Essays on Corporate Finance in Continuous Time



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To my parents
Manuel da Ressurreição Cardoso & Maria de Lourdes Cruz Poço
for their unending love and support.

In memory of my beloved grandmother
Ana da Conceição Mendes da Cruz
1927-2020

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Abstract

This thesis provides three contributions to theoretical corporate finance in continuous time. The first contribution focus on regulation over patent litigation, the second on management dismissal, and the third on the capital structure of financial institutions.

First, I study the effects of the Hatch-Waxman Integrity Act over the set of admissible financial positions available to an informed agent that requests an inter partes review to challenge a patent but wants to prevent expected losses of litigation. This Act requires that a person that short-sells the shares of a patent-holding company must wait ninety days before requesting an inter partes review. Enacting this Act does not harm informed agents. Instead, it decreases the percentage of the defendant's equity that a plaintiff must short-sell in order to hedge her expected loss of requesting the inter partes review. Also, short-selling hedging positions are smaller if the patent can inexpensively be converted into a commercial product, and if the value of this commercial product has low volatility.

Second, I develop a continuous time model to study the Leicester-Pearson phenomenon, that is, the atypical cases of managerial dismissal triggered by high firm performance. My explanation proposes that differences in the knowledge and exerted effort between a firm's incumbent manager and an outsider candidate can cause this phenomenon. The firm has an option to replace the incumbent which is exercised when the outsider is considered better for the existing cash-flow level after paying replacement costs. I derive the level of cash-flows that triggers dismissal and identify the key condition that sustains dismissal with increasing cash-flow performance. The incumbent always faces this phenomenon if the outsider has comparative advantage in the effort attribute for which it suffices to have lower outsider's risk aversion and effort costs.

Third, I study the relationship between three regulatory measures imposed in Europe after the 2008 crisis and the capital structure decisions of regulated banks. I analyse the increase of the level of coverage of Deposit Insurance Schemes, the introduction of the Private Sector Acquisition resolution tool, and the introduction of the bail-in tool. The introduction of these measures incentivises banks to decrease the ratio of equity over assets and to increase the ratio of deposits over total debt. Lastly, introducing the bail-in tool generates a recapitalisation option, that is, banks unable to rebalance their capital structure may have an incentive to increase ex-ante the weight of bail-inable debt over total debt if they expect that a potential bail-in will result in a value-enhancing recapitalisation.

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Chapter 1

Introduction

1.1 Thesis Overview

This thesis contributes to the field of corporate finance in continuous time. In detail, Chapter Two studies changes in regulation over patent litigation by building on real options theory, Chapter Three focus on management dismissal and builds on the fields of contract theory and real options theory, and Chapter Four focus on the capital structure of banks and it builds on the trade-off theory in continuous time and banking theory.

In Chapter One I study the impact of the Hatch-Waxman Integrity Act over the set of admissible financial positions of an informed agent that initiates an inter partes review and assumes a long or short financial position over the shares of the patent-holder firm.

The inter partes review, enacted in 2012, was designed to provide a fast and inexpensive procedure to verify the property and validity of a patent. However, reduced costs of litigation and lower petition requirements created speculative opportunities. Uninformed speculators could request inter partes reviews over patents and short the stocks of the patent-holder firm anticipating that the information asymmetry would lead market participants to price the patent litigation negatively. In response, regulators proposed the Hatch-Waxman Integrity Act 2019 which requires a waiting period of ninety days between the date of short-selling and the date of the inter partes review request. This legislation is directed, in essence, to

ill-intended uninformed speculators who profit from price swings caused by the revelation of an inter partes review. The problem is that this legislation does not protect informed agents who request the inter partes review to trigger the revelation of the true quality and ownership of a patent but, at the same time, want to protect themselves from potential litigation losses by assuming a financial position in the equity of the defendant company.

I present a framework with two players: a plaintiff and a defendant. The defendant owns a patent modeled as an investment option which, after exercise, generates value from a commercially viable product. The patent's quality is measured by the size of the investment costs, i.e. the patent is good if the investment costs are low or bad if they are so high that the patent value approximates zero. This quality is known by the defendant and the plaintiff but unknown by financial markets. The plaintiff wants to request an inter partes review to reveal the true type of the patent and its property rights. She can avoid expected losses of litigation by defining a financial position, i.e. buying or shorting shares of the defendant, hence, exploiting the asymmetry of information against the market.

I then compare the set of feasible financial positions available to the plaintiff before and after introducing the Hatch-Waxman Integrity Act. This Act imposes that, conditional on the plaintiff assuming a short-selling position, she is obliged to wait ninety days before requesting the inter partes review. In fact, the Hatch-Waxman Integrity Act introduces an European call exercisable in ninety days after the short-sale position where the plaintiff can choose between requesting the inter partes review or close the initial position.

The results show, first, that introducing the Hatch-Waxman Integrity Act reduces the number of shares that must be shorted in order to hedge the plaintiff's expected value of litigation. The wedge between the time of the short-selling and the litigation time introduced by this Act generates positive time value that increases the plaintiff's net value and, consequently, permits a reduction in the hedging short-selling position. Second, if a patent is good, the plaintiff has a larger range of profitable financial positions than if the patent is bad because the quality of the patent affects the plaintiff's expected value of litigation. Third, the hedging short-selling position decreases if the value of the commercial product is highly volatile because higher uncertainty increases the time value of the patent and the time value of the option to litigate.

Chapter Two investigates the conditions that prompt involuntary managerial replacement when firms start to perform exceptionally well. I call this the Leicester-Pearson phenomenon in reference to the relationship between the football coach Nigel Pearson and Leicester City. Pearson guided Leicester City through one of the the most successful comebacks in football to be replaced, in the end, with a more awarded coach just to win the Premier League. This phenomenon contrasts with the majority of involuntary managerial dismissals caused by poor firm performance, public scandals or by the CEO's proximity to the retirement age.

I develop a continuous-time framework to study this phenomenon where an unlevered firm generates continuous cash-flows subject to exogenous shocks. The firm is owned by a risk-neutral shareholder who has an agency relationship with a risk-averse incumbent manager and has an option to replace the incumbent with a risk-averse outsider manager. The shareholder faces moral hazard, managerial hidden savings and costly replacement.

I propose an explanation for the Leicester-Pearson phenomenon based on the differences in the skills between these managers. In detail, I consider that each manager has a skill that combines a knowledge attribute and an effort attribute and these attributes are employed to improve cash-flow performance. The knowledge attribute is costless, is independent of cash-flow performance and its contribution is limited as the firm grows. I consider that knowledge has a limited contribution because, as a firm grow larger, it is increasingly harder for members of the upper echelon of a firm to pass their personal insights throughout the organisation. Differently, the effort attribute is costly, depends on on cash-flow performance and its impact increases as the firm grows. I derive the level of cash-flows that triggers dismissal and identify how differences in the attributes of the incumbent manager and the outsider candidate interact in order to generate a dismissal decision that sustains the Leicester-Pearson phenomenon.

My results demonstrate that a dismissal trigger that sustains the Leicester-Pearson phenomenon emerges under two conditions. First, the outsider manager must always be more efficient than the incumbent in the effort attribute, that is, the outsider can generate a higher optimal effort contribution than the incumbent. This is always the case when the outsider's risk aversion and effort costs parameters are lower than the incumbent manager's risk aversion and effort costs.

Second, either the incumbent manager has comparative advantage in the knowledge attribute or the shareholder faces very high replacement costs, or both. Under these conditions, the incumbent manager is irreplaceable while cash-flows are low because she either better at the knowledge attribute or she is entrenched due to high replacement costs. However, as cash-flows increase, the outsider's comparative advantage in the effort attribute becomes more advantageous because it escalates with cash-flows and, eventually, the incumbent is dismissed.

The dismissal trigger that sustains this phenomenon decreases when the agents' discount rate decreases, the incumbent's risk aversion and effort costs increase, the outsider's risk aversion and costs of effort decrease. Uncertainty has a non-monotonic effect over the dismissal decision.

In Chapter Three I study the impact of the regulatory changes imposed in Europe after the 2008 crisis and the capital structure decisions of regulated banks. I analyse the increase of the level of coverage of Deposit Insurance Schemes, the introduction of the Private Sector Acquisition resolution tool, and the introduction of the bail-in tool.

I set up a continuous-time framework in which a bank earns stochastic cash-flows and is financed by non-deposit liabilities and by deposits subject to the usual bank-run characteristics. The bank faces a regulator that imposes a Basel-style insolvency regulation, provides deposit insurance, limits bankruptcy costs through the private sector acquisition tool and, if the bank holds non-deposit liabilities, determines the bail-in by converting this type of debt into equity. As in Leland (1994), the financial institution must select a capital structure to maximise its total value given these regulatory actions.

My results show, first, that the regulatory changes under study cause an increase in the leverage of financial institutions. An increase in the levels of deposit insurance transfers expected bankruptcy costs out of the balance sheet to deposit insurance funds. As a consequence, debt finance becomes less expensive which increases the leverage in equilibrium. The private sector acquisition tool decreases the costs of bankruptcy which also prompts debt finance. Lastly, a bail-in allows the conversion of non-deposit liabilities to equity in exchange of postponing the closure of the institution. This reduces the expected costs of financial fragility which boosts the leverage ratio. Second, the model shows that these regulatory

changes should increase the ratio of deposits over non-deposit liabilities because more deposit insurance subsidises depositors in the event that the bank closes while non-deposit liabilities remain exposed to bankruptcy costs. Third, my analysis shows that, in the presence of the bail-in tool, the increase in the level of coverage of deposits does not always fully crowd out non-deposit liabilities from the debt structure. The bail-in tool can be useful when a bank is closer to insolvency since it offers the chance to recapitalise the institution when it is unable to change its capital structure. Therefore, the bank can take advantage of the bail-in tool by selecting an ex-ante level of bail-inable liabilities that can later trigger a bail-in by the regulator. I call this the recapitalisation option of bail-ins.

1.2 Literature Overview

I present a brief overview of the literature underlying my thesis, namely, on capital structure, real options theory, contract theory, and banking literature. Each one of the upcoming chapters contains a more detailed and specific account of relevant research.

Continuous-time stochastic modelling provides essential frameworks to study economic phenomena that require a careful treatment of time dynamics, intra-period optimisation and stopping time optimisation. Corporate finance research benefitted substantially with the advancements of these tools after the seminal works of Black and Scholes (1973) and Merton (1973) which triggered the general interest for continuous-time models and prompted a crescendo of research in the following decades.

Research in corporate finance flourished, particularly in the fields of security pricing and capital structure, benefitting from new insights over the long-standing static Modigliani-Miller propositions. For example, Merton (1974) applied option-pricing techniques to provide a model that values a firm's equity and debt. This work was later supplemented by Black and Cox (1976), allowing shareholders to default on their debt obligations. Other developments followed with emphasis on the frictions commonly faced by firms when selecting their capital structure. The model of Leland (1994), which I make use of in Chapter Four, extended the trade-off theory between default costs and debt tax shields proposed by Kraus and Litzenberger (1973) to a dynamic setup which determines the optimal amount of debt that

maximises the firm value. Later, Leland and Toft (1996) allowed for the debt maturity to be also endogenously determined with the default time. Sundaresan (2013) provides a review on the literature on capital structure and summarises the main stylised facts. More recently, He and Milbradt (2016) dropped the ex-ante commitment of the firm to a debt maturity structure and to a default time, and Cheng and Milbradt (2012) studied the optimal capital structure of debt when a firm faces debt rollover restrictions and risk-shifting problems in the asset side.

Real options theory is built, originally, on the work of Black and Scholes (1973) but focus instead on the problem of choosing the optimal time to make an investment decision where the decision maker faces uncertainty and the investment decision is, at least, partially irreversible. The key technical condition to make a real options problem treatable is that, under adequate conditions, we are able to transform the problem of finding an optimal stopping time into a free-boundary problem (Peskir and Shiryaev 2006). Brennan and Schwartz (1985) and McDonald and Siegel (1986) provided the first remarkable contributions. The first assessed an investment opportunity in a framework comprising commodity price uncertainty while the second considered two sources of uncertainty, the cash-flows and the costs of the project. These preliminary works were later synthesised and extended in Dixit and Pindyck (1994).

Real options theory flourished to numerous other areas. Three excellent overviews of real options literature can be found in Lambrecht (2017) who emphasises finance applications, in Azevedo and Paxson (2014) who focus on game theory applications and in Trigeorgis and Tsekrekos (2018) who provide a broader discussion of this field. Recent examples are Morellec et al. (2015) and Shibata and Nishihara (2015) who investigate the joint investment decision and the investment's financing decision, or Cardoso and Pereira (2015) and Gryglewicz et al. (2020) who studies the investment decision with the problem of aligning the interests between shareholders and manager.

Game theory and contract theory literature benefitted from the introduction of continuous-time frameworks. Grenadier and Wang (2005) provided one of the primary contributions by bringing moral hazard and adverse selection to real options, later extended by Shibata (2009) with the introduction of an audit technology. Grenadier and Malenko (2011) followed with

a continuous-time signalling game between insiders and outsiders. Grenadier et al. (2016) consider a problem where a principal chooses the time to invest based on information conveyed by a biased agent. Sannikov (2008) created a moral-hazard framework in continuous-time that innovated by representing the incentive compatible condition as an adapted process. Recent developments tackled increasingly complex frictions as seen, for example, in He et al. (2017) who introduces bayesian learning in managerial ability, Miao and Rivera (2016) who includes a principal with ambiguous beliefs and Koskinen and Maeland (2016) who study the issue of investment timing in a screening model where an investor invites several applicants to compete over proposed projects.

New research in banking theory literature developed continuous-time models that proved particularly useful to study risk and liquidity management problems in financial institutions. Milne and Whalley (n.d.) provides one of the earliest contributions by studying the impact of capital regulation over the dividend distribution and cash-flows risk level selected by a financial institution. Rochet and Villeneuve (2005) takes a portfolio management approach to study the investment policy and the dividend distribution rule of a financial institution. More recently, Sundaresan and Wang (2017) studied the debt structure of a financial institution subject to deposit insurance in a classic trade-off capital structure model with deposits and non-deposit liabilities, Lambrecht and Tse (2019) presented a continuous-time model where a risk-averse manager dynamically optimises the capital structure and the risk level of assets, and Hugonnier and Morellec (2017) studied the effect of liquidity and leverage requirements over default decisions of banks.

Chapter 2

Hedging Patent Litigation under the Hatch-Waxman Integrity Act

The Hatch-Waxman Integrity Act is designed to reduce abusive short-selling from speculators that petition for an inter partes review of a patent while shorting the shares of the patent holder firm. However, this legislation does not distinguish uninformed speculators from informed agents. While the former profit from price swings caused by the public announcement of an inter partes review, the latter profit from the public revelation of the true quality of a patent. My model studies the effects of introducing this Act over the set of admissible financial positions available to an informed agent that requests an inter partes review but wants to prevent expected losses of litigation. I show that enacting this Act does not harm informed agents. Instead, it decreases the percentage of the defendant's equity that a plaintiff must short-sell in order to hedge her expected loss of requesting the inter partes review. Also, the hedging short-selling position is smaller for patents that can be easily converted into profitable products because patent quality increases the plaintiff's expected litigation value. Finally, for patents in highly uncertain product markets, the hedging short-selling position is even smaller because higher volatility increases the time value of the patent and the time value of the litigation option.

2.1 Introduction

The inter partes review is a legal procedure established in the 2012 America Invents Act that allows a person to challenge the validity of a patent issued in the United States. The plaintiff requests the inter partes review to the Patent Trial and Appeal Board (PTAB), an administrative law body of the Patent and Trademark Office (PTO). This procedure was created to be fast and inexpensive[1] in order to reduce the prolific number of existing bogus patents approved by the PTO[2].

The inter partes review has one key consequence. Lower litigation costs and a hastened litigation process encourage informed players to capitalise on the existing asymmetric information over equity markets. These agents can hedge the litigation process or even make a profit by assuming a financial position over the publicly traded shares of a firm that they know holds an invalid patent and by filling an inter partes review to challenge the patent's validity. However, lower litigation costs also attract uninformed speculators who profit from the impact of announcing an inter partes review over the share price of the patent holder.

Consider the case of Hayman Capital Management managed by Kyle Bass:

> "A hedge fund manager who bet successfully on a housing market crash in the United States, is aiming at pharmaceutical companies. Acorda Therapeutics stock fell almost 10% on news of his challenge (...). Hayman is using a newish process called "inter partes review". If the US Patent Office accepts the petition, one of Acorda's patents may be disallowed within the next year for the obviousness of its technology."[3]

Moreover, this first inter partes review filled by Bass against Acorda was followed by a

[1]Under the inter partes review, the time of the litigation decision cannot surpass eighteen to twenty four months (Dubis 2015), the litigation costs are reduced from millions of dollars to thousands of dollars and, given the absence of standing requirement, the spectrum of entities that can file a petition increases considerably when compared with court litigation (Vishnubhakat et al. 2016). Details on the Inter partes review rules can be found in the Chapter 42, Title 37 CFR.

[2]The soaring number of bogus patents is, in part, related to the alignment of interests between the PTO and the patent applicant. This moral hazard issue is, in part, caused by the PTO budget dependency on post-allowance fees. For instance, as noted by Frakes and Wasserman (2017a), more than half of the PTO's patent revenues obtained in 2016 were fees from issuing new patents and from renewing previously conceded patents. Moreover, Frakes and Wasserman (2017b) suggests that patent reviewers become time constrained as they are promoted, which increase the propensity to concede patent grants.

[3]New York Times article "Kyle Bass Wields New Weapon in Challenging Drug Makers" available https://dealbook.nytimes.com/2015/02/11/kyle-bass-wields-new-weapon-in-challenging-drug-makers/.

second petition which resulted in an additional decrease of 5% in this biotech firm stock price[4]. As of March of 2017, nine out of eighteen patents challenged by Hayman had been invalidated by the patent office (Multak 2017).

The pressure to crowd-out uninformed speculators prompted north-american legislators to present the Hatch-Waxman Integrity Act of 2019. This Act proposes the following amendment to the Security Exchange Act:

> "A person shall be considered to be using a manipulative or deceptive device if [he/she](...) files a petition to institute an inter partes review (...) with respect to a patent and, during the 180-day period beginning on the date that is 90 days before the date on which the person files the petition (...), engages in a short sale of any publicly traded security of the owner of the patent that is the subject of the petition"

This legislation amendment aims at reducing abusive short-selling from short-term uninformed speculators who focus on profiting from price swings caused by the revelation of an inter partes review. The stock market reacts when these agents announce the inter partes review by accounting for the possibility that the patent-holder will lose the patent. These speculators close the position profitably afterwards.

However, this legislation amendment does not separate these short-term uninformed agents from informed agents who profit from the revelation of the true quality of a patent validity. While the former introduce noise in the market, the latter remove the market mispricing by inducing patent quality revelation [5].

With this in mind, this chapter studies the effects of introducing the Hatch-Waxman Integrity Act of 2019 over the set of admissible financial positions of an informed agent that initiates an inter partes review and assumes a long or short financial position over the shares of the patent holder firm.

[4]See "Litigation Trading: An Introduction to Wall Street's Interest in Patent Cases" available in https://www.americanbar.org/groups/intellectual_property_law/publications/landslide/2015-16/september-october/litigation_trading_introduction_wall_streets_interest_patent_cases/

[5]The "patent troll" is a third possible category. This agent does not know the true patent quality but initiates the inter partes review and waits for the court decision. I choose to not address this category in the present chapter because, in my model, this agent is a sub-case of the informed agent but with a probability distribution underlying the patent quality.

This chapter contributes to the litigation literature in two ways. First, my model builds on the seminal work of Choi and Spier (2018) on patent litigation under financial hedging to provide a detailed account of the implications of the Hatch-Waxman 2019 amendment over the set of financial strategies that produce profitable litigations against patent holding firms. Moreover, I refine Choi and Spier's framework by using a continuous time setup that allows us to model the lag between the time when a financial position is established and the moment when the inter partes review is requested, and grants us closed form solutions for my value functions. Moreover, the presence of stochastic shocks allows us to show the implications of uncertainty which is not considered in this recent literature.

Second, I extend the real options literature to the problem of patent litigation initiated by an outsider. Existing literature only considers the case where the owner of a patent has the option to litigate against a challenger who captures market share protected by the patent (Marco 2005, Jeon 2015, Lin et al. 2019).

Mine is a continuous time model between a defendant and a plaintiff. The defendant is an unlevered firm with equity shares traded publicly without frictions other than transaction costs. The firm owns fixed assets and a patent. I model the patent as an investment option which generates a stochastic stream of value after it is converted into a commercial product. I consider that the patent has high quality or low quality depending on how expensive it is to convert it into a commercial product, i.e. the quality is high if the investment costs are small and low if the investment costs are unbearably large. Theranos is an example of this cost-quality relationship in patents. The company claimed to have developed and implemented a small automatic technology that could perform fast blood tests with minimum blood samples. However the technology was later proved to be underdeveloped and requiring more investment to become feasible[6].

The defendant and the plaintiff are privately informed about the true quality of the patent. Since this information is not publicly available, the defendant's equity market value is the sum of the fixed asset value and the expected value of the patent given the two possible patent qualities.

[6]Barron's article "Theranos: From Unicorn Hype to 'Massive Fraud'" available in https://www.barrons.com/articles/theranos-from-unicorn-hype-to-massive-fraud-1521052226

The plaintiff can explore the information asymmetry against other equity market participants by taking a long or short financial position over the defendant's equity and initiating an inter partes review. The plaintiff can choose a financial position that generates a profitable expected value of litigation. However, she always has an incentive to choose, at least, a hedging financial position to cover the litigation costs and protect her from the possibility that the PTAB rules in favor of the defendant. After the inter partes review request, the court decides who owns the patent and reveals its true quality. The defendant's equity market value adjusts after this information is publicly available.

I study and compare two cases: the base case and the Hatch-Waxman case. In the base case, the plaintiff takes a long or short financial position over the defendant's equity and simultaneously fills for an inter partes review against the defendant. In the Hatch-Waxman case, the plaintiff takes a financial position over the defendant's equity and, if that position is a short-sale, she must wait a mandatory period of ninety days if she is considering requesting the inter partes review. During this period, the plaintiff has an option to litigate since, at the end of the mandatory period, she must decide between requesting the inter partes review or closing her initial financial position without litigating. I then compare the two cases to understand how the introduction of the Hatch-Waxman Integrity Act affects the plaintiff's hedging financial position, that is, the share of equity that the plaintiff must either acquire or short-sell in order to cover her expected value (gain or loss) of requesting the inter partes review.

My results show that, first, after the enactment of the Hatch-Waxman Integrity Act, the plaintiff can hedge the expected loss of an inter partes review by shorting a smaller share of the defendant's equity. This is a consequence of the negative relationship between the plaintiff's financial position and the time to the inter partes review request. In detail, the plaintiff sees an increase in the time value of her option to litigate if the legislator forcefully imposes a larger wedge between the moment that a person short-sells a patent-holding company and the moment she requests an inter partes review over that patent. The increase in the litigation option value decreases the expected loss of litigation and, therefore, the plaintiff can reduce the size of the short-selling position over the defendant's equity that hedges her expected loss of litigation. By construction, this effect is not extended to long

financial positions.

Second, I show that, if a patent is good, the plaintiff has a larger range of profitable financial positions than if the patent is bad. This occurs because the quality of the patent affects the plaintiff's expected value of active litigation. If the patent is good, the plaintiff has more possibilities to profit because she can derive value from the possibility of winning a valuable patent and from the possibility of extracting a financial gain through the establishment of a financial position over the inter partes review process. Differently, if the patent is bad, she cannot derive a high value from winning the patent because its value approximates zero. Therefore, in this situation, she can only gain from establishing a financial position and, hence, the range of profitable financial positions will be comparatively smaller.

Third, I observe that the short-selling financial position that hedges the plaintiff's expected loss of litigation decreases if the value of the commercial product is highly volatile. This effect is processed via two channels. In the first channel, higher uncertainty increases the time value of the patent. This effect occurs in the base case and in the Hatch-Waxman case. In detail, the owner of the patent has an incentive to delay the conversion of the patent into a product because more volatility increases the value of the option to convert the patent into a product and its investment trigger. Consequently, the expected loss of litigation decreases because the ex-post defendant's equity value increases and, hence, the defendant will need a smaller hedging short-selling position.

The second channel only occurs in the Hatch-Waxman case. Here, higher uncertainty increases the time value of the option to litigate. In detail, higher uncertainty increases the upper-bound of the range of admissible values for the commercial product which ultimately raises the time value of the litigation or, more accurately, it decreases the loss of litigation. Consequently, an increase in the product's value volatility leads to an even smaller hedging short selling position under the Hatch-Waxman Act.

I provide comparative statics for several parameters in my model. I find that the number of shares that plaintiff should short sell to hedge the inter partes review process decreases if instantaneous commercial value growth rate is higher, if the rate of arrival of a PTAB decision increases, if the probability that the PTAB will favour the plaintiffs request is

higher, and when the plaintiff's share of value over a competing patent is higher. Moreover, it increases if the discount rate increases.

The rest of the chapter is organised as follows. Section 2.2 presents the literature review; Section 2.3 introduces the model and details the Base case and the Hatch-Waxman case; Section 2.4 discusses the main results and provides comparative statics for the most interesting parameters; Section 2.5 presents the conclusions and deliberates on possible research avenues.

2.2 Literature Review

This chapter is closely related with Choi and Spier (2018) who provides a primer investigation on the problem of financial hedging of litigations. Their findings suggest that taking a short position in a stock of a defendant provides two strategic advantages: if the lawsuit has negative expected value, a short position allows to turn it into a profitable position and, the short position increases the bargaining position of the plaintiff by raising the minimum amount that he is willing to receive to settle the position. I take a further step by addressing the specific case of patent litigation under the Hatch Waxman Integrity Act and by selecting a continuous time framework which provides the flexibility to study the stopping time problem introduced by this regulatory change. In summary, I contribute to the modelling aspect of this subject by taking into account time-varying patent value subject to uncertainty, by considering that the market is uninformed about the quality of the patent and that the arrival of the court's decision is subject to a jump process.

According to Lemley and Shapiro (2005), a patent is intangible and has probabilistic value because it is subject to uncertainty of future commercial value and to uncertainty about the validity of the patent which can be a target of litigation. From this perspective, a patent is not as a realised stream of cash-flows but a real option that can generate value only if it is integrated or converted into a product. I take this into consideration in my model where the patent in question is an unexercised investment option.

Real options[7] have been extensively used in patent valuation. Schwartz (2004) evaluates

[7]This literature was introduced in the work of Brennan and Schwartz (1985) who provides a model to study the problem of optimal investment timing under commodity price uncertainty. This was later extended

R&D projects protected by patents while accounting for uncertainty in cash-flows and in costs, as in McDonald and Siegel (1986). The value of the patent depends on the investment costs of R&D and the expiration date of the patent. Regulation can be used to improve the conversion of patents to actual products either by increasing the expiration date or reducing research costs. However, this requires a balance between the benefits of promoting innovative developments with the costs of reducing market competition. Meng (2008) studies a patent race where two firms compete in R&D spending to obtain an exclusive patent. Investment rates are selected by weighting the benefits of the waiting value of real options with the costs of suffering a setback against the opponent. The results show that firms under R&D competition pressures tend to over-invest and to present high return volatility due to technological risks. Leung and Kwok (2012) presents a duopolistic patent-investment race where the incumbent firm faces information asymmetry on the entrant's cost. The informational disadvantage triggers a strong response by the incumbent which can lower the investment option value of the entrant. However, they find that the entrant's real option value may increase if the incumbent has a strong belief that the entrant has high cost. Other papers relating real options and R&D investment are Weeds (2002), Garlappi (2004), Miltersen and Schwart (2004) and Hsu and Lambrecht (2007).

My model focuses on the inter partes review process initiated by an outsider against the owner of the patent. However, several works have focused instead on the litigation option held by the owner of a patent against potential infringers. For example, Marco (2005) uses a real options model and shows that the litigation value depends on the property rights cost, on the technology and on the different believes about the litigation. This paper shows that imperfect enforceability reduces the lifetime of a patent, increased uncertainty about the patent's validity delays litigation and that the litigation rate lowers with higher relative profits and with the hazard rate of patent litigations. Jeon (2015) followed with a model where litigation can be preempted with a ex-ante settlement or followed by an ex-post settlement between the challenger and the incumbent. This model explains why litigation

by McDonald and Siegel (1986) who considers dual uncertainty in a firm revenues and in its costs. Presently, this field intersects with several other branches of economic literature such as game theory (Grenadier and Wang 2005, Bouvard 2012, Grenadier et al. 2016, Orlov et al. 2020), environmental economics (Cortazar et al. 1998, Insley 2002), labor economics (Jacobs 2007), or economic policy (Gries et al. 2012).

ruling by trial is so scarce. The threat to litigate exerts pressure over the challenger to pay royalties. If these royalties are sufficiently high, they settle the feud and prevent settlement by court. Lin et al. (2019) has a litigation real options model between a challenger and an incumbent but they add outside options to each party, that is, the incumbent can dismiss the litigation process and the challenger can quit the market during the litigation. The results show that when the relationship between the challenger's gain to the incumbent's loss is sufficiently low, the opposing parties wait for the court's decision. In this case, the incumbent knowingly attempts to wipe out the challenger and regain the monopoly conceded by the patent.

2.3 Model

2.3.1 Setup

Consider a setting with two parties: a plaintiff (p) and a defendant (d). Unlike Schwartz (2004) and Meng (2008), the defendant already owns a patent but can be challenged by the plaintiff through litigation. Litigation is processed by an inter partes review. First, the plaintiff submits the petition challenging the validity of the patent to the PTAB, that is, she requests a review of the property of the patent and of the quality of the patent. When the PTAB makes a decision, the true quality of the patent is publicly announced. Moreover, if the PTAB decides in favour to the defendant, he maintains the ownership of the patent but, if the PTAB decides against the defendant, the defendant loses the property of the patent to the plaintiff.

As described by Lemley and Shapiro (2005), patents have two sources of uncertainty: the uncertainty underlying the commercial value of the product after the patent is exercised and the uncertainty about the property rights of the patent. I take these elements into consideration. First, I treat the patent as an investment option that, when exercised, is transformed into a commercial product. Here, the value of the commercial product behaves stochastically. Second, I assume probability distributions over the quality of the patent and over the patent's property rights subsequent to the PTAB decision.

The value of the monopolistic commercial product is represented by an Ito diffusion

$X = \{X_t : 0 \leq t < \infty\}$, defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ and satisfying the stochastic differential equation with the following geometric Brownian Motion form:

$$dX_t = \alpha X_t dt + \sigma X_t dW_t \tag{2.1}$$

where $X_0 > 0$, $\{W_t\}_{t\geq 0}$ is a Wiener process, α is the instantaneous conditional expected growth rate of X and σ is the instantaneous conditional standard deviation. Also, $\alpha = r - \delta$ $(r > \delta)$, where r is an exogenous discount rate and δ (> 0) corresponds to the opportunity cost from deferring. I drop the subscript t for the rest of the chapter.

I write the true quality of a patent as $\theta \in \{\underline{\theta}, \overline{\theta}\}$ with $0 \leq \underline{\theta} < \overline{\theta} < +\infty$. A patent with high quality $\overline{\theta}$ can be commercially explored. Differently, a type $\underline{\theta}$ is a bogus patent and it can hardly be converted into an actual cash-flow generating product. The plaintiff and the defendant know the true quality but the equity market cannot observe the quality. I assume the market confers a probability ξ that the patent has high quality and probability $(1 - \xi)$ that it has low quality. The court reveals the true quality of the patent at the end of the lawsuit if the patent lawsuit is filled.

Furthermore, I assume the quality of the patent directly affects the investment cost $K(\theta)$, that is, the cost that the owner of the patent pays to convert it in a cash-flow generating product. I write the total investment cost as:

$$K(\theta) := \frac{1}{\theta} \tag{2.2}$$

This expression states that the investment costs are inversely related with the patent quality. In my model, the good and the bad patents can both be converted to the same product, in the sense that they yield a product of value X with instantaneous growth rate α and instantaneous standard deviation σ. However, they are distinct in the technology costs: the good patent has a cheaper technology and the bad patent is more expensive to be implemented. In an extreme scenario, a quality parameter θ close to zero means that the patent exists but it is virtually impossible to construct that product because of the exorbitant cost of its technological components[8].

[8] An alternative specification for different patent quality can be to set up different instantaneous growth

I write T^* as the first passage time at which X crosses the investment trigger X^* (Peskir and Shiryaev 2006). Formally, T^* is:

$$T^* = \inf \{t \in [0, +\infty) : X \notin (0, X^*)\} \tag{2.3}$$

Equation (2.3) shows that the patent is converted into a product when the value of the commercial product hits a single investment trigger. This is a simplifying feature of this model which can be improved in the future by considering the presence of positive and negative jumps in equation (2.1) or by considering an underlying bimodal distribution[9]. Moreover, I consider that the patent does not generate cash-flows before the owner converts it into a commercial product[10].

Lemma 1. *The value of patent $P(X)$ is*

$$P(X) = \frac{1}{\theta(\beta - 1)} \left(\frac{X}{X^*(\theta)} \right)^{\beta}, \qquad \forall\, X < X^* \tag{2.4}$$

where the investment trigger X^* is:

$$X^*(\theta) = \frac{\beta}{\beta - 1} \frac{1}{\theta} \tag{2.5}$$

and the constant β is:

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} \quad > 1 \tag{2.6}$$

The value of the patent is an investment option with value $P(X)$ represented by equation (2.4). When X crosses the investment trigger X^*, the patent is converted into a commercial

rate α which would change the underlying commercial product value. In this case, the two patent types would yield two different products. This can be considered in future research.

[9] A different approach would be to consider multiple investment stages as in Sundaresan et al. (2015). They consider a setup where growth options evolve endogenously with its investment opportunity set. In this case, when an investment option is exercised, it generates further investment options that can be reverted to previous stages if the underlying stochastic process becomes too low. A patent could be modelled by following this approach combined with a death process where the product R&D is terminated if the research suddenly becomes inviable.

[10] I leave this feature for future research development. I note that this could be modelled by considering, for example, royalties behaving similarly to a dividend which would affect the option value and the conversion strategy.

product. In this case the owner pays the lump sum investment cost $K(\theta)$ and receives the commercial product's value X. The second term is the expected discount factor $\mathbb{E}_t(e^{-rT^*}) = (X/X^*(\theta))^{\beta}$ and it is used to discount the patent's net value at investment from a random time T^* to the present. As in Dixit and Pindyck (1994), the parameter β is the elasticity[11] of the patent value with respect to the value X. Since $\beta > 1$, the value of the patent $P(X)$ increases with as the product value X increases.

The defendant and the plaintiff have exclusive private information about the true quality of the patent[12]. Therefore, the market is not informed about the true equity value and has to estimate its expected value. I write the defendant's market value of equity as $V_d^M(X)$, where:

$$\begin{aligned} V_d^M(X) &= Y + \mathbb{E}^{\theta}\left[P(X|\tilde{\theta})\right] \\ &= Y + \xi P(X|\overline{\theta}) + (1-\xi)P(X|\underline{\theta}) \end{aligned} \tag{2.7}$$

The defendant's market value of equity comprises the constant firm's asset value Y and the expected patent value. Here, the uninformed market estimates the true quality of the patent by weighting the patent value under good type $\overline{\theta}$ with subjective probability ξ and the patent value under bad type $\underline{\theta}$ with probability $(1-\xi)$. The plaintiff will use the informational advantage over the market to extract a financial gain.

The truthful type of the patent is only revealed by an inter partes review or if the patent owner converts the patent into a commercial product. Since $X^*(\overline{\theta}) < X^*(\underline{\theta})$, the market learns that the patent is good if the owner converts it into a commercial product when X reaches $X^*(\overline{\theta})$. If X reaches $X^*(\overline{\theta})$ and the patent remains idle, then the market knows the patent is bad. At the same time, the owner of a bad patent cannot pretend the patent is good by investing in $X^*(\overline{\theta})$. That is because the quality of the patent affects the cost structure and the high investment cost would be exposed in the accounting balance sheet[13].

[11] The elasticity can be written as

$$\frac{dP}{P(X)}\frac{X}{dX} = \beta$$

For instance, for a 1% increase in the product value X, the value of the patent has an increase equal to β%.

[12] This assumption can be relaxed in future research.

[13] A different situation happens when the patent type is reflected on the ability of the patent to generate value, i.e. when the growth rate of X is either high or low. The accounting balance sheet would not reflect the true expected value of the commercial product and the differences in the quality of the patent would take

Throughout this chapter, I will restrict my analysis to $X < X^*(\overline{\theta})$.

In the base case, I set up the following steps at time $t = 0$. First, the plaintiff establishes a stake Δ in the defendant's equity. If $\Delta > 0$ the defendant has a long position and, if $\Delta < 0$, the defendant has a short position.

Second, the plaintiff challenges the validity of the patent by requesting an inter partes review. The plaintiff faces a subjective probability ϕ of winning and $(1-\phi)$ of losing the inter partes review request. Moreover, since the legal framework allows any person to challenge the patent's validity, I consider that there is a $\gamma \in [0, 1]$ parameter that that measures the percentage of the plaintiff's property share over a competing patent. For example, if $\gamma = 1$, the plaintiff is the sole owner of a competing patent and, if $\gamma = 0$, the plaintiff has no competing patent.

The inter partes review process is then initiated and the PTAB's final decision has a rate of arrival λ. If the plaintiff wins and she has a competing patent ($\gamma \in (0, 1]$) she takes ownership over the defendant's patent in the percentage of her share, if she wins and she does not have a competing patent ($\gamma = 0$), the plaintiff does not gain the property of the patent, the defendant loses the patent to a third party exogenous to the model and the true type θ is revealed. Finally, if she loses, the defendant maintains the ownership of the patent but the true type θ is revealed.

After time $t = 0$, the inter partes review is under evaluation until hitting the random time $\overline{T}$ when the PTAB decides the ownership of the patent and publicly reveals the quality of the patent. At this time, the plaintiff closes her position. At time T^*, the patent is optimally converted into a commercially viable product by the owner of the product.

2.3.2 Inter Partes Review

The inter partes review procedure allows any individual (within certain limits) to dispute the validity of an existing patent by requesting a review to the PTAB. This review simultaneously challenges the ownership and the quality of the patent since the grounds on which the patentability is challenged are often related with claims and detailed features that are protected by previous patents. This means that the PTAB may decide that the patent

several years to be reflected in the revenues of the company.

belongs to someone else and that it has features that turns the patent into an unusable patent, that is, into a bogus patent.

So, the PTAB has two functions: to assess and reveal the true patent quality θ at the end of the lawsuit, and to decide the ownership of the patent with probability ϕ. I consider that the plaintiff has continuous costs of litigation κ_p and the defendant has continuous costs of litigation κ_d. My underlying assumption here is that firm size affects the litigation costs that the defendant is willing to incur.

I write $L_p(X)$ as the value of the active litigation for the plaintiff. I model the court's decision arrival using a Poisson jump process where λ is the mean arrival rate of the PTAB's decision. The instantaneous change in the plaintiff's value of active litigation given the decision arrival is:

$$dI = \begin{cases} 0 & , \text{w}/p = 1 - \lambda dt \\ \gamma\phi P(X) - L_p(X) & , \text{w}/q = \lambda dt \end{cases} \tag{2.8}$$

with $\mathbb{E}\left(dIdX\right) = 0$. At every dt after the litigation starts, the PTAB either postpones the decision or makes a final decision. In the first situation, the value active of the litigation stays unchanged. In the second case, litigation is terminated and the value of active litigation $L_p(X)$ is replaced with the value of the patent $P(X)$ with winning probability ϕ and in the proportion of ownership of a competing patent γ.

As shown in Appendix 5.1.2, I can then write $L_p(X)$ as:

$$L_p(X) = \gamma\phi P(X) - \frac{\kappa_p}{r + \lambda} \tag{2.9}$$

Equation (2.9) shows that the plaintiff's litigation value $L_p(X)$ is the expected value of the patent before the PTAB's decision $\gamma\phi P(X)$ net of litigation costs $\kappa_p/(r + \lambda)$. The rate arrival of a decision λ does not affect the plaintiff's expected value because the option is not exercised and, therefore, the litigants are not losing cash-flows while they wait for the decision. However, it affects the litigation costs which converge to zero if the PTAB is very fast at making inter partes decision and it tends to κ_p/r if the PTAB is very slow.

I write $L_d(X)$ as the value of active litigation for the defendant. Using the same procedure,

I obtain:

$$L_d(X) = (1 - \phi)P(X) - \frac{\kappa_d}{r + \lambda} \tag{2.10}$$

The litigation value for the defendant follows the same structure as presented for the plaintiff. As it is the difference between the expected value of the patent for the defendant net of litigation costs.

2.3.3 Taking a Financial Position in the Defendant's Equity

The plaintiff explores the information asymmetry against other equity market participants by assuming a financial position Δ in the defendant's equity. This position is long if $\Delta > 0$ or it is short if $\Delta < 0$. A profitable financial position guarantees a positive plaintiff's expected value of litigation while a hedging financial position generates a null expected value of litigation. The plaintiff will always select, at least, a hedging financial position because she must cover the litigation costs and insure the possible unfavourable PTAB ruling. A hedging financial position requires the defendant's equity shares to be publicly traded and the equity market to have the necessary supply of shares available. Furthermore, the patent can never represent a very small fraction of the defendant's firm value.

The plaintiff pays a transaction cost $\tau|\Delta|S$ whenever she opens or closes a financial position and this cost only depends on the quantity of shares transacted but not its price. Here, τ is the cost fraction parameter, S is the number of outstanding shares and $|\Delta|$ is the modular function of Δ that guarantees that the plaintiff always has a cost regardless of the shorting or longing the defendant's shares.

Both parties wait for the PTAB to decide on the inter partes review request. When the decision arrives, the plaintiff closes her position. So, I define the rate of arrival of a decision by the PTAB as λ and write the expected transaction cost of closing the financial position at the moment of the PTAB's decision as $L_\tau|\Delta|S$, where:

$$L_\tau = \frac{\lambda\tau}{r + \lambda} \tag{2.11}$$

▷ **Benchmark Case.** At time t, the value of the commercial product is X. Then, I write

the plaintiff's expected profit of litigation as $B(X)$, where:

$$B(X) = -\tau|\Delta|S - \Delta V_d^M(X) + \Delta\left(Y + L_d(X)\right) + L_p(X) - L_\tau|\Delta|S \tag{2.12}$$

The first two terms of the right-hand side of equation (2.12) are related with the establishment of the financial strategy while the remaining three terms are related with the strategy outcome. Hence, the plaintiff sustains transaction costs τ and assumes a position Δ over the defendant's equity market value $V_d^M(X)$. Equity market value assumes that the defendant has full ownership over the investment option because the position Δ is taken before the inter partes review petition. The third term is the expected value of a Δ position in the defendant's equity value after the information about the lawsuit becoming publicly available which correspond to the defendant's asset value Y and the value of active litigation faced by the defendant firm $L_d(X)$. The last term is expected transaction cost of closing the initial position.

I define $\Delta_B(X)$ as the percentage of equity that hedges the plaintiff's profit function $B(X)$, that is:

$$\Delta_B(X) = \{\Delta : B(X) = 0\} \tag{2.13}$$

I can write $\Delta_B(X)$ as

$$\Delta_B(X) := \begin{cases} \frac{L_p(X)}{V_d^M(X) - (Y + L_d(X)) + (\tau + L_\tau)S}\,, & \Delta_B(X) \geq 0 \\ \frac{L_p(X)}{V_d^M(X) - (Y + L_d(X)) - (\tau + L_\tau)S}\,, & \Delta_B(X) < 0 \end{cases} \tag{2.14}$$

Inspecting equation (2.14), I observe that the plaintiff always has a long hedging position if she expects a positive value of active litigation ($L_p(X) > 0$) under Assumption 1. Moreover, higher transaction costs will required a larger long financial position to hedge the litigation.

▷ **Hatch-Waxman Case.** The Hatch-Waxman Integrity Act imposes a waiting time before the inter partes review if the plaintiff chooses a short-selling position ($\Delta < 0$) over the defendant's. Let us write T_{HW} as the time of litigation in the Hatch-Waxman case. If, at the initial time $t = 0$, the plaintiff assumes a short-selling position, then she needs to wait

until T_{HW} to request the inter partes review.

For $t \in [0, T_{HW})$ I define the function $H(X,t)$ to be the plaintiff's expected profit of litigation if the plaintiff is forced to wait T_{HW} periods between the moment she defines a financial strategy and the moment she petitions for an inter partes review. That is:

$$H(X,t) = -\tau|\Delta|S - \Delta V_d^M(X) + C(X,t) \tag{2.15}$$

The first term in equation (2.15) is the transaction cost associated to the plaintiff's financial position. The second term is $C(X,t)$ is an European call option to be exercised at time T_{HW} and written as:

$$C(X,t) = e^{-r(T_{HW}-t)}\mathbb{E}_t\left[\max\left\{\Delta\left(Y + L_d(X_{T_{HW}})\right) + L_p(X_{T_{HW}}) - L_\tau|\Delta|S\,,\, \Delta V_d^M(X_{T_{HW}}) - \tau|\Delta|S\right\}\right] \tag{2.16}$$

As shown in Appendix 5.1.3, I can rewrite equation (2.15) as:

$$H(X,t) = -(1 + e^{-r(T_{HW}-t)})\tau|\Delta|S + N\left(d_1\right)D_1X^\beta - e^{-r(T_{HW}-t)}N(d_2)D_2 \tag{2.17}$$

where D_1 and D_2 are constants that aggregate the terms respectively related to value X and the fixed cost terms. These constants are written as:

$$\begin{aligned} D_1 &= \left(\frac{\beta}{\beta-1}\right)^{1-\beta}\frac{\left(\gamma\underline{\theta}\overline{\theta}\phi\theta^\beta + \Delta\overline{\theta}\left(\theta(\xi-1)\underline{\theta}^\beta - \underline{\theta}(\phi-1)\theta^\beta\right) - \Delta\theta\underline{\theta}\xi\overline{\theta}^\beta\right)}{\beta\theta\underline{\theta}\overline{\theta}} \\ D_2 &= \Delta\frac{\kappa_d}{r+\lambda} + \frac{\kappa_p}{r+\lambda} - \frac{r}{r+\lambda}\tau|\Delta|S \end{aligned} \tag{2.18}$$

and $N(.)$ corresponds to the cumulative standard normal distribution giving us the risk adjusted probability with d_1 and d_2 constants written as:

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{D_1X^\beta}{D_2}\right) + \left(r + \frac{1}{2}\beta^2\sigma^2\right)(T_{HW}-t)}{\beta\sigma\sqrt{T_{HW}-t}} \\ d_2 &= d_1 - \beta\sigma\sqrt{T_{HW}-t} \end{aligned} \tag{2.19}$$

I define $\overline{\Delta}(X,t)$ as the percentage of equity that hedges the profit function $H(X,t)$:

$$\overline{\Delta}(X,t) = \{\Delta : H(X,t) = 0\} \tag{2.20}$$

The solution to $\overline{\Delta}(X,t)$ is numerical and, similarly to equation (2.14), it comprises two branches in order to guarantee that establishing a financial position is always costly regardless of being a long or short position.

Finally, I define $\Delta_{HW}(X,t)$ as the percentage of equity that hedges the inter partes review request under the Hatch Waxman Integrity Act:

$$\Delta_{HW}(X,t) = \Delta_B(X)\,(1-\mathbb{I}_{[\Delta_B<0]})(1-\mathbb{I}_{[\overline{\Delta}<0]})+\overline{\Delta}(X,t)\,\left(\mathbb{I}_{[\Delta_B<0]}\mathbb{I}_{[\overline{\Delta}<0]} + (1-\mathbb{I}_{[\Delta_B<0]})\mathbb{I}_{[\overline{\Delta}<0]}\right) \tag{2.21}$$

Here, $\mathbb{I}_{[\Delta_B<0]}$ is an indicator function with value one if the benchmark case $\Delta_B(X)$ is negative, i.e. a short position, and zero if Δ_B is positive, and $\mathbb{I}_{[\overline{\Delta}<0]}$ is an indicator function with value one if $\overline{\Delta}(X,t)$ is negative and zero otherwise. Equation (2.21) is written this way to account for the asymmetric impact that the Hatch-Waxman Integrity Act has over short selling and long positions.

Assumption 1. *The model parameterisation respects the following inequalities:*

$$\begin{aligned} L_d(X) &> 0 \\ V_d^M(X) &> (1-\phi)P(X) + Y \\ \mathbb{E}^{\theta}\left[P(X|\tilde{\theta})\right] &> L_d(X) + (\tau + L_\tau)\,S \end{aligned} \tag{2.22}$$

This assumption guarantees two main features in the model: first, it restricts to the cases where the defendant has an incentive to participate in the inter partes review litigation and, second, it guarantees that transaction costs do not limit the plaintiff's choice of financial positions. In detail, the first inequality ($L_d(X) > 0$) guarantees that the defendant participates in the litigation by ensuring that his expected litigation value is positive. Without this restriction, he would never present his case during the inter partes review hearing and would just hand over the patent to the plaintiff before the litigation occurs.

The second inequality ($V_d^M(X) > (1-\phi)P(X) + Y$) also guarantees that the defendant participates in the litigation. Differently from the first inequality that ensures that participation is not detrimental, in this case, this inequality guarantees that the expected value of the defendant's assets is not so good that he just prefers to let the plaintiff go through the litigation without opposition.

The third inequality ($\mathbb{E}^{\theta}\left[P(X|\tilde{\theta})\right] - (\tau + L_{\tau})\, S > L_d(X)$) guarantees that transaction costs do not prevent short selling strategies. The plaintiff prefers a short-selling position when her value of active litigation is negative ($L_p(X) < 0$). However, if transaction costs are very high, she cannot implement this strategy. This first restriction excludes scenarios where short-selling is not feasible.

2.4 Numerical Results

2.4.1 Parameters

In this subsection, I provide a picture of the results by selecting values for the model parameters. Whenever possible, I select values that are in accordance with reality, e.g. the annualised time to litigation T_{HW} and the rate of arrival λ of PTAB's decision. However, since this is not always possible, I provide comparative statics later on for a wide range of parameters' values to stress test different choices.

I consider that the plaintiff establishes a financial position Δ over the equity of the defendant at time $t = 0$. The existing asset (Y) of the defendant has value \$20 and the patent's commercial value at the initial time $t = 0$ (X_0) is \$9. The patent's commercial value improves at a growth rate (α) of 3% annually, while suffering uncertainty (σ) shocks of 40%. The plaintiff faces a lump sum transaction cost (τ) of \$0.005 whenever she opens or closes a Δ position over one outstanding share ($S = 1$). I assume that the outstanding share is infinitely divisible. I consider that the plaintiff has a competing patent ($\gamma = 1$). The annualised rate of interest (r) is 4%.

The equity market does not have information about the type of patent and assume, subjectively, that the patent is good with 25% of probability (ξ) and bad with 75% of probability ($1 - \xi$). A good patent ($\overline{\theta}$) faces a lump sum exercise cost of $K(\overline{\theta}) = 1/0.1 = 10$.

A bad patent faces a quality parameter $\underline{\theta} = 0.001$ which translates into a prohibitive investment cost of $K(\underline{\theta}) = 1000$.

The parties face an instantaneous rate of arrival of $\lambda = 0.5$ during litigation. This means that the PTAB's decision with respect to the inter partes review petition is expected to arrive in two years[14] which is consistent with the time limit for a decision under the inter partes review system as the PTAB's final decision cannot surpass eighteen to twenty four months (Dubis 2015). I stress test this number in the comparative statics. The plaintiff faces an instantaneous cost of litigation $\kappa_p = 2$ and the defendant faces litigation costs $\kappa_d = 1$ while litigation continues. The probability of a plaintiff win (ϕ) or loss ($1 - \phi$) is 50%. In the Hatch-Waxman case, the plaintiff needs to wait 90 days resulting in an annualised time to litigation (T_{HW}) of 0.25.

2.4.2 Key Results and Discussion

In Figures 2.1 to 2.3, the darker regions correspond to the feasible profitable financial positions (shares of equity Δ) as a function of key parameters. The dashed line displays the financial positions Δ that hedge the inter partes review process in the base case. If the plaintiff wants to establish a position that generates a positive expected profit in the base case ($B(X_0) > 0$), she must select a Δ below this dashed line. The dotted line represents $\overline{\Delta}(X_0, 0)$, that is, the financial position that hedges the plaintiff's expected value of requesting an inter partes review in ninety days. Any financial position below this line generates profit in expectation ($H(X_0, 0) > 0$) in the Hatch-Waxman case. Since the Hatch-Waxman Integrity Act only limits short sales, the plaintiff is not forced to satisfy the waiting period if she establishes a long position. This is represented by the bold line which corresponds to the hedging financial position $\Delta_{HW}(X_0, 0)$. The non-shaded regions are either unprofitable and/or, as clarified in the figures, they violate Assumption 1 which restricts my results to feasible outcomes[15].

[14] The expected time of arrival of a judicial decision is:

$$\mathbb{E}(\overline{T}) = \int_0^{+\infty} \lambda \overline{T} e^{-\lambda \overline{T}} d\overline{T} = \frac{1}{\lambda}$$

[15] I acknowledge that, in practice, implementing a long or short financial position has other restrictions such as limited market liquidity or regulatory restrictions to short-selling. Even though these restrictions are

▷ **The impact of introducing the Hatch-Waxman Integrity Act.** How does the set of profitable short positions changes now that the investor is forced to face a ninety days waiting period? Without loss of generality, let us focus on Figure 2.1 to explain this result.

First, consider regions A_3 and A_4. Here, the hedging financial positions $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$ are positive and, therefore, the introduction of the Hatch-Waxman Integrity Act does not change the hedging financial position ($\Delta_{HW}(X_0, 0) = \Delta_B(X_0)$).

Moreover, in order for a long position to be admissible (and region A_4 to exist), the plaintiff's value of active litigation must be positive ($L_p(X_0) > 0$). The plaintiff must then have a competing patent[16] and her expected patent value ($\gamma\phi P(X_0)$) must supersede her present value of litigation costs ($\kappa_p/(r + \lambda)$). This is the best possible scenario for the plaintiff because the Hatch-Waxman Integrity Act does not impede the immediate request of the inter partes review and her expected value from litigation is positive.

Second, consider regions A_2, A_6 and A_7. In this range, the introduction of the Hatch-Waxman Integrity Act expands the subset of profitable short-selling positions ($\Delta_{HW}(X_0, 0) > \Delta_B(X_0)$) and, therefore, the plaintiff can now select a financial position inside the area A_6. However, why will the plaintiff only consider financial positions in A_6 but ignore positions in the area A_7? The plaintiff cannot select financial positions within the area A_7 because these are long positions and so, they are not constrained by the mandatory waiting period of ninety days. Also, it is not reasonable to expect that the plaintiff would wait ninety days with a long position locked in an uncertain investment. Therefore, the plaintiff can either wait for the commercial value X to quit these regions or she can establish a short financial position Δ that requires a mandatory waiting period.

Third, consider regions A_1 and A_5. Here, the hedging financial position increases from $\Delta_B(X_0)$ to $\Delta_{HW}(X_0, 0)$ allowing for short-selling positions within region A_5. The plaintiff hedges the inter partes review litigation by selecting a smaller short selling position[17].

important they do not change the meaning of my key results since such restriction would only reduce the set of available financial positions but would not affect the effect of introducing the Hatch-Waxman Integrity Act.

[16] Observing equation (2.9), the presence of a competing patent ($\gamma \in (0, 1]$) is a necessary (but not sufficient) condition for the plaintiff to have a positive value of active litigation.

[17] To be precise, I say that a financial position is smaller if its absolute size is tinier. If a position is short, I call it smaller if it is less negative. For example, for a shorting position, I say that $\Delta = -3\%$ is smaller than

But what makes region A_5 and A_6 profitable regions under the Hatch-Waxman Integrity Act? This jump in the hedging financial position from $\Delta_B(X_0)$ to $\Delta_{HW}(X_0, 0)$ is caused by the positive relationship between $\overline{\Delta}(X_0, 0)$ and T_{HW}.

I explain this relationship here and provide a demonstration in Appendix 5.1.4. First, function $H(X,t)$ increases with time to litigation T_{HW}, i.e. $\partial H(X,t)/\partial T_{HW} > 0$. An increase in the time to litigation (T_{HW}) increases the call option's time value beyond its intrinsic value, that is, the value that the litigation option would have if the plaintiff had to make a decision today. Since the call option $C(X,t)$ is the only term depending on T_{HW}, the profit function $H(X,t)$ consequently increases.

Second, the profit function $H(X,t)$ increases with lower Δ positions, i.e. $\partial H(X,t)/\partial \Delta < 0$. This always occurs because Assumption 1 excludes the less realistic cases where the defendant could benefit ex-ante from the inter partes review. Since profit increases with the time to litigation T_{HW} and decreases with Δ, then the zero profit boundary $\overline{\Delta}(X_0)$ has to increase to offset the gain from the higher time to litigation T_{HW} and, so, to guarantee that $H(X,t) = 0$.

To see what happens when I consider extreme cases of the time to litigation, I take limits over the profit function[18]:

$$\begin{aligned} &\lim_{T_{HW}\to 0} H(X,t) = B(X) \\ &\lim_{T_{HW}\to +\infty} H(X,t) = -\tau|\Delta|S + \Delta(1-\phi)P(X) - \Delta\mathbb{E}^{\theta}\left[P(X|\tilde{\theta})\right] + \gamma\phi P(X) > B(X) \end{aligned} \tag{2.23}$$

If the regulator reduces the mandatory waiting period towards zero, the plaintiff's profit function in the Hatch-Waxman case approximates the base case profit function. If, instead, the regulator increases the waiting period, then the profit function approximates the plaintiff's expected change in value of establishing a financial position and litigating deducted of transaction costs[19]. An increase in the mandatory waiting period gives more time for the option to litigate to become in-the-money.

Enacting the Hatch-Waxman Integrity Act has one practical implication. For sufficiently

a position $\Delta = -4\%$.

[18] See Appendix 5.1.4 for details.

[19] This is similar to the European call option without dividend payout. Taking the limit over the price of this call option with respect to the time to maturity, one gets the price of the underlying asset.

low levels of X_0, the increase in the hedging financial position from $\Delta_B(X_0)$ to $\Delta_{HW}(X_0, 0)$ facilitates inter partes review requests by individuals that lack the financial firepower to assemble large short selling strategies. Agents that are informed about the true quality of a patent but do not have the financial firepower may now assume a smaller shorting financial position Δ because, under the Hatch-Waxman Integrity Act, their financial position Δ is selected to cover the value of the option to litigate ($C(X, t)$) which is lower than the sum between the value of the litigation and the value of closing the financial position.

▷ **The impact of the patent's true type.** Overall, the feasible hedging financial positions $\Delta_B(X_0)$ and $\Delta_{HW}(X_0, 0)$ are higher if the patent is good than if the patent is bad. That is:

$$\Delta_B(X_0|\overline{\theta}) \geq \Delta_B(X_0|\underline{\theta}) \qquad\qquad \Delta_{HW}(X_0, 0|\overline{\theta}) \geq \Delta_{HW}(X_0, 0|\underline{\theta})$$

This implies that, in the good patent state, the plaintiff has a larger range of profitable financial positions than in the bad patent state.

Without loss of generality, let us compare subfigures 2.1.a and 2.1.b to explain this result. First, for regions where the plaintiff can only profitably choose short positions (A_1 and A_2 in the base case and A_1, A_2, A_5 and A_6 in the Hatch-Waxman case), the lower hedging financial positions in the bad patent state occurs because, if the patent is bad, the plaintiff has a lower expected value of active litigation $L_p(X_0)$ leading to larger hedging short-selling financial positions.

Moreover, the unfeasible region is larger in the bad patent state which reduces the available financial short-selling positions in this state. In detail, the feasible profitable region occurs for values of X_0 greater than 5.3 in the good patent state while, in the bad patent state, it only occurs for values of X_0 that are greater than 7.6. Higher investment costs in the bad patent state ($K(\underline{\theta}) > K(\overline{\theta})$) generates unfeasible cases more easily because the patent value is so low that the defendant does not participate in the litigation.

Second, in the region where long positions are possible (A_4), the subset of profitable long financial positions is larger if the patent is good. In detail, the value of the bad patent is dominated by comparatively higher investment costs $K(\underline{\theta}) > K(\overline{\theta})$ that also make the

patent insensitive to commercial value. So, high commercial values X_0 does not improve the bad patent value significantly and the wedge between the value of a good patent and the value of a bad patent ($P(X_0|\overline{\theta}) > P(X_0|\underline{\theta})$) persists. This originates a higher value of active litigation for the plaintiff and for the defendant when the patent is good. In this case, the hedging long financial positions will be higher in the good patent case because, on the one hand, the plaintiff's value of active litigation is higher and, on the other hand, the gap between the defendant's equity market value $V_d^M(X_0)$ and the his expected value of litigation $(1-\phi)P(X_0)+Y$ will be lower.

▷ **The Effect of Uncertainty.** What is the effect of uncertainty over the admissible hedging financial positions? As observed in Subfigures 2.2.a and 2.2.b, the hedging financial positions $\Delta_B(X_0)$ and $\Delta_{HW}(X_0,0)$ both increase with higher commercial value uncertainty σ. In the base case, higher uncertainty σ increases the patent value $P(X)$ through its negative effect over β, the elasticity of the patent value to changes in the commercial value X, seen in equation (2.6). More uncertainty increases the investment trigger $X^*(\theta)$ which, by condition (2.3), translates into an increase in the investment time T^* (the optimal time when the patent will be converted into a commercially viable product). In other words, the greater the volatility, the greater the patent value and the greater the owner's incentive to keep this patent unexercised. To guarantee zero profit in the hedging condition presented in equation (2.12), $\Delta_B(X_0)$ must increase to offset the gains in profit driven by the higher uncertainty.

In the Hatch-Waxman case, uncertainty affects $\overline{\Delta}(X_0,0)$ via two channels. The first is the same described in the previous paragraph: uncertainty increases patent value $P(X)$ via elasticity β. The second is the effect of uncertainty over the litigation option. The value of the litigation option increases with uncertainty because volatility increases the upper-bound of the range of possible values that the commercial product can attain at the time to litigation T_{HW}, thus increasing the time value of the call option to litigate. Moreover, this second channel is responsible for the expanding gap between $\overline{\Delta}(X_0,0)$ and $\Delta_B(X_0)$ as σ increases. In conclusion, the hedging financial position $\overline{\Delta}(X_0,0)$ has to increase to guarantee zero profit and offset the effect of higher uncertainty.

The change in the hedging position from $\Delta_B(X_0)$ to $\Delta_{HW}(X_0,0)$ is more pronounced

in the good type state than in the bad type state. This is because increases in volatility impact the profit function through the patent value. A good patent has low investment costs which allows uncertainty to impact the value of the patent. In the bad type this is hard to obtain because the patent value is close to zero due to the high investment costs and, hence, less sensitive to changes in uncertainty. The true type of the patent affects the component $D_1 X^\beta$ and the risk neutral probabilities $N(d_1)$ and $N(d_2)$ by changing the term d_1 in equation (2.19) which affects equation (2.17).

2.4.3 Other Comparative Statics

Consider Figures 2.2 and 2.3. Higher growth rate α increases the hedging financial positions $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$. That is because a positive shock in the growth rate of commercial value increases the patent's value and, consequently, requires a higher $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$ to offset the effect of higher growth. The growth rate α has an additional effect in the Hatch-Waxman case. It does not only affect the value of the patent via the elasticity of the patent's value to the commercial value (β), but it also has an effect over the value of the option to litigate which experiences an increase via the higher likelihood that this option will be in-the-money with higher growth rate.

The discount rate r behaves symmetrically with the growth rate α. An increase in the discount rate r decreases the hedging financial positions $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$. The patent value $P(X_0)$ decreases in value and becomes more sensitive to changes in X_0, since it underlies an increase in elasticity β. The Hatch-Waxman case has an additional channel where the discount rate r affects the option to litigate. In detail, a lower discount rate represents a lower opportunity cost and, hence, there is a higher likelihood that the option to litigate will be in-the-money in the future.

The hedging positions $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$ increases with higher γ which means that the plaintiff can have a smaller short-selling position as her share of property over a competing patent decreases. That is because owning a competing patent increases the plaintiff's value of active litigation $L_p(X_0)$ represented in equation (2.9) and, therefore, the plaintiff can select smaller short-selling positions in order to hedge (or profit) from the litigation.

Regarding the effect of arrival rate λ, I consider a range within the limits of legistlation.

That is, a rate of arrival of $\lambda \in [0.5, 1]$ that correspond to an expected time of arrival of the litigation decision of $\overline{T} \in [1, 2]$ years. An increase in the rate of arrival λ of the inter partes review decision increases the hedging financial position $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$. The hedging financial positions increase because a more efficient PTAB decision process decreases the litigation costs for the parties which increases value functions $B(X_0)$ and $H(X_0, 0)$ and, so, it creates a cushion for the plaintiff to select smaller short-selling positions and, in some cases, long positions. In conclusion, when the PTAB deliberates rapidly in inter partes review processes, the plaintiff has access to hedging or profitable financial position that would not be available otherwise. In some cases, these are long positions which is aligned with the goal of legislators of mitigating short-selling under the Hatch-Waxman Integrity Act.

The hedging positions $\Delta_B(X_0)$ and $\overline{\Delta}(X_0, 0)$ increase with higher ϕ, i.e. they increase if the PTAB is more likely to favour the plaintiff in the inter partes review. This occurs because, a higher ϕ increases the plaintiff's value of active litigation $L_p(X_0)$ while decreasing the defendant's value of active litigation $L_d(X_0)$. So, the plaintiff expects to earn more (or lose less), and the defendant expects to earn less (lose more), increasing the difference between the defendant's market value $V_d^M(X_0)$ and the defendant's expected true value $Y + L_d(X_0)$. Consequently, the plaintiff is not required to have such a large short-selling position to hedge (or profit from) the litigation as ϕ increases. Moreover, for very high ϕ, the plaintiff can even profit by selecting a long position because her active litigation value becomes positive ($L_p(X_0) > 0$). Lastly, by Assumption 1, if ϕ is very low, the defendant observes a jump in the equity value with the litigation and does not participate in the litigation. However, if ϕ is very high, the defendant finds that he loses if he participates in the litigation and so everyone is better off if he just hands the patent to the plaintiff.

2.5 Conclusion

This chapter studies the impact of the 2019 Hatch-Waxman Integrity Act over the financial position selected by a plaintiff that wants to hedge or profit from requesting an inter partes review to challenge the validity of a patent. In my model, a defendant possesses a patent

that can be converted into a commercial product. The true value of the patent is privately known by the defendant and the plaintiff but unknown by the market. The plaintiff has a chance to profit by challenging the patent's validity while taking a financial position over the defendant's market value. This model approaches the introduction of the Hatch-Waxman Integrity Act by considering that the plaintiff must wait 90 days to request the inter partes review if she assumes a short-selling position over the equity of the defendant.

My model yields several predictions. First, introducing the Hatch-Waxman Integrity Act reduces the hedging financial position, that is, it reduces the percentage of equity market value that is necessary to short-sell so that the plaintiff hedges her stake in the inter partes review process. By establishing a short-selling position over the defendant's firm, the plaintiff earns a call option exercisable in 90 days that allows her to choose between requesting the inter partes review or closing the financial position. The wedge between the time of the short-sale and the time of the inter partes review request under the Hatch-Waxman Integrity Act causes an increase in the value of the call option to litigate. This increases the plaintiff's expected profit at the short-sale time and relaxes the hedging restriction since the plaintiff does not need to short so many shares to attain the same expected value.

Second, a good patent provides a larger range of profitable financial positions than a bad patent. This occurs because the quality of the patent affects the plaintiff's expected value of active litigation. If the patent is good, the plaintiff has more possibilities to profit because she can derive value from the possibility of winning a valuable patent and from the possibility of extracting a financial gain through the establishment of a financial position. Differently, if the patent is bad, she cannot derive a high value from winning the patent because its value approximates zero. Therefore, in this case, she can only gain from setting up a financial position but not so much from winning the patent.

Third, I show that uncertainty impacts the hedging position. All else equal, higher commercial value uncertainty causes a reduction in the short-selling position necessary to hedge the litigation. More uncertainty increases the value of the patent and, in the Hatch-Waxman case, it increases the value of the call option to litigate.

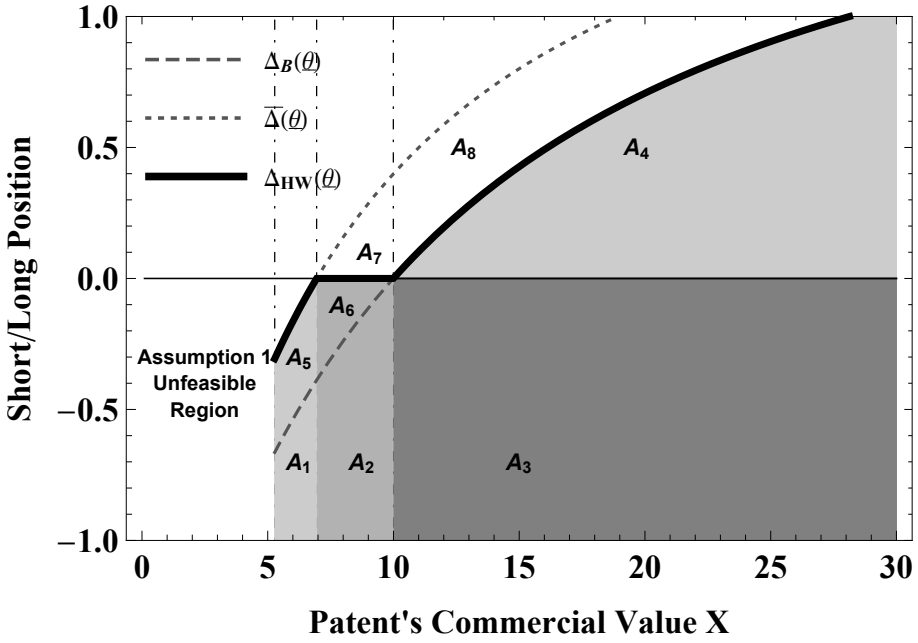
Moreover, my comparative statics show that the plaintiff's short-selling position that hedges the inter partes review process decreases with higher instantaneous commercial value

growth rate, higher rate of arrival of a PTAB decision, with higher probability that the PTAB will favour the plaintiffs request, and with higher plaintiff's share of value over a competing patent. Moreover, it increases if the discount rate jumps.

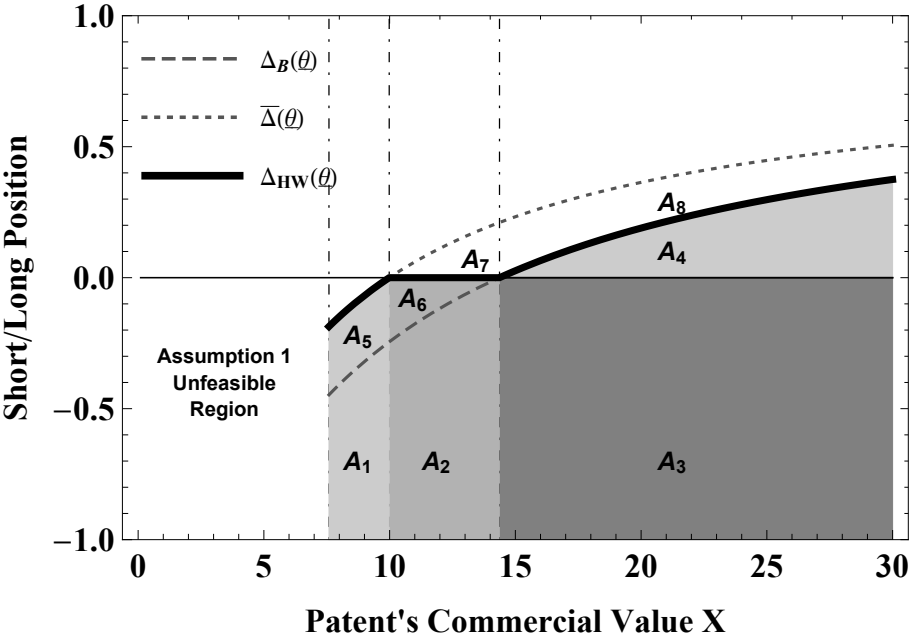
I suggest two paths of future research. First, relax the assumption that the plaintiff must choose, after the waiting period, between requesting the inter partes review or close the position. If prompting litigation is not profitable but closing the position generates a loss, the plaintiff can just wait and make the decision afterwards in an unspecified time.

Second, future research could also focus on studying the impact of this regulatory change over a settlement agreement between the plaintiff and the defender that could prevent the litigation. According to US PTO data on patent trial, on average approximately 13.3% of patent trials are subject to pre-institution settlements[20]. In my model, I limit my analysis to the impact of the Hatch-Waxman Integrity Act over the financial position taken by an investor that requests an inter partes review. However, it would be interesting to understand if these players can find an equilibrium partition of the firm value that guarantees that the information regarding the true type of the patent is maintained private. Papers such as Sarkar (2013) and Sundaresan and Wang (2007a) integrate Nash bargaining games in their models to find the equilibrium share of value between two parties.

[20]For more details on US PTO trial statistics see here.

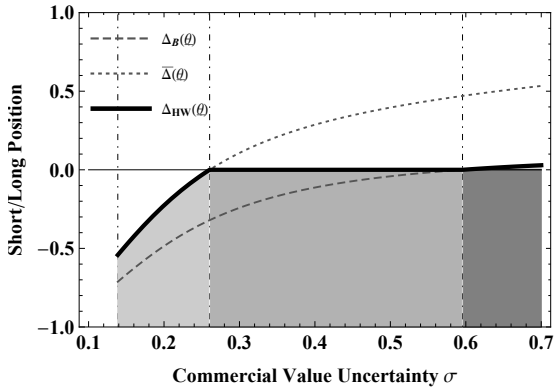


(a) True Type of Patent is Good ($\theta = \overline{\theta}$)

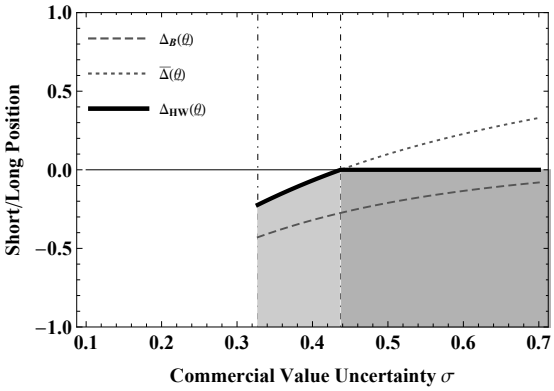


(b) True Type of Patent is Bad ($\theta = \underline{\theta}$)

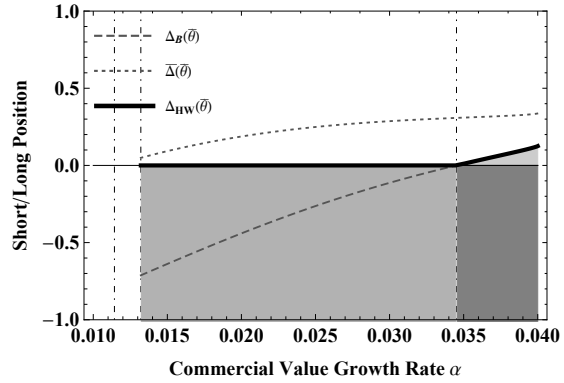
Figure 2.1: Financial positions that hedge the inter partes review litigation as a function of commercial value X_0 at time $t = 0$. Δ_B is the hedging financial position in the base case, $\overline{\Delta}$ is the hedging position if the plaintiff has to wait 90 days to request the inter partes review, and Δ_{HW} is the hedging position in the Hatch-Waxman case. Coloured regions are the admissible profitable Δ positions. Plots are parameterised with the values described in Subsection 2.4.1



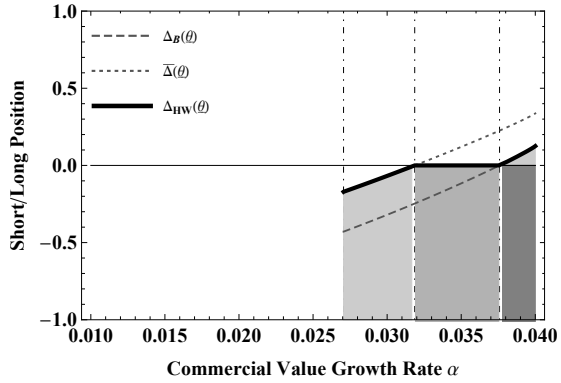
(a) True Type of Patent is Good ($\theta = \overline{\theta}$)



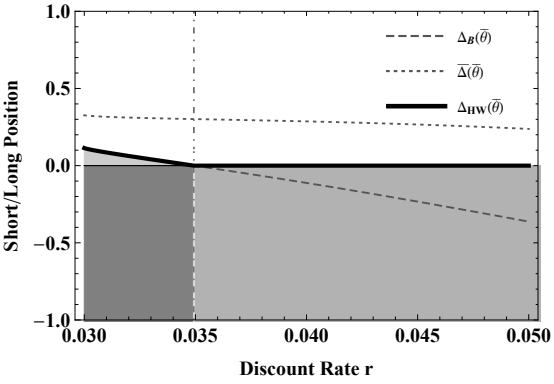
(b) True Type of Patent is Bad ($\theta = \underline{\theta}$)



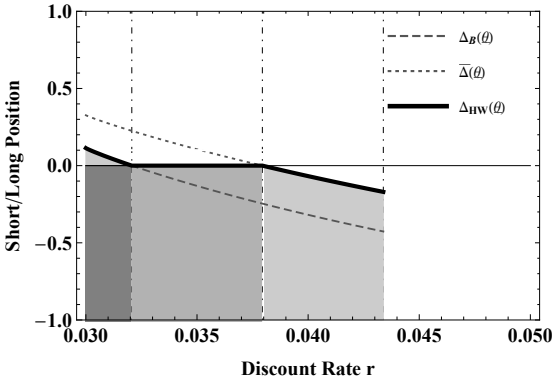
(c) True Type of Patent is Good ($\theta = \overline{\theta}$)



(d) True Type of Patent is Bad ($\theta = \underline{\theta}$)

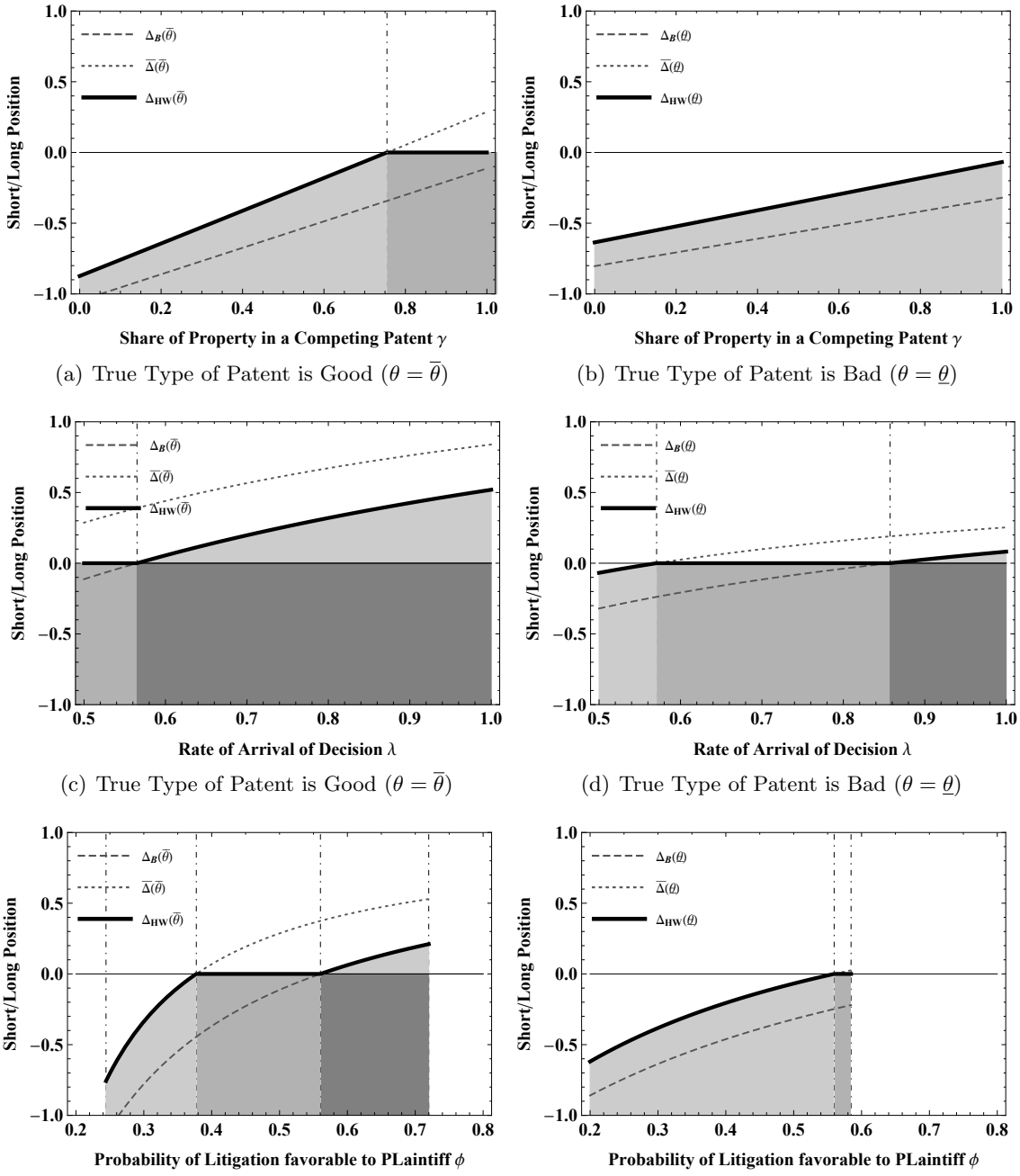


(e) True Type of Patent is Good ($\theta = \overline{\theta}$)



(f) True Type of Patent is Bad ($\theta = \underline{\theta}$)

Figure 2.2: Comparative statics for the share of equity Δ that hedges the plaintiff in the inter partes review litigation. Δ_B is the hedging stake in the base case, $\overline{\Delta}$ is the hedging stake if the plaintiff has to wait 90 days to request the inter partes review, and Δ_{HW} is the hedging stake in the Hatch-Waxman case. Plots are parameterised with the values described in Subsection 2.4.1. .



