



DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

OPTIMAL MONETARY POLICY UNDER HYSTERESIS

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Number 250

October 2005

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Optimal Monetary Policy under Hysteresis

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Abstract

This paper analyses a new-Keynesian model incorporating hysteresis in output. Specifically, we assume that the natural rate of output sluggishly adjusts towards current output. We also assume that the natural rate has an upper bound and that, in addition to having standard objectives, the policymaker seeks to minimise deviations of actual output from this upper bound. We then solve for optimal monetary policy under a range of Phillips curve specifications. Our results suggest that despite increasing inflation temporarily, gradual demand expansions are usually desirable when the natural rate is low. Our model also offers a new explanation for inflation persistence.

Keywords: Monetary Policy, Hysteresis, Unemployment, Inflation Persistence, Demand Expansions.

JEL Classification Numbers: E52, E24, E31.

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*I am indebted to John Muellbauer and Richard Mash for their numerous suggestions and detailed comments on this paper. I would also like to thank Elizabeth Foote for helpful comments on an earlier draft. At various stages, I have benefited from useful discussions with a number of other people: I am grateful to Jennie Castle, Martin Galpin, Godfrey Keller, Roland Meeks, Adrian Pagan, Natalie Quinn, Mathan Satchi, Jari Stehn, David Vines and seminar participants at Oxford University for their thoughts, suggestions, and help with specific points. Finally, I am grateful to the ESRC for providing financial support while this paper was written.

"What should policymakers do if unemployment has settled at a high level? The experiences of the success countries suggest that some kind of demand expansion is desirable. The details of the ideal policy are not clear; however. The expansions in the success countries were largely accidental – they began with shocks to spending that were not caused by policy. Perhaps an intentional monetary easing could be designed to produce even better outcomes, such as a fall in unemployment with a smaller runup in inflation. Future research should address this issue." (Ball, 1999, p. 236)

1 Overview

Over the past 35 years, equilibrium unemployment rates have been subject to considerable and lasting changes over time in a wide range of countries. A large body of theoretical and empirical literature attributes part of these movements to hysteresis effects, whereby the equilibrium unemployment rate may be permanently affected by changes in the actual unemployment rate driven by fluctuations in demand.¹ However, the implications of hysteresis for optimal monetary policy have not been previously analysed. This is despite the fact that if hysteresis operates and the equilibrium rate of unemployment is initially at a high level, it seems obvious to consider whether a demand expansion might be desirable and, if so, whether it should be gradual or rapid, and whether it can be designed in such a way to avoid a very large increase in inflation. It also seems interesting to consider how policymakers should respond to cost-push shocks in the presence of hysteresis.

This paper attempts to address these questions by modifying a standard new-Keynesian monetary policy model to incorporate hysteresis in output (we implicitly assume that the standard link between unemployment and output carries over to hysteresis in these variables). Specifically, we assume that the natural rate of output sluggishly adjusts towards the current output level. We also assume that the natural rate of output has an upper bound and augment the standard loss function to capture the idea that the policymaker may wish to minimise deviations of actual output from this upper bound. We then solve our model for optimal monetary policy under a range of Phillips curve specifications.

Our results suggest that if the initial natural rate of output is below its upper bound, it is usually desirable to run a gradual demand expansion to increase it. Moreover, although inflation does pick up temporarily during the transition to equilibrium, the

¹The usage of the term "hysteresis" varies in the literature (see the discussions in Roed, 1997, and O'Shaughnessy, 2000). In this paper, we define it as referring to the case where a deviation of the actual unemployment (or output) rate from the existing equilibrium unemployment (or output) rate causes a *permanent* change in the equilibrium rate (some authors refer to this case as "pure hysteresis"). By contrast, we view cases where the equilibrium unemployment rate may be affected temporarily, but not permanently, by changes in the actual unemployment rate as being clearly distinct, and refer to them using the term "persistence" (other authors sometimes use the term "partial hysteresis" to describe these cases).

expansion can sometimes be designed so that the increase is minimal. We also show that it is optimal for policymakers to respond less actively to cost-push shocks when there is hysteresis in the natural rate. In addition, we show that the introduction of hysteresis into the new-Keynesian model can explain persistence in both inflation and output in a way which is new in the literature. Finally, we argue that, despite its relative simplicity, our model can broadly replicate the actual movements of inflation, output and the natural rate of output in several countries which reduced equilibrium unemployment significantly during the 1990s.

The policy implications of our results are clear. In countries suffering from high equilibrium rates of unemployment, policymakers should expand output gradually. Meanwhile, in low-unemployment countries, care should be taken to ensure that actual output does not fall too far below its natural rate.

2 Introduction

Since the early 1970s, unemployment has risen substantially in most developed countries. Having said this, unemployment experiences have been diverse, especially since the late 1980s. In particular, there were several "success stories" in the 1990s: the equilibrium rate of unemployment fell substantially in the United Kingdom, the Netherlands, Ireland, Portugal and Denmark. However, unemployment remains a major concern in a number of European countries.

Much of the existing literature (e.g. Layard, Nickell and Jackman, 1991; OECD, 1994, 1999; Siebert, 1997; Nickell, 1997; Nickell and Layard, 1999; Blanchard and Wolfers, 2000; IMF, 2003; Nickell, Nunziata and Ochel, 2005) tries to explain these experiences by focussing on changes in labour market institutions or on the interactions between shocks and institutions. However, a range of empirical evidence (e.g. Blanchard and Jimeno, 1995; Ball, 1997, 1999; Fitoussi, Jestaz, Phelps and Zoega, 2000; Schettkat, 2003; Baker, Glyn, Howell and Schmitt, 2005; Glyn, 2005) suggests that it may be necessary to look much beyond labour market institutions to fully explain the facts and, in particular, that labour market reforms probably cannot account for the extent to which equilibrium unemployment rates fell in the 1990s "success stories".²

In light of this evidence, Ball (1999) argues that hysteresis induced by demand contractions and expansions had an important role in explaining what happened during the 1980s and 1990s.³ In particular, he argues that in the 1980s, equilibrium unemployment rates rose by less in countries which pursued a relatively expansionary monetary policy and quickly increased actual output levels following disinflation (on this, see also Ball, 1997). He also presents evidence suggesting that the countries which were successful during the 1990s generally benefited from demand expansions. Although these led to

²For further discussion of this debate, see Kapadia (2005a).

³As argued in Kapadia (2005a), variations in the capital stock may be another important factor.

temporary increases in inflation, he argues that these were relatively short-lived and the long-term impact was to permanently reduce equilibrium unemployment rates through positive (i.e. reverse) hysteresis mechanisms. Moreover, he argues that a lack of similar expansions helps to explain outcomes in countries which performed poorly over the same period.

Ball's (1999) discussion is largely case study based. However, there is also a wide range of econometric evidence which can shed light on whether there is hysteresis in output and/or unemployment (for an extensive discussion of the evidence relating to unemployment hysteresis, see Roed, 1997). For example, many authors (e.g. Nelson and Plosser, 1982; Campbell and Mankiw, 1987) argue that output follows a unit root process. There are competing explanations for this result but, as Mankiw (2001) notes, the presence of hysteresis in output is one clear possibility.

In terms of unemployment, some authors try to find evidence for particular theoretical hysteresis mechanisms (see the studies cited in our discussion of these mechanisms below). However, much of the empirical literature tries to test for the existence of (pure) hysteresis against the alternative of persistence, whereby the equilibrium unemployment rate always eventually returns to its old level following a change in the actual unemployment rate (for a discussion of our usage of the terms "hysteresis" and "persistence", see footnote 1). Since the observed difference between persistence and hysteresis becomes smaller as the degree of persistence increases, it can be econometrically difficult to distinguish between the two cases, especially in small samples. Nevertheless, many studies find support for hysteresis across a wide range of countries (e.g. Mitchell, 1993; Jaeger and Parkinson, 1994; Roed, 1996).⁴ The most convincing evidence is probably provided by Stanley (2004). He performs a meta-regression analysis of 24 different studies, containing 99 separate estimates for various nations. His results support a hysteresis interpretation of the data and, perhaps more importantly, he finds (p. 590) that: "There are empirical reasons to suspect that evidence to the contrary is a reflection of small-sample, misspecification and publication bias". Finally, note that Mankiw (2001) also appears to favour the hysteresis interpretation in his discussion of the evidence, concluding (p. C48) that: "The data's best guess is that monetary shocks leave permanent scars on the economy".

There are also several theoretical models which suggest that changes in the current level of economic activity may have a permanent impact on potential activity (Carlin and Soskice, 1990, Bean, 1994, and Roed, 1997, all survey some of these mechanisms). Although most of these models were designed to explain negative hysteresis, many of them can be modified so that they also work in reverse. Ball's (1999) preferred mechanism

⁴As noted below, in addition to testing for a unit root in unemployment, Jaeger and Parkinson (1994) directly test whether, as proposed in this paper, deviations of the actual unemployment rate from the existing natural (i.e. equilibrium) rate of unemployment cause a permanent change in the natural rate. They find strong support for this proposition in Canada, the United Kingdom and Germany, but not in the United States.

relates to the idea that the long-term unemployed may not put much downward pressure on wages because their skills may have depreciated, their search intensity may have decreased, and they may be viewed suspiciously by employers.⁵ As a result, inflationary pressures may be relatively similar for different amounts of long-term and hence overall unemployment. However, since sufficiently high levels of demand may lead firms to hire even the long-term unemployed, there may be scope for positive hysteresis.

Insider-outsider mechanisms are another well-known potential source of hysteresis (Blanchard and Summers, 1986; Gottfries and Horn, 1987; Lindbeck and Snower, 1988). Under these theories, wages are assumed to be primarily determined by employed insiders, perhaps because wage bargainers (e.g. unions) are mainly concerned about these individuals, and because turnover costs may make it difficult for firms to replace existing workers, especially if they possess a high level of firm-specific skills. Therefore, demand shocks which change actual unemployment (and hence the number of insiders) may have permanent effects on the equilibrium unemployment rate. Having said this, the empirical evidence on these models is probably mixed at best (Bean, 1994).

If the equilibrium rate of unemployment depends on the capital stock over a certain range (as in Kapadia, 2005a), then hysteresis may also operate through investment channels over a certain range. In particular, if the capital stock is initially at a relatively low level and, in the spirit of Keynes, the investment rate can be temporarily influenced by the level of aggregate demand via its impact on current and expected future profitability, a demand expansion could raise the level of the aggregate capital stock and hence lead to a permanent reduction in the equilibrium unemployment rate. Meanwhile, in an open economy, if it is easier to exit than to enter foreign markets, there may be trade hysteresis associated with exchange rate fluctuations. Finally, as shown by Weitzman (1982) and Manning (1990), there may be multiple unemployment equilibria in the presence of increasing returns to scale (for empirical evidence from the United Kingdom which supports this idea, see Manning, 1992). Although this is slightly different from the other mechanisms in that small shocks may not cause an equilibrium shift, it does provide another channel through which large changes in the actual unemployment rate could cause the equilibrium unemployment rate to change.

