The harmony of the spheres in English musical mathematics, 1650–1750

Benjamin Wardhaugh

> The outermost Circle represents the Zodiack, and the Aspects of the Planets, to which you see the *Diapason* with its Intersections exactly agreeing; as *viz.* The two Terms thereof, to a Conjunction and Opposition; the middle Section (which generates a Fifth on one side, and a Fourth on the other) to a [quartile]. A Third and a Sixth compleating also the Compass of an Octave, as a [trine] and [sextile] do a Semicircle or the two opposite points of an Orbe. To which may be added, that a *Diapason* is divided into Twelve Semitones, as the Zodiack into Twelve Signes or Sections.[1]

Introduction

This is one of the clearer echoes of the harmony of the spheres to be heard in English musical writing in the seventeenth century. This passage, describing a diagram in which the musical scale was superimposed on the zodiac, responded to Ptolemy's *Harmonics* (book 3, sections 9–16), and it clearly asserted that the numbers and ratios involved in describing musical harmony could have significance outside music, in the cosmos. Its appearance in *The Division-Viol* by Christopher Simpson (c.1602 – 1669), one of England's most prominent practical musicians in the Restoration period, illustrates how seriously the trope of the harmony of the spheres could be taken by both theorists and practitioners at that time.

The period 1650–1750 saw a distinct flowering of mathematical studies of music in England.[2] They included wholly mathematical responses to Descartes's *Compendium musicæ* of 1650 such as are to be found in the manuscripts of Nicolaus Mercator and Isaac Newton, where music was conceived as posing problems wholly mathematical in character. They included the experimental investigations of the Royal Society and the mechanical explanations for musical phenomena provided by Francis

---

[1] Christopher Simpson, *Chelys / The Division-Viol* (London, 1667), p. 24.

[2] Benjamin Wardhaugh, *Music, experiment and mathematics in England, 1653–1705* (Farnham, 2008); Penelope M. Gouk, *Music, Science and Natural Magic in Seventeenth-Century England* (New Haven and London, 1999).

and Roger North. They included writings such as those of Robert Hooke (1635 – 1703) on cosmic vibration in which music seemed to be appealed to as a model for otherwise inexplicable effects on a cosmic scale. They also included sections such as that in Simpson's *Division-Viol*, in which practitioners engaged with the possibility of a meaningful mathematical description of music, and with the possible extra-musical meanings of that description.

As John Hollander has shown, this was a period when the English literary imagination was very much aware of the tradition of the harmony of the spheres.[3] Such an awareness was certainly not absent from the scientific imagination: Robert Boyle (1627 – 1691) and Isaac Newton (1642 – 1727), for instance, can be found referring to cosmic harmony and heavenly music in their writings (see below). It is therefore natural to ask how far mathematical writings about music made contact with the tradition of the music of the spheres, of an extra-musical significance for musical mathematics; how far musical mathematics was 'cosmic', and in what sense.

Initially, it seems that the conclusion of such an enquiry must be largely a negative one. Only a few English writings on the mathematics of music make any explicit reference to the music of the spheres, even though most of them depend heavily upon a corpus of ancient Greek writings which does. The passage from Simpson quoted above is quite exceptional, particularly among the writings of practitioners. Yet there were, as we will see, a range of more subtle ways mathematical studies of music made contact with the idea of cosmic harmony: Dixon also discusses some of these in this volume.

The Greek corpus of special interest to mathematical writers on music comprised works by Aristoxenus, Cleonides, Nicomachus, Alypius, Gaudentius, Bacchius, Aristides Quintilianus, Martianus Capella, Ptolemy, Porphyry and Bryennius. Several had been edited during the sixteenth century; all appeared in new editions during the seventeenth. The first seven authors appeared in Marcus Meibom's 1652 Latin–Greek collection of musical *auctores*, the final three in editions by the English mathematician John Wallis in the 1680s and 90s.[4] Thus by the later

---

[3] John Hollander, *The untuning of the sky: ideas of music in English poetry, 1500–1700* (Princeton, 1961).

[4] Marcus Meibom, *Antiquæ musicæ auctores septem* (2 vols, Amsterdam, 1652); John Wallis, *Claudii Ptolemaei harmonicorum libri tres* (Oxford, 1682); idem, *Operum mathematicorum volumen tertium* (Oxford, 1699), pp. 183–355, 357–508.

seventeenth century many of the important texts had been printed more than once. Some of the content of the ancient texts was also available at second hand through the works of more recent scholars such as Marin Mersenne (1588–1648), Athanasius Kircher (1601 or 2 – 1680) and René Descartes (1596–1650), not to mention those of Gioseffo Zarlino (1517 – 1590). As a result, the specific sources of information used by English writers do not always prove to be clear.

The seventeenth century was the period when some attempted to replace the ‘sounding number’ with the ‘sounding body’, in Paolo Gozza’s apt formulation: to use mechanics and experiment to replace numerology in accounting for the role of small whole numbers and their ratios in musical tuning theory.[5] Others abandoned altogether the exclusive reliance on whole-numbered ratios in the description of musical sound. As Vanhaelen and Prins argue in the introduction to this volume, different models of the music of the spheres existed concurrently, and even within the works of individual writers we can discern multiple views of the subject apparently in tension with one another. By focussing on mathematical writings about music in particular, and on the English case, the present chapter attempts to illustrate how such changes could take place, and in particular how authors with less – or no – interest in a numerological basis could continue to produce purely mathematical studies of music.

The ‘sounding number’ and its meanings

> I held the Paper so that the Spectrum might fall upon this delineated Figure, and agree with it exactly, whilst an Assistant whose Eyes for distinguishing Colours were more critical than mine, did by right lines … drawn cross the Spectrum, note the confines of the Colours that is of the red … of the orange …, of the yellow …, of the green …, of the blue …, of the indico …, and of the violet …. And this operation being divers times repeated both in the same and in several Papers, I found that the Observations agreed well enough with one another, and that the rectilinear lines … were by the said cross lines divided after the manner of a musical Chord.[6]

---

[5] Paolo Gozza (ed.), *Number to Sound: the Musical Way to the Scientific Revolution* (Dordrecht, 2000).

