

# NON-COOPERATIVE AND COOPERATIVE CLIMATE POLICIES

## WITH ANTICIPATED BREAKTHROUGH TECHNOLOGY<sup>Ⓟ</sup>

Niko Jaakkola<sup>§</sup> and Frederick van der Ploeg<sup>†</sup>

### Abstract

Global warming can be curbed by pricing carbon emissions and thus substituting fossil fuel with renewable energy consumption. Breakthrough technologies (e.g., fusion energy) can reduce the cost of such policies. However, the chance of such a technology coming to market depends on investment. We model breakthroughs as an irreversible tipping point in a multi-country world, with different degrees of international cooperation. We show that international spill-over effects of R&D in carbon-free technologies lead to double free-riding, strategic over-pollution and underinvestment in green R&D, thus making climate change mitigation more difficult. We also show how the demand structure determines whether carbon pricing and R&D policies are substitutes or complements.

**Keywords:** global warming, carbon pricing, renewable R&D, tipping point, international cooperation, non-cooperative policies, feedback Nash equilibrium.

**JEL codes:** D62, D90, H23, Q35, Q38, Q54, Q58

**Running title:** CLIMATE POLICY WITH BREAKTHROUGH TECHNOLOGY

*March 2018*

---

<sup>Ⓟ</sup> We are very grateful for helpful discussions with Armon Rezai, Christian Traeger and Aart de Zeeuw on this topic and for the comments of the participants in the 2016 SURED conference in Banyuls-sur-Mer, as well as of participants in EAERE 2016, the 2016 Tinbergen Conference on Combating Climate Change, and the 2016 CESifo Area Conference on Energy and Climate Economics. Support from the BP funded OxCarre and the European Research Council (FP7-IDEAS-ERC grant no. 269788: ‘Political economy of Green Paradoxes’) are gratefully acknowledged.

<sup>§</sup> Corresponding author. Ifo Institute at the University of Munich. Email: [jaakkola@ifo.de](mailto:jaakkola@ifo.de). Tel: +49 89 9224 1400. A large part of the work in this paper was conducted while visiting VU University Amsterdam. Financial support from the European Research Council (FP7-IDEAS-ERC grant no. 269788), and the gracious hospitality offered by VU University and by Cees Withagen, are warmly acknowledged.

<sup>†</sup> OXCARE, Department of Economics, University of Oxford. Also affiliated with VU University Amsterdam, The Netherlands.

## 1. Introduction

Breakthrough technologies may dramatically reduce the cost of carbon-free alternatives to fossil fuel and fundamentally change the global problem of combating climate change. By investing in a breakthrough technology, each country can bring forward the expected date of arrival of such technologies. The effectiveness of the technology, however, also depends on by how much carbon is priced. What happens if countries do not cooperate in international negotiations? The answer is that each one of them prices carbon too low and does not invest enough in the breakthrough technology, with the result that the planet has to endure excessive global warming as the expected arrival of the breakthrough takes longer. This corresponds to a double free-riding problem.

Our objective is to shed new light on these dilemmas in a world where technology spillovers are prevalent and fossil fuel is abundant. We do this by modelling the breakthrough technology as an endogenous regime shift where the probability of the breakthrough increases in the globally accumulated investments in the technology. As investments in R&D make the breakthrough technology more imminent, and as the benefits of the breakthrough represent a public good, this results in an additional international free-rider problem. We distinguish three outcomes: (1) international cooperation among all countries concerned; (2) non-cooperative behaviour, in feedback (subgame-perfect) Nash equilibrium; and (3) second-best outcomes with international cooperation on R&D investments only.

We make three main contributions. First, we demonstrate how the outcome of partial international cooperation – on R&D efforts only – depends on the response of fossil fuel demand to technological breakthroughs. In the absence of international agreement on carbon pricing, internationally coordinated R&D efforts will exceed the socially optimal level if technological breakthroughs have their biggest impact of fossil demand when fossil prices are low. The reason is intuitive: a global R&D coalition will then expect the return to investment to be higher when carbon taxes are low. In this case, carbon pricing and policies to clean R&D are substitutes. We show this analytically for the special case where marginal damages from global warming (i.e., from a higher stock of carbon in the atmosphere) are constant and R&D stocks can be adjusted at a linear cost. Interestingly,

for this case the open-loop Nash equilibrium outcome coincides with a feedback equilibrium outcome.

Second, we demonstrate the importance of strategic effects in our multi-country framework. To capture richer strategic interactions, we allow for more general functional forms, in particular rising marginal damages from global warming and sluggish adjustment of green R&D stocks. We employ newly developed numerical methods to solve the required Hamilton-Jacobi-Bellman equations for the feedback Nash equilibrium of the calibrated, fully non-cooperative model, and compare the outcome with the cooperative (efficient) equilibrium. The incentives to free-ride on both abatement and R&D are compounded by dynamic strategic incentives to over-pollute and underinvest (to encourage others to undertake more effort). We characterise these effects analytically, and find numerically that the dynamic strategic incentives can be very strong indeed. We thus contribute to the highly policy-relevant topic of joint free-riding with respect to both R&D efforts and carbon emissions.<sup>1</sup>

Our third contribution is technical, and related to the non-cooperative analysis of regime shifts under endogenous, state-dependent tipping probability. To the best of our knowledge, this paper is the first to consider technological regime shifts dependent on R&D stocks in a non-cooperative setting, and the first to consider non-cooperative endogenous regime shifts with more than one state variable.<sup>2</sup>

Section 2 first sets out how to deal with a breakthrough technology with an uncertain date of arrival in the context of a single country or of the global economy. Section 3 sets up the general non-cooperative problem and the feedback Nash equilibrium. Section 4 obtains analytical results for the case of constant marginal damages of global warming

---

<sup>1</sup> Our analysis builds on an earlier literature on dynamic R&D games (e.g., Dasgupta et al., 1983; Harris and Vickers, 1987, 1995; Harris et al., 2010; Doraszelski, 2003). In patent races, excessive investment can arise as firms try to pre-empt their competitors from obtaining the prize of monopoly rights to an innovation (Harris and Vickers, 1987). In our setup, the chance of a breakthrough innovation depends on the joint stock of green R&D and the innovation spills over immediately and completely once it has occurred, so that such pre-emption motives are absent. We thus get the opposite result, i.e., *under*-investment in green R&D in a non-cooperative setup.

<sup>2</sup> van der Ploeg and de Zeeuw (2016) analyse and compare the open-loop Nash equilibrium and the cooperative outcome of a North-South climate game with tipping, but do not investigate the feedback Nash equilibrium outcome. Nkuiya (2015) has recently considered climate tipping points in a multi-country framework where the hazard rate depends on the atmospheric carbon stock, in a univariate framework. The optimal (i.e., cooperative) management of renewable resources and pollution under endogenous tipping probabilities has been studied by, for example, Polasky et al. (2011) and de Zeeuw and Zemel (2012).

and no investment adjustment costs. Section 5 discusses the details of our illustrative calibration. Section 6 obtains numerical results for the general model under cooperation. Section 7 solves numerically the feedback Nash equilibrium and compares the outcomes with those under international policy cooperation. Section 8 concludes.

## 2. Climate Policy with Anticipated Breakthrough in the Global Economy

Before we discuss the problem of climate policy among competing countries, we consider the case of a single country, or what will turn out to be equivalent to the cooperative outcome for climate policy in the global economy. To focus on the anticipated arrival of a breakthrough carbon-free technology, we abstract from general equilibrium considerations. In view of the abundance of coal and shale gas, we suppose for purposes of our analysis that fossil fuel is not scarce.

Normalise world population to unity. Let  $A > 0$  be the stock of carbon in the atmosphere and  $D(A)$ ,  $D' > 0$ ,  $D'' \geq 0$ , be the per-capita damages from global warming, where the function  $D(A)$  is weakly convex. The rate of decay of atmospheric carbon is  $\gamma > 0$ . The per-capita rate of fossil fuel use is denoted by  $F \geq 0$ , so that the stock of carbon in the atmosphere evolves according to

$$(1) \quad \dot{A}(t) = F(t) - \gamma A(t) \text{ with } A(0) = A_0 > 0,$$

where fossil fuel use is measured in tons of carbon per unit of time. The exogenous cost of producing one unit of fossil fuel, or equivalently one ton of emitted carbon, is  $d > 0$ .

The rate of carbon-free renewable energy use (e.g. solar, or fusion following the breakthrough) is denoted by  $R \geq 0$ . The breakthrough in renewable technology occurs at some unknown future time  $T > 0$ . The cost of producing one unit of renewable energy is  $b(t) = \beta > 0$ ,  $t < T$ , before the breakthrough and  $0 < b(t) = \tilde{\beta} < \beta$ ,  $t \geq T$ , after the breakthrough. The hazard rate for the breakthrough is a concave, increasing function of the stock of renewable R&D,  $B$ , i.e.,  $h = H(B) > 0$ ,  $H' > 0$ ,  $H'' \leq 0$ . More formally, the equation governing the evolution of  $b(t)$  is, for small  $dt$ ,

$$(2) \quad b(t+dt) = \begin{cases} \beta & \text{with probability } 1-H(B)dt, \text{ if } b(t) = \beta, \\ \tilde{\beta} & \text{with probability } H(B)dt, \text{ if } b(t) = \beta, \\ \tilde{\beta} & \text{if } b(t) = \tilde{\beta}. \end{cases}$$

The probability of the breakthrough in a short interval  $dt$  is thus  $H(B)dt$  if the breakthrough has not occurred yet, and 0 otherwise. The cumulative probability of the breakthrough occurring sometime in the interval  $[0, T]$  equals  $1 - \exp(-\int_0^T H(B(t))dt)$ . The stock of accumulated R&D knowledge evolves according to

$$(3) \quad \dot{B} = I - \delta B \text{ with } B(0) = B_0, 0 \leq B \leq \bar{B} \equiv \bar{I} / \delta,$$

where  $I \in (-\infty, \bar{I}]$  denotes per-capita investment in renewable R&D and  $\delta > 0$  stands for the depreciation rate of the stock of renewable R&D. The upper bound on the investment rate  $\bar{I}$  is given by a budget constraint unrelated to energy use (e.g. due to political constraints on the spending available for this use). We assume this constraint is quite high but finite.

Utility of fossil fuel and renewable energy use is given by the utility function  $U(F, R)$  which is concave and homogeneous of degree  $\mu \in (0, 1)$ , and which satisfies the Inada conditions  $\lim_{F \rightarrow 0} U_F = \lim_{R \rightarrow 0} U_R = \infty$ . The two types of energy are imperfect substitutes. Utility of all goods and services is quasi-linear in consumption of other commodities and services,  $c$ , so the total utility flow is  $U(F, R) + c$ . Non-energy consumption is what is left over of exogenous income,  $Y > 0$ , after paying for the cost of fossil fuel and renewable energy use and for renewable R&D, namely  $c = Y - dF - bR - C(I, B)$ , with the last term denoting the per-capita cost of investment in renewable R&D. As the level of  $Y$  is immaterial to the problem, we without loss of generality omit it henceforth.<sup>3</sup> The discount rate is given by the constant  $\rho > 0$ .

The government seeks to maximise total expected utility:

$$(4) \quad \max_{F(t), R(t), I(t)} \mathbb{E} \int_0^\infty e^{-\rho t} (U(F, R) - dF - bR - C(I) - D(A)) dt$$

---

<sup>3</sup> We thus implicitly assume  $Y$  is large enough so that corner solutions never arise.

subject to the dynamics of carbon (1) and R&D (3) and the stochastic process governing breakthrough arrival (2), and where we use  $\mathbb{E}$  to denote the expectations operator.

### 2.1. After-breakthrough problem

Using the principle of dynamic programming, we start with the after-breakthrough problem. After the breakthrough there is no further role for investment in clean R&D, i.e.,  $I = 0$ , and the only state variable is the stock of carbon in the atmosphere,  $A$ . Hence, the Hamilton-Jacobi-Bellman (HJB) equation for this problem is

$$(5) \quad \rho W(A) = \max_{F,R} [U(F,R) - dF - \tilde{\beta}R - D(A) + W_A(A)(F - \gamma A)].$$

We denote socially optimal fossil fuel and renewable energy use, respectively, by  $F^*, R^*$ .

The optimality conditions for energy use are thus

$$(6) \quad U_F(F^*, R^*) = d + \tau^*(A) \text{ and } U_R(F^*, R^*) = \tilde{\beta} \text{ with } \tau^*(A) \equiv -W_A(A) > 0,$$

and give rise to the following demands for fossil fuel and renewable energy:

$$(7) \quad F^* = F(d + \tau^*, \tilde{\beta}) \text{ and } R^* = R(d + \tau^*, \tilde{\beta}),$$

where we omit that  $\tau^*$  depends on  $A$ . Hence, the price of fossil fuel is set to its user cost which consists of the production cost plus the after-breakthrough social cost of carbon (SCC), denoted by  $\tau^*$ , where the latter is the marginal disvalue of an extra ton of carbon in the atmosphere. The social optimum can be attained in a market economy by levying a specific carbon tax equal to the SCC and rebating the revenue in lump-sum manner to private agents. Alternatively, it can be attained via a competitive market for carbon emission rights. The price of renewable energy is set to the post-tip unit production cost. We have  $F_{d+\tau^*} < 0$  and  $R_{\tilde{\beta}} < 0$ , and assume that the two types of energy are gross substitutes, so that  $F_{\tilde{\beta}} > 0$  and  $R_{d+\tau^*} > 0$ .

The HJB equation (5) can be rewritten in a more succinct form as

$$(8) \quad \rho W(A) = U^*(d + \tau^*, \tilde{\beta}) + \gamma \tau A - D(A),$$

where

$$(9) \quad U^*(d + \tau, b) \equiv \left[ \max_{F, R} U(F, R) - (d + \tau)F - bR \right] - (\tau^* - \tau)F(d + \tau, b)$$

denotes the maximum utility obtained from energy use, including the social disvalue of emitted carbon, given an arbitrary ‘carbon tax’  $\tau$ . Carbon emissions thus lower utility if the carbon tax is below the SCC. Upon differentiating equation (8) with respect to time, using equation (1), and ensuring that coefficients on  $\dot{A}$  are always equal to zero, we get

$$(10) \quad \dot{\tau}^* = (\rho + \gamma)\tau^* - D'(A) \quad \text{or}$$

$$(10') \quad \tau^*(t) = \int_t^\infty D'(A(s)) e^{-(\rho + \gamma)(s-t)} ds.$$

The after-breakthrough SCC is (10'). It corresponds to the present discounted value of all future marginal damages resulting from burning one unit of carbon today. Decay of atmospheric carbon, at rate  $\gamma$ , alleviates global warming and thus implies a lower SCC.

If the damage function is linear, i.e.,  $D(A) = \alpha A$ , marginal damages from global warming are constant and from equation (10) the SCC is constant too:  $\tau^* = \alpha / (\rho + \gamma)$ .<sup>4</sup> The post-

tip value function is then also linear,  $W(A) = w - \frac{\alpha}{\rho + \gamma} A$ , with a constant term

$$w \equiv \frac{U^*(d + \tau^*, \tilde{\beta})}{\rho}.$$

With convex instead of linear damages, the dynamics of the stock of atmospheric carbon and the SCC are given by the saddle-path system consisting of equations (1) and (10) with fossil fuel use given by (7). We can write the policy rule as  $\tau^* = \tau^A(A, \tilde{\beta})$  with  $\tau_A^A > 0$  and  $\tau_{\tilde{\beta}}^A > 0$ ; this corresponds to the stable manifold of the system, so that the post-tip SCC is higher if the stock of atmospheric carbon is higher (so that global warming is more intense) and the cost reduction resulting from the breakthrough in renewable energy is smaller.

---

<sup>4</sup> Equation (10) implies that any other SCC explodes to infinity or collapses to zero as long as the breakthrough has not occurred; neither can be optimal. As the SCC falls to zero, the marginal benefit of increasing the SCC is strictly positive, while the marginal cost goes to zero. If the SCC goes to infinity, the benefit of lowering the tax becomes arbitrarily large as the marginal product of fossil fuels explodes.

## 2.2. Before-breakthrough problem

For now, we abstract from investment adjustment costs. In this formulation, the R&D stock follows a most rapid approach path to its optimal level. If at time  $t$  the R&D stock  $B(t)$  is above this optimal level, it jumps down immediately with any excess stock immediately converted to consumption, yielding a discrete lump of utility of magnitude  $\hat{U}(t)$ . More precisely, denoting the target level of the R&D stock by  $B^+(t) \leq B(t)$ , this lump is given by the limit of flow utility as the investment rate is allowed to go to negative infinity, or  $\hat{U}(t) = \lim_{I \rightarrow -\infty} \int_t^{t+T(I)} -I dt = B(t) - B^+(t)$ , with  $T(I) \equiv \frac{B(t) - B^+}{-I}$ .

Let the (undiscounted) before-breakthrough value function be  $V(A, B)$ . The corresponding HJB equation is<sup>5</sup>

$$(11) \quad \begin{aligned} \rho V(A, B) = & \text{Max}_{F, R, B} [U(F, R) - dF - \beta R - I - D(A)] \\ & + V_A(A, B)(F - \gamma A) + V_B(A, B)(I - \delta B) + H(B)[W(A) - V(A, B)]. \end{aligned}$$

The final term in equation (11) is the expected instantaneous welfare gain from the breakthrough. Note that the breakthrough makes the remaining R&D stock worthless.

**Lemma 1:** The optimality conditions for the before-breakthrough problem are

$$(12) \quad U_F(F^{B^*}, R^{B^*}) = d + \tau^{B^*} \quad \text{and} \quad U_R(F^{B^*}, R^{B^*}) = \beta \quad \text{with} \quad \tau^{B^*} \equiv -V_A(A) > 0,$$

$$(13) \quad I^* = \begin{cases} \bar{I} & B < \Psi(A), \\ -\infty & \text{for } B > \Psi(A), \\ \delta \Psi(A) + \Psi'(A) \dot{A} & B = \Psi(A), \end{cases}$$

where  $\Psi(A)$  is the unique optimal R&D stock, implicitly defined by

$$(14) \quad H'(\Psi(A))[W(A) - V(A, \Psi(A))] = \rho + \delta + H(\Psi(A)).$$

**Proof:** The optimality conditions for energy use are immediate. We give the intuition for the investment rate and steady-state R&D stock here, and refer the reader to the Online Appendix for the full proof. The linearity of the maximand in  $I$  implies a bang-bang

---

<sup>5</sup> A textbook derivation of the continuous time HJB equation with Poisson regime switching is given in Dixit and Pindyck (1994).

solution. The ability to convert R&D stock immediately to consumption, and the linear utility in the latter, means that the shadow value of the R&D stock  $V_B$  is never below unity (the planner can always jump down and convert R&D stock to utility one-for-one). Where  $V_B = 1$ , we show that the economy immediately consumes a lump of the R&D stock and jumps to some trajectory along which it continues to evolve gradually thereafter. We show that the only trajectory consistent with gradual evolution is given by (14), by constructing profitable deviations to any other possible locus. As the economy cannot jump up, below the locus given by (14) we must have  $V_B > 1$ , implying investment at the maximal rate.  $\square$

The superscript on  $\tau^{B^*}, F^{B^*}, R^{B^*}$  refers to the optimal choice in the regime ‘before’ the breakthrough has occurred (we denote the optimal R&D stock before the breakthrough by an asterisk only, as this stock plays no role more after the breakthrough). The first part of (12) states that the marginal benefit of fossil fuel use must equal its marginal production cost plus the shadow value of the carbon stock. The second part states that the marginal benefit of optimal renewable energy equals its production cost. Equation (13) states that investment follows a most rapid approach path (Spence and Starrett, 1975) until it reaches the optimal R&D stock given by (14). From above, this stock is reached immediately. The interpretation of the optimal stock (14) is that the increase in the expected value of a breakthrough for a marginal unit of green technology (allowing for the R&D stock becoming obsolete after the tip) must equal the user cost of capital. The tip increases the value by  $W(A) - V(A, B)$ , and a marginal unit of R&D increases the probability of tipping by  $H'(B)$ . The user cost of capital consists of discounting, depreciation and the risk of the capital stock losing its value at breakthrough, or  $\rho + \delta + H(B)$ . It follows from the implicit function theorem that equation (14) can be solved to give the stock of green R&D as an increasing function of  $W(A) - V(A)$ . With a cheaper renewable energy substitute available, the carbon tax before the breakthrough typically exceeds that afterwards. Hence,  $W'(A) - V'(A) = \tau^{B^*} - \tau^* \geq 0$ , and therefore equation (14) can be solved for  $B^* = B(A)$  with  $B'(A) \geq 0$ . The stock of green R&D thus increases in the stock of carbon in the atmosphere and the degree of global warming.

