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VERTICAL RELATIONAL CONTRACTS AND TRADE CREDIT

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Vertical Relational Contracts and Trade Credit

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Abstract

This paper uses a vertical relational contract between two firms to explore the implications of trade credit when the ability to repay is not observed by the supplier.

Trade credit limits the supplier's possibilities to punish the cashless downstream firm and termination may be used in equilibrium. We find that the supplier always sells too little despite having enough instruments to fix the double marginalization problem. The downward distortion in the quantity results from the need to make the contract self-enforced and/or to tackle the asymmetric information problem. The distortion remains even as the firms become arbitrarily patient and a larger discount factor does not necessarily translate into a larger welfare.

We show that the optimal contract resembles a simple debt contract: if the fixed repayment is met, the contract continues to the next period. Otherwise, the manufacturer asks for the highest possible repayment and terminates for a number of periods. The toughness of the termination policy decreases with the repayment.

JEL Classification: C73, D82, L14.

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1 Introduction

Trade credit plays an important role in market supply chains. See for example, Giannetti, Burkart and Ellingsen (2011). Vertically-related firms find it costly to conduct all their business-to-business transactions on pure cash-and-carry basis and the capacity to enter into contracts with delayed obligations is essential for a good business environment. Trade credit is rarely secured on collateral. Moreover, enforcing repayment through the courts can be problematic. The legal cost may be too high relative to the size of the transaction or the buyer may have been adversely affected by a shock leaving nothing to the supplier to foreclose on. This raises the issue of what determines when trade credit is paid back.

In this paper, we study supplier trade credit within a vertical relational contract in an environment where adverse shocks may make the cashless downstream firm unable to honor the credit agreement. Since these shocks are not observable to the supplier, trade credit, by postponing the payment until the shock is realized, makes asymmetric information matter. To induce repayment, the supplier ensures that it is worthwhile for the downstream firm to repay the credit rather than face retaliation (the simplest form being the refusal to transact further).

We identify a new mechanism that makes the supplier distort the quantity of the good downwards despite having enough instruments to set the final quantity. The reasons behind this distortion are twofold. First, suppose the shocks are observable. The surplus the supplier needs to leave to the downstream firm for repayment to be honored increases with the quantity supplied. Thus, the supplier faces a trade off between maximizing and sharing the surplus and the resulting quantity is too small. It is as if the supplier was facing an additional marginal cost as a result of the relational contract. Second, suppose the shocks are not observable but, even in a low revenue state, the downstream firm does not want to walk away from the contract. Because the downstream firm is cashless, to tackle the asymmetric information problem, the supplier has to leave enough surplus in the high revenue states so that the true state is reported. A tougher retaliation policy is accompanied by a smaller quantity distortion because it can be used, instead of giving away surplus, to provide incentives to report the truth.

The model makes it possible to identify subtle effects between the need to make the contract enforceable and to address the asymmetric information problem. When the supplier addresses both problems at the same time, a tension emerges between them. Increasing the repayment in low revenue states or toughening the punishment following small repayments decreases the incentives for the downstream firm to underreport revenues but at the same time make it less worthwhile continuing the relationship.

Because of the downstream firm's credit constraint, the quantity distortion remains even as the level of trust increases. This suggests that when the level of trust (or quality enforcement) reaches a minimum level, policy efforts are best spent at relaxing firms' credit constraints. This is reinforced by the finding that more trust does not necessarily translate into a larger welfare as it enables the manufacturer to inefficiently increase her share of the surplus at the expense of decreasing the total surplus.

It has been extensively documented, both in developed and developing countries, that firms make deals with each other and get finance using ongoing relationships and trade credit. For example, trade credit accounts for about 11.5 to 17 percent of the assets of G-7 traded non-financial firms (Rajan and Zingales (1995)). Similarly, credit provision associated with relational contracts has been documented, for instance, by Bernstein (1992 and 1996) for the New York diamond trade and the US grain markets and by Uchida *et al.* (2006) for the Japanese small and mid-sized enterprises. In developing countries, where the absence of formal institutions forces firms to use informal substitutes instead, the evidence is even more prominent. For instance, McMillan and Woodruff (1999a and 1999b), Johnson, McMillan and Woodruff (2002), Fafchamps (1997 and 2000) and Macchiavello and Morjaria (2012) documents how firms in Vietnam, post-comunist and African countries use relational contracts to substitute for missing laws of contract and trade credit to make up for a lack of access to financial markets. Trade credit provision within a relational contract is also expected to be pervasive when transactions are not entirely legal. For example, when firms operate in the shadow economy or in black markets, such as drug trade.

The model uses an agency setting where an upstream firm supplies a good and offers trade credit to a cashless downstream firm. For instance, let the upstream firm ("she") be a manufacturer and the downstream firm ("he") a retailer. The manufacturer's machinery is used as collateral making her less credit constrained than the retailer[1]. The manufacturer proposes a quantity forcing contract which establishes the volume of trade and a repayment[2]. The retailer sells the good and pays back to the manufacturer. However, a shock may occur before the payment, making him unable to repay either part or the whole amount. The manufacturer cannot observe if the failure of payment is due to the shock or to the retailer stealing the money[3] and she punishes him by suspending trade for a number of periods. It is widely accepted that the "extra enforceability power of suppliers (as compared to banks) comes from the fact that they can threaten to stop supplying intermediate goods to their customers" (Cuñat and Garcia-Appendini (2012), page 545). McMillan and Woodruff (1999a and 1999b) find that such retaliation occurs in Vietnam. Fafchamps (1997) highlights the "importance of regular purchases as a determinant of supplier credit and the use of stopped deliveries when payment is not forthcoming" in Zimbabwean manufacturing (page 812). More generally, Fafchamps (2004) finds that 52% of a sample of Sub-Saharan African manufacturers suspend trade following a (late or non) payment dispute.[4]

The repeated game approach formalizes an economic concept of trust and the discount factor is a good proxy for the trust level in the relationship. As we mentioned earlier,

[1] High credit quality suppliers have comparative advantage in securing outside finance that they can pass on small, credit-constrained buyers (Boissay and Gropp (2007)).

[2] Since the quantity is delivered before the uncertainty is resolved, the quantity distortion is not due to screening purposes. Using a two-part tariff or a more complex nonlinear scheme is equivalent to offering a quantity forcing contract. We use this contract to eliminate the vertical externalities coming from the retailer's market power.

[3] The asymmetric information problem considered is reminiscent of the model of Green and Porter (1984) where an oligopoly is colluding in a market with noisy prices. When a low price is observed, firms do not know with certainty if this bad outcome is due to a market shock or a firm deviating to a larger quantity.

[4] The punishment we consider is similar to how credit reference agencies operate: they "simplify the information about each agent i with a credit report showing when the agent last 'cheated' (e.g., paid late or not at all). This information is kept on the agent's record for a set number of years T, after which time it is erased." Fafchamps (2010), page 57.

It could also be interpreted as the manufacturer choosing a probability of a permanent termination.

we find two surprising results regarding the level of trust. First, as the relationship becomes arbitrarily trustworthy[5], the quantity distortion remains constant (even when no termination is used). When the value of the future increases, the manufacturer would like to increase all the repayments in the same proportion to profit from the increased level of trust leaving the asymmetric information problem unaffected. However, the manufacturer cannot increase the low state repayments because they are bounded by the retailer's revenues due to the limited liability. Increasing only the large repayments would imply longer termination periods to maintain truthtelling but this is very costly for the manufacturer when the future is important. Thus, when the level of trust is high enough, any further increase in trust translates into softer termination policies rather than larger quantities which makes the dynamic enforceability problem stop mattering. Second, a larger level of trust does not necessarily increase welfare. This is because it enables the manufacturer to inefficiently increase her share of the surplus at the expense of decreasing the total surplus (by imposing an excessively tough termination policy).

Finally, we find that the optimal contract is a debt contract. The manufacturer asks for a fixed repayment that if met guarantees the continuation of the contract. Otherwise, the manufacturer asks for the highest possible repayment and suspends trading for a number of periods. McMillan and Woodruff (1999b) find evidence of debt contracts being used for their sample in Vietnam: "One (manager) (case #12) sent employees to visit a customer every day to ask for a late payment. "After a few weeks of negotiation, the firm got back part of the debt and stopped selling to this customer." Another (case #10) after some negotiation accepted 70% of the amount owed, and another (case #8), owed money by a firm in Taiwan, after protracted negotiations was paid a year late. What is important, another said (case #4), "is to forget about the debt and keep social relations with the customer." (page 642). To provide incentives to repay as much as possible, larger repayments are associated with shorter trade suspension. The optimality of the debt contract comes from extracting the retailer's surplus in the default states, which allows the manufacturer to soften the termination policy and, hence to decrease the

[5] The discount factor becomes arbitrarily close to 1.

inefficiency. This finding may help to understand the conclusion reached by McMillan and Woodruff (1999b) about the retaliation not being "as forceful as in the standard repeated-game story" (page 637).

1.1 Related literature

This paper belongs to the literature on inter-firm relational contracts with asymmetric information initiated by Levin (2003). Trade credit bundled with limited liability imposes important restrictions on the contracts that this literature considers[6]. Li and Matouschek (forthcoming) is the closest related paper. They consider a setup where a manager rewards a worker for exerting an observable (but not contractible) effort with an ex-ante fixed wage and an ex-post discretionary positive bonus. With some probability, there is a shock that makes it costly (and inefficient) to reward the worker, but whether this has occurred is not observed by the agent. Unlike in our framework with a cashless retailer, the manager can borrow without limits. The optimal contract does not involve termination, as the manager can be punished for failing to pay the bonus with larger future payments. If the manager can only borrow a proportion of the output, the characterization of the contract becomes too complicated[7] and the authors assume that in the event of a shock, the manager cannot pay anything to the worker[8]. In this case the optimal contract may involve termination when the amount that can be borrowed is small and the expected profits of the manager are small. The second difference concerns the timing of the shock. In Li and Matouschek's paper the manager always receive the output and the shock occurs at the payment stage, while in our paper the manufacturer is not sure about how much revenues the retailer has obtained.[9]

This paper is also related to the literature where an investor finances entrepreneur who can divert the investment cashflows. Incentives to repay are given by liquidating the

[6] See Malcomson (2012) for a recent survey.

