# Appendix – Online Supplementary Material

We sketch a simple field-theoretic example that instantiates the axioms of CBQFT-1 and CBQFT-2. The aim is not to give a realistic model, but to make concrete how context enters the Hamiltonian, jump rates, and Bohmian dynamics, and to highlight the additional structure of CBQFT-2.

## Basic scalar-field set-up

Consider a real scalar field $\phi(x)$ in Minkowski spacetime, with conjugate momentum $\pi(x)$ and free Hamiltonian

$$H_0 = \frac{1}{2} \int d^3x \left( \pi(x)^2 + |\nabla \phi(x)|^2 + m^2 \phi(x)^2 \right).$$

At the algebraic level, one may regard $H_0$ and $\phi(x)$ as elements of the quasi-local field algebra $\mathcal{A}$. In concrete models, these are represented on Hilbert spaces that may depend on the macroscopic context.

For CBQFT-1 we work in a single fixed Fock representation of $\mathcal{A}$:

$$\mathcal{F} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n,$$

where $\mathcal{H}_n$ is the $n$-particle subspace. The field admits a mode decomposition in terms of creation and annihilation operators $a^\dagger(k), a(k)$, and in the one-particle sector $\mathcal{H}_1 \cong L^2(\mathbb{R}^3)$ the configuration is $q = x \in \mathbb{R}^3$ with wavefunction $\Psi_t(x)$. Bohmian guidance then takes the usual form (Axiom A3)

$$\frac{dQ_t}{dt} = \frac{\hbar}{m} \Im \left( \frac{\nabla \Psi_t(Q_t)}{\Psi_t(Q_t)} \right).$$

In CBQFT-2 the underlying field algebra and Hamiltonian density are the same, but different macroscopic contexts $\Lambda$ are represented by (in general disjoint and unitarily inequivalent) representations $\pi_\Lambda$ of $\mathcal{A}$ on Hilbert spaces $\mathcal{H}_\Lambda$. In the *particle-ontology* regime one assumes that, for each fixed $\Lambda$, the active $\mathcal{H}_\Lambda$ admits a *Fock-like* particle-number grading.[40] For the present toy model we keep the formulas transparent by working sectorwise with a particle-number grading in each $\mathcal{H}_\Lambda$, so that the formal expressions for $H_0$ and the guidance equation apply *within* each fixed-$\Lambda$ interval (Axiom B3).

[40] "Fock-like" here means only that $\mathcal{H}_\Lambda$ admits a sector decomposition $\mathcal{H}_\Lambda = \bigoplus_{n \geq 0} \mathcal{H}_{\Lambda,n}$ supporting the particle configuration beable and the Bell-type jump law. It does *not* require that $(\pi_\Lambda, \mathcal{H}_\Lambda)$ be unitarily equivalent to a single fixed Fock representation of $\mathcal{A}$.

Finally, in both cases, we introduce a bounded "detector region" $D \subset \mathbb{R}^3$ in which detection events occur. The contextual structure in CBQFT-1 and CBQFT-2 will control how the field interacts with this region. The difference between these approaches, as we shall see, concerns how context $\Lambda$ is treated.

## CBQFT-1: external context

In CBQFT-1 the macroscopic context is encoded by a classical parameter $\Lambda \in \mathcal{C}$ (here: detector ON/OFF), and the Hamiltonian takes the context-sensitive form

$$H(\Lambda) = H_0 + H_I + H_C(\Lambda),$$

as in Axiom A2. Here $H_I$ contains genuine field self-interactions, while $H_C(\Lambda)$ encodes coupling to the classical context.

### Contextual absorber in CBQFT-1

To model a simple absorbing detector in a fixed context $\Lambda_{\text{on}}$, let us include a non-Hermitian contribution to the Hamiltonian:

$$H_C(\Lambda_{\text{on}}) = -\frac{i}{2}\Gamma_{\Lambda_{\text{on}}}, \qquad \Gamma_{\Lambda_{\text{on}}} = \gamma \int_{\mathbb{R}^3} d^3x \, \chi_D \, \phi(x)^2,$$

with $\gamma > 0$ and $\chi_D$ the characteristic function of the detector region $D \subset \mathbb{R}^3$,