If the hazard function is linear, i.e.,  $H(B) = h_0 + h_1 B > 0$  with  $h_0 \geq 0$ ,  $h_1 > 0$ , we have

$$(14') \quad B^*(A) = W(A) - V(A, B^*) - h_1^{-1}(\rho + \delta + h_0).$$

R&D knowledge falls with the user cost of capital,  $\rho + h_0 + \delta$ .

Using equations (12) and (14) in the pre-tip HJB equation (11), differentiating with respect to time and ensuring that the coefficient on  $\dot{A}$  is zero, we get the forward-looking dynamics of the pre-breakthrough pre-tip social cost of carbon,

$$(15) \quad \dot{\tau}^{B^*} = [\rho + \gamma + H(B^*)] \tau^{B^*} + H(B^*) W_A(A) - D'(A).$$

The pre-tip SCC is then  $\tau^{B^*}(t) = \int_t^\infty (D'(A(s)) - H(B^*)[W'(A) - V'(A)]) e^{-(\rho + \gamma)(s-t)} ds$ , which consists of the usual discounted value of marginal damages from global warming plus the increase, due to a marginal unit of pollution, in the expected gain in welfare from when the breakthrough in renewable energy finally occurs (using the discount rate including the rate of decay of atmospheric carbon).

### 2.3. Case: linear damages from global warming

With constant marginal damages from global warming, equation (15) boils down to

$$(15') \quad \dot{\tau}^{B^*} = [\rho + \gamma + H(B)](\tau^{B^*} - \tau^*) \quad \text{with} \quad \tau^* = \frac{\alpha}{\rho + \gamma},$$

where  $\tau^*$  is the usual SCC without tipping. The next proposition establishes that the optimal stock of green R&D is constant. Equation (15') then indicates that the optimal SCC is not affected by the breakthrough (i.e.,  $\tau^{B^*} \equiv -W_A = -V_A \equiv \tau^*$  is constant).

**Proposition 1:** If the hazard function and the damage function are linear and adjustment costs for R&D are zero, the pre-tip value function is linear in  $A$ , i.e.  $V(A) = \Omega(B) - \tau^{B^*} A$ ,  $\tau^{B^*}(t) = \tau^*, \forall t \geq 0$ , and the optimal stock of green R&D is constant and given by

$$(16) \quad B^* = \frac{1}{h_1} \left( \sqrt{h_1 \Delta U^*(\tau^*)} - \delta(\rho + h_0) - (\rho + h_0) \right)$$

with  $\Delta U^*(\tau) \equiv U^*(d + \tau, \tilde{\beta}) - U^*(d + \tau, \beta) > 0$ . The function  $\Omega(B)$  satisfies: (i)

$$\Omega(B^*) = \rho^{-1} U^*(d + \tau^*, \tilde{\beta}) - h_1^{-1} \left[ \delta + \sqrt{h_1 \Delta U^*(\tau^*) - \delta(\rho + h_0)} \right]; \text{ (ii) for } B > B^*,$$

$$\Omega'(B) = 1; \text{ (iii) for } B \leq B^*, \quad \Omega''(B) - \frac{\rho + \delta + H(B)}{\bar{I} - \delta B} \Omega'(B) - \frac{H'(B)}{\bar{I} - \delta B} \Omega(B) + \frac{w H'(B)}{\bar{I} - \delta B} = 0.$$

**Proof:** As in the after-breakthrough regime, the carbon tax must be constant by (15') (see footnote 4) and equal to the after-breakthrough tax:  $\tau^{*B} = \tau^*$ . Equation (14') then implies that  $B^*$  is independent of  $A$ , i.e.  $\Psi' = 0$ ; and that  $V_{AB} = 0, V_{AA} = 0$ . We thus conjecture a value function of the form  $V(A, B) = \Omega(B) - \tau^{B^*} A$ , with  $\Omega(B)$  a function to be characterised. We substitute the conjectured  $V$  into (11) to get

$$\rho(\Omega(B) - \tau^{B^*} A) = U^*(d + \tau^{B^*}, \beta) - \delta B - \alpha A + \tau^{B^*} \gamma A + h(B) \left[ W(A) - (\Omega(B) + \tau^* A) \right].$$
 We

then substitute  $W(A) = w - \tau^* A$  into (8). Subtracting the post-breakthrough HJB equation from the above, and substituting the optimal R&D stock  $B(A)$  from (14'), we obtain a quadratic in the difference  $\Omega(B) - w$ , which then yields the solution

$$(17) \quad w - \Omega(B) = \frac{1}{h_1} \left[ \delta + 2 \sqrt{h_1 \left( U^*(d + \tau^*, \tilde{\beta}) - U^*(d + \tau^*, \beta) - \delta(\rho + h_0) \right)} \right]$$

Then,  $B(A) = w - \Omega(B) - h_1^{-1}(\rho + \delta + h_0)$  from equation (14'), which yields (16). To get the properties of  $\Omega(B)$ , we can obtain (i) immediately from (17) and the definition of  $w$ . The proof of lemma 1 demonstrates that the value function involves  $V_B = 1$  for  $B > B^*$ , yielding (ii). We then partially differentiate (11) with respect to  $B$  and note that  $V_B(A, B) = \Omega'(B)$  and  $dV_B / dt = V_{BB} \dot{B}$  (as  $V_{AB} = 0$ ) to get (iii).  $\square$

Linear hazard, damage and investment cost functions thus lead to a linear before- and after-breakthrough value function and constant carbon tax. Furthermore, the stock of green R&D knowledge is independent of the degree of global warming, and is increasing in the difference in post- and pre-breakthrough gross surplus flows  $U^*(d + \tau^*, \tilde{\beta}) - U^*(d + \tau^*, \beta) > 0$ . The stock of green R&D clearly decreases in the discount rate  $\rho$ , in the exogenous component of the hazard rate  $h_0$ , and in the

depreciation rate  $\delta$ . The effect of an increase in the marginal effect of investment on the breakthrough probability,  $h_1$ , is ambiguous. If the marginal effect is small, then increasing it makes additional investment worthwhile. However, if the marginal effect is already large, then increasing it reduces the need for investment, as the breakthrough probability is already quite large so that the social planner prefers to convert part of the capital stock into consumption. A further interpretation is offered by (14'): a more responsive hazard rate increases the marginal profitability of the investment, given the value of the breakthrough  $W - V$ . At the same time, by making the breakthrough more likely, an increase in  $h_1$  increases the pre-tip value  $V$ , reducing the value of the breakthrough itself  $W - V$ .<sup>6</sup> Equation (17) implies that  $V < W$  so that value to go is less before than after the breakthrough technology has come to market. A solution with a strictly positive  $B$  exists if the marginal expected flow benefit of the capitalised value of the breakthrough occurring exceeds the user cost of capital for the first unit of R&D stock, i.e., if  $\frac{h_1}{\rho + h_0} [U^*(d + \tau^*, \tilde{\beta}) - U^*(d + \tau^*, \beta)] > \delta + \rho + h_0$  holds.<sup>7</sup>

### 2.3. Second-best green R&D without carbon taxation

Politicians typically prefer the carrot to the stick and therefore shy away from carbon taxation and prefer to facilitate green R&D instead. Let us therefore consider the second-best case where carbon taxes are ruled out and focus on the question of what the optimal level of green R&D is given this second-best political constraint. More specifically, given that fossil fuel is not scarce and that there are no Hotelling rents that can be grabbed (cf.,

---

<sup>6</sup> To see this, note  $\frac{\partial B^*}{\partial h_1} = \sqrt{h_1 \Delta U^{\text{SB}} - \delta(\rho + h_0)}^{-1} \left( -\frac{\Delta U^{\text{SB}}}{2h_1} + \frac{\rho + h_0}{h_1^2} \left[ \delta + \sqrt{h_1 \Delta U^{\text{SB}} - \delta(\rho + h_0)} \right] \right)$  is

positive for small and positive  $B^*$  and negative for large and positive  $B^*$ . Further,

$$\begin{aligned} \left. \frac{\partial^2 B^*}{\partial h_1^2} \right|_{\frac{\partial B^*}{\partial h_1} = 0} &= \frac{\rho + h_0}{h_1^2 \sqrt{h_1 \Delta U^* - \delta(\rho + h_0)}} \left( -\frac{\delta + \sqrt{h_1 \Delta U^* - \delta(\rho + h_0)}}{h_1} + \frac{\Delta U^*}{2\sqrt{h_1 \Delta U^* - \delta(\rho + h_0)}} \right) \\ &= \frac{\rho + h_0}{h_1^2 \sqrt{h_1 \Delta U^* - \delta(\rho + h_0)}} \left( -\delta B^* - \frac{\Delta U^*}{2} \right) < 0. \end{aligned}$$

Thus,  $B^*$  is a strictly concave function of  $h_1$ , increasing for low and decreasing for high values of  $h_1$ .

<sup>7</sup> This condition follows upon substitution of (16) into (14') and making use of  $W(A) - V(A) = w - \tilde{w}$ .

Rezai and van der Ploeg, 2017), the second-best problem is thus to solve the pre-tip HJB equation (11) subject to the additional constraint  $U_F = d$ . We retain in this sub-section the assumption of a linear hazard rate for the breakthrough and linear damages, and abstract from adjustment costs when investing in green R&D.

The problem is solved exactly as in the optimal case. We denote the optimal choices for the controls in the second-best case by the superscript ‘SB’. The optimality condition for renewable energy is  $U_R(F^{\text{SB}}, R^{\text{SB}}) = \beta$  before the breakthrough and  $U_R(F^{\text{SB}}, R^{\text{SB}}) = \tilde{\beta}$  after. The optimal R&D stock is still given by (14) with value functions taking into account the constraint on taxes, so that the optimal stock of green R&D is given by equation (14’). The second-best problem has essentially an identical structure to the first-best one, except that fossil energy use is higher, and renewable energy use adjusts accordingly. The post-breakthrough value function has exactly the same structure as in the first-best case,  $W^{\text{SB}}(A) = \Omega^{\text{SB}}(B) - \alpha(\rho + \gamma)^{-1}A$ , although the function  $\Omega^{\text{SB}}(B)$  is of course different. Solving for the before-breakthrough case, we get in similar fashion the second-best optimal green R&D stock:

$$(16') \quad B^{\text{SB}} = h_1^{-1} \left( \sqrt{h_1 \Delta U^{\text{SB}} - \delta(\rho + h_0)} - (\rho + h_0) \right),$$

where  $\Delta U^{\text{SB}} \equiv U^*(d, \tilde{\beta}) - U^*(d, \beta)$  is the difference in surplus from energy use after and before the breakthrough, given that carbon is not taxed, and including the full social disvalue of emitted carbon. This stock of green technology is positive if the expected benefit of the breakthrough exceeds the effective user cost of the first unit of capital, given that there is no carbon pricing, i.e., if  $h_1 \Delta U^{\text{SB}} (\rho + h_0)^{-1} \geq \rho + h_0 + \delta$  holds. The comparative statics are qualitatively identical to the first-best capital stock. The following proposition compares the optimal second-best stock of green R&D to the first-best stock.

**Proposition 2:** Suppose that, for all  $F, R$  which satisfy  $p_F \equiv U_F(F, R) \in [d, d + \alpha(\rho + \gamma)^{-1}]$  and  $p_R \equiv U_R(F, R) \in [\tilde{\beta}, \beta]$ ,  $F_{p_F p_R}$  is positive (negative). Then, the second-best optimal stock of green R&D in the global economy falls short of (exceeds) the first-best R&D stock.

**Proof:** See Appendix A.

We have thus established sufficient conditions, on how the fossil fuel demand curve responds to changes in the renewable energy price, which imply unambiguous effects of the fossil fuel price on the value of the breakthrough. If  $F_{p_F p_R} < 0$ , the breakthrough curbs fossil fuel demand more at lower fossil fuel prices. Thus, the breakthrough is more valuable if fossil fuel is taxed at less than the efficient rate. Consequently, the second-best R&D stock is higher if carbon taxes are precluded. However, if  $F_{p_F p_R} > 0$ , the breakthrough has a bigger impact on fossil fuel demand at higher fossil fuel prices, so that second-best R&D effort in green technology is below the first-best level. Second-best R&D investment can thus exceed or fall short of the first-best level if carbon pricing differs from the efficient level.

**Example 1 (Cobb-Douglas utility function):**

We have  $U(F, R) = \zeta(F^\alpha R^{1-\alpha})^\mu + M$ ,  $\sigma = 1, 0 < \mu < 1, \zeta > 0$ , for the Cobb-Douglas utility function, so  $U_F = \alpha\mu\zeta F^{\alpha\mu-1} R^{(1-\alpha)\mu} = p_F$  and  $U_R = (1-\alpha)\mu\zeta F^{\alpha\mu} R^{(1-\alpha)\mu-1} = p_R$ . Hence,

we get  $F = \left( \frac{\alpha\mu\zeta}{p_F} \left( \frac{(1-\alpha)p_F}{\alpha p_R} \right)^{(1-\alpha)\mu} \right)^{\frac{1}{1-\mu}}$  and thus  $F_{p_F} = - \left( \frac{1-(1-\alpha)\mu}{1-\mu} \right) \frac{F}{p_F} < 0$  and

$F_{p_F p_R} = \left( \frac{1-(1-\alpha)\mu}{1-\mu} \right) \left( \frac{(1-\alpha)\mu}{1-\mu} \right) \frac{F}{p_F p_R} > 0$ . Thus the globally optimal second-best stock

of green R&D is below the first-best optimal stock.  $\square$

**Example 2 (CES utility function):**

We have  $U(F, R) = \zeta \left( \alpha^{\frac{1}{\sigma}} F^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} R^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\mu\sigma}{\sigma-1}} + M$ ,  $\sigma > 0, 0 < \mu < 1, \zeta > 0$ , for the

CES utility function that is used in the numerical illustrations (see Sections 5.4 and 6).

This gives in a similar fashion  $F = \left( \mu\zeta\alpha^{1-\mu} \left( \alpha + (1-\alpha)(p_F / p_R)^{\sigma-1} \right)^{\frac{1-(1-\mu)\sigma}{\sigma-1}} / p_F \right)^{\frac{1}{1-\mu}}$ .

Hence,  $F_{p_F} = - \frac{F}{(1-\mu)p_F} \left[ 1 - (1-(1-\mu)\sigma) \left( \frac{(1-\alpha)(p_F / p_R)^{\sigma-1}}{\alpha + (1-\alpha)(p_F / p_R)^{\sigma-1}} \right) \right] < 0$ . With  $\sigma = 1$ ,

this boils down to  $F_{p_F} = -\left(\frac{1-(1-\alpha)\mu}{1-\mu}\right)\frac{F}{p_F} < 0$  as in Example 1. If  $\sigma > 1$ , the term in the square brackets in  $F_{p_F}$  increases in  $p_R$  and as  $F$  decreases in  $p_R$ , we have that  $F_{p_F p_R} > 0$ , as in the Cobb-Douglas case, and the second-best stock of green R&D is lower than the first-best level. If  $\sigma < 1$ ,  $F$  still decreases in  $p_R$  and  $F_{p_F}$  decreases in  $p_R$ , so that  $F_{p_F p_R}$  is ambiguous. There is a cut-off level of  $\sigma$ , say  $0 < \bar{\sigma} < 1$ , so that  $F_{p_F p_R} > 0$  for all  $\sigma > \bar{\sigma}$ ,  $F_{p_F p_R} < 0$  for all  $0 < \sigma < \bar{\sigma}$ , and  $F_{p_F p_R} = 0$  for all  $\sigma = \bar{\sigma}$ . Hence, for all  $\sigma > \bar{\sigma}$  the second-best stock of green R&D exceeds the first-best level.  $\square$

Empirically, Papageorgiou et al. (2017) have estimated values for the elasticity of substitution between fossil fuel and renewable energy,  $\sigma$ , significantly above one (about 2.5). Hence, the empirically relevant case implies that the optimal second-best level of the stock of green R&D is below the first-best level.

### 3. Non-Cooperative Outcomes in the Global Economy with Adjustment Costs for Investments in Green R&D and Convex Damages of Global Warming

We now derive the conditions for the non-cooperative feedback Nash equilibrium. This equilibrium is derived by dynamic programming and thus satisfies subgame perfection. We allow for multiple countries, indexed by  $i = 1, \dots, N$ , each of them concerned with fighting global warming. For simplicity, we focus on symmetric outcomes. We have fixed the aggregate population size of the global economy to unity, so that each country has a population size of  $1/N$ . Importantly, as country  $i$  fossil fuel and renewables consumption  $F_i$  and  $R_i$ , the consumption of the numeraire good, and investment rates  $I_i$  are all expressed in per-capita terms, we can compare the non-cooperative outcomes with  $N > 1$  to the internationally cooperative outcome with  $N = 1$  derived in Section 2.<sup>8</sup>

---

<sup>8</sup> This bias can be decomposed in terms of the gap between international policy cooperation and the non-cooperative open-loop Nash equilibrium outcome and the gap between the non-cooperative feedback and open-loop Nash equilibrium outcomes. The former increases as the number of countries rises whilst keeping the size of the global economy constant; the latter can go the other way as the aggregate effects of rivals may be large (cf., Fudenberg and Levine, 1988).

We describe here the general model with strictly convex damages from global warming. In addition, we allow investment in clean R&D to adjust smoothly as we suppose from now on that investment by country  $i$  incurs a per-capita adjustment cost. To this end, the vector of green R&D stocks of the  $N$  countries is denoted by  $\{B_i\}$  and the global aggregate green R&D stock by  $B(t) \equiv \sum_{i=1}^N B_i(t)$ . The investment adjustment cost function is convex in the investment rate and decreases in the global R&D stock. We use the quadratic specification  $C(I_i, B) = I_i + \frac{\phi}{2} \frac{I_i^2}{B}$  with  $\phi \geq 0$ .<sup>9</sup> This implies that we assume immediate and full spill-over of green knowledge externalities. This is relevant if international patent markets for this type of technology are absent or ineffective.

We suppose that each country can observe the green R&D stock of the other countries, so that the value function for any country depends on the atmospheric carbon stock  $A$  and, in principle, on the vector of R&D stocks  $\{B_i\}$ . However, as it is the global stock of green R&D that both increases the chance of a breakthrough for all countries, and makes further R&D investments less costly, the payoff-relevant R&D stock (in the sense of Maskin and Tirole (2001)) is the global aggregate rather than the country-specific R&D stock. We focus on the feedback Nash equilibrium in Markovian strategies. The state variables for our general problem with convex damages and adjustment costs for investment in green R&D are thus  $A$  and  $B$ .

