

1 Introduction

The gravity equation in international trade is one of the most robust empirical regularities in economics. A remarkable numeric similarity of estimated coefficients over time, space and for different types of goods has puzzled economists for some time. In this paper I provide a geometric argument why a large class of data generating processes lead to the observation of the coefficients typically found, and thus provide a natural and simple explanation for the numeric similarity of estimates provided in this literature. I provide examples of trade that are consistent with this model.

In the notation of Jan Tinbergen (1962), gravity equations in economics are written as

$$E_{ij} = \alpha_0 Y_i^{\alpha_1} Y_j^{\alpha_2} D_{ij}^{\alpha_3}, \quad (1)$$

where E_{ij} denotes exports of country i to country j , Y_i GNP of country i , Y_j GNP of country j and D_{ij} the distance between the two countries. Tinbergen provides estimates suggesting that α_1, α_2 are close to 1 and α_3 is close to -1. Similar gravity equations have frequently been estimated since, and estimates that don't reject these parameter values have been observed for aggregate trade flows as well as a wide range of goods or services, and they have been remarkably persistent over time (Disdier and Head 2008). While the reason for the persistence of the coefficients indicating economic size (α_1 and α_2) is predicted by many of the current standard models of international trade (see discussion in Evenett and Keller 1998), the persistent role of distance α_3 continues to puzzle economists. For example, a paper by Disdier and Head (2008) is titled "The puzzling persistence of the distance effect on bilateral trade" and argues that point. Chaney (2013) provides a paper that offers a theoretical explanation for this puzzle.¹

The main contribution of this paper is to emphasize the analogy of gravity equations in inter-

¹An occasional confusion in the gravity literature worth clarifying is that the distance coefficient does not measure trade costs. It merely measures the decline of trade with distance, for example relative trade levels at 100 vs. 10 kilometers distance. Ceteris paribus there is no reason for the distance coefficient to decline if transport costs fall, as a constant fall of the marginal trade cost would be absorbed by the constant α_0 (see Yotov 2012) and leave the distance cost ratio constant.

national trade with gravity in physics, and to take it further than the literature has so far. I describe the geometric reason that leads to the inverse square relationship for Newtonian gravity in three dimensional space and argue that a similar mechanism in two dimensional space can explain this puzzle. I argue that there is a large class of processes that lead to these mathematical relationships, and many of these processes have an interpretation as trade. The two most important examples are (i) if transport networks branch out like random trees and sales are equally likely along any road, (ii) if sales connections take place like Levy flight processes, the standard way to model search in two dimensional space.

Trade takes place in two dimensional space, and thus it is not surprising that the answer to some of the empirical regularities related to distance are found in elementary two dimensional geometry. Head and Mayer (2013) write that gravity equations “emerge from mainstream modeling frameworks in economics and should no longer be thought of as deriving from some murky analogy with Newtonian physics”. Yet the analogy with physics has not fully been exploited, and it is helpful to explain the persistence of estimates of -1 on the log of distance, which is not typically part of trade models that predict gravity.

This simple geometric argument, explained in greater detail further below, is based on the geometry of two-dimensional space. A ray sent out randomly from the center of a circle is twice as likely to hit a given arc of fixed length on a circle with radius r than on a circle with radius $2r$. As I show below, this elementary observation can be used to show that shapes of arbitrary shape and size, intersected by rays sent from a point outside them will lead to lengths of intersections that approximately obey Tinbergen’s equations. This relationship emerges quite generally from intersections of one dimensional lines with two dimensional areas, and a large class of processes in two dimensional space will thus mechanically conform to gravity with the coefficients estimated. A robust empirical regularity established by data that aggregate a very large number of individual and independent relationships would invite an explanation of that type: of a simple, general mathematical relation that encompasses a large number of different

processes². As discussed below, this approach is robust to aggregate economic activity that is unevenly distributed in space.

Given the similarity of the intuition to gravity in physics I find it very likely that Tinbergen, who named gravity equations in economics after having studied physics, was aware of this relationship. Yet this explanation seems not to be known widely in economics, answers a question frequently raised, and to my knowledge has no reference in the existing literature.

