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# Memory of Recessions

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**Abstract**

This paper reviews the evidence on the effects of recessions on potential output. In contrast to the assumption in mainstream macroeconomic models that economic fluctuations do not change potential output paths, the evidence is that they do in the case of recessions. A model is proposed to explain this phenomenon, based on an analogy with water flows in porous media. Because of the discrete adjustments made by heterogeneous economic agents in such a world, potential output displays hysteresis with regard to aggregate demand shocks, and thus retains a memory of the shocks associated with recessions.

**Keywords:** Recessions, Permanent Effects, Hydraulic Keynesianism, Porous Media, Hysteresis.

**JEL Classification Numbers:** E12, E32, A12.

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## 1 Introduction

The financial crisis that has preoccupied much of the economic world since 2007 has exposed the flaws of the "new consensus" in macroeconomics (Blanchard, 2008) organised around the dynamic stochastic general equilibrium (DSGE) account of economic fluctuations. In this account the behaviour of the macro economy is deduced from axioms specifying the behaviour of a rational representative economic agent. The axioms do not fit the micro evidence on how actual economic agents behave; the key issue of how heterogeneous economic agents interact with each other is ignored; there is no meaningful notion of unemployment in such models; financial intermediation plays no important role, on the misplaced assumption that financial markets are "efficient"; and so on (see Colander et al., 2009, for a lucid critique).

A key issue arising from this dual crisis in finance and mainstream economics is whether the recessions following in the wake of the financial crisis will have permanent effects on output, employment and unemployment. In mainstream models, of the DSGE type, equilibrium or trend time paths for output are invariant with regard to fluctuations around these paths. The present paper points to a corpus of evidence that indicates violations of such invariance assumptions, and suggests some analytical steps that can aid the understanding of economic systems in which recessions, and maybe booms, change the trend or potential level of output.

There is extensive evidence, going back to at least the 1930s, on the debilitating effects of the long spells of unemployment experienced during recessions (Pilgrim Trust, 1938). The idea that recessions leave in their wake higher equilibrium rates of unemployment is both plausible and well documented (see the papers in Cross, 1995, for example), even if such effects have been largely ignored in mainstream models. If the equilibrium or sustainable rate of unemployment is haunted by recessions, and maybe by booms on the plus side, it would be surprising if the potential level of output were not also haunted by economic fluctuations. The present paper confines itself to this relatively neglected issue, of how recessions affect the potential level of output.

This paper proceeds as follows. The next section offers a taxonomy of possibilities as to how economic fluctuations might affect potential levels of output. A third section provides a brief review of empirical tests of the hypothesis that recessions displace trend or potential levels of output. A fourth outlines a mathematical model, based on porosity in macroeconomic flows, in which fluctuations in aggregate demand displace potential output levels. A concluding section discusses areas in which this line of research might be extended.

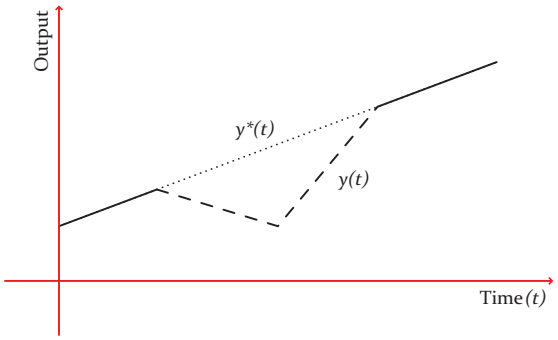


**Figure 1** – No change in potential output level. After recessionary period the output returns to its previous trend.

## 2 A taxonomy of possibilities

In the "new consensus" mainstream models the growth path for potential output is not permanently affected by recessions or by booms. In the "plucking" model (Friedman, 1993), for example, a recession sees output fall below potential, capital per effective worker falls below the steady state level, diminishing returns to capital imply a higher marginal productivity of capital and generate an investment spurt, leading to a higher rate of growth of output than potential growth during the recovery phase. Figure 1 illustrates this, where $y(t)$ and $y^*(t)$ are the natural logarithms of the actual and potential levels of output respectively, and $t$ is time.