[7] The Public Perfect Equilibrium frontier is non-differentiable.

[8] This is akin to setting the size of the shock, s, to zero in our Example in Section 3.

[9] This different becomes starker in the case with an extreme shock. In this case the manager keeps all the output and pays nothing the worker, while the retailer has no profits at all.

entrepreneur's assets (Hart and Moore (1998)), threatening to withhold the second (and last) period investment (Bolton and Scharfstein (1990)) or carrying on an audit as in the costly state verification models (Townsend (1979) and Gale and Hellwig (1985)). As in this literature, we find that the optimal contract is a debt contract[10]. Povel and Raith (2004) allow for the size of the investment to be secretly chosen by the entrepreneur as well as how much to repay and the probability of liquidating. Their focus is on showing that the optimal contract is still a debt contract. As in our model, they also find that the entrepreneur under-invests to decrease the fixed repayment and hence the likelihood of defaulting[11]. However, in our model, because the manufacturer has the bargaining power, the distortion is not only motivated to soften the (inefficient) termination policy but also to increase her share of the profits. As a result, the quantity distortion is even larger.[12] Another fundamental difference is the use of a relational contract in the analysis. The repeated game approach formalizes an economic concept of trust, which has been shown to play a crucial role in trade credit provision. Formally, we endogenize the future value that accrues to the retailer if he does not default, as it corresponds to the potential profits generated within the relationship.

Finally, DeMarzo and Fishman (2007a, 2007b) and the literature thereafter, considers a multi-period model of Bolton and Scharfstein (1990) and find the optimal long-term contract can be implemented by a combination of equity, long-term debt and a line of credit. Important differences with this literature is that our manufacturer can choose the quantity he gives to the retailer as well as the length of the termination.

The paper proceeds as follows. Section 2 introduces the model. Section 3 explores a simple example where an unlucky retailer may lose part of his revenues before paying to

[10] Innes (1990) finds that debt contracts are optimal in an environment with moral hazard with limited liability and observable output. A debt contract gives the best incentives as it makes the agent residual claimant in the good times and penalises him in the bad times by extracting all the surplus.

[11] In Green and Porter (1984), when quantities are chosen from a sufficiently fine grid of points, a similar result emerges. In particular, firms "produce quantities larger than the monopoly output to reduce the incentives to deviate from equilibrium play, which in turn allows equilibrium punishments to be less severe. Because punishments actually occur in equilibrium, this reduced severity is valuable". (Mailath and Samuelson (2006), p. 353).

[12] See footnote 19.

the manufacturer. Section 4 generalizes the previous example to a continuum of possible demand states. Finally, Section 5 concludes. The proofs are relegated to the Appendix.

2 The Setup

A manufacturer and a retailer have the opportunity to trade at dates $t = 0, 1, 2, \ldots.$ In each period, the manufacturer produces a good at a marginal cost $c > 0$ and needs a retailer to market the product to the final consumer. The retailer can sell the good at no cost but he is completely credit constrained and needs to be fully financed by the manufacturer in order to sell. The manufacturer offers him trade credit, that is, he pays the manufacturer back after selling the good but within the same period (i.e., no interest rate is charged). To keep the analysis simple, we assume that the retailer is not able to save (i.e. any profits are consumed within the same period) and therefore we can restrict attention to perfect public equilibria.

To remove double marginalization distortions, we let the manufacturer offer[13] a quantity forcing contract. Since the retailer is cashless, there are not many instruments available to the manufacturer to punish a default. We let the manufacturer use the threat of a T period termination as a mean to provide incentives. Choosing the length of termination is equivalent to choosing a probability of a permanent termination.[14] An alternative interpretation to the termination is to keep trading but in less profitable terms (for instance by diminishing the quality of the good).[15]

We denote by $0 < \delta < 1$ the discount factor and we assume that in the periods of no trade, both firms get a constant outside option which is normalized to 0.

The timing is summarized in Figure 1. In each period, the manufacturer offers a contract to the retailer. The retailer rejects or accepts and if he accepts, he places an order in the market. Then an *iid* shock is realized (and observed only by the retailer)

[13] We assume that the manufacturer has all the bargaining power. We discuss in footnote 19 the implications of this assumption.

[14] Denote by t the probability of terminating forever with the retailer. Then $\delta^T = 1 - t$.

[15] See Baker, Gibbons and Murphy (1994) for an example in the employer-employee relationship framework.

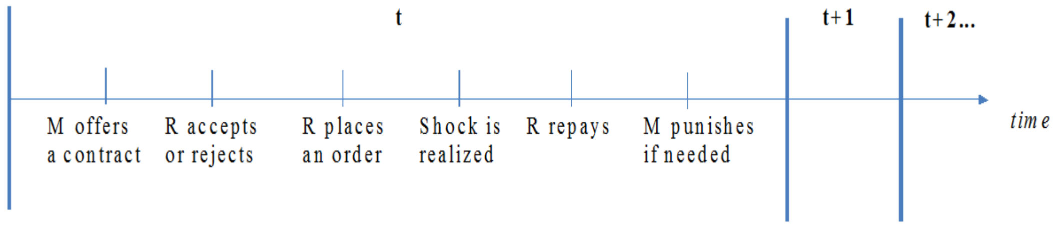

Figure 1: Timing of the game. M stands for Manufacturer and R for Retailer.

which determines the size of the revenues. Finally, the retailer decides how much to repay and the contract is terminated for a number of periods if it is specified in the contract.[16]

The effect of the shock is to add randomness to the revenues of the retailer. For instance, there may be uncertainty with the respect to the willingness to pay of final consumers or the demand may be certain but either the goods or the revenues are stolen now and then (for instance, by an organized crime group). Similarly, the setup could also represent a situation where the uncertainty refers to how many units of a perishable or non-perishable but non-returnable good are demanded every period in the market. If selling the good would be costly for the retailer, the shock could be on the retailer's costs as well.

3 Example

As an example, we consider the case where the retailer receives revenues $R(q)$ from selling the good. However with probability p, there is a shock and the revenues are $sR(q)$ instead, where $0 \leq s < 1$.[17] The manufacturer offers a quantity forcing contract $\{q, D_H, D_L, T\}$ where q is the quantity, D_H is the repayment if demand is high and D_L if it is low. In order to give incentives to repay the true amount, the manufacturer terminates the

[16] Termination involves inefficiencies just as in the subjective performance evaluation of Levin (2003) or in Fuchs (2007). One simple way to make the contract renegotiation-proof is to give the bargaining power to the retailer. In this case, the manufacturer receives no rents and is indifferent between terminating or keep on trading. See footnote 19 for more details.

[17] We use this multiplicative functional form for the revenues because of its simplicity, but the results are robust to using a general function. This functional form is, for instance, used in Green and Porter (1984).

contract for T periods following D_L and forever following a smaller payment[18].

In a benchmark situation with law enforcement and symmetric information, the parties maximize their joint expected one-period profits: $E(s)R(q) - cq$, where $E(s) = 1 - p + ps$. The resulting first best quantity is then determined by:

$$q^{FB} : R'(q) = \frac{c}{E(s)} = \widetilde{c}$$

where $\widetilde{c}$ is the effective marginal cost, which accounts for the likelihood of the shock. This is the relevant marginal cost against which we make comparisons. The parties sell less than in the absence of the shock because with some probability they do not receive the entire revenues but incur the production costs anyway.

Let us now explore the implications of asymmetric information and the absence of legal contract enforcement. Let π_R denote the retailer's present discounted value from date t on when repaying as much as he can:

$$\pi_R = (1-p)\left[R(q) - D_H + \delta\pi_R\right] + p\left[sR(q) - D_L + \delta^{T+1}\pi_R\right]$$

The previous equation says that with probability $1-p$ there is no shock and the retailer pays back D_H to the manufacturer. In this case, she renews the contract and the game remains in the cooperative phase the next period. However, with probability p, there is a shock that "destroys" part of the revenues. The retailer can only pay back D_L and the game moves to the trade suspension phase the next period. In this case, the retailer will earn again π_R only after the end of the punishment phase of T periods.

In a similar way, let π_M be the present discounted value for the manufacturer:

$$\pi_M = (1-p)\left[D_H - cq + \delta\pi_M\right] + p\left[D_L - cq + \delta^{T+1}\pi_M\right]$$

[18] When $s > 0$, the retailer can repay some amount following a shock. Not repaying anything or repaying something smaller than D_L implies stealing, in which case it is optimal to impose the maximum punishment because it relaxes the dynamic enforcement constraint and it is not imposed in equilibrium. When $s = 0$, then $D_L = 0$ and the manufacturer punishes for T periods following no repayment.

With probability $1-p$, the retailer repays D_H so the relationship move on to the next period and with the complementary probability, the retailer only repays D_L and the manufacturer terminates the contract for T periods.

Since the shocks are not observable to the manufacturer, she needs to ensure that the retailer has incentives to report the true demand. Because the retailer is cashless, only a high demand retailer can pretend to have low demand revenues:

$$R(q) - D_H + \delta \pi_R \geq R(q) - D_L + \delta^{T+1} \pi_R \tag{IC}$$

The incentive compatibility condition, IC, reflects the following retailer's trade-off: if he does not pay back the appropriate amount, he keeps $D_H - D_L$ but this will automatically trigger the termination phase, which yields valuation π_R only after T periods. For the retailer to pay back D_H, the manufacturer needs to ensure that tomorrow's gains from not being terminated are larger than the difference in payments today: $\delta\left(\pi_R - \delta^T \pi_R\right) \geq D_H - D_L$. The IC is not satisfied if the repayments are different and the manufacturer never punishes. Similarly, the tougher the punishment, the easier it is for the manufacturer to satisfy the constraint.

