Interpretations of Marks from Draughting Tools in Some Italian Renaissance Drawings

Evidence for the Use of Geometrical and Numerical Design Systems

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Part I

A Survey of the Historical Background for Design Systems

Introduction

Design is a process of trial and error, or, to be more precise, of trial, judgment and modification. The designer first makes an attempt at creation and then judges the results. If the results do not satisfy, the designer then makes modifications or undertakes new trials which are subsequently judged. The sequence of trial, judgment and modification cycles until the designer is satisfied. During the judgment stages the criteria by which a work in progress is judged will consist of intuitive components along with, perhaps, intellectual components. The intuitive component, where the designer makes decisions based on what looks or feels right, obviously cannot be analyzed with forensic certainty. This is because of the overwhelming number of possible influences that impinge on the designer, both conscious and unconscious. Such influences as emotional preferences for certain types of compositional elements affect the designer in ways that we cannot analyze. Hence, there is no way of retracing the designer's subjective feelings.

When evidence permits, however, analysis may reveal, with much greater certainty, the intellectual criteria for certain design choices. These intellectual components in a design process involve systems or, in other words, self-imposed

restrictions, such as rules of thumb, constructive geometry, numerical ratios or linear perspective.

The subject of this inquiry involves the process of making works of art and architecture in the Italian Renaissance.[1] Specifically, it focuses on that aspect of the process where artists design their works. By *design*, I am referring to the process by which an artist makes the formal decisions and mechanical operations that result in a finished composition. Some of these decisions which determine the size, shape and position of design elements are, of course, completely intuitive. The disposition of visual elements might be the result of a guess or some other non-system related choice, and consequently, we can never know for certain why they were made.

Other decisions, however, are system-based. These systems may be either traditional workshop procedures resulting from the commercial aspect of the artistic enterprise or intellectual procedures learned from contemporary treatises on methodology or original investigation. Some drawings contain evidence of the use of systems. By analyzing that evidence one can determine what the artist was thinking, at least with regards to the system.

This thesis is the result of a search for evidence of Renaissance artists' application of system-based procedures in producing their designs. Examples deemed suitable for analysis have been limited to those drawings where the evidence of a design system, in the form of draughting tool marks indicating geometry or other system, is clear and the analytical method of determining the artists' procedures was successful. Pictorial and architectural designs are considered together because many of the same organizational systems were used in both.

The research method of this thesis is to examine drawings that contain marks from draughting tools and interpret the marks in an attempt to understand the design processes that produced the evidence. Analyses of these marks reveal patterns in the usage of certain systems that relate to the process of design or artists' methods. These systemized design operations are here categorized under the headings of constructive geometry and numerical ratios. In order to provide a context for interpreting the use of the two types of systems in these drawings, the first part of this inquiry outlines the historical design

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[1]The word "Renaissance," throughout, refers to the 15th and 16th Centuries in Italy.

traditions that were familiar to Renaissance artists and architects. This outline is limited to those design traditions that are relevant to the works and systems analyzed. In Chapter 1, the historical background for geometrical and numerical design systems is traced through primary and secondary sources with references to works from ancient Egypt, Greece and Rome to works of the Middle Ages. Chapter 2 consists of the literary input of treatises and other written sources by Cennino Cennini, Leon Battista Alberti, Antonio Averlino (called Filarete), Francesco di Giorgio Martini, Leonardo da Vinci and Andrea Palladio. Chapter 3 provides background information on the investigation itself: the research method of examining drawings and the drawing instruments that made the marks. There is also a summary of Renaissance design traditions, the knowledge of which, makes it possible to interpret the draughting marks in the context of a tradition of design procedures.

The second part of this inquiry consists of observations and analyses of the physical evidence from draughting tools found in the original drawings of Italian Renaissance artists and architects. The analyzed drawings are by Leonardo da Vinci, Antonio da Sangallo the Younger and Andrea Palladio. The interpretations of the evidence will involve the systems of constructive geometry and harmonic numbers, linear perspective, the crossed arcs method of locating reference axes, pre-planning verses improvisation, and the process of trial, judgment and modification.

Case studies of additional drawings that are of interest to this investigation are included in Chapter 8. These more concise studies provide examples of draughting tool marks in drawings from which information may be obtained on various artists' working methods. The drawings analyzed in Chapter 8 are by Leonardo, Paolo Uccello, Michelangelo and Peruzzi. Also included is an anonymous Italian architectural drawing from the 16th or 17th Centuries which demonstrates the continued application of the kind of constructive geometry, for architecture, advocated by Francesco di Giorgio in the 15th Century.

Design systems are employed in a step-by-step sequence. When employing a design system, a designer typically has to make steps that are dependent upon previous steps. If each step is dependent on those that precede

it, then it is sometimes possible to reconstruct the order of the entire sequence of operations from the tracks left behind by the designer. Knowing the sequence of operations that a designer performs makes it possible to speculate on the motives for each step and hence to have a better understanding of the design process. This thesis attempts to initiate an investigatory method whereby this sequence of operations can be revealed, given sufficient evidence.

In order to discover the tracks left behind by these draughtsmen it was necessary to examine very closely a large number of drawings. The evidence in these drawings consists of marks incised into the paper by the artists with draughting tools such as a compass, or dividers and stylus. Note that, for the drawings analyzed here, these tools do not usually apply ink or chalk or metal to the paper. They mark by pressing into the paper leaving a barely visible impression to be used as a guide. Ink, chalk or metal may or may not be applied afterwards. These marks may record some of the systematic steps of the design process. Unfortunately for illustration purposes, the marks tend to be visible only in the original drawings and seldom in printed reproductions or even in original photographs. This is because the shallow, colorless incisions and indentations are only visible when light is directed from a low angle creating highlights and shadows within these distortions of the paper surface. As a consequence, diagrams are necessary to show the positions of these marks and a number of such diagrams are included in this thesis. I have observed all of the marks, mentioned in the second section of this thesis, in the original drawings, under raking light and through magnifying lenses.

The reason the title of this thesis begins with the word “interpretations” is because the draughting tool marks that are under scrutiny do not speak for themselves. Without a context in which to interpret the marks they can reveal nothing about the artists’ working methods. Many previous observers of the drawings analyzed in Part II either ignored the marks or categorized them as “construction marks,” and left it at that. In order to derive some meaning from these marks, they must be seen as evidence of a process whereby a designer employed a system as a means to an end. Discovering the details of that process requires that the evidence be interpreted in the context of the systems employed. I believe that the design systems of the Renaissance should be construed as being

part of a tradition that goes back to antiquity. Therefore, it is necessary to know the possibilities of the systems and their historical contexts.

It is for this reason that the historical record regarding design techniques and systems that were available to Renaissance artists is included in Part I. The literature on artists' methods from the Renaissance period is helpful in establishing some of the means for interpreting the evidence, but the literature is lacking. Italian Renaissance designers did not typically make the practical aspects of their design techniques known. Even those few who chose to write treatises only rarely came close to revealing an occasional secret of their working procedures. For example, we see in Chapter 2, on Renaissance sources, that Cennini recommended drawing with charcoal and erasing with a feather the parts that did not come out right. Leonardo wrote about looking at clouds and patchy walls for compositional inspiration. Although there were many different means for arriving at compositions at the artists' disposal, in most cases there is little or no record of the process.

