi}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{3.A.11}$$

which yields the marginla utility of consumption in logs as,

$$\ln u_c = -\sigma \ln\left(c - \chi\frac{n^{1+\varphi}}{1+\varphi}\right). \tag{3.A.12}$$

At steady state one has,

$$\frac{\partial \ln u_c}{\partial \ln c} = \frac{\partial \ln u_c}{\partial c}\frac{\partial c}{\partial \ln c} = \frac{-\sigma c}{c - \chi \frac{n^{1+\varphi}}{1+\varphi}} \tag{3.A.13}$$

Combining all of these one arrives at the following definition for IES under GHH preferences,

$$IES = \frac{c - \chi \frac{n^{1+\varphi}}{1+\varphi}}{\sigma c}. \tag{3.A.14}$$

Thus one sees that the IES depends on more than the parameters that define the utility function, as in the case with CRRA preferences. In this sense GHH preferences decouple IES from risk aversion as knowledge of risk parameters are not sufficient to pin down the IES.

Part II

CHAPTER 4

Asset Pricing in a New Keynesian Model with Nominal Price and Wage Rigidities

4.1 Introduction

New Keynesian models have become an important tool in macroeconomic policy analysis and as a means of informing macroeconomic policy advice. (De Paoli et al., 2010) The asset pricing implications of New Keynesian models, however, have not been studied in any detail; the most relevant papers being Wei (2009), De Paoli et al. (2010), Challe and Giannitsarou (2014) and Swanson (2015).

The asset pricing implications of Real Business Cycle models have been studied extensively, particularly with reference to the "Equity Premium Puzzle" of Mehra and Prescott (1985) and the "Risk Free Rate Puzzle" of Weil (1989). This literature focuses on the idea of joint replication of business cycle and asset pricing results. It highlights that there is a fundamental trade-off between the two objectives, i.e. any friction (or combination of frictions) introduced to improve either is to the detriment

of the other. As shown in previous chapters, this literature has had some success at joint replication with the combination of capital adjustment costs and habits in consumption. In this chapter I take the model developed in the previous chapter and extend it into a New Keynesian framework.

The recent financial crisis and policy responses to it have highlighted how little is understood, particularly in a theoretical sense, about how policy affects asset prices. This chapter begins to fill this gap in the literature. I focus on monetary policy since it reacts much faster and more frequently to real economic disturbances than fiscal policy. Furthermore monetary policy is most commonly modelled as controlling the nominal interest rate which has a much more direct link to financial variables than fiscal policy instruments. The current chapter serves as a first step towards the ultimate goal of understanding the asset pricing implications of monetary policy by providing the necessary extension of asset pricing models into a New Keynesian setting. The monetary policy effect on asset prices has been studied in the empirical macroeconomics literature, e.g. the empirical impact of monetary policy shocks on asset prices in the tradition of Rigobon and Sack (2004). There is also a significant empirical literature in the opposite direction, i.e. on how monetary policy reacts to asset prices. Bernanke and Gertler (2000), Rigobon and Sack (2003) and more recently Christiano et al. (2010) are examples of this literature.

The limited literature on the asset pricing implications of New Keynesian models appears to have been precluded by the dominance of log-linearisation (and other local approximation techniques) as a solution technique in New Keynesian models. Log-linearisation implies linearity between the certainty equivalents of assets (Jermann, 1998), which is not an a priori desirable feature when studying asset prices. If we assume that some portion of agent wealth is tied up in financial assets then

the linearity imposed on the system under log-linearisation implies that agents face a symmetric response of marginal utility when asset prices change. The concavity of the utility function due to risk aversion means that this is not the case for a fall in asset prices (and hence wealth) as for an equal rise in asset prices.[1] Agents must be compensated for this asymmetric response of utility via a higher return leading to the existence of risk premia. The use of log-linearisation removes such premia by assuming risk averse agents are in fact risk neutral across contemporaneous gambles among different assets.[2] This makes the log-linear approach, and its variants, methodologically unsuited to the study of asset prices within New Keynesian models, and indeed DSGE models in general.

While Wei (2009), De Paoli et al. (2010) and Challe and Giannitsarou (2014) provide a good starting point for the analysis of asset prices in New Keynesian models, they all rely on the log-linearisation of the model around the steady state - the modus operandi of the New Keynesian literature and indeed much of macroeconomics, the unsuitability of which for asset pricing has already been discussed in detail in the previous chapters. The common response in the macro-finance literature, as noted in previous chapters, is to use log-linear/log-normal approximations whereby they assume joint log-normality of terms within the Euler equation, the equation central to asset pricing in DSGE models. Indeed, the limited literature on asset pricing in New Keynesian models follows the macro-finance literature and uses either the log-linear/log-normal approximation (Wei, 2009; De Paoli et al., 2010) or some higher order variant (Challe and Giannitsarou, 2014). Swanson (2015) uses a higher order solution approach which is a "more non-linear approach". This work differs from

[1]The fall in marginal utility is much larger than the rise in marginal utility for opposite (but otherwise identical) movements in the asset price.

[2]By this I mean that agents treat investment in different asset classes as different gambles. The implicit contemporaneous risk neutrality means agents are indifferent between investing in risky equity and the risk free bond.

Swanson (2015) as it delves into the relationship linking underlying economic variables to asset prices.

The vast majority of New Keynesian models also ignore endogenous capital accumulation. In previous chapters it has been shown that it is the capital gains channel that drives the return on equity, and the price of capital is the main driver of this channel. Thus the absence of endogenous capital accumulation, and the resulting investment dynamics, have prevented a complete analysis/understanding of asset prices. Despite the tendency to ignore capital accumulation there are various papers that do allow capital accumulation, and a variable Tobin's Q, to play a role.[3] In all such models it is possible to report equity returns[4] but one cannot delve deeper into the drivers of this return due to both the use of local approximation methods, and (more importantly) the lack of any relationship that links the equity return to the underlying economy. This chapter demonstrates that a relationship can be derived between the return on equity and the real economy in a simple New Keynesian model with sticky wages, sticky prices and endogenous firm specific capital investment. Thus allowing one to gain finer insights into the drivers of the equity return than hitherto explored.

The central result of this chapter shows that an equity return relationship can be derived for New Keynesian models which does not rely on the log-linear/log-normal approach. This provides a general relationship that does not suffer the detrimental asset pricing implications of log-linearisation or the need for specific distributional assumptions. In this sense, the relationship derived is considered more general and robust than the previous results in the literature. I find that in the presence of

[3]Wei (2009), De Paoli et al. (2010) and Challe and Giannitsarou (2014) are notable exceptions in the limited literature looking at asset pricing in New Keynesian models. While, Weinke and Sveen (2003), Sveen and Weinke (2004), Smets and Wouters (2007), Sveen and Weinke (2009) and Justiniano et al. (2010) provide a small set of examples in the more general New Keynesian literature.

[4]In models that ignore capital accumulation it is still possible to study asset prices but they miss an important driver of equity prices.

endogenous firm-specific capital investment the return on equity is made up of two components (i) the return on capital and (ii) the expectation of future profit streams of the firm. This relationship is standard and expected. I do not contend that such a mechanism is not at work in the background of other models that allow for variable Tobin's Q in New Keynesian models. Rather, this is the first time that this relationship been explicitly derived and explored.

In line with previous chapters, the model economy continues to have capital adjustment costs, habits in consumption and preferences of the GHH form (Greenwood et al., 1988). Nominal rigidities are introduced in the form of sticky wages and sticky prices. The asset pricing literature has settled on the combination of strong internal habits and strong capital adjustment costs as necessary for any viable quantitative replication of asset pricing and business cycle results. Together, capital adjustment costs and internal consumption habits mute two of the three channels open to agents to smooth consumption. As previously discussed, in response agents exploit the labour channel to smooth income and thereby smooth consumption. This results in the countercyclical movement of labour, which is troubling in Real Business Cycle models and the result of the income effect dominating the substitution effect. I have shown, in the previous chapter, that one can generate the correct cyclical response of labour by using the GHH preferences of Greenwood et al. (1988). Within the confines of the RBC, this is at the cost of weaker habit intensity as the income effect becomes habit induced and must be weakened to generate the appropriate cyclical response. Thus a trade-off between strong habit intensity (and the consequent improvement in asset pricing results) and a desire to avoid countercyclical labour exists.

The response of labour to a technology shock is much more ambiguous in the New Keynesian framework - there is indeed much empirical debate surrounding the cycli-

cal movement of labour in New Keynesian models. Thus the use of GHH preferences with internal consumption habits, and their ability to generate any cyclical response of labour, provides a model which can be agnostic to this debate when used by other researchers and can adapt should the literature ever reach an (unlikely) consensus. In this chapter I follow Cantore et al. (2014) in the view that theoretically the response of labour to a positive technology shock will be negative in a New Keynesian model with sticky prices.[5]

The quantitative results indicate that the introduction of nominal price and nominal wage rigidities with GHH preferences leads to a re-emergence of countercyclical labour. The countercyclical response of labour in the present chapter is attributed to the presence of both nominal rigidities and their interacting intertemporally in a non-linear fashion. This non-linear interaction is explicitly accounted for by the non-linear solution methods used. Nominal rigidities are introduced as an intangible cost to changing the nominal wage and the nominal price in the spirit of Rotemberg (1982). I also find that firm specific capital investment leads to a large reduction in labour volatility as firms seek to keep marginal costs sufficiently stable. I do not comment on the magnitude of the countercyclical response of labour and leave a study of fine tuning 'how much' labour responds to future work using the calibration scheme discussed in the previous chapter.[6]

The quantitative asset pricing results highlight that the simple New Keynesian model studied in this chapter continues to suffer from the same quantitative asset pricing shortcoming of its RBC brethren. However, this was a priori expected given the sim-

[5]See the Thesis Introduction for a more detailed discussion.

[6]The previous chapter showed that labour could be made to move with the cycle for a range of habit parameters. In simulation results I find that this is again possible in the New Keynesian model but over a very small range in the vicinity of zero habit intensity.

plicity of the model studied.[7] There is a reduction in the asset pricing volatilities in the presence of nominal rigidities which I attribute to the stabilisation of the economy by the central bank.

4.2 Model Set-Up

The model studied in this chapter is a simple New Keynesian model with nominal rigidities in prices and wages. There exist a continuum of households, indexed on the unit interval $i \in (0, 1)$, each of whom supply differentiated labour enduing them with a degree of market power in the labour market. Households set wages in a monopolistically competitive labour market. Nominal rigidities for households are introduced in the form of sticky wages. Households incur a cost when they change their nominal wage in any period *à la* Rotemberg pricing (Rotemberg, 1982). The introduction of a Rotemberg style cost when adjusting wages induces inertia in the wage setting process. This inertia allows households to be off their "fully flexible wage" labour supply curve for periods of time. Sticky wages in the spirit of Rotemberg pricing have been examined by Chugh (2006) in the study of optimal fiscal and monetary policy.

Households sell their labour to perfectly competitive employment agencies. The employment agencies costlessly aggregate the differentiated labour of the individual households into an aggregate labour supply. This aggregate labour supply is then sold as an input to intermediate goods firms.

There exist a continuum of monopolistically competitive intermediate goods firms, indexed on the unit interval $j \in (0, 1)$, each of whom produce a differentiated good. This endues them with some degree of price-setting power for their goods in the goods market. Nominal rigidities for firms are introduced in the form of sticky prices *à la*

[7] See Introduction for a discussion

Rotemberg pricing (Rotemberg, 1982). Analogously to households, the resulting inertia allows firms to be off their "fully flexible price" labour demand curve. Firms are assumed to own the capital in the economy in contrast to previous chapters where households owned capital. As a consequence firms make the capital investment decisions. The capital itself is homogeneous; while the capital investment decision is firm-specific and referred to as the firm-specific capital investment assumption. The intermediate goods firms combine their firm-specific capital with aggregate labour supply to produce output using the production technology. Production uncertainty exists via exogenous technology productivity shocks.

The intermediate goods firms sell their differentiated output to perfectly competitive final goods firms. These final goods firms costlessly aggregate the output of intermediate goods firms. The aggregate output is used for both consumption and investment purposes, i.e. it may be costlessly transformed into either the consumption good or the investment good. This costless transformation assumption prevents the firm investment decision from distorting output demand, i.e. the demand function is unchanged for the firm regardless of the use of their output. (Woodford, 2003)

The presence of nominal rigidities requires the introduction of a central bank to close the economy. The central bank sets the nominal interest rate to influence the real economy and reduce the distortionary impact of inflation resulting from nominal rigidities. Nominal uncertainty exists in the economy via exogenous shocks to the nominal interest rate.

4.2.1 Households

Employment Agencies

Perfectly competitive employment agencies buy the labour supply of each household, $n_t^s(i)$. This is costlessly aggregated into aggregate labour which is used as an input by the intermediate goods firms.

The employment agencies aggregate labour supply using a Dixit-Stiglitz aggregator,

$$n_t^s = \left[\int_0^1 n_t^s(i)^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di\right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}, \tag{4.2.1}$$

where $\varepsilon^w > 0$ measures the degree of substitutability between different labour inputs. Employment agencies seek to maximise their profits leading to the demand function for labour,

$$n_t^s(i) = \left[\frac{w_t(i)}{W_t}\right]^{-\varepsilon^w} n_t^s, \tag{4.2.2}$$

where n_t^s is the aggregate labour supply and w_t is the aggregate wage level defined as,

$$w_t = \left[\int_0^1 w_t(i)^{1-\varepsilon^w} di\right]^{\frac{1}{1-\varepsilon^w}}. \tag{4.2.3}$$

Households

Households seek to maximise lifetime utility,

$$U_t(i) = E_t\left\{\sum_{h=0}^{\infty} \beta^h u\left(c_{t+h}(i) - \nu c_{t+h-1}(i), n_{t+h}^s(i)\right)\right\} \tag{4.2.4}$$

where β is the time discount factor.

The period utility of the household depends on consumption, c, and labour sup-

ply n^s. It has the same form across all households. It exhibits internal habits in consumption, the strength/intensity of which is governed by the parameter ν.

Following the previous chapter, the period utility function is of the GHH form (Greenwood et al., 1988) and has functional form,

$$u\left(c_t, c_{t-1}, n_t\right) = \frac{1}{1-\sigma}\left[\left(c_t - \nu c_{t-1}\right) - \chi\frac{n_t^{1+\varphi}}{1+\varphi}\right]^{1-\sigma} \tag{4.2.5}$$

where σ measures the degree of risk aversion, χ is the utility weight of the disutility of labour and φ is the inverse of the Frisch elasticity of labour supply.

The period budget constraint for the household i is,

$$c_t\left(i\right)+\frac{B_{t+1}^n\left(i\right)}{\mathcal{R}_t^n P_t}+v_t S_{t+1}\left(i\right) = \frac{w_t\left(i\right)}{P_t}n_t^s\left(i\right)+\frac{B_t^n\left(i\right)}{P_t}+\left(d_t+v_t\right)S_t\left(i\right)-\frac{\chi^w}{2}\left(\frac{w_t\left(i\right)}{w_{t-1}\left(i\right)}-1\right)^2\frac{w_t n_t^s}{P_t} \tag{4.2.6}$$

Household income is used to purchase an aggregate consumption good, c_t, from the final goods firm and to purchase assets for the subsequent period. Household income is derived from the supply of labour and the return on asset holdings.

As each household is a monopolistic supplier of differentiated labour, $n_t^s\left(i\right)$, they can set their nominal wage, $w_t\left(i\right)$, in the labour market. Households face an intangible cost, measured as lost income, from changing their nominal wage across periods. This cost is modelled in the spirit of Rotemberg costs to ensure the wage Phillips curve has a closed form.[8] The parameter χ^w governs the degree of wage stickiness.

The intangible nature of the cost of changing wages is in the spirit of De Paoli et al.

[8] A closed non-linear form for either the wage or price Phillips curves is in general not possible under the Calvo style of price and wage inertia.

(2010) who similarly assume an intangible cost of adjusting prices in the sticky price context. The intangible cost may be considered as modelling the psychological effect of changing prices for households. When changing wages households anticipate that there will be productive time lost in bargaining over wages which cannot be used to supply labour[9]. This cost in terms of lower effective working hours is accounted for by the household via a lower income in the budget constraint. The intangible nature of the cost implies no impact on the aggregate resource constraint but rather affects the equilibrium by inducing households to make decisions based on lower effective income.

There are two broad classes of assets in which the household may invest: bonds and equity. Nominal bonds have a variable cost of $v_t^B = \frac{1}{\mathcal{R}_t^n}$, where $\mathcal{R}_t^n$ is the nominal return. The nominal bonds provide a known return in the subsequent period of one unit of the nominal consumption good. The nominal interest rate is known in advance and set by the central bank. This allows the central bank to influence real demand via monetary policy.

Equity has a variable cost and provides a real return comprised of a real dividend and capital return. Equity investment is in the form of shares in a broad equity index,

$$S_t = \int_0^1 q_t(i) S_t(i) di,$$

where $S_t(i)$ is the shares in the intermediate firms and $q_t(i)$ their market weights. The market weights are calculated as the proportion of output produced by each intermediate goods firm i.e.,

$$q_t(i) = \frac{y_t(i)}{y_t}. \tag{4.2.7}$$

[9]This is an un-modelled process in this set up

This equity index provides an aggregate real dividend,

$$d_t = \int_0^1 q_t(i)\, d_t(i)\, di,$$

and real capital return at the prevailing price for the stock index, v_t.

The preferences of households exhibit local non-satiation so all income is used for either consumption or asset purchases, hence the budget constraint holds with equality. It is further assumed that households have no productive activity at home so that all labour is supplied to firms.

As suppliers of differentiated labour the household is demand constrained and therefore faces the demand curve for labour,

$$n_t^s(i) = \left[\frac{w_t(i)}{w_t}\right]^{-\varepsilon^w} n_t^s \tag{4.2.8}$$

Household optimisation of lifetime utility subject to their budget constraint and labour demand yields,

$$\frac{w_t}{P_t} = -\frac{\varepsilon^w}{\varepsilon^w - 1}\frac{u_{n,t}}{u_{c,t} - \beta\nu E_t\left[u_{c,t+1}\right]} + E_t\left[m_{t,t+1}x_{t+1}\frac{w_{t+1}}{P_{t+1}}\frac{n_{t+1}}{n_t}\right] - x_t\frac{w_t}{P_t} \tag{4.2.9}$$

$$1 = E_t\left\{\mathcal{D}_{t,t+1}\mathcal{R}_t^n\right\} \tag{4.2.10}$$

$$1 = E_t\left\{m_{t,t+1}\left(\frac{d_{t+1} + v_{t+1}}{v_t}\right)\right\} \tag{4.2.11}$$

The auxiliary variable x_t has been introduced to simplify notation and is defined as,

$$\begin{aligned} x_t &= \frac{\chi^w}{\varepsilon^w - 1}\frac{w_t}{w_{t-1}}\left(\frac{w_t}{w_{t-1}} - 1\right) \\ &= \frac{\chi^w}{\varepsilon^w - 1}\pi_t^w\left(\pi_t^w - 1\right) \end{aligned}$$

where $\pi_t^w = \frac{w_t}{w_{t-1}}$ is wage inflation. Finally under Rotemberg costs all households face the same problem and so the household indexation may be dropped once the first order conditions have been derived. The equation (4.2.9) is the wage Phillips curve, (4.2.10) defines the Fisher relationship and (4.2.11) is the Euler equation governing equity holdings. In conditions (4.2.10) and (4.2.11), $m_{t,t+1}$ and $\mathcal{D}_{t,t+1}$ are the real and nominal stochastic discount factors, defined as,

$$m_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$$
$$\mathcal{D}_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}}$$
$$\Lambda_t = u_{c,t} - \beta \nu E_t \left[u_{c,t+1} \right]$$

The wage Phillips curve, (4.2.9), defines how households set wages. It is the labour supply curve in this economy. Let us consider the "flexible wage" case ($\chi^w = 0$), where households face no cost to changing their nominal wage. In this case, the labour supply curve becomes,

$$\frac{w_t}{P_t} = -\frac{\varepsilon^w}{\varepsilon^w - 1} \frac{u_{n,t}}{u_{c,t} - \beta \nu E_t \left[u_{c,t+1} \right]}. \tag{4.2.12}$$

So households set their nominal wage at a mark-up from the RBC case where all households supply identical labour. Thus the supply of differentiated labour leads to a second best supply of labour where wages are higher and labour supply lower than in the corresponding RBC case studied in previous chapters. The wage Phillips Curve shows that the presence of sticky wages, i.e $\chi^w > 0$, leads to a further wedge in the labour supply function preventing the wage from reaching the "fully flexible" wage. Thus sticky wages allow the household to remain off the "fully flexible" labour supply curve for extended periods of time.

4.2.2 Firms

Final Goods Firms

The output of intermediate goods firms, $y_t(j)$, is bought by perfectly competitive final goods firms which costlessly aggregate the output. This aggregate output may be costlessly transformed into either consumption or investment goods.

The final goods firms aggregate output using the Dixit-Stiglitz aggregator,

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon^p - 1}{\varepsilon^p}} dj\right]^{\frac{\varepsilon^p}{\varepsilon^p - 1}}, \tag{4.2.13}$$

where $\varepsilon^p > 0$ measures the degree of substitutability between different goods. The final goods firms maximise their profits leading to the standard demand function for the intermediate goods firms,

$$y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon^p} y_t, \tag{4.2.14}$$

where y_t is the aggregate demand and P_t is the aggregate price level defined as,

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon^p} dj\right]^{\frac{1}{1-\varepsilon^p}}. \tag{4.2.15}$$

Intermediate Goods Firms

Monopolistically competitive intermediate goods firms seek to maximise lifetime profit. They dislike changing their price, $P_t(j)$, between periods since doing so incurs a cost *à la* Rotemberg (1982) . Following De Paoli et al. (2010) this cost is assumed to be an intangible cost that enters the firms optimisation problem as a form of 'disutility', i.e. it doesn't affect cash flow. The intermediate goods firms own the capital in the

economy and hence are responsible for making the capital investment decisions. This assumption on the ownership of capital[10], and the resulting capital investment decision, is referred to as the firm specific capital investment assumption.

Intermediate goods firm j seeks to maximise lifetime real profit defined as,

$$\Gamma = E_t \left\{ \sum_{h=0}^{\infty} m_{t,t+h} \left[\begin{array}{l} \frac{P_{t+h}(j) y_{t+h}(j)}{P_{t+h}} - \frac{w_{t+h}}{P_{t+h}} n_{t+h}^{d}(j) \\ -I_{t+h}(j) - \frac{\chi^P}{2} \left(\frac{P_{t+h}(j)}{P_{t+h-1}(j)} - 1 \right)^2 y_{t+h} \end{array} \right] \right\} \tag{4.2.16}$$

where $y_t(j)$ is its output, $n_t^d(j)$ is the labour demand and $I_t(j)$ is capital investment made by the firm. All variables without firm indexation are aggregate variables. Following the Rotemberg pricing literature the cost of price adjustment is assumed to be quadratic.[11] The parameter χ^P determines the strength of the disutility arising from adjusting prices, and hence the degree of price stickiness. If $\chi^P = 0$ then there is no price stickiness and the model collapses to one with monopolistically competitive firms and fully flexible prices. Hereafter referred to as the flexible price model.

Intermediate goods firms production must be feasible given the production technology available to the firm, characterised by the neo-classical production function $f(\cdot)$ and takes the Cobb-Douglas form for each firm j,

$$zf(k(j), n(j)) = zk^{\alpha}(j) n^{1-\alpha}(j) \tag{4.2.17}$$

where $\alpha \in (0,1)$ is the capital share of production. This technology is assumed to be subject to non-idiosyncratic random shocks to productivity, z_t, which follows an

[10] As noted previously, the capital is homogeneous.

[11] The impact of non-symmetric functions are an interesting extension left for future work. Such non-symmetric functions would capture the idea that price changes in one direction is more palatable than the other. For example firms may prefer reducing prices over rising prices by the same amount.

auto-regressive process, i.e.

$$\ln z_{t+1} = \rho \ln z_t + \upsilon_t^y \tag{4.2.18}$$

where ρ measures the persistence of the shock and υ_t^y is a random shock.