The manufacturer also needs to give the retailer incentives to stay within the relational contract. The dynamic enforcement constraints ensure that a low (respectively, high) demand retailer does not want to walk away with all the revenues. These constraints are:

$$sR(q) - D_L + \delta^{T+1} \pi_R \geq sR(q) \tag{DE}$$

and

$$R(q) - D_H + \delta \pi_R \geq R(q) \tag{DE_H}$$

respectively. The future expected value of being within the non-cooperative (cooperative) phase, rather than being terminated forever, needs to be larger than the low (high) demand payment. Note that if DE is satisfied, an $L-type$ retailer does not want to walk away from the relation and that if IC holds, an $H-type$ retailer gets some rent to report a high demand, therefore, an $H - type$ does not want to walk away either and we can ignore DE_H. Note as well that DE remains relevant even if there was no asymmetric information (for instance, if s were to be 1) as this constraint addresses the lack of ability to enforce repayment rather than the ability to observe outcomes and actions.

In addition to the IC and DE constraints, a cashless retailer cannot be forced to make a repayment exceeding the revenues he reports in the current period: $D_H \leq R(q)$ and $D_L \leq sR(q)$. We call the limited liability constraints LLs.

Using the above information, the manufacturer's problem becomes:

$$\max_{q,D_H,D_L,T} \pi_M = \frac{(1-p)\,D_H + pD_L - cq}{1-\delta\,(1-p) - p\delta^{T+1}}$$
$$\text{s.t.} \quad (IC),\ (DE),\ (LL_H) \text{ and } (LL_L)$$

where $\pi_R = \frac{E(s)R(q)-[(1-p)D_H+pD_L]}{1-\delta(1-p)-p\delta^{T+1}}$.

By inspection of π_M, the manufacturer always wants to increase D_H and D_L as much as possible, therefore at least two constraints need to bind to bound D_H and D_L from above. We show in the Appendix that the IC is always binding and that the LL_H is never binding. The asymmetric information problem is always present and since the manufacturer need to give rents in at least one state to make the relationship valuable, she prefers to do so in the high state in order to relax the informational problem. Therefore, we are left with three possible combinations of binding constraints: $IC-DE$, $IC-DE-LL_L$ and $IC-LL_L$.

By rewriting π_M in terms of π_R: $\pi_M = \frac{E(s)R(q)-cq}{1-\delta(1-p)-p\delta^{T+1}} - \pi_R$, we see that the manufacturer maximizes the total surplus minus how much she gives to the retailer. She cannot appropriate all the surplus and the ultimate amount of surplus left to the retailer depends on which of the constraints are binding. To the extent that π_R increases with q, it is as if the manufacturer had an extra marginal cost and, hence, we expect quantity distortions to emerge.[19]

[19] When the retailer has the bargaining power, he maximizes: $\pi_R = \frac{E(s)R(q)-cq}{1-\delta(1-p)-p\delta^{T+1}} - \pi_M$ subject to (IC), (DE), (LL_H), (LL_L) and the participation constraint of the manufacturer, $\pi_M = 0$. We conjecture

In what follows, we characterize each of the three regimes relegating tedious computations to the Appendix. At the end of this section, we discuss more precisely how the occurrence of each regime depends on the size of the shock and illustrate it with a numerical example.

3.1 Flat contract

In this section we show that when the demands are very similar (s large) or the future is not very valuable (δ small), the optimal contract establishes a unique payment regardless of the demand state (that is, a flat or pooling contract) and a permanent termination if this repayment is not met[20].

This case occurs when $\delta E(s) \leq s$[21] and involves the IC and DE binding. These constraints bound the repayments by how much the retailer can obtain next time he has the opportunity to walk away with the revenues: $D_H = \delta E(s)R(q)$ and $D_L = \delta^{T+1}E(s)R(q)$. If he repays D_H, that would be tomorrow while if he repays D_L, it would be the period after the termination ends.

Using this information, the manufacturer's problem is:

$$\max_{q,T} \pi_M = \frac{\left((1-p)\,\delta + p\delta^{T+1}\right) E(s)R(q) - cq}{1 - \delta\,(1-p) - p\delta^{T+1}}$$

Note that T decreases the objective function: not only it increases the inefficiency from terminating following a shock (effect in the denominator) but it also reduces D_L (effect in the numerator). As a result, the manufacturer never punishes following D_L, that is $T = 0$, which implies by the IC a unique repayment: $D_H = D_L = \delta E(s)R(q)$. A flat contract removes the asymmetric information problem as the retailer can always

the quantity distortion to be significantly smaller. The reason is twofold: first, the retailer does not need to share the surplus with the manufacturer (i.e. the second part of the objective function is zero), and second, since the retailer keeps all the profits, we expect (DE) not to be binding (i.e. the retailer does not have incentives to walk away from the contract).

[20] Termination does not occur in equilibrium.

[21] This condition is implied by a slack LL_L once the optimal D_L is introduced.

repay regardless of the state of the demand. The optimal quantity is given by:

$$R'(q) = \frac{c}{\delta E(s)} > \widetilde{c} \tag{1}$$

Note that a larger δ or a smaller p increases the total repayment and the quantity sold.

Proposition 1 *When the shock or the discount factor are small, the manufacturer only needs to address the dynamic enforcement problem. The manufacturer offers a flat contract and, despite the use of a quantity forcing contract, the quantity is distorted downwards.*

The intuition behind this result is the following. When the low demand is nonetheless large, inducing separation is too costly and the manufacturer asks for the same repayment to avoid wasteful termination periods. This is also the case when δ is small and the enforceability problem is very tight as a result. The dynamic enforceability is then relaxed by both decreasing D_L (below the limit established by LL_L) and getting rid of the retaliation policy. The implication of asking for the same repayment is that the asymmetric information problem disappears. This case is equivalent to the one where the shocks are observable (and LL_L does not bind).[22] The quantity distortion is exclusively the result of the rents that the manufacturer needs to give to the retailer so he does not walk away from the relationship.[23]

When $s < \delta E(s)$, the limited liability constraint, LL_L, always binds. Then we can have two possible cases depending on whether the dynamic enforcement binds (Section 3.2) or not (Section 3.3).

[22] To see this, imagine that the shocks are observable. The manufacturer asks for D_H if demand is high, D_L if demand is low and terminate forever if the appropriate payment is not made. Therefore, the manufacturer needs to replace (IC) by (DE_H) and transform (DE) in the following way: $\delta\pi_R \geq D_L$. These two constraints imply: $D_H = D_L = \delta E(s)R(q)$, and hence we obtain the same solution.

[23] Note that if LL_L is satisfied, LL_H is also satisfied because the high revenues are larger for any q.

3.2 Debt contract

The manufacturer always offers a debt contract when the three constraints bind ($IC - DE - LL_L$). In the corporate finance literature, a debt contract establishes a fixed repayment and leaves no money to the borrower when he defaults. In this example, the debt contract consist in a repayment D_H. When this repayment is not met, the retailer has to give back all the low demand revenues and is subject to a termination policy.

More precisely, LL_L pins down the payment in the low state: $D_L = sR(q)$ and the IC and DE jointly determine D_H and the termination length: $D_H = \delta E(s)R(q)$ and $\delta^{T+1}E(s) = s$ where $0 < T$[24]. As in the previous section, the high repayment is bounded by the amount obtained from walking away the next period. The difference in repayments, $D_L < D_H$, requires the manufacturer to terminate the contract for a positive number of periods after a low payment. The termination length is chosen so that the retailer is indifferent between stealing the low revenues today, $sR(q)$, or the expected revenues the next chance he has to do it, $\delta^{T+1}E(s)R(q)$. Termination forever occurs when the shock destroys all the revenues ($s = 0$).

The optimal quantity maximizes:

$$\max_q \pi_M = \frac{((1-p)\,\delta E(s) + ps)\,R(q) - cq}{1 - \delta(1-p) - \frac{ps}{E(s)}}$$

and it is determined by:

$$R'(q) = \frac{c}{(1-p)\,\delta E(s) + ps} > \widetilde{c} \tag{2}$$

Proposition 2 *When the shock or the discount factor are large, the limited liability constraint binds. When DE binds, the manufacturer needs to address both the asymmetric information and the dynamic enforcement problems. The manufacturer offers a debt contract and, despite the use of a quantity forcing contract, the quantity is distorted downwards.*

Note that there is a tension between addressing the asymmetric information problem

[24] T is positive because the parameter specification is such that $\delta < \frac{s}{E(s)}$.

(IC) and giving incentives to the retailer to keep trading (DE). Indeed, from IC, the manufacturer would like to increase D_L[25] and T[26] as much as possible to give incentives to the high demand retailer to say the truth. At the same time, from DE, a larger D_L and T decreases the value of the relationship for a low demand retailer and hence increases his incentives to walk away.

Finally, a larger δ directly increases D_H and hence a larger quantity is offered. Instead, when p increases, both repayment amounts as well as q decrease. Since D_H decreases proportionally more a smaller T is needed (see Appendix).[27]

3.3 Flat or debt contract

When DE does not bind, the manufacturer can offer either a debt or a flat contract depending on the parameter specification. In any case, because of the limited liability constraint, the manufacturer recovers as much revenues as possible from the low demand state: $D_L = sR(q)$. The IC bounds the high demand payment to: $D_H = \alpha(T)\delta E(s)R(q)$, where $\alpha(T) = \frac{1-\delta^T + \frac{(1-\delta)s}{\delta E(s)}}{1-\delta^{T+1}}$. The $\alpha(T)$ defined for $T \in \left[0, \log_\delta\left(\frac{s}{E(s)}\right) - 1\right)$[28] is an increasing function[29]. When $T = 0$, the manufacturer offers a flat contract with a unique repayment equal to $sR(q)$. A larger T is associated with a larger D_H. However, $\alpha(T) < 1$ for DE to be slack.

Using this information, the manufacturer maximizes:

$$\max_{q,T} \pi_M = \frac{((1-p)\alpha(T)\delta E(s) + ps)\, R(q) - cq}{1 - \delta(1-p) - p\delta^{T+1}}$$

[25] An increase in D_L decreases the RHS of IC. It also decreases the LHS indirectly through a decrease in π_R but to a less extent so overall the first effect dominates.

[26] Note that $\frac{1-\delta^T}{1-\delta(1-p)-p\delta^{T+1}}$ is increasing in T.

[27] Note that LL_H is always satisfied as it requires: $\frac{\delta ps}{1-\delta(1-p)} < 1$.

[28] We show in the Appendix that when $T = \log_\delta\left(\frac{s}{E(s)}\right) - 1$, which is the optimal punishment of Section 3.2, the DE constraint binds.

