SOME ASPECTS OF THE INTERACTION OF FIELDS WITH MATTER.

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D. M. Brink.

Balliol College
Oxford.

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ABSTRACT

Part I. THE SHELL MODEL NEAR ^{208}Pb.

The nucleus ^{208}Pb has a double closed shell of protons and neutrons. Because of the stability of this structure it is possible that a nucleus with a few extra particles outside the double closed shell can be described as a system in which these few particles move in an average central field produced by the ^{208}Pb core. It is assumed that the individual particle levels are those predicted by the shell model, and that the nuclear forces acting between the extra particles can be treated as a perturbation on their motion in the central field. The behaviour of vacancies in the closed shell is equivalent to the behaviour of particles in the same states. Thus nuclei with a few holes in the closed shell and nuclei with a few extra particles can be treated by identical methods. Similar considerations hold for nuclei with a few holes and a few extra particles except for some details in the coupling of the angular momenta of holes and particles.

To investigate the effect of internucleon forces it is necessary to calculate matrix elements of the internucleon potential between two-particle states. It has been hoped that in heavy nuclei, where the radius of the nucleus is large compared with the range of nuclear forces, the effect of the range of the forces would be small so that zero range forces could be used. The effect of the range of

the forces has been investigated and a relatively simple expansion obtained for the matrix elements of the internucleon potential as a power series in the ratio of the range of the nuclear forces to the nuclear radius. Calculations have been extended to take the second term of the series into account, and it is found that, in many cases, the contribution of this term is very important. This is particularly so for Majorana forces in a two-particle state coupled to give a low spin. The zero range approximation gives two large an interaction in states of low spin, while it gives a good estimate of the interaction in states of high spin. In particle hole configurations this situation is exactly reversed.

As an example of the application of the model the energy levels of the four nuclei ^{210}Bi, ^{208}Te, ^{204}Pb and ^{202}Pb have been investigated. In ^{210}Bi it is possible to classify the known low lying levels in terms of two particle neutron-proton configurations and in particular to predict a low lying isomeric state with $I \sim 8$. ^{208}Te is of interest as it gives an example of a particle hole configuration. The even-even nuclei ^{204}Pb and ^{202}Pb have very similar level schemes each with a 9-isomeric state. These schemes can be understood on the basis of the above model.

Part II. SOME REACTIONS WITH NEUTRONS

Reactions with neutrons are of considerable importance in the study of heavy nuclei. The interaction producing the reaction is the short range nuclear force, the cross sections are large, and there is no Coulomb barrier to complicate the interpretation of the experiments. Two types of reaction have been considered. The first is a process of inelastic scattering in the 1 MeV region and the second resonance scattering at slow neutron energies.

The inelastic scattering of neutrons is usually thought of as a compound nucleus process. In regions of atomic weight A = 170 and A = 230, however, the nucleus is highly distorted and there is the possibility of a direct inelastic process which can be described as follows. The neutron strikes the target nucleus and on reflection from its surface the neutrons set the target nucleus into rotation (i.e. excite a surface rotational mode) and are scattered with reduced energy. The interaction of the neutron with the nucleus has been represented by a complex potential well (the 'optical model') the complex parts of the potential giving an absorption to be identified with compound nucleus formation. It is known that the "optical model" gives large cross sections in the region of atomic weights A = 150 and A = 240 corresponding to S, D and P partial wave resonances inside the nucleus. In these regions of atomic weight it is found that the scattering cross section predicted by the direct process is comparable with the compound inelastic cross section, while in intermediate regions of atomic weight the direct process is unimportant. The two processes are incoherent and

give different neutron and γ-ray angular distributions.

In recent years there has been an intensive experimental investigation of the properties of slow neutron resonances in many nuclei. These resonances correspond to virtual levels of the compound nucleus and the experiments tell us something about high energy nuclear states. In general these states will be a complicated mixture of single particle configurations and there is a possibility that their properties can be treated statistically. On this assumption the distribution function for reaction widths has been investigated and it is found that reduced partial widths have a distribution which is approximately exponential.

Photo capture of neutrons is the inverse process to the nuclear photo effect. The dipole photo effect is known to have a broad resonance at about 13 → 20 MeV where the width and position of this resonance very slowly with atomic weight and appear insensitive to details of nuclear structure. An equation to the resonance is assumed and used to calculate the expectation values of the dipole radiation widths of slow neutron resonances. The theory predicts radiation widths too large by a factor of 3; but gives the right variation with atomic weight. The variation of radiation width with the spin of the emitting state is considered and it is found, using a modified sum rule, that the total radiation with Γ_γ^{λ} should be independent of spin. The spectral distribution of the emitted γ-rays and the dependence of the total width on the energy of the radiating state are investigated.

PREFACE.

The work presented in this thesis was carried out in Oxford during the years 1952 - 55 under the supervision of Prof. M. H. L. Pryce.

It is divided into two distinct parts; Part I contains a theoretical investigation into the level structure of heavy nuclei near the double closed shell of ^{208}Pb, while Part II is concerned with an account of the theory of a number of reactions induced by neutrons.

Some of the results of Part I were described in a paper published in the Proceedings of the Physical Society and the first part of Part II has been accepted for publication in the same Journal.

I should like to express my appreciation to my supervisor for his continued interest and encouragement in this work, and also to Prof. Weisskopf for stimulating discussions. I am indebted to the Rhodes Trustees and to the Royal Society for the award of Scholarships which have made this study possible.

D. M. Brink.

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Part I.

THE NUCLEAR SHELL MODEL NEAR ^{208}Pb.

Part I

THE SHELL MODEL NEAR ^{208}Pb.

Contents

Chapter 1.

The Shell Model.

1(i): Introduction.

Soon after the hypothesis that nuclei are composed of protons and neutrons was proposed, attempts were made to understand nuclear structure on the basis of an independent particle model. Early developments in this direction were made by Bartlett (1932), Elsasser (1933, 34) and Guggenheimer (1934); but poor results obtained in the calculation of nuclear binding energies discouraged further attempts for a time. During the intervening period N. Bohrs' (1936) theory of nuclear reactions, emphasising collective aspects of motion, enjoyed considerable success and it was supposed that an independant particle model of nuclear structure could not be consistent with Bohrs' ideas.

In recent years the independant particle model has come back into vogue. The nuclear shell model proposed by Mayer (1949, 1950) and Haxel, Jensen and Suess (1949) has been very successful in explaining many regularities in the structure of nuclei and in accounting for the properties of their ground states. The model has given insight into the nature of isomeric states found in some nuclei. It has even proved possible to give a detailed account of the level structure and

properties of excited states of light nuclei, taking the independent particle model as a first approximation (Inglis 1953, Lane 1953, 54).

In the remainder of this chapter we will give an introduction to those aspects of the shell model relevant to the work presented in this thesis. Reviews of nuclear shell structure, or certain of its aspects, have been given by Mayer, Moskowski and Nordheim (1951), Nordheim (1951), Rosenfeld (1951), Flowers (1952), Goldhaber and Hill (1952), Klinkenberg (1952) and Pryce (1954).

1(ii): The Independent Particle Model.

This model assumes that a nucleon inside the nucleus moves relatively freely and that its interaction with the remaining nucleons of the nucleus can be replaced to a good approximation by an average potential field. This potential field is common to all nucleons of a given type and, in the approximation of the model, the particles of the nucleus are considered to move in independent orbits in this potential, subject only to the restrictions of the Pauli exclusion principle. It is difficult to understand the physical basis of the model, though an explanation founded on the Pauli exclusion principle has been given (Weisskopf 1950). It is adequately justified, however, by the usefulness of its results.

It is assumed that the average potential field acting on the nucleons of the nucleus is produced by the nucleons themselves and could, in principle, be determined by a Hartree-Foch self consistent field calculation, Too little is known about the nature of nuclear forces for such a calculation to be made; but fortunately quite qualitative arguments are sufficient to deduce the nature of the potential. There is evidence that the density of nucleons inside the nucleus is uniform and the same for all nuclei and since nuclear forces have a short range, the potential should be described adequately as a "square well with rounded edges". The depth of the well will be the same for all nuclei and its radius approximately equal to the nuclear radius. The coulomb potential must be added to this nuclear potential to give the self consistent field acting on a proton.

Adopting the independent particle model, a stationary state of the nucleus is represented as a stationary state of a system of protons and neutrons moving without interaction in a potential field of the form described above. The wave function of such a state is the sum of products of individual particle wave functions, correctly antisymmetrized to satisfy the Pauli exclusion principle.

The single particle states involved are highly degenerate and in this thesis a group of states with the

same energy is termed a level. The energy of the system of protons and neutrons is specified when we give the number of protons and neutrons in each level and we will use the term "configuration" to describe that group of states with the same total number of protons and neutrons and the same total energy. In general a configuration contains a large number of single particle states, but those configurations in which all occupied single particle levels are filled are non degenerate. Such a configuration is called a closed sub-shell configuration. If further, the closed sub-shell is separated by a large energy gap from other configurations it is termed a closed shell. As in the electronic structure of atoms, a nucleus with a closed shell of protons or neutrons has a high stability, and there is a discontinuity in nucleon binding energies as the shell is filled.

Having solved the problem of the independent particle model without interactions, the specific nucleon-nucleon forces can be introduced as a perturbation. To the first order of perturbation the interparticle interaction will split a configuration up into a number of levels of different energies. In general these levels will correspond to particular stationary values of the total angular momentum I of the nucleus. In higher orders of perturbation linear combinations of the

wave functions of levels with the same I and belonging to the same or different configurations, must be considered and the energy matrix diagonalized. In this way a solution of any desired accuracy can be built up.

The angular momentum coupling of vacancies in a closed subshell is almost identical with the coupling for particles in the same shell (Condon and Shortley T.A.S.) and to the first order of perturbation in the inter-nucleon interactions, vacancies in a closed shell have similar properties to particles in the same shell. Making use of the above equivalence property, excited states of nuclei with almost closed shell configurations can be investigated profitably in terms of the excitation of the vacancies or "holes" in the closed shell.

1(iii): Individual Nucleon Levels.

Almost any spin independent nucleon-nucleus interaction potential intermediate between a square well and a harmonic oscillator well leads to the following order of single particle levels. The lowest level in the well is always an s state with orbital angular momentum $\ell = 0$. This is followed by a p state and then by the groups $1d, 2s$ with $\ell = 2, 0$; $1f, 2p$ with $\ell = 3, 1$; $1g, 2d, 3s$ with $\ell = 4, 2, 0$; $1h, 2f, 3p$ with $\ell = 5, 2, 1$; and so on to the top of the well. For a harmonic oscillator these groups of levels are degenerate while for a

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square well they split up and the levels of high angular momentum in any group are depressed relative to the levels with low angular momentum.

A model based on such a level system will have closed shell configurations when each of the above groups is filled i.e. for

N, Z = 2, 8, 20, 40, 70, 112.

There is, however, evidence for closed shell configurations at the nucleon numbers

N, Z = 2, 8, 20, 28, 50, 82, 126.

(Elsasser (1934)). The first three numbers of this sequence coincide with the closed shell numbers for a potential well. The departure from this sequence has been explained independently by Haxel, Jensen and Suess (1949) and by Mayer (1949, 50), by introducing a strong spin orbit coupling which depresses the individual particle states with $j = l + 1/2$ and raises those with $j = l - 1/2$. A new grouping of levels results which follows that indicated by experiment. ~~Figure 1 shows a diagram of the individual particle level groupings for protons and neutrons taken from Klinkenberg (1952).~~

1(iv): The Shell Model Near ^{208}Pb.

The ground state of the nucleus ^{208}Pb with

126 neutrons and 82 protons is a double closed shell configuration separated by an energy gap of ~ 3 MeV from other possible configurations. Because of the stability of this structure it should be possible to describe nuclei with a small number of extra nucleons outside a ^{208}Pb core, very simple on the basis of the individual particle model. We assume that in the low excited states of these nuclei the double closed shell of 126 neutrons and 82 protons is not disturbed much, so that these low states arise from the relatively simple configurations of the extra-core particles. In the simplist case, the low levels of the nuclei ^{209}Pb and ^{209}Bi are the states of one extra neutron or proton moving in the average central field produced by the double closed shell core. In the nuclei ^{210}Pb, ^{210}Bi and ^{210}Po with two extra nucleons, the low excited states arise from two particle configurations which are split by some nuclear force acting between the extra nucleons. Nuclei with more than two extra nucleons can be treated similarly, though in practice the complication increases rather rapidly with number of extra particles.

As mentioned before, vacencies or holes in a closed shell behave rather like extra particles and nuclei with an almost complete double closed shell, have level systems which can be thought of in terms of the

"excitation of holes". The single hole states follow the single particle levels ~~of the closed shell~~ in approximately the inverse order. A nucleus deficient in two nucleons have a level system corresponding to the excitation of two holes, where again ~~the~~ a two hole configuration is split into states of definate total angular momentum by the internucleon interactions. There are some nuclei with particles outside the core and holes in the core. The states of these nuclei are treated in terms of combined particle hole configurations. The coupling of the angular momentum of particle and hole configurations requires some core and will be treated in Chapter II, where matrix elements of the nuclear forces are calculated for particle-hole configurations.

In order to investigate the level structure of nuclei differing by more than one particle from the ^{208}Pb nucleus we need to know something of the order and spacing of single particle and single hole levels. We can obtain this information by looking at the level structure of the four nuclei ^{209}Pb, ^{209}Bi, ^{207}Te and ^{207}Pb, since, on the individual particle model, the states of these nuclei are single particle or hole levels.

According to the shell model the lowest levels available for the odd neutron in ^{209}Pb are $g_{9/2}$, $i_{11/2}$, $g_{7/2}$, $d_{5/2}$ $d_{3/2}$ and $k_{15/2}$, where the $k_{15/2}$ level may be depressed strongly

by spin orbit coupling. ^{209}Pb decays to ^{209}Bi by decay with an ft value for the transition compatible with a first forbidden transition $\Delta I=0$ or 1 and change of parity. Since the ground state of ^{209}Bi has spin $9/2$ and odd parity the odd neutron in ^{209}Pb should be in a $g_{9/2}$ or $i_{11/2}$ level. An investigation by Lee Whitting (1955) favours the $g_{9/2}$ assignment. Information about the excited states of ^{209}Pb is available from the reaction ^{208}Pb (d,p) ^{209}Pb Harvey (1953). Proton groups are observed corresponding to the ground state and to excited states at energies of 0.75, 1.56, 2.03 and 2.54 MeV. Harvey makes the level assignments $g_{9/2}$ for the ground state and $i_{11/2}$, $d_{5/2}$, $g_{7/2}$ and $d_{3/2}$ respectively on the basis of relative cross section measurements and shell model considerations. The neutron binding energy in the ground state, determined from the Q value of the reaction, is 3.87 MeV.

The shell model gives $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$ and $i_{13/2}$ as possible states for the 83rd proton in ^{209}Bi. As the ground state spin is $9/2$ we assume that this corresponds to the $h_{9/2}$ level. Inelastic scattering of 2.5 MeV neutrons indicates levels at 0.9 MeV and 1.6 MeV excitation (Poole 1953, Eliot, Hicks, Beghian and Halban 1954), but no spin assignments can be made at present. Analogy with the closed shell of 82 neutrons

would suggest that the first excited state of ^{209}Bi is the $f_{7/2}$ level.

The low levels of ^{207}Tl and ^{207}Pb arise from core excitations in which the vacent neutron or proton state moves from one configuration to another. The energy required to remove a particle from a closed subshell will, roughly speaking, be the one particle energy of that state plus a "pairing energy" or energy of interaction of the particle with other nucleons in the same subshell. Thus the level order in these nuclei will follow the one particle states in approximately the inverse order.

Low states in ^{207}Te will arise from holes in the levels $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $f_{7/2}$ and $h_{11/2}$ ^{207}Tl is known to have an excited state at .35 MeV (short range α_s in AcC). There is no α-γ angular correlation (Gorodetzky, Gallmann, Knipper, Armbruster 1953) indicating that perhaps the excited state is an hole. On the other hand all other odd-even isotopes of Te appear to have a spin $\frac{1}{2}$ ground state, while ^{203}Tl is known to have a level sequence $1/2$, $3/2$, $5/2$ (Wapstra 1953, Prescott 1953). On the individual particle model it is unlikely that the $d_{3/2}$ and $s_{1/2}$ levels would be inverted in going from ^{203}Tl to ^{207}Tl, so it is probable

that the ground state is the $s_{1/2}$ level and the first excited state the $d_{3/2}$ level. The absence of angular correlation would then have to be explained by a mixing of orbital angular momenta in the α transition or a M1, E2 mixture in the γ transition.

Pryce (1952) has given a complete discussion of the levels of ^{207}Pb and shows that a level sequence $p_{1/2}$, $f_{5/2}$ (.56 MeV), $p_{3/2}$ (.8 MeV) and $i_{13/2}$ (1.6 MeV) is consistent with the experimental details of the decay schemes of ^{207}Tl and ^{207}Bi. There is a group of levels at an excitation of $\sim$ 2 MeV, to which it is not possible to assign definite orbits. Two of them may correspond to the $f_{7/2}$ and $h_{9/2}$ neutron holes. Figure II gives the four proposed level sequences in diagramatic form.

Before going on to consider the more detailed application of the independent particle model to other nuclei near ^{208}Pb, we will mention one piece of experimental evidence which points strongly to the individual particle nature of at least some of the states of nuclei in this region. Harvey (1953) has made a number of experiments with (d,p) and (d,t) reactions and obtains the following interesting results. He finds, with ^{206}Pb, ^{207}Pb, ^{208}Pb and ^{208}Bi as target nuclei in a (d,p) reaction, groups of states with approximately the same

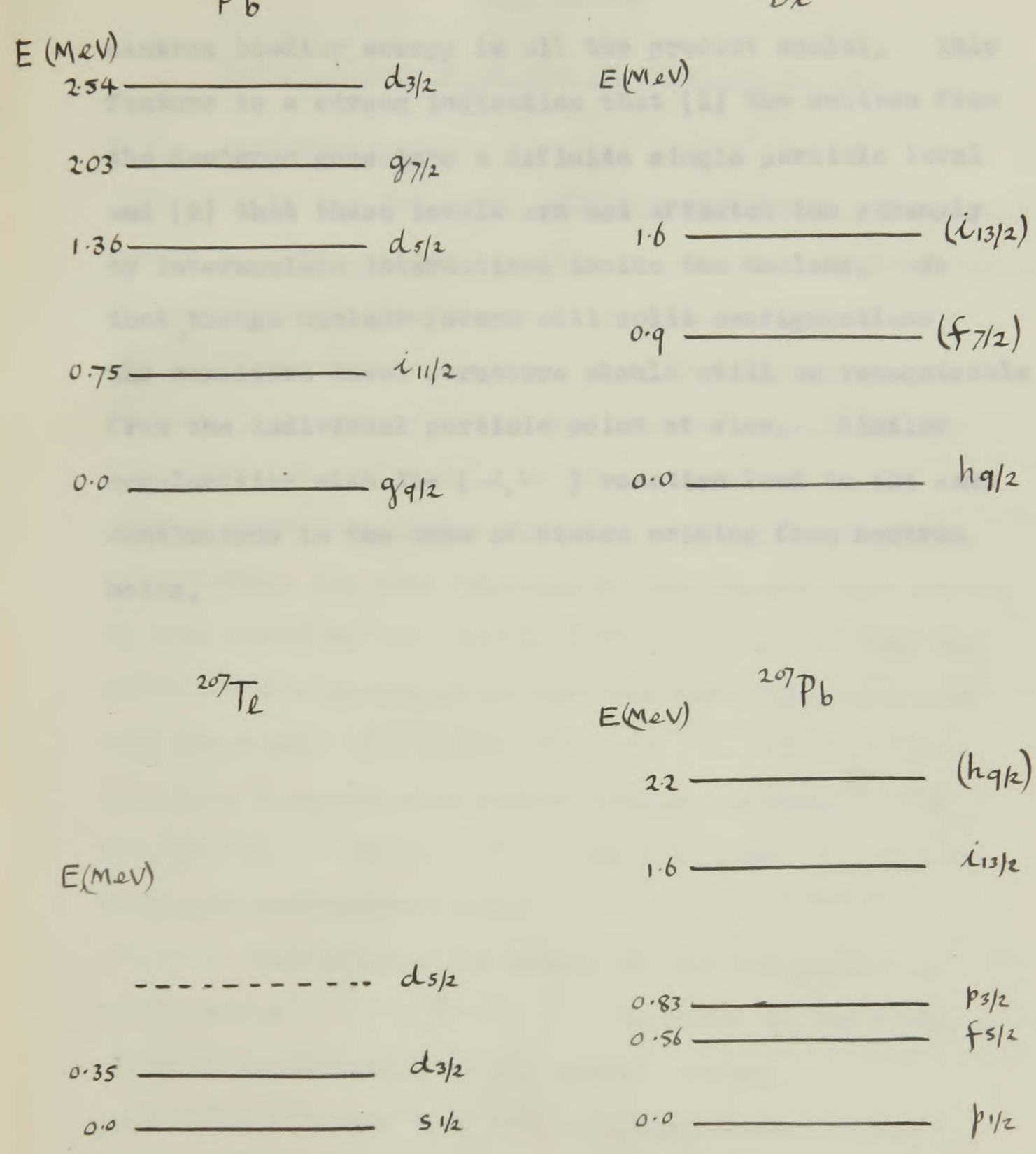

Figure I The low lying states of ^{209}Pb, ^{207}Pb, ^{209}Bi and ^{207}Tl with possible spin assignments

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neutron binding energy in all the product nuclei. This feature is a strong indication that (1) the neutron from the deuteron goes into a definite single particle level and (2) that these levels are not affected too strongly by internucleon interactions inside the nucleus. So that, though nuclear forces will split configurations, the resultant level structure should still be recognisable from the individual particle point of view. Similar regularities with the (d, t) reaction lead to the same conclusions in the case of states arising from neutron holes.