In contrast to this are models in which recessions, and perhaps booms, change potential output growth paths. With constant returns to capital per effective worker, a shock to capital (or structural parameters or policy variables) will change the growth path for potential output. In a non-linear framework a switch from a positive to a negative permanent drift term as an economy moves from a growth to a recessionary state results in a permanently lower level of potential output after a recession (Hamilton, 1989). Figure 2 illustrates the case of a recession curse on the level of potential output, the growth rate for $y^*(t)$ remaining the same, but the level being displaced downwards.

If the "creative destruction" aspects of a recession (Schumpeter, 1942) were sufficient to outweigh the destructive aspects, the outcome would be the increase in potential output level indicated in figure 3. This would obviously involve a stronger growth in output during recovery from recession than in the mainstream case in which the potential output path is unchanged.

The cases where recessions change the growth rates for potential output complete the taxonomy. A recession could entail lower productivity growth if learning-by-doing is important in production, or if there are recession-induced effects on the growth rate of

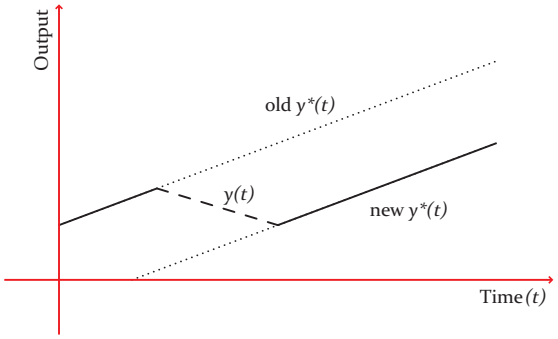


**Figure 2** – Fall in potential output level. The potential output $y^*(t)$ drops to a lower level (original level shown as dashed line) after the recessionary period. The rate of growth remains as before.

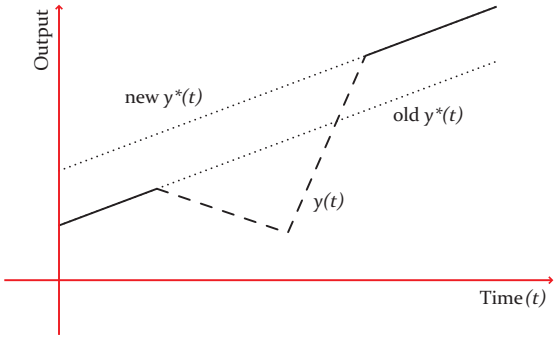


**Figure 3** – Rise in potential output level. The level of the potential output is higher after the recession.

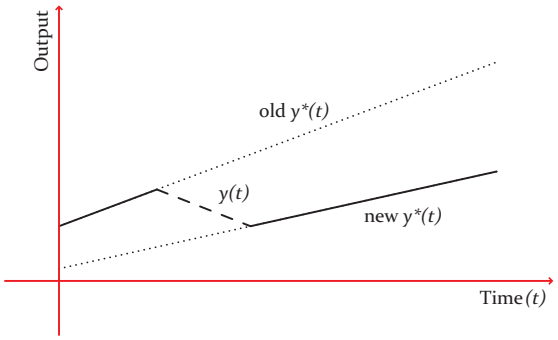


**Figure 4** – Fall in potential output growth rate. In this case it is the *rate of growth*, post-recession, that is permanently altered.

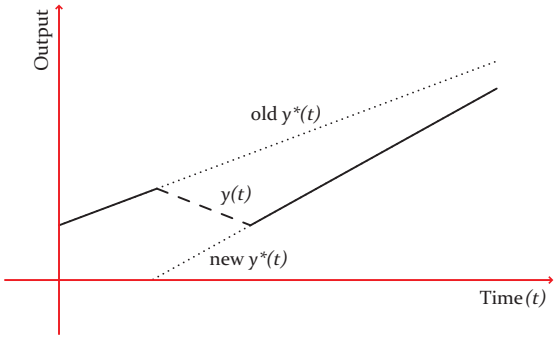


**Figure 5** – Rise in potential output growth rate. Here the rate of growth is higher after the recession.

the labour supply, by way of lower migration flows or participation rates, for example. Figure 4 illustrates the case of a reduction in the growth rate of potential output.

If a recession were to bring in its wake a Schumpeterian super recession blessing in the form of higher productivity growth the case illustrated in figure 5 would apply.