This paper proceeds as follows: Section 2 describes the geometric process that leads to this result, Section 3 discusses three classes of examples of trade that are consistent with this model, Section 4 concludes.

2 Geometry

This section gives the main geometric argument of this paper, first in the form of a simple, intuitive argument, and then in a more formal treatment. The end of the section links the central finding to gravity equations.

The reason for the persistence of the parameters estimated in gravity equation in trade may be found in the geometry of two dimensional space, yet it is instructive to start with the three dimensional case commonly used in physics. The intuition is explained graphically in Figure 1. Four rays labeled a , b , c and d are sent out from a point O and intersect three areas at distances r , $2r$ and $3r$. The same four rays that span one square in distance r span four squares at distance $2r$ and nine squares at distance $3r$. Thus the areas that the intersections of the rays span increase quadratically in the distance of the areas to O . If the rays represent some physical force in three dimensional space this force will thus decrease in the distance squared, a common finding called the ‘inverse square law’ in physics. It applies to gravitation as well as electric, magnetic, light, sound or radiation phenomena.

²See discussions in Gabaix 1999, Kirman 1993 or Rauch 2014.

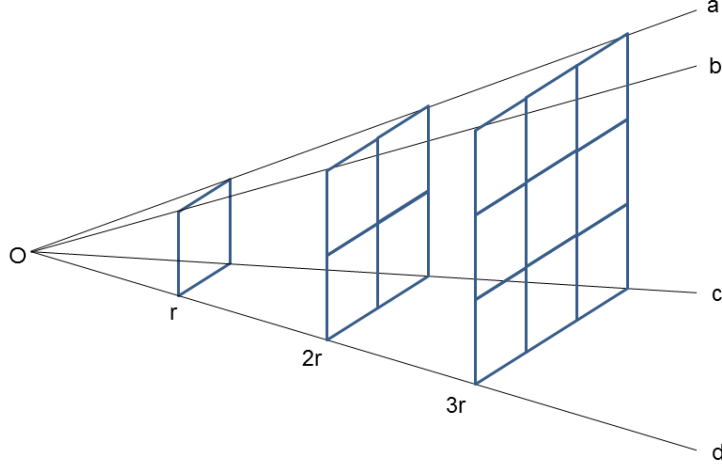


Figure 1: Rays a , b , c , and d emanating from O intersect areas at distances r , $2r$ and $3r$.

An analogous observation can be made for phenomena in 2-dimensional space. Consider the area created by the rays c and d . The lines they intersect at distances r , $2r$ and $3r$ now decrease linearly in distance, not in squared distance, as evident from looking at the lines they intersect. We thus expect radiation or similar phenomena in 2-dimensional space to weaken linearly in distance.

The point O in the above example can represent a very small origin country. The ray can represent trade. Assume that the angle at which the ray radiates out of O is random. Then we can use the above geometric example as the basis for a thought experiment in which the size of the origin country and the size of the destination country do not change, both are very small, but the distance between the two countries doubles. Following the notation in Equation 1, if we normalize units such that at distance $D = r$ value $E = e$, where e denotes the probability that a pixel at that distance is intersected by the ray, then we would observe at $D = 2r$ a measure of $E = e/2$. This is consistent with Equation 1 if and only if $\alpha_3 = -1$.³ We may think of a destination country as a number of small squares, to which I refer as pixels, grouped together.

³This is clear from solving the system of equations $e = cr^{\alpha_3}$ and $e/2 = c(2r)^{\alpha_3}$, where c is a constant in this thought experiment.

If all of these pixels are at the same distance from the origin, doubling the number of pixels doubles the probability that the ray intersects these pixels. This is consistent with Equation 1 if and only if $\alpha_2 = 1$. Equally assuming that there are twice the number of origins of rays at constant distance to one pixel, the probability that it is hit by at least one ray will be double. This is consistent with Equation 1 if and only if $\alpha_1 = 1$. This interpretation of a country as a set of pixels introduces some noise, as a pixel might intersect a ray with longer or shorter length of intersection. This type of distortion can be made arbitrarily small by reducing the size of pixels.