$$\chi_D(x) := \begin{cases} 1, & x \in D, \\ 0, & x \notin D. \end{cases}$$

In the one-particle sector $\Gamma_{\Lambda_{\text{on}}}$ acts (up to the usual approximations) as multiplication by $\gamma \, \chi_D(x)$ on $\Psi_t(x)$, yielding an effective Schrödinger equation

$$i\partial_t \Psi_t(x) = \Big(H_0^{(1)} - \frac{i}{2}\gamma \, \chi_D(x)\Big)\Psi_t(x),$$

where $H_0^{(1)}$ is the one-particle free Hamiltonian. A short calculation yields

$$\frac{d}{dt}\|\Psi_t\|^2 = -\gamma \int_D |\Psi_t(x)|^2 \, d^3x,$$

so the norm decreases at a rate determined by the probability density in the detector region. This is the familiar "absorbing potential" picture: probability flows into $D$ and never comes back (see Proposition 4.1).

More generally, CBQFT-1 allows context to influence not only the *absorption* of trajectories but also their *shape*. One can include, in addition to the purely anti-Hermitian term above, a real detector potential

$$H_C(\Lambda_{\rm on}) = V_{\rm det}(x) \; - \; \frac{i}{2}\Gamma_{\Lambda_{\rm on}},$$

with $V_{\rm det}(x)$ supported in $D$. In the one-particle sector this yields

$$i\partial_t \Psi_t(x) = \Big(H_0^{(1)} + V_{\rm det}(x) - \tfrac{i}{2}\gamma\,\chi_D(x)\Big)\Psi_t(x).$$

The real part $V_{\rm det}$ and the imaginary part $-\frac{i}{2}\gamma\,\chi_D$ both enter the Schrödinger evolution and hence can reshape $\Psi_t$ in the detector region and bend the particle trajectories via the Bohmian law. One may represent a "click" in CBQFT-1 by a *stopping convention*: define the first-entry time $\tau_D := \inf\{t \geq t_0 : Q_t \in D\}$ and, in the effective open-system regime, treat $\tau_D < \infty$ as the detection time of that run, while the norm loss of $\Psi_t$ tracks (at the ensemble level) the probability weight flowing into this detection channel.[41]

### Guidance and detection in CBQFT-1

In CBQFT-1 the wavefunction always evolves *deterministically* according to the Schrödinger-type equation (10); there is no stochastic collapse of $\Psi_t$. The stochasticity lies entirely in the configuration process $Q_t$. In the one-particle sector, the Bohmian trajectory $Q_t$ is guided by the Bohmian guidance equation with $\Psi_t$ evolving under the non-Hermitian Hamiltonian above. In the *open-system* regime of CBQFT-1, a non-Hermitian absorber provides a convenient way to encode detection phenomenology. A simple convention is: once the trajectory first enters the detector region $D$ (at $\tau_D$), we record a "click" and cease tracking that run at the level of the explicit degrees of freedom. The accompanying norm loss of $\Psi_t$ then reflects, at the ensemble level, the probability weight flowing into that coarse-grained detection channel.

At this point CBQFT-1 does not, by itself, continue the story, as the theory has no internal dynamics for the context $\Lambda$ and no rule that changes the Hamiltonian after the detector clicks. One may, of course, *externally* prescribe a piecewise-constant $\Lambda(t)$ to mimic a detector that is switched off after a detection, or reset at some later time, and then renormalise $\Psi_{t_*}$ to

[41] This is an effective modelling convention rather than an additional configuration jump law: the anti-Hermitian term is *not* fed into the Bell-type jump-rate prescription for $Q_t$.

describe the conditional state of the subensemble in which no detection has yet occurred. Such changes of context, however, are not generated by the microscopic dynamics of $(\Psi_t, Q_t)$ itself. If no such intervention is made, the non-Hermitian term causes $\|\Psi_t\|$ to decay whenever there is amplitude in $D$.

## CBQFT-2: dynamic context

CBQFT-2 upgrades this picture by making the macroscopic context itself a stochastic dynamical variable. In the general formulation, the context may be represented by a spacetime field $\Lambda(x,t)$ encoding, for example, spatially varying phases, boundary conditions, or detector settings. In the present toy model we compress this structure, at each time $t$, into a single label $\Lambda_t$ that records the relevant macroscopic status of the detector.