Drawings that reveal how artists arrive at compositions are hard to find. The drawings selected for this thesis had to be among those few that had draughting tool marks that could be interpreted to reveal some aspect of the artists' working methods. A cursory investigation of the extant Italian Renaissance drawings reveals that the overwhelming majority are studies for figures or for other portions of compositions. There are relatively few drawings that deal with entire compositions. In the case of architecture, drawings that record the struggle to arrive at final compositions are outnumbered by finished presentation drawings and sketches of ornaments. One must also remember that the number of surviving drawings, in all categories, is a minute fraction of the total production. Among those drawings that are studies for figural or architectural compositions are ones that can be described as sketches. These works represent an intuitive approach, and since the approach is not systematic there can be no step-by-step reconstructions. This thesis does not attempt to draw conclusions regarding Italian Renaissance design processes or other working methods unless the evidence allows a successful reconstruction of the step-by-step procedures.

The methodology of examining the physical evidence from original drawings is calculated to provide the firmest ground upon which to base

hypotheses about the artists' working methods used in these drawings. This is in contrast to other methods of investigating artists' design procedures that are more speculative.[2]

One of these other methods, that of overlaying hypothetical geometrical analyses upon reproductions of finished works, is flawed.[3] An artist's composition, whether it be architectural or pictorial contains many formal elements. The disposition and character of those elements could theoretically have been arrived at by any number of means, systematic or otherwise. Investigators who rely only on overlaying grids or geometric figures run into problems. Firstly, the assumption that a particular work was designed using geometry may be erroneous. Secondly, given a geometrically assisted design, determining which geometrical procedure was used is problematic. Users of this method cannot rule out the possibility that hypothetical reconstructions, regardless of the true method of design, may correspond to a reproduction's forms simply by coincidence. There is the possibility that the correspondence that an analysis purports to discover is merely the result of the fortuitous alignment of elements exaggerated by wide grid lines that hide inaccuracies in the hypothetical reconstruction. The problem of line widths verses the scale of the original is magnified when dealing with architectural designs. Furthermore, even if an investigator chanced upon a reconstruction that matched exactly the artist's design procedure, there is no way of knowing, without other supporting evidence, that the reconstruction was the correct one.

Another investigatory method involves numerical systems. Suppose that an investigator began an analysis of a composition assuming that a numerical system was used. Measurements are made of the design, painting or building. The resulting ratios and proportions are then analyzed to see if they correspond to known numerical systems. This method of approaching an artist's design procedure is also flawed. What appears to be a consistent numerical system may actually be the result of a geometrical procedure. The manipulation of geometric figures can easily result in measurements that relate either harmonically or in the manner of some other numeric system. As with geometrical analyses, without

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[2]See James Elkins, 'The Case Against Surface Geometry,' *Art History*, (June, 1991), 143-174 for a discussion of this problem, detailed more fully in Chapter 3 of this thesis.

[3]Kemp, Martin, *The Science of Art*, (Yale, 1990), 20. "With sufficiently thick lines and a certain amount of wishful thinking almost anything can be proved."

knowing in advance what system, if any, was used, there is no way to judge the validity of the analysis with regards to the artist's actual design procedure.[4]

The method presented in this thesis also differs from investigations that deal with the functions of drawings. This thesis does not address matters of drawing styles, attribution, or methods of transfer from drawing to painting.[5] While these subjects are important for a complete understanding of the function of drawings, they are outside of the focus of this thesis. The focus here is on interpreting the tool marks in order to learn something about the artists' working methods.

The subject of draughting tool marks in drawings has not received a great deal of attention from art historians. For most of the drawings considered here, there has been little mention in the literature of the construction marks and of the implications of those marks with regard to the design process or other artists' working methods. To my knowledge, no one has attempted to reconstruct the step–by-step operations implied by the marks in these drawings. In addition to my art historical research, I also work as a designer and my interest in the study of art is to discover the design processes behind successful works of art. My approach of recreating designers' steps in the process of design by following the physical evidence is unusual. As a consequence of the absence of supporting material, there is little opportunity to recount past investigations by others who have examined the same drawings and reached similar or dissimilar conclusions. My hope is that the originality of the observations and analyses will add to the body of art historical knowledge. For the most part, all of the analysis in Part II represent my own observations and analyses of the works themselves.

Included in this thesis are a number of diagrammatic illustrations. I have drawn many in-text diagrams that illustrate evidence that could be

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[4]Robin Evans, *The Projective Cast*, (MIT Press, 1995), 248. Evan's example is "two equally convincing analyses of Alberti's façade for Santa Maria Novella . . . External evidence might lead us to think [one] more likely original, but what, otherwise is there to chose between them? They both rely on auxiliary lines (imaginary lines) to reveal privileged relations and virtual figures that could not easily be inferred from direct inspection."

[5]See Francis Ames-Lewis, *Drawing in Early Renaissance Italy*, (Yale, 1981), for techniques in the various drawing media and general background. For methods of transfer see Carmen Bambach Capell, *The Tradition of Pouncing Drawings in the Italian Renaissance Workshop: Innovation and Derivation,* 2 vols., Ph.D. dissertation, (Yale, 1988), and *Drawing and Painting in the Italian Renaissance Workshop, Theory and Practice, 1300-1600*, (Cambridge, Mass., 1999).

communicated in no other way. It will be necessary when reading to study the diagrams and to keep them in mind when following the text. Because the reader may not have access to the original drawings, I have made every effort to insure the accuracy of these diagrams. It should also be said that, in contrast to most theses in the history of art, the diagrams constitute part of the narrative of the thesis and are, if they are worked through systematically, the equivalent of many pages of writing.

The artists of the Renaissance left us little written information regarding their design methods. The written sources rarely come close to an explanation of how designs were made in theory or in practice. We usually have only the finished products. Some of these finished works, however, appear to be so rigorously composed that it seems inconceivable for them to have been created without the aid of systems. Fortunately, there are clues within Renaissance textual sources that shed some light on the process of design, in the context of Medieval and Antique traditions. Also, there is the occasional drawing that contains, in the form of tool marks within the paper, a record of a particular designer's thoughts on a specific design. This thesis presents an outline of the traditional design methods and interpretations of the physical marks in an attempt to reveal a little of the magic which is the design process.

Chapter 1

Pre-Renaissance Design Systems

> *Of all the arts or sciences that pertain to architecture, there is nothing which can offer more certainty and render such honour to the architect as arithmetic and geometry.*[6]
> Pietro Cataneo

For the Italian Renaissance artist or architect the design process sometimes involved systems, tools that helped the designer organize the disparate parts of the composition into a unified whole. Unity was desirable because when a composition is unified, all of its parts have something in common, making the parts belong together either visually or conceptually. The use of systems was advantageous because systems can help alleviate some of the uncertainty that would hamper a creative endeavor if the process had to be invented from scratch every time. The evidence from draughting tool marks that is analyzed in the second part of this thesis reveals patterns of two such design systems that were in use among various artists and architects of the Italian Renaissance. These two systems involve the use of numbers and the use of geometry.

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[6]"Di tutti l'arti o scienza che appartengano all'architettura, non è cosa che possa porgere piu sicurtà, e rendere cosi honorato l'architetturo quanto l'arismetica, e geometria," L'Architettura, (Venetia, 1567), Bk. VII, 150.