Chapter II.

Matrix Elements of the Interaction Energy.

2(i): Introduction.

In the following sections we consider the energy levels of a two-particle system in a central nuclear field. If the two particles do not interact they will move in orbits characterized by total angular momentum quantum numbers j_1 and j_2. The angular momenta of the two particles will couple to give a resultant total angular momentum quantum number I, with z component M. We can write the wave function of the two-particle system in this state as $|j_1 j_2, IM\rangle$. If the two particles are identical we must use the antisymmetrized wave function $N[|j_1 j_2, IM\rangle - (-1)^{j_1+j_2+I}|j_2 j_1, IM\rangle]$ Here N is a normalizing factor taking the value if $j_1 \neq j_2$. If $j_1 = j_2$ and I is even $|j_1 j_2, IM\rangle$ is itself antisymmetrical.

The interaction energy of the two particles in the state $|j_1 j_2, IM\rangle$ is given to the first order of perturbation by the matrix element $\langle j_1 j_2, IM|U|j_1 j_2, IM\rangle$ where U is the potential energy of the interaction. The wave function of the system without interaction may not, however, be a good approximation to the stationary state of the interacting system; in this case configuration interaction must be considered, requiring

the calculation of matrix elements of the type

$$\langle j_1 j_2, I M | V | j_1' j_2', I M \rangle$$

We will use a central two-body interaction potential of the following exchange type:

$$V = V(r_{12})(w + m P_x + h P_\tau + b P_\sigma)$$

where r_{12} ir the internucleon distance; w, m, h and b are constants such that $w + m + b - h = -1$; P_x is the coordinate exchange operator, P_σ is the spin exchange operator and P_τ ($= - P_x P_\sigma$) the isotopic spin exchange operator.

When evaluating matrix elements of exchange potentials it is convenient to work in the LS coupling scheme. Explicit expressions for the transformation coefficients from jj to LS coupling ~~scheme~~ are given by Pryce (1952) and Blin-Stoyle (1953).

Each term in the above potential can be written as a product of a spin and a coordinate operator. In the LS representation a matrix element of a term in the potential breaks up into the product of a spin matrix element and a coordinate matrix element. The spin matrix element has the value unity for a Wigner or a Majorana force and, for a Bartlett or Heisenberg force, takes the value 1 in a triplet spin state and - 1 in a singlet state. For a coordinate exchange potential the

coordinate matrix element has the form

$$\langle l_1' l_2', LM | V | l_1 l_2, LM \rangle$$

where $l_1 (\in j_1 \pm 1/2)$ etc. are the orbital angular momentum quantum numbers of the single particle states and reduces to $(-1)^{l_1+l_2-L} \langle l_1' l_2', LM | V | l_2 l_1, LM \rangle$ Thus the calculation of matrix elements of an interaction potential of any exchange type reduces to the calculation of coordinate matrix elements of a function of internucleon distance.

2(ii): Coordinate Matrix Elements.

The coordinate matrix element $\langle l_1' l_2', LM | V | l_1 l_2, LM \rangle$ of a function V of internucleon distance is independent of the magnetic quantum number. By averaging over all values of the magnetic quantum number we obtain an expression for the matrix element independent of the axis quantization. If we denote the value of the matrix element by F(L), then

$$F(L) = \frac{1}{2L+1} \sum_{M=-L}^{L} \langle l_1' l_2', LM | V | l_1 l_2, LM \rangle.$$

We now make a transformation to a coordinate representation of the state vectors of the two-particle

system. The wave functions will be products of a radial component and an angular component consisting of appropriate linear combinations of spherical harmonics.

Thus we have

$$\langle \underline{r}_1 \underline{r}_2 | l_1 l_2, L M \rangle = \phi_1(r_1)\phi_2(r_2) \sum_m C(l_1 l_2 L, m\, M-m) Y_{l_1}^{m}(\theta_1,\phi_1) Y_{l_2}^{M-m}(\theta_2,\phi_2) \quad (1)$$

The phases of the spherical harmonics are chosen to agree with the work of Racah (1942) and Condon and Shortley (1935). $C(l_1 l_2 L, m\, M-m)$ is the usual vector coupling coefficient. The matrix element $\langle \underline{r}_1 \underline{r}_2 | l_1' l_2', L M \rangle$ can be similarly expressed replacing l_1 and l_2 by l_1' and l_2', and ϕ by ψ in the radial wave functions.

Introducing the density function

$$\rho_L(\underline{r}_1 \underline{r}_2) = \frac{1}{2L+1} \sum_{M=-L}^{L} \langle l_1' l_2', L M | \underline{r}_1 \underline{r}_2 \rangle \langle \underline{r}_1 \underline{r}_2 | l_1 l_2, L M \rangle \quad (2)$$

the expression for $F(L)$ reduces to a double integral over the coordinates $\underline{r}_1$ and $\underline{r}_2$ of the two nucleons,

$$F(L) = \iint V(|\underline{r}_1 - \underline{r}_2|)\, \rho_L(\underline{r}_1 \underline{r}_2)\, d\underline{r}_1\, d\underline{r}_2 . \quad (3)$$

2(ii).

The density function is independent of the z axis of the coordinate representation and depends only on the magnitudes of the vectors $\underline{r}_1$ and $\underline{r}_2$ and on the angle between them. In order to evaluate the integral in (3) in an approximate form, suitable for short range forces, we make a transformation to relative coordinates. Put

$$\underline{r}_1 - \underline{r}_2 = \underline{s} \quad \text{and} \quad \rho_L(\underline{r}_1, \underline{r}_2) = \rho'_L(\underline{r}, \underline{s})$$

then

$$F(L) = \iint V(s)\, \rho'_L(\underline{r}, \underline{s})\, d\underline{r}\, d\underline{s}. \tag{4}$$

Now expand $\rho'_L(\underline{r}, \underline{s})$ as a power series in $\underline{s}$ and put this expansion into the integral for F(L). V(s) is a function of s only, and because of this symmetry, terms arising from odd powers of s_x, s_y or s_z will vanish on integration over $\underline{s}$. Using identies of the form

$$\int V(s)\, s_x^{2n}\, d\underline{s} = \frac{4\pi}{2n+1} \int V(s)\, s^{2n+2}\, ds$$

the expression for F(L) reduces to

$$\begin{aligned} F(L) = {} & 4\pi \int V(s)\, s^2\, ds \cdot \int \rho'_L(\underline{r}, 0)\, d\underline{r} \\ & + \frac{4\pi}{3!} \int V(s)\, s^4\, ds \cdot \int \left[\nabla^2 \rho'_L(\underline{r}, \underline{s})\right]_{\underline{s}=0} d\underline{r} \\ & + \frac{4\pi}{5!} \int V(s)\, s^6\, ds \cdot \int \left[(\nabla^2)^2 \rho'_L(\underline{r}, \underline{s})\right]_{\underline{s}=0} d\underline{r} \\ & + \text{higher terms}. \end{aligned} \tag{5}$$

2(ii).

In the integrals $\int [(\nabla^2)^n \rho_L'(\underline{r},\underline{s})]_{\underline{s}=0} d\underline{r}$, ∇^2 is the Laplacian operating on $\underline{s}$ in the density function. The first two of these integrals are evaluated in the Appendix A

$$4\pi \int \rho'(\underline{r},0)\, d\underline{r} = \int \psi_1^*(r)\, \psi_2^*(r)\, \phi_1(r)\, \phi_2(r)\, r^2\, dr \qquad (A5)$$

reducing to the results of Bryce (1952) in the special case $l_1' = l_1$ and $l_2' = l_2$, and

$$4\pi \int \left[\nabla^2 \rho_L'(\underline{r},\underline{s})\right]_{\underline{s}=0} d\underline{r} = P_1(L) \int \psi_1^*(r)\, \psi_2^*(r)\, \phi_1(r)\, \phi_2(r)\, dr$$
$$- P_0(L) \int \frac{d}{dr}\left[\psi_1^*(r)\, \phi_1(r)\right] \frac{d}{dr}\left[\psi_2^*(r)\, \phi_2(r)\right] r^2\, dr.$$
$$(A6, 7, 11)$$

Higher terms can be evaluated by similar methods.

2(iii): Expressions for Matrix Elements in the jj Representation.

We will specialize in the case where $j_1' = j_1$ and $j_2' = j_2$ and calculate explicit expressions for the matrix elements of the nucleon-nucleon interaction potential in the jj representation. Only the first two terms in the expansion for the coordinate matrix element will be retained.

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For convenience write

$$I_0 = R^3 \int \psi_1^2(r)\, \psi_2^2(r)\, r^2\, dr$$

$$I_{10} = R^5 \int \psi_1^2(r)\, \psi_2^2(r)\, dr$$

$$I_{11} = R^5 \int \frac{d}{dr}\left(\psi_1^2(r)\right) \frac{d}{dr}\left(\psi_2^2(r)\right) r^2\, dr$$

Here is a scale factor equal to the nuclear radius and introduced so that I_0, and I_{10} and I_{11} depend only on the form of the radial wave functions and not on the nuclear radius. Also let us write

$$a_s = |\langle I\,0, I | j_1 j_2, I\rangle|^2 P_0(I) \quad ; \quad b_s = |\langle I\,0, I | j_1 j_2, I\rangle|^2 P_1(I)$$

$$a_t = \sum_{L=I-1}^{I+1} |\langle L\,1, I | j_1 j_2, I\rangle|^2 P_1(L) \quad , \quad b_t = \sum_{L=I-1}^{I+1} |\langle L\,1, I | j_1 j_2, I\rangle|^2 P_1(L)$$

The matrix elements occurring here are the transformation coefficients from LS to jj coupling. Subscripts s and t distinguish the singlet and triplet parts of the interaction. The quantities defined refer only to Wigner and Bartlett forces. Corresponding quantities for a coordinate exchange force will be indicated by a superscript

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M. The coordinate exchange operator has no effect in the zero range approximation. Thus $a_t^M = a_t$, $a_s^M = a_s$.

The matrix elements of the interaction energy can be written in terms of the second and fourth moments of the interaction potential and the quantities defined above, containing the dependence on I and on the nuclear radius. We can separate the effect of coordinate exchange by writing the interaction potential as

$$V(r) = V(r)(w + b P_\sigma) + V(r) P_x (m - h P_\sigma).$$

The interaction energy matrix element then splits into two terms. The first arises from the Wigner and Bartlett interactions, and the second from the interactions containing coordinate exchange.

$$E_{12} = \langle j_1 j_2, IM | V | j_1 j_2, IM \rangle = (w+b) E^\circ_{12} + (m-h) E^M_{12}.$$

Here

$$E^\circ_{12} = \left[\frac{I_0}{R^3} \int V(r) r^2 dr - \frac{I_1}{R^5} \int V(r) r^4 dr \right] (a_s + \varepsilon a_t) + \frac{I_1}{R^5} \int V(r) r^2 dr \cdot (b_s + \varepsilon b_t) \qquad (6)$$

with $\varepsilon = \frac{w-b}{w+b}$.

E_{12}^{M} has a similar expression with b_t^M , b_s^M and ε^M replacing b_t , b_s and ε . In this case $\varepsilon^M = \frac{m+h}{m-h}$ In these expressions ε and ε^M are factors containing the spin dependence of the force and take the value 1 if there is no spin dependence and - 1 for spin exchange forces.

Blatt and Jackson (1949) have interpreted low energy scattering data in terms of a well depth parameter s and an intrinsic range b, which can be simply related to any particular interaction potential. The values of these parameters, determined from the binding energy of the deuteron and from low-energy scattering data, depend only weakly on the shape of the potential assumed, so we will express the moments of the potential occurring in (6) in terms of s and b. For a square well potential b is the radius of the well in centimetres and if V_o is the depth of the well, $V = 102 \times 10^{-26} s b^{-2}$ MeV. For a Yukawa well $V(r) = -U_o \exp(-r/\beta)/r/\beta$, $b = 2.12\beta$ cm, and $U_o = 312 \times 10^{-26} s b^{-2}$ MeV. Substituting these values in the expressions for the moments of the potentials we get

$$\int V(r) r^2 dr = 34 \times 10^{-26} s b \quad \text{for both Yukawa and square well potentials and}$$

$$\int V(r) r^4 dr = 3.4 \times 10^{-26} s b^3 \quad \text{(square well)}$$

$$= 7.56 \times 10^{} s b^3 \quad \text{(Yulawa well).}$$

Hence we get the interaction energy expressed in terms of s and b,

$$E_{12}^{o} = -\frac{34\times10^{-26}\,sb}{R^3}\left[\left(I_o - \frac{0.1b^2}{R^2}I_{11}\right)(a_t+\varepsilon a_s) + \frac{0.1\,b^2}{R^2}I_{10}(b_t+\varepsilon b_s)\right]$$

for a square well potential and

$$E_{12}^{o} = -\frac{34\times10^{-26}\,sb}{R^3}\left[\left(I_o - \frac{0.22\,b^2}{R^2}I_{11}\right)(a_t+\varepsilon a_s) + \frac{0.22\,b^2}{R^2}I_{10}(b_t+\varepsilon b_s)\right]$$

for Yukawa well. There are similar expressions for the coordinate exchange term E_{12}^{M}. E_{12} is given in MeV and R and b are measured in centimetres. It is seen that the dependence of the interaction energy on the intrinsic range of the potential is very much stronger for a Yukawa interaction than for a square well interaction. The binding energy of the deuteron and low energy neutron-proton scattering data suggests a value s = 1.3 and $b = 2.4\times10^{-13}$ cm. The nuclear radius in the neighbourhood of ^{208}Pb is of the order of $R = 10^{-3}$cm. Thus we get an estimate for $34\times10^{-26}sb/R^3 = 0.2$MeV.

2(iv): Matrix Elements for Particular Configurations.

By application of first-order perturbation

theory, the dependence of the splitting of a two-particle configuration on the total angular momentum quantum number I has been investigated for three particular configurations of interest in the level structure of ^{210}Bi, namely ($i_{11/2}$; $i_{13/2}$), ($g_{9/2}$, $h_{9/2}$) and ($g_{9/2}$, $i_{13/2}$). The matrix elements of the interaction potential have been computed using the expressions derived in Chapter 2(i) for a square well neutron-proton interaction. In order to illustrate the effect of range and exchange on the proton-neutron interaction energy, E_{12} has been computed for all possible I-values in each of the three configurations ($i_{11/2}$, $i_{13/2}$), ($g_{9/2}$, $h_{9/2}$) and ($g_{9/2}$, $i_{13/2}$). The following cases have been considered:

(i) A zero range force with no spin dependence.

(ii) A Wigner force with b/R = 0.2.

(iii) A Majorana force with b/R = 0.2.

(iv) A Rosenfeld mixture giving saturation of nuclear binding energies (Rosenfeld 1948)

again with b/R = 0.2. The results are given in graphical form in ~~the~~ figure $\frac{11}{\lambda}$ with E_{12} plotted against I. The energy scale corresponds to choosing the interaction parameter 34×10^{-26} sb/R = 0.2 MeV in agreement with low-energy scattering data. The small value of ration

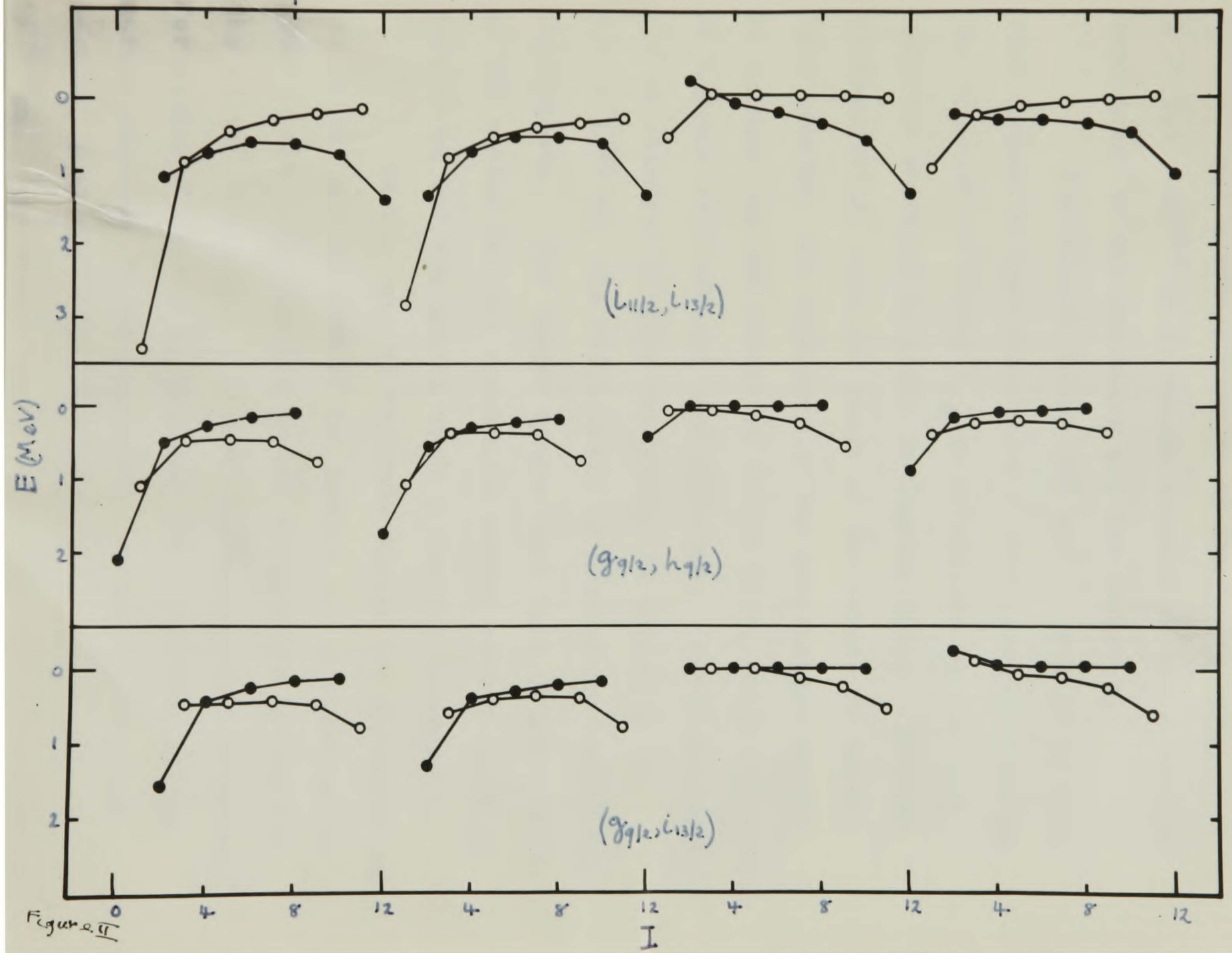


Figura n. 11

b/R = 0.2 corresponding to an intrinsic range of $b \doteq 1.6 \times 10^{-13}$ cm is chosen because of the essential inadequacy of the expansion (6) for larger b.

For Wigner forces with b/R = 0.2 it is seen that forces of zero range give a good approximation to the interaction energy E_{12} as calculated by the more accurate expressions given in Chapter 2(i). In each configuration, when the range of the forces is taken into account, the magnitude of the interaction energy is reduced for the states of lowest spin, while the states of highest spin are not much affected. These conclusions can be expected to hold for intrinsic ranges up to 2.4×10^{-13}cm, the value given by low-energy scattering experiments. For longer ranges than this, higher terms in the series for the coordinate matrix element must be considered and the method rapidly becomes unworkable.

In the case of Majorana forces the expansion as far as the second term of the series can be expected to give a fair approximation to the interaction energy for the states of highest I in the configurations considered. For states of lowest I, however, the magnitudes of the matrix elements of the interaction potential are much reduced, even for an intrinsic range as short as $b = 1.6 \times 10^{-13}$ cm. For ranges longer than this, and in particular

for ranges of the order of b = 2.4×10^{-13} cm predicted from scattering experiments, the zero range approximation to the value of the matrix element is completely inadequate and the expansion as a power series in the range of the potential is no longer useful. In all cases the effect of the finite range of the potential is to reduce the interaction energy in the states of low total angular momentum I, but for Majorana forces this effect is much stronger than for Wigner forces.