From this discussion, we can see that there are a wide range of potential sources of hysteresis. Moreover, the empirical evidence presented above strongly suggests that the phenomenon is observed in practice. Therefore, it seems obvious to consider the theoretical implications of hysteresis for optimal monetary policy. However, in the existing literature, discussion of this issue is extremely limited. In particular, the few papers that do introduce hysteresis equations into small macroeconomic models (e.g. O'Shaughnessy,

⁵Hargreaves Heap (1980) and Nickell (1987) also present these arguments. Moreover, Nickell (1987) finds that in the U.K. data, the long-term unemployed do indeed exert less downward pressure on wages than the short-term unemployed, while, in a more recent study, Llaudes (2005) reaches similar conclusions for most of the 19 OECD countries which he considers.

2000; Mankiw, 2001) only consider the effects of contractionary monetary shocks and do not discuss optimal policy, either in response to low initial natural rates of output or cost-push shocks.⁶ Moreover, their results may not be particularly robust since they do not consider a range of Phillips curve specifications in their analysis.

Having said this, several authors do consider the implications of output persistence for monetary policy (e.g. Lockwood and Philippopoulos, 1994; Lockwood, Miller and Zhang, 1998; Svensson, 1997; Jonsson, 1997). Since the persistence parameters in some (but not all) of these papers are permitted to equal one, some of them are therefore implicitly modelling hysteresis as well. However, in this extreme case, hysteresis is immediate: the natural rate of output in period t is equal to actual output in period $t - 1$. In particular, the possibility of a sluggishly adjusting natural rate is precluded. Perhaps more importantly, these papers are not really directed towards analysing the implications of hysteresis. Instead, they are primarily concerned with fairly abstract theoretical issues relating to improvements over discretionary equilibria, optimal delegation, and the gains from commitment, all with regard to reducing inflation bias and/or stabilisation bias. As a result, they do not discuss how the economy might evolve under the optimal monetary policy when the initial natural rate of output is low, or how this might depend on parameter values which are permitted to vary over empirically plausible ranges. Finally, these papers also all consider only one type of Phillips curve specification in their analysis.

Therefore, in contrast to the existing literature, we introduce hysteresis into a standard new-Keynesian model and, abstracting from issues relating to delegation (our model does not generate an average inflation bias in any case), analyse what happens under the optimal monetary policy both in response to low initial natural rates of output and cost-push shocks. To model hysteresis, we adopt the functional form employed by Hargreaves Heap (1980) and Mankiw (2001), and assume that the natural rate of output sluggishly adjusts towards the current output level.⁷ As well as being more general, sluggish adjustment probably reflects a more realistic description of how hysteresis is likely to work in practice, especially if, as in this paper, the model is taken to be quarterly. Moreover, when applied to unemployment hysteresis, Jaeger and Parkinson (1994) find strong empirical support for this precise functional form. Since hysteresis is only likely to operate over a certain range, we also assume that the natural rate of output has an upper bound and augment the standard loss function to capture the idea that the policymaker may wish to have actual output as close as possible to this upper bound.

⁶Mankiw (2001) simply combines a backward-looking Phillips curve with a hysteresis equation and calculates a theoretical impulse-response function for unemployment following an assumed disinflation; O'Shaughnessy (2000) considers the impact of reducing the growth of the money supply in a more sophisticated open economy model containing hysteresis.

⁷It is straightforward to link output hysteresis to unemployment hysteresis by introducing proportional relationships linking actual output and the natural rate of output to actual unemployment and the equilibrium rate of unemployment respectively. Doing this would not affect any of our results.

To try to make our results more robust, we also solve our model for a range of different Phillips curve specifications.

Our results suggest that it is optimal for policymakers to respond less actively to cost-push shocks when there is hysteresis in the natural rate. We also find that if the natural rate is initially below its upper bound, a demand expansion is almost always desirable, despite causing inflation to increase temporarily. However, the optimal speed of adjustment and the extent to which inflation rises during the expansion can vary considerably, depending on both the parameter values and the assumed Phillips curve specification. Nevertheless, in most cases, output and the natural rate of output both increase fairly gradually under the optimal policy, while inflation picks up by a fairly small amount before slowly returning back to its initial level. These dynamics also indicate how our model can explain persistence in both inflation and output without appealing to serially correlated shocks or incorporating lagged variables in either the Phillips curve or the IS curve. Moreover, we argue that they broadly match the data from the 1990s "success stories".

The remainder of this paper is structured as follows. Section 3 introduces our model and describes how it departs from the standard new-Keynesian monetary policy model. Section 4 solves our model for optimal monetary policy under discretion, while section 5 discusses our results and shows how our model offers a new explanation for inflation and output persistence. Section 6 attempts to relate our model to the experiences of the 1990s "success stories". Finally, section 7 concludes and discusses the policy implications of our results.

3 Introducing the Model

We work within the new-Keynesian framework for analysing monetary policy. Specifically, we build on the simple closed economy macroeconomic model used by Clarida, Gali and Gertler (1999), Walsh (2003, chapter 11), Woodford (2003, chapter 7) and McCallum and Nelson (2004). This model consists of three components: an expectational IS curve, a Phillips curve, and a loss function for the policymaker. However, for simplicity, we follow a common approach in the literature and assume that the interest rate may be varied freely and costlessly, and that the goods market adjusts instantaneously. Under these circumstances, the IS curve imposes no real constraint on the policymaker. As a result, we abstract from it in what follows, assuming instead that the policymaker simply chooses output directly.

3.1 Hysteresis Equation

This setup normally takes the natural rate of output as exogenous. We dispense with this assumption and introduce an equation designed to model output hysteresis in a simple

way. Specifically, we adopt the functional form employed by Hargreaves Heap (1980) and Mankiw (2001), and assume that the natural rate of output, y_t^* , evolves according to:

$$\begin{aligned} y_{t+1}^* &= y_t^* + \gamma(y_t - y_t^*) + v_{t+1} \\ y_{t+1}^* &= (1 - \gamma)y_t^* + \gamma y_t + v_{t+1} \end{aligned} \quad (1)$$

where y_t is output at time t and v_{t+1} is an unforecastable natural rate shock term, the interpretation of which is discussed in section 5.4 below.⁸

The incorporation of this equation into the standard new-Keynesian model represents the main innovation of this paper. It implies that the natural rate of output sluggishly adjusts towards actual output, with the speed of adjustment increasing in the parameter γ . When $\gamma = 0$, there is no hysteresis and our model reduces to the standard setup in the literature; when $\gamma = 1$, the natural rate of output equals actual output in the previous period.

This specification also implies that hysteresis can operate regardless of the initial level of the natural rate of output. It may seem more plausible to suggest that hysteresis only operates over some range and, in particular, that there may exist an upper bound on the natural rate (consider, for example, the capital stock hysteresis mechanism discussed above, for which this is the case). However, imposing an upper bound directly on (1) would introduce a non-linearity into the model. Therefore, for simplicity, we implicitly introduce the idea of an upper bound on the natural rate by specifying it as a bliss point in the loss function below.

3.2 Policy Objective

We assume that the policymaker attempts to minimise the loss function:

$$L_t = \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[(\pi_{t+i} - \pi^T)^2 + \chi (y_{t+i} - y_{t+i}^*)^2 + \phi (\bar{y}^* - y_{t+i})^2 \right] \right\} \quad (2)$$

where β is a discount factor, π_t is inflation at time t , π^T is the inflation target (which, for simplicity, will be taken as zero in all of what follows), and $\bar{y}^*$ may be interpreted as a natural rate upper bound which reflects the highest possible sustainable natural rate of output (for given technology, labour market institutions, etc.).⁹ The inclusion

⁸Technically, we also assume that the distribution of the shock is truncated to prevent the natural rate of output rising above the natural rate upper bound discussed in section 3.2.

⁹Technically, there is no formal upper bound on the natural rate of output in the model. However, the incorporation of $\bar{y}^*$ in the loss function implies that the policymaker would only ever want to exploit hysteresis over a certain range. This could be justified on the grounds that the policymaker understands that hysteresis only operates over some range even though this condition is not actually imposed on (1). In other words, the policymaker understands that $\bar{y}^*$ is the highest possible sustainable natural rate of output and therefore has no desire to attempt to raise the natural rate above this level.

of the $(\bar{y}^* - y_{t+i})$ term in (2) represents our second modification to the standard new-Keynesian model – with $\phi = 0$, (2) would reduce to a standard quadratic loss function. Its inclusion is designed to capture the idea that when the natural rate of output is endogenous, in addition to the usual objectives, the policymaker is likely to have some desire for actual output to be close to the highest possible sustainable natural rate.¹⁰ Having said this, since it is a non-standard term, the weight on it could be fairly low. Therefore, when the parameters are calibrated below, ϕ will be permitted to take a broad range of values.

3.3 Phillips Curve Specifications

Since there is considerable debate over the correct form of the Phillips curve, we consider a range of specifications in our analysis. This is important for robustness since the form of the Phillips curve is likely to have significant implications for the optimal choice of output and the dynamics of inflation as the natural rate of output adjusts.

3.3.1 The New-Keynesian Phillips Curve

Despite its well-documented problems (Mankiw, 2001), we use the new-Keynesian Phillips curve (NKPC) as one of our specifications since it still represents the benchmark theoretical model in the literature. The foundations of the NKPC are discussed by a range of authors (e.g. Galí and Gertler, 1999; Walsh, 2003, chapter 5; Woodford, 2003, chapter 3). Under this specification, inflation is given by:

$$\pi_t = \kappa (y_t - y_t^*) + \beta E_t \{\pi_{t+1}\} + \bar{\eta} e_t \quad (3)$$

where $\kappa = \delta (1 - \lambda) (1 - \beta\lambda) / \lambda$, $\bar{\eta} = (1 - \lambda) (1 - \beta\lambda) / \lambda$, λ is the probability that firms must keep their prices fixed in any given period, δ is the output elasticity of real marginal cost, and e_t is a cost-push shock. In much of the discussion in this paper, we ignore the cost-push shock. However, when solving our model, we assume that it evolves according to:

$$e_t = \rho e_{t-1} + \varepsilon_t \quad (4)$$

where $\rho \in [0, 1)$ is a parameter measuring the degree of persistence in the shock and ε_t is an independent and identically distributed random variable with zero mean and variance σ_ε^2 (i.e. an unforecastable error term).

¹⁰It could be contended that, in practice, policymakers may find it difficult to estimate $\bar{y}^*$. However, in any given country, it could probably be inferred with reference to historical experience (since labour market institutions are fairly stable over time in most countries) and/or by comparing with the most successful countries amongst those which have fairly similar labour market institutions.

3.3.2 The Inflation-Target Expectations Phillips Curve

To capture the potential impact of expectations anchoring on inflation, we also consider the inflation-target expectations Phillips curve (ITEPC) developed by Kapadia (2005b). As explained there, this Phillips curve is based on the assumption that instead of forming a rational expectation when setting prices, a proportion ω of firms (described as "*inflation-targeters*") simply expect future inflation to always equal its target and the future output gap to always be zero (though they do respond to the current output gap). Introducing inflation-targeters into the NKPC gives the ITEPC. With an inflation target of zero, this may be written as:

$$\pi_t = \kappa [(y_t - y_t^*) - \omega\beta\lambda E_t \{ (y_{t+1} - y_{t+1}^*) \}] + \beta [1 - \omega(1 - \lambda)] E_t \{ \pi_{t+1} \} + \eta e_t \quad (5)$$

where $\eta = (1 - \lambda)(1 - \beta\lambda)(1 - \omega\beta\lambda\rho) / \lambda$. Note that when $\omega = 0$, (5) reduces to (3). Extensive discussion of the ITEPC is provided in Kapadia (2005b). This paper also solves for optimal monetary policy under the ITEPC but in the absence of hysteresis.