The question is then one of which of these possibilities is the most likely to be relevant on the basis of evidence regarding past recessions.

## 3 Evidence on past recessions

Keynes (1934) posed the question: "Is the economic system self-adjusting?" The conventional wisdom of Keynes' time, and the "new consensus" at the time of the onset of the present financial crisis in 2007, answered this question in the affirmative. The evidence, however, when allowed to speak by not imposing the self-adjusting assumption on the data, supports the negative answer given by Keynes. DSGE models, which assume that the self-adjustment property holds, have been subjected to extensive empirical calibration. Even in the linear frameworks used, however, cointegrated vector autoregression (CVAR)

tests indicated that "…most assumptions underlying the DSGE model and, hence, the real business cycle model were rejected when properly tested (Hoover, Johansen, and Juselius, 2007, p.254). So the confidence placed in the DSGE macroeconomics of self-adjustment to invariant equilibria was more a matter of assumption than of evidence.

The maximum likelihood estimates of the processes driving US GNP post-1945 by Hamilton (1989) pioneered the practice of allowing the data to speak on the issue of the effects of recession on potential output. The Hamilton specification allows for a Markov process whereby the economy switches between a positive output growth rate in normal times, and a negative growth rate in recessions, there being a switch between positive and negative permanent drift terms for output. This proved to provide the best empirical fit to the data, beating alternative specifications, such as that output was driven by longer term trends in which the switching was between faster and slower growth rate regimes. "The estimated parameter values suggest that a typical economic recession is associated with a 3% permanent drop in the level of GNP" (Hamilton, 1989, p.357). It would be difficult to extend the Hamilton framework to allow for switching between booms as well as recessions and normal periods, so the evidence just pertains to a recession curse on the level of permanent potential output.

More informal methods were used in the Dow (1998) estimates of the effects of the major 20th century recessions in the UK on potential output. Dow identifies three major post-1945 recessions: 1973–75, 1979–82 and 1989–93. The estimate is that in the course of these recession "…productive capacity got destroyed, but for which economic activity by the end of the period would have been 15 per cent higher than it was," with the downward displacements of capacity output being 2.2%, 5.3% and 8.4% in the successive recessions (Dow, 1998, pp.385,386). Dow also postulates effects of major recessions on the growth rate, the 1920–21 and 1929–32 recessions reducing the UK output growth rate from 2.5% to 2%,and the three recessions from 1973 reducing the growth rate from 3% to 2% (Dow, 1998, p.27).

The most comprehensive evidence on whether the output losses arising from recessions are permanent is the Cerra and Saxena (2008) study of the recoveries from financial and political crises in 190 countries over the period 1960–2001. The impulse response functions for each type of crisis are estimated from

$$\dot{y}_{i,t} = a_i + \sum_{j=1}^{4} b_j \dot{y}_{i,t-j} + \sum_{s=0}^{4} c_s D_{i,t-s} + e_{i,t} \tag{1}$$

where $\dot{y}$ is the change in the natural log of GDP, $D$ is a dummy variable for each financial or political crisis, $i$ indicates the country and $j$ and $s$ the number of time lags. "Impulse response functions show that less than 1 percentage point of the deepest output loss is regained by the end of ten years following a currency crisis, banking crisis, deterioration in political governance, twin financial crises or twin political crises. Of the large negative

shocks examined, a partial rebound in output is observed only for civil wars. Moreover, the magnitude of persistent output loss ranges from around 4% to 16% for the various shocks" (Cerra and Saxena, 2008, p.456).

Of particular interest are the output losses arising from banking crises, which average 8% over the full sample at the ten year horizon. These persistent output losses were lowest, at around 4%, for Latin America and lower-middle income countries; highest at over 15% for Middle Eastern countries; and just less than 15% for high income, upper-middle income and transition economies (Cerra and Saxena, 2008, Figure 4, p.444).

These results confirm earlier findings that recoveries from crises are characterised by lower growth in the recovery phases than in average expansion years, implying that when output drops it remains below its previous trend (Cerra and Saxena, 2005c); and that there were permanent output losses arising from the 1997–98 Asian financial crises (Cerra and Saxena, 2005a) and from Sweden's banking crisis of the early 1990s (Cerra and Saxena, 2005b). In the "Phoenix miracle" study of Calvo et al. (2006), recovery from financial crises is quicker than in the Cerra and Saxena estimates, but it still takes a long time for output to recover to the trend levels that would have prevailed in the absence of crises.