Intermediate goods firms are demand constrained. The demand for output of intermediate goods firm j is defined by,

$$y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon^p} y_t. \tag{4.2.19}$$

Finally, as owners of capital the intermediate goods firms face the capital accumulation process,

$$k_{t+1}(j) = (1-\delta)\, k_t(j) + I_t(j), \tag{4.2.20}$$

where δ is the fixed depreciation rate. Investment is subject to capital adjustment costs. The process governing the conversion of investment into capital is defined as,

$$I_t(j) = \phi\left(\frac{I_t(j)}{k_t(j)}\right) k_t(j). \tag{4.2.21}$$

The non-linear function $\phi(\cdot)$ is assumed to be concave. For ease of notation we further define $g_t(j) = \frac{I_t(j)}{k_t(j)}$. It continues to have the same properties and functional form as in previous chapters.[12]

[12]The function $\phi(\cdot)$ is assumed to have the following properties,

$$\phi'(\cdot) > 0, \quad \phi''(\cdot) < 0,$$
$$\phi(0) = 0, \quad \phi(\delta) = \delta.$$

The functional form for the adjustment cost function is,

$$\phi\left(\frac{i}{k}\right) = \begin{cases} \frac{b_1}{1-\frac{1}{\xi}}\left(\frac{i}{k}\right)^{1-\frac{1}{\xi}} + b_2 & \text{if } i > 0 \\ 0 & \text{if } i = 0 \end{cases} \tag{4.2.22}$$

The intermediate goods firm j solves the following optimisation problem,

$$\max E_t \left\{ \sum_{h=0}^{\infty} m_{t,t+h} \left[\begin{array}{l} \frac{P_{t+h}(j) y_{t+h}(j)}{P_{t+h}} - \frac{w_{t+h}}{P_{t+h}} n_{t+h}^d (j) \\ -I_{t+h}(j) - \frac{\chi^P}{2} \left(\frac{P_{t+h}(j)}{P_{t+h-1}(j)} - 1 \right)^2 y_{t+h} \end{array} \right] \right\}$$

s.t.

$$y_{t+h}(j) = \left[\frac{P_{t+h}(j)}{P_{t+h}} \right]^{-\varepsilon^p} y_{t+h} \cdots\cdots \lambda_{t+h}(j)$$

$$y_{t+h}(j) = z_t f\left(k_{t+h}(j), n_{t+h}^d(j) \right) \cdots\cdots \mu_{t+h}(j)$$

$$k_{t+h+1}(j) = (1-\delta) k_{t+h}(j) + \phi \left(\frac{I_{t+h}(j)}{k_{t+h}(j)} \right) k_{t+h}(j) \cdots\cdots \gamma_{t+h}(j)$$

This leads to the following first order conditions,

$$\frac{w_t}{P_t} = \mu_t z_t f_{n,t}, \tag{4.2.23}$$

$$E_t \left\{ m_{t,t+1} \chi^P \frac{P_{t+1}}{P_t} \left(\frac{P_{t+1}}{P_t} - 1 \right) y_{t+1} \right\} = (\varepsilon^p - 1) y_t + \chi^P \frac{P_t}{P_{t-1}} \left(\frac{P_t}{P_{t-1}} - 1 \right) y_t - \varepsilon^p \frac{w_t}{P_t} \frac{y_t}{z_t f_{n,t}} \tag{4.2.24}$$

$$\frac{1}{\phi'(g_t)} = E_t \left\{ m_{t,t+1} \left[\frac{w_{t+1} f_{k,t+1}}{P_{t+1} f_{n,t+1}} + \frac{1}{\phi'(g_{t+1})} \left(1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1}) \right) \right] \right\}. \tag{4.2.25}$$

Analogously to the household problem, under Rotemberg pricing all firms face the same problem so that once the first order conditions have been derived one can drop firm indexation. Thus the aggregate relationships have the same form as at the firm level. The relationship (4.2.23) is the labour demand condition, (4.2.24) is the non-linear Phillips Curve and (4.2.25) is the optimality condition for capital investment.

Let us first consider (4.2.23), the optimality condition governing labour demand. The presence of firm specific capital implies that the marginal cost for the firm is

given by[13],

$$\mathrm{mc}_t = \frac{w_t}{P_t}\frac{1}{z_t f_{n,t}}, \tag{4.2.26}$$

so μ_t is equal to the effective marginal cost of producing the marginal unit of output.

Let $\pi_t = \frac{P_t}{P_{t-1}}$ be the inflation rate at t and let $x_t \equiv \chi^P \pi_t (\pi_t - 1) y_t$ so that the non-linear Phillips Curve, (4.2.24), may be rewritten as,

$$x_t = E_t \{m_{t,t+1} x_{t+1}\} - (\varepsilon^p - 1) y_t \left[1 - \frac{\varepsilon^p}{\varepsilon^p - 1}\mathrm{mc}_t\right]. \tag{4.2.27}$$

The term $\frac{\varepsilon^p}{\varepsilon^p - 1} > 1$ is the mark-up and the term in the square brackets is the real gross marginal profit. Thus the non-linear Phillips Curve continues to have the usual interpretation where prices are set as a function of expected future inflation and current profits.

The optimality condition for capital investment is given by (4.2.25). This differs from the Euler equation that governs investment in models without firm specific capital investment as it is no longer the marginal product of capital that matters but rather $\frac{w_{t+1} f_{k,t+1}}{P_{t+1} f_{n,t+1}}$. Since capital investment is firm specific the firm does not pay a rental cost for capital so that the marginal cost for the firm is given by the real wage. Note that what matters now is not how productive capital is per se but rather its ability to reduce effective marginal cost. The effective marginal cost is given by $\frac{w_{t+1}}{P_{t+1} z_{t+1} f_{n,t+1}}$ since each unit of labour is paid the real wage and produces $\frac{1}{z_{t+1} f_{n,t+1}}$units of output. Consider the firms decision to invest in capital. This means that it can produce at least the same amount of output using less labour since it has more capital, so its

[13]Total cost in the presence of firm specific capital is given by,

$$\mathcal{TC}_t = \frac{w_t}{P_t} n_t.$$

Marginal cost is defined as $\mathrm{mc}_t = \frac{d\mathcal{TC}_t}{dy_t}$, which when applied to the above yields the required result

effective marginal cost falls. Finally this saved effective marginal cost is invested in capital which can produce $z_{t+1}f_{k,t+1}$ units of output. We refer to $MS_{t+1} = \frac{w_{t+1}f_{k,t+1}}{P_{t+1}f_{n,t+1}}$ as the marginal savings from capital investment which is brought about by making production more capital intensive. (Sveen and Weinke, 2004) The remainder of the investment condition continues to have the same interpretation as in previous chapters.[14]

4.2.3 Central Bank

The economy is closed by specifying how the central bank sets the nominal interest rate. The central bank sets the gross nominal interest rate $\mathcal{R}_t^n$ according to the Taylor rule,

$$\frac{\mathcal{R}_t^n}{\overline{\mathcal{R}^n}} = (\pi_t^p)^{\rho_\pi} \left(\frac{y_t}{\overline{y}}\right)^{\rho_y} \eta_t, \tag{4.2.28}$$

where the constants ρ_π and ρ_y control the degree to which the central bank responds to price inflation and output deviations. The central banks rule is subject to uncertainty via the nominal interest rate shock η_t which follows an autoregressive process similar to the aggregate technology shocks,

$$\ln \eta_{t+1} = \iota \ln \eta_t + \upsilon_t^{\mathcal{R}^n} \tag{4.2.29}$$

where ι is the degree of persistence of the shock and $\upsilon_t^{\mathcal{R}^n}$ is a random shock. The impact of different central bank policy rules is the focus of the final chapter; here it is used merely to close the economy.

[14]The capital adjustment cost function has a split function form given by (4.2.22). The arguments above provide reasons for ignoring the lower branch of this split function. With arguments similar to household consumption smoothing, the economy set-up considered in the remainder of this thesis firms use investment in capital to smooth their marginal costs. This ensures that there is always positive investment in the economy. Simulations were conducted with the full split function, i.e. with the possibility of zero investment, and I find that the lower branch is never used, i.e. investment is always positive, as is the case in previous chapters.

4.2.4 Equilibrium

In equilibrium all markets clear. The goods market clearing condition requires that consumption and investment equal the aggregate output produced by intermediate firms, i.e.

$$c_t + I_t = y_t. \tag{4.2.30}$$

Goods market clearing further requires that demand by final goods firms be equal to the total supply by the intermediate goods firms.

Labour market clearing requires that the labour supplied by households equals the labour demand of firms, i.e.

$$n_t^s = n_t^d = n_t. \tag{4.2.31}$$

Equilibrium in the bond markets requires that nominal bonds are in zero net supply, i.e. the nominal interest rate clears the nominal bond market. Thus,

$$B_t^n = 0. \tag{4.2.32}$$

Finally equilibrium in the equity market requires that all shares are held, i.e.

$$S_t = S_t(j) = 1, \tag{4.2.33}$$

where $S_t(j)$ is the outstanding shares in firm j.

All firms solve the same problem which implies that in equilibrium the dividends are identical across firms. This means that aggregate dividends are equal to firm level dividends, i.e.

$$d_t = d_t(j). \tag{4.2.34}$$

4.2.5 The Evolution of Real Wages

In a model with sticky wages, regardless of whether prices are sticky or fully flexible, an additional regulatory condition is required to pin down the evolution of real wages (Chugh, 2006). I follow Chugh (2006) and impose the following identity,

$$\frac{\widetilde{w}_t}{\widetilde{w_{t-1}}} = \frac{\pi_t^w}{\pi_t^p}, \tag{4.2.35}$$

where $\widetilde{w}_t = \frac{w_t}{P_t}$ is the real wage. This additional identity is required as there is no guarantee that it holds in equilibrium. I follow the exposition of Chugh (2006) to highlight the need for this additional constraint.

Consider an economy with flexible wages where the real wage is determined by real factors in the economy, as is the case in our model. In such an economy (whether it has flexible or sticky prices) the rate of wage inflation adjusts freely to ensure that the identity holds. If prices are sticky then there exists a price setting function which determines the rate of inflation and hence flexible wages adjust to ensure the identity holds.

In the presence of sticky wages Chugh (2006) highlights that there now exists an independent force which affects wage inflation, namely the wage Phillips curve. Processes also exist which determine price inflation. As there are independent processes that drive wage and price inflation there is no longer a "free" variable that brings about the inequality; previously this was accomplished by the "free" movement of wage inflation. Together the ratio of wage and price inflation will not be consistent with the identity above and hence it must be imposed on the equilibrium. Chugh (2006) highlights that this condition is assumed in the literature with sticky prices but not ever explicitly referenced.

4.3 Calibration

4.3.1 Technology

The introduction of monopolistic competition does not change the specification of technology parameters from Part One. Rather it requires the calibration of two additional parameters - ε^p, the degree of substitutability between different intermediate goods, and χ^P, the "disutility" of changing prices.

The degree of substitutability parameter is calibrated by making an assumption on the mark-up charged by firms, $\frac{\varepsilon^p}{\varepsilon^p-1}$. In accordance with other literature, I assume a mark-up of 1.2 which implies a parameter value of $\varepsilon^p = 6$. This assumption is standard in the literature given its empirically plausibility as shown in Christiano et al. (2005), among others.

I follow De Paoli et al. (2010) and Ireland (2001) in setting $\chi^P = 77$. Keen and Wang (2007) provide a correspondence between mark-up and the percentage of re-optimising firms under Calvo pricing and the resulting appropriate calibration for χ^P under Rotemberg pricing. Using a mark-up of 1.2, the calibration $\chi^P = 77$ implies that under Calvo pricing somewhere between $20\% - 25\%$ of firms re-optimise their prices. This is reasonable given the literature using Calvo pricing often yields point estimates of price re-optimisation frequency around 18 months which implies that $20\% - 25\%$ of firms re-optimise their prices under Calvo. This range is commonly found in the empirical literature as shown in Christiano et al. (2005) and Altig et al. (2011) among others and is relatively standard. In addition, Ascari and Rossi (2009) show that in the case of zero trend inflation, an assumption used in this thesis, the following relationship exists between frequency of adjustment in Calvo models and

the parameter governing the adjustment cost in Rotemberg,

$$\chi^p = \frac{(\varepsilon^p - 1)\,\theta^p}{(1-\theta^p)\,(1-\beta\theta^p)}, \tag{4.3.1}$$

where θ^p is the proportion of firms that cannot change prices under Calvo, β is the discount factor and ε^p governs the degree of substitutability between intermediate goods. The empirical estimates provided by Keen and Wang (2007) are in accordance with this relationship.

Table 4.1: Calibrated Technology Parameters for U.S.

Capital Share	$\alpha = 0.36$
Depreciation Rate	$\delta = 0.025$
Degree of Substitutability	$\varepsilon^p = 6$
Price 'Disutility' Parameter	$\chi^P = 77$
Autocorrelation Coefficient	$\rho = 0.95$
Standard Deviation of Innovations	0.007

4.3.2 Preferences

The calibration of the preference parameters remains unchanged from the model presented in Chapter 2. The introduction of sticky wages requires the calibration of two additional parameters - ε^w, the degree of substitutability between different labour types and χ^w, the parameter governing the cost of adjusting the nominal wage.

The degree of substitutability parameter is calibrated in a similar fashion to that for intermediate goods, i.e. an assumption is made about the wage mark-up of households $\left(\frac{\varepsilon^w}{\varepsilon^w - 1}\right)$. I assume a mark-up of 1.05 which implies a parameter value of $\varepsilon^w = 21$. The absence of empirical work similar to Keen and Wang (2007) I assume an analo-

gous theoretical relationship exists for χ^w as for χ^p in order to obtain a calibration for χ^w,

$$\chi^w = \frac{(\varepsilon^w - 1)\,\theta^w}{(1-\theta^w)\,(1-\beta\theta^w)}, \tag{4.3.2}$$

where θ^w is the proportion of households that cannot change prices under Calvo and ε^w governs the degree of substitutability between different labour types. Under Calvo pricing, where measure $(1-\theta^w)$ of households may change wages, the average wage duration is given by $\frac{1}{1-\theta^w}$. There is some variation in estimates of average wage duration in the literature, varying from 3 quarters (Christiano et al., 2005; Chugh, 2006) to 5.6 quarters (Barattieri et al., 2014). I assume average wage duration of 4 quarters as assumed in (Gali, 2008, Ch. 6, p.129) and Furlanetto et al. (2013), indeed this is similar to Altig et al. (2011) and Smets and Wouters (2007) both of whom assume average wage duration of 3.6 quarters. The assumption of average wage duration of 4 quarters implies a parameter value of $\chi^w = 237.515$ when $\varepsilon^w = 21$.

There is much debate surrounding the response of labour in New Keynesian models to a technology shock. From an empirical perspective, Gali (1999) sparked a debate in the literature by showing that the data indicates that labour response to a technology shock is countercyclical. A flurry of activity followed Gali (1999) which either showed the robustness of his results or attempted to disprove Gali (1999).[15] Given the response of labour to a technology shock is the subject of such debate both in the empirical and theoretical literature I do not take a specific stance on the response of labour in New Keynesian models in this chapter. Rather I calibrate the habit intensity parameter at $\nu = 0.65$ in line with the estimates obtained in Christiano et al. (2005). I note that this calibration for the habit intensity parameter is at the low end of estimates. The asset pricing literature highlights that strengthening the habit intensity should lead to improved asset pricing results. As such, by using

[15]The reader is referred to Smets and Wouters (2007) for a detailed discussion of this debate.

this low end calibration I am actually biasing the results against better asset pricing replication.

Table 4.2: Calibrated Preference Parameters for U.S.

Time Preference	$\beta = 0.9963$
Frisch Elasticity of Labour Supply	$\frac{1}{\varphi} = 0.75$
Curvature Parameter	$\sigma = 2$
Habit Intensity	$\nu = 0.65$
Degree of Substitutability	$\varepsilon^w = 21$
Nominal Wage Adjustment Parameter	$\chi^w = 237.515$

4.3.3 Central Bank

The final set of parameters that need to be calibrated are those governing the central bank's policy rule. I follow Challe and Giannitsarou (2014) in the calibration of ρ_π and ρ_y for the Taylor rule, both of which are relatively standard in the literature. In line with the literature I calibrate the volatility of monetary policy shocks to generate a $100bp$ deviation in the nominal interest rate from a surprise shock to the central banks monetary policy rule.

Table 4.3: Calibrated Central Bank Parameters for U.S.

Inflation Response	$\rho_\pi = 1.5$
Output Response	$\rho_y = 0.6$
Autocorrelation Coefficient	$\iota = 0.65$
Standard Deviation Monetary Policy Shock	0.0028

Challe and Giannitsarou (2014)

4.4 Business Cycle Results

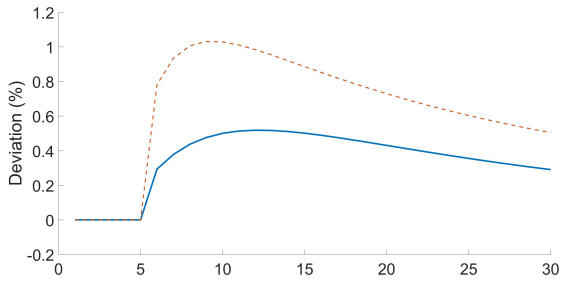
The business cycle results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique.

4.4.1 Business Cycle Dynamics

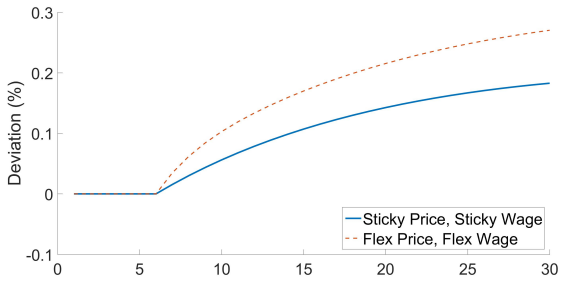
The impulse responses to a technology shock for the Fully Flexible Price Model (henceforth referred to as the "Flex Price Model"), i.e. flexible prices and flexible wages, and the Sticky Wage, Sticky Price Model (henceforth referred to as the "Sticky Price Model") are presented in Figure 4.1. The impulse responses for the Sticky Price Model are presented in Figure 4.2 are for a $100bp$ shock to the nominal interest rate.

The Flex Price model is the 'RBC equivalent' model in a setting with nominal rigidities. When firms can reset their prices in every period and households their wages in every period the small differences between the impulse responses of a 'pure RBC model' can be attributed to the presence of monopolistic competition and firm-specific capital investment on the economy. Such differences manifest themselves in the impulse responses for capital and investment and are very small, i.e. the dynamics are essentially identical.

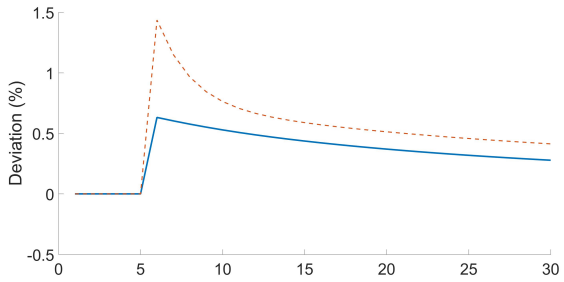
In the presence of sticky wages households respond less vigorously to a technology shock by setting a lower nominal wage so as to incur a smaller cost from adjusting their nominal wage. The dynamics of the economy may be understood via an analysis of the response of labour and the real wage as this effectively drives the response of all other contemporaneous variables - i.e. consumption and investment. The response of labour and the real wage determine household income thereby impacting directly on



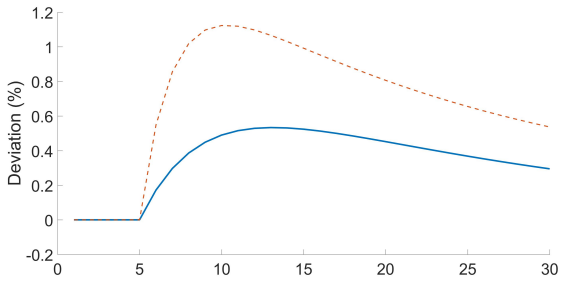
(a) Output



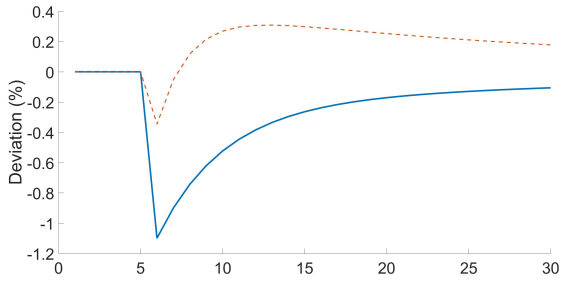
(b) Capital



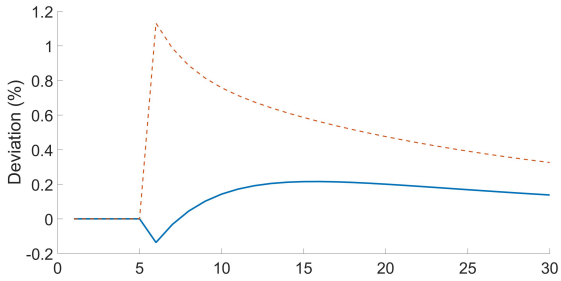
(c) Investment



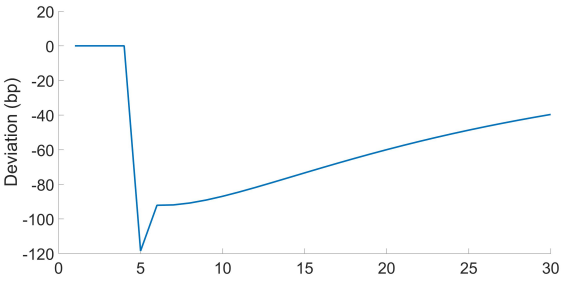
(d) Consumption



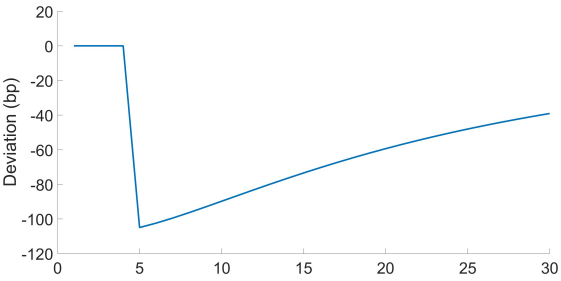
(e) Labour



(f) Real Wage

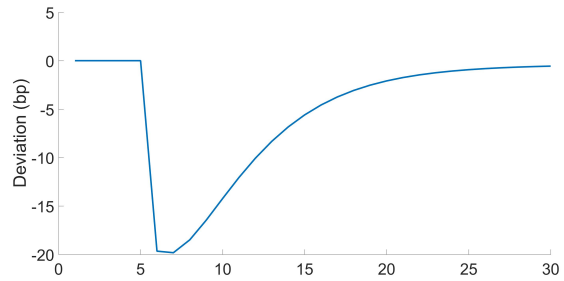


(g) Wage Inflation (Basis Point)

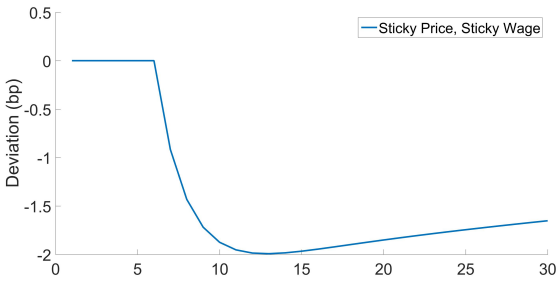


(h) Price Inflation (Basis Point)

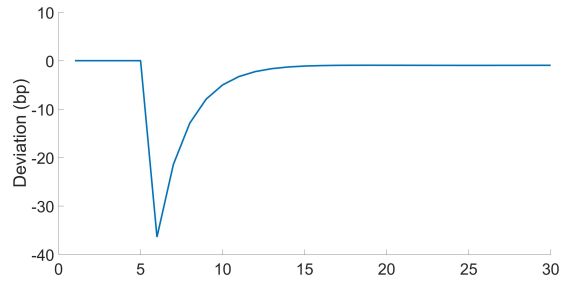
Figure 4.1: Impulse Responses - Technology Shock



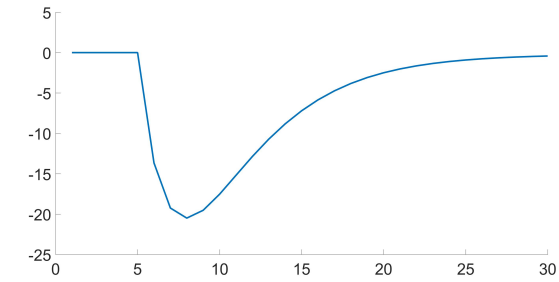
(a) Output



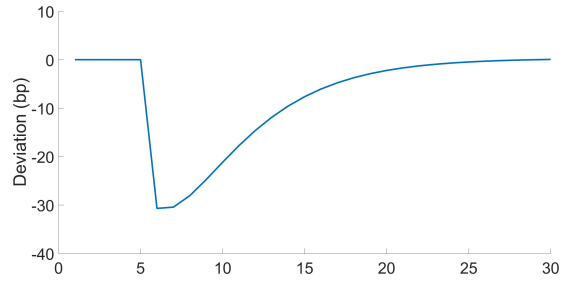
(b) Capital



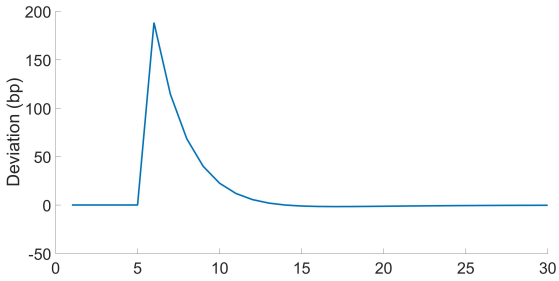
(c) Investment



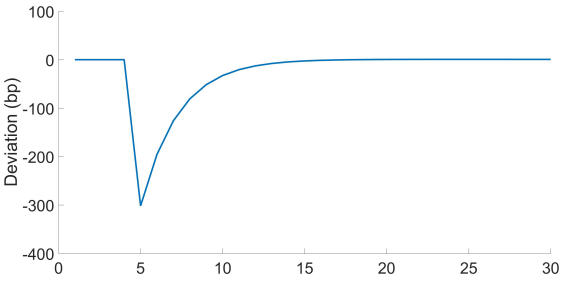
(d) Consumption



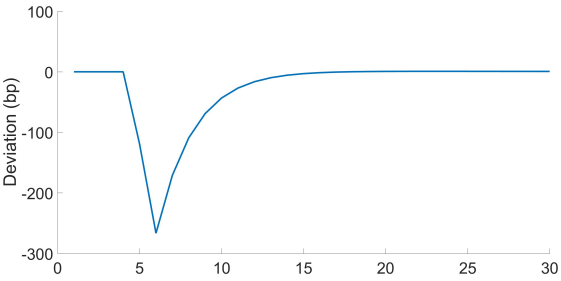
(e) Labour



(f) Real Wage



(g) Wage Inflation



(h) Price Inflation

Figure 4.2: Impulse Responses - 100bp Nominal Interest Rate Shock

household consumption and financial asset purchase decisions. They also completely determine the marginal cost of firms thereby influencing investment decisions. The response of labour and the real wage requires further analysis of labour supply and labour demand in the presence of sticky wages and sticky prices.