3.3.3 The Hybrid and Accelerationist Phillips Curves

Empirical evidence frequently suggests that inflation (and output) are fairly persistent, in the sense that they can remain away from their steady-state values for many periods, with their autoregressive coefficients being fairly close to one.¹¹ Although based on clear microfoundations, the NKPC has therefore been criticised by many authors (e.g. Fuhrer and Moore, 1992; Mankiw, 2001; Estrella and Fuhrer, 2002) on the grounds that it cannot capture this feature of the data unless shocks are assumed to be serially correlated. As we shall see below, in the presence of hysteresis, inflation and output persistence *can* be accounted for when using the NKPC, even if shocks are not persistent. Nevertheless, since it is often suggested that a hybrid Phillips curve (HPC) incorporating both forward and backward looking inflation terms is empirically more consistent than the NKPC, we also consider the HPC as one of our specifications. This takes the form:

$$\pi_t = \kappa' (y_t - y_t^*) + \alpha_1 E_t \{ \pi_{t+1} \} + \alpha_2 \pi_{t-1} + \eta' e_t \quad (6)$$

where, depending on how the backward-looking term is motivated, κ' , η' , α_1 and α_2 may be functions of structural parameters. Note that with $\alpha_1 = 0$ and $\alpha_2 = 1$, the HPC reduces to a purely backward-looking (accelerationist) Phillips curve (APC).

Although there is no clear consensus on the microeconomic source of the lagged inflation term in (6), it is frequently motivated by assuming that between optimal price

¹¹See, for example, Nelson and Plosser (1982), Fuhrer and Moore (1995) and Nelson (1998). Note, however, that some authors (e.g. Alogoskoufis and Smith, 1991; Cogley and Sargent, 2001, 2005; Levin and Piger, 2003; Benati, 2005) have argued that inflation persistence is only observed in the data during certain periods rather than being an inherent characteristic of the economy.

adjustments, all firms index their prices either fully (Christiano, Eichenbaum and Evans, 2005) or partially (Smets and Wouters, 2003; Woodford, 2003) to lagged inflation. At first, the former approach may seem more plausible. However, as shown by Al-Haschimi (2005) and Mash (2005), partial indexation can be loosely interpreted as arising from a situation in which some firms index fully to lagged inflation while other firms do not index at all. Since microeconomic evidence suggests that many prices remain fixed for more than one period (Taylor, 1999), this approach may well be more realistic. Given that it is also more general, we therefore adopt the partial indexation assumption when using the HPC. Under this approach, $\kappa' = \delta (1 - \lambda) (1 - \beta\lambda) / [\lambda (1 + \beta\zeta)] = \kappa / (1 + \beta\zeta)$, $\eta' = (1 - \lambda) (1 - \beta\lambda) / [\lambda (1 + \beta\zeta)]$, $\alpha_1 = \beta / (1 + \beta\zeta)$ and $\alpha_2 = \zeta / (1 + \beta\zeta)$, where ζ measures the degree of indexation to lagged inflation ($\zeta = 0$ yields the NKPC).

3.4 Initial Conditions

We assume that inflation in period $t - 1$ is zero. In much of our discussion, we will be focussing on the transition to equilibrium under the assumption that the initial natural rate of output in period t is below its upper bound (i.e. $y_t^* < \bar{y}^*$): this can be viewed either as simply a starting condition or as the product of a negative v shock to the hysteresis equation in period t .¹²

4 Solving for Optimal Monetary Policy

For each Phillips curve specification, we now solve our model for optimal monetary policy in response to both suboptimal initial natural rates of output and cost-push shocks. The policymaker's problem is to choose y_{t+i} for all $i \geq 0$ to minimise (2) subject to (1) and either (3), (5) or (6), where the constraints are taken to apply in all future periods.

For simplicity, we restrict attention to the discretionary case, in which it is assumed that the policymaker is unable to make commitments over the path of future policy.¹³ Despite this, the policymaker still faces a dynamic optimisation problem. This is because the inclusion of the hysteresis equation means that there is an intertemporal link between time periods: the decision made in period $t + i$ will affect y_{t+i+1}^* and hence the period $t + i + 1$ decision, even though the policymaker cannot directly commit to the period $t + i + 1$ decision. This connection between periods must be taken into account

¹²The initial condition on the natural rate is specified in period t rather than $t - 1$ because y_t^* cannot be affected by policy in period t : it is only the natural rate in subsequent periods which is endogenous.

¹³Since our model does not generate an average inflation bias, the only gains from commitment would be the small ones associated with the elimination of stabilisation bias. Moreover, as shown in Kapadia (2005b), these gains are even more minimal if a high-proportion of firms are using inflation-target expectations, and non-existent if $\omega = 1$ since the solutions under discretion and commitment coincide in this case. Therefore, although the details would be slightly different, our main results and broad conclusions would almost certainly be unchanged in the commitment case. Nevertheless, solving for optimal monetary policy under commitment is clearly a potential extension.

by the policymaker.

To solve, we use the minimal state variable (MSV) approach discussed by McCallum (1983, 1999) and McCallum and Nelson (2004). This involves determining the relevant endogenous state variables and ‘guessing’ the reduced form processes for inflation and output in terms of these state variables and arbitrary coefficients.¹⁴ These ‘guesses’ are used to substitute out for the forward-looking term(s) in the Phillips curve before writing down the Lagrangian. The first order conditions from the Lagrangian then provide restrictions on the coefficients in the reduced form processes. Further restrictions are provided by substituting the ‘guesses’ into the original Phillips curve and comparing terms. Together these restrictions can be used to determine the coefficients in the ‘guesses’ and hence the solution. Since the restrictions are non-linear, analytical solutions for the coefficients are either very complicated (e.g. deriving from the solutions to a quartic) or not available at all. Therefore, for calibrated parameter values, we solve for the coefficients numerically.¹⁵

Appendices A and B illustrate how this method is implemented in practice: Appendix A solves the model under the NKPC and ITEPC; Appendix B solves it under the HPC.¹⁶ The reduced form solutions for inflation and output in each case are presented below.

4.1 The New-Keynesian Phillips Curve

Under the NKPC, the solutions for inflation and output are given by:

$$\pi_{t+i} = a_1 (\bar{y}^* - y_{t+i}^*) + a_2 e_{t+i} \quad (7)$$

$$y_{t+i} = \bar{y}^* + c_1 (\bar{y}^* - y_{t+i}^*) + c_2 e_{t+i} \quad (8)$$

where a_1 , a_2 , c_1 and c_2 are coefficients which can all be solved for numerically using restrictions specified in Appendix A.

4.2 The Inflation-Target Expectations Phillips Curve

Under the ITEPC, the solutions take the same reduced form as under the NKPC but the coefficients (which can be solved for numerically using restrictions specified in Appen-

¹⁴Despite also being endogenous, a ‘guess’ does not need to be made for the natural rate of output since its evolution is completely determined by (1).

¹⁵Following McCallum (1983, footnote 9), we restrict the solutions to be real. Nevertheless, this procedure can still sometimes lead to multiple solutions for the coefficients. Where this is the case, we use the MSV criterion (see McCallum, 1983, 1999) to select the appropriate solution. In our model this is also always a stable solution.

¹⁶When solving, we assume that the v shock to equation (1) is zero in expectation. Technically, since the distribution of this shock is truncated (see footnote 8), this assumption becomes unrealistic when the natural rate of output is close to its upper bound. However, this detail becomes irrelevant if we assume that there are no v shocks and instead just view the initial conditions as given.

dix A) are different:

$$\pi_{t+i} = h_1 (\bar{y}^* - y_{t+i}^*) + h_2 e_{t+i} \quad (9)$$

$$y_{t+i} = \bar{y}^* + f_1 (\bar{y}^* - y_{t+i}^*) + f_2 e_{t+i} \quad (10)$$

4.3 The Hybrid and Accelerationist Phillips Curves

Under the HPC, lagged inflation is an additional endogenous state variable. Since this makes the model much more difficult to solve, we ignore cost-push shocks for simplicity in this case. The solutions for output and inflation are given by:

$$\pi_{t+i} = m_1 (\bar{y}^* - y_{t+i}^*) + m_2 \pi_{t+i-1} \quad (11)$$

$$y_{t+i} = \bar{y}^* + n_1 (\bar{y}^* - y_{t+i}^*) + n_2 \pi_{t+i-1} \quad (12)$$

where the coefficients can be solved for numerically using restrictions deriving from equations specified in Appendix B. The solution for the APC is a special case of the HPC solution.

5 Discussion of Results

All of our results are based on simulations for the calibrated parameter values listed below. Robustness checks were performed by varying the parameters over a range of plausible values. Unless stated otherwise, the outcomes from this analysis indicate that the broad results discussed below are not sensitive to the particular parameter values used.¹⁷

5.1 Calibration

Table 1 lists the parameter values used in our baseline calibration. We interpret the time interval as one quarter and therefore set λ to be 0.75 (implying that, consistent with the discussion in Taylor, 1999, firms reset their price once a year on average) and β to be 0.99. We set δ to be 0.28, implying that κ is approximately 0.024, which is the value used by Woodford (2003, p. 431). We also follow Woodford (2003, p. 431) in setting χ to be 0.048. When discussing the ITEPC and HPC, we report our results for different values of ω and ζ . However, in the baseline calibration, we set ζ to be 0.46 (as estimated by Smets and Wouters, 2003) and ω to be 0.5. The remaining two parameters, ϕ and γ , are more specific to our model. Since it represents the weight on a non-standard term in the loss function, we set ϕ at the fairly low value of 0.01 in the

¹⁷To obtain the numerical solutions and perform the robustness checks, we used *Maple 9*. The relevant codes and the associated spreadsheets which allow the paths of the variables to be plotted are available on request from the author.

Parameter	Value in Baseline Calibration
β	0.99
λ	0.75
δ	0.28
$\kappa = \delta (1 - \lambda) (1 - \beta\lambda) / \lambda$	0.024
χ	0.048
ζ	0.46
ω	0.5
ϕ	0.01
γ	0.25

Table 1: Parameter Values under the Baseline Calibration

baseline calibration. However, since it is a preference parameter as well, we allow it to vary over a broad range. In particular, we also consider the $\phi = 0.0625$ case, under which, in annual terms, the policymaker gives equal weight to the non-standard upper bound output gap term and the inflation term. Finally, although we consider different values in our discussion, we assume that γ is 0.25 in the baseline calibration. This implies that if actual output is initially one percent above the natural rate of output and stays at that level, then the natural rate will rise by 0.25 percent in the first quarter and by approximately 0.68 percent by the end of one year. As we shall see below, in the context of the solution, this results in a fairly gradual adjustment process which lasts for a plausible length of time. Moreover, this choice of γ is broadly consistent with the econometric results of Jaeger and Parkinson (1994) in their estimation of the unemployment equivalent of equation (1): using quarterly data over the period 1961(1)-1991(4), they estimate the equivalent parameter to γ to be 0.244 in Canada, 0.218 in the United Kingdom, and 0.179 in Germany.