(a) True Type of Patent is Good ($\theta = \overline{\theta}$)

(b) True Type of Patent is Bad ($\theta = \underline{\theta}$)

(c) True Type of Patent is Good ($\theta = \overline{\theta}$)

(d) True Type of Patent is Bad ($\theta = \underline{\theta}$)

(e) True Type of Patent is Good ($\theta = \overline{\theta}$)

(f) True Type of Patent is Bad ($\theta = \underline{\theta}$)

Figure 2.3: Comparative statics for the financial positions that hedge the inter partes review litigation. Δ_B is the hedging financial position in the base case, $\overline{\Delta}$ is the hedging position if the plaintiff has to wait 90 days to request the inter partes review, and Δ_{HW} is the hedging position in the Hatch-Waxman case. Coloured regions are the admissible profitable Δ positions. Plots are parameterised with the values described in Subsection 2.4.1. .

Chapter 3

The Leicester-Pearson Phenomenon

The Leicester-Pearson phenomenon occurs when good firm performance triggers managerial dismissal. I provide a continuous time model with uncertainty and moral hazard to study this phenomenon and propose an explanation based on the differences in managerial skills between competing managers. Managers can apply their knowledge and exert effort. Knowledge does not depend on cash-flow performance and its impact is limited as the firm grows while the effort contribution depends on cash-flow performance and its impact increases as the firm grows. I derive the level of cash-flows that triggers dismissal and identify the key condition that sustains dismissal with increasing cash-flow performance. The incumbent always faces this phenomenon if the outsider has comparative advantage in the effort attribute for which it suffices to have lower outsider's risk aversion and effort costs. The dismissal trigger that sustains this phenomenon decreases when the agents' discount rate decreases, the incumbent's risk aversion and effort costs increase, the outsider's risk aversion and costs of effort decrease. Uncertainty has a non-monotonic effect over the dismissal decision.

3.1 Introduction

The literature on CEO turnover suggests that involuntary managerial dismissal is mostly associated with poor firm performance, the CEO's proximity to retirement age, or public scandals bringing such unprecedented upheaval that require a swift replacement to ease shareholder concerns[1]. All of these factors produce an adverse environment for the manager to remain in her job. However, few real-world cases suggest that, sometimes, firms performing extremely well under CEOs far from retiring and absent of notable scandals are willing to replace the incumbent manager with someone else with executive attributes[2] more adequate for the new level of firm performance.

I name this as Leicester-Pearson phenomenon in reference to the relationship between Leicester City football club and the manager Nigel Pearson. Pearson led Leicester through one of the most successful comebacks in the history of english football, from League One (third tier) in the 2008-09 season to the 14th place of Premier League (tier one) in the 2013-14 season. The club replaces him in 2015 with the awarded coach Claudio Ranieri to win the Premier League in the following season for the first time in its history[3]. As discussed in the news:

[1] CEO replacement is negatively related with firm performance as described in Warner et al. (1988), Gibbons and Murphy (1990), Murphy and Zimmerman (1993), and Jenter and Lewellen (2020). Recent literature suggests that performance relative to industry or market also influences CEO replacement (Kaplan and Minton 2012, Eisfeldt and Kuhnen 2013, Jenter and Kanaan 2015).

CEOs have a higher replacement probability if they are closer to retirement age (Sabac 2007, Brickley 2003, Jenter and Lewellen 2020). See de Vries (1988) for an informal discussion on the reasons why CEO retirement and succession are involuntary. Note that retirement age is also linked to voluntary turnovers in the CEO turnover literature (Jenter and Lewellen 2020).

Agrawal and Cooper (2017) finds an increased probability of CEO dismissal after scandals that resulted in accounting restatements. Anecdotal evidence: Scotia Assurance CEO Peter Cruikshank was replaced after a securities fraud investigation detected nearly GBP 500 million in liabilities that had been incorrectly recorded (Fryer 2009); Siemens' CEO Klaus Kleinfeld and chairman Heinrich von Pierer were replaced in 2003 after investigations uncovered bribery schemes to promote the sales in Italy, Nigeria, and Argentina (Loscher 2012).

[2] The importance of outside competition for managerial skills has been highlighted by (Fama 1980) as a mechanism that disciplines incumbent managers.

[3] In detail, Leicester hired Nigel Pearson in the 2008-09 campaign after descending to the worst standing in the history of the club, the League One (third tier) of the English football system. Pearson follows by winning League One and leading Leicester City in 2009-10 to the fifth place of the Championship (second tier). However, after disagreements with the board, Pearson quits Leicester in mid-2010 and the club enters a period of one year and a half of bad performance under four different managing directions. Pearson returns in 2011 and takes Leicester to the to the 14th place Premier League (first tier) in the 2013-14 season. In 2015, Leicester sacks Pearson and exchanged him with Chelsea's former coach Claudio Ranieri, a more experienced and awarded manager who ends up winning the Premier League in the following season. It is worth noting that, officially, Pearson was fired following a racism scandal issue involving his son but not himself directly.

> "Of the 11 players who featured in [Pearson's] final game (...) only N'Golo Kante and Danny Drinkwater are the outstanding additions this season. Ranieri may have massaged this team into champions, but Pearson may well deserve a winners' medal as well."[4]

The Leicester's case is exemplary but is not an isolated case. Consider, in the business world, the case of Barclays in July 2015:

> "Barclays fired Antony Jenkins as its chief executive (...) three years after he was promoted from the retail bank (...). Sir Mike Rake, deputy chairman, said: 'Notwithstanding Antony's significant achievements, it became clear to all of us that a new set of skills were required for the period ahead.' "[5]

or Lego's resolution in August 2017:

> "(...) Sales increased 6 per cent last year to a record USD 6bn. [However], Lego is jettisoning its chief executive after only eight months in the job. Bali Padda (...) will be replaced by Niels Christiensen [who] made a remarkable turnaround in Danfoss, taking it from industrial technology to digital technology."[6]

These cases propose that, despite the indications of excelling performance, firms can still decide to replace their incumbent manager with someone holding different attributes. Moreover, the change of performance can be itself a driver of managerial turnover by making the set of attributes of a replacer candidate more appealing when the firm performance increases. In light of these elements, this chapter provides a framework to answer the question "in what circumstances is a shareholder more willing to replace a manager with increasing cash-flow performance?".

[4] Newsweek opinion article "Why Nigel Pearson deserves more credit for Premier League Title" available in https://www.newsweek.com/leicester-city-premier-league-tottenham-hotspur-455165.

[5] Financial Times article "Barclays fires Jenkins after clash over investment banking unit" available in https://www.ft.com/content/74fed7c6-2537-11e5-9c4e-a775d2b173ca.

[6] Financial Times article "Lego removes CEO after 8 months for younger replacement" available in https://www.ft.com/content/28eea626-2ec6-3c90-acc4-bdeab9f536a3. Even though the age of Bali Padda was highlighted as the official reason to replace the incumbent CEO, the fact is that Lego's 2016 press release appointing Padda as the new CEO never identified him as an interim CEO. As Thomas Kirk Kristiansen commented in the same press release: *"Bali possesses the right LEGO mindset. He has a deep knowledge of our entire company, a very clear sense of what we need to do to take the company to the next level, and he is a true ambassador for the LEGO Idea and the LEGO brand."* The press release can be found in https://www.lego.com/en-us/aboutus/news/2019/october/bali-padda-new-ceo.

In particular, I build on the work of Hamori and Koyuncu (2015) and propose a combination of knowledge and effort attributes and then study how differences in attributes between the incumbent manager and the outsider candidate interact in order to identify how the replacement decision is more likely to occur as cash-flows increase[7]. My analysis generates predictions that are consistent with the aforementioned examples and contrast with the consensual narrative of managerial dismissal literature.

This chapter extends existing literature in several ways. First, it contributes to corporate governance literature by addressing the link between competing executive attributes and the decision to dismiss the incumbent manager. Existing literature has used differences in attributes, such as skillsets, to explain cross-sectional variation in managerial compensation but it has paid little attention to the way that differences in attributes and firm performance simultaneously affect dismissal decisions. For example, the super-star theory is a widely accepted explanation for the growth of executive pay during recent decades, particularly in the top tier of the skills distribution (Kaplan and Rauh 2010, 2013). As argued by Rosen (1981) and Baranchuk et al. (2011), highly qualified individuals have a comparative advantage when capital investment increases, because the marginal value of talent is greater at larger firms, which are therefore willing to pay more for that talent[8].

Second, this chapter advances the contract theory literature by studying the dismissal decision jointly with the optimal compensation decision in the presence of moral hazard. This joint analysis departs from the existing works that study either the CEO compensation decision (Bebchuk and Fried 2005, Becker 2006, Gao 2010, Palia 2015, Alok and Gopalan 2018) or, separately, the turnover decision (Parrino 1997, Brickley 2003, Kato and Long 2006, Eisfeldt and Kuhnen 2013, Jenter and Lewellen 2020). For example, the existing literature establishes that firms should compensate risk-averse CEOs for the risk faced when exercising

[7]Instead of two attributes in a single skill, I could have considered that each manager has a skillset composed by two skills that impact the firm performance differently. In this case, it would be necessary to identify the managerial skills that have comparative advantages as the firm changes. This can be considered in future research.

[8]In the empirical literature, Kaplan et al. (2012) study the characteristics of 316 CEO candidates and how these relate with subsequent firm performance. Using factor analysis they identify two main dimensions of variation in CEO characteristics: The first factor aggregates general talent of a CEO while the second loads negatively on overconfidence while loading positively on communication. Custódio et al. (2013) tested whether the CEO compensation scheme is determined by the type of managerial skills. They develop a general ability index that allows to compare general managerial skills against firm specific skills and conclude that the supply of generalist managers has steadily increased in the past 15 years.

the required unobservable effort (Holmstrom and Milgrom 1987, Sannikov 2008, He et al. 2017). These works have nothing to say about the effects of risk aversion on the agent's dismissal decision while this chapter shows that this decision depends on the risk aversions of the incumbent manager and of the replacer candidate.

Third, this chapter extends the real options literature by studying the case where a firm exchanges two individuals. The existing real options literature has been prolific on the study of asset replacement (Reindorp and Fu 2011, Zambujal-Oliveira and Duque 2011, Adkins and Paxson 2011, Store et al. 2018), but is still absent on the case where a firm has a contract with an individual and replaces it with a contract with another person. The key feature of managerial replacement present in my model that differs from the general case of asset replacement is the inclusion of optimal incentive alignment to mitigate the executive moral hazard issues. The literature treats the productivity of assets as independent from human action. Consequently, assets are immune to moral hazard frictions. However, when modelling the replacement of self-interested agents, like managers, one must take into consideration the incentive alignment problem that eliminates the moral hazard issues. My model shows that the agent replacement decision depends on the incentive alignment costs faced by the firm with respect to the incumbent manager and to the replacer candidate.

I present a general continuous time model where a risk neutral shareholder owns an unlevered firm with stochastic cash-flows sensitive to the attributes of the manager in place. The firm starts with an incumbent manager whom can be later replaced with an outside candidate. As in He (2011), managers are risk-averse with a CARA utility function and the firm faces moral hazard and hidden savings.

The attributes of managers are their knowledge and their effort and each attribute contributes differently to the growth rate of cash-flows. When applied, these attributes increase the cash-flow growth. I consider that applying knowledge is costless for both managers and it contributes with a fixed inflow. I consider that the effort attribute is costly but it contributes with an inflow that varies positively with the current level of cash-flows. I consider that cash-flows do not grow with knowledge because, as a firm grow larger, it is increasingly harder for members of the upper echelon of a firm to pass their personal insights throughout the organisation. The moral hazard and the hidden savings friction

require the shareholder to pay an instantaneous compensation that optimally incentivises the manager to exert the recommended effort. Therefore, the effort contributions depend on their managers' specific risk aversion and effort costs.

The shareholder has a real option to replace the incumbent manager with the outsider manager after paying replacement costs, such as search and legal costs. A firm chooses to retain or replace the incumbent manager by comparing the firm value if it retains the incumbent to the firm value under the replacer management deducted of replacement costs.

I specify a structure for the model and solve it in two steps: first, I solve for the optimal compensation that guarantees that the manager in place always exerts the required effort and I find the required effort that maximises the shareholders continuation value function; second, I solve for the optimal time to dismiss the incumbent and replace her with the outsider manager which, under adequate conditions (Peskir and Shiryaev 2006), is reduced to a free boundary problem where I solve for the level of cash-flows that triggers replacement. My solution of the dismissal problem relates to the real option literature stemming from Dixit and Pindyck (1994).

Most of the prior theoretical works on compensation and replacement decisions analyse static models. This approach has generated important insights, but it ignores the fact that these decisions are inherently different in nature. The former problem ("how much to pay?") has to be solved intra-period for each time; the latter ("when should a manager be fired?") is an optimal stopping time problem. The continuous time approach used in this chapter overcomes the limitations of static frameworks by solving for the optimal managerial compensation together with the optimal dismissal decision. The continuous time problem necessarily requires some simplifying assumptions: for example, compensation in my model is paid instantaneously. But, in contrast to discrete time stopping models, the optimal stopping problem is tractable in my continuous time model.

In terms of results, I derive the equation that sustains the existence of a Leicester-Pearson phenomenon in this model. In an environment where the incumbent is either more efficient than the outsider in the knowledge attribute or she is entrenched due to high replacement costs, the Leicester-Pearson phenomenon exists if and only if the outsider manager is more efficient than the incumbent in the effort attribute. That is, the outsider's

effort contribution to the expected cash-flow growth is greater than the incumbent's effort contribution. Moreover, a sufficient (but not necessary) condition for the outsider manager to always have this comparative advantage is for the outsider's risk aversion and effort costs to be lower than the incumbent manager's risk aversion and effort costs.

The mechanics of the model in the presence of the Leicester-Pearson phenomenon are as follow. The incumbent manager runs the firm by contributing with her knowledge and with her effort. The shareholder lets the incumbent run the firm but he is always comparing the contribution of the incumbent to the cash-flow growth against the possible contribution of an outsider candidate. For the Leicester-Pearson phenomenon to exist here, it is necessary for the outsider to have comparative advantage in the effort attribute. Comparative advantage in the knowledge attribute is not required. In fact, the Leicester-Pearson phenomenon exists when the incumbent has comparative advantage in the knowledge attribute or when the incumbent is entrenched due to sufficiently high replacement costs.

Comparative advantage in the effort attribute is not very useful for the shareholder while the firm cash-flows are low. In this case, the shareholder benefits more from knowledge attribute, which provides a fixed contribution to cash-flow growth. So, the incumbent stays in place either because she has comparative advantage in the knowledge attribute or because high replacement costs prevent profitable replacement while cash-flows remain low. The shareholder will then wait until the outsider's effort contribution is sufficiently high to justify replacement.

I provide comparative statics for all relevant parameters. First, the dismissal trigger has a non-monotonic relationship with uncertainty, i.e. for low levels of volatility, a positive shock in uncertainty decreases the dismissal trigger but the effect is opposite if volatility continues to increase. This is due to the different impact that changes in volatility have over the required effort of the incumbent manager and over the required effort of the outsider manager.

Second, the optimal dismissal trigger increases with the discount rate due to its effect over the present value of the managers' knowledge contributions and via its effect over the costs of managerial incentive alignment. Third, the optimal dismissal trigger decreases if the cash-flow growth component that is independent of management increases because

it improves the value of the firm under the outsider's management. Fourth, an increase in the incumbent's effort costs decreases the optimal dismissal trigger while an increase in the outsider's effort costs increases the dismissal trigger due to their effects over the costs of incentivising the risk-averse manager to be aligned with the shareholder. Also, an increase in the incumbent's knowledge contribution and a decrease in the outsider's knowledge contribution increases the optimal dismissal trigger.

My analysis also provides a new result by demonstrating that managerial risk aversion affects the dismissal decision. The moral hazard literature established long time ago that risk aversion in effort inducing contract models between a risk-neutral principal and a risk-averse agent require additional compensation (Holmstrom and Milgrom 1987, Sannikov 2008). However, this literature has been silent on the effects over the agent's dismissal decision. In this model, risk aversion increases the incentive alignment costs between the shareholder and the manager in place which makes that manage more expensive and forces the shareholder to lower the optimal required effort. More importantly, they show that the optimal dismissal trigger decreases with higher incumbent risk aversion and lower outsider candidate risk aversion.

My framework builds on the principal-agent theory works of Gryglewicz and Hartman-Glaser (2014) and He (2011) which extend Holmstrom and Milgrom (1987) to a continuous time setup. I share with these models the presence of moral hazard, the CARA utility preferences and the existence of private savings for the manager. He (2011) studies the optimal compensation policy between shareholders and a manager and finds that the effect of debt-overhang on managerial incentives decreases the level of debt when determining the optimal capital structure. Gryglewicz and Hartman-Glaser (2014) studies the impact of moral hazard between shareholders and a manager on the firm's investment policy. They find that that the intensity of the moral hazard problem affects the productivity growth of the firm and the investment decision. I depart from these models by focusing on the optimal managerial dismissal decision instead of studying the optimal capital structure and investment decisions.

The rest of the chapter is organised as follows. Section 3.2 presents the literature review; Section 3.3 introduces the general model; Section 3.4 provides a model application; Section

3.5 discusses the main results and provides comparative statics for the model application provided in Section 3.4; Section 3.6 presents some preliminary conclusions, and delves into possible future research avenues.

3.2 Literature Review

▷ **CEO Turnover Literature.** The literature on CEO turnover and compensation documents the negative link between the likelihood of executive turnover and firm performance, i.e. Gibbons and Murphy (1990), and Barro and Barro (1990). Specifically, my work relates the most with Eisfeldt and Kuhnen (2013) who account for the issue of CEO turnover in an assignment framework derived from Tervio (2008). Their model captures the interaction between firms and CEOs in the labour market, where parties optimise between maintaining the employment contract or choosing their respective outside options. These outside options are key to understand how industry conditions affect turnover decisions. Specifically these are affected by factors that, in practice, are dependent from the state of nature in industry. For example, during recessions, the skillset required by firms may change which in turn may generate the replacement of the CEO. Their empirical analysis conclude that the proxy for skill-weights shock is statistically significative and indicates that if an industry is unproductive, there is approximately an increase of 49% on the likelihood of a turnover[9]. My work complements Eisfeldt and Kuhnen (2013) who do not consider the endogenous derivation of the turnover triggers nor do they consider the agency friction as I do.

▷ **Assortative Matching Literature.** An increasingly large literature building on from the work of Sattinger (1993) relies on assortative models to explore the matching between firms and CEOs with different characteristics. These models are advantageous as they are often empirically tractable and, for simplest constructions, I can obtain closed form solutions. My model is indirectly connected to this literature as I consider that my incumbent CEO

[9] Consistent with Brickley (2003), I can further see that, in these unproductive industries, a CEO susceptible to a turnover is likely to be replaced by an industry outsider (21.78%), specially when the turnover is a firing (29.57%). In one hand, industry outsiders have a positive impact on the firm's productivity with a median residual gain of 0.68% of ROA in the year after turnover, but they also derive a higher compensation with a median pay increase of 0.91USD per 1000USD of firm assets value.

faces competition from the CEO market.

In Tervio (2008) firms with different sizes and managers with heterogeneous skills are matched, further generating a market equilibrium compensation. This work takes a leap from traditional body that strictly focus on firm size as the source of higher CEOs compensation by introducing competition for talent as an explanatory source. Data calibration shows that shocks to equilibrium matches, i.e. replacing equilibrium-matched CEOs with other types, have a negligent impact in total economic surplus. In contrast, firm size has an overwhelming impact in differences of managerial compensation, specially amongst the highest levels of CEOs compensation, which can be interpreted has being a result of the exogenous firm scale component.

While Tervio (2008) obtains the distribution of CEO quality from the observed distribution of compensation and firm market value, Gabaix and Landier (2008) goes one step further and develops a framework that allows for the derivation of the equilibrium CEO compensation in closed form. The main prediction indicates that the compensation of managers, in equilibrium, should vary positively with the size of the CEO's firm and the average industry firm size. Also, their calibration results hints on what was later described as the power rule of CEO compensation[10]: Talent amongst CEOs is bounded while firm size is unbounded, which leads to increasingly high pay premiums for firms with higher valuations.

Edmans et al. (2009) builds on the competitive assignment framework of Gabaix and Landier (2008) by embedding a risk-neutral principal-agent conflict onto the market equilibrium mechanism while using multiplicative preferences instead of the usual additive preferences. This construction allows for the endogenous determination of the compensation and incentives. The multiplicative preferences lead to the conclusion that dollar-dollar incentives may vary negatively with firm size, contradicting the previous results of the related literature with additive preferences. Moreover, Edmans and Gabaix (2011) later

[10] Gabaix (2016) describes one of the most stable regularities in executive compensation. The power rule is defined for the wage of the CEO as: $w(n) = D(n^*)S(n^*)^{1-b}S(n)^b$. Where n is the ranking of the firm we're looking at, n^* is the ranking of the reference firm, e.g. the median firm. $D(n^*)$ is a constant such as skill scarcity, and $S(.)$ is the size of firm, e.g. market capitalisation. The CEO pay is proportional to the size of the company through a power rule with a power of 1/3. Moreover, see Gabaix et al. (2014) for an application of Gabaix and Landier (2008) to the crisis and after-crisis periods.

accounts for risk aversion in the talent assignment model with moral hazard. They find that cross-sectional variation in risk and disutility have a positive impact in the level of pay. Differently from my model, agents share a CRRA utility function and hold the same cost of effort. Heterogeneity is instead measured by the level of outside options, i.e. their reservation wage.

While these works stress the importance of differences in quality among CEOs as a source of huge differences in compensation premium, they do provide evidence. Falato et al. (2015) is the first work to actually use the assortative matching model for the CEO labor market and derive direct measures of the skill premium in pay. They encode information on CEOs and form three measures of credentials (quality of managerial skills): One proxy for reputation, other for career progression and another for academic track-record. They find that new CEOs can earn up to 5% (280,000 USD) of compensation premium per decile of skill difference. This highest scenario of compensation-skills premium occurs for the CEOs with the best career and reputation and for those that manage larger firms. Furthermore, they quantify the returns for equity and conclude that performance differential can be up to 0.5% of credential decile. Note that this comparatively high difference in results estimation between firm performance and CEO compensation given skill quality differentials is consistent with the idea that manager skills are bounded unlike firm value and, consequently, differences in skills will at the upper top imply significant larger differences in compensation than in firm performance, as in Gabaix et al. (2014) and Gabaix and Landier (2008).

All of these aforementioned works fail to account the following firm's idiosyncrasies: They do not account for differences in the match composition between the firm needs and the portfolio of skills available in the market, nor do they account for changes in quality in the supply of CEO skills.

▷ **Dynamic Principal- Agent Literature.** My model fits in the existing theoretical literature on dynamic agency theory which focus on the problem of finding incentive compatible contracts that secure optimal executive effort and often does not pay much attention to the CEO turnover issue. Among the existing theory, I relate with Sannikov (2008) who pioneered the use of incentive compatible compensation under the form of an

adapted process to solve for the continuous time hidden action problem in continuous time, He (2009) whom restricts to geometric brownian motions and considers executive contract termination coincidently with the firm's closure, and Gryglewicz and Hartman-Glaser (2014) who considers an infinitely lived incumbent manager.

Continuous time agency models vary substantially in applications and results. For instance, He (2009) sets up a framework where the stochastic firm size increases throughout time and reduces the moral hazard problem as a consequence of the agent's stake increase. Stronger incentives are guaranteed throughout the optimal path with performance-based stocks. Furthermore, He (2011) targets the problem of a firm's capital structure, extending the traditional Leland (1994) and, consequently, providing the theoretical support for the differences in the capital structure between small and large firms. Specifically, debt-overhang motivates a lower required executive effort which, in return, generates the lower optimal leverage proportion typical in smaller firms.

I particularly relate with the work of He (2012) which also considers turnover, though as a threatening mechanism and not as a consequence of market characteristics. This framework considers unobservable actions with an agent that can save privately while controlling the firm's profitability. Savings allow for compensation cuts' cushioning after bad performance, which consequently leads to incentive distortion. To solve this, the principal has the a firing mechanism in place combined with an optimal compensation with downside-protection to mitigate the cushion behaviour.

Recent examples in this literature follow: Gryglewicz and Hartman-Glaser (2014) and Demarzo and Sannikov (2017) focus on investment decisions, He et al. (2014) considers that profitability is multiplicative with managerial effort, He et al. (2017) introduces Bayesian learning of managerial ability, and Miao and Rivera (2016) considers ambiguous beliefs for the principal. Moreover, it is common to simplify by restricting the firm to a unique uncertainty source, i.e. size (He (2009)), cash-flows (Gryglewicz and Hartman-Glaser (2014)) or profitability (He (2012)) are subject to a single stochastic process.

The reader may find detailed reviews on executive compensation in Edmans and Gabaix (2016), Murphy (2013) and Frydman and Jenter (2010), and a brief review on CEO turnover in Brickley (2003).

3.3 Model

I present a continuous time principal-agent model with agent dismissal. The model has three players: a shareholder who acts as a principal, an incumbent CEO who has been the agent in place since the firm's inception, and an outsider CEO, who replaces the incumbent manager after her dismissal.

The shareholder cares about two key governance decisions. At every moment, he has to solve a compensation problem and, hence, to decide the optimal effort to be induced. Moreover, the shareholder must choose the optimal time to dismiss the incumbent CEO.

The shareholder is risk-neutral and owns a firm that generates cash-flows represented by an Ito diffusion $x = \{x_t : t \geq 0\}$ that is defined on a filtered probability space $(\mathbb{O}, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ and with growth dx_t that satisfies the following differential equation:

$$dx_t = \delta(x_t)\, dt + \alpha(a_t, x_t)\, dt + \beta(b_t, x_t)\, dt + \sigma(x_t)\, dw_t \tag{3.1}$$

Here, $\sigma(x_t) > 0$ is the instantaneous conditional standard deviation, and $\delta(x_t) + \alpha(a_t, x_t) + \beta(b_t, x_t)$ is the instantaneous conditional expected change in cash-flows, where $a_t > 0$ and $b_t > 0$ are effort levels. Furthermore, $\partial\alpha(a_t, x_t)/\partial a_t > 0$, $\partial\beta(b_t, x_t)/\partial b_t > 0$, and $\partial^2\alpha(a_t, x_t)/\partial a_t^2 \leq 0$, $\partial^2\beta(b_t, x_t)/\partial b_t^2 \leq 0$.

I consider that the firm cash-flows can be affected by two managerial skills, α and β. For example, a skill related with finance and a skill focused on technology. I further assume:

Assumption 2. *The incumbent manager has skill α and the outsider manager has skill β.*

In my model, managers are specialised in their own skill. Moreover, they both have attributes that contribute to the firm performance. First, managers have a knowledge attribute which provides a fixed contribution to cash-flow growth and, second, they have an effort attribute which provides a variable contribution that depends on the manager's effort and on the firm cash-flow level. I justify the independence of the knowledge attribute from cash-flows with the fact that it is increasingly difficult for the insight of a manager to pass throughout the organisation as it grows larger.

Therefore, I write the contribution of the incumbent's skill α to the growth of the firm cash-flows as $\alpha(a_t, x_t)$, which I decompose as:

$$\alpha(a_t, x_t) = \underline{\alpha}(a_t, x_t) + \overline{\alpha}$$

Here, $\underline{\alpha}(a_t, x_t)$ is the incumbent's variable contribution to the firm cash-flow growth due to the effort attribute and $\overline{\alpha}$ is the incumbent's fixed contribution to the firm cash-flow due to the knowledge attribute.

After the incumbent dismissal, the shareholder hires the outsider who can now apply her skill β. This skill contributes to the growth of the firm cash-flows with load $\beta(b_t, x_t)$, where:

$$\beta(b_t, x_t) = \underline{\beta}(b_t, x_t) + \overline{\beta}$$

Here, $\underline{\beta}(b_t, x_t)$ is the outsider's variable contribution to the firm cash-flow growth due to the effort attribute and $\overline{\beta}$ is the outsider's fixed contribution to the firm cash-flow due to the knowledge attribute.

When no effort is exerted, cash-flows are expected to increase in the amount of the fixed contribution. That is, respectively, $\alpha(0, x_t) = \overline{\alpha}$ if the incumbent is the manager in place or $\beta(0, x_t) = \overline{\beta}$ if the outsider is in place. Finally, I consider that $\delta(x_t)$ is the change in cash-flows occurring independently of any managerial skill and it occurs even in the absence of a manager. Since $\delta(x_t)$ is the expected cash-flow growth independent of managerial contributions. This is the growth rate that would occur if the firm functions without a manager. This term does not have fixed contribution dependent on managerial expertise nor does it depend on managerial effort. Finally, all agents discount their future flows at the exogenous rate r.

This model has two main frictions. First, the shareholder faces a moral hazard problem with respect to managerial effort. This is a classic principal-agent setup where the shareholder cannot see the chosen managerial action and can only infer it from a noisy signal. To overcome this, the contracts offered to the incumbent and outsider managers must guarantee the alignment of interests, that is, that managers are willing to exert the effort requested by the

shareholder.

Second, the shareholder faces hidden savings. Managers can have wealth in this model and they can save and borrow at rate r in a private savings account that is unknown to the shareholder. Savings affect the consumption decision and, since they are private, they indirectly affect the moral hazard problem by distorting the managers' incentives. For example, hidden savings lead to accumulated hidden wealth which allows for compensation cuts' cushioning after bad performance. So, the contracts must additionally deal with this problem. The shareholder achieves this by securing that saving is done inside the firm (through an inside fund) and not privately. Subsequently, the shareholder can set up a compensation scheme that respects the managers' optimal consumption-savings plan while preventing private savings.

3.3.1 Incumbent Manager

The incumbent manager manages the firm from time 0 until her dismissal, which occurs at the optimal dismissal time T_d^*. She experiences instantaneous utility $u(x_t, \tilde{p}_t)$ with the following constant absolute risk aversion (CARA) functional form:

$$u(x_t, \tilde{p}_t) = -\frac{1}{\gamma} e^{-\gamma \tilde{p}_t} \tag{3.2}$$

where $\gamma > 0$ is the risk aversion coefficient and $\tilde{p}_t$ is the agent's instantaneous consumption.

The incumbent CEO has a wealth balance $W^I = \{W_t^I :\geq 0\}$ that capitalises instantaneously at rate r and is defined as:

$$W_t^I = \int_0^t e^{r(t-s)} \left(p_s - \tilde{p}_s - g(x_t, \tilde{a}_t)\right) ds,$$

where I assume $W_0^I = 0$. This expression can be written in differential form as follows:

$$dW_t^I = rW_t^I dt + p_t dt - \tilde{p}_t dt - g(x_t, \tilde{a}_t) dt \tag{3.3}$$

The incumbent CEO's wealth increases because she earns interest on her existing wealth level (rW_t^I) and because she earns the wage p_t. It decreases because the CEO consumes

$\tilde{p}_t$ and because she incurs in labor costs $g(x_t, \tilde{a}_t)$.The CEO's wealth W^I is subject to the following transversality condition: $\lim_{T\to+\infty} \mathbb{E}\left[e^{-rT} W_T^I\right] = 0$.

The shareholder and incumbent manager sign a contract $\mathbb{C} = (p, a)$ that specifies a wage rule $p = \{p_t : t \geq 0\}$ and a required managerial effort rule $a = \{a_t : t \geq 0\}$. The wages and the required effort are $\mathcal{F}_t$-measurable at each time t: that is, they are functions of the CEO's performance history.

Following He (2011), the contract $\mathbb{C}$ is incentive-compatible and no-savings ($dW_t^I = 0$). That is, given $\mathbb{C}$, the agent always finds it optimal to exert the recommended managerial effort ($\tilde{a} = a$) and to spend the entire wage ($\tilde{p} + g(x, \tilde{a}) = p$). Working backwardly, I determine the path of realised efforts and consumptions that are optimal for the CEO.

I write the CEO's instantaneous labor cost function as follows:

$$g(x_t, \tilde{a}_t) = x_t \theta \bar{g}(\tilde{a}_t) \tag{3.4}$$

The parameter $\theta > 0$ represents the inefficiency of the incumbent CEO in exerting effort. To secure the alignment of interests, the shareholder will A higher θ means that the CEO is more inefficient and, hence, effort exertion is more expensive.

As a result, the shareholder finds it more expensive to induce effort if the incumbent manager is more inefficient. This function also depends on the cash-flow level x_t because it should be harder to improve cash-flows as they grow larger. Moreover, $\bar{g}(\tilde{a}_t)$ holds the usual desirable economic properties $\partial \bar{g}(a_t)/\partial a_t > 0 \quad \text{and} \quad \partial^2 \bar{g}(a_t)/\partial a_t^2 \geq 0$. Furthermore, the manager does not support effort costs when she shirks, that is, $\bar{g}(x_t, 0) = 0$.

I consider that the shareholder establishes a fund representing the incumbent CEO's inside stake in the firm. This insider fund has value $N_t^I \equiv N^I(M_t^I)$ with the following functional form:

$$N_t^I = -\frac{1}{r\gamma} \ln(-r\gamma M_t^I) \tag{3.5}$$

Here, $M_t^I \equiv M_t^I(\mathbb{C}, x_t)$ is the incumbent CEO's continuation value given recommended effort a and wage rule p. The internal fund is contingent on firm performance and so, the shareholder adjusts the level of the insider fund N_t^I as the level of cash-flows x_t changes.

Furthermore, the fund accrues at the discount rate r. At the dismissal time, the shareholder transfers the internal fund out of the balance sheet to the incumbent manager[11]. This purpose of this fund is to guarantee that savings are done inside the firm and not privately.

Assumption 3. *The shareholder imposes a non-compete clause that prevents the incumbent CEO from returning to the labor market after being dismissed.*

Assumption 4. *The CEO does not expect to be dismissed.*

These assumptions are helpful solve the optimal stopping time problem by allowing a stable optimal dismissal decision. Assumption 3 rules out the cases where the CEO's continuation value is potentially affected by an outside option of employment. Non-compete clauses are commonly used in CEO contracts to minimise the spill-over of sensitive information to competitors[12]. This assumption prevents different continuation value structures after the dismissal decision[13].

Assumption 4 considers that the incumbent manager is myopic about the contract termination. This assumption bypasses the case where the termination of the manager's tenure causes a discontinuity in her continuation value which would change her effort incentives. In detail, if the manager learns that she is close to being fired and if that firing is followed by a negative discontinuity in her continuation value, then she will be willing to work more for the same money. This would change the optimal compensation problem and the optimal dismissal decision[14,15].

[11] See He (2011) for further details on the practical implementation of this fund scheme.

[12] From a random sample of 500 firms Garmaise (2009) finds that more than 70% have non-competing clauses in their C-suite contracts. Similarly, Bishara et al. (2015) studies 874 labor contracts between S&P 1500 firms and their managers and finds that 80% of the contracts contain non-competing clauses.

[13] If the manager goes to a different company after the dismissal, she will have a different continuation value structure due to the different compensation and required effort schedule in her new job. Since the managers' continuation value function is the expectation over all discounted future instantaneous utility, the optimal contract selected by our shareholder will necessarily be affected by the future possible jobs that the manager can have, which constitutes an additional layer of difficulty outside the scope of this chapter.

[14] The issue here is the loop: if she is about to be fired and lose value, she will be willing to work more for the same money. So, the shareholder can increase the optimal required effort, which will change the optimal dismissal decision by making it less likely to be fired. This reverts the process and so she will demand more money for that same amount of effort which will eventually make her more likely to be fired.

[15] Instead of having these two assumptions, I can have a single one that requires the manager to have the exact same job at an exactly equal company after being dismissed. I find this less realistic than having assumptions 3 and 4.

Optimisation Problem 1 (Incumbent Manager)**.** *The optimal continuation value of the Incumbent CEO given a contract* $\mathbb{C}$ *is:*

$$\begin{aligned} M_t^I(\mathbb{C}, x_t) = \max_{\tilde{p},\tilde{a}} \quad & \mathbb{E}_t\left[\int_t^{+\infty} -\frac{1}{\gamma} e^{-r(s-t)-\gamma\tilde{p}_s} ds\right] \\ s.t. \quad & dW_t^I = rW_t^I dt + p_t dt - \tilde{p}_t dt - g(x_t, \tilde{a}_t) dt \\ & dx_t = \delta(x_t)dt + \alpha(\tilde{a}_t, x_t)\, dt + \sigma(x_t)\, dw_t, \qquad t \in [0, T) \end{aligned} \tag{3.6}$$

The CEO attains her maximum value by choosing spending $\tilde{p}$ and effort $\tilde{a}$. These decisions are subject to her wealth balance presented in equation (3.3). During the incumbent's tenure, cash-flows behave in accordance with equation (3.1) which depends on the realised effort $\tilde{a}_t$ and is restricted to the incumbent CEO's skill as assumed in Assumption 2. The optimal instantaneous consumption of the incumbent CEO is $\tilde{p}_t^*$, and the optimal instantaneous effort is $\tilde{a}_t^*$. The manager's present value evaluates the integral from t to infinity because of Assumption 4.

Lemma 2. *Under a contract that is incentive-compatible and no-savings, the manager's optimisation problem can be restricted to the no-wealth case, when the CEO has* $W_t^I = 0$ *for all* $t \geq 0$.

Lemma 2 collapses the CEO's optimisation problem to the case where she never holds private wealth. Under a direct contract $\mathbb{C}$ and given the CARA utility function in equation (3.2), the presence of wealth W_t^I changes the consumption decision $\tilde{p}_t^* = p_t + rW_t^I - g(x_t, \tilde{a}_t)$ under the no-savings compensation p_t but it does not change the choice of optimal effort a_t^*. This means that, if the CEO is suddenly endowed with wealth, she will simply consume the extra term, i.e. the interest over wealth rW_t^I, but she will still hold the same optimal choice of effort a_t^*. Therefore, I restrict to the simplest case where $W_t^I = 0$.

Proposition 1. *For a contract* $\mathbb{C} = (p, a)$ *to be incentive compatible and no-savings, the manager's continuation value must have the following dynamics:*

$$dM_t^I = -r\gamma\Gamma_t\sigma(x_t)M_t^I dw_t \tag{3.7}$$

where Γ_t *is an adapted process respecting the square-integrability condition* $\mathbb{P}\left[\int_0^t (\Gamma_s)^2\, ds < \infty\right] = 1$ *and is written as:*

$$\Gamma_t = \frac{\partial g(x_t, a_t)}{\partial a_t}\left(\frac{\partial \alpha(x_t, a_t)}{\partial a_t}\right)^{-1}$$

and the shareholder must pay the optimal instantaneous wage:

$$p_t = g(x_t, a_t) + rN_t^I \tag{3.8}$$

Proposition 1 presents the necessary conditions to secure incentive-compatibility between the shareholder and the incumbent, and to prevent changes in wealth W^I (no private savings). Equation (3.7) shows the dynamics of the manager's continuation value that guarantee that her consumption-savings optimisation problem and effort optimisation problem are satisfied.

Furthermore, equation (3.8) shows the optimal managerial compensation that avoids private savings. The first term in the right-hand side of equation (3.8) is the labor cost faced by the incumbent manager and the second term is the interest from the inside fund established by the shareholder (see equation (3.5)). By setting up this inside fund, the shareholder essentially moves the savings process from the hands of the manager to the firm. Consequently, the optimal compensation includes an interest over the fund that removes the incentive for the manager to save privately. In other words, the shareholder saves on behalf of the manager and pays her an interest so that the paid compensation is fully consumed.

3.3.2 Outsider Manager

The outsider manager manages the firm from time T_d^* onwards. Similarly to the incumbent CEO, the outsider CEO has a CARA utility function:

$$v(x_t, \tilde{q}_t) = -\frac{1}{\varphi}e^{-\varphi \tilde{q}_t} \tag{3.9}$$

where $\varphi > 0$ is the risk aversion coefficient and $\tilde{q}_t$ is the outsiders instantaneous consumption.

The outsider CEO has a wealth balance $W^O = \{W_t^O :\geq 0\}$. The outsider CEO's balance grows instantaneously at the rate r, the same savings rate faced by the incumbent CEO.

The outsider's wealth balance is written as:

$$dW_t^O = rW_t^O dt + q_t dt - \tilde{q}_t dt - h(x_t, \tilde{b}_t)dt \tag{3.10}$$

subject to the savings transversality condition $\lim_{T\to+\infty} \mathbb{E}\left[e^{-rT}W_T^O\right] = 0$. In equation (3.10), the term rW_t^O is the interest earned over the existing outsider's wealth, q_t is the wage paid by the shareholder, $\tilde{q}_t$ is the outsider's consumption, and $h(x_t, \tilde{b}_t)$is the outsider's instantaneous labor costs

$$h(x_t, \tilde{b}_t) = x_t \xi \bar{h}(\tilde{b}_t) \tag{3.11}$$

The labour costs represented in equation (3.11) increase with the outsider's inefficiency over her skillset as measured by constant $\xi > 0$, and with the cash-flow level x_t. Furthermore, $\bar{h}(\tilde{b}_t)$ is a function of the exerted effort $\tilde{b}_t$ and has the following properties: $\partial\bar{h}(b_t)/\partial b_t > 0$ and $\partial^2\bar{h}(b_t)/\partial b_t^2 \geq 0$. As in the incumbent case, the outsider does not support effort costs when she shirks: $\bar{h}(x_t, 0) = 0$.

The shareholder and the outsider CEO agree on a contract $\mathbb{D} = (q, b)$ that describes a wage rule $q = \{q_t : t \geq T_d\}$ and a required managerial effort rule $b = \{b_t : t \geq T_d\}$. Parties initiate the contract at time T_d, that is, the dismissal time of the incumbent CEO which coincides with the outsider's hiring time.

The firm sets up an insider fund for the outsider CEO when she starts working at the dismissal time. I write the outsider CEO's continuation value function process in its certainty equivalent form:

$$N_t^O = -\frac{1}{r\varphi}\ln(-r\varphi M_t^O) \tag{3.12}$$

Here, $M_t^O \equiv M_t^O(\mathbb{D}, x_t)$ is the incumbent CEO's continuation value given the recommended effort b and the wage rule q. The shareholder adjusts the level of the insider fund with changes in the cash-flows, the fund accrues at rate r and, at the dismissal time, the shareholder transfers the insider fund out of the balance sheet to the outsider manager.

I extend Assumption 3 to the outsider CEO case to rule out outside employment options, and impose the following assumption for tractability reasons:

Assumption 5. *The outsider CEO is never replaced.*

Analogously to the incumbent case, I restrict to a contract $\mathbb{D}$ that is incentive-compatible and no-savings: the incumbent CEO always finds optimal to exert the recommended managerial effort ($\tilde{b} = b$) and to spend all the resources available ($dW_t^O = 0$).

Optimisation Problem 2 (Outsider Manager)**.** *The optimal continuation value of the Outsider CEO given a contract $\mathbb{D}$ solves the following problem:*

$$\begin{aligned} M_t^O(\mathbb{D}, x_t) = \max_{\tilde{q},\tilde{b}} \quad & \mathbb{E}_t\left[\int_t^{+\infty} -\frac{1}{\varphi} e^{-r(s-t)-\varphi\tilde{q}_s} ds\right] \\ s.t. \quad & dW_t^O = rW_t^O dt + q_t dt - \tilde{q}_t dt - h(x_t, \tilde{b}_t) dt \\ & dx_t = \delta(x_t)\, dt + \beta(\tilde{b}_t, x_t)\, dt + \sigma(x_t)\, dw_t, \qquad t \in [T, +\infty) \end{aligned} \tag{3.13}$$

$M_t^O(\mathbb{D}, x_t)$ is the outsider CEO's continuation value given recommended effort b and wage rule q. Henceforth, I write $M_t^O \equiv M_t^O(\mathbb{D}, x_t)$. After being hired, the outsider CEO maximises her future value by selecting an optimal consumption $\tilde{q}^*$ and optimal effort $\tilde{b}^*$ subject to the wealth balance dynamics represented in equation (3.10) and the cash-flow diffusion process represented in equation (3.1). Lemma 2 holds for the outsider CEO and so I restrict to the no-wealth case, $W_t^O = 0$.

Proposition 2. *For a contract $\mathbb{D} = (q, b)$ to be incentive compatible and to prevent changes in private wealth, the outsider manager's continuation value must have the following dynamics:*

$$dM_t^O = -r\varphi\Phi_t\sigma(x_t)M_t^O dw_t \tag{3.14}$$

where Φ_t is an adapted process respecting the square-integrability condition $\mathbb{P}\left[\int_0^t (\Phi_s)^2\, ds < \infty\right] = 1$ and is written as:

$$\Phi_t = \frac{\partial h(x_t, b_t)}{\partial b_t}\left(\frac{\partial \beta(x_t, b_t)}{\partial b_t}\right)^{-1}$$

and the shareholder must pay the optimal instantaneous wage:

$$q_t = h(x_t, b_t) + rN_t^O \tag{3.15}$$

Proposition 2 shows the conditions that sustain the incentive-compatible and no-savings contract $\mathbb{D}$ where equation (3.14) represents the dynamics of the outsider's continuation value and equation (3.15) is the optimal managerial compensation. In these equations, b_t is the incentive-compatible effort that I will determine in the shareholder's optimisation problem. The outsider's optimal wage has the same structure of the incumbent's optimal wage presented in equation (3.8). In detail, the outsider's optimal wage equals the labor cost faced by the CEO and and the interest over her continuation value, that is, the interest over the inside fund that removes the incentive of the manager to save privately.

3.3.3 The Shareholder's Decision

The shareholder earns the firm's cash-flows after paying the managerial compensation to the CEO in place. Moreover, during the incumbent manager tenure, the shareholder has a real option to dismiss the incumbent and replace her with the outsider. One of the fundamental aspects of a real option is the irreversibility nature[16]. I use this approach in the context of managerial dismissal because a firm that dismisses a manager that is performing well, will most likely find it hard to rehire her again.

Given the equilibrium contract $\mathbb{C}$, I define the equity value assuming the firm operates in continuity under the incumbent's management as:

$$\begin{aligned} E^I(x_t, M_t^I) = \max_a \quad & \mathbb{E}_t\left[\int_t^{+\infty} e^{r(s-t)}\,(x_s - p_s)\,ds\right] \\ \text{s.t.} \quad & dx_t = \delta(x_t)\,dt + \alpha(a_t, x_t)\,dt + \sigma(x_t)\,dw_t \\ & dM_t^I = -r\gamma\Gamma_t\sigma x_t M_t^I\,dw_t \end{aligned} \tag{3.16}$$

Moreover, I write $E^O(x_t, M_t^O)$ for the equity value when the firm operates in continuity

[16] Dixit and Pindyck (1994) explain that an investment decision is irreversible because, once the investment is exercised, the project can hardly be reverted to its original state. For example, an investment in ore extraction can never revert the mine to its original aspect.

under the outsider's CEO tenure:

$$\begin{aligned} E^O(x_t, M_t^O) = & \max_b \quad \mathbb{E}_t \left[\int_t^{+\infty} e^{r(s-t)} \left(x_s - q_s \right) ds \right] \\ & \text{s.t.} \quad dx_t = \delta(x_t)\, dt + \beta(b_t, x_t)\, dt + \sigma(x_t)\, dw_t \\ & \qquad\quad dM_t^O = -r\varphi\Phi_t\sigma x_t M_t^O dw_t \end{aligned} \tag{3.17}$$

The shareholder has an option to dismiss the incumbent and replace her with the outsider manager. I formally write the optimal dismissal time as T_d^*:

Definition 3.3.1 (Dismissal Time)**.** *The dismissal trigger x_d^* is a cash-flow level such that the optimal dismissal time T_d^* is the first passage time at which a cash-flow x_t crosses x_d^*:*

$$T_d^* = \inf \left\{ t \in [0, +\infty) : x_t \notin (x_d^*, +\infty) \right\} \tag{3.18}$$

As discussed in detail by Peskir and Shiryaev (2006), the optimal dismissal time in equation (3.18) defines T_d^* and introduces x_d^*. T_d^* occurs when x_t crosses the optimal boundary value x_d^*. For consistency of my exposure, I also write $M_d^{I*} \equiv M_{T_d^*}^I$ for the incumbent manager's continuation value and $M_d^{O*} \equiv M_{T_d^*}^O$ for the outsider manager's continuation value at the dismissal time T_d^*.

The shareholder's value for $t \in [0, T_d^*)$ is:

$$S^I(x_t, M_t^I) = E^I(x_t, M_t^I) + P_d^I(x_t) \tag{3.19}$$

This function is the sum of the equity value $E^I(x_t, M_t^I)$ and the real option $P_d^I(x_t)$ to dismiss the incumbent manager.

The shareholder's value function for $t \in [T_d^*, +\infty)$ is:

$$S^O(x_t, M_t^O) = E^O(x_t, M_t^O) \tag{3.20}$$

This function equals the equity value $E^O(x_t, M_t^O)$ assuming the firm operates in continuity under the outsider manager's control.

Optimisation Problem 3 (Shareholder's Decision)**.** *The shareholder's dismissal trigger is obtained by solving the following sub-problems in the presented order.*

1. *Find the specific solution of equity value $E^I(x_t, M_t^I)$ under the incumbent tenure, and the specific solution of equity value $E^O(x_t, M_t^O)$ under the outsider's tenure.*

2. *Find the maximum valued dismissal option $P_d^I(x_t) \equiv P_d^I(x_t|x_d^*)$ by maximising the dismissal option for the dismissal trigger x_d. The optimal dismissal trigger is:*

$$x_d^* \in \arg\max_{x_d} P_d^I(x_t|x_d) \tag{3.21}$$

where the following condition must be respected at the dismissal time T_d^*:

$$S^I(x_d^*, M_d^{I*}) = S^O(x_d^*, M_d^{O*}) - K(M_d^{I*}, M_d^{O*}) \tag{3.22}$$

I solve the shareholder's dismissal decision in two steps. First, I find the specific solution of $E^I(x_t, M_t^I)$, that is, the shareholder's value assuming the firm operates in continuity under the incumbent management. The shareholder selects an optimal incumbent effort rule $a^* = \{a_t^*\}_{t=0}^{\infty}$ that respects the incentive-compatible, no-savings contract $\mathbb{C}$ and Assumption 4. Furthermore, I find the specific solution for function $E^O(x_t, M_d^O)$, that is, the shareholder's value when the firm operates in continuity under the outsider's management. In this case, for $t \geq T_d^*$ and under the incentive-compatible and no-savings contract $\mathbb{D}$, the shareholder selects an optimal outsider effort rule $b^* = \{b_t^*\}_{t=T_d^*}^{\infty}$.

Second, I solve indirectly for the optimal dismissal time T_d^* by finding the trigger cash-flow level x_d^* that maximises the dismissal option while respecting equation (3.22). This condition states that the dismissal time T_d^* is the moment at which the shareholder is indifferent between maintaning the incumbent manager and guarantee an expected value of $S(x_d^*, M_d^{I*})$, or hire the outsider manager and get an expected value $S^O(x_d^*, M_d^{O*})$ after paying costs $K(M_d^{I*}, M_d^{O*})$.

I consider that $K(M_d^{I*}, M_d^{O*})$ is a function that incorporates the flows related with the

replacement of managers. In detail:

$$K(M_d^{I*}, M_d^{O*}) = N_d^{I*} - N_d^{O*} + \overline{K} \tag{3.23}$$

The first two terms in equation (3.23) represent the net changes in the insider fund since the incumbent manager's fund is closed and, at the same time, the outsider manager's fund is initiated. A different way of seeing this is thinking about CEO's pension funds as transfers, that is, the funds can be transferred from firm to firm as managers move on to new jobs. The last term of the dismissal costs is a lump sum constant $\overline{K}$ corresponding to replacement costs such as searching and legal costs. As described by Taylor (2010), it is common for a firm to endure searching, entrenchment and legal costs of firing an executive.

3.4 Solution with Mean-Reverting Cash-Flow Growth and Quadratic Effort Costs

3.4.1 Application Structure

In this section, I use a tractable structure to find an analytical solution for the optimisation problem 3. To achieve this, first, I write the cash-flow diffusion process in equation (3.1) as a mean reverting process[17]. That is:

$$dx_t = (\Delta - \kappa x_t)dt + (a_t x_t + \overline{\alpha})\mathbb{I}_{t<T_d^*}dt + (b_t x_t + \overline{\beta})\mathbb{I}_{t\geq T_d^*}dt + \sigma\sqrt{x_t}dw_t \tag{3.24}$$

The first term in the right-hand side of equation (3.24) is the expected change in the level of cash-flows independent of managerial action, i.e. $\delta(x_t) = \Delta - \kappa x_t/$ Here, $\Delta > 0$ is a continuous constant increment in revenue and $\kappa > 0$ is a cost increment rate over x_t. Without managerial guidance and without exogenous shocks, the firm will suffer from decreasing cash-flows if cash-flows become sufficiently high ($x_t > \Delta/\kappa$). The second term is the expected change in cash-flows during the incumbent's tenure. This term increases by $\alpha(a_t, x_t) = a_t x_t + \overline{\alpha}$, where the first term is the variable contribution of skillset α that

[17] Mean reverting processes have been used in previous literature. See, for example, Dixit and Pindyck (1994) and the Appendix of He (2009).

depends on the incumbent's effort and on the cash-flows level while the second term is the fixed contribution of skillset α. The third and last term is the expected change in cash-flows during the outsider's tenure. Here, the instantaneous cash-flow increases by $\beta(b_t, x_t) = b_t x_t + \overline{\beta}$ where the first term is the variable contribution of skillset β related to effort, and the second term is the fixed contribution of skillset β related with her expertise.

Second, I write the incumbent manager's effort cost function as $\bar{g}(a_t) := {a_t}^2/2$ resulting in instantaneous labor cost function:

$$g(x_t, a_t) = \frac{\theta {a_t}^2}{2} x_t \tag{3.25}$$

Finally, I write the outsider CEO's effort cost function as $\bar{h}(b_t) := {b_t}^2/2$ for which the outsider's instantaneous labor cost function follows as:

$$h(x_t, b_t) = \frac{\xi {b_t}^2}{2} x_t \tag{3.26}$$

In summary, the skill α is put into practice when the incumbent manager exerts effort a which contributes to an increase in the cash-flow growth of $\overline{\alpha} + a_t x_t$. The skill β is applied when the incumbent is replaced with the outsider manager and the latter exerts effort b, increasing the instantaneous cash-flow growth in $\overline{\beta} + b_t x_t$.

3.4.2 Model Solution

I start by tackling the first step in the optimisation problem 3 and find the specific solutions of equity functions $E^I(x_t, M_t^I)$ and $E^O(x_t, M_t^O)$. The shareholder is concerned about setting the optimal required effort rule a^* under the incentive-compatible and no-savings contracts $\mathbb{C}$ and to choose the optimal time to close the firm.

▷ **Incumbent CEO.** Let me assume the following functional form for the equity value under the incumbent's management

$$E^I(x_t, M_t^I) = A^I(x_t) - N_t^I \tag{3.27}$$

I consider that $E^I(x_t, M_t^I)$ is separable with respect to the cash-flow x_t and the incumbent CEO's continuation value M_t^I. Here, $A^I(x_t)$ is an equation yet to be determined that represents the firm's asset value and that depends strictly on the cash-flows, and N_t^I is the certainty equivalent value of the incumbent manager which, as defined in equation (3.5), only depends on the incumbent CEO's continuation value.

Lemma 3. *The equity value in continuity under the incumbent's management is:*

$$E^I(x_t, M_t^I) = \theta\left(\theta r\sigma^2\gamma + 1\right) a_t^* \left(\frac{\overline{\alpha} + \Delta}{r} + x_t\right) + \frac{1}{r\gamma}\ln(-r\gamma M_t^I) \tag{3.28}$$

where the optimal effort required to the outsider CEO is:

$$a_t^* = \kappa + r - \sqrt{(\kappa + r)^2 - \frac{2}{\theta + \theta^2 r\sigma^2\gamma}} \tag{3.29}$$

The first term on the right-hand side of equation (3.28) is the specific solution of the function $A^I(x_t)$. This function is a positive affine function of the cash-flow level x_t, and it varies positively with the fixed cash-flow growth $\overline{\alpha} + \Delta$ and negatively with the variable cost parameter κ, the effort cost parameter θ, the risk-aversion parameter γ, and the discount rate r.

The firm operates for as long as the value of its assets is nonnegative, that is, $A^I(x_t) \geq 0$. Unlike the usual geometric brownian motion process, this condition can be violated under the process represented in equation (3.24) which admits negative cash-flows. Therefore, I write the closing trigger of the firm as the level x_c^I that fulfils the expression $A^I(x_c^I) = 0$. That closing trigger is:

$$x_c^I = -\frac{\overline{\alpha} + \Delta}{r} \tag{3.30}$$

▷ **Outsider CEO.** Similarly to the incumbent CEO stage, I assume that the equity value function during the outsider CEO's tenure is written as:

$$E^O(x_t, M_t^O) = A^O(x_t) - N_t^O \tag{3.31}$$

The shareholder's value after the dismissal is the difference between the firm's asset value $A^O(x_t)$ under the outsider's management and the insider fund of the outsider manager's N_t^O, where N_t^I is defined in equation (3.12).

Lemma 4. *The equity value in continuity under the outsider CEO's management is:*

$$E^O(x_t, M_t^O) = \xi\left(\xi r\sigma^2\varphi + 1\right) b_t^* \left(\frac{\overline{\beta} + \Delta}{r} + x_t\right) + \frac{1}{r\varphi}\ln(-r\varphi M_t^O) \tag{3.32}$$

where the optimal effort required to the outsider CEO is:

$$b_t^* = \kappa + r - \sqrt{(\kappa + r)^2 - \frac{2}{\xi + \xi^2 r\sigma^2\varphi}} \tag{3.33}$$

As in the incumbent case, the first term on the right-hand side of equation (3.28) is the solution to the asset value $A^O(x_t)$ and it depends positively and linearly on the cash-flow level x_t. The asset value function increases with higher fixed cash-flow growth $\overline{\beta} + \Delta$ and decreases with higher κ, effort cost parameter ξ, the risk-aversion parameter φ, and the discount rate r. To guarantee that the firm operates only with nonnegative asset value, I write the closing trigger of the firm under the as the level x_c^O that fulfils the expression $A^O(x_c^O) = 0$. This closing trigger is:

$$x_c^O = -\frac{\overline{\beta} + \Delta}{r} \tag{3.34}$$

Furthermore, I impose the following technical assumption that serves to guarantee that the solution of $E^I(x_t, M_t^I)$ and $E^O(x_t, M_t^O)$ are well defined in the real numbers:

Assumption 6. *The instantaneous firm cost rate κ is restricted to $\kappa > \kappa_{min}$, where:*

$$\kappa_{min} := \max\left\{\sqrt{\frac{2}{\theta(1 + \theta r\sigma^2\gamma)}} - r, \sqrt{\frac{2}{\xi(1 + \xi r\sigma^2\varphi)}} - r\right\} \tag{3.35}$$

The shareholder decides to dismiss the incumbent by weighting at every $t \in [0, T_D^*]$, the

expected value of $S^I(x_d^*, M_d^{I*})$ against paying dismissal costs $K(M_d^{I*}, M_d^{O*})$ and earning expected value $S^O(x_d^*, M_d^{O*})$ under the outsider tenure. In the current application, the assumption stated in equations (3.27) and (3.31) allows us to separate the equity values in the asset function that is strictly dependent on cash-flows and the insider fund level that is dependent on the manager's continuation value. Therefore, the value matching condition in Equation (3.22) simplifies to:

$$A^I(x_d^*) + P_d^I(x_d^*) = A^O(x_d^*) - \overline{K}$$

The dismissal option only depends on the cash-flow level x_t since all terms related with the manager's continuation values M_t^I and M_t^O disappear at the optimal dismissal trigger x_d^*.

Furthermore, I impose the following Assumption:

Assumption 7. *The cash-flow level x_t tends to the dismissal trigger x_d from below:*

$$\lim_{x_t \uparrow x_d} P_d^I(x_t|x_d) = A^O(x_d) - A^I(x_d) - \overline{K}$$

This assumption restricts to the cases where the cash-flow level x_t approaches the dismissal trigger x_d from bellow. To replicate the Leicester-Pearson phenomenon, it must be the case that the incumbent manager is more likely to be dismissed as cash-flows increase. I will later present and study the relationship between the model parameters that sustain the existence of an optimal trigger x_d^* where the likelihood of dismissal increases with cash-flows.

The following Lemma presents the specific solution for the dismissal option given the mean reverting process represented in equation (3.24) and the effort cost functions (3.25) and (3.26):

Lemma 5. *The optimal dismissal option $P_d^I(x_t) \equiv P_d^I(x_t|x_d^*)$ is:*

$$P_d^I(x_t) = C_d^I(x_d^*) \frac{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_t\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d^*\right)} \tag{3.36}$$

where constant $C_d^I(x_d^*)$ is the defined as:

$$C_d^I(x_d^*) = b_t^*\xi\left(\xi r\sigma^2\varphi + 1\right)\left(\frac{\overline{\beta}+\Delta}{r} + x_d^*\right) - a_t^*\theta\left(\gamma\theta r\sigma^2 + 1\right)\left(\frac{\overline{\alpha}+\Delta}{r} + x_d^*\right) - \overline{K} \quad (3.37)$$

and the optimal dismissal trigger x_d^* respects the following condition:

$$b_t^*\xi\left(\xi r\sigma^2\varphi + 1\right) - a_t^*\theta\left(\gamma\theta r\sigma^2 + 1\right) - \frac{rC_d^I(x_d^*)}{\Delta+\overline{\alpha}}\frac{{}_1\mathbf{F}_1\left(1+\frac{r}{\kappa-a_t^*}; 1+\frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa-a_t^*)}{\sigma^2}x_d^*\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa-a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa-a_t^*)}{\sigma^2}x_d^*\right)} = 0 \quad (3.38)$$

The optimal dismissal option represented in equation (3.36) is the product of two terms. The first term is the net replacement value at dismissal $C_d^I(x_d^*) > 0$, a constant that represents the net value earned by the firm at the optimal dismissal time T_d^*, when the cash-flow x_t hits the boundary x_d^*. The second term is the expected discount factor $\mathbb{E}\left[e^{-\rho T_d^*}\right]$ used to discount payoffs earned at the optimal dismissal time T_d^*. Here, ${}_1\mathbf{F}_1(.)$ is the Kummer confluent hypergeometric function. This function is uncommon in real options but it is widely used, for example, in Cox-Ingersoll-Ross processes and in the pricing of Asian options. The optimal dismissal trigger x_d^* is the dismissal trigger that solves the First Order Condition presented in equation (3.38). The optimal trigger does not have a closed form solution and, hence, I will derive and study a numerical solution in the next section.

Proposition 3. *The incumbent manager faces increasing likelihood of dismissal with higher cash-flows if and only if*

$$b_t^*\xi\left(\xi r\sigma^2\varphi + 1\right) - a_t^*\theta\left(\gamma\theta r\sigma^2 + 1\right) > 0 \quad (3.39)$$

Proposition 3 identifies the necessary condition between the model parameters that guarantee the existence of an optimal dismissal trigger x_d^* in the context of the Leicester-Pearson phenomenon, that is, increasing cash-flows lead to dismissal. In other words, condition (3.39) is necessary for an incumbent manager to be more likely to be fired as cash-flows increase. Taking limits of this expression with respect to its parameters, I find that this condition is violated for a sufficiently high outsider effort cost ξ, high outsider risk

aversion φ, low incumbent effort cost θ, and low incumbent risk aversion γ.

Differently, firm characteristics, such as uncertainty σ and firm cost rate κ, do not cause a direct violation of this condition but they facilitate it when combined with other parameters. That is, if the cash-flow volatility σ and the firm cost rate κ are sufficiently high, the left-hand side of equation (3.39) approximates zero asymptotically, limiting the range by which the managers' characteristics can change. For example, consider the existence of a firm 1 and a firm 2, where firm 1 has lower costs ($\kappa_1 < \kappa_2$) and lower cash-flow uncertainty ($\sigma_1 < \sigma_2$) than firm 2. Assume these firms have incumbent managers with the same attributes of risk aversion ($\gamma_1 = \gamma_2$) and effort costs ($\theta_1 = \theta_2$) and that they face the same outsider candidate. In this case, if the incumbent of firm 2 is exposed to the Leicester-Pearson phenomenon, then the incumbent of firm 1 is also exposed to potential dismissal with increased performance. However, the opposite relationship is not always true.

Furthermore, I define the breakeven trigger $\overline{x}$ as:

$$\overline{x} = \frac{b_t^* \xi(\overline{\beta} + \Delta)\left(\xi r\sigma^2\varphi + 1\right) - a_t^* \theta(\overline{\alpha} + \Delta)\left(\gamma\theta r\sigma^2 + 1\right) - \overline{K}r}{r\left(a_t^*\theta\left(\gamma\theta r\sigma^2 + 1\right) - b_t^*\xi\left(\xi r\sigma^2\varphi + 1\right)\right)} \tag{3.40}$$

The breakeven trigger is the level of cash-flow x_t that equalises the value of assets under the incumbent management $A^I(\overline{x})$ to the value of assets under the outsider management net of replacement costs $A^O(\overline{x}) - \overline{K}$. When the cash-flow x_t surpasses the breakeven trigger, the dismissal decision is profitable. In our model, if the Leicester-Pearson phenomenon is present, the optimal dismissal trigger and the breakeven trigger have the following relationship:

Proposition 4. *The shareholder may only dismiss the manager after achieving the breakeven trigger, i.e.* $x_d^* > \overline{x}$.

Proposition 4 shows that the incumbent manager is never dismissed until the decision is monetarily profitable. In the next section, I show how the difference between the dismissal trigger and the breakeven trigger depends on the parameters of the model [18].

[18] The gap between the breakeven trigger and the dismissal trigger is also called hysteresis region. This is related with the concept of economic hysteresis defined by Dixit (1992). In detail, economic hysteresis occurs if active investment projects are not reverted immediately when the underlying causes that originated the investment dissipate. That is, divestment is economically profitable but does not occur immediately. My case is similar because the shareholder does not fire the incumbent immediately when the economic conditions that justify having her in place disappear.

Assumption 8. *The cost of replacing the incumbent is restricted to* $\overline{K} > \overline{K}_{min}$*, where*

$$\overline{K}_{min} := \frac{\overline{\beta} - \overline{\alpha}}{r} b_t^* \xi \left(\xi r \sigma^2 \varphi + 1 \right) \tag{3.41}$$

This assumption excludes the trivial cases where $\overline{x} < x_c^I$ in which the outsider is always more profitable than the incumbent manager. Without this assumption, the shareholder could be better off by hiring the outsider at the initial time $t = 0$, for any $x_0 \in [x_c^I, x_d^*)$ and avoid replacement costs.

Note from equation (3.41) that, if the fixed contribution $\overline{\alpha}$ is greater than $\overline{\beta}$, replacement can occur even without replacement costs. In this case, the difference $(\overline{\alpha} - \overline{\beta})/r$ is the friction that prevents replacement since skillset α contributes more to cash-flows than β in the . Hence, the incumbent can remain in place while cash-flows x_t remain below x_d^* either because she has a higher knowledge contribution than the outsider or because she is entrenched by the replacement costs.

3.5 Numerical Results and Discussion

In this section, I use the values in Table 3.1 to parameterise the model application solved in the previous section. My goal is to study the sensitivity of the optimal dismissal trigger to these parameters in order to understand how the dismissal decision under the Leicester-Pearson phenomenon is affected. Analytical solutions are computationally intensive and hard to interpret and, hence, I choose to parameterise and plot the model in order to analyse the reaction of the optimal dismissal trigger to changes in the parameters. I choose these parameters solely with the purpose of illustrating the mechanics of the game rather than providing an actual real world case study. The ranges of parameters are selected to be as broad as possible but constrained at those limits where extreme values lead to unstable solutions.

General Parameters

Parameter	Value	Description
r	0.2	Instantaneous discount rate
x_0	0.5	Initial cash-flow level

Firm Parameters

Parameter	Value	Description
Δ	1.8	Revenue increment independent of managers
κ	2.5	Cost increment rate independent of managers
σ	0.5	Instantaneous standard deviation

Incumbent Manager Parameters

Parameter	Value	Description
γ	5	Risk aversion parameter
θ	0.45	Effort cost parameter
$\overline{\alpha}$	0.2	Fixed revenue associated to skillset α
M_0^I	-0.5	Incumbent Continuation Value at $t = 0$

Outsider Manager Parameters

Parameter	Value	Description
φ	4	Risk aversion parameter
ξ	0.4	Effort cost parameter
$\overline{\beta}$	0.1	Fixed revenue associated to skillset β
M_0^O	-0.5	Outsider Continuation Value at $t = 0$
$\overline{K}$	0.01	Replacement costs

Table 3.1: Input parameters.

3.5.1 Dismissal Decision

The incumbent manager is hired at time $t = 0$ to use her skill α to improve the firm's cash-flows. She faces a risk-aversion level $\gamma = 5$ and an effort cost rate $\theta = 0.45$. The shareholder has a real option to dismiss the incumbent and replace her with an outsider. The outsider faces lower risk-aversion and effort cost rate than the incumbent. She is an expert in skill β, has a risk-aversion level of $\varphi = 4$, and an effort cost rate $\xi = 0.4$.

Players discount flows at a common discount rate $r = 0.2$, cash-flows have uncertainty $\sigma = 0.5$, and the firm begins operations with an initial cash-flow level of $x_0 = 0.5$. The growth of cash-flows has a component that is independent from managerial intervention[19], that is, cash-flows increase continuously with $\Delta = 1.8$ and decrease with $\kappa x_t = 2.5 x_t$.

The growth of cash-flows is also affected by the managerial skill in place. During the incumbent's tenure, the shareholder requires an instantaneous effort $a_t^* = 1.028$ that respects

[19]This occurs, for example, in firms that are monopolistic and have contractual agreements requiring a fixed monthly fee payment from their clients where the contract price increases with inflation.

the optimality condition presented in equation (5.48). Through the effort attribute, the incumbent manager increases the growth of cash-flows in a proportion $1.028x_t$ and, through her knowledge attribute, the incumbent manager generates an additional continuous constant contribution $\overline{\alpha} = 0.2$.

The incumbent manager's tenure ends when the level of cash-flows x_t crosses the optimal dismissal trigger $x_d^* = 2.93$. At that moment, the shareholder pays replacement costs $\overline{K} = 0.01$ and substitutes the incumbent with the outsider manager, effectively replacing managerial skill α with skill β. Now, with the outsider in place, the shareholder requires an instantaneous optimal effort $b_t^* = 1.23$ presented in equation (5.57). He expects the growth of cash-flows to increase by $b_t^* x_t = 1.23x_t$ as a consequence of the outsider's effort attribute and to increase by $\overline{\beta} = 0.1$ due to her knowledge contribute.

The input parameters in Table 3.1 guarantee, by Proposition 3, the existence of a Leicester-Pearson phenomenon, that is, the incumbent manager is more likely to face dismissal if cash-flows x_t increase towards the optimal dismissal trigger[20]. In this model, this phenomenon arises because the incumbent is more competitive in the knowledge attribute and the outsider is more competitive in the effort attribute.

In detail, the incumbent is more competitive in the knowledge attribute and so, she delivers a higher fixed contribution to the growth of cash-flows ($\overline{\alpha} > \overline{\beta}$)[21]. The fixed contribution does not escalate with the level of cash-flows which is useful when the firm is small and the cash-flows are low as the manager is able to provide a certain contribution to the growth of cash-flows despite the level of cash-flows being low.

However, managerial knowledge is harder to spread throughout a firm that grows larger. In this case, the variable contribution becomes more relevant than the fixed contribution for the growth of cash-flows. The shareholder then considers replacing managers given that the outsider's lower effort cost and risk aversion makes her more competitive in the effort attribute. The firm can then benefit from a higher optimal effort ($b_t^* > a_t^*$) that is

[20]The phenomenon does not occur in the cases where the parameterisation does not respect Proposition 3. I ignore these cases since they are not relevant for this paper.

[21]In this parameterised example, the friction that sustains the incumbent in place for lower values of cash-flows is the comparative advantage in the knowledge attribute. The same could have been achieved if I considered very high replacement costs that entrench the incumbent manager. As discussed in the previous section, high replacement cost can generate a scenario where the incumbent is in place and has comparative disadvantage in both attributes.

particularly advantageous when cash-flows are high, since effort escalates with cash-flows.

Moreover, in Figure 3.1 and in accordance with Proposition 4, we see that the dismissal trigger is higher than the breakeven trigger $\overline{x} = 2.79$. This means that, after reaching this indifference trigger $\overline{x}$, the shareholder chooses to hold the incumbent for an additional time until the cash-flow level x_t crosses x_d^*. This is a usual feature in real options model that, in this case, reflect the irreversibility aspect of the contractual relationship between the firm and the incumbent manager joint with the effect of uncertainty over the dismissal decision. Dismissing the incumbent while she is performing well is an irreversible decision since it is hard to rehire that manager in the future under the same contractual agreement. At the same time, cash-flows are exposed to uncertainty shocks that can lower the current cash-flow level. Therefore, in order for the shareholder to be willing to dismiss the incumbent, it must be the case that cash-flows are very high, to the point that hiring the outsider is sufficiently advantageous against maintaining the incumbent.

Figure 3.1 presents the three key regions of this model. The first region occurs when cash-flows are extremely low, $x_t < \overline{x}$. In this case, the shareholder prefers the incumbent manager because the fixed contribution of skillset α relative to β more than compensates the comparative disadvantage this manager has in terms of effort induction. The second region occurs when cash-flows are moderately low $x_t \in [\overline{x}, x_d^*)$. In this case, the incumbent's comparative advantage dissipates since the firm benefits more from the variable contribution of managerial effort than from the fixed contribution of knowledge. However, the shareholder will not dismiss the incumbent in this region since the cash-flow x_t is not sufficiently high and, hence, a potential decrease in cash-flows could reposition again the firm in region 1. The shareholder dismisses the incumbent when the firm enters the third region, that is, $x_t \geq x_d^*$. In this region the outsider is not only monetarily advantageous but also the level of cash-flows is sufficiently high to ensure, in expectation, that the shareholder does not see the firm cash-flows decreasing to the point where the dismissal decision becomes a bad decision ex-post.

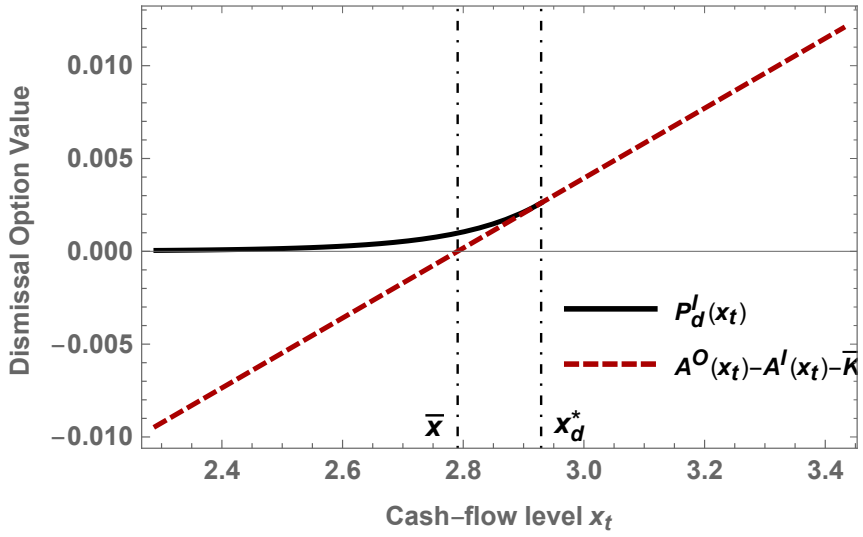


Figure 3.1: This plot is parameterised using the data in Table 3.1. The black bold line is the value of the dismissal option $P_d^I(x_t)$ as a function of cash-flow x_t, as defined in equation (3.36). The dashed diagonal line is the difference between the asset value under the outsider, the asset value under the incumbent and the replacement costs.

3.5.2 Comparative Statics

To study the effects of different parameters over the optimal dismissal trigger x_d^*, I parameterise the model application with values from Table 3.1, except for the parameter under analysis for which I define a range of values.

▷ **Effect of uncertainty σ.** Uncertainty has two key effects over the equilibrium solution. First, the optimal dismissal trigger x_d^* has a non-monotonic relationship with uncertainty (Subfigure 3.2.a). Second, the wedge between the optimal dismissal trigger and the breakeven trigger increases with uncertainty (Subfigure 3.2.b).

The first result is a consequence of the effect of uncertainty over the net replacement value $C_d^I(x_t)$ that affects the calculation of the optimal dismissal trigger x_d^* and of the breakeven trigger $\overline{x}$. The second result occurs because changes in uncertainty also affect the expected discount factor $\mathbb{E}\left[e^{-rT_d}\right]$ which consequently affects the optimal dismissal trigger x_d^* without changing the breakeven trigger $\overline{x}$.

If cash-flows have low uncertainty σ, the optimal dismissal trigger and the breakeven trigger vary negatively with uncertainty but, if cash-flows have high uncertainty σ, they

vary positively. This non-monotonic relationship is caused by the difference between the effect of uncertainty over the optimal incumbent effort a_t^* and the effect over the optimal outsider effort b_t^*. This is more discernible in Subfigure 3.2.e which shows the difference of optimal efforts, and in Subfigure 3.2.c which plots the condition for the existence of a Leicester-Pearson phenomenon (equation (3.39)).

The incumbent optimal effort a_t^* and the outsider optimal effort b_t^* decrease with higher uncertainty ($\partial a_t^*/\partial\sigma < 0$ and $\partial b_t^*/\partial\sigma < 0$) because riskier cash-flows increase the costs of incentive alignment and, hence, force the shareholder to request a lower optimal effort[22]. In detail, the shareholder has two managerial costs: the first is the manager's cost of exerting effort which is covered by the firm and the second is the cost of securing the incentives alignment. Inspecting equation (5.43) for the incumbent, we see that the effort cost term is $\theta {a_t}^2 x_t/2$ and the cost of incentive alignment term is $r\gamma\theta^2 a_t^2\sigma^2 x_t/2$. For the outsider, in equation (5.52), we see that the shareholder covers the effort cost by paying $\xi b_t^2 x_t/2$ and incurs in further costs of incentive alignment equal to $r\varphi\xi^2 b_t^2\sigma^2 x_t/2$. It is clear that more uncertainty σ results in higher costs of incentive alignment for both managers which, consequently, lower the optimal efforts a_t^* and b_t^*.

However, the optimal efforts decrease at different rates and that causes the non-monotonic behaviour of x_d^*. Subfigure 3.2.f shows that the optimal incumbent effort decreases faster than the optimal outsider effort if uncertainty is low. That is because, for low levels of uncertainty, the relatively higher effort costs and higher risk aversion of the incumbent manager makes the optimal effort a_t^* very sensitive to uncertainty shocks. However, for extremely high levels of uncertainty σ, this disadvantage becomes irrelevant. This leads to a peak in the difference between the two required efforts (Subfigure 3.2.e) and the described non-monotonic effect over the triggers (Subfigure 3.2.a).

Regarding the second result, the increasing wedge between the optimal dismissal trigger and the breakeven trigger is caused by the impact of uncertainty over the expected discount factor in the dismissal option which is usually observed in canonic real options models (Dixit and Pindyck 1994). More uncertainty increases the shareholder's optimal dismissal trigger

[22] Ultimately, this results in a decrease of managerial compensation paid by the shareholder (See Subfigures 3.2.g and 3.2.h).

x_d^* over the breakeven trigger $\overline{x}$. The shareholder is more cautious when deciding to dismiss the incumbent and selects a higher optimal dismissal trigger so that, when dismissal occurs, cash-flows are so high that they will hardly revert to the unprofitable region.

▷ **Effect of discount rate r.** Higher discount rate r decreases the value of the dismissal option $P_d^I(x_t)$ (Subfigure 3.3.d) and causes an increase in the optimal dismissal trigger x_d^* (Subfigure 3.3.a). The discount rate causes an increase in the optimal dismissal trigger via its effect over the present value of the managers' knowledge contributions and via its effect over the costs of managerial incentive alignment.

First, observing the net replacement value at dismissal $C_d^I(x_d^*)$, we see that the present value of the incumbent's knowledge contribution $\overline{\alpha}/r$ and of the outsider's knowledge contribution $\overline{\beta}/r$ decrease with higher discount rate r. That is, the shareholder becomes more impatient when waiting for these future flows. Since we have $\overline{\alpha} > \overline{\beta}$ then, all else equal, the net replacement value increases because we have a reduction in the opportunity cost[23] of replacing the incumbent. In other words, the incumbent's comparative advantage in the knowledge attribute becomes less relevant for the dismissal decision as the discount rate increases. Note that this model admits $\overline{\alpha} < \overline{\beta}$ for a sufficiently high replacement cost $\overline{K}$ and, in that case, the effect would be opposite: with higher discount rate r, the outsider's comparative advantage in the knowledge attribute would be reduced and that would press the trigger down, all else equal.

Second, the discount rate r affects the incentive alignment problem. A higher discount rate increases the costs of aligning the interests between the shareholder and the manager in place. Consequently, as shown in Subfigure 3.3.f, this forces the shareholder to decrease the optimal efforts a_t^* for the incumbent ($\partial a_t^*/\partial r < 0$) and b_t^* for the outsider candidate ($\partial b_t^*/\partial r < 0$). However, the outsider candidate's optimal effort decreases at a greater rate than the incumbent's optimal effort. In other words, the outsider can exert more effort than the incumbent but, as r increases the gap between b_t^* and a_t^* fades away (Subfigure 3.3.e). So, the incumbent becomes relatively less inefficient in the effort attribute, the net

[23]The opportunity cost related with the knowledge attribute is the loss stop receiving a higher knowledge contribution $\overline{\alpha}$ to receive a lower knowledge contribution $\overline{\beta}$.

replacement value decreases and the shareholder chooses a higher optimal dismissal trigger.

▷ **Effect of revenue increment Δ and cost increment rate κ.** The revenue increment Δ and the cost increment κx_t are the two components of the expected cash-flow growth that are independent from management.

Focusing first on Δ, we observe that a higher revenue increment increases the value of the dismissal option (Subfigure 3.4.d) and decreases the optimal dismissal trigger x_d^* (Subfigure 3.4.a). The effect is processed through the different impact that changes in Δ have over the value of assets $A^I(x_t)$ and $A^O(x_t)$. In detail, the value of assets under the outsider manager $A^O(x_t)$ is more sensitive to Δ than the value of assets under the incumbent manager $A^I(x_t)$. The revenue increment Δ escalates with the expression $b_t^*\xi\left(\xi r\sigma^2\varphi + 1\right)$ which is always larger than $a_t^*\theta\left(\gamma\theta r\sigma^2 + 1\right)$ by Proposition 3 and given the parameter set in Table 3.1. Consequently, the net replacement value at dismissal $C_d^I(x_d)$ increases with Δ, increasing the value of the dismissal option and decreasing the optimal dismissal trigger. Note that, unlike the previous parameters, changes in Δ do not affect the levels of optimal effort a_t^* and b_t^* (see Subfigures 3.4.e and 3.4.f).