The effect of a spin exchange term in the interaction potential has been investigated by Pryce (1952) for forces of zero range. If a spin exchange component is included in the interaction potential, states in which the spins of the two particles are most nearly parallel are not affected, while the magnitude of the interaction energy is reduced for states in which the spins tend to be antiparallel, provided the spin exchange force is attractive in the triplet state. This result can be simply expressed in terms of the Nordheim number. If the two particles have total angular momentum quantum numbers j_1 and j_2 and orbital angular momentum numbers l_1 and l_2 we define this number as $N = (j_1 - l_1) + (j_2 - l_2)$. If N is odd the state of maximum I corresponds to parallel alignment of spins and the interaction energy is unaffected by the inclusion of a spin exchange term in the force,

while it is reduced in the state of minimum I. For N even the state of minimum I is unaffected and the interaction energy in the state of maximum I is reduced. These conclusions are not altered when the finite range of the nuclear force is included in the calculations. For N odd the Nordheim rule (Nordheim 1950) predicts that, in any configuration, a state of high angular momentum should have the lowest energy, while the state of lowest I should have the lowest energy for N even. This rule will hold for the configurations considered above provided the nuclear forces contain coordinate and spin exchange components of sufficient strength.

2(v): Interaction of Holes and Particles.

The method for calculating the matrix elements of internucleon potentials for a configuration of a closed shell plus a few particles, is well known and in the last sections we have considered the case of two extra unlike particles in detail. Because of the equivalence of particles and holes, the method of calculation of these matrix elements in a configuration consisting of a number of holes is identical to the particle case. In this section we will use the formalism of Appendix B to calculate the interaction energy of a proton (or neutron) outside a closed shell with a neutron (or proton) hole. We write

the state vector of the system as

$$|j_1 j_2, JM\rangle = \sum_{m_1 m_2} C(j_1 j_2 J, m_1 m_2)(-1)^{j_2-m_2} \eta_{j_1 m_1} \xi^{+}_{j_2 m_2} |\rangle$$

The vector $|\rangle$ represents the double closed shell, while $\eta_{j_1 m_1}$ and $\xi_{j_2 m_2}$ are creation operators for proton and neutron respectively. The neutron-proton interaction potential can be expressed in terms of creation and annihilation operators as

$$U = \sum_{np} u(|r_p - r_n|)$$

$$= \sum_{ab\alpha\beta} \eta_a \eta_b^{+} \xi_\alpha \xi_\beta^{+} \langle a\alpha|u|b\beta\rangle \qquad (7)$$

where the terms $\langle a\alpha|u|b\beta\rangle$ in (7) are the two particle matrix elements of the neutron-proton interaction. The matrix elements of U between two states $\eta_a \xi_\alpha^{+}|\rangle$ and $\eta_b \xi_\beta^{+}|\rangle$ can be evaluated using the usual methods of second quantization and yields the result

$$\langle|\xi_\beta \eta_b^{+} U \eta_a \xi_\alpha^{+}|\rangle = (E_c + E_p - E_n)\,\delta(a,b)\,\delta(\alpha,\beta) - \langle b\alpha|u|a\beta\rangle$$

E_c is the internal interaction energy of the closed shell, while E_p and E_n are the interaction energies

of the proton and the neutron hole respectively with the closed shell core. Thus for the coupled system we get

$$\langle j_1 \bar{j}_2, JM | V | j_1 \bar{j}_2, JM \rangle$$

$$= E_c + E_p - E_n - \sum_{\substack{m_1, m_2 \\ m_1', m_2'}} C(j_1 j_2 J, m_1 m_2) C(j_1 j_2 J, m_1' m_2')$$

$$\times (-1)^{2j_2 - m_2 - m_2'} \langle j_1 m_1, j_2 - m_2' | V | j_1 m_1', j_2 - m_2 \rangle$$

$$= E_c + E_p - E_n - \sum_{JJ'} (2J'+1) W(j_1 j_2 j_2 j_1, JJ')$$

$$\times \langle j_1 j_2, J'M | V | j_1 j_2, J'M \rangle$$

$$= E_c + E_p - E_n - E_{int}. \qquad (8)$$

In (8) the term $E_c + E_p - E_n$ is the energy of the unperturbed configuration, which is evaluated in this work from the known single particle and hole levels. The term E_{int} determine the splitting of the particle-hole configuration and this is the term of interest. It should be noted that the term E_{int} occurs with the opposite sign to the corresponding term in the case of the interaction of two extra-core particles or of two holes in a closed shell.

It is shown in Appendix B that the particle-

hole interaction energy, Hint, can be written in the same form as a particle particle interaction provided the following transformations are made:

(1) $P_\sigma \rightarrow 1 - P_\sigma$

(2) For an ordinary central interaction

$$\langle l_1 l_2, LM | V | l_1 l_2, LM \rangle \rightarrow \langle l_1 l_2, LM | V | l_1 l_2, LM \rangle$$

(3) For a coordinate exchange or Majorana potential

$$\langle l_1 l_2, LM | V | l_1 l_2, LM \rangle \rightarrow [C(l_1 l_2, L)]^2 \times \int V(r) Y^{*N}_L(\theta_1, \phi_1) Y^N_L(\theta_2, \phi_2) R(r_1, r_2) dr_1 dr_2$$

$R(r_1, r_2)$ is the product of radial wave functions introduced in Appendix A. The transformation for P_σ reduces to the results of Pryce (1953a). The expansion (5) of the coordinate matrix elements as a power series in the range of the nuclear forces can be carried out just as before, by making the substitutions

$P_0(L) \rightarrow P_0(L)$

$P_1(L) \rightarrow P_1(L)$ for an ordinary force.

$P_1(L) \rightarrow L(L+1) P_0(L)$ for a Majorana force.

In the last sections we have discussed the splitting of the levels of a particle-hole configuration

arising from states of unlike particles. When the particle and hole are states of like nucleons the results given above remain unchanged if we replace V in (8) by $V(1 - P_x P_\sigma)$.

Chapter III.

Some Applications of the Model.

We will discuss the excited states of several nuclei on the basis of the shell model and compare the predicted level systems with experiment. In this work we use individual particle levels given from a study of the states of ^{207}Pb, ^{207}Te, ^{209}Pb and ^{209}Bi Chapter I. Calculations of configuration splitting are made with zero-range nuclear forces or with the approximation for short range forces developed in Chapter II. The nuclei considered ^{210}Bi, ^{208}Te, ^{209}Pb and ^{207}Pb were chosen because they provide a variety of particle configurations. ^{210}Bi has a proton and a neutron outside double closed shells, ^{208}Te has a proton hole and an extra neutron, while ^{208}Pb and ^{207}Pb have four and six neutron holes respectively.

3(i): The Lower Levels of ^{210}Bi.

^{210}Bi exists in two isomeric states, a beta decaying state of low spin, in the following referred to as β, formed in the beta decay of ^{210}Pb (RaD); and a long-lived alpha decaying state denoted by α The state α probably has a high spin. It is formed following slow neutron capture in ^{209}Bi (Neumann, Howland and Perlman 1950). Goldhaber and Hill (1953) give the

energy of α, estimated from a closed cycle, as 25 ± 45 keV below the state β. The binding energy of the last neutron in ^{210}Bi, obtained from a closed cycle, is 4.54 MeV (Kinsey, Bartholomew and Walker 1951). Comparing this with the neutron binding energy in ^{209}Pb (3.87 MeV) we obtain a value of approximately 0.67 MeV for the proton-neutron interaction energy in both the isomeric states of ^{210}Bi.

The gamma-ray spectrum following the capture of thermal neutrons in ^{209}Bi consists of a broad line of maximum energy 4.17 MeV indicating ransitions to a group of states within an energy range of about 70 keV (Kinsey, Bartholomew and Walker 1951). This group of states lies about 0.37 MeV above the isomeric ground states α and β of ^{210}Bi, as shown by the difference between the neutron binding energy and the maximum gamma-ray energy following the neutron capture. The neutron capture state should have spin I = 4 or 5 and odd parity. Thus the group of states reached by the 4.17 MeV gamma emission can have spins in the range I = 2 to I = 7 if the gamma-rays correspond to electric dipole, magnetic dipole or electric quadrupole transitions. States formed in this group following gamma radiation from the neutron capture state would decay by a cascade of soft gamma-rays to one of the states α or β. Neither

of the states α or β is reached strongly by direct gamma transitions from the capture state.

The reaction $^{209}Bi(d,p)^{210}Bi$ shows levels in ^{210}Bi with neutron binding energies of 4.16, 2.52, 1.95 and 1.40 MeV (Harvey 1953). These levels should be compared with the states found in ^{209}Pb with the $^{208}Pb(d,p)^{209}Pb$ reaction. The relevant levels in ^{209}Pb have neutron binding energies of 3.87, 2.31, 1.84 and 1.27 MeV. The reaction cross sections for the various levels show a similar correspondence. This correspondence of levels and cross sections has been interpreted by Pryce (1952) and Harvey (1953) as indicating that these "levels" found in ^{210}Bi are the unresolved configurations formed by a proton in the ground state and the captured neutron in states corresponding to the independent particle states of ^{209}Pb. It should be noted that the neutron binding energy deduced by this method corresponds to the value obtained from the gamma-ray energies following slow neutron capture in ^{209}Bi. Thus it is probable that the group of states reached by gamma emission from the slow neutron capture state is the same as the lowest group of states reached in the $^{209}Bi(d,p)^{210}Bi$ reaction.

^{210}Pb(RaD) decays by beta emission to an excited state of ^{210}Bi. The transition energy is low, probably

about 18 keV, and the ft-value (log ft = 5.5, Bannermann and Curran 1952) is consistent with an allowed or first-forbidden transition. The excited state of ^{210}Bi formed in this beta transition decays to the state β by gamma emission. Formerly, about six soft gamma rays following the beta decay had been reported, of which the strongest and most energetic had an energy of 46.7 keV. The level diagram to account for these gamma rays was extremely complicated, requiring five or six levels of low spin very close together near the β state. More recent work by Wu, Boehm and Nagel (1953) confirms only the 46.7 keV. gamma ray and suggests that the other gamma rays previously reported were spurious. They estimate that the 46.7 keV gamma ray is emitted following 92 $\pm$ 5% of the beta transitions and suggest that the remainder of the transitions go directly to the β state. This work indicates that there is only one level of low spin near the β state. Measurements on the ratio of the L conversion line intensities suggest that the 46.7 keV gamma ray is an M1 transition. The β state of ^{210}Bi having odd parity, the 46.7 keV excited state should have spin I=1 or 0 and odd parity.

Recent experiments by Fred, Tomkins and Barnes (1953) and Smith (private communication) indicate that the state β of ^{210}Bi has spin I = 1 and a very small

magnetic moment, while theoretical studies of the shape of the beta spectrum of RaE are consistent with the assignment I = 1 and odd parity for this state (Yamada 1953, Rose and Osborn 1954). Lee-Whiting (1955) has made calculations with explicit individual particle assignments, and has shown that the shape of the spectrum of RaE is explainable with ratios of matrix elements calculated from the assumption that the state of ^{210}Bi is $(\pi i_{9/2}, \nu g_{9/2})_{1-}$. On the other hand the ft value for the β decay of ^{210}Pb is consistent with $(\pi g_{9/2}; \nu h_{9/2})_{0-}$ for the 46.7 keV excited state formed in the decay.

Levy and Perlman 1953 have investigated the half life for β decay of the α isomeric state. Their experiment was made by observing the growth of ^{210}Po. This could be due either to β decay of the α isomer or to a isomeric γ transition to the β state which decay to ^{210}Po in the normal 3.5 day β transition. The half life was found to be 7×10^8 years. It is shown in the next paragraphs that the isomeric state is expected to have a spin of at least I = 8 on theoretical grounds. Accepting this assignment, it is difficult to understand a half life for formation of ^{210}Po as short as 7×10^8 years.

We will now consider possible classifications of the low levels of ^{210}Bi consistent with the above

experimental information on the basis of a two-particle model. We have concluded from the calculated proton-neutron interaction energy matrix elements given above that a good estimate of proton-neutron interaction energies for the state of highest spin in any configuration is given by the zero range approximation to the interaction potential. For these states we can calculate the proton-neutron interaction energy from the expression

$$E_{12} = -u_0 I_0 (a_t + \varepsilon a_s)$$

where I_0 is the radial overlap integral defined above and

$$u_0 \simeq \cdot 2 \text{ MeV}$$

for nuclear forces consistent with low-energy proton-neutron scattering data. The above work also shows that it is difficult to make an accurate estimate of the matrix elements of the proton-neutron potential starting from a short range approximation for the states of low spin in any configuration. The results indicate, however, a possible theoretical basis for Nordheim's rule in a two-particle configuration. We will use this rule to make estimates of the interaction energy for such states. It appears from the diagram of energy levels given above that the interaction energy is small for states of spin intermediate between the maximum and

minimum values. In the following work values of a_s and a_t are taken from Pryce (1952).

We have seen that beta-decay are consistent with the assignment of the or single-particle configurations to the ground state of ^{209}Pb. Let us first assume the ground state of ^{209}Pb to be the $g_{9/2}$ level. Then the lowest two-particle configuration in ^{210}Bi would be the $(g_{9/2}, h_{9/2})$. This configuration is split by proton-neutron interaction and the levels of spin 0 and spin 9 depressed. The diagram of levels given for the $(g_{9/2}, h_{9/2})$ configuration shows that for a Rosenfeld mixture of range 1.6×10^{-13} MeV i/, the interaction energy is approximately 1 MeV in the state with spin I = 0 and 0.3 MeV in the state with spin I = 9. A natural choice of parameters could give an interaction energy in the state I = 0 of 0.7 MeV indicated by experiment and it is possible that the state $(g_{9/2}, h_{9/2})_{I=0}$ corresponds to the experimental state β. The state $(g_{9/2}, h_{9/2})_{I=9}$ could correspond to the experimental state α if depressed by configuration interaction with states $(i_{11/2}, h_{9/2})_{I=9}$ and $(i_{11/2}, f_{7/2})_{I=9}$.

Assigning the $g_{9/2}$ level to the ground state of ^{209}Pb the 0.75 MeV excited state is probably the $i_{11/2}$ level. Also it is probable that the 0.8 MeV excited

state in ^{209}Bi corresponds to the $f_{7/2}$ proton level. Making these identifications in ^{209}Pb and ^{209}Bi, we then have the $(g_{9/2}, f_{7/2})$ and $(i_{11/2}, h_{9/2})$ configurations lying close together at an excitation of about 0.8 MeV in ^{210}Bi before the proton-neutron interaction is considered. The proton-neutron interaction in each of the two states $(g_{9/2}, f_{7/2})_{I=8}$ and $(i_{11/2}, h_{9/2})_{I=10}$ is amut 1.4 MeV as calculated from (7) using square well wave functions to estimate I_0. Thus both these states might be expected to lie near the $(g_{9/2}, h_{9/2})_{I=0}$ state.

There are three possible low-lying states of spin I = 1, namely $(g_{9/2}, h_{9/2})_{I=1}$, $(g_{9/2}, f_{9/2})_{I=1}$, $(i_{11/2}, h_{9/2})_{I=1}$. All these states have odd parity and would be expected to lie close together at about 0.5 MeV above the $(g_{9/2}, h_{9/2})_{I=0}$ state before configuration interaction is considered. Configuration mixing could bring one state of spin I = 1 close to, above or below the $(g_{9/2}, h_{9/2})_{I=0}$ state. This doublet would then correspond to the state β and the 46.7 keV excited state. The present work cannot distinguish between the two possibilities for the spin of β, but an assignment of I = 0 or I = 1 is consistent with the predictions of the model. There may be a low-lying state of spin I = 2 formed from the above three configurations. All these states have odd parity.

3(i).

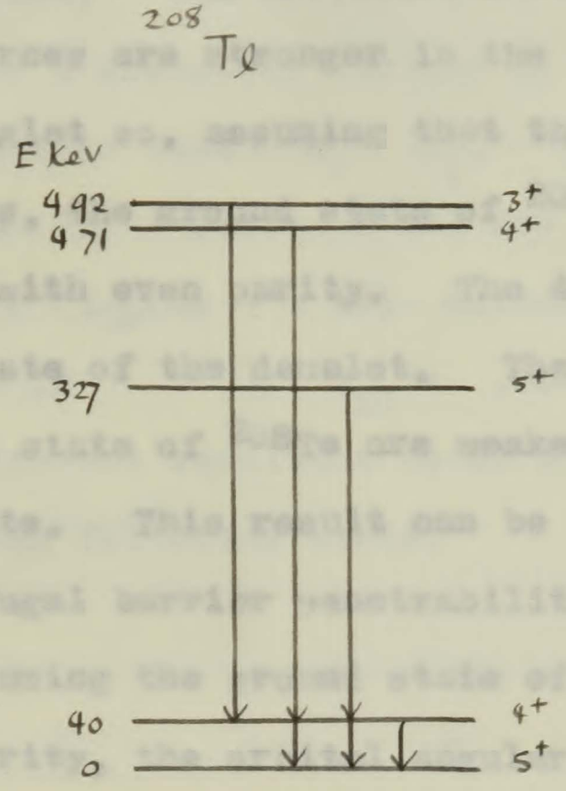
The group of levels reached by gamma emission following neutron capture in ^{209}Bi, should be the group of levels in the $(g_{9/2}, h_{9/2})$ configuration with intermediate spins I= 2 to I = 7. The group of states with neutron binding energy 4.16 MeV formed in the ^{209}Bi(d,p) ^{210}Bi reaction corresponds to the formation of ^{210}Bi in the $(g_{9/2}, h_{9/2})$ configuration. The probability of reaching a state of spin I with this reaction should be proportional to 21+1 if the interaction between the 83rd proton and the incoming neutron is small. The total number of states in the $(g_{9/2}, h_{9/2})$ configuration is 100, so the state of this configuration with spin I 0 would be formed with probability 0.01 and the state with I 1 with probability 0.03. Thus the state β of ^{210}Bi would not be reached strongly in the ^{209}Bi(d,p)^{210}Bi reaction. Similar considerations show that the experimental state α should not be reached with probability greater than .2 in the (d,p) reaction on ^{209}Bi.

3(ii): ^{208}Te.

The nucleus ^{208}Te has one proton hole in the double closed shell plus one additional neutron. It is formed following the α decay of ^{212}Bi and a study of the short range α particles and of the accompanying γ-rays yields information about the level structure

3(ii).

in this nucleus. This is summarised in the following diagram.



A study of internal conversion coefficients indicates that all the γ rays shown are emitted in M1 transitions. (Nielsen, private communication).

The low excited states of ^{208}Te will arise from configurations with one proton hole and one extra neutron. On the basis of our discussion in Chapter I of the available levels for a proton hole and an extra neutron, we expect that the lowest levels of ^{208}Te should come from the $(s_{1/2}^{-1}, g_{9/2})$ configuration, giving rise

to a doublet with spins I = 4 and 5. This doublet is degenerate to the first order for spin independent interactions. It is known, however, that in the deutron the nuclear forces are stronger in the triplet state than in the singlet so, assuming that this still holds inside the nucleus, the ground state of ^{208}Te should have a spin I = 5 with even parity. The 40 KeV level will be the 4+ state of the doublet. The α transitions to the ground state of ^{208}Te are weaker than those to the 40 KeV state. This result can be understood on the basis of centrifugal barrier penetrabilities for the α particles since, assuming the ground state of ^{212}Bi has spin I = 1 and odd parity, the orbital angular momentum of the particle in the ground state transition is ℓ = 5, while in the transition to the 40 keV state it is only ℓ = 3.

If the ground state and first excited states have spins I = 5 and 4 respectively then it is expected that the higher states should have the following spin assignments,

327 KeV, I = 4 or 5; 471 KeV, I = 4 or 5;

492 KeV, I = 3.

Further excited states come from the configurations $(d_{3/2}^{-1}, g_{9/2})$, $(d_{5/2}^{-1}, g_{9/2})$ and $(s_{1/2}^{-1}, i_{11/2})$; with energies of 350 KeV, ~ 700 KeV and 750 KeV respectively

before interparticle interaction is considered. The lowest group of states comes from the $(d_{3/2}^{-1}, g_{9/2})$ configuration. The splitting of this configuration calculated for a Rosenfeld mixture by the methods of Chapter II has the form $E_{int} = E_0 + E_1$. E_0 comes from the term $.2(I_0 - I_{11} a^2/R^2)(as + \varepsilon at)$ and E_1 from the term $.2 \times a^2/R^2 I_{10}(bs + \varepsilon bt)$. Assuming values $.2(I_0 - I_{11} a^2/R^2) = .5$ MeV and $.2 \times a^2/R^2 I_{10} = .005$ MeV values of E_0 and E_1 for different resultant total angular momenta I are given in the following table.