5.2 Optimal Paths During the Transition to Equilibrium

For each Phillips curve specification, we now consider what happens to inflation, output and the natural rate of output under the optimal policy during the transition to equilibrium when $y_t^* < \bar{y}^*$. For the purposes of this discussion, we assume that there are no cost-push shocks at time t or in subsequent periods and no natural rate shocks after period t (or in period t either, if the initial condition on the natural rate is viewed as just being given). We also normalise $\bar{y}^*$ to be zero and set y_t^* to be -0.05.¹⁸ This implies that the natural rate of output is initially five percent below its upper bound. If Okun's law applies in this situation, this would be equivalent to the equilibrium rate of unemployment being approximately 2.5 percentage points above its lower bound (though this precise figure should perhaps be treated with slight caution since the law is normally viewed as linking actual output to actual unemployment over the business cycle).

¹⁸Apart from having a proportionate scaling effect on all variables, changing the value of y_t^* does not affect our results.

In Figures 1-3, we plot our results for each Phillips curve under a variety of different parameter constellations. These results are obtained by feeding the initial conditions into the relevant first-period ($i = 0$) solutions for inflation and output, and then letting the system evolve according to (1) and either (7) and (8), (9) and (10), or (11) and (12), as appropriate. The black lines plot the path of output under the optimal policy; the dashed lines plot the natural rate of output; and the grey lines plot *quarterly* inflation. The title of each panel lists the parameters in that chart which differ from the relevant baseline calibration: parameters which are not listed take the values listed in Table 1.

Although a visible adjustment process normally takes place under the optimal policy, Figure 1(b) illustrates that this is not always the case (though plotted for the NKPC, this is a general result which also applies for the ITEPC and HPC). However, this exception only arises when both γ and ϕ take extremely low values (or when one of them takes a value of zero). When γ is very small, the natural rate can hardly change over time; when ϕ is also very small, the policymaker barely attaches any weight to the upper bound output gap term and therefore chooses to set actual output close to the natural rate, thus allowing inflation to be maintained at almost zero. This result is not surprising: if both γ and ϕ are zero, our model reduces to the standard new-Keynesian model, in which it is optimal to set output at the (exogenous) natural rate in the absence of cost-push shocks. However, since our model is only really of interest when it differs substantively from the standard setup, we assume in all of what follows that γ and ϕ are both sufficiently greater than zero for adjustment to take place under the optimal policy.¹⁹

5.2.1 The New-Keynesian Phillips Curve

Under the NKPC, the key policy coefficient which determines the speed of adjustment is c_1 in equation (8). If $c_1 = -1$, output is always set at the existing natural rate and there is no adjustment: the natural rate remains at its initial level forever and inflation is always zero (when $c_1 = -1$, $a_1 = 0$). For higher values of c_1 , adjustment does take place, with the speed of adjustment increasing in c_1 . In what follows, we describe policy as becoming more "aggressive" as the speed of adjustment increases.

Panel (a) of Figure 1 depicts what happens under the baseline calibration. As can be seen, it is optimal to run a gradual demand expansion by setting output above the initial natural rate and increasing it slowly as the natural rate itself increases. Inflation picks up temporarily (to about 1.8 percent in annual terms in the first quarter) but then declines and is almost back to zero after five years. After seven and a half years, the adjustment of the natural rate is practically complete.

Panels (c)-(f) of Figure 1 illustrate how changes in the parameters affect the solution. Since a rise in ϕ increases the loss associated with deviations of actual output from the

¹⁹Mathematically, when both γ and ϕ are slightly greater than zero, the non-adjustment solution becomes complex and is therefore ruled out.

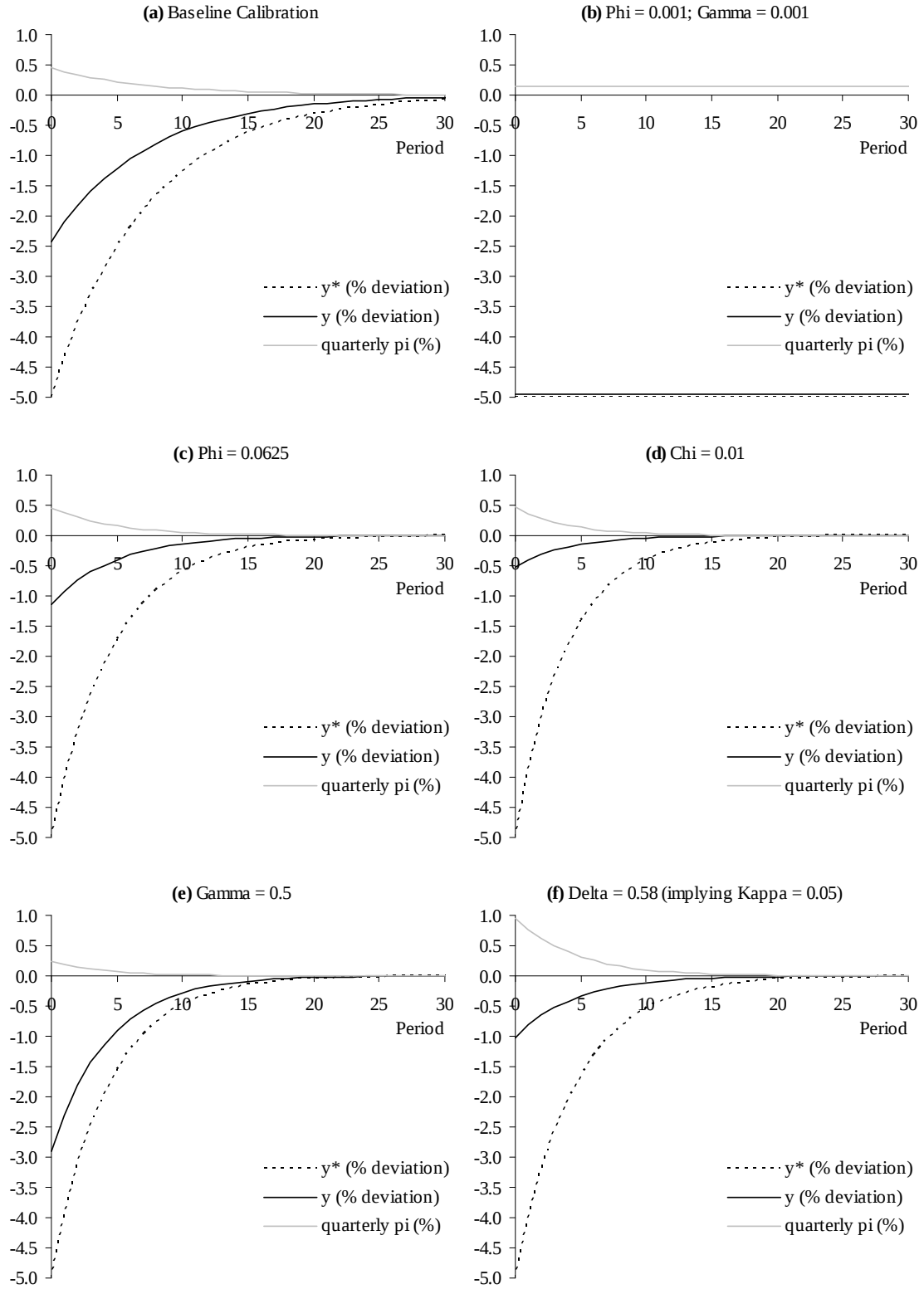


Figure 1: NKPC Transition Paths

upper bound (i.e. the loss associated with the difference between the black line and zero in the charts), output is set increasingly close to the upper bound as ϕ increases. This is depicted in panel (c). Meanwhile, χ represents the weight on the $(y_t - y_t^*)$ term in the loss function (i.e. the weight on the deviation of the black line from the dashed line). Therefore, as illustrated in panel (d), policy becomes more aggressive as χ falls (i.e. c_1 is decreasing in χ). Note, however, that inflation is relatively unaffected by changes in these preference parameters, especially in early periods. This is because, with γ fixed, the sum of positive output gaps over the future needed to increase the natural rate to its upper bound is a constant. Forward-looking firms realise this fact and take it into account in their current pricing decisions. As a result, the speed with which adjustment is actually carried out has relatively little impact on inflation, especially in early periods when the amount of future adjustment required is similar across different adjustment paths.

The numerical solutions also suggest that c_1 is decreasing in γ .²⁰ Intuitively, a rise in γ is beneficial for the policymaker since it increases the speed with which the natural rate adjusts. Even if policy were held fixed, this would reduce loss. However, though the policymaker does take some of the gains from a higher γ in terms of achieving faster overall adjustment, it is optimal for her to adjust policy so that she can also take some of the gains in terms of lowering first-period inflation and output gaps during the transition. As a result, the policymaker chooses to act less aggressively when γ increases. Despite this, she still acts aggressively enough for the natural rate to adjust more quickly than previously. For example, when $\gamma = 0.5$ (panel (e)), adjustment is practically complete after only five years. Moreover, due to the faster adjustment, inflation during the transition is much lower than under the baseline calibration (less than one percent in annual terms in the first quarter). By contrast, when $\gamma = 0.15$ (not illustrated), adjustment takes approximately ten years and inflation initially rises to about 2.9 percent.

Finally, the results indicate that c_1 is increasing in δ . When δ (and hence κ) increases, setting output above the natural rate becomes more inflationary. This has a substantial impact on inflation during the transition, which is compounded by the fact that under the NKPC, all firms are forward-looking and therefore take account of expected future inflation when resetting their prices. To mitigate against this, it is optimal for the pol-

²⁰There is one exception to this: for implausibly low values of the discount factor, β , the result can sometimes be overturned. This stems from the fact that, holding all else equal, there is a small incentive when $\beta < 1$ to be less aggressive than when $\beta = 1$, because even though the associated slower adjustment raises future costs, current inflation is reduced. For a given $\beta < 1$, an increase in γ reduces this incentive since the future costs of being less aggressive are higher when the natural rate can adjust more quickly (i.e. the tendency when $\beta < 1$ to be less aggressive than when $\beta = 1$ is offset as γ increases). So, when $\beta < 1$, this provides a mechanism by which an increase in γ encourages the policymaker to be slightly more aggressive. For plausible values of β (e.g. $\beta > 0.96$), this effect is swamped by the opposing effect described in the main text. This is because the initial tendency to be less aggressive is minimal, meaning that there is little to offset as γ increases. However, for lower values of β , it can sometimes be significant enough to make the relationship between c_1 and γ non-monotonic.

icymaker to act more aggressively. Although this will marginally increase inflation in the first period and raise the loss associated with deviations of actual output from the natural rate, it will also allow inflation to decline more quickly and can therefore be beneficial up to a certain point. Panel (f) illustrates what happens when $\delta = 0.58$ (implying $\kappa = 0.05$): note that there is a very sharp increase in inflation at the start of the demand expansion in this scenario.

5.2.2 The Inflation-Target Expectations Phillips Curve

Under the ITEPC, the key adjustment coefficient is f_1 in equation (10). As noted above, when there are no inflation-targeters (i.e. when $\omega = 0$), the ITEPC reduces to the NKPC. In this case, $f_1 = c_1$.

The effects of introducing inflation-targeters can be analysed by comparing panel (a) in Figures 1 and 2. As we would expect, inflation picks up by less when some firms use inflation-target expectations, since these firms help to anchor inflation to the target. In addition, we can see that it is optimal for the policymaker to act slightly less aggressively when ω is positive. Intuitively, this is because inflation-targeters only raise their prices in response to the current output gap – since they are not fully forward-looking, the speed with which the natural rate adjusts and inflation declines is irrelevant for their current pricing decision. In particular, in the presence of inflation-targeters, it matters less if inflation and the output gap are expected to be positive for a long time into the future, since this will not affect current inflation as much as when all firms are using rational expectations. Therefore, reducing the current output gap at the expense of slightly increasing output gaps in the fairly distant future (i.e. running a more gradual demand expansion) is worthwhile up to a certain point when some firms are using inflation-target expectations. Because of this, the natural rate may increase relatively slowly under the ITEPC, sometimes taking nine or ten years to adjust fully.