We define global fossil fuel use as  $F(t) \equiv \sum_{i=1}^N F_i(t) / N$ , and the global investment rate in green R&D as  $I(t) \equiv \sum_{i=1}^N I_i(t) / N$ . In other words, each country  $i$ 's per-capita rate of fossil fuel use  $F_i$  and per-capita investment rate  $I_i$  have both been weighted with the country's population mass  $N^{-1}$  and then summed up to obtain the aggregate rate of build-up of the carbon and green R&D stocks, respectively. The dynamics of the atmospheric carbon stock and the individual stocks of green R&D thus become

---

<sup>9</sup> Since R&D is assumed to be freely available throughout the world and spills over even before the breakthrough is realised (in terms of lowering adjustment costs for other countries), it makes sense that adjustment costs are relative to global R&D knowledge. If the denominator of the R&D cost featured the local R&D stock, instead of the global stock, then symmetrically dividing the world into smaller countries (by increasing  $N$ ) would increase the total R&D cost, as a more finely divided world would lose out on economies of scale to innovation.

$$(1') \quad \dot{A}(t) = \left[ \sum_{i=1}^N N^{-1} F_i(t) \right] - \gamma A(t) \quad \text{with } A(0) = A_0 > 0, \text{ and}$$

$$(3') \quad \dot{B}_i(t) = N^{-1} I_i(t) - \delta B_i(t) \quad \text{with } B_i(0) = B_{i0}, \quad i = 1, \dots, N,$$

Equation (1') boils down to equation (1). Equations (3') can be summed across countries to get the global aggregate (3).

We want to compare equilibria under different degrees of international cooperation: full cooperation, no cooperation, and partial cooperation. To do this, we now characterise the solution concepts associated with these cases. The state of the system at any time  $t$  is summarised by  $(t, A(t), B(t), b(t))$ . Denote the  $N$ -vectors mapping this state to the control variables by  $\lambda^F, \lambda^R, \lambda^I : \mathbb{R}^3 \times \{\beta, \tilde{\beta}\} \rightarrow \mathbb{R}^N$ . With (1'), (3'), and (2), these induce time paths of the control and state variables in the obvious way. Denote the expected (unmaximised) value for country  $i$ , by

$$(18) \quad \begin{aligned} & \tilde{V}_i(\lambda^F, \lambda^R, \lambda^I) = \\ & \mathbb{E} \int_0^\infty e^{-\rho t} \frac{1}{N} \left( U(\lambda_i^F, \lambda_i^R) - d\lambda_i^F - \tilde{\beta}\lambda_i^R - D(A) - C(\lambda_i^I) \right) dt \end{aligned}$$

with  $\dot{A}$  and  $\dot{B}$  given by (1') and (3'), and subject to the stochastic process governing the regime shift (2). The aggregate welfare of country  $i$  is just the per-capita utility flow multiplied by its population size  $1/N$ . Now assume the functions  $\lambda^F, \lambda^R, \lambda^I$  are stationary, i.e., independent of the calendar date  $t$  (in other words, we consider subgame-perfect Nash equilibria in Markovian strategies). The solution concepts associated with different degrees of international cooperation between the  $N$  countries are then:

1. Full international cooperation, where governments cooperatively set both carbon taxes and investments in green R&D. This is the global first best and solves

$$\max_{\lambda^F, \lambda^R, \lambda^I} \sum_{j=1}^N \tilde{V}_j(\lambda^F, \lambda^R, \lambda^I)$$

In our framework, the first best can conveniently be obtained by setting  $N = 1$ . Its properties have been discussed in Section 2.

2. No international policy cooperation, so each country sets both the carbon tax and investment green R&D unilaterally conditional on the states of the economy. That is, every country  $i$  solves

$$\max_{\lambda_i^F, \lambda_i^R, \lambda_i^I} \tilde{V}_i(\lambda^F, \lambda^R, \lambda^I)$$

given beliefs over the other countries' policies  $\lambda_{-i}^F, \lambda_{-i}^R, \lambda_{-i}^I$ . This is a differential game between  $N$  countries.

3. International policy cooperation on R&D efforts only, with all countries choosing unilaterally their carbon tax rates, given cooperatively agreed R&D efforts. This form of partial international policy cooperation yields a second-best outcome, similar to that in Section 2.3, but in a multi-country framework. This corresponds to a differential game between  $N$  countries and a coalition consisting of all of them, solving sequentially

$$\begin{aligned} \text{coalition:} \quad & \max_{\lambda^I} \sum_{j=1}^N \tilde{V}_j(\lambda^F, \lambda^R, \lambda^I), \\ \text{countries:} \quad & \max_{\lambda_i^F, \lambda_i^R} \tilde{V}_i(\lambda^F, \lambda^R, \lambda^I), i \in \{1, \dots, N\}, \quad i = 1, \dots, N. \end{aligned}$$

We consider the case in which the coalition 'leads', by setting R&D rates first in each short time interval. We will be more specific about this second-best setup in Section 4.3.

We abstract in this symmetric setup from coalitions among a subset of countries.

### 3.1. Equilibrium after the breakthrough

Following the breakthrough, there is no further use for green R&D as the cost reduction has materialised. Thus, from then on the carbon stock  $A$  is the only state variable. Aggregate welfare of country  $i$  is denoted  $W_i$ , and is obtained by summing individual welfares over the population mass of  $1/N$ . Furthermore, by assumption, country  $i$  believes the  $N-1$  other countries follow a Markovian strategy (i.e., with actions after the breakthrough conditioned on the current carbon stock  $A$  only). We denote country  $i$ 's belief over country  $j$ 's strategy by  $\xi_{ij}(A)$ , so that country  $i$  expects that other countries'

rates of energy use after the breakthrough are  $F_j = \xi_{ij}^F(A)$  for  $j \neq i$ .<sup>10</sup> The after-breakthrough HJB equation for country  $i$  is then

$$(5') \quad \begin{aligned} \rho W_i(A) = & \max_{F_i, R_i} N^{-1} \left( U(F_i, R_i) - dF_i - \tilde{\beta} R_i - D(A) \right) \\ & + W_i'(A) \left( N^{-1} \left[ F_i + \sum_{j \neq i} \xi_{ij}(A) \right] - \gamma A \right). \end{aligned}$$

We look for a symmetric equilibrium, so that  $\xi_{ij}^F = \xi_{ik}^F \equiv \xi_{-i}^F$  for all  $j, k \neq i$ . We denote the optimal post-tip variables with a superscript ‘N’. Then,  $F_i^N, R_i^N$  are given by

$$(6') U_{F_i}(F_i^N, R_i^N) = d + \tau_i^N \quad \text{and} \quad U_{R_i}(F_i^N, R_i^N) = \tilde{\beta}, \quad \text{with} \quad \tau_i^N \equiv -W_i'(A), \quad i = 1, \dots, N.$$

We obtain  $W'(A)$  by partially differentiating the maximand of (5') with respect to  $A$ :

$$(19) \quad \begin{aligned} \rho W_i'(A) = & N^{-1} \left( \left( U_{F_i^N} - d + W_i'(A) \right) \frac{\partial F_i^N}{\partial A} + \left( U_{R_i^N} - \tilde{\beta} \right) \frac{\partial R_i^N}{\partial A} - D'(A) \right) \\ & + W_i'(A) \left( N^{-1} \left[ \sum_{j \neq i} \xi_{ij}'(A) \right] - \gamma \right) + W_i''(A) \dot{A}. \end{aligned}$$

The brackets multiplying  $\partial F_i^N / \partial A$  and  $\partial R_i^N / \partial A$  equal zero by the envelope theorem. Rearranging and noting that  $\dot{\tau}_i^N = -W_i''(A) \dot{A}$ , this gives the equation of motion for the after-tip carbon tax:

$$(20) \quad \dot{\tau}_i^N = (\rho + \gamma) \tau_i^N - N^{-1} D'(A) - \tau_i^N \frac{N-1}{N} \xi_{-i}'(A), \quad i = 1, \dots, N.$$

Beliefs have to be correct, so that in symmetric equilibrium  $F_i^N = \xi_{ji}(A) \equiv F^N$  for all carbon stocks  $A$  and for all  $j, i$ . Thus, the optimal carbon tax is set to the sum of the marginal damages on country  $i$ 's population resulting from its own emissions; minus the degree to which a marginal ton emitted by country  $i$  induces other countries to cut back on their emissions. This is seen more clearly from integrating out (20):

---

<sup>10</sup> Country  $i$ 's strategy also involves a renewables use policy but this has no strategic effect.

$$\begin{aligned}
(20') \quad \tau_i^N(t) &= \int_t^\infty e^{-(\rho+\gamma)(s-t)} \left[ N^{-1} D'(A(s)) + \tau_i^N(s) \frac{N-1}{N} \frac{\partial F^N(A(s))}{\partial A(s)} \right] ds \\
&< \tau_i^O(t) = \int_t^\infty e^{-(\rho+\gamma)(s-t)} N^{-1} D'(A(s)) ds \\
&< \tau^*(t) = \int_t^\infty e^{-(\rho+\gamma)(s-t)} D'(A(s)) ds, \quad i = 1, \dots, N.
\end{aligned}$$

where we denote by  $\tau_i^O(t)$  the country  $i$  tax in an open-loop Nash equilibrium. There are two biases in the post-tip Markov-perfect non-cooperative taxes,  $\tau_i^N(t)$ . The first one corresponds to the first term in the square brackets in (20') and is the traditional *free-rider* bias resulting if countries do not internalise the adverse effects of their carbon emissions on the global warming damages of the other countries. They thus only internalise their own damages,  $N^{-1} D'(A(s))$ , instead of global damages,  $D'(A(s))$ . The second one is the *strategic* bias and corresponds to the second term in the square brackets. It vanishes if there is only country ( $N = 1$ ), but for multiple countries ( $N > 1$ ) it is negative (see below) and results from the strategic effects induced in the feedback Nash equilibrium (cf., van der Ploeg and de Zeeuw, 1992). Higher emissions by country  $i$  increase the stock of atmospheric carbon, and with convex damages from global warming, lead to a higher carbon tax, lower fossil fuel consumption and lower carbon emissions of the other countries. This strategic effect implies that country  $i$  on the margin will set a lower carbon tax (as the second term in the square brackets in (19') is negative). Given  $A(t)$ , country  $i$  thus sets its carbon tax below the sum of discounted private marginal damages on country  $i$ ,  $\tau_i^O(t)$  (which *inter alia* corresponds to the non-cooperative carbon taxes for the open-loop Nash equilibrium albeit evaluated along the feedback Nash equilibrium path), and *a fortiori* below the internationally cooperative carbon tax,  $\tau^*(t)$ .

### 3.2. Equilibrium before the breakthrough

The fully non-cooperative outcomes for the before-breakthrough problem correspond to a Markov-perfect equilibrium in which all countries condition their strategies on the aggregate global green R&D stock  $B$  as well as the atmospheric carbon stock  $A$ . To reiterate, strategies are not conditioned on the distribution of R&D stocks. Given that adjustment costs depend on the aggregate global green R&D stock, this is a Markov-

perfect equilibrium.<sup>11</sup> The memoryless strategies of this equilibrium imply value functions that do not depend on *past* aggregate carbon and green R&D stocks (nor on past *actions*), but only on the *current* stocks. We therefore suppose that country  $i$  believes that country  $j \neq i$  follows Markovian strategies  $F_j = \zeta_{ij}^F(A, B)$  and  $I_j = \zeta_{ij}^I(A, B)$ . Denoting the before-breakthrough value function of country  $i = 1, \dots, N$  by  $V_i(A, B)$ , the HJB equation of country  $i$  is given by

$$\begin{aligned}
 (21) \quad \rho V_i(A, B) = & \max_{F_i, R_i, I_i} N^{-1} \left[ U(F_i, R_i) - dF_i - \beta R_i - I_i - \frac{\phi}{2} \frac{I_i^2}{B} - D(A) \right] \\
 & + V_{iA}(A, B) \left[ \frac{1}{N} \left( F_i + \sum_{j \neq i} \zeta_{ij}^F(A, B) \right) - \gamma A \right] \\
 & + V_{iB}(A, B) \left[ \frac{1}{N} \left( I_i + \sum_{j \neq i} \zeta_{ij}^I(A, B) \right) - \delta B \right] + H(B) [W_i(A) - V_i(A, B)]
 \end{aligned}$$

We denote the optimal choices for country  $i$  in the before-breakthrough Nash equilibrium by ‘NB’. The static optimality conditions for the before-breakthrough problem are

$$(12') \quad U_{F_i}(F_i^{\text{NB}}, R_i^{\text{NB}}) = d + \tau_i^{\text{NB}} \quad \text{and} \quad U_{R_i}(F_i^{\text{NB}}, R_i^{\text{NB}}) = \beta \quad \text{with} \quad \tau_i^{\text{NB}} \equiv -V_{iA}(A, B) > 0,$$

$$(22) \quad I_i^{\text{NB}} = \frac{1}{\phi} (q_i^{\text{NB}} - 1)B \quad \text{with} \quad q_i^{\text{NB}} \equiv V_{iB}(A, B) > 0, \quad i = 1, \dots, N.$$

Equation (12') is as equation (12) except that country  $i$ 's optimal carbon tax depends on the marginal value of atmospheric carbon to country  $i$ , which now depends also on the aggregate green R&D stock  $B$ . Equation (22) shows that the rate of green R&D investment increases in the marginal value of R&D capital to country  $i$ ,  $q_i^{\text{NB}}$  (cf., Tobin's stock market value  $Q$ ) and is proportional to the global stock of green R&D.

Using equations (12') and (22) in the HJB equation (21), differentiating with respect to time and ensuring that the coefficients on  $\dot{A}$  and  $\dot{B}$  are zero, we find that the forward-looking dynamics of country  $i$ 's pre-breakthrough tax on carbon follow from

---

<sup>11</sup> As adjustment costs are not a function of the domestic R&D stock, the distribution of the R&D stocks is not payoff-relevant (in the sense of Maskin and Tirole (2001)) given that other countries do not condition their strategies on the distribution.

$$(23) \quad \dot{\tau}_i^{\text{NB}} = [\rho + \gamma] \tau_i^{\text{NB}} - \frac{1}{N} D'(A) + H(B) (\tau_i^{\text{NB}} - \tau_i^{\text{N}}) - \tau_i^{\text{NB}} \frac{N-1}{N} \zeta_A^{F,-i} + q_i^{\text{NB}} \frac{N-1}{N} \zeta_A^{I,-i}.$$

Integrating this, we get country  $i$ 's optimal Markov-perfect non-cooperative carbon tax:

$$(23') \quad \tau_i^{\text{NB}}(t) = \int_t^\infty e^{-(\rho+\gamma)(s-t)} \left[ \frac{1}{N} D'(A) + H(B) (\tau_i^{\text{N}} - \tau_i^{\text{NB}}) + \tau_i^{\text{NB}} \frac{N-1}{N} \zeta_A^{F,-i} - q_i^{\text{NB}} \frac{N-1}{N} \zeta_A^{I,-i} \right] ds.$$

The first term in the square brackets in (23') again gives the impact of marginal climate damages on country  $i$  resulting from an extra ton of emissions (one  $N$ -th of global marginal damages as the damages caused to the other countries are not internalised). The second term in the square brackets reflects the fact that an extra ton of emissions affects the payoff from the breakthrough, increasing it as the carbon tax typically falls when the breakthrough reduction in the cost of clean energy occurs. This second term tends to increase the carbon tax set by country  $i$ . The last two terms in the square brackets reflect two strategic motives: an extra ton of carbon in the atmosphere by country  $i$  makes other countries emit less (the third term which is akin to the second term in the square brackets of (20') for the post-tip case), and invest more in green R&D (the fourth term), both of which are beneficial to country  $i$ . Thus, under the expected signs of the strategic effects and of the change in carbon taxes when the breakthrough cost reduction in clean energy occurs, these last two terms push towards lower carbon taxes and higher carbon emissions.

Similarly, the forward-looking dynamics of the marginal value of R&D capital are

$$(24) \quad \dot{q}_i^{\text{NB}} = [\rho + \delta + H(B)] q_i^{\text{NB}} - H'(B) [W_i(A) - V_i(A, B)] - \frac{1}{N} \frac{(q_i^{\text{NB}} - 1)^2}{2\phi} + \tau_i^{\text{NB}} \frac{N-1}{N} \zeta_B^{F,-i} - q_i^{\text{NB}} \frac{N-1}{N} \zeta_B^{I,-i}, \quad i = 1, \dots, N.$$

Integrating (24), we get the marginal value of an extra unit of global R&D as

$$\begin{aligned}
(24') \quad q_i^{\text{NB}}(t) = & \int_t^\infty e^{-\int_t^s [\rho + \delta + H(B(s'))] ds'} \left[ H'(B)(W_i - V_i) + \frac{1}{N} \frac{(q_i^{\text{NB}} - 1)^2}{2\phi} \right] ds + \\
& \int_t^\infty e^{-\int_t^s [\rho + \delta + H(B(s'))] ds'} \left[ -\tau_i^{\text{NB}} \frac{N-1}{N} \zeta_B^{F,-i} + q_i^{\text{NB}} \frac{N-1}{N} \zeta_B^{I,-i} \right] ds.
\end{aligned}$$

Thus, the value of a marginal unit of the R&D stock consists of the present discounted value of the expected increase in value from the breakthrough (due to the increased probability of a breakthrough), plus the benefit of lower future investment adjustment costs where country  $i$  only considers its own share of these benefits.<sup>12</sup> From this should be subtracted the present discounted value of costs of other countries increasing their emissions and decreasing their R&D investment in response to a higher R&D stock.

In summary, with many countries, the economy suffers from a *double free-riding problem*: on the one hand, polluting carbon emissions are too high, and, on the other hand, beneficial R&D investment rates too low. Furthermore, such international free-riding problems are aggravated by strategic attempts of individual countries to incentivise other countries to cut back on their carbon emissions and to invest more in green R&D, by polluting even more and investing even less. These strategic effects are absent in the open-loop Nash equilibrium outcomes (denoted by the superscript ‘OB’ in the before-breakthrough regime):

$$(23'') \quad \tau_i^{\text{OB}}(t) = \int_t^\infty e^{-(\rho + \gamma)(s-t)} \left[ \frac{1}{N} D'(A) - H(B)(\tau_i^{\text{OB}} - \tau_i^{\text{O}}) \right] ds, \text{ and}$$

$$(24'') \quad q_i^{\text{OB}}(t) = \int_t^\infty e^{-\int_t^s [\rho + \delta + H(B(s'))] ds'} \left[ H'(B)(W_i - V_i) + \frac{1}{N} \frac{(q_i^{\text{O}} - 1)^2}{2\phi} \right] ds.$$

Beliefs have to be correct, so that in symmetric feedback Nash equilibrium  $F_i^{\text{NB}} \equiv F_i = \zeta_{ij}^F$ ,  $I_i^{\text{N}} = I_i = \zeta_{ij}^I$ , and  $V_i(A, B) = V_j(A, B)$  for all  $i, j$ .

---

<sup>12</sup> Note that  $W_i$  and  $V_i$  refer to country  $i$ 's total, not per-capita, value. In symmetric equilibrium,  $V_i = V^{\text{N}} / N$  where  $V^{\text{N}} \equiv \sum V_i$ . The free-riding motive is thus captured in the term  $W_i - V_i = (W^{\text{N}} - V^{\text{N}}) / N$  (with  $W^{\text{N}} \equiv \sum W_i$ ).

