



DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

**THE WELFARE EFFECTS OF THIRD-DEGREE PRICE
DISCRIMINATION WITH NON-LINEAR DEMAND FUNCTIONS**

Simon Cowan

Number 364
October 2007

Manor Road Building, Oxford OX1 3UQ

The welfare effects of third-degree price discrimination with non-linear demand functions

Simon Cowan*

The welfare effects of third-degree price discrimination are analyzed when demand in one market is an additively shifted version of demand in the other market and both markets are served with uniform pricing. Social welfare is lower with discrimination if the slope of demand is log-concave or the convexity of demand is non-decreasing in the price. The demand functions commonly used in models of imperfect competition satisfy at least one of these sufficient conditions.

JEL classification: D42, L12, L13

Keywords: price discrimination, monopoly.

* University of Oxford; simon.cowan@economics.ox.ac.uk.

I am grateful to Igal Hendel, Katy Graddy, Tom Holmes, David Myatt, John Thanassoulis, John Vickers and two anonymous referees for helpful comments on earlier versions of this paper. This is a revised version of Department of Economics Working Paper 205 (October 2004).

1. Introduction

■ In 1999 Coca-Cola admitted that it was developing a vending machine that would raise the price of a Coke when the external temperature increased above a certain level. The subsequent negative publicity, however, induced Coca-Cola to abandon any plans it might have had to introduce this type of machine. This paper addresses the general question: what are the welfare effects of allowing a monopolist to practise third-degree price discrimination rather than requiring it to set a uniform price in all markets? A firm practises third-degree price discrimination when it classifies customers into separate markets using observed characteristics and sets different prices in these markets. When price discrimination is allowed a monopolist earns higher profits and individual consumers gain or lose depending on whether the discriminatory prices in their markets are below or above the uniform price. The impact of discrimination on total welfare, defined as aggregate consumer surplus plus profits, can go either way. In one well-known case discrimination definitely lowers welfare. If all markets are served with uniform pricing, demand functions are linear and marginal cost is constant then total welfare is lower with price discrimination than with uniform pricing. This is because total output is the same in the two cases. Pigou (1929) and Robinson (1969) gave early proofs of this result.

This paper analyzes the welfare effects of discrimination with a more general demand function. Suppose that demand is $Q = a + bq(p)$ where $a \geq 0$ and $b > 0$, $q(p)$ is the underlying demand function and p denotes the price. The additive and multiplicative terms, a and b respectively, vary across markets. Linear demand is a special case. The price elasticity of demand falls as a/b increases so the monopoly price is an increasing function of a/b . There are two interpretations of the demand function. Friedman (1987) presents a model of the demand for heating where the external temperature enters the demand function additively, so the shift factor, a , might represent the effect of the external temperature on demand. An alternative interpretation involves geographical discrimination. Suppose that a single firm, say a supermarket, restaurant or cinema chain, sells in different towns. In a given town there are a committed consumers who always buy the product and b price-sensitive consumers, each with demand of $q(p)$. The demographic composition of each town is defined by the ratio of committed to price-sensitive customers, a/b . Discrimination is not feasible within a town, maybe because of arbitrage possibilities, but it is feasible to discriminate across towns and a profit-maximizing firm will want to do so if a/b varies across towns. The UK's Competition Commission investigated supermarket pricing and found that "pricing might also respond to local demographics" in addition to local competitive pressures (Competition Commission, 2000, p 125). For simplicity the analysis is presented with a alone varying, but all the results also hold when b also varies across markets.

The main result is that discrimination lowers welfare for all underlying demand functions typically used in theoretical and econometric models of imperfect competition, as long as all markets are served. With this demand structure social welfare with discrimination is lower than that with uniform pricing if welfare with discrimination is a concave function of a/b . Two sufficient conditions for concavity of the welfare function are presented. An important role is played by the ratio of the curvature of the *slope* of the demand function to the curvature of the demand function itself. If this ratio is at most 1, which is equivalent to the slope of demand being log-concave, then welfare falls with discrimination. Many demand functions have log-concave slopes. Some demand functions, though, such as those in the iso-elastic class, do not satisfy this condition. Nevertheless welfare is also lower with discrimination for a general class of such functions, which are characterized by curvature that does not decrease with the price. Welfare can rise with discrimination when a large concentration of low-value consumers induces the firm to cut price substantially when discrimination is allowed and two examples are given. In general, though, welfare only rises with discrimination under very delicate assumptions in this model.