[6] Isaac Newton, *Opticks* (London, 1704), p. 92.

This, from Isaac Newton himself, is another clear case of musical mathematics with extra-musical significance. But it is perhaps less clear here than in Simpson's musical astrology exactly what that significance was, partly because of the lack of precedents in the ancient literature with which to compare this remarkable passage.[7]

The point was that Newton's preferred division of the musical string turned out, on observation by a disinterested third party, to be also the most satisfactory division of the spectrum of sunlight into a set of separate colours. In the *Opticks,* published in 1704, the resulting divided line would also be bent round into a circle and made to provide a way of determining in detail how different colours would combine. This was a novel meaning to assign to musical mathematics; in the absence of any mechanical explanation of why the division of the musical string should be relevant to the division of the spectrum it seems natural to call it a numerological one, though we will see that there was much more to Newton's understanding of musical sound and musical mathematics.

Both Simpson and Newton worked with mathematical descriptions of the musical scale which bore a family resemblance, and quite a close one, to that of Gioseffo Zarlino. That is, they presented versions of the syntonic diatonic or 'just intonation', first described in a modern work by Pietro Aaron in 1523 and promoted in Zarlino's important *Istitutioni harmoniche* in 1558.[8] In both cases the transmission may have been indirect – through Mersenne or Descartes – but it is worth noting that for Zarlino himself there was a numerological significance to the numbers involved. As Gozza summarises:

> The 'interval' between the string and its half, 2/1, is the mathematical form of the musical octave interval, the Greek diapason, "Mother and source of all Intervals;" from this "Sounding whole divisible in parts" Zarlino

---

[7] A very important new examination of Newton's musical analogies appeared shortly before the present volume went to press: Niccolò Guicciardini, 'The Role of Musical Analogies in Newton's Optical and Cosmological Work', *Journal of the History of Ideas* 74 (2013), pp. 45–68.

[8] See J. Murray Barbour, *Tuning and Temperament: a historical survey* (East Lansing, 31 1951; New York, 2004), pp. 89–105, citing Bartolomeus Ramis de Pareja, *Musica practica* (Bologna, 1482) and Gioseffo Zarlino, *Istitutioni harmoniche* (Venice, 1558); see also Wardhaugh, *Music, Experiment, and Mathematics*, pp. 13–15.

> mathematically deduces, and verifies through experience, that the “natural forms” of the consonances ordered in the scale of just intonation are defined by ratios between the first six integers, the *senarius*, the perfect number.[9]

Zarlino looked back, with his ‘senarius’, to the ‘tetractys’ of the ancient Pythagoreans, described for example by Sextus Empiricus in a work frequently printed from the mid-sixteenth century onwards:

> By ‘*tetraktys*’ they [*sc*. ‘the Pythagoreans’] mean a number which, being constituted out of the first four numbers, fits together the most perfect number, as for instance ten: for one and two and three and four becomes ten. This number is the first *tetraktys*, and is described as the ‘fount of ever-flowing nature’ in as much as the whole universe is organised on the basis of these numbers according to *harmonia*; and *harmonia* is a *systēma* of three concords, the fourth, the fifth and the octave; and the proportions [*analogiai*] of these three concords are found in the found numbers previously mentioned, in one, two, three and four.[10]

This was a mode of explanation in which relevant properties were ascribed directly to the numbers themselves in order to account for the effects produced when they were instantiated in such phenomena as music. Thus in the Euclidean *Sectio canonis*, printed at least three times by the seventeenth century, one could read that

> Among notes we also recognise some as concordant, others as discordant, the concordant making a single blend out of the two, while the discordant do not. In view of this, it is to be expected that the concordant notes, since they make a single blend out of the two, are among those numbers which are spoken of under a single name in relation to one another, being either multiple or epimoric

---

[9] Gozza, *Number to Sound*, p. 38, quoting Gioseffo Zarlino, *Sopplimenti musicali* (Venice, 1588), p. 98 and 27–28.

[10] Sextus Empiricus, *Adversus Mathematicos* vii.94–5, trans. in Barker, *Greek Musical Writings II*, p. 30. See also Aristotle, *Metaphysics* 985b23ff.

> [i.e. the first term being either a multiple of, or one greater than, the second].[11]

Similarly, a passage from the Pythagorean Philolaus quoted in Boethius's book on music (a basic musical text – in theory – in the universities) stated that:

> [T]he tone had its origins in the number that constitutes the first cube of the first odd number, for that number was greatly revered among the Pythagoreans. Since 3 is the first odd number, if you multiply 3 by 3, and then this by 3, 27 necessarily arises; and this stands at the distance of a tone from the number 24, the same 3 being the difference.[12]

Many individuals interested in the mathematical basis of musical harmony in the early modern world made it clear that they had read such texts as these and assimilated this mode of explanation from them. As Paolo Gozza has put it, 'the subject of music is the "sounding number," where number is the form and sound the matter'.[13] And that same, musically-significant form, might also be instantiated in other ways than sound: the cosmos, the spectrum, the soul.

By the time Simpson and Newton were writing it was becoming less usual to give musical mathematics an extra-musical significance. But this was a period when several models of thought on these matters were available, and were not necessarily mutually exclusive, as we will see. An example is provided by John Birchensha (c.1605–1681?), one of the more colourful English mathematical music theorists. Composer, performer and teacher of music, he taught Samuel Pepys and perhaps the Duke of Buckingham during his heyday in the 1660s, but he also succeeded in interesting the Royal Society in his ideas about musical theory. As a theorist he stands out for the austerity of his approach. He was committed to a rather primitive form of tuning theory in which every perfect fifth was supposed to be mathematically perfect

---

[11] Euclid, *Sectio canonis* 149, trans. in Barker, *Greek Musical Writings II*, p. 193. See also Andrew Barker, *The Science of Harmonics in Classical Greece* (Cambridge, 2007), pp. 263–327.

[12] Philolaus, quoted in Boethius, *De institutione musica*, 3.5; trans. in Barker, *the Science of Harmonics*, p. 281 (based on the translation of Calvin M. Bower and Claude V. Palisca in *Fundamentals of Music* (New Haven and London, 1989)).