Let me give an example of such a model with an economic interpretation. The world is a 2 dimensional orchard with trees distributed on the plain. Each tree has an exogenous number of fruits. There is some variety in fruits, and people value variety. The owner of the tree goes out and visits other locations, exchanging fruits whenever she comes near another tree. We call this exchange “trade”. Other owners visit the tree and also engage in trade. Fruits that are not exchanged are consumed as domestic consumption. In this example we would expect gravity to hold. As the following discussion shows, this is robust to having trees of different size, as well as aggregation of trees (countries) of different size.

This problem can be set up as follows more formally: Consider a two dimensional vector space. It contains point O , from which a force originates, as in Figure 1. This point is normalized to have coordinates $[0, 0]$. A force is defined as a set of 2-dimensional vectors that describe the path of movement. As examples from the sciences we may think of a ray of light, sound, smell, gravity, a foraging animal, etc., in the interpretation of trade we may think of a traveling salesman. The first vector $\mathbf{v}_1 = [x_1, y_1]$ describes the first step, the second vector $\mathbf{v}_2 = [x_2, y_2]$ the second step, and so on. Variables x_i and y_i are random variables.

$$\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \dots \\ \mathbf{v}_n \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \dots & \dots \\ x_n & y_n \end{bmatrix}$$

The path, denoted V is described by the sum $V_n = \mathbf{v}_1 + \mathbf{v}_2 + \dots + \mathbf{v}_n$. After two steps the vectors arrive at point $[x_1 + x_2, y_1 + y_2]$. The distance travelled after i steps is denoted d_i , which is defined as the norm $d_i \equiv \|\sum_{j=1}^i \mathbf{v}_j\|$, and can be computed as Euclidian distance. The space in which we measure gravity is described by circle with radius a around O . Then $c_{\lambda a}$ is a circle with radius λa around O , with $0 < \lambda < 1$. I make two assumptions on V .

Assumption 1: $\max\{d_1, d_2, \dots, d_n\} > a$ for some large n .

This is an assumption of participation, it ensures that the force covers the whole area studied.

Assumption 2: $P(V_n \cap c_{\lambda a}) = k \forall \lambda$, where k is a constant for some large n .

The probability that V_n intersects a circle of radius λa around the origin is constant for all λ . This is an assumption of stationarity, it implies that the force behaves similarly at any distance from the origin. Examples that fulfill this assumption include a straight line from the origin in any direction, a sine curve, a Levy flight path, and distributions where vectors \mathbf{v} are drawn iid from any probability density function.

There is some empirical evidence for stationarity of trade flows. Hillberry and Hummels (2008) conduct the most detailed study on the relationship between trade volumes and distance, and conclude that while at short distances trade values fall steeply in distance, they are relatively flat after 200 miles. In the standard CEPII trade dataset 99.75 of distances are longer than 200 miles. For the remaining distances this is thus evidence for a flat relationship. Head and Mayer (2013) compute a cumulative plot of trade values over distance to nonparametrically analyze the relationship between trade volumes and distance. On a two dimensional plane the model

would predict a linear relationship between distance and aggregate trade flows. On a sphere such as the surface of the earth the model predicts that cumulative trade flows follow the sine function (at least on the half of the earth on which the trade flow originates).⁴ In Figure 2 I recompute Figure 2 from Head and Mayer (2013) for distances up to 10,000km, and compare it to a sine function, a linear function, or an iceberg trade cost (the best fit to the data in a regression of log trade on log distance). The sine curve provides the best fit, sum of squared differences from the data are a quarter compared to linear, and close to the iceberg fit.