Price discrimination has the undesirable effect of ensuring that marginal utilities differ between consumers and thus output is distributed inefficiently, but this negative effect may be offset if total output is higher with discrimination. Varian (1985), building on the analysis of Schmalensee (1981), shows that a necessary condition for discrimination to raise welfare above the uniform-pricing level is that total output increases.¹ The output effects in the model can be found by applying the general formula given by Holmes (1989), who corrected the “adjusted-concavity” criterion of Robinson (1969) and also pioneered the analysis of price discrimination in oligopoly. The problem with the output test is that it does not always produce conclusive results. When output is known to increase this does not imply that welfare rises since an output increase is necessary for welfare to rise, but not sufficient. For some demand functions the effect of discrimination on output cannot be determined.

If price discrimination opens up new markets then welfare is likely to increase, and indeed weak Pareto improvements can be achieved if one market is served with uniform pricing and a new one is opened when discrimination is allowed (Hausman and Mackie-Mason, 1988). The focus in the present paper is on the case where all markets are served with uniform pricing, and assumptions are made to rule out the important possibility considered by Hausman and Mackie-Mason. The model considers only a pure monopolist. Price discrimination is also common in industries with competition, such as airlines, and Coca-Cola itself is subject to significant competitive pressure. Stole (forthcoming) surveys models of

¹ Adachi (2005) shows that when there are consumption externalities total welfare can increase with discrimination even if total output remains constant.

competitive price discrimination (see also Armstrong and Vickers, 2001, and Armstrong, 2006).

The structure of the paper is as follows. Section 2 contains the model of discriminatory and uniform pricing and the welfare framework. Section 3 provides general sufficient conditions for welfare to fall with discrimination. The effect of discrimination on output is considered in Section 4. The possibility of discrimination raising welfare is covered in Section 5. Conclusions are in Section 6.

2. The model

■ The utility function is $U(Q - a) = U(q)$ where Q is the quantity consumed, $a \geq 0$ is the shift factor and $q \equiv Q - a \geq 0$. This function is increasing, strictly concave and differentiable four times. The price is $p \geq 0$. Utility maximization implies that $U'(Q - a) = p$, so demand is $Q = a + q(p)$ and a acts as an additive shift factor. Call $q(p)$ the underlying demand function, which satisfies $q'(p) < 0$ because of the strict concavity of utility. There is a choke price, \bar{p} , which satisfies the following properties: $q(p) = 0$ and $Q = 0$ for $p > \bar{p}$, $Q = a + q(\bar{p}) \geq 0$ when $p = \bar{p}$, and $q(p) > 0$ for $p < \bar{p}$. At any price above the choke price demand is zero and underlying demand is strictly positive for prices below the choke price. The choke price is the maximum feasible price that the monopolist can set, and it may be thought of as the price of a substitute for the product.² Without a choke price the profit function would be unbounded as profits are at least $(p - c)a$, where c is marginal cost, and with a positive a it would pay to send the price to infinity. Consumer surplus is $U(q(p)) - pq(p) - pa$.

One interpretation of the demand function, following Friedman (1987), is that a is minus one times the external temperature, Q is the amount of energy purchased for heating and $q = Q - a$ is the consumer's comfort level, which falls if the weather becomes colder but can be restored if the consumption of energy for heating rises to match. The extension to the case where the underlying demand function is also subject to multiplicative shifts, so $Q = a + bq(p)$ for $b > 0$, is straightforward and is discussed in Appendix 1. What matters is the ratio a/b , rather than a itself. There is no loss of generality in assuming that b is constant and equal in all markets. The second version has b price-sensitive customers, each with demand function $q(p)$, and a committed customers who each buy one unit. The underlying demand function, $q(p)$, can be thought of as the probability that a price-sensitive customer buys a single unit. The choke price, in this interpretation, is the reservation price of the price-sensitive customer with the highest valuation, which also equals the reservation price of the committed customers.

² In many developing countries the ability of water companies to raise prices for piped water is constrained by the existence of competitive vendors of water in containers.

There are two markets, a weak one with $a = 0$ and a strong one with $a = A > 0$. The assumption of two markets is made for simplicity and nothing depends on it: all results apply for any number of markets above two as long as the shift factors are in the interval $[0, A]$. The firm has a constant marginal cost of $c \geq 0$. When the firm discriminates on price it chooses p to maximize profit, $(p - c)(a + q(p))$, where $a = \{0, A\}$. The next section discusses the solution to this problem in full. For the moment note that the discriminatory price is an increasing function of a , denoted by $p(a)$. The strong market has a lower price elasticity of demand at any given price, and standard monopoly theory implies that the profit-maximizing price rises as the elasticity falls.