I	3	4	5	6
E_0	−.66	−.32	−.16	−.77
E_1	.04	.06	.03	.21
E_{int}	−.62	−.24	−.13	−.56

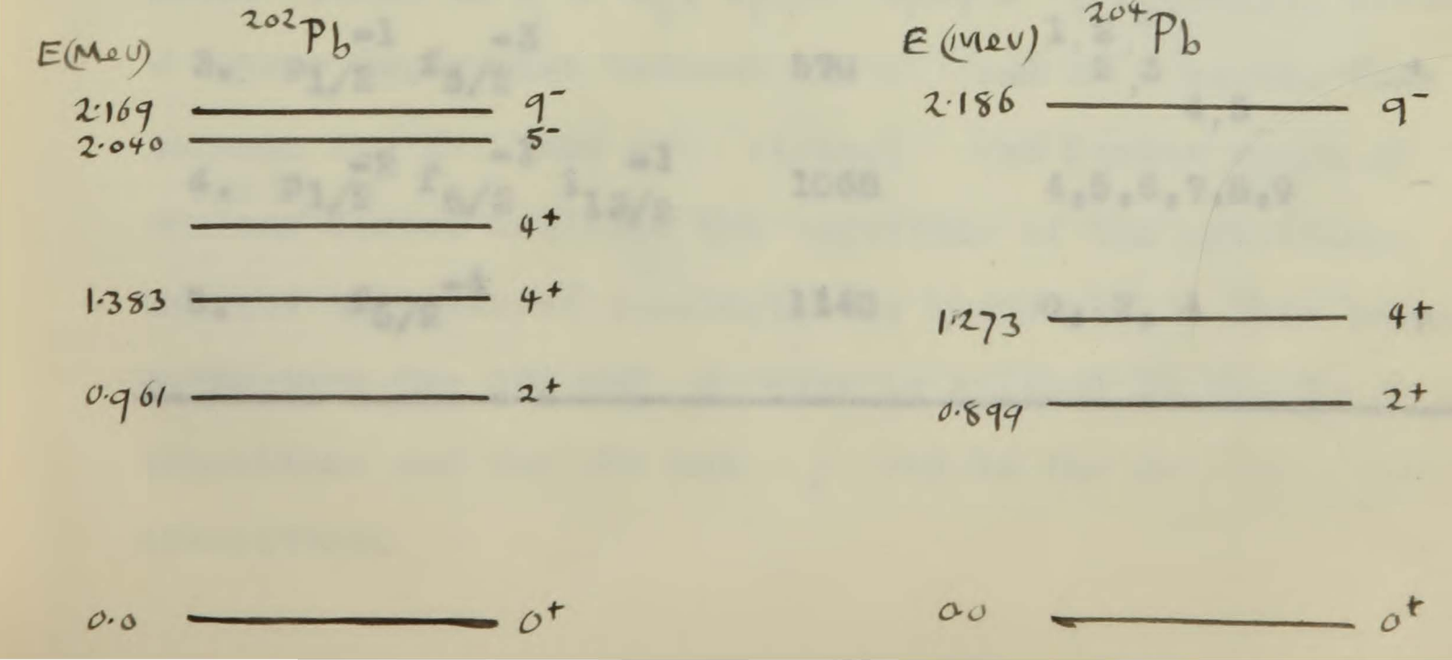
To this approximation the independent particle model predicts a level sequence I = 5, 4, 3 and 6 where the 3 and 6 levels may be interchanged for slightly different parameters. This results is in quantitative agreement with the observed sequence; but it seems impossible to get

3(ii).

agreement on the level spacing and in particular to bring the I= 3 level down close to the I = 4 level. However the $(d_{5/2}^{-1}, g_{9/2})$ configuration is close to the $(d_{3/2}^{-1}, g_{9/2})$. Since the two configurations have the same orbital wave functions strong configuration interaction is expected. This could change the level spacing considerably.

3(iii): ^{204}Pb and ^{202}Pb.

These two nuclei, deficient in four and six neutrons respectively, have very similar level structures. They can be formed in an isomeric state with I 9 and odd parity in the reactions ^{205}Te(d, 3n) ^{204}Pb, ^{203}Te(d,n) ^{204}Pb and the isomeric states decay to ground by a cascade of γ transitions. Level diagrams proposed by Bergstrom and Wapstra (1955) and Krohn and Raboy (1954) are given below.



3(iii).

In both nuclei the order of the two lowest transitions is uncertain; but theoretical considerations given later favour the order shown.

Applying the independent particle model to ^{204}Pb and ^{202}Pb, the low energy levels of these nuclei are described in terms of the excitation of the neutron holes. The configurations which contribute, together with their energies before perturbation by the inter-nucleon forces, can be found from the single hole levels known from the study of ^{207}Pb (1,4). The following table gives these configurations with their energies, and the possible spins and parities of their levels.

^{204}Pb.

		Energy	Spins	Parity
1.	$p_{1/2}^{-2} f_{5/2}^{-2}$	0	0, 2, 4	+
2.	$p_{1/2}^{-2} f_{5/2}^{-1} p_{3/2}^{-1}$	300	1,2,3,4	+
3.	$p_{1/2}^{-1} f_{5/2}^{-3}$	570	1, 2, 2, 3, 4, 5	+
4.	$p_{1/2}^{-2} f_{5/2}^{-1} i_{13/2}^{-1}$	1068	4,5,6,7,8,9	−
5.	$f_{5/2}^{-4}$	1140	0, 2, 4	+

3(iii).

^{202}Pb.

		Energy	Spins	Parity
1.	$p_{1/2}^{-2} f_{5/2}^{-4}$	0	0, 2, 4	+
2.	$p_{1/2}^{-2} f_{5/2}^{-3} p_{3/2}^{-1}$	300	0,1,2,3 1,2,3,4 3,4,5,6	+
3.	$p_{1/2}^{-1} f_{5/2}^{-5}$	570	2, 3	+
4.	$p_{1/2}^{-2} f_{5/2}^{-3} i_{13/2}^{-1}$	1068	5,6,7,8 4,5,6,7,8,9 2,3,4,5,6,7,8,9,10,11	−
5.	$f_{5/2}^{-6}$	1140	0	+

To the first order of perturbation in the inter-particle interactions, the ground state 0+ and the first two excited states (2+ and 4+) in ^{204}Pb are states of the configuration $(p_{1/2}^{-2} \; f_{5/2}^{-2})$. The splitting of this configuration (for nuclear forces of zero range) is proportional to $\langle I \; 0, \; I | 5/2 \; 5/2; I \rangle^2 P_0(33.1)$, giving a larger separation between the 0+ and 2+ states than between the 2+ and 4+ states. The finite range of nuclear forces modifies the magnitude of the splitting; but the qualitative conclusion is unchanged. This indicates that the 899 KeV γ rays is emitted in the $2+ \Rightarrow 0+$ transition and the 374 KeV γ ray in the $4+ \Rightarrow 2+$ transition.

3(iii).

The independent particle model predicts a group of states arising from the $(p_{1/2}^{-2}\ f_{5/2}^{-1}\ p_{3/2}^{-1})$ configuration with spins I 1,2,3,4, lying close together at an excitation of about 300 KeV above the 4 state of the $(p_{1/2}^{-2}\ f_{5/2}^{-2})$ configuration. The 4 state of this group should be reached in the decay of the 9- isomeric state with an intensity of about 1%; but the ray emitted in this transition has not been reported.

There should be a group of levels from the $p_{1/2}^{-2}\ f_{5/2}^{-1}\ i_{13/2}^{-1}$ configuration at an excitation of 1MeV above the 1273 KeV 4+ state. This configuration is split by internucleon forces and, for zero range forces, the splitting is proportional to $2a_S = 2\,|\langle I\,0,I|\,5/2\ 13/2,I\rangle|^2\ P_0(36,I)$

I	4	5	6	7	8	9
as	0	0.15	0	0.31	0	0.78

The lowest level of the configuration is 9- and evidently corresponds to the observed isomeric state at 2186 KeV.

The ground state and the first two excited states of ^{202}Pb should be the three states of the $p_{1/2}^{-2}\ f_{5/2}^{-4}$ configuration. In ^{204}Pb there are two holes in the $f_{5/2}$ level and in ^{202}Pb two particles in the same level. Because of the symmetry of particles and holes the splitting

of the configuration $(p_{1/2}^{-2}\ f_{5/2}^{-4})$ in ^{202}Pb should be the same as the splitting of the $p_{1/2}^{-2}\ f_{5/2}^{-2}$ in ^{204}Pb. The experiments show that this is true to a good approximation.

The splitting of the $(p_{1/2}^{-2}\ f_{5/2}^{-3}\ p_{3/2}^{-1})$ configuration can be viewed in the following way. The three $f_{5/2}$ holes can couple to spins I= 5/2, 3/2 and 9/2. Of these the 5/2 level is lowest, and it can be shown that the lowest levels of the above configuration come from a $p_{3/2}$ hole coupled to this 5/2 state. The 4^+ level from this group is observed in the decay of the 5^- state, at an energy of 450 KeV above the $(p_{1/2}^{-2}\ f_{5/2}^{-4})_{1\ 4}$ state.

Similar considerations apply to the $(p_{1/2}^{-2}\ f_{5/2}^{-3}\ i_{13/2}^{-1})$ configuration. Again with zero range forces it is expected that the 9^- state should be lowest. Presumably the 5^- state comes from this configuration though calculation with zero range forces always put it above the 9^- state.

The assignments given in this Chapter differ in detail from those made by Maeder et al.(1954); but the general features agree.

Conclusion.

New experimental evidence available since Pryce (1952) was written has greatly simplified the theoretical

task of classifying the low levels of ^{210}Bi. Now that the γ ray spectrum following the β decay of ^{210}Pb has been reinvestigated and simplified, it is no longer necessary to postulate a large splitting of high configurations ot account for a number of closely spaced levels of low spin near the ground state. A much more natural assignment to the values of the various constants will explain the experimental picture. Pryce's calculations with zero range forces suggested that the ground state of ^{210}Bi should have a low spin. However consideration of the range and exchange properties of the nuclear forces has shown that the lowest state is of odd parity but may have a low spin I = 1 or a high spin I = 8,9,10.

A study of the experimental level structure of ^{208}Te, ^{204}Pb and ^{202}Pb in the light of the independent particle model gives considerable insight into the structure of these nuclei. It can be seen how the various levels appear and brings home rather strikingly the point that studies of γ cascades in nuclei often reveals only a very limited number of their possible excited states. For example in ^{204}Pb there must be at least 10 states lying below the 9$^-$ isomeric state while only three of them have been observed. On the other hand in ^{208}Te it seems fairly certain that all states, with an excitation of less than 500 KeV are known.

The fact that single particle wave functions may not give a good approximation to the low state of ^{209}Pb, ^{209}Bi, ^{207}Pb and ^{207}Te; together with the inadequacy of the zero range approximation to nuclear forces and of the first order perturbation treatment make it difficult and unprofitable to make exact theoretical calculations on the independent particle model. On the other hand the model can explain in a semi-quantitative way the observed level structure of nuclei near the ^{208}Pb double closed shell. It is concluded that the independent particle model with the simple refinements presented in this thesis can have useful application in conjunction with experimental investigations made in this region.

Appendix A.

Evaluation of the Integrals over the Density Function.

First we transform the expression (2) for the density function to a form more suitable for further calculation. Spherical harmonics of the same argument occurring in (2) can be coupled using vector coupling coefficients, and then the sum over vector coupling coefficients reduces to a Racah coefficient (Racah 1942). Putting $\psi_1^*(r_1)\,\psi_2^*(r_2)\,\phi_1(r_1)\,\phi_2(r_2) = R(r_1, r_2)$ the density function reduces to

$$\rho_L(r_1, r_2) = R(r_1, r_2)\,(-1)^{\ell_1+\ell_1'+L} \times \sum_{J,N} \left[W(\ell_1\ell_1'\ell_2\ell_2', JL)\, Y_J^{*N}(\theta_1,\phi_1)\, Y_J^{N}(\theta_2,\phi_2)\, C(\ell_1\ell_1', J)\, C(\ell_2\ell_2', J) \right] \tag{A 1}$$

The coefficient $C(\ell_1\ell_1', J)$ is closely related to the vector coupling coefficient and has the simple expression

$$C(\ell\ell', J) = \left[\frac{(2\ell+1)(2\ell'+1)}{4\pi(2J+1)}\right]^{1/2} C(\ell\ell' J, 00)$$

$$= 0 \quad \text{if } \ell+\ell'+J \text{ is odd}$$

$$= (-1)^{g-J}\left[\frac{(2\ell+1)(2\ell'+1)(2g-2J)!\,(2g-2\ell)!\,(2g-2\ell')!}{4\pi(2g+1)!}\right]^{1/2} \times \frac{g!}{(g-J)!\,(g-\ell)!\,(g-\ell')!} \quad \text{if } 2g = \ell+\ell'+J \text{ is even.}$$

The product $C(\ell_1 \ell_1', J)\ C(\ell_2 \ell_2', J)$ can be expanded as a series in the products $C(\ell_1 \ell_2, \lambda)\, C(\ell_1' \ell_2', \lambda)$ the coefficients in the expansion being Racah coefficients

$$C(\ell_1 \ell_1', J)\, C(\ell_2 \ell_2', J) = (-1)^{\ell_1 + \ell_2'} \sum_{\lambda} \left[(2\lambda+1)\, W(\ell_1 \ell_1' \ell_2 \ell_2', J\lambda) \right.$$

$$\left. \times\ C(\ell_1 \ell_2, \lambda)\, C(\ell_1' \ell_2', \lambda) \right] \qquad (A2)$$

The expansion (A2) follows from the definition of the Racah coefficients in terms of vector coupling coefficients, and from the relation connecting $C(\ell_1 \ell_2, \lambda)$ with the vector coupling coefficient $C(\ell_1 \ell_2 \lambda, 00)$ Finally (A1) and (A2) give

$$\rho_L(\underline{r}_1, \underline{r}_2) = R(r_1, r_2)(-1)^{L+\ell_1'+\ell_2'} \sum_{\lambda J N} \left[(2\lambda+1)\, W(\ell_1 \ell_1' \ell_2 \ell_2', JL)\, W(\ell_1 \ell_1' \ell_2 \ell_2', J\lambda) \right.$$

$$\left. \times\ C(\ell_1 \ell_2, \lambda)\ C(\ell_1' \ell_2', \lambda)\ Y_J^{*N}(\Omega_1, \phi_1)\ Y_J^{N}(\Omega_2, \phi_2) \right] \qquad (A3)$$

The integrals to be evaluated are of the form

$$4\pi \int \left[(\nabla^2)^k \rho_L'(\underline{r}, \underline{s}) \right]_{\underline{s}=0} d\underline{r}$$

where ∇^2 is the Laplacian operating on $\underline{s}$ in the density function $\rho_L'(\underline{r}, \underline{s})$.

We now go on to consider how the integrals (A4)

may be evaluated and so calculate the first few terms of the series (5) for F(L). In the first term of the series we have the integral $4\pi \int \rho_L(\underline{r},\underline{r})\, d\underline{r}$ Substituting the expression (A3) for $\rho_L(\underline{r}_1,\underline{r}_2)$ and carrying out the integration we get

$$4\pi \int \rho_L(\underline{r},\underline{r})\, d\underline{r} = (-1)^{L+\ell_1'+\ell_2'} \int R(r,r) r^2 dr$$

$$\times \sum_{J\lambda} (2J+1)(2\lambda+1) W(\ell_1 \ell_1' \ell_2 \ell_2', J\lambda) W(\ell_1 \ell_1' \ell_2 \ell_2', JL) P_0(\lambda)$$

$$= P_0(L) \int R(r,r) r^2 dr \qquad (A5)$$

Equation (A5) follows from the orthogonality relation for Racah coefficients (Racah 1942). Here

$$P_0(L) = 4\pi\, C(\ell_1 \ell_2, L)\, C(\ell_1' \ell_2', L)$$

$$= 0$$

unless $\ell_1+\ell_2+L$ and $\ell_1'+\ell_2'+L$ are both even.

The second term in the series for F(L) contains the integral

$$4\pi \int \left[\nabla^2 \rho_L'(\underline{r},\underline{s})\right]_{\underline{s}=0} d\underline{r}$$

We can write the Laplacian in the form

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - \frac{1}{r^2} \underline{L}^2$$

where $\underline{L}^2$ is the square of the orbital angular momentum operator of the second particle. Putting this form of the Laplacian into the integral splits into the sum of two parts. The first reduces to

$$-P_0(L)\int \frac{d}{dr}\left(\psi_1^*(r)\,\phi_1(r)\right)\frac{d}{dr}\left(\psi_2^*(r)\,\phi_2(r)\right)r^2\,dr \qquad (A6)$$

on integration by parts. The second part is

$$P_1(L)\int R(r,r)\,dr \qquad (A7)$$

with

$$P_1(L) = (-1)^{L+l+l_1'+l_2'}\sum_{\lambda J} P_0(\lambda)\,(2\lambda+1)(2J+1)$$
$$\times\, J(J+1)\,W(l_1 l_1' l_2 l_2', JL)\,W(l_1 l_1' l_2 l_2', J\lambda) \qquad (A8)$$

$P_1(L)$ is not yet in a form suitable for computation because the Racah coefficients have complicated sum expressions and are not tabulated for high values of the angular momentum quantum numbers. The expression for $P_1(L)$ may be simplified however, by using a recurrence relation for Racah coefficients. This recurrence relation, obtained by specialization of parameters from a formula given by Biedenharn, Blatt and Rose (1953) is

$$W(\ell_2 J\,1\,\ell_2', \ell_2\ell_2')\; W(\ell_1 J\,1\,\ell_2', \ell_1'\ell_2)$$

$$= \sum_\lambda (2\lambda+1)\; W(\ell_1\lambda J\ell_2', \ell_2\ell_2')\; W(1\lambda\,\ell_2'\ell_1', L\ell_2')\, W(1\lambda\ell_2\ell_1, L\ell_2)$$

or using symmetry relations for Racah coefficients (Racah 1942)

$$W(\ell_1\ell_1'\ell_2\ell_2', JL)\; W(\ell_2\,1\,J\ell_2', \ell_2\ell_2')$$

$$= \sum_\lambda (-1)^{L+\lambda} (2\lambda+1)\; W(\ell_1\ell_1'\ell_2\ell_2', J\lambda)\, W(1\lambda\,\ell_2'\ell_1', L\ell_2')\, W(1\lambda\ell_2\ell_1, L\ell_2) \qquad (A9)$$

Now

$$W(\ell_2\,1\,J\ell_2', \ell_2\ell_2') = -2\left[\frac{(2\ell_2-1)!\,(2\ell_2'+1)!}{(2\ell_2+2)!\,(2\ell_2'+2)!}\right]^{1/2}\left[J(J+1) - \ell_2(\ell_2+1) - \ell_2'(\ell_2'+1)\right]$$

and putting

$$\Delta(\lambda, L) = W(1\lambda\ell_2'\ell_2', L\ell_2')\; W(1\lambda\ell_2\ell_1, L\ell_2)\left[\frac{(2\ell_2+2)!\,(2\ell_2'+2)!}{(2\ell_2+1)!\,(2\ell_2'-1)!}\right]^{1/2}$$

$$- \frac{\ell_2(\ell_2+1) + \ell_2'(\ell_2'+1)}{(2L+1)}\,\delta(\lambda, L)$$

(A9) gives

$$J(J+1)\, W(\ell_1\ell_1'\ell_2\ell_2', JL) = \sum_\lambda (-1)^{L+\lambda+1}\, \Delta(\lambda, L)\, W(\ell_1\ell_1'\ell_2\ell_2', J\lambda). \qquad (A10)$$

Substituting (A10) in (A8) and using the orthogonality relations for Racah coefficients we obtain an expression for $P_1(L)$

$$P_1(L) = \sum_{\lambda=L-1}^{L+1} (2\lambda+1)\ \Delta(\lambda,L)\ P_0(\lambda). \qquad (A\,11)$$

The coefficients $\Delta(\lambda,L)$ have the following simple form and can be easily evaluated

(i) $\Delta(\lambda,L) = \Delta(L,\lambda)$.

(ii) $\Delta(\lambda,L) = 0$ unless $\lambda+1 \geq L \geq \lambda-1$

(iii) $$\Delta(L,L-1) = \frac{(2L-2)!}{(2L+1)!}\Big[(L+\ell_1+\ell_2+1)(L+\ell_1-\ell_2)(L+\ell_2-\ell_1)(\ell_1+\ell_2-L+1)$$
$$\times\ (L+\ell_1'+\ell_2'+1)(L+\ell_1'-\ell_2')(L+\ell_2'-\ell_1')(\ell_1'+\ell_2'-L+1)\Big]^{1/2}$$

$$\Delta(L,L) = \frac{2(2L-1)!}{(2L+2)!}\Big[(\ell_1(\ell_1+1)-\ell_2(\ell_2+1))(\ell_1'(\ell_1'+1)-\ell_2'(\ell_2'+1))$$
$$- L(L+1)[\ell_1(\ell_1+1)+\ell_2(\ell_2+1)+\ell_1'(\ell_1'+1)+\ell_2'(\ell_2'+1)]$$
$$+ [L(L+1)]^2\Big]$$

There is an alternative expression for $P_1(L)$ which is particularly simple and useful in the special case $\ell_1 = \ell_1'$ and $\ell_2 = \ell_2'$. We have the expression for $P_1(L)$ from A8

$$P_1(L) = (2\ell_1+1)(2\ell_2+1)(-1)^L \sum_J J(J+1)\, W(\ell_1\ell_1\ell_2\ell_2, JL)$$

$$\times\, C(\ell_1\ell_1 J, 0\,0)\, C(\ell_2\ell_2 J, 0\,0) \qquad (A12)$$

Recurrence relations for Clebsh Gorden Coefficients (Racah 1942) give

$$J(J+1)\; C(\ell_1\ell_1 J, 0\,0)\; C(\ell_2\ell_2 J, 0\,0)$$

$$= [\ell_1(\ell_1+1)\ell_2(\ell_2+1)]^{1/2}\left[C(\ell_1\ell_1 J, 1\,0) + C(\ell_1\ell_1 J, 0\,1)\right]$$

$$\times\left[C(\ell_2\ell_2 J, 1\,0) + C(\ell_2\ell_2 J, 0\,1)\right]$$

and substituting in A(12) we get

$$P_1(L) = 2[\ell_1(\ell_1+1)\ell_2(\ell_2+1)]^{1/2}\frac{(2\ell_1+1)(2\ell_2+1)}{2L+1}\left[C(\ell_1\ell_2 L, 0\,0)\, C(\ell_1\ell_2 L, 1\,{-1})\right.$$

$$\left. + C(\ell_1\ell_2 L, 0\,1)\, C(\ell_1\ell_2 L, 1\,0)\right] \qquad A13$$

Similarly for exchange forces when we have $\ell_1' = \ell_2$, $\ell_2' = \ell_1$,

and a multiplicative factor $(-1)^{L+l_1+l_2}$

$$P_1^{M_1}(L) = (2l_1+1)(2l_2+1)(-1)^{l_1+l_2}\sum_J (J)(J+1)\,W(l_1 l_2 l_2 l_1, J L)$$

$$\times \left[C(l_1 l_2 J, 0\,0)\right]^2 \Big].$$

which reduces to

$$P_1^{M}(L) = \frac{(2l_1+1)(2l_2+1)}{2L+1}\Big[2\left[l_1(l_1+1)\,l_2(l_2+1)\right]^{1/2} C(l_1 l_2 L, 0\,0)\, C(l_1 l_2 L, 1\,-1)$$

$$+ l_1(l_1+1)\left[C(l_1 l_2 L, 1\,0)\right]^2 + l_2(l_2+1)\left[C(l_1 l_2 L, 0\,1)\right]^2\Big]$$

A(14)

Appendix B.