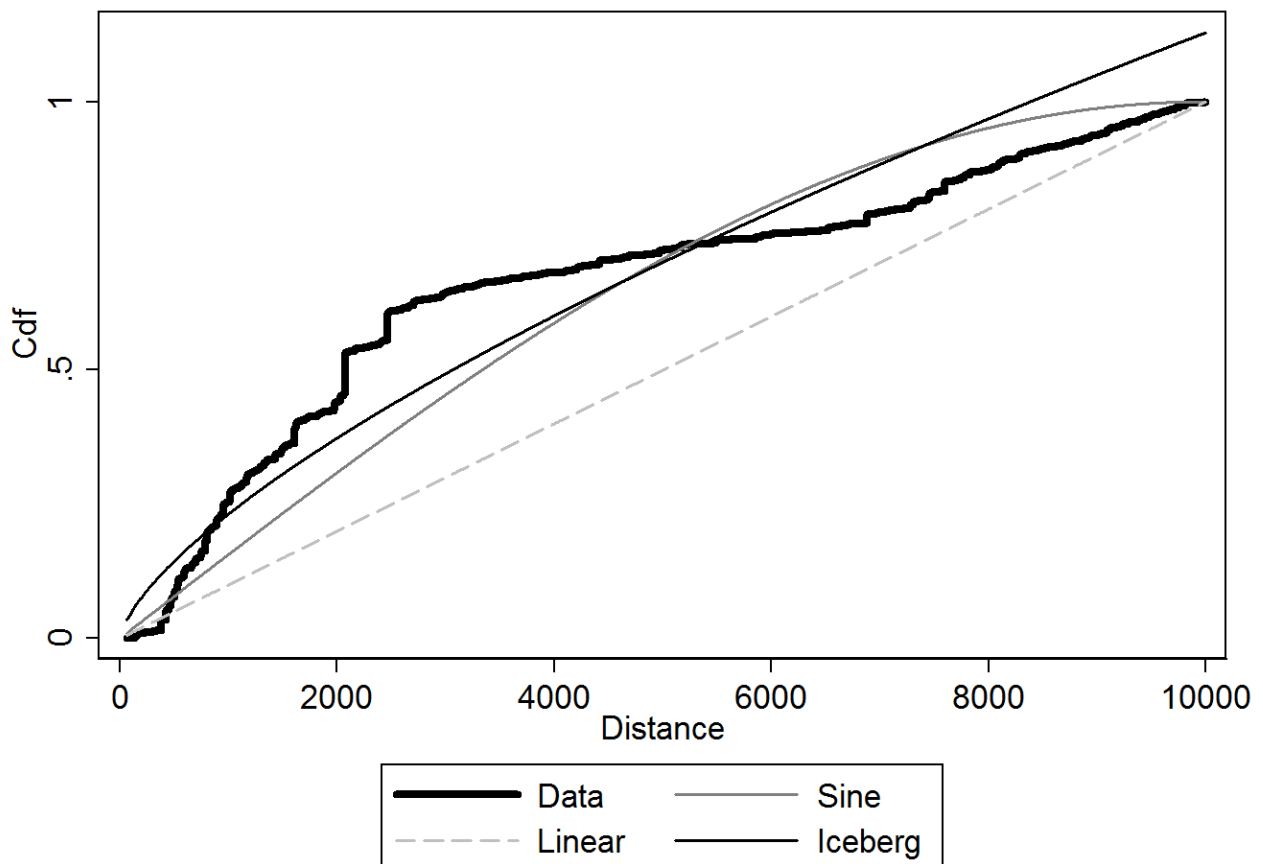


Figure 2: This Figure recreates Figure 2 of Head and Mayer (2013). It shows cumulative trade flows by distance, compared to a sine curve, a linear curve, or the best fitted linear approximation (iceberg).

Lemma 1: The probability to find an intersection of V_n with a small arc of a circle with radius

⁴To see this, consider an observer who walks straight from the North Pole to the equator keeping the same longitude. At regular intervals he measures the latitude of the point he finds himself at. These correspond to the radius of the circle above on a sphere. These radii increase following the sine function until the equator.

λa decreases linearly in λ .

From assumption 2 it is clear that a data generating process by which distances to point O are computed from points taken in regular distances along one or many simulations of V will result in a uniform distribution of distances. The probability of an intersection decreases as the number of arcs increases with the radius.

Assume now that there are multiple points like O that send forces in all directions, and equally are intersected by forces originating at other points. Define trade between two points to take place if a force originating in one intersects the other. Define a country as a set of such points. The size of a country, its GDP or GNP, is simply the sum of the points it consists of. Each trade event is associated with the Euclidian distance between the sender and receiver point. Aggregate distances to the country level using the harmonic mean.

Proposition 1: Trade flows generated by this process will conform in expected value to equation 1.