Numerical and Geometrical systems were not new to the Renaissance period. They were part of a long tradition that goes back to Antiquity. In order to provide the necessary background for interpreting the tool marks in the selected Renaissance drawings of Part II, it is necessary here to trace the history of these design systems. The following historical outline will be limited to the occurrences of, or literary references to, those systems found in the drawings analyzed in Part II. The historical outline will also contain some philosophical background on the use of the systems to clarify how and why they were used. The first section addresses numerical systems, the second addresses geometrical systems.

Numerical Systems

Numerical systems, as a design process, consist of the repetition of a unit or module. A module is a unit of measure used as a standard. It is sometimes taken from a prominent feature of the work, such as in architecture, where the width of a column at its base is often the module from which all the other parts are measured. Otherwise, an arbitrary unit of measure, such as a foot or meter, is the module. Other parts of the composition relate to the module in that they are either of the same size, a multiple or an even fraction of the standard. When modules are used, relationships among the parts can be expressed as numerical relationships and, as such, they can be understood within a numerical system. When a designer composes with modules, he accepts a self-imposed restriction on the choices of sizes for compositional elements. The length of walls in a room, for examples, must be chosen from a limited set of numerical alternatives.

Designing is naturally an ordering process and numbers are useful in a design because of their ability to order. Numbers can do this by becoming abstractions of the parts of a composition; these abstractions can then be composed according to a numerical system. For example, the formal structure of the lines and words of a sonnet or the patterns of notes in a fugue contain a numerical component. Likewise, the dimensions of elements within a painting or an architectural ground plan offer the designer an opportunity to order the composition using numbers.

Numerical systems can be simple, as when a designer chooses to make a painting measuring one unit by two. These systems can also be complex, as in

the program for creating a particular species of Corinthian capital, which is complex because it comprises and organizes many elements. However, numerical systems, even complex ones, only make use of simple, whole number ratios. This is because of the traditions from which these systems arose (see below) and the limited utility of unwieldy fractional proportions in a creative endeavor.

To illustrate the utility of simple, whole number ratios in the context of creative endeavors, consider the numbers that underlie measuring systems. Those in English speaking countries are familiar with the differences between the imperial and metric systems of measurement. In the former, the principal units of feet, yards, gallons, and pounds, are divided into smaller units, (12, 36, 128, and 16 respectively) that can easily factor into simple ratios. In the latter, all of the principal units are divided by base 10 units, making calculations easier because we are all familiar with base 10 arithmetic. The two systems are tools that are optimized for different activities. The numerical divisions in the imperial system are optimized for the manipulation of materials in a creative way. Those who create things by apportioning materials in measured quantities tend to use simple ratios and multiples. (This may be because the creative side of the brain differs from the mathematical side.) In the case of length, for example, a yard of cloth can be divided into halves, thirds, quarters, sixths, ninths and twelfths. It practically invites divisions into simple ratios. Also consider any baking recipe written for the imperial system. Divisions tend to be halves, thirds and quarters and multiples tend to count no more than twelve. Having names for multiples, such as pints and quarts, also tends to keep the numbers smaller. The base 10 numbers that inform the metric systems are optimized for objective observational and accounting activities. The metric system is ideal for empirical sciences because the goal there is to observe without changing the object. The simple ratios inherent in imperial measures make that system ideal for changing the object in a creative way. The same ratios that enhance the creative potential of the imperial system are also found in numerical design systems.

Numerical systems are conceptual, rather than perceptual. By this I mean that it is the design, or the abstract *idea* of the work that contains the expression of the numerical system, as opposed to the actual visual phenomena. In the case of architecture, for example, it is sometimes difficult or impossible to visualize

the numerical proportions of a room one is standing in, even though the room's design may be following the prescriptions of a numerical system.[7]

One reason for designing with numbers is that some numbers have special religious or metaphysical properties. It is a tendency of numbers to sometimes acquire significance or meaning from the things that they number. When a number has been associated with an idea or event, subsequent occurrences of the number re-express the significance or meaning of the idea or event. For example, biblical scholars in the Middle Ages gave meaning to numbers in their exegetical writings in order to show the inter-relatedness, and hence the inherent design, of God's works.

By calling this a design process I do not mean that numerical systems of any kind automatically result in successfully composed designs or that there is anything inherently special, from a modern point of view, about a composition designed in this way. Rather, I suggest that there have been successful designers that have used numerical systems while designing. The reason for this is, perhaps, that numerical systems facilitate the intuitive part of the design process by limiting the possibilities from an overwhelming number of choices. The belief, in the minds of Renaissance designers, that the system had the power to produce designs in harmony with cosmic order, might also have inspired their creative efforts.

The numerical systems, discussed in Part II of this thesis, originated in Greece in the Classical period.[8] The Greek philosophers created a world view where numbers formed an integral part of their cosmology. They valued the abstract quality of the numerical systems of mathematics because it was, to them, "permanent, ideal, and perfect as opposed to the short-lived, imperfect, and corruptible nature of physical objects."[9] To the Greeks, knowledge of special numerical relationships could unlock the mysteries of the universe.

Numbers can carry special significance when they are combined with other numbers to create proportional relationships. Proportion has the power to

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[7]For a thorough philosophical discussion of the issue of conceptual proportions and those perceived in buildings see Branko Mitrovic, 'Objectively Speaking,' *Journal of the Society of Architectural Historians*, (March, 1993), 59-67. See also, below the section on Vitruvius.

[8]Kline, Morris, *Mathematics in Western Culture*, (Oxford, 1953), 24.

[9]Kline, *Mathematics*, 31.

organize disparate parts because of the bonding effect that three or more related numbers can have. The three numbers in a proportional relationship are called the first term, the mean and the last term; they are in proportion when the first term is to the mean as the mean is to the last term. That is, they all share the same numerical relationship. Examples of proportions used in numerical systems are arithmetic, geometric and harmonic. These are discussed below in Chapter 7 on Palladio.

Plato expressed how numerical proportion can create unity: "the mean becoming first and last, and the first and last both becoming means, they will all of them of necessity come to be the same, and having become the same with one another will be all one."[10] In the *Timaeus*, Plato composed a numerical system to describe the whole of creation, using numbers to represent the abstract concepts of existence, essence, the same and the different (35a-b):

> First of all, he took away one part of the whole, and then he separated a second part which was double the first, and then he took away a third part which was half as much again as the second and three times as much as the first, and then he took a fourth part which was twice as much as the second, and a fifth part which was three times the third, and a sixth part which was eight times the first, and a seventh part which was twenty-seven times the first. After this he filled up the double intervals and the triple, cutting off yet other portions from the mixture and placing them in the intervals, so that in each interval there were two kinds of means, the one exceeding and exceeded by equal parts of its extremes, the other being that kind of mean which exceeds and is exceeded by an equal number.[11]

In Plato's creation story the creator is employing a numerical design process. He divides up a whole into a specific set of ratios which relate to each other by their proportion. Proportion is the unifying quality.