Second Quantization Methods.

When investigating nuclei with nearly closed shells it is often useful to use the formalism of second quantization and write the independent particle state vector for the system as creation and annihilation operators operating on the closed shell state vector. We will write the creation operator (Fermi-Dirac) for a nucleon of one particular type, either proton or neutron, in a state with angular momentum quantum numbers (j,m), as $\eta_{j,m}^{\dagger}$. Normally it is necessary to give other quantum numbers besides (j,m) in order to specify the individual particle state completely; but for the sake of simplicity of notation we will not write these in explicity. In terms of these creation operators the state vector of a system with a closed shell plus an extra particle in the nucleon state $|j, m\rangle$ is $\eta_{jm}^{\dagger}|\rangle$ where $|\rangle$ is the closed shell state vector. 2j nucleons in a level, coupled up to give a total angular momentum (j,m) are equivalent to an almost closed subshell with one hole in the single particle state (j,m). Thus we can write the state vector of such a system as an annihilation operator for the state $|j-m\rangle$, operating on the closed shell state, i.e. $(-1)^{j-m}\eta_{jm}^{\dagger}|\rangle$.

The operator η^{+}_{jm} is the adjoint of η_{jm} while the factor $(-1)^{j-m}$ must be included in order that the resulting state vector may have the right transformation properties under the rotation group.

For two like particles outside a closed shell coupled up to give a total angular momentum $(J.M)$ we can write the state of the system as

$$|j_1 j_2, JM\rangle = N \sum_{m_1+m_2=M} C(j_1 j_2 J, m_1 m_2)\, \eta_{j_1 m_1} \eta_{j_2 m_2} |\rangle$$

N is a normalization factor which is equal to 1 unless the two particles are in the same level, when it is equal to $1/\sqrt{2}$ Or again for a proton outside a closed shell plus a neutron hole coupled up to give a total angular momentum $(J, M.)$ we have

$$|j_1 \bar{j}_2, JM\rangle = \sum_{m_1+m_2=M} (-1)^{j_2-m_2} C(j_1 j_2 J, m_1 m_2)\, \eta_{j_1 m_1} \xi^{+}_{j_2 m_2} |\rangle$$

where now η is a creation operator for protons and ξ a creation operator for neutrons. The wave functions of more complicated configurations can be built up in an analogous way. It should be noted that wave functions described in the formalism of second quantization are antisymmetrical from the start and need not be combined with fractional parentage coefficients. In fact it is

possible to write the fractional parentage coefficient for the formation of the state $|j^n, J M\rangle$ with n identical Fermi-Dirac particles coupled up to give a total angular momentum (J M), from the states $|j^{n-1}, J' M'\rangle$ as the reduced matrix element of a creation operator. In the notation of Flowers (1952)

$$\langle j^n, J \{| j^{n-1} J', j J\rangle = \frac{1}{\sqrt{n}} \langle J || \eta_j || J' \rangle$$

The definition of the reduced matrix element used here is that of Biedenharn and Rose (1952) where

$$\langle J M | \eta_{jm} | J' M' \rangle = C(J' j J, M' m) \langle J || \eta_j || J' \rangle$$

The second quantization formalism is particularly useful when we are dealing with an independent particle model and wish to evaluate matrix elements of operators which are sums of single particle operators or of two particle operators. It is well known for systems of identical Fermi-Dirac particles (Dirac) that if $U = \sum_{\text{particles}} u_i$ and $V = \sum_{ij} v_{ij}$ then the operators U and V can be expanded in the form

$$U = \sum_{a,b} \eta_a \eta_b^+ \langle a | u | b \rangle$$

$$= \sum_{abcd} \eta_a \eta_b \eta_d \eta_c \langle a b | v | c d \rangle .$$

η_a and η_a^+ are the creation and annihilation operators for single particle state $|a\rangle$ and $\langle a|u|b\rangle$ and $\langle ab|v|cd\rangle$ are simply matrix elements of the single and two particle operators u and v respectively between independent particle states. If the state vectors of the system are expressed in terms of creation and annihilation operators then matrix elements can be evaluated using the well known commutation relations for these operators:

$$\eta_a \eta_b + \eta_b \eta_a = 0$$

$$\eta_a^+ \eta_b^+ + \eta_b^+ \eta_a^+ = 0$$

$$\eta_a \eta_b^+ + \eta_b^+ \eta_a = \delta_{ab}$$

Matrix elements of sum operators for mixed systems e.g. of protons and neutrons can be evaluated similarly.

Formulae for matrix elements expressed in terms of fractional parentage coefficients can be found easily with the formalism of second quantization. For example if $T_{kq} = \sum_i t_{ikq}$ is a sum of independent particle tensor operators and $|J,M\rangle$ and

$|J^1, M^1\rangle$ are two n particle states. We can write

$$\langle J'M'|T_{kq}|JM\rangle = \sum_{\substack{jm \\ j'm'}} \langle J'M'|\eta_{j'm'}\eta^+_{jm}|JM\rangle \times \langle j'm'|t_{kq}|jm\rangle$$

$$= \sum_{\substack{mm'M_0 \\ jj'J_0}} \langle J'M'|\eta_{j'm'}|J_0M_0\rangle\langle J_0M_0|\eta^+_{jm}|JM\rangle \times \langle j'm'|t_{kq}|jm\rangle$$

The matrix elements in this equation can be replaced by products of reduced matrix elements and Clebsh-Gordon coefficients and the latter summed to give a Racah coefficient. Thus we have

$$\langle J'\|T_k\|J\rangle = \sum_{jj'J_0} U(J'kJ_0j, Jj')\langle j'\|t_k\|j\rangle \times \langle J'\|\eta_{j'}\|J_0\rangle\langle J\|\eta_j\|J_0\rangle^*$$

with $U(J'kJ_0j, Jj') = [(2J+1)(2j'+1)]^{1/2} W(J'kJ_0j, Jj')$

Similarly it can be shown that matrix elements of a sum of two particle interactions can be expanded in terms of two particle matrix elements by the equation

$$\langle \alpha JM|V|\beta JM\rangle = \sum_{\substack{k,J_1,J_1',J_2 \\ \alpha'\alpha''\beta'}} U(j_1j_2JJ_2, kJ_1)U(j_1'j_2'JJ_2, kJ_1')\langle j_1j_2, kM|V|j_1'j_2', kM\rangle \times \langle \alpha J\|\eta_{j_1}\|\alpha'J_1\rangle\langle \alpha'J_1\|\eta_{j_2}\|\alpha''J_2\rangle \langle \beta J\|\eta_{j_1'}\|\beta'J_1'\rangle^*\langle \beta'J_1'\|\eta_{j_2'}\|\alpha''J_2\rangle^*$$

Appendix C.

In 2.4 we obtained an expression for the energy of interaction of a particle and a hole in the form,

$$E_{int} = \sum_{J'} (2J'+1)\, W(j_1 j_2 j_2 j_1, J J')\, \langle j_1 j_2, J'N | U | j_1 j_2, J'N \rangle \quad (C\,1)$$

Transforming from the jj to the (L,S) coupling scheme, and, assuming the potential U is a product spin and an orbital components $U = V_x V_\sigma$ its matrix element reduce to

$$\langle j_1 j_2, JN | U | j_1 j_2, JN \rangle$$

$$= \sum_{LS} \left[X(l_1 l_2 \tfrac{1}{2} \tfrac{1}{2}, LS, j_1 j_2, J) \right]^2 \langle l_1 l_2, LM | V_x | l_1 l_2, LM \rangle \langle S | V_\sigma | S \rangle \quad (C\,2)$$

The transformation coefficients from jj to (L,S) coupling in (C 2) are written in the notation of Jahn and Hope 1954 and are related to $9\text{-}j$ symbol of Fano (1953) by the equation

$$X(l_1 l_2 s_1 s_2, L S, j_1 j_2, J) = \left[(2j_1+1)(2j_2+1)(2L+1)(2S+1)\right]^{1/2} \times \begin{Bmatrix} l_1 & s_1 & j_1 \\ l_2 & s_2 & j_2 \\ L & S & J \end{Bmatrix}$$

The expression for Eint can be simplified by using a relation between the X coefficients and Racah coefficients

$$\sum_j \Big[(-1)^{j_1+j_2-(j_1'+j_2')}(2j+1)\,W(j_1 j_2 j_2' j_1', Jj)$$

$$\times X(l_1 s_1 l_2' s_2', j_1 j_2', l_3, j)\,X(l_1' s_1' l_2 s_2, j_1' j_2, l_3, j)\Big]$$

$$=\sum_{LS}\Big[(-1)^{l_1+l_2-(l_1'+l_2')}(2l+1)\,W(l_1 l_2 l_2' l_1', Ll)$$

$$\times(-1)^{s_1+s_2-(s_1'+s_2')}(2s+1)\,W(s_1 s_2 s_2' s_1', Ss)$$

$$\times X(l_1 s_1 l_2 s_2, j_1 j_2, LS, J)\,X(l_1' s_1' l_2' s_2', j_1' j_2', LS, J)\Big] \qquad (C3)$$

Specializing this relation by putting $j_1=j_1'$, $j_2=j_2'$ $l_1=l_1'$, $l_2=l_2'$, $s_1=s_2=s_3=s_4=1/2$ we get for the interaction energy

$$E_{int}=\sum_{LS}\Big[X(l_1 l_2\, \tfrac12\tfrac12, LS, j_1 j_2, J)\Big]^2 \times\Big[\sum_l (2l+1)\,W(l_1 l_2 l_2 l_1, Ll)$$

$$\times\langle l_1 l_2, l\,N|V_x|l_1 l_2, l\,N\rangle\times\Big[\sum_s (2s+1)\,W(\tfrac12\tfrac12\tfrac12\tfrac12, Ss)\langle s\|V_d\|s\rangle\Big]$$

C4

Thus, apart from a change in sign, the splitting of a particle-hole configuration due to the inter particle interactions is the same as the splitting of a particle-particle configuration with the matrix elements of the interaction potentials transformed according to the rule

$$\langle l_1 l_2, LM | V_x | l_1 l_2, LM \rangle \rightarrow \sum_l (2l+1) W(l_1 l_2 l_2 l_1, Ll) \langle l_1 l_2, lN | V_x | l_1 l_2, lN \rangle \qquad (C5)$$

$$\text{and} \quad \langle S | V_\sigma | S \rangle \rightarrow \sum_s (2s+1) W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}, Ss) \langle s | V_\sigma | s \rangle \qquad (C6)$$

The transformation (C 6) can be effected by replacing P_σ, the spin exchange operator, by $(1 - P_\sigma)$. If V_x is an ordinary central force the sum in (C 5) reduces simply to the matrix element $\langle l_1 l_2, LM | V | l_1 l_2, LM \rangle$ so that, for these forces, the coordinate part of the particle-hole interaction is the same as for a particle - particle interaction. For a cordinate exchange operator we get the transformation

$$\langle l_1 l_2, LM | V P_x | l_1 l_2, LM \rangle = [C(l_1 l_2, L)]^2 \int V(r) R(r_1, r_2) Y_L^{*N}(\theta_1, \phi_1) Y_L^{N}(\theta_2, \phi_2) \, dr_1 \, dr_2 \qquad (C7)$$

References.

Bannermann, R.C., and Curran, S.C., 1952, Phys.Rev., 85, 134.

Bartlett, T.H., 1932, Phys. Rev., 41, 370.

Phys. Rev., 42, 145.

Bergstrom, I., and Wapstra, A.H., 1955, Phil.Mag., 46, 61.

Biedenharn, L.C., Blatt, J.M., and Rose, M.E., 1953,

Rev.Mod.Phys., 24, 251.

Blatt, J.M., and Jackson,J.D., 1949, Phys.Rev., 76, 18.

Blin-Stoyle, R.J., 1953, Proc.Phys.Soc. A, 66, 729.

Bohr, N., 1936, Nature, 137, 344.

Condon, E.U., and Shortley, G.H., 1935, "Theory of Atomic

Spectra" (C.U.P.).

Dirac, P.A.M., 1935,"The Principles of Quantum Mechanics"

O.U.P.

Eliot, E.A., Hicks, D., Beghian, L.E., and Halban, H., 1954

Phys.Rev., 94, 144.

Elsasser, W., 1933, J.Phys. Radium, 4, 549.

1934, J.Phys. Radium, 5, 389, 635.

Flowers, B.H., 1952a, "Progress in Nuclear Physics", 2, 235.

1952b, Proc.Roy.Soc., 212, 248.

215, 398.

Fano, U., 1951, N.B.S. Report, 1214.

Fred, M., Tomkins, F.S., and Barnes, R.F. 1953, Phys.Rev.,

92,1324.

Goldhaber, M., and Hill, R.D., 1952, Rev.Mod.Phys., 24, 179.

Gorodetzky, S., Gallmann, A., Knipper, A., Ambruster, R.,
1953, C.R.Acad.Sci.,Paris, 237, 245.

Guggenheimer, K.M., 1934, J.Phys. Radium, 5, 253, 475.

Harvey, J.A., 1953, Canad.J.Phys., 31, 278.

Haxel, O., Jensen, J.H.D., and Suess, H.E., 1949, Phys.Rev.,
75, 1766.

Inglis, D.R., 1953, Rev.Mod.Phys., 25, 390

Kinsey, B.B., Bartholomew, G.A., and Walker, W.H., 1951,
Phys.Rev., 82, 380.

Klinkenberg, P.F.A., 1952, Rev.Mod.Phys., 24, 63.

Krohn, V.E., and Raboy, S., 1954, Phys.Rev., 88, 680.

Jahn, H.A., and Hope, J., 1954, Phys.Rev., 93, 318.

Lane, A.M., 1953, Proc.Phys.Soc., 66, 997.

1955, Proc.Phys.Soc., 68, 197.

Lee-Whiting, G.E., 1955, Phys.Rev., 97, 463.

Levy, H.B., and Perlman, I., 1954, Phys.Rev., 94, 152.

Maeder, D., Wapstra, A.H. Nijgh, G.J. and Ornstein, L.T.L.M.
1954, Physica, 20, 521.

Mayer, M.G., 1949, Phys.Rev., 75, 1969.

1950, Phys.Rev., 78, 16, 22.

Mayer, M.G., Moskowski, S.A., and Nordheim, L.W., 1951,
Rev.Mod.Phys., 23, 315.

Neumann, H.M., Howland, JJ., and Perlman, I., 1950,
Phys.Rev., 77, 720.

Nordheim, L.W., 1950, Phys.Rev., 78, 294.

1951, Rev.Mod.Phys., 23, 322.

Poole, M.J., 1953, Phil.Mag., 44, 1398.

Prescott, J.R., 1954, Proc.Phys.Soc.A, 67, 254.

Pryce, M.H.L., 1952, Proc.Phys.Soc.A., 65, 773.

Racah, G., 1942, Phys.Rev., 62, 438.

Rose, M.E., and Osborn, R.K., 1954, Phys.Rev., 93, 1315.

Rosenfeld, L., 1948, "Nuclear Forces", 243.

1951, Physica, 17, 461.

Wapstra, A.H., 1950, Phys.Rev., 86, 562.

Weisskopf, V.F., 1950, Helv.Phys.Acta., 23, 187.

1951, Science, 113, 101.

Wu, C.S., Boehm, F., and Nagel, E., 1953, Phys.Rev., 91, 319.

Yamada, M., 1953, Prog.Theor.Phys., 10, 252.

Part II

SOME REACTIONS WITH NEUTRONS.

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Contents

Chapter I.

Inelastic Scattering of Neutrons by a Deformed Nucleus.

1(i): Introduction.

Many of the features of the process of inelastic scattering of neutrons by medium and heavy nuclei have been explained on the basis of Bohr's (1936) compound nucleus model. This model assumes that when a neutron of moderate energy strikes a target nucleus it may be absorbed to form a compound system of some stability. Subsequently this compound system can decompose emitting a neutron and leaving the target nucleus in an excited state. The theory supposes that the formation and the decay of the compound nucleus may be treated as independent processes, so that the mode of decay of the compound system depends only on its energy, total angular momentum and parity, but not specifically on the way in which it was formed. Hauser and Feshbach (1953) have considered this theory in some detail and the results of their calculations account for most of the observed features of inelastic scattering for neutron energies up to about 5 MeV.

The collective model of the nucleus proposed

by Bohr (1952) has been very successful in describing the properties of low excited states in heavy nuclei. These states are assumed to be the various collective modes of motion of the nuclear surface and, in many nuclei, they arise from the rotation of a large elipsoidal surface deformation. Consider the scattering of a neutron by such a nucleus. The incident neutron may be elastically scattered or it may be absorbed to form a compound system; but there is also the possibility that direct interaction with the nuclear surface will inelastically scatter the neutron and excite a surface mode. One can picture this process in the following way. The neutron enters the nucleus and is reflected within the walls of the nucleus. If the nucleus is non spherical, there is a chance that, by these reflections the nucleus is set in rotation or vibration. Evidently the cross section for this process will be highest for those nuclear radii for which a standing wave develops inside the nucleus, thus giving rise to many reflectors. In the present paper we investigate this process and compare its cross section with the cross section for compound nucleus formation. We will be interested chiefly in the excitation of the rotational surface modes arising in strongly deformed nuclei.

1(ii): The Collective Model.

The surface coordinates $\alpha_{\lambda\mu}$ of a nucleus are defined by the equation to the nuclear surface

$$R(\theta,\phi) = R_0\left(1+\sum_{\lambda\mu}\alpha_{\lambda\mu}\,Y_\lambda^\mu(\theta,\phi)\right) \qquad (1)$$

where R_0 is the radius of the undeformed nucleus and $Y_\lambda^\mu(\theta\phi)$ is a normalised spherical harmonic with Condon and Shortley (T.A.S) phases. The rotational levels considered by Bohr (1952) are described by the coordinates $\alpha_{\lambda\mu}$ and the nuclear rotational Hamiltonian H_0^1 is a function of these coordinates. Deformations with $\lambda \neq 2$ do not contribute appreciably, thus need not be considered.

The Hamiltonian for the scattering of a neutron by a Bohr nucleus can be written $H_0 + T + V$, where T is the neutron kinetic energy and V is the neutron nucleus interaction potential. Following Bohr we approximate the potential V by a square well,

$$\begin{aligned} V(\underline{r},\alpha_{2\mu}) &= -V_0 \quad \text{if } r<R(\theta,\phi) \\ &= 0 \quad \text{if } r>R(\theta,\phi) \end{aligned} \qquad (2)$$

Because V depends on the neutron coordinate $\underline{r}$ and the nuclear surface coordinates $\alpha_{2\mu}$, it contains a direct interaction between the neutron and the surface deformations, and it is this coupling which gives rise

to inelastic scattering with the excitation of surface modes. In order to calculate the cross section for the scattering, we split V into two parts by expanding as a power series in the surface deformations $\alpha_{2\mu}$. For small deformations, powers of $\alpha_{2\mu}$ higher than the first may be neglected, so that

$$V(\underline{r}, \alpha_{2\mu}) = \bar{V}(r) + V_0 R_0 \delta(r - R_0) \sum_{\mu} \alpha_{2\mu} Y_2^{\mu}(\theta, \phi)$$

$$= \bar{V}(r) + H' . \qquad (3)$$

with $\bar{V}(r) = -V_0$ if $r < R_0$

$= 0$ if $r > R_0$

and $\delta(r - R_0)$ the Dirac delta function.

The averaged central potential $\bar{V}(r)$ represents the interaction of a neutron with an undeformed nucleus while the term H', containing both neutron and surface coordinates, is the coupling between the neutron and the surface deformations.