[13] Gozza, *Number to Sound*, p. 12.

– with a ratio of string lengths of 2 to 3 – come what may. What came was a set of 33 pitches within the octave, required in order to play in a large number of keys and scales – including reconstructions of ancient Greek modes – without any impure fifths.[14] A fragment of his ‘Grand Scale’, as constructed by his contemporary the mathematician John Pell, is as follows[15]

1	51597,80352	…	A
2	52977,99168	…	G # ‘diesis’
3	54358,17984	…	G #
4	55099,80288	…	A @
	…		

It was necessary to adopt a string length of more than ten billion units (specifically, 10,319,560,704) in order to ensure that all the fret positions in this ‘scale’ could be given as whole numbers without any approximation or rounding. This represented one extreme in the search for a correct musical mathematics, and it illustrates the character of much English musical mathematics from this period: the mathematics were elaborate, accurate, and internally coherent, but their relationship to musical practice was far from clear. String lengths were specified with an accuracy that could not possibly be realized in practice, and Birchensha’s scheme was one in which both pure thirds and circular modulation were impossible. Birchensha was a professional performer, composer and teacher, and he must in his own practice have used a tuning very different from that described in his theoretical works.

Birchensha was following the lead, presumably, of his theoretical predecessors, although his near-silence about his sources makes it hard to be confident which ancient or modern works he had seen. We should perhaps be reminded of the *Sectio canonis* ascribed to Euclid, in which problems in musical ratio theory were stated and solved but the premise – that music should embody certain ratios of small whole numbers – was neither examined nor justified. For Birchensha the premise and the set of permissible ratios were substantially the same, but the context of seventeenth-

---

[14] Christopher D.S. Field and Benjamin Wardhaugh, *John Birchensha: Writings on Music* (Aldershot, 2010), pp. 59–63.

[15] British Library, Add. MS 4388, fol. 37^{r}. See Field and Wardhaugh, *John Birchensha,* pp. 62–3.

century harmonic practice meant that his solutions could bear little relation to musical reality.

Was this a genuine musical numerology, in which ‘musical’ ratios were important not for musical but for extra-musical reasons? Birchensha could well have had access to models such as Robert Fludd’s cosmic monochord or Johannes Kepler’s harmonic cosmology. Yet he did not propose or allude to any correspondence of his musical numbers with other parts of the cosmos. His work seems closer to the austerities of Descartes’ *Compendium musicæ*: the correspondence of musical intervals with mathematical ratios was still largely a matter of assertion – though both men certainly knew that such things could be demonstrated by experiment – but no extra-musical significance was attached to them. (The difference in Descartes’s case was that the result was a practicable scale: that of Zarlino.) The motivation of Birchensha’s mathematics may have embodied an echo of the harmony of the spheres in its insistence on an ultimately Pythagorean set of ratios, but its meaning was musical, not cosmic.

A somewhat similar example was Thomas Salmon (1647–1706), a musical amateur who published three books on musical topics and corresponded with such figures as Wallis and Newton on the details – the very fine details – of musical tuning. He seems to have known Birchensha and perhaps was taught by him, but he did not adopt his ideas about tuning in detail. Salmon instead worked out an elaboration of Zarlino’s just intonation – specifically, he proposed a way of constructing not just the diatonic but also the chromatic scale according to consistent mathematical principles.[16]

The justification for such elaborations was a little more explicit in Salmon’s writings than it was in Birchensha’s. It was essential to Salmon’s project, which he conceived as a practical one to ‘perform musick in perfect and mathematical proportions’ that it would ‘produce Performances, much more exact and powerful’.[17] That is, it would improve the experience of music for listeners. Near the end of his life he put on a demonstration performance at the Royal Society to prove it. The

---

[16] Benjamin Wardhaugh, *Thomas Salmon: Writings on Music* (2 vols, Aldershot, 2013), esp. vol. 2; Thomas Salmon, ‘The Theory of Musick reduced to Arithmetical and Geometrical Proportions’, *Philosophical Transactions*, 24 (1705), pp. 2072–7.

[17] Thomas Salmon, *A Proposal to Perform Musick, in Perfect and Mathematical Proportions* (London, 1688), p. 5.

result: ‘the most compleat Harmony was heard’.[18]

Just what that meant exercised Salmon, and he had suggested in an earlier work that the apprehension of musical harmony was the work of a special faculty, distinct from the ordinary hearing. ‘This peculiar faculty doth not meerly arise from an excellency of the common hearing, and consequently … they are not the same’.

> But whether the distinction comes from a different formation of the little intrigues of the ear, or only from an improvement that some mens souls are able to make of sounds so qualifyed and represented to them; it is hard to determine, and needless for my purpose, so long as we find *de facto*, that there is such a Musical hearing, and that God hath given some men such a particular faculty, wheresoever it pleased him to place it.[19]

This was much more than Birchensha ever said, and it conferred a corresponding sense of purpose on Salmon’s equally impracticable musical mathematics. We seem, in fact, to be close once again to a kind of musical numerology, in which numbers functioned directly as explanations of phenomena and of effects upon the human soul. Salmon would have found support for such a view in ancient hints about the essential harmoniousness of the soul.[20]

With one of Salmon’s associates we face a similar situation. Salmon worked in close association with John Wallis (1616 – 1703), Savilian Professor of geometry at Oxford from 1649 until his death in 1703. Wallis associated himself closely with the ancient sources for musical mathematics, considering it ‘a Task well agreeing with his Province’ as Savilian professor to undertake detailed textual work on them.[21] As mentioned above, he published editions of the musical works of Ptolemy (1682) and of Porphyry and Briennius (1699).[22] And he published in the *Philosophical Transactions* his own system of mathematically-defined musical tuning, closely

---

[18] Salmon, ‘The Theory of Musick reduced’, p. 2069 [*sic*].

[19] Thomas Salmon, *An Essay to the Advancement of Musick* (London, 1672), p. 3.

[20] Aristotle, *De anima*, 407b27–408a33.

