Analysis of the asymptotic response of complete contacts - MATLAB code

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This document is intended as a guide for users to reproduce relevant figures from chapters 6-8 of the thesis 'Analysis of the asymptotic response of complete contacts.' These come from the three papers:

- **Riddoch, D. J.**, Cwiekala, N. and Hills, D. A., *Dislocations in an arbitrary angle wedge. Part II: Cracks in the wedge*, The Journal of Strain Analysis for Engineering Design: 03093247211047785, October 2021.
- **Riddoch, D. J.** and Hills, D. A., *Slip at the edge of complete contacts*, Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, https://doi.org/10.1177/09544062221142695, January 2023
- **Riddoch, D. J.** and Hills, D. A., *Distributed dislocation problems in wedge shaped domains* European Journal of Mechanics A / Solids, *in press*

1 Single crack

In this section, all results provided in section 3 of the first of these papers can be reproduced. For all the following steps to work properly, all 5 scripts should be downloaded and placed in the same destination folder; those being: 'SingleCrack.m,' 'SingleCrackCalib.m,' 'SCVaryAlpha.m,' SCVaryTheta.m,' and 'WilliamsEigenSol.m.' All codes were originally written in MATLAB R2017b, and have been run in R2022b. It is hoped they will continue to work in future releases but cannot be guaranteed.

1.1 Calibration matrices

In order to reproduce the calibration matrix for a single case, with a given combined contact half-angle α, and a given crack angle θ, begin by opening 'SingleCrack.m.' In the 'Inputs' section, changes can be made to specify precisely the wedge angle and crack angle(remember this is measured from the wedge bisector, with anti-clockwise being positive) required by the user. These are the 'alpha' and 'theta' variables defined in lines 20 and 21 respectively. All other vriables can be left at default values or changed as required. The code can then be run, and the values 'a, b, c, d,' in the workspace correspond to the matrix entries described in equation 13. The values 'lam1' and 'lam2' are the Williams eigenvalues, λ_I, and λ_{II}, respectively. The default values should reproduce the results given in equation 14, for an orthogonal crack in a three quarter plane.

1.2 Varying angles

The codes needed to reproduce figures 2 and 3 are functionally identical and so we will describe them at the same time. To reproduce figure 2, open 'SCVaryAlpha.m.' Again 'SingleCrack-Calib.m,' and 'WilliamsEigenSol.m' are required. The'Inputs' section allows the user to change the lower and upper bounds of α, 'alphaLower'(line 27) and 'alphaUpper'(line 28) respectively. Note that for $2\alpha < 257.4^o$, the mode II term is not singular so the results are not valid, and for $2\alpha \rightarrow 2\pi$, numerical errors will likely occur as the distributions of dislocations on the free surfaces approach one another. The value of θ can be changed in line 30, and the number of points along the x-axis is given by 'Pts,' line 26.

To reproduce figure 3, open 'SCVaryTheta.m.' Once again both 'SingleCrackCalib.m,' and 'WilliamsEigenSol.m' are required. The'Inputs' section allows the user to change the lower and upper bounds of θ, 'thetaLower'(line 28) and 'thetaUpper'(line 29) respectively. Note that for $\theta \rightarrow \alpha$, numerical errors will likely occur as the distributions of dislocations on the crack faces and the free surface approach one another. The value of α can be changed in line 27, and the number of points along the x-axis is given by 'Pts,' line 26.

In both cases, a numerical error occurs at a certain point in the default numerical range. This is due to a local failure of the solution to converge, and is easily rectified by changing the values of the constants in this region. This is done by the 'Patch' section in both codes, results are calculated and then inserted into the existing solution. The size of the patched area can be changed, as can the number of points used in the patch.

The default values in both codes will reproduce the figures 2 and 3.

2 Two crack

This section is intended to reproduce results provided in section 4 of the same paper, except the single calibration matrix where the incorrect results were published, these were later corrected by Riddoch and Hills: **Riddoch, D. J.** and Hills, D. A., *Distributed dislocation problems in wedge shaped domains*, European Journal of Mechanics - A/Solids, *in press*. For all the following steps to work properly, all six scripts should be downloaded and placed in the same destination folder, those being: 'TwoCrack.m,' 'TwoCrackCalib.m,' 'TCVarylR.m,' 'TCVaryAlpha.m,' 'TCVaryTheta.m,' and 'WilliamsEigenSol.m.' All codes were originally written in MATLAB R2017b, and have been run in R2022b. It is hoped they will continue to work in future releases but cannot be guaranteed.