#### 4. Special case: Non-cooperative outcomes with linear damages and no investment adjustment costs

We can solve analytically the special case with linear global warming damages, a linear hazard rate ( $H(B) = h_0 + h_1 B$ ) and no investment adjustment costs ( $\varphi = 0$ ), as we did for the global economy in Section 2.3. Ignoring equilibria in non-linear strategies, we show that in equilibrium all players expect their counterparts to choose strategies independent of the state variables.<sup>13</sup>

##### 4.1. After the breakthrough of the new carbon-free technology

With linear damages from global warming, we consider the obvious equilibrium in which the other players impose carbon taxes which do not depend on  $A$ . Each country's best response is then also to implement a constant carbon tax, namely

$$(25) \quad \tau_i^N = \tau_i^O = \frac{1}{N} \frac{\alpha}{\rho + \gamma} < \tau^* = \frac{\alpha}{\rho + \gamma}.$$

This is an equilibrium outcome, because no player has strategic leverage.<sup>14</sup> The carbon taxes are set a factor  $N$  lower than under international policy cooperation because the non-cooperative carbon taxes do not internalise the international global warming externality. Hence, fossil fuel consumption and carbon emissions are higher and global warming is more severe. Given (25), the value functions take the form

$$(26) \quad W_i^N(A) = w_i^N - \frac{1}{N} \frac{\alpha}{\rho + \gamma} A, \quad i = 1, \dots, N,$$

where the capitalised surplus flow under non-cooperative carbon taxes is given by:

$$(27) \quad w_i^N \equiv \frac{1}{\rho N} U^*(d + \tau^N, \tilde{\beta}), \quad i = 1, \dots, N,$$

---

<sup>13</sup> We report the equilibrium outcome in continuous time as in the other sections. However, the underlying equilibrium is technically the limit of a discrete-time formulation as the period length becomes infinitesimally small. It is more convenient to develop the equilibrium precisely using a discrete-time set-up. We elaborate on this in the Online Appendix, where the equilibrium is obtained.

<sup>14</sup> A player with strategic leverage recognises that, by affecting the evolution of the state, she is able to affect other players' actions in the future. This requires that other players' equilibrium strategies depend on the state to begin with.

where the function  $U^*$  is given by (9). Hence, the surplus flow reflects private benefits from per-capita fossil fuel and renewables consumption in country  $i$ , with its low carbon taxes, but incorporates the full per-capita disvalue of all countries' carbon emissions.

Note that the post-breakthrough feedback Nash equilibrium coincides with the open-loop Nash equilibrium: the absence of strategic leverage means equilibrium strategies are (degenerate) functions of time only, so that the best response to these is also the open-loop best Nash response. This is easily seen by comparing the open-loop Nash carbon tax and the feedback Nash carbon tax in (20'): for the strategies given in (20'),  $\partial F^N / \partial A = \partial \tau^N / \partial A = 0$ , so that the carbon tax rules coincide. This outcome depends on the assumption of linear damages from global warming. If marginal damages, in contrast, rise with global warming, strategies of each country will depend on the state variables:  $\partial \tau^N / \partial A \neq 0$ . This then introduces full dynamic strategic interactions (see also Section 6). However, it should be mentioned that there may also exist other feedback equilibria with linear damages, in which the countries use non-linear strategies. We do not investigate potential multiplicity of equilibria further.

#### 4.2. Before the breakthrough of the new carbon-free technology

With linear damages, the pre-breakthrough regime has an intuitive equilibrium. The HJB equations for each of the countries in the absence of investment adjustment costs and for the fully non-cooperative case are

$$\begin{aligned}
 \rho V_i^N(A, \underline{B}) = \max_{F_i, R_i, I_i} \frac{1}{N} & \left[ U(F_i^N, R_i^N) - dF_i^N - \beta R_i^N - D(A) - I_i \right] \\
 (28) \quad & + H(B) [W_i^N(A) - V_i^N(A)] \\
 & + V_{iA}^N(A, \underline{B}) (\Sigma_j N^{-1} F_j - \gamma A) + V_{iB}^N(A, \underline{B}) \cdot (\Sigma_j N^{-1} \underline{I} - \delta \underline{B}),
 \end{aligned}$$

for  $i \in 1, \dots, N$ . The first line gives the aggregate utility flow of country  $i$ , minus the effective user cost of capital. Note that country  $i$  owns the capital stock  $B_i$ , and only considers the cost of capital applied to this stock, but still takes into account that the breakthrough probability is determined by the aggregate stock  $B$ .

We now construct a symmetric non-cooperative Markov-perfect equilibrium.

**Lemma 2:** Define the steady state country  $i$  R&D stock

$$(29) \quad B^{N,i} \equiv (Nh)^{-1} \left( \sqrt{N^{-1} (h_1 \Delta U^N - \delta [\rho + h_0]) + \left( \frac{N-1}{2N} \delta \right)^2} - (\rho + h_0) - \frac{N-1}{2N} \delta \right),$$

with  $\Delta U^N \equiv U^*(d + \tau^N, \tilde{\beta}) - U^*(d + \tau^N, \beta)$ . Consider the following strategies: set  $F_i, R_i$  to satisfy  $U_{iF_i} = d - \tau^N$ ,  $U_{iR_i} = \beta$ , and set

$$I_i(t) = \begin{cases} \bar{I} & \text{if } B_j(t) = B_k(t) < B^{N,i} \quad \forall j, k, \\ \delta B^{N,i} & \text{if } B_j(t) = B_k(t) = B^{N,i} \quad \forall j, k, \\ \delta B_i(t) & \text{if } B_i(t) < B^{N,i} \text{ and } \exists j : B_j(t) > B_i(t), \\ -\infty & \text{otherwise.} \end{cases}$$

If  $I_i = -\infty$ , then the target of the downward jump is  $B_i^+(t) = \min \left\{ B^{N,i}, \min_k B_k(t) \right\}$ . These strategies yield an equilibrium value function  $V_i^N(A, B) = \Omega^N(\underline{B}) - N^{-1} \alpha / (\rho + \gamma)$ . As long as this satisfies

$$(30) \quad h_1(B_i - B^{\min})(W_i(A) - V_i^N(A, B^{\min} \underline{1})) \leq \delta(B_i - B^{\min})B^N + \left( \frac{d\Omega^N(b \underline{1})}{db} \bigg|_{b=B^{\min}} - 1 \right) [\bar{I} / N - B^{\min}],$$

for all  $\underline{B}(t)$  with  $B^{\min}(t) \equiv \min_k B_k(t)$  and  $\underline{1}$  the unit vector, the above strategies constitute a Markov-perfect Nash equilibrium.

**Proof.** We outline the proof here; for a detailed derivation, see the Online Appendix. We set up a discrete-time game with periods of length  $\Delta t$  and consider equilibria as the period length becomes very small. We confirm that it is an equilibrium for the strategies to have no cross-dependence between the two state variables as conjectured, i.e., optimal R&D investment is independent of the carbon stock  $A$ , and energy use is independent of the R&D stocks  $\underline{B}$ . Equilibrium energy use policies are then derived. We calculate payoffs for the equilibrium R&D strategies, and for possible deviations: from on the symmetric investment path, from above the steady-state stock, or from asymmetric distributions. On the symmetric equilibrium path, any deviation will at the very least cause joint R&D efforts to pause, and at worst for the joint R&D stocks to fall before the pause, which is never optimal below  $B^N$ . From an asymmetric distribution of R&D stocks, condition (30)

guarantees that a country required to equalise its stock to the level of the country with the lowest stock prefers to do this immediately, rather than delaying for one period as a one-shot deviation.  $\square$

The equilibrium in this lemma features Markovian trigger strategies. As long as the countries have identical R&D stocks, they all invest together at the maximal rate until they reach the steady-state R&D stock. If the countries have unequal R&D stocks, these are immediately equalised to the lowest stock level; any country which has stocks above this level will consume the difference. No country has an incentive to deviate from the symmetric R&D path, because an individual deviation will cause the other countries to stop investing and follow suit instead. The interpretation of condition (30) is as follows: suppose in an asymmetric distribution country  $i$  has R&D stocks above the minimal level. The strategies prescribe immediate equalisation to the lowest individual stock level. If country  $i$  delays equalising the stock by a short time interval, the flow benefit is given by the left-hand side of (30): this consists of the higher probability of the breakthrough, times the value of the breakthrough. The cost is given by the right-hand side, and consists of the higher depreciation in the short interval, plus the delay in starting the joint investment build-up. Due to the one-shot deviation principle, it is sufficient to ensure this is not profitable. The condition is straightforward to check, but as we only want to make a simple analytical point, we assume from now on that it holds.

The equilibrium stock of green R&D of an individual country, denoted  $B^{N,i}$ , satisfies

$$(31) \quad B^{N,i} = N^{-1} \left[ W_i^N(A) - V_i^N(A) - (\rho + h_0 + \delta) / h_1 \right].$$

Green R&D investments thus rise with the gain associated with the breakthrough drop in the cost of clean energy, given the degree of international cooperation on carbon taxes, fall with the degree of non-cooperation, and fall with the baseline effective user cost of capital  $\rho + h_0 + \delta$ . Note that the individual-country value functions are a fraction  $1/N$  of aggregate value; so the aggregate stock is lower than it would be in a centrally planned case (cf. equation (14')). Substituting the equilibrium values yields (29), which depends on the difference in the per-capita surplus flows before and after the breakthrough,  $\Delta U^N \equiv U^*(d + \tau^N, \tilde{\beta}) - U^*(d + \tau^N, \beta)$ . The flows, again, involve private benefits from resource use given the relatively low carbon taxes, but also the full environmental impacts

of all other countries' carbon emissions. The equilibrium stock of green R&D for country  $i$  in the non-cooperative outcome is positive if  $h_1 \Delta U^N / ((\rho + h_0)N) > \rho + h_0 + \delta$ , so the benefit of the first unit of R&D capital must exceed the effective cost of capital (given countries only consider their own benefit). The term  $N^{-1}$  outside the square brackets in (31) refers just to the fact that the global stock of green R&D is divided symmetrically between the countries. Inside the square root, the term  $N^{-1}$  reflects the fact that each country only considers its share of the global utility flow. From equation (29), it is easy to show that  $B_i^N$  is a negative function of  $(N-1)/N$  and of  $\delta$ , so the stock of green R&D decreases in the depreciation rate of green R&D and in the non-cooperative bias (measured by the extent to which  $N$  is bigger than 1).<sup>15</sup> It is also clear from (29) that the stock of green R&D decreases in the pure rate of time preference,  $\rho$ , and the exogenous component of the hazard rate,  $h_0$ . The sensitivity of the hazard rate to R&D investment,  $h_1$ , has an ambiguous effect on the non-cooperative stocks of green R&D, as in the socially optimal case. The symmetric open-loop Nash equilibrium yields the same equilibrium outcome as the feedback Nash equilibrium, because the latter features constant emissions and a bang-bang R&D investment which is only a function of the R&D stocks (not of carbon stocks), with the same equilibrium R&D stock level. This results from the assumption of linearity of both climate damages and the hazard rate, and the absence of investment adjustment costs.

#### 4.3. Second best: Cooperation on R&D efforts but not on carbon taxes

We now consider the second-best case of partial international policy cooperation where countries set green R&D stocks cooperatively but carbon taxes unilaterally. We assume that the international R&D coalition acts as a leader, moving first each period. We use a continuous-time Stackelberg equilibrium set-up (e.g. Benckekroun and Long, 2002).<sup>16</sup> At each moment in time, the coalition of all countries first maximises joint welfare given the strategies of the individual countries. Second, the countries then maximise individual

<sup>15</sup> This is done by differentiating (29), and substituting into the resulting expression (29) and rearranging.

<sup>16</sup> We do not need to resort to a discrete-time framework as in section 4.2, because the countries make R&D investment decisions as a unitary actor.

welfare similarly, given beliefs over the strategies employed by the other countries and the coalition. We limit all parties to Markovian strategies.

International cooperation on green R&D affects whether countries consider the value of the breakthrough only to themselves or to all the other countries as well when making their green R&D decisions. We again focus on a linear equilibrium, in which the optimal level of green R&D is independent of the degree of global warming,  $A$ . We solve for the Nash equilibrium of the partial cooperation case in Appendix A.

We denote the optimal variables in the R&D coalition case by the superscript ‘R&D’. Clearly the post-breakthrough phase is identical to the non-cooperative case, as there is no reason to conduct R&D, so the international coalition has no role. Thus  $W_i^{\text{R\&D}} = W_i^{\text{N}}$ .

Before the breakthrough, setting  $\tau^{\text{R\&D}} = \tau^{\text{N}}$  is an equilibrium. The stationary R&D stock has the same structure as in the social optimum, i.e., expressing the R&D stock of country  $i$  in terms of the values to individual countries,  $B_i^{\text{R\&D}} = N^{-1} \left( N(W_i^{\text{R\&D}} - V_i^{\text{R\&D}}) - h_1 (\rho + h_0 + \delta) \right)$ . Compared to the non-cooperative case (see equation (28)), the international coalition conducts R&D as a unitary agent, internalising the positive R&D spill-over effects (note the term  $N$  multiplying the value of the breakthrough to an individual country). The change in value to country  $i$  with the arrival of the breakthrough of course differs from the non-cooperative case. Solving as before, the aggregate steady state R&D stock is  $B^{\text{R\&D}} = h_1^{-1} \left( \sqrt{h_1 \Delta U^{\text{N}}} - \delta(\rho + h_0) - (\rho + h_0) \right)$ , which is positive as long as  $h_1 \Delta U^{\text{N}} / (\rho + h_0) > \rho + h_0 + \delta$ . The coalition thus considers the global benefits of green R&D investment, but recognises that the benefits depend on the difference in the surplus flows given non-cooperative carbon taxation. To reiterate, the globally aggregated stock of green R&D departs from the socially optimal stock only in that the global difference in utility flows  $\Delta U^{\text{N}}$  is calculated using non-cooperative carbon taxes instead of the optimal taxes. The following proposition compares green R&D stocks under different degrees of international policy cooperation:

**Proposition 3:** *Suppose that, for all  $F$  and  $R$  such that  $p_F \equiv U_F(F, R) \in [d, d + \alpha(\rho + \gamma)^{-1}]$  and  $p_R \equiv U_R(F, R) \in [\tilde{\beta}, \beta]$ ,  $F_{p_F p_R}$  is positive*

(negative). Denote aggregate global stocks of green R&D under full international cooperation, only international cooperation on R&D, and no international cooperation at all by the superscripts  $*$ , R&D and N, respectively. Then,  $B_i^* > B_i^{\text{R\&D}} > B_i^{\text{N}}$   $\left( B_i^{\text{R\&D}} > \max \{ B_i^*, B_i^{\text{N}} \} \geq \min \{ B_i^*, B_i^{\text{N}} \} \right), i = 1, \dots, N$ .

**Proof:** See Appendix A.

The conditions on how fossil fuel demand responds to changes in the renewable energy cost are the same as in Proposition 2. With many countries, there are two market failures. First, in the absence of international policy cooperation, countries pollute too much and invest too little in green R&D. Second, if countries only cooperate in setting green R&D, the direction of the R&D distortion depends on how the fossil fuel demand curve responds to the breakthrough. If fossil fuel demand is more responsive to renewable energy prices when fossil prices are high ( $F_{p_F p_R} > 0$ ), the value of the breakthrough is higher under full cooperation than without carbon taxation and second-best R&D falls short of the efficient outcome. In other words, R&D and carbon taxation are complements.<sup>17</sup> As we have shown in Examples 1 and 2 above, this holds for the often-used Cobb-Douglas utility specification (raised to the power of  $\mu$  to ensure diminishing marginal utility to  $F$  and  $R$  jointly), and for the CES utility specification with an elasticity of substitution between the two types of energy above unity (the empirically relevant case).

Finally, we can also solve the equilibrium for the case of partial cooperation on carbon pricing only, with countries choosing their R&D policies unilaterally. This second-best equilibrium is exactly symmetric with the equilibrium discussed in Proposition 3. One interesting result is that, for  $F_{p_F p_R} < 0$ , so that fossil fuel demand responds more at low prices, the second-best R&D stock with tax cooperation is below the fully non-cooperative R&D stock, as carbon taxes reduce the value of the breakthrough. We do not report the details as partial cooperation only on carbon pricing seems a less plausible case

---

<sup>17</sup> If  $F_{p_F p_R} < 0$ , cooperative R&D stocks tends to exceed non-cooperative R&D unless the breakthrough is extremely valuable in the non-cooperative outcome relative to the cooperative case. A necessary condition for non-cooperative R&D to be higher is  $N^{-1} (h_1 \Delta U^{\text{N}} - \delta(\rho + h_0)) \geq (h_1 \Delta U^* - \delta(\rho + h_0))$ .

than partial cooperation on green R&D: politicians typically prefer the carrot of R&D subsidies to the stick of tax instruments.

## 5. Illustrative calibration

We calibrate our model under the assumption that the base year 2013 corresponds to business as usual with zero (or ‘negligible’) carbon taxes and renewable subsidies.

### 5.1. Atmospheric decay, temperature and climate damages

Let  $A$  and  $F$  be measured in Gigatons of carbon (GtCs). We use a simple one-box carbon cycle with all excess carbon eventually leaving the atmosphere, of the form

$$(1') \quad \dot{A} = F - \gamma(A - 581), \quad \gamma \geq 0,$$

where 581 GtC is the pre-industrial stock of carbon in the atmosphere and we set  $\gamma = 1/200$  implying that carbon stays on average 200 years in the atmosphere.<sup>18</sup> The initial 2013 stock of carbon is 841 GtC, so  $A(0) = 841$ .

The climate damage flow is an increasing function of the atmospheric stock given by

$$(32) \quad D(A) = \alpha A^\theta, \quad \alpha > 0, \quad \theta > 1,$$

We set  $\alpha = 6 \times 10^{-11}$  \$trillion / (GtC) <sup>$\theta$</sup>  and  $\theta = 3.5$ , which implies a current annual flow of global warming damages of just over \$1 trillion. This is roughly in line with the specification suggested by Ackerman and Stanton (2012) albeit slightly less convex.<sup>19</sup>

### 5.2. Knowledge accumulation and breakthrough technology

Investment and stocks of green R&D are measured in 2013 US dollars. Green R&D depreciates at 5 percent per year, so  $\delta = 0.05$ . We set the current stock of green R&D as a ballpark estimate to \$310 billion (IEA, 2015).<sup>20</sup> The cost of investment in green R&D

<sup>18</sup> A two-box carbon cycle would be more accurate (Maier-Reimer and Hasselmann, 1987; Golosov et al., 2012). Our key points are more easily made with a simplified carbon cycle.

<sup>19</sup> Dietz and Stern (2015) and Rezai and van der Ploeg (2016, 2017) also add more realism by allowing for a more convex damage specification than in DICE.

<sup>20</sup> For numerical convenience, we set  $B(0) = \$1$  trillion in the simulations. The form of adjustment costs we use makes it infinitely costly to build up the clean R&D stock from zero. This is why it is important to start from a non-zero stock of clean R&D.

including adjustment costs is  $C(I, B) = \left(1 + \frac{\phi}{2} \frac{I}{B}\right) I = \left[1 + \frac{1}{2}(q-1)\right] I$ . We assume that the current investment rate is \$26 billion year, or double the rate of current public investment (Frankfurt School, 2016; NSF, 2014). We assume that only 60% of current investment expenditures get delivered, and thus infer  $\phi = 1.1$ .