The character and philosophy of Greek mathematics was determined, to a large extent, by the Pythagoreans. According to Aristotle, the Pythagoreans "were the first to take up mathematics, . . .[and] thought its principles were the principles of all things."[12] All things including not just the physical but also the abstract concepts such as justice, soul and reason were "numerically

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[10]*Timaeus*, (ed. *The Dialogues of Plato*, *Great Books*, vol. 7, trans. by Benjamin Jowett, University of Chicago, 1952), 448.
[11]*Timaeus*, (ed. 1952), 449.
[12] *Metaphysics*, I. 5; 985b (ed. *Great Books*, vol. 8, trans. by W. D. Ross, University of Chicago, 1952, 499-630), 503-504.

expressible." Pythagoras and his followers had developed a cult based on their ascription of religious properties to numbers. Plutarch relates a Pythagorean practice of sacrificing to celestial gods using odd numbers and to terrestrial gods using even numbers.[13] Pythagoreans saw numerical analogies throughout nature and thought that everything could be reduced to number. According to George Hersey, "to the Pythagoreans, who were essentially number magicians, numbers were not only quantities, they were qualities as well."[14]

Pythagoreans discovered that in music the consonantal relationships of tones made by plucking taut strings had a numerical basis. If two strings are of the same length, diameter and tension and one string is depressed at the mid point then the resulting tones are one octave apart. Other tonal pairs which sound to the ear as harmonic or resonant when played together can be expressed as ratios of whole units when measured as lengths of strings. The ratio of 2 to 3, for example, is known as a musical perfect fifth interval and 3 to 4 is a perfect fourth.[15] Because these ratios manifest themselves as physical properties which result in musical harmony, their significance increases. The fact that two notes sound right together adds significance to the fractional quantities which define the notes. Because of the association with musical harmony, the numerical ratios appeared to the mathematical philosophers to have some kind of primal connection to the way the world is structured.

In the Third Century BC the intellectual center of the world moved from Athens to Alexandria. There, mathematics, which had been treated by the philosophers of Plato's academy as abstract, was turned to practical investigations of scientific and engineering problems.[16] The Alexandrians found, by indirect measurement, the radius of the earth and the distance from the earth to the moon. Their accomplishments in astronomy culminated in a reasoned theory of the cosmos that accounted for the movements of heavenly bodies, called the Ptolemaic theory. This theory held that the earth was at the center of

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[13]*Numa Pompilius*, (ed. *Plutarch, The Lives of the Noble Grecians and Romans*, Dryden translation, *Great Books*, vol. 14, University of Chicago, 1952, 49-60) 57.

[14]Hersey, George, *Pythagorean Palaces*, (Cornell, 1976), 1.

[15]The Pythagorean scale is not the same as the scale used today which was standardized in the eighteenth century. This is because of problems occurring in some tonal relationships. According to Robin Evans, "[T]he ratios between notes within the system get distressingly complicated and cannot be arranged in such a way that all whole tones hold the same relation to one another." *The Projective Cast*, (London 1995), 263.

[16]Kline, *Mathematics*, 62.

the universe and the heavenly bodies moved around it in complex but orderly cycles. With this theory "the evidence for design in the universe was complete. . . [t]he universe was rational and the principles that determined its motions were mathematical."[17] The theological implications of the Ptolemaic theory motivated Medieval Christians to create a theory of the cosmos with God as architect using numbers and geometry.

Medieval Christians also expanded on the Greek concept of numbers as God's compositional elements. St. Augustine was able to tease out the numerical composition of the days of creation, related in the Book of Genesis, based on his interpretation of scripture and the numerology of Pythagoras.[18] The number three stood for the soul and was especially holy because of the Trinity. Four represented the body and had multiple associations with scripture as well as nature. Six, the number of days God labored, was the perfect number because it was the sum of its own factors. Seven, the day of rest, was the sum of three and four, the soul and body, and therefore represented completeness in man and creation.[19] The same composition is then repeated in the ages of man beginning with the first age from Adam to the Deluge, the Deluge to Abraham, and so on until the sixth age which we are currently experiencing. The seventh age will begin sometime in the future and will represent completeness again.[20] "[M]any other numbers and combinations of numbers are used in the sacred writings, to convey instruction under a figurative guise, and ignorance of numbers often shuts out the reader from this instruction."[21]

Medieval art and architecture abounds with examples of forms that were determined, at least in some part, by numerology. In Gothic churches, three, expressing the Trinity, is the number of portals on the façade and gives meaning to the three-part elevation of the interior: colonnade, triforium and clerestory. The twelve-part division of a rose window in a Medieval cathedral would be an example of an architectural design where the significance of a number, in this

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[17]Kline, *Mathematics*, 84-85.

[18]Will Durant, *The Age of Faith, A History of Medieval Civilization – Christian, Islamic, and Judaic – from Constantine to Dante: A. D. 325-1300*, *The Story of Civilization*, vol. 4 , (New York, 1950), 985. For the numerology of Pythagoras, see below.

[19]St. Augustine, *The City of God*, Bk. XI, Chapter 30 (ed. *Great Books*, vol. 18., trans. by Marcus Dods, University of Chicago, 1952) 339.

[20]St. Augustine, *City*, Bk. XXII, Chapter 30 (ed. *Great Books*, vol. 18) 617-618.

[21]St. Augustine, *On Christian Doctrine*, Bk. II Chapter 16 (ed. *Great Books*, vol. 18., trans. by J. F. Shaw, University of Chicago, 1952) 645.

case the number of disciples or the signs of the zodiac, was an integral part of the design. Baptismal fonts are usually hexagonal or octagonal or both as a reminder of the relationship of the numbers six (man) and eight (eternity) to the sacrament of Baptism. Everything from the number of bays in the nave, transept and choir to the number of steps leading to the front door were probably informed by some numerological rationale. The same numerological opportunities occur in manuscript illumination where the monastic artists could indulge in abstract patterns to express the religious aspect of numbers. Designing with numerical systems reflects God's ordering process in the composition of creation.

Rudolf Wittkower, in his seminal book, *Art and Architecture in the Age of Humanism*, develops the thesis that architects of the Renaissance used numerical systems that had "remained alive" from the ideas of Pythagoras and Plato:

> The conviction that architecture is a science, and that each part of a building, inside as well as outside, has to be integrated into one and the same system of mathematical ratios, may be called the basic axiom of Renaissance architects. …[T]he ratios have to comply with conceptions of a higher order and that a building should mirror the proportions of the human body; a demand which became universally accepted on Vitruvius' authority. As man is the image of God and the proportions of his body are produced by divine will, so the proportions in architecture have to embrace and express the cosmic order.[22]

Wittkower cites a Renaissance source for his conviction regarding artists' preoccupation with numbers and their mystical interpretations. In 1534, a Franciscan monk, commenting on a design for a church under consideration in Venice, argued for certain proportional relationships:

> I should like the width of the nave to be 9 paces which is the square of three, the first and divine number. The length of the nave, which will be 27, will have a triple proportion which makes a diapason [the octave} and a diapente [the perfect fifth]. And this mysterious harmony is such that when Plato in the *Timaeus* wished to describe the wonderful consonance of the parts and fabric of the world, he took this as the first foundation of his description, multiplying as far as necessary these same proportions and figures according to the fitting rules and consonances until he had

[22]Wittkower, Rudolf, *Architectural Principles in the Age of Humanism*, (1949), (ed. New York, 1971), 101.