[29]
$$\frac{\partial \alpha(T)}{\partial T} = \frac{-\delta^T(\ln\delta)(1-p)(1-s)(1-\delta)}{E(s)\left(1-\delta^{T+1}\right)^2} > 0$$

The quantity is determined by:

$$R'(q) = \frac{c}{(1-p)\alpha(T)\delta E(s) + ps} > \widetilde{c} \tag{3}$$

where T is the optimal punishment (defined in the Appendix). A flat contract (i.e. $T = 0$) is offered when:

$$\frac{1-p}{p} \frac{E(s) - s}{s} \leq \frac{1}{\varepsilon} \tag{4}$$

where ε is the elasticity of demand at the optimal quantity[30]. Otherwise a debt contract is offered. The proof is in the Appendix where we also show that there is downward distortion in the quantity.[31]

Proposition 3 *When the shock or the discount factor are large, the limited liability constraint binds. When DE does not bind, the manufacturer only needs to address the asymmetric information problem. The manufacturer offers a debt contract if the probability and the size of the shock are small enough and a flat contract otherwise. Despite the use of a quantity forcing contract, the quantity is distorted downwards. When the manufacturer offers a debt contract, a shorter punishment is associated with a larger quantity distortion.*

To tackle the asymmetric information problem, the supplier has to leave surplus in the high revenue state so that this state is reported, which explains the quantity distortion. There is a trade-off between a tougher punishment and a smaller quantity, because a longer termination decreases the incentives of the retailer to steal, allowing the manufacturer to keep more surplus. With a tougher punishment, the manufacturer can ask for a larger D_H and offer a larger q. However, increasing the punishment comes at a cost: it increases the inefficiency following a shock as she terminates profitable trading with the

[30] We break down the revenue function as follows: $R(q) = p(q)q$ where $p(q)$ is the inverse demand function. The elasticity of demand is defined as:

$$\frac{1}{\varepsilon} = -\frac{p'(q)q}{p(q)}$$

[31] The LL_H is always satisfied as it requires: $s < 1$.

unlucky retailer. When p is large or s large, the cost associated with termination (i.e. losing the expected revenues of using a flat contract, $sR(q) - cq$) is larger than the benefit of increasing the repayment in the high state by: $(E(s) - s) R(q)$ and a debt contract is offered.[32]

3.4 The value of the future

From the previous analysis, it emerges that even though the manufacturer has enough instruments to set the quantity, she chooses to sell less than the efficient amount in each of the three regimes. This result remains even as the parties become arbitrarily patient.

Proposition 4 *As the discount factor tends to* 1*, the outcome is always bounded away from efficiency.*

Proof. See Appendix. ■

Intuitively, when LL_L does not bind, the manufacturer is better off asking for the same quantity regardless of the demand realization to avoid using a termination policy to separate the types. Since only the dynamic enforcement problem is present in this regime, one would expect the quantity to converge to the efficient level as δ tend to 1. However, as δ tends to 1 this regime never occurs. As the discount value increases, the retailer values more the relationship and the dynamic enforcement constraint stops binding. The manufacturer would like to increase D_L to make DE binding again, but cannot because of the limited liability. She could increase D_H instead, but since D_L is fixed by the limited liability, she would need to increase T to satisfy IC again. However, this is very costly for the manufacturer because δ is large, as a result, she does not increase the repayments and the quantity distortion persists (regardless of whether trade suspension is used or not)[33].

It is worth highlighting that it is the impatience of the retailer the one that is creating the downward distortion in the quantity. Indeed, if the discount factor for the retailer

[32] Note that $\varepsilon < 1$, because of the downward distortion. Using the elasticity where a monopolist prices, $\varepsilon = 1$, we provide a sufficient (but not necessary) condition for a flat contract to be offered: $\frac{(1-p)^2}{p+(1-p)^2} \leq s$.

[33] There is quantity distortion in Section 3.3 even when $T = 0$.

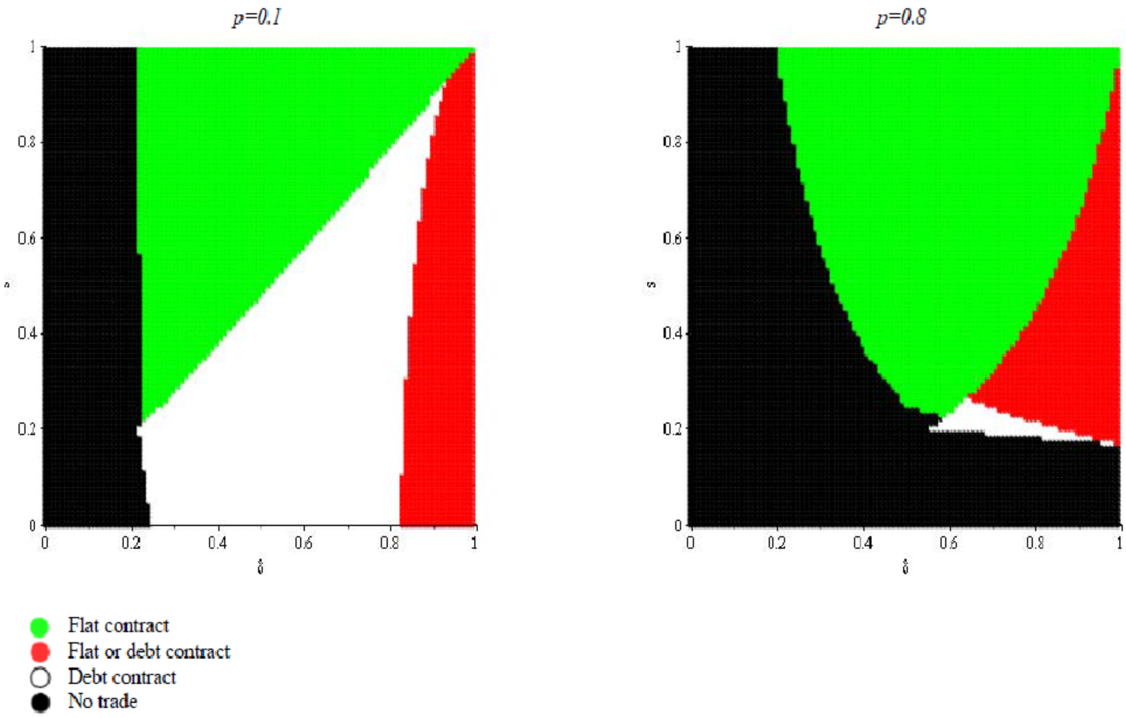

Figure 2: Optimal regime when $R(q) = (10 - q)\, q$ and $c = 2$.

were different from the one for the manufacturer, then it would be retailer's discount factor appearing in conditions (1), (3) and (2). In contrast, both discounts factors would affect the choice of the termination policy.

Finally, to illustrate how the interaction between the different parameters determines the regimes, we use the linear demand. Figure 2 depicts which regime yields the largest profits for the manufacturer in the space δ and s for different likelihoods of the shock.[34]

The left hand side graph in Figure 2 depicts the case where the likelihood of the shock is low and equal to $p = 0.1$. When the value of the future is small no contract can be implemented and hence no trade occurs.[35] If the demands in both states are similar (or δ small, i.e. $\delta E(s) < s$), then the manufacturer asks for the same payment. As the

[34]The line that separates the "Flat contract" from the "Debt contract" area is $s = \frac{\delta(1-p)}{1-\delta p}$ and the line that separates the "Debt contract" from the "Flat/Debt" contract area is $s = \frac{\delta^{T+1}(1-p)}{1-p\delta^{T+1}}$ where T is the optimal termination policy at the border between the regimes, $T = \log_\delta \left(\frac{s}{E(s)}\right) - 1$.

[35]Although this is out of the scope of this model, the no trade situation could also correspond to a manufacturer that vertically integrates downwards and serves the consumers.

low demand becomes smaller (or δ larger), then LL_L starts binding. The manufacturer can either ask for a debt contract and punish when D_L is repaid or possibly ask for a flat contract as δ increases because the value of the relationship increases and DE stops binding. The right hand side graph depicts a shock of likelihood $p = 0.8$. As p increases, less contracts can be self-enforced (for low s and δ). Since the shock is more likely and the manufacturer does not want to terminate with an unlucky retailer who cannot pay, a flat contract becomes more common.

4 The Model

In this Section, we generalize the example by exploring the scenario where the retailer faces a continuum of demand states. The size of the shock s is an *iid* random variable distributed on the interval $[\underline{s}, \overline{s}]$ with $h(s)$ and $H(s)$ as the density and cumulative distribution functions, respectively. Denote the revenues for a given state s as $R(q; s)$, where $\frac{\partial R(q;s)}{\partial s} > 0$ and $R(q; 0) = 0$, and the expected revenues as $R_E(q) = \int_{\underline{s}}^{\overline{s}} R(q; s)h(s)ds$. We are going to assume $\underline{s} = 0$, that is, the largest shock "destroys" all the revenues. We do this to rule out regime where the limited liability does not matter (Section 3.1) so to focus on the effect that liquidity constraints have on the optimal contract.[36] As in the example, the manufacturer offers a quantity q, a repayment $D(\widetilde{s})$ and a termination policy $T(\widetilde{s})$, for each particular shock reported $\widetilde{s}$.

We first find the conditions under which the contract is incentive compatible and/or dynamically enforceable and then we proceed to characterize the optimal contract. We conclude the Section with an example.

4.1 Incentive compatibility and dynamic enforceability

First, we establish the structure of the contract and then we lay down the conditions under which the retailer reports the true state of demand and does not have incentives

[36]We conjecture that if $\underline{s}$ is large enough, the limited liability constraint will not bind. The manufacturer nonetheless has to address the asymmetric and dynamic enforceability problem and will ask for a unique repayment equal to $D = \delta R_E(q)$ where q is the optimal quantity.

to walk away from the contract.