Non-Linear Labour Dynamics: Shifts and Twists

In this section I proceed with an analysis of the labour market. It highlights that in the presence of sticky prices and sticky wages there are non-linear dynamics at work that lead not just to shifts in labour supply and labour demand, but also to rotations of these curves. The rotation of these curves has not been highlighted in the literature previously.

The income effect is defined as any change in the response of labour supply due to movements in non-labour components of the labour supply function.[16] As shown in the previous chapter, standard GHH preferences remove the income effect but that it reappears in the presence of internal habits in consumption. The introduction of sticky wages leads to the following labour supply condition in equilibrium,

$$\frac{w_t}{P_t} = \left(\frac{1}{\Theta_t^w}\right)\left\{-\frac{\varepsilon^w}{\varepsilon^w - 1}\frac{u_{n,t}}{\Lambda_t} + \frac{1}{n_t}E_t\left[m_{t,t+1}\frac{\chi^w}{\varepsilon^w - 1}\pi_{t+1}^w\left(\pi_{t+1}^w - 1\right)\frac{w_{t+1}}{P_{t+1}}n_{t+1}\right]\right\},$$

where,

$$\Theta_t^w = 1 + \frac{\chi^w}{\varepsilon^w - 1}\pi_t^w\left(\pi_t^w - 1\right). \tag{4.4.1}$$

When there is no stickiness in wages ($\chi^w = 0$) the equilibrium labour supply function collapses to one similar to that studied in previous chapters. The difference being that household power in the labour market leads to a magnification of the labour supply response by the multiplier $\frac{\varepsilon^w}{\varepsilon^w - 1} > 1$. Thus absent sticky wages there will be a

[16] This is predominantly the result of movements in consumption which does not react directly to the shock but rather in response to the movement of other variables.

stronger income effect induced due to the assumption of household market power in labour supply. The presence of sticky wages introduces two additional factors into the labour supply function - the multiplier Θ_t^w and the term incorporating expectations. As I will demonstrate below, both of these terms act to alter the income effect under sticky wages, so that one needs a broader interpretation of the income effect that accounts for wage inflation effects.

The multiplier, Θ_t^w, acts to either mute or magnify all shifts in the labour supply depending on the gross wage inflation. In particular since $\frac{\chi^w}{\varepsilon^w - 1} > 0$,

$$\pi_t^w > 1 \Rightarrow \Theta_t^w > 1,$$
$$\pi_t^w < 1 \Rightarrow \Theta_t^w \in [0, 1).$$

Thus if gross wage inflation is greater than one (less than one) then the shift in the labour supply curve is muted (magnified).[17]

The expectation term shifts the labour supply curve in a non-uniform manner as it acts as a multiplier on $\frac{1}{n_t}$. The multiplier is strongest (largest shift) at smaller values for n_t and weakens as $n_t \to 1$. As it changes the slope of the labour supply curve, this term acts to change the relative elasticity of the labour supply curve.[18] The direction in which the elasticity is changed is determined by expected gross wage inflation in the next period, i.e. π_{t+1}^w. If $\pi_{t+1}^w > 1$ then the labour supply shifts upwards and becomes more elastic since there is a greater shift at lower values of n_t. Conversely, if $\pi_{t+1}^w < 1$ then the labour supply curve shifts downwards and becomes

[17] One can ignore the case of $\Theta_t^w < 0$ by noting that π_t^w is always very close to unity so that $\pi_t^w (\pi_t^w - 1)$ will always be close to zero and one would require a very large value for $\frac{\chi^w}{\varepsilon^w - 1}$ to generate a term large enough to force $\Theta_t^w < 0$. Sensible calibration prevents the cases where sticky wages lead to positive income effects.

[18] Note, the multiplier above will also act on this term as well.

Table 4.1: Income Effect, Labour Supply Elasticity Cases

	$\pi_t^w > 1$	$\pi_t^w < 1$
$\pi_{t+1}^w > 1$	Shift Muted; Labour Supply More Elastic; Income Effect Muted	Shift Amplified; Labour Supply More Elastic; Income Effect Amplified
$\pi_{t+1}^w < 1$	Shift Muted; Labour Supply Less Elastic; Income Effect Muted	Shift Amplified; Labour Supply Less Elastic; Income Effect Amplified

less elastic. The strength of the income effect in the presence of sticky wages, therefore, depends on the behaviour of gross wage inflation both in the current period and the next period, i.e. by considering four distinct cases. The overall impact on the income effect can be completely understood using this partial equilibrium analysis. As I will show below, this is no longer the case for the substitution effect.

The labour demand function under the assumption of monopolistic competition in the goods market by firms and the presence of sticky prices is given by,

$$\begin{aligned}\frac{w_t}{P_t} = &\left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \Theta_t^p\right) z_t \left(1-\alpha\right) k_t^{\alpha} n_t^{-\alpha} \\ &- \frac{1-\alpha}{n_t} E_t \left[m_{t,t+1} \frac{\chi^p}{\varepsilon^p} \pi_{t+1}^p \left(\pi_{t+1}^p - 1\right) y_{t+1} \right],\end{aligned}$$

where,

$$\Theta_t^p = \frac{\chi^p}{\varepsilon^p} \pi_t^p \left(\pi_t^p - 1\right).$$

The substitution effect in this model is no longer just the direct response of labour demand to the contemporaneous shock but requires an expanded interpretation that incorporates price inflation. In the absence of sticky prices, ($\chi^p = 0$), firms only face a fraction, $\frac{\varepsilon^p - 1}{\varepsilon^p}$ of the technology shock thereby weakening the shift in labour supply

due to the shock. The introduction of sticky prices, however, introduces two additional factors - Θ_t^p and the additional expectation term - both of which will affect the overall size of the substitution effect.

Unlike the labour supply function, Θ_t^p affects only the impact of the technology shock and had no impact on the additional expectation term. There are two possible cases[19] for the impact of Θ_t^p on the substitution effect,

1. Amplification of the technology shock: $\Theta_t^p > 0 \Leftrightarrow \pi_t > 1$
2. Muting of the technology shock: $\Theta_t^p \in \left[-\frac{\varepsilon^p - 1}{\varepsilon^p}, 0\right) \Leftrightarrow \pi_t \in \left[\frac{1}{2} - \frac{1}{2}\frac{\sqrt{(\chi^P)^2 - 4\chi^P(\varepsilon^p - 1)}}{\chi^P}, 1\right)$

The expectation term shifts the labour demand curve in a non-uniform manner thereby rotating the demand function. The shift is largest at lower values for n_t and declines as $n_t \to 1$. Similar to the labour supply function the impact on demand elasticity is affected by the behaviour of gross price inflation in the next period, π_{t+1}^p. If $\pi_{t+1}^p > 1$ then labour demand shifts downwards hence leading to a more elastic labour demand function. Conversely if $\pi_{t+1}^p < 1$ then labour demand is shifted upwards causing labour demand to become less elastic. The expectations term also shifts the demand curve leading to a broadening of the interpretation of substitution effect compared to a model without sticky prices.

In the case of labour supply, both the shift and the rotation of the curve are magnified (muted) by the same multiplier so that the overall impact of the income effect is completely understood in the partial equilibrium setting. The same partial equilibrium analysis is not sufficient to pin down the overall substitution effect since the shift and rotation of the labour demand curve occur independently so the impact on the substitution effect is indeterminate. This means that one needs a quantitative

[19]The third case of a reversal of the substitution effect is ruled out by similar arguments as for the case of $\Theta_t^w < 0$, i.e. sensible calibration again rules out situations of large disinflation.

Table 4.2: Substitution Effect, Labour Demand Elasticity Cases

	$\pi_t^p > 1$	$\pi_t^p \in \left[\frac{1}{2} - \frac{1}{2}\frac{\sqrt{(\chi^P)^2 - 4\chi^P(\varepsilon^p - 1)}}{\chi^P}, 1\right]$
$\pi_{t+1}^p > 1$	Technology Shock Amplified; Labour Demand More Elastic	Technology Shock Muted; Labour Demand More Elastic
$\pi_{t+1}^p < 1$	Technology Shock Amplified; Labour Demand Less Elastic	Technology Shock Muted; Labour Demand Less Elastic

analysis to determine the overall impact on the substitution effect. In the analysis below I continue to use the quantitative decomposition of the income and substitution effects suggested by King (1991).[20]

In the Flex Price model there is an income effect that is negative and dominates a weaker substitution effect. This results in the negative contemporaneous response of labour to a technology shock and one gets the negative response of labour and an increase in the wage, as expected. In the Sticky Price model the response of labour continues to be negative but the real wage falls. Contemporaneous gross nominal wage inflation is less than unity in the period of the shock and also less than unity in the period after the shock so there is an amplification of the shift on a less elastic labour supply curve. Thus leading to an amplification of the income effect. Gross price inflation in the period after the shock is also less than unity thereby leading to a less elastic labour demand which lies sufficiently below the pre-shock demand curve to generate the fall in the real wage. Beyond this, there is little that can be said about the substitution effect. In order to get a more complete understanding of the substitution effect one must look to the quantitative decomposition of the income and substitution effects presented in Appendix B to this chapter. It shows that the

[20]The decomposition of King (1991) has been discussed in the previous chapter.

substitution effect is much stronger under the Sticky Price Model than under the Flex Price Model. Further it shows that the income effect is magnified (as per the conclusion of the partial equilibrium analysis) in the Sticky Price Model. This leads to the observed dynamics for the Sticky Price model.

Mechanically there is a large shift out and steepening of the labour demand curve and a shift left and steepening of the labour supply curve, this is depicted graphically in the figure below. The rotation of the labour demand and labour supply curves

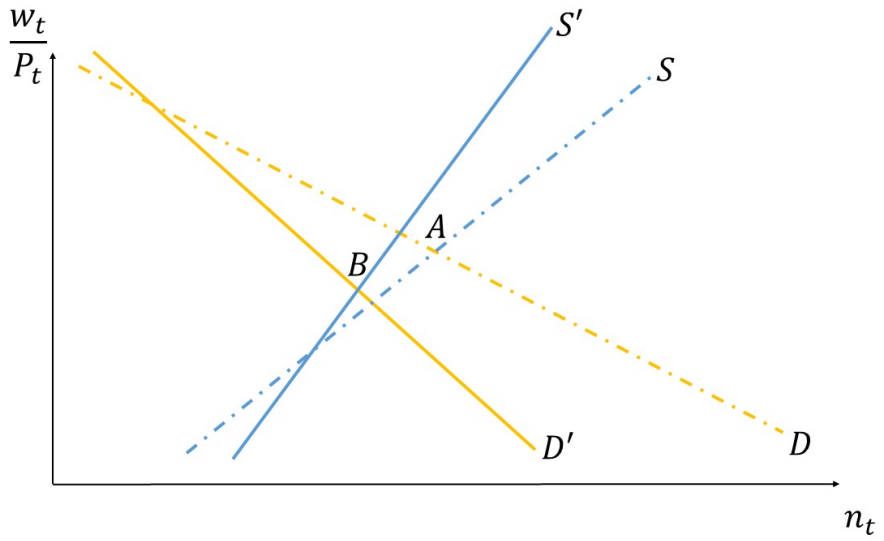


Figure 4.3: Rotation and Shift of Labour Supply and Labour Demand Curves to a Technology Shock. The dashed lines are pre-shock curves and the solid lines are the curves post-shock, i.e. after the curves have been rotated and shifted.

has intuitive appeal. Sticky wages lead households to dislike altering their nominal wage. The labour supply is less elastic as households seek to recoup some part of their intangible loss. Consider a rise in the real wage. The intangible menu cost faced by households leads them to increase wages more and reduce labour by less than if wages were not sticky, i.e. the labour supply becomes steeper and hence less elastic. Households focus on the margin that doesn't induce greater menu costs, i.e.

they use the labour channel rather than real wage changes. However, since they do face a menu cost regardless they also require a larger response in the wage for any change in labour supply to recoup these costs. Similarly firms face an intangible cost to profits due to the presence of sticky prices and are therefore reluctant to change prices. Consequently they seek to recoup any cost associated with changing prices by smoothing their wage bill via less elastic labour demand. Again, consider a rise in the real wage. In the absence of sticky wages firms would like to reduce their workforce in order to smooth their marginal costs and hence maximise profits. The intangible menu cost faced by firms from changing prices makes firms reduce their demand for labour by more than otherwise in an attempt to minimise the impact on profits via a reduction in their wage bill. This less elastic demand curve has the effect of laying below the pre-shock labour demand curve for a significant range of values for n_t thereby leading to an overall fall in the equilibrium real wage. Thus the desire of both firms and households to recoup their intangible costs from price and wage stickiness leads to the observed rotation in the labour demand and supply curves. This analysis adds to the general view in the literature (see Cantore et al. (2014)) that labour falls in response to an expansionary technology shock in New Keynesian models. The general reasoning being that firms that cannot freely adjust prices to meet demand must reduce labour demanded (and hence household income) to quell any excess demand. This response is only exacerbated when there are sticky wages as labour must fall by more to generate the appropriate fall in income to quell the excess demand for consumption caused by the expansionary technology shock.

Other Business Cycle Dynamics

The fall in the real wage coupled with the decline in labour reduces household income. This leads to the lower response of consumption and the pronounced hump shape is a characteristic of internal consumption habits. That consumption doesn't respond

much to the shock and output increases due to the technology shock leads to the observed dynamics of investment. Firms have a smaller wage bill and smaller demand leading to the excess output having to be transformed into capital via investment. Investment falls as demand slowly increases leading to less excess output.

A positive shock to the nominal interest rate leads induces a fall in consumption via general equilibrium channels that induce the household to defer consumption in response to the higher interest rate. The fall in demand reduces output and flows on to affect the dynamic response of all other variables. The increase in the nominal rate is classified as a 'contractionary' shock as it leads to an overall reduction in economic activity. The impact of a contractionary nominal shock follows the dynamics expected and seen throughout the literature.

The nominal interest rate shock leads to a fall in equilibrium labour coupled with a rise in the real wage. This is despite the movements of the labour supply and demand curves being in the same directions as for the technology shock. The rise in the real wage is the result of the labour demand curve having a stronger substitution effect coupled with a supply curve that has a stronger income effect and is less elastic. The quantitative decomposition presented in Appendix C bears out this analysis. The larger shift outwards means that the steepening of the demand curve causes it to cross the pre-shock demand curve at a larger value of n_t than the pre-shock equilibrium level. Thus the rotated demand curve lies above the pre-shock demand curve in the region where it intersects the labour supply curve thereby leading to an increase in the equilibrium real wage coupled with a decline in equilibrium labour.[21]

This analysis highlights the issues with linearising the labour supply and demand

[21] A proof that the intersection of the pre-shock and post-shock demand curves depends on the size of the deviations of the gross price inflation rate is provided in an appendix to this chapter.

Table 4.3: Business Cycle Results

	Data	RBC - GHH Preferences	Flex Price Model	Sticky Price Model
Standard Deviation (%)				
Output	1.72	0.95	0.96	0.42
Standard Deviation Relative to Output				
Consumption	0.79	0.98	1.03	0.96
Hours Worked	0.63	0.40	0.43	2.32
Investment	3.24	0.99	1.20	1.41
Real Wage	0.47	1.03	1.00	0.36
Contemporaneous Correlation with Output				
Consumption	0.87	0.95	0.97	0.96
Hours Worked	0.93	0.13	0.21	-0.63
Investment	0.86	0.87	0.83	0.85
Real Wage	0.21	0.92	0.91	0.81
Euler Equation Error		-4.6995×10^{-5}	-6.2984×10^{-5}	-7.9882×10^{-6}

curves in a New Keynesian model since it is the interplay of the shifts and twists in response to price and wage inflation dynamics that lead to the two vastly different labour market outcomes. Log-linearising New Keynesian models ignores this "twisting" of the labour demand and labour supply curves from the change in elasticities. Indeed, log-linearising the model treats both the shift and rotation of the curves as shifts in the curves. The diversity of outcomes in the impulse responses can be parsimoniously rationalised in a non-linear model where the responses are due to more than just shifts in the labour market curves. The analysis presented here complements the views held in the literature by looking at both the household side, i.e supply side, and the firm side, i.e. the demand side, of the labour market to shed light on the observed joint response of labour and the real wage.

4.4.2 Quantitative Business Cycle Results

The "RBC-GHH" model in the quantitative results refers to an RBC equivalent model to that studied, i.e. one with capital adjustment costs, internal habits in consumption and GHH preferences but without monopolistically competitive firms and households.

The "RBC-GHH" model is introduced as a benchmark model in order to assess the quantitative impact of moving to a world with monopolistically competitive firms and households. It is the model of the previous chapter.

The Flex Price Model results are very similar to the RBC-GHH benchmark with the exception of investment. This is a priori expected since the Flex Price Model is the 'RBC equivalent' in a nominal world due to the absence of any effect of nominal variables on real variables. The response of investment differs since there is firm specific capital investment in our model set-up. As discussed, the presence of firm specific capital changes the investment decision in the economy as it is the ability of investment to reduce effective marginal cost rather than the absolute productivity of capital. In the Flex Price model, this leads to more volatile investment relative to output. The more volatile investment series no longer has a direct impact on the consumption series since both are no longer controlled by households. Thus firm specific capital investment divorces the strong link between investment and consumption.

The quantitative results from the Sticky Price model are a consequence of forward looking households and firms having a weaker response to the contemporaneous productivity shock and the interplay of both technology shocks and shocks to the nominal interest rate. Of particular note in the Sticky Price model is the significantly smoother output volatility and the much larger relative volatility of hours.

The re-emergence of countercyclical labour is expected in the New Keynesian model. However, study of both the impulse responses and quantitative results suggests that this response is again too extreme. One could use the alternative calibration scheme proposed in the previous chapter where it was shown that habit intensity could determine the strength of the income effect. In simulation results I find that a positive

response of labour is again possible in the New Keynesian model but only over a very small (and implausible) range in the vicinity of zero habit intensity. A study of the interaction between the habit intensity, and income and substitution effects in a New Keynesian model with GHH preferences is an interesting avenue left for future work, however it is outside the scope of this thesis.

4.5 Asset Pricing Implications

There are two assets in the economy: equity and the nominal bond. The nominal bond is priced by the central bank according to its policy function (4.2.28). It both compensates for aggregate uncertainty and protects the household from inflation risk by taking this into account. Thus we may use the central bank's policy rule to understand movements in the nominal risk free rate.

The return on equity, however, does not have an obvious form related to the real economy. It is the purpose of this section to derive a relationship between the return on equity and the variables of the real economy. Such a relationship, while operating in the background in many models, has not previously been brought to the fore and derived for a New Keynesian model. The relationship one is interested in is that that links R^e_{t+1} in the equity Euler equation, $1 = E_t\left[m_{t,t+1}R^e_{t+1}\right]$, to the underlying economic variables. That is one is interested in a relationship of the form,

$$R^e_{t+1} = f\left(\text{real economic variables}, \text{nominal economic variables}, \text{deep model parameters}\right),$$

which not only gives a value for R^e_{t+1} but also informs one about the deeper mechanism by which agents price assets. By relating the real and financial sides of the economy this relationship allows one to understand how the underlying economic variables impact on asset prices and how policy, via an impact on the real economy, affects

asset prices.

4.5.1 Equity Pricing

I adapt the methodology of Altug and Labadie (2008, Ch. 10) to derive an equilibrium pricing relationship for equity in the model. Altug and Labadie (2008) only consider the simplest case of a perfectly competitive firm without capital adjustment costs. I extend this analysis to the more general case of monopolistically competitive firms where capital adjustment costs, and nominal price and wages rigidities are present.

The derivation of the relationship consists of three distinct steps. First the financial structure of the firm as outlined by Altug and Labadie (2008, Ch. 10) is derived. Second, a profit aggregation result needs to be derived in order to move from the individual firm to that of the equity index, which is an aggregate of the individual firms. Finally, the households Euler Equation for equity and the firms investment first order condition are invoked to arrive at a stochastic difference equation which is solved to get the desired relationship between the return on equity and the underlying economic variables.

The Financial Structure of the Firm

We begin by outlining the financial structure of the intermediate goods firms as it is shares in these firms which are held by households. This financial structure is sufficiently general but it does assume that the Modigliani-Miller theorem holds, i.e. that the form of financing does not matter so that one can consider the firm as completely equity financed.[22] The gross real profits of the firm are given by,

$$\frac{\Xi_t(j)}{P_t} = \frac{P_t(j)\, y_t(j)}{P_t} - \frac{w_t}{P_t} n_t^d(j) \tag{4.5.1}$$

[22]While Modigliani-Miller has been assumed here, future work will seek to understand how (and whether) firm leverage affects the results presented here.

The firm can disburse real profits as dividends to stock, $d_t(j)$, or hold them as retained earnings, $RE_t(j)$, i.e.

$$\frac{\Xi_t(j)}{P_t} = d_t(j)\, s_t(j) + RE_t(j). \tag{4.5.2}$$

Similarly real investment is financed via retained earnings, $RE_t(j)$, or new equity issue, $v_t(S_{t+1}(j) - S_t(j))$, i.e.

$$I_t(j) = RE_t(j) + v_t(S_{t+1}(j) - S_t(j)). \tag{4.5.3}$$

We can combine (4.5.2) and (4.5.3) to yield,

$$\frac{\Xi_t(j)}{P_t} = d_t(j) + I_t(j), \tag{4.5.4}$$

where we have used the equilibrium condition $S_t = 1 \quad \forall j, t$ to get to the relationship above. According to (4.5.4) real profits are either used to pay real dividends or used to finance real investment. In particular note that the firm does not retain any earnings.

Profit Aggregation

In order to consider the return on the share index we derive a form for aggregate real profit. Let us define this as

$$\frac{\Xi_t}{P_t} = \int_0^1 \frac{\Xi_t(j)}{P_t} dj. \tag{4.5.5}$$

The aggregate profit is defined as[23],

$$\begin{aligned}
\Xi_t &= \int_0^1 P_t(j)\, y_t(j)\, dj - \int_0^1 w_t n_t^d(j)\, dj \\
&= \int_0^1 P_t(j)\, y_t(j)\, dj - w_t n_t^d \\
&= P_t y_t - w_t n_t
\end{aligned}$$

[23]For profit aggregation it does not matter if we use real or nominal since the aggregate price level can be moved outside the integral

where we have used the intermediate goods firms demand condition, (4.2.14), and the equilibrium labour condition. However, note that here y_t is given by the Dixit-Stiglitz aggregate of intermediate goods given by,

$$Y_t = \left[\int_0^1 \left(F(k_t(j), n_t(j)) \right)^{\frac{\varepsilon^p - 1}{\varepsilon^p}} dj \right]^{\frac{\varepsilon^p}{\varepsilon^p - 1}}. \tag{4.5.6}$$

In general this is not useful for linking aggregate output to aggregate factors in a simple, and mathematically tractable, manner in the presence of price dispersion and firm specific capital. In the presence of price dispersion, firm specific capital leads to complete heterogeneity between firms. This is because when only some fraction of firms can reset prices, hence leading to price dispersion, heterogeneity will very quickly appear as previous pricing decisions will impact capital investment decisions via an impact on marginal costs. In such a case all firms will eventually have different capital stocks.

Under Rotemberg pricing all firms are, in equilibrium, ex-post identical and so charge the same price in all periods, so that there is no price dispersion. Thus all firms will, ex-post and in equilibrium, make identical capital investment decisions. If firms are ex-post identical then for measure Ω of intermediate firms one has,

$$k_t(j) = \frac{k_t}{\Omega} \quad \text{and} \quad n_t(j) = \frac{n_t}{\Omega}$$

where k_t and n_t are aggregate capital and labour respectively. The production function exhibits constant returns to scale so one gets,

$$F(k_t(j), n_t(j)) = \frac{1}{\Omega} F(k_t, n_t).$$

Thus under Rotemberg pricing one gets,

$$\begin{aligned} y_t &= \left[\int_0^1 \left(\frac{1}{\Omega}F(k_t, n_t)\right)^{\frac{\varepsilon^p-1}{\varepsilon^p}} dj\right]^{\frac{\varepsilon^p}{\varepsilon^p-1}} \\ &= F(k_t, n_t), \end{aligned}$$

where the final equality comes from the fact that there are measure Ω of intermediate firms on the unit interval $j \in (0, 1)$.