> included the whole world and each of its members and parts.
> We, being desirous of building the church, have thought it necessary, and most appropriate to follow that order of which God, the greatest architect, is the master and author.[23]

Wittkower makes a strong argument insofar as the intellectual climate of the period was desirous of harmonic designs in theory. However, the focus of his argument is on Andrea Palladio, and the central principles of that architect's design process. In Chapter 7, I argue from my interpretation of Palladio's tool marks, that there are reasons to doubt some aspects of Wittkower's thesis. That Renaissance architects, (and pictorial artists as well) had numerical design systems at their disposal, among other systems, is probably nearer to the truth.

With justification from the structure of the physical world as well as religion and philosophy, it is easy to see how the use of numerical systems to assist design could "remain alive" and become a tradition in Western culture. Numerical systems organize and add meaning in both Christian and pagan world views. Although the utilization of a numerical system does not automatically result in a work of art, numbers assist the designer by providing unifying choices and create an element of intellectual satisfaction in a design.

Geometrical Systems

The other traditional design process that Renaissance artists and architects had at their disposal was the geometrical system of design or "constructive geometry." It involves building the structure of a composition with lines and arcs, using a compass or dividers and a straight edge. These tools can also be used to execute a design based on numbers, but, for our purposes, designing with geometry means using the tools to locate points by means of intersecting lines and arcs. The points are connected with lines to make shapes. The points are the footings for geometric shapes whose parts interconnect and reinforce each other, forming a stable framework. The compass makes arcs and circles and can transfer a measurement from one point or line to another. The straightedge is used only for connecting points with straight lines and not for measuring.

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[23] Wittkower, *Architectural Principles*, (1971), Appendix 1, 155, trans. from Gianantonio Moschini, *Guida per la Cittá di Venezia*, (1815), I, i, 55-61.

With a given straight line and an arbitrarily chosen point upon which to start, an endless variety of shapes can be constructed. These various interconnected shapes, then, can undergird architectural or pictorial compositions.

In contrast to the conceptual nature of numerical systems, geometrical systems are fundamentally perceptual. That is, the act of designing with geometry requires that the designer have a visual connection to the design. It requires a coordination between the hand and the eye. Judgments are made based on the designer's response to perceived relationships between the elements of the design. A numerical system, on the other hand, could theoretically be employed in the designer's mind without ever drawing the design.

The two systems, however, are not incompatible. In fact, as we see in the examples analyzed in Part II, the two systems can be used together. The system of generating geometrical relationships can sometimes preclude a close tie-in to numerical systems, though, because shapes which are geometrically plotted often cannot be expressed in whole or rational numbers. To see this clearly, consider two kinds of rectangles. One, a 1 x 2 unit rectangle, can be conceived just as easily numerically or geometrically. Another kind of rectangle common to the geometrical design process is generated from a square by taking the measure of the diagonal with dividers from corner to corner. The dividers are then pivoted forty-five degrees to align with one side thus extending that side and forming a longer rectangle. This rectangle is known as a root-two rectangle because its numerical ratio is one by the square root of two. Since numerical design systems of the Renaissance period consisted of simple ratios of whole numbers, such as halves, quarters and thirds, the irrational square root of two would not have been considered as an option within a numerical system. We see in Chapter 2 that Francesco di Giorgio made efforts to reconcile the incompatibility of the two systems.

To illustrate the essential character of constructive geometry consider an excerpt from an architectural treatise written in the later Renaissance. In the seventh book of *L'Architettura* (Venice, 1567), Pietro Cataneo (c1510-c1574), writes, "Of all the arts or sciences that pertain to architecture, there is nothing

which can offer more assurance and render such honor to the architect as arithmetic and geometry."[24] He then gives demonstrations on how to create some geometric figures, using a compass and straight edge. One of the demonstrations is for making polygons of five, seven and ten sides:

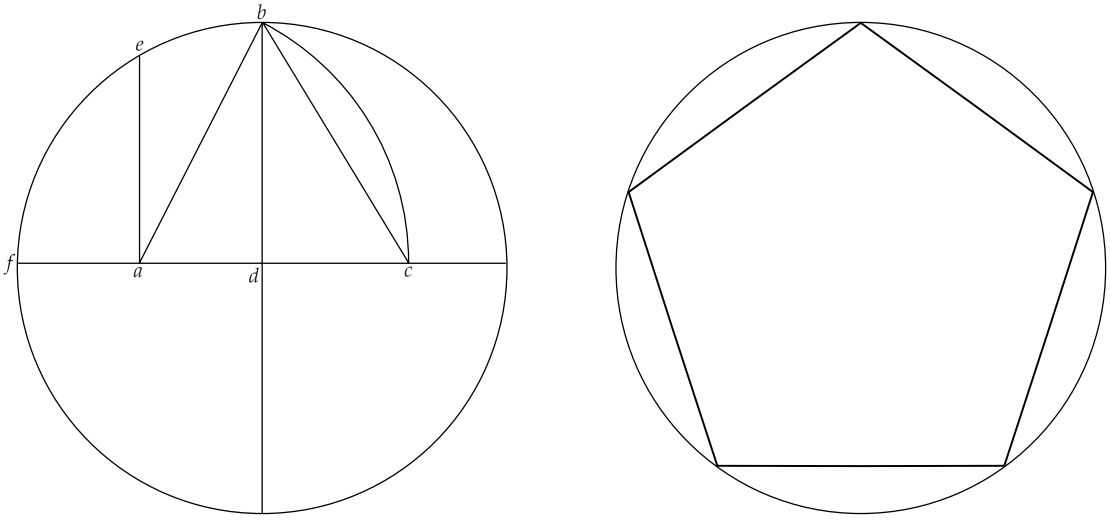


FIGURE 1 (AUTHOR'S DRAWING)

> Beginning with a circle and two right angled diameters, start at 'a' which is half a radius, and draw a line to 'b.' Then "planting one leg of the compass on 'b', arcing, make an arc to point c." The line 'bc' is the side of a pentagon measured inside the same circle. The line 'cd' is the side of a decagon. The line 'ae' is a heptagon.[25]

Once a measure, such as 'bc', is plotted, the dividers are opened to the equivalent width and walked around the circle, each indentation marking the corner of a pentagon. The straightedge is used to connect the corners and draw the pentagon. Other measures when walked around the circle in the same way create the other polygons.

With constructions such as this one, the merits of geometry as a design tool is made evident to anyone who draws the figure. There is a feeling of rightness, or precise symmetry that comes when one walks the dividers around the circle and has the point of the compass land in the same indentation as the starting point. In the words of Plato: "What I mean [by the beauty of shape] is rather something straight or round and is what is constructed out of these with a compass, rule, and square, such as plane figures and solids. Those things I take it

[24]Pietro Cataneo, (Venetia, 1567), Book 7, 150.
[25]Cataneo, (Venetia, 1567), Book 7, 151.

are not beautiful in a relative sense, as others are, but are by their very nature forever beautiful by themselves." (Philebus, 51c).[26]

Design processes which utilize such procedures are called "constructive" because the physical act of manipulating tools, inherent in constructing networks of two-dimensional geometric shapes, is analogous to the act of constructing buildings in three dimensions. Stability is dependent upon a tangible reference that is true, in the geometric sense, such as two axes that cross at precise right angles. Beginning with a precise starting point is important because initial errors will be carried on and possibly magnified throughout the process. Both constructive geometry and actual building involve putting parts together in such a way that they hold each other up.