[21] Notice of John Wallis, *Claudii Ptolemæi harmonicorum libri tres*, in *Philosophical Transactions* 13, pp. 20–21, at p. 20.

[22] John Wallis, *Claudii Ptolemæi harmonicorum libri tres* (Oxford, 1682); idem, *Opera mathematica* (3 vols, Oxford, 1699), vol. 3, pp. 183–355, 357–508.

related to Salmon's. At the same time, he was closer to the new world of mechanical explanation than Salmon – he was a founding member of the Royal Society – and his reading in the continental authors of the first half of the century had made him aware of the possibility of a mechanical explanation for music's effects. Writing to Henry Oldenburg, secretary of the Royal Society, he pondered

> whether two strings that are unisons, be therefore Harmonious, because (supposing them to have one common beginning) the Vibrations of the one doe exactly answere those of the other: And, next unto those, Octaves; because that the vibrations of the more Acute, being twice as many in the same time, every vibration of the more Grave or slower string, is coincident with every second Vibration of the quicker or more Acute

and so on for the other consonances.[23] This explanation of consonance was plausible to some, but it had problems both in its own terms and in terms of musical practice.[24] Sound's speed was known to be finite, and as Newton would point out in a letter of 1677, one would therefore have to stand in the right spot in relation to a pair of sound sources in order to experience the effect of coincidence of pulses mentioned here.[25] Worse, such an explanation as this made it incomprehensible that the mathematically imperfect tunings widely used in musical practice were tolerable to the ear at all. Wallis was not convinced, and he refused to commit himself:

> Whether this, I say, be the true Physicall cause of Harmony & Discord in Sounds, (though it be a very specious and promising account of it) I will not positively determine; because I would not, even here exclude a strict examination by Experiment.[26]

---

[23] Henry Oldenburg, *Correspondence*, ed. A Rupert Hall and Marie Boas Hall (13 vols, Madison and London, 1965–77), vol. 1, pp. 190–203: John Wallis to Henry Oldenburg, 14 May 1664), at p. 192.

[24] See H. Floris Cohen, *Quantifying Music: the science of music at the first stage of the scientific revolution, 1580–1650* (Dordrecht, 1984), pp. 90–92, 94–97, 103–111.

[25] Jamie C. Kassler, *The Beginnings of the Modern Philosophy of Music in England: Francis North's* A Philosophical Essay of Musick *(1677) with comments of Isaac Newton, Roger North and in the Philosophical Transactions* (Aldershot, 2004), pp. 175–8.

[26] Oldenburg, *Correspondence*, vol. 1, p. 193.

For Wallis, then, the meaning of musical mathematics, including his own, might be sought in mechanical explanation, but it was not quite clear that it would turn out to lie there, at least in the form in which mechanical explanations of consonance were available to him. Thus at least the possibility of numerological explanation for music's effects seems still to have been visible.[27]

There were those, however, for whom this 'coincidence theory' (the modern term for it) did provide an adequate explanation for music's effects on the human person, and who made it their justification for an investigation of musical tuning. Francis North (1637–85) was one, whose *Philosophical Essay of Musick* appeared anonymously in 1677: 'when the Pulses of *tones* are coincident one with the other, there is an Union of the *sounds*'; 'chords are more or less perfect, according as they are more or less coincident'.[28] North gave some quite detailed discussions of the mechanics of particular forms of sound production, particularly in pipes. He used this mechanical basis to support a theory of tuning broadly similar to Salmon's and Holder's: a modification of Zarlino's just intonation, with a detailed specification for the positions of semitones.

With Francis North, fully committed to the coincidence theory of consonance, we have reached a point where musical mathematics had a distinctively mechanical meaning, even though many of his detailed numerical conclusions were similar to those of Wallis and Salmon. The coincidence theory of consonance had rescued mathematical ratio theory for the mechanical world. But extra-musical interpretations of musical ratio theory were a casualty. For North the point of the numbers was not that they corresponded directly with anything in the human soul, but rather that they described the mechanical processes of sound's production and reception. There was still the possibility that the human sensory apparatus was peculiarly attuned to musical sound, that, for instance, the tympanum 'can be soe tuned … or stretch'd, that it becomes harmonicall or unison to whatsoever sound is heard',[29] as Robert Hooke

---

[27] See further David Cram and Benjamin Wardhaugh, *John Wallis: Writings on Music* (Aldershot, 2013).

[28] Francis North, *A Philosophical Essay of Musick, Directed to a Friend* (London, 1677), pp. 7 and 9.

[29] London, Royal Society Library MS Classified Papers II no.1: Robert Hooke, 'A Curious Dissertation concerning the Causes of the Power & Effects of Music', transcribed in Penelope M. Gouk, 'The Role

speculated. But music's correspondence with, and ability to influence, the human person, would in such a view have to be mediated by mechanical effects and explanations.

A digression: the 'sounding body' and its uses

Taking such a view made it harder to argue that musical ratios could have any significance outside music: harder to make a cosmic connection like Simpson's or a natural philosophical one like Newton's. Harder, but not impossible.

The phenomena of vibration and particularly of sympathetic resonance were a frequent topic of discussion by English experimentalists. Francis Bacon (1561 – 1626) devoted part of his *Sylva sylvarum* to musical and vibrational topics,[30] and his treatment received a detailed unpublished commentary by Edmund Chilmead (1610 – 1654): a comprehensive account of musical sound, how it behaved and what could be said about its mechanical nature from the observation of how it behaved.[31]

Later in the century such discussions continued, for instance around the various musical experiments performed at the Royal Society in 1664.[32] John Wallis published in the *Philosophical Transactions* for 1677 the results of the experimental investigations of others into the existence of vibrational nodes.[33] This example illustrates how the phenomena of vibration and particularly of resonance were, as has been discussed by Penelope Gouk, a classic instance of an effect whose causes were unseen and therefore 'occult' in a sense current during the seventeenth century.[34] For some authors, as she suggests, this may have been a route linking magical concerns to

---

of Acoustics and Music Theory in the Scientific Work of Robert Hooke', *Annals of Science* 37 (1980), pp. 573–605, at p.601.