Under Rotemberg pricing one can therefore relate the aggregate output to aggregate factors which is not possible under Calvo pricing without the need to resort to alternative aggregation schemes or additional restricting assumptions. In this sense the result presented here is sufficiently general as it only relies on well known characteristics that appear with the Rotemberg pricing assumption. So one can write the relationship between output and the marginal products as,

$$y_t = z_t f_{k,t} k_t + z_t f_{n,t} n_t.$$

So we get,

$$\Xi_t = P_t z_t z_t f_{k,t} k_t + P_t z_t f_{n,t} n_t - w_t n_t. \tag{4.5.7}$$

In real terms this becomes,

$$\frac{\Xi_t}{P_t} = f_{k,t} k_t + \left[f_{n,t} - \frac{w_t}{P_t}\right] n_t. \tag{4.5.8}$$

From the intermediate firm's first order condition, (4.2.24), we have,

$$\frac{w_t}{P_t} = \left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \Theta_t\right) f_{n,t}. \tag{4.5.9}$$

where,

$$\Theta_t = \frac{\chi_p}{\varepsilon^p}\pi_t\left(\pi_t - 1\right) - E_t\left[m_{t,t+1}\frac{\chi_p}{\varepsilon^p}\pi_{t+1}\left(\pi_{t+1} - 1\right)\frac{y_{t+1}}{y_t}\right], \tag{4.5.10}$$

has been introduced for notational simplicity. Thus the aggregate profit may be written as,

$$\frac{\Xi_t}{P_t} = f_{k,t}k_t + \left(\frac{1}{\varepsilon^p} - \Theta_t\right)f_{n,t}n_t. \tag{4.5.11}$$

This relationship states that aggregate profit in this case is given by the revenue product of capital plus that portion of the revenue product of labour all firms retain by virtue of being able to set prices. Re-writing (4.5.11) as,

$$\frac{\Xi_t}{P_t} = \left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \Theta_t\right)f_{k,t}k_t + \left(\frac{1}{\varepsilon^p} - \Theta_t\right)y_t.$$

In this form, the first term is the the saving that all firms make from owning capital. The second term is the pure profit all firms makes by being able to set prices, i.e. due to the presence of market power.

The Return on Equity Relationship

Aggregating (4.5.4) and combining with (4.5.11) yields the following relationship for aggregate dividends,

$$d_t = f_{k,t}k_t + \left(\frac{1}{\varepsilon^p} - \Theta_t\right)f_{n,t}n_t - I_t. \tag{4.5.12}$$

This may be substituted into the households Euler equation for equity to yield,

$$v_t = E_t\left\{m_{t,t+1}\left[v_{t+1} + f_{k,t+1}k_{t+1} + \left(\frac{1}{\varepsilon^p} - \Theta_{t+1}\right)f_{n,t+1}n_{t+1} - I_{t+1}\right]\right\}. \tag{4.5.13}$$

We now establish that,

$$E_t\left\{m_{t,t+1}\left[f_{k,t+1}k_{t+1}-I_{t+1}\right]\right\}=\frac{k_{t+1}}{\phi'\left(g_t\right)}+E_t\left\{m_{t,t+1}\left(\frac{1}{\varepsilon^p}-\Theta_{t+1}\right)f_{k,t+1}k_{t+1}\right\}-E_t\left\{m_{t,t+1}\frac{k_{t+2}}{\phi'\left(g_{t+1}\right)}\right\}$$

This removes the investment term and expresses the terms in terms of the capital stock. We first establish that,

$$E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right]k_{t+1}\right\}=\frac{E_t\left[k_{t+1}\right]}{\phi'(g_t)} \tag{4.5.14}$$

Proof.

$$\begin{aligned}
&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right]k_{t+1}\right\}\\
=&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right]\right\}E_t\left\{k_{t+1}\right\}\\
-&\mathrm{cov}_t\left(m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right],k_{t+1}\right)\\
=&\frac{E_t\left[k_{t+1}\right]}{\phi'(g_t)}
\end{aligned}$$

□

where we have used the firm's first order condition in the final step to get to the result and that in equilibrium we have $E_t\left[k_{t+1}\right]=k_{t+1}$.[24] We next establish that,

$$\begin{aligned}
&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right]k_{t+1}\right\}\\
=&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}k_{t+1}-I_{t+1}\right]\right\}+E_t\left[\frac{k_{t+2}}{\phi'(g_{t+1})}\right]
\end{aligned}$$

[24]This is because capital is predetermined via the capital accumulation equation

Proof.

$$
\begin{aligned}
&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}+\frac{1}{\phi'(g_{t+1})}\left(1-\delta+\phi(g_{t+1})-g_{t+1}\phi'(g_{t+1})\right)\right]k_{t+1}\right\}\\
=&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}k_{t+1}-I_{t+1}\right]\right\}+E_t\left\{m_{t+1}\frac{(1-\delta)k_{t+1}+\phi(g_{t+1})k_{t+1}}{\phi'(g_{t+1})}\right\}\\
=&E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}k_{t+1}-I_{t+1}\right]\right\}+E_t\left[\frac{k_{t+2}}{\phi'(g_{t+1})}\right]
\end{aligned}
$$

□

Where we get to the second step by expanding expanding the expectation and noting that $\frac{g_{t+1}\phi'(g_{t+1})k_{t+1}}{\phi'(g_{t+1})}=I_{t+1}$ and that $k_{t+2}=(1-\delta)\,k_{t+1}+\phi\left(g_{t+1}\right)k_{t+1}$. So that we have established that,

$$
E_t\left\{m_{t+1}\left[\left(\frac{\varepsilon^p-1}{\varepsilon^p}+\Theta_{t+1}\right)f_{k,t+1}k_{t+1}-I_{t+1}\right]\right\}=\frac{E_t\left[k_{t+1}\right]}{\phi'(g_t)}-E_t\left[\frac{k_{t+2}}{\phi'(g_{t+1})}\right] \quad (4.5.15)
$$

The final result comes from rearranging the left hand side to get the desired result. Substituting back into households Euler equation for equity yields,

$$
\begin{aligned}
v_t&=\frac{k_{t+1}}{\phi'\left(g_t\right)}+E_t\left\{m_{t,t+1}\left[v_{t+1}+\left(\frac{1}{\varepsilon^p}-\Theta_{t+1}\right)f_{n,t+1}n_{t+1}+\left(\frac{1}{\varepsilon^p}-\Theta_{t+1}\right)f_{k,t+1}k_{t+1}-\frac{k_{t+2}}{\phi'\left(g_{t+1}\right)}\right]\right\}\\
&=\frac{k_{t+1}}{\phi'\left(g_t\right)}+E_t\left\{m_{t,t+1}\left[v_{t+1}+\left(\frac{1}{\varepsilon^p}-\Theta_{t+1}\right)y_{t+1}-\frac{k_{t+2}}{\phi'\left(g_{t+1}\right)}\right]\right\}
\end{aligned}
$$

This is a stochastic difference equation which may be solved forward to yield,

$$
v_t=\frac{k_{t+1}}{\phi'\left(g_t\right)}+E_t\left\{\sum_{i=1}^{\infty}m_{t,t+i}\left(\frac{1}{\varepsilon^p}-\Theta_{t+i}\right)y_{t+i}\right\}. \quad (4.5.16)
$$

Where we have used the tansversality condition which guarantees that the value of capital falls to zero as $t\to\infty$. Let us consider each part of this in turn. The first term, $\frac{k_{t+1}}{\phi'(g_t)}$, is the value of capital holdings at the end of the period evaluated at the current price of capital. This is the value of capital that the firm will begin the

subsequent period with. The first part of the conditional expectation is the aggregate 'pure profit' intermediate goods firms earn in the given period by virtue of being able to set prices. To see this note that under flexible prices, i.e. when $\chi^P = 0$, $\Theta_t = 0$ in all periods so that the above solution becomes,

$$v_t = \frac{k_{t+1}}{\phi'(g_t)} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,t+i} \left(\frac{1}{\varepsilon^p} \right) y_{t+i} \right\}. \tag{4.5.17}$$

Under flexible prices the real profit is given by,

$$\begin{aligned} \frac{\Xi_t}{P_t} =& f_{k,t} k_t + \left(\frac{1}{\varepsilon^p} \right) f_{n,t} n_t \\ =& \left(\frac{\varepsilon^p - 1}{\varepsilon^p} \right) f_{k,t} k_t + \frac{1}{\varepsilon^p} y_t \end{aligned}$$

Real profit under flexible prices is therefore composed of two components. The first is the savings the firm makes by virtue of owning capital and hence not having to pay rent to capital and the second is the 'pure profit' firms earn by virtue of being able to set prices. If firms did not own capital then the first term would disappear as the saving on rental costs no longer exists however the second term remains hence its definition as the 'pure profit'. Having noted that the first term of the expectation is the 'pure profit' and is present regardless of sticky prices then we can say that the term Θ_t is the loss of 'pure profit' due to the presence of sticky prices. Since prices are sticky firms lose some of their potential 'pure profit' as they choose not to reset their price to the flexible level price in any given period and hence does not realise all of the potential 'pure profit'. The price of equity (4.5.16) therefore is comprised of two channels - a capital channel and an expectations channel. This is in contrast to the RBC model where equity prices only depend on the capital channel.

Finally one may derive a form for the equity payoff, i.e. $v_{t+1} + d_{t+1}$, which then

allows an equity return form to be derived.

$$\begin{aligned} v_{t+1} + d_{t+1} = & \left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \Theta_{t+1} \right) z_{t+1} f_{k,t+1} k_{t+1} + \left(\frac{1}{\varepsilon^p} - \Theta_{t+1} \right) y_{t+1} \\ & + \frac{k_{t+1}}{\phi'(g_{t+1})} \left[1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1}) \right] \\ & + E_{t+1} \left\{ \sum_{i=1}^{\infty} m_{t+1,t+1+i} \left(\frac{1}{\varepsilon^p} - \Theta_{t+1+i} \right) y_{t+1+i} \right\} \end{aligned}$$

The return on equity is defined as,

$$R^e_{t+1} = \frac{v_{t+1} + d_{t+1}}{v_t}, \tag{4.5.18}$$

so we have found a relationship between the return on equity and variables in the real economy. The advantage of such a relationship is that it does not rely on distributional assumptions required when one insists on log-linearising. That we don't need to assume joint log-normality of the stochastic discount factor and the return on equity makes this result more general than those hitherto presented in the literature. Furthermore the advantage of this relationship from one that relies on the log-normal/log-linear assumption is that the dynamics of the real economy can directly be traced to the dynamics of the ex-post return. Such a relationship allows one to understand how policy that affects real variables, e.g. monetary policy, affects asset pricing fundamentals.

4.6 Asset Pricing Results

The asset pricing results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique. The "RBC-GHH" model continues to refer to an RBC equivalent model to that studied, i.e. one with capital adjustment costs, internal

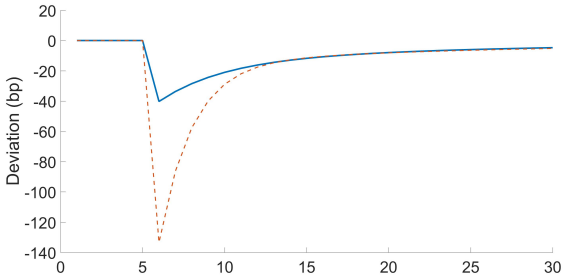
habits in consumption and GHH preferences but without nominal rigidities.

4.6.1 Asset Pricing Dynamics

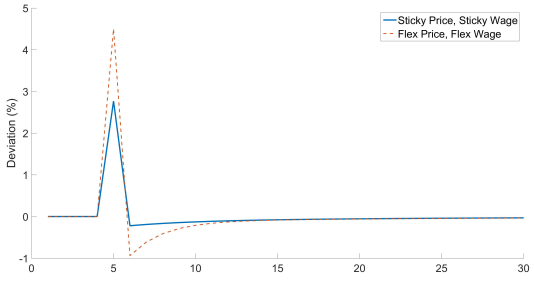
The impulse responses for asset prices and their components are presented in Figure 4.1 and Figure 4.2 for technology shocks and shocks to the nominal interest rate respectively. The technology impulse responses are for a one percentage point technology shock. The nominal interest rate shock impulse responses are for a $100bp$ shock to the nominal interest rate.

The consumption response drives the response in the risk-free interest rate - the rate at which households transfer consumption across periods. Habits induce consumption inertia causing households to prefer excessively smooth consumption so they are willing to transfer their consumption into future periods at a lower return. The large initial fall in the risk free rate is a consequence of this extreme desire to smooth consumption, especially in the Flex Price model. The muted response in the Sticky Price model is due to the muted consumption response as nominal rigidities induce demand inertia. The risk-free rate approaches steady state from below in response to a technology shock as agents progressively value contemporaneous consumption more once consumption begins to fall. A similar analysis, albeit in the opposite direction, holds for monetary policy shocks.

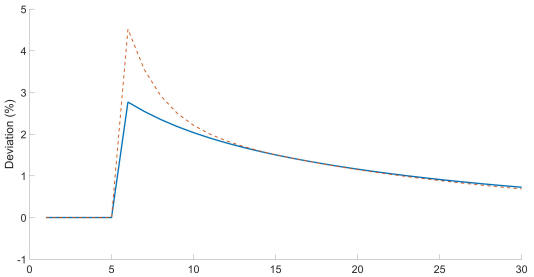
The return on equity impulse has a characteristic zig-zag shape regardless of the nature of the shock. The contemporaneous response of the return is driven by the fact that, at impulse, the return is determined by the price of equity in the period before impulse which remains at steady state level. The precipitous reversal results from returns post-impulse using the price of equity affected by the shock. The characteristic zig-zag shape may also be understood as lagged household response to the



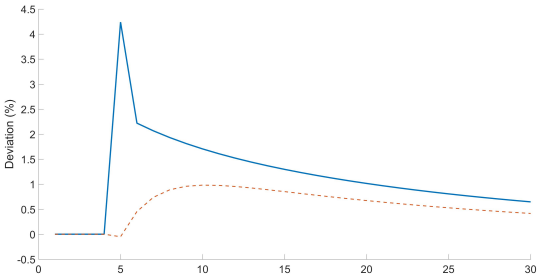
(a) Risk Free Return (bp)



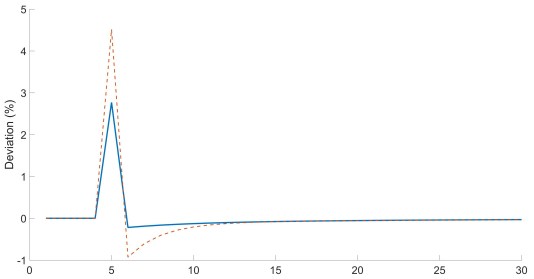
(b) Ex-Post Return on Equity



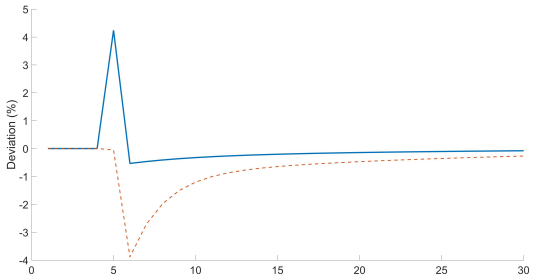
(c) v_t



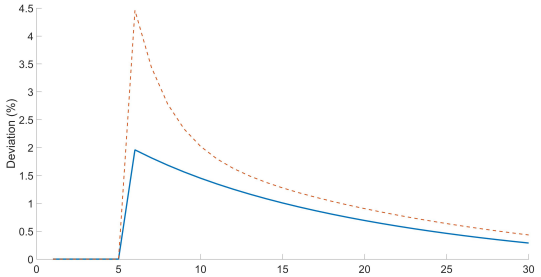
(d) d_{t+1}



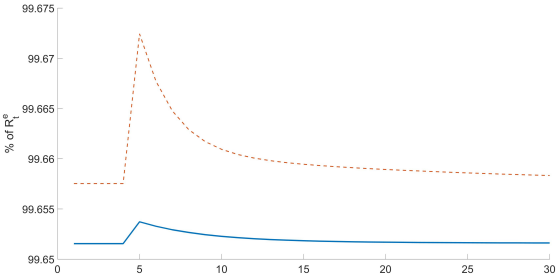
(e) Capital Gain: $\frac{v_{t+1}}{v_t}$



(f) $\frac{d_{t+1}}{v_t}$

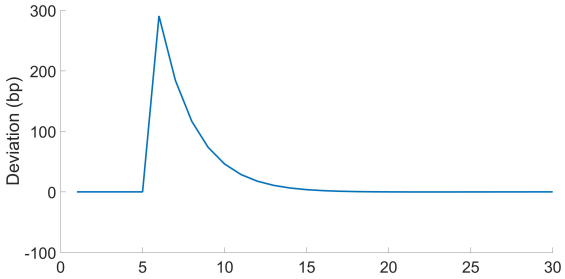


(g) Tobin's Q: $\frac{1}{\phi'(g_t)}$

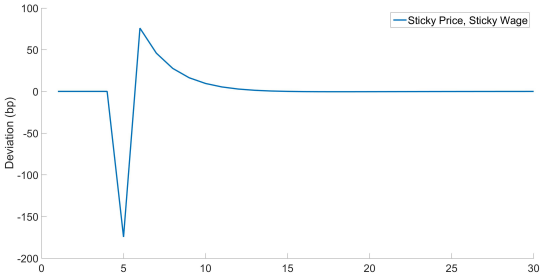


(h) Capital Gain Proportion of Ex-Post Return

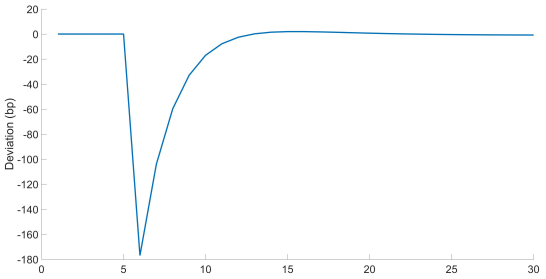
Figure 4.1: Asset Pricing Impulse Responses - Technology Shock



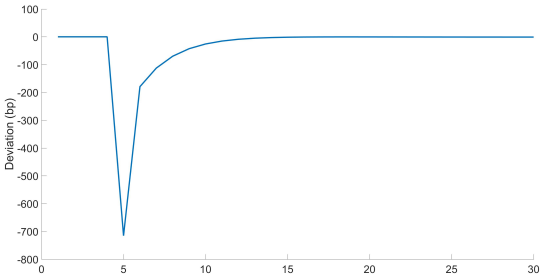
(a) Risk Free Return (bp)



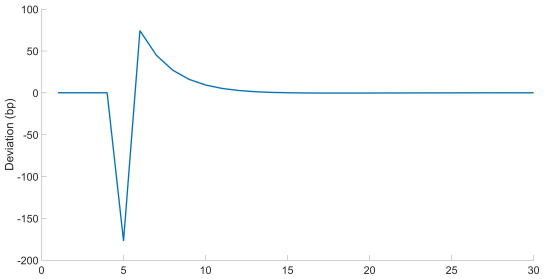
(b) Ex-Post Return on Equity



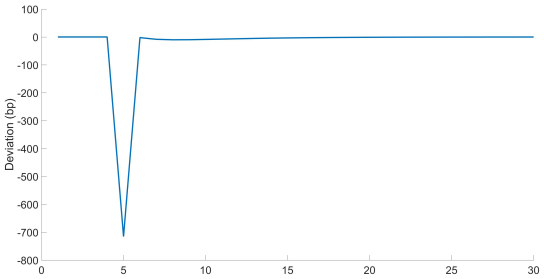
(c) v_t



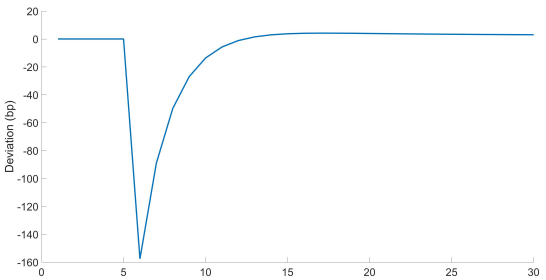
(d) d_{t+1}



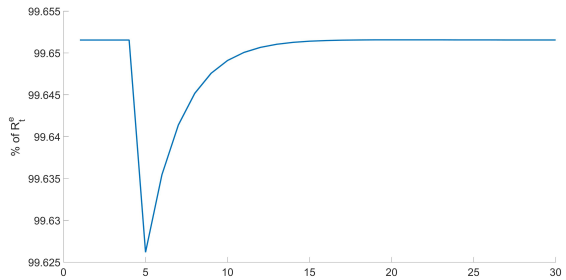
(e) Capital Gain: $\frac{v_{t+1}}{v_t}$



(f) $\frac{d_{t+1}}{v_t}$



(g) Tobin's Q: $\frac{1}{\phi'(g_t)}$



(h) Capital Gain Proportion of Ex-Post Return

Figure 4.2: Asset Pricing Impulse Responses - Nominal Interest Rate Shock

equity return. In the period of the technology shock, households experience an unexpected increase in the return and so over-invest in equity, driving up prices and causing the precipitous decline in return in the subsequent period. A similar analysis holds, in reverse, for the contractionary monetary policy shock. The response of the ex-post return is also much larger for the technology shock in the sticky wage model.

The dynamics of the return on equity can be decomposed into two channels - the capital gain channel and the dividend channel. The capital gain is defined as the ratio of adjacent equity prices, i.e. $\frac{v_{t+1}}{v_t}$, and this captures the return on equity due to the movement of equity price. The dividend channel is the value of dividends, i.e. $\frac{d_{t+1}}{v_t}$, and captures the return on equity due to the movement of dividends. The capital gain channel constitutes the vast majority of the return on equity[25] and this is reflected in the dynamics of the equity return being essentially identical to that for the capital gain.

Let us consider the evolution of the equity price, which is the only determinant of the dynamics of the capital gain. A technology shock leads to a rise in the price of equity, while a contractionary monetary policy shock leads to a fall in the equity price. The price of equity is defined as,

$$v_t = \frac{k_{t+1}}{\phi'(g_t)} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,t+i} \left(\frac{1}{\varepsilon^p} - \Theta_{t+i} \right) y_{t+i} \right\}. \tag{4.6.1}$$

which as already discussed is the sum of the value of capital at the end of the period and the pure profit of the firm adjusted for sticky prices. One can see that the response of the price of equity therefore follows the impulse responses of the appropriate real variables - capital and output - both of which increase (decrease) in response to a positive technology (contractionary monetary policy) shock. Tobin's Q, defined as

[25] See Figure: 5.1h and Figure: 5.2h

$\frac{1}{\phi'(g_t)}$ in the presence of capital adjustment costs, is the price of capital in this model. Thus Tobin's Q acts purely as a multiplier on the response of capital. The initial response of Tobin's Q in each model may be understood by that of investment, which rises (falls) in response to a positive technology (contractionary monetary policy) shock. Subsequent dynamics depend on the evolution of the investment-capital ratio as capital only evolves with a lag.

The evolution of dividend may also be understood in terms of the evolution of the real business cycle variables. The dividend is defined as,

$$d_{t+1} = \left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \Theta_{t+1}\right) f_{k,t+1} k_{t+1} + \left(\frac{1}{\varepsilon^p} - \Theta_t\right) y_{t+1} - I_{t+1}, \tag{4.6.2}$$

which may also be expressed as,

$$d_{t+1} = \frac{\Xi_{t+1}}{P_{t+1}} - I_{t+1}, \tag{4.6.3}$$

where $\frac{\Xi_{t+1}}{P_{t+1}}$ is the aggregate real profit as previously defined. Thus the dynamics of dividends may be understood via the dynamics of aggregate real profit and investment. The initial positive spike in the dividend response to a technology shock for the Sticky Price model is driven by the dynamics of aggregate real profit, and in particular the response of the wage bill. As discussed in detail previously the Sticky Price model sees the wage bill unambiguously decline in response to a technology shock leading to an increase in aggregate real profits which drives the large initial increase in dividends. The wage bill rises in the Flex Price model but it is small due to offsetting movements of labour and the real wage in that model. This causes the small decline in the aggregate dividend observed for the Flex Price model. Finally in response to a contractionary monetary policy shock there is a large (relative) rise in the wage bill, which more than dominates the falling investment, which drives the fall

Table 4.1: Asset Pricing Results

	Data	RBC - GHH Preferences	Flex Price Model	Sticky Price Model
Mean (% p.a)				
Risk Free	1.49	1.15	1.19	1.36
Equity	7.87	3.38	3.17	2.07
Equity Premium	6.52	2.22	1.98	0.70
Standard Deviations (% p.a)				
Risk Free	2.62	2.61	2.64	0.89
Equity	16.72	9.65	9.18	5.39
Equity Premium	15.69	9.31	8.85	5.31

in the dividend. However, as noted previously, the dividend channel does not play a significant role in the determination of the return of equity.

4.6.2 Quantitative Asset Pricing Results

The quantitative asset pricing results indicate that the simple New Keynesian model studied here suffers the same drawbacks in the quantitative replication of asset pricing results as its RBC brethren.

For ease of comparison to previous results, the risk-free rate results reported are for the real risk free bond which is priced according to,

$$R_t^f = \frac{1}{E_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\right]}.$$

This also facilitates the calculation of a real equity premium since the equity return is in real terms. The introduction of sticky prices leads to quantitatively superior risk free results with a simultaneous increase in both the mean rate and a decrease in the volatility. The Flex Price model continues to suffer from the excess risk free rate volatility problem. In the sticky price model the central bank controls the movement of the nominal risk free rate which then influences the real risk free rate defined above.

The impact of the central bank is evidenced by the fall in the volatility of the risk free rate. Thus the there is a route to overcoming the excess risk free rate volatility problem via the use of an appropriate monetary policy rule, e.g. a central bank which smooths the nominal rate via inertia in the monetary policy rule.

There is a deterioration of the equity return, this is due to the equity return under New Keynesian models being a claim not just on capital return as in previous chapters, but also a claim on future profit streams of the firm. The flexible price model has a slightly lower return on equity than the RBC-GHH model. As this is the 'RBC-equivalent' model in the New Keynesian framework one can conclude that simply moving to the New Keynesian framework cannot simply improve the joint replication results beyond the those of their RBC-equivalent models.