We want to compare the magnitude of the direct scattering with compound nucleus scattering so it is useful to include compound nucleus formation in our model. Feshbach, Porter and Weisskopf (1954) have proposed a simple description of neutron scattering in which they replace the neutron-nucleus interaction by a complex

potential well. The imaginary part of the potential causes an absorption to be identified with compound nucleus formation. We use the results of their paper by taking V_o, the strength of the neutron, nucleus interaction (2) to be complex with a small imaginary part. The compound nucleus scattering can then be calculated from Hauser and Feshbach (1953) theory. The decay of the compound nucleus is assumed to be independent of its mode of formation and, in particular, the phase of the inelastically scattered neutron wave is indeterminate. In contrast to this, the phase of the neutron waves inelastically scattered from the nuclear surface deformations is completely determined by the phase of the incident wave. Thus the contributions to the scattering by direct interaction and due to compound nucleus formation (averaged over nuclear resonances) are incoherent.

1(iii): The Calculation of the Cross Section.

The cross section for inelastic scattering has been calculated treating the potential $\bar{V}(r)$ exactly, and the term H^1 as a perturbation on the scattering by this potential. As there is no spin interaction in the potential, we can write the unperturbed solution as a pure orbital wave function corresponding to scattering of plane waves by $\bar{V}(r)$. The solution with wave number k

can be written

$$\psi(\underline{r}) = \frac{i\pi^{1/2}}{k} \sum_{\ell} (-1)^{\ell} (2\ell+1)^{1/2} \psi_{\ell 0}(\underline{r}) \qquad (4)$$

where the functions $\psi_{\ell m}$ are the bounded partial wave solutions for the potential $\bar{V}r$. e have

$$\psi_{\ell m} = \frac{1}{r} u_{\ell}(kr)\, Y_{\ell}^{m}(\theta,\phi) \qquad (5)$$

where $u_{\ell}(kr) \sim e^{-ikr} - \eta_{\ell} e^{ikr}$ for large r .

η_{ℓ} is the complex reflection factor of Feshbach, Porter and Weisskopf with $|\eta_{\ell}| < 1$ because of the absorption term in the potential.

Suppose we are considering neutron scattering from an initial nuclear state $|I_0 M_0\rangle$ to a final state $|I, M\rangle$, with an incident neutron wave ψ_0 with wave number k_0 and a scattered neutron wave ψ_s of energy E and wave number k. To the first order of perturbation ψ_s is the solution of the inhomogeneous equation,

$$\left(-\frac{\hbar^2}{2M}\nabla^2 + \bar{V}(r) - E\right)\psi_s = -\langle I M | H' | I_0 M_0 \rangle \psi_0(\underline{r}) \qquad (6)$$

corresponding to outgoing waves. This equation can be solved by Greens Function methods (Mott and Massey) and for large r the wave function ψ_s has the asymptotic form

1(iii).

$$\psi_s \simeq \frac{e^{ikr}}{2ikr}\frac{2M}{\hbar^2}\sum_{lm} Y_l^m(\theta,\phi)\left[\int(-1)^m \psi_{lm}(r')\langle IM|H'|I_0M_0\rangle\psi_0(r')dr'\right] \quad (7)$$

If we substitute for H' from (3) and evaluate the integrals we get the explicit form

$$\psi_s \simeq \frac{e^{ikr}}{2ikr}\frac{V_0R_0 2M}{\hbar^2}\sum_{ll_0\mu}\left[\frac{[(2l_0+1)(2l+1)]^{1/2}}{4kk_0}u_{l_0}(k_0R_0)\,u_l(kR_0)\right.$$
$$\left.\times Y_l^{\mu}(\theta,\phi)\,C(2ll_0,-\mu\mu_0)\,C(ll_03,00)\langle IM|H'|I_0M_0\rangle\right] \quad (8)$$

where the $C(2ll_0,-\mu\mu)$ and Clebsh-Gordon vector addition coefficients in the notation of Biedenharn, Blatt and Rose (1953).

In the following it is convenient to write $K^2=\frac{2M}{\hbar^2}|V_0|$ then K is approximately the wave number of the scattered neutron inside the nucleus for low incident neutron energies. Put $KR_0=X$, $k_0R_0=x_0$ and $kR=x$. The differential cross section for inelastic scattering is given in terms of by

$$\sigma(\theta)=\frac{k}{k_0}\left[|\psi_s|^2r^2\right]_{r\to\infty}$$

so, averaging over initial nuclear orientations and summing over final orientations we get for the differential cross section

$$\sigma(\theta)=\frac{x^4}{k_0^2}\frac{M(I_0,I)}{16x_0x}\sum_{\mu}\left|\sum_{ll_0}\left[[(2l_0+1)(2l+1)]^{1/2}u_{l_0}(x_0)\,u_l(x)\right.\right.$$
$$\left.\left.\times C(2ll_0,-\mu\mu)\,C(ll_03,00)\,Y_l^{\mu}(\theta,\phi)\right]\right|^2 \quad (9)$$

1(iii).

and for the total cross section

$$\sigma = \frac{M(I_0 I)\, x^4}{k_0^2\, 16\, x\, x_0} \sum_{l\, l_0} (2l_0+1)(2l+1)\, |\mu_{l_0}(x)|^2 |\mu_l(x)|^2 \left[C(l l_0 2, 0 0)\right]^2 \tag{10}$$

The coefficient $M(I_0 I) = \frac{1}{2I_0+1} \sum_{M_0} |\langle I M | H' | I_0 M_0 \rangle|^2$ is the averaged square of the nuclear matrix element of the deformations $\alpha_{2\mu}$, and it is seen that the dependence of the cross section on the structive of the initial and final nuclear states entered only through this factor. In even even nuclei we are interested in a $0^+ \Rightarrow 2^+$ transition and for this transition

$$M(0,2) = \beta^2/5 \tag{11}$$

if the two states have a rotational character. Here β is the nuclear deformation parameter defined by Bohr. In odd A nuclei with a rotational sequence I_0, I_0+1, I_0+2 $(I \neq \frac{1}{2})$

$$M(I_0, I_0+1) = \frac{3}{5} \frac{\beta^2 I_0}{(I_0+1)(I_0+2)}$$

$$\text{and} \quad M(I_0, I_0+2) = \frac{6}{5} \frac{\beta^2}{(I_0+1)(2I_0+1)} \tag{12}$$

Transitions with $\Delta I > 2$ are forbidden on this model by angular momentum considerations if we neglect $\lambda \neq 2$ in (1).

1(iv): The Angular Distribution of Reaction Products

Neutrons.

Equation (9) for the differential cross section yields the following expression for the angular distribution of the inelastically scattered neutrons.

$$W(\theta) \propto \sum_{l_0 l_0' l l' J} \Big[u^*_{l_0'}(x_0')\, u^*_{l'}(x')\, u_{l_0}(x_0)\, u_l(x)\, (2l_0+1)(2l+1)(2l_0'+1)(2l+1)$$

$$\times\, C(l_0 l 2, 00)\, C(l_0' l' 2, 00)\, C(l_0 l_0' J, 00)\, C(l l' J, 00)$$

$$\times\, W(l_0 l_0' l l', J 2)\, P_J(\cos\theta) \Big]$$

Here $W(l_0 l_0' l l', J 2)$ is a Racah (1942) coefficient.

γ-Rays.

The density matrix for the excited state of the nucleus after scattering can be found from the final state wave function ψ_s by integrating $|\psi_s|^2$ over the directions of neutron emission and by summing over initial nuclear orientations. Putting

$$S(l l_0) = \sqrt{2l_0+1}\, C(l_0 l 2, 00)\, u_{l_0}(x_0)\, u_l(x),$$

and using the methods of Biedenharn and Rose (1953) we get for the angular distribution of the γ-rays

$$W(\gamma) = \sum_k R_{k0}(\Gamma)\, F_k(2 I_0 I)\, P_k(\cos\gamma)$$

where

$$R_{k0} = \sum_{l l_0 l_0'} \Big[(-1)^{I_0 - I + l_0} \sqrt{(2I+1)(2l_0+1)(2l_0'+1)}\, S(l l_0)\, S^*(l l_0')$$

$$\times\, C(l_0 l_0' k, 00)\, W(\Gamma I 2 2; k I_0)\, W(2 2 l_0 l_0', k l) \Big]$$

1(iv).

The angular correlation coefficients F_2 (2 I l_0) are tabulated by Biedenharn and Rose (1953).

1(v): Results.

In this section we give the results of some calculations of the inelastic cross section by the direct process and compare them with cross sections for the compound nucleus scattering calculated from the formula of Hauser and Feshbach 1953. First, however, we express the direct cross section (10), explicitly in terms of optical model parameters introduced by Feshbach, Porter and Weisskopf 1954. We write for the logarithmic derivatives at the nuclear boundary

$$f_l + i g_l = R_0 \left[\frac{d u_l}{dr} / u_l \right]_{r=R_0}$$

$$\Delta_l + i s_l = R_0 \left[\frac{d u_l^+}{dr} / u_l^+ \right]_{r=R_0}$$

Here u_l^+ is the partial wave solution for potential $\bar{V}$ (r) with only outgoing waves. If we put

$$M_l = s_l - g_l$$

$$N_l = f_l - \Delta_l$$

it can be shown that

$$|u_l(x)|^2 = \frac{4x\, s_l}{M_l^2 + N_l^2}$$

and substituting into (10) the total cross section for the direct inelastic process is

$$\sigma = \frac{M(I_0 I)}{k_0^2} \sum_{l l_0} (2l_0+1)(2l+1)\left[C(l l_0 2, 0 0)\right]^2 \frac{X^2 S_{l_0}}{M_{l_0}^2 + N_{l_0}^2} \cdot \frac{X^2 S_l}{M_l^2 + N_l^2}$$

Hauser and Feshbach (1953) give their results in terms of a "barrier penetrability" $T_l(E)$ which is the probability that a neutron in a partial wave l and with energy E will be absorbed and form a compound nucleus. For the "optical model" T_l has the expression

$$T_l = 1 - |M_l|^2 = \frac{-4 S_l g_l}{M_l^2 + N_l^2}$$

In calculating both the direct scattering and the compound nucleus scattering we have chosen a potential well with $V_0 = 40(1 + 0.05i)$ MeV and $R_0 = 1.45 \times 10^{-13} A^{1/3}$ cm.

We have made calculations of the inelastic cross section for an even even nucleus, i.e. for a $0^+ \rightarrow 2^+$ transition. The 2^+ state is assumed to have a pure rotational character and to lie 0.2 MeV above the 0^+ ground state. Incident neutron energies from zero to 1.5 MeV have been considered and the range of atomic weights $A \sim 100$ to $A \sim 240$ has been covered.

The nuclear distortion parameter β ranges from zero near closed shells to a maximum value of $\beta = .3$ in the regions $A \sim 170$ and $A \sim 240$. A value of $\beta^2 = 0.05$ has

been used throughout, but as the direct cross section is proportional to β^2 the results given here can easily be adapted to nuclei with different values of β. Values of β derived from quadrupole moments or γ-ray transition probabilities should be used rather than those derived from consideration of energy level spacings, since it is probable that the electromagnetic values give a truer picture of the nuclear slope (Ford 1954).

Figure I gives the direct inelastic cross section σ_d plotted against $X = K R_0$ and against A the atomic weight for an incident neutron energy of 1 MeV. The same figure shows, for comparison, values of σ_{ci}, the compound inelastic cross section for the same $0^+ \rightarrow 2^+$ transition. In Figure II σ_d is plotted against the incident neutron energy E for values of E ranging from zero to 1.5 MeV and for A = 150, 165, 230, 240.

Compound nucleus scattering theory gives a differential cross section which is symmetrical about 90° to the direction of the incident neutrons. In the present theory we can estimate the forward-backward asymmetry by calculating the quantity $\left| \frac{W(\pi) - W(0)}{W(\pi) + W(0)} \right|$. In the region of atomic weights A ~ 150 and A ~ 240 the forward-backward asymmetry is of the order of 10% on the basis of optical model wave functions. The

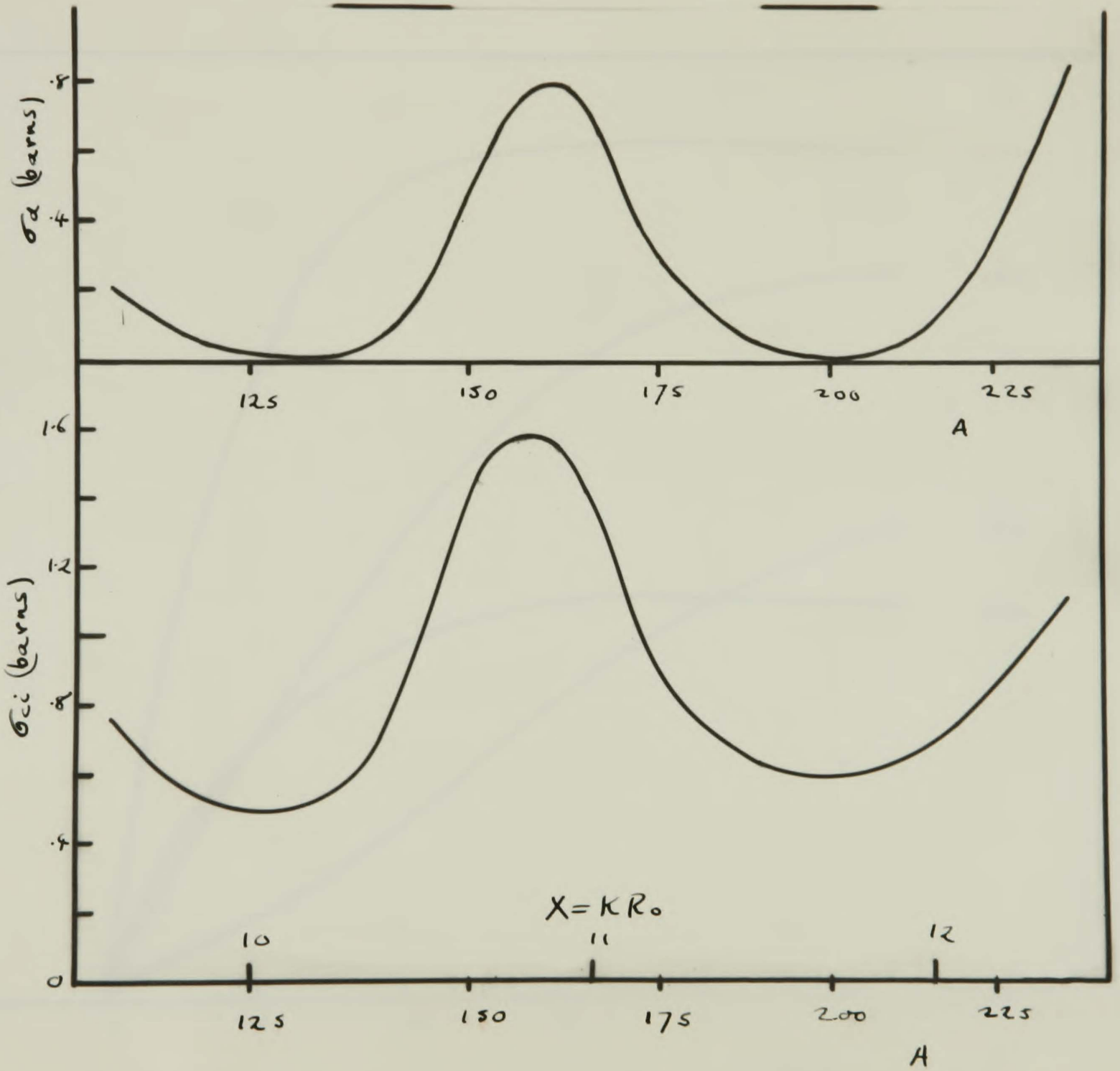

σd (barns)
.8
.4
125
150
175
200
225
A
σci (barns)
1.6
1.2
.8
.4
0
X = KR₀
10
11
12
125
150
175
200
225
A

Figure I

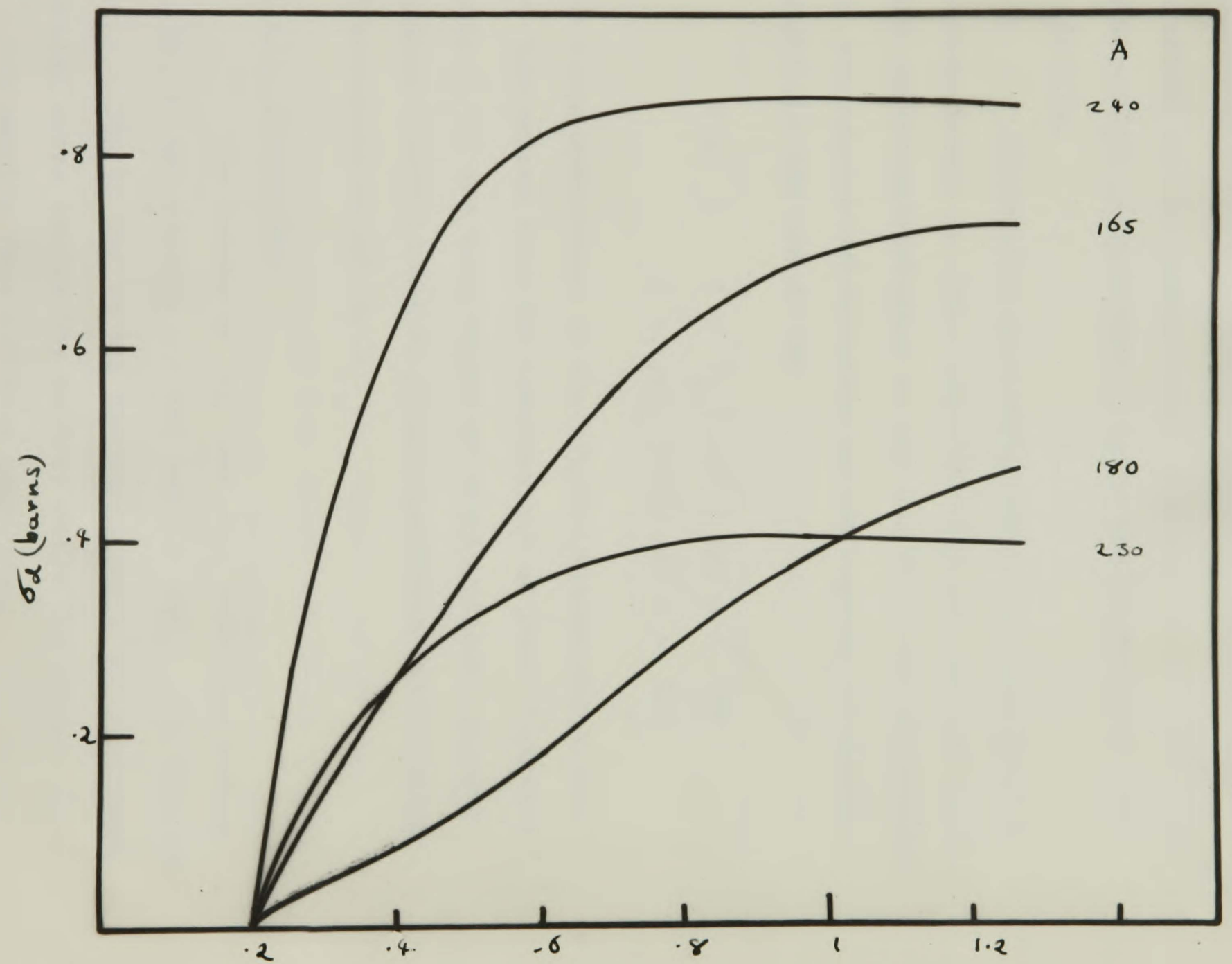


Figure II

effect is small because the asymmetry arises from interference of P and S or D neutron partial waves, and according to the optical model where S and D waves give an important contribution P waves are unimportant and conversely.

Neglecting odd parity terms and assuming the neutron energy is large compared with the nuclear excitation under consideration we get the following expressions for the angular distribution in the regions of atomic weight $A \sim 150$ and $A \sim 240$

$$W(\theta) = \begin{cases} 1 + P_2(\cos\theta) & \text{for } A \sim 150 \\ 1 + 0.2P_2(\cos\theta) & \text{for } A \sim 240 \end{cases}$$

The large anisotropy in the angular distribution for $A \sim 150$ arises from the interference of S and D partial waves. In the same region of atomic weight compound nucleus theory yields an almost isotropic distribution of scattered neutrons.

1(vi): Discussion.

The curves of σ_d and σ_{ci} both show strong maxima in the regions $A \sim 150$ and $A \sim 240$. The maximum for $A \sim 150$ is due to the resonance for S and D neutron partial waves inside the nucleus while the maxima at $A \sim 240$ results from a P wave resonance. The cross

sections σ_d and σ_{ci} are both of the same order of magnitude for A ~150 and A ~ 240 while in between maxima, at A ~130 and A ~ 200, σ_d falls almost to zero. Higher order effects will make the peaks less sharp, but in any case the general features of the variation of cross section with atomic weight should remain. It is clear that for A ~ 200 the direct process should be quite unimportant.

In Figure II the curves of σ_d against energy show a different behaviour in the regions A ~ 150 and A ~ 240. For A ~ 150 and 165 the increase in σ_d with E is almost linear while for A ~ 230 and A ~ 240, the initial rise just above threshold is much sharper. Only S and D partial waves contribute appreciably to for A ~150 and 165 and one expects a sharp rise in cross section just above threshold due to the emergence of S waves. The opposite behaviour shown in Figure II for A ~ 150 is due to the fact that at .2 MeV the barrier penetrability for D waves is very small, so that almost the entire rise in σ_d from .2 to 1 MeV is due to the increase in D wave penetrabilities. The cross section for A 230 and 240 rise steeply above threshold because penetrabilities for P waves increase more rapidly with energy than for D waves. For A ~240 and for energies above 1 MeV F

partial waves give an appreciable contribution to the cross section.