The dismissal option value decreases with higher cost increment parameter κ (Subfigure 3.5.d) which leads to an increase in the optimal dismissal trigger x_d^* (Subfigure 3.5.a). The increase in the optimal dismissal trigger is caused by the different sensitivities of the optimal efforts to changes in κ. As seen in Subfigures 3.5.e and 3.5.f, the optimal efforts a_t^* and b_t^* decrease with higher κ. Higher κ increases the cost structure independently of who is managing the firm. Even though the actual managers' effort costs and incentive-alignment costs remain barely unchanged, the leeway to request managerial effort is reduced. The shareholder is then forced to decrease the optimal required efforts. However, the decrease in the outsider's optimal effort b_t^* is stronger than the decrease in the incumbent's optimal effort a_t^* (Subfigure 3.5.f). That is because the outsider is more competitive in the effort attribute and is already being paid more (see Subfigures 3.5.g and Subfigures 3.5.h).

▷ **The effect of Managerial Effort Costs θ and ξ.** The effort cost parameters θ for the incumbent and ξ for the outsider have opposite effects over the optimal dismissal trigger.

Looking first to the incumbent's effort cost parameter θ, we see that the dismissal option value rises if the incumbent effort cost parameter θ increases (Subfigure 3.6.d) which leads to a declining optimal dismissal trigger x_d^* (Subfigure 3.6.a). A jump in θ increases the shareholder's costs of compensating the incumbent: it increases not only the effort cost term $\theta {a_t}^2 x_t/2$ but also the cost of incentive alignment $r\gamma\theta^2 a_t^2\sigma^2 x_t/2$ (see equation (5.43)). The shareholder is then forced to adjust his effort choice by selecting a lower optimal effort a_t^* that ends up lowering the incumbent's final compensation (Subfigures 3.6.g). As a result, the wedge between the outsider's optimal effort and the incumbent's optimal effort (Subfigure 3.6.e) increases and the outsider's comparative advantage over the incumbent with respect to the effort attribute expands (Subfigure 3.6.c). In summary, higher θ increases the net replacement value at dismissal and, consequently, the shareholder has an incentive to dismiss the incumbent sooner (in expectation) so that he can benefit from the comparative advantage of the outsider in the effort attribute.

The value of the dismissal option drops with the outsider's effort cost parameter ξ (Subfigure 3.7.d) leading to an increase in the dismissal trigger x_d^* (Subfigure 3.7.a). Higher ξ translate into higher effort cost $\xi b_t^2 x_t/2$ and higher incentive alignment costs $r\varphi\xi^2 b_t^2\sigma^2 x_t/2$ (as seen in equation (5.52)). The shareholder responds to this increase in managerial costs by lowering his optimal effort level b_t^* (Subfigures 3.7.e and 3.7.f). This erodes the comparative advantage of the outsider in the effort contribution (Subfigure 3.7.c) and decreases the net replacement value at dismissal. The shareholder is then forced to select a higher optimal dismissal trigger to accommodate the loss of profit at dismissal.

▷ **The effect of Managerial Risk Aversion γ and φ.** The dismissal option value increases with the incumbent's risk aversion parameter γ (Subfigure 3.8.d) causing a decrease in the optimal dismissal trigger x_d^* (Subfigure 3.8.a). Increased risk aversion exacerbates the moral hazard problem by increasing the cost of incentive alignment $r\gamma\theta^2 a_t^2\sigma^2 x_t/2$. In other words, the incumbent manager requires a higher compensation to secure the incentive-alignment with the shareholder. The shareholder is forced to decrease the required effort level a_t^* (Subfigure 3.8.e) to face the higher salary costs, decreasing the expected growth rate of cash-flows during the incumbent tenure and the asset value under the incumbent

management $A^I(x_t)$. The comparative advantage of the outsider in the effort attribute increases (Subfigure 3.8.c), the net replacement value increases and, hence, the shareholder decreases his optimal dismissal trigger.

The dismissal option value decreases with the outsider's risk aversion φ (Subfigure 3.10.d) leading to an increase in the optimal dismissal trigger x_d^* (Subfigure 3.10.a). More risk aversion aggravates the moral hazard issue by increasing the shareholder's cost of compensating the risk-averse outsider in order to bear incentives, that is, it increases $r\varphi\xi^2 b_t^2\sigma^2 x_t/2$. The shareholder decreases the outsider's optimal effort b_t^* which reduces the wedge between the outsider's optimal effort b_t^* and the incumbent's optimal effort (Subfigure 3.10.e) and lowers the outsider's comparative advantage over the incumbent (Subfigure 3.10.c). This translates into a lower asset value under the outsider management $A^O(x_t)$ which lowers the net replacement value at dismissal and causes the increase in the optimal dismissal trigger.

▷ **The effect of the managers' fixed contributions $\overline{\alpha}$ and $\overline{\beta}$.** An increase in the incumbent manager's knowledge contribution $\overline{\alpha}$ decreases the dismissal option value (Subfigure 3.10.d) and increases the dismissal trigger x_d^* (Subfigure 3.10.a). If the incumbent can contribute more through his knowledge attribute, she is able to generate a higher continuous expected cash-flow return $\Delta - \kappa x_t + a_t x_t + \overline{\alpha}$ which improves the asset value under the incumbent management $A^I(x_t)$ and reduces the net replacement value at dismissal. The fixed contribution $\overline{\alpha}$ bears no costs for the manager and, hence, it does not affect the shareholder's cost structure. So, the required effort stays unchanged (Subfigures 3.10.e and 3.10.f) and so do the agents' compensations (Subfigures 3.10.g and 3.10.h). In summary, the incumbent becomes more competitive in the knowledge attribute which makes replacement harder to occur in the short-term.

An increase in the outsider's knowledge contribution $\overline{\beta}$ boosts the value of the dismissal option (Subfigure 3.11.d) and decreases the optimal dismissal trigger x_d^* (Subfigure 3.11.a). A higher knowledge contribution increases the expected growth rate of cash-flows during his tenure $\Delta - \kappa x_t + b_t x_t + \overline{\beta}$, and yields a higher asset value under the outsider's management $A^O(x_t)$ which expands the net replacement value at dismissal. Consequently, the shareholder

has an incentive to dismiss the incumbent for a lower dismissal trigger so that he can access the higher asset value $A^O(x_t)$ sooner (in expectation). Fixed contributions are costless for the manager. Therefore, a change in $\overline{\beta}$ bears no effect over the optimal required effort b_t^* (Subfigure 3.11.e and 3.11.f) nor does it change the shareholder's managerial costs (Subfigure 3.11.g and 3.11.h).

3.6 Conclusion

This chapter studies the Leicester-Pearson phenomenon which occurs when managers of a firm that performs exceptionally well are dismissed and replaced with someone else with different skills. I propose an explanation based on how the knowledge and realised effort of managers affect the firm performance.

I develop a principal-agent model in continuous time with moral hazard in which a firm has a risk-averse incumbent manager in place and holds a real option to dismiss the incumbent and replace her with a risk-averse outsider manager. Managers contribute in two ways to the expected growth of cash-flows. First, they have a knowledge attribute that allows them to generate a costless fixed contribution to the cash-flow growth. Second, they have an effort attribute in which they can exert effort and increase the cash-flow growth with a variable contribution that escalates with the current level of cash-flows.

The shareholder has two decisions to make. First, he decides the optimal managerial effort to be required at every moment in time and, second, he decides when to dismiss the incumbent. The dismissal decision is determined by weighting the asset value under the incumbent management against the asset value under the outsider's management deducted of replacement costs. The shareholder is implicitly comparing the fixed and variable contributions of the incumbent manager against the fixed and variable contributions of the outsider candidate throughout time.

I derive the condition that sustains the Leicester-Pearson phenomenon in my model. Implicitly, dismissal is more likely to occur as cash-flows increase if the outsider is more efficient than the incumbent manager in the effort attribute and that is always the case if the outsider has lower risk-aversion and lower effort cost parameters than the incumbent.

The model dynamics are as follows. The incumbent manager stays in place while cash-flows are low either because she is more competitive in the knowledge contribution or because she is entrenched due to high replacement costs. However, as cash-flows increase, replacement is more likely as the shareholder needs instead someone that is more competitive in exerting effort that escalates with the higher cash-flow level.

I provide comparative statics for all the relevant parameters. Under the Leicester-Pearson phenomenon, the dismissal trigger is lower when the agents' discount rate decreases, the share of cash-flows growth independent of managers increase, the incumbent's risk aversion and effort costs increase, the incumbent's fixed contribution decrease, the outsider's risk aversion and costs of effort decrease, and when the outsider's fixed contribution increase. Uncertainty has a non-monotonic effect: for low levels of volatility, a positive shock in uncertainty makes it more likely to dismiss the manager but the effect reverses if volatility is very high.

I suggest three paths of future research. The first is to extend the model to a more realistic financial structure by considering that the firm is levered, faces taxation and the possibility of default on credit. It would be interesting to see how this firm chooses the optimal required effort and the optimal dismissal trigger, and to compare it with the firm presented in the present chapter. The second path is to generalise the frictions of the model by studying the presence of different intensities of moral hazard as in Gryglewicz and Hartman-Glaser (2014), and considering that the shareholder does not have perfect information about the outsider's characteristics. The third path is to solve the model without the assumptions of no-competition and expectation of no-dismissal. These are fairly strong assumptions that avoid the existence of a discontinuity in the manager's continuation value. However, overcoming these assumptions would allow us to pin-down the effects of outside employment options and rational expectations of dismissal simultaneously over the optimal incentive-compatible compensation, the optimal effort level, and over the optimal dismissal decision.

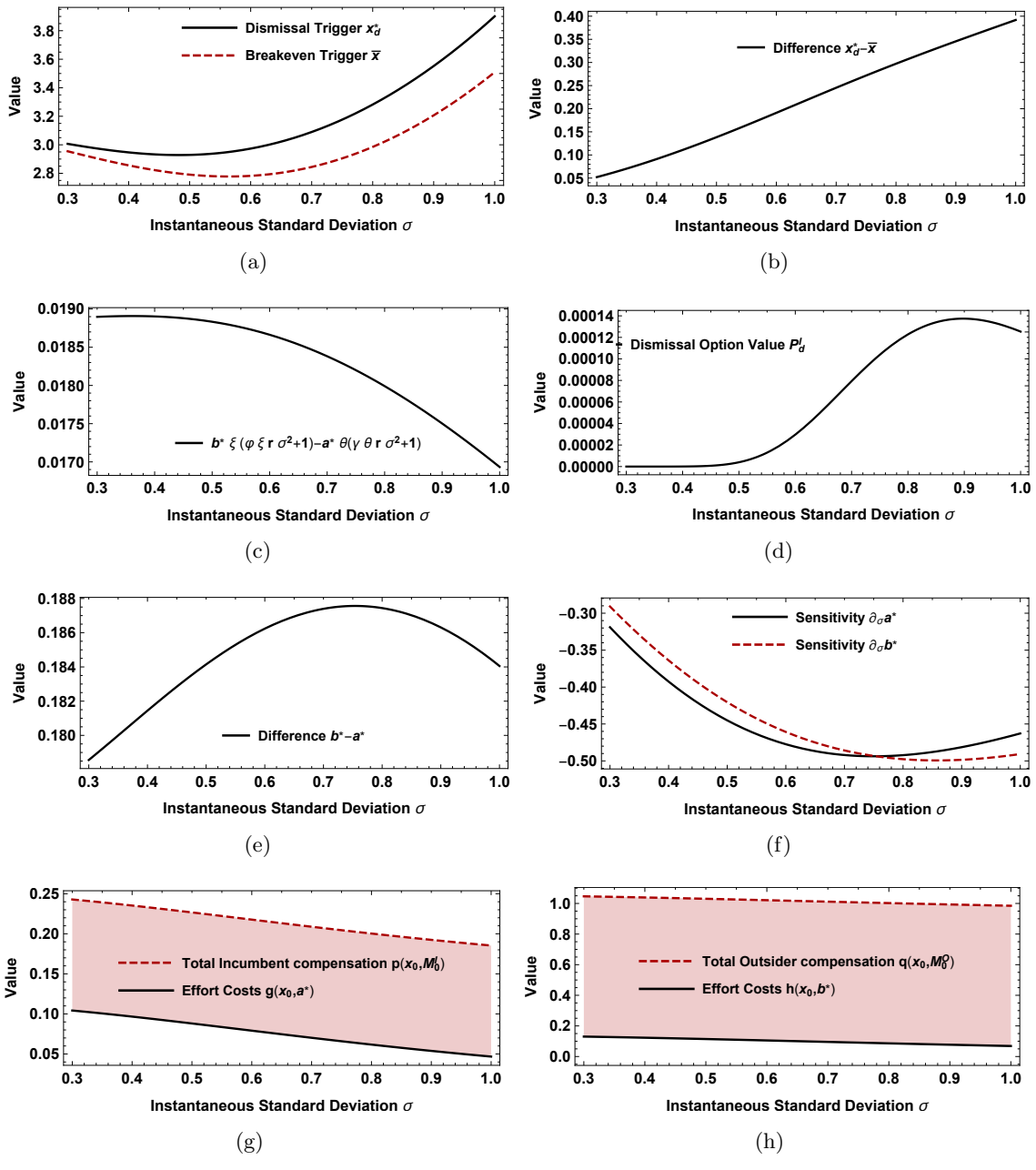


Figure 3.2: Comparative statics for cash-flow uncertainty $\sigma \in [0.3, 1]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

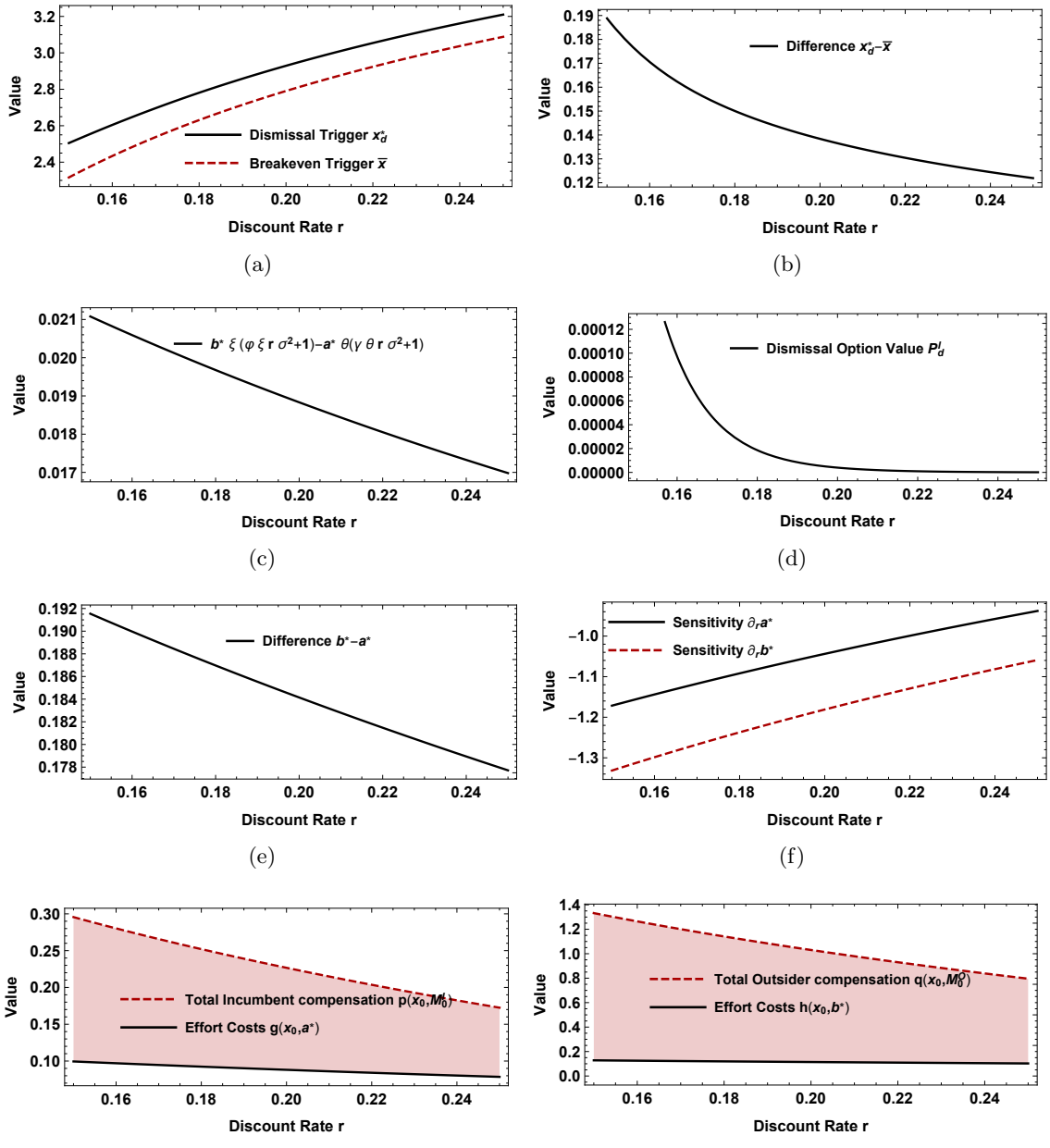


Figure 3.3: Comparative statics for discount rate $r \in [0.15, 25]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

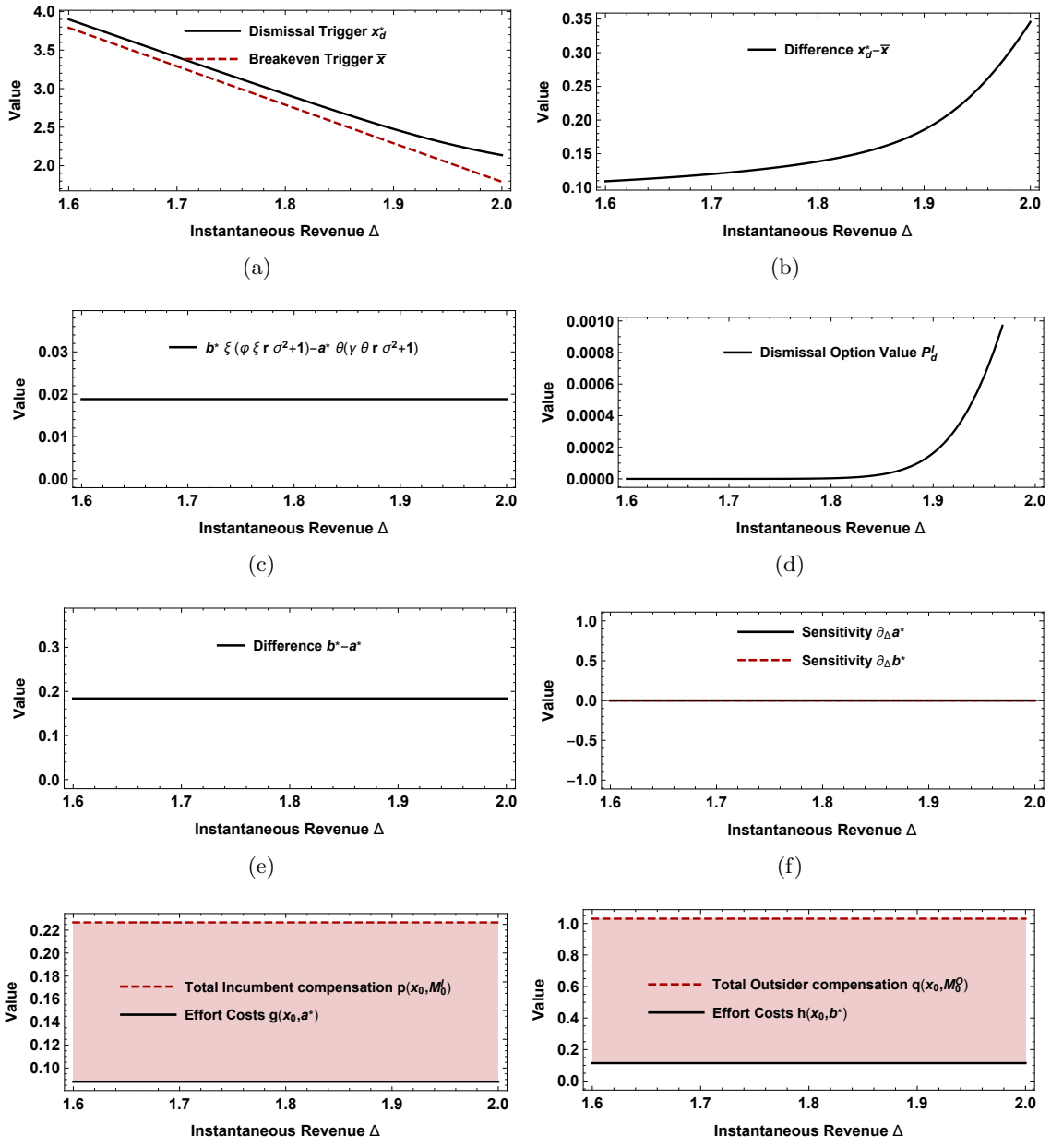


Figure 3.4: Comparative statics for the revenue increment $\Delta \in [1.6, 2.0]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

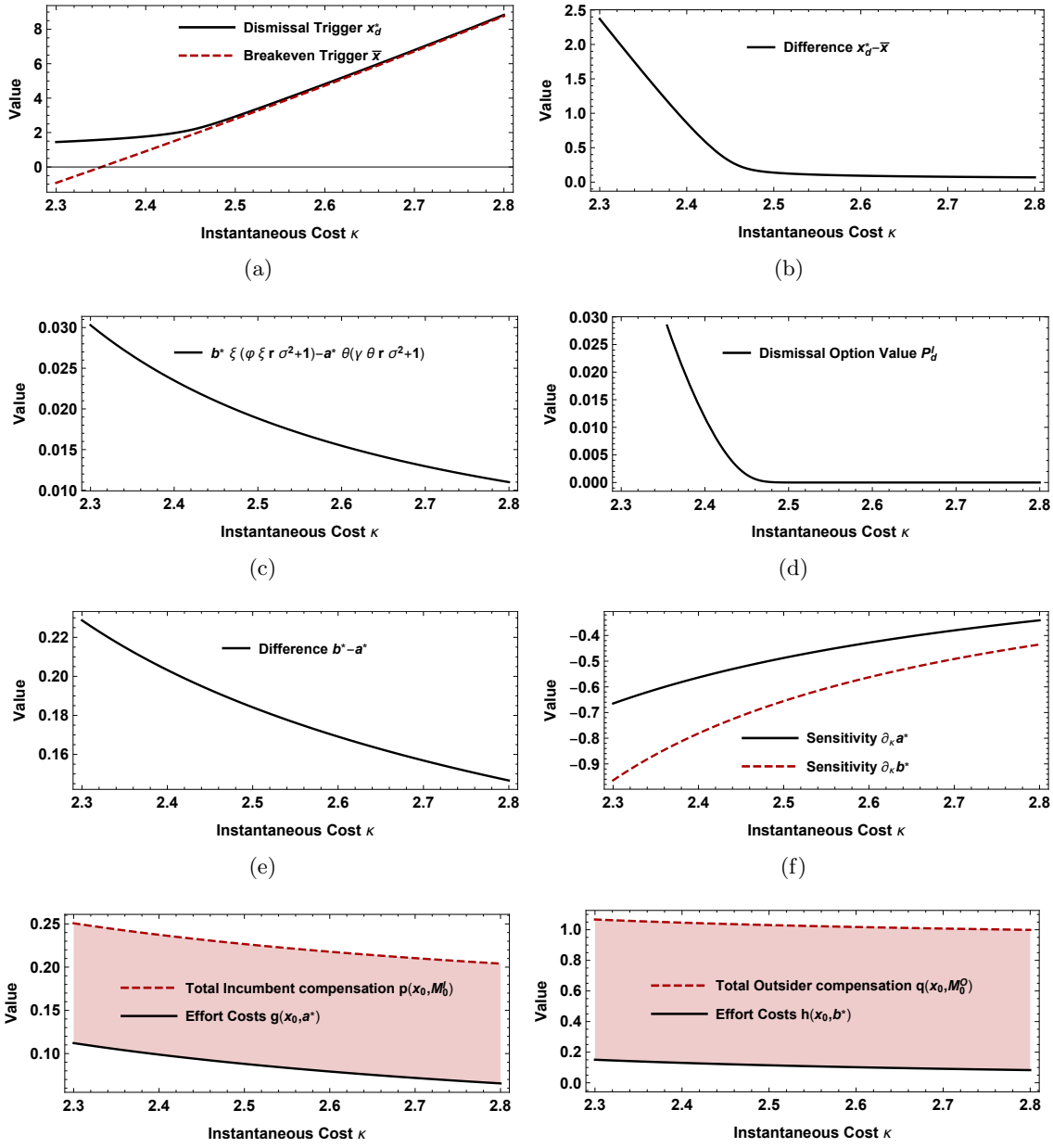


Figure 3.5: Comparative statics for the cost increment rate $\kappa \in [2.3, 2.8]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

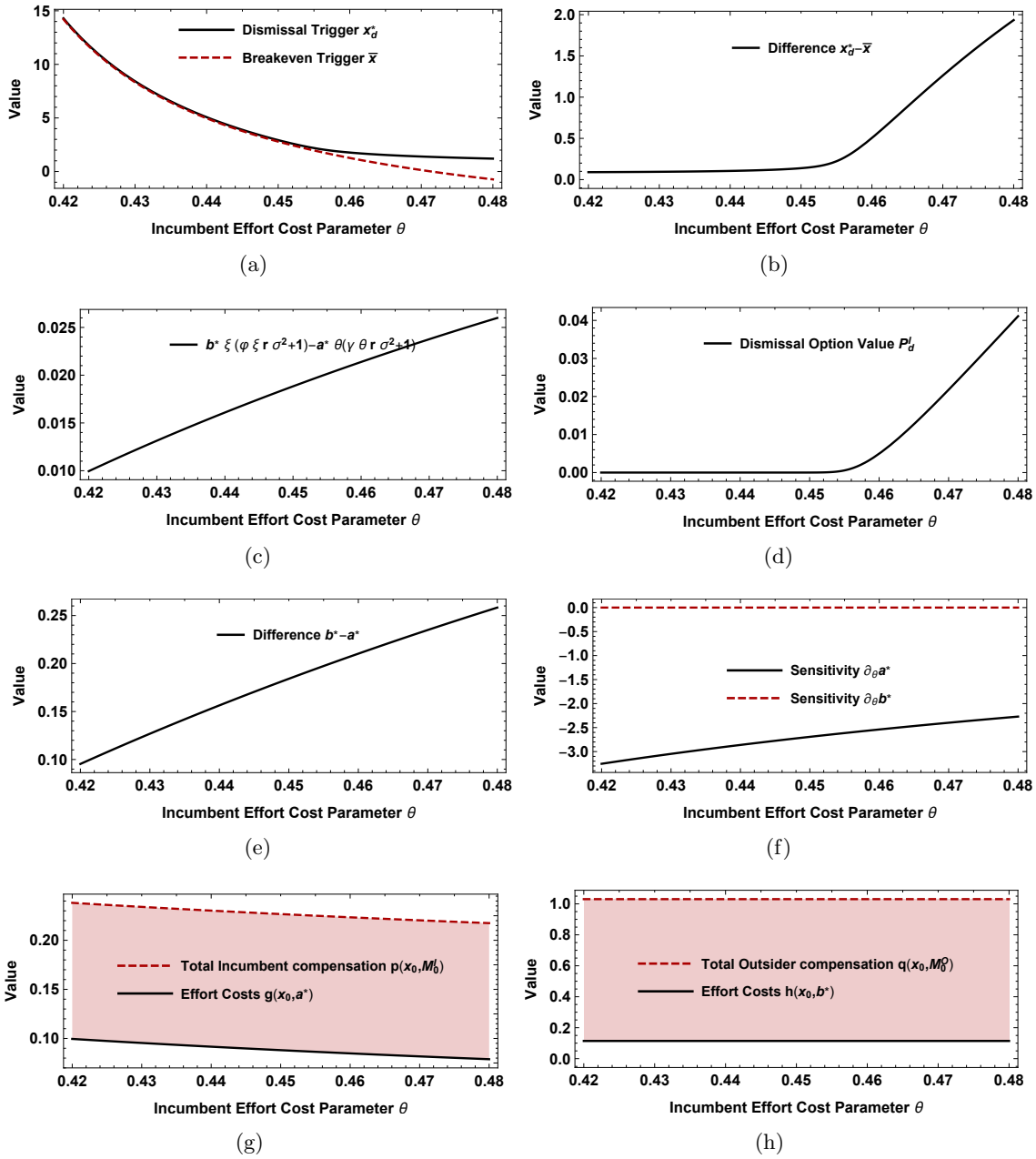


Figure 3.6: Comparative statics for instantaneous cost $\theta \in [0.42, 0.48]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

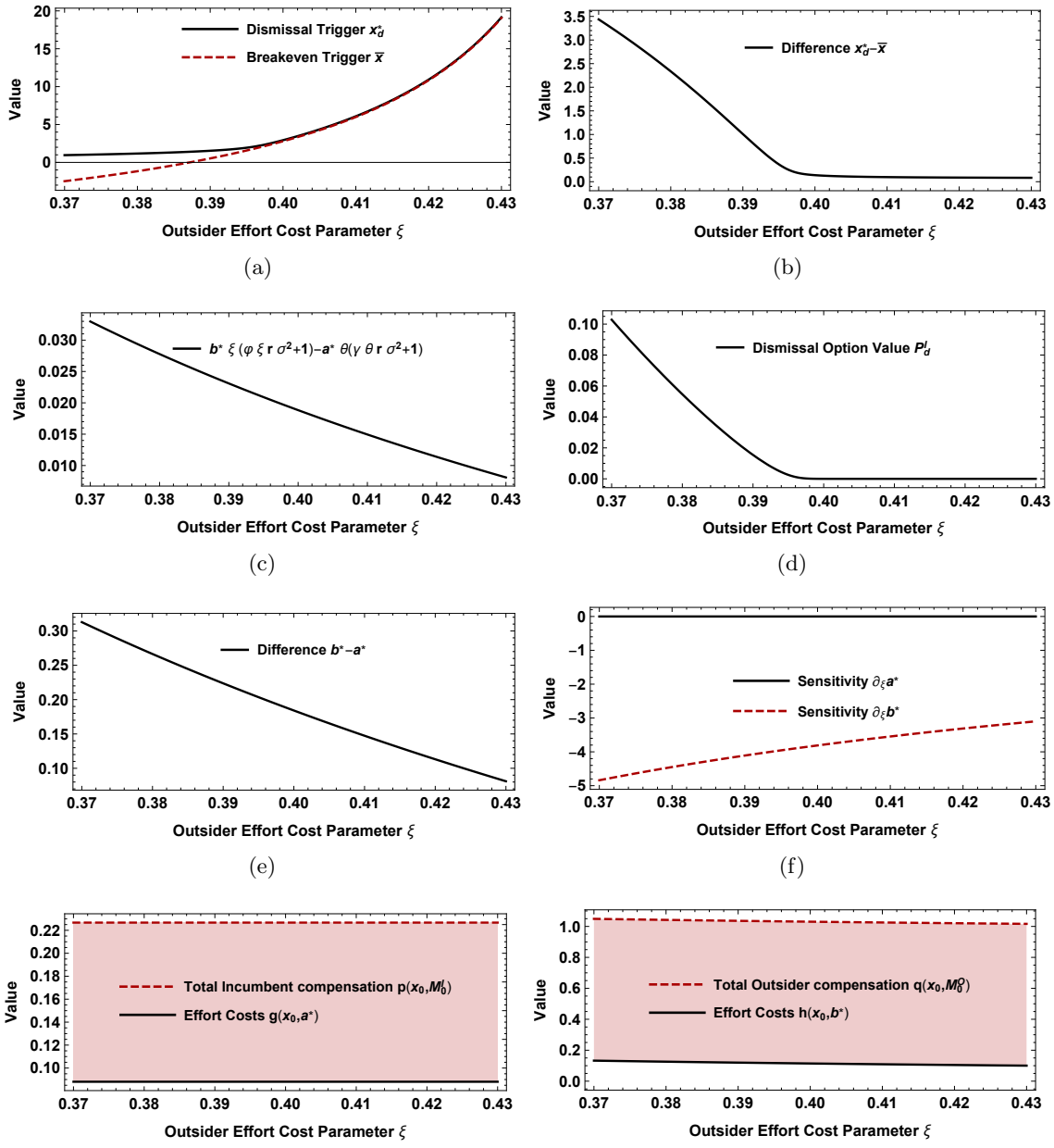


Figure 3.7: Comparative statics for instantaneous cost $\xi \in [0.37, 0.43]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

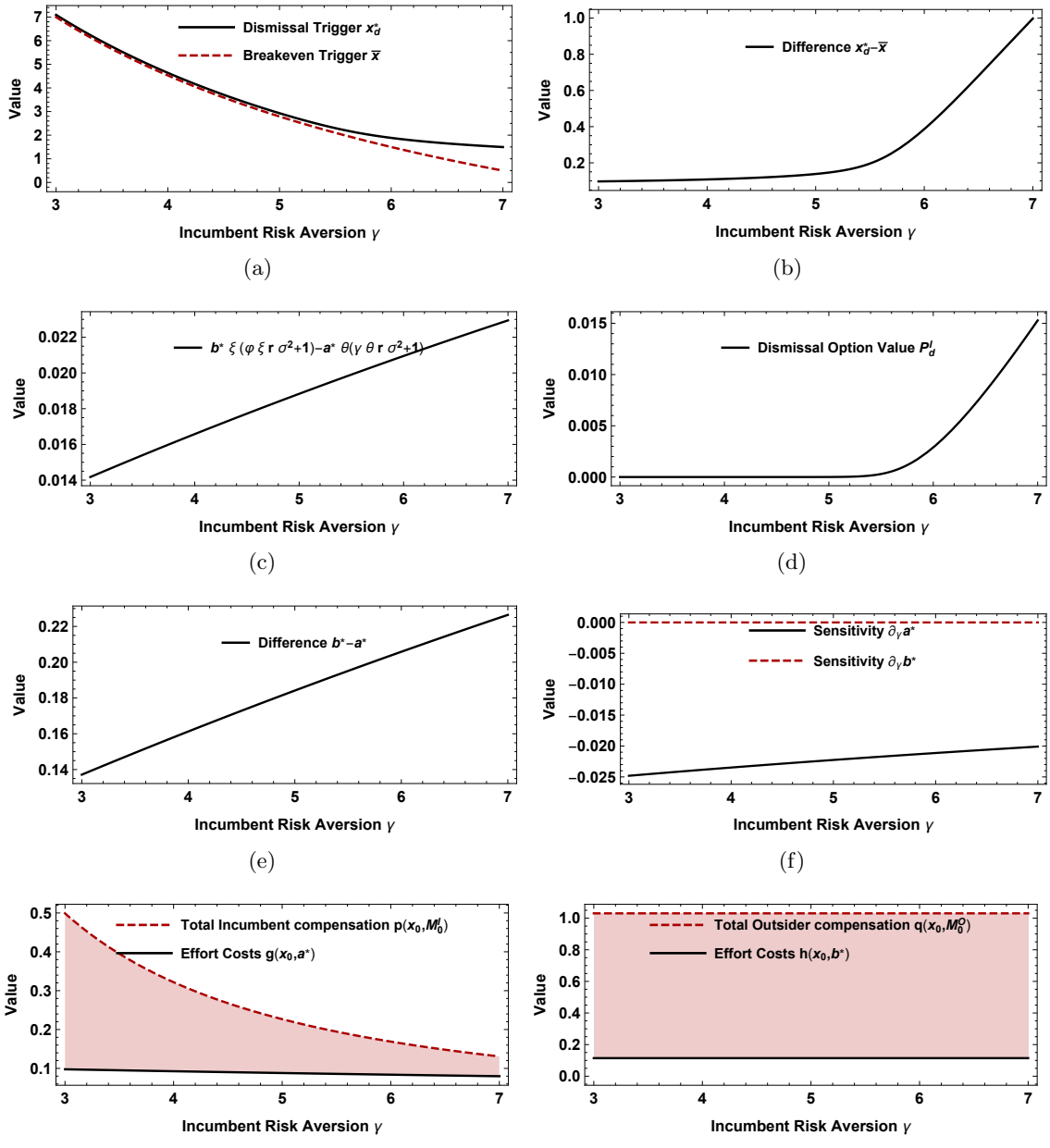


Figure 3.8: Comparative statics for risk aversion $\gamma \in [3, 7]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

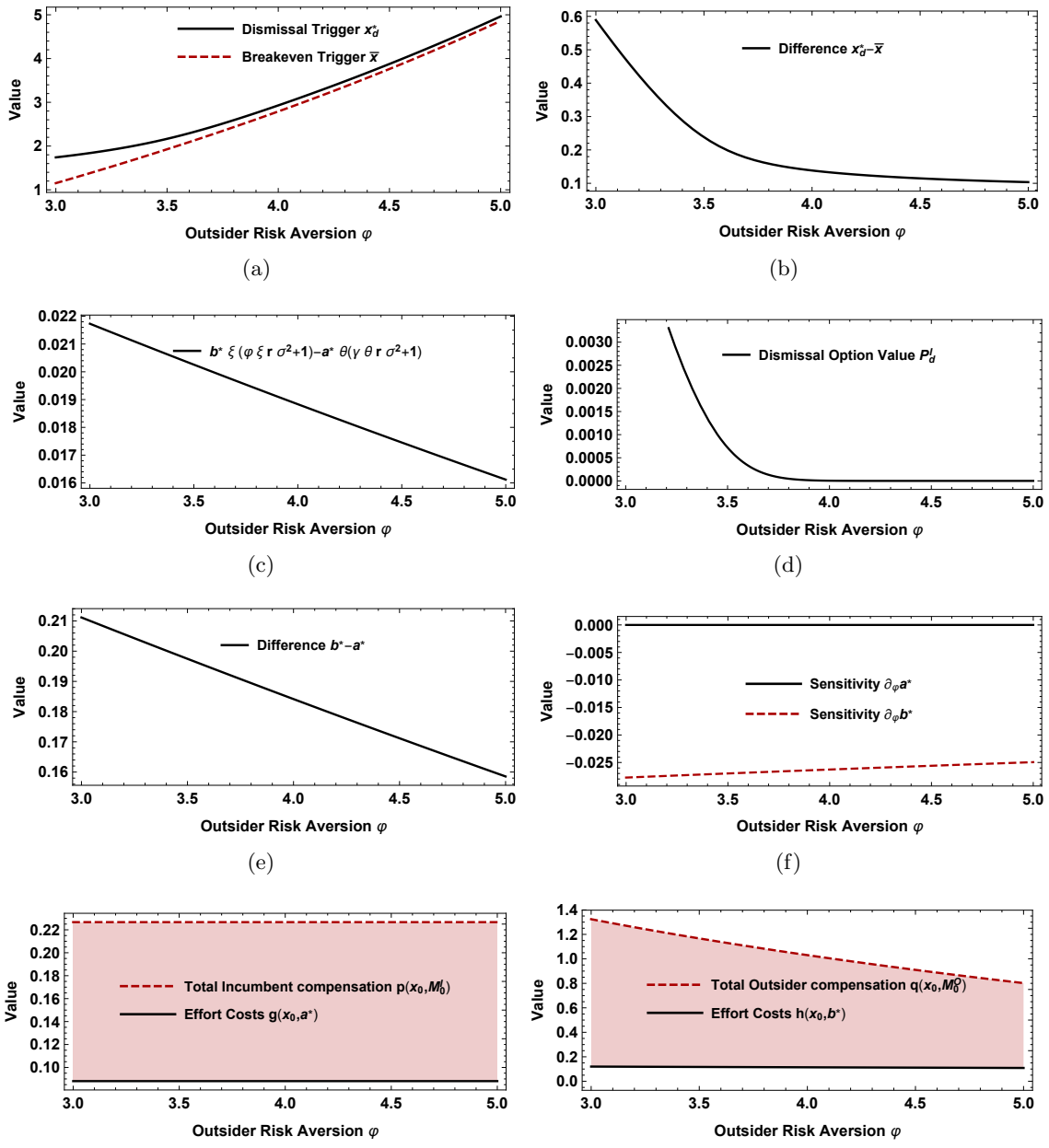


Figure 3.9: Comparative statics for risk aversion $\varphi \in [3, 5]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

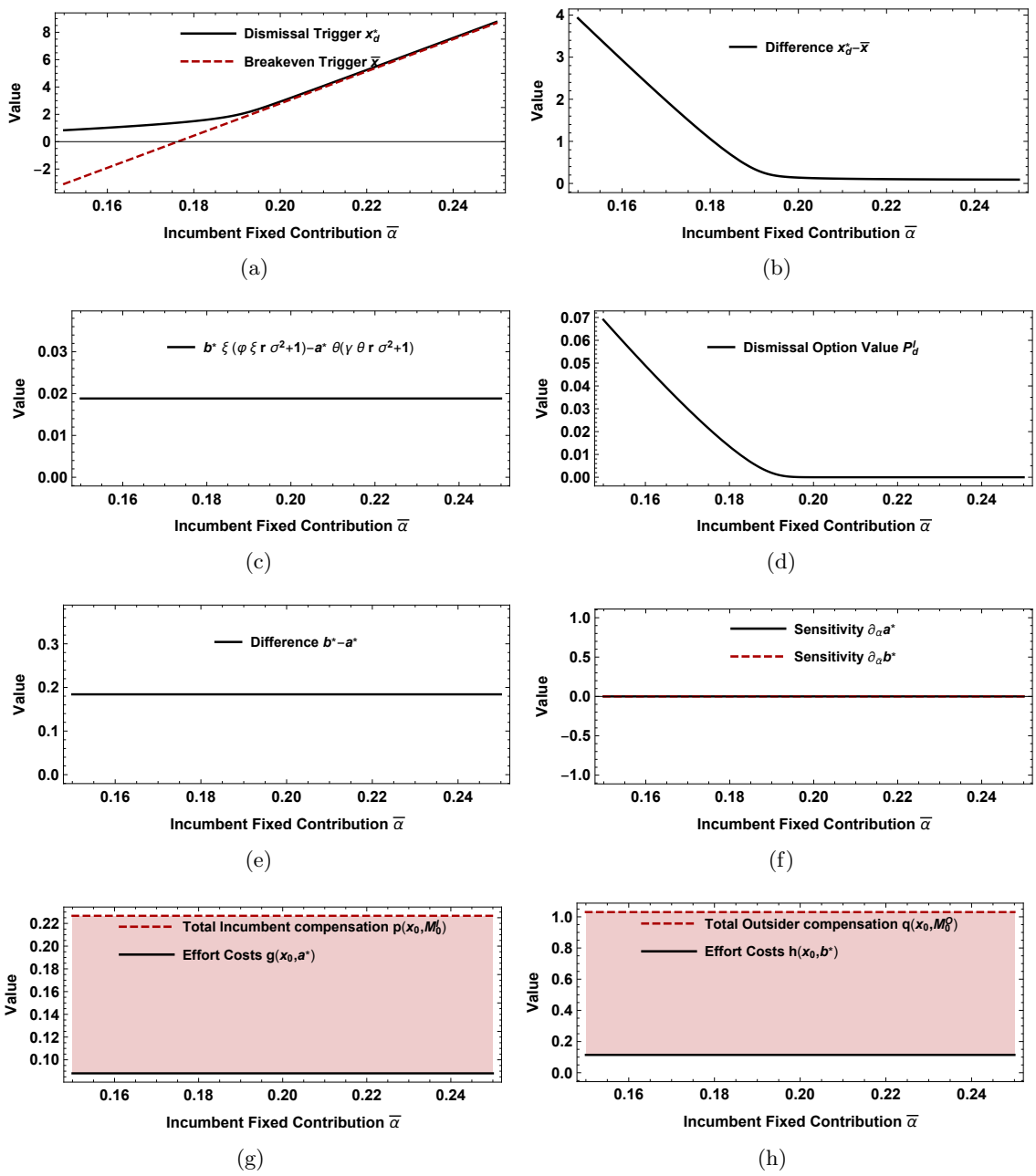


Figure 3.10: Comparative statics for incumbent's fixed contribution $\overline{\alpha} \in [0.15, 0.25]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

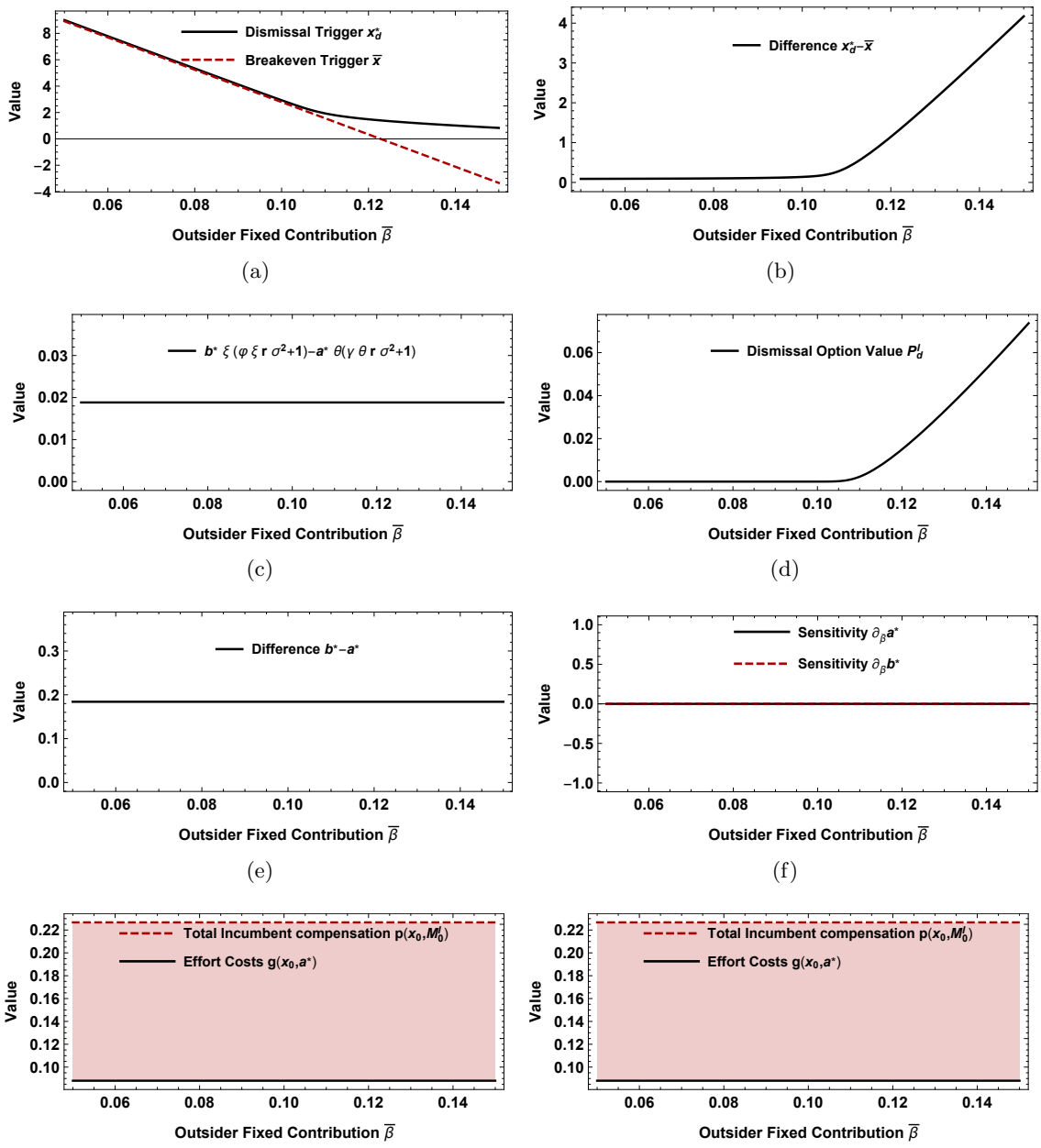


Figure 3.11: Comparative statics for outsider's fixed contribution $\overline{\beta} \in [0.05, 0.15]$. Subfigure (a) presents the values of the optimal dismissal trigger and breakeven trigger; Subfigure (b) shows the difference between these triggers; Subfigure (c) shows the left-hand side of equation (3.39) in Proposition 3; Subfigure (d) presents the value of the dismissal option; Subfigure (e) presents the difference between the required efforts b_t^* and a_t^*; Subfigure (f) shows the sensitivity of the optimal efforts to changes in the parameter; Subfigure (g) presents decomposes the value of the instantaneous compensation paid to the incumbent manager at time $t = 0$; Subfigure (h) shows the decomposition of the instantaneous compensation to be paid to the outsider manager had she been in place at time $t = 0$. Remaining parameters respect Table 3.1.

Chapter 4

Financial Regulation Reforms and Bank Capital Structure

I study the relationship between three regulatory measures imposed in Europe after the 2008 crisis and the capital structure decisions of regulated banks. I analyse the increase of the level of coverage of Deposit Insurance Schemes, the introduction of the Private Sector Acquisition resolution tool, and the introduction of the bail-in tool. For that, I develop a continuous time model with asset uncertainty where a bank selects the optimal capital structure while facing a regulator with a real option to impose a resolution measure. This model relies upon a set of parameters that can be estimated with empirical data and can easily accommodate additional extensions. My results show that the introduction of these regulatory measures incentivises banks to decrease the ratio of equity over assets and to increase the ratio of deposits over total debt. Lastly, introducing the bail-in tool generates a recapitalisation option, that is, banks unable to rebalance their capital structure may have an incentive to increase ex-ante the weight of bail-inable debt over total debt if they expect that a potential bail-in will result in a value-enhancing recapitalisation.

4.1 Introduction

The financial crisis of 2007–08 exposed a number of weaknesses in the regulatory framework[1]. Financial institutions were revealed to be poorly capitalised and several banks experienced entirely unanticipated liquidity crises. Moreover, the legislation in place to manage bank resolution and to wind up failing banks proved completely inadequate in many jurisdictions.

New regulations were introduced around the world in response to these problems. Basel III strengthened capital requirements and introduced a countercylical buffer, deposit insurance schemes were reinforced, and a number of new approaches to bank resolution were developed. Figure 4.1 illustrates the effect of these regulatory changes on the capital structure of financial institutions supervised by the European Central Bank. Two major changes occurred after 2009-10. First, the ratio of equity over assets started to increase. Second, the ratio of non-deposit liabilities over total liabilities decreased: that is, deposits became an increasingly important element part of the liability structure of financial institutions.

In light of these observations, this chapter asks what the relationship is between the new regulatory measures and the capital structure decisions of regulated financial institutions. Specifically, I examine the effect of a significant increase in the level of coverage provided by deposit insurance schemes; the introduction of a Private Sector Acquisition resolution tool, which reduces the costs of bank resolution by enabling the transfer of assets from troubled banks without shareholder approval; and the introduction of a Bail-in resolution tool that allows regulators to convert bail-inable debt to equity without any debt-holder vote. My analysis generates predictions that are consistent with the facts presented in Figure 4.1. The regulatory tools that I study are detailed in Appendix 5.3.1, where I present three case studies of resolution intervention.

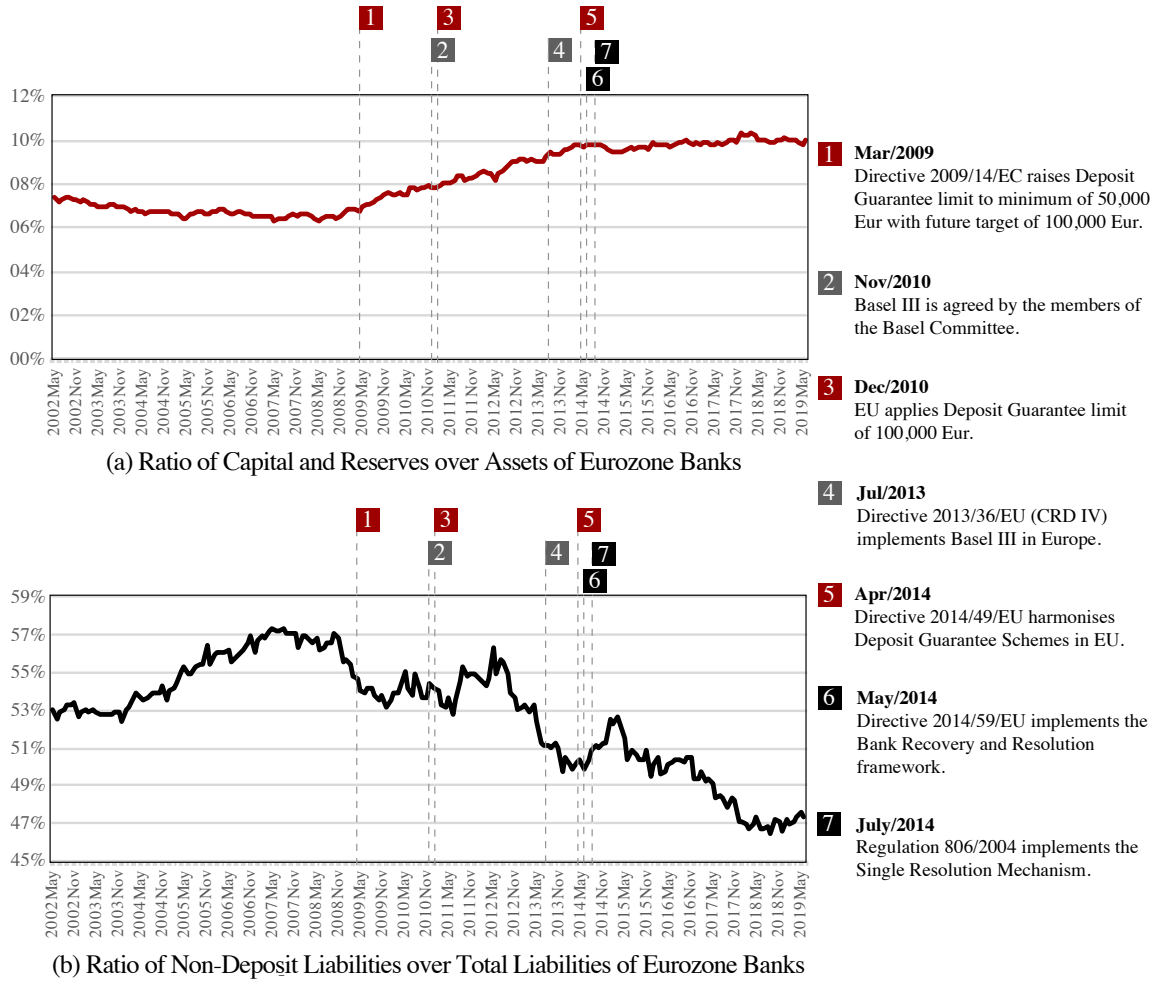
A natural framework for considering the effects of regulatory changes on capital structure is the standard trade-off theory (Kraus and Litzenberger 1973). This theory argues that firms should trade off the tax advantages of debt against the potential costs of financial fragility. The trade-off theory captures some of the most salient determinants of a firm's in capital structure choice, and several important papers have argued that it should form the basis

[1] See Dewatripont (2014) for an overview of the main issues in the banking sector and the changes in the regulatory framework that occurred after the European crisis.

of my analysis of capital structure regulation (Admati et al. 2013). I deploy the trade-off theory, but I make some necessary modifications to the basic theory to reflect the fact that banks have access to insured deposits, which renders this form of debt more attractive. Banks are further complicated because their failure generates massive systemic costs that are absent from standard models of corporate finance; those costs are not considered by bank managers, and they justify some capital regulation.

This chapter makes several contributions. First, by studying how banks select their capital structure today while anticipating the future impact of potential policy interventions, it addresses the ex-ante impact of new regulations. This is particularly relevant because the regulatory changes I analyse only affect financial institutions in case regulatory intervention occurs. In contrast, recent research focusses more on ex-post implications. For example, Walther and White (2019) focus on how the set of resolution policies changes with uncertain news and possible bank-runs[2].

[2]Walther and White (2019) use a signalling game between a regulator and uninformed depositors to study the set of bail-in policies available when facing uncertain news and potential subsequent bank-runs. Bad information constrains effective resolution as the regulator chooses weaker bail-ins because it fears a potential bank run.



(a) Ratio of Capital and Reserves over Assets of Eurozone Banks

(b) Ratio of Non-Deposit Liabilities over Total Liabilities of Eurozone Banks

Figure 4.1: Subfigure (a) shows that Eurozone banks are highly levered with a ratio of equity over assets stable between 6% and 10%. This is consistent with Gropp and Heider (2010) and far below the capital ratio of non-financial firms (Frank and Goyal 2008). Bank capital capital ratios remain above the regulatory minimum, a phenomenon already documented by Flannery and Rangan (2008). Subfigure (b) shows a decrease in the ratio of non-deposit liabilities to total liabilities starting in 2012. This is in contrast to prior work that documents a bank preference for non-deposit liabilities (Gropp and Heider 2010). From 2002 until the end of 2012, this ratio stayed above 53%.Deposits became the dominant form of debt finance from the beginning of 2016. Banks now have more than 53 Euros of deposits for every 100 Euros of total liabilities. The pattern is approximately the same if the denominator is restricted to issued debt securities. **Data:** Balance sheet data for the total sample of banks subordinated to the European Central Bank, from May-2002 to May 2019. Source: ECB Statistical Data Warehouse.

Second, my results shed new light on the effect that regulation has upon bank capital structure. Gropp and Heider (2010) provide empirical evidence that, in the past, financial and non-financial firms faced the same incentives when defining their capital structure. My analysis suggests that, as a result of recent regulatory changes, this is no longer the case. This chapter therefore identifies new directions for empirical work on the capital structure of banks.

Third, my model relies upon a set of parameters that can be estimated with empirical data; the model therefore yields estimates of the relative magnitude and importance of a variety of economic effects. Moreover, the setup is simple and flexible and it is easy to add additional features to the base case studied below. The model should therefore be useful for regulators, think tanks, and other policy institutes.

Most of the prior theoretical work on bank capital structure analyses discrete-time, finite-horizon models. This approach has generated many important insights. However, it cannot address questions that relate to optimal closure times; nor can it generate meaningful calibrations. These lacuna can be addressed only in a continuous time framework. The continuous time framework employed in this chapter draws heavily upon the real options literature (Dixit and Pindyck 1994) and is closely related to Leland (1994) and Sarkar (2013). As in these works, my capital structure depends on the trade-off between debt tax shields and bankruptcy costs. However, those papers are directed to non-financial firms and, therefore, I depart from their models by accounting also for deposits besides credit; by having a regulatory entity that determines, endogenously, the intervention trigger and, more importantly, by considering that the bank faces the aforementioned resolution policies in case of intervention[3].

I present a model in which a financial institution earns stochastic cash-flows[4] and must

[3]The differences between financial and non-financial institutions are more complex than these three distinct modelling aspects. In detail, banks are subject to microprudential supervision, namely, tight regulation including several different measures of capital requirements, liquidity requirements, risk coverage measures and additional rules for global systemically important financial institutions (SIFIs); moreover, banks are subject to macroprudential supervision where the supervisory entity analises the risks of a generalised collapse in the financial system and imposes specific measures to reduce the co-integration between banks; and third, banks are exposed to a wider range of operational and credit risks.

[4]I consider the archetypical definition of bank as a firm that allocates financial resources from those who have to those who need. Hence, cash-flows are the underlying variable because they are the periodic payments that the bank receives from its loaning activity. These flows can fluctuate when new loans are issued, i.e. new periodic payments will occur, when loans are fully paid or when debtors default on their payments.

select a capital structure to maximise its total value. The financial institution has access to deposit finance, which is subject to the usual Diamond and Dybvig (1983)-style runs, and also to non-deposit debt finance, upon which runs are impossible. I consider a regulator that imposes a formal Basel-style insolvency regulation; it also provides deposit insurance protection, controls the cost of resolution by allowing a private sector acquisitions tool, enables bail-inable debt, and decides when to trigger the conversion of that bail-inable debt to equity.

The continuous time framework that I employ enables me to determine the optimal intervention time and to analyse in detail the ways in which banks trade off debt against equity finance, and deposit against uninsured debt finance. I am also able to perform meaningful calibration in my set-up and, hence, to generate clear regulatory guidance.

I start by considering a base case in which deposits are uninsured, resolution is privately managed, and there is a standard tax advantage to debt. My model is a sequential game with a single stage in which the bank operates until it is forced to close by the regulator. At time zero, the bank determines its optimal choice of deposit and non-deposit debt financing. The bank operates until being closed by the regulator, which happens as soon as the bank fails to comply with the capital regulation. At closure, assets are sold privately and used to pay the bank creditors and, if possible, the shareholders.

I derive a closed-form solution for the optimal capital structure by trading off the costs of bankruptcy at closure against the tax benefits of debt finance. Moreover, I derive an optimal intervention trigger for the regulator's closure decision by trading off the benefits of stopping a violation of capital requirements against the possible benefits of letting the bank recover by itself. In the base case, I impose a no-run constraint upon the capital structure: the bank is required to hold a minimum level of non-deposit finance to cushion depositors from bankruptcy and disincentive bank runs; beyond that level, banks are indifferent between deposit and non-deposit debt finance.

I introduce additional elements to the base case model in stages. First, I consider the effect that costless deposit insurance has upon the optimal capital structure and, second, I examine the effect of introducing a private sector acquisitions tool. In both cases, the mechanics of the game remain unchanged: the bank selects the optimal capital structure at time zero and

the regulator decides to close the institution when the endogenous capital regulation rule is violated. However, these new regulatory tools change the pay-offs. First, deposit insurance shields depositors from any loss at closure and, second, the private sector acquisition tool decreases closure costs by avoiding a disordered privately managed bankruptcy.

Finally, I add a bail-in tool that allows the regulator to impose a debt-to-equity swap. The bail-in tool introduces a new stage into my model. The first stage runs from the bank's choice of capital structure (at time zero) to the time the regulator forces the conversion of unprotected creditor claims into equity. Conversion occurs at a rate that respects the "no worse off" condition required by current resolution frameworks[5]. The second stage runs from the conversion until the time when the regulator closes the firm. Closure occurs using the private sector acquisition tool combined, if necessary, with deposit insurance coverage. Regulatory intervention at both stages occurs when the endogenous capital regulation rule is violated. I provide closed form solutions for each type of debt.

My analysis yields a number of empirical implications. First, the model predicts that each of the regulatory changes that I consider causes an increase in the leverage of financial institutions. An increase in deposit insurance protection renders debt cheaper and so should cause an increase in leverage; a private sector acquisition tool reduces the cost of closure and, hence, also pushes banks towards debt financing; and finally, a bail-in postpones or entirely avoids formal insolvency or resolution procedures and, hence, by reducing the costs of financial fragility, also serves to increase bank indebtedness.

Second, calibrating the model suggests that the observed increase in banks' equity/assets ratios since 2010 (Figure 4.1.a) can only be explained by the increase in regulatory capital requirements introduced by Basel III[6]. Moreover, the sensitivity of bank's equity/assets ratio to increases in capital requirement ratios may be higher than apparent. Introducing these resolution tools and increasing the limits of deposit insurance push the institution towards debt financing, which undermines the effect of a capital requirement increase.

[5] The No Worse-Off Principle is defined in Article 74 of Directive 2014/59/EU. The Bank Recovery and Resolution Directive is available in https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32014L0059.

[6] The underlying ideas of Basel III were initially discussed in 2009. The European Union adopted a legislative package in 2013 increasing some capital requirement ratios and introducing new types of capital requirements. A summary of Basel III requirements can be found in https://www.bis.org/bcbs/basel3/b3summarytable.pdf.

Third, the model also generates predictions for the mix of deposit and non-deposit debt finance that banks use. The predictions are consistent with the debt structure trend for European banks presented in Figure 4.1.b: between 2012 and 2018, the long-standing bank preference for non-deposit liabilities over deposits was reversed. In line with this fact, my model predicts that recent regulatory changes should leave banks with more deposits than non-deposit liabilities, because deposit insurance relaxes the no-run constraint and subsidises depositors in the event that the bank closes.

Interestingly, my analysis demonstrates that deposit insurance need not cause deposits completely to crowd out non-deposit liabilities. It is hard for any firm to change its capital structure in a near-death situation (Gertner and Scharfstein 1991). The bank therefore takes advantage of the bail-in tool by selecting an ex-ante level of bail-inable liabilities that can be later converted to equity. This policy relaxes the closure constraint for lower firm performances, and so gives the bank's assets time to recover. That is, bail-ins generate a *recapitalisation option*. The recapitalisation option is valuable because the bank selects a fixed capital structure up-front. More generally, the recapitalisation option is valuable whenever the bank's capital structure is sticky; this could occur either because of adjustment costs or because of regulatory strictures. My calibration results indicate that banks with high growth rates of earnings, low volatility of earnings, low taxation and facing high capital requirements always hold non-deposit liabilities. Moreover, my model predicts that, conditional on high growth rates of earnings, financial institutions with high uncertainty will choose very high levels of non-deposit liabilities.

My analysis suggests that policy makers wanting to introduce the bail-in tool with a deposit insurance scheme should also impose a minimum volume of bail-inable liabilities[7]. My calibration shows that different growth rates of cash-flows, volatility, taxation and regulatory requirements may entail different optimal debt structures for financial institutions. I find that it is not guaranteed that all financial institutions will choose to hold bail-inable debt at the levels deemed sufficient by the regulator to make this tool useful. Some banks

[7] The BRRD indicates that the resolution policies are conditional on the minimum requirement for own funds and eligible liabilities (MREL). In detail, banks in the European Union for which liquidation is not the preferred strategy may be required to hold a minimum level of bail-inable debt to be suitable candidates for bail-ins. MREL requirements are still under development at the time of this chapter. The 2008 guidelines for the first wave of resolution plans can be found here.

simply do not hold bail-inable debt and, hence, have no recapitalisation option.

Moreover, imposing a minimum volume of bail-inable liabilities attenuates the solvency and liquidity risks that high levels of deposits may pose to Deposit Insurance Funds. My model does not incorporate systemic risk but I hypothesise that, if a majority of banks exhibit the preference for deposits that the banks in my model do, then the deposit insurance fund may be threatened by negative shocks to the banking sector. In this case, resolution tools aimed at sterilising bailout needs are innocuous because the sovereign will respond to them by bailing out the deposit insurance fund instead of the financial institutions.

The roadmap for this chapter is as follows. Section 4.2 presents a review of related literature; Section 4.3 introduces the base case, that is, before the regulatory changes; Section 4.4 introduces costless deposit insurance to the base model; Section 4.5 adds private sector acquisitions; Section 4.6 adds the bail-in tool; Section 4.7 provides quantitative results and comparative statics; Section 4.8 discusses some of the limitations of my framework; and Section 4.9 concludes the chapter and suggest future research avenues.

4.2 Literature Review

▷ **Trade-Off Theory of Capital Structure.** My model builds on the trade-off theory of capital structure, first studied by Kraus and Litzenberger (1973). This theory identifies the optimal corporate capital structure as arising from a trade-off between the tax shield of debt and deadweight costs[8]. See Frank and Goyal (2008) for an overview of trade-off theory, Sundaresan (2013) for a review on capital structure and a summary of stylised empirical facts, and Graham et al. (2015) for an empirical overview of corporate financing policy over the last century.

The importance of tax benefits of debt for the capital structure decisions of financial institutions was often neglected in the prior banking literature. That changed recently with several empirical works confirming the importance of tax shields for the capital structure of banks. Schandlbauer (2017) adjusted the setup of Heider and Ljungqvist (2015) for US financial institutions subjected to 13 different state tax increases between 2000 and 2011. They find that well-capitalised banks increase their non-depository debt to benefit from an enlarged tax shield of debt. Financial constraints over worse-capitalised banks prevent these institutions from making the same size and type of adjustment. Instead, they partially adjust their hybrid securities to benefit from tax deductions. Furthermore, Schepens (2016) studies an exogenous decrease in the relative tax benefit of debt to equity for Belgium banks in 2006. This effective increase in the tax shield of equity increased the capital structure (equity over total value). In my model this would correspond to to a negative shock in the tax rate parameter. Lower marginal benefit of debt leads to a higher capital structure ratio. For additional evidence on the importance of taxation to banks' capital and debt structure decisions see Ashcraft (2008), Horvarth (2013), Hemmelgarn and Teichmann (2014), de Mooij and Gaetan (2015), and de Mooij and Keen (2016). For general reviews on the impact of taxation in non-financial and financial firms see Hanlon and Heitzman (2010) and Graham and Leary (2011).

[8]This theory originally rose from the discussion about the extreme predictions of Modigliani and Miller (1963), namely that, a firm with a linear value function will finance with 100% debt when subject to corporate income taxation.

The second important pillar of the trade-off theory is the magnitude of deadweight costs. Empirical evidence for financial institutions is not extensive but it is well established since the Savings and Loan Crisis. For instance, James (1991) studied the realised losses of bank failures from 1985 and 1988 using FDIC data. They find that bankruptcy costs amount to approximately 30% of failed bank's assets. Furthermore, whole bank transactions, where total assets are purchased by another institution, yield the lowest average loss among the different types of resolution tools available to the FDIC. Evidence on non-financial firms is particularly abundant. Andrade and Kaplan (1998) studied a sample of financially distressed highly leveraged transactions. Using market data, they estimated distress costs of around 10 to 20% of firm value, with a maximum cap at 23%. Korteweg (2010) estimates 15% to 30% of firm value for a sample of highly distressed LBO firms. Davydenko et al. (2012) proposes a novel approach that is not limited to highly leveraged transactions. They account for shocks in equity and debt value caused by unanticipated information in default announcements. They estimate an average default cost of around 21.7% of the market value of assets, with costs of 30.5% in the case of bankruptcies. More recently, Glover (2016) used a structural model with state dependency and calculates default cost ex-ante and ex-post bankruptcy. For a sample of 2505 firms, the average company faces an expected default loss of 45% of firm value. However, conditional on default the average cost amounts to only 25%, in line with previous research. For additional studies on the problematic of financial and economic distress and corporate bankruptcy see, for example, Asquith et al. (1994), Bris et al. (2006), Acharya et al. (2007) and Almeida and Philippon (2007). For related surveys, see Hotchkiss et al. (2008) and Senbet and Wang (2012).

From a modelling perspective, capital structure literature is divided in two groups: Static and dynamic. The static trade-off theory supposes that the firm's debt level is optimised at a single period. The model of Bradley et al. (1984) provides the standard setting for the static capital structure under opposing effects of taxation and bankruptcy costs. Under a dynamic approach firms adjust the capital structure dynamically throughout time. The inception of dynamic models occurred with Brennan and Schwartz (1984) who provided a dynamic trade-off theory framework under stochastic shocks. The absence of adjustment costs generates a dynamic adjustment to the optimal levels every time the firm suffers a

shock. However, this adjustment should not occur at every point in time in order to be realistic. Hence, Fischer et al. (1989) introduced adjustment costs. In this case, leverage ratios are only adjusted when the benefit from rebalancing the capital structure exceeds adjustment costs. They find even when these costs are small, the firm's debt ratio can have significant changes over time. More recently, Strebulaev (2007) builds and calibrates a dynamic capital structure model with infrequent adjustment to generate data and compare with existing empirical literature. Standard cross-sectional results seem to corroborate a number of empirical studies and the model is rejected using established methodology to test results. Overall, the paper reinforces the need to expand the research on capital structure. Examples of other developments in capital structure models includes endogenous investment (Hennessy and Whited 2005), a principal-agent setting with agency costs (Demarzo and Sannikov 2006), effects of shifts in macroeconomic states of the economy (Hackbarth et al. 2006).

▷ **Distress and Private-side Restructuring Literature.** The sequence of Bail-In and Private Sector Acquisition in this model is closely related to private-side[9] restructuring literature. Restructuring decisions are usually endogenous and depends on a bargaining distressed exchange of claims between stakeholders. I do not account for bargaining between players. I respect the BRRD regulation and consider that unprotected creditors do not have an ex-ante value loss associated to a future Bail-in conversion as the regulator is independent from stakeholders' lobbying.

However, private-sided restructuring measures can erode credit claims' value. For example, Mella-Barral and Perraudin (1997) shows that strategic debt service affects corporate debt valuation. In this model, shareholders have negotiation power. They confront creditors with a "take-it-or-leave-it" coupon payment when the firm is in financial distress. Creditors unable to confront this suffer a value transfer to shareholders. In Mella-Barral (1999) shareholders and creditors face a distressed renegotiation in the form of a debt swap. High-leveraged firms can experience credit reorganisation before liquidation, while low leverage firms are directly liquidated. Creditor's bargaining power affects the price of their claims. Lambrecht

[9] The voluntary measures used by the firm's stakeholders to deal with bankruptcy.

(2001) extends the debt swap problem to a duopolistic framework. Consistent with Zingales (1998), they show that the last firm to go bankrupt has higher relative operating profits combined with lower debt repayments, and higher expected value of becoming a monopolist, i.e. the "survivor".

Sundaresan and Wang (2007b) considers a Nash bargaining negotiation between shareholders and creditors while the firm is an ongoing concern. Creditors with high bargaining power can capture some of the firm value before reaching bankruptcy. Coupon reduction depends on fire sales costs, bargaining power and taxes. In the model of Sarkar (2013), shareholders make a distressed exchange offer to creditors near financial distress in order to postpone costly bankruptcy reorganisation. Parties negotiate via a Nash bargaining game where the bargaining power is split between shareholders and creditors, thus allowing for the possibility of renegotiation to split the restructuring surplus. They show that greater bargaining power resulting in an early DE offer, smaller debt reduction and smaller equity stake. See Hotchkiss et al. (2008) for an empirical research survey on the use of private measures for resolving default and reorganising financially distressed firms.

▷ **New Regulation and Contractual Arrangements.** My article fits within a larger literature that investigates the impact of recent regulatory changes on the decisions of the bank's stakeholders. Post-financial crisis literature on capital structure paid initial attention to the use of contingent capital, namely contingent convertible (CoCos) debt. The conversion threshold is a central concern in these models as this parameter will dictate the conversion of CoCo debt to equity. For example, Glasserman and Nouri (2012) study contingent capital in the form of debt subject to equity conversion upon reaching a threshold based on book value variables. Their results indicate that the fair yield for contingent capital is sensible to the convertible tranche, asset's volatility and recovery rate. Chen et al. (2017) follows with a model with endogenous default, debt rollover and a combination of jump and diffusion processes. They conclude that bankruptcy induced by excessive debt, which occurs when CoCos are combined with endogenous default for low threshold conversion, can be overcome if parties agree to hold sufficiently high debt-to-equity conversion thresholds. My intervention triggers are close in spirit to the conversion thresholds presented in these two

papers. However, I derive it optimally from the regulator's free-boundary problem in which the regulator has to choose the best time to step in the bank. As a result, uncertainty plays a key role in every intervention trigger which induces the regulator to delay intervention if the bank's assets are more volatile. Chen et al. (2017) has a similar mechanism for one of the thresholds, i.e. the bankruptcy threshold chosen by the shareholders. However, I differ in the way I treat the order of triggers. In my model, default is impossible to occur before conversion when the firm has junior debt in its debt structure. That is because the regulator is in charge of the stopping times and will impose a bail-in in that case. This may not occur in a model with contractually agreed contingent convertible's thresholds. Differently, Sundaresan and Wang (2015) focused on CoCos with market equity triggers in which shareholders and junior creditors are do not hold an option to select their optimal conversion policies. As a result, this restriction of interests lead to the existence of multiple equilibrium. For a unique equilibrium to hold, a conversion rate must not lead to a value transfer from shareholders to junior creditors. However, that prevents punishing shareholders for firm mismanagement and leads to problems when designing the CoCo contract with market triggers. Also, the issue of equilibrium multiplicity worsens in the presence of discrete jumps and increasingly costly bankruptcy. Additional discussions of trigger designs can be found in Baily et al. (2009) for a combination of a systemic crisis indicator and bank-specific accounting information, in Glasserman and Wang (2011) and Bolton and Samama (2012) for bank management's decision, and in Dickson (2010) for regulatory decision.

Furthermore, post-financial crisis literature on capital structure focused more recently on the new regulatory environment of financial institutions. For example, my chapter is closely related with Sundaresan and Wang (2017). They study the case of liability structure in a classic capital structure model with deposits and non-deposit liabilities. Optimal funding decision builds on the trade-off between debt tax shield and bankruptcy costs. This shares some features with my chapter though I opt to extend to the impact of resolution tools, effects of the bail-in tool, instead just the introduction of deposits in the capital structure. We both find that, in equilibrium, financial institutions want to hold more deposits in their capital structure once deposit insurance is introduced. Lambrecht and Tse (2019) present a continuous time model where a risk-averse manager dynamically optimises the capital

structure and the asset's riskiness. They study how liquidation, bailout and bail-in policies affect these decisions. My model is distinct in two aspects: First, I focus on a static capital structure optimally determined at time $t = 0$. A static capital structure is, I believe, more adequate since firms do not actively correct their capital structure when facing shocks in their market value (Welch 2004) and, in a near bankruptcy situation capital structure changes are difficult to implement (Gertner and Scharfstein 1991). Also, the model becomes tractable with a static capital structure, allowing me to solve for the intervention triggers and to obtain closed form solutions for the equilibrium controls. Second, I optimise for total bank value instead of the managerial value (which depends on the equity value). This impacts optimal decisions if the bank faces liquidation in bad states of the nature. In Lambrecht and Tse (2019) model, the decision maker increases exposure by issuing bad loans. In my case, the bank decreases leverage because optimisation accounts for the creditors loss in default. I also relate with the work of Hugonnier and Morellec (2017), They study the impact of liquidity and leverage requirements on default conditions of financial institutions. They show that liquidity requirements decrease the size of losses conditioned to default but increase the probability of default. They show also that leverage requirements decrease the probability of default. Therefore, a combination of both requirements, as required by Basel III, can simultaneous decrease losses in default and the likelihood of default.

▷ **Bank Runs and Deposit Insurance.** My framework imposes a minimum non-deposit liabilities to prevent bank-runs in the absence of a deposit insurance scheme. This is supported on the extensive literature of depositors runs. For instance, the seminal paper Diamond and Dybvig (1983) studies how different types of depositors act in the presence of demand-deposit contracts. In the welfare improving equilibrium, impatient depositors withdraw earlier at the cost of patient depositors. In the bad equilibrium, a bank run occurs when both type of investors try to simultaneously withdraw their holdings from the bank. Nonetheless, this work leaves unanswered the question of which factors trigger the run. Information about the lack of quality of the bank is not necessarily the trigger as shown by Breeden (1984). In this model, highly averse depositors may choose to consume less now to invest more in an uncertain future, regardless of bank asset's quality. Intuitively, Jacklin and

Bhattacharya (1988) shows that bank runs do not threaten the bank if long-term assets are liquid and depositors are not significantly risk averse. Goldstein and Pauzner (2005) extend Diamond and Dybvig's work by exploring the impact of uncertain economic conditions on equilibria. The bank equilibrium contract emerges as a function of the bank-run probability. Allen et al. (2018) follows by considering deposit insurance as a public policy. They find that government guarantees mitigate the inefficiencies created from lack of coordination between the types of depositors. They note that not all types of guarantee schemes are equally effective. Guarantees against panic runs are the most effective in the mitigation of bank runs at no public expense. Political environment tend to affect the adoption and design of government deposit guarantee systems (Demirguc-Kunt et al. 2008). More democratic countries with fewer underlying risks increase government deposit insurance. This seems to correlate positively with the influence of international institutions.Deposit insurance can, however, trigger moral hazard frictions as demonstrated by Chernykh and Cole (2011) in an experiment in Russia[10]. Demirguc-Kunt and Detragiache (2002) argues that effective prudential regulation and supervision mitigates moral hazard frictions and other bank fragilities exacerbated by deposit insurance. Furthermore, Morrison and White (2011)'s model of moral hazard and adverse selection study the ideal deposit insurance. Contrary to the idea that banks should pay for deposit insurance, they show that general taxation combined with tough capital requirements is the best configuration. See Allen et al. (2011) and Calomiris and Jaremski (2016) for additional reviews on deposit insurance.

4.3 Base Case

4.3.1 Elements of the Model

I consider a model with two active players: a Bank and a Regulator. The players move sequentially: The Bank selects its capital structure, and then the Regulator chooses when to close the bank. My base case is close to the standard Leland (1994) framework.

[10]They concluded that banks entering the scheme increased risk-taking, sustaining the theory that deposit insurance can motivate moral hazard. Also, banks observed an increment on their level of deposits in absolute terms and relative to total assets, which benefitted smaller banks. State-owned banks presented a decrease in deposits, suggesting that the shift in risk did not affect depositors.

▷ **The Bank.** The bank receives cash-flows[11, 12] represented by an Ito diffusion $x = \{x_t : 0 \leq t < \infty\}$ that satisfies the following Geometric Brownian Motion:

$$dx_t = \mu x_t dt + \sigma x_t dw_t, \tag{4.1}$$

where $x_0 > 0$, $\{w_t\}_{t \geq 0}$ is a Wiener process, μ is the instantaneous conditional expected growth rate of x and $\sigma > 0$ is the instantaneous conditional standard deviation. I write $\rho > 0$ for the discount rate and, for convergence, I assume that $\rho > \mu$.

The bank's assets are financed with liabilities and equity. The liability part of the bank's balance sheet comprises two types of perpetual debt: non-deposit liabilities and deposits. The bank decides its liability structure at time $t = 0$ by selecting coupons c for non-deposit creditors and d for deposit creditors. I write $l = c + d$ for the total coupon paid by the bank.

Assumption 9 (Non-Negative Coupons)**.** *The bank selects non-negative coupons for non-deposit liabilities and deposits.*

In my model, debt is used to finance assets and, therefore, the bank is expected to pay a positive interest payment every time it receives cash-flows.

The value $V(x_t)$ of the bank's assets is the present value of future cash-flows:

$$V(x_t) = \mathbb{E} \int_t^{+\infty} e^{-\rho(s-t)} x_s ds = \frac{x_t}{\rho - \mu} \tag{4.2}$$

and the accounting value L of the bank's liabilities is the present value of future coupon payments:

$$L = \int_t^{+\infty} e^{-\rho(s-t)} (c + d) ds = \frac{c + d}{\rho} \tag{4.3}$$

The regulator may force the bank to close at any time, in which case I say that the bank

[11] I model cash-flows through a Geometric Brownian Motion for its simplicity because, with this diffusion process, I can obtain closed form solutions for the optimal parameters of the model. However, I consider a single GBM instead of a sum of GBMs because these processes are not additive and, therefore, a linear combination of GBMs will not yield a composite GBM. See, for example, Petersen and Mukuddem-Petersen (2005) and Witbooi et al. (2011) for frameworks that consider composites of risky assets.