Geometry feels structurally sound because the parts fit together in a manner that is tactile and substantial. Each part rests on the previous one and they all rest upon a sound footing. Shapes created in this manner appear unified because each part is based on the last. The tools of compass and straightedge give the designer and the builder the power of exactness, similar to that of a plumb and string at a building site. Precision drafting tools are an essential part of a design process based on constructive geometry. The appeal of the visual system to the designer is in the concrete nature of the constructive procedures which is more akin to building than the abstract, calculating nature of numbers.

Constructive geometry is ideal for composing the plan of a building, which is typically an arrangement of geometrical shapes. These geometric shapes and their disposition can be composed by using a system that is analogous to the act of building itself. Despite this affinity with architecture, however, constructive geometry is not the only means of composing an architectural plan. Determining that constructive geometry was used in a particular design requires more solid evidence than the presence of geometrical shapes.

In the case of pictorial compositions, the lines and shapes of constructive geometry have a much more subtle relationship to the finished composition (except, for example, in the case of carpet pages in illuminated Medieval manuscripts where the whole image may be constructive geometry). The effects of constructive geometry on a painting may be as unassuming as the

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[26]Plato, *Philebus*, trans. by Dorothea Frede, Cambridge, 1993, p. 60.

determination of the vertical and horizontal axes. Only in rare cases have construction marks, still visible in the painting medium, shown the disposition of elements in a figural composition to have been determined by constructive geometry.[27] The use of linear perspective provides the most visible example of constructive geometry. Although, to some extent, perspective may be a means of creating the illusion of depth, it is also a means of organizing a composition on the surface of the picture.

Historically the practical purpose of geometry as a science was to measure and quantify plane surfaces and volumes. Euclid wrote *The Elements*, circa 300 BC that detailed the definitions, postulates and propositions of the science of geometry and this book served as the standard geometry textbook for centuries.

The kind of constructive geometrical design procedures that were used by Renaissance artists, however, bear little resemblance to Euclid's exercises. Occasionally there was an octagon that needed to be constructed or a midpoint of an arc that needed to be located, but typical designing required only the most elementary procedures. For example, the construction of two lines that meet at right angles to be used as the central axes of a composition is the most common procedure. This is because it establishes the initial vertical and horizontal axes. Also common is the process of dividing a line in half. The reason that there was little need for the procedures of *The Elements* is that Renaissance artists were not trying to measure or quantify, but were designing. Constructive geometry provides the designer with visual choices that relate to each other in such a way that the parts of the composition will be unified.

The traditional origin of geometry, according to Herodotus, can be traced back to ancient Egypt. After the Nile flooded it was necessary to measure the earth to reestablish property lines. Hence the word "geometry" is from *geo* meaning earth and *metron* meaning measure.[28] It was during the reign of Sesostris in the fourteenth century BC that a geometrical system was devised to apportion land for tax purposes.

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[27]White, John, *The Birth and Rebirth of Pictorial Space*, (1957), (ed. Cambridge, Mass, 1987), 204.
[28]Dilke, O. A. W., *Mathematics and Measurement*, (British Museum Pub., 1987), 7-8 and A. F. Scott, *A History of Mathematics*, (London, 1958), 7.

Early examples of the use of geometry for compositional purposes, consisting of a grid of counted squares, have been found in ancient Egypt.[29] Beginning in the Middle Kingdom and continuing through to the Roman occupation, Egyptian mural painters and relief sculptors routinely used squared grids as guides for drawing figures and laying out compositions. Generations of artists employed these grids as aids in design with little variation for centuries. The grids were made with string, dipped in red paint, and snapped horizontally and vertically against the wall. The subsequent process of carving or painting usually removed or covered most of these red lines. The grids are visible, however, in many murals that are either unfinished or the deteriorated state of the paintings is such that the lines are uncovered.

The main reason for using grids was to maintain the correct proportions of the figures. The very conservative style of Egyptian mural figure drawing dictated that when a figure was drawn on a grid, the knee, for example, had to lie on a specific grid line counting from the bottom. The hips, shoulders, head etc. had their respective locations on the grid. The numbers for these locations occasionally changed over the centuries, (because of a change in dynasty perhaps), but the design process was generally consistent.

Those familiar with modern day model sheets for cartoon characters will recognize the Egyptian design system. Horizontal lines indicate the size of the head, for example, and its ratio to other body parts. The process of using measuring lines to locate the main features of a figure is still used today and used for the same reason: standard grids maintain consistent proportion from drawing to drawing.

The Egyptian system, however, may have had more sophisticated reasons for the choices of numerical relationships. There may have been significance, for example, in the proportional relationship from one form to another. A change in proportions may have signaled a break with the proportions of a previous ruler. The grid also gave the artist countable choices for the sizes and positions of other forms in the compositions. This feature of the process is similar to that used, until recently, by layout artists in the modern printing process. The grid

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[29]Gay Robins, *Proportion and Style in Ancient Egyptian Art*, (Univ. of Texas Press, 1994). Professor Robins records the history of scholarship in the area of Egyptian grids as well as adds analyses of the use of grids in compositions and the change in proportions over time.

constrains choices by limiting the infinite possibilities of size and position to those sizes and positions that correspond to locations in the grid.

The grids in Egyptian art were not always followed rigidly, however. Artists drew freely over the lines using the grids as guides rather than mechanical crutches. On occasion, figures were drawn in proportion without the use of grids. Such draftsmanship must have been the result of highly skilled artist who had internalized the proportions after a considerable amount of experience.

In Greek art there is little evidence of the kind of grids found Egypt. This does not mean that the Greeks used no design system, only that there is no direct evidence of the use of grids. Some have tried to divine the systems after-the-fact by measuring the finished products of Greek art. In the 1930s Jay Hambidge invented a method for analyzing design systems which he called dynamic symmetry and submitted ancient artifacts to the test.[30] Although Hambidge's diagrams are interesting as hypotheses, the geometry of dynamic symmetry does not correspond to any known design process. The overlay of geometry on a photograph of a painted vase, for example, may correspond in many striking ways to the vase and its painting, but is not clear how the potter or the painter could have caused these correspondences to occur given the three-dimensional nature of the object.

There is also little evidence for how Greek architects designed their buildings. One rare example of a Greek geometric design procedure was discovered at the Temple of Apollo at Didyma.[31] The temple was begun sometime after 334 BC, on the southwestern coast of present day Turkey. Luthar Haselberger, in the early 1980s, discovered an almost complete set of architectural plans incised into the remaining walls and floors of the now ruined temple. The purpose of these incised plans was mainly practical. They provided full scale images for stone masons to use in shaping the stones of the building. The preservation of these finely etched lines is fortuitous, since they would have been erased in the final polishing of the walls had the temple been completed.

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[30]Hambidge, Jay, *Practical Applications of Dynamic Symmetry*, (Yale, 1932); *The Parthenon and other Greek Temples: Their dynamic Symmetry*, (Yale 1924); *Dynamic Symmetry: The Greek Vase*, (Yale, 1920).