Panels (b) and (c) of Figure 2 illustrate how the solution changes as the proportion of firms using inflation-target expectations varies. Even when ω is only 0.2 (panel (b)), inflation is much lower during the transition than under the NKPC, rising to only 1.1 percent in annual terms; when $\omega = 1$ (panel (c)), inflation is almost non-existent, never being more than 0.2 percent in annual terms.

In terms of the other parameters, changes in ϕ (panel (d)), χ (not illustrated) and γ (panel (e)) all affect the ITEPC solution in the same way that they affect the NKPC solution (but changes in the preference parameters and the associated optimal speeds of adjustment now have a slightly greater impact on inflation since not all firms are forward-looking). However, the effect on f_1 of changing δ (and hence κ) depends on the value of ω . This is because, as noted above, inflation in each period is increasingly determined by the output gap in that period and decreasingly by future output gaps as ω rises. Therefore, as ω rises, there is an increasing incentive to act less aggressively

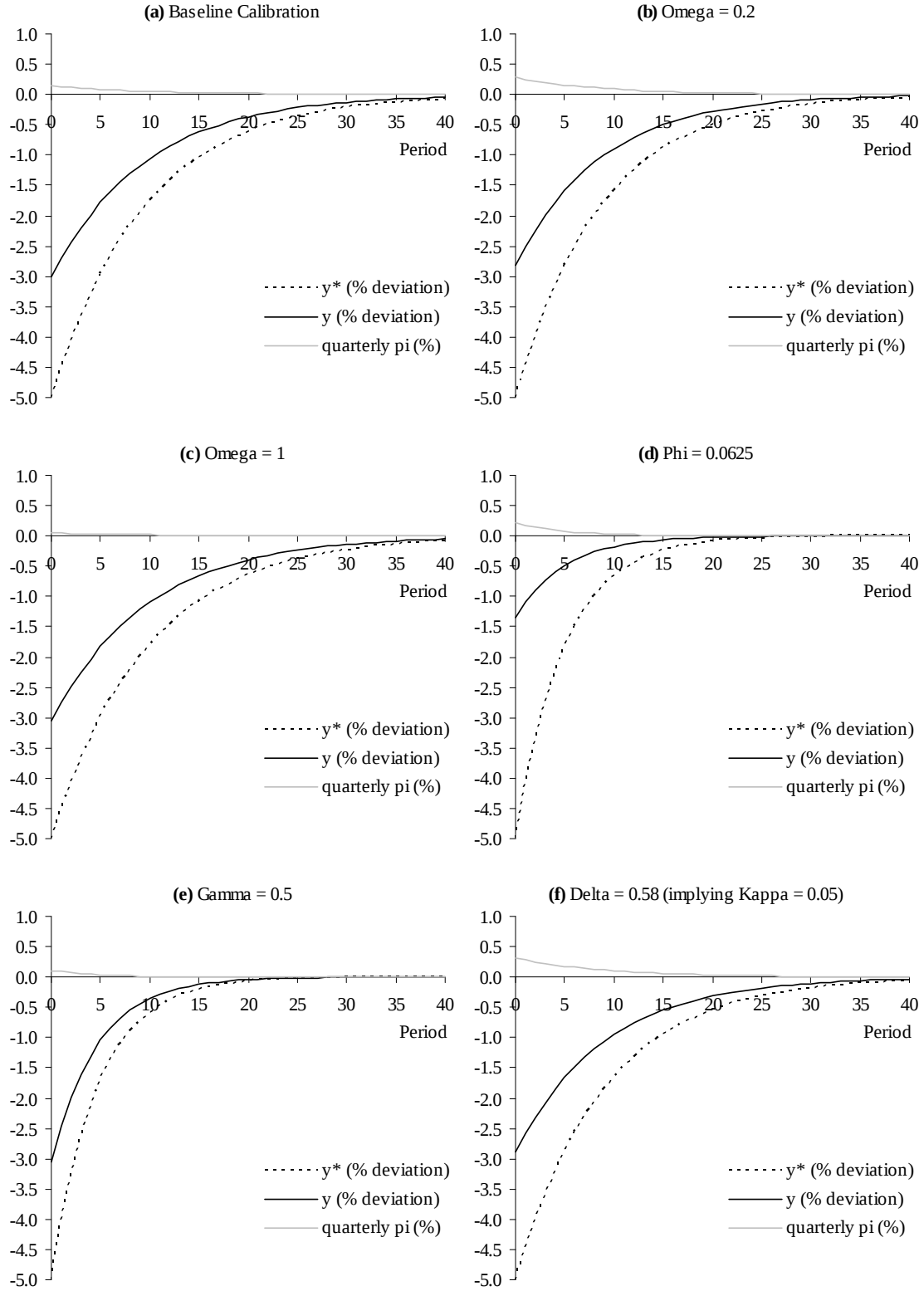


Figure 2: ITEPC Transition Paths

to mitigate against the greater current inflationary pressures associated with a rise in δ . Eventually, when ω is sufficiently high (greater than about 0.7 when the other parameters take the values listed in Table 1), this incentive is strong enough to outweigh the opposing incentive (which is dominant under the NKPC) to act more aggressively as δ increases. Having said this, we should note that regardless of the sign of the derivative, f_1 is relatively insensitive to changes in δ for medium and high values of ω . This can be seen by comparing the paths of output in panels (a) and (f) of Figure 2.

5.2.3 The Accelerationist Phillips Curve

Under the APC, firms are entirely backward-looking. The lack of a forward-looking term in the Phillips curve means that if inflation has increased because of an expansion in output, it can never be reduced again without contracting output. Therefore, as pointed out by Hargreaves Heap (1980), there is a long-run trade-off between inflation and the natural rate of output in the presence of hysteresis under the APC.²¹ As a result, even under the optimal policy (which was not considered by Hargreaves Heap, 1980), there can never be full adjustment in the sense of output, the natural rate of output and inflation all eventually converging to zero. Instead, after a period of transition, the solution will converge onto a steady-state in which the natural rate is permanently below its upper bound and inflation is permanently above its target of zero (see, for example, panels (a) and (b) of Figure 3).

The precise steady-state outcomes for inflation and the natural rate of output are obviously dependent on both preferences and the parameter values. Under the baseline calibration depicted in panel (a), inflation settles at approximately one percent in annual terms, with the natural rate being about 2.4 percent below its upper bound. As we would expect, when ϕ increases, the natural rate of output settles at a higher value and hence so does inflation; while when δ (and hence κ) increases, the reverse is true. Though not illustrated, both of these effects can be quite significant. By contrast, since actual output equals the natural rate in steady-state, long-term outcomes for the variables are relatively insensitive to χ (though the optimal speed of adjustment does increase substantially as χ falls). Finally, as illustrated in panel (b), the benefits of a rise in γ manifest themselves in terms of both a higher natural rate and slightly lower inflation in steady-state.

5.2.4 The Hybrid Phillips Curve

The impact of using the HPC rather than the NKPC can be seen by comparing Figure 3(c) with Figure 1(a). As is clear from these transition paths, the optimal policy is more aggressive when there is a lagged inflation term in the Phillips curve. Intuitively, this stems from the fact that the incorporation of a such a term introduces additional inertia

²¹This can be shown by combining the APC (i.e. (6) with $\alpha_1 = 0$ and $\alpha_2 = 1$) with the hysteresis equation, (1). Assuming that there are no shocks, this gives $\pi_t - \pi_{t-1} = (\kappa'/\gamma) (y_{t+1}^* - y_t^*)$.

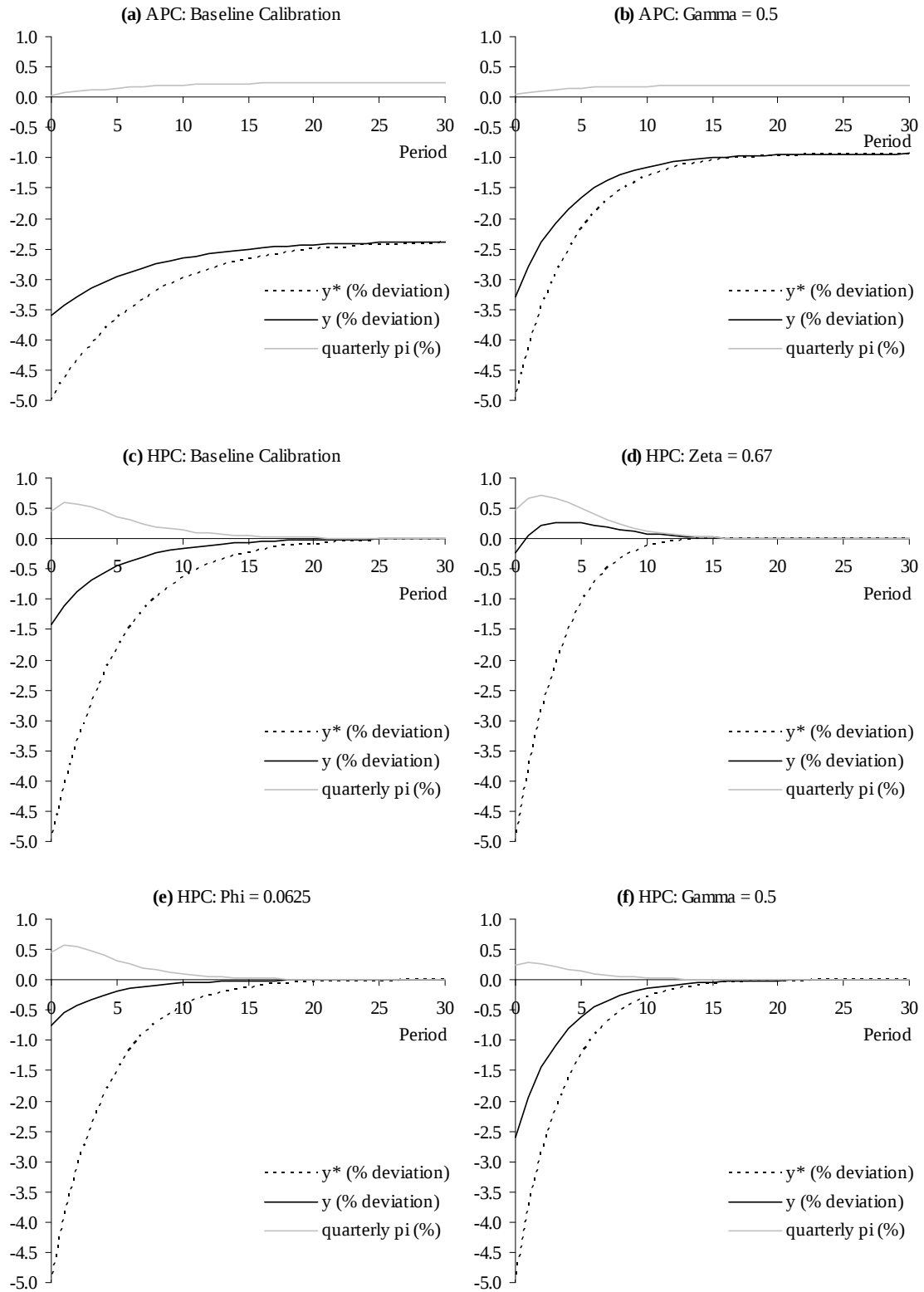


Figure 3: Accelerationist and Hybrid Phillips Curve Transition Paths

into inflation. As a result, compared to the case where all firms are forward-looking, inflation is higher during the transition. So, an increase in ζ (and hence α_2) has a similar effect to the increase in δ discussed in section 5.2.1 above. For reasons explained there, to mitigate against the higher inflationary pressures associated with an increase in ζ , it is optimal for the policymaker to act more aggressively. Indeed, as illustrated in Figure 3(d), it may be optimal for the policymaker to be so aggressive that actual output temporarily rises above the natural rate upper bound of zero. In this case, adjustment is very quick: it is almost complete after only three years.