When uniform pricing is required the aggregate demand function is $A + 2q(p)$, and the profit function is $(p - c)[A + 2q(p)]$. An assumption is made later to guarantee that both markets are served with the uniform price. Since division by 2 does not affect the value of p that maximizes this function it follows that the optimal uniform price is $p(A/2)$, which is the discriminatory price that would be set if the shift parameter equalled $A/2$. This is the crucial link between the uniform price and the discriminatory prices: the uniform price is the discriminatory price that applies when a equals its average value.

Social welfare in a market with price p and parameter a is the sum of consumer surplus and profits, or utility minus costs, so $W = U(q(p)) - cq(p) - ac$. Define $w(a) \equiv U(q(p(a))) - cq(p(a))$ as the component of welfare that depends on price (and then on a) when the firm discriminates. This is social welfare derived from serving the price-sensitive customers (or part of demand). Welfare with discrimination is $W(p(a), a) = w(a) - ac$. Total welfare with discrimination in the two markets is

$$W^D = w(0) + w(A) - Ac. \quad (1)$$

When the firm sets the uniform price, $p(A/2)$, welfare in a market with parameter a is $w(A/2) - ac$, so total welfare is

$$W^U = 2w(A/2) - Ac. \quad (2)$$

The difference between welfare with uniform pricing and with price discrimination is

$$W^U - W^D = 2 \left(w(A/2) - \frac{1}{2}w(0) - \frac{1}{2}w(A) \right). \quad (3)$$

Equation (3) is the key. The term in large brackets is the difference between $w(\cdot)$ evaluated at the average value of a and the average value of $w(\cdot)$. If $w(a)$ is strictly concave then (3) is positive and price discrimination cuts welfare, while convexity implies that uniform pricing is worse. This is simply an application of Jensen's inequality. The conditions for concavity are now explored.

3. Sufficient conditions for discrimination to reduce welfare

■ To find conditions for the welfare function to be concave or convex it is necessary to characterize discriminatory pricing in more detail and to define measures of the degree of non-linearity of the demand function. The curvature of direct demand is $\alpha \equiv -pq''(p)/q'(p)$. This has the same sign as q'' . The shift factor a does not affect α directly, though a change in a causes the firm to adjust the price and this may affect α . The price elasticity of demand is $\eta \equiv -pq'/Q$, which does depend on a because Q increases linearly with a .

The curvature of the slope of demand is $\beta \equiv -pq'''/q''$. This is analogous to the concept of relative prudence applied to utility functions – see Kimball (1990). The ratio of the slope curvature to demand curvature, $\beta/\alpha \equiv q'q'''/[q'']^2$, plays an important role in the welfare analysis.³ A critical value of β/α is 1. To illustrate, define the class of functions with constant α by $q = A - Bp^{1-\alpha}/(1-\alpha)$ for $A \geq 0$, $B > 0$ and $\alpha \neq 1$. The logarithmic form, $q = A - B\ln(p)$, has $\alpha = 1$. The slope curvature is $\beta = 1 + \alpha$ and the ratio of the slope curvature to demand curvature is $\beta/\alpha = 1 + 1/\alpha$. Thus $\beta/\alpha < 1$ if and only if demand is strictly concave ($\alpha < 0$). Underlying demand is iso-elastic when $A = 0$ and $\alpha > 1$, with the elasticity being $-pq'/q = \alpha - 1$ and β/α exceeding 1.

The first-order condition for the discriminatory pricing problem is

$$a + q(p) + (p - c)q'(p) = 0. \quad (4)$$

The second-order condition is $2q'(p) + (p - c)q''(p) < 0$, which can be written as $2\eta - \alpha > 0$, where all terms are evaluated at the price that satisfies (4). In terms of standard monopoly theory the second-order condition requires marginal revenue to be a locally decreasing function of output.⁴ The second-order condition is assumed to hold whenever the first-order

³ The equivalent ratio for a utility function can be important in the economics of risk. Carroll and Kimball (1996) show that consumption is a concave function of wealth when the ratio of prudence to risk aversion is constant and non-negative.

⁴ A slightly stronger assumption is that marginal revenue declines in output everywhere. Caplin and Nalebuff (1991b) assume this in their analysis of existence.

condition does so the profit function (which is continuous) is quasi-concave. This guarantees uniqueness and that the price rises with a .