2.1 Calibration matrix

In order to reproduce the calibration matrix for a single case, with a given combined contact half-angle α, and a given crack angle θ, begin by opening 'TwoCrack.m.' In the 'Inputs' section, changes can be made to specify precisely the wedge angle and crack angles(remember this is measured from the wedge bisector, with anti-clockwise being positive) required by the user. These are the 'alpha,' 'theta1,' and 'theta2,' variables defined in lines 20, 21 and 22 respectively. All other vriables can be left at default values or changed as required. The code can then be run, and the values 'a, b, c, d, e, f, g, h' in the workspace correspond to the matrix entries described in equation 25. The values 'lam1' and 'lam2' are the Williams eigenvalues, λ_I, and λ_{II}, respectively. The default values will reproduce the corrected results given in; **Riddoch, D. J.** and Hills, D. A., *Distributed dislocation problems in wedge shaped domains*, European Journal of Mechanics - A/Solids, *in press*; not those present in equation 26.

2.2 Crack length ratio

The results shown in figure 6 can be reproduced using the code 'TCVarylR.m.' Again, both 'TwoCrackCalib.m' and 'WilliamsEigenSol.m' are required. The inputs section allows the user to change the wedge angle and both crack angles independently. The crack lengths can be varied, and a vector is expected to be provided for 'cl2,' the length of the second crack. The number of Gauss points, and a multiplicative contant used in normalisation can also be changed, but this is not necessary and will affect the results.

2.3 Varying angles

The results shown in figures 7 and 8 can be reproduced using the codes 'TCVaryAlpha.m' and 'TCVaryTheta.m' respectively. Again, both 'TwoCrackCalib.m' and 'WilliamsEigenSol.m' are required. The inputs section of both codes allows the user to change crack lengths, number of Gauss points, multiplicative constant and the number of points in the scan. Also this is where the vectors for α and θ are created. By default when varying α, the θ values are set to depend on alpha, this can be changed if desired. When varying θ we have chosen to fix θ_1 and vary θ_2, although this can be changed if desired.

3 Slip

This section is intended to guide the user to recreate three figures from the paper: **Riddoch, D. J.** and Hills, D. A., *Slip at the edge of complete contacts*, Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, https://doi.org/10.1177/09544062221142695, January 2023. These being figures 6-8. For all steps to work properly the seven scripts must be downloaded and placed in the same destination folder, these being: 'SingleSlip.m,' 'SlipVaryCoF.m,' 'SlipVaryAlpha.m,' 'FindTolSeq.m,' 'CalculatePhiSeq.m,' 'FindStresses.m,' and 'WilliamsEigenSol.m.'

3.1 Traction plot

Figure 6 of the above paper can be reproduced using the code 'SingleSlip.m.' The code works by optimising the error produced from calculating the discarded equation. The parameter specification section allows the user to change the coefficient of friction, the wedge angle and the interface angle, as well as deciding which equation to discard, and setting the number of Gauss points desired. The default values are fairly slow, but fall within the converged range of the solution. The default values will reproduce figure 6.

3.2 Varying coefficient of friction

Figure 7 can be reproduced using 'SlipVaryCof.m.' The code works by optimising the error produced from calculating the discarded equation. The parameter specification section allows the user to change the wedge angle and the interface angle, as well as deciding which equation to discard, and setting the number of Gauss points desired. The default values are fairly slow, but fall within the converged range of the solution. The 'Varying the Coefficient of Friction', section, allows the user to select upper and lower bounds for the coefficients of friction to be considered, recall the bounds set out on these values in the paper. The default values will reproduce figure 7.

3.3 Varying angles

Figure 8 can be reproduced using 'SlipVaryAlpham.' The code works by optimising the error produced from calculating the discarded equation. The parameter specification section allows the user to change the coefficient of friction and the interface angle, as well as deciding which equation to discard, and setting the number of Gauss points desired. The default values are fairly slow, but fall within the converged range of the solution. The 'Varying the wedge angle', section, allows the user to select upper and lower bounds for the wedge angle to be considered, recall the bounds set out on these values in the paper. The default values will reproduce figure 8.

4 Dislocation methods

This section allows the user to recerate figure 4 from the paper **Riddoch, D. J.** and Hills, D. A., *Distributed dislocation problems in wedge shaped domains* European Journal of Mechanics A / Solids, *in press*. For all steps to work properly the six scripts must be downloaded and placed in the same destination folder, these being: 'SlipConvergenceTest.m,' 'FindTolSim.m,' 'FindTolSeq.m,' 'CalculatePhiSeq.m,' 'FindStresses.m,' and 'WilliamsEigenSol.m.'

To reproduce figure 4, open 'SlipConvergenceTest.m.' The parameter specification section allows the user to change the loading, the wedge and interface angles, the coefficient of friction and the multiplicative constant. The number of Gauss points can be changed in the 'Varying N' section, with a minimum and maximum value to be set. The ratio can be changed in the Paramter section within the loop. The default values will reproduce figure 4.