This implies that the gravity equation is met by countries of arbitrary shape and location with respect to one another. Aggregation does not create any distortion provided aggregate distance between countries is measured by the harmonic mean of all pairwise distances between two countries. To see this consider an origin and a destination country, both as a number of grouped pixels that are at varying distances from one another. Denote by r_1, r_2, \dots, r_n all pairwise distances between the full set of pixels in the origin and the full set of pixels in the destination country. Denote the number of pixels in both countries Y_1 and Y_2 , then $n = Y_1 Y_2$. The gravitational pull generated by the two points associated with r_1 is c/r_1 , inverse to distance as above, where c is a gravitational constant. The full gravitational pull between the two countries is then $E_{ij} = c Y_1 Y_2 (1/r_1 + 1/r_2 \dots + 1/r_n)$. If we measure distance by the harmonic mean of all pairwise distances $D_{ij} = (1/r_1 + 1/r_2 \dots + 1/r_n)^{-1}$, this corresponds exactly to equation 1 with $\alpha_1 = 1$, $\alpha_2 = 1$ and $\alpha_3 = -1$, where Y_1 and Y_2 are interpreted as GNP. Note that this measure only requires pairwise distances and is independent of the shape of both countries. This measure of distance between two countries has the advantage over the commonly applied

distance measures, typically using either the distance between capitals or the largest cities, of having this theoretical foundation.

Next I discuss how violations of these assumptions influence the result. This discussion shows that even some violations of the assumptions above would not alter the distance coefficient. I also use a simulation in this exercise to verify the discussion. In this simulation I create random countries shaped as circles, defined by a centroid and a radius in a unit square. Locations and sizes are drawn from uniform probability distributions. I simulate forces as described, and count trade between two simulated countries the force originating in one intersecting the other. I verify that the distance coefficient in this simulated data is close to -1 at five percent level of significant in around 95 percent of cases under the assumptions above.⁵

First I explore the consequences of Assumption 1 being violated. Consider the force V moving as described above, but stopping if it reaches a distance d from origin O . The consequence would be that no trade of distance larger than d would be observed. The logarithm applied in equation 1 converts zero trade flows into missing observations. For the remaining units of observed trade flows the distance coefficient remains at -1 . I verify this assertion using a simulation as described above. Hence violations of this assumption can still lead to distance coefficients of -1 .

Second, I explore how a violation of Assumption 2 affects the result using a simulation. The two main reasons why this would happen is either if the movement V changes after some time (such that the step frequency is different at greater distances from the origin) or that the intersections of V and a destination country generates trade of different magnitude at different distances from the origin (such that the origin country trades more with a closer country at the same frequency of intersection). Both cases can be generally expressed such that trade volume E is a function of distance λ . Using simulations I can verify that if trade volumes change in distance from the origin by function $E(\lambda) = f(\lambda)$, then distance will enter the gravity equation in the functional form of $f(\lambda)/\lambda$.

⁵In 10 iterations of 100 simulations I can reject a coefficient of -1 for the distance coefficient the following number of times at five percent level of significance: 96, 93, 90, 93, 93, 94, 97, 94, 97, 95.

Third, a distortion is introduced by the world not being flat, and measurable trade taking place on a sphere. These distortions however are negligible at short distances.⁶ Most trade takes place at distances that are small relative to the circumference of the Earth, thus empirically this is not a concern that likely introduces a great bias in practice.

Forth, economic activity is not evenly distributed in space, both across as well as within countries. GNP control variables included in estimations of gravity serve to level the playing field in estimations of gravity equations. Their inclusion flattens the world as if economic activity was evenly distributed. If economic activity was uniformly distributed within countries, there would be no bias, even if countries had different GNPs (this point is developed further in the example of transport networks below). If economic activity is unevenly distributed within two countries, such that the points are of different size, the correct way to measure distance between them is to use a weighted harmonic mean, $D_{ij} = c(w_1/r_1 + w_2/r_2 \dots + w_n/r_n)^{-1}$ where the weights w are the product of the share of economic activity within the country of both points associated with the distance. To see why this must hold note that the gravitational pull in this case would be proportional to $(w_1/r_1 + w_2/r_2 \dots + w_n/r_n)$. Thus if distance between two countries was measured by this harmonic mean, and GNP controls included, the model described would hold in a world with economic activity that is unevenly distributed within and across countries. Note that even zero activity such as on the location of oceans or deserts does not introduce a bias, as a GNP control of zero would lead to a prediction of zero trade for such pixels.