[12] Moreover, there can be an issue of having unobservable firm's asset value. In this case, to determine intervention triggers using real world data can be problematic if a firm has unobservable asset's cash-flows. That can be overcome by using credit spreads or the CDS risk premium as seen, for instance, in Yang and Zhao (2015) and Pelger (2012).

has entered bankruptcy. If that happens, the bank's assets are sold to another owner, who continues to receive the cash flow stream x_t. If the bank goes bankrupt, then it experiences a cost equal to a fraction $\epsilon \in (0,1)$ of the value $V(x_t)$ of its assets at the time of its bankruptcy: that is, it experiences a cost $\epsilon V(x_t)$. I refer to the value of bank assets net of the costs of bankruptcy as the *impaired asset value*[13], $(1-\epsilon)V(x_t)$.

The bank also pays corporate tax at a rate[14] $\tau \in (0,1)$ on cash-flows above its debt payments. It follows that equity-holders receive a continuous income stream after interest and tax of $(x_t - c + d)(1-\tau)$.

The bank selects c and d at time $t = 0$ in order to maximise the value function $B(x_t)$. $B(x_t)$ is the sum of the values of the claims on the bank:

$$B(x_t) = E(x_t) + C(x_t) + D_t$$

where $E(x_t)$ is the value of the claim held by the equity-holders (or shareholders), $C(x_t)$ is the value function for non-deposit creditors and D is the value function[15] of depositors. These value functions are specified in Subsection (4.3.3). I write $CS(x_t)$ for the capital structure ratio: that is for the ratio between equity value and bank value. I define the debt ratio $DS(x_t)$ to be the ratio between the non-deposit creditors' value function and the total value of all liabilities:

$$CS(x_t) := \frac{E(x_t)}{B(x_t)} \qquad\qquad DS(x_t) := \frac{C(x_t)}{C(x_t) + D}$$

I impose the following assumption:

Assumption 10. *All claimants have perfect information about the bank's cash-flow.*

Assumption 10 ensures that bank runs are possible in this model. Depositors worry

[13]This chapter treats bankruptcy costs as fire sales costs, in a similar manner as Leland (1994) and Strebulaev (2007). Empirical literature shows that selling assets at discount is frequently used by distressed firms (Asquith et al. 1994, Pulvino 1998). More recently, Ma et al. (2019) have shown that innovative firms under distress tend to sell first intangible assets like patents before liquidating fixed assets.

[14]It is simple, though outside my goal, to extend this model in order to account also for a dividend tax rate, as in Tserlukevich (2008).

[15]The depositors' value function does not depend on x_t.

about solvency[16] and so, if depositors observe that the present value of the bank's cash-flows is insufficient to cover their claim, they withdraw their funds. The private cost of bank runs is so high that the bank prefers to avoid it. Hence, I assume:

Assumption 11 (No Bank-Run)**.** *The bank selects a capital structure that does not induce bank-runs.*

▷ **Regulation.** The bank is subject to capital regulation. Two elements of the bank's balance sheet are important for capital regulation: the value of assets presented in condition (4.2) and the accounting value of liabilities in condition (4.3). The regulation stipulates a safe value $\xi \in (0, 1)$ for the percentage by which the value $V(x_t)$ of assets exceeds the accounting value of liabilities. That is:

$$\frac{V(x_t) - L}{V(x_t)} \geq \xi \tag{4.4}$$

The regulator's only possible action is to close the bank. Banks are socially valuable and, ceteris paribus, regulators prefer not to close them. The regulator need not close a bank as soon as the capital requirement (4.4) is violated[17]. However, keeping a bank open when the capital regulation is not satisfied is risky. I capture this effect by assuming that the regulator experiences a flow of utility $R(x_t)$ given by the margin by which Equation (4.4) is satisfied:

$$R(x_t) := V(x_t)(1 - \xi) - L \tag{4.5}$$

Hence, when capital regulation condition (4.4) is violated, the regulator experiences a

[16] I focus on solvency concerns rather than liquidity concerns which is more usual in bank-run literature. Nonetheless, these two options should not differ significantly in my setup.

[17] I use the current European banking crisis management framework as inspiration for the actions of my regulator. In Europe, banks are subject to early requirements spanning internal governance, business strategy, capital and liquidity adequacy, as detailed in Art. 27 of Directive 2014/59/EU and in EBA Guidelines EBA/GL/2015/03. If a bank is not fulfilling these requirements, the regulator asks the bank to implement early intervention measures. These are managerial and operational changes that can recover the bank and avoid triggering a harsh resolution measure such as closure. A resolution is only triggered when the regulator determines the bank is failing or likely to fail (Art. 32 of Directive 2014/59/EU). Failing own fund requirements (Art. 104 of Directive 2014/59/EU) and having less assets than liabilities are objective measures that contribute to triggering resolution but, according to EBA Guidelines EBA/GL/2015/07, such decision is "an expert judgement and should not be automatically derived from any of the objective elements". In general, the regulator decides to trigger a resolution when no early measures are expected to restore the bank to viability within a reasonable timeframe.

negative utility. It can stop the negative utility flow by closing the bank. However, if the regulator chooses to leave the bank open, there is a chance that it will recover, and that the utility flow $R(x_t)$ will turn positive. The regulator therefore faces a real options problem: it must select the optimal time to close the bank so as to trade off the benefits of immediately preventing further loss of utility when condition (4.4) is violated against the possible benefits to be earned if condition (4.4) is satisfied again in the future. I write T_b for the optimal closure time. Peskir and Shiryaev (2006) demonstrate that T_b is the first passage time at which x_t crosses a boundary value x_b:

$$T_b = \inf \{t \in [0, +\infty) : x_t \notin (x_b, +\infty)\} \tag{4.6}$$

I write the closure option value as $O(x_t)$ and I value $O(x_t)$ in subsection 4.3.2. More importantly, I also derive the closed form of the regulator's closure trigger x_b, which the bank incorporates in its optimal capital structure problem.

I impose the following assumption:

Assumption 12 (Initial Capital Requirement)**.** *The bank respects regulatory condition (4.4) when it chooses the capital structure.*

▷ **Timeline and Equilibrium.** Consider Figure 4.2. At $t = 0$, the bank selects coupons $(c, d) \in [0, +\infty)^2$ so as to maximise its value function $B(x_0)$. The bank operates until the regulator chooses to close the bank[18]. I therefore consider two stages. During stage 1, $t \in [0, T_b)$ and the bank is operational. The second, *closure* stage, obtains for all $t \in [T_b, +\infty)$. I use a subscript $n \in \{1, \mathcal{C}\}$ to represent each stage. For example, $B_{\mathcal{C}}(x_t)$ is the bank value in the closure stage.

The model of this chapter is a sequential game with perfect information and I therefore

[18]I depart from Sundaresan and Wang (2007b), Sarkar (2013) and related works by having a separate entity that makes the intervention decision instead of allowing the shareholders to define the moment of bankruptcy. In these works, conflicts of interests would lead to the typical Leland (1994) bankruptcy trigger, later than the timing that would fulfil the typical interests of a regulator. In my case, the capital requirement buffer contributes to an earlier preemption than the equivalent setup of these models.

solve it by backward induction. I write the equilibrium tuple as $\mathcal{E}$:

$$\mathcal{E} := (c^*, d^*, x_b^*)$$

At time 0, the bank chooses the optimal periodic coupons c^* paid to non-deposit creditors and d^* paid to depositors. The regulator closes the bank at time T_b, when the cash-flow x_t first crosses the closure trigger $x_b^* := x_b(c^*, d^*)$.

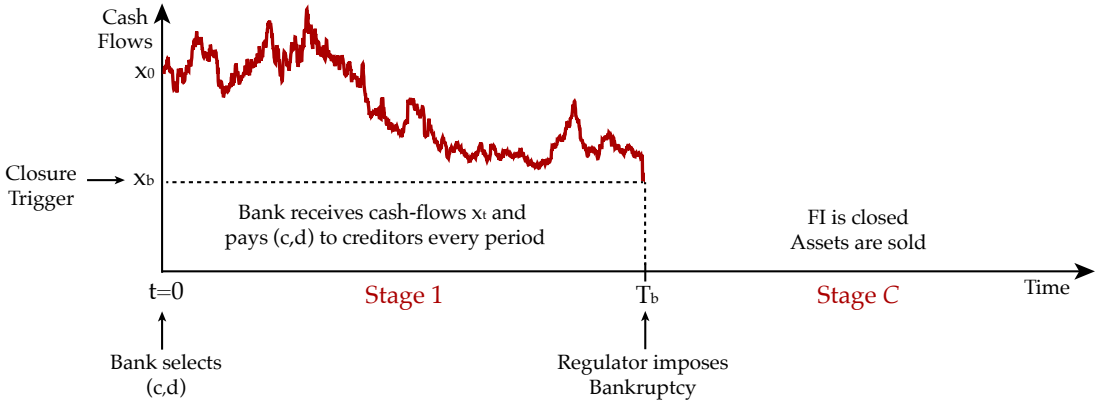


Figure 4.2: **Base Case Timeline.** Cash-flows evolve stochastically while gradually declining until they reach the closure trigger x_b. That occurs at time T_b, when the regulator closes the bank and it enters bankruptcy.

4.3.2 The Regulator's Decision

In this section, I solve for the regulator's closure option $O(x_t)$ and the optimal closure trigger x_b. The regulator closes the bank when option value $O(x_b)$ net of the disutility $R(x_b)$ is equal to the utility 0 of closing the bank; that is, when the following expression is satisfied:

$$O(x_b) + R(x_b) = 0$$

When the expression is satisfied, the regulator is indifferent between keeping the bank alive and closing it.

I determine the value $O(x_t)$ of the regulator's closure option in Appendix 5.3.2, using standard real options techniques laid out by Dixit and Pindyck (1994). I find that, for a

given closure trigger x_b,

$$O(x_t) = -R(x_b)\left(\frac{x_t}{x_b}\right)^{\beta}, \qquad \forall x_t \geq x_b \tag{4.7}$$

where

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0 \tag{4.8}$$

Condition (4.7) states that the value of the regulator's closure option is the product between the regulator's payoff if the option is exercised ($-R(x_b)$) and the expected discount factor[19] $(x_t/x_b)^{\beta}$ that discounts the payoff from time T_b to the current time.

In this framework, β is the elasticity[20] of the closure option with respect to the cash-flow x_t: that is, β measures the relative change of the option's value to relative changes in cash-flows. β is negative so that, as one would expect, the value of the closure option decreases as cash-flows increase.

The regulator closes the bank when the closure option has the highest value. The closure trigger x_b therefore solves the following problem:

$$x_b \in \arg\max_{x_b} O(x_t),$$

The following Proposition, which I prove in Appendix 5.3.3, presents the solution to this problem:

Proposition 5 (Optimal Bank Closure)**.** *For a given combination of non-deposit coupon c and deposit coupon d, the regulator closes the bank at the time T_b when the bank's cash-flow*

[19] The expected discount factor $\mathbb{E}\left[e^{-\rho T_b}\right]$ is used to discount payoffs earned at a random time T_b. As shown by Dixit and Pindyck (1994), I can use the fact that T_b is the first time x_t crosses x_b to write it as $(x_t/x_b)^{\beta}$.

[20] The option elasticity can be written as

$$\frac{dO}{O(x)}\frac{x}{dx} = \beta$$

For example, for a 1% increase in cash-flows x_t, there is a decrease in the closure option's value equal to β%.

x_t first crosses the following closure trigger x_b:

$$x_b = \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c + d}{\rho} \tag{4.9}$$

The closure trigger x_b declines with the growth rate of assets μ and standard deviation σ. The regulator is more lenient with high growth and high uncertainty banks since these institutions are more likely to recover from a near bankruptcy scenario. Banks with negative growth and low uncertainty will, almost surely, continue to shrink cash-flows as time passes and the regulator is therefore less likely to wait for them to recover. Furthermore, a regulator facing a higher discount rate ρ selects a higher trigger x_b. In this case, the regulator becomes more impatient[21] and closes the bank sooner.

A higher regulatory rate ξ increases the closure trigger. This reflects a tighter capital regulation since it enlarges the percentage by which the value of assets should exceed the accounting value of liabilities. Higher non-deposit coupon c and deposit coupon d also increase the closure trigger because they reduce the value of assets available after debt payments.

Figure 4.3 presents an example that illustrates value functions $O(x_t)$, $R(x_t)$ and the closure trigger x_b. In this example, the regulator decides to close the bank when cash flows hit the trigger $x_b = 10.45$. When the closure option is exercised at this cash flow level, the regulator's flow of utility $R(x_t)$ from the bank climbs to zero.

4.3.3 The Bank's Problem

This subsection derives the bank's optimisation problem. To do so, I start by deriving the bank's value function. I further present the No Bank-Run condition necessary to enforce Assumption 12.

[21] Cardoso and Pereira (2015) and Grenadier and Wang (2005) study how different degrees of impatience between a principal and an agent affect the exercise of investment options.

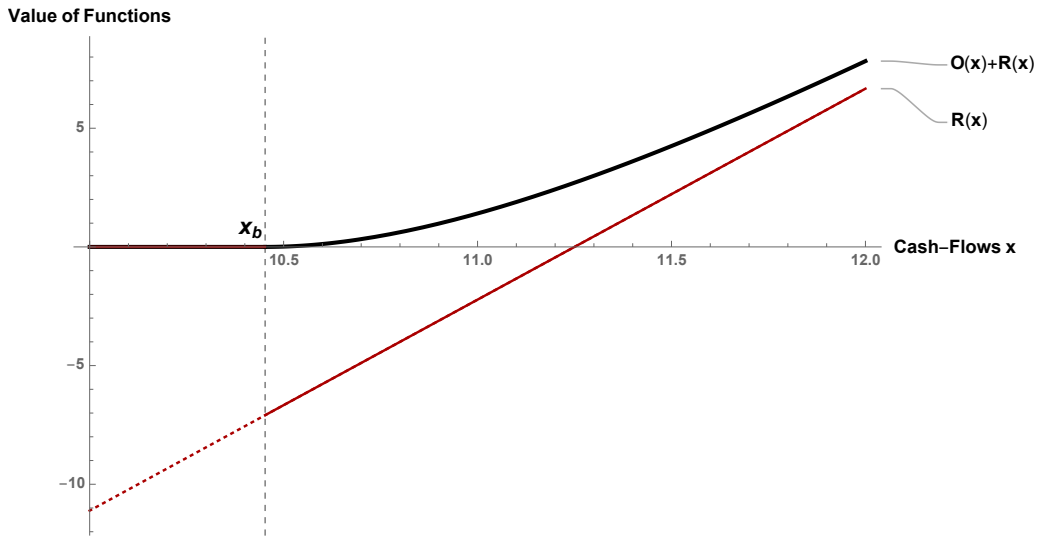


Figure 4.3: **The Decision to Close the Bank.** The regulator experiences positive utility for as long as $R(x)$ is positive. Capital regulation is violated when $R(x)$ becomes negative (x drops below 11.25) but the regulator will not close the bank immediately. Only when cash-flows deteriorate to the point of crossing trigger $x_b = 10.45$ will the regulator shut down the institution. **Parameters:** $\rho = .1$, $\mu = .01$, $\sigma = .05$, $\xi = .2$.

4.3.3.1 Value Functions

The bank's value function is given by the following accounting identity:

$$B(x_t) := E(x_t) + C(x_t) + D \quad , \forall t \in [0, T_b)$$

I therefore start by determining each of the value functions in this identity.

Shareholders For $t \in [0, T_b)$, the shareholders' value is given by the following expression:

$$\begin{aligned} E(x_t) := \ & \mathbb{E}_t \left[\int_t^{T_b} e^{-\rho(s-t)} (1-\tau) \left(x_s - c - d \right) ds \right] + \\ & + \mathbb{E}_t \left[\int_{T_b}^{+\infty} e^{-\rho(s-t)} \max \left\{ 0, (1-\tau) \left((1-\epsilon) x_b - c - d \right) \right\} ds \right] \end{aligned} \tag{4.10}$$

Shareholders receive periodic cash-flows after interest payments and taxes $(1-\tau)(x_t - c - d)$ while the financial institution is operational.

When the bank is closed, assets are sold to another institution, where they will continue to generate cash-flows. The impaired asset value obtained from this sale is distributed among

creditors first and only then shareholders given their status of residual claimants.

Shareholders are therefore entitled to the present value of $(1-\tau)((1-\epsilon)x_b - c - d)$ from time T_b until infinity as represented in function (4.10). Furthermore, shareholders are protected by limited liability. It follows that their income from the bank's assets is never negative and, hence, that they receive the maximum of the after-tax impaired cash flows and zero, as in Equation (4.10).

Using standard procedures described in Dixit and Pindyck (1994), the shareholders' value function $E(x_t)$ can be written as follows:

$$E(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c+d}{\rho}\right) - \left(\frac{x_t}{x_b}\right)^{\beta}(1-\tau)\min\left\{\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}, \epsilon\frac{x_b}{\rho-\mu}\right\} & , \text{ for } x_t > x_b \\ (1-\tau)\max\left\{0, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right\} & , \text{ for } x_t \leq x_b \end{cases} \tag{4.11}$$

The value function (4.11) has several components. When the bank is operational ($x_t > x_b$), the equity value is the sum of two terms: the first term, $(1-\tau)(x_t/(\rho-\mu) - (c+d)/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to creditors; the second term is the expected change in value if the regulator closes the bank.

The expected change in equity value is the product of the expected discount factor $(x_t/x_b)^{\beta}$ and the change in equity value if the institution is closed. This last term is the difference between the closure equity value $(1-\tau)\max\{0, (1-\epsilon)x_b/(\rho-\mu) - (c+d)/\rho\}$ and the after tax present value of cash-flows net of debt payments $(1-\tau)(x_t/(\rho-\mu) - (c+d)/\rho)$.

Because the shareholders have limited liability, their payoff when the bank is closed ($x_t \leq x_b$) is at least zero. They receive a zero payoff when the impaired asset value $(1-\epsilon)V(x_b)$ is insufficient to cover the present value of coupons L. If the impaired asset value covers the present value of coupons, shareholders obtain the residual value after paying creditors and paying taxes[22].

[22] I account for taxation over this residual value as it constitutes an asset sale profit for shareholders.

Non-Deposit Creditors The value function for non-deposit creditors is as follows:

$$C(x_t) := \mathbb{E}_t \left[\int_t^{T_b} e^{-\rho(s-t)} c ds + \int_{T_b}^{+\infty} e^{-\rho(s-t)} \min \Big\{ c, (1-\epsilon)x_b - d \Big\} ds \right] \tag{4.12}$$

Non-deposit creditors earn a periodic coupon c while the bank is alive. When the regulator closes the bank at time T_b, these creditors receive the full amount of the present value of future coupons if the impaired asset value is sufficient to cover their claim after paying depositors.

However, if the impaired asset value is lower than future payments to all creditors, non-deposit creditors suffer a haircut. They only get the impaired asset value after paying depositors, that is, the present value of $((1-\epsilon)x_b - d)$ from time T_b until infinity.

Recall that the closure time T_b is the first time at which the cash flow x_t crosses the trigger x_b. Using this fact, I re-write the value function (4.12) as follows:

$$C(x_t) = \begin{cases} \frac{c}{\rho} + \left(\frac{x_t}{x_b}\right)^{\beta} \min \left\{ 0, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho} \right\} & , \text{ for } x_t > x_b \\ \\ \min \left\{ \frac{c}{\rho}, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{d}{\rho} \right\} & , \text{ for } x_t \leq x_b \end{cases} \tag{4.13}$$

While the bank is alive ($x_t > x_b$), the value of non-deposit credit comprises two terms: the first term, c/ρ, is the present value of coupons; the second term is the expected change in the non-deposit credit value function if the bank closes.

The expected change in value is the product between the expected discount factor $(x_t/x_b)^{\beta}$ and the change in creditors' value function at closure. Creditors lose c/ρ when the institution is closed and receive in exchange the closure payoff $\min\{c/\rho, (1-\epsilon)x_b/(\rho-\mu) - d/\rho\}$.

When the institution closes ($x_t \leq x_b$), non-deposit creditors are fully paid if the impaired asset value covers all creditors, in which case, they receive the present value of coupons c/ρ. If the impaired asset value is too low, non-deposit creditors take a loss by receiving the impaired asset value net of deposits $(1-\epsilon)x_b/(\rho-\mu) - d/\rho$.

Depositors By Assumption 11, the bank selects its capital structure to ensure that runs cannot occur. Consequently, the depositor value function is invariant to the state variable

x_t. It can be written as follows:

$$D = \int_t^{+\infty} e^{-\rho(s-t)} d \; dt = \frac{d}{\rho} \tag{4.14}$$

Total Bank Value Summing the claims of shareholders, non-deposit creditors and depositors, I obtain the bank's value function.

$$B(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_t}{x_b}\right)^{\beta}\left(B_{\mathcal{C}}(x_b) - (1-\tau)\frac{x_b}{\rho-\mu} - \tau\frac{c+d}{\rho}\right) & , \text{ for } x_t > x_b \\ \\ B_{\mathcal{C}}(x_b) & , \text{ for } x_t \leq x_b \end{cases} \tag{4.15}$$

Here $B_{\mathcal{C}}(x_b)$ is the value of the bank when it is closed:

$$B_{\mathcal{C}}(x_b) := \frac{d}{\rho} + \min\left\{\frac{c}{\rho}, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{d}{\rho}\right\} + (1-\tau)\max\left\{0, (1-\epsilon)\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right\}$$

So long as the bank is alive ($x_t > x_b$), its total value is the sum of three components: the present value of cash-flows after taxes, the debt tax shield, and the expected change to the bank's value given the regulator's closure option.

The expected change of value is the product of the expected discount factor $(x_t/x_b)^{\beta}$ and the change in the bank's value upon closure: that change is equal to the difference between the closure value $B_{\mathcal{C}}(x_b)$ and the after-tax present value of cash flows net of interest payments.

At closure, the bank value $B_{\mathcal{C}}(x_b)$ comprises the certain deposit value added to the value at closure of the claims of non-deposit creditors and shareholders.

I define a *strong* bank to be one whose asset value upon closure, net of bankruptcy costs, is sufficient to cover the value of the coupon payments promised to all creditors. That is, a strong bank satisfies the following condition:

$$(1-\epsilon)V(x_b) \geq L$$

I define a *weak* bank to be any bank that does not satisfy this condition. Lemma 6 demonstrates that the strength of a bank is determined by the capital adequacy regulatory

ratio, ξ and, hence, that it can be selected by the regulator. Specifically, there is a boundary value for ξ that separates strong from weak banks:

Lemma 6 (Bank's Type Boundary)**.** *Let*

$$\overline{\xi} = \frac{\beta\epsilon - 1}{\beta - 1} \tag{4.16}$$

Then any bank whose capital regulation $\xi \geq \overline{\xi}$ is strong; any bank for which $\xi < \overline{\xi}$ is weak.

The boundary $\overline{\xi}$ is the regulatory ratio that sets the bankruptcy asset value equal to the present value of coupons at the bank's closure. Banks with tighter capital requirements are closed sooner and, hence, they are better able to meet their liabilities.

Figure 4.4 illustrates this feature of my model. In this example, the bank's type boundary is $\overline{\xi} = 0.4$. For any regulatory requirement ratio $\xi \in (0, 0.4)$, the impaired asset value $(1-\epsilon)V(x_b)$ upon bank closure is insufficient to support the value of coupon payments L promised to all creditors. For a $\xi \in (0.4, 1)$, the bank generates an impaired asset value at closure that always covers all creditors.

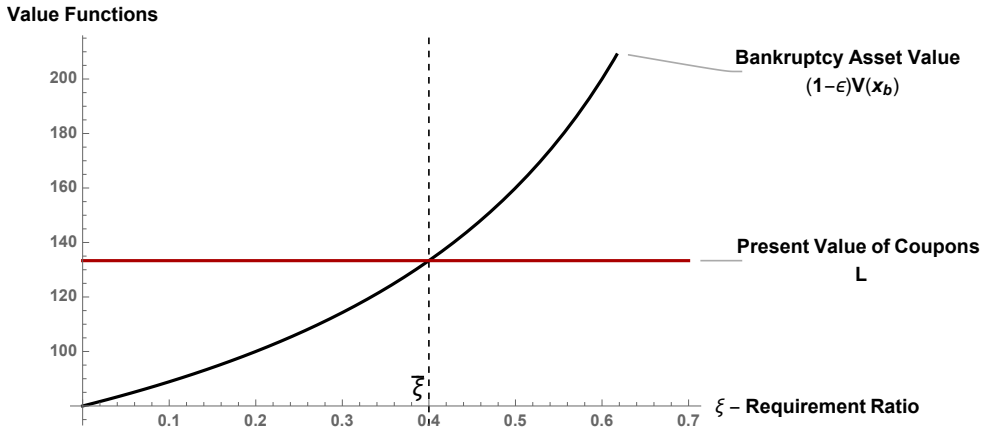


Figure 4.4: **The type of bank. Fixed Parameters:** $\mu = .01$, $\sigma = 0.1$; $\rho = 0.15$; $\epsilon = 0.3$; $c = 10$; $d = 10$. $\overline{\xi} = 0.4$. For some exogenous regulatory ratio ξ located on the left of $\overline{\xi}$, the bank will be considered weak since the impaired asset value of assets is lower than the present value of interest payments. If $\xi \geq \overline{\xi}$, the bank is considered strong.

4.3.3.2 Preventing Bank Runs

I now discuss my treatment of deposit runs. Assumptions 10 and 11 establish the behaviour of banks when the financial system lacks deposit insurance. With these assumptions, and using the results obtained in Proposition 5, I can derive the "No Bank-Run" condition of this model; the condition imposes a restriction on the optimal mix of deposits and non-deposit coupons.

Lemma 7 (No Bank-Run Condition)**.** *For any feasible pair of coupons* (c, d)*, depositor bank runs never occur if and only if condition (4.17) is satisfied:*

$$\frac{c}{d} \geq \gamma \equiv \frac{\beta - 1}{\beta} \frac{1 - \xi}{1 - \epsilon} - 1 \tag{4.17}$$

Ratio γ in condition (4.17) is the minimum quotient of non-deposit coupon c to the deposit coupon d that prevents a run by depositors. By respecting $c/d \geq \gamma$, the bank secures the impaired asset value at closure $(1 - \epsilon)V(x_b)$ exceeds the present value of deposit coupons D. This is sufficient for two reasons: first, depositors are senior over non-deposit creditors when the impaired asset value is distributed among claimants; second, the closure trigger x_b is the lowest level of cash-flows attained during the bank's operation and so, the impaired asset value always covers depositors for any level $x_t > x_b$.

Ratio γ declines with growth rate of assets μ, increases with standard deviation σ and increases with bankruptcy costs fraction ϵ. A bank with high growth, low volatility of assets and low bankruptcy costs has a high impaired asset value $(1 - \epsilon)V(x_b)$. More value is therefore available to cushion deposits at closure leading to a low γ ratio. This institution has more flexibility to select a mix of non-deposit liabilities and deposits that prevents bank runs than a bank with low growth rate μ of assets, high standard deviation σ and high bankruptcy costs fraction ϵ.

The ratio γ decreases if the regulatory requirement ratio ξ increases. This identifies a trade-off between regulatory requirements and bank self-protection. The regulator chooses higher closure triggers for higher regulatory ratio ξ which guarantees that more debt can

be covered in bankruptcy. Since depositors are senior, more resources are available for depositors at closure which relaxes the minimum ratio of non-deposit over deposits coupons that prevents runs.

4.3.3.3 Initial Capital Requirement

Assumption 12 sets a maximum level for the optimal coupons selected at time 0. This assumption prevents the bank from deliberately harming the regulator when defining the capital structure. Rearranging the regulation condition (4.4) at $t = 0$, I obtain the Initial Capital Requirement (ICR) condition:

$$c + d \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 \tag{4.18}$$

4.3.3.4 Non-Negative Coupons

By Assumption 9, I require each type of liabilities to be non-negative:

$$c \geq 0 \qquad \text{and} \qquad d \geq 0 \tag{4.19}$$

4.3.3.5 Bank's Capital Structure Optimisation

The bank's optimal capital structure problem 4 is set up as follows. The bank selects coupons (c, d) to pay creditors that maximise the total bank value $B(x_0)$ while respecting the regulator's optimal closure trigger condition (4.9), the NBR condition (4.17), the ICR condition (4.18), and the non-negative coupons condition (4.19).

Optimisation Problem 4 (Capital Structure of Base Case)**.** *The bank's optimal capital*

structure problem is:

$$
\begin{aligned}
(c^*, d^*) \quad & \in \arg\max_{\{c,d\}} B(x_0) \\
s.t. \qquad & x_b := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c+d}{\rho} && \textit{[Closure Trigger]} \\
& \frac{c}{d} \geq \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 && \textit{[No Bank-Run]} \\
& c + d \leq \frac{\rho(1-\xi)}{\rho-\mu}x_0 && \textit{[Initial Capital Requirement]} \\
& c \geq 0 \quad , \quad d \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}
$$

4.3.4 The Base Case Equilibrium

I now solve Optimisation Problem 4 to determine the equilibrium of the base case. The solution does not pin down unique values[23] for c and d: I solve instead for the total coupon $l = c + d$ and determine the acceptable ranges for c and d.

Proposition 6 (Base Case Equilibrium)**.** *The equilibrium of the game is:*

$$\mathcal{E} = (c^*, d^*, x_b^*)$$

Bank Optimal Decision. *The total periodic coupon $l^* = c^* + d^*$ is optimal iff:*

$$l^* = \min\left\{l^{**}, \bar{l}\right\} \tag{4.20}$$

where

$$l^{**} = \begin{cases} \frac{(\beta-1)(1-\xi)\rho\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)}x_0, & \textit{for } \xi < \overline{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)}x_0, & \textit{for } \xi \geq \overline{\xi} \end{cases} \tag{4.21}$$

and

$$\bar{l} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{4.22}$$

[23] Deposits and non-deposit liabilities are indistinguishable when I get the first order condition of the bank's value function (4.15) with respect to c and d. The differentiating aspect is the NBR condition which allows me to setup the range of minimum optimal non-deposit coupon c^*.

Any combination of c^* and d^* that respects the following conditions is optimal:

$$d^* := l^* - c^* \qquad \textit{and} \qquad c^* \in \begin{cases} (\hat{c}, \overline{c}) & , \quad \textit{for} \quad l^{**} \geq \bar{l} \\ (\underline{c}, \tilde{c}) & , \quad \textit{for} \quad l^{**} < \bar{l} \end{cases} \tag{4.23}$$

with

$$\hat{c} = \begin{cases} -\frac{\rho(\beta(\epsilon-\xi)+\xi-1)}{(\beta-1)(\mu-\rho)} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{4.24}$$

$$\overline{c} = \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{4.25}$$

$$\underline{c} = \begin{cases} \frac{\rho}{\rho-\mu} \frac{\beta(\epsilon-\xi)+\xi-1}{\beta} \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau} \right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{4.26}$$

$$\tilde{c} = \begin{cases} \frac{\beta-1}{\beta} \frac{\rho}{\rho-\mu} (1-\xi) \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau} \right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ \frac{(\beta-1)(\xi-1)\rho \left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon} \right)^{-1/\beta}}{\beta(\mu-\rho)} x_0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{4.27}$$

Regulator Optimal Decision. *At time T_b, cash-flows x_t hit the trigger x_b^*, the Regulator steps in and imposes a bankruptcy. That trigger is:*

$$x_b^* = \frac{\beta}{\beta-1} \frac{\rho-\mu}{1-\xi} \frac{c^*+d^*}{\rho} \tag{4.28}$$

At time 0, the bank selects an optimal total coupon l^* according to condition (4.20). In detail, the bank selects $l^* = l^{**}$ if the initial capital requirement is not violated. That is the case if $l^{**} < \bar{l}$. Otherwise, the institution's optimal total coupon has the corner solution $l^* = \bar{l}$. In this case, the solution binds at the ICR condition and the regulator has zero utility at $t = 0$.

The bank must also choose a combination of deposits d^* and non-deposit liabilities c^*. This is characterised in condition (4.23). The sum of deposit and non-deposit coupons must always equal the total coupon l^* as formulated in $d^* := l^* - c^*$ in condition (4.23).

I use the non-deposit coupon to write the interval of feasible optimal solutions. The upper-bound of the c^* interval is the optimal total coupon l^*. A non-deposit coupon c^* closer to the total coupon l^* requires a deposit coupon d^* closer to zero. The lower-bound of c^* depends on the type of bank. A weak bank selects a c^* that respects the NBR condition to prevent depositors' runs. A strong bank is not limited by the NBR condition and only needs to guarantee that the lower-bound of c^* is always zero to respect the non-negativity constraint. Figure 4.5 illustrates the interval for the present value of non-deposit liabilities c^*/ρ in equilibrium.

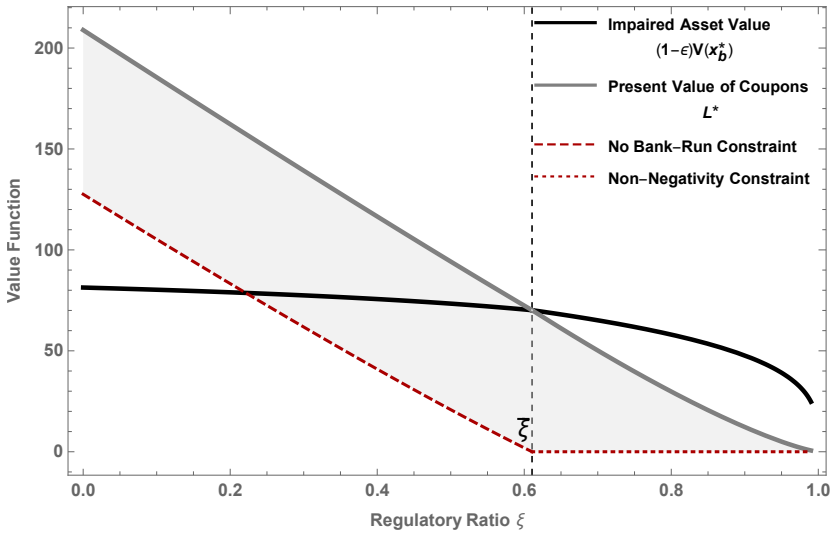


Figure 4.5: **Example of c^*/ρ interval. Parameters:** $\mu = -.02$, $\sigma = 0.1$; $\rho = 0.15$; $\epsilon = 0.5$. $\overline{\xi} = 0.61$. The grey area defines the interval for c^*/ρ. A strong bank ($\xi \geq \overline{\xi}$) has an impaired asset value that covers the present value of coupons for all creditors. The strong bank can choose any level of non-deposit liabilities as long as it is positive. A weak bank ($\xi < \overline{\xi}$) cannot cover the present value of coupons with the impaired asset value. To prevent runs the weak bank should hold at least the minimum level of c^*/ρ as represented in the red dashed line. Any coupon c^* that satisfies that condition prevents bank runs. Beyond that level, the bank is indifferent between deposit and non-deposit debt finance.

4.4 Costless Deposit Insurance

4.4.1 Elements of the Model

In this section, I add a Deposit Insurance Scheme (DIS) to my framework. I use the subscript "di" to distinguish the value functions, parameters and controls in this section.

▷ **The Bank.** The bank's assets generate cash flow x_t that follows the GBM defined in equation (4.1) with drift $\mu \in (-\infty, \rho)$, volatility $\sigma > 0$ and exogenous discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (4.2).

The bank selects coupon c_{di} for non-deposit creditors and a coupon d_{di} for depositors at time 0. These coupons are paid until the bank is closed. I write the total coupon as $l_{di} = c_{di} + d_{di}$. The accounting value of liabilities L_{di} is the present value of total future coupon payments:

$$L_{di} = \int_t^{+\infty} e^{-\rho(s-t)}(c_{di} + d_{di})ds = \frac{c_{di} + d_{di}}{\rho}, \tag{4.29}$$

At closure time, the assets of the failing bank are bought by another institution. As in Section 4.3.1, the impaired asset value after asset sales is $(1 - \epsilon)V(x_t)$ and, also as in Section 4.3.1, the corporate tax rate τ is levied on cashflows net of interest payments.

The bank's value function can be formulated as the sum of the value of the claims:

$$B_{di}(x_t) = E_{di}(x_t) + C_{di}(x_t) + D_{di}$$

where $E_{di}(x_t)$ is the value of the claim held by the equity-holders, $C_{di}(x_t)$ is the value function for Non-deposit Creditors and D_{di} is the value function for depositors. In line with Section 4.3.1, I write the capital structure ratio $CS_{di}(x_t)$ and the debt structure ratio $DS_{di}(x_t)$ as:

$$CS_{di}(x_t) := \frac{E_{di}(x_t)}{B_{di}(x_t)} \qquad\qquad DS_{di}(x_t) := \frac{C_{di}(x_t)}{C_{di}(x_t) + D_{di}}$$

▷ **Bank's Closure.** The regulator closes the bank at time T_{di} when cash-flows x_t first cross

the trigger x_{di}:

$$T_{di} = \inf \{t \in [0, +\infty) : x_t \notin [x_{di}, +\infty)\} \tag{4.30}$$

As in Subsection 4.3.1, the regulator trades off the utility benefit of immediate enforcement against the potential benefits achieved when the bank's cash flow recovers. Following Proposition 5, the closure trigger x_{di} in the DIS case is:

$$x_{di} = \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_{di} + d_{di}}{\rho} \tag{4.31}$$

where $\xi \in (0, 1)$ is an exogenous regulatory requirement ratio depicted in equation (4.4) and β is the elasticity of the closure option value to cash-flows outlined in equation (4.8).

▷ **Deposit Insurance.** Now assume that all depositors are fully protected by a costless deposit insurance scheme[24]. I assume that:

Assumption 13. *The deposit insurance fund never defaults.*

The deposit insurance fund covers the difference between the present value of deposits and the impaired asset value, if the latter is insufficient to pay the depositors. The deposit insurance scheme therefore makes the following transfer to bank depositors upon the bank's closure:

$$\max \left\{ 0, \frac{d_{di}}{\rho} - (1 - \epsilon) V(x_{di}) \right\} \tag{4.32}$$

Full deposit insurance prevents runs, and so it renders the No Bank-Run condition (4.17) irrelevant.

▷ **Equilibrium.** As in Section 4.3, the equilibrium has two stages: stage 1 runs from time 0 until the bank closes at time T_{di}, and stage 2 runs from time T_{di} onwards.

At time 0, the bank selects the non-deposit coupon c_{di} and the deposit coupon d_{di} so as to maximise the value function $B_{di}(x_0)$. When the bank is closed at time T_{di}, its impaired

[24] A natural extension is to account for an insurance fee in proportion of deposits as, for example, seen in the liability structure models of Sundaresan and Wang (2017) and Gambacorta et al. (2017). This fee increases the bank's cost of owning deposits. Banks will consequently be less willing to accept deposits as a source of debt. Morrison and White (2011) suggest however that capitalising the insurance fund through bank contributions is welfare-neutral. A better alternative is instead to use general taxation on agents outside the financial system prevents banks from crowding-out deposits from its liability structure.

asset value is distributed in line with the claimant hierarchy. The deposit insurance scheme is called upon only if the impaired asset value is insufficient to meet depositor claims.

As in Section 4.3, I solve for a sequential equilibrium $\mathcal{E}_{di}$:

$$\mathcal{E}_{di} := (c^*_{di}, d^*_{di}, x^*_{di})$$

At $t = 0$, the bank selects the value maximising pair of coupons (c^*_{di}, d^*_{di}). The regulator closes the bank at time T_{di}, when the cash-flow x_t first crosses the closure trigger $x^*_{di} := x_{di}(c^*_{di}, d^*_{di})$.

4.4.2 The Bank's Problem

This subsection presents the bank's optimisation problem under costless deposit insurance. I derive the value function of the bank and explain how the DIS affects the bank's claimants.

4.4.2.1 Value Functions

The value of the bank satisfies the following accounting identity at all times:

$$B_{di}(x_t) := E_{di}(x_t) + C_{di}(x_t) + D_{di} \quad , \forall t \in [0, T_{di})$$

I therefore start by determining the value functions for each one of these claims.

Shareholders For $t \in [0, T_{di})$, the value function of shareholders can be written as follows:

$$\begin{aligned} E_{di}(x_t) := \; & \mathbb{E}_t \left[\int_t^{T_{di}} e^{-\rho(s-t)} (1-\tau) \left(x_s - c_{di,s} - d_{di,s} \right) ds \right] + \\ & \mathbb{E}_t \left[\int_{T_{di}}^{+\infty} e^{-\rho(s-t)} \max \left\{ 0, (1-\tau) \left((1-\epsilon) x_{di} - c_{di} - d_{di} \right) \right\} ds \right] \end{aligned} \tag{4.33}$$

The first term in Equation (4.33) is the value to equity holders of the cash flows the bank generates while it is operating: at every $t \in [0, T_{di})$ shareholders receive after tax cash-flows net of interest payments $(1-\tau)\left(x_s - c_{di,s} - d_{di,s}\right)$.

The second term in Equation (4.33) is the value that equity holders derive from the bank when it is closed. The bank's assets are sold upon closure to another institution that receives all of their subsequent cash flows. As payment from the asset sale, the equity holders of

the closed bank receive the impaired asset value net of liabilities and taxes. That is, they receive the present value of the cash flow stream $(1-\tau)\left((1-\epsilon)x_{di} - c_{di} - d_{di}\right)$ from time T_{di} until infinity if that value is non-negative; if it is not, then, because they are protected by limited liability, they receive zero.

It is immediate from a comparison of Equation (4.10) with Equation (4.33) that the only effect that deposit insurance can have upon the value of equity is through its effect on the closure time T_{di}. That is because deposit insurance pays out only after default and, hence, it does not affect cash flows before time T_{di}. It follows immediately that, by analogy with Equation (4.11), $E_{di}(x_t)$ can be written as follows:

$$E_{di}(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right) - \left(\frac{x_t}{x_{di}}\right)^{\beta}(1-\tau)\min\left\{\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}, \epsilon\frac{x_{di}}{\rho-\mu}\right\} & , \text{ for } x_t > x_{di} \\ (1-\tau)\max\left\{0, (1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right\} & , \text{ for } x_t \leq x_{di} \end{cases} \tag{4.34}$$

The intuition for Equation (4.34) is identical to the intuition for Equation (4.11).

Non-Deposit Creditors Non-deposit creditors earn no direct benefit from a deposit insurance scheme. However, these creditors experience an indirect effect because deposit insurance alters the closure time T_{di} and, hence, affects the closure payoff. In particular, if closure is delayed so long that the impaired asset value at time T_{di} is below the present value of deposits, non-deposit creditors may receive a zero payoff upon closure.

The following expression for the value $C_{di}(x_t)$ of their claim follows by analogy with Equation (4.12)

$$C_{di}(x_t) := \mathbb{E}_t\left[\int_t^{T_{di}} e^{-\rho(s-t)}c_{di,s}ds + \int_{T_{di}}^{+\infty} e^{-\rho(s-t)}\min\left\{c_{di}\ , \max\left\{(1-\epsilon)x_{di} - d_{di}\ , 0\right\}\right\}ds\right] \tag{4.35}$$

Equation (4.35) follows the same intuition of Equation (4.12) with the sole difference that, with deposit insurance, non-deposit creditors can get a zero payoff at closure. The reason that this could happen with deposit insurance is as follows. Deposit insurance releases the bank from having a capital structure that prevents bank runs and, hence, it can select pairs

c_{di} and d_{di} for which the impaired asset value is lower than the present value of cash-flows: that is, for which $(1-\epsilon)V(x_{di}) < d_{di}/\rho$. If this happens, then, because non-deposit creditors are junior claimants, they receive a zero payoff.

Again by analogy with Section 4.3, the value function in Equation (4.35) can be written as follows:

$$C_{di}(x_t) = \begin{cases} \frac{c_{di}}{\rho} + \left(\frac{x_t}{x_{di}}\right)^{\beta} \min\left\{0\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\ , -\frac{c_{di}}{\rho}\right\}\right\} & ,\ \text{for } x_t > x_{di} \\ \\ \min\left\{\frac{c_{di}}{\rho}\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\ , 0\right\}\right\} & ,\ \text{for } x_t \leq x_{di} \end{cases} \tag{4.36}$$

The intuition for Equation (4.36) is identical to the intuition of Equation (4.13).

Depositors Deposit insurance protects depositors from losses when the impaired asset value $(1-\epsilon)V(x_{di})$ is lower than the present value of future deposit payments d_{di}/ρ. Therefore, the value of deposits does not depend upon the cash flow process x_t:

$$D_{di} = \mathbb{E}_t\left[\int_t^{+\infty} e^{-\rho(s-t)} d_{di}\, dt\right] = \frac{d_{di}}{\rho} \tag{4.37}$$

Total Bank Value Summing the claims of shareholders, non-protected creditors and depositors, I obtain the following value function for the bank in the presence of costless deposit insurance.

$$B_{di}(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{di}+d_{di}}{\rho} + \left(\frac{x_t}{x_{di}}\right)^{\beta}\left(B_{di,\mathcal{C}}(x_{di}) - (1-\tau)\frac{x_{di}}{\rho-\mu} - \tau\frac{c_{di}+d_{di}}{\rho}\right) & ,\ \text{for } x_t > x_{di} \\ \\ B_{di,\mathcal{C}}(x_{di}) & ,\ \text{for } x_t \leq x_{di} \end{cases} \tag{4.38}$$

Here $B_{di,\mathcal{C}}(x_{di})$ is the bank's closure value:

$$\begin{aligned} B_{di,\mathcal{C}}(x_{di}) := \frac{d_{di}}{\rho} + \min\left\{\frac{c_{di}}{\rho}\ , \max\left\{(1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\ , 0\right\}\right\} \\ +(1-\tau)\max\left\{0, (1-\epsilon)\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right\} \end{aligned}$$

While the bank is open ($x_t > x_{di}$), its value function is the sum of the present value of cash-flows after taxes, the debt tax shield and the expected change in value when the closure option is exercised. That change is the product of the expected discount factor $(x_t/x_{di})^{\beta}$ at the closure date and the difference between the bank's closure value $B_{di,\mathcal{C}}(x_{di})$ and the combined present value of cashflows $(1-\tau)x_{di}/(\rho-\mu)$ and the tax rebate $\tau(c_{di}+d_{di})/\rho$.

The three terms in the bank's closure value $B_{di,\mathcal{C}}(x_{di})$ are, respectively, the present value of payments to depositors, the non-deposit credit value at closure and the equity value at closure.

In Section 4.3.3.1, I defined a strong bank to be one whose impaired asset value is guaranteed to be sufficient to cover the present value of all future interest payments at closure $(1-\epsilon)V(x_{di}) \geq L_{di}$. A weak bank is one that does not satisfy this condition. Because banks no longer have to satisfy a no-run condition with a DIS, it is useful to distinguish between banks that do, and do not, need to call upon the DIS upon failure.

I define an *independent* bank to be a weak bank whose impaired asset value is sufficient to cover the present value of coupons promised to depositors; independent banks therefore do not draw upon the deposit insurance fund. An independent bank satisfies the following condition:

$$(1-\epsilon)V(x_b) \geq D_{di} \tag{4.39}$$

I define a *dependent* bank to be a weak bank that violates Condition (4.39). Dependent banks are obliged to draw upon the deposit insurance fund when they are closed. Lemma 8 relates the capital adequacy of a bank to its dependency upon the deposit insurance fund.

Lemma 8 (DIS Dependence Boundary)**.** *Let*

$$\underline{\xi} := \underline{\xi}(c_{di}, d_{di}) = \frac{c_{di}\beta(1-\epsilon) + d_{di}(1-\beta\epsilon)}{d_{di}(1-\beta)} \tag{4.40}$$

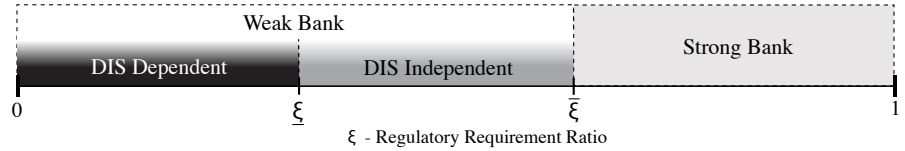
Then any weak bank whose capital regulation $\xi \geq \underline{\xi}$ is independent; any bank for which $\xi < \underline{\xi}$ is dependent.

The boundary $\underline{\xi}$ is the regulatory ratio that sets the bankruptcy asset value equal to the present value of deposits at the bank's closure. Banks with tighter capital requirements are

closed earlier and, hence, are better able to meet depositors payments.

Figure 4.6 illustrates the boundaries in the DIS case. Unlike the $\overline{\xi}$ boundary, $\underline{\xi}$ depends on the composition of debt. In Subfigure 4.6.a the weak bank has both types of debt and so, depending on the exogenous regulatory ratio ξ, it may be DIS dependent or independent. In Subfigure 4.6.b, the weak bank uses deposits as the single source of debt and, consequently, it will need the intervention of the deposit insurance fund at the bank's closure. In this situation, $\underline{\xi} = \overline{\xi}$. Finally, in Subfigure 4.6.c, the weak bank only uses non-deposit liabilities and so, when it closes, the non-deposit creditors will fully absorb the bankruptcy losses. In this situation, $\underline{\xi} = 0$.

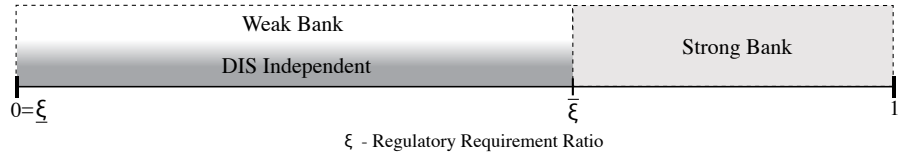
Since $\underline{\xi}$ depends upon coupon levels c_{di} and d_{di}, it is determined in equilibrium. I write the equilibrium DIS dependence boundary as $\underline{\xi}^* = \underline{\xi}(c^*_{di}, d^*_{di})$.



(a) Non-deposit coupon $c_{di} > 0$ and deposit coupon $d_{di} > 0$



(b) Non-deposit coupon $c_{di} = 0$ and deposit coupon $d_{di} = l_{di}$



(c) Non-deposit coupon $c_{di} = l_{di}$ and deposit coupon $d_{di} = 0$

Figure 4.6: Weak bank boundaries in the presence of costless deposit insurance.

4.4.2.2 Initial Capital Requirement

I continue to impose Assumption 12: that is, that the bank respects the capital requirement ξ when it selects its initial capital structure. The Assumption yields the following Initial Capital Requirement:

$$c_{di} + d_{di} \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{4.41}$$

4.4.2.3 Non-Negative Coupons

By Assumption 9, each type of liability must be non-negative:

$$c_{di} \geq 0 \qquad \text{and} \qquad d_{di} \geq 0 \tag{4.42}$$

4.4.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_{di}, d_{di}) so as to maximise the total bank value $B_{di}(x_0)$ subject to the regulator's optimal closure trigger in condition (4.31), the Initial Capital Requirement condition (4.41) and the non-negative coupon condition (4.42).

Optimisation Problem 5 (Capital Structure with deposit insurance)**.** *The bank's optimal capital structure problem is:*

$$\begin{aligned}
(c^*_{di}, d^*_{di}) \quad & \in \underset{\{c_{di}, d_{di}\}}{\arg\max}\ B_{di}(x_0) \\
s.t. \qquad & x_{di} := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_{di}+d_{di}}{\rho} && \textit{[Closure Trigger]} \\
& c_{di} + d_{di} \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 && \textit{[Initial Capital Requirement]} \\
& c_{di} \geq 0 \quad , \quad d_{di} \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}$$

4.4.3 The Deposit Insurance Equilibrium

Proposition 7 (Costless Deposit Insurance Case Equilibrium)**.** *The equilibrium of the game with costless deposit insurance is:*

$$\mathcal{E}_{di} = (c^*_{di}, d^*_{di}, x^*_{di}) \tag{4.43}$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}$$

Bank Optimal Decision. *The total periodic coupon* $l^*_{di} = c^*_{di} + d^*_{di}$ *is optimal iff:*

$$l^*_{di} = \min\left\{l^{**}_{di}, \bar{l}_{di}\right\} \tag{4.44}$$

where

$$l^{**}_{di} = \begin{cases} \frac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0, & \text{for } \xi < \overline{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi \geq \overline{\xi} \end{cases} \tag{4.45}$$

and

$$\bar{l}_{di} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{4.46}$$

Any combination of c^*_{di} *and* d^*_{di} *that respects the following conditions is optimal:*

$$d^*_{di} := l^*_{di} - c^*_{di} \qquad \text{and} \qquad c^*_{di} = \begin{cases} 0 & , \quad \xi < \overline{\xi} \\ c^{**}_{di} & , \quad \xi \geq \overline{\xi} \end{cases} \tag{4.47}$$

with

$$c^{**}_{di} \in \begin{cases} \left(0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right) & , \quad \text{for} \quad l^{**} \geq \bar{l} \\ \left(0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right) & , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{4.48}$$

Regulator Optimal Decision. *At time* T_{di}*, cash-flows* x_t *hit the trigger* x^*_{di}*, the Regulator steps in and closes the bank. That trigger is:*

$$x^*_{di} = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c^*_{di}+d^*_{di}}{\rho} \tag{4.49}$$

The bank chooses total coupon l^*_{di} according to condition (4.44). It selects the interior solution $l^*_{di} = l^{**}_{di}$ if the initial capital requirement is respected. Otherwise, the bank must select the total coupon $l^* = \bar{l}$ that enforces a regulatory utility of zero at $t = 0$.

The bank selects an optimal combination of deposits d^* and non-deposit liabilities c^* in accordance to condition (4.47). To guarantee that the sum of c^*_{di} and d^*_{di} respects the

equilibrium total coupon l^* we set $d^* := l^* - c^*$.

Weak banks always select $c^*_{di} = 0$ when they have access to costless deposit insurance. This means that the equilibrium dependence boundary $\underline{\xi}^*$ equals the bank's type boundary $\overline{\xi}$ and, so, weak banks choose to become DIS dependent and crowd-out all non-deposit debt. So, in equilibrium, weak banks choose the case presented in Figure 4.6.b.

A strong bank selects the optimal non-deposit coupons c^*_{di} from the interval of admissible c^*_{di} defined in condition (4.48). The strong bank must always select a positive non-deposit coupon and, at most, should select a coupon close but lower than the total coupon l^*_{di} which implicitly secures that the regulatory utility is never negative when the bank chooses the capital structure.

4.4.4 Discussion on the effects of introducing a DIS

I discuss the impact of introducing costless deposit insurance in the equilibrium by comparing the DIS equilibrium in Proposition 7 against the base case equilibrium in Proposition 6. I write the equilibrium capital structure at time 0 for the base case as $CS^*(x_0) \equiv CS(x_0|\mathcal{E})$ and for the DIS case as $CS^*_{di}(x_0) \equiv CS_{di}(x_0|\mathcal{E}_{di})$. I also write the equilibrium debt structure at time 0 for the base case as $DS^*(x_0) \equiv DS(x_0|\mathcal{E})$ and for the DIS case as $DS^*_{di}(x_0) \equiv DS_{di}(x_0|\mathcal{E}_{di})$.

Proposition 8 (Change in the Equilibrium Total Coupon)**.** *Introducing costless deposit insurance weakly increases the equilibrium total coupon paid by the bank:*

$$l^*_{di} \geq l^* \tag{4.50}$$

Proposition 9 (Change in the Equilibrium Debt Structure)**.** *Introducing costless deposit insurance weakly decreases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS^*_{di}(x_0) \leq DS^*(x_0) \tag{4.51}$$

Proposition 10 (Change in the Equilibrium Capital Structure)**.** *Introducing costless deposit*

insurance weakly decreases the equilibrium ratio between equity value and total bank value:

$$CS^*_{di}(x_0) \leq CS^*(x_0) \tag{4.52}$$

Propositions 8 to 10 state that, in equilibrium, banks will neither decrease leverage nor decrease deposits if the regulator introduces costless deposit insurance. Precisely what the bank does depends upon its type.

▷ **The Effect of DIS in Weak Banks.** By Lemma 8, the weak bank can be DIS-dependent or independent depending on the combination of c_{di} and d_{di} it selects. In equilibrium, it takes advantage of costless deposit insurance and chooses to be DIS dependent.

With a costless DIS, deposits become more attractive than non-deposits: deposits are covered by the DIS while non-deposit liabilities experience losses at closure (because $(1-\epsilon)V(x_b) < c_{di}/\rho$) if the institution is weak. Therefore, in equilibrium, the weak bank choses to use deposits as its sole debt source:

$$DS^*_{di}(x_0|\xi < \overline{\xi}) = 0 < DS^*(x_0|\xi < \overline{\xi})$$

Then, because it has only deposits in its liability structure, the bank increases its equilibrium leverage since the costs of closure are now born by the insurance fund and the institution can therefore take greater advantage of the debt tax shield. The equilibrium total coupon l^*_{di} is therefore weakly higher than the base case total coupon l^*, and the bank's equity capital level weakly decreases:

$$CS^*_{di}(x_0|\xi < \overline{\xi}) \leq CS^*(x_0|\xi < \overline{\xi})$$

The lower equilibrium equity/bank value ratio reflects a higher debt value and a lower equity value relative to the case without a DIS. The increase in debt is caused by the higher coupon l^*_{di}. Higher leverage lowers the expected value of equity by decreasing the present value of cash-flows after interest payments $(1-\tau)(V(x_t) - (c_{di} + d_{di})/\rho)$, and by increasing

the closure trigger ($\partial x_{di}/\partial l_{di} > 0$) so that the bank is closed earlier.

▷ **The Effect of DIS in Strong Banks.** Strong banks do not change their funding decision when the regulator introduces costless deposit insurance. This is because these institutions do not trigger a payout by the deposit insurance fund. If a strong bank is closed, the impaired asset value always covers the promised future payments to all creditors and, so, its expected payoffs stay unchanged.

A strong bank is unaffected by the introduction of costless deposit insurance; its optimal total debt coupon l^*_{di} is the same as the base case total coupon l^*. Then, as in the proofs of Propositions 9 and 10, the capital structure ratio and the debt structure ratio are unchanged:

$$CS^*_{di}(x_0|\xi \geq \overline{\xi}) = CS^*(x_0|\xi \geq \overline{\xi}) \qquad DS^*_{di}(x_0|\xi \geq \overline{\xi}) = DS^*(x_0|\xi \geq \overline{\xi})$$

The following corollary follows from Propositions 8 to 10:

Corollary 10.1. *Costless deposit insurance incentivises weak financial institutions to use deposits as the exclusive source of debt and to increase total leverage at the expense of the insurance fund. Higher leverage decreases equity value and, hence, the capital structure ratio of weak banks decreases. The incentives of strong bank are not affected by costless deposit insurance.*

▷ **Implementing the change in the Capital Structure.** I conclude this Section by examining the way in which the bank implements the new capital structure when deposit insurance is introduced, and with an examination of the wealth effects of the introduction. The executive team of the bank will never make a capital structure decision that renders the existing shareholders worse-off.

When deposit insurance is introduced, the bank's executive team buys back equity, and pays for it with a partial claim over deposits. The value of this claim over deposits should preserve the existing shareholders' value. I use η_{di} to represent the share of deposits held by

existing shareholders after the capital structure changes. We must have:

$$E_{di}(x_0|\mathcal{E}_{di}) + \eta_{di} D_{di}(\mathcal{E}_{di}) \geq E(x_0|\mathcal{E})$$

That is, the final value of the existing shareholder's claim after the change in capital structure should be at least as high as equity value before the DIS is introduced.

Moreover, the value of the shareholders's claim over deposits must be bounded by the bank value increase that results from the change in the capital structure. That is:

$$\eta_{di} D_{di}(\mathcal{E}_{di}) \leq B_{di}(x_0|\mathcal{E}_{di}) - B(x_0|\mathcal{E})$$

Using these two inequalities, I can find the admissible range of η_{di} that maintains existing shareholders satisfied with the capital structure change in response to the introduction of a DIS.

$$\eta_{di} \in \left[\frac{E(x_0|\mathcal{E}) - E_{di}(x_0|\mathcal{E}_{di})}{D_{di}(\mathcal{E}_{di})}, \frac{B_{di}(x_0|\mathcal{E}_{di}) - B(x_0|\mathcal{E})}{D_{di}(\mathcal{E}_{di})} \right] \tag{4.53}$$

In summary, the introduction of a costless deposit insurance changes the incentives of weak financial institutions but not strong institutions. Weak banks use deposits as their exclusive source of debt because they can transfer closure costs to the deposit insurance fund at no expense (insurance fees). Hence, weak banks choose to be DIS-dependent. At the same time, this gives an incentive to leverage up the institution. However, this leverage increase causes a decrease in the value of equity. Existing shareholders will see the value of the claim decrease because higher debt coupons lower the expected residual claim over cash-flows and increase the intervention trigger. Nonetheless, the capital structure is designed by the bank and, hence, existing shareholders should be compensated for the loss of expected value. This is attained through a buyback of shares in exchange for a claim over deposits.

4.5 Private Sector Acquisition

4.5.1 Elements of the Model

The post-financial crisis European regulation grants authorities the right to apply resolution tools to failing financial institutions. In this section, I add the Private Sector Acquisition (PSA) resolution tool to my framework. I use the subscript "p" to distinguish the value functions, parameters and controls in this section.

▷ **The Bank.** The bank's assets generate cash flow x_t following the GBM defined in equation (4.1) with drift $\mu \in (-\infty, \rho)$, standard deviation $\sigma > 0$ and exogenous discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (4.2).

The bank selects coupon c_p for non-deposit creditors and a coupon d_p for depositors at time 0. These coupons are paid until the bank is closed. The total coupon is $l_p = c_p + d_p$. The accounting value of liabilities L_p is the present value of total future coupon payments:

$$L_p = \int_t^{+\infty} e^{-\rho(s-t)}(c_{di} + d_{di})ds = \frac{c_{di} + d_{di}}{\rho}, \tag{4.54}$$

Under the PSA, the sale of assets generates costs of magnitude $\epsilon_p V(x_t)$, with $\epsilon_p \in (1, 0)$. The *impaired asset value* is written as $(1 - \epsilon_p)V(x_t)$. The corporate tax rate $\tau \in (0, 1)$ is levied on cash-flows net of interest payments $(1 - \tau)(x_t - c_{di} - d_{di})$.

The bank selects coupons (c_p, d_p) so as to maximise its continuation value function $B_p(x_t)$:

$$B_p(x_t) = E_p(x_t) + C_p(x_t) + D_p$$

where $E_p(x_t)$ is the value of equity under a PSA, $C_p(x_t)$ is the value function for Non-deposit Creditors and D_p is the value function for depositors. Following Section 4.3, I write the capital structure ratio $CS_p(x_t)$ as the ratio between equity value and bank value and the debt structure ratio $DS_{di}(x_t)$ as the ratio between the non-deposit creditors value and total liabilities value:

$$CS_p(x_t) := \frac{E_p(x_t)}{B_p(x_t)} \qquad\qquad DS_p(x) := \frac{C_p(x_t)}{C_p(x_t) + D_{p,t}}$$

▷ **Bank's Closure.** Following Subsection 4.3.1, the bank is closed at time T_p, when cash-flows x_t first cross the trigger x_p:

$$T_p = \inf \{t \in [0, +\infty) : x_t \notin [x_p, +\infty)\} \tag{4.55}$$

Similarly to Proposition 5, the closure trigger x_p in the PSA case is:

$$x_p = \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_p + d_p}{\rho} \tag{4.56}$$

where $\xi \in (0, 1)$ is the exogenous regulatory requirement ratio in equation (4.4) and β is the elasticity of the closure option value to cash-flows in equation (4.8).

▷ **Private Sector Acquisitions.** Authorities use the Private Sector Acquisition (PSA) tool to intermediate the sale of the assets of a failing bank to other financial institution, regardless of stakeholders' approval[25]. The only way that the PSA changes my model is through its effect upon the closure costs:

Assumption 14. *Closure costs are always lower under the PSA. That is:*

$$\epsilon > \epsilon_p$$

The PSA tool decreases closure costs because a resolution authority can avoid externalities that occur in private-sector resolutions (i.e., in bankruptcies managed by creditors). For example, the PSA avoids "rush to the courthouse" and "hold out" externalities (Roubini 2004) that would cause high fire sales costs. The "rush to the courthouse" externality occurs when a group of creditors initiate a lawsuit and attach some of the assets to the lawsuit, preventing other creditors from accessing those assets.

The "holdout" (or "free-riding") externality happens when an advantageous resolution

[25] For example, under the European Central Bank supervision, this operation comprehends assets, liabilities or even shares (See Art. 38 and 39 of BRRD and Art. 24 of SRM for further details) whereas, for example, under the supervision of the Bank of England, the equivalent tool is restricted to assets.

plan does not reach the required quorum in a creditors' assembly. In this circumstance, some creditors may want to haste the resolution process by agreeing to pay these holdouts. However, that ultimately creates an incentive for more creditors to hold out as well. This stalls the sale of assets and erodes the chance for an orderly and less costly resolution.

▷ **Type of Equilibrium.** As in Sections 4.3 and 4.4, the equilibrium has two stages: stage 1 runs from time 0 until the bank closes at time T_p, and stage 2 runs from time T_p onwards.

The bank selects the optimal coupons (c_p^*, d_p^*) and the regulator then chooses the closure time T_p. I write $x_p^* := x_p(c_p^*, d_p^*)$ for the bank's closure trigger. The equilibrium $\mathcal{E}_p$ of this sequential game is:

$$\mathcal{E}_p := \left(c_p^*, d_p^*, x_p^*\right)$$

4.5.2 The Bank's Problem

In this subsection, I present the bank's optimisation problem under the private sector acquisition tool. I derive the value function of the bank and explain the impact that this resolution tool has over the bank's claimants.

4.5.2.1 Value Functions

The value of the bank satisfies the accounting identity:

$$B_p(x_t) := E_p(x_t) + C_p(x_t) + D_p \quad , \forall t \in [0, T_p)$$

The value functions for the three claimants have the same structure as in Section 4.4. However, the PSA affects the payoffs at closure for some of the claimants.

Shareholders For $t \in [0, T_p)$, the shareholders' value function can be written as follows

$$E_p(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right) - \left(\frac{x_t}{x_p}\right)^{\beta}(1-\tau)\min\left\{\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}, \epsilon_p\frac{x_p}{\rho-\mu}\right\} & , \text{ for } x_t > x_p \\ (1-\tau)\max\left\{0, (1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right\} & , \text{ for } x_t \leq x_p \end{cases} \tag{4.57}$$

Equation (4.57) has the same intuition as Equation (4.34). The change in the closure cost fraction from ϵ to ϵ_p increases the payoffs at closure for the shareholders of strong banks. Shareholders of weak banks are unaffected at closure because, irrespective of the introduction of the PSA tool, their payoff is zero.

Non-Deposit Creditors The value function for non-deposit creditors can be written as follows:

$$C_p(x_t) = \begin{cases} \frac{c_p}{\rho} + \left(\frac{x_t}{x_p}\right)^{\beta}\min\left\{0\ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}\ , -\frac{c_p}{\rho}\right\}\right\} & , \text{ for } x_t > x_p \\ \min\left\{\frac{c_p}{\rho}\ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{d_p}{\rho}\ , 0\right\}\right\} & , \text{ for } x_t \leq x_p \end{cases} \tag{4.58}$$

The components of Equation (4.58) have a similar interpretation to those of Equation (4.36). The decrease in closure costs only affects the closure payoffs for non-deposit creditors of weak independent banks; in those banks, non-deposit creditors receive the residual value after paying deposits: that is, they receive $(1-\epsilon_p)x_p/(\rho-\mu) - d_p/\rho$.

If the bank is not a weak-independent bank, non-deposit creditors do not suffer changes in their closure payoffs: creditors of strong banks receive their full promised claim c_p/ρ, while creditors of weak-dependent banks have a payoff of zero since impaired asset value is entirely used to pay depositors.

Depositors The value function of depositors is unaffected by the introduction of a PSA.

$$D_p = \left[\int_t^{+\infty} e^{-\rho(s-t)} d_p dt\right] = \frac{d_p}{\rho}, \quad \text{for } t \geq 0 \tag{4.59}$$

Total Bank Value The total value function for the financial institution is:

$$B_p(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_p+d_p}{\rho} + \left(\frac{x_t}{x_p}\right)^{\beta}\left(B_{p,\mathcal{C}}(x_p) - (1-\tau)\frac{x_p}{\rho-\mu} - \tau\frac{c_p+d_p}{\rho}\right) & , \text{ for } x_t > x_{di} \\ \\ B_{p,\mathcal{C}}(x_p) & , \text{ for } x_t \leq x_p \end{cases} \tag{4.60}$$

where $B_{p,\mathcal{C}}(x_p)$ is the bank's value at the closure stage and is written as:

$$B_{p,\mathcal{C}}(x_p) := \frac{d_p}{\rho} + \min\left\{\frac{c_p}{\rho}\ , \max\left\{(1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{d_p}{\rho}\ , 0\right\}\right\}(1-\tau)\max\left\{0, (1-\epsilon_p)\frac{x_p}{\rho-\mu} - \frac{c_p+d_p}{\rho}\right\}$$

While the financial institution is open ($x_t > x_p$), its value function is the sum of the present value of cash-flows after taxes, the debt tax shield and the expected change in value when the closure option is exercised. That change is the product of the expected discount factor $(x_t/x_p)^{\beta}$ at the closure date and the difference between the bank's closure value $B_{p,\mathcal{C}}(x_p)$ and the combined present value of cashflows $(1-\tau)x_p/(\rho-\mu)$ and the tax rebate $\tau(c_p+d_p)/\rho$.

At closure ($x_t \geq x_p$), the bank has value $B_{p,\mathcal{C}}(x_p)$ which comprises the closure value for depositors, non-deposit creditors and shareholders.

The change in the closure cost fraction from ϵ to ϵ_p affects the boundary $\overline{\xi}$ that separates strong from weak banks. Lemma 9 characterises the boundary with a PSA tool:

Lemma 9 (Bank's Type Boundary under PSA)**.** *Let:*

$$\overline{\xi}_p = \frac{\beta\epsilon_p - 1}{\beta - 1} \tag{4.61}$$

Then any bank whose capital regulation $\xi \geq \overline{\xi}_p$ is strong; any bank for which $\xi < \overline{\xi}_p$ is weak.

The intuition for Lemma 9 is similar to Lemma 6. The boundary $\overline{\xi}_p$ is the regulatory ratio that sets the impaired asset value under PSA equal to the present value of coupons at the bank's closure. Given Assumption 14, the bank's type boundary under PSA $\overline{\xi}_p$ is always lower than $\overline{\xi}$.

Figure 4.7 illustrates the change in boundaries along the range of the regulatory require-

ment ratio ξ. The introduction of the PSA increases efficiency at closure: it lowers the closure costs which consequently lowers the threshold that separates strong from weak banks. In this case, a subset of institutions that were previously deemed as weak banks are now considered strong. Hence, introducing the PSA tool makes some institutions better off.

Figure 4.8 illustrates the impact of PSA upon $\overline{\xi}$ for different growth rates μ of assets. For example, consider a bank that has a growth rate of assets $\mu = 0.01$ and faces a regulatory ratio $\xi = 0.25$. The decrease in closure costs from $\epsilon = 0.4$ to $\epsilon = 0.1$ moves the type boundary from $\overline{\xi} = 0.63$ to $\overline{\xi}_p = 0.16$ after the PSA. This weak bank becomes a strong institution with the introduction of the PSA tool.

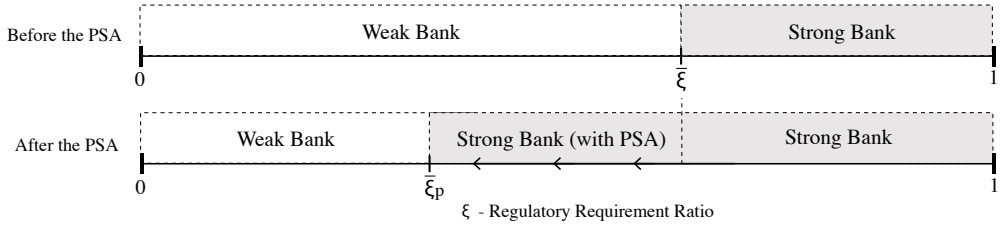


Figure 4.7: **Impact of a change in the closure cost fraction from ϵ to ϵ_p.**

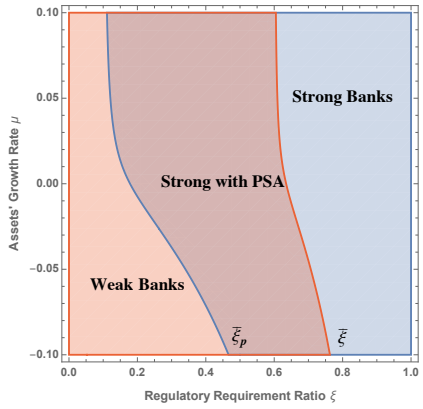


Figure 4.8: **Example of impact of a change of closure cost fraction for different growth rates $\mu \in [-.1, .1]$.** The right region (blue area) comprises the banks that are strong before the PSA. The left region (red area) includes the weak banks that remain weak after the PSA. The intermediate region corresponds to banks that were weak when closures were privately managed but became strong after the PSA is introduced.**Fixed Parameters:** $\sigma = 0.05$; $\rho = 0.15$; $\epsilon = 0.6$; $\epsilon_p = 0.1$.

4.5.2.2 Initial Capital Requirement

Under Assumption 12, the following Initial Capital Requirement (ICR) must hold:

$$c_p + d_p \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{4.62}$$

4.5.2.3 Non-Negative Coupons

By Assumption 9, each type of liability must be non-negative:

$$c_p \geq 0 \qquad \text{and} \qquad d_p \geq 0 \tag{4.63}$$

4.5.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_p, d_p) so as to maximise the total bank value $B_p(x_0)$ subject to the regulator's optimal closure trigger in condition (4.56), the ICR condition (4.62) and the non-negative coupons condition (4.63).

Optimisation Problem 6 (Bank)**.** *The optimal capital structure problem for the bank is:*

$$\begin{aligned}
&\left(c_p^*, d_p^*\right) \quad \in \underset{\{c_p, d_p\}}{\arg\max}\ B_p(x_0) \\
&s.t. \qquad x_p := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} && \textit{[Closure Trigger]} \\
&\qquad\quad c_p + d_p \leq \frac{\rho(1-\xi)}{\rho-\mu} x_0 && \textit{[Initial Capital Requirement]} \\
&\qquad\quad c_p \geq 0 \quad , \quad d_p \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}$$

4.5.3 The Private Sector Acquisition Equilibrium

Proposition 11 (Private Sector Acquisition Case Equilibrium)**.** *The equilibrium of the game with PSA is:*

$$\mathcal{E}_p = \left(c_p^*, d_p^*, x_p^*\right) \tag{4.64}$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}_p$$

Bank Optimal Decision. *The total periodic coupon* $l_p^* = c_{di}^* + d_p^*$ *is optimal iff:*

$$l_p^* = \min\left\{l_p^{**}, \bar{l}_p\right\} \tag{4.65}$$

where

$$l_p^{**} = \begin{cases} \frac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0, & \text{for } \xi < \overline{\xi}_p \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{4.66}$$

and

$$\bar{l}_p = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{4.67}$$

Any combination of (c_p^*, d_p^*) *is a feasible optimal solution, provided that it respects the following conditions:*

$$d_p^* := l_p^* - c_p^* \qquad \text{and} \qquad c_p^* = \begin{cases} 0 & , \quad \xi < \overline{\xi}_p \\ c_p^{**} & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{4.68}$$

with

$$c_p^{**} \in \begin{cases} \left(0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right) & , \quad \text{for} \quad l_p^{**} \geq \bar{l}_p \\ \left(0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right) & , \quad \text{for} \quad l_p^{**} < \bar{l}_p \end{cases} \tag{4.69}$$

Regulator Optimal Decision. *At time* T_p*, cash-flows* x_t *hit the trigger* x_p^**, the Regulator steps in and imposes a bankruptcy. That trigger is:*

$$x_p^* = \frac{\beta}{\beta-1} \frac{\rho-\mu}{1-\xi} \frac{c_p^* + d_p^*}{\rho} \tag{4.70}$$

4.5.4 Discussion on the effects of introducing a PSA

I discuss the impact of introducing private sector acquisition tool in the equilibrium by comparing the PSA equilibrium in Proposition 11 against the costless deposit insurance

equilibrium in Proposition 7. I write the equilibrium capital structure at time 0 for the DIS case as $CS^*_{di}(x_0) \equiv CS_{di}(x_0|\mathcal{E}_{di})$ and for the PSA case as $CS^*_p(x_0) \equiv CS_p(x_0|\mathcal{E}_p)$. I also write the equilibrium debt structure at time 0 for the DIS case as $DS^*_{di}(x_0) \equiv DS_{di}(x_0|\mathcal{E}_{di})$ and for the PSA case as $DS^*_p(x_0) \equiv DS_p(x_0|\mathcal{E}_p)$.

Proposition 12 (Change in the Equilibrium Total Coupon)**.** *Introducing the PSA tool weakly increases the equilibrium total coupon paid by the bank:*

$$l^*_p \geq l^*_{di}$$

Proposition 13 (Change in the Equilibrium Debt Structure)**.** *Introducing the PSA tool weakly increases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS^*_p(x_0) \geq DS^*_{di}(x_0) \tag{4.71}$$

Proposition 14 (Capital Structure Change)**.** *Introducing the PSA tool weakly decreases the equilibrium ratio between equity value and total bank value:*

$$CS^*_p(x_0) \leq CS^*_{di}(x_0) \tag{4.72}$$

Propositions 12 to 14 state that, in equilibrium, banks will not decrease leverage if the regulator introduces the private sector acquisition tool. These results are contingent on the type of the bank.

▷ **The Effect of PSA in Weak Banks.** If the bank is weak ($\xi < \overline{\xi}_p$), costless deposit insurance neutralises the impact of the PSA over the capital structure decision. This is because the bank only uses deposits in its liability structure. As discussed in Subsection 4.4.4, having deposits as an exclusive source of debt transfers the closure costs to the insurance fund. Therefore, introducing the PSA benefits the insurance fund by decreasing closure

costs but it does not affect the bank's claimants. Hence, the weak bank does not change its capital structure:

$$CS_p^*(x_0|\xi < \overline{\xi}_p) = CS_{di}^*(x_0|\xi < \overline{\xi}_p)$$

▷ **The Effect of PSA in Strong Banks.** The decrease in closure costs from ϵ to ϵ_p incentivises the strong bank to increase the total coupon from l_{di} to l_p which leads to an increase in total debt. The bank does so to explore the debt tax shield.

However, an increase in debt has two important effects for shareholders: first, it decreases the residual present value of cash-flows available for shareholders $(1-\tau)\left(V(x_t) - L_p\right)$ and, second, it increases the optimal closure trigger $x_p > x_{di}$ (earlier closure). These two effects cause a decrease in the equity value.

Therefore, the strong bank lowers the equity over total bank value ratio:

$$CS_p^*(x|\xi > \overline{\xi}) \leq CS_{di}^*(x|\xi > \overline{\xi})$$

The PSA does not change the debt structure of the strong bank. Despite the increase in leverage, the mix between deposits and non-deposit liabilities should not change as a consequence of PSA. So, the debt structure in equilibrium should be:

$$DS_p^*(x_0|\xi > \overline{\xi}) = DS_{di}^*(x_0|\xi > \overline{\xi})$$

In the intermediate region ($\xi \in [\overline{\xi}_p, \overline{\xi}]$), where a bank is only strong under the presence of the PSA tool, the capital structure results are similar to the strong bank region. The bank increases leverage and suffers a decrease in the value of the equity claim; as a result, its equity/value ratio decreases:

$$CS_p^*(x_0|\xi \in [\overline{\xi}_p, \overline{\xi}]) \leq CS_{di}^*(x_0|\xi \in [\overline{\xi}_p, \overline{\xi}])$$

Banks in the intermediate region change their debt structure ratio so that they use both types of debt. The bank can always select a residual amount of non-deposit liabilities so

that the debt structure does not materially differ significantly from the pre-PSA case. If a weak bank becomes strong with the PSA, it no longer has an incentive to use deposits as the exclusive source of debt.

From Propositions 12 to 14 and the above description, I get the following corollary:

Corollary 14.1. *Introducing the private sector acquisition tool incentivises all strong banks to increase leverage. Higher leverage decreases equity value and, hence, the capital structure ratio of strong banks decreases. Strong banks that were weak before the PSA no longer have an incentive to hold deposits as the only source of debt. The incentives of weak banks are not affected by the PSA.*

▷ **Implementing the change in the Capital Structure.** As in Subsection 4.4.4, the executive team must preserve the existing shareholders' value after the capital structure change. They can achieve this by buying back equity shares and paying with a claim over total liabilities.

If the bank wishes to maintain the debt structure, the claim over total liabilities must maintain the existing proportion of deposits and non-deposit liabilities. Using η_p to represent the share of ex-post liabilities held by existing shareholders, I write:

$$E_p(x_0|\mathcal{E}_p) + \eta_p \left(C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)\right) \geq E_{di}(x_0|\mathcal{E}_{di})$$

This expression requires the total value of the shareholders' claim after the change in capital structure, to be weakly greater than the equity value before the PSA is introduced.

Furthermore, shareholders should not receive more than the change in value caused by the new capital structure:

$$\eta_p \left(C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)\right) \leq B_p(x_0|\mathcal{E}_p) - B_{di}(x_0|\mathcal{E}_{di})$$

Using these two inequalities, I can find the admissible range of η_p that maintains existing shareholders satisfied with the capital structure change in response to the introduction of a

PSA resolution tool.

$$\eta_p \in \left[\frac{E_{di}(x_0|\mathcal{E}_{di}) - E_p(x_0|\mathcal{E}_p)}{C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)}, \frac{B_p(x_0|\mathcal{E}_p) - B_{di}(x_0|\mathcal{E}_{di})}{C_p(x_0|\mathcal{E}_p) + D_p(\mathcal{E}_p)} \right] \tag{4.73}$$

In summary, the introduction of a private sector acquisition tools changes the incentives of all strong banks. This includes the banks that were strong before the PSA and the banks that became strong only after the PSA. The banks that remain weak are not affected by this resolution tool.

In terms of capital structure, the decrease in closure costs incentivises strong banks to leverage up the institution. However, this increase in leverage causes a decrease in the value of equity because higher debt coupons lower the shareholders' residual claim over cash-flows and increase the intervention trigger.

Regarding the debt structure, I see that banks that were strong before the PSA maintain the same debt structure. Banks that became strong only after the PSA no longer have an incentive to hold deposits as the sole source of debt. So, I expect these banks to add non-deposit liabilities though my model does not prescribe an optimal change. Finally, weak banks are unaffected by the PSA and do not change their debt structure.