We specify a linear function for the dependence of the breakthrough hazard rate on the aggregate green R&D stock:

$$(33) \quad H(B) = h_0 + h_1 B, \quad h_0 > 0, \quad h_1 > 0.$$

There is no possibility of a breakthrough if there is no green R&D stock, so  $h_0 = 0$ . We set  $h_1 = 0.0042$ , so that the steady state expected arrival time of the technological breakthrough for the socially optimal benchmark is around 10.5 years. The probability of a breakthrough at this steady state in a given year is approximately  $1/10.5 = 9.5\%$  and the cumulative probability of a breakthrough in the next ten years is  $1 - \exp(-10(h_0 + h_1 B^*)) = 61\%$ . We assume a radical breakthrough in that production costs drop by a factor 10:  $\tilde{\beta} = 0.1\beta$ . We also consider an even more radical breakthrough of a factor 20:  $\tilde{\beta} = 0.05\beta$ .

### 5.3. Fossil fuel and renewable energy: production and costs

Initial global fossil fuel and renewable energy use in 2013 are  $F(0) = 9.9$  GtC or 560 million Giga BTU and  $R(0) = 0.212$  GtC equivalents or 12 million Giga BTU, respectively.<sup>21</sup> We take a production cost of \$8.9/BTU or \$504/tC and for renewable energy of \$17.8/BTU, so renewable energy is currently twice as costly as fossil fuel. Since energy use is measured in GtC, we have a cost of  $d = \$504/\text{tC}$  and  $\beta = \$1008/\text{tC}$ .

### 5.4. Utility and demand for fossil fuel and renewable energy

Utility is quasilinear and given by

$$(34) \quad U(F, R) = \zeta \left( \alpha^{\frac{1}{\sigma}} F^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} R^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\mu\sigma}{\sigma-1}} + M, \quad \sigma > 0, \quad 0 < \mu < 1, \quad \zeta > 0$$

---

<sup>21</sup> To avoid scaling problems, we measure renewable energy in terms of GtC-equivalents.

where  $M$  is non-energy consumption and  $\mu < 1$  indicates the concavity of the utility function. The empirical results of Papageorgiou et al. (2015) suggest to set an elasticity of substitution between fossil fuel and renewable energy of  $\sigma = 2.5 > 1$ , which implies that the two energy types are gross substitutes. The efficiency conditions yield  $F/R = (\beta/d)^\sigma \alpha / (1-\alpha)$ , so given base year values for energy consumption and values for the unit pre-breakthrough production costs of fossil fuel,  $d$ , and renewable energy,  $\beta$ , we calibrate the utility share parameter as

$$\alpha = \frac{1}{1 + (R(0)/F(0))(\beta/d)^\sigma} = \frac{1}{1 + (.212/9.9)(1008/504)^{2.5}} = .891887.$$

We set the concavity parameter to  $\mu = 0.5$ , which implies a long-run price elasticity of the energy aggregate of 2. To match the levels of energy use, we require  $\zeta = 3.318$ . Finally, we use a social discount rate of 3% per year, so  $\rho = 0.03$ .

## 6. Simulation of Globally Cooperative Outcomes<sup>22</sup>

### 6.1. First best

Figure 1 illustrates time paths for the social optimum, and the effects of increasing the discount rate. All panels illustrate the breakthrough as occurring at the moment its cumulative probability reaches 50%. The carbon tax is an increasing and convex function of the carbon stock. As the initial carbon stock is 841 GtC and the quasi-steady state value is 1280 GtC, the carbon tax rises with time. The initial carbon tax starts at 154 \$/tC and then rises to its quasi-steady state value of 340 \$/tC. Once the breakthrough occurs, the carbon tax can be relaxed slightly. However, note from figure 1 that the renewable substitute becomes sufficiently cheaper, so that carbon emissions fall regardless. The post-breakthrough steady-state carbon stock is lower, namely 1145 GtC. If the breakthrough is delayed sufficiently, we have overshooting with the carbon stock and the tax decreasing following the breakthrough.

Green R&D effort is a non-monotonic function of the existing R&D stock. A higher existing stock reduces the marginal benefit on an extra unit of R&D; however, it also reduces the adjustment costs associated with developing this extra unit. The latter effect

---

<sup>22</sup> The computational algorithm that we have developed for these simulations is discussed in Appendix B.

dominates for low  $B$ , including in the early years for our calibration. Thus, before the breakthrough reduction in the cost of clean energy, green R&D efforts first rise for a few years, and then fall as the R&D stock on which later efforts build is sufficiently high.<sup>23</sup> A higher discount rate of 4% per year ( $\rho = 0.04$ ) is associated with a lower carbon tax and lower green R&D investment efforts. As a result, the quasi steady-state carbon stock is higher and the expected arrival of the breakthrough is delayed slightly.

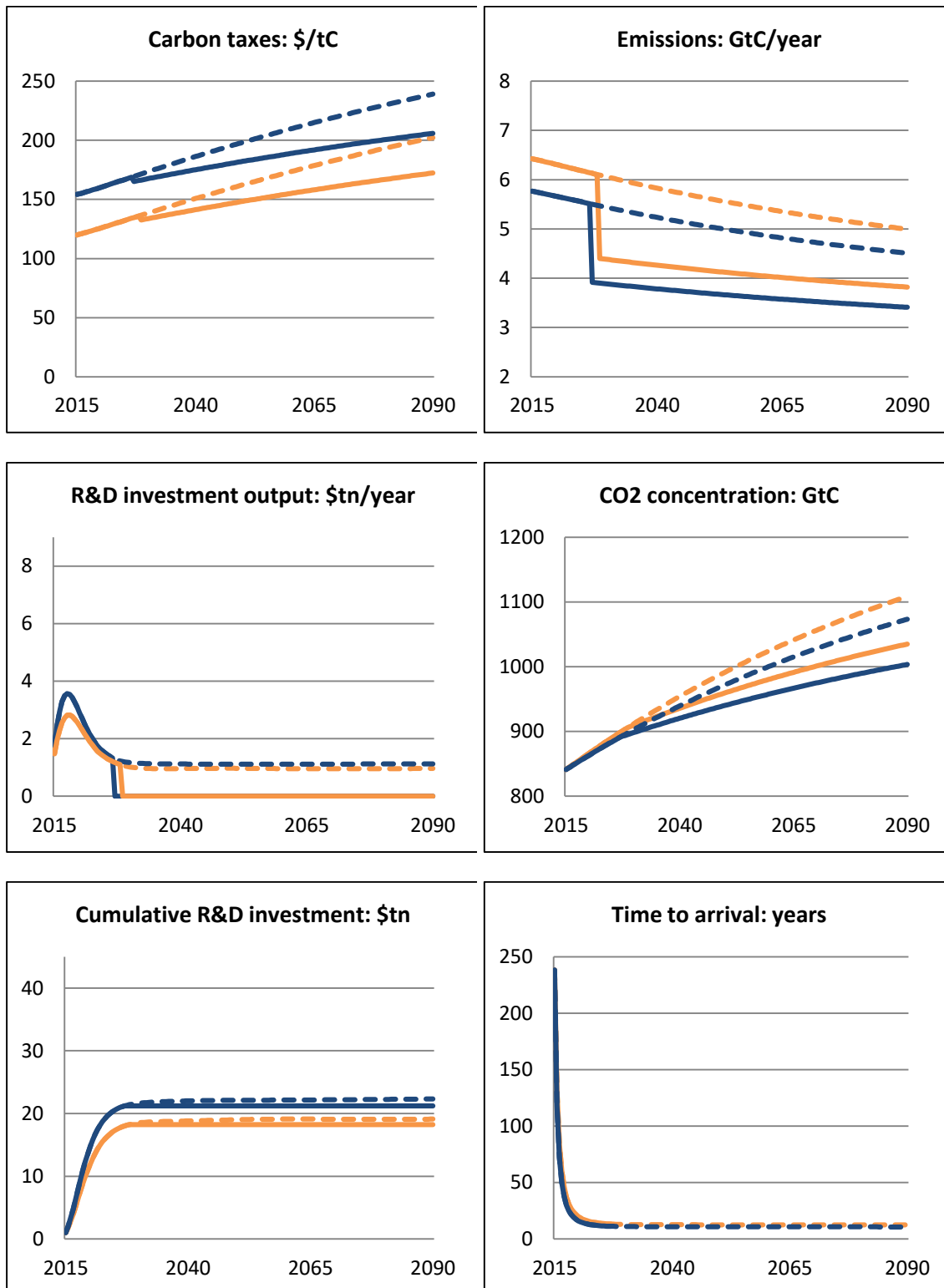
Figure 2 displays the effect of a more radical breakthrough. When the breakthrough reduces the renewable cost by 95%, instead of 90%, the immediate carbon tax can be relaxed in expectation of the technological solution with slightly higher carbon emissions before the breakthrough. Post-breakthrough carbon taxes are also lower, but the more drastic drop in the cost of clean energy ensures that carbon emissions are much lower following the breakthrough and thus the carbon stock after the breakthrough will fall below that in the base case. Green R&D investment is higher with the more drastic eventual cut in the cost of clean energy. It roughly doubles compared to the benchmark case, so that the breakthrough is expected to arrive earlier, in 8 years rather than 12.

Figure 3 shows the simulations for the cooperative outcome if the expected breakthrough technology is more imminent, with the hazard rate doubled for any level of the stock of green R&D. Carbon taxes are almost unaffected by the more promising technology prospects. This is a property of all our calibrated solutions: the marginal cost of the carbon stock hardly depends on the green R&D stock, and the marginal value of the green R&D stock hardly depends on the carbon stock. Even though green R&D is twice as effective in increasing the probability of the breakthrough, clean R&D investment is barely above the benchmark case for the first years, and then falls below. The marginal benefit of the R&D stock increases, so that in the globally cooperative outcome investment is curbed to save on the marginal cost.

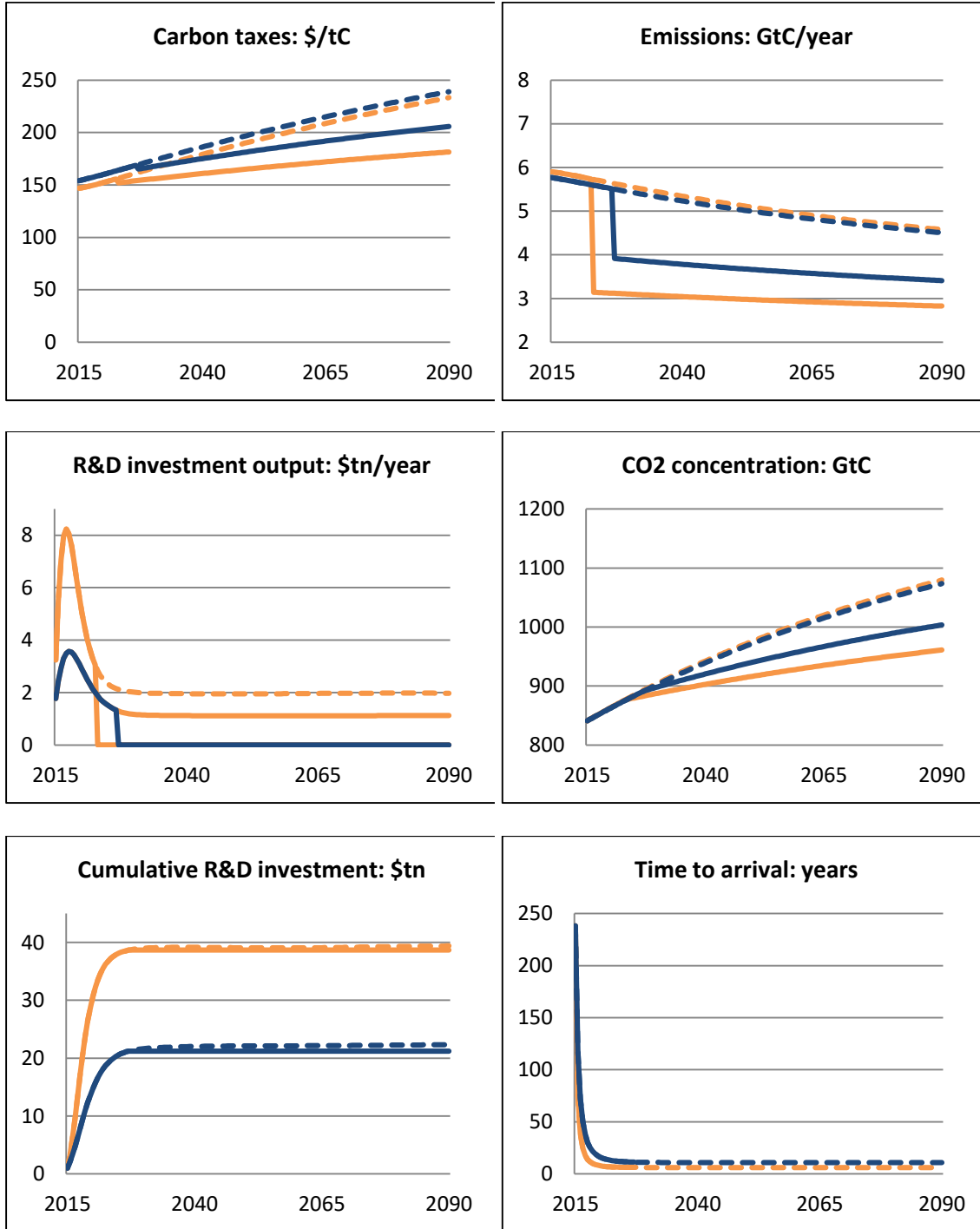
---

<sup>23</sup> The cross-derivative of the value function is very small: optimal green R&D effort is nearly independent of the carbon stock, and the optimal carbon tax does not depend much on the R&D stock.

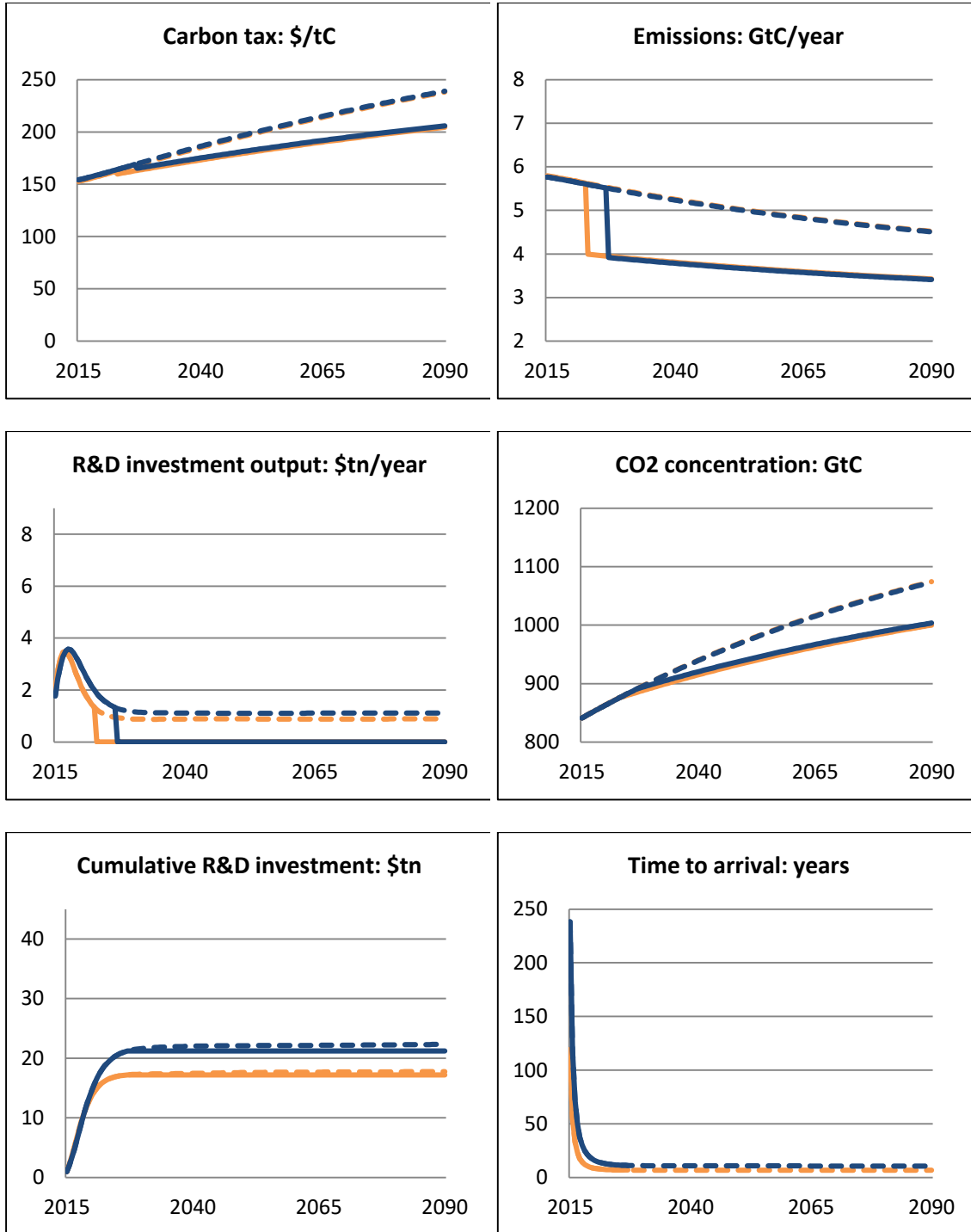
**Figure 1: Higher Discounting and the Globally Cooperative Outcome**



**Key:** Base case (*blue*) and high discounting,  $\rho = 0.04$  (*orange*), for pre-breakthrough regime (*dashed*) and given breakthrough at  $t = 15$  (*solid*). The horizontal axis gives calendar time in years.

**Figure 2: Higher Efficacy of Breakthrough and the Globally Cooperative Outcome**

**Key:** Base case (*blue*) and high R&D efficacy,  $\Delta = 0.95\beta$  (*orange*), for pre-breakthrough regime (*dashed*) and given breakthrough at  $t = 15$  (*solid*). The horizontal axis gives calendar time in years.