The European Commission *Quarterly Report on the Euro Area* (2009, Volume 8, No. 2) has taken this evidence seriously, outlining the possibilities that the present financial crisis could have a permanent effect on the level of output or its growth rate (p.28), and reviewing the evidence on permanent effects from past financial crises (pp.29–36). The European Commission simulations indicate that a three year 200 base point increase in risk premiums arising from the financial crisis would have a permanent effect on the level of potential output, and a lasting but not permanent effect on the growth rate (Box 3, p.35). The conclusion is a more guarded one that "…although the adjustment phase could be protracted, the most likely scenario for the Euro area is for a return of potential growth to its pre-crisis long-term trend…nevertheless, risks of a moderate crisis-induced reduction in long-run potential growth cannot be ruled out in the absence of adequate policy responses" (European Commission, 2009, p.36). In relation to the role of policy responses in the recovery from recessions associated with banking crises, an IMF study finds that post-trough recoveries are more sluggish in response to banking-crisis recessions in industrial countries compared to other recessions, but that fiscal policy is particularly effective in encouraging stronger recovery paths from recessions following banking crises in such countries (Panizza et al., 2009).

## 4 A model of porosity in macroeconomic flows

Keynes' *General Theory* framework provided insight into how fragile "animal spirits" could lead to a fall in business and consumer confidence about economic prospects, driv-

ing the decline in private sector investment and rise in private sector saving usually seen as the hallmarks of a Keynesian recession (Akerlof and Shiller, 2009). And Keynes' *Treatise on Money* analysis of liquidity preference during deleveraging processes associated with a "credit crunch" can provide some insight into the balance sheet adjustments taking place during the current recession (Leijonhufvud, 2009). But as Leijonhufvud argues, there are aspects of the current financial crisis and ensuing recessions that do not fit in easily with Keynes' analysis. Interest rates were high as the UK strove to return to gold at an overvalued exchange rate in the build up to the 1920s recession, whereas interest rates were low and there was little pressure on the US dollar exchange ratein the build up to the post-2007 recession. And Keynes' income-expenditure analysis fails to deal systematically with balance sheet distortions of the kind experienced during the build up to, and unravelling of, the post-2007 recession (see Minsky, 1977, for a framework in which such distortions are endemic). As Leijonhufvud argues, the intellectual bankruptcy of the mainstream DSGE model "does not mean that we should revert to the old Keynesian theory that preceeded it, or adopt the New Keynesian theory that has tried to compete with it" (Leijonhufvud, 2009, p.755).

The evidence that recessions have permanent effects on output suggests a need to abandon the sharp dichotomy between the analysis of cyclical fluctuations and potential output paths in macroeconomics. Keynes, and some aspects of Keynesian economics, have provided insights into the impulse and propagation mechanisms that might underly such permanent effects. But as Leijonhufvud argues, the greatest insight from Keynes is that the complex system of a modern economy is a "delicate machine the working of which we do not understand" (Leijonhufvud, 2009, p.755). What follows can be regarded as a modest attempt to outline some equations that allow recessions to have permanent effects on output, in a framework that allows for differences in the behavioural propensities of economic agents.

The tradition of visualising flows in economic systems as similar, in some important sense, to flows of water goes back at least to the metaphors of neoclassical economics (Mirowski, 1989). Irving Fisher, in his Ph.D. thesis submitted to Yale University in 1891 under the guidance of the physicist J. Willard Gibbs, built a hydraulic model of equilibirum prices in which marginal utility and marginal cost are reflected in the depth of the liquid in cisterns connected by levers and pistons (Fisher, 1925). The physical embodiment of the "hydraulic" tradition in Keynesian economics was in the Phillips machine, in which coloured water flows through pipes connecting reservoir tanks representing various stocks, with mechanical coupling through valves providing feedback from the various parts of the system. Papers in the volume edited by Leeson (2000) document the history of the Phillips machine, and Bissell (2007) discusses its significance as an early hydromechanical analog computer. In these hydraulic systems water flows freely through pipes,

mechanical coupling by way of stoppers (Fisher) or valves (Phillips) providing the system with an economic interpretation.