It is assumed that the price in the strong market, $p(A)$, is strictly below the choke price \bar{p} . From the earlier assumption about the choke price it follows that $q(p(A)) > 0$. This ensures that both markets are served when a uniform price is set because the uniform price, $p(A/2)$, is below $p(A)$. For the assumption to hold A cannot be too large. To see this let underlying demand be $q = 1 - p$, with the choke price being 1, and let marginal cost be zero. The discriminatory price is the lower of $(a + 1)/2$ and 1. The necessary and sufficient condition for $p(A) < 1$ is that $A < 1$. With a much larger A , such as $A = 2$, the firm would want to set both the uniform price and $p(A)$ equal to 1. No price-sensitive consumer would purchase when the uniform price is set. When discrimination is allowed the price remains at 1 in the strong market but drops to $1/2$ in the weak market, so discrimination provides a weak Pareto improvement because it opens a new market. To rule out this market-opening effect the size of A is restricted so the differences between the markets are assumed to be not too large.

A useful feature of the function $p(a)$ is that it is twice-differentiable on the compact interval $[0, A]$. The effect of a rise in a on the price is always positive and is

$$p'(a) = \frac{-1}{2q' + (p - c)q''} = \frac{p}{Q(2\eta - \alpha)}. \quad (5)$$

The second version of (5) follows from the definitions of η and α and the first-order condition. When marginal cost is positive the price elasticity exceeds 1 (by the first-order condition), and the total effect of a rise in a on the elasticity is $\eta'(a) = -(\eta - 1)^2 p'(a)/c$.⁵ Since $p(a)$ and its derivative are both differentiable the welfare function $W(p(a), a) = U(q(p(a))) - cq(p(a)) - ac = w(a) - ac$ is also twice-differentiable. If the second derivative of W can be signed concavity or convexity can be established. The effect of an increase in a on welfare is

$$w'(a) - c = -Qp'(a) - c = -\frac{p(a)}{2\eta - \alpha} - c. \quad (6)$$

The middle version of equation (6) is interpreted as follows. An increase in a raises profits by $p - c$ (by the envelope theorem) and costs consumers the price of the extra unit and their

⁵ Differentiating the discriminatory price formula $p(a) = \eta c / (\eta - 1)$, which is obtained from the first-order condition, gives this expression.

existing demand multiplied by the price increase. Differentiating again, noting that $\alpha = \alpha(p(a))$ and using the derivatives of $p(a)$ and $\eta(a)$, gives

$$w''(a) = \frac{-p'(a)}{(2\eta - \alpha)^2} [2\eta^2 + p\alpha'(p) - \alpha]. \quad (7)$$

With linear demand $\alpha = 0$ so expression (7) becomes $-p'(a)/2 < 0$, which confirms that welfare is lower with discrimination. When demand is non-linear the sign of the expression in square brackets must be determined.

The first-order and second-order conditions imply that the price elasticity is at least unity and that $2\eta - \alpha > 0$. No restriction, however, for the sign of the term $p\alpha'(p)$ is implied by optimization alone. It may seem unlikely that expression (7) can be signed generally. The main result of the paper is that, nevertheless, two simple conditions are sufficient for the term in square-brackets in (7) to be positive and thus for welfare to fall with discrimination, and these conditions hold for a very wide set of demand functions.

One condition uses the concept of log-concavity. A function $f(x) > 0$ is log-concave (log-convex) if $\ln f(x)$ is concave (convex).⁶ If $f(x)$ is twice differentiable then log-concavity holds if and only if $ff''/(f')^2 \leq 1$ for $f' \neq 0$. Defining the slope of demand to be $-q'(p)$, log-concavity of the slope of demand holds when $\beta/\alpha \leq 1$.

Proposition 1. Total welfare with discrimination is lower than with uniform pricing if (i) the slope of demand, $-q'(p)$, is log-concave, or (ii) the curvature of direct demand is non-decreasing in the price.