The behaviour of inflation under the HPC during the transition to equilibrium is also interesting. As can be seen in panels (c)-(f) of Figure 3, the inertia caused by the lagged inflation term means that inflation can follow a hump-shaped path, with its peak not being reached until the second or third period. It is also clear that inflation during the transition is much higher and slightly more persistent than under the NKPC, peaking at about 2.3 percent in annual terms under the baseline calibration (panel (c)) and at almost three percent in panel (d) (and remaining fairly high for several periods in both cases). Moreover, if δ is increased to 0.58 (not illustrated), the inflation peak is approximately 4.5 percent.

Finally, note that under the HPC, changes in the parameters affect the solution in the same way as under the NKPC. For illustration, the effects of varying the non-standard parameters in the model (ϕ and γ) are depicted in panels (e) and (f) respectively.

5.2.5 General Comments

We can see from the transition paths depicted in Figures 1-3 that, in the presence of hysteresis, it is almost always optimal to run a demand expansion if the natural rate of output is initially below its upper bound. Moreover, in almost all cases, inflation and the natural rate both eventually converge onto their respective bliss points when the adjustment process is complete. However, the speed of adjustment under the optimal policy sometimes varies considerably, both with the parameter values and the assumed Phillips curve specification: in some cases, adjustment takes as little as three years; in other cases, it takes almost ten years. Nevertheless, in most cases, demand expansions which are fairly gradual are optimal.

It is also clear that the improvement in the natural rate comes at the expense of a temporary increase in inflation. This is particularly noticeable in the case of the HPC, under which, even in the baseline calibration, inflation peaks at 2.3 percent in annual terms and remains above one percent for almost two years. However, we may question the suitability of using the HPC in the context of this particular model. The incorporation of a lagged inflation term into the NKPC to obtain the HPC has no clear microfoundations: it is normally justified on empirical grounds. In particular, it is often claimed that its inclusion is necessary to explain the persistence in inflation and output frequently ob-

served in the data. However, as is clear from Figure 1, our model can predict inflation and output persistence even under the NKPC (this result is discussed further in section 5.4). Therefore, the motivation for including a lagged inflation term in the Phillips curve is weakened in the context of our model. In light of this, it may well be the case that the NKPC and ITEPC scenarios considered above are more relevant. Moreover, in these cases, inflation rises by much less than under the HPC and does not stay high for so long. In particular, even if only a very low proportion of firms use inflation-target expectations, the increase in inflation during the transition is quite small (see, for example, Figure 2(b)). Therefore, we could interpret our results as suggesting not only that gradual demand expansions are desirable, but also that they can often be designed in such a way that the associated increase in inflation during the transition to equilibrium is minimal. The policy implications of this are clear and will be discussed further in our conclusion.

5.3 Optimal Response to Cost-Push Shocks

Thus far, we have focussed on the question of how best to deal with suboptimal initial natural rates of output (possibly interpreted as resulting from adverse natural rate shocks). However, we can also use the solution to our model to determine the optimal response to cost-push shocks in the presence of hysteresis (i.e. we can consider what happens when the e shocks in (3) and (5) are no longer assumed to be zero). The key result here is that the policymaker should respond less actively to cost-push shocks when there is hysteresis in the natural rate. To see why this is the case, consider a non-persistent positive cost-push shock. In the standard new-Keynesian model, it is optimal to contract output in response to this type of shock: though this output contraction has a direct cost, inflation is reduced and overall loss is lowered as a result. However, when there is hysteresis, the output contraction lowers the natural rate of output and is therefore associated with additional costs in the future. In other words, the output contraction is much more costly than in the standard setup. Therefore, although output should still be contracted slightly following the shock, the response should be smaller when there is hysteresis. Consequently, inflation will increase by more than is normally the case. Moreover, as we would expect, the size of the optimal response is reduced (and inflation is increased) when either the natural rate adjusts more quickly or the policymaker places greater weight on the upper bound output gap term (i.e. the magnitude of c_2 and equivalent coefficients is decreasing in γ and ϕ , while the magnitude of a_2 and equivalent coefficients is increasing in γ and ϕ).

5.4 Hysteresis as a Source of Inflation and Output Persistence

As noted above, the NKPC has often been criticised on empirical grounds for being unable to generate inflation and output persistence in the absence of serially correlated shocks. However, in the presence of hysteresis, it is clear from the transition paths in Figure 1 that, even under the NKPC, both inflation and output can be highly persistent.

We can also see this mathematically by combining the NKPC solutions for inflation and output, (7) and (8), with the hysteresis equation, (1). Doing this, we show in Appendix C that, if there are no shocks during the transition to equilibrium, inflation and output evolve according to:

$$\pi_{t+i+1} = (1 - \gamma - \gamma c_1) \pi_{t+i} \quad (13)$$

$$y_{t+i+1} - \bar{y}^* = (1 - \gamma - \gamma c_1) (y_{t+i} - \bar{y}^*) \quad (14)$$

From (13) and (14), it is clear that if inflation and output are away from their steady-state values, they will remain so in subsequent periods. It is also clear that the degree of persistence is determined by the value of $1 - \gamma - \gamma c_1$. Under the baseline calibration, this term takes a value of 0.87, illustrating that the model is capable of generating quite high levels of persistence. By contrast, if $\chi = 0.01$, $\phi = 0.0625$, $\delta = 0.58$ and $\gamma = 0.5$ (changes which all serve to reduce the size of the term), it takes a value of 0.52.

From this discussion, we can see how our model is able to generate significant persistence in inflation and output without appealing to serially correlated shocks or including lagged terms in either the Phillips curve or the IS curve. Moreover, so long as adjustment takes place, this will still be the case even if c_1 is not set optimally. Having said this, it is necessary for the initial natural rate of output to be below its upper bound. Therefore, it could be argued that inflation and output persistence are not inherent to the model, only being generated under certain circumstances. However, in light of the growing empirical literature mentioned above which suggests that inflation persistence may be episodic (e.g. Alogoskoufis and Smith, 1991; Cogley and Sargent, 2001, 2005; Levin and Piger, 2003; Benati, 2005), this could actually be a desirable feature. Moreover, if the economy is frequently subject to natural rate shocks (i.e. v shocks), then this would provide a clear channel through which non-persistent shocks could generate persistence in inflation and output in our model.²²

This then raises the question of how the v shock in (1) may be interpreted in practice. It is clear that for it to be relevant, it must affect the current natural rate of output relative

²²Although we have abstracted from the possible non-zero expectation of the v shock when solving our model (see footnote 16), this result would almost certainly continue to apply if it were taken into account. Moreover, although it is usual to view some of the example shocks discussed below as normally not being truncated, they can still generate a v shock with a truncated distribution under certain non-linear mappings (e.g. threshold rules). For example, white noise investment shocks could map into a truncated v shock if, as suggested below, the equilibrium rate of unemployment depends on the capital stock, but only over a certain range.

to its upper bound. Therefore, it could be interpreted as an unanticipated demand shock not linked to policy – the demand expansions in some of the 1990s "success stories" which, according to Ball (1999, p. 236), were "largely accidental", could be viewed as possible examples. Alternatively, it could be interpreted as some kind of sectoral shock. For example, a negative export shock is likely to reduce the current potential of the economy since certain workers and firms are likely to be geared towards that sector. In this situation, an increase in domestic demand might therefore temporarily generate inflation if other sectors are already at capacity, but might also act as a mechanism for positive hysteresis by inducing workers to retrain, thus allowing the natural rate to eventually return to its old level. Similarly, a negative investment shock could reduce the current natural rate: indeed, Kapadia (2005a) has shown that the equilibrium rate of unemployment can depend on the capital stock over a certain range. If the investment rate can be temporarily influenced by the level of aggregate demand, a demand expansion could then help to restore the natural rate to its old level by raising the level of the aggregate capital stock. The key point, though, is that in all of these cases, the optimal response to the initial shock will generate persistence in both inflation and output.

6 Comments on the 1990s "Success Stories"

This paper was partially motivated by a desire to explain the economic performance of the 1990s "success stories". All of these countries significantly reduced their equilibrium unemployment rates during some kind of demand expansion which lasted for several years. Moreover, broadly speaking, inflation rose in the early stages of these expansions (both in absolute terms and relative to the industrial country average), at around the time or just before equilibrium unemployment rates started to fall. However, although the degree to which inflation increased varied across countries, these increases were all only temporary.

For example, as argued by Ball (1999), Portugal and the Netherlands both experienced fairly rapid demand expansions in the mid to late 1980s, continuing into the early 1990s in the case of the Netherlands. In both countries, equilibrium unemployment rates started to fall substantially in the late 1980s but inflation increased temporarily between 1988 and 1990/1991 before declining again. Moreover, the rise in inflation (of approximately five percentage points in both cases) was considerably greater than the 2.5 percentage point average increase in industrial countries over a similar period.²³ Meanwhile, the United Kingdom experienced a sustained gradual demand expansion following its exit from the Exchange Rate Mechanism in late 1992.²⁴ The equilibrium

²³All inflation data quoted in this section are based on annual percentage changes in the CPI, taken from International Monetary Fund, *International Financial Statistics*.

²⁴When Ball (1999) discusses the United Kingdom, he refers to the late 1980s demand expansion as being important in reducing the equilibrium rate of unemployment. However, by mid-1992, the U.K.

unemployment rate fell gradually for the rest of the decade, though there was a small runup in inflation in the early stages of the expansion, with inflation temporarily increasing by just over two percentage points between 1993 and 1995 (a period when the industrial country average barely changed). A similar pattern was repeated during the Danish demand expansion of the late 1990s, with inflation temporarily rising by approximately 1.5 percentage points between 1998 and 2000 (slightly more than the average increase in industrial countries). Finally, Ireland, which had the largest percentage fall in equilibrium unemployment of all the "success stories", saw inflation increase by almost four percentage points more than the industrial country average between the start of 1999 and the end of 2000, though this runup did come further into its long demand expansion.

It seems reasonable to assume that the falls in equilibrium unemployment rates in all of these countries approximately translated into proportionate increases in their natural rates of output. In light of this, we can see from the transition paths in Figures 1-3 that our model can broadly replicate the actual movements of inflation, output and the natural rate of output in the 1990s "success stories". In particular, in response to demand expansions lasting for several years, it can predict temporary runups in inflation and gradual but permanent improvements in the natural rate, both of which are of the correct order of magnitude. Moreover, by varying parameter values across countries, it is likely that the match can be improved further. For example, the model may be able to explain the relative stability of U.K. inflation during its transition by appealing to the more gradual nature of its demand expansion and the possibility that inflation expectations may have been better anchored as a result of inflation targeting.