A final limitation of the approach compared to the empirical literature is that the predicted gravity equations do not include terms for multilateral resistance (Anderson and Van Wincoop 2003), and would not predict such terms empirically. These terms however would appear in a version that adds heterogeneous border effects to the theory as additional frictions to the radiation effect described.

⁶The distortions follow the sine function.

3 Examples and relationship to existing models

Existing models of trade generate gravity equations related to equation 1, in which the trade elasticity is related to technology parameters (following Eaton and Kortum 2002), or preference parameters (following Armington 1969, Melitz 2003). These models and their differences are summarized in Arkolakis et al (2009). The current model abstracts from both, technology and preferences, the magnitude of the distance coefficient arises from transportation or search costs in 2-dimensional space.

To illustrate the difference between this and existing models, I sketch a simple version of an Armington (1969) model. Suppose there are different farms scattered on a plain with no transport costs. They each produce a different variety, and consume a homothetic aggregator of all available varieties. Such a model could predict that trade between two farms of size Y_1 and Y_2 is just $Y_1 Y_2 / \sum_i Y_i$. This model is closely related to the ‘balls-and-bins model of trade’ (Armenter and Koren 2014). In such a frictionless world an estimate of the gravity model would result in an insignificant gravity coefficient. What would introduce a unit trade elasticity would be if trade happened along rays going from one farm to the other. There are two main examples how we can think of a ray: information and transport networks. I discuss both below.

Information and search

This model of trade as radiation can be interpreted as a model of information, similar in its interpretation of trade costs to the model by Chaney (2013). We could think of trade deriving from random encounters between a seller and a buyer. Any functions describing the movement of information sent from the seller or the buyer that is stationary up to a distance of d would lead to gravity equations for information up to that distance d .

Let me give one example that may be particularly relevant. A useful way to model the flow of information is to turn to the large and growing literature on optimal search in two dimensional space. Many animal species including albatrosses, bony fishes, bumblebees, deer, microbes, penguins, sea turtles and sharks seem to conform to the same search behavior when searching

for food, a model labeled the Lévy flight foraging hypothesis. One could argue that these animals ‘trade’ according to gravity equations.⁷ By Lévy flight search behavior animals searching for food would choose its step length l from the probability density function $P(l) \approx l^{-\mu}$, with $1 < \mu \leq 3$. This fat tailed distribution leads to occasional long steps, which imply that a given spot is revisited with much lower probability than under Brownian motion. Under some conditions this type of search behavior is thought optimal (Viswanathan et al 1999), with the optimal search parameter close to $\mu = 2$. This process fulfils the condition of stationarity required for the geometrical argument above to be valid. Thus foraging behavior described by this theory, or search by the optimal search rule in two dimensional space are consistent with the geometrical reasoning in the previous section and thus lead to gravity equations with $\alpha_1 = 1$, $\alpha_2 = 1$, and $\alpha_3 = -1$. Note further that Lévy flight search predicts movement with an infinite expected length.⁸ There is some empirical evidence that humans conform to Lévy flight movement patterns in some circumstances (Edwards et al 2007, Schuster et al 1996, Sims et al 2008). Noteworthy is Brockman et al (2006), who show that banknotes circulate consistent with the Lévy flight theory. Banknotes exchange in most cases involves direct contact between people, and thus provide an estimate of the geography of human interaction, and casual human information flow. This type of explanation is especially relevant to explain local trade, where this type of information exchange is expected, and indeed there is evidence that the decline of trade with distance is strongest at short distances (Hillberry and Hummels 2008), as equation 1 implies.⁹

⁷Is it reasonable to speak of trade in the case of animals? Merriam Webster’s Dictionary (2014) defines trade among other definitions as “the activity or process of buying, selling, or exchanging goods and services” and “the act of exchanging one thing for another”. When a bee visits a flower it provides the flower the service of pollination, while receiving the good nectar in return. By the dictionary’s definition the bee trades with the flower. (Adrian Wood suggested to me that the definition in the dictionary should include the word ‘voluntary’. I agree, but feel that it is still justified to speak of trade in the case of a bee and a flower.)