In terms of implementation, the bank's executive team has to compensate existing shareholders if a change in the equilibrium capital structure decreases the value of the equity claim. Thence, it is expected them to be given a claim over the new debt that is at least as valuable as the equity claim value prior to the PSA.

4.6 The Bail-In Case

4.6.1 Elements of the Model

In this section, I introduce the Bail-in resolution tool, which I model as a debt to equity exchange imposed by the regulator.

▷ **The Bank.** The assets of the bank generate cash-flows represented by x_t following the GBM defined in equation (4.1) with drift $\mu \in (-\infty, \rho)$, volatility $\sigma > 0$ and discount rate $\rho > 0$. The value of assets $V(x_t)$ is the present value of future cash-flows as in Equation (4.2).

The bank selects coupon c_{bi} for non-deposit creditors and a coupon d_{bi} for depositors at time 0. I write the total coupon as $l_{bi} = c_{bi} + d_{bi}$. The accounting value of total liabilities L_{bi} is the present value of total future coupon payments:

$$L_{bi} = \int_t^{+\infty} e^{-\rho(s-t)}(c_{bi} + d_{bi})ds = \frac{c_{bi} + d_{bi}}{\rho}, \tag{4.74}$$

The assets of the failing bank are bought by another institution at the closure time. As in Section 4.5.1, the impaired asset value after asset sales is $(1 - \epsilon_p)V(x_t)$, where ϵ_p is the closure cost fraction under the PSA tool. Furthermore, as in the previous sections, the corporate tax rate τ is levied on cashflows net of interest payments.

The bank's value function $B_{bi}(x_t)$ is the sum of the value of the claims:

$$B_{bi}(x_t) = E_{bi}(x_t) + C_{bi}(x_t) + D_{bi}$$

where $E_{bi}(x_t)$ is the equity claim value, $C_{bi}(x_t)$ is the non-deposit credit value and D_{di} is the value of deposits. When needed, I identify value functions according to the stage ti which they refer: I write $B_{bi,n}(x_t)$ for the bank's value function at stage $n \in \{1, 2, \mathcal{C}\}$. Stage 1 is the stage preceding the bail-in resolution; stage 2 succeeds the bail-in; and stage $\mathcal{C}$ is the closure stage. I use the same notation for value functions $E_{bi}(x_t)$, $C_{bi}(x_t)$ and D_{bi}.

The capital structure ratio $CS_{bi}(x_t)$ and the debt structure ratio $DS_{bi}(x_t)$ are defined as

follows:

$$CS_{bi}(x_t) := \frac{E_{bi}(x_t)}{B_{bi}(x_t)} \qquad\qquad DS_{bi}(x) := \frac{C_{bi}(x_t)}{C_{bi}(x_t) + D_{bi}}$$

▷ **Bail-In.** The regulator imposes a Bail-in over the capital structure of the bank at the bail-in time T^1_{bi}, when cash-flows x_t first cross the trigger x^1_{bi}:

$$T^1_{bi} = \inf\left\{t \in [0, +\infty) : x_t \notin [x^1_{bi}, +\infty)\right\} \tag{4.75}$$

Following Proposition 5, the bail-in trigger x^1_{bi} is written as:

$$x^1_{bi} = \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_{bi} + d_{bi}}{\rho} \tag{4.76}$$

where $\xi \in (0, 1)$ is the regulatory requirement ratio presented in equation (4.4) and β corresponds to the elasticity of the bail-in option value to cash-flows depicted in equation (4.8).

I restrict the bail-in policy to a unique total conversion of non-deposit debt to new equity[26]. This implies that only non-deposit debt is considered as bail-inable, the entire claim is converted to equity, and that the bail-in conversion only occurs once. Consequently, the bank's debt comprises only deposits after the bail-in.

When bail-in occurs, the regulator issues new equity and converts non-deposit liabilities to new equity at a rate $\theta \in [0, 1]$; old equity claims are converted to new equity at a rate $(1 - \theta)$. That is, as in Sarkar (2013), the following identities hold at the time of the bail-in:

$$E_{bi,1}(x_{bi}) = (1 - \theta)E_{bi,2}(x_{bi}) \tag{4.77}$$

and

$$C_{bi,1}(x_{bi}) = \theta E_{bi,2}(x_{bi}) \tag{4.78}$$

[26]My definition of bail-in can be relaxed to partial debt reduction with equity compensation. See Sarkar (2013) for a consideration on this extension.

▷ **Bank's Closure.** The bank continues to operate after the bail-in until it reaches closure time T_{bi}^2, when the regulator imposes a PSA over the assets of the bank. Time T_{bi}^2 is the time when cash-flows x_t first cross the trigger x_{bi}^2:

$$T_{bi}^2 = \inf\left\{t \in [0, +\infty) : x_t \notin [x_{bi}^2, +\infty)\right\} \tag{4.79}$$

Following Proposition 5 the closure trigger x_{bi}^2 in the Bail-in case is:

$$x_{bi}^2 = \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{d_{bi}}{\rho} \tag{4.80}$$

▷ **Equilibrium.** Figure 4.9 depicts the three stages of the equilibrium. At $t = 0$, the bank defines the capital structure by maximising the value function $B_{bi}(x_0)$ for a pair of coupons $(c_{bi}, d_{bi}) \in (0, +\infty)^2$. The bank operates from time 0 until time T_{bi}^1, when the regulator imposes a bail-in by converting non-deposit debt to equity. After the bail-in, the institution continues to operate until T_{bi}^1, when the regulator closes the bank, sells the assets through a PSA and pays depositors. The deposit insurance fund will cover the missing part of deposits if the asset sale is insufficient to cover this claim.

This is a sequential game with perfect information between these two players. I write equilibrium triggers ${x_{bi}^1}^* \equiv x_{bi}^1(c_{bi}^*, d_{bi}^*)$ and ${x_{bi}^2}^* \equiv x_{bi}^2(d_{bi}^*)$. The equilibrium tuple of the game is:

$$\mathcal{E}_{bi} := \left(c_{bi}^*, d_{bi}^*, {x_{bi}^1}^*, {x_{bi}^2}^*\right)$$

4.6.2 The Bank's Problem

4.6.2.1 Value Functions

The value of the bank satisfies the following accounting identity:

$$B_{bi}(x_t) = E_{bi}(x_t) + C_{bi}(x_t) + D_{bi} \quad \forall t \in [0, T_{bi}^2)$$

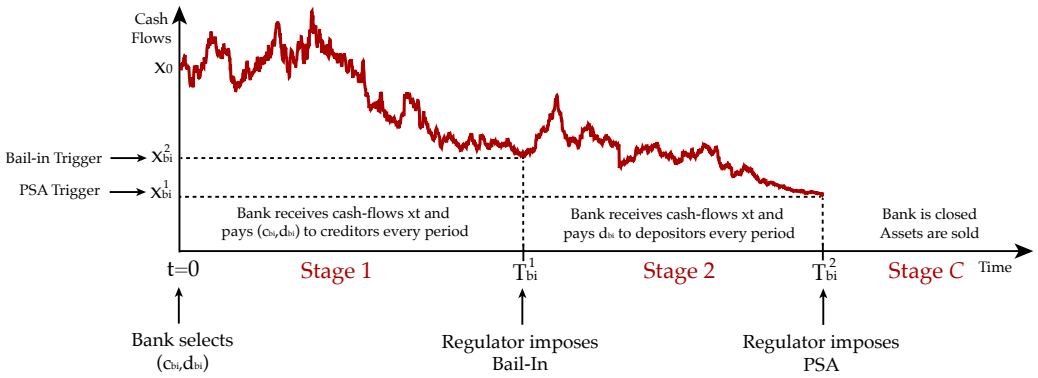


Figure 4.9: **Model timeline the Bail-In Case.** Cash-flows evolve stochastically while gradually declining until reaching the bail-in trigger x_{bi}^1 at time T_{bi}^1. The regulator converts non-deposit creditors to shareholders at the Bail-In trigger $x_{bi}^1$and prolong the firm's life until reaching the respective Private Sector Acquisition trigger x_{bi}^2 at which bank is closed and the assets are sold.

Equity. For $t \in [0, T_{bi}^1)$, the value of the equity claim is given by the following expression:

$$E_{bi}(x_t) := \mathbb{E}_t\left[\int_t^{T_{bi}^1} e^{-\rho(s-t)}(1-\tau)\left(x_s - c_{bi} - d_{bi}\right)ds\right] + \mathbb{E}_t\left[\int_{T_{bi}^1}^{T_{bi}^2} e^{-\rho(s-t)}(1-\tau)\left(x_s - d_{bi}\right)ds\right] + \\ + \mathbb{E}_t\left[\int_{T_{bi}^2}^{+\infty} e^{-\rho(s-t)}\max\left\{0, (1-\tau)\left((1-\epsilon_p)x_{bi}^2 - d_{bi}\right)\right\}ds\right] \tag{4.81}$$

The three terms in Equation (4.81) are, respectively, the value that shareholders derive from the bank in stage 1 (before bail-in), stage 2 (after bail-in), and stage $\mathcal{C}$ (after closure by the regulator). Stage 1 runs from time t until bail-in occurs at time T_{bi}^1. During this stage, shareholders receive instantaneous cash-flows after interest payments and taxes equal to $(1-\tau)(x_t - c_{bi} - d_{bi})$. After bail-in occurs, the shareholders are augmented by the non-deposit creditors, who become shareholders as a result of the bail-in. Hence, because $c_{bi} = 0$ after bail-in, shareholders receive cash flow $(1-\tau)(x_t - d_{bi})$. Shareholders are protected by limited liability. Hence, during stage $\mathcal{C}$, which runs from closure at time T_{bi}^2 to infinity, shareholders receive the larger of zero and the after-tax present value of the impaired cash flow less the deposit payment: that is, the larger of 0 and the present value of $(1-\tau)((1-\epsilon_p)x_{bi}^2 - d_{di})$.

Using standard techniques developed in Dixit and Pindyck (1994), the equity value

function $E_{bi}(x_t)$ can be written as follows:

$$E_{bi}(x_t) = \begin{cases} (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right) + (1-\tau)\left(\frac{x_t}{x_{bi}^1}\right)^{\beta}\left[\frac{c_{bi}}{\rho} - \theta\left(\frac{x_{bi}^1}{\rho-\mu} - \frac{d_{bi}}{\rho}\right)\right] - \\ \qquad -(1-\tau)(1-\theta)\left(\frac{x_t}{x_{bi}^2}\right)^{\beta}\min\left\{\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}, \epsilon_p\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_t > x_{bi}^1 \\ (1-\tau)\left(\frac{x_t}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) - (1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^{\beta}\min\left\{\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}, \epsilon_p\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_{bi}^1 \geq x_t > x_{bi}^2 \\ (1-\tau)\max\left\{0, (1-\epsilon_p)\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}\right\} & \text{for } x_t \leq x_{bi}^2 \end{cases} \tag{4.82}$$

Like Equation (4.81), value function (4.82) has three cases that correspond to the three stages of this version of the model. While the bank is operational and has not yet been subjected to a bail-in ($x_t > x_{bi}^1$), the value of the claim of initial shareholders equals the equity value. In this case, the equity value is the sum of three terms: the first term, $(1-\tau)(x_t/(\rho-\mu) - (c_{di}+d_{di})/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to all creditors; the second term is the expected change in value if the regulator imposes a bail-in to the bank; the third term is the expected change in value if the regulator closes to the bank.

The expected change in equity value caused by the bail-in is the product of the expected discount factor $(x_t/x_{bi}^1)^{\beta}$ and the change in equity value if the bank suffers the bail-in. This change in value is the difference between the present value of future non-deposit coupons $(1-\tau)c_{bi}/\rho$ and the share of residual cash-flows to be given to new shareholders $(1-\tau)\theta\left(x_{bi}^1/(\rho-\mu) - d_{bi}/\rho\right)$. This means that, at the bail-in, old shareholders no longer owe future coupon payments to non-deposit credit claimers but they lose a share θ of residual cash-flows to equity that is given to these credit claimers.

The expected change in equity value for old shareholders if the regulator closes the bank is the product of the expected discount factor $(x_t/x_{bi}^2)^{\beta}$ and a share $(1-\theta)$ on the change in equity value at closure. In detail, the change in equity value for old shareholders is the difference between the closure equity value $(1-\tau)(1-\theta)\max\left\{0, (1-\epsilon_p)x_{bi}^2/(\rho-\mu) - d_{bi}/\rho\right\}$ and the after tax present value of cash-flows net of debt payments $(1-\tau)(1-\theta)\left(x_{bi}^2/(\rho-\mu) - d_{bi}/\rho\right)$.

After the bail-in and before the regulator closes the bank ($x_{bi}^1 \geq x_t > x_{bi}^2$), equity is

owned by both old and new shareholders. Its value is divided in two terms: the first term, $(1-\tau)(x_t/(\rho-\mu)-d_{di}/\rho)$, is the after tax present value of cash-flows excluding the present value of coupons promised to depositors; The second term is the expected change in value if the regulator closes to the bank.

When the bank is closed ($x_t \leq x_{bi}^2$), shareholders receive the closure payoff. Because shareholders have limited liability, this payoff is at least zero. They receive a zero payoff when the impaired asset value from the PSA $(1-\epsilon_p)V(x_{bi}^2)$ is insufficient to cover the present value of deposit coupons d_{bi}/ρ. If the impaired asset value covers the present value of coupons, all shareholders obtain the residual value after paying depositors and taxes.

Non-deposit Liabilities. The value function for the non-deposit credit claim is:

$$C_{bi}(x_t) := \mathbb{E}_t\left[\int_t^{T_{bi}^1} e^{-\rho(s-t)} c_{bi} ds\right] \tag{4.83}$$

Non-deposit creditors earn a periodic coupon c_{bi} until the bank is bailed-in at time T_{bi}^1. The non-deposit claim ceases to exist upon bail-in, when non-deposit creditors are converted to new equity owners.

$$C_{bi}(x_t) = \begin{cases} \frac{c_{bi}}{\rho} + \left(\frac{x_t}{x_{bi}^1}\right)^{\beta}\left[\theta(1-\tau)\left(\frac{x_{bi}^1}{\rho-\mu}-\frac{d_{bi}}{\rho}\right)-\frac{c_{bi}}{\rho}\right]- \\ \qquad -\theta(1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^{\beta}\min\left\{\frac{x_{bi}^2}{\rho-\mu}-\frac{d_{bi}}{\rho}, \epsilon_p\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_t > x_{bi}^1 \\ 0 & \text{for } x_t \leq x_{bi}^1 \end{cases} \tag{4.84}$$

Before the bail-in is implemented ($x_t > x_{bi}^1$) the value of non-deposit credit comprises three terms: the first term, c_{bi}/ρ, is the present value of coupons. The second term is the expected change in the value for non-deposit creditors if the regulator imposes a bail-in. This is the product between the expected discount factor $(x_t/x_{bi}^1)^{\beta}$ and the change in creditors' value if a bail-in occurs. The change in value if a bail-in occurs is the difference between the share of cash-flows net of deposits and taxation $\theta(1-\tau)(x_{bi}/(\rho-\mu)-d/\rho)$ received by non-deposit creditors and the present value of coupons c_{bi}/ρ they forfeit.

The third term in the case where $x_t < x_{bi}^1$ is the expected change in the non-deposit creditors value if the bank closes. This is the product between the expected discount factor $(x_t/x_{bi}^2)^\beta$ and the change in the creditors' value at closure. Creditors become the new shareholders after T_{bi}^1 and have a share θ of the equity value. The creditors' value at closure is the difference between the closure equity value $\theta(1-\tau)\max\left\{0,(1-\epsilon_p)x_{bi}^2/(\rho-\mu)-d_{bi}/\rho\right\}$ and the after tax present value of cash-flows net of debt payments $\theta(1-\tau)\left(x_{bi}^2/(\rho-\mu)-d_{bi}/\rho\right)$.

After the bail-in time T_{bi}^1, the financial institution does not hold non-deposit liabilities and this claim ceases to exist.

Deposits. Deposits have a constant value since they are protected by the deposit insurance scheme and because bail-ins are not extended to this class of debt. That value is:

$$D_{bi} = \frac{d_{bi}}{\rho}, \qquad \text{for } t \geq 0 \tag{4.85}$$

Bank Value. I obtain the following value function for the financial institution by summing the equity, the non-deposit credit, and the deposits claims:

$$B_{bi}(x_t) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_t}{x_{bi}^1}\right)^\beta \frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^\beta \min\left\{\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}, \epsilon_p \frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_t > x_{bi}^1 \\ (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{d_{bi}}{\rho} - (1-\tau)\left(\frac{x_t}{x_{bi}^2}\right)^\beta \min\left\{\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}, \alpha\frac{x_{bi}^2}{\rho-\mu}\right\} & \text{for } x_{bi}^1 \geq x_t > x_{bi}^2 \\ (1-\tau)\max\left\{0,(1-\epsilon_p)\frac{x_{bi}^2}{\rho-\mu} - \frac{d_{bi}}{\rho}\right\} + \frac{d_{bi}}{\rho} & \text{for } x_t \leq x_{bi}^2 \end{cases} \tag{4.86}$$

While the financial institution is open ($x_t > x_{bi}^1$) its value function is the sum of the present value of cash-flows after taxes, the tax shield of debt, the expected change in total value when the bail-in option is exercised, and the expected change in value when the closure option is exercised.

The expected value change associated to the bail-in option is the product between the expected discount factor $(x_t/x_{bi}^1)^\beta$ and the loss of the tax shield of non-deposit liabilities $\tau c_{bi}/\rho$ at the bail-in time T_{bi}. The expected value change associated to the bank's closure is the product between the expected discount factor $(x_t/x_{bi}^2)^\beta$ and the closure value.

After the bail-in ($x^1_{bi} \geq x_t > x^2_{bi}$), the bank generates after tax cash-flows and a deposit tax shield $\tau d_{bi}/\rho$. The last term in this expression is expected change in value given closure.

The institution is subjected to a PSA at closure ($x_t \leq x^2_{bi}$) which will yield a total value $(1-\tau)(1-\epsilon_p)V(x^2_{bi})$, provided all depositors are fully covered by the impaired asset value. If the impaired asset value is lower than deposits, the bank value is equal to the deposits value D_{bi}, since these claimants are covered by the deposit insurance scheme.

In the Bail-in case, a strong bank is a bank whose impaired asset value is always sufficient to cover the deposits obligations given the PSA closure costs. From Lemma 9, the boundary that separates strong banks from weak banks is:

$$\overline{\xi}_p = \frac{\beta\epsilon_p - 1}{\beta - 1}$$

where the institution is considered strong if $\xi \geq \overline{\xi}_p$ and weak if $\xi < \overline{\xi}_p$. This boundary stays unchanged from Section 4.5.2.

Consider the following assumption:

Assumption 15 (No Worse-Off Principle)**.** *No shareholder or creditor shall incur greater losses under a bail-in than they would have incurred if the bank had been subjected to normal bankruptcy proceedings.*

Assumption 15 accounts for the No Worse-Off (NWO) principle as defined in Article 74 of Directive 2014/59/EU[27]. The Bail-In conversion share θ must be such that each class of bail-inable investors can only accept a Bail-In if they receive a share of value that is weakly better than suffering a private-sided bankruptcy.

Lemma 10. *The No Worse-Off Principle is satisfied when*

$$\theta \in \left[\frac{\min\left\{(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\}}{E_{bi,2}(x^1_{bi}) + C_{bi,2}(x^1_{bi})}, 1 - \frac{\max\left\{(1-\tau)\left[(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\}}{E_{bi,2}(x^1_{bi}) + C_{bi,2}(x^1_{bi})} \right] \cap [0,1] \tag{4.87}$$

[27] See Bank Recovery and Resolution Directive.

Lemma 10 states that the bail-in conversion share θ must respect the participation constraints of equity-owners and non-deposit creditors at the bail-in trigger time. That is, it must respect these claimants' alternative of facing a bank's closure at time T_{bi}^1 while suffering a closure cost fraction ϵ.

▷ **Bail-in Conversion Share.** Now consider the partition of equity value between old and new shareholders when the bank suffers the bail-in.

The existing literature, such as Sarkar (2013) and Pawlina (2010) relies upon a Nash bargaining game to determine the optimal share of value. This literature assigns exogenous bargaining power to every party and determines the way that the value is divided. However, this approach does not work in my model because, to be consistent with reality, the conversion rule must be defined by financial authorities that are completely independent of the failing banks' stakeholders.

There are several alternatives to determine the conversion share θ under a bail-in intervention[28]. I adopt $\overline{\theta}$ as the conversion share rule for my model:

Assumption 16. *The bail-in conversion share is:*

$$\overline{\theta} \equiv \frac{c_0}{\rho\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right)} \tag{4.88}$$

Assumption 16 states that the regulator selects a bail-in conversion share $\overline{\theta}$ that is equal to the ratio between the present value of future coupon payments to non-deposit creditors and the value of the bank excluding deposits. Conversion share must respect the NWO principle in Lemma 10 and, so, the proposed share $\overline{\theta}$ must always be limited by the boundaries presented in condition (4.87).

[28] For example, following Moulin (2004), one can use an egalitarian rule. In this case, I would select a share θ that solves the equality:

$$(1-\theta)\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right) - \max\left\{(1-\tau)\left[(1-\epsilon_p)\frac{x_{bi}^1}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\} = \theta\left(E_{bi,2}(x_{bi}^1) + C_{bi,2}(x_{bi}^1)\right) - \min\left\{(1-\epsilon_p)\frac{x_{bi}^1}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\}$$

This condition sets the share of value of non-deposit creditors equal to the share of value of shareholders excluding their respective value had they entered a bankruptcy. Hence, this alternative considers the shareholders' and non-deposit creditors' opportunity cost of the bail-in, i.e. the bankruptcy alternative.

This rule is consistent with the financial regulator's goal of guaranteeing that shareholders are the first to suffer the consequences of a bail-in. With the rule in Equation (4.88), the value of non-deposit liabilities is unaffected ex-ante by the bail-in rule: they receive $\overline{\theta}E_{bi,2}(x_{bi}^1)$, which is equal to c_{bi}/ρ. For the period $t \in [0, T_{bi}^1)$, the expected loss of value caused by bail-in is experienced by shareholders, while the value of non-deposit liabilities depends only upon the expected present value of coupons. Hence, conversion rule $\overline{\theta}$ secures that non-deposit creditors never suffer a decrease in the expected value of the claim until time T_{bi}^1 nor capture any rents beyond the promised present value of the coupons until T_{bi}^1. When non-deposit creditors become shareholders they may absorb losses if cash-flows continue to erode ($x_t < x_{bi}^1$).

4.6.2.2 Initial Capital Requirement

Under Assumption 12, the Initial Capital Requirement (ICR) must hold:

$$c_{bi} + d_{bi} \leq \frac{\rho(1-\xi)}{\rho - \mu} x_0 \tag{4.89}$$

4.6.2.3 Non-Negative Coupons

By Assumption 9, each type of liability must be non-negative:

$$c_{bi} \geq 0 \qquad \text{and} \qquad d_{bi} \geq 0 \tag{4.90}$$

4.6.2.4 Bank's Capital Structure Optimisation

The bank selects coupons (c_{bi}, d_{bi}) so as to maximise the total bank value $B_{bi}(x_0)$ subject to the regulator's optimal bail-in trigger in condition (4.76), his optimal closure trigger in condition (4.80), the ICR condition (4.89) and the non-negative coupons condition (4.90).

Optimisation Problem 7 (Bank)**.** *The optimal capital structure problem for the bank is:*

$$
\begin{aligned}
(c_{bi}^*, d_{bi}^*) \quad & \in \underset{\{c_{bi}, d_{bi}\}}{\arg\max}\ B_{bi}(x_0) \\
s.t. \qquad & x_{bi}^1 := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} && \textit{[Bail-In Trigger]} \\
& x_{bi}^2 := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_p+d_p}{\rho} && \textit{[Closure Trigger]} \\
& c_{bi}+d_{bi} \leq \frac{\rho(1-\xi)}{\rho-\mu}x_0 && \textit{[Initial Capital Requirement]} \\
& c_{bi} \geq 0 \quad , \quad d_{bi} \geq 0 && \textit{[Non-Negative Coupons]}
\end{aligned}
$$

4.6.3 The Bail-in Equilibrium

Proposition 15 (Bail-in Case Equilibrium)**.** *The equilibrium of the game with Bail-in is:*

$$\mathcal{E}_{bi} := \left(c_{bi}^*, d_{bi}^*, {x_{bi}^1}^*, {x_{bi}^2}^*\right)$$

The equilibrium DIS dependence boundary is:

$$\underline{\xi}^* = \overline{\xi}_p$$

Bank Optimal Decision. *The total periodic coupon $l_{bi}^* = c_{bi}^* + d_{bi}^*$ is optimal iff:*

$$l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}_{bi}\} \tag{4.91}$$

where the interior solution is $l_{bi}^{**} = c_{bi}^{**} + d_{bi}^{**}$ with

$$
d_{bi}^{**} = \begin{cases}
(1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \textit{for } \xi < \overline{\xi}_p \\
(1-\xi)\dfrac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}x_0 & \textit{for } \xi \geq \overline{\xi}_p
\end{cases} \tag{4.92}
$$

and

$$c_{bi}^{**} = \begin{cases} (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{4.93}$$

Otherwise, the bank selects $\overline{l}_{bi} = \overline{c}_{bi} + \overline{d}_{bi}$ *where*

$$\overline{d}_{bi} = \begin{cases} \frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & \text{for } \xi < \overline{\xi}_p \\ \frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{4.94}$$

and

$$\overline{c}_{bi} = \begin{cases} \frac{\rho}{\rho-\mu}(1-\xi)\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}\right)x_0 & \text{for } \xi < \overline{\xi}_p \\ \frac{\rho}{\rho-\mu}(1-\xi)\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{4.95}$$

Regulator Optimal Decision. *At time* T_{bi}^1*, cash-flows* x_t *hit the trigger* ${x_{bi}^1}^*$*, the Regulator steps in and imposes a Bail-in. That trigger is:*

$${x_{bi}^1}^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c_{bi}^* + d_{bi}^*}{\rho} \tag{4.96}$$

At time T_{bi}^2*, cash-flows* x_t *hit the trigger* ${x_{bi}^2}^*$*, the Regulator steps in and imposes a Private Sector Acquisition. That trigger is:*

$${x_{bi}^2}^* = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{d_{bi}^*}{\rho} \tag{4.97}$$

4.6.4 Discussion on the effects of introducing Bail-Ins

I write the equilibrium capital structure at time 0 for the PSA case as $CS_p^*(x_0) \equiv CS_p(x_0|\mathcal{E}_p)$ and for the Bail-In case as $CS_{bi}^*(x_0) \equiv CS_{bi}(x_0|\mathcal{E}_{bi})$. I also write the equilibrium debt structure at time 0 for the PSA case as $DS_p^*(x_0) \equiv DS_p(x_0|\mathcal{E}_p)$ and for the Bail-In case as $DS_{bi}^*(x_0) \equiv DS_{bi}(x_0|\mathcal{E}_{bi})$.

Proposition 16 (Change in the Equilibrium Total Coupon)**.** *Introducing the Bail-in tool weakly increases the equilibrium total coupon paid by the bank:*

$$l_{bi}^* \geq l_p^*$$

Proposition 17 (Change in the Equilibrium Debt Structure)**.** *Introducing the Bail-in tool weakly increases the equilibrium ratio between non-deposit debt value and total liabilities:*

$$DS_{bi}^*(x_0|\xi < \overline{\xi}_p) \geq DS_p^*(x_0|\xi < \overline{\xi}_p) \tag{4.98}$$

The change in debt structure is undefined for strong banks.

Proposition 18 (Capital Structure Change)**.** *Introducing the Bail-in tool weakly decreases the equilibrium ratio between equity value and total bank value:*

$$CS_{bi}^*(x_0) \leq CS_p^*(x_0) \tag{4.99}$$

Propositions 16 to 18 state that, in equilibrium, the introduction of a bail-in tool does not cause banks to decrease leverage. The precise effect depends upon whether the bank is weak or strong.

▷ **The Effect of Bail-Ins in Weak Banks.** Introducing bail-ins affects the equilibrium debt structure and capital structure of weak banks (that is, banks for which $\xi < \overline{\xi}_p$). Without bail-ins, weak banks only have deposit debt. With them, they may also choose to assume some non-deposit liabilities. Hence, the weak banks' debt structure weakly increases with

the introduction of the bail-in:

$$DS^*_{bi}(x_0|\xi < \overline{\xi}_p) \geq DS^*_p(x_0|\xi < \overline{\xi}_p)$$

The bail-in tool can be valuable for weak banks because it creates a *recapitalisation option*. The reason is that, if the bank holds non-deposit liabilities, the regulator has an option to change the capital structure by converting non-deposit credit to equity. Doing so prolongs the life of the institution and gives the bank a second chance to recover the cash-flows and possibly to prevent a future closure.

In the PSA case, the weak bank was restricted to select the optimal leverage by balancing the expected costs of closure against the benefit of the tax shield over deposits. The bail-in tool incentivises the weak bank to increase the total coupon from l^*_p to l^*_{bi} so as to use the recapitalisation option to increase the bank's debt capacity and, hence, the tax shield it derives from borrowing.

Because it increases debt capacity and, hence, the tax shield of debt, the bail-in causes a weak decrease in the bank's equilibrium capital/value ratio:

$$CS^*_{bi}(x_0|\xi < \overline{\xi}_p) \leq CS^*_p(x_0|\xi < \overline{\xi}_p)$$

▷ **The Effect of Bail-Ins in Strong Banks.** In the PSA case, the institution is free to define any mix of positive deposits and non-deposit coupons provided that the identity $l^*_p = c^*_p + d^*_p$ is respected. In contrast, when the bail-in tool is introduced, the strong bank must select a unique bail-in equilibrium coupon mix (c^*_{bi}, d^*_{bi}). My model therefore yields no predictions regarding the change in the debt structure.

Similarly to the weak bank's case, the introduction of the bail-in tool increases the total leverage of the strong institution causing an increase in value and a decrease in the equity value due to the lower after-taxes residual cash-flow $(1-\tau)(V(x_t) - L_{bi})$. Hence, I have:

$$CS^*_{bi}(x_0|\xi \geq \overline{\xi}_p) \leq CS^*_p(x_0|\xi \geq \overline{\xi}_p)$$

From Propositions 16 to 18 and the above description, I get the following corollary:

Corollary 18.1. *Introducing the bail-in tool incentivises all banks to increase leverage. Higher leverage decreases equity value and increases value and, hence, the capital structure ratio of banks decreases. Weak banks will not decrease the debt structure ratio. Strong banks can increase, decrease or maintain their debt structure.*

4.7 Quantitative Results

In this section, I compare the bail-in case against the base case. I calibrate the model, provide plots for the equilibrium coupons, equilibrium capital structure and equilibrium debt structure (see Appendix).All claims are evaluated at time $t = 0$ in equilibrium. That is, under the equilibrium tuple $\mathcal{E}$ in the base case or the equilibrium tuple $\mathcal{E}_{bi}$ in the bail-in case.

▷ **Capital and Debt Structure Ratios.** For the base case, the equilibrium capital structure is $CS^*(x_0)$ and the equilibrium debt structure is $DS^*(x_0)$. Explicitly:

$$CS^*(x_0) := \frac{E(x_0|\mathcal{E})}{B(x_0|\mathcal{E})} \qquad\qquad DS^*(x_0) := \frac{C(x_0|\mathcal{E}_i)}{D(\mathcal{E}_i) + C(x_0|\mathcal{E})}$$

As in the previous section, I write the equilibrium Capital Structure for the Bail-In case as $CS^*_{bi}(x_0)$ and the equilibrium Debt Structure as $DS^*_{bi}(x_0)$:

$$CS^*_{bi}(x_0) := \frac{E_{bi}(x_0|\mathcal{E}_{bi})}{B_{bi}(x_0|\mathcal{E}_{bi})} \qquad\qquad DS^*_{bi}(x_0) := \frac{C_{bi}(x_0|\mathcal{E}_{bi})}{D_{bi}(\mathcal{E}_i) + C_{bi}(x_0|\mathcal{E}_{bi})}$$

To understand the role of the options of regulatory intervention (closure and bail-in) over the capital and debt structures of the bank, I write the accounting versions of the capital structure ratio and the debt structure ratio using the accounting value of the claims. To recall, the accounting value of a claim is the present value of all future receivables to be earned by the owner of the claim. For example, in the base case, the accounting value of equity is the after tax present value of cash-flows net of interest payments $(1-\tau)(V(x_0) - L)$, the accounting value of deposits is D and the accounting value of non-deposit liabilities is $L - D$.

Hence, for the base case, I write the equilibrium Accounting Capital Structure ratio as $ACS^*(x_0)$ and the Accounting Debt Structure ratio as $ADS^*(x_0)$, where:

$$ACS^*(x_0) := \frac{1-\tau)\left(V(x_0|\mathcal{E}) - L(\mathcal{E})\right)}{(1-\tau)\left(V(x_0|\mathcal{E}) - L(\mathcal{E})\right) + L(\mathcal{E})} = \frac{(\tau-1)(\mu(l^*) - \rho(l^* - x_0))}{\tau(\mu-\rho)l^* + \rho(\tau-1)x_0}$$
$$ADS^*(x_0) := \frac{L(\mathcal{E}) - D(\mathcal{E})}{L(\mathcal{E})} = \frac{c^*}{l^*}$$

For the bail-in case, the equilibrium Accounting Capital Structure ratio is $ACS^*_{bi}(x_0)$ and the Accounting Debt Structure ratio is $ADS^*_{bi}(x_0)$, where:

$$ACS^*_{bi}(x_0) := \frac{1-\tau)\left(V_{bi}(x_0|\mathcal{E}_{bi}) - L_{bi}(\mathcal{E}_{bi})\right)}{(1-\tau)\left(V_{bi}(x_0|\mathcal{E}_{bi}) - L_{bi}(\mathcal{E}_{bi})\right) + L_{bi}(\mathcal{E}_{bi})} = \frac{(\tau-1)(\mu(l^*_{bi}) - \rho(l^*_{bi} - x_0))}{\tau(\mu-\rho)l^*_{bi} + \rho(\tau-1)x_0}$$

$$ADS^*_{bi}(x_0) := \frac{L_{bi}(\mathcal{E}_{bi}) - D_{bi}(\mathcal{E}_{bi})}{L_{bi}(\mathcal{E}_{bi})} = \frac{c^*_{bi}}{l^*_{bi}}$$

▷ **Calibration Values.** Table 4.1 summarises the values used for the model calibration. I study two different setups for the bank's assets. I define the "good state" bank as the bank that has assets with a positive growth rate of cash-flows $\mu = 0.01$ and low uncertainty $\sigma = 0.01$. Differently, the bank in the "bad state" has assets that generate cash-flows with negative growth rate $\mu = -0.01$ and high uncertainty $\sigma = 0.05$. My choice of a growth rate μ close to 0 is consistent with the recent estimations of Detragiache et al. (2018). They identify that the assets' profitability of European banks in the periods are very close to 0 and often bellow 1%, during and after the crisis. My choice for the uncertainty σ is also in line with Chen et al. (2017) who estimate the standard deviation from the model calibration for 17 financial institutions to range from 0.038 to 0.086.

Consistent with previous literature (Hugonnier and Morellec 2017, de Mooij and Gaetan 2015), I set the corporate tax rate to $\tau = 0.25$. This is in line with Langedijk et al. (2014) who estimate that corporate income tax rates among European banks range from 0.10 to 0.40.

The closure costs fraction $\epsilon = 0.20$ for the base case (private-sided closure) is close to the estimates of James (1991) for financial firms and to Andrade and Kaplan (1998) and Korteweg (2010) for non-financial firms. In summary, these papers estimate that bankruptcy costs range from 10% to 30% of firm value. Furthermore, I define a PSA closure cost $\epsilon_p = 0.15$ which is cautiously close to ϵ due to the lack of research on banks' asset sales intermediated by resolution authorities.

The values of the other parameters are as follows. The regulatory requirement ratio is $\xi = 0.15$ and it is inspired by the EBA guidelines on capital and liquidity adequacy requirements for early intervention triggers[29]. The exogenous discount rate is $\rho = 0.15$ which

[29]See Art. 27 of Directive 2014/59/EU and in EBA Guidelines EBA/GL/2015/03. In summary, these

is slightly higher than, for example, Hugonnier and Morellec (2017). I choose a higher rate to ensure the comparative statics presented in Figures 4.10 to 4.17 do not have indeterminate results across the ranges studied. Moreover, lower discount rates do not change my results significantly. Finally, I arbitrarily set the initial cash-flow level to $x_0 = 50$.

Parameter	Common	Good State	Bad State	Description
μ	-	0.01	-0.01	Expected instantaneous growth of cash-flows
σ	-	0.01	0.05	Instantaneous volatility
τ	0.25	-	-	Tax rate
ϵ	0.20	-	-	Closure cost fraction in the base case
ϵ_p	0.15	-	-	Closure cost fraction under the PSA
ξ	0.15	-	-	Regulatory requirement ratio
ρ	0.15	-	-	Exogenous discount rate
x_0	50	-	-	Initial cash-flow level

Table 4.1: Calibration parameters.

4.7.1 Equilibrium Coupons

▷ **Base Case - Coupon.** Consider Figure 4.10. The equilibrium total coupon l^* increases with the cash-flows growth rate μ and decreases with the regulatory requirement ratio ξ. For the calibration values described in Table 4.1, changes in the cash-flows' uncertainty σ and changes in the corporate tax rate τ do not seem to affect the equilibrium total coupon significantly.

The equilibrium total coupon level l^* always presents an interior solution when the bank is in the bad state. That is because the negative cash-flow growth and the higher uncertainty increase the costs of closure which forces the bank to choose a lower equilibrium total coupon when maximise the bank's value.

Moreover, the bad state has a higher minimum levels of non-deposit coupons than the good state (orange dashed lines). As described in Subsection 4.3.3, banks with lower growth rate of assets and higher standard deviation σ must hold a higher proportion of non-deposit liabilities in its coupon structure in order to prevent a bank-run.

guidelines suggest that national regulators can select regulatory capital ratios by imposing a buffer over the capital adequacy ratio imposed to banks.

▷ **Bail-in Case - Coupon.** Consider Figure 4.13. Similarly to the base case, calibration shows that the equilibrium total coupon l_{bi}^* increases with the cash-flows growth rate μ and decreases with the regulatory requirement ratio ξ, while changes in uncertainty σ and in the tax rate τ do not seem to affect the significantly the equilibrium total coupon.

In the bail-in case, the good state bank is almost exclusively financed by deposits while the bad state bank tends to include non-deposit liabilities in their coupon structure for positive growth rates. Non-deposit liabilities are included in the coupon structure of the bad state bank because they can be used to induce the regulator to recapitalise the institution through the Bail-in tool and gain time for the cash-flows to recover. This is particularly relevant for institutions with high volatility of cash-flows σ and cash-flows' growth rates μ that are not extremely negative. A bank with these characteristics has a high likelihood of recovering from a near closure situation and, so, it has an incentive to hold non-deposit liabilities that can be converted to equity at the bail-in time.

Subfigures 4.13.e and 4.13.f show that changes in the tax rate do not seem to affect the equilibrium coupon structure with the exception of the bad state for very low tax rates. In this case, the lower tax rates causes a decrease in the bail-in costs for the bank $-\tau c_{bi}/\rho(x_0/x_{bi}^1)^\beta$ and, so, it is cheaper for the bank to induce a recapitalisation via the bail-in resolution tool.

The equilibrium total coupon l_{bi}^* decreases if the regulatory requirements ratio ξ increases (subfigures 4.13.g and 4.13.h). In this case, higher capital requirements increase the bail-in and closure triggers. This forces the bank to select a lower equilibrium total coupon to avoid earlier interventions. The bank in the bad state includes non-deposit liabilities if the regulatory requirement ratio is high, that is, if $\xi > 15.4\%$. A higher ξ ratio increases the sensitivity of the closure trigger to changes in deposits $\partial^2 x_{bi}^1/\partial d_{bi}\partial\xi > 0$. Therefore, having one additional unit of deposit coupon d_{bi} becomes very expensive if the regulatory requirement ratio is high. This prompts the bad state bank to reduce the deposit coupon and increase the non-deposit coupon c_{bi} as ξ increases.

4.7.2 Equilibrium Capital Structure

▷ **Base Case - Capital Structure.** Consider Figure 4.11. The capital structure ratio varies negatively with growth μ and tax rate τ and it varies positively with uncertainty σ and capital requirement ratio ξ.

As shown in subfigures 4.11.a and 4.11.b, lower growth rate μ prompts the firm to decrease leverage and to increase equity causing the capital structure ratio to increase. This is particularly the case for negative values of μ. Furthermore, the capital structure ratio $CS^*(x_0)$ is lower than the accounting capital structure $ACS^*(x_0)$ for growth rates that are not extremely negative. However, this reverts as μ takes extremely negative value. In this case, the closure option becomes positively valued for equity-holders because, for extremely negative growth rates, the limited liability property limits the losses of shareholders at closure.

The positive relationship between the equity/value ratio and cash-flow uncertainty σ is represented in Subfigures 4.11.c and 4.11.d. In the good state, the capital structure ratio $CS^*(x_0)$ is always lower than the accounting capital structure $ACS^*(x_0)$ while, in the bad state, high σ values inverts the relationship inverts. This is because the closure option becomes positively valued for equity-holders.

Furthermore, subfigures 4.11.e and 4.11.f show that higher taxation causes a decrease in the ratio of equity over value in both states. Both equity and total value functions decrease as the tax rate increase. However, the total value function decreases at a slower pace due to the tax shield of debt.

Subfigures 4.11.g and 4.11.h reveal that higher capital requirements increase the capital structure ratio. Higher capital requirements ratio ξ increase the intervention trigger x_b. To prolong the bank's lifetime the bank decreases the equilibrium total coupon. As a result, the weight of equity in the capital structure increases.

▷ **Bail-In Case - Capital Structure.** Consider Figure 4.14. In the bail-in case to growth (μ) and uncertainty (σ) do not affect the capital structure significantly. Overall, the ratio of equity over value remains stable around 10.5%-12.5%.

As presented in subfigure 4.14.a and 4.14.b, the capital structure ratio $CS^*_{bi}(x_0)$ stays bellow the accounting capital structure $ACS^*_{bi}(x_0)$ for growth rates $\mu > -0.026$ in the good state and $\mu > -0.018$ in the bad state. For lower values of μ, the limited liability property of equity limits the losses of shareholders at closure which makes the closure option positively valued for shareholders and, hence, it leads to $CS^*_{bi}(x_0) > ACS^*_{bi}(x_0)$.

The gap between the capital structure ratio and the accounting capital structure ratio widens significantly for $\mu \in [-0.026, 0]$ in the good state and for $\mu \in [-0.018, 0.05]$ in the bad state. This is a consequence of a surge of non-deposit liabilities in the equilibrium coupon structure, observable in subfigures 4.13.a and 4.13.b. All else constant, a higher level of equilibrium non-deposit coupon increases the conversion rate $\overline{\theta}$ in condition (4.87) which translates into a lower share of new equity available to initial shareholders.

Subfigures 4.14.c and 4.14.d show that the bank's equity over value ratio does not change significantly given changes in uncertainty. The difference between the capital structure ratio $CS^*_{bi}(x_0)$ and the accounting capital structure $ACS^*_{bi}(x_0)$ is essentially caused by the presence of non-deposit liabilities in the equilibrium coupon structure which lowers the conversion share of new equity offered to existing shareholders at the bail-in time.

Subfigures 4.14.e and 4.14.f show the negative effect of taxes on capital structure. In fact, higher tax rates decrease equity and total bank value. However, equity decreases faster than total value with higher tax rates. The slower decrease of bank's total value function is a consequence of the tax shield of debt. Higher tax rates increase tax shields in the bank's total value, alleviating the negative impact of taxation in the after-tax cash-flows.

Finally, subfigures 4.14.g and 4.14.h show that the equilibrium capital structure increases with the regulatory requirement ratio ξ. This is a consequence of the negative relationship between equilibrium total coupon l^*_{bi} and the regulatory requirement ratio ξ observed in subfigures 4.13.g and 4.13.h. Higher requirement ratios ξ increases the closure trigger, forcing the bank to decrease the equilibrium coupon. As a result, the residual cash-flow is higher available to shareholders increases causing an increase in the equity value function $E_{bi}(x_0|\mathcal{E}_{bi})$ and an increase in the capital structure ratio $CS^*_{bi}(x_0)$ and the accounting capital structure $ACS^*_{bi}(x_0)$.

▷ **Base Case vs Bail-In Case - Capital Structure Comparison.** Figure 4.16 combines Figures 4.11 and 4.14 to show the aggregate effect of transiting from the base case (before the regulatory changes) to the bail-in case (after the regulatory changes). Consistent with Propositions 10, 14 and 18, the main result of Figure 4.16 is:

Result 1. *The introduction of the Costless Deposit Insurance, Private Sector Acquisition tool and the Bail-In tool causes a decrease in the ratio of Equity over Total Bank Value in equilibrium. This decrease is greater if the bank is in a bad state than in a good state.*

In equilibrium, the financial institution selects higher total coupons after the regulatory changes. Higher coupons increases the debt value and decreases the weight of Equity in the Total Bank Value. As discussed in the previous sections, each one of these tools contributes to this effect. First, the introduction of costless deposit insurance makes the "No Bank-Run" condition obsolete for weak banks. Any closure costs that would be supported by depositors are transferred to the deposit insurance fund. Strong banks are not affected by this first regulatory change. Second, the decrease of closure costs fraction ϵ to ϵ_p under the Private Sector Acquisition tool decreases the termination costs for strong banks, prompting these institutions to increase leverage. Third, the bail-in offers a recapitalisation option that, when exercised, prolongs the life of the bank. Bail-Ins can affect both types of banks but not all banks will choose to hold non-deposit liabilities. Only those banks that benefit from a recapitalisation option will choose to include non-deposit liabilities. This is translated in a weak increase in the total liabilities of the bank.

Figure 4.16 demonstrate the combined effects of these regulatory changes. Subfigures 4.16.a to 4.16.d show that the capital structure decrease from the base case to the bail-in case is smaller if the growth rate of cash-flows μ is greater and the cash-flow uncertainty σ is lower. Furthermore, observing 4.16.e and 4.16.f, the gap between the capital structures of the base case and the bail-in case diminishes as tax rates τ and regulatory ratios ξ increase.

In summary, anks in a bad state show a sensitivity of their equilibrium capital structure to changes in regulation that is higher than the banks in a good state banks. A major factor for this difference is the expected discount factor of closure in the base case $(x_0/x_b^*)^\beta$ which increases significantly with lower growth rate μ and higher volatility σ.

4.7.3 Equilibrium Debt Structure

▷ **Base Case - Debt Structure.** My model does not provide a unique debt structure ratio in the base case. Figure 4.12 shows the minimum debt structure ratios admissible in the base case while Figure 4.17 shows the region of feasible equilibrium debt structures. The debt structure has an upper-bound of 100% non-deposit liabilities and a lower-bound debt structure defined by the lowest amount of non-deposit liabilities necessary to avoid a depositors-run, as defined in Lemma 7.

The debt structure region in the bad state is always contained in the good state debt structure region for all studied parameters. The No-Bank Run condition, discussed in Subsection 4.3.3.2, forces the bank in the bad state to hold a higher level of minimum non-deposit coupons. The bad state bank has less freedom when choosing its debt structure.

▷ **Bail-In Case - Debt Structure.** As displayed in Figure 4.15, the debt structure ratio $DS^*_{bi}(x_0)$ is always equal to the accounting debt structure $ADS^*_{bi}(x_0)$ This is a consequence of the chosen bail-in conversion rate $\overline{\theta}$ defined in condition (4.88). A higher conversion rate ($\theta > \overline{\theta}$) would lead to a debt structure ratio higher than the accounting debt structure. In this hypothetical case, non-deposit claimants would capture a higher share of new equity at the bail-in intervention. This would increase their expected total value but not their present value of cash-flows. The opposite case, $\theta < \overline{\theta}$, holds with opposite results.

Debt structure follows the same change pattern as the coupon structure. As seen in subfigures 4.15.a) and 4.15.b, the debt structure ratio is highly sensitive to changes in the growth rate μ in critical subregions. These regions are coincident with a higher share of value $\overline{\theta}$. For instance, in both subfigures, small differences of 2 percentage points around the growth rate of 0% can lead to changes in the debt structure of more than 10p.p. The bank chooses to include non-deposit liabilities as the growth rate moves improves. For every additional unit of deposits, the expected value of the closure option becomes comparatively more expensive for the bank than the expected value of the bail-In option and, hence, the bank can increase value by including non-deposit liabilities in the debt structure.

As seen in subfigures 4.15.c and 4.15.d, the reaction of debt structure to changes in the

uncertainty parameter σ depends on the state of the bank. Higher uncertainty in the good state, increases the debt structure ratio. Differently, higher uncertainty in a bad state regime decreases the debt structure ratio. The specific combination of high uncertainty and high growth induces the highest levels of non-deposit liabilities in the coupon structure which causes a high debt structure.

The debt structure ratios $DS^*_{bi}(x_0)$ and $ADS^*_{bi}(x_0)$ vary negatively with tax rate τ. This ratio is less sensible to shocks in the good state than in the bad state. Observing subfigures 4.15.e and 4.15.e, we see that a tax rate increase of 10p.p. causes a reduction of less than 0.5 p.p. in the debt structure ratio for a bank in the good state and of, approximately, 7p.p. in the bad state

The debt structure ratio varies positively with the requirement ratio ξ but the state of the bank is crucial for the sensitivity of $DS^*_{bi}(x_0)$ to ξ. Subfigure 4.15.f shows that a shock of 10p.p. in the capital requirement ratio yields a maximum increase of 1p.p.. This sensitivity is significantly higher in the bad state where, as shown in subfigure 4.15.g, an increase in the regulatory requirement ratio can result, at most, in an increase of nearly 20p.p., depending on the region under study.

▷ **Base Case vs Bail-In Case - Debt Structure Comparison.** Figure 4.17 combines Figures 4.12 and 4.15 to show the aggregate effect of transiting from the base case (before the regulatory changes) to the bail-in case (after the regulatory changes). Consistent with Propositions 9, 13 and 17, the main result of Figure 4.17 is:

Result 2. *For low values of regulatory requirement ratio ξ, the introduction of regulatory changes causes a decrease in the ratio of equilibrium Non-Deposit Liabilities over Total Liabilities. The effect is undefined for high levels of ξ.*

In the base case, the range is bounded below by the No Bank-Run condition and above by the Initial Capital Requirement condition. The bail-in case has a unique solution for the debt structure ratio $DS^*_{bi}(x_0)$. Inspecting subfigures 4.17.a to 4.17.f, one sees that the equilibrium debt structure in the bail-in case is always lower than the minimum bound for the equilibrium Debt Structure in the base case. This is a consequence of the chosen

regulatory requirement ratio $\xi = 0.15$.

However, for higher levels of ξ, approximately $\xi \geq .18$, I cannot corroborate this effect (subfigures 4.17.g and 4.17.h) because the debt structure ratio in the bail-in case is contained within the boundaries of the base case debt structure ratio. In the base case, a higher regulatory requirement ratio ξ expands the range of admissible debt structures by making the no-bank run condition obsolete while, in the bail-in case, a higher regulatory requirement ratio ξ increases the relative costs of deposits over non-deposit liabilities in the debt structure. That is because, the sensitivity of the closure option cost to changes in deposits increases with ξ. So, in order to increase increase debt, the bank prefers to use non-deposit liabilities instead of deposits because the firsts do not affect the closure of the institution.

4.8 Model Limitations

In this section, I highlight the main limitations of my framework which, I believe, constitute possible areas of extension in the future. These limitations arise from the assumptions and modelling building blocks chosen to prevent unnecessary modelling complexity.

First, I consider the bank as an isolated institution subject to one regulatory requirement instead of an institution inside an interconnected financial system and subject to several regulatory controls. Financial regulation has two key supervisory dimensions in reality: macroprudential and microprudential. In their macroprudential role, regulators focus on the interconnectedness of banks and analyse if the system is resilient enough to prevent pass-through effects of idiosyncratic shocks or resist to systemic shocks. In their microprudential role, regulators verify if each bank is individually respecting key safety metrics, such as, capital adequacy ratios, liquidity ratios, risk coverage measures and, for global systemically important financial institutions (SIFIs), additional restrictive measures. My model ignores the macroprudential role of regulation and is very restrictive in the microprudential role. I believe that my framework could benefit from the consideration of multiple regulatory triggers, namely, liquidity triggers such as the "liquidity coverage ratio" or the "net stable funding ratio". Furthermore, it would be interesting to see an equilibrium systemic intervention trigger in which the regulator would step inside a financial firm if it fails to comply with a

trigger that is determined for an interconnected group of financial institutions.

Second, I consider cash-flows as the bank's main value driver. This choice comes from the classic concept of banks as intermediaries that allocate financial resources from lenders to borrowers. Here, cash-flows correspond to the periodic payments that the bank receives from its loaning activity. These flows can fluctuate when new loans are issued which leads to new periodic payments, when loans are fully amortised or when borrowers occasionally default on their payments. Alternatively, one could consider the bank's assets as the key value driver as seen, for example, in Panageas (2010) and Lambrecht and Tse (2019).

Third, I use a single Geometric Brownian Motion to model the bank's cash-flows. I consider this functional form because it is proven to be very flexible and because it yields closed form solutions for the equilibrium conditions of the model. It is important to note that, even though it is not necessary to have closed form solutions, these facilitate calibration since one just needs to plug in parameter values to get the equilibrium solutions. I consider a single GBM instead of a sum of GBMs because these processes are not additive and, therefore, a linear combination of GBMs does not yield a composite GBM. This is a simplification of reality since, nowadays, banks have multiple divisions and complex asset structures subject to multiple value drivers. A setup like Petersen and Mukuddem-Petersen (2005) and Witbooi et al. (2011) with composites of risky assets could be more adequate to tackle the complexity of banks' assets.

Lastly, a key criticism of my framework is that all players have perfect information about each others at every moment in time. This is particularly relevant for depositors whom my model assumes that can know the state of the bank's cash-flows and decide whether to run on their deposits in the institution. It would be interesting to consider that these players cannot perfectly see the bank's current cash-flows.

4.9 Conclusion

This chapter studies the impact of three key regulatory changes over the capital structure of financial institutions: a strong increase in the deposit limits in Deposit Insurance Schemes; the introduction of Private Sector Acquisitions, that is, the sale of the bank's assets by the

regulator; and the Bail-In tool which converts non-deposit creditors to equity.

I build a capital structure model in continuous time in which a financial institution trades-off debt tax shields and bankruptcy costs in the shadow of possible regulatory intervention. The regulator decides when to close the bank and, if the bail-in tool is available, it also decides when to impose a bail-in. I use this framework to examine the relationship between the three regulatory changes and the ratio of equity over total value as well as the ratio of non-deposit liabilities over total liabilities. . Both of these ratios changed after the financial crisis.

My model yields several predictions. First, my analysis suggests that each of the above regulatory changes leads to an increase in bank leverage. Costless deposit insurance incentivises the use of deposits as a single source of debt so that any potential closure costs are met by the insurance fund; the private sector acquisition tool reduces bank closure costs and so incentivises the bank to assume more debt; finally, the bail-in tool postpones the bank's closure pushing and so causes the bank to borrow more.

Second, my results suggest that the increase in the equity/value ratio observed in Europe after 2010 was not caused by the above regulatory changes, because all induce the bank to borrow more as a proportion of its asset value. The observed increase in banks' equity/value ratios can therefore be explained by the higher capital requirements introduced by Basel III. Moreover, it is likely that the actual sensitivity of the equity/value ratio to changes in capital requirements may be higher than apparent given the aforementioned effect of the three regulatory changes under study.

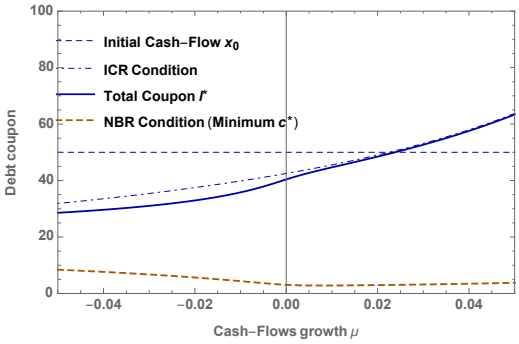
Third, my results indicate that the decrease in the ratio of non-deposit liabilities to total liabilities can be explained by the changes in the regulatory environment, namely, by the strengthening of deposit insurance. My analysis suggests that, for a realistic parameter calibration, deposits become the bank's preferable financing source.

Despite the preference for deposits, my results suggest that deposit insurance does always crowd-out non-deposit liabilities from the debt structure of banks. That is because bail-ins generate a valuable *recapitalisation option* when the bank's capital structure is hard-to, or impossible, to change. The institution can induce a future recapitalisation by adding non-deposit liabilities to the debt structure that will be converted to equity when

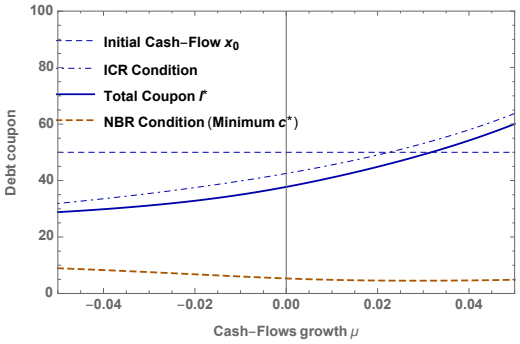
the regulator imposes the bail-in.

I suggest two paths of future research. The first is to study the other dimensions of the bail-in tool. In my model, I restrict this resolution tool to a debt-to-equity exchange. However, bail-ins comprise other actions, for example, managerial turnover. It would be interesting to study the incentives of a manager to define the capital structure of his bank knowing that, in the future, if the bank is bailed-in he can be fired and incur in reputation costs. I conjecture that this would reduce the incentives of this agent to include non-deposit liabilities in the debt structure.

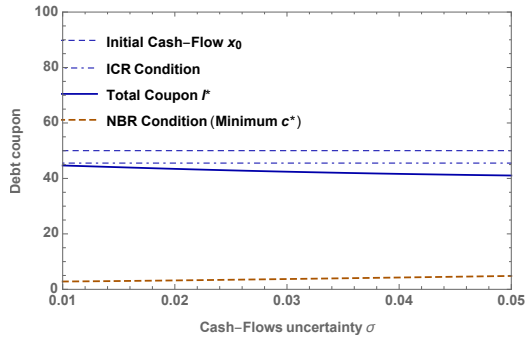
A second research path relates with the systemic implications of Global Systemically Important Banks (GSIFIs). Each GSIFI bank is subject to specific capital requirements aiming at curbing the increased systemic risk. Hence, it would be interesting to study the GSIFI case by modelling the expected spillover effect of a GSIFIs performance over the financial system and make endogenous the bank-specific regulatory requirement ratio.



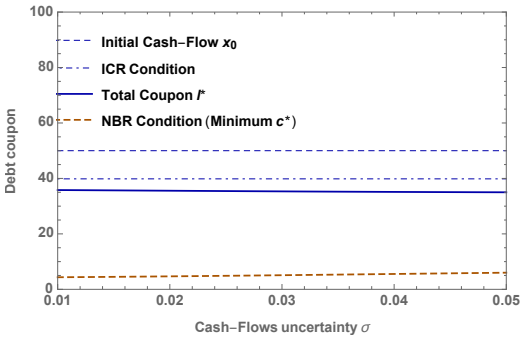
(a) **Good State - Growth** $\mu \in [-.05, .05]$



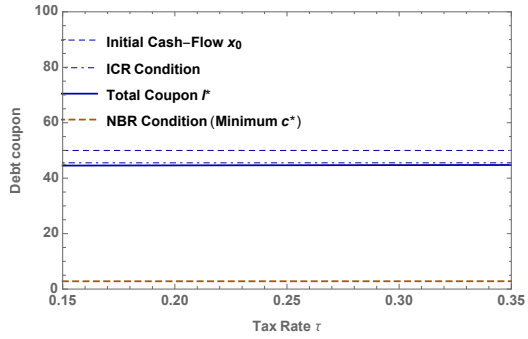
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



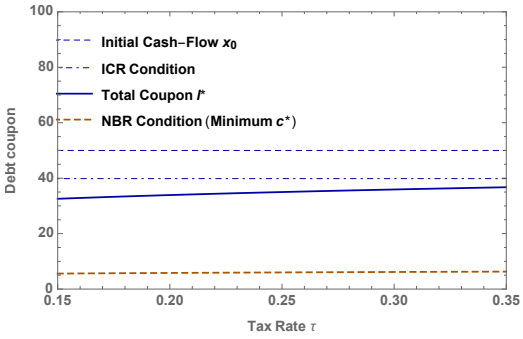
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



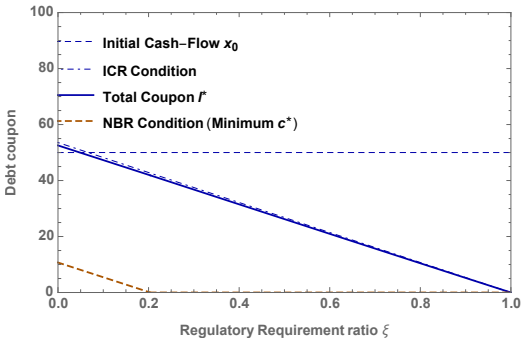
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



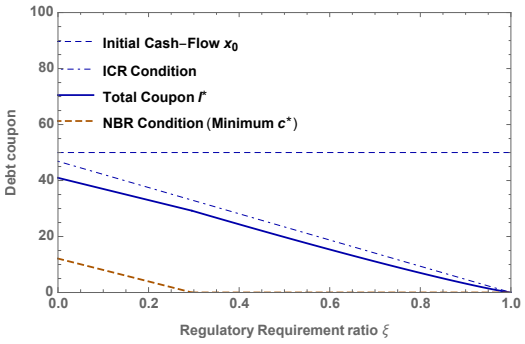
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

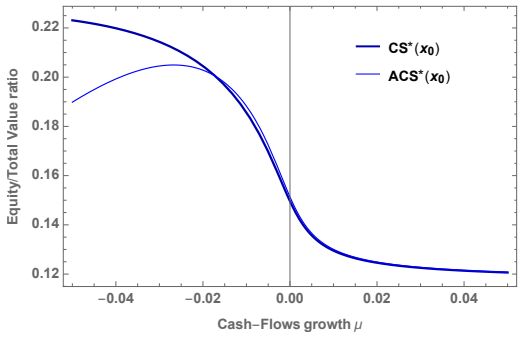


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

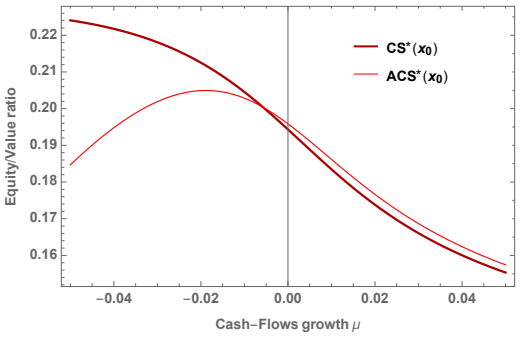


(h) **Bad State - Cap. Req. ratio** $\xi \in [0, 1]$

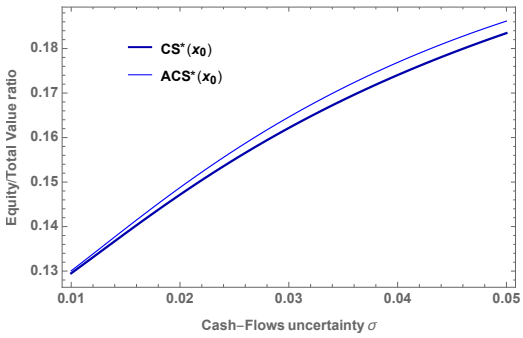
Figure 4.10: **Base Case - Equilibrium Coupon Structure. Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon = .2$; $\tau = 0.25$; $x_0 = 50$.



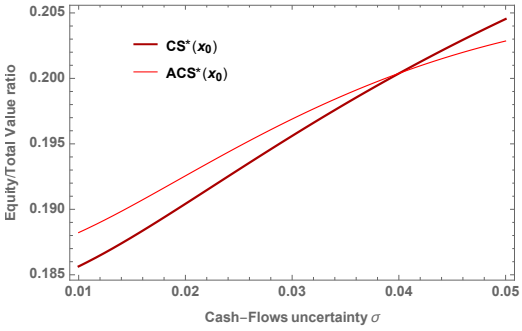
(a) **Good State - Growth** $\mu \in [-.05, .05]$



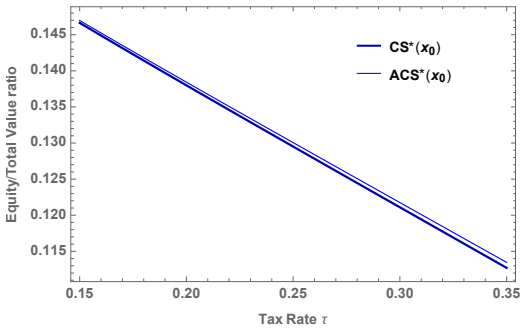
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



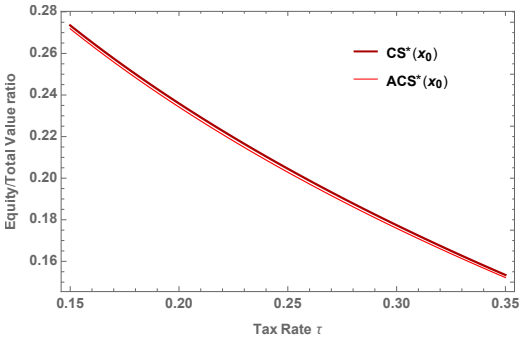
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



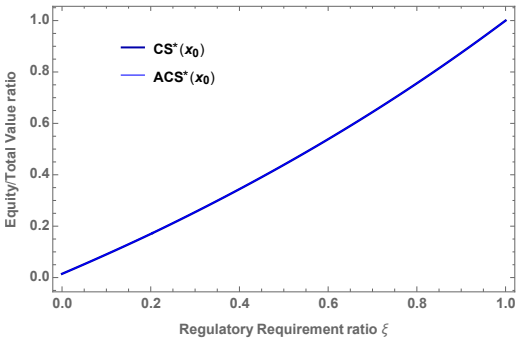
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



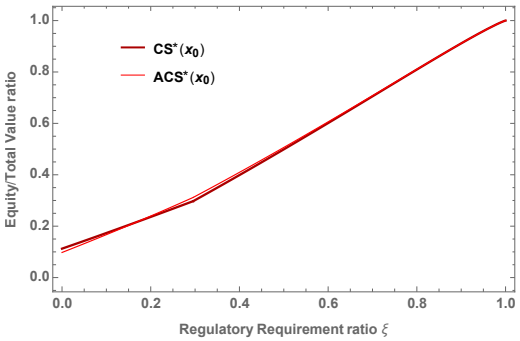
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

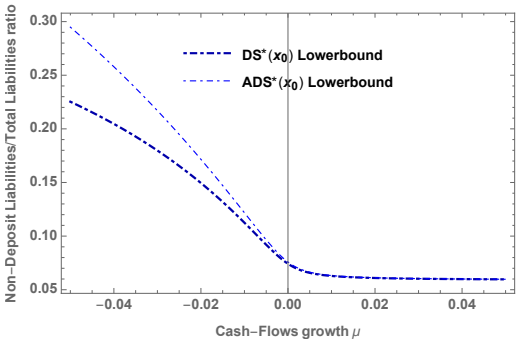


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

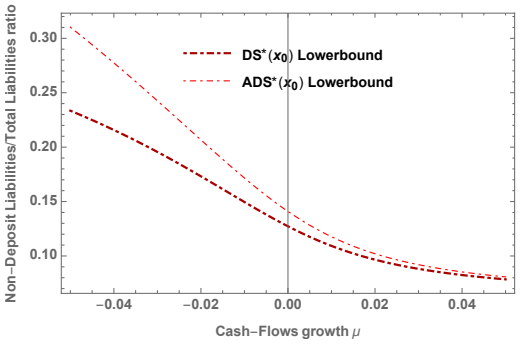


(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

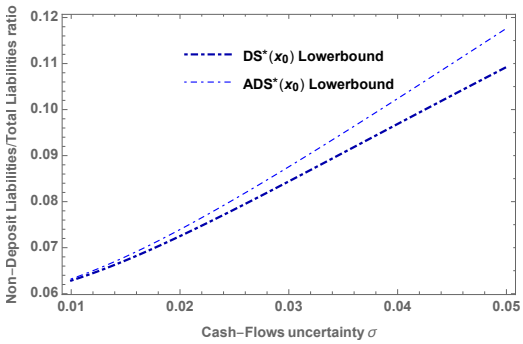
Figure 4.11: **Base Case - Equilibrium Capital Structure. Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon = 0.2$; $\tau = 0.25$; $x_0 = 50$.



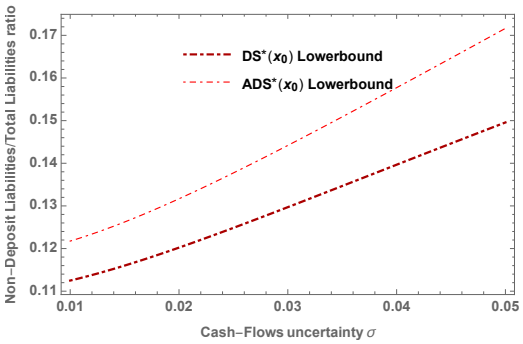
(a) **Good State - Growth** $\mu \in [-.05, .05]$



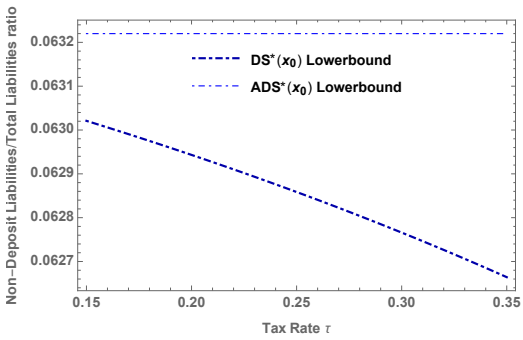
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



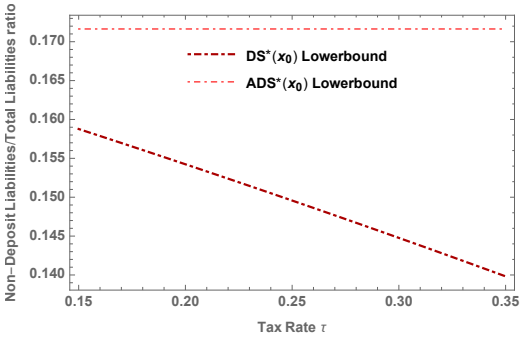
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



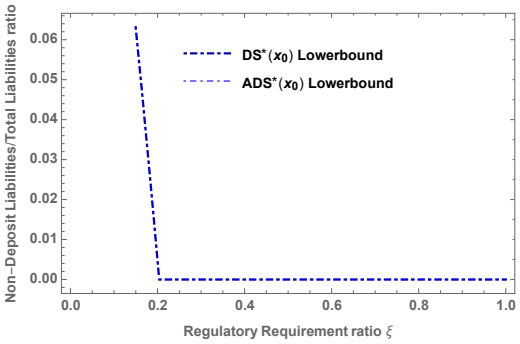
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



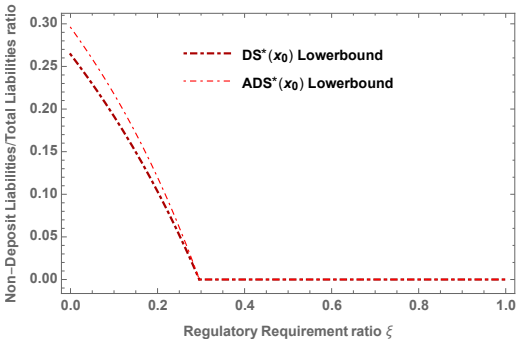
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

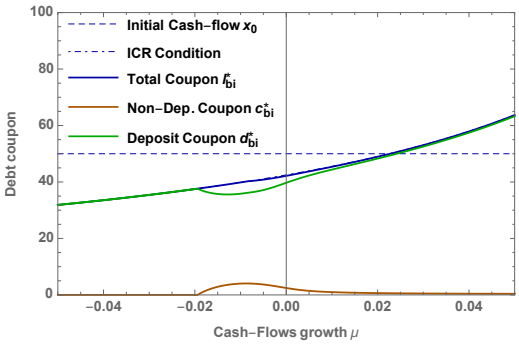


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

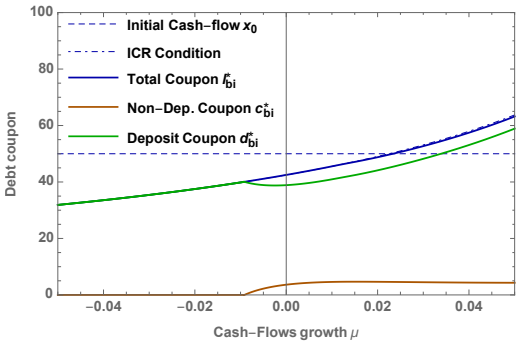


(h) **Bad State -Cap. Req. ratio changes** $\xi \in [0, 1]$

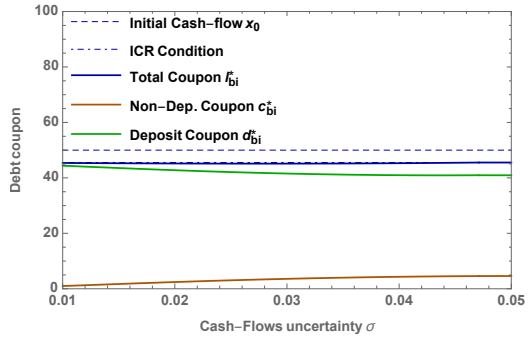
Figure 4.12: **Base Case - Equilibrium Debt Structure: Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon = 0.20$; $\tau = 0.25$; $x_0 = 50$.



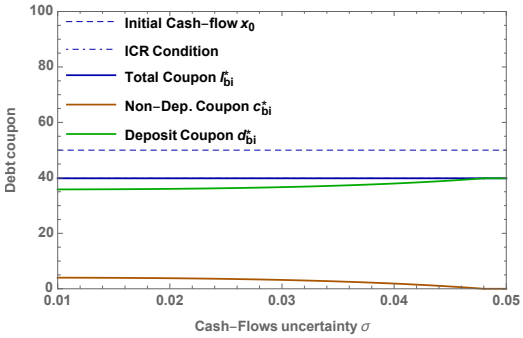
(a) **Good State - Growth** $\mu \in [-.05, .05]$



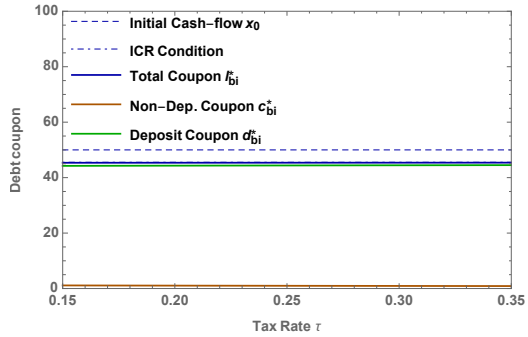
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



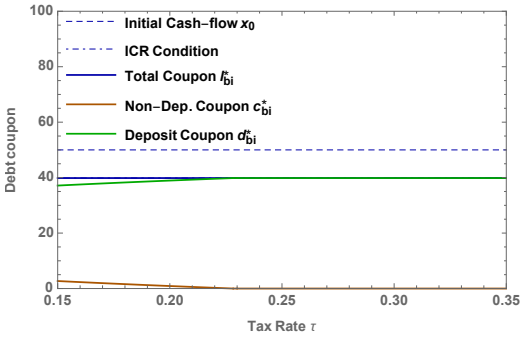
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



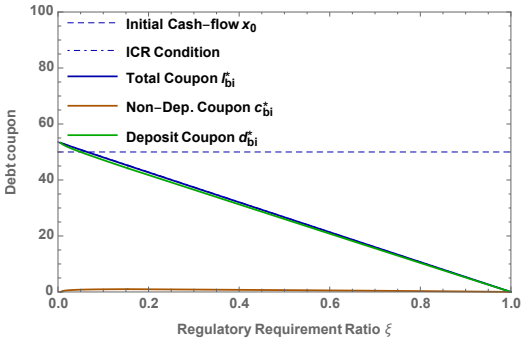
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



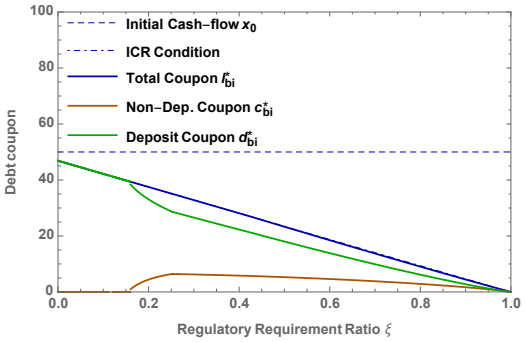
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

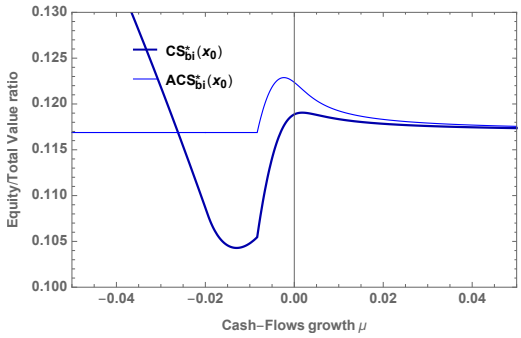


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

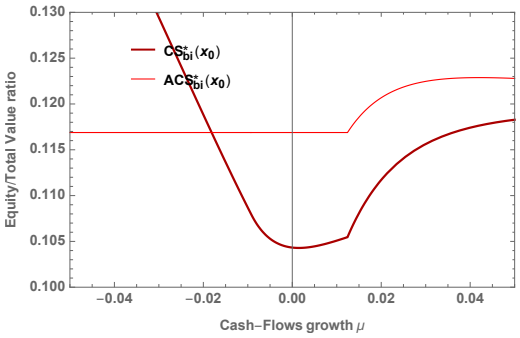


(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

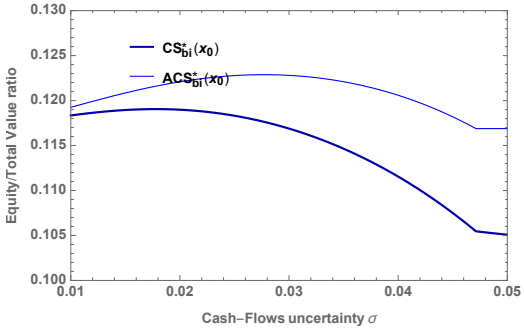
Figure 4.13: **Bail-In Case - Equilibrium Coupon Structure. Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



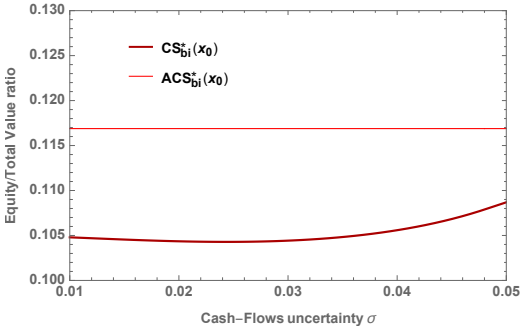
(a) **Good State - Growth** $\mu \in [-.05, .05]$



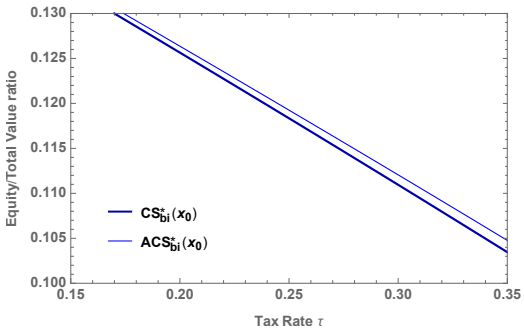
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



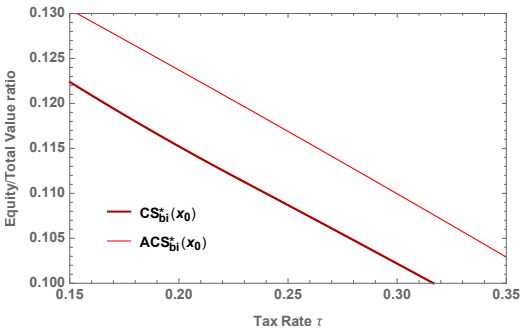
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



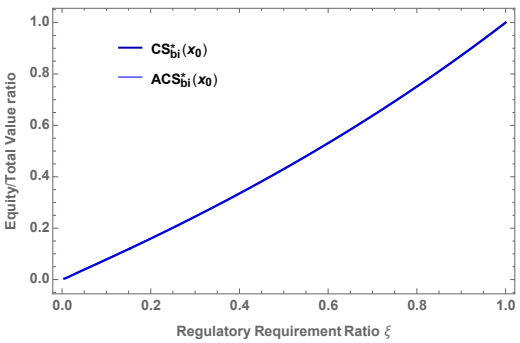
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



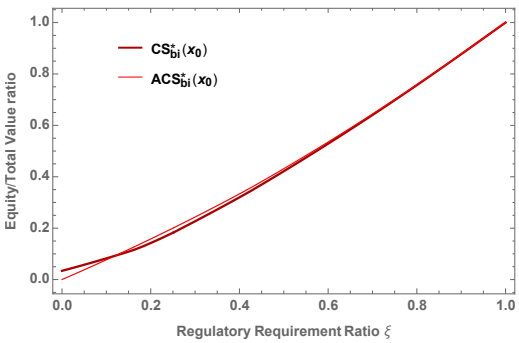
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

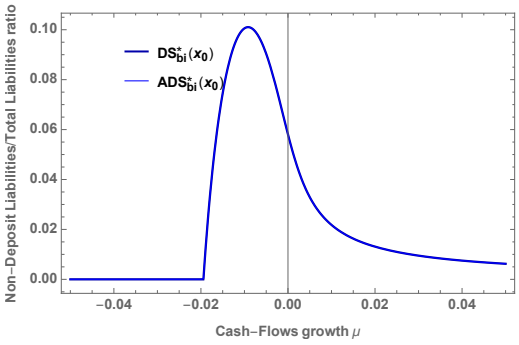


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

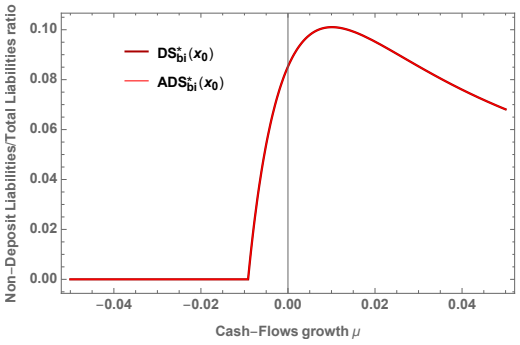


(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

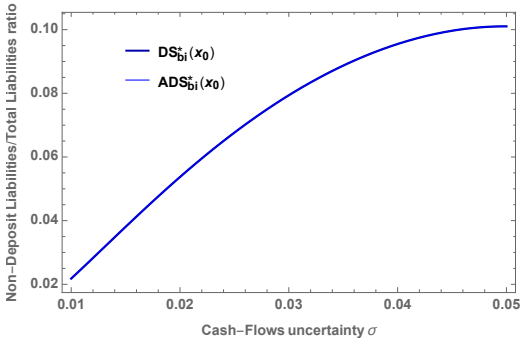
Figure 4.14: **Bail-In Case - Equilibrium Capital Structure. Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



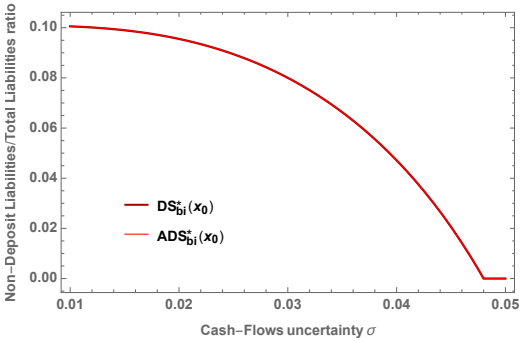
(a) **Good State - Growth** $\mu \in [-.05, .05]$



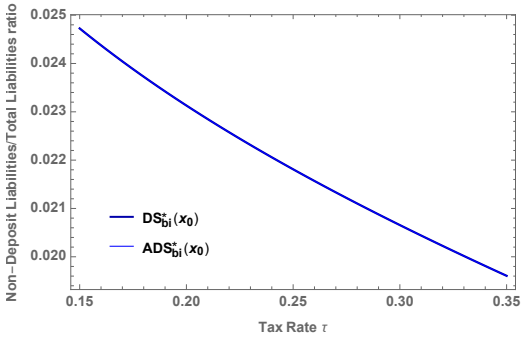
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



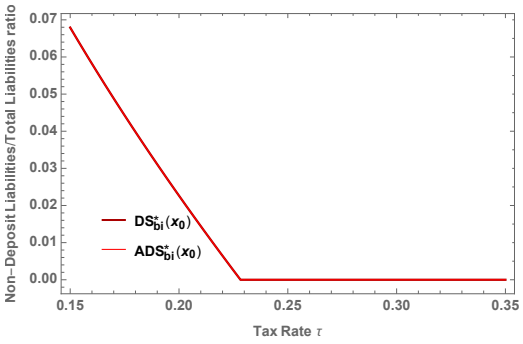
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



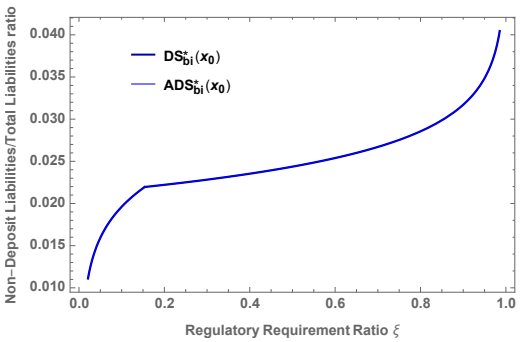
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



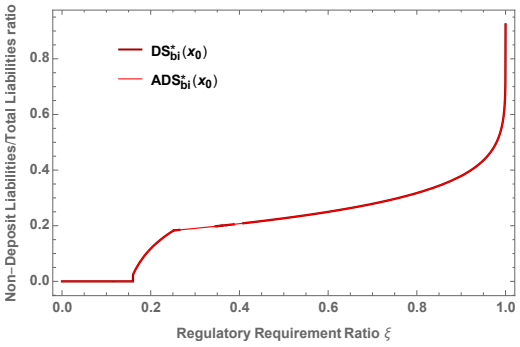
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

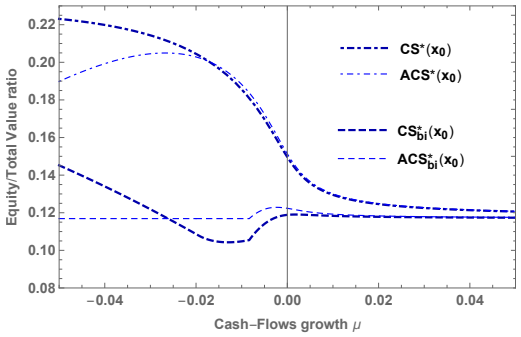


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

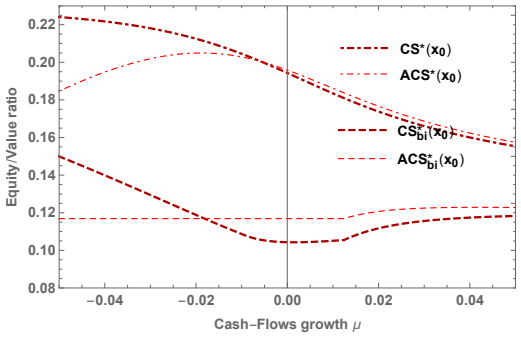


(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

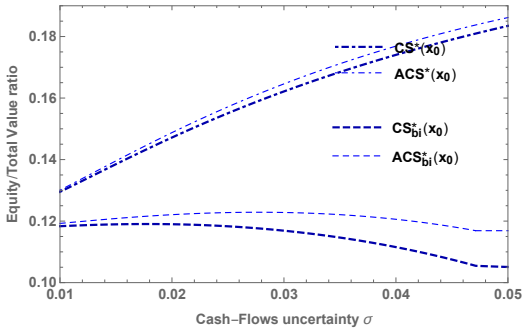
Figure 4.15: **Bail-In Case - Equilibrium Debt Structure: Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$ $\epsilon_p = 0.15$; $\tau = 0.25$; $x_0 = 50$.