[31]Haselberger, Lothar, 'The Construction Plans for the Temple of Apollo at Didyma,' *Scientific American*, (December, 1985), 126-132.

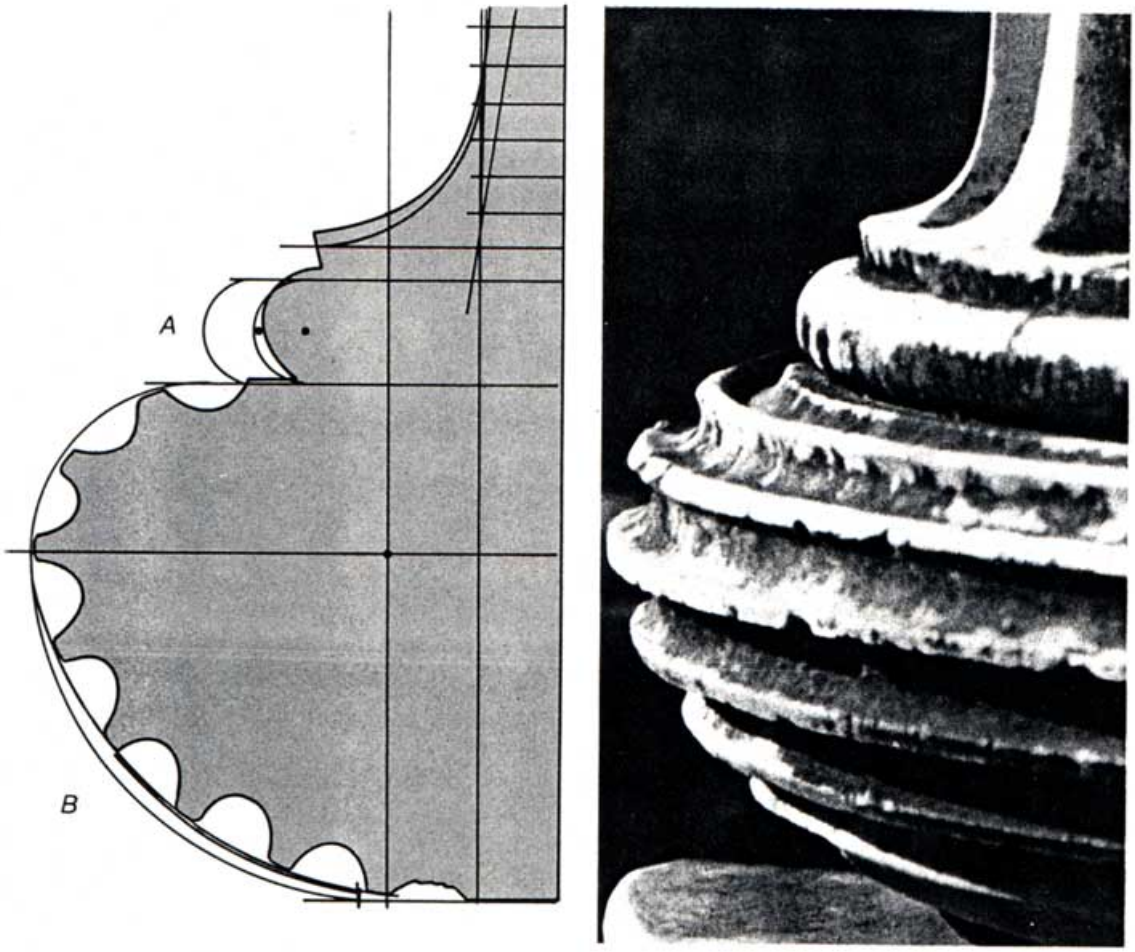


FIGURE 2 DESIGN OF A GREEK COLUMN BASE (HASELBERGER DIAGRAM AND PHOTO)

The temple had been designed, as had many other Greek temples, with what are called "refinements." These are the small deviations on straight or plumb lines. The columns swell in the middle and the outer columns lean inward, the base of the temple also arches upward, in the middle. These adjustments are miniscule in relation to the gigantic scale of the whole temple and they are not easily perceived without precise measurements. They are intended to compensate for optical illusions and make the building more beautiful. One can imagine that for the stone masons these tiny deviations from straight and plumb would be difficult to execute at such a large scale. The full scale drawings on the site were to help accomplish this. The entasis, or swelling, of a column, for example, was accomplished with a series of horizontal lines at regular intervals between the outer curve and central axis of the column. A stone cutter could refer to the drawing, measure the horizontal line, and cut the stone by that width for that height.

Some of the incised lines had another purpose. In an area of the temple wall where the form of the 19.7 meter columns was drawn to scale, the profile of a prototype column base was designed on the wall (see figure 2). An armature of straight constructions lines was drawn with horizontal and vertical axes. Compass arcs with points on the intersections trace out the form of the torus (the large convex molding at the base), a smaller molding and the upper curve joining the base to the vertical shaft of the column. Haselberger believes that the architect designed the profile of the base using geometrical constructions because there are multiple arcs and lines that do not conform to the base profile as built. Apparently, one set of lines conforms to the original plan as the base profile had been conceived, but the designer decided to change the profile. By moving the compass foot to other parts of the construction, adjustments could be made to the base's contour. The final profile matches that of the extant column bases.

The etched lines surviving at the Temple of Apollo go a long way toward helping us explain the way Greek builders executed large scale projects. But, we cannot determine, from this one example of the column base design, how the overall design of the temple was constructed. There may have been similar geometrical constructions to determine the shape of the plan, the height of the pediments or the interior arrangement of the columns. These compositional elements might, instead, have been determined by numerical systems. We do not know because there is so little evidence. However, we do have evidence, from the Renaissance, that the geometrical design process that the Greeks performed on the column base lived on. A Renaissance architect, Antonio da Sangallo il Giovanni, modified the profile of a column base template, destined for St. Peter's in Rome, using a procedure surprisingly similar to that of the unknown Greek temple designer. A detailed analysis of that procedure comprises Chapter 5 of this thesis.

Vitruvius' Systems

The artists and architects of the Renaissance had available to them, through any number of sources, instruction in Euclidean geometry as well as the antique concepts relating to numerical proportion.[32] However, the design

[32]A typical textbook: Steven Victor, *Practical Geometry in the High Middle Ages: An Edition with Translation and Commentary of the art of the* Artis cuius libet consummatis, (Harvard, 1973). For an overview of Italian Renaissance educational practices, see Paul F. Grendler, 'What Piero

process was not usually linked to these subjects in the normal course of instruction. There was only one ancient book that linked geometry and numbers with the design process. The Fifteenth Century witnessed the re-discovery of a treatise, written in Antiquity, which expressed, to some degree, how constructive geometry and numerical ratios should be applied to design. It was written by a Roman, known to us as Vitruvius, in the First Century BC and dedicated to Caesar Augustus. Vitruvius was an architect about whom few biographical details are known and whose architectural career is generally judged to be of little distinction.[33] Upon retiring from his profession in c. 33 BC he wrote a treatise, *De architectura libri decem* which summed up in ten books his notions of Greek and Roman Republican architecture. The ten books immortalized his own architectural theory, including prescriptions on how one should go about designing and building the various projects that a contemporary architect would be called upon to execute. Writing in the treatise genre that had been previously reserved for the better-recognized arts of rhetoric and music, Vitruvius was attempting, in his crude way, to raise the status of architecture to a subject of learning. It was also his intent to raise the quality of contemporary building practice, which he considered to be without theoretical basis.[34] In actuality, *De architectura*, did not always accurately reflect the practices of Greek and Roman builders nor did Vitruvius' contemporaries pay much heed to his prescriptions. The book was virtually ignored for fourteen centuries. But, due to the vicissitudes of chance *De architectura* was the only treatise on architecture to survive from Antiquity to the Renaissance. The architects of the Renaissance, consequently, looked upon Vitruvius as the authority on architectural matters.[35]

De architectura addresses many aspects of the building process but the present investigation will focus on those occasions where Vitruvius discusses the design process. In the introduction where he lists the things that the architect should know, he opens with the relative importance of geometry and numbers:

Learned in School: Fifteenth-Century Vernacular Education,' in *Piero della Francesca and His Legacy*, Marilyn Aronberg Lavin, ed., (National Gallery of Art, Washington, 1995), 161-176.