The impact on the mean equity return is attributed to the significantly higher equity price in New Keynesian models due to the presence of the profit channel in addition to the capital channel present in RBC models.

$$v_t = \underbrace{\frac{k_{t+1}}{\phi'(g_t)}}_{\text{Capital Channel}} + \underbrace{E_t\left\{\sum_{i=1}^{\infty} m_{t,t+i}\left(\frac{1}{\varepsilon^p} - \Theta_{t+i}\right) y_{t+i}\right\}}_{\text{Profit Channel}}.$$

The decomposition of the equity price into these two channels for the three models is provided in Table: 4.8. One notes that the equity price is significantly higher in the New Keynesian variants and that this is driven by the presence of the profit channel. The economy in both the New Keynesian and RBC models continues to be subject to the same shocks, as such these will have a smaller proportional impact on the equity return in the New Keynesian model than in the RBC model due to the equity prices being higher in the former.

Table 4.2: Equity Price Decomposition

	RBC - GHH Preferences	Flex Price Model	Sticky Price Model
Mean			
Equity Price	17.61	74.44	73.09
Capital Channel	17.61	13.24	13.19
Profit Channel	0	61.2	59.9
Standard Deviation			
Equity Price	1.61	7.14	6.03
Capital Channel	1.61	1.22	0.77
Profit Channel	0	5.92	5.29
Capital Channel and Profit Channel Correlation			
	0	1.00	0.96

There is a fall in the capital channel, which may be considered the RBC-equivalent equity price. The fall in the capital channel is driven by a lower Tobin's Q, combined with a lower capital stock, in the New Keynesian model when compared to the RBC model. The difference between the capital channel for the Flex Price model and the RBC model suggests that capital investment is on average lower in the New Keynesian model than in the RBC model resulting from the presence of monopolistic competition which causes output to be below the RBC level, hence leading to a lower quantum of output and investment.[26]

There is a precipitous fall in the volatility of the equity return in the Sticky Price model. This is attributed to the smoothing of the real economy resulting from the presence of nominal rigidities that introduce inertia into the response of the economy. It may also be attributed to the presence of a central bank that acts to stabilise the economy via monetary policy. This excessively smooth return volatility could be remedied by the introduction of other sources of shocks. One that would be instructive would be investment specific shocks to the capital accumulation equation in the

[26]Note that in the current set-up there are no tariffs that allow this 'contraction effect' of monopolistic competition to be offset. Hence the flexible price model, while having identical dynamics differ in their overall level of output.

spirit of Justiniano et al. (2010). Such investment specific shocks would add volatility to the equity price via Tobin's Q and also to the investment series, both of which would lead to an increase in the volatility of the equity return.

The lower mean return in the Sticky Price model is attributed to the fact that not all of the pure profit of the Flex Price model materialises due to the presence of sticky prices. However the size of the fall suggests that this is only part of the mechanism at work and that there is a general equilibrium mechanism operating simultaneously. This general equilibrium mechanism operates through the central bank attempting to stabilise the economy. The central bank, through policy, insulates the household from the full impact of the shocks hitting the economy. Consequently, this stability reduces the precautionary savings desire of households, evidenced by the rise in the risk free rate. This facilitates greater risky investment in equity which drives down equity returns beyond what would be expected purely in the face of sticky prices.

The quantitative results and discussion above provides further motivation for the research question of this thesis, i.e. does the form of the central bank policy affect asset prices. The impact of different central bank policy rules on asset pricing results is the focus of the next chapter.

4.7 Conclusion

This chapter has served as a "proof of concept" that one can price assets in New Keynesian models without the need to resort to log-linear/log-normal solution methods. It provides the assumptions that are needed to derive the link between the underlying macroeconomic variables and the financial economy - the key assumptions are Rotemberg pricing and the Modigliani-Miller theorem. Relaxing these assumptions

provide avenues for future research arising from this chapter. Furthermore, this chapter shows that one can solve New Keynesian models with purely non-linear methods.

In this chapter I have shown that one can relate the return on equity to the real economy without the need to log-linearise or assume joint log-normality. The relationship derived has not been proposed in the literature previously. The relationship derived here provides the necessary theoretical link needed for the study of central banking policy rules on asset prices in the next chapter, and more broadly on the macroeconomic policy impact on asset prices towards which this research agenda aims.

I proceed to provide quantitative business cycle and asset pricing results. I find that in the presence of sticky prices, sticky wages and GHH preferences the countercyclical labour problem that plagues many macro-finance models reappears. I also show that the presence of countercyclical labour, far from being the problem it is in the RBC framework, makes intuitive sense in the New Keynesian framework. I conclude that sticky wages and sticky prices are a necessary component of any model which aims to successfully jointly replicate asset pricing and business cycle results. Finally, the quantitative asset pricing results show that the simple New Keynesian model studied in this chapter continues to suffer from the same quantitative shortcoming of the RBC models studied in the previous chapters. This is not a cause for concern as it was a priori expected given the simplicity of the models studied in this thesis.

Appendix

4.A Crossing Point of Labour Demand

This appendix derives the condition that must be satisfied for the labour demand curve subject to a monetary policy shock to cross the pre-shock labour demand curve at an equilibrium labour level that is less than that for the crossing of the labour demand curve subject to technology shocks.

The pre-shock labour demand curve is given by,

$$\frac{w}{P} = \frac{\varepsilon^p - 1}{\varepsilon^p} (1 - \alpha) k^{\alpha} n^{-\alpha}.$$

Let τ denote the period of the shock. The new labour demand curve in the period of the technology shock is given by,

$$\begin{aligned}\left(\frac{w_\tau}{P_\tau}\right)^T = &\left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \frac{\chi_p}{\varepsilon^p}\pi_\tau^T \left(\pi_\tau^T - 1\right)\right) z_\tau^T (1 - \alpha) k^{\alpha} \left(n_\tau^T\right)^{-\alpha} \\ &- \frac{1 - \alpha}{n_\tau^T} E_\tau \left[m_{\tau,\tau+1} \frac{\chi_p}{\varepsilon^p} \pi_{\tau+1}^T \left(\pi_{\tau+1}^T - 1\right) y_{\tau+1}^T\right],\end{aligned}$$

where the price superscripts have been dropped from the gross price inflation rate for notational simplicity. As capital does not move in the period of the shock it remains at k, the pre-shock level. Similarly the new labour demand curve in the period of the

monetary policy shock is given by,

$$\left(\frac{w_\tau}{P_\tau}\right)^M = \left(\frac{\varepsilon^p - 1}{\varepsilon^p} + \frac{\chi_p}{\varepsilon^p}\pi_\tau^M\left(\pi_\tau^M - 1\right)\right)(1-\alpha)\,k^\alpha\left(n_\tau^M\right)^{-\alpha} - \frac{1-\alpha}{n_\tau^M}E_\tau\left[m_{\tau,\tau+1}\frac{\chi_p}{\varepsilon^p}\pi_{\tau+1}^M\left(\pi_{\tau+1}^M - 1\right)y_{\tau+1}^M\right].$$

At the crossing point with the pre-shock labour demand curve $\frac{w}{P} = \left(\frac{w_\tau}{P_\tau}\right)^T$ and $n = n_\tau^T = n^T$ for the technology shock and $\frac{w}{P} = \left(\frac{w_\tau}{P_\tau}\right)^M$ and $n = n_\tau^M = n^M$ for the monetary policy shock. One may solve for the crossing points n^T and n^M to yield,

$$\left(n^T\right)^{1-\alpha} = -\frac{E_\tau\left[m_{\tau,\tau+1}\frac{\chi_p}{\varepsilon^p}\pi_{\tau+1}^T\left(\pi_{\tau+1}^T - 1\right)y_{\tau+1}^T\right]}{\left(\frac{\varepsilon^p-1}{\varepsilon^p}\right)\left(1-z_\tau^T\right) - \frac{\chi_p}{\varepsilon^p}\pi_\tau^T\left(\pi_\tau^T - 1\right)z_\tau^T}k^{-\alpha},$$

and,

$$\left(n^M\right)^{1-\alpha} = \frac{E_\tau\left[m_{\tau,\tau+1}\frac{\chi_p}{\varepsilon^p}\pi_{\tau+1}^M\left(\pi_{\tau+1}^M - 1\right)y_{\tau+1}^M\right]}{\frac{\chi_p}{\varepsilon^p}\pi_\tau^M\left(\pi_\tau^M - 1\right)}k^{-\alpha},$$

We note that is it sufficient to compare $\left(n^T\right)^{1-\alpha}$ and $\left(n^M\right)^{1-\alpha}$ since,

$$\left(n^T\right)^{1-\alpha} > \left(n^M\right)^{1-\alpha} \Rightarrow n^T > n^M.$$

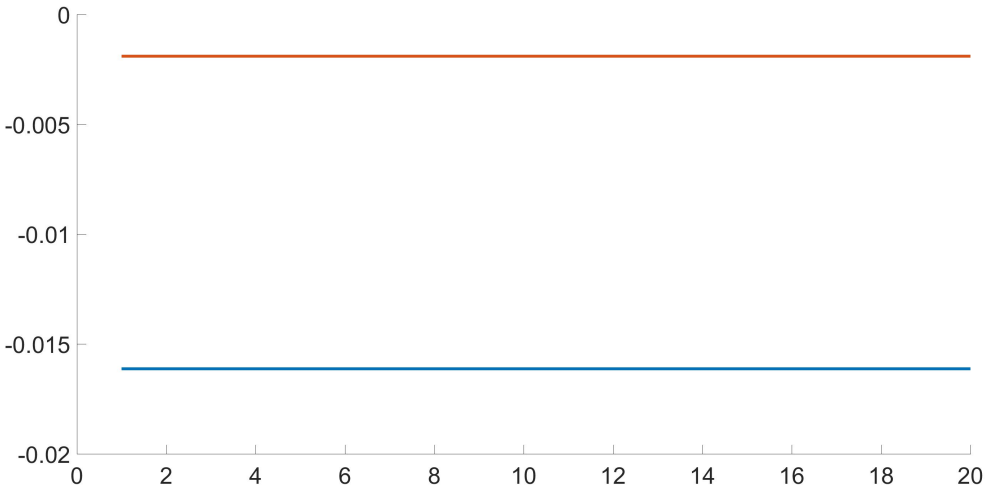
Thus for $n^T > n^M$ the following condition must be satisfied,

$$\frac{E_\tau\left[m_{\tau,\tau+1}\frac{\chi_p}{\varepsilon^p}\pi_{\tau+1}^T\left(\pi_{\tau+1}^T - 1\right)y_{\tau+1}^T\right]}{E_\tau\left[m_{\tau,\tau+1}\frac{\chi_p}{\varepsilon^p}\pi_{\tau+1}^M\left(\pi_{\tau+1}^M - 1\right)y_{\tau+1}^M\right]} < \frac{\left(\frac{\varepsilon^p-1}{\varepsilon^p}\right)\left(1-z_\tau^T\right) - \frac{\chi_p}{\varepsilon^p}\pi_\tau^T\left(\pi_\tau^T - 1\right)z_\tau^T}{\frac{\chi_p}{\varepsilon^p}\pi_\tau^M\left(\pi_\tau^M - 1\right)}$$

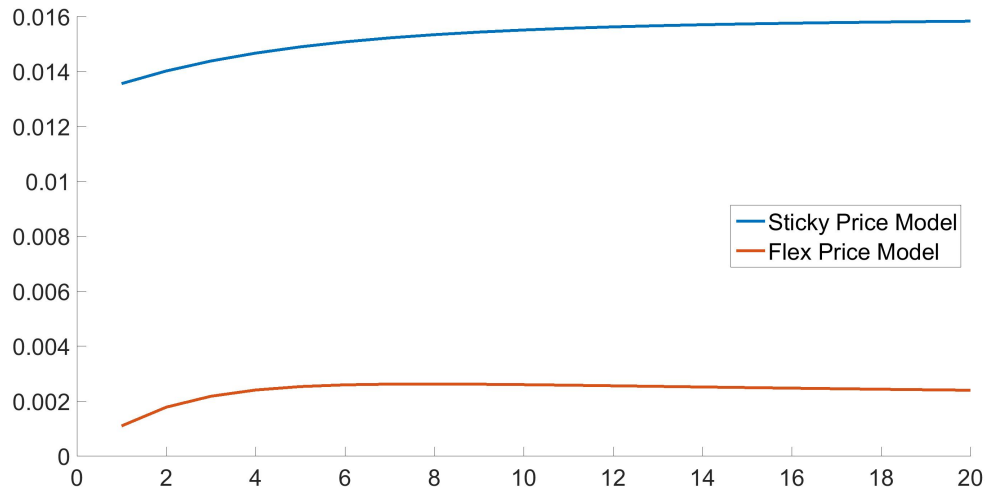
The two sides of this expression have a simple geometric interpretation. The right hand side of the expression is the ratio of the shifts in the labour demand in response to each of the shocks, while the left hand side is the ratio of the flattening of the labour demand in response to each of the shocks. If the right hand side is less than unity, then the monetary policy shock shifts the labour demand by more than the technology shock. If the left hand side is less than unity then the monetary policy shock flattens

the labour demand more than the technology shock. Thus if the condition holds then the monetary policy shock both shifts the demand curve further and flattens it more than the technology shock, leading to $n^T > n^M$. The quantitative results show that this is indeed the case.

4.B King (1991) Decomposition: Flex Price Model vs Sticky Price Model



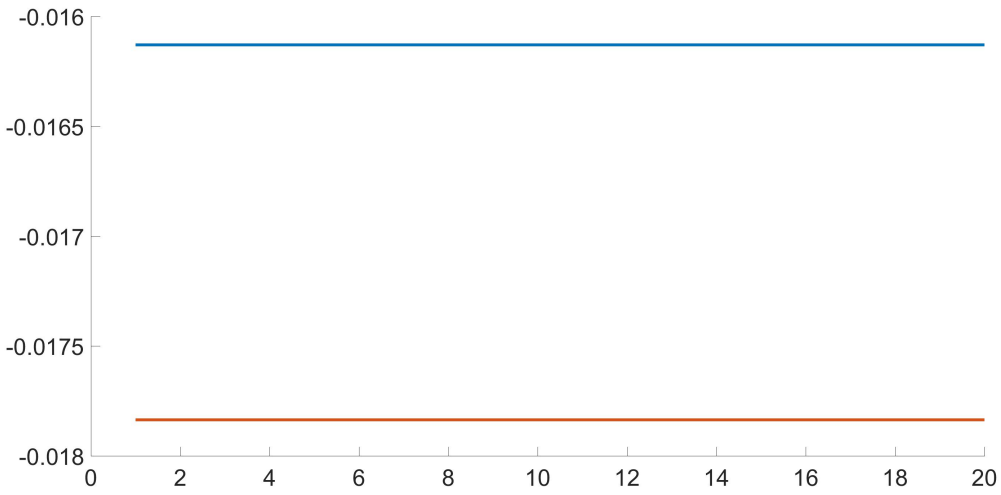
(a) Income Effect



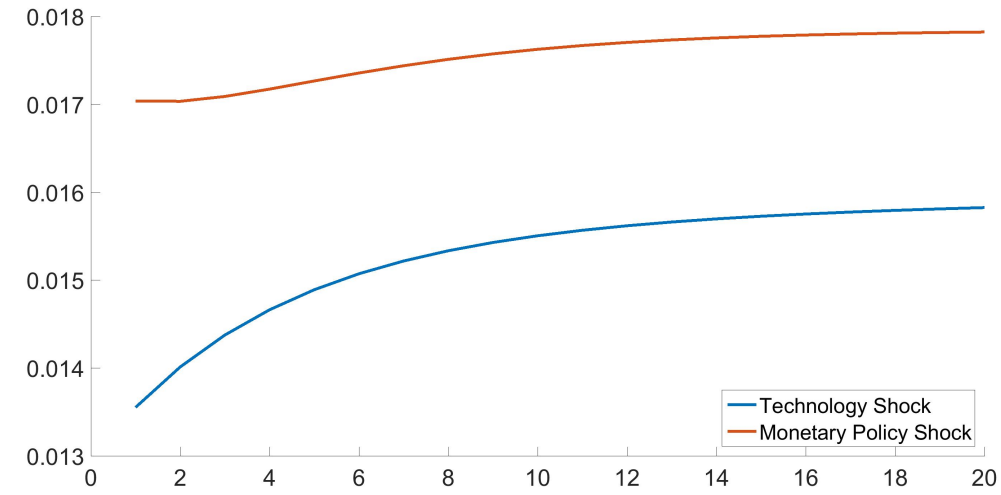
(b) Substitution Effect

Figure 4.B.1: King (1991) Decomposition: Flex Price Model vs Sticky Price Model

4.C King (1991) Decomposition: Technology Shock vs Monetary Policy Shock



(a) Income Effect



(b) Substitution Effect

Figure 4.C.1: King (1991) Decomposition: Technology Shock vs Monetary Policy Shock

Part III

CHAPTER 5

Simple Taylor Rules and Asset Pricing Dynamics

> *...even though we may know our destination and the general route by which we must get there, conducting monetary policy...is akin to driving without full vision - perhaps like driving in a rainstorm with defective windshield wipers. It can be done, but only very carefully*
>
> — **John Crow**
>
> Crow (1988)

5.1 Introduction

The recent financial crisis and policy responses to it have highlighted how little is understood, particularly in a theoretical sense, about how central banking policy affects asset prices. This chapter begins to fill this gap in the literature. The monetary pol-

icy effect on asset prices has been studied in the empirical macroeconomics literature, e.g. the empirical impact of monetary policy shocks on asset prices in the tradition of Rigobon and Sack (2004). There is also a significant empirical literature in the opposite direction, i.e. on how monetary policy reacts to asset prices. Bernanke and Gertler (2000), Rigobon and Sack (2003) and more recently Christiano et al. (2010) are examples of this literature.

I study the impact of the choice of policy regime in the simple New Keynesian model with nominal price and wage rigidities of the previous chapter. The presence of nominal rigidities in both prices and wages means that a central bank fully stabilising price inflation is no longer the optimal policy response. (Gali, 2008, Ch. 6) The equity pricing relationship developed in the previous chapter established the link between real economic variables and equity prices, the asset of central interest in this thesis. Since the specific choice of policy rule has consequences for how the real economy behaves it should have a flow on effect, via the equity pricing relationship, on asset prices. It is this impact that I study in this chapter. In studying how asset prices respond to central bank policy this chapter begins to answer the questions posed at the beginning of this thesis along two dimensions,

1. How does monetary policy influence asset prices?

2. Does the choice of policy regime have a differential impact on asset prices?

This is not an exhaustive study of all possible policy regimes that the central bank could implement. Rather I study the choice between two central banking regimes - one where a simple Taylor Rule is followed (the 'simple regime') and another where the simple Taylor Rule is augmented with some degree of interest rate smoothing (the 'smoothing regime'). I focus on Taylor Rules as this characterisation of monetary policy has become the dominant approach to the modelling of central bank policy in New

Keynesian models. In addition I also study how asset prices behave as the degree of smoothing varies.

When the central bank regime contains a smoothing component one can decompose the impact of central banking policy into two channels. The first is a direct channel which captures the contemporaneous impact of a change in the nominal interest rate. The second channel, only present in the smoothing regime, is an expectations channel that allows the change in the nominal rate to have more than just a contemporaneous effect on the economy. This expectations channel operates by providing agents with credible information about the future range of nominal interest rates due to the inertial response of the central bank.

I find that the choice of policy regime does not fundamentally change the mean risk free return or mean return on equity. Rather I find that the choice of regime has a large impact on the volatility of the return on equity - as the degree of smoothing is increased there is a fall in the volatility of the return on equity. This fall in the volatility of returns, with little impact on mean returns, is understood via the the presence, and strengthening, of the expectations channel of the smoothing regime. As agents have a greater degree of certainty surrounding the information they act upon they see further and through short-term shocks and anomalies.

5.2 Economy Set-Up

I study the impact of different monetary policy rules and policy environments in the New Keynesian model developed in the previous chapter. A brief overview of the model economy and it's salient features is presented below; for a more detailed exposition the reader is referred to the previous chapter.

There exist a continuum of households, indexed on the unit interval $i \in (0, 1)$, each of whom supply differentiated labour enduing them with a degree of market power in the labour market. Households set wages in a monopolistically competitive labour market. Nominal rigidities for households are introduced in the form of sticky wages. Households incur a cost when they change their nominal wage in any period *à la* Rotemberg pricing (Rotemberg, 1982).

Households sell their labour to perfectly competitive employment agencies. The employment agencies costlessly aggregate the differentiated labour of the individual households into an aggregate labour supply. This aggregate labour supply is then sold as an input to intermediate goods firms.

There exist a continuum of monopolistically competitive intermediate goods firms, indexed on the unit interval $j \in (0, 1)$, each of whom produce a differentiated good. This endues them with some degree of price-setting power for their goods in the goods market. Nominal rigidities for firms are introduced in the form of sticky prices *à la* Rotemberg pricing (Rotemberg, 1982). Firms own the capital in the economy. As a consequence firms make the capital investment decisions - the firm specific capital investment assumption. The intermediate goods firms combine their capital with aggregate labour supply to produce output using the production technology. Production uncertainty exists via exogenous technology productivity shocks.

The intermediate goods firms sell their differentiated output to perfectly competitive final goods firms. These final goods firms costlessly aggregate the output of intermediate goods firms. The aggregate output is used for both consumption and investment purposes, i.e. it may be costlessly transformed into either the consumption

good or the investment good. This costless transformation assumption prevents the firm investment decision from distorting the output demand function, i.e. the demand function is unchanged for the firm regardless of the use of their output (Woodford, 2003).

The final component of the model is the central bank. The central bank conducts monetary policy through the setting of the nominal interest rate. The nominal interest rate is set according to a Taylor rule. In the previous chapter a simple Taylor rule was assumed for the purposes of closing the model. The purpose of this chapter is the study of different monetary policy rules and their impact on asset prices. The focus will be on different formulations of the Taylor rule.[1]

5.2.1 Calibration

This chapter looks at policy rule experiments of the model introduced in the previous chapter. As such I keep the calibration of the previous chapter, for ease of reference the calibration is presented in the tables below.

Table 5.1: Calibrated Model Parameters (Quarterly Frequency)

Capital Share	$\alpha = 0.36$	Curvature Parameter	$\sigma = 2$
Depreciation Rate	$\delta = 0.025$	Habit Intensity	$\nu = 0.65$
Goods Degree of Substitutability	$\varepsilon^p = 6$	Labour Degree of Substitutability	$\varepsilon^w = 21$
Price 'Disutility' Parameter	$\chi^P = 77$	Nominal Wage Adjustment Parameter	$\chi^w = 237.515$
Technology Shock Autocorrelation Coefficient	$\rho = 0.95$	Inflation Response	$\rho_\pi = 1.5$
Standard Deviation of Innovations	0.007	Output Response	$\rho_y = 0.6$
Time Preference	$\beta = 0.9963$	Policy Shock Autocorrelation Coefficient	$\iota = 0.65$
Frisch Elasticity of Labour Supply	$\frac{1}{\varphi} = 0.75$	Standard Deviation Monetary Policy Shock	0.0028

[1] In the Conclusion I provide sketch studies of policy environments where there is uncertainty surrounding the smoothing parameter and zero lower bound environments.

5.3 Monetary Policy Rule Experiments

The presence of nominal rigidities in both prices and wages means that a central bank fully stabilising price inflation is no longer the optimal policy response. (Gali, 2008, Ch. 6) The central bank faces a "tradeoff between stabilising the output gap, the price inflation rate, and the wage inflation rate: it is impossible for more than one of the three variables g_t[output gap], π_t [price inflation] and ω_t [wage inflation] to have zero inflation."(Erceg et al., 2000) In fact both Erceg et al. (2000) and (Gali, 2008, Ch. 6) demonstrate that in the presence of both sticky prices and sticky wages there exists no policy rule that allows the central bank to simultaneously achieve the flexible price equilibrium, i.e. no policy rule exists that can fully remove the impact of both nominal rigidities.

The intuition behind this result is best explained by Erceg et al. (2000), I follow that exposition closely in what follows. The flexible price equilibrium may be achieved in an economy with both sticky prices and sticky wages via policy that results in the nominal rigidities not distorting the equilibrium. Let us consider a central bank that can achieve the flexible price equilibrium, i.e. a policy which leads to the constraints due to nominal rigidities no longer binding. Such a policy must simultaneously set price and wage inflation to zero so that there is no need for adjustment. The absence of adjustment arises from the fact that both households and firms view current prices and wages as the desired ones, so that the constraints imposed by nominal rigidities do not bind. In this case the flexible price equilibrium is achieved even though nominal rigidities are present. This implies a constant real wage from the real wage identity,

$$\frac{\widetilde{w}_t}{\widetilde{w_{t-1}}} = \frac{\pi_t^w}{\pi_t^p}, \tag{5.3.1}$$

where $\widetilde{w}_t = \frac{w_t}{P_t}$ is the real wage. But the real wage is set by the intersection of the labour supply,

$$\frac{w_t}{P_t} = -\frac{\varepsilon^w}{\varepsilon^w - 1} \frac{u_{n,t}}{u_{c,t} - \beta\nu E_t \left[u_{c,t+1}\right]}, \tag{5.3.2}$$

and labour demand,

$$\frac{w_t}{P_t} = \frac{\varepsilon^w - 1}{\varepsilon^w} z_t f_{n,t}. \tag{5.3.3}$$

Under this the policy that leads to no adjustment forces a contradiction, i.e. the real wage is simultaneously fixed and variable, and so no such policy can exist. Thus in the presence of nominal rigidities in both prices and wages the central bank cannot achieve the flexible price equilibrium. The fundamental idea underlying this result is that the central bank is attempting to achieve two objectives, price and wage stability[2], via a single instrument, the nominal interest rate. Such a trade off does not arise in the presence of a single nominal rigidity as the central bank is in an environment where it is using a single instrument to achieve a single objective.