The model in the present paper follows this "hydraulic" tradition but introduces an important innovation by allowing for porosity in the medium through which the water flows. In terrestrial hydrology there is a well documented difference between wetting and drying curves of soil, wetting requiring less energy than drying, and so the paths are not reversible (see Appelbe et al., 2009, Figure 5, p.51). Less mechanical work or energy is required to place moisture into an unsaturated medium than to remove it. A sponge is easily saturated by insertion in bathwater, but requires a lot of squeezing to remove the water content. The sponge displays "effects of retentiveness".

It is interesting that this phrase arose in the early literature on hysteresis. *J.A.* Ewing (1881) coined the term "hysteresis" to refer to the persistence of past states observed in electromagnetic fields in ferric metals, the field characteristics not returning to the previous states when magnetising forces were applied and then removed. Sir William Thomson (Lord Kelvin), Ewing's assessor for the Royal Society of London paper in which the term "hysteresis" was coined, wanted Ewing to use the phrase "effects of retentiveness" instead of the somewhat mysterious new term. Ewing stuck to his guns, arguing that the new term would be relevant to a wider range of contexts than ferromagnetism (see *A.W.* Ewing, 1939). Ewing proved to be right, not just in relation to terrestrial hydrology (Appelbe et al., 2009) but also in a wide range of applications (Bertotti and Mayergoyz, 2006), including economic systems (Cross et al., 2009). In what follows the equations used to explain hysteresis effects in terrestrial hydrology are applied to the determination of potential output in economic systems.

In relation to soil-water hysteresis, heterogeneity at the micro level is provided by the particles that make up the soil being different in size. The pore spaces between the particles can either be made up of water or air. The water content of pore spaces is subject to what are termed "Haines jumps" in response to variation in water pressure, the response being flat up to a critical point at which the pore becomes full of water, and up to a critical point at which the pore empties of water. If the processes in question are rate-independent, being dependent on the level of the input but not on its rate of change, this hydrological system can be represented by a Preisach-type model (first introduced in Preisach, 1935) of hysteresis (Flynn et al., 2006). The key features of this model can be represented in a modified form of Darcy's Law (Darcy, 1856) whereby water flows in response to gradients in water potential (equations taken from Appelbe et al., 2009):

$$\dot{\theta}(t) = k\left(\psi_{\text{ref}}(t) - \psi(t)\right) \tag{2}$$

$$\theta(t) = \left(\mathcal{P}\,\psi\right)(t). \tag{3}$$

Here $\theta(t)$ is the water content of the soil; $\dot{\theta}(t)$ is the rate of change of water content;

$\psi_{\text{ref}}$ is the reference or external water potential, the input variable reflecting disturbances that can change the water content, such as an increase in precipitation; $\psi(t)$ is the matric water potential, the matric being derived from the matrix of particles that make up the soil, indicating the thermo-mechanical work that must be performed to bring the soil water to a common reference state against all surface forces at the same reference level as the gravitational potential; and $(\mathcal{P}\,\psi)\,(t)$ is the Preisach operator describing the pore characteristics of the soil. In equations (2) and (3), $\theta(t)$ and $\psi(t)$ are the output variables, and $\psi_{\text{ref}}(t)$ is the input variable. Differences between the reference and matric potentials drive water flows and the Preisach operator links water content to matric potential.

The question is whether this model is a good metaphor for output flows in economic systems. At the macro level think of $\theta(t)$ as corresponding to the level of potential output $y^*(t)$; $\psi_{\text{ref}}(t)$ as being the level of actual aggregate demand $AD(t)$; and $\psi(t)$ as corresponding to an "equilibrium" level of aggregate demand, $AD^*(t)$, that could be consistent with a constant level of potential output. Thus we have Table 1, in the spirit of Fisher's mechanical analogies (Fisher, 1925, pp.85-86).