[30] See Penelope M. Gouk, 'Music in Francis Bacon's Natural Philosophy', in Gozza, *Number to Sound*, pp. 135–152.

[31] Oxford, Bodleian Library, MS Tanner 204.

[32] Wardhaugh, *Music, Experiment and Mathematics*, pp. 102–6.

[33] John Wallis, '*Dr.* Wallis*'s Letter to the Publisher, concerning a new Musical Discovery*', *Philosophical Transactions* 12 (1677), pp. 839–842.

[34] Gouk, *Music, Science and Natural Magic*; on occult qualities in this period see also Keith Hutchison, 'What Happened to Occult Qualities in the Scientific Revolution?', *Isis* 73 (1982), pp. 233–53; John Henry, 'Occult Qualities in the Experimental Philosophy: active principles in pre-Newtonian matter theory', *History of Science* 24 (1986), pp. 335–81.

musical ones (and see Burnett’s paper in this volume).[35] Here we are concerned with the different possibility that vibrational effects may have been a route linking the music of the spheres to musical mathematics.

Robert Hooke (1635–1703), for example, suggested that parts of the apprehension, judgement, or memory might be attuned in their mechanical properties to the properties of musical sound:

> …somewhat like those Bells or Vases which *Vitruvius* mentions to be placed in the antient Theaters, which did receive and return the Sound more vigorous and strong; or like the Unison-toned Strings, Bells or Glasses, which receive Impressions from Sounds without, and retain that Impression for some time, answering the Tone by the same Tone of their own.[36]

Similarly, William Briggs (1642–1704) raised in the *Philosophical Collections* in 1682 the possibility that the combining of visual data from the two eyes might be explained by an appeal to the analogy of harmonious sound.[37] Newton wrote a discouraging reply, and as far as we know the idea went no further.[38] But such speculations indeed gave vibration and resonance new meaning and possibilities. These examples might still be construed as ‘cosmic’ (compare the examples in Burnett’s paper in this volume), but possibly only in a somewhat attenuated sense. On the other hand, they clearly show the significance that could be attached to musical phenomena as mechanical effects and sources of mechanical explanations.

---

[35] For instance Penelope M. Gouk, ‘Newton and Music: from the microcosm to the macrocosm’, *International Studies in the Philosophy of Science*, 1 (1986), pp. 36–59; eadem, ‘The Harmonic Roots of Newtonian Science’, in John Fauvel, Raymond Flood, Michael Shortland, and Robin Wilson (eds), *Let Newton Be! a new perspective on his life and works* (Oxford: Oxford University Press, 1988), pp. 101–25.

[36] Robert Hooke, ‘An Hypothetical Explication of Memory; how the organs made use of by the mind in its operation may be mechanically understood’ (Waller’s title), in ‘Lectures of Light’ VII, in Robert Hooke, *The Posthumous Works of Robert Hooke*, ed. Richard Waller (London, 1705), pp. 138–48, at p. 141. The paper was read to the Royal Society in 1682: see Thomas Birch, *A History of the Royal Society of London* (4 vols, London, 1756–57), vol. 4, pp. 153, 154.

[37] William Briggs, ‘Theory of Vision’, *Philosophical Collections*, 6 (1682): 167–78.

[38] Isaac Newton, *Correspondence*, ed. H. W. Turnbull (7 vols, Cambridge, 1959–77), vol. 2, pp. 377–8: Isaac Newton to William Briggs, 20 June 1682.

Robert Boyle, too, devoted attention to vibrational effects and their ability to produce substantial results: '*the Great Effects of even Languid and Unheeded Motion*', as he put it in one title page:

> Perhaps 'twill not be absurd to enquire, whether, in bodies of a very differing appearance from strings, the various Textures, Connexions, and Complications, that Nature or Art, or both, may make of the parts, may not bring them to a state equivalent to the Tensions of the strings of Musical Instruments, whereby divers of the mentioned parts may be stretched in the manner requisite to dispose them to receive a vibrating motion from some peculiar Sounds.[39]

And for Hooke the possibilities of vibration and resonance as distinctive types of mechanical phenomenon included the tantalizing speculation that they might be used to explain the motions of heavenly bodies:

> The Experiment was very considerable, though plain, giving a further Explanation of Gravity, by making a large Glass vibrate, with a Viol Bow … he thought, that it might contribute to explain the cause of gravity, and suggest an hypothesis for explaining the motion of gravity by.[40]

This referred to an experiment performed for the Royal Society in 1683. Hooke speculated on several occasions that a mechanical quality called 'congruity', which operated in a way analogous to sympathetic resonance, might be responsible for both cosmic and terrestrial effects.[41]

Thus there did seem to some to be a possible link between musical effects and cosmic ones, mediated by the idea of resonance as a mechanical effect or mechanical model. The availability of a plausible mechanical account of music's effects seems to have raised the possibility that music might be of use in accounting mechanically for other effects in other parts of the world. Although no seventeenth-century author

---

[39] Robert Boyle, *An Essay of the Great Effects of Even Languid and Unheeded Motion* (London, 1685), p. 82.

[40] William Derham (ed.), *Philosophical Experiments and Observations of the late Eminent Dr Robert Hooke* (London, 1726), p. 88; cf. Birch, *History*, vol. 4, pp. 48, 194.

[41] Wardhaugh, *Music, Experiment and Mathematics*, pp. 113–18.

seems to have connected that idea to a developed musical mathematics, it seems that a route was still open through which the harmony of spheres might be heard in the world of mechanical explanation.