This broad empirical consistency comes despite the fact that our model is relatively simple: it only really contains three equations and does not touch on the potential role of investment booms, which, as argued in Kapadia (2005a), may be particularly important in explaining the successes of Ireland and Portugal. A more sophisticated model, perhaps incorporating a sluggishly adjusting IS curve, an explicit role for the capital stock, and open economy considerations, could probably replicate the data even more closely, especially if the effects of cost-push shocks were also taken into account. Although labour market reforms may have had some role, our results therefore suggest that positive hysteresis and expectations anchoring may together go a long way towards explaining the performance of the 1990s "success stories".

economy was stagnant, with unemployment at a high rate and seemingly poor prospects for recovery. Therefore, although the late 1980s expansion may have brought about some positive hysteresis, it seems that the real stimulus to the economy came from the export boom which ensued following the enforced devaluation of late 1992.

7 Conclusion

7.1 Summary of the Paper and its Main Results

The main objective of this paper was to analyse the implications of hysteresis for the conduct of monetary policy. We modified the standard new-Keynesian model to incorporate hysteresis in output and then solved our model for optimal monetary policy under a range of different Phillips curve specifications. We showed that gradual demand expansions are usually desirable when the natural rate of output is fairly low, and that they can sometimes be designed so that the associated increase in inflation during the transition to equilibrium is minimal. We also found that policymakers should respond less actively to cost-push shocks in the presence of hysteresis. In addition, we showed how introducing hysteresis into the new-Keynesian model offers a new explanation for inflation and output persistence. Finally, we argued that our model could broadly replicate the dynamics of inflation, output and the natural rate of output in the 1990s "success stories".

7.2 Possible Extensions

As the model is currently specified, all types of demand expansions (and contractions) are assumed to generate the same amount of hysteresis. However, if hysteresis is mainly associated with capital stock or trade effects, we may expect investment-led or export-led expansions to be more successful, while consumption-led expansions may simply generate inflation without improving the natural rate. To address this issue, it may therefore be interesting to build a more sophisticated open economy version of our model which incorporates different sectors. In addition, as suggested above, it may be worthwhile to solve our model for optimal monetary policy under commitment (although our main results would probably not be affected – see footnote 13), and extend it to incorporate sluggish adjustment in the goods market. Finally, it may be interesting to test our model against the standard new-Keynesian monetary policy model without hysteresis to see whether it can match the data more successfully. In particular, relative to the standard model, the reduced form solutions in our model have an additional term relating to the deviation of the current natural rate of output from its upper bound. This distinction could be used to compare the models empirically.

7.3 Policy Implications

The policy implications of our results are clear. Firstly, in countries where the equilibrium unemployment rate is already relatively low, care should be taken to ensure that actual output does not fall too far below its natural rate. In particular, policymakers may wish to respond less actively to adverse cost-push shocks than the standard

new-Keynesian monetary policy model suggests. Perhaps more importantly, in countries suffering from high equilibrium rates of unemployment, policymakers should consider running gradual demand expansions. These could help to bring about a permanent reduction in equilibrium unemployment with only a moderate, temporary inflationary cost. Moreover, if the process is sufficiently gradual and an inflation targeting regime is in place, the inflation expectations of some firms in the economy may remain anchored and the increase in inflation during the transition could be minimal. In the current climate, the potential welfare gains for certain European countries of pursuing gradual demand expansions could therefore be considerable.

Appendix A

This appendix solves our model for optimal monetary policy under the ITEPC; setting $\omega = 0$ throughout provides a derivation of the NKPC solution. The policymaker's minimisation problem under the ITEPC is to choose y_{t+i} for all $i \geq 0$ to minimise (2) subject to (1) and (5), where the constraints are taken to apply in all future periods. We start by defining x_{t+i} to be the output gap relative to the natural rate upper bound:

$$x_{t+i} = y_{t+i} - \bar{y}^* \quad (15)$$

Using (15), we can rewrite the minimisation problem as:

$$\min_{x_{t+i}} \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[(\pi_{t+i})^2 + \chi (x_{t+i} + \bar{y}^* - y_{t+i}^*)^2 + \phi (x_{t+i})^2 \right] \right\} \quad (16)$$

subject to (for all $i \geq 0$):

$$y_{t+i+1}^* = (1 - \gamma) y_{t+i}^* + \gamma x_{t+i} + \gamma \bar{y}^* + v_{t+i+1} \quad (17)$$

and:

$$\begin{aligned} \pi_{t+i} = & \kappa \left[(x_{t+i} + \bar{y}^* - y_{t+i}^*) - \omega \beta \lambda E_t \{ (x_{t+i+1} + \bar{y}^* - y_{t+i+1}^*) \} \right] \\ & + \beta [1 - \omega (1 - \lambda)] E_t \{ \pi_{t+i+1} \} + \eta e_{t+i} \end{aligned} \quad (18)$$

In what follows, we assume that $E_t (v_{t+i+1}) = 0$ (see footnote 16). Since all decisions are made in period $t + i$, we can therefore ignore the v_{t+i+1} shock in our analysis. (Alternatively, we can simply assume that there are no natural rate shocks at all.)

To solve for the optimal monetary policy under discretion, we use the minimal state variable (MSV) approach discussed by McCallum (1983, 1999) and McCallum and Nelson (2004). It is clear that the relevant state variables which will affect the solution are $(\bar{y}^* - y_{t+i}^*)$ (note that y_{t+i}^* is completely determined in period $t + i$) and e_{t+i} (if the

natural rate of output is at its upper bound and there are no cost-push shocks, the policymaker can simply set actual output equal to the natural rate and achieve zero loss). Therefore, we conjecture that, in the solution, the reduced form processes for inflation and the (upper bound) output gap are given by:

$$\pi_{t+i} = h_1 (\bar{y}^* - y_{t+i}^*) + h_2 e_{t+i} \quad (9)$$

$$x_{t+i} = y_{t+i} - \bar{y}^* = f_1 (\bar{y}^* - y_{t+i}^*) + f_2 e_{t+i} \quad (10)$$

where f_1 , f_2 , h_1 and h_2 are all coefficients to be determined. From these equations:

$$E_t \{ \pi_{t+i+1} \} = h_1 E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} + h_2 \rho e_{t+i} \quad (19)$$

$$E_t \{ x_{t+i+1} \} = f_1 E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} + f_2 \rho e_{t+i} \quad (20)$$

where we have used the fact that $E_t \{ e_{t+i+1} \} = \rho e_{t+i}$, which follows from (4). Using (19) and (20), we can substitute out for the forward-looking inflation and output terms in (18). This gives:

$$\begin{aligned} \pi_{t+i} = & \kappa x_{t+i} + \kappa (\bar{y}^* - y_{t+i}^*) + \beta \Omega E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} \\ & + \{ \eta - \beta \rho [\kappa \omega \lambda f_2 - (1 - \omega (1 - \lambda)) h_2] \} e_{t+i} \end{aligned} \quad (21)$$

where:

$$\Omega = h_1 [1 - \omega (1 - \lambda)] - \kappa \omega \lambda (1 + f_1) \quad (22)$$

The Lagrangian can then be formed using (16), (17) and (21). The first order conditions for π_{t+i} , x_{t+i} and y_{t+i+1}^* are respectively given by:²⁵

$$\pi_{t+i} - \mu_{t+i} = 0 \quad (23)$$

$$\phi x_{t+i} + \chi (x_{t+i} + \bar{y}^* - y_{t+i}^*) - \gamma \lambda_{t+i} + \kappa \mu_{t+i} = 0 \quad (24)$$

$$E_t \{ \lambda_{t+i} - \beta [(1 - \gamma) \lambda_{t+i+1} + \Omega \mu_{t+i} + \kappa \mu_{t+i+1} + \chi (x_{t+i+1} + \bar{y}^* - y_{t+i+1}^*)] \} = 0 \quad (25)$$

where λ_{t+i} and μ_{t+i} are the period $t + i$ Lagrange multipliers on (17) and (21) respectively. Using (23), we may rewrite (24) as:

$$\lambda_{t+i} = (1/\gamma) [(\phi + \chi) x_{t+i} + \kappa \pi_{t+i} + \chi (\bar{y}^* - y_{t+i}^*)] \quad (26)$$

We can then use (23) and (26) to eliminate the Lagrange multipliers from (25). Doing

²⁵The first order condition relating to the natural rate is taken with respect to y_{t+i+1}^* rather than y_{t+i}^* because in period $t + i$, only y_{t+i+1}^* is endogenous: y_{t+i}^* is already completely determined and therefore not relevant to the optimisation problem.

this and simplifying gives:

$$(\phi + \chi) x_{t+i} + (\kappa - \Omega\gamma\beta) \pi_{t+i} + \chi (\bar{y}^* - y_{t+i}^*) - E_t \{ \beta [\phi (1 - \gamma) + \chi] x_{t+i+1} + \beta \kappa \pi_{t+i+1} + \beta \chi (\bar{y}^* - y_{t+i+1}^*) \} = 0 \quad (27)$$

Now note that (17) implies that:

$$E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} = (1 - \gamma) (\bar{y}^* - y_{t+i}^*) - \gamma x_{t+i} \quad (28)$$

Substituting (28) into (27) and collecting terms gives:

$$(\phi + \chi + \gamma\beta\chi) x_{t+i} + (\kappa - \Omega\gamma\beta) \pi_{t+i} + \chi [1 - \beta (1 - \gamma)] (\bar{y}^* - y_{t+i}^*) - \beta E_t \{ [\phi (1 - \gamma) + \chi] x_{t+i+1} + \kappa \pi_{t+i+1} \} = 0 \quad (29)$$

We can now obtain our first restrictions on the undetermined coefficients in our conjectured solutions by substituting them into (29) and equating terms in the endogenous state variables, $(\bar{y}^* - y_{t+i}^*)$ and e_{t+i} . However, first note that substituting (28) and then (10) into (19) and (20) gives:

$$E_t \{ \pi_{t+i+1} \} = h_1 [1 - \gamma (1 + f_1)] (\bar{y}^* - y_{t+i}^*) - (h_1 f_2 \gamma - h_2 \rho) e_{t+i} \quad (30)$$

$$E_t \{ x_{t+i+1} \} = f_1 [1 - \gamma (1 + f_1)] (\bar{y}^* - y_{t+i}^*) - f_2 (f_1 \gamma - \rho) e_{t+i} \quad (31)$$

Substituting (9), (10), (30) and (31) into (29) and equating terms in $(\bar{y}^* - y_{t+i}^*)$ gives:

$$\beta\gamma [\phi (1 - \gamma) + \chi] f_1^2 + \{ \phi [1 - \beta (1 - \gamma)^2] + \chi [1 + \gamma\beta - \beta (1 - \gamma)] \} f_1 + \beta\kappa\gamma f_1 h_1 + \{ \kappa [1 - \beta (1 - \gamma)] - \gamma\beta\Omega \} h_1 + \chi [1 - \beta (1 - \gamma)] = 0 \quad (32)$$

Meanwhile, equating terms in e_{t+i} gives:

$$\{ \phi + \chi + \gamma\beta\chi + \beta [\phi (1 - \gamma) + \chi] (\gamma f_1 - \rho) + \beta\kappa\gamma h_1 \} f_2 + [\kappa (1 - \beta\rho) - \Omega\gamma\beta] h_2 = 0 \quad (33)$$

To obtain two further restrictions on the undetermined coefficients, we substitute (28) into (21), followed by (9) and (10). Doing this and equating terms in $(\bar{y}^* - y_{t+i}^*)$ gives:

$$h_1 + (\beta\Omega\gamma - \kappa) f_1 - \kappa - \beta\Omega (1 - \gamma) = 0 \quad (34)$$

Meanwhile, equating terms in e_{t+i} gives:

$$h_2 - \kappa f_2 + \beta\Omega\gamma f_2 - \eta + \beta\rho [\kappa\omega\lambda f_2 - (1 - \omega (1 - \lambda)) h_2] = 0 \quad (35)$$

Together, (32) and (34) can be used to obtain numerical solutions for f_1 and h_1 for particular parameter values (recall that Ω is given by (22)). Given these solutions, f_2 and h_2 can be solved for numerically using (33) and (35), thus providing the complete solution under the ITEPC. Meanwhile, under the NKPC, the restrictions on the undetermined coefficients in the solution are still given by (32)-(35), but with $\omega = 0$ (which implies that $\Omega = h_1$), and f_1, f_2, h_1 and h_2 replaced by c_1, c_2, a_1 and a_2 respectively.