Chapter II.

Resonance Reactions with Neutrons.

2(i): Introduction.

Although the characteristic features of the low lying levels of many nuclei are known from a study of radioactive decay schemes, and of various nuclear reactions, there is not much information available on the properties of highly excited levels. At present such information as we have is obtained from experiments made on the resonant scattering of neutrons.

It is well known that at low energies the cross section for neutron scattering does not vary uniformly, but shows sharp resonances. In medium nuclei ($A \sim 50$) these resonances have an average spacing between 1 KeV and 10 KeV, while in heavier nuclei the spacing may be as small as a few e.v. As a general rule the resonance spacing is much larger in near magic number nuclei, than in nuclei with proton and neutron configurations which are far from closed shells. These resonances are assumed to correspond to virtual states of the compound nucleus formed from the target nucleus plus the scattered neutron. According to the Bohr compound nucleus picture these states have a complicated many particle structure. Only a small part of the wave function corresponds to the

emission of a neutron, hence the virtual states are metastable against neutron emission. In general radiative capture and elastic scattering are competing processes.

Since a virtual state of the compound nucleus can emit a particle or radiation its energy can not be well defined, because a state of definate energy must be a stationary state. This lack of definition can be described qualitatively as a width Γ_λ in energy of a virtual state λ, defined as the energy width at half maximum cross section (of the resonance) From the complementary relation between energy and time it is expected that Γ_λ is connected to the mean life of the virtual state by the uncertainty relation

$$\Gamma_\lambda \Delta t_\lambda \simeq \hbar$$

Thus Γ_λ provides a measure of the transition probability for the emission of a particle or a photon from the virtual state. If the state λ can decay into several channels, we can ascribe a probability for the decay into a particular channel S which can be expressed as a partial width $\Gamma_{\lambda s}$. The law that the total probability of decay per unit time is the sum of the probabilities for decay into the various channels can be written as

$$\Gamma_\lambda = \sum_s \Gamma_{\lambda s}$$

2(i).

It is found that at low energies (upto a few tens of KeV in medium nuclei, and up to a few hundred eV in heavy nuclei) the resonances are distinct and have level widths small compared with their spacing. At higher energies the resonances overlap and the resonance structure becomes confused. In this work we want to use information obtained from neutron resonance scattering to give information on the properties of excited states of the compound nucleus, so we are interested in those regions where the neutron resonances are distinct with levels spacings large compared with level widths. Thus we confine ourselves to low neutron energies, where only S partial waves contribute to the cross section.

As stated above the virtual states of the compound nucleus have a complicated structure, and in this work we assume that this structure can be treated statistically. On the basis of this assumption predictions of the partial widths are made and compared with experiment.

2(ii): Statistics of Particle Widths.

It is well known (Feshbach and Weisskopf 1949) that the partial width for S wave neutron emission, averaged over virtual states of the compound nucleus, is related in a general way to the average resonance spacing

D in the energy region near the resonance by the equation

$$\Gamma_n \simeq \frac{2}{\pi} k/K D$$

Here k is the wave number of the emitted neutron and K is its wave number inside the nucleus ($\simeq 10^{13}$ cm^{-1}). Different theories (e.g. Feshbach, Porter and Weisskopf 1954) may give a different numerical coefficient of proportionality; but the general form of the relation remains. It is known, however, that the neutron width fluctuates from resonance to resonance. In this chapter we investigate these fluctuations and find the distribution function for level widths.

The partial width for the emission of a particle from a virtual level λ of the compound nucleus into a given channel s with orbital angular momentum ℓ depends on the height of the potential or centrifugal barrier it must penetrate in order to escape. It is useful to separate out the barrier dependence of the partial width by defining a "reduced width" $\gamma_{\lambda s}$ by the equation

$$\Gamma_{\lambda s} = 2 k v_\ell \gamma_{\lambda s}$$

where k is the wave number of the emitted particle and v_ℓ is a barrier penetration factor (Blatt and Weisskopf). The reduced width so defined depends only on conditions inside the nucleus. If we consider resonances, corresponding to virtual states with a total angular momentum I

2(ii).

I the reduced width will fluctuate from level to level, but on the average should vary slowly with energy. In this chapter we consider the statistics of reduced widths in an energy interval containing a large number of resonances; but over which the average values of the reduced widths do not vary appreciably.

The general theory of nuclear reactions has been investigated by many authors (Wigner and Eisenbud 1947; Teichmann and Wigner 1952) and it has been found possible to express the scattering matrix for any particle reaction in terms of a derivative matrix $R_{ss'} = \sum_{\lambda} \frac{\gamma_{\lambda s}\gamma_{\lambda s'}}{E_\lambda - E}$. The quantities $\gamma_{\lambda s}$ are constants related to the reduced widths by the approximate equations

$$\gamma_{\lambda s} = \gamma_{\lambda s}^2 .$$

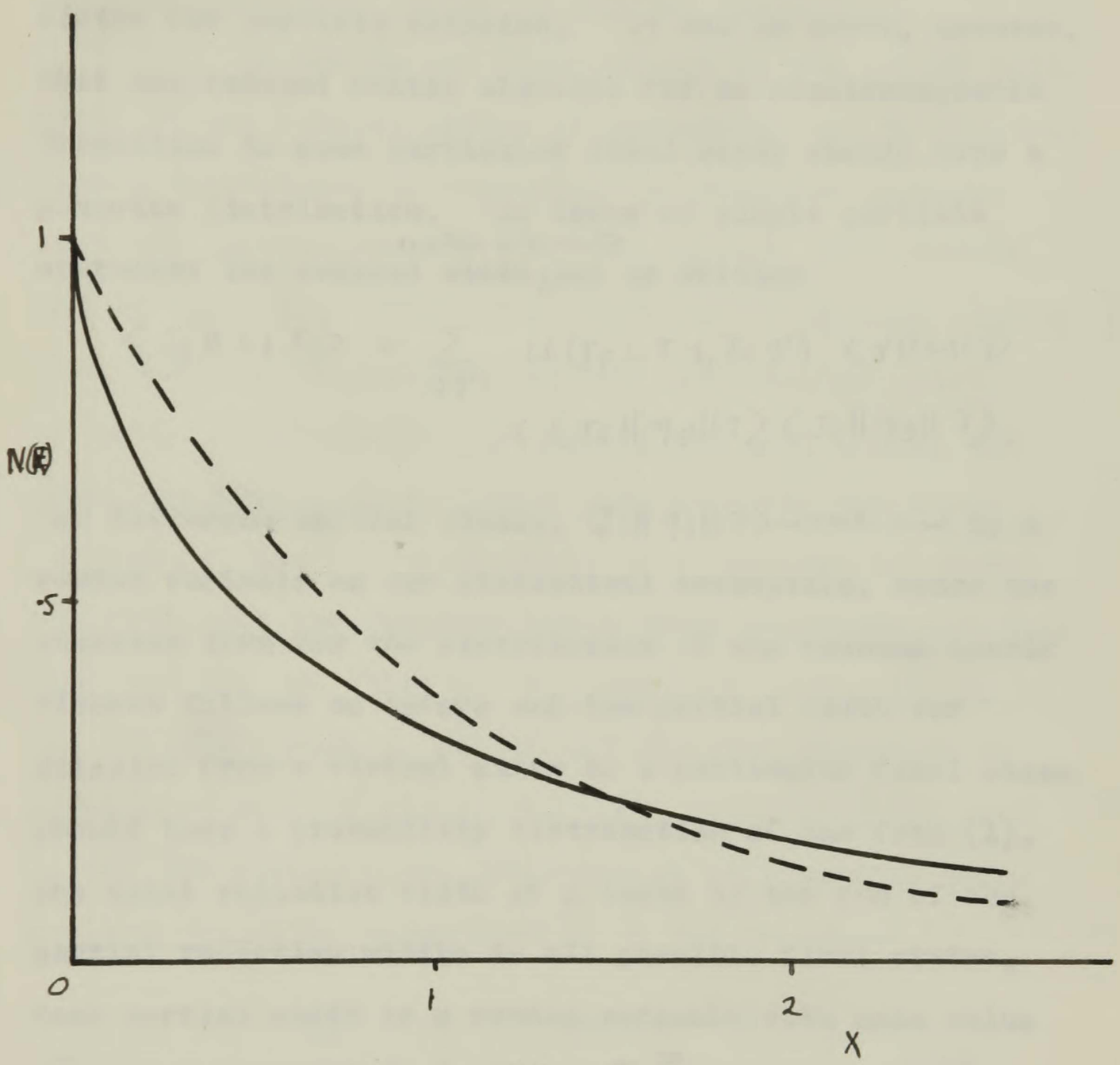
They are real and take both positive and negative values. Teichmann and Wigner 1952 have shown that they can be expressed as a sum of single particle terms with amplitudes determined by the single particle structure of the virtual state. On our statistical assumption these amplitudes should be random variables. Thus the $\gamma_{\lambda s}$ can be expressed as a sum of random variables, thus might be expected to have a gaussian distribution with mean value zero. (The central limit theorem of probability theory gives that the distribution of a sum of independent random variables is asymptotically gaussian).

2(ii).

In a particular nucleus, the reduced partial width for states with the same total angular momentum and parity is a random variable. If $\overline{\gamma}_{\lambda s}$ is the mean value of the reduced width $\gamma_{\lambda s}$ and $P(\gamma_{\lambda s}/\overline{\gamma}_{\lambda s} \geq X)$ is the probability that $\gamma_{\lambda s}/\overline{\gamma}_{\lambda s} \geq X$ then assuming a gaussian distribution for $y_{\lambda s}$ we have

$$P\left(\gamma_{\lambda s}/\overline{\gamma}_{\lambda s} \geq X\right) = \frac{1}{\sqrt{\pi}} \int_{X}^{\infty} \frac{e^{-x}}{\sqrt{x}}\, dx \qquad (12)$$

The statistics of neutron widths has been investigated by Hughes (1955). With the present experimental accuracy, the distributions of reduced widths in 10 nuclides are exponential. There seems to be no correlation between the neutron widths of specific levels and the spacing of adjacent levels. The distribution function (12) is plotted in Figure III. A comparison plot of an exponential distribution with the same mean shows that the distribution derived in this chapter is approximately exponential with a slightly longer tail. It can be shown that the exponential distribution has standard deviation $\sqrt{2}$ while the distribution function (12) has standard deviation $\sqrt{3}$ The present experiments are consistent with either distribution; but a more detailed investigation of the standard deviations could distinguish between the two.



$\frac{1}{\sqrt{\pi}} \int_x^\infty \frac{e^{-x}}{\sqrt{x}}\, dx$

e^{-x}

Figure III The Distribution Function for Reduced Widths : $P(\theta/\bar{\theta} \geq x)$

2(ii).

The above argument applies only to partial widths for particle emission. It can be shown, however, that the reduced matrix elements for an electromagnetic transition to some particular final state should have a gaussian distribution. In terms of single particle operators the reduced ~~width~~ matrix elements can be written

$$\langle J_f \| L \| J_i \rangle = \sum_{jj'J} u(J_f L J j, J_i j') \langle j' \| L \| j \rangle \times \langle J_f \| \eta_{j'} \| J \rangle \langle J_i \| \eta_j \| J \rangle .$$

For different initial states, $\langle J_i \| \eta_j \| J \rangle$ is real and is a random variable on our statistical assumption, hence the gaussian form for the distribution of the reduced matrix element follows as before and the partial width for emission from a virtual state to a particular final state should have a probability distribution of the form (1). The total radiation width of a level is the sum of the partial radiation widths to all possible final states. Each partial width is a random variable with mean value $\overline{\Gamma}_s$ and standard deviation $\sqrt{3}\,\overline{\Gamma}_s$ thus, by the central limit theorem, the total radiation width of a virtual state has a gaussian distribution with mean value $\sum_s \overline{\Gamma}_s$ and standard deviation $\left[3 \sum_s \overline{\Gamma}_s^2\right]^{1/2}$.

2(iii): Radiative Widths of Neutron Resonances.

In medium and heavy nuclei we can obtain estimates of the partial widths of resonances due to electric dipole transitions by comparing the emission process with the inverse process i.e. the nuclear photo effect. The nuclear photo effect has been studied by many authors from both the theoretical and the experimental point of view. Montalbetti, Goldemberg and Katz (1953) and Jones and Terwillinger (1953) have measured the cross section for the ( ) reaction as a function of energy and find that the reaction has a broad resonance and that the position and width of the resonance varies slowly and uniformly with A, and is not strongly dependent on details of nuclear structure. Theposition of the peak E_m and the width of the resonance Γ at half maximum cross section are given approximately by the relations

$$E_m = 38.5\ A^{-.186}$$
$$\Gamma = 15\ A^{-.186}, \qquad (14)$$

whereA is the atomic weight. The equations (14) give the general trend of the variation of the resonance parameters with atomic weight. There are fluctuations in the position and width of the resonance, and in particular Γ appears to be rather smaller than the above

estimate in magic nuclei (Nathans and Halpern 1954).

Levinger and Bethe (1950) have calculated the integrated cross section for the photo effect due to electric dipole transitions, $\sigma_{int} = \int_0^\infty \sigma dE$ and find that

$$\sigma_{int} = 0.015A(1 + 0.8x)$$

where x is the fraction of attractive exchange force in the neutron-proton interaction potential. This theoretical results is in good agreement with the experimental values found by Montalbetti, Goldemberg and Katz (1953) for medium and heavy nuclei, if x is chosen somewhere between 0 and 0.5. Estimates of the cross-sections for magnetic dipole and electric quadrupole transitions indicate that these processes should contribute only a few percent to the total cross section (Blatt and Weisskopf, Levinger and Bethe 1950).

The photo effect in medium and heavy nuclei is largely a compound nucleus process. The target nucleus absorbs the photon, and is left in a highly excited state which can decay by γ emission or, if the excitation energy is above neutron or proton threshold, by particle emission. The photo effect has been discussed by Goldhaber and Teller (1948) and by Steinwedel, Jensen (1952) on the basis of a collective model. The particular

normal modes reached by the absorbtion of electric dipole radiation are those in which there is a mass motion of the neutrons against the protons. These vibrations are strongly damped due to interaction with other modes of oscillation. If this collective motion can be described in terms of a damped simple harmonic oscillator, the calculation of the dependence of cross section on energy can be performed classically (Steinwedel, Jensen 1952). With this assumption the shape of the photoelectric resonance is given by

$$\sigma \propto \frac{(\Gamma E)^2}{(E^2 - E_m^2)^2 + (\Gamma E)^2} \qquad (15)$$

In the above paragraphs we have considered the nuclear photo effect from the ground state of the nucleus. Now we assume that the energy dependence of the photo effect is independent of the detailed structure of the initial state so that, if it were possible to perform the photo effect on an excited state, the cross section for absorbtion of a photon of energy E would still have an energy dependence given by (15). In the following paragraphs a calculation of the radiative widths of neutron resonances will be made on the basis of this assumption.

(a): Calculation of Radiation Widths.

Levinger and Bethe (1950) derive their expression for the integrated photo electric cross section by using the dipole sum rule;

$$\sum_{\beta} (E_\beta - E_\alpha) |K_\beta| Q_1^m |\alpha\rangle|^2 = \frac{3}{4\pi} \frac{NZ}{A} \frac{e^2\hbar^2}{2M} (1+0.8x)$$

$$= X \qquad (16)$$

where Q_1^m is the electric dipole operator

$$Q_1^m = e \sum_{k=1}^{Z} r_k Y_1^{*m}(\theta_k, \phi_k)$$

and E_α and E_β are the energies of the states α and β respectively. If we are interested in transitions to final states β with a particular total angular momentum we can find a modified sum rule as follows. The matrix elements of the dipole operator Q_1^m can be written as reduced matrix elements (Biedenharn and Rose 1953).

$$\langle \beta I_\beta M_\beta | Q_1^m | \alpha I_\alpha M_\alpha \rangle = C(I_\alpha 1 I_\beta, M_\alpha m) \times \langle \beta I_\beta || Q_1 || \alpha I_\alpha \rangle$$

Now consider a restricted sum over those states with a total angular momentum I and write

$$\sum_{\substack{\beta \\ I_\beta = I}} (E_\beta - E_\alpha) \; |\langle \beta I_\beta || Q_1 || \alpha I_\alpha \rangle|^2 = X_I$$

In terms of X_I, the sum rule (16) can be written as

$$\sum_I \left[C(I\alpha\, I, M\alpha\, m)\right]^2 X_I = X \qquad (17)$$

Equation (17) holds for all M_α and m, and it can be shown that this implies $X_I = X$. Hence we have the sum rule over states with total angular momentum I

$$\sum_{\substack{\beta \\ I_\beta = I}} (E_\beta - E_\alpha) \left|\langle \beta I_\beta \| Q_1 \| \alpha I_\alpha \rangle\right|^2 = \frac{3}{4\pi} \frac{NZ}{A} \frac{e^2\hbar^2}{2M} (1 + .8x) \qquad (18)$$

Let us write $M(\alpha I_\alpha, I E)$ for the quantity $|\langle \beta I_\beta \| Q_1 \| \alpha I_\alpha \rangle|^2$ averaged over those states β with energy E_β near E and with $I_\beta = I$. In terms of this average, the sum rule (18) becomes

$$\int_0^\infty (E - E_\alpha)\, M(\alpha I_\alpha, I\, E + E_\alpha)\, \rho_I(E + E_\alpha)\, dE = \frac{3}{4\pi} \frac{NZ}{A} \frac{e^2\hbar^2}{2M} (1 + .8x) \qquad (19)$$

where $\rho_I(E)$ is the density of states with angular momentum I and parity opposite to α at energy E.

The average cross section for the photo effect to a final state with angular momentum I and for an incident photon of energy E, is proportional to

$$E\left(M \alpha I_\alpha, I(E + E_\alpha)\right) \rho_I(E + E_\alpha)$$

where E_α is the energy of the state α . According

to the discussion of the photo effect given in the above paragraphs, this cross section is proportional to

$$\frac{(\Gamma E)^2}{(E^2 - E_m^2)^2 + (\Gamma E)^2}$$

Thus, choosing the factors of proportionality in order to satisfy the sum rule (19), we get the following expression for the averaged square of the reduced matrix elements of the dipole operator;

$$M(\alpha I_\alpha, I E + E_\alpha) = \frac{2}{\pi} \frac{X}{\rho_I(E + E_\alpha)} \quad \frac{\Gamma E}{(E^2 - E_m^2)^2 + (\Gamma E)^2} \qquad (20)$$

The radiation width for a dipole transition from a state β to a state α is given in terms of the square of the reduced matrix element of the electric dipole operator by the equation

$$\Gamma_{\gamma,\beta\alpha} = \frac{16\pi}{9\hbar^3 c^3} (E_\beta - E_\alpha)^3 |\langle \beta I_\beta \| Q_1 \| \alpha I_\alpha \rangle|^2$$

Using the average value of the square of the reduced matrix element given in equation (20) we get an expression for the partial width, averaged over initial states, for radiation of a photon of energy E to a final state α with energy E_α .

It turns out that this radiation width depends only on the energies E_0 of the initial state and E_α of the final state, and on the spin I of the initial state. Thus we can write the radiation width as a function of these three quantities. We have

$$\Gamma_{\gamma I}(E_0, E_\alpha)\,\rho_I(E_0) = \frac{4}{3\pi}\,\frac{NZ}{A}\,\frac{e^2}{\hbar c}\,\frac{1+.8x}{Mc^2}\,\frac{\Gamma E^4}{(E^2-E_m^2)^2+(\Gamma E)^2}$$

$$= 10^{-6}\,\frac{A\Gamma E^4}{(E^2-E_m^2)^2+(\Gamma E)^2} \qquad (21)$$

with $E = E_0 - E_\alpha$ and choosing $x = .2$.

According to Bethe (1936) and others the density of levels $\rho_I(E)$ of a given spin I is proportional to (2I + 1) provided I is not too big. (Each level is still degenerate with (2I + 1) magnetic substates), and we can write

$$\rho_I(E) = (2I+1)\,\rho_0(E).$$

The evidence for this dependence is entirely theoretical and, so far, experiments have not given a conclusive verdict. Hughes, Garth and Levin (1953) have estimated level spacings in neighbouring nuclei of different spins, from .1 MeV neutron captive cross sections and find no correlation between level densities and spins. The level densities fluctuate, however, so much from nucleus to nucleus that it is impossible to draw any definate

conclusion from their results. If the level density is proportional to (2I + 1), then the mean radiation width for a transition from an initial state of spin I to a final state of definate energy is inversely proportional to (2I + 1).