Sound and number in the Newtonian world

The culmination of the mechanical study of sound in the seventeenth century brings us back to Isaac Newton, who in 1687 published his analysis of the sound wave in book 2 of the *Principia mathematica*. Its conclusions were such as the following:

> If pulses are propagated thro' a fluid, the several particles of the fluid, going and returning with the shortest reciprocal motion, are always accelerated or retarded according to the law of the oscillating pendulum.[42]

This was a discussion of sound in characteristically Newtonian style, in which the mechanical analogies of an earlier generation of English natural philosophers were replaced by mathematical analogies. Where Hooke and Boyle argued for the correspondence of one mechanical system with another, Newton would argue for the correspondence of a physical system with a specially-devised mathematical (usually geometrical) one.[43] For example Hooke presented a model for the motion of the planets around the sun consisting of a conical pendulum whose motion he asserted was analogous to that of the planets; Newton introduced one consisting of a series of geometrical constructions whose properties he showed were analogous to those of the planets' motions.[44]

The development of the Newtonian account of sound, which would continue over the following century and more, did not bear directly on the theory that consonance resulted from the coincidence of strokes or pulses: the 'conicidence' theory discussed above. That theory could be made compatible with Newton's ideas

---

[42] Isaac Newton, *The Mathematical Principles of Natural Philosophy*, trans. Andrew Motte (London, 1729), vol. 2, p. 173 (Book 2, Proposition XLVII, Theorem XXXVII).

[43] See Benjamin Wardhaugh, '"Analogy" and mathematical models: from Bacon to the Newtonians', forthcoming.

[44] Hooke, *Posthumous Works*, p. 547 ('Lectures concerning Navigation and Astronomy … read in the Year 1683'); Newton, *Mathematical Principles*, books 1 and 3, *passim*.

about the nature of sound, or with others. Newton's description of sound was not therefore incompatible with the study of musical tuning through pure ratios. The case of Newton's disciple Brook Taylor (1685 – 1731) shows that it was possible to do Newtonian-style work on sound – in Taylor's case on the vibration of strings[45] – while taking seriously the tradition of musical tuning in terms of pure ratios derived ultimately from ancient sources: Taylor drafted a treatise on the subject during the same period as his work on vibrating strings.[46] The case of Newton himself shows that it was possible to combine Newtonian work on sound with accepting the possibility of an extra-musical interpretation of tuning theories.

Yet for some in the eighteenth century that juxtaposition was an embarrassment. Brook Taylor published in his book on perspective an account of the 'colour wheel' which Newton had derived from his musical division of the spectrum, without comment on the meaning or validity of the scale–spectrum correspondence.[47] But one Newtonian tried to justify the correspondence in terms of an underlying similarity of mechanical cause: Jean Jacques Dortous de Mairan (1678–1771), whose 'Discourse on the propagation of sound' was read to the French Royal Academy of Science in 1737 and published in the *Memoires* of the Academy. Mairan attempted to tease out the implications of what Newton said in the *Opticks* in order to set out explicitly how the forces, speeds, or perhaps vibrational frequencies, of different colours of light must vary so as to produce the observed refractive effects. And he tried to explain the appearance of musical ratios in the division of the spectrum, by suggesting that the perceptibility of qualitative difference might be governed by the same laws for different senses. But this did not satisfy him, and he remarked that what Newton had observed still lacked any proper explanation.[48] In the long run Mairan's

---

[45] Brook Taylor, 'De motu nervi', *Philosophical Transactions*, 28 (1713), pp. 26–32; see further John T. Cannon and Sigalia Dostrovsky, *The Evolution of Dynamics: vibration theory from 1687 to 1742* (New York, 1981).

[46] Cambridge, St. John's College, Brook Taylor Papers (uncatalogued).

[47] For another example see Brook Taylor, *New Principles of Linear Perspective* (London, 1719), pp. 62–70 with fig. 25. Facsimile edition in Kirsti Anderson, *Brook Taylor's Work on Linear Perspective* (London, 1992), pp. 145–243: see pp. 222–30 and 243; see also Anderson's discussion at pp. 48–51.

[48] Jean Jacques Dortous de Mairan, 'Discours Sur la Propagation du Son dans les différents Tons qui le modifient', *Memoires de Mathématique et de Physique, tirés des registres de l'Academie Royale des Sciences de l'année MDCCXXXVII,* (Paris, 1737), pp. 1–59, esp. pp. 22–4, 'Sur l'Analogie du Son &

dissatisfaction with a correspondence unfounded on any similarity of mechanical causes was to become the usual view.

So traditional musical mathematics, and therefore an echo of the harmony of the spheres such as we identified in the work of John Birchensha, Thomas Salmon, and others, could co-exist with a Newtonian account of the nature of sound; but it is not clear that the stronger strain of cosmic harmony represented by numerological interpretations of musical numbers could do so. Like the coincidence theory of consonance, the Newtonian account of sound could salvage musical ratio theory more easily than its extra-musical interpretations.

Mathematical perfection in music

There was, finally, another route, a different reinterpretation of musical mathematics, and one which silenced the music of the spheres completely. This was to reinterpret the idea of mathematical perfection in music, substituting some other criterion for the traditional one of instantiating small ratios of whole numbers.

In practice such elaborations usually involved the equal division of some musical interval into some number of equal parts, an operation which had become feasible early in the seventeenth century with the invention of logarithms. Such an operation produced complex ratios for its intervals, with consequences for the adequacy of the coincidence theory to account for the experience of consonance. The devisers of such schemes were typically silent about the reasons why such mathematical perfection was important.

Newton himself used both combinatorial techniques and the novel mathematical tool of logarithms to investigate the properties of some systems of ratios in more detail.[49] This was how he arrived at the specific form of the scale which he used in his division of the spectrum. Others went still further, and used these new mathematical techniques to transform the investigation of musical harmony wholesale.

Both the mathematician and first President of the Royal Society William Brouncker (1620–84) and the Danish mathematician Nicolaus Mercator (c.1620–87),

---

des differents Tons avec la Lumiére & les Couleurs en général', and pp. 24–33, 'Sur l'Analogie particuliére des Tons & des Couleurs prismatiques', at pp. 25–6.

[49] Benjamin Wardhaugh, 'Musical logarithms in the seventeenth century: Descartes, Mercator, Newton', *Historia mathematica* 35 (2008), pp. 19–36.

resident mostly in England during 1654–82, responded to the *Compendium musicæ* of Descartes with elaborate mathematical schemes which left behind any direct consideration of simple pure ratios in the quest to divide the octave in accordance with novel principles of mathematical perfection. Mercator arrived at a division of the octave into 31 equal parts, whose corresponding string lengths he worked out using logarithms and set out to six decimal places.[50] He was interested in the fact that such a scheme could be used to produce a rough approximation to the mean-tone intonation of practitioners.

Brouncker, meanwhile, penned an elaborate discussion of the mathematics of tuning in which, as well as criticising Descartes's approach, he developed the novel mathematical concept of 'ratio-harmonicall division'.[51] This was the division of a ratio into two parts whose logarithmic sizes formed a ratio equal to the original ratio (thus the octave, with ratio 1:2, could be divided into two parts with sizes of 4 and 8 semitones). Brouncker employed this tool to divide up a basic interval equivalent to the 'golden proportion' (the ratio which divides a line into two parts so that the whole is to the larger part as the larger is to the smaller), arriving eventually at a set of positions for the frets on a chromatically-tuned instrument which he, too, worked out using logarithms and gave to eight decimal places.[52]

Each scheme had both advantages and disadvantages as a practical proposition; each embodied a particular idea of mathematical perfection in music. By contrast with those discussed earlier in this chapter, each scheme manifested a striking lack of interest in pure ratios of small whole numbers. This had two implications. First, it implicitly asserted the inadequacy of the coincidence theory as an account of consonance. The intervals of a scheme like Brouckner's or Mercator's would never produce frequent coincidences of pulses, and any claim that they would be acceptable to the ear implied that the ear was doing something other than count coincidences when it assessed consonance. Neither Brouncker nor Mercator, unfortunately, set out any theory of the mechanism of consonance or its perception.

---

[50] ibid.

[51] René Descartes, *Renatus Des-Cartes Excellent Compendium of Musick and Animadversions of the Author*, ed. and trans. anon. [trans. Walter Charleton, ed. William Brouncker] (London, 1653), pp. 87–8. See further Benjamin Wardhaugh, *The* Compendium Musicæ *of René Descartes: early English responses* (Brepols, forthcoming).

[52] Descartes, *Excellent Compendium*, p. 90.

Secondly, the disinterest of these authors in small pure ratios also put their musical mathematics at a rather greater distance from the music of the spheres than the other authors we have seen. The attitude here seems to have been that mathematical perfection was important for its own sake – and they consistently set out their results to a degree of precision which could not possibly have been realized in practice – but that the traditional form of that perfection was to be discarded. Thus these most mathematical of responses to music proceeded in a manner very largely indifferent to the possibility that harmony might have something cosmic about it.

An example from the following century illustrates the point in a Newtonian context, through a rare attempt to develop the idea of musical harmony itself using Newtonian methods, by Robert Smith (1689–1768), Master of Trinity College, Cambridge, Fellow of the Royal Society and Plumian Professor of Astronomy. His book on *Harmonics* appeared in 1749.[53] Smith wished to determine quantitatively what was the best tuning for a musical instrument. Not content with any of the various solutions that had been offered by previous theorists or that he knew were in practical use by musicians, he established, like Newton and Mercator, an abstract mathematical criterion. For Smith this was nothing so simple as the equal division of a given interval into equal parts. Instead, he developed a way to quantify the degree of harmonic perfection possessed by a given scale.

He proceeded by first quantifying the harmoniousness of pairs of simultaneous sounds. In effect he ranked the harmoniousness of pure consonances by the proportion of their pulses which would coincide, and the harmoniousness of any other pair of sounds by its closeness to the nearest pure consonance. This done, it was possible to compute the total harmoniousness of the important intervals in a given scale. He asked for thirds, fifths and sixths to be equally ‘harmonious’ in his sense.

In true Newtonian style, Smith now constructed a geometrical diagram which was analogous to the situation of concern. It had fixed lines representing thirds, fifths, and sixths, and a movable line whose intersection with them indicated the sizes of those intervals in a given musical scale. The movable line was constrained geometrically so that it could only show combinations of interval sizes that were actually possible (that is, it pivoted in such a way that if the thirds increased in size,

---

[53] Robert Smith, *Harmonics, or the Philosophy of Musical Sounds* (Cambridge, 1749; London, 1759). The discussion which follows refers principally to pp. 14–23, 56–160 in the second edition.

fifths increased by a smaller amount and sixths decreased). The problem was now to place that line so as to minimize the total detuning of thirds, fifths and sixths from pure: a geometrical problem which he could solve by geometrical means producing a scale embodying what he called ‘equal harmony’.

> Hence in the system of equal harmony the temperaments of the Vth, VIth and IIId are $^{-5}/_{18}$, $^{+3}/_{18}$ and $^{-2}/_{18}$ of a comma respectively …
> Does it not follow then, that the system of equal harmony, as above derived from the best system of perfect intervals, is the best tempered and most harmonious system that the nature of sounds is capable of?[54]

Having thus determined the sizes of the thirds, fifths and sixths in the most harmonious possible musical scale, Smith devoted most of the rest of his book to detailed instructions on how to tune instruments, particularly keyboard instruments, to it. There is some evidence that he actually tuned real instruments in this way, though his ideas do not seem to have become influential.

One might argue that by applying to it mathematical techniques and a style of scientific explanation that were being widely applied to other subjects, Smith’s Newtonian harmonics made music cosmic again. But this would be to stretch a point: it seems that Smith was scarcely interested in such wider questions. Rather, his work seems to represent the definitive overthrow of the music of the spheres tradition, in the sense that a musical mathematics was being elaborated here in a way which excluded the possibility of a detailed correspondence with other phenomena.

Conclusion

We have identified three possible ways in which mathematical and mechanical studies of music could be cosmic in our period: they may be roughly characterized as numerological, mechanical, and Newtonian. These different approaches existed in the same period in later seventeenth-century England and in some cases co-existed within the work of individuals; the evidence presented in this chapter does not really display a chronological trend, although it is tempting, and possibly correct, to see the Newtonian approach as a sort of culmination. They are readily distinguished by the

---

[54] Ibid., pp. 140–142.

different factors which mediated between music and the cosmos. In the numerological approach the mediation was carried out by numbers and ratios: Gozza's 'sounding number'. In the mechanical case the mediator was mechanical analogy, typically the claim that vibration or sympathetic resonance was analogous to some other phenomenon.