[33]Kruft, Hanno-Walter, *A History of Architectural Theory from Vitruvius to the Present*, trans. by Ronald Taylor, Elsie Callander and Antony Wood, (Princeton Architectural Press, 1994), 21 ff.

[34]Onians, John, *Bearers of Meaning*, (New Jersey, 1988), 33.

[35]Onians, *Bearers*, 41, 17.

> Geometry, also, is of much assistance in architecture, and in particular it teaches us the use of the rule and compasses, by which especially we acquire readiness in making plans for buildings in their grounds, and rightly apply the square, the level, and the plummet . . . it is by arithmetic that the total cost of buildings is calculated and measurements are computed, but difficult questions involving symmetry are solved by means of geometrical theories and methods.[36]

Here, Vitruvius expresses the fundamental differences between the two systems. Arithmetic is for practical applications but geometry takes precedence over arithmetic in the design process. Geometry requires the use of a compass and straightedge or rule (*regula*) and is employed in making the groundplan (*descriptio*). The composition requires "careful and laborious thought and watchful attention directed to the agreeable effect of one's plan."[37] That was his definition of reflection (*cogitatio*) which in designing should be combined with invention (*inventio*) which "is the solving of intricate problems and the discovery of new principles by means of brilliancy and versatility."[38] The picture that this portrays is of a designer who uses tools while designing and tries out all possibilities before choosing the best one. In opposition to this picture of the architect is the more theoretical and numerical aspects of architectural design.

Vitruvius speaks of numbers in the context of design in two areas. One is in his theory of proportion and the other is in descriptions of the formulae of architectural ornament. To Vitruvius, proportion refers to the relationships among numbers and not to an aesthetic concept. The aesthetic qualities arise from the correct application of proportion. The well-designed building has *venustas* (beauty) which Vitruvius divided into six concepts, some of which arise from proportion: *Ordinatio* results when the parts are consistently proportioned according to a module (*modulus*); *Dispositio* is the fitting arrangement of parts in the design of a building in plan, the forms of which are groundplan (*ichnographia*)

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[36]"Geometria autem plura praesidia praestat architecturae; et primum ex euthygrammis circini tradi usum, e quo maxime facilius aedificiorum in areis expediuntur descriptiones. . . Per arithmeticen vero sumptus aedificiorum consummantur, mensurarum rationes explicantur difficilesque symmetriarum quaestiones geometricis rationibus et methodis inveniuntur." *Vitruve de L'Architecture*, I, 1, 4, Livre 1, *Texte Établi*, *Traduit et Commenté* par Philippe Fleury, (Paris 1990), 5. Trans. Kruft, *Architectural Theory*, (1994).

[37]"Cogitatio est cura studii plena et industriae vigilantiaesque effectus propositi cum voluptate." *Vitruve*, I, 2, 2. Trans. 25, note 24.

[38]Inventio autem est quaestionum obscurarum explicatio ratioque novae rei vigore mobili reperta. Vitruve, I, 2, 2, Trans. Kruft, *Architectural Theory*, (1994), 25, note 24.

elevation (*orthographia*) and perspective (*scaenographia*).[39] *Eurythmia* results from a harmony between the parts when heights are in proportion to the widths. *Symmetria* results from proportion. *Decor* results when the forms are appropriate. *Distributio* refers to the appropriateness and suitability of the building to its purpose.[40]

The formulae for architectural ornament that Vitruvius recommends are examples of pre-designed forms that have *ordinatio* and *symmetria*. The architect is advised to use these forms, when appropriate, and to follow Vitruvius' prescriptions as one would follow a recipe. Vitruvius wanted to establish official proportions for the purpose not only of insuring that future architects applied the traditional orders with proportion but, also, that they would use Vitruvius' proportions.

Vitruvius finds his justification for the importance of numerical relationships in the proportions of the human body. He sees the same design scheme of interrelated measures that he advocates in buildings in the measurements of a "well formed human body," (*hominis bene figurati membrorum*). In the same way that the length of a foot, for example, divides into a man's height an even number of times, the width of the base of a column should divide evenly into the column's height. It should be noted that it is not clear how a human body was to be determined "well formed" if the theory of proportions had not already been established.

The human body analogy has two characteristics which are important for Vitruvius' design theory. One is that parts relate to each other by having a common measure, just as a building that is designed from a module. The other characteristic is the numerical relationships between the corresponding measures. The measurements of the parts of the body relate according to simple fractions in the same way that musical tones did for Pythagoras.

It should also be noted that the human body analogy is a theoretical concept and is problematic in practice. The interpretation of human proportions as

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[39]"the delineation of the facade and the receding sides of the building the lines meeting at the point of the compass in the center." "Item scaenographia est frontis et laterum abscedentium adumbratio ad circinique centum omnium linearum responsus," *Vitruve*, I, 2, 2. Trans. Kruft, *Architectural Theory*, (1994), p. 25, note 24. Unless the Romans had a version of one point perspective, *scaenographia* must refer to something such as isometric or empirical perspective or, according to Daniele Barbaro's edition of Vitruvius it refers to a cut-away section view. See Vitruvius, *De architectura*, ed. Daniele Barbaro, (1567), p. 21.

[40]Kruft, *Architectural Theory*, (1994), 25-27.

simple fractions maintains consistency only if one does not try to measure a human body, since the human body is organic and is not a precise or consistent object to measure. Although the length of a foot, for example, can be ascertained with relative precision, exactly where does one begin to measure the palm, the finger or the face? Clearly there is a methodological problem with using proportions of the human body as a justification for a design theory. If one needs to have even fractions then it is easy to adjust the measurement procedure to locate where the wrist meets the hand or the point where the top of the nose begins; otherwise one can simply find a model with the right proportions. Individual variations in people make measuring insufficiently accurate to ascertain what "well formed" means.

In a similarly theoretical vein, Vitruvius points out the praiseworthy aspects of the correspondence of the human figure with geometrical shapes. When standing, a man's height equals his arm span just as the height of a square equals its width. When lying on his back a man's navel locates the center of a circle that his stretched out arms and legs touch. Again, there are problems resulting from the variances in the proportion from one person to another. Also, there are variations in how one stands or how much one stretches one's arms. But, the idea conveyed is that the square and the circle, the