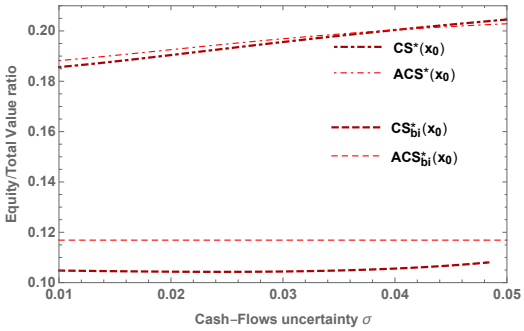
(a) **Good State - Growth** $\mu \in [-.05, .05]$



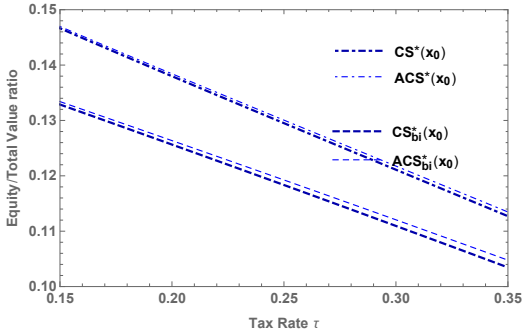
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



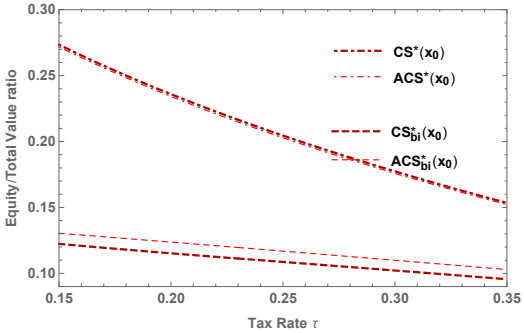
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



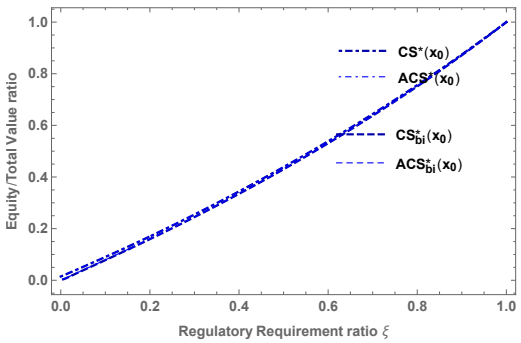
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



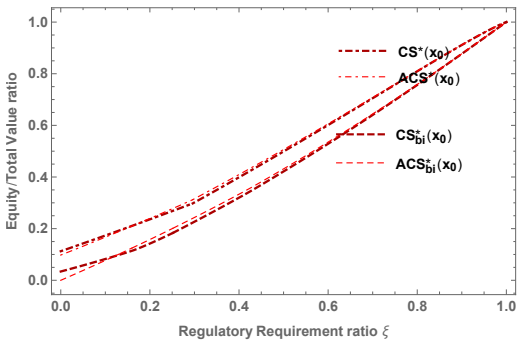
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$

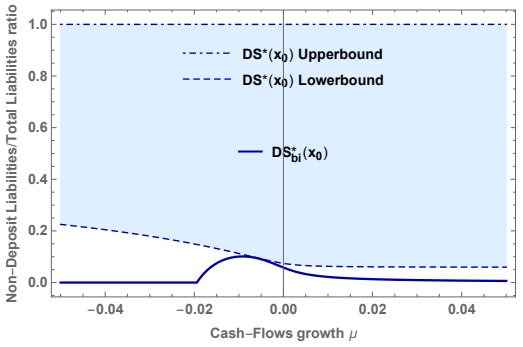


(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$

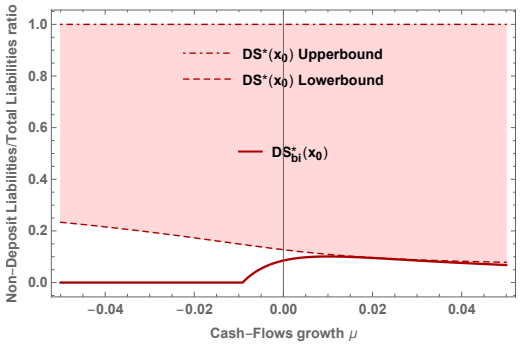


(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

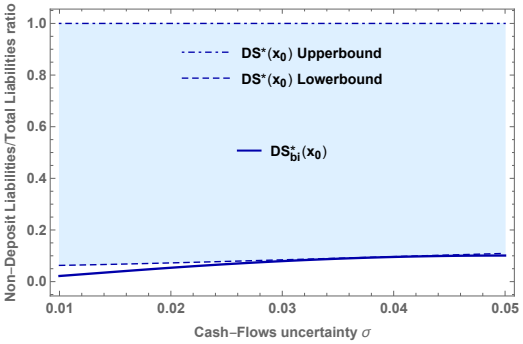
Figure 4.16: **Capital Structure Transition:** Base Case equilibrium capital structure (dot-dashed lines) vs Bail-In Case equilibrium capital structure (dashed lines). **Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$; $\epsilon_p = 0.15$; $\epsilon = 0.2$; $\tau = 0.25$; $x_0 = 50$.



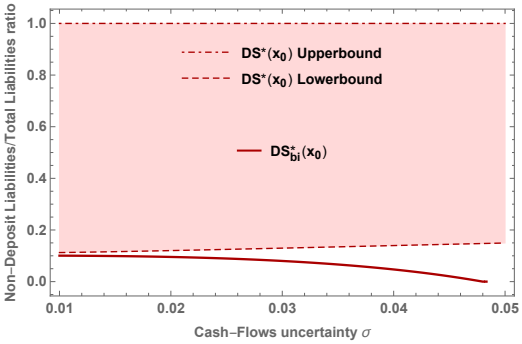
(a) **Good State - Growth** $\mu \in [-.05, .05]$



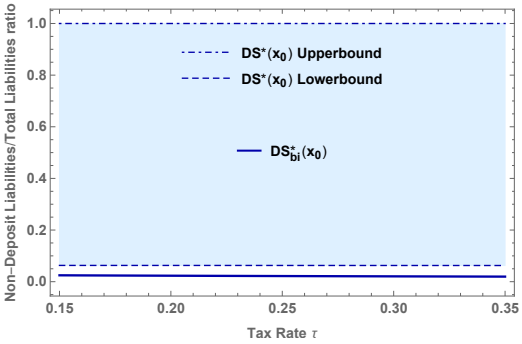
(b) **Bad State - Growth** $\mu \in [-.05, .05]$



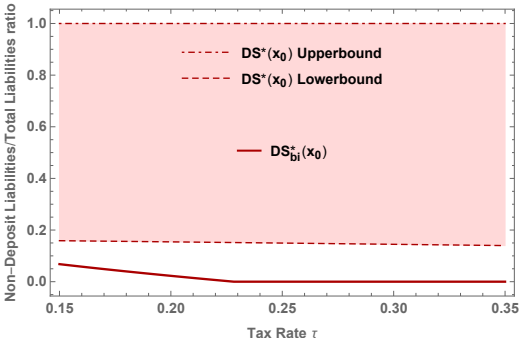
(c) **Good State - Uncertainty** $\sigma \in [.01, .05]$



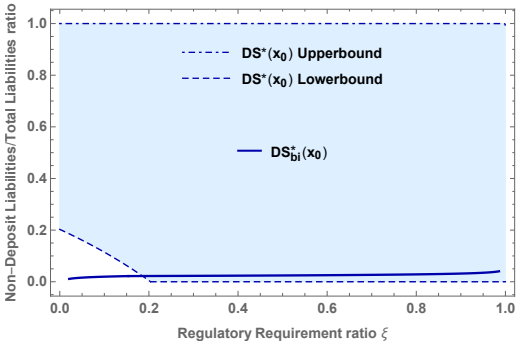
(d) **Bad State - Uncertainty** $\sigma \in [.01, .05]$



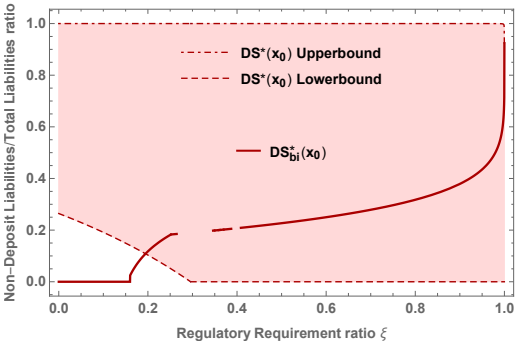
(e) **Good State - Taxes** $\tau \in [.15, .35]$



(f) **Bad State - Taxes** $\tau \in [.15, .35]$



(g) **Good State - Cap. Req. ratio** $\xi \in [0, 1]$



(h) **Bad State -Cap. Req. ratio** $\xi \in [0, 1]$

Figure 4.17: **Debt Structure Transition:** The coloured region is the admissible equilibrium debt structure for the Base Case. The bold line is the Bail-in Case equilibrium Debt Structure. **Parameters:** Good State: $\sigma = 0.01$; $\mu = 0.01$; Bad State: $\sigma = 0.5$; $\mu = -0.01$; Common: $\rho = 0.15$; $\xi = 0.15$; $\epsilon_p = 0.15$; $\epsilon = 0.2$; $\tau = 0.25$; $x_0 = 50$.

Chapter 5

Appendix

5.1 Appendix of Chapter 2

5.1.1 Patent Value

I model the decision to convert a patent into a cash-flow generating product as an investment opportunity. I write the value of the patent as $P(X)$. Following Dixit and Pindyck (1994), the bellman equation of the closure option is:

$$rP(X)dt = \mathbb{E}_t[dP]$$

Expanding the RHS using Ito's Lemma and rearranging, I obtain the following differential equation that must be satisfied by $P(X)$:

$$\frac{1}{2}\sigma^2 X^2 P''(X) + \alpha X P'(X) - rP(X) = 0 \tag{5.1}$$

In addition $P(X)$ must satisfy the following conditions:

$$\lim_{X \to 0} P(X) = 0 \tag{5.2}$$

$$P(X^*) = X^* - K(\theta) \tag{5.3}$$

Condition (5.2) reflects the property of (2.1), that is, when value X reaches zero, it stays there. Condition (5.3) is the value matching condition. At the exercise of the patent, the owner receives value X^* incurs in investment cost $K(\theta)$ and loses investment opportunity $O(X^*)$. I propose the following solution for this ODE:

$$P(X) = C_1 X^{\beta} + C_2 X^{\beta -}$$

where C_1, C_2, $\beta_- < 0$ and $\beta > 0$ are constants. I replace this solution proposal into ODE (5.1). Using conditions (5.2) and (5.3), I arrive to the following results:

$$C_1 = (X^* - K(\theta)) \left(\frac{1}{X^*}\right)^{\beta} \tag{5.4}$$

$$C_2 = 0 \tag{5.5}$$

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{5.6}$$

In detail, $C_2 = 0$ is imposed since condition (5.2) requires the closure option to converge to zero when cash-flows go to zero. I combine this with the value matching condition (5.3) to obtain the solution for constant C_1. Finally, β is the positive root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta-r=0$ that arises when I replace the proposed solution $P(X)$ in the ODE. β_- is the positive root which is inconsequential since $C_2 = 0$.

I must always exercise so as to give the option its highest value. Hence, $P(X)$ should be the highest value function for investment trigger X^*:

$$\max_{X^*} \; P(X)$$

Resulting in:

$$P(X) = \frac{1}{\theta(1-\beta)} \left(\frac{X}{X^*}\right)^{\beta} \tag{5.7}$$

$$X^* = \frac{\beta}{\beta - 1}\frac{1}{\theta} \tag{5.8}$$

5.1.2 Litigation Value

Setting the Bellman equation

$$rL_p(X)dt = \mathbb{E}_t[dL_p] - \kappa_p dt$$

and expanding yields the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 L_p''(X) + \alpha X L_p'(X) - rL_p(X) - \kappa_p + \lambda\left(\gamma\phi P(X) - L_p(X)\right) = 0$$

Proposing a solution of the form

$$L_p(X) = C_3 P(X) + C_4\kappa_p$$

where C_3 and C_4 are constants, I obtain the value of active litigation for the plaintiff:

$$L_p(X) = \gamma\phi P(X) - \frac{\kappa_p}{r+\lambda} \tag{5.9}$$

Let $L_d(X)$ be the value of active litigation for the defendant. Using the same procedure, I obtain:

$$L_d(X) = (1-\phi)P(X) - \frac{\kappa_d}{r+\lambda} \tag{5.10}$$

Following the same procedure, I need to determine the present value of transaction costs, since the plaintiff only closes her position when the judge's decision arrives. Following the same procedure, I write the present value of closing the position after the litigation as L_τ:

$$rL_\tau dt = \lambda\left(\tau - L_\tau\right)dt$$

Rearranging, it yields:

$$L_\tau = \frac{\lambda\tau}{r+\lambda} \tag{5.11}$$

5.1.3 Hatch-Waxman Condition

Under the Hatch-Waxman Act, the plaintiff considers the following profit condition $H(X)$:

$$H(X,t) = -\tau|\Delta|S - \Delta V_d^M(X) + C(X,t) \tag{5.12}$$

The first two terms in the right-hand side correspond to the total cost of the plaintiff's financial position in the defendant's equity (accounting for transaction costs $\tau|\Delta|S$) and the second term is an European call option since, after THW periods, the plaintiff chooses between requesting an inter partes review or closing the financial position. The European call option $C(X,t)$ is written as:

$$C(X,t) = e^{-r(T_{HW}-t)}\mathbb{E}\left[\max\left\{\Delta\left(Y + L_d(X_{T_{HW}})\right) + L_p(X_{T_{HW}}) - L_\tau|\Delta|S\,,\, \Delta V_d^M(X_{T_{HW}}) - \tau|\Delta|S\right\}\right] \tag{5.13}$$

I assume that the plaintiff makes a final decision at time T_{HW}, choosing the highest value between initiating the inter partes review or closing the financial position opened at time $t = 0$ at market prices.

Let us define the following variable $Q = X^\beta$. I impose a temporary change of variable in the model from X to $Q \equiv Q(X)$ because there is a non-linear relationship between the call option and X. So, to use the Black-Scholes option pricing model, I need a state variable that follows a geometric brownian motion. Using Ito's lemma, instantaneous changes in Q are represented by:

$$dQ = \frac{\partial Q}{\partial t}dt + \frac{1}{2}\frac{\partial^2 Q}{\partial X^2}(dX)^2 + \frac{\partial Q}{\partial X}dX \tag{5.14}$$

I use equation (2.4) and take derivatives of $Q(X)$ with respect to X. Replacing and rearranging expression (5.14) yields the following geometric brownian motion:

$$dQ = rQdt + \beta\sigma Q\, dW \tag{5.15}$$

Imposing the change of variable in equations (2.7), (2.9) and (2.10), I obtain:

$$\begin{aligned} L_d(Q) &= \frac{1-\phi}{\beta}\left(\frac{\beta}{(\beta-1)\theta}\right)^{1-\beta} Q - \frac{\kappa_d}{r+\lambda} \\ L_p(Q) &= \frac{\gamma\phi}{\beta}\left(\frac{\beta}{(\beta-1)\theta}\right)^{1-\beta} Q - \frac{\kappa_p}{r+\lambda} \\ V_d^M(Q) &= \left(\frac{\xi}{\beta}\left(\frac{\beta}{(\beta-1)\overline{\theta}}\right)^{1-\beta} + \frac{1-\xi}{\beta}\left(\frac{\beta}{(\beta-1)\underline{\theta}}\right)^{1-\beta}\right) Q + Y \end{aligned} \tag{5.16}$$

Furthermore, let us define the following constants:

$$D_1 = \left(\frac{\beta}{\beta-1}\right)^{1-\beta} \frac{\left(\gamma\underline{\theta}\overline{\theta}\phi\theta^\beta + \Delta\overline{\theta}\left(\theta(\xi-1)\underline{\theta}^\beta - \underline{\theta}(\phi-1)\theta^\beta\right) - \Delta\theta\underline{\theta}\xi\overline{\theta}^\beta\right)}{\beta\theta\underline{\theta}\overline{\theta}} \tag{5.17}$$

$$D_2 = \Delta\frac{\kappa_d}{r+\lambda} + \frac{\kappa_p}{r+\lambda} - \frac{r}{r+\lambda}\tau|\Delta|S \tag{5.18}$$

Rewriting expression (5.13), I get:

$$\begin{aligned} C(Q,t) &= e^{-r(T_{HW}-t)}\mathbb{E}\left[\max\left\{\Delta\left(Y + L_d(Q_{T_{HW}})\right) + L_p(Q_{T_{HW}}) - L_\tau|\Delta|S\,,\, \Delta V_d^M(Q_{T_{HW}}) - \tau|\Delta|S\right\}\right] \\ &= e^{-r(T_{HW}-t)}\mathbb{E}\left[\max\left\{\Delta\left(L_d(Q_{T_{HW}}) - V_d^M(Q_{T_{HW}})\right) + L_p(Q_{T_{HW}}) + \Delta Y - (L_\tau-\tau)|\Delta|S\,,\, 0\right\} - \tau|\Delta|S + \Delta V_d^M(Q_{T_{HW}})\right] \\ &= e^{-r(T_{HW}-t)}\mathbb{E}\left[\max\left\{\Delta\left(L_d(Q_{T_{HW}}) - V_d^M(Q_{T_{HW}})\right) + L_p(Q_{T_{HW}}) + \Delta Y - (L_\tau-\tau)|\Delta|S\,,\, 0\right\} - \tau|\Delta|S + \Delta V_d^M(Q_{T_{HW}})\right] \\ &= -e^{-r(T_{HW}-t)}\tau|\Delta|S + \Delta V_d^M(Q) + e^{-r(T_{HW}-t)}\mathbb{E}\left[\max\left\{D_1 Q_{T_{HW}} - D_2, 0\right\}\right] \end{aligned} \tag{5.19}$$

Using (Black and Scholes 1973) and changing the state variable back to X, I get:

$$C(X,t) = -e^{-r(T_{HW}-t)}\tau|\Delta|S + \Delta V_d^M(X) + N\left(d_1\right) D_1 X^\beta - e^{-r(T_{HW}-t)}N(d_2)D_2 \tag{5.20}$$

where $N(.)$ corresponds to the cumulative standard normal distribution giving us the risk adjusted probability, and d_1 and d_2 are constants:

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{D_1 X^\beta}{D_2}\right) + \left(r + \frac{1}{2}\beta^2\sigma^2\right)(T_{HW}-t)}{\beta\sigma\sqrt{T_{HW}-t}} \\ d_2 &= d_1 - \beta\sigma\sqrt{T_{HW}-t} \end{aligned} \tag{5.21}$$

Replacing equation (5.20) in the hedging equation (5.12), I get:

$$H(X,t) = -(1+e^{-r(T_{HW}-t)})\tau|\Delta|S + N\left(d_1\right) D_1 X^\beta - e^{-r(T_{HW}-t)}N(d_2)D_2 \tag{5.22}$$

5.1.4 Effect of Hatch-Waxman Integrity Act

▷ **Relationship between T_{HW} and $\overline{\Delta}$.** Consider an open neighbourhood around $(\Delta, T_{HW}) = (\Delta', T'_{HW})$ and let us apply a Taylor expansion over $H(X,t)$. Without loss of generality, I ignore terms of order equal and higher than 2. I then have a linear approximation:

$$H(X,t|\Delta,T_{HW}) = H(X,t|\Delta',T'_{HW}) + \left.\frac{\partial H(X,t)}{\partial \Delta}\right|_{(\Delta',T'_{HW})}(\Delta-\Delta') + \left.\frac{\partial H(X,t)}{\partial T_{HW}}\right|_{(\Delta',T'_{HW})}(T_{HW}-T'_{HW}) \tag{5.23}$$

where $\partial H(X,t)/\partial\Delta < 0$ and $\partial H(X,t)/\partial T_{HW} > 0$. Now, let us restrict the financial position Δ to hedging ones, that is, $\overline{\Delta}$ around point $\overline{\Delta}'$. I rewrite equation (5.23) as:

$$\left.\frac{\partial H(X,t)}{\partial \Delta}\right|_{(\overline{\Delta}',T'_{HW})}(\overline{\Delta}'-\overline{\Delta}) = \left.\frac{\partial H(X,t)}{\partial T_{HW}}\right|_{(\overline{\Delta}',T'_{HW})}(T_{HW}-T'_{HW}) \tag{5.24}$$

Consider T''_{HW} a sufficiently small increase over the time to litigation T'_{HW}, i.e. $T''_{HW} > T'_{HW}$. In this case, the hedging financial position $\overline{\Delta}''$ that secures equality presented in equation (5.24) must necessarily be such that $\overline{\Delta}'' > \overline{\Delta}'$.

▷ **Limits of function $H(X,t)$ with respect to T_{HW}.** Let us first take the limit of $H(X,t)$ when the mandatory litigation period $T_{HW}-t$ tends to zero. In this case, I have:

$$\begin{aligned}
&\lim_{T_{HW}\to t} H(X,t) \\
&= \lim_{T_{HW}\to t} -\tau|\Delta|S - \Delta V_d^M(X) + \max\left\{\Delta\left(Y + L_d(X_{T_{HW}})\right) + L_p(X_{T_{HW}}) - L_\tau|\Delta|S\,,\, \Delta V_d^M(X_{T_{HW}}) - \tau|\Delta|S\right\} \\
&= \max\left\{B(X), -2\tau|\Delta|S\right\} = B(X)
\end{aligned} \tag{5.25}$$

Second, let us consider the less realistic scenario where the mandatory waiting period $T_{HW}-t$ goes to infinity. It is useful to remind that:

$$\lim_{T_{HW}\to+\infty} N(d_1) = 1 \qquad\qquad \lim_{T_{HW}\to+\infty} e^{-r(T_{HW}-t)}N(d_2) = 0$$

In this case, I have:

$$\begin{aligned}
&\lim_{T_{HW}\to+\infty} H(X,t) \\
&= -\tau|\Delta|S - \Delta V_d^M(X) + \lim_{T_{HW}\to+\infty} C(X,t) \\
&= -\tau|\Delta|S + \Delta(1-\phi)P(X) - \Delta\mathbb{E}^\theta\left[P(X|\tilde{\theta})\right] + \gamma\phi P(X) \quad > B(X)
\end{aligned} \tag{5.26}$$

5.2 Appendix of Chapter 3

5.2.1 The Managers' Optimal Decision

Proof of Lemma 2. I show this auxiliary Lemma in two steps.

First, I make use of the mechanism design literature to restrict the problem to contracts that are incentive-compatible and no-savings, that is, they prevent private savings such that $dW_t^I = 0$. As standard in contract theory, an indirect contract that induces an optimal consumption-effort policy $(\tilde{p}^*, \tilde{a}^*)$ can be replaced, by the revelation principle, with a direct contract that expresses the incentive-compatible effort and incentivises the manager to spend all resources available.

In detail, let $\mathbb{C}' = (p', a')$ induce the optimal spending-effort policy $(\tilde{p}^*, \tilde{a}^*)$ and yield an initial continuation value M_0' and wealth dynamics:

$$dW_t^I = rW_t^I dt + p_t' dt - \tilde{p}_t^* dt - g(x_t, \tilde{a}_t^*)dt$$

This contract can be replaced by a direct contract $\mathbb{C} = (\tilde{p}^* + g(\tilde{a}^*, x) - rW, \tilde{a}^*)$ provided that the consumption-savings optimal condition is fulfilled (see Equation (5.33)). The manager's continuation value is the same, that is, M_0' and the wealth balance stays unchanged, i.e. $dW_t^I = 0$.

Now consider that, under the direct contract $\mathbb{C}$, the manager finds that policy $(\hat{p}, \hat{a})$ actually yields a strictly higher continuation value $\hat{M}_0 > M_0'$. In this case, the manager's wealth follows:

$$dW_t^I = (\tilde{p}_t^* - \hat{p}_t)dt + (g(x_t, \tilde{a}_t^*) - g(x_t, \hat{a}_t))dt \neq 0$$

If the shareholder had offered the initial indirect contract $\mathbb{C}' = (p', a')$ then the manager would have achieved a higher continuation value $\hat{M}_0 > M_0'$ had she chosen initially the policy $(\hat{p}, \hat{a})$ with wealth dynamics represented by:

$$dW_t^I = rW_t^I dt + p_t' dt - \hat{p}_t dt - g(x_t, \hat{a}_t)dt$$

However, this would be inconsistent with the optimality of the spending-effort policy $(\tilde{p}^*, \tilde{a}^*)$ when the manager faces the indirect contract $\mathbb{C}'$.

So, I can restrict the problem to direct contracts that are incentive-compatible and no-savings. In this case, the principal requires (or recommends) an effort a that equals the manager's optimal effort $\tilde{a}^*$ and pays a salary $p = \tilde{p}^* + g(\tilde{a}^*, x) - rW$ that secures:

$$dW_t^I = 0 \tag{5.27}$$

Second, I show that the incentive compatibility problem (the effort alignment problem) does not depend on the wealth balance W. Consider that the CEO is endowed at time t with an exogenous wealth, that is, $W_t^I = \mathcal{W}$. I write the manager's continuation value as $M_t^I(\mathbb{C}, x_t, \mathcal{W})$, under incentive-compatible and no-savings contract $\mathbb{C} = (p, a)$. Under this contract, condition (5.27) is fulfilled and, by the *substitution method*, we can restrict to the incentive-compatibility problem:

$$M_t^I(\mathbb{C}, x_t, \mathcal{W}) = \max_{\tilde{a}} \mathbb{E}_t \left[\int_t^{+\infty} -\frac{1}{\gamma} e^{-r(s-t)-\gamma(p_s + r\mathcal{W} - g(\tilde{a}_s, x_s))} ds \right] \tag{5.28}$$

Due to the exponential functional form of CARA utility, I can then write:

$$\begin{aligned} M_t^I(\mathbb{C}, x_t, \mathcal{W}) &= e^{-\gamma r \mathcal{W}} \max_{\tilde{a}} \mathbb{E}_t \left[\int_t^{+\infty} -\frac{1}{\gamma} e^{-r(s-t)-\gamma(p_s + r\mathcal{W} - g(\tilde{a}_s, x_s))} ds \right] \\ &= e^{-\gamma r \mathcal{W}} M_t^I(\mathbb{C}, x_t, 0) \end{aligned} \tag{5.29}$$

This shows the effort incentives are independent from wealth level $\mathcal{W}$ under a CARA utility function. I can restrict the incentive alignment problem solution to the no-wealth case $W_t^I = 0$ since, when $W_t^I = \mathcal{W}$, the manager will just consume the interest $r\mathcal{W}$. □

Proof of Proposition 1. Recall the manager's optimisation problem:

$$\begin{aligned} M_t^I(\mathbb{C}, x_t) = \max_{\tilde{p}, \tilde{a}} \quad & \mathbb{E}_t \left[\int_t^{+\infty} -\frac{1}{\gamma} e^{-r(s-t)-\gamma \tilde{p}_s} ds \right] \\ \text{s.t.} \quad & dW_t^I = rW_t^I dt + p_t dt - \tilde{p}_t dt - g(x_t, \tilde{a}_t) dt \\ & dx_t = \delta(x_t)\, dt + \alpha(a_t, x_t)\, dt + \sigma x_t\, dw_t, \qquad t \in [0, T) \end{aligned} \tag{5.30}$$

where $M_t^I(\mathbb{C}, x_t)$ is the manager's continuation value for her optimal consumption strategy $\tilde{p}^*$ and optimal effort strategy $\tilde{a}^*$ when she faces an incentive-compatible contract $\mathbb{C}$ and the cash-flow level is x_t. Moreover, let me restrict to solutions that respect the following integrability and transversality conditions:

$$\begin{aligned} \mathbb{E}_t \left[\int_t^{+\infty} -\frac{1}{\gamma} e^{-r(s-t)-\gamma \tilde{p}_s} ds \right] < \infty \\ \lim_{T \to +\infty} \mathbb{E}\left[e^{-rT} W_T^I \right] = 0 \end{aligned} \tag{5.31}$$

My goal is to obtain the diffusion process dM_t^I, that is, the dynamics of the manager's continuation value under the incentive-compatible and no-savings contract $\mathbb{C}$.

For the remainder of the proof, let me write $M_t^I \equiv M_t^I(\mathbb{C}, x_t)$. Now, I write the Hamilton-Jacobi-Bellman partial differential equation for the manager's optimisation problem 5.30 as:

$$rM_t^I = \max_{\tilde{p}, \tilde{a}} \left\{ u(\tilde{p}) + \frac{\partial M_t^I}{\partial x_t} \left(\delta(x_t) + \alpha(\tilde{a}_t, x_t) \right) + \frac{1}{2} \frac{\partial^2 M_t^I}{\partial x_t^2} (\sigma x_t)^2 + \frac{\partial M_t^I}{\partial W_t^I} \left(rW_t^I + p_t - \tilde{p}_t - g(\tilde{a}_t, x_t) \right) \right\} \tag{5.32}$$

I find the conditions that maximise the expression inside brackets. I fix first an effort policy $\tilde{a} = a$ and write the first order condition over the realised consumption control. The consumption-savings policy must respect:

$$\frac{\partial u(\tilde{p}_t)}{\partial \tilde{p}_t} = \frac{\partial M_t^I}{\partial W_t^I} \Rightarrow u\left(\tilde{p}_t^*\right) = r M_t^I \tag{5.33}$$

This condition states that the marginal utility derived from consumption must equal the marginal value derived from a change in the savings balance W_t^I. Recalling that we restrict, by Lemma 2, to no-savings and incentive-compatible contracts, we know that $\mathbb{C}$ is such that it prevents the manager from saving ($dW_t^I = 0$) and requires effort an effort $a = \tilde{a}^*$. I obtain the following differential equation:

$$\frac{\partial M_t^I}{\partial x_t}\left(\delta(x_t) + \alpha(a_t, x_t)\right) + \frac{1}{2}\frac{\partial^2 M_t^I}{\partial x_t^2}(\sigma x_t)^2 = 0 \tag{5.34}$$

If differential equation (5.34) is respected under a fixed $a = \tilde{a}^*$, the manager is satisfied with her choice of consumption and has no incentives to incur in hidden savings. By backward induction, this allows me to know the equilibrium compensation under $\mathbb{C}$ that secures no-savings and adequate choice of effort incentives. However, we still have to find the condition that secures the optimal choice of effort since equation (5.34) only assumes that the manager is optimising the effort.

Therefore, consider now the incentive-compatibility optimisation problem. Akin to the consumption-savings optimality problem, I fix now a consumption policy $\tilde{p} = rW + p - g(\tilde{a}, x)$ at the recommended level. I write the FOC for effort and obtain:

$$\frac{\partial M_t^I}{\partial x_t}\frac{\partial \alpha(x_t, \tilde{a}_t)}{\partial \tilde{a}_t} = \frac{\partial M_t^I}{\partial W_t^I}\frac{\partial g(\tilde{a}_t, x_t)}{\partial \tilde{a}_t} \Rightarrow \frac{\partial M_t^I}{\partial x_t} = -r\gamma\frac{\partial g(\tilde{a}_t, x_t)}{\partial \tilde{a}_t}\left(\frac{\partial \alpha(x_t, \tilde{a}_t)}{\partial \tilde{a}_t}\right)^{-1} M_t^I \tag{5.35}$$

Condition (5.35) must be respected in order to have choices of effort that are optimal for the manager.

I focus now on the dynamics of the manager's continuation value $M_t^I(p_t, a_t)$. Applying Ito's lemma I have:

$$dM_t^I = \frac{\partial M_t^I}{\partial x_t}\left(\delta(x_t) + \alpha(a_t, x_t)\right)dt + \frac{\partial M_t^I}{\partial x_t}\sigma x_t dw_t + \frac{1}{2}\frac{\partial^2 M_t^I}{\partial x_t^2}(\sigma x_t)^2 dt \tag{5.36}$$

Replacing equation (5.34) and equation (5.35) in the differential equation (5.36), gives me the dynamics of the manager's continuation value when she is choosing the optimal consumption and effort when facing the incentive-compatible and no-savings contract $\mathbb{C} = (p, a)$. This yields the diffusion process:

$$dM_t^I = -r\gamma\Gamma_t\sigma x_t M_t^I dw_t \tag{5.37}$$

where Γ_t is an adapted process respecting the square-integrability condition $\mathbb{P}\left[\int_0^t (\Gamma_s)^2\, ds < \infty\right] = 1$ and it is written as:

$$\Gamma_t = \frac{\partial g(x_t, a_t)}{\partial a_t}\left(\frac{\partial \alpha(x_t, a_t)}{\partial a_t}\right)^{-1}$$

Furthermore, I rearrange condition (5.33) and take into consideration Lemma 2 to get the equilibrium wage received by the manager:

$$p_t = g(x_t, a_t) - \frac{1}{\gamma}\ln\left(-r\gamma M_t^I\right) \tag{5.38}$$

Verification of Optimality. I now verify that a manager is indeed better-off when she follows a strategy path that does not deviate from the recommended strategy under $\mathbb{C} = (p, a)$, the contract that is incentive compatible and prevents change in wealth.

Consider a case where, before time t, the CEO follows a discretionary strategy where she consumes $\hat{p}$ and exerts some effort $\hat{a}$. Moreover, consider that, after time t, the CEO decides to changes the strategy, by consuming the recommended consumption $p_t - g(x_t, a_t) + rW_t^I$ and exerting effort a_t, consistent with the optimal contract $\mathbb{C}$. Therefore, after t, the continuation value for the manager $M_t^I \equiv M_t^I(\mathbb{C}, x_t)$ is the equilibrium value. I write the time-t expectation of the agent's total gain as

$$\mathcal{M}_t = \int_0^t e^{-rs}u(\hat{p})ds + e^{-rt}e^{-\gamma r W_t^I} M_t^I \tag{5.39}$$

I apply Ito's lemma to get $d\mathcal{M}_t$ and then impose a change of measure since the diffusion processes under $\mathbb{P}$ (recommended effort under contract $\mathbb{C}$) and under $\mathbb{P}^{\hat{a}}$ (discretionary effort $\hat{a}$) are related by $\sigma x_t w_t = \sigma x_t w_t^{\hat{a}} + \int_0^t (\alpha(x_s, \hat{a}_s) - \alpha(x_s, a_s))ds$. The dynamics of condition (5.39) can be written as:

$$e^{rt+\gamma r W_t^I} d\mathcal{M}_t = \left(e^{\gamma r W_t^I} u(\hat{p}_t) - rM_t^I - r\gamma\Gamma_t M_t^I\left(\alpha(x_t, \hat{a}_t) - \alpha(x_t, a_t)\right)\right)dt - \gamma r M_t^I dW_t^I - \gamma r \Gamma_t \sigma x_t M_t^I dw_t^{\hat{a}} \tag{5.40}$$

where the CEO's wealth is $W_t^I = \int_0^t e^{r(t-s)}\left(p_s - \hat{p}_s - g(x_t, \hat{a}_t)\right)ds$, when she chooses a different consumption plan.

Taking first order conditions over deviating effort $\hat{a}_t$ and consumption $\hat{p}_t$, and setting $\hat{a}_t = a_t$, $\hat{p}_t = (p_t - g(x_t, a_t) + rW_t^I)$, I get a null drift in condition (5.40). For all other choices of consumption and effort, the drift is weakly negative and, hence, the total expected gain process $\mathcal{M}_t$ is a supermartingale.

□

5.2.2 Mean Reverting Growth and Quadratic Costs

The shareholder's optimisation problem 3 has two problems. The first is to find a solution for $E^I(x_t, M_t^I)$ and $E^O(x_t, M_t^O)$ and, implicitly, find the optimal required effort to be asked to the manager in place. The second is to find the optimal dismissal time by treating the optimal stopping time problem as a free-boundary problem, i.e. a "real options" problem as portrayed in Dixit and Pindyck (1994).

I solve the optimisation problem 3 in three Lemmas. Lemma 3 solves for the equity value function assuming the firm operates in continuity under the incumbent management, Lemma 4 solves for the equity value function assuming the firm operates in continuity under the outsider management, and Lemma 5 solves for the dismissal option value function.

Proof of Lemma 3. I write the HJB for the equity value under the incumbent management $E^I(x_t)$, solve for the intra-period optimal required effort $a^* \equiv \{a_t^*\}_{t=0}^{T_d}$ under contract $\mathbb{C}$, and use the optimal effort a^* to find the specific solution for the equity value function.

I find now the specific solution for the equity value under the incumbent management. I first use equations (3.24) and (3.25) to obtain the adapted process $\Gamma_t = \theta a_t$, that is, the specific form of the adapted process in Proposition 1. The equity value under the equilibrium contract $\mathbb{C}$ is:

$$\begin{aligned} E^I(x_t, M_t^I) = \max_a \quad & \mathbb{E}_t \left[\int_t^{+\infty} e^{r(s-t)} \left(x_s - p_s\right) ds \right] \\ \text{s.t.} \quad & dx_t = \delta(x_t)\, dt + \alpha(a_t, x_t)\, dt + \sigma(x_t)\, dw_t \\ & dM_t^I = -r\gamma\Gamma_t \sigma x_t M_t^I dw_t \end{aligned} \tag{5.41}$$

Writing it in HJB-PDE form yields:

$$\begin{aligned} rE_t^I = \max_{a_t} \quad & x_t - p_t + \frac{\partial E_t^I}{\partial x_t}(\Delta - \kappa x_t + \overline{\alpha} + a_t x_t) + \frac{1}{2}\frac{\partial^2 E_t^I}{\partial x_t^2}\sigma^2 x_t \\ & + \frac{1}{2}\frac{\partial^2 E_t^I}{\partial {M_t^I}^2}\left(r\gamma\theta a_t \sigma\sqrt{x_t} M_t^I\right)^2 - \frac{\partial^2 E_t^I}{\partial x_t \partial M_t^I} r\gamma\theta a_t (\sigma\sqrt{x_t})^2 M_t^I \end{aligned} \tag{5.42}$$

Similarly to the first step, I simplify with $\partial^2 E_t^I / \partial x_t \partial M_t^I = 0$, with $\partial E_t^I / \partial x_t = \partial A_t^I / \partial x_t$ and $\partial^2 E_t^I / \partial x_t^2 = \partial^2 A_t^I / \partial x_t^2$. I replace with the diffusion loading $\Gamma_t = \theta a_t$ and with the optimal compensation $p_t = x_t \theta a_t^2/2 - \gamma^{-1}\ln(-r\gamma M_t^I)$. The HJB becomes:

$$rA_t^I = \max_{a_t} \left\{ \frac{\partial A_t^I}{\partial x_t}(\Delta - \kappa x_t + \overline{\alpha} + a_t x_t) - \frac{\theta a_t^2}{2} x_t - \frac{r\gamma\theta^2 a_t^2 \sigma^2}{2} x_t \right\} + \frac{1}{2}\frac{\partial^2 A_t^I}{\partial x_t^2}\sigma^2 x_t + x_t \tag{5.43}$$

The maximisation problem equation (5.43) yields following condition for optimal required effort a_t^*:

$$a_t^* = \frac{\partial A_t^I}{\partial x_t}\frac{1}{\theta + r\gamma\theta^2\sigma^2} \tag{5.44}$$

Replacing with the optimal required effort a_t^* back in equation (5.43), we obtain:

$$rA_t^I = \frac{\partial A_t^I}{\partial x_t}(\Delta - \kappa x_t + \overline{\alpha}) + \left(\frac{\partial A_t^I}{\partial x_t}\right)^2 \frac{1}{2}\frac{x_t}{\theta + r\gamma\theta^2\sigma^2} + \frac{\partial^2 A_t^I}{\partial x_t^2}\frac{1}{2}\sigma^2 x_t + x_t \tag{5.45}$$

I propose the following solution to the ODE:

$$A_t^I = C_1 + C_2 x_t \tag{5.46}$$

where C_1, C_2 are constants to be determined. This yields the following solution for the asset value function under the incumbent management:

$$A^I(x_t) = \left(\theta + \theta^2 r\sigma^2\gamma\right) a_t^* \left(\frac{\overline{\alpha} + \Delta}{r} + x_t\right) \tag{5.47}$$

where the optimal effort required to the incumbent CEO is:

$$a_t^* = \kappa + r - \sqrt{(\kappa + r)^2 - \frac{2}{\theta + \theta^2 r\sigma^2\gamma}} \tag{5.48}$$

The firm value function $A^I(x_t)$ under the incumbent's management is an affine function of the instantaneous cash-flow x_t. The firm's value varies positively with the fixed cash-flow growth $\overline{\alpha} + \Delta$ and varies negatively with the variable cost parameter κ, the effort cost parameter θ, the risk-aversion parameter γ, and the discount rate r. Substituting with equation (5.47) in equation (3.27), the specific solution for $E^I(x_t, M_t^I)$ is:

$$E^I(x_t, M_t^I) = \theta\left(\theta r\sigma^2\gamma + 1\right)\left(\kappa - \sqrt{(\kappa + r)^2 - \frac{2}{\theta + \theta^2 r\sigma^2\gamma}} + r\right)\left(\frac{\overline{\alpha} + \Delta}{r} + x_t\right) - N_t^I \tag{5.49}$$

□

Proof of Lemma 4. I write the Hamilton-Jacobi-Bellman Equation for the equity value under the outsider management $E^O(x_t)$ and solve for effort $b^* \equiv \{b_t^*\}_{t=T_d}^{+\infty}$, that is, the optimal effort path that the shareholder finds optimal to request under contract $\mathbb{D}$. Then, using the optimal effort b^*, I find the specific solution of the equity value function.

I first use equations (3.24) and (3.26) to obtain $\Phi_t = \xi b_t$, that is, the specific form of the adapted process in Proposition 2. Recall, the equity value under the equilibrium contract $\mathbb{D}$ is:

$$\begin{aligned} E^O(x_t, M_t^O) = \max_b \quad & \mathbb{E}_t\left[\int_t^{+\infty} e^{r(s-t)}\left(x_s - q_s\right) ds\right] \\ \text{s.t.} \quad & dx_t = \delta(x_t)\, dt + \beta(b_t, x_t)\, dt + \sigma(x_t)\, dw_t \\ & dM_t^O = -r\varphi\Phi_t\sigma x_t M_t^O dw_t \end{aligned} \tag{5.50}$$

I write temporarily $E_t^O \equiv E^O(x_t, M_t^O)$ and construct the Hamilton-Jacobi-Bellman partial differential equation (HJB-PDE):

$$\begin{aligned} rE_t^O = \max_{b_t} \quad & x_t - q_t + \frac{\partial E_t^O}{\partial x_t}(\Delta - \kappa x_t + \overline{\beta} + b_t x_t) + \frac{1}{2}\frac{\partial^2 E_t^O}{\partial x_t^2}\sigma^2 x_t \\ & + \frac{1}{2}\frac{\partial^2 E_t^O}{\partial M_t^{O2}}\left(r\varphi\xi b_t \sigma\sqrt{x_t} M_t^O\right)^2 - \frac{\partial^2 E_t^O}{\partial x_t \partial M_t^O} r\varphi\xi b_t (\sigma\sqrt{x_t})^2 M_t^O \end{aligned} \tag{5.51}$$

I simplify the HJB-PDE with the following elements: first, I note that, given the functional form assumed in equation (3.31), the crossed derivative $\partial^2 E_t^O / \partial x_t \partial M_t^O = 0$, the derivative $\partial E_t^O / \partial x_t = \partial A_t^O / \partial x_t$ and $\partial^2 E_t^O / \partial x_t^2 = \partial^2 A_t^O / \partial x_t^2$; second, I replace the diffusion loading $\Phi_t = \xi b_t$ and the optimal compensation $q_t = x_t \xi b_t^2 / 2 - \varphi^{-1} \ln(-r\varphi M_t^O)$. The HJB becomes:

$$rA_t^O = \max_{b_t} \left\{ \frac{\partial A_t^O}{\partial x_t}(\Delta - \kappa x_t + \overline{\beta} + b_t x_t) - \frac{\xi b_t^2}{2} x_t - \frac{r\varphi\xi^2 b_t^2 \sigma^2}{2} x_t \right\} + \frac{1}{2}\frac{\partial^2 A_t^O}{\partial x_t^2}\sigma^2 x_t + x_t \tag{5.52}$$

The maximisation problem equation (5.52) yield following condition for optimal required effort b_t^*:

$$b_t^* = \frac{\partial A_t^O}{\partial x_t}\frac{1}{\xi + r\varphi\xi^2\sigma^2} \tag{5.53}$$

Replacing with the optimal required effort b_t^* back in equation (5.52), we obtain:

$$rA_t^O = \frac{\partial A_t^O}{\partial x_t}(\Delta - \kappa x_t + \overline{\beta}) + \left(\frac{\partial A_t^O}{\partial x_t}\right)^2 \frac{1}{2}\frac{x_t}{\xi + r\varphi\xi^2\sigma^2} + \frac{\partial^2 A_t^O}{\partial x_t^2}\frac{1}{2}\sigma^2 x_t + x_t \tag{5.54}$$

Inspecting equation (5.54), I propose the following solution to the ODE:

$$A_t^O = C_3 + C_4 x_t \tag{5.55}$$

where C_3 and C_4 are constants to be determined. Replacing the proposed solution (5.55) in equation (5.54), I obtain:

$$A^O(x_t) = \left(\xi + \xi^2 r\sigma^2\varphi\right) b_t^* \left(\frac{\overline{\beta} + \Delta}{r} + x_t\right) \tag{5.56}$$

where the optimal effort required to the outsider CEO is:

$$b_t^* = \kappa + r - \sqrt{(\kappa + r)^2 - \frac{2}{\xi + \xi^2 r\sigma^2\varphi}} \tag{5.57}$$

The firm value function $A^O(x_t)$ under the outsider's management is an affine function of the instantaneous cash-flow x_t. The firm's value varies positively with the fixed cash-flow growth $\overline{\beta} + \Delta$ and varies negatively with the variable cost parameter κ, the effort cost parameter ξ,

the risk-aversion parameter φ, and the discount rate r. Substituting with equation (5.56) in equation (3.31), the specific solution for $E^O(x_t, M_t^O)$ is:

$$E^O(x_t, M_t^O) = \xi\left(\xi r\sigma^2\varphi + 1\right)\left(\kappa - \sqrt{(\kappa+r)^2 - \frac{2}{\xi + \xi^2 r\sigma^2\varphi}} + r\right)\left(\frac{\overline{\beta}+\Delta}{r} + x_t\right) - N_t^O \tag{5.58}$$

□

Proof of Lemma 5. Replacing the optimal required effort a_t^* in the cash-flow process, we get:

$$dx_t = (\Delta - \kappa x_t)dt + \left(\kappa + r - \sqrt{(\kappa+r)^2 - \frac{2}{\theta + \theta^2 r\sigma^2\gamma}}\right)x_t dt + \overline{\alpha}dt + \sigma\sqrt{x_t}dw_t \tag{5.59}$$

Using this process, I define $P_d^I \equiv P_d^I(x_t|x_d)$. I follow Dixit and Pindyck (1994) and write the Bellman equation of the dismissal option as:

$$rP_d^I dt = \mathbb{E}_t[dP_d^I]$$

Expanding the RHS using Ito's Lemma and rearranging, I obtain the following differential equation that must be satisfied by $P_d^I(x_t)$:

$$\frac{1}{2}\sigma^2 x\frac{\partial P_d^I}{\partial x_t} + \left(\Delta + \overline{\alpha} + rx_t - \sqrt{(\kappa+r)^2 - \frac{2}{\theta + \theta^2 r\sigma^2\gamma}}x_t +\right)\frac{\partial^2 P_d^I}{\partial x_t^2} - rP_d^I = 0 \tag{5.60}$$

In addition $P_d^I(x_t|x_d)$ must satisfy the following conditions:

$$\lim_{x_t \to -\infty} P_d^I(x_t|x_d) = 0 \tag{5.61}$$

$$A^I(x_d) + P_d^I(x_d|x_d) = A^O(x_d) - \overline{K} \tag{5.62}$$

Condition (5.61) states that the option to dismiss the manager approximates zero if cash-flows are very low. This is restriction I impose to limit the cases where the incumbent manager faces a Leicester-Pearson phenomenon. Therefore, if cash-flows are very low, the shareholder is not interested in replacing the incumbent. Condition (5.62) is the value matching condition presented in equation (3.22) after replacing with equations (3.19),(3.20) and (3.23).

I propose the following solution to the ODE represented in equation (5.60):

$$P_d^I(x_t|x_d) = C_5\,{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2}x_t\right) + C_6\mathbf{L}\left(-\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2} - 1; \frac{2(\kappa - a_t^*)}{\sigma^2}x_t\right) \tag{5.63}$$

where C_5 and C_6 are constants to be determined, ${}_1\mathbf{F}_1(.)$ is the Kummer confluent hypergeometric function where $\partial_1\mathbf{F}_1(.)/\partial x_t > 0$. and $\mathbf{L}(.)$ is the generalised Laguerre polynomial

where $\partial \mathbf{L}(.)/\partial x_t > 0$ The term $C_3\, {}_1\mathbf{F}_1(.)$ corresponds to a real option that increases in value with x_t and $C_4\, \mathbf{L}(.)$ corresponds to a real option that decreases in value with x_t.

Using condition (5.62) and since $\partial_1 \mathbf{F}_1(.)/\partial x_t > 0$, I define $C_6 = 0$. From the value matching condition (5.61), I find the solution for C_5. Therefore, I write:

$$P_d^I(x_t|x_d) = \left(A^O(x_d) - A^I(x_d) - \overline{K}\right) \frac{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_t\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d\right)} \tag{5.64}$$

I am interested in obtaining $P(x_t) \equiv P_d^I(x_t|x_d^*)$, the dismissal option value that is exercised when x_t crosses the optimal dismissal trigger x_d^* at the optimal dismissal time T_d^*. To do so, I maximise the dismissal option $P_d^I(x_t|x_d)$ in equation (5.64) for x_d:

$$x_d^* \in \arg\max_{x_d} P_d^I(x_t|x_d)$$

Therefore, the maximum value dismissal option is $P(x_t)$:

$$P_d^I(x_t) = C_d^I(x_d^*) \frac{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_t\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d^*\right)} \tag{5.65}$$

with constant $C_d^I(x_d^*)$ is defined as:

$$C_d^I(x_d^*) = b_t^* \xi \left(\xi r \sigma^2 \varphi + 1\right) \left(\frac{\overline{\beta} + \Delta}{r} + x_d^*\right) - a_t^* \theta \left(\gamma \theta r \sigma^2 + 1\right) \left(\frac{\overline{\alpha} + \Delta}{r} + x_d^*\right) - \overline{K}$$

and the optimal dismissal trigger respects the following identity:

$$b_t^* \xi \left(\xi r \sigma^2 \varphi + 1\right) - a_t^* \theta \left(\gamma \theta r \sigma^2 + 1\right) - \frac{r C_d^I(x_d^*)}{\Delta + \overline{\alpha}} \frac{{}_1\mathbf{F}_1\left(1 + \frac{r}{\kappa - a_t^*}; 1 + \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d^*\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d^*\right)} = 0 \tag{5.66}$$

□

Proof of Proposition 3. Before proving the result, let me specify two elements. First, for easiness of exposure, I define function $L(x_d)$ as:

$$L(x_d) := b_t^* \xi \left(\xi r \sigma^2 \varphi + 1\right) - a_t^* \theta \left(\gamma \theta r \sigma^2 + 1\right) - \frac{r C_d^I(x_d)}{\Delta + \overline{\alpha}} \Lambda(x_d) \tag{5.67}$$

where

$$\Lambda(x_d) := \frac{{}_1\mathbf{F}_1\left(1 + \frac{r}{\kappa - a_t^*}; 1 + \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa - a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa - a_t^*)}{\sigma^2} x_d\right)}$$

This function is the left-hand side of the First Order Condition represented in equation (3.38), that is, if x_d maximises $P_d^I(x_t|x_d)$ then $L(x_d) = 0$. I write the following properties of $\Lambda(x_d)$:

$$\begin{aligned}
&\lim_{x_d \to +\infty} \Lambda(x_d) = \lim_{x_d \to +\infty} \frac{\sum_{n=0}^{+\infty} \frac{\left(1+\frac{r}{\kappa-a_t^*}\right)_n \left(\frac{2(\kappa-a_t^*)}{\sigma^2} x_d\right)^n}{\left(1+\frac{2(\overline{\alpha}+\Delta)}{\sigma^2}\right)_n n!}}{\sum_{n=0}^{+\infty} \frac{\left(\frac{r}{\kappa-a_t^*}\right)_n \left(\frac{2(\kappa-a_t^*)}{\sigma^2} x_d\right)^n}{\left(\frac{2(\overline{\alpha}+\Delta)}{\sigma^2}\right)_n n!}} = \overline{C}, \qquad \overline{C} \in \mathbb{R}^+ \\
&\lim_{x_d \to -\infty} \Lambda(x_d) = 0 \\
&\frac{\partial \Lambda(x_d)}{\partial x_d} > 0
\end{aligned} \tag{5.68}$$

where the operator $(z)_n$ is the Pochhammer's symbol for the rising factorial. It decomposes as $(z)_n = z(z+1)...(z+n-1)$.

Second, in order to discuss the x_d optimality, I take the second order derivative over $P_d^I(x_t|x_d)$ with respect to x_d and decompose it into the product of a strictly positive function and a non-strictly positive function:

$$\frac{\partial^2 P_d^I(x_t|x_d)}{\partial x_d^2} = \hat{H}(x_d) H(x_d)$$

Here, $\hat{H}(x_d) \in \mathbb{R}^+$ and, hence, does not affect the sign of the second order condition (SOC). For the remainder of the proof I restrict to studying the sign of $H(x_d) \in \mathbb{R}$, which has the following form:

$$H(x_d) = -\Lambda(x_d)\left(b\xi\left(\xi r\sigma^2\varphi + 1\right) - a\theta\left(\gamma\theta r\sigma^2 + 1\right)\right) + \frac{rC_d^I(x_d)\,\Lambda(x_d)^2}{\bar{\alpha}+\Delta} - \frac{(\kappa + r - a_t^*)C_d^I(x_d)}{2(\bar{\alpha}+\Delta)+\sigma^2}\Xi(x_d) \tag{5.69}$$

where

$$\Xi(x_d) := \frac{{}_1\mathbf{F}_1\left(2+\frac{r}{\kappa-a_t^*}; 2+\frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa-a_t^*)}{\sigma^2} x_d\right)}{{}_1\mathbf{F}_1\left(\frac{r}{\kappa-a_t^*}; \frac{2(\overline{\alpha}+\Delta)}{\sigma^2}; \frac{2(\kappa-a_t^*)}{\sigma^2} x_d\right)}$$

I write the following properties of $\Xi(x_d)$:

$$\begin{aligned}
&\lim_{x_d \to +\infty} \Xi(x_d) = \hat{C}, \qquad \hat{C} \in \mathbb{R}^+ \\
&\lim_{x_d \to -\infty} \Xi(x_d) = 0 \\
&\frac{\partial \Lambda(x_d)}{\partial x_d} > 0
\end{aligned} \tag{5.70}$$

I prove the result by contradiction. Assume the existence of a trigger $\hat{x}_d^*$ that respects the FOC ($L(\hat{x}_d) = 0$), and that fulfils the following equality:

$$b_t^* \xi\left(\xi r\sigma^2\varphi + 1\right) = a_t^* \theta\left(\gamma\theta r\sigma^2 + 1\right)$$

Equation (5.67) becomes:

$$L(\hat{x}_d^*) = -\frac{r\left(b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)\frac{\overline{\beta}-\overline{\alpha}}{r}-\overline{K}\right)}{\Delta+\overline{\alpha}}\Lambda(\hat{x}_d)$$

Using properties (5.68), this expression is always positive when $b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)(\overline{\beta}-\overline{\alpha})/r < \overline{K}$, it is always negative when $b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)(\overline{\beta}-\overline{\alpha})/r > \overline{K}$, and it is always null when $b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)(\overline{\beta}-\overline{\alpha})/r = \overline{K}$. Hence, it is the case that function $P(x_t) = 0$ without inflection points for all $\hat{x}$. Therefore, the FOC $L(\hat{x}_d) = 0$ is violated and $\hat{x}_d^*$ cannot be optimal.

Assume now the existence of a trigger $\hat{\hat{x}}_d^*$ that respects the FOC ($L(\hat{\hat{x}}_d) = 0$), and that fulfils the following condition:

$$b_t^*\xi\left(\xi r\sigma^2\varphi+1\right) < a_t^*\theta\left(\gamma\theta r\sigma^2+1\right)$$

Moreover, let me write $a_t^*\theta\left(\gamma\theta r\sigma^2+1\right) = b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)+\epsilon$ where constant $\epsilon \in \mathbb{R}^+$. Equation (5.67) becomes:

$$L(\hat{\hat{x}}_d) = -\epsilon - \frac{r}{\Delta+\overline{\alpha}}\left(b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)\left(\frac{\overline{\beta}-\overline{\alpha}}{r}\right)-\epsilon x_d-\overline{K}\right)\Lambda(\hat{\hat{x}}_d) \tag{5.71}$$

If $\hat{\hat{x}}_d$ is optimal then the FOC is:

$$\left(b_t^*\xi\left(\xi r\sigma^2\varphi+1\right)\left(\frac{\overline{\beta}-\overline{\alpha}}{r}\right)-\epsilon x_d-\overline{K}\right)\Lambda(\hat{\hat{x}}_d) = -\frac{\Delta+\overline{\alpha}}{r}\epsilon \tag{5.72}$$

Replacing this expression in equation (5.69) yields $H(\hat{\hat{x}}_d) > 0$. This violates the maximality of $\hat{\hat{x}}_d$ in the second order condition and, hence, $\hat{\hat{x}}_d$ cannot be optimal.

A trigger x_d^* exists and is optimal only if $b_t^*\xi\left(\xi r\sigma^2\varphi+1\right) > a_t^*\theta\left(\gamma\theta r\sigma^2+1\right)$, which can be shown in a similar fashion by admitting an $\epsilon \in \mathbb{R}^-$. □

Proof of Proposition 4. It suffices to show that:

$$\left.\frac{\partial P(x_t|x_d)}{\delta x_d}\right|_{x_d=\overline{x}} > 0$$

Taking first order derivative of $P(x_t|x_d)$ with respect to x_d and replacing $x_d = \overline{x}$ yields

$$b_t^*\xi\left(\xi r\sigma^2\varphi+1\right) - a_t^*\theta\left(\gamma\theta r\sigma^2+1\right)$$

which is always positive by Proposition 3. □

5.3 Appendix of Chapter 4

5.3.1 Overview of the Regulatory Changes

In this chapter I consider three major regulatory changes: Bail-in tool, Liquidation tool and the increased limit for Deposit Insurance. I now detail and characterise these new elements.

▷ **Bail-In.** Bail-ins empower the regulator (or resolution authority) with the legal right of changing the property and control rights across different (bailinable) claimants, e.g. unsecured senior liabilities, subordinated debt and equity, without the need for court authorisation (Chennells and Wingfield 2015). In practice, the authority can erase the claims of shareholders and unprotected creditors or even change the nature of these claims, e.g. debt-to-equity swap. Sometimes, bail-ins comprise as well a set of restructuring measures (e.g. change in management, or sale of parts of the business) to improve the bank's viability, which is a unique trait unavailable to contingent convertibles.

The embryonic case of a bail-in occurred in Cyprus after its sovereign crisis of 2012 where, given the levels of bank capital necessities and the public debt refinancing needs, the most plausible solution to avoid a banking collapse was a bail-in of unprotected depositors[1] (Michaelides 2014).

▷ **Private Sector Acquisition.** A Private Sector Acquisition (European Central Bank) or Private Sector Purchase (Bank of England)[2]. The resolution authority uses this tool to apply an orderly sale of the assets, using its proceedings to cover the liabilities of the bank.

This tool is distinct from the pure bankruptcy because the resolution proceedings are managed by the resolution authority and not by an assembly of creditors. In theory, this control leads to lower costs of fire sales as the authority can avoid negative externalities associated with resolutions managed by creditors, i.e. "rush to the courthouse" and "hold-out" externalities (Roubini 2004). The authority has sufficient control to search the best opportunity to sell the assets hence generating higher sales proceedings for the creditors.

[1] In detail, for a total write-down of 5.8 billion Euros, the best option involved imposing a stability levy over residents and non-residents' depositors. In spite of the existing opposition and the lack of a clearly defined legal framework within the EU, this bail-in to banks was carried out for the first time for the first time in Eurozone. A levy was determined at 6.75% for deposits up to 100 thousand euros, and 9.9% above that level (Zenios 2015). For a critical discussion of bail-ins as a resolution mechanism, check Avgouleas and Goodhart (2014).

[2] In detail, the BoE defines a Private Sector Purchase as a partial or total alienation of the assets of a financial institution to a private sector buyer, whereas the ECB defines Private Sector Acquisition as the transfer of parts of the business (assets, shares, liabilities) to one or more purchasers, regardless of shareholders? approval (Art. 38 and 39 of BRRD and Art. 24 of SRM). Note that the ECB considers the possibility of transferring the liabilities to the private acquirer. I choose to ignore this later feature in my model and restrict to the asset sale.

▷ **Increase in the limit for Deposit Insurance.** Deposit insurance is a mechanism by which a guarantor (usually an institution funded by the government or by its associated banks) can cover the losses of depositors in the event of a bankruptcy of some financial institution. If the insured limit is sufficiently high, such that most depositors are covered, this mechanism can avert bank-runs given that depositors are not at risk of supporting bankruptcy costs in their role of creditors (Iyer and Puri 2012). Deposit Insurance Schemes in developed economies are not particularly new. However, they were subject to significant increases in the deposit insurance limit during and after the financial crisis. For some European countries, the regulatory change[3] Because these increments in the limit for deposit insurance were in most countries non-trivial, I treat this aspect in my model in an extreme version, i.e. the bank under study goes from no deposit insurance to full deposit insurance.

[3]EU member states were obliged to increase their deposit insurance level to a minimum of EUR50,000 in 2009, and then to an homogenised level of EUR100,000 in 2011.

Table 1: Summary of Resolution Frameworks

Area[a]	USA	United Kindgom	European Union
Interveners	(1) The chartering authority of a financial institution, i.e. a state banking department or the US Office of the Comptroller of the Currency, decides when to put a firm under resolution. (2) The Federal Deposits Insurance Corporation (FDIC) receives the firm under legally independent functioning from its deposit insurance role.	After consultation of the HM Treasury, the supervisor (Prudential Regulation Authority) and the resolution authority (Bank of England) decide on the resolution. For investment firms the prudential supervisor is usually the Financial Conduct Authority.	Each member state has an administrative organism holding full resolution powers, e.g., Finanzmarktaufsicht in Austria or Banca d'Italia in Italy. The Single Resolution Board is an independent european authority which can enforce resolution for banks within the jurisdictions of the the Single Resolution Mechanism.
Resolution Trigger	Banks are required to be adequately capitalized with risk-based capital ratios of 8% or higher[b] which triggers a warning it is is breached. If the ratio lowers to 6%, the regulator steps in and impose corrective actions. The bank is considered to be critically undercapitalized at 2%, for which the regulator will close it and transfer the assets to the FDIC.	Resolution can occur before a firm is balance sheet insolvent. It occurs when the financial institution: (1) is failing or on the verge of failing, for instance, when it cannot meet the minimum requirements in terms of quality capital and liquidity levels; (2) is reasonably unlikely to recover with actions taken outside the resolution regime.	Occur when: (1) a bank is failing; and/or (2) no other measure taken by the stakeholders (e.g. voluntary debt-to-equity swap) is likely to ensure the institution's viability in a timely matter or without causing instability in the financial system[c]. Some events can trigger early resolution[d], namely, operational risk events (e.g. rogue trading), deterioration in the level of eligible liabilities, strong outflow of funds (e.g. deposits) due to reputation damage, rating downgrades.
Goals of Resolution[e]	The decision of the receiver should aim at maximizing the return on the assets of the bank under resolution, and to minimize the costs of the insurance fund that may arise in the case of a liquidation, while preserving the interests of protected creditors.	To safeguard banking services while maintaining financial stability, public confidence, and avert using public funding. If an orderly resolution cannot exist, the BoE aims to mitigate the effects of failure, e.g. fire-sales costs, preserve credit operations and securing access to deposits.	Respond in an efficient manner to a bank under financial distress, ensure financial stability and mitigate the costs for society, i.e. public resources supported by taxpayers.
Available Tools	**(1) Purchase and Assumption (P&A):** FDIC sells assets to an Assuming Institution, e.g. a healthy bank. **(a) Whole Bank P&As:** The FDIC asks for bids on all assets of the failing bank on a "take is as it is" basis, without guarantees. **(b) P&As with Optional Shared Loss:** The FDIC, as a receiver, shares partial assets' losses with the AI. **(c) Bridge Bank P&As:** The failed institution is transferred to a temporary structure controled by the FDIC and chartered by the OCC. **(2) Deposit Payoff (DP)** occurs when the FDIC acts as a deposit insurer, and thus pays these claimants up to the insured maximum amount (see next table). **(a) Straight DP:** FDIC determines the amount of insured deposits and pays it directly. Used when liquidation is the least costly resolution. **(b) Insured Dep. Transfer:** FDIC transfers secured depositors to a local bank which directly pays the claims. **(c) Deposit Insurance National Bank:** It is a temporary bank created to ensure that depositors can access and manage their funds.	**(1) Stabilization Tools** to promote financial stability and the continuation of critical business functions: **(a) Private Sector Purchase** consists on a partial or total alienation of the assets of a financial institution to a private sector buyer; **(b) Bridge Bank** is used to transfer part/total of the business to a subsidiary of the BoE. This is an intermediary step with the goal of future sale. **(c) Bail-In** use the claims of the failing institution as a resource to recapitalize and absorb its losses. **(2) Complementary Tools** can only be used in conjunction with the previous three stabilization tools: **(a) Asset Separation** is used to transfer assets and credits to a separated asset management vehicle. **(b) Bank Administration Procedure** When part of the business is not immediately transferred to a private buyer or into a bridge bank, the "residual bank" is put into administration, so that basic services are continued. **(3) Insolvency** If the public interest is not met, the an insolvency expert is appointed to manage the wind-down of the assets of the bank.	**(1) Private Sector Acquisition** (Art. 38 and 39 of BRRD and Art. 24 of SRM), consisting on the transfer of parts of the business (assets, shares, liabilities) to one or more purchasers, regardless of shareholders' approval; **(2) Bridge Bank** (Art. 40 and 41 of BRRD, and Art. 25 of SRM) is a controlled temporary structure, used by the resolution entity to encompass parts of the business; **(3) Asset Separation** (Art. 42 of BRRD and Art 26 of SRM) of assets that can affect market stability to an asset management vehicle[f] controlled by the resolution authority.; **(4) Bail-In** (Art. 43 to 55 of BRRD) Recapitalization of the financial institution through mandatory debt-to-equity swap; **(4) Government Stabilization** (Art 56 to 58 of BRRD) Temporary (and last resort) public support.

[a] The resolution framework of the USA is consigned in the Dodd-Frank Act - Title II (2009), of the UK in the Banking Act 2009 (amended by the BRRD), and in the European Union by the Bank Recovery and Resolution Directive (BRRD) and the Single Resolution Mechanism Regulation (SRMR) in 2014.

[b] For additional details on risk-based capital ratios see: https://www.fdic.gov/regulations/safety/manual/section2-1.pdf

[c] For details on when a bank is considered as failing see: https://www.eba.europa.eu/documents/10180/1085517/EBA-GL-2015-07+GL+on+failing+or+likely+to+fail.pdf

[d] See https://www.eba.europa.eu/regulation-and-policy/recovery-and-resolution/guidelines-on-early-intervention-triggers

[e] The FSB provides a set of common key attributes and goals for an effective resolution of financial institutions at http://www.fsb.org/2011/11/r_111104cc/

[f] During the sovereign crisis, AMVs often implied that losses were absorbed by taxpayers via government support (**?**).

Table 2: Summary of Bail-In Tools and Depositors Protection Schemes

Area	USA	United Kindgom[a]	European Union[b]
Hierarchy of Creditors	The allocation of proceedings (or responsibilities, in the inverse order) should obey the following hierarchy: (1) Secured Depositors and Preferred Creditors; (2) Depositor Class on a pro-rata basis (this includes uninsured depositors and FDIC in the event of proceedings allocation); (3) General Creditors; (4) Subordinated Creditors; (5) Shareholders.	The insolvency hierarchy is: (1) Fixed charge holders, i.e. mortgage securities, covered bonds, collateralised positions; (2) Fees and expenses held by liquidators and by resolution authority; (3) Ordinary Preferential Creditors, i.e. protected depositors; (4) Secondary Preferential Creditors, i.e. uninsured depositors; (5) Floating charge holders that do not fit in the first category; (5) Unsecured Senior Creditors; (6) Unsecured Subordinated creditors; (7) Interest post-insolvency; (8) Shareholders.	Member states have discretionary powers to govern the ranking in insolvency hierarchy provided that, as established in Art. 108 of BRRD, the following general order is followed: (1) Covered deposits and "deposit guarantee schemes subrogating to the rights and obligations of covered depositors in insolvency"; (2) Deposits from "natural persons and MSM-sized firms" which exceeds the coverage level; (3) Ordinary unsecured, non-preferred Creditors.
Bail-In Specifics	The Dodd-Frank Act - Title II defines new approaches to banking resolution, allowing for an orderly resolution through instruments that may, in some specific cases, replicate the outcome of a bail-in. However, this legislation does not define or characterize a bail-in tool in the 'sensu stricto' created by the BRRD tool.	The bail-in tool defined in BRRD was included in the UK legislation under the Financial Services Act in 2013 (Schedule 2) and thus most of the rules are, for now, similar to the European case. **Solutions:** Open Bank and Closed Bank Resolutions, though the first is more likely to be used in the UK[c]. **Exceptions:** Certain financial arrangements may be excluded from a bail-in, e.g. collateral, or . **Requirements (MREL):** Calibration of MREL will be done on a firm-specific basis. On the first phase, financial Institutions will comply with BoE's Interim Requirements[d] by 2020, with different timings and requirements depending on their category (G-SIB, D-SIB, or others), e.g. from Jan-2019, UK G-SIBs must hold at least 16% of RWAs or 6% of leverage exposures. On the second phase, the BoE will impose tougher requirements by 2022. **Minimum Bail-In:** The write-off of liabilities myst reach a threshold of 8%.	A bail-in is applied, without needing court approval, by (1) writing-off equity and (2) writing-off or imposing a debt-to-equity swap to liabilities that are not ex-ante expressly excluded (e.g. covered deposits, employee remuneration). **Solutions:** (1) An Open Bank Resolution where a failing institution is recapitalized via the write-down of liabilities or a debt-to-equity swap ensuring its continuation; (2) A Closed Bank Resolution where toxic assets stay in the existing institution (bad bank) and a new entity (good bank) is created to encompass clean assets. Credit claims with no systemic impact can either stay in the old bank or move into the new one, being swapped to equity or written-down. **Exceptions:** Critical creditors may be excluded from a bail-in when it is impossible to consider them without creating market instability or affecting the continuity of core operations. **Requirements (MREL):** Banks should, at all times, hold a minimum level of own funds and bail-in eligible liabilities. Such level is determined by each member-state resolution authority[e]. **Minimum Bail-In:** To avoid instability, the BRRD permits ending the burden upon reaching a threshold of 8% of total liabilities (plus capital). After securing the minimum bail-in, resolution funds may assume 5% of the losses.
Depositors Protection	The FDIC acts as an insurance on amounts up to 250.000 USD per depositor on each ownership category, e.g., Retirement accounts, Employee Benefit Plan accounts, per FDIC-secured financial institution.	The Financial Services Compensation Scheme (FSCS) provides insurance coverage up to 85.000 GBP per deposit claims (personal or firm) against financial institutions under resolution from 30th of January of 2017[f].	The Directive 2014/49/EU[g] legislates that each member state is responsible for ensuring, within its territory, the existence of at least one Deposit Guarantee Scheme, with a coverage amount of 100.000 Euros per depositor.

[a] See http://www.prarulebook.co.uk/ and the BoE's guide on resolution at www.bankofengland.co.uk/financialstability/Documents/resolution/apr231014.pdf
[b] This table is based on the Directive 2014/59/EU available at http://data.europa.eu/eli/dir/2014/59/oj
[c] As conjectured by (**?**)
[d] As of November 2016. For full details go to http://www.bankofengland.co.uk/publications/Pages/news/2016/082.aspx
[e] The general criteria used to determine the minimum level of bail-in eligible securities is defined at the Art.45 (6) of BRRD.
[f] Recently updated at https://www.fscs.org.uk/what-we-cover/compensation-limits/
[g] See http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:32014L0049

Table 3: Examples of Bail-Ins (and other resolution tools)

Example 1:	**The Cypriot Depositors Bail-in (2012 onwards)**
Institutions	Bank of Cyprus (BoC) and Cyprus Popular Bank (Laiki).
Authorities	The Central Bank of Cyprus (CBC) acted initially as the supervisor and later on as the resolution authority, after the Cyprus Resolution Law. The Troika (ECB, IMF, and EC) also intervened in the process.
Tools	Sale of Business from Laiki to BoC and Bail-In of BoC, complemented with a Bailout from Troika.
Concerns	How to implement unavoidable restructuring measures using untested tools (bail-in) without a carefully carved legal ground to support them. This was "patient zero" for bail-ins of financial institutions.
Summary	By the beginning of the crisis, BoC and Laiki's were highly exposed to greek sovereign bonds. In 2011, the Greek Private Sector Involvement originated a write-off of around 4 Billion Euros on their balance sheets. Cyprus delayed the financial assistance request up until June of 2012, after increasing economic imbalances, and after almost 1 year of failure to access capital markets. The agreement with Troika, conditional on stress tests to banks, was only accomplished by March 2013 and involved a bailout of EUR 10 Billion without support to BoC and Laiki (which needed approximately EUR 7 Billion). This delay in negotiations allowed capital (deposits) outflows of around 10 to 17 Billion Euros. Subsequently, and after failing to legally approve a tax of 9.9% for uninsured depositors and of 6.75% for insured depositors, the parliament passed the Cyprus Resolution Law on March 22, endowing the CBC with resolution powers. Laiki was hastily resolved with write-offs of shareholders, bondholders and uninsured depositors. Critical assets, insured depositors and 9 Billion Euros from the Emergency Liquidity Assistance were transferred to BoC. All greek related assets and liabilities, e.g. loans and deposits, were sold at discount to Piraeus Bank at around 3.2 Billion Euros. Approximately 37.5% of BoC's uninsured deposits were then converted to equity. Temporary capital controls were further imposed to avoid additional deposits outflows.
Example 2:	**Spanish Cajas Bail-In (2009 onwards)**
Institutions	Ten Savings banks: Caixa Bank, BBVA, Ibercaja, Banco Sabadell, Unicaja, Mare Nostrum, BFA, NCG Banco, Novacaixa Galicia, Liberbank, and BFA Bankia.
Authorities	The Fund for Orderly Bank Restructuring (FROB) managed the restructuring processes, and the Deposit Guarantee Fund (DGF) acts as a deposit insurer, though it had a further role in this process.
Tools	Public Funding, Bail-Ins and Asset Separation via an asset management company (SAREB).
Concerns	Include the BRRD in legislation and carefully implement the resolution. Minimize the public financial burden.
Summary	Cajas are savings banks where the figure of shareholder is replaced by the "mutualist" associate, without dividends, and governed by private and public stakeholders. Cajas needed public support and thus, from 2009 to 2012, the FROB provided financial aid of approximately EUR 15 Billion to eight of the listed banks, combined with EUR 7.9 Billion from the DGF. In mid 2012, the EU facilitated a total of EUR 41.3 Billion to recapitalize the 10 banks through the FROB and to inject capital in SAREB. By February 2013, approximately 94% of that amount had been used. All banks were obliged to transfer critical assets, at discount, to SAREB, e.g. illiquid or non-performing real state loans. The operation totaled EUR 50.8 Billion exchanged against sovereign guaranteed SAREB bonds. Shareholders were wiped out, preference shares and perpetual subordinated creditors were converted to common equity. Other subordinated creditors could choose between equity or senior debt. All subordinated claimants received an haircut. The bail-in totaled EUR 13.6 Billion. To reduce litigation with private claimants, the DGF set up a liquidation mechanism, offering to buy at discount the equity instruments from converted claimants.
Example 3:	**Danish Bail-In (2015 onwards)**
Institutions	Andelskassen Bank.
Authorities	The Danish Financial Supervisory Authority (FSA) acts as regulator, and the Finansiel Stabilitet Company (FSC) holds the functions of receiver and resolution managing entity.
Tools	Open Bank Resolution combining a Bail-In with Bridge Bank.
Concerns	Secure an orderly and smooth resolution by granting uninterrupted critical services and full access to deposits. Minimize the litigation costs of the uninsured claims affected by the bail-in.
Summary	From 2008 to 2012, Denmark introduced 5 bank packages focused on securing financial stability through structured resolution framework for these institutions. In 2015, the BRRD was legally embodied. FSA asked Andelskassen to submit a recovery plan by March 2015. In October 5, 2015 the FSA concluded that a private restructuring was no longer feasible. To prevent failure and given that resolution was in the public interest, the regulator intervened immediately. In that same day, FSC took over, changed the board and the management team with immediate effect, and created a bridge bank, called Broinstitut I A/S, fully detained by the resolution authority. Resolution was initiated on the afternoon of that day and thus critical banking services stayed uninterrupted. The FSA proceeded with a write-off of all equity, all subordinated credit claims and part of the senior credit claims. While some uninsured depositors had to support a write-off, the Danish Deposit Guarantee Scheme protected insured depositors up to 100.000 Euros. In succession, the FSA used the Resolution Fund to inject DKK 37.5 million of new capital and ensure solvency. No claimant affected by the write-off proceeded with litigation against the resolution procedure.