Since the quantity is chosen before the state of the demand is realized, it is not used for sorting purposes and hence does not depend on the report $\widetilde{s}$. For a given state of the demand s and a given quantity q, the retailer chooses a report $\widetilde{s}$ to maximize his profits:

$$\pi_R(\widetilde{s}; s) = \underbrace{R(q; s) - D(\widetilde{s})}_{\text{Today's payoff of reporting } \widetilde{s}} + \underbrace{\delta^{T(\widetilde{s})+1}\pi_R}_{\text{Expected payoff of reporting } \widetilde{s}}$$

where π_R is the expected present discounted profits from staying in the relationship. Note that if the retailer were not credit constrained, the choice of $\widetilde{s}$ would not depend on the true s as it is equally costly for any type of retailer to report any $\widetilde{s}$. In other words, there is no sorting condition in this model, $\frac{\partial^2 \pi_R(\widetilde{s}; s)}{\partial \widetilde{s} \partial s} = 0$, as in Levin (2003). Because the retailer is credit constrained, however, he cannot repay a larger $D(\widetilde{s})$ than his actual revenues $R(q; s)$:

$$D(\widetilde{s}) \leq R(q; s) \quad \forall s, \widetilde{s} \tag{5}$$

The limited liability condition (5) links the choice of the report with the true state of demand.

For a given q, the manufacturer has two instruments to deter the retailer from under-reporting the state of the demand: following a low $\widetilde{s}$, she can either increase the payment today $D(\widetilde{s})$ or increase the length of the termination period $T(\widetilde{s})$, which decreases the retailer's continuation value. Given that increasing $T(\widetilde{s})$ is also costly for the manufacturer (because she loses future trade), whenever it is possible, the manufacturer asks for the largest repayment: $D(\widetilde{s}) = R(\widetilde{s}, q)$.

In equilibrium, however, the manufacturer cannot extract all the revenues from the retailer in all the states, as this makes the relationship worthless to the retailer (i.e., $\pi_R = 0$). There must be a report s^* for which the manufacturer does not ask for all the revenues and, consequently, does not terminate, $T(s^*) = 0$[37]. The retailer's profits in this

[37] If the manufacturer were to terminate for some periods following the report s^*, she would strictly prefer to increase the repayment to decrease the termination length (and hence to reduce the inefficiency generated by the no-trade situation).

case are:

$$\pi_R(s^*; s) = R(q; s) - R(q; s^*) + \delta\pi_R \tag{6}$$

Hence for $\widetilde{s} \geq s^*$, the manufacturer can only offer the contract: $T(\widetilde{s}) = 0$ and $D(\widetilde{s}) = R(q; s^*)$, because she can no longer decrease the punishment period to compensate the retailer for a larger repayment. Since the largest shock leads to zero revenues, the limited liability constraint (5) is always binding for some states and hence $s^* > 0$. To summarize, the manufacturer offers the following repayment schedule:

$$D(\widetilde{s}) = \begin{cases} R(q; \widetilde{s}) \text{ if } \widetilde{s} < s^* \\ R(q; s^*) \text{ if } \widetilde{s} \geq s^* \end{cases}$$

and termination policy:

$$T(\widetilde{s}) = \begin{cases} T(\widetilde{s}) \text{ if } \widetilde{s} < s^* \\ 0 \text{ if } \widetilde{s} \geq s^* \end{cases}$$

where $T(\widetilde{s})$ is to be defined[38]. The contract offered by the manufacturer resembles to what it is known in the corporate finance literature as a debt contract. Debt contracts leave no money to the borrower in the bad states while making the borrower the residual claimant in the good states.

Lemma 1 *The optimal contract is a debt contract.*

A debt contract is optimal in this model because it minimizes the inefficiency associated with the termination (by trading-off larger repayments for lower termination periods in the default states) while inducing the retailer to report the true.

Let us proceed to lay down the conditions under which this contract induces truth-telling. Since $\frac{\partial^2 \pi_R(\widetilde{s}; s)}{\partial \widetilde{s} \partial s} = 0$, let us denote by $u(\widetilde{s})$ the part of the retailer's payoff that does not depend on his type s:

$$u(\widetilde{s}) = -R(q; \widetilde{s}) + \delta^{T(\widetilde{s})+1}\pi_R \tag{7}$$

[38] In equation (8).

The independence between the incentives to report a demand state and the actual demand state makes the task of inducing truth-telling simple. Intuitively, if it were feasible, the retailer would always report the demand state $\widetilde{s}$ with the highest $u(\widetilde{s})$, regardless of the true s. To induce the retailer to report the true state of demand, he needs to be indifferent between reporting the truth or underreporting the demand. Thus, truth-telling is achieved when $u(\widetilde{s})$ is constant: $u'(\widetilde{s}) \mid_{\widetilde{s}=s}= 0 \; \forall \widetilde{s} \leq s$. Furthermore, the contract needs to be self-enforceable. The retailer does not want to walk away from the contract if the repayment is weakly smaller than the continuation value: $u(\widetilde{s}) \mid_{\widetilde{s}=s} \geq 0 \; \forall s$. In terms of the Example in Section 3, the first constraint corresponds to the incentive compatibility constraint, IC, and the second to the dynamic enforcement constraint, DE.

Proposition 5 *Truth-telling is achieved when $\pi_R(\widetilde{s}; s)$ is constant as a function of the report $\widetilde{s}$:*

$$\frac{\partial \pi_R(\widetilde{s}; s)}{\partial \widetilde{s}} \mid_{\widetilde{s}=s}= 0 \; \forall_{\widetilde{s} \leq s} \tag{IC_s}$$

Given that the retailer can always run away with the current revenues, $R(q; s)$, dynamic enforcement is ensured if:

$$\left(R(q; \widetilde{s}) \leq \delta^{T(\widetilde{s})+1} \pi_R \right) \mid_{\widetilde{s}=s} \forall \; s \tag{DE_s}$$

Proof. See Appendix. ■

From Lemma 1, when $s^* \leq s$, telling the truth results in $u = -R(q; s^*) + \delta \pi_R$ and misreporting $\widetilde{s} < s^*$[39] results in $u = -R(q; \widetilde{s}) + \delta^{T(\widetilde{s})+1} \pi_R$. The retailer chooses to tell the truth if he is indifferent between both options, which give us condition (8).

$$\delta^{T(\widetilde{s})+1} \mid_{\widetilde{s}=s}= \frac{R(q; s) - R(q; s^*) + \delta \pi_R}{\pi_R} \tag{8}$$

Equation (8) uniquely defines $T(s)$. Note that when the retailer reports $s = 0$, if $u > 0$, the manufacturer terminates for a finite number of periods[40], while if $u = 0$, she

[39] Note that misreporting $s^* < \widetilde{s} < s$ would lead to the same payoff for the retailer.
[40] That is, $\delta^{T(0)+1} > 0$.

terminates forever. In the last case, equation (8) becomes $\delta^{T(s)+1} = \frac{R(q;s)}{\pi_R}$. In both cases, a larger report is coupled with a shorter punishment period.

Before solving the problem of the manufacturer, let us find π_R. Using Lemma 1:

$$\pi_R = \int_0^{s^*} \left[R(q;s) - R(q;s) + \delta^{T(s)+1}\pi_R\right] h(s)ds + \int_{s^*}^{\overline{s}} \left[R(q;s) - R(q;s^*) + \delta\pi_R\right] h(s)ds$$

The retailer gives back all his revenues if the shock is smaller than s^* and has the contract terminated for $T(s)$ periods. Otherwise, he repays the constant amount $R(q;s^*)$ and keeps trading with the manufacturer the next period. Using condition (8), π_R can be simplified to:[41]

$$\pi_R = R_E(q) - R(q;s^*) + \delta\pi_R$$

because, by (ICs), the repayment plus the continuation value should be constant and equal to u (or equivalently, $-R(q;s^*) + \delta\pi_R$) for all the possible states. Consider a first best world where the retailer is not credit constrained. Then an optimal contract could be a quantity (chosen so that the expected revenues minus the production costs are maximized) and a fixed up-front payment from the retailer. If the manufacturer has all the bargaining power, then this payment would be equal to the expected revenues (and hence the manufacturer would extract all the rents, $\pi_R = 0$) and the retailer would never have his contract terminated.[42] We can interpret the above equation in these terms, where the retailer keeps the expected profits minus a fixed payment, $R(q;s^*)$, and he never has his contract terminated. The difference from the first best world comes in terms of a different quantity and a smaller repayment amount in order to leave some rents to the retailer ($\pi_R > 0$) so it is worth for him to stay in relationship. Solving for π_R yields:

$$\pi_R = \frac{R_E(q) - R(q;s^*)}{1-\delta} \tag{9}$$

Hence, for the expected discounted profits to be positive, it needs to be the case that

[41] If $u = 0$, then $\pi_R = R_E(q)$.

[42] An ex-ante strategically equivalent possibility is for the manufacturer to ask for the obtained revenues in all the states.

the revenues at s^* are smaller than the expected revenues.

Finally, using (8) and (9), the dynamic enforcement condition of Proposition 5 becomes:

$$u = \frac{\delta R_E(q) - R(q; s^*)}{1-\delta} \geq 0 \tag{10}$$

In order to ensure the repayment, the manufacturer needs to guarantee that the retailer obtains at least the difference between what he can walk away with tomorrow if he stays in the relationship and the maximum repayment that he can potentially face today.

4.2 Optimal contract

Using Lemma 1, the profits of the manufacturer when the retailer reports the true revenues are:

$$\pi_M = \int_0^{s^*} \left[R(q; s) + \delta^{T(s)+1}\pi_M\right] h(s)ds + (1 - H(s^*)) \left[R(q; s^*) + \delta\pi_M\right] - cq$$

that is, the manufacturer receives all the revenues in the bad states ($s < s^*$) and terminates the contract with the retailer for $T(s)$ periods. Otherwise, the manufacturer charges a fixed repayment, $R(q; s^*)$, and never terminates the contract. Finally, the manufacturer incurs the production costs.