The state at time $t$ is the triple $(\Psi_t, Q_t, \Lambda_t)$, as in Axiom B1, where $\Lambda_t$ is a piecewise constant jump process taking values in a finite set

$$\mathcal{C} = \{\Lambda_{\text{ready}}, \Lambda_{\text{fired}}\}.$$

These correspond to a detector that is initially *ready* to detect and then becomes *saturated* (or, in a slightly richer variant, fired and subsequently reset) after click events. Each value of $\Lambda$ selects a representation $\pi_\Lambda$ of the field algebra on a Hilbert space $\mathcal{H}_\Lambda$, and the triple $(\Psi_t, Q_t, \Lambda_t)$ evolves according to Axioms B1–B6. In particular, the Hamiltonians $H_\Lambda$ are self-adjoint in each sector, and context changes are implemented by isometries, so the wavefunction is always normalised along each realised trajectory (Proposition 7.1) and full Bohmian equivariance is maintained (Proposition 7.2).[42]

### Sectors and re-expression

In the simplest implementation one may take

$$\mathcal{H}_{\Lambda_{\text{ready}}} \cong \mathcal{F}_{\text{free}}, \qquad \mathcal{H}_{\Lambda_{\text{fired}}} \cong \mathcal{F}_{\text{det}},$$

where $\mathcal{F}_{\text{free}}$ denotes a Hilbert space carrying a representation appropriate to a non-triggered detector and $\mathcal{F}_{\text{det}}$ denotes a (possibly disjoint) Hilbert space/representation appropriate to the fired detector (for instance, different macroscopic boundary conditions or additional pointer degrees of freedom).[43]

[42] If an amplification is present, $H_\Lambda$ is understood to act as $H_\Lambda \otimes I_{\mathcal{K}}$.

[43] For the particle-ontology toy model we assume each admits a particle-number grading, but we do *not* assume that they are unitarily equivalent as global representations of $\mathcal{A}$.

On a common local algebra $\mathcal{A}_0$ of observables supported in a bounded region outside $D$, local quasiequivalence ensures that the restrictions of $\pi_{\Lambda_{\text{ready}}}$ and $\pi_{\Lambda_{\text{fired}}}$ to the overlap algebra $\mathcal{A}_0$ have the same normal folium, even though Assumption 7.1 allows the global representations to be disjoint. Axiom B6 then supplies an isometry

$$U_{\text{ready}\to\text{fired}} : \mathcal{H}_{\Lambda_{\text{ready}}} \longrightarrow \widetilde{\mathcal{H}}_{\Lambda_{\text{fired}}}$$

such that

$$\langle \Psi, \pi_{\Lambda_{\text{ready}}}(A)\Psi\rangle = \langle U_{\text{ready}\to\text{fired}}\Psi, \widetilde{\pi}_{\Lambda_{\text{fired}}}(A)\, U_{\text{ready}\to\text{fired}}\Psi\rangle \quad \forall A \in \mathcal{A}_0.$$

At a detection event the state is re-expressed according to $\Psi_{t^+} = U_{\text{ready}\to\text{fired}}\Psi_{t^-}$, preserving local expectations and the norm. This is a concrete instance of the isometric re-expression principle of Axiom B6 (see also Lemma 7.1).

### A macroscopic detector observable

We may now introduce within our toy model a macroscopic "pointer" observable associated with the detector. Let $R \subset \mathbb{R}^3$ be a region containing the detector hardware (including $D$), and let $M \in \mathcal{A}$ be a coarse-grained observable localised in $R$ that distinguishes the *ready* and *fired* macroscopic configurations of the detector.