Proof. (i) Differentiating α gives $p\alpha'(p) - \alpha = \alpha^2[1 - \beta/\alpha]$. Log-concavity of $-q'$ implies that $\beta/\alpha \leq 1$ and thus $2\eta^2 + p\alpha'(p) - \alpha > 0$. (ii) By the first-order condition $\eta \geq 1$ and by the second-order condition $2\eta - \alpha > 0$, so $2\eta^2 - \alpha > 0$. Thus $\alpha'(p) \geq 0$ is sufficient for $2\eta^2 + p\alpha'(p) - \alpha > 0$. *Q.E.D.*

Which underlying demand functions have log-concave slopes? Here existing results on log-concave density functions and associated distributions can be used. Underlying demand $q(p)$ can be written as a positive constant (which can be normalized to 1) multiplied by $[1 - F(p)]$ where $F(p)$ is a cumulative distribution function. The interpretation is that $1 - F(p)$ is the

⁶ See An (1998) for a discussion of log-concavity and log-convexity.

probability of purchase for an individual consumer or the fraction of price-sensitive consumers with valuations of at least p . The slope of demand, $-q'(p)$, is then $f(p)$, i.e. the associated density function. Bagnoli and Bergstrom (2005) show that many commonly used distributions have log-concave densities. Examples (with associated demand functions in brackets where the names differ) include the uniform (linear), normal (probit), exponential, logistic (logit) and the extreme value distributions. The constant- α function has a log-concave slope when demand is concave. Demand functions derived from the Weibull, Gamma, Chi-Squared and Beta distributions have log-concave slopes under some parameter restrictions. A log-concave slope implies that there is no more weight in the tails of the demand distribution than with an exponential function, for which the slope is just log-concave (and log-convex). The slope of demand is single-peaked. Log-concavity implies that, in a sense, preferences are not too diverse.⁷ This helps to give some intuition for the fact that welfare is lower when the demand slope is log-concave: with the consumers having similar tastes the uniform price, which ensures that marginal utilities are equal, is particularly valuable.

The second condition in Proposition 1 is useful because some standard demand functions do not have log-concave slopes. An example is iso-elastic demand, which has a log-convex slope and constant curvature. Similarly the following demand functions do not have log-concave slopes everywhere but always have increasing curvatures: demand derived from the lognormal, F , Weibull and Gamma distributions (when their slopes are log-convex), and from the t distribution with two or more degrees of freedom (see Bagnoli and Bergstrom, 2005, for discussion of these distributions).⁸ Thus part (ii) of the Proposition significantly expands the set of demand functions for which it is known that discrimination reduces welfare. Naturally there is some overlap between the sets of demand functions to which the two conditions in the Proposition apply. For example the normal and exponential demand functions satisfy both, and the constant- α function also does when demand is concave.

Why does welfare fall with discrimination if the curvature or convexity of demand is non-decreasing in the price? At a given monopoly price both the output distortion and the deadweight loss associated with monopoly pricing are more severe the more convex demand becomes.⁹ In addition as the demand function shifts out the price rises and the distortion increases even if the convexity of demand remains the same simply because the divergence of

⁷ See Caplin and Nalebuff (1991a) for an interpretation of log-concavity as similarity of preferences in the context of social choice.

⁸ Demand derived from a t -distribution with one degree of freedom – a Cauchy distribution – has too much weight in the upper tail for an interior monopoly price to exist.

⁹ See Malueg (1994) for a clear diagrammatic explanation of the role of convexity and concavity of demand in the measurement of the deadweight cost of monopoly pricing, and Malueg (1993) for an application to price discrimination.

price from marginal cost is greater. Additional insight can be obtained for some demand functions by considering the effect of discrimination on output.

4. Output and price discrimination

■ The traditional analysis of the welfare effects of price discrimination focuses on the effect on total output. A sufficient condition for welfare to fall with discrimination is that output does not rise. Holmes (1989) discusses the effect of discrimination on total output as part of his analysis of competitive discrimination, and his condition can be applied directly in the current model to find the output effect. The technique is to constrain the price difference between the two markets to be no more than a parameter $r \geq 0$, so $p_s - p_w \leq r$ where p_s is the price in the strong market (with $a = A$ here) and p_w is the price in the weak market (where $a = 0$). If $r \geq p(A) - p(0)$ the constraint does not bind and the firm sets the discriminatory prices. If $r = 0$ then the uniform price, $p(A/2)$, is obtained. The condition is that if

$$\left(\frac{p_w - c}{p_w} \right) \alpha(p_w) - \left(\frac{p_s - c}{p_s} \right) \alpha(p_s) \quad (8)$$

is non-positive (positive) for all values of r between 0 and $p(A) - p(0)$ then total output does not rise (increases) with discrimination.¹⁰ Note that as r rises p_s increases and p_w falls since relaxation of the constraint moves both prices away from the uniform price towards their unconstrained levels.

The most general set of sufficient conditions for output (and thus welfare) to fall, or at most stay constant, with discrimination is in the next result.

Proposition 2. Total output does not rise with discrimination if the curvature of direct demand is non-decreasing ($\alpha'(p) \geq 0$) and demand is strictly convex everywhere ($\alpha > 0$).