The inability of the central bank to achieve the first best equilibrium allocation means that there are real consequences to the rule that the central bank chooses to follow. This is because the central bank must now choose between different second best equilibria resulting from the rule that it chooses to implement. That an optimal rule does not exist in this model allows one to study the differential impact of different policy rules on both business cycle results and, more crucially for this thesis, asset pricing results.

5.3.1 Taylor Rules

I use non-linear Taylor rules to characterise the way in which central banks make policy decisions in response to the information available to it. The simple non-linear

[2]There are two objectives since stabilising both wages and prices leads to stability of the output gap via the Philips Curve relationships.

Taylor rule used in this chapter takes the form,

$$\frac{\mathcal{R}_t^n}{\overline{\mathcal{R}^n}} = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\rho_\pi} \left(\frac{y_t}{\overline{y}}\right)^{\rho_y} \eta_t, \tag{5.3.4}$$

where R_t^n is the gross nominal interest rate (the policy instrument), π_t is the inflation measure used by the central bank, y_t is output, and η_t is an exogenous monetary policy shock. Variables in steady state are denoted with bars above them. The ratio $\frac{\pi_t}{\overline{\pi}}$ measures the deviation of inflation from its steady state value, I assume $\overline{\pi_t} = 1$. The ratio $\frac{y_t}{\overline{y}}$ is the deviation of output from its steady state, it may be seen as a measurable proxy for the output gap used by the central bank since it cannot measure the latent natural rate of output. The two parameters ρ_π and ρ_y measure the degree to which the central bank responds to inflation and output deviations respectively. I follow the 'Taylor principle' in assuming that the central bank response to inflation deviation is more than one-for-one and that it is more important than deviations in output.

The Taylor rule is subject to uncertainty via the nominal interest rate shock η_t which follows an autoregressive process similar to the aggregate technology shocks,

$$\ln \eta_{t+1} = \iota \ln \eta_t + \upsilon_t^{\mathcal{R}^n} \tag{5.3.5}$$

where ι is the degree of persistence of the shock and $\upsilon_t^{\mathcal{R}^n}$ is a random shock.

The measure of inflation used by the central bank may be the price inflation rate, the wage inflation rate or some combination of the two. A significant concern is that with two nominal rigidities the inflation measure targeted by the central bank may have consequences for equilibrium. This may be easily studied by using the following measure for inflation,

$$\pi_t = (\pi_t^p)^{\gamma_\pi} (\pi_t^w)^{\gamma_\pi}, \tag{5.3.6}$$

where π_t^p is price inflation, π_t^w is wage inflation and $\gamma_\pi \in [0,1]$ measures the relative importance of each type of inflation in the measure. I find that there is negligible quantitative difference and no impact on the dynamics as γ_π varies in its range so that the form of inflation targeted does not matter. As a consequence I assume that the central bank does not use a hybrid inflation measure and focuses only on price inflation, i.e. $\pi_t = \pi_t^p$.

A related concern arising from the presence of two nominal rigidities is what the central bank should target. There are three possible regimes - 1) price inflation-output gap, 2) wage inflation-output gap, and 3) price inflation-wage inflation. Given that I find the type of inflation measure does not matter, this leaves two possible regimes - 1) and 3). Erceg et al. (2000) show that the difference between these two regimes is small from a welfare perspective and the first regime appears to be the regime of choice both in the literature and the real world.[3] I therefore focus on the impact the first regime has on equilibrium dynamics and quantitative results. There are also regimes that ignore the output gap that I do not pursue in this analysis. It is widely accepted that central bank policy responds to variability in both inflation and output, thus to ignore it would be patently counter-factual.

5.3.2 Taylor Rules, Interest Rate Smoothing and Commitment

The simple Taylor rules assume that the central bank faces no constraints in setting the nominal rate, i.e. the central bank may set the nominal rate immediately to that level the simple rule prescribes. As an alternative to the simple rule above I consider the impact of rules with some degree of interest rate smoothing, or policy inertia.

[3]In the proposed set-up the study of a regime that targets only price and wage inflation is easy to implement. I leave it here as an avenue for future work as it is only of passing theoretical interest.

These rules take specific form,

$$\frac{\mathcal{R}_t^n}{\overline{\mathcal{R}^n}} = \left[\frac{R_{t-1}^n}{\overline{R^n}}\right]^{\gamma_R} \left[(\pi_t^p)^{\rho_\pi} \left(\frac{y_t}{\overline{y}}\right)^{\rho_y}\right]^{1-\gamma_R} \eta_t \tag{5.3.7}$$

where γ_R controls the degree of interest rate smoothing. The policy experiments run on these rules look not just at comparisons with the simple rules but also how the results change as a function of the degree of smoothing.

Numerous empirical studies highlight that there is a degree of inertia in the setting of nominal rates by central banks.[4] Woodford (1999) contends that policy inertia is, in fact, essential for a central bank that faces agent behaviour that is forward-looking. Inertia allows central banks to anchor agent expectations, to some degree, of the path of future policy actions and thus allows policy to have more than just a passing, contemporaneous effect on the economy.

In following a rule with a degree of interest rate smoothing the central bank is committing to changing the nominal interest rate by less than the optimal amount. Thus if it increases interest rates it is committing to keeping rates at a higher level for a prolonged period once raised. This allows it to anchor agent expectations about future interest rates. Indeed, following Goodfriend (1990), this ability to use the expectations channel to anchor future rates allows the central bank to affect longer rates via the short nominal rate. By affecting such long rates the central bank can achieve stabilisation on a much larger scale. The simple rules do not allow the central bank to influence future rates as any rate change carries no information about the future while interest rate smoothing rules do contain such information.

[4] e.g. see Goodfriend (1990), Rotemberg and Woodford (1998), Rudebusch and Svensson (1999), Woodford (1999), and Gertler et al. (1999)

Woodford (1999) shows that interest rate smoothing acts as the commitment device that lends credibility to central bank pronouncements. It prevents the central bank from manipulating agent expectations and then failing to carry though on its promises. This is because the credibility achieved by commitment, "which ultimately frees the central bank to act more effectively, can be maintained only if in the short run the bank regards itself as constrained to fulfil previous (implicit or explicit) commitments."(Woodford, 1999) This requires the central bank behaviour to depend, at least partially, on past actions because such actions have directly led to agent expectations that bring about a particular equilibrium at present. This means that there exists a value for the smoothing parameter in the Taylor Rule that would bring about the commitment equilibrium.[5]

Interest rate smoothing Taylor rules may be viewed as monetary policy under commitment. The commitment policy is characterised by a smaller deviations in interest rates due to a credible commitment by the central bank to a specific program of interest rates. In this case the presence of interest rate smoothing allows the central bank to achieve stabilisation goals with a smaller initial change because it can commit credibly to a future program of interest rates. While it cannot proffer a specific path it can guarantee that future interest rates will reside in a smaller range, as shown in the figure below. Thus by following an interest rate smoothing regime the central bank has a second channel - the expectations channel - via which it can influence the economy. This channel exists in addition to the direct channel the central bank has available to it in regimes without interest rate smoothing. Indeed, to use a term currently in vogue at central banks, one may consider the smoothing regime as providing a degree of implicit "forward guidance".

[5]The commitment solution is not calculated for the non-linear model. Rather I proceed on a positive analysis of how the smoothing parameter affects the economy with the implicit understanding that there exists a parameter that yields the commitment solution.

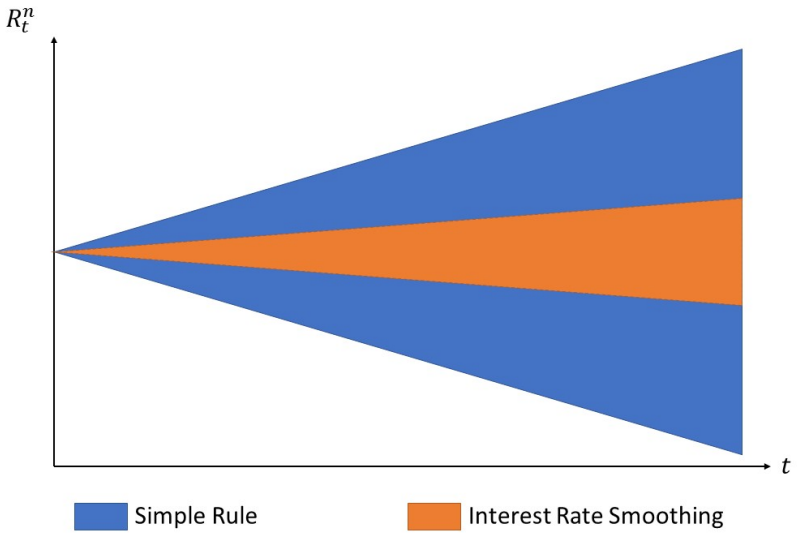


Figure 5.1: Simple vs Interest-Smoothing Taylor Rules

Figure: 5.1 illustrates how interest rate smoothing Taylor rules may be viewed as monetary policy under commitment. The commitment policy is characterised by a smaller possible range of future interest rates as the central bank is committing itself to only slowly changing interest rates from past values. In contrast, under the simple rule the central bank can change rates in a much wider range.

5.3.3 Smoothing or Autocorrelated Shocks?

Both simple Taylor rule (5.3.4) and the Taylor rule with smoothing (5.3.7) are subject to the same autocorrelated shock process. This approach differs from much of the literature where one often has simple Taylor rules being subject to autocorrelated shocks, while Taylor rules with smoothing are subject to i.i.d shocks. The idea being that both smoothing and autocorrelated shocks capture the same feature of policy inertia. However, this approach is not followed in this chapter as I am interested in studying the behaviour across different policy regimes. Keeping the shock processes

identical ensures that the central bank is operating in the same economic environment. Thus keeping the policy shock process the same one can isolate the impact of moving from a no smoothing regime to a smoothing regime. Smets and Wouters (2007), Carrillo et al. (2007) and Challe and Giannitsarou (2014) all develop models with both smoothing and autocorrelated shocks.

Rudebusch (2002), Gerlach-Kristen (2004) and more recently Coibion and Gorodnichenko (2012) find evidence that both policy inertia and autocorrelated shocks matter empirically. Indeed all of these papers, beginning with Rudebusch (2002), find that the empirical results are not robust in tests that attempt to differentiate between policy smoothing and autocorrelated shocks as the driver behind observed policy inertia. These papers conclude that both smoothing and smoothing are playing a role in observed policy inertia.

Rudebusch (2002) and Rudebusch (2006) argue that observed policy inertia may be the result of omitted variables in the central bank's reaction function. If one is to use the stylised Taylor rules, as in this thesis, then one is necessarily omitting additional factors that the central bank may be responding to.[6] If these factors happen to be persistent then they will induce persistence in the shock process. In so far as the shock process may be interpreted to capture unmodelled components of the central bank's reaction function then such persistence in the process is justified. Finally, Coibion and Gorodnichenko (2012) state that "allowing for persistent shocks significantly lowers the degree of monetary policy inertia." This informs the range of the smoothing parameter in the reported results, with analysis being carried out for smoothing parameters in the range $\gamma_R \in [0, 0.64]$.

[6]e.g. Coibion and Gorodnichenko (2012) suggest some of these factors include asset prices, liquidity conditions and general market conditions.

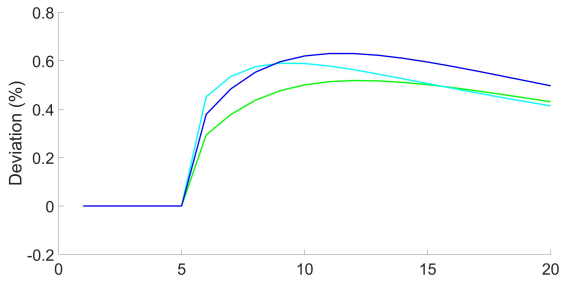
5.4 Business Cycle Results

The business cycle results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique. The "No Smoothing" model in the quantitative results section refers to the Sticky Price New Keynesian model studied in the previous chapter, i.e. the monetary policy rule is a simple Taylor Rule.

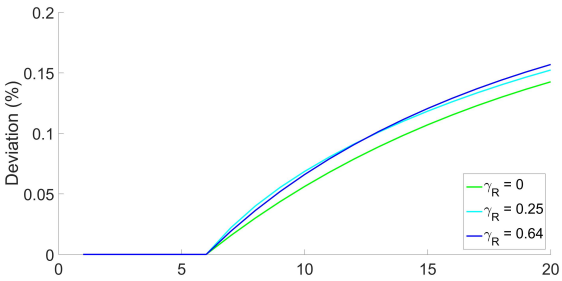
5.4.1 Business Cycle Dynamics

The presence of interest rate smoothing by the central bank does not fundamentally change the dynamics of the economy to technology or monetary policy shocks. The vast majority of the impulse responses continue to have the same sign and shape despite the shock; except for the real wage, wage inflation and price inflation in response to a nominal interest rate shock. While the presence of interest rate smoothing does not change the direction of most of the responses it does have an impact on their magnitude. The pattern appears to be that once smoothing is introduced there is a large shift away from the no smoothing impulse, with maximum reached somewhere in the region of $\gamma_R = 0.25$, and then a gradual move back toward the no smoothing impulse.

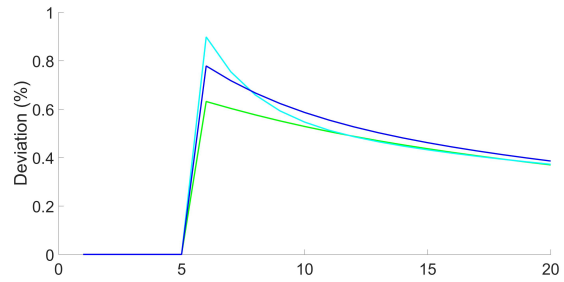
Interest rate smoothing introduces inertia into the nominal interest rate. This inertia helps to smooth income so that the contemporaneous variables - consumption, labour and investment - do not have to react as much to achieve the household aim of smoothing consumption. As in previous chapters the response of most variables can be traced back to the response of labour. The response of labour is less negative under the interest rate smoothing regime than under the regime with no smoothing. This leads directly to the magnitude of responses seen for output, consumption and



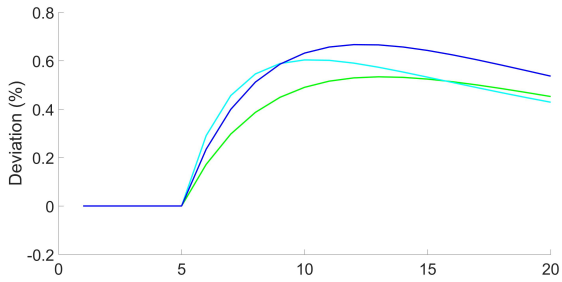
(a) Output



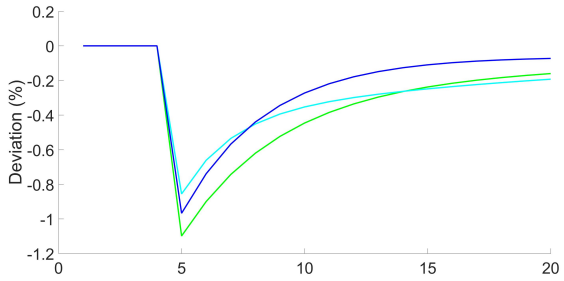
(b) Capital



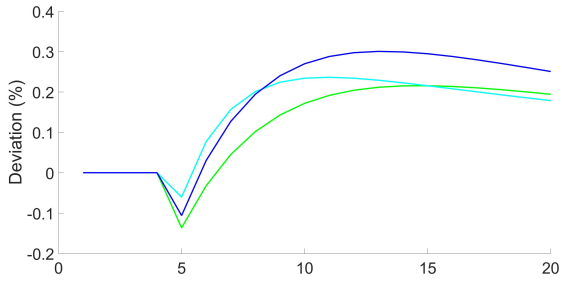
(c) Investment



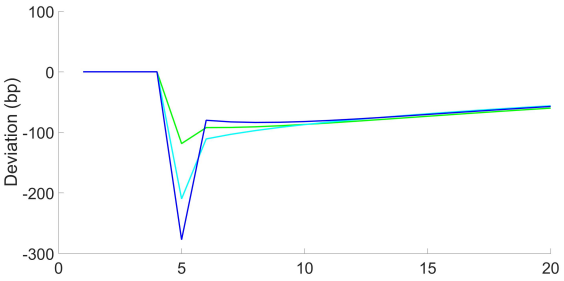
(d) Consumption



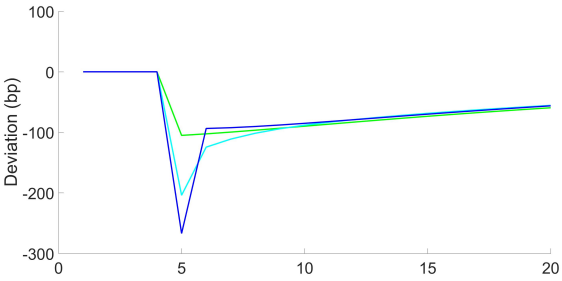
(e) Labour



(f) Real Wage

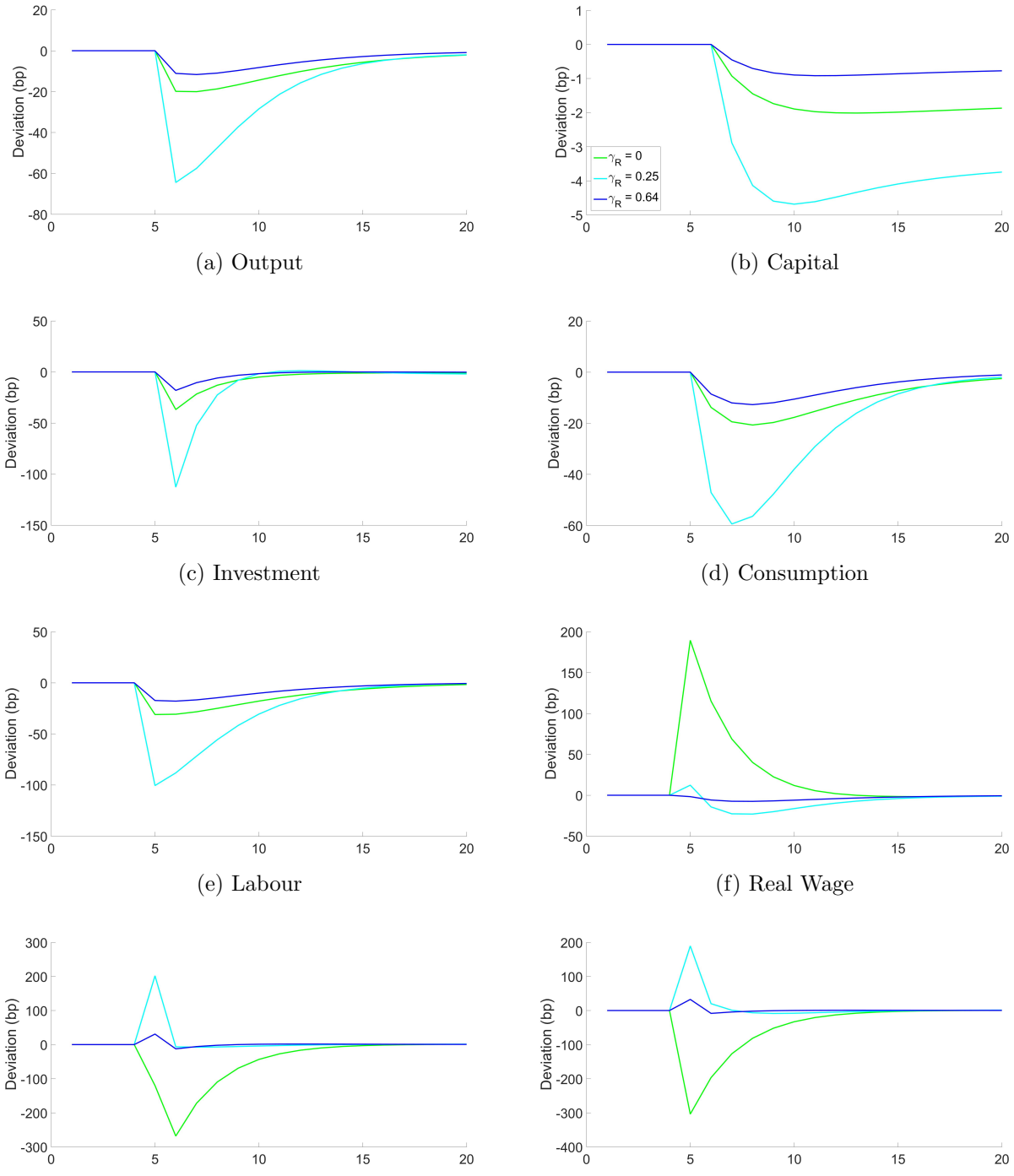


(g) Wage Inflation (Basis Point)



(h) Price Inflation (Basis Point)

Figure 5.1: Impulse Responses - Technology Shock



(a) Output (b) Capital

(c) Investment (d) Consumption

(e) Labour (f) Real Wage

(g) Wage Inflation (h) Price Inflation

Figure 5.2: Impulse Responses - Nominal Interest Rate Shock

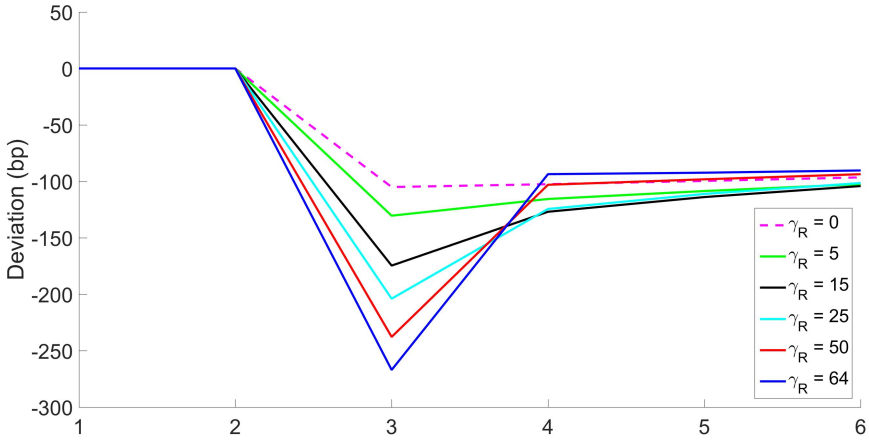
investment. As agents work more (less negative response) there is an increase in consumption and investment resulting from higher income relative to the no smoothing regime.

The response of output can be decomposed into two components - a direct response to the shock and an indirect response driven by labour. Completely differentiating output with respect to the technology shock leads to,

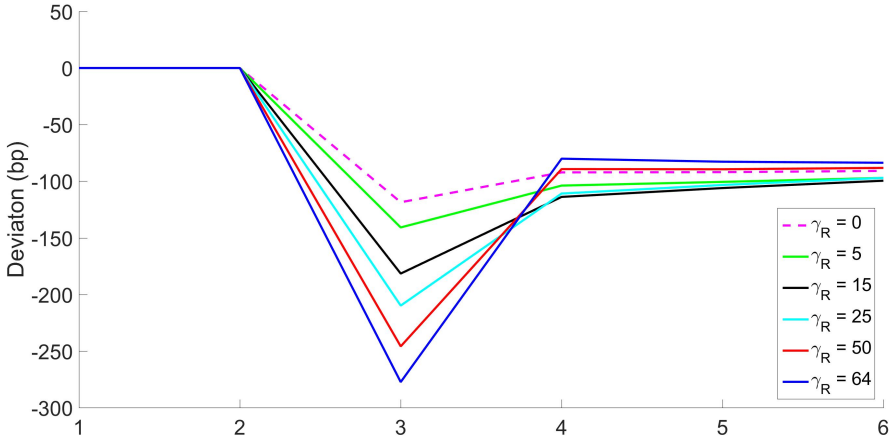
$$\begin{aligned}\frac{dy_t}{dz_t} =& f\left(k_t, n_t\right) + z_t f_{n,t}\frac{dn_t}{dz_t} + z_t f_{k,t}\frac{dk_t}{dz_t} \\ =& f\left(k_t, n_t\right) + z_t f_{n,t}\frac{dn_t}{dz_t},\end{aligned}$$

where $f(\cdot)$ is the production function and $f_{n,t}$ is the marginal product of labour. The second equality comes from the fact that capital does not respond contemporaneously to shocks. The labour impulse response shows that $\frac{dn_t}{dz_t} < 0$ so that the impact of labour mutes the response of output. This leads to the observed response of output as the smoothing parameter is varied.