**Table 1** – Analogies between economic and hydrological quantities

| In hydrology | In economics |
|---|---|
| Water content, $\theta(t)$ | Potential output level, $y^*(t)$ |
| Water flow, $\dot{\theta}(t)$ | Potential output growth, $\dot{y}^*(t)$ |
| Reference potential, $\psi_{\text{ref}}(t)$ | Aggregate demand, $AD(t)$ |
| Matric potential, $\psi(t)$ | "Equilibrium" aggregate demand, $AD^*(t)$ |

It must be stressed here that "equilibrium" aggregate demand is simply the level that would be consistent with an unchanging level of potential output if matched by actual aggregate demand. Equations (2) and (3) thus become:

$$\dot{y}^*(t) = k\,(AD(t) - AD^*(t)) \tag{4}$$

$$y^*(t) = (\mathcal{P}\,AD^*)\,(t). \tag{5}$$

The input variable is $AD(t)$, with equation (4) specifying that potential output increases (decreases) when this is greater than (less than) some "equilibrium" level of aggregate demand $AD^*(t)$. Equation (5) indicates that the two output variables of the system, $y^*(t)$ and $AD^*(t)$, are linked by a Preisach operator that summarises the non-linear reactions of the heterogeneous economic agents in the system to shocks affecting the actual level of aggregate demand. Again, it should be noted that $AD^*(t)$ is an output of the system, not some equilibrium time path determined exogenously to the system, and is only an "equilibrium" in the sense that $y^*(t)$ would not change if $AD^*(t)$ were equal to $AD(t)$.

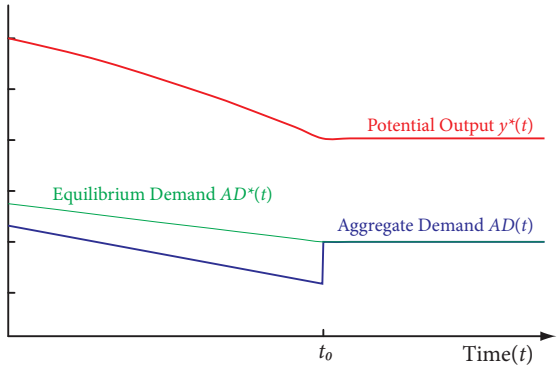


**Figure 6** – Illustration of the "equilibrium" aggregate demand variable $AD^*(t)$. If the actual aggregate demand were to jump at an instant $t_0$ such that $AD(t_0) = AD^*(t_0)$, then the system will remain in equilibrium, with constant output.

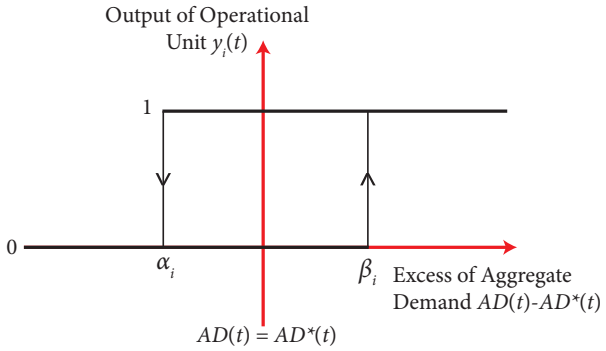


**Figure 7** – The output of operational unit $i$ is determined by the excess of aggregate demand via a "lazy switch". Distinct thresholds, $\alpha_i$ and $\beta_i$, determine switching off and on (0 and 1) respectively.

The jumps in economic activity at the micro level can be illustrated in a simple framework in which one unit of capital is required to produce one unit of output. Each actual or possible operational unit of a firm requires an excess of actual over "equilibrium" aggregate demand of at least $\beta_i$ to become active and produce output. A shortfall of actual in relation to "equilibrium" demand of at least $\alpha_i$ is required for an operational unit that was previously active to become inactive As illustrated in figure 7, in the range from $\alpha_i$ to $\beta_i$ the operational unit can be either inactive or active depending on whether this range has been approached from below or above. Note that the operational units of firms are heterogeneous in the sense that they have different $\beta$ and $\alpha$ triggers for activity and inactivity, respectively, providing the foundations for a Preisach model representation of their behaviour.

Extending the metaphor to the micro level involves inquiring into the workings of

a "delicate machine the working of which we do not understand" (Leijonhufvud, 2009, p.755). The different soil particles and pore spaces correspond to economic agents who are heterogeneous in their "animal spirits" or expectations about the uncertain future, face different chances of becoming unemployed, are differentially exposed to increases in interest rate premiums arising from a financial crisis, and so on. The moisture analogy is that the economic agents are either "wet" in the sense of being employed and producing output, or "dry" in that they are unemployed or do not produce output. Corresponding to the "Haines jumps", between wet and dry states in the soil pores, are the discrete changes in economic activity at the micro level. Producing output usually involves fixed costs that are sunk in the sense that they cannot be recovered should production be discontinued. As well as the obvious costs sunk in physical capital investment are the costs of hiring labour, marketing outlays and the costs of arranging finance. Such sunk costs, along with the tension between striking while the profit opportunities are hot and waiting to gather more information about inherently uncertain market conditions, imply separate triggers for proceeding with and abandoning investment projects (Dixit and Pindyck, 1994), as in the Preisach model. Such considerations imply that many economic adjustments at the micro level are more associated with discrete changes in large doses than with the continuous changes in smallish doses that characterise mainstream models in economics.

At the macro level the crucial question is whether the "wetting" and "drying" processes that underly the system described in equations (4) and (5) can provide a coherent account of economic fluctuations in which periods of "normal" expansion of potential output are disturbed by recessions that displace downwards paths for potential output. In this model recessions are driven by shocks to aggregate demand, which cause $AD(t)$ to fall below $AD^*(t)$, so that $\dot{y}^*(t) < 0$ for $k > 0$. In what follows we report simulations that indicate how the endogenous variables $y^*(t)$ and $AD^*(t)$ react to shocks in the exogenous variable $AD(t)$.

## 5 Simulations

Demonstrations of the behaviours that arise from the model introduced in (4) and (5) have been presented elsewhere, particularly in the case where the "input" function (aggregate demand in our framework) is an oscillating function of time, perhaps including a short-run perturbation, see for example Cross et al. (2007).

We will consider the situation when the aggregate demand, $AD(t)$, is close to a saturation, at a level $AD_1$, before the time moment $\tau$ when a recession begins. We also suggest that after the end of the recession, say for $t > \sigma > \tau$, the aggregate demand is saturated close to another level $AD_2$. The natural assumption is that

$$AD_1 > AD_2 > \min_{\tau<t<\sigma} AD(t) \tag{6}$$

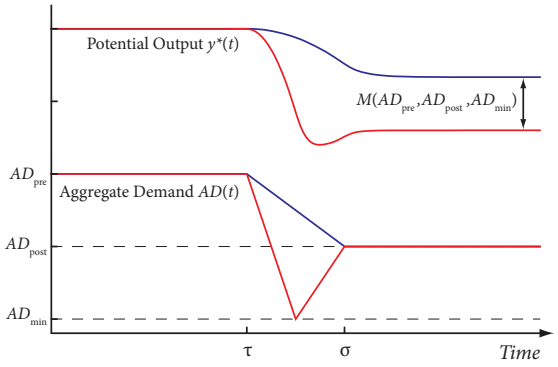


**Figure 8** – Illustration of the "recession curse", quantified by $M\left(AD_{\text{pre}}, AD_{\text{post}}, AD_{\text{min}}\right)$. Potential output $y^*(t)$ and aggregate demand $AD(t)$ are shown on the same axes. While $AD(t)$ follows the same path after the chages, $y^*(t)$ displays a lasting effect due to the recession. The effect will depend on the values of $AD(t)$ before and after the recession, as well as the minimum of $AD(t)$ during the recession.

The last quantity $AD_{\text{min}} = \min_{\tau<t<\sigma} AD(t)$ characterizes the overall depth of the recession. The memory of recession in this layout is in the fact that the level $y^*_{\text{post}}$, to which the output variable $y^*(t)$ stabilizes after the the recession is over, depends not only on the "pre-recession" and "after-recession" levels $AD_{\text{pre}}$, $AD_{\text{post}}$ of the agregate demand, but also on the maximal depth $AD_{\text{min}}$ of the recession.

A natural quantitative measure $M\left(AD_{\text{pre}}, AD_{\text{post}}, AD_{\text{min}}\right)$, of the corresponding "recession curse" is defined by

$$M\left(AD_{\text{pre}}, AD_{\text{post}}, AD_{\text{min}}\right) = y^*_{\text{post}} - z^* \tag{7}$$

where $z_*$ is the (saturated) output level for the hypothetical "mild" recession when the inequalities (6) are swapped by the relationships

$$AD_1 > AD_2 = \min_{\tau<t<\sigma} AD(t)$$

The function (7) is expected to be increasing in the last argument, $AD_{\text{min}}$. This is demonstrated in figure 8 using results from numerical experiments.