5.3.2 Regulator Option Value

I derive the regulator's closure option $O(x_t)$ presented in section 4.3.2. Recall the regulator's utility function defined in (4.5):

$$R(x_t) = (1-\xi)V(x_t) - L \tag{5.73}$$

Following Dixit and Pindyck (1994), I write the Bellman equation of the closure option as:

$$\rho O(x_t)dt = \mathbb{E}_t[dO]$$

Expanding the RHS using Ito's Lemma and rearranging, I obtain the following differential equation that must be satisfied by $O(x_t)$:

$$\frac{1}{2}\sigma^2 x^2 O''(x_t) + \mu x O'(x_t) - \rho O(x_t) = 0 \tag{5.74}$$

In addition $O(x_t)$ must satisfy the following conditions:

$$\lim_{x_t \to +\infty} O(x_t) = 0 \tag{5.75}$$

$$O(x_b) = -R(x_b) \tag{5.76}$$

Condition (5.75) comes from the observation that the regulator would never close an over-performing bank. The closure option becomes worthless if the bank's cash-flows are exceedingly high. Condition (5.76) is the value matching condition which states that, at x_b, the regulator exercises the closure option and stops facing (dis)utility $R(x_b)$.

I propose the following solution for this ODE:

$$O(x) = Ax^{\beta} + Bx^{\beta_+} \tag{5.77}$$

where A, B, $\beta < 0$ and $\beta_+ > 1$ are constants. I replace this solution proposal into ODE (5.74). Using conditions (5.75) and (5.76), I arrive to the following results:

$$A = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{1}{x_b}\right)^{\beta} \tag{5.78}$$

$$B = 0 \tag{5.79}$$

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \tag{5.80}$$

In detail, $B = 0$ is imposed since condition (5.75) requires the closure option to converge to zero when cash-flows go to infinity. I combine this with the value matching condition (5.76) to obtain the solution for constant A. Finally, β is the negative root of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-\rho=0$ that arises when I replace the proposed solution $O(x)$ in the ODE. β_+ is the positive root which is inconsequential since $B = 0$. Replacing these solutions in (5.77), I get the closure option's value:

$$O(x_t) = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{x_t}{x_b}\right)^{\beta}$$

5.3.3 Base Case

Proof of Proposition 5. Recall from function (4.7), the value of the closure option:

$$O(x_t) = \left(\frac{c+d}{\rho} - \frac{(1-\xi)x_b}{\rho-\mu}\right)\left(\frac{x_t}{x_b}\right)^{\beta}$$

My problem is:

$$x_b \in \arg\max_{x_b} O(x_t)$$

Obtaining the FOC over $O(x_t)$ and rearranging, I obtain:

$$x_b = \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{c+d}{\rho}$$

□

Proof of Lemma 6. Consider the present value of all future payments to all creditors (non-deposit creditors and depositors):

$$\mathbb{E}_{T_b}\left[\int_{T_b}^{+\infty} e^{-\rho(s-T_b)}\,(c+d)\,dt\right] = \frac{c+d}{\rho}$$

From Proposition (5), the optimal intervention trigger selected by the Regulator is:

$$x_b(\overline{\xi}_1) := \frac{\beta(c+d)(\rho-\mu)}{(\beta-1)(1-\overline{\xi})\rho}$$

Set the value of the assets' proceedings under bankruptcy at the trigger x_b equal to the total creditors' present value of payments, i.e. solve $(1-\epsilon)\frac{x_b(\overline{\xi})}{\rho-\mu} = \frac{c+d}{\rho}$ for $\overline{\xi}$. □

Proof of Lemma 7. Consider the present value of all future payments to depositors:

$$\mathbb{E}_t\left[\int_t^{+\infty} e^{-\rho(s-t)}d\;dt\right] = \frac{d}{\rho}$$

and the Regulator's optimal intervention trigger:

$$x_b := \frac{\beta(c+d)(\rho-\mu)}{(\beta-1)(1-\xi)\rho}$$

Because of assumption (10), depositors know if the value of the assets' proceedings under bankruptcy are sufficient to cover their claims. Therefore, for the bank, it suffices to set up a level of non-deposit claims that, at the trigger x_b (selected by the regulator), secures that the present value of depository payments are fully covered. Specifically, I set:

$$(1-\epsilon)\frac{x_b}{\rho-\mu} \geq \frac{d}{\rho}$$

which, after rearranging, yields the no bank-run condition in equation (4.17).

Consider the parameters that define the financial institution: (i) the growth rate of cash-flows (μ), (ii) the uncertainty of cash-flows (σ), and (iii) the bankruptcy costs (ϵ). Taking first order derivatives on γ:

$$\begin{aligned}
\frac{\mathrm{d}\gamma}{\mathrm{d}\mu} &= \tfrac{1-\xi}{1-\epsilon}\tfrac{\mathrm{d}}{\mathrm{d}\mu}\left(\tfrac{\beta(\mu)-1}{\beta(\mu)}\right) < 0, \quad \forall\mu \in]-\infty, \rho[\\
\frac{\mathrm{d}\gamma}{\mathrm{d}\sigma} &= \tfrac{1-\xi}{1-\epsilon}\tfrac{\mathrm{d}}{\mathrm{d}\mu}\left(\tfrac{\beta(\sigma)-1}{\beta(\sigma)}\right) > 0, \quad \forall\sigma \in]0, +\infty[\\
\frac{\mathrm{d}\gamma}{\mathrm{d}\epsilon} &= \tfrac{(\beta-1)(1-\xi)}{\beta}\tfrac{\mathrm{d}}{\mathrm{d}\epsilon}\left(\tfrac{1}{1-\epsilon}\right) > 0, \quad \forall\epsilon \in]0, 1[
\end{aligned}$$

I also take the first derivative of the multiplier of non-deposit liabilities over deposits to the capital requirement buffer ratio ξ, I have:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\xi} = \frac{(\beta-1)}{\beta(1-\epsilon)}\frac{\mathrm{d}}{\mathrm{d}\xi}(1-\xi) < 0, \quad \forall\xi \in]0, \overline{\xi}_1[$$

Therefore, banks with simultaneous low or negative growth, high volatility, and high bankruptcy costs will have to hold a higher ratio of non-deposit coupons over deposit coupons than banks with high growth, low volatility, and low bankruptcy costs. Moreover, higher □

Proof of Proposition (6). The initial optimisation problem is

$$\begin{aligned}
(c^*, d^*, x_b^*) \quad &\in \arg\max_{\{c,d\}} B(x_0) \\
\text{s.t.} \qquad & x_b := \frac{\beta}{\beta-1}\frac{\rho-\mu}{1-\xi}\frac{d+c}{\rho} && \text{[Closure Trigger]} \\
& c \geq d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right) && \text{[NBR Condition]} \\
& x_0 \geq \frac{(\rho-\mu)(c+d)}{\rho(1-\xi)} && \text{[Rationality Constraint]} \\
& c \geq 0 \quad , \quad d \geq 0 && \text{[Non-Negative Constraints]}
\end{aligned}$$

I must restrict the problem to the feasible cases. First notice that I have two major cases: Case A when $\xi < \overline{\xi}$ and Case B when $\xi \geq \overline{\xi}$. For Case A, the no bank-run condition (equation (4.17)) eliminates the lowest firm value outcome case in which depositors are not fully paid. Case B however is not affected by the NBR Condition because a capital requirement buffer $\xi \geq \overline{\xi}$ always ensures that all creditors, and hence depositors, are fully paid. I take the NBR constraint into the bank's value function (4.15) and rewrite it for time $t = 0$ as:

$$B(x_0) = \begin{cases}
(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_0}{x_b}\right)^{\beta}\left(\tau\left(\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right) - \epsilon\frac{x_b}{\rho-\mu}\right) & , \quad \xi < \overline{\xi} \qquad \text{[Case A]} \\
(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} - (1-\tau)\left(\frac{x_0}{x_b}\right)^{\beta}\left(\epsilon\frac{x_b}{\rho-\mu}\right) & , \quad \xi \geq \overline{\xi} \qquad \text{[Case B]}
\end{cases}$$

▷ **Feasible Set solution.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_1 = \left\{(c,d) \in \mathbb{R}^2 | g(c,d) \leq 0\right\},$$

I have two possible cases: Case A for $\xi < \overline{\xi}$ and Case B for $\xi \geq \overline{\xi}$. Depending on the case, some constraints are not relevant or redundant. In detail, I will have the following vector of constraints:

$$g^A(c,d) = \begin{pmatrix} c + d - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -d \\ -c + d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right) \end{pmatrix} \qquad g^B(c,d) = \begin{pmatrix} c + d - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -d \\ -c \end{pmatrix} \tag{5.81}$$

Regarding Case A, for a family of constants that secures $m = \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 > 0$, the NBR Constraint leads to $c > 0$ for all points except $(c,d) = (0,0)$ where the NBR constraint and the coupon's non-negative constraints are redundant. Therefore, I can eliminate $c \geq 0$ from the constraint set in Case A. Combining the NBR constraint with the Rationality constraint and $d \geq 0$, I guarantee that $\mathcal{G}_1$ is compact in this first case. In Case B, the NBR constraint is unnecessary because $\xi \geq \overline{\xi}$ always guarantees that NBR is fulfilled. Now, $c \geq 0$ is introduced to secure, jointly with the Rationality Constraint and $d \geq 0$, that $\mathcal{G}_1$ is compact for case B. Consequently, with a compact set $\mathcal{G}_1$ in both cases, the Weierstrass Theorem guarantees the existence of a maximising solution.

▷ **Constraint Qualification.** Consider the Constraint Qualification matrix:

$$CQ_A := \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 \end{pmatrix} \qquad CQ_B := \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{5.82}$$

In Case A, $\xi < \overline{\xi} \Rightarrow \frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1 > 0 \Rightarrow rank(CQ_A) = 2$. In Case B, $rank(CQ_B) \leq 2$ as the three conditions cannot bind at the same time. I have full rank for all candidates to binding constraints.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers $(\chi_1^A, \chi_2^A, \chi_3^A)$ for Case A and $(\chi_1^B, \chi_2^B, \chi_3^B)$ for Case B. For Case A, the Lagrangian is written as:

$$\begin{aligned} \mathscr{L}(c,d)|_{\xi<\overline{\xi}} =& (1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c+d}{\rho} + \left(\frac{x_0}{x_b}\right)^{\beta}\left(\tau\left(\frac{x_b}{\rho-\mu} - \frac{c+d}{\rho}\right) - \epsilon\frac{x_b}{\rho-\mu}\right) \\ &+ \chi_1^A\left(\rho(1-\xi)x_0 - (\rho-\mu)(c+d)\right) + \chi_2^A d + \chi_3^A\left(c - d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right)\right) \end{aligned} \tag{5.83}$$

And for Case B:

$$\begin{aligned}\mathscr{L}(c,d)|_{\xi\geq\bar{\xi}} =&(1-\tau)\frac{x_0}{\rho-\mu}+\tau\frac{c+d}{\rho}-(1-\tau)\left(\frac{x_0}{x_b}\right)^{\beta}\left(\epsilon\frac{x_b}{\rho-\mu}\right)\\ &+\chi_1^B\left(\rho(1-\xi)x_0-(\rho-\mu)(c+d)\right)+\chi_2^B d+\chi_3^B c\end{aligned} \tag{5.84}$$

▷ **Kuhn-Tucker Conditions.**

Case A: First Order Conditions are:

$$\frac{dB(x_0)}{dc}=0\Rightarrow \quad \tfrac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{\rho(\xi-1)}\left(\tfrac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}-\tfrac{\chi_1^A}{\rho}+\chi_3^A+\tfrac{\tau}{\rho}=0 \tag{5.85}$$

$$\frac{dB(x_0)}{dd}=0\Rightarrow \quad \tfrac{\chi_3^A(-\beta\xi+\beta\epsilon+\xi-1)}{\beta(\epsilon-1)}+\tfrac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{\rho(\xi-1)}\left(\tfrac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}-\tfrac{\chi_1^A}{\rho}+\chi_2^A+\tfrac{\tau}{\rho}=0 \tag{5.86}$$

with complementary slack conditions:

$$\chi_1^A\left(\rho(1-\xi)x_0-(\rho-\mu)(c+d)\right)=0 \tag{5.87}$$

$$\chi_2^A d=0 \tag{5.88}$$

$$\chi_3^A\left(c-d\left(\tfrac{\beta-1}{\beta}\tfrac{1-\xi}{1-\epsilon}-1\right)\right)=0 \tag{5.89}$$

And Lagrange multiplier constraints:

$$\chi_1^A\geq 0 \qquad \chi_2^A\geq 0 \qquad \chi_3^A\geq 0 \tag{5.90}$$

Only solutions satisfying (5.87)-(5.90) are admissible.

Case B: First Order Conditions are:

$$\frac{dB(x_0)}{dc}=0\Rightarrow \quad \tfrac{\beta(\tau-1)\epsilon\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}}{\rho(\xi-1)}-\tfrac{\chi_1^B}{\rho}+\chi_3^B+\tfrac{\tau}{\rho}=0 \tag{5.91}$$

$$\frac{dB(x_0)}{dd}=0\Rightarrow \quad \tfrac{\beta(\tau-1)\epsilon\left(\frac{(\beta-1)(\xi-1)\rho x_0}{\beta(c+d)(\mu-\rho)}\right)^{\beta}}{\rho(\xi-1)}-\tfrac{\chi_1^B}{\rho}+\chi_2^B+\tfrac{\tau}{\rho}=0 \tag{5.92}$$

with complementary slack conditions:

$$\chi_1^B\left(\rho(1-\xi)x_0-(\rho-\mu)(c+d)\right)=0 \tag{5.93}$$

$$\chi_2^B d=0 \tag{5.94}$$

$$\chi_3^B c=0 \tag{5.95}$$

And Kuhn-Tucker multiplier constraints:

$$\chi_1^B\geq 0 \qquad \chi_2^B\geq 0 \qquad \chi_3^B\geq 0 \tag{5.96}$$

Only solutions satisfying (5.93)-(5.96) are admissible.

▷ **Corner Solutions.** I have six possible candidates to corner solutions for each Case A and B . To distinguish them I use, just for the sake of exposure in this proof, the superscript (ij). Here, $i \in A, B$ is the Case and $j \in \{1, ..., 6\}$ is the solution candidate under study. For example, in the first solution candidate for Case A, I need to find $c_2^{(A1)}$, $d_2^{(A1)}$ and $\chi_1^{(A1)}$.

▷ **Solution Candidates.**

A1. $g_1^A(c,d)$ **is binding, i.e.** $(\chi_1^A > 0, \chi_2^A = \chi_3^A = 0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c^{(A1)}, d^{(A1)})$ and the Kuhn-Tucker multiplier $\chi_1^{(A1)}$. Recalling that $l = c + d$, I have:

$$l^{(A1)} = \frac{\rho(1-\xi)}{\rho - \mu} x_0$$

$$\chi_1^{(A1)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta} (\beta\xi\tau - \beta\epsilon - \xi\tau + \tau)}{\xi - 1} + \tau$$

This solution candidate does not allow me to define a full solution for each coupon. However, any par of coupons (c,d) is optimal if it satisfies equation (5.3.3), the inequality:

$$c > d\left(\frac{\beta-1}{\beta}\frac{1-\xi}{1-\epsilon} - 1\right)$$

To guarantee $\chi_1^{(A1)} > 0$, it suffices to have:

$$\xi > \frac{\tau + \left(\frac{\beta-1}{\beta}\right)^{\beta} (\beta\epsilon - \tau)}{\beta\tau}$$

One way to guarantee $\chi_1^{(A1)} > 0$ is to restrict my tax rate τ to

$$\tau > -\frac{\beta\epsilon \left(\frac{\beta-1}{\beta}\right)^{\beta}}{1 - \left(\frac{\beta-1}{\beta}\right)^{\beta}} > 0 \tag{5.97}$$

where $\left(\frac{\beta-1}{\beta}\right)^{\beta} > 0$. This ensures that any $\xi \in [0, \bar{\xi})$ is feasible.

A2. $g_2^A(d)$ **is binding, i.e.** $(\chi_1^A = 0, \chi_2^A > 0, \chi_3^A = 0)$ In this solution candidate, the Non-Negative Constraint for Deposits is the only constraint binding. I solve for the optimal coupons $(c^{(A2)}, d^{(A2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(A2)}$. I find however that

$$\chi_2^{(A2)} = 0$$

This implies that constraint $g_2^A(d)$ does not bind and the candidate solution is not feasible. However, for a d sufficiently small to be negligible, I have

$$c < \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta} x_0 =: c_1^{(A2)}$$

A3. $g_3^A(c,d)$ **is binding, i.e.** $(\chi_1^A = 0, \chi_2^A = 0, \chi_3^A > 0)$ I am considering that NBR Constraint binds. The optimisation problem yields an optimal multiplier solution that suggests, however, that the candidate solution never binds.

$$\chi_3^{(A3)} = 0$$

The NBR Constraint is never binding. Nonetheless, I can find the non-binding solution for the NBR Constraint. That boundary is:

$$c > \frac{\rho}{\rho-\mu}\frac{\beta(\epsilon-\xi)+\xi-1}{\beta}\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-1/\beta} x_0 =: c^{(A3)}$$

A4. $g_1^A(c,d)$ **and** $g_2^A(d)$ **bind, i.e.** $(\chi_1^A > 0, \chi_2^A > 0, \chi_3^A = 0)$ In this solution candidate, I check the case in which the Non-Negative Constraint for Deposits and the Rationality Constraint bind. This candidate solution does not however hold at the Non-Negative Constraint for Deposits:

$$\chi_2^{(A4)} = 0$$

Even though

$$\chi_1^{(A4)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}(\beta\xi\tau-\beta\epsilon-\xi\tau+\tau)}{\xi-1}+\tau$$

Therefore, I must always fulfil the condition:

$$c < \frac{\rho(1-\xi)}{\rho-\mu}x_0 := c^{(A4)}$$

A5. $g_1^A(c,d)$ **and** $g_3^A(c,d)$ **bind, i.e.** $(\chi_1^A > 0, \chi_2^A = 0, \chi_3^A > 0)$ I check if binding the Rationality Constraint and the NBR Constraint leads to a feasible candidate solution. However, I find

$$\chi_3^{(A5)} = 0$$

$$\chi_1^{(A5)} = \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}{\xi - 1} + \tau$$

The null multiplier tells me, as expected, that the NBR condition is not binding. In this case, I have for the total debt coupon:

$$l^{(A5)} = \frac{\rho(1-\xi)}{\rho-\mu} x_0$$

and

$$c > -\frac{\rho(\beta(\epsilon-\xi)+\xi-1)}{(\beta-1)(\mu-\rho)} x_0 =: c^{(A5)}$$

A6. $g_2^A(c,d)$ **and** $g_3^A(c,d)$ **bind, i.e.** $(\chi_1^A = 0, \chi_2^A > 0, \chi_3^A > 0)$ I check if binding both Non-negative Constraints lead to a feasible candidate solution. This solution is not a feasible candidate, as the multipliers are negative:

$$\chi_2^{(A6)} = -\frac{(\beta-1)(\xi-1)\tau}{\beta\rho(\epsilon-1)} < 0$$
$$\chi_3^{(A6)} = -\frac{\tau}{\rho} < 0$$

B1. $g_1^B(c,d)$ **is binding, i.e.** $(\chi_1^B > 0, \chi_2^B = \chi_3^B = 0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c^{(B1)}, d^{(B1)})$ and the Kuhn-Tucker multiplier $\chi_1^{(B1)}$. Recalling that $l_1 = c + d$, I have:

$$l^{(B1)} = \frac{\rho(1-\xi)}{\rho-\mu} x_0$$

$$\chi_1^{(B1)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}(1-\tau)\beta\epsilon}{1-\xi} + \tau > 0$$

This solution candidate does not allow me to define a full solution for each coupon. However, any par of coupons (c,d) is optimal if it satisfies equation (5.3.3).

B2. $g_2^B(d)$ **is binding, i.e.** $(\chi_1^B = 0, \chi_2^B > 0, \chi_3^B = 0)$ In this solution candidate, the Non-Negative Constraint for Deposits is the only constraint binding. I solve for the optimal coupons $(c^{(B2)}, d^{(B2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(B2)}$. I find however that

$$\chi_2^{(B2)} = 0$$

This implies that constraint $g_2^B(d)$ does not bind and the candidate solution is not feasible. However, for a d sufficiently small to be negligible, I have

$$c < \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0 =: c_1^{(B2)}$$

B3. $g_3^B(c,d)$ **is binding, i.e.** $(\chi_1^B = 0, \chi_2^B = 0, \chi_3^B > 0)$ I am considering that the Non-Negative Constraint for Non-Deposit Liabilities binds. The optimisation problem yields an optimal multiplier where the candidate solution never binds.

$$\chi_3^{(B3)} = 0$$

The Non-Negative Constraint is never binding. Nonetheless, I can find the non-binding solution for the Non-Negative Constraint. That boundary is:

$$\begin{aligned} c &> 0 =: c^{(B3)} \\ d &< \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0 =: d^{(B3)} \end{aligned}$$

B4. $g_1^B(c,d)$ **and** $g_2^B(d)$ **bind, i.e.** $(\chi_1^B > 0, \chi_2^B > 0, \chi_3^B = 0)$ In this solution candidate, I check the case in which the Non-Negative Constraint for Deposits and the Rationality Constraint bind. This candidate solution does not however hold at the Non-Negative Constraint for Deposits:

$$\chi_2^{(B4)} = 0$$

Even though

$$\chi_1^{(B4)} = \frac{\beta\left(\frac{\beta-1}{\beta}\right)^{\beta}(\tau-1)\epsilon}{\xi-1} + \tau > 0$$

The following conditions must hold close to the aforementioned boundaries:

$$\begin{aligned} c &< \frac{\rho(1-\xi)}{\rho-\mu} x_0 := c^{(B4)} \\ d &> 0 =: d^{(B4)} \end{aligned}$$

B5. $g_1^B(c,d)$ **and** $g_3^B(c,d)$ **bind, i.e.** $(\chi_1^B > 0, \chi_2^B = 0, \chi_3^B > 0)$ I check if binding the Rationality Constraint and the Non-negative Constraint for Non-Deposit Liabilities lead to a feasible candidate solution. However, I find

$$\begin{aligned} \chi_3^{(B5)} &= 0 \\ \chi_1^{(B5)} &= \frac{\beta\left(\frac{\beta-1}{\beta}\right)^{\beta}(\tau-1)\epsilon}{\xi-1} + \tau > 0 \end{aligned}$$

The null multiplier tells me, as expected, that the Non-negative Constraint is not binding. In this case, I must always have a positive level of non-deposit liabilities. As for Case A, the total debt coupon:

$$l^{(B5)} = \frac{\rho(1-\xi)}{\rho-\mu} x_0$$

and the coupons must respect

$$c > 0 =: c^{(B5)}$$
$$d < \frac{(\xi - 1)\rho x_0}{\mu - \rho} =: d^{(B5)}$$

B6. $g_2^B(c,d)$ **and** $g_3^B(c,d)$ **bind, i.e.** $(\chi_1^B = 0, \chi_2^B > 0, \chi_3^B > 0)$ I check if binding both Non-negative Constraints lead to a feasible candidate solution. This solution is not a feasible candidate, as the multipliers are negative:

$$\chi_2^{(B6)} = -\frac{\tau}{\rho} < 0$$
$$\chi_3^{(B6)} = -\frac{\tau}{\rho} < 0$$

▷ **Interior Solution** $(\chi_1^A = \chi_2^A = \chi_3^A = 0)$ **and** $(\chi_1^B = \chi_2^B = \chi_3^B = 0)$ **- First Order Conditions.** In this case, the solution for the first order result cannot specify a specific optimal solution for each type of liability but only for the total debt coupon. Recall from the beginning that I define $l = c + d$ as the periodic total coupon to be paid.

Case A So, any pair (c,d) is optimal if $c^{**} + d^{**} = l^{**}$, with:

$$l^{**} = \frac{(\beta - 1)(1 - \xi)\rho \left(\frac{(\xi-1)\tau}{\beta(\epsilon - \xi\tau) + (\xi - 1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho - \mu)} x_0 \tag{5.98}$$

Case B **Case B.1** $(\chi_1^B = \chi_2^B = \chi_3^B = 0)$. In this case:

$$l^{**} = \frac{(\beta - 1)(1 - \xi)\rho \left(-\frac{\tau - \xi\tau}{\beta\epsilon - \beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho - \mu)} x_0 \tag{5.99}$$

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm $B(x_0)$ is concave for the unconstrained situation. This is relevant to guarantee interior solutions are maximising arguments.

Case A: Taking second order conditions and rearranging, I guarantee that l^{**} is a maximising argument for the firm value function conditional on subset $\xi \in [0, \overline{\xi})$ iff:

$$\xi < \frac{\beta\epsilon - \tau}{(\beta - 1)\tau}$$

However, since Case A is conditional on subset $\xi \in [0, \overline{\xi})$, I can note that $\frac{\beta\epsilon-\tau}{(\beta-1)\tau} > \overline{\xi}$ always for $\tau < 1$. Given that the tax rate $\tau \in (0,1)$, then SOC is always negative:

$$\underbrace{\beta}_{<0} \underbrace{\left(\frac{x_0}{x_b}\right)^{\beta} \frac{1}{\rho(1-\xi)l^{**}}}_{>0} \underbrace{((1-\xi)\tau - \beta(\epsilon - \xi\tau))}_{>0} < 0$$

Case B: Taking second order conditions and rearranging, I guarantee that l_1^{**} is a maximising argument for the firm value function conditional on subset $\xi \in [\overline{\xi}, 1]$:

$$\underbrace{-\frac{\beta^2(1-\tau)\epsilon}{\rho(1-\xi)l^{**}}}_{<0} \underbrace{\left(\frac{x_0}{x_b}\right)^{\beta}}_{>0} < 0$$

▷ **Equilibrium Solution.** By solving the constrained optimisation problem, I can synthesise the main equilibrium result. First, let me denominate some of the key parameters as:

$$\overline{c} := \begin{cases} c^{(A4)} & , \quad \xi < \overline{\xi} \\ c^{(B4)} & , \quad \xi \geq \overline{\xi} \end{cases} \qquad \hat{c} := \begin{cases} c^{(A5)} & , \quad \xi < \overline{\xi} \\ c^{(B5)} & , \quad \xi \geq \overline{\xi} \end{cases}$$

$$\underline{c} := \begin{cases} c^{(A3)} & , \quad \xi < \overline{\xi} \\ c^{(B3)} & , \quad \xi \geq \overline{\xi} \end{cases} \qquad \tilde{c} := \begin{cases} c^{(A2)} & , \quad \xi < \overline{\xi} \\ c^{(B2)} & , \quad \xi \geq \overline{\xi} \end{cases}$$

$$c^{**} := \begin{cases} c^{(A3)} & , \quad \xi < \overline{\xi} \\ c^{(B3)} & , \quad \xi \geq \overline{\xi} \end{cases}$$

and, taking the Rationality Constraint at its binding point, let me write:

At time $t = 0$, the bank selects the coupons (c^*, d^*) to be paid every time $t \in [0, T_b[$. The total periodic coupon $l^* = c^* + d^*$ is optimally defined as:

$$l^* = \min\left\{l^{**}, \bar{l}\right\} \tag{5.100}$$

where

$$l^{**} = \begin{cases} \frac{(\beta-1)(1-\xi)\rho\left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi < \overline{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)} x_0, & \text{for } \xi \geq \overline{\xi} \end{cases} \tag{5.101}$$

and

$$\bar{l} = \frac{\rho(1-\xi)x_0}{\rho - \mu} \tag{5.102}$$

I cannot find a specific solution for coupons (c^*, d^*). However, I let the deposits coupon be defined as the difference between the total coupon and the non-deposit liabilities coupon and define the interval of feasible solutions for the non-deposit liabilities coupons. Therefore, I have:

$$d^* := l^* - c^* \qquad \text{and} \qquad c^* \in \begin{cases}]\hat{c}, \overline{c}[& , \quad \text{for} \quad l^{**} \geq \bar{l} \\ ]\underline{c}, \tilde{c}[& , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{5.103}$$

where $\hat{c}$ is the lower boundary for the optimal c^*. In this case, the equilibrium l^* binds at the Rationality Constraint and is weakly bounded by the No Bank-Run Condition if $\xi < \overline{\xi}$ (Case A), and by the Non-negative constraint for Non-Deposit Liabilities if $\xi \geq \overline{\xi}$ (Case B). $\hat{c}$ is written as:

$$\hat{c} = \begin{cases} -\frac{\rho(\beta(\epsilon-\xi)+\xi-1)}{(\beta-1)(\mu-\rho)} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{5.104}$$

$\overline{c}$ is the upper boundary when l^* binds at the Rationality Constraint and c^* is weakly bounded by the Non-negative constraint for Deposits. $\overline{c}$ is written as:

$$\overline{c} = \frac{\rho(1-\xi)}{\rho-\mu} x_0 \tag{5.105}$$

$\underline{c}$ is the lower bound when the interior solution l^{**} holds and is weakly bounded by the No Bank-Run Condition if $\xi < \overline{\xi}$ (Case A), and by the Non-negative constraint for Non-Deposit Liabilities if $\xi \geq \overline{\xi}$ (Case B).

$$\underline{c} = \begin{cases} \frac{\rho}{\rho-\mu} \frac{\beta(\epsilon-\xi)+\xi-1}{\beta} \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau} \right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ 0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{5.106}$$

$\tilde{c}$ is the upper boundary when the interior solution l^{**} holds and c^* is weakly bounded by the Non-negative constraint for Deposits. $\tilde{c}$ is written as:

$$\tilde{c} = \begin{cases} \frac{\beta-1}{\beta} \frac{\rho}{\rho-\mu} (1-\xi) \left(\frac{(\xi-1)\tau}{\beta(\epsilon-\xi\tau)+(\xi-1)\tau} \right)^{-1/\beta} x_0 & , \quad \xi < \overline{\xi} \\ \frac{(\beta-1)(\xi-1)\rho \left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon} \right)^{-1/\beta}}{\beta(\mu-\rho)} x_0 & , \quad \xi \geq \overline{\xi} \end{cases} \tag{5.107}$$

□

5.3.4 Costless Deposit Insurance Case

Proof of Lemma 8. Recall the present value of all future payments to depositors d/ρ and the optimal closure trigger (4.31) at the threshold $\underline{\xi}$:

$$x_{di}(\underline{\xi}) := \frac{\beta(c_{di}+d_{di})(\rho-\mu)}{(\beta-1)(1-\underline{\xi})\rho}$$

Then solving $(1-\epsilon)\frac{x_{di}(\underline{\xi})}{\rho-\mu} = \frac{d}{\rho}$ for $\underline{\xi}$ yields equation (4.40). □

Proof of Proposition 7. Before solving the equilibrium, I can refine the problem by excluding the cases already solved in the base case. In the presence of a DIS, I have three branches possible depending on the exogenous parameter ξ.

▷ **.** I divide the bank's value function $B_{di}(x_t)$ at the lower threshold. I call $B_{di}(x_t|\xi \leq \underline{\xi})$

when the bankruptcy proceedings cannot fully cover all deposits and $B_{di}(x_t|\xi > \underline{\xi})$ when the bankruptcy proceedings can always cover all deposits. In my setup it is the case that

$$B_{di}(x_t|\xi > \underline{\xi}) := B(x_t)$$

For the cases in which depositors are always covered, the value function of the financial institution in the presence of a DIS is the same as in the case without deposit insurance which results in the following remark:

Remark 1. *The equilibria for $\xi > \underline{\xi}$ in the presence of a DIS is the same as in the base case, i.e.*

$$(c^*_{di}, d^*_{di}, x^*_{di}) = (c^*, d^*, x^*_b), \qquad \forall \xi \in]\underline{\xi}, 1] \tag{5.108}$$

Hence, I restrict to study the optimisation problem for $B_{di}(x_t|\xi \leq \underline{\xi})$. In this case the value functions for Equity-holders, Non-Deposit Creditors and Depositors are respectively:

$$E_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} (1-\tau)\left[\frac{x_t}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right] - \left(\frac{x_t}{x_{di}}\right)^{\beta}(1-\tau)\left(\frac{x_{di}}{\rho-\mu} - \frac{c_{di}+d_{di}}{\rho}\right) & , \text{ for } x_t > x_{di} \\ 0 & , \text{ for } x_t \leq x_{di} \end{cases}$$

$$C_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} \frac{c_{di}}{\rho} - \left(\frac{x_t}{x_{di}}\right)^{\beta}\frac{c_{di}}{\rho} & , \text{ for } x_t > x_{di} \\ 0 & , \text{ for } x_t \leq x_{di} \end{cases} \qquad \text{and} \qquad D_{di}(\xi < \underline{\xi}) = \frac{d_{di}}{\rho}$$

Summing each claimers value functions I obtain the bank's total value is then:

$$B_{di}(x_t|\xi < \underline{\xi}) = \begin{cases} (1-\tau)\frac{x_t}{\rho-\mu} + \tau\frac{c_{di}+d_{di}}{\rho} - \left(\frac{x_t}{x_{di}}\right)^{\beta}\left((1-\tau)\left(\frac{x_{di}}{\rho-\mu} - \frac{d_{di}}{\rho}\right) + \tau\frac{c_{di}}{\rho}\right) & , \text{ for } x_t > x_{di} \\ \frac{d_{di}}{\rho} & , \text{ for } x_t \leq x_{di} \end{cases} \tag{5.109}$$

Moreover, by inspecting the bank's value function in (5.109), I see that any marginal increase in value can be performed by deposits instead of non-deposit liabilities at a lower functional cost. I can make an additional simplification for this optimisation problem:

Remark 2. *For $\xi < \underline{\xi}$, only solutions that bind at the non-deposit creditors non-negative constraint are feasible. Formally:*

$$c_{di} = 0$$

▷ **Optimisation Problem.** Taking into consideration Remarks (1) and (2), I rewrite the

optimisation problem for the bank as:

$$\begin{aligned}
(c^*_{di}, d^*_{di}) \quad & \in \underset{\{c_{di}, d_{di}\}}{\arg\max}\ B_{di}(x_0 | \xi \leq \underline{\xi}) \\
\text{s.t.} \qquad & x_{di} := \frac{\beta}{\beta - 1} \frac{\rho - \mu}{1 - \xi} \frac{c_{di} + d_{di}}{\rho} && \text{[Closure Trigger]} \\
& x_0 \geq \frac{(\rho - \mu)(c_{di} + d_{di})}{\rho(1 - \xi)} && \text{[Rationality Constraint]} \\
& c_{di} = 0 \quad , \quad d_{di} \geq 0 && \text{[Non-Negative Constraints]}
\end{aligned}$$

▷ **Constraint Set Feasibility.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_{di} = \left\{ (c_{di}, d_{di}) \in \mathbb{R}^2 | g(c_{di}, d_{di}) \leq 0 \right\}$$

with

$$g(c_{di}, d_{di}) = \begin{pmatrix} g_1(c_{di}, d_{di}) \\ g_2(d_{di}) \\ g_3(c_{di}) \end{pmatrix} = \begin{pmatrix} c_{di} + d_{di} - \frac{(1-\xi)\rho}{\rho - \mu} x_0 \\ -d_{di} \\ 0 \end{pmatrix}$$

Let $Q_1 := \frac{(1-\xi)\rho}{\rho - \mu} x_0$ be a family of constants and let (ρ, μ, x_0, ξ) be such that $Q_1 > 0$. Then, I secure the feasible set $\mathcal{G}_{di}$ is compact which, by the Weierstrass Theorem, guarantees the existence of a maximising solution.

▷ **Nondegenerate Constraint Qualification.** Consider the Constraint Qualification matrix

$$CQ := \nabla g(c_{di}, d_{di}) = \begin{pmatrix} 1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{5.110}$$

For any possible combinations of binding constraints, $Rank(CQ) \in \{1, 2\}$. Therefore, I can exclude the case where three constraints are binding. I can imediately see that, in this case, I have two possible solutions either an interior solution for d^*_{di} (with $c^*_{di} = 0$) or a corner solution where d^*_{di} binds at the rationality constraint. A solution with $(c_{di}, d_{di}) = (0, 0)$ is excluded.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers $(\chi^C_1, \chi^C_2, \chi^C_3)$ and take first order conditions.

$$\begin{aligned}
\mathscr{L}(c_{di}, d_{di})|_{\xi \leq \underline{\xi}} =& (1 - \tau) \frac{x_t}{\rho - \mu} + \tau \frac{c_{di} + d_{di}}{\rho} - \left(\frac{x_t}{x_{di}} \right)^{\beta} \left((1 - \tau) \left(\frac{x_{di}}{\rho - \mu} - \frac{d_{di}}{\rho} \right) + \tau \frac{c_{di}}{\rho} \right) \\
& + \chi^C_1 \left(\rho(1 - \xi) x_0 - (\rho - \mu)(c_{di} + d_{di}) \right) + \chi^C_2 d_{di} + \chi^C_3 c_{di}
\end{aligned} \tag{5.111}$$

▷ **Kuhn-Tucker Conditions.**

$$\chi_1^C \left(\rho(1-\xi)x_0 - (\rho-\mu)(c_{di}+d_{di})\right) = 0 \tag{5.112}$$

$$\chi_2^C d_{di} = 0 \tag{5.113}$$

$$c_{di} = 0 \tag{5.114}$$

and Kuhn-Tucker multiplier constraints are:

$$\chi_1^C \geq 0 \quad , \qquad \chi_2^C \geq 0 \tag{5.115}$$

▷ **Solutions Candidates.** Only solutions satisfying (5.112)-(5.115) are admissible. Hence, I reduce to two possible candidates of admissible solutions.

C1. Interior Solution ($\chi_1^C = 0, \chi_2^C = 0$). For this solution candidate, no inequality constraint will bind. I have:

$$d_{di}^{(C1)} = \frac{(\beta-1)(\xi-1)\rho x_0 \left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)}$$

with

$$\chi_3^{(C1)} = \frac{\tau-\xi\tau}{\rho(\tau-1)((\beta-1)\xi+1)}$$

The solution multiplier is always $\chi_3^{(C1)} > 0$ because the Second Order Condition for the interior solution requires $\xi > 1/(1-\beta)$.

C2. Corner Solution ($\chi_1^C = 0, \chi_2^C > 0$). For this solution candidate, the Rationality Constraint binds. I have:

$$d_{di}^{(C2)} = \frac{(1-\xi)\rho x_0}{\rho-\mu}$$

The solution multipliers are:

$$\chi_1^{(C2)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}(1-\tau)((\beta-1)\xi+1)}{1-\xi} + \tau$$

$$\chi_3^{(C2)} = \frac{\left(\frac{\beta-1}{\beta}\right)^{\beta}}{\rho} > 0 \quad \text{(always)}$$

I require that $\chi_1^{(C2)} > 0$ for the solution to hold.

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm the concavity of $B_{di}(x_0|\xi < \underline{\xi})$ for the unconstrained situation. Taking second order conditions and

rearranging, I guarantee d^*_{bi} is a maximising arguments for the firm value function conditional on subset $\xi \in [0, \underline{\xi})$. The SOC condition is:

$$\frac{\mathrm{d}B_{bi}(x_0|\xi \leq \underline{\xi})}{\mathrm{d}d}\Big| < 0 \Leftrightarrow \underbrace{-\beta\frac{1-\tau}{1-\xi}\frac{1}{d^*_{di}}}_{>0}\underbrace{\left(\frac{x_0}{x_{di}}\right)^{\beta}}_{>0}\underbrace{(1+(\beta-1)\xi)}_{<0} \qquad \text{iff} \qquad \xi > \frac{1}{\beta-1}$$

To secure that the interior solution is a maximum, I must impose $\xi > \frac{1}{1-\beta}$.

▷ **Equilibrium Capital Requirement Ratio Threshold.** At last, take into consideration that $c^*_1 = 0$. Replacing in $\underline{\xi}(c_{di}, d_{di})$ I see:

$$\underline{\xi}(c^*_{di}, d^*_{di}) = \overline{\xi}$$

In summary, for all $\xi < \overline{\xi}_1$, if the regulator introduces a costless deposit insurance scheme in Scenario 1 (without resolution tools), the bank will preempt the intervention decision by selecting a level of liabilities that is made of 100% deposits. If $\xi \geq \overline{\xi}_1$, the equilibrium choice of debt follows Condition (5.100), the equilibrium result for Scenario 1 without DIS.

▷ **Equilibrium Coupon.** At time $t = 0$, the bank selects the coupons (c^*_{di}, d^*_{di}) to be paid every time $t \in [0, T_{di}[$. As mentioned in Remark (1), for $\xi > \overline{\xi}$, the equilibrium does not change with the introduction of a DIS. Henceforth, all equilibrium results for $\xi > \overline{\xi}$ correspond to Case B in the proof of Proposition (6). Combining all equilibria, the total periodic coupon $l^*_{di} = c^*_{di} + d^*_{di}$ can be defined as:

$$l^*_{di} = \min\left\{l^{**}_{di}, \bar{l}_{di}\right\} \tag{5.116}$$

where the interior solution is

$$l^{**}_{di} = \begin{cases} \frac{(\beta-1)(\xi-1)\rho\left(\frac{\tau-\xi\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{-1/\beta}}{\beta(\mu-\rho)}x_0, & \text{for } \xi < \overline{\xi} \\ \frac{(\beta-1)(1-\xi)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-\frac{1}{\beta}}}{\beta(\rho-\mu)}x_0, & \text{for } \xi \geq \overline{\xi} \end{cases} \tag{5.117}$$

and the corner solution (binding at the rationality constraint) is:

$$\bar{l}_{di} = \frac{\rho(1-\xi)x_0}{\rho-\mu} \tag{5.118}$$

Any combination of $(c^*_{di}d^*_{di})$ is a feasible optimal solution, provided that it respects the following conditions:

$$d^*_{di} := l^*_{di} - c^*_{di} \qquad \text{and} \qquad c^*_{di} = \begin{cases} 0 & , \quad \xi < \overline{\xi} \\ c^{**}_{di} & , \quad \xi \geq \overline{\xi} \end{cases} \tag{5.119}$$

with

$$c_{di}^{**} \in \begin{cases} \left]0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right[& , \quad \text{for} \quad l^{**} \geq \bar{l} \\ \left]0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon-\beta\tau\epsilon}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right[& , \quad \text{for} \quad l^{**} < \bar{l} \end{cases} \tag{5.120}$$

□

Proof of Proposition 8. I compare the optimal total coupon for the DIS case in condition (4.44) with the optimal total coupon for the base case in condition (4.20). In Case (A) I study the equilibria for weak banks and in Case (B) I study equilibria for strong banks.

Remark 3. *For $\beta < 0$, a function $f : z \mapsto z^{\frac{1}{\beta}} - 1$ is always positive in the domain $z \in [0, 1)$.*

▷ **Case (A): $\xi \in [0, \overline{\xi})$.** Consider a transition from interior to interior solution. That is:

$$(1) \quad l^* = l^{**} \quad \text{and} \quad l_{di}^* = l_{di}^{**}$$

Writing the ratio between the new equilibrium and the old equilibrium total coupon, I have:

$$\frac{l_{di}^{**}}{l^{**}} := \left(\frac{(\tau-1)((\beta-1)\xi+1)}{\beta\xi\tau - \beta\epsilon - \xi\tau + \tau}\right)^{\frac{1}{\beta}}, \qquad \forall \xi < \overline{\xi} \tag{5.121}$$

Rearranging condition (5.121) and using Remark 3, the ratio l_{di}^{**}/l^{**} is greater than 1 iff:

$$\beta\epsilon < (1 + (\beta - 1)\xi)$$

which holds for all $\xi < \overline{\xi}$. Therefore the interior solution for the equilibrium total coupon always increases when the regulator introduces the costless deposit insurance. That is

$$\frac{l_{di}^{**}}{l^{**}} > 1, \qquad \forall \xi < \overline{\xi} \tag{5.122}$$

With this result, and given that, the equilibrium total coupon conditions are $l^* = \{l^{**}, \bar{l}\}$ and $l_{di}^* = \{l_{di}^{**}, \bar{l}\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l^{**} < \bar{l} < l_{di}^{**} \qquad \Rightarrow \qquad l^* = l^{**} \quad \text{and} \quad l_{di}^* = \bar{l} \\ &(3) \quad \bar{l} < l^{**} < l_{di}^{**} \qquad \Rightarrow \qquad l^* = \bar{l} \quad \text{and} \quad l_{di}^* = \bar{l} \end{aligned} \tag{5.123}$$

Consider Scenario (2): $l^* = l^{**}$ and $l_{di}^* = \bar{l}$. Since it must be the case $l^{**} < \bar{l} < l_{di}^{**}$, the result presented in condition (5.122) guarantees that the following condition always holds:

$$\frac{\bar{l}_{di}}{l^{**}} = \frac{\beta}{\beta-1}\left(\frac{\tau(1-\xi)}{-\beta\epsilon + \tau(1+(\beta-1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi < \overline{\xi} \tag{5.124}$$

In Scenario (3), the optimal coupon does not change: $l^* = \bar{l}$ and $l^*_{di} = \bar{l}$.

▷ **Case (B):** $\xi \in [\overline{\xi}, 1]$**.** Comparing condition (4.44) with the condition (4.20) we see that the total coupon does not change:

$$l^*_{di} = l^*, \qquad \forall \xi \geq \overline{\xi}$$

The financial institution has no incentive to change its equilibrium total leverage coupon if deposit insurance is introduced.

□

Proof of Proposition 9. Included in the Proof of Proposition (7). □

Proof of Proposition 10. Let me represent the absolute change in the claims with a Δ. The change in equilibrium bank value after the introduction of costless deposit insurance is

$$\Delta B_{di}(x|\mathcal{E}_{di}) = B_{di}(x|\mathcal{E}_{di}) - B(x|\mathcal{E}_{di})$$

Furthermore, the change in equity value is $\Delta E_{di}(x|\mathcal{E}_{di})$ when I introduce the costless deposit insurance, the change in non-deposit liabilities is $\Delta C_{di}(x|\mathcal{E}_{di})$ and the change in deposits is $\Delta D_{di}(\mathcal{E}_{di})$. All these represent the change in the claims value function from the base case to the costless deposit insurance case. Moreover, note that

$$\Delta B_{di}(x|\mathcal{E}_{di}) = \Delta E_{di}(x|\mathcal{E}_{di}) + \Delta C_{di}(x|\mathcal{E}_{di}) + \Delta D_{di}(\mathcal{E}_{di})$$

For the remaining of this proof I will consider equilibria will hold respectively in the base and DIS case and, so, I omit the equilibrium notation, e.g. $B_{di}(x) \equiv \Delta B_{di}(x|\mathcal{E}_{di})$.

Furthermore, consider the following Remarks:

Remark 4. *A function $f : (l, \beta) \mapsto l^{1-\beta}$ with $l \in \mathbb{R}_+$ and $\beta \in \mathbb{R}_-$ is monotonically increasing in l.*

Remark 5. *I restrict to the tuple of parameters (μ, σ, ρ, ξ) that secures $\xi > \frac{1}{1-\beta}$*

These remarks aim at easing the present proof. Remark (4) characterises a function that appears recurrently and Remark (5) imposes a necessary condition in my parameterisation. I note that this last remark is necessary to obtain maximising solutions in the costless deposit insurance case, as demonstrated in the Proof of Proposition (7).

▷ **Capital structure when $\xi \geq \overline{\xi}$.** I can see that the model does not change if the bank is positioned in $\xi \geq \overline{\xi}$ by comparing the value function (4.38), for the deposit insurance case,

with value function (4.15) for the base case. Moreover, this is also the case for the equity value function (4.34) and (4.11), respectively. Hence, I have:

$$\Delta E_{di}(x|\xi \geq \overline{\xi}) = 0 \qquad \text{and} \qquad \Delta B_{di}(x|\xi \geq \overline{\xi}) = 0$$

which leads to

$$\Delta CS_{di}(x|\xi \geq \overline{\xi}) = 0$$

▷ **Capital structure when $\xi < \overline{\xi}$.** Let me write the value functions for this subcase. The difference value function for equity in equilibrium as:

$$\begin{aligned}
&\Delta E_{di}(x|\xi < \overline{\xi}) := E_{di}(x|\xi < \overline{\xi}) - E(x|\xi < \overline{\xi}) = \\
&= \frac{1-\tau}{\rho}\left(l^* - l^*_{di}\right) + \frac{1-\tau}{\rho}\left(\frac{x}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}}\right)^{\beta}\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi} - 1\right)\left((l^*)^{1-\beta} - (l^*_{di})^{1-\beta}\right)
\end{aligned} \tag{5.125}$$

Given Remark (4) it is easy to show that $\Delta E_{di}(x|\xi < \overline{\xi}) \leq 0$ but I will discuss this next in greater detail for each one of the possible cases. Moreover, the difference value function for total bank value is:

$$\begin{aligned}
&\Delta B_{di}(x|\xi < \overline{\xi}) := B_{di}(x|\xi < \overline{\xi}) - B(x|\xi < \overline{\xi}) = \\
&= \frac{\tau}{\rho}\left(l^*_{di} - l^*\right) - \frac{x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}{\rho(1-\beta)(1-\xi)}\left((l^*_{di})^{1-\beta}(\tau-1)((\beta-1)\xi+1) - (l^*)^{1-\beta}\left(-\beta\epsilon + \tau(1+(\beta-1)\xi)\right)\right)
\end{aligned} \tag{5.126}$$

Now,given that $\overline{l}$ is an upperbound for interior solutions l^{**}_{di} or l^{**}, I restrict my analysis to three cases:

$$\begin{aligned}
&\text{(A)} \quad \overline{l} < l^{**} < l^{**}_{di} \qquad \Rightarrow \qquad l^* = \overline{l} \quad \text{and} \quad l^*_{di} = \overline{l} \\
&\text{(B)} \quad l^{**} < \overline{l} < l^{**}_{di} \qquad \Rightarrow \qquad l^* = l^{**} \quad \text{and} \quad l^*_{di} = \overline{l} \\
&\text{(C)} \quad l^{**} < l^{**}_{di} < \overline{l} \qquad \Rightarrow \qquad l^* = l^{**} \quad \text{and} \quad l^*_{di} = l^{**}_{di}
\end{aligned} \tag{5.127}$$

Case (A): $\overline{l} < l^{**} < l^{**}_{di}$. In this case, the tuple of parameters $(\mu, \sigma, \rho, \tau, \xi, \epsilon)$ is such that, in the base case and in the costless deposit insurance case, the bank always chooses a level of total coupon that binds at the rationality constraint. Specifically, $l^* = \overline{l}$ for the base case and $l^*_{di} = \overline{l}$ after I introduce the costless deposit insurance.

Therefore, I have:

$$\Delta E^{(A)}_{di}(x|\xi < \overline{\xi}) = 0 \tag{5.128}$$

and

$$\Delta B_{di}^{(A)}(x|\xi<\overline{\xi}) := \underbrace{-\frac{x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\overline{l}^{1-\beta}}{\rho(1-\beta)(1-\xi)}}_{<0}\underbrace{((\tau-1)((\beta-1)\xi+1)-(-\beta\epsilon+\tau(1+(\beta-1)\xi)))}_{\substack{<0\\ \text{by } \xi<\overline{\xi}}}>0 \tag{5.129}$$

As a result, the capital structure decreases due to an increase in the denominator, i.e.

$$\Delta CS^{(A)}(x|\xi<\overline{\xi})<0$$

Case (B): $l^{**}<\overline{l}<l_{di}^{**}$**.** In this case, the tuple of parameters $(\mu,\sigma,\rho,\tau,\xi,\epsilon)$ is such that, in the base case the bank always chooses an interior solution for the optimal level of total coupon and in the costless deposit insurance case the bank's optimal total coupon binds at the rationality constraint. In detail, $l^{*}=l^{**}$ for the base case and $l_{di}^{*}=\overline{l}$ for the costless deposit insurance case.

Using condition (5.124) with Remarks (4) and (5), I have:

$$\Delta E_{di}^{(B)}(x|\xi<\overline{\xi})=\underbrace{\frac{1-\tau}{\rho}\left(l^{**}-\overline{l}\right)}_{<0}+\underbrace{\frac{1-\tau}{\rho}\left(\frac{x}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}}\right)^{\beta}}_{>0}\underbrace{\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi}-1\right)}_{\substack{>0\\ \text{by Remark (5)}}}\underbrace{\left((l^{**})^{1-\beta}-(\overline{l})^{1-\beta}\right)}_{\substack{<0\\ \text{by Remark (4)}}}<0 \tag{5.130}$$

and

$$\Delta B_{di}^{(B)}(x|\xi<\overline{\xi})=\underbrace{\frac{\tau}{\rho}\left(\overline{l}-l^{**}\right)}_{>0}-$$

$$\underbrace{-\frac{x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}{\rho(1-\beta)(1-\xi)}}_{<0}\underbrace{\left(\left(\overline{l}\right)^{1-\beta}(\tau-1)((\beta-1)\xi+1)-(l^{**})^{1-\beta}(-\beta\epsilon+\tau(1+(\beta-1)\xi))\right)}_{<0}>0 \tag{5.131}$$

In case B, the capital structure decreases due to a simultaneous decrease in the numerator (equity value function) and an increase in the denominator (bank's value function), i.e.

$$\Delta CS^{(B)}(x|\xi<\overline{\xi})<0$$

Case (C): $l^{**}<l_{di}^{**}<\overline{l}$**.** In this case, the tuple of parameters $(\mu,\sigma,\rho,\tau,\xi,\epsilon)$ is such that, in the base case and in the costless deposit insurance the bank always chooses an interior solution. In this case, $l^{*}=l^{**}$ for the base case and $l_{di}^{*}=l_{di}^{**}$ for the costless deposit insurance case. Using condition (5.121) with Remarks (4) and (5), I have:

$$\Delta E_{di}(x|\xi<\overline{\xi})=\underbrace{\frac{1-\tau}{\rho}l_{di}^{**}\left(1+\left(\frac{x}{x_{di}}\right)^{\beta}\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi}-1\right)\right)}_{>0}\underbrace{\left(\left(\frac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{\frac{1}{\beta}}-1\right)}_{\substack{<0\\ \text{by Condition (5.121)}}}<0 \tag{5.132}$$

and

$$\Delta B_{di}(x|\xi<\overline{\xi}) = \underbrace{\frac{l_{di}^{**}}{\rho}\left(1-\left(\frac{\beta\xi\tau-\beta\epsilon-\xi\tau+\tau}{(\tau-1)((\beta-1)\xi+1)}\right)^{\frac{1}{\beta}}\right)}_{>0}\underbrace{\left(\tau-\left(\frac{x}{x_{di}}\right)^{\beta}\frac{(\tau-1)((\beta-1)\xi+1)}{(1-\beta)(1-\xi)}\right)}_{>0}>0 \tag{5.133}$$

In case C, the capital structure decreases due to a simultaneous decrease in the numerator (equity value function) and an increase in the denominator (bank's value function), i.e.

$$\Delta CS^{(C)}(x|\xi<\overline{\xi})<0$$

□

5.3.5 Private Sector Acquisitions Case

Proof of Lemma 9. This Proof is identical to the Proof of Lemma 6. □

Proof of Proposition 11. This Proof is identical to the Proof of Proposition 6 provided ϵ is replaced with ϵ_p. □

Proof of Proposition 12. Consider three cases referring each partition of the bank's type: Case (A) corresponds to the weakest banks, the group of institutions that respect the condition $\xi<\overline{\xi}_p$. Case (B) corresponds to the intermediate region, i.e. the institutions that were once deemed as weak but, with the introduction of the PSA, they are now considered strong. Specifically, $\xi\in[\overline{\xi}_p,\overline{\xi}]$. At last, Case (C) corresponds to the banks that were always strong, i.e. $\xi\geq\overline{\xi}$.

▷ **Case (A):** $\xi\in[0,\overline{\xi}_p)$**.** In this case the bank's value function and payoff structure never changes. The proceedings from the bank's closure will continue to be insufficient to cover liabilities and, in this circumstance, the bank benefits from deposit insurance which it will explore by selecting a maximum level of deposits in the liability structure. Hence, for $\xi<\overline{\xi}_p$, I have:

$$l_p^*=l_{di}^*$$

▷ **Case (B):** $\xi\in[\overline{\xi}_p,\overline{\xi}]$**.** I start by studying one transition: Interior to interior solution. That is:

$$(1)\quad l_{di}^*=l_{di}^{**}\quad\text{and}\quad l_p^*=l_p^{**}$$

Let me write now the ratio between the new equilibrium and the old equilibrium. Since any function $f : x \mapsto x^{\frac{1}{\beta}}$ is positive in the domain $x \in [0, 1]$ and since I am in the subset $\xi \geq \overline{\xi}_p$, I have:

$$\frac{l_p^{**}}{l_{di}^{**}} = \left(\frac{\beta\epsilon_p}{1 + (\beta - 1)\xi}\right)^{\frac{1}{\beta}} > 1 \quad \forall \epsilon_p < \epsilon \tag{5.134}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{di, p\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l_{di}^{**} < \bar{l} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = \bar{l} \\ &(3) \quad \bar{l} < l_{di}^{**} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = \bar{l} \quad \text{and} \quad l_p^* = \bar{l} \end{aligned} \tag{5.135}$$

Consider Scenario (2): $l_{di}^* = l_{di}^{**}$ and $l_p^* = \bar{l}$. Since $l_p^* = \min\{l_p^{**}, \bar{l}\}$ and given condition (5.137), then it must be the case that

$$\frac{\bar{l}}{l_{di}^{**}} = \frac{\beta}{\beta - 1}\left(\frac{(1 - \xi)\tau}{(\tau - 1)(1 + (\beta - 1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi \in [\overline{\xi}_p, \overline{\xi}], \quad \forall \epsilon_p < \epsilon \tag{5.136}$$

In Scenario (3) the optimal coupon does not change: $l_{di}^* = \bar{l}$ and $l_p^* = \bar{l}$.

▷ **Case (C): $\xi > \overline{\xi}$.** I start by studying one transition: Interior to interior solution. That is:

$$(1) \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = l_p^{**}$$

which leads to

$$\frac{l_p^{**}}{l_{di}^{**}} = \left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} > 1 \quad \forall \epsilon_p < \epsilon \tag{5.137}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{di, p\}$, I can restrict the rest of the problem to two additional possible scenarios:

$$\begin{aligned} &(2) \quad l_{di}^{**} < \bar{l} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = l_{di}^{**} \quad \text{and} \quad l_p^* = \bar{l} \\ &(3) \quad \bar{l} < l_{di}^{**} < l_p^{**} \quad \Rightarrow \quad l_{di}^* = \bar{l} \quad \text{and} \quad l_p^* = \bar{l} \end{aligned} \tag{5.138}$$

Consider Scenario (2): $l_{di}^* = l_{di}^{**}$ and $l_p^* = \bar{l}$. Since $l_p^* = \min\{l_p^{**}, \bar{l}\}$ and given condition (5.137), then it must be the case that

$$\frac{\bar{l}}{l_{di}^{**}} = \frac{\beta}{\beta - 1}\left(\frac{(1 - \xi)\tau}{(\tau - 1)(1 + (\beta - 1)\xi)}\right)^{\frac{1}{\beta}} > 1, \qquad \forall \xi \in [\overline{\xi}_p, \overline{\xi}], \quad \forall \epsilon_p < \epsilon \tag{5.139}$$

In Scenario (3) the optimal coupon does not change: $l_{di}^* = \bar{l}$ and $l_p^* = \bar{l}$.

Hence, for all scenarios the equilibrium post-PSA total liability coupon is always equal or greater than the equilibrium total liability coupon chosen before this tool being introduced.

□

Proof of Proposition 13. Restrict to $\xi < \overline{\xi}_p$. By comparing conditions (4.68) and (4.69) in Proposition 11 with conditions (4.47) and (4.48) in Proposition 7, I conclude the combination of deposit coupons and non-deposit coupons stays unchanged:

$$c^*_{di} = c^*_p = 0$$
$$d^*_{di} = l^*_{di} \qquad \text{and} \qquad d^*_p = l^*_p$$

The leverage increase is absorbed by deposits and the weak bank will maintain the debt structure.

Restrict to $\xi > \overline{\xi}$. By comparing conditions (4.68) and (4.69) in Proposition 11 with conditions (4.47) and (4.48) in Proposition 7, I conclude that the range of deposit and non-deposit liabilities changes due to the increase in the total leverage coupon. However, the bank can choose any combination of deposits and non-deposit liabilities such that the existing mix stays unchanged.

Restrict to $\xi \in [\overline{\xi}_p, \overline{\xi}]$. In this case, I have:

$$c^*_{di} = 0 \qquad \text{and} \qquad d^*_{di} = l^*_{di}$$
$$c^*_p = c^{**}_p \qquad \text{and} \qquad d^*_p = l^*_p$$

with

$$c^{**}_p \in \begin{cases} \left]0, \frac{\rho(1-\xi)}{\rho-\mu} x_0\right[& , \quad \text{for} \quad l^{**}_p \geq \bar{l}_p \\ \left]0, \frac{(\beta-1)(\xi-1)\rho\left(-\frac{\tau-\xi\tau}{\beta\epsilon_p-\beta\tau\epsilon_p}\right)^{-1/\beta}}{\beta(\mu-\rho)} x_0\right[& , \quad \text{for} \quad l^{**}_p < \bar{l}_p \end{cases} \tag{5.140}$$

The domain of c^{**}_p is an open set as the Non-Negative Constraints do not bind in the optimisation problem. Hence, the optimal solution imposes the presence of both types of debt for $\xi > \overline{\xi}_p$, i.e. $d^*_p > 0$. As a result the existing debt structure with 100% of deposits will have to change, after the PSA, to include non-deposit liabilities as well. □

Proof of Proposition 14. Recall that $\overline{\xi}_p > \overline{\xi}$, i.e. the introduction of the PSA decreases the threshold that separates the strong from weak banks. Hence, I define three partitions: In Case (A), the bank is positioned in $\xi \in [0, \overline{\xi}_p)$, i.e. this is a "very weak bank"; In Case (B), the bank is in $\xi \in [\overline{\xi}_p, \overline{\xi})$ which means that the bank that was weak before the introduction of the PSA and now became strong. I call this the "intermediate bank". Finally, in Case (C), the bank is in $\xi \in [\overline{\xi}, 1]$ which means that this is a "strong bank" and the introduction of the PSA will not affect the structure of the bank's value function, even though it will affect the payoffs.

I may need to combine these partitions with the feasible equilibria transitions. Given that $\bar{l}$ is an upper-bound for interior solutions l^{**}_{di} or l^{**}_p, I have three possible equilibria as

seen in Proposition (12):

$$\begin{aligned}
&(1) \quad l_{di}^{**} < l_p^{**} < \bar{l} \qquad \Rightarrow \qquad l_{di}^{*} = l_{di}^{**} \quad \text{and} \quad l_p^{*} = l_p^{**} \\
&(2) \quad l_{di}^{**} < \bar{l} < l_p^{**} \qquad \Rightarrow \qquad l_{di}^{*} = l_{di}^{**} \quad \text{and} \quad l_p^{*} = \bar{l} \\
&(3) \quad \bar{l} < l_{di}^{**} < l_p^{**} \qquad \Rightarrow \qquad l_{di}^{*} = \bar{l} \quad \text{and} \quad l_p^{*} = \bar{l}
\end{aligned} \tag{5.141}$$

To study the changes in capital structure, I must understand first the changes in the equity value claim and in the bank's total value. Let me represent the absolute change in the claims with a Δ. The change in equilibrium bank value after the introduction of private sector acquisition tool is

$$\Delta B_{di,p}(x) = B_p(x|\mathcal{E}_p) - B_{di}(x|\mathcal{E}_{di})$$

This is the change in bank's value as a consequence of a regulatory change ($B_{di} \to B_p$) and a simultaneous change in the capital structure towards the new equilibria ($\mathcal{E}_{di} \to \mathcal{E}_p$).

The changes in the claims value function from the costless deposit insurance case to the private sector acquisition case are as follows: the change in equity value is $\Delta E_{di,p}(x)$, the change in non-deposit liabilities is $\Delta C_{di,p}(x)$ and the change in deposits is $\Delta D_{di,p}$. Moreover, note that

$$\Delta B_{di,p}(x) = \Delta E_{di,p}(x) + \Delta C_{di,p}(x) + \Delta D_{di,p}$$

Finally, the capital structure ratio change is:

$$\begin{aligned}
\Delta CS_{di,p}(x) &= \frac{E_p(x|\mathcal{E}_p)}{B_p(x|\mathcal{E}_p)} - \frac{E_{di}(x|\mathcal{E}_{di})}{B_{di}(x|\mathcal{E}_{di})} = \frac{E_p(x|\mathcal{E}_p)}{E_{di}(x|\mathcal{E}_{di})} - \frac{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)}{C_{di}(x|\mathcal{E}_{di}) + D_{di}(\mathcal{E}_{di})} \\
&= \frac{E_p(x|\mathcal{E}_p)}{E_{di}(x|\mathcal{E}_{di})} - \frac{L_p(\mathcal{E}_p)}{L_{di}(\mathcal{E}_{di})}
\end{aligned} \tag{5.142}$$

And let me set the following remark useful for the rest of the proof:

Remark 6. *A function $f : x \mapsto x^{\frac{1}{\beta}}$ defined in the domain $x \in [0, 1]$ is always greater than 1.*

▷ **Case (A):** $\xi \in [0, \overline{\xi}_p)$ **.** The bank's value function and payoff structure never changes. The bank explores deposit insurance by only having deposits in its debt structure. So, closure costs are transferred to the insurance fund and become irrelevant for the bank's value function. As a result, the decrease in closure costs introduced with the PSA do not affect the weak bank. The equilibrium total leverage coupon (which equals the equilibrium deposit coupon) will not change, i.e. $l_p^* = l_{di}^*$ as discussed in Proposition (12). In conclusion, I have:

$$\Delta B_{di,p}^{(A)}(x) = 0 \qquad \wedge \qquad \Delta E_{di,p}^{(A)}(x) = 0 \qquad \Rightarrow \qquad \Delta CS_{di,p}^{(A)}(x) = 0$$

▷ **Case (B):** $\xi \in [\overline{\xi}_p, \overline{\xi})$**.** A bank deemed weak prior to the introduction of the PSA is now cosidered strong. So, from Proposition (7) and Proposition (11), the relevant equilibrium optimal total leverage coupons are:

$$\begin{aligned} l_{di}^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}\left(\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}} \\ l_{p}^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon_p)^{\frac{1}{\beta}} \\ \bar{l} &= \frac{(1-\xi)\rho x_0}{\rho-\mu} \end{aligned} \tag{5.143}$$

The difference equations for equity and total value are, respectively:

$$\Delta E_{di,p}^{(B)}(x) = \frac{1-\tau}{\rho}\left(l_{di}^{*}-l_{p}^{*}\right)+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^*)^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^*)^{1-\beta}\right) \tag{5.144}$$

$$\Delta B_{di,p}^{(B)}(x) = \frac{\tau}{\rho}\left(l_{p}^{*}-l_{di}^{*}\right)+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^*)^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^*)^{1-\beta}\right) \tag{5.145}$$

Subcase (B1): $l_{di}^{*} = l_{di}^{**}$ **and** $l_{p}^{*} = l_{p}^{**}$.

$$\begin{aligned} \Delta E_{di,p}^{(B1)}(x) &= \frac{1-\tau}{\rho}\left(l_{di}^{**}-l_{p}^{**}\right)+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^{**})^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right) \\ &= \underbrace{\frac{1-\tau}{\rho}\left(1-\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}\right)l_{di}^{**}}_{<0}+ \\ &+\underbrace{\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}-1\right)}_{>0} \end{aligned} \tag{5.146}$$

I conjecture that expression (5.146) is negative:

$$\Delta E_{di,p}^{(B1)}(x) < 0 \Rightarrow 1-\beta x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta} l_{di}^{**} > 0$$

and confirm it since, given the domains of the parameters, that is always the case.

Recall condition (5.134) from Proposition (12):

$$\begin{aligned} \Delta B_{di,p}^{(B1)}(x) &= \frac{\tau}{\rho}\left(l_{p}^{**}-l_{di}^{**}\right)+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_p(l_p^{**})^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right) \\ &= \underbrace{\left(\frac{\tau}{\rho}l_{di}^{**}+\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\right)}_{>0}\underbrace{\left(\left(\frac{\beta\epsilon_p}{1+(\beta-1)\xi}\right)^{-\frac{1}{\beta}}-1\right)}_{>0}<0 \end{aligned} \tag{5.147}$$

The change in the bank's value is always positive. I can conclude that in Case (B1), the capital structure will always decrease, i.e.

$$\Delta CS_{di,p}^{(B1)}(x) < 0 \tag{5.148}$$

Subcase (B2): $l_{di}^{*}=l_{di}^{**}$ **and** $l_{p}^{*}=\bar{l}$. Recall functions $E_{p}^{(B)}(x)$ and $B_{p}^{(B)}(x)$ and notice that, in this subcase, the optimal total coupon after the introduction of the PSA is restricted to domain $\bar{l}\in[l_{di}^{**},l_{p}^{**}]$ by Proposition (12). I study the bounds of the domain of $\bar{l}$ to show that equity value function has a root that yields no change in equity value.

Fix a family of parameters $(\mu,\sigma,\rho,\epsilon_{p},\xi)$ securing the new optimal total coupon level binds at the lower bound of the domain, i.e. $\bar{l}=l_{di}^{**}$. I have in this circumstance

$$\Delta E_{di,p}^{(B2)}(x|\bar{l}=l_{di}^{**})=\underbrace{\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}}_{>0}\underbrace{(\beta\epsilon_{p}-(1+(\beta-1)\xi))}_{>0}>0 \tag{5.149}$$

and

$$\Delta B_{di,p}^{(B2)}(x|\bar{l}=l_{di}^{**})=\underbrace{\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}}_{>0}\underbrace{(\beta\epsilon_{p}-(1+(\beta-1)\xi))}_{>0}>0 \tag{5.150}$$

Since $\bar{l}=l_{di}^{**}$, I have $\Delta L_{di,p}^{(B2)}(x|\bar{l}=l_{di}^{**})=0$. Then, by condition (5.142) and given $\xi>\overline{\xi}_{p}$, the change in the capital structure is always positive:

$$\Delta CS_{di,p}^{(B2)}(x|\bar{l}=l_{di}^{**})>0 \tag{5.151}$$

Now fix a family of parameters $(\mu,\sigma,\rho,\epsilon_{p},\xi)$ securing the upper bound of the domain, i.e. $\bar{l}=l_{p}^{**}$. Making use of condition (5.134) from Proposition (12) and Remark (6), I get:

$$\begin{aligned}\Delta E_{di,p}^{(B2)}(x|\bar{l}=l_{p}^{**})&=\frac{1-\tau}{\rho}\left(l_{di}^{**}-l_{p}^{**}\right)\\&\quad+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_{p}(l_{p}^{**})^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right)\\&=\underbrace{\frac{1-\tau}{\rho}\left(1-\left(\frac{\beta\epsilon_{p}}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}\right)}_{<0}l_{di}^{**}+\\&\quad+\underbrace{\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\left(\left(\frac{\beta\epsilon_{p}}{1+(\beta-1)\xi}\right)^{\frac{1}{\beta}}-1\right)}_{>0}\end{aligned} \tag{5.152}$$

I conjecture that expression (5.152) is negative:

$$\Delta E_{di,p}^{(B2)}(x|\bar{l}=l_{p}^{**})<0\Rightarrow 1-\beta x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}l_{di}^{**}>0$$

and confirm it since, given the domains of the parameters, that is always the case.

Recall condition (5.134) from Proposition (12):

$$\begin{aligned}\Delta B_{di,p}^{(B2)}(x|\bar{l}=l_{p}^{**})&=\frac{\tau}{\rho}\left(l_{p}^{**}-l_{di}^{**}\right)+\frac{1-\tau}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}\left(\beta\epsilon_{p}(l_{p}^{**})^{1-\beta}-(1+(\beta-1)\xi)(l_{di}^{**})^{1-\beta}\right)\\&=\underbrace{\left(\frac{\tau}{\rho}l_{di}^{**}+\frac{(1-\tau)(1+(\beta-1)\xi)}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}(l_{di}^{**})^{1-\beta}\right)}_{>0}\underbrace{\left(\left(\frac{\beta\epsilon_{p}}{1+(\beta-1)\xi}\right)^{-\frac{1}{\beta}}-1\right)}_{>0}<0\end{aligned} \tag{5.153}$$

The change in the bank's value is always positive. In this case, if the bank's optimal leverage binds at the rationality constraint and the rationality constraint is equal (or sufficiently close) to the optimal unconstrained total coupon level l_p^{**}, then the capital structure will always decrease, i.e.

$$\Delta CS_{di,p}^{(B2)}(x|\bar{l} = l_p^{**}) < 0 \tag{5.154}$$

I conclude that, for financial institutions located in the region $\xi \in [\overline{\xi}_p, \overline{\xi}]$, i.e. that went from weak to strong with the PSA's introduction, and are forced to bind at the the rationality constraint can observe an increase or a decrease in the optimal capital structure. The change will be positive if the total coupon ($\bar{l}$) is closer to the DIS equilibrium total coupon (l_{di}^{**}) or negative if the total coupon ($\bar{l}$) is closer to the unconstrained equilibrium total coupon in the PSA case (l_p^{**})

Subcase (B3): $l_{di}^* = l_p^* = \bar{l}$. I now consider that both optimal solutions, before and after PSA's introduction, bind at the rationality constraint. The difference equations for equity and total value are, respectively:

$$\Delta E_{di,p}^{(B3)}(x) = \underbrace{\frac{(1-\tau)(\bar{l})^{1-\beta}}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} \tag{5.155}$$

$$\Delta B_{di,p}^{(B3)}(x) = \underbrace{\frac{(1-\tau)(\bar{l})^{1-\beta}}{\rho(1-\beta)(1-\xi)}\left(x^{-1}\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{-\beta}}_{>0}\underbrace{(\beta\epsilon_p - (1+(\beta-1)\xi))}_{>0} \tag{5.156}$$

Since $l_{di}^* = l_p^* = \bar{l}$, I have $\Delta L_{di,p}^{(B3)}(x) = 0$. Then, by condition (5.142) and given $\xi > \overline{\xi}_p$, the change in the capital structure is always positive:

$$\Delta CS_{di,p}^{(B3)}(x) > 0 \tag{5.157}$$

▷ **Case (C):** $\xi \in [\overline{\xi}, 1]$ **.** In this case, the bank is a strong bank before and after the introduction of the PSA. The bank's value function does not change as in the intermediate case (weak banks become strong banks), but the payoff structure changes and impacts the capital structure. In essence, the decrease in closure costs introduced with the PSA allows for an increase in leverage by the existing trade-off between the tax shield of debt and the closure costs. So, from Proposition (7) and Proposition (11), the relevant equilibrium optimal total leverage coupons are:

$$\begin{aligned} l_{di}^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon)^{\frac{1}{\beta}} \\ l_p^{**} &= \frac{(1-\xi)\rho x_0}{\rho-\mu}\frac{\beta-1}{\beta}\left(\frac{\tau}{\tau-1}\right)^{-\frac{1}{\beta}}(\beta\epsilon_p)^{\frac{1}{\beta}} \\ \bar{l} &= \frac{(1-\xi)\rho x_0}{\rho-\mu} \end{aligned} \tag{5.158}$$

The difference functions are in this case:

$$\Delta E^{(C)}_{di,p}(x) = \frac{1-\tau}{\rho}\left(l^*_{di} - l^*_p\right) - \frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p(l^*_p)^{1-\beta} - \epsilon(l^*_{di})^{1-\beta}\right) \tag{5.159}$$

$$\Delta B^{(C)}_{di,p}(x) = \frac{\tau}{\rho}\left(l^*_p - l^*_{di}\right) - \frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p(l^*_p)^{1-\beta} - \epsilon(l^*_{di})^{1-\beta}\right) \tag{5.160}$$

Subcase (C1): $l^*_{di} = l^{**}_{di}$ and $l^*_p = l^{**}_p$. Replacing with the equilibrium solutions for the total coupons in the difference functions, I have:

$$\Delta E^{(C1)}_{di,p}(x) = \frac{1-\tau}{\rho}\left(1 - \frac{\beta}{\beta-1}\left(\frac{x}{\frac{\beta(\rho-\mu)}{(\beta-1)(1-\xi)\rho}}\right)^\beta\frac{\epsilon(l^{**}_{di})^{-\beta}}{(1-\xi)}\right)\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} - 1\right)l^{**}_{di} \tag{5.161}$$

Now, I postulate that the difference in the equity's value function (5.161) is negative:

$$\Delta E^{(C1)}_{di,p}(x) < 0 \Rightarrow \left(\frac{x}{x_0}\right)^\beta\frac{\tau}{(\beta-1)(1-\tau)} - 1 < 0$$

which always hold for the domains of the parameters.

$$\Delta B^{(C1)}_{di,p}(x) = \left(\frac{\tau}{\rho} - \frac{1-\tau}{\rho}\frac{\beta}{\beta-1}\left(\frac{x}{\frac{\beta(\rho-\mu)}{(\beta-1)(1-\xi)\rho}}\right)^\beta\frac{\epsilon(l^{**}_{di})^{-\beta}}{(1-\xi)}\right)\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\frac{1}{\beta}} - 1\right)l^{**}_{di} \tag{5.162}$$

I conjecture that the difference in the bank's value function (5.162) is positive:

$$\Delta B^{(C1)}_{di,p}(x) < 0 \Rightarrow 1 + \frac{1}{\beta-1}\left(\frac{x}{x_0}\right)^\beta > 0$$

which always hold for the parameters' domains. Then, by condition (5.142) and given $\xi > \overline{\xi}_p$, the change in the capital structure is always negative:

$$\Delta CS^{(C1)}_{di,p}(x) < 0 \tag{5.163}$$

Subcase (C2): $l^*_{di} = l^{**}_{di}$ and $l^*_p = \bar{l}$. Recall functions $E^{(B)}_p(x)$ and $B^{(B)}_p(x)$ and notice that, in this subcase, the optimal total coupon after the introduction of the PSA is restricted to domain $\bar{l} \in [l^{**}_{di}, l^{**}_p]$ by Proposition (12). I study the bounds of the domain of $\bar{l}$ to show that equity value function has a root that yields no change in equity value.