**Figure 3: Doubling Hazard Rates and Globally Cooperative Outcome**

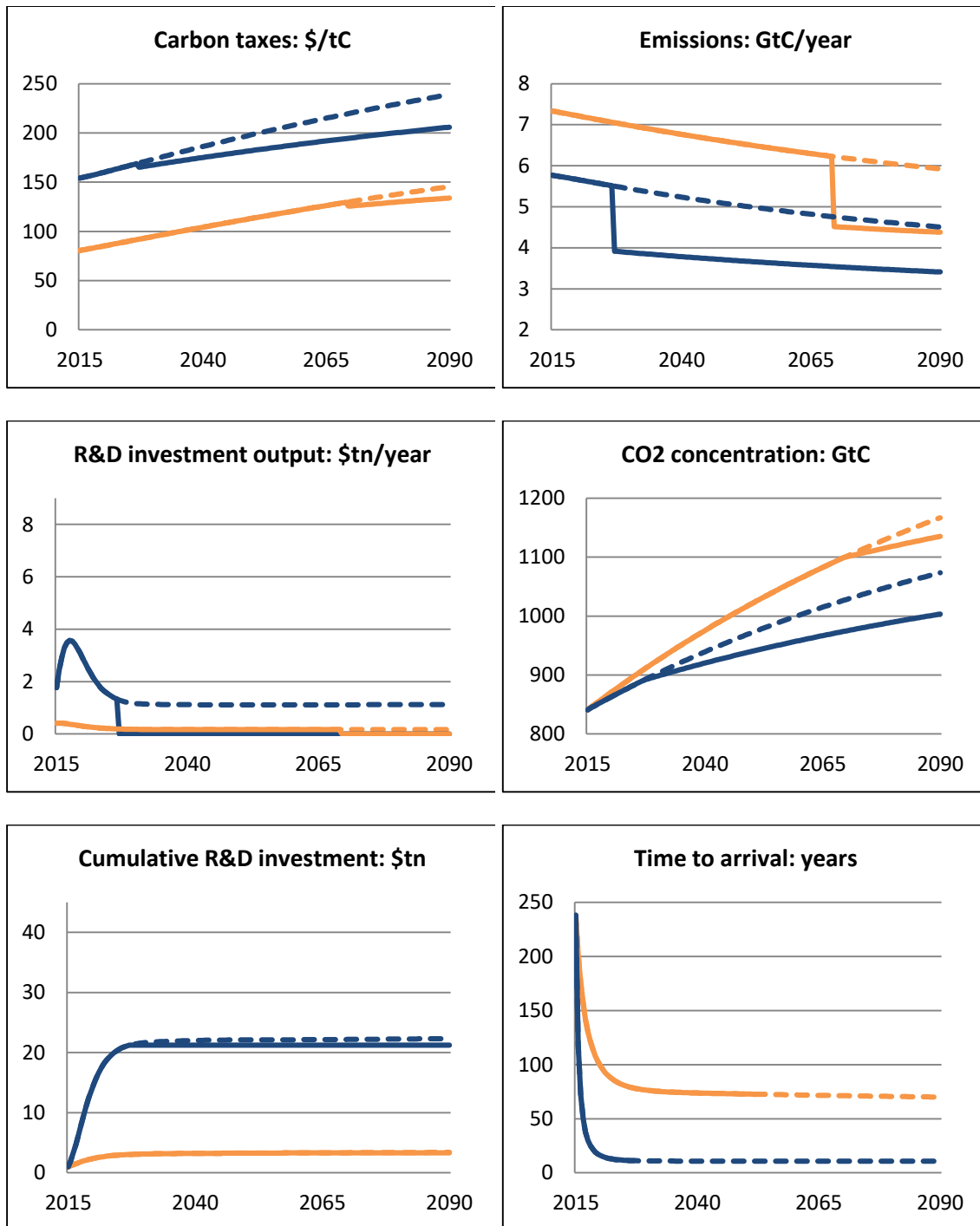
**Key:** Base case (*blue*) and double the hazard rate, at each level of the green R&D stock, i.e.,  $h_0 = 1/15$  and  $h_1 = .0084$  (*orange*), for pre-breakthrough regime (*dashed*) and realised breakthrough at  $t = 15$  (*solid*). The horizontal axis gives calendar time in years.

## 7. Simulation of Non-Cooperative Outcomes for the Global Economy

We compute the feedback Nash (Markov-perfect) equilibrium for the global economy and for illustrative purposes we consider the case of two countries,  $N = 2$ . The results are displayed in figure 4. The blue curves indicate the globally cooperative outcome with  $N = 1$  and the orange curves indicate the feedback Nash equilibrium for the non-cooperative outcome for the global economy with  $N = 2$ . The double international free-riding incentives lower both carbon taxes and investments in green R&D. Initial non-cooperative carbon taxes are almost 50% below those in the globally cooperative outcome, and emissions almost 30% higher. Non-cooperative investments in green R&D fall over 75% short. As a result, the global stock of green R&D ends up much lower and the expected breakthrough under the non-cooperative outcome occurs much later than in the globally optimal outcome, i.e., around 2070 rather than 2027. The combined effect of higher carbon emissions and the long delay until the breakthrough drop in the cost of clean energy mean that the rise in atmospheric carbon concentrations will be much higher in the non-cooperative outcome. In fact, the quasi-steady state carbon stock is 1490 GtC (compared to 1280 GtC under the socially optimal outcome).

The various components of the carbon taxes and investment rates in green R&D can be separated into the free-riding and strategic components (see equation (23') for the carbon tax, and (24') for the marginal value of R&D stock). Carbon emissions in the non-cooperative equilibrium are 27% higher than the socially optimal emissions. By themselves, the free-riding incentives would raise emissions by 23%. The remaining 4% is due to strategic attempts to encourage lower future emissions (about two thirds of the effect) and higher future investment (the remaining one third). Hence, strategic pollution has a small but non-negligible impact on the non-cooperative outcome.

The equilibrium investment rate is 76% lower than efficient; only 19 %-points is accounted for by free-riding. The remainder is almost fully accounted for by a strategic attempt to encourage other countries to invest more in the future, with underinvestment to encourage lower future emissions being negligible. Thus, the strategic effect of current investment on other countries' future investment is very strong: all countries

**Figure 4: Non-Cooperative Outcome with Two Countries**

**Key:** Base case (*blue*) and the 2-player Nash equilibrium (*orange*), for pre-breakthrough regime (*dashed*) and realised breakthrough at  $t = 15$  (*solid*). The horizontal axis gives calendar time in years.

underinvest heavily, in a futile hope to encourage others to invest more in the future. The result is a long delay in the arrival of the breakthrough technology.

## 8. Conclusion

We have studied the economics of breakthrough energy technologies in the context of climate change where such breakthroughs are modelled as once-and-for-all regime shifts. Such breakthrough technologies can be made more imminent by investing in green R&D knowledge. Since this type of knowledge typically spills over to a large extent from one country to another, R&D investment represents a second international free-rider problem which interacts with the free-rider problem resulting from the global warming externality. Individual countries invest too little in green R&D as they want to free ride on other countries investing in green R&D, but still benefit from lower costs once the breakthrough drop in the cost of clean energy has occurred. Without international policy cooperation, we thus find that carbon emissions are priced too low and governments invest too little in green R&D. International spill-over effects of R&D in carbon-free technologies thus lead to double free-riding problems resulting from failing to internalise the carbon and green R&D externalities. These international free-rider problems are in a feedback (Markov-perfect) Nash equilibrium further exacerbated by strategic effects that do not occur in the open-loop Nash equilibrium. Individual countries increase pollution because they also know that this raises the marginal costs of global warming of other countries, who thus take more action to mitigate carbon emissions and increase the chances of a breakthrough; and they invest too little also as the lower global stock of green R&D incentivises other countries to invest more in green R&D and to pollute less.

If governments find it for political reasons difficult to price carbon, they might rely on green R&D instead, in which case partly cooperative second-best climate policies in the global economy result. We have shown that, if there is international coordination of green R&D whilst carbon taxes are set unilaterally, R&D investment may exceed the level under full international policy cooperation. This case arises if fossil demand is more responsive at low prices, so that the breakthrough is more effective when carbon taxes are low (or even absent). Hence, international cooperation on green R&D efforts may partly offset the failure to agree on a global carbon price, leading to higher investment in clean technologies. The converse of this is that, with international cooperation only on

carbon pricing, unilateral green R&D efforts may fall even below the level under fully non-cooperative policies.

Finally, we have used numerical simulations to illustrate the different cooperative and non-cooperative equilibria of a general version of our model with nonlinear global warming damages and adjustment costs for investments in green R&D. We have contrasted for the first time non-cooperative feedback Nash equilibrium outcomes with cooperative outcomes in economies with technological tipping points and a global warming externality. We have shown that without international policy cooperation, the free-riding problem on green R&D investments may be quite severe, with countries underinvesting sharply. This may lead to a very long delay in the arrival of the breakthrough. Optimal R&D efforts are initially much higher, and even rise over time, as the accumulating knowledge stock makes further investment cheaper.

There are two future directions of research. First, we have assumed that fossil fuel is abundant. If it is scarce, there will be Hotelling rents and second-best climate policies might lead to an acceleration of global warming as fossil fuel producers try to avoid having too many stranded assets when the breakthrough technologies finally come to market. This is a version of the Green Paradox, which arises when carbon taxes rise too rapidly or renewable energy is subsidised (e.g., Sinn; 2008; and for surveys, see Long, 2015 and van der Ploeg and Withagen, 2015) but also when there is an imminent breakthrough of a new technology (van der Ploeg, 2012; Winter, 2014). Second, apart from strategic behaviour among countries concerned with climate change and bringing forward breakthroughs of carbon-free energy technologies, it is also of interest to investigate strategic resource dependence resulting from the livelihood of fossil fuel producers being threatened by the strategic investment by fossil fuel producers in the R&D of a carbon-free substitute for fossil fuel (e.g., Gerlagh and Liski, 2011; Michielsen, 2012; Jaakkola, 2015).

## References

Benchechrone, H. and N.V. Long (2002): Transboundary fishery: A differential game model, *Economica*, 69, 274, 207-221.

- Dasgupta, P., R. Gilbert and J. Stiglitz (1983). Strategic considerations in invention and innovation: the case of natural resources, *Econometrica*, 51, 1439-1448.
- Dietz, S. and N.H. Stern (2015). Endogenous growth, convexity of damages and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. *Economic Journal*, 125, 583, 574-602.
- Dixit, A.K. and R.S. Pindyck (1994). *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.
- Doraszelski, U. (2003). An R&D race with knowledge accumulation, *Rand Journal of Economics*, 20-42.
- Frankfurt School (2016). *Global Trends in Renewable Energy Investment 2016*. Frankfurt School-UNEP Centre / Bloomberg New Energy Finance, Frankfurt. Available at <http://fs-unep-centre.org/publications/global-trends-renewable-energy-investment-2016>.
- Fudenberg, D. and D.K. Levine (1988). Open-loop and closed-loop equilibria in dynamic games with many players, *Journal of Economic Theory*, 44, 1, 1-18.
- Gerlagh, R. and M. Liski (2011). Strategic resource dependence, *Journal of Economic Theory*, 146, 2, 699-727.
- Harris, C. and J. Vickers (1987). Racing with uncertainty, *The Review of Economic Studies*, 54, 1, 1-21.
- Harris, C. and J. Vickers (1995). Innovation and natural resources: a dynamic game with uncertainty, *RAND Journal of Economics*, 26, 418-430.
- Harris, C., S. Howison and R. Sircar (2010). Games with exhaustible resources, *SIAM Journal on Applied Mathematics*, 70, 7, 2556-2581.
- Hotelling, H. (1931). The economics of exhaustible resources, *Journal of Political Economy*, 39, 2, 137-175.
- IEA, (2015). *IEA Energy Technology RD&D Budgets (2015 edition)*, International Energy Agency, Paris.
- Jaakkola, N. (2015). Putting OPEC out of business, revised Research Paper 99, OxCarre, Department of Economics, University of Oxford.
- Judd, K. (1998). *Numerical Methods in Economics*, MIT University Press, Cambridge, Mass.
- Kamien, M.I. and N.L. Schwartz (1971). Optimal maintenance and sale age for a machine subject to failure, *Management Science*, 17, 495-504.
- Long, N.V. (2015). The Green Paradox in open economies: lessons from static and dynamic models, *Review of Environmental Economics and Policy*, 9, 2, 266-284.
- Michielsen, T. (2012). Strategic resource extraction and substitute development, *Resource and Energy Economics*, 36, 2, 455-468.
- Nkuiya, B. (2015). Transboundary pollution game with potential shift in damages, *Journal of Environmental Economics and Management*, 72, 1-14.

- Ploeg, F. van der (2012). Breakthrough renewables and the Green Paradox, Research Paper 91, OxCarre, Department of Economics, University of Oxford.
- Ploeg, F. and A.J. de Zeeuw (1992). International aspects of pollution control, *Environmental and Resource Economics*, 2, 2, 117-139.
- Ploeg, F. van der and A.J. de Zeeuw (2016). Non-cooperative and cooperative responses to climate catastrophes in the global economy: a North-South perspective, *Environmental and Resource Economics*, 65, 3, 519-540.
- Ploeg, F. van der and C. Withagen (2015). Global warming and the Green Paradox: a review of adverse effects of climate policies, *Review of Environmental Economics and Economic Policy*, 9, 2, 285-302.
- Polasky, S., A. de Zeeuw and F. Wagener (2011). Optimal management with potential regime shifts, *Journal of Environmental Economics and Management*, 62, 2, 229-240.
- Popp, D. (2008). R&D subsidies and climate policy: is there a “free lunch”?, *Climatic Change*, 77, 311-341.
- Rezai, A. and F. van der Ploeg (2016). Intergenerational inequality aversion, growth and the role of damages: Occam’s rule for the global carbon tax, *Journal of the Association of Environmental and Resource Economists*, 3, 2, 493-522.
- Rezai, A. and F. van der Ploeg (2017). Second-best renewable policies to decarbonize the economy: commitment and the Green Paradox, *Environmental and Resource Economics*, 66, 3, 409-434.
- Sinn, H.-W. (2008). Public policies against global warming: a supply side approach, *International Tax and Public Finance*, 15, 4, 360-394.
- Spence, M and D. Starrett (1975). Most rapid approach paths in accumulation problems, *International Economic Review*, 16, 2, 388-403.
- Stern, N. (2007). *The Economics of Climate Change. The Stern Review*, Cambridge University Press, Cambridge, U.K.
- Winter, R. (2014). Innovation and the dynamics of global warming, *Journal of Environmental Economics and Management*, 68, 1, 124-140.
- Zeeuw, A. de and A. Zemel (2012). Regime shifts and uncertainty in pollution control, *Journal of Economic Dynamics and Control*, 36, 7, 939-950.

## Appendix A: Derivation of and proofs

**Proof of Proposition 2:** From (16), (15') and (16'),  $B^{SB} \geq B^*$  is equivalent to

$$(A8) \quad U^*(d, \tilde{\beta}) - U^*(d, \beta) \geq U^*(d + \tau^*, \tilde{\beta}) - U^*(d + \tau^*, \beta),$$

which can be written as

$$(A9) \quad U^*(d + \tau^*, \beta) - U^*(d, \beta) \geq U^*(d + \tau^*, \tilde{\beta}) - U^*(d, \tilde{\beta})$$

So  $B^{\text{SB}} \geq B^*$  if the increase in the social benefit flow of energy use from raising carbon taxes from zero to the socially optimal level is higher before the breakthrough than after. A sufficient condition for this is if the benefit of imposing socially optimal taxes is increasing in the cost reduction for all renewable cost levels  $b \in (\tilde{\beta}, \beta)$ . We first calculate this benefit by integrating the marginal benefit of increasing taxes from  $\tau = 0$  to  $\tau = \tau^*$ . Taking the optimal consumption levels as a function of an arbitrary  $\tau$  and of the reduction in renewables cost  $\Delta\beta$ , the marginal increase in utility flow due to a small increase in taxes is  $dU^*/d\tau = (\tau - \alpha(\rho + \gamma)^{-1})dF^*/d\tau$ . We can then express the increase in utility from implementing socially optimal taxes as

$$(A10) \quad U^*(d + \tau^*, \beta - \Delta\beta) - U^*(d, \beta - \Delta\beta) = \int_0^{\tau^*} \left( \tau - \frac{\alpha}{\rho + \gamma} \right) \frac{dF^*}{d\tau} d\tau$$

This can be differentiated with respect to the reduction in renewables cost  $\Delta\beta$  to get

$$(A11) \quad \frac{d}{d\Delta\beta} (U^*(d + \tau^*, \beta - \Delta\beta) - U^*(d, \beta - \Delta\beta)) = \int_0^{\tau^*} \left( \tau - \frac{\alpha}{\rho + \gamma} \right) \frac{d^2 F^*}{d\tau d\Delta\beta} d\tau$$

which is positive (negative) if  $F_{p_{RPF}}^* > (<) 0$  in  $\tau \in [0, \tau^*], \Delta\beta \in [0, \beta - \tilde{\beta}]$ .  $\square$

**Derivation of the partial cooperation case:** It is straightforward to obtain the partial cooperation case. Markovian strategies, with the coalition leading, take the form  $\underline{I} = \varsigma^I(A, \underline{B})$ ,  $F_i = \varsigma_i^F(A, \underline{B}, I)$ ,  $R_i = \varsigma_i^R(A, \underline{B}, I)$ . We conjecture the equilibrium strategies involve constant  $F_i$  and  $R_i$ , and  $\underline{I}$  which depends only on  $\underline{B}$ . The individual countries solve a problem identical to (28), but taking the investment rate to be a function as given above. The coalition solves  $\rho V^{\text{R\&D}} = \max_{\underline{I}} \sum_{j \in \{1, \dots, N\}} V_j$ , where the individual value function have the structure (28), and assumes the energy use policies are given by  $\varsigma_i^F, \varsigma_i^R$ . For the former problem, we obtain first-order conditions for the energy use variables identical to those in Lemma 2. Again, we can confirm that the value function must be linear in  $A$  (see footnote 4). In particular, as  $F_i$  is constant (and so independent of  $\underline{B}$ ), the coalition cannot manipulate energy use, and the evolution of the R&D stocks is independent of the evolution of the carbon stock. For the coalition, the solution still has

a bang-bang structure, but the aggregate marginal cost of investment by an individual country  $1/N$  is compared to the aggregate value of this investment,  $V_B^{\text{R\&D}} = NV_{iB}^{\text{R\&D}}$ . Calculation identical to those in section 2.3 yields the required outcome.

**Proof of Proposition 3:** We first show that  $B^{\text{R\&D}} \geq B^N$ . First observe that, for a constant  $q > 0$ ,  $-a + \sqrt{a^2 + q}$  is decreasing in  $a$ . Then,

$$\begin{aligned}
 h_1 B^N &= \sqrt{N^{-1} \left( h_1 \Delta U^N - \delta(\rho + h_0) \right) - \frac{(N-1)^2}{4N^2} \delta^2 - (\rho + h_0) - \frac{N-1}{2N} \delta} \\
 (A12) \quad &\leq \sqrt{N^{-1} \left( h_1 \Delta U^N - \delta(\rho + h_0) \right) - (\rho + h_0)} \\
 &\leq \sqrt{h_1 \Delta U^N - \delta(\rho + h_0)} - (\rho + h_0) \\
 &= B^{\text{R\&D}}
 \end{aligned}$$

We then order  $B^{\text{R\&D}}$  and  $B^*$ . Note that

$$(A13) \quad h_1 (B^{\text{R\&D}} - B^*) = \sqrt{h_1 \Delta U^N - \delta(\rho + h_0)} - \sqrt{h_1 \Delta U^* - \delta(\rho + h_0)}$$

so that  $B^{\text{R\&D}} \geq B^* \Leftrightarrow \Delta U^N \geq \Delta U^*$ . This follows easily using the argument developed in the proof of Proposition 1.  $\square$

## Appendix B: Numerical Algorithm

We solve the general case with nonlinear hazard and damage functions and investment adjustment costs numerically. We use projection methods to approximate the value functions. We need to obtain the solution to two HJB equations: for the before- and after-breakthrough problem, respectively. This involves solving for the fixed points  $V(A, B)$  and  $W(A)$  of the HJB equations (5) and (11) for the cooperative case with  $N = 1$ , and for the fixed points  $V_i(A, B)$  and  $W_i(A)$  of the HJB equations (5') and (19) for the non-cooperative case with  $N > 1$ . We thus need an algorithm for solving these two simultaneous dynamic programming problems. This algorithm proceeds by solving the full model in two steps using GAMS and the KNITRO 9.0 solver. In the first step, we solve the post-breakthrough regime by numerically solving the HJB equation. We approximate the value function  $W(A)$  with the  $m$ th degree Chebyshev polynomial

$W(A) = \sum_{i=0}^{m-1} a_i \phi_i(A)$  in which the  $a_i$  are coefficients to be determined and  $\phi_i(A)$  is the

Chebyshev polynomial of degree  $i$  (see Judd, 1998). We then solve the system of equations consisting of the HJB equation (with the true value function and its derivative replaced by the approximation and its derivative), the optimality conditions, and the equation of motion for the carbon stock. This system of equations is set to hold exactly at the Chebyshev nodes for a given region in the state space. We verify the accuracy of the approximation of the feasible solution: the HJB equation has relative errors at most of order (and for most solutions much higher)  $10^{-4}$ .

In the second step, we solve the pre-breakthrough regime in the same manner. We first calculate the quasi-steady state before the breakthrough. We start with solving the HJB equations in a small region around this steady state, and use homotopy to gradually expand this region. We approximate  $V(A,B)$  with the Chebyshev tensor polynomials

$V(A,B) = \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} a_{ij} \phi_i(A) \phi_j(B)$ . We then solve the system to minimise the sum of

squared relative deviations of the HJB equation and its derivatives with respect to  $A$  and  $B$  at a set of Chebyshev nodes. Including deviations in the derivatives increases the accuracy of the solution, as it reduces the error in the first-order conditions. We verify again the relative error in the HJB equation, which is always at most of order  $10^{-4}$  (and mostly much less). Once we have obtained the solutions to the HJB equations, we can calculate the carbon taxes and Tobin's  $q$  and obtain the simulations reported in Sections 6 and 7 using standard Runge-Kutta methods for numerical integration of ordinary differential equations.

## ONLINE APPENDIX

**Proof of Lemma 1.** We adapt the strategy used by Polasky et al. (2011) to a bivariate context, and characterise the structure of any value function which solves problem (11). The bivariate nature of our model prevents us from obtaining an explicit, closed-form statement for the value, but we can still characterise optimal policies.

With  $C(I) \equiv I$ , it is clear that the maximand is linear in the control variable  $I$ . We consider the outcome if investment is required to satisfy  $I \in [-\infty, \bar{I}]$ . We choose  $\bar{I} > 0$  sufficiently large but finite; this will be made more precise. Then, the optimal investment rate is

$$I^* = \begin{cases} -\infty & V_B < 1, \\ \text{indeterminate} & \text{if } V_B = 1, \\ \bar{I} & V_B > 1. \end{cases}$$

We consider the outcome with an infinite disinvestment rate by looking at the limit of the system dynamics as  $I$  falls without bound.<sup>24</sup> We assume a solution to the problem exists and induces a piecewise-differentiable value function. The value function must be continuous: as the economy can be controlled in all directions, any jumps in value would necessarily be equalised by the optimal strategy, sufficiently close to an upward discontinuity, of driving the economy to the high-value region.

We will immediately define the crucial locus  $B = \Psi(A)$  implicitly by the relation

$$(OA1) \quad \rho + \delta + H(\Psi(A)) = H'(\Psi(A))(W(A) - V(A, B)).$$

We assume the parameters are such that  $\rho + \delta + H(0) < H'(0)(W(A) - V(A, 0))$ , to focus on the interesting case in with positive R&D investment.

**Step 1:**  $V_B \geq 1$ .

We first show that  $V_B \geq 1$ , i.e. that the slope of the value function in  $B$  cannot be less than unity.

The full HJB equation is

---

<sup>24</sup> Polasky *et al.* (2011) construct the value function for any bound on the disinvestment rate, and then take the limit as the bound becomes less and less binding. In our bivariate setting, constructing the value function is more difficult. We assume the existence of a value function under an infinite disinvestment rate, but derive the dynamics by taking the limit as the disinvestment rate is allowed grow arbitrarily. An alternative is to give the planner a second instrument of converting a discrete quantity of the R&D stock into utility (Polasky *et al.*, 2006), which would give the same result.

$$\begin{aligned}
(OA2) \quad \rho V(A, B) = & \max_{F, R, I} U(F, R) - dF - \beta R - D(A) - C(I) + V_A(F - \gamma A) \\
& + V_B(I - \delta B) + H(B)(W(A) - V(A, B)).
\end{aligned}$$

Suppose that there exists some region in which  $V_A$  is finite and  $V_B < 1$ .<sup>25</sup> Define a square in the region  $\Phi \equiv \{(A, B) : A \in (A^L, A^H), B^L < B < B^H\}$ , such that  $V$  is differentiable and has finite derivatives in  $\Phi$ . It is optimal to set  $I = -\infty$  for  $(A, B) \in \Phi$ . Consider a point  $(A_0, B_0) \in \Phi$  and a sufficiently negative  $\underline{I}$  so that setting  $I = \underline{I}$  and optimal energy use rates  $F^*, R^*$  given  $V_A$ , the economy will hit the lower boundary  $B = B_L$ . Denote the time at which the economy reaches this boundary as  $\tilde{t}(\underline{I})$  and define  $(\tilde{A}, \tilde{B})_{\underline{I}} \equiv (A(\tilde{t}(\underline{I})), B(\tilde{t}(\underline{I}))) = (A(\tilde{t}(\underline{I})), B^L)$ . We can integrate the equation of motion (3) in  $t \in [0, \tilde{t}]$  to obtain

$$(OA3) \quad B^L = e^{-\delta \tilde{t}} B_0 + \underline{I} \frac{1 - e^{-\delta \tilde{t}}}{\delta}.$$

As  $\underline{I}$  becomes arbitrarily negative,  $\tilde{t}$  falls to zero. The value at the initial point is

$$\begin{aligned}
(OA4) \quad V(A_0, B_0) = & \int_0^{\tilde{t}} e^{-\rho t} [U(F^*, R^*) - dF^* - \beta R^* - D(A) - \underline{I}] dt + e^{-\rho \tilde{t}} V(\tilde{A}, \tilde{B}) \\
= & \int_0^{\tilde{t}} e^{-\rho t} [U(F^*, R^*) - dF^* - \beta R^* - D(A)] dt \\
& - \frac{\delta}{\rho} \frac{1 - e^{-\rho \tilde{t}}}{1 - e^{-\delta \tilde{t}}} (B(\tilde{t}) - e^{-\delta \tilde{t}} B_0) + e^{-\rho \tilde{t}} V(\tilde{A}, \tilde{B}),
\end{aligned}$$

where the equality substitutes for  $\underline{I}$  using (OA3) and solving the integral explicitly.

We can now differentiate the above with respect to  $B_0$  :

---

<sup>25</sup> This need not be the *maximal* such region, i.e. the proof only requires us to investigate some convex region which might be a subregion of some larger region in which  $V_B < 1$ .

$$\begin{aligned}
V_{B_0}(A_0, B_0) = & \int_0^{\tilde{t}} e^{-\rho t} \frac{d}{dB_0} \left[ U(F^*, R^*) - dF^* - \beta R^* - D(A) \right] dt \\
& + e^{-\rho \tilde{t}} \left[ U(F^*, R^*) - dF^* - \beta R^* - D(A) \right] \Big|_{t=\tilde{t}} \frac{d\tilde{t}}{dB_0} \\
& - \frac{\delta}{\rho} \frac{1 - e^{-\rho \tilde{t}}}{1 - e^{-\delta \tilde{t}}} \left( \left[ \frac{\rho e^{-\rho \tilde{t}}}{1 - e^{-\rho \tilde{t}}} - \frac{\delta e^{-\delta \tilde{t}}}{1 - e^{-\delta \tilde{t}}} \right] \left[ B^L - e^{-\delta \tilde{t}} B_0 \right] \frac{d\tilde{t}}{dB_0} + \delta e^{-\delta \tilde{t}} B_0 \frac{d\tilde{t}}{dB_0} - e^{-\delta \tilde{t}} \right) \\
& - \rho e^{-\rho \tilde{t}} V(\tilde{A}, B^L) \frac{d\tilde{t}}{dB_0} + e^{-\rho \tilde{t}} \left( V_A(\tilde{A}, B^L) \frac{d\tilde{A}}{dB_0} \right).
\end{aligned}$$

From (OA3), we observe that  $d\tilde{t} / dB_0 = -(I - \delta B_0)^{-1} = -\frac{1 - e^{-\delta \tilde{t}}}{\delta} \frac{1}{B^L - B_0}$ , and integrating

the equation of motion for  $A$  and then differentiating this with respect to  $B_0$ , we obtain

$$\begin{aligned}
\frac{dA(t)}{dB_0} &= \int_0^t \frac{d \left[ F^*(A(s), B(s)) - \gamma A(s) \right]}{dB_0} ds \\
&= \int_0^t \left[ \frac{U_{FF} U_{RR} - U_{FR}^2}{U_{RR}} \right]^{-1} \left[ -V_{AA} \frac{dA(s)}{dB_0} - V_{AB} \frac{dB(s)}{dB_0} \right] - \gamma \frac{dA(s)}{dB_0} ds.
\end{aligned}$$

The key fact to observe is that, as the derivatives of the value and utility functions are bounded in  $\Phi$ , then as  $\lim_{I \rightarrow -\infty} \tilde{t} = 0$ , also  $\lim_{I \rightarrow -\infty} dA(\tilde{t}) / dB_0 = 0$ . In other words, the optimal trajectory with an unbounded disinvestment rate is parallel to the  $B$ -axis. Using these result, we obtain

$$\lim_{I \rightarrow -\infty} V_{B_0}(A_0, B_0) = 1.$$

But this is a contradiction, since we supposed  $V_B < 1$ . Thus, we must have  $V_B \geq 1$  everywhere where the derivative exists.

**Step 2:**  $W(A) \geq V(A, B)$  where  $I > \underline{I}$ .

Any  $(A, B)$  such that  $W(A) < V(A, B)$  must involve  $I = \underline{I}$ ; the pre-breakthrough value can be lower than the post-breakthrough value only if it involves greatly excessive R&D stocks such that it would be optimal to convert a part of them to consumption immediately, rather than risk a breakthrough before this could be achieved.

**Step 3:**  $V_B = 1$  implies  $B$  jumps down, unless  $B = \Psi(A)$ .

Suppose now that there is some region  $\Phi'$  with  $V_B = 1$  and  $I$  finite. The HJB equation (A2) must hold in this region. As the economy evolves gradually, at any point in the interior of the region  $dV_B / dt$  is constant. Differentiating (A2),

$$\begin{aligned} \frac{dV_B}{dt} &= V_{BA}\dot{A} + V_{BB}\dot{B} \\ &= (\rho + \delta + H(B))V_B - H'(B)(W(A) - V(A, B)) \end{aligned}$$

which can only equal zero on  $B = \Psi(A)$ . Thus, gradual evolution in  $\Phi'$  is not possible unless the economy is on this locus. If it is not, the economy must immediately jump down to the boundary of the region  $\Phi'$  or to the locus  $B = \Psi(A)$ . Denote the location after the jump by  $(A_0, B_0^+)$ . Taking the limit of (OA4) as  $\underline{I} \rightarrow -\infty$ , we get for  $(A_0, B_0) \in \Psi'$ ,

$$(OA5) \quad V(A_0, B_0) = B_0 - B_0^+ + V(A_0, B_0^+).$$

This implies that the state space can be divided into regions which satisfy  $V_B > 1, I = \bar{I}$ ; regions which satisfy  $V_B = 1$  and involve immediate jumps down, either to the locus  $B = \Psi(A)$  or all the way to the boundary of the region given  $A$ . Note that the economy cannot jump across a region with  $V_B > 1$ : the value thus obtained cannot be consistent with (OA5), as the posited value at the starting point exceeds the value at the target point plus the difference in  $B$ . Note that we have not ruled out the possibility that there might be loci other than  $B = \Psi(A)$  at which the value function is not differentiable, and along which the economy might evolve gradually.

**Step 4a:** Jumps must end at  $B \leq \Psi(A)$ .

Suppose there exists a locus  $B = \psi(A)$  (at least in the neighbourhood of some  $A$ ) along which it is optimal to allow the economy to evolve gradually, satisfying  $\rho + \delta + H(\psi(A)) > H'(\psi(A))[W - V^D]$ , implying  $\psi(A) > \Psi(A)$  (as the LHS is increasing, and the RHS decreasing, in  $\psi(A)$ ).

As, by assumption, it is optimal to allow  $B$  to evolve along the locus, the value can be stated as a function of  $A$  only; we denote it  $V^\psi(A)$ . The value satisfies

$$\begin{aligned} \rho V^\psi(A) &= \max_{F, R} U(F, R) - dF - \beta R - D(A) - \delta \psi(A) - \psi'(A)\dot{A} \\ &\quad + V_A^\psi(A)\dot{A} + H(\psi(A))(W(A) - V^\psi(A)), \end{aligned}$$

where the last two terms on the top line refer to the investment cost along the locus. The first-order conditions are

$$(OA6) \quad \begin{aligned} U_F(F^\psi, R^\psi) &= d - (V_A^{\Psi_i}(A) - \psi'(A)), \\ U_R(F^\psi, R^\psi) &= \beta. \end{aligned}$$

Now construct a deviation from the optimal plan as follows. Convert  $\Delta B$  units of the knowledge stock into an immediate lump of consumption. Thereafter set energy use according to (OA6) and invest at rate  $\bar{I}$  until the economy reaches the locus  $B = \psi(A)$  again. This occurs at date  $\Delta t$  (we normalise the initial date to  $t=0$ ), determined by

$$(OA7) \quad \begin{aligned} B^D(\Delta t) &= B_0 - \Delta B + \int_0^{\Delta t} \bar{I} - \delta B^D(t) dt = \psi(A(\Delta t)), \\ A^D(\Delta t) &= A_0 + \int_0^{\Delta t} F^\psi - \gamma A^D(t) dt, \end{aligned}$$

where the superscripts on the state variables indicate that we refer to the paths under the deviation. Note that the first line actually contains two equations, allowing us to pin down the unknowns  $A^D(\Delta t)$ ,  $B^D(\Delta t)$  and  $\Delta t$ . As the evolution of  $A(t)$  is identical under the supposed equilibrium strategy and the deviation strategy, clearly  $\lim_{t \rightarrow \Delta t} A^D(t) = A(\Delta t)$ ,  $\lim_{t \rightarrow \Delta t} B^D(t) = \psi(A(\Delta t))$ , and the deviation value  $V^D(A, B)$  satisfies

$$(OA8) \quad \lim_{t \rightarrow \Delta t} V^D(A^D(t), B^D(t)) = V^\psi(A(\Delta t)).$$

Along the deviation path, the deviation value satisfies

$$\begin{aligned} V^D(A^D(t), B^D(t)) &= \int_t^{\Delta t} e^{-\rho(s-t)} \left( U(F^\psi, R^\psi) - dF^\psi - \beta R^\psi - \bar{I} - D(A^D) + H(B^D)(W - V^D) \right) ds \\ &\quad + e^{-\rho(\Delta t-t)} V^\psi(A(\Delta t)), \end{aligned}$$

and this holds, in particular, for  $A^D(0) = A(0)$ ,  $B^D(0) = B(0) - \Delta B$ . We can thus calculate  $\Delta V \equiv V^D(A(0), B(0)) - V^\psi(A(0))$ :

$$\begin{aligned} \Delta V &= V^D(A(0), B(0) - \Delta B) + \Delta B - V^\psi(A(0)) \\ &= \Delta B + \int_0^{\Delta t} e^{-\rho t} \left( -\bar{I} + \delta \psi(A) + \psi'(A) \dot{A} \right. \\ &\quad \left. + H(B^D) [W(A^D) - V^D(A^D, B^D)] - H(B) [W(A) - V^\psi(A, B)] \right) dt \end{aligned}$$

where we have cancelled terms due to the fact that the deviation strategy satisfies (OA6).

Using the Leibniz integral rule, we can now calculate  $d\Delta V / d\Delta B$ :

$$\begin{aligned} \frac{d\Delta V}{d\Delta B} = & 1 + \int_0^{\Delta t} e^{-\rho t} \left( H'(B^D) [W - V^D] - H(B^D) V_B^D \right) \frac{dB^D(t)}{d\Delta B} dt \\ & + e^{-\rho \Delta t} \left( -\bar{I} + \delta B(\Delta t) + \psi'(A(\Delta t)) \dot{A}(\Delta t) \right) \frac{d\Delta t}{d\Delta B}, \end{aligned}$$

where by (OA6) the energy use terms are independent of  $B^D(t)$ , and because of (OA8) the value terms have cancelled out on the second line.

We now want to determine the terms  $dB^D(t) / d\Delta B$  and  $d\Delta t / d\Delta B$  to consider whether an increase in magnitude of the jump  $\Delta B$  might increase value. Totally differentiating the second line of (OA7) yields

$$\frac{dA^D(\Delta t)}{d\Delta t} = F - \gamma A^D(\Delta t) = \dot{A}(\Delta t).$$

Integrating the equation of motion for  $B$ , we obtain

$$B^D(t) = e^{-\delta t} (B(0) - \Delta B) + \int_0^t e^{-\delta(t-s)} \bar{I} ds,$$

so that  $dB^D(t) / d\Delta B = -e^{-\delta t}$  and  $dB^D(t) / dt = \bar{I} + \delta B^D(t) = \dot{B}^D(t)$ . Now, implicitly differentiating  $B^D(\Delta t) = \psi(A(\Delta t))$  yields

$$\frac{d\Delta t}{d\Delta B} = - \frac{-e^{-\delta \Delta t}}{-\psi'(A(\Delta t)) \frac{dA(\Delta t)}{d\Delta t} + (\bar{I} - \delta B^D(\Delta t))} = \frac{e^{-\delta \Delta t}}{\dot{B}(\Delta t) - \psi'(A(\Delta t)) \dot{A}(\Delta t)}.$$

Hence,

$$\begin{aligned} \frac{d\Delta V}{d\Delta B} = & 1 - \int_0^{\Delta t} e^{-(\rho+\delta)t} \left( H'(B^D) [W - V^D] - H(B^D) V_B^D \right) dt - e^{-(\rho+\delta)\Delta t} \\ & \approx \Delta t \left( \frac{1 - e^{-(\rho+\delta)\Delta t}}{\Delta t} - H'(B^D) [W - V^D] + H(B^D) \right) \\ & \rightarrow \lim_{\Delta t \rightarrow 0} \Delta t \left( \rho + \delta + H(\psi(A)) - H'(\psi(A)) [W - V^D] \right), \end{aligned}$$

where the approximation holds for small  $\Delta B$  (and thus small  $\Delta t$ ), and the limit is taken as  $\Delta B \rightarrow 0$ . Clearly if (A4) holds, the term in the brackets is strictly positive. We can then find a small enough deviation  $\Delta B'$  associated with a  $\Delta t$  such that  $d\Delta V / d\Delta B > 0$  for all

$\Delta B \in (0, \Delta B')$ , so that deviating non-marginally to  $\Delta B$  yields strictly higher welfare than the posited optimal path along  $B = \psi(A)$ , and thus the path cannot be an equilibrium.

**Step 4b:** Jumps must end at  $B \geq \Psi(A)$ .)

Suppose instead that the candidate target locus  $B = \psi(A)$  satisfies  $\rho + \delta + H(\psi(A)) < H'(\psi(A))[W - V^D]$ , or  $\psi(A) < \Psi(A)$ . Consider a point  $(A(0), B(0)) = (A_0, \psi(A_0) + \Delta B)$  with  $\Delta B > 0$  and a deviation such that, instead of jumping immediately, the planner sets  $F^D(t) = F^\psi(A(t))$  and steers the economy along  $(A^\psi(t), \psi(A^\psi(t)) + \Delta B)$  for  $t \in [0, \Delta t]$ . At  $t = \Delta t$ , the economy jumps to the locus  $B = \psi(A)$ . The value for this deviation is

$$(OA9) \quad V^D(A^D(t), B^D(t)) = \int_t^{\Delta t} e^{-\rho(s-t)} \left( U(F^\psi, R^\psi) - dF^\psi - \beta R^\psi - D(A^D) + H(B^D)(W - V^D) - \delta(B^\psi + \Delta B) - \psi'(A^D)\dot{A}^D \right) ds + e^{-\rho(\Delta t-t)} (V^\psi(A(\Delta t)) + \Delta B).$$

Again denoting the benefit of deviating by  $\Delta V \equiv V^D(A(0), B(0)) - (V^\psi(A(0)) + \Delta B)$ ,

$$\begin{aligned} \Delta V &= \int_t^{\Delta t} e^{-\rho(s-t)} \left( H(B^D)(W(A) - V^D(A^D, B^D)) - H(B^\psi)(W(A) - V^\psi(A^\psi, B^\psi)) \right. \\ &\quad \left. - \delta \Delta B \right) ds + e^{-\rho(\Delta t-t)} \Delta B - \Delta B \\ &= \int_t^{\Delta t} e^{-\rho(s-t)} \left( H(B^\psi + \Delta B)(W(A) - V^D(A^\psi, B^\psi + \Delta B)) - H(B^\psi)(W(A) - V^\psi(A^\psi, B^\psi)) \right. \\ &\quad \left. - \delta \Delta B \right) ds + e^{-\rho(\Delta t-t)} \Delta B - \Delta B. \end{aligned}$$

where we have cancelled terms as the paths of energy use and the carbon stock  $A$  are identical on the two paths. We can take the derivative of this with respect to  $\Delta B$ :

$$\begin{aligned} \frac{d\Delta V}{d\Delta B} &= \int_t^{\Delta t} e^{-\rho(s-t)} \left( H'(B^\psi + \Delta B)(W(A) - V^D(A^\psi, B^\psi + \Delta B)) \right. \\ &\quad \left. - H(B^\psi)V_B^D(A^\psi, B^\psi + \Delta B) - \delta \right) ds + e^{-\rho(\Delta t-t)} - 1. \end{aligned}$$

Taking the limit as  $\Delta B \rightarrow 0$ ,

$$\begin{aligned} \lim_{\Delta B \rightarrow 0} \frac{d\Delta V}{d\Delta B} &= \int_t^{\Delta t} e^{-\rho(s-t)} \left( H'(B^\psi)(W(A) - V^D(A^\psi, B^\psi)) \right. \\ &\quad \left. - H(B^\psi)V_B^D(A^\psi, B^\psi) - \delta \right) ds - (1 - e^{-\rho(\Delta t-t)}) \\ &\approx \Delta t \left( H'(B^\psi)(W(A) - V^D(A^\psi, B^\psi)) - H(B^\psi)V_B^D(A^\psi, B^\psi) - (\delta + \rho) \right), \end{aligned}$$

where the approximation holds as  $\Delta t$  becomes very small. From (OA9) we observe that, for small  $\Delta t$ ,  $V_B^D \approx 1$ . Thus, the deviation has a positive value for some small  $\Delta t$  as the term in the brackets goes to  $H'(\psi(A)) [W - V^D] - [\rho + \delta + H(\psi(A))] > 0$ , so that the purported optimal path  $B = \psi(A)$  in fact allows a profitable deviation.

**Step 5:**  $V_B > 1$  for  $B < \Psi(A)$ ,  $V_B = 1$  for  $B > \Psi(A)$ .

The above arguments imply any jumps must end up exactly on  $B = \Psi(A)$ . As regions satisfying  $V_B = 1$  require an immediate jump, and as the economy cannot jump up, we have that  $V_B > 1$  for  $B < \Psi(A)$ . Now suppose there is some region with  $B > \Psi(A)$  satisfying  $V_B > 1$ . This region must extend all the way to the natural upper bound on  $B$ , given by  $\bar{B} = \bar{I} / \delta$ , as otherwise the economy would jump from the region above this to the locus separating the two, which lies (by supposition) above  $B = \Psi(A)$ ; this cannot be optimal. But the deviation argument above (step 4a) holds for paths arbitrarily close to  $B = \bar{B}$ , implying driving the economy (asymptotically) to maximum possible  $B$  cannot be optimal as long as this is sufficiently large (i.e.  $\bar{I}$  is sufficiently large). Thus, for  $B > \Psi(A)$  we must have  $V_B = 1$ .

We have thus shown that the economy follows a most rapid approach path to the locus given by (14).

**Proof of Lemma 2:** We work in a discrete-time set-up, to be able to more precisely deal with ‘immediate’ responses to any deviations, in particular when the deviation and/or response may involve a jump in the state variable. With a period length  $\Delta t$ , the pre-breakthrough Bellman equation for country  $i$  is

$$\begin{aligned} V_i^N(A(t), \underline{B}(t)) = \max_{F_i, R_i, B_i} \int_0^{\Delta t} e^{-\rho t} N^{-1} (U(F_i, R_i) - dF_i - \beta R_i - I_i - D(A)) dt \\ + e^{-\rho \Delta t} \left( [1 - H(B(t)) \Delta t] V_i^N(A(t + \Delta t), B(t + \Delta t)) \right. \\ \left. + [H(B(t)) \Delta t] W_i^N(A(t + \Delta t), B(t + \Delta t)) \right). \end{aligned}$$

The equations of motion are  $A(t + \Delta t) = A(t) + \Delta t \left( \sum_j [N^{-1} F_j] - \gamma A(t) \right)$ ,  $\underline{B}(t + \Delta t) = \underline{B}(t) + \Delta t (N^{-1} \underline{I} - \delta \underline{B}(t))$ . Using standard manipulations (see Dixit and Pindyck, 1994), the continuous-time HJB equation (28) is the limit of this as  $\Delta t \rightarrow 0$ . The post-

breakthrough Bellman equation is written down and analysed similarly, but of course omitting the state  $\underline{B}$ , the control  $\underline{I}$  and the breakthrough payoffs:

$$W_i^N(A(t)) = \max_{F_i, R_i} \int_0^{\Delta t} e^{-\rho t} N^{-1} (U(F_i, R_i) - dF_i - \beta R_i - D(A)) dt + e^{-\rho \Delta t} W_i^N(A(t + \Delta t)).$$

Given the purported strategies after tipping (in particular that optimal energy is independent of the level of emissions), the FOCs after tipping are  $U_{F_i} = d - W_{iA}(A)$ ,  $U_{R_i} = \tilde{\beta}$ , which are easily shown (following the analysis in section 2.1) to yield  $\tau^N \equiv -W_{iA} = N^{-1} \alpha / (\rho + \gamma)$ . Given the purported equilibrium strategies, the pre-breakthrough stage also involves  $\tau = \tau^N$ , confirming the strategies on energy use are consistent with a Nash equilibrium.

We now proceed to show that there is no profitable deviation in terms of investment. We consider a sequence of games, indexed by  $\Delta t$ , and are only interested in the tail  $\Delta t \rightarrow 0$ : we want to show that, as long as  $\Delta t$  is sufficiently small, the proposed investment strategies form a Nash equilibrium. In discrete time, we only need to consider one-shot deviations.

In terms of investment, the equilibrium path will feature all countries having identical R&D stocks, building these jointly at the maximal rate, up until  $B^{N,i}$ . We will find useful the expression  $V_{i,t}^{\text{SYM}}(b) \equiv V_i^N(A, b \underline{1})$ , with  $\underline{1}$  the unit vector, which gives the value to an individual country when all countries hold the stock level  $b$ . Denote by  $T$  the period at which  $b$  reaches  $B^{N,1}$ . Then in the previous period  $I_{i,T-\Delta t} = N(B^{N,i} - B_{i,T-\Delta t}) / \Delta t + \delta N B_{i,T-\Delta t}$ , and using this, we obtain  $V_{i,T-\Delta t}^{\text{SYM}}(b) \approx 1 + \Delta t(-[\rho / 2 + \delta]) + h_1[W_T - V_{i,T}^{\text{SYM}}(B^{N,1})]$ , which is greater than unity for any small but finite  $\Delta t$ . For some earlier period on the symmetric investment path  $I_{i,T-m\Delta t} = \bar{I}$ , and

$$\begin{aligned} \text{(OA10)} \quad V_{i,T-m\Delta t}^{\text{SYM}}(b_{T-m\Delta t}) &\approx V_{i,T-(m-1)\Delta t}^{\text{SYM}}(b_{T-(m-1)\Delta t}) [1 - \Delta t(\rho + \delta + H(B_{T-m\Delta t}))] \\ &\quad + \Delta t [h_1(W_{T-m\Delta t} - V_{i,T-m\Delta t}^{\text{SYM}})]. \end{aligned}$$

The definition of  $B^{N,i}$  ensures that that  $h_1(W_i - V_i) - (\rho + \delta + H(B)) > 0$  for  $B < B^{N,i}$ , as can be confirmed using the approach taken in Proposition 1. By recursive substitution of (OA10), we ascertain that, in the linear approximation,  $V_{i,T-m\Delta t}^{\text{SYM}}(b_{T-m\Delta t}) > 1$  for any finite  $\Delta t$ .

Suppose first the economy is on the symmetric equilibrium path with  $B_j = B_k, \forall j, k$ . Further, suppose that  $t < T - \Delta t$ , so that the equilibrium strategies call for maximal investment. Consider a downward deviation by country  $i$  of setting  $I_i^D < \bar{I}$  (which is the only possible deviation). As

the evolution of  $A$  is independent of the investment path, we can ignore all terms which either depend on  $A$  or are constant. Thus dropping these, and denoting time with subscripts to reduce clutter, we write the deviation value as

$$\begin{aligned} V_i^D(A_t, \underline{B}_t) &= \int_0^{\Delta t} e^{-\rho t} N^{-1} (-I_i^D) dt \\ &\quad + e^{-\rho \Delta t} \left( [H(B_t) \Delta t] W_i(A_{t+\Delta t}) + [1 - H(B_t) \Delta t] V_i(A_{t+\Delta t}, \underline{B}_{t+\Delta t}^D) \right), \\ V_i(A_{t+\Delta t}, \underline{B}_{t+\Delta t}^D) &= \int_0^{\Delta t} e^{-\rho t} N^{-1} (-\delta B_{i,t+\Delta t}^D) dt \\ &\quad + e^{-\rho \Delta t} \left( [H(B_{t+\Delta t}^D) \Delta t] W_i(A_{t+2\Delta t}) + [1 - H(B_{t+\Delta t}^D) \Delta t] V_i(A_{t+2\Delta t}, \underline{B}_{t+2\Delta t}^D) \right), \end{aligned}$$

where  $\underline{B}_{t+\Delta t}^D$  is the vector which has all elements equal to  $B_{i,t+\Delta t}^D = (1 - \delta \Delta t) B_{i,t} + I_i^D \Delta t$ , as all countries follow the deviation by disinvestment, setting their R&D stocks equal to the deviating country's R&D stock.

Similarly, not deviating yields value

$$\begin{aligned} V_i(A_t, \underline{B}_t) &= \int_0^{\Delta t} e^{-\rho t} N^{-1} (-\bar{I}) dt \\ &\quad + e^{-\rho \Delta t} \left( [H(B_t) \Delta t] W_i(A_{t+\Delta t}) + [1 - H(B_t) \Delta t] V_i(A_{t+\Delta t}, \underline{B}_{t+\Delta t}) \right), \\ V_i(A_{t+\Delta t}, \underline{B}_{t+\Delta t}) &= \int_0^{\Delta t} e^{-\rho t} N^{-1} (-\bar{I}) dt \\ &\quad + e^{-\rho \Delta t} \left( [H(B_{t+\Delta t}) \Delta t] W_i(A_{t+2\Delta t}) + [1 - H(B_{t+\Delta t}) \Delta t] V_i(A_{t+2\Delta t}, \underline{B}_{t+2\Delta t}) \right). \end{aligned}$$

Using these, we can calculate the benefit to country  $i$  of deviating:

$$\begin{aligned} V_{i,t}^D - V_{i,t} &= \int_0^{\Delta t} e^{-\rho t} N^{-1} (\bar{I} - I_i^D) dt + e^{-\rho \Delta t} \left( 1 - [H(B_t) \Delta t] \right) \left\{ \int_0^{\Delta t} e^{-\rho t} N^{-1} (\bar{I} - \delta B_{i,t+\Delta t}^D) dt \right. \\ &\quad + e^{-\rho \Delta t} \left[ (H(B_{t+\Delta t}^D) - H(B_{t+\Delta t})) \Delta t W_{i,t+2\Delta t} + (1 - [H(B_{t+\Delta t}^D) \Delta t]) V_{t+2\Delta t}^D \right. \\ &\quad \left. \left. - (1 - [H(B_{t+\Delta t}) \Delta t]) V_{t+2\Delta t} \right] \right\} \\ &= \Delta t \left\{ \frac{1 - e^{-\rho \Delta t}}{\rho \Delta t} \frac{\bar{I} - I_i^D}{N} + e^{-\rho \Delta t} (1 - [H(B_t) \Delta t]) \left\{ \frac{1 - e^{-\rho \Delta t}}{\rho \Delta t} \frac{\bar{I} - \delta B_{i,t+\Delta t}^D}{N} \right. \right. \\ &\quad + e^{-\rho \Delta t} \left[ h_1 \frac{I_i^D - \bar{I}}{N} \Delta t W_{i,t+2\Delta t} + (1 - [H(B_{t+\Delta t}^D) \Delta t]) \frac{V_{i,t+2\Delta t}^D - V_{i,t+2\Delta t}}{\Delta t} \right. \\ &\quad \left. \left. \left. + h_1 \frac{\bar{I} - I_i^D}{N} \Delta t V_{i,t+2\Delta t} \right] \right\} \right\}, \end{aligned}$$

where we omit the arguments of the value functions to reduce clutter, using the superscript ‘D’ to denote the value being evaluated along the deviation path and denoting time by subscripts. We have also used the linearity of the hazard function and the fact that the post-tip value is the same under both paths, as the deviation does not affect the carbon stock  $A$ .

We can now consider the profitability of deviations as  $\Delta t \rightarrow 0$ . For a downward deviation,  $\lim_{\Delta t \rightarrow 0} V_{i,t}^D - V_{i,t} = V_{i,t+2\Delta t}^D - V_{i,t+2\Delta t} < 0$ , as  $B_{k,t+2\Delta t}^D < B_{k,t+2\Delta t}$  for any finite  $\Delta t$ . This holds even for an infinitesimal deviation, as the deviation causes all countries to take a one-period break from investment.

Consider next a deviation by player  $i$  downward from the symmetric steady state. For  $I_{i,t}^D < \delta B^{N,i}$ , the payoff from deviating is

$$\begin{aligned} V_{i,t}^D = & \int_0^{\Delta t} e^{-\rho t} \left( -\frac{I_{i,t}^D}{N} \right) dt + e^{-\rho \Delta t} \left( H(B^N) \Delta t W_{t+\Delta t} + [1 - H(B^N) \Delta t] \left\{ \int_0^{\Delta t} e^{-\rho t} (-\delta B_{i,t+\Delta t}^D) dt \right. \right. \\ & + e^{-\rho \Delta t} \left( H(B_{t+\Delta t}^D) \Delta t W_{t+2\Delta t} + [1 - H(B_{t+\Delta t}^D) \Delta t] \left\{ \int_0^{\Delta t} e^{-\rho t} \left( -\frac{I_{i,t+2\Delta t}^D}{N} \right) dt \right. \right. \\ & \left. \left. \left. + \left( H(B_{t+2\Delta t}^D) \Delta t W_{t+3\Delta t} + [1 - H(B_{t+2\Delta t}^D) \Delta t] V_{i,t+3\Delta t}^D \right) \right\} \right\} \right). \end{aligned}$$

The expected value given a deviation by country  $i$  in period  $t$  takes into account that the deviation is followed by  $i$  staying put in period  $t + \Delta t$  with the other countries mimicking the deviation by setting  $I_{-i,t+\Delta t}^D = I_{i,t}^D$ , then all countries investing sufficiently in period  $t + 2\Delta t$  to return to the steady state, so that  $V_{j,t+3\Delta t} = V_j^{\text{SYM}}(B^{N,i})$ ,  $\forall j$ . Thus we calculate  $B_{i,t+\Delta t}^D = B_{j,t+2\Delta t}^D = (1 - \delta \Delta t) B^{N,i} + I_{i,t}^D \Delta t$ , and we can similarly obtain the required  $I_{i,t+2\Delta t}^D$ . We write down the value of following the equilibrium strategy instead and take the difference, and after some Taylor expansions we get

$$V_{i,t}^D - V_{i,t} \approx (\Delta t)^2 \frac{\delta N B^{N,i} - I_{i,t}^D}{N} \left( -(N-1) h_1 [W_t - V_{i,t}] \right) < 0.$$

The cost of the deviation by  $i$  is that other countries follow suit. At  $B^{N,i}$ , the marginal effect of country  $i$ 's own stock on the breakthrough probability is just equal to the associated saving on the user cost of capital, but the reaction by other countries is not similarly offset. Thus, even a small deviation is never profitable. This holds *a fortiori* for larger deviations.

An upward deviation at the symmetric steady state  $B^{N,i}$  is dealt with similarly, although we now need to take second-order Taylor expansions to obtain that the cost of deviation is proportional to  $(\Delta t)^3$  as the other countries do not follow suit in deviating. For a deviation  $I_{i,t}^D > \delta B^{N,i}$ , the resulting benefit is

$$V_{i,t}^D - V_{i,t} \approx (\Delta t)^3 \frac{\delta N B^{N,i} - I_{i,t}^D}{N} \left( (\rho + H(B^{N,i}))^2 + \rho(\delta - H(B^{N,i})) / 2 \right) < 0.$$

This also yields the benefit of deviating upwards when within one period of reaching  $B^{N,i}$ , as the linearity of investment cost means that such a deviation can be viewed as a sum of the step to  $B^{N,i}$  (the costs of which cancel out) and the deviation to above  $B^{N,i}$ .

Next, consider a deviation given an asymmetric distribution of stocks. A deviation would mean some player not equalising stocks downward, or the player with minimal stock changing her stock (instead of keeping it constant). We again follow the same approach to obtain the benefit of the former deviation. Denoting  $B^{\min} = \min_k B_k$ , this is proportional, in the limit, to

$$(\Delta t)^2 \left( h_1 [B_i - B^{\min}] [W_i(A) - V_i(A, B^{\min})] + \bar{I} - \delta B_{i,t} - \frac{d\Omega(B^{\min} \underline{1})}{db} [\bar{I} - \delta B^{\min}] \right).$$

Note that the derivative is taken at the symmetric allocation where all countries have jumped to  $B^{\min}$ , symmetrically raising all countries' R&D stocks (with  $\underline{1}$  denoting the unit vector). This must be positive for  $B_i = N B^{N,i}$ ,  $B_{-i} = 0$  for a deviation (not equalising downwards) to never be profitable, and condition (30) ensures this.

The deviation by the country with the minimal stock cannot be optimal for the same reason a deviation by a player off the symmetric investment path is not optimal: it will at best delay coordination, and the continuation of joint investment, by an additional period; and, if downward, it will also cause the other players to reduce their stock further.

## References

- Polasky, S., A. de Zeeuw and F. Wagener (2011). Optimal management with potential regime shifts, *Journal of Environmental Economics and Management*, 62, 2, 229-240.
- Polasky, S., N. Tarui, G.M. Ellis, and C.F. Mason (2006): Cooperation in the commons, *Economic Theory*, 29, 71-88.