⁸I conduct a test of the gravity model for bees, for which I found the required data. Zurbuchen et al (2010) develop a methodology to study flight distances of bees that only forage a single plant genus. Host plants were successively placed in increasing distances from the fixed nesting stands. They provide counts of bees by distance, and indeed, for none of the sets of numbers provided in their paper did a regression of the log number of bees by distance on log distance allow to reject a coefficient of -1 on log distance at 5 percent level of significance.

⁹It is not clear if the numbers in Hillberry and Hummels (2008) suggest that the distance coefficient of -1 holds at short distances or not.

The encounter of a seller and a buyer is a sufficient condition to trade, not a necessary one. This explanation requires that these informational encounters are important quantitatively relative to other functions of distance such as the increase of trade costs or the divergence of preferences in distance, as stated above. Yet there is evidence that information frictions are very costly. Head et al. (2014) shows that information flows even measured as citations in highly specialized fields of mathematics are much more likely if there is evidence for personal interaction between the cited and citing mathematician. Lendle et al (2016) find that distance coefficients for an online trading platform are much smaller in absolute value than one, which points to the importance of search costs to generate geographic friction.

Transport networks

It is plausible to think of trade to take part along rays going out of a center, for example trucks going out in various directions along a road network from a factory, a ship going down a river or river delta selling at every harbor it encounters, water or gas being sold along a pipeline. All these will lead to gravity equations empirically up to a distance d if the goods that are traded are shipped on a network that stretches out at a distance larger than d , if sales are equally likely or frequent at any point along that road, at any distance to the origin, and if economic activity is uniformly distributed in space. This is the direct interpretation of trade as radiation. Note that the theory applies if a particular trade is decomposed into several branches of inputs or outputs, that are entangled in a complex supply chain and shipped independently. This example also includes networks combining various local searches into larger supply chains.¹⁰

The implicit assumption of evenly distributed economic activity might be invalidated in these examples, if a navigable river attracts economic activity (for quantifications see Michaels and Rauch 2014). Yet even if this condition of even economic activity is invalidated the gravity equations might apply. Consider the extreme example where all the population lives along a

¹⁰Dedrick et al (2009) decompose the main inputs for computers by source country, and show that products such as the Lenovo ThinkPad in 2005, the HP notebook PC in 2005, or the fifth-generation iPod in 2005 use inputs from various countries in Asia, Europe and America alike. Examples for global supply chains are numerous and their importance is growing (see among many other examples Baldwin and Venables 2013, Yi 2003, Klier and Rubenstein 2009, Johnson and Noguera 2012).

big river uniformly distributed along its shore, and all other areas are barely inhabited, as in the example of the river Nile. Then there will nevertheless be twice as much land at distance $2r$ from the origin than at r , albeit uninhabited. There may be the same level of trade at distances r and $2r$, yet relative to the land that exists, observed by the econometrician, at these distances trade is half at $2r$ to what it is at r . Even the world in this extreme example would conform to Tinbergen's gravity, since gravity equations include linear terms for GDP or GNP. If a pixel near the Nile has x times the economic activity to a pixel in the desert, these GNP controls will absorb this difference of activity, and scale the expected trade of the people in the desert down by this factor of x . The GNP controls even out the economic activity of the space considered.¹¹

4 Conclusion

Any process of human exchange of goods, information, or services by which traded items or information originate at one point and randomly spread out from there by a stationary process in two dimensional space up to a distance d leads to gravity equations up to d where trade levels are exactly proportional to the size of both countries and exactly inversely proportional to distance. We would find the same properties if the origin and destination of an input or final good are chosen independently from one another. This fact is equivalent to Newtonian gravity in three dimensional space. The relationships holds unbiased for countries of arbitrary shape, size and relative location, and can theoretically absorb the unevenness of economic activity in space. This model can explain why coefficients on log distance in estimates of gravity equations have been so persistent near -1 over time. The theory also implies that distances between two countries in empirical gravity papers should be measured by the harmonic mean of pairwise distances of local economic activity in both countries.

¹¹As stressed above, even zero economic activity such as over oceans or deserts would not introduce a bias, as zero GNP controls for pixels on the right hand side would lead to predictions of zero trade on the left.

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