Proof. Expression (8) is zero when $r = 0$ and negative for $0 < r \leq p(A) - p(0)$ when both conditions hold since $p_s \geq p_w$ for all r , $p'_s(r) > 0$ and $p'_w(r) < 0$. *Q.E.D.*

Proposition 2 is encompassed by condition (ii) of Proposition 1, because Proposition 2 only holds for strictly convex demands. Demand functions satisfying the conditions of Proposition 2 include the iso-elastic, exponential, the Weibull and Gamma functions (when their

¹⁰ Robinson's adjusted-concavity criterion compares the curvatures of inverse demand in the two markets at the uniform price (see Robinson, 1969, pp. 193-195). She assumed that the price elasticities of demand are equal, or very close, at the uniform price.

parameters imply log-convex slopes), and demand derived from the t distribution. Demand functions that do not satisfy the conditions in Proposition 2, and for which the output test in (8) is indeterminate, include the log-normal, normal and logistic, all of which have regions of both concavity and convexity. If both prices are in the convex region of demand then Proposition 2 does apply, but if prices are in the concave region neither Proposition 2 nor the general expression (8) can be applied.

The output test in (8) can be used to provide jointly sufficient conditions for output to rise. These conditions are the reverse of those in Proposition 2. Total output increases if demand is concave and $\alpha'(p) \leq 0$. An example is the constant- α function when demand is concave and marginal cost is positive. When output rises with discrimination the welfare effect is unclear in general. Proposition 1, however, applies directly for the concave, constant- α demand function. Demand curvature is constant and the slope of demand is strictly log-concave. The output increase does not offset the distributional inefficiency caused by price discrimination and welfare falls. Thus the same conditions that imply that output rises are also sufficient for welfare to fall with this demand function.

5. Price discrimination and welfare increases

■ Welfare can increase with discrimination, but the conditions for this effect are rather special. Two examples are given in this section. In the first example the welfare function is not concave everywhere, while in the other welfare is convex everywhere. Suppose first that underlying demand is $q = 1 - p$ for $0.26 < p \leq 1$, and $q = 1$ for $p \leq 0.26$. Marginal cost is zero. Figure 1 illustrates the inverse demand function. The demand function is not differentiable, so one of the basic assumptions of the model in the previous sections no longer holds.

The weak market has $a = 0$. A market-power strategy involves pricing above 0.26 and exploiting the firm's monopoly power. The best price strictly above 0.26 is 0.5, which yields demand of 0.5 and profits of 0.25. The mass-market strategy involves pricing at 0.26 to increase demand to 1. This gives higher profits of 0.26. The profit function is not quasi-concave. As a increases from 0 the relative advantage from the mass-market pricing strategy declines. The profit from mass-market pricing is $0.26(a + 1)$, while profits from the market-power strategy are $(a + 1)^2/4$ since the market-power price is $p(a) = (a + 1)/2$. For $a > 0.04$ higher profits are earned by the market-power strategy. Let the strong market have $A = 0.4$ so the price is $p(0.4) = 0.7$. In the weak market, where $a = 0$, the firm prices at 0.26. The uniform price is $p(0.2) = 0.6$, which is close to the discriminatory price in the strong market.