In the Newtonian case mathematics re-entered the picture, but with a new function, replacing the mechanical model as the vehicle for analogies. This increased the possibilities for the analysis of sound and vibration by rendering it analogous to specific geometric or algebraic constructions, but it largely removed from view the possibility that it might also correspond to the cosmos: such mathematical analogies or models tended in practice to be of use only for modelling one particular physical system, and even when they were applied, like Smith's, directly to the study of harmony they did not re-open the possibility of a correspondence with the cosmos. Thus through a change in the meaning of mathematics in scientific explanations the possibility of a cosmic interpretation of musical mathematics, a music of the spheres, was lost. Although the methods at work had wider applicability, the specific geometrical or algebraic models used by such as Smith could have no meaning outside music. In this sense, then – and ironically in view of Newton's own interests – the Newtonian approach was the one in which the music of the spheres was most nearly silent.

Select Bibliography

Barbour, J. Murray, *Tuning and Temperament: a historical survey* (East Lansing, 31 1951; New York, 2004).

Barker, Andrew, *Greek Musical Writings: II. Harmonic and Acoustic Theory* (Cambridge, 1989).

Barker, Andrew, *The Science of Harmonics in Classical Greece* (Cambridge, 2007).

Birch, Thomas, *A History of the Royal Society of London* (4 vols, London, 1756–57).

Boyle, Robert, *An Essay of the Great Effects of Even Languid and Unheeded Motion* (London, 1685).

Cannon, John T. and Sigalia Dostrovsky, *The Evolution of Dynamics: vibration theory from 1687 to 1742* (New York, 1981).

Cohen, H. Floris, *Quantifying Music: the science of music at the first stage of the scientific revolution, 1580–1650* (Dordrecht, 1984).

Cram, David and Benjamin Wardhaugh, *John Wallis: Writings on Music* (Aldershot, 2013).

de Mairan, Jean Jacques Dortous, 'Discours Sur la Propagation du Son dans les différents Tons qui le modifient', *Memoires de Mathématique et de Physique, tirés des registres de l'Academie Royale des Sciences de l'année MDCCXXXVII,* (Paris, 1737), pp. 1–59.

Derham, William (ed.), *Philosophical Experiments and Observations of the late Eminent Dr Robert Hooke* (London, 1726).

Descartes, René, *Renatus Des-Cartes Excellent Compendium of Musick and Animadversions of the Author,* ed. and trans. anon. [trans. Walter Charleton, ed. William Brouncker] (London, 1653).

Field, Christopher D.S. and Benjamin Wardhaugh, *John Birchensha: Writings on Music* (Aldershot, 2010).

Gouk, Penelope M., 'The Role of Acoustics and Music Theory in the Scientific Work of Robert Hooke', *Annals of Science* 37 (1980), pp. 573–605.

Gouk, Penelope M., 'Newton and Music: from the microcosm to the macrocosm', *International Studies in the Philosophy of Science*, 1 (1986), pp. 36–59.

Gouk, Penelope M., 'The Harmonic Roots of Newtonian Science', in John Fauvel, Raymond Flood, Michael Shortland, and Robin Wilson (eds), *Let Newton Be! a new perspective on his life and works* (Oxford: Oxford University Press, 1988), pp. 101–25.

Gouk, Penelope M., *Music, Science and Natural Magic in Seventeenth-Century England* (New Haven and London, 1999).

Gozza, Paolo (ed.), *Number to Sound: the Musical Way to the Scientific Revolution* (Dordrecht, 2000).

Guicciardini, Niccolò, 'The Role of Musical Analogies in Newton's Optical and Cosmological Work', *Journal of the History of Ideas* 74 (2013), pp. 45–68.

Hooke, Robert, *The Posthumous Works of Robert Hooke*, ed. Richard Waller (London, 1705).

Kassler, Jamie C. , *The Beginnings of the Modern Philosophy of Music in England: Francis North's* A Philosophical Essay of Musick *(1677) with comments of Isaac Newton, Roger North and in the Philosophical Transactions* (Aldershot, 2004).

Meibom, Marcus, *Antiquæ musicæ auctores septem* (2 vols, Amsterdam, 1652).

Newton, Isaac, *Opticks* (London, 1704).

Newton, Isaac, *The Mathematical Principles of Natural Philosophy*, trans. Andrew Motte (London, 1729).

Newton, Isaac, *Correspondence*, ed. H. W. Turnbull (7 vols, Cambridge, 1959–77).

North, Francis, *A Philosophical Essay of Musick, Directed to a Friend* (London, 1677).

Oldenburg, Henry, *Correspondence*, ed. A Rupert Hall and Marie Boas Hall (13 vols, Madison and London, 1965–77).

Salmon, Thomas, *An Essay to the Advancement of Musick* (London, 1672).

Salmon, Thomas, *A Proposal to Perform Musick, in Perfect and Mathematical Proportions* (London, 1688).

Simpson, Christopher, *Chelys / The Division-Viol* (London, 1667).

Smith, Robert, *Harmonics, or the Philosophy of Musical Sounds* (Cambridge, 1749; London, 1759).

Taylor, Brook, ‘De motu nervi’, *Philosophical Transactions*, 28 (1713), pp. 26–32.

Wallis, John, ‘*Dr.* Wallis*’s Letter to the Publisher, concerning a new Musical Discovery*’, *Philosophical Transactions* 12 (1677), pp. 839–842.

Wallis, John, *Claudii Ptolemaei harmonicorum libri tres* (Oxford, 1682).

Wardhaugh, Benjamin, ‘Musical logarithms in the seventeenth century: Descartes, Mercator, Newton’, *Historia mathematica* 35 (2008), pp. 19–36.

Wardhaugh, Benjamin, *Music, experiment and mathematics in England, 1653–1705* (Farnham, 2008).

Wardhaugh, Benjamin, *Thomas Salmon: Writings on Music* (2 vols, Aldershot, 2013).

Wardhaugh, Benjamin, *The* Compendium Musicæ *of René Descartes: early English responses* (Brepols, forthcoming).