Having related the response of all other variables to the response of labour and the real wage one can see that the response of these two variables as the degree of smoothing changes drives the dynamic response of the economy. In particular it is these variables that drive the observed pattern of a large shift away from the no smoothing impulse and gradual movement back to the no smoothing impulse. The analysis here is identical to that presented in the previous chapter and the reader is referred to that chapter for a more detailed exposition. As shown in the previous chapter the response of labour and the real wage can be understood by the response of wage and price inflation, and how these interact to move the labour supply and labour demand curves. The response of price and wage inflation as the smoothing parameter



(a) Price Inflation



(b) Wage Inflation

Figure 5.3: Price and Wage Impulse Responses as γ_R Changes

changes is given in Figure: 5.3. There is consistency in these responses, i.e. there is no reversal in the response as the smoothing parameter changes. The response of labour can be explained by the relative strength of the shift or the rotation ("twist") of the relevant curves. The shift of the curves is governed by the contemporaneous response of the respective inflation measures while the inflation measure in the subsequent period governs the "twist" (or rotation) of the curves.[7] In particular one notices that the movement from $\gamma_R = 0$ to $\gamma_R = 0.25$ accounts for the vast majority

[7]A detailed description of this analysis is provided in the previous chapter

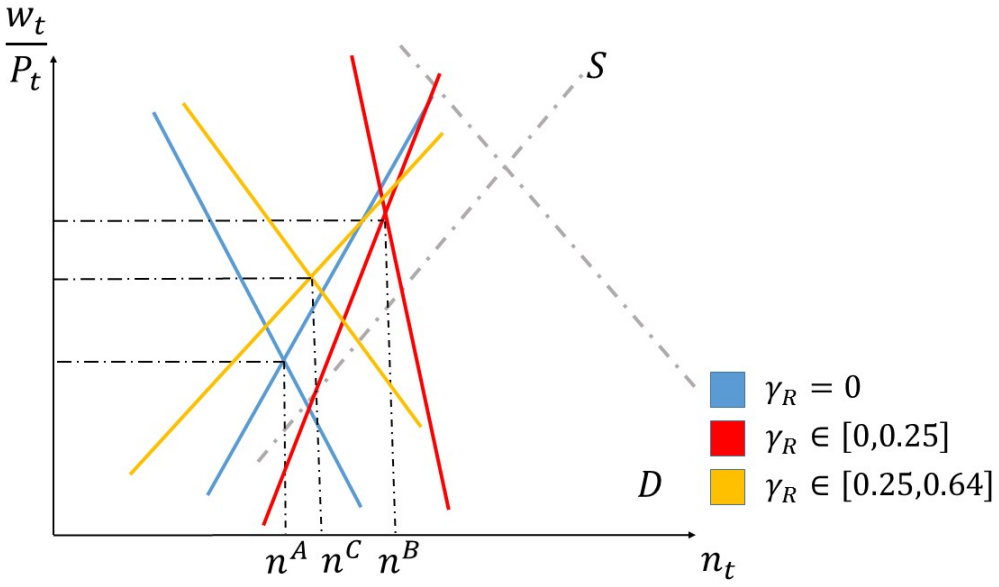


Figure 5.4: Labour Market as γ_R Changes for a Technology Shock

of the response in labour. There is shift of roughly $200bp$ in each inflation measure over this range while over the larger range $\gamma_R \in (0.25, 0.64]$ there is a total movement of less than $100bp$. The converse is true for the inflation measure in the subsequent period, i.e. there is a large response and overtaking of the $\gamma_R = 0$ response in the range $\gamma_R \in (0.25, 0.64]$ while $\gamma_R \in (0, 0.25]$ sees a much more muted response that remains below that for $\gamma_R = 0$. Thus there is greater shift in the curves coupled with a greater amount of steepening in the range $\gamma_R \in (0, 0.25]$ and there are smaller shifts coupled with greater flattening in the range $\gamma_R \in (0.25, 0.64]$, see Figure 5.4. The dashed dotted lines depict pre-shock equilibrium, while the blue lines depict the no smoothing response, the red lines are representative of the smoothing response for $\gamma_R \in (0, 0.25]$, and the yellow lines are representative of the smoothing response for $\gamma_R \in (0.25, 0.64]$. One sees that whether the shift or the rotation dominates leads to different outcomes and these are in line with those seen in the labour and wage responses. This analysis also explains the presence of this phenomenon in the impulse responses for the monetary policy shock. A quantitative decomposition of the income

Table 5.1: Business Cycle Results

	Data	No Smoothing	Smoothing				
			$\gamma_R = 0.05$	$\gamma_R = 0.15$	$\gamma_R = 0.25$	$\gamma_R = 0.5$	$\gamma_R = 0.64$
Standard Deviation (%)							
Output	1.72	0.42	0.42	0.42	0.41	0.37	0.37
Standard Deviation Relative to Output							
Consumption	0.79	0.96	0.97	0.97	0.98	1.04	1.15
Hours Worked	0.63	2.32	2.30	2.40	2.56	3.47	4.19
Investment	3.24	1.41	1.40	1.39	1.38	1.39	1.32
Real Wage	0.47	0.36	0.37	0.37	0.39	0.57	0.87
Contemporaneous Correlation with Output							
Consumption	0.87	0.96	0.96	0.96	0.96	0.95	0.94
Hours Worked	0.93	-0.63	-0.61	-0.57	-0.50	-0.13	0.35
Investment	0.86	0.85	0.85	0.85	0.84	0.75	0.58
Real Wage	0.47	0.81	0.81	0.79	0.75	0.60	0.67
Euler Equation Error		$-7.9882\text{x}10^{-6}$	$-8.0022\text{x}10^{-6}$	$-7.9248\text{x}10^{-6}$	$-7.8607\text{x}10^{-6}$	$-7.8400\text{x}10^{-6}$	$-8.6002\text{x}10^{-6}$

and substitution effects using the decomposition of King (1991) is presented in the Appendix to this chapter. It bears out the same conclusions as the analysis above. Thus having analysed the mechanism by which labour and the real wage respond to shocks one can easily understand the remainder of the impulse responses.

Finally the impulse responses to a nominal interest rate shock exhibit a reversal of the response of inflation, of either type, once interest rate smoothing regimes are followed. This can be understood as a consequence of greater amount of commitment by the central bank. As the interest rate is smoothed any response to a shock is less within the control of the central bank and as such one sees inflation rise. This rise, and the subsequent fall, is of smaller magnitude than the no smoothing case. This is due to the ability of the central bank to credibly commit to keeping interest rates somewhat steady.

5.4.2 Quantitative Business Cycle Results

The quantitative results show that the monetary policy regime has small impact on the volatility of output. As agents face smoother interest rates there is a reduction of output volatility. This makes intuitive sense as agents expect any given interest rate in the immediate future to remain near the current interest rate thereby leading to less uncertainty in their decisions. This in turn leads to smaller volatility of output. The increased volatility of hours is due to labour becoming procyclical as the degree of interest rate smoothing is strengthened. This leads to more volatile labour, relative to output, when combined with the falling volatility of output.

The quantitative results for consumption and investment may be understood by the interplay between these variables in the aggregate resource constraint. The aggregate resource constraint requires that $c_t + I_t = y_t$, so that the variance of output may be decomposed as,

$$\sigma_y^2 = \sigma_c^2 + \sigma_I^2 + 2\text{cov}\left(c, I\right), \tag{5.4.1}$$

which may be rearranged to yield,

$$\frac{\sigma_c^2}{\sigma_y^2} = 1 - \frac{\sigma_I^2}{\sigma_y^2} - \frac{2\text{cov}\left(c, I\right)}{\sigma_y^2}. \tag{5.4.2}$$

The volatility of investment does not change relative to output as interest rates are smoothed. Holding output constant the covariance between consumption and investment is negative as an increase in one must necessarily lead to a decrease in the other. Combined with the fact that output volatility is falling this leads to the observed quantitative results for output and investment.

5.5 Asset Pricing Results

The asset pricing results, both impulse responses and quantitative results, have been obtained from simulations obtained using the Parameterised Expectations Algorithm as the solution technique. The "No Smoothing" model in the quantitative results section refers to the Sticky Price New Keynesian model studied in the previous chapter, i.e. the monetary policy rule is a simple Taylor Rule.

As in the previous chapter I continue to define the risk free rate as,

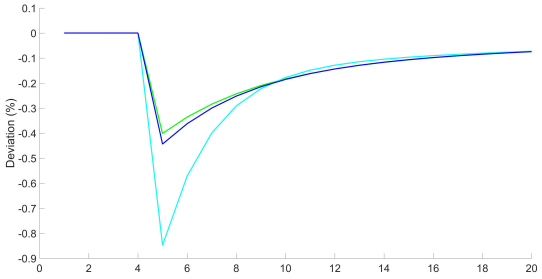
$$R_t^f = \frac{1}{E_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\right]}.$$

This also facilitates the calculation of a real equity premium as the equity return is in real terms.

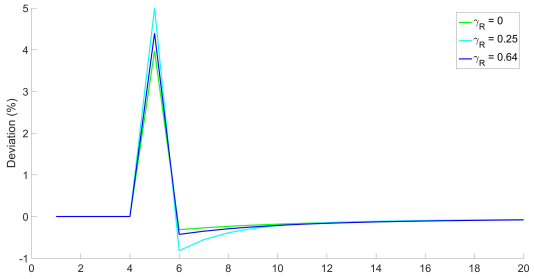
5.5.1 Asset Pricing Dynamics

The degree of interest rate smoothing has little impact on the dynamics of asset prices in response to a technology shock. One continues to see the pattern observed in the business cycle impulse response to a technology shock, namely a move away from the no smoothing case and then move back as the smoothing parameter is strengthened. This pattern continues to be driven by the labour market as explained in the previous section. The dynamics of the equity return, as seen in previous chapters, is completely determined by the dynamics of the capital gain and continues to be so as the degree of smoothing parameter varies.

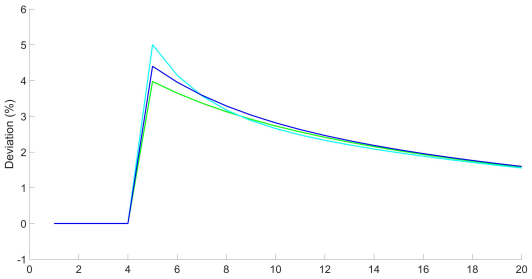
The choice of policy regime and the degree of interest rate smoothing does have some apparently interesting consequences for the dynamic response of asset prices to



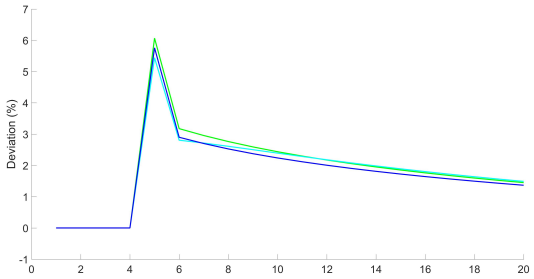
(a) Risk Free Return (bp)



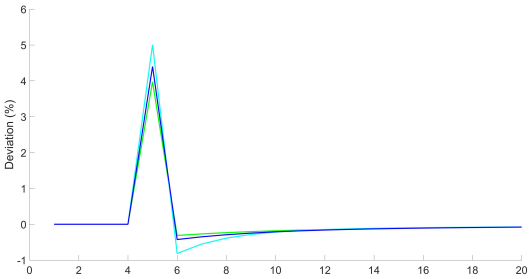
(b) Ex-Post Return on Equity



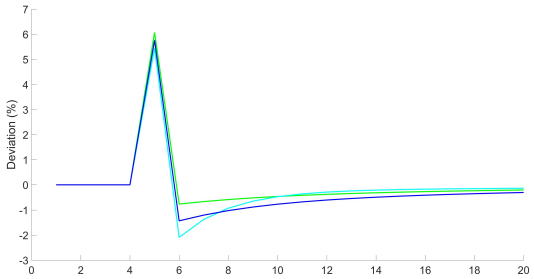
(c) v_t



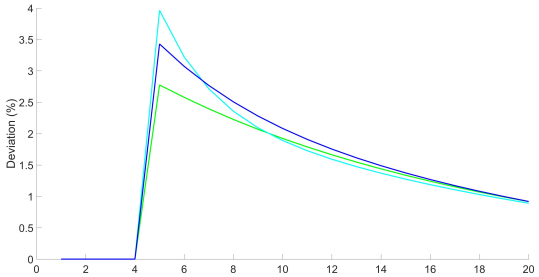
(d) d_{t+1}



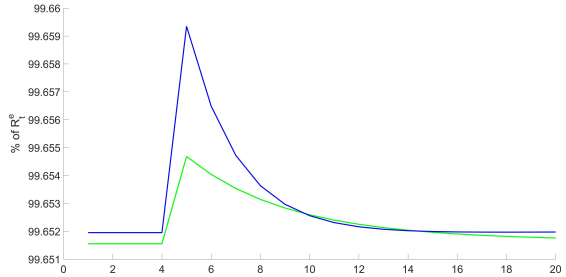
(e) Capital Gain: $\frac{v_{t+1}}{v_t}$



(f) $\frac{d_{t+1}}{v_t}$

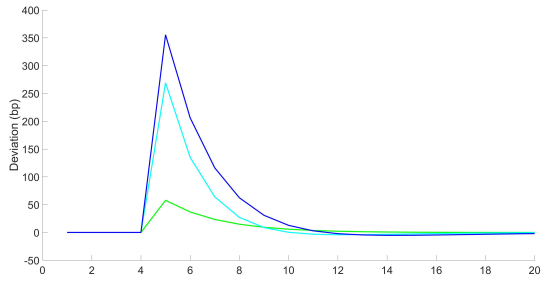


(g) Tobin's Q: $\frac{1}{\phi'(g_t)}$

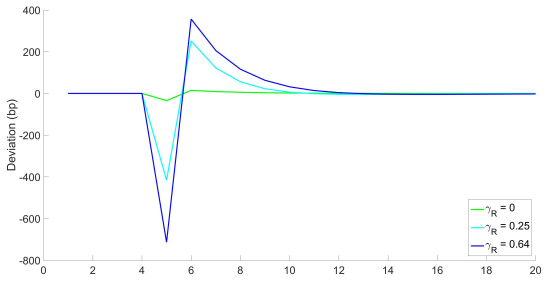


(h) Capital Gain Proportion of Ex-Post Return

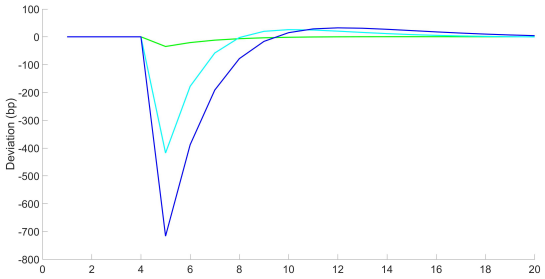
Figure 5.1: Asset Pricing Impulse Responses - Technology Shock



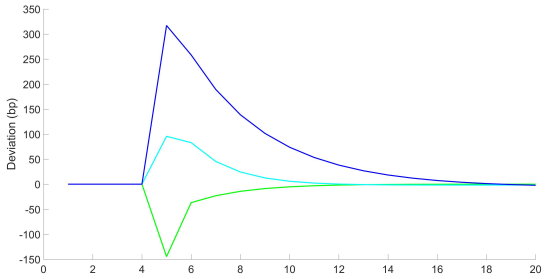
(a) Risk Free Return (bp)



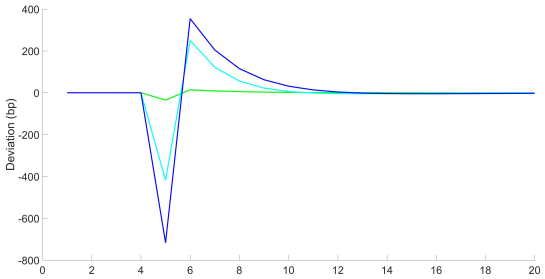
(b) Ex-Post Return on Equity



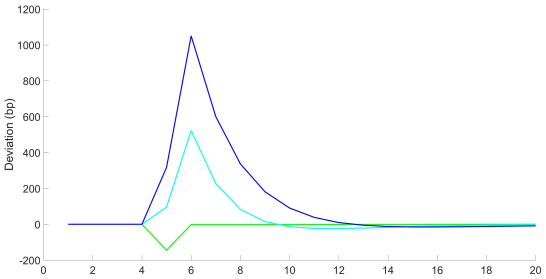
(c) v_t



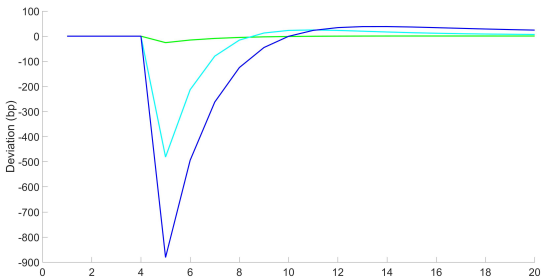
(d) d_{t+1}



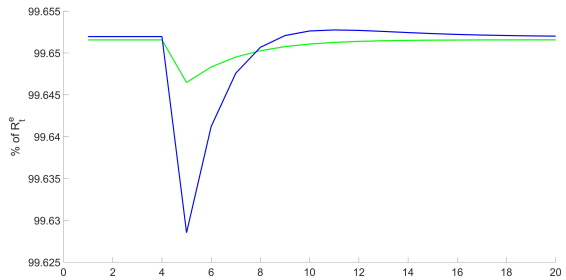
(e) Capital Gain: $\frac{v_{t+1}}{v_t}$



(f) $\frac{d_{t+1}}{v_t}$



(g) Tobin's Q: $\frac{1}{\phi'(g_t)}$



(h) Capital Gain Proportion of Ex-Post Return

Figure 5.2: Asset Pricing Impulse Responses - Nominal Interest Rate Shock

Table 5.1: Asset Pricing Results

	Data	No Smoothing	Smoothing				
			$\gamma_R = 0.05$	$\gamma_R = 0.15$	$\gamma_R = 0.25$	$\gamma_R = 0.5$	$\gamma_R = 0.64$
Mean (% p.a)							
Risk Free	1.49	1.36	1.37	1.38	1.40	1.46	1.52
Equity	7.87	2.07	2.04	1.98	1.92	1.72	1.62
Equity Premium	6.52	0.70	0.67	0.60	0.51	0.26	0.10
Standard Deviations (% p.a)							
Risk Free	2.62	0.89	0.86	0.84	0.87	1.24	1.68
Equity	16.72	5.39	5.26	4.96	4.62	3.47	2.64
Equity Premium	15.69	5.31	5.19	4.89	4.55	3.26	2.06

a monetary policy shock. The dynamics of the equity return continue to be driven by the capital gain channel but there is a change in sign of the dividend channel which deserves some mention despite its minor role in dynamics of the equity return. In the previous chapter it was shown that the expected dividend may be decomposed as,

$$d_{t+1} = \frac{\Xi_{t+1}}{P_{t+1}} - I_{t+1}, \tag{5.5.1}$$

where $\frac{\Xi_{t+1}}{P_{t+1}}$ is the real profit and I_{t+1} is investment. As shown in the previous chapter, under the no smoothing regime, the fall in dividend is due to the large increase in the wage bill dominating the fall in investment. Under an interest rate smoothing regime, however, it has been shown that the labour market behaves somewhat differently. In particular the wage bill only rises marginally under interest rate smoothing so that the falling investment can mildly dominate the fall in real profit. This leads to the observed change in sign of dividend dynamics.

5.5.2 Quantitative Asset Pricing Results

The quantitative results show that the policy regime chosen by the central bank does not have a significant impact on mean asset returns. Over the range of γ_R studied one sees a rise in the mean risk free rate of $16bp$ and a fall in the equity return of

45bp, both of which are relatively small. This (very) gradual move together of the mean rates explains the fall in the equity premium.

The asset pricing statistic of more interest are the standard deviations. Here one sees a marked difference as the policy regime changes to one with interest rate smoothing and again as the degree of smoothing increases. The risk free rate becomes more volatile - the standard deviation rises by 79bp - while the equity return becomes significantly less volatile - the standard deviation falls by 275bp.

The nominal interest rate is given by,

$$R_t^n = \frac{1}{E_t\left[m_{t,t+1}\frac{1}{\pi_{t+1}^p}\right]}, \tag{5.5.2}$$

which is the non-linear Fisher relationship in this economy. The log-linear approximation of this relationship sheds light on the observed trend in the volatility of the risk free rate. The first order log-linear approximation of the non-linear Fisher equation gives the standard linear Fisher equation,

$$\widehat{R_t^n} = \widehat{R_t^f} + \widehat{\pi_{t+1}^p}, \tag{5.5.3}$$

where the hats denote log-deviations of the variable concerned. Rearranging (5.5.3) for the risk free rate and taking the variance gives,

$$\mathrm{Var}\left(\widehat{R_t^f}\right) = \mathrm{Var}\left(\widehat{R_t^n}\right) + \mathrm{Var}\left(\widehat{\pi_{t+1}^p}\right) - 2\mathrm{Cov}\left(\widehat{R_t^n}, \widehat{\pi_{t+1}^p}\right). \tag{5.5.4}$$

Under the smoothing regime there is an increase in the volatility of the risk free rate despite a smoothing of the nominal rate (i.e. lower volatility) and smoother resulting inflation. This increase is due to a fall in the covariance between the nominal rate

and price inflation. The covariance falls as the smoothing of the interest rate acts as a wedge to weaken the strict relationship between the nominal rate and price inflation.

The precipitous fall in the volatility of the equity return as the smoothing parameter is increased is attributed to the presence of the expectations channel of monetary policy. Monetary policy, under smoothing, has two channels via which it interacts with the economy. The first is the direct channel that affects the economy via the contemporaneous movement of the interest rate. The second channel is an expectations channel that conveys information to agents about the future path of interest rates. In the previous chapter it was shown that the profit channel is the main determinant of equity prices in the New Keynesian model, it is through this profit channel of equity prices that the expectations channel of central banking policy under interest rate smoothing operates. The expectations channel is not present when the central bank does not smooth interest rates - any interest rate decision does not commit the central bank to a certain interest rate path in subsequent periods. However, the interest rate smoothing regime provides agents with information about the range of future interest rates as the central bank has committed to some degree of inertia in their policy response. This implicit "forward guidance" has been shown to lead to an overall smoother economy by providing greater certainty over the path of interest rates over a longer period. The greater certainty in future interest rate paths allows a greater certainty of future firm profits, hence dividends, and also future capital prices - all of which are central to the equity investment decision. As the degree of smoothing is increased there is, therefore, greater certainty in the real economy which flows on to greater certainty in the equity market and hence a fall in the volatility of equity investment.

The greater certainty surrounding the nominal rate, together with the expectations

channel of monetary policy, may also be used to explain the trend in the mean asset returns. Greater inertia in the nominal rate leads to household income that is less volatile so that agents have less of a precautionary saving desire, as evidenced by the low returns in the risk free asset under the no smoothing regime and subsequent rise with the degree of smoothing. The weakening of the precautionary saving motive promotes more risky investment as evidenced by the fall in the equity return. Thus the expectations channel of monetary policy not only affects the volatility of asset prices but also the means, but one sees that quantitatively the majority of the expectations impact of monetary policy is on asset volatility.

5.6 Conclusion

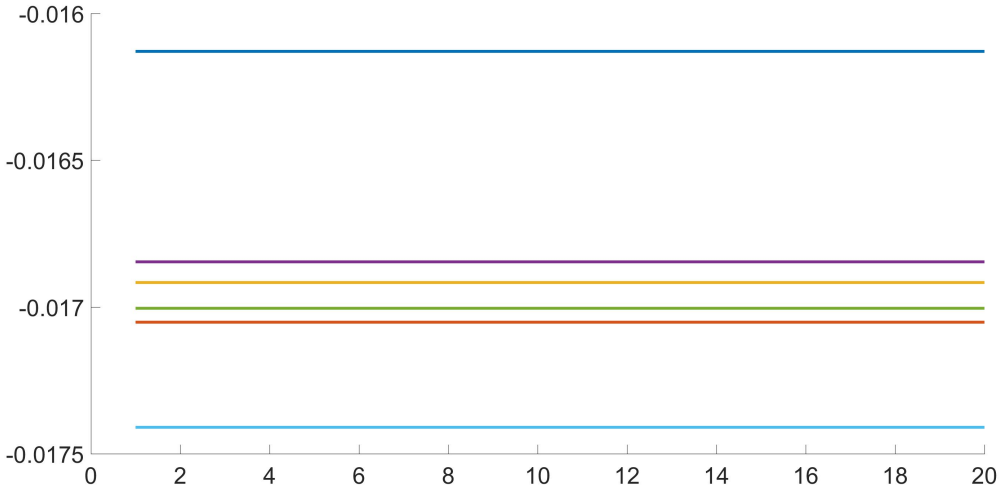
In this chapter I have studied how the choice of central banking policy regimes affect asset prices. I find that the choice of policy regime does not matter for mean asset returns but it does have significant consequences for the volatility of asset returns. This is attributed to the presence of an expectations channel in the smoothing regime that is not present in the simple regime. As a secondary result, I also find that the introduction of the smoothing regime has implications for the dynamic responses in the labour market which depend on the degree of interest rate smoothing.

The results in this chapter show that the choice of central bank policy does matter for asset pricing results, in particular for the volatility of asset returns. From a research perspective this means that one should consider seriously the form of the policy regime used to model the central bank within the New Keynesian framework when studying asset pricing. From a policy perspective, the results of this chapter are interesting insofar as they show that the central bank affects the asset markets not through contemporaneous policy action but rather via the information such pol-

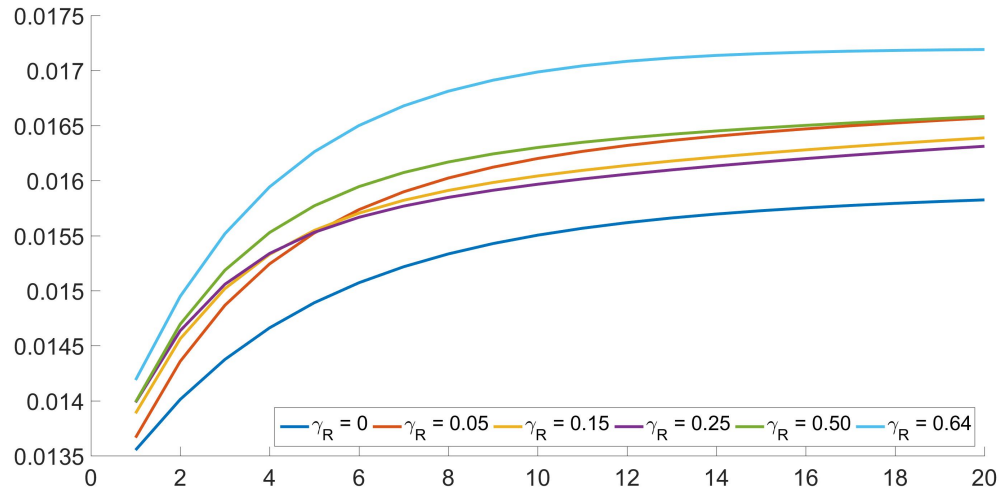
icy action conveys. This is not surprising as efficient markets should already have priced in expectations of the current policy move. Consequently central bank policy, through the expectations channel, does not significantly affect the mean return in the market but rather the volatility the returns. Thus the central bank can use policy to calm asset price volatility by some amount of forward guidance to inform agent expectations of the future. Such guidance, however, must be accompanied with some form of commitment mechanism like interest smoothing to provide a degree of credibility to anchor agent expectations. Thus smoothing regimes aid central banks to achieve their dual goals (post-crisis) of financial market stability and macroeconomic stability.