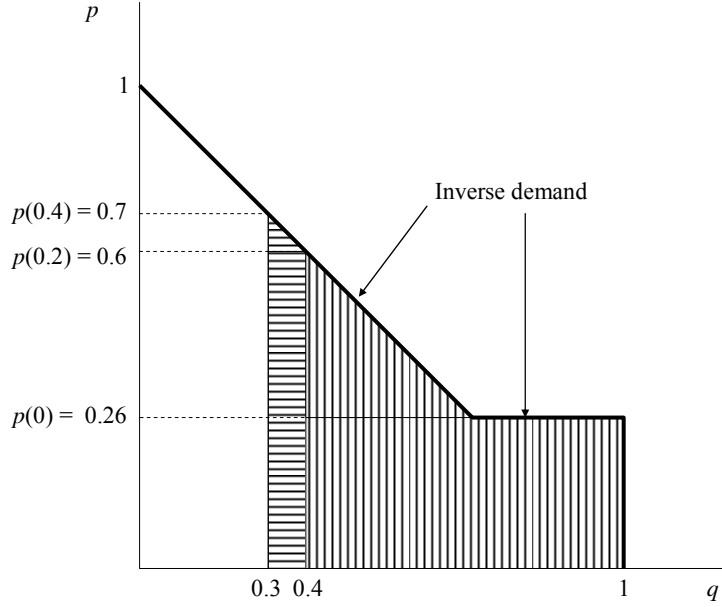


Figure 1
Inverse demand where discrimination raises welfare

The welfare effects are illustrated in Figure 1. The loss in social welfare from setting the discriminatory price of 0.7 in the strong market rather than the uniform price of 0.6 is the horizontally shaded area, while the gain from setting a price of 0.26 in the weak market rather than the uniform price is the much larger area that is vertically shaded. It is the large jump down in price when $a = 0$ that ensures that price discrimination raises welfare.¹¹ In this example a revealed preference argument shows that aggregate consumer surplus also rises with discrimination – the total cost of buying the quantities purchased at the uniform price exceeds the cost of buying the same quantities at the discriminatory prices. Consumers in the strong market, as usual, lose from discrimination, but the gains to consumers in the weak market outweigh this.

Welfare, as a function of a , is constant for $0 \leq a \leq 0.04$. For higher values of a the firm switches to the market-power strategy and welfare jumps down, and thereafter is concave and decreasing. Overall the welfare function is quasi-concave rather than concave. While the type of extreme convexity of the demand slope illustrated in the example can cause welfare to rise with discrimination, it need not do so. If the kink in demand was at 0.24, or the maximum number of customers was 0.96 rather than 1, the market-power strategy would always be

¹¹ The welfare loss in the strong market from discrimination is $0.1 \times 0.6 + 0.5 \times 0.1^2 = 0.065$, while the welfare gain in the weak market from discrimination is $0.5 \times 0.34^2 + 0.6 \times 0.26 = 0.2138$.

adopted, and the standard analysis with linear demand would apply: welfare would be concave everywhere in a , and discrimination would keep output constant and cut welfare.

The second example has a welfare function that is convex in a . Let demand come from a symmetric beta function. When the parameter of the demand function is below 1 the slope of demand is log-convex and is U-shaped, so demand is concentrated in the tails. Demand curvature falls with the price when the slope is log-convex, so neither condition in Proposition 1 applies. Details are in Appendix 2. When the slope of demand is sufficiently U-shaped the welfare function is always convex and discrimination raises welfare.

6. Conclusions

■ This paper has outlined two conditions for third-degree price discrimination to lower welfare when all markets are served and the underlying demand function is subject to additive and multiplicative shifts. Although the welfare effects could go either way, the conditions for discrimination to raise welfare are rather stringent in this model. The expectation is that discrimination will reduce welfare. The paper shows the value of a direct analysis of welfare using a parameterization of the source of demand differences between markets, rather than just using the output test (which is inconclusive when output rises with discrimination and when demand has regions of both convexity and concavity).

Of course the paper does not provide the whole story about the welfare effects of third-degree price discrimination. The assumption of interior solutions excludes the case where very high values of the demand-shifting variable induce the firm to price at the maximum level. It is, though, straightforward to extend the model to allow for this possibility. The welfare effects are then essentially the same as those characterized by Hausman and Mackie-Mason (1988): with discrimination the firm cuts the price in the weak market and keeps price at the maximum level in the strong market, so a Pareto improvement is generated. An intermediate case is also possible, where the uniform price is high but not at the maximum level, while in the strong market the price equals the maximum. This has ambiguous welfare effects. Finally other parameterizations of demand shifts are of interest. A topic for further research is to model upward shifts in the inverse demand function, so willingness to pay increases by the same amount at each output level.

Appendix 1

■ This Appendix discusses the generalization that allows for multiplicative shifts of demand.

Propositions 1 and 2 apply for any demand function that can be written in the form $Q = a + bq(p)$ where $b > 0$, and both b and a vary across markets. The argument is sketched here but formal proofs are not given. The first-order condition for profit maximization implies that the discriminatory price is a function of a and b that is homogeneous of degree zero, so the price depends on the ratio a/b . This is intuitive: if a and b both rise in the same proportion then the demand function shifts multiplicatively and there is no incentive to change the price. From this observation everything else follows. Output ($Q = a + bq(p)$), consumer surplus, profits and welfare are all homogeneous of degree one in a and b . The discriminatory price, the price elasticity of demand and α and β (which both depend only on p) are all homogeneous of degree zero in a and b . Since discriminatory welfare is homogeneous of degree one the function is concave (convex) in both its arguments if and only if it is concave (convex) in one of them. Both Propositions thus hold when both a and b vary across markets.