Appendix B

This appendix solves our model for optimal monetary policy under the HPC, and hence the APC, since this is just a special case of the HPC. Using (15), the policymaker's minimisation problem under the HPC is given by:

$$\min_{x_{t+i}} \frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[(\pi_{t+i})^2 + \chi (x_{t+i} + \bar{y}^* - y_{t+i}^*)^2 + \phi (x_{t+i})^2 \right] \right\} \quad (16)$$

subject to (for all $i \geq 0$):

$$y_{t+i+1}^* = (1 - \gamma) y_{t+i}^* + \gamma x_{t+i} + \gamma \bar{y}^* + v_{t+i+1} \quad (17)$$

and:

$$\pi_{t+i} = \kappa' (x_{t+i} + \bar{y}^* - y_{t+i}^*) + \alpha_1 E_t \{ \pi_{t+i+1} \} + \alpha_2 \pi_{t+i-1} + \eta' e_{t+i} \quad (36)$$

Under the HPC, lagged inflation is an additional endogenous state variable. Since this makes the model more difficult to solve, we ignore cost-push shocks for simplicity. Therefore, we conjecture that, in the solution, the reduced form processes for inflation and the (upper bound) output gap are given by:

$$\pi_{t+i} = m_1 (\bar{y}^* - y_{t+i}^*) + m_2 \pi_{t+i-1} \quad (11)$$

$$x_{t+i} = y_{t+i} - \bar{y}^* = n_1 (\bar{y}^* - y_{t+i}^*) + n_2 \pi_{t+i-1} \quad (12)$$

where m_1, m_2, n_1 and n_2 are all coefficients to be determined. From (11):

$$E_t \{ \pi_{t+i+1} \} = m_1 E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} + m_2 \pi_{t+i} \quad (37)$$

Using (37), we can substitute out for the forward-looking inflation term in (36). Dropping the cost-push shock, this gives:

$$\pi_{t+i} = \kappa' x_{t+i} + \kappa' (\bar{y}^* - y_{t+i}^*) + \alpha_1 [m_1 E_t \{ (\bar{y}^* - y_{t+i+1}^*) \} + m_2 \pi_{t+i}] + \alpha_2 \pi_{t+i-1} \quad (38)$$

The Lagrangian can then be formed using (16), (17) and (38). The first order conditions

for π_{t+i} , x_{t+i} and y_{t+i+1}^* are respectively given by:

$$\pi_{t+i} - (1 - \alpha_1 m_2) \mu_{t+i} + \beta \alpha_2 E_t \{ \mu_{t+i+1} \} = 0 \quad (39)$$

$$\phi x_{t+i} + \chi (x_{t+i} + \bar{y}^* - y_{t+i}^*) - \gamma \lambda_{t+i} + \kappa' \mu_{t+i} = 0 \quad (40)$$

$$E_t \{ \lambda_{t+i} - \alpha_1 m_1 \mu_{t+i} - \beta [(1 - \gamma) \lambda_{t+i+1} + \kappa' \mu_{t+i+1} + \chi (x_{t+i+1} + \bar{y}^* - y_{t+i+1}^*)] \} = 0 \quad (41)$$

where λ_{t+i} and μ_{t+i} are the period $t + i$ Lagrange multipliers on (17) and (38) respectively. Following similar steps to those presented in Appendix A, we can then use (40) to eliminate λ_{t+i} and λ_{t+i+1} from (41) to obtain:

$$\begin{aligned} (\phi + \chi + \gamma \beta \chi) x_{t+i} + (\kappa' - \alpha_1 m_1 \gamma) \mu_{t+i} + \chi [1 - \beta (1 - \gamma)] (\bar{y}^* - y_{t+i}^*) \\ - \beta E_t \{ [\phi (1 - \gamma) + \chi] x_{t+i+1} + \kappa' \mu_{t+i+1} \} = 0 \end{aligned} \quad (42)$$

Now note that from (39):

$$E_t \{ \mu_{t+i+1} \} = (1/\beta \alpha_2) [(1 - \alpha_1 m_2) \mu_{t+i} - \pi_{t+i}] \quad (43)$$

Substituting (43) into (42) and collecting terms gives:

$$\begin{aligned} \alpha_2 (\phi + \chi + \gamma \beta \chi) x_{t+i} + \kappa' \pi_{t+i} + [\alpha_2 \kappa' - \alpha_1 \alpha_2 m_1 \gamma - \kappa' (1 - \alpha_1 m_2)] \mu_{t+i} \\ + \alpha_2 \chi [1 - \beta (1 - \gamma)] (\bar{y}^* - y_{t+i}^*) - \alpha_2 [\beta \phi (1 - \gamma) + \chi] E_t \{ x_{t+i+1} \} = 0 \end{aligned} \quad (44)$$

To proceed, we need to derive an expression for μ_{t+i} in terms of the endogenous state variables. To do this, first note that (39) implies that:

$$\mu_{t+i} = \psi E_t \{ \mu_{t+i+1} \} + [1/(1 - \alpha_1 m_2)] \pi_{t+i} \quad (45)$$

where $\psi = \beta \alpha_2 / (1 - \alpha_1 m_2)$. Assuming that $\psi < 1$ (which will be satisfied in the actual solution), we can use the forward operator to rewrite (45) as:

$$\mu_{t+i} = \frac{1}{1 - \alpha_1 m_2} \sum_{j=0}^{\infty} \psi^j E_t \{ \pi_{t+i+j} \} \quad (46)$$

Now, it is clear that by substituting (11), (28) and then (12) into (37), we can obtain an expression for $E_t \{ \pi_{t+i+1} \}$ solely in terms of the endogenous state variables, $(\bar{y}^* - y_{t+i}^*)$ and π_{t+i-1} . By the same principle, although the number of substitutions increases, we can find an expression for expected inflation in every future period in terms of the endogenous state variables. Moreover, if we substitute all of these expressions into (46) and collect terms, the coefficients on $(\bar{y}^* - y_{t+i}^*)$ and π_{t+i-1} both form infinite geometric series which have common ratios less than one, meaning that both series have a finite

sum. Therefore, after some tedious algebra, it can be shown that:

$$\sum_{j=0}^{\infty} \psi^j E_t \{ \pi_{t+i+j} \} = \frac{m_1 (\bar{y}^* - y_{t+i}^*) + [m_2 (1 - \psi p_1) + \psi m_1 p_2] \pi_{t+i-1}}{(1 - \psi m_2) (1 - \psi p_1) - \psi^2 m_1 p_2} \quad (47)$$

where $p_1 = 1 - \gamma - \gamma n_1$ and $p_2 = -\gamma n_2$. Substituting (47) into (46) gives an expression for μ_{t+i} in terms of the endogenous state variables. Moreover, using the method explained in Appendix A, it is straightforward to obtain an expression for $E_t \{ x_{t+i+1} \}$ in terms of the endogenous state variables. Substituting these expressions, along with (11) and (12), into (44) and equating terms in $(\bar{y}^* - y_{t+i}^*)$ and π_{t+i-1} then gives us two restrictions on the undetermined coefficients. Obtaining the other two restrictions is more straightforward: we substitute (28), followed by (11) and (12), into (38) and equate terms in $(\bar{y}^* - y_{t+i}^*)$ and π_{t+i-1} . This gives:

$$m_1 (1 - \alpha_1 m_2) - \kappa' (1 + n_1) - \alpha_1 m_1 (1 - \gamma - \gamma n_1) = 0 \quad (48)$$

$$m_2 (1 - \alpha_1 m_2) - (\kappa' - \alpha_1 m_1 \gamma) n_2 - \alpha_2 = 0 \quad (49)$$

Together, these four restrictions can be used to solve numerically for m_1, m_2, n_1 and n_2 , thus providing the complete solution under the HPC. To obtain the APC solution, we can just set $\alpha_1 = 0$ and $\alpha_2 = 1$.

Appendix C

We wish to show how (13) and (14) may be derived. Assuming that there are no shocks during the transition to equilibrium, the solutions for output in successive periods under the NKPC follow from (8):

$$y_{t+i} - \bar{y}^* = c_1 (\bar{y}^* - y_{t+i}^*) \quad (50)$$

$$y_{t+i+1} - \bar{y}^* = c_1 (\bar{y}^* - y_{t+i+1}^*) \quad (51)$$

Substituting the $t+i$ version of the hysteresis equation, (1), into (51), dropping the shock term, and simplifying gives:

$$y_{t+i+1} - \bar{y}^* = c_1 [(1 - \gamma) (\bar{y}^* - y_{t+i}^*) - \gamma (y_{t+i} - \bar{y}^*)]$$

Assuming that it is non-zero (as we will throughout this appendix), we can then substitute out for the $(\bar{y}^* - y_{t+i}^*)$ term using (50). Doing this and collecting terms gives equation (14):

$$y_{t+i+1} - \bar{y}^* = (1 - \gamma - \gamma c_1) (y_{t+i} - \bar{y}^*) \quad (14)$$

To derive (13), first note that, in the absence of shocks, the solutions for inflation in

successive periods under the NKPC follow from (7):

$$\pi_{t+i} = a_1 (\bar{y}^* - y_{t+i}^*) \quad (52)$$

$$\pi_{t+i+1} = a_1 (\bar{y}^* - y_{t+i+1}^*) \quad (53)$$

Substituting the $t + i$ version of (1) into (53), dropping the shock term, and simplifying gives:

$$\pi_{t+i+1} = a_1 [(1 - \gamma) (\bar{y}^* - y_{t+i}^*) - \gamma (y_{t+i} - \bar{y}^*)] \quad (54)$$

Substituting (50) into (54) gives:

$$\pi_{t+i+1} = a_1 [(1 - \gamma - \gamma c_1) (\bar{y}^* - y_{t+i}^*)]$$

Finally, substituting out for $(\bar{y}^* - y_{t+i}^*)$ using (52) gives:

$$\pi_{t+i+1} = (1 - \gamma - \gamma c_1) \pi_{t+i} \quad (13)$$

which is what we have in the main text.

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