When choosing s^*, the manufacturer faces the following trade-off: a larger s^* allows the manufacturer to extract a larger expected repayment from the retailer today (and hence π_R decreases); however, by (8), and in order to keep the truth-telling incentives unchanged for all other types, this comes at a cost of a longer termination period.[43] On the other hand, an increase in the quantity leads to an increase in the total payment as well as an increase in the production costs. Since it also leads to an increase in π_R, the

[43] After substituting for $\pi_R(s^*, q)$, the sign of the derivative is negative:

$$\frac{\partial \delta^{T(s,q)}}{\partial s^*} = -\frac{1-\delta}{\delta}\frac{\partial R(s^*, q)}{\partial s^*}\frac{R_E(q) - R(s,q)}{\left(R_E(q) - R(s^*, q)\right)^2} < 0 \ \forall s \leq s^*$$

where the inequality follows from the fact that in order to have $\pi_R(s^*, q) > 0$, it needs to be the case that $R_E(q) > R(s^*, q)$.

effect on the termination policy is less clear and depends on the particular form of the demand function.[44]

Rewriting π_M in terms of π_R, illustrates the fact that the manufacturer cannot appropriate all the surplus:

$$\pi_M = \frac{R_E(q) - cq}{1 - \delta \int_{s^*}^{\bar{s}} h(s)ds - \int_0^{s^*} \delta^{T(s)+1} h(s)ds} - \pi_R$$

As in the Example of Section 3, to the extent that π_R increases with q, it is as if the manufacturer had an extra marginal cost associated with the asymmetric information and enforceability problem. As a result, quantity distortions may emerge.

The manufacturer chooses s^* and q to maximize π_M subject to dynamic enforcement constraint in (10). After replacing π_R, the problem of the manufacturer becomes[45]:

$$\max_{q,s^*} \pi_M = \frac{\int_0^{s^*} R(q;s)h(s)ds + (1 - H(s^*)) R(q;s^*) - cq}{(1-\delta) \frac{\int_{s^*}^{\bar{s}} (R(q;s) - R(q;s^*))h(s)ds}{R_E(q) - R(q;s^*)}} \tag{11}$$

$$\text{s.t.} \quad \frac{\delta R_E(q) - R(q;s^*)}{1-\delta} \geq 0$$

As the future becomes more and more valuable ($\delta \to 1$), the dynamic enforcement constraint (10) no longer binds. Interestingly, when this is the case, the optimal s^* and q do not depend on δ. The optimal termination length, $T(s)$, do depend on δ by the incentive compatibility constraint in (8). The manufacturer would like to increase the all the repayments by the same proportion to profit from the increase in the value of the future while leaving the incentives to misreport unaffected. However, because the limited liability is binding for the lower states ($s \leq s^*$), she can only enlarge the set of states where she asks for all the revenues (i.e., implement a larger s^*). To give the retailer incentives to report the truth, she will need to implement longer termination periods which are very costly when the future is valuable. Thus, overall, once δ is large enough,

[44] In the multiplicative example that we present below, $T(s)$ does not depend on q.

[45] The retailer's participation constraint is never binding as he always has the option of walking away with the current revenues $R(q;s)$ by reporting $\widetilde{s} = 0$ and hence not repaying anything.

she prefers to translate any further increase in δ into shorter termination periods,[46] rather than larger s^* and q.

In contrast, when (10) binds, both choices depend on δ. The fixed repayment for the states above s^* is equal to what the retailer expects to obtain tomorrow if he does not repay: $R(q; s^*) = \delta R_E(q)$. Therefore, a larger discount factor allows the manufacturer to implement a larger s^*. An increase in s^* does not unambiguously increase the expected repayment because the fixed repayment $R(q; s^*)$ is constrained by $\delta R_E(q)$ and hence remains unchanged. Since the expected repayment remains unchanged, the termination policy does not change either. Thus, the only instrument available for the manufacturer is the choice of q. Note as well that the manufacturer will terminate with the retailer forever if there is no repayment at all.

The following Proposition summarizes this discussion.

Proposition 6 *The optimal s^* and q are independent of the discount factor, δ, if and only if δ is large enough: $\delta > \frac{R(q;s^*)}{R_E(q)}$, where $R(q; s^*)$ is the fixed repayment and $R_E(q)$ the expected revenues.*[47]

Proof. See Appendix. ■

Since the choice of q and s^* depends on the particular revenue function, in what follows, we illustrate these results with an example.

4.3 Example

For simplicity and comparability with the Example in Section 3, we use the multiplicative revenue function: $R(q; s) = sR(q)$. The first best quantity that maximizes the one-period joint profits, $E(s)R(q) - cq$, is determined by:

$$R'(q) = \frac{c}{E(s)} = \widetilde{c}$$

[46] We show this in the next Section.

[47] The full version of this Proposition in the Appendix contains the first-order conditions from which s^* and q are found both for this and lower δ.

where $E(s) = \int_0^{\overline{s}} sh(s)ds$. If on average, the shocks are detrimental, there is reduction in quantity as in the first example.

With this revenue function, the dynamic enforcement constraint (10) becomes (12) and the retailer's profits in (9) become: $\pi_R = \frac{(E(s)-s^*)R(q)}{1-\delta}$, which means that for $\pi_R > 0$, s^* needs to be smaller than the expected shock. The problem of the manufacturer (11) becomes:

$$\max_{q,s^*} \pi_M = \frac{\widehat{E}(s,s^*)R(q) - cq}{(1-\delta)\,\frac{E(s)-\widehat{E}(s,s^*)}{E(s)-s^*}}$$

$$\text{s.t.} \qquad \frac{\delta E(s) - s^*}{1-\delta} \geq 0 \tag{12}$$

where $\widehat{E}(s,s^*) = H(s^*)\,E(s \mid s \leq s^*) + (1 - H(s^*))\,s^*$ is the expected shock that matters to the manufacturer in terms of the repayment established by the debt contract. Since for any $s^* < \overline{s}$, $\widehat{E}(s,s^*) < E(s)$, the manufacturer cannot appropriate all the surplus and there is under-supply of q.

The constraint (12) does not bind if s^* is smaller than the discounted value of the expected shock tomorrow, which happens for a large δ. When the dynamic enforcement constraint binds, s^* is bounded by it so the manufacturer only chooses q. In particular, the manufacturer sells the quantity that maximizes her profits taking into account the repayment that she is expected to obtain given s^*. Conversely, when (12) does not bind, the manufacturer also chooses s^*. A larger s^*, on one hand, increases the expected repayment[48] and hence π_M. On the other, it also decreases π_R and the retailer's incentives to truthfully report the demand state. Hence, a tougher termination policy is needed[49], which increases the inefficiency from the trade suspension.

Corollary 2 *If $R(q;s) = sR(q)$, the optimal s^* and q are determined by these first order*

[48]Note that $\frac{\partial \widehat{E}(s,s^*)}{\partial s^*} > 0$.

[49]This is reflected in the denominator of π_M, which is increasing in s^* because $\frac{\partial \widehat{E}(s,s^*)}{\partial s^*} < 1$.

conditions when $\delta E(s) > s^*$:

$$q : R'(q) = \frac{c}{\widehat{E}(s, s^*)} > \widetilde{c} \tag{13}$$

$$s^* : \frac{R(q)}{cq} = \frac{H(s^*)}{1 - H(s^*)} \frac{E(s) - E(s \mid s \leq s^*)}{\widehat{E}(s, s^*)(E(s \mid s \geq s^*) - s^*) - E(s)(E(s) - s^*)} \tag{14}$$

Otherwise, $s^* = \delta E(s)$ and q is determined by (13). In both cases, the quantity is distorted downwards. The optimal s^* and q are strategic complements.

Despite using a quantity forcing contract, the quantity sold by the manufacturer is always distorted downwards. To see this, imagine that the dynamic enforceability constraint does not bind. The manufacturer sells the first best quantity only if $s^* = \overline{s}$[50], which would make the retailer walk away with the current revenues as his expected profits are then zero. With a binding constraint, an efficient quantity will require $\delta > 1$[51], which is not possible either. Since $\frac{\partial \widehat{E}(s,s^*)}{\partial s^*} > 0$, the downward distortion in the quantity decreases with s^*, regardless of whether the constraint binds or not. Therefore, q and s^* are strategic complements, that is, a tougher punishment[52] policy is accompanied by a smaller quantity distortion. This is because a tougher punishment can be used, instead of giving away surplus, to provide incentives to report the truth. Figures 3 and 4 depict s^* and q as a function of δ for $R(q) = (10 - q)\, q$, $c = 2$ and $s \sim U(0, 1)$.

Trade occurs if the value of the future or level of trust is large enough, $0.45 \leq \delta$. Provided that trade occurs, if the value of the future is intermediate, $\delta < \frac{s^*}{E(s)} = 0.82$, then the dynamic enforcement constraint (12) binds. Both s^* and q increases with δ. This is because an increase in the value of the future relaxes the dynamic enforceability constraint. The manufacturer can appropriate a larger share of the total surplus (s^* increases) and hence offers a larger quantity. For a larger value of the future, the constraint (12) does not bind. Since (13) and (14) do not contain δ, the choice of q and s^* are not affected by δ as in Proposition 6. Therefore, the quantity is distorted downwards

[50] Because only then: $E(s) = \widehat{E}(s, s^*)$.

[51] Indeed, $\delta = \frac{\overline{s}}{E(s)} > 1$.

[52] Termination occurs for a larger set of states.

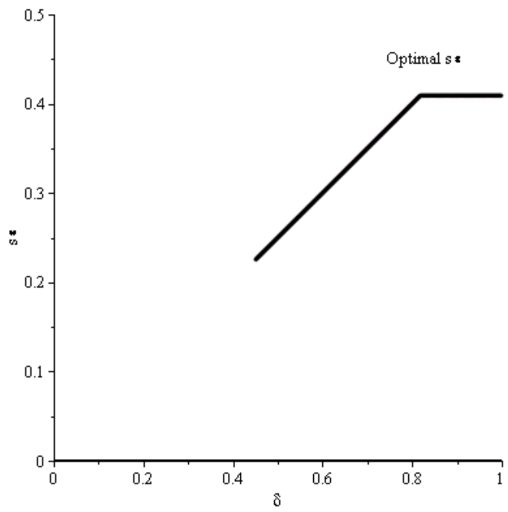

Figure 3: s^* when $R(q) = (10 - q)\,q$, $c = 2$ and $s \sim U\,(0, 1)$

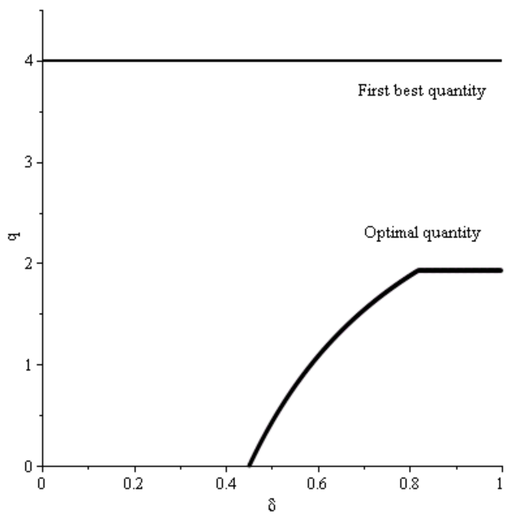

Figure 4: Optimal quantity when $R(q) = (10 - q)\,q$, $c = 2$ and $s \sim U\,(0, 1)$

even for δ arbitrarily close to 1. This is because the manufacturer does not want to keep translating increases in the value of the future into larger s^* because the unavoidable termination (due to limited liability) becomes too costly. Instead, increases in the value of future translate into shorter expected termination periods[53], as it is depicted in Figure 5. The implications of this result are important for policy makers. It suggests that when the level of trust is intermediate, policy efforts at increasing the level of trust or making the contracts more enforceable (that is, increasing the discount factor) will be ineffective at increasing the volume of trade. This is because the limited liability bundled with the asymmetric information problem makes it very costly for the manufacturer to implement a larger quantity. Without the limited liability constraint, the manufacturer could increase all the repayments in the same proportion and appropriate more surplus leaving the asymmetric information problem unchanged. However, the presence of limited liability prevents the manufacturer from increasing lower repayments (for states below s^*). Increasing the repayment for large states only (above s^*) needs to be accompanied with a tougher termination so the the retailer still reports the truth. But a tougher punishment becomes too costly for the manufacturer when the future is valuable and she decides not to increase s^* and q. As a result the dynamic enforceability constraint stops binding. An implication of this result is that when the value of the future is intermediate, policy efforts should be targeted at relaxing credit constraints if the quantity in the market is to be increased.

Finally, we look at the surplus generated in the market and how it is divided between the manufacturer and the retailer. Figure 6 depicts the total per period average profits as well as the per period average profits of the retailer and the manufacturer.

For the manufacturer, an increase in the (inefficiently small) quantity unambiguously increases her profits[54]. Instead, an increase in s^* increases the repayment but also the inefficiency for terminating with an unlucky retailer that is not able to repay $s^*R(q)q$.

[53] Defined as $\int_0^{s^*} T(s)h(s)ds = \int_0^{s^*} \left[\frac{\ln\left(\frac{\delta E(s) - s^* + (1-\delta)s}{E(s)-s^*}\right)}{\ln \delta} - 1 \right] h(s)ds$

[54] The manufacturer's per period average profits are: $(1-\delta)\pi_M = \frac{s^*\left(1-\frac{1}{2}s^*\right)p(q)q - cq}{\frac{(1-s^*)^2}{1-2s^*}}$.

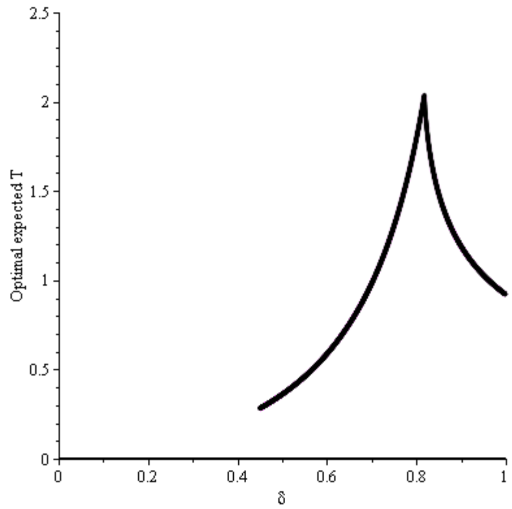

Figure 5: Expected termination when $R(q) = (10 - q)\, q$, $c = 2$ and $s \sim U\,(0, 1)$

Since the manufacturer has all the bargaining power, as δ increases, she keeps increasing s^* and q as long as her profits increases (which happens up to $\delta = 0.82$). After that, she keeps her choices of s^* and q fixed.

For the retailer, and increase in s^* means keeping a smaller share of the surplus, so his profits[55] decreases as a result. However, a larger q means larger expected revenues and hence larger profits for the retailer. Since both s^* and q increase with δ, the retailer's per period profits are non-monotonic in the discount factor and are maximized when $\delta = 0.65$.

Overall, the total surplus[56] is non-monotonic in the discount factor. An increase in s^* decreases the total surplus because of the inefficiency created with the termination while an increase in q increases the total surplus because the quantity is under-supplied. As a result, the total surplus is inverse U-shaped in the discount factor and it is maximized when $\delta = 0.69$.

Corollary 3 *The expected welfare is non-monotonic in the discount factor.*

For intermediate level of trust below $\delta = 0.69$, further increases in δ makes the man-

[55] The retailer's per period average profits are: $(1-\delta)\pi_R = \left(\frac{1}{2} - s^*\right) p(q)q$.

[56] The total per period average profits are: $(1-\delta)(\pi_R + \pi_M) = \frac{\frac{1}{2}p(q)q - cq}{\frac{(1-s^*)^2}{1-2s^*}}$.

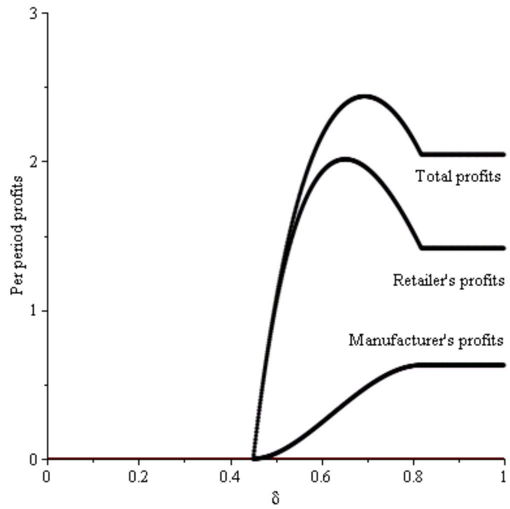

Figure 6: Per period profits when $R(q) = (10 - q)\, q$, $c = 2$ and $s \sim U\,(0, 1)$

ufacturer increase q and terminate for longer periods. In this region, the increase in efficiency from a larger volume of trade compensates the inefficiency from a longer termination. After $\delta = 0.69$, the opposite is true. The manufacturer, nonetheless keeps increasing both q and s^* because this allows her to keep a larger share of the albeit smaller surplus.

5 Conclusions

The goal of this paper has not been to explain why and how much trade credit is offered by a supplier[57]. We take this decision as given, and rather we explore how trade credit affects the different contract characteristics, such as late payment penalty, quantity and price

[57] Answers to these questions can, for instance, be found in Burkart and Ellingsten (2004) that show how trade credit and bank lending are complements because goods are less divertable to private benefits than money. In the same way, Cuñat (2007) shows how suppliers of services and differentiated goods are more willing to sell on credit than suppliers of standarized goods because they may be harder to replace and hence the downstream firm is more reluctant to default. Also, Smith (1987) finds that trade credit may be a consequence of an agency problem. Indeed, if there is quality variation in the good supplied, the downstream firm may be more reluctant to pay before having had the time to inspect the good. Another instance is Daripa and Nilsen (2011) who point out that trade credit mitigates the negative externality on the manufacturer from the retailer's trade off between loss sales and inventory costs.

of the good sold. The main prediction of this analysis is that trade credit does have an important impact on the market outcome. In particular, the quantity sold in the market is expected to be lower than the efficient one. The supplier shares the surplus with the downstream firm to give him incentives to reveal the true demand and/or stay within the contract. This result has implications for the structural empirical industrial organization literature. It shows that having enough instruments to remove vertical externalities from market power may not be enough to assume that efficient quantities are sold in the market when trade credit and limited liability play an important role.

Another important result is that the quantity distortion remains even as the value of the future becomes arbitrarily large. This suggests that policy interventions that aim at making the contracts more enforceable when the level of trust is intermediate will not be successful in restoring efficiency in the market if downstream firms are credit constrained and their ability to repay is unobservable to the supplier. Instead, policy efforts are better expend at relaxing credit constraints.

Finally, we find that the optimal contract resembles a debt contract. Debt contracts are successful in keeping the termination policy to the minimum while still providing the downstream firm incentives to repay the appropriate amount.

In our analysis, we have assumed that the quantity offered by the downstream firm does not change depending on the past repayment history. This framework would be suitable for industries where it is very costly to adjust the production quantity from one period to the other. It would be interesting to explore the form of non-stationary contracts and determine in which way the upstream firm will increase or decrease the quantity in each period as a function of the previous period repayment as well as how this choice interacts with the termination policy. We nonetheless expect quantity distortions to remain as the manufacturer still needs to share the total surplus with the retailer.

6 Appendix

Computations for the Example. We first show that no other regimes exist. Suppose that LL_H binds, then $D_H = R(q)$ and either DE or LL_L need to bind to bound D_L. LL_L binding implies $\pi_R = 0$, which triggers the strategic default by the retailer. DE binding makes the low repayment equal $D_L = \frac{p\delta^{T+1}sR(q)}{1-\delta(1-p)}$. With these two repayments, IC is satisfied only if $s > \frac{1-\delta(1-p)}{p\delta}$, which leads to a contradiction as $\frac{1-\delta(1-p)}{p\delta} > 1$. Suppose that only DE and LL_L bind. An increase in the punishment period not only decreases π_M but also makes DE more binding. Therefore, the manufacturer chooses $T = 0$ and as a result IC is violated. Hence, IC is always binding.

- **Section 3.2:** We compute the comparative statics when IC, DE and LL_L bind. The sign of the comparative statics of q with respect to p is determined by:

$$\frac{\partial^2 \pi_M}{\partial q \partial p} \propto -\frac{c\left(\delta\left[E(s) + (1-p)(1-s)\right] - s\right)}{\left(\delta(1-p)E(s) + ps\right)^2}$$

Note that q decreases with p if:

$$\delta > \frac{s}{E(s) + (1-p)(1-s)}$$

which is always true because the RHS is increasing in s, and the inequality holds at the upper bound of s, $\frac{\delta(1-p)}{1-\delta p}$. By inspection, it is easy to see that $\frac{\partial D_H}{\partial p} < 0$, $\frac{\partial D_L}{\partial p} < 0$ and $\frac{\partial T}{\partial p} < 0$.

- **Section 3.3:** We show that when IC and LL_L bind, the optimal punishment is bounded by the optimal punishment in Section 3.2. We plug $D_L = sR(q)$ and $D_H = \alpha(T)\delta E(s)R(q)$ into DE to obtain:

$$s < \delta^{T+1}\left[\frac{E(s) - (1-p)\frac{\left(1-\delta^T\right)\delta E(s) + (1-\delta)s}{1-\delta^{T+1}}}{1-\delta(1-p)}\right]$$

If we plug this inequality in the optimal punishment of Section 3.2, $\delta^{T+1}E(s) = s$, it is easy to show that the equation is violated.

Simple algebra shows that the RHS of condition (3) is larger than $\widetilde{c}$ when $1-\delta(1-p)-p\delta^{T+1}>0$, which is always true, and that it is decreasing in T:

$$\frac{\partial \text{RHS of (3)}}{\partial T}=\frac{c\ln\delta(1-p)^2(1-\delta)(1-s)\delta^{T+1}}{\left(\delta(1-p)^2+\left(1-\delta(1-p)^2\right)s-((1-p)E(s)+ps)\delta^{T+1}\right)^2}<0$$

The first order condition for T is:

$$\frac{\partial\pi_M}{\partial T}=\frac{-\delta^{T+1}\ln\delta}{1-\delta(1-p)-p\delta^{T+1}}\left[(1-p)\frac{1-\delta\alpha(T)}{1-\delta^{T+1}}E(s)R(q)-p\pi_M\right]$$

It is easy to check that this condition, evaluated at $T=0$, is negative if:

$$\frac{cq}{R(q)}\leq\frac{s-(1-p)E(s)}{p}$$

which give us condition (4) when replacing c using (3). ■

Proof of Proposition 4. When $\delta\to1$, the constraint $s<\delta E(s)$ is more likely to be satisfied, so either the regime in Section 3.2 or Section 3.3 occur. However, as $\delta\to1$, no punishment in Section 3.2 could satisfy $\delta^{T+1}=\frac{s}{E(s)}$ so only the regime of a large shock in Section 3.3 emerges (indeed, when $\delta\to1$, DE is slack). Using l'Hopital's rule, equation (3) becomes:

$$\lim_{\delta\to1}R'(q)=\frac{c}{\frac{s+[(1-p)E(s)+ps]T}{T+1}}\tag{15}$$

Note that at $T=0$, the RHS of (15) is larger than $\widetilde{c}$ and that it is decreasing in T. Noting that as $T\to+\infty$, the RHS of (15) tends to $\frac{c}{(1-p)E(s)+ps}$ completes the proof. ■

Proof of Proposition 5. Because of condition (5), a retailer can lie only downwards. To avoid underreporting of revenues, the manufacturer needs to ensure that $u'(\widetilde{s})\geq0$, that is, the payoff $u(\widetilde{s})$ increases with $\widetilde{s}$. However, the manufacturer cannot design a contract that satisfies $u'(\widetilde{s})>0$ for $s\in[s^*,\overline{s}]$ because she has no means to increase $u(\widetilde{s})$ (i.e., she is already not terminating with the retailer and asking for a smaller repayment $R(q;s\prime)<R(q;s^*)$ would make types $s\in[s',s^*]$ overreport their types). Furthermore, for $s\in[0,s^*)$ the contract cannot satisfy $u'(\widetilde{s})>0$ either, because the manufacturer is already extracting the maximum payment and increasing the payoff with $\widetilde{s}$ would imply a costly (for the manufacturer) increase in the termination period. Indeed, the manufacturer wants to set $T(s)$ as low as possible as long as there is truth-telling. Finally, to prevent the retailer walking away with the earnings, the manufacturer need to ensure that $u(\widetilde{s})$ is non-negative, which completes the proof. ■

Proposition 6. The optimal $s^*$and q are determined by

$$(1 - H(s^*))\,\pi_R - H(s^*)\pi_M \frac{E(s) - E(s \mid s \leq s^*)}{E(s) - s^*} = \widehat{\lambda} \tag{16a}$$

$$\pi_R \frac{\frac{\partial R(q;s^*)}{\partial q} - c}{\frac{\partial R(q;s^*)}{\partial q} - \delta \frac{\partial R_E(q)}{\partial q}} + \frac{\int_0^{s^*} \left(\frac{\partial R(q;s)}{\partial q} - \frac{\partial R(q;s^*)}{\partial q} \right) h(s)ds}{\frac{\partial R(q;s^*)}{\partial q} - \delta \frac{\partial R_E(q)}{\partial q}} (\pi_R + \pi_M) \tag{16b}$$

$$+\pi_M \left(\frac{\int_0^{s^*} (R(q;s^*) - R(q;s))\, h(s)ds}{R_E(q) - R(q;s^*)} \frac{\frac{\partial R_E(q)}{\partial q} - \frac{\partial R(q;s^*)}{\partial q}}{\frac{\partial R(q;s^*)}{\partial q} - \delta \frac{\partial R_E(q)}{\partial q}} \right) = \widehat{\lambda}$$

where λ is the shadow cost of the dynamic enforcement constraint and $\widehat{\lambda}$ is adjusted by the second best loss of revenues, $\widehat{\lambda} = \lambda \left(R_E(q) - \int_0^{s^*} R(q;s)h(s)ds - (1 - H(s^*))\, R(q;s^*) \right)$. If and only if δ is large enough, $\delta > \frac{R(q;s^*)}{R_E(q)}$, then $\widehat{\lambda} = 0$ and the optimal $s^*$and q are independent of δ. ■

Proof of Proposition 6. The Lagrangian function is:

$$\mathcal{L} = \frac{\int_0^{s^*} R(q;s)h(s)ds + (1 - H(s^*))\, R(q;s^*) - cq}{(1-\delta) \left(1 + \frac{H(s^*)R(q;s^*) - \int_0^{s^*} R(q;s)h(s)ds}{R_E(q) - R(q;s^*)} \right)} - \lambda \left(R(q;s^*) - \delta R_E(q) \right)$$

where λ is the shadow cost associated with the constraint. The first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial s^*} &= \pi_R \frac{\partial R(q;s^*)}{\partial s^*} \frac{1-H(s^*)}{R_E(q) - \int_0^{s^*} R(q;s)h(s)ds - (1-H(s^*))\, R(q;s^*)} \\
&\quad -\pi_M \frac{\partial R(q;s^*)}{\partial s^*} \frac{\frac{H(s^*)R_E(q) - \int_0^{s^*} R(q;s)h(s)ds}{R_E(q)-R(q;s^*)}}{R_E(q) - \int_0^{s^*} R(q;s)h(s)ds - (1-H(s^*))\, R(q;s^*)} \\
&\quad -\lambda \frac{\partial R(q;s^*)}{\partial s^*} = 0 \\
\frac{\partial \mathcal{L}}{\partial q} &= \pi_R \frac{\int_0^{s^*} \frac{\partial R(q;s)}{\partial q} h(s)ds + (1-H(s^*)) \frac{\partial R(q;s^*)}{\partial q} - c}{R_E(q) - \int_0^{s^*} R(q;s)h(s)ds - (1-H(s^*))\, R(q;s^*)} \\
&\quad -\pi_M \frac{H(s^*) \frac{\partial R(q;s^*)}{\partial q} - \int_0^{s^*} \frac{\partial R(q;s)}{\partial q} h(s)ds - \frac{\left(H(s^*)R(q;s^*) - \int_0^{s^*} R(q;s)h(s)ds\right)\left(\frac{\partial R_E(q)}{\partial q} - \frac{\partial R(q;s^*)}{\partial q}\right)}{R_E(q)-R(q;s^*)}}{R_E(q) - \int_0^{s^*} R(q;s)h(s)ds - (1-H(s^*))\, R(q;s^*)} \\
&\quad -\lambda \left(\frac{\partial R(q;s^*)}{\partial q} - \delta \frac{\partial R_E(q)}{\partial q} \right)
\end{aligned}$$

and

$$\lambda \left(R(q;s^*) - \delta R_E(q) \right) = 0$$

Setting the first two conditions equal to zero, give us equation (16a) and (16b), respectively.

We assume that the second order conditions hold. Note that the optimal s^* cannot be in a corner as this would give zero profits to either the manufacturer (if $s^* = 0$) or to the retailer (if $s^* = 1$). We assume that the demand and the cost function are such that production takes place, i.e., $0 < q < +\infty$. ■

Proof of Corollary 2. Suppose that the constraint does not bind, the first order conditions are:

$$\frac{\partial \pi_M}{\partial q} = \frac{\widehat{E}(s,s^*)R'(q) - c}{(1-\delta)\, \frac{E(s) - \widehat{E}(s,s^*)}{E(s)-s^*}} = 0$$

$$
\begin{aligned}
\frac{\partial \pi_M}{\partial s^*} &= (1-H(s^*))\,\frac{E(s)-s^*}{E(s)-\widehat{E}(s,s^*)}R(q) \\
&\quad + H\left(s^*\right)\frac{E\left(s \mid s\leq s^*\right)-E(s)}{\left(E(s)-\widehat{E}(s,s^*)\right)^2}\left(\widehat{E}(s,s^*)R(q)-cq\right)=0
\end{aligned}
$$

The cross derivative is:

$$
\begin{aligned}
\frac{\partial^2 \pi_M}{\partial s^* \partial q} &= (1-H(s^*))\,\frac{E(s)-s^*}{E(s)-\widehat{E}(s,s^*)}R'(q) \\
&\quad + H\left(s^*\right)\frac{E\left(s \mid s\leq s^*\right)-E(s)}{\left(E(s)-\widehat{E}(s,s^*)\right)^2}\left(\widehat{E}(s,s^*)R'(q)-c\right)=0
\end{aligned}
$$

Using $\frac{\partial \pi_M}{\partial q}=0$, it is easy to see that $\frac{\partial^2 \pi_M}{\partial s^* \partial q}>0$. ■

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