In order to calculate the total radiation width of a state we must sum over all possible final states α The density of final states is $\rho_I(E)$ thus

$$\Gamma_{\gamma I}(E_0) = \sum_{I_\alpha} \int_0^{E_0} \Gamma_{\gamma_I}(E_0, E_\alpha)\, \rho_{I_\alpha}(E_\alpha)\, dE_\alpha$$

$$= 10^{-6} \sum_{I_\alpha} \int \frac{A\,\Gamma E^4}{(E^2 - E_m^2)^2 + (\Gamma E)^2} \frac{\rho_{I_\alpha}(E_0 - E)}{\rho_I(E_0)}\, dE$$

Possible spins of the final state are $I_\alpha = I, I \pm 1$, and, when the sum over final states is carried out, the dependence of $\rho_I(E)$ on I sums to give a factor 3 irrespective of whether $\rho_I(E)$ is proportional to (2I + 1) or is independent of I provided $I \neq 0, \frac{1}{2}$. Thus we get

$$\Gamma_\gamma(E_0) = 3 \times 10^{-6}\, A\Gamma \int_0^{E_0} \frac{E^4}{(E^2 - E_m^2)^2 + (\Gamma E)^2} \frac{\rho_\alpha(E_0 - E)}{\rho(E_0)}\, dE \qquad (22)$$

with the total radiation width spin independent. In the special case when the radiating state has I = 0 there is only one possible final spin i.e. I = 1. In this case, if the level density of states of spin I is proportional to 2 I + 1, the result (22) given above remains

unchanged, while if the level density is spin independent it must be reduced by a factor of 3. Similar considerations hold if the initial state has spin $I = \frac{1}{2}$. The results for the radiation width must be multiplied by a factor 2/3 for a spin independent level density.

As discussed in the previous section the partial radiation width $\Gamma_{\gamma I}(E_o, E)$ and the total width $\Gamma_\gamma(E_o)$ of a state of energy Eo are random variables whose values fluctuate from level to level. In the above paragraphs we have calculated the mean values of these quantities averaged over initial states with energies in the neighbourhood of Eo. The fluctuations in the partial width $\Gamma_{\gamma I}(E_o, E)$ have been discussed fully; but it is of interest to write down an expression for the standard deviation μ of the total width $\Gamma_\gamma(E_o)$. We have

$$\mu^2 = 3 \sum_\alpha \left[\Gamma_{\gamma I}(E_o, E_\alpha)\right]^2$$

$$= \frac{9 \times 10^{-12}}{\rho_I(E_o)} \int_0^{E_o} \frac{A \Gamma E^4}{(E_o^2 - E_m^2)^2 + (\Gamma E)^2} \frac{\rho_o(E_o - E)}{\rho_o(E_o)} dE \text{ MeV} \qquad (23)$$

(b): Results and Discussion.

In order to calculate radiative widths from the above formulae we must know the energy dependence of the density of levels with zero angular momentum. Semi-

empirical expressions have been given for $\rho_0(E)$ by many authors and, in this work, we will use one given by Le Couteur and Lang (1954):

$$\rho_0 = \frac{\alpha}{A^2 E^2} \exp 2\sqrt{\frac{AE}{f}} \quad /\text{MeV} \qquad (24)$$

with f = 8 MeV and α = 10 MeV. This expression is, at best, only a first approximation to $\rho_0(E)$. It takes no account of the effect of nuclear shell structure and gives a uniform variation of level density with A, whereas it is known that nuclei with almost closed shells have an abnormally low level density. It is hoped, however, that equation (24) will give a reasonable account of the variation of level density with E and A, for nuclei from closed shells. We assume as above that $\rho_I(E) = (2I + 1)\ \rho_0(E)$. Also we assume that both odd and even parity states have the same density distribution.

Using equation (24) for the energy density and the expressions (14) for the resonance parameters Γ and E_m, we have calculated the radiation widths $\Gamma_\gamma(E_0)$ as a function of Eo and A. The results of these calculations are given in graphical form. Figure IV shows the total radiation width as a function of Eo for A = 100 and Eo varying from 5 to 10 MeV, while Figure V gives $\Gamma_\gamma(Eo)$ plotted against A for A from 50 to 200 and

$E_o \simeq 8$ MeV. The spectral distribution of γ rays, emitted from a state of energy $E_o \simeq 8$ MeV, is shown in Figure VI for $A \simeq 50$ and $A = 150$.

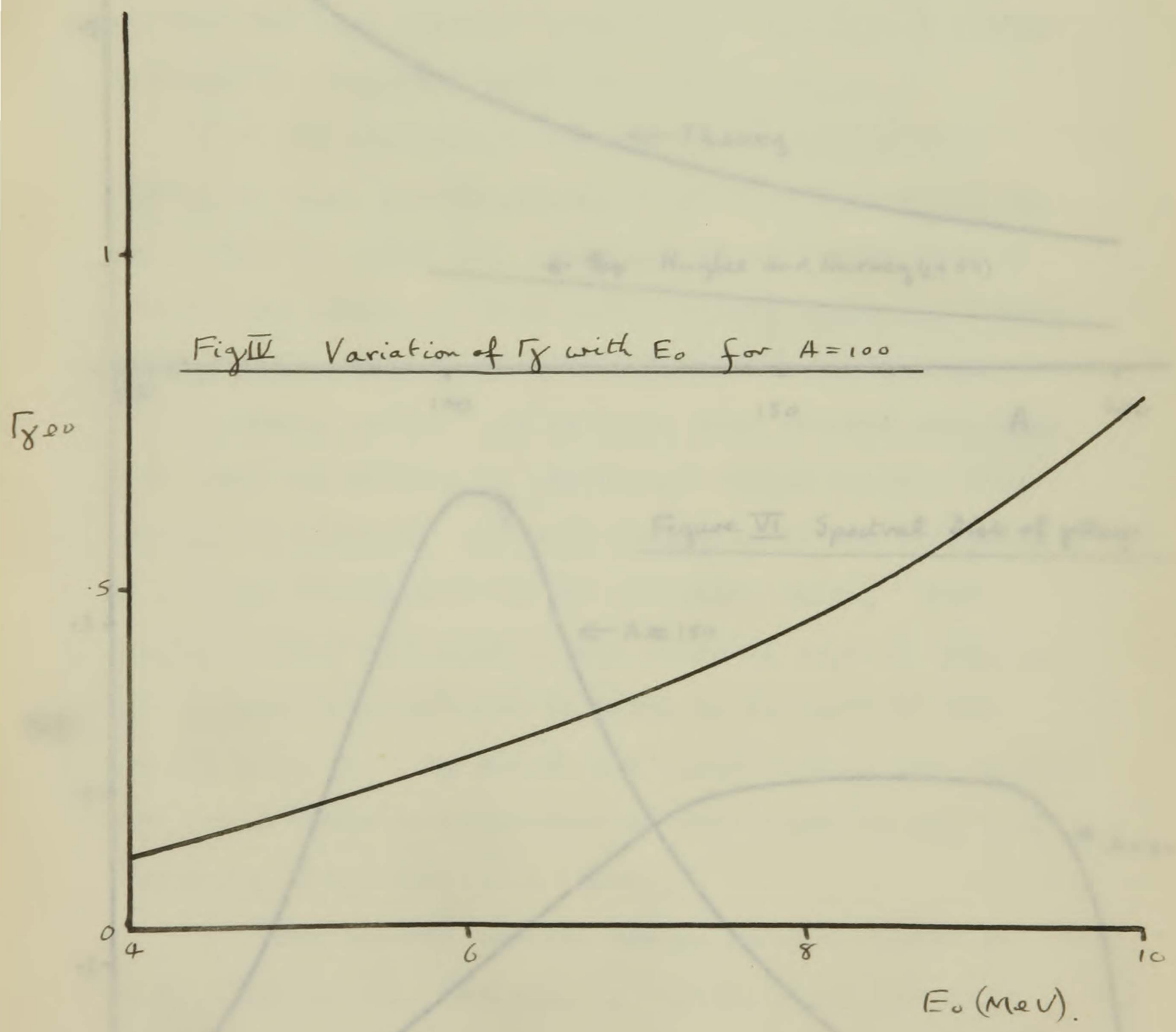


Fig VI Variation of Γ_γ with E_o for $A = 100$

Figure V Radiation width Γ_γ ~ atomic weight A

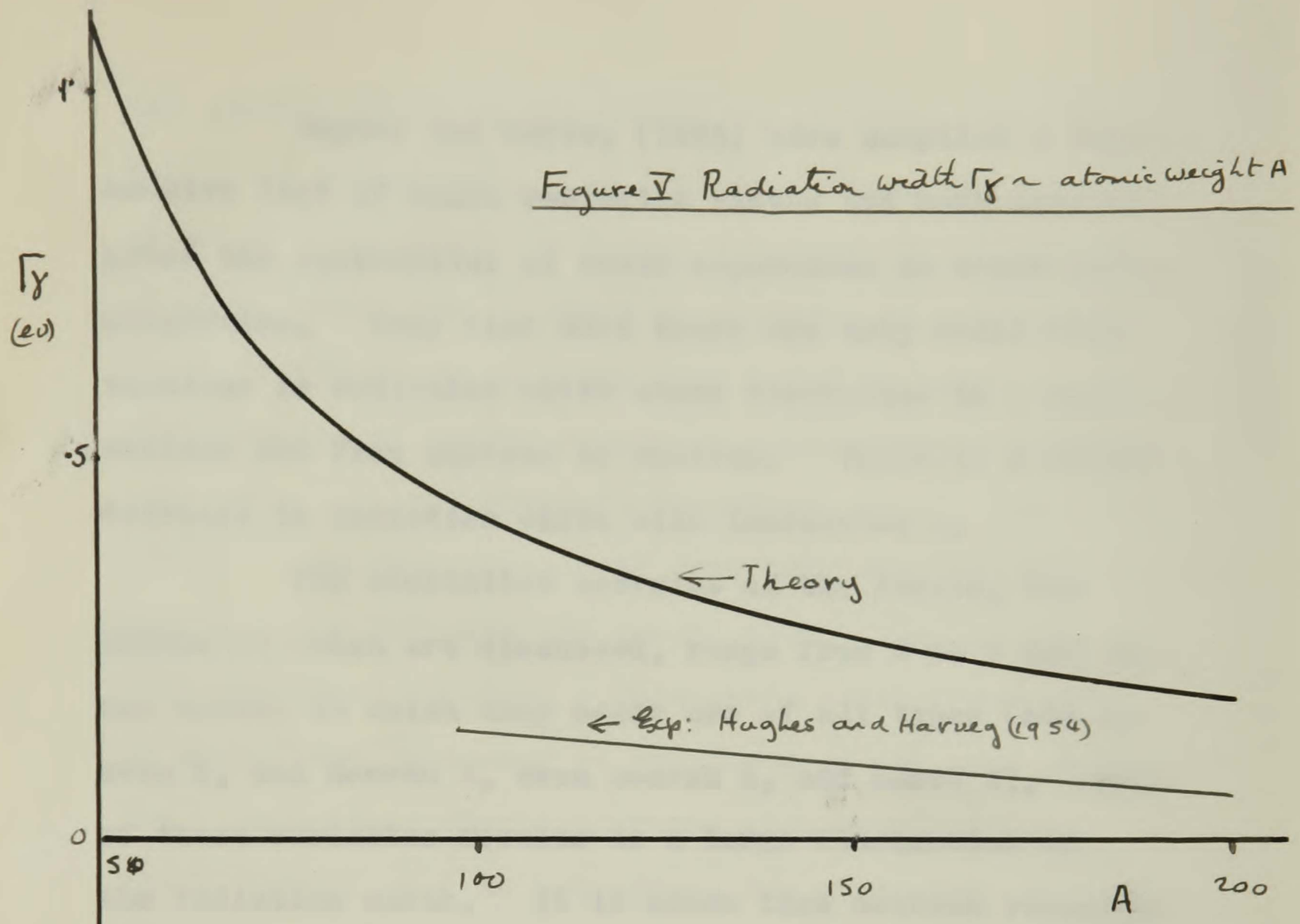
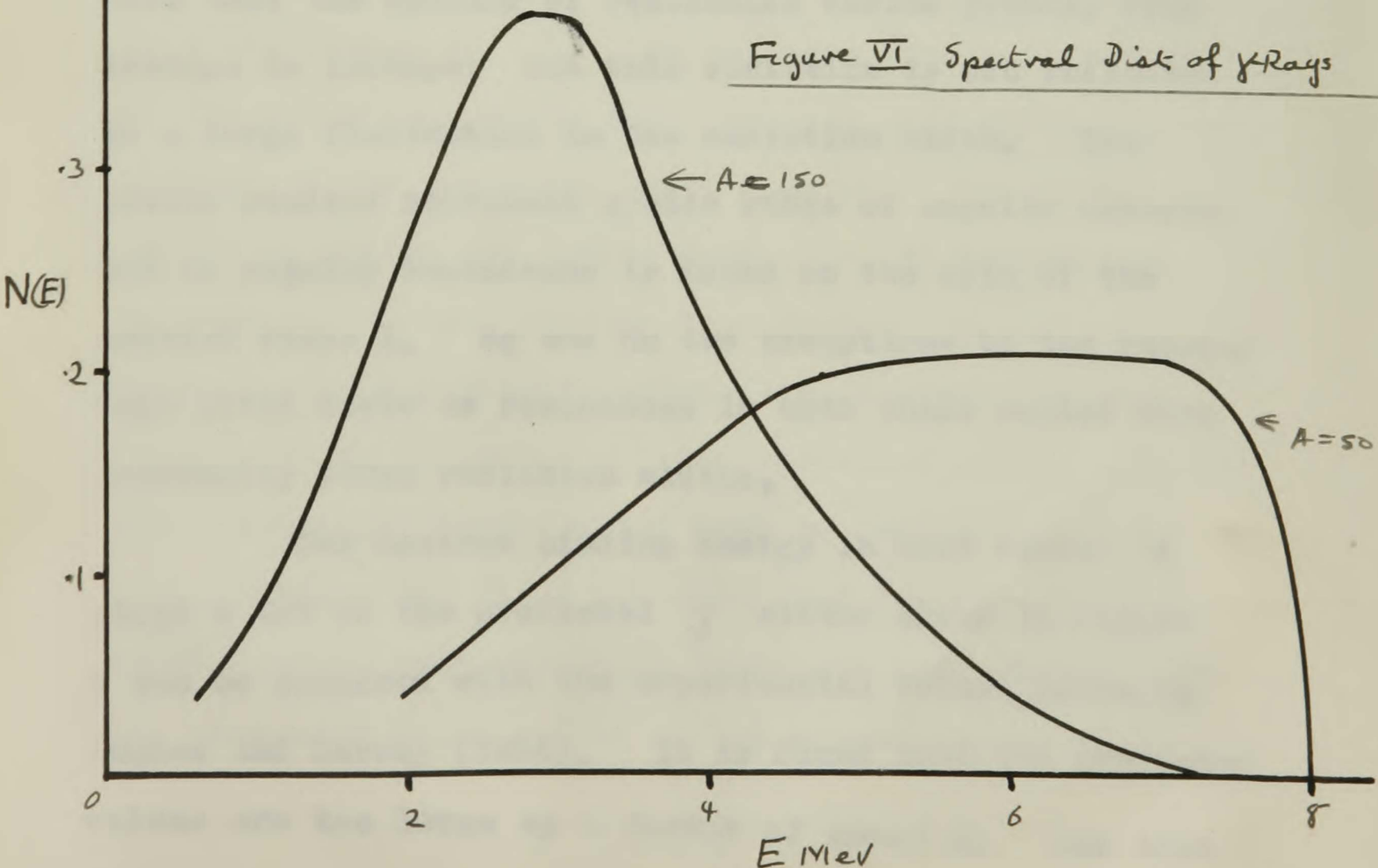


Figure VI Spectral Dist. of γ-Rays



Hughes and Harvey (1954) have compiled a comprehensive list of known radiation widths and have investigated the systematics of their dependence on other nuclear properties. They find that there are only small fluctuations in radiation width among resonances in a single nucleus and from nucleus to nucleus. There is a slight decrease in radiation width with increasing A.

The excitation energies of the levels, the widths of which are discussed, range from 6 to 9 MeV and the nuclei in which they occur are of all types (odd Z, even N, odd N-even Z, even Z-even N, odd N-odd Z). None of these variables results in a large fluctuation of the radiation width. It is known from neutron resonance work that the spacing of resonances varies greatly from isotope to isotope; but this variation is not reflected in a large fluctuation in the radiation width. The levels studied represent a wide range of angular momenta; but no regular dependence is found on the spin of the initial state I. Hg and Ru are exceptions to the general rule given above as resonances in both these nuclei have abnormally large radiation widths.

The neutron binding energy in most nuclei is about 8 MeV so the predicted γ widths shown in Figure V can be compared with the experimental values given by Hughes and Harvey (1954). It is found that the predicted values are too large by a factor of about 3; but that

the general trend of the variation of width with atomic weight is reproduced. In a given nucleus the radiation width of a resonance is random variable on the present picture, hence the γ width is expected to suffer statistical fluctuations in going from level to level. The distribution of widths should be approximately gaussian with mean Γ_γ and standard deviation μ. The ratio μ/Γ_γ has been calculated from (22) and (23) for A 100 and $E_0 =$ 8 MeV with the result that $\mu/\Gamma_\gamma = .3$. Thus the model predicts a relatively small fluctuation in going from resonance to resonance in the same nucleus. The fluctuations should decrease with increasing A. Small changes in f do not affect the predicted radiative widths appreciably and even if f is changed by a factor of 2 the change in Γ_γ is not more than a factor of 2. On the other hand quite small changes in f produce large changes in the predicted level spacing. Hence the result of Hughes and Harvey that Γ_γ is insensitive to changes in level spacing from nucleus to nucleus. There would be a slight correlation, however, a large level spacing favouring a large radiation width. It is known that near magic nuclei have an abnormally large level spacing and it is probable that the variation of level density with energy is anomolous near closed shells. The proximity of Ag and Hg to the double closed shell of

^{208}Pb could account for the large radiation widths of neutron resonances in these nuclei.

In the present model it has proved possible to resolve the problem of the dependence of radiation widths on the spin of the radiating state by using the modified sum rule (18). If the density of levels of spin I is proportional to (2 I + 1) the model predicts a total radiation width independent of I. This conclusion is unchanged for a level density independent of I provided that the spin of the target nucleus is $J > 1$. For $J \leqslant 1$ there are anomolies, the most striking occuring for a target nucleus of spin $\frac{1}{2}$. Since the captured neutrons are in S partial waves the resonances reached by slow neutron capture will have $I \geqslant 0$ or 1. It was shown for a spin independent level density that the $I = 0$ states have a radiative width only of the $I = 1$ states. This difference should be detectable in resonance capture experiments.

Figure VI shows that for $A = 50$, high energy rays are favoured over low energy γs in the decay of the capture state. For heavier nuclei ($A = 150$) medium energy γs are favoured. Further calculations show that in the decay of an excited state of low energy maximum energy γ transitions predominate, while for highly excited states transitions to intermediate states

give the main contribution to the radiation width.

As mentioned in a previous paragraph the predictions of the model presented here are too large by a factor of 3. The weakest link in the argument is probably the assumed form of the dipole photo electric cross section. This expression was derived by supposing that the collective motion of the protons against the neutrons could be treated as a linearly damped harmonic oscillation. The presence of non linear terms would modify the energy dependence of the cross section at low energies sufficiently to bring the results into line with experiment. Throughout the work it has been assumed that M1 and E2 transitions do not contribute much to the radiation widths of the highly excited states we have considered. Some evidence for this comes from work of Kinsey, Bartholomew and Walker (1954) on the γ rays emitted from nuclei following thermal neutron capture.

References.

Bethe, H.A., 1937, Rev.Mod.Phys., 9, 69.

Biedenharn, L.C., Blatt, J.M., and Rose, M.E., 1953, Rev.Mod.Phys., 24, 251.

Blatt, J.M., and Weisskopf, V.F., 1952, "Theoretical Nuclear Physics".

Bohr, A., 1952, Dan.Mat.Fys.Medd., 26, No.14.

Bohr, N., 1936, Nature, 137, 344.

Condon, E.U., and Shortley, G.H., 1935, "Theory of Atomic Spectra", (C.U.P).

Le Couteur, K.J., and Lang, J.M.B., 1954, Proc.Phys.Soc.A. 67, 585.

Feshback, H., Porter, C.E., and Weisskopf, V.F., 1954, Phys.Rev., 96, 448.

Feshbach, H., and Weisskopf, V.F., 1949, Phys.Rev., 76,1550.

Ford, K.W., 1954, Phys.Rev., 95, 1250.

Goldhaber, M., and Teller, E., 1948, Phys.Rev., 74, 1046.

Hauser, W., and Feshbach, H., 1952, Phys.Rev., 87, 366.

Hughes, D.J., 1955, Bull. of Am.Phys.Soc., Jan.1955, 22.

Hughes D.J., and Harvey, J.A., 1954, Nature, 173, 942.

Hughes, D.J., Garth, and Levin 1953, Phys.Rev., 91,1423

Jones, L.W., and Terwillinger, K.M., 1953, Phys.Rev., 91, 699.

Kinsey, V.V., and Bartholomew, G.A., 1954, Phys.Rev., 93, 1260.

Levinger, J.S., and Bethe, H.A., 1950, Phys.Rev., 78, 115.

Montalbetti, R., Katz, L., and Goldemberg, J., 1953, Phys. Rev., 91, 659.

Mott, N.F. and Massey, H.S.W., 1950, "Theory of Atomic Collisions".(C.U.P).

Nathans, R., and Halpern, J., 1954, Phys.Rev., 93, 437.

Racah, G., 1942, Phys.Rev., 62, 438.

Steinwedel, H. and Jensen, J.H.D., 1950, Z.Naturforsch.,5a, 413.

Teichmann, T. and Wigner E.P., (1952) Phys.Rev., 87, 123.

Wigner, E.P., and Eisenbud, L., 1947, Phys.Rev., 72, 29.