Appendix

5.A King (1991) Decomposition: Technology Shocks



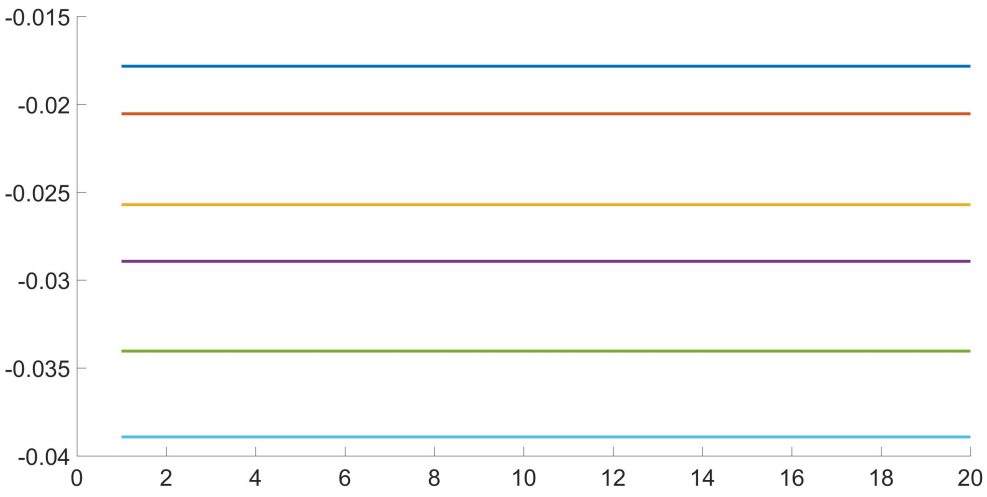
(a) Income Effect



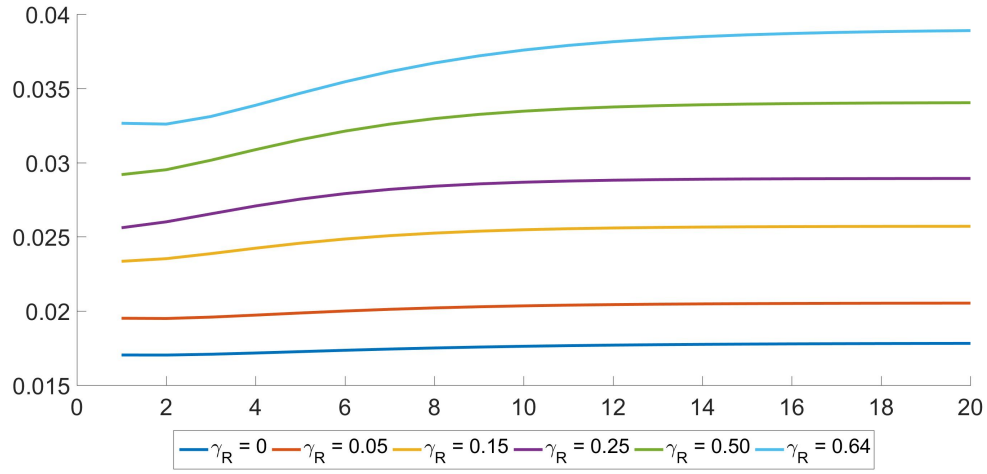
(b) Substitution Effect

Figure 5.A.1: King (1991) Decomposition: Technology Shocks

5.B King (1991) Decomposition: Monetary Policy Shock



(a) Income Effect



(b) Substitution Effect

Figure 5.B.1: King (1991) Decomposition: Monetary Policy Shock

CHAPTER 6

Conclusions

> *The theory that can absorb the greatest number of facts, and persist in doing so, generation after generation, through all changes of opinion and detail, is the one that must rule all observation.*
>
> — **Adam Smith**

I conclude this thesis in two parts. The first presents some general conclusions and concluding remarks on the thesis as a whole. The second part of this conclusion presents presents some extensions to Part III and some avenues for future research that extend the model in various ways to study the response of policy in other environments and also to allow the study of a broader family of macroeconomic policies.

6.1 Final Conclusions

This thesis was motivated by a desire to understand how central banking policy affects financial markets. This required an understanding of how asset pricing works in the current state-of-the-art theoretical macroeconomic model that permits the study of central banking policy - the New Keynesian model.

I see the contribution of this thesis to the literature as three-fold. First, it highlights the advantages of moving from log-linearisation to non-linear methods, and the advantages of a fully endogenous labour market. Second, I show that if one is willing to move beyond the log-linearisation strait jacket of the New Keynesian tradition then one can indeed delve deeper into the asset pricing mechanism than has hitherto been studied in the literature. Third, and finally, the model developed provides a theoretical laboratory in which to begin studying monetary policy, and with appropriate extensions broader macroeconomic policy, impact on asset prices.

The first contribution was achieved in Part I. In it I have dealt with the constraints imposed by tradition in both the New Keynesian and equilibrium asset pricing literatures. The New Keynesian tradition, and macroeconomics more generally, is to use log-linearisation to solve inherently non-linear systems. I have argued that this approach prevents New Keynesian models, and DSGE models more generally, from adequately explaining asset pricing results. This shortcoming, for it is a serious shortcoming conceded by the literature, severely limits our understanding of the pricing mechanism in these models and our ability to probe its workings. The equilibrium asset pricing literature, in attempts to provide better quantitative replication of asset pricing facts while limiting the deterioration of business cycle results, have resorted to frictions that lead to labour market misbehaviour. I showed that in the real models

used in this literature this leads to counter-factual labour market behaviour in the sense that such labour market behaviour cannot be reconciled with the structure of these models. As such there is misbehaviour in the labour market mechanism which this literature deals with by effectively making labour inelastic or imposing implausibly sophisticated frictions on the labour market. I showed that one can achieve superior joint replication results with an appropriate change in the preference structure of agents. Thus, Part I serves to dispel the constraints imposed by the literature which stand in the way of any successful attempt to pursue the study proposed in this thesis.

The thesis began with claims that one must use non-linear methods when studying asset prices and one cannot ignore the endogenous movement of labour. I demonstrated first that non-linear methods allow for an improvement in quantitative asset pricing results. Second I provided a diagnosis of the countercyclical movement of labour in the RBC models that aim to simultaneously study asset prices and business cycle fluctuations. In the presence of capital adjustment costs and internal habits in consumption labour falls in response to a positive technology shock. This was attributed to the strengthening of the income effect such that it dominates the substitution effect, leading to the observed dynamics of labour in this model. I demonstrated that when the countercyclical movement of labour is restricted via the introduction of GHH preferences there is a significant improvement in quantitative asset pricing results without much deterioration in the business cycle. This was due to households no longer being able to smooth consumption via the smoothing of income, which forced them to use the costly inter-temporal investment channel. However, this did not get us far enough in terms of quantitative replication of asset pricing results, nor did it leave one with a model that can be used to answer the question this thesis seeks to explore.

Part I shone a light on the dynamics of labour in the presence of frictions needed for the quantitative replication of asset prices. It demonstrated that endogenous labour provides households an alternative to the costly inter-temporal channel (investment) via an exploitation of the costless intra-temporal channel made possible by labour. This sheds further light on the response of the literature to this problem - introduce labour market frictions. I eschewed the introduction of frictions since these frictions make it more difficult to move to a New Keynesian framework in a seamless manner.

The study of labour in Part I is the necessary first step in this thesis since the labour market plays a central role in New Keynesian models - nominal rigidities in prices and wages are most evident in the labour market in these models. Thus understanding how labour behaves in the face of capital adjustment costs and internal habits in consumption in asset pricing models without the features necessary for New Keynesian models allows one to isolate what the introduction of New Keynesian features cause.

Having dealt with the constraints imposed by the literature, Part II provided both the second and third contributions of this thesis. Part II moved away from the RBC models of Part I to a New Keynesian framework. However this was not motivated by any shortcoming or feature of the model of Part I, though there are many. Rather the move to New Keynesian models was motivated by the more pragmatic aim that it is only in New Keynesian models that one may study how central banking policy affects asset prices; i.e. this is the appropriate model to study the question at hand. The model developed, through the analysis of Part II, and the example of Part III provides the foundations of a theoretical laboratory with which to begin the study of monetary policy on asset prices and the real economy within a single model. This is

the third and final contribution of this thesis.

In Part II, I derived the relationship providing the link between the financial economy, in this case the equity market, and the underlying fundamental economic variables in a simple New Keynesian model. This link has hitherto lacked explicit study in the literature despite operating in the background in New Keynesian models, and I contend it is due to the inability of the macroeconomic profession, until very recently, to embrace non-linear solution methods. I also showed that the use of non-linear methods provided deeper insights into the operation of the labour market. Thus, by embracing non-linear methods, one can probe deeper into not just the pricing mechanism but also the labour market. Using this relationship and a non-linear solution to the model I showed that the simple New Keynesian model continues to suffer from the quantitative asset pricing shortcomings of the models of Part I. This leads me to conclude that merely moving to the New Keynesian model is not sufficient to achieve a large improvement in quantitative asset pricing results, one will need more complex New Keynesian models than studied in this thesis.

The model of Part II contains nominal rigidities in both prices and wages. In the presence of both nominal rigidities it is well understood that the central bank cannot pursue an optimal policy that allows it to achieve the flexible price equilibrium. This has implications for the choice of policy as it results in a choice over different possible macroeconomic equilibria. Part III completed the analysis of this thesis by conducting a series of numerical experiments surrounding the choice of monetary policy regime and how this choice affects asset prices. It showed how the model developed throughout the thesis may be used to gain further insight into both the asset market and real economy. In particular it showed that, when using Taylor Rules, the central bank achieves its goal of macroeconomic stability with lower asset market volatility, i.e. it

can achieve some degree of asset market stability, when following a regime with some degree of inertia. This asset market stability was shown to be achieved with little impact on the mean returns, making it a superior outcome to that from achieved in a regime with no inertia.

So, how does monetary policy affect financial markets? This was the question posed in the introduction. This thesis has gone some of the way to answering this question, but it has only just scratched the surface. The next section provides a plethora of possible extensions and questions this work leaves unexplored and unanswered. The study carried out in this thesis provides the foundations from which one can answer the questions and explore the extensions briefly discussed below. In this sense, and in light of the many possible extensions provided below, this thesis has been an extended "proof of concept" for analysis to be carried out in future work. As Lao Tzu famously wrote, "[a] journey of a thousand miles begins with one step" - this thesis provides the first steps in the journey to better understand how macroeconomic policy affects financial markets.

6.2 Extensions and Avenues for Future Work

6.2.1 Extensions

In this section I briefly sketch two extensions that warrant further and deeper analysis; these are presented here and provide an idea of other questions specifically about simple policy rules that the model developed in this thesis can answer. These sketches shed additional light on the impact of monetary policy on asset pricing results from Part III. The first extension considers the impact uncertainty about the smoothing parameter, γ_R, in the Taylor Rule. While the second extension considers the impact Zero Lower Bound concerns. In the analysis that follows I focus solely on the quanti-

tative asset pricing results, i.e. I do not consider business cycle results or dynamics.[1] Both extensions are sketches of analysis that can easily be studied in the model developed in this thesis. Indeed there is significant scope for future work arising from these extensions.

Smoothing Parameter Uncertainty

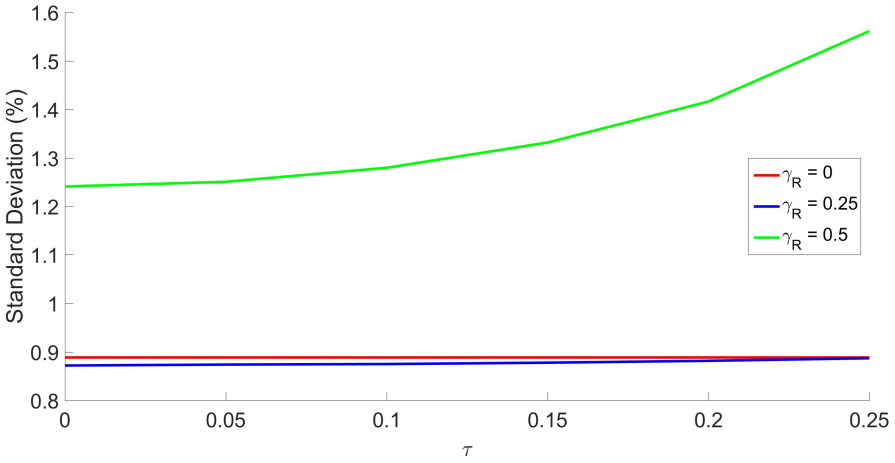
The quantitative asset pricing results of the previous section show that a central bank following an interest rate smoothing regime leads to lower equity return volatility. The equity return volatility falls as the degree of smoothing is increased. This strengthens the expectations channel of monetary policy by giving agents greater certainty of the range where future interest rates will reside. The extension here considers what happens if there is some uncertainty surrounding the smoothing parameter. It relaxes the assumption that agents know what the degree of smoothing will be a priori in order to understand whether it is merely the act of smoothing that leads to reduced volatility of the equity return or whether stability of the smoothing parameter also matters. That is, I ask if uncertainty surrounding γ_R has any economic impact.

I assume that the smoothing parameter follows a Normal Distribution,

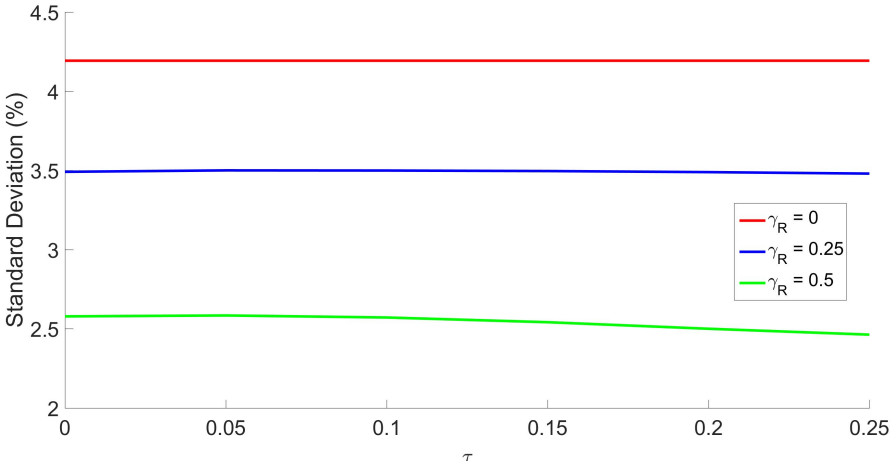
$$\gamma_R \sim N\left(\overline{\gamma_R}, \kappa_R \overline{\gamma_R}\right), \tag{6.2.1}$$

where $\overline{\gamma_R}$ is the mean smoothing parameter and $\kappa_R \in (0, 1)$. The parameter κ_R governs the variance of the smoothing parameter, which is assumed to be proportional to the mean for this extension. At the beginning of each period the central bank receives a random draw from this distribution for the smoothing parameter. The agents in the model know the distribution of the smoothing parameter. The figure below shows the

[1] The results suggest that both of these extensions do not fundamentally impact on the dynamics of the economy or quantitative business cycle results. Future work would focus also on the dynamic responses.



(a) Risk Free Rate Volatility



(b) Equity Return Volatility

Figure 6.1: Return Volatility as κ_R Changes

impact on return volatility as the parameter κ_R varies. The uncertainty surrounding γ_R has little impact on return volatility for low levels of κ_R. The volatility of the asset return averages the range of volatilities over which the smoothing parameter varies. As the volatility of the degree of smoothing increases there is a large increase in the risk free volatility rises somewhat significantly for $\gamma_R = 0.50$ as more of the 'extreme' smoothing values are chosen with higher probability. There is a small impact on the equity return volatility as the variation in γ_R changes over all possible values of γ_R.

The general stability of the volatilities as the variation in γ_R increases suggests that what matters is the average value announced by the central bank and not the uncertainty surrounding the actual value of γ_R in any given period. This is because the monetary policy shock leads to uncertainty surrounding the cause of the change in the nominal rate. It is impossible for the agents to disentangle the change in the nominal rate due to parameter smoothing uncertainty and that due to the monetary policy shock. Thus, from a policy perspective, the central bank benefits from the uncertainty surrounding monetary policy shocks as it allows it some latitude in the degree of smoothing implemented in any given period.

The study carried out in this extension considers a very specific type of uncertainty, in particular a type of uncertainty where the agent knows the specific distribution of possible smoothing parameters. A fuller analysis would consider various distributions, especially distributions that are non-symmetric. A second avenue would consider cases were there is ambiguity surrounding $\overline{\gamma_R}$, i.e. there is not just uncertainty surrounding γ_R but also the distribution from which $\overline{\gamma_R}$ is drawn. Finally, one could also consider a central bank following a distribution that depends on past policy actions and the state of the economy, i.e. κ_R tied to the history of γ_R and the nominal interest rate.

Zero Lower Bound

The analysis of policy rules in this thesis has presumed that the central bank is unconstrained in its choice of the nominal interest rate as prescribed by the Taylor Rule it follows. In particular, the central bank is permitted to set a negative interest rate. The extension of this section seeks to understand how asset pricing results are affected if the central bank is restricted to setting non-negative interest rates.

Table 6.1: Asset Pricing Results - Zero Lower Bound

	Data	No Smoothing	Smoothing				
			$\gamma_R = 0.05$	$\gamma_R = 0.15$	$\gamma_R = 0.25$	$\gamma_R = 0.5$	$\gamma_R = 0.64$
Mean (% p.a)							
Risk Free	1.49	1.34	1.34	1.35	1.37	1.43	1.48
		(1.36)	(1.37)	(1.38)	(1.40)	(1.46)	(1.52)
Equity	7.87	2.20	2.17	2.11	2.03	1.82	1.68
		(2.07)	(2.04)	(1.98)	(1.92)	(1.72)	(1.62)
Equity Premium	6.52	0.86	0.82	0.75	0.66	0.39	0.19
		(0.70)	(0.67)	(0.60)	(0.51)	(0.26)	(0.10)
Standard Deviations (% p.a)							
Risk Free	2.62	1.07	1.02	0.93	0.88	1.02	1.36
		(0.89)	(0.86)	(0.84)	(0.87)	(1.24)	(1.68)
Equity	16.72	5.95	5.82	5.54	5.21	4.06	3.11
		(5.39)	(5.26)	(4.96)	(4.62)	(3.47)	(2.64)
Equity Premium	15.69	5.85	5.73	5.46	5.14	3.94	2.81
		(5.31)	(5.19)	(4.89)	(4.55)	(3.26)	(2.06)

The values in the parentheses are the non-zero lower bound asset pricing results presented in the previous section and reproduced here for ease of comparison

The quantitative asset pricing results in the presence of the zero lower bound show that there is a quantitative impact of the constraint but it does not affect the general trends highlighted in the previous section. The results for the mean returns are quantitatively similar to the unconstrained case. There is a significant quantitative impact of the zero lower bound on the volatilities of the returns, which increase with respect to their unconstrained counterparts. The inability of the central bank to stimulate the economy at the lower bound means there is greater volatility in the economy which also manifests itself in the asset pricing results.

The quantitative result of most interest is the volatility of the equity return. One notices a large impact of the zero lower bound at low levels of smoothing but at higher levels of smoothing there is a progressively smaller difference. The zero lower bound weakens the expectations channel of central banking policy, when compared to the unconstrained policy. The fall in the volatility, however, is much larger in the

presence of the constraint which suggests that the expectations channel is strengthened much faster in the presence of the zero lower bound as the smoothing parameter is increased. This makes intuitive sense as the greater the level of smoothing the less "bite" has the zero lower bound constraint. A greater degree of smoothing acts to provide greater certainty to agents in the economy over a longer period and this greater certainty acts to mitigate the greater volatility due to the zero lower bound.

The results presented in this extension suggest that in the presence of the zero lower bound it is even more advantageous to follow a policy regime with interest rate smoothing. Indeed, the central bank can significantly reduce asset market volatility induced by the zero lower bound by following a policy that has a large degree of smoothing. This makes economic sense since a greater degree of smoothing provides agents with a greater degree of certainty over a longer horizon, i.e. expectations are formed for a longer period. This allows agents to see through the zero lower bound in the short term.

6.2.2 Avenues for Future Work

I conclude this thesis with two additional avenues for future work that extend the model developed in this thesis. The first considers extensions to additional shocks while the second considers extensions to a more nuanced financial system by introducing a banking sector to study the more nuanced macroeconomic policy responses of macro-prudential policy.

Investment Specific Shocks

Part III provides more than insight into the design of monetary policy regimes when both macroeconomic stability and financial market stability are objectives; it also acts as an objective test of the model developed in this thesis. The data on movements in

the nominal interest rate suggest that the nominal interest rate exhibits significant inertia, i.e. a smoothing regime is a better depiction of reality than non-smoothing regimes. When faced with this test the model continues to perform well with respect to mean asset returns but moves away from the data with respect to asset market volatility. This result suggests that, although the current model provides a superior joint explanation of asset prices and business cycles, there is still work to do. In particular one needs more than the two shocks in the current model. There is considerable empirical evidence that suggest that the two shocks studied here, i.e. shocks to productivity and monetary policy, are only minor drivers of the business cycle.

Justiniano et al. (2010) show that shocks to the effectiveness of investment are important drivers of the business cycle. Conceptually, investment specific shocks affect the real economy via the return on capital. Suppose that a shock leads to more effective investment. This will drive up the capital return inducing agents to invest more, consume less and work more. Justiniano et al. (2010) showed that, while such a channel plays a limited role in the Real Business Cycle model, the frictions associated with the New Keynesian model allow this channel to gain greater prominence. The study of investment specific shocks serves a dual purpose. First investment specific shocks are a natural extension given the focus on asset prices. In Part II I show that asset pricing in New Keynesian models relies heavily on the price of capital. Given the importance of such shocks to the cycle they should have a non-negligible impact on asset prices in the model. Secondly, investment specific shocks provide a shock which the central bank cannot smooth away via policy. The central bank indirectly smooths the impact of technology shocks by setting the nominal interest rate via the Taylor Rule. The inability to smooth the shock will provide a source of volatility that will aid in quantitative replication of the stylised facts. In particular this inability should lead to greater asset return volatility as the current generation of models suffer from

excessively smooth asset return volatility.

A Banking System and Macro-Prudential Policy

The provision of debt by banks allows one to analyse how leverage constraints impinge on the behaviour of agents with respect to asset pricing results. Thus one may study how leverage concerns affect risk premia on equity. This extension provides a richer, and more realistic, environment in which to continue the analysis of policy on asset prices.The extension allows one to study more nuanced, and realistic monetary policy regimes in order to further analyse the impact of monetary policy on asset prices. Monetary policy conducted via changes to the nominal interest rate is a very blunt instrument, akin to using a sledgehammer to kill a fly. In order to study more targeted policy regimes one requires a more nuanced model. The introduction of a banking sector allows one to study macro-prudential policy as an addition to the policy toolkit of central banks. Macro-prudential policy has been of interest in the current literature and popular press given its ability to produce more nuanced and targeted policy responses.

Macro-prudential policy regimes have become an important part of the central bank policy toolkit. It is a set of policy measures which aim to guarantee the safety and soundness of the financial system, and help mitigate the systemic risks associated with the banking system. One widely advocated macro-prudential policy tool is the targeting of bank balance sheets. It is on this approach to macro-prudential policy that the extension focuses. The model developed can be used to understand not just the impact of macro-prudential policy but also its interaction with simple monetary policy.

The recent financial crisis has highlighted not only the crucial role the banking sec-

tor plays in the transmission of shocks, via the credit channel, but also how little is understood about this mechanism. The financial sector in my current model is very simple; consisting only of an equity market and a market for risk free bonds. Introducing a simple model of the banking sector where bank lending will subject to default in the spirit of Bernanke et al. (1999) adds more realism to this simple financial sector. The likelihood of default depends on macroeconomic conditions allowing credit to both move with macroeconomic conditions and endogenously interact with it. Consider an economic downturn. This will see loans default and banks, seeking to recoup their capital, will increase lending rates and reduce the quantum of lending ,and in turn there will be less production as firms need loans to finance their productive activities; i.e. the financial accelerator will affect the return on equity. Thus macro-prudential policy targeting bank balance sheets gives the central bank more nuanced policy options by allowing it to respond directly to the cause of instability, the credit channel, and not rely on a more nebulous response via monetary policy.

These extensions allow me to build on the foundational work carried out in this thesis and extend it to provide answers to policy questions that are both topical and important for current policy makers.

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