Subcase (C2.1): Lower bound $\bar{l} = l^{**}_{di}$. Fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the new optimal total coupon level binds at the lower bound of the domain, i.e. $\bar{l} = l^{**}_{di}$. I have, in this circumstance, a positive change in the equity claim value because, as assumed, $\epsilon_p < \epsilon$. Specifically:

$$\Delta E^{(C2)}_{di,p}(x|\bar{l} = l^{**}_{di}) = -\frac{1-\tau}{\rho-\mu}x^\beta\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p - \epsilon)\,(l^{**}_{di})^{1-\beta} > 0 \tag{5.164}$$

Similarly, the Bank's Total Value Function observes an increase with the introduction of the PSA:

$$\Delta B_{di,p}^{(C2)}(x|\bar{l}=l_{di}^{**}) = -\frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p-\epsilon)\,(l_{di}^{**})^{1-\beta} > 0 \tag{5.165}$$

In conclusion, considering the lower bound of the domain, i.e $\bar{l} = l_{di}^{**}$. Since I have $\Delta L_{di,p}^{(C2)}(x|\bar{l} = l_{di}^{**}) = 0$, by condition (5.142) and given $\xi > \overline{\xi}$, I find a positive change in capital structure:

$$\Delta CS_{di,p}^{(C2)}(x|\bar{l}=l_{di}^{**}) > 0 \tag{5.166}$$

Subcase (C2.2): Lower bound $\bar{l} = l_p^{**}$. Now fix a family of parameters $(\mu, \sigma, \rho, \epsilon_p, \xi)$ securing the upper bound of the domain, i.e. $\bar{l} = l_p^{**}$. Making use of condition (5.134) from Proposition (12) and Remark (6), I get:

$$\Delta E_{di,p}^{(C2)}(x|\bar{l}=l_p^{**}) = \frac{\tau}{\rho}\left(1-\left(\frac{\epsilon_p}{\epsilon}\right)^{\beta}\right)l_{di}^{**} - \frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\beta}-1\right)\epsilon(l_{di}^{**})^{1-\beta} \tag{5.167}$$

I postulate that this equation is negative and find that:

$$\Delta E_{di,p}^{(C2)}(x) < 0 \Rightarrow \left(\frac{x}{x_0}\right)^{\beta}\frac{\tau}{(\beta-1)(1-\tau)} - 1 < 0$$

which always hold for the domains of the parameters. When the corner solution $\bar{l}$ is sufficiently closer to the interior solution l_p^{**}, the equity claim suffers a decrease in value. That occurs because the value increase of facing lower bankruptcy costs (with the PSA) is superseded by the increase in leverage which decreases the cash-flows available for shareholders and increase the intervention trigger, .i.e. the regulator is expected to close the bank sooner than in the previous case.

$$\Delta B_{di,p}^{(C2)}(x|\bar{l}=l_p^{**}) = \frac{\tau}{\rho}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\beta}-1\right)l_{di}^{**} - \frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\left(\frac{\epsilon_p}{\epsilon}\right)^{\beta}-1\right)\epsilon(l_{di}^{**})^{1-\beta} \tag{5.168}$$

I need to confirm if the first term (positive) can always cover the second term (negative). Hence, I conjecture that the difference in the bank's value function (5.168) is positive:

$$\Delta B_{di,p}^{(C2)}(x|\bar{l}=l_p^{**}) > 0 \Rightarrow 1 + \frac{1}{\beta-1}\left(\frac{x}{x_0}\right)^{\beta} > 0$$

This last expression always hold for the parameters' domains. As a result, by condition (5.142) and given $\xi > \overline{\xi}$, the change in the capital structure is always negative:

$$\Delta CS_{di,p}^{(C2)}(x|\bar{l}=l_p^{**}) < 0 \tag{5.169}$$

Threshold leverage coupon: $\bar{l} = \tilde{l}$. If the optimal leverage coupon $\bar{l}$ changes signs within its domain, by Bolzano's Theorem, there exists a level of total leverage coupon that generates no change in the equity claim's value. I let that level be represented by a tilde, i.e. $\tilde{l}$. With a few standard calculations, I obtain:

$$\Delta E^{(C2)}_{di,p}(x|\bar{l}=\tilde{l})=0 \Rightarrow \tilde{l}^{(C2)} = \left(\frac{\frac{\rho}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}\left(\epsilon_p-\epsilon\left(\left(\frac{\tau}{\tau-1}\frac{1-\xi}{\beta\epsilon}\right)^{-\frac{1}{\beta}}\frac{\beta-1}{\beta}\right)^{1-\beta}\right)}{\left(\frac{\tau}{\tau-1}\frac{1-\xi}{\beta\epsilon}\right)^{-\frac{1}{\beta}}\frac{\beta-1}{\beta}} \right)^{\frac{1}{\beta}} \tag{5.170}$$

Subcase (C3): $l^*_{di} = \bar{l}$ and $l^*_p = \bar{l}$. In this subcase, the bank maintains the corner solution, binding at the rationality constraint maximum leverage coupon. Since $\epsilon_p < \epsilon$ I have, respectively, for equity and total value:

$$\Delta E^{(C3)}_{di,p}(x) = -\frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p-\epsilon)\,(\bar{l})^{1-\beta} > 0 \tag{5.171}$$

$$\Delta B^{(C3)}_{di,p}(x) = -\frac{1-\tau}{\rho-\mu}x^{\beta}\left(\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}\frac{1}{1-\xi}\right)^{1-\beta}(\epsilon_p-\epsilon)\,(\bar{l})^{1-\beta} \tag{5.172}$$

In conclusion, considering the lower bound of the domain, i.e $\bar{l} = l^{**}_{di}$. Since I have $\Delta L^{(C3)}_{di,p}(x) = 0$, by condition (5.142) and given $\xi > \overline{\xi}$, I find a positive change in capital structure:

$$\Delta CS^{(C3)}_{di,p}(x) > 0 \tag{5.173}$$

□

5.3.6 Bail-in Case

Proof of Lemma 10. At the bail-in trigger x^1_{bi}, the participation constraint for shareholders is:

$$(1-\theta)\left(E_{2,1}(x^1_{bi}) + C_{2,1}(x^1_{bi})\right) \geq \max\left\{(1-\tau)\left[(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{c_{bi}+d_{bi}}{\rho}\right], 0\right\} \tag{5.174}$$

and for non-deposit creditors is:

$$\theta\left(E_{2,1}(x^1_{bi}) + C_{2,1}(x^1_{bi})\right) \geq \min\left\{(1-\epsilon)\frac{x^1_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}, \frac{c_{bi}}{\rho}\right\} \tag{5.175}$$

I get condition (4.87) by solving these inequalities for θ and restricting the boundaries to $\theta \in [0,1]$.

□

Proof of Proposition 15. After the regulatory changes, the optimisation problem for Scenario 2 is defined as:

$$
\begin{aligned}
(c^*_{bi}, d^*_{bi}) \quad & \in \underset{\{c_{bi}, d_{bi}\}}{\arg\max}\ B_{bi}(x_0) \\
\text{s.t.} \qquad & x^1_{bi} := \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_p + d_p}{\rho} && \text{[Bail-In Trigger]} \\
& x^2_{bi} := \frac{\beta}{\beta - 1}\frac{\rho - \mu}{1 - \xi}\frac{c_p + d_p}{\rho} && \text{[Closure Trigger]} \\
& c_{bi} + d_{bi} \leq \frac{\rho(1-\xi)}{\rho - \mu}x_0 && \text{[Initial Capital Requirement]} \\
& c_{bi} \geq 0 \quad , \quad d_{bi} \geq 0 && \text{[Non-Negative Coupons]}
\end{aligned}
$$

▷ **Value Function.** I must restrict the problem to the feasible cases. First notice that I have two major cases: Case A when $\xi < \overline{\xi}_p$ and Case B when $\xi \geq \overline{\xi}_p$. In this Scenario the Bank's Value Function will then be:

$$
B_{bi}(x_0) = \begin{cases}
(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^{\beta}\frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^{\beta}\left(\frac{x^2_{bi}}{\rho-\mu} - \frac{d_{bi}}{\rho}\right) & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\
(1-\tau)\frac{x_0}{\rho-\mu} + \tau\frac{c_{bi}+d_{bi}}{\rho} - \tau\left(\frac{x_0}{x^1_{bi}}\right)^{\beta}\frac{c_{bi}}{\rho} - (1-\tau)\left(\frac{x_0}{x^2_{bi}}\right)^{\beta}\left(\epsilon_p\frac{x^2_{bi}}{\rho-\mu}\right) & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]}
\end{cases}
$$

▷ **Constraint Set Feasibility.** Consider the feasible set of the general constrained maximisation problem:

$$\mathcal{G}_{bi} = \left\{(c_{bi}, d_{bi}) \in \mathbb{R}^2 | g(c_{bi}, d_{bi}) \leq 0\right\}$$

with

$$g(c_{bi}, d_{bi}) = \begin{pmatrix} g_1(c_{bi}, d_{bi}) \\ g_2(c_{bi}) \\ g_3(d_{bi}) \end{pmatrix} = \begin{pmatrix} c_{bi} + d_{bi} - \frac{(1-\xi)\rho}{\rho-\mu}x_0 \\ -c_{bi} \\ -d_{bi} \end{pmatrix}$$

Let $Q_{bi} := \frac{(1-\xi)\rho}{\rho-\mu}x_0$ be a family of constants and let (ρ, μ, x_0, ξ) be such that $Q_{bi} > 0$. Then, I secure the feasible set $\mathcal{G}_{bi}$ is compact which, by the Weierstrass Theorem, guarantees the existence of a maximising solution.

▷ **Constraint Qualification.** Consider the Constraint Qualification matrix

$$CQ := \nabla g(c_{bi}, d_{bi}) = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.176}$$

I immediately see that, for any possible combinations of binding constraints, $Rank(CQ) \in \{1, 2\}$. Therefore, I can exclude the case where three constraints are binding.

▷ **Lagrangian.** Form the Lagrangian with Kuhn-Tucker multipliers χ_1 for the NBR

Condition and χ_2 for the Rationality Constraint. For Case A:

$$\begin{aligned}\mathscr{L}(c_{bi},d_{bi})|_{\xi<\overline{\xi}_p} =&(1-\tau)\frac{x_0}{\rho-\mu}+\tau\frac{c_{bi}+d_{bi}}{\rho}-\tau\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}\frac{c_{bi}}{\rho}-(1-\tau)\left(\frac{x_0}{x_{bi}^2}\right)^{\beta}\left(\frac{x_{bi}^2}{\rho-\mu}-\frac{d_{bi}}{\rho}\right)\\&+\chi_1\left(\rho(1-\xi)x_0-(\rho-\mu)(c_{bi}+d_{bi})\right)+\chi_2 c_{bi}+\chi_3 d_{bi}\end{aligned} \tag{5.177}$$

And for Case B:

$$\begin{aligned}\mathscr{L}(c_{bi},d_{bi})|_{\xi\geq\overline{\xi}_p} =&(1-\tau)\frac{x_0}{\rho-\mu}+\tau\frac{c_{bi}+d_{bi}}{\rho}-\tau\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}\frac{c_{bi}}{\rho}-(1-\tau)\left(\frac{x_0}{x_{bi}^2}\right)^{\beta}\left(\epsilon_p\frac{x_{bi}^2}{\rho-\mu}\right)\\&+\chi_1\left(\rho(1-\xi)x_0-(\rho-\mu)(c_{bi}+d_{bi})\right)+\chi_2 c_{bi}+\chi_3 d_{bi}\end{aligned} \tag{5.178}$$

▷ **Kuhn-Tucker Conditions.** For both cases, the complementary slack conditions are:

$$\chi_1\left(\rho(1-\xi)x_0-(\rho-\mu)(c_{bi}+d_{bi})\right)=0 \tag{5.179}$$

$$\chi_2 c_{bi}=0 \tag{5.180}$$

$$\chi_3 d_{bi}=0 \tag{5.181}$$

and Kuhn-Tucker multiplier constraints are:

$$\chi_1\geq 0 \quad , \qquad \chi_2\geq 0 \quad , \qquad \chi_3\geq 0 \tag{5.182}$$

Only solutions satisfying (5.179)-(5.182) are admissible.

▷ **Corner Solutions.** I have six possible candidates to admissible corner solutions. To distinguish them I use, just for the sake of exposure in this proof, the superscript (j). Here, $j\in\{1,...,6\}$ is the solution candidate under study. For example, in the first solution candidate, I need to find $c_{bi}^{(1)}$, $d_{bi}^{(1)}$ and $\chi_1^{(1)}$.

1. $g_1(c_{bi},d_{bi})$ **is binding, i.e.** $(\chi_1>0,\chi_2=\chi_3=0)$ For this solution candidate, the Rationality Constraint is the only constraint binding. I solve for the optimal coupons $(c_{bi}^{(1)},d_{bi}^{(1)})$ and the Kuhn-Tucker multiplier $\chi_1^{(1)}$:

$$c_{bi}^{(1)}=\begin{cases}(1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)x_0 & , \quad \xi<\overline{\xi}_p \qquad \text{[Case A]}\\ (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & , \quad \xi\geq\overline{\xi}_p \qquad \text{[Case B]}\end{cases}$$

$$d_{bi}^{(1)}=\begin{cases}(1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi<\overline{\xi}_p \qquad \text{[Case A]}\\ (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}x_0 & , \quad \xi\geq\overline{\xi}_p \qquad \text{[Case B]}\end{cases}$$

$$\chi_1^{(1)} = \begin{cases} \tau + \tau\left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\beta - 1 - \beta\left(\frac{\tau-1}{\tau}\frac{((\beta-1)\xi+1)}{1-\xi}\right)^{1/\beta}\right) & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\ \tau + \tau\left(\frac{\beta-1}{\beta}\right)^{\beta}\left(\beta - 1 - \beta\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right) & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]} \end{cases}$$

Solution is admissible if and only $\chi_1^{(1)} > 0$, which is guaranteed when:

$$\left(\frac{\tau-1}{\tau}\frac{((\beta-1)\xi+1)}{1-\xi}\right)^{1/\beta} > \frac{\beta-1}{\beta}, \quad \text{for} \quad \xi < \overline{\xi}_{bi} \qquad \text{[Case A]}$$

$$\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} > \frac{\beta-1}{\beta}, \quad \text{for} \quad \xi \geq \overline{\xi}_{bi} \qquad \text{[Case B]}$$

If these conditions are guaranteed, the solution candidate 1 is admissible as a valid solution.

2. $g_2(c_{bi})$ **is binding, i.e.** $(\chi_1 = 0, \chi_2 > 0, \chi_3 = 0)$ In this solution candidate, the Non-Negative Constraint for Non-Deposit Liabilities is the only constraint binding. I solve for the optimal coupons $(c_{bi}^{(2)}, d_{bi}^{(2)})$ and the Kuhn-Tucker multiplier $\chi_2^{(2)}$:

$$c_{bi}^{(2)} = 0$$

$$d_{bi}^{(2)} = \begin{cases} \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\ \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]} \end{cases}$$

$$\chi_2^{(2)} = \begin{cases} \frac{\tau}{\rho}\left(\frac{\tau(1-\xi)}{(\tau-1)((\beta-1)\xi+1)} - 1\right) & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\ \frac{\tau}{\rho}\left(\frac{\tau(1-\xi)}{\epsilon_p\beta(\tau-1)} - 1\right) & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]} \end{cases}$$

Let me restrict to $\xi > \frac{1}{1-\beta}$ (as I will see, this is a useful restriction for the unconstrained problem as well). A solution is admissible if and only $\chi_2^{(2)} > 0$, which is guaranteed when:

$$\xi < \frac{1}{\beta(\tau-1)+1}, \quad \text{for} \quad \xi < \overline{\xi}_p \qquad \text{[Case A]}$$

$$\xi < 1 - \frac{\tau-1}{\tau}\epsilon_p\beta, \quad \text{for} \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]}$$

Then, the solution candidate 2 is a valid solution.

3. $g_3(d_{bi})$ **is binding, i.e.** $(\chi_1 = 0, \chi_2 = 0, \chi_3 > 0)$ In this solution, the Non-Negative Constraint for Deposits is the only constraint binding. I need to solve for the optimal coupons $(c_{bi}^{(3)}, d_{bi}^{(3)})$ and the Kuhn-Tucker multiplier $\chi_3^{(3)}$. However, I find that, for $\xi \in [0,1]$:

$$\chi_2^{(3)} = \frac{\tau}{(\beta-1)\rho} < 0$$

This candidate solution is never an admissible solution.

4. $g_1(c_{bi}, d_{bi})$ **and** $g_2(c_{bi})$ **bind, i.e.** $(\chi_1 > 0, \chi_2 > 0, \chi_3 = 0)$ In this solution, the Rationality Constraint and the Non-Negativity Constraint for Non-Deposit Liabilities are both binding. I solve for the optimal coupons $(c_{bi}^{(4)}, d_{bi}^{(4)})$ and the Kuhn-Tucker multipliers $\chi_1^{(4)}$ and $\chi_2^{(4)}$:

$$c_{bi}^{(4)} = 0$$

$$d_{bi}^{(4)} = \frac{\rho(1-\xi)}{\rho - \mu} x_0$$

$$\chi_1^{(4)} = \begin{cases} \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{1-\tau}{1-\xi}((\beta-1)\xi + 1) + \tau & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\ \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{1-\tau}{1-\xi}\epsilon_p \beta + \tau & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]} \end{cases}$$

$$\chi_2^{(4)} = \begin{cases} \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{\beta\xi(\tau-1)+\xi-1}{(\xi-1)\rho} & , \quad \xi < \overline{\xi}_p \qquad \text{[Case A]} \\ \left(\frac{\beta-1}{\beta}\right)^{\beta} \frac{\epsilon_p\beta(\tau-1)+(\xi-1)\tau}{(\xi-1)\rho} & , \quad \xi \geq \overline{\xi}_p \qquad \text{[Case B]} \end{cases}$$

The solution candidate is admissible if and only if $\chi_1^{(4)} > 0$ and $\chi_2^{(4)} > 0$ for any acceptable range of parameters.

5. $g_1(c_{bi}, d_{bi})$ **and** $g_3(d_{bi})$ **bind, i.e.** $(\chi_1 > 0, \chi_2 = 0, \chi_3 > 0)$ I check if it is possible to bind the Rationality Constraint and the Non-Negative Constraint for Deposits. I solve for the optimal coupons $(c_{bi}^{(5)}, d_{bi}^{(5)})$ and the Kuhn-Tucker multipliers $\chi_1^{(5)}$ and $\chi_3^{(5)}$ but find that:

$$\chi_3^{(5)} = -\frac{\left(\frac{\beta-1}{\beta}\right)^{\beta} \tau}{\rho} < 0$$

The solution candidate is not admissible for $\chi_2^{(5)} < 0$.

6. $g_2(c_{bi})$ **and** $g_3(d_{bi})$ **bind, i.e.** $(\chi_1 = 0, \chi_2 > 0, \chi_3 > 0)$ I check if it is possible to bind the two Non-Negative Constraints. I solve for the optimal coupons $(c_{bi}^{(6)}, d_{bi}^{(6)})$ and the Kuhn-Tucker multipliers $\chi_2^{(6)}$ and $\chi_3^{(6)}$ but find that:

$$\chi_2^{(6)} = -\frac{\tau}{\rho} < 0$$

The solution candidate is not admissible for $\chi_2^{(6)} < 0$.

▷ **Interior Solution** ($\chi_1 = \chi_2 = \chi_3 = 0$) **- First Order Conditions.** Fixing the Kuhn-Tucker multipliers at zero and rearranging, I obtain:

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}d_{bi}}\Big|_{\xi\in[0,\overline{\xi}_p)} = 0 \Rightarrow d_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}x_0$$

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}d_{bi}}\Big|_{\xi\in[\overline{\xi}_p,1]} = 0 \Rightarrow d_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}x_0 \tag{5.183}$$

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}c_{bi}}\Big|_{\xi\in[0,\overline{\xi}_p)} = 0 \Rightarrow c_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0$$

$$\frac{\mathrm{d}B_{bi}(x_0)}{\mathrm{d}c_{bi}}\Big|_{\xi\in[\overline{\xi}_p,1]} = 0 \Rightarrow c_{bi}^{**} = (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right]x_0 \tag{5.184}$$

▷ **Interior Solution - Second Order Conditions.** In this part, I confirm the concavity of $B_{bi}(x_0)$ for the unconstrained situation. Taking second order conditions and rearranging, I guarantee d_{bi}^* and c_{bi}^* are maximising arguments for the firm value function conditional on subset $\xi \in [0, \overline{\xi}_p)$ and restricting to one of the following pairs of necessary and sufficient conditions:

$$|\beta| \geq 1 \wedge \xi < \min\left\{\frac{(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1} - \tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2}}{\tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2} + (\beta-1)(\tau-1)\left(\frac{1}{d_{bi}}\right)^{\beta+1}}, 1 + \frac{\epsilon_p\beta(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{-\beta-2}}{(\beta+1)c_{bi}\tau}\right\}$$

or, instead

$$|\beta| \leq 1 \wedge \xi > \max\left\{\frac{(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1} - \tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2}}{\tau(\beta+1)c_{bi}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{\beta+2} + (\beta-1)(\tau-1)\left(\frac{1}{d_{bi}}\right)^{\beta+1}}, 1 + \frac{\epsilon_p\beta(1-\tau)\left(\frac{1}{d_{bi}}\right)^{\beta+1}\left(\frac{1}{c_{bi}+d_{bi}}\right)^{-\beta-2}}{(\beta+1)c_{bi}\tau}\right\}$$

Then, the SOC for the firm is, for $\xi \in [0, \overline{\xi}_p)$:

$$\frac{\mathrm{d}^2B_{bi}(x_0)}{\mathrm{d}{d_{bi}}^2} = \underbrace{-\frac{\beta}{\rho}}_{>0}\left(\underbrace{\frac{1+(\beta-1)\xi}{d}\frac{1-\tau}{1-\xi}\left(\frac{x_0}{x_{bi}^2}\right)^{\beta} + \frac{\tau c_{bi}}{(c_{bi}+d_{bi})^2}(1+\beta)\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}}_{<0}\right) < 0$$

and for $\xi \in [\overline{\xi}_p, 1]$, the SOC comes as:

$$\frac{\mathrm{d}^2B_{bi}(x_0)}{\mathrm{d}{d_{bi}}^2} = \underbrace{-\frac{\beta}{\rho}}_{>0}\left(\underbrace{\frac{\epsilon_p\beta}{d_{bi}}\frac{1-\tau}{1-\xi}\left(\frac{x_0}{x_{bi}^2}\right)^{\beta} + \frac{\tau c_{bi}}{(c_{bi}+d_{bi})^2}(1+\beta)\left(\frac{x_0}{x_{bi}^1}\right)^{\beta}}_{<0}\right) < 0$$

The SOC for c_{bi} is the same for all $\xi \in [0,1]$:

$$\frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{c_{bi}}^2}\Big|_{\xi\in[0,\overline{\xi}_p)} = \frac{\mathrm{d}^2 B_{bi}(x_0)}{\mathrm{d}{c_{bi}}^2}\Big|_{\xi\in[\overline{\xi}_p,1]} = \underbrace{\frac{\beta\tau}{\rho}}_{<0}\underbrace{\frac{2d_{bi}+(1-\beta)c_{bi}}{(c_{bi}+d_{bi})^2}}_{>0}\underbrace{\left(\frac{x_0}{x^1_{bi}}\right)^{\beta}}_{>0} < 0$$

For negative SOCs for the sufficient stronger pair of condition that heavily simplifies my parameterisation:

$$\xi > \frac{1}{1-\beta} \quad \wedge \quad |\beta| \geq 1 \tag{5.185}$$

This eliminates, however, several cases where the maximisation property of controls (d_{bi}, c_{bi}) would be secured.

▷ **Equilibrium Solution.** By solving the constrained optimisation problem, I can synthesise the main equilibrium result. First, let me denominate the optimal parameters as:

$$\begin{aligned} \overline{c}_{bi} &:= c^{(1)}_{bi} & \overline{d}_{bi} &:= d^{(1)}_{bi} \\ \underline{c}_{bi} &:= c^{(2)}_{bi} & \underline{d}_{bi} &:= d^{(2)}_{bi} \\ \hat{c}_{bi} &:= c^{(4)}_{bi} & \hat{d}_{bi} &:= d^{(4)}_{bi} \end{aligned}$$

and, taking the Rationality Constraint at its binding point, let me write:

$$\overline{l}_{bi} := \frac{(1-\xi)\rho}{\rho-\mu}x_0$$

and also let me write the total periodic coupon for the interior solution as:

$$l^{**}_{bi} = c^{**}_{bi} + d^{**}_{bi}$$

Then, the equilibrium solution (c^*_{bi}, d^*_{bi}) is defined by the periodic coupon c^*_{bi} for non-deposit liabilities and by the periodic coupon d^*_{bi} for deposits as:

$$(c^*_{bi}, d^*_{bi}) = \begin{cases} (\hat{c}_{bi}, \hat{d}_{bi}) & \text{for} \quad l^{**}_{bi} \geq \overline{l}_{bi} \quad \wedge c^{**}_{bi} \leq 0 \\ (\overline{c}_{bi}, \overline{d}_{bi}) & \text{for} \quad l^{**}_{bi} \geq \overline{l}_{bi} \quad \wedge c^{**}_{bi} > 0 \\ (\underline{c}_{bi}, \underline{d}_{bi}) & \text{for} \quad l^{**}_{bi} < \overline{l}_{bi} \quad \wedge c^{**}_{bi} \leq 0 \\ (c^{**}_{bi}, d^{**}_{bi}) & \text{for} \quad l^{**}_{bi} < \overline{l}_{bi} \quad \wedge c^{**}_{bi} > 0 \end{cases} \tag{5.186}$$

where, when the Rationality Constraint and Non-Negative Constraint for Non-Deposit Liabilities bind, I have:

$$\hat{c}_{bi} = 0 \qquad \text{and} \qquad \hat{d}_{bi} = \frac{(1-\xi)\rho}{\rho-\mu}x_0 \tag{5.187}$$

when only the Rationality Constraint binds, I have:

$$\overline{c}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)x_0 & , \quad \xi < \overline{\xi}_p \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta}\right)x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{5.188}$$

$$\overline{d}_{bi} = \begin{cases} (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi < \overline{\xi}_p \\ (1-\xi)\frac{\rho}{\rho-\mu}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases}$$

when only the Non-Negative Constraint for Non-Deposit Liabilities binds:

$$\underline{c}_{bi} = 0 \qquad \text{and} \qquad \underline{d}_{bi} = \begin{cases} \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{(\beta-1)\xi+1}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi < \overline{\xi}_p \\ \frac{\beta-1}{\beta}\frac{\rho}{\rho-\mu}(1-\xi)\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{1/\beta} x_0 & , \quad \xi \geq \overline{\xi}_p \end{cases} \tag{5.189}$$

and, at last, the interior solution (no constraint is binding) yields:

$$d^{**}_{bi} = \begin{cases} (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}} x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}} x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases} \tag{5.190}$$

$$c^{**}_{bi} = \begin{cases} (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right] x_0 & \text{for } \xi < \overline{\xi}_p \\ (1-\xi)\frac{\left[1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{\beta}}}{\frac{\beta}{\beta-1}\frac{\rho-\mu}{\rho}}\left[1-\left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right] x_0 & \text{for } \xi \geq \overline{\xi}_p \end{cases}$$

□

Proof of Proposition 16. Consider two cases concerning each type of bank: Case (A) studies weak banks ($\xi < \overline{\xi}_p$) while Case (B) studies strong banks ($\xi \geq \overline{\xi}_p$).

Remark 7. *A function* $f : \kappa \mapsto \kappa^{-\frac{1}{\beta}}\left(1-\beta\left(1-\kappa^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}}$ *with* $\kappa \in \mathbb{R}_+$ *and* $\beta \in \mathbb{R}_-$ *is always equal or greater than 1:*

$$\lim_{\kappa\downarrow 0} f(\kappa) = +\infty \qquad \lim_{\kappa\uparrow+\infty} f(\kappa) = +\infty$$
$$\lim_{\kappa\downarrow 1} f(\kappa) = 1 \qquad \lim_{\kappa\uparrow 1} f(\kappa) = 1$$

Remark 8. *I restrict to the tuple of parameters* (μ, σ, ρ, ξ) *that secures* $\xi > \frac{1}{1-\beta}$

▷ **Case (A):** $\xi \in [0, \overline{\xi}_p)$**.** I start by studying the interior to interior transition.That is:

$$(1) \quad l^*_p = l^{**}_p \quad \text{and} \quad l^*_{bi} = l^{**}_{bi}$$

Write the ratio between the new equilibrium total debt coupon and the old equilibrium total debt coupon:

$$\frac{l^{**}_{bi}}{l^{**}_p} = \left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{-\frac{1}{\beta}}\left(1-\beta\left(1-\left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}} \tag{5.191}$$

Considering a change of variables $\kappa = \left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{-\frac{1}{\beta}}$ and using the result obtained in Remark 7, I conclude that

$$\frac{l_{bi}^{**}}{l_p^{**}} \geq 1, \qquad \forall \xi < \overline{\xi}_p \tag{5.192}$$

With this result and given that, by the equilibrium total coupon condition: $l_i^* = \min\{l_i^{**}, \bar{l}\}$ for $i \in \{p, bi\}$, I can restrict the rest of the problem to two other scenarios:

$$\begin{aligned} &(2) \quad l_p^{**} < \bar{l} < l_{bi}^{**} \qquad \Rightarrow \qquad l_p^* = l_p^{**} \quad \text{and} \quad l_{bi}^* = \bar{l} \\ &(3) \quad \bar{l} < l_p^{**} < l_{bi}^{**} \qquad \Rightarrow \qquad l_p^* = \bar{l} \quad \text{and} \quad l_{bi}^* = \bar{l} \end{aligned} \tag{5.193}$$

Consider Scenario (2): $l_p^* = l_p^{**}$ and $l_{bi}^* = \bar{l}$. Since $l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}\}$ and given condition (5.192), then it must be the case that

$$\frac{\bar{l}}{l_p^{**}} = \left(1 - \beta\left(1 - \left(\frac{\tau-1}{\tau}\frac{\xi(\beta-1)+1}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right)^{-\frac{1}{\beta}} \geq 1 \tag{5.194}$$

In Scenario (3) the optimal coupon does not change: $l_p^* = \bar{l} \quad \text{and} \quad l_{bi}^* = \bar{l}$.

▷ **Case (B):** $\xi \in [\overline{\xi}_p, 1]$**.** I start by studying the interior to interior solution. That is:

$$(1) \quad l_p^* = l_p^{**} \quad \text{and} \quad l_{bi}^* = l_{bi}^{**}$$

I write the ratio

$$\frac{l_{bi}^{**}}{l_p^{**}} = \left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{-\frac{1}{\beta}}\left(1 - \beta\left(1 - \left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\beta}} \tag{5.195}$$

Considering a change of variables $\kappa = \left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{-\frac{1}{\beta}}$ and using the result obtained in Remark 7, I conclude that

$$\frac{l_{bi}^{**}}{l_p^{**}} \geq 1, \qquad \forall \xi \geq \overline{\xi}_p \tag{5.196}$$

In Scenario (2): $l_p^* = l_p^{**}$ and $l_{bi}^* = \bar{l}$. Since $l_{bi}^* = \min\{l_{bi}^{**}, \bar{l}\}$ and given condition (5.196), then it must be the case that

$$\frac{\bar{l}}{l_p^{**}} = \left(1 - \beta\left(1 - \left(\frac{\tau-1}{\tau}\frac{\epsilon_p\beta}{1-\xi}\right)^{\frac{1}{\beta}}\right)\right)^{-\frac{1}{\beta}} \geq 1 \tag{5.197}$$

In Scenario (3) the optimal coupon does not change: $l_p^* = \bar{l} \quad \text{and} \quad l_{bi}^* = \bar{l}$. In summary, the total debt coupon will never decrease with the introduction of the Bail-in tool:

$$l_{bi}^* \geq l_p^* \tag{5.198}$$

□

Proof of Proposition 17. The debt structure ratio change is:

$$\Delta DS_{p,bi}(x) = \frac{C_{bi}(x|\mathcal{E}_{bi})}{C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi})} - \frac{C_p(x|\mathcal{E}_p)}{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)} = \frac{C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) - C_p(x|\mathcal{E}_p)D_{bi}(\mathcal{E}_{bi})}{(C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi}))\,(C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p))} \tag{5.199}$$

I can restrict the problem to study the difference

$$C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) - C_p(x|\mathcal{E}_p)D_{bi}(\mathcal{E}_{bi}) \tag{5.200}$$

I study the two cases concerning each type of bank: Case (A) is focused on weak banks ($\xi < \overline{\xi}_p$) while Case (B) is focused on strong banks ($\xi \geq \overline{\xi}_p$).

▷ **Case (A): $\xi \in [0, \overline{\xi}_p)$.** Before the bail-in is introduced, the debt structure of the weak bank is entirely constituted with deposits. In detail, $d_p^* = l_p^*$ and $c_p^* = 0$. This implies that the value function of non-deposit liabilities of weak banks is:

$$C_p(x|\mathcal{E}_p, \xi < \overline{\xi}_p) = 0$$

With the bail-in, I always have $d_{bi}^* > 0$ and $c_{bi}^* \geq 0$ which implies $D_{bi}(\mathcal{E}_{bi}) > 0$ and $C_{bi}(x|\mathcal{E}_{bi}) \geq 0$. Hence, from expression (5.200), I conclude:

$$C_{bi}(x|\mathcal{E}_{bi})D_p(\mathcal{E}_p) \geq 0 \Rightarrow \Delta DS_{p,bi}(x) \geq 0$$

▷ **Case (B): $\xi \in [\overline{\xi}_p, 1]$.** In the strong bank case, I can simplify expression (5.200) by substituting the value of non-deposit liabilities in the PSA case $C_p = c_p/\rho$, the value of deposits in the PSA case $D_p = d_p/\rho$ and the value of deposits in the Bail-in case $D_{bi} = d_{bi}/\rho$. So, I have:

$$\frac{d_p^*}{\rho}\left(C_{bi}(x|\mathcal{E}_{bi}) - \frac{c_p^* d_{bi}^*}{\rho d_p^*}\right) \tag{5.201}$$

The debt structure of strong banks is undefined for the PSA case. The strong bank can select any pair (c_p^*, d_p^*) provided it respects the ranges presented in Proposition 11.

Without loss of generality, I write $C_{bi}(x|\mathcal{E}_{bi}) \equiv \overline{C}^* \geq 0$ as a constant under the equilibrium capital structure in the bail-in case. Furthermore, from condition (4.68) I write $d_p^* = l_p^* - c_p^*$. Finally, I replace these identities in condition (5.201) and, use the range defined in condition (4.69) to take limits over $c_p^* \in (0, l_p^*)$:

$$\begin{aligned} \overline{C}^* - \lim_{c_p^* \to 0} \frac{c_p^* d_{bi}^*}{\rho(l_p^* - c_p^*)} = \overline{C}^* \geq 0 \quad &\Rightarrow \Delta DS_{p,bi}(x) \geq 0 \\ \overline{C}^* - \lim_{c_p^* \to l_p^*} \frac{c_p^* d_{bi}^*}{\rho(l_p^* - c_p^*)} = -\infty < 0 \quad &\Rightarrow \Delta DS_{p,bi}(x) < 0 \end{aligned} \tag{5.202}$$

Condition (5.202) shows that, depending on the undefined mix (c_p^*, d_p^*), a strong bank can increase, decrease or maintain its debt structure when it changes towards the unique bail-in

equilibrium coupon mix (c^*_{bi}, d^*_{bi}). Therefore, the change in debt structure of strong banks is undefined when bail-ins are introduced.

□

Proof of Proposition 18. The change in equilibrium bank value after the introduction of the bail-in tool is

$$\Delta B_{p,bi}(x) = B_{bi}(x|\mathcal{E}_{bi}) - B_p(x|\mathcal{E}_p)$$

This is the change in bank's value as a consequence of a regulatory change ($B_p \to B_{bi}$) and a simultaneous change in the capital structure towards the new equilibria ($\mathcal{E}_p \to \mathcal{E}_{bi}$).

The change in equity value with the introduction of a Bail-in is $\Delta E_{p,bi}(x)$, the change in non-deposit liabilities is $\Delta C_{p,bi}(x)$ and the change in deposits is $\Delta D_{p,bi}$. Moreover,

$$\Delta B_{p,bi}(x) = \Delta E_{p,bi}(x) + \Delta C_{p,bi}(x) + \Delta D_{p,bi}(x)$$

Finally, the capital structure ratio change is:

$$\Delta CS_{p,bi}(x) = \frac{E_{bi}(x|\mathcal{E}_{bi})}{B_{bi}(x|\mathcal{E}_{bi})} - \frac{E_p(x|\mathcal{E}_p)}{B_{di}(x|\mathcal{E}_p)} = \frac{E_{bi}(x|\mathcal{E}_{bi})}{E_p(x|\mathcal{E}_p)} - \frac{C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi})}{C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p)} \tag{5.203}$$

This expression can be further simplified. Consider first the PSA case. By Proposition 11, the weak bank only uses deposits, i.e. $d^*_{bi} = l^*_{bi}$. This means that the entire credit claim is protected by the DIS and so the value of the credit claim is not a function of the the closure trigger and the state variable. If, instead, the institution is a strong bank, then it always pays creditors at closure. Hence, the total credit claim also does not change with the closure decisions. For both types of banks under the PSA, I can write:

$$C_p(x|\mathcal{E}_p) + D_p(\mathcal{E}_p) = L_p(\mathcal{E}_p) = \frac{l^*_p}{\rho}, \qquad \forall \xi \in [0,1]$$

Consider now the Bail-in case. From Proposition 15, the weak bank is not restricted to 100% of deposits since, in some cases, having non-deposit liabilities is desirable. Deposits are always protected by the DIS and so and non-deposit liabilities have their claim preserved ex-ante by Assumption 4.88. If the bank is instead a strong bank, then it always pays creditors at closure. Hence, the total credit claim does not change with the closure decisions. Hence, I can write for both types of banks in the Bail-in case:

$$C_{bi}(x|\mathcal{E}_{bi}) + D_{bi}(\mathcal{E}_{bi}) = L_{bi}(\mathcal{E}_{bi}) = \frac{l^*_{bi}}{\rho}, \qquad \forall \xi \in [0,1]$$

I simplify condition (5.203):

$$\Delta CS_{p,bi}(x) = \frac{E_{bi}(x|\mathcal{E}_{bi})}{E_p(x|\mathcal{E}_p)} - \frac{l^*_{bi}}{l^*_p} = \frac{1}{E_p(x|\mathcal{E}_p)} \left(E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p) \frac{l^*_{bi}}{l^*_p} \right) \tag{5.204}$$

and restrict the problem to the study of expression:

$$E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} \tag{5.205}$$

From Proposition 16, I have three possible equilibrium transitions:

$$\begin{aligned}
&(1) \quad l^{**}_p < l^{**}_{bi} < \bar{l} \qquad \Rightarrow \qquad l^*_p = l^{**}_p \quad \text{and} \quad l^*_{bi} = l^{**}_{bi} \\
&(2) \quad l^{**}_p < \bar{l} < l^{**}_{bi} \qquad \Rightarrow \qquad l^*_{di} = l^{**}_{di} \quad \text{and} \quad l^*_p = \bar{l} \\
&(3) \quad \bar{l} < l^{**}_{di} < l^{**}_p \qquad \Rightarrow \qquad l^*_{di} = \bar{l} \quad \text{and} \quad l^*_p = \bar{l}
\end{aligned} \tag{5.206}$$

▷ **Case (A):** $\xi \in [0, \overline{\xi}_p)$ **.** I write the parameters κ_1 and $\overline{\kappa}_1$ as

$$\kappa_1 := \frac{\tau - 1}{\tau}\frac{\xi(\beta - 1) + 1}{1 - \xi} \tag{5.207}$$

and

$$\overline{\kappa}_1 := \max\{1, \kappa_1\} \tag{5.208}$$

From Propositions 11 and 15, I can write the following relationships between optimal coupons in the bail-in and the PSA cases regardless of having an interior or corner solution:

$$\frac{l^*_{bi}}{l^*_p} = \left(\frac{1 - \beta(1 - \overline{\kappa}_1^{\frac{1}{\beta}})}{\overline{\kappa}_1}\right)^{\frac{1}{\beta}} \qquad \in [1, +\infty) \tag{5.209}$$

$$\frac{d^*_{bi}}{l^*_p} = \left(1 - \beta(1 - \overline{\kappa}_1^{\frac{1}{\beta}})\right)^{\frac{1}{\beta}} \qquad \in [0, 1] \tag{5.210}$$

$$\frac{c^*_{bi}}{l^*_p} = \left(1 - \beta(1 - \overline{\kappa}_1^{\frac{1}{\beta}})\right)\left(\frac{1 - \beta(1 - \overline{\kappa}_1^{\frac{1}{\beta}})}{\overline{\kappa}_1}\right)^{\frac{1}{\beta}} \qquad \in [0, +\infty) \tag{5.211}$$

$$\frac{d^*_{bi}}{l^*_{bi}} = \overline{\kappa}_1^{\frac{1}{\beta}} \qquad \in [0, 1] \tag{5.212}$$

$$\frac{c^*_{bi}}{l^*_{bi}} = 1 - \overline{\kappa}_1^{\frac{1}{\beta}} \qquad \in [0, 1] \tag{5.213}$$

Condition (5.205) in the weak banks' case is:

$$\begin{aligned}
E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} =& \frac{1 - \tau}{\rho - \mu}x_0\left(1 - \frac{l^*_{bi}}{l^*_p}\right) - \left(\frac{x}{x^1_{bi}}\right)^{\beta}\tau\frac{c_{bi}}{\rho} \\
&- \left(\frac{x}{x^2_{bi}}\right)^{\beta}(1 - \tau)\left(\frac{x^2_{bi}}{\rho - \mu} - \frac{d_{bi}}{\rho}\right) + \left(\frac{x}{x_p}\right)^{\beta}(1 - \tau)\left(\frac{x^1_{bi}}{\rho - \mu} - \frac{l_{bi}}{\rho}\right)
\end{aligned} \tag{5.214}$$

which can be simplified using conditions (5.209) to (5.213):

$$\begin{aligned} &E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} = \\ &= \underbrace{\frac{1-\tau}{\rho-\mu}x_0\left(1-\frac{l^*_{bi}}{l^*_p}\right)}_{\leq 0} - \underbrace{\frac{1-\tau}{\rho}\left(\frac{\beta}{\beta-1}\frac{1}{1-\xi}-1\right)\left(\frac{x}{x^2_{bi}}\right)^{\beta}(\beta-1)(1-\overline{\kappa}_1^{\frac{1}{\beta}})\left(\frac{\overline{\kappa}_1}{\kappa_1}-1\right)}_{=0} \leq 0 \end{aligned} \tag{5.215}$$

By condition (5.204), we have:

$$\Delta CS_{p,bi}(x|\xi < \overline{\xi}_p) \leq 0 \tag{5.216}$$

▷ **Case (B):** $\xi \in [\overline{\xi}_p, 1]$ **.** I write the parameters κ_2 and $\overline{\kappa}_2$ as

$$\kappa_2 := \frac{\tau-1}{\tau}\frac{(\beta-1)\epsilon_p}{1-\xi} \tag{5.217}$$

and

$$\overline{\kappa}_2 := \max\{1, \kappa_2\} \tag{5.218}$$

Using a similar approach as in case (A) but now for strong banks, I get:

$$E_{bi}(x|\mathcal{E}_{bi}) - E_p(x|\mathcal{E}_p)\frac{l^*_{bi}}{l^*_p} = \underbrace{\frac{1-\tau}{\rho-\mu}x_0\left(1-\frac{l^*_{bi}}{l^*_p}\right)}_{\leq 0} \leq 0 \tag{5.219}$$

Recalling condition (5.204), this implies:

$$\Delta CS_{p,bi}(x|\xi \geq \overline{\xi}_p) \leq 0 \tag{5.220}$$

□

Bibliography

Acharya, V. V., Bharath, S. T. and Srinivasan, A.: 2007, Does industry-wide distress affect defaulted firms? evidence from creditor recoveries, *Journal of Financial Economics* **85**(3), 787 – 821.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X07000645*

Adkins, R. and Paxson, D.: 2011, Renewing assets with uncertain revenues and operating costs, *The Journal of Financial and Quantitative Analysis* **46**(3), 785–813.
URL: *http://www.jstor.org/stable/23018548*

Admati, A. R., DeMarzo, P. M., Hellwig, M. F. and Pfleiderer, P. C.: 2013, Fallacies, irrelevant facts, and myths in the discussion of capital regulation: Why bank equity is not socially expensive, *Stanford University Graduate School of Business Research Paper* (13-7).
URL: *https://ssrn.com/abstract=2349739*

Agrawal, A. and Cooper, T.: 2017, Corporate governance consequences of accounting scandals: Evidence from top management, cfo and auditor turnover, *Quarterly Journal of Finance* **07**(01), 1650014.
URL: *https://doi.org/10.1142/S2010139216500142*

Allen, F., Carletti, E., Goldstein, I. and Leonello, A.: 2018, Government guarantees and financial stability, *Journal of Economic Theory* **177**, 518 – 557.
URL: *http://www.sciencedirect.com/science/article/pii/S0022053118303235*

Allen, F., Carletti, E. and Leonello, A.: 2011, Deposit insurance and risk taking, *Oxford Review of Economic Policy* **27**(3), 464–478.
URL: *http://www.jstor.org/stable/23607112*

Almeida, H. and Philippon, T.: 2007, The risk-adjusted cost of financial distress, *The Journal of Finance* **62**(6), 2557–2586.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2007.01286.x*

Alok, S. and Gopalan, R.: 2018, Managerial compensation in multidivision firms, *Management Science* **64**(6), 2856–2874.
URL: *https://doi.org/10.1287/mnsc.2016.2672*

Andrade, G. and Kaplan, S. N.: 1998, How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed, *The Journal of Finance* **53**(5), 1443–1493.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/0022-1082.00062*

Ashcraft, A. B.: 2008, Does the market discipline banks? new evidence from regulatory capital mix, *Journal of Financial Intermediation* **17**(4), 543 – 561.
URL: *http://www.sciencedirect.com/science/article/pii/S104295730700037X*

Asquith, P., Gertner, R. and Scharfstein, D.: 1994, Anatomy of financial distress: An examination of junk-bond issuers, *The Quarterly Journal of Economics* **109**(3), 625–658.
URL: *http://dx.doi.org/10.2307/2118416*

Avgouleas, E. and Goodhart, C.: 2014, A critical evaluation of bail-in as a bank recapitalisation mechanism, *CEPR Discussion Paper* (DP10065).
URL: *https://ssrn.com/abstract=2501539*

Azevedo, A. and Paxson, D.: 2014, Developing real option game models, *European Journal of Operational Research* **237**(3), 909 – 920.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221714001179*

Baily, M., Bernard, A., Campbell, J., Diamond, D., Duffie, D., French, K., Kashyap, A., Mishkin, F., Rajan, R., Scharfstein, D., Shiller, R., Shin, H., Slaughter, M. and Stulz, R.: 2009, An expedited resolution mechanism for distressed financial firms: Regulatory hybrid securities, *Squam Lake Working Group on Financial Regulation Working Paper* .
URL: *https://www.cfr.org/report/expedited-resolution-mechanism-distressed-financial-firms*

Baranchuk, N., MacDonald, G. and Yang, J.: 2011, The economics of super managers, *Review of Financial Studies* **24**(10), 3321–3368.
URL: *https://doi.org/10.1093/rfs/hhr059*

Barro, J. R. and Barro, R. J.: 1990, Pay, performance, and turnover of bank CEOs, *Journal of Labor Economics* **8**(4), 448–481.
URL: *http://www.jstor.org/stable/2535267*

Bebchuk, L. and Fried, J.: 2005, Pay without performance: Overview of the issues, *Journal of Applied Corporate Finance* **17**(4), 8–23.
URL: *http://dx.doi.org/10.1111/j.1745-6622.2005.00056.x*

Becker, B.: 2006, Wealth and executive compensation, *The Journal of Finance* **61**(1), 379–397.
URL: *http://www.jstor.org/stable/3699344*

Bishara, N., Martin, K. and Thomas, R.: 2015, An empirical analysis of noncompetition clauses and other restrictive postemployment covenants, *Vanderbilt Law Review* **68**(1), 1–51.
URL: *http://www.vanderbiltlawreview.org/2015/01/an-empirical-analysis-of-noncompetition-clauses-and-other-restrictive-postemployment-covenants/*

Black, F. and Cox, J. C.: 1976, Valuing corporate securities: Some effects of bond indenture provisions, *The Journal of Finance* **31**(2), 351–367.
URL: *http://dx.doi.org/10.1111/j.1540-6261.1976.tb01891.x*

Black, F. and Scholes, M.: 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* **81**(3), 637–654.
URL: *http://www.jstor.org/stable/1831029*

Bolton, P. and Samama, F.: 2012, Capital access bonds: contingent capital with an option to convert, *Economic Policy* **27**(70), 275–317.
URL: *http://dx.doi.org/10.1111/j.1468-0327.2012.00284.x*

Bouvard, M.: 2012, Real option financing under asymmetric information, *Review of Financial Studies* .
URL: *http://rfs.oxfordjournals.org/content/early/2012/06/12/rfs.hhs068.abstract*

Bradley, M., Jarrell, G. A. and Kim, E. H.: 1984, On the existence of an optimal capital structure: Theory and evidence, *The Journal of Finance* **39**(3), 857–878.
URL: *http://www.jstor.org/stable/2327950*

Breeden, D. T.: 1984, Futures markets and commodity options: Hedging and optimality in incomplete markets, *Journal of Economic Theory* **32**(2), 275 – 300.
URL: *http://www.sciencedirect.com/science/article/pii/0022053184900553*

Brennan, M. and Schwartz, E.: 1984, Optimal financial policy and firm valuation, *The Journal of Finance* **39**(3), 593–607.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1984.tb03647.x*

Brennan, M. and Schwartz, E.: 1985, Evaluating natural resource investments, *Journal of Business* **58**(2), 135–157.
URL: *http://www.jstor.org/stable/2352967*

Brickley, J. A.: 2003, Empirical research on CEO turnover and firm-performance: a discussion, *Journal of Accounting and Economics* **36**(13), 227 – 233.
URL: *http://www.sciencedirect.com/science/article/pii/S0165410103000570*

Bris, A., Welch, I. and Zhu, N.: 2006, The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization, *The Journal of Finance* **61**(3), 1253–1303.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2006.00872.x*

Calomiris, C. W. and Jaremski, M.: 2016, Deposit insurance: Theories and facts, *Annual Review of Financial Economics* **8**(1), 97–120.
URL: *https://doi.org/10.1146/annurev-financial-111914-041923*

Cardoso, D. and Pereira, P. J.: 2015, A compensation scheme for optimal investment decisions, *Finance Research Letters* **14**, 150 – 159.
URL: *http://www.sciencedirect.com/science/article/pii/S1544612315000483*

Chen, N., Glasserman, P., Nouri, B. and Pelger, M.: 2017, Contingent capital, tail risk, and debt-induced collapse, *The Review of Financial Studies* **30**(11), 3921–3969.
URL: *http://dx.doi.org/10.1093/rfs/hhx067*

Cheng, I.-H. and Milbradt, K.: 2012, The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts, *The Review of Financial Studies* **25**(4), 1070–1110.
URL: *https://doi.org/10.1093/rfs/hhr142*

Chennells, L. and Wingfield, V.: 2015, Bank failure and bail-in: an introduction, *Bank of England Quarterly Bulletin* **55**(3), 228–41.
URL: *https://ssrn.com/abstract=2667477*

Chernykh, L. and Cole, R. A.: 2011, Does deposit insurance improve financial intermediation? evidence from the russian experiment, *Journal of Banking & Finance* **35**(2), 388 – 402.
URL: *http://www.sciencedirect.com/science/article/pii/S0378426610003237*

Choi, A. H. and Spier, K. E.: 2018, Taking a financial position in your opponent in litigation, *American Economic Review* **108**(12), 3626–50.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.20161863*

Cortazar, G., Schwartz, E. S. and Salinas, M.: 1998, Evaluating environmental investments: A real options approach, *Management Science* **44**(8), 1059–1070.
URL: *https://pubsonline.informs.org/doi/abs/10.1287/mnsc.44.8.1059*

Custódio, C., Ferreira, M. A. and Matos, P.: 2013, Generalists versus specialists: Lifetime work experience and chief executive officer pay, *Journal of Financial Economics* **108**(2), 471–492.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X13000020*

Davydenko, S. A., Strebulaev, I. A. and Zhao, X.: 2012, A market-based study of the cost of default, *The Review of Financial Studies* **25**(10), 2959–2999.
URL: *http://dx.doi.org/10.1093/rfs/hhs091*

de Mooij, R. and Gaetan, N.: 2015, *Taxation and Regulation of the Financial Sector*, MIT Press, MIT Press.
URL: *http://dx.doi.org/10.7551/mitpress/9780262027977.001.0001*

de Mooij, R. and Keen, M.: 2016, Debt, taxes, and banks, *Journal of Money, Credit and Banking* **48**(1), 5–33.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/jmcb.12289*

de Vries, M.: 1988, The dark side of CEO succession, *Harvard Business Review* .
URL: *https://hbr.org/1988/01/the-dark-side-of-ceo-succession*

Demarzo, P. M. and Sannikov, Y.: 2006, Optimal security design and dynamic capital structure in a continuous-time agency model, *The Journal of Finance* **61**(6), 2681–2724.
URL: *http://www.jstor.org/stable/4123442*

Demarzo, P. M. and Sannikov, Y.: 2017, Learning, termination, and payout policy in dynamic incentive contracts, *The Review of Economic Studies* **84**(1), 182–236.
URL: *http://dx.doi.org/10.1093/restud/rdw029*

Demirguc-Kunt, A. and Detragiache, E.: 2002, Does deposit insurance increase banking system stability? an empirical investigation, *Journal of Monetary Economics* **49**(7), 1373 – 1406.
URL: *http://www.sciencedirect.com/science/article/pii/S030439320200171X*

Demirguc-Kunt, A., Kane, E. J. and Laeven, L.: 2008, Determinants of deposit-insurance adoption and design, *Journal of Financial Intermediation* **17**(3), 407 – 438.
URL: *http://www.sciencedirect.com/science/article/pii/S1042957308000107*

Detragiache, E., Tressel, T. and Turk-Ariss, R.: 2018, Where have all the profits gone? european bank profitability over the financial cycle, *IMF Working Papers* (18/99).
URL: *https://www.imf.org/en/Publications/WP/Issues/2018/05/09/Where-Have-All-the-Profits-Gone-European-Bank-Profitability-Over-the-Financial-Cycle-45865*

Dewatripont, M.: 2014, European banking: Bailout, bail-in and state aid control, *International Journal of Industrial Organization* **34**, 37 – 43.
URL: *http://www.sciencedirect.com/science/article/pii/S0167718714000289*

Diamond, D. W. and Dybvig, P. H.: 1983, Bank runs, deposit insurance, and liquidity, *Journal of Political Economy* **91**(3), 401–419.
URL: *http://dx.doi.org/10.1086/261155*

Dickson, J.: 2010, Too-big-to-fail and embedded contingent capital, *The Office of the Superintendent of Financial Institutions, Canada* .
URL: *http://www.osfi-bsif.gc.ca/Eng/Docs/jdlh20100506.pdf*

Dixit, A.: 1992, Investment and hysteresis, *Journal of Economic Perspectives* **6**(1), 107–132.
URL: *https://www.aeaweb.org/articles?id=10.1257/jep.6.1.107*

Dixit, A. and Pindyck, R.: 1994, *Investment Under Uncertainty*, Princeton University Press, New Jersey.
URL: *http://www.jstor.org/stable/j.ctt7sncv*

Dubis, J. W.: 2015, Inter partes review: A multi-method comparison for challenging patent validity, *Cybaris Intell. Prop. L. Rev.* **6**(2), 107.
URL: *https://open.mitchellhamline.edu/cybaris/vol6/iss2/4*

Edmans, A. and Gabaix, X.: 2011, The effect of risk on the CEO market, *The Review of Financial Studies* **24**(8), 2822–2863.
URL: *http://dx.doi.org/10.1093/rfs/hhq153*

Edmans, A. and Gabaix, X.: 2016, Executive compensation: A modern primer, *Journal of Economic Literature* **54**(4), 1232–1287.
URL: *http://pubs.aeaweb.org/doi/10.1257/jel.20161153*

Edmans, A., Gabaix, X. and Landier, A.: 2009, A multiplicative model of optimal CEO incentives in market equilibrium, *The Review of Financial Studies* **22**(12), 4881–4917.
URL: *http://dx.doi.org/10.1093/rfs/hhn117*

Eisfeldt, A. L. and Kuhnen, C. M.: 2013, CEO turnover in a competitive assignment framework, *Journal of Financial Economics* **109**(2), 351–372.
URL: *http://dx.doi.org/10.1016/j.jfineco.2013.02.020*

Falato, A., Li, D. and Milbourn, T.: 2015, Which skills matter in the market for CEOs? evidence from pay for CEO credentials, *Management Science* **61**(12), 2845–2869.
URL: *https://doi.org/10.1287/mnsc.2014.2024*

Fama, E. F.: 1980, Agency problems and the theory of the firm, *Journal of Political Economy* **88**(2), 288–307.
URL: *http://www.jstor.org/stable/1837292*

Fischer, E. O., Heinkel, R. and Zechner, J.: 1989, Dynamic capital structure choice: Theory and tests, *The Journal of Finance* **44**(1), 19–40.
URL: *http://www.jstor.org/stable/2328273*

Flannery, M. J. and Rangan, K. P.: 2008, What caused the bank capital build-up of the 1990s?, *Review of Finance* **12**(2), 391–429.
URL: *http://dx.doi.org/10.1093/rof/rfm007*

Frakes, M. D. and Wasserman, M. F.: 2017a, Decreasing the patent office's incentives to grant invalid patents.
URL: *https://www.brookings.edu/research/decreasing-the-patent-offices-incentives-to-grant-invalid-patents/*

Frakes, M. D. and Wasserman, M. F.: 2017b, Is the time allocated to review patent applications inducing examiners to grant invalid patents? Evidence from microlevel application data, *The Review of Economics and Statistics* **99**(3), 550–563.
URL: *https://doi.org/10.1162/REST_a_00605*

Frank, M. Z. and Goyal, V. K.: 2008, Chapter 12 - Trade-off and pecking order theories of debt, *in* B. E. Eckbo (ed.), *Handbook of Empirical Corporate Finance*, Handbooks in Finance, Elsevier, San Diego, pp. 135 – 202.
URL: *http://www.sciencedirect.com/science/article/pii/B9780444532657500044*

Frydman, C. and Jenter, D.: 2010, CEO Compensation, *Annu. Rev. Fin. Econ.* **2**(1), 75–102.
URL: *http://dx.doi.org/10.1146/annurev-financial-120209-133958*

Fryer, B.: 2009, Time for an end run?, *Harvard Business Review* .
URL: *https://hbr.org/2009/11/time-for-an-end-run*

Gabaix, X.: 2016, Power laws in economics: An introduction, *Journal of Economic Perspectives* **30**(1), 185–205.
URL: *http://www.jstor.org/stable/43710016*

Gabaix, X. and Landier, A.: 2008, Why has CEO pay increased so much?, *The Quarterly Journal of Economics* **123**(1), 49–100.
URL: *http://dx.doi.org/10.1162/qjec.2008.123.1.49*

Gabaix, X., Landier, A. and Sauvagnat, J.: 2014, CEO pay and firm size: An update after the crisis, *The Economic Journal* **124**(574), F40–F59.
URL: *http://dx.doi.org/10.1111/ecoj.12084*

Gambacorta, L., Ricotti, G., Sundaresan, S. and Wang, Z.: 2017, The effects of tax on bank liability structure, *BIS Working Paper* (611).
URL: *https://ssrn.com/abstract=2923758*

Gao, H.: 2010, Optimal compensation contracts when managers can hedge, *Journal of Financial Economics* **97**(2), 218 – 238.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X1000070X*

Garlappi, L.: 2004, Risk premia and preemption in R&D ventures, *The Journal of Financial and Quantitative Analysis* **39**(4), 843–872.
URL: *http://www.jstor.org/stable/30031887*

Garmaise, M. J.: 2009, Ties that truly bind: Noncompetition agreements, executive compensation, and firm investment, *The Journal of Law, Economics, and Organization* **27**(2), 376–425.
URL: *https://doi.org/10.1093/jleo/ewp033*

Gertner, R. and Scharfstein, D.: 1991, A theory of workouts and the effects of reorganization law, *The Journal of Finance* **46**(4), 1189–1222.
URL: *http://www.jstor.org/stable/2328856*

Gibbons, R. and Murphy, K. J.: 1990, Relative performance evaluation for chief executive officers, *ILR Review* **43**(3), 30–S–51–S.
URL: *http://dx.doi.org/10.1177/001979399004300303*

Glasserman, P. and Nouri, B.: 2012, Contingent capital with a capital-ratio trigger, *Management Science* **58**(10), 1816–1833.
URL: *https://doi.org/10.1287/mnsc.1120.1520*

Glasserman, P. and Wang, Z.: 2011, Valuing the treasury's capital assistance program, *Management Science* **57**(7), 1195–1211.
URL: *https://doi.org/10.1287/mnsc.1110.1351*

Glover, B.: 2016, The expected cost of default, *Journal of Financial Economics* **119**(2), 284 – 299.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X15001749*

Goldstein, I. and Pauzner, A.: 2005, Demand-deposit contracts and the probability of bank runs, *The Journal of Finance* **60**(3), 1293–1327.
URL: *http://dx.doi.org/10.1111/j.1540-6261.2005.00762.x*

Graham, J. R. and Leary, M. T.: 2011, A review of empirical capital structure research and directions for the future, *Annual Review of Financial Economics* **3**(1), 309–345.
URL: *https://doi.org/10.1146/annurev-financial-102710-144821*

Graham, J. R., Leary, M. T. and Roberts, M. R.: 2015, A century of capital structure: The leveraging of corporate america, *Journal of Financial Economics* **118**(3), 658 – 683. NBER Symposium on New perspectives on corporate capital structures.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X14001809*

Grenadier, S. R. and Malenko, A.: 2011, Real options signaling games with applications to corporate finance, *Review of Financial Studies* **24**(12), 3993–4036.
URL: *http://rfs.oxfordjournals.org/content/24/12/3993.abstract*

Grenadier, S. R., Malenko, A. and Malenko, N.: 2016, Timing decisions in organizations: Communication and authority in a dynamic environment, *American Economic Review* **106**(9), 2552–81.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.20150416*

Grenadier, S. R. and Wang, N.: 2005, Investment timing, agency, and information, *Journal of Financial Economics* **75**(3), 493 – 533.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X04001540*

Gries, T., Prior, U. and Sureth, C.: 2012, A tax paradox for investment decisions under uncertainty, *Journal of Public Economic Theory* **14**(3), 521–545.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9779.2012.01547.x*

Gropp, R. and Heider, F.: 2010, The determinants of bank capital structure, *Review of Finance* **14**(4), 587–622.
URL: *http://dx.doi.org/10.1093/rof/rfp030*

Gryglewicz, S. and Hartman-Glaser, B.: 2014, Dynamic agency and real options, *Working Paper presented at the USC-UCLA-UCI Finance Day* .
URL: *https://financetheory.org/wp-content/uploads/2016/08/SS01.Hartman-Glaser.pdf*

Gryglewicz, S., Hartman-Glaser, B. and Zheng, G.: 2020, Growth options, incentives, and pay for performance: Theory and evidence, *Management Science* **66**(3), 1248–1277.
URL: *https://doi.org/10.1287/mnsc.2018.3267*

Hackbarth, D., Miao, J. and Morellec, E.: 2006, Capital structure, credit risk, and macroeconomic conditions, *Journal of Financial Economics* **82**(3), 519 – 550.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X06001115*

Hamori, M. and Koyuncu, B.: 2015, Experience matters? the impact of prior CEO experience on firm performance, *Human Resource Management* **54**(1), 23–44.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1002/hrm.21617*

Hanlon, M. and Heitzman, S.: 2010, A review of tax research, *Journal of Accounting and Economics* **50**(2), 127 – 178.
URL: *http://www.sciencedirect.com/science/article/pii/S0165410110000340*

He, Z.: 2009, Optimal executive compensation when firm size follows geometric brownian motion, *Review of Financial Studies* **22**(2), 859–892.
URL: *https://doi.org/10.1093/rfs/hhn037*

He, Z.: 2011, A model of dynamic compensation and capital structure, *Journal of Financial Economics* **100**(2), 351 – 366.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X11000183*

He, Z.: 2012, Dynamic compensation contracts with private savings, *The Review of Financial Studies* **25**(5), 1494–1549.
URL: *http://dx.doi.org/10.1093/rfs/hhs054*

He, Z., Li, S., Wei, B. and Yu, J.: 2014, Uncertainty, risk, and incentives: Theory and evidence, *Management Science* **60**(1), 206–226.
URL: *https://doi.org/10.1287/mnsc.2013.1744*

He, Z. and Milbradt, K.: 2016, Dynamic debt maturity, *The Review of Financial Studies* **29**(10), 2677–2736.
URL: *https://doi.org/10.1093/rfs/hhw039*

He, Z., Wei, B., Yu, J. and Gao, F.: 2017, Optimal long-term contracting with learning, *The Review of Financial Studies* **30**(6), 2006–2065.
URL: *http://dx.doi.org/10.1093/rfs/hhx007*

Heider, F. and Ljungqvist, A.: 2015, As certain as debt and taxes: Estimating the tax sensitivity of leverage from state tax changes, *Journal of Financial Economics* **118**(3), 684 – 712.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X15000057*

Hemmelgarn, T. and Teichmann, D.: 2014, Tax reforms and the capital structure of banks, *International Tax and Public Finance* **21**(4), 645–693.
URL: *https://doi.org/10.1007/s10797-014-9321-4*

Hennessy, C. and Whited, T.: 2005, Debt dynamics, *The Journal of Finance* **60**(3), 1129–1165.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2005.00758.x*

Holmstrom, B. and Milgrom, P.: 1987, Aggregation and linearity in the provision of intertemporal incentives, *Econometrica* **55**(2), 303–328.
URL: *http://www.jstor.org/stable/1913238*

Horvarth, B.: 2013, The impact of taxation on bank leverage and asset risk, *CentER Discussion Paper Series* **2013**(076).
URL: *https://ssrn.com/abstract=2366359*

Hotchkiss, E. S., John, K., Mooradian, R. M. and Thorburn, K. S.: 2008, Chapter 14 - bankruptcy and the resolution of financial distress, *in* B. E. Eckbo (ed.), *Handbook of Empirical Corporate Finance*, Handbooks in Finance, Elsevier, San Diego, pp. 235 – 287.
URL: *http://www.sciencedirect.com/science/article/pii/B9780444532657500068*

Hsu, Y.-W. and Lambrecht, B.: 2007, Preemptive patenting under uncertainty and asymmetric information, *Annals oOperational Research* (151), 5–28.
URL: *https://link.springer.com/article/10.1007/s10479-006-0125-5*

Hugonnier, J. and Morellec, E.: 2017, Bank capital, liquid reserves, and insolvency risk, *Journal of Financial Economics* **125**(2), 266 – 285.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X17300971*

Insley, M.: 2002, A real options approach to the valuation of a forestry investment, *Journal of Environmental Economics and Management* **44**(3), 471 – 492.
URL: *http://www.sciencedirect.com/science/article/pii/S0095069601912094*

Iyer, R. and Puri, M.: 2012, Understanding bank runs: The importance of depositor-bank relationships and networks, *The American Economic Review* **102**(4), 1414–1445.
URL: *http://www.jstor.org/stable/23245460*

Jacklin, C. J. and Bhattacharya, S.: 1988, Distinguishing panics and information-based bank runs: Welfare and policy implications, *Journal of Political Economy* **96**(3), 568–592.
URL: *http://www.jstor.org/stable/1830360*

Jacobs, B.: 2007, Real options and human capital investment, *Labour Economics* **14**(6), 913 – 925.
URL: *http://www.sciencedirect.com/science/article/pii/S0927537107000474*

James, C.: 1991, The losses realized in bank failures, *The Journal of Finance* **46**(4), 1223–1242.
URL: *http://www.jstor.org/stable/2328857*

Jenter, D. and Kanaan, F.: 2015, CEO turnover and relative performance evaluation, *The Journal of Finance* **70**(5), 2155–2184.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12282*

Jenter, D. and Lewellen, K.: 2020, Performance-induced CEO turnover, *The Review of Financial Studies* .
URL: *https://doi.org/10.1093/rfs/hhaa069*

Jeon, H.: 2015, Patent infringement, litigation, and settlement, *Economic Modelling* **51**, 99 – 111.
URL: *http://www.sciencedirect.com/science/article/pii/S0264999315002023*

Kaplan, S., Klebanov, M. and Sorensen, M.: 2012, Which CEO characteristics and abilities matter?, *The Journal of Finance* **LXVII**(3), 973–1007.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/j.1540-6261.2012.01739.x/full*

Kaplan, S. N. and Minton, B. A.: 2012, How Has CEO Turnover Changed?, *International Review of Finance* **12**(1), 57–87.

Kaplan, S. N. and Rauh, J.: 2010, Wall street and main street: What contributes to the rise in the highest incomes?, *The Review of Financial Studies* **23**(3), 1004–1050.
URL: *http://dx.doi.org/10.1093/rfs/hhp006*

Kaplan, S. N. and Rauh, J.: 2013, It's the market: The broad-based rise in the return to top talent, *Journal of Economic Perspectives* **27**(3), 35–56.
URL: *http://www.aeaweb.org/articles?id=10.1257/jep.27.3.35*

Kato, T. and Long, C.: 2006, Executive turnover and firm performance in china, *The American Economic Review* **96**(2), 363–367.
URL: *http://www.jstor.org/stable/30034674*

Korteweg, A.: 2010, The net benefits to leverage, *The Journal of Finance* **65**(6), 2137–2170.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2010.01612.x*

Koskinen, Y. and Maeland, J.: 2016, Innovation, competition, and investment timing, *Review of Corporate Finance Studies* **5**(2), 166–199.
URL: *https://doi.org/10.1093/rcfs/cfw002*

Kraus, A. and Litzenberger, R. H.: 1973, A state-preference model of optimal financial leverage, *The Journal of Finance* **28**(4), 911–922.
URL: *http://www.jstor.org/stable/2978343*

Lambrecht, B. M.: 2001, The impact of debt financing on entry and exit in a duopoly, *The Review of Financial Studies* **14**(3), 765–804.
URL: *https://doi.org/10.1093/rfs/14.3.765*

Lambrecht, B. M.: 2017, Real options in finance, *Journal of Banking & Finance* **81**, 166 – 171.
URL: *http://www.sciencedirect.com/science/article/pii/S0378426617300560*

Lambrecht, B. and Tse, A.: 2019, Liquidation, bailout, and bail-in: Insolvency resolution mechanisms and bank lending, *Working Paper* .
URL: *https://ssrn.com/abstract=3123170*

Langedijk, S., Pagano, N. and Rossi, A.: 2014, Debt bias in corporate taxation and the costs of banking crises in the EU, *EU Taxation Papers* (50), 1725–7565.
URL: *https://ideas.repec.org/p/tax/taxpap/0050.html*

Leland, H.: 1994, Corporate debt value, bond covenants, and optimal capital structure, *The Journal of Finance* **49**(4), 1213–1252.
URL: *http://dx.doi.org/10.1111/j.1540-6261.1994.tb02452.x*

Leland, H. and Toft, K.: 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *The Journal of Finance* **51**(3), 987–1019.
URL: *http://dx.doi.org/10.1111/j.1540-6261.1996.tb02714.x*

Lemley, M. A. and Shapiro, C.: 2005, Probabilistic patents, *Journal of Economic Perspectives* **19**(2), 75–98.
URL: *http://www.aeaweb.org/articles?id=10.1257/0895330054048650*

Leung, C. M. and Kwok, Y. K.: 2012, Patent-investment games under asymmetric information, *European Journal of Operational Research* **223**(2), 441 – 451.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221712004882*

Lin, D., Liu, D. and Whalley, E.: 2019, The impact of product market characteristics on firms? strategies in patent litigation, *Real Options Conference 2018 Working Paper* .
URL: *http://www.realoptions.org/openconf2019/data/papers/399.pdf*

Loscher, P.: 2012, The CEO of siemens on using a scandal to drive change, *Harvard Business Review* .
URL: *https://hbr.org/2012/11/the-ceo-of-siemens-on-using-a-scandal-to-drive-change*

Ma, S., Tong, J. T. and Wang, W.: 2019, Selling innovation in bankruptcy.
URL: *https://ssrn.com/abstract=2903003*

Marco, A. C.: 2005, The option value of patent litigation: Theory and evidence, *Review of Financial Economics* **14**(3), 323 – 351.
URL: *http://www.sciencedirect.com/science/article/pii/S1058330005000054*

McDonald, R. and Siegel, D.: 1986, The value of waiting to invest, *The Quarterly Journal of Economics* **101**(4), 707–727.
URL: *http://www.jstor.org/stable/1884175*

Mella-Barral, P.: 1999, The dynamics of default and debt reorganization, *Review of Financial Studies* **12**(3), 535–578.
URL: *http://rfs.oxfordjournals.org/content/12/3/535.abstract*

Mella-Barral, P. and Perraudin, W.: 1997, Strategic debt service, *The Journal of Finance* **52**(2), 531–556.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1997.tb04812.x*

Meng, R.: 2008, A patent race in a real options setting: Investment strategy, valuation, capm beta, and return volatility, *Journal of Economic Dynamics and Control* **32**(10), 3192 – 3217.
URL: *http://www.sciencedirect.com/science/article/pii/S0165188908000158*

Merton, R. C.: 1973, Theory of rational option pricing, *The Bell Journal of Economics and Management Science* **4**(1), 141–183.
URL: *http://www.jstor.org/stable/3003143*

Merton, R. C.: 1974, On the pricing of corporate debt: The risk structure of interest rates, *The Journal of Finance* **29**(2), 449–470.
URL: *http://dx.doi.org/10.1111/j.1540-6261.1974.tb03058.x*

Miao, J. and Rivera, A.: 2016, Robust contracts in continuous time, *Econometrica* **84**(4), 1405–1440.
URL: *http://dx.doi.org/10.3982/ECTA13127*

Michaelides, A.: 2014, Cyprus: from boom to bail-in, *Economic Policy* **29**(80), 639–689.
URL: *http://economicpolicy.oxfordjournals.org/content/29/80/639*

Milne, A. and Whalley, A. E.: n.d., Bank capital regulation and incentives for risk-taking, *Cass Business School Research Paper* (17).
URL: *https://ssrn.com/abstract=299319*

Miltersen, K. R. and Schwart, E. S.: 2004, R& D Investments with Competitive Interactions, *Review of Finance* **8**(3), 355–401.
URL: *https://doi.org/10.1007/s10679-004-2543-z*

Modigliani, F. and Miller, M. H.: 1963, Corporate income taxes and the cost of capital: A correction, *The American Economic Review* **53**(3), 433–443.
URL: *http://www.jstor.org/stable/1809167*

Morellec, E., Valta, P. and Zhdanov, A.: 2015, Financing investment: The choice between bonds and bank loans, *Management Science* **61**(11), 2580–2602.
URL: *https://doi.org/10.1287/mnsc.2014.2005*

Morrison, A. D. and White, L.: 2011, Deposit insurance and subsidized recapitalizations, *Journal of Banking & Finance* **35**(12), 3400 – 3416.
URL: *http://www.sciencedirect.com/science/article/pii/S0378426611001841*

Moulin, H.: 2004, *Fair Division and Collective Welfare*, MIT Press.
URL: *https://books.google.pt/books?id=qQXtEnb2B2cC*

Multak, A. D.: 2017, The big patent short: Hedge fund challenges to pharmaceutical patents and the need for financial regulation, *Fordham J. Corp. & Fin. L.* **23**, 301.

Murphy, K. J.: 2013, Executive compensation: Where we are, and how we got there, *in* G. Constantinidis, M. Harris and R. Stulz (eds), *Handbook of the Economics of Finance, Vol. 2b*, Elsevier Inc., chapter 4.

Murphy, K. and Zimmerman, J.: 1993, Financial performance surrounding CEO turnover, *Journal of Accounting and Economics* **16**(1), 273 – 315.
URL: *http://www.sciencedirect.com/science/article/pii/0165410193900147*

Orlov, D., Skrzypacz, A. and Zryumov, P.: 2020, Persuading the principal to wait, *Journal of Political Economy* **128**(7), 2542–2578.
URL: *https://doi.org/10.1086/706687*

Palia, D.: 2015, The endogeneity of managerial compensation in firm valuation: A solution, *The Review of Financial Studies* **14**(3), 735–764.
URL: *https://dx.doi.org/10.1093/rfs/14.3.735*

Panageas, S.: 2010, Bailouts, the incentive to manage risk, and financial crises, *Journal of Financial Economics* **95**(3), 296–311.
URL: *https://www.sciencedirect.com/science/article/pii/S0304405X09002384*

Parrino, R.: 1997, CEO turnover and outside succession a cross-sectional analysis, *Journal of Financial Economics* **46**(2), 165 – 197.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X97000287*

Pawlina, G.: 2010, Underinvestment, capital structure and strategic debt restructuring, *Journal of Corporate Finance* **16**(5), 679 – 702.
URL: *http://www.sciencedirect.com/science/article/pii/S0929119910000428*

Pelger, M.: 2012, Contingent convertible bonds: Pricing, dilution costs and efficient regulation., *Working Paper* .
URL: *http://ssrn.com/abstract=2055825*

Peskir, G. and Shiryaev, A. N.: 2006, *Optimal stopping and free-boundary problems*, Springer Science & Business Media.
URL: *https://link.springer.com/book/10.1007/978-3-7643-7390-0*

Petersen, M. A. and Mukuddem-Petersen, J.: 2005, Stochastic behaviour of risk-weighted bank assets under the basel ii capital accord, *Applied Financial Economics Letters* **1**(3), 133–138.
URL: *http://dx.doi.org/10.1080/17446540500101978*

Pulvino, T. C.: 1998, Do asset fire sales exist? an empirical investigation of commercial aircraft transactions, *The Journal of Finance* **53**(3), 939–978.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/0022-1082.00040*

Reindorp, M. J. and Fu, M. C.: 2011, Capital renewal as a real option, *European Journal of Operational Research* **214**(1), 109 – 117.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221711002797*

Rochet, J.-C. and Villeneuve, S.: 2005, Corporate portfolio management, *Annals of Finance* **1**(3), 225–243.
URL: *https://doi.org/10.1007/s10436-005-0018-7*

Rosen, S.: 1981, The economics of superstars, *American Economic Review* **71**(5), 845–858.
URL: *https://www.jstor.org/stable/1803469*

Roubini, N.: 2004, Private sector involvement in crisis resolution and mechanisms for dealing with sovereign debt problems, *in* A. Haldane (ed.), *Fixing Financial Crises in the 21st Century*, Routledge, Taylor & Francis Group.
URL: *https://www.routledge.com/Fixing-Financial-Crises-in-the-21st-Century/Haldane/p/book/9780415647809*

Sabac, F.: 2007, Dynamic agency with renegotiation and managerial tenure, *Management Science* **53**(5), 849–864.
URL: *https://doi.org/10.1287/mnsc.1060.0638*

Sannikov, Y.: 2008, A continuous- time version of the principal: Agent problem, *The Review of Economic Studies* **75**(3), 957–984.
URL: *http://www.jstor.org/stable/20185061*

Sarkar, S.: 2013, Distressed exchange, bargaining power, and prior capital structure, *Journal of Economic Dynamics and Control* **37**(12), 2695 – 2709.
URL: *http://www.sciencedirect.com/science/article/pii/S0165188913001504*

Sattinger, M.: 1993, Assignment models of the distribution of earnings, *Journal of Economic Literature* **31**(2), 831–880.
URL: *http://www.jstor.org/stable/2728516*

Schandlbauer, A.: 2017, How do financial institutions react to a tax increase?, *Journal of Financial Intermediation* **30**, 86 – 106.
URL: *http://www.sciencedirect.com/science/article/pii/S1042957316300286*

Schepens, G.: 2016, Taxes and bank capital structure, *Journal of Financial Economics* **120**(3), 585 – 600.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X16000209*

Schwartz, E. S.: 2004, Patents and R&D as real options, *Economic Notes* **33**(1), 23–54.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.0391-5026.2004.00124.x*

Senbet, L. W. and Wang, T. Y.: 2012, Corporate financial distress and bankruptcy: A survey, *Foundations and Trends in Finance* **5**(4), 243–335.
URL: *https://ssrn.com/abstract=2034646*

Shibata, T.: 2009, Investment timing, asymmetric information, and audit structure: A real options framework, *Journal of Economic Dynamics and Control* **33**(4), 903 – 921.
URL: *http://www.sciencedirect.com/science/article/pii/S0165188908001851*

Shibata, T. and Nishihara, M.: 2015, Investment timing, debt structure, and financing constraints, *European Journal of Operational Research* **241**(2), 513 – 526.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221714007322*

Store, K., Fleten, S.-E., Hagspiel, V. and Nunes, C.: 2018, Switching from oil to gas production in a depleting field, *European Journal of Operational Research* **271**(2), 710 – 719.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221718304545*

Strebulaev, I.: 2007, Do tests of capital structure theory mean what they say?, *The Journal of Finance* **62**(4), 1747–1787.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2007.01256.x*

Sundaresan, S.: 2013, A Review of Merton's Model of the Firm's Capital Structure with Its Wide Applications, *Annual Review of Financial Economics* **5**(1), 21–41.
URL: *http://www.annualreviews.org/doi/abs/10.1146/annurev-financial-110112-120923*

Sundaresan, S. and Wang, N.: 2007a, Investment under uncertainty with strategic debt service, *American Economic Review* **97**(2), 256–261.
URL: *https://www.aeaweb.org/articles?id=10.1257/aer.97.2.256*

Sundaresan, S. and Wang, N.: 2007b, Investment under uncertainty with strategic debt service, *American Economic Review* **97**(2), 256–261.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.97.2.256*

Sundaresan, S., Wang, N. and Yang, J.: 2015, Dynamic investment, capital structure, and debt overhang, *Review of Corporate Finance Studies* **4**(1), 1–42.
URL: *http://rcfs.oxfordjournals.org/content/4/1/1.abstract*

Sundaresan, S. and Wang, Z.: 2015, On the design of contingent capital with a market trigger, *The Journal of Finance* **70**(2), 881–920.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12134*

Sundaresan, S. and Wang, Z.: 2017, Bank liability structure, *Columbia Business School Research Paper* (14-41).
URL: *https://www8.gsb.columbia.edu/researcharchive/articles/6375*

Taylor, L.: 2010, Why are CEOs rarely fired? evidence from structural estimation, *The Journal of Finance* **65**(6), 2051–2087.
URL: *http://www.jstor.org/stable/23324405*

Tervio, M.: 2008, The difference that CEOs make: An assignment model approach, *American Economic Review* **98**(3), 642–68.
URL: *http://www.aeaweb.org/articles?id=10.1257/aer.98.3.642*

Trigeorgis, L. and Tsekrekos, A. E.: 2018, Real options in operations research: A review, *European Journal of Operational Research* **270**(1), 1 – 24.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221717310664*

Tserlukevich, Y.: 2008, Can real options explain financing behavior?, *Journal of Financial Economics* **89**(2), 232 – 252.
URL: *http://www.sciencedirect.com/science/article/pii/S0304405X08000925*

Vishnubhakat, S., Rai, A. K. and Kesan, J. P.: 2016, Strategic decision making in dual ptab and district court proceedings, *Berkeley Technology Law Journal* **31**(1), 45–116.
URL: *http://www.jstor.org/stable/43917619*

Walther, A. and White, L.: 2019, Rules versus discretion in bank resolution, *CEPR Discussion Paper* (DP14048).
URL: *https://ssrn.com/abstract=3471242*

Warner, J. B., Watts, R. L. and Wruck, K. H.: 1988, Stock prices and top management changes, *Journal of Financial Economics* **20**, 461 – 492.
URL: *http://www.sciencedirect.com/science/article/pii/0304405X88900542*

Weeds, H.: 2002, Strategic delay in a real options model of R&D competition, *The Review of Economic Studies* **69**(3), 729–747.
URL: *http://www.jstor.org/stable/1556717*

Welch, I.: 2004, Capital structure and stock returns, *Journal of Political Economy* **112**(1), 106–131.
URL: *http://www.jstor.org/stable/10.1086/379933*

Witbooi, P. J., van Schalkwyk, G. J. and Muller, G. E.: 2011, An optimal investment strategy in bank management, *Mathematical Methods in the Applied Sciences* **34**(13), 1606–1617.
URL: *http://dx.doi.org/10.1002/mma.1467*

Yang, Z. and Zhao, Z.: 2015, Valuation and analysis of contingent convertible securities with jump risk, *International Review of Financial Analysis* **41**, 124 – 135.
URL: *http://www.sciencedirect.com/science/article/pii/S1057521915001118*

Zambujal-Oliveira, J. and Duque, J.: 2011, Operational asset replacement strategy: A real options approach, *European Journal of Operational Research* **210**(2), 318 – 325.
URL: *http://www.sciencedirect.com/science/article/pii/S0377221710006077*

Zenios, S.: 2015, Fairness and reflexivity in the cyprus bail-in, *Empirica* pp. 1–28.
URL: *http://dx.doi.org/10.1007/s10663-015-9306-2*

Zingales, L.: 1998, Survival of the fittest or the fattest? exit and financing in the trucking industry, *The Journal of Finance* **53**(3), 905–938.
URL: *http://dx.doi.org/10.1111/0022-1082.00039*