For example, $M$ could be a suitably normalised coarse-grained energy, charge, or magnetisation operator associated with the detector. In an algebraic setting one typically constructs such macroscopic quantities as limits of local observables supported in $R$ and then takes the corresponding weak-operator limit in the relevant GNS representation. In the present schematic model we simply assume that $M \in \mathcal{A}$ has two macroscopically distinct expectation values corresponding to the ready and fired states. In each context sector we represent $M$ via

$$M_\Lambda := \pi_\Lambda(M) \in \mathcal{B}(\mathcal{H}_\Lambda), \qquad \Lambda \in \{\Lambda_{\text{ready}}, \Lambda_{\text{fired}}\}.$$

Let us choose $M$ and the reference states in each sector such that

$$\langle \Psi_{\text{ready}}, M_{\Lambda_{\text{ready}}}\Psi_{\text{ready}}\rangle \approx 0, \qquad \langle \Psi_{\text{fired}}, M_{\Lambda_{\text{fired}}}\Psi_{\text{fired}}\rangle \approx 1,$$

with small fluctuations around 0 and 1 reflecting microscopic noise. Thus $M$ acts as a macroscopic observable whose expectation value records whether the detector has fired and depends on the macroscopic context. By construction, $M$ is *not* an element of the overlap algebra $\mathcal{A}_0$. In contrast, the overlap algebra $\mathcal{A}_0$ used in the isometry $U_{\text{ready}\to\text{fired}}$ was chosen to be supported in a bounded region *outside* $R$ (and hence outside $D$). For all $A \in \mathcal{A}_0$ we have

$$\langle \Psi, \pi_{\Lambda_{\text{ready}}}(A)\Psi\rangle = \langle U_{\text{ready}\to\text{fired}}\Psi, \widetilde{\pi}_{\Lambda_{\text{fired}}}(A)\, U_{\text{ready}\to\text{fired}}\Psi\rangle,$$

so local expectations away from the detector are preserved at the instant of the context jump, whereas the expectation of the macroscopic pointer $M$ changes dramatically (from $\approx 0$ to $\approx 1$) because $M \notin \mathcal{A}_0$. This illustrates explicitly how CBQFT-2 treats macroscopic contextual observables: they are encoded by algebra elements whose representations differ between sectors, and their expectations encode the value of the stochastic context $\Lambda_t$.

### A context kernel with intrinsic reset

The context process $\Lambda_t$ is a continuous-time jump Markov process with transition rates $W_{\Lambda_i\to\Lambda_j}(Q_t, \mathcal{J}_t)$ as in B5, where $\mathcal{J}_t$ collects any additional coarse-grained data that may influence the context transition. To model a detector that both saturates and *intrinsically resets*, we tie the transition from $\Lambda_{\text{ready}}$ to $\Lambda_{\text{fired}}$ to the arrival of *any* particle in the detector region, and we add a slow, endogenous reset from $\Lambda_{\text{fired}}$ back to $\Lambda_{\text{ready}}$. In the $N(t)$-particle sector, write $q = (x_1, \dots, x_{N(t)})$ and let $Q_t$ be the full Bohmian configuration. Fix rate constants $\kappa > 0$ and $\kappa_{\text{reset}} > 0$ and define

$$W_{\text{ready}\to\text{fired}}(Q_t, \mathcal{J}_t) = \kappa \sum_{i=1}^{N(t)} \chi_D\big(x_i(t)\big), \tag{26}$$

$$W_{\text{fired}\to\text{ready}}(Q_t, \mathcal{J}_t) = \kappa_{\text{reset}}. \tag{27}$$

Thus:

- While the detector is *ready* and at least one Bohmian trajectory lies in $D$, the context process acquires a large switching rate $W_{\text{ready}\to\text{fired}}$. In the limit $\kappa \to \infty$, the flip from $\Lambda_{\text{ready}}$ to $\Lambda_{\text{fired}}$ occurs essentially at the first time any particle world-line enters $D$.

- Once $\Lambda_t = \Lambda_{\text{fired}}$, the detector remains fired for a random retention time of order $1/\kappa_{\text{reset}}$, after which it flips back to $\Lambda_{\text{ready}}$ with rate $W_{\text{fired}\to\text{ready}}$. No external intervention is required: the reset is an intrinsic stochastic effect encoded in the context kernel itself.

A slightly more refined variant replaces the indicator $\chi_D(x_i)$ by the flux of Bohmian probability into $D$, e.g.

$$W_{\text{ready}\to\text{fired}}(Q_t, \mathcal{J}_t) = \kappa \int_{\partial D} n(x) \cdot J(x,t)\, dS_x,$$

with $J(x,t)$ the total Bohmian current and $n(x)$ the outward normal, and lets $\kappa_{\text{reset}}$ depend on coarse-grained environmental variables included in $\mathcal{J}_t$. Both choices are compatible with Axiom B5 and illustrate how the context kernel can depend directly on microscopic quantities while still producing an effective cycle of *ready* $\to$ *fired* $\to$ *ready* dynamics.