The dependence on the minimum of $AD(t)$ is shown in figure 9. Note also that by definition

$$M\left(AD_{\text{pre}}, AD_{\text{post}}, AD_{\text{post}}\right) = 0.$$

It can be also proven that

$$M\left(AD_{\text{pre}}, AD_{\text{post}}, AD_{\text{post}}\right) = 0.$$

for any $AD_{\text{min}}$, if $AD_{\text{post}} \geq AD_{\text{pre}}$, that is the "recession curse is over" if the post-recession aggregate demand $AD_{\text{post}}$ overcomes the pre-recession level $AD_{\text{pre}}$. In this case the new "boom blessing" suppresses the recession curse.

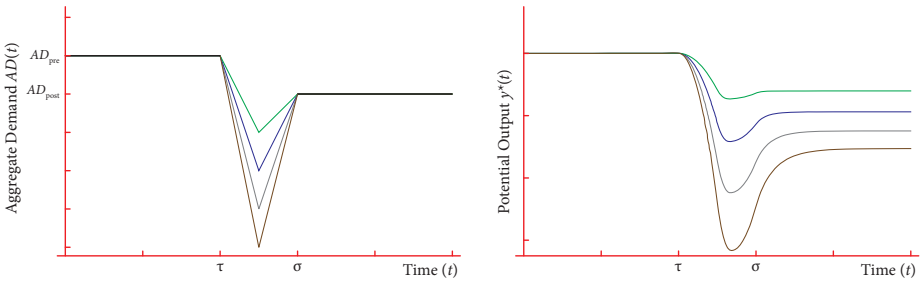


**Figure 9** – Dependence of the recession curse on depth of recession. The level of aggregate demand before and after the recession are the same in each of the four cases, just the depth of the recession is different, as shown on the right. The corresponding potential output curves are shown on the right.

The model (4),(5) described here is not immediately applicable to the choice from the taxonomy of possibilities outlined in Section 2, and shown in figures 1–5. However, a natural modification of the porous flow metaphor supports figure 4. The simplest modification of such type may be described by the system

$$\dot{y}(t) = R(t)y(t) \tag{8}$$

$$\dot{R}(t) = k\left(z_{\text{ref}}(t) - z(t)\right) \tag{9}$$

$$R(t) = \left(\mathcal{P}\, z\right)(t) \tag{10}$$

Here the new variable $R(t)$ is the variable spectral radius of a corresponding Liontief matrix (as described in Takayama, 1985) which characterizes the current trend in the economy. The value $R(t)$ depends on the rate of change of the aggregate demand, $z$, and, repeating with appropriate modifications the previous discussions, we conclude that a natural model of this last dependence is given by the equations (9),(10). Experiments with the model (8)–(10) are in progress; they tentatively support the scenario shown at Figure 4.

## 6 Concluding remarks

A key issue is whether the recessions following the post-2007 financial crisis will have permanent effects on output. Mainstream macroeconomic models assume that the effects are not permanent, whereas the evidence suggests that they are. It is more important to introduce regulatory and other measures to try and avoid financial crises, and to apply fiscal and monetary stimuli at the onset of recessions, if the effects of the recessions on potential output are permanent than if they are merely temporary.

The present paper has outlined a hydraulic model of the determination of potential output based on an analogy with water flows through porous media. While this model

can produce the permanent effects of potential output arising from the negative shocks to aggregate demand associated with recessions, there are several limitation. One is that a richer specification of the Preisach operator $\mathcal{P}$ is required to capture the supply side of the economy and how this interacts with the demand side. The introduction of a financial sector into the model would allow the activity rates of operational units within firms to depend on how lending to firms is affected by changes in interbank lending rates as well as by central bank base rates. Another limitation is that the model needs to be extended to allow for the interaction of potential output and aggregate demand with the rate of inflation. The present model can be regarded as a first pass at attempting to explain a key aspect of "a delicate machine the workings of which we do not really understand."

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