Appendix 2

■ This Appendix illustrates the conditions under which discrimination raises welfare when demand is derived from a beta distribution.

The demand function derived from a symmetric beta distribution is

$$q = 1 - \int_0^p [x(1-x)]^{v-1} dx / \int_0^1 [x(1-x)]^{v-1} dx \quad (\text{A1})$$

where $0 \leq p \leq 1$, $v > 0$, and the denominator of the second term is the beta function. For $v \geq 1$ the slope is log-concave and Proposition 1 applies. When $v = 1$ demand is linear. Assume that $v < 1$, so the slope is strictly log-convex and is U-shaped. The curvature of direct demand is $\alpha = (1-v)(1-2p)/(1-p)$, so demand is convex for $0 < p \leq 0.5$ and concave for $0.5 \leq p < 1$, and α falls as p rises. Let marginal cost be zero. From equation (7) welfare is convex in a if $2 + p\alpha'(p) - \alpha$ is negative. This yields a quadratic in p from which it follows that if $p > 0.5 + 0.5(1 - (1-v^2)^{0.5})/v$ welfare is convex. Take a low value of v such as 0.4. The critical value of p for convexity is 0.604. Numerical optimization shows that the profit-maximizing price when $a = 0$ is 0.678, and since $p(a)$ rises the welfare function is convex for all higher values of a .

References

- Adachi, T. "Third-Degree Price Discrimination, Consumption Externalities and Social Welfare." *Economica*, Vol. 72 (2005), pp. 171-178.
- An, M.Y. "Logconcavity versus Logconvexity: A Complete Characterization." *Journal of Economic Theory*, Vol. 80 (1998), pp. 350-369.
- Armstrong, M. "Recent Developments in the Economics of Price Discrimination" In R. Blundell, W. Newey and T. Persson, eds., *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress*. Cambridge: Cambridge University Press, 2006.
- AND Vickers, J. "Competitive Price Discrimination." *RAND Journal of Economics*, Vol. 32 (2001), pp. 579-605.
- Bagnoli, M. AND Bergstrom, T. "Log-Concave Probability and its Applications." *Economic Theory*, Vol. 26 (2005), pp. 445-469.
- Caplin, A. AND Nalebuff, B. "Aggregation and Social Choice: A Mean Voter Theorem." *Econometrica*, Vol. 59 (1991a), pp. 1-23.
- AND ——— "Aggregation and Imperfect Competition: On the Existence of Equilibrium." *Econometrica*, Vol. 59 (1991b), pp. 25-29.
- Carroll, C. AND Kimball, M. "On the Concavity of the Consumption Function." *Econometrica*, Vol. 64 (1996), pp. 981-992.
- Competition Commission *Supermarkets: A report on the supply of groceries from multiple stores in the United Kingdom*. London: The Stationery Office, Cm 4842, 2000.
- Friedman, D. "Cold Houses in Warm Climates and Vice Versa: A Paradox of Rational Heating." *Journal of Political Economy*, Vol. 95 (1987), pp. 1089-1097.
- Hausman, J. AND Mackie-Mason, J. "Price Discrimination and Patent Policy." *Rand Journal of Economics*, Vol. 19 (1988), pp. 253-265.
- Holmes, T. "The Effect of Third-Degree Price Discrimination in Oligopoly." *American Economic Review*, Vol. 79 (1989), pp. 244-250.
- Kimball, M. "Precautionary Saving in the Small and in the Large." *Econometrica*, Vol. 58 (1990), pp. 53-73.
- Malueg, D. "Bounding the Welfare Effects of Third-Degree Price Discrimination." *American Economic Review*, Vol. 83 (1993), pp. 1011-1021.
- Malueg, D. "Monopoly Output and Welfare: The Role of Curvature of the Demand Function." *Journal of Economic Education*, Vol. 25 (1994), pp. 235-250.
- Pigou, A.C. *The Economics of Welfare*. London: Macmillan, Third Edition, 1920.
- Robinson, J. *The Economics of Imperfect Competition*. London: Macmillan, Second Edition, 1969.
- Schmalensee, R. "Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination." *American Economic Review*, Vol. 71 (1981), pp. 242-247.

Stole, L. "Price Discrimination and Imperfect Competition" In M. Armstrong and R. Porter, eds., *The Handbook of Industrial Organization* Volume 3. New York: North-Holland, forthcoming.

Varian, H.R. "Price Discrimination and Social Welfare." *American Economic Review*, Vol. 75 (1985), pp. 870-875.