### Feedback in the micro-dynamics

To make the feedback loop explicit, suppose the one-particle part of the Hamiltonian depends on the context as

$$H^{(1)}_{\Lambda_{\text{ready}}} = H^{(1)}_0 + V_{\text{ready}}(x), \qquad H^{(1)}_{\Lambda_{\text{fired}}} = H^{(1)}_0 + V_{\text{fired}}(x),$$

with, for example,

$$V_{\text{ready}}(x) \approx 0 \qquad \text{(transparent detector window)}, \tag{28}$$

$$V_{\text{fired}}(x) = V_0\, \chi_D(x) \qquad \text{(activated barrier)}, \tag{29}$$

where $V_0 \gg m$ and $\chi_D$ is the indicator of $D$. Then:

- Initially, $\Lambda_t = \Lambda_{\text{ready}}$ and all particles are guided by Bohmian velocities derived from $\Psi_t$ evolving under $H_{\Lambda_{\text{ready}}}$. The region $D$ is effectively transparent: trajectories can enter and cross $D$, and when any particle does so, $W_{\text{ready}\to\text{fired}}$ becomes large.

- At the first detection time $t_*$, the context flips to $\Lambda_{\text{fired}}$, the state is re-expressed as $\Psi_{t_*^+} = U_{\text{ready}\to\text{fired}}\Psi_{t_*^-}$, and from then on $\Psi_t$ evolves under $H_{\Lambda_{\text{fired}}}$. All subsequent particles in the beam are guided by Bohmian velocities associated with the *new* Hamiltonian $H_{\Lambda_{\text{fired}}}$; for instance, they may now be reflected from $D$ or trapped.

- After a random retention time of order $1/\kappa_{\text{reset}}$, the detector relaxes and the context stochastically returns to $\Lambda_{\text{ready}}$, at which point $H_{\Lambda_{\text{ready}}}$ is reinstated and the detector is again able to fire.

In this way a single microscopic event (a trajectory entering $D$) induces a persistent change in the macroscopic context $\Lambda_t$, and that changed context in turn affects the micro-dynamics of *all* later particles via $H_{\Lambda_t}$ and the jump structure. Unlike our model using CBQFT-1, no external "manual reset" of the context is required: the feedback from $Q_t$ to $\Lambda_t$ is encoded directly in the kernel $W$, and the resulting irreversibility of the context process can be analysed using the entropy production methods of Section 7.5.2.

### Top–down influence on Bohmian trajectories

It is useful to summarise, in explicitly Bohmian terms, how the macroscopic detector in CBQFT-2 bends trajectories and records detection events via the context variable. For each context value $\Lambda \in \{\Lambda_{\text{ready}}, \Lambda_{\text{fired}}\}$ and particle number sector, the Schrödinger-type equation with Hamiltonian $H_\Lambda$ determines a context-dependent velocity field

$$v_\Lambda(x,t) = \frac{\hbar}{m} \Im\left(\frac{\nabla\Psi_t(x)}{\Psi_t(x)}\right),$$

where $\Psi_t$ is the restriction of the global state to the relevant sector in $\mathcal{H}_\Lambda$. The actual configuration $Q_t$ follows the associated Bohmian trajectory $dQ_t/dt = v_\Lambda(Q_t, t)$ between particle jumps. In the *ready* context, we chose

$$H^{(1)}_{\Lambda_{\text{ready}}} = H^{(1)}_0 + V_{\text{ready}}(x), \qquad V_{\text{ready}}(x) \approx 0,$$

so the detector region $D$ is effectively transparent and the velocity field $v_{\Lambda_{\text{ready}}}(x,t)$ is very close to the free-field velocity. The macroscopic detector in $R$ is "present" in the ontology via $\Lambda_t$, but while it remains ready it only weakly perturbs the microscopic trajectories. By contrast, in the *fired* context we take

$$H^{(1)}_{\Lambda_{\text{fired}}} = H^{(1)}_0 + V_{\text{fired}}(x), \qquad V_{\text{fired}}(x) = V_0\,\chi_D(x), \quad V_0 \gg m,$$

so the effective potential in $D$ changes dramatically. The wavefunction $\Psi_t$ is then evolved under $H_{\Lambda_{\text{fired}}}$, and the resulting velocity field $v_{\Lambda_{\text{fired}}}(x,t)$ bends trajectories away from $D$ (e.g. by reflection at the activated barrier) or traps them there if $V_{\text{fired}}$ is chosen to support bound states inside $D$. In either case, the macroscopic change in context—encoded in the single label $\Lambda_t = \Lambda_{\text{fired}}$—is manifested as a systematic, lawlike modification of the microscopic velocity field in the detector region.

Detection is represented in two complementary ways. First, the *first* particle to enter $D$ triggers a stochastic context jump $\Lambda_{\text{ready}} \to \Lambda_{\text{fired}}$ via the large rate $W_{\text{ready}\to\text{fired}}$ in (26), so the detection event is recorded in the macroscopic sector label $\Lambda_t$ and in the pointer observable $M$. Second, once the detector has fired, the modified potential $V_{\text{fired}}$ can be chosen so that the trajectory of the absorbed particle is effectively trapped in $D$ (or removed from the relevant configuration sector in a full Bell-type QFT), and all later particles are deflected or blocked. In this sense the detector both *bends* and *absorbs* Bohmian trajectories in a top–down manner: a change in the macroscopic context $\Lambda_t$ reshapes the microscopic velocity field and the configuration space transitions available to the particles.

**Locality of the context update.** The construction above also illustrates how a context jump can be genuinely local. Suppose, in addition to the detector in $R$, there is a second, spatially separated detector with hardware region $R' \subset \mathbb{R}^3$, described by its own macroscopic pointer observable $M' \in \mathcal{A}$ and corresponding sector-dependent operators $M'_\Lambda := \pi_\Lambda(M')$. As with $M$, one may think of $M'$ as arising from limits of local observables supported entirely in $R'$. We may choose the overlap algebra $\mathcal{A}_0$ to contain all observables localised in $R'$ (and, in an idealised sense, the macroscopic pointer $M'$), while excluding $M$, which is supported in $R$. Then at a context jump $\Lambda_{\text{ready}} \to \Lambda_{\text{fired}}$ triggered by a particle entering $D \subset R$, Axiom B6 guarantees that for all $A \in \mathcal{A}_0$,

$$\langle \Psi, \pi_{\Lambda_{\text{ready}}}(A)\Psi \rangle = \langle U_{\text{ready}\to\text{fired}}\Psi, \widetilde{\pi}_{\Lambda_{\text{fired}}}(A)\, U_{\text{ready}\to\text{fired}}\Psi \rangle.$$

In particular, the expectations of all observables associated with the remote detector in $R'$ — including its macroscopic pointer $M'$ — are unchanged across the jump, while the local pointer $M$ in $R$ can flip from $\approx 0$ to $\approx 1$. Thus a context update modelling saturation of the first detector does not "drag along" other macroscopic systems: the isometry acts trivially on observables in spacelike separated regions, and only the local macroscopic structure in $R$ is reconfigured.

## Comparison: CBQFT-1 versus CBQFT-2

This simple scalar-field model highlights the differences between CBQFT-1 and CBQFT-2:

- In CBQFT-1 the context $\Lambda$ is *external*: it modulates the Hamiltonian and the jump rates, and non-Hermitian contributions can model absorption, but there is no dynamical law within the theory that updates $\Lambda$ in response to $Q_t$. Saturation or reset of the detector must be encoded in an externally prescribed schedule $\Lambda(t)$.

- In CBQFT-2, however, the context $\Lambda_t$ is a genuine dynamical variable, evolving stochastically according to a Markov kernel $W_{\Lambda_i \to \Lambda_j}(Q_t, \mathcal{J}_t)$ that can depend on microscopic quantities. In this example, a single detection event drives a permanent context change that reshapes the subsequent Bohmian micro-dynamics without external intervention: the context dynamics is fully *endogenous* to the model.

Thus CBQFT-2 not only reproduces the kind of effective absorption modelled by non-Hermitian terms, but embeds it in a richer framework where macroscopic context, microscopic dynamics, and feedback between them are described by a single, internally consistent dynamical scheme.