

# Bayesian adaptive design: Improving the effectiveness of monitoring of the Great Barrier Reef

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## Abstract

Monitoring programs are essential for understanding patterns, trends and threats in ecological and environmental systems. However, such programs are costly in terms of dollars, human resources and technology, and complex in terms of balancing short- and long-term requirements. In this work, we develop new statistical methods for implementing cost-effective adaptive sampling and monitoring schemes for coral reef that can better utilize existing information and resources, and which can incorporate available prior information. Our research was motivated by developing efficient monitoring practices for Australia's Great Barrier Reef. We develop and implement two types of adaptive sampling schemes, static and sequential, and show that they can be more informative and cost-effective than an existing (non-adaptive) monitoring program. Our methods are developed in a Bayesian framework with a range of utility functions relevant to

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environmental monitoring. Our results demonstrate the considerable potential for adaptive design to support improved management outcomes in comparison to set-and-forget styles of surveillance monitoring.

**Key Words:** adaptive design; Bayesian inference; coral reef ecosystems; reef monitoring; utility functions.

## Introduction

Monitoring programs are essential for understanding patterns, trends and threats in ecological and environmental systems (Lovett et al., 2007). They also underpin the design and performance assessment of adaptive management regimes. Monitoring programs, however, are notoriously expensive in terms of monetary, human and technological resources, and are complex in terms of balancing short- and long-term requirements. In addition to these constraints, it is of utmost importance for a monitoring program to have a robust design that is appropriate for addressing designated monitoring objectives and high quality data collection (Chapman, 2012). As knowledge, monitoring methods and technologies, and objectives evolve, monitoring programs should ideally adapt with new and better designs and/or the addition of variables to the suite of variables already being monitored. To be effective, a monitoring program should also have clearly specified and measurable information objectives and while adapting to new knowledge and techniques, maintain as much as possible the integrity of the long-term data record (Nichols and Williams, 2006).

The Great Barrier Reef (GBR) is the world’s largest coral reef ecosystem, extending

for over 2300 kilometers along the north-east coast of Queensland, Australia. The GBR is listed as a World Heritage Site by the United Nations Educational, Scientific and Cultural Organization (UNESCO) for its unique natural properties and enormous scientific and environmental importance. It is also of significant economic value worth nearly 6 billion dollars per annum (Deloitte Access Economics, 2013). In light of the ecological and economic significance of the GBR, the Australian Institute of Marine Science’s Long-Term Monitoring Program (LTMP) has monitored the status and trends in the distribution and abundance of reef biota on a large spatial scale for approximately 3 decades (Sweatman et al., 2008). As a result, the LTMP now provides the longest continuous record of change in reef communities over such a large geographical area. A perceived strength of this program is that since its inception it has used standardized sampling protocols with calibrated observers and a sampling design that has remained unchanged over considerable periods of time.

Like any other monitoring program, an important constraint on the LTMP is the high cost of collecting data. In this case, monitoring costs include the labour costs of people with the necessary skills and qualifications to do the monitoring and curate the data, as well as the costs of equipment and ship running costs. Clearly, these costs increase with the size of the reef area monitored, the number of transects surveyed, and how these transects are distributed among the number of sites within reefs, and reefs within locations sampled. Therefore, given a set of monitoring objectives it is important to design a program that maximizes the information gained for the investment made in monitoring against those objectives. One possible way to increase the return on investment in monitoring would be to move away, to some degree, from a design which is fixed in space

and time to one that can be adapted based on current knowledge of the system and current information priorities. Motivated by this possibility, we explore here adaptive sampling schemes for the GBR whose adaptation is informed by statistical models and analyses generated from historical data.

While adaptive designs have been used in other areas, including computer experiments (Williams et al., 2000), social and survey analyses (Creswell et al., 2003), and clinical trials (Müller and Schäfer, 2001), the application of adaptive designs to monitoring in ecological and environmental sampling and monitoring is still in its infancy, but see Falk et al. (2014). While adaptive ecological monitoring has received some attention, the sorts of approaches proposed tend not to rely on adaptive design but rather rely on changes in a sampling design when new questions arise (Lindenmayer et al., 2011). In contrast, here we use current knowledge and ongoing sampling to estimate model parameters to address specific objectives and use these data to design the next sampling points. Such decisions are based on maximizing a defined utility function that quantifies the usefulness of a particular design in estimating some value of interest related to the monitoring objective being considered. Here, the design could include any aspects of the data collection that can be controlled such as where and when to collect data. Such utility functions could also be used to estimate the usefulness of a sampling design in estimating model parameters and generating accurate predictions (Berger, 1985, Chapter 2). The challenge in this quantification is integrating over uncertainty which may exist in, for example, the parameter values and the data which will be observed.

A significant challenge in the application of such utility functions is their estimation under potentially considerable uncertainty. Such uncertainty can arise from numerous

sources including the estimation of parameter values and the data which are observed. Here, in order to account for such uncertainties, we use a Bayesian linear regression model to describe percentage coral cover as a measure of the state of monitored reef communities. Focusing on a single sub-region (the Cooktown-Lizard Island sub-region) of the GBR sampled by the LTMP, we illustrate how adaptive designs might be useful in providing better estimates of coral cover more efficiently. Two types of monitoring schemes are developed and implemented, namely static and sequential sampling schemes. Changes in expected utilities, where the expected utility is the weighted average utility of the possible outcomes associated with these two types of design adaptation are also explored.

We begin by providing details regarding the nature of the dataset used, the statistical model applied to these data, plus additional information regarding monitoring frameworks and objectives of the LTMP. Next, several Bayesian utility functions that may be of interest in such sampling designs are discussed. We then detail the two alternative Bayesian adaptive sampling designs and algorithms: the static and the sequential design. Finally, we discuss the results of this case study regarding the potential for adaptive sampling designs to provide better utility across a range of monitoring objectives for estimating coral cover the GBR.

## Methods

### The design of the LTMP

The LTMP tracks changes in reef communities over time in six sub-regions of the GBR arrayed north to south. Reefs in a sub-region lie in one of 3 positions across the shelf (inshore, mid-shelf, and outer-shelf) (Figure 1). These shelf positions are referred to hereafter as habitats. Inner-reefs are the closest to the coast and are most exposed to terrestrial and human influences (Bellwood et al., 2004). The mid-shelf habitat lies between the inner and outer-shelf habitats, extending over a large part of the GBR lagoon. The outer-reef habitat is exposed to more oceanic conditions. The LTMP sampled the benthic communities of 47 reefs on the GBR annually from 1991 to 2004 and every second year since. The surveys sample 5 permanent  $50 \times 1\text{m}^2$  transects located between 6 and 9m depth at 3 sites per reef using video (prior to 2006) and high-resolution digital still images (from 2006). From these samples, percentage of hard coral cover was estimated for all genera (Jonker et al., 2008). During intervening years from 2005 onwards, a series of paired reefs, open and closed to fishing, were monitored using the same methods as the LTMP to assess the effects of the re-zoning of the GBR World Heritage Area. These surveys constitute the Representative Areas Program (RAP) monitoring program.

Here we use LTMP and RAP data from a single sub-region of the GBR, the Cooktown-Lizard Island sub-region, collected from 1991 to 2010 inclusive. This sub-region is the northern-most section of the GBR sampled by the LTMP, spanning latitudes from  $14^\circ \text{ S}$  to  $15^\circ 50' \text{ S}$ . In this sub-region, 2 reefs are sampled in the inner-shelf habitat and 3 reefs are sampled per habitat in the mid-shelf and outer-shelf habitats.

## A model of coral cover

In order to explore the effects of adaptive sampling designs on various monitoring objectives, these coral cover data were fitted to a statistical model that captured the dynamics of coral cover over 20 years (Osborne et al., 2011). The response of utility values could then be examined as the sampling regime was modified. The estimates of percentage cover of corals across the Cooktown Lizard Island sub-region were converted into proportions which ranged from 0.0037 to 0.6150, with a median of 0.0100 and a mean of 0.0226. Let  $y_i$  denote the coral cover proportions observed for a site  $i$ . To ensure that the response variable approximately followed a Normal distribution, a logit transformation was applied to the coral cover proportions, i.e.,  $\text{logit}(y_i) = x_i^T \boldsymbol{\beta} + \epsilon_i$ . The histogram plot of  $\text{logit}(y_i)$  confirmed that the proportions were approximately Normally distributed. A logit transformation is appropriate for analyzing proportional data in ecology (Warton and Hui, 2011) as it maps proportions to the whole real line and has a natural interpretation. Here,  $x_i$  is a  $k \times 1$  predictor vector,  $\boldsymbol{\beta}$  is a  $k \times 1$  vector of coefficients, and  $\epsilon_i$  are the independent and identically distributed random variables, i.e.  $\epsilon_i \sim N(0, \sigma^2)$ . Bayesian posterior inference for the described model can be found in Gelman et al. (2014) and Appendix S1.

We fitted a range of candidate models for estimating coral cover (Cover) with different combinations of covariates, including Year, Year<sup>2</sup>, Shelf, Reef, Site, and Opened/Closed (O/C) Reef. The model chosen for further exploration was the one containing covariates that were deemed to have a substantive effect on coral cover, in that the 95% credible intervals of the associated regression coefficients did not encompass zero.

The linear model adopted was,

$$\text{logit}(\text{Cover}_i) = \beta_0 + \beta_1 \cdot \text{Year}_i + \beta_2 \cdot \text{Year}_i^2 + \beta_3 \cdot \text{Mid shelf}_i + \beta_4 \cdot \text{Outer shelf}_i + \beta_5 \cdot \text{Opened Reef}_i + \epsilon_i. \quad (1)$$

The baseline categories for Shelf and O/C Reef are inshore and closed reef, respectively, and are incorporated in  $\beta_0$ . Reefs within shelf positions did not appear to have a substantive effect on coral cover. Therefore, our adaptive designs chose among options for sampling different combinations of habitats represented by reefs within habitats.

We conducted posterior predictive checks for model (1) by simulating replicated data under the fitted model and then comparing these to the observed data (Gelman and Hill, 2007). Posterior predictive checks allowed us to assess if the fitted model was appropriate for the observed data. The posterior predictive samples fitted the observed percentage coral cover well on both the logit and the natural scales (see Figure 2). We note that in our analysis we have used a Gaussian approximation to the logit of coral cover, hence on the logit scale we obtained a symmetric posterior predictive distribution. The posterior predictive samples matched the observed data very well on the scale of percentage coral cover. We also found that around 96% of the observed data lay inside its respective 95% credible interval of the posterior predictive distribution. This suggests that model (1) is adequate in estimating coral cover. In addition, we performed sensitivity analysis (Saltelli et al., 2000) on the priors for the model parameters and found them to be insensitive to different prior values.

In order to explore the adaptive design of the LTMP, a true model had to be assumed so that data could be generated for selected design choices. A summary of the



posterior distributions of the parameters of model (1) is presented in Table 1. For simulating data in the adaptive sampling design algorithms discussed later in this paper, this model was assumed to be the true model as were the posterior means assumed to be true parameter values. This model, however, does not account for spatial variability as the reefs were located sufficiently far apart for there to be negligible spatial dependence when spatial analyses were performed on the data.

## Monitoring frameworks and objectives

Based on model (1), we identified a number of sampling or monitoring objectives relevant to the LTMP. We note that the monitoring objectives of the LTMP are rather informal; its main purpose being the surveillance of the state and trends of the GBR. Consequently, the LTMP was designed to sample sub-regions and shelf positions (habitats) to represent biotic and abiotic gradients along and across the continental shelf. Here we explore two sampling frameworks for implementing these objectives, namely a static and a sequential design.

Using 20 years of data, the static design framework informs the sampling plan for a 6-year period into the future without considering new information that becomes available during those six years (see Algorithm 1 in Appendix S2). In this sense, this approach is time invariant in that the same set of reefs in all 3 habitats is sampled in all of the 6 future years. It is, however, still adaptive compared to the present strategy for the LTMP since it incorporates historical data to make decisions about how data should be collected in the future. In contrast, in addition to using the existing 20 years of data collected by the LTMP, the sequential design framework uses new information that becomes available at

each sampling time to determine the sampling plan for the next sampling year. That is, which habitats and reefs within habitats are visited are optimized based on all available data. The design algorithm iteratively updates existing information using newly collected data while taking into account information gain and monitoring costs. Here, travel costs from one habitat to another and reefs within habitats are considered as monitoring costs, but other costs could also be incorporated.

To implement these monitoring frameworks, a range of design choices needs to be considered. In the first design choice (Design D1), each LTMP habitat in the Cooktown-Lizard Island sector is sampled every year (2011, 2012, 2013, 2014, 2015, 2016), resulting in 6 visits to each habitat during these 6 years. Design D2 samples each habitat every 1.5 years (mid-2011, 2013, mid-2014, 2016), resulting in 4 visits over the 6 years. The third design choice mirrors the current practice of the LTMP where sampling of the GBR is conducted every second year (2012, 2014, 2016), resulting in 3 visits to each habitat during these 6 years. We name this design the ‘LTMP design’. For Design D3, each habitat is sampled every 3 years (2013, 2016), resulting in 2 visits to each habitat over the 6 years. Design D4 samples from each habitat once only; in the 6<sup>th</sup> year (2016).

In order to compare the sequential design approach with the static design approach, we performed 300 simulation runs for the sequential sampling algorithm outlined in Algorithm 2 in Appendix S2 for Design D1 and the LTMP design. Note that 500 observations were collected at each habitat per visit for Design D1 whereas 1000 observations were collected at each habitat per visit for the LTMP design. This gave rise to the equal sample sizes for the two design choices at the end of 6 years. The main difference here between the sequential and the static framework is that the habitats and reefs within

habitats were adaptively and sequentially chosen whereas habitats and reefs within habitats in the static design were predetermined.

To illustrate situations in which adaptive designs may be beneficial, five objectives were investigated and contrasted via these two sampling frameworks. Objective (a) aims to maximize the precision of all parameters in estimating coral cover in the 6<sup>th</sup> year. This objective may be useful say for assessing improvements in water quality, where the effects of remediation on land are not expected to be detectable for some considerable period. Objective (b) aims to maximize the precision of the time-related parameters in estimating coral cover in the 6<sup>th</sup> year. This objective may be useful when the effect of a subset of parameters is of interest, e.g. the effect of a few main factors in improving water quality. Objective (c) aims to maximize the overall precision in predicting coral cover in the 6<sup>th</sup> year. This objective may be suitable for assessing the impact of an intervention, say fishing closure on the biomass of predatory fishes, the effect of which may be detectable after a relatively short period. Objective (d) aims to maximize the precision of all parameter estimates in the 6<sup>th</sup> year while also maximizing the precision of prediction in the 6<sup>th</sup> year. This objective may be useful for assessing the joint effects of several simultaneous interventions, e.g. assessing concurrent improvements in water quality and the impact of fishing closure. Lastly, the combined objective (e) aims to maximize the overall precision in predicting coral cover and minimize the monitoring costs within a certain year. For this objective, a sequential sampling algorithm was designed to sequentially update existing information using new information (data). The algorithm takes into account sequential information gain as well as the sequential monitoring costs of visiting and sampling a reef given its location relative to other reefs and reefs within habitats.

Given the monitoring objectives described above, we know intuitively that in order to obtain the best estimates of coral cover in the 6<sup>th</sup> year, all the sampling effort should most likely be allocated to that year. In this context, this objective may not seem very informative from a practitioner’s point of view. Our intent is not to provide a trivial example in presenting this design. Instead, our intent in presenting this design is to provide a baseline case against which the strength the other designs can be compared. Alternatively, all sampling could be allocated to an intermediate year, say the 3rd year, if a different baseline was of greater interest.

## Bayesian utility functions

A utility function is required to quantify the usefulness of sampling designs. In Bayesian design, the expected utility,  $U(\mathbf{d}) = \mathbb{E}[u(\mathbf{d}, \mathbf{y})]$ , is maximized, with respect to a given design  $\mathbf{d}$ , for response data  $\mathbf{y}$ , modeled by the likelihood function  $p(\mathbf{y}|\boldsymbol{\theta}, \mathbf{d})$  and prior  $p(\boldsymbol{\theta})$  with  $p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) \propto p(\mathbf{y}|\boldsymbol{\theta}, \mathbf{d})p(\boldsymbol{\theta})$ . An optimal design  $\mathbf{d}^*$  can therefore be expressed as

$$\mathbf{d}^* = \arg \max_{\mathbf{d} \in D} U(\mathbf{d}), \text{ where } U(\mathbf{d}) = \int_{\mathbf{y}} u(\mathbf{d}, \mathbf{y})p(\mathbf{y}|\mathbf{d})d\mathbf{y},$$

where  $\mathbf{y}$  is generated from the likelihood  $p(\mathbf{y}|\boldsymbol{\theta})$  and  $\boldsymbol{\theta}$  is generated from the prior  $p(\boldsymbol{\theta})$ , such that  $U(\mathbf{d})$  is the expected utility for design  $\mathbf{d} \in D$ , the set of possible designs. Bayesian utility functions are typically functions of the posterior  $p(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})$ , so the above integral will not generally have an analytic form, and hence will need to be approximated. Here, we use Monte Carlo integration (Geweke, 1989) to approximate the expectation as follows.

$$\hat{U}(\mathbf{d}) = \frac{1}{M} \sum_{m=1}^M u(\mathbf{d}, \mathbf{y}^{(m)}),$$

where  $\mathbf{y}^{(m)}$  is drawn from  $p(\mathbf{y}|\boldsymbol{\theta}^{(m)}, \mathbf{d})p(\boldsymbol{\theta}^{(m)})$ .

A number of utility functions relevant to the present case study were identified.

Corresponding to objective (a), where the aim was to maximize the (joint) posterior precision of all of the model parameters  $\boldsymbol{\theta}$  in estimating coral cover in the 6<sup>th</sup> year, the inverse of the determinant of the posterior variance-covariance matrix is a useful utility. The utility is also known as the ‘Bayesian D-posterior precision’ (Drovandi et al., 2013) and is given by:

$$u(\mathbf{d}, \mathbf{y}) = \frac{1}{\det(\text{var}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}))},$$

Note that  $\boldsymbol{\theta} = \boldsymbol{\beta}$  and the parameter  $\sigma^2$  is not considered here. This particular criterion is very important in the context of the LTMP as it allows us to estimate model parameters as well as possible from monitoring data from the GBR. This criterion allows us to assess the relative importance of the corresponding covariates, which can be used for allocating future sampling effort. The uncertainty of these parameter estimates can also help guide future decisions about effort allocation. In turn, by modifying the sampling design through time, the model should improve through time as the process is iterative.

The ‘Bayesian D-posterior precision’ utility is related to the ‘D-optimality’ utility (John and Draper, 1975). We note that other utilities may be applicable in the Bayesian framework applied here. For instance, ‘A-optimality’ which minimizes the average variance

of the estimates of the regression coefficients, ‘C-optimality’ that minimizes the variance of a best linear unbiased estimator of a predetermined linear combination of model parameters, and ‘E-optimality’ that maximizes the smallest eigenvalue of the information matrix (Atkinson et al., 2007). A fuller exploration of the application of these alternative utilities is beyond the scope of the present paper, but such exploration in the future should assist in achieving a much fuller appreciation of the benefits that might be derived through the application of different utilities in the design of monitoring programs.

With respect to objective (b), where the aim was to maximize the precision of the time-related parameters in estimating coral cover in the 6<sup>th</sup> year, a useful utility is the ‘Bayesian D<sub>s</sub>-posterior precision’. The utility maximizes the precision of parameter estimates for a reduced number of the complete set of unknown model parameters. It is appropriate when the goal is to estimate a subset of  $s$  parameters as precisely as possible (Box, 1971). The posterior variance-covariance matrix is partitioned into

$$\text{var}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) = \begin{bmatrix} \text{var}_{11}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) & \text{var}_{12}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) \\ \text{var}'_{12}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) & \text{var}_{22}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d}) \end{bmatrix},$$

where  $\text{var}_{11}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})$  is an  $s \times s$  submatrix which denotes the variance-covariance of the parameters of interest and  $\text{var}'_{12}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})$  is the transpose of  $\text{var}_{12}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})$ . The utility thus aims to maximize the following,

$$u(\mathbf{d}, \mathbf{y}) = \frac{\det\{\text{var}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})\}}{\det\{\text{var}_{22}(\boldsymbol{\theta}|\mathbf{y}, \mathbf{d})\}}.$$

With regard to objective (c), where the aim was to maximize the overall precision in

289 predicting coral cover in the 6<sup>th</sup> year, the corresponding prediction utility is similar to that  
 290 in Diggle and Lophaven (2006). Under each design option ( $\mathbf{d} \in D = 1, 2, 3, 4$ ), the utility is  
 291 the inverse of the averaged prediction variance at each habitat ( $h = 1, 2, 3$ ), with  $\hat{\mathbf{y}}_h$   
 292 referring to predictions at habitat  $h$ , i.e.

$$u(\mathbf{d}, \mathbf{y}) = \frac{1}{\sum_h \text{var}(\hat{\mathbf{y}}_h | \mathbf{y}, \mathbf{d})}.$$

293 Corresponding to objective (d), where the aim was to maximize the precision of all  
 294 parameter estimates in the 6<sup>th</sup> year while also maximizing the precision of prediction in the  
 295 6<sup>th</sup> year, combining the ‘Bayesian D-posterior precision’ and the prediction utility results in  
 296 the following utility,

$$u(\mathbf{d}, \mathbf{y}) = -\alpha \cdot \log(\det(\text{var}(\boldsymbol{\theta} | \mathbf{y}, \mathbf{d}))) - (1 - \alpha) \cdot \log\left(\sum_h \text{var}(\hat{\mathbf{y}}_h | \mathbf{y}, \mathbf{d})\right), \quad (2)$$

297 where  $0 \leq \alpha \leq 1$  determines the weight given to each of the utilities. The weight can be  
 298 altered according to the importance of each utility. Examples of compound criteria in the  
 299 non-Bayesian design context can be found in Atkinson (2008) and McGree et al. (2008).

300 Similarly, for objective (e), where the aim was to maximize the overall precision in  
 301 predicting coral cover and minimize the monitoring costs within a certain year, the utility  
 302 is,

$$u(\mathbf{d}, \mathbf{y}) = -\alpha \cdot \log\left(\sum_h \text{var}(\hat{\mathbf{y}}_h | \mathbf{y}, \mathbf{d})\right) - (1 - \alpha) \cdot \log(\text{Cost}_h), \quad (3)$$

303 where the subscript  $h = 1, 2, 3$  refers to the 3 habitats. Here  $\text{Cost}_h$  is calculated using the

distance between habitats while assuming a fixed cost per unit of distance.

In this paper, compound utility functions were constructed through a weighted linear combination of utility functions. For such utilities, the difficulties in appropriately defining the weight parameter have been noted in the literature, see Clyde and Chaloner (1996). Indeed, here, the variance of model predictions is on the same scale as the monitoring costs, so it may be difficult to weight each utility appropriately. Other approaches could be explored, including maximizing the precision of parameter estimates conditional on a cost constraint, see Clyde and Chaloner (1996) and McGree et al. (2008).

## Bayesian adaptive sampling designs and algorithms

### Static design

In the static sampling framework, we are interested in implementing sampling designs for collecting coral cover data for a period of 6 years (2011 to 2016) beyond the end of the time series used here (2010). Various designs for data collection have been discussed above.

These design choices allow us to compare the information gain under 4 hypothetical sampling scenarios, described below. Most importantly, we are able to compare our proposed sampling strategies with the current sampling practice implemented by the LTMP.

In order to explore implications for changing levels of total sampling effort, four hypothetical sampling scenarios for data collection were designed and named Case 1 to 4. In Case 1, the total samples collected over the 6-year period is fixed at  $N = 9000$



observations. For Cases 2 to 4, the samples collected during each visit to each habitat are fixed at different sizes, i.e.,  $n = 250, 500, 750$  observations respectively, resulting in different overall sample sizes for different designs. We note that Design D4 and Case 4 may be viewed as potentially unrealistic; waiting 6 years before discovering the state of the reef may be unacceptable and it may not be possible to sample 3000 observations in the same habitat at once. However, the inclusion of these extreme scenarios here is important as they allow us to understand the impact of the frequency of sampling and the total samples collected with respect to different monitoring objectives. They also allow us to investigate the extent to which the information about these reefs has been gained over different frequencies of sampling and samples sizes and, most importantly, contrast the current LTMP design with other monitoring approaches.

It is not necessary to predetermine the sampling years at the start of monitoring. A plausible design scenario would be to adopt an algorithm to determine the optimal sampling frequency within 6 years and the associated optimal sampling times. For instance, for the LTMP design in which the sampling frequency is 3 times within 6 years, the optimal sampling years could be (2011, 2013, 2015), or (2012, 2013, 2014), etc. Similarly, there are different possibilities of sampling years for Designs D3 and D4. For simplicity, here we focus on the fixed sampling designs and sampling scenarios illustrated in Table 2.

Since the true model is not known *a priori*, we assume that model (1) is the true model for predicting coral cover. A static sampling algorithm is outlined in Algorithm 1 in Appendix S2, and is used to design the sampling plan for the forthcoming 6 years and based on the existing 20 years of coral reef data. Several utility functions are calculated using the static sampling algorithm, where each utility corresponds to a specific objective.

For a given design, the algorithm firstly simulates parameters from the posteriors of model (1). Given the drawn parameters and the design, data are then simulated. The simulated data are appended to the historical data, and model (1) is fitted to the combined data to estimate the posterior distribution of the parameters of model (1). If required, parameters of interest and predictions are generated from the respective posterior distributions. The estimated expected utility is found as the average of the utilities obtained from the Monte Carlo simulations.

## Sequential design

The sequential design makes use of the existing 20 years of coral reef data to inform the choice about the best habitat to visit sequentially, taking into account information gain and monitoring costs. Similar to the static design, model (1) is used in this sampling algorithm (see Algorithm 2 in Appendix S2). The design is iterative and evolves within an annual sampling campaign conditional on the observed data at any point in time. Here we calculate all utility functions discussed earlier, with a special focus on utility (3) where the objective is to learn about the precision of coral cover prediction at each habitat while considering traveling costs from one habitat to another.

For a given habitat, the algorithm firstly samples parameter values from the posterior given  $\mathbf{y}_{(kept)}$  and  $\mathbf{d}_{(kept)}$  of the assumed true model. Given the drawn parameter values, an equal amount of new data are simulated for each reef within the habitat and appended to the historical data. Model (1) is then fitted to the combined data to estimate the posterior distributions of the parameters. Predictions are generated using the

estimated parameters and the cost of visiting each habitat calculated. For each habitat, a utility is calculated using the information on predictions and costs. Under each visit, the estimated utility for each habitat is obtained. The best habitat to visit is the one with the greatest estimated expected utility while taking travel cost into consideration. Sometimes the next chosen habitat might turn out to be the same one as the one just sampled. Under the assumption that model (1) is the true model with parameters given by the posterior mean based on  $\mathbf{y}_{(kept)}$ , the data collected at the next habitat are simulated under model (1) and appended to the historical data. The updated data are brought forward to the next iteration of the algorithm.

The sequential design approach could be extended and made more realistic by using more realistic monitoring costs. Such costs are not recorded in the LTMP database and we were reluctant to try to reconstruct them retrospectively. Intuitively the greatest monitoring cost would be travel time and its associated staff and ship running costs. Hence, in the absence of more direct estimates of costs, distance is likely to be a useful surrogate for cost in our investigations here where the primary goal was to explore relative benefits of different design choices in a single sub-region of the GBR. Future studies may wish to incorporate more direct estimates, where available, to assess absolute differences in costs, including the costs of transit between sub-regions.

## Results

For Case 1 where  $N$  was fixed at 9000, the utility increased gradually as the frequency of sampling decreased (Figure 3(a)). Designs D3 and D4 were better sampling choices than

the LTMP design because the information gained, or the ‘Bayesian D-posterior precision’ of these designs, was larger than that of the LTMP design. Therefore, if the objective is to obtain the best possible estimate in the 6<sup>th</sup> year, all resources should be allocated to sampling in the 6<sup>th</sup> year. While this result may be intuitive, it presents a baseline against which other designs can be compared. For Cases 2 ( $n = 250$ ), 3 ( $n = 500$ ) and 4 ( $n = 750$ ), the expected utility reduced gradually as the number of visits decreased, since the decreasing amount of data collected resulted in less information gain (Figure 3(a)).

For objective (b), where the precision of the time-related parameters (Year + Year<sup>2</sup>) were maximized, ‘Bayesian D<sub>s</sub>-posterior precision’ reported in Figure 3(b) shows a similar pattern to Figure 3(a). For Case 1, Designs D3 and D4 were better sampling choices than the LTMP design. For Cases 2, 3 and 4, Design D1 was the best choice due to the larger size of the samples collected. It is interesting to note that increasing  $n$  from 500 to 750 under various design choices improves the information gain by a smaller degree, as compared to the improvement incurred by changing  $n$  from 250 to 500. This indicates the possible existence of a threshold level for the amount of data required to maximize the information with respect to the precision of all the time-related parameters. The extra or unallocated resources could thus be spent on other aspects of monitoring.

For all sampling scenarios that maximize the overall precision in predicting coral cover in the 6<sup>th</sup> year (Figure 3(c)), Design D1 appeared to be the best choice, that is, it had the largest utility. Evidently, precision of prediction decreased as the number of visits decreased. The change in sample size ( $n$ ) had a considerable effect on the precision of prediction for Design D1 but the least effect with Design D4.

The combined expected utility produced under the multiple objectives (i.e., utility

(2)), to maximize the precision of all parameters in estimating coral cover at the same time maximizing the overall precision in predicting coral cover in the 6<sup>th</sup> year, is given in Figure 3(d). Interestingly, the combined expected utility of the ‘Bayesian D-posterior precision’ and the prediction utility showed a very similar pattern to Figure 3(c). Using the combined expected utility, we calculated the efficiency of each design with respect to the optimal approach in order to better understand the performance of this utility. To evaluate this efficiency, the design found by the combined utility was evaluated under the ‘Bayesian D-posterior precision’ and prediction utilities. An efficiency under each of these utilities was then estimated by dividing each value by the maximum possible utility value (as given by optimal approaches under each utility function). These efficiencies are shown in Figure 4. Given that all efficiencies were around one, it shows that the combined design is highly efficient under both utilities, and thus gave rise to similar performance as shown in Figures 3(c) and 3(d).

The LTMP design under the static framework outperformed the other 3 scenarios for objective (a) (Figure 5(a)). Under the sequential framework, Bayesian D-posterior precision increased gradually as the sampling year increased, suggesting that more samples gave rise to greater information. The LTMP design outperformed Design D1 under both the static and sequential frameworks.

With respect to objective (b), Design D1 and the LTMP design under the sequential framework produced higher expected utilities than the static framework (Figure 5(b)). The same level of information gain achieved in 2016 using the static framework was achieved in 2014 using the sequential framework, indicating that the static framework was less efficient in that it required two extra years of sampling to achieve the same amount of information

collected under the sequential framework. These significant resource savings could be allocated to other aspects of monitoring.

Similarly, Figure 5(c-d) shows that Design D1 and the LTMP design under the sequential framework were better designs than the static framework for objectives (c) and (d). Here the differences between Design D1 and the LTMP design under the same framework were not substantial. The same level of information gain achieved in 2016 under the static framework was achieved as early as 2013 or 2014 for the sequential framework. This, again, indicates that the sequential framework was more efficient than the static framework for this particular set of objectives.

Under the sequential framework, each habitat was equally likely to be chosen as the best habitat for sampling and the choice of the best reef was independent of each other (Figure 6). In general, the information gained from sampling the inner-shelf habitat was higher than in the two other habitats due to a smaller travel cost incurred when sampling inshore as this habitat is also closest to shore. The inshore was followed by the mid-shelf habitat and then outer-shelf habitat in terms of the amount of information gained.

## Discussion

In this research, we have developed and explored static and sequential sampling approaches in a Bayesian design framework for monitoring coral cover on the GBR. We have shown that given the chosen objectives, our adaptive sampling designs consistently performed at least as well as, and in some cases substantially better than, the current non-adaptive LTMP design. This research demonstrates the first instance of monitoring coral reef

communities in a Bayesian experimental design context, where reduced cost or resources and improvement in information gain were observed when addressing specific objectives. Several utility functions that are new to Bayesian design have also been introduced.

We have used the static sampling design to make decisions about the best sampling choice in the next 6 years for different monitoring objectives, using information obtained from the historical data. Given the results, the LTMP design does not appear to be the ideal sampling choice for the objectives investigated here. Based on these results and depending on the objectives selected in the future, the sampling design for the LTMP could be adapted to achieve greater utility. In particular, Design D4 was the best choice given objectives (a) and (b) whereas Design D1 was a better choice for objectives (c) and (d).

The purpose of our proposed sequential sampling algorithm was to develop a sampling plan that sequentially updates existing information using newly collected data, while taking into account the costs incurred during data collection. At present, the LTMP samples all 3 habitats in the Cooktown-Lizard Island sector every alternate year, without considering past information and costs. However, the parameter estimates in Table 1 suggest that some habitats are more informative in terms of the information gained from sampling them, and hence, it may not be necessary to visit all 3 habitats in a particular sampling year.

A limitation of our algorithm is that for each proposed visit, it only aims to minimize the cost incurred during the next visit without considering the total costs incurred for all planned visits. This single-step approach may lead to some inefficiency in terms of habitat selection. Although it is conceptually feasible to design an exhaustive search algorithm that explores all possible candidates for all desired visits, taking into

account the total costs incurred, doing so would be far more computationally challenging. In view of this, we suggest that the proposed one-step-ahead algorithm is sufficient as it combines computational feasibility with capability in sequentially updating existing information using new data.

The incorporation of costs into the sequential design framework has also enabled us to explore the allocation of resources more effectively to habitats that require the most attention and sampling. The sequential framework has demonstrated its advantages and flexibility in monitoring as compared to the static framework. For instance, for objectives (c) and (d) the same level of information gain achieved in 2016 under the static framework was achieved as early as 2013 or 2014 under the sequential framework.

The approaches developed here can be readily adapted to other monitoring objectives in the context of the LTMP and corresponding adaptive sampling designs and to other monitoring programs. These objectives could involve other aspects of coral cover, crown-of-thorns starfish population estimation, and comparisons of the responses of fish and benthic communities to opened and closed reef-use zoning. It may also be desirable to build more flexible models that are capable of capturing the changes in coral cover following disturbances. Any of these possible designs could also be extended to account for model uncertainty using appropriate utility functions.

The sequential sampling framework proposed in this paper could also be extended in a number of ways. For example, assuming that we start with some available resources (say \$100k), we can use the sequential sampling algorithm to decide the first habitat to visit. The remaining resources then become \$100k minus the cost of visiting the first habitat. We could then use the algorithm to decide on the second habitat to visit. The remaining



resources are \$100k minus the cost of visiting the first habitat and the second habitat. A third habitat could be visited if all resources had not yet been consumed. Such an iterative process would continue until all resources are consumed by maximizing information gain while prioritizing the habitats that reveal the most information. A constraint of the resources allocated to each visit could be imposed to ensure that a roughly equal amount of resources are expended for each visit.

By using a relatively simple model, some computational issues common in Bayesian design are mitigated. We note that a more complex statistical model might be able to improve predictions of coral cover in the future. For instance, a generalized additive model (Hastie and Tibshirani, 1990) allows more complex relationships between the response and the predictor variables to be included by using smoothers such as regression splines and local regression smoothers. Alternatively, generalized linear mixed models taking into account the hierarchical structure of spatial scales (i.e. habitats, reefs, sites, and transects), also known as multilevel models, provide a useful modeling framework for the expression of uncertainty at several levels of aggregation (Osborne et al., 2011; Sweatman et al., 2011). In the same vein, Vercelloni et al. (2014) combined Bayesian hierarchical and semi-parametric methods for simultaneously quantifying uncertainties across a four-tiered spatial hierarchy of coral cover from the GBR, and scale-specific variability over time. However, by using a more complex model, one would have to deal with the increased computational costs in finding optimal designs as model complexity increases.

In this study, our aim was not to build a definite model but rather to illustrate why adaptive design might be useful if the aim was to estimate, as in this case, coral cover. Our work here is a relatively simple first step in demonstrating the potential importance of

adaptive design in monitoring but that needs further development in the future. Because it was our intent to use observations recorded in an actual, long-term monitoring study, we confined the covariates to those recorded in the LTMP. Some covariates are available (e.g. disturbances) for the LTMP, but many are not and many others are interpolated through space (0.1 degree grid) and therefore are not dynamic on the scales considered here. Thus, these missing covariates could justifiably be ignored here. Of course, as other relevant covariates become available, they can and should be added to the model to at least test whether their addition would benefit model performance.

In conclusion, our research has shown that Bayesian adaptive design could provide a beneficial alternative for monitoring the GBR compared to the unchanging design that is currently implemented. Similar adaptive sampling schemes could be easily altered to suit other ecological or environmental monitoring programs. The broad adoption of these and similar design methods could offer opportunities for more cost effective monitoring against specific objectives in a wide variety of ecological and environmental monitoring programs.

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618 **List of tables**

619 **Table 1**

Table 1: Summary of the posterior distributions of the parameters of the model of interest.

	<b>Mean</b>	<b>Std dev</b>	<b>2.5th %</b>	<b>50th %</b>	<b>97.5th %</b>
Intercept	-4.1410	0.0178	-4.1789	-4.1410	-4.1069
Year	-0.0106	0.0044	-0.0198	-0.0106	-0.0016
Year <sup>2</sup>	-0.0302	0.0040	-0.0388	-0.0301	-0.0221
Mid-shelf	-0.3200	0.0153	-0.3506	-0.3207	-0.2877
Outer-shelf	0.0187	0.0187	-0.0185	0.0186	0.0575
Opened reef	-0.1331	0.0158	-0.1654	-0.1332	-0.0992
$\sigma$	0.4587	0.0028	0.4535	0.4586	0.4645

Table 2: Total samples collected ( $N$ ) for each choice of sampling designs and sampling scenarios across 3 habitats over the 6-year period.

Design	Case 1 ( $N = 9000$ )	Case 2 ( $n = 250$ )	Case 3 ( $n = 500$ )	Case 4 ( $n = 750$ )
<b>D1</b>	3 habitats $\times$ 6 visits $\times 500 = \mathbf{9000}$	3 habitats $\times$ 6 visits $\times \mathbf{250} = 4500$	3 habitats $\times$ 6 visits $\times \mathbf{500} = 9000$	3 habitats $\times$ 6 visits $\times \mathbf{750} = 13500$
<b>D2</b>	3 habitats $\times$ 4 visits $\times 750 = \mathbf{9000}$	3 habitats $\times$ 4 visits $\times \mathbf{250} = 3000$	3 habitats $\times$ 4 visits $\times \mathbf{500} = 6000$	3 habitats $\times$ 4 visits $\times \mathbf{750} = 9000$
<b>LTMP</b>	3 habitats $\times$ 3 visits $\times 1000 = \mathbf{9000}$	3 habitats $\times$ 3 visits $\times \mathbf{250} = 2250$	3 habitats $\times$ 3 visits $\times \mathbf{500} = 4500$	3 habitats $\times$ 3 visits $\times \mathbf{750} = 6750$
<b>D3</b>	3 habitats $\times$ 2 visits $\times 1500 = \mathbf{9000}$	3 habitats $\times$ 2 visits $\times \mathbf{250} = 1500$	3 habitats $\times$ 2 visits $\times \mathbf{500} = 3000$	3 habitats $\times$ 2 visits $\times \mathbf{750} = 4500$
<b>D4</b>	3 habitats $\times$ 1 visit $\times 3000 = \mathbf{9000}$	3 habitats $\times$ 1 visit $\times \mathbf{250} = 750$	3 habitats $\times$ 1 visit $\times \mathbf{500} = 1500$	3 habitats $\times$ 1 visit $\times \mathbf{750} = 2250$



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Figure 1:

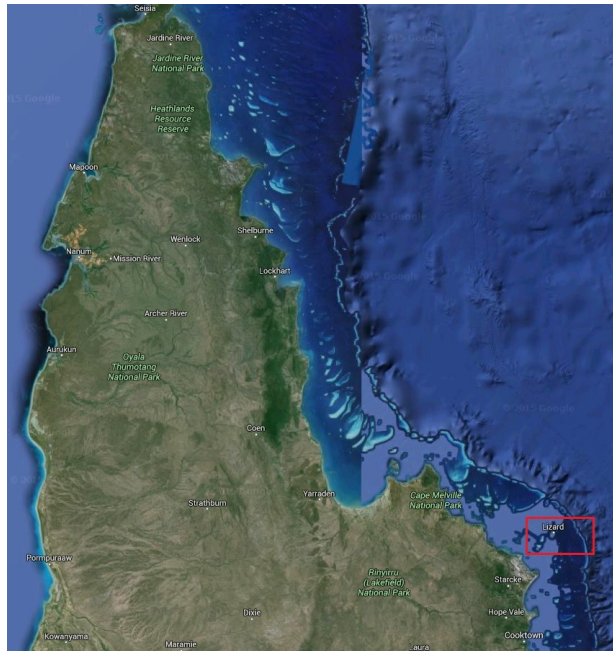


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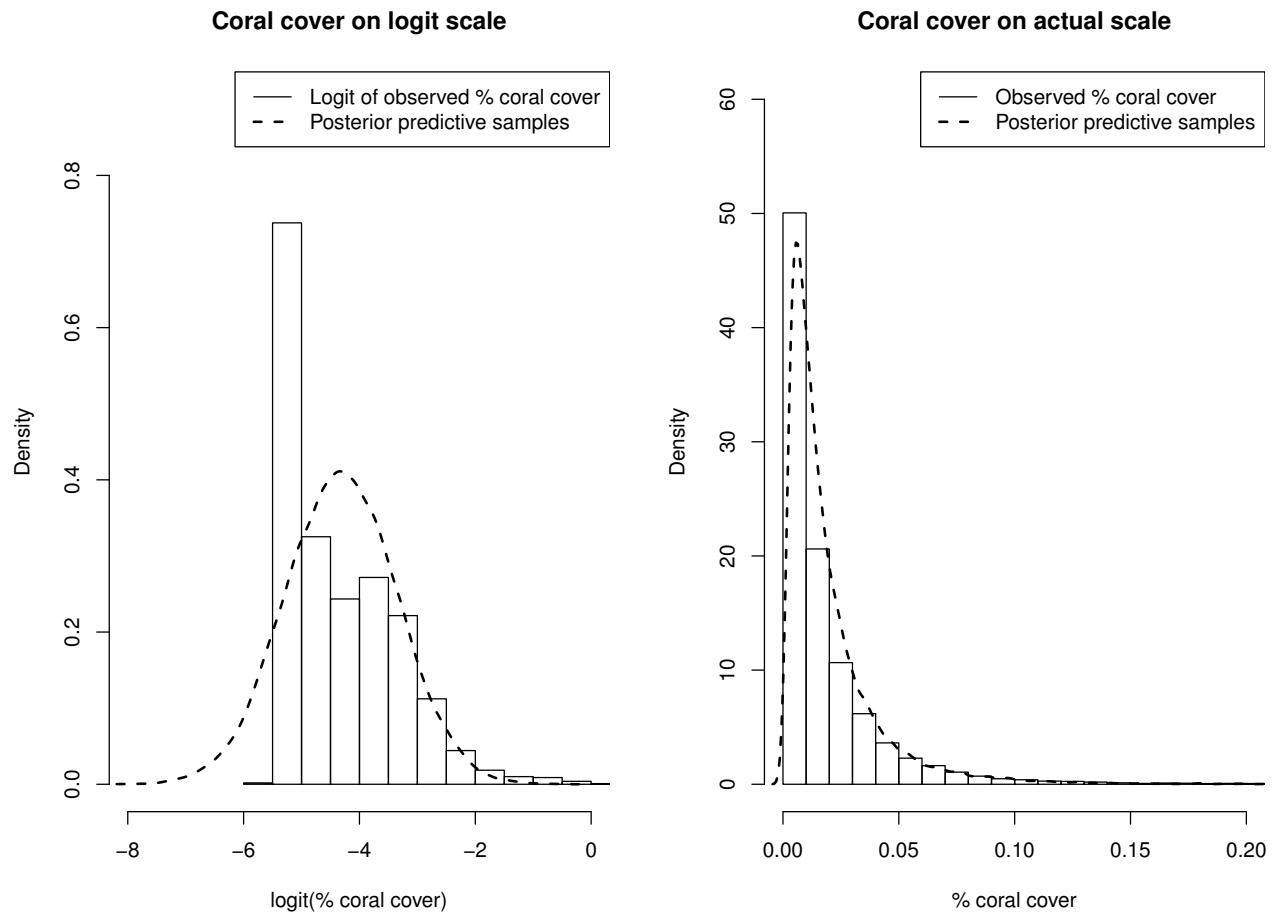
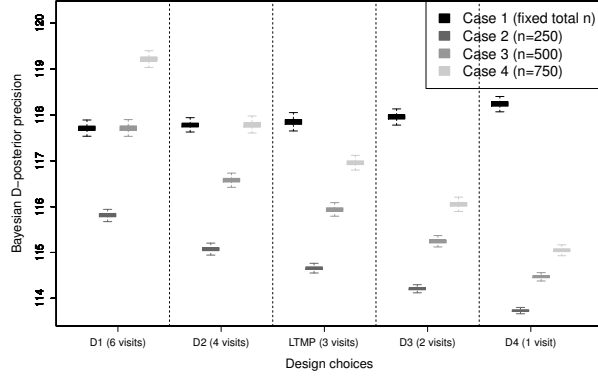
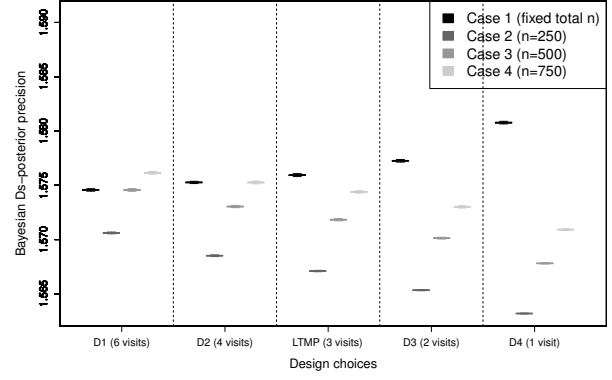


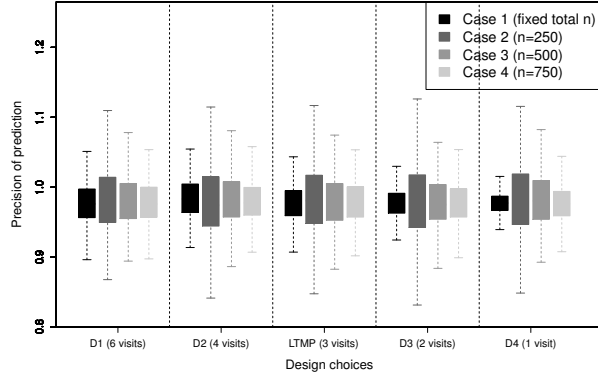
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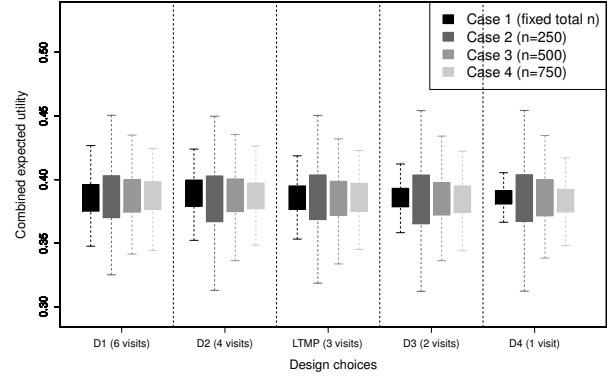
(a) Bayesian D-posterior precision



(b) Bayesian  $D_s$ -posterior precision



(c) Precision of prediction



(d) Combined expected utility

Figure 4:

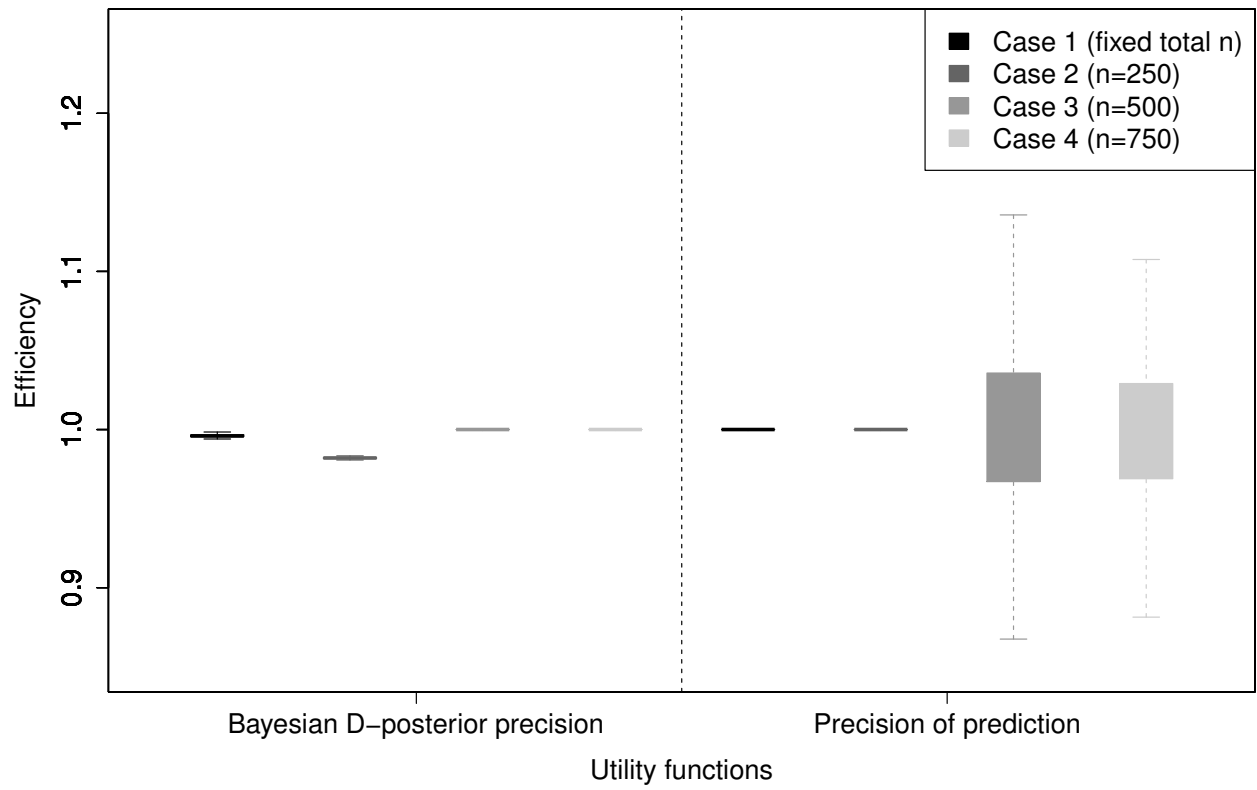
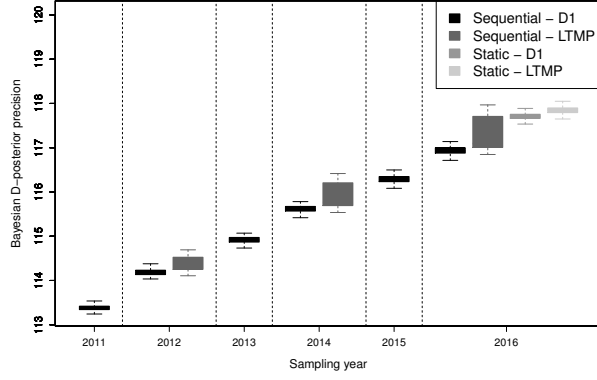
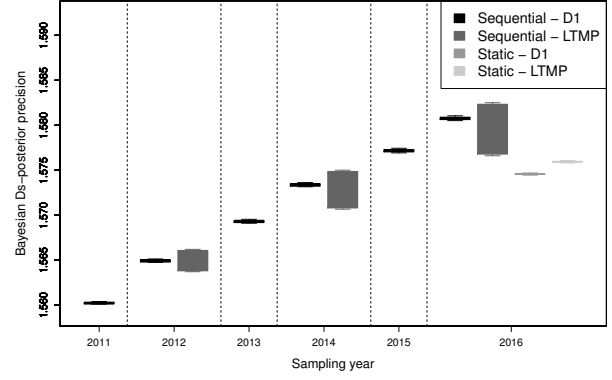


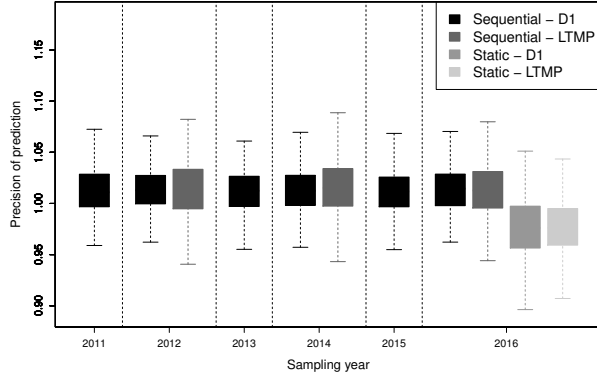
Figure 5:



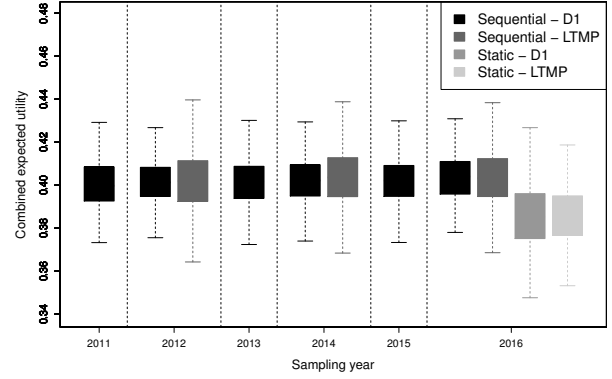
(a) Bayesian D-posterior precision



(b) Bayesian  $D_s$ -posterior precision



(c) Precision of prediction



(d) Combined expected utility

Figure 6:

