CHANGES IN MATHEMATICAL CULTURE FOR POST-COMPULSORY MATHEMATICS STUDENTS

The Roles of Questions & Approaches to Learning



Eleanor Darlington

Kellogg College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Trinity Term 2013

Acknowledgements

Immeasurable thanks to Anne Watson for the opportunity to come to Oxford to write this thesis, for fostering my thinking and development as a researcher, for showing me the benefits of working barefoot, for finding my confidence and making me hold onto it… and for teaching me long division. It has been a wonderful experience and I hope that this DPhil gives you a good send-off into a well-earned 'retirement'.

Thank you to the ESRC for the financial support to undertake this project.

In the Mathematical Institute, special thanks to Charlotte Turner-Smith, the teaching and administrative staff for helping me to conduct my research, as well as the hundreds of undergraduate mathematicians who took part in the questionnaires and interviews.

I am very grateful to Sonya Milanova who, as well as being a wonderful team mate and friend, proof-read much of this thesis.

Whilst this PhD concerns itself with the enculturation of new undergraduate mathematicians, *my* enculturation process and the communities of practice which I have become part of throughout the last four years have unwillingly, unwittingly or unknowingly contributed towards the thinking, composure and sense of urgency which was required to write this thesis. Special thanks to:

Kellogg College's students and fellows, especially Sarah Gauntlett and Jonathan Michie, Ana Nacvalovaite and James Chanter.

The 'strong independent women' of my Torpids and Eights, Sandra Kotzor, Laurence Birdsey, the Very Reverend David Border, Ian Maconnachie, Sahil Sinha, Helen Popescu, Jarms, Liz Jamie, Karl Offord and Susana Hancock.

The students of N1a Mathematics Education, John Mason, Andy Ragatz and Gabriel Stylianides.

Osiris, especially my heroes Sophie Shawdon and Karolina Chocian.

The International Commission on Mathematical Instruction and the University of Oxford Disability Advisory Service.

Beccy Preece, Lotti Trigle, Annika Bruger, Dieuwertje Kooij, Becky Pawley, Flo Morton, Caitlin Goss, Alex Dix, Mary Foord-Weston and Mike Genchi for a truly life-changing year with some truly remarkable people.

Dan Moulin, Mairéad McKendry and Steve Puttick.

Ellen Border, Judy Gleen, Mary Foord-Weston and Caitlin Goss for more love, understanding, support and time than I probably deserved. Thank goodness I'm such a ray of sunshine.

Abstract

Since there are insufficient mathematicians to meet economic and educational demands and many well-qualified, successful mathematics students exhibit signs of disaffection, the student experience of undergraduate mathematics is high on the political agenda. Many undergraduates struggle with the school-university transition, which has been associated with students' prior experiences of mathematics which, at A-level, are regularly criticised for being too easy and too different to undergraduate mathematics. Furthermore, the University of Oxford administers a Mathematics Admissions Test (OxMAT) as a means of identifying those best prepared beyond the limited demands of A-level.

Consequently, a study was conducted into the mathematical enculturation of Oxford undergraduates, specifically in terms of examination questions and students' approaches to learning. Analysis of the Approaches and Study Skills Inventory for Students (ASSIST) (Tait *et al.*, 1998) revealed the majority of students to adopt strategic approaches to learning (ATLs) in all four year-groups, though the descriptions given by students in interviews of the nature of their ATL highlighted some shortcomings of the ASSIST as the motivation for memorisation appeared to be an important factor. The MATH taxonomy (Smith *et al.*, 1996), revealed that most A-level questions require routine use of procedures, whereas the OxMAT tested a variety of skills from applying familiar mathematics in new situations to justifying and interpreting information to form proofs. This is more in-line with the requirements of undergraduate assessment, although the MATH taxonomy and student interviews revealed that these still allowed for rote memorisation and strategic methods. Thus, the changing nature of mathematics and questions posed to students at the secondary-tertiary interface appears to affect students' ATLs, though this is not reflected by the ASSIST data.

Contents

Chapter 1: The State of Progression from A-Level to University Mathematics Study

1.1 – Rationale 13

1.2 – Examination of the Literature 21

1.2.1 – Critique 23

1.2.2 – Transmaths 24

1.2.3 – Key Research Questions 25

Chapter 2: The Nature of Post-Compulsory Mathematics Questions & Students' Responses

2.1 – Approaches to Learning 27

2.1.1 – Deep & Surface Approaches 27

2.1.2 – Correlates with ATLs 29

2.1.2.1 – Attainment 29

2.1.2.2 – Personality 30

2.1.2.3 – Teaching 30

2.1.2.4 – Assessment 31

2.1.3 – Alternative Suggestions 32

2.1.3.1 – Terminology 32

2.1.3.2 – Meaningful Learning vs. Rote Memorisation 33

2.1.3.3 – Holist vs. Serialist 33

2.1.3.4 – Generative vs. Reproductive Processing 34

2.1.3.5 – Deep- vs. Surface-Level Processing 35

2.1.3.6 – Instrumental vs. Relational Understanding 35

2.1.3.7 – Holistic vs. Atomistic Cognitive Approaches 36

2.1.3.8 – Utilising vs. Internalising vs. Achieving 37

2.1.3.9 – Transformational vs. Reproductive Learning 37

2.1.4 – Strategic Approach to Learning 37

2.1.5 – Limitations 38

2.2 – Question Analysis 44

2.2.1 – Routine & Non-Routine Questions 44

2.2.2 – Taxonomies 47

2.2.3 – MATH Taxonomy 49

2.2.3.1 – Categories 50

2.2.3.2 – Uses 52

2.2.3.3 – Use in Empirical Research 53

2.2.3.4 – Example Questions 54
2.2.3.5 – Limitations 56
2.2.4 – Alternative Suggestions 58
2.2.4.1 – Bloom's Taxonomy 59
2.2.4.2 – Galbraith & Haines 62
2.2.4.3 – Assessment Component Taxonomy 65
2.2.4.4 – SOLO Taxonomy 67
2.3 – Secondary-Tertiary Mathematics Transition 70
2.3.1. – The Nature of Mathematics 70
2.3.1.1 – Advanced Mathematics 70
2.3.1.2 – The Undergraduate Curriculum 73
2.3.2 – Conceptions of Mathematics 75
2.3.3 – A-Level Criticism 76
2.3.3.1 – A-Level Administration 77
2.3.3.2 – Participation 77
2.3.3.3 – Examination Boards 79
2.3.3.4 – Reforms 79
2.3.3.5 – Standards 80
2.3.3.6 – Criticism 82
2.3.3.7 – Relationship with Universities 95
2.3.4 – Pedagogy 96
2.3.4.1 – Students' Expectations 96
2.3.4.2 – Didactic Contract 97
2.3.4.3 – Understanding 99
2.3.4.4 – Approaches to Learning 100
2.3.4.5 – Renegotiating the Didactic Contract 101
2.3.4.6 – Constructivism 103
2.3.4.7 – A 'Perfect Pedagogy'? 104
2.3.4.8 – Social & Sociomathematical Norms 105
2.3.4.9 – Pedagogical Re-Engineering 107

Chapter 3: Enculturation into the Undergraduate Mathematics Community

3.1 – Communities of Practice 110
3.1.1 – Roots of the Concept in Social Theory 110
3.1.2 – Legitimate Peripheral Participation 111

3.1.3 – Communities of Practice at the Secondary-Tertiary Interface 112
3.1.4 – Problems within Communities of Practice 113
3.1.5 – Communities of Practice in the Context of Undergraduate Mathematics Learning. 116
3.2 – Self-Efficacy & Self-Concept 117
3.2.1 – Impact of Undergraduate Study 117
3.2.2 – Big Fish Little Pond 120
3.2.3 – Relationship with Communities of Practice 122
3.2.4 – Gender 122

Chapter 4: Methodology

4.1 – Overview of Methods Employed 126
4.2 – Learning Mathematics as a Sociocultural Experience 132
4.3 – Mixed Methods 137
4.4 – Student Interviews 144
4.4.1 – Description 144
4.4.2 – Justification 144
4.4.3 – Procedure & Sampling 146
4.4.4 – Strengths & Limitations 152
4.4.5 – Analysis 156
4.4.5.1 – Data Organisation 157
4.4.5.2 – Saxe's Four Parameter Model 158
4.5 – ASSIST 163
4.5.1 – Description 163
4.5.2 – Justification 165
4.5.3 – Procedure & Sampling 166
4.5.4 – Strengths & Limitations 166
4.5.4.1 – Sample 166
4.5.4.2 – Reliability 169
4.5.4.3 – Validity 169
4.6 – Mathematical Assessment Test Hierarchy 171
4.6.1 – Description 171
4.6.2 – Justification 173
4.6.4 – Strengths & Limitations 174
4.6.4.1 – Validity 174
4.6.4.2 – Reliability 175

4.6.5 – Analysis 175
4.7– General Study Strengths & Limitations 176
4.7.1 – Oxford 176
4.7.1.1 – Case Study 177
4.7.1.2 – 'Insider' Research 178
4.7.2 - Self Report 179
4.8 – Ethics 182

Chapter 5: Student Approaches to Learning throughout Undergraduate Study at the University of Oxford

5.1 – Factor Analysis 183
5.2 – Descriptive Statistics 184
5.3 – Approaches to Learning 185
5.3.1 – Sweep 1 185
5.3.2 – Sweep 2 187
5.3.2.1 – Likert Scales & Comparing Groups of Data 187
5.3.2.2 – Years 1-4 187
5.3.2.3 – Contrasting BA & MMath Years 188
5.3.2.4 – Summary 190
5.4 – First-Years' ATL 192
5.4.1 – Matching Students 192
5.4.2 – Comparing Sweeps 193
5.4.2.1 – Differences by ATL 194
5.4.2.2 – Difference by Subscale Score 195
5.4.3 – Summary 196
5.5 ATL & Year Group 197
5.5.1 – Differences by ATL 197
5.5.2 – Differences by Scale Score 199
5.5.3 – Summary 201
5.6 – ATL & Gender 203
5.6.1 – Differences by ATL 203
5.6.2 – Differences by Subscale Mean 204
5.6.2.1 – All Years 204
5.6.2.2 – Individual Year Groups 205
5.6.3 – Summary 206

5.7 – Individual ASSIST Items 208
5.7.1 – Year Group Differences 208
5.7.2 – Gender Differences 211
5.7.3 – General Responses 214
5.7.3.1 – Course Satisfaction 214
5.7.3.2 – Worries 215
5.7.3.3 – Success 216
5.7.3.4 – Memorisation 216
5.7.4 – Summary 217
5.8 – Conclusion 219

Chapter 6: Contrasts in Challenges Presented by A-Level Mathematics, the Oxford Admissions Test & First Year Undergraduate Examinations

6.1 – MATH Taxonomy 223
6.2 – A-Level Examinations 223
6.3 – University of Oxford Mathematics Admissions Test 227
6.4 – Undergraduate Examinations 229
6.5 – Observations 235
6.6 – Conclusion 240

Chapter 7: Student Reports of Mathematics Study at the University of Oxford

7.1 – Students' Stories 242
7.2 – Overview 246
7.3 – Prior Understandings 248
7.3.1 – Prior Understandings Fostered by School Study 249
7.3.2 – Prior Understandings Fostered by the Admissions Process 261
7.3.2.1 – Choosing to Apply 261
7.3.2.2 – The Admissions Test 261
7.3.2.3 – The Oxford Interview 268
7.3.3 – Summary 275
7.4 – Conventions & Artefacts 277
7.4.1 – Conventions 279
7.4.1.1 – Conventions of the Institution 279
7.4.1.2 – Conventions of the Student Body 284
7.4.2 – Artefacts 286
7.4.2.1 – Problem Sheets 286

7.4.2.2 – Examinations 287
7.4.3 – Summary 290
7.5 – Social Interactions 292
7.5.1 – Formal Social Interactions 293
7.5.2 – Informal Social Interactions 297
7.5.3 – Extra-Curricular Social Interactions 299
7.5.4 – Summary 301
7.6 – Activity Structures 302
7.6.1 – Activity Structures Involved in Completing Problem Sheets 302
7.6.2 – Activity Structures Involved in Preparing for Examinations 305
7.6.3 – Summary 311
7.7 – Perceptions of Self & Others 313
7.7.1 – Emotional Impact of Transition 314
7.7.2 – Perceptions of Ability 316
7.7.3 – Enjoyment of Undergraduate Mathematics 318
7.7.4 – Summary 320
7.8 – Conclusion 321

Chapter 8: Synthesis of Undergraduate Mathematicians' Experiences of their Course Relating to the Mathematics, the Assessment & the Community

8.1 – Summary of Study Aims 327
8.2 – Summary of Research Findings 328
8.3 – Confidence, Guilt & Despair: The Approaches to Learning Framework in the Context of Undergraduate Mathematics at Oxford 331
8.4 – Deep or Cheat: The Differing Role of Problem Sheets & Examinations 338
8.5 – Challenging but not Meeting Expectations: The Contribution of the Oxford Admissions Process to Students' Experiences 344
8.6 – Limitations 350
8.7 – Further Research 353

Chapter 9: Conclusion

..... 359

Bibliography 362

Appendices

2.1 – MATH Taxonomy 408
2.2 – Entry Requirements 413

2.3 – AQA C1 January 2006 415
2.4 – OCR FP3 June 2007 428
4.1 – ASSIST Questionnaire 442
4.2 – Request for ASSIST Participation 445
4.3 – ASSIST Official Information 446
4.4 – Electronic form of ASSIST 448
4.5 – Request for Interview Participation 450
4.6 – Interview Notesheet 451
4.7 – Interview Information & Consent Form 452
4.8 – Commonly-Asked Interview Questions 454
4.9 – University of Oxford Admissions Statistics 457
4.10 – MATH Taxonomy Questions 459
5.1 – Factor Analysis 461
5.2 – Comparing ATLs of BA Students with MMath Students 464
5.3 – Comparing Scale Scores of BA Students with MMath Students 465
5.4 – Year-Group Differences by ASSIST Item 466
5.5 – Comparing ATLs of First-Years in Sweep 1 & Sweep 2 469
5.7 – Comparing ATLs of Students Across Year-Groups 471
5.8 – Investigating Year Group Differences (Men Only) 472
5.9 – Investigating Year-Group Differences (Women Only) 473
5.10 – Comparing ATLs between Year Group Pairings 474
5.11 – Comparing Scale Scores between Year-Group Pairings 478
5.12 – Investigating Gender Differences (Sweep 1) 480
5.13 – Investigating Gender Differences in ATL (Sweep 2) 481
5.14 – Comparing ATLs between Genders in Each Year-Group 482
5.15 – Investigating Gender Differences in Scale Scores (Sweep 2) 484
5.16 – Differences in Individual Items between Year 1 & Year 2 485
5.17 – Differences in Individual Items by Gender (Sweep 2) 494
5.18 – Proportional Responses to Individual ASSIST Items 496
6.1 – AQA C1 January 2006 – Application of MATH Taxonomy 498
6.2 – Edexcel FP3 June 2006 – Application of MATH Taxonomy 503
6.3 – Oxford MAT 2007 – Application of MATH Taxonomy 509
6.4 – Oxford Pure Mathematics I 2008 – Application of MATH Taxonomy 518
6.5 – Oxford Pure Mathematics II 2011 – Application of MATH Taxonomy 523

7.1 – Brian's Story 530
7.2 – Camilla's Story 536
7.3 – Christina's Story 540
7.4 – Juliette's Story 543
7.5 – Mandy's Story 547
7.6 – Malcolm Interview Transcript 552
7.7 – Qualifications Offered for Entry 568
9.1 – Linear Algebra II Problem Sheet 569

List of Abbreviations

The following abbreviations are used throughout this thesis:

A2	The full A-level
ACME	Advisory Committee on Mathematics Education
AEA	Advanced Extension Award
AINS	Application in new situations
AQA	Assessment & Qualifications Alliance
AS	Advanced Subsidiary Level – the first year of A-level
ASI	Approaches to Studying Inventory
ASSIST	Approaches & Study Skills Inventory for Students
ATL	Approach to learning
COMP	Comprehension
COP	Community of practice
ESRC	Economic & Social Research Council
EVAL	Evaluation
FKFS	Factual knowledge & fact systems
FMSP	Further Mathematics Support Network
GCSE	General Certificate of Secondary Education
HEI	Higher education institution
ICC	Implications, conjectures & comparisons
HESA	Higher Education Statistics Agency
ICMI	International Commission on Mathematical Instruction
IT	Information transfer
J&I	Justifying & interpreting
JCQ	Joint Council for Qualifications
LMS	London Mathematical Society
MATH	Mathematical Assessment Task Hierarchy
OCR	Oxford, Cambridge & Rutland Examinations
OxMAT	Oxford Mathematics Admissions Test
PME	International Group for the Psychology of Mathematics Education
QCA	Qualifications & Curriculum Authority
RUOP	Routine use of procedures
SOLO	Structure of Observed Learning Outcome taxonomy
STEM	Science, technology, economics & mathematics
STEP	Sixth-Term Extension Paper
TIMSS	Trends in International Mathematics & Science Study
WJEC	Welsh Joint Education Committee

Chapter 1: Introduction

The State of Progression from A-Level to University Mathematics Study

Currently, approximately nine per cent of A-level Mathematics students go on to study the subject at university (ACME, 2012). A-level Mathematics serves two purposes – it acts both as a stand-alone qualification and a university preparation examination. This creates a challenge for examiners who must produce a syllabus and assessment which responds to the needs of both possible A-level Mathematics student (see Chapter 2.3.3). This results in a challenge for new undergraduate mathematicians in transitioning to tertiary study – not only do they have to adapt to a new autonomy in learning and general independence away from their family (Anderson *et al.*, 2000; Fisher & Hood, 1987; Kantanis, 2000; Peel, 2000), but they have to respond to a subject which, arguably, changes in its nature and form between secondary and tertiary level (see Chapter 2.3).

1.1 – Rationale

At present, there are many challenges and difficulties being experienced by educators and students alike in the field of undergraduate mathematics. Student challenges have been high up on the agenda for educational research and policymaking for years, right up until the birth of this thesis. This came to greater prominence in 1997 after the publication of the Dearing Report (National Committee of Inquiry into Higher Education, 1997) wherein recommendations concerning tuition fees, expansion of available courses and teaching were made.

Policies aimed at increasing participation in higher education – particularly in STEM[1] subjects – have inadvertently caused problems, as universities begin to find it harder to teach their students due to increased numbers (Baumslag, 2000). Students' academic backgrounds also continue to diversify both in terms of qualification type and standard of qualification (Kitchen, 1999), with increasing numbers of schools, colleges and sixth forms offering alternatives to the A-level to British students[2].

Porkess (2008) commented that over 100,000 students sat A-level Mathematics each year in the early 1980s, falling to little more than half this number in 2008. The number of students studying the subject at this level has often been a matter of concern (Hoyles *et al.*, 2001), although this trend has changed in recent years (see Figure 1.1). However, uptake of A-level Mathematics has been found to be relatively low when making international comparisons (Hodgen *et al.*, 2010).

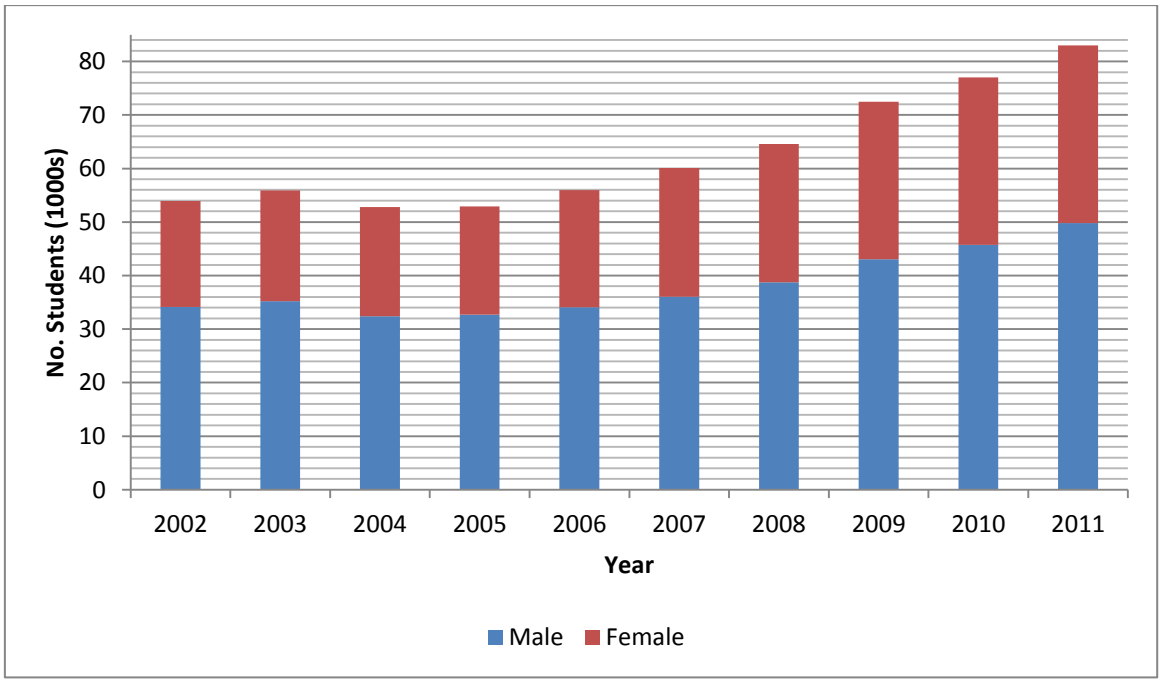


Figure 1.1 - Number of candidates for A-level Mathematics in the UK.
Source: JCQ (2012)

[1] Science, technology, engineering and mathematics.
[2] In 2011-2012, the IB Diploma Programme was offered by 208 UK schools, with 5114 British students studying for examinations in it in 2011 (International Baccalaureate Organisation, 2011).

These data from JCQ (2012) show an increase in the number of mathematics candidates since 2004, which Paton (2011) attributes to the changing economic climate. That is, students are beginning to recognise that mathematics has a high exchange value in the workplace and in higher education, and therefore study it in order to increase their chances of job prosperity. Last year saw the highest number of candidates sitting the examination since the modular system[3] was introduced in 2002. Incidentally, 2005 saw a shift in the modules offered, with the 'pure' modules being replaced by 'core' modules. This involved an amount of restructuring of the material in each module, which resulted in less content being covered and, argues Porkess (2003), more difficult concepts being moved to more advanced modules.

A-level Mathematics has been through several revisions in recent years (see Figure 1.2), complementing its move from being an examination taken by students who wanted to go to university to a stand-alone qualification which can even be studied by the module.

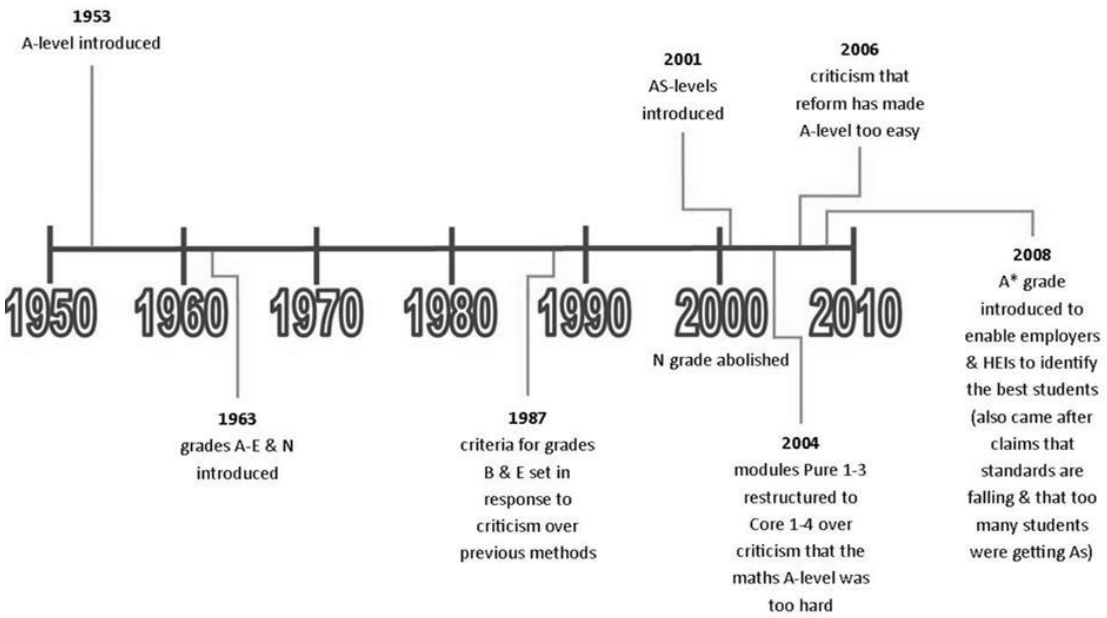


Figure 1.2 - Timeline of A-level Mathematics

[3] From 2002, A-level Mathematics was divided into six modules for which pupils take one examination each over the course of two years (see Chapter 2.3.3.1).

Savage *et al.* (2000) refer to the 1960s as the 'golden age' of undergraduate mathematics preparation, where pupils "were inspired and stretched… [and] acquired the all-important study skills together with sound mathematical knowledge and understanding which prepared them for Higher Education" (p. 2). The revisions that have been made to the A-level in the meantime are evidence that these criticisms are being responded to; however, the nature of the material studied in A-level Mathematics has also come under fire. *Tackling the Mathematics Problem* (Savage, 2003) reports on the notion that it is becoming increasingly common for students to appear comparatively unprepared for mathematics degrees despite achieving good grades at A-level. Suggestions that the amount of proof covered in the current syllabus is inadequate are commonplace, with students' experiences of proof being a common topic of research in tertiary mathematics education (see Chapter 2.3.1).

Concerns have been raised over the past decades that new students are arriving in their first year at university without sufficient mathematics knowledge (ACME, 2011b; Williams, 2011). The levels of mathematical competency of these students has also been found by Smith (2004) to be decreasing over the years, with scores on a diagnostic test for new students decreasing with each new cohort. Diagnostic testing is now used in many mathematics departments across the UK (Edwards, 1996; MathsTEAM, 2003; Williams *et al.*, 2010), with many universities conceding that "the idea that the final year of school should fit the students for the first year of mathematics is no longer automatic" (Baumslag, 2000, p. 6; see also Chapter 2.3.3). These tests are normally of students' knowledge of mathematical identities and methods such as differentiation, integration, inequalities and trigonometry. The departments hope that this will ensure the students' fluency in these areas in order to take on new mathematical concepts without the added burden of experiencing difficulty in technical aspects. The diagnostic tests represent the beginning of a move by universities to act themselves to help lessen the impact of the misalignment of syllabuses at the secondary-tertiary interface. In fact, Hawkes and Savage (2000) found that over sixty mathematics,

physics and engineering departments in the UK engaged in diagnostic testing of new students. This suggests widespread acknowledgement of the problem by universities, with some universities continuing to help to bridge the gap between each level of study by offering extra help and optional support classes to their students (Sutherland & Dewhurst, 1999).

Whilst they have been criticised for failing to keep up-to-date with A-level syllabus changes (Chetwynd, 2004; Hawkes & Savage, 2000), universities have conceded and made changes to what they are teaching students. In the past, it was possible that students did not begin learning material appropriate to their current level of understanding and competence when they arrived, with Clark and Lovric (2009) describing the secondary-tertiary mathematics transition as "a modern-day rite of passage" (p. 755). This can involve repetition as well as the more dangerous "omission of essential mathematical background from first-year university courses. This is particularly problematic because of the incremental nature of mathematics, whereby new topics assume understanding of preceding ones as background" (Chetwynd, 2004, p. 30).

Baumslag (2000) recommends that universities regularly check syllabi and textbooks to keep up-to-date. However, this becomes more difficult as the International Baccalaureate, for example, grows in popularity within British schools and the educational backgrounds of incoming students continues to diversify as international students come to study in the UK (Hoyles *et al.*, 2001). This advice is clearly being taken on board, as constant revisions to the university curriculum have become the norm (Savage *et al.*, 2000), with ACME (2012) recently making recommendations to Ofqual and the Education Select Committee that a 'national subject committee' be established in order to more closely tie universities and examination boards. This comes partially as a consequence of the fact that school syllabi and examinations are failing to prepare students for mathematics degrees, or those which involve a large proportion of mathematics (Sutherland & Dewhurst, 1999).

The skills taught at school are beginning to be considered an insufficient basis for further study, and "what is sometimes referred to as a 'gap'… between school and tertiary mathematics, may be increasing" (Thomas, 2008, p. 1; see also de Guzman *et al.*, 1998). Alongside this, cultural discontinuities between the two levels of study have been identified (Perrenet & Taconis, 2009) in that students become part of an entirely different mathematical 'culture' and environment, which presents challenges of its own.

The need for a large number of mathematically competent graduates continues to increase, as they are necessary to ensure economic development and success (Gago, 2004; Petocz & Reid, 2005; Wolf, 2002). Furthermore, transition problems and attrition in all degree subjects have considerable economic costs (Pargetter, 1995), meaning that failing to support students in their studies can have huge financial costs. The problem is by no means unique to the UK, with much research in this area being produced in countries such as the USA (US Department of Education, 2000), the Netherlands (Heck & van Gastel, 2006) and many others.

The term 'Mathematics Problem' is used to describe concerns regarding the relatively small number of students choosing to study the subject at tertiary level, not just at home but on an international scale (e.g. ACME, 2009, in the UK; Engelbrecht & Harding, 2003, in South Africa; European Commission, 2005, in Sweden, Norway, Poland and Germany; Hillel, 2001, and Jackson, 2000, worldwide; Spellings Commission, 2006, in the USA). This has been attributed to increased numbers of students having negative experiences of the subject at school (Smith, 2004). Furthermore, once students advance to this level of study, many are failing to succeed in the new environment, with low pass rates in mathematical subjects being common in the first year of study (LMS, 1995).

Mathematics has a "special status and exchange value as a 'strong subject'" (Williams, 2011, p. 217) and therefore requires specialists to teach it to children in schools. Unfortunately, the number of mathematics graduates who go on to teach is decreasing (French, 2004; LMS,

1995), which might affect children's experiences and appreciation of the subject (Perkins, 2005). At the school level, it is possible that there are some students who are left disadvantaged by being taught by teachers who lack a passion for the subject, as they fail to inspire students to pursue the subject further. In fact, Chetwynd (2004) reports that, former Secretary of State for Education and Skills and mathematics graduate, Charles Clarke stated that we need to "refresh the teachers' enthusiasm for mathematics that led them into teaching in the first place" (p. 29).

Much research on the student experience, transition, approaches to learning and so forth neglects to include mathematics students in its samples. Furthermore, that which does often blindly groups mathematics with 'other sciences' with which, one could argue, it shares few commonalities. There is much to be said on whether it is fair to consider mathematics alongside biology, chemistry, engineering and physics with respect to research questions in these fields. This makes it contentious to attempt to apply any research findings from one subject to another (Becher, 1994), particularly when mathematics is involved. *Mathematics* education is an area which receives more attention and focus thanks to various issues which have not been highlighted to anywhere near as great an extent elsewhere. For example, there are much-documented issues with the nature of the subject, differences between material covered at school and university, students' understanding of particular concepts and the ways in which students go about learning it. The lack of research into these topics in other subjects suggests that this is not as great of a concern in other scientific subjects.

According to Savage (2003), incoming students are lacking in three areas:

1. They are unable to fluently and consistently perform algebraic manipulations and simplifications.
2. Their analytical powers are weak in instances where they are required to solve multi-step problems.

3. They are ignorant of the fact “that mathematics is a precise discipline in which exact, reliable calculation, logical exposition and proof pay essential roles” (p. 8). More broadly, students are not clear on the nature of mathematics and, more specifically, undergraduate mathematics.

Thus, as a consequence of a multitude of factors, the experience of undergraduate mathematicians is high on the political and educational research agenda. Furthermore, the increasingly complex and ever-changing nature of student fees mean that there is a new impetus for rivalry amongst universities for offering the best student experience (Ertl & Wright, 2008) and even the best value for money.

A significant attempt to positively impact the number of students studying mathematics at a higher level was made through the development of the ‘Further Mathematics Support Programme’ (FMSP) which aims to increase the uptake of A-level Further Mathematics through various initiatives. For example, they work with current teachers to provide continual professional development to help those who are not confident teaching Further Mathematics, as well as providing online tuition, interactive lectures and revision schedules for pupils. Whilst it is not possible to say whether it was the FMSP which led to Further Mathematics becoming the fastest-growing A-level subject last year (JCQ, 2012), the funding from the Department of Education to the FMSP supports their attempts to increase the mathematical competency of school-leavers.

1.2 – Examination of the Literature

Selden and Selden (1993) have commented that Alan Schoenfeld's address at a 1990 meeting of the American Mathematical Society was a significant occasion when mathematicians and tertiary-level educators became particularly interested in research into undergraduate mathematics education[4]. At the time, they commented that empirical research and its responsive actions were "quietly, but surely, overtaking a largely unsuspecting mathematics community" (p. 432). Shortly after, David Tall's (1991a) *Advanced Mathematical Thinking* was published to great reception. Inspired by an International Group for the Psychology of Mathematics Education (PME) working group which focused on advanced mathematical thinking, this is a key text in the field of undergraduate mathematics education, as it charts the development of interest and research in this complex area.

More recently, studies have been commissioned on national and international scales into tertiary mathematics education, as the research and teaching communities have become increasingly aware of problems faced by teachers and learners alike. For example, Holton (2001) describes the findings of a study conducted by the International Commission on Mathematical Instruction and confirms the existence of a range of issues which are, or ought to be, researched. It has been claimed that such interest has stemmed from the increasingly diverse student population admitted to universities to study mathematics and mathematics-related subjects. Holton (2001) claims that this has meant that "universities have begun to adopt a role more like that of the old school system and less like the elite institutions of the past" (p. v). In the UK, this is compounded by the gradual evolution of A-level Mathematics to accommodate the changing student base. It would perhaps therefore be necessary for universities to respond by changing their syllabuses and pedagogy; however, this is not a simple change which may be instigated with guaranteed positive effects and requires in-depth

[4] This is based on English-speaking and European mathematics education.

research to ensure positive changes. For example, Alcock and Simpson (2001) attempted a teaching experiment at the University of Warwick whereby first-year undergraduates were taught real analysis[5] using workbook-based approaches rather than the standard lecture format. A similar approach is still used today; however, it has undergone revision over the years to help it further address the problems that new undergraduates have been found to experience with this area of study.

In the last decade, there has been a rise in published research about the disaffection of undergraduate mathematicians. In the UK, Daskalogianni and Simpson (2001, 2002) have published conference papers identifying and describing disaffection. In Australia, considerable interest was taken by Kath Crawford and her colleagues (Crawford *et al.*, 1994, 1998a, 1998b) in undergraduate students' conceptions of mathematics, which they related to students' personal relationships with the subject and its content, as well as their previous experiences and understanding of mathematics. Students' emotional response to advanced mathematics is a topic which has been extensively explored by Melissa Rodd (Rodd, 2002; Brown & Rodd, 2004; Rodd & Bartholomew, 2006). Using survey data and interviews with prospective mathematics undergraduates on an open day at a Russell Group university, Darlington (2009) investigated and identified gender differences at the secondary-tertiary mathematics interface with particular reference to students' coping styles in their first year of undergraduate study. However, it is important that these research areas are married in order that a comprehensive picture of the undergraduate experience may be drawn; one cannot possibly understand the undergraduate mathematics students' plight without exploring students' prior experiences and conceptions of the subject, as well as their changing attitudes and feelings about mathematics.

[5] Real analysis is a branch of mathematics which focusses on topics such as limits, infinite series and calculus.

1.2.1 – Critique

The proposed research into Oxford undergraduates' experiences comes at a time when the existing research is often international or irrelevant given the curriculum changes introduced over the last decade.

Much of the available literature was conducted abroad and therefore one could argue has limited applicability in the UK. There has been a great deal of research conducted on this topic in the USA (e.g. Selden & Selden, 1993; 2005; Thurston, 1994), Australia (e.g. Crawford *et al.*, 1994; 1998a; 1998b; Reid *et al.*, 2005), New Zealand (e.g. Anthony, 2000), Sweden (e.g. Filipsson & Thunberg, 2008), France (e.g. Gueudet, 2008), Canada (e.g. Kajander & Lovric, 2005), South Africa (e.g. Maguire *et al.*, 2001) and the Netherlands (e.g. Perrenet & Taconis, 2009). International literature's applicability to this field is questionable since education systems abroad are often very different to that in the UK. For example, American universities offer students a much broader field of options early on, leading them to eventually 'major' in one particular subject, rather than have them study one particular subject all the way through. This means that those majoring in mathematics will not have had a reasonably comparable subject experience with a British student who studies only mathematics for three years. Moreover, different secondary schooling means that one could argue that there is the potential for students to be better equipped for undergraduate mathematics in certain countries, thanks to exposure to different aspects of the subject earlier on. Indeed, the most recent report of the Trends in International Mathematics and Science Study (TIMSS) (Mullis *et al.*, 2012) ranked England and Northern Ireland within the top ten high-achieving countries, with Singapore ranking the highest.

As well as research which focuses on study outside of the UK, there is much research on the student experience of mathematics as a service subject (e.g. Faulkner *et al.*, 2009), for example the courses studied as part of an engineering or chemistry degree. Such mathematics is very

different in its focus and, by acting only as a means by which to conduct specific analyses or apply mathematics in specific situations, such research is not necessarily useful or relevant to a project which focuses on an entirely different student experience.

Moreover, much of the literature reviewed for this project is from the 1990s and 2000s. The age of this research limits the applicability of its findings since the secondary curriculum and A-level Mathematics have evolved since this time (see Chapter 2.3.3). This means that students entering tertiary study have different mathematical backgrounds now, in 2013, to those in the 1990s. Such a difference in mathematical knowledge has resulted in some universities introducing bridging courses or changes in topics covered at different levels in order that students may be 'eased' into the study of advanced mathematics more than they may have been in the past. This means that students are likely to have had a different experience of their degree to those in previous decades.

1.2.2 – Transmaths

In more recent years, special editions of research journals have been dedicated to topics such as:

- 'Deepening Engagement in Mathematics in Pre-University Education' (Wake *et al.*, 2011); and
- 'Enhancing the Participation, Engagement & Achievement of Young People in Science & Mathematics Education' (Reiss & Ruthven, 2011).

Furthermore, a large, ESRC[6]-funded research project concerning the student experience of post-compulsory mathematics is being conducted by the 'Transmaths' team at the University of Manchester. While the publications from this study provide a broad-brush view of some of

[6] Economic & Social Research Council is a research council in the UK which provides funding for research in social and economic issues.

the current issues, upon close examination it appears that they are skirting around several central ideas that I propose to cover in greater depth and detail. They report on:

- A-level as preparation for university mathematics[7], e.g. Pepin (2009);
- students' poor self-efficacy, e.g. Pampaka & Williams (2010);
- student identity, e.g. Black (2010);
- institutional practices, e.g. Davis *et al.* (2009); and
- students' identification with mathematics, e.g. Jooganah & Williams (2010).

In this thesis, I supplement the sociological context, which they describe by focusing on the cognitive demands of mathematics, and how students adapt, if at all, their study methods:

- the demands of questions at A-level and university;
- students' preparation for university by the A-level;
- the nature of questions and entry requirements at the transition; and
- the way in which students consequentially approach the learning of mathematics at Oxford.

I also aim to conjecture why mathematics study at Oxford may differ from student experiences elsewhere, and the potential benefits and limitations of the University's unique teaching structure.

1.2.3 – Key Research Questions

Through analysis of the literature in Chapters 2 and 3, the following research questions are posed:

1. How do undergraduates' experiences of studying mathematics at Oxford change throughout their university career? Specifically,
    a. What challenges do students report facing in each year of study?

[7] They comment that questions are easier, that teachers teach towards the examination, and that students study towards the examination.

b. How do students report their approaches to learning and studying mathematics?

2. Based on previous experience of mathematics, what challenges lie in Oxford students' enculturation into a new mathematical environment? Specifically,
    a. What types of skills and challenge are tested by A-level Mathematics and Further Mathematics questions?
    b. How does the OxMAT's assessment of students' mathematical understanding compare to A-level Mathematics and Further Mathematics?
    c. How do undergraduate mathematics examinations compare to the A-level and the OxMAT?
3. What is the relationship between students' approaches to learning and the challenges they perceive in undergraduate mathematics assessment at the University of Oxford?

Chapter 2: Literature Review

The Nature of Post-Compulsory Mathematics Questions & Students' Responses

2.1 – Approaches to Learning

> *'Approaches to learning' refers to [the] individual differences in intentions and motives when facing a learning situation, and the utilisation of corresponding strategies.*
>
> (Diseth & Martinesen, 2003, p. 195)

2.1.1 – Deep & Surface Approaches

The distinction between deep and surface approaches to learning (ATLs) has been debated and written about for over twenty years. It has been subjected to refinement, with some definitions being concise and others further reaching. They can be concisely explained and distinguished as follows:

Table 2.1 - Contrasts between deep & surface ATLs

	Deep Approach	Surface Approach
Intention	Understanding	Memorisation
Strategy	Seeking comprehension	Rote learning

Deep approaches are characterised by learning strategies that focus on meaning, directed towards understanding by critically relating new ideas to previous knowledge and experience (Ramsden, 1983). In mathematics, this would lead to the construction of a network of ideas. Conversely, surface approaches focus on memorising without reflecting on the task or thinking about its implications in relation to other knowledge (Trigwell & Prosser, 1991a). Such approaches jeopardise success if what is learnt by rote is forgotten, or cannot be adapted to

be used in mathematical problem-solving (Novak, 1978) because it is detached from mathematical meaning.

This distinction may be over-simplistic. A student with a deep approach seeks to understand a particular concept and, whilst they may remember it as a consequence, "this is viewed as an almost unintentional by-product" of their actions (Kember, 1996, p. 343). Consequently, it has been suggested that a deep ATL is "a necessary, but not a sufficient, condition for productive studying" (Lonka *et al.*, 2004, p. 307). For example, a student who learns with an intention to understand may not always *achieve* a deep understanding if "the subject matter is unfamiliar or too difficult" (Entwistle *et al.*, 1979a, p. 367). Moreover, memorisation is not purely characteristic of a surface approach; it has been found to play a role in deep ATLs (Kember, 1996; Watkins & Biggs, 1996). For example, a mathematician may need to memorise definitions in order to then be able to fully understand the reasoning behind a theorem or proof. Therefore, memorisation can act as "a necessary precursor to understanding, and for other purposes it is a way of reinforcing understanding" (Entwistle, 1997, p. 216).

However, those who adopt deep ATLs may not necessarily achieve higher grades than those using surface approaches. It is possible that "a student with high orientation towards a deep approach, but who is not particularly competent, may perform less well than a student with a 'highly polished' surface approach" (Cuthbert, 2005, p. 244). On the whole, deep approaches to learning mathematics "generate high quality, well-structured, complex outcomes; they produce a sense of enjoyment in learning and commitment to the subject; they are related to... higher grades" (Lipinskienė & Glinskiene, 2005, pp. 11-12), with surface approaches only allowing learners to remember fragments of information in the short-term. This permits students to "memory dump" (Anderson *et al.*, 1998, p. 417) what they have learnt, thus preventing the construction of solid foundations from which to build the understanding of new concepts.

2.1.2 – Correlates with ATLs

A wide variety of research has been conducted on the relationship between ATLs and contextual factors such as attainment, assessment and teaching (Trigwell *et al.*, 1999), as well as personal factors such as age and gender (Regan & Regan, 1995). For example, quantitative research has suggested that women have a greater propensity to adopt surface ATLs, although such claims are often based on statistics with low significance (Severiens & Ten Dam, 1994). Such work has established that ATLs involve "elements of both individual stability and contextual variability" (McCune & Entwistle, 2000, p. 1), with institutional and departmental variations suggested by inventory-based research (Ramsden, 1983; Ramsden & Entwistle, 1981). It is important to understand any links in these areas, since an experiment by Gibbs (1994) suggested that manipulation of the learning context can alter students' ATLs.

2.1.2.1 – Attainment

Positive correlations between attainment and a self-reported deep approach have been found (Cano, 2005; Entwistle *et al.*, 2000; Lindblom-Ylänne & Lonka, 1999; Meyer *et al.*, 1990; Newstead, 1992; Reid *et al.*, 2007; Ramsden, 1983; Marton & Säljö, 1984; Sadler-Smith, 1997), suggesting that those who adopt a deep ATL tend to perform best. Conversely, negative relationships have been identified between self-reported surface approaches and performance (Beishuizen *et al.*, 1994; Cano, 2005; Lindblom-Ylänne & Lonka, 1999; Marton & Säljö, 1984; Meyer *et al.*, 1990; Provost & Bond, 1997; Ramsden, 1983; Reid *et al.*, 2007) and measurements indicating low levels of surface learning have been found to correlate with academic success (Diseth, 2002; Diseth & Martinesen, 2003; Watkins, 2001). Furthermore, having poor study techniques is something recognised by students and lecturers alike as being something that can contribute to failure (Anthony, 2000).

2.1.2.2 – Personality

Entwistle and Ramsden (1983) suggested that relationships exist between a learner's personality and their ATL; specifically, those deemed 'unstable extraverts' by inventory-based psychological testing tend to adopt poor study methods, whereas their 'stable introvert' counterparts' methods are much more appropriate. In a study involving psychology undergraduates, Diseth (2003) used the Approaches and Study Skills Inventory for Students (Tait *et al.*, 1998; see Chapter 4.5) to establish whether there were any relationships between self-reported ATLs, inventory-assessed personality and academic achievement. He found "significant positive relations between deep approaches and openness [and] surface approaches and neuroticism" (Diseth, 2003, p. 151). Those with surface ATLs have been identified as generally having lower self-concepts as learners (Dart *et al.*, 1999), whilst those adopting deep approaches tend to be more confident in their abilities, and more self-motivation to learn (Bruinsma, 2003, cited in Heijne-Penninga *et al.*, 2008).

2.1.2.3 – Teaching

The literature suggests that the relationship between teaching and ATL is reactive, with pedagogy influencing ATLs (Biggs, 1999; Prosser & Trigwell, 1999; Ramsden, 1992), and preferences for particular ATLs influencing students' routes of study. Those who adopt particular ATLs have been found to prefer certain courses, teaching styles and assessment methods (Entwistle & Tait, 1990; Trigwell & Prosser, 1991a, 1991b). For example, those with surface ATLs tend to prefer "methods of teaching which 'spoonfeed' them what they need to pass exams, while students with predominantly deep approaches want to be challenged and stimulated" (Entwistle & Meyer, 1992, p. 594). It has also been suggested that "game playing by teachers of the kind that encourages cynicism in students" can encourage them to have surface ATLs (Biggs, 1988, p. 199).

Hence, it is possible that a student will adapt their ATL to the teaching environment (Eley, 1992), since ATLs have been described by Trigwell and Prosser (1991a) as "a function of both the student and the context" (p. 254). However, one should question whether the interplay between ATLs and choice of course can become problematic; that is, being required to study a course whose pedagogy is not complimentary of one's deep or surface approach has the potential to result in poor academic performance.

2.1.2.4 – Assessment

Students have been found to adopt their study methods according to the assessment they face (Ramsden, 1988; Thomas, 1986, cited by Entwistle, 1989; Thomas & Bain, 1984). For example, open-book examinations can stimulate a deep learning approach, and closed-book exams a surface approach (Heijne-Penninga *et al.*, 2008). Factors which have been found to encourage a surface ATL can affect all groups of students, "even those with a predilection towards deep learning" (Biggs, 1988, p. 199). Ramsden (1983) suggests that "perceived excessive workload, emphasis on accurate recall, threatening learning situations, lack of intrinsic interest in the subject-matter combined with a need to pass" (p. 696) are possible causes of anxiety which trigger surface ATLs. Specifically, if the student perceives that the workload they face is too great, they tend to regress to surface approaches (Entwistle & Ramsden, 1983; Lizzio *et al.*, 2002; Newble & Entwistle, 1986; Newble *et al.*, 1988; Ramsden & Entwistle, 1981; Trigwell & Prosser, 1991b).

Reid *et al.* (2007) caution that "these approaches are not mutually exclusive and an individual may switch between them" (p. 754). The contextual dependence of ATLs which has been suggested in the literature (e.g. Campbell *et al.*, 2001; Cassidy, 2004; Entwistle, 2001; Lucas & Mladenovic, 2004; Ramsden, 1987) means that one can consider ATLs dynamic and able to meet with the demands of the academic situation (Byrne *et al.*, 2009). However, approaches adopted for a particular task are influenced by "pre-existing beliefs about knowledge and

learning, and gerneral pre-disposition towards particular approaches to learning" (Campbell *et al.*, 2001, p. 175). Therefore, an element of consistency and stability in one's ATL can be reasonably assumed owing to the interplay between approaches and assessment methods:

> *Perhaps the more workable view is that a [learning] style may well exist in some form, that is it may have structure, but the structure is, to some degree, responsive to experiences and the demands of the situation... to allow change and to enable adaptive behaviour.*
>
> (Cassidy, 2004, p. 428)

Mathematics students are often required to give "the correct statement of definitions in examinations... [and hence become] liable to degenerate into learning by rote... [leaving them] unable to relate directly to the form of the definition" (Robert & Schwarzenberger, 1991, p. 130). Therefore, undergraduate mathematics students may believe that merely learning by rote is necessary in order to succeed, something which could prevent them from developing knowledge and relating it to higher-order concepts later on.

2.1.3 – Alternative Suggestions

2.1.3.1 – Terminology

It has been suggested that there are too many conceptual frameworks in the ATLs domain (Sadler-Smith, 2001). Terminology has evolved throughout time, as writers sought to redefine and clarify their assertions based on their own – or others' – research. However, many are based on different theoretical backgrounds, which initially resulted in a broad phraseology.

The deep/surface dichotomy, as originally defined by Marton and Säljö (1984), was developed based on empirical research and was influenced by other work in this area. Different terminology was used by different writers, often with the nature of their definitions differing slightly – for example, referring to *cognitive* approaches rather than *learning* approaches.

Eventually, terms 'converged' to the deep/surface dichotomy (Ford, 1981; Schmeck, 1983), contrasting slightly in their background and nature.

However, one could suggest parallels with the definitions of 'deep' and 'surface' approaches as they have come to be known:

Table 2.2 - Similar terminology to the deep/surface dichotomy

Deep	Surface	Author
Meaningful learning	Rote learning	Ausubel (1963)
Holists	Serialists	Pask and Scott (1972)
Generative processing	Reproductive processing	Wittrock (1974)
Deep-level processing	Surface-level processing	Marton (1976)
Relational understanding	Instrumental understanding	Skemp (1976)
Holistic cognitive approach	Atomistic cognitive approach	Svensson (1976)
Internalising	Utilising	Biggs (1978)
Transformational learning	Reproductive learning	Thomas and Bain (1984)
Deep memorisation	Surface memorisation	Tang (1991)[8]

2.1.3.2 – Meaningful Learning vs. Rote Memorisation

Influenced by the work of Jean Piaget, Ausubel (1963) contrasted meaningful learning with rote memorisation. Looking into cognitive structure, he wrote of meaningfully learnt concepts as being those which are "related to existing concepts in cognitive structure in ways making possible the understanding of various kinds of significant relationships" (p. 217). Conversely, rote learnt concepts "are discrete and relatively isolated entities which are only relatable to cognitive structure in an arbitrary, verbatim fashion not permitting the establishment of... relationships" (*ibid.*). For this reason, such learning has a greater chance of being forgotten, it being part of an "unstable, ambiguous, disorganized" cognitive structure (*ibid.*).

2.1.3.3 – Holist vs. Serialist

Pask and Scott (1972) distinguish between holists and serialists. Serialists are learners who remember information by constructing low order relations, having "a tendency to examine less data and use a step-by-step approach" which means that they "put much more emphasis on...

[8] Cited by Kember (1996).

separate topics and... logical sequences, connecting them only late in the process" (Riding & Cheema, 1991, p. 203).

On the other hand, holists "learn, remember and recapitulate [information] as a whole" (p. 218) through looking at the wider picture and "searching for patterns and relationships" as they "perceive the learning task in an overall context from the start" (Riding & Cheema, 1991, p. 203). Holists may be sub-divided into those who are 'irredundant' and those who are 'redundant'. Whilst both sub-groups adhere to the same overall view of learning, irredundant holists only focus on "relevant and essential constituents" of the concept whilst redundant holists seek "irrelevant or over specific material, commonly derived from data used to "enrich" the curriculum" to add to their schema of knowledge on that topic (Pask & Scott, 1972, p. 218).

Pask (1976) discusses contrasting pathologies of learning, describing serialists as incapable of taking a global view and having an 'improvidence' pathology, whereas holists tend towards 'globetrotting' wherein they "make hasty decisions from insufficient evidence" (Riding & Cheema, 1991, p. 204). A striking difference between Pask's (1976) descriptions of holists and serialists ,when compared to other authors' suggestions of categories of ATLs, is his suggestion that both groups of learners are capable of sharing the same understanding and that it is just their means of 'getting there' that are different.

2.1.3.4 – Generative vs. Reproductive Processing

Working predominantly with primary school children, Wittrock (2010) proposed a generative model of learning which "predicts that learning is a function of the abstract and distinctive, concrete associations which the learner generates between his prior experience as it is stored in long-term memory, and the stimuli" (p. 41). Conversely, "reproductive processing is the rehearsal or repetition of semantic, phonological, or distinctive information when the learner's previous experience indicates that the construction of semantic or distinctive associations is not probable" (Wittrock & Carter, 1975, p. 490). Wittrock (1974) claimed that only generative

processing could result in effective recall and understanding of what is being learnt, and that "the learner must actively construct meaning if he is to learn with understanding" (p. 195).

2.1.3.5 – Deep- vs. Surface-Level Processing

The distinction between these two ways in which an individual can process material was made by Marton (1976). With the latter, "the student is concerned with reproducing the *signs of learning* – i.e. the words used in the original text – rather than mastering *what is signified* – i.e. the meaning" (Biggs, 1979, p. 383). Deep-level processing, conversely, takes place on occasions when the learner looks at the 'bigger picture', aiming to connect related concepts together. Processing is heavily related to the orientation of the learner; if they are learning for learning's sake, then they will use deep-level processing. However, if their orientation is extrinsic, their "approach is more likely to be surface level" (Laurillard, 1979, p. 401). Importantly, Laurillard (1979) comments that surface-level processing is not necessarily solely adopted by lazy students, but can also be rationally chosen based on the demands of the task.

2.1.3.6 – Instrumental vs. Relational Understanding

A distinction was made by Skemp (1976) between an instrumental and a relational understanding of a mathematical concept. Crudely, an instrumental understanding involves the learner rote-learning rules and procedures, whereas a relational understanding is consistent with an awareness of the basis for the concept itself, and the reasoning behind it.

When considering mathematics at more advanced levels, "an individual has an instrumental understanding of a concept if he or she can state the definition of the concept, is aware of the important theorems connected with that concept, and can apply those theorems in specific instances", whereas a learner with a relational understanding "understands the informal notion this concept was created to exhibit, why the definition is a rigorous demonstration of this intuitive notion, and why the theorems associated with this concept are true" (Weber, 2002, p. 2).

Weber (2002) also distinguishes between instrumental and relational proofs:

> *An instrumental proof is a proof in which one primarily uses definitions and logical manipulations without referring to his or her intuitive understanding of a concept. A relational proof is a proof in which one uses his or her intuitive understanding of a concept as a basis for constructing a formal argument.*
>
> (p. 2)

2.1.3.7 – Holistic vs. Atomistic Cognitive Approaches

Svensson (1976) distinguished between these two approaches which, as with deep and surface ATLs, were defined by a task which required participants to read some text and answer questions about it afterwards. Students exhibiting a holistic approach "showed indications of a general direction towards understanding text as a whole... The indications of an atomistic approach were: focusing on specific comparisons of the text, focusing on the sequence of the text, but not the main parts, memorising details and, in contrast, clear evidence of a lack of orientation towards the message as a whole" (Marton & Säljö, 1984, p. 47).

It was the work of Svensson (1976) that led to the development of the terms 'deep ATL' and 'surface ATL' (Marton & Säljö, 1984). Concerned that the term 'processing' did not convey the intentional aspect of learning, Entwistle *et al.* (1979a) then began using the term 'approach', as defined by Svensson (1976), combining this with Marton's (1976) original deep/surface dichotomy.

2.1.3.8 – Utilising vs. Internalising vs. Achieving

In his Study Behaviour Questionnaire, Biggs (1978) examined three different dimensions to learning: achieving, utilising and internalising:

Table 2.3 - Biggs' (1978) Dimensions of Learning

Dimension	Motive	Strategy
Achieving	Achievement: Obtain highest grades, play to win	Organised study: Schedule time, behave like the 'model student'
Utilising	Extrinsic: Gain qualification	Reproducing: Limit learning to course essentials and rote learn them
Internalising	Intrinsic: Study to actualise interest and competence in academic subjects	Meaning assimilation: Interrelate knowledge, read widely, discuss academic issues

Adapted from Gano-Garcia & Justicia-Justicia (1994, p. 254) and Ramsden (1985, p. 57)

Such terminology was used when research on ATLs first began; however, it was later changed to fit the deep-surface-achieving 'trichotomy' in order to prevent confusion (Eklund-Myrskog, 1999).

2.1.3.9 – Transformational vs. Reproductive Learning

These different learning approaches can be distinguished through particular assessment methods. Thomas and Bain (1984) identified two different types of learning carried out by students, depending on whether they were studying for assessment with closed- or open-ended questions. Assessments with closed-ended questions appeared to be approached with reproductive learning methods, whereas those with open-ended questions had transformational approaches applied to them.

2.1.4 – Strategic Approach to Learning

As the dichotomies in Chapter 2.1.3 were being written about, another 'type' of approach to studying emerged: that of a strategic approach. Described by Ertmer and Newby (1996) as 'expert' learners, those with a strategic ATL have been found to use a combination of deep and surface ATLs, "supported by a competitive form of motivation... combined with vocational motivation within an achieving motivation" (Entwistle & Tait, 1990, p. 171). This combination

of approaches means that "it is possible simultaneously to be both a deep and achieving learner and a surface and achieving learner but not a deep and surface learner; the latter are mutually exclusive" (Scouller & Prosser, 1994, p. 267) because "it is not possible to focus and not to focus on meaning at the same time" (Diseth & Martinesen, 2003, p. 196).

Table 2.4 - Comparison between deep, surface and strategic ATLs

	Deep	Surface	Strategic
Intention	Understanding	Memorisation	Success
Strategy	Seeking comprehension	Rote learning	Mixture of deep/surface

Reid *et al.* (2007) describe a strategic approach as involving "organised studying and good time management and is driven by the desire for high achievement" (p. 754). Those who play "the assessment game" (Entwistle *et al.*, 1979a, p. 366) might be less academically capable, but overcome this by developing means by which to succeed (Furnham *et al.*, 2003). It is for this reason that many have commented that strategic ATLs – and, to a lesser extent, surface ATLs – tend to be instigated by institutional demands (Biggs, 1993; Lindblom-Ylänne & Lonka, 1999, 2000, 2001).

Those who commonly utilise strategic ATLs tend to report a conscientious personality (Diseth, 2003; Heinström, 2000). Moreover, the notion that a strategic approach is based on a desire to achieve the highest grades has been supported by positive correlations between those with a strategic approach and attainment (Diseth & Martinesen, 2003; Newstead, 1992; Ramsden, 1983; Reid *et al.*, 2007; Sadler-Smith, 1997; Schouwenburg & Kossowska, 1999).

2.1.5 – Limitations

The 'approaches to learning' theory has been subjected to a lot of criticism (e.g. Haggis, 2003, 2009; Malcolm & Zukas, 2001; Webb, 1996, 1997) based on both the concept of an ATL, and whether it is measurable, as well as what could and should be 'done' with ATL data.

Despite being described by Richardson (2000) as “a cliché in discussions about teaching and learning in higher education” (p. 27), Haggis (2009) queries why concerns over the proportion of students who take surface ATLs “remain largely unanswered” (p. 378). One of the problems associated with the ATLs research becoming so very commonplace in research in higher education is that “there has been an inevitable degree of conceptual slippage” (Marshall & Case, 2005, p. 258). That is, for example, the term ‘surface approach to learning’ has been bastardised into ‘surface learning’ and then to ‘surface learners’. Indeed, Lucas and Mladenovic (2004) assert that “there is no such thing as a student who is necessarily assumed to be either a ‘surface’ or a ‘deep’ learner” (p. 400).

This is because a student’s ATLs are context-dependent. The theory of ATLs “capture students’ responses and adaptations to course contexts, rather than representing innate cognitive characteristics of a student” (Case & Marshall, 2004, p. 606). The contextual nature of a student’s ATL is something which is criticised in the literature in the sense that a number of writers believe that the ATL theory oversimplifies something specific and context-dependent (Barnett, 1990; Haggis, 2003; Malcolm & Zukas, 2001; Volet & Chalmers, 1992). Whilst the theory is “a simplified version of reality, in which the minutiae and detail are stripped away, leaving what are assumed to be important factors” (Bean, 1982, p. 18), Malcolm and Zukas (2001) argue that the theory implies that the student is “an anonymous, decontextualized, degendered being” (p. 38), with Haggis (2003) claiming that it “avoids any real engagement with the complexities of location and context” (p. 101). Indeed, she also goes on to say that concepts related to ATL such as ‘meaning’ and ‘understanding’ vary “according to discipline, subdiscipline, and tutor” (p. 95). However, Coffield *et al.* (2004) describe the deep/strategic/surface trichotomy as one of the few approaches to learning theories which actually takes context into account.

Whilst this criticism has some merit, none of the writers acknowledge the amount of empirical research using the ATL theory which concentrates solely on one particular discipline or subject area, and even goes so far as to revise instruments to measure ATL for the particular subject under question. It is true that there is research which bundles students of a variety of degree disciplines together as if it is assumed that the subject itself had no bearing on their ATLs, which suggests an ignorance of the implications of context on ATL in spite of the research regarding the influences of factors such as subject matter, attainment, assessment, engagement and personality. Indeed, Meyer and Eley (1999) argue that "perceptions and experiences of learning contexts might be shaped… by the epistemology of a discipline and they might therefore vary considerably from one discipline to another" (p. 198). Furthermore, there is also research which groups mathematics with other subjects, such as physics, engineering or statistics, as if they present the same challenges to students and therefore would have the same influences on ATLs. I contend that this is not the case and, as such, interpretations of such research for the mathematics-specific context should be made with extreme care and caution.

In mathematics, the notion of the 'Chinese paradox' (Kember, 2000; Kember & Gow, 1990), Haggis (2003) argues, contradicts claims that surface ATLs result in poorer learning outcomes. Research by Kember and colleagues in Asia found that some high-achieving pupils who were able to use memorisation as a route to understanding. Furthermore, Lucas and Mladenovic (2004) contend that there is research in accounting education which suggests that strategies such as memorisation are necessary "to support students' progression to higher levels of understanding" (p. 405). However, to use such findings to question research which suggests a negative correlation between surface ATLs and attainment is another example of a failure to take context into account, something which Haggis (2003) herself criticises others for. In the context of mathematics or in the context of certain assessment, it may be that memorisation can result in a cohesive understanding of a concept, but it is important to describe and analyse

the context specifics alongside such claims. This might not be the same in other subjects, or might not be the same in other groups of students. Furthermore, the definition of a strategic ATL would encompass the use of memorisation as a vehicle for understanding as a student makes use of both deep and surface approaches in order to achieve the best possible outcome in assessment.

Haggis (2003) argues that the strategic ATL does not represent its own, distinct ATL but that it is just "seen as the ability to switch between deep/surface approaches" (p. 91). Whilst it is true that the concept of a strategic ATL is based on the student making use of approaches involved in purely deep or purely surface ATLs, the decision and ability to do this are important factors which make the strategic ATL one which is distinct from the other two rather than being merely a mixture. Case and Marshall (2004) suggest that there is a deep-surface continuum, rather than a dichotomy or trichotomy, because neither deep nor surface ATLs alone fully take into account the complexity of the student's learning process. Again, this is something which can be explored in a context-specific situation, and is one of the reasons why it appears that ATLs research may be best used in situations when there is supportive data from other sources in order to enrich the dry data collected from multiple-choice instruments. It is all well and good being able to analyse students' ATLs, but there is little to go on without being able to describe the contextual factors and influences in the particular situation. This requires research using other methods or samples in order to make use of one group of ATL data.

Indeed, Marshall and Case (2005) claim that administering self-report inventories does not "adequately address contextual subtleties", as "there are contextual nuances and unexpected findings that cannot be captured by this method" (p. 260). This contributes towards an advocation for using such inventories as part of a mixed methods research study which probes the issues regarding ATLs further through other methods. Furthermore, ATL surveys measure *impressions*, rather than *actual* behaviour (Haggis, 2003), which gives them limited

applicability. This compounds general criticisms which have been made of Likert scales[9] research (see Cohen *et al.*, 2007), such as the assumption that the intervals between 'strongly agree' and 'agree' are perceived as the same by a participant as those between 'agree' and 'neither agree nor disagree' (Mitchell, 1997).

The development of instruments to investigate students' ATLs means that the prevalence of ATLs research in the literature encourages some "to see it as encapsulating 'the truth' about student learning" (Marshall & Case, 2005, p. 258). Malcolm and Zukas (2001) criticise the implication in the literature that it is possible to control and/or predict student ATLs, and a number of writers criticise the ATLs framework for becoming increasingly prescriptive (Malcolm & Zukas, 2001; Haggis, 2003, 2009; Webb, 1997). Haggis (2003) bemoans suggestions in the literature that a deep ATL is preferable. She describes encouraging students towards a deep ATL as being akin to encouraging them towards 'elite' goals, which all students may not aim for. However, Marshall and Case (2005) criticise this claim, saying that these goals are crucial "even though many students may find these goals or aims difficult to attain" (p. 262). In the higher education context, students should be endeavouring to take their learning and understanding to the next level and, as such, to be able to engage with academic concepts to an advanced level. How else might they do this than incorporating deep ATLs into their working habits?

Alternative suggestions to the ATLs theory have been suggested, such as that by Mann (2001) who proposed a distinction between experiences of alienation and engagement. She compares alienation with a surface ATL, describing it as an expression of "alienation from the subject and process of study itself" (p. 7). However, whilst this idea does resonate with descriptions of the surface ATL, this does not take into consideration a strategic ATL. Would a student who could be described, on one hand, as adopting strategic ATLs as being partially engaged and partially

[9] Most instruments used to assess students' ATLs use Likert scales.

alienated? That is, a student could tend towards a particular way of learning or studying in order to maintain or promote engagement with the subject, or to avoid alienation. This alienation/engagement distinction is not helpful in a number of contexts, particularly if one wishes to consider relationships between students' *actions* in response to contextual factors such as assessment. Knowing that a student feels alienated or engaged does not tell us anything about what consequence this might have on their working habits. An engaged student might be a student who adopts a deep ATL, but it also might be a student who adopts a strategic ATL.

Therefore, whilst a number of limitations do exist in terms of the theory of ATLs, these can be avoided and controlled for by a careful understanding, and use, of the concepts:

> *We do not see the need to 'throw the baby out with the bathwater', but feel that it is very important for student learning researchers to engage critically with the way in which these concepts are being used.*
>
> (Marshall & Case, 2005, p. 257)

Indeed, debating the terminology and the utility of the ATLs theory in higher education, Coffield *et al.* (2004) write:

> *On the grounds of robustness and econological validity, we recommend that the concepts developed by Entwistle… and others, of deep, surface and strategic approaches to learning… be adopted for general use in post-16 learning rather than any of the other competing languages.*
>
> (p. 52)

2.2 – Question Analysis

The nature of students' approaches to learning is related to the material that they are learning in the first place. That is, it is necessary to consider the types of questions which mathematics students are required to answer in examinations in order to understand the ways in which they approach the learning of mathematics. There are a number of ways of categorising mathematics questions, ranging from broad distinctions to more specific taxonomies.

2.2.1 – Routine & Non-Routine Questions

At different levels of schooling, students are asked questions of varying and increasing levels of complexity and difficulty. In this sense, one could distinguish between 'non-routine' and 'routine' questions[10]. This term appears to have originated with Pólya (1945), who defined a routine problem as follows:

> *a problem is a 'routine problem' if it can be solved either by substituting special data into a formally solved general problem, or by following step by step, without any trace of originality, some well-worn conspicuous example*.
>
> (p. 171)

Routine questions have been found by Berry *et al.* (1999) to form the basis for the majority of marks awarded in A-level Mathematics examinations. Furthermore, when they redistributed the marks on pure mathematics papers to give more reward for solutions to non-routine problems, they found that 297/311 of the scripts analysed would have had reduced marks.

[10] See also Boaler (1997) who describes a conceptual/procedural dichotomy which is very similar to the non/routine distinction.

For example, a routine A-level Mathematics question might be:

The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t \qquad 0 \leq t \leq 4$$

(a) Find:

i. $\frac{dy}{dt}$; *(3 marks)*

ii. $\frac{d^2y}{dt^2}$. *(2 marks)*

(AQA Pure Core 1 January 2010, question 3)

Alternatively, one might consider the following to be a routine undergraduate mathematics question:

Consider the 2×2 *real square matrix* $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$

Show that A has at least one real eigenvalue; and that if $a \neq d$ or $b \neq 0$, then it has two distinct real eigenvalues.

Deduce that A is diagonalisable.

(University of Oxford Algebra I 2010, question 3)

As such questions do not require original thought or application of knowledge in new situations, it is perhaps understandable to find that such questions are those which lower-attaining students are better at (Berry *et al.*, 1999). This is because they do not require the respondent to have a conceptual understanding of the material employed, as it is possible for them to practise, drill-style, similar questions in advance of formal assessment.

Conversely, a non-routine question requires "creative thinking and the application of a certain heuristic strategy to understand the problem situation and find a way to solve the problem" (Elia *et al.*, 2009, pp. 606-607). In order to solve such a problem, a thorough understanding of all of the component concepts would be necessary. Worryingly, studies have found that first-year undergraduate students struggle with non-routine calculus questions despite possessing the knowledge necessary to be able to solve them (Selden *et al.*, 1989, 1994).

However, it is important to caution that a question may be considered routine by one person but not by another as "routineness has to do with what the solver is used to" (Hughes *et al.*, 2006, p. 91). Furthermore, one question may be considered routine in one particular instance, and yet non-routine in another. Undergraduate mathematicians at Oxford have described how they have been set questions in their weekly assignments which require original thought and advanced use of concepts learnt during lectures, only to have identical or similar questions asked of them in final examinations (Darlington, 2010). This then means that the approach to answering such a question becomes very different second time around, i.e.

> *A student who succeeds in proving an unseen theorem is demonstrating an ability to apply knowledge to new situations, but may only be demonstrating factual recall when proving it for a second time.*
>
> (Smith *et al.*, 1996, p. 68)

This means that, whilst students have admitted[11] to 'proving' theorems given in examinations using rote-learning (Darlington, 2010), they may also be capable of doing this in apparently non-routine questions, had they seen similar questions posed in advance. Moreover, it seems likely that the provision of past paper solutions as students revise would increase the possibility of them adopting such a convoluted rote-learning approach.

Rote learning is not of benefit to the student in answering further, non-routine questions, as this knowledge cannot be adapted or manipulated for use in problem-solving contexts (Novak, 1978). Should students resort to memorising for reproduction, they will lack the conceptual understanding necessary to use such a definition or theorem in an attempt to answer more involved, advanced questions which require their application. These questions would be considered non-routine as they cannot be answered through stating what has already been learnt or by carrying out a series of tried-and-tested steps in search of the answer.

[11] These students reported that they believed that this was not what their lecturers and tutors wanted.

This is further compounded by the idea that many mathematics students start their first year viewing the subject as a rote-learning task (Crawford *et al.*, 1994; 1998a; 1998b), something which has been found to continue as they progress through their degree (Anderson *et al.*, 1998; Maguire *et al.*, 2001).

2.2.2 – Taxonomies

In response to criticisms of current assessment formats, specific means by which questions may be classified have been developed. Such classifications are referred to as 'taxonomies', and may be used to help create questions which assess particular skills and concepts according to guidelines set forth by governing bodies or suggested and encouraged by research.

Kadijević (2002) strongly encourages the operationalisation of taxonomies when designing assessment, describing it as "a useful framework", as opposed to "a dogmatic recipe" (p. 101), which can be used to "guide and foster an adequate mathematics learning as well as achieve a comprehensive evaluation of its outcome" (p. 97). Various different taxonomies have been proposed; some designed for general assessment, some for mathematics assessment, and some for tertiary-level mathematics assessment. However, as with most frameworks in education, it is necessary to exercise an amount of caution.

Little published empirical research in this area has validated the use of taxonomies using statistical analysis, meaning that its trustworthiness cannot be properly reflected in students' scores (Kadijević, 2002). However, the most significant difficulty associated with using taxonomies relates to the classification process itself, namely:

- **It is difficult to put certain questions into just one category.**

  Sometimes, more involved questions can include more routine and procedural calculations as part of a more complicated solution.

- **It is difficult to know what skills and thinking are employed by individual students to answer a question.**

  For example, when asked to prove a theorem, a student may do this by one of two means:

  1. Learn the proof by rote and reproduce it from memory when assessed. This would be akin to employing a surface ATL.
  2. Understand the principles, concepts and definitions, and use these to independently develop a proof. This would be achievable by a deep ATL.

Furthermore, there is the additional complexity of revision and pre-examination assessment wherein, if a student is asked to construct a proof on two occasions, they may achieve this via different means on each occasion (Smith *et al.*, 1996). Initially, they may genuinely construct the proof from understanding and, when asked to later, the student may have committed it to memory or internalised what they had done as merely a routine use of familiar procedures which can be adapted for slightly different tasks.

Resnick (1987) cautions against taxonomies which either explicitly state, or suggest, a hierarchy of skills. That is, taxonomies which claim that factual and procedural knowledge is necessary before one can answer a question requiring deeper understanding. Many are cumulative, in that "each class of behaviours was presumed to include all the behaviours of the less complex classes" (Kreitzer & Madaus, 1994, p. 66). Resnick (1987) claims that this type of practice and the "relative ease of assessing people's knowledge, as opposed to their thought processes, further feeds this tendency in educational practice" (pp. 48-49).

More specific to mathematics, where there are gradually increasing moves towards diversifying types of assessment made available to students – for example through coursework or questions which utilise computers or graphic calculators – Huntley *et al.* (2009) have

claimed that general assessment taxonomies are not applicable in this area. In requiring such a different skillset to the humanities and sciences, he claims that general taxonomies "are not pertinent to mathematics" (p. 3) and its idiosyncratic demands and topics. For instance, Duval (2006) asks "Is the way of thinking the same in mathematics as in the other areas of knowledge?" (p. 105).

2.2.3 – MATH Taxonomy

A modification of Bloom's (1956) taxonomy (see Chapter 2.2.4.1) for undergraduate mathematics was made by Smith *et al.* (1996), where their focus was on using a taxonomy to classify the skills required to complete a particular mathematical task. They designed the Mathematical Assessment Task Hierarchy to assist the development and construction of advanced mathematics assessments in order to ensure that students are assessed on a range of knowledge and skills. The taxonomy ensures that students have the opportunity to demonstrate their understanding of mathematical concepts at different levels. Consequently, it appears that it was also designed in order to encourage students to adopt deep ATLs.

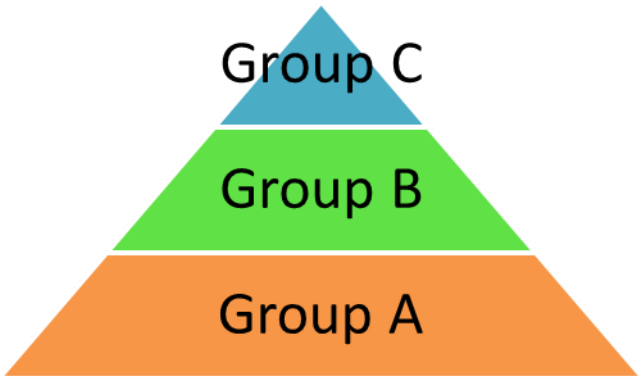


Figure 2.5 - Hierarchy of categories in the MATH taxonomy

The mathematical skills associated with Group C – "those that we associate with a practising mathematician and problem solver" (Pountney *et al.*, 2002, p. 15) – are those which, unfortunately, have been found to be most lacking amongst undergraduate mathematicians (see Chapter 2.3.1).

2.2.3.1 – Categories

The categories in the MATH taxonomy are designed in order to describe the "*nature* of the activity… not the degree of difficulty" (Smith *et al.*, 1996, p. 68). That is, a Group A task may be considered more difficult than a Group C task by a particular student, depending on their perception of difficulty, as well as the particular challenges associated with the task. Example 1 (Group A – Routine Use of Procedures) may be perceived to be more difficult than Example 2 (Group C – Evaluation) by virtue of requiring more complex, time-consuming calculation which, should a mistake be made, can have a detrimental effect on the accuracy of the solution. Further examples and descriptions of each group may be found in Appendix 2.1.

Example 1: Find the inverse of matrix A.

$$A = \begin{pmatrix} 1 & 4 & 3 & 4 & 4 & 4 & 2 & 8 & 1 \\ 0 & 6 & 6 & 6 & 1 & 2 & 1 & 5 & 6 \\ 2 & 0 & 3 & 0 & 1 & 0 & 6 & 0 & 3 \\ 5 & 5 & 1 & 4 & 7 & 8 & 5 & 2 & 3 \\ 1 & 3 & 2 & 2 & 7 & 1 & 6 & 0 & 1 \\ 2 & 7 & 6 & 9 & 0 & 2 & 8 & 7 & 3 \\ 5 & 9 & 0 & 4 & 3 & 2 & 3 & 7 & 0 \\ 3 & 9 & 7 & 1 & 4 & 1 & 0 & 8 & 8 \\ 1 & 6 & 2 & 8 & 0 & 4 & 5 & 5 & 0 \end{pmatrix}$$

Example 2 (Wood & Smith, 2002): The Mean Value Theorem is a powerful tool in calculus. List 3 consequences of the Mean Value Theorem and show how the theorem is used in the proofs of these consequences.

Table 2.6 – The MATH taxonomy

Group A	*Routine procedures*	
	Factual Knowledge & Fact Systems	Bring to mind previously learnt information in the form that it was given.
	Comprehension	Decide whether conditions of a simple definition are satisfied, understand the significance of symbols in a formula & substitute into that, recognise examples & counterexamples.
	Routine Use of Procedures	Using a procedure/algorithm in a familiar context. When performed properly, all people solve the problem correctly and in the same way. Students will have been previously exposed to these in drill exercises.
Group B	*Using existing mathematical knowledge in new ways*	
	Information Transfer[12]	Transferring information from verbal to numerical or vice versa, deciding whether conditions of a conceptual definition are satisfied, recognising applicability of a generic formula in particular contexts, summarising in non-technical terms, framing a mathematical argument from a verbal outline, explaining relationships between component parts, explaining processes, resembling given components of an argument in their logical order.
	Application in New Situations	Choose and apply appropriate methods/information in new situations.
Group C	*Application of conceptual knowledge to construct mathematical arguments*	
	Justifying & Interpreting	Proving a theorem in order to justify a result/method/model, finding errors in reasoning, recognising limitations in a model, ascertaining appropriateness of a model, discussing significance of given examples, recognition of unstated assumptions.
	Implications, Conjectures & Comparisons	Given or having found a result/situation, draw implications and make conjectures and the ability to justify/prove these. The student also has the ability to make comparisons, with justification, in various mathematical contexts.
	Evaluation	Judge the value of material for a given purpose based on define criteria – the students may be given the criteria or may have to determine them.

Adapted from Smith *et al.* (1996)

One of the reasons why Group C tasks have very little attention paid to them may be the amount of time it would take to introduce such skills and nurture their development in students (Leinch *et al.*, 2002). Time pressures, deadlines and targets are frequently blamed for

[12] This category covers a type of understanding consistent with Duval's (2006) description of mathematical activity as consisting "in the transformation of [semiotic] representations" (p. 111).

tendencies to teach for assessment, which can be to the detriment of students' learning. This is even more troubling should students go from one environment where their Group C skills are not fostered into one where they are required and are presumed to be well-engrained in their view of mathematics. Indeed, Leinch *et al.* (2002) argue that "all students deserve the opportunity to attempt projects that develop the Group C-level skills" (pp. 4-5).

2.2.3.2 – Uses

The MATH taxonomy was designed with assessment construction in mind – that is, it was devised in order to provide a framework for ensuring that assessment is varied and tests a variety of skills. Smith *et al.* (1996) claim that assessment tasks "show students what we value and how we expect them to direct their time. Good questions are those which help to build concepts, alert students to misconceptions and introduce applications and theoretical ideas" (p. 66). Therefore, the relationship between learning approaches and assessment (see Chapter 2.1.2.4) is a particularly important factor to consider when designing assessment tasks. During such a process, Smith *et al.* (1996) propose that, when writing assessments, examiners should make a table so that they can see the balance of the tasks that they are setting (see Table 2.7).

Table 2.7 – Designing assessment with the MATH taxonomy

MATH Taxonomy	Q1	Q2	Q3	…
Group A				
Factual Knowledge & Fact Systems	✓			
Comprehension				
Routine Use of Procedures			✓	
Group B				
Application in New Situations		✓		
…				

Leinch *et al.* (2002) complement the taxonomy, describing it "as a useful tool when determining the role of problems posed to students in the development of their mathematical skills" (p. 13). This is particularly the case in areas such as algebra, where computer algebra systems (CAS) can be used to perform the more routine procedures and algorithms that otherwise would engulf the majority of students' study time on particular concepts. The use of such categories as those in the MATH taxonomy therefore is helpful in order to ensure that

students may progress onto studying and developing skills in solving higher level learning tasks (Wood, 2011).

The development of such skills is closely related to the development of particular approaches to learning associated concepts. One of the things that the MATH taxonomy facilitates is the development of deep ATLs (Wood *et al.*, 2002), with students who have a holistic view of a task being more likely to be able to answer Group C questions than those who work in a sequential fashion (Malabar & Pountney, 2002). D'Souza and Wood (2003) suggest that weaker students who gravitate towards surface ATLs may be stretched into developing deep approaches through the development of appropriate assessment tasks as they would be seen "as necessary in order to succeed" (p. 297).

2.2.3.3 – Use in Empirical Research

Ball *et al.* (1998) and Smith *et al.* (1996) applied the MATH taxonomy to a study of existing tertiary-level examination papers and found that the majority that they analysed were "heavily biased towards Group A tasks" (p. 828). Similarly, Etchells and Monaghan (1994; cited by Pountney *et al.*, 2002) found that A-level Mathematics examinations awarded marks mainly for Group A tasks. Indeed, Crawford (1983, 1986) and Crawford *et al.* (1993) found that most new entrants to higher education in Australia were most familiar with Group A tasks, with virtually no experience of Group C tasks.

Whilst the MATH taxonomy was not designed to imply that associated questions were of an increasing level of difficulty, research suggests that students do perceive tasks in Group C to be more difficult than those in Group B, and those in Group B more difficult than Group A (Wood & Smith, 2002). Perceived difficulty appeared to be associated with conceptual difficulty (see also d'Souza & Wood, 2003). Furthermore, familiarity is not necessarily related to perceptions of difficulty, as the students participating in the Wood and Smith (2002) study were able to use

their familiarity with certain question types in order to be aware of inherent difficulties associated with such tasks.

It appears to be the general consensus in the literature that Group C skills should be a consequence of higher education, as these higher-order skills are synonymous with a ""higher education" [which] resides in the higher order states of mind" (Barnett, 1990, p. 202). It has been debated whether assessment which allows students who only have Group A and B skills to pass serves its purpose because "high marks should be reserved for those who have demonstrated that they have acquired Group C level skills" (Pountney *et al.*, 2002, p. 16). Leinch *et al.* (2002) suggest that Group B and C skills should be gradually introduced and utilised in such a fashion that "they become *for that individual* student Group A… tasks because they have developed the insight into the problem-solving process that makes the solution of the problem straightforward" (p. 13).

2.2.3.4 – Example Questions

Published research which makes use of the MATH taxonomy provides various examples of questions which would fit into each group, many of which are applicable to upper-secondary, and many to tertiary-level, mathematics.

Table 2.8 - MATH taxonomy applied to secondary & tertiary mathematics questions

Category	A-level Example	Undergraduate Example[13]
Group A		
FKFS	State the cosine rule. *No examples of FKFS could be found in post-2000 papers.*	Let $f: X \to Y$ be a function and $B \subseteq Y$. Define what is meant by $f^{-1}(B)$, the inverse image of B under f. *University of Bristol Analysis examination 2010*
COMP	Given that $zz^* = 9$, describe the locus of z. *OCR FP3 June 2007*	If the function f is continuous on the interval (a, b) but not bounded then $\int_a^b f(x)\ dx$ does not make sense as a proper Riemann integral. Briefly explain why not. *University of Oxford Analysis III assignment 2012*
RUOP	The equation of a curve is $y = f(x)$, where $f(x) = \frac{3x+1}{(x+2)(x-3)}$. Express $f(x)$ in partial fractions. *OCR FP1 June 2007*	Use L'Hôpital's Rule to find the limit of the sequence $\left(\frac{\log(n^3+1)}{\log(2n^5-1)}\right)_{n\in\mathbb{N}}$ *University of Manchester Sequences & Series examination 2010*
Group B		
IT	Describe a sequence of two geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 50 - x^2$. *AQA C3 June 2009*	Describe, in about 10 lines, the ideas of the Mean Value Theorem. Imagine that you are describing the theorem to a student about to start university. *Wood and Smith (2002)*
AINS	Show, with the aid of a sketch, that $y > \tanh\left(\frac{y}{2}\right)$ for $y > 0$ and deduce that $\operatorname{arcosh} x > \frac{x-1}{\sqrt{x^2-1}}$ for $x > 1$. *STEP III 2008*	Prove that $\max(a, b) = \frac{1}{2}(a+b) + \frac{1}{2}\lvert a-b\rvert$ *University of Oxford Analysis assignment 2012*

[13] Example questions were obtained either online or through personal communication with directors of undergraduate studies in the relevant mathematics departments.

Group C		
J&I	The matrix A is given by $A = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geq 1$, $A^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}$ *OCR FP1 June 2008*	Explain why the Mean Value Theorem does not apply to the function $f(x) = \lvert x + 1 \rvert$ on the interval $[-3, 1]$. *Wood and Smith (2002)*
ICC	The fact that $6 \times 7 = 42$ is a counter-example of which of the following statements? (a) The product of any two integers is odd; (b) If the product of two integers is not a multiple of 4 then the integers are not consecutive; (c) If the product of two integers is a multiple of 4 then the integers are not consecutive; (d) Any even integer can be written as the product of two even integers. *OxMAT specimen test 1 2009*	Prove that if f and g are continuous at x_0, then $\max\{f, g\}$ is continuous at x_0. What about $\min\{f, g\}$? *University of Oxford Analysis II assignment 2 2012*
EVAL	Given a particular function, discuss the accuracy of the trapezium rule in finding the area under the curve. *No examples could be found in post-2000 papers.*	The Mean Value Theorem is a powerful tool in calculus. List 3 consequences of the Mean Value Theorem and show how the theorem is used in the proofs of these consequences. *Wood & Smith (2002)*

2.2.3.5 – Limitations

The main limitations associated with the MATH taxonomy are also common to most other taxonomies. In particular, some tasks involve the use of more than one type of knowledge or activity – "even in the higher-level skills there are some mechanical parts" (Leinch *et al.*, 2002, p. 6). That is, before coming to a greater conclusion, it may be necessary for a student to perform a routine use of procedures or demonstrate comprehension in order to proceed.

Ball *et al.* (1998) give the following example:

> *In your own words, describe each of the following and give an example of each:*
>
> *(i) A vector space*
> *(ii) A subspace of a vector space*
> *(iii) A spanning set*
> *(iv) A linearly independent set*
> *(v) A basis of a vector space V*

Here, asking students to use their own words requires students to demonstrate a *comprehension* of the terms, and then to use information transfer to give their own explanation. The second part of the question which asks for an example then requires the student to use *implications, conjectures and comparisons*.

Furthermore, there may be occasions when answering one particular question may call on different skills from different students. This is particularly the case when constructing proofs, as students with a surface ATL may answer this through rote learning, whereas those with a deep understanding may construct the proof based on an appreciation of definitions and following through an argument. So this could involve Group A skills from the former and Group C skills from the latter. As individual students may view the same mathematical task differently in terms of perceptions of difficulty and approach, perhaps due to their own mathematical experience, classification can also be problematic (Berry *et al.*, 1999). For example, one student may approach proof by induction as a RUOP through having practiced similar versions of the question previously[14] or as J&I through understanding the way in which proof by induction works and using these principles to establish that a statement is true. However, since the MATH taxonomy focuses on mathematical demands rather than difficulty, the impact of student perceptions of difficulty on classification is minimised with this particular taxonomy. Students who are well-prepared for questions of a more demanding type may not find them as difficult as a student who is not.

[14] This is possible at A-level as all specifications have students prove generic sums of series by induction.

The occasion on which the student is asked to prove something may also have an impact. The first time around, the student may demonstrate Group B skills when doing so in an assignment or a tutorial, whereas if the same task was set in an examination later in the year, Group A skills may be called upon.

It is possible that one person will think that a question belongs in one category, whereas another may disagree. In fact, a task may not fit easily into one particular category. Smith *et al.* (1996) address this issue, explaining that they do not aim to permit the classification of every task, but to provide examiners with a means by which to design assessments which call upon a variety of skills and knowledge from students. They say, "the examiner's judgement, objectives and experience… determine the final evaluation of an assessment task" (p. 68). However, this will be the same of any taxonomy as there are bound to be subjective cases in all instances where two individuals may not agree on what category the question belongs in, save for the most basic distinctions.

2.2.4 – Alternative Suggestions

It appears that the MATH taxonomy is the most appropriate to adopt in classifying types of mathematical question at post-compulsory level through its development stemming from classifying questions of this standard, which the other reviewed taxonomies lack either in terms of the level of study or mathematics altogether.

2.2.4.1 – Bloom's Taxonomy

Perhaps the earliest taxonomy developed for educational assessment was that produced by Bloom *et al.* (1956). Describing the purpose behind the taxonomy, they postulated:

> *Some teachers believe their students should 'really understand', others desire their students to 'internalize knowledge', still others want their students to 'grasp the core or essence' or 'comprehend'. Do they all mean the same thing?*
>
> (p. 1)

An outline of Bloom's Taxonomy of Educational Objectives is given in Table 2.11.

Designed for general application across all school subjects, many have deemed it particularly ill-fitting to mathematics (Kilpatrick, 1993; Romberg *et al.*, 1990). Ormell (1974) describes the categories as being "extremely amorphous in relation to mathematics", claiming that they "cut across the natural grain of the subject, and to try to implement them… is a continuous exercise in arbitrary choice" (p. 7).

More generally, it has also been suggested that Bloom's taxonomy fails to identify levels of *learning* as opposed to designing different *types* of question (Freeman & Lewis, 1998), as well as criticisms of claims that the taxonomy is indeed hierarchical (Anderson & Sosniak, 1994).

Furthermore, Pring (1971) has claimed that the cumulative hierarchical nature is flawed, as certain levels in Bloom's taxonomy may be considered, to an extent, interdependent:

> *For something to be recognised as fact requires some comprehension of the concepts employed and thus of the conceptual framework within which the concepts operate. Similarly with regard to the knowledge of terminology, it does not make sense to talk of knowledge of terms or of symbols in isolation from the working knowledge of these terms or symbols, that is, form a comprehension of them and thus an abiilty to apply them.*
>
> (p. 90)

Furthermore, Travers (1980) claims that it lacks sufficient theoretical basis.

A revised version of the taxonomy was developed by Anderson and Sosniak (1994), responding to some of the general criticisms of the original.



Figure 2.9 - Revised version of Bloom's taxonomy



Figure 2.10 - Original version of Bloom's taxonomy

The new version, left, varies through being concerned with the *action* of the student as opposed to the *thing* that they are expected to have at each level. Here,

- **Remembering** is concerned with whether the student is able to recall certain information.
- **Understanding** requires the student to describe and paraphrase certain ideas or concepts.
- **Applying** takes place when a student employs known information in a new situation.
- **Analysing** asks whether the student can discriminate between different parts.
- **Evaluating** requires the student to justify or evaluate something.
- **Creating** a new construct and developing a point of view then falls at the top of the hierarchy in this new version.

Therefore, Bloom *et al.* (1956) classified six different types of educational objectives:

Table 2.11 - Bloom's Taxonomy of Educational Objectives

Category		***Sub-Category***		***Description***
Knowledge	Knowledge of specifics	Knowledge of terminology	Hierarchical	Recall of facts, patterns, methods
		Knowledge of specific facts		
	Knowledge of ways and means of dealing with specifics	Knowledge of conventions		
		Knowledge of trends and sequences		
		Knowledge of classifications and categories		
		Knowledge of criteria		
		Knowledge of methodology		
	Knowledge of universals and abstractions in a field	Knowledge of principles and generalisations		
		Knowledge of theories and structures		
Comprehension		Translation		Understanding
		Interpretation		
		Extrapolation		
Application				Using learnt information in new situations to solve problems
Analysis		Analysis of elements		Breaking information/concepts down
		Analysis of relationships		
		Analysis of organisational principles		
Synthesis		Production of a unique communication	Higher order thinking	Combining different parts to form a whole
		Production of a plan, or proposed set of operations		
		Derivation of a set of abstract relations		
Evaluation		Evaluation in terms of internal evidence		Judging the value of information/methods
		Judgements in terms of external criteria		

Adapted from Krathwohl (2002)

2.2.4.2 – Galbraith & Haines

A taxonomy designed by Galbraith and Haines (1995) was devised with post-design classification in mind, rather than as a tool for designing diverse assessment papers. They described three types of task:

1. **Mechanical** – the use of routine calculations and algorithms;
2. **Interpretive** – constructing conceptual conclusions; and
3. **Constructive** – reaching conclusions through using the skills required to answer mechanical and interpretive tasks without being given the tools through which to do this by the question.

Analysing assessment using such categories can then be used to map the improvement of students' skills, as well as acting "as a measure of mathematical competence", to measure "the overall impact of a teaching programme in deepening the understandings and capacities of students on basic concepts and procedures that underpin its structure" (Galbraith *et al.*, 1996, p. 218).

This taxonomy relies upon an understanding of a dichotomy between conceptual and procedural knowledge. Galbraith and Haines (2000) describe conceptual knowledge as being a consequence of the development of schemas which link existing knowledge together, and then to new concepts. They assume that conceptual knowledge is "stored as a linked network of units, where the more elaborate the network, the more nodes there are for activation to be initiated" (p. 652). Conversely, procedural knowledge is that which is based on repeated attempts at drill exercises "in response to an activating condition" (p. 653). These two types of knowledge are interrelated, as procedures may be added to the schema of knowledge required to develop a conceptual understanding of that particular area of mathematics. Students who lack sufficient conceptual knowledge struggle to retrieve information necessary to complete the task, or the knowledge which they bring to mind is incomplete or inaccurate.

Students who lack sufficient procedural knowledge are those who do not remember rules correctly or make trivial mistakes during the procedure.

Stronger students are better able to do this than their weaker counterparts, who suffer "a greater memory load in consequence of attempting to operate on separate pieces of data" (Galbraith & Haines, 2001, p. 653).

Therefore it may be possible to view mechanical tasks as those which can enable a student to call upon procedural knowledge as specified by the question. Interpretive tasks involve students to use and apply conceptual knowledge, and constructive tasks link both procedural and conceptual knowledge. Consequently, they believe that the order of success in tasks in each of these categories by students is:

Mechanical > Constructive > Interpretive

Furthermore, it is claimed that the constructive and interpretive categories of task are those which are necessary to possess at undergraduate level and, because many students have been found to be lacking in higher-order skills such as those detailed here, "it may be that they need to be articulated and demonstrated explicit for the benefit of [these] students who have experienced only mechanical approaches" (Galbraith & Haines, 2000, p. 667).

Galbraith and Haines (2000) recognise the similarities between their taxonomy and the MATH taxonomy (Smith *et al.*, 1996), describing broad correspondence:

Table 2.12 - Galbraith & Haines' taxonomy

MATH Taxonomy		Galbraith & Haines	Example
A	Factual Knowledge & Fact Systems Comprehension Routine Use of Procedures	Mechanical	$x^2 - \alpha x + 12 = 0$ represents a family of equations. Four members of the family are obtained by giving α the values 5, 6, 7 and 8. For what values of α can the equation be solved by factorizing the left-hand side? a) None b) 5 only c) 6 and 7 d) 8 only e) 7 and 8
B	Information Transfer Application in New Situations	Interpretive	Which of the following could be the equation of the graph shown? a) $y = (x-2)^2(1-x)$ b) $y = (2-x)^2(1-x)$ c) $y = (x-2)^2(x-1)$ d) $y = (x-1)^2(x-2)$ e) none of these
C	Justifying & Interpreting Implications, Conjectures & Comparisons Evaluation	Constructive	The equations of two graphs are $y = 3/x$ and $y = x^2 - 4x$. Obtain a cubic equation whose solution gives the x-coordinate of the point(s) of intersection of these two graphs. How many positive roots does this equation have?

Adapted from Galbraith & Haines (2000)

The development of Galbraith and Haines' (2000) taxonomy as a tool for analysing existing assessment tasks was the reason behind its use by Brown (2010) in analysing mathematics examinations. Its similarity to the MATH taxonomy increased Brown's belief that it was reliable, and he found that very few questions were posed at the constructive level. For Brown, this was even more concerning, as "the testing of mechanical skills predominated both prior to and after the introduction of the graphics calculator" (p. 201), despite the belief that their introduction would remove the tendency to test students' mechanical skills rather than their

ability to apply them. Such findings can then aid the development of more appropriate assessment tasks and act as a means of communicating to departments and institutions that the skills that they are developing in students are perhaps not those which are the most constructive. Its adoption may then enable examiners to target "specific attributes involved in the co-ordination of algebraic and graphical representations" and to tackle "inadequate knowledge networks, improperly learned connections, or faulty production rules" which may stand in the way of students developing their mathematical thinking (Galbraith *et al.*, 1996, p. 219).

2.2.4.3 – Assessment Component Taxonomy

This was designed for use when classifying tasks given by alternative assessment formats such as those involving multiple choice questions and CAS.

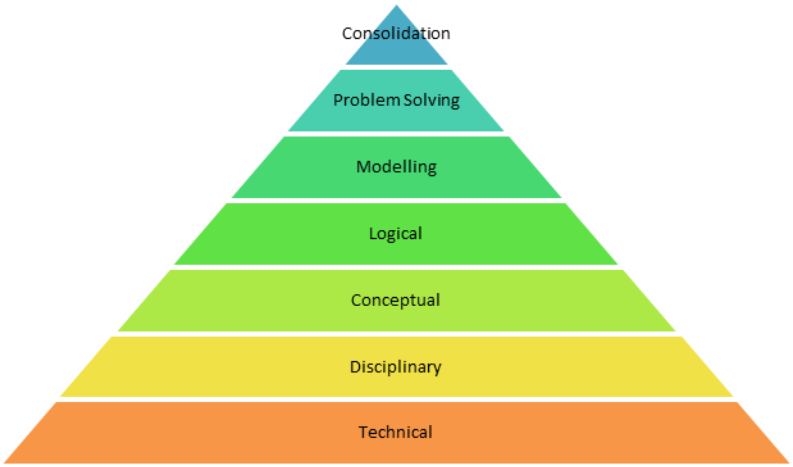


Figure 2.13 - Hierarchy of categories in the Assessment Component Taxonomy

Huntley *et al.* (2009) claimed that this was necessary as more 'traditional' taxonomies do not take into account such questions. With it becoming increasingly common in mathematics education – at all levels – to assess students' understanding using less traditional means, they wanted to be able to find which mathematical components would be best assessed using which means. They found that multiple-choice questions do not necessarily have to be solely used to assess lower-level cognitive problems which can be solved using surface ATLs.

However, attempts to assess students' ability to utilise higher-level thinking using such question types can prove difficult. In fact, "the more cognitively demanding conceptual and problem solving assessment components are better for" more traditional questions which require students to find a solution themselves (p. 14).

The authors propose the 'assessment component taxonomy' of seven 'mathematics assessment components' which are in a cognitive level-based hierarchy which also considers associated mathematical tasks.

In this taxonomy, skills which require a deep ATL are found at the opposite 'end' to those which require a surface approach.

Table 2.14 - Assessment Component Taxonomy

Mathematics Assessment Component	Cognitive Skills	Related MATH Taxonomy Components	Related Bloom's Taxonomy Components
Technical	Manipulation Calculation	Group A	Knowledge
Disciplinary	Recall (memory) Knowledge (facts)		
Conceptual	Comprehension: algebraic, verbal, numerical, visual (graphical)	Group B	Comprehension
Logical	Ordering Proofs		Application
Modelling	Translating words into mathematical symbols	Group C	Analysis
Problem Solving	Identifying and applying a mathematical method to arrive at a solution		Synthesis
Consolidation	Analysis Synthesis Evaluation		Evaluation

Adapted from Huntley *et al.* (2009, pp. 5-6), and Huntley (2008, p. 136)

2.2.4.4 – SOLO Taxonomy

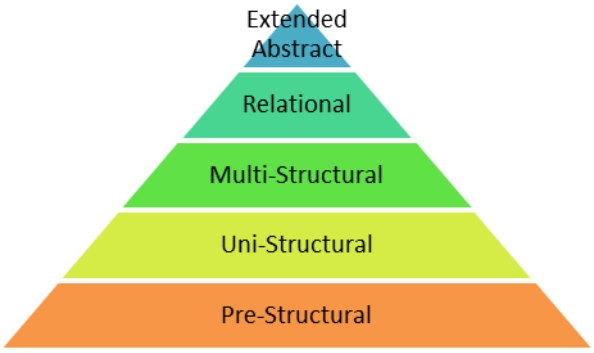


Figure 2.15 - Hierarchy of categories in the SOLO taxonomy

Biggs and Collis (1982) proposed the Structure of the Observed Learning Outcome (SOLO) taxonomy which implies increasing levels of structural complexity in their understanding. The SOLO taxonomy's hierarchy extends from pre-structural to extended abstract knowledge. Designed to be applicable to a variety of secondary-level subjects, they proposed that extension was possible in order to apply it to higher-level and different thinking and skillsets. Concerning itself with categorising cognitive performance rather than skills required to answer questions, Boulton-Lewis (1994) suggests that undergraduate students "should ideally develop knowledge in their discipline areas that is organised structurally at the relational… level" on the SOLO taxonomy (p. 389). This taxonomy can be used in order to establish what learners "know and believe about their own learning" and to determine their "knowledge of a discipline at the time they enter" (Boulton-Lewis, 1995, p. 146).

Table 2.16 - SOLO taxonomy

Pre-Structural	The response has no logical relationship to the display, being based on inability to comprehend, tautology or idiosyncratic relevance.
Uni-Structural	The response contains one relevant item from the display but misses others that might modify or contradict the response. There is a rapid closure that oversimplifies the issue.
Multi-Structural	The response contains several relevant items, but only those that are consistent with the chosen conclusion are stated. Closure is selective and premature.
Relational	Most or all of the relevant data are used, and conflicts resolved by the use of a relating concept that applies to the given context of the display, which leads to a firm conclusion.
Extended Abstract	The context is seen only as one instance of a general case. Questioning of basic assumptions, counter examples and new data are often given that did not form part of the original display. Consequently, a firm closure is often seen to be inappropriate.

Adapted from Biggs (1979, p. 387)

As with many other assessment task taxonomies, the SOLO taxonomy has been found to be associated with student ATLs. As learners progress from pre-structural to extended abstract levels of knowledge, a deep ATL becomes increasingly necessary (Boulton-Lewis, 1994; Trigwell & Prosser, 1991b; Watkins, 1983). In fact, the SOLO taxonomy has been used in conjunction with the Study Processes Questionnaire in order to establish whether relationships exist between study processes and the types of questions posed to students in a diverse number of subjects including educational studies (Biggs, 1979), medicine (Pandey & Zimitat, 2007), physics (Prosser *et al.*, 2000), accounting (Ramburuth & Mladenovic, 2004), physiotherapy (Tang, 1994), nursing (Cholowski & Chan, 1992) and biology (Hazel *et al.*, 2002; van Melle & Tomalty, 2000; Zimitat & McAlpine, 2003). Furthermore, the categories in the SOLO taxonomy have been found to be related to ability and student motivations (Chan *et al.*, 2001). From such connections, the SOLO taxonomy can then be used in order to devise and develop curricula in order that students are assessed in such a way that they may achieve the desired learning outcomes of their educators.

Whilst the SOLO taxonomy has not been applied to mathematics education as frequently as it has other disciplines, advanced topics in algebra have had the SOLO taxonomy applied to them (Chick, 1988; Coady & Pegg, 1994, 1995), as well as postgraduate study of mathematics (Chick, 1998). However, the literature in this area is very sparse. Biggs (1979) describes the SOLO taxonomy as being best suited to situations when one is required to learn "the meaning of a finite display of information and [make] judgements about that information – a piece of prose, a map, a moral dilemma, a poem, a mathematical problem, etc." (p. 384). However, in reality, a mathematical problem is not always as involved as Biggs may have been thinking – problem solving is a type of mathematical task, rather than the only thing required of learners in mathematics. The reader may note that the language used to describe each category in Table 2.16 is not especially relevant to the study of mathematics.

More generally, the structure of the SOLO taxonomy has been criticised for being overly ambiguous in its descriptions of its categories and having rather 'blurred' divisions between each category (Chan *et al.*, 2001). It has been suggested that the introduction of sub-categories "would reduce ambiguity and increase… inter-rater reliability when applying it" (Chan *et al.*, 2002, p. 518) which, in reality, would bring the taxonomy more in line with the MATH taxonomy (see Chapter 2.2.3).

2.3 – Secondary-Tertiary Mathematics Transition

2.3.1. – The Nature of Mathematics

2.3.1.1 – Advanced Mathematics

Literature about the difference between secondary and tertiary mathematics is littered with words such as 'rigour', 'abstract', 'conceptual', 'creative', 'deductive' and 'formal'. Indeed, university mathematics – or, rather, advanced mathematics – has been frequently identified as presenting students with a very different challenge to that taught to them at school. For example, Gueudet (2008) argues that, at school, "students just have to produce results. At university, they seem to have an increasing responsibility towards the knowledge taught" (p. 240). This takes the form of applying what they have been taught in a creative fashion which should ultimately allow them to construct proofs of mathematical statements and conjectures, this being the basis and aim of advanced mathematics. Indeed, Edwards *et al.* (2005), who define advanced mathematical thinking as "thinking that requires deductive and rigorous reasoning about mathematical notions that are not entirely accessible to us through our five senses" (p. 17).

The secondary-tertiary mathematics transition has been described as definable by the formalisation of mathematical concepts, something which "involves the construction of... new mental object[s] which... [are] different from, and therefore may conflict with, the old objects. It causes the long period of confusion which first-year university students meet and is a significant barrier to formal mathematical thinking" (Robert & Schwarzenberger, 1991, p. 129). Such a change involves cognitive reconstruction to enable students to deal with the increasingly abstract nature of the mathematics being studied. It has been found that some students can fall victim to transitional changes, resulting in the exhibition of signs of disaffection with mathematics (Daskalogianni & Simpson, 2002).

In a study of the secondary-tertiary mathematics interface, Kajander and Lovric (2005) found that students' school experiences often shape study approaches at undergraduate level. These stemmed from their beliefs about mathematics which were that mathematics is a rule-based subject requiring the learner to memorise facts and algorithms (Anderson *et al.*, 1998; Crawford *et al.*, 1994, 1998a, 1998b; Schoenfeld, 1985). This mismatch may then cause students to hit a stumbling block when faced with atypical questions and problems (Schoenfeld, 1985). It also means that students cannot progress successfully whilst continuing to rely on "the acceptance of definitions and the recollection of procedures", since "more abstract courses require a deeper understanding of relevant concepts" (Entwistle & Meyer, 1992, p. 593). Lithner (2003) describes how students find it much more difficult to succeed through merely memorising formulae and procedures at undergraduate level, as those involved are far more complex, and their applications much more involved.

Tall (1991a) established a body of work which attempts to describe more precisely what is involved in mathematical thinking at university level. He describes advanced mathematical thinking as a cyclical process where "the creative act of considering a problem context in mathematical research... leads to the creative formulation of conjectures and on to the final stage of refinement and proof" (p. 3).

Specific difficulties have been found to lie in students' understanding of mathematical proof (Bell, 1976; Harel & Sowder, 1998; Selden & Selden, 2003; Tall & Vinner, 1981; Weber, 2001), particularly in real analysis, and even with the best students (Selden *et al*, 1994). The introduction given to secondary school pupils to undergraduate-level mathematics of proof – generally taking the form of proof by induction of sums of series – has been criticised for failing to develop a deep understanding of the concept in students' minds, as "few examination questions are set which demand any depth of understanding or which require any creativity in the process of justification" (Anderson, 1996, p. 129). Indeed, students describe the

production of such proofs as an algorithm to be followed (Darlington, 2010) as "it has been taught (and rote-learned) as a formal technique" (Anderson, 1996, p. 134). At university, "the concepts themselves are also radically different from the student's previous experience; they often involve not merely a generalisation but also an abstraction and a formalisation" (Brousseau, 1997, p. 128). This often results in students "memorising theorems and proofs at the possible expense of meaning or significance" (Jones, 2000, p. 58), taking a surface approach which is far from what Tall describes.

Indeed, the abstract nature of advanced mathematics has the potential to cause students great difficulty in adapting their ways of thinking. Learners are required to change their ways of thinking "from *describing* to *defining*, from *convincing* to *proving* in a logical manner based on those definitions. This transition requires a cognitive reconstruction which is seen during the university students' initial struggle with formal abstractions as they tackle the first year of university" (Tall, 1991a, p. 20). Thus, students who failed to construct appropriate concept images (Tall & Vinner, 1981) for algebraic concepts were found by Ioannou and Nardi (2009) to feel overwhelmed by their courses, finding that their excitement and enjoyment began to subside and evolve into "puzzlement and, to some extent, resignation from effort to understand" (p. 39).

Rodd (2002) describes mathematics as 'hot' because it stirs up emotions in the learner, particularly at undergraduate level. Reflecting on her longitudinal research with undergraduate mathematicians in the UK, she comments that undergraduate mathematics is initially presented "without fuzziness or debatable results", with "adrenalin-producing" assessment which leaves students more intellectually and emotionally exposed than in other subjects (p. 2). Rodd and Bartholemew (2006) comment that "mathematics is a troublesome subject: it has an aura of being important, hard, boring, high status and challenging" (p. 35). Indeed,

mathematics is amongst a group of undergraduate courses which has been found to have the heaviest workloads (Ramsden, 1983), and such courses have high entry requirements.

Perrenet and Taconis (2009) accuse school mathematics of hindering undergraduates' mathematical careers as its culture "differs [so] significantly from the professional culture of mathematics" (p. 183)[15]. Moreover, Watson (2008) contrasts mathematics as a discipline and a subject to be studied:

> *Mathematics as a discipline, by contrast to school mathematics, is concerned with thought, structure, alternatives, abstract ideas, deductive reasoning and an internal sense of validity and authority. It is also concerned with uncertainties about ways forward in its own realms of enquiry.*
>
> (p. 6)

School mathematics could be viewed as "inculcating a purely knee-jerk response in students" (Bibby, 1991, p. 43) because it places a stronger emphasis on the rehearsal, calculation and the implementation of algorithmic procedures than undergraduate mathematics (Bibby, 1991; Boesen, 2006). This then may result in a "(false) belief that, given sufficient time and study, there will be an algorithm that will solve any given problem" (Ervynck, 1991, p. 52). Since students' ability to apply what they have learnt at school in terms of their mathematical understanding, learning approaches and conceptions of mathematics to the undergraduate setting is essential in their success with the subject at tertiary level (Wood, 2001), they often understandably experience a 'bump' in their educational path (Perrenet & Taconis, 2009).

2.3.1.2 – The Undergraduate Curriculum

As a consequence of the mathematics covered by the secondary curriculum and criticisms of it as preparation for undergraduate study, the university mathematics curriculum has had to evolve in order to take account of new students' understanding of mathematics. This often

[15] See also papers from the 'Disciplinary Mathematics & School Mathematics' working group at ICMI Rome 2008.

takes the form of a renewed focus on topics away from the pure to the applied, the introduction of bridging courses and the introduction of continuous assessment (Kahn & Hoyles, 1997). However, students will inevitably eventually realise that "the central question [of the mathematics they are studying] changes from "What is the result?" to "Is it true that…?"" (Dreyfus, 1999, p. 106). Whenever this comes, it is necessary for learners to experience a cognitive change whereby they are no longer required to merely use "visual conviction and proceptual manipulation", but to work with "defined objects and formal deduction" (Tall, 1995, p. 173).

For example, Lewis and White (2002) found that presenting familiar material to new undergraduates can be problematic as "they already have a successful framework for dealing with the material to answer questions and unless the new information or formalism complements this, students often find it hard to understand why they can no longer answer questions that they could at A-level" (p. 14). They illustrate this with an example of a question which may be taught and answered in different ways at school and university:

> *Question*
>
> *A family has two children. Given that at least one of the children is a boy, what's the probability that both are boys?*
>
> *A-level solution*
>
> *Two children families, assume* $\mathrm{P}(B) = \mathrm{P}(G) = \frac{1}{2}$ *and gender of the children is independent. Families are BB, BG, GB, GG.*
>
> $$\mathrm{P}(BB|\text{at least 1 boy}) = \frac{\mathrm{P}(BB \cap \text{at least 1 boy})}{\mathrm{P}(\text{at least 1 boy})} = \frac{\mathrm{P}(BB)}{\mathrm{P}(\text{at least 1 boy})} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$
>
> *Undergraduate level solution*
>
> *Let* $\Omega = \{(BG), (BB), (GB), (GG)\}$ *where* (BG) *means oldest is a boy, youngest is a girl etc.*
>
> *Assume all outcomes are equally likely,*

Let E be event both are boys, so $E = \{(BB)\}$

Let F be the event there is at least one boy so $F = \{(BG), (BB), (GB)\}$

$$\mathrm{P}(E|F) = \frac{\mathrm{P}(E \cap F)}{\mathrm{P}(F)} = \frac{|E \cap F|}{|F|} = \frac{1}{3}$$

(p. 16)

This suggests that secondary mathematics, questions and solutions appear to use less notation and rigour. Therefore, the thinking and assessment which must go with this material must follow accordingly.

2.3.2 – Conceptions of Mathematics

Mathematics students have been found to enter undergraduate study with qualitatively different conceptions of mathematics, with many viewing it as a rote learning task (Crawford *et al.*, 1994, 1998a, 1998b), even during later years of study (Anderson *et al.*, 1998; Maguire *et al.*, 2001) based on earlier experiences of a mathematics which they defined as an exercise in memorising rules and repeating procedures. Such methods encourage surface ATLs, which should not be transferred across to undergraduate territory. At this level, the material studied necessitates cognitive reconstruction, as suggested by Tall (1991a) and Lithner (2003).

Asking students to define mathematics based on their experiences so far, Crawford *et al.* (1994) were able to separate responses into five categories:

Mathematics is:
- numbers, rules and formulae;
- numbers, rules and formulae which can be used to solve problems;
- a complex logical system and way of thinking;
- complex logical system which can be used to solve problems;
- a complex logical system which can be used to solve

> problems and provides new insights used for understanding the world.
>
> (p. 335)

The majority of students' responses fell into the first two rather fragmented definitions of mathematics (see also Solomon, 2007b), with a later study finding that those students with more cohesive conceptions tend to adopt more appropriate approaches to learning undergraduate mathematics (Crawford *et al.*, 1998). The categorisation above is not a clear-cut distinction, as there are many studies which suggest that even talented students lack a fundamental knowledge of mathematics (LMS, 1995).

Such a misunderstanding is perhaps understandable when the contrasting nature of school and undergraduate mathematics is explored. The *academic* transition can be viewed as a significant transition itself notwithstanding the general life transition as its formalisation involves a change "from describing to defining, from convincing to proving in a logical manner based on these definitions" (Tall, 1991a, p. 20).

2.3.3 – A-Level Criticism

Caution should be exercised when reading this section in terms of interpreting comments from referenced sources. Think tank reports and opinion pieces are amongst the literature referred to when discussing the present state of A-level Mathematics, a subject of hot debate.

Furthermore, the situation with A-level Mathematics is very fluid and constantly evolving as a consequence of the work of the Education Select Committee and advice from bodies such as the Advisory Committee on Mathematics Education (ACME). For example, at the time of writing, plans have recently been put in place to re-introduce linear A-levels. Therefore, this section of the literature review describes the state of post-compulsory mathematics at one point in time and, as such, references to it may become moot in the future.

2.3.3.1 – A-Level Administration

The majority of English, Welsh and Northern Irish candidates applying to UK universities receive offers based on their A-level results. A-levels are comprised of the Advanced Subsidiary (AS) level, typically sat in Year 12 of secondary school, and the corresponding 'A2' sat in Year 13. Together, the component marks from three modules in each year are combined to form an A-level grade; however, students are able to walk away after the first year of study with an AS qualification if they wish. The majority of offers for undergraduate places are based on three, full A-levels (see Appendix 2.2), but it is becoming normal practice for students to study four subjects in their AS year, before 'dropping' one subject after AS so that they may concentrate on three subjects through to the full A-level.

Up until this academic year, the Qualifications and Curriculum Authority stipulated what should be covered in the A-level, allowing the exam boards to draw up their own specifications for each subject. These had to be approved by Office of Qualifications and Examination Regulation (Ofqual) before they could be rolled out in order to ensure that standards remain consistent (Bassett *et al.*, 2009). As of 2013, examination administration will be conducted by the Standards and Testing Agency following the government's educational reforms. These reforms followed research commissioned by Ofqual which resulted in the publication of the 'Fit For Purpose?' report (Higton *et al.*, 2012) which concluded, amongst other things, that modular A-level Mathematics does not test students in their use of mathematical tools on a synoptic level, and that students are not permitted the time to properly develop their mathematical skills.

2.3.3.2 – Participation

Whilst past research and statistics have regularly commented that there are problems with participation in mathematics post-16, leading to the production of various reports (e.g. Smith, 2004) and development of schemes and initiatives, science and mathematics have recently

enjoyed increases in participation as a consequence of what Vasagar (2011) calls the 'Brian Cox effect'. These increases are shown in Figure 2.17. Levels have not yet climbed back to those in the mid-1980s when mathematics was at its most popular (Matthews & Pepper, 2007); however, it should be remembered that the A-level now serves a very different function to that which it did then:

> *Ever since their introduction, A-levels have been associated with entry into higher education. This remains a valid and useful application. But over time they have also acquired a broader significance as a precursor to employment and as one strand in a qualifications framework which is designed to recognise the full range of advanced achievement of which young people are capable, ranging from the purely academic and theoretical learning through to the skills and knowledge associated with specific jobs.*
>
> (Tomlinson, 2002, p. 10)

The 'mathematics problem' (see Chapter 1.1) also intensified after Curriculum 2000 when the A-level was 'split' into AS and A2 (Brown *et al.*, 2008).

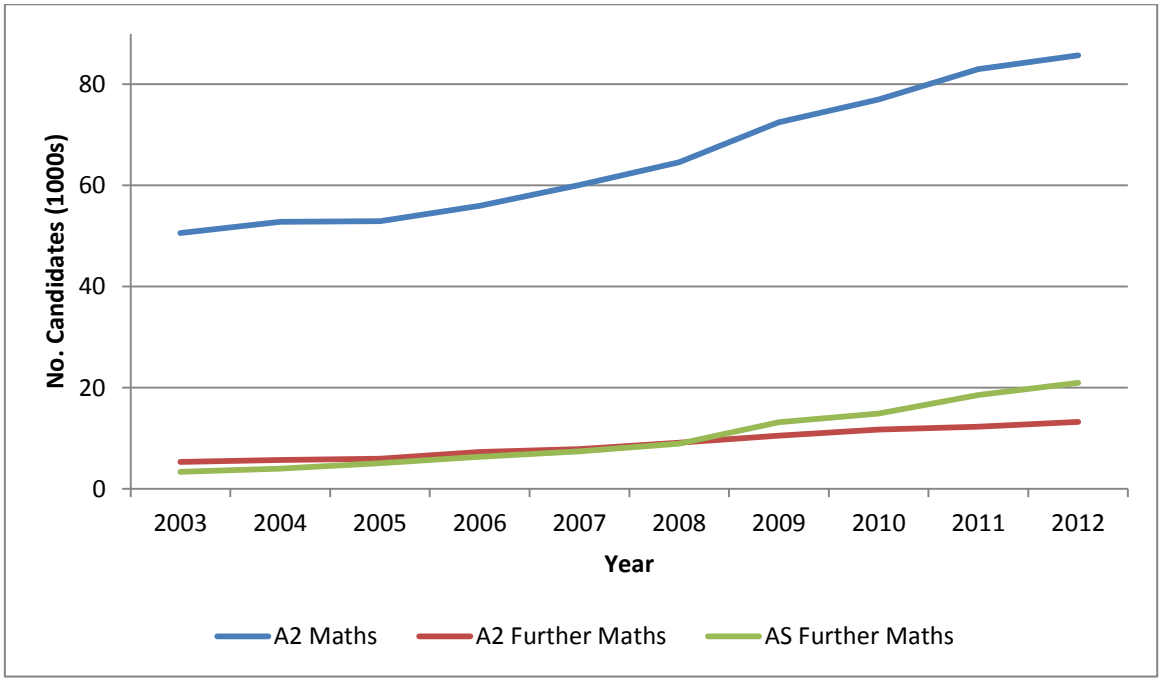


Figure 2.17 - Candidates of A-Level Mathematics over the last decade
Source: JCQ (2012)

Approximately one in seven A-level Mathematics students also study Further Mathematics (Smith, 2012), part of a steady increase over the last decade.

2.3.3.3 – Examination Boards

The three most commonly-used examination boards in England are Edexcel, the Assessment and Qualifications Alliance (AQA) and the Oxford, Cambridge and RSA Examinations (OCR) (House of Commons Education Committee, 2012), each of which offer very similar syllabi and examination types as a consequence of the A-level administration system. Newton *et al.* (2007) conducted a review on the standards and demands of each examination board which concluded that there are no discernible differences between those currently in operation. This was also concluded by Taverner (1996), who compared students' A-Level Information Systems[16] (ALIS) scores with their A-level results in order to see whether there were any differences. None were identified; however, this study investigated different examination boards to those which are in operation today (when there were eight), as well as being conducted well before Curriculum 2000 and the changes which followed it in terms of the format and syllabi offered by the boards.

However, the House of Commons Education Committee (2012) recently published a report which concluded that standards in A-level examinations are decreasing because of competition between the different examination boards to be selected for use in schools and colleges. They suggest that Ofqual should play a more central role to ensure that one syllabus is dictated to the boards, from which they may set questions[17].

2.3.3.4 – Reforms

In the 1980s, A-level Mathematics was assessed by examinations taken at the end of the second year of study which looked at a mixture of the topics covered over the course of the

[16] ALIS tests are run by the Centre for Evaluation and Monitoring at the University of Durham. The scores act as performance indicators for post-16 students, using data from GCSE grades and their own baseline tests.

[17] This situation remains very fluid and related reports and comments are constantly produced.

two years. However, a modular system was later introduced, which has been widely criticised for a variety of reasons (see Chapter 2.3.3.6).

One of the most recent adaptations made to A-level Mathematics in 2004 involved the redefinition of the pure mathematics modules on offer. Old specifications described Pure Mathematics 1-3; however, these have now been restructured and renamed as *Core* Pure Mathematics 1-4. There is also now the requirement to study only two applied[18] mathematics modules instead of three, compounding the already diminishing content of the Mathematics A-level (Bassett *et al.*, 2009). Porkess (2003) described this as a "disaster for mathematics" (p. 12), also claiming that the content of A-level Mathematics had decreased by one-sixth in the new syllabus (Porkess, 2006). Criticisms abounded that the new A-level was failing to educate students in sufficient mathematical concepts which, in turn had a negative impact on students' preparedness for undergraduate study (Smith, 2004).

2.3.3.5 – Standards

A-level standards are inevitably debated on an annual basis when examination results are released (Warmington & Murphy, 2007). It was in 1982 that this became possible, when a cap on the percentage of students who were allowed to achieve each grade was scrapped, meaning that it became possible for more than ten per cent of candidates to achieve an A-grade (Fee *et al.*, 2009). Therefore, the concept of 'grade inflation' was born as an increasing proportion of students began to achieve the highest grades.

[18] Statistics 1-3, Mechanics 1-3 and Decision Mathematics 1-2 are the available options.

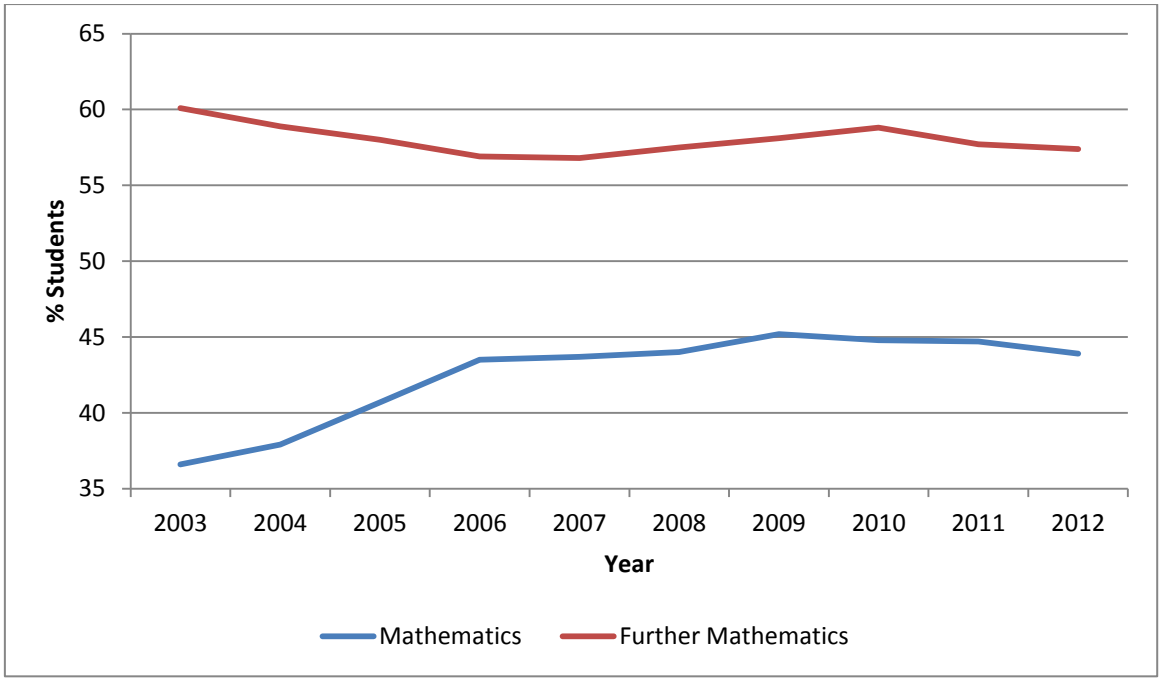


Figure 2.18 - A* & As in A-Level Mathematics
Source: JCQ (2012)

The 'decrease' in inflation from 2010 visible in Figure 2.18 comes in the same year that the A* was introduced at A-level for students awarded over 90% on the uniform mark scale across their A2 units. Figures in the graph post-2009 are for cumulative A* and A-grades.

It has been argued that it is neither practical nor actually possible to ascertain whether examinations are becoming easier because more students are achieving A-grades (Newton, 1997; Patrick, 1996). Furthermore, Fee *et al*. (2009) caution that "although today's candidates might perform very poorly on an exam of 10 years ago, candidates prepared for the older examination could equally struggle with today's papers" (p. 44).

Despite this, various studies have suggested that students with similar abilities would have achieved very different A-level grades now to in the past. Coe (2011) claims that A-level grades for equally able students have increased by a tenth of a grade each year since 1988. Research regarding such a change often stems from comparison of ALIS test scores with actual A-level grades; however, Lawson (1997) utilised the diagnostic tests already in use at Coventry University in order to ascertain whether students' competence was changing throughout time.

He found that “there has been a noticeable decline over the period from 1991 to 1997 in the competence of students in certain fundamental mathematical topics” (pp. 156-157), claiming that a student achieving a grade N in 1991 would be of a similar ability to a student achieving a C-grade in 1997. Similarly, in 2002 he drew similarities between B- and N-grade students from 1999 and 1991, respectively.

2.3.3.6 – Criticism

Criticism of A-level Mathematics in educational research literature and the media is rife, and was heightened after the syllabus changes after Curriculum 2000. Unprecedented coverage of examination results was reported in 2002, with the media’s concern over standards becoming increasingly prolific since 2003 (Warmington & Murphy, 2007). Bassett *et al.* (2009) describe the new A-levels as ‘ersatz’, arguing that they have the potential to stifle independent study and thinking. They claim that Curriculum 2000 “damaged their intellectual integrity” (p. 5) through an increasing mechanisation of students and examination, which caused further difficulties at the secondary-tertiary interface when students are more likely than ever before to struggle in transitioning to tertiary study because of their lack of subject knowledge upon leaving school. They describe the students of such A-levels as ‘high maintenance’, unable to properly learn and understand thanks to ‘backwash’ (see Chapter 2.3.3.6) and the type of assessment they face.

Bassett *et al.* (2009) argue that the current examinations at A-level are described as allowing “candidates less scope for using their own mind”, with the failure of including universities in designing subject specifications leading to inappropriate subject content which will not sufficiently prepare students for university study (p. 10).

Syllabus & Specification

The revised syllabus introduced in 2004, as well as being criticised for the dissemination of pure mathematics modules, was considered by many teachers to be easier than its

predecessor (QCA, 2007). Additionally, the change in syllabus had an impact on the ways that examiners marked students' examination scripts. Bassett *et al.* (2009) contrast the 1952 and 2008 syllabi, describing a shift from a syllabus "written in normal text, in paragraphs", to one with "a numbered list, with very precise details as well as "curriculum objectives"" (p. 14). They describe the new syllabus as more of a contract[19] the students which "cannot promote genuine learning" (p. 14), as it describes a set of things that students must do and know for reward, rather than requiring of them a conceptual understanding of various topics in mathematics.

A report by *Reform* (Bassett *et al.*, 2009), an independent, right-wing think tank, described the decreasing emphasis on geometry and proof at A-level through examining the 1952, 1960, 1980, 1990 and 2000 specifications. Asking R.A. Bailey, Professor of Statistics at Queen Mary, University of London for comment, *Reform* reported her condemnation of the questions set for students, saying that they "seem to have been set by people who do not understand the subject" (p. 14). Questions, she said, are not practical in terms of demonstrating the practicality of applying particular mathematical concepts in the real world, with some statistics questions liable to encourage school-leavers to apply certain concepts inappropriately and poorly later on.

In 2002, Advanced Extension Awards (AEAs) were introduced to stretch more able students and allow them the opportunity to show their understanding of various mathematical concepts. However, the mathematics AEA will be withdrawn in 2015 due to the introduction of the A*, with AEAs in all other subjects having ceased in 2009 due to low demand (Baird & Lee-Keeley, 2009). This response relies on the A* giving universities a better indication of potential students' mathematical abilities[20]; however, it removes a source for students to demonstrate further understanding which could set them apart from other applicants to undergraduate

[19] This is consistent with the idea of a 'didactic contract' (see Chapter 2.3.4.2).
[20] This is something which is disputed by ACME (2012).

courses, as well as denying them the opportunity to explore mathematics more deeply and begin to develop a synoptic understanding of the subject and the topics that they were covering for their A-level. Grades – a distinction, merit or fail – in AEAs were once used as part of entry requirements from certain universities who would decrease their typical offer in favour of success in the AEA.

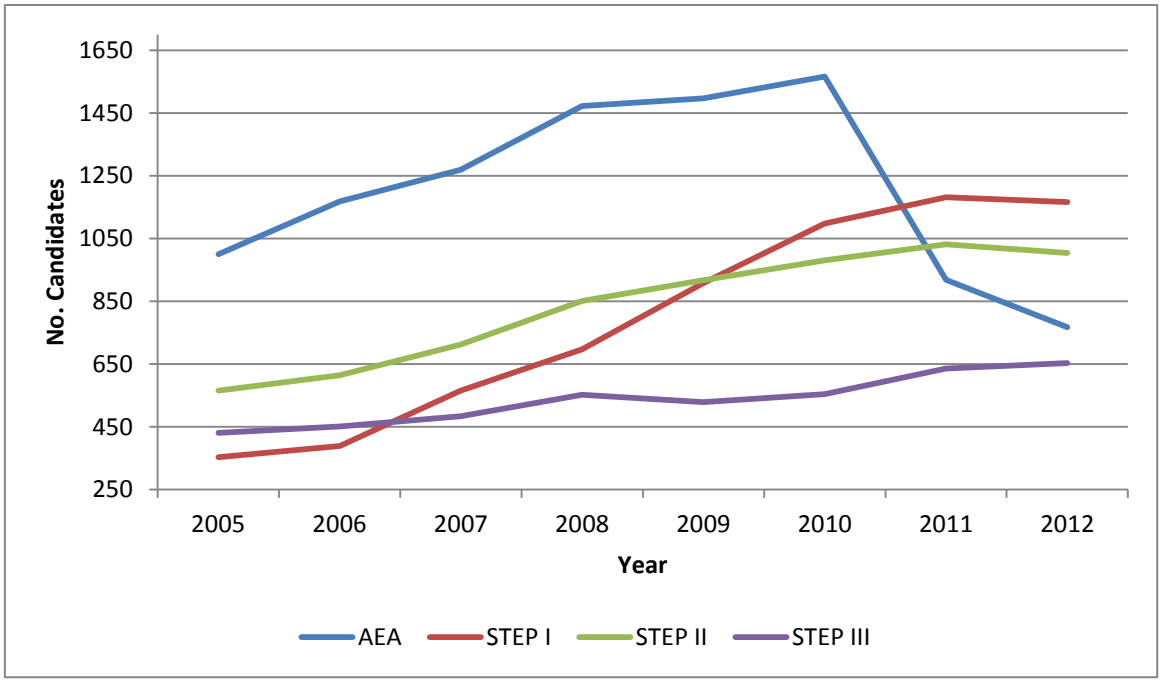


Figure 2.19 - Candidates sitting STEP & AEA examinations
Sources: Cambridge Assessment (2012b) & JCQ (2012)

However, the Sixth Term Extension Papers (STEP) will continue to function as a means of stretching the most able students. There are three different levels of STEP, with Cambridge Assessment, who administer the tests, describing them as serving three functions:

1. *It acts as a hurdle*
   A good STEP grade is generally considered indicative of mathematical potential.
2. *It acts as preparation*
   Students' understanding of the mathematical concepts covered in the papers is tested in a similar fashion to tertiary mathematics.
3. *It tests motivation*
   To do well in a STEP examination, it is important that students do plenty of preparation.

(Cambridge Assessment, 2012a)

Figure 2.19 shows the sharp decline in candidates of AEA mathematics after the introduction of the A*, whereas uptake for STEP examinations has continued to increase. This may be a consequence of some universities beginning to include STEP grades as part of a typical requirement for entry onto their mathematics courses (see Appendix 2.2).

Question Layout

University professor R.A. Bailey's criticism, reported by *Reform*, of the new A-level extends to the way in which questions are posed to students. She describes new examinations as being "more like using a sat-nav system than reading a map" (Bassett *et al.*, 2009, p. 12); that is, she believes that the new papers are heavily structured in such a way that students are spoon-fed the particular calculations and steps required to answer the question, without them requiring a more holistic understanding of *what* to do before doing it to find the answer. She describes an evolution of A-level questions since 1951 as examinations moved from posing single questions to those split into increasingly small sub-questions which indicated the marks awarded for each part (see Table 2.21). The modular system, she claims, caused a further degeneration in which questions posed became "orders: do this, do that", instructing students on what methods to employ in order to answer the question (Bassett *et al.*, 2009, p. 12).

Consequently, students of the newer syllabi have been found "to be able to do bite-size, piecemeal mathematics, but seem unable to see the bigger picture" (Quinney, 2008, p. 5). Indeed, Quinney compared A-level questions posed four decades apart and concludes that "It would be easy to draw the conclusion that A-level is getting easier" (p. 3). Both of the questions in Table 2.20 are worth a similar number of marks.

Table 2.20 - Comparison of A-Level Mathematics questions in 1968 & 2006

1968 Mathematics 1 Advanced Level, Cambridge Examination Board	2006 Core Mathematics 1 A-Level, OCR Examinations
i) Differentiate with respect to x: (a) $\log_e \sin^2 3x$, (b) $\frac{\sqrt{(x^2-2)}}{x}$ ii) If, at time t sec., the velocity v ft./sec. of a particle moving along the axis of x is given by the formula $v = e^t + 4e^{-2t}$, and if at time $t = 0$ the particle is at the origin, find an expression in terms of t for its distance from the origin at time t. Find the time at which the acceleration of the particle will be zero, and the velocity and position of the particle at this instant.	i) Solve the equation $x^4 - 10x^2 + 25 = 0$ ii) Given that $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$ find $\frac{dy}{dx}$ iii) Hence find the number of stationary points on the curve $y = \frac{2}{5}x^5 - \frac{20}{3}x^3 + 50x + 3$

Adapted from Quinney (2008)

Furthermore, Fee *et al.* (2009) concur with Bailey's comments (Bassett *et al.*, 2009), claiming that A-level examinations "are much shorter and far more highly structured than those they will face in higher education" (p. 50).

Caution should be taken when making interpretations based on Table 2.21. Their analysis of the evolution of question types posed to students at A-level was not grounded in any particular taxonomy or theory and these are only specific examples.

Table 2.21 - Evolution of A-Level questions, 1951-2008

Year	Question
June 1951	Prove the formula $s = ut + \frac{1}{2}at^2$ for uniformly accelerated motion in a straight line. The motion of a train between two stations A and B is in three stages. In the first stage the train starts from rest at A and moves with constant acceleration. In the second stage it moves with constant speed and in the last stage it has constant retardation and comes to rest at B. If the times taken over the three stages are in the ratio $6:14:4$, show that the average speed is four-fifths of the maximum speed and that the distance travelled with constant speed is three-quarters of the distance AB.
June 1970	Three fixed buoys A, B and C form an equilateral triangle of side 8 kilometres. The buoy B is due east of A and the buoy C is to the north of the line AB. A steady current of speed 4 kilometres per hour flows from west to east. A motor-boat which has a top speed of 12 kilometres per hour in still water does the triangular journey $ABCA$ at top speed. Find, graphically or otherwise, the time taken on each leg of the journey giving your answers in minutes.
June 1990	[In this question take the value of g to be $10m/s$] From a point O on horizontal ground a particle is projected with speed $200\ m/s$ at an angle of elevation α above the horizontal. The particle moves freely under gravity. The horizontal lower surface of a cloud is $500\ m$ above the ground. Find the smallest value of α such that the particle would reach the cloud and, for this value of α, find the distance from O at which the particle would strike the ground. It is given that $\alpha = 60°$. Find the length of time for which the particle is above the lower surface of the cloud. Find also the speed of the particle at the instant when it enters the cloud.
June 2000	Two railway trucks A and B are moving in the same direction on the same straight horizontal track, and they collide. Truck A has mass $20{,}000\ kg$ and truck B has mass $10{,}000\ kg$, and immediately before the collision their speeds are $1.5\ m/s$ and $1\ m/s$ respectively. When the trucks collide they are automatically coupled together. Find the speed of the trucks immediately after they collide. After they collide, a braking mechanism exerts a resisting force of magnitude $5000\ N$ on the leading truck B, and the trucks slow down. Calculate i) the deceleration of the trucks ii) the magnitude of the force exerted on the leading truck B by the second truck A while the trucks are decelerating.
June 2008	A model train travels along a straight track. At time t seconds after setting out from station A, the train has velocity $v\ m/s$ and displacement x metres from A. It is given that, for $0 \leq t \leq 7$, $x = 0.01t^4 - 0.1t^3 + 0.72t^2$. After leaving A the train comes to instantaneous rest at station B. i) Express v in terms of t. Verify that when $t = 2$ the velocity of the train is $1.28\ m/s$. ii) Express the acceleration of the train in terms of t, and hence show that when the acceleration of the train is zero, $t^2 - 8t + 12 = 0$. iii) Calculate the minimum value of v. iv) Sketch the (t, v) graph for the train, and state the direction of motion of the train when it leaves B. v) Calculate the distance AB.

Adapted from Fee *et al.* (2009, pp. 45-48)

Such comparisons do appear to be fair when looking at the evolution of examination questions over the years, with the more recent questions certainly being typical of those analysed for the

purposes of this thesis (see Appendices 2.3 and 2.4). The compartmentalised nature of questions posed to students at present would appear to favour weaker students through offering them the opportunity to answer part-questions which dictate exact methods and requirements, which means that they may earn marks for answering easier parts of the available questions. Interestingly, however, Porkess (2006) claims that those students who "obtained low grades actually knew quite a lot of mathematics but were not given the opportunity to show it" (p. 8). If we look at the examples of questions posed pre-Curriculum 2000, that seems to be a surprising assertion as the nature of those questions requires students to think for themselves and decide upon a method by which to find the answer – problem-solving – whereas the more recent examples require students to operate on a step-by-step basis as they are guided through part-questions which can often be answered independently of each other.

Modular System

Hirst and Meacock (1999) describe the views on a modular examination system in A-level Mathematics as varying "from "the best thing since sliced bread" to "the work of the devil"" (p. 122). The literature certainly appears to be very critical of assessing students' understanding in this way, through failing to ask of students a synoptic understanding of the concepts that they have studied, as well as permitting them to resit examinations and 'play the system' in order to achieve grades which may not be an accurate representation of their overall mathematical ability. Wilde *et al.* (2006) argue that the modular system encourages "a commodification of knowledge" amongst A-level pupils in the "sense that they want to move on, get the badge" (p. 9). Furthermore, "anecdotal evidence" exists which suggests that one of the driving forces behind the introduction of the modular system was "that it is easier to attain higher grades" (Taverner & Wright, 1997, p. 111). In fact, the changes to the structure of A-level Mathematics in 2006 decreased the content considerably, leading Porkess (2003) to

assert that “There will be those who try to claim that this is not a diminution in standards”, but “I am not one of them” (p. 15). Furthermore, the presentation of A-level Mathematics in modules, argues Smith (2004), “makes it virtually impossible to set genuinely thought-provoking examination questions that assess the full range of mathematical skills” (p. 94). Moreover, Hodgson and Spours (2004) claim that modular examination negative impacts teaching because it encourages teaching to the test. Furthermore, the introduction of linear examination from 2015 gives an indication of the popular viewpoint on modular examination.

However, whilst the number of criticisms which could be made of the modular A-level, it would be unfair to suggest that it is without its merits:

> *There is of course a danger in developing a critique of modularization of both throwing the baby out with the bathwater and of over-romanticizing the past as some golden age when teachers did not teach to the test, and learners did not learn to pass an exam but simply struggled to understand.*
>
> (Hayward & McNicholl, 2007, p. 345)

Indeed, a modular A-level does have a number of advantages:

- The ability to only do a subject just to AS-level encourages “young people to attempt subjects that would otherwise have seemed too daunting, especially science and mathematics” (Hayward & McNicholl, 2007, p. 340).
- Assessment can be spread over a longer period of time, reducing the burden and pressure on students at the end of their second A-level year.
- Teachers can give better predictions of final A-level grades based on results from previous A-level examinations which can “help students make more informed choices when applying for higher education courses” (Taverner & Wright, 1997, p. 105).
- Resits give students a second chance to prove themselves.

- Modular A-level examinations "make qualifications transparent, reduce barriers to progression and maximize access, flexibility and portability" (Hayward & McNicholl, 2007, p. 336).

Synoptic Assessment

Higton *et al.* (2012) describe one particular mathematics admissions tutor as not favouring the modular system on account of it not encouraging a synoptic understanding, failing to require of students "complex problem solving requiring the application of several complimentary mathematical techniques" (p. 81). Indeed, if the examples of piecemeal questions described earlier and in Table 2.21 are believed to be typical, it is understandable how this may propagate such a failure to develop a holistic understanding, even within each module. Furthermore, admissions tutors interviewed in Higton *et al.*'s (2012) study expressed a belief that the modular system "meant that there could be no surprises and the examinee could 'learn the exam' rather than the subject'" (p. 58). Suggestions made to introduce a synoptic paper appear to be sensible in forcing students to synthesise their knowledge and understanding of a group of concepts in order to demonstrate a conceptual understanding, a practice which used to be implemented by science A-levels until 2010.

Choice of Modules

One of the consequences of the modular system is that students then have the opportunity to choose the modules that they study, within reason. Certain pure mathematics modules are core to the subject and must be studied by all students; however, the mixture of applied modules is a choice left to the student[21]. Bassett *et al.* (2009) say that a negative consequence of this for each new undergraduate cohort – and undergraduate teachers – is that students then enter university having covered a different number of modules in certain areas, with differing depths of understanding of certain mathematical consequence. This "presents a

[21] This may be impacted by the teaching offered at the school, or the restrictions of timetabling.

headache for universities" that are then forced to cover non-compulsory areas of the mathematics in the first year which some students may already be familiar with (p. 13). However, most A-level Mathematics students tend to do particular groups of modules, with Mechanics being the most popular optional module (Ward-Penny *et al.*, 2013).

Resits

Students' opportunity to resit individual modular examinations has been criticised for allowing students to give an inaccurate representation of their mathematical understanding. Fee *et al*. (2009) claim that "there is no doubt that judicious early takes and retakes boost an individual's changes of fulfilling potential" (p. 44). This is compounded by the fact that grades may be achieved through the total 'score' earned in each module, this aggregation system meaning that a high score on an easier, earlier module sat at AS-level can then help to even out a lower score at A2 in a more involved module.

Resitting is common, as is "mark-grubbing" (ACME, 2012, p. 15), where students resit papers that they have already gained reasonable scores on in order to gain even higher marks to present themselves with a buffer. Bassett *et al*. (2009) claim that such a practice then leaves new undergraduates with the mistaken belief that resits are normal practice at higher levels, expecting "to be able to retake exams in which they feel they have underperformed" (p. 18).

Ability

Hirst and Meacock (1999) looked at entrants to Southampton University when modular and non-modular A-level examinations were available for study to see if there was any difference in new students' mathematical ability, by looking at scores in assessment once they got to university. They found no significant difference between those who sat modular examinations and those who did not.

However, Taverner (1996) compared ALIS data for students doing modular and non-modular examinations to see what grades they predicted and if they were different between the two groups:

> *for students of equal prior achievement, as measured by their average GCSE score, following an A-level course which offers modular assessment allows them to gain a result half a grade higher than if they had followed a non-modular course.*
>
> (Taverner, 1996, p. 109)

This study appears to be more trustworthy in the sense that ALIS scores are calculated on a very consistent basis and they are routinely used to ascertain whether there are changes in standards amongst university entrants, with scores on such tests acting as reasonably accurate predictors of A-level grades achieved by students. Furthermore, their study drew from a larger pool of data, looking at a greater number of students going on to study mathematics at a range of universities, not relying on results of undergraduate assessment which may differ from university to university.

Backwash

There are widespread claims that current forms of A-level assessment in mathematics facilitate rote learning in students (Alton, 2008). Furthermore, new undergraduate students are believed to lack the capacity to do wider reading and independent study in order to develop an understanding of the concepts that they are studying on account of the fact that they are so used to being taught to the test and being bound by very prescriptive syllabi (Reisz, 2008).

The notion of 'teaching to the test' is something which has already been discussed as being a negative impact on students' understanding, as well as a potential consequence of the forms of assessment that secondary school pupils do. Daly *et al*. (2012) discuss this in terms of 'backwash', an expression used by Popham (1987) to describe measurement-driven instruction. Backwash in the UK education system has been attributed to targets being set by

schools which are driven by league tables of public examination results. Bassett *et al.* (2009) contend that this target-driven teaching strives for "results at any cost rather than widening students' understanding of a particular subject" (p. 26). The increasingly compartmentalised nature of A-level Mathematics – in terms of modules and the piecemeal layout of questions – and the resulting backwash "encourages short-term learning and not long-term understanding and students are unlikely to see the many connections between different areas of mathematics" (Taverner, 1997, p. 198), which can only go on to have severely negative consequences at the undergraduate level where students are expected to draw on a combination of mathematical concepts in their studies of both new and unfamiliar topics.

Students themselves have described characteristics of backwash in their own accounts of A-level study, recounting a reliance on past papers and their being "trained by their teachers to perform in examinations, with teachers conscious that this training was an important aspect of their role" (Daly *et al.*, 2012, p. 151). In fact, Gordon and Rees (1997) believe that teachers are able to teach students to pass any type of test. It is a small consolation to know that both teachers and students in Daly *et al*.'s (2012) study were aware that this strategic approach to teaching and learning had many limitations, as this sets up students with various expectations of the way in which they would be expected to behave in their learning at undergraduate level. It is an example of a secondary school didactic contract which must be broken at tertiary level (see Chapter 2.3.4.2).

Marking

Backwash may even be impacted by the transparency of assessment procedures. A-level mathematicians are "heavily directed in answering questions with rigid marking schemes and "assessment objectives" making it clear exactly what is expected" (Bassett *et al.*, 2009, p. 5). Since students are able to access subject specifications, which in turn facilitates rote learning on the part of the students and 'coaching' on the part of the teachers, Torrance (2007) claims

that students are challenged even less. Furthermore, he claims that this results in an 'assessment *as* learning', in that the majority of students' learning experiences are defined by practising sample questions, taking part in regular assessment and rehearsing procedures ahead of these.

In her interviews with *Reform* (Bassett *et al.*, 2009), Professor Bailey describes the mark schemes for A-level Mathematics as being problematic:

> *The mark scheme is rigidly broken down into single marks. One cannot avoid the suspicion that the main reason for the change to the "sat-nav" type of question is to enable consistent marking from people who may not be trusted to actually understand the mathematics.*
>
> (p. 14)

In fact, an investigation into the strategies that examiners used when marking GCSE Mathematics by Greatorex and Suto (2006) revealed some worrying trends. The most common strategy adopted was 'matching'. This method is characterised by the examiner comparing "the letter(s)/number(s)/single word/part of diagram written by the candidate on the short answer line/pre-determined spot in the answer space with those given in the mark scheme" (p. 8). Rather than 'evaluating', 'scrutinising' or 'scanning' students' responses, this has the potential to permit non-specialists to mark examination scripts, leaving students at risk of not being properly rewarded for their use of mathematics. It is perhaps the case that this type of marking and mark scheme is adopted as a time-saving measure, as examiners often have hundreds of scripts to mark at any one time. This may also account for why students are not stretched with deeper, more probing questions which require them to explain and evaluate particular mathematical concepts. Borrowing terminology from the MATH taxonomy (see Chapter 2.2.3), it could be that Group C tasks are more time-consuming to mark, and would require deeper, more specialised knowledge of the topic being tested in order to properly assess the students' answers.

2.3.3.7 – Relationship with Universities

Porkess (2006) hits out at claims that the A-level continues to serve universities in providing them with students with a sound mathematical basis and a reliable indication of their mathematical understanding, claiming that "it is manifestly not doing so" (p. 8). The apparent discrepancy between what students actually know post-A-level and what their lecturers expect them to know when they begin university study "will, at the very least, impair the quality of their education and, at the worst, may prove too difficult for them to bridge" (Lawson, 1997, p. 151). As the content of A-level Mathematics has continued to change throughout the decades, universities have made a number of concessions to change, such as course restructures becoming common in many universities after Curriculum 2000. The four-year undergraduate MMath in mathematics was introduced in 1992 on the recommendation of the LMS and Neumann report, which claimed that changes were necessary in order to respond to: (1) changes in the secondary mathematics curriculum; (2) the continuing growth of mathematics; and (3) to ensure that undergraduate qualifications in the UK could remain comparable with those in other countries (Neumann, 1992, p. 186). Furthermore, Porkess (2003) recommended that attention should be paid by universities to A-level reforms, and that they should be prepared to respond to them if only in terms of their entry requirements, if not in terms of their teaching and syllabi. This is something which will apply when the changes come into force for the 2015 examination sessions.

University involvement in A-level syllabi has steadily decreased since the 1980s "due to changing priorities and a shifting of academics' financial incentives, primarily to research" (Bassett *et al.*, 2009, p. 24). The apparent consequences of this and complaints raised by academics over the content of A-level Mathematics and the way in which students are assessed clearly indicate that their re-involvement is necessary. This sentiment is shared by Fee *et al*. (2009) and Bassett *et al*. (2009) and such wishes appear to have been granted by the government: as of the 2014-2015 academic year, universities will begin to play an active part

in determining the content of A-levels as well as the way in which they are assessed (House of Commons Education Committee, 2012). *Reform* even go so far as suggesting that universities "be able to veto exam boards' specifications if they are not sufficiently rigorous or do not require the right content or the development of the correct skills" (Bassett *et al.*, 2009, p. 27), although the general involvement of universities in A-level curriculum is supported wholeheartedly by ACME (2012). Whether or not this would be a practical outcome that could serve the needs of students at either side of the secondary-tertiary interface remains to be seen, as a compromise over such issues is unlikely to be easily achieved.

2.3.4 – Pedagogy

2.3.4.1 – Students' Expectations

Many of the problems experienced by undergraduate mathematicians stem from inaccurate expectations they have of the subject, with various studies such as that by Hirst *et al.* (2004) highlighting the impact of expectations in terms of students' experiences of the secondary-tertiary interface. It has therefore been suggested that universities be aware of the challenges faced by would-be undergraduates so that they may take action in alleviating some of the strain on students by providing them with "timely and effective assistance... during the transition period from school to university to assist them with the management of stress associated with academic concerns" (Jones & Frydenberg, 1999, p. 3).

Various factors have been found to fail to meet with students' expectations, particularly those relating to undergraduate pedagogy. The majority of students will go from having interactive classroom experiences to going to lecture theatres where they are seldom asked to contribute or interact with the lecturer and other students which can be, for some, a very frustrating change (Sierpinska *et al.*, 2008). At most universities, lectures are the main form of teaching and delivery of material, perhaps supplemented by support classes or tutorials. However, at Oxford, the regular, small-group tutorial forms the seminal basis for study.

2.3.4.2 – Didactic Contract

Didactics refers to activities which are used in order to teach, with Vergnaud (1990; cited by Egéa-Kuehne, 2003) defining it as "the study of teaching and learning processes pertaining to a particular domain of knowledge: for example a discipline, a trade, or a profession. It rests on pedagogy, psychology, [epistemology] and of course the discipline studied. But it cannot be reduced to that" (p. 348). The undergraduate experience is synonymous with the use of lectures, which have often come under fire (e.g. Bligh, 1972; Fritze & Nordkvelle, 2003; Holton, 2001; Leron & Dubinsky, 1995). For example, Laurillard (1993) criticises them for failing to offer an interactive or adaptive learning environment, burdening students with the task of reflecting on the content of the lecture and their prior knowledge, then resolving any differences between these themselves. This, of course, is no mean feat when the undergraduate mathematics lecture often involves students copying notes from the blackboard whilst trying to synthesise this physical process with the mental process expected of them. It then seems that lectures have little to offer other than a method by which information may be transferred, leaving students little or no opportunity to think about the concepts concerned (Bligh, 1972).

Hence, the concept of the lecture and what it requires from the lecturer and the student are part of the 'didactic contract'. This term was coined by Guy Brousseau (1988) at the beginning of the 1970s and developed by Chevallard (1988) to refer to a series of mutual understandings and reciprocal expectations held by both teacher and learner concerning any knowledge to be taught. The contract is rarely made explicit between both parties, with it constantly evolving through their interactions with each other (Brousseau, 1997). From this, it can be understood that the didactic contract established between teacher and pupil whilst learning school mathematics is quite different to that required between lecturer and student at university.

In their younger years, learners may come to expect what Boaler and Greeno (2000) call 'didactic teaching', which is commonplace in many schools, whereby "students come to class, watch teachers demonstrate procedures, and then practice the procedures – alone" (p. 177). More recently, teaching in England has become increasingly interactive and exploratory; indeed, to do well in inspections lessons have to contain significant discussion and student participation (Ofsted, 2010). The extent to which this happens in higher-level mathematics classes is unknown but, with the best school teaching, there is likely to be a difference between typical lesson expectations at school and typical lecture expectations at university.

Traditionally, at undergraduate level "the teacher is usually obliged to present the notions in a lecture course before getting the students to work with them: there seems to be no question of allowing, or making, the students (re)discover certain aspects of the notions before they are formalised" (Robert & Schwarzenberger, 1991). Whilst this often suffers from criticism (Maclellan, 2005), such as for "presenting the subject as if it was just a set of rules that needed to be learnt" (Thomas & Holton, 2003, p. 351), it is important to note that at this level it is not always possible to rediscover concepts in an accessible way, with it being seemingly impossible to make a rigorous concept of convergence "accessible to students in which the $\varepsilon - \delta$ definition is likely to be constructed spontaneously" (Robert & Schwarzenberger, 1991, p. 129). Perry (1970) looked at the development of didactic contracts from the student perspective, finding that they vary from (1) "my teacher knows the truth, and is responsible for telling it to me clearly; to (2) in some areas there is a certainty, and in some, only opinion is backed up by reasoning; my job is to learn how to justify my opinions, and to examine critically those of others" (Mason, 1988, p. 169). The different didactic contracts drawn up by different students in the same classroom means that it is important to consider the reasons behind this. For example, might students belonging to one particular 'unofficial' community of practice (see Chapter 3) have a different perception of the didactic contract to those in another community of practice?

The didactic contract is also written with other factors in mind which constitute what Brousseau (1997) referred to as the *situation didactique* (didactical situation) which is based on a constructivist approach whereby "the didactic contract is the rule of the game and the strategy of the didactical situation" (p. 31). This involves "the classroom ethos, as well as the social and institutional forces acting upon that situation, including government directives such as a National Curriculum statement, inspection and testing regimes, parental and community pressures, and so on" (Mason & Johnston-Wilder, 2004, p. 79). Consequently, the need instilled into teachers to meet targets often has an influence on the ways in which they teach and as such influences the didactic contract forged between themselves and their pupils.

2.3.4.3 – Understanding

Part of this unspoken contract, which may be induced by a need on the part of teachers to get pupils to get an answer correct and pass an examination, may involve a need to encourage an instrumental understanding (Skemp, 1976; see Chapter 2.1.3.6). Having an instrumental understanding of mathematical concepts at this high level means that learners are often unable to effectively construct proofs about that concept, with Weber's (2002) study finding that doctoral students were able to construct all proofs about group isomorphisms that were set whereas undergraduates with instrumental understandings were not. This suggests that, in order to advance to higher levels in mathematics study, it is necessary to have a full relational understanding of mathematical concepts.

The difficulties encountered by some undergraduates may stem from the instrumental nature of the school didactic contract, which is difficult to leave behind when entering the undergraduate environment. It has even been suggested that such a contract may be *established* at university since the examinations often ask students to correctly state definitions and theorems, which can be rote-learnt.

2.3.4.4 – Approaches to Learning

At university level, it is unsurprising to find that students who adopt surface ATLs or have instrumental understanding have a greater tendency to sink than swim; Ramsden (1992) found that students who fitted this description tended to find it more difficult to adapt to undergraduate life. This is not always helped by the pedagogy implemented by teachers in higher education. Biggs (1999) criticises this as a form of 'institutionalisation', which inadvertently encourages students to learn in more superficial ways in response to the modes of assessment they expect to be given. However, it is possible that the tutorial-driven approach at the University of Oxford may go some way to prevent students from responding to assessment in such a superficial way, through regularly meeting with a mathematics research fellow who is duty-bound tutorial to provide their tutees with mathematical stimulation, questioning them and developing their mathematical thinking.

Students can finish school, and hence enter university study, with a belief that a type of learning akin to a surface ATL is expected of them and that it gets the right results; they have seen it work by getting a high grade in A-level Mathematics to gain them entry onto an undergraduate mathematics course. This is not always the fault of the student who has never known any different; but it will cause them to find it difficult to get to grips with tertiary-level mathematics. Daskalogianni and Simpson (2001) would classify this as an instance of when students' beliefs about mathematics overhang (see also Chapter 2.3.2). Their research of new undergraduate mathematicians found that students often found it difficult to adapt their styles of working and learning, yet were more likely to edit their beliefs concerning the didactic contract and the new context. It is this 'breaking' of the contract that often results in problems for students.

2.3.4.5 – Renegotiating the Didactic Contract

Breaking, or sometimes *renegotiating*, the didactic contract (Brousseau & Otte, 1991) comes when the expectations set out in the contract are challenged either by the teacher or by the learner. This frequently results in distress on the part of the learner who is unprepared and ill-equipped to deal with this change. This seemed to have occurred after the implementation of a new first year analysis course as part of the Warwick analysis project (Alcock & Simpson, 2001) whereby students embarked on a course which had them working in small groups in small classes with a teacher and two peer tutors. Meeting twice a week in two hour blocks, students were given questions to answer as part of an assessed portfolio based on a text by Burn (1992), with questions encouraging students "to develop the mathematical content and argument for themselves" (Alcock & Simpson, 2001, p. 109). This teaching experiment was met with much praise, with evidence to suggest that students in the experimental group had come away from the course with a deeper understanding of analysis than their lecture-taught counterparts. However, there was an element of resistance by participants which suggests the need to negotiate a new didactic contract. Upon assessing the course, one student said "What I would really like is if we could have a lecture, and then be given a set of questions based on the lecture, and do it in class" (Alcock & Simpson, 2001, p. 104). This student clearly looks back on the school classroom didactic with fondness, preferring a particular method of teaching and learning. The course aimed to 'bridge the gap' between school and university mathematics, though apparently did not meet the expectations of some. In doing so, this module creates an opportunity for renegotiating the contract with students, encouraging them "to amend their evolutionarily developed general cognitive strategy" (p. 109) in order to begin to get to grips with the rigorous nature of advanced mathematics.

It is in mathematics, both at secondary and tertiary levels, that the didactic contract seems to have the greatest impact and significance, with Stodoksly *et al.* (1991) suggesting that the heavy reliance on teachers by pupils is much greater here than in other subjects. Mathematics

has been highlighted as being a very different subject to the arts at undergraduate level, with students having very different expectations of lectures. Mathematics and physics students have alternative motives for going to lectures to their arts counterparts (Evans & Abbott, 1998) which means that they have very different didactic contracts; hence, mathematics students do not expect discussion and debate in their lectures and so may rebel if met with this.

Students may also be let down, to a certain extent, by any didactic contract that is formed at school. This comes from a paradox associated with the concept whereby "everything the teacher attempts in order to make the student produce the behaviour" the teacher expects; hence, living up to their end of the contract "tends to deprive the student of the necessary conditions for the understanding of the target notion" (Blanc, 1995, p. 10). Therefore, a situation develops whereby a student accepting the contract means that learning cannot take place. "To learn, for him, implies [rejecting] the contract, and to accept being himself engaged in the problem. In fact learning will not be based on the correct functioning of the contract, but rather on breaching it" (Brousseau, 1984, p. 115). In a sense, it encourages behaviourist tendencies:

> *The teacher looks for certain tell-tale behaviour, as does the examiner. The pupil seeks to provide that behaviour. Soon the focus is on the behaviour, not on the inner state which gives rise to behaviour. The dilemma is then that everything the teacher does to make the pupil produce the behaviour the teacher expects, tends to deprive the pupil of the conditions necessary for producing the behaviour as a by-product of learning: the behaviour sought and the behaviour produced become the focus of attention.*
>
> (Mason, 1988, p. 168).

This then promotes the adoption of inappropriate ATLs since the development of a didactic contract is synonymous with learners having a clear idea of what they are to do; hence "they will find ways of doing it that will avoid internalising, appreciating, or realising... what the task was intended to do... in their urge to complete tasks" (Mason *et al.*, 2005, p. 151). It is such

tendencies that educators should aim to avoid, with emphasis in recent years coming from the need to have learners *construct* knowledge for themselves. This was an idea picked up on in the design of the Warwick analysis project (Alcock & Simpson, 2001), with an emphasis placed on students developing an ability to construct meaning for themselves, which is more in line with typical sixth-form teaching which tends to be more interactive than undergraduate teaching.

2.3.4.6 – Constructivism

Connections between the didactic contract and constructivism are also evident when renegotiating contracts, with the notion of 'match' and 'fit' (von Glasersfeld, 1987) seeming significant. Reading a set of correct answers from the learner may suggest an understanding of a particular mathematical concept which may not actually exist. Here, the understanding demonstrated by the learner *fits* with that of the teacher, but it does not necessarily *match* it. At undergraduate level, this works slightly differently. The lecturer can never assume that the student's knowledge matches theirs due to their advanced academic background; however, they can ask that the student's knowledge matches with their expectations as set out in the course syllabus and implied in an examination. A student with knowledge *fitting* what is expected of them might be able to reproduce proofs that they have memorised and recite definitions that they have been given, as well as following familiar procedures in questions requiring application of certain theorems. The student gets a good examination result so, on the surface, they appear to understand the concept. What they might actually have is a surface ATL; however, this may require greater effort on their part since this approach is associated with a rapidly expanding schema which involves an increasing number of facts, rules and procedures to remember. Conversely, a student whose understanding comes close to *matching* that of the lecturer can write similar proofs and can derive an understanding of certain concepts by going back to first principles.

2.3.4.7 – A 'Perfect Pedagogy'?

This is the understanding that academics want students to develop, with various teaching experiments devised in order to ease students into accepting the new didactic contract. The Warwick analysis project (Alcock & Simpson, 2001) acted as a way of bridging the gap, with the structure of lectures and introduction of supervisors being helpful since students on the normal course often "felt that, unlike in school, there was no-one easily accessible to help them" (p. 105). The notion of the didactic contract was used largely in some studies which looked at the training of lecturers (e.g. Hardy & Hanley, 2002; Legrand, 2001), encouraging them to reflect on personal experience in order to improve their own practice. The researchers had them try to make a "distinction between the 'game of the teacher' and 'the game of the learner'" (Legrand, 2001, p. 525) by introducing the notion of the didactic paradox using an absurd problem paradigm:

> *In an elementary school, the teachers ask Paul who is 8 years old the following question: "You have ten pens in each of your pockets, how old are you?" Paul answered, "I am 20".*
>
> (p. 525)

The impact of the contract is clear in this instance, with the one to which Paul thinks he is adhering to being built on the fact that he has recently been taught addition and as such expects questions from the teacher to relate to this – numbers mentioned in a problem are presumably those that he must add. Sessions concerning pedagogy and didactics were found to be of the greatest influence and importance to the participants in Legrand's (2001) study, it being the opinion of the researchers that it is important for would-be teachers to see the impact of a didactic contract "in order to help them avoid a simplistic analysis of their students' responses or behaviour... [which cause] them to definitively categorise students as good or bad" (p. 526). Hardy and Hanley's (2002) study supports the need for such reflection

on the part of teachers, finding that “unenergising replications and a sense of contradictions are astonishingly persistent in our students and teachers” (p. 6).

The undergraduate mathematics didactic contract was taken into consideration when research took place designing a first-year course on improper integration with an aim of reinforcing the use of a graphic register (González-Martín & Camacho, 2004). Being aware of the fact that the particular didactic contract which was about to be entered into was new for the students, they “began with situations close to them to provoke a gradual acceptance of this new contract” (p. 480). This awareness of the impact that renegotiating a didactic contract can have was successful in that it prevented rebellion among the students who were later successfully able to recognise and accept the register. Research on attempts at renegotiating the didactic contract using technology have also involved teaching of the limit concept (Delos Santos & Thomas, 2002) but they have been found to be difficult, with it necessary for the teacher to be open to a new style of investigative teaching.

2.3.4.8 – Social & Sociomathematical Norms

The concept of the didactic contract is not only difficult for teachers to grapple with in terms of renegotiation, but it also creates a paradoxical situation in itself. The didactic contract is based on what is known as a ‘double bind’ (Bateson, 1973) – a situation where an individual unwittingly receives conflicting information. The didactic contract does this because “pupil and teacher are locked in an expectation of growth which can be established by circumstances beyond their control” (Mellin-Olsen, 1987, p. 185). It has been suggested that the “contradiction between examination systems calling for individualistic strategies and the ideology calling for co-operative methods” (Mellin-Olsen, 1987, p. 187) in schools is a particularly damaging example of a double bind in education due to its reinforcement at all levels up to and including university.

Such contradictions become part of a 'social norm', a term which refers to "those aspects of classroom social interactions that become normative" (Yackel *et al.*, 2000, p. 278). These norms are developed irrespective of the didactic contract established between teacher and learner, but also distinguish between classroom micro-cultures. This applies to undergraduate mathematics, where many approaches may be taken in the teaching of, for example, applications of differential equations. In a study by Yackel *et al.* (2000), one lecturer encouraged discussion on the possible solutions to equations such as $y'' - 4y' + 3y = e^{2x}$. Whilst this did facilitate discussion, students were not encouraged to explain reasons for guesses they made; rather, he only told them if they were right or wrong. Consequently, it would sometimes be difficult for other students in the class to be able to understand the reasoning behind such guesses, leaving them blind to other possibilities and thought patterns (Rasmussen, 1998). Conversely, researchers had found a lecturer of a project class who had established a norm within his classroom which expected discussion between students who were encouraged to try to understand the mathematical reasoning of their peers. Such a norm is associated with the classroom didactic contract since "while it is the teacher who typically initiates the constitution of norms, all participants in the interaction contribute to their on-going negotiation" (Yackel *et al.*, 2000, p. 281).

A *sociomathematical* norm relates specifically to circumstances when it is the mathematics that sets apart the norms of two different classrooms. This often refers to the norm being either an emphasis on procedural methods and abiding by the rules, or explanation of a solution in terms of the mathematical principles governing the question. The work by Yackel *et al*. (2000) is clearly in support of norms which involve discussion of the reasoning behind one's mathematical thinking and understanding of rules and algorithms. It also highlights that, whilst many suggestions for reform have been made regarding the establishment of an appropriate didactic contract, it is important to consider the social aspect of the learning environment.

Such classroom social norms mean that teaching and learning methods often reach a 'normal desirable state' of activity. This has been found to occur in school classrooms whereby "steady states of activity [are] seen by teachers as appropriate for pupils at different stages of lessons" (Brown & McIntyre, 1993, p. 67) and as such become governed by didactic contracts. Specific to the context of undergraduate learning environments, Taylor (1983; cited by Entwistle & Marton, 1984) found that students establish their own 'study contract' independent of the lecturer's pedagogic approach, perhaps participating in 'unofficial' communities of practice (see Chapter 3.1). This can cause conflict since lecturers often "believe that [their students] should all be chanelling their energies towards the goals which are valued most highly by the academic staff" (Entwistle & Marton, 1984, p. 221) whereas – as we have seen – students can see the didactic contract as a way of putting minimal effort in for maximal gain, often using surface ATLs to achieve their own personal goals. This is reiterated by Schoenfeld (1985) who concluded that students who come from having a didactic contract which involves learning procedures feel that an understanding or ability to apply what they have learned is unwarranted and pointless. It is necessary for both the teacher and learner's goals to be similar in order for effective learning to take place.

2.3.4.9 – Pedagogical Re-Engineering

Whilst it has been suggested that having didactic, and study, contracts can be damaging – due to associated paradoxes and the effects of breaking/renegotiating them – it may be possible to act on suggestions from researchers to develop pedagogical strategies through the use of 'didactical engineering'. This is "a set of classroom sequences which are conceived, linked together and organised in time by an *engineer teacher* in order to carry out a learning project" which "evolves under the reaction of the pupils, and also under the choices and decisions of the teacher" (Douady, 1997, p. 373).

It involves a constant reassessment of the teaching methods until a desirable pedagogy is implemented:

> *The didactiaian... [is] faced with a teaching object that has already been implemented. Why should it be changed? What aims should be included in this reform? What difficulties can be expected, and how can they be overcome? How can the field of validity for the solutions proposed be determined? This set of questions must be answered.*
>
> (Artigue, 1993, p. 30)

This approach was adopted by Artigue (1991) to develop an appropriate and effective method of teaching differential equations at undergraduate level. The method adopted encouraged students "to conjecture and debate ideas in groups within a large class, where arguments were proposed and addressed to other students rather than the teacher" (Artigue, 1991, p. 191). This supports the ideas of Yackel *et al*. (2000) in that a classroom with discussion between the learners as well as with the the teacher is the most effective approach in promoting a good learning outcome. Artigue's method of engineering the didactic involved the use of what is a traditionally French method of scientific debate; this requires a renegotiation of the didactic contract "so that students come to understand and accept their responsibilities as active participants in the knowledge-building process" (Selden & Selden, 2001, p. 244) and has been found to be more effective in helping students solve certain problems than traditional lecture methods.

Describing university teachers as dividing their time between doing a finite set of tasks in preparing and executing their lectures, a system of 'pedagogical re-engineering' was introduced by Collis (1998) which acts in a similar way to didactic engineering. He claimed that lecturers tend to divide their time between: (1) general organisational aspects of the course; (2) instructor presentations; (3) students' on-going study; (4) assignments; (5) examinations; and (6) general communication within the course (p. 381). Similar to didactical engineering, each of the above components was examined "for flexibility and enrichment possibilities" (p.

381) in order to make a change to those didactics, which were then examined and refined whilst aiming to establish the new didactics[22] as concrete.

Mason (2001) comments that a revised didactic contract has been discussed on many occasion in order "to balance developing competency with enculturation into mathematical thinking, rather than succumbing to student desire to minimise effort and simply be trained in requisite behaviour" (p. 72). It is important to remember that first-year students vary in their mathematical backgrounds, abilities and goals and as such will have different expectations of what the undergraduate didactic contract will be. For those students who are used to having their mathematics handed to them as 'rules without reason', it is important to ease them into a revised didactic contract so that they neither rebel against it nor fail to adapt to it without resorting to an inappropriate approach to learning advanced mathematics. This requires an awareness on the part of undergraduate educators of the difficulties faced by students in this transition so that appropriate gap-bridging pedagogies – such as that established by the Warwick analysis project (Alcock & Simpson, 2001) – may be implemented.

[22] The course was redesigned to make greater use of the internet.

Chapter 3: Literature Review

Enculturation into the Undergraduate Mathematics Community

3.1 – Communities of Practice

At university, there are a number of 'communities' that students can become members of. For example, there will be social groupings which they can join, as well as academic groupings to do with their chosen courses and degrees. Students' approaches to learning may be affected by their membership of such groups if they choose to learn and work with each other.

3.1.1 – Roots of the Concept in Social Theory

Wenger (1998a) postulated a social theory of learning wherein students learn as part of a social community, the members of which are engaged in the same practice.

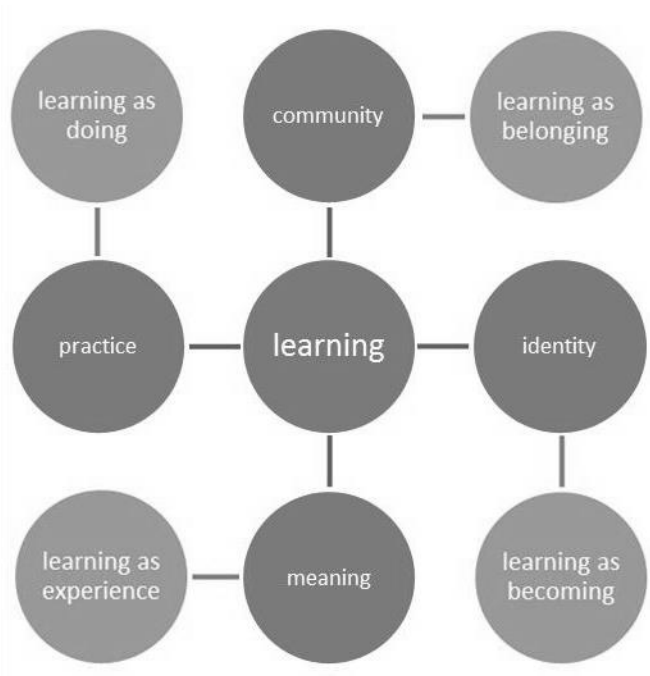


Figure 3.2 - Components of learning
Adapted from Wenger (1998a, p. 5)

For example, undergraduate mathematicians would be part of their own community of practice (COP) as they all are encouraged to engage in the same learning activities, e.g. going to lectures, tutorials and classes, in order to becoming masters of the subject and earn a mathematics degree.

The social aspect of the community involves the members providing "scaffolding for each other to acquire the skills and knowledge for participation" (Olitsky, 2007,

p. 34). The students are able to work together to "collaborate over an extended period to share ideas, find solutions, and build innovations" (Savin-Baden *et al.*, 2008, p. 224). Their common identity as undergraduate mathematicians means that they can socially connect and forge relationships with each other in order that they may produce a new capability (Wenger, 1998b) to become better mathematicians. In order to become members of the community, it is necessary for the "newcomers" to mutually engage with an "old-timer" (Hunter *et al.*, 2007, p. 38) – this often being students' lecturers and tutors. Furthermore, within the more general community of undergraduate mathematicians, it is possible for students to forge identities within what one may call sub-communities – 'unofficial' COPs. These may include the tutor groups to which students are assigned at Oxford and, by being smaller, they may encourage students to develop a stronger identity and role.

3.1.2 – Legitimate Peripheral Participation

Lave and Wenger (1991) argue that it is essential for new members of a COP to view themselves as 'legitimate peripheral participants'. By this, they mean that newcomers are not full members of the community; however, they have the potential to become one if they adhere to the community's standards and expectations. It is necessary for newcomers to participate in the practices of the community and "learn to function" there in order to establish full membership (Brown & Duguid, 1991, p. 48). The extent to which new students experience mathematics as a legitimate peripheral participant can then dictate the quality of the relationship that they have with mathematics.

In order to become a member of a COP, it is necessary for a student to learn to function within it appropriately. For example, they learn to speak the language of the community and become familiar with the community's goals. This involves them becoming enculturated (Brown *et al.*, 1989). Unfortunately, the imposed standards and expectations by the COP (Lave & Wenger, 1991) may be unattainable by certain members. It is possible for an undergraduate

mathematician to fail to understand the mathematics that they are taught, find no joy in learning it and thus ultimately reject membership (Herzig, 2004a).

3.1.3 – Communities of Practice at the Secondary-Tertiary Interface

At course level during the secondary-tertiary mathematics transition, it is important for departments to make students feel welcome in their new community and ease them into new modes of study. Gillespie and Noble (1992) suggest that academic integration has far more influence on attrition than social integration, at least during the immediate transition into tertiary study, with Halpin's (1990) research finding that concern shown by, and interaction with, the student's faculty were more influential on persistence than peer group relations. This idea is reinforced by Peel (2000) who believes that, for students "experiencing failure and disillusionment, the offer of personal contact and assistance can also make a significant difference" (p. 29). The tutorial system at Oxford means that students are in regular, compulsory contact with a member of teaching staff. Therefore, anyone who struggles is more likely to be identified and helped than at other universities. This may explain why drop-out rates at Oxford are amongst the lowest in the UK (HESA, 2011).

McInnis and James (1995; cited by Lawrence, 2003) suggest that students who actively participate in 'learning communities' are more likely to complete the course and even perform better. This is because their participation develops both their social and their academic skills, which in turn makes them more involved in extracurricular activities which act "as gateways to greater student involvement with... [lecturers] and with other people on campus" (Tinto, 1998, p. 7).

Establishing their position in a general social COP is very important for new students to feel a sense of belonging in their new environment (Wilcox *et al.*, 2005) since a "need to belong is one of the core desires that shapes human behaviour" (Kantanis, 1997, p. 102). The transition involves both academic and social hurdles, and students must find a way of enjoying each of

these aspects of university life without compromising the other. The social factors associated with the transition can be highly influential, with Kantanis (2000) commenting that "social transition underpins a successful academic transition to university" (p. 102). Her research at an Australian university found that social aspects of the transition were those considered to be most important and most difficult by new students who seek new friends to share and experience university life with. Hence those students who are not active members of mathematics COPs may feel content with their role in generic communities, having friends with whom they may journey through university life.

Social integration can have different impacts on the experiences of males and females at university. Stage (1989) found that, for certain men, "social integration was an important indirect influence on persistence... academic integration was not" (p. 396). Conversely, Pascarella and Terenzini (1983) found women to be more heavily influenced by social integration (that with peers from all different courses of study) than academic integration (that with peers within their own course of study), whereas the opposite appeared true for men. This suggests that male undergraduates would be more interested in integrating with other members of the undergraduate mathematics population than women, making a stronger attempt to engage in the undergraduate mathematics COP than women.

3.1.4 – Problems within Communities of Practice

> *When we are with a community of practice of which we are a full member, we are in familiar territory. We can handle ourselves competently. We experience competence and we are recognised as competent. We know how to engage with others. We understand why they do what they do because we understand the enterprise to which participants are accountable. Moreover, we share the resources they use to communicate and go about their activities.*
>
> (Wenger, 1998a, p. 152)

Success in entering a new community is not guaranteed. It depends on "students' levels of epistemic fluency in terms of their awareness of the existence of epistemic games (ways of co-

constructing knowledge) which involve different kinds of epistemic forms (target knowledge structures which are characteristic of the community), and their metacognitive awareness of their own success in accessing these new ground rules" (Solomon, 2006, p. 376). Failure to become legitimate participants in the mathematics COP has the potential to ultimately result in attrition. The involvement and integration of students in their departmental COPs is key to their persistence (Bair & Haworth, 1999; Girves & Wemmerus, 1988; Herzig, 2002; Lovitts, 2001; Tinto, 1993)[23].

There are various sub-communities that the students can both identify with, and socially be part of:

> *There are more immediate communities of practice which also figure in… students' identities: the undergraduate community in general, the mathematics undergraduate community and the first-year community within it, and the classroom community of learners and tutors. The students' identities and their relationships to mathematics are also shaped by their membership of these often more visible communities.*
>
> (Solomon, 2007b, p. 84)

Rodd (2003) also describes the practices of the undergraduate mathematics community:

> *Their community is established by practices, such as attending (or skiving off) lectures, doing 'homework', participating in tutorials, sitting exams, joining the 'maths society' and just being seen around the department and being familiar with its personalities.*
>
> (p. 19)

The sub-communities joined by individual students can be telling of their experiences. For example, students who struggle with the subject may form their own community since they share similar experiences and seek the assistance and support of others sympathetic to their situation. Conversely, an enthusiastic, successful and highly-motivated student who regularly

[23] All cited by Herzig (2004a).

attends lectures and seminars and establishes relationships with staff forms a basis for success and involvement by actively participating in their community (Brown & Rodd, 2004).

It is possible to distinguish between two different groups of COPs – namely formal (imposed) and informal (spontaneous) COPs. An informal COP is one which a student might join as a consequence of choosing to belong to a particular community, such as a sports club. A formal COP is thrust upon the students by imposed practices, such as lectures and tutorials, or the existence of the Mathematical Institute as a means of gathering students. Nearly all undergraduate mathematicians at Oxford go to and make notes in lectures, which are imposed practices. Compulsory tutorials are a smaller COP, whereas study groups that students make, perhaps as a consequence of this, are informal, though they are structured. That is, they are spontaneous in the sense that the students choose to study together, but the students are brought together in the first place by the college/tutorial system.

It is also important to consider that students may also be members of other COPs outside of their academic studies. Participation in extra-curricular activities means that students may be part of other social groups within which they play a larger (or smaller), more successful (or weaker) role than within their department. In this way, they are successfully integrating in one area more than another (Tinto, 1975). Membership of such communities may also have an impact on attrition.

Research conducted by Solomon (2007b) found that women find it difficult to “acknowledge themselves as successful mathematicians” (p. 84), resulting in difficulties in integrating into mathematics COPs. If a student does not believe that they belong and that they are worthy of membership, it is unlikely that they will be successful in becoming legitimate participants without sufficient encouragement and support.

3.1.5 – Communities of Practice in the Context of Undergraduate Mathematics Learning

Communities of practice have been described here as a means of explaining the social aspects of learning in academic settings. At the University of Oxford, undergraduate mathematicians are assigned membership to formal communities of practice – tutorial groups – which are intended by the University to act as a means of learning.

The majority of students' contact time with the Mathematics Institute is through the medium of lectures. Such environments are not known for their interactivity, meaning that students' opportunities to ask questions and seek clarification must come from interactions in tutorials, with lecturers on a one-to-one basis or with their peers. Students may also choose to work with each other on an informal basis when revising or doing weekly assignments.

Whilst it is possible that students may go about their learning of mathematics on a solo basis, working alone and referring to texts and seeking to develop their understanding independently, the unique nature of the Oxford tutorial system means that all undergraduates are members of at least one social learning community. Therefore, the language of 'a community of practice' is important in this context as it acts as a means of describing students' working and learning, whilst providing means of describing students' general experiences as they interact with others within their college, their department and the wider university community. It is these relationships which have the potential to shape students' experiences and, ultimately, their working practices, and therefore the COP framework can be used as a way of describing and explaining this.

3.2 – Self-Efficacy & Self-Concept

3.2.1 – Impact of Undergraduate Study

> *Efficacy expectations determine how much effort people will expend and how long they will persist in the fact of obstacles and aversive experiences.*
>
> (Bandura, 1977, p. 194)

In the context of challenges associated with the study of undergraduate mathematics, a student's self-efficacy will impact upon how much time and persistence they are prepared to give and whether are able to make to overcome them. It is possible that students will have negative reactions to their courses for a number of possible reasons, so the way in which they respond to this is related to the students' self-efficacy. Moreover, a student's self-efficacy determines whether they will choose to tackle a challenging situation at all.

Therefore, it could be argued that self-efficacy is related to approaches to learning. If a student is prepared to persist in learning something using a deep ATL, then this suggests higher self-efficacy than someone who gives up, fails to understand it deeply and reverts to surface approaches in order to pass examinations. Since "an efficacy expectation is the conviction that one can successfully execute the behaviour required to produce the outcomes" (Bandura, 1977, p. 193), those who fail in attempts to have a thorough understanding of a mathematical concept as a means of achieving the desired outcome – a good score on assessment and/or the understanding of the concept – may not have lowered self-efficacy as a consequence. Readjustment of an ATL to a strategic one may not leave the student feeling bad, but realising that there may be another means of achieving their ends. Their expectation that a deep understanding may achieve might be shattered, but their emotional response to this reality when considered alongside the notion that they may achieve the same desired outcome may not necessarily be negative.

Self-efficacy has been found to decrease at school-level transitions (Blyth *et al.*, 1983; Eccles *et al.*, 1987). In the undergraduate context a number of new, threatening experiences could occur which students are unfamiliar with or unprepared for – namely, failing to understand the mathematics to their satisfaction or to attain high marks. Such a threatening experience when studying the subject could be damaging to a student's confidence. Should this happen, impacts can then occur on a number of levels.

Since it has been found that "successes raise mastery expectations; repeated failures lower them, particularly if the mishaps occur early in the course of events" (Bandura, 1977, p. 195), students' experiences at the secondary-tertiary interface are particularly relevant in this context. Success with the subject at school – guaranteed for all undergraduates since proof of this is required for admission – means that a certain level of this will be expected at university. Repeated failures with respect to their own personal expectations would then result in a readjustment of future expectations based on past events. However, as Bandura (1977) argues, impacts on someone's self-efficacy due to negative experiences may not always be negative themselves. That is, should a new undergraduate overcome mathematical challenges through persisting in their attempts to understand and be successful until they achieve it, then their self-efficacy may be strengthened. The satisfaction associated with achieving something difficult then has a positive impact on expectations for future success.

Lumsden (1994) suggested that children who are confident in their abilities are more likely to choose to participate in more challenging tasks than their less confident counterparts. In the undergraduate context this may extend to completion of non-compulsory work, reading around the subject and attendance and participation in the mathematics community such as in mathematics societies or student committees. Active participation in the undergraduate mathematics COP may therefore require an amount of confidence in one's abilities and a feeling of worthiness.

Failure to adjust and continued negative impacts on self-efficacy may result in a feeling that there is no solution and signs of 'learned helplessness' (Peterson *et al.*, 1995). Someone exhibiting symptoms of learned helplessness feels that they are helpless to control the outcome of a particular situation. For example a feeling that, after attempts to succeed and understanding have failed, they are powerless to turn things around. Expanding on the work of Dweck (1975) and Dweck and Repucci (1973), one could suggest that students who persist in their attempts to understand are more likely than the learned helpless to attribute their academic failures to a lack of effort rather than a lack of ability. In mathematics in particular, achievement is strongly related to self-concept (Marsh, 1986).

Conversely, Williams (2008) describes the 'optimistic' student as one who creates more problem solving opportunities. In their case,

> *Inclination to explore is associated with optimism because exploring what is unknown (present failure) is consistent with the perception that 'not knowing' is temporary and 'knowing' can result from personal effort […] Optimistic students look for ways to overcome problems they encounter by examining what can be altered to increase chances of succeeding.*
>
> (p. 582)

This suggests that an optimistic student might be more likely to be successful because the associated perseverance increases their chances of finding the correct answer to a mathematics problem.

Reciprocal relationships between performance and self-efficacy have been widely documented (Bandura, 1986, 1997; Lenney, 1977; Pajares, 1996; Pajares & Graham, 1999; Pajares & Kranzler, 1995; Pampaka *et al.*, 2011; Schunk, 1991; Williams & Williams, 2010; Vollmer, 1975; Watson, 1988; Zimmerman *et al.*, 1992), with suggestions that this is also connected to personal attributions of success. The belief that either effort or ability is the cause of success has an impact (e.g. Pajares & Graham, 1999; Weiner, 1974). Specifically, the requirement of

minimal effort to be successful "fosters ability ascriptions that reinforce a strong sense of self-efficacy", whereas success attributed to effort "connote a lesser ability and are thus likely to have a weaker effect on perceived self-efficacy" (Bandura, 1977, p. 201). That is, students who claim that they do well in mathematics because they work hard are more likely to have weaker self-efficacy than students who find that success in mathematics assessment comes more easily. Furthermore, the notion that self-efficacy may remain 'intact' if a strategic ATL enables a successful, quantifiable outcome in place of a deep understanding would relate to effort. Should this adaptation to the approach involve no more effort than previous methods utilised, the student may maintain/develop a strong self-efficacy. Conversely, expending great amounts of time and effort on achieving a deep understanding may be interpreted as being synonymous with failure, negatively impacting upon self-efficacy.

Moreover, for students unused to circumstances when significant effort is required to be successful, an impact on self-confidence and self-efficacy can result (Weiner, 1974). Furthermore, there is a risk that feelings of disaffection may result since these occasions may lead the student to believe that "failure is inevitable, resulting in behaviours that encourage limited student motivation to learn in favour of avoidance attitudes and behaviours" (Nichols, 2006, p. 151). Such behaviours may be considered synonymous with lack of engagement, levels of which have been attributed to personal efficacy beliefs and performance (Miller *et al.*, 1996; Pintrich & DeGroot, 1990; Pintrich & Schrauben, 1992; Schunk, 1984).

3.2.2 – Big Fish Little Pond

A further impact of undergraduate study on new mathematicians' confidence could come from their arrival in a situation where they are no longer the best at their craft. For many of the students, they were the brightest at school and were top of the class in mathematics, where they were able to pick up new concepts and do very well in examinations with relative ease. This would contribute to a strong feeling of confidence in their mathematical ability. Indeed,

many students cite 'being good at it' as the main reason for studying mathematics at university (Darlington, 2009). Furthermore, at A-level, 76% of A-level students participating in a Qualifications and Curriculum Authority (QCA) study claimed that they chose to study A-level Mathematics because they had coped well with it at GCSE, and 43% cited being better at mathematics than other subjects as a strong influence on their decision to do it (QCA, 2006).

However, this may not continue to be the case at university when they are joined by others with much the same experience. It may continue, but the ease with which the desired outcome may be achieved may be different to previous experience.

Such a change of fortunes shares commonalities with the big-fish-little-pond effect, a phenomenon described by Marsh (1987) as when "equally able students have lower academic self-concepts in high ability schools than in low-ability schools" (p. 280) because of their perceived relative position in the 'ranking' of success and ability in school.

New undergraduates then may become a small fish in a big pond wherein they become 'average' or even 'poor' at mathematics in comparison to their peers. The change to their circumstance comes as a consequence of two factors:

1. The grouping they are now in is significantly larger than previously – in the hundreds rather than the tens.
2. Many members of the new group are equally or more talented than them.

Sax (1994) comments that, "regardless of actual ability, a student will feel more academically confident among a relatively lower-ability peer group than among a higher ability grouping" (p. 144), the chances of which are higher at school than at university. He argues that even the most initially-confident mathematics students become less so at the undergraduate level. It is possible for some students to maintain their position as the most successful. Equally, some may not perceive a change in their status and may come from highly competitive

environments where their fellow students were also very able and where they had to work very hard to succeed.

In many schools, streaming is commonplace in setting pupils according to ability. Many Oxford undergraduate mathematicians will have experienced this, with the transition from GCSE to A-level acting as a means of streaming as students advance further into the subject. Arrival to university could be described as a very severe example of streaming, where after students have been found to display signs of lower self-concepts (Kulik, 1985).

3.2.3 – Relationship with Communities of Practice

Marsh (1987) has claimed that "group membership influences the values and standards of performance used by people in their self-evaluations" (p. 281). This suggests that membership in certain COPs can affect confidence and self-efficacy. Those who feel incapable of gaining membership in the undergraduate mathematics COP may feel dejected – perhaps even rejected – by this experience if they feel like they do not belong with the current members. Consequently, they may seek membership in a COP of fellow 'rejects' wherein they feel that they are able to meet the standards and expectations of their peers. Within here, they may even be able to revert from being a small fish in a big pond back to a big fish in a little pond, reinforcing their confidence.

3.2.4 – Gender

Sax (1994) claims that "greater interaction with faculty ultimately has a small negative effect on women's self-confidence in math" (p. 153). One could suggest that this is perhaps due to the nature of their relationship with the faculty; that is, women are more likely than men to work collaboratively and seek help when needed, meaning that this interaction may only come from negative circumstances. However, women have been found to have poor relationships with faculty members in mathematics because these are dominated by men. The lack of female role models for undergraduate women – because the majority of university

mathematics professors and researchers are men – means that they struggle to identify themselves as strongly as mathematicians rather than outsiders, which has contributed to high drop-out rates and low uptake of female PhD students (Herzig, 2004b). High-level female mathematicians "provide a powerful effect as role models, counsellors, and advocates for students and for junior women faculty" (Stage & Maple, 1996, p. 24).

Women have been repeatedly found in empirical research to be less confident in their mathematical abilities than men, even the high achievers (Astin, 1978; Brown *et al.*, 2008; Ethington, 1988; Kyriacou & Goulding, 2006; MacCorquodale, 1984; Marsh *et al.*, 1985; Meece *et al.*, 1982; Pampaka *et al.*, 2011; Sax, 1994; Sherman, 1982, 1983; Williams & Williams, 2010). Such gender differences with respect to confidence increase towards the end of secondary school (Hyde *et al.*, 1990; Meece *et al.*, 1982) when women begin to form the minority of mathematics students (see Chapter 1.1). At tertiary level, self-efficacy has been found in some research to decrease for both men and women (Astin, 1977, 1993; Drew, 1992); however, suggestions that women are less confident in their abilities than their male counterparts at this level are more common (Astin, 1977; Higher Education Research Institute, 1991; Sax, 1994; Smart & Pascarella, 1986). This is perhaps, in the context of undergraduate mathematics, due to the increasingly competitive nature of futher study (Sax, 1994) or because women find themselves without many strong female role-models in their department and feel out of place in a male-dominated COP.

In everyday society, women do not tend to outwardly display signs of confidence in their mathematical abilities because of societal pressures and stereotypes of women in mathematics (Caporrimo, 1990). They have been found to have less self-confidence in their talents (Ware *et al.*, 1985) and are more susceptible to mathematics anxiety (Betz, 1978; Ho *et al.*, 2000) which can have a serious impact on their confidence. Mathematics anxiety has been associated with poor performance (Hembree, 1990) which is in itself related to confidence

(Reyes, 1984). Anxieties of this nature can have a serious negative impact on students' learning and understanding (Hembree, 1990), even impacting upon students' perceptions of their own mathematical abilities (Hannula, 2002b).

Mura (1987) found that men are more likely to overestimate their final grade than women. Mathematics students demonstrate "a different pattern of performance expectancies" (Shah & Burke, 1996, p. 23) to students of other disciplines, yet this overestimation has been identified in other academic departments (e.g. Vollmer, 1975, for psychology). In fact, when studying tertiary-level mathematics, women have been found to be tougher on themselves than men when they do not understand certain concepts to their satisfaction (Solomon, 2007b). Differences in confidence levels are also evident here, where women appear to have less faith in themselves at postgraduate levels (Becker, 1990), and are less likely to complete research degrees than men (Shah & Burke, 1996). It is also at this advanced, postgraduate level that mathematics anxiety is still identified in students (Hembree, 1990).

Solomon *et al.* (2011) claim that female undergraduate mathematicians suffer from 'fragile identities' in that they often fail to achieve legitimate participation (Wenger, 1998b) in it, remaining on the periphery. They claim that "girls appear to lack a niche in this particular world" (p. 1) – one traditionally regarded as masculine. As a consequence, it is understandable that talented women mathematicians would not feel comfortable choosing to study it beyond compulsory levels. Mendick (2005) claims that, in the current society, "doing mathematics is doing masculinity" (p. 237), which can be off-putting to potential female mathematicians and demoralising for current female students.

Evans' (1999) comprehesive literature review came up with mixed findings about the existence of a significant relationship between gender and academic persistence. Whilst some studies have found that women are more likely to persist in tertiary study and do so more quickly than men, Evans cautions that this is not the case in all subject areas. She claims that men are more

likely to persist in some areas, the ‘closest’ of which to mathematics she describes being engineering.

Chapter 4: Methodology

4.1 – Overview of Methods Employed

The attempt to answer the research questions of this thesis was achieved through three strands of data collection, namely:

1. semi-structured interviews with mathematics undergraduates at the University of Oxford (see Chapter 4.4);
2. questionnaire data from the Approaches and Study Skills Inventory for Students (ASSIST; see Appendix 4.1) collected from mathematics undergraduates at the University of Oxford (see Chapter 4.5); and
3. categorisation of question types in a selection of A-level pure mathematics examinations, University of Oxford Admissions Tests (OxMAT) and first-year examinations at Oxford using the MATH taxonomy (see Appendix 2.1 and Chapter 4.6).

The research questions were:

1. How do undergraduates' experiences of studying mathematics at Oxford change throughout their university career?
    a. What challenges do students report facing in each year of study?
    b. How do students report their approaches to learning and studying mathematics?
2. Based on previous experience of mathematics, what challenges lie in Oxford students' enculturation into a new mathematical environment?
    a. What types of skills and challenge are tested by A-level Mathematics and Further Mathematics questions?

b. How does the OxMAT's assessment of students' mathematical understanding compare to A-level Mathematics and Further Mathematics?
c. How do undergraduate mathematics examinations compare to the A-level and the OxMAT?

3. What is the relationship between students' approaches to learning and the challenges they perceive in undergraduate mathematics assessment at the University of Oxford?

Question 1 was answered through coordination of data from the ASSIST and interviews, Question 2 was answered using data from the MATH taxonomy, and Question 3 was answered using data from the ASSIST and interviews. This mixed methods approach aims to answer the questions using a combination of methods which complement each other and go towards providing rich, detailed data and descriptions of the undergraduate mathematics experience at Oxford.

An overview of the methods employed to answer each of the research questions is given in Table 4.1.

Table 4.1 - Overview of methods employed by research question

Data Source	Sample	Access	Approach	Analysis	Literature Review Section	Data Chapter
1. How do undergraduates' experiences of studying mathematics at Oxford change throughout their university career?						
a) What challenges do students perceive facing in each year of study?						
ASSIST	Over 300 current Oxford undergraduate mathematicians	Departmental mailing list	ASSIST scoring key (Tait & McCune, 2001)	Statistical analysis on SPSS – descriptive statistics (median, mode, proportion), analysis of dominant ATLs (Fisher's exact test or Pearson's chi-square test), analysis of subscale scores (independent-samples Mann-Whitney U test or Kruskal-Wallis test).	2.3: The secondary-tertiary mathematics transition 3.1: Communities of practice 3.2: Self-efficacy and self-concept	7: Student reports of Mathematics study at the University of Oxford
Interviews	13 students[24]: 4xY1, 3xY2, 4xY3, 2xY4	Students asked to participate in interviews via email. All students who replied were contacted for interview.	One hour, semi-structured interview	Thematic analysis		
b) How do students report their approach to learning and studying mathematics?						
ASSIST	Over 300 current Oxford undergraduate mathematicians	Departmental mailing list	ASSIST scoring key	See method of analysis for question 1 (a).	2.1: Approaches to Learning	5: Student Approaches to Learning throughout Undergraduate Study at the University of Oxford

[24] See also Table 4.5.

Interviews	12 students: 4xY1, 3xY2, 4xY3, 2xY4	Students asked to participate in interviews via email. All students who replied were contacted for interview.	One hour, semi-structured interview	Thematic analysis		
2. Based on previous experience of mathematics, what challenges lie in Oxford students' enculturation into a new mathematical environment?						
a) What types of skills and challenge are tested by A-level Mathematics and Further Mathematics questions?						
A-level past papers	C1: January 2006 AQA, OCR, WJEC, Edexcel FP3: January 2007 AQA, OCR, WJEC, Edexcel	Online – exam board websites Large-scale quantitative analysis was not intended for this part of the thesis. The application of the MATH taxonomy aims to provide an illustrative means of describing the differences between secondary and tertiary mathematics questioning.	Apply MATH taxonomy (Smith *et al.*, 1996)	MATH taxonomy used on all papers by myself, and validated by a recent Oxford MMath graduate, a current Oxford MMath undergraduate, a practising mathematics teacher and a professor of mathematics education. This triangulation by experts aimed to ensure that the questions were categorised in the most consistent way possible.	2.2: Question Analysis 2.3.3: A-level Criticism	6: Contrasts in Challenges Presented by A-Level Mathematics, the Oxford Admissions Test & First-Year Undergraduate Examinations
b) How does the OxMAT's assessment of students' mathematical understanding compare to A-level Mathematics and Further Mathematics?						
OxMAT past papers	2007-2011 OxMATs	Online – Oxford Mathematical Institute website (University of Oxford, 2013a)	Apply MATH taxonomy	See above.	2.2: Question Analysis	6: Contrasts in Challenges Presented by A-Level Mathematics, the Oxford Admissions Test & First-Year Undergraduate Examinations
c) How do undergraduate mathematics examinations compare to the A-level and the OxMAT?						
Past Pure	2008 and 2011 P1	Online – (University of	Apply	See above.	2.2: Question	6: Contrasts in

Mathematics 1 & Pure Mathematics 2 papers	& P2 examinations	Oxford, 2013b)	MATH taxonomy		Analysis	Challenges Presented by A-Level Mathematics, the Oxford Admissions Test & First-Year Undergraduate Examinations
3. What is the relationship between students' approaches to learning and the challenges they perceive in undergraduate mathematics assessment at the University of Oxford?						
ASSIST	Over 300 current Oxford undergraduate mathematicians	Departmental mailing list	ASSIST scoring key	See method of analysis for question 1 (a).	2.1: ATLs 2.3: The secondary-tertiary mathematics transition 3.1: COPs 3.2: Self-efficacy & self-concept 3.3: Student motivation	7: Student reports of Mathematics study at the University of Oxford
Interviews	12 students: 4xY1, 3xY2, 4xY3, 2xY4	Students asked to participate in interviews via email. All students who replied were contacted for interview.	One hour, semi-structured interview	Thematic analysis		

Participants comprised students from all four years of undergraduate study, with one phase of ASSIST data collection in the autumn term ('Sweep 1') involving only first-year undergraduates in order that the second phase in the summer term ('Sweep 2') involving all year groups could 'map' the first-years' ATLs over time. Students were contacted via their departmental mailing lists (see Appendix 4.2) and invited to take part by completing the ASSIST online via a link for a Google Docs form (see Appendices 4.3 and 4.4). Later in the year, an email requested participation in the interviews (see Appendices 4.5 and 4.6), asking students to get in touch if they were interested. Both emails included a brief background to the study, as well as its possible implications, and a comment about my own background in mathematics and mathematics education.

4.2 – Learning Mathematics as a Sociocultural Experience

When attempting to describe and analyse undergraduate mathematicians' experiences of learning, it is important to consider the theories of learning which act as the basis for someone's personal understanding of what it is to learn mathematics.

Broadly speaking, I shall be analysing the data for this thesis using the perspective that learning mathematics is a sociocultural experience. This theory mainly relies on the work of Vygotsky (1896-1934), who described learning as a social process wherein "the individual emerges from a socio-cultural context" (Confrey, 1995, p. 38). Learning is a social activity which Lave and Wenger (2005) describe as involving "the whole person… it involves becoming a full participant, a member" of a social community (p. 152). Lave and Wenger (1991) write of legitimate peripheral participation (see Chapter 3.1.2) in a COP which enables novices and newcomers to the community to become experienced in that practice. However, their work is not specific to learning mathematics and is a more general theory. It is therefore important that I contextualise this for my data and purpose. That is, in this context, a new undergraduate's legitimate peripheral participation in the undergraduate mathematics community[25] can foster their development into a more experienced member.

However, it is not the case that there is just one COP for the new undergraduate mathematician. The most important distinction in this context is between two different, though similar, communities:

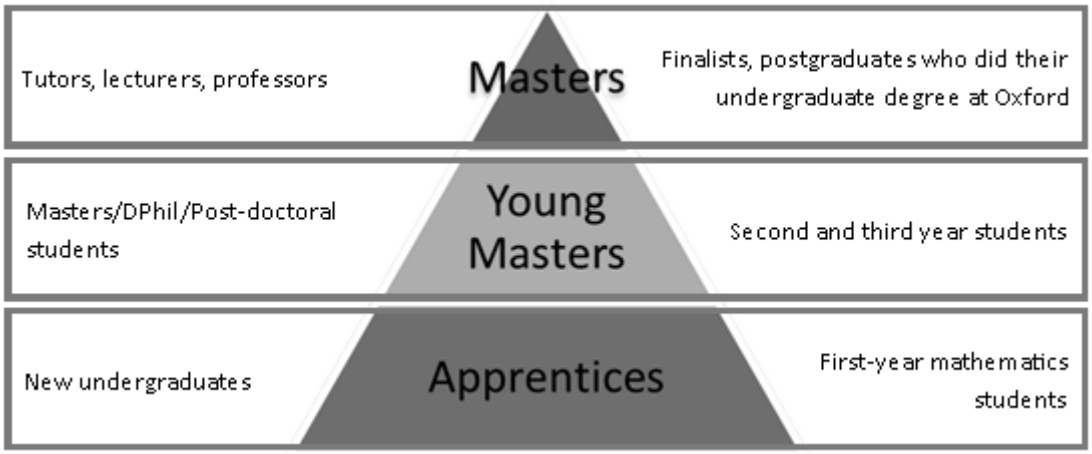
1. The community of mathematicians at the University of Oxford
2. The community of undergraduate mathematics students at the University of Oxford

The priorities of the students are important to consider when analysing their self-report data. Furthermore, they are also engaging in other COPs, for example the community of students at

[25] Or in the COP that is their tutorial group, their self-selected peer groups, their cohort of mathematicians, etc.

their college, the community of residents of their accommodation, the community of members of their sports team, and so on (see also Chapter 3.1). Lave and Wenger (2005) describe a COP as encompassing "apprentices, young masters with apprentices, and masters some of whose apprentices have themselves become masters" (p. 155). Therefore, in the two contexts of mathematicians and mathematics students, different groups of people fall into these categories:

Figure 4.2 - Figures in the university mathematics COP



A legitimate peripheral participation in a community "means that learning is not merely a condition for membership, but is itself an evolving form of membership" (Lave & Wenger, 2005, p. 152). That is, learning and understanding undergraduate mathematics is both a requirement for participation in the undergraduate mathematics COP and a consequence of this participation and membership.

The idea is that different students in the same situation may have different goals in a COP, depending on their primary concern. If a student prioritises learning mathematics and becoming a successful mathematician, their goals may be different to the student who strives to be a good undergraduate mathematics student. In the context of mathematics:

> *since the teaching of mathematics is a widespread and highly organised social activity, and even allowing for the possibility of divergent multiple aims and goals*

> *among different persons, ultimately these aims, goals, purposes, rationales, and so on, need to be related to social groups and society in general*
>
> (Ernest, 2004, online)

At the undergraduate mathematics level, his comments refer to students' individual aims and goals in their degree; these might be these general ones regarding the development of understanding of mathematical concepts, or the reasons for them studying mathematics to begin with. More specifically, one could consider individuals' aims and goals in terms of the understanding of mathematics – does the student strive to develop a thorough understanding of all mathematical concepts, or do they strive to be successful in examinations and therefore adapt their learning approaches in order that they may do this in the most efficient way possible? This is where a distinction between the undergraduate mathematician and the undergraduate mathematics student becomes important.

In a general context, not specific to mathematics, Wertsch (1985) describes internalisation as "the process of gaining control over external sign forms" (p. 65). These signs could mean mathematical symbols, but they could also mean speech. The process of internalising a concept would enable the student to be able to discuss it with someone else. However, in mathematics, an internalisation might not necessarily mean that someone has understood meaning. For example, someone might be able to draw $y = \sin x$, select points on it and be able to manipulate it; however, they might not have an understanding of why $y = \sin 2x$ looks as it does, even though they know this to be true. Therefore it is important to consider this in a mathematical context.

Applying this to the work of Duval (2006) for the mathematics-specific context, signs primarily serve the purpose of permitting the substitution of some signs for other signs.

> *The part that signs play in mathematics is not to be substituted for objects but for other signs! What matters is not representations but their transformation. Unlike*

> *the other areas of scientific knowledge, signs and semniotic representation transformation are at the heart of mathematical activity.*
>
> (p. 107)

To this end, a syntactical approach to problem-solving is one which involves the use of the words and symbols given in the question; conversely, a semantic approach[26] is one which involves making use of other examples and representations in order to aid the solving of the problem. For example, a question might ask:

Make x the subject of $3xy = 4 - 2x$

Someone might then:

Put everything with x on the same side of the equation	$3xy + 2x = 4$
Take x out of the expression	$x(3y + 2) = 4$
Make x the subject	$x = \frac{4}{3y+2}$

Some students might manipulate both sides of this equation and factorise it without ever thinking about why they are performing these manipulations and what the formula actually says. This would be a syntactic approach to answering the question, which might be indicative of a surface ATL. However, it may also be the method used by a student who tends to use deep ATLs because they are so familiar with the concepts and the required method.

Therefore, in this terminology, someone with a surface ATL would not have 'control' over the concepts that they are using. If one merely memorises and reproduces information which has been given to them without understanding it and being able to apply it in new situations without trivialising it through drill exercises and reducing it to an algorithmic process, can they *know* it? Piaget (1970) argues that "knowing an object does not mean copying it – it means acting on it" (p. 15), which is certainly not synonymous with the surface approach definition.

[26] See also Weber & Alcock (2004), who describe syntactic and semantic knowledge and proofs.

The social context of learning mathematics as an undergraduate involves various possible social interactions such as those with tutors, coursemates and lecturers. Whilst some – arguably most – learning is a consequence of discussion with, and instruction of, the student from these different people, an amount of this learning might be 'passive'. That is, in social situations undergraduate mathematicians may see other mathematicians at work and learn from their practices. Instances of lecturers writing proofs or doing worked examples on the blackboard in a lecture only to realise that they are wrong give students an insight into what it is to be a mathematician and the thinking which it requires. Witnessing mathematicians at work in the common room, discussing mathematics with each other, or presenting and discussing mathematics at seminars or as part of presentations by mathematical societies also do this. In terms of social learning which can be done in the context of those who are learning to become good undergraduate mathematics students, new undergraduates may learn from older current undergraduates about their practices and establish for themselves what the 'norms' are in this context, which will then influence their actions and beliefs.

4.3 – Mixed Methods

The research questions for this thesis (see Chapter 4.1) stemmed out of an interest in the undergraduate mathematician's experience, which was a consequence of my own experiences of the subject at another Russell Group university. In this particular instance, my interpretation of 'undergraduate experience' is as a combination of their general enculturation into the new mathematics learning environment of the University, as well as the undergraduates' responses and adaptations to it, which may be influenced by their prior experience of mathematics at school. Consequently, two types of change have been investigated:

1. Background – changes in culture
2. Foreground – changes for the individual

Data collected to investigate this was of three kinds: a picture of how students change (ASSIST) in the context of the change of culture (MATH taxonomy), and self-report of current undergraduates (semi-structured interviews).

The theoretical orientation for this study was of a pragmatist paradigm. For this study and the research questions it poses, the centrality of the problem led to the development of all approaches employed to try and understand the problem (Creswell, 2003). For me, it was important to first pose the research questions of interest before beginning to consider the philosophical and methodological approach required to answer the research questions. In this sense, I believe that "the research question should drive the method(s) used" (Onwuegbuzie & Leech, 2005, p. 377). Such an approach is supported by Onwuegbuzie and Leech (2005), who describe pragmatic researchers as being "in a better position to use qualitative research to inform the quantitative portions of research studies, and vice versa" (p. 383). Furthermore, Tashakkori and Teddlie (2003) take the perspective that pragmatism is the most effective paradigm for driving mixed methods research (see also Hanson *et al.*, 2005).

Despite claims by a multitude of writers – such as Howe (1988), who gave his 'incompatibility thesis' – that quantitative and qualitative paradigms should not be mixed, the 'quant/qual debate' has continued to rumble on for decades. However, despite its relative infancy, the increasing volumes of literature about mixed methods fill me with confidence in employing them for the purpose of this thesis. The juxtaposition of the two methods and stances in mixed methods research forms the basis of a description of mixed methods research. This permits qualitative data "to expand and elaborate on quantitative findings" (Creswell *et al.*, 2006, p. 5). That is, the interview data can be used to find more data specific to individuals' experience whist the ASSIST acts as a means of broadly investigating the student body's ATLs. The findings of the MATH taxonomy can be elaborated through interviews, whilst comments made in interviews can be substantiated using data from the ASSIST and MATH taxonomy. Mixed methods are a method which is growing in popularity and prominence, with many writers supporting it over either a qualitative or quantitative approach on the grounds that each method can be used to strengthen the use of the other. Indeed, Rocco *et al.* (2003) claim that:

> *Purely quantitative research tends to be less helpful through its oversimplification of casual relationships; purely quantitative research tends to be less helpful through its selectivity in reporting.*
>
> (p. 24)

Onwuegbuzie and Leech (2005) describe reliance on purely qualitative/quantitative approaches as "extremely limiting" (p. 384), restricting the researcher in a successful attempt to answer their research question. In the case of this thesis, it is important that the student 'story' be told. Such an aim would be seen by Greene (2008) as appropriate in driving a mixed methods approach, as mixed methods convey "magnitude and dimensionality… [and] contextual stories about lived experiences" (p. 7). These lived experiences are those of particular concern – the experiences that the individual students live through as they make the transition between secondary and tertiary mathematics and the ways in which they are

enculturated into the undergraduate mathematics community. Furthermore, the most effective way to probe deeply into these experiences is by discussing them with the student, allowing them to embellish on any factors and experiences that they or I deem to be important or interesting, and cannot be quantified. However, some aspects of their experience which can be quantified can go towards supporting the qualitative data and permitting more generalisable statements due to their being collected from a larger population.

Greene *et al.* (1989) describe five possible purposes of mixed methods studies. The purpose of its use for this thesis would be defined, in their terms, as for *complementarity*. By this, they mean that mixed methods is selected in order to elaborate on the findings of one method with another, enhancing the findings of qualitative data with quantitative data (or vice versa).

Amongst Bryman's (2006) list of sixteen rationales for mixed methods, many of them closely resonate with this study:

Table 4.3 - Rationales for mixed methods research

Rationale		Definition	Example for this thesis
1	**Triangulation**		N/A
2	**Offset**	Combining quantitative/qualitative methods to offset the weaknesses of each in certain contexts with the strength of the other.	Qualitative methods could not reach as many participants successfully as quantitative in the ASSIST/ATL data collection. Furthermore, the student experience could not be properly explored in depth through quantitative methods as one can never really predict or quantify certain feelings and experiences, particularly in the context of this thesis where their very idiosyncrasies are compounded by the individuality of the university under research.
3	**Completeness**	Combining methods enhances the comprehensiveness of the research.	Having a quantitative description of students' ATLs can then be used with interviews to illustrate how these might come about and students' own descriptions of these.
4	**Process**	Whilst "quantitative research provides an account of structures in social life… qualitative research provides a sense of progress" (p. 106).	Qualitative interviews describe students' personal experiences and progress on an individual basis whereas the quantitative data gives a snapshot of one moment in time. Whilst the ASSIST was redistributed in order to map progress and comparisons were made across the year group, thick descriptions of this could not be made without student narratives.
5	**Different Research Questions**		There are 3 questions in the study which, independently, require different methods of attack by virtue of the answers they seek.
6	**Explanation**	Use of one method to explain the findings of the other.	Student interviews can be used to explain and elaborate on the ASSIST and MATH taxonomy data.
7	**Unexpected Results**		N/A

8	**Instrument Development**		N/A
9	**Sampling**	Using one method to facilitate the sampling of participants for the other.	Generally N/A here, except the inclusion in the ASSIST of a question asking for contact details from participants should they wish to take further part in the study (i.e. in interviews).
10	**Credibility**	"enhancing the integrity of findings" (p. 106) in using multiple methods.	Students' reports of their ATLs based on their own experiences enhance the descriptions given by the ASSIST in terms of a subject-specific dimension.
11	**Context**	Combining methods provides contextual understanding.	Student interviews provide descriptions of the Oxford undergraduate experience which cannot be gleaned from any quantitative data.
12	**Illustration**	Mixed methods puts "'meat on the bones' of 'dry' quantitative findings" (p. 106).	Student interviews colour in the picture drawn of their experiences and behaviour provided by the ASSIST and MATH taxonomy.
13	**Utility**	Making research more accessible and useful for parties involved in policy and development.	Quantitative findings of the ASSIST and categorisation from the MATH taxonomy may appeal to mathematics educators who are not familiar with educational research, whist the qualitative findings provide a more detailed explanation of that which can provide a greater insight.
14	**Confirm & Discover**		N/A
15	**Diversity of Views**	"uncovering relationships between variables through quantitative research while also revealing meanings among research participants through qualitative research" (pp. 106-107).	Student interviews describe the background to quantitative findings in the ASSIST and MATH taxonomy.

Adapted from Bryman (2006)

The different research questions posed for this study are suited to different types of research methods. In order to explore students' experiences of studying mathematics throughout their university careers (Research Question 1), the use of a questionnaire to compare and contrast students' ATLs across year-groups provides quantitative data from which comparisons may be drawn. The student interviews combine with this in order to be able to describe the evolving

experience of particular individuals which could not be captured in a non-longitudinal study such as this.

The challenges of enculturation into tertiary mathematics study (Research Question 2) were explored on the question level, which could be achieved using the MATH taxonomy and further articulated by students in qualitative interviews. Contrasts between secondary and tertiary questions, as well as secondary and pre-university questions could be made using this method, which also acted as a means of describing the unique nature of the Oxford admissions process in terms of the utility of the admissions test.

Research Question 3, regarding relationships between students' ATLs and the challenges they perceive in university mathematics assessment, is addressed using a mixture of qualitative and quantitative approaches. Quantitative data collection using the ASSIST means that students' ATLs can be described on a cohort level, whilst qualitative data can be used from student interviews in order to question and probe students' perceptions of the challenges that they face, their ways of learning and working, and how these might relate to assessment.

It can therefore be seen through the method outline and Table 4.3 that this study and its aims are suited to this approach, driven by a pragmatic viewpoint. The rest of this chapter will look at the particulars of how data were collected and analysed within the mixed methods framework.

In this next section, the three methods of data collection will be discussed, specifically relating to:

- a description of the instrument/method;
- justification of its use;
- procedure;
- sampling;

- validity;
- reliability;
- credibility; and
- analysis.

4.4 – Student Interviews

4.4.1 – Description

Semi-structured interviews were conducted with a selection of undergraduate mathematicians across all year groups (see Table 4.4).

Such students offered their participation after the entire student body was contacted via the departmental mailing list seeking participation in this part of the study (see Appendix 4.5). Questions asked and topics covered focused primarily on the students' experiences of studying mathematics at Oxford, specifically relating to four areas: (1) their experience of school mathematics; (2) their preparation for entry to Oxford and the admissions process; (3) their current (and past, if a student beyond the first year) experiences of the subject and pedagogy; and (4) any changes experienced or anticipated in their mathematics studying and learning. Questions were asked regarding any differences between, for example, methods of revision at school and university or the types of mathematics studied. Participants were not forced to talk about changes when they may not have experienced any, and a number of them claimed there to have been no differences in their experiences at certain stages. Students were asked if there were any differences, rather than what differences were. Interviews were conducted one-to-one, with a dictaphone recording the interview. All of the students had previously completed the ASSIST, and so their scores on each of the deep, surface and strategic scales were known (see Chapter 4.5).

4.4.2 – Justification

The research questions posed required the student voice to be heard, and an account of their experiences to be given – something which could only reasonably be achieved through student interviews. The learning experiences of individual students within the same university, studying the same course, will be different despite the common experiences shared by the

hundreds of students in the same situation. This belief was supported through the data collected in Darlington (2010) which revealed students to display signs of a range of ATLs, which was reinforced by the data collected by the ASSIST prior to interviews being conducted.

Use of a questionnaire for the purpose of answering the research questions would be inappropriate for a number of reasons, namely:

- Questions about personal experience would undoubtedly require lengthy answers, which could be off-putting to potential participants.
- Further probing of comments made by students cannot be facilitated.
- Whilst certain topics were planned to be addressed, one can never really anticipate all of the different things which will be described and discussed. It is possible that discussion with one student in one interview could inspire questions which could be posed to students in later interviews. This relates to the 'saturation' of data in that continual collection of data might result in the emergence of new ideas which may then be tested and compared between participants. Therefore the data from a semi-structured interview has two features:
    1. A great deal of common ground from the core questions posed.
    2. An accumulation of issues which were not initially anticipated but which arose in interviews.
- It would be inappropriate to attempt to quantify issues concerning the student experience, particularly given that the Likert scale of the ASSIST has done that to a certain degree. It is the purpose of the interview to provide explanatory detail, as well as confirmatory evidence of the process of enculturation that can be deduced from the analysis of questions. Furthermore, it permitted the ability to uncover new issues and insights which were not anticipated or sought at the beginning of the research process.

4.4.3 – Procedure & Sampling

Interviews were conducted with a selection of undergraduate mathematicians in Trinity Term[27] before their end-of-year examinations. Students were contacted via their departmental mailing list, asking for their participation. Students from both joint and single honours degrees from all year groups were requested. In all, 13 students were interviewed, comprising of:

Table 4.4 - Interview participation by year group

Course	1st Year	2nd Year	3rd Year	4th Year	Total
Mathematics	2	3	3	1	9
+ Philosophy			1	1	2
+ Computer Science	1				1
+ Statistics	1				1
Total	4	3	4	2	13

Every student who responded was interviewed. Cohen *et al.* (2007) comment "that the parameters of generalisability in this type of sample are negligible" (p. 114). Of course, this means that the sample is inevitably biased. It is possible that the students who replied did so because they are interested in being part of the research, and want to share their experiences. They may have had an agenda; for example, they may have had particular experiences that they wish to share in the hope that they may be addressed in the future for the benefit of either themselves or other students. Therefore, the representativeness of the sample used may be brought into question.

However, this was unavoidable in this particular instance. Upon further analysis of the interviewees and their data, the spread of ATLs which they showed predominant leanings towards in the ASSIST appeared to be approximately in proportion with the whole cohort (or, at least, the group of students who completed the ASSIST). Specifically,

[27] Oxford names its three terms as Michaelmas Term (September – December), Hilary Term (January – March) and Trinity Term (April – July).

Table 4.5 - Interview participants by ATL

Year	Deep ATL	Strategic ATL	Surface ATL	Total
1	3	1	0	4
2	1	1	1	3
3	1	3	0	4
4	1	1	0	2
Total	6	6	1	13

The participants, according to their ATL, approximately reflect the proportions in each year group as ascertained by the ASSIST (see Table 5.1). That is, similar proportions of students with deep and strategic approaches, and surface approaches being the least common.

Interviews were semi-structured so as to provide a framework for discussion whereby certain topics would be covered but without the restriction of having a script to keep to. This prevents discussion from being stifled and potentially interesting issues and experiences from being recorded and explored. Semi-structured interviews are suited for instances like this when participants' perceptions and personal experiences are the subject of enquiry, and permit the interviewer to "seek both clarification and elaboration on the answers given", arming the interviewer with "more latitude to probe beyond the answers" (May, 1993, p. 93). The ability to probe participants' responses allows the interview to collect relevant data which might not otherwise have been picked up through a structured interview with a fixed set of questions. Furthermore, probing may also act as a means of ensuring reliability (see Table 4.8).

Kvale (1996) suggests the semi-structured interviewer begins with an initial "sequence of themes to be covered, as well as suggested questions" (p. 124). Therefore, an initial framework for the interview (see Table 4.6) was decided in advance of speaking to all participants, which would have the students discuss and describe their experiences of mathematics learning in a chronological order. This means that "the interview can be shaped by the interviewee's own understandings as well as the researcher's interests, and unexpected themes can emerge" (Mason, 2004, p. 1021).

Table 4.6 - Interview framework

School experience of mathematics	Oxford entry
Now/degree progression	Changes

Within that framework, key aspects were outlined to be discussed with participants, with an "openness to changes of sequence and forms of questions in order to follow up the answers given and the stories told by the subjects" (Kvale, 1996, p. 124). These were:

- **School experience of mathematics:**
    - What mathematics was like
    - A-level subjects studied
    - Modules in mathematics studied
    - Why Mathematics A-level
    - Enjoyment of mathematics
    - How mathematics was taught
    - How mathematics was learnt and revised
- **Oxford entry:**
    - Why a mathematics degree
    - Why Oxford
    - Memories of the OxMAT
    - Memories of their interview
- **Now/degree progression:**
    - What was the secondary-tertiary transition like?
    - Experience of the tutorial system

- How they do tutorial work
- Revision
- Perceived ATLs
- COPs

- **Changes:**
  - In enjoyment, revision, ATL, work ethic
  - Perceptions of why
  - Plans for the future
  - Expectations of academic future/outcome

Any notes taken during the interview were recorded on a sheet with a layout akin to Table 4.6, above (see Appendix 4.6).

Participants were "provided with a context for the interview by a briefing before and a debriefing afterward" (Kvale, 1996, p. 127). The briefing read as follows:

> *Before we begin, I'll just outline what we're going to do here, what we're going to talk about today, and the purpose of the interview. This interview is about undergraduate maths students' experiences of undergraduate maths here at Oxford, and I'm hoping to find out about your experiences of the subject and learning it both at A-level and in your first year and subsequent years. So that's what you've learnt, how you've learnt it, what you've liked, disliked, been good at, been bad at, and so on. The data I collect from the interviews will be used and analysed as part of my doctorate, which I'm in the second year of, in conjunction with some other data. What we talk about today will be recorded on this dictaphone, and in the consent form you've read about starting and stopping the recording. Do you have any questions before we begin?*

Students were also provided with a consent form (see Appendix 4.7) prior to the interview which outlined issues regarding tape recording the interview, as well as an outline of the interview and its purpose. After the interview, a short debrief of the student was given where what was discussed was summed briefly summed up. The students were also asked if they had any questions, or if there was anything else that they wanted to add.

The types of questions asked and the way in which they are asked is important in an interview. Tuckman (1972) recommends that interviewers consider in advance whether the interviewees might give answers which give an overly-positive impression of themselves, something which is possible in a situation like this when study habits and perceptions of difficulty are under discussion. Such topics, also in common with those covered in the ASSIST, may inadvertently cause the subject to give answers which they think the interviewer is looking for.

Table 4.7 - Types of question in semi-structured interviews

Type of Question	Description	Example
Introducing Questions	Opening questions which "may yield spontaneous, rich, descriptions where the subjects themselves provide what they experience as the main dimensions of the phenomena investigated" (p. 133).	Tell me about your Oxford interview.
Follow-up Questions	An attempt to extend answers already given to previous questions. This may be done through directly questioning a previous utterance, or repeating significant words.	What do you mean by 'editing' proofs?
Probing Questions	Pursuit of answers through "probing their content but without stating what dimensions are to be taken into account" (p. 133).	Can you think of any other examples?
Specifying Questions	Follow-up questions which are 'operational'.	What do you actually do when you're working with other students on a problem sheet?
Direct Questions	Direct introduction of new topics and dimensions.	Did you do any resits in your A-levels?
Indirect Questions	Projective questions which may relate to others' attitudes.	How does that compare to how your peers revise?
Structuring Questions	Questions asked to introduce a new topic.	OK, so going back to your first year, what mathematics did you find difficult or easy?
Silence	Use of silence to further the interview. I also used 'OK…' as a means of prompting students to continue talking about a topic.	OK…
Interpreting Questions	Rephrasing an answer or attempts to clarify.	So you mean that you memorised the material?

Adapted from Kvale (1996, pp. 133-135)

As each student is an individual, each interview will also be so and, in that, the trust of the interviewer by the interviewee will be different (Cicourel, 1964). It was hoped that, by giving students an idea of my background in mathematics, they would feel more relaxed in my presence. Furthermore, my lack of personal experience with Oxford undergraduate mathematics would mean that the students would feel a sense of control over their situation and its discussion. However, the absence of a shared institutional background has the potential to be of a hindrance, when interviewees may have described modules, examinations, lecturers and other experiences without consideration of the interviewer's understanding and knowledge of these. However, my experience as a lecturer and tutor in the department and a note-taker for mathematics undergraduates minimises this handicap.

A shared background in mathematics was a means of establishing a rapport with the interviews, along with the asking of some 'descriptive questions' (Spradley, 1979). Questions such as 'what college are you at?', 'what year are you in?' and 'what A-levels did you do?' acted as an easy means of 'breaking the ice' and beginning the interview.

Whilst it is possible that different interviewees could interpret the same interview question in a different way, or fail to understand it, this was prevented by preparing to rephrase questions to suit the individual participant (Oppenheim, 1992). This was further aided by a preparedness to be less specific with questions by encouraging students to discuss experiences rather than asking questions formulaically. It was hoped that this would encourage them to be more frank (Tuckman, 1972).

Whilst a list of questions was not strictly adhered to in the interview, Appendix 4.8 lists a set of commonly-asked questions which were posed to most students in some capacity.

4.4.4 – Strengths & Limitations

Caution was taken when designing and carrying out the interviews due to the various potential pitfalls associated with such a method of data collection. Whilst Cohen *et al.* (2007) caution that "interviewers and interviewees alike bring biographical baggage with them into the interview situation" (p. 150), the purpose of the student interviews is to discuss their personal experience so this should not have proven problematic. It was, however, important for me not to attempt to influence the students' responses based on my personal experience of studying the subject at university (or, indeed, to seek them during analysis). This was aided to a certain extent by the fact that I earned my degree at a different institution, which had a different academic and pastoral structure.

One of the problems associated with interviewing is the assumption that the participant has an awareness of their experience. May (1993) comments that, "while accounts may be a genuine reflection of a person's experiences, there might be circumstances or events which the person was not aware" of (p. 109). For example, in discussing their experiences of tutorials, students can only infer the intentions of their tutors in their questioning from their experience. What they report is their perception.

In the context of qualitative research, 'reliability' and 'validity' are not relevant terms considering the nature of data collection (Altheide & Johnson, 1998). The most commonly-referenced alternative terminology is Lincoln and Guba's (1985) 'trustworthiness', which refers to a measure of testing the quality of qualitative research whilst considering four different areas:

1. Credibility
2. Transferability
3. Dependability
4. Confirmability

Indeed, Seale (1999) describes the some data's trustworthiness as "lying at the heart of issues conventionally discussed as validity and reliability" (p. 266). To that end, the trustworthiness of this research was ensured and safeguarded through the use of a framework implied by Guba (1981), as outlined in Table 4.8.

Table 4.8 - Endeavours to make this study trustworthy

Element of trustworthiness	Question to ask oneself	What this avoids	Means by which to avoid it	Methods employed for this study
Credibility	How can one establish confidence in the "truth" of the findings of a particular inquiry for the subjects with which, and the context in which, the inquiry was carried out?	Non-interpretability	Use prolonged engagement	Over the course of 4 years I became familiar with the Mathematical Institute, through MSc research based there, as well as teaching undergraduate courses and note-taking for disabled undergraduate students.
			Use persistent observation	
			Use peer debriefing	Findings were discussed with my doctorate supervisor and peers, as well as research being presented in formal settings at seminars and conferences to the educational research community.
			Do triangulation	Elements of the interview were related to the data collected by the ASSIST and MATH taxonomy, providing an overlap. Data collected were also contrasted with existing literature.
			Collect referential adequacy materials	Audio recordings were kept and listened to multiple times before, during and after transcription.
			Do member checks	Audio transcripts were available to participants for them to check if they wished.
Transferability	How can one determine the degree to which the findings of a particular inquiry may have applicability in other contexts or with other subjects?	Non-comparability	Collect thick descriptive data	A range of questions were posed to students, and relevant comments probed further where possible. Contextual factors are described in depth.
			Do theoretical/purposive sampling	Opportunistic sampling brought together a group of participants whose profile approximately matched that of the wider population (i.e. gender, course studied, ATL). This 'wider population' was not that of the whole (i.e. all undergraduate

				mathematicians at Oxford); however, its participants' profile was similar to that of the student body (see Appendix 4.9).
Dependability	How can one determine whether the findings of an inquiry would be consistently repeated if the inquiry were replicate with the same (or similar) subjects in the same (or similar) context?	Instability	Use overlapping methods	Use of mixed methods in this study helps to reinforce the findings of each one, as the topics covered in the interviews are also covered to a certain degree using the ASSIST and MATH taxonomy. Indeed, the interviews are used to colour in the picture drawn by the other two methods.
			Use stepwise replication	N/A – Stepwise replication is only used when multiple researchers are involved.
			Leave audit trail	Detailed documentation and a running account of the research process were kept. This thesis acts as an audit trail itself.
Confirmability	How can one establish the degree to which the findings of an inquiry are a function solely of subjects and conditions of the inquiry and not of the biases, motivations, interests, perspectives, and so on of the inquirer?	Bias	Do triangulation	As before.
			Practice reflexivity	Use of an audit trail.

Adapted from Guba (1981)

4.4.5 – Analysis

After transcription, the interviews were subjected to thematic analysis. This appeared to be the most appropriate method of analysing the interviews as the interviews themselves were undertaken in order to explore the phenomenon that is the undergraduate mathematics learning experience. This method of analysis is used to identify emerging patterns in the interview transcripts which may then be used to organise and describe the students' comments in rich detail (Braun & Clarke, 2006). Thematic analysis infers a description of the students' "truth space" (Onwuegbuzie, 2003, p. 400); that is, the students' feelings, experiences and opinions. Specifically,

> *A theme is an abstract entity that beings meaning and identity to a recurrent experience and its variant manifestations. As such, a theme captures and unifies the nature or basis of the experiences into a meaningful whole.*
>
> (Desantis & Ugarriza, 2000, p. 362)

Thematic analysis is often criticised for being so frequently used without apparent consultation of guidelines (Braun & Clarke, 2006). Indeed, it is described in handbooks such as Tashakkori and Teddlie (2003) as if it were an obvious and trivial method and concept. However, for this thesis, the guidelines set out by Braun and Clarke (2006) were followed closely in order to provide a consistent, reliable framework to follow:

Table 4.9 - Framework for thematic analysis

Phase		Description of the Process
1	Familiarising yourself with your data	Transcribing data (if necessary), reading and re-reading the data, noting down initial ideas.
2	Generating initial codes	Coding interesting features of the data in a systematic fashion across the entire data set, collating data relevant to each code.
3	Searching for themes	Collating codes into potential themes, gathering all data relevant to each potential theme.
4	Reviewing themes	Checking if the themes work in relation to the coded extracts (Level 1) and the entire data set (Level 2), generating a thematic 'map' of the analysis.
5	Defining and naming themes	Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for

		each theme.
6	Producing the report	The final opportunity for analysis. Selection of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report on the analysis.

Taken from Braun & Clarke (2006, p. 87)

4.4.5.1 – Data Organisation

After transcription, students' utterances were liberally coded into many categories. This first step involved careful analysis and resulted in many statements made by the participants fitting into multiple categories. In all, these categories totalled over 120 once all interviews were analysed. These were then grouped within themselves down to 27 categories.

These were:

1. A-level
2. ATLs
3. Assessment
4. Challenges
5. Collegiate system
6. Contrasts and comparisons
7. Degree structure
8. Degree-related activity
9. Examination
10. Expectations
11. External motivators
12. Interview
13. Mathematics peers
14. Nature of mathematics
15. Negative emotions
16. OxMAT
17. Pedagogy
18. Perceptions of success
19. Positive emotions
20. Preferences
21. Resources available to students
22. Response to failure
23. Social activities
24. Social ATLs
25. Teachers
26. Topics in mathematics
27. Undergraduate mathematics

In order to utilise the data collected and the themes uncovered in the transcripts, a framework for organising this and interpreting it in the context of enculturation into an academic community was sought.

4.4.5.2 – Saxe's Four Parameter Model

Saxe's (1991) Four Parameter Model (see Figure 4.8) was used in order to explore the relationship between cognition and culture. It grouped categories of utterances together in order to describe how the enculturation of undergraduate mathematicians has an impact on them, specifically relating to questioning and assessment.

Even though the model was created with a view of describing mathematical practices in culturally-specific circumstances, such as the candy sellers he described, the model is believed to also be general in its application (Lagrange & Monaghan, 2009).

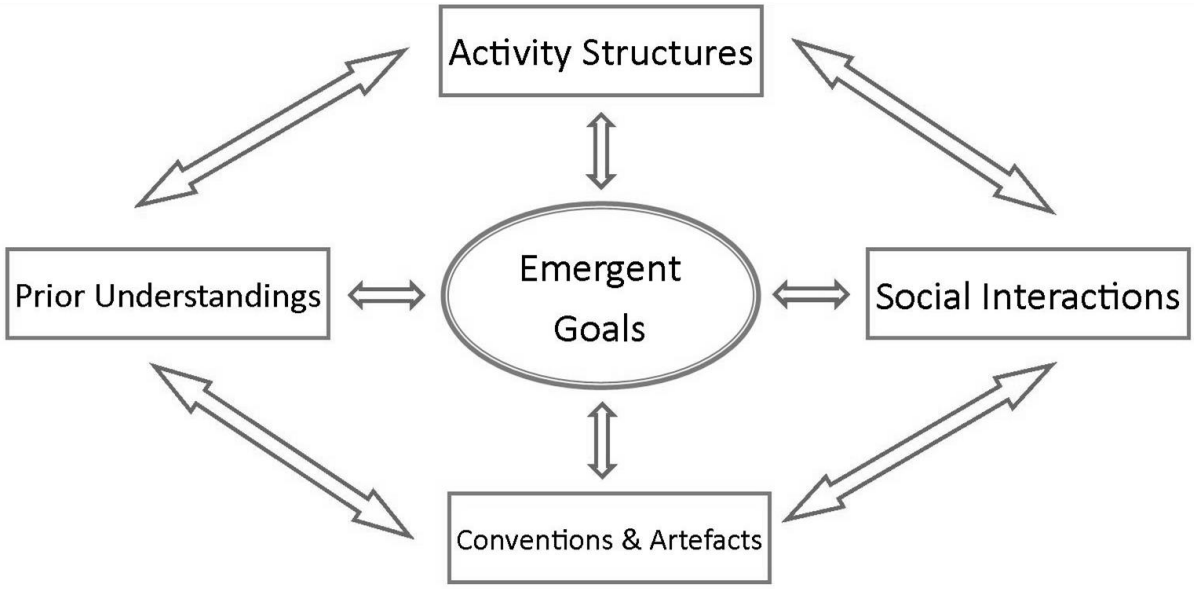


Figure 4.10 - Saxe's Four Parameter Model

Saxe's model actually focuses on how the four parameters impact upon the emergent goals of the individual; however, this is not the primary way in which I will be using the model. The role of this model is to act as a means of organising the data that I have in order that it may be analysed and discussed in a way which is meaningful to the topic that it is about. However,

students' emergent goals may be considered to be central to their experience, as these dictate their actions. Indeed, Cole (1991) asserts that "goals are not static forms that exist ready made in the minds of subjects" but they emerge as someone brings "to bear in their own understanding to organising and accomplishing problems that emerge during their participation in cultural practices" (p. 241). If one 'recontextualises' this for the new undergraduate student, it is possible to consider their emergent goals as occurring as they are enculturated into a new environment, with new mathematics, new practices and new conventions. Their goal might be to become good at mathematics, to do well in mathematics examinations, to become an active member of the Oxford mathematics COP, and so on.

After examining the 27 categories to see whether they may fit within the four-parameter model, it was noticed that Saxe's model lacked an affective domain which would be an important consideration in this research. Emotional response to circumstances involved in their enculturation is something which appeared to be a significant part of the enculturation process for students and as such this category was added for the purposes of this analysis.

Table 4.11 - Saxe's Four Parameter Model

Parameter	Prior Understandings	Social Interactions	Activity Structures	Conventions & Artefacts	Affective Responses
Definition	• Perception • Used to "both constrain and enable the goals they construct". • Fundamental to the kind of goals that emerge in a practice.	• Other people may influence the goals a person sets themselves, and assist in their achievement. • Social interactions that emerge in a practice – may simplify some goals and complicate others.	• Tasks people perform in everyday life that are culturally-determined. • General tasks that must be accomplished in a practice and the general motives for practice participation.	• Accepted ways of doing things in the culture • Tools used in the culture, both concrete and mental. • Artefacts which are interwoven with the practice.	Emotional responses to the circumstances.
Example in Undergraduate Mathematics Context	• What it is to revise mathematics. • What mathematics is. • How they perceived their performance in the OxMAT.	• Means by which mathematics is communicated socially to students. • Social ways of learning and doing mathematics. • Students' experiences of the collegiate system.	• How students study during term-time. • Revision and examination preparation. • Response to failure.	• Assessment • Pedagogy • Degree structure	• Preferences • Perceptions of success • Negative emotions

Adapted from Saxe (1991)

For the purposes of this study, Saxe's model "simplifies the intricate relationships between the participants' actions and the practice in which the tasks are performed" (Magajna, 1997), allowing the data to be meaningfully described, analysed and discussed. To that end, the 27 themes were distributed into the five parameters as follows:

Table 4.12 - Adapted model

Parameter	Theme	Sub-Themes
Social Interactions	Teachers	Class tutors, lecturers, tutor, interview prompts
	Mathematics peers	Comparison with peers, Invariants, gender
	Collegiate system	Friends, college
	Social activities	Extra-curricular activities
	Social ATLs	Collaboration, support, types of help
Conventions & Artefacts	Pedagogy	Classes, lectures, lecture notes, tutorials, notes
	What undergraduate mathematics is	Theorems, definitions
	Degree structure	MMath, passing an exam, current modules
	Assessment	Demands of university exams, contrasting exam formats, unseen questions, repetitive questions
	Resources available to students	Past papers, problem sheets
Affective Responses	Positive emotions	Enjoyment, easy, success
	Preferences	Favourite mathematics, blame, rationale, mathematical preferences, interest
	Negative emotions	Hard, intimidating, dislikes, tired, criticism, shock to the system
	Perceptions of success	OxMAT perception, interview perception, confidence
Activity Structures	Examination	University mathematics questions, university revision, revision notes, essays
	Response to failure	Adapt and adjust, learning from mistakes, stuck, attainment
	Degree-related activity	Effort, independent work, studying, work ethic
	Topics in mathematics	Algebra, decision mathematics, computer science, analysis, calculus, differential equations, probability, dynamics, pure mathematics, statistics, proof, applied mathematics, optional modules, philosophy
	ATL: Strategic	Practice, relating/editing proofs
	ATL: Deep	Visualisation, necessity of understanding
	ATL: Surface	Copying, memorising, pattern-spotting

Prior Understandings	OxMAT	OxMAT preparation, OxMAT utility, response to OxMAT
	A-level	A-level subjects, demands of A-level Mathematics, A-level Mathematics revision, A-level Mathematics, A-level as university preparation, A-level Mathematics modules, science, STEP, resits
	Contrasts and comparisons	Difference between A-level and university mathematics, OxMAT vs. A-level, changes throughout years, A-level vs. university revision
	Nature of mathematics	Procedural, computation, abstract, mathematical thinking
	Expectations	Expectations of university mathematics, degree preparation, what Oxford wants
	Challenges	Transition, workload, time pressure, mathematical struggles, understanding, attainment, pace
	External motivators	Career, gap year, money, choosing mathematics, aspirations, choosing Oxford
	Interview	Interview questions, response to interview questions, interview utility, interview preparation

4.5 – ASSIST

4.5.1 – Description

The Approaches and Study Skills Inventory for Students (ASSIST) (Tait *et al.*, 1998) is a multiple choice, Likert scale questionnaire developed by a team of writers over the course of nearly twenty years as a means of quantifying, categorising and measuring student ATLs (see also Chapter 2.1). One of the major benefits of this particular scale over others is that it was developed for use with tertiary students, unlike many other scales described in the educational research literature. Furthermore, its current state is a consequence of gradual change and evolution over time; the ASSIST is a descendant of the Approaches to Studying Inventory (ASI; Entwistle & Ramsden, 1983), and such testing and revisions increase its reliability.

A number of other well-known options were considered but ultimately disregarded for a variety of reasons, although mainly to do with the number of studies which questioned their reliability and validity:

Table 4.13 - ATL instruments considered

Scale	Source	Why Not
Study Processes Questionnaire	Biggs (1987a)	Reliability and validity questioned (Burnett & Dart, 2000).
Learning Styles Questionnaire	Honey & Mumford (1986)	Poor construct validity (Swailes & Senior, 1999).
Learning Styles Inventory (LSI)	Kolb (1976)	Duff & Duffy (2002) call the LSI inappropriate in the higher education context as it does not reflect the sophisticated nature of undergraduate learning. It is "criticised on the conceptual grounds that it put together the unrelated elements of cognitive process, cognitive style and cognitive level" (Cuthbert, 2005, p. 243).
ASI	Entwistle & Ramsden (1983)	The precursor to the ASSIST.

A number of lesser-known options were considered. These are not regularly used in empirical research and are either based on conceptual frameworks which were not related to this study or they lacked sufficient statistical reinforcement:

- Dunn and Price Learning Styles Inventory (Dunn, 1990)
- Inventory of Learning Processes (Schmeck *et al.*, 1977; Schmeck *et al.*, 1991)
- Learning and Study Strategies Inventory (Weinstein *et al.*, 1987)
- Inventory of Learning Strategies (Vermunt & van Rijswijk, 1988)
- Inventory of General Study Orientations (Mäkinen & Vainiomaki, 2002)
- Reflections on Learning Inventory (Meyer *et al.*, 1990)

Analysis of the ASSIST reveals someone to display a 'dominant' ATL out of deep, strategic and surface approaches as defined in Chapter 2.1. A comprehensive review of the ATLs literature by Coffield *et al.* (2004) recommended the adoption of the deep/surface/strategic trichotomy of terms as described by Entwistle over the other dichotomies described in the literature (see Chapter 2.1.3). Each statement in the ASSIST relates to one of these three, with related subscales as follows:

- **Deep ATL**:
    - Seeking meaning
    - Relating ideas
    - Use of evidence
    - Interest in ideas
- **Strategic ATL**:
    - Organised studying
    - Time management
    - Alertness to assessment demands
    - Achieving

- Monitoring effectiveness

- **Surface ATL**:
  - Lack of purpose
  - Unrelated memorising
  - Syllabus boundedness
  - Fear of failure

4.5.2 – Justification

Data collected from the ASSIST can be used in three main ways, namely for:

1. calculating the proportions of a given sample who dominantly display characteristics typical of deep/surface/strategic ATLs;
2. examining individual scale items to see the array of responses provided by a given sample; and
3. examining the average scores on each of the three scales.

Results from this can be used to:

1. *Compare results in one sample to another sample*
   - comparing students of English Literature with those of medicine; or
   - comparing students of one subject in one university to those in another.
2. *Compare results of one given sample over time*

   For example, the possibility of students changing from displaying one particular dominant ATL to another over a period of time may be investigated by re-administering the questionnaire at a later date.
3. Examine the experience of a group of students in order to identify any common characteristics of the group that may be considered problematic or of concern in order to act on these to improve the situation for students.
4. Have students reflect on their own ATL and its consequences.

For the purpose of this thesis, it is the second of these possible uses of the ASSIST which will be used. However, its analysis and combination of that with other data could be used by policymakers in the context of the third possibility.

4.5.3 – Procedure & Sampling

Potential participants for completion of the ASSIST were contacted via the departmental mailing list for the Mathematics Institute with an outline of the research (see Appendix 4.2) and a link to an online form of the questionnaire (see Appendix 4.4). Use of the email meant that the whole of the undergraduate population could be reached without bias; however, it could not be guaranteed that all of the students will have actually opened the email and read its contents. Approximately 65% of the first-year cohort completed the questionnaire in Sweep 1, and over 40% of all year-groups completed the questionnaire in Sweep 2 (see Chapter 5).

4.5.4 – Strengths & Limitations

4.5.4.1 – Sample

The nature of the sampling means that not all of the students completed the questionnaire. There are a number of reasons why students may not have completed the questionnaire:

- **They find themselves 'spammed' with emails from various sources asking for their participation in questionnaires relating to their course, which they find tiresome.** Some of the interview participants remarked that they often receive course feedback questionnaires from the department and their undergraduate mathematics association, which can be overwhelming, along with questionnaires from their colleges.
- **They were put off the questionnaire by its length.**

  However, the questionnaire was spread out over a few pages, so it was broken down into 20-question sections so as not to be off-putting. This meant that it was possible for participants to get through answering some questions before quitting. Any

responses that had been submitted until the point of quitting were still submitted. This only occurred in one instance.

- **They were not interested in the topic of the research.**

  An attempt was made to ensure that the topic was sold well to the students as a means of providing feedback which can improve the student situation in the future, as well as acting as a way for them to think about their own learning.

No financial incentive was given to participate in the study.

Online surveys can reach the entire population being researched without limiting the sample to being just students in attendance at a particular time and place. Whilst paper-and-pencil methods have been found to be better received than other means, potential methods for return of the questionnaires were deemed to be suitably inappropriate to not to attempt this. Students could have either been handed questionnaires in lectures or via pigeon post to their colleges. The limitations of each method are rather severe.

Specifically, the limitations of the lecture method are related to it limiting the sample to students in attendance in the lecture. One could hypothesise that students who are less satisfied with their course are less likely to attend their lectures. This would then mean that their thoughts and experiences are not recorded and do not contribute to the data collected. Furthermore, compulsory modules common to all students may only be found in the first year. Students of subsequent years do not all study the same modules. Lecture-based questionnaire distribution also relies on the permission of the lecturer to finish early in order that students may complete the questionnaire. This would require a maximum of ten minutes to do the questionnaire, plus an additional ten minutes for distribution and collection at the end. Therefore, a 50 minute lecture would be seriously cut short – something which lecturers were

very reluctant to do when I was piloting the ASSIST when completing my dissertation for MSc Educational Research Methodology.

The main problem with postal distribution is regarding questionnaire return. Whilst distribution could be conducted with relative ease, using the University's pigeon post system to send questionnaires to students' college pigeon holes, the means by which they would have to return them could be off-putting. Having to go to the trouble to answer the questionnaire before then returning it back through the pigeon post is an extra effort which many students may not be prepared to make. An additional problem with this is establishing a list of undergraduate mathematicians to send the questionnaires to. This is not something that the Mathematical Institute was prepared to provide.

Therefore, a decision was taken to distribute an online link to the questionnaire to students via email. The Mathematical Institute kindly agreed to use their undergraduate mailing list to do this, and also sent a subsequent reminder one week later. All students have internet access and are expected to check their emails daily because important communications reach them this way. Furthermore, in an age when smartphones are becoming increasingly popular, it is expected that all students will have had the opportunity to receive and respond to the email within a couple of days of it being sent. Distribution via email therefore does not limit the scope of access to the link; however, the decision taken by students to respond to it would be the decider on the overall number of participants.

Internet-based distribution also has an advantage in terms of data analysis. Use of Google Docs to create an online survey meant that data was immediately presented in spreadsheet form which could be coded using the click of a button. Conversely, paper-and-pencil questionnaires would have had to have been manually entered into the statistical software program SPSS, which would increase the potential for human error. It also creates an added expense through paper and printing costs.

4.5.4.2 – Reliability

As with any other quantitative measure, there are a number of limitations to the use of the ASSIST. Whilst its validity and reliability have been tested on multiple occasions and has been found (Speth *et al.*, 2007) to be stronger than alternative scales such as the ASI (Entwistle & Ramsden, 1983; Tait & Entwistle, 1996), Coffield *et al.* (2004) criticise the ASSIST for having less reliable sub-scales. They also comment that it as not been tested as a "basis for pedagogical interventions" (p. 25); however, this is not of concern here as this is not the intention of this study. Students have been found to make mistakes in their responses to items on similar scales to the ASSIST (Mogashana *et al.*, 2012). In order to minimise this risk, the questionnaire was divided over three pages, with large gaps between questions and frequent reminders of what each option on the Likert scale stood for. Furthermore, participants were not able to accidentally miss a question because an error message was produced before submission to alert them to this. After submission, participants are able to edit their responses if they realise a mistake.

The ASSIST has good test-retest reliability, with correlation coefficients at one week, two week and three month follow-ups being at least 0.65 (Clarke, 1986; Richardson, 1990), as well as good validity and reliability (Byrne *et al.*, 1999, 2004; Diseth, 2001; Entwistle *et al.*, 2000; Kreber, 2003; Reid *et al.*, 2005). Its factor structure "is clear-cut and has been confirmed with other samples and at different levels of performance" (McCune & Entwistle, 2000, p. 1). During its development, Cronbach's α for the ASSIST was 0.84 for deep, 0.80 for strategic and 0.87 for surface ATLs (Tait & McCune, 2001), an improvement on that of the ASI.

4.5.4.3 – Validity

Its applicability and suitability for use with undergraduate mathematicians is something which has already been confirmed and discussed at length owing to its use in a dissertation three years ago (Darlington, 2010; see also Darlington, 2011). However, its use in relevant empirical

research appears to be limited. This is surprising given that ATLs were very much a hot topic in educational research in the 1980s. It was developed with psychology students as a sample base (Entwistle & Ramsden, 1983), and has been used to research students of other subjects such as accounting (Byrne *et al.*, 1999, 2002, 2004, 2009; Marriott, 2002), biological sciences (Speth *et al.*, 2005, 2007), geography (Maguire *et al.*, 2001), medicine (Reid *et al.*, 2007), psychology (Diseth & Martinesen, 2003; Huws *et al.*, 2009) and the social sciences (Spada *et al.*, 2006). However, it has yet to be applied to the mathematics student. It is therefore important to note that some "subscales are more likely to vary in their relationships across different samples. Relationships thus need to be checked in the particular sample used for the study" (Tait & McCune, 2001, p. 1). Factor analysis conducted in Darlington (2010) before use confirmed that the ASSIST measures, in mathematics undergraduates at Oxford, what it purports to do for others.

Furthermore, in their analysis of instruments which measure ATLs, Coffield *et al.* (2004) comment that:

> *ASSIST is useful as a sound basis for discussing effective and ineffective strategies for learning and for diagnosing students' existing approaches, orientations and strategies. It is an important aid for course, curriculum and assessment design, including study skills support. It is widely used in universities for staff development and discussion about learning and course design… It is crucial, however, that the model is not divorced from the inventory, that its complexity and limitations are understood by users, and that students are not labelled as 'deep' or 'surface' learners.*
>
> (p. 56)

Therefore, thanks to mixed literature concerning the stability of ATLs (Vermetten *et al.*, 1999) and successful use in my previous research, I believe that this scale has been used effectively to aid the answering of the research questions (see Chapter 4.1).

4.6 – Mathematical Assessment Test Hierarchy

4.6.1 – Description

The MATH taxonomy (Smith *et al.*, 1996) is a framework which may be used in order to categorise the nature of a mathematics question in terms of the skills and knowledge required to answer it (see Chapter 2.2). Smith and his colleagues do not profess that it measures how *difficult* a question is, rather what the question requires of the learner in terms of knowledge. The categories in the taxonomy (see Table 4.14) allow for two different questions of the same difficulty to be classified differently.

Table 4.14 – MATH taxonomy groups

Group A		Group B		Group C	
Factual Knowledge & Fact Systems	FKFS	Information Transfer	IT	Justifying & Interpreting	J&I
Comprehension	COMP	Application in New Situations	AINS	Implications, Conjectures & Comparisons	ICC
Routine Use of Procedures	RUOP			Evaluation	EVAL

Examination of past A-level Mathematics questions can be used to illustrate the types of question which may fall into different groups and subgroups, as well as how difficulty might not be implied by the groups.

Table 4.15 - Example A-level Mathematics questions

	Question	Source
1	Find all values of θ in the range $\theta° \leq \theta \leq 360°$ satisfying $3 - 7\cos\theta = 6\sin^2\theta$	WJEC C2, January 2010, q. 2 (a)
2	Use de Moivre's theorem to show that $\sin 5\theta = \sin\theta\,(16\cos^4\theta - 12\cos^2\theta + 1)$	Edexcel FP3, June 2006, q. 3
3	The functions f and g are defined with their respective domains by $f(x) = x^2$ for all real values of x $g(x) = \frac{1}{2x-3}$ for real values of x, $x \neq \frac{3}{2}$ State the range of f.	AQA C3, May 2008, q. 4 (a)
4	The inverse of g is g^{-1}. Find $g^{-1}(x)$.	AQA C3, May 2008, q. 4 (b) (i)
5	Simplify $(2x + 5)^2 - (x - 3)^2$, giving your answer in the form $ax^2 + bx + c$.	OCR C1, January 2007, q. 5

For example, one might consider Question 1 to be easier than Question 2; indeed, Question 1 is from Pure Core Mathematics 2 (C2) and Question 2 from Further Pure Mathematics 3 (FP3), and the mathematics is more advanced. However, the skills required are similar according to the definitions in the MATH taxonomy (RUOP).

Question 3 would be classed as Computation, and Question 4 would be RUOP because it requires the student to calculate the inverse of a function, something which they will have been able to have practised multiple times. The difference in Questions 3 and 4 in terms of the way in which they may be categorised according to MATH taxonomy comes despite the fact that they could be considered to be of similar difficulty in terms of the number of students who tend to find the correct answer.

Perceptions of difficulty are all relative to the stage of learning that a student is in. A question such as Question 5 might be considered at A-level to be easy, and would fall into the category of RUOP. However, at GCSE, it would not be so easy, but would still be considered RUOP.

4.6.2 – Justification

The MATH taxonomy may be used by educators in two ways:

1. Teachers may use the taxonomy in their classroom planning so that they can have their students use a variety of different skills.
2. Examiners can use the MATH taxonomy as a guideline when designing examinations. This means that the assessment will be of a variety of skills, requiring different levels of understanding and its demonstration.

For the purpose of this thesis, the MATH taxonomy will be used in order to examine the types of skills assessed in past A-level Mathematics examinations, OxMATs and undergraduate mathematics examinations to establish whether the types of questioning at each level are any different.

A-level mathematics papers were selected on the basis that they are, by far, the most-commonly studied pre-university mathematics qualification amongst new Oxford undergraduates. According to university admissions statistics (University of Oxford, 2012; see also Appendix 7.7), of those undergraduates beginning in September 2011, 74.4% had done A-levels, with the second most popular pre-university qualification being the International Baccalaureate (IB). Only 5.2% of new students had done the IB, so it was not felt that this was a sufficient proportion to warrant analysis as part of this thesis; however, this is certainly something which can be considered in further research in this area.

4.6.3 – Procedure & Sampling

A-level Mathematics papers from the modules 'Core 1' (the first compulsory pure mathematics module at A-level) and 'Further Pure 1' (the first pure mathematics module in A-level Further Mathematics) from one year across all examination boards (AQA, Edexcel, OCR, WJEC) were examined. No significant differences have been identified between examination boards in the

past (Newton *et al.*, 2007; Taverner, 1996; Tymms & Fitz-Gibbon, 1991); however, doing this means that a snapshot of one year with one examination specification could be made. Only five OxMATs were available online at the time of writing, so all of these were subjected to the MATH taxonomy. Undergraduate examinations in analysis and algebra were analysed because these are two courses which are studied as standard in undergraduate mathematics courses at all universities, with analysis often being a topic which proves difficult for new undergraduates (see Chapter 2.3.1.1).

4.6.4 – Strengths & Limitations

4.6.4.1 – Validity

In order to check the categorisation of the questions examined, my categorisation was verified by a number of experts. A current mathematics undergraduate, recent mathematics graduate, mathematics teacher and professor of mathematics education were all consulted to ascertain their dis/agreement with my categorisation. The consultants were all given the same sample of eight questions blind (see Appendix 4.10), and their categorisations compared to mine. Had there been situations where there was any disagreement, each party would have put forward their reasons for selecting their category in the hope that all parties could justify one category to use. Fortunately, this did not happen, although there were two disagreements which transpired to be purely due to mistakes or misunderstandings/misinterpretation of each of the categories. The most important consideration to make in classifying the questions is of the student's experience of the questions, which is why mark schemes were consulted when classifying the questions. Furthermore mark schemes indicate the degree of difficulty that the examination setter expects for the intended cohort of students.

The consultants looked at a total of 12 questions, which equated to 8% of all questions analysed.

4.6.4.2 – Reliability

A number of disadvantages of the MATH taxonomy are given in Chapter 2.2.3.5. More specifically to this study, the MATH taxonomy has only been used on A-level examinations which were set nearly 20 years ago (Crawford, 1983, 1986; Crawford *et al.*, 1993; Etchells & Monaghan, 1994), and of tertiary-level examinations over fifteen years ago (Ball *et al.*, 1998; Smith *et al.*, 1996), which will have been to some extent influenced by the A-level syllabus at the time. A-level syllabi and design have constantly changed since their introduction in 1953 (see Chapter 2.3.3). This study will not be seeking to examine a large sample of papers in order to perhaps quantify the proportion of each question type and to pass comment on this. It is not the study's specific intention to make generalisations about the nature of post-compulsory mathematics examinations in the UK, but to use a few examples to examine their nature and to gain a basic insight into this as a topic of research. Further research may choose to investigate this further. Whilst recent educational reform concerning students aged 16-19 means that extensive use of the MATH taxonomy on A-level examinations has restricted use, implications for policy will become no less relevant because the content, style and specific nature of the linear examinations has yet to be seen.

4.6.5 – Analysis

Use of the MATH taxonomy on a small, select sample is not intended to be quantified or examined to a great extent. The findings from the analysis will act as an illustrative tool in the description of the different types of questioning and thinking tested and required at the three different levels.

4.7– General Study Strengths & Limitations

4.7.1 – Oxford

This particular study focuses on the students of the University of Oxford and, in that sense, can be viewed as a case study. This particular case, it is hypothesised, has the potential to be different to other cases at other universities in both domestically and internationally. The University is regularly considered to be amongst the best in the world (e.g. Times Higher Education, 2012), and distinguishes itself from other institutions through its pedagogical focus on the use of the tutorial system. The tutorial system is highly regarded by educators and academics (Palfreyman, 2008), with it affording students the opportunity to go over topics and concepts with which they struggle (Batty, 1994). Indeed, Rose and Ziman (1964; cited by Ashwin, 2005) contend that the tutorial system possesses "some special and unique method for getting intellects to sparkle, for filling heads with knowledge, for making undergraduates big with wisdom" (p. 59). Whilst other universities use lectures as the predominant form of teaching, Oxford supports lectures with compulsory, weekly, small-group tutorials for the students which aid to develop the students' understanding of mathematical concepts. At most other universities, tutorials are non-compulsory, non-existent, or not the main focus of teaching and learning.

Oxford students are subject to amongst the highest entry requirements of all UK universities to study mathematics in terms of A-level (or their alternative) grades (see Appendix 2.2). Furthermore, students are presented with two additional hurdles which are not typical in other universities – the interview and the OxMAT. Students are interviewed at the first- and second-choice colleges to which they apply, with the possibility of further interviews at other colleges should they be 'pooled'[28]. The OxMAT is an examination set by the University for

[28] 'Pooling' occurs when a candidate interviewed at one college is not offered a place because it is oversubscribed with exceptional applications. Therefore, that college places the candidate in the 'pool' so that other colleges which are not already full can consider their application.

students applying to study the subject, either as single or joint honours, and is sat by students in the autumn term of the year of their application to the University. There is no standardised pass rate, but the scores from these tests, along with A-level grades and interview feedback are used by admissions tutors to decide the students' fate.

The University of Cambridge is perhaps the closest university to Oxford in terms of its academic and pastoral structure, and requires similar entrance criteria to the University to Oxford. However, their 'additional' examination requirement comes in the form of the STEP examinations (see Chapter 1), which are not explicitly written to act as an entrance examination to the University, unlike the OxMAT.

Whilst the enculturation of students into tertiary study in terms of questioning, learning communities and ATLs could be investigated at any university, the Oxford case is of particular interest since it could be claimed that it is very different. This means that there is the possibility that results from this study could conflict with results from studies conducted elsewhere on account of such differences. However, the results could fall in line with other evidence in spite of these differences and this study will certainly be of interest within Oxford.

4.7.1.1 – Case Study

Much that is written in the research methods literature appears to refer to case studies as more being concerned with ethnographic research than a general focus on one particular sample, as in this case. However, the cautions such literature makes are of vital importance in this particular study.

Much is written on the limitations of case studies in terms of their apparent non-generalisability across wider populations and contexts. However, Stake (1995) asserts that there is nonetheless much to be learnt from individual case studies. Readers may learn through their existing familiarity with existing cases, enabling them to "add this one in, thus making a slightly new group from which to generalise", creating "a new opportunity to modify

old generalisations" (p. 85). Furthermore, Flyvbjerg (2006) asserts that if "knowledge cannot be formally generalised, [it] does not mean that it cannot enter into the collective produces of knowledge accumulation in a given field or society" (p. 227). Readers who are familiar with research conducted at anonymous institutions across the UK and abroad, may be able to see for themselves any similarities or differences with this study (see Chapter 9). Should there be a similarity of difference, this coming because of or in spite of the 'Oxfordness' of this particular case would be of particular interest. Whether Oxford is a 'black swan' or a 'white swan' (Popper, 1959) in this respect will be of interest in itself.

Whilst it is true that, in the context of this thesis, one could claim that "there are too many elements that are *specific*" (Gilham, 2000, p. 6), these in themselves are the focus of interest. Furthermore, Flyvbjerg (2006) writes about five misunderstandings of case study research, one of which being that its non-generalisability renders it sub-par. He states that "social science has not succeeded in producing general, context-independent theory" at all (p. 223), in that studies of this type will always be conducted in institutions which have their own independent, idiosyncratic quirks which make them different from others. Furthermore, within that the participation from individuals will always vary and a truly representative sample can never be honestly drawn.

4.7.1.2 – 'Insider' Research

Whilst this study is not 'insider' research in the traditional sense – an academic studying their institution, perhaps with participation from their peers – the close relationship that I have with the University has the potential to give data collection and analysis both strengths and limitations. Whilst I did not study for my undergraduate degree in mathematics at Oxford, I am still a student here and therefore am familiar with the academic situation – I am culturally literate, having been myself enculturated into the general University environment. As a note-taker for disabled students at the University, I have attended numerous mathematics lectures

aimed at each of the four year-groups and so am familiar with the institution-specific terminology. As a lecturer and tutor for an undergraduate mathematics course in mathematics education, I am familiar with institutional practices.

There are, however, pitfalls which I aimed to avoid throughout the study. Common experiences shared between myself and participants are easy to overlook (Powney & Watts, 1987), through seeming obvious and assumed to be typical. Perhaps the biggest limitation in this particular context would be closeness between me as a researcher and the position of power that it puts me in in this instance. The fear of judgement when discussing topics such as those covered in this study have the potential to impact what information is shared (Shah, 2004). I was careful to explain to students that I am not personally connected to the Mathematical Institute and would not be communicating their individual comments and results to their tutors, lecturers, advisors or any other academic staff. Conversely, the students' awareness that I am familiar with their situation through being a student myself may have been an advantage in the sense that it could increase their confidence and trust in me (Hockey, 1993).

4.7.2 - Self Report

Some factors about self-report were considered when designing and conducting this research. Self-report interviews are the only way to investigate and understand others' life worlds (Kvale, 2008) and truth-spaces (Onwuegbuzie, 2003). My aim to understand and learn of students' experiences can only be accessed via this method. Indeed, Kvale (2008) comments that

> *The force of the interview is its privileged access to the subjects' everyday world. The deliberate use of the subjective perspective need not be a negative bias; rather, the personal perspectives of the interviewees and interviewer can provide a distinctive and sensitive understanding of the everyday life world.*
>
> (p. 87)

Student self-report has strengths and limitations when considering its use in students self-reporting in both questionnaires and interviews.

Some criticism of the use of self-report has come from problems associated with accurate recall of information and events by participants (Wetland & Smith, 1993). This would perhaps be more of a problem in students in later years of their degree when being interviewed and asked questions regarding their school experiences of mathematics and their interviews for entry to the University. Indeed, a couple of students did seem to struggle to remember precise details regarding their interviews, for example, but this data was nonetheless useful and made a valuable contribution toward the study. It was not the particular questions that they were asked which were of paramount importance – it was their perception of how the interview went and what processes were involved in their attempts to answer the questions that they were asked.

Thorndike (1920) commented on a possible 'halo effect' which might be created by respondents when they feel that a particular type of response would be viewed more favourably by the researcher or would present them in a more positive light. In the context of this study, this could involve students describing experiences with mathematics as being easier than in reality – they may have found adapting to a new academic environment very difficult but may wish to portray themselves as having found the transition easy so that they do not appear weak. This is possible in interview situations, where experiences may be mis-described, or in Likert scale data where relative agreement or disagreement with a statement could be affected. The ASSIST questionnaires were anonymous which means that this would result in the potential for the student to feel less embarrassment because they are not facing the researcher directly; however, in the interview situation, the personal contact has the potential to affect responses on a greater level. I went to great pains to explain to the students my academic background as well as the nature of the study in order they did not feel threatened

by me as a mathematician or as a researcher. Indeed, the data collected from the interviews certainly seemed frank, as students willingly offered up descriptions of their negative experiences and insecurities in most cases.

4.8 – Ethics

Looking into students' experience of their course, which may be negative, is quite a sensitive issue. It is important that care be taken in the way in which questions are posed without making them feel uncomfortable and without overly encouraging negative responses.

Voluntary, informed consent was required from all participants before commencing (see Appendices 4.4 and 4.6).

Data was locked up – physically in terms of physical copies in a cabinet and electronically by a password once data have been entered on a computer for analysis in SPSS – and anonymised through the use of pseudonyms and assigning questionnaires codes. Data may only be matched back to the specific participant by the researcher. When publishing any findings of the research to the Mathematical Institute, it is imperative that care is taken to prevent individual students from being identified through small details such as their college. Students' backgrounds will therefore be disguised as much as possible without compromising the integrity of data.

Participants were afforded the opportunity to see what has been written about them prior to publication. This acts as a method of triangulation since it enables them to comment further on what has been reported and help to remove any ambiguity in what was said/written. Participants could drop out at any time. Results have been published anonymously and were presented at conferences and Mathematics Education Research Group seminars in the first instance. Data will be destroyed four years after completion of the DPhil.

Chapter 5: Data

Student Approaches to Learning throughout Undergraduate Study at the University of Oxford

Data from the ASSIST (see Appendix 4.1) was collected on two occasions, which will be referred to as 'sweeps'. To distinguish between them:

- **Sweep 1** – Data collected from first-year students only at the beginning of their first term at Oxford.
- **Sweep 2** – Data collected from students across all year groups at the end of the year. First-year students who participated in Sweep 1 were encouraged to do so again in Sweep 2 in order that comparisons between their earlier and later responses could be made.

Reminders of the particular groups of students being discussed will be given throughout this chapter using a simple flow-diagram. Sweep 1 will always refer to only first-year students. The reader should assume that Sweep 2 consists of students across all of the four year groups unless otherwise stated or indicated.

5.1 – Factor Analysis

Tait and McCune (2001) recommend that confirmatory factor analysis on data collected from the ASSIST is conducted on account of the fact that different groups of respondents may yield different overall results. This meant that it was important to check whether the ASSIST was as applicable to mathematics undergraduates as it was to the psychology undergraduates used to test and develop the questionnaire. Factor analysis conducted in Darlington (2011) confirms

that the ASSIST is appropriate for use amongst undergraduate mathematicians at the University at Oxford (see also Appendix 5.1 for factor analysis on Sweep 2).

5.2 – Descriptive Statistics

Based on ASSIST data:

Table 5.1 - Descriptive statistics, Sweeps 1 & 2

	Sweep 1		Sweep 2		Whole Population
	Count	Percentage	Count	Percentage	Percentage
Gender					
Male	118	*67.0*	150	*65.8*	*71.1*
Female	58	*33.0*	78	*34.2*	*28.9*
Course					
Mathematics	149	*84.7*	201	*88.2*	*72.3*
Mathematics & Statistics	5	*2.8*	9	*3.9*	*11.2*
Mathematics & Computer Science	16	*9.1*	6	*2.6*	*8.1*
Mathematics & Philosophy	6	*3.4*	12	*5.3*	*8.5*
Year					
1st	176	*100.0*	69	*30.3*	*26.4*
2nd	0	*0.0*	66	*28.9*	*27.0*
3rd	0	*0.0*	50	*21.9*	*26.1*
4th	0	*0.0*	43	*19.0*	*20.5*
Qualifications					
A-Levels	136	*77.3*	198	*86.8*	*74.4*
International Baccalaureate	10	*5.7*	9	*3.9*	*5.2*
Other	30	*17.0*	21	*9.2*	*20.4*
ATL					
Deep	18	*10.2*	39	*17.1*	
Surface	2	*1.1*	21	*9.2*	
Strategic	156	*88.6*	168	*73.7*	
Total	**176**	***100.0***	**238**	**100.0**	
Average Age	***18.6***		**20.82**		

Here, the sample is reasonably representative of the entire undergraduate cohort at the University in terms of gender, the proportions of students on each course, prior qualifications (University of Oxford, 2012) and the proportions of students in each year[29].

[29] Course and year group data from personal communication with Nia Roderick of the Mathematical Institute (figures as of 8 May 2013).

5.3 – Approaches to Learning

The ASSIST scoring key (Tait & McCune, 2001) may be used to calculate a score for each student on each of the deep, surface and strategic scales. The largest score indicates a student has a greater propensity towards the associated ATL. Unless stated otherwise, references henceforth made regarding a student's ATL as ascertained by the ASSIST imply that this was the scale on which that student scored highest. For example, a student who scored 54 on surface, 40 on strategic and 34 on deep would be referred to as someone with a predominantly surface ATL, or similar.

5.3.1 – Sweep 1



The first phase of data collection revealed the most common approach to be a strategic ATL, with 88.6% of respondents' scores being indicative of this. A very small minority (1.1%) had a surface ATL, and 10.2% a deep ATL (see Figure 5.2).

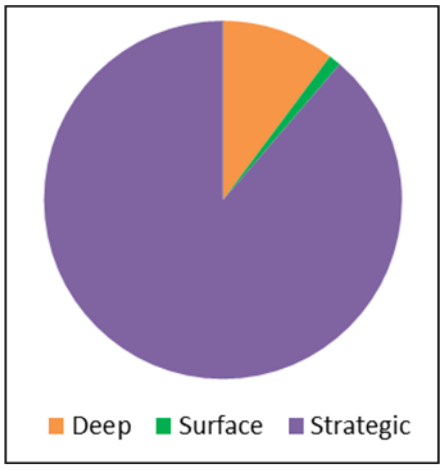


Figure 5.2 - ATLs, Sweep 1

This data corroborates with that in the literature which suggests that students of secondary mathematics, owing to their exposure to certain types of mathematics and assessment questions, would likely demonstrate actions and beliefs consistent with a strategic ATL.

As the vast majority of participants had studied A-level Mathematics and Further Mathematics, it could be argued that there would not be a great deal of variation in their ATLs. This is reflected in the data since the vast majority of participants in Sweep 1 (and 2) predominantly use a strategic ATL. Prosser and Trigwell (1999) suggest that differences in ATLs may be a consequence of students' perceptions of learning and the subject, their prior experiences of studying, and their prior ATLs. Hence, since undergraduates at the University of Oxford come from a variety of

different academic backgrounds, this may account for some of the differences evident in the data[30]. Data concerning the students' schooling was not collected for the purposes of this study, but could be considered in the future.

The data here, with the vast majority of students displaying dominance of strategic ATLs, contradicts existing research which suggests that deep ATLs are more likely to lead to academic success and better learning outcomes (Cano, 2005; Entwistle *et al.*, 2000; Heikkila & Lonka, 2006; Lindblom-Ylänne & Lonka, 1999; Marton & Säljö, 1976, 1984; Meyer *et al.*, 1990; Newstead, 1992; Ramsden, 1983; Reid *et al.*, 2007; Sadler-Smith, 1997; Struyven *et al.*, 2003; Prosser & Trigwell, 1999; Trigwell & Prosser, 1991b; Watkins, 2001). These students all achieved top grades at A-level and gained a place at Oxford, yet do not exhibit signs of predominantly using deep ATLs according to the data collected from the ASSIST. Instead, these data support studies which found positive correlations between strategic ATLs and attainment (Diseth & Martinsen, 2003; Newstead, 1992; Ramsden, 1983; Reid *et al.*, 2007; Sadler-Smith, 1997; Schouwenburg & Kossowska, 1999). These findings also provide additional evidence which suggests that students can use memorisation as a vehicle for conceptual understanding, and that this method can be used by the highest achievers (Dhalin & Regmi, 1997; Kember, 2000; Kember & Gow, 1990; Marton & Trigwell, 2000) – the 'Chinese paradox'. However, Marton *et al.* (1993) suggest that this is successful because Chinese students distinguish between two types of memorisation – memorisation as rote learning, and memorisation as a means of understanding. Moreover,

> *students may appear to use attributes of surface learning approaches to achieve a short-term objective. However, knowing when to be 'strategic' is often a necessary skill needed by students using deep approaches to learning.*
>
> (Rollnick *et al.*, 2008, p. 30)

[30] For 2012 entry, 57.5% of places went to state sector applicants and 42.5% to independent sector applicants (University of Oxford, 2012).

5.3.2 – Sweep 2

5.3.2.1 – Likert Scales & Comparing Groups of Data



Likert data, as collected in the ASSIST, is ordinal, so *t*-tests are not normally used to compare two groups because one cannot assume that respondents perceive the intervals in the scale as being equidistant. Whilst Mendenhall *et al*. (1993) claim that *t*-tests can be used in instances when the data distribution be at least 'mound-shaped', the Kolmogorov-Smirnov test conducted on the data suggest that the ASSIST data do not meet these requirements (see Appendix 5.2). Therefore, a combination of Fisher's Exact Test and independent-samples Mann Whitney U-tests were used to establish whether any differences existed in the students' ATLs between groups as determined by the ASSIST.

5.3.2.2 – Years 1-4

The second phase of the ASSIST involved students from all four year groups. Overall for this group, a 9.2% minority of respondents had a surface ATL, with a slightly larger proportion with a deep ATL (17.1%) and 73.7% of participants with a strategic ATL.

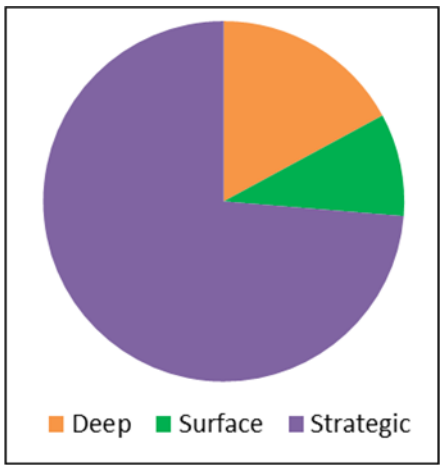


Figure 5.3 - ATLs, Sweep 2

The number of students with a surface ATL may be attributed towards assessment and institutional demands on students in later years of their degree, which has been found to influence students' ATLs (Biggs, 1993; Lindblom-Ylänne & Lonka, 1999, 2000, 2001; Ramsden 1988; Thomas, 1986; Thomas & Bain, 1984). Indeed, Gijbels and Dochy (2006) contest that "students relate studying to the assessment requires in a manipulative, even cynical, manner" (p. 400). In mathematics, students who have fragmented conceptions of the subject are more likely to adopt surface ATLs than their peers

with more cohesive conceptions of mathematics, who are more likely to adopt deep ATLs (Crawford *et al.*, 1994), which suggests that there are students who are struggling to develop their understanding of mathematical concepts in an interconnected fashion.

5.3.2.3 – Contrasting BA & MMath Years



It is important to note that students in their fourth year of their degree are in the masters year – that is, they have been awarded a BA degree in Mathematics, but are continuing onto an additional year in order to be awarded an 'MMath'. Students are only permitted to continue on to the fourth year if they achieve a minimum of a second class honours in their final examinations in their third year, which means that the students in this year group are subjected to selection. This may act as a source of bias or influence over their responses to the ASSIST and the analysis of their responses.

For students in the first three years of the degree, their dominant ATLs were as follows:

Table 5.4 - Proportions of students by ATL in Years 1, 2 & 3, Sweep 2

	Year 1		Year 2		Year 3	
	Count	*%*	Count	*%*	Count	*%*
Deep	12	*17.4*	13	*19.7*	4	*8.0*
Strategic	50	72.5	45	*68.2*	42	*84.0*
Surface	7	10.1	8	*12.1*	4	*8.0*
Total	69	100.00	66	*100.00*	50	*100.00*

Therefore, of these students in Years 1-3, a smaller proportion displayed dominance in a deep ATL, whereas there were larger proportions of strategic and surface ATLs compared to examining all four year groups together. These differences are all very small and therefore the data from students in Years 1-3 closely resembles that to students in Years 1-4.

However, when examining Year 4 in isolation compared to Years 1-3, slightly more substantial differences can be seen, in that there is a higher proportion of students with predominantly deep ATLs in Year 4 than in Years 1-3, and lower proportions adopting surface ATLs in Year 4.

Table 5.5 - Proportions of students by ATL in Years 1-3 & Year 4, Sweep 2

	Years 1-3		Year 4	
	Count	*%*	Count	*%*
Deep	29	*15.7*	10	*23.3*
Strategic	137	74.1	31	*72.1*
Surface	19	10.3	2	*4.7*
Total	185	100.00	43	*100.00*

It is possible to investigate differences in the data in two ways:

1. looking at degree stage vs. ATL (i.e. deep, surface, strategic); or
2. looking at degree stage vs. median scores on each of the ATL scales.

However, these differences were not statistically significant ($p = .348$; see Appendix 5.2). This suggests that the absences of large differences in Chapter 5.3.2.2 might be due to the fact that the number of fourth-year participants in the study was small (*N*=43) compared to the numbers in other year groups (185 in Years 1-3).

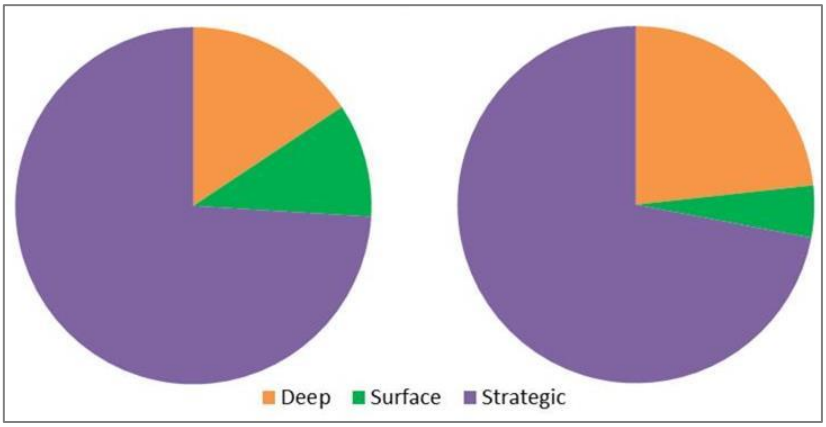


Figure 5.6 - Proportions of dominant ATLs for Years 1-3 (left) & Year 4 (right), Sweep 2

Contrasting the median scores on each of the deep, strategic and surface scales using a Mann-Whitney U Test revealed no significant differences (see Appendix 5.3) between the responses of students in Years 1-3 compared to those in Year 4:

Table 5.7 - Average scale scores for Years 1-3 & Year 4, Sweep 2

	Year	N	Median	Variance	Minimum	Maximum
Deep	1-3	185	61	92.360	16	77
	4	43	60	69.388	42	76
Strategic	1-3	185	71	125.225	29	95
	4	43	71	127.869	49	94
Surface	1-3	185	46	129.607	20	75
	4	43	44	122.994	22	76

Furthermore, there were no significant differences in the responses by students in Years 1-3 to those in Year 4 on the individual items on the ASSIST (see Appendix 5.4), with the exception of '*22 – I often worry about whether I'll ever be able to cope with the work properly*' where more fourth-year students tended to somewhat disagree or disagree with the statement than students in years 1-3. This shows an increased confidence in students in the MMath year in terms of their ability to deal with the workload, which perhaps is a consequence of experience.

Consequently, despite the implications of selection and a desire to go on to further study on students' ATLs, there is no statistically significant difference between the outcomes of the fourth-year participants' responses and those of students in Years 1-3. Whilst this is so, tables and graphs displaying data from Years 1-4 will have Year 4 highlighted as a reminder that these students' experiences and backgrounds may be different to those of their less-experienced peers thanks to the selection and election to study the masters year.

5.3.2.4 – Summary

Initial analyses in 5.3.2.3 into the ATLs of participants in Sweeps 1 and 2 indicate no statistical differences, though there are some differences in the raw data. It was thought that there may be statistically significant difference in terms of the proportions of ATLs or in the scores on each of the deep, surface and strategic scales between students in the fourth year of their

degree compared to those in earlier years due its elective and selective nature. However, this did not transpire to be the case. This may be attributed to the fact that a very large proportion of undergraduate mathematicians at Oxford go on to the MMath (87%) compared to those in other institutions (see Appendix 4.9), and so the difference in the characteristics of the students between each level of study is not all that different. One might expect the proportion of students adopting surface ATLs to be smaller in the fourth year because surface ATLs have been associated with poor attainment. Whilst this was true for this sample – median scores on the surface scale decreased from 46 in Years 1-3 to 44 in Year 2, and proportions of students predominantly adopting surface ATLs decreased from 10.3% in Years 1-3 to 4.7% in Year 4 – the differences were not statistically significant. Reasons for the insignificant changes in ATLs and scores across these groups may also be attributed to the applicability of the ASSIST, and relevance of the deep/surface/strategic 'trichotomy' to this population.

As anticipated based on previous research conducted in this area on similar samples (Darlington, 2010, 2011), the vast majority of participants in both Sweep 1 (first-years upon arrival at Oxford) and Sweep 2 (Years 1-4) were found in 5.3.2.2 to have predominantly adopted strategic ATLs. The nature of school mathematics and the tendency of pupils to practise rehearsed procedures and repeat such actions in examinations reflects the nature of a strategic ATL, as well as the tendency of undergraduate students to selectively memorise course material for reproduction in examinations (see Chapter 2.3.1). The change between the secondary and tertiary level in terms of what constitutes a strategic ATL is significant and should be noted (see Chapter 7 for descriptions) as this brings into question the utility of describing a student's ATL as strategic when it could reflect two different practices.

5.4 – First-Years' ATL



This Chapter looks at any changes in first-year students' ATLs between Sweep 1 (very beginning of their first year) and Sweep 2 (end of their first year).

5.4.1 – Matching Students

In distributing the ASSIST twice during the academic year, it was hoped that some of the first-year students' ATLs could be 'tracked' from the beginning of their undergraduate life to the end of their first year and their first set of university examinations. In order to match the questionnaires between the two sweeps, respondents were asked to give their individual student card[31] numbers as these numbers are unique between students and are something that the majority will know off-hand. It was not possible for me to trace the numbers back to find the identity of the individual students.

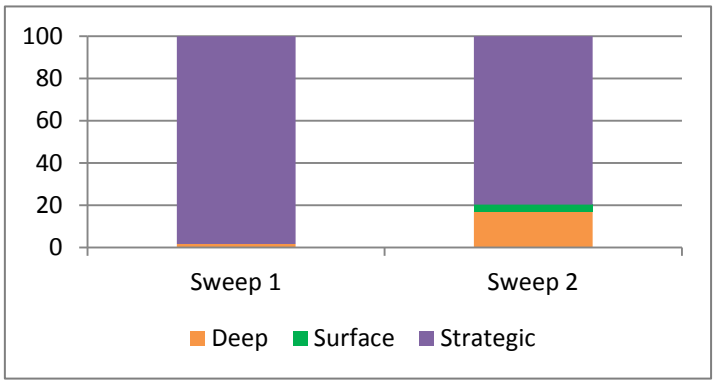


Figure 5.8- First-years' ATLs in each sweep

Of the 176 respondents in Sweep 1, 59 could be tracked into Sweep 2. Whilst this was less than I had hoped, it did provide some data which proved interesting. The proportion of students with predominantly strategic ATLs decreased from 98.3% in Sweep 1 to 79.7% in Sweep 2. This was a consequence of the first sweep yielding no data indicating a predominantly surface ATL in any of the participants, whereas this rose to 3.4% in Sweep 2. This was compounded by an increase in the proportion of students with deep ATLs by 15.2% from 1.7% to 16.9%.

[31] These are also referred to as 'bod cards'.

Of the 59 'returning' students to the ASSIST, 46 (78.0%) maintained a 'consistent' ATL. Of the changes, ten students (77.0%) went from deep to strategic, one (7.7%) changed from strategic to deep and two (15.4%) changed from strategic to surface. This data suggests a strong level of stability in the students' ATLs, although the size of the dataset in this instance is very small.

If this small amount of data is to be analysed, one might suggest similar reasons to those given in Chapter 5.3.2 for the increase in the proportion of strategic ATLs across the years. It is possible that new students may respond and adapt to new mathematics positively by studying the subject in such a way which supports its learning and understanding, as well as achieving well in assessment; however, this data suggests that old habits die hard, something which Doyle (2008) suggests is because students avoid changing their ATLs as they are afraid of taking learning risks. Indeed, Gijbels *et al.* (2008) suggest that the stronger the initial ATL, the harder it is to change it. As students have likely worked in a particular way throughout their whole school career, their ATLs may be very embedded.

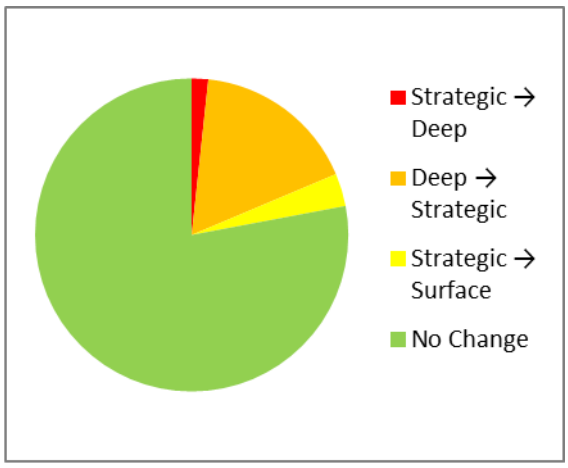


Figure 5.9 - Changes in ATLs of first-year students between sweeps

5.4.2 – Comparing Sweeps

Whilst comparing bod card numbers suggested that there were a small number of 'returning' students to the study, a large number of students in both sweeps did not give their bod card number – possibly because they did not know it off by heart and did not have the card to hand, or because they did not trust that they would not be identifiable through using this means of matching data. As data collected in Sweep 1 was from first-year students upon arrival at Oxford and Sweep 2 data from students of all years collected at the end of the academic year,

it meant that comparisons could be made between first-year students in Sweep 1 and Sweep 2.

It is possible to investigate differences in the data in two ways:

3. looking at sweep vs. ATL (i.e. deep, surface, strategic); or
4. looking at sweep vs. median scores on each of the ATL scales.

5.4.2.1 – Differences by ATL

Between Sweeps 1 and 2, the proportion of students with deep ATLs increased by 7.5%, strategic ATLs decreased by 17.1% and surface ATLs increased by 9.5%.

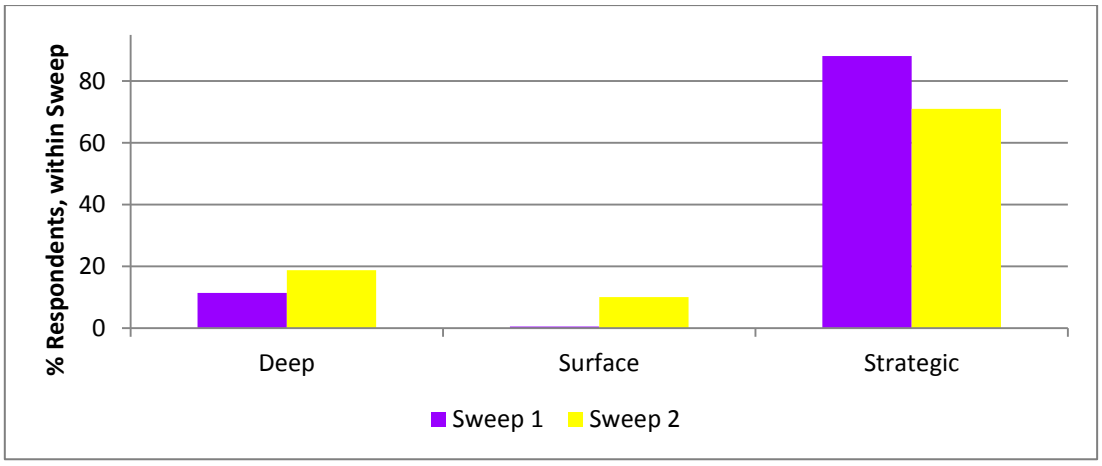


Figure 5.10 - Proportion of first-years in each sweep with predominantly deep, strategic & surface ATLs

This was found to be significantly different between the two sweeps, using Fisher's Exact Test ($p = .000$), suggesting that students' ATLs shift between entering university (Sweep 1) and completing their first year (Sweep 2; see Appendix 5.5). These results support those of Garrison and Cleveland-Innes (2005), for example, who found the ATLs of students of an online course to change between the beginning and end, and of Zeegers (2001), who used Biggs' Study Process Questionnaire (Biggs, 1987a) and found the ATLs of new undergraduate chemists to change between 4, 8, 16 and 30 month intervals. Furthermore, it supports studies which suggest that it is possible for ATLs to change over time in response to a number of factors such as curriculum design (English *et al.*, 2004).

This suggests a change in the students' behaviour between the beginning and end of the year, after having been exposed to undergraduate mathematical concepts and study for the first time, which may account for these changes. Perhaps those students who found it difficult went on to develop surface ATLs, whereas some students endeavoured to work hard and make an effort to understand mathematical concepts and so developed deep ATLs.

5.4.2.2 – Difference by Subscale Score

Owing to the change in the distribution of dominant ATLs of first-year students between the two sweeps, tests were conducted in order to establish whether there were statistically significant differences in the first-year students' scores on each of the deep, strategic and surface scales between Sweep 1 and Sweep 2.

Table 5.11 - Average scale scores of first-year students in Sweep 1 & Sweep 2

	Sweep	N	Median	Variance	Minimum	Maximum
Deep	1	176	48	35.7	31	62
	2	69	60	97.6	31	77
Strategic	1	176	71	77.0	34	89
	2	69	71	130.7	29	93
Surface	1	176	49	100.7	20	67
	2	69	45	133.2	24	75

Whilst median scores were shown to increase for the deep scale, remain consistent for the strategic scale and decrease for the surface scale, an independent-samples Mann-Whitney U test revealed there to be no statistical significance in these differences between the sweeps (see Appendix 5.6). Coupled with the significant difference in students' dominant ATLs between the two sweeps, this suggests that the increase in the proportion of students who predominantly use surface ATLs may come as a consequence of small increases in their scores on this scale, relative to the strategic scale such that these two scores are similar and similarly so for the increase in proportion of students who rely on deep ATLs. This brings into focus the utility of scales such as this where small differences to participants' responses can make large differences in the outcome of group data.

5.4.3 – Summary

Data in Chapter 5.4.2 comparing first-year students' responses to the ASSIST in Sweeps 1 and 2 indicates that students' predominant ATLs change between the beginning and end of their first year. Proportions of students predominantly adopting deep ATLs increased between 7.5% between the two sweeps, and an increase in 9.5% for surface ATLs. This indicates that students respond to the change in the nature of mathematics between secondary and tertiary levels (see Chapter 2.3.2) by adjusting their ATLs. Though the differences were not significant, the increase in the median score on the deep scale between Sweep 1 and Sweep 2, and the decrease in surface median by 4 are indicative of the reasons behind this change in ATLs. This suggests that the majority of students whose ATLs evolve do so towards a deep ATL, which has been associated with better learning outcomes (see Chapter 2.1.2.1). This may be a consequence of the nature of the mathematics being studied, or the assessment (see Chapter 2.1.2.4) at the end of their first year of study at Oxford. These findings support those of Prosser and Trigwell (1999), who suggested that ATLs are related to students' prior experiences and perceptions – the students' ATLs at the beginning of the year will have been influenced by those that they adopted at the secondary level, but were subject to change based on revised conceptions of mathematics once they began to study it at tertiary level.

5.5 ATL & Year Group



As each year of study presents different challenges to students as they acclimatise to undergraduate study in their first year, become more familiar with it in their second year, face final examinations in their third year, and potentially go on to a fourth year, statistical analysis was conducted in order to establish whether any significant differences existed between the year groups in terms of students' ATLs.

Participants across all four year groups were involved in Sweep 2. In Figure 5.12, it can be seen that the percentage of students tending towards each particular ATL does not seem to remain consistent across the years.

5.5.1 – Differences by ATL

The proportions of each ATL fluctuate with each year group. Strategic ATLs form the vast majority of those of the students in each year group; however, these decrease from 72.5% in Year 1 to 68.2% in Year 2, before increasing to 84.0% in Year 3 (see Table 5.13). The change in proportion in the second year may be attributed to students attempting to 'turn over a new leaf' given their previous year's experiences, aiming to adopt deep ATLs in order to better understand the mathematics that they are learning. Indeed, Chin and Brown (2000) suggest that changes to students' ATLs are a consequence of self-reflection, as much as changes in course context. They contend that metalearning – self-control of learning

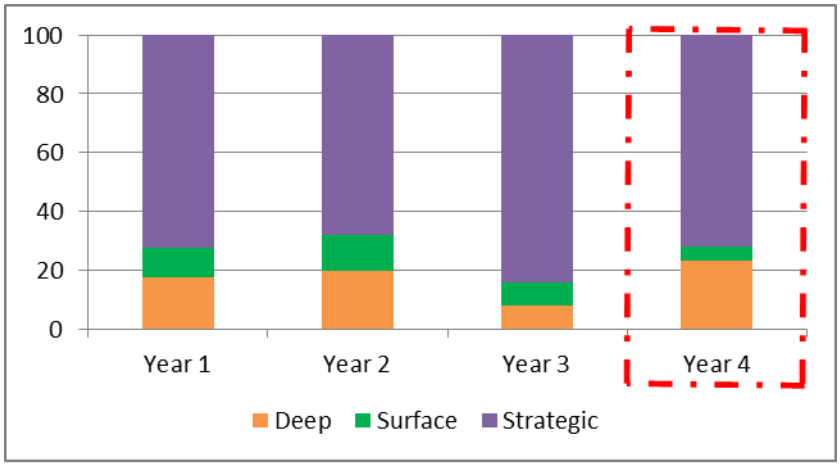


Figure 5.12 - ATL by year group, Sweep 2

approaches – can play a substantial role in shifts in students' ATLs (see also Cano, 2005, and English & Ihnatko, 1998).

Table 5.13 - ATLs by year group, Sweep 2

	Year 1		Year 2		Year 3		Year 4	
	Count	*%*	Count	*%*	Count	*%*	Count	*%*
Deep	12	*17.4*	13	*19.7*	4	*8.0*	10	*23.3*
Strategic	50	72.5	45	*68.2*	42	*84.0*	31	*72.1*
Surface	7	10.1	8	*12.1*	4	*8.0*	2	*4.7*
Total	69	100.00	66	*100.00*	50	*100.00*	43	*100.00*

The increase in Year 3 to 84.0% of students displaying signs of a strategic ATL is likely due to the pressures of their final examinations which form the whole of their degree class for the BA. This supports the work of Case and Gunstone (2003) and Entwistle (1997), who argue that work and time pressures can influence students' abilities to adopt deep ATLs. However, Lucas and Mladenovic (2004) suggest, like Chin and Brown (2000), that this requires students to have "an *awareness* of their motives, the task demands and their cognitive abilities; as well as [the] ability to *control* the strategies deemed appropriate for the task and with respect to their personal motives" (p. 404). In the fourth year, the smaller proportion of students with strategic ATLs may be due to the fact that this is the MMath year; it may attract more enthusiastic students, as well as requiring students to have reached a minimum standard in their BA examinations to continue. Should deep ATLs result in better assessment outcomes, as suggested by Reid *et al.* (2007; see also Chapter 2.1.2.1), this could explain this shift.

Furthermore, the increase in the proportion of students with surface ATLs from Year 1 (10.1%) to Year 2 (12.1%) may be a response generated by students towards assessment demands becoming overwhelming (Ramsden, 1979, 1983; Newble & Jaeger, 1983; Thomas & Bain, 1984; Wilson & Fowler, 2005). This decreasing again in Year 3 may be a consequence of a recognition that surface approaches may not serve them well in final examinations.

5.5.2 – Differences by Scale Score

However, while the raw data suggests some changes, Fisher's Exact Test (see Appendix 5.7) suggests that there is no statistically significant difference in Sweep 2 between the year groups in terms of their ATL ($p = .362$). An independent-samples Kruskal-Wallis test also suggests no statistical difference in terms of ATL scores:

Table 5.14 – Independent-samples Kruskal-Wallis test for differences in ATL scores between year groups, Sweep 2

Null Hypothesis	Sig.	Decision
The distribution of deep subscale means is the same across categories of year.	.530	Retain the null hypothesis
The distribution of strategic subscale means is the same across categories of year.	.767	Retain the null hypothesis
The distribution of surface subscale means is the same across categories of year.	.115	Retain the null hypothesis

These conclusions also hold when data is split into just men (see Appendix 5.8) and just women (see Appendix 5.9).

Since significant differences were not identified between the year groups when considering all four at once, tests were conducted in order to establish whether there were any differences in ATL between each pair of groups. That is, data were compared between:

- Year 1 and Year 2;
- Year 1 and Year 3;
- Year 1 and Year 4;
- Year 2 and Year 3;
- Year 2 and Year 4; and
- Year 3 and Year 4

No significant differences were found when comparing dominant ATLs (see Table 5.15 and Appendix 5.10).

Table 5.15 - Fisher's exact test data comparing year groups for ATL differences, Sweep 2

	Year 1	Year 2	Year 3	Year 4
Year 1				
Year 2	.303			
Year 3	2.466	4.072		
Year 4	1.376	1.701	4.291	

Scores on each of the deep, strategic and surface scales were compared between each year group pairing using an independent-samples Mann-Whitney U test (see Table 5.16 and Appendix 5.11).

Table 5.16 - Independent-samples Mann-Whitney U test data comparing scale scores by year groups, Sweep 2

Null Hypothesis	Independent-samples Mann-Whitney U test					
	Years 1 & 2	Years 1 & 3	Years 1 & 4	Years 2 & 3	Years 2 & 4	Years 3 & 4
The distribution of deep is the same across the categories of year.	.289	.161	.363	.708	.946	.769
The distribution of strategic is the same across the categories of year.	.399	.976	.574	.371	.865	.560
The distribution of surface is the same across the categories of year.	.269	.357	.117	.093	.479	.032

The null hypothesis was rejected in all cases except for the surface scale comparison between Year 3 and Year 4, where there was a significant difference in the subscale scores between students in Year 3 and Year 4 (see Table 5.17).

Table 5.17 - Average surface scale scores of third & fourth-year students, Sweep 2

	Year	N	Median	Variance	Minimum	Maximum
Surface	3	50	50.5	137.847	20	69
	4	43	44.0	122.994	22	66

That is, students in Year 3 scored significantly higher on this scale than those in Year 4, showing a greater tendency towards surface ATLs in the third than the fourth year. This is unsurprising given the selective and elective nature of this year.

Much existing research concerns whether it is possible for students' ATLs to change; for example, Trigwell and Prosser (1991a) have suggested that ATLs are the result of education and therefore are not fixed. However, this has concerned whether it is possible to 'induce'

different ATLs through the students' educational experiences. The educational experiences of the first-year students changed between Sweep 1 and Sweep 2. However, students experience slight differences later on because, in their third year, they no longer attend tutorials and instead go to classes for their courses. This change in experience is something which was commented on in the students' interviews (see Chapter 7). Therefore, the results here support claims that it is not possible for students' ATLs to change freely (Haggis, 2003), although it is clearly the case that some students' responses have changed. The individual nature of such change is explored further in the interviews.

5.5.3 – Summary

Despite fluctuating proportions of students predominantly adopting each ATL across the four year groups (see Figure 5.12), which see 11.6% fewer students adopting deep ATLs in Year 3 compared to Year 2, and 15.3% more students adopting deep ATLs in Year 4 compared to Year 3, there were no statistically significant differences in the proportions of ATLs and median scale scores across each year group. However, when year groups were compared in pairs, there was a significant difference in the surface ATL scales of third- and fourth-year students. Specifically, the median surface scale score in Year 4 was 6.5 less than in Year 3. This may be attributed to the selective and elective fourth year – that is, surface ATLs have been correlated with poor attainment (see Chapter 2.1.2.1) and lack of enthusiasm and interest (see Chapter 2.1.2.2), which are not traits which one would presume are found amongst students in a masters year of a mathematics degree.

The absence of statistically significant change between all other year group pairings suggests that students' ATLs do not change between each year, in spite of some differences in pedagogy, the nature of mathematics and assessment demands that they might experience. Students have weekly tutorials in the first two years of their degree, therefore receiving support from their tutor and having an opportunity for feedback and to ask questions,

whereas they instead have less personal, larger classes in their third and fourth years. As the mathematics that they study becomes more advanced, students may begin to find their courses easier or more difficult to understand – the opportunity to choose optional courses increases after the first year, so students may be able to study mathematics which they find more interesting, and perhaps more easy, which may contribute to them adopting deep ATLs as they engage with the material. Examinations in the third and fourth years contribute towards the students' degree outcomes, which means that assessment pressures are significantly greater in those years, which has the potential to affect students' ATLs (see Chapter 2.1.2.4) in that they may choose to adopt strategic ATLs in order to increase their probability of examination – and therefore, degree – success. Whilst the data here cannot support these assertions with statistical significance, there are indications that this may be the case (see also Chapter 7).

5.6 – ATL & Gender

As many writers have argued that gender plays a role in a students' ATL (Severiens & Ten Dam, 1994), analyses were conducted in order to establish whether they existed in this dataset. Furthermore, Darlington (2010) argued that gender differences exist in first-year Oxford undergraduate mathematicians' scores on the surface scale, in that women scored significantly higher than men. This analysis serves to extend that across all year groups.

5.6.1 – Differences by ATL

Whilst no gender differences were found in Sweep 1 (see Appendix 5.12), data for Sweep 2 suggest that the proportion of women with deep ATLs is much less than for men in each of Year 1 (-13.9%), Year 2 (-11.8%) and Year 3 (-14.3%) (see Figures 5.18 & 5.19). Furthermore, none of the third-year women who participated in Sweep 2 displayed a deep-dominant ATL. This is not to say that there were very few women in this year group; indeed, they formed 44% of the 52 participants in that year group.

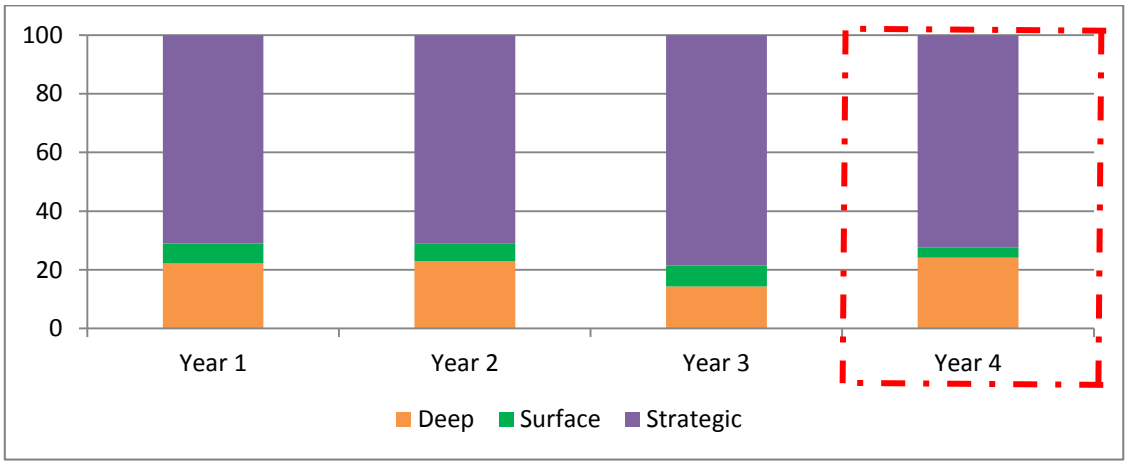


Figure 5.18 - Male ATLs, Sweep 2

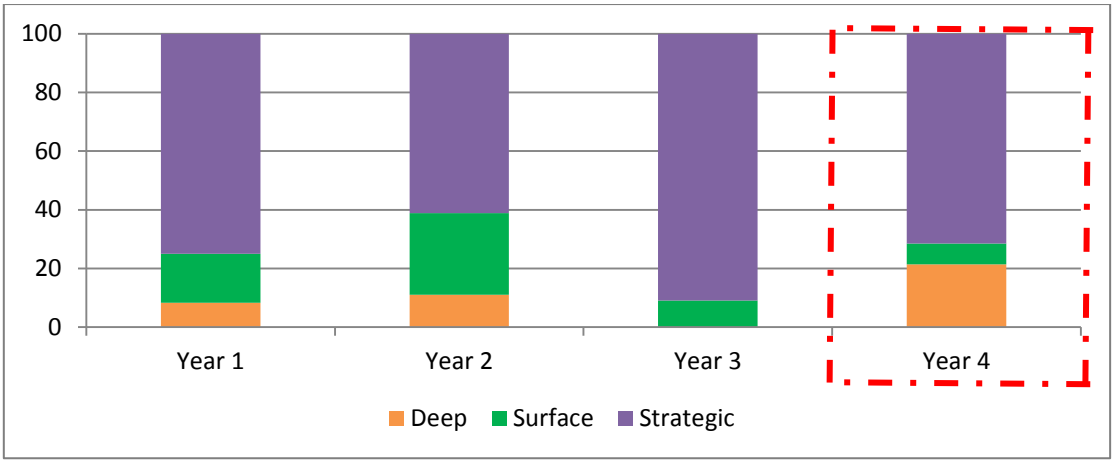


Figure 5.19 - Female ATLs, Sweep 2

The proportion of participants of each gender with a surface ATL was much higher in women than in men. For Years 1-4, this comprised of 6.7%, 6.2%, 7.1% and 3.4% of the male participants, respectively, whereas this was 16.7%, 27.8%, 9.1% and 7.1% for women. These gender differences were significant $(\chi^2(2) = 9.551, p = .008 < .05)$ (see Appendix 5.13). However, there were no significant differences when looking at individual year groups in Sweep 2 (see Appendix 5.14).

5.6.2 – Differences by Subscale Mean

5.6.2.1 – All Years

Across all of the year groups as a whole, an independent-samples Mann Whitney U-test (see Appendix 5.15) revealed no significant gender differences in the strategic ATL scores $(p = .328)$, but significant differences in deep and surface scores $(p = .000)$. That is, women tended to score significantly higher on the surface scale, and significantly lower on the deep scale.

Table 5.20 - Average scale scores of men & women, Sweep 2

	Gender	N	Median	Variance	Minimum	Maximum
Deep	M	150	63.0	80.022	16	77
	F	78	55.0	71.270	31	72
Strategic	M	150	71.0	119.879	35	95
	F	78	71.5	133.812	29	89
Surface	M	150	42.5	122.737	20	75
	F	78	50.5	116.227	22	73

5.6.2.2 – Individual Year Groups

Across Years 1-3, further independent-samples Mann Whitney U-tests suggested significant gender differences in the deep and surface scales, but not in the strategic scale. In the fourth year, however, no significant gender differences were identified across any of the scales. This is consistent with the data in Table 5.20, which suggest that women score higher on the surface scale and lower on the deep scale than their male counterparts.

Table 5.21 - Mann Whitney U-test for gender differences on subscale means, Sweep 2

Null Hypothesis	Year	Sig.	Decision
The distribution of deep subscale means is the same across categories of gender.	1	.002	Reject the null hypothesis
	2	.000	Reject the null hypothesis
	3	.000	Reject the null hypothesis
	4	.056	Retain the null hypothesis
The distribution of strategic subscale means is the same across categories of gender.	1	.930	Retain the null hypothesis
	2	.051	Retain the null hypothesis
	3	.953	Retain the null hypothesis
	4	.990	Retain the null hypothesis
The distribution of surface subscale means is the same across categories of gender.	1	.009	Reject the null hypothesis
	2	.032	Reject the null hypothesis
	3	.037	Reject the null hypothesis
	4	.204	Retain the null hypothesis

Gender differences across the individual year groups are evident in box plots:

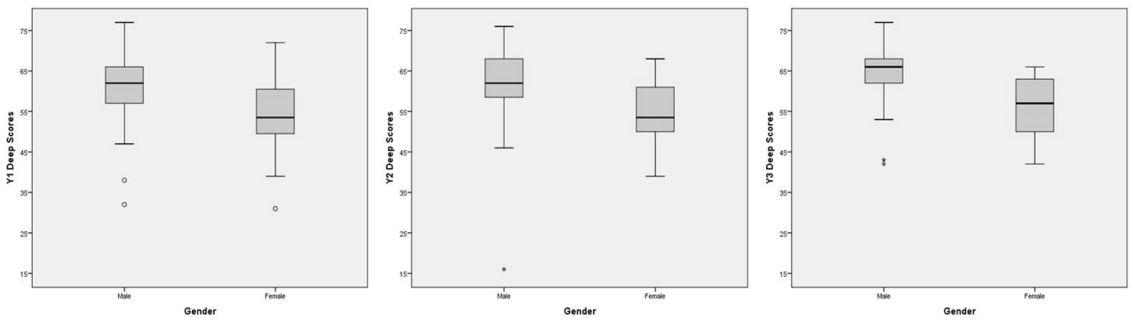


Figure 5.22 - Box plots of scores on the deep scale by gender, Sweep 2

These plots show that women's scores on the deep scale of the ASSIST tend to be lower than men's, with their scores on surface scales being significantly higher than men's.

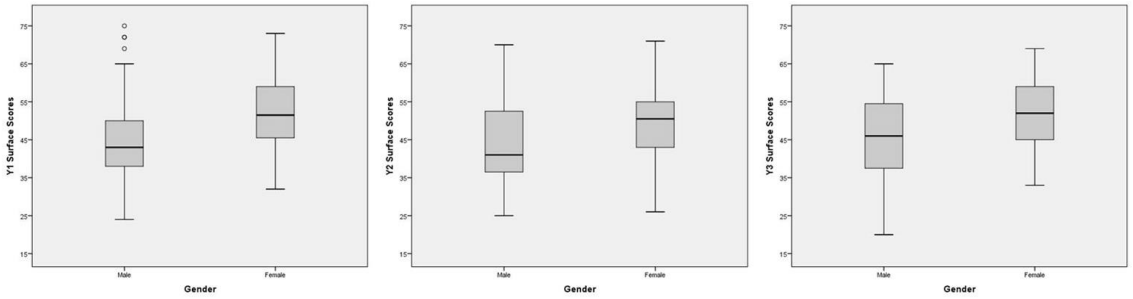


Figure 5.23 - Box plots of scores on the surface scale by gender, Sweep 2

It is important to note that this data is self-reported, which could be impacted upon by an amount of the participants' self-deprecation in their responses, something which has been found to be displayed more by college women than men (Padesky & Hammen, 1981). Therefore, responses to a statement such as 'Often I feel I'm drowning in the sheer amount of material we're having to cope with' could be answered in such a way that responses would tend to the higher end of the scale for women more so than men. Furthermore, women have been found to be less confident in their academic talents in mathematics (Becker, 1990; Solomon, 2007b), with Solomon *et al. (*2011) commenting that female undergraduates often suffer from 'fragile identities' in this environment.

5.6.3 – Summary

Whilst Sweep 1 data indicated there were no significant gender differences in terms of the proportions of students predominantly adopting each ATL or in terms of the scores on each scale, differences were found across the board in Sweep 2. This may be attributed to the small sample size in Sweep 1 relative to Sweep 2. In Sweep 2, the proportion of women predominantly adopting deep ATLs was significantly lower than men across all of the four year groups, and the proportion of surface ATLs was significantly higher. Furthermore, significant differences were found in the deep and surface scale scores for men and women in Years 1-3. There were no statistically significant differences in the strategic scale, and no significant

differences in any of the deep, surface and strategic scales in Year 4. This may be attributed to the fact that women's confidence, both in general and in tertiary mathematics, has been found to be lower than men's, as well as women tending to be self-depreciating in their responses to questions regarding their experiences, which would affect the outcome of a self-report questionnaire such as the ASSIST. The lack of gender differences in Year 4 suggests that, at this level, women's confidence may be reinforced by success in the first three years of their degree, which would affect their responses to a number of ASSIST items.

5.7 – Individual ASSIST Items

Year 1 Year 2 Year 3 Year 4

Whilst the ASSIST is a useful tool in itself to assess individual students' – and groups of students' – ATLs, looking closer at participants' responses to individual items on the scale can itself prove interesting, as well as providing an insight into where the year group and gender differences identified in Chapters 5.5 and 5.6.2 came from.

5.7.1 – Year Group Differences

A small number of significant differences can be identified across the year group pairings.

Table 5.24 – Year group differences identified in individual items of ASSIST, Sweep 2

Year Groups	Item	Difference	Appendix
1 & 2	[SU45] I often have trouble in making sense of the things I have to remember.	Second-years tend to disagree more with this statement than first-years $(\chi^2(4) = 11.039, p = .025)$.	5.16.1
1 & 3	[DE30] When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it.	Third-years tend to agree with this statement more than first-years $(p = .022)$. The median first-year response is 'neither agree nor disagree', whereas in Year 3 it is 'somewhat agree'.	5.16.2
1 & 4	[SU22] I often worry about whether I'll ever be able to cope with the work properly.	First-years tend to 'somewhat agree' with this statement more than fourth-years, who tend to 'somewhat disagree' $(\chi^2(4) = 11.318, p = .022)$.	5.16.3
	[ST40] I usually plan out my week's work in advance, either on paper or in my head.	First-years tend to agree with this statement more than fourth-years $(\chi^2(4) = 9.613, p = .047)$. The median first-year response was 'somewhat agree', whereas it was 'neither agree nor disagree' in Year 4.	5.16.4
	[SU45] I often have trouble in making sense of the things I have to remember.	Fourth-years are more likely to disagree with this statement than first-years, who are more neutral $(p = .008)$. The median first-year response students was 'neither agree nor disagree', whereas it was 'somewhat disagree' in Year 4.	5.16.5

2 & 4	[ST15] I look carefully at tutors' comments on problem sheets to see how to get higher marks next time.	A greater proportion of second-years 'agreed' or 'somewhat agreed' (77.3%) with this statement than fourth-years (58.1%) $(p = .016)$. That is, second-years were more likely to agree in some capacity with this statement than fourth-years, who were more neutral (30.2% of them neither agreed nor disagreed).	5.16.6
3 & 4	[DE04] I often set out to understand for myself the meaning of what I have to learn.	A significantly greater proportion of fourth-years disagreed with this statement (25.6%) than third-years, 6% of whom 'somewhat disagreed' with it $(\chi^2(3) = 8.108, p = .043)$. No students fully disagreed with the statement.	5.16.7
	[SU22] I often worry about whether I'll ever be able to cope with the work properly.	Whilst third-years tended to 'somewhat agree' with this statement, fourth-years were more likely to 'somewhat disagree' $(\chi^2(4) = 11.751, p = .018)$.	5.16.8
	[DE26] I find that studying academic topics can be quite exciting at times.	Whereas the median response by third-years was 'agree', that of fourth-years was 'somewhat agree'. That is, third-years tended to more strongly agree with this statement than fourth-years $(p = .032)$.	5.16.9

Overall, these year group differences suggest some significant differences between the responses of fourth-year students and those in other years in a number of respects, namely:

- Fourth-year students find their mathematics studies easier than first-year students. Furthermore, second-years find it easier than first-years.
- Fourth-year students worry less than first-year and third-year students about workload.
- First-year students make the specific effort to plan their time more than fourth-year students.
- Second-year students are more interested in tutors' feedback than fourth-year students.

- Fourth-year students do not set out to understand for themselves as much as third-year students.
- Third-year students find academic topics more exciting than fourth-year students.
- Third-year students tend to read and reflect more on what they learn than first-years.

These differences mainly relate to the students' confidence in their ability to do and engage in mathematics study at university, something which would likely increase with time throughout their degree as they become more familiar with its study and its nature. Furthermore, in Year 4, when students already have achieved a BA degree in mathematics and will have reached a minimum level of attainment in order to continue, it is understandable that they will be more confident, having reached this level, and be engaged in the subject because they have chosen to continue for an additional year. First-years will likely be new and enthusiastic, eager to do well in their first year at Oxford, as well as to be nervous and unsure about their ability to be successful as they are not yet familiar with the nature of the mathematics, the institution and the assessment formats and practices. The pressures of final examinations in Year 3 appear to have an impact on students' responses to some of the items in the ASSIST, as students in that year appear to spend more time reading and reflecting on material than those in the first year, and worry about their workload more than fourth-years.

5.7.2 – Gender Differences

Data here go towards identifying the particular aspects of the ASSIST which contributed towards the gender differences identified between men and women's scores on the deep and surface scales in Sweep 2 (see Chapter 5.6.2). On half of the surface items, women more strongly agreed with the statements than men (see Appendix 5.17).

The female participants' responses to items on the surface scale suggest that they are:

- **less confident in their abilities;**
    - Often I feel I'm drowning in the sheer amount of material we're having to cope with ($p < .001$).
    - I often worry about whether I'll ever be able to cope with the work properly ($p < .05$).
    - Often I lie awake worrying about work I think I won't be able to do ($p < .01$).
- **less enthusiastic about their subject; and**
    - There's not much of the work here that I find interesting or relevant ($p < .01$).
- **more likely to struggle with understanding undergraduate mathematical concepts.**
    - Much of what I'm studying makes little sense: it's like unrelated bits and pieces ($p < .001$).
    - I'm not really sure what's important in lectures, so I try to get down all I can ($p < .01$).
    - I often have trouble in making sense of the things I have to remember ($p < .05$).
    - I like to be told precisely what to do in essays or other assignments ($p < .05$).

Furthermore, their responses to items on the deep scale are significantly lower than their male counterparts, with women agreeing significantly less with 82% of the deep scale items. Their responses suggest that women are:

- **less likely to engage with mathematical concepts. Significantly more women than men tended to disagree with the following statements than men:**
    - I often set out to understand for myself the meaning of what we have to learn ($p < .01$).
    - I try to relate ideas I come across to those in other topics whenever possible ($p < .01$).
    - Regularly I find myself thinking about ideas from lectures when I'm doing other things ($p < .001$).
    - When I read lecture notes or a book, I try to find out for myself exactly what the author means ($p < .05$).
    - When I'm working on a new topic, I try to see in my own mind how all the ideas all fit together ($p < .01$).
    - Often I find myself questioning things I hear in lectures or read in books ($p < .001$).
    - When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it ($p < .01$).
    - Ideas in course books or lecture notes often set me off on long chains of thought on my own ($p < .001$).
    - When I read, I examine the details carefully to see how they fit in with what's being said ($p < .05$).
    - I like to play around with ideas of my own even if they don't get me very far ($p < .001$).

- **less enthusiastic about their subject. Men were significantly more likely than women to agree with these statements than women.**
    - I find that studying academic topics can be quite exciting at times ($p < .05$).
    - Some of the ideas I come across on the course, I find really gripping ($p < .01$).
    - I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them ($p < .001$).

These results correspond with Dart *et al.*'s (1999) contention that those with surface ATLs tend to have lower self-concepts, as well as Bruinsma's (2003, cited by Heijne-Penninga *et al.*, 2008) study which suggested that students with deep ATLs tend to be more confident in their abilities as learners. The gender differences suggested by the students' responses to the ASSIST may be a consequence of the fact that women have been found to "express… fragile identities more often or at least more readily" (Solomon, 2007a, p. 1). Women undergraduate mathematicians have been found to describe senses "of constant danger of feeling out of their depth" (Solomon, 2007b, p. 91).

5.7.3 – General Responses

The responses given by the students as a whole are of interest in terms of examining their perceptions of their learning environment.

The results in the Appendix 5.18 give indications of the perceptions of the overall cohort of their learning. Specifically, this gives an insight into students (1) satisfaction with their course; (2) levels of struggles with their course; (3) perceptions of success; and (4) reliance on memorisation. The results suggest that students exhibit mixed feelings in terms of how happy they are studying undergraduate mathematics at Oxford, whether they are struggling or not and whether they perceive themselves to be successful or not, but that the majority rely on memorisation as a form of learning.

5.7.3.1 – Course Satisfaction

A number of statements on the ASSIST provide an insight into the students' satisfaction with their course. The vast majority of these statements revealed students to be enjoying their studies and finding what they are studying to be interesting. Specifically,

- **16 – There's not much of the work here that I find interesting or relevant.**

  Most students disagreed with this statement, and 83.3% either disagreed or somewhat disagreed.

- **26 – I find that studying academic topics can be quite exciting at times.**

  Most students agreed with this statement, with over 85% of respondents either somewhat agreeing or agreeing with it.

- **39 – Some of the ideas I come across on the course, I find really gripping.**

  Most students somewhat agreed with this statement, and only 5.3% disagreed in some capacity.

- **52 – I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them.**

  Most students agreed with this statement, and only 7% disagreed in some capacity.

However, there were significant gender differences in terms of responses to items 26 ($p = .011$), 39 ($p = .006$) and 52 ($p = .000$), wherein women tended to disagree with the statement more than men (see Chapter 5.7.1).

For more negatively-phrased statements, the participants' responses were largely positive:

- **16 – There's not much of the work here that I find interesting or relevant.**

  Most respondents disagreed with this statement (49.1%), and only 7.0% disagreed in some capacity. However, women tended to score more highly on this item than men ($p = .003$).
- **29 – When I look back, I sometimes wonder why I ever decided to come here.**

  Whilst 67.6% of participants disagreed with this statement in some capacity, over 20% agreed in some capacity. Whilst other statements regarding satisfaction are mainly positive, this is one which indicates that there are a reasonable number of students who are not happy in their studies.
- **3 – Often I find myself wondering whether the work I'm doing here is worthwhile.**

  Equal numbers of participants somewhat disagreed and somewhat agreed with this statement (25.0%), which shows a strong mixture in terms of the feelings of the students.

5.7.3.2 – Worries

Whilst the majority of participants either disagreed or somewhat disagreed with *'48 – Often I lie awake worrying about work I think I won't be able to do'* (55.2%), significantly more women agreed with the statement than men ($p = .007$). This may be attributed to gender differences

in terms of confidence; however, 50% of students either agreed or somewhat agreed with '22 – *I often worry about whether I'll ever be able to cope with the work properly*' with less statistical significance in gender differences ($p = .015$). Therefore, this suggests that the majority of participants are not confident in their ability to cope with the workload and content. Furthermore, the majority of respondents somewhat agreed with '*35 – I often seem to panic if I get behind with my work*', further indicating that students are experiencing a highly-pressured environment. Responses to this statement were also free of gender differences, suggesting that this pressure is not only felt by women.

5.7.3.3 – Success

Despite indications that students are not confident in their mathematical ability and worry about whether they will be successful, most participants somewhat agreed with '*24 – I feel that I'm getting on well, and this helps me put more effort into work'*. Nearly 70% of respondents either agreed or somewhat agreed with this statement. This is encouraging, as it suggests that the students perceive themselves as performing reasonably well, and this inspiring them to work hard.

5.7.3.4 – Memorisation

As suggested by the literature, mathematics undergraduates have a tendency to memorise mathematics, only to 'memory dump' it later on, even into their third year (Anderson *et al.*, 1998). Many undergraduate mathematicians view the subject as a rote-learning task (Crawford *et al.*, 1994, 1998a, 1998b). This is supported by the responses to *'6 – I find I have to concentrate on just memorising a good deal of what I have to learn'*, a statement with which the majority of the students somewhat agreed. Nearly 62% of respondents either agreed or somewhat agreed with the statement, and only 6.1% disagreed with it.

However, Entwistle (2001) argues that memorisation "makes an essential contribution to understanding" (p. 599), and reports made by participants in the student interviews (see

Chapter 7) will go some way to establishing the reasons behind this and the form(s) that it takes.

5.7.4 – Summary

Individual items in the ASSIST were analysed for differences between groups in order to establish the questions which resulted in either significant or insignificant gender differences in scale scores or ATL distribution being identified in Chapters 5.3, 5.4, 5.5 and 5.6. The significant differences in terms of men's and women's ATLs identified in 5.6 (women scored significantly higher on surface scales and significantly lower on deep scales in Years 1-3, and significantly greater proportions of women had surface ATLs and lower proportions had deep ATLs) were further investigated in the 52 items in the ASSIST. The reasons behind these differences were identified in 22 statements which had significantly different responses from men and women. The differences suggested that women were less confident in their abilities, less enthusiastic about mathematics and learning mathematics, and that they struggle more with new concepts than men. This falls in line with existing literature regarding mathematics and confidence in women, and explains the reasons behind the gender differences identified in Chapter 5.6.

The differences in responses to the ASSIST given by different year groups outlined in Chapter 5.5, and the lack of expected differences, were examined in each ASSIST item. Significant differences were identified in nine items on the questionnaire. Such differences suggested that students in Year 4 were more confident than those in earlier years in terms of the workload that they had, and felt more relaxed about their studying in that they did not make special study plans as much as first-years and are less concerned about feedback on their work than second-years. Interestingly, third-year students were found to respond significantly more positively than fourth-year students to an item concerning their enthusiasm for new mathematical topics. This was unexpected as one might expect students in Year 4 to be more

enthusiastic about mathematics because they elected to study the subject for an additional year; however, this does not appear to be the case. First-year students were found to find new concepts more difficult than second-year students, which is indicative of their 'newness' to tertiary mathematics and the inherent differences between school and university mathematics (see Chapter 2.3.1).

General responses to items on the ASSIST were analysed in order to establish whether students overwhelmingly responded to certain items either positively or negatively. Those items which students tended to mainly agree or disagree with tended to relate to course satisfaction, their worries, their work ethic and memorisation. Specifically, students' opinions of their degree were largely very positive, with most of them agreeing that they put a lot of time and effort into their studies. However, most of them were found to worry about their ability to cope with the workload and were afraid of getting behind on their work. This suggests that the undergraduate mathematics environment at Oxford is stressful for many students, something which might explain why most of the students agreed that they spent a lot of time on memorisation of concepts for examinations.

5.8 – Conclusion

ASSIST data were collected from students across all of the four year groups in order to ascertain whether there were any obvious shifts in their approaches to learning undergraduate mathematics between each year. Furthermore, repetitious participation of first-year students in Sweeps 1 and 2 acted as a means of comparing students' ATLs upon arrival at the University, before they had had to do any examinations or many problem sheets, and their ATLs after having revised for their first-year examinations.

Whilst some of the literature suggests that it is possible for students' ATLs to change in response to changes in environment, pedagogy and the assessment types (see Chapter 2.1.2), the data collected here contradicts that. No statistically significant differences were identified between students' responses in different year groups, though there were overall changes in the proportions of students who adopted particular ATLs. The proportion of students with strategic ATLs reached its maximum in the third year, which is possibly in response to final examinations. The proportional decrease in strategic approaches and increase in deep ATLs in the fourth year is indicative of the selective nature of this year, and its increased difficulty as a masters year. Indeed, owing to the relationship which has been found between attainment and a surface ATL (see Chapter 2.1.2.1), it seems likely that the decrease in the proportion of students adopting surface ATLs between Year 3 and Year 4 is likely because such students do not want to or are unable to continue to the fourth year.

Whilst there were difference in the overall composition of dominant ATLs in Years 1-3 compared to Year 4, the difference was not statistically significant. The elective and selective nature of the fourth year is attributed to any changes in dominant ATLs or scale scores, although it was surprising to see that this difference was not statistically significant. However, when comparing pairs of year groups (such as comparing Year 1 with Year 4, Year 2 with Year 3 or Year 1 with Year 2, for example), there were significant differences in the surface scale

scores of students in Year 3 compared to Year 4. Third-year students tended to score significantly higher on the surface scale than their MMath year counterparts, which supports the idea that the elective and selective nature of Year 4 results in a reduction in the proportion of students with predominantly surface ATLs.

Furthermore, when responses to individual items on the ASSIST were compared between the year group pairings, a number of significant differences suggested that fourth-years were more confident in their abilities as mathematicians and as students of mathematics. This contrasts with first-year students who were found to be significantly more likely to worry about their ability to cope and the workload that they faced than students in later years, who were more experienced with mathematics study at the University of Oxford.

First-year students tracked between Sweep 1 and Sweep 2 did not show a significant change in their ATLs from one response to the next. Over 78% of respondents' overall ATLs did not change between the two sweeps. However, statistically significant differences were found to exist between Sweeps 1 and 2 in terms of the distribution of dominant ATLs between each group. Whilst there was no statistically significant difference between the scores on each scale between each sweeps, the increase in the proportion of a mainly surface ATL from 0.6% to 10.1%, increase in deep ATLs from 11.4% to 18.8% and decrease in strategic ATLs from 88.1% to 71.0% was significant. These changes may arise from students' adaptations to undergraduate mathematics, specifically relating to the nature of mathematics and mathematics learning at the advanced level. Between Sweeps 1 and 2, students may have established that merely rote learning mathematics was no longer appropriate or necessary, and as such either established deeper ATLs or, conversely, found the mathematics too difficult to engage with on a conceptual level and so resorted to surface ATLs. Regardless, this significant change warrants further investigation than was possible in this thesis.

Across all years, the strategic ATL was by far the most commonly dominant ATL, which may be due to a number of factors. Much research has suggested that secondary students study mathematics in a very procedural way, being assessed on it in such a way that they practise performing routine calculations as the main body of their study, before being examined on familiar questions in A-level examinations (see Chapter 2.3.3). Research in undergraduate mathematics education has suggested a reliance by students on memorising mathematics – which may be a surface ATL or act as part of a strategic ATL – in order to repeat it in examinations, often failing to understand the underlying mathematical concepts (see Chapter 2.3). Therefore, this would explain the significant dominance of this ATL across all groups (ranging from a minimum of 68% in Year 2 to 84% in Year 3, and 89% for first-year students participating in Sweep 1). However, the lack of significant change in ATLs and deep, surface and strategic scores between the year groups warrants further attention. The specific form that students report these ATLs take is described and discussed in Chapter 7.

In response to research which suggests that women are more likely to adopt surface ATLs than men, and women's confidence in their mathematical abilities is lower, tests were conducted in order to establish whether there were any gender differences in terms of students' ATLs and their scores on each of the deep, strategic and surface scales. The findings corroborate with much of the literature in the sense that, in the second sweep, women were found to score significantly higher on the surface scale ($p = .000$) than their male counterparts, and significantly lower on the deep scale ($p = .000$). Furthermore, significant differences were identified in Sweep 2 in men and women's dominant ATL ($p = .008$). Engaging with specific questions in the ASSIST highlighted the items which led to such different outcomes, with these often being related to confidence in one's ability and enjoyment of the subject. This is further supported by the fact that the data collected in Sweep 1 – when students first arrive at university, fresh from secondary study when they have been very successful in their mathematics studies – did not show any significant differences between the genders. This

raises concern over women's experience of undergraduate mathematics and why it might be so different to that of their male counterparts.

The presence and absence of statistical differences which were tested for in this chapter suggest that some attention should be paid to the definition of a strategic ATL in the context of the secondary-tertiary mathematics interface. Whilst one could describe the strategic ATL of a secondary mathematics pupil as being rote learning through repeatedly practising procedures in the knowledge that similar questions will appear in an examination, the strategic ATL of an undergraduate mathematics students takes a different form – memorisation of statements, proofs and previously seen material for verbatim reproduction in an examination. Clearly these are two very different actions, and so care should be taken when making comparisons in terms of the ATLs of pupils and students on each side of the transition. That is not to say that the ASSIST is not applicable in this situation – though both it and the concept of ATLs do have their limitations (see Chapter 2.1.5) – but that care should be taken in its interpretation in this context.

Chapter 6: Data

Contrasts in Challenges Presented by A-Level Mathematics, the Oxford Admissions Test & First-Year Undergraduate Examinations

6.1 – MATH Taxonomy

In order to ascertain whether there are any differences in the challenges presented by mathematics assessment at A-level, undergraduate level and in the University of Oxford mathematics admissions test (OxMAT), the MATH taxonomy (Smith *et al.*, 1996; see Chapter 2.2.3) was applied to a selection of papers from each of the three.

6.2 – A-Level Examinations

At A-level, students have the option of studying a variety of modules which include pure mathematics, mechanics, statistics, discrete mathematics and 'further' pure mathematics. These optional modules form one-third of the A-level Mathematics curriculum. There are two exam sittings each year for each module (one in January and one in the summer), so there is a large number of possible papers that students can take at A-level, and there are four different examination boards offering them (AQA, OCR, Edexcel, WJEC) (see also Chapter 2.3.3.6). In order to gain an insight into the challenges posed by the questions in A-level Mathematics and Further Mathematics, I decided to analyse only the modules Core Mathematics 1 and Further Pure Mathematics 3. These were chosen

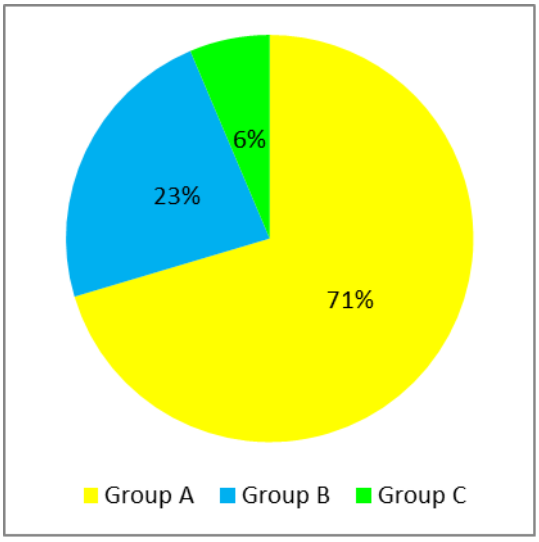


Figure 6.1 - Proportions of Group A, B and C questions in A-level examinations

because C1 is, for most students, the first A-level Mathematics module they study, and FP3 is the most advanced module in pure mathematics for students of A-level Further Mathematics. This means that concentrating on these two modules was an opportunity to analyse questions at both novice and advanced levels in this qualification. One paper from each examination board for each of FP3 and C1 were analysed, with these being chosen at random. Only papers from 2006 or later were considered, because this is when the A-level Mathematics and Further Mathematics syllabi and examination structures underwent their most recent change (see Figure 1.2).

Table 6.2 - Total marks available for Group A, B & C questions in each A-level paper analysed

Module	Exam Board	Year	MATH		
			Group A	Group B	Group C
C1	AQA	2006	84.7	12.0	5.3
C1	Edexcel	2006	68.0	29.3	2.7
C1	OCR	2007	88.9	11.1	0.0
C1	WJEC	2010	88.0	5.3	6.7
Mean C1			82.4	17.4	3.7
FP3	AQA	2007	76.0	24.0	0.0
FP3	Edexcel	2006	66.7	33.3	0.0
FP3	OCR	2007	48.6	22.2	29.2
FP3	WJEC	2010	53.3	40.0	6.7
Mean FP3			61.2	29.9	9.0
Mean Both			71.8	22.2	6.5

Each individual question was analysed using the MATH taxonomy (see Appendix 2.1). The mark schemes were used for reference in order to ascertain the methods by which examiners expected students to answer the questions, which served to provide an insight into the skills that were anticipated and required in the examinations. An example of the MATH taxonomy applied to the 2006 AQA C1 and 2006 Edexcel FP3 papers are given in Appendices 6.1 and 6.2. The proportions of marks available in each paper were calculated for each of Groups A, B and C. As OCR papers total 72 marks and the other examination boards 75, percentages have been included in this instance.

The data here show that the majority of marks in both C1 and FP3 papers are awarded for responses to Group A questions (see Figure 6.1 and Table 6.2), with this being particularly high in the case of C1. Very few marks are awarded for responses to Group C questions in both

modules; however, more marks are awarded for Group C questions than Group B questions in the 2007 OCR FP3 paper. This appears to be an anomaly.

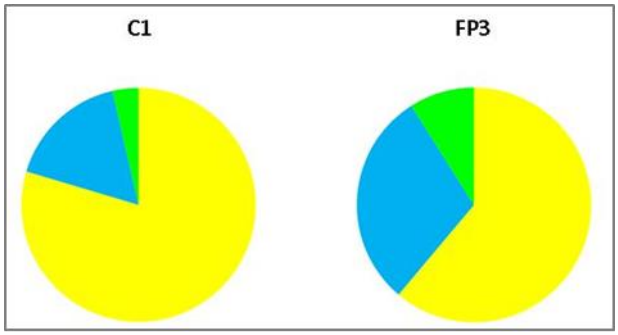


Figure 6.3 - Differences in proportion of Group A, B & C questions in Core 1 & Further Pure 3 A-level Mathematics modules

The main difference which is apparent between C1 and FP3 papers in terms of the MATH taxonomy is that FP3 papers award proportionally fewer marks for Group A questions than C1, with many more Group B questions (see Figure 6.3). As FP3 is the most advanced module in A-level Further Mathematics, it was expected that it would therefore involve more advanced questions which test pupils on their ability to make justifications, comparisons, interpretations and conjectures. However, despite predictions that FP3 would contain more Group C questions than C1 due to the fact that the further pure modules introduce proof and group theory, this transpired not to be the case other than in the 2007 OCR paper. Mathematical proof is not generally something which can be done by use of Group A or Group B skills; however it is possible that some can be reduced to a 'routine use of procedures' or reproduced as 'factual knowledge and fact systems' should the syllabus and question type permit it.

Amongst A-level papers, the majority of Group A marks consisted of 'routine use of procedures' (88.7%), the majority of Group B marks from 'application in new situations' (93.9%), and all of the Group C marks were for 'justifying and interpreting' (see Figure 6.4). This supports claims and empirical studies concerning the procedural nature of A-level Mathematics (Alton, 2008; Crawford *et al.*, 1994, 1998a, 1998b; Bassett *et al.*, 2009; Taverner, 1997). However, it should be noted that more able students of A-level Mathematics (such as future Oxford undergraduates) are better able to turn rehearsed procedures akin to Group A questions into conceptual knowledge and incorporate them into their schema of mathematical

concepts (Tall & Razali, 1993). Therefore, the reliance on procedures at A-level does not necessarily mean that students will have no conceptual understanding of the mathematics that they have studied.

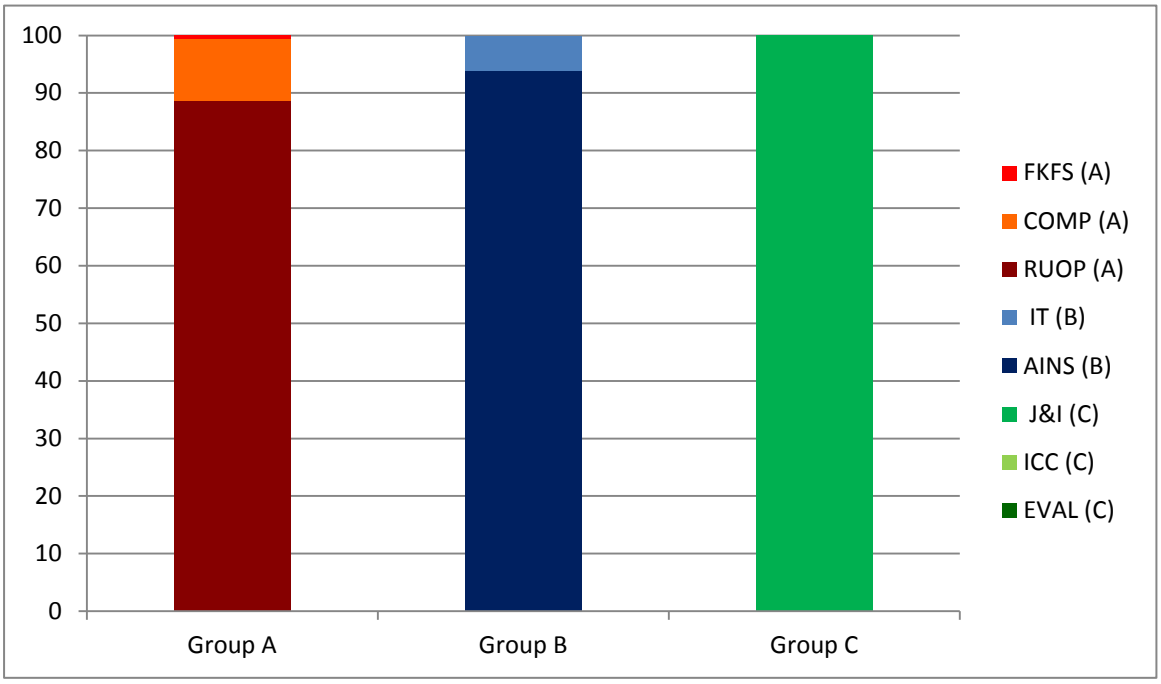


Figure 6.4 - Make-up of questions within each of Group A, B & C in A-level Mathematics

These findings support the work of Berry *et al.* (1999) and Monaghan (1998), whose categorisation of A-level mathematics questions as either 'routine' or 'non-routine' concluded that the majority of available marks in the examinations were awarded for answers to routine questions. Furthermore, it corroborates with Etchells and Monaghan's (1994) research using the MATH taxonomy which found that most marks awarded in A-level Mathematics were for Group A tasks.

6.3 – University of Oxford Mathematics Admissions Test

Past papers for the OxMAT are available online from 2007. Since this is only a small number of examinations, the MATH taxonomy was applied to all of them.

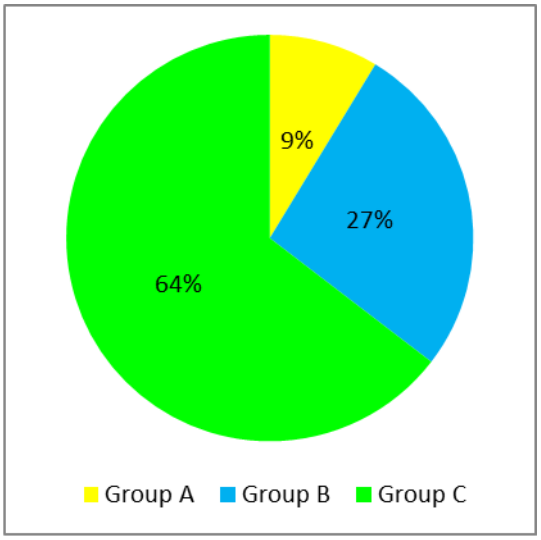


Figure 6.5 - Proportions of Group A, B and C questions in OxMATs

Four marks are awarded for each multiple choice question (there are 10 of these) and the four subsequent questions are worth a total of 15 marks each. However, the marks for each part of questions 2-5 are not indicated. Therefore, in order to gain a quantitative insight into the number of marks awarded for questions testing skills of each of Group A, Group B and Group C, the marks for each part of the question were estimated based on what each part entailed. An example of the MATH taxonomy applied to the 2007 OxMAT is given in Appendix 6.3. The average number of these estimated marks awarded for questions of each type (A, B, C) were calculated for each examination.

The vast majority of marks awarded in each OxMAT were for Group C questions, and Group A questions providing a very small number of available marks to the student (see Figure 6.5 and Table 6.6).

Table 6.6 - Total marks available for Group A, B & C questions in each OxMAT paper analysed

Paper	Question	Marks Available			Paper	Question	Marks Available		
		Group A	Group B	Group C			Group A	Group B	Group C
2007	1	0	20	20	2008	1	8	4	28
	2	5	2	8		2	5	4	6
	3	6	3	6		3	0	6	9
	4	0	9	6		4	2	8	5
	5	4	5	6		5	0	0	15
Total		*15*	*39*	*46*	*Total*		*15*	*22*	*63*

2009	**1**	4	12	24	**2010**	**1**	0	4	36
	2	2	3	10		**2**	0	3	12
	3	2	3	10		**3**	0	7	8
	4	2	8	5		**4**	2	10	3
	5	0	4	11		**5**	0	0	15
Total		*10*	*30*	*60*	***Total***		*2*	*24*	*74*
2011	**1**	0	12	28	**2012**	**1**	4	0	36
	2	2	5	8		**2**	0	1	14
	3	0	10	5		**3**	0	3	12
	4	1	4	10		**4**	0	8	7
	5	1	0	14		**5**	2	3	10
Total		*4*	*31*	*65*	***Total***		*6*	*15*	*79*

In the OxMAT, the majority of Group A marks came from 'routine use of procedures' (75.8%), the majority of Group B marks from 'application in new situations' (86.4%), and the majority of Group C marks from 'justifying and interpreting' (60.8%) (see Figure 6.7).

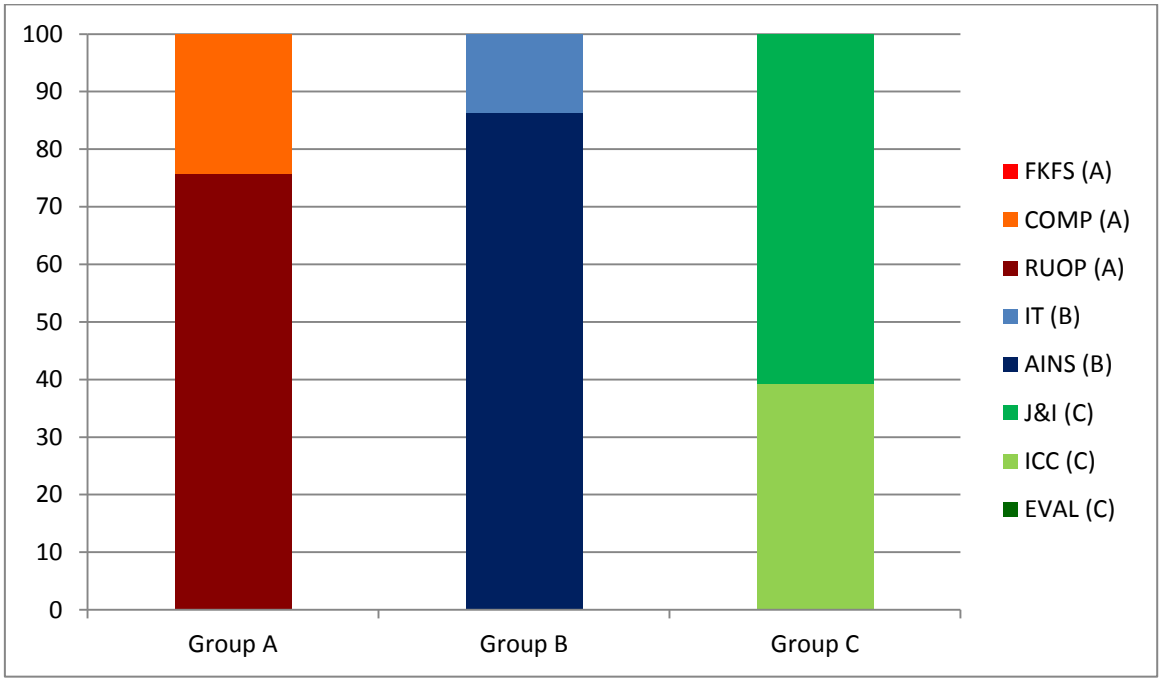


Figure 6.7 - Make-up of questions within each of Group A, B & C in OxMATs

6.4 – Undergraduate Examinations

Past papers for undergraduate examinations in mathematics at the University are available online from 2003. It was only in 2006 that the Mathematics Institute began to state the available marks for each part of the questions in the paper; prior to this, students were aware that each question was worth 20 marks but not, for example, what proportion of the marks for question 2 were from question 2 (a) and what proportion from 2 (b).

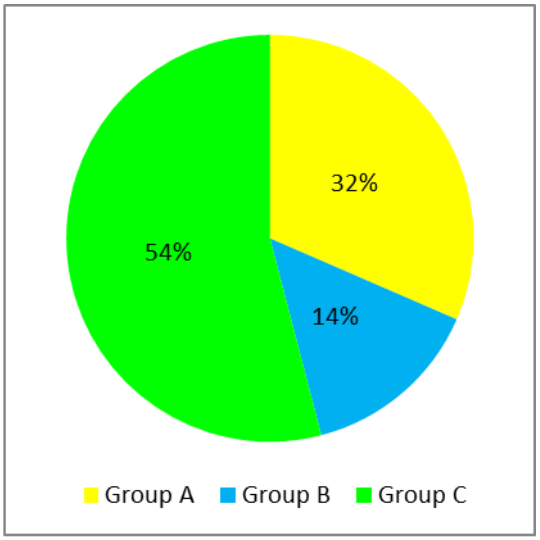


Figure 6.8 - Proportions of Group A, B and C questions in undergraduate examinations

Consequently, any quantitative analysis of undergraduate examinations is based only on the years 2006-2012. Based on the definitions of each group in the MATH taxonomy, each part question of each examination was classified accordingly, which enabled me to calculate the average number of marks available for each examination in each group. An example of the MATH taxonomy applied to the 2008 Pure Mathematics I and 2011 Pure Mathematics II papers are given in Appendices 6.4 and 6.5.

First-year students do a number of examinations in the summer, but I decided to only analyse questions from Pure Mathematics I and Pure Mathematics II because the A-level examinations analysed were also pure mathematics. Both of these topics are core to undergraduate courses nationwide. Pure Mathematics I focused on topics in algebra and Pure Mathematics II on topics in analysis.

The majority of marks awarded in first-year pure mathematics examinations are for questions requiring students to demonstrate skills from Group C of the MATH taxonomy – that is, 'justifying and interpreting' and 'implications, conjectures and comparisons'. The minority of

marks were awarded for questions requiring Group B skills, and approximately a third of marks available for the demonstration of Group A skills (see Figure 6.8 and Table 6.9).

Table 6.9 - Total marks available for Group A, B & C questions in each undergraduate paper analysed

Paper	Question	Marks Available			Paper	Question	Marks Available		
		Group A	Group B	Group C			Group A	Group B	Group C
Pure Mathematics I 2006	**1**	11	2	7	Pure Mathematics II 2006	**1**	9	0	11
	2	0	10	10		**2**	15	0	5
	3	9	0	11		**3**	6	3	11
	4	2	0	18		**4**	14	0	6
	5	12	0	8		**5**	9	0	11
	6	2	0	18		**6**	11	0	9
	7	1	12	7		**7**	14	2	4
	8	6	10	4		**8**	1	0	19
Total		*43*	*34*	*83*	***Total***		*79*	*5*	*76*
Pure Mathematics I 2007	**1**	11	4	5	Pure Mathematics II 2007	**1**	8	4	8
	2	1	11	8		**2**	10	2	8
	3	11	4	5		**3**	3	0	17
	4	14	2	4		**4**	12	0	8
	5	6	3	11		**5**	9	0	11
	6	9	0	11		**6**	1	0	19
	7	2	4	16		**7**	0	9	11
	8	1	13	6		**8**	3	3	14
Total		*55*	*41*	*66*	***Total***		*46*	*18*	*96*
Pure Mathematics I 2008	**1**	4	0	16	Pure Mathematics II 2008	**1**	15	0	5
	2	3	11	6		**2**	6	0	14
	3	7	9	4		**3**	12	4	4
	4	4	0	16		**4**	12	0	8
	5	11	0	9		**5**	13	0	7
	6	2	7	11		**6**	8	0	12
	7	4	0	16		**7**	11	0	9
	8	0	8	12		**8**	0	8	12
Total		*35*	*35*	*90*	***Total***		*77*	*12*	*71*
Pure Mathematics I 2009	**1**	2	0	18	Pure Mathematics II 2009	**1**	8	6	6
	2	15	0	5		**2**	7	0	13
	3	7	3	10		**3**	7	3	10
	4	1	6	13		**4**	2	0	18
	5	7	5	8		**5**	9	0	11
	6	14	0	6		**6**	0	6	14
	7	3	4	13		**7**	3	0	17
	8	0	14	6		**8**	2	6	12
Total		*49*	*32*	*79*	***Total***		*38*	*21*	*101*

Paper	Question	Marks Available			Paper	Question	Marks Available		
		A	B	C			A	B	C
Pure Mathematics I 2010	**1**	17	0	3	Pure Mathematics II 2010	**1**	5	0	15
	2	2	18	0		**2**	6	0	14
	3	1	10	9		**3**	10	5	5
	4	8	0	12		**4**	3	0	17
	5	12	2	6		**5**	10	0	10
	6	7	10	3		**6**	2	0	18
	7	4	0	16		**7**	9	3	8
	8	0	6	14		**8**	0	14	6
Total		*51*	*46*	*63*	***Total***		*45*	*22*	*93*
Pure Mathematics I 2011	**1**	5	1	14	Pure Mathematics II 2011	**1**	11	0	9
	2	9	1	10		**2**	8	0	12
	3	6	14	0		**3**	9	0	11
	4	10	0	10		**4**	6	0	14
	5	0	0	20		**5**	9	0	11
	6	17	0	3		**6**	4	0	16
	7	6	0	14		**7**	2	0	18
	8	0	0	20		**8**	0	0	20
Total		*53*	*16*	*91*	***Total***		*49*	*0*	*111*
Pure Mathematics I 2012	**1**	12	0	8	Pure Mathematics II 2012	**1**	8	0	12
	2	11	6	3		**2**	4	0	16
	3	2	0	18		**3**	2	4	14
	4	1	11	8		**4**	8	0	12
	5	1	0	19		**5**	12	0	8
	6	7	8	5		**6**	4	0	16
	7	5	0	15		**7**	7	0	13
	8	1	8	11		**8**	2	4	14
Total		*40*	*33*	*87*	***Total***		*47*	*8*	*105*
Pure I Average		**46.6**	**33.9**	**79.9**	**Pure II Average**		**54.4**	**12.2**	**93.3**

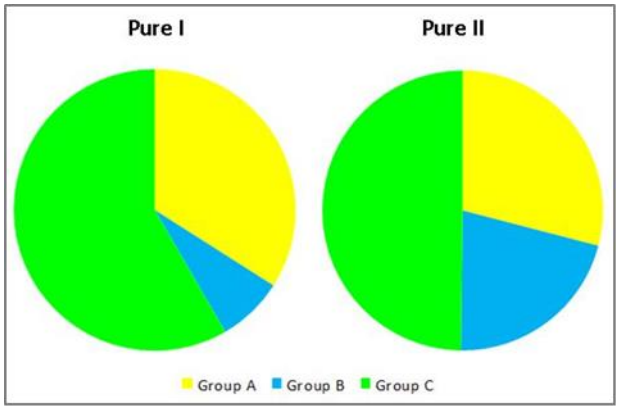


Figure 6.10 - Differences in proportion of Group A, B & C questions in Pure I & Pure II undergraduate mathematics modules

The spread of marks by group was slightly different between Pure Mathematics I and Pure Mathematics II (see Figure 6.10). Pure Mathematics II is much more abstract than Pure Mathematics I as it focuses on analysis, whereas topics in algebra have the potential to be assessed through asking students to demonstrate their understanding of certain procedures, or applying them, in this

topic area. This means that the possibility of being awarded marks for Group A and B questions might be greater in algebra. This is reflected in the proportion of marks available for correct responses to Group B questions being higher in Pure Mathematics I than II.

Whilst there are eight questions in each examination, candidates have to submit answers to five questions. Therefore, it is possible that some combinations of questions might result in heavier reliance on one group over another. In order to establish whether this could change the proportion of questions posed in each group, the five questions from each paper which awarded the least marks to Group C skills were totalled (see Table 6.11). Whilst this had no impact on the general spread and rank of each group within Pure Mathematics II examinations, the spread of marks awarded across each group for questions in Mathematics I became much more even, with the majority of marks tending to be from Group A questions, should students select those questions which permitted this. However, it is not necessarily the case that these students will have performed best with such a combination of questions – it is not necessarily true that Group A questions are the easiest and Group C questions are the most difficult. Indeed, my analysis of the questions posed in undergraduate examinations highlighted the different possibilities for Group A questions – many of these required 'comprehension', but for this to be demonstrated, students necessarily must have had an impressive understanding of the topics concerned.

In Table 6.9, highlighted cells indicate the five questions where the least marks are awarded for answers to Group C questions.

Table 6.11 - Total marks available for Group A, B & C questions in each undergraduate paper analysed when maximising the number of Group A questions answered

Pure I Paper	Marks for 5 Questions, Minimising Group C			Pure II Paper	Marks for 5 Questions, Minimising Group C		
	Group A	Group B	Group C		Group A	Group B	Group C
2003	Mark breakdown not given			**2003**	Mark breakdown not given		
2004	Mark breakdown not given			**2004**	Mark breakdown not given		
2005	Mark breakdown not given			**2005**	Mark breakdown not given		
2006	30	34	36	**2006**	63	2	35
2007	28	34	28	**2007**	39	15	46
2008	23	35	42	**2008**	63	4	33
2009	43	22	35	**2009**	33	15	52
2010	39	40	21	**2010**	35	22	43
2011	48	15	37	**2011**	43	0	57
2012	32	33	35	**2012**	37	4	59
Average	*35*	*30*	*33*	**Average**	*45*	*9*	*46*

Unlike in A-level Mathematics where the majority of Group A marks were for a 'routine use of procedures' (88.7%), in undergraduate mathematics examinations, the majority of Group A marks were for 'factual knowledge and fact systems' (90.0%), with RUOP forming a very small percentage of these marks (4.0%) (see Figure 6.12). The majority of Group B marks were for 'application in new situations' (83.0%), and the majority of Group C marks for 'justifying and interpreting' (87.1%).

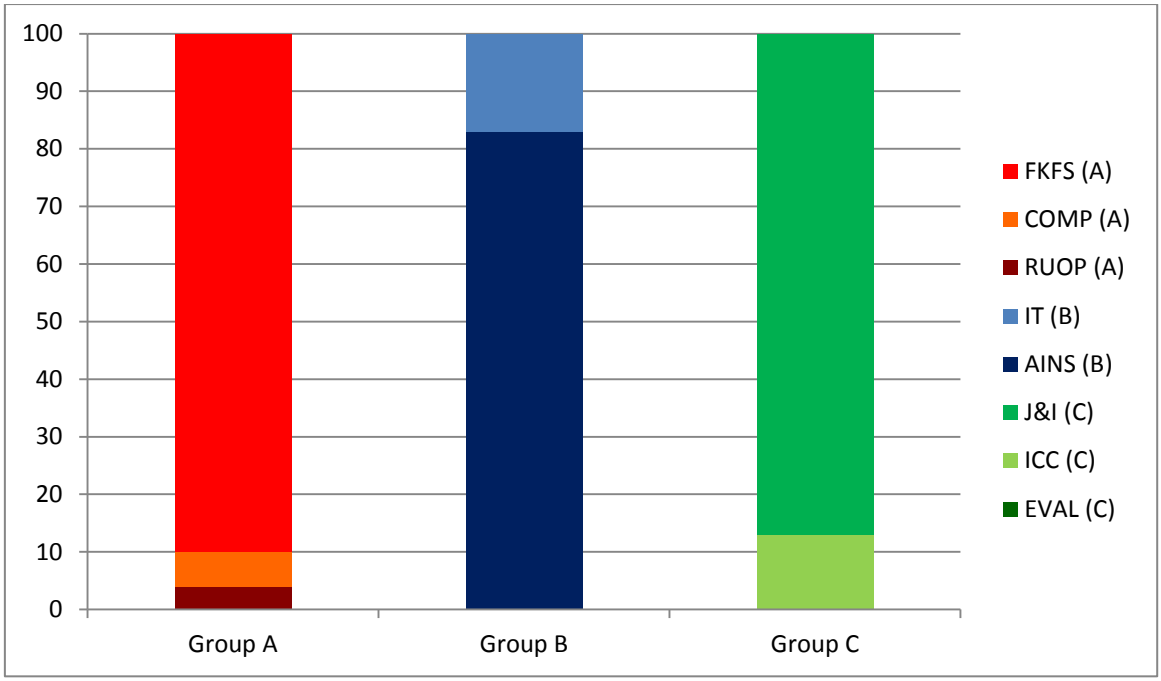


Figure 6.12 - Make-up of questions within each of Group A, B & C in undergraduate examinations

6.5 – Observations

The analysis of these three different examination types has revealed substantial differences in terms of the types of questions posed to students at each level.

The vast majority of A-level questions are Group A, in particular most of these questions asking students to perform a routine use of procedures in answering the questions. This is common to both C1 and FP3. The most at extreme instance of this is in the 2007 OCR paper, where 88.9% of the available marks were awarded for answers to Group A questions. All but a few of these questions were in the subcategory 'routine use of procedures'; that is, students are able to answer the majority of questions through the use of well-practised procedures which they will be familiar with from doing work during lessons (see Appendices 2.2 and 2.3, which give examples of how tightly some textbooks work with examination question styles), and which are themselves similar in nature to previous examinations. Both C1 and FP3 have the majority of their marks in Group A which suggests that, despite FP3 being a more advanced module, the examinations do not challenge students in a different way to earlier A-level Mathematics and Further Mathematics modules. It is this reliance on routine, mechanical skills at A-level which Cox (1994) suggests encourages strategic ATLs. Furthermore,

> *These routine, mechanical skills appear to be well drilled and retained quite well, but even the best students appear to optimise their performance by strategic learning; that is their better marks are often achieved by greater facility with routine material rather than by deeper knowledge of the fundamental topics.*
>
> (p. 11)

Hence, the nature of the questions posed at this level can have an impact on students' approaches to learning mathematics and, consequently, their understanding of mathematical concepts.

Between A-level and the OxMAT, the proportion of marks awarded for responses to Group C questions increases sharply (from 7.5% to 65%, respectively). This reflects the desire of

admissions tutors to use the OxMAT as a means of identifying students who are able to use mathematical thinking, to answer questions which are not familiar and rehearsed, to solve problems. That is not to say that the mathematics involved is more difficult in the OxMAT than A-level; indeed, the OxMAT only requires knowledge up to mid-AS-standard, but it is the way in which students use this knowledge that is of interest. The data collected here using the MATH taxonomy go towards confirming the merits of the University administering such an examination for candidates in that the OxMAT does test students' understanding of mathematical concepts and their ability to use them in a very different way to A-level. Furthermore, one may question whether studying the A-level prepares students at all for passing the OxMAT as the skillset required is indeed so different. Are students who do A-level Mathematics and who pass the OxMAT able to be successful because they have had this skill somehow developed during school mathematics – though it is not apparently examined – or is it an innate ability of theirs to be able to solve mathematical problems as well as correctly answer mathematics questions?

However, student interviews and anecdotal evidence suggests that a large number of students – particularly those who hail from private schools – are prepared for the OxMAT by their school. Such students form 41.5% of Oxford undergraduates (University of Oxford, 2012), and there are various other external people and organisations which can be used by applicants to prepare for the OxMAT (see Section 7.3.2.3 for examples). The availability of past papers and partial solutions online means that students can do past papers in order to prepare for the examination, and their teachers may be able to help them with solutions and give them guidance regarding preparation. However, the prevalence of Group C questions in these examinations, and the limited number of questions requiring routine uses of procedures means that such preparation might have a limited impact compared to engaging in similar preparation for A-level examinations, which vary very little year-to-year and consist primarily

of questions which students will have had the opportunity to practice many similar examples until their skills are fine-honed.

Undergraduate mathematics examinations continue this trend in focusing on students' responses to Group C questions, although the proportion of marks awarded here is lower (54%) than in the OxMAT. This is possibly due to the fact that a greater number of marks are awarded for responses to Group A questions (32%). These questions either take the form of statements of definitions and theorems or of proofs of statements which students will have seen in their lecture notes. In such instances, for the purpose of this analysis, the students have been assumed to have answered those questions as a consequence of memorisation (FKFS) than through using proof techniques to do it themselves. This leaves a fairly large proportion of the marks available for reproduction of knowledge – something which could be achieved by anyone who took the time to memorise it, not necessarily an undergraduate mathematician. In all of the examinations analysed, being able to answer these questions alone would be sufficient to earn a candidate at least a pass (30% or higher), with most examinations analysed having the potential to reward students with only Group A skills with a third-class mark (40% or higher). In two cases, both in Pure Mathematics II, a candidate only demonstrating Group A skills could have earned a first-class mark (70% or higher).

In advanced mathematics, knowledge of precise definitions and theorems is key, hence their being tested in undergraduate examinations; however, it seems that there are instances when too great a proportion of questions asked are from Group A and not enough in Group C. The make-up of Group A questions is very different at A-level and undergraduate level (see Figure 6.13); most Group A questions in A-level Mathematics are 'routine use of procedures', whereas most Group A questions at undergraduate level are 'factual knowledge and fact systems'. At undergraduate level, there is little opportunity for students to demonstrate routine use of procedures, as the mathematics being studied is very abstract and any

computational processes which could be assessed do not form a large part of what is studied. However, this is more likely to take place in algebra than analysis as algebra directly lends itself to applied processes more so than analysis.

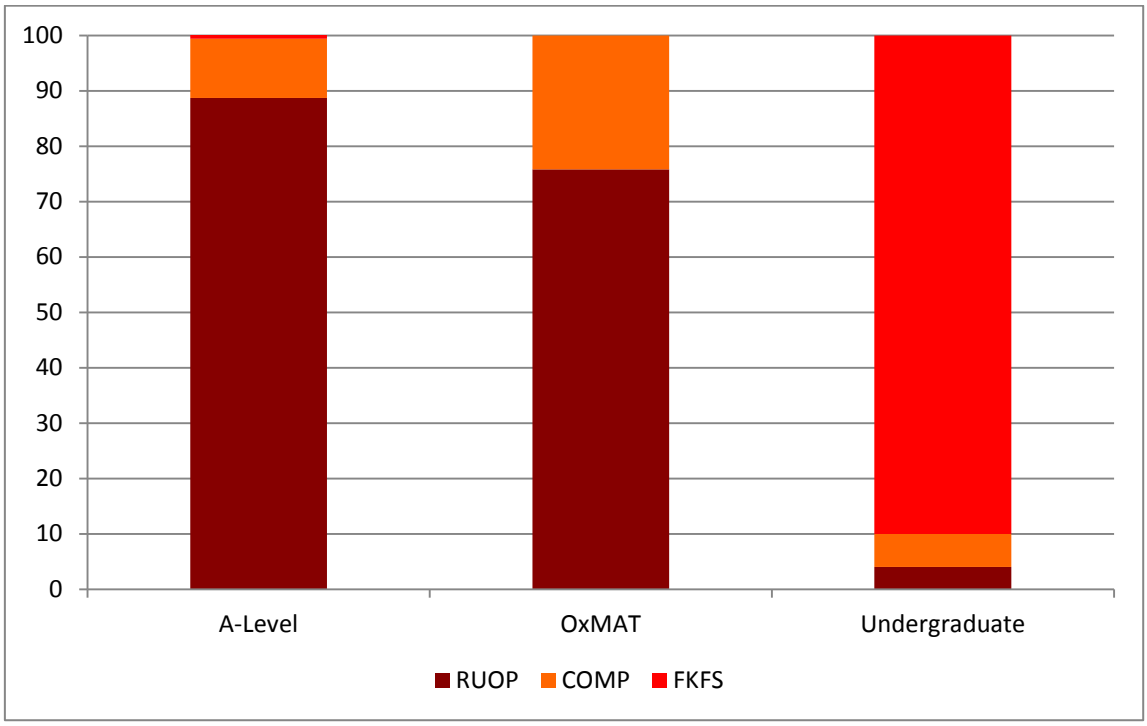


Figure 6.13 - Differences in the composition of Group A questions across A-levels, OxMATs & undergraduate examinations

The nature of advanced mathematics is reflected in the higher concentration of Group C questions at the undergraduate level compared to A-level. At the tertiary level, mathematics is formal, rigorous and deductive, unlike secondary mathematics which inculcates "a purely knee-jerk response in students" (Bibby, 1991, p. 43). Furthermore, the type of mathematics studied at school and university "changes from "What is the result?" to "Is it true that...?"" (Dreyfus, 1999, p. 106).

An obvious difference in the types of questions are asked between each of these three stages of assessment comes from the grouping of questions within each of Groups A, B and C. For example, at A-level and in the OxMAT, the majority of Group A questions are 'routine use of procedures', whereas at undergraduate level these are 'factual knowledge and fact systems'. Unlike at A-level, where RUOP questions form the majority of those asked in the examination

in general, this is not the case at undergraduate level, where Group C questions tend to constitute the majority of the marks on offer. This difference in the make-up of the Group A questions stems from the fact that A-level Mathematics appears to be very procedural, and that undergraduate mathematics is not, but the questions which require Group A skills tend to just be those which involve factual recall of definitions or, to a lesser extent, the construction of proofs which could be memorised from lecture notes. The 'definitions' questions give students who do not believe themselves capable of answering other questions which require application of these definitions the opportunity to gain some marks, whilst the questions which require students to reproduce proofs do not necessarily require students to memorise them parrot fashion without understanding what they are learning.

6.6 – Conclusion

The difference between secondary and tertiary mathematics in terms of the types of questions that are posed appears to be great (see Figure 6.14), notwithstanding the difficulty of the questions themselves and the difficulty of the material being examined. The increasing necessity to have students demonstrate that they can answer Group C questions is indicative of the changing nature of mathematics between these two points, and the heavy use of Group C questions in the OxMAT may highlight the importance of these skills in being a successful undergraduate mathematician.

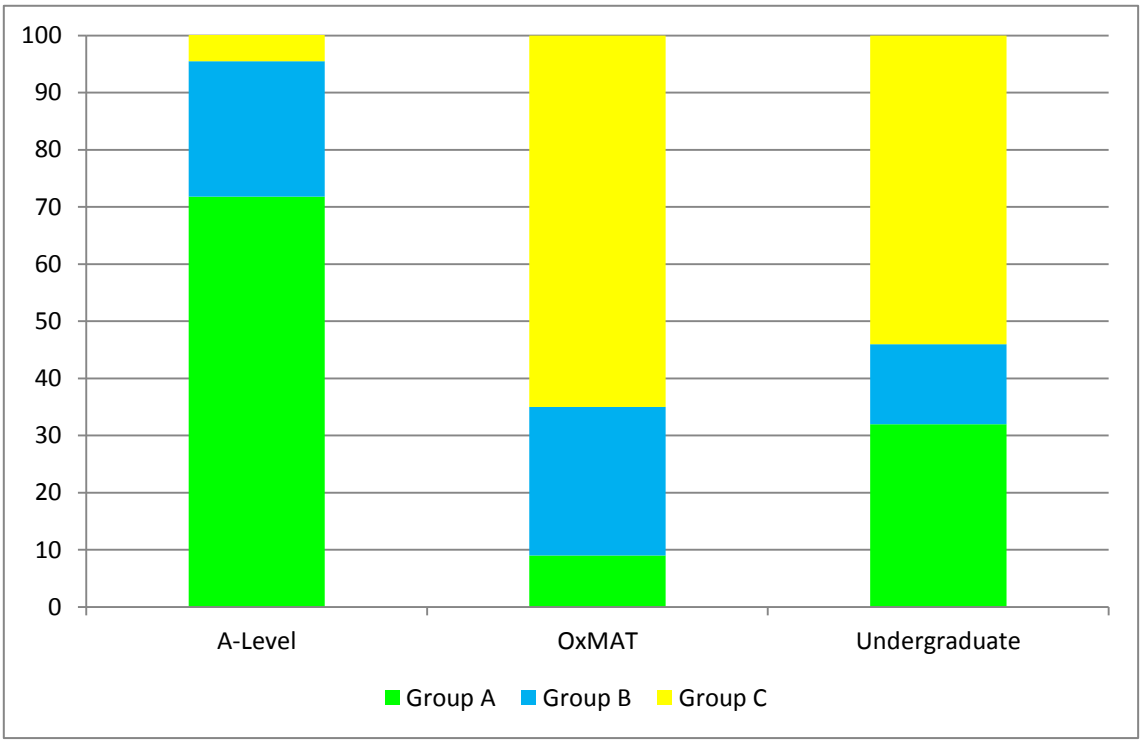


Figure 6.14 - Proportion of Group A, B & C questions in A-levels, OxMATs & undergraduate examinations

The existence of such a gulf in the proportion of Group C questions asked at school and university is clearly something which should be addressed, and bridged, at A-level so as to prepare students for future undergraduate mathematics study. Whilst the A-level is not intended purely for these pupils, it nonetheless has a responsibility in terms of preparing them for tertiary study in mathematics. Further Mathematics has a greater responsibility in terms of exposing pupils to advanced mathematics; however, the analysis here suggests that it does not

provide pupils with a broader conception of mathematics than the Mathematics course (see also Chapter 7.3). It is important that A-level pupils are exposed to the type of mathematics and mathematics questions which can give them an understanding of how secondary and tertiary mathematics might differ, as well as having them develop sufficient technical skills in calculus, algebra and trigonometry which they require as the basis for undergraduate study. This has become a matter of concern at a number of UK universities, as increasing numbers are conducting diagnostic tests on new undergraduates in these topics in order to ensure that they are sufficiently proficient in their use of basic concepts (Edwards, 1996; MathsTEAM, 2003; Williams *et al.*, 2010). The nature of the OxMAT shows what skills prospective mathematics undergraduates should possess if they are to be successful – and, indeed, gain entry to the University – and so could be useful for examiners of secondary mathematics. Furthermore, the difference in the challenges presented by A-level Mathematics and the OxMAT support suggestions that the OxMAT is used by the University because "the top universities no longer trust standards of A-level [Mathematics] as a reliable indicator of a pupil's abilities" (Kounine *et al.*, 2008, p. 21).

Whilst it is not the sole purpose of A-level Mathematics and Further Mathematics to prepare students for university study, it is important that they serve to develop students' problem-solving abilities as well as their ability to repeat a procedure, something which appears to currently be its main focus. For students to be successful in other subjects at undergraduate level, and in the workplace in general, it is necessary that they are able to solve problems, to interpret information that they are given and choose methods for a purpose, to justify why things may be true, to deduce the implications of results and to make judgements about the value of material that they are given for a purpose. All of these skills are Group C skills, and very few of them are tested in A-level students, most likely to their detriment.

Chapter 7: Data

Student Reports of Mathematics Study at the University of Oxford

7.1 – Students' Stories

Thirteen current undergraduate mathematicians were interviewed as part of the study. They comprised of a mixture of year groups, courses, genders and ATLs.

Table 7.1 - Participants in the student interviews

Name	Gender	Year	Degree	ATL
Ethan	M	1	Mathematics & Computer Science	Deep
Jacob	M	1	Mathematics	Strategic
Priya	F	1	Mathematics & Statistics	Deep
Ryan	M	1	Mathematics	Deep
Brian	M	2	Mathematics	Surface
Juliette	F	2	Mathematics	Strategic
Sabrina	F	2	Mathematics	Deep
Camilla	F	3	Mathematics & Philosophy	Strategic
Isaac	M	3	Mathematics	Strategic
Katie	F	3	Mathematics	Deep
Mandy	F	3	Mathematics	Strategic
Christina	F	4	Mathematics & Philosophy	Deep
Malcolm	M	4	Mathematics	Strategic

In order to get a sense and overview of the comments made by the students interviewed, the stories of five undergraduates will be told within the analysis:

- Mandy Section 7.3.1
- Brian Section 7.3.2.3
- Camilla Section 7.4.1.1
- Christina Section 7.5.2
- Juliette Section 7.5.3

Out of the thirteen students who were interviewed, I have chosen to tell their stories because, between them, they describe a broad range of the different possible experiences. All of the interviews were interesting, although some of the students whose stories are not being told here were less articulate or offered nothing additional to what is expressed by these five. That is, the descriptions they gave of their experience were generic for the group of thirteen and telling multiple similar stories would not be worthwhile. Ryan, Ethan and Priya had studied for qualifications other than A-levels[32] (IB, Singapore A-level, CIE A-level, respectively) so their experiences prior to attending Oxford will have been different. For these students, only their experiences transitioning between years through Oxford is relevant to the whole study but entry experiences had unique aspects which will not be included. Finally, first-year students were excluded from this storytelling because they had not been fully through the initial process of enculturation. The eight students interviewed, analysed, but not reported in detail each contributed to the overall 'generic' story, but individually would only overlap with parts of the stories represented here.

As ATLs are central to the initial conception of the thesis, it was important to ensure that at least one story was told for each of the deep, surface and strategic approaches. This meant that Brian's story had to be told – that of someone with a predominantly-surface ATL. Brian had a positive experience of secondary mathematics, yet a negative experience of undergraduate mathematics and felt 'cheated' by A-level because he felt it gave him a false impression of what undergraduate mathematics might involve, and his questionnaire revealed a surface ATL. His experiences are similar to those of Juliette, but her ASSIST suggested she had a strategic ATL. It was therefore important to tell Juliette's story because this could highlight that people with similar attitudes to the subject can have different ATLs. It could be argued

[32] Such pre-university qualifications form such a small minority of the qualifications offered for entry to the University that descriptions of these students' experiences of them were excluded from analysis because it is not the intention of this study to make comparisons between different upper-secondary mathematics qualifications.

that Christina had quite the opposite experience to both Brian and Juliette in that she was heavily involved in the mathematics community at Oxford by being active in the Invariants – the student mathematics society at Oxford. Christina had a deep ATL and was planning on going on to study mathematics at doctoral level, therefore it is important to tell her story as it is quite the opposite to Brian and Juliette's. Having one each of students with deep, strategic and surface ATLs, it is also important to include Mandy because she described a method of working which was different to the other students. Mandy created mind maps, and considered her university and school revision to not be all that different. Moreover, it was important to include someone who was in their third year as this is when tutorials end and classes begin, so the experience of having an extra level of support removed is something which is worth exploring in her story. Furthermore, as a Mathematics and Philosophy student, Camilla spoke of how she found the mathematics side of her degree to be too much of an exercise in memorisation and not particularly interesting, justifying her decision to only study philosophy modules in her fourth year. The opportunity for her to choose to continue or stop studying mathematics at that level, and the decision she made, are interesting and make a story worth telling.

It is the 'worth' of the stories that were considered when making the selection of these five students. The five students are not being presented as a representative sample of the thirteen interviewees; however, they are representative of what they said. That is, though three of the five stories told are about women, this was not a deliberate intention, as women form a minority of the undergraduate mathematicians at Oxford (and half of the number of students interviewed for this thesis). The value added by these women's stories comes from the fact that, together with Brian's, they are representative of the comments made by the general group of students interviewed, though they might not be a representative sample of undergraduate mathematicians at Oxford. Solomon (2007a) claims that her previous research suggest that women "find it difficult to remain positive in the university learning

environment", whilst men "tell more positive stories" (p. 3). However, her implication that women have more negative experiences does not hold true for those women interviewed as part of this study, who do not fit those stereotypes. There is a mixture of positive and negative experiences of both the men and the women who took part, without one gender tending towards either the positive or negative. Therefore, the reader should not be concerned by the fact that detailed stories are told about more women than men.

Stories are told within the text of the analysis when they become relevant. More detailed stories for Brian, Camilla, Christina, Juliette and Mandy are given in Appendices 7.1-5, and the transcript for Malcolm is given in Appendix 7.6 as an example.

7.2 – Overview

The analysis of the students' interviews will be told using Saxe's Four Parameter Model as a framework (see Chapter 4.4.5). That is, the students' reports of mathematics study at school and university will be told with reference to the four parameters, with an additional parameter in the form of the affective domain – 'perceptions of self and others'. The parameters are:

1. **Prior understandings**: This concerns students' prior understandings of what it is to learn, know, study and be assessed on mathematics prior to becoming enculturated into the undergraduate mathematics community. This includes:
    a. students' school experiences of mathematics, learning mathematics and being assessed on their knowledge and understanding of mathematics;
    b. students' experiences of mathematics assessment during admissions tests and interviews at Oxford; and
    c. students' experiences of mathematics, its learning and its assessment at the secondary-tertiary interface.
2. **Conventions and artefacts**: The conventions in a culture are the accepted ways of doing things in that culture, such as accepted ways of learning, working and interacting, which may or may not be influenced by any of the four parameters. Artefacts in a culture are the materials which students are provided with in order to facilitate their learning, such as documents associated with assessment, pedagogy and pastoral care.
3. **Social interactions**: This concerns the way in which students socialise, work and communicate with other members of the undergraduate mathematics community of practice, in particular in relation to their learning and working. Social interactions with non-mathematicians are also of importance.

4. **Activity structures**: This concerns what students *do* in their culture, and how they participate in it. Specifically, this involves the ways in which they learn and work, as well as participate in the undergraduate mathematics community of practice.
5. **Perceptions of self and others**: This concerns students' perceptions of themselves as undergraduate mathematicians, which may be related to the other four parameters, as well as their perceptions of other students in terms of their learning, understanding and working.

Each of the five parameters is related to the other four, as will be described and discussed throughout the analysis.

Any names of schools, colleges or specific details which could be used to identify the students have been either omitted or pseudonyms utilised.

7.3 – Prior Understandings

Students' prior understandings of mathematics, based on previous experiences, are key in their experiences of new mathematics and the learning of new mathematics. It is these experiences which shape their conceptions of what it is to learn and do mathematics, as well as their conceptions of what mathematics is and what professional mathematicians do. For the majority of applicants to the University, their prior understandings are shaped by A-level Mathematics study[33], its modules, its assessment and its content. Their perceptions of their personal understanding of, and their confidence in their ability to do, mathematics will be shaped by their performance in A-level examinations which, for all applicants, will be very impressive owing to the high entry requirements set forth by Oxford. Their prior experiences of the subject will play a role in their decision to apply to study mathematics at undergraduate level. Of course, some students' prior understandings may also be shaped whilst at school through factors extraneous to the A-level, such as their teacher and any extra-curricular or additional mathematics that they study; however, it is the A-level which is the most common and most dominant feature of the new undergraduates' experiences of the subject leading up to their first day at university.

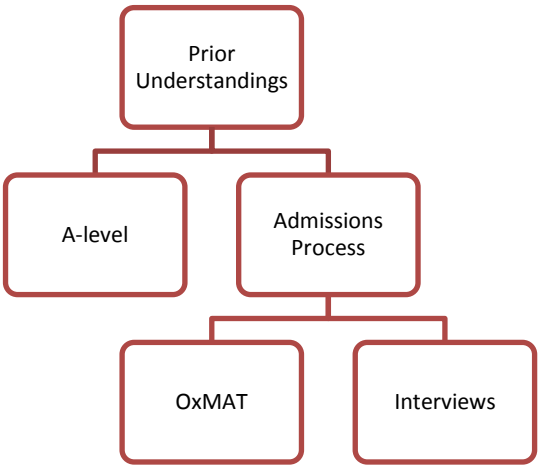


Figure 7.3 – Types of, and bases for, prior understandings of undergraduate mathematicians at Oxford

The majority of UK universities do not interview applicants for mathematics degrees for a place, which means that school experiences will form the vast majority of these students' prior understandings of the subject. However, the Oxford admissions process actually adds a new dimension to the students' experience and may serve to further shape their perceptions and

[33] 74% of those offered places to begin study in 2011 applied with A-levels (University of Oxford, 2012).

understandings of the subject. That is, the experiences that they have during the interview and the admissions test have the potential to alter: (1) their impression of what it would be to learn and study undergraduate mathematics; (2) their impression of their own understanding of mathematics; and (3) their impression of what mathematics is at this higher level. For this reason, the prior understandings of undergraduate mathematicians at the University of Oxford were found to be mainly formed based on:

1. school experience, specifically A-level Mathematics and Further Mathematics content, study and assessment;
2. the admissions process, specifically the OxMAT and Oxford interview; and
3. the transition into undergraduate mathematics that they experience in the first year of their degree (see Figure 7.3).

7.3.1 – Prior Understandings Fostered by School Study

Of the thirteen students interviewed, ten had standard UK A-levels; however, Ryan studied for the International Baccalaureate, Ethan for the Singaporean A-level and Priya for CIE[34] A-level. All but Priya and Ethan did Further Mathematics to A-level, although this was because Further Mathematics is not offered as a subject in Singapore or for the CIE. The most common other A-level studied was Physics; three-quarters of the A-level students also do this subject, which was in keeping with previous findings (Darlington, 2009). One-third of students cited its complementarity to mathematics as being the primary reason for choosing it. Indeed, Sabrina believed that her transition into undergraduate mathematics studies would have been easier if she had done A-level Physics. Five students studied at least one language at A-level, and six students studied Chemistry at least until AS-level. The most common reasons cited by students for their A-level choices were that they enjoyed it and/or were good at it, as evidenced by their earning a place at Oxford. Indeed, these were the primary reasons that the students cited for

[34] These are Cambridge International Examinations A-levels, an international qualification available to students throughout the world, which is accepted by UK universities.

studying mathematics to A-level, and Sabrina also said that she did A-level Mathematics because she "liked getting things right or wrong, and getting marked more for knowing and understanding, than necessarily just expressing yourself well"[35].

The participants studied a mixture of modules in their A-level Mathematics and Further Mathematics, and one-third of them were given an element of choice in their non-compulsory modules. Neither mechanics nor statistics appeared to be more popular than the other, although only about half of the students studied a decision mathematics module. Modules in decision mathematics were criticised by the students, with many of those who did not do this module claiming that it was because their teachers believed it to have little value. Juliette said that her teachers "hated decision maths" and Sabrina's said that it was "too easy and a waste of time". However, Ward-Penny *et al.* (2013) argue that this might be because few "teachers will have encountered decision mathematics during their own school education", making them less "comfortable teaching decision mathematics modules" (p. 3). Indeed, Camilla described it as being "a doss", with Brian reflecting that, whilst it "was a massive waste of time", he did not "complain about it [at the time] because it was so freaking easy". Indeed, a review by the QCA (2007) found that a combination of Statistics 1 and Decision Mathematics 1 were perceived as the easiest by teachers of A-level Mathematics and experts in this field.

As well as having similar prior understandings of mathematics in the sense that they studied similar combinations of A-level Mathematics and Further Mathematics modules, the students interviewed were also similar in terms of the revision techniques that they adopted for the subject at this level. All of the students used past papers as their primary means of revising for their examinations, although the number attempted by the students ranged from none to all of them, with Christina commenting that she "probably did all of the past papers more than once", and Jacob doing as "as many papers as I could bear to do". These students were not

[35] Line numbers for quotes embedded in the main body of the text will not be given so as to facilitate ease of reading.

alone in their claims, as reliance on past papers for revision is something which Daly *et al*. (2012) found to be commonplace across the board in A-level Mathematics. Furthermore, students are engaged in this culture of doing past papers to prepare for assessment even at Key Stage 2 (Reay & William, 1999). Two students, Juliette and Malcolm, described their mathematics revision as being minimal, with many students saying that they spent far less time on their mathematics revision than that for other A-level subjects. Juliette "didn't really bother with revising for some because they were so straightforward", something which did not transfer to her university revision methods. Revision at the school level, for most of the students, appeared to take the form of 'practice as preparation', with most students commenting that they understood the material that they studied as they went along, and so did not need to refer back to their notes to consolidate their understanding and knowledge, and *learn* anything, as part of their revision:

> *I just sort of knew everything and knew I could do it from doing questions in class so there wasn't much to be gained from doing any proper revision.*
>
> (Camilla)

> *At school I'd pretty much absorbed all the material as it was taught and never really had to think about it outside of the classroom.*
>
> (Jacob)

> *You get your head around that stuff in five seconds!*
>
> (Isaac)

> *It's almost embarrassing how little I did […] It sounds really cocky, but I basically didn't need to revise properly because I already knew it and had picked it up as I'd gone along*
>
> (Malcolm)

The modular system of the A-level, similar to that of undergraduate mathematics in the sense that different topics were taught and examined separately, was something which the

interviewees all appreciated as it meant that they "could concentrate on one lot of stuff for each exam sitting rather than learning everything later on. It took the pressure off" (Brian). Amongst academics, views on the modular system vary "from "the best thing since sliced bread" to "the work of the devil"" (Hirst & Meacock, 1999, p. 122), although the views students in this study very much aligned with the former. Some participants recognised the utility of the provision of resits at A-level for those "who have an exam they do badly on when they normally do well" (Sabrina), although a similar number thought that the system was too open to students taking advantage and using it unfairly. Juliette claimed that it made "a bit of a mockery of the exam" when pupils resat modules for reasons other than extenuating circumstances, and Malcolm said that "the grade you have at the end is kind of fake if you do that". Furthermore, Katie expressed a belief that resits should not be permitted for pupils who achieve an A-grade the first time around, particularly in mathematics because of its lower content relative to some other A-level subjects. Only one of the students interviewed resat a mathematics module, and two students had at least one paper re-marked on account of the fact that their results seemed far too low[36].

As well as the re-marking of A-levels attracting controversy, the content of A-level Mathematics as preparation for undergraduate study has been widely criticised by teachers and educational researchers, alike. A multitude of studies have claimed that it is poor university preparation (e.g. Lawson, 1997; Porkess, 2006; Savage, 2003; Smith, 2004), and that it fails to develop students' mathematical thinking (Higton *et al.*, 2012). Students are "unable to see the bigger picture" (Quinney, 2008, p. 5) and the A-level allows "candidates less scope for using their own mind" (Bassett *et al.*, 2009, p. 10). Studies which seek to understand the students' views on the appropriateness of A-level Mathematics and Further Mathematics as a basis for university study in the subject have not been conducted before as far as I have been able to ascertain; however, this was a topic broached with the students interviewed for this

[36] All of the remarks were successful in increasing their grade, sometimes substantially.

study, as prior understandings of mathematics could play a key role in their enculturation into the undergraduate mathematics environment. This relates to both the mathematics that they studied, as well as the conceptions of mathematics that they developed as a consequence. All of the participants believed that A-level Mathematics and Further Mathematics were insufficient preparation for undergraduate mathematics study, although recognised the difficulties associated with ensuring it meets *their* needs as future mathematics undergraduates as well as the needs of *others* who study the subject for different reasons and purposes. For example, their needs will be very much different to that of a student who goes on to study undergraduate physics, as well as the student who studies the subject at A-level but does not take it any further. The criticisms of the A-level given by the students fell into three strands, namely that:

1. it was not sufficiently challenging, which meant that they struggled to adapt to this becoming the case at university;
2. it does not give students "a true sense of what maths actually is at a higher level" (Katie), causing a conceptual shock in the first year; and
3. it does not teach students mathematical problem-solving skills nor develop their mathematical thinking, which makes answering questions at the undergraduate level more difficult.

The students described this as limiting their perceptions of mathematics, as well as their ability to be successful in the transition to university study. The overly-routine nature of A-level Mathematics questions (Berry *et al*., 1999; Etchells & Monaghan, 1994) means that students experience a difficult transition, "unable to use their mathematics knowledge outside the narrow confines of textbook exercises and short examination questions" (Osmon, 2011, pp. 125-126).

All of the participants suggested ways in which the A-level could be improved in order to make it more suitable for their purposes, mainly pertaining to making it more difficult and having it contain advanced mathematical concepts more in-line with those that they would later study at university. Most students proposed that more abstract mathematics and proof be covered at A-level as "the gap between what we did then and what we did in Michaelmas [term of the first year] was really big in some cases" (Jacob). Indeed, a significant decreasing emphasis on proof has been identified throughout the years (Bassett *et al.*, 2009); however, it remains to see whether the new A-level curriculum reforms will do anything to counter that. As all of the participants studied Further Mathematics, there was also recognition that "Further Maths wasn't really 'further' enough to make it useful for people doing maths degrees" (Brian), something which they believed should be par for the course in the second A-level. A number of students also suggested that there was not a great deal of content in A-level Mathematics and Further Mathematics, which is supported by Bassett *et al*. (2009) and Porkess (2003). Furthermore, a large number of the participants believed that it was important that the topics covered at A-level be more closely-related to analysis and proof because "It would change people's perspectives of maths" (Brian), which impacted upon many of the students' experiences in the transition as they struggled to adjust to the mathematics that they were studying being quite different to what they had expected. Indeed, Juliette questioned "How else are we to know what's coming up?" if A-level appears to teach and examine nothing of this kind. Some of the participants felt that A-level was insufficiently challenging which meant adapting to an environment where they did not find mathematics to be easy a difficult adjustment to make:

> *I do not believe that it was challenging and so this makes coming to university and being challenged even more difficult because it is not a familiar experience […] For many students, I believe that university is the first time that they have found mathematics difficult*
>
> (Ethan)

Unlike most Russell Group universities (see Appendix 2.2), the University of Oxford's typical offer involves an A-level in Further Mathematics. In line with this, all of the UK A-level participants had A-level Further Mathematics. Whilst a legitimate case could be made for the fact that A-level Mathematics might not be intended as preparation for undergraduate mathematics study, and as such it might not serve students as well as it could, these students have all studied the double A-level. This means that their comments about the suitability of the A-level for their purposes are based on a wider experience of the subject at this level than the majority of their peers. Approximately one-seventh of A-level Mathematics students also do Further Mathematics (Smith, 2012), so the typical Oxford mathematics student very much came from a minority at A-level.

All thirteen students were successful at the secondary level, with a number of them commenting that they regularly achieved full marks in examinations when they were at school. This is a great contrast to their current experience, with Juliette describing the prospect of getting 100% in a problem sheet as "unfathomable". This was something which Sabrina reported as being "demoralising at first", and was a difficult aspect of her transition into undergraduate study, because it was so unfamiliar. A number of the students described getting stuck as a rare occurrence in their experience of A-level Mathematics, and something which was often a result of a mistake, rather than a failure to comprehend a new concept.

The differing nature of secondary and tertiary mathematics was something which all of the participants recognised, with most of the students describing A-level Mathematics and Further Mathematics as being primarily based on a rehearsal of familiar procedures during lessons, followed by examinations which asked them to answer similar, direct questions to those which they had practised. Conversely, "in undergraduate mathematics you will sometimes be asked to apply a procedure without having seen *any worked examples at all*" (Alcock, 2013, p. 8). Furthermore, "all the past papers were similar" (Jacob, see also Appendices 2.3 and 2.4), which

contributed towards their finding mathematics examinations at this level reasonably easy. The students then described undergraduate study as something more abstract and more centred around proof, as did the students interviewed by Furinghetti *et al.* (2011). The Oxford students also described tertiary mathematics as requiring a different way of thinking:

> *I think probably the most obvious difference is the rigour and focus on proofs. At school we did some simple derivations and […] a bit of mathematical induction, but I had never attempted a solution to something along the lines of a* reductio ad absurdum *argument before I came to university.*
>
> (Ryan)

This is compounded by the fact that these proofs are of specific mathematical objects, as opposed to the categories of objects, as they would have to at university. At school, "students are asked to prove by induction *this* formula gives the sum of the first n terms of *this* series or prove that *this* trigonometric identity is equivalent to *that* one" (Alcock & Simpson, 2002, p. 28). Indeed, this had an impact on the students' expectations of what undergraduate study would be and meant that many of them were introduced to topics that they were not expecting. Specifically, analysis came under fire from all of the students as being unexpected. The majority of the participants' expectations were not in line with what they met when they came to Oxford. Most of them did not research what their degree would entail, the only research for those who did it coming from browsing the Mathematical Institute website

Mandy's Story

Now in her third year, studying without having support from the tutorial system, Mandy balances many extra-curricular activities with her studies, preferring to have a lot to do because it forces her to manage her time efficiently. Despite not being overly-confident in her abilities, she earnt a first-class result in her first year and estimates that she spends 40 hours/week on her studies. Her revision practices differ to the other participants – and, she believes, the rest of her peers in general – in that she constructs mind maps as an active means of ensuring that she can make connections between mathematical concepts.

In contrast to the other participants, she heavily researched what undergraduate mathematics Oxford might be like, on advice of her school mathematics teacher. Having done some research, she found "the prospect of doing this kind of stuff much more appealing than […] carrying on with the same level and type of stuff as at school". Mandy was expecting her degree to be "very logical and to involve lots of proofs" unlike, she believes, many of her peers who "weren't expecting this level of proof and abstractness".

before they came. The only student who did something other than this was Mandy, whose teacher recommended that she researched online and in books in order to see what was in store. This, he believed, would see her in good stead when it came to her Oxford interview, as well as preparing her for what was to come. Fortunately, she "found the prospect of doing this kind of stuff much more appealing than […] carrying on with the same level and type of stuff as school". Conversely, a quarter of the participants believed that, had they known what was in store for them, they would likely not have applied to do mathematics. Malcolm conjectured that anyone who saw analysis lecture notes before coming to university "might run away screaming" and, discouragingly, Juliette said that if she had been more informed, she "might not be sitting here now". However, at the time, the students did not feel the need to do so ahead of coming to Oxford:

> *I didn't do anything else, really, because it's just maths, isn't it?*
>
> (Christina)

> *It just seemed a bit pointless at the time. At school, I did maths, I wanted to do more of it… ergo I applied to do it at uni. I only thought it's worth researching if it's a degree that you haven't done an A-level subject in. That way, you might wonder what you would study. Or if you did history or something like that, you might have a particular interest so you'd do a bit of research to see if that was covered in the particular uni. Maths is the same everywhere. Maths is maths.*
>
> (Juliette)

> *I didn't really see the point because I knew I wanted to do it and we didn't have to do any work before we started.*
>
> (Brian)

On the contrary, the necessity of researching and preparing for an undergraduate degree (and the lack of preparedness of new undergraduates) is highlighted by the recent publication of Lara Alcock's (2013) book, 'How to Study for a Mathematics Degree'. Aimed at students themselves, the book covers a wide variety of topics, aiming to introduce students to the

nature of undergraduate mathematics, with one chapter entitled 'Mathematics is Not Just Procedures'. Interestingly, whilst the lack of research led to these students feeling ill-prepared for what was ahead and many experiencing a "shock to the system" – a phrase used by six participants – when they came to study analysis, in particular, this did not necessarily have a negative impact in everyone's experiences of the subject. Camilla said that she "thought all of analysis was really cool" and was excited about it; however, this was not a sentiment shared by most of the other interviewees. Camilla remarked that "There's just nothing to prepare you for it at school", and Juliette explained that this meant that she and her peers "weren't really to know that that sort of maths existed". Consequently, the majority of participants described themselves as having found studying analysis both difficult and even so much as traumatising:

> *Analysis was really awful to begin with*
>
> (Katie)

> *The whole analysis thing was just awful and I didn't know what was going on at all. Really awful.*
>
> (Brian)

> *[Analysis] frightened me*
>
> (Camilla)

More generally, some of the students were not expecting there to be such an emphasis on proofs at undergraduate level. As we have seen, their prior experiences of mathematics were of a computation-driven subject, meaning that many of them had not anticipated the dominant position that proof takes, which led Juliette to assert that "I don't think it was unreasonable for me to think that [computational mathematics] would be the primary activity at university". Having this new mathematics to 'adjust' to was an additional challenge for the participants when they arrived at Oxford, as well as the mathematics being studied being generally more difficult and abstract than what they had encountered before. Ethan, who *was*

expecting an emphasis on proofs at university level, said that "mathematics is now difficult primarily because it is so different to anything which I have encountered before".

Those students who were met by a mathematics which did not fit their expectations reported that they were, for the most part, expecting undergraduate mathematics to be "a natural continuation of the maths that we'd done at school" (Katie). This is an instance of their beliefs about mathematics 'overhanging' (Daskalogianni & Simpson, 2001), wherein students take their conceptions of mathematics and doing mathematics from secondary school into the tertiary setting, causing them difficulties in the secondary-tertiary mathematics transition. The descriptions that the participants gave of the differences between secondary and tertiary mathematics were very often extremely perceptive, drawing upon factors associated with the types of questioning they experienced, as well as the nature of the mathematics that they were studying. Furthermore, some of their descriptions of school-level mathematics were often littered with negativity, either in tone or in description, inferring that secondary mathematics is inferior to tertiary mathematics:

> *At school you can become a mindless drone which just processes things and bashes out an answer after practising it hundreds of times, whereas at university you have to be very precise and link things together and be able to understand a lot of concepts which are often really abstract and not necessarily straightforward.*
>
> (Katie)

> *The concern isn't about using numbers and doing calculations any more… Instead, it's about the abstract theory behind everything and why things work. And as you go more and more into it, things become more and more abstract and detached from things that you think are actually useful to a normal person. You spend a lot of time learning about why things are true, whereas at school you didn't really care about that.*
>
> (Isaac)

> *When I was a school student, I believed that the mathematical thinking that I required was to be able to answer the questions that the teacher gave us, which I would be good at through practising […] However, now I am a university student I*

> *believe mathematical thinking to be something much more than this […] Mathematical thinking means you must have a cohesive understanding of the mathematics that you know so that you can use it. You must necessarily understand the roots of all that you know if you are to be successful, whereas school understanding only needed to be the understanding that you gain from doing a lot of practice of simple calculations.*
>
> (Ethan)

It appears that the majority of the participants were only influenced in their perceptions of what mathematics was by their school studies, with the exception of a few. This then meant that later exposure to mathematics throughout the admissions process and initial stages of their degree served to cause them to revisit and readjust their conceptions and expectations of what they would study. All of the participants described analysis as being one of the most difficult aspects of the transition between secondary and tertiary mathematics, the vast majority citing it as *the* most difficult aspect. Students have been found to struggle considerably with analysis (Selden *et al.*, 1994), and it has been suggested that students' difficulties and negative experiences with analysis can cause negative attitudes about advanced mathematics (Alibert & Thomas, 1991; Pinto & Tall, 1999). Indeed, the transition to undergraduate study marks the beginning of a mathematical thinking which "requires deductive and rigorous reasoning about mathematical notions that are not entirely accessible to us through our five senses" (Edwards *et al*., 2005, p. 17), which is a difficult shift for most students. The readjustment and adaptation in terms of their conceptions of what it was to do mathematics further served to be a source of challenge for them, compounded by the challenges associated with moving away from home, becoming an independent learner and studying something more difficult than they had studied before.

7.3.2 – Prior Understandings Fostered by the Admissions Process

7.3.2.1 – Choosing to Apply

The decision to apply to study undergraduate mathematics is the beginning of students' enculturation into the university mathematics environment, in that the way in which they qualify their decision will undoubtedly shape their expectations of the subject and their ability to be successful and enjoy it. The main influences cited by the students who were interviewed were their previous success and their enjoyment of the subject, with others citing an interest. Three students described a mathematics degree's potential to fuel a successful, lucrative career as being a driving factor, with the potential earning power of the degree being identified as the driving factor in many students' decisions (Montmarquette *et al.*, 2002). Two students described parental pressures to study such a subject at university. To study mathematics at Oxford, in particular, was a decision triggered by the fact that it is one of the best universities in the world, and that they should aim high, as well as teacher influence, which was the main motivator behind Isaac's decision to apply to Oxford, as his school was not high-performing at A-level:

> *I didn't know if I was good or they [his A-level Mathematics classmates] weren't*

He would soon find out whether or not he was an exceptional mathematics student after going through the Oxford admissions process.

7.3.2.2 – The Admissions Test

Past OxMAT papers are available on the Mathematical Institute's website dating back to 2007 (University of Oxford, 2013a). This is something which all but two of the students took advantage of when preparing for their admissions test. Neither Camilla nor Isaac – whose ASSISTs both suggested that they had a strategic ATL – did any past papers, both citing that the variation in the questions posed each year was such that "there wasn't a great deal to be had from doing them" (Isaac). Ethan only did one past paper for this reason; however, the rest of

the students interviewed all practised a number of papers. Whilst there might not have been any patterns in the past papers in terms of the questions posed, as all of the students suggested, four of the students did at least one of them in timed conditions in order to ascertain how they should pace themselves for their admissions test. Their previous experience of mathematics examinations had been of ninety-minute examinations for each A-level module, whereas the OxMAT is a three-hour test. Juliette believed that doing a paper to time was important "so you know whether you're going to be really pressed for time or if you can afford to sit and think for a little while to get through a question".

Juliette was fortunate enough to have been given special preparatory classes for the test at her school. This was something that her school offered because so many pupils were applying to study mathematics at Oxford and, she speculated, because they wanted to do all they could to help them to secure a place because it would reflect well on the school. In these classes, she would do past papers and her teachers would help her and the other pupils with questions that they could not do. Ryan was also able to talk to his teachers about any of the questions that he could not do. Whilst one might expect that this gave them an advantage over their arguably less well-prepared peers, this does not appear to be the case. For example, Camilla and Isaac were both successful in their application, yet reported that they did not do any significant preparation. However, the small sample here means that it would be inappropriate to make generalisations. Isaac and Camilla may well be exceptional in this sample, but their actions may not be in such a minority in the overall student population.

Mark schemes are available online for students to see how to answer any questions, and some of the participants used them to gauge how well they might be able to do in the test.

However, this variation in question type meant that there was the potential for the students to perform differently from one paper to the next:

> *Sometimes I was good at them, and sometimes I didn't get anywhere with it. Hit and miss.*
>
> (Katie)

This was why Brian "got very stressed out in the exam" when he found it to be much more difficult than the practice papers which he had done to prepare.

Whilst Camilla did not do any past papers, she did revise the first two core A-level Mathematics modules, which were stipulated as being the most advanced mathematics that would be required to do the test, as did Mandy. Isaac was the only student who did not do any preparation for the test, although one could argue that by looking at the past papers and identifying that there were no obvious types of question or topics that he was likely to be tested on acted as a means of preparation in itself. His usual revision practices at A-level involved completing lots of past papers, which he said was key in being able to do well in the examination, reinforcing the claims made by many of the students that the OxMAT was very different to their A-level Mathematics examinations.

Christina described the OxMAT as "a breath of fresh air", with three others describing it as enjoyable. Despite not preparing for the test, Isaac did enjoy it and found it interesting, as did some of the other participants. Camilla enjoyed "being challenged properly" by the test, unlike in her A-level Mathematics examinations. However, the difficulty that Brian had in answering the questions compared to the practice papers that he tried meant that it "scared the hell" out of him. Whilst most of the students described the OxMAT as being difficult – and none described it as easy – they gave varying descriptions of how well they thought that they had performed. These ranged from feeling that they had not done very well (Malcolm), to performing averagely (Brian, Christina, Mandy, Juliette, Katie) to feeling that they had done

well (Ethan, Jacob). For many of the students who described the test as being difficult and themselves as not having done particularly well, this was likely the first time that they were properly challenged by mathematical assessment in a long time, with many of them citing being good at mathematics as one of the reasons for them deciding to study mathematics at university. Brian did not answer all of the questions in the OxMAT completely to his satisfaction which was something alien to him, as he usually answered all of the questions in a mathematics examination, and only lost marks when he made a careless mistake. For him, this "wasn't easy to deal with".

All of the participants described the OxMAT as being significantly different to A-level, with the exception of Mandy who identified two similarities in spite of this – the use of language and the style of the questions, insofar as them being split into sub-parts was similar to A-level. The main differences between the two cited by the students interviewed were that:

- the OxMAT is more about problem solving;
- the OxMAT was more difficult; and
- the types of questions were asked were different in that students are required to apply their existing knowledge to a new problem and not told what mathematics to use and what to do.

Seven students alluded to the OxMAT requiring students to solve problems and puzzles using logic, with Camilla comparing them to the types of question asked in the UKMT challenges[37], although with more difficult mathematics. Three students reported that the mathematics required to answer the questions was not particularly difficult – indeed, students are told that only knowledge to C1 and C2 level would be required – but many of the students remarked

[37] "The United Kingdom Mathematics Trust individual mathematics challenges are lively, intriguing multiple choice questions, which are designed to stimulate interest in maths in a large number of pupils. The three levels cover the secondary school range 11-18 and together they attract over 600,000 entries from over 400 schools and colleges" (UKMT, 2013, online).

that the OxMAT overall was more difficult than A-level or IB. Priya likened its difficulty to STEP III, the most advanced STEP examination (see Chapter 2.3.3.6). The source of this difficulty appears to be the style of question and the requirement that, in order to answer them correctly, students must "think more deeply" (Ryan) so as to have "a deep understanding of what was going on in the problem and an idea of what to do to find the answer" (Malcolm), using "the techniques [that] you have in your arsenal" (Jacob) because it "wasn't explicit what technique you had to use" (Mandy). This was something which all of the participants commented on, with this being the key difference for most of them between A-level and the admissions test. This is particularly important because "at university, you will have responsibility for deciding which procedure to apply" (Alcock, 2013, p. 4). Many of the students made direct comparisons between A-level and the OxMAT, with Ethan offering a comparison which encapsulates the others made by the other participants:

> *OxMAT is more about using the maths you know, but A-level is about doing lots of examples to show you can do what you're asked to do.*

The students explained this difference to be the reason for the University administering the test in the first place. All of them gave explanations of its use in terms of the A-level failing to provide either sufficient assessment of students' mathematical abilities, to separate those students at the higher end of the ability range, or to give them an opportunity to see the students' thought processes because they cannot see the examination scripts. Katie expressed a belief that the admissions tutors would want to "see the different avenues that you tried to take", with Isaac commenting that being able to see students' attempts at answers themselves would mean that they could see "how a student communicates what they are thinking", something which is then further explored in the interviews. This means that the OxMAT is better able to "help them spread out people in the 30%" (Mandy). That is, the previous entry requirement of an A-grade at A-level (awarded for over 70%) left quite a range of possible

marks between the lowest- and highest-achieving students applying for places, whereas there was only a 10% difference between lower grades.

The distinction between those who are good at mathematics and those who are good at answering A-level Mathematics questions correctly was made between many students, and also used as a reason for the OxMAT being implemented. For example,

> *It's easy for a bright but not particularly mathematical person to do well at A-level through being able to repeat procedures, so I suppose this identifies people who can do mathematics without having the question guide you through all the steps*
>
> (Juliette)

> *It's pretty easy to practise all of the A-level computations and get good at those, whereas you need to have the right kind of mathematical mind to do the questions in the entrance exam*
>
> (Camilla)

> *It helps find the best people to study university maths rather than the people who are the best at studying school maths*
>
> (Ryan)

Isaac also speculated that the admissions tutors use the responses that students give in their OxMAT as a means of identifying what kinds of question they should ask in their interview, something which would be confirmed by Sabrina's interview experience, when she went through the answers to her OxMAT with the interviewer. One comment which stood out from the rest regarding the purpose of the OxMAT was one made by Mandy, who implied that success in the OxMAT was next to revising and practising past papers. She believed that those who are successful are those who "put the hours in revising and sitting the past paper questions before the OxMAT", and that Oxford is interested in accepting students who are prepared to put in effort to their studies. This is something which is reflected in her working practices (see Mandy's Story in Chapter 7.3.1), as her approach involved spending a large

amount of time studying, employing slightly different revision approaches to her peers in order to ensure that she understood the connections between all of the concepts, rather than launching into memorisation and past papers straight away, as she suggested her peers did. However, this is not supported by some of the participants' reports of how they prepared for the test.

Of course, the utility of the OxMAT is not solely for the admissions tutors, but the students themselves remarked that it had proven useful for them in terms of developing their mathematical understanding, problem solving skills and giving them an insight into university study. Three participants claimed that their doing the admissions test had helped them to think mathematically in a different way. Mandy said that it was good "for pushing my brain to think about different maths", and Jacob said that it caused him to "write mathematics more cogently" and that it gave him "a better understanding of the utility of the [A-level] techniques". For Brian, he later realised that it had prepared him for university study:

> *it was interesting and it was strange being in a situation where I wasn't just blasting out all of the answers in the maths exam without really having to think. Kind of prepared me for now, when I definitely can't do that anymore!*

What Brian is saying here is very interesting, as he is alluding to the fact that all of the students who come to Oxford will have gained top grades in their A-level Mathematics and Further Mathematics, and likely never really struggled to answer mathematics questions. This could be the first time that they were properly challenged, and that they did not answer all of the questions fully, which may have acted as a way of 'bringing them back down to earth', to potentially readjusting their expectations in the sense that they would no longer find mathematics to be easy, and should expect to be challenged by their degree.

A number of students likened the OxMAT to university study, with Ryan claiming that it was "more representative of the skills required for maths at university level". Priya felt that it had prepared her for what undergraduate mathematics might "look like", and Ethan said that it

gave him “an idea of what tutorials and mathematics at university entailed”. Furthermore, Brian thought that they were more like problem sheets than A-levels, teaching him that it is necessary to persevere in order to be successful in undergraduate mathematics assessment. Conversely, however, three students did not think that the OxMAT was useful in terms of preparing them for university study. Therefore, the students were divided in terms of what they took from the OxMAT and whether or not it affected their prior understanding of mathematics and mathematics study before they came to Oxford.

7.3.2.3 – The Oxford Interview

For any degree subject, the Oxford admissions interview is world-renowned as being very difficult, aimed at finding the brightest and the best. Indeed, the public’s perception of them as being particularly difficult and oftentimes unusual has meant that there are now a number of resources available to potential applicants. Indeed, Christina described her fears in going to interview at Oxford, having heard “so many horror stories about tough Oxford interviews”. Books such as Christopher See’s 2012 publication, ‘How to get into Oxbridge’, are available, and a number of organisations such as Oxbridge Interviews, Oxbridge Applications and the Oxford Royale Academy all offer interview preparation courses and help, sometimes charging over £1000 for their services. An essential part of the entry process, interviews often take place at multiple colleges for each student – sometimes multiple times at the same college – normally over the course of two days, when they will be resident in one of the colleges to which they have applied.

The Mathematical Institute is open with students about the nature of the interviews, with the undergraduate prospectus informing them that

> *If shortlisted for interview, then these will be predominantly academic. You may be asked to look at problems of a type that you have never seen before. Don't worry; we will help you! We want to see if you can respond to suggestions as to how to tackle new things, rather than find out simply what you have been taught. Ultimately, we are most interested in a candidate's potential to think imaginatively, deeply and in a structured manner about the patterns of mathematics.*
>
> (University of Oxford, 2013a, online)

Owing to its reputation and its nature, all but three of the students who were interviewed for this study engaged in mock interviews in one form or another, with these normally taking place either at their own school or, in the case of some students at state schools, at a local private school offering the service. Most of them found the interviews useful, with Brian asserting that it "filled me with confidence"[38], although some of them described it as having limited utility in that the questions asked in the mock were unlike those that they were asked in their actual interview. Indeed, Camilla claimed that this meant that her mock interview was only useful in the sense that it exposed her to a similarly stressful situation. Two students' mock interviews did not reflect the expected nature of an Oxford interview, in that they were not challenged mathematically but were instead engaged in discussion about their interests and background.

A number of the students prepared for their interview using additional means such as by doing STEP questions (Katie and Christina), Olympiad questions[39] (Brian), NRICH questions[40] (Jacob), A-level Mathematics revision (Priya), OxMAT questions (Mandy) or by reading books (Ryan).

[38] This was an interesting observation because Brian's mock interview success gave him confidence going in to his interviews. These went well, which filled him with confidence for his degree. At university, his struggles were quite surprising to him probably because he had, until then, not been challenged in such a way.

[39] International mathematics competitions attended by winners of national mathematics tests.

[40] Questions available online which test students' problem solving, mathematical reasoning and critical thinking.

For these students, the preparation was not about *learning* mathematics, but about the thinking processes required to be successful. Ethan described what he did as being a means of "freeing my thought process", and Katie's use of STEP questions meant that she could "use maths a bit differently", ready for a similar experience in the interview.

Between them, the thirteen students interviewed described four main types of interview that they were involved in, namely:

1. Strictly-mathematical interviews in which they engaged directly with the interviewer on one or more questions which were posed directly to them there and then;
2. Interviews which closely resembled tutorials in that the applicants were given a problem sheet to do the night before, which they would discuss with the interviewers the next day;
3. Interviews which involved a discussion about a mathematical topic that they were interested in, or a "philosophical chat about mathematics" (Malcolm); and
4. Interviews where the student and interviewer would discuss their answers to the entrance examination.

In the interviews, regardless of the type, students are exposed to an interaction which closely represents that of an Oxford tutorial. This is something which was recognised by most of the participants, with Camilla commenting that the similarity lay in the fact that "you're sat with the tutor and trying to work through the maths and talk about it in quite an intense setting". This is an example of the Oxford interview working in a way which assists the student, as well as the admissions tutors. Whilst the admissions tutors use the interviews as a means for identifying the students who they think would be the most suited and the most successful on the course, the Oxford interview style gives students the opportunity to experience an environment which closely represents that which they would be part of in the undergraduate mathematics community. This is their first exposure to the culture which they could be part of,

should they be successful. This insight is not something which can be gained from school study, or from an entrance examination. Furthermore, the residential nature of the interviews was something which many participants appreciated as they described it as giving them an insight into college life. For the majority of the students interviewed for this study, Oxford was the only university at which they were interviewed, with the exception of two students, whose interviews at Princeton University and the University of Bath neither tested their mathematical thinking nor gave them a sense of what mathematical study and interaction might involve.

All participants were interviewed at multiple colleges and, for the most part, described some interviews as being more successful and enjoyable than others. Interestingly, these were, invariably, the colleges at which they were later offered a place. These were the colleges where they had successfully convinced the admissions tutors that they would be successful in engaging with them and with mathematics. Admissions tutors have "room for discretion which means that admissions decisions [at Oxford] are not formulaic" (Zimdars, 2010, p. 319), and the custom of interviewing students at multiple colleges, coupled with the admissions test, goes a long way towards counteracting what have been found to be relatively low reliability and validity of admissions interviews (Kreiter *et al*., 2004)[41]. Many students described the interviews as being more enjoyable than they had expected, and Isaac described the questions as "actually quite cool". However, there were also those who had negative experiences during the interview process, although these were often followed by more positive experiences of interviews at other colleges. Juliette, in particular, was very shaken by the experience of a couple of her interviews, and cried after two of them because she felt that she had done very badly and had been so nervous.

[41] This study was conducted in the medical sciences. The vast majority of research papers on university admissions interviews are for medical schools; however, this might suggest why the Oxford admissions process is more robust than procedures at other UK universities which merely require students to achieve certain A-level grades.

Jacob and Isaac found the interviews “pretty scary”, although Jacob conceeded that hindsight had an impact on his perceptions:

> *I was terrified the whole way through but once I’d found out I’d got in, I looked on the experience fondly! Haha.*

The way in which the admissions tutors used prompts, hints and guidance was also something which caused Juliette distress, when she felt that “it became quite awkward as they started to try and dumb it down more and more” to no avail. However, the other students spoke of this assistance far more fondly, with many saying that they appreciated that the tutors were not expecting them to be able to reach an answer quickly without being given any help. Such circumstances are examples of a type of ‘collaboration’ with their tutor which is akin to that which they would have in undergraduate tutorials, where their learning and understanding can be fostered by the tutor’s use of guidance and questioning. This very much acts as a type of scaffolding, in the sense of Wood *et al.* (1976):

> *Well-executed scaffolding begins by luring the child into actions that produce recognisable-for-him solutions. Once that is achieved, the tutor can interpret discrepancies to the child. Finally, the tutor stands in a confirmatory role until the tutee is checked out to fly on his own*
>
> (p. 96)

It is perhaps also an opportunity for the tutors to see how students respond to getting stuck, as “not being able to do maths isn’t normal for a lot of people when they get here” (Isaac), and how, as apprentices, they can work and interact with a master (see Chapter 4.2).

Whilst four of the students believed that the Oxford interview did not give them a sense of what undergraduate mathematics might entail because it was not university-level topics that were covered, Jacob thought that he got to “do a bit of ‘real’, like, university maths in the interview”. Furthermore, Isaac claimed that the type of thinking required to be successful in the interview is akin to that which they should need to do upon arrival at university, although

the content was not the same. Isaac was not alone in describing perceptions of why the University conducts these interviews, and the benefits it provides for both parties. Katie described the interviews as being "a test of how your mind works" and of your potential to study undergraduate mathematics; Priya recognised the importance of the tutor-student interaction in the context, perhaps over the students' ability to reach a correct answer; Camilla said that "they're just trying to see how you think"; and Camilla asserted that "getting the answers wasn't the thing at all", but their "understanding, the way of thinking" was the most important aspect. The students' descriptions fell in line with those given by the Mathematical Institute of what they look for in a new student:

> *Tutors will, in addition to assessing aptitude and technical skills, seek in successful candidates*
>
> *A) a capacity to absorb and use new ideas,*
> *B) the ability to think and work independently, and*
> *C) perseverance and enthusiasm.*
>
> (University of Oxford, 2013c, online)

The interviews also proved useful for the students as it afforded them an opportunity to perhaps reassess their expectations of undergraduate mathematics study. Priya commented that the interview made her realise that university mathematics was going to be different to school mathematics, and Ryan described the interviews as having a significant impact on his expectations and perceptions:

> *The interviews went some way to making me realise just how little maths I actually knew but at the same time I had a much more profound realisation of that when I actually arrived and started taking courses.*

Mandy's expectations were somewhat altered by her experience, later believing that undergraduate mathematics would "be more logical and more about proofs" than she had anticipated. However, Juliette felt that the interview did not give her a sense of how "boringly… rigorous and arbitrary" it would be, a description which may go towards explaining

her negative experiences of the subject upon arrival at Oxford. Perhaps the most interesting comment about the interview and undergraduate expectations came from Brian, whose struggles with the subject at university resulted in him adopting a surface ATL. His attempts to 'hide' in tutorials so that his tutor could not see the extent of his lack of understanding contrast with his comment that the interviews met his expectations of the subject as being based on "interestingly posed questions with doable maths underneath". Like Juliette, this describes a disparity between expectations and reality, and support their assertions for disliking undergraduate mathematics based on such misconceptions. Furthermore, as tutorials act as a means of engaging students in discussion about mathematical concepts, it seems that Brian's unwillingness to participate further hinders the possibility of him ever adopting a deep ATL:

Brian's Story

Brian's reports were consistent with the definition of a surface ATL – he is struggling with and dislikes mathematics, and works in ways which achieve limited success without revealing his self-perceived poor understanding.

Despite school and interview success, Brian soon felt "left behind" by his peers at Oxford and continues to struggle today, in his second year, to the extent that he often resorts to copying his friends' problem sheet answers. Brian "hides" in tutorials, and frequently asks questions so as to divert his tutor's attention away from asking *him* questions, which he does not believe he could answer. He has not sought help from his tutor as he is embarrassed and fears that he will be "thrown out". Not only does Brian find his studies too difficult, but:

> *I hate not understanding what I'm studying and I don't find anything interesting. I kind of can't find it interesting because to do that I'd need to have an understanding of what they're talking about.*

Brian believes that the content of A-level Mathematics and Further Mathematics gave him "a false impression of what university maths would be", though he gained top grades. At University, he was awarded a 2:2 in his first-year examinations, although attributes this to luck. To revise, he memorises proofs from his lecture notes and problem sheets in the hope that these will be examined, as he does not believe himself capable of doing these of his own accord. This memorisation is something which has resulted in feelings of depression.

> *a deep approach learner was also more likely to engage in talk at the conceptual, analytical, and metaconceptual levels, beyond the procedural and observational levels that the surface approach learner typically engaged in.*

(Chin & Brown, 2000, p. 126)

Such actions correspond with Mann's (2001) description of a student who alienates themselves from the rest of the community as a strategy for self-preservation – he is hiding in

tutorials so that his perceived shortcomings as a mathematician cannot be uncovered by his tutor. Conversely, Camilla's decision to specialise in philosophy in her fourth year (see Chapter 7.4.1.1) is an example of alienation as a result of a context which requires compliancy rather than creativity.

7.3.3 – Summary

Therefore, it appears that the Oxford admissions process had an impact on most of the participants' expectations of undergraduate mathematics study, relating both to the nature of mathematics and what it might be like to do mathematics at Oxford. The interview served to further shape and develop their prior understandings of mathematics and of the culture, which certainly seems to have been valuable for most of the participants. The OxMAT challenged students, often for the first time – it taught Brian that he was perhaps no longer going to be able to answer every question perfectly – and it gave many of them an insight into what it might be to answer undergraduate mathematics questions. Indeed, the analysis of OxMAT questions using the MATH taxonomy in Chapter 6 shows that the nature of questions posed at A-level and in the OxMAT is very different, and that the challenges of the OxMAT share many similarities with undergraduate mathematics examinations. The admissions process is unique to Oxford and the University of Cambridge and, as such, might give students there a better insight into undergraduate mathematics study than students of mathematics at other universities. Their prior understandings are shaped by this experience and should be advantageous. Therefore, if they experience a challenging transition into tertiary study, we must wonder about the difficulties experienced by students at other institutions.

Furthermore, the participants of this study all came from a similar secondary mathematics background, studying the maximum amount of mathematics possible at A-level as preparation for their undergraduate degrees. Whilst the qualification does serve to prepare students to a certain extent, it also has many shortcomings. As Alcock (2013) cautions:

> *University mathematics has much in common with school mathematics, and students who have been accepted onto a degree programme already have an array of mathematical skills that will serve them. On the other hand, university mathematics also differs from school mathematics in some important respects.*
>
> (p. 3)

The comments made by students regarding the differing nature of secondary and tertiary mathematics, as well as how well they perceived how well A-level Mathematics and Further Mathematics prepared them for their degree, are all confirmed by the literature in this area. However, these students' study of Further Mathematics provides a new contribution to the literature concerning the suitability of A-level as university preparation.

7.4 – Conventions & Artefacts

The accepted ways of doing things in the culture, i.e. the conventions, are those which pertain to the actions of the students within it. That is, the things that most students do – and are expected to do – in terms of their learning and engagement with the subject. In the case of the undergraduate mathematician at the University of Oxford, the conventions of the institution as well as the conventions of the student body, as a result of cultural influences, are paramount in their enculturation and involvement in the COP. Conventions of the institution identified through analysis of the students' comments include pedagogical practices, as well as the degree structure – the flexibility of the degree in terms of modules studied – and the nature of the mathematics being studied. Conventions of the student body include the practices of the students on a day-to-day basis which are culturally accepted and seen as the norm. These were identified through students' comments regarding their activity structures (see Chapter 7.6), and are those which they described as being commonplace amongst themselves and their peers. Specifically, this refers to the standard ways of working and revising in undergraduate mathematics.

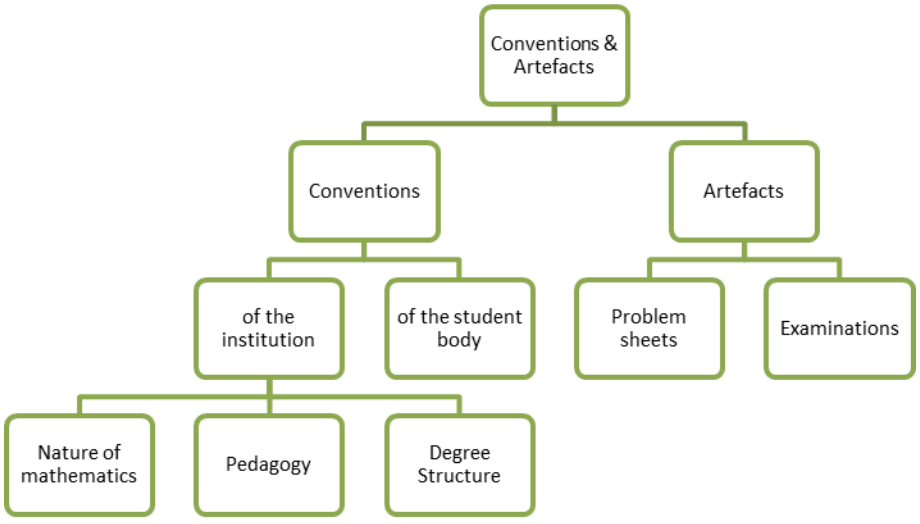


Figure 7.4 - Types of conventions & artefacts in the undergraduate mathematics context at the University of Oxford

The artefacts of the Oxford undergraduate mathematics COP are tangible objects which are available to the students in order to engage with the culture (see Figure 7.4). Analysis of the participants' transcripts revealed these to be problem sheets and examinations, as well as supplementary materials such as examiners' reports and model answers. Such artefacts undoubtedly influenced the students' activity structures in the culture in that their approaches to learning and studying mathematics were affected by the assessment which they faced throughout their degree. The outcomes of these then had an affective impact, as well as one on their social interactions, through working together on problem sheets, and discussions with their tutor and class teachers about the answers.

In the following sections, references will be made to a study by Trigwell and Ashwin (2003). Whilst their study also focussed specifically on the student learning experience at the University of Oxford, it was conducted on students across all subject areas. Students in the mathematical and physical sciences comprised 27.4% of their sample and, of these, it is unknown what proportion were students of mathematics or joint honours with mathematics. Additionally, they make references to a deep/surface ATL dichotomy, not taking into account the strategic approach outlined in this thesis so far. Furthermore, they did not use as robust a scale as the ASSIST to test the students' alignments with the deep or surface approaches. That said, their study provides support of some of the claims made by students about the Oxford tutorial system.

7.4.1 – Conventions

7.4.1.1 – Conventions of the Institution

Nature of Mathematics

> *The move from elementary to advanced mathematical thinking involves a significant transition: that from describing to defining, from convincing to proving in a logical manner based on those definitions… It is the transition from the coherence of elementary mathematics to the consequence of advanced mathematics, based on abstract entities which the individual must construct through deductions from formal definitions.*
>
> (Tall, 1991c, p. 20)

The interview participants were asked to make contrasts between the nature of mathematics that they had studied at the secondary and tertiary level, specifically the nature of the questions that were posed to them in assessment. The students were quick to reference the fact that the mathematics that they learnt and were assessed on at school was very different to their first-year courses, particularly analysis, and felt that they were ill-prepared for such a change by A-level Mathematics and Further Mathematics (see Chapter 7.3.1). Juliette described the difference between the nature of the two 'types' of mathematics:

> *I came to understand that maths is about the fundamentals of the calculations that we performed at school. It's about being precise and proving things to be true, no matter how obvious.*

Undergraduate mathematics is interested in "what works and why it works and how you can prove it works" (Sabrina), unlike secondary mathematics which all of the participants described as being centred around calculations and procedures fundamental – yet also far removed – from advanced mathematics. Indeed, Robert and Schwarzenberger (1991) comment that "The concepts [studied at university] themselves are also radically different from the students' previous experience; they often involve nor merely a generalisation but also an abstraction and a formalisation" (p. 128). Whilst the students were asked to contrast the two levels of mathematics, the vast majority of them used descriptions pertaining to the types of questions

that they had to answer in assessment rather than the nature of the subject itself. This is understandable as they have been through an education system which is very assessment-driven, doing public examinations in years 2, 6, 9, 11, 12 and 13 of their schooling, often resulting in them experiencing 'assessment *as* learning' (Torrance, 2007). The questions that they had to answer at A-level were posed in such a way that they were primarily asked to perform calculations which were similar to those which they had practised in class (see Chapter 7.3.1), rather than making creative use of the mathematics that they were studying. This was something which all of the participants highlighted when discussing the nature of the OxMAT (see Chapter 7.3.2.2), and was remarked as a fundamental difference between the A-level and more advanced mathematics.

The nature of the mathematics being studied has, without doubt, an impact on the nature of the questions which are posed to assess its understanding (see also Chapter 6). Chapter 6 illustrates the change in the nature of questions posed at each level, and points towards a change in the nature of mathematics, confirmed by students' reports of their prior understandings of mathematics.

Pedagogy

Conventional teaching practices in undergraduate mathematics at Oxford are centred around the tutorial system, something which sets it apart from other universities around the world (see also Chapter 4.7.1). Students must attend weekly tutorials in order to engage with mathematics and receive feedback on their weekly assignments, as well as attending lectures, though these are not compulsory. For undergraduate mathematicians at Oxford, the primary purpose of tutorials is go review problem sheets and discuss any concepts which the students find difficult (Batty 1994; Jaworski, 2000).

The transmissionist approach adopted by lecturers is well-documented and long-standing in the literature (e.g. Bligh, 1972; Sierpinska, 2008; Laurillard, 1993), and is often criticised for failing to engage students in actively learning about the topics, as opposed to providing them with material from which they are expected to work with themselves. Indeed, Sabrina suggested that lectures were not for teaching, but for providing students with notes:

> *Lectures are just a way to get loads of you in a room to write some notes. Then you go away and try and understand it.*

However, the close and personal nature of the Oxford tutorial is only something which students experience in the first two years of their degree. After this point, the opportunity to choose to study more specialised modules means that students are assigned to classes for their options, typically of 8-10 students, which are led by a specialist in that area. Postgraduate and postdoctoral students mark their problem sheets for these classes. The interactive element of these is diminished compared to the tutorials (see Chapter 7.5.1), and leaves students utilising them as more of a means of assessing the accuracy of their answers in problem sheets than engaging with the mathematical concept:

> *That's just answer collection. It doesn't help you with what you haven't done yet, just shows you what you should have done if you didn't do it right.*
>
> (Sabrina)

This is something for which the majority of the participants criticised classes, as it left little time for them to explore the concepts, taking them away from the supportive setting of the tutorial. This supports the findings of Trigwell and Ashwin (2003), who found that students held a mixed view of classes, which "may reflect some tutors' lack of experience of teaching" them (p. 7), and that "however high the quality of classes, a large proportion of students will always prefer tutorials" (p. 8). Many of the participants described problem sheets as integral to their learning and understanding mathematics. Ryan asserted that "any understanding of mathematics purely from lectures is superficial until you have attempted some problems". The

opportunity to explore the topics in small groups was something described as important to their learning and, often, essential in the transition. Ten of the participants spoke of the help that they were given by their tutors as they began to explore new topics, often struggling to understand new concepts from lectures alone. Furthermore, Mandy described her difficulties in the third year associated with the removal of tutorials from her learning environment, commenting that "having less support from college is really challenging". However, Katie claimed there to be little difference between classes and tutorials, and that tutorials were "overrated". Such a belief may be perpetuated by her own experiences with her own tutor. Consistency in the support and teaching of students across tutorial groups cannot be guaranteed when each student has a different tutor. This could be compounded by the potential for some tutors to be disengaged and uninterested, seeing their role as tutor secondary to that as an academic. Conversely, other students may enjoy a tutor with a particular skill, enjoyment and enthusiasm for tutoring and expanding their mathematical horizons and nurturing their mathematical thinking.

Degree Structure

At Oxford, the degree is structured such that the first year consists of only compulsory modules, the second year introduces a small amount of flexibility and choice, and the third year allows students to pursue any mathematical direction that they choose out of a selection of available courses. This permits students to specialise in a particular area of mathematics, if they wish, as well as – or instead of – pursuing topics of interest outside of mathematics, such as mathematics education, computer science or philosophy.

The students interviewed all chose to focus on different areas once they were able, for reasons of interest, as well as a couple of the students making a strategic decision to choose to do courses with a strong coursework element:

> *doing the essays and coursework means that you get that out of the way, and there's no exams, which alleviates some of the stress later on. Less exams.*
>
> (Malcolm)

Four of the thirteen students interviewed were reading for joint honours courses in mathematics – Camilla and Christina studied Mathematics and Philosophy, Ethan studied Mathematics and Computer Science, and Priya studied Mathematics and Statistics – and as such were permitted to study non-mathematics modules earlier on their degree. For Ethan and Priya, who are both first-year students, there had not yet been a great opportunity for them to specialise in either mathematics or their other subject; however, Camilla and Christina are in their third and fourth years of their degree, respectively, which means that Camilla was on the cusp of making decisions, and Christina had already decided to specialise. Interestingly, whilst Christina is only studying mathematics modules in her final year, Camilla has made the decision not to study any mathematics next year, but instead focus on philosophy.

Camilla's Story

As a third-year student of Mathematics and Philosophy, Camilla has decided to specialise in philosophy in her fourth year. Similar to her experiences of A-level Chemistry, which she dropped after AS because she believed it over-relied on rote-learning, she prefers the thinking required for philosophy.

Camilla's first year was difficult as much of the mathematics was "totally alien"; however, in spite of this, she thought that "analysis was really cool" and she was "really excited about it", even though it was "way too hard".

She adopts a strategic approach to revising for her examinations. An unexpected repercussion of this was that, last year, one of her papers contained a question about what she thought was non-examinable material, which meant that she was unable to answer the question. She was also relying on a particular type of question to come up, based on previous papers.

She believes herself to be more conscientious than many of her peers, and works hard at both the mathematics and philosophy parts of her degree, whilst also balancing being part of her college JCR alongside her studies.

The reasons given by both students are completely contrasting and exemplify the different attitudes that

different students can have about mathematics study at the university level. Christina is only studying mathematics courses this year because she struggled with essay-writing in philosophy in the last three years. Furthermore, she spent proportionally far more time working on philosophy earlier in her degree, despite the fact that she was doing fewer philosophy courses than mathematics. Conversely, Camilla has chosen to specialise in philosophy because of negative experiences when learning mathematics for examinations:

> *In maths, all I seem to be doing is memorising a series of proofs and things, you know, on each topic, and then in the exam just reproducing it. Whereas… In philosophy you're thinking and showing your personal thinking and understanding and I much prefer that. It's much more interesting.*

Camilla's comments here reflect the different responses that the students' activity structures can illicit, and whether they have a positive or negative experience in doing them. Whilst they both adopted what appear to be conventional approaches to revising and had similar experiences of the subject, Christina's ASSIST responses revealed her to have a deep-dominant ATL, whereas Camilla's was of a strategic ATL. Perhaps it is that Christina was more engaged in the mathematics than Camilla. This certainly seems to be the case from Christina's regular involvement in the Invariants[42] (see Chapter 7.5.2), whereas Camilla was more detached from the undergraduate mathematics COP. She also enjoyed the subject, even reporting that she initially thought "analysis was really cool" and "was excited by it" when she first arrived at Oxford, although had performed worse than Christina in her examinations. Camilla attributed what she perceived to be her poor performance – an average of 64 in in her second year examinations – to the examination questions that she was hoping for, based on patterns in previous papers, not coming up, something which is indicative of a strategic ATL.

7.4.1.2 – Conventions of the Student Body

The comments made by the participants regarding their activity structures in examination revision at Oxford suggested a very clear convention in terms of the students' practices. A

[42] The mathematics society at the University of Oxford.

number of the students asserted that what they did was similar, or the same, to their peers, often citing that there was no other possible way of revising for their examinations.

> *Everyone else does basically the same thing. I think the only difference is how much time people put in to revision and doing their work earlier in the year.*
>
> (Juliette)

The common method of revision adopted by the students involved re-reading lecture notes, making revision notes, practising past papers and problem sheet questions, and committing definitions, theorems, and sometimes proofs, to memory. However, the order in which the students did this differed. The majority of students memorised mathematical material before attempting example questions, although a number of them claimed that this was not perhaps the best way for them to do that, given they had no way of testing their understanding until much later on. Only Mandy indicated that she revised in a different way to her peers, through constructing mind maps (see Mandy's Story in Chapter 7.3.1), speculating that they were not interested in developing conceptual understanding and relating ideas:

> *it seems to be that everyone just ploughs straight into writing lots of notes and definitions, theorems and proofs and just do lots of cramming to remember it. Nobody else does mind maps and things like that. I guess they just understand how the relationships work as they go along or… Haha… Or they just don't care!*

This certainly seemed to be the case for Brian, in particular, who described his surface approach to learning mathematical proofs as being a means to an end, having identified the types of questions which he would have to answer in examinations (see Brian's Story in Chapter 7.3.2.3). A number of the students confessed to memorising certain proofs verbatim because they "don't have the conceptual understanding to be able to derive it in the exam" (Juliette). These students are "memorising theorems and proofs at the possible expense of meaning or significance" (Jones, 2000, p. 58), however, which may have damaging consequences later on.

7.4.2 – Artefacts

The artefacts in this culture are the problem sheets and examinations that students do. Whilst problem sheet marks do not count towards their degree, they act as a formative assessment for both students and tutors. Only examinations in the third and fourth year contribute towards the total degree classification. Students on the MMath receive separate classifications for the BA part of their degree, and the masters year. The types of questions posed to students in problem sheets and examinations appeared to share a number of similarities; however, there was a marked difference in the number of 'factual knowledge and fact systems' questions posed in undergraduate examinations (see Chapter 6). Whilst it appears that students were not asked to define mathematical terms or state known theorems in problem sheets, this was commonplace in undergraduate examinations.

Whilst Oxford operates in unconventional means through the reliance on the tutorial and college system, it seems that it very much adopts traditional means of undergraduate mathematics assessment in the UK (Iannone & Simpson, 2011a).

7.4.2.1 – Problem Sheets

From the participants' descriptions, the weekly problem sheets seemed to involve testing questions which they often struggled to answer. All of the students described themselves as having, at one time or another, to seek help from their peers in answering some problem sheet questions. Whilst this was often resolved through their peers explaining how to reach the answer, sometimes a number of the participants resorted to copying answers because they did not understand how it worked or, despite understanding the ideas behind it, believed that they never would have been able to do it themselves (see Chapter 7.6.1).

The majority of participants believed the problem sheets to be an important part of their learning mathematics, with a number of them claiming that if they did not have such

assessment throughout the year then they would find revision very difficult, and would be unable to assess their understanding of the mathematical concepts earlier on in the year.

> *Doing the problem sheets are key to understanding of a topic and then the tutes*[43] *reinforce these.*
>
> (Ryan)

Without the understanding achieved through answering problem sheets, the students would struggle to be able to comprehend how to answer certain examination questions, as well as to develop their understanding of mathematical concepts. Furthermore, a number of the participants remarked that significant similarities existed between certain problem sheet questions and examination questions, meaning that revision of problem sheets was an important part of their revision process. As indicated in his story in Chapter 7.3.2.3, Brian's surface ATL extended to memorising the answers to certain problem sheet questions in anticipation of something the same – or similar – appearing in his examination. Analysis in Chapter 6 revealed that this occurred on a few occasions, although this was rare.

7.4.2.2 – Examinations

The descriptions given by the participants of the types of questions posed to them in undergraduate examinations were consistent with those which were arrived at in Chapter 6, with Katie's descriptions of the types of questions even appearing to mimic the ideas of the groups in the MATH taxonomy:

> *Most of the exams follow some kind of definition-theorem-proof-crazy proof system!*

By this, Katie was referencing the idea that most undergraduate mathematics questions began with the statement of a definition, followed by a statement of a theorem, requesting a proof, before then asking the students to use their understanding of these concepts and related concepts to construct a proof themselves. Thurston (1994) refers to this as the "definition-

[43] Colloquialism for 'tutorials'.

theorem-proof (DTP) model of mathematics" (p. 163), a model which is evident in examination questions as well as lectures. Davis and Hersh (1983) comment that "a typical lecture in advanced mathematics… consists entirely of definition, theorem, proof, definition, theorem, proof, in solemn and unrelieved concatenation" (p. 151). This is all the more important when one considers the comments made by Griffiths and McLone in 1984 about undergraduate mathematics examinations, who conjectured that "whether or not one possesses the technical expertise to answer the questions on a conventional British mathematics examination paper, the *structure* of the questions can be readily seen" (p. 300). Moreover, the findings reflect the nature in which the students were taught in lectures, as "the model for teaching mathematics to undergraduates appears to be 'definition, theorem, proof' and this disjunction presents a problem to the undergraduate engaging with proof" (Almeida, 2000, p. 869). Proofs of theorems that they had met in lecture notes were often learnt by the students word-for-word as part of their revision process, limiting the amount of mathematical creativity required of students to later parts of the question. Indeed, Mandy speculated that a student could "probably get a low 2:1 just with bookwork questions", with various students making claims that such questions are amongst the easiest in the examination and do not require an understanding of the mathematical concepts:

> *The easy marks to get in those exams are those which are like 'state this theorem and give the proof'.*
>
> (Malcolm)

> *you're normally asked to state definitions in the questions so as long as you learn all of those, then you have some easy marks because you don't have to understand anything. Like, any old person could memorise a definition and write it down.*
>
> (Katie)

This is something commonly identified as being problematic by educational researchers and educators alike, with Vinner (1991) suggesting that questions which require the statement of

definitions can reduce students to rote-learning mathematics, failing to grasp the meaning of the definition.

However, many of the participants noted that knowledge of the definitions and statements of the theorems were necessary in order that they are able to answer more complex questions:

> *If you don't remember definitions and theorems and proofs, then you won't be able to do any of the more advanced options, so you'll have nowhere to go*
>
> (Camilla)

> *in some cases, it is possible that you can lose many marks in an exam if you cannot do the beginning of a question.*
>
> (Ethan)

Furthermore, Priya commented that actively memorising definitions, theorems and proofs actually served to deepen her understanding. This is confirmed by Kember (1996), Watkins and Biggs (1996) and Entwistle (1997), who states that it is possible for memorisation to act as "a necessary precursor to understanding, and for other purposes it is a way of reinforcing understanding" (p. 216). However, these students are right in their claims that knowing a definition precisely is very important for their studies in that

> *a mathematical definition does have the property that everything satisfying it belongs to the corresponding category and that everything belonging to the category satisfies the definition. Deductions made from the definitions provide us with theorems that hold for every member of the category and, in the context of the problems provided by those lecturing to first year undergraduates, any theorem a student is asked to prove can be deduced from the definitions.*
>
> (Alcock & Simpson, 2002, p. 228)

The students' revision practices were clearly shaped by the nature of the examinations set by the University, particularly because they identified clear patterns in the examination questions asked over the years, with particular questions being more common than others, instigating in the participants a response to ensure that they revised particular things as a priority:

> *the structure of the exams hasn't changed apart from once, I think they all changed […] Some courses will have very similar questions year to year, er, and then some courses will have very different questions year to year based on the same sort of ideas, but very different.*
>
> (Isaac)

This aligns with the students' prior understandings of mathematics examinations (see Chapter 7.3.1), and has a significant impact on their working practices as a consequence.

7.4.3 – Summary

The conventions and artefacts in the University of Oxford mathematics COP have been shown to have a significant impact on the students' actions (activity structures, social interactions), self-efficacy (perceptions of themselves and others), and their contrasts with their prior understandings provide additional to challenges to students at the secondary-tertiary interface. In Chapter 7.3.1, criticisms of the A-levels in Mathematics and Further Mathematics were given by the students, and served to back up those given in the literature (see Chapter 2.3.3); however, the conventions and artefacts at undergraduate level are also guilty of serving the students with predictable ways of being assessed which can affect their approaches to learning and revising for examinations. It is true that "the conventional system has been successful for the best British mathematicians, who assiduously practised the technique of 'working old papers' and learned a lot of mathematics in the process. However, what worked for them may be quite inappropriate for the average contemporary undergraduate" (Griffiths & McLone, 1984, p. 302). Whilst Griffiths and McLone's comments were made 29 years ago, it has even more applicability in the current context when the A-level in Mathematics has been

subjected to such vicious criticism for failing to effectively prepare pupils for tertiary study, something which was not such a hot topic in the 1980s.

The linkage between the factors associated with the four parameters in Saxe's (1991) model is becoming evident through this analysis. Prior understandings, conventions and artefacts describe influences to the students' behaviour and beliefs, with the conventions and artefacts at secondary and tertiary level sharing a number of similarities as well as significant differences[44]. The impact of this on social interactions, activity structures and students' perceptions of themselves and others follows. For example, the way in which students are taught in lectures, tutorials and classes influences the students' process of building their identity as members of a mathematics community of practice with consequences in the students' self-efficacy and self-confidence (Furinghetti *et al.*, 2012).

[44] The reports made by students here confirm findings from Darlington (2010), in which questionnaire data found that 82% of first-year students believed that their ATL had changed between school and university.

7.5 – Social Interactions

Initial analysis of the interviews suggested that there are two different types of social interaction for the undergraduate mathematician in the context of mathematics at the University of Oxford (see Figure 7.5).

These are:

1. **Formal social interactions**: *tutorials, lectures, classes*

   those which are formally organised and structured by the University as a means of learning and studying mathematics. These mechanisms themselves also act as some of the artefacts in the culture.

2. **Informal social interactions**: *study groups, the Invariants*

   those social interactions which stem from unofficial relationships established between students independent of their formal social interactions. These interactions may or may not be with those students who they interact on a formal basis.

Additionally, there are what shall be termed 'extra-curricular social interactions'. This term refers to the social interactions and experiences that the students engage with outside of the Mathematical Institute, out of mathematics, and out of the studying environment. For example, this would include their participation in sports clubs and societies, and the friendships they establish extraneous to studying purposes.

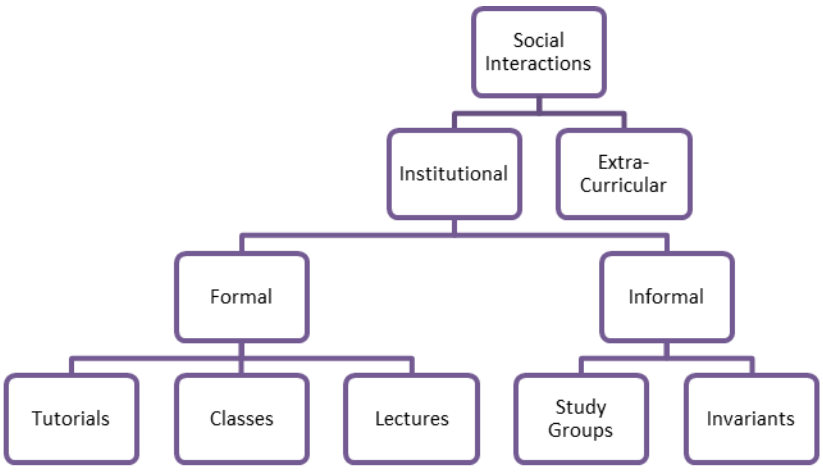


Figure 7.5 - Types of social interactions in the undergraduate mathematics context at the University of Oxford

The University of Oxford is slightly different to other universities in the sense that its compulsory formal social environment – and teaching pedagogy – for students is that of the tutorial. Students' social interactions are further 'engineered' by the University through the collegiate system, which is used to assemble students in tutor groups. Therefore, the COPs that students engage in are, to an extent, influenced by such a teaching and pastoral structure. Tutors have two distinct roles – academic and pastoral. They exist to foster their students' understanding of the mathematical concepts that they are studying, as well as to mark and give feedback on assignments set throughout the term. In addition, they are a connection between the student and their college/department in such a way that they can provide assistance for students if they are having problems, either personal or academic. Perhaps this is part of the reason that the University of Oxford has, at 0.4%, one of the lowest drop-out rates in the country (HESA, 2011).

A student's enculturation into the new mathematical environment will undoubtedly be affected by their social interactions with other members of the culture, be they lecturers, tutors or other students. The way in which they engage with other members of the community and participate in their practice will also have a significant impact (see Chapter 3.1.3) on them at any stage of their degree

7.5.1 – Formal Social Interactions

Tutorials perform a specific role for students in their first year of university study, when all of the participants reported that their tutors provided them with invaluable help in adapting to tertiary mathematics (see also Chapter 7.4.1.1). Indeed, various studies have found that small group learning is "effective in promoting greater educational achievement, more favourable attitudes toward learning, and increased persistence" (Springer *et al.*, 1999, p. 21). In order to supplement and support what they were provided with in lectures, the students described their tutors as having taken the time to explain mathematical concepts with them more

closely, fostering their understanding and appreciation of the rigorous nature of mathematical proof, as well as giving them feedback on their problem sheets.

> *He initially said he'd have some extra hours available for us in addition to our actual tutorials where, if we were having problems or wanted to ask questions, then we could go and find him and he'd help us. So, er, that was really useful.*
>
> (Isaac)

> *He was definitely really patient with us at first when we were trying to get to grips with everything.*
>
> (Malcolm)

> *getting us used to the idea of formal proofs and how they work. He would show us them in tutes on the blackboard and explain what was going on in all of the steps and then we would join in and see if we could work together on them at first. That was really helpful.*
>
> (Camilla)

Camilla was not the only participant who found this helpful, with Ethan stating that "Without this help I feel that I would not find my studies as manageable as I do now". However, Katie was quick to criticise her tutorial experience by saying that her tutor was not supportive of her when she was experiencing difficulties with mathematics in her first year.

> *He seemed puzzled about why I was reacting the way I was, and getting upset, and just suggested that I worked hard until it got easier. It wasn't very helpful of him.*

Isaac described the role his tutor played in encouraging him to quit rowing after his marks decreased year-upon-year, and he "unashamedly" let his sport get in the way of his studies. He speculated that his tutor showed a great interest in his tutee's extra-curricular endeavours "partially so he'd have something to blame it on if we started underperforming!" Furthermore, half of the participants described their fears of being 'discovered' as struggling in their studies during a tutorial. Four students made comments regarding a fear that they would be asked a question that they were unable to answer, this being something which Brian described himself

as regularly engaging in as he 'hid' in tutorials so as not to be found out to be struggling. This manifested itself in his working practices as he engaged in a surface ATL, and yet he was fearful of admitting his problems to his tutor in a bid to seek help, instead worrying that he would be "thrown out" if discovered, and was embarrassed by what he perceived to be his inability in mathematics. This is not uncommon, as a study in an Irish mathematics department (Grehan *et al.*, 2010) found that students, like Brian, who were struggling "were often not aware that they had a problem or were unwilling to admit it (to themselves or others) until it was too late. Students were also reluctant to ask for help and feared embarrassment" (p. 35).

In addition to providing support and guidance for the courses that the students were studying, a third of the students described their tutors as having taught them "a cool bit of maths" (Malcolm) alongside that of their normal courses when time permitted. This is something which tutorials allow for, unlike lectures where, quite often, "Interesting digressions have to be avoided" due to time constraints (Griffiths & McLone, 1984, p. 298). Tutors also offered many of them assistance with understanding anything that they struggled with in lectures, which was particularly problematic in the first year.

An attempt to ease new students into the undergraduate mathematics context was something which their lecturers also endeavoured to do during introductory lectures, with more than half of the participants describing their appreciation of the reduced pace with which the lectures ran. Sabrina speculated that, particularly in analysis, this was because "they know that everyone's going to respond to it by going 'huh?!'", although this was short-lived:

> *I'd say that they did that maybe for two or three weeks and then it was full steam ahead! They had this incredible ability to cover what was basically everything we'd ever done about calculus […] during the course of about a quarter of a lecture.*
>
> (Malcolm)

A quarter of the participants expressed a particular fondness for a particular lecturer – though not the same lecturer – on the basis that they made the material that they were teaching more accessible and enjoyable for them. The students appreciated their evident enthusiasm for the subject, something which can only be properly communicated in a social environment. Whilst undergraduate lectures are not typically social in their nature, and students are not encouraged to talk to each other or the lecturer, the undergraduate lecture does have the potential to use a social element to enculturate students into the environment. For example, the lecturers may communicate with the students their passion for the subject and, perhaps most importantly, their actions whilst writing on the blackboard can provide students with an insight into what it is to be a mathematician. Quite often, lecturers will make a mistake when doing an example on the board, or proving a statement, which serves to make them seem more approachable, and the changes that they make to a mistake afterwards demonstrate to students the thinking process behind what they are doing. Without much of a social interaction, "when you're in lectures, you're not doing maths. You're just listening to an old man saying something and writing it down on a board for you to copy" (Sabrina).

The third-year classes are attended by more students than an undergraduate tutorial. More than half of the participants described this as making them less effective in terms of fostering their understanding as it slows down the pace of the class:

> *So if everyone has understood a question then you just skip it [in a tutorial], but in a class with 8-10 people you can't do that because there's inevitably going to be one person who didn't understand. It means that you can't talk about random cool stuff as much.*
>
> (Christina)

Conversely, Katie claimed that this was not a problem and that, as a matter of fact, she believed that tutorials did not provided "any real benefit over classes, especially if you have a good class tutor".

7.5.2 – Informal Social Interactions

In Chapter 7.5.2, all of the participants described the social ways in which they worked on problem sheets together if they became stuck on a question. For all of them, most of the time, this meant working with other students from their college and tutor group, these being the mathematicians that they first met at Oxford. Whilst the collegiate environment has its advantages in this respect by providing students with a ready-made study group, half of the students complained that the system as it is makes it difficult for them to meet people on their course outside of their college, and the students themselves do not make an effort to mix during later years when they are studying optional courses:

> *If there's a lecture with only, like, 12 people then there'll be 12 people dotted around the lecture room, rather than a group of four here, and a group of four there, and things like that.*
>
> (Malcolm)

> *There aren't a lot of mixing opportunities in the department because you go to the lectures and there's nothing social about that, and then you talk to other mathmos [sic] when you're in a tute.*
>
> (Mandy)

> *The social side you get comes from any interaction with other students and that's only really going to happen in tutorials, first off, and then maybe later on in classes.*
>
> (Camilla)

However, in later years, a number of the participants described themselves as having made friends with other mathematicians through meeting them at classes for their optional courses. Therefore, the possibility of students broadening their involvement in wider or different COPs within the undergraduate mathematics community at Oxford appears to increase as students begin to study increasingly advanced mathematics. Whilst Malcolm's comments (above) suggest that students become very involved in their pre-existing COPs and do not socialise

with students from other colleges very readily, Christina and Katie described their friendships and working groups with other students through meeting them in classes.

The Invariants are a mathematical society at the University which organise formal and informal seminars and discussions in the department, both internal academics and mathematicians from outside of the University. In this way, the society acts in a way which serves to promote socialisation between students and staff, or apprentices and masters (see Chapter 4.2). However, this was an opportunity only taken by two of the students who were interviewed. Priya had been to one Invariants meeting, and Christina was an active member; however, the rest of the participants had never been to a seminar or expressed any interest in joining.

Christina's Story

Christina is a finalist of joint honours with philosophy. She is heavily involved in the Invariants and enjoys mathematics, and has applied to do a doctorate in mathematics, philosophy or computer science when she has finished at Oxford.

Her initial experience of undergraduate study was harder than she had expected, although she did not find it as "upsetting" as she believes many of her peers did. This is something Christina attributes to the fact that many of them were used to finding mathematics easy when they were at school and were still expecting to be the best when they got to Oxford, only to find themselves wrong. This is something she believes to be "a bit stupid", though Christina herself has performed well throughout her degree and was awarded with first class honours for the first three years of her degree. That is not to say that she did not find the transition to tertiary study difficult.

Christina is a committee member of the Invariants, spending a lot of her time with other mathematicians who she has met through the society. She joined during the secondary-tertiary mathematics transition when she was experiencing difficulties with mathematics. She found the Invariants' enthusiasm for mathematics to be inspiring and, through her involvement, was reminded that "maths is cool".

Moreover, my questioning whether they were a member of the Invariants was met with laughter by four of the participants, with a number of them describing the members of this society as being "geeky" (Juliette), and therefore off-putting:

> *The kind of people you meet there are the kind of people who want to go to a maths lecture, and, er… Haha. The type of people who go to optional maths lectures, which might put some people off.*
>
> (Malcolm)

Isaac said that when a representative of the Invariants came to introduce new students to the society in his first year, he was "the strangest guy you could've possibly imagined", which "just put everyone in the room completely off joining it".

However, when probed further, many of the students qualified them not being part of the Invariants as being because they were not particularly enthusiastic about mathematics and therefore not interested in learning more.

> *They're very enthusiastic about maths. That's something I can't relate to anymore!*
>
> (Brian)

> *As much as I love maths, the prospect of adding to the amount I do depresses me.*
>
> (Jacob)

> *They're all so passionate about maths and are really good at it, which is intimidating.*
>
> (Juliette)

It is not the case that only the more able students, or those with deep ATLs, were members of the Invariants, as Christina described herself as joining after having found the first year of her degree very difficult. Going there, and seeing that "it was all 'maths is cool!'" increased her enthusiasm and served to increase her interest, as well as earning her new friends on her course, who she later studied with. It appears that the COP created by regular membership of the Invariants resulted in mathematical discussion, and may have an impact on students' engagement and success in the subject.

7.5.3 – Extra-Curricular Social Interactions

Of course, undergraduate mathematicians at Oxford are also able to become members of a number of non-academic COPs (see Chapter 3.1.3) which can impact upon or be impacted by academic experiences. Isaac's tutor's attempts to stop him from rowing in his final year of his degree came after his performance was compromised by his participation in too many extra-curricular activities. Whilst he was not alone in doing a sport outside of his degree – seven participants were also members of various college and university sports teams – he was alone in having that negatively impact on his performance. The majority of the other participants

described their non-mathematical activities as not getting in the way of their studies, although Katie did not run for a second year on her Junior Common Room[45] committee, saying it was because "I had such a lot of work to do for my degree", and Jacob and Ryan admitted that they should spend more time studying and less time doing other things.

Both Juliette and Brian's decision to continue with their studies was undoubtedly affected by the friendships which they had established whilst at Oxford, with Juliette describing her studies as being a "kind of sacrifice" necessary to mean that she could continue to live in Oxford and have fun with her friends. In a sense, these students' social interactions outside of their studies have taken priority over their social interactions within, to the extent that they dictate their presence. This serves to further limit their enculturation into the mathematical community, and means that they are not fully participating in the undergraduate mathematics COP. Whilst an argument could be made for the case that only those students who are regular members of the Invariants are active participants in the COP, these students in particular appear to have rejected it as a consequence of feeling that they do not belong. Like most of the other participants, the majority of Brian's friends are not mathematicians, and he mixes with people from different courses, although these people are mainly at his college. Jacob described this mixture of friends as being "not ideal in a practical sense" in that "it's always better to know as many people as

Juliette's Story

Having made a significant breakthrough at the end of her first year upon realising "what maths is", Juliette is now adjusting to undergraduate mathematics, though does not particularly enjoy it. She was disappointed with her 60% in her first-year examinations. Whilst she thinks it was a fair reflection of her understanding, she was nonetheless dissatisfied: "I used to pride myself on getting high marks in exams, but I just don't think that kind of performance is possible from me anymore".

Juliette does not socialise in the department and is not a member of the Invariants. She is intimidated by their passion for mathematics, which she does not share, saying that they are "geeky" and "not really my kind of people". In fact, Juliette credits coming to Oxford with her becoming more confident and outgoing, saying that the experience has been "really life-changing". This is the reason why she perseveres because, despite the fact that she dislikes her studies, she sees them as a necessary and acceptable "sacrifice" for being able to live in Oxford and be with her friends.

[45] A body which represents undergraduates in the organisation of college life and to operate certain services within the college for the students.

you can on your course because you can help each other with problem sheets and stuff like that" (Jacob).

7.5.4 – Summary

Therefore, it seems that these formal social interactions have a significant impact on the students' experiences of the subject. The social nature of the tutorial serves to give them an opportunity to discuss mathematics and broaden their understanding of mathematical concepts – something which would be less likely without this environment. This is seen by students as a disadvantage in Year 3 when they do not have tutorials; however, the potential for some social interactions in lectures might increase as the number of students in attendance decreases and the learning environment becomes more intimate. The formal social interactions arranged through tutorials then go on to affect students' informal social interactions when they work with each other on problem sheets and develop friendships with other students on their course. The majority of students were only friends with other mathematicians at their college, whilst those who had friends elsewhere met them through classes in their third year, and Christina met her friends through the Invariants. Extra-curricular social interactions, for the majority of participants, were college-based, although a number of them participated in blues sports or university-wide societies. For the majority, these activities did not negatively impact upon their studies, in two cases even contributing towards the students' good time-management.

7.6 – Activity Structures

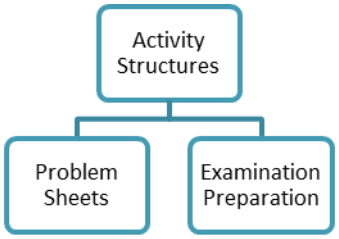


Figure 7.6 - Types of activity structures of undergraduate mathematicians at the University of Oxford

In the context of the enculturation of new undergraduate mathematicians at the University of Oxford, 'activity structures' are defined as the activities that the students actually undertake in relation to the mathematics community. That is, the actions involved in being an undergraduate mathematician at the University, as opposed to anything that they do more generally associated with university life. Specifically, this pertains to their ways of learning, studying and revising (see Figure 7.6). These will be affected by their prior understandings of what it is to learn, study and revise mathematics, as well as the mathematics that they must learn, and the assessment that they must prepare for and go through at the undergraduate level. As we have seen, the students perceive secondary and tertiary mathematics to be "completely different creatures" (Brian) which suggests that there is the potential for its learning, studying and assessment to also be different.

7.6.1 – Activity Structures Involved in Completing Problem Sheets

Discussions with the participants revealed that the time taken to finish a problem sheet was much shorter in the first year, taking approximately four hours, but significantly longer in later years, and could require students to spend days on them in order to finish them to their satisfaction.

However, it is often the case that the students feel unable to answer the problem sheet to the best of their ability with or without help:

> *a lot of the time, I really think that I'd get the same mark on a problem sheet if I spent five hours or five months on it. Haha. Sometimes, I just don't know what to do at all and end up at a dead end*
>
> (Juliette)

All of the participants bar one began work on their problem sheets alone, and only discussed them with their peers if they had trouble answering the questions. This was commonplace for all of the students, as they described their ability to finish a problem sheet entirely on their own as being weak. Brian said that he "normally can't do many, if any, like that", and other participants bemoaned the fact that they were no longer able to achieve the perfect scores that they were capable of at school. Indeed, Sabrina exclaimed that "If I get over 75 it feels like I've won the lottery. I think most people feel that way". The participants tended to approach their peers, often those who were in their tutorial group, and for the most part other mathematicians at their college, after making initial attempts at the questions themselves. Often, the participants described what they did with their peers as a form of 'collaboration', more than anything else:

> *Then it's like a coming together of the brains to try and make sure that we can both answer the questions*
>
> (Ryan)

> *once it becomes more difficult, I'll call in back-up if I need it*
>
> (Jacob)

> *It's sort of collaborating. So if one person is really behind on a problem sheet then there's a bit of, not copying, but reading and using it. So it can be quite one-sided but it normally goes both ways.*
>
> (Isaac)

However, half of the participants described themselves as resorting to copying the answers of their peers "every now and then" (Sabrina) if they were unable to understand how they had reached the answer. This was not due to laziness or a reluctance to understand, however, with a third of the students bemoaning the fact that they could often understand what their peers had done to reach the answer, but did not believe themselves to be capable of writing such an answer themselves.

> *a lot of the time I'll understand what they did but there's definitely no way I would have been able to have come up with it myself. Sometimes that kind of thing is completely beyond my capabilities, even though I can see what's going on.*
>
> (Brian)

This was not limited to Brian, whose struggles with the subject were documented in Chapter 7.3.2.3, but also extended to the reports given by students whose ASSIST questionnaire responses identified them as having predominantly deep ATLs, and to students who described a positive and successful learning experience. Multiple students expressed a belief that the problem sheets were an integral part of their learning which would help them to understand mathematical material ready for their examinations, as well as providing them with a means of revisiting lecture material which they often struggled to keep up with.

> *I think that any understanding of maths purely from lectures is superficial until you have attempted some problems and so problem sheets are key to actually understanding the material.*
>
> (Ryan)

> *For me, it was the only way you can really learn what you're doing as you go along because the lectures are so fast-paced you need to have assignments to make sure that you're keeping up with it. If you left it all until later in the year to be tested on it, I think you would really struggle because of the volume of material.*
>
> (Christina)

Furthermore, it is the re-reading of lecture notes which forms the basis for any studying that students do outside of their lectures, classes and tutorials, with the exception of completing problem sheets. None of the participants described themselves as ever reading around the subject or pursuing any mathematical topics to find out more and explore their interests, but merely using their time to go over what they had been told in lectures in a bid to understand what had been communicated to them.

7.6.2 – Activity Structures Involved in Preparing for Examinations

The consolidation of understanding and efforts made to *begin* to understand lecture material were reported by the participants as forming the basis for their revision practices. This was often the first step involved in their revision process, as they strove to understand the mathematical concepts in order that they would later be able to be able to answer examination questions involving them. This was in stark contrast to their school experiences, where they did not struggle to understand what they were taught, and certainly did not need to spend any of their revision time to get to grips with any of the material (see Chapter 7.3.1):

> *I try and understand what I'm doing by reading through everything carefully, but I didn't really need to even try to do that at school because it just… happened.*
>
> (Malcolm)

The revision practices that the participants described themselves as engaging in at school are very different to those which they describe at the university level. Whilst their school revision consisted, for the most part, merely of practising answering questions similar to those which they expected in their examinations owing to the similar nature of papers from year-to-year and the vast quantity of example questions available to them, this was far from what they described as doing at Oxford. At school, they described no effort to *learn* mathematics, but to *practise* it. The students interviewed described themselves or their peers as engaging in revision methods which involved revisiting lecture material which they would "have to read a

million times to get your head around" (Isaac) before engaging in some 'memorisation' and practising of past questions. The participants' comments indicated that, rather than forming the largest part of revision, completing past papers was actually secondary to consolidating understanding and memorisation:

> *So now, before you do any [practice] questions, you have to… So there's two steps before that now.*
>
> (Katie)

> *I have to spend a lot more time working through the material, mainly… because I didn't understand it the first time around.*
>
> (Juliette)

> *Uni maths, I spent most of my revision trying to understand it, which sadly doesn't leave much time for actually getting used to questions.*
>
> (Sabrina)

The word 'memorised' punctuated students' descriptions of their revision process, with very few of the students describing themselves as 'learning' the material. This is perhaps due to their previous experiences of the subject, where they did not describe themselves as having to make an active effort to learn the mathematics during revision, instead immediately doing past papers and practising what they felt that they had already internalised. The notion of 'learning' mathematics was not something which any of them described themselves as having to engage in, and therefore might, for them, be a foreign word to associate with mathematics.

> *You have to commit a lot of stuff to memory these days, whereas at school you just learn what to do with the thing and do it when the question asks you to.*
>
> (Ryan)

Three distinct purposes of memorisation were described by the students, the first of which being legitimised and justified by all of the students as the only way to be able to understand mathematical concepts, to be able to answer in-depth questions, and because it was necessary

to in order to answer questions which required them to state definitions or theorems precisely (see, for example, question 1 (a), Appendix 6.4). According to Christina, this is the first time that she had "to remember… stuff" for a mathematics examination, which means that "You have to memorise the definitions. You don't have any other choice. You just need to know exactly what it is" (Isaac). However, more than half of the participants described themselves as having memorised proofs from either their lecture notes or problem sheets because these often come up in examinations:

> *Everything, really. Just, like, knowing all of the proofs so you have to remember them. And the theorems. And the definitions. And just… everything, you know?*
>
> (Camilla)

> *You need to memorise so-and-so's theorem and so-and-so's lemma because a lot of the questions in the exams ask you to state those.*
>
> (Ryan)

> *Sometimes I just had to rewrite the answers [to problem sheet questions] again and again until I could remember*
>
> (Brian)

The latter of these descriptions, Brian's attempt to memorise the answers to past papers and problem sheets in anticipation of these questions appearing in his examinations, was something only he reported, which is something consistent with a surface ATL. His apparent "resignation from effort to understand" (Ioannou & Nardi, 2009, p. 39) meant that he was blindly memorising the mathematics in the hope that he would be able to reproduce it in an examination without having to understand any of the underlying concepts. This is something which Tall (1991b) describes as 'disastrous', and defines as a 'disjunctive generalisation' in that learning by rote in this way "simply adds to the number of disconnected pieces of information in the student's mind without improving the student's grasp of the broader abstract

implications" (p. 12). This is something which Mandy actively avoids by constructing mind maps.

Unfortunately, the nature of the examination questions posed at this level mean that students are able to get away with this kind of practice:

> *I can tell you it and prove to you that I know the proof. But I can't prove to you that I understand the proof if you just get me to write the proof*[46]
>
> (Isaac)

Memorisation of proofs was not something which was only reported by Brian, however, as many of the students described it as being necessary because of their complexity, and because they believed that they were not capable of writing the proof for themselves without knowing it off by heart:

> *I also spend more time memorising bookwork, again probably because I don't have the conceptual understanding to be able to derive it in the exam.*
>
> (Juliette)

> *the only way I'm going to be able to prove a theorem is by remembering it from the notes.*
>
> (Ryan)

Learning in this way is not without its pitfalls, however, with more than half of the students reporting that they had forgotten what they had previously memorised when they were in the middle of an examination. Furthermore, Anderson *et al*. (1998) claim that students who rely on memorisation for reproduction can have "such fragile understanding that reconstructing forgotten knowledge seemed alien to many" (p. 418). However, it is not possible to answer all of the questions in undergraduate mathematics examinations merely through memorisation of others' work, which means that it is necessary that students have a thorough understanding of

[46] What Isaac is describing here is similar to the 'match' vs. 'fit' of knowledge as described by von Glasersfeld (1987; see Chapter 2.3.4.6).

the mathematical concepts in order that they may use them for other purposes themselves. Such questions were not met with enthusiasm by most of the participants, with a number of those with strategic ATLs commenting that they would "probably avoid" them (Isaac), if possible. Furthermore, perhaps owing to the fact that his surface approaches meant that he did not develop a conceptual understanding of some mathematical topics, Brian "couldn't do any of the other questions which you had to do from scratch and you hadn't seen before".

Isaac described himself as engaging in a practice which he called 'editing'. In order to be able to answer certain questions which demanded a proof, he identifies similar questions and proofs to those which are in his lecture notes, and adapts them to fit new instances. By aligning two particular statements, he claimed that it was possible "to see the kinds of ways that they're changed slightly from the ones you already have in your notes", meaning that "you can try and 'edit' the content of what's in the proof you have in your notes so that it makes it true for the question you're looking at". This was something which he likened to a cheating of sorts; however, the way in which he identified similarities and could understand how the two proofs might be similar and related, and that the arguments in the 'new' proof which he had constructed held, are not indicative of a surface approach to doing mathematics. Far from being cheating, this seems to actually demonstrate an understanding of the statement and what is necessary to prove its truth. The description that Isaac gave is of a strategic approach, yet it in no way should be viewed as a negative or undesirable approach to answering such a question.

The strategic nature of the students' revision processes is further exemplified in their descriptions of their construction of revision notes. Owing to the fact that they all perceive the volume of material that they need to know as being very large, many of them were very selective in what they included in their revision notes in a bid to be more successful in their examinations. A number of them described themselves as omitting longer proofs from their

notes on the basis that “I know that we’ll never be examined on them” (Juliette) because “they’d take forever or they’d be impossible for you to do and remember” (Isaac), as well as totally disregarding any non-examinable material. Specifically strategic techniques also included searching for, and identifying, patterns in past papers as a guide to revision. Indeed, Mason (1989) comments that “students are interested, as are we all, in minimising the energy they need to invest in order to get through events” (p. 7). This backfired for Camilla when she had revised in a way which relied on “one type of question to come up” in an examination, only for it not to. However, for the most part, past papers reveal “the types of things they want you to be able to recite” (Mandy), and act as a starting point for some of the students’ revision in the sense that they can then use what they see in the past papers to guide what specifically they will memorise and, in the case of computational questions, practise. One-third of students described the use of problem sheets as a revision aid, as they had seen similar questions appear in examinations, and because they act as an indicator of “the types of questions that you can be asked about certain topics” (Mandy).

> *sometimes you have to use it in a way that you’ve used it before, like, on a problem sheet, so as long as you know what you did in your problem sheets and you make sure that you can do those when you’re revising then you’re good.*
>
> (Katie)

Whilst students may initially employ memorisation as a cynical ploy to answer questions which they expect in examinations without actually having a thorough understanding of the mathematical concepts and an ability to prove a mathematical truth, some of the participants reported that they used memorisation as a route to understanding mathematics:

> *Regurgitating the maths each time helps me deepen my understanding of it because I think about the maths each time I read it and write it down*
>
> (Priya)

All of the participants described their memorisation process as being a consequence of repeatedly writing out what they needed to learn again and again, meaning that "it'll eventually sink in" (Christina).

> *I have a 'definitions' list and a 'theorems' list and things like that, and then once something becomes really obvious and I know it, then it doesn't need to go on the next draft of the list.*
>
> (Katie)

7.6.3 – Summary

The ways of working that the students described were all connected to the ways in which they described the form of examination at the end of the academic year. The nature of the examination paper and the availability of past papers shaped their revision process, with the use of past papers acting as a familiar method of revision to that which they adopted at the secondary level. Whilst none of the participants worked with others on their assignments *per se*, they did seek assistance from their peers if and when they became stuck, quite often copying other people when they failed to understand how to answer a particular question. This had the potential to impact upon students' confidence in their ability to do mathematics, with Brian's confidence shattered by his perceived inability to understand and perform well without resorting to copying and avoiding being tested. His method of proving mathematical statements was not to prove, but instead to regurgitate what he had seen before. Should this not be successful, he had nowhere to go. Indeed,

> *[a] student's process of proving not only relies on beliefs about mathematics, in particular about the approach to theoretical thinking, but also generates hot feelings.*
>
> (Furinghetti *et al.*, 2011, p. 8)

These 'hot feelings' were also apparent in Juliette's report of undergraduate mathematics study and, to a lesser extent, some of the other participants. Camilla's choice to cease studying

mathematics in the final year of her joint honours degree serve to demonstrate the impact of her 'proving process' on her decisions and beliefs. Isaac's guilty description of 'editing' proofs suggest that his process of proving and his beliefs about mathematics do not necessarily align as his perceived "cheating" using this approach did not match his belief that he should be able to do such proofs without resorting to this method. This was surprising given his descriptions of this process giving indications that he necessarily must have understood the mathematical concepts and the function of the related proving techniques in order to be assured of its accuracy.

7.7 – Perceptions of Self & Others

The fifth parameter used to describe the enculturation of new undergraduates into the Oxford mathematics COP is not one of the four parameters described by Saxe (1991), but instead a dimension which I added (see Chapter 4.4.5) as during analysis it became apparent that the emotional responses experienced by students during the transition and throughout their degree were of great importance in considering the culture and the nature of the student body. All of the participants were very forthcoming in the way in which they reported their experiences of Oxford mathematics thus far, candidly describing their triumphs as well as their perceived failures, their likes as well as their dislikes, and any changes in their emotional experience.

The three areas of discussion which stood out in particular in the analysis are:

1. the emotional impact of the secondary-tertiary transition;
2. the students' perceptions of their own ability; and
3. the students' enjoyment of undergraduate mathematics study (see Figure 7.7).

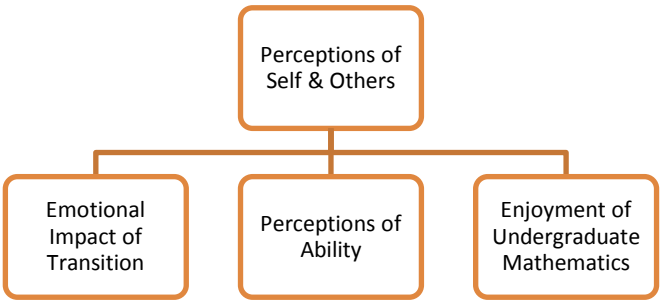


Figure 7.7 - Observational & emotional perceptions of self & others in the undergraduate mathematics context at the University of Oxford

These areas all relate to the four parameters already discussed. Often, the students' descriptions of their emotional responses hinged on their prior experiences of the subject, when they were very successful. The activities which they engaged in as part of their degree often had an emotional response. For example, Brian described his revision processes as having a negative impact, reporting that "It actually made me depressed". The conventions in the culture have the potential to support students' personal experiences of the subject, as the pastoral role of

the tutor can serve to ensure that the students are being successful in their studying, as well as enjoying it. The social interactions which the students have with fellow apprentices, as well as masters (see Chapter 4.2), impact on their own perceptions of self, with the vast majority of participants reporting their performance and perceived understanding compared to that of their peers, with whom they socially interact in tutorials.

7.7.1 – Emotional Impact of Transition

As described in Chapter 7.3 the transition between secondary and tertiary mathematics study can be an emotional time for new undergraduates as they adjust to a life away from home, studying an unfamiliar subject. Clark and Lovric (2009) describe the secondary-tertiary mathematics transition as "a modern-day rite of passage" (p. 755) in the sense that "A rite of passage is a sequence of events that enables an individual to deal with, and overcome, a 'life crisis'" (p. 756). Students must go through a separation from high school, a liminal phase (a.k.a. the transition) from high school to university, and incorporation to university study, both mathematically and socially. All of the participants described the transition as being difficult for them, mainly citing the unfamiliarity of topics such as analysis for this. Robert and Schwarzenberger (1991) describe this as a "long period of confusion" which "gives rise to a fundamental discontinuity in the difficult transition from elementary to advanced mathematics" (p. 129). This was compounded by the fact that the difficulty they were experiencing with mathematics was an entirely new phenomenon for them, having previously been very successful in their mathematics studies whilst at school (see Chapter 7.3.1), something which Malcolm found "demoralising". Indeed, the majority of mathematics undergraduates at Oxford no doubt transitioned from a situation "where they easily excelled their school-fellows whereas at university they compete with their peers" (Hall, 1982, pp. 600-601). Having once been a big fish in a small pond (see Chapter 3.2.2), students become subjected to a situation where there is a possibility of them becoming very small fish in a big

pond. Not all new undergraduates at Oxford will be able to continue to be 'top of the class' at university as they were at school. Approximately half of the participants described this as having a negative impact on their confidence, for example:

> *because I was so shell-shocked with all of the proof and analysis and things like that, that it made it very daunting. Going to lectures was something I dreaded because I knew I would get there and not understand, and I dreaded problem sheets and tutes because I knew that I wouldn't be able to do it all*
>
> (Juliette)

> *Horrible! I'm not as confident as I was, although I know why it's harder and why I'm not doing as well, so I don't get wound up about it. I know that if I work harder then I'll be able to do better, which is why I'm making more of an effort this year.*
>
> (Isaac)

> *It was awful. I spent a lot of time in tears at the beginning because I just didn't know what was going on and I wasn't doing very well, so I was worried that I'd fail or get kicked out.*
>
> (Katie)

However, the majority of the students described themselves as having been able to recover from this initial shock to the system once they had adapted to the 'new' mathematics that they were studying, and accepting that they were no longer going to be able to get the perfect score on their assessment. This is something which has been identified in the literature, with the secondary-tertiary mathematics interface being a highly-researched area in mathematics education. Furthermore, Rodd (2002) has written of the 'hot' nature of undergraduate mathematics, which provides many challenges for new students, including on a personal and emotional level. For a quarter of the students, it seemed that the process of revision in their first year served to increase their confidence after their understanding of the mathematical concepts was developed and consolidated. Juliette, for example, credited it with helping her to get "over the initial panic" that she experienced. Ryan also cited examples of ways in which he began to adapt to the new culture, saying that it was necessary that he "took some time to get

used to work taking longer and requiring deeper thought" before he could feel more 'at home' in his studies.

7.7.2 – Perceptions of Ability

All of the participants described themselves as being less successful at university than they were at school, with a large number of them reporting difficulties in understanding mathematical concepts as well as performing to a standard that they would like to in assessment. For example, Juliette ascribed her memorisation of certain proofs ready for examinations as being due to the fact that she did not believe that she had the necessary conceptual understanding to be able to reconstruct these proofs herself.

The perceived abilities and understandings of other students were discussed by the majority of the students interviewed, with most of the students claiming that they believed themselves to be weaker than most of their fellow tutees. For many of the participants, the tutorial environment facilitated their ability to make comparisons between themselves and the other students in their group. More than half of the students interviewed made comments pertaining to their inability in mathematics compared to their peers, believing themselves to be amongst the weaker members of their tutorial group:

> *I think I probably struggled more at the start than most […] It seemed that way from knowing other people in my tutor group*
>
> (Malcolm)

> *I think I'm normally towards the less understanding and less confident end of the group.*
>
> (Isaac)

> *I still didn't quite understand what was happening properly but everyone else did so I kind of got left behind without having a way of getting back in line.*
>
> (Brian)

All of the students made such comparisons without prompt, and none of them claimed to believe that they were at the higher end of the achievement spectrum, although Mandy did suggest that her use of mind maps as part of revision – and her peers' apparent neglect to use such techniques – meant that they were either uninterested in understanding relationships or unable to do so, implying that few of them were able to see the relationships between the different mathematical concepts that they were studying.

All of the students in Years 2-4 discussed their previous year's examination results, which ranged from a lower second-class result to a first-class result. Implications in the literature (including Trigwell & Ashwin, 2003) that a deep ATL may result in students being more successful in assessment do not appear to hold in the case of this particular sample as, of the four who achieved a 2:2 in the previous year, one was deep, two were classified as having strategic ATLs and one as surface. The only first-class result, obtained by Christina, was associated with a deep ATL; however, Katie and Sabrina's deep ATLs corresponded with a 2:2 and 2:1, respectively. This may be attributed to the idea that "a student with high orientation towards a deep approach, but who is not particularly competent, may perform less well than a student with a 'highly polished' surface approach" (Cuthbert, 2005, p. 244). For example, time spent understanding the mathematical concepts may be done at the expense of learning them for an examination, thus hindering the students' potential to be successful in the examination.

7.7.3 – Enjoyment of Undergraduate Mathematics

As well as ATL not appearing to show any correlation with attainment, there was no apparent relationship between ATL and students' enjoyment of their degree. Brian and Juliette both described a particular dislike for undergraduate mathematics, describing themselves only refraining from dropping out because of enjoying the other aspects of being at Oxford, and having made it this far through:

> *I hate it. I hate not understanding what I'm studying and don't find anything interesting. I kind of can't find it interesting because to do that I'd need to have an understanding of what they're talking about.*
>
> (Brian)

> *I'm enjoying being at Oxford and being a student but I don't like what I'm doing. I don't hate all of it, but I don't actively enjoy myself. I don't look forward to lectures and tutes and enjoy doing problem sheets. They're just a… kind of sacrifice. Like, to be here and enjoy being at Oxford and spending time with my friends, I have to do this work. It's a condition of me doing that, and I just get on with it.*
>
> (Juliette)

For both of these students, it appears that a mismatch between expectations and reality in terms of the content of the degree played a part in this, with the disillusion and impact on their confidence which came as a consequence of a performance they considered to be unsatisfactory also influencing their emotional experience. They have both 'cooled-out' of mathematics (Daskalogianni & Simpson, 2002), through arriving at university with inaccurate conceptions of undergraduate mathematics study, and finding it so difficult that they become disengaged.

Brian's ASSIST revealed a surface ATL, consistent with his descriptions of his activity structures, whereas Juliette's revealed a strategic ATL, which also correlated with her accounts. All of the students with deep ATLs described themselves as enjoying their degree and the mathematics

that they were studying, whereas the descriptions given by the students with surface ATLs were more mixed. For example, Sabrina (deep ATL) said:

> *I'm actually really enjoying it – it's challenging in a good way, and I really like most of my options.*

Conversely, some of the students with strategic ATL described themselves as finding mathematics more interesting and enjoyable when they began to revise for their examinations. Isaac said that he enjoyed "the maths when I'm reading about it and understanding what's happening because it's really cool", but that his revision practices turn mathematics into something which he no longer enjoys. Conversely, Jacob described the opposite process, claiming that the revision process, when he began to familiarise himself with the concepts, resulted in his interest peaking "as I find this is when I tend to actually understand it!" Mandy described what she studied as being fairly enjoyable, although conceded that "Some of it is a bit crap", whereas Katie was very enthusiastic about mathematics and her enjoyment of the subject.

Students in later years of their course, who are able to take advantage of the ability to specialise in particular areas in mathematics, spoke fondly of this opportunity:

> *I'm enjoying everything much more now that I can choose the modules that I want to study. It means that you can do something that you like rather than having to do certain things. It makes for a much more enjoyable time! Haha.*
>
> (Katie)

> *if you like it, you're more likely to spend time on it because it's not a chore, so then you're more likely to understand and do well. I did a lot better the last couple of years on pure maths than applied maths, so I think that's why*
>
> (Camilla)

7.7.4 – Summary

The participants' reports of their experiences of mathematics at the secondary and tertiary level revealed the different perceptions that they had of themselves as mathematics students at each stage. Whilst all of the participants were very confident in their mathematical ability at A-level, performing at the very top-end, they all found undergraduate mathematics – at least initially – to be very difficult to adapt to due to the differences which they perceived between school and university mathematics. Whilst most of them were able to overcome this, Brian and Juliette, in particular, found the transition and their experiences and difficulties to be too great. Consequently, neither of them has a strong self-belief as a mathematician – Brian, in particular[47] – and they show no liking for their course or their subject. It appears that the only thing preventing attrition on their parts is the social relationships that they have developed with their peers, as well as the fear of embarrassment. The participants were all able to make comparisons between themselves and their peers in terms of their mathematical ability, something reinforced through their experiences in tutorials when they could see for themselves others' understanding of mathematics, as well as in social learning situations when doing problem sheets.

[47] This is consistent with claims by Dart *et al.* (1999), who found that students with a lower self-concept were more likely to have surface ATLs.

7.8 – Conclusion

The majority of participants believed that they were towards the weaker end of the ability range in their year group; however, this gives just cause to question the motives for the thirteen students in this sample for taking part in the study. It could be that they were interested in taking part in mathematics education research, it could have been because they were not having a pleasant time and wanted to 'vent' their frustration, it could have been because they were having a wonderful time and wanted to share that with someone, or many other reasons. There was a wide range in the students' beliefs and perceptions in this respect, from Brian's complete disengagement and severe struggles, to Christina's active participation in the undergraduate mathematics COP and success in examinations. The range of the students' experiences and perceptions of themselves was wide, yet their reports of both present and past mathematics study shared many similarities. Whilst all of the participants came from very similar backgrounds in terms of their secondary mathematics qualifications and study, they came from a range of school backgrounds – from state comprehensives, to grammar schools, to independent schools and public schools. Therefore, the consistency in the participants' responses regarding their prior understandings and activity structures should be of great interest to educators and suggest that inaccurate conceptions of mathematics are also fostered in very privileged environments, as well as more mainstream ones. However, this is in contrast to work by Crawford *et al.* (1994) who found that students with more fragmented conceptions of mathematics were more likely to adopt surface ATLs.

As previously mentioned, all of the participants except those with non-standard A-level qualifications or the IB studies Further Mathematics to A-level and achieved at least an A-grade in both mathematics courses. Therefore, the concerns raised by these students about the suitability of either or both A-levels in preparing students for undergraduate mathematics (see Chapter 7.3.1) have great weight in the discussions relating to the content and structure

of the A-level. The inaccurate conceptions which many students described – in particular a lack of awareness of proof, and no experience of doing them themselves – are in line with those described in the literature, and are compounded by the fact that these students had an additional level of experience in terms of their prior understandings. For the admissions tutors, the Oxford admissions process serves to help them to identify the brightest mathematical minds, those students with the greatest enthusiasm and capacity to learn mathematics, for university study. For students, whilst it might not be apparent to them at the time, the admissions process serves to highlight to them that undergraduate mathematics might present very different challenges to A-level Mathematics and Further Mathematics. Specifically, it implies that the nature of undergraduate mathematics might be very different to secondary mathematics, and the difficulties that they all experienced in answering the admissions test and interview questions served as a 'wake up call' for a number of them, making it apparent to them that they were no longer necessarily going to be the best student, able to answer all of their problem sheet and examination questions correctly and with relative ease.

The conventions of the University in terms of the provisions for teaching and learning were described by the participants as having shaped their learning and working practices. For example, the predictable nature of undergraduate mathematics examinations caused many of the students to adopt strategic approaches to learning mathematics in order to answer the examination questions with ease. Most of the participants described their revision processes at A-level as being minimal and, for the most part, revision was more of a 'practice as preparation' than a revisiting of material to consolidate understanding and commit it to memory. It is perhaps because the revision techniques that students adopted at each level were so different that many of the students appeared to think that actively memorising definitions, theorems and proofs was not the 'proper' way of doing mathematics. However, the practices that they described themselves as engaging in when it came to answering atypical and unseen questions – such as editing – show that many of the students did in fact

have some level of mathematical understanding of the concepts. For Brian, however, this did not appear to exist as he memorised mathematics from his lecture notes in a bid to reproduce it in examinations, attributing any success he had in answering examination questions to being lucky in terms of the questions asked and what he had managed to learn. Indeed, a significant proportion of even the more engaged students who rely on merely reproducing proofs in examinations are unable to follow through and use such proofs in more involved questions (Schoenfeld, 1989).

The students all described themselves as engaging with each other socially in a learning setting in a very similar way, although the degree to which they actively participated in the undergraduate mathematics community of practiced varied greatly. Whilst Christina was heavily involved in the Invariants and had a large number of friends who also studied mathematics who were from other colleges, the majority of the other participants did not engage with fellow mathematicians in the same way. The collegiate system at the University, in this way, was criticised by a number of the participants for stunting their ability to make friends outside of their own colleges, as most of them only socialised with other mathematicians who they met in their own tutorial groups. However, all of the participants except Christina engaged in extra-curricular activities which were based outside of the Mathematical Institute, exposing themselves to a wide range of experiences and groups of friends from different courses, colleges and universities.

For this reason, there are a number of significant differences and similarities in the experiences of all of the participants which are of great interest. If it is that these students all came from different academic backgrounds, but studied the same qualification, but went on to have similar difficulties in the secondary-tertiary mathematics transition and describe the A-level as being insufficient, then there is something very important to consider regarding the nature of the A-level. Despite having different tutors and different groups of friends in mathematics, the

majority of participants described themselves as engaging in very similar study and revision practices, which were heavily influenced by the examination and problem sheets ('artefacts') and the conventional ways in which other students studied and revised. The ways in which they described their activity structures were also described as being very conventional within the culture, and also align with the descriptions of student study in mathematics which can be found in the literature.

The students' comments regarding the nature of A-level, OxMAT and undergraduate mathematics questions support the findings in Chapter 6 from applying the MATH taxonomy to these three types of assessment. Furthermore, the descriptions used by students regarding their working practices go some way to supporting the definitions of deep, surface and strategic ATLs as identified by the ASSIST in Chapter 5. However, one must question whether the similarity in students' descriptions of their revision methods is of significance when coupled with the students' ASSIST results which (with the exception of Brian) indicate that some of them predominantly adopt deep ATLs and others strategic ATLs. Whilst Oxford may be considered to be 'special' in many ways and for many reasons, the similarity between some of these findings and those from empirical research conducted at other institutions across the country and the world suggest that even the teaching methods – and the calibre of students in attendance – cannot prevent the Oxford student population from experiencing similar difficulties in undergraduate mathematics and engaging in similar activities when learning and doing it.

Descriptions by students of their own ATLs were interesting when considering them alongside the results of the ASSIST. For example, whilst Mandy's ASSIST responses suggested a predominantly-strategic ATL, her ways of working suggested otherwise. Her use of mind maps as a means of actively endeavouring to understand the basis of the mathematical concepts that she was studying suggest a deep approach; a student with a deep approach primarily

seeks to understand and any memorisation could be "viewed as an unintentional by-product" of their actions (Kember, 1996, p. 343). Conversely, Brian's descriptions of his ways of learning were very consistent with definitions and suggestions in the literature (see Chapter 2.1.1). The reports of activity structures by the other students whose ASSISTs suggested a predominantly-strategic ATL (Camilla, Malcolm, Isaac, Jacob, Juliette) were also consistent with the traditional definitions of a strategic ATL (see Chapter 2.1.4). The ways in which these students described themselves as learning and practising mathematics in ways which reflected the assessment they would face could be used to describe them as 'expert' learners, in the sense of Ertmer and Newby (1996), as they play the 'assessment game' (Entwistle *et al.*, 1979a). However, there was nothing significantly different in most of the descriptions given by these students and those whose ASSISTs suggested an orientation towards a deep ATL (Priya, Sabrina, Katie, Christina, Ryan, Ethan). What appears to be different between these two groups of students is the level of enthusiasm for the subject that they displayed – one of them has mathematial tattoos! – as opposed to their ways of working on it and learning it. There also appeared to be a difference between the students in terms of whether the difficulties they experience in the mathematical transition are seen as a pleasurable challenge, which they have suitable ways to address, or an insurmountable challenge, which they respond to by taking a surface ATL.Whilst they might behave in the same way as their peers, their responses to the following statements generally scored more highly than the students identified as having predominantly strategic ATLs.

For example:

Table 7.8 - ASSIST items relevant to interview comments

Question	Statement	Difference
3	Often I find myself wondering whether the work I'm doing here is worthwhile.	Strategic scored higher
12	I tend to read very little beyond what is actually required to pass.	Strategic scored higher
26	I find that studying academic topics can be quite exciting at times.	Deep scored higher
39	Some of the ideas I come across on the course I find really gripping.	Deep scored higher

This is only based on a small sample size so it would be inappropriate to conduct statistical testing; however, this does provide a possible challenge to assumptions about ATLs, particularly in relation to the results in Chapter 5.

Chapter 8: Discussion

Synthesis of Undergraduate Mathematicians' Experiences of their Course Relating to the Mathematics, its Assessment & the Community

8.1 – Summary of Study Aims

This study was concerned with the changes in mathematical culture for post-compulsory mathematics students at the University of Oxford, specifically pertaining to the role of questions and approaches to learning. The research questions posed for this study were:

4. How do undergraduates' experiences of studying mathematics at Oxford change throughout their university career?
    a. What challenges do students report facing in each year of study?
    b. How do students report their approaches to learning and studying mathematics?
5. Based on previous experience of mathematics, what challenges lie in Oxford students' enculturation into a new mathematical environment?
    a. What types of skills and challenge are tested by A-level Mathematics and Further Mathematics questions?
    b. How does the OxMAT's assessment of students' mathematical understanding compare to A-level Mathematics and Further Mathematics?
    c. How do undergraduate mathematics examinations compare to the A-level and the OxMAT?
6. What is the relationship between students' approaches to learning and the challenges they perceive in undergraduate mathematics assessment at the University of Oxford?

8.2 – Summary of Research Findings

Using a mixed methods approach, bringing together data from a self-report survey on students' ATLs, the application of a taxonomy to categorise the type of questions posed by A-level Mathematics and Further Mathematics, the OxMAT and undergraduate mathematics examinations, and interviews with current Oxford undergraduate mathematicians, the findings of this research contribute a wealth of knowledge concerning the suitability of the ATL framework to the undergraduate mathematics context, the importance and contribution of the Oxford admissions process on students' experiences, and the impact of problem sheets as opposed to examinations on students' ATLs, understanding and conceptions.

Chapter 5 presented data collected from the ASSIST (Tait *et al.*, 1998), which supports claims that students' ATLs can change when encountering new teaching approaches, but that they can also remain stagnant. Analysis suggests that students' ATLs are significantly different at the beginning and the end of their first year of undergraduate mathematics study at Oxford, but the difference between ATLs at the end of Years 1-4 are not significantly different. Women were found to score significantly higher on the surface scale, with this mainly relating to their confidence in their ability to do mathematics and their engagement with the subject. Significant differences between the year-groups when considering individual items in the ASSIST were chiefly as a consequence of fourth-year students' familiarity in studying mathematics, increasing their confidence compared to less experienced students. Furthermore, the elective and selective nature of the fourth year means that it is more likely that these students will be successful and engaged than those in earlier years. Whilst not statistically significant, there were differences between Year 1 and Year 3 in the sense that a smaller proportion of students in Year 3 predominantly adopted deep ATLs (8.0%) than those in Year 1 (17.4%), which may be attributed to the fact that, in the third year, students are experiencing considerable pressure on their studies because of their final examinations. First-

years are yet to experience the examination system insofar as seeing what their ATLs result in in terms of examination performance, whereas this will be something that third-years will be much more familiar with.

Whilst data regarding students' examination results were not collected with the ASSIST, this was a topic which was broached during the student interviews. No correlation appeared to exist between ATLs as determined by the ASSIST and academic outcomes; for example, of the students who predominantly adopted deep ATLs, one achieved a first, one a 2:1 and one a 2:2 in their examinations.

Data collected using the MATH taxonomy (Smith *et al.*, 1996) in Chapter 6 suggested that A-level Mathematics and Further Mathematics, the OxMAT and undergraduate pure mathematics examinations at Oxford provide very different challenges for students. Whilst A-level Mathematics and Further Mathematics primarily task students with performing routine use of procedures, the OxMAT mainly challenges their ability to make justifications, interpretations, conjectures and comparisons, and undergraduate mathematics examinations predominantly require students to demonstrate factual knowledge and fact systems on top of those types of skills tested in the OxMAT.

Interviews with current students covered a myriad of topics relating to their experience of studying mathematics at the secondary level, applying to Oxford to read mathematics and actually studying mathematics at Oxford. Chapter 7 reported the experiences of 13 students using a revised version of Saxe's four parameter model (Saxe, 1991), also drawing on the specific stories of five students to illustrate the varying experiences in the interview participants. The majority of students described their experiences of school mathematics as being very successful and the revision practices they engaged in acting as a form of examination preparation as opposed to knowledge review. Whilst interview and admissions test experiences unsettled a number of students through challenging their conceptions and

understanding of mathematics, they were nonetheless left surprised and disturbed by first-year studies of analysis and proof. Students attributed this to their lack of exposure to mathematical proof at the school, describing themselves as unprepared for undergraduate study in that respect, a finding that accords with anecdotal and other comments from universities and with current government policy at A-level. This is also something which mathematics professors in UK universities recognise as being a problem, with 65% of them blaming differences in the culture of mathematics at school and university for this problem (Thomas *et al.*, 2013). Though the ATLs of the interview participants as determined by the ASSIST comprised a mixture of predominantly deep, surface and strategic approaches, the descriptions that the students gave of their working and revision methods were remarkably similar. The differences between the students' reports of study at Oxford tended to relate to their confidence in their ability to be successful, as well as their understanding of mathematical concepts.

8.3 – Confidence, Guilt & Despair: The Approaches to Learning Framework in the Context of Undergraduate Mathematics at Oxford

Whilst there is a vast quantity of literature regarding empirical studies in higher education of students' ATLs (see Chapter 2.1), there is very little which concerns undergraduate mathematicians. Furthermore, there is not a great deal with a longitudinal element (e.g. Kell & Van Deursen, 2002). Though this study does not incorporate a longitudinal element as such, the comparison between the year-groups was intended as a means of achieving a similar effect. Whilst Watkins and Hattie (1985) reported students' scores on the deep scale of an ATL inventory as declining over the course of three years at university, this was not the case for this sample. The only difference identified between the years was a decrease in the average score on the surface scale between the third and fourth years, which may be attributed to the elective and selective nature of the MMath year. The stagnant nature of students' ATLs across the four years – with the exception of changes identified in Year 1 students between Sweep 1 and Sweep 2, wherein there were significant increases in the proportions of students adopting predominantly deep and surface ATLs ($p = .000$) – may be reflective of the stability in the students' learning environment and pedagogy across their degree. The most important distinction between the participants was highlighted by the large proportion of students identified as mainly using strategic ATLs (73.7% in Sweep 2) which contrasted with the descriptions given by the 13 interview participants.

Women were found to have scored significantly higher on the surface scales and significantly lower on the deep scales (p=.008) in Years 1-3, although not in Year 4. This confirms the findings of Greasley (1998), who claimed that it was more likely for women to adopt strategic and surface approaches to men. Furthermore, use of Entwistle and Tait's (1995) revised version of the ASI also found that men scored higher on deep scales and women higher on

surface scales (Duff, 2002; Sadler-Smith, 1996). However, there are inconsistent findings in research investigating relationships between ATLs and gender (Richardson & King, 1991; Severiens & Ten Dam, 1994). For example, Anthony (2000) said that there was no difference in ATLs of men and women in undergraduate mathematics, and Macbean (2004) argued the same of courses where mathematics was a service subject[48]. Similar could be said for studies by Zeegers (2001) and Wilson *et al.* (1996), although their participants were science and psychology students, respectively, thus severely limiting the applicability of their findings to this undergraduate mathematics context. Furthermore, the higher score of men on the deep scale and of women on the surface scale identified in Chapter 5 is in direct contrast to studies by Biggs (1987b) and Watkins and Hattie (1981), which found the exact opposite to be true, although with undergraduate psychologist participants.

When individual items of the ASSIST were examined, it revealed that the source of the gender differences in scale medians was primarily due to responses to certain questions which concerned confidence. This may be attributed towards women being less mathematically confident than men at the undergraduate level (Astin, 1977, 1993; Becker, 1990; Higher Education Research Institute, 1991; Pascarella, 1985; Pascarella *et al*., 1987; Sax, 1994; Smart & Pascarella, 1986), and more likely to answer questions in a self-deprecating manner. They are more likely to underestimate their abilities (Drew, 1992), and they "express… fragile identities more often or at least more readily" in the context of tertiary mathematics (Solomon *et al.*, 2011, p. 1). However, whilst this was a finding in the quantitative data of the ASSIST, the women who participated in the interviews (7 of the 13) did not all describe themselves as lacking confidence or as suffering considerable difficulties. In fact, the only negative feelings and experiences reported were by Brian and Juliette, with a number of the female participants

[48] These were courses in the research institution's Physics and Astronomy Department and Biochemistry Department.

describing themselves as enjoying their subject and being successful at Oxford as well as at school.

At the secondary level, students' strategic ATLs took on a different form to the strategic ATLs of undergraduate mathematicians. This related in particular to the role of past papers in the culture, and the understanding of what constitutes learning. At A-level, students reported themselves as having completed past papers as their main method of revision – sometimes doing tens of papers in each module – because the papers were so similar each exam session and because the culture of 'doing mathematics' at school was characterised by mechanistic processes and drill exercises of familiar problems. All of the participants described themselves as being able to understand mathematical processes at school without difficulty, which contrasted with their reports of undergraduate mathematics study where concentrated attempts to understand formed a large part of the revision process. This, and the necessity to *memorise* mathematics, were the primary revision activity for most of the participants, which were then supported by past papers. However, in the context of undergraduate mathematics, the past papers took on a different role wherein they acted as guidance for what students needed to memorise and revise instead of being used to practise what they already understood. Whilst secondary revision often solely consisted of practising mathematics questions, tertiary revision consists of reviewing, refreshing, consolidation and learning.

This may be attributed to the differing nature of the questions posed at each level – whilst 71% of A-level Mathematics and Further Mathematics marks came from Group A questions (Smith *et al.*, 1996), the majority of which required routine use of procedures, the majority of undergraduate mathematics examination marks come from Group C questions. However, on average, 32% of marks are for Group A questions. In this context, however, this is for reproduction of factual knowledge, often in the form of asking students to state definitions or theorems. These 'bookwork' questions are described by students as requiring rote

memorisation, though a number of them described themselves as engaging in memorisation of existing proofs in their notes or on problem sheets in order to reproduce them in examinations because they did not believe themselves capable of reconstructing the proofs themselves. It is this motivation for memorisation which appears to be the distinguishing feature of undergraduate mathematicians' ATLs, rather than choosing from one of deep, surface and strategic to describe their approaches. Such a practice is something which has already been identified as problematic in tertiary mathematics education, as students have been found to be "memorising theorems and proofs at the possible expense of meaning or significance" (Jones, 2000, p. 58).

As Group A questions are synonymous with factual recall and routine procedures, it might be easy to suggest that Group A questions all foster a surface ATL. However, the 'comprehension' questions which are part of Group A questions require students to be able to demonstrate an understanding of a concept, something which they may not be able to do merely by rote-learning something. Furthermore, questions in Group C which might appear to be challenging students in a particular way could lead someone to conclude that a deep ATL is required to answer them. However, should these questions be similar or the same to something which the student has seen before, a surface ATL could be employed in order to answer such questions successfully. Therefore, whilst there is a crude relationship between Groups A and C and surface and deep ATLs, respectively, there are exceptions where genuine understanding must be demonstrated and situations when students may strategically use memorisation in order to answer them correctly.

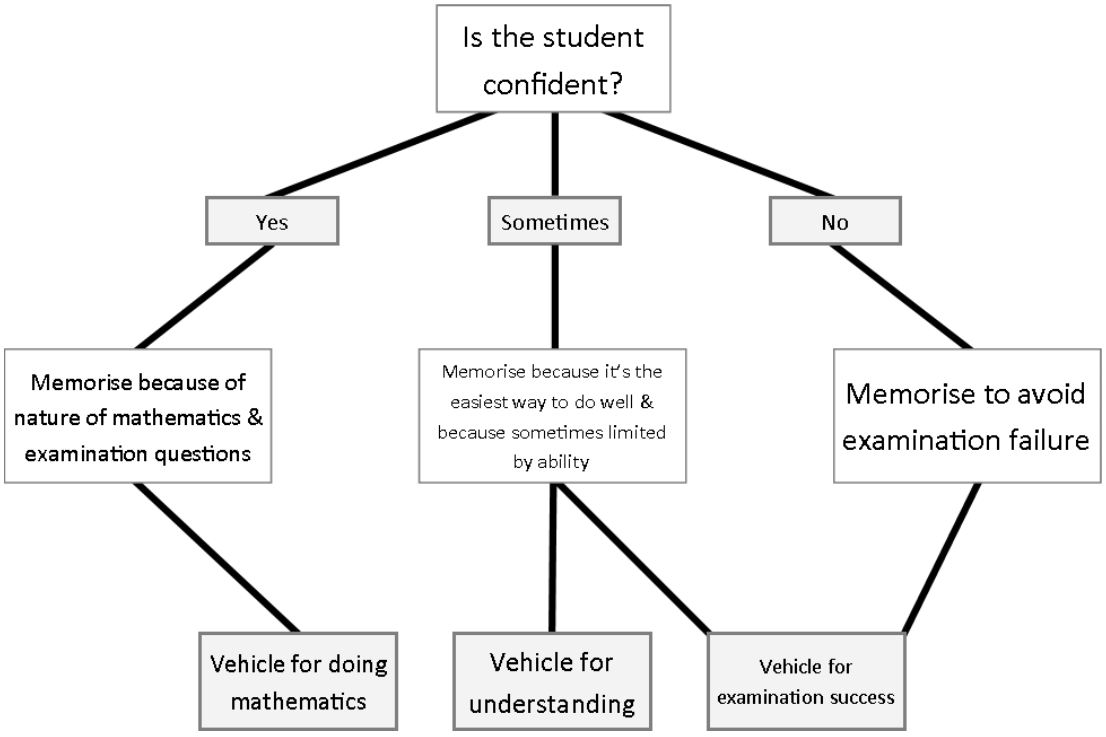
All interview participants described a strategic element to their learning, though only six of them were described as taking predominantly strategic ATLs by the ASSIST. Within descriptions of their revision strategy, there were differences in students' confidence in their ability to do mathematics which appeared to make a difference. There appeared to be students who were

confident in their use of memorisation, perceiving it to be a necessary part of being a mathematician. These students memorise definitions and theorems in order that they may then use them to do mathematics. Other students communicated a feeling of guilt at using memorisation, as they committed existing proofs of well-known theorems from their lecture notes to memory because they did not feel capable of doing them themselves. For these students, such 'deep memorisation' (Meyer, 2000; Tang, 1991; Marton *et al.*, 1997) could act as a vehicle to understanding, much in the same way as the Chinese paradox (Kember, 2000; Marton *et al.*, 1992). Their motivation is examination performance, although understanding may be a by-product of their memorisation. Indeed, Baumslag (2000) argues that, in tertiary mathematics, "Deep and sophisticated ideas take time to sink in, and require repetition and contemplation to be absorbed" (p. 99), and Entwistle (2001) claims that memorisation "makes an essential contribution to understanding". Finally, some students chose to engage in extensive memorisation of proofs and answers to some problem sheet questions out of despair, failing to understand sufficient mathematics to be able to answer examination questions otherwise. A lack of understanding of the mathematical concepts means that understanding is a very unlikely consequence of memorisation.

However, rote learning is a "potentially safe strategy" (Diseth & Martinsen, 2003, p. 204); indeed, it is important to remember that a student who is better at memorisation than a student who is weaker at trying to understand on the conceptual level may be able to perform better in examinations. That is, memorisation out of despair might achieve better assessment outcomes than memorisation for being a mathematician. This is practically possible in undergraduate mathematics examinations, when the proportion of Group A questions requiring 'factual knowledge and fact systems' can be sufficient that a student may achieve an upper second-class honours result for an examination purely based on demonstrating skills in this area – it is possible for an average of 45% of available marks in undergraduate examinations to be awarded for Group A questions, with two of the papers subjected to the

MATH taxonomy in Chapter 6 permitting students to achieve 63% of marks for reproduction of facts.

Crucially, the confident students are responding to the question types and a *requirement* to memorise mathematics to answer questions, whereas guilty or desperate students are responding to the *facilitation* of memorisation given by the nature of the examination questions.



The utility of memorisation in understanding is an important distinction to make when considering these motivations. It has the power to be the root of understanding, but this is not always possible.

> *Usually it is best to understand something before committing it to memory, but it can be a useful tactic when one has failed to understand something, to learn it off by heart. Understanding can then follow subsequently… when one cannot make further progress in studying something, one can profitably learn the item off by heart. It is then available for use, and furthermore, a subsequent attempt to*

> *understand the new idea is then often successful. This is better than the alternative, of simply giving up when one just cannot understand something.*
>
> (Baumslag, 2000, p. 68)

As students' ATLs are important to consider in the sense that they can affect their understanding and performance, this 're-characterisation' of students' ATLs in the context of undergraduate mathematics highlights the role of confidence in the type of memorisation that they engage in. This is an aspect which is largely ignored as being a major factor in the literature, with Campbell *et al*. (2001) describing a student's strategy as being dependent upon "a complex interaction between, first, the student's pre-existing beliefs about knowledge and learning, and general pre-disposition towards particular ATLs, and, second, the student's perceptions of the learning approach that is required by the educational context" (p. 175). Use of past papers shapes the latter, and school experiences the former. The description given by the interview participants suggest that there is very little variation in the approaches that they use, although the motivations behind them are key. It is impossible to deny that all genuine students want to perform well in examinations, so assessment will always play a role in the approaches they take, but the affective and cognitive factors of confidence and understanding are of paramount importance. Confidence may, of course, be impacted by understanding and associated attainment, meaning that ensuring that students are equipped with the tools to understand mathematics is the key to steering them towards using memorisation in a legitimate way and opening themselves up to *doing* mathematics rather than *reproducing* it.

8.4 – Deep or Cheat: The Differing Role of Problem Sheets & Examinations

> *a degree programme should offer students the (somewhat nebulous) idea of what it is to be a mathematician and the possibility of becoming one… In particular, students should realise that mathematics is not just a formulaic process… a mathematics degree should challenge its students.*
>
> (Good, 2011, p. 15)

The comments made by the 13 interviewees about their revision processes and the types of examination questions that they were posed suggest that there was a very 'formulaic process' involved in their mathematical activities. Whilst the majority of marks awarded in undergraduate examinations were found to be for Group C questions – the most desirable possibility (Smith *et al.*, 1996) because it presents students with challenges which they may not have had the opportunity to rehearse, instead using their mathematical understanding and thinking to be able to answer the question – a significant proportion of marks were available for answering questions which required the statement of definitions and theorems, as well as for giving proofs which students already had in their lecture notes. Furthermore, as well as it being found to be the case for Pure Mathematics 2 examinations in 2006 and 2008 in Chapter 6, the students themselves are aware that a significant proportion of marks are available in examinations for doing 'bookwork' – that is, for memorising the content of lecture notes and reproducing it in an examination. A number of students justified their use of memorisation outside of the 'legitimate' context of memorising the statement of definitions and theorems based on the types of questions posed in examinations.

Since "Assessment drives what students learn" (Smith & Wood, 2000, p. 126), past papers have a very important role in shaping students' approaches to revising for examinations. According to the students' reports of their and their peers' studying practices, the shape that these approaches take in the context of undergraduate mathematics at Oxford involves

strategically memorising mathematics in order to reproduce it. This may be because they feel unable to do the mathematics required on their own and so this provides a way of answering a question whilst by-passing the need to understand. Whilst it is possible that the process of memorisation can aid students' understanding of the mathematical concepts, it is also possible that the memorisation can be 'blind' and that students are reproducing something that they do not understand. The way in which such questions are posed means that the marker cannot know either. Indeed, in Chapter 7.6.2, Isaac commented that, in these questions:

> *I can tell you it and prove to you that I know the proof. But I can't prove to you that I understand the proof if you just get me to write the proof*

Conversely, when discussing the role of problem sheets, most of the interview participants described them as being integral to their learning and understanding mathematics. These weekly assignments act as formative (William & Black, 1990) assessment and the marks do not count towards the students' degree outcome and, as such, all of the students described themselves as having either worked with or having consulted another student or students regarding the problem sheets when they became stuck. Whilst some students asked their peers for hints when stuck, the majority confessed that they had, at some point, copied what someone else had done when they were unable to. However, for the most part, they made an effort to understand the solution, and they would be able to explore this further in the accompanying tutorial.

The support of the tutorial means that whether or not the student has engaged with the question 'deeply' and been able to answer it themselves or has cheated and copied one of their peers, they will be able to learn more about the concept and the solution from a 'master'. Unlike end-of-year examinations, problem sheets do not require students to state definitions or theorems or to construct proofs which already appear in their lecture notes (University of Oxford Mathematical Institute, 2008; see also Appendix 8.1); therefore, there are no 'factual knowledge and fact systems' questions to be answered in them. This means that it is necessary

for students to engage with the course material in order to come up with a solution to the questions and, as such, memorisation can play no role. Anything which might constitute a 'routine use of procedures' would not necessarily be so because the problem sheet would act as the first occasion when they could practise a procedure, and as such it would not be routine. Furthermore, problem sheets do not usually contain several, similar questions requiring students to perform calculations or do any mechanistic mathematics.

Interaction such as that in tutorials at this level has been found to be "successful in fostering students' participation in mathematical augmentation and their acquisition of important concepts and methods" (Yackel *et al.*, 2000, p. 278), with interaction between students and lecturers being found to be "more conducive for students to develop conceptual understandings of the material" covered (Yoon *et al.*, 2011, p. 1107). Reflection is facilitated in these sessions, and allows students to engage in 'cognitive apprenticeships' (Farmer *et al.*, 1992) with their tutors in order to develop their mathematical understanding.

> *relationships between undergraduate mathematicians and their tutors might be perceived as contributing to the constitution of a mathematical community of which the undergraduates become a part. Such a community has characteristics of mathematics, the doing of mathematics, the social structures of the academic community of mathematics, the university structures within which the mathematical academy grows, and so on. Tutors, as experienced practitioners in this community, might be seen as encouraging the students (newcomers, peripheral participants) to fully participate within the community*
>
> (Nardi *et al.*, 2005, p. 289)

However, this is not to say that problem sheets do not assess students' mathematical understanding – in fact, assuming that a student completes a problem sheet on their own, the solutions to a problem sheet may give a better indication of their mathematical understanding than an examination script. If it is possible to earn enough marks to pass (or perform well) through a memorisation which could be done by someone who had no mathematical background in examinations, then mathematical understanding is not being assessed.

As described in Section 8.3, the motivations for memorisation by the students appeared to stem from three different backgrounds: memorisation as a necessary part of doing mathematics, memorisation as a route to understanding, memorisation as a last resort in examination preparation. It is only in instances when a student has used memorisation in one of the first two ways that they may be able to demonstrate mathematical understanding. When a student responds to a question which demands 'factual knowledge and fact systems' (something identified in Chapter 6 as being common in undergraduate mathematics examinations), they are demonstrating factual recall. However, it is possible for some questions to be designed to test Group C skills but which are answered by students using factual recall. This is possible when a question asks for a student to produce a proof which has already been given in lecture notes. One student might be able to construct the proof themselves, remembering precise definitions as a consequence of memorisation as a means of doing mathematics. Another student might be able to reproduce the proof as a consequence of factual recall from an active process of memorisation which they employed in order to be able to understand the mathematics. However, another student might be able to reproduce the proof as a consequence of rote learning, without understanding the mathematics or the meaning of what they have written. Such students will have engaged in such a practice as a consequence of revision which has highlighted the nature of the questions posed in examinations, and the likelihood of such questions appearing or topics being examined.

James *et al.* (2002) argue that, "For most students, assessment requirement literally defines the curriculum" and that it is (perhaps indirectly) used "to spell out the learning that will be rewarded and to guide students" into a particular ATL (p. 7). Consequently, if students are required to engage with and do mathematics in their problem sheets, but to perceive this as a secondary activity in their examination revision then examiners do not get to see a great deal of evidence of their mathematical understanding. It is also possible that students might "memory dump" what they have learned at the end of one year for their examinations,

resulting in "such a fragile understanding that *reconstructing* forgotten knowledge [seems] alien to many taking part" (Anderson *et al.*, 1998, p. 418).

At Oxford, closed-book examinations form the vast majority of summative assessment methods, as has been found to be the case in the majority of other UK undergraduate mathematics programmes (Iannone & Simpson, 2011a, 2011b, 2012). In a study at a high-ranking UK university, Iannone & Simpson (2013) used a modified version of the Assessment Preferences Inventory (Birenbaum, 1994) and found that students perceive such examinations to be the most accurate measure of mathematical ability when contrasted with other forms of assessment. Furthermore, their participants' descriptions of the nature of the assessment suggested that "students perceive assessment of memory[49] to be dominant over assessment of understanding for closed-book examinations" (p. 28). This reflects the comments made by the interview participants in Chapter 7 and supports the interpretation of data collected using the MATH taxonomy in Chapter 6. Hence, there is a growing amount of literature which suggests that closed-book exainations in tertiary mathematics might not be the best way forward for assessing students' understanding of mathematics in certain areas. Alternative forms of assessment include presentations, group projects, online quizzes, portfolios, group projects, library tasks, computer aided assessment, multiple choice questions and essays (Iannone & Simpson, 2012), although the suitability of these would be very context- and content-dependent.

The roles played by problem sheets and end-of-year examinations are very different, as are the challenges that they present to the students doing them. Whilst students are forced to engage with mathematical concepts in order to be successful in answering questions on problem sheets, well-executed use of memorisation can make someone who is bad at mathematics but

[49] See also Bergqvist (2007), who found that 70% of examination questions in Swedish universities can be solved using 'imitative reasoning', i.e. that which is "founded on recalling answers or remembering algorithms" (p. 348).

good at memorisation successful in answering questions on examination papers. Oxford students appear to be *doing* mathematics throughout the academic year, and have this ability fostered by their tutors as they help students to engage with the mathematics and reflect on their practices, but the nature of the end-of-year examinations means that they then do a great deal of *reproducing* mathematics.

This means that students do not seem to be fairly rewarded for their ability to do mathematics, as the problem sheets act only formatively, though this is reasonable given these are much more open to cheating than closed book examinations. Though

> *closed book exams are relatively easy to set, administer and mark; they are seen as harder to cheat in than most other forms of assessment and they require a balance of memory, application and understanding which many mathematicians may feel suits the subject*
>
> (Iannone & Simpson, 2011a, p. 194)

it is important that the suitability of the examinations in terms of whether they give an accurate representation of a students' ability to do mathematics should be given great consideration.

8.5 – Challenging but not Meeting Expectations: The Contribution of the Oxford Admissions Process to Students' Experiences

Whilst the Oxford admissions process serves as a means of identifying the best students and selecting them for study at the University, interviews conducted with current students in Chapter 7 suggest that the process also serves a purpose for the students – whether or not this is intentional is not known. The participants described the interviews and OxMAT as being useful for them in a number of ways, namely:

- The residential nature of the interview days meant that they got to experience college life.
- The nature of the interviews gave them an impression of the Oxford tutorial.
- The OxMAT questions gave them an experience of more challenging mathematics questions, an experience which many of them were not used to.

A minority of the participants claimed that the interviews gave them a sense of what university mathematics would be like, though a number of other interviewees explicitly said that they did not think that the interview or admissions test gave them any mathematical insight or advantage. The nature of the questions posed in the OxMAT is very different to that in A-level Mathematics and Further Mathematics (see Chapter 6), something which all of the interview participants remarked on, claiming that they differed in the sense that the OxMAT is not explicit about the mathematics and procedures that are required to solve the problem, whereas A-level mathematics specifically states what pupils must do to answer a question. That is, whilst A-level Mathematics and Further Mathematics mainly comprise of Group A (specifically RUOP) questions, the OxMAT offers a more diverse range of questions, though the majority of marks tend to come from Group C questions. In that sense, it teaches students that "mathematics is not simply about doing what you are told" (Lesh, 2000, p. 73). Comparing the undergraduate mathematics examinations and the OxMAT, it appears that the OxMAT acts as

a bridging gap between A-level and undergraduate questions in the sense that it introduces a greater proportion of Group C questions for the students to solve. However, the mathematics required to solve the questions is no more difficult than A-level.

Furthermore, the OxMAT questions do not introduce tertiary-level topics such as analysis or formal proof, areas in mathematics that the interviewees described as being unexpected, surprising and even traumatising. If students' expectations of university mathematics study are shaped by A-level Mathematics and Further Mathematics, they are challenged by their experiences of the admissions process where the types of things that they have to do with mathematics that they know is very different and, for most students, is perceived to be more difficult than school experiences. Indeed, a number of participants described how the admissions process was the first time that they found mathematics challenging, though most of them commented that they found the OxMAT enjoyable because of its difference and challenge compared to A-level. The interview process, conducted one-on-one between the applicant and an admissions tutor who is usually a research fellow in the Mathematical Institute, can vary greatly depending on the college, the admissions tutor and the applicant – interview participants described a number of types of interview in Chapter 7. The common factor in all of the admissions interviews is that they all require the applicant to convince some very critical people (the admissions tutors) that they are able to engage deeply with mathematical concepts.

Therefore one might ask two questions:

1. If the admissions process at Oxford and other HEIs which use interviews and written tests allege and attempt to find the students with the most potential to be successful undergraduate mathematicians, why do some students fail to reach their potential at this level?

2. If the admissions process is intended to give students an insight into undergraduate mathematics study, how is it that these students with high achievement and high potential are surprised and unsettled by analysis and mathematical proof?

Whilst undergraduate students are selected on the basis of academic promise and achievement, a number of them go on to struggle with the mathematics being studied, as described by the students in the interviews. The proportion of students describing surface ATLs in the ASSIST suggest that there are a number of students who struggle considerably (assuming that surface ATLs are indicative of academic difficulties), and the comments made by students such as Juliette and Brian suggest that there are students who are able to pass examinations whilst failing to understand or engage with the mathematics. The questions raised in Chapter 8.3 regarding the applicability of the ASSIST to the undergraduate mathematics context and the data collected in Chapter 5 suggest that it is difficult to identify the number of students who employ memorisation in a negative fashion (i.e. because they cannot otherwise answer mathematics questions as opposed to using memorisation as a means of understanding or solely in order to answer questions which require students to state facts) mean that there might be a greater proportion of students who adopt inappropriate (in the sense that they use memorisation because they are otherwise unable to answer mathematics questions) ATLs than it appears. However, Oxford has one of the lowest drop-out rates out of all UK universities, and a very high proportion of students go on to do the MMath year, which is indicative of a high level of student engagement and success given its elective and selective nature. Therefore it is difficult to ascertain whether the proportion of students who cannot and do not engage with tertiary-level mathematical concepts is of concern in this particular sample.

The majority of interview participants claimed that they had not prepared for undergraduate study after gaining a place. The most that any did to research what their degree would involve was to peruse the Mathematical Institute website. It is possible to find lecture notes for first-

year courses such as Analysis, so it was interesting to hear from all but a couple of interviewees that they were not expecting it and, furthermore, that most interviewees were not expecting such a reliance on proof at this level. In common with research conducted by Hirst *et al.* (2004), those students who were expecting an element of proof found it more difficult than they had expected. For a number of participants, it seemed that their beliefs about mathematics 'overhung' (Daskalogianni & Simpson, 2001). The data here support studies which found that many students go into university with inaccurate conceptions of mathematics, often viewing mathematics as a rote-learning task (Crawford *et al.*, 1994, 1998a, 1998b), even further into their degree (Anderson *et al*., 1998; Maguire *et al*., 2001).

All of the interview participants who had A-levels in Mathematics and Further Mathematics said that they did not think that these subjects were adequate preparation for undergraduate mathematics study in the sense that it did not challenge them sufficiently and that it did not give them sufficient experience of proof to have them believe that this would be a significant part of further study. Even Further Mathematics papers had very few Group C questions, the opposite of undergraduate examinations. Therefore, it seems that the inaccurate perceptions of mathematics that the students perceived themselves to have had upon arrival at Oxford stemmed from their prior experience of mathematics, which shaped their prior understandings of what mathematics is. If this is the case of students who do the double A-level and go on to study it at one of the best universities in the world, then it is likely that students who go to other universities experience similar difficulties. This is something which ACME have already touched upon:

> *the shift from performing techniques to proving properties is not one for which they are prepared, and they may feel overwhelmed and demotivated. This is also true for universities that attract learners with the best grades at A-level*
>
> (ACME, 2011a, p. 15)

Consequently, the data here suggest that students' prior understandings are either misunderstood by admissions tutors or that these are not taken into consideration; Petocz and Reid (2005) argue that "the way that students understand the discipline are often assumed" (p. 91) and evidence here suggests that their prior understandings may not align with those of university mathematics departments. It is therefore important for universities to consider whether it is acceptable for students to arrive at university with inaccurate conceptions of mathematics, and whether there is something that they could and/or should do to address the misalignment. Government policy will introduce, for the 2014-2015 academic season, the requirement for universities to be more involved with the setting of A-level syllabi with one intention being that it leads to a better alignment in terms of content and assessment (House of Commons Education Committee, 2012), something which is supported by ACME (2012).

The unique aspects of undergraduate admissions processes and undergraduate study at the University of Oxford make the data collected for this thesis similarly unique. However, it is not the case that the findings presented here are only relevant to this particular university and context. The students who participated in this research were amongst the brightest and most successful A-level students in the country when they applied to study at the University, also convincing admissions tutors of their suitability for studying their chosen course. It could be argued that these students were, therefore, amongst the best prepared for undergraduate mathematics. However, the findings presented in Chapters 5, 6 and 7 suggest that some of these students might actually not have been as prepared as one might expect. They may have had inaccurate conceptions about the nature of undergraduate mathematics study. They may have not had much experience of being challenged academically. They may have found learning and understanding undergraduate mathematics to be very difficult indeed. They may not have felt that they had been prepared for undergraduate study by their secondary study. Therefore, this research is very pertinent to those involved in institutions which do not employ similarly extensive and stringent admissions processes, as well as those with different systems

of teaching. It is also bound to be of interest to those organisations who produce final mathematics qualifications for school-leavers ahead of their commencing undergraduate study.

8.6 – Limitations

Due to the fact that the Department of Educational Studies and the Mathematical Institute are separate entities in the University, there was considerable dependency on administrative action from the Mathematical Institute in collecting data for this thesis. This often came at times of the academic year when administrative staff were busy focussing on new students arriving or examinations, and meant that my requests for assistance were not a priority. Whilst this meant that some emails were not sent out at the correct time, this did not impact upon the timing of data collection or the number of students who were contacted.

As with many studies of this kind, the number of participants was lower than was hoped. Sweep 1 (*N*=176, 78.6% of students) was participated in by, proportionally, many more students than Sweep 2 (*N*=238, 28.03% of students). This is likely because Sweep 1 was at the beginning of the first term of undergraduate study for new students, when they are enthusiastic about their new surroundings, and are yet to have received many requests for participation in surveys via email. Personal communication with current undergraduates suggests that students become bombarded by a large number of surveys from the university, the department, their colleges and other sources throughout the year, which results in many students feeling overwhelmed and failing to do the surveys. Furthermore, Sweep 2 took place around the examination period such that students may have been far too preoccupied with that to be able to dedicate any time to completing questionnaires. Reminder emails were sent out in both Sweeps 1 and 2 in order to increase numbers, which were successful, though participation in Sweep 2 remained lower than hoped. The summer term was chosen for Sweep 2 because this is when students will have been engaging in examination revision and will likely have been reflecting on their practices and felt in touch with their learning and studying, making them more able to answer the questions. However, perhaps collecting the data in the spring term may have been advisable as students would not have been as preoccupied with

revision and may not have had their email inboxes full of other requests for survey participation.

Unfortunately, the timescale of the DPhil meant that it was not possible to conduct a longitudinal study which tracked students from one year to the next. However, the small amount of data matching from Sweep 1 to Sweep 2 (see Chapter 5.4.2) suggest that it might have been difficult to track many students from one year to the next. This may, however, be something which could have been counteracted by distributing questionnaires online at a different point in the academic year, or finding a way to use paper and pencil methods successfully[50].

Before conducting and analysing interviews, it was hoped that a sample of at least 25 students could be achieved, with a mixture amongst the year-groups. All students who offered to participate in the interviews were interviewed; however, this amounted to only 13 participants. Whilst this was initially a matter of concern, analysis of the interviews in Chapter 7 found that there was saturation of data, particularly in respect to students' prior understandings and conventions and artefacts. Morse (1995) claims that "saturation is the key to excellent qualitative research" (p. 147), with Guest *et al.* (2006) claiming that saturation in social sciences tends to occur by the twelfth interview – one less than the sample here. Furthermore, the sample size for the student interviews falls within the desirable ranges for qualitative social science interviews suggested by Creswell (1998) who recommends between 5-25 participants, and Morse (1994) who recommends sample sizes greater than 6.

The number of papers analysed using the MATH taxonomy in Chapter 6 could have been increased in order to increase the reliability of the claims made. Whilst all of the available

[50] Attempts to have students complete the ASSIST during a lecture in Darlington (2010) did not yield a very high proportion of returned questionnaires despite there being sufficient for every student in attendance to do so. Furthermore, many of the questionnaires were incomplete or spoilt. Finding lecturers who are willing to cooperate by setting aside some of the end of one of their lectures was also a difficult task.

OxMATs were analysed, there were other possibilities for the A-level and undergraduate examinations. However, since the student interviews suggested that papers rarely changed in their nature from year-to-year, this was not of a great concern. There were no papers which varied greatly in terms of the proportions of questions from Groups A, B and C to each other, and there would likely be very negative reactions from pupils and teachers if there were significant changes from one year to the next given they prepare for examinations based on past papers. Furthermore, since the A-level is on the verge of significant change into a linear examination, the merits of conducting wide-scale research into the nature of its questions has limited practical applicability at the moment.

This study was conducted at Oxford – and Oxford alone – for a number of reasons (see Chapter 4.7.1) and so it is not generalisable to other universities. However, the point of a case study such as this is to study "the particularity and complexity of a single case, coming to understand its activity within important circumstances" (Stake, 1995, p. xi). There are a number of important, idiosyncratic characteristics which set Oxford apart from other universities – the tutorial system, the entry requirements, the admissions process, the examination system, to name a few – although similar could be said of any other institution. All universities operate within their own individual 'bubbles' wherein they have different pastoral systems, examination and degree structures, teaching staff and different students. Therefore, whilst Oxford is certainly a special case in comparison to other UK universities, similar limitations would apply to any other case. Though case study research "has long been… stereotyped as a weak sibling among social science research methods" (Yin, 2003, p. xiii), the point of using Oxford as a case study in this instance was to investigate whether an institution like this suffers from difficulties and therefore, if there are any problems or issues, then there are implications for other universities given the more demanding selection criteria and small-group teaching at Oxford.

8.7 – Further Research

From the interview data collected regarding the Invariants, it is a shame that the topic of students' involvement in this society could not have been probed further. That is, whilst there seemed to be a divide between those students who perceived the Invariants as being geeky and requiring more mathematical engagement than they had, and those who were involved themselves, only two interview participants were active members. To gain an insight into the different COPs and their impact on students' engagement and practices would certainly have been relevant to this study and perhaps warrants further research.

Having identified an alternative view of undergraduate mathematics students' ATLs based on the motivation for using, and extent of use of, memorisation, the relationship between this and attainment calls for further research. Weber and Alcock (2004) distinguished between syntactic and semantic proof production wherein more complex knowledge is required for semantic proof production whereas syntactic production is mainly based around manipulating definitions, recalling associated theorems and making derivations from those. However, whilst semantic knowledge grants deeper understanding of mathematical concepts and "grants flexibility in applying known concepts to new situations", syntactic knowledge can also be effective (Ioannou & Nardi, 2009, p. 2307). Therefore, it would be interesting to see whether students who memorise mathematics for reproduction are able to perform better in examinations than those who use memorisation as a means of doing mathematics. Furthermore, the proportion of students with each motivation for memorisation who go on to study mathematics at postgraduate levels is also of interest. Based on anecdotal evidence, Oxford is also a unique case in the sense that the vast majority of undergraduate mathematicians go on to do the MMath; therefore the motivations of postgraduate students would be of great interest.

Since this study has suggested that mathematical confidence may have an impact on their motivation for memorisation, research may be conducted which tests this hypothesis. On a large scale such as, for example, the self-efficacy scale in the Student Readiness Inventory (Le *et al.*, 2005) could be used to measure mathematics confidence and contrasted with results from a questionnaire which investigates the impetus for students' memorisation, or this being probed more deeply in interviews.

Finally, extending this research to other universities across the UK could serve to compare them with Oxford to see whether the data collected here are significantly different to those elsewhere. There is the possibility that using the ASSIST on another student population may yield different results in terms of ATL proportions and subscale means, although the descriptions given by students in the interviews and existing empirical research regarding students' ATLs and proving in undergraduate mathematics suggest that there are not likely to be differences elsewhere. However, the calibre of students at Oxford means that it may be the case that the proportion of students adopting predominantly surface ATLs at other institutions may be higher, if we assume that the demands of the courses are the same.

Chapter 9: Conclusion

This study highlights the importance of internal decisions about pedagogy and assessment made by universities, as well as the importance of prior understandings on students' enculturation into a new mathematical environment. Furthermore, specific to the context of the University of Oxford, the relationships between admissions processes, students' expectations of undergraduate mathematics study and the types of questions posed in examinations were uncovered.

The University of Oxford strikes up quite a different didactic contract with its students than those at many other universities; the tutorial system and interview process are exclusive to Oxford and Cambridge, and the OxMAT is unique to Oxford. Many[51] of the students who make it through the admissions process and are offered a place end up going from being a big fish in a small pond to a small fish in a big pond (Marsh, 1987). These students were all high-performers in school mathematics, and were deemed by admissions tutors at Oxford to have the potential to be successful undergraduate mathematicians. However, this does not always transpire to be the case. For example, the student interviews revealed Brian and Juliette, in particular, to be struggling with their studies, and the other participants recounted a number of challenges that they had experienced during the secondary-tertiary transition and beyond. Studies of student transition problems are widely-reported (Hawkes & Savage, 2000), as are those about mismatches between the expectations of universities and new undergraduates' actual competencies (LMS, 1995).

What this study revealed was that, despite admissions processes which expose students to new mathematical experiences quite different to those they had at school, and which challenge their abilities more so than A-level, students nonetheless perceive themselves as

[51] However, this will continue to be the case for a very few students – each pond must have its own big fish.

coming to Oxford ill-prepared for the mathematics that they meet (see Chapter 7.3). All of the students interviewed found analysis and mathematical proof to be a shock to the system, and did not think that the interview process did anything to indicate that this would be the main feature of undergraduate study, though the majority did nothing to research what they would be studying in advance of going to Oxford. A quick perusal of the departmental website (University of Oxford Mathematical Institute, 2008) gives the public access to lecture notes for the different courses run in all of the four years, and reading these gives a clear indication that proof will be a focal point of tertiary study. The fact that future undergraduates feel no need to research the content of their upcoming courses, compounded by the lack of mathematical proof covered in A-level Mathematics and Further Mathematics, leaves students in a position where they arrive at university unprepared. Furthermore, the types of challenge posed by questions at the secondary, intermediate and tertiary levels differ to such an extent (see Chapter 6) that, ignoring the difference in the nature of mathematics, students might perhaps benefit from exposure to more Group C[52] questions before beginning their university careers. Furthermore, the utility of the OxMAT in terms of preparing students for university mathematics study goes largely unnoticed by the students, with most of the interview participants commenting that they did not find it useful preparation in terms of mathematics and did not think that the mathematics covered was anything like undergraduate mathematics (though they did report finding it useful in the sense that it challenged them mathematically, something which they were not used to with A-level Mathematics and Further Mathematics).

Whilst the nature of the questions posed at A- and undergraduate-level, and the approaches to learning (ATLs) reported by students in interviews, are very different, the responses to the questionnaire about these (the ASSIST) did not highlight these differences (see Chapter 5). Students appeared to work strategically at both levels – preparing for A-level examinations

[52] Those which require 'justification and interpretation', 'implications, conjectures and comparisons' and 'evaluation' (Smith *et al*., 1996; see Chapter 2.2).

through repeated, exhaustive practice of similar questions, and revising for undergraduate examinations by combining efforts to understand with varying kinds of memorisation – the difference in the form that memorisation took is very important. The study practices that students engaged in at A-level, which did not usually include reviewing and learning the concepts they had been taught as well as practising past questions, were very different to those described at undergraduate level (see Chapter 7). At school, pupils memorise procedures, whereas at university, students memorise facts or someone else's mathematics. Students' beliefs in their own capabilities resulted in them resorting to different kinds of memorisation. Students who were more mathematically confident engaged in memorisation as a means of doing mathematics, whereas those who were not so confident actively memorised some mathematics to reproduce in examinations because they did not believe themselves capable of understanding it enough to be able to do it any other way (see Chapter 8.3). These less confident students felt guilty about their use of such memorisation, although knew that it would be sufficient for them to perform well in examinations. Conversely, there were also students who memorised large quantities of their lecture notes and problem sheets because it was the only way for them to answer any questions in examinations. It appears to be possible for students to *pass* undergraduate examinations using only 'factual knowledge and fact systems' (see Chapter 6.4), the students themselves are aware of this and believe that memorisation alone can ensure that they can *do well* in some examinations (see Chapter 7.4.2.2).

Strategic ATLs are themselves encouraged by those lecturers who, for example, suggest to students what might be on the examination, as well as the similar nature of past papers[53] encouraging students to look for patterns in, or recurring, questions. For this reason it is important to question whether students become *enculturated* into what it is to do

[53] As well as past papers being perceived to be predictable and similar by students, analysis in Chapter 6 using the MATH taxonomy showed very little difference in the proportions of questions in Groups A, B and C between papers.

mathematics, or if they are *institutionalised* into doing undergraduate mathematics at Oxford. The latter is certainly not the intention of the pedagogical structure, where compulsory tutorials are used in order to develop students' understanding of mathematical concepts and to engage them in mathematical discussion, involving them in the mathematics community of practice (COP). Undergraduates also have the opportunity of involving themselves in the mathematics society, the Invariants, although this was something which only two of the 13 interview participants took up. There was no apparent division in the interviewed students' enthusiasm for mathematics based on their ATL – that is, students with deep ATLs were not necessarily more mathematically engaged than those with strategic ATLs, and there did not appear to be a correlation between ATL and attainment. Consequently, the data here suggest that a strategic ATL in the context of undergraduate mathematics does not have to be a bad thing, or that it has to mean that a student is disaffected. The distinction in the motivations for students' utilisation of memorisation is the factor which should be considered when investigating correlates with attainment and engagement, enthusiasm and mathematics self-efficacy.

Whilst certain types of questions posed in undergraduate examinations may be answered purely by memorising the content of lecture notes for reproduction in the examination, this is not possible in students' weekly problem sheets. In these, students are not asked to state definitions or theorems or to do merely reproduce part of their notes (see, for example, Appendix 9.1), but instead have their mathematical understanding tested. Indeed, a number of interview participants described them as being crucial for their mathematical understanding (see Chapter 7.4.2.1). Since problem sheets are used as formative assessment, the fact that some students cheat on them is not necessarily of a concern as the interview participants who admitted to doing this said that they also endeavoured to understand what they had copied. Furthermore, tutorials act as a means for students to seek clarification and enhance their understanding and so it is not important for students to understand everything first time. At

some UK universities, marks for problem sheets account for a small percentage of the available marks in a module; however, this is not the case at Oxford.

The idea of *open*-book assessment (as opposed to closed-book) could be considered as an alternative means of summative assessment for undergraduate mathematicians in order to have examinations focus more on solving unfamiliar problems. It would mean that examinations no longer have to contain questions which ask students to state definitions and theorems, but rather to use these in solving unfamiliar mathematical problems. However, when analysing the types of assessment used across undergraduate mathematics courses in the UK, Iannone and Simpson (2011a) only found open-book examinations in statistics modules. Whilst open-book examinations have been found to be better discriminators among students than closed-book examinations, and results in open-book examinations have been found to be comparable with those of closed-book examinations (Phiri, 1993), undergraduate mathematicians do not think that they are good tests of their understanding (Iannone & Simpson, 2013). Although this contradicts the findings of a study by Struyven *et al.* (2005), Iannone and Simpson (2013) conjecture that the first-year students they interviewed in their study perceived closed-book examinations to be the best tests of their understanding because of "their enculturation into mathematics" (p. 29). That is, their previous experiences of mathematics assessment were almost exclusively in closed-book examinations, and so they may consequently believe these to be the way to examine mathematical understanding. The specific reasons for the participants making their choices were not probed; however, this would certainly be of interest.

The practicalities associated with such assessment are complex, as students would have to be simultaneously isolated from each other whilst also having access to books and notes. Furthermore, the benefits are unknown, although could be considered as a focus of further research. Other messages for further research include consideration of undergraduate

mathematics societies and their relationship with student engagement and attainment – does membership of the Invariants, or similar, cause or be caused by engagement and are these students typically more successful? Belonging to a COP like this has the potential to expose its members to more and different mathematics to those who are not involved – might this impact upon their experiences? The extent and motivation of memorisation in revision, its relationship to confidence and its impact on results are important to consider in terms of student performance, engagement and understanding. Messages for Oxford include that there are a number of shortcomings of the types of questions posed in summative assessment, though a great importance and utility of the problem sheet/tutorial system. This research demonstrates how significant an impact such artefacts have on students' activity structures, meaning that they should be an important source of consideration for mathematics departments. Furthermore, consideration should be made regarding the admissions process and, whilst it finds students who are better able to engage deeply with mathematics and respond well to mathematical challenges, it neglects to give students an indication of the mathematics that they study. Students are not prepared by the admissions process (or previous study) for the role of bookwork and the role of proof in advanced mathematics, and so are shocked by the prevalence of real analysis and proof. Therefore they are not prepared for a need for new kinds of study, such as the role of memorisation. They come to realise that the nature of mathematical knowledge is different as they are enculturated into the new mathematical environment, something which some students respond negatively to. If beliefs 'overhang' (Daskalogianni & Simpson, 2001), and those beliefs were a motivating factor for choosing to study mathematics further, there is a great potential for students to become disaffected and 'cool off' mathematics (Daskalogianni & Simpson, 2002). However, this is not necessarily the responsibility of the universities. Students do not feel the need to investigate the nature of undergraduate mathematics as it fits a very specific mould at secondary-level: procedural, mechanistic and predictable. Therefore, universities must take heed of the fact

that mathematical competency at A-level might not necessarily translate into mathematical competency at the undergraduate level.

Bibliography

Abel, W. (1966) Attrition & the Student who is Certain. *Personnel & Guidance Journal, 44*(10), 1042-1045.

Advisory Committee on Mathematics Education (2009) *The Mathematics Education Landscape in 2009.* Retrieved May 13, 2011, from http://www.acme-uk.org/news/position-statements

Advisory Committee on Mathematics Education (2011a) *Mathematical Needs: The Mathematical Needs of Learners.* London: ACME.

Advisory Committee on Mathematics Education (2011b) *Mathematical Needs: Mathematics in the Workplace & Higher Education.* London: ACME.

Advisory Committee on Mathematics Education (2012) *ACME Response to Ofqual Consultation on A-Level Reform - September 2012.* Retrieved November 18, 2012, from http://www.acme-uk.org/media/10163/acme%20response%20to%20ofqual%20consultation%20on%20a-level%20-%20final%20submitted.pdf

Alcock, A., & Simpson, A. (2002) Definitions: Dealing with Categories Mathematically. *For the Learning of Mathematics, 22*(2), 28-34.

Alcock, L. (2013) *How to Study for a Mathematics Degree.* Oxford: Oxford University Press.

Alcock, L., & Simpson, A. (2001) The Warwick Analysis Project: Practice & Theory. In D. Holton (Ed.), *The Teaching & Learning of Mathematics at University Level: An ICMI Study* (pp. 99-111). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Alibert, D., & Thomas, M. (1991) Research on Mathematical Proof. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 215-229). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Almeida, D. (2000) A Survey of Mathematics Undergraduates' Interaction with Proof: Some Implications for Mathematics Education. *International Journal of Mathematical Education in Science & Technology, 31*(6), 869-890.

Altheide, D., & Johnson, J. M. (1998) Criteria for Assessing Interpretive Validity in Qualitative Research. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Collecting & Interpreting Qualitative Materials* (pp. 283-312). London: Sage Publications.

Alton, A. (2008) When is Predictability a Problem in an Examination? *Paper Presented at the 34th International Association for Educational Assessment (IAEA) Annual Conference.* Cambridge, UK.

Anderson, J. (1996) The Legacy of School - Attempts at Justifying & Proving Among New Undergraduates. *Teaching Mathematics & its Applications, 15*(3), 129-134.

Anderson, J., Austin, K., Barnard, T., & Jagger, J. (1998) Do Third-Year Mathematics Undergraduates Know What They Are Supposed To Know? *International Journal of Mathematical Education in Science & Technology, 29*(3), 401-420.

Anderson, L. W., Jacobs, J., Schramm, S., & Splittgerber, F. (2000) School Transitions: Beginning of the End or a New Beginning? *International Journal of Educational Research, 33*(4), 325-339.

Anderson, L., & Sosniak, L. (Eds.) (1994) *Bloom's Taxonomy: A Forty-Year Retrospective.* Chicago, USA: National Society for the Study of Education.

Anthony, G. (2000) Factors Influencing First-Year Students' Success in Mathematics. *International Journal of Mathematical Education in Science & Technology, 31*(1), 3-14.

Artigue, M. (1991) Analysis. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 167-198). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Artigue, M. (1993) Didactical Engineering as a Framework for the Conception of Teaching Products. In R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Eds.), *Didactics of Mathematics as a Scientific Discipline* (pp. 27-40). London: Springer.

Ashwin, P. (2005) Variation in Students' Experiences of the 'Oxford Tutorial'. *Higher Education, 50*(4), 531-644.

Astin, A. (1964). Personal & Environmental Factors Associated with College Dropouts Among High Aptitude Students. *Journal of Educational Psychology, 55*(4), 218-227.

Astin, A. W. (1977) *Four Critical Years.* San Francisco, USA: Jossey-Bass.

Astin, A. W. (1993) *What Matters in College? Four Critical Years Revisited.* San Francisco, USA: Jossey-Bass.

Ausubel, D. G. (1963) Cognitive Structure & the Facilitation of Meaningful Verbal Learning 1. *Journal of Teacher Education, 14*(2), 217-222.

Bair, C. R., & Haworth, J. G. (1999) Doctoral Student Attrition & Persistence: A Meta-Synthesis of Research. *Paper presented at the annual meeting of the Association for the Study of Higher Education.* San Antonio, USA.

Baird, J. A., & Lee-Keeley, L. (2009) The Dearth of Managerialism in Implementation of National Examinations Policy. *Journal of Education Policy, 24*(1), 55-81.

Ball, B., Smith, G., Wood, L., Coupland, M., & Crawford, K. (1998) Creating a Diversity of Mathematical Experiences for Tertiary Students. *International Journal of Mathematical Education in Science & Technology, 29*(6), 827-841.

Bandura, A. (1977) Self-Efficacy: Toward a Unifying Theory of Behavioral Change. *Psychological Review, 84*(2), 191-215.

Bandura, A. (1986) *Social Foundations of Thought & Action: A Social Cognitive Theory.* Englewood Cliffs, USA: Prentice Hall.

Bandura, A. (1997) *Self-Efficacy: The Exercise of Control.* New York, USA: W.H. Freeman.

Bandura, A., & Cervone, D. (1983) Self-Evaluative & Self-Efficacy Mechanisms Governing Motivational Effects of Goal Systems. *Journal of Personality & Social Psychology, 45*(5), 1017-1028.

Bandura, A., & Locke, A. (2003) Negative Self-Efficacy & Goal Effects Revisited. *Journal of Applied Psychology, 88*(1), 87-99.

Barnett, R. (1990) *The Idea of Higher Education.* Buckingham: Open University Press.

Bassett, D., Cawston, T., Thraves, L., & Truss, E. (2009) *A New Level.* London: Reform.

Bateson, G. (1973) *Steps to an Ecology of Mind.* San Francisco, USA: Chandler Publishing Company.

Batty, C. J. (1994) *How do Undergraduates Learn Mathematics? A Guide to Studying Mathematics at Oxford.* Oxford: University of Oxford Mathematical Institute Study Guide.

Baumslag, B. (2000) *Fundamentals of Teaching Mathematics at University Level.* London: Imperial College Press.

Bean, J. P. (1982) Conceptual Models of Student Attrition: How Theory Can Help the Institutional Researcher. *New Directions for Institutional Research, 36*(1), 17-33.

Becher, T. (1994) The Significance of Disciplinary Differences. *Studies in Higher Education, 19*(2), 151-161.

Becker, J. R. (1990) Graduate Education in the Mathematical Sciences: Factors Influencing Women & Men. In L. Burton, *Gender & Mathematics: An International Perspective* (pp. 119-130). London: Cassell.

Beishuizen, J., Stoutjesdijk, E., & van Putten, K. (1994) Studying Textbooks: Effects of Learning Styles, Study Tasks, & Instruction. *Learning & Instruction, 4*(2), 151-174.

Bell, A. W. (1976) A Study of Pupils' Proof-Explanations in Mathematical Situations. *Educational Studies in Mathematics, 7*, 23-40.

Bergqvist, E. (2007) Types of Reasoning Required in University Examinations in Mathematics. *Journal of Mathematical Behavior, 26*(4), 348-370.

Bergsten, C. (2011) Why do Students go to Lectures? In M. Pytlak, T. Rowland, & E. Swoboda (Ed.), *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education*, (pp. 1960-1970). Rzeszów, Poland.

Berry, J., Maull, W., Johnson, P., & Monaghan, J. (1999) Routine Questions & Examination Performance. In O. Zaslavsky (Ed.), *Proceedings of the 23rd International Conference of the International Group for the Psychology of Mathematics Education*, *2*, pp. 105-112. Haifa, Israel.

Betz, N. (1978) Prevalence, Distribution, & Correlates of Math Anxiety in College Students. *Journal of Counseling Psychology, 25*(5), 441-448.

Bibby, N. (1991) Wherefore "Plug-and-Chug"?: Computer Algebra vs. A-Level Mathematics. *The Mathematical Gazette, 75*(471), 40-48.

Biggs, J. B. (1978) Individual & Group Differences in Study Processes. *British Journal of Educational Psychology, 48*(3), 266-279.

Biggs, J. B. (1979) Individual Differences in Study Processes & the Quality of Learning Outcomes. *Higher Education, 8*(4), 381-394.

Biggs, J. B. (1987a) *Study Process Questionnaire: Manual.* Hawthorn, Australia: Australian Council for Educational Research.

Biggs, J. B. (1987b) *Student Approaches to Learning & Studying.* Hawthorn, Australia: Australian Council for Educational Research.

Biggs, J. B. (1988) Assessing Student Approaches to Learning. *Australian Psychologist, 23*(2), 197-206.

Biggs, J. B. (1993) What do Inventories of Students' Learning Processes Really Measure? A Theoretical Review & Clarification. *British Journal of Educational Psychology, 63*(1), 3-19.

Biggs, J. B. (1999) *Teaching for Quality Learning in University.* Buckingham: Open University Press.

Biggs, J. B., & Collis, K. F. (1982) *Evaluating the Quality of Learning: The SOLO Taxonomy.* New York, USA: Academic Press.

Birenbaum, M. (1994) Towards Adaptive Assessment - The Students' Angle. *Studies in Educational Evaluation, 20*(2), 239-255.

Black, L. (2010) Developing a Leading Identity Across the Transition to Mathematically Demanding Programmes at University. In M. Joubert, & P. Andrews (Ed.), *Paper presented at the British Congress for Mathematics Education*, (pp. 33-41). Vienna, Austria.

Blanc, P. (1995) Investigating the Mathematics Learning of Student Teachers: Explorations & Discoveries. *Proceedings of the British Society for Research into Learning Mathematics*, (pp. 9-13). Birmingham, UK.

Bligh, D. A. (1972) *What's the Use of Lectures?* (3rd ed.). Harmondsworth: Penguin.

Bloom, B., Engelhart, M., Furst, E., Hill, W., & Krathwohl, D. (Eds.) (1956) *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook I: Congitive Domain.* New York: David McKay.

Blyth, D. A., Simmons, R. G., & Carlton-Ford, S. (1983) The Adjustment of Early Adolescents to School Transition. *Journal of Early Adolescence, 3*(1-2), 105-120.

Boaler, J. (1997) *Experiencing School Mathematics: Teaching Styles, Sex & Setting.* Buckingham: Open University Press.

Boaler, J., & Greeno, G. G. (2000) Identity, Agency and Knowing in Mathematics World. In J. Boaler (Ed.), *Multiple Perspectives on Mathematical Teaching and Learning* (pp. 171-184). London: Ablex Publishing.

Boesen, J. (2006) *Assessing Mathematical Creativity.* Retrieved January 6, 2011, from http://202.198.141.77/upload/soft/0-article/+00/101.pdf

Boulton-Lewis, G. M. (1994) Tertiary Students' Knowledge of their Own Learning & SOLO Taxonomy. *Higher Education, 28*(3), 387-402.

Boulton-Lewis, G. M. (1995) The SOLO Taxonomy as a Means of Shaping & Assessing Learning in Higher Education. *Higher Education Research & Development, 14*(2), 143-154.

Braun, V., & Clarke, V. (2006) Using Thematic Analysis in Psychology. *Qualitative Research in Psychology, 3*(2), 77-101.

Brint, S., Cantwell, A. M., & Hanneman, R. A. (2008) The Two Cultures of Undergraduate Academic Engagement. *Research in Higher Education, 49*(5), 383-402.

Brousseau, G. (1984) The Crucial Role of the Didactic Contract in the Analysis & Construction of Situations in Teaching & Learning Mathematics. In J. G. Steiner (Ed.), *Theory of Mathematics Education* (pp. 110-119). Bielefeld, Germany: Institut für Didactik der Mathematik der Universität Bielefeld.

Brousseau, G. (1988) Le Contrat Didactique: Le Milieu. *Recherches en Didactiques des Mathématiques, 7*(2), 33-115.

Brousseau, G. (1997) *Theory of Didactical Situations in Mathematics.* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans.) Dordrecht, the Netherlands: Kluwer Academic Publishers.

Brousseau, G., & Otte, M. (1991) The Fragility of Knowledge. In A. J. Bishop, S. Mellin-Olsen, & J. van Dormolen (Eds.), *Mathematical Knowledge: Its Growth Through Teaching* (pp. 13-36). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Brown, J. S., & Duguid, P. (1991) Organizational Lerning & Communities-of-Practice: Toward a Unified View of Working, Learning, & Innovation. *Organization Science, 2*(1), 40-57.

Brown, J. S., Collins, A., & Duguid, P. (1989) Situated Cognition & the Culture of Learning. *Educational Researcher, 18*(1), 32-42.

Brown, M., & Rodd, M. (2004) Successful Undergraduate Mathematicians: A Study of Students in Two Universities. In M. J. Høines, & A. B. Fuglestad (Ed.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, *4*, pp. 97-104. Bergen, Norway.

Brown, M., Brown, P., & Bibby, T. (2008) "I Would Rather Die": Reasons Given by 16-Year Olds for Not Continuing their Study of Mathematics. *Research in Mathematics Education, 10*(1), 3-8.

Brown, R. G. (2010) Does the Introduction of the Graphics Calculator into System-Wide Examinations Lead to Change in the Types of Mathematical Skills Tested. *Educational Studies in Mathematics, 73*(2), 181-203.

Brown, S., & McIntyre, D. (1993) *Making Sense of Teaching.* Birmingham: Open University Press.

Bruinsma, M. (2003) *Effectiveness of Higher Education. Factors that Determine Outcomes of University Education.* Groningen, the Netherlands: GION, Rijksuniversiteir Groningen.

Bryman, A. (2006) Integrating Quantitative & Qualitative Resarch: How is it Done? *Qualitative Research, 6*(1), 91-113.

Burn, R. P. (1992) *Numbers & Functions: Steps into Analysis.* Cambridge: Cambridge University Press.

Burnett, P. C., & Dart, B. C. (2000) The Study Process Questionnaire: A Construct Validation Study. *Assessment & Evaluation in Higher Education, 25*(1), 93-99.

Bye, D., Pushkar, D., & Conway, M. (2007) Motivation, Interest, & Positive Affect in Traditional & Non-Traditional Undergraduate Students. *Adult Education Quarterly, 57*(2), 141-158.

Byrne, M., Flood, B., & Willis, P. (1999) Approaches to Learning: Irish Students of Accounting. *Irish Accounting Review, 6*(2), 1-29.

Byrne, M., Flood, B., & Willis, P. (2002) The Relationship Between Learning Approaches & Learning Outcomes: A Study of Irish Accounting Students. *Accounting Education: An International Journal, 11*(1), 27-42.

Byrne, M., Flood, B., & Willis, P. (2004) Validation of the Approaches & Study Skills Inventory for Students (ASSIST) Using Accounting Students in the USA & Ireland: A Research Note. *Accounting Education, 13*(4), 449-459.

Byrne, M., Flood, B., & Willis, P. (2009) An Inter-Institutional Exploration of the Learning Approaches of Students Studying Accounting. *International Journal of Teaching & Learning in Higher Education, 20*(2), 155-167.

Cambridge Assessment (2012a) *Why Use STEP?* Retrieved October 5, 2012, from STEP Background & Research: http://www.admissionstests.cambridgeassessment.org.uk/adt/step/research

Cambridge Assessment (2012b) *Candidate Numbers*. Retrieved October 8, 2012, from STEP About The Test: http://www.admissionstests.cambridgeassessment.org.uk/adt/step/about#Results

Campbell, J., Smith, D., Boulton-Lewis, G., Brownlee, J., Burnett, P., Carrington, S., *et al*. (2001) Students' Perceptions of Teaching & Learning: The Influence of Students' Approaches to Learning & Teachers' Approaches to Teaching. *Teachers & Teaching: Theory & Practice, 7*(2), 173-187.

Cano, F. (2005) Epistemological Beliefs & Approaches to Learning: Their Change Through Secondary School & their Influence on Academic Performance. *British Journal of Educational Psychology, 75*(2), 203-221.

Caporrimo, R. (1990) Gender, Confidence, Math: Why Aren't the Girls "Where the Boys Are?". *Paper presented at the Annual Meeting of the American Psychological Association.* Boston, USA.

Case, J., & Gunstone, R. (2003) Going Deeper than Deep & Surface Approaches: A Study of Students' Perceptions of Time. *Teaching in Higher Education, 8*(1), 55-70.

Case, J., & Marshall, D. (2004) Between Deep & Surface: Procedural Approaches to Learnings in Engineering Education Context. *Studies in Higher Education, 29*(5), 605-615.

Cassidy, S. (2004) Learning Styles: An Overview of Theories, Models, & Measures. *Educational Psychology, 24*(4), 419-444.

Chan, C. C., Hong, J. H., & Chan, M. Y. (2001) *Applying the Structure of the Observed Learning Outcomes (SOLO) Taxonomy on Students' Learning Outcomes: A Comprehensive Review.* Unpublished Manuscript: Hong Kong Polytechnic University.

Chetwynd, A. (2004) Enriching Learning Opportunities in Maths at the Interface between School & University. *MSOR Connections, 4*(2), 29-30.

Chevallard, Y. (1988) *Sur l'analyse didactique. Deux études sur les notions de contrat et de situation.* Publications de l'IREM d'Aix-Marseille.

Chick, H. (1988) Student Responses to a Polynomial Problem in the Light of the SOLO Taxonomy. *Australian Senior Mathematics Journal, 2*(1), 91-110.

Chick, H. (1998) Cognition in the Formal Modes: Research Mathematics & the SOLO Taxonomy. *Mathematics Education Research Journal, 10*(2), 4-26.

Chin, C., & Brown, D. E. (2000) Learning in Science: A Comparison of Deep & Surface Approaches. *Journal of Research in Science Teaching, 37*(2), 109-138.

Cholowski, K. C., & Chan, L. K. (1992) Diagnostic Reasoning Among Second-Year Nursing Students. *Journal of Advanced Nursing, 17*(9), 1171-1181.

Cicourel, A. V. (1964) *Method & Measurement in Sociology.* New York, USA: The Free Press.

Clark, M., & Lovric, M. (2009) Understanding Secondary-Tertiary Transition in Mathematics. *International Journal for Mathematical Education in Science & Technology, 40*(6), 755-776.

Clarke, R. M. (1986) Students' Approaches to Learning in an Innovative Medical School: A Cross-Sectional Study. *British Journal of Educational Psychology, 56*(3), 309-321.

Coady, C., & Pegg, J. (1994) Tertiary Students' Understanding of Second Order Relationships in Function Notation. In G. Bell, B. Wright, N. Leeson, & J. Geake (Eds.), *Challenges in Mathematics Education: Constraints on Construction* (pp. 179-186). Lismore, Australia: Mathematics Education Research Group of Australasia.

Coady, C., & Pegg, J. (1995) Students' Use of Second-Order Relationships in Algebra. In B. Atweh, & S. Flavel (Ed.), *Proceedings of the 18th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 189-194). Darwin, Australia: MERGA.

Coe, R. (2011) *Durham University Expert Comments on Grade Inflation.* Retrieved October 4, 2012, from Durham University: http://www.dur.ac.uk/news/newsitem/?itemno=12970

Coffield, F., Moseley, D., Hall, E., & Ecclestone, K. (2004) *Should We Be Using Learning Styles? What Research Has to Say to Practice.* Towbridge: Learning & Skills Research Centre/Cromwell Press Ltd.

Cohen, L., Manion, L., & Morrison, K. (2007) *Research Methods in Education* (6th ed.). London: Routledge.

Cole, M. (1991) Conclusion. In L. B. Resnick, J. M. Levine, & S. D. Teasley (Eds.), *Perspectives on Socially Shared Cognition* (pp. 398-417). Washington DC, USA: American Psychological Association.

Collins, J. L. (1982) Self-Efficacy & Ability in Achievement Behavior. *Paper presented at the meeting of the American Educational Research Association.* New York, USA.

Collis, B. (1998) New Didactics for University Instruction: Why & How? *Computers & Education, 31*(4), 373-393.

Confrey, J. (1995) A Theory of Intellectual Development - Part II. *For the Learning of Mathematics, 15*(1), 38-48.

Cornu, B. (1991) Limits. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 153-166). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Cox, W. (1994) Strategic Learning in A-Level Mathematics? *Teaching Mathematics & its Applications, 13*(1), 11-21.

Crawford, K. (1983) Past Experiences & Mathematical Problem Solving. *Proceedings of the 7th Annual Conference of the International Group for the Psychology of Mathematics Education.* Haifa, Israel.

Crawford, K. (1986) Cognitive & Social Factors in Problem Solving Behaviour. *Proceedings of the 10th Annual Conference of the International Group for the Psychology of Mathematics Education*, (pp. 415-421). London, UK.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1993) Learning Mathematics at University Level: Initial Conceptions of Mathematics. *Proceedings of the 16th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA)*, (pp. 209-214). Brisbane, Australia.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1994) Conceptions of Mathematics & How it is Learned: The Perspectives of Students Entering University. *Learning & Instruction, 4*(4), 331-345.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998a) Qualitatively Different Experiences of Learning Mathematics at University. *Learning & Instruction, 8*(5), 455-468.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998b) University Mathematics Students' Conceptions of Mathematics. *Studies in Higher Education, 23*(1), 87-94.

Creswell, J. W. (1998) *Qualitative Inquiry & Research Design: Choosing Among 5 Traditions.* Thousand Oaks, USA: Sage Publications.

Creswell, J. W. (2003) *Research Design: Qualitative, Quantitative, & Mixed Methods Approaches* (2nd ed.). Thousand Oaks, USA: Sage Publications.

Creswell, J. W., Shope, R., Plano Clark, V. L., & Green, D. O. (2006) How Interpretive Qualitative Research Extends Mixed Methods Research. *Research in the Schools, 13*(1), 1-11.

Cuthbert, P. F. (2005) The Student Learning Process: Learning Styles or Learning Approaches? *Teaching in Higher Education, 10*(2), 235-249.

Daly, A. L., Baird, J. A., Chamberlain, S., & Meadows, M. (2012) Assessment Reform: Students' & Teachers' Responses to the Introduction of Stretch & Challenge at A-Level. *Curriculum Journal, 23*(2), 139-155.

Darlington, E. (2009) *Gender Differences on the Secondary-Tertiary Mathematics Interface.* University of Warwick: Unpublished MSc Dissertation.

Darlington, E. (2010) *Undergraduate Mathematicians' Approaches to Learning.* University of Oxford: Unpublished MSc Dissertation.

Darlington, E. (2011) Undergraduate Mathematicians' Approaches to Learning. In C. Smith (Ed.), *Proceedings of the British Society for the Learning of Mathematics*, *31*, pp. 41-47. Oxford, UK.

Dart, B. C., Burnett, P. C., Boulton-Lewis, G., Campbell, J. L., Smith, D., & McCrindle, A. (1999) Classroom Learning Environments & Students' Approaches to Learning. *Learning Environments Research, 2*(2), 137-156.

Daskalogianni, K., & Simpson, A. (2001) Beliefs Overhang: The Transition from School to University. In J. Winter (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, *21*, pp. 97-108. Keele University, UK.

Daskalogianni, K., & Simpson, A. (2002) "Cooling-Off": The Phenomenon of a Problematic Transition from School to University. *Proceedings of the Second International Conference on Teaching Mathematics at the Undergraduate Level*, (pp. 103-110). Hersonissos, Crete.

Davis, P. J., & Hersh, R. (1983) *The Mathematical Experience.* Harmondsworth: Penguin.

Davis, P., Black, L., Farnsworth, V., Harris, D., Hernandez-Martinez, P., Pampaka, M., *et al.* (2009) Students' Experiences of First Year University & the Mediation of Institutional Discourses on their Transition into Mathematics in STEM. *Paper presented at the conference of the British Educational Research Association Transition Symposium*, (pp. 1-3). University of Manchester.

Davis, R. B., & Vinner, S. (1986) The Notion of Limit: Some Seemingly Unavoidable Misconception Stages. *Journal of Mathematical Behavior, 5*(3), 281-303.

de Guzman, M., Hodgson, B. R., Robert, A., & Villani, V. (1998) Difficulties in the Passage from Secondary to Tertiary Education. *Proceedings of the International Congress of Mathematics*, (pp. 747-762). Berlin, Germany.

Deci, E. L. (1975) *Intrinsic Motivation.* New York, USA: Plennum Press.

Delos Santos, A. G., & Thomas, M. O. (2002) Teaching Derivative with Graphic Calculators: The Role of a Representational Perspective. In W.-C. Yang, S.-S. Chu, T. de Alwis, & F. M. Bhatti (Ed.), *Proceedings of the 7th Asian Technology Conference in Mathematics* (pp. 349-358). Melaka, Malaysia: Program Committee.

Desantis, L., & Ugarriza, D. N. (2000) The Concept of Theme as Used in Qualitative Nursing Research. *Western Journal of Nursing Research, 22*(3), 351-372.

Dhalin, B., & Regmi, M. P. (1997) Conceptions of Learning Among Nepalese Students. *Higher Education, 33*(4), 471-493.

Diener, C. I., & Dweck, C. S. (1978) An Analysis of Learned Helplessness: Continuous Changes in Performance, Strategy, & Achievement Cognitions Following Failure. *Journal of Personality & Social Psychology, 36*(5), 451-462.

Diseth, Å. (2001) Validation of a Norwegian Version of the Approaches & Study Skills Inventory for Students (ASSIST): Application of Structural Equation Modelling. *Scandinavian Journal of Educational Research, 45*(4), 381-394.

Diseth, Å. (2002) The Relationship between Intelligence, Approaches to Learning & Academic Achievement. *Scandinavian Journal of Educational Research, 46*(2), 219-230.

Diseth, Å. (2003) Personality & Approaches to Learning as Predictors of Academic Achievement. *European Journal of Personality, 17*(2), 143-155.

Diseth, Å., & Martinesen, Ø. (2003) Approaches to Learning, Cognitive Style, & Motives as Predictors of Academic Achievement. *Educational Psychology, 23*(2), 195-207.

Douady, R. (1997) Didactic Engineering. In T. Nunes, & P. Bryant (Eds.), *Learning & Teaching Mathematics: An International Perspective* (pp. 373-401). Hove: Psychology Press Ltd.

Doyle, T. (2008) *Helping Students Learn in a Learner-Centred Environment: A Guide to Facilitating Learning in Higher Education.* Sterling, USA: Stylus Publishing, LLC.

Drew, D. E. (1992) Affective & Behavioral Components of Undergraduate Science Education. *Paper presented at the Annual Meeting of the American Educational Research Association.* San Francisco, USA.

Dreyfus, T. (1999) Why Johnny Can't Prove. *Educational Studies in Mathematics, 38*(1-3), 85-109.

d'Souza, S., & Wood, L. (2003) *Tertiary Students' Views about Group Work in Mathematics*. Retrieved September 26, 2012, from Proceedings of the Australian & New Zealand Associations of Research in Education Joint Conference. Auckland, New Zealand.: http://www.aare.edu.au/03pap/dso03154.pdf

Duff, A. (2002) Approaches to Learning: Factor Invariance Across Gender. *Personality & Individual Differences, 33*(6), 997-1010.

Duff, A., & Duffy, T. (2002) Psychometric Properties of Honey & Mumford's Learning Styles Questionnaire. *Personality & Individual Differences, 33*(1), 147-163.

Dunn, R. (1990) Understanding the Dunn & Dunn Learning Styles Model & the Need for Individual Diagnosis & Prescription. *Journal of Reading, Writing & Learning Disabilities, 6*(3), 223-247.

Duval, R. (2006) A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational Studies in Mathematics, 61*(2), 103-131.

Dweck, C. S. (1986) Motivational Processes Affecting Learning. *American Psychologist, 41*(10), 1040-1048.

Dweck, C. S., & Repucci, N. D. (1973) Learned Helplessness & Reinforcement Reponsibility in Children. *Journal of Personality & Social Psychology, 25*(1), 109-116.

Dweck, C., & Leggett, E. L. (1988) A Social Cognitive Approach to Motivation & Personality. *Psychological Review, 95*(2), 256-273.

Eccles, J., Wigfield, A., & Reuman, D. (1987) Changes in Self-Perceptions & Values at Early Adolescence. *Paper presented at the annual meeting of the American Educational Research Association.* San Francisco, USA.

Edwards, B. E., Dubinsky, E., & McDonald, M. A. (2005) Advanced Mathematical Thinking. *Mathematical Thinking & Learning, 7*(1), 15-25.

Edwards, P. (1996) *Implementing Diagnostic Testing for Non-Specialist Mathematics Courses.* London: Open Learning Foundation.

Egéa-Kuehne, D. (2003) Understanding Curriculum in France: A Multifaceted Approach to Thinking Education. In *International Handbook of Curriculum Research* (pp. 329-365). London: Lawrence Erlbaum Associates.

Eklund-Myrskog, G. (1999) Students' Approaches to Learning in Finnish General Upper Secondary School. *Scandinavian Journal of Educational Research, 43*(1), 5-18.

Eley, M. G. (1992) Differential Adoption of Study Approaches within Individual Students. *Higher Education, 23*(3), 231-254.

Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009) Exploring Strategy Use & Strategy Flexibility in Non-Routine Problem Solving by Primary School High Achievers in Mathematics. *ZDM Mathematics Education, 41*(5), 605-618.

Engelbrecht, J., & Harding, A. (2003) Is Mathematics Running Out of Numbers? *South African Journal of Science, 99*(1-2), 17-20.

English, L., & Ihnatko, T. (1998) Curriculum Innovation: Assessing the Effectiveness of the Explicit Teaching of Communication Skills in a Large First-Year Accounting Course. In C. Rust (Ed.), *Improving Students as Learners.* Oxford: Oxford Centre for Staff & Learning Development, Oxford Brookes University.

English, L., Luckett, P., & Mladenovic, R. (2004) Encouraging a Deep Approach to Learning through Curriculum Design. *Accounting Education: An International Journal, 13*(4), 461-488.

Entwistle, N. (1989) Approaches to Studying & Course Perceptions: The Case of the Disappearing Relationship. *Studies in Higher Education, 14*(2), 155-156.

Entwistle, N. (1997) Reconstituting Approaches to Learning: A Response to Webb. *Higher Education, 33*(2), 213-218.

Entwistle, N. (2001) Styles of Learning & Approaches to Studying in Higher Education. *Kybernetes, 30*(5-6), 593-602.

Entwistle, N. J., & Meyer, J. H. (1992) Findings & Implications from Research on Student Learning. *South African Medical Journal, 81*(12), 593-595.

Entwistle, N. J., & Ramsden, P. (1983) *Understanding Student Learning.* London: Croom Helm.

Entwistle, N. J., & Tait, H. (1990) Approaches to Learning, Evaluations of Teaching, & Preferences for Contrasting Academic Environments. *Higher Education, 19*(2), 169-194.

Entwistle, N. J., & Tait, H. (1995) *The Revised Approaches to Studying Inventory.* Edinburgh: Centre for Research on Learning & Instruction, University of Edinburgh.

Entwistle, N., Hanley, M., & Hounsell, D. (1979a) Identifying Distinctive Approaches to Studying. *Higher Education, 8*(4), 365-380.

Entwistle, N. J., Hanley, M., & Ratcliffe, G. (1979b) Approaches to Learning & Levels of Understanding. *British Journal of Educational Psychology, 5*(1), 99-114.

Entwistle, N., & Marton, F. (1984) Changing Conceptions of Learning & Research. In F. Marton, D. Hounsell, & N. Entwistle (Eds.), *The Experience of Learning* (pp. 211-236). Edinburgh: Scottish Academic Press.

Entwistle, N., Tait, H., & McCune, V. (2000) Patterns of Response to an Approaches to Studying Inventory Across Constrasting Groups & Contexts. *European Journal of Psychology of Education, 15*(1), 33-48.

Ernest, P. (2004) *What is the Philosophy of Mathematics Education?* Retrieved February 11, 2013, from http://people.exeter.ac.uk/PErnest/pome18/PhoM_%20for_ICME_04.htm

Ertl, H., & Wright, S. (2008) Reviewing the Literature on the Student Experience in Higher Education. *London Review of Education, 6*(3), 195-210.

Ertmer, P. A., & Newby, T. J. (1996) The Expert Learner: Strategic, Self-Regulated, & Reflective. *Instructional Science, 24*(1), 1-24.

Ervynck, G. (1991) Mathematical Creativity. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 42-53). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Etchells, T., & Monaghan, J. (1994) Hand Held Technology: Assessment Issues. *Acta Didactica Universitatis Comenianae: Mathematics, 3*(2), 119-130.

Ethington, C. A. (1988) Differences Among Women Intending to Major in Quantitative Fields of Study. *Journal of Educational Research, 81*(6), 354-359.

European Commission (2005) *Students as Journeymen Between Communities of Higher Education & Work: Final Report.* Retrieved February 17, 2012, from http://www.hewl.net/HPSE_CT-2001-0068__final_ju.pdf

Evans, L., & Abbott, I. (1998) *Teaching & Learning in Higher Education.* London: Continuum International Publishing Group.

Evans, M. (1999) *School Leavers' Transition to Tertiary Study: A Literature Review (Working Paper).* Department of Econometrics & Business Statistics: Monash University.

Farmer, J. A., Buckmaster, A., & LeGrand, B. (1992) Cognitive Apprenticeship: Implications for Continuing Professional Development. *New Directions for Adult & Continuing Education, 55*(1), 41-49.

Faulkner, F., Hannigan, A., & Gill, O. (2009) The Changing Profile of Third-Level Service Mathematics Students (1997-2008). *Proceedings of the 3rd National Conference on Research in Mathematics Education*, (pp. 355-366). Drumcondra, Ireland.

Fee, H., Greenan, K., & Wall, A. (2009) An Investigation into Secondary School Exit Standards: Implications for University Lecturers. *International Journal of Management Education, 8*(2), 43-53.

Filipsson, L., & Thunberg, H. (2008) *Aims Versus Expectations - A Swedish Study of Problems Related to the Transition from Secondary to Tertiary Education in Mathematics.* Retrieved January 3, 2011, from http://www.math.kth.se/~thunberg/full-pdf/ICME11_aims_vs_expect.pdf

Fisher, S., & Hood, B. (1987) The Stress of the Transition to University: A Longitudinal Study of Psychological Disturbance, Absent-Mindedness & Vulnerability to Homesickness. *British Journal of Psychology, 78*(3), 425-441.

Flyvbjerg, B. (2006) Five Misunderstandings About Case-Study Research. *Qualitative Inquiry, 12*(2), 219-245.

Ford, N. (1981) Recent Approaches to the Studying & Teaching of Effective Learning in Higher Education. *Review of Educational Research, 51*(3), 345-377.

Freeman, R., & Lewis, R. (1998) *Planning & Implementing Assessment.* London: Kogan Page.

French, D. (2004) The DfES Reponse to the Smith Report. *Newsletter of the Mathematical Association, 125*, 7.

Fritze, Y., & Nordkvelle, Y. T. (2003) Comparing Lectures: Effects of the Technological Context of a Studio. *Education & Information Technologies, 8*(4), 327-343.

Furinghetti, F., Maggiani, C., & Morselli, F. (2012) How Mathematics Students Perceive the Transition From Secondary to Tertiary Level with Particular Reference to Proof. *Nordic Studies in Mathematics Education, 17*(3-4), 101-116.

Furnham, A., Chamorro-Premuzic, R., & McDougall, F. (2003) Personality, Cognitive Ability, & Beliefs About Intelligence as Predictors of Academic Performance. *Learning & Individual Differences, 14*(1), 49-66.

Gago, J. M. (2004) *Increasing Human Resources for Science & Technology in Europe.* Brussels: European Commission.

Galbraith, P. L., & Haines, C. R. (1995) Students' Mathematical Characteristics: Some Developmental Skills for Undergraduate Programs. *Bulletin of the Institute of Mathematics & its Applications, 31*(2), 175-179.

Galbraith, P., & Haines, C. (2000) Conceptual (Mis)Understandings of Beginning Undergraduates. *Journal of Mathematical Education in Science & Technology, 31*(5), 651-678.

Galbraith, P., & Haines, C. (2001) Conceptual & Procedural Demands Embedded in Modelling Tasks. In J. F. Matos, W. Blum, K. S. Houston, & S. P. Carreira (Eds.), *Modelling & Mathematics Education: ICTMA9: Applications in Science & Technology* (pp. 342-353). Chichester: Horwood Publishing.

Galbraith, P., Pemberton, M., & Haines, C. (1996) Testing to a Purpose: Assessing the Mathematical Knowledge of Entering Undergraduates. *Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australasia*, (pp. 215-220). University of Melbourne, Australia.

Gano-Garcia, F., & Justicia-Justicia, F. (1994) Learning Strategies, Styles & Approaches: An Analysis of their Interrelationships. *Higher Education, 27*(2), 239-260.

García-Ros, R., Pérez, F., & Talaya, I. (2008) New University Students' Instructional Preferences & how these Relate to Learning Styles & Motivational Strategies. *Journal of Research in Educational Psychology, 6*(3), 547-570.

Garner, I. (2000) Problems & Inconsistencies with Kolb's Learning Styles. *Educational Psychology, 20*(3), 341-348.

Garrison, D. R., & Cleveland-Innes, M. (2005) Facilitating Cognitive Presence in Online Learning: Interaction is Not Enough. *American Journal of Distance Education, 19*(3), 133-148.

Geraniou, E., & Tall, D. (2004) *Motivation: Is The Student Expected To Have It Or Should It Be Provided?* Retrieved February 12, 2009, from http://www.warwick.ac.uk/staff/David.Tall/pdfs/dot2004z-geraniou-pme.pdf

Gibbs, G. (1994) Improving Student Learning: Theory & Practice. *Proceedings of the First International Improving Student Learning Symposium.* Oxford: Oxford Centre for Staff Development.

Gijbels, D., & Dochy, F. (2006) Students' Assessment Preferences & Approaches to Learning: Can Formative Assessment Make a Difference? *Educational Studies, 32*(4), 399-409.

Gijbels, D., Segers, M., & Struyf, E. (2008) Constructivist Learning Environments & the (Im)possibility to Change Students' Perceptions of Assessment Demands & Approaches to Learning. *Instructional Science, 36*(5-6), 431-443.

Gilham, B. (2000) *Case Study Research Methods* (Vol. 6). London: Continuum.

Gillespie, M., & Noble, J. (1992) Factors Affecting Student Persistence: A Longitudinal Study. *Retrieved July 14, 2009 from: http://www.act.org/research/researchers/reports/pdf/ACT_RR92-04.pdf.* Iowa: American College Testing Program.

Girves, J. E., & Wemmerus, V. (1988) Developing Models of Graduate Student Degree Progress. *Journal of Higher Education, 59*(2), 163-189.

Glastra, F. J., Hake, B. J., & Schedler, P. E. (2004) Lifelong Learning as Transitional Learning. *Adult Education Quarterly, 54*(4), 291-307.

González-Martín, A. S., & Camacho, M. (2004) Legitimization of the Graphic Register in Problem Solving at Undergraduate Level: The Case of the Improper Integral. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, *2*, pp. 479-486. Bergen, Norway.

Good, C. (2011) We Can't Let Them Graduate Unless... In P. Rowlett (Ed.), *HE Mathematics Curriculum Summit: A Report on the Summit Held at the University of Birmingham on 12 January 2011* (pp. 15-16). London: Mathematical Sciences HE Curriculum Innovation Project.

Gordon, N. (2005) Crisis, What Crisis? *MSOR Connections, 5*(3), 1-10.

Gordon, S. P., & Rees, M. (1997) High-Stakes Testing: Worth the Price? *Journal of School Leadership, 7*(3), 345-368.

Gottfried, A. E. (1985) Academic Intrinsic Motivation in Elementary & Junior High Students. *Journal of Educational Psychology, 77*(6), 631-645.

Greasley, K. (1998) Does Gender Affect Students' Approaches to Learning? In S. Brown, S. Armstrong, & G. Thompson (Eds.), *Motivating Students.* London: Kogan Page.

Greatorex, J., & Suto, W. M. (2006) An Empirical Exploration of Human Judgement in the Marking of School Examinations. *Paper presented at the 32nd Conference for the International Association for Educational Assessment.* Singapore.

Green, J., Martin, A. J., & Marsh, H. W. (2005) Academic Motivation & Engagement: A Domain Specific Approach. In P. L. Jeffery (Ed.), *Proceedings of the Australian Association for Research in Education Conference*, (pp. 1-11). Parramatta, Australia.

Greene, J. C. (2008) Is Mixed Methods Social Inquiry A Distinctive Methodology? *Journal of Mixed Methods Research, 2*(1), 7-22.

Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989) Toward a Conceptual Framework for Mixed-Method Evaluation Designs. *Educational Evaluation & Policy Analysis, 11*(3), 255-274.

Grehan, M., O'Shea, A., & Mac an Bhaird, C. (2010) How do Students Deal with Difficulties in Mathematics? In D. Green (Ed.), *Proceedings of the CTEL-MSOR Conference*, (pp. 34-38). University of Birmingham, UK.

Griffiths, H. B., & McLone, R. R. (1984) A Critical Analysis of University Examinations in Mathematics Part I: A Problem of Design. *Educational Studies in Mathematics, 15*(3), 291-311.

Guba, E. G. (1981) Criteria for Assessing the Trustworthiness of Naturalistic Inquiries. *Educational Communication & Technology Journal, 29*(2), 75-92.

Guest, G., Bunce, A., & Johnson, L. (2006) How Many Interviews Are Enough? An Experiment with Data Saturation & Variability. *Field Methods, 18*(1), 59-82.

Gueudet, G. (2008) Investigating the Secondary-Tertiary Transition. *Educational Studies in Mathematics, 67*(3), 237-257.

Hackett, G., & Betz, N. E. (1989) An Exploration of the Mathematics Self-Efficacy/Mathematics Performance Correspondence. *Journal for Research in Mathematics Education, 20*(3), 261-273.

Haggis, T. (2003) Constructing Images of Ourselves? A Critical Investigation into 'Approaches to Learning' Research in Higher Education. *British Educational Research Journal, 29*(1), 89-105.

Haggis, T. (2009) What Have We Been Thinking Of? A Critical Overview of 40 Years of Student Learning Research in Higher Education. *Studies in Higher Education, 34*(4), 377-390.

Hall, G. G. (1982) Motivation Among Mathematics Undergraduates. *International Journal of Mathematical Education in Science & Technology, 13*(5), 599-602.

Halpin, R. L. (1990) An Application of the Tinto Model to the Analysis of Freshman Persistence in a Community College. *Community College Review, 17*(4), 22-32.

Hannula, M. S. (2002a) Goal Regulation: Needs, Beliefs & Emotions. *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, *4*, pp. 73-80. Norwich, UK.

Hannula, M.S. (2002b) Attitude Towards Mathematics: Emotions, Expectations & Values. *Educational Studies in Mathematics, 49*(5), 25-46.

Hannula, M. S. (2006) Motivation in Mathematics: Goals Reflected in Emotion. *Educational Studies in Mathematics, 63*(2), 165-178.

Hanson, W. E., Creswell, J. W., Plano Clark, V. L., Petska, K. S., & Creswell, J. D. (2005) Mixed Method Research Designs in Counseling Psychology. *Journal of Counseling Psychology, 52*(2), 224-235.

Harackiewicz, J. M., Barron, K. E., & Elliot, A. T. (1998) Rethinking Achievement Goals: When Are They Adaptive for College Students & Why? *Educational Psychologist, 33*(1), 1-21.

Hardy, T., & Hanley, U. (2002) A Re-examination of Reflective Practice: Is it a Viable Frame for Mathematics Teacher Education at the Present Time & Was it Ever? In P. Valero, & O. Skovsmose (Ed.), *Proceedings of the 3rd International Conference* (pp. 1-12). Copenhagen, Denmark: Centre for Research on Learning Mathematics.

Harel, G., & Sowder, L. (1998) Students' Proof Schemes. *Research in Collegiate Mathematics Education, 3*, 234-283.

Hawkes, T., & Savage, M. (Eds.) (2000) *Measuring the Mathematics Problem.* London: Engineering Council.

Hayward, G., & McNicholl, J. (2007) Modular Mayhem? A Case Study of the Development of the A-Level Science Curriculum in England. *Assessment in Education: Principles, Policy & Practice, 14*(3), 335-351.

Hazel, E., Prosser, M., & Trigwell, K. (2002) Variation in Learning Orchestration in University Biology Courses. *International Journal of Science Education, 24*(7), 737-751.

Heck, A., & van Gastel, L. (2006) Mathematics on the Threshold. *International Journal of Mathematical Education in Science & Technology, 37*(8), 925-945.

Hedegaard, M. (2005) The Zone for Proximal Development as Basis for Instruction. In H. Daniels (Ed.), *An Introduction to Vygotsky* (pp. 227-251). Hove: Routledge.

Heijne-Penninga, M., Kuks, J. B., Hofman, W. H., & Cohen-Schotanus, J. (2008) Influence of Open- & Closed-Book Tests on Medical Students' Learning Approaches. *Medical Education, 42*(10), 967-974.

Heikkila, A., & Lonka, K. (2006) Studying in Higher Education: Students' Approaches to Learning, Self-Regulation & Cognitive Strategies. *Studies in Higher Education, 31*(1), 99-119.

Heinström, J. (2000) The Impact of Personality & Approaches to Learning on Information Behaviour. *Information Research, 5*(3), Accessed via http://informationr.net/ir/5-3/paper78.html.

Hembree, R. (1990) The Nature, Effects, & Relief of Mathematics Anxiety. *Journal for Research in Mathematics Education, 21*(1), 33-46.

Herzig, A. H. (2002) Where have all the Students Gone? Participation of Doctoral Students in Authentic Mathematical Activity as a Necessary Condition for Persistence Toward the PhD. *Educational Studies in Mathematics, 50*(2), 177-212.

Herzig, A. H. (2004a) Becoming Mathematicians: Women & Students of Color Choosing & Leaving Doctoral Mathematics. *Review of Educational Research, 74*(2), 171-214.

Herzig, A. H. (2004b) 'Slaughtering this Beautiful Math': Graduate Women Choosing & Leaving Mathematics. *Gender & Education, 16*(3), 379-395.

Hidi, S. (2000) An Interest Researcher's Perspective: The Effects of Extrinsic & Intrinsic Factors on Motivation. In C. Sansone, & J. M. Harackiewicz (Eds.), *Intrinsic & Extrinsic Motivation: The Search for Optimal Motivation & Performance* (pp. 373-404). London: Academic Press.

Higher Education Research Institute (1991) *The American College Student, 1989: National Norms for 1985 & 1987.* Los Angeles, USA: Higher Education Research Institute, UCLA.

Higher Education Statistics Agency (2011) *Non-Continuation Following Year of Entry: Full-Time First Degree Entrants by Age Marker & Continuation Status (T3a).* Retrieved February 10, 2012, from http://www.hesa.ac.uk/index.php?option=com_content&task=view&id=2064&Itemid=141

Higton, J., Noble, J., Pope, S., Boal, N., Ginnis, S., Donaldson, R., *et al.* (2012) *Fit for Purpose? The View of the Higher Education Sector, Teachers & Employers on the Suitability of A Levels.* London: Ipsos MORI/Ofqual.

Hirst, K., & Meacock, S. (1999) Modular A-Levels & Undergraduates. *Teaching Mathematics & its Applications, 18*(3), 122-128.

Hirst, K., Meacock, S., & Ralha, E. (2004) Student Expectations of Studying Mathematics at University. *Mathematics Education Research Group of Australasia, 3*(4), 295-302.

Ho, H., Senturk, D., Lam, A. G., Zimmer, J. L., Hong, S., Okamoto, Y., *et al.* (2000) The Affective & Cognitive Dimensions of Math Anxiety: A Cross-National Study. *Journal for Research in Mathematics Education, 31*(3), 362-379.

Hockey, J. (1993) Research Methods - Researching Peers & Familiar Settings. *Research Papers in Education, 8*(2), 199-225.

Hodgen, J., Pepper, D., Sturman, L., & Ruddock, G. (2010) *Is the UK an Outlier? An International Comparison of Upper Secondary Mathematics Education.* London: Nuffield Foundation.

Hodgson, A., & Spours, K. (2004) *Beyond A Levels: Curriculum 2000 & the Reform of 14-19 Qualifications.* London: Kogan Page.

Holton, D. (Ed.). (2001) *The Teaching & Learning of Mathematics at University Level: An ICMI Study.* Dordrecht, the Netherlands: Kluwer Academic Publishers.

Honey, P., & Mumford, A. (1986) *The Manual of Learning Styles.* Maidenhead: Peter Honey.

House of Commons Education Committee (2012) *The Administration of Examinations for 15-19 Year Olds in England: First Report of Session 2012-13.* London: The Stationery Office Limited.

Howe, K. R. (1988) Against the Quantitative-Qualitative Incompatibility Thesis or Dogmas Die Hard. *Educational Researcher, 17*(8), 10-16.

Hoyles, C., Newman, K., & Noss, R. (2001) Changing Patterns of Transition from School to University Mathematics. *International Journal of Mathematical Education in Science & Technology, 32*(6), 829-845.

Hughes, R., Monaghan, J., Shingadia, E., & Vaughan, S. (2006) Revisiting Routine Questions. *Teaching Mathematics & its Applications, 25*(2), 90-96.

Hunter, A. B., Laursen, S. L., & Seymour, E. (2007) Becoming a Scientist. The Role of Undergraduate Research in Students' Cognitive, Personal, & Professional Development. *Science Education, 91*(1), 36-74.

Huntley, B. (2008) *Comparing Different Assessment Formats in Undergraduate Mathematics.* University of Pretoria: Unpublished Doctoral Thesis.

Huntley, B., Engelbrecht, J., & Harding, A. (2009) Can Multiple Choice Questions be Successfully Used as an Assessment Format in Undergraduate Mathematics? *Pythagoras, 69*(1), 3-16.

Huws, N., Reddy, P. A., & Talcott, J. B. (2009) The Effects of Faking on Non-Cognitive Predictors of Academic Performance in University Students. *Learning & Individual Differences, 19*(4), 476-480.

Hyde, J. S., Fennema, E., Ryan, M., Frost, L. A., & Hopp, C. (1990) Gender Comparisons of Mathematics Attitudes & Affect: A Meta-Analysis. *Psychology of Women Quarterly, 14*(2), 299-324.

Iannone, P., & Nardi, E. (2007) The Interplay Between Syntactic & Semantic Knowledge in Proof Production: Mathematicians' Perspectives. In D. Pitta-Pantazi, & G. Philippou (Ed.), *Proceedings of the 5th Congress of the European Society for Research in Mathematics Education*, (pp. 2300-2309). Larnaca, Cyprus.

Iannone, P., & Simpson, A. (2011a) The Summative Assessment Diet: How We Assess Mathematics Degrees. *Teaching Mathematics & its Applications, 30*(4), 186-196.

Iannone, P., & Simpson, A. (2011b) How Do We Assess Our Students? *MSOR Connections, 12*(1), 7-10.

Iannone, P., & Simpson, A. (Eds.) (2012) *Mapping University Mathematics Assessment Practices.* Norwich: University of East Anglia.

Iannone, P., & Simpson, A. (2013) Students' Perceptions of Assessment in Undergraduate Mathematics. *Research in Mathematics Education, 15*(1), 17-33.

Iben, M. F. (1991) Attitudes & Mathematics. *Comparitive Education, 27*(2), 135-151.

International Baccalaureate Organisation (2011) *The IB Diploma Programme Statistical Bulletin.* Retrieved February 10, 2012, from http://www.ibo.org/facts/statbulletin/dpstats/documents/statistical_bulletin_may_2011.pdf

Ioannou, M., & Nardi, E. (2009) Engagement, Abstraction & Visualisation: Cognitive & Emotional Aspects of Year 2 Mathematics Undergraduates' Experience in Abstract Algebra. *Proceedings of the British Society for Research into the Learning of Mathematics*, *29(2)*, pp. 35-40. Bristol University, UK.

James, R., McInnis, C., & Devlin, M. (2002) *Assessing Learning in Australian Universities*. Retrieved September 26, 2012, from Centre for Higher Education Studies & Australian University Teaching Committee: http://www.cshe.unimelb.edu.au/assessinglearning/docs/AssessingLearning.pdf

Jaworski, B. (2000) Collaborating with Mathematics Tutors to Explore Undergraduate Teaching. *Paper presented at the 1st ESRC Teaching & Learning Research Programme Conference*, (pp. 1-17). Leicester, UK.

Joint Council for Qualifications. (2012) *Examination Results*. Retrieved October 8, 2012, from http://www.jcq.org.uk/national_results/alevels/

Jones, B., & Frydenberg, E. (1999) Who Needs Help & When: Coping with the Transition from School to University. *Paper presented at the Annual Conference of the American Educational Research Association*, (pp. 1-27). Montreal, Canada.

Jones, K. (2000) The Student Experience of Mathematical Proof at University Level. *International Journal of Mathematical Education in Science & Technology, 31*(1), 53-60.

Kadijević, D. (2002) TIMSS 2003 Mathematics Cognitive Domains: Examination, Operationalization & Implications. *Zbornik Instituta za Pedagoŝka Istraživanja, 34*(2), 96-102.

Kahn, P. E., & Hoyles, C. (1997) The Changing Undergraduate Experience: A Case Study of Single Honours Mathematics in England & Wales. *Studies in Higher Education, 22*(3), 349-362.

Kajander, A., & Lovric, M. (2005) Transition from Secondary to Tertiary Mathematics: McMaster University Experience. *International Journal of Mathematical Education in Science & Technology, 36*(2-3), 149-160.

Kantanis, T. (1997) Factors that Affect Student Transition from Year 12 to the First Year of University: A Case Study of Arts Undergraduates at Monash University. *Paper presented at the conference for the Australian Association for Research in Education.* Brisbane, Australia.

Kantanis, T. (2000) The Role of Social Transition in Students' Adjustment to the First-Year of University. *Journal of Institutional Research, 9*(1), 100-110.

Kaplan, A., & Maehr, M. L. (1999) Achievement Goals & Student Well-Being. *Contemporary Educational Psychology, 24*(4), 330-358.

Kell, C., & van Deursen, R. (2002) Undergraduate Curricula Facilitate Student Learning Profile Development. *Paper presented at the British Educational Research Association Conference.* Exeter, UK.

Kember, D. (1996) The Intention to Both Memorise & Understand: Another Approach to Learning? *Higher Education, 31*(3), 341-354.

Kember, D. (2000) Misconceptions About the Learning Approaches, Motivation & Study Processes of Asian Students. *Higher Education, 40*(1), 99-121.

Kember, D., & Gow, L. (1990) Cultural Specificity of Approaches to Study. *British Journal of Educational Psychology, 60*(2), 356-363.

Kilpatrick, J. (1993) The Chain & The Arrow: From the History of Mathematics Assessment. In M. Niss (Ed.), *Investigations into Mathematics Education: An ICMI Study* (pp. 31-46). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Kinkead, J. (2003) Learning Through Inquiry: An Overview of Undergraduate Research. *New Directions for Teaching & Learning, 93*(1), 5-17.

Kitchen, A. (1999) The Changing Profile of Entrants to Mathematics at A Level & to Mathematical Subjects in Higher Education. *British Educational Research Journal, 25*(1), 57-74.

Kloosterman, P. (1988) Self-Confidence & Motivation in Mathematics. *Journal of Educational Psychology, 80*(3), 345-351.

Kolb, D. A. (1976) *Learning Style Inventory: Technical Manual.* Boston, USA: McBer.

Kounine, L., Marks, J., & Truss, E. (2008) *The Value of Mathematics.* London: Reform.

Krathwohl, D. R. (2002) A Revision of Bloom's Taxonomy: An Overview. *Theory Into Practice, 41*(4), 212-218.

Kreber, C. (2003) The Relationship Between Students' Course Perception & their Approaches to Studying Undergraduate Science Courses: A Canadian Experience. *Higher Education Research & Development, 22*(1), 57-75.

Kreiter, C. D., Yin, P., Solow, C., & Brennan, R. L. (2004) Investigating the Reliability of the Medical School Admissions Interview. *Advances in Health Sciences Education, 9*(2), 147-159.

Kreitzer, A., & Madaus, G. (1994) Empirical Investigations of the Hierarchical Structure of the Taxonomy. In L. Anderson, & L. Sosniak (Eds.), *Bloom's Taxonomy: A Forty-Year Retrospective* (pp. 64-81). Chicago, USA: National Society for the Study of Education.

Kulik, C. L. (1985) Effects of Inter-Class Ability Grouping on Achievement & Self-Esteem. *Paper presented at the annual meeting of the American Psychological Association.* Los Angeles, USA.

Kvale, S. (1996) *Interviews.* London: Sage Publications.

Kvale, S. (2008) *Doing Interviews.* London: Sage Publications.

Kyriacou, C., & Goulding, M. (2006) *A Systematic Review of Strategies to Raise Pupils' Motivational Effort in Key Stage 4 Mathematics. Technical Report.* Research Evidence

in Education Library, London: EPPI-Centre, Social Science Research Unit, Institute of Education, University of London.

Lagrange, J., & Monaghan, J. (2009) On the Adoption of a Model to Interpret Teachers' Use of Technology in Mathematics Lessons. *Proceedings of CERME 6*, (pp. 1605-1614). Lyon, France.

Laurillard, D. (1979) The Process of Student Learning. *Higher Education, 8*(4), 395-409.

Laurillard, D. (1993) *Rethinking University Teaching: A Framework for the Effective use of Educational Technology.* London: Routledge.

Lave, J., & Wenger, E. (1991) *Situated Learning: Legitimate Peripheral Participation.* Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (2005) Practice, Person, Social World. In H. Daniels (Ed.), *An Introduction to Vygotsky* (pp. 149-156). Hove: Routledge.

Lawrence, J. (2003) *The 'Deficit-Discourse' Shift: University Teachers & their Role in Helping First Year Students Persevere & Succeed in the New University Culture.* Retrieved July 14, 2009, from a Paper first presented at the 6th Pacific Rim First Year in Higher Education Conference 2002: Changing Agendas: http://ultibase.rmit.edu.au/Articles/march03/lawrence1.pdf

Lawson, D. (1997) What Can We Expect from A-Level Mathematics Students? *Teaching Mathematics & its Applications, 16*(4), 151-157.

Le, H., Casillas, A., Robbins, S. B., & Langley, R. (2005) Motivational & Skills, Social, & Self-Management Predictors of College Outcomes: Constructing the Student Readiness Inventory. *Educational & Psychological Measurement, 65*(3), 1-28.

Legrand, M. (2001) On the Training of French Prospective University Teachers. In D. Holton (Ed.), *The Teaching & Learning of Mathematics at University Level: An ICMI Study* (pp. 519-528). London: Springer.

Leinch, C., Pountney, D. C., & Etchells, T. (2002) Appropriate Use of a CAS in the Teaching & Learning of Mathematics. *International Journal of Mathematical Education in Science & Technology, 33*(1), 1-14.

Lenney, E. (1977) Women's Self-Confidence in Achievement Settings. *Psychological Bulletin, 84*(1), 1-13.

Lepper, M. R. (1988) Motivational Considerations in the Study of Instruction. *Cognition & Instruction, 5*(4), 289-309.

Leron, U., & Dubinsky, E. (1995) An Abstract Algebra Story. *American Mathematical Monthly, 102*(3), 227-242.

Lesh, R. (2000) Beyond Constructivism: Identifying Mathematical Abilities that are Most Needed for Success Beyond School in an Age of Information. *Mathematics Education Research Journal, 12*(3), 177-195.

Lewis, H., & White, J. (2002) Teaching & Learning Across the A-Level - University Transition. *MSOR Connections, 2*(4), 14-18.

Lin, Y. G., McKeachie, W. J., & Kim, Y. C. (2003) College Student Intrinsic &/Or Extrinsic Motivation & Learning. *Learning & Individual Differences, 13*(3), 251-258.

Lincoln, Y. S., & Guba, E. G. (1985) *Naturalistic Inquiry.* London: Sage Publications.

Lindblom-Ylänne, S., & Lonka, K. (1999) Individual Ways of Interacting with the Learning Environment - Are they Related to Study Success? *Learning & Instruction, 9*(1), 1-18.

Lindblom-Ylänne, S., & Lonka, K. (2000) Individual Study Orchestrations of High Achieving University Students. *European Journal of Educational Psychology, 15*(1), 19-32.

Lindblom-Ylänne, S., & Lonka, K. (2001) Medical Students' Perceptions of the Assessment Practices. *Advances in Health Sciences Education, 6*(1), 121-140.

Linnenbrink, E. A. (2006) Emotion Research in Education: Theoretical & Methodological Perspectives on the Integration of Affect, Motivation, & Cognition. *Educational Psychology Review, 18*(4), 307-314.

Lipinskienė, D., & Glinskiene, R. (2005) The Factors of Educational Environment: Their Influences on Students' Approach to Learn. *Pedagogika, 78*, 10-16. Retrieved July 16, 2009 from: www.ceeol.com/aspx/getdocument.aspx?logid=5&id=fc9ff043-ad34-4321-9ff1-55f6e01f4524.

Lithner, J. (2003) Students' Mathematical Reasoning in University Textbook Exercises. *Educational Studies in Mathematics, 52*(1), 29-55.

Lizzio, A., Wilson, K., & Simons, R. (2002) University Students' Perceptions of the Learning Environment & Academic Outcome: Implications for Theory & Outcome. *Studies in Higher Education, 27*(1), 27-50.

London Mathematical Society (1995) *Tackling the Mathematics Problem.* Birmingham: University of Birmingham.

Lonka, K., Olkinuora, E., & Mäkinen, J. (2004) Aspects & Prospects of Measuring Studying & Learning in Higher Education. *Educational Psychology Review, 16*(4), 301-323.

Lovitts, B. E. (2001) *Leaving the Ivory Tower: The Causes & Consequences of Departure from Doctoral Study.* Lanham, USA: Rowman & Littlefield.

Lucas, U., & Mladenovic, R. (2004) Approaches to Learning in Accounting Education. *Accounting Education: An International Journal, 13*(4), 399-407.

Lumsden, L. (1994) *Student Motivation to Learn.* Retrieved January 26, 2012, from http://www.ericdigests.org/1995-1/learn.htm

Macbean, J. (2004) Students' Conceptions of, & Approaches to, Studying Mathematics as a Service Subject at Undergraduate Level. *International Journal of Mathematical Education in Science & Technology, 35*(4), 553-564.

MacCorquodale, P. (1984) *Self-Image, Science, & Math: Does the Image of the "Scientist" Keep Girls & Minorities from Pursuing Science & Math.* Washington, USA: National Institute of Education.

Maclellan, E. (2005) Conceptual Learning: The Priority for Higher Education. *British Journal of Educational Studies, 53*(2), 129-147.

Maehr, M. L. (1974) Culture & Achievement Motivation. *American Psychologist, 29*(12), 887-895.

Magajna, Z. (1997) Researching Geometric Thinking in Out of School Contexts. *Proceedings of the British Society for Research into the Learning of Mathematics. 17 (1-2)*, pp. 69-74. Nottingham & Oxford, UK: BSRLM.

Maguire, S., Evans, S. E., & Dyas, L. (2001) Approaches to Learning: A Study of First-Year Geography Undergraduates. *Journal of Geography in Higher Education, 25*(1), 95-107.

Mäkinen, J., & Vainiomaki, P. (2002) One Step Towards Expertise: The Role of the Attachment Period in General Practice. *Medical Teacher, 24*(1), 102-104.

Malabar, I., & Pountney, D. C. (2002) Using Technology to Integrate Constructivism & Visualisation in Mathematics Education. In I. Vakalis, D. Hughes Hallett, D. Quinney, & C. Kourouniotis (Ed.), *Proceedings of the 2nd International Conference on the Teaching of Mathematics at the Undergraduate Level.* Heronissos, Crete.

Malcolm, J., & Zukas, M. (2001) Bridging Pedagogic Gaps: Conceptual Discontinuities in Higher Education. *Teaching in Higher Education, 6*(1), 33-42.

Mann, S. J. (2001) Alternative Perspectives on the Student Experience: Alienation & Engagement. *Studies in Higher Education, 26*(1), 7-19.

Marriott, P. (2002) A Longitudinal Study of Undergraduate Accounting Students' Learning Style Preferences at two UK Universities. *Accounting Education: An International Journal, 11*(1), 43-62.

Marsh, H. W. (1986) Verbal & Math Self-Concepts: An Internal/External Frame of Reference Model. *American Educational Research Journal, 23*(1), 129-149.

Marsh, H. W. (1987) The Big-Fish-Little-Pond Effect on Academic Self-Concept. *Journal of Educational Psychology, 79*(3), 280-295.

Marsh, H. W., Smith, I. D., & Barnes, J. (1985) Multidimensional Self-Concepts: Relations with Sex & Academic Achievement. *Journal of Educational Psychology, 77*(5), 581-596.

Marshall, D., & Case, J. (2005) 'Approaches to Learning' Research in Higher Education: A Response to Haggis. *British Educational Research Journal, 31*(2), 257-267.

Martin, A. J., & Marsh, H. W. (2005) Student Motivation & Engagement in Mathematics, Science, & English: Multilevel Modelling. In P. L. Jeffery (Ed.), *Proceedings of the Australian Association for Research in Education Conference*, (pp. 1-13). Parramatta, Australia.

Marton, F. (1976) What Does it Take to Learn? Some Implications of an Alternative View to Learning. In N. J. Entwistle (Ed.), *Strategies for Research & Development in Higher Education* (pp. 32-42). Amsterdam, the Netherlands: Swets & Zeitlinger.

Marton, F., & Säljö, R. (1976) On Qualitative Differences in Learning: I. Outcome & Process. *British Journal of Educational Psychology, 46*(1), 4-11.

Marton, F., & Säljö, R. (1984) Approaches to Learning. In F. Marton, D. J. Hounsell, & N. J. Entwistle (Eds.), *The Experience of Learning* (pp. 36-55). Edinburgh: Scottish Academic Press.

Marton, F., & Trigwell, K. (2000) Variato est mater studorium. *Higher Education, 19*(3), 381-395.

Marton, F., Dall'Alba, G., & Beaty, E. (1993) Conceptions of Learning. *International Journal of Educational Research, 19*(3), 277-300.

Marton, F., Dall'Alba, G., & Tse, K. T. (1992) The Paradox of the Chinese Learner. *Paper given at a symposium on student learning in a cross-cultural context.* Nepal: 4th South-East Asian Regional Conference, the International Association of Cross Cultural Psychology.

Marton, F., Watkins, D., & Tang, C. (1997) Discontinuities & Continuities in the Experience of Learning: An Interview Study of High-School Students in Hong Kong. *Learning & Instruction, 7*(1), 21-48.

Mason, J. (1988) Tensions. In D. Pimm (Ed.), *Mathematics, Teachers & Children* (pp. 164-169). Tyne & Wear: Hodder & Stoughton.

Mason, J. (1989) Mathematical Abstraction as the Result of a Delicate Shift of Attention. *For the Learning of Mathematics, 9*(2), 2-8.

Mason, J. (2000) Asking Mathematical Questions Mathematically. *International Journal of Mathematical Education in Science & Technology, 31*(1), 97-111.

Mason, J. (2001) Mathematical Teaching Practices at Tertiary Level: Working Group Report. In D. Holt (Ed.), *The Teaching & Learning of Mathematics at University Level: An ICMI Study* (pp. 71-86). Dordrecht, the Netherlands: Kluwer Academic Publishing.

Mason, J. (2004) Semistructured Interviews. In M. S. Lewis-Beck, A. Bryman, & T. F. Liao (Eds.), *The SAGE Encyclopedia of Social Science Research Methods* (pp. 1021-1022). London: Sage Publications.

Mason, J., & Johnston-Wilder, S. (Eds.) (2004) *Fundamental Constructs in Mathematics Education.* London: RoutledgeFalmer.

Mason, J., Graham, A., & Johnston-Wilder, S. (2005) *Developing Thinking In Algebra.* London: Paul Chapman Educational Publishing.

MathsTEAM (2003) *Diagnostic Testing for Mathematics.* Birmingham: Learning & Teaching Support Network.

Matthews, A., & Pepper, D. (2007) *Evaluation of Participation in A-Level Mathematics: Final Report.* London: Qualifications & Curriculum Authority.

May, T. (1993) *Social Research: Issues, Methods & Process.* Buckingham: Open University Press.

McCune, V., & Entwistle, N. (2000) The Deep Approach to Learning: Analytic Abstraction & Idiosyncratic Development. *Paper presented at the Innovations in Higher Education Conference.* Helsinki, Finland.

McInnis, C., & James, R. (1995) First Year on Campus — Diversity in the Initial Experiences of Australian Undergraduates. University of Melbourne, Australia: Centre for the Study of Higher Education.

Meece, J. L., Parsons, J. E., Kaczala, C. M., Goff, S. B., & Futterman, R. (1982) Sex Differences in Mathematics Achievement: Toward a Model of Academic Choice. *Psychological Bulletin, 91*(2), 324-348.

Mellin-Olsen, S. (1987) *The Politics of Mathematics Education.* Lancaster: D. Reidel Publishing Company.

Mendick, H. (2005) Mathematical Stories: Why Do More Boys than Girls Choose to Study Mathematics at AS-Level in England? *British Journal of Sociology of Education, 26*(2), 235-251.

Meyer, J. H. (2000) Variation in Contrasting Forms of 'Memorising' & Associated Observables. *British Journal of Educational Psychology, 70*(2), 163-176.

Meyer, J. H., & Eley, M. (1999) The Development of Affective Subscales to Reflect Variation in Students' Experiences of Studying Mathematics in Higher Education. *Higher Education, 37*(2), 197-216.

Meyer, J. H., Parsons, P., & Dunne, T. T. (1990) Individual Study Orchestrations & their Association with Learning Outcome. *Higher Education, 20*(1), 67-89.

Middleton, J. A., & Spanias, P. A. (1999) Motivation for Achievement in Mathematics: Findings, Generalizations, & Criticisms of the Research. *Journal for Research in Mathematics Education, 30*(1), 65-88.

Miller, R. B., Greene, B. A., Montalvo, G. P., Ravindran, B., & Nichols, J. D. (1996) Engagement in Academic Work: The Role of Learning Goals, Future Consequences, Pleasing Others, & Perceived Ability. *Contemporary Educational Psychology, 21*(4), 388-422.

Mitchell, P. D. (1997) The Impact of Educational Technology: A Radical Reappraisal of Research Methods. *Journal for the Association of Learning Technology, 5*(1), 48-54.

Mogashana, D., Case, J. M., & Marshall, D. (2012) What Do Student Learning Inventories Really Measure? A Critical Analysis of Students' Responses to the Approaches to Learning & Studying Inventory. *Studies in Higher Education, 37*(7), 783-792.

Monaghan, J. (1999) *An Investigation into Students' Global A-Level Performance & their Performance in Routine & Non-Routine Questions.* Retrieved August 22, 2013, from http://www.education.leeds.ac.uk/research/projects/global-alevel-mathematics-performance

Montmarquette, C., Cannings, K., & Mahseredjian, S. (2002) How Do Young People Choose College Majors? *Economics of Education Review, 21*(6), 543-556.

Morse, J. (1994) Designing Funded Qualitative Research. In N. Denzin, & Y. Lincoln (Eds.), *Handbook for Qualitative Research* (pp. 220-235). London: Sage Publications.

Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012) *TIMSS 2011 International Results in Mathematics.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Munns, G., & Martin, A. J. (2005) It's All About MeE: A Motivation & Engagement Framework. In P. L. Jeffrey (Ed.), *Proceedings of the Australian Association for Research in Education Conference*, (pp. 1-8). Parramatta, Australia.

Mura, R. (1987) Sex-Related Differences in Expectations of Success in Undergraduate Mathematics. *Journal for Research in Mathematics Education, 18*(1), 15-24.

Nakamura, J. (1988) Optimal Experience & the Uses of Talent. In M. Csikszentmihalyi, & I. S. Csikszentmihalyi (Eds.), *Optimal Experience: Psychological Studies of Flow in Consciousness* (pp. 150-171). Cambridge: Cambridge University Press.

Nardi, E. (2008) *Amongst Mathematicians: Teaching & Learning Mathematics at University Level.* New York, USA: Springer.

Nardi, E., Jaworski, B., & Hegedus, S. (2005) A Spectrum of Pedagogical Awareness for Undergraduate Mathematics: From "Tricks" to "Techniques". *Journal for Research in Mathematics Education, 36*(4), 284-216.

National Committee of Inquiry into Higher Education. (1997) *Higher Education in the Learning Society (The Dearing Report).* London: Her Majesty's Stationery Office.

Neumann, P. M. (1992) The Future of Honours Degree Courses in Mathematics. *Journal of the Royal Statistical Society. Series A (Statistics in Society), 155*(2), 185-189.

Newble, D. I., & Entwistle, N. J. (1986) Learning Styles & Approaches: Implications for Medical Education. *Medical Education, 20*(3), 162-175.

Newble, D. I., & Jaeger, K. (1983) The Effects of Assessments & Examinations on the Learning of Medical Students. *Medical Education, 17*(1), 165-171.

Newble, D. I., Entwistle, N. J., Hejka, E. J., Jolly, B. C., & Whelan, G. (1988) Towards the Identification of Student Learning Problems: The Development of a Diagnostic Inventory. *Medical Education, 22*(6), 518-526.

Newstead, S. E. (1992) A Study of Two "Quick-And-Easy" Methods of Assessing Individual Differences in Student Learning. *British Journal of Educational Psychology, 62*(3), 299-312.

Newton, P., Baird, J., Goldstein, H., Patrick, H., & Tymms, P. (Eds.) (2007) *Techniques for Monitoring the Comparability of Examination Standards.* London: QCA.

Nichols, J. D. (2006) Empowerment & Relationships: A Classroom Model to Enhance Student Motivation. *Learning Environment Research, 9*(2), 149-161.

Novak, J. D. (1978) An Alternative to Piagetian Psychology for Science & Mathematics Education. *Studies in Science Education, 5*(1), 1-30.

O'Brien, V., Martinez-Pons, M., & Kopala, M. (1999) Mathematics Self-Efficacy, Ethnic Identity, Gender, & Career Interests Related to Mathematics & Science. *Journal of Educational Research, 92*(4), 231-235.

Ofsted (2010) *Generic Grade Descriptors & Supplementary Subject-Specific Guidance for Inspectors on Making Judgements During Subject Survey Visits to Schools.* Retrieved April 4, 2011, from http://www.ofsted.gov.uk/Ofsted-home/Forms-and-guidance/Browse-all-by/Other/General/Generic-grade-descriptors-and-supplementary-subject-specific-guidance-for-inspectors-on-making-judgements-during-subject-survey-visits-to-schools

Olitsky, S. (2007) Promoting Student Engagement in Science: Interaction Rituals & the Pursuit of a Community of Practice. *Journal of Research in Science Teaching, 44*(1), 33-56.

Onwuegbuzie, A. J. (2003) Effect Sizes in Qualitative Research: A Prolegomenon. *Quality & Quantity: International Journal of Methodology, 37*, 393-409.

Onwuegbuzie, A. J., & Leech, N. L. (2005) On Becoming a Pragmatic Researcher: The Importance of Combining Quantitative & Qualitative Research Methodologies. *International Journal of Social Research Methodology, 8*(5), 375-387.

Oppenheim, A. N. (1992) *Questionnaire Design, Interviewing & Attitude Measurement.* London: Pinter.

Ormell, C. P. (1974) Bloom's Taxonomy & the Objectives of Education. *Educational Research, 17*(1), 3-18.

Osmon, P. (2011) Applied Mathematics = Modeling > Problem Solving? In C. Smith (Ed.), *Proceedings of the British Society for the Learning of Mathematics*, *31(3)*, pp. 125-131. University of Bristol.

Padesky, C. A., & Hammen, C. L. (1981) Sex Differences in Depressive Symptom Expression & Help Seeking Among College Students. *Sex Differences, 7*(3), 309-320.

Pajares, F. (1997) Current Directions in Self-Efficacy Research. In M. Maehr, & P. R. Pintrich (Eds.), *Advances in Motivation & Achievement* (pp. 1-49). Greenwich: JAI Press.

Pajares, F., & Graham, L. (1999) Self-Efficacy, Motivation Constructs, & Mathematics Performance of Entering Middle School Students. *Contemporary Educational Psychology, 24*(2), 124-139.

Pajares, F., & Kranzler, J. (1995) Self-Efficacy Beliefs & General Mental Ability in Mathematical Problem-Solving. *Contemporary Educational Psychology, 20*(4), 426-443.

Palfreyman, D. (2008) *The Oxford Tutorial.* Witney: Aston.

Pampaka, M., & Williams, J. (2010) Measuring Mathematics Self-Efficacy of Students at the Beginning of their Higher Education Studies. In M. Joubert, & P. Andrews (Ed.), *Proceedings of the 7th conference of the British Congress for Mathematics Education*, (pp. 159-166). University of Manchester.

Pampaka, M., Kleanthous, I., Hutcheson, G. D., & Wake, G. (2011) Measuring Mathematics Self-Efficacy as a Learning Outcome. *Research in Mathematics Education, 13*(2), 169-190.

Pandey, P., & Zimitat, C. (2007) Medical Students' Learning of Anatomy: Memorisation, Understanding & Visualisation. *Medical Education, 41*(1), 7-14.

Pargetter, R. (1995) Transition Research, Analysis & Strategy. *Keynote address at the Monash University Conference on the Transition from Secondary School to University.* Monash University, Melbourne, Australia.

Parker, J. D., Hogan, M. J., Eastabrook, J. M., Oke, A., & Wood, L. M. (2006) Emotional Intelligence & Student Retention: Predicting a Successful Transition from High School to University. *Personality & Individual Differences, 41*(7), 1329-1336.

Pascarella, E. T. (1985) Students' Effective Development Within the College Environment. *Journal of Higher Education, 56*(6), 640-663.

Pascarella, E. T., & Terenzini, P. T. (1983) Predicting Voluntary Freshman Year Persistence/Withdrawal Behavior in a Residential University: A Path Analytic Validation of Tinto's Model. *Journal of Educational Psychology, 75*(2), 215-226.

Pascarella, E. T., Smart, J. C., Ethington, C. A., & Nettles, M. T. (1987) The Influence of College on Self-Concept: A Consideration of Race & Gender Differences. *American Educational Research Journal, 24*(1), 49-77.

Pask, G. (1976) Styles & Strategies of Learning. *British Journal of Educational Psychology, 46*(2), 128-148.

Pask, G., & Scott, B. C. (1972) Learning Strategies & Individual Competence. *International Journal of Man-Machine Studies, 4*(3), 217-253.

Paton, G. (2011) *A-Level Results: Maths & Science Surge 'Due to Brian Cox Effect'.* Retrieved February 13, 2012, from The Telegraph: http://www.telegraph.co.uk/education/universityeducation/8708742/A-level-results-maths-and-science-surge-due-to-Brian-Cox-effect.html

Patrick, H. (1996) Comparing Public Examination Standards Over Time. *Paper Presented at the Annual Conference of the British Educational Research Association.* Lancaster, UK.

Peel, M. (2000) 'Nobody Cares': The Challenge of Isolation in School to University Transition. *Journal of Institutional Research, 9*(1), 22-34.

Pepin, B. (2009) Mathematics at the Boundary Between School & University Mathematics: The Case of 'Worms' Versus 'The Writing Hand'. *Paper presented at the European Conference on Educational Research: Theory & Evidence in European Educational Research*, (pp. 1-9). Vienna, Austria.

Perkins, M. (2005) If You Could Teach the World Just One Thing... *Newsletter of the London Mathematical Society, 339*, 31-32.

Perrenet, J., & Taconis, R. (2009) Mathematical Enculturation from the Students' Perspective: Shifts in Problem-Solving Beliefs & Behaviour During the Bachelor Programme. *Educational Studies in Mathematics, 71*(2), 181-198.

Perry, W. G. (1970) *Forms of Intellectual Development in the College Years: A Scheme.* New York, USA: Holton, Rineheart & Winston.

Peterson, C., Maier, S. F., & Seligman, M. E. (1995) *Learned Helplessness: A Theory for the Age of Personal Control.* Oxford: Oxford University Press.

Petocz, P., & Reid, A. (2005) Rethinking the Tertiary Mathematics Curriculum. *Cambridge Journal of Education, 35*(1), 89–106.

Phiri, P. A. (1993) A Comparison of Assessment by Closed Book & Open Book Tests. *International Journal of Mathematics Education in Science & Technology, 24*(1), 23-26.

Piaget, J. (1970) *Genetic Epistemology.* New York, USA: Norton & Norton.

Pinto, M., & Tall, D. (1999) Student Constructions of Formal Theory: Giving & Extracting Meaning. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference for the International Group for the Psychology of Mathematics Education*, *3*, pp. 281-288. Haifa, Israel.

Pintrich, P. R. (2000) Multiple Goals, Multiple Pathways: The Role of Goal Orientation in Learning & Achievement. *Journal of Educational Psychology, 92*(3), 544-555.

Pintrich, P. R., & DeGroot, E. (1990) Motivational & Self-Regulated Learning Components of Classroom Academic Performance. *Journal of Educational Psychology, 82*(1), 33-40.

Pintrich, P. R., & Schrauben, B. (1992) Students' Motivational Beliefs & their Cognitive Engagemrnt in Classroom Academic Tasks. In D. H. Schunk, & J. L. Meece (Eds.), *Student Perceptions in the Classroom* (pp. 149-183). Hove: Erlbaum.

Pólya, G. (1945) *How To Solve It.* New York, USA: Anchor Books.

Popham, W. J. (1987) The Merits of Measurement-Driven Instruction. *Phi Delta Kappan, 68*(9), 679-682.

Popper, K. (1959) *The Logic of Scientific Discovery.* New York, USA: Basic Books.

Porkess, R. (2003) The New AS & A Levels in Mathematics. *MSOR Connections, 3*(4), 12-17.

Porkess, R. (2006) Unwinding the Vicious Circle. *Proceedings of the 5th Conference of the Institute of Mathematics & its Applications*, (pp. 1-18). Loughborough, UK.

Porkess, R. (2008) More Mathematically Competent Undergraduates. *MSOR Connections, 7*(4), 30-34.

Pountney, D., Leinbach, C., & Etchells, T. (2002) The Issue of Appropriate Assessment in the Presence of a CAS. *International Journal of Mathematical Education in Science & Technology, 33*(1), 15-36.

Powney, J., & Watts, M. (1987) *Interviewing in Educational Research.* London: Routledge & Kegan Paul.

Pring, R. (1971) Bloom's Taxonomy: A Philosophical Critique. *Cambridge Journal of Education, 1*(2), 83-91.

Pritchard, D. (2010) Where Learning Starts? A Framework for Thinking About Lectures in University Mathematics. *International Journal of Mathematical Education in Science & Technology, 41*(5), 609-623.

Prosser, M., & Trigwell, K. (1999) *Understanding Learning & Teaching: The Experience of Higher Education.* Buckingham: Society for Research into Higher Education & Open University Press.

Prosser, M., Trigwell, K., Hazel, E., & Waterhouse, F. (2000) Students' Experience of Studying Physics Concepts: The Effects of Disintegrated Perceptions & Approaches. *European Journal of the Psychology of Education, 15*(1), 61-74.

Provost, S. C., & Bond, N. W. (1997) Approaches to Studying & Academic Performance in a Traditional Psychology Course. *Higher Education Research & Development, 16*(3), 309-320.

Qualifications & Curriculum Authority (QCA) (2007) *Evaluation of Participation in GCE Mathematics: Executive Summary.* Retrieved September 30, 2012, from

http://www.ofqual.gov.uk/downloads/category/106-gq-monitoring?download=303%3Aevaluation-of-participation-in-gce-mathematics-final-report-executive-summary

Quinney, D. (2008) So Just What is Conceptual Understanding of Mathematics? *MSOR Connections, 8*(3), 2-7.

Ramburuth, P., & Mladenovic, R. (2004) Exploring the Relationship Between Students' Orientations to Learning, the Structure of Students' Learning Outcomes & Subsequent Academic Performance. *Accounting Education, 13*(4), 507-527.

Ramsden, P. (1979) Student Learning & Perceptions of the Academic Environment. *Higher Education, 8*(4), 411-427.

Ramsden, P. (1983) Institutional Variations in British Students' Approaches to Learning & Experiences of Teaching. *Higher Education, 12*(6), 691-705.

Ramsden, P. (1985) Student Learning Research: Retrospect & Prospect. *Higher Education Research & Development, 4*(1), 51-69.

Ramsden, P. (1987) Improving Teaching & Learning in Higher Education: A Case for a Relational Perspective. *Studies in Higher Education, 12*(3), 275-286.

Ramsden, P. (1988) Context & Strategy: Situational Influences on Learning. In R. R. Schmeck (Ed.), *Learning Strategies & Learning Styles* (pp. 159-184). New York, USA: Plenum Press.

Ramsden, P. (1992) *Learning to Teach in Higher Education.* London: Routledge.

Ramsden, P., & Entwistle, N. J. (1981) Effects of Academic Departments on Students' Approaches to Studying. *British Journal of Educational Psychology, 51*(3), 368-383.

Rasmussen, C. (1998) Reform in Differential Equations: A Case Study of Students' Understandings and Difficulties. *Paper presented at the annual meeting of the American Educational Research Association.* San Diego, USA.

Reay, D., & William, D. (1999) 'I'll be a Nothing': Structure, Agency & the Construction of Identity Through Assessment. *British Educational Research Journal, 25*(3), 343-354.

Regan, J., & Regan, L. (1995) Changes in Students' Study Processes during the First Year of their Undergraduate Courses in Relation to Age, Gender & Faculty. *Paper presented at the 25th Conference of the Australian Association for Research in Education.* Hobart, Australia.

Reid, A., Wood, L. N., Smith, G. H., & Petocz, P. (2005) Intention, Approach & Outcome: University Mathematics Students' Conceptions of Learning Mathematics. *International Journal of Science & Mathematics Education, 3*(4), 567-586.

Reid, W. A., Duvall, E., & Evans, P. (2005) Can We Influence Medical Students' Approaches to Learning? *Medical Teacher, 27*(5), 401-407.

Reid, W. A., Duvall, E., & Evans, P. (2007) Relationship Between Assessment Results & Approaches to Learning & Studying in Year Two Medical Students. *Medical Education, 41*(8), 754-762.

Reiss, M., & Ruthven, K. (Eds.) (2011) Enhancing the Participation, Engagement & Achievement of Young People in Science & Mathematics Education. *International Journal of Science & Mathematics Education, 9*(2).

Reisz, M. (2008) *Hits & Misses.* Retrieved October 1, 2012, from Times Higher Education: http://www.timeshighereducation.co.uk/story.asp?storycode=402225

Resnick, L. B. (1987) *Education & Learning to Think.* Washington, USA: National Academy Press.

Reyes, L. H. (1984) Affective Variables & Mathematics Education. *Elementary School Journal, 84*(5), 558-581.

Richardson, J. T. (1990) Reliability & Replicability of the Approaches to Studying Questionnaire. *Studies in Higher Education, 15*(2), 155-167.

Richardson, J. T. (2000) *Researching Student Learning: Approaches to Studying in Campus-Based & Distance Education.* Buckingham: The Society for Research in Higher Education.

Richardson, J. T., & King, E. (1991) Gender Differences in the Experience of Higher Education: Quantitative & Qualitative Approaches. *Educational Psychology, 11*(3-4), 363-382.

Riding, R., & Cheema, I. (1991) Cognitive Styles - An Overview & Integration. *Educational Psychology, 11*(3-4), 193-215.

Robert, A., & Schwarzenberger, R. (1991) Research in Teaching & Learning Mathematics at an Advanced Level. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 127-139). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Rocco, T. S., Bliss, L. A., Gallagher, S., & Pérez-Prado, A. (2003) Taking the Next Step: Mixed Methods Research in Organizational Systems. *Information Technology, Learning & Performance Journal, 21*(1), 19-29.

Rodd, M. (2002) Hot & Abstract: Emotion & Learning in Undergraduate Mathematics. In M. J. Høines, & A. B. Fuglestad (Ed.), *Proceedings of the 2nd International Conference on the Teaching of Mathematics at Undergraduate Level.* Hersonissos, Crete.

Rodd, M., & Bartholomew, H. (2006) Invisible & Special: Young Women's Experiences as Undergraduate Mathematics Students. *Gender & Education, 18*(1), 35-50.

Rollnick, M., Davidowitz, B., Keane, M., Bapoo, A., & Magadla, L. (2008) Students' Learning-Approach Profiles in Relation to their University Experience & Success. *Teaching in Higher Education, 13*(1), 29-42.

Romberg, T. A., Zarinnia, E. A., & Collis, K. F. (1990) A New World View of Assessment in Mathematics. In G. Kulm (Ed.), *Assessing Higher Order Thinking in Mathematics* (pp. 21-38). Washington DC, USA: American Association for the Advancement of Science.

Rose, J., & Ziman, J. (1964) *Camford Observed.* London: Victor Gollancz.

Ryan, R. M., & Deci, E. L. (1989) Bridging the Research Traditions of Task/Ego Involvement & Intrinsic/Extrinsic Motivation: Comment on Butler (1987). *Journal of Educational Psychology, 81*(2), 265-268.

Ryan, R. M., & Deci, E. L. (2000) Self-Determination Theory & the Facilitation of Intrinsic Motivation, Social Development, & Well-Being. *American Psychologist, 55*(1), 68-78.

Sadler-Smith, E. (1996) Approaches to Studying: Age, Gender & Academic Performance. *Educational Studies, 22*(3), 367-379.

Sadler-Smith, E. (1997) "Learning Style": Frameworks & Instruments. *Educational Psychology, 17*(1), 51-63.

Sadler-Smith, E. (2001) A Reply to Reynolds's Critique of Learning Styles. *Management Learning, 32*(3), 291-304.

Sansone, C., & Smith, J. L. (2000) Interest & Self-Regulation: The Relation Between Having to & Wanting to. In C. Sansone, & J. M. Harackiewicz (Eds.), *Intrinsic & Extrinsic Motivation: The Search for Optimal Motivation & Performance* (pp. 341-372). London: Academic Press.

Savage, M. (2003) Tackling the Maths Problem: Is it Far More Extensive than We Thought? In S. Hibberd, & L. Mustoe (Ed.), *Proceedings of the 4th IMA Conference on the Mathematical Education of Engineering.* Loughborough University, UK.

Savage, M., Kitchen, A., Sutherland, R., & Porkess, R. (2000) Mathematics at the Interface (1960-2000). In T. Hawkes, & M. Savage (Eds.), *Measuring the Mathematics Problem* (pp. 2-4). London: Engineering Council.

Savin-Baden, M., McFarland, L., & Savin-Baden, J. (2008) Learning Spaces, Agency & Notions of Improvement: What Influences Thinking & Practices About Teaching & Learning in Higher Education? An Interpretive Meta-Ethnography. *London Review of Education, 6*(3), 211-227.

Sax, L. J. (1994) Mathematical Self-Concept: How College Reinforces the Gender Gap. *Research in Higher Education, 35*(2), 141-166.

Saxe, G. (1991) *Culture & Cognition: Studies in Mathematical Understanding.* Hillsdale, USA: Lawrence Erlbaum Associates.

Schmeck, R. R. (1983) Learning Styles of College Students. In R. Dillon, & R. R. Schmeck (Eds.), *Individual Differences in Cognition* (pp. 233-279). New York, USA: Academic Press.

Schmeck, R., Geisler-Brenstein, E., & Cercy, S. P. (1991) Self-Concept & Learning: The Revised Inventory of Learning Processes. *Educational Psychology, 11*(3), 343-362.

Schmeck, R., Ribich, F., & Ramanaiah, N. (1977) The Development of a Self-Report Inventory for Assessing Individual Differences in Learning Processes. *Applied Psychological Measurement, 1*(3), 413-431.

Schoenfeld, A. H. (1985) *Mathematical Problem Solving.* London: Academic Press.

Schoenfeld, A. H. (1989) Explorations of Students' Mathematical Beliefs & Behavior. *Journal for Research in Mathematics Education, 20*(4), 338-355.

Schouwenburg, H. C., & Kossowska, M. (1999) Learning Styles: Differential Effects of Self-Control & Deep-Level Information Processing on Academic Achievement. In I. Mervielde, I. Deary, F. De Fruyt, & F. Ostendorf (Eds.), *Personality Psychology in Europe* (pp. 261-281). Tilburg, the Netherlands: University Press.

Schunk, D. H. (1991) Self-Efficacy & Academic Motivation. *Educational Psychologist, 26*(3-4), 207-231.

Scouller, K. M., & Prosser, M. (1994) Students' Experiences in Studying for Multiple Choice Question Examinations. *Studies in Higher Education, 19*(3), 267-279.

Seale, C. (1999) Quality in Qualitative Research. *Qualitative Inquiry, 5*(4), 465-478.

Secretary of Education's Commission on the Future of Higher Education (Spellings Commission) (2006) *A Test of Leadership: Charting the Future of U.S. Higher Education.* Washington: US Department of Education.

See, C. (2012) *How to get into Oxbridge.* London: Kogan Page.

Selden, A., & Selden, J. (1993) Mathematics Education Research: What Would That Be Like? *The College Mathematics Journal, 24*(5), 431-445.

Selden, A., & Selden, J. (2001) Tertiary Mathematics Education Research & its Future. In D. Holton (Ed.), *The Teaching & Learning of Mathematics at University Level: An ICMI Study* (pp. 237-254). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Selden, A., & Selden, J. (2003) Validations of Proofs Written as Texts: Can Undergraduates Tell Whether an Argument Proves a Theorem? *Journal for Research in Mathematics Education, 34*(1), 4-36.

Selden, A., & Selden, J. (2005) Perspectives on Advanced Mathematical Thinking. *Mathematical Thinking & Learning, 7*(1), 1-13.

Selden, J., Mason, A., & Selden, A. (1989) Can Average Calculus Students Solve Nonroutine Problems? *Journal of Mathematical Behaviour, 8*(1), 45-50.

Selden, J., Selden, A., & Mason, A. (1994) Even Good Calculus Students Can't Solve Nonroutine Problems. *Research Issues in Undergraduate Mathematics Learning, MAA Notes, 33*(1), 19-26.

Severiens, S. E., & Ten Dam, G. T. (1994) Gender Differences in Learning Styles: A Narrative Review & Quantitative Meta-Analysis. *Higher Education, 27*(4), 487-501.

Shah, C., & Burke, G. (1996) Student Flows in Australian Higher Education. *Monash University Centre for the Economics of Education & Training.* Canberra, Australia: Australian Government Publishing Service.

Shah, S. (2004) The Researcher/Interviewer in Intercultural Context: A Social Intruder! *British Educational Research Journal, 30*(4), 549-575.

Sherman, J. (1982) Continuing in Mathematics: A Longitudinal Study of the Attitudes of High School Girls. *Psychology of Women Quarterly, 7*(2), 132-140.

Sherman, J. (1983) Factors Predicting Girls' & Boys' Enrollment in College Preparatory Mathematics. *Psychology of Women Quarterly, 7*(3), 272-281.

Sierpinska, A., Bobos, G., & Knipping, C. (2008) Sources of Students' Frustration in Pre-University Level Prerequisite Mathematics Courses. *Instructional Science, 36*(4), 289-320.

Sivian, A., Wong Leung, R., Woon, C., & Kember, D. (2000) An Implementation of Active Learning & its Effects on the Quality of Student Learning. *Innovations in Education & Training International, 37*(4), 381-389.

Skemp, R. R. (1971) *The Psychology of Learning Mathematics.* London: Penguin.

Skemp, R. R. (1976) Relational Understanding & Instrumental Understanding. *Mathematics Teaching, 77*(1), 20-26.

Skinner, E. A., & Belmont, M. J. (1993) Motivation in the Classroom: Reciprocal Effects of Teacher Behavior & Student Engagement Across the School Year. *Journal of Educational Psychology, 85*(4), 571-581.

Skinner, E. A., Wellborn, J. G., & Connell, J. P. (1990) What it Takes to Do Well in School & Whether I've Got it: The Role of Perceived Control in Children's Engagement & School Achievement. *Journal of Educational Psychology, 82*(1), 22-32.

Smart, J. C., & Pascarella, E. T. (1986) Self-Concept Development & Educational Degree Attainment. *Higher Education, 15*(1-2), 3-15.

Smith, A. (2004) *Making Mathematics Count (Report of an inquiry into post-14 mathematics education following appointment by the Secretary of State for Education & Skills).* Retrieved August 1, 2011, from http://www.mathsinquiry.org.uk/report/

Smith, C. (2012) 'Going It Alone' Within Further Mathematics? *Proceedings of the British Society for Research into Learning Mathematics*, *32*, pp. 47-52. Manchester, UK.

Smith, G., & Wood, L. (2000) Assessment of Learning in University Mathematics. *International Journal of Mathematical Education in Science & Technology, 31*(1), 125-132.

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996) Constructing Mathematical Examinations to Assess a Range of Knowledge & Skills. *International Journal of Mathematical Education in Science & Technology, 27*(1), 65-77.

Solomon, Y. (2006) Deficit or Difference? The Role of Students' Epistemologies of Mathematics in their Interactions with Proof. *Educational Studies in Mathematics, 61*(3), 373-393.

Solomon, Y. (2007a) Theorising Fragile Identities: What Can We Learn from Mathematics Students? *Proceedings of the 2nd conference in Sociocultural Theory*, (pp. 1-11). Manchester, UK.

Solomon, Y. (2007b) Not Belonging? What Makes a Functional Learner Identity in Undergraduate Mathematics? *Studies in Higher Education, 32*(1), 79-96.

Solomon, Y., Lawson, D., & Croft, T. (2011) Dealing with 'Fragile' Identities: Resistance & Refiguring in Women Mathematics Students. *Gender & Education, 23*(5), 565-583.

Spada, M. M., Nikcevic, A. V., Moneta, G. B., & Ireson, J. (2006) Metacognition as a Mediator of the Effect of Test Anxiety on a Surface Approach to Studying. *Educational Psychology: An International Journal of Experimental Educational Psychology, 26*(5), 615-624.

Spence, K. W. (1960) *Behavior Theory & Learning: Selected Papers.* Englewood Cliffs, USA: Prentice Hall.

Speth, C. A., Namuth, D. M., & Lee, D. J. (2007) Using the ASSIST Short Form for Evaluating an Information Technology Application: Validity & Reliability Issues. *Informing Science Journal, 10*(1), 107-118.

Speth, C., Lee, D., & Hain, P. (2005) Get an ASSIST & Support Your Students' Learning Online. *Proceedings of the 19th Annual Conference on Distance Teaching & Learning*, (pp. 1-7). University of Wisconsin, USA.

Spradley, J. (1979) *The Ethnographic Interview.* New York, USA: Holt, Rinehart & Winston.

Springer, L., Stanne, M. E., & Donovan, M. S. (1999) Effects of Small-Group Learning on Undergraduates in Science, Mathematics, Engineering, & Technology: A Meta-Analysis. *Review of Educational Research, 69*(1), 21-51.

Stage, F. K. (1989) Motivation, Academic & Social Integration, & the Early Dropout. *American Educational Research Journal, 26*(3), 385-402.

Stage, F. K., & Maple, S. A. (1996) Incompatible Goals: Narratives of Graduate Women in the Mathematics Pipeline. *American Educational Research Journal, 33*(1), 23-51.

Stake, R. E. (1995) *The Art of Case Study Research.* London: Sage Publications.

Stodoksly, S. S., Salk, S., & Glaessner, B. (1991) Student Views About Learning Math & Social Studies. *American Educational Research Journal, 28*(1), 89-116.

Struyven, K., Dochy, F., & Janssens, S. (2003) Students' Perceptions About New Modes of Assessment: A Review. In M. Segers, F. Dochy, & E. Cascallar (Eds.), *Optimising New Modes of Assessment: In Search for Qualities & Standards* (pp. 171-223). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Sutherland, R., & Dewhurst, H. (1999) *Mathematics Education. Framework for Progression from 16-19 to HE.* University of Bristol: Graduate School of Education.

Svensson, L. (1976) *Study Skill & Learning.* Gothenburg, Sweden: Acta Universitatis Gothoburgensis.

Swailes, S., & Senior, B. (1999) The Dimensionality of Honey & Mumford's Learning Styles Questionnaire. *International Journal of Selection & Assessment, 7*(1), 1-11.

Tait, H., & Entwistle, N. (1996) Identifying Students at Risk Through Ineffective Study Strategies. *Higher Education, 31*(1), 91-116.

Tait, H., & McCune, V. (2001) *Scoring Key for the Approaches & Study Skills Inventory for Students (ASSIST).* Retrieved December 14, 2009, from Edinburgh: ETL Project: http://www.ed.ac.uk/etl/publications.html

Tait, H., Entwistle, N., & McCune, V. (1998) ASSIST: A Reconceptualisation of the Approaches to Studying Inventory. In C. Rust (Ed.), *Improving Student Learning: Improving Students As Learners* (pp. 262-271). Oxford: Oxford Centre for Staff & Learning Development.

Tall, D. (1991a) *Advanced Mathematical Thinking.* Dordrecht, the Netherlands: Kluwer Academic Publishers.

Tall, D. (1991b) Intuition & Rigour: The Role of Visualisation in the Calculus. In W. Zimmerman, & S. Cunningham (Eds.), *Visualisation in Teaching & Learning Mathematics* (pp. 105-119). Washington, USA: Mathematical Association of America.

Tall, D. (1991c) The Psychology of Advanced Mathematical Thinking. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 3-21). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Tall, D. (1995) Cognitive Growth in Elementary & Advanced Mathematical Thinking. *Proceedings of the Conference of the International Group for the Psychology of Learning Mathematics*, (pp. 161-175). Recife, Brazil.

Tall, D., & Razali, M. R. (1993) Diagnosing Students' Difficulties in Learning Mathematics. *International Journal of Mathematical Education in Science & Technology, 24*(2), 209-222.

Tall, D., & Vinner, S. (1981) Concept Image & Concept Definition in Mathematics with Particular Reference to Limits & Continuity. *Educational Studies in Mathematics, 12*(2), 151-169.

Tang, C. (1991) *Effects of Different Assessment Methods on Tertiary Students' Approaches to Studying.* Unpublished PhD Dissertation: University of Hong Kong.

Tang, C. (1994) Assessment & Student Learning: Effects of Modes of Assessment on Students' Preparation Strategies. In G. Gibbs (Ed.), *Improving Student Learning: Theory & Practice* (pp. 151-170). Oxford: Oxford Brookes University, The Oxford Centre for Staff Development.

Tashakkori, A., & Teddlie, C. B. (2003) *Handbook of Mixed Methods in Social & Behavioral Research.* London: Sage Publications.

Taverner, S. (1996) Comparing Boards in A Level Mathematics: Who is the Fairest of them All? *The Mathematical Gazette, 80*(488), 362-366.

Taverner, S. (1997) Modular Courses - A Drip Feed Approach to Teaching & Learning? *Teaching Mathematics & its Applications, 16*(4), 196-199.

Taverner, S., & Wright, M. (1997) Why Go Modular? A Review of Modular A-Level Mathematics. *Educational Research, 39*(1), 104-112.

Taylor, E. (1983) *Orientations to Study: A Longitudinal Interview Investigation of Students on Two Human Studies Degree Courses at Surrey University.* Unpublished Doctoral Thesis: University of Surrey.

Thomas, M. O. (2008) The Transition from School to University & Beyond. *Mathematics Education Research Journal, 20*(2), 1-4.

Thomas, M., & Holton, D. (2003) Technology as a Tool for Teaching Undergraduate Mathematics. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. Leung (Eds.), *Second International Handbook in Mathematics Education* (pp. 351-394). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Thomas, M., de Freitas Druck, I., Huillet, D., Ju, M. K., Nardi, E., Rasmussen, C., *et al*. (2013) Key Mathematical Concepts in the Transition from Secondary School to University. *Proceedings of the 12th International Congress on Mathematical Education*, (pp. 90-136). Seoul, Korea.

Thomas, P. R. (1986) *The Structure & Stability of Learning Approaches.* University of Queensland: Unpublished doctoral thesis.

Thomas, P., & Bain, J. (1984) Contextual Dependence of Learning Approaches: The Effects of Assessments. *Human Learning, 3*(2), 227-240.

Thorndike, E. L. (1920) A Constant Error in Psychological Ratings. *Journal of Applied Psychology, 4*(1), 25-29.

Thurston, W. P. (1994) On Proof & Progress in Mathematics. *The Bulletin of the American Mathematical Society, 30*(2), 161-177.

Times Higher Education (2012) *World University Rankings 2012-2013*. Retrieved May 24, 2013, from http://www.timeshighereducation.co.uk/world-university-rankings/2012-13/world-ranking

Tinto, V. (1975) Dropout from Higher Education: A Theoretical Synthesis of Recent Research. *Review of Educational Research, 45*(1), 89-125.

Tinto, V. (1993) *Leaving College: Rethinking the Causes & Cures of Student Attrition* (2nd ed.). Chicago, USA: University of Chicago Press.

Tinto, V. (1998) Learning Communities: Building Gateways to Student Success. *The National Teaching and Learning Forum, 7*(4), Retrieved July 14, 2009 from: http://www2.aacc.nche.edu/ATDPDFS/Building%20Learning%20Communities/LEARNINGCOMMUNITIES2-BuildingGateways.pdf.

Tomlinson, M. (2002) *Inquiry into A Level Standards.* London: Department for Education.

Torrance, H. (2007) Assessment as Learning? How the Use of Explicit Learning Objectives, Assessment Criteria & Feedback in Post-Secondary Education & Training Can Come to Dominate Learning. *Assessment in Education, 14*(3), 281-294.

Travers, R. M. (1980) Taxonomies of Educational Objectives & Theories of Classification. *Educational Evaluation & Policy Analysis, 2*(1), 5-23.

Trigwell, K., & Ashwin, P. (2003) *Undergraduate Students' Experience of Learning at the University of Oxford.* Oxford: Institute for the Advancement of University Learning.

Trigwell, K., & Prosser, M. (1991a) Improving the Quality of Student Learning: The Influence of Learning Context & Student Approaches to Learning on Learning Outcomes. *Higher Education, 22*(3), 251-266.

Trigwell, K., & Prosser, M. (1991b) Relating Approaches to Study & Quality of Learning Outcomes at the Course Level. *British Journal of Educational Psychology, 61*(3), 265-275.

Trigwell, K., Prosser, M., & Waterhouse, F. (1999) Relations Between Teachers' Approaches to Teaching & Students Approaches to Learning. *Higher Education, 37*(1), 57-70.

Tuckman, B. W. (1972) *Conducting Educational Research.* New York, USA: Harcourt Brace Jovanovich.

Tymms, P. B., & Fitz-Gibbon, C. T. (1991) A Comparison of Examination Boards: A Levels. *Oxford Review of Education, 17*(1), 17-32.

UKMT (2013) *Individual Competitions*. Retrieved August 7, 2013, from http://www.mathcomp.leeds.ac.uk/individual-competitions/

University of Oxford (2012) *Undergraduate Admissions Statistics: 2012 Entry.* Oxford: University of Oxford.

University of Oxford (2013a) *Information on the Admissions Test*. Retrieved August 8, 2013, from http://www.maths.ox.ac.uk/prospective-students/undergraduate/specimen-tests

University of Oxford (2013b) *Past Papers*. Retrieved January 2, 2013, from http://www.maths.ox.ac.uk/current-students/undergraduates/examinations/past-papers

University of Oxford (2013c) *Admissions Criteria*. Retrieved July 23, 2013, from http://www.maths.ox.ac.uk/prospective-students/undergraduate/admissions-criteria

University of Oxford Mathematical Institute (2008) *Course Materials*. Retrieved September 16, 2013, from http://www.maths.ox.ac.uk/courses/material

US Department of Education (2000) *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics & Science Teaching for the 21st Century.* Retrieved November 1, 2012, from http://www.ed.gov/inits/math/glenn/report/pdf

Valås, H., & Søvik, N. (1993) Variables Affecting Students' Intrinsic Motivation for School Mathematics: Two Empirical Studies Based on Deci & Ryan's Theory of Motivation. *Learning & Instruction, 3*(4), 281-298.

Vallerand, R. J., & Bissonnette, R. (1992) Intrinsic, Extrinsic, & Amotivational Styles as Predictors of Behavior: A Prospective Study. *Journal of Personality, 60*(3), 599-620.

van Etten, S., Pressley, M., Freebern, G., & Echevarria, M. (1998) An Interview Study of College Freshmen's Beliefs About their Academic Motivation. *European Journal of Psychology & Education, 13*(1), 105-130.

van Melle, E., & Tomalty, L. (2000) Using Computer Technology to Foster Learning for Understanding. *Microbiology Education, 1*(1), 7-13.

Vansteenkiste, M., Simon, J., Lens, W., Soenens, B., Matos, L., & Lacante, M. (2004) Less is Sometimes More: Goal Content Matters. *Journal of Educational Psychology, 96*(4), 755-764.

Vasagar, J. (2011) *A-Levels Boom in Mathematics & Science Credited to 'Brian Cox Effect'.* Retrieved September 30, 2012, from The Guardian: http://www.guardian.co.uk/education/2011/aug/18/a-levels-boom-maths-science

Vergnaud, G. (1990) La Théorie des Champs Conceptuels. *Recherches en Didactique des Mathématiques, 10*(2-3), 133-170.

Vermetten, Y. J., Lodewijks, H. G., & Vermunt, J. D. (1999) Consistency & Variability of Learning Strategies in Different University Courses. *Higher Education, 37*(1), 1-21.

Vermunt, J. D., & van Rijswijk, F. A. (1988) Analysis & Development of Students' Skill in Self-Regulated Learning. *Higher Education, 17*(5), 647-682.

Vinner, S. (1991) The Role of Definitions in the Teaching & Learning of Mathematics. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 65-81). London: Kluwer Academic Publishers.

Volet, S. E., & Chalmers, D. (1992) Investigation of Qualitative Differences in University Students' Learning Goals, Based on an Unfolding Model of Stage Development. *British Journal of Educational Psychology, 62*(1), 17-34.

Vollmer, F. (1975) Sex Differences in Expectancy of Examination Results. *Scandinavian Journal of Psychology, 16*(1), 152-155.

von Glasersfeld, E. (1987) Learning as a Constructive Activity. In C. Janvier (Ed.), *Problems of Representation in the Teaching & Learning of Mathematics.* Hillsdale, USA: Lawrence Erlbaum Associates.

Vygotsky, L. (1978) *Mind in Society.* (M. Cole, S. Scribner, V. John-Steiner, & E. Souberman, Trans.) Cambridge, USA: Harvard University Press.

Wake, G. (2011) Introduction to the Special Issue: Deepening Engagement in Mathematics in Pre-University Education. *Research in Mathematics Education, 13*(2), 109-118.

Wake, G., Williams, J., & Drake, P. (Eds.) (2011) Deepening Engagement in Mathematics in Pre-University Education. *Research in Mathematics Education, 13*(2).

Ward-Penny, R., Johnston-Wilder, S., & Johnston-Wilder, P. (2013) Discussing Perception, Determining Provision: Teachers' Perspectives on the Applied Options of A-level Mathematics. *Teaching Mathematics & its Applications, 32*(1), 1-18.

Ware, N. C., Steckler, N. A., & Leserman, J. (1985) Undergraduate Women: Who Chooses a Science Major? *Journal of Higher Education, 56*(1), 73-84.

Warmington, P., & Murphy, R. (2007) 'Read All About It!' UK News Media Coverage of A-Level Results. *Policy Features in Education, 5*(1), 70-83.

Watkins, D. (1983) Depth of Processing & the Quality of Learning Outcomes. *Instructional Science, 12*(1), 49-58.

Watkins, D. (2001) Correlates of Approaches to Learning: A Cross-Cultural Meta-Analysis. In R. J. Sternberg, & L. F. Zhang (Eds.), *Perspectives on Thinking, Learning & Cognitive Styles* (pp. 139-166). Mahwah, USA: Psychology Press.

Watkins, D. A., & Biggs, J. B. (1996) *The Chinese Learner: Cultural, Psychological & Contextual Influences.* Hong Kong: Comparative Education Research Centre & Australian Council for Educational Research.

Watkins, D., & Hattie, J. (1981) The Learning Processes of Australian University Students: Investigations of Contextual & Personological Factors. *British Journal of Educational Psychology, 51*(3), 384-393.

Watkins, D., & Hattie, J. (1985) A Longitudinal Study of the Approaches to Learning of Australian Tertiary Students. *Human Learning, 4*(2), 127-141.

Watson, A. (2008) School Mathematics as a Special Kind of Mathematics. *For the Learning of Mathematics, 28*(3), 3-7.

Watson, J. M. (1988) Student Characteristics & Prediction of Success in a Conventional University Course. *Journal of Experimental Education, 56*(4), 203-212.

Webb, G. (1996) *Understanding Staff Development.* Buckingham: Open University Press.

Webb, G. (1997) Deconstructing Deep & Surface: Towards a Critique of Phenomenography. *Higher Education, 33*(2), 195-212.

Weber, K. (2001) Student Difficulty in Constructing Proofs: The Need for Strategic Knowledge. *Educational Studies in Mathematics, 48*(1), 101-119.

Weber, K. (2002) The Role of Instrumental & Relational Understanding in Proofs About Group Isomorphisms. *Proceedings of the 2nd International Conference on the Teaching of Mathematics at the Undergraduate Level.* Hersonisoss, Crete.

Weber, K. (2004) Traditional Instruction in Advanced Mathematics Courses: A Case Study of One Professor's Lectures & Proofs in an Introductory Real Analysis Course. *Journal of Mathematical Behavior, 23*(2), 115-133.

Weber, K. (2008) The Role of Affect in Learning Real Analysis: A Case Study. *Research in Mathematics Education, 10*(1), 71-85.

Weber, K., & Alcock, L. (2004) Semantic & Syntactic Proof Productions. *Educational Studies in Mathematics, 56*, 209-234.

Weiner, B. (1974) *Achievement Motivation & Attribution Theory.* Morristown, USA: General Learning Press.

Weiner, B. (1984) Principles of a Theory of Student Motivation & their Application within an Attributional Framework. In R. E. Ames, & C. Ames (Eds.), *Student Motivation* (pp. 15-38). London: Academic Press.

Weiner, B. (1985) An Attributional Theory of Achievement, Motivation & Emotion. *Psychological Review, 92*(4), 548-573.

Weinstein, C. E., Schulte, A., & Palmer, D. (1987) *Learning & Studying Strategies Inventory (LASSI).* Clearwater, USA: H. & H.

Wenger, E. (1998a) *Communities of Practice: Learning, Meaning, & Identity.* Cambridge: Cambridge University Press.

Wenger, E. (1998b) *Communities of Practice: Learning as a Social System*. Retrieved July 27, 2011, from http://www.open.ac.uk/ldc08/sites/www.open.ac.uk.ldc08/files/Learningasasocialsystem.pdf

Wertsch, J. V. (1985) *Vygotsky & the Social Formation of Mind.* Cambridge, USA: Harvard University Press.

Wetland, E. J., & Smith, K. W. (1993) *Survey Responses: An Evaluation of their Validity.* San Diego, USA: Academic Press.

Wilcox, P., Winn, S., & Fyvie-Gauld, M. (2005) 'It was Nothing to do with the University, it was just the People': The Role of Social Support in the First-Year Experience of Higher Education. *Studies in Higher Education, 30*(6), 707-722.

William, D., & Black, P. (1990) *Inside the Black Box: Raising Standards through Classroom Assessment.* London: GL Assessment Ltd.

Williams, G. (2008) How Group Composition Can Influence Opportunities for Spontaneous Learning. In M. Goos, R. Brown, & K. Makar (Ed.), *Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia*, (pp. 581-588). University of Queensland, Australia.

Williams, J. (2011) Looking Back, Looking Forward: Valuing Post-Compulsory Mathematics Education. *Research in Mathematics Education, 13*(2), 213-221.

Williams, J., Hernandez-Martinez, P., & Harris, D. (2010) Diagnostic Testing in Mathematics as a Policy & in Practice in the Transition to Higher Education. *Paper Presented at the Conference for the British Educational Research Association.* University of Warwick, Coventry, UK.

Williams, T., & Williams, K. (2010) Self-Efficacy & Performance in Mathematics: Reciprocal Determinism in 33 Nations. *Journal of Educational Psychology, 102*(2), 453-466.

Wilson, K. L., Smart, R. M., & Watson, R. J. (1996) Gender Differences in Approaches to Learning in First Year Psychology Students. *British Journal of Educational Psychology, 66*(1), 59-71.

Wilson, K., & Fowler, J. (2005) Assessing the Impact of Learning Environments on Students' Approaches to Learning. *Assessment & Evaluation in Higher Education, 30*(1), 85-99.

Wittrock, M. C. (1974) A Generative Model of Mathematics Learning. *Journal of Research in Mathematics Education, 5*(4), 181-196.

Wittrock, M. C. (1986) Students' Thought Processes. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching* (pp. 297-314). London: Collier Macmillan.

Wittrock, M. C. (2010) Learning as a Generative Process. *Educational Psychologist, 45*(1), 40-45.

Wittrock, M. C., & Carter, J. F. (1975) Generative Processing of Hierarchically Organized Words. *American Journal of Psychology, 88*(3), 489-501.

Wolf, A. (2002) *Does Education Matter? Myths About Education & Economic Growth.* London: Penguin.

Wood, D., Bruner, J., & Ross, G. (1976) The Role of Tutoring in Problem Solving. *Journal of Child Psychology, 17*(2), 89-100.

Wood, L. (2001) The Secondary-Tertiary Interface. In D. Holton (Ed.), *The Teaching & Learning of Mathematics at University Level* (pp. 87-98). London: Kluwer Academic Publishers.

Wood, L. (2011) Power & Passion: Why I Love Mathematics. (B. Barton, & M. Thomas, Eds.) *CULMS Newsletter*, 5-14.

Wood, L., & Smith, G. H. (2002) Perceptions of Difficulty. *Proceedings of the 2nd International Conference on the Teaching of Mathematics at Undergraduate Level.* Hersonissos, Crete.

Wood, L., Smith, G. H., Petocz, P., & Reid, A. (2002) Correlations Between Students' Performance in Assessment & Categories of a Taxonomy. In M. Boezi (Ed.), *Proceedings of the 2nd International Conference on the Teaching of Mathematics at the Undergraduate Level.* Hersonissos, Crete.

Yackel, E., Rasmussen, C., & King, K. (2000) Social & Sociomathematical Norms in an Advanced Undergraduate Mathematics Course. *Journal of Mathematical Behavior, 19*(2), 275-287.

Yin, R. (2003) *Case Study Research: Design & Methods* (3rd ed.). London: Sage Publications.

Yoon, C., Kensington-Miller, B., Sneddon, J., & Bartholemew, H. (2011) It's Not The Done Thing: Social Norms Governing Students' Passive Behaviour in Undergraduate Mathematics Lectures. *International Journal of Mathematical Education in Science & Technology, 42*(8), 1107-1122.

Zeegers, P. (2001) Approaches to Learning in Science: A Longitudinal Study. *British Journal of Educational Psychology, 71*(1), 115-132.

Zimdars, A. (2010) Fairness & Undergraduate Admission: A Qualitative Exploration of Admissions at the University of Oxford. *Oxford Review of Education, 38*(3), 307-323.

Zimitat, C., & McAlpine, A. (2003) Student Use of Computer-Assisted Learning (CAL) & Effects on Learning Outcomes. *Biochemistry & Molecular Biology Education, 31*(2), 146-150.

Zimmerman, B. J., Bandura, A., & Martinez-Pons, M. (1992) Self-Motivation for Academic Attainment: The Role of Self-Efficacy Beliefs & Personal Goal Setting. *American Educational Research Journal, 29*(3), 663-676.

Appendices

Appendices are numbered to correspond with the Chapter they relate to, i.e. Appendix 2.3 is the third appendix referred to in Chapter 2.

2.1 – MATH Taxonomy

Descriptions of the MATH Taxonomy below are taken from Smith *et al.* (1996).

Group A

Factual Knowledge & Fact Systems

The difficulty and depth of the material may cover a wide range from remembering a specific formula or definition (factual knowledge) to learning a complex theorem (a fact system), but the only skill required is to bring to mind previously learnt information in the form that it was given.

For example,

- *State Cramer's rule for solving a system of equations.*
- *State and prove the Hahn-Banach theorem.*

Comprehension

It is quite possible to reproduce knowledge without understanding. To demonstrate comprehension of factual knowledge, students should:

- be able to decide whether or not conditions of a simple definition are satisfied. By a 'simple definition', I mean one which is a matter of terminology, making use of previously acquired knowledge or skills. The student has merely learnt a new term, but not one which requires a significant conceptual change in their mathematical understanding.
  For example,

  Decide, giving reasons, whether or not the differential equation $xy' + y = exp(x)$ *is linear.*

  Answer true or false:

  - *All continuous functions are differentiable*
  - *Some continuous functions are not differentiable*
  - *All differentiable functions are continuous*

- understand the significance of symbols in a formula and show an ability to substitute into a formula.
- be able to recognise examples and counterexamples.

For example,

Identify the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Routine Use of Procedures

This requires the ability to use material in a way which goes beyond simple factual recall. The essential feature is that when the procedure or algorithm is properly used, all people solve the problem correctly and in the same way. This does not preclude the possibility that there may be more than one routine procedure applicable to a given problem. Students would have been expected to have worked on problems using these procedures in drill exercises.

In some cases, there may be several distinct processes underlying a particular procedure and although students may be able to state the general procedure to be followed and understand its principles, they may be unable to carry out the detail. As an example, a student may know that the area under a curve can be obtained by integration and may be able to set up the integral correctly, but be unable to do all but the simplest integrations.

For example,

Solve the initial value problem

$$\frac{dQ}{dt} = kQ \qquad Q(0) = Q_0$$

Given that $Q(\tau) = Q_0/2$ *where* $\tau = 5568$.

Let C *be the circle* $|z - 1| = 1$. *Evaluate the integral* $\int_C \frac{dz}{z^2-1}$

Group B

Information Transfer

This may be shown by the ability to perform the following tasks:

- transformation of information from one form to another – verbal to numerical or vice versa;
  For example,

  Here is an attempted proof of a form of L'Hôpital's rule:

  Statement:

  If $f(a) = g(a) = 0$ *then* $lim_{x\to a} \frac{f(x)}{g(x)} = lim_{x\to a} \frac{f\prime(x)}{g\prime(x)}$

  Proof:

  (a) $lim_{x\to a} \frac{f(x)}{g(x)} = lim_{x\to a} \frac{f(x)-f(a)}{g(x)-g(a)}$

(b) $= lim_{x \to a} \frac{f(x)-f(a)/(x-a)}{g(x)-g(a)/(x-a)}$

(c) $= lim_{x \to a} \frac{f\prime(x)}{g\prime(x)}$

Explain carefully what is happening in each of the steps, labelled (a), (b) and (c). Explain where there could be difficulties with the proof. What conditions should be added to the statement to make the proof valid?

- deciding whether or not conditions of a conceptual definition are satisfied. A conceptual definition is one whose understanding requires a significant change in the student's mode of thought or mathematical knowledge, for example, the definition of a limit or of linear independence. Deciding whether or not a definition is simple or conceptual will often be a subjective judgement, however.
- recognising the applicability of a generic formula in particular contexts.
  For example,

  A function is defined by $f(x) = \begin{cases} sin\, x & x \neq 0 \\ 1 & x = 0 \end{cases}$

  Find $f'(x)$.

- summarising in non-technical terms for a different audience, or paraphrasing;
- framing a mathematical argument from a verbal outline of the method;
- explaining the relationships between component parts of the material;
- explaining processes;
- reassembling the [given] component parts of an argument in their logical order.
  For example,

  For any finite set S*, a field* F *of subsets of* S *and a real-valued function* P *on* F*, a probability space is defined by the following 3 axioms:*

  (i) $P(A) \geq 0 \; \forall \; A \in F$
  (ii) $P(S) = 1$
  (iii) $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ $A_i \in F$ *with* $A_i \cap A_j = \emptyset \qquad i \neq j$

  We want to prove the theorem $P(\emptyset) = 0$*. The steps of the proof are given, but they are not in the correct order. Arrange them to form a logically valid proof.*

  (a) So $P(\emptyset) = 2P(\emptyset)$
  (b) Now, $P(A_1 \cup A_2 = P(A_1) + P(A_2)$ *(by axiom (iii))*
  (c) Take $A_1 = A_2 = \emptyset$
  (d) So $P(\emptyset) = 0$
  (e) $\emptyset \in F$ *(since* F *is a field)*
  (f) Then $A_1 \cup A_2 = A_1 \cap A_2 = \emptyset$

Application in New Situations

Ability to choose and apply appropriate methods or information in new situations, including the following:

- modelling real life settings;
- proving a previously unseen theorem or result which goes beyond the routine use of procedures;
  For example,

  Use the method outlined below to show that if the function f defined by $f(z) = u(x,y) + iv(x,y)$ has a derivative at $z_0 = x_0 + iy_0$ then the first partial derivatives of u and v exist at z_0 and

  $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \text{ at } z_0.$$

  These are the Cauchy-Riemann equations.

  Step 1 Write the derivative of f as a limit.

  Step 2 Express this limit in terms of u and v.

  Step 3 Evaluate this limit in 2 ways and compare the results.

- extrapolation of known procedures to new situations;
  For example,

  Solve the following two equations by showing that the indicated substitution transforms the equation to one which is linear in x and v.

  1. $y' + 2y = x/y^2 \quad v = y^3$
  2. $y' - 5y = -\frac{5}{2}xy^3 \; v = y^{-2}$

  Generalise the method in 1 and 2 above to solve $y' + P(x)y = Q(x)y^n$

- choosing and applying appropriate statistical techniques or algorithms

Group C

Justifying & Interpreting

Ability to justify and/or interpret a given result or a result derived by the student. This includes:

- proving a theorem in order to justify a result, method or model;
- the ability to find errors in reasoning;
- recognising the limitations in a model and being able to decide if a model is appropriate;
- recognition of computational limitations and sources of error;
- interpreting a regression model;
- discussing the significance of given examples and counterexamples;
- recognition of unstated assumptions.

Implications, Conjectures & Comparisons

Given or having found a result/situation, the student has the ability to draw implications and make conjectures and the ability to justify or prove these. The student also has the ability to make comparisons, with justification, in various mathematical contexts. Examples are:

- the ability to make conjectures based, for example, on inductive or heuristic arguments, and then to prove these conjectures by rigorous methods;
  For example,

  > *Take the expression $n^2 + n + 17$, let $n = 1$, and evaluate the result. Is it a prime number? Substitute $n = 2$. Is the result a prime number? Substitute values of n from 3 to 10. Are the results all prime numbers? Can you come to a general conclusion? Are using deductive or inductive arguments? Are you certain of your conclusion? Is the conclusion actually true?*

- comparisons between algorithms;
  For example,

  > *Compare the method of undetermined coefficients with variation of parameters for second order linear DEs.*

- the ability to deduce the implications of a given result;
- the construction of examples and counterexamples.

Evaluation

Evaluation is concerned with the ability to judge the value of material for a given purpose based on definite criteria. The students may be given the criteria or may have to determine them. This includes the following:

- the ability to make judgements;
  For example,

  > *Write a short exposition evaluating the relative merits of Leibniz's and Newton's notation for differentiation.*

- the ability to select for relevance;
- the ability to coherently argue the merits of an algorithm;
  For example,

  > *Explain why the method of Laplace transforms works so well for linear DEs with constant coefficients and integro-differential equations involving a convolution.*

- creativity, which includes going beyond what is given, restructuring the information into a new whole and seeing implications of the information which is not apparent to others.

2.2 – Entry Requirements

Entry requirements for courses in BSc Mathematics and MSc/MMath Mathematics at Russell Group universities in the United Kingdom are as follows:

University	Entry Requirements	
	BSc	MSc
Birmingham	AAB FM advantageous[i]	A*AA FM advantageous – A2 FM typically results in a lower offer[ii]
Bristol	A*AA +1 of Physics, Chemistry, Biology, Economics or Computer Science AAA (inc. FM)[iii]	
Cambridge	A*AA (min AS FM) + min grade 1 in two STEP papers + interviews[iv]	
Cardiff	AAB[v]	AAA[vi]
Edinburgh	AAA in one sitting FM recommended[vii]	
Glasgow	BBB, preferably with two science subjects[viii]	
Imperial	A*A*A (A* FM)[ix]	
KCL	AAAa at (for A2 FM) OR A*AAa (for AS FM) AAaaa (inc. A in FM) OR A*Aaaa (inc. AS FM)[x]	
Leeds	AAA or A*AB AAB or A*BB or A*AC, inc. FM at A2 AAB or A*BB or A*AC, + A in AS FM[xi]	
Liverpool	ABB[xii]	
LSE	LSE do Mathematics with Economics, and Mathematics & Economics.	
Manchester	AAB (min. B in A2 FM) AABa (A in AS FM) + min. A and B in C3 and C4 AAA (excl. FM) + A in C3 and C4[xiii]	
Newcastle	AAB[xiv]	
Nottingham	AAA or A*AB inc. FM at A2[xv]	
Oxford	A*A*A with A*A2 FM A*AAa with A*AS FM A*AA + admissions test + interview[xvi]	
Queen's	ABB[xvii]	AAB[xviii]
Sheffield	AAA[xix]	
Southampton	AAA AAB (inc. FM)[xx]	
UCL	A*A*A (A* FM) A*AA (A* and A for M and FM, or vice versa) + 1 in STEP[xxi]	
Warwick	A*A*A (A* FM) + grade 2 STEP A*AA + grade 1 STEP[xxii]	

In all cases, the first grade stated is A-level Mathematics unless stated otherwise.

Further Mathematics not required unless stated otherwise.

Requirements as found on cited websites, 23 July 2012:

i http://www.birmingham.ac.uk/students/courses/undergraduate/maths/mathematics.aspx

ii http://www.birmingham.ac.uk/students/courses/undergraduate/maths/mathematics-MSci.aspx

iii http://www.bris.ac.uk/prospectus/undergraduate/2012/sections/MATH/200/admissions#entry

iv http://www.maths.cam.ac.uk/undergrad/admissions/guide.pdf
v http://coursefinder.cardiff.ac.uk/undergraduate/course/detail/84.html
vi http://coursefinder.cardiff.ac.uk/undergraduate/course/detail/1082.html
vii http://www.ed.ac.uk/studying/undergraduate/degrees?id=G102&cw_xml=degree.php
viii http://www.gla.ac.uk/media/media_126858_en.pdf
ix http://www.gla.ac.uk/media/media_126858_en.pdf
x http://www.kcl.ac.uk/prospectus/undergraduate/mathematics/entryrequirements
xi http://www.maths.leeds.ac.uk/undergraduate/degree-courses/mathematics.html
xii http://www.liv.ac.uk/study/undergraduate/courses/mathematics-mmath/entry-requirements/
xiii http://www.maths.manchester.ac.uk/undergraduate/ugadmission/entry-requirements.html
xiv http://www.ncl.ac.uk/undergraduate/degrees/g103/entryrequirements/
xv http://www.nottingham.ac.uk/mathematics/prospective/undergraduate/admissionsapplications.aspx
xvi http://www.maths.ox.ac.uk/prospective-students/undergraduate/admissions-criteria
xvii http://www.qub.ac.uk/home/StudyatQueens/CourseFinder/UCF2013-14/?y=1314&id=G1&rp=az
xviii http://www.qub.ac.uk/home/StudyatQueens/CourseFinder/UCF2013-14/?y=1314&id=G1&rp=az
xix http://maths.dept.shef.ac.uk/maths/prospectiveug/entrygrades.php
xx http://www.southampton.ac.uk/maths/undergraduate/courses/g103_mmath.page?#entry
xxi http://www.ucl.ac.uk/mathematics/undergraduates/prospective_undergrad/degree_desc.htm
xxii http://www2.warwick.ac.uk/fac/sci/maths/admissions/ug/offer

2.3 – AQA C1 January 2006

Textbook: The School Mathematics Project (2004) *Core 1 for AQA*. Cambridge: Cambridge University Press

	Question	Worked Example	Similar Question	Past Paper
QUESTION 1				
a	Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$.	**Example 7** Expand and simplify $(\sqrt{3}+1)(3\sqrt{3}-5)$. **Solution** $(\sqrt{3}+1)(3\sqrt{3}-5)$ $= \sqrt{3}\times 3\sqrt{3} - \sqrt{3}\times 5 + 3\sqrt{3} - 5$ $= 9 - 5\sqrt{3} + 3\sqrt{3} - 5$ $= 4 - 2\sqrt{3}$	Expand the brackets and write each result as simply as possible. (a) $(\sqrt{2}+9)(3\sqrt{2}-1)$ (b) $(\sqrt{2}+\sqrt{3})(\sqrt{5}-\sqrt{2})$ (c) $(\sqrt{5}+6)(4-3\sqrt{5})$ (d) $(\sqrt{3}+7)^2$	Express $(4\sqrt{5}-1)(\sqrt{5}+3)$ in the form $p+q\sqrt{5}$ where p and q are integers.
		p. 24	q. 9, p. 25	q. 4 (a), May '06
b	Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer.		Hence write $\sqrt{75}-\sqrt{12}$ in the form $n\sqrt{3}$ where n is an integer.	Express $5\sqrt{8}+\frac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer.
			q. 5 (b), p. 25	q. 3 (a), Jan '08
QUESTION 2				
The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$. The line AB has equation $3x + 4y = 7$.				

a ii	Show that $k = -2$		The line AB has the equation $5x - 2y = 7$, and the point A has coordinates $(1, -1)$ and the point B has coordinates $(3, k)$. Find the value of k. p. 20, q. 9 (a) (i)	
a ii	Hence find the coordinates of the mid-point of AB.	If a straight line is drawn between the points $(1, 2)$ and $(5, 8)$ it is easy to see that their mid-point (the point halfway between them) is $(3, 5)$. Adding the two given x-coordinates and dividing by 2 gives the x-coordinate of the mid-point; similarly with the y-coordinates. The mid-point of the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ p. 11	Find the mid-point of each of these line segments. (a) From $(1, 6)$ to $(7, 2)$ (b) From $(-3, 1)$ to $(5, 3)$ (c) From $(-3, -5)$ to $(4, -1)$ q. 3, p. 12	
b	Find the gradient of AB.	**Example 1** Find the gradient of the straight line graph $4x + 5y - 3 = 0$. **Solution** Make y the subject of the equation. $5y = -4x + 3$ $y = -\frac{4}{5}x + \frac{3}{5}$ Look at the coefficient of x. So the gradient is $-\frac{4}{5}$. p. 7	The line AB has equation $5x - 2y = 7$, and the point A has coordinates $(1, -1)$ and the point B has coordinates $(3, k)$. (a) (i) Find the value of k. (ii) Find the gradient of AB. q. 9, p. 20	The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$. Find the gradient of AB. q. 2 (a) (i), Jan '07

The line AC is perpendicular to the line AB.				
c i	Find the gradient of AC.	If two lines with gradients m_1 and m_2 are perpendicular, $m_2 = -\frac{1}{m_1}$ or $m_1 m_2 = -1$.	Which of these lines are perpendicular to the line $y = 3x + 1$? (a) $y = \frac{1}{3}x - 1$ (b) $6x - 2y + 3 = 0$ (c) $y = -\frac{1}{3}x + 2$ (d) $x + 3y = 1$	The points A and B have coordinates $(6, -1)$ and $(2, 5)$ respectively. Show that the gradient of AB is $-\frac{3}{2}$.
		p. 8	q. 8, p. 9	q. 1 (a) (i), May '07
c ii	Hence find an equation of the line AC.			Hence find an equation of the line AB, giving your answer in the form $ax + by = c$ where a, b and c are integers.
				q. 1 (a) (ii), May '07
c iii	Given that the point C lies on the x-axis, find its x-coordinate.		For each of these equations, (i) rearrange it into the form $y = mx + c$ (ii) give the gradient (iii) give the intercept on the y-axis (a) $3x + y + 7 = 0$ (b) $x + 2y - 8 = 0$ (c) $4x + 5y + 1 = 0$ (d) $3x - 2y - 6 = 0$ (e) $-7x + 2y + 3 = 0$ (f) $-4x - 6y + 9 = 0$	The line AB intersects the line with equation $2x + 3y = 8$ at the point C. Find the coordinates of C.
			q. 4, p. 9	q. 1 (b), May '09

<table>
<tr><th colspan="5">QUESTION 3</th></tr>
<tr><td>a
i</td><td>Express $x^2 - 4x + 9$ in the form $(x - p)^2 + q$ where p and q are integers.</td><td rowspan="2">Example 8
Write $x^2 - 8x + 3$ in completed-square form.
Hence find the minimum value of the expression $x^2 - 8x + 3$.
State the value of x that gives this minimum value.

Solution
Start with the first two terms.
The constant inside the brackets is found by halving the coefficient of x.
$x^2 - 8x = (x - 4)^2 - 16$
Adjust your answer to take into account the constant term.
$x^2 - 8x + 3 = (x - 4)^2 - 16 + 3$
$\Rightarrow x^2 - 8x + 3 = (x - 4)^2 - 13$
You can check this by multiplying out the brackets and simplifying.
$(x - 4)^2 \geq 0$ for all values of x so the minimum value of $x^2 - 8x + 3$ is 13.
The minimum value occurs when $(x - 4) = 0$, i.e. $x = 4$.</td><td>Express $x^2 + 12x + 11$ in the form $(x + a)^2 + b$, finding the values of a and b.</td><td>Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers.</td></tr>
<tr><td></td><td></td><td>q. 7 (a), p. 43</td><td>q. 3 (a) (i), May ‘07</td></tr>
<tr><td>a
ii</td><td>Hence, or otherwise, state the coordinates of the minimum point of the curve with equation
$y = x^2 - 4x + 9$.</td><td>p. 41</td><td>State the minimum value of the expression $x^2 + 12x + 11$.</td><td>Write down the coordinates of the vertex (minimum point) of the curve with equation
$y = x^2 + 10x + 19$.</td></tr>
<tr><td></td><td></td><td></td><td>q. 7 (b), p. 43</td><td>q. 3 (a) (ii), May ‘07</td></tr>
<tr><td colspan="5">The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.</td></tr>
<tr><td>b
i</td><td>Show that the x-coordinates of the points of intersection of L and C satisfy the equation
$x^2 - 2x - 3 = 0$</td><td>Example 8
Find the point of intersection of these graphs
$y = 2x - 1$
$y = 4 - x$

Solution (by equating the expressions for y)</td><td>Solve these pairs of equations by ‘equating the expressions for y’.
(a) $y = 2 - \frac{1}{3}x$
$y = 3(x - 1)$
(b) $y = \frac{2}{3}x$</td><td>Show that the x-coordinate of any point of intersection of the line and circle satisfies the equation $x^2 - 4x + 4 = 0$.</td></tr>
</table>

		At the point of intersection, both graphs have the same y value. Therefore the expression for y in the first graph must equal the expression for y in the second graph. $2x - 1 = 4 - x \implies 3x = 5 \implies x = \frac{5}{3}$ Substitute into the simpler equation. $y = 4 - \frac{5}{3} = \frac{7}{3}$ So the point of intersection is $\left(\frac{5}{3}, \frac{7}{3}\right)$	$y = 2(2 - x)$ (c) $s = 2t - 3$ $s = 1 + \frac{1}{2}t$	
		p. 16	q. 4, p. 17	q. 4 (c) (i), Jan '08
b ii	Hence find the coordinates of the points of intersection of L and C.			
QUESTION 4				
The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.				The quadratic equation $(k + 1)x^2 + 4kx + 9 = 0$ has real roots.
a	Show that $m^2 - 8m + 12 = 0$.	The expression $b^2 - 4ac$ is called the **discriminant** of the equation $ax^2 + bx + c = 0$. • If the value of the discriminant is less than zero, the equation has **no real roots**. • If the value of the discriminant is zero, the equation has one real root, sometimes called a **repeated root**, as both factors of the quadratic give rise to it. • If the value of the discriminant is greater than zero, the equation has two different (**distinct**) real roots.	Determine the values of k for which the equation $x^2 + 2(k - 3)x + (5k - 1) = 0$ has equal roots.	Show that $4k^2 - 9k - 9 \geq 0$.
		p. 50	q. 9 (b), p. 51	q. 8, May '08
b	Hence find the possible values of m.			Hence find the possible values of k.

QUESTION 5

A circle with centre C has equation

$$x^2 + y^2 - 8x + 6y = 11.$$

a	By completing the square, express this equation in the form $(x-a)^2 + (y-b)^2 = r^2$	**Example 1** Write the circle equation $x^2 - 2x + y^2 + 4y + 1 = 0$ in the form $(x-a)^2 + (y-b)^2 = r^2$ and hence give the coordinates of its centre and its radius. **Solution** *Complete the square for the x^2 and x terms.* $x^2 - 2x = (x-1)^2 - 1$ *Complete the square for the y^2 and y terms.* $y^2 + 4y = (y+2)^2 - 4$ *So the original circle equation is* $(x-1)^2 - 1 + (y+2)^2 - 4 + 1 = 0$ *Tidy the numerical values.* $(x-1)^2 + (y+2)^2 - 4 = 0$ *Rearrange so the radius term is visible.* $(x-1)^2 + (y+2)^2 = 2^2$ So the centre is at $(1, -2)$ and the radius is 2 units.	Find the radius and coordinates of the centre of the circle given by each of these equations. (a) $x^2 - 4x + y^2 - 2y + 1 = 0$ (b) $x^2 - 2x + y^2 + 6y + 1 = 0$ (c) $x^2 + 10x + y^2 - 4y + 18 = 0$ (d) $x^2 + y^2 - 10y - 24 = 0$ (e) $x^2 + y^2 + 8x - 6y - 11 = 0$ (f) $x^2 - x = y^2 - 7y + 8\frac{1}{2} = 0$	By completing the square, express this equation in the form $(x-a)^2 + (y-b)^2 = r^2$
			q. 7, p. 101	q. 4 (a), Jan '07
b i	Write down the coordinates of C.		For each of these circle equations, give the coordinates of the centre and the radius. (a) $(x-2)^2 + (y-6)^2 = 16$ (b) $(x+3)^2 + (y-4)^2 = 25$ (c) $(x-4)^2 + y^2 = 7$ (d) $(x+1.5)^2 + (y-0.5)^2 = 4$	Write down the coordinates of C.
				q. 4 (b) (i), Jan '07
b ii	Write down the radius of the circle.			Write down the radius of the circle.
		p. 100	q. 2, p. 100	q. 4 (b) (ii), Jan '07
The point O has coordinates $(0,0)$.				
c i	Find the length of CO.	**Example 2** A circle has its centre at $C(3,3)$ and has a radius of 2 units. From the point $P(10,-3)$ a tangent is drawn that	A circle has the equation $x^2 + y^2 - 2x - 16y + 49 = 0$.	A circle with centre C has equation $(x+3)^2 + (y-2)^2 = 25$.

		touches the circle at Q. Find the length of PQ. **Solution** *Draw and label a sketch.* Using Pythagoras, $CP = \sqrt{7^2 + 6^2} = \sqrt{85}$. *Notice that triangle CPQ is right-angled and has two if its sides given and one to be found.* Using Pythagoras, $PQ = \sqrt{CP^2 - CQ^2} = \sqrt{85 - 4} = 9$ units.	Find the length of a tangent drawn from the origin to the circle.	The point P has coordinates $(2, 6)$. Find the distance PC, leaving your answer in surd form.
		p. 102	q. 3 (c), p. 103	q. 5 (c) (i), May '07
c ii	Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer.		State, with reasons, whether the point $(-1, 2)$ lies inside, on or outside the circle $(x - 1)^2 + (y - 4)^2 = 9$.	A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$. Prove that the point $Q(-1, 4)$ lies inside the circle.
			q. 6, p. 101	q. 4 (d), Jan '08
QUESTION 6				
The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 + x^2 - 10x + 8$				The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 7x - 6$.
a i	Using the factor theorem, show that $x - 2$ is a factor of $\mathrm{p}(x)$.	**Example 9** A polynomial is given by $\mathrm{p}(x) = x^3 + 4x^2 + x - 6$. Show that $(x + 3)$ is a factor of $\mathrm{p}(x)$ and express $\mathrm{p}(x)$ as a product of three linear factors. **Solution** *To show that* $(\mathrm{x} + 3)$ *is a factor, evaluate* $\mathrm{p}(-3)$. $\mathrm{p}(-3) = (-3)^3 + 4(-3)2 + (-3) - 6$ $= -27 + 36 - 3 - 6$	A polynomial is given by $\mathrm{f}(x) = x^3 - 4x^2 - 7x + 10$. (a) By finding the value of $\mathrm{f}(-2)$, show that $(x + 2)$ is a factor of $\mathrm{f}(x)$.	Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$.
			q. 3, p. 93	q. 7 (a) (i), Jan '08
a	Hence express $\mathrm{p}(x)$ as a product		(b) Factorise $\mathrm{f}(x)$ into the	Express $\mathrm{p}(x)$ as the

ii	of three linear factors.	$= 0$ So $(x + 3)$ is a factor of $\mathrm{p}(x)$. *Now write* $\mathrm{p}(x)$ *as the product of* $(x + 3)$ *and a quadratic factor.* $\mathrm{p}(x) = (x + 3)(x^2 + x - 2)$ *With practice, you will be able to write down the quadratic actor straight away by realising that the coefficient of* x^2 *must be* 1*, the constant term must be* -2 *and hence the coefficient of* x *must be* 1 *(to achieve* $4x^2 + x$ *in the expansion).* $\mathrm{p}(x) = (x + 2)(x + 2)(x - 1)$ *The quadratic factorises. This will not always be the case.*	product of three linear factors.	product of three linear factors.
		p. 92	q. 3, p. 93	q. 7 (a) (ii), Jan '08
b	Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes. (You are not required to calculate the coordinates of the stationary points.)		A polynomial is given by $\mathrm{f}(x) = (2x + 1)(x + 2)(x - 3)$. Sketch the graph of $y = \mathrm{f}(x)$, showing clearly where the graph crosses both axes.	The polynomial $p(x)$ is given by $p(x) = x^3 + x^2 - 8x - 12$. Sketch the graph of $y = x^3 + x^2 - 8x - 12$, indicating the values of x where the curve touches or crosses the x-axis.
			q. 2 (b), p. 96	q. 6 (c) (i), May '08

QUESTION 7

The volume, $V\ m^3$, of water ina tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2$$

for $t \geq 0$.

A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t$$
$$0 \leq t \leq 4$$

a i Find $\frac{dV}{dt}$

a ii Find $\frac{d^2V}{dt^2}$

Example 4

Find the stationary points of the graph of $y = x^3 - 3x^2 - 9x + 5$ and determine their types.

Solution

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

At stationary points $\frac{dy}{dx} = 0$, so

$3x^2 - 6x - 9 = 0$

$\Rightarrow 3(x^2 - 2x - 3) = 0$

$\Rightarrow (x + 1)(x - 3) = 0$

So $x = -1$ or $x = 3$.

Find the value of y at each of these values of x.

When $x = -1$,

$y = (-1)^3 - 3 \times (-1)^2 - 9 \times -1 + 5$

$= 10$

When $x = 3, y = 3^2 - 3 \times 3^2 - 9 \times 3 +$

Given that $y = (x - 5)(2x^2 + 1)$, find

(a) $\frac{dy}{dx}$

(b) $\frac{d^2y}{dx^2}$

Find $\frac{dy}{dt}$

q. 4 (a) (i), May '07

Find $\frac{d^2y}{dt^2}$

		$5 = 22$ So the stationary points are $(-1, 10)$ and $(3, -22)$. *Differentiate $\frac{dy}{dx}$ to get the second derivative. Then find its value at each stationary point.* $$\frac{d^2y}{dx^2} = 6x - 6$$ When $x = -1$, $\frac{d^2y}{dx^2} = 6 \times -1 - 6 = -12$. This is negative, so $x = 1$ gives a maximum. When $x = 3$, $\frac{d^2y}{dx^2} = 6 \times 3 - 6 = 12$. This is positive, so $x = 3$ gives a minimum. So $(-1, 10)$ is a maximum and $(3, -22)$ is a minimum.		
		p. 128	q. 2, p. 132	q. 4 (a) (ii), May '07
b	Find the rate of change of the volume of water in the tank, in m^3s^{-1}, when $t = 2$.			Find the rate of change of y with respect to t when $t = 1$. q. 4 (c), May '07
c i	Verify that V has a stationary value when $t = 1$.	**Example 1** The function f is defined by $f(x) = x^3 - 5x^2 + 1$. Is the function increasing, decreasing or stationary at the point where $x = 3$?	An office worker can leave home at any time between 6:00 a.m. and 10:00 a.m. each morning. When he leaves home x hours after 6:00 a.m. $(0 \leq x \leq 4)$, his journey time to the office is y minutes, where $$y = x^4 - 8x^3 + 16x^2 + 8$$ Show that y has a maximum value when	Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value.
c ii	Determine whether this is a maximum or minimum value.			

		First differentiate to get $\mathrm{f}'(x)$. $\mathrm{f}'(x) = 3x^2 - 10x$ *Substitute* $x = 3$. $\mathrm{f}'(3) = 27 - 30 = -3$ *Look at the sign of* $\mathrm{f}'(3)$. $\mathrm{f}'(3)$ is negative, so f is decreasing at $x = 3$.	$x = 2$.	
		p. 124	q. 6 (c), p. 133	q. 4 (b), May '07
QUESTION 8				
The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L. The points A and B have coordinates $(-1, 0)$ and $(2, 0)$, respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.				
a	Find the area of the rectangle ABC.			
b i	Find $\int (3x^2 - x^3)\mathrm{d}x$	**Example 3** Find $\int (x + 3)(x^2 - 5)\,\mathrm{d}x$. **Solution** *Multiple out the brackets to get a polynomial.* $\int (x + 3)(x^2 - 5)\,\mathrm{d}x$ $= \int (x^3 + 3x^2 - 5x - 15)\,\mathrm{d}x$ $= \left(\frac{x^4}{4}\right) + 3\left(\frac{x^3}{3}\right) - 5\left(\frac{x^2}{2}\right) - 15x + c$ $= \frac{1}{4}x^4 + x^3 - \frac{5}{2}x^2 - 15x + c$	Find the following integrals. (a) $\int (2 - 3x + x^2)\,\mathrm{d}x$ (b) $\int (5x^3 + 2x^5)\,\mathrm{d}x$	Find $\int (x^3 - 10x^2 + 28x)\,\mathrm{d}x$.
		p. 139	q. 4, p. 140	q. 5 (b) (i), May '06
b ii	Hence find the area of the shaded region	**Example 1** Calculate the area under the graph of	The diagram shows the graph of $y = 12 - 3x^2$ and the tangent to the curve at the point	The curve with equation $y = x^3 - 10x^2 + 28x$ is

	bounded by the curve and the line L.	$y = x^2$ between $x = 3$ and $x = 6$. **Solution** Area $= \int_3^6 x^2\,dx = \left[\frac{1}{3}x^3\right]_3^6$ $= \left(\frac{1}{3} \times 6^3\right) - \left(\frac{1}{3} \times 3^2\right)$ $= 72 - 9 = 63$	$P\,(2, 0)$. The region enclosed by the tangent, the curve and the y-axis is shaded. Find the area of the shaded region.	sketched below. The curve crosses the x-acix at the origin O and the point $A(3, 21)$ lies on the curve. Hence determine the area of the shaded region bounded by the curve and the line OA.
		p. 147	q. 4 (c), p. 152	q. 5 (b) (iii), May '06
For the curve above with equation $y = 3x^2 - x^3$:				
c i	Find $\frac{dy}{dx}$	**Example 1** If $f(x) = x^4 + 2x^3$, find $f'(x)$. **Solution** $f'(x) = 4x^3 + 2(3x^2) = 4x^3 + 6x^2$	Given that $y = (x^2 + 4)(x - 3)$, find $\frac{dy}{dx}$.	The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below. The curve crosses the x-acix at the origin O and the point $A(3, 21)$ lies on the curve. Find $\frac{dy}{dx}$.
		p. 118	q. 3, p. 122	q. 5 (a) (i), May '06
c ii	Hence find an equation of the tangent at the point on the curve where $x = 1$.	**Example 4** The graph of $y = x^3 - x^2$ passes through the point $P(2, 4)$. Find (a) the gradient of the tangent to the graph at P (b) the equation of the tangent to the graph at P **Solution** (a) *First differentiate.* $\frac{dy}{dx} = 3x^2 - 2x$	Find the equation of the tangent to $y = x(x^2 - 5)$ at the point where $x = 2$.	Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. Hence find an equation of the tangent to the curve at the point A.

		Then substitute $x = 2$. When $x = 2$, $\frac{dy}{dx} = 12 - 4 = 8$. So the gradient of the tangent at P is 8. (b) *The line through* (x_1, y_1) *with gradient* m *has equation* $y - y_1 = m(x - x_1)$. Equation of tangent at P is $y - 4 = 8(x - 2) \Rightarrow y = 8x - 12$.		
		p. 120	q. 5, p. 121	q. 6 (b) (ii), Jan '07
c iii	Show that y is decreasing when $x^2 - 2x > 0$.	**Example 3** The function $f(x)$ given by $f(x) = 2x^3 - 3x^2 - 36x$ is decreasing over the interval $a < x < b$. Calculate the values of a and b. **Solution** $f'(x) = 6x^2 - 6x - 36$ For $f(x)$ to be decreasing, $f'(x) < 0$, so $6x^2 - 6x - 36 < 0$ $\Rightarrow x^2 - x - 6 < 0$ $\Rightarrow (x + 2)(x + 3) < 0$ So $a = -2$ and $b = 3$.	The function $f(x)$ defined by $f(x) = x^3 - 6x^2$ is decreasing over the interval $p < x < q$. Calculate the values of p and q.	The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M. Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$.
		p. 126	q. 1, p. 126	q. 2 (d), Jan '08
d	Solve the inequality $x^2 - 2x > 0$.			

2.4 – OCR FP3 June 2007

Textbook: Neill, H. & Quadling, D. (2005) *Further Pure 2 & 3*. Cambridge: Cambridge University Press

Question		Worked Example	Similar Question	Past Paper
QUESTION 1				
i	By writing z in the form $re^{i\theta}$, show that $zz^* = \|z\|^2$			
ii	Given that $zz^* = 9$, describe the locus of z.			
QUESTION 2				
A line l has equation $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mathrm{t}(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and a plane Π has equation $8x - 7y + 10z = 7$. Determine whether l lies in Π, is parallel to Π without intersecting it, or intersects Π at one point.		***Find the coordinates of the point of intersection of*** $\boldsymbol{r = \begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}2\\1\\-3\end{pmatrix}}$ ***with the plane*** $\boldsymbol{3x + 2y + 4z = 11}$. Rewriting the line in the form $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} + t\begin{pmatrix}2\\1\\3\end{pmatrix}$ and taking components yields the equations $x = 1 + 2t$, $y = 0 + t$ and $z = 1 - 3t$. Substituting these into the equation of the plane gives $3(1 + 2t) + 2t + 4(1 - 3t) = 11$, which gives $t = -1$. So the line meets the plane at the point with parameter -1, namely $(-1, -1, 4)$.	Verify that the line with equation $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \mathrm{t}(-4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ lies wholly in the plane with equation $3x - 2y + 4z = 2$.	Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \mathrm{t}(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i}+3\mathbf{k}+2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.
		Example 2.5.2 p. 251	p. 259, q. 15	q. 2, Jun '08
QUESTION 3				
Find the general solution of		***Find the general solution of the differential***	Find the general solution of each of the	Find the general solution of the

the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{3x}$	***equation*** $\boldsymbol{\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}}$ It was shown in Example 3.3.4 that the complementary function is $(Cx + D)e^{3x}$. The usual procedure suggests trying a particular integral of the form ae^{3x}. But that will not work in this case: since ae^{3x} is part of the complementary function, substituting $y = ae^{3x}$ in the left side produces 0 on the right side. There is a general rule which works in such cases: *If the usual trial integral $y = F(x)$ does not work because it is part of the complementary function, try $y = xF(x)$ instead.* In this example a trial integral $y = axe^{3x}$ will not work either, because that too is part of the complementary function. So try $y = ax^2e^{3x}$. Then $\frac{dy}{dx} = a(2x)e^{3x} + ax^2(3e^{3x})$ $= a(2x + 3x^2)e^{3x}$ $\frac{d^2y}{dx^2} = a(2 + 6x)e^{3x} + a(2x + 3x^2)(3e^{3x})$ $= a(2 + 12x + 9x^2)e^{3x}$.	following differential equations: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$	differential equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 4x$

	Substituting these into the left side of the equation gives $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y$ $= a(2 + 12x + 9x^2)e^{3x} - 6a(2x + 3x^2)e^{3x}$ $+ 9ax^2e^{3x}$ $= a(2 + (12 - 6 \times 2)x$ $+ (9 - 6 \times 3 + 9)x^2)e^{3x}$ $= 2ae^{3x}$. To get e^{3x} on the right side you have to make $2a = 1$, so $a = \frac{1}{2}$, and the general solution is $y = \frac{1}{2}x^2e^{3x} + (Cx + D)e^{3x}$, or $y = \left(\frac{1}{2}x^2 + Cx + D\right)e^{3x}$.		
	Example 3.5.3, p. 274	q. 2 (f), p. 275	q. 2, Jan '08

QUESTION 4

Elements of the set $\{p, q, r, s, t\}$ are combined according to the operation table shown below.

	p	q	r	s	t
p	t	s	p	r	q
q	s	p	q	t	r
r	p	q	r	s	t
s	r	t	s	q	p
t	q	r	t	p	s

i	Verify that $q(st) = (qs)t$.		In each of the following combination tables, identify the products $s \circ t$, $t \circ s$, $(p \circ q) \circ s$ and $p \circ (q \circ t)$, find the identity element and the inverse of s. Find also the solution for the equations $x \circ t = s$ and $u \circ y = t$.	A group G of order 6 has the combination table shown below. $\begin{array}{c\|cccccc} & e & a & b & p & q & r \\ \hline e & e & a & b & p & q & r \\ a & a & b & e & r & p & q \end{array}$

			$\begin{array}{c\|cccccc} & p & q & r & s & t & u \\ \hline p & q & r & p & t & u & s \\ q & r & p & q & u & s & t \\ r & p & q & r & s & t & u \\ s & u & t & s & r & p & q \\ t & s & u & t & q & r & p \\ u & t & s & u & p & q & r \end{array}$ q. 1 (a), p. 375	$\begin{array}{c\|cccccc} b & b & e & a & q & r & p \\ p & p & q & r & e & a & b \\ q & q & r & p & b & e & a \\ r & r & p & q & a & b & e \end{array}$ State, with a reason, whether or not G is commutative. q. 1, Jan '08 & q. 1 (a) (i), Jan '08
ii	Assuming that the associative property holds for all elements, prove that the set $\{p, q, r, s, t\}$, with the operation table shown, forms a group G.	***Prove that the set*** $\{\mathbf{1}, \boldsymbol{i}, -\mathbf{1}, -\boldsymbol{i}\}$ ***with the operation of multiplication is a group.*** For a set as small as this, it is often easiest to show that the binary operation is closed by constructing a table. $\begin{array}{c\|cccc} \times & 1 & i & -1 & -i \\ \hline 1 & 1 & i & -1 & -i \\ i & i & -1 & -i & 1 \\ -1 & -1 & -i & 1 & i \\ -i & -i & 1 & i & -1 \end{array}$ There are four properties to establish, namely the four properties of the group. **1 Closure**: The table shows that the operation of multiplication is closed since every possible product is a member of the set $\{1, i, -1, -i\}$. **2 Associativity**: Multiplication of complex numbers is associative, so multiplication of these elements is associative. **3 Identity**: The element 1 is the identity element, since $1 \times z = z \times 1 = z$ for every	Show that this table is not a group table. $\begin{array}{c\|cccc} & e & a & b & c \\ \hline e & e & a & c & b \\ a & a & c & b & e \\ b & c & b & e & a \\ c & b & e & a & c \end{array}$	The operation $*$ is defined by $x * y = x + y - a$, where x and y are real numbers and a is a real constant. Prove that the set of real numbers, together with the operation $*$, forms a group.

		complex number z. **4 Inverse**: The inverses of 1, -1, i and $-i$ are of 1, -1, $-i$ and i respectively, so every element has an inverse which is in the set $\{1, i, -1, -i\}$. Therefore $\{1, i, -1, -i\}$ with the operation of multiplication is a group.		
		Example 9.6.1, p. 377	q.2, p. 382	q. 7 (i) (a), Jan '09
iii	A multiplicative group H is isomorphic to the group G. The identity element of H is e and another element is d. Write down the elements of H in terms of e and d.			

QUESTION 5				
i	Use de Moivre's theorem to prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$	***Find*** $\int \sin^5\theta\, d\theta$ As $(2i\sin\theta)^5 = \left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$, $32i\sin^5\theta = 2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)$ Therefore $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$. So $\int \sin^5\theta\, d\theta = \frac{1}{16}\left(-\frac{1}{5}\cos 5\theta + \frac{5}{3}\cos 3\theta + 10\cos\theta\right) + k$ $= \frac{-1}{80}\cos 5\theta + \frac{5}{48}\cos 3\theta - \frac{5}{8}\cos\theta + k$	Express $\cos^6\theta$ in terms of sines and/or cosines of multiples of θ. Check your answers by substituting a suitable value for θ in the original expression and in the answer.	Use de Moivre's theorem to prove that $\sin 6\theta \equiv 2\sin 2\theta\,(16\cos^4\theta - 16\cos^4\theta + 3)$
		Example 7.4.2, p. 346	q. 1 (c), p. 346	q. 7 (ii), Jan '08
ii	Hence find the largest possible root of the equation	***Write*** $\cos 5\theta = 0$ ***as a polynomial equation in*** $c = \cos\theta$ ***and solve it.***	Write the equation $\cos 3\theta = -\frac{1}{2}$ as a polynomial equation in $c = \cos\theta$. Show that the roots can be written as $\cos\alpha$,	Hence show that one of the roots of the equation $x^2 - 6x + 1 = 0$ is $\cot^2\left(\frac{1}{8}\pi\right)$

	$64x^6 - 96x^4 + 36x^2 - 3 = 0$, giving your answer in trigonometrical form.	Using $\cos 5\theta = 16c^5 - 20c^3 + 5c$ from the previous example, $\cos 5\theta = 0 \Leftrightarrow 16c^5 - 20c^3 + 5c = 0$. Now $\cos 5\theta$ is equal to 0 if 5θ is an odd multiple of $\frac{1}{2}\pi$, so that $\theta = \frac{1}{10}\pi, \frac{3}{10}\pi, \frac{5}{10}\pi$ and so on. Since $\cos\theta$ is a decreasing function over the interval $0 \leq \theta \leq \pi$ you get different values for $c = \cos\theta$ by taking $\theta = \frac{1}{10}r\pi$ for $r = 1, 3, 5, 7, 9$. As expected, the quantic polynomial equation $16c^5 - 20c^3 + 5c = 0$ has five roots. Now the root $c = 0$ corresponds to $r = 5$, since $\cos\frac{5}{10}\pi = \cos\frac{1}{2}\pi = 0$. It follows that $\cos\frac{1}{10}\pi$, $\cos\frac{3}{10}\pi$, $\cos\frac{7}{10}\pi$ and $\cos\frac{9}{10}\pi$ are the roots of $16c^4 - 20c^2 + 5 = 0$. Notice that: $\cos\frac{7}{10}\pi = \cos\left(\pi - \frac{3}{10}\pi\right) = -\cos\frac{3}{10}\pi$ and $\cos\frac{9}{10}\pi = \cos\left(\pi - \frac{1}{10}\pi\right) = -\cos\frac{1}{10}\pi$. The roots can therefore be written as $\pm\cos\frac{1}{10}\pi$ and $\pm\cos\frac{3}{10}\pi$. Now the equation for $16c^4 - 20c^2 + 5 =$	$\cos\beta$ and $\cos\gamma$, where α, β and γ are all between 0 and π. Use your equation to show that $\cos\frac{2}{9}\pi + \cos\frac{4}{9}\pi = \cos\frac{1}{9}\pi$, and to find the value of $\cos\frac{1}{9}\pi\cos\frac{2}{9}\pi\cos\frac{4}{9}\pi$.	

		0 is a quadratic in c^2 with roots $c^2 = \frac{20 \pm \sqrt{80}}{32} = \frac{5 \pm \sqrt{5}}{8}$ Since $\cos\frac{1}{10}\pi > \cos\frac{3}{10}\pi > 0$ it follows that $\cos\frac{1}{10}\pi = \frac{5+\sqrt{5}}{8}$ and $\cos\frac{3}{10}\pi = \frac{5-\sqrt{5}}{8}$.		
		Example 7.3.1, p. 339	q. 10, p. 344	q. 8 (iii), Jan '07
QUESTION 6				
Lines l_1 and l_2 have equations $\frac{x-3}{2} = \frac{y-4}{-1} = \frac{z+1}{1}$ and $\frac{x-5}{4} = \frac{y-1}{3} = \frac{z-1}{2}$ respectively.				
i	Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$.	***Let l be the line in Π which passes through P and which is perpendicular to OP, where O is the origin. Find the vector equation of l.*** The direction of OP is $\overrightarrow{OP} = \begin{pmatrix}3\\2\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}3\\2\\0\end{pmatrix}$. As l is perpendicular to $\overrightarrow{OP}$ and to the normal Π it is in the direction $\begin{pmatrix}3\\2\\0\end{pmatrix} \times \begin{pmatrix}-1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\-3\\5\end{pmatrix}$. Its vector equation is $\mathbf{r} = \begin{pmatrix}3\\2\\0\end{pmatrix} + t\begin{pmatrix}2\\-3\\5\end{pmatrix}$	Find the equation of the plane through $(1, 2, -1)$ parallel to the plane $5x + y + 7z = 20$.	Two lines have equations $\frac{x-k}{\frac{2}{z}} = \frac{y+1}{-5} = \frac{z-1}{-3}$ and $\frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2}$ Where k is a constant. For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$.
ii	Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$.			
		Example 4.5.3b, p. 290	q. 16, p. 259	q. 5 (ii), June '08
iii	Find the distance between the planes Π_1 and Π_2.	***Find the perpendicular distance of the point $P(4, 5, 6)$ from the plane $x + 2y - 2z = 9$.***	The line l_1 passes through the point A, whose position vector is $\mathbf{i} - \mathbf{j} - 5\mathbf{k}$, and is parallel to the vector $\mathbf{i} - \mathbf{j} - 4\mathbf{k}$. The	A line l has equation

		The normal $\mathbf{n}$ is given by $\begin{pmatrix}1\\2\\-2\end{pmatrix}$. Notice that you could use any multiple of this. M is the foot of the perpendicular from P, T is any point in the plane, and $\mathbf{p}$ is the vector PT. The perpendicular distance is then $PM = PT\cos\theta$, which you can find from the scalar product $\mathbf{n}\cdot\mathbf{p} = \lvert\mathbf{n}\rvert\lvert\mathbf{p}\rvert\cos\theta$ by dividing by $\lvert n\rvert$, which in this case is $\sqrt{1^2+2^2+(-2)^2} = 3$. A simple way to find the coordinates of a point T lying in the plane is to put $y = z = 0$, giving $x = 9$, so T is $(9,0,0)$. Then $\overrightarrow{PT} = \begin{pmatrix}9\\0\\0\end{pmatrix} - \begin{pmatrix}4\\5\\6\end{pmatrix} = \begin{pmatrix}5\\-5\\-6\end{pmatrix}$, so $\mathbf{p} = \begin{pmatrix}5\\-5\\-6\end{pmatrix}$. Finally, $PM = \dfrac{\mathbf{n}\cdot\mathbf{p}}{\lvert\mathbf{n}\rvert}$ $= \dfrac{\begin{pmatrix}1\\2\\-2\end{pmatrix}\cdot\begin{pmatrix}5\\-5\\-6\end{pmatrix}}{3}$ $= \frac{1}{3}\left(1\times 5 + 2\times(-5) + (-2)\times(-6)\right)$ $= \frac{1}{3}(5 - 10 + 12)$ $= \frac{7}{3}$	line l_2 passes through the point B, whose position vector is $2\mathbf{i} - 9\mathbf{j} - 14\mathbf{k}$, and is parallel to the vector $2\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Find the perpendicular distance from A to Π.	$\mathbf{r} = \begin{pmatrix}-7\\-3\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-2\\3\end{pmatrix}$. A plane Π passes through the points $(1,3,5)$ and $(5,2,5)$, and is parallel to l. Find the distance between l and Π.

		The length of the perpendicular is $\frac{7}{3}$.		
		Example 2.5.3, p. 252	q. 15, p. 293	q 7 (ii), May '10
iv	State the relationship between the answer to part (iii) and the lines l_1 and l_2.			
QUESTION 7				
i	Show that $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2\cos\phi)z + 1.$		Show that $\frac{e^{n\theta i}-1}{e^{\theta i}-1} = \frac{\sin\frac{1}{2}n\theta}{\sin\frac{1}{2}\theta} e^{\frac{1}{2}(n-1)\theta i}$ provided that $\theta$is not a multiple of 2π. q 3, p. 351	
ii	Write down the 7 roots of the equation $z^7 = 1$ in the form $e^{i\phi}$ and show their positions in an Argand diagram.		Write down in the form $a + bi$, to 3 decimal places, all the 7th roots of 1. q. 3 (b), p. 326	
iii	Hence express $z^7 - 1$ as the product of one real linear factor and three real quadratic factors.	***Write $z^8 + z^4 + 1$ as the product of 4 quadratic factors with real coefficients.*** The method is to solve the equation $z^8 + z^4 + 1$ and then to find real quadratic factors by combining conjugate pairs of roots. A simple way to begin is to notice that, since $\cos\frac{2}{3}\pi = -\frac{1}{2}$, $z^8 + z^4 + 1$ can be written as $(z^4)^2 - 2z^4\cos\frac{2}{3}\pi + 1$. It can therefore be split into factors as		

		$\left(z^4 - e^{\frac{2}{3}\pi i}\right)\left(z^4 - e^{-\frac{2}{3}\pi i}\right)$. So the roots of $z^8 + z^4 + 1 = 0$ are a combination of the roots of $z^4 = e^{\frac{2}{3}\pi i}$ with those of $z^4 = e^{-\frac{2}{3}\pi i}$ Using the result in the blue box with $a = 1$, $\alpha = \frac{2}{3}\pi$ and $n = 4$ the first equation has roots $\exp\left(\left(\frac{1}{6}\pi + \frac{2r\pi}{4}\right)i\right)$ for $r = 0, 1, 2, 3$; that is, $e^{-\frac{1}{6}\pi i}, e^{-\frac{2}{3}\pi i}, e^{-\frac{7}{6}\pi i}, e^{-\frac{5}{3}\pi i}$ You then get 4 pairs of factors of $z^8 + z^4 + 1$: $\left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{-\frac{1}{6}\pi i}\right)$ $= z^2 - 2z\cos\frac{1}{6}\pi + 1$ $= z^2 - \sqrt{3}z + 1$ $\left(z - e^{\frac{2}{3}\pi i}\right)\left(z - e^{-\frac{2}{3}\pi i}\right)$ $= z^2 - 2z\cos\frac{2}{3}\pi + 1$ $= z^2 + z + 1$ $\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{-\frac{7}{6}\pi i}\right)$ $= z^2 - 2z\cos\frac{7}{6}\pi + 1$ $= z^2 + \sqrt{3}z + 1$		

		$\left(z - e^{\frac{5}{3}\pi i}\right)\left(z - e^{-\frac{5}{3}\pi i}\right)$ $= z^2 - 2z\cos\frac{5}{3}\pi + 1$ $= z^2 - z + 1$ That is, $z^8 + z^4 + 1$ $= (z^2 - \sqrt{3}z + 1)(z^2 + z + 1)(z^2 + \sqrt{3}z + 1)(z^2 - z + 1)$ Example 6.5.3, p. 332		
QUESTION 8				
i	Find the general solution of the differential equation $\frac{dy}{dx} + y\tan x = \cos^3 x$, expressing y in terms of x in your answer.	***Solve the differential equation*** $\frac{dy}{dx} + 4y = \sin 2x$. If $y = a\sin 2x + b\cos 2x$, the left side of the differential equation is $(2a\cos 2x - 2b\sin 2x)$ $+ 4(a\sin 2x + b\cos 2x)$ $= (4a - 2b)\sin 2x + (2a + 4b)\cos 2x$ For the cosines to go out, you need $2a + 4b = 0$, or $a = -2b$. The expression in the line above then reduces to $(4(-2b) - 2b)\sin 2x = -10b\sin 2x$ The goal is to get $\sin 2x$ on the right, so choose $b = -0.1$. this gives $a = -2 \times (-0.1) = 0.2$ So a particular integral is $0.2\sin 2x - 0.1\cos 2x$.	Solve the following differential equation, and identify a particular integral and the complementary function $\frac{dy}{dx}\sin x + y\cos x = 1$ q. 5 (d), p. 267	Find the general solution of the differential equation $x^3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65\sin 2x$ q. 4, Jan '09
ii	Find the particular solution for which $y = 2$ when $x = \pi$.			The variables x and y satisfy the differential equation $\frac{d^2y}{dx^2} + 16y = 8\cos 4x$ Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$.

		You have to add this to the complementary function, which is Ce^{-4x}. The general solution of the differential equation is therefore $y = 0.2\sin 2x - 0.1\cos 2x + Ce^{-4x}$		
		Example 3.5.1, p. 273		q. 6 (iii) Jan '10
QUESTION 9				
The set S consists of the numbers 3^n, where $n \in \mathbb{Z}$. ($\mathbb{Z}$ denotes the set of integers $\{0, \pm 1, \pm 2, \ldots\}$.)				
ii	Prove that the elements of S, under multiplication, form a commutative group G. (You may assume that **addition** of integers is associative and commutative.)	***Prove that the set of non-singular 2×2 matrices with the operation of matrix multiplication is a group.*** **1 Closure**: The product of two non-singular 2×2 matrices $\mathbf{M}$ and $\mathbf{N}$ is a 2×2 matrix $\mathbf{MN}$. Since $\mathbf{M}$ and $\mathbf{N}$ are non-singular, $\det \mathbf{M} \neq 0$ and $\det \mathbf{N} \neq 0$, and since $\det \mathbf{MN} = \det \mathbf{M} \det \mathbf{N}$, $\det \mathbf{MN} \neq 0$, so $\mathbf{MN}$ is non-singular. The operation of matrix multiplication is therefore closed. **2 Associativity**: Since matrix multiplication is associative the group operation is associative. **3 Identity**: The 2×2 identity matrix $\mathbf{I}$ is non-singular and is a member of the set. It has the property that for any matrix $\mathbf{M}$ in the set $\mathbf{IM} = \mathbf{MI} = \mathbf{M}$. **4 Inverse**: If $\mathbf{M}$ is non-singular, then $\mathbf{M}^{-1}$ exists and is non-singular, so there exists an element in the set such that $\mathbf{M}^{-1}\mathbf{M} = \mathbf{MM}^{-1} = \mathbf{I}$.	Prove that $(\mathbb{Z} - \{0\}, \times)$ is not a group.	Show that the set of numbers $\{3, 5, 7\}$, under multiplication modulo 8, does not form a group.
			q. 5, p. 382	q. 1 (i), Jan '07

<table>
<tr><td>ii</td><td colspan="4">Determine whether or not each of the following subsets of S, under multiplication, forms a subgroup of G, justifying your answers.</td></tr>
<tr><td>ii
a</td><td>The numbers 3^{2n}, where $n \in \mathbb{Z}$.</td><td rowspan="3">Explain why $(\mathbb{Q} - \{0\}, \times)$ is not a subgroup of $(\mathbb{Q}, +)$.

Although $\mathbb{Q} - \{0\}$ is a subset of $\mathbb{Q}$, the operations in the two groups are different, so $(\mathbb{Q} - \{0\}, \times)$ is not a subgroup of $(\mathbb{Q}, +)$.</td><td rowspan="3">G is a commutative group, and $H = \{a \in G : x^3 = e\}$.

Prove that H is a subgroup of G.</td><td rowspan="3">A multiplicative group Q of order 8 has elements $\{e, p, p^2, p^3, a, ap, ap^2, ap^3\}$, where e is the identity. The elements have the properties $p^4 = e$ and $a^2 = p^2 = (ap)^2$.

Prove that $\{e, a, p^2, ap^2\}$ is a subgroup of Q.</td></tr>
<tr><td>ii
b</td><td>The numbers 3^n, where $n \in \mathbb{Z}$ and $n \geq 0$.</td></tr>
<tr><td>ii
c</td><td>The numbers $3^{(\pm n^2)}$, where $n \in \mathbb{Z}$.</td></tr>
<tr><td></td><td></td><td>Example 10.4.1, p. 397</td><td>q. 2, p. 405</td><td>q. 8 (iii), May ‘09</td></tr>
</table>

4.1 – ASSIST Questionnaire

UNIVERSITY OF OXFORD
DEPARTMENT OF EDUCATION

15 Norham Gardens, Oxford OX2 6PY
Tel: +44(0)1865 274024 Fax: +44(0)1865 274027
general.enquiries@education.ox.ac.uk
www.education.ox.ac.uk

Director Professor Anne Edwards

Approaches & Study Skills Inventory for Students

Approaches to Learning

The next part of this questionnaire asks you to indicate your relative agreement or disagreement with comments about studying undergraduate maths. Please work through the comments, giving your immediate response. In deciding your answers, think in terms of **UNDERGRADUATE MATHS**.

Please circle:

5 agree
4 somewhat agree
3 neither agree nor disagree
2 somewhat disagree
1 disagree

		Agree	*Somewhat Agree*		*Somewhat Disagree*	*Disagree*
1)	I manage to find conditions for studying which allow me to get on with my work easily.	5	4	3	2	1
2)	When working on a problem set, I'm keeping in mind how best to impress the marker.	5	4	3	2	1
3)	Often I find myself wondering whether the work I'm doing here is worthwhile.	5	4	3	2	1
4)	I often set out to understand for myself the meaning of what we have to learn.	5	4	3	2	1
5)	I organise my study time effectively to make the best use of it.	5	4	3	2	1
6)	I find I have to concentrate on just memorising a good deal of what I have to learn.	5	4	3	2	1
7)	I go over the work I've done carefully to check the reasoning and that it makes sense.	5	4	3	2	1
8)	Often I feel I'm drowning in the sheer amount of material we're having to cope with.	5	4	3	2	1
9)	I look at definitions and go back to first principles when constructing proofs.	5	4	3	2	1

10)	It's important for me to feel that I'm doing as well as I really can on the courses here.	5	4	3	2	1
11)	I try to relate ideas I come across to those in other topics or other courses whenever possible.	5	4	3	2	1
12)	I tend to read very little beyond what is actually required to pass.	5	4	3	2	1
13)	Regularly I find myself thinking about ideas from lectures when I'm doing other things.	5	4	3	2	1
14)	I think I'm quite systematic and organised when it comes to revising for exams.	5	4	3	2	1
15)	I look carefully at tutors' comments on my work to see how to get higher marks next time.	5	4	3	2	1
16)	There's not much of the work here that I find interesting or relevant.	5	4	3	2	1
17)	When I read lecture notes or books, I try to find out for myself exactly what the author means.	5	4	3	2	1
18)	I'm pretty good at getting down to work whenever I need to.	5	4	3	2	1
19)	Much of what I'm studying makes little sense: it's like unrelated bits and pieces.	5	4	3	2	1
20)	I think about what I want to get out of this course to keep my studying well focused.	5	4	3	2	1
21)	When I'm working on a new topic, I try to see in my own mind how all the ideas fit together.	5	4	3	2	1
22)	I often worry about whether I'll ever be able to cope with the work properly.	5	4	3	2	1
23)	Often I find myself questioning things I hear in lectures or read in books.	5	4	3	2	1
24)	I feel that I'm getting on well.	5	4	3	2	1
25)	I concentrate on learning just those bits of information I have to know to pass.	5	4	3	2	1
26)	I find that studying academic topics can be quite exciting at times.	5	4	3	2	1
27)	I'm good at following up alternative approaches to answering questions suggested by lecturers/tutors.	5	4	3	2	1
28)	I keep in mind who is going to mark my work and what they're likely to be looking for.	5	4	3	2	1
29)	When I look back, I sometimes wonder why I ever decided to come here.	5	4	3	2	1
30)	When I am reading, I stop from time to time to reflect on what I am trying to learn from it.	5	4	3	2	1
31)	I work steadily through the term, rather than leave it all until the last minute.	5	4	3	2	1
32)	I'm not really sure what's important in lectures so I get down all I can.	5	4	3	2	1
33)	Ideas in course books or lecture notes often set me off on long chains of thought of my own.	5	4	3	2	1
34)	Before starting work on an assignment or exam question, I think first how best to tackle it.	5	4	3	2	1
35)	I often seem to panic if I get behind with my work.	5	4	3	2	1
36)	When I read, I examine the details carefully to see how they fit in with what I already know.	5	4	3	2	1
37)	I put a lot of effort into studying because I'm determined to do well.	5	4	3	2	1
38)	I gear my studying closely to just what seems to be required for assignments and exams.	5	4	3	2	1
39)	I find some of the mathematical ideas I come across in my degree really gripping.	5	4	3	2	1
40)	I usually plan out my week's work in advance, either on paper or	5	4	3	2	1

	in my head.					
41)	I keep an eye open for what lecturers seem to think is important and concentrate on that.	5	4	3	2	1
42)	I'm not really interested in mathematics, but I'm mainly studying it for other reasons (e.g. job prospects).	5	4	3	2	1
43)	Before tackling a problem or assignment, I first try to work out what lies behind it.	5	4	3	2	1
44)	I generally make good use of my time during the day.	5	4	3	2	1
45)	I often have trouble in making sense of the things I have to remember.	5	4	3	2	1
46)	I like to play around with ideas of my own even if they don't get me very far.	5	4	3	2	1
47)	When I finish a piece of work, I check it through to see if it really meets the requirements.	5	4	3	2	1
48)	Often I lie awake worrying about work I think I won't be able to do.	5	4	3	2	1
49)	It's important for me to be able to follow the argument, or to see the reason behind things.	5	4	3	2	1
50)	I find it easy to motivate myself.	5	4	3	2	1
51)	I prefer being guided through maths questions step-by-step, rather than being given one big question and having to figure out how to answer it myself.	5	4	3	2	1
52)	I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them.	5	4	3	2	1

4.2 – Request for ASSIST Participation

From: Mathematical Institute
Sent: 2 October 2011 12:50
To: [redacted]
Cc: ellie.darlington@education.ox.ac.uk
Subject: Undergraduate Maths Research

Dear Students,

Congratulations on being accepted to Oxford! The next few weeks will be a very exciting time for you as you begin to settle into university life and expand your mathematical horizons, something which all new undergraduates go through. The transition to undergraduate mathematics is something which is heavily researched in the education community in order to ensure that the student experience and mathematical understanding are both optimised.

I am currently running a project at Oxford University about the student experience of university mathematics, with the first phase being concerned with students' approaches to learning. If you could spare a few minutes over the next couple of days, I would be very appreciative if you could fill in this short multiple-choice questionnaire:

https://docs.google.com/spreadsheet/viewform?hl=en_US&formkey=dE0wMXdTTGtkaGRZM2xjYlA1Q2EwZ2c6MQ#gid=0

The outcomes of this research will be extremely powerful in terms of aiding students' understanding of mathematical concepts, as well as improving the student experience. Therefore, I ask you to take some time to complete the questionnaire since your responses can improve both your experiences, and those of future maths students.

An information sheet is attached for you to read before you complete the questionnaire.

Thank you for your time, and best of luck with the rest of term.

Best wishes,

Ellie

_____
Ellie Darlington
Mathematics Education Research Group | Department of Education | 15 Norham Gardens | OX2 6PY
ellie.darlington@education.ox.ac.uk

4.3 – ASSIST Official Information

UNIVERSITY OF OXFORD
DEPARTMENT OF EDUCATION

15 Norham Gardens, Oxford OX2 6PY
Tel: +44(0)1865 274024 Fax: +44(0)1865 274027
general.enquiries@education.ox.ac.uk
www.education.ox.ac.uk

Director Professor Anne Edwards

Dear Student,

This questionnaire has been designed to allow you to describe, in a systematic way, how you go about learning and studying **undergraduate maths**. The technique involves asking you some questions which overlap to some extent to provide good overall coverage of different ways of studying.

You are being asked to complete this questionnaire as part of research into undergraduate mathematicians' academic experiences. It is hoped that the results of this research can go towards helping future maths undergrads in the transition between school and university. Your participation has the potential to help future undergrads develop appropriate and effective learning strategies which can help them to achieve highly and enjoy studying the subject.

Your responses will NOT be passed on to third parties, including your tutors and any other academic staff at the university. Therefore, you are asked to respond as truthfully as possible so that your responses give an accurate reflection of your studying habits.

Your responses will be anonymised. The questionnaires will be kept in a locked cabinet which only I will have access to, and will be destroyed after the study has been written up (by September 2013). Anonymous, coded data from the questionnaires entered into a statistical software package will be kept securely on disk in the hope that Oxford students' responses may be compared with those of students at other universities at a later date.

Should you wish to withdraw from the study or have any further questions regarding the questionnaire or the research project in general, please feel free to contact me via email. Any complaints regarding this research should be made to Dr Lars Malmberg, Chair of DREC (lars-erik.malmberg@education.ox.ac.uk, 01865 274047).

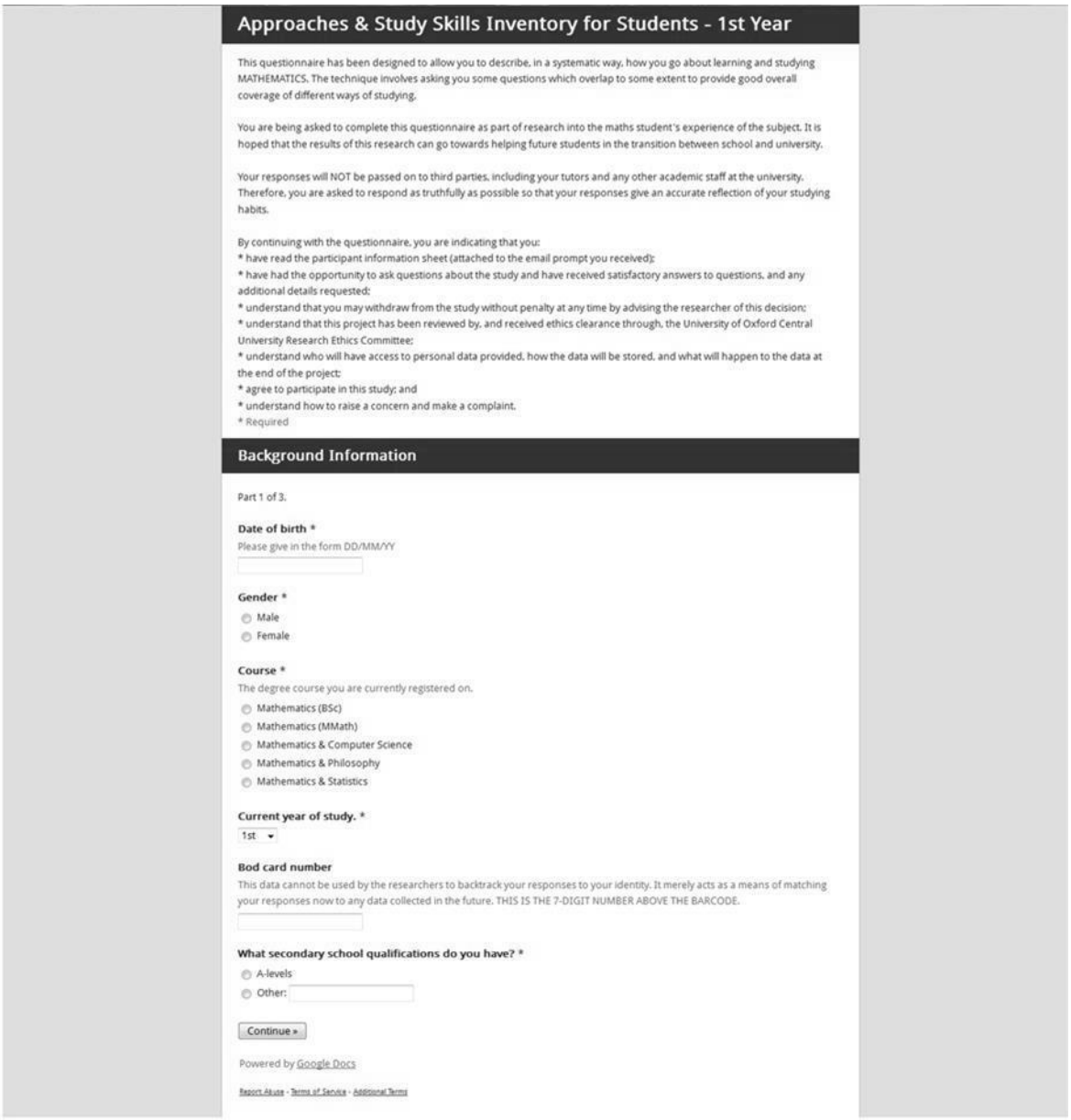
Ellie Darlington
ellie.darlington@education.ox.ac.uk

In completing the online questionnaire, you are indicating that you:

- have read the participant information sheet;
- have had the opportunity to ask questions about the study and have received satisfactory answers to questions, and any additional details requested;
- understand that I may withdraw from the study without penalty at any time by advising the researcher(s) of this decision;
- understand that this project has been reviewed by, and received ethics clearance through, the University of Oxford Central University Research Ethics Committee;
- understand who will have access to personal data provided, how the data will be stored, and what will happen to the data at the end of the project;
- agree to participate in this study; and
- understand how to raise a concern and make a complaint.

4.4 – Electronic form of ASSIST

This shows the homepage of the ASSIST, followed by the first set of questions as illustration.



Approaches & Study Skills Inventory for Students - 1st Year

This questionnaire has been designed to allow you to describe, in a systematic way, how you go about learning and studying MATHEMATICS. The technique involves asking you some questions which overlap to some extent to provide good overall coverage of different ways of studying.

You are being asked to complete this questionnaire as part of research into the maths student's experience of the subject. It is hoped that the results of this research can go towards helping future students in the transition between school and university.

Your responses will NOT be passed on to third parties, including your tutors and any other academic staff at the university. Therefore, you are asked to respond as truthfully as possible so that your responses give an accurate reflection of your studying habits.

By continuing with the questionnaire, you are indicating that you:
* have read the participant information sheet (attached to the email prompt you received);
* have had the opportunity to ask questions about the study and have received satisfactory answers to questions, and any additional details requested;
* understand that you may withdraw from the study without penalty at any time by advising the researcher of this decision;
* understand that this project has been reviewed by, and received ethics clearance through, the University of Oxford Central University Research Ethics Committee;
* understand who will have access to personal data provided, how the data will be stored, and what will happen to the data at the end of the project;
* agree to participate in this study; and
* understand how to raise a concern and make a complaint.

* Required

Background Information

Part 1 of 3.

Date of birth *
Please give in the form DD/MM/YY

Gender *
- Male
- Female

Course *
The degree course you are currently registered on.
- Mathematics (BSc)
- Mathematics (MMath)
- Mathematics & Computer Science
- Mathematics & Philosophy
- Mathematics & Statistics

Current year of study. *
1st

Bod card number
This data cannot be used by the researchers to backtrack your responses to your identity. It merely acts as a means of matching your responses now to any data collected in the future. THIS IS THE 7-DIGIT NUMBER ABOVE THE BARCODE.

What secondary school qualifications do you have? *
- A-levels
- Other:

Continue »

Powered by Google Docs

Report Abuse - Terms of Service - Additional Terms

Approaches & Study Skills Inventory for Students - 1st Year

Approaches to Learning

Part 2 of 3.

The next part of this questionnaire asks you to indicate your relative agreement or disagreement with comments about studying MATHEMATICS.

Please work through the comments, giving your immediate response.

	Agree	Agree Somewhat	Neither Agree Nor Disagree	Disagree Somewhat	Disagree
I manage to find conditions for studying which allow me to get on with my work easily.	○	○	○	○	○
When working on a problem set, I'm keeping in mind how best to impress the marker.	○	○	○	○	○
I have found myself wondering whether the work I'm doing here is worthwhile.	○	○	○	○	○
I often set out to understand for myself the meaning of what we have to learn.	○	○	○	○	○
I organise my study time effectively to make the best use of it.	○	○	○	○	○
I find I have to concentrate on just memorising a good deal of what I have to learn.	○	○	○	○	○
I go over the work I've done carefully to check the reasoning and that it makes sense.	○	○	○	○	○
I feel I'm drowning in the sheer amount of material we're having to cope with.	○	○	○	○	○
I look at definitions and go back to first principles when constructing proofs.	○	○	○	○	○
It's important for me to feel that I'm doing as well as I really can.	○	○	○	○	○
I try to relate ideas I come across to those in other topics whenever possible.	○	○	○	○	○
I have read very little beyond what is actually required to pass.	○	○	○	○	○
I have found myself thinking about ideas from lectures when I'm doing other things.	○	○	○	○	○
I think I'm quite systematic and organised when it comes to revising for exams.	○	○	○	○	○
I look carefully at tutors' comments on my work to see how to get higher marks next time.	○	○	○	○	○

4.5 – Request for Interview Participation

From: Mathematical Institute
Sent: 23 May 2011 11:03
To: [redacted]
Cc: ellie.darlington@education.ox.ac.uk
Subject: Undergraduate Maths Research

Dear Students,

As part of a research project into students' experiences of undergraduate mathematics, I am looking for students to take part in interviews which explore your experiences of the subject so far in terms of:

- any challenges you have been faced with;
- the nature of the maths you've been studying; and
- any changes experienced as you have progressed through your studies.

Students across all degree streams, year groups and colleges are invited to take part. This research will then go on to inform government and departmental policy and actions regarding undergraduate mathematics teaching across the UK.

Interviews will last approximately 30 minutes and refreshments will be provided, and I am entirely flexible on where and when the interview takes place.

If you would be interested in taking part, please get in touch with me via email: ellie.darlington@education.ox.ac.uk

Best wishes,

Ellie

Ellie Darlington
Mathematics Education Research Group | Department of Education | 15 Norham Gardens | OX2 6PY
ellie.darlington@education.ox.ac.uk

4.6 – Interview Notesheet



	DATE	☐☐/☐☐/2012
	TIME	☐☐:☐☐
GENDER	☐ F ☐ M	
COURSE	☐ Mx ☐ +S ☐ +P ☐ +CS	
QUALS	☐ A ☐ _____	
YEAR	☐ 1ST ☐ 2ND ☐ 3RD	☐ 4TH
ATL	☐ DE ☐ SU ☐ ST	
BOD	|_|_|_|_|_|_|_|_|	

School Experience of Mathematics	**Oxford Entry**
Now/Degree Progression	**Changes**

4.7 – Interview Information & Consent Form

UNIVERSITY OF OXFORD
DEPARTMENT OF EDUCATION

15 Norham Gardens, Oxford OX2 6PY
Tel: +44(0)1865 274024 Fax: +44(0)1865 274027
general.enquiries@education.ox.ac.uk
www.education.ox.ac.uk

Director Professor Anne Edwards

Dear Student,

You are being asked to consent to your involvement in a short interview as part of research into undergraduate maths students' experiences of their course.

It is hoped that the results of this research can go towards helping future maths undergrads both in the transition between school and university, and throughout their course. Your participation has the potential to help future undergrads develop appropriate and effective learning strategies, bring awareness to both schools and universities of the struggles which are faced by undergraduate mathematicians and to prepare prospective undergraduates for the course, which can help them to achieve highly and enjoy studying the subject.

The interview will centre around discussion of your experiences of secondary and tertiary mathematics, as well as the challenges you have perceived with these. This will include any differences you believe exist between the studying environment at school and university, the challenges which you have faced and how – if applicable – you overcame them, and your conceptions of mathematics. The interview is by no means a test, and you will not be judged in any way based on your responses.

You are free to:

- withdraw from this interview at any time;
- ask for the tape recording to be stopped;
- play back the tape recording; or
- have parts or all of the recording erased.

The recordings will be transcribed within a week of collection, and then the digital recordings kept on a secure computer with password protection. No identifiers (e.g. your name) will be kept alongside these files. Your participation is entirely voluntary and anonymous. I will be the only person with access to the recordings, and will delete the recordings after the project has been written up (by September 2015).

Should you have any further questions regarding the questionnaire or the research project in general, please feel free to contact me via email. Any complaints regarding this research should be made to Dr Lars Malmberg, Chair of DREC (lars-erik.malmberg@education.ox.ac.uk, 01865 274047).

Please sign below to indicate that you:

- have read the participant information sheet;
- have had the opportunity to ask questions about the study and have received satisfactory answers to questions, and any additional details requested;
- understand that I may withdraw from the study without penalty at any time by advising the researcher(s) of this decision;
- understand that this project has been reviewed by, and received ethics clearance through, the University of Oxford Central University Research Ethics Committee;
- understand who will have access to personal data provided, how the data will be stored, and what will happen to the data at the end of the project;
- agree to participate in this study; and
- understand how to raise a concern and make a complaint.

Signed: _____ **Date**: ____/____/2012

4.8 – Commonly-Asked Interview Questions

Why did you apply to Oxford?

Why did you choose to study maths at university?

Did you research what your degree would involve in advance of coming to oxford?

What happened in your interview?

Do you remember any of the interview questions that you were asked? What were they?

How did you respond to the interviewers' prompts?

What was your general impression of how the interview felt and how it went afterwards?

Did you enjoy the maths questions that you were asked in the interview?

Did you do any interview preparation?

What was your impression of what maths at Oxford would be like based on the interview?

How did the interview compare to any you had elsewhere?

Were you expecting maths to be like it is when you first started at Oxford?

How did you find the OxMAT?

Did you do any preparation for the OxMAT?

Why do you think Oxford uses the OxMAT?

Did you get anything out of doing the test? Was it useful?

How does the OxMAT compare to A-level exams?

How are things going at the minute?

What non-compulsory courses are you taking?

What made you pick those courses?

What working and studying do you normally do during term-time?

How does your work ethic compare to that of your peers?

Do you perform better in some courses than others?

What did you get in your exams in last year/in previous years?

Did your results spark any change in your working habits?

Are you going to do the MMath or BSc? [Or, why did you decide to continue on to the MMath?]

How did your first year at Oxford go?

What was the hardest part about your first year?

What did Oxford do to make the transition manageable?

Are there any differences between school and university maths?

Do you think a different kind of mathematical thinking is needed at school and university?

Do you think that A-level maths could do anything to better prepare you for university maths?

What do you do in tutorials?

Do you find tutorials helpful?

How does the format of A-level maths questions compare to the format of university maths questions?

How did you find A-level maths?

What A-level maths modules did you do? Why?

What A-levels did you do? Why?

What do you think of the modular A-level system?

Did you resit any A-level maths odules?

How did you revise for A-level maths?

What is your current revision technique?

How does your revision at university compare to school?

How does your revision technique compare now to in earlier years of your degree?

How does your way of revising compare to your peers' ways?

Is your revision technique successful?

Do you know any mathematicians outside of your college?

Are you a member of the Invariants?

Who do you spend most of your time with outside of your degree?

Are you involved any clubs or societies?

What's the most challenging maths you've done so far at Oxford?

Are any challenges/hurdles you've experienced similar to those of your peers?

What did you do to overcome any challenges/hurdles?

How does this year compare to previous years of your degree?

What part of your degree have you enjoyed the most?

Are you enjoying your studies at the moment? Has that always been the case?

What do you do in classes?

Do you find classes helpful?

How do you do problem sheets?

Do you work with other people on problem sheets?

What does doing problem sheets do for your understanding of maths?

Thinking about school maths… What is that?

Thinking about university maths… What is that?

What are your plans for after your degree?

4.9 – University of Oxford Admissions Statistics

Unless stated otherwise, the following data are taken from the statistics for 2011 entry, this being the start date for the first-year students who participated in data collection for this thesis.

Nearly 45,000 people achieved AAA nationally in their A-levels in 2010 (UCAS data).

42.3% of UK applicants were from independent sector schools.

4.9.1 – Number of Applicants

In 2011, the third most oversubscribed course at Oxford was mathematics.

Mathematics has the fourth lowest success rate of all large courses[54] at Oxford:

	Applications	Shortlisted for Interview	Acceptances	Success Rate (%)	Applicants Per Place
Mathematics	1133	51.7%	173	15.3	6.5
Mathematics & Philosophy	95	48.5%	16	16.8	5.9
Mathematics & Computer Science	99	66.3%	26	26.3	3.8
Mathematics & Statistics	209	56.7%	26	12.4	8.0
Total	1536	53.4%	241	15.7	6.4

4.9.2 – Qualifications Offered

The success rate of applicants increased with the number of A* grades they offered. Of all applicants offering A-levels:

A*A*A*	2266
A*A*A	2509
A*AA	2879
Total	9362

A-level accounted for 2295 of 2778 acceptances (82.6%). IB accounted for 172 acceptances (6.2%). These are the two most-often offered qualifications.

[54] Those courses with over 70 places.

4.9.3 – Gender

In terms of gender by course,

	Acceptances			Success Rate (%)		
	M	F	T	M	F	T
Mathematics	123	50	173	17.4	11.7	15.3
Mathematics & Philosophy	13	3	16	23.6	7.5	16.8
Mathematics & Computer Science	23	3	26	28.8	15.8	26.3
Mathematics & Statistics	13	13	26	11.8	13.1	12.4

4.9.4 – Inconsistencies

Admissions statistics cannot be used to presume the size of the cohort being requested to participate. For example, comparing admissions statistics for 2009 with data provided by the Mathematical Institute in May of 2010 show significant discrepancies due to those being made offers not meeting them, those being made offers not taking them up, drop-outs and course changes.

	Admissions Statistics (May 2010)			Departmental Statistics			Change		
	M	F	T	M	F	T	M	F	T
Mathematics	123	51	174	126	50	176	+3	-1	+2
Mathematics & Philosophy	16	8	24	12	3	15	-4	-5	-9
Mathematics & Computing	13	5	18	14	8	22	+1	+3	+5
Mathematics & Statistics	13	10	23	13	10	23	0	0	0

4.9.5 – Students by Year & By Course

Official statistics released by the Mathematical Institute on 8 May 2013 show the number of students in each year within each degree course.

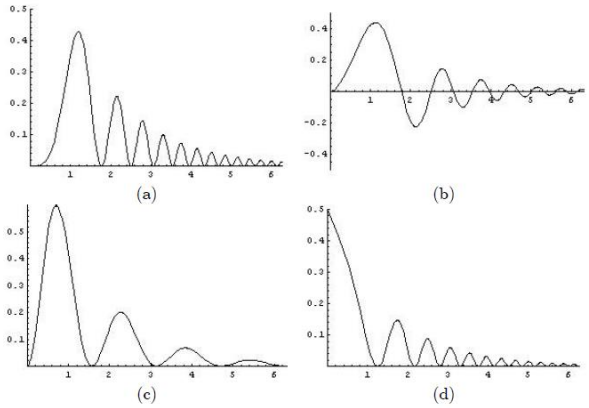
Degree Course	Year 1	Year 2	Year 3	Year 4	Total
MMath Mathematics	172	162	137	119	590
BA Mathematics	0	0	24		24
MMath Mathematics & Statistics	11	32	21	23	87
BA Mathematics & Statistics	0	0	8		8
MMath Mathematics & Philosophy	18	16	13	21	68
BA Mathematics & Philosophy	0	0	3		3
MMath Mathematics & Computer Science	23	19	11	11	64
BA Mathematics & Computer Science	0	0	5		5
Total	**224**	**229**	**222**	**174**	**849**

All students enrol on MMath courses at the beginning of their degree and must only make the decision about whether to continue on this route or to change to the BA in their third year.

4.10 – MATH Taxonomy Questions

The following sample questions from A-level, OxMAT and undergraduate examinations were given to a number of mathematics and mathematics education specialists to check against my classifications.

Question	MATH	Explanation
Oxford P2 2007, Q 6(a)		
Give an example to show that a power series need not converge uniformly in $\|x\| < \rho$.	ICC	Give an example.
Oxford P1 2011, Q1 (c)		
Why does your definition make sense?	IT	Explaining in words.
OCR FP3 June 2007, Q4 (i)		
Verify that $q(st) = (qs)t$.	COMP	Substituting into a formula.
WJEC FP3 June 2010, Q1 (b)		
$f'(x) = 2(2\cosh^2 x - 7\cosh x + 3)$. Show that there is one stationary point on the graph of f. Find its x-coordinate, giving your answer correct to two decimal places.	AINS	Unrehearsed question based on familiar procedures.
AQA FP3 June 2007, Q3		
By using an integrating factor, find the solution of the differential equation $$\frac{\mathrm{d}y}{\mathrm{d}x} + (\tan x)y = \sec x$$ given that $y = 3$ when $x = 0$.	AINS	Need to decide which integrating factor to use.
OxMAT 2009, Q2		
A list of real numbers $x_1, x_2, x_3, \ldots$ is defined by $x_1 = 1, x_2 = 3$ and then for $n \geq 3$ by $$x_n = 2x_{n-1} - x_{n-2} + 1$$ So, for example, $x_3 = 2x_2 - x_1 + 1 = 2 \times 3 - 1 + 1 = 6$. Find the values of $x_4$and x_5.	COMP	Substituting into a formula.
Oxford P1 2008, Q 1 (a)		
What does it mean to say that $B \subseteq V$ is a basis of V?	FKFS	Definition recall.
Oxford P2 2008, Q4 (b)		
Give an example of a continuous function $g: \mathbb{R} \to \mathbb{R}$ which is bounded but does not attain its lower bound.	ICC	Giving an example.
OCR C1 June 2007, Q 2 (b)		
Describe a transformation that transforms the curve $y = x^3$ to the curve $y = 8x^3$.	IT	Describing a transformation with words.
OxMAT 2011, Q1 (b)		
A rectangle has perimeter P and area A. The values P and A must satisfy $P^3 > A$ (b) $A^2 > 2P + 1$ (c) $P^2 \geq 16A$ (d) $PA \geq A + P$	J&I	Need to go through the different options.
OxMAT 2007, Q1 (g)		
On which of the axes below is a sketch of the graph $$y = 2^{-x}\sin^2(x^2)$$	J&I	Justify why certain items are disregarded.

		
Edexcel C1 January 2006, Q1		
Factorise completely $x^3 - 4x^2 + 3x$.	RUOP	Routine, likely-rehearsed procedure.

5.1 – Factor Analysis

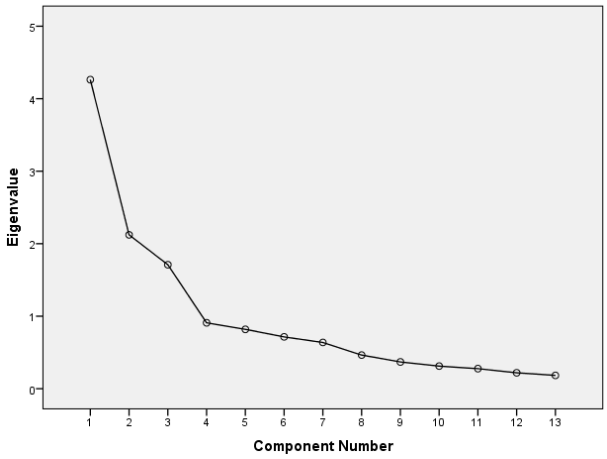
Initial factor extraction revealed 13 components, which correspond to the 13 different subscales[55] in the ASSIST. Subsequent extraction then revealed three factors, which correspond to the three different approaches to learning that the ASSIST measures.

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings[a]
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	4.263	32.789	32.789	4.263	32.789	32.789	3.248
2	2.122	16.326	49.115	2.122	16.326	49.115	2.977
3	1.710	13.155	62.270	1.710	13.155	62.270	2.739
4	.909	6.994	69.264				
5	.819	6.297	75.561				
6	.715	5.498	81.060				
7	.638	4.906	85.966				
8	.464	3.568	89.534				
9	.369	2.839	92.373				
10	.312	2.400	94.772				
11	.277	2.128	96.900				
12	.220	1.691	98.591				
13	.183	1.409	100.000				

Extraction method: principal component analysis

[a] When components are correlated, sums of squared loading cannot be added to obtain a total variance.

The data in the above table is further substantiated by a scree plot, where a point of inflexion can be seen at component four:



[55] The three scales:
Deep ATL – seeking meaning, relating ideas, use of evidence, interest in ideas
Strategic ATL – organised studying, time management, alertness to assessment demands, achieving
Surface ATL – monitoring effectiveness, lack of purpose, unrelated memorising, syllabus boundedness, fear of failure

The structure matrix then identifies which component corresponds to which approach to learning.

	Component		
	1	2	3
Seeking Meaning	**.832**	-.093	-.087
Relating Ideas	**.882**	-.129	-.134
Use of Evidence	**.742**	.099	.127
Interest in Ideas	**.700**	.033	-.247
Organised Studying	-.010	**.715**	-.331
Time Management	-.164	**.890**	-.085
Alertness to Assessment Demands	.107	**.503**	.345
Achieving	-.030	**.811**	-.167
Monitoring Effectiveness	.393	**.582**	.257
Lack of Purpose	-.094	-.338	**.650**
Unrelated Memorising	-.093	-.042	**.788**
Syllabus Boundedness	-.221	-.003	**.553**
Fear of Failure	.008	-.003	**.780**

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalisation.
Rotation converged in 11 iterations.

This is an indication that:

- component 1 corresponds with the deep ATL;
- component 2 corresponds with the strategic ATL; and
- component 3 corresponds with the surface ATL.

The relationship between each of the components can be seen in a component correlation matrix:

Component	Deep	Strategic	Surface
Deep	1.000		
Strategic	.244	1.000	
Surface	-.211	-.079	1.000

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalisation.

Figures in this table are sensible indications of the relationship between the different approaches. For example, the negative relationship between deep and surface approaches is apparent if one considers the almost polar means by students of each approach may attack their learning and studying. Strategic approaches share elements of deep and surface approaches in order to make the most of both methods in order to achieve highly. Here, the numbers suggest a positive relationship with deep approaches and a small negative relationship with surface approaches, implying that it has more in common with a deep ATL than a surface ATL.

Finally, when analysing the sub-categories of each ATL, there can be seen to be relationships between the factors:

	Seeking Meaning	Relating Ideas	Use of Evidence	Interest in Ideas	Organised Studying	Time Management	Alertness to Assessment Demands	Achieving	Monitoring Effectiveness	Lack of Purpose	Unrelated Memorising	Syllabus Boundedness	Fear of Failure
Seeking Meaning	1.000												
Relating Ideas	.668*	1.000											
Use of Evidence	.515**	.483**	1.000										
Interest in Ideas	.506**	.725**	.399**	1.000									
Organised Studying	.221*	.155*	.190*	.284**	1.000								
Time Management	.041	.000	.064	.175*	.632**	1.000							
Alertness to Assessment Demands	.002	.068	.125	.001	.110	.227**	1.000						
Achieving	.104	.111	.286**	.222**	.516**	.668**	.304**	1.000					
Monitoring Effectiveness	.323**	.290*	.348**	.252**	.428**	.437**	.381**	.329**	1.000				
Lack of Purpose	-.219*	-.331**	-.228**	-.427**	-.420**	-.297**	-.032	-.465**	-.137	1.000			
Unrelated Memorising	-.273**	-.293**	-.156	-.330**	-.339**	-.034	-.015	-.223**	.002	.509**	1.000		
Syllabus Boundedness	-.333*	-.335**	-.152	-.333**	-.187*	-.170*	.070	-.217*	.006	.304**	.411**	1.000	
Fear of Failure	-.207*	-.226**	-.038	-.206*	-.272**	-.058	.122	-.064	-.107	.471**	.592**	.308**	1.000

[a] *Determinant = .003*

5.2 – Comparing ATLs of BA Students with MMath Students

Comparing students from Years 1-3 with students in Year 4 in order to see whether there is any statistically significant difference in their ATLs given the fourth year is both selective and elective.

Degree Stage * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Degree Stage	Years 1-3	Count	29	137	19	185
		Expected Count	31.6	136.3	17.0	185.0
		% within Degree Stage	15.7%	74.1%	10.3%	100.0%
		% within ATL	74.4%	81.5%	90.5%	81.1%
		% of Total	12.7%	60.1%	8.3%	81.1%
	MMath	Count	10	31	2	43
		Expected Count	7.4	31.7	4.0	43.0
		% within Degree Stage	23.3%	72.1%	4.7%	100.0%
		% within ATL	25.6%	18.5%	9.5%	18.9%
		% of Total	4.4%	13.6%	0.9%	18.9%
Total		Count	39	168	21	228
		Expected Count	39.0	168.0	21.0	228.0
		% within Degree, ie 1-3 or 4	17.1%	73.7%	9.2%	100.0%
		% within Dominant	100.0%	100.0%	100.0%	100.0%
		% of Total	17.1%	73.7%	9.2%	100.0%

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	2.386[a]	2	.303	.309		
Likelihood Ratio	2.505	2	.286	.323		
Fisher's Exact Test	2.237			.348		
Linear-by-Linear Association	2.356[b]	1	.125	.135	.085	.041
N of Valid Cases	228					

a. 1 cells (16.7%) have expected count less than 5. The minimum expected count is 3.96.
b. The standardized statistic is -1.535.

This Fisher's Exact Test revealed no significant differences between students in their BA years and students in the MMath year.

5.3 – Comparing Scale Scores of BA Students with MMath Students

An independent-samples Mann-Whitney U Test revealed there to be no significant differences between BA (Years 1-3) and MMath (Year 4) students in their subscale scores.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of degree stage.	Independent-Samples Mann-Whitney U Test	.791	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of degree stage.	Independent-Samples Mann-Whitney U Test	.715	Retain the null hypothesis.
3	The distribution of surface is the same across categories of degree stage.	Independent-Samples Mann-Whitney U Test	.093	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.4 – Year-Group Differences by ASSIST Item

Owing to the fact that significant differences were identified in the surface scale scores of students in Year 3 and Year 4, individual items of the ASSIST were compared between the six different year-group pairs:

Item	Scale	Statement	Year-Group Pairing						
			Years 1-3 & 4	Years 1 & 2	Years 1 & 3	Years 1 & 4	Years 2 & 3	Years 2 & 4	Years 3 & 4
			Fisher's Exact Test Value						
01	ST	I manage to find conditions for studying which allow me to get on with my work easily.	3.201	3.626	5.795	1.841	2.363	3.601	5.452
02	ST	When working on an assignment, I'm keeping in mind how best to impress the marker.	4.327	2.877	.752	3.382	1.553	3.087	4.261
03	SU	Often I find myself wondering whether the work I'm doing here is worthwhile.	6.632	5.945	7.483	9.101	6.401	7.676	2.557
04	DE	I often set out to understand for myself the meaning of what we have to learn.	5.175	2.752	6.519	2.106	5.007	6.034	8.108*
05	ST	I organize my study time effectively to make the best use of it.	6.223	2.011	6.372	5.847	6.002	4.587	6.533
06	SU	I find I have to concentrate on just memorising a good deal of what I have to learn.	4.094	4.990	1.342	3.315	7.043	3.602	5.757
07	ST	I go over the work I've done carefully to check the reasoning and that it makes sense.	1.292	6.213	6.232	1.731	3.722	2.995	3.089
08	SU	Often I feel I'm drowning in the sheer amount of material we're having to cope with.	4.532	3.003	4.353	4.014	9.159	2.925	7.594
09	DE	I look at the evidence carefully and try to reach my own conclusion about what I'm studying.	1.149	3.389	3.447	2.992	4.419	1.060	2.014
10	ST	It's important for me to feel that I'm doing as well as I really can.	1.094	6.648	5.077	2.748	3.547	1.361	3.269
11	DE	I try to relate ideas I come across to those in other topics whenever possible.	3.099	3.866	2.834	4.770	5.130	2.380	2.312
12	SU	I tend to read very little beyond what is actually required to pass.	7.694	6.688	3.935	7.847	8.974	6.280	8.987
13	DE	Regularly I find myself thinking about ideas from lectures when I'm doing other things.	1.179	2.699	1.440	2.212	2.855	1.518	1.000
14	ST	I think I'm quite systematic and organised when it comes to revising for exams.	4.689	.463	4.101	4.743	3.749	3.651	3.914

15	ST	I look carefully at tutors' comments on problem sheets to see how to get higher marks next time.	7.353	4.301	1.376	3.321	5.204	11.279*	3.406
16	SU	There's not much of the work here that I find interesting or relevant.	1.937	1.782	2.143	2.123	1.456	1.833	2.895
17	DE	When I read lecture notes or a book, I try to find out for myself exactly what the author means.	4.080	.846	3.737	4.278	2.066	3.160	3.835
18	ST	I'm pretty good at getting down to work whenever I need to.	1.457	5.937	3.026	2.659	2.307	2.440	.867
19	SU	Much of what I'm studying makes little sense: it's like unrelated bits and pieces.	5.669	6.603	2.216	6.374	7.854	1.854	8.656
20	ST	I think about what I want to get out of this course to keep my studying well-focused.	2.160	1.716	1.327	1.252	2.809	3.416	1.715
21	DE	When I'm working on a new topic, I try to see in my own mind how all the ideas all fit together.	1.897	7.298	3.424	4.445	4.473	1.447	2.751
22	SU	I often worry about whether I'll ever be able to cope with the work properly.	12.752**	3.399	2.213	11.318*	3.447	6.869	11.751*
23	DE	Often I find myself questioning things I hear in lectures or read in books.	3.296	3.655	1.337	4.022	3.659	4.623	1.448
24	ST	I feel that I'm getting on well, and this helps me put more effort into work.	4.594	2.461	1.699	4.374	1.083	3.247	4.977
25	SU	I concentrate on learning just those bits of information that I have to know to pass.	4.249	4.140	3.699	6.662	.951	2.224	2.869
26	DE	I find that studying academic topics can be quite exciting at times.	2.760	1.343	7.125	2.093	6.278	1.655	8.791*
27	ST	I'm good at following up some of the reading suggested by lecturers or tutors.	1.779	2.833	4.274	2.509	2.321	1.029	3.410
28	ST	I keep in mind who is going to make a problem sheet and what they're likely to be looking for.	1.570	5.992	6.424	.285	4.524	3.586	4.771
29	SU	When I look back, I sometimes wonder why I ever decided to come here.	3.056	2.157	2.620	1.942	5.228	3.481	4.637
30	DE	When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it.	1.935	8.121	10.713*	3.451	1.045	2.530	4.107
31	ST	I work steadily through the term, rather than leave it all until the last minute.	8.053	7.051	5.486	8.360	2.949	6.098	5.136
32	SU	I'm not really sure what's important in lectures, so I try to get down all I can.	5.036	2.005	1.876	4.640	1.925	3.475	4.988
33	DE	Ideas in course books or lecture notes often set me off on long chains of thought on my own.	1.648	3.463	2.438	1.779	2.976	4.045	.566
34	ST	Before starting work on a problem sheet or exam question, I think first how best to tackle it.	4.437	4.073	2.924	5.124	4.704	6.249	1.080
35	SU	I often seem to panic if I get behind with my work.	6.651	3.155	1.035	6.685	4.960	4.751	5.371
36	DE	When I read, I examine the details carefully to see how they fit in with what's being said.	2.661	6.549	2.531	2.348	4.820	7.397	1.389
37	ST	I put a lot of effort into studying because I'm determined to do well.	6.589	2.379	1.647	4.574	1.436	6.046	4.586
38	SU	I gear my studying closely to just what seems to be required for problem sheets and exams.	1.795	5.241	5.891	3.923	2.922	1.878	2.682

39	DE	Some of the ideas I come across on the course, I find really gripping.	1.994	2.571	3.336	4.202	1.874	2.104	1.757
40	ST	I usually plan out my week's work in advance, either on paper or in my head.	8.013	2.589	3.884	9.613*	2.071	5.461	4.002
41	ST	I keep an eye open for what lecturers seem to think is important and concentrate on that.	4.654	1.409	2.900	5.729	2.783	4.145	2.823
42	SU	I'm not really interested in my degree, but I have to take it for other reasons.	4.607	3.128	2.433	4.466	1.806	3.980	2.719
43	DE	Before tackling a mathematics question, I first try to work out what lies behind it.	4.568	3.022	3.164	2.982	2.580	5.744	5.012
44	ST	I generally make good use of my time during the day.	4.728	2.848	.657	4.861	3.947	3.533	4.221
45	SU	I often have trouble in making sense of the things I have to remember.	7.883	11.039*	6.402	13.375**	6.183	3.283	7.517
46	DE	I like to play around with ideas of my own even if they don't get me very far.	1.355	2.705	4.307	3.585	2.826	1.227	1.351
47	ST	When I finish a piece of work, I check it through to see if it really meets the requirements.	2.152	4.854	8.675	3.320	3.791	2.215	4.583
48	SU	Often I lie awake worrying about work I think I won't be able to do.	5.696	1.377	1.183	5.980	.530	3.537	4.112
49	DE	It's important for me to be able to follow the argument, or to see the reason behind things.	0.768	3.156	3.757	.450	3.324	1.677	2.084
50	ST	I don't find it at all difficult to motivate myself.	7.950	3.873	3.781	5.710	6.925	8.693	7.162
51	SU	I like to be told precisely what to do in essays or other assignments.	5.523	1.876	1.748	5.683	2.815	4.450	4.785
52	DE	I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them.	1.384	6.997	2.477	2.096	1.980	2.826	1.867

** Significant at the 0.05 level*

*** Significant at the 0.01 level*

**** Significant at the 0.001 level*

5.5 – Comparing ATLs of First-Years in Sweep 1 & Sweep 2

This tests whether differences exist in the distribution of dominant ATLs in first-year students between Sweep 1 and Sweep 2.

ATL * Sweep Crosstabulation

			Sweep		Total
			1	2	
ATL	Deep	Count	20	13	33
		Expected Count	23.7	9.3	33.0
		% within ATL	60.6%	39.4%	100.0%
		% within Sweep	11.4%	18.8%	13.5%
		% of Total	8.2%	5.3%	13.5%
		Std. Residual	-.8	1.2	
	Strategic	Count	155	49	204
		Expected Count	146.5	57.5	204.0
		% within ATL	76.0%	24.0%	100.0%
		% within Sweep	88.1%	71.0%	83.3%
		% of Total	63.3%	20.0%	83.3%
		Std. Residual	.7	-1.1	
	Surface	Count	1	7	8
		Expected Count	5.7	2.3	8.0
		% within ATL	12.5%	87.5%	100.0%
		% within Sweep	0.6%	10.1%	3.3%
		% of Total	0.4%	2.9%	3.3%
		Std. Residual	-2.0	3.2	
Total		Count	176	69	245
		Expected Count	176.0	69.0	245.0
		% within ATL	71.8%	28.2%	100.0%
		% within Sweep	100.0%	100.0%	100.0%
		% of Total	71.8%	28.2%	100.0%

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	17.711[a]	2	.000	.000		
Likelihood Ratio	16.087	2	.000	.000		
Fisher's Exact Test	15.716			.000		
Linear-by-Linear Association	.139[b]	1	.710	.725	.427	.134
N of Valid Cases	245					

a. 1 cells (16.7%) have expected count less than 5. The minimum expected count is 2.25.
b. The standardized statistic is .372.

5.6 – Comparing Deep, Surface & Strategic Scale Scores of First-Years in Sweep 1 & Sweep 2

Testing to see whether there are is a significant difference in students' scores on each of the deep, strategic and surface scales between Sweep 1 and Sweep 2.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of sweep.	Independent-Samples Mann-Whitney U Test	.455	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of sweep.	Independent-Samples Mann-Whitney U Test	.850	Retain the null hypothesis.
3	The distribution of surface is the same across categories of sweep.	Independent-Samples Mann-Whitney U Test	.469	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.7 – Comparing ATLs of Students Across Year-Groups

Investigating differences between year groups in terms of dominant ATL:

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	12	50	7	69
		Expected Count	11.8	50.8	6.4	69.0
		% within Year	17.4%	72.5%	10.1%	100.0%
		% within ATL	30.8%	29.8%	33.3%	30.3%
		% of Total	5.3%	21.9%	3.1%	30.3%
	2nd	Count	13	45	8	66
		Expected Count	11.3	48.6	6.1	66.0
		% within Year	19.7%	68.2%	12.1%	100.0%
		% within ATL	33.3%	26.8%	38.1%	28.9%
		% of Total	5.7%	19.7%	3.5%	28.9%
	3rd	Count	4	42	4	50
		Expected Count	8.6	36.8	4.6	50.0
		% within Year	8.0%	84.0%	8.0%	100.0%
		% within ATL	10.3%	25.0%	19.0%	21.9%
		% of Total	1.8%	18.4%	1.8%	21.9%
	4th	Count	10	31	2	43
		Expected Count	7.4	31.7	4.0	43.0
		% within Year	23.3%	72.1%	4.7%	100.0%
		% within ATL	25.6%	18.5%	9.5%	18.9%
		% of Total	4.4%	13.6%	0.9%	18.9%
Total		Count	39	168	21	228
		Expected Count	39.0	168.0	21.0	228.0
		% within Year	17.1%	73.7%	9.2%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	17.1%	73.7%	9.2%	100.0%

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	6.381[a]	6	.382	.387		
Likelihood Ratio	7.014	6	.320	.343		
Fisher's Exact Test	6.526			.362		
Linear-by-Linear Association	.466[b]	1	.495	.512	.267	.038
N of Valid Cases	228					

a. 2 cells (16.7%) have expected count less than 5. The minimum expected count is 3.96.
b. The standardized statistic is -.682.

5.8 – Investigating Year Group Differences (Men Only)

Comparing predominant ATLs of male participants across year groups:

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	10	32	3	45
		Expected Count	9.6	32.7	2.7	45.0
		% within Year	22.2%	71.1%	6.7%	100.0%
		% within ATL	31.2%	29.4%	33.3%	30.0%
		% of Total	6.7%	21.3%	2.0%	30.0%
	2nd	Count	11	34	3	48
		Expected Count	10.2	34.9	2.9	48.0
		% within Year	22.9%	70.8%	6.2%	100.0%
		% within ATL	34.4%	31.2%	33.3%	32.0%
		% of Total	7.3%	22.7%	2.0%	32.0%
	3rd	Count	4	22	2	28
		Expected Count	6.0	20.3	1.7	28.0
		% within Year	14.3%	78.6%	7.1%	100.0%
		% within ATL	12.5%	20.2%	22.2%	18.7%
		% of Total	2.7%	14.7%	1.3%	18.7%
	4th	Count	7	21	1	29
		Expected Count	6.2	21.1	1.7	29.0
		% within Year	24.1%	72.4%	3.4%	100.0%
		% within ATL	21.9%	19.3%	11.1%	19.3%
		% of Total	4.7%	14.0%	0.7%	19.3%
Total		Count	32	109	9	150
		Expected Count	32.0	109.0	9.0	150.0
		% within Year	21.3%	72.7%	6.0%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	21.3%	72.7%	6.0%	100.0%

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	1.418[a]	6	.965	.971		
Likelihood Ratio	1.548	6	.956	.964		
Fisher's Exact Test	1.612			.969		
Linear-by-Linear Association	.011[b]	1	.915	.941	.487	.059
N of Valid Cases	150					

a. 4 cells (33.3%) have expected count less than 5. The minimum expected count is 1.68.
b. The standardized statistic is -.107.

Comparing subscale scores across year groups for male participants:

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.323	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.948	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.509	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.9 – Investigating Year-Group Differences (Women Only)

Investigating whether differences exist between year groups in terms of ATL (women only):

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	2	18	4	24
		Expected Count	2.2	17.8	4.0	24.0
		% within Year	8.3%	75.0%	16.7%	100.0%
		% within ATL	28.6%	31.0%	30.8%	30.8%
		% of Total	2.6%	23.1%	5.1%	30.8%
	2nd	Count	2	11	5	18
		Expected Count	1.6	13.4	3.0	18.0
		% within Year	11.1%	61.1%	27.8%	100.0%
		% within ATL	28.6%	19.0%	38.5%	23.1%
		% of Total	2.6%	14.1%	6.4%	23.1%
	3rd	Count	0	19	3	22
		Expected Count	2.0	16.4	3.7	22.0
		% within Year	0.0%	86.4%	13.6%	100.0%
		% within ATL	0.0%	32.8%	23.1%	28.2%
		% of Total	0.0%	24.4%	3.8%	28.2%
	4th	Count	3	10	1	14
		Expected Count	1.3	10.4	2.3	14.0
		% within Year	21.4%	71.4%	7.1%	100.0%
		% within ATL	42.9%	17.2%	7.7%	17.9%
		% of Total	3.8%	12.8%	1.3%	17.9%
Total		Count	7	58	13	78
		Expected Count	7.0	58.0	13.0	78.0
		% within Year	9.0%	74.4%	16.7%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	9.0%	74.4%	16.7%	100.0%

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	7.582[a]	6	.270	.274		
Likelihood Ratio	8.865	6	.181	.268		
Fisher's Exact Test	7.231			.267		
Linear-by-Linear Association	1.056[b]	1	.304	.358	.179	.049
N of Valid Cases	78					

a. 8 cells (66.7%) have expected count less than 5. The minimum expected count is 1.26.
b. The standardized statistic is -1.027.

Comparing subscale scores across year-groups for female participants:

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.668	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.343	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Kruskal-Wallis Test	.553	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.10 – Comparing ATLs between Year Group Pairings

5.10.1 – Y1 & Y2

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	12	50	7	69
		Expected Count	12.8	48.6	7.7	69.0
		% within Year	17.4%	72.5%	10.1%	100.0%
		% within ATL	48.0%	52.6%	46.7%	51.1%
		% of Total	8.9%	37.0%	5.2%	51.1%
	2nd	Count	13	45	8	66
		Expected Count	12.2	46.4	7.3	66.0
		% within Year	19.7%	68.2%	12.1%	100.0%
		% within ATL	52.0%	47.4%	53.3%	48.9%
		% of Total	9.6%	33.3%	5.9%	48.9%
Total		Count	25	95	15	135
		Expected Count	25.0	95.0	15.0	135.0
		% within Year	18.5%	70.4%	11.1%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	18.5%	70.4%	11.1%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	.303[a]	2	.859	.891		
Likelihood Ratio	.303	2	.859	.891		
Fisher's Exact Test	.351			.891		
Linear-by-Linear Association	.001[b]	1	.972	1.000	.549	.126
N of Valid Cases	135					

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 7.33.
b. The standardized statistic is -.035.

5.10.2 – Year 1 & Year 3

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	12	50	7	69
		Expected Count	9.3	53.3	6.4	69.0
		% within Year	17.4%	72.5%	10.1%	100.0%
		% within ATL	75.0%	54.3%	63.6%	58.0%
		% of Total	10.1%	42.0%	5.9%	58.0%
	3rd	Count	4	42	4	50
		Expected Count	6.7	38.7	4.6	50.0
		% within Year	8.0%	84.0%	8.0%	100.0%
		% within ATL	25.0%	45.7%	36.4%	42.0%
		% of Total	3.4%	35.3%	3.4%	42.0%
Total		Count	16	92	11	119
		Expected Count	16.0	92.0	11.0	119.0
		% within Year	13.4%	77.3%	9.2%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	13.4%	77.3%	9.2%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	2.545[a]	2	.280	.301		
Likelihood Ratio	2.665	2	.264	.286		
Fisher's Exact Test	2.466			.301		
Linear-by-Linear Association	.671[b]	1	.413	.442	.267	.112
N of Valid Cases	119					

a. 1 cells (16.7%) have expected count less than 5. The minimum expected count is 4.62.
b. The standardized statistic is .819.

5.10.3 – Year 1 & Year 4

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	1st	Count	12	50	7	69
		Expected Count	13.6	49.9	5.5	69.0
		% within Year	17.4%	72.5%	10.1%	100.0%
		% within ATL	54.5%	61.7%	77.8%	61.6%
		% of Total	10.7%	44.6%	6.2%	61.6%
	4th	Count	10	31	2	43
		Expected Count	8.4	31.1	3.5	43.0
		% within Year	23.3%	72.1%	4.7%	100.0%
		% within ATL	45.5%	38.3%	22.2%	38.4%
		% of Total	8.9%	27.7%	1.8%	38.4%
Total		Count	22	81	9	112
		Expected Count	22.0	81.0	9.0	112.0
		% within Year	19.6%	72.3%	8.0%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	19.6%	72.3%	8.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	1.459[a]	2	.482	.525		
Likelihood Ratio	1.531	2	.465	.472		
Fisher's Exact Test	1.371			.551		
Linear-by-Linear Association	1.286[b]	1	.257	.265	.172	.079
N of Valid Cases	112					

a. 1 cells (16.7%) have expected count less than 5. The minimum expected count is 3.46.
b. The standardized statistic is -1.134.

5.10.4 – Year 2 & Year 3

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	2nd	Count	13	45	8	66
		Expected Count	9.7	49.5	6.8	66.0
		% within Year	19.7%	68.2%	12.1%	100.0%
		% within ATL	76.5%	51.7%	66.7%	56.9%
		% of Total	11.2%	38.8%	6.9%	56.9%
	3rd	Count	4	42	4	50
		Expected Count	7.3	37.5	5.2	50.0
		% within Year	8.0%	84.0%	8.0%	100.0%
		% within ATL	23.5%	48.3%	33.3%	43.1%
		% of Total	3.4%	36.2%	3.4%	43.1%
Total		Count	17	87	12	116
		Expected Count	17.0	87.0	12.0	116.0
		% within Year	14.7%	75.0%	10.3%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	14.7%	75.0%	10.3%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	4.072[a]	2	.131	.119		
Likelihood Ratio	4.266	2	.119	.126		
Fisher's Exact Test	3.970			.126		
Linear-by-Linear Association	.652[b]	1	.419	.458	.268	.108
N of Valid Cases	116					

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 5.17.
b. The standardized statistic is .808.

5.10.5 – Year 2 & Year 4

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	2nd	Count	13	45	8	66
		Expected Count	13.9	46.0	6.1	66.0
		% within Year	19.7%	68.2%	12.1%	100.0%
		% within ATL	56.5%	59.2%	80.0%	60.6%
		% of Total	11.9%	41.3%	7.3%	60.6%
	4th	Count	10	31	2	43
		Expected Count	9.1	30.0	3.9	43.0
		% within Year	23.3%	72.1%	4.7%	100.0%
		% within ATL	43.5%	40.8%	20.0%	39.4%
		% of Total	9.2%	28.4%	1.8%	39.4%
Total		Count	23	76	10	109
		Expected Count	23.0	76.0	10.0	109.0
		% within Year	21.1%	69.7%	9.2%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	21.1%	69.7%	9.2%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	1.797[a]	2	.407	.465		
Likelihood Ratio	1.951	2	.377	.445		
Fisher's Exact Test	1.701			.464		
Linear-by-Linear Association	1.088[b]	1	.297	.364	.195	.085
N of Valid Cases	109					

a. 1 cells (16.7%) have expected count less than 5. The minimum expected count is 3.94.
b. The standardized statistic is -1.043.

5.10.6 – Year 3 & Year 4

Year * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Year	3rd	Count	4	42	4	50
		Expected Count	7.5	39.2	3.2	50.0
		% within Year	8.0%	84.0%	8.0%	100.0%
		% within ATL	28.6%	57.5%	66.7%	53.8%
		% of Total	4.3%	45.2%	4.3%	53.8%
	4th	Count	10	31	2	43
		Expected Count	6.5	33.8	2.8	43.0
		% within Year	23.3%	72.1%	4.7%	100.0%
		% within ATL	71.4%	42.5%	33.3%	46.2%
		% of Total	10.8%	33.3%	2.2%	46.2%
Total		Count	14	73	6	93
		Expected Count	14.0	73.0	6.0	93.0
		% within Year	15.1%	78.5%	6.5%	100.0%
		% within ATL	100.0%	100.0%	100.0%	100.0%
		% of Total	15.1%	78.5%	6.5%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	4.394[a]	2	.111	.112		
Likelihood Ratio	4.473	2	.107	.112		
Fisher's Exact Test	4.291			.121		
Linear-by-Linear Association	3.812[b]	1	.051	.068	.041	.028
N of Valid Cases	93					

a. 2 cells (16.7%) have expected count less than 5. The minimum expected count is 2.77.
b. The standardized statistic is -1.952.

5.11 – Comparing Scale Scores between Year-Group Pairings

5.11.1 – Year 1 & Year 2

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.289	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.399	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.269	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.11.2 – Year 1 & Year 3

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.161	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.976	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.357	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.11.3 – Year 1 & Year 4

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.363	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.574	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.117	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.11.4 – Year 2 & Year 3

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.708	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.371	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.093	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.11.5 – Year 2 & Year 4

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.946	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.865	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.479	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.11.6 – Year 3 & Year 4

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.769	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.560	Retain the null hypothesis.
3	The distribution of surface is the same across categories of year.	Independent-Samples Mann-Whitney U Test	.032	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.12 – Investigating Gender Differences (Sweep 1)

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	14	102	2	118
		Expected Count	12.1	104.6	1.3	118.0
	Female	Count	4	54	0	58
		Expected Count	5.9	51.4	.7	58.0
Total		Count	18	156	2	176
		Expected Count	18.0	156.0	2.0	176.0

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	2.116[a]	2	.347	.382		
Likelihood Ratio	2.798	2	.247	.329		
Fisher's Exact Test	1.600			.430		
Linear-by-Linear Association	.393[b]	1	.531	.626	.357	.165
N of Valid Cases	176					

a. 2 cells (33.3%) have expected count less than 5. The minimum expected count is .66.
b. The standardized statistic is .627.

In the first sweep, Fisher's test was negative for a relationship between ATL and gender $(p = .430)$.

	Gender	N	Median	Variance	Minimum	Maximum
Deep	M	118	48.5	39.3	31	62
	F	58	48.0	28.0	34	59
Strategic	M	118	72.0	82.1	34	89
	F	58	70.0	67.9	51	87
Surface	M	118	48.0	112.0	24	67
	F	58	50.5	74.9	20	63

An independent samples Mann Whitney U-test confirmed no significant gender differences in terms of the scores across each of the three subscales.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.176	Retain the null hypothesis.
2	The distribution of strategic is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.816	Retain the null hypothesis.
3	The distribution of surface is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.145	Retain the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.13 – Investigating Gender Differences in ATL (Sweep 2)

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	32	109	9	150
		Expected Count	25.7	110.5	13.8	150.0
	Female	Count	7	59	12	78
		Expected Count	13.3	57.5	7.2	78.0
Total		Count	39	168	21	228
		Expected Count	39.0	168.0	21.0	228.0

Expected counts are all above 5, so Pearson's chi-square may be used.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	9.551[a]	2	.008	.008		
Likelihood Ratio	9.766	2	.008	.009		
Fisher's Exact Test	9.488			.009		
Linear-by-Linear Association	9.402[b]	1	.002	.002	.001	.001
N of Valid Cases	228					

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 7.18.
b. The standardized statistic is 3.066.

5.14 – Comparing ATLs between Genders in Each Year-Group

5.14.1 – Year 1

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	10	32	3	45
		Expected Count	7.8	32.6	4.6	45.0
	Female	Count	2	18	4	24
		Expected Count	4.2	17.4	2.4	24.0
Total		Count	12	50	7	69
		Expected Count	12.0	50.0	7.0	69.0

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	3.312[a]	2	.191	.216		
Likelihood Ratio	3.444	2	.179	.200		
Fisher's Exact Test	3.208			.216		
Linear-by-Linear Association	3.259[b]	1	.071	.091	.057	.039
N of Valid Cases	69					

a. 3 cells (50.0%) have expected count less than 5. The minimum expected count is 2.43.
b. The standardized statistic is 1.805.

5.14.2 – Year 2

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	11	34	3	48
		Expected Count	9.5	32.7	5.8	48.0
	Female	Count	2	11	5	18
		Expected Count	3.5	12.3	2.2	18.0
Total		Count	13	45	8	66
		Expected Count	13.0	45.0	8.0	66.0

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	6.113[a]	2	.047	.047		
Likelihood Ratio	5.545	2	.063	.106		
Fisher's Exact Test	5.378			.060		
Linear-by-Linear Association	4.585[b]	1	.032	.047	.027	.020
N of Valid Cases	66					

a. 2 cells (33.3%) have expected count less than 5. The minimum expected count is 2.18.
b. The standardized statistic is 2.141.

5.14.3 – Year 3

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	4	22	2	28
		Expected Count	2.2	23.5	2.2	28.0
	Female	Count	0	20	2	22
		Expected Count	1.8	18.5	1.8	22.0
Total		Count	4	42	4	50
		Expected Count	4.0	42.0	4.0	50.0

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	3.425[a]	2	.180	.257		
Likelihood Ratio	4.919	2	.085	.227		
Fisher's Exact Test	3.248			.257		
Linear-by-Linear Association	1.989[b]	1	.158	.291	.145	.110
N of Valid Cases	50					

a. 4 cells (66.7%) have expected count less than 5. The minimum expected count is 1.76.
b. The standardized statistic is 1.410.

5.14.4 – Year 4

Gender * ATL Crosstabulation

			ATL			Total
			Deep	Strategic	Surface	
Gender	Male	Count	7	21	1	29
		Expected Count	6.7	20.9	1.3	29.0
	Female	Count	3	10	1	14
		Expected Count	3.3	10.1	.7	14.0
Total		Count	10	31	2	43
		Expected Count	10.0	31.0	2.0	43.0

Some expected counts <5 so Fisher's Exact Test to be used instead of Pearson's chi-square.

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	.308[a]	2	.857	1.000		
Likelihood Ratio	.291	2	.865	1.000		
Fisher's Exact Test	.681			1.000		
Linear-by-Linear Association	.155[b]	1	.694	.754	.475	.238
N of Valid Cases	43					

a. 3 cells (50.0%) have expected count less than 5. The minimum expected count is .65.
b. The standardized statistic is .393.

5.15 – Investigating Gender Differences in Scale Scores (Sweep 2)

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of deep is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.000	Reject the null hypothesis.
2	The distribution of strategic is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.328	Retain the null hypothesis.
3	The distribution of surface is the same across categories of gender.	Independent-Samples Mann-Whitney U Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

5.16 – Differences in Individual Items between Year 1 & Year 2

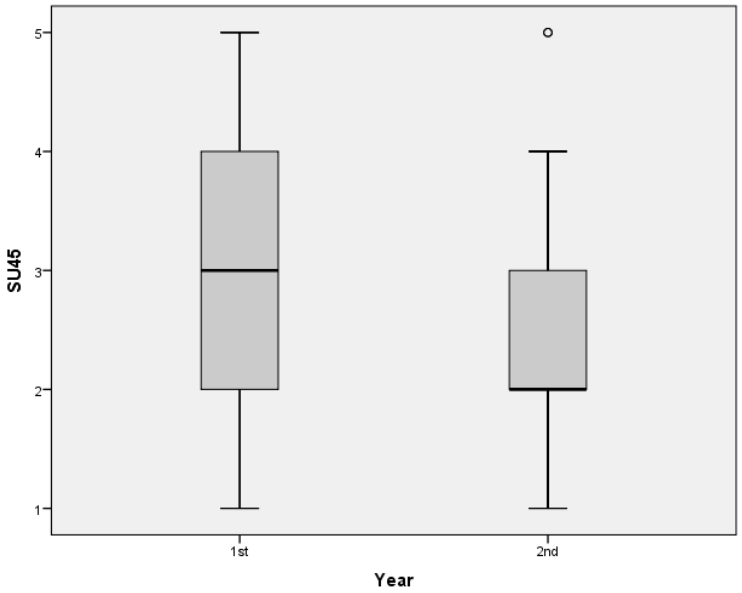
5.16.1 – I often have trouble in making sense of the things I have to remember

Second-year students tend to disagree more with this statement than first-year students ($\chi^2(4) = 11.039, p = .025$). The median response in Year 1 is 'neither agree nor disagree', whereas it is 'somewhat disagree' in Year 2. That is, new undergraduates find it more difficult to make sense of mathematics than in the second year.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
1	Count	10	16	18	16	9	69
	% within year	14.5%	23.2%	26.1%	23.2%	13.0%	100.0%
2	Count	6	30	18	10	2	66
	% within year	9.1%	45.5%	27.3%	15.2%	3.0%	100.0%
Total	Count	16	46	36	26	11	135
	% within year	11.9%	34.1%	26.7%	19.3%	8.1%	100.0%

This is understandable, because first-year students will be newer to the advanced mathematics and the type of mathematics that they are studying. This 'culture shock' can then have an impact on how easy or difficult they find the topics that they are studying.

SU45	Year 1	Year 2
Mean	2.97	2.58
Median	3.00	2.00
Variance	1.587	.925
Std. dev	1.260	.962
Min.	1	1
Max.	5	5

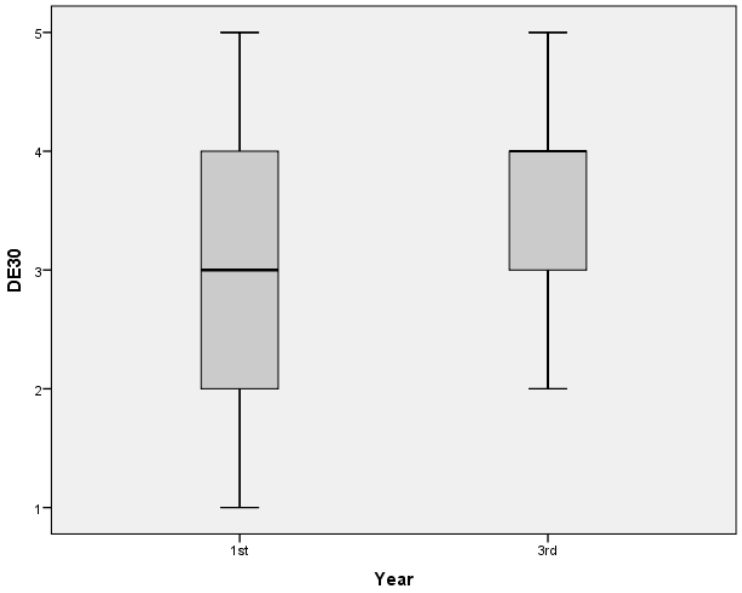


5.16.2 – When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it

Third year students tend to agree with this statement more than first-years ($p = .022$). The median response in Year 1 is 'neither agree nor disagree', whereas in Year 3 it is 'somewhat agree'. That is, third-years tend to read and reflect more than first-years.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
1	Count	3	15	24	17	10	69
	% within year	4.3%	21.7%	34.8%	24.6%	14.5%	100.0%
3	Count	0	7	14	26	3	50
	% within year	0.0%	14.0%	28.0%	52.0%	6.0%	100.0%
Total	Count	3	22	38	43	13	119
	% within year	2.5%	18.5%	31.9%	36.1%	10.9%	100.0%

This may be attributed to the fact that students sit final examinations in the Year 3 and so may be inclined to more closely study the material in order that they may be successful in their examinations.



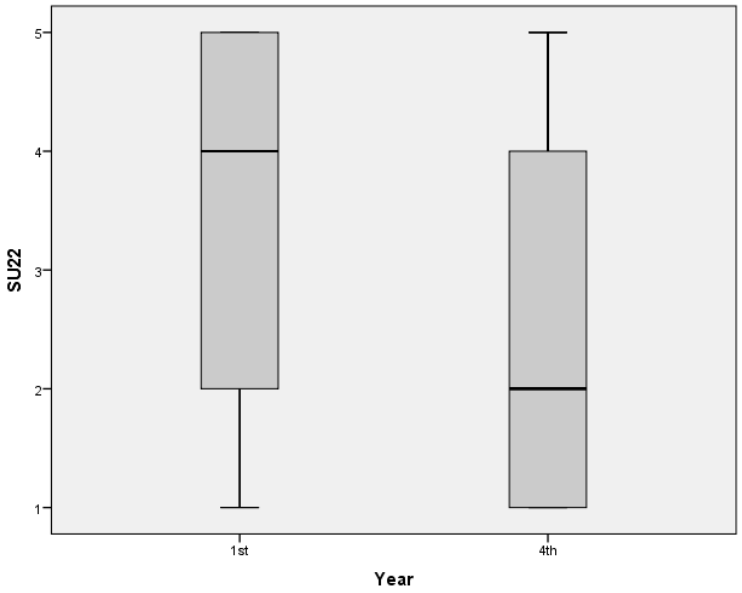
DE30	Year 1	Year 2
Mean	3.23	3.50
Median	3.00	4.00
Variance	1.181	.663
Std. dev	1.087	.814
Min.	1	2
Max.	5	5

5.16.3 – I often worry about whether I'll ever be able to cope with the work properly

Year 1 students tend to 'somewhat agree' with this statement more than Year 4 students, who tend to 'somewhat disagree' ($\chi^2(4) = 11.318, p = .022$). That is, first-year students tend to worry more about the workload than their fourth-year counterparts.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
1	Count	7	12	9	21	20	69
	% within year	10.1%	17.4%	13.0%	30.4%	29.0%	100.0%
4	Count	12	12	6	8	5	43
	% within year	27.9%	27.9%	14.0%	18.6%	11.6%	100.0%
Total	Count	19	24	15	29	25	112
	% within year	17.0%	21.4%	13.4%	25.9%	22.3%	100.0%

This may because, for new students, the first year might be overwhelming because it is a new experience, whereas students in their fourth year will be more experienced with the workload and content and therefore be able to have accurate expectations about whether they can cope with the workload.



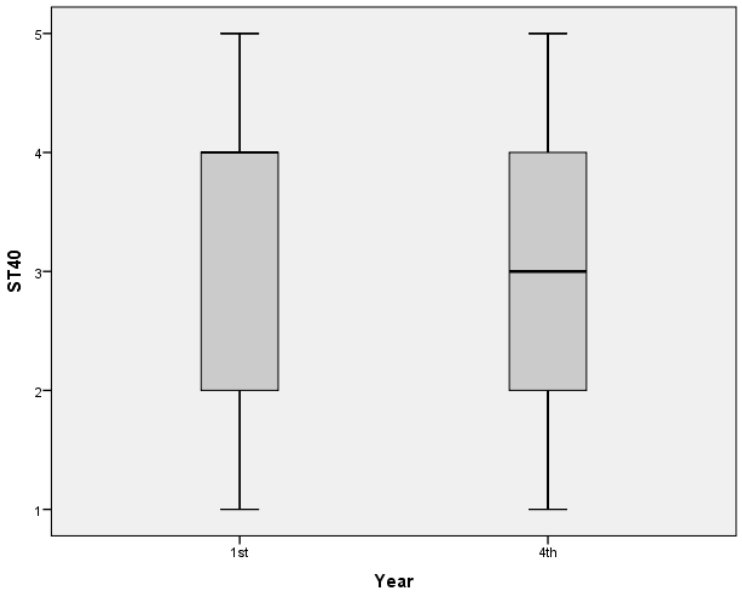
SU22	Year 1	Year 4
Mean	3.51	2.58
Median	4.00	2.00
Variance	1.812	1.916
Std. dev	1.346	1.384
Min.	1	1
Max.	5	5

5.16.4 – I usually plan out my week's work in advance, either on paper or in my head

Students in their first year at Oxford tend to agree with this statement than fourth-year students ($\chi^2(4) = 9.613, p = .047$). The median response in Year 1 was 'somewhat agree', whereas it was 'neither agree nor disagree' in Year 4. That is, first-year students tend to make a more concentrated effort to plan their time than students in their fourth year.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
1	Count	12	10	5	25	17	69
	% within year	17.4%	14.5%	7.2%	36.2%	24.6%	100.0%
4	Count	6	11	9	7	10	43
	% within year	14.0%	25.6%	20.9%	16.3%	23.3%	100.0%
Total	Count	18	21	14	32	27	112
	% within year	16.1%	18.8%	12.5%	28.6%	24.1%	100.0%

This is perhaps because fourth-year students are more experienced with the workload and what they can achieve within a certain period of time and therefore do not feel the need to plan their time, whereas new undergraduates are not yet familiar with the amount of work required and the amount of time that it takes, therefore feeling the need to carefully plan their time.



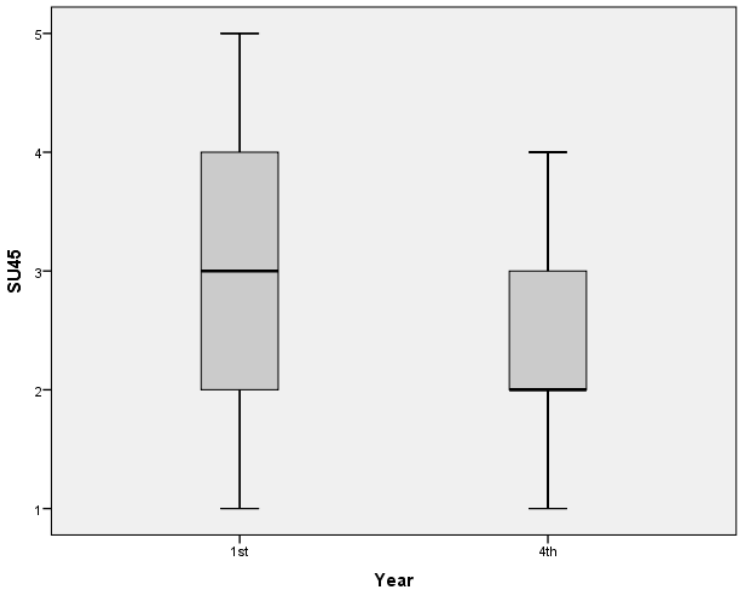
ST40	Year 1	Year 4
Mean	3.36	3.09
Median	4.00	3.00
Variance	2.087	1.944
Std. dev	1.445	1.394
Min.	1	1
Max.	5	5

5.16.5 – I often have trouble in making sense of the things I have to remember

Fourth-year students are more likely to disagree with this statement than first-year students, who are more neutral ($p = .008$). The median response for Year 1 students is 'neither agree nor disagree', whereas it is 'somewhat disagree' in Year 4. That is, first-years are more likely to struggle to make sense of mathematical material than fourth-years.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
1	Count	10	16	18	16	9	69
	% within year	14.5%	23.2%	26.1%	23.2%	13.0%	100.0%
4	Count	6	22	7	8	0	43
	% within year	14.0%	51.2%	16.3%	18.6%	0.0%	100.0%
Total	Count	16	38	25	24	9	112
	% within year	14.3%	33.9%	22.3%	21.4%	8.0%	100.0%

This is likely due to the fact that first-year students have just been introduced to tertiary-level mathematics, which has been found to be quite different in its nature to secondary mathematics, as well as very difficult. On the other hand, students in the fourth year will have achieved highly in the first three years of their degree, and so are likely to be more comfortable with university mathematics.



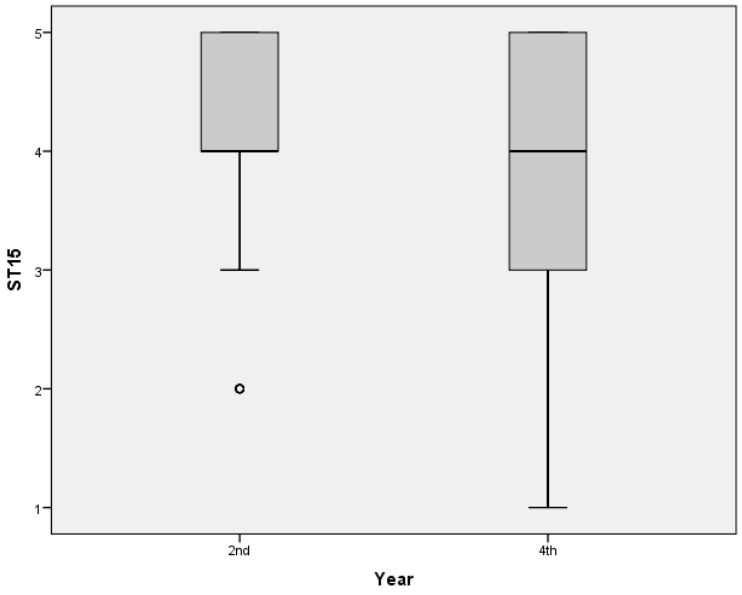
ST40	Year 1	Year 4
Mean	2.97	2.40
Median	3.00	2.00
Variance	1.587	.911
Std. dev	1.260	.955
Min.	1	1
Max.	5	4

5.16.6 – I look carefully at tutors' comments on problem sheets to see how to get higher marks next time.

A greater proportion of second-years 'agreed' or 'somewhat agreed' (77.3%) with this statement than fourth-years (58.1%) ($p = .016$). That is, Year 2 students were more likely to agree in some capacity with this statement than Year 4 students, who were more neutral (30.2% of them neither agreed nor disagreed). This suggests that students in their second year are more likely to be interested in feedback than those in the fourth year.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
2	Count	0	8	7	34	17	66
	% within year	0.0%	12.1%	10.6%	51.5%	25.8%	100.0%
4	Count	2	3	13	13	12	43
	% within year	4.7%	7.0%	30.2%	30.2%	27.9%	100.0%
Total	Count	2	11	20	47	29	109
	% within year	1.8%	10.1%	18.3%	43.1%	26.6%	100.0%

This difference between the two year-groups may be due to students in their second year becoming more conscientious in response to their examination results the previous year, whereas those in the fourth year are more relaxed about their studies and results. However, the difference in responses here is also likely due to the participants' interpretations of the question – second-years have tutorials to supplement their learning whereas fourth-years go to classes. This may account for the fact that such a large number of students in Year 4 responded to this statement neutrally.



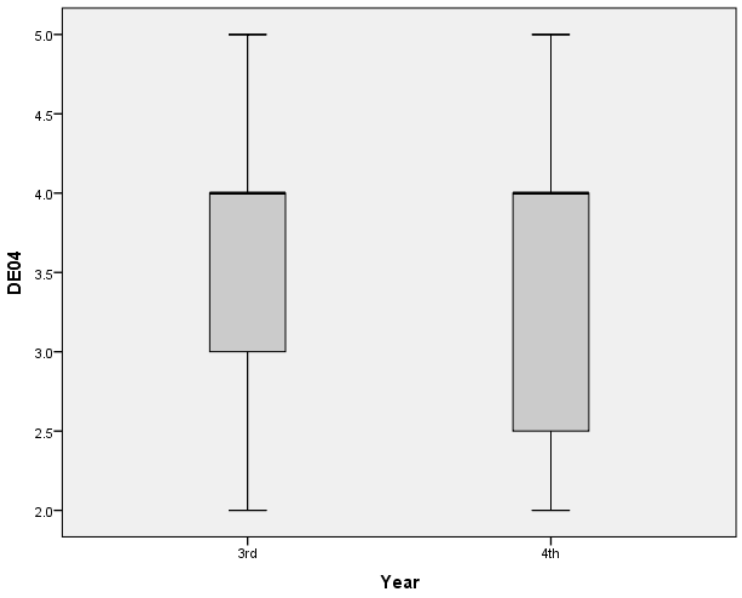
ST15	Year 2	Year 4
Mean	3.91	3.70
Median	4.00	4.00
Variance	.853	1.216
Std. dev	.924	1.103
Min.	2	1
Max.	5	5

5.16.7 – I often set out to understand for myself the meaning of what we have to learn

A significantly greater proportion of Year 4 students disagreed with this statement (25.6%) than Year 3 students, 6% of whom 'somewhat disagreed' with it ($\chi^2(3) = 8.108, p = .043$). No students fully disagreed with the statement. That is, fourth-years are less likely to actively seek to understand mathematical concepts for themselves than those in their third year.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
3	Count	0	3	14	23	10	50
	% within year	0.0%	6.0%	28.0%	46.0%	20.0%	100.0%
4	Count	0	11	7	15	10	43
	% within year	0.0%	25.6%	16.3%	34.9%	23.3%	100.0%
Total	Count	0	14	21	38	20	93
	% within year	0.0%	15.1%	22.6%	40.9%	21.5%	100.0%

This is an interesting outcome, which cannot be explained using existing literature. This may be attributable to the types of teaching that students experience at the two levels – perhaps fourth-years are guided more closely in their classes than third-years.



DE04	Year 3	Year 4
Mean	3.80	3.56
Median	4.00	4.00
Variance	.694	1.252
Std. dev	.833	1.119
Min.	2	2
Max.	5	5

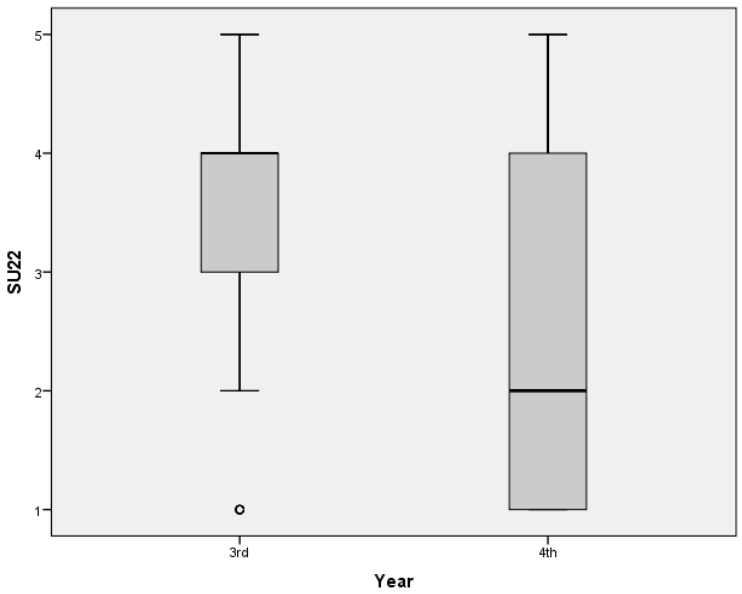
5.16.8 – I often worry about whether I'll ever be able to cope with the work properly

Whilst Year 3 students tended to 'somewhat agree' with this statement, Year 4 students were more likely to 'somewhat disagree' ($\chi^2(4) = 11.751, p = .018$). That is, third-years are more likely to worry about their workload than fourth-years.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
3	Count	6	5	9	18	12	50
	% within year	12.0%	10.0%	18.0%	36.0%	24.0%	100.0%
4	Count	12	12	6	8	5	43
	% within year	27.9%	27.9%	14.0%	18.6%	11.6%	100.0%
Total	Count	18	17	15	26	17	93
	% within year	19.4%	18.3%	16.1%	28.0%	18.3%	100.0%

This difference is likely to occur because students in their third year are nearing their final examinations and a very stressful time, whereas those in their fourth year have already experienced final examinations and so know that they were able to cope with the work they had to do in their BA years. This will likely give them confidence into their fourth year.

SU22	Year 3	Year 4
Mean	3.50	2.58
Median	4.00	2.00
Variance	1.684	1.916
Std. dev	1.298	1.384
Min.	1	1
Max.	5	5

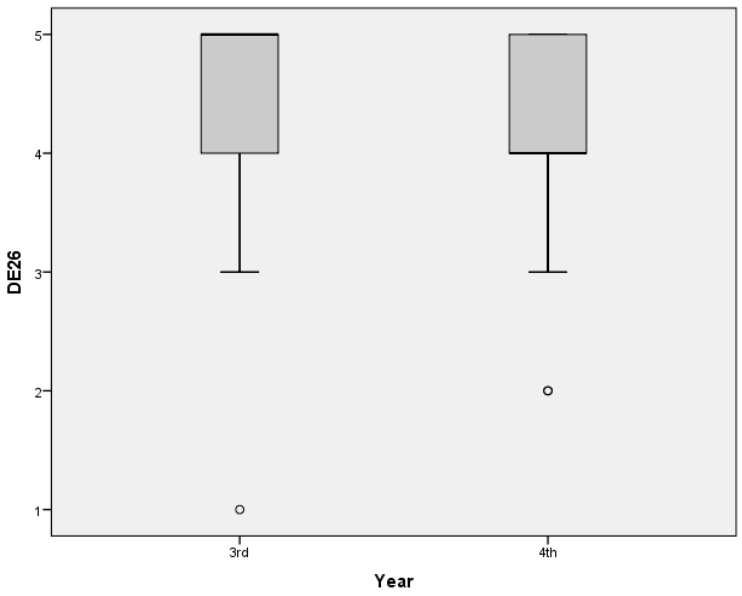


5.16.9 – I find that studying academic topics can be quite exciting at times

Whereas the median response by students in Year 3 was 'agree', that of students in Year 4 was 'somewhat agree'. That is, third-year students tended to more strongly agree with this statement than fourth-year students ($p = .032$). This suggests that third-year students find mathematical topics more exciting than fourth-years.

Year		Disagree	Somewhat Disagree	Neither Agree Nor Disagree	Somewhat Agree	Agree	Total
3	Count	1	0	6	13	30	50
	% within year	2.0%	0.0%	12.0%	26.0%	60.0%	100.0%
4	Count	0	2	2	21	18	43
	% within year	0.0%	4.7%	4.7%	48.8%	41.9%	100.0%
Total	Count	1	2	8	34	48	93
	% within year	1.1%	2.2%	8.6%	36.6%	51.6%	100.0%

Again, this is a surprising outcome. One might assume that, in Year 4, students might be more enthusiastic because this year is non-compulsory and they choose to stay on. However, this is only a difference between agreeing and somewhat agreeing. Furthermore, most students do stay on to the fourth year, something interview participants described as the norm. A number of students expressed a desire to do the MMath because it is what everyone else does.



DE26	Year 3	Year 4
Mean	4.42	4.28
Median	5.00	4.00
Variance	.738	.587
Std. dev	.859	.766
Min.	1	2
Max.	5	5

5.17 – Differences in Individual Items by Gender (Sweep 2)

Item	Scale	Statement	Fisher's Exact Test Value
01	ST	I manage to find conditions for studying which allow me to get on with my work easily.	5.753
02	ST	When working on an assignment, I'm keeping in mind how best to impress the marker.	5.636
03	SU	Often I find myself wondering whether the work I'm doing here is worthwhile.	6.548
04	DE	I often set out to understand for myself the meaning of what we have to learn.	13.288**
05	ST	I organize my study time effectively to make the best use of it.	1.330
06	SU	I find I have to concentrate on just memorizing a good deal of what I have to learn.	3.037
07	ST	I go over the work I've done carefully to check the reasoning and that it makes sense.	6.678
08	SU	Often I feel I'm drowning in the sheer amount of material we're having to cope with.	19.800***
09	DE	I look at the evidence carefully and try to reach my own conclusion about what I'm studying.	6.025
10	ST	It's important for me to feel that I'm doing as well as I really can.	2.222
11	DE	I try to relate ideas I come across to those in other topics whenever possible.	14.585**
12	SU	I tend to read very little beyond what is actually required to pass.	6.072
13	DE	Regularly I find myself thinking about ideas from lectures when I'm doing other things.	20.932***
14	ST	I think I'm quite systematic and organized when it comes to revising for exams.	2.810
15	ST	I look carefully at tutors' comments on problem sheets to see how to get higher marks next time.	1.785
16	SU	There's not much of the work here that I find interesting or relevant.	15.330**
17	DE	When I read lecture notes or a book, I try to find out for myself exactly what the author means.	11.465*
18	ST	I'm pretty good at getting down to work whenever I need to.	2.317
19	SU	Much of what I'm studying makes little sense: it's like unrelated bits and pieces.	28.634***
20	ST	I think about what I want to get out of this course to keep my studying well-focused.	6.275
21	DE	When I'm working on a new topic, I try to see in my own mind how all the ideas all fit together.	15.018**
22	SU	I often worry about whether I'll ever be able to cope with the work properly.	12.226*
23	DE	Often I find myself questioning things I hear in lectures or read in books.	39.714***
24	ST	I feel that I'm getting on well, and this helps me put more effort into work.	4.518
25	SU	I concentrate on learning just those bits of information that I have to know to pass.	4.161
26	DE	I find that studying academic topics can be quite exciting at times.	12.271*
27	ST	I'm good at following up some of the reading suggested by lecturers or tutors.	8.756

28	ST	I keep in mind who is going to make a problem sheet and what they're likely to be looking for.	4.838
29	SU	When I look back, I sometimes wonder why I ever decided to come here.	7.328
30	DE	When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it.	13.488**
31	ST	I work steadily through the term, rather than leave it all until the last minute.	2.398
32	SU	I'm not really sure what's important in lectures, so I try to get down all I can.	13.317**
33	DE	Ideas in course books or lecture notes often set me off on long chains of thought on my own.	28.732***
34	ST	Before starting work on a problem sheet or exam question, I think first how best to tackle it.	10.698*
35	SU	I often seem to panic if I get behind with my work.	9.114
36	DE	When I read, I examine the details carefully to see how they fit in with what's being said.	11.427*
37	ST	I put a lot of effort into studying because I'm determined to do well.	2.626
38	SU	I gear my studying closely to just what seems to be required for problem sheets and exams.	5.556
39	DE	Some of the ideas I come across on the course, I find really gripping.	13.245**
40	ST	I usually plan out my week's work in advance, either on paper or in my head.	7.812
41	ST	I keep an eye open for what lecturers seem to think is important and concentrate on that.	6.420
42	SU	I'm not really interested in my degree, but I have to take it for other reasons.	3.158
43	DE	Before tackling a mathematics question, I first try to work out what lies behind it.	8.631
44	ST	I generally make good use of my time during the day.	6.840
45	SU	I often have trouble in making sense of the things I have to remember.	12.711*
46	DE	I like to play around with ideas of my own even if they don't get me very far.	41.266***
47	ST	When I finish a piece of work, I check it through to see if it really meets the requirements.	6.222
48	SU	Often I lie awake worrying about work I think I won't be able to do.	13.885**
49	DE	It's important for me to be able to follow the argument, or to see the reason behind things.	5.182
50	ST	I don't find it at all difficult to motivate myself.	8.537
51	SU	I like to be told precisely what to do in essays or other assignments.	10.861*
52	DE	I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them.	21.015***

5.18 – Proportional Responses to Individual ASSIST Items

Item	Scale	Statement	Mode[56]	Median	Variance	% Responses, Sweep 2				
						Disagree	Somewhat Disagree	NAND	Somewhat Agree	Agree
01	ST	I manage to find conditions for studying which allow me to get on with my work easily.	5	4	.935	1.3	7.5	10.1	37.7	43.4
02	ST	When working on an assignment, I'm keeping in mind how best to impress the marker.	4	3	1.462	9.2	23.2	18.4	35.5	13.6
03	SU	Often I find myself wondering whether the work I'm doing here is worthwhile.	2/4	3	1.895	20.6	25.9	13.6	25.9	14.0
04	DE	I often set out to understand for myself the meaning of what we have to learn.	4	4	1.124	2.2	15.8	24.6	36.0	21.5
05	ST	I organize my study time effectively to make the best use of it.	4	3	1.070	3.9	19.3	29.8	35.5	11.4
06	SU	I find I have to concentrate on just memorising a good deal of what I have to learn.	4	4	1.140	6.1	14.9	17.5	36.8	24.6
07	ST	I go over the work I've done carefully to check the reasoning and that it makes sense.	4	4	1.167	3.9	16.2	24.1	38.2	17.5
08	SU	Often I feel I'm drowning in the sheer amount of material we're having to cope with.	4	3	1.549	8.3	23.7	22.8	25.9	19.3
09	DE	I look at the evidence carefully and try to reach my own conclusion about what I'm studying.	4	4	1.003	2.2	10.5	14.0	46.9	26.3
10	ST	It's important for me to feel that I'm doing as well as I really can.	5	5	1.752	1.3	2.6	10.5	32.5	53.1
11	DE	I try to relate ideas I come across to those in other topics whenever possible.	4	4	1.019	2.2	11.8	16.7	46.1	23.2
12	SU	I tend to read very little beyond what is actually required to pass.	4	4	1.574	12.3	14.5	18.9	38.2	16.2
13	DE	Regularly I find myself thinking about ideas from lectures when I'm doing other things.	4	4	1.233	4.8	14.5	19.7	40.8	20.2
14	ST	I think I'm quite systematic and organised when it comes to revising for exams.	4	4	1.336	5.3	15.8	16.7	39.5	22.8
15	ST	I look carefully at tutors' comments on problem sheets to see how to get higher marks next time.	4	4	.964	2.2	8.8	19.7	44.3	25.0
16	SU	There's not much of the work here that I find interesting or relevant.	1	2	.928	49.1	34.2	9.6	4.8	2.2
17	DE	When I read lecture notes or a book, I try to find out for myself exactly what the author means.	4	4	.915	1.3	11.4	22.8	44.7	19.7
18	ST	I'm pretty good at getting down to work whenever I need to.	4	4	2.241	3.9	16.7	18.4	39.9	21.1
19	SU	Much of what I'm studying makes little sense: it's like unrelated bits and pieces.	2	2	1.039	36.4	37.7	15.8	7.9	2.2
20	ST	I think about what I want to get out of this course to keep my studying well-focused.	4	3	1.313	11.8	22.4	28.5	29.4	7.9

[56] Where 1= Disagree, 2 = Somewhat Disagree, 3 = Neither Agree Nor Disagree, 4 = Somewhat Agree, 5 = Agree

21	DE	When I'm working on a new topic, I try to see in my own mind how all the ideas all fit together.	4	4	.877	1.8	8.3	14.5	50.4	25.0
22	SU	I often worry about whether I'll ever be able to cope with the work properly.	4	3.5	1.906	14.9	18.0	17.1	27.2	22.8
23	DE	Often I find myself questioning things I hear in lectures or read in books.	4	4	1.122	3.9	14.5	22.4	42.1	17.1
24	ST	I feel that I'm getting on well, and this helps me put more effort into work.	4	4	1.213	4.4	11.4	15.8	42.1	26.3
25	SU	I concentrate on learning just those bits of information that I have to know to pass.	2	3	1.651	14.5	29.4	18.0	25.0	13.2
26	DE	I find that studying academic topics can be quite exciting at times.	5	4	.809	1.8	3.9	8.8	39.5	46.1
27	ST	I'm good at following up some of the reading suggested by lecturers or tutors.	2	3	1.386	15.8	27.6	23.7	26.8	6.1
28	ST	I keep in mind who is going to make a problem sheet and what they're likely to be looking for.	4	3	1.551	17.5	23.2	22.8	27.6	8.8
29	SU	When I look back, I sometimes wonder why I ever decided to come here.	1	2	1.891	49.6	18.0	11.8	11.4	9.2
30	DE	When I'm reading, I stop from time to time to reflect on what I'm trying to learn from it.	4	4	.930	2.2	16.7	30.3	39.9	11.0
31	ST	I work steadily through the term, rather than leave it all until the last minute.	5	4	1.581	6.1	14.5	8.8	32.0	38.6
32	SU	I'm not really sure what's important in lectures, so I try to get down all I can.	4	4	1.396	6.1	11.8	15.8	37.3	28.9
33	DE	Ideas in course books or lecture notes often set me off on long chains of thought on my own.	3	3	1.062	7.5	25.0	33.8	28.1	5.7
34	ST	Before starting work on a problem sheet or exam question, I think first how best to tackle it.	4	4	.946	3.1	8.8	18.0	50.4	19.7
35	SU	I often seem to panic if I get behind with my work.	4	3	1.574	8.3	23.2	18.9	29.8	19.7
36	DE	When I read, I examine the details carefully to see how they fit in with what's being said.	4	4	.776	1.3	10.5	26.3	49.6	12.3
37	ST	I put a lot of effort into studying because I'm determined to do well.	4	4	.938	1.3	7.5	15.8	39.5	36.0
38	SU	I gear my studying closely to just what seems to be required for problem sheets and exams.	4	4	1.269	4.4	15.4	17.5	39.9	22.8
39	DE	Some of the ideas I come across on the course, I find really gripping.	4	4	.762	1.8	3.5	9.2	45.2	40.4
40	ST	I usually plan out my week's work in advance, either on paper or in my head.	4	4	1.988	14.5	16.7	13.2	28.5	27.2
41	ST	I keep an eye open for what lecturers seem to think is important and concentrate on that.	4	4	.944	2.6	11.0	20.2	49.1	17.1
42	SU	I'm not really interested in my degree, but I have to take it for other reasons.	1	1	.733	62.3	26.3	7.0	3.1	1.3
43	DE	Before tackling a mathematics question, I first try to work out what lies behind it.	4	4	.922	3.9	17.5	27.2	45.6	5.7
44	ST	I generally make good use of my time during the day.	4	4	1.285	6.6	19.7	22.4	37.7	13.6
45	SU	I often have trouble in making sense of the things I have to remember.	2	3	1.202	11.0	36.8	24.1	22.4	5.7
46	DE	I like to play around with ideas of my own even if they don't get me very far.	4	4	1.122	3.5	18.0	22.4	41.2	14.9
47	ST	When I finish a piece of work, I check it through to see if it really meets the requirements.	4	3	1.282	10.1	27.2	22.4	33.8	6.6
48	SU	Often I lie awake worrying about work I think I won't be able to do.	1	2	1.837	32.0	23.2	18.0	16.7	10.1
49	DE	It's important for me to be able to follow the argument, or to see the reason behind things.	5	5	.489	4.0	1.3	5.3	34.6	58.3
50	ST	I don't find it at all difficult to motivate myself.	4	4	1.235	7.5	10.5	28.1	37.7	16.2
51	SU	I like to be told precisely what to do in essays or other assignments.	5	4	1.256	3.5	11.0	25.9	28.5	31.1
52	DE	I sometimes get 'hooked' on academic topics and feel I would like to keep on studying them.	5	4	1.001	3.5	3.5	15.4	38.6	39.0

6.1 – AQA C1 January 2006 – Application of MATH Taxonomy

Group A

FKFS	Factual Knowledge & Fact Systems
RUOP	Routine Use of Procedures
COMP	Comprehension

Group B

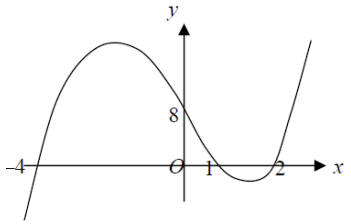
IT	Information Transfer
AINS	Application in New Situations

Group C

J&I	Justifying & Interpreting
ICC	Implications, Conjectures & Comparisons
E	Evaluation

Question		Mark Scheme Answer	MATH	Justification	Marks
QUESTION 1					
a	Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$.	$(\sqrt{5})^2 + 2\sqrt{5} - 2\sqrt{5} - 4 = 1$	RUOP	Rehearsed procedure; familiar type of question	2
b	Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer.	$\sqrt{8} = 2\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$ Answer $= 5\sqrt{2}$	RUOP	Rehearsed procedure; familiar type of question	2
QUESTION 2					
The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$. The line AB has equation $3x + 4y = 7$.					
a i	Show that $k = -2$	$15 + 4k = 7 \Rightarrow 4k = -8 \Rightarrow k = -2$	RUOP	Rehearsed procedure; familiar type of question	1
a ii	Hence find the coordinates of the mid-point of AB.	$\frac{1}{2}(x_1 + x_2)$ or $\frac{1}{2}(y_1 + y_2)$ Midpoint coordinates $\left(3, -\frac{1}{2}\right)$	RUOP	Rehearsed procedure; familiar type of question	2
b	Find the gradient of AB.	Attempt at $\Delta y/\Delta x$ or $y = -\frac{3}{4}x + \frac{7}{4}$ Gradient $AB = -\frac{3}{4}$	RUOP	Rehearsed procedure; familiar type of question	2
The line AC is perpendicular to the line AB.					
c i	Find the gradient of AC.	$m_1 m_2 = -1$ used or stated Hence gradient $AC = \frac{4}{3}$	RUOP	Rehearsed procedure; familiar type of question	2

c ii	Hence find an equation of the line AC.	$y-1=\frac{4}{3}(x-1)$ or $3y=4x-1$ etc.	RUOP	Rehearsed procedure; familiar type of question	1
c iii	Given that the point C lies on the x-axis, find its x-coordinate.	$y=0 \Rightarrow x-1=-\frac{3}{4}$ $x=\frac{1}{4}$	RUOP	Rehearsed procedure; familiar type of question	2
QUESTION 3					
a i	Express x^2-4x+9 in the form $(x-p)^2+q$ where p and q are integers.	$(x-2)^2+5$	RUOP	Rehearsed procedure; familiar type of question	2
a ii	Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y=x^2-4x+9$.	Minimum point $(2,5)$ or $x=2, y=5$	COMP	Understanding equation to be able to state coordinates	2
The line L has equation $y+2x=12$ and the curve C has equation $y=x^2-4x+9$.					
b i	Show that the x-coordinates of the points of intersection of L and C satisfy the equation $x^2-2x-3=0$	$12-2x=x^2-4x+9$ $\Rightarrow x^2-2x-3=0$	RUOP	Rehearsed procedure; familiar type of question	1
b ii	Hence find the coordinates of the points of intersection of $L$and C.	$(x-3)(x+1)=0$ $x=3,-1$ Substitute one value of x to find y. Points are $(3,6)$ and $(-1,14)$	RUOP	Rehearsed procedure; familiar type of question	4
QUESTION 4					
The quadratic equation $x^2+(m+4)x+(4m+1)=0$, where m is a constant, has equal roots.					
a	Show that $m^2-8m+12=0$.	$(m+4)^2=m^2+8m+16$ $b^2-4c=(m+4)^2-4(4m+1)=0$ $m^2+8m+16-16m-4=0$ $\Rightarrow m^2-8m+12=0$	AINS	Non-standard question; only one vaguely similar question in textbook	3
b	Hence find the possible values of m.	$(m-2)(m-6)=0$ $m=2, m=6$	RUOP	Rehearsed procedure; familiar type of question	2
QUESTION 5					
A circle with centre C has equation $x^2+y^2-8x+6y=11$.					

a	By completing the square, express this equation in the form $(x-a)^2+(y-b)^2=r^2$	$(x-4)^2+(y+3)^2$ $(11+6+9-36)$ RHS= 6^2	RUOP	Rehearsed procedure; familiar type of question	3
b i	Write down the coordinates of C.	Centre $(4,-3)$	COMP	Understanding equation to be able to state coordinates	1
b ii	Write down the radius of the circle.	Radius $=6$	COMP	Understanding equation to be able to state radius	1
c i	Find the length of CO.	$CO^2=(-4)^2+3^2$ $CO=5$	RUOP	Rehearsed procedure; familiar type of question	2
c ii	Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer.	Considering CO and radius $CO<r \Rightarrow O$ is **inside** the circle	J&I	Justifying choice of inside/outside	2
QUESTION 6					
The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x)=x^3+x^2-10x+8$					
a i	Using the factor theorem, show that $x-2$ is a factor of $\mathrm{p}(x)$.	$p(2)=8+4-20+8$ $=0, \Rightarrow x-2$ is a factor	RUOP	Substitution, but factor theorem not given	2
a ii	Hence express $\mathrm{p}(x)$ as a product of three linear factors.	Attempt at quadratic factor x^2+3x-4 $p(x)=(x-2)(x+4)(x-1)$	RUOP	Rehearsed procedure; familiar type of question	3
b	Sketch the curve with equation $y=x^3+x^2-10x+8$, showing the coordinates of the points where the curve cuts the axes. (You are not required to calculate the coordinates of the stationary points.)	Gap through $(0,8)$ marked 3 roots marked on x-axis Cubic curve through their 3 points 	RUOP	Use understanding of cubics and basic shapes of graphs; rehearsed	4
QUESTION 7					
The volume, $V\ m^3$, of water in a tank at time t seconds is given by $V=\frac{1}{3}t^6-2t^4+3t^2$ for $t \geq 0$.					

a i	Find $\frac{\mathrm{d}V}{\mathrm{d}t}$	$\frac{\mathrm{d}V}{\mathrm{d}t} = 2t^5 - 8t^3 + 6t$	RUOP	Rehearsed procedure; familiar type of question	3
a ii	Find $\frac{\mathrm{d}^2V}{\mathrm{d}t^2}$	$\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 10t^4 - 24t^2 + 6$	RUOP	Rehearsed procedure; familiar type of question	2
b	Find the rate of change of the volume of water in the tank, in m^3s^{-1}, when $t = 2$.	Substitute $t = 2$ into their $\frac{\mathrm{d}V}{\mathrm{d}t}$ $(= 64 - 64 + 12) = 12$	RUOP	Rehearsed procedure; familiar type of question	2
c i	Verify that V has a stationary value when $t = 1$.	$t = 1 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 2 - 8 + 6$ $= 0 \Rightarrow$ Stationary value	COMP	Substituting into an equation	2
c ii	Determine whether this is a maximum or minimum value.	$t = 1 \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}t^2} = -8$ Maximum value	RUOP	Rehearsed procedure; familiar type of question	2
QUESTION 8					
The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L. The points A and B have coordinates $(-1, 0)$and $(2, 0)$, respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at $C$where $x = 2$.					
a	Find the area of the rectangle ABC.	$y_D = 3 + 1 = 4$ or $y_C = 12 - 8 = 4$ Area $ABCD = 3 \times 4 = 12$	RUOP	Rehearsed procedure; familiar type of question	2
b i	Find $\int(3x^2 - x^3)\mathrm{d}x$	$x^3 - \frac{x^4}{4}(+C)$	RUOP	Rehearsed procedure; familiar type of question	3
b ii	Hence find the area of the shaded region bounded by the curve and the line L.	Sub limits -1 and 2 into their (b) (i) and $[8 - 4] = \left[-1 - \frac{1}{4}\right] = 5\frac{1}{4}$ Shaded area = "their" (rectangle – integral) $= 12 - 5\frac{1}{4} = 6\frac{3}{4}$	RUOP	Rehearsed procedure; familiar type of question	4

For the curve above with equation $y = 3x^2 - x^3$:					
c i	Find $\frac{dy}{dx}$	$\frac{dy}{dx} = 6x - 3x^2$	RUOP	Rehearsed procedure; familiar type of question	2
c ii	Hence find an equation of the tangent at the point on the curve where $x = 1$.	When $x = 1$, $y = 2$ when $x = 1$, $\frac{dy}{dx} = 3$ as 'their' grad of tangent Tangent is $y - 2 = 3(x - 1)$	RUOP	Rehearsed procedure; familiar type of question	3
c iii	Show that y is decreasing when $x^2 - 2x > 0$.	Decreasing when $\frac{dy}{dx} = 6x - 3x^2 < 0$ $3(2x - x^2) < 0$ $\Rightarrow x^2 - 2x > 0$	J&I	Justifying given statement	2
d	Solve the inequality $x^2 - 2x > 0$.	Two critical points 0 and 2 $x > 2$, $x < 0$ ONLY	RUOP	Rehearsed procedure; familiar type of question	2

6.2 – Edexcel FP3 June 2006 – Application of MATH Taxonomy

Group A		Group B		Group C	
FKFS	Factual Knowledge & Fact Systems	IT	Information Transfer	J&I	Justifying & Interpreting
RUOP	Routine Use of Procedures	AINS	Application in New Situations	ICC	Implications, Conjectures & Comparisons
COMP	Comprehension			E	Evaluation

Question	Mark Scheme Answer	MATH	Justification	Mark
QUESTION 1				
$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Prove, by induction, that for all positive integers n, $\mathbf{A}^n = \begin{pmatrix} 1 & n & \frac{1}{2}(n^2+3n) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$	$\mathbf{A}^1 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{A}$ (Hence true for $n = 1$) $\mathbf{A}^{k+1} = \mathbf{A}^k.\mathbf{A} = \begin{pmatrix} 1 & k & \frac{1}{2}(k^2+3k) \\ 0 & 1 & k \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 1 & k+1 & 2+k+\frac{1}{2}(k^2+3k) \\ 0 & 1 & k+1 \\ 0 & 0 & 1 \end{pmatrix}$ $+k+\frac{1}{2}(k^2+3k) = \frac{1}{2}(k^2+5k+4)$ $= \frac{1}{2}(k^2+2k+1+3k+3)$ $= \frac{1}{2}((k+1)^2+3(k+1))$ (Hence, if result is true for $n = k$, then it is true for $n = k+1$.) By induction, implies true for all positive integers.	AINS	Proof by indication can be reduced to a procedure, meaning that there does not need to be any justificatory effects of proving; but this is a new context (matrices)	5

QUESTION 2					
a	Find the Taylor expansion of $\cos 2x$ in ascending powers of $\left(x-\frac{\pi}{4}\right)$ up to and including the term in $\left(x-\frac{\pi}{4}\right)^5$	$f(x)=\cos 2x \quad f\left(\frac{\pi}{4}\right)=0$ $f'(x)=-2\sin 2x \quad f'\left(\frac{\pi}{4}\right)=-2$ $f''(x)=-4\cos 2x \quad f''\left(\frac{\pi}{4}\right)=0$ $f'''(x)=8\sin 2x \quad f'''\left(\frac{\pi}{4}\right)=8$ $f^{(iv)}(x)=16\cos 2x \quad f^{(iv)}\left(\frac{\pi}{4}\right)=0$ $f^{(v)}(x)=-32\sin 2x \quad f^{(v)}\left(\frac{\pi}{4}\right)=-32$ $\cos 2x = f\left(\frac{\pi}{4}\right)+f'\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right)+\frac{f''\left(\frac{\pi}{4}\right)}{2}\left(x-\frac{\pi}{4}\right)^2 + \frac{f'\left(\frac{\pi}{4}\right)}{3!}\left(x-\frac{\pi}{4}\right)^3+\cdots$ Three terms are sufficient to establish method. $\cos 2x = -2\left(x-\frac{\pi}{4}\right)+\frac{4}{3}\left(x-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(x-\frac{\pi}{4}\right)^5+\cdots$	RUOP	Rehearsed procedure; familiar type of question	5
b	Use your answer to (a) to obtain an estimate of cos 2, giving your answer to 6 decimal places.	Substitute $x=1\left(1-\frac{\pi}{4}\approx 0.21460\right)$ $\cos 2=-2\left(1-\frac{\pi}{4}\right)+\frac{4}{3}\left(1-\frac{\pi}{4}\right)^3-\frac{4}{15}\left(1-\frac{\pi}{4}\right)^5+\cdots \approx -0.416147$	AINS	Substitution into a formula	3
QUESTION 3					
a	Use de Moivre's theorem to show that	In this solution $\cos\theta = c$and $\sin\theta = s$.	RUOP	Rehearsed procedure; familiar type of question	5

	$\sin 5\theta = \sin\theta(16\cos^4\theta - 12\cos^2\theta + 1)$	$\cos 5\theta + i\sin 5\theta = (c+is)^5$ $(= c^5 + 5c^4 is + 10c^3(is)^2 10c^2(is)^3 + 5c(is)^4 + (is)^5)$ $\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$ $= 5c^4 s - 10c^2(1-c^2)s$ $+ (1-c^2)^2 s = s(16c^4 - 12c^2 + 1)$ $(s^2 = 1 - c^2)$			
b	Hence, or otherwise, solve, for $0 \le \theta \le \pi$, $\sin 5\theta + \cos\theta \sin 2\theta = 0$	$\sin\theta\,(16\cos^4\theta - 12\cos^2\theta + 1) + 2\cos^2\theta \sin\theta = 0$ $\sin\theta = 0 \Rightarrow \theta = 0$ $16c^4 - 10c^2 + 1 = (8c^2 - 1)(2c^2 - 1) = 0$ $c = \pm\frac{1}{2\sqrt{2}},\ \ c = \pm\frac{1}{\sqrt{2}}$ $\theta \approx 1.21, 1.93; \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$	RUOP	Rehearsed procedure; familiar type of question	6
QUESTION 4					
	$\frac{d^2x}{dt^2} + 3\sin x = 0$ At time $t = 0, x = 0$ and $\frac{dx}{dt} = 0.4$				
a	Use approximations of the form $\left(\frac{dx}{dt}\right)_0 \approx \frac{x_1 - x_0}{h}$ and $\left(\frac{d^2x}{dt^2}\right)_0 \approx \frac{x_1 - 2x_0 + x_{-1}}{h^2}$, with $h = 0.1$ to obtain estimates of x at $t = 0.1$, $t = 0.2$ and $t = 0.3$.	$\left(\frac{dx}{dt}\right)_0 = 0.4 \approx \frac{x_{0.1} - 0}{0.1} \Rightarrow x_{0.1} \approx 0.04$ $\left(\frac{d^2x}{dt^2}\right)_{0.1} = -3\sin x_{0.1} \approx \frac{x_{0.2} - 2x_{0.1} + 0}{0.01}$ $x_{0.2} \approx 0.0788$ $\left(\frac{d^2x}{dt^2}\right)_{0.2} = -3\sin x_{0.2} \approx \frac{x_{0.3} - 2x_{0.2} + x_{0.1}}{0.01}$ $x_{0.3} \approx 0.115$	AINS	Non-rehearsed method	5
b	Find a series solution for x, in	$f''(t) = -3\sin x, \qquad f''(0) = 0$	AINS	Non-rehearsed method	4

	ascending powers of t, up to and including the term in t^3.	$f'''(t) = -3\cos x \frac{dx}{dt}, \quad f'''(0) = -3 \times 0.4 = -1.2$ $f(t) = f(0) + tf'(0) + \frac{t^2}{2}f''(0) + \frac{t^3}{3!}f'''(0) + \cdots$ $= 0.4t - 0.2t^3$			
c	Use your answer to (b) to obtain an estimate of $x$at $t = 0.3$.	Substituting $t = 0.3$ into (b) gives $f(0.3) \approx 0.1146$	COMP	Substituting into a formula	2
QUESTION 5					
The eigenvalues of the matrix $\boldsymbol{M}$, where $\boldsymbol{M} = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$ are λ_1 and λ_2, where $\lambda_1 < \lambda_2$					
a	Find the value of λ_1 and the value of λ_2.	$(4-\lambda)(1-\lambda) + 2 = 0$ $\lambda^2 - 5\lambda + 6 = (\lambda - 3)(\lambda - 2) = 0$ $\lambda_1 = 2, \quad \lambda_2 = 3$	RUOP	Rehearsed procedure; familiar type of question	3
b	Find $\boldsymbol{M}^{-1}$.	$\boldsymbol{M}^{-1} = \frac{1}{6}\begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}$	RUOP	Rehearsed procedure; familiar type of question	2
c	Verify that the eigenvalues of $\boldsymbol{M}^{-1}$ are $\lambda_1^{-1}$and λ_2^{-1}.	$\begin{vmatrix} \frac{1}{6} & \frac{1}{3} \\ -\frac{1}{6} & \frac{2}{3} \end{vmatrix} = -\frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{6} = 0$	COMP	Substitution into a formula	3
A transformation T: $\mathbb{R}^2 \to \mathbb{R}^2$ is represented by the matrix $\boldsymbol{M}$. There are two lines, passing through the origin, each of which is mapped onto itself under the transformation T.					
d	Find Cartesian equations for each of these lines.	Using eigenvalues $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix}$ $4x - 2y = 2x \Rightarrow y = x$ $\begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix}$ $4x - 2y = 3x \Rightarrow y = \frac{1}{2}x$	AINS	Non-rehearsed method; using previously-found information	4

QUESTION 6

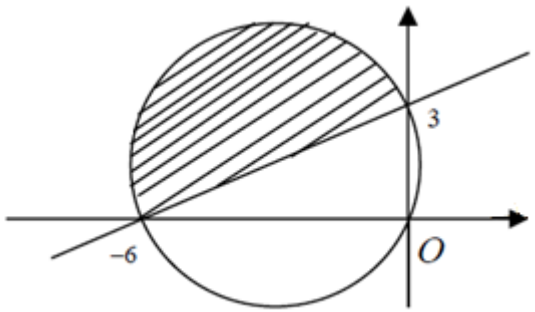
The point P represents a complex number z on an Argand diagram, where

$$|z-6+3i| = 3|z+2-i|$$

a	Show that the locus of P is a circle, giving the coordinates of the centre and the radius of this circle.	Let $z = x + iy$ $(x-6)^2 + (y+3)^2 = 9[(x+2)^2 + (y-1)^2]$ Leading to $8x^2 + 8y^2 + 48x - 24y = 0$ This is a circle; the coefficients of x^2 and y^2 are the same and there is no xy term. $(x^2 + 6x + y^2 - 3y = 0)$ Leading to $(x+3)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{45}{4}$ Centre $\left(-3, \frac{3}{2}\right)$ Radius $\frac{3}{2}\sqrt{5}$	RUOP	Rehearsed procedure; familiar type of question	7

The point Q represents a complex number z on an Argand diagram, where

$$\tan[\arg(z+6)] = \frac{1}{2}$$

b	On the same Argand diagram, sketch the locus of P and the locus of Q.		AINS	Unusual instance of standard question	5
c	On your diagram, shade the region which satisfies both $\|z-6+3i\| > 3\|z+2-i\|$ and $\tan[\arg(x+6)] > \frac{1}{2}$		IT	Representation in graphical form	2

QUESTION 7

The points A, B and C lie on the plane Π_1 and, relative to a fixed origin O, the have position vectors

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} \text{ and } \mathbf{c} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$$

respectively.

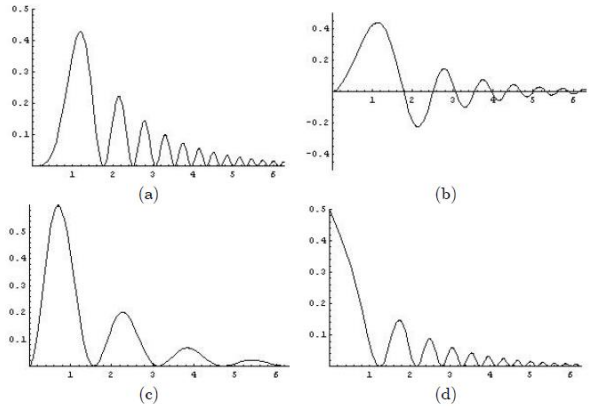
a	Find $(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})$.	$(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&0&-3\\4&-5&-1\end{vmatrix}=-15\mathbf{i}-10\mathbf{j}-10\mathbf{k}$	RUOP	Rehearsed procedure; familiar type of question	4
b	Find an equation for Π_1, giving your answer in the form $\mathbf{r}.\mathbf{n}=\mathbf{p}$.	$\mathbf{r}\cdot(3\mathbf{i}+2\mathbf{j}+2\mathbf{k})=(3\mathbf{i}+2\mathbf{j}+2\mathbf{k})\cdot(\mathbf{i}+3\mathbf{j}-\mathbf{k})$ $\mathbf{r}\cdot(3\mathbf{i}+2\mathbf{j}+2\mathbf{k})=7$	RUOP	Rehearsed procedure; familiar type of question	2
The plane Π_2 has cartesian equation $x+z=3$ and Π_1 and Π_2 intersect in the line l.					
c	Find an equation for l, giving your answer in the form $(\mathbf{r}-\mathbf{p})\times\mathbf{q}=\mathbf{0}$.	Let $x=\lambda,\ z=3-\lambda$ Then $2y=7-3\lambda-2(3-\lambda)\ \Rightarrow y=\frac{1}{2}-\frac{1}{2}\lambda$ The direction of l is any multiple of $(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$ $\left(\mathbf{r}-\left(\frac{1}{2}\mathbf{j}+3\mathbf{k}\right)\right)\times(2\mathbf{i}-\mathbf{j}-2\mathbf{k})=\mathbf{0}$ The general form is $\{\mathbf{r}-[\mathbf{i}+2\mathbf{k}+c_1(2\mathbf{i}-\mathbf{j}-2\mathbf{k})]\}\times c_2(2\mathbf{i}-\mathbf{j}-2\mathbf{k})=\mathbf{0}$	RUOP	Rehearsed procedure; familiar type of question	4
The point P is the point on l that is the nearest to the origin O.					
d	Find the coordinates of P.	$\left(\lambda\mathbf{i}+\left(\frac{1}{2}-\frac{1}{2}\lambda\right)\mathbf{j}+(3-\lambda)\mathbf{k}\right)\cdot(2\mathbf{i}-\mathbf{j}-2\mathbf{k})=\mathbf{0}$ $2\lambda-\frac{1}{2}+\frac{1}{2}\lambda-6+2\lambda=0$ Leading to $\lambda=\frac{13}{9}$ $P:\left(\frac{13}{9},-\frac{2}{9},\frac{14}{9}\right)$	RUOP	Rehearsed procedure; familiar type of question	4

6.3 – Oxford MAT 2007 – Application of MATH Taxonomy

Group A		Group B		Group C	
FKFS	Factual Knowledge & Fact Systems	IT	Information Transfer	J&I	Justifying & Interpreting
RUOP	Routine Use of Procedures	AINS	Application in New Situations	ICC	Implications, Conjectures & Comparisons
COMP	Comprehension			E	Evaluation

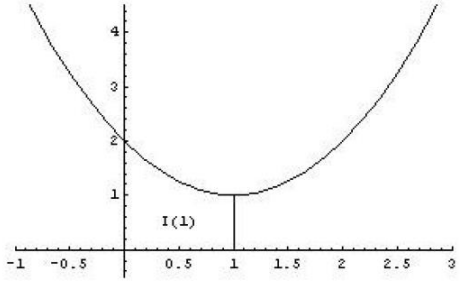
	Question	Mark Scheme Answer	MATH	Justification	Mark
QUESTION 1					
For each part of these questions, you will be given four possible answers, just one of which is correct.					
A	Let r and s be integers. Then $$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$ is an integer if (a) $r + s \leq 0$ (b) $s \leq 0$ (c) $r \leq 0$ (d) $r \geq s$.	Separating out the powers of 2 and 3 we have $$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}} = 2^{(r+s+2r-2s-3r)} \times 3^{(r+s+r-s-2r-4s)}$$ $$= 2^{-s} \times 3^{-4s}$$ which is an integer if $s \leq 0$. **The answer is (b)**.	AINS	Standard procedure but recognising integer not explicit	4
B	The greatest value which the function $$f(x) = (3\sin^2(10x + 11) - 7)^2$$ takes, as x varies over all real values, equals (a) -9 (b) 16 (c) 49 (d) 100	• $\sin(10x + 11)$ takes values between -1 and 1 as x varies; • $\sin^2(10x + 11)$ takes values between 0 and 1 as x varies; • $3\sin^2(10x + 11)$ takes values between 0 and 3 as x varies; • $3\sin^2(10x + 11) - 7$ takes values between -7 and -4 as x varies; • $(3\sin^2(10x + 11) - 7)^2$ takes values between 16 and 49 as x varies. **The answer is (c)**.	AINS	Can also be done algebraically but explicit means not given	4
C	The number of solutions x to the equation	Using the identity $\sin^2 x + \cos^2 x = 1$ we see	AINS	Algebraic	4

	$7\sin x + 2\cos^2 x = 5$ in the range $0 \le x < 2\pi$, is (a) 1 (b) 2 (c) 3 (d) 4	$7\sin x + 2\cos^2 x = 5$ $\Leftrightarrow 2\sin^2 x - 7\sin x + 3 = 0$ $\Leftrightarrow (2\sin^2 x - 1)(\sin x - 3) = 0$ Now $\sin x = 3$ has no solutions, and in the range $0 \le \pi < 2\pi$ we note $\sin x$ takes the value $1/2$ twice(at $\pi/6$ and at $5\pi/6$). **The answer is (b)**.		manipulation but not explicit	
D	The point on the circle $(x-5)^2 + (y-4)^2 = 4$ which is closest to the circle $(x-1)^2 + (y-1)^2 = 1$ is (a) $(3.4, 2.8)$ (b) $(3,4)$ (c) $(5,2)$ (d) $(3.8, 2.4)$	The circle with equation $(x-5)^2 + (y-4)^2 = 4$ has centre $(5,4)$ and radius 2. The circle with equation $(x-1)^2 + (y-1)^2 = 1$ has centre $(1,1)$ and radius 1. The vector from the circle's centre to the second circle's centre is $(-4,-3)$ which has length $\sqrt{(-4)^2 + (-3)^2} = 5$. So the point on the first circle, closest to the second is $(5,4) + \frac{2}{5}(-4,-3) = (5,4) + (-1.6,-1.2) = (3.4, 2.8)$ **The answer is (a)**.	AINS	Familiar algebra but processes not given	4
E	If x and n are integers then $(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$ Is (a) Negative when $n > 5$ and $x < 5$ (b) Negative when n is odd and $x > 5$ (c) Negative when n is a multiple of 3 and $x > 5$ (d) Negative when n is even and $x < 5$.	Let $f_n(x) = (1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$ • If $x = 4$ then $f_n(x) = 0$ and so (a) and (d) are false. • If $n = 6$ then each exponent in $f_n(x)$ is even and so (c) is false. • If $x > 5$ then each bracket is negative, and if n is odd then $f_n(x)$ =(negative)(positive)(negative)(positive)(negative)< 0 **The answer is (b)**.	J&I	Need to assess truth of each option	4
F	The equation	If we set $y = 2^x$ then the equation $8^x + 4 = 4^x + 2^{x+2}$ can be	J&I	Whilst these are	4

	$8^x + 4 = 4^x + 2^{x+2}$ has (a) no real solutions (b) one real solutions (c) two real solutions (d) three real solutions	rewritten as $y^3 + 4 = y^2 + 4y$ $\Leftrightarrow y^3 - y^2 - 4y + 4 = 0$ $\Leftrightarrow (y-1)(y^2-4) = 0$ So $y = 1, 2, -2$ are the possible values for y. But as $y = 2^x > 0$ then only positive values for y will lead to real values for x. Hence $y = 1, 2$ and $x = 0, 1$ are the only possible x-values. **The answer is (c)**.		familiar algebraic techniques, the possibility of solutions requires justification beyond familiar calculation	
G	On which of the axes below is a sketch of the graph $y = 2^{-x}\sin^2(x^2)$  	If $y = 2^{-x}\sin^2(x^2)$ then note that $y > 0$, which discounts (b). Also $y(0) = 0$ which discounts (d). Finally, the points where graph (c) meets the x-axis arise regularly – this is not the case with $y = 2^{-x}\sin^2(x^2)$ where $y = 0$ at $x = \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \ldots$ **The answer is (a)**.	J&I	Must justify option selected based on discounting others	4
H	Given a function $f(x)$, you are told that $\int_0^1 3f(x)\,\mathrm{d}x + \int_1^2 2f(x)\,\mathrm{d}x = 7$ $\int_0^2 f(x)\,\mathrm{d}x + \int_1^2 f(x)\,\mathrm{d}x = 1$ It follows that $\int_0^2 f(x)\ dx$ equals	If we set $A = \int_0^1 f(x)\,\mathrm{d}x\,, \quad B = \int_1^2 f(x)\,\mathrm{d}x$ then we have the equations $3A + 2B = 7 \qquad (A + B) + B = 1$ Solving these simultaneous equations we find $A = 3$ and $B =$	AINS	Creating simultaneous equations but unfamiliar context which doesn't make this the obvious action	4

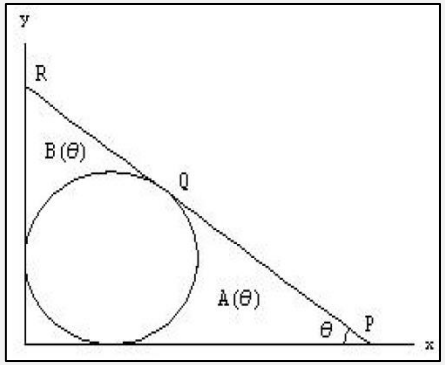
	(a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 2	-1. Hence $$\int_0^2 f(x)\,\mathrm{d}x = A + B = 3 - 1 = 2$$ **The answer is (d)**.			
I	Given that a and b are positive and $$4(\log_{10} a)^2 + (\log_{10} b)^2 = 10$$ then the greatest possible value of a is (a) $\frac{1}{10}$ (b) 1 (c) $\sqrt{10}$ (d) $10^{\sqrt{2}}$	Note that a is largest when $\log_{10} a$ is largest. As $4(\log_{10} a)^2 + (\log_{10} b)^2 = 1$ then $(\log_{10} a)^2$ is largest when $b = 1$ and $\log_{10} b = 0$. So $$4(\log_{10} a)^2 = 1 \implies \log_{10} a = \frac{1}{2} \implies a = \sqrt{10}$$ **The answer is (c)**.	J&I	Noticing the relationship which makes a largest	4
J	The inequality $$(n+1) + (n^4+2) + (n^9+3) + (n^{16}+4) + \cdots + (n^{10000}+100) > k$$ is true for all $n \geq 1$. It follows that (a) $k < 1300$ (b) $k^2 < 101$ (c) $k \geq 101^{10000}$ (d) $k < 5150$	Note that $$(n+1) + (n^4+2) + (n^9+3) + (n^{16}+4) + \cdots + (n^{10000}+100)$$ increases as n increases. So the inequality will hold for all $n \geq 1$ if it holds for $n = 1$. So we need $$(1+1) + (1+2) + (1+3) + (1+4) + \cdots + (1+100) > k$$ $$\Leftrightarrow 2 + 3 + 4 + \cdots + 101 > k$$ $$\Leftrightarrow \frac{100}{2}(2 + 101) > k$$ $$\Leftrightarrow 5150 > k$$ **The answer is (d)**.	ICC	The implications of the increase with n	4
QUESTION 2					
Let $$f_n(x) = (2 + (-2)^n)x^2 + (n+3)x + n^2$$ where n is a positive integer and x is any real number.					
i	Write down $f_3(x)$.	So $$f_3(x) = -6x^2 + 6x + 9 = -6\left(x^2 - x - \frac{3}{2}\right)$$ $$= -6\left(\left(x - \frac{1}{2}\right)^2 - \frac{7}{4}\right) = -6\left(x - \frac{1}{2}\right)^2 + \frac{21}{2}$$	COMP	Substituting into a formula	1

	Find the maximum value of $f_3(x)$.	So the maximum is $\frac{21}{2} = 10.5$ achieved at $x = \frac{1}{2}$	AINS	Using detail of preceding question without being explicitly given what to do	2
	For what values of n does $f_n(x)$ have a maximum value (as x varies)? [Note you are not being asked to calculate the value of this maximum.]	For any n, $f_n(x)$ is a quadratic in x which has a maximum when the lead coefficient is negative. If $2 + (-2)^n < 0$ then n is an odd number greater than 1.	ICC	Implications of what results in the maximum value	3
ii	Write down $f_1(x)$.	Setting $n = 1$ we have $f_1(x) = 4x + 1$.	COMP	Substituting into a formula	1
	Calculate $f_1(f_1(x))$ and $f_1\big(f_1(f_1(x))\big)$.	So $f_1(f_1(x)) = 4(4x + 1) + 1 = 16x + 5$ $f_1\big(f_1(f_1(x))\big) = 4(16x + 5) + 1 = 64x + 21$	COMP	Substituting into a formula	2
	Find an expression, simplified as much as possible, for $f_1\left(f_1\big(f_1(\ldots f_2(x))\big)\right)$ where f_1 is applied k times. [Here k is a positive integer.]	More generally $f_1^k(x) = 4^k x + \left(1 + 4 + \cdots + 4^{k-1}\right) = 4^k x + \frac{4^k - 1}{3}$	ICC	Conjecturing a formula	4
iii	Write down $f_2(x)$.	Setting $n = 2$ we have $f_2(x) = 6x^2 + 5x + 4$.	COMP	Substituting into a formula	1
	The function $f_2\left(f_2\big(f_2(\ldots f_2(x))\big)\right)$ where f_2 is applied k times, is a polynomial in x. What is the degree of this polynomial?	So $f_2^k(x)$ is a polynomial of degree 2^k.	ICC	Implications of previous part, noticing relationship	1

QUESTION 3					
Let $$I(c) = \int_0^1 ((x-c)^2 + c^2 \, \mathrm{d}x$$ where c is a real number.					
i	Sketch $y = (x-1)^2 + 1$ for the values $-1 \leq x \leq 3$ on the axes below and show on your graph the area represented by the integral $I(1)$.		RUOP	Simple graph sketching (plus COMP – understanding meaning of integral)	3
ii	Without explicitly calculating $I(c)$, explain why $I(c) \geq 0$ for any value of c.	As $(x-c^2) + c^2 \geq 0$ for all x then $I(c) \geq 0$.	J&I	Explanation	2
iii	Calculate $I(c)$.	$$I(c) = \int_0^1 ((x-c)^2 + c^2) \; dx = \left[\frac{(x-c)^3}{3}\right]_0^1 + c^2 = \frac{(1-c)^3}{3} + \frac{c^2}{3} + c^2$$ $$= 2c^2 - c + \frac{1}{3}$$	RUOP	Explicit request of a familiar calculation	3
iv	What is the minimum value of $I(c)$ (as c varies)?	Completing the square $$I(c) = 2\left(c^2 - \frac{c}{2} + \frac{1}{6}\right) = 2\left(\left(c - \frac{1}{4}\right)^2 + \frac{5}{48}\right) = 2\left(c - \frac{1}{4}\right)^2 + \frac{5}{24}$$ So the minimum is $\frac{5}{24}$	AINS	Unstated algebraic manipulation	3
v	What is the maximum value of $I(\sin\theta)$ as θ varies?	If c can only vary between ± 1 then the minimum is at $I(-1)$ as -1 is furthest from $1/4$. In this case $$I(-1) = 2\left(\frac{5}{4}\right)^2 + \frac{5}{24} = \frac{50}{16} + \frac{5}{24} = \frac{150+10}{48} = \frac{160}{48} = \frac{10}{3}$$	J&I	Recognition of variation of c	4

QUESTION 4

In the diagram below is sketched the circle with centre $(1,1)$ and radius 1 and a line L. The line L is tangential to the circle at Q; further L meets the y-azis at R and the x-axis at P in such a way that the angle OPQ equals θ where $0 < \theta < \pi/2$.



i	Show that the co-ordinates of Q are $(1+\sin\theta\,,1+\cos\theta)$ and that the gradient of PQR is $-\tan\theta$.	Let $C=(1,1)$ denote the centre of the circle, then CQ makes angle θ with the vertical and is of length 1. So $Q=C+\overrightarrow{CQ}=(1,1)+(\sin\theta,\cos\theta)=(1+\sin\theta\,,1+\cos\theta)$ The gradient of the line is $\frac{-RO}{PO}=-\tan\theta$ by looking at the triangle OPR.	AINS	Standard, unstated trigonometric processes	3
	Write down the equation of the line PQR and so find the co-ordinates of P.	So using the formula $y-y_Q=m(x-x_Q)$ we have $y-1-\cos\theta=-\tan\theta\,(x-1-\sin\theta)$ At P we have $y=0$ and so we have $x=\cot\theta\,(1+\cos\theta)+1+\sin\theta=\frac{\cos\theta+\cos^2\theta+\sin\theta+\sin^2\theta}{\sin\theta}$ $=1+\cot\theta+\csc\theta$	AINS	Recognition of formula to begin	4
ii	The region bounded by the circle, the x-axis and PQ has	If we consider the diagram with $\frac{\pi}{2}-\theta$ as the angle OPR rather than θ, then this is just a reflection of the θ-diagram in the $y=x$ line. Hence, comparing	J&I	Explanation	3

	area $A(\theta)$; the region ounded by the circle, the y-axis and QR has area $B(\theta)$. (See diagram.) Explain why $$A(\theta) = B\left(\frac{\pi}{2} - \theta\right)$$ for any θ.	areas, $$A(\theta) = B\left(\frac{\pi}{2} - \theta\right)$$			
	Calculate $A(\pi/4)$.	So when $\theta = \pi/4$ we have, dividing up the triangle $$A\left(\frac{\pi}{4}\right) + B\left(\frac{\pi}{4}\right) + \frac{3\pi}{4} + 1 = \frac{1}{2}P_{\frac{\pi}{4}}R_{\frac{\pi}{4}}$$ But $A\left(\frac{\pi}{4}\right) = B\left(\frac{\pi}{4}\right)$ and $P_{\frac{\pi}{4}} = R_{\frac{\pi}{4}} = 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$. Hence $$2A\left(\frac{\pi}{4}\right) + \frac{3\pi}{4} + 1 = \frac{1}{2}\left(2 + \sqrt{2}\right)^2 = 3 + 2\sqrt{2}$$ giving $$A\left(\frac{\pi}{4}\right) = 1 + \sqrt{2} - \frac{3\pi}{8}$$	AINS	Substituting into a formula that goes beyond normal manipulation	2
iii	Show that $$A\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3}$$	Let $D = (1, 0)$. When $\theta = \pi/3$ we can calculate $A\left(\frac{\pi}{3}\right)$ as the area of the congruent right-angled triangles DCP and PCQ minus $1/3$ of the circle. So $$A\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2} \times \left(P_{\frac{\pi}{3}} - 1\right) \times 1\right) - \frac{\pi}{3} = \left(1 + \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} - 1\right) - \frac{\pi}{3} = \sqrt{3} - \frac{\pi}{3}$$	J&I	Using given formula in particular way	3
QUESTION 5					
Let $f(n)$ be a function defined, for any integer $n \geq 0$, as follows: $$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ (f(n/2))^2 & \text{if } n > 0 \text{ and } n \text{ is even} \\ 2f(n-1) & \text{if } n > 0 \text{ and } n \text{ is odd} \end{cases}$$					
i	What is the value of $f(5)$?	$$f(5) = 2f(4) = 2\big(f(2)\big)^2 = 2\big((f(1)^2)\big)^2 = 2((2^2)^2) = 32$$	COMP	Substituting into a formula	1
The *recursion depth* of $f(n)$ is defined to be the number of other integers m such that the value of $f(m)$ is calculated whilst computing the value of $f(n)$. For example, the recursion depth of $f(4)$ is 3, because the values of $f(2)$, $f(1)$, and $f(0)$ need to be calculated on the way to computing the value of $f(4)$.					

ii	What is the recursion depth of $f(5)$?	As we had to calculate $f(4), f(2), f(1), f(0)$ on the way then $f(5)$ has recursion depth 4.	COMP	Substituting into a formula	2
Now let $g(n)$ be a function, defined for all integers $n \geq 0$, as follows: $$f(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 + g(n/2) & \text{if } n > 0 \text{ and } n \text{ is even} \\ 1 + g(n-1) & \text{if } n > 0 \text{ and } n \text{ is odd} \end{cases}$$					
iii	What is $g(5)$?	$g(5) = 1 + g(4) = 1 + 1 + g(2) = 1 + 1 + 1 + g(1)$ $= 1 + 1 + 1 + 1 + g(0) = 4$	COMP	Substituting into a formula	1
iv	What is $g(2^k)$, where $k \geq 0$ is an integer? Briefly explain your answer.	For any natural number k $g(2^k) = 1 + g(2^{k-1}) = \cdots = k + g(2^0) = k + g(1) = k + 1$	ICC	Conjecturing based on rigour	3
v	What is $g(2^l + 2^k)$ where $l > k \geq 0$ are integers? Briefly explain your answer.	For natural numbers $l > k \geq 0$ $g(2^l + 2^k) = k + g(2^{l-k} + 1) = k + 1 + g(2^{l-k}) = k + 1 + l - k + 1$ $= l + 2$	ICC	Conjecturing based on rigour	3
vi	Explain briefly why the value of $g(n)$ is equal to the recursion depth of $f(n)$.	In the definition of $g(n)$ a further 1 is added to previously calculated values at each stage whether n is even or odd; as $g(0) = 0$ then $g(n)$ is a measure of the number of previously calculated values, i.e. $g(n)$ is a measure of previously calculated values, i.e. $g(n)$ equals the recursion depth.	IT	Explaining relationships	5

6.4 – Oxford Pure Mathematics I 2008 – Application of MATH Taxonomy

Group A		Group B		Group C	
FKFS	Factual Knowledge & Fact Systems	IT	Information Transfer	J&I	Justifying & Interpreting
RUOP	Routine Use of Procedures	AINS	Application in New Situations	ICC	Implications, Conjectures & Comparisons
COMP	Comprehension			E	Evaluation

		Question	MATH	Justification	Mark
QUESTION 1					
Let V be a finite-dimensional real vector space.					
a		What does it mean to say that $B \subseteq V$ is a basis of V?	FKFS	Definition	1
		What is meant by the dimension of V?	FKFS	Definition	1
		For $S \subseteq V$, what is meant by the span $< S >$ of S?	FKFS	Definition	1
Let S, T be subsets of V, and let $A: V \to V$ be a linear map. Show that:					
	i	$< S >$ is a subspace of V;	J&I	Unseen; proof	2
	ii	$< S \cup T > = < S > + < T >$;	J&I	Unseen; proof	3
	iii	$< A(S) > = A(< S >)$.	J&I	Unseen; proof	3
Let n be a natural number, let $\{e_1, e_2, \dots, e_n\}$ be the standard basis for $\mathbb{R}^n$, and let B be the $n \times n$ matrix $B = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$					
b	i	For each integer k in the range $1 \leq k \leq n$ show that $\dim < e_k, Be_k, B^2e_k, B^3e_k, \dots, B^{n-1}e_k \geq k$	J&I	Unseen; proof	4
Suppose now that $v_1, v_2, \dots, v_n$ are vectors in $\mathbb{R}^n$ such that $\dim < v_k, Bv_k, B^2v_k, B^3v_k, \dots, B^{n-1}v_k > = k$ for $1 \leq k \leq n$					
	ii	Show that $\{v_1, v_2, \dots, v_n\}$ is a basis of $\mathbb{R}^n$.	J&I	Unseen; proof	5
QUESTION 2					
a		Describe three types of elementary row operation which may be performed on a system of simultaneous linear equations.	FKFS	Definition	3

	Applying an elementary row operation to the system $A\mathbf{x} = \mathbf{y}$ changes the system to $EA\mathbf{x} = E\mathbf{y}$ where E is a matrix. For each type of elementary row operation give an example of such a matrix E.	ICC	Unseen; example	3
Let $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the system of linear equations $A\mathbf{x} = \mathbf{b}$ in $\mathbf{x} \in \mathbb{R}^3$ where $$A = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ 3 & 2 & 1 \\ 4 & 1 & 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$				
b	Find conditions which α, β, γ must satisfy for the system $A\mathbf{x} = \mathbf{b}$ to be consistent.	AINS	Unseen; not RUOP	4
	Find the general solution when these conditions in α, β, γ are met.	AINS	Unseen; not RUOP	4
Let A be as in part (b) and let $$B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \\ 4 & 7 & 10 \end{pmatrix}$$				
c	Show that the equation $AX = B$ in the 3×3 matrix X is consistent.	J&I	Unseen; proof	3
	Find a 3×3 *singular* matrix X which solves the system $AX = B$.	AINS	Unseen; not RUOP	3
QUESTION 3				
Let $T: V \to W$ be a linear map between finite-dimensional real vector spaces V and W.				
a	Define the kernel, $\ker T$, and the image, $\mathrm{Im} T$.	FKFS	Definition	3
	Show that $\ker T$ is a subspace of V and $\mathrm{Im}\ T$ is a subspace of W.	FKFS	Lemma 5.11, Linear Algebra I	3
	State the Rank-Nullity Theorem.	FKFS	Definition	1
Throughout the remainder of this question let $$A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 3 & 7 & 0 \\ 1 & 0 & 3 & 2 \end{pmatrix}$$ and let $S: \mathbb{R}^4 \to \mathbb{R}^3$ be the map given by $S\mathbf{v} = A\mathbf{v}$.				
b	Find a basis for the kernel of S.	AINS	Unseen; not RUOP	2
	Find the rank of S and verify the Rank-Nullity Theorem for S.	AINS	Unseen; not RUOP	3
c	Show that there does *not* exist a 4×3 matrix B such that $BA = I_4$.	ICC	Unseen; counterproof	4
	Find all 4×3 matrices C such that $AC = I_3$.	AINS	Unseen; not RUOP	4

[*Here I_n denotes the $n \times n$ identity matrix.*]					
QUESTION 4					
a		Let V be a real finite-dimensional vector space and $T: V \to V$ be a linear map. Let $\mathcal{E}$ be a basis of V. What does it mean to say that A is the matrix of T with respect to $\mathcal{E}$?	FKFS	Definition	2
		Let $S: V \to V$ be a second linear map and let B be the matrix of S with respect to $\mathcal{E}$. Show that BA is the matrix of ST with respect to $\mathcal{E}$.	J&I	Unseen; proof	6
Let V be the space of polynomials in x with real coefficients of degree two or less, and let $\mathcal{E}$ be the basis $1, x, x^2$.					
b		Let $T: V \to V$ be the linear map $(Tp)(x) = p(x+1)$. Write down the matrix A of T with respect to $\mathcal{E}$.	COMP	Unseen; understanding	2
		Let k be a positive integer. What is the map T^k? Write down the matrix for A^k. Show that A^k is not diagonalizable.	J&I	Unseen; proof	7
Let B be a real square matrix and let r be a positive integer. For each of the following statements either prove the statement or provide a counter-example.					
c	i	If B is diagonalizable then B^r is diagonalizable.	ICC	Unseen; counterexample	2
	ii	If B^r is diagonalizable then B is diagonalizable.	ICC	Unseen; counterexample	2
QUESTION 5					
Let S be a set and $R \subseteq S \times S$ is a binary relation on S.					
a		What does it mean to say that R is an equivalence relation?	FKFS	Definition	1
		What is meant by the equivalence class of $x \in S$?	FKFS	Definition	1
		Show that the equivalence classes of R partition S.	FKFS	Theorem 121, Groups & Group Actions	6
Let G be a finite group. We define the relation $\sim$ on G by $h \sim k$ if and only if $h = g^{-1}kg$ for some $g \in G$					
b		Show that $\sim$ is an equivalence relation.	FKFS	Proposition 114, Groups & Group Actions	3
		Show that if $h \sim k$ then the order of h equals the order of k.	J&I	Unseen; proof	3
Now let $G = A_4$ denote the group of even permutations of $\{1, 2, 3, 4\}$.					
c		Show that $(1\ 2\ 3)$ and $(1\ 3\ 2)$ are not $\sim$-equivalent.	J&I	Unseen; proof	6
[*You may assume for any $\sigma \in S_4$ that $\sigma^{-1}(1\ 2\ 3)\sigma = (1\sigma\ 2\sigma\ 3\sigma)$.*]					
QUESTION 6					
a		Let G be a group. What does it mean to say that $H \subseteq G$ is a subgroup of G?	FKFS	Definition	1
Let H be a subgroup of G. The binary relation $\sim$ on G is defined (for $a, b \in G$) by					

$a \sim b$ if and only if $a^{-1}b \in H$				
	Show that $\sim$ is an equivalence relation on G and that the equivalence class of a is the left coset aH.	J&I	Unseen; proof	4
	Show that the map $\phi: H \to aH$ given by $\phi(h) = ah$ is a bijection.	J&I	Unseen; proof	3
Let S be the set $$S = \left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}-1\\0\end{pmatrix}, \begin{pmatrix}0\\-1\end{pmatrix}\right\} \subseteq \mathbb{R}^2$$ and let G denote the set of 2×2 invertible matrices A such that $A(S) = S$, where $A(S) = \{A\mathbf{v} : \mathbf{v} \in S\}$				
b	Show that G is a group under matrix multiplication and write down the eight elements of G.	AINS	Unseen Unseen; not RUOP	3
	Show that $f: G \to \{\pm 1\}$ given by $f(A) = \det A$ is a group homomorphism.	J&I	Unseen; proof	3
	List the cosets of $\ker f$.	AINS	Unseen; not RUOP	3
	Describe briefly, in geometric terms, the difference between the elements of the cosets.	IT	Unseen; explain difference	3
[*You may assume any standard properties of determinants provided they are stated clearly.*]				
QUESTION 7				
a	Let R and S be commutative rings with identity. What does it mean to say that $I \subseteq R$ is an ideal of R?	FKFS	Definition	1
	Let $x \in R$. Show that $$xR = \{xr : r \in R\}$$ is an ideal of R.	J&I	Unseen; proof	4
	Show that every ideal of $\mathbb{Z}$ is of the form $d\mathbb{Z} = \{dn : n \in \mathbb{Z}\}$ for some $d \in \mathbb{Z}$.	J&I	Unseen; proof	3
b	What does it mean to say that $\phi: R \to S$ is a ring homomorphism ?	FKFS	Definition	1
	Let $\mathbb{Z}[x]$ denote the ring of polynomials with integer coefficients. Let $a \in \mathbb{Z}$. Show that the map $$f_a: \mathbb{Z}[x] \to \mathbb{Z} \text{ given by } f_a(p(x)) = p(a)$$ is a ring homomorphism. Show that the kernel of f_a is $(x-a)\mathbb{Z}[x]$. What is the image of f_a?	ICC	Unseen; implications of given information	7
Let $$R_1 = \frac{\mathbb{Z}[x]}{(x-1)\,\mathbb{Z}[x]} \qquad R_2 = \frac{\mathbb{Z}[x]}{(2x-1)\,\mathbb{Z}[x]}$$				
c	Show that R_1 is isomorphic to $\mathbb{Z}$, but that R_2 is not isomorphic to $\mathbb{Z}$.	J&I	Unseen; proof	4
QUESTION 8				
Let ABC be a triangle and let the position vectors of A, B, C be $\mathbf{a}, \mathbf{b}, \mathbf{c}$ with respect to some origin O.				
a	Let P, Q, R be the midpoints of AB, BC, CA. Write down the position vector of P and the position	COMP	Unseen; understanding	4

	vector of a general point on CP.			
	Show that the three lines CP, AQ, BR are concurrent at a point X, whose position vector you should determine.	AINS	Unseen; not RUOP	4
Let N_1, N_2, N_3 be the three points on AB, BC, CA such that CN_1, AN_2, BN_3 are perpendicular to AB, BC, CA respectively.				
b	Show, when we take the origin to be the intersection of AN_2 and BN_3, that $$\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{c}=\mathbf{c}\cdot\mathbf{a}$$	J&I	Unseen; proof	3
	Deduce that OC is perpendicular to AB and that CN_1, AN_2, BN_3 are concurrent at the origin.	J&I	Unseen; proof	2
c	Using the same origin as in part (b) show that the point Y with position vector $$\frac{1}{2}(\mathbf{a}+\mathbf{b}+\mathbf{c})$$ is equidistant from each of A, B, C.	J&I	Unseen; proof	4
	Show that X, O, Y are collinear, where X is as in part (a).	J&I	Unseen; proof	2
	Find the equation of this line when $A=(0,0), B=(4,0), C=(0,3)$ in $\mathbb{R}^2$.	AINS	Unseen; not RUOP	1

6.5 – Oxford Pure Mathematics II 2011 – Application of MATH Taxonomy

Group A		Group B		Group C	
FKFS	Factual Knowledge & Fact Systems	IT	Information Transfer	J&I	Justifying & Interpreting
RUOP	Routine Use of Procedures	AINS	Application in New Situations	ICC	Implications, Conjectures & Comparisons
COMP	Comprehension			E	Evaluation

		Question	MATH	Justification	Mark
QUESTION 1					
Let S be a nonempty subset of $\mathbb{R}$, (x_n) a sequence of real numbers and $\ell \in \mathbb{R}$.					
a	i	Define each of the terms • ℓ is a lower bound for S • ℓ is a maximum for S • ℓ is an infimum for S	FKFS	Definition	3
		Prove that if S has an infimum, then it is unique.	J&I	Unseen; proof	2
	ii	State the Completeness Axiom for $\mathbb{R}$, and the Approximation Property for infima.	FKFS	Definition	2
	iii	Define what it means to say that (x_n) is monotone, and (x_n) converges to ℓ.	FKFS	Definition	2
	iv	Prove that the limit of a convergent sequence is unique.	FKFS	Theorem 6.8, Analysis I	2
		Prove that a bounded monotone decreasing sequence is convergent.	FKFS	Theorem 8.1, Analysis I	2
Let $\alpha > 0$ and let $A = \{x \in (0, \infty) : x^2 \geq \alpha\}$. The sequence (x_n) is defined recursively by $x_1 = 1$ and $$x_{n+1} = \frac{x_n^2 + \alpha}{2x_n}$$ for integers $n \geq 1$.					
b	i	Prove that $x_n \in A$ for all $n \geq 2$, and that $\inf A$ exists and is strictly positive.	J&I	Unseen; proof	2
[You may not use square roots unless you prove that they exist.]					
	ii	Prove that $x_{n+1} \leq x_n$ for all $n \geq 2$.	J&I	Unseen; proof	2
	iii	Prove that $x_n \to \inf A$ as $n \to \infty$.	J&I	Unseen; proof	2

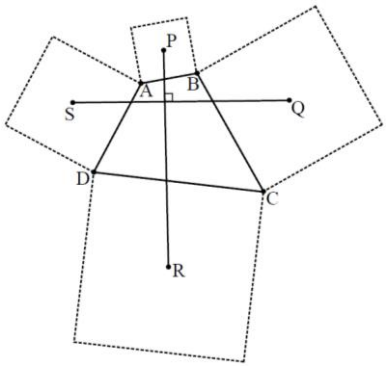
QUESTION 2					
Let (a_n) and (b_n) be two sequences of complex numbers, let S be a subset of $\mathbb{C}$ and $\ell \in \mathbb{C}$.					
a	i	What does it mean to say that (a_n) is a Cauchy sequence?	FKFS	Definition	1
		Prove that if $a_n \to \ell$ then (a_n) is a Cauchy sequence.	FKFS	Theorem 10.1, Analysis I	3
	ii	What does it mean to say that (b_n) is a subsequence of (a_n)?	FKFS	Definition	1
		State, without proof, the Bolzano Weierstrass Theorem for sequences of complex numbers.	FKFS	Definition	1
We say that ℓ is a cluster point for (a_n) if (a_n) has a subsequence that converges to ℓ.					
	iii	Prove that if a bounded sequence (a_n) of complex numbers has precisely one cluster point ℓ, then for each $\varepsilon > 0$ the set $$\{n \in \mathbb{N} : |a_n - \ell| \geq \varepsilon\}$$ is either empty or is finite, and hence deduce that $a_n \to \ell$ as $n \to \infty$.	J&I	Unseen; proof	4
Let $\alpha \in \mathbb{C}$ be a complex number with modulus $\|\alpha\| = 1$, and consider the sequence $$(\alpha^n) = (\alpha, \alpha^2, \dots, \alpha^n, \dots)$$ of powers of α. Let A be the set of cluster points of (α^n). *[In the remainder of this question you may use standard properties of the modulus function and Algebra of Limits without proof, but if you use exponential or logarithmic functions, then you must first define them and prove that they have the required properties.]*					
b	i	Prove that if A has exactly one element, that is, if the sequence (α^n) has exactly one cluster point, then $\alpha = 1$.	J&I	Unseen; proof	4
In the remainder of this question we assume that A has exactly $m > 1$ elements.					
	ii	Show that if $c \in A$, then $c^k \in A$ for all positive integers k.	J&I	Unseen; proof	2
		Deduce that there exists a positive integer j such that $c^j = 1$, and hence that $1 \in A$.	ICC	Unseen; deduction	1
	iii	Show that $\alpha^k \in A$ for all positive integers k, and deduce that $\alpha^m = 1$.	ICC	Unseen; deduction	3
QUESTION 3					
Let (a_n) be a sequence of real numbers.					
a	i	Define what it means to say that $\sum_1^\infty a_n$ is a series, and that the series $\sum_1^\infty a_n$ is convergent with sum $\sum_1^\infty a_n$.	FKFS	Definition	2
		What does it mean to say that the series $\sum_1^\infty a_n$ is absolutely convergent?	FKFS	Definition	1
	ii	State carefully the Ratio Test and the Leibniz Alternating Series Test.	FKFS	Definition	2

	iii	Show, for instance by use of (ii), that the series $$\sum_{n=1}^{\infty}\frac{n!}{n^n} \text{ and } \sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$$ are both convergent. [You may use, without proof, any standard limit.]	J&I	Unseen; proof	8
Let p_n denote the nth prime, so $p_1=2, p_2=3, p_3=5, \ldots$ We consider the series $$\sum_{1}^{\infty}\frac{1}{p_n}$$					
b	i	Assume that the series $\sum_1^\infty \frac{1}{p_n}$ is convergent. Show that then there exists an integer k such that $$\sum_{n=k+1}^{\infty}\frac{1}{p_n}<\frac{1}{2}$$	J&I	Unseen; proof	3
	ii	With the assumption and notation from (i) above we put $Q=p_1p_2\ldots p_k$. Show that for each integer $N\geq 1$, $$\sum_{n=1}^{N}\frac{1}{1+nQ}\leq\sum_{j=1}^{\infty}\left(\sum_{n=k+1}^{\infty}\frac{1}{p_n}\right)^j$$ for instance by considering the possible prime factorisations of $1+nQ$.	J&I	Unseen; proof	3
		Deduce that $\sum_1^\infty \frac{1}{p_n}$ must be divergent.	ICC	Unseen; deduction	1
[Standard facts about prime factorisations and series may be used without proof provided they are clearly stated. You may also use, without proof, the Multinomial Formula stating that for a particular integer n and m real numbers $x_1, \ldots, x_m$, $$(x_1+\cdots+x_m)^n=\sum_{\alpha=(\alpha_1,\ldots,\alpha_m)}c_\alpha x_1^{\alpha_1}x_2^{\alpha_2}\ldots x_m^{\alpha_m}$$ *where the summation is over all m-tuples $\alpha=(\alpha_1,\ldots,\alpha_m)$ of nonnegative integers satisfying $\alpha_1+\cdots+\alpha_m=n$, and the c_α are positive integers.]*					
QUESTION 4					
Let $f\colon[0,1]\to\mathbb{R}$ be a function, x_0 be an element of $[0,1]$ and S be a subset of $[0,1]$.					
a	i	Define the terms • f is continuous at x_0 • f is continuous • f is uniformly continuous on S	FKFS	Definition	3

	ii	Prove that if $f: [0,1] \to \mathbb{R}$ is continuous, then it is uniformly continuous.	FKFS	Theorem 3.2, Analysis II	3
[The Bolzano-Weierstrass Theorem and standard properties of limit points for sets may be used without proof.]					
	iii	Which of the following statements are true? Brief explanations are required. (A) If $f: [0,1] \to \mathbb{R}$ is uniformly continuous on $(0,1]$ and continuous at 0, then it is uniformly continuous on $[0,1]$. (B) $g(x) = \frac{\sin x}{x}$, $x \in (0,1]$, is uniformly continuous on $(0,1]$. (C) $h(x) = \sqrt{x}$, $x \in [0,\infty)$, is uniformly continuous on $[0,\infty)$.	J&I	Unseen; justifying choice	6
Let $\{q_n : n \in \mathbb{N}\}$ be an enumeration of the rational numbers in $(0,1)$. Define $$f_n(x) = \begin{cases} 0 & \text{if } x \in (0, q_n) \\ 2^{-n} & \text{if } x \in [q_n, 1) \end{cases}$$ and $$f(x) = \sum_{n=1}^{\infty} f_n(x), \quad x \in (0,1)$$					
b	i	Show that $0 \leq f(x) \leq 1$ for all $x \in (0,1)$, and that f is strictly increasing.	J&I	Unseen; proof	2
	ii	Show that when $x_0 \in (0,1)$ is rational, then there exists an $\varepsilon > 0$ such that $$f(x) < f(x_0) - \varepsilon$$ For all $x \in (0, x_0)$.	J&I	Unseen; proof	2
		Deduce that f is discontinuous at all the rational numbers in $(0,1)$.	ICC	Unseen; deduction	1
	iii	Show that f is continuous at each irrational $x_0 \in (0,1)$.	J&I	Unseen; proof	3
QUESTION 5					
Let $f: \mathbb{R} \to \mathbb{R}$ be a function and $x_0 \in \mathbb{R}$.					
a	i	Define what it means for f to be differentiable at x_0.	FKFS	Definition	1
		Show that f is differentiable at x_0 if and only if there exist $\alpha \in \mathbb{R}$ and a function $\varepsilon: \mathbb{R} \to \mathbb{R}$ satisfying $\varepsilon(x) \to 0$ as $x \to x_0$ such that $$f(x) = f(x_0) + \alpha(x - x_0) + \varepsilon(x)(x - x_0)$$ for all $x \in \mathbb{R}$.	J&I	Unseen; proof	4
	ii	State and prove the Mean Value Theorem.	FKFS	Definition Theorem 12.5,	1 3

				Analysis II	
[You may use standard results about continuous functions without proof, but if you use Fermat's or Rolle's Theorems you must prove them.]					
	iii	Assume that $f:\mathbb{R}\to\mathbb{R}$ is differentiable. Prove that f is increasing, in the sense that $f(x)\le f(y)$ whenever $x<y$, if and only if $f'(x)\ge 0$ for all $x\in\mathbb{R}$.	FKFS	Corollary 12.10, Analysis II	4
b	i	Assume that $f:\mathbb{R}\to\mathbb{R}$ is differentiable at x_0 with $f'(x_0)>0$. Show that there exists $\delta>0$ such that $f(x)>f(x_0)$ for all $x\in(x_0,x_0+\delta)$	J&I	Unseen; proof	2
	ii	Let $$g(x)=\begin{cases}x+2x^2\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$ Show that g is differentiable and that $g'(0)=1$.	J&I	Unseen; proof	3
[Standard properties of trigonometric functions and of differentiable functions may be used without proof provided they are clearly stated.]					
	iii	Does it follow in (i) that f is increasing on an interval $(x_0,x_0+\varepsilon)$ for some $\varepsilon>0$? An explanation is required.	J&I	Unseen; justifying choice	2
QUESTION 6					
Let $a_n\in\mathbb{C}$, $n\in\mathbb{N}_0$ be complex numbers and consider the power series $\sum_0^\infty a_n x^n$.					
a	i	Define the radius of convergence ρ for power series.	FKFS	Definition	1
		Show that when $\rho>0$ and $\lvert x\rvert<\rho$, then the series $\sum_0^\infty a_n x^n$ converges absolutely.	J&I	Unseen; proof	3
[Standard results about series may be used without proof provided they are clearly stated.]					
	ii	State, without proof, the Weierstrass M-Test.	FKFS	Definition	1
	iii	Assume $\rho>0$. Show that for each $r<\rho$ the power series $\sum_0^\infty a_n x^n$ converges uniformly in $\lvert x\rvert\le r$.	FKFS	Theorem 9.3, Analysis II	2
		Give an example to show that a power series need not converge uniformly in $\lvert x\rvert<\rho$.	ICC	Unseen; example	2
Assume that the radius of convergence for the power series $\sum_0^\infty a_n x^n$ is ρ, where $0<\rho<\infty$. Let $$S=\left\{x\in\mathbb{C}:\lim_{n\to\infty}a_n x^n=0\right\}$$					
b	i	Show that if $\lvert x\rvert<\rho$, then $x\in S$.	J&I	Unseen; proof	2
	ii	Let $x_0\in S$ with $x_0\neq 0$. Show that if $x\in\mathbb{C}$ and $\lvert x\rvert<\lvert x_0\rvert$, then there exists an integer $N\ge 0$ such that $$\lvert a_n x^n\rvert<\left\lvert\frac{x}{x_0}\right\rvert^n$$ For all $n\ge N$.	J&I	Unseen; proof	2

		Deduce that $\sum_0^\infty a_n x^n$ is absolutely convergent.	ICC	Unseen,; deduction	1
	iii	Show that $\rho = \sup\{\|z\| : z \in S\}$.	J&I	Unseen; proof	2
c		Determine the radius of convergence for the power series $\sum_0^\infty nc(n)x^n$, where $c(n)$ is the nth coefficient of the expansion of $\sqrt{2}$ in base 2, that is, $\sqrt{2} = \sum_0^\infty c(n)2^{-n}$ with $c(n)$ equal to 0 or 1.	J&I	Unseen; proof	4
[You may assume that $\sqrt{2}$ is irrational.]					
QUESTION 7					
a	i	Define the integral $$\int_0^1 f(x)\,\mathrm{d}x$$ For a continuous function $f: [0,1] \to \mathbb{R}$.	FKFS	Definition	2
[You can assume without proof that the integral exists, and may use elementary properties of step functions and their integrals.]					
	ii	Prove that if $f, g: [0,1] \to \mathbb{R}$ are continuous and $f(x) \le g(x)$ for all $x \in [0,1]$, then $$\int_0^1 f(x)\,\mathrm{d}x \le \int_0^1 g(x)\,\mathrm{d}x$$	J&I	Unseen; proof	4
	iii	Prove that if $f: [0,1] \to \mathbb{R}$ is continuous and $f(x) \ge 0$ for all $x \in [0,1]$, then $$\int_0^1 f(x)\,\mathrm{d}x = 0$$ implies that $f(x) = 0$ for all $x \in [0,1]$.	J&I	Unseen; proof	4
Let $f: [0,1] \to \mathbb{R}$ be a continuous function, and assume that $$\int_0^1 f(x)\varphi(x)\,\mathrm{d}x = 0$$ for all continuous functions $\varphi: [0,1] \to \mathbb{R}$ with $\varphi(0) = \varphi(1) = 0$.					
b	i	Prove that $f(x) = 0$ for all $x \in [0,1]$.	J&I	Unseen; proof	5
Let $g: [0,1] \to \mathbb{R}$ be a continuous function, and assume that $$\int_0^1 g(x)\varphi'(x)\,\mathrm{d}x = 0$$ for all continuously differentiable functions $\varphi: [0,1] \to \mathbb{R}$ with $\varphi(0) = \varphi(1) = 0$.					
	ii	Prove that $g(x) = c$ for all $x \in [0,1]$, where $c \in \mathbb{R}$ is a constant.	J&I	Unseen; proof	5

[You may use the Fundamental Theorem of Calculus without proof.]					
QUESTION 8					
a	i	Let $z, w \in \mathbb{C} \setminus \{0\}$. Precisely when is it true that $$\lvert z+w\rvert = \lvert z\rvert + \lvert w\rvert$$ holds?	J&I	Unseen; proof	5
	ii	Prove Ptolemy's Theorem: any four distinct points A, B, C, D in the plane satisfy $$\lvert AB\rvert\lvert CD\rvert + \lvert BC\rvert\lvert DA\rvert \geq \lvert AC\rvert\lvert BD\rvert$$ and are cocyclic in the given order if and only if $$\lvert AB\rvert\lvert CD\rvert + \lvert BC\rvert\lvert DA\rvert = \lvert AC\rvert\lvert BD\rvert$$	J&I	Unseen; proof	8
[Note that $(b-a)(d-c)+(c-b)(d-a)=(c-a)(d-b)$ for complex numbers $a, b, c, d \in \mathbb{C}$.]					
b		Let $ABCD$ be a quadrilateral in the plane, see figure below. Prove that the line-segments, PR and QS on the figure, joining the centres of opposite squares, are perpendicular and of equal length. 	J&I	Unseen; proof	7

7.1 – Brian's Story

Gender	Male
Course	MMath Mathematics
Qualifications	A-level
Year	2
ATL	Strategic

Brian's ASSIST results suggest that he has a surface ATL. The comments that he made in the interview were consistent with the definition of a surface ATL, as he reported to be struggling considerably with the subject, disliking it very much and working in ways which permitted him to have some success without revealing what he considered to be low levels of understanding and appreciation of the subject.

It actually made me depressed

Prior Understandings

A former comprehensive state school pupil in his second year of a single honours degree at Oxford, Brian's prior understandings of the subject come from his study of mathematics and further mathematics at A-level. He was near the top of his class in further mathematics, this being one of the reasons that he applied to study it at degree level. Furthermore, he enjoyed studying mathematics, though he claims not to have been "hugely inspired by it". The latter is something he said when he was describing his current mathematical study to show that this feeling had no changed with studying the subject at a higher level. Brian does not enjoy tertiary mathematics, saying that he hates it:

> *I hate it. I hate not understanding what I'm studying and I don't find anything interesting. I kind of can't find it interesting because to do that I'd need to have an understanding of what they're talking about.*

Conversely, he was "good" at school mathematics, keeping up with the lessons and understanding what he was learning. However, Brian describes how, at the secondary level, it is sufficient "to understand how to do the question", rather than "to show any conceptual understanding" in the questions posed in assessment. This appears to contribute towards the roots of his misconception of the nature of mathematics upon entering university, with him "almost feel[ing] cheated" by what he studied at A-level. He believes that what he studied and how he was examined gave him "a false impression of what university maths would be". For that reason, he thinks that further mathematics A-level should contain "something more conceptual, more abstract… Something more like uni maths". Brian suggested that the content of previous modules should be increased, which would leave more "room" for later further mathematics modules to teach other material. Specifically, he thinks that "the idea of an epsilon-delta proof should […] be introduced" so as to "change people's perspectives of maths". Whilst he did learn to prove by induction sums of geometric and arithmetic sums of series at A-level, Brian claims that he found those "easy" after having "practised a couple". Furthermore, the small amount of exposure to proof he had at school meant that he was unaware that he would later on "be relying so much on proof for anything".

As with A-level mathematics revision, Brian prepared for his OxMAT by doing some past papers and marking them in order to give him an idea of the time constraints in the examination, and of how successful he could be in passing the examination. Sitting the OxMAT

“scared the hell out of” him, as he found the examination more difficult than the practice ones which he had attempted. Brian was unable to answer all of the questions – something which he was not used to from school experiences of mathematics examinations. On reflection, he thinks that his performance was perhaps average, although his initial recognition that the OxMAT questions vary considerably year-to-year and deduction that success in past examinations might not necessarily equate with success in the examination that he sat, suggests that expectations of success were based on past performance in similar examinations. Indeed, A-level mathematics and further mathematics revision merely consisted of doing past papers, and success in those was sufficient to expect success in his final examinations.

Brian contrasted the nature of the mathematics and demands of the questions in the OxMAT and A-levels by saying that “the method isn’t explicit in the question” for the entrance examination and that, despite the mathematics itself not being difficult or particularly advanced, the questions were “cleverly cloaked in context”. For that reason, he thought that the OxMAT questions were a “good representation of a problem sheet”. So this experience of the OxMAT gave him an impression of what problem sheets would be like to tackle, adjusting his prior understanding and expectations. His claim that “the ability to adapt to problems like that is important” suggests that his struggles with the subject once he arrived at Oxford were related to his being unable to do that himself.

After doing the OxMAT, Brian was invited to interview three times – twice at the same college. Having prepared for the interview with a practice interview with his teacher, and one at a nearby private school, he felt that the interviews met with his expectations of university mathematics at the time. That is, “interestingly posed questions with doable maths underneath”, something which contrasts with how he reported his experiences of the subject in his first and second year of study at Oxford. His interviews at Osiris Hall both went well, although he did find that working through problems aloud with the interviewer felt “artificial”; however, his teacher had warned him before he went to interview about this, saying that “the way you respond to hints was something they look out for”. Whilst his interview at St. Sophia’s College “went terribly”, he was not fazed because he had already decided that he wanted to go to Osiris Hall. Brian found the interviews useful, on reflection, as he later realised that “it’s important not to be afraid to share whatever pops into your head, even if it’s wrong” and not to worry about looking “silly in front of everyone”, an interesting observation given this is now what he claims to fear and actively avoid during tutorials.

Further to his comments regarding feeling that A-level mathematics and further mathematics did him a disservice in terms of shaping his expectations of undergraduate study, Brian did not do anything in the way of research or preparation for his degree after being offered a place, other than briefly looking at the Mathematical Institute website, because he “didn’t really see the point”. Following on from his comments regarding the absence of abstract mathematics and proof at A-level, his initial experiences of analysis were “just awful”. He found the “change in the type of maths” very difficult, and this was compounded by him feeling that he “was being left behind whilst everyone else was adapting”. He was not alone, as “Students starting out in advanced mathematics have great difficulty with proof before they attain familiarity with the workings of the mathematical culture” (Tall, 1991d, p. 19). Despite his tutor

attempting to help the students bridge the gap between secondary and tertiary mathematics at the beginning of his first year, Brian felt that "everyone else seemed to understand it a lot quicker" than him, meaning that he started to "hide" in tutorials and try to make it through them without appearing to lack understanding, as he perceived himself to. He was embarrassed about not being able to understand as much as he thought he should due to the fact that he passed the interview and admissions test and gained a place at Oxford:

> *I'd just got into Oxford so it would have looked ridiculous if I told someone I didn't understand anything.*

Brian believes that he never overcame a feeling of being left behind, although does try to make the most of tutorials. Whilst he believes that his fellow tutees are "developing their conceptual understanding" in the tutorials, he uses them as a means of finding out "what the answers were to the problem sheets". In his first-year examinations, he was awarded a 2:2, although he believes that this was because he "was pretty lucky with some of them and happened to know the right stuff".

Social Interactions

Whilst Brian freely spoke of struggling with undergraduate mathematics, disliking it and making an effort to "hide" in tutorials, he also remarked that he actively diverts attention away from his perceived lack of understanding in these learning situations by asking lots of questions himself, so that he can prevent himself from being asked to answer any himself. His tutor is not aware of his struggles, which means that one of the useful aspects of the tutorial in monitoring students' progress and understanding outside of formal assessment is compromised by Brian's actions. His friends know that he is struggling, but he has not told his tutor about this because he does not "want to get thrown out", and he believes that his tutor would be "horrified" if he knew how little Brian knows. Furthermore, given his marks on problem sheets have been satisfactory, he fears that his tutor would accuse him of being "a cheat" if his lack of understanding was exposed.

This would not be an unreasonable conclusion for his tutor to make, given Brian's main social interactions with his fellow mathematics undergraduates at his college are when he seeks help from them on problem sheets. The majority of his spare time is spent with friends he has made through sport or non-mathematicians, and he does not actively engage in the mathematics communities of practice at the Mathematical Institute, such as the MURC[57] or the Invariants. Brian is not a member of the Invariants because he feels that he does not understand enough mathematics to actively participate. Furthermore, he describes its members as "not exactly my kind of people" – he cannot relate to the fact that they are enthusiastic about mathematics as he no longer has "such a good relationship with it" any more.

[57] The Mathematics Undergraduate Representative Committee – a student group which acts as a medium for communication between current undergraduates and the governing body of the Mathematical Institute

Activity Structures

Seeking assistance from other mathematicians at his college in order to answer the questions on his problem sheets is a last resort for Brian, before which he attempts to answer the questions of his own accord using his lecture notes. However, he claims that he "normally can't do many, if any" without needing help. Typically, his friends try to help him understand, but this normally is not successful which results in Brian "copying what they've done". What he achieves through doing this is, he believes, sufficient that he gets marks and answers which do not flag him up to his tutor as a student having significant problems. The times that he does understand what his friends have done to answer a question, he often believes that he would not have been able to come up with the answer himself. He claims that there are things which are "completely beyond my capabilities, even though I can see what's going on". An exception to this is when confronted by a proof which is "similar to one in a book", as he is then sometimes able to "just change the numbers or whatever", which can yield the correct answer.

The only studying that Brian does outside of lectures and tutorials is completing problem sheets because he does not have enough time to be able to do more than that. His time is constrained by playing rugby and working in a part-time job. However, even with enough time to spare, Brian postulates that, if "I started trying to properly go back and understand everything from the beginning [of the first year], I think I'd never leave my room". This is compounded by the fact that he goes to analysis and calculus lectures with the preconception that he will not learn anything because he will not understand what is said.

Unlike using past papers as a means of revision on their own at school, Brian now uses them initially as an aid and means of identifying what he should revise and learn in order to pass examinations:

> *I looked at past papers first to see what kind of thing we had to do, and then I learned all of the definitions and proofs. Then I also looked at problem sheets to see if anything was the same in those and the exams, and learned the answers to the ones which were the same or similar.*

His method of 'learning' these definitions and proofs is to write them out repeatedly until he is able to remember them, something he describes as being both time-consuming and dull. Similarly, utilising this technique with problem sheet answers, he also endeavours to understand the answers, testing himself on answering the questions without assistance once he has tried to 'learn' the answers. This attempt at understanding the mathematics in the answers is something which Brian feels sets secondary and tertiary revision apart for him since, at university, he believes that one must "worry about understanding what's happening in examples or existing proofs and then show that you understand it. There's not a great deal that you have to do that you can practise", unlike at A-level.

His method of revision, which involves "a lot of writing things out", arose out of his belief that this was the logical choice based on the types of questions posed in university examinations. Furthermore, he believes that his peers do the same as he does; however, he thinks that they are able to understand the mathematics involved faster and better than he can. Brian thinks

that his revision technique was fairly successful last year because he "could state definitions and theorems fine", but occasionally "misremembered" parts of proofs, which he thinks lost him marks in the examinations. Most importantly, he describes himself as being unable to do "questions which you had to do from scratch and you hadn't seen before". When faced with those questions, he either did nothing or "kind of define[d] any terms" in the question, possibly followed by "a couple of lines of pointless drivel". This means of revision and response to questions is synonymous with a surface approach to learning, where the learner does not or cannot seek to understand the meaning of what they learn, and as such memorises facts in a bid to ask questions which explicitly ask that they are stated.

Conventions & Artefacts

Such questions exist in undergraduate examinations, as Brian describes some of the standard questions which are posed as being those which ask the candidate to state a definition or proof that they have already seen in lecture notes. The questions are all sub-divided into smaller parts, and it is the "more involved questions which ask you to prove something yourself" which he struggles with the most. Describing the revision practices of his peers and the types of questions posed in examinations, it very much seems that this revision technique is an accepted way of studying in this particular culture. The nature of the end-of-year examination in shaping the practices of students is evident through Brian's description of his revision techniques – the types of questions posed have affected the way in which Brian studies, and his belief that he is incapable of answering certain questions at all has affected his self-belief and confidence. This also applies to the nature of problem sheets as, on a more regular basis, the nature of the questions posed also affect his social interactions with other members of the mathematics community of practice at Oxford – feeling incapable of answering the questions properly means that his main interaction with his peers in a study situation is through seeking their help to answer them and his contribution towards discussion in tutorials is minimal.

Descriptions he gives of tutorials suggest that he believes that the point of undergraduate tutorials is to foster the development of the students' conceptual understanding of the mathematics, and that he is not able to utilise this artefact in the way in which it was intended because he does not feel that he can actively participate in the tutorial out of fear that he will appear incapable and, consequently, fears that he could face rejection as a result of this transpiring. Being this position and experiencing these feelings, and the feeling that he does not understand, mean that he is unable to enjoy or understand lectures, which therefore creates a self-perpetuating cycle if he is unable to break out of it through finding some enjoyment in what he is studying, or beginning to strengthen his understanding of the mathematics. For Brian, an understanding and an enjoyment of mathematics are very much related, although his feelings that he is "beyond help" mean that the chances of anything changing in this respect are small. Brian only intends on doing the BA, rather than carrying on to study the fourth year for an MMath, because he does not want "to spend any more time doing something I hate and potentially making my grades even worse by doing so", as he believes that the mathematics will increase in difficulty, and his ability to be successful in assessment on it will decrease, in the fourth year.

Perceptions of Self & Others

Brian's self-belief and perception of himself as a mathematician appears to have stemmed from the change in environment from secondary to tertiary study and, consequently, his ability to enjoy and understand the subject. He found the OxMAT stressful because he found it more difficult than the past papers that he tried, something which he was not used to. Studying mathematics that he was not used to led him to struggle to keep up, being so unfamiliar with the feeling of not understanding mathematics and the perception of others that he must be capable because he gained a place at Oxford, that he shied away from situations where his understanding could truly be tested without him being able to copy others or find means of answering questions correctly without actually understanding the mathematics involved.

He perceives many of his peers to be more successful in assessment and have a better understanding of mathematical concepts, describing other students as being more able to "legitimately" write proofs in examinations. That is, they are able to use their understanding of the mathematical concepts to write a proof themselves, whereas he believes himself only capable of memorising and reproducing ones that he has seen before. Brian says that this act of memorising through repeatedly writing out his notes has "actually made me depressed", a negative emotion often synonymous with a student adopting a surface approach to learning mathematics.

7.2 – Camilla's Story

Gender	Female
Course	Mathematics & Philosophy
Qualifications	A-levels
Year	3
ATL	Strategic

As a third year student of the joint honours degree with philosophy, Camilla has decided that she will specialise in her final year in philosophy and cease studying mathematics at the end of this year. She believes herself to be more conscientious than her peers, and works hard at both the mathematics and philosophy aspects of her degree, whilst also balancing being part of her college JCR alongside her studies.

> ***in maths, all I seem to be doing is memorising a series of proofs and things, you know, on each topic, and then in the exam just reproducing it. Whereas… In philosophy you're thinking and showing your personal thinking and understanding and I much prefer that. It's much more interesting.***

Prior Understandings

Choosing to study mathematics at university because she was good at it at school, enjoyed it and believes that a mathematics degree would give her good job prospects, Camilla did very little revision other than 3-4 past papers because she felt that she "sort of knew everything" and did not think that there was "much to be gained from doing any proper revision". However, she recognises some limitations in A-level mathematics and further mathematics, having studied the subject at degree level, suggesting that an 'introduction to proof' module be constructed which could concentrate on "the more abstract stuff" in order to better prepare students for undergraduate mathematics study.

With her previous experience of A-level mathematics and further mathematics being that mathematics was very much centred around computations, Camilla found the OxMAT fun because she "had to solve actual problems, rather than answer questions like at A-level". She enjoyed "being challenged properly", recognising that the admissions test acts as a way for Oxford admissions tutors to find people with "the right kind of mathematical mind", making a distinction between the ability to solve problems and the ability to answer a standard mathematics question.

Whilst Camilla did not think that the OxMAT was useful for her before she went to Oxford, she likened the interview to a tutorial and the residential nature of the interview – staying overnight in a college room and having a college dinner – as being very useful insights into what undergraduate study might be like at Oxford. She likened the mathematics examined during the interviews to "those maths challenge things", the UKMT challenges, as they act as a means of the admissions tutors seeing how the students think. Whilst she had prepared for the interview by doing a practice one at a local private school[58], she did not think that it was useful other than it put her "in a semi-stressful situation" because the question that she was asked in the practice interview was completely unlike those which she was asked in the actual interviews. Camilla found the questions that she was asked during the interviews to be very challenging, reporting that her interview at Scone College in particular "was really hard core". She was given guidance during her interview at Cardinal's Hall towards the answer after she

[58] Camilla was a pupil at a state comprehensive.

was unable to answer the question on her own, which she found “encouraging”. However, her interview at Chaucer College was the opposite in that she “couldn’t do it at all” and the interview “didn’t really go to plan”.

Camilla did not research what her mathematics degree would involve before she came; however, she was “more concerned with the philosophy side” and was given some pre-reading for her course. This meant that being introduced to analysis was “a shock to the system” as it was “totally alien” – in fact, epsilon-delta proofs actually “frightened” her initially. However, in spite of this, she thought that “analysis was really cool” and she “was really excited about it”, even though it was “way too hard”. During the secondary-tertiary mathematics transition, Camilla found time management to be one of the most difficult aspects of her studies, which was compounded by the problems that she had in understanding what was going on “just from lectures”, although this was something that she found that she got used to with time. Her transition between secondary and tertiary mathematics was facilitated by the help of her tutor, who assisted her in getting “used to the idea of formal proofs and how they work”, and the way in which he had her and her fellow tutees work together on some proofs during the tutorials was “definitely really helpful” for her.

The adjustments that she has been making throughout her degree, she thinks, are reflected by the increase in her end-of-year examination results, going from a 55 in her first year to 64 in her second year. She has learned how long it will take “to achieve certain revision goals”, having expected in her first year to know and understand what she needed to for her examinations quicker than she did. Camilla’s time management for her revision was also affected by the fact that she did not plan on memorising so much material, and so in her second year she “started with memorising a bit earlier on to make sure that” she “had some knowledge there in the bank”.

Activity Structures

Camilla describes her revision approach as being the opposite to that of her friend Jack. Whilst Jack’s revision process began with him going through his lecture notes and attempting problem sheets and past papers before he contemplated memorising any of the mathematical material, she began with the memorisation process after having written her lecture notes, which left her little time to test her understanding. On reflection, she has decided that it is “better to understand what you’re memorising” and so will change her revision practices this year as a consequence. Camilla maintains that doing past papers is the best way to revise, and wishes that there were more practice questions available so that she could develop her understanding further that way. She has been very strategic in the way in which she writes her revision notes, missing out large parts of the material she has covered in lectures because it is deemed non-examinable, saying that she works to ensure “that I know as much as I can so that I can do the best as I can in the exam, so of course I’m not going to revise” non-examinable material. An unexpected repercussion of this was that, last year, one of her examination papers contained a question about what she thought was non-examinable material, which meant that she was unable to answer the question. Camilla was also relying on a particular type of question to come up, based on previous papers, but it did not. She memorises “everything, really”, and does this by “writing it out again and again”.

Unlike her peers, Camilla works on her problem sheets as soon as she is given them because this means that she has plenty of time to work on them if she finds them very difficult. If she cannot answer a question, she first consults books and the internet, before then asking her friend Grace, another mathematics student at her college, how she answered the question. If Grace is unable to help her, then Camilla attempts to answer the question herself and then goes to classes to find out how to answer the question properly.

Conventions & Artefacts

Camilla does not like classes as much as she did tutorials because "they're less personal" and, as such, means that she cannot command as much attention during them. She admits that she used to dominate discussions in tutorials so that she could make the most of them, whereas in larger classes this is not possible. Furthermore, she does not find them to be as "efficient" as tutorials because a greater number of students in attendance means that a greater amount of time is spent addressing each question in turn as there is normally one person in the group who struggles with each question, which ultimately slows the class down. However, Camilla acknowledges that she is "often the person everyone else would find annoying by being the only person who doesn't understand!"

The conventions that Camilla identified in the past examinations for her degree guided her revision, such as with her ignoring chunks of her lecture notes when revising because it would be non-examinable, and revising certain material because she was expecting a particular question to be posed about it, based on patterns in previous papers. Her method of revision – specifically, the reliance on memorisation – is something which she describes as being commonplace amongst undergraduate mathematicians, and as something which is successful, claiming that "There's not really another way" of revising such things.

The structure of her degree and the way in which it permits joint honours students to specialise, if they wish, in one of their two subjects in the final year of their degree is something which Camilla is going to take full advantage of. As with her experiences of chemistry A-level, which she dropped after AS because it over-relied on rote learning, she is choosing to only study philosophy in her fourth year. She finds mathematics to involve too much memorising and reproduction of someone else's work, whereas, in philosophy, she is assessed on her "personal thinking and understanding", which she much prefers.

Perceptions of Self & Others

Camilla regularly compared herself to her peers, describing herself as being more conscientious than many of them. She thinks that they "seem to let their work get on top of them so they end up doing a lot of work last minute", whereas she likes to be able to work consistently to allow herself the time to complete work that she finds difficult before the deadline. She thinks that her method of revision is the same as everyone else's at Cardinal's Hall, although identifies herself as being different to her friend Jack in the order in which she does each part of her revision – understanding, memorising and practising.

Failing to be able to achieve full marks on her work at university, when she was used to doing that at school in mathematics, makes her feel "very sad", although this is not something which

she was unfamiliar with, as she sometimes struggled with her chemistry AS-level. However, whilst she is aware of the struggles that she has with performing as well as she would like to in undergraduate mathematics, it appears that the reason that she has chosen to specialise in philosophy in her fourth year is not because she thinks that she will be more successful, but because she finds it more interesting and rewarding, and because she wants to get away from the unfulfilling working practices that she has developed in mathematics.

Social Interactions

Camilla has not involved herself greatly in the undergraduate mathematics community of practice at Oxford, sticking to working only with her college friends on her mathematics, and not engaging with other mathematicians in her department. She is "not that bothered" about belonging to a group such as the Invariants, particularly because the nature of her joint honours degree means that "going to extra maths stuff isn't really on my list of things to do", though she has been to some special interest seminars in philosophy.

Camilla has involved herself in college life, becoming an active member of her college JCR committee, and has many friends at her college who do not do mathematics. An insight into the reason for her and other undergraduate mathematicians at Oxford failing to interact with many other mathematicians outside of their college is suggested by Camilla:

> *It's just not very practical or possible for us to know a great number of other mathmos[59] because when you're in a lecture, you're only there for the lecture and then you go. The social side you get comes from any interaction you have with other students and that's only really going to happen in tutorials, first off, and then maybe later on in classes.*

Social interaction when working towards learning and understanding mathematics does appear to be something which Camilla valued during the first two years of her degree, as she now misses having tutorials and the benefits that they brought her understanding.

[59] This is a colloquial term for a mathematics undergraduate.

7.3 – Christina's Story

Gender	Female
Course	Mathematics & Philosophy
Qualifications	A-level
Year	4
ATL	Deep

Christina is a finalist studying the joint honours degree with philosophy. She is heavily involved in the Invariants and enjoys mathematics, and has applied to study a doctorate in the subject after she has finished at Oxford.

> ***I looked on the Maths Institute website and it listed all of the modules, which sounded interesting. I remember being excited about it because I had no idea what it meant, which meant that eventually I would understand a lot of really hard-sounding, interesting stuff!***

Prior Understandings

A former state comprehensive pupil, Christina's enjoyment of the subject and the fact that she was good at it led to her decision to study it at university. Unlike many of her peers, she had exposure to the subject outside of the normal school syllabus; for example, she went to a summer school when she was in year 8 which spurred on her interest in mathematics. It was this interest which led to her researching the degree on the Mathematical Institute website before she went to Oxford, which itself increased her interest and excitement in the subject further after seeing the types of mathematics that she would study. In fact, not understanding what she was reading spurred her interest because she found exciting the prospect that she would eventually understand something so complicated.

School mathematics was not something she found complicated, reaching a point during revision where she did so many past papers that "you can pretty much do it in your sleep", also being able to do one in half the maximum allowed time. Acknowledging that A-level mathematics and further mathematics are "very focussed on the exam" and that it has to be accessible to everyone taking it and not just prospective mathematics undergraduates, Christina nonetheless feels that "you don't know what maths is just from doing A-levels". This is because it does not rely on proof in the same way as undergraduate mathematics. Furthermore, she says that the proofs by induction that she did at A-level were not popular with her and her classmates: "everyone pretty much hated them". At school, she thinks that most students are given the impression that mathematics is "more about calculating something and finding an answer to an explicit question", whereas it is in actual fact "more about proving things".

This notion that undergraduate mathematics might not be the same as school mathematics was something which was highlighted to her during her Oxford interview, when she found that the admissions tutors were more interested in her "understanding, the way of thinking". Whilst many students find and found the interviews "scary", and she had heard "many horror stories about tough Oxford interviews", hers at Osiris Hall "was fine". Whilst her practice interview at school was not helpful as it was more 'chatty' than her interview turned out to be, she performed well in an interview which was similar to an Oxford tutorial – she was given some questions to attempt the night before, and asked to bring her answers to the interview.

This, combined with the experience of staying in a college room whilst she was at Oxford, gave Christina the opportunity to experience how Oxford life might be.

Christina believes that there is a link between the OxMAT and the Oxford interviews in the sense that both methods attempt to find students who are good problem-solvers. She found the OxMAT questions to be "a breath of fresh air", compared to the questions that she was asked to answer at A-level, and had practised some of the past questions before her actual examination.

Her initial experience of undergraduate study was harder than she had expected, although she did not find it as "upsetting" as she believes that many of her peers did. This is something Christina attributes to the fact that many of her peers were used to finding mathematics easy when they were at school and still expecting to be the best when they got to Oxford, only to find themselves wrong. This is something she believes to be "a bit stupid", though Christina herself has performed well throughout her degree and was awarded with first class honours for the first three years of her degree. That is not to say that she did not find the transition to tertiary study difficult, partially due to how fast-paced it was – "You do in an hour what you did in A-level in six months!" – though she found that her enthusiasm and interest in the subject was uplifted when she was struggling after she joined the Invariants in her first year.

Social Interactions

Christina is actually a committee member of the Invariants, spending a lot of her time with other mathematicians that she has met through the society. It is these friends and other mathematicians at her college that she sometimes discusses problem sheets with if and when she gets stuck on some questions.

Conventions & Artefacts

Now in her fourth year, Christina does not have tutorials but instead has classes. Whilst she is still offered this support and is given problem sheets to do for her mathematics modules, there have been times recently when she has not handed in her work either at all or by the deadline for classes because there are "no bad consequences" if she does that, and she does not find the marking very helpful. She finds that classes have limited utility compared to tutorials because of the larger number of students in them than in tutorials:

> *if everyone has understood a question then you just skip it [in a tutorial], but in a class with 8-10 people you can't do that because there's inevitably going to be one person who didn't understand. It means that you can't talk about random cool stuff as much*

This is something that she can explore as a member of the Invariants, and her enjoyment of mathematics over philosophy – the other half of her joint honours – means that she has chosen, in her final year, to specialise in mathematics and no longer study philosophy. This ability to specialise is something which Christina thinks works to all of the students' advantages because it means that students can choose courses which they are more interested in, which makes their studies more enjoyable. Furthermore, this has meant that she can study computer science alongside her mathematics.

Computer science is something which she became interested in recently, and is something that she is considering pursuing further in a doctorate after she has finished her degree. Assessment in computer science is slightly different to that in her mathematics courses – students are given 'take-home exams' to do over the holidays as opposed to weekly problem sheets. She describes the questions in her mathematics examinations as following a particular pattern:

> *The questions tend to involve you starting off by defining a term and then some kind of standard theorem, which you have to prove, and then the question gets harder and you have to prove something you won't necessarily have seen before.*

She describes the conventional response of students to this type of question in terms of their revision as being similar to her approach, which involves making revision notes, memorising definitions and doing practice questions.

Activity Structures

Christina comments that undergraduate revision differs to school revision because, at this level, students are "having to remember… stuff" whereas school revision focuses on practice questions. She works alone in her room when revising, and spends a lot of time writing "everything down lots of times" during her revision, which helps her to remember it, claiming that "The more you write down a definition, the more likely you are to learn it… same with a proof or a theorem". Christina describes proofs as being the act of "trying to get from one definition at the beginning to another at the end" using different steps.

Whilst she revises alone in her room, sometimes she consults her peers when working on problem sheets. Though she discusses the content of problem sheets with her friends, she "never actually work[s] with people as such" unless she gets stuck. In those instances, she merely asks her friends about how they answered the question and they tell her "the little trick they used", which is sufficient for her to be able to go and answer the question herself.

Perceptions of Self & Others

Christina mixes with a wide variety of undergraduate mathematicians, including those from different year groups, thanks to involving herself both in her college community but also that of the Invariants. She is aware of the practices of other students and compares herself to them and their actions favourably. She is successful and enjoys the subject, which is further indicated by her decision to pursue it at the expense of the opportunity to do philosophy. Indeed, she is intending on studying for a doctorate in mathematics after she has finished her degree.

She was prepared for what her degree would entail, though it was more difficult than she expected. Christina's comments about other students at the beginning of her time at Oxford suggest that she thinks that they were misguided in their expectations, and her prior experiences of the subject were an advantage to her.

7.4 – Juliette's Story

Gender	Female
Course	MMath Mathematics
Qualifications	A-levels
Year	2
ATL	Strategic

Juliette is a second year student whose ASSIST questionnaire identified as having a strategic approach to learning mathematics. Having made a breakthrough at the end of her first year upon realising "what maths is", she is now adjusting to the subject, though does not enjoy it very much.

> ***I'm enjoying being at Oxford and being a student but I don't like what I'm doing […] to be here and enjoy being at Oxford and spending time with my friends, I have to do this work. It's a condition of me doing that, and I just get on with it.***

Prior Understanding

As a student at a prestigious private school which endeavoured to have as many students as possible go to Oxbridge, Juliette was well-prepared for her interviews and admissions test at Oxford. She had two mock interviews, one with an "external organisation contracted by the school", and one with the head of mathematics at another school. To prepare for the OxMAT, her school put on special classes where the pupils could do past papers with a teacher to prepare for the examination. She found this preparation useful as it familiarised her with the question style and allowed her to get herself "in that mind-set" where "your brain is on the right wavelength", as well as being useful in preparing her for that length of examination "so you know whether you're going to be really pressed for time or if you can afford to sit and think for a little while to get through a question". Whilst Juliette did not find the OxMAT useful for herself, she thinks that they test students' "ability to think mathematically rather than just apply a formula or method", which is important because "it's easy for a bright but not particularly mathematical person to do well at A-level through being able to repeat procedures". She thought that she had performed OK, having attempted all of the questions to the best of her ability.

However, at her interviews she was left in tears after an interview Beaumont College because she had felt it had gone so badly, something which she also did after one of her practice interviews for the same reason. Juliette was nervous during the interview and did not understand the question, despite the guidance the tutors gave her, which "was horrible" and was "quite awkward". However, her other interviews went much better, saying that the interviews at St. Seraphina's College and St. Michael's Hall were "actually quite nice". She got a place at St. Michael's, which was where she felt "most positive" after her interview. Whilst the interviews did test her mathematical ability and understanding, they did not give her the impression of how "rigorous and arbitrary the pure maths at university would be".

Juliette had not done any research before going or applying to Oxford about what her degree would entail, although she wishes she had: "I might not be sitting here now!" She did not feel the need to do it:

> *At school, I did maths, I wanted to do more of it… ergo I applied to do it at uni*

She thought that research was only worth doing "if it's a degree that you haven't done an A-level subject in" or if you had a particular interest in something and wanted to see whether it would be on the course. Since A-level did not cover topics in analysis, she was "shell-shocked with all of the proof and analysis and things like that", and thinks that it would be better if A-level mathematics or further mathematics should contain some basic analysis in one of the modules, otherwise "How else are we to know what's coming up?" She found A-level to be an exercise in "applying methods that you'd learned and practised lots of examples of" and, even though she found some of the later further pure modules more challenging and had to work harder to understand some of the concepts, she "didn't really bother with revising for some [of the modules] because they were so straightforward". At school, she was used to achieving marks above 90, only losing marks because of careless mistakes, whereas "now the idea of getting 100 in a problem sheet or exam is just unfathomable!"

In her first year, she averaged 60% in her examinations, which she thinks was a fair reflection of her understanding and performance throughout the year, although she was disappointed, saying "I used to pride myself on getting high marks in exams, but I just don't think that kind of performance is possible from me anymore". Juliette found analysis to be, by far, the most difficult thing that she was faced with during her first year because she had no prior knowledge of it, and "had no idea it would be so dependent upon proofs". Indeed, she believes that the amount of time she spent on calculations at A-level means that she does not think it unreasonable that she would expect it to "be the primary activity at university". Her tutor, however, was very helpful in hear first year because he explained new concepts to her and her fellow tutees carefully. It was all-the-more helpful for Juliette because she struggled to keep up with writing notes in lectures, never mind understanding what she was being taught.

Conventions & Artefacts

Tutorials generally involve going over problem sheet questions and discussing the answers, which Juliette finds very useful, although she "lives in terror" that her tutor will ask her about a question that she did not do herself. This happens if she has been unable to answer a question, and copies one of her peers' answers, although she endeavours to understand the answer she writes in case it comes up for discussion in her tutorial. This kind of working process on problem sheets appears to be quite common amongst all of the students who were interviewed.

Juliette also describes herself as engaging in a generally accepted way of revising for her examinations, saying that everyone revises in a similar way, speculating that "the only difference is how much time people put in to revision and doing their work earlier in the year".

Activity Structures

After writing up lecture notes, Juliette works through as many past papers as possible. Her revision notes are condensed versions of lecture notes, which exclude "really long proofs" from lecture notes because she 'knows' she will not be examined on them, because "that's a lot to know and remember and be able to do yourself" and because there have not been "questions in the past which test you on anything as in-depth as that". Her use of past papers to guide her revision methods is further exemplified as she describes how she spends a lot of

time “memorising bookwork” because she does not feel that she has “the conceptual understanding to be able to derive it in the exam”. As well as rote-learning, Juliette also endeavours to understand what she memorises because of the possibility that she might forget it during the examination, which means that she will stand a chance of being able to attempt to answer the question. She describes rote learning as “not ideal, but I think it’s still kind of OK” based on the fact that her peers also do it, and that sometimes she cannot see another way of knowing the material for the examination. Much of her revision time is spent on trying to understand her notes, as she “didn’t understand it the first time around”.

Juliette uses problem sheets to guide her revision, and tries to focus on material that is not assessed in them at the beginning of her revision process because it is this mathematics which she has not already had to try to understand in order to answer a question. Quite often, Juliette finds herself “at a dead end” when doing her problem sheets, claiming that she thinks she would get the same mark on them whether she spent “five hours or five months on it”. As well as sometimes finding problem sheets “quite tedious”, she also resorts to copying her course mates if she cannot answer a question on a problem sheet. As with her revision, however, she does attempt to understand the answer, although is not confident in her ability to have been able to answer the question herself at all:

> *A lot of the time, I can understand things that I couldn’t do to begin with but I would never have been able to come up with it myself.*

Social Interactions

The friends from whom she occasionally copies problem sheet answers are those who she has met through her college, particularly in her tutorial group. Juliette has a mixture of friends from mathematics and non-mathematics courses at her college, having become friends with other mathematicians at her college “automatically” after having shared tutorials and lectures with them. This is the only social interaction she has with other undergraduate mathematicians at Oxford, as she does not socialise in the department and is not a member of the Invariants. She is intimidated by their passion for mathematics, which she does not share, saying that they are “not really my kind of people”.

Conceptions of Self & Others

Juliette credits coming to Oxford with her becoming more confident and outgoing, saying that the experience has been “really life-changing” for her. This is the reason why she perseveres with her studies as, despite the fact that she dislikes her studies, she sees them as a “sacrifice” for being able to live at Oxford and be with her friends. She has found this year to be “a more enjoyable experience as a whole” compared to her first year now that she is more “comfortable” with her studies, although she is not expecting her results to be any better at the end of the year. Juliette is not “finding anything less difficult”, although has been less upset by what she has studied this year than last. In her first year, she “cried more about applied maths than pure” maths, so chose to specialise in pure mathematics as much as she could. She finds that she understands pure mathematics “more easily” and finds applied mathematics more “messy”. Furthermore, she is more inspired by her pure mathematics lecturers than her applied mathematics lecturers, as she finds them to be more passionate about their subject.

Her confidence in mathematics has increased this year, particularly after she “got over the initial panic” and did some revision at the end of her first year. Previously, she felt that she was “clinging on for dear life”, but during revision, “some things began to click”. Juliette believes that coming “to terms with ‘what maths is’” was a major breakthrough for her as an undergraduate mathematician.

7.5 – Mandy's Story

Gender	Female
Course	MMath Mathematics
Qualifications	A-levels
Year	2
ATL	Strategic

A second-year single honours student, Mandy balances a lot of extra-curricular activities with her studies, preferring to have a lot to do because it forces her to manage her time efficiently. Despite not being overly-confident about her abilities, she earned a first class result in her first year and estimates that she spends approximately 40 hours per week this year on her studies.

> ***everyone around a table, and then we'd sit there kind of quietly all doing the sheet. We kind of think out loud and so we all know when everyone's confused or stuck or angry… There's often swearing!***

Prior Understandings

Like many of her peers, Mandy's decision to study mathematics at university was based on her enjoyment of it at school. Furthermore, after having made the decision to apply to university, her interest in the subject was strengthened after her school mathematics teacher encouraged her to research what undergraduate mathematics involved. He believed that it was important that Mandy was not led to believe that undergraduate mathematics was the same as secondary mathematics, and that having an awareness of what a mathematics degree might entail would do her service when it came to being interviewed for a place at Oxford. Furthermore, having researched the content of undergraduate mathematics courses, she found "the prospect of doing this kind of stuff much more appealing than […] carrying on with the same level and type of stuff as at school". Mandy was expecting her degree "to be very logical and to involve lots of proofs and things like that", unlike, she believes, many of her peers, who "weren't expecting this level of proof and abstractness".

Reflecting on the difference between secondary and tertiary mathematics, Mandy thinks "that they sometimes feel like completely different subjects". Her experience of school mathematics was as something "very procedure-driven", where the mathematics that they "did was something that you could practise a lot until it was really engrained in you and you didn't really have to think whilst you were doing it". In order to revise for A-level mathematics and further mathematics, the main tool Mandy used was past papers, practising answering these, as well as writing out her notes, making definition cards for each module and constructing mind maps. Clearly a conscientious student, she appreciated the modular examination system as this meant that she felt that she "could properly spend time on everything and know it rather than having a lot to do at the end".

Mandy describes her interviews as being useful in two ways: (1) they were "a very good representation of a tutorial", and (2) the requirement to stay in a college room overnight gave her "a feel for what studying and living there would be like". Like many of her peers, she had interviews at two colleges – two interviews at Halifax College and one at Wolsey College. Her preparation for her interviews went beyond what most of her peers did in that, as well as doing a practice interview with her school mathematics teacher, she re-familiarised herself

with the OxMATs which were available online, and she read the book that she referenced in her UCAS personal statement "inside out". The interviews met with her expectations in that they were "more logical and more about proofs" than any mathematics that she had done before, although she did find it "quite strange doing some maths by thinking out loud in front of someone else rather than sitting there and quietly doing it" herself.

The interview followed her passing the OxMAT. Again, she appears to have prepared for this more thoroughly than some of her peers since, in addition to practising past papers, she also did revision of what she had learned in the Core 1 and Core 2 modules in A-level mathematics, also doing one of the papers under timed conditions because she had never done an examination of that length before. The amount of preparation that she did reflects her belief that the University utilises these examinations as a means of identifying a particular type of student:

> *the students who are more committed to maths, who will put the hours in revising and sitting the past paper questions before the OxMAT. They want people who put the effort in and everything… they want to be there.*

She also believes that the OxMAT helps admissions tutors to "spread out people in the 30%" – that is, the 30% difference in marks available to be awarded an A at A-level[60]. A student who fell into this 30% herself, Mandy believed she had performed averagely in the examination, having found it challenging such that she was unable to completely answer each question at first attempt. She recalled "only getting so far through each question, then moving onto the next question until [she] had done as much as possible, then flicking backwards and forwards trying to get as far through each question as possible". She found that the questions posed in the OxMAT contrasted with those at A-level because, whilst A-level questions "say 'do this calculation, find this thing' and so on", the OxMAT questions "said, like, 'what is this, find that' but it didn't tell you what you had to do".

Despite her preparation for the entrance examination and interview, A-level grades and research regarding what her degree would entail, Mandy's experiences of first year mathematics were still a "shock to the system" as she found that she had to work a lot harder to be successful at university than she did at school. This is something that she does not think A-level mathematics and further mathematics challenge students with. Furthermore, she did not think that her A-level sufficiently prepared her for undergraduate study at Oxford. Mandy thought that A-level was adequate for other institutions but, because Oxford is one of the best universities in the world and because she believes studying there is more difficult than elsewhere[61], the nature of the A-level meant that she was "not used to having to really work at maths which is something you definitely have to do at Oxford". Having found tutorials and mock examinations very helpful when she had these in the first two years of her degree, the reduction in the amount of support offered has made her learning and understanding more difficult.

[60] As are awarded for scores between 70-100%.

[61] There was no other foundation for her believing that the A-level was sufficient for other universities. Her belief was clearly based on a speculation that Oxford is significantly more difficult than anywhere else and that she found that what she had studied wasn't sufficient for what she encountered at Oxford.

Social Interactions

Mandy found tutorials to be very useful in her first year. They put her mind at ease as they showed her that she shared "similar concerns and things with everyone else in the tute" and giving her a forum to ask questions about mathematics. Whilst she no longer has tutorials, she continues to work in ways which allow her to discuss mathematics. When working on problem sheets, Mandy often works with her peers in a group in the library:

> *When anyone gets stuck we all stop and talk about it and help each other out by either sharing what we did or we sit with our notes and everyone has a look to see what they think to do, and we talk about it. Then once we've, like, agreed on what to do, we all write it down.*

This is about the limit of the time that she currently spends with other mathematicians, as she is involved in many extra-curricular activities – she plays two sports, works in her college bar and is the treasurer of a university society. Whilst this means that she is forced to manage her time effectively, she is "always busy" and it has made it challenging for her to successfully maintain social relationships with her friends and family. Mandy describes herself as being "pretty unusual" in the sense that the majority of her mathematician friends are actually those from outside of her college. She remarks that "There aren't a lot of mixing opportunities in the department", but once she began to do optional courses, she became friends with students from other colleges by meeting them in classes. None of her mathematics friends are members of the Invariants, a society that she has never been involved with. Their meetings often clashed with her other commitments, and the later and later in her university career it became, the more she lost "the momentum to go to things like that", as she believes that if she went to an Invariants seminar, then everyone else in attendance would be friends, and she would be an outsider. She also claims not to have been interested in anything that they have put on so far.

Activity Structures

Mandy is looking forward to her third year when she will be able to choose more optional courses and she will be able to follow her interests, which are currently more in applied mathematics than pure. She estimates that she spends approximately 40 hours per week on her studies, generally working between 10am and 6pm in her college library. Not all of her courses allow her to work in a group on problem sheets, which means that for some of them, she works independently as much as possible before consulting books. However, she has at least one friend on each of her courses, so she can always call upon someone to help her if she needs to. Whilst she used to be able to finish a problem sheet the night before a tutorial, she now finds that they can take up to 2½ days to complete.

Consulting past problem sheets forms part of her revision for examinations, as she claims that it is possible to identify similar questions in the sheets to past examination papers. Therefore, she believes that it is important to ensure "that you could do the ones that are on there". Otherwise, her revision technique shares many similarities with those of her peers, but also some significant differences to the others reported, and the others which she has witnessed. Like many of her peers, Mandy re-writes lecture notes, but then she constructs mind maps

before moving on to making lists of definitions and theorems and practising past papers. These mind maps

> *Separate everything into topics and the key theorems and things and then show how there's a link between everything. Have, like, a definition name and then draw lines from it to different theorems and things to show how they're related, and how different theorems require knowledge of other theorems to work*

She does this to facilitate her understanding of the relationships between different concepts, rather than disjoint bits and pieces – something, which she thinks would make it more difficult to learn and understand "because you don't have a concept of the […] big picture". Like the other students interviewed, Mandy also "spend[s] a lot of revision" actively memorises definitions, theorems and some proofs in response to the types of questions that she presumes will be posed in her examinations. The volume of material that she had to revise took Mandy a long time to understand, and she was not sure that her "memory was good enough to remember everything", something which affected her confidence going into her exams.

Conventions & Artefacts

Of the five stories being told, Mandy's is the only one which is told shortly after she has stopped having weekly tutorials to support her learning. Furthermore, in the third year, students at her college are no longer given collections or mock examinations, which has left her feeling "left on your own to get on with it without them doing anything to guide you in your revision". This has had a big impact on her learning and understanding, particularly at a time when she has begun to find the content and questions posed in her new modules more challenging than previously, which makes her studying more time-consuming.

Mandy struggled to keep up in her first year once the pace of teaching increased in her lectures, something which made revision difficult for her later on. She does not think that a lot of her peers do sufficient revision, speculating that this is because their previous experiences of being examined on the subject did not require her to do a great amount of studying in order to be successful. Her unusual revision technique is something which she also recognises as not being the norm, as she reports that "everyone just ploughs straight into writing lots of notes and definitions, theorems and proofs and just do lots of cramming", often neglecting to "understand how the relationships work" if they do not understand them already. That is, Mandy recognises the convention in the culture in terms of research methods, but believes herself to work in an exceptional way.

This is something facilitated by the conventions used in undergraduate assessment, as Mandy describes the types of questions posed in the examinations

> *A lot of the time there'll be a question which asks you to prove a certain theorem that you will have had in your lecture notes, so you just need to make sure that you commit those to memory. If you look at the past papers, you can see the types of things they want you to be able to recite.*

Indeed, she speculates that it might be possible for someone to be awarded a 2:1 just by answering questions which require students to do such work, although this varies between modules and years. Additionally, one of the reasons she revises problem sheets is because questions which are identical or similar appear in examinations “loads of the time”, and it is important to revise those because they are “some of the easy marks”.

Perceptions of Self & Others

Mandy consistently demonstrated an awareness that she works differently to her peers, apparently believing that her methods are more appropriate if she is to have a thorough understanding of what she has been taught. Whilst she initially struggled with the volume of work, she is now able to effectively manage her time well such that she can engage in a lot of extra-curricular activities whilst still achieving a first class mark in her examinations last year. She prefers pure mathematics to applied mathematics, and finds it more challenging, and finds pure mathematics difficult as she struggles to “mentally imagine the ideas and concepts” and finds it “a little boring and pointless because it’s so far from the application”. Mandy is aware of the investment that she makes at the moment by studying courses which are prerequisites for ones in later years, meaning that whilst “some of it is a bit crap”, she appreciates that “you just have to put up with it" as she looks forward to specialising in the future.

7.6 – Malcolm Interview Transcript

ED: OK, Malcolm, so you're fourth year straight maths, is that right?

MALCOLM: Yes, final year.

ED: what was it that made you decide to apply to do maths at Oxford?

MALCOLM: Maths because… I enjoyed it, I was good at it, and I think as a non-specialist subject it probably has the best career prospects out of most degree subjects.

ED: In what way?

MALCOLM: Obviously, if you do something specific like law, medicine or dentistry, you're going to be able to earn a lot of money, but you've kind of decided your job by doing that. Maths as a normal subject, I think, probably has the best earning power. It definitely did out of all of the A-levels I did, I think.

ED: Which A-levels did you do?

MALCOLM: Maths, Further Maths, so, you know… Then I did Physics and German.

ED: German?

MALCOLM: Haha, yeah. I really liked having a language. It's a really useful practical thing. My family went on holiday there one year and I wanted to be able to speak it properly so since then I've loved going to Germany. I thought it would be really useful, but actually, in most of the places you go, people speak English. Haha! But it's fun being able to speak to them in their own language.

ED: Yeah, that's great. I've not managed to hold onto my A-level French, though!

MALCOLM: Haha.

ED: Then again, I haven't been to France in years to be able to practise.

MALCOLM: Yeah, that helps.

ED: What made you go for Physics?

MALCOLM: I thought it complemented the maths well, and it was interesting. I liked all of the different components to it… Like… The electronics stuff, the astronomy… Everything. I wasn't quite as good at it, though.

ED: No?

MALCOLM: No, I had to resit a couple of modules because I messed them up.

ED: In what way?

MALCOLM: Oh, I'm not sure, the marks just weren't very good in two of them, so I did them again. I didn't have anything to lose.

ED: And your mark improved?

MALCOLM: Substantially, yeah, and it wasn't like I really had to do any extra revision for them, so it didn't disrupt my revision for other things in that sitting… Thinking about it… I don't think I would've got an A overall if I hadn't resat them both.

ED: But you didn't have to do that for Maths or Further Maths?

MALCOLM: No, I think I got As in everything first time.

ED: So what do you think of the modular system?

MALCOLM: Well, that was really helpful, although I think some people took the piss with those a little bit, resitting things multiple times. It's like the grade you have at the end is kind of fake if you do that. But the module thing is good because it means that you can focus on one topic at a time and be examined on it like that. It's basically like going to lectures when you're at Oxford for different courses, so you're distinguishing between the different types of maths.

ED: So why Oxford?

MALCOLM: Aim high! I wanted to see if I could get into Oxford or Cambridge because it means that you're at the best university, and I was really good at maths and thought I'd have a good shot at it. Basically… I, er, I went for Oxford over Cambridge because Cambridge is apparently harder to get into than Oxford so I wanted to increase my chances of getting in to one of them, so this seemed the sensible option.

ED: OK, Malcolm, so tell me about your interview for Oxford.

MALCOLM: I came up in December of... 2007. The year before I got rejected. I, er, applied to Wykeham College for no particular reason, really, just at random and had three scheduled interviews at Wykeham and then a whole bunch more extra ones were added after I'd done those three.

ED: OK

MALCOLM: I think... I think what happened was the people I'd spoken to at Wykeham liked me enough to offer me a place but they didn't want to have me there because they had better people at Wykeham. So they sent me around elsewhere, at three other colleges.

ED: OK...

MALCOLM: I had more interviews and... Fernham College and Lady Matilda's College.

ED: So what happened in the actual interviews that you had?

MALCOLM: They were largely the same, so basically they sort of introduce some sort of mathematical idea unlike anything you'd probably thought about before, like, at A-level and... They ask a really massive question that just made me go blank and mumble stuff! Then they helped me break it down into small parts until it became small enough for me to begin

answering and... er... that was pretty much how it worked for about four of them. One at Fernham wasn't quite like that, it was more of a philosophical chat about mathematics.

ED: OK...

MALCOLM: But, er, in general it was a big hard question and then they were trying to see how well I could cope with something like that.

ED: Can you remember what the questions were, at all? I know it was a long time ago...

MALCOLM: Yeah, I have no idea. There was something about, it was like, a piece of paper and I knew that each of... The paper had something written on the other side and I had to work out how many pieces of paper I'd have to turn over to work out which one had that thing written on it. Something like that... Does it make sense?

ED: Yes, that's a pretty classic maths problem, yeah.

MALCOLM: Oh...

ED: OK, so how did you find your first interview in general, your general reaction?

MALCOLM: Erm... Not really scary, but I think I wasn't really prepared for what was actually happening because my sixth form had done what they thought was mock interviews but they'd just been, like, "so you want to study maths..?", "why do you want to study maths here?" and things, and that wasn't at all what the interviews are like. But it was... They were quite enjoyable. It wasn't intimidating or anything, just interesting. But it wasn't what I was expecting because it wasn't what my college had prepared me for.

ED: Did you get interviewed anywhere else that you applied?

MALCOLM: No, just Oxford.

ED: How well did you respond to the prompts from the interviewers when they broke down the questions?

MALCOLM: Well, there was one question. It was about particles moving on a string or something. He started off with an infinite number of particles and... Oh, OK, I remember, I remember... So there's an infinite number of particles on a string or something, and they move sort of randomly and when they hit one another, they move back in the opposite direction and something like that. I had no idea what to do, and he broke it down for me really well to, like, two, three, four particles, and I think one of those small, integer number of particles gets the correct answer and he seemed quite shocked about how well I managed, I think.

ED: How did you get on with the entrance test?

MALCOLM: That was interesting. I didn't think I'd done very well on it because I couldn't answer the last part on all of the longer questions, so I thought that was the end, but then I got an interview, so I must have done OK!

ED: Did you do any preparation for the OxMAT?

MALCOLM: I had a look at a few of the past papers and tried to do the questions in it which looked interesting, and like I wouldn't definitely be able to do them easily.

ED: There were some that you thought were easy?

MALCOLM: Maybe 'easy' is too dismissive! Haha. Ones which were more straightforward, where the method was clear to me. Some of the multiple choice ones were like that. Some of them were definitely not like that and were really hard or abstract.

ED: How did the OxMAT compare to the A-levels you did?

MALCOLM: It was totally different. The longer questions were completely unlike anything we'd ever had to do before, and they required fairly basic maths but then a deep understanding of what was going on in the problem and an idea of what to do to find the answer. The multiple-choice questions were hard, and they looked like they'd be easy at first because it'd be an integral or something which seemed to look for you to do a calculation, but there was more to it than that.

ED: So why do you think Oxford uses the entrance test?

MALCOLM: For those reasons, really… The A-level gets you to do calculations and use different methods that you've tried before, just with different numbers. The papers are all the same but different. So everyone applies with good AS-level grades, but Oxford don't know that that means that you're good at maths and can think mathematically.

ED: No?

MALCOLM: No, they know that you're good at answering maths questions correctly. You're good at doing calculations and using an… an algorithm that you've used before. The entrance test forces you to think out of the box and to use maths that you know to solve a problem. It doesn't tell you what maths to use, either.

ED: Did you find it useful?

MALCOLM: I think it made me realise that I was capable of doing things like that… Once I knew I had an interview, that is! Haha. It was interesting and it was strange being in a situation where I wasn't just blasting out all of the answers in a maths exam without having to really think. Kind of prepared me for now, when I definitely can't do that any more!

ED: So what was your impression on what studying maths at Oxford would be like, based on the interview?

MALCOLM: I guess, yeah, the problem-solving aspect and how there's big questions. I think the interview was useful in terms of getting you to kind of work with a tutor to get an answer, and see what kind of help they could be, without just telling you the answer and you going 'oh yeah'. Because at school, I found whenever I was stuck at maths, I'd ask the teacher and they'd just basically show me what to do, and the reason I normally needed help was because I'd done something stupid to not get the answer right and missed a step in a calculation or something, rather than needing to be guided through the question.

ED: So there weren't really any 'big' questions to be broken down at A-level?

MALCOLM: No, they're all broken up into tiny sub-parts, which generally just tell you exactly what to do. But the interview was different because they said "here's a problem, what's the answer".

ED: But at A-level..?

MALCOLM: That's more "here's a problem... let's find the answer by doing all of these specific things one by one".

ED: Had you done any research on what a degree in maths would involve before you came up?

MALCOLM: Apart from having a look on the department website and the past OxMATs and things like that, no, not really.

ED: OK…

MALCOLM: Actually… Yeah, so I think that probably wasn't the best idea… In hindsight…

ED: In what sense?

MALCOLM: Well, I wasn't prepared for quite how formal and abstract it was going to be, and it was quite a shock to the system, so I think it might have been a better idea if I'd looked at past lecture notes online or looked at some books, maybe.

ED: Oh, I see…

MALCOLM: Although, I think that might have put me off! Haha.

ED: Why do you say that?

MALCOLM: I think if you looked at, like, Analysis notes without having seen anything like that before, you might run away screaming. Actually… To be fair, I think if I'd seen it I probably would have thought it looked hard but found it exciting to think that I would understand it when I got there and that I'd be able to sit and do an exam with things like that on the page.

ED: So you've got your final year exams coming up?

MALCOLM: Yes... Looming!

ED: So how are things going at the minute?

MALCOLM: Yeah, they're alright. The courses I'm doing vary on how hard and demanding and how much work they require, which is a little strange. There's one I'm doing at the moment that's largely based around this one textbook and one MATLAB program thing...

ED: Ah, MATLAB...

MALCOLM: Yeah! Haha. So that's... Lots of the questions that we're set to do for that each week are using this MATLAB program and we all know none of the exam questions will be on that level... It's a little, erm... It... It does help to understand the type of mathematics going on but I think it takes too much time working on the MATLAB code than actually understanding it.

ED: And are those compulsory?

MALCOLM: Well, no, but they're really useful to do, I think. Keeps you up to date with the course.

ED: So what work do you normally do in term-time apart from that… I take that the MATLAB thing is something that you do throughout term time and not just in the holidays?

MALCOLM: Yes, so that's each week. During the week, I normally just work on the problem sheets that I have to do, the ones that I have to hand in. Then that's it, really. I don't do a lot else. If I really struggle on a problem sheet I might try and devote some time to that topic and read about it much closer, but that only normally happens after I've handed in the problem sheet, anyway.

ED: Is that similar to your peers, do you think? The work that you're doing in a normal week?

MALCOLM: I think most other people probably devote more time to general study than I do. They certainly seem to understand what they're doing, or at least, they did last year and the year before, so I assume that can only come from working hard outside of lectures. There are… Of course there are some really lazy people who don't seem to do anything all year and then when it comes to exams and they lock themselves in their room to revise and you don't see them until the morning after the last exam.

ED: Ah, I see. So what are the other courses you're doing apart from this MATLAB one?

MALCOLM: There's a course in probability... Combinatorics. It's really interesting stuff and it's lectured by the head of maths at my college and, erm... Continuous Optimisation is the other one. They're pretty good courses.

ED: What made you pick them?

MALCOLM: Erm... They follow on from modules that I did last term so I thought that made sense and... In previous terms, I've picked up far too many modules and then not known which ones to drop so I didn't want to do that again! Last term I was doing graph theory, which was a part 2 module, and a course called numerical linear algebra. About half the modules I've done are quite a numerical approach type ones. I've done courses like that before in second year and enjoyed them, so I guess that played a part. It's a little bit... I have a friend in college who often picks the same modules as me, I've found, so doing the same modules is really helpful for us when it comes to revision. You can work together with that and it's really great.

ED: Are there certain modules or types of maths that you tend to perform better on than others?

MALCOLM: I think, to begin with, I struggled with analysis considerably. It was so hard and so different that it was really difficult. However, once I think I started to adapt to everything it didn't become so daunting. Now, because I'm picking the modules that I want to do and I like, there's not really anything that I'm necessarily better at than anything else. Not that I can tell, at least. Before this year, I think I was a bit better on probability-type stuff than other things.

ED: Why was that?

MALCOLM: I think I liked the applicability of some of it and it involved a different kind of thinking, in a way. Everything else was so abstract. Although, saying that, probability courses now are really abstract, but I think I held on to the understanding I had of it earlier on enough to appreciate what's going on now.

ED: Did you do a lot of probability modules at A-level?

MALCOLM: Yeah, I did all of them.

ED: Which other modules, do you remember?

MALCOLM: Just all of the normal pure ones, the statistics ones and I think a mechanics one or something like that.

ED: Did you get to pick which ones you did?

MALCOLM: We had to do certain ones in the AS-level and then they asked us what we wanted to keep going with. I really enjoyed probability so I pushed for that… the statistics modules… and I think everyone else was really into pure maths and so we did some more of that and a bit of mechanics.

ED: And you said that you liked the probability courses you've done so far in your degree?

MALCOLM: Yes, I guess I must like doing that. I hadn't thought of that before. I definitely had a natural flair for the stats stuff that we did at school, definitely. It came really easily to me.

ED: Lovely. How did you get on in your exams the last three years?

MALCOLM: Erm... At the end of last year I got one mark off a 2:1 overall...

ED: Oh no!

MALCOLM: It was all OK but then there was one really awful exam that kind of ruined it. Then the year before that I got something similar. I can't remember what it was in first year. Nothing drastically different, I don't think.

ED: What was it that made you decide to continue to the fourth year instead of just finishing with a three-year bachelors?

MALCOLM: Well, I thought of approaching my fourth year as a year before a PGCE to help me decide and then... I changed my mind. It was a whole bunch of things. OK... On quite a personal level, I was dating a girl who lived in America and then I was thinking about moving to America to train to teach out there, so I thought that I couldn't do that yet because I hadn't

applied yet, so I thought I could just stay and do another year of maths whilst I was dealing with the application process and then, erm... And also, I didn't want to leave after having got one mark off a 2:1 as my final grade so I wanted to do another year to...

ED: Rectify it!

MALCOLM: Definitely.

ED: And how is this year going compared to the previous few?

MALCOLM: I think it's a lot better. Because I've got to pick modules that I definitely want to do and am interested in, going to lectures has been a lot more fun and I'm understanding everything better than I was. I've, er, certainly learnt from what happened last year and I've started working harder, I think.

ED: So you're still planning on doing a PGCE?

MALCOLM: Definitely, yeah. Although I broke up with my girlfriend, so it'll be a normal PGCE in the UK.

ED: Oh, OK. I'm sorry.

MALCOLM: Yeah, don't worry.

ED: How did your first year go, do you remember?

MALCOLM: I can't remember the mark I got, but I definitely found first year really tough. I think... The way the maths course works, it gets you into options really early, which I think is good because in the first year I was trying to work out which bits of the degree that I actually enjoyed. There wasn't much! Haha. But then after that I found it a lot easier and a lot more enjoyable. First year was really, really tough but...

ED: What was the hardest part?

MALCOLM: I'm not sure. It was probably all the analysis stuff and the calculus stuff. It was... Yeah, it was sort of just getting my head around the mathematical way of thinking being nothing like the way it works at A-level.

ED: OK...

MALCOLM: It... At A-level it's a much more prescriptive thing, like, to answer these problems you do this kind of thing and so on. But when you get to degree level and it's a whole other thing. It's "apply this theorem that you learnt four weeks ago to this thing that is vaguely related, and then another lemma from last term..." and things like that. It's much more... You've got to work, yourself, through the problem, which is probably a better skill to have, but it just took me a while to pick that up, I think.

ED: So were you expecting it to be that way when you first came up?

MALCOLM: No, well, I was expecting it to be different.

ED: In that way?

MALCOLM: I don't know, really. It was just sort of a bit of a shock to the system. Maybe the workload as well was a bit greater as well. I barely did any work outside of set school maths at A-level so...

ED: Was that just maths, or other subjects too?

MALCOLM: Haha, I think it was everything, really.

ED: How well do you think that your A-level set you up for uni maths?

MALCOLM: Not very well. It's just not the same way of thinking. But I'm not sure how they could be any better. Like, there was a whole series of introductory lectures at the start of term, which were about how proofs work and things like that. I think perhaps more of those, or more in-depth versions of those would have been helpful. I don't know if Oxford really understands how little we'd all done of that before we came. Or maybe they didn't care...

ED: In what sense?

MALCOLM: Well proof is such a big part of everything, but the introduction to it is quite short but sweet, as if you're meant to know that anyway. But then, I guess, maybe they're expecting us to have gone and researched it and read books and prepared ourselves, you know?

ED: And the A-level?

MALCOLM: I think it's good at getting you to know all of the general ideas behind everything and being able to do calculations, and maybe kind of getting you to think in the right way… Actually, no. I think… I guess it could, although the Oxford maths test definitely did that. I think maybe… No… It might just be that maths students tend to be people who are good at thinking mathematically when you're at school and it's not necessarily the A-level that makes you that way. You know what I mean?

ED: Are you saying that the A-level doesn't develop mathematical thinking?

MALCOLM: Yeah, like, it helps you learn more maths but not how to do it. I can't phrase it quite the way I want to. But I don't really know what it could do to be more useful in terms of getting you ready for university because they've got to make it so that people who want to do other subjects at university can get something out of it. Most people who do it probably don't go on to do undergrad maths… Maybe… I think there were some of the further pure modules… They were trickier and got you to think a bit more than the other ones, so maybe more stuff like that? More proof. I think all we did was proof by induction as a tiny section of one of the modules.

ED: I see. So how did you revise for A-level Maths?

MALCOLM: I didn't do a great deal, to be honest. It all sank in pretty well throughout the year, so when we got to the end of the module, we'd do a mock and I'd always do well in that. Before the exams, I did a couple of past papers and wrote down things that I sometimes forgot to look at on, like, the bus on the way to school.

ED: OK…

MALCOLM: It's almost embarrassing how little I did. Compared to revision for Physics and German, it was minimal. It sounds really cocky, but I basically just didn't really need to revise properly because I already knew it and had picked it up as I'd gone along.

ED: OK. So you mentioned proof as being problematic when you came to Oxford?

MALCOLM: Yeah, because, like, most of the maths and the mathematical reasoning and the problems were... I could understand. I just couldn't come up with it myself. I sort of picked up techniques for helping me to come up with those kinds of ideas as time went on. It just took a while.

ED: Did your tutor play a role in that?

MALCOLM: My tutor did help, but when you're just seeing them for one hour a week or something, there's only so much they can do. There's more pressing issues than things like that in the tutorials, I think.

ED: So what role did your tutor play for you in your first couple of years?

MALCOLM: Just standard, really. We had tutorials whenever we had to have them, and we all went and we'd go through problem sheets and he'd throw in maybe a cool bit of maths which was related to what we were doing and we'd talk about it. It normally depended on how easy we all found the problem sheet and if there was any time left for other stuff to talk about.

ED: Was he helpful in the transition you made between school and university maths?

MALCOLM: Well… Er… I guess so, maybe. He was definitely really patient with us at first when we were trying to get to grips with everything. The lecturers we had for some of the introductory courses were more important for that, I think.

ED: In what sense?

MALCOLM: They went to a big effort to slowly explain concepts and formalised things to us at the beginning so we were led into things carefully without getting lost instantly. Well… Not instantly. I'd say that they did that maybe for two or three weeks and then it was full steam ahead! They had this incredibly ability to cover what was basically everything we'd ever done about calculus or whatever during the course of about a quarter of a lecture, and then we'd steam through some new stuff that we'd never come across before. But you get used to it.

ED: OK, I see.

MALCOLM: Classes are also really useful this year and last year. You can go to those and they give you back your problem sheets and talk through the answers, which is really useful. Sometimes it's not as good as a tutorial, though…

ED: Why not?

MALCOLM: There's more people there, so there's more people to ask questions which means that sometimes they spend time talking about something that some of you are, inevitably, going to find easy and unhelpful, when you actually have another question you need answering. But, on the whole, they're good.

ED: So these problem sheets… How do you go about doing them, typically?

MALCOLM: For normal courses, not like the MATLAB one because that's completely different because you're using a computer? So I normally just sit with my lecture notes and try and work through it using the notes. Obviously, there will come a point where I get stuck and can't do a question, which is when I'd ask a friend what they did. I used to take my work around to my girlfriend's house and talk about it with her, or maybe in the library at college with other people on my course.

ED: OK, so you often work with other people on them?

MALCOLM: In an ideal world, I wouldn't have to but, yeah, I end up going to someone and asking them for help on most sheets.

ED: And do you help anyone with theirs?

MALCOLM: Yeah, sometimes I can do questions that other people haven't managed but that's normally because I've noticed the little trick you have to do, and they haven't realised yet. I also help people who are in the year below at college if I can and they get really stuck and ask me.

ED: Do you enjoy the problem sheets and find them useful, at all?

MALCOLM: It depends on the module as to whether I enjoy them to be honest! Haha. Useful? Definitely. I think if I didn't do them then it'd get to exam time and I wouldn't have a clue what's happened the whole year and have to learn everything from scratch! Haha. Doing the problem sheets means that you've had a chance to try and understand everything. Then, later on, there'll be bits and pieces that you need to brush up on your understanding of, and things to commit to memory, but you have the initial learning from the problem sheets.

ED: So you said that you went from doing your A-level and doing barely any work and, I presume getting As and really high marks?

MALCOLM: Yeah...

ED: Maybe full marks sometimes?

MALCOLM: Yeah... Those were the days!

ED: To then coming to university and scoring in the 60s in your first year, did you say?

MALCOLM: Yeah.

ED: What was that like?

MALCOLM: Oh, yeah, that was really demoralising. I just didn't really know how to deal with it, I think. It's not like I got a result at the end of the year and was horrified at how I'd done, it was more like... We had collections at the start of Hilary and Trinity and, like, those were... They definitely indicated how much work I needed to do to get anywhere near a good mark in my degree. I think I found it really tough, basically. Collections made me realise the bits I didn't really know anything about and then I just revised like crazy at the end of the year. Maybe I should have revised like crazy initially and then worked out the bits I didn't know but I sort of did it in a bit of a lazy kind of way. It was a really helpful thing to have a go at finding out how much I didn't know, but it wasn't great for the ego.

ED: How would you say your understanding of maths now compares to your understanding when you were at school?

MALCOLM: I presume you mean of the maths that we're doing at the time, because I understand more maths overall now! Haha.

ED: Yes…

MALCOLM: Oh, it's way worse. At school, the teacher would just show us something new and explain how to do it, and I was, like, "OK. Done." And then I just tried a couple of questions, could do it, and then it was fine. Now, they just define something, and it's a struggle to write it down and digest it whilst the lecturer is talking, without being left behind. It's a totally different thing.

ED: Thing?

MALCOLM: The maths. The understanding. Eventually by the exams, I try and understand what I'm doing by reading through everything carefully, but I didn't really need to even try to do that at school because it just… happened.

ED: I think I see what you're saying. So when you did your revision for your exams, what was your technique?

MALCOLM: I'm not sure, really. Well... In first year, it was just cramming, learning theorems, proofs, learning direct from lecture notes, rather than actually, like, thinking, engaging. It's not always easier to remember, like, parrot fashion, a proof. I think if you understand it and you can, like, rebuild it from scratch in an exam it's much better. That's how I go about it now, or at least I try. Yeah. Trying to work out a theorem and its proof and, like, see if I can do it from memory or something like that. And then look up some past questions and see if I can do those.

ED: And they're online, aren't they?

MALCOLM: Yes. You can access all of the examiners' reports and mark schemes online, which can be quite helpful. It says things like "most students tried doing this, but actually they should have used this approach" so you see that when you're trying the past paper and you go "ah, OK" and use that when you're working.

ED: That seems useful, OK…

MALCOLM: The easy marks to get in those exams are those, which are like "state this theorem and give the proof". I'm never really tempted to answer longer, more constructive questions, but I wouldn't have... I tended to, in fact, in first and second year, try to answer really wordy answers to anything that's really mathematical. Like "it looks a bit like this, so if you do this, then this happens, which makes this happen" which... Obviously that's not how maths works! Haha.

ED: But you gave it a go!

MALCOLM: Exactly.

ED: But now you work differently in your revision and technique?

MALCOLM: Yeah, I think I find it quite hard to motivate myself in the first half of the year, probably because up until the end of A-level, outside of being in the classroom I rarely had to motivate myself, which is kind of a problem! Haha. But yeah, once I realised that it wasn't going to happen unless I was constantly working outside of my lectures and tutes and everything, then I had to be constantly working and it helped a lot.

ED: How was it for other people in your year in that respect?

MALCOLM: Yeah, definitely similar. It probably took me slightly longer to realise than everyone else, though! I think I probably struggled more at the start than most, yeah... Although that might not be the case. It's hard to tell how well other people are going. Yeah. I feel like I got to the place where I was able to motivate myself and do the work and things later than I should have done. It seemed that way from knowing other people in my tutor group.

ED: Oh?

MALCOLM: They seemed a bit more confident and able than I was to begin with, I think. They were certainly more studious than I was, I think.

ED: Were they people you made friends with and worked with throughout the years?

MALCOLM: Erm... Yeah. I think so. Three of us are really good friends.

ED: The friend you said who does similar modules to you, are they one of those?

MALCOLM: Yeah. But, er... Last year I did a lot of revision with my girlfriend at another college. We met through just going to the same classes for revision, consultation periods…

ED: Consultation periods?

MALCOLM: In Trinity there's a, they call it a consultation period where, er, each week the lecturer and your tutor for the module will have a specific time to answer your questions. It's like a class or a tute without any structure to it. It's as many people turn up as are going to turn up. So if you turn up and don't have any questions then there's no point. So that's good for motivating revision as well. But other than that, generally, lecturers are really good at replying to specific questions by email and things like that. They clarify things.

ED: Oh, OK. Sorry, you were saying about your girlfriend…

MALCOLM: Yeah, and that was really helpful because we were doing a bunch of the same courses as well. Yeah, that's kind of a problem in a way with the course.

ED: What is?

MALCOLM: The whole college thing. It, like... For my first, like, two years I was only really with mathematicians from St. Matilda's and if they were choosing different modules to me then I had nobody that I could work with. But if it had been less, sort of, collegiate in that sense, I might have found a more social study group with people who were into the same things as me. Plus people don't really tend to hang out at the Maths Institute so you're kind of restricted with the types of people you'd meet. There's less mixing with mathematicians and things. If there's a lecture with only, like, 12 people, then there'll be 12 people dotted around the lecture room, rather than a group of four here, and a group of four there, and things like that.

ED: Isn't there like a maths society which could, effectively, bring more people together? What are they called..?

MALCOLM: The Invariants?

ED: Yeah, that's it.

MALCOLM: They do a termly newsletter that goes out to everyone on their, like, list of people and... I've been twice, I think. They meet up, I think, every week, and they'll have a lecture on something interesting, a maths topic, and they'll meet up in the common room for tea and biscuits afterwards. The two I went to... The first one was... Argh, I can't remember... They had a lecturer on a topic... The second one they had an improvised comedy from Cambridge who do maths comedy which was...

ED: Was it good?

MALCOLM: Yeah, it was actually quite good. They did it in a lecture hall and one of the things that they had... They had the Greek alphabet written up on the board behind them and they had to do a sketch where each line started with the next letter of the Greek alphabet. Haha. It was pretty complicated but it was pretty...

ED: Oh, OK.

MALCOLM: I don't think there are *many* people who go every week. It's... I have a friend who goes every week and he's made a whole bunch of friends at other colleges but I think the down side is... The kind of people you meet there are the kind of people who want to go to a maths lecture and, er...Haha. The type of people who go to optional maths lectures, which might put some people off.

ED: Do most people give it a try at least once, or..?

MALCOLM: I think there's quite a lot who've been at least once to see what it is, mainly at the beginning of the first year. I think, in theory, it's a good idea but, er, it doesn't work quite as well as it could.

ED: Are you involved in any clubs or societies, at all?

MALCOLM: I play in a band, but that's about it.

ED: A band?

MALCOLM: Yeah.

ED: Who are you in the band with?

MALCOLM: Just some guys that I met either at college or when I was at gigs and things like that. We get together a few nights a week and sometimes play gigs in pubs.

ED: Is the practicing and gigging quite time-consuming, or…

MALCOLM: Haha. We're not very good! So it's only a few hours in the week and a bit at the weekend. It's not stopping me from doing any work, if that's what you mean! Haha. There's plenty of other spare time that I could spend in the library but I don't. But playing in the band is a lot of fun.

ED: That's really cool. So are any of them mathematicians?

MALCOLM: No, none of them. Some of them aren't even students.

ED: So who do you spend most of your time hanging out with, outside of the lecture theatre?

MALCOLM: I think mainly those guys and some others from college, really.

ED: What has been the most challenging aspect of your degree so far, do you think?

MALCOLM: I think it's like I was saying earlier on. It's like the counterintuitive thing... Like, it should be getting harder but... I feel like it hasn't worked in that way. The maths hasn't been harder, but the degree hasn't been getting any easier. Like, the transition... So... But definitely the earlier analysis modules were hardest but I think not because of the maths but because it's a different mathematics that you're trying to figure out. Once you adapt to what this maths is then the challenge is different. The challenge becomes working out how to do it and revise and do well. Then the challenge is doing well. The third term of second year was probably worst. The modules that I didn't enjoy kind of got to their hardest point and... I didn't know that it was going to get more enjoyable or anything after that, but it did and it was good. I guess from that point, it was a lot more enjoyable.

ED: And what have you enjoyed the most, do you think?

MALCOLM: I think definitely the education course last year.

ED: Oh, you're too kind! Haha.

MALCOLM: Haha, yeah. I found that I... I was actually talking to friends and totally just drop in a cool thing that I'd learnt in that class, which is something I hadn't done at any other point. Like "oh, today, there was this cool theorem!" Haha. I found it really interesting and the kind

of thing that other people find interesting as well. The lectures and classes were really good, too.

ED: In what way?

MALCOLM: I thought the lecturing was really good. I liked that it was interactive and you wanted to hear our opinions and find out what we knew about things. Most lectures aren't interactive, and they're really boring. You might as well just have the notes printed online and just get them from there. Although, I guess that's another thing where people might not look at them until it's really late.

ED: That's true, yes. And what did you think about the assessment of the Maths Ed module?

MALCOLM: It was a little intimidating having to write essays again after so long but it turned out OK.

ED: It's quite different to what you were doing in your other courses, isn't it?

MALCOLM: Yeah. That makes it a good thing. And then doing the essays and coursework means that you get that out of the way, and there's no exams, which alleviates some of the stress later on. Less exams.

7.7 – Qualifications Offered for Entry

The table below shows the numbers of offers made to, and final acceptances for, candidates not taking A-levels for 2011 entry:

Type of Qualification	Offers	Acceptances
A-levels	2596	2405
A-level/Pre-U combination	74	63
Pre-U	13	13
International Baccalaureate	208	168
Scottish (Advanced Highers or Highers)	46	41
Singaporean SIPCAL	37	27
English Language requirements (as sole condition)	22	19
US APTs or SATs	31	23
Romanian Diploma de Bacalaureat	14	9
German Abitur	24	22
French Baccalaureate (inc. International Option)	18	16
Polish Matura	10	7
Irish Leaving Certificate	2	1
Dutch VWO	6	6
Australian ATAR/UAI	18	14
European Baccalaureate	8	7
Hong Kong DSE	6	6
Other	98	86
Unconditional Offer	317	300
TOTAL	***3548***	***3233***

Source: (University of Oxford, 2012)

The educational background of Oxford undergraduates:

	Acceptances 2012	
	Total	%
Comprehensive	703	26.1
Grammar	495	18.4
Sixth-Form Colleges	232	8.6
FE Institutions	51	1.9
Other Maintained	29	1.1
Maintained Sector	1,510	56.0
Independent	1,118	41.5
Other UK Institutions	6	0.2
Overseas Schools	7	0.3
Individuals	54	2.0
All Other Category	67	2.5
TOTAL	2,695	100.0

Source: University of Oxford (2012)

9.1 – Linear Algebra II Problem Sheet

Linear Algebra II, Problem Sheet 4

1. Show that a symmetric bilinear form is nondegenerate if and only if its matrix in a basis is invertible.

2. Let A be a symmetric matrix. Prove by using row and column operations that there is an invertible matrix P such that P^TAP is diagonal.

3. Find an orthogonal basis for the form on $\mathbb{R}^3$ given by the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

4. Find an orthonormal basis for the vector space P of all real polynomials of degree at most 3, with the symmetric form defined by $< f, g >= \int_{-1}^{1} f(x)g(x)dx$.

5. Let V be the vector space of real 2×2 matrices. Let $< A, B >= tr(AB)$. Determine the matrix of this form with respect to the standard basis of matrix units. Find an orthogonal basis for this form.

6. (a) Let W_1 and W_2 be subspaces of a vector space V with a symmetric bilinear form. Prove that $(\mathbf{a})(W_1 + W_2)^{\perp} = W_1^{\perp} \bigcap W_2^{\perp}$, $(\mathbf{b})W_1 \subset W_1^{\perp\perp}$, $(\mathbf{c})$ If $W_1 \subset W_2$, then $W_2^{\perp} \subset W_1^{\perp}$.

   (b) Let W be a subspace of a real finite dimensional vector space with a symmetric positive definite bilinear form. Show that $W^{\perp\perp} = W$.

7. Find a real orthogonal matrix P such that P^TAP is diagonal when A is the matrix: $(\mathbf{a})\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $(\mathbf{b})\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $(\mathbf{c})\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

8. Write $x^2 + xy + y^2 = 1$ in the form $(x, y)A\begin{pmatrix} x \\ y \end{pmatrix} = 1$ where A is a symmetric 2×2 matrix. Find an orthogonal matrix P such that P^TAP is diagonal. Use this to show that $x^2 + xy + y^2 = 1$ defines an ellipse. Sketch it and find its area.

Extra Problems:

9. Use the Spectral Theorem to prove that a positive definite real symmetric $n \times n$ matrix A has the form $A = P^TP$ for some P.

10. Let A, B be Hermitian matrices that commute ($AB = BA$). Prove that there is a unitary matrix P such that P^*AP and P^*BP are both diagonal.

11. Let A be a square complex matrix. Prove that $kerA = (imA^*)^{\perp}$. Deduce that if A is Hermitian, $\mathbb{C}^n = kerA \oplus imA$.

12. Show that if A is a nonsingular complex matrix, there is a positive definite Hermitian matrix B such that $B^2 = A^*A$, and that B is uniquely determined by A.

    Show also that A is a product $A = UP$ where P is positive definite Hermitian and U is unitary. Prove that this decomposition is unique.

1

Source: University of Oxford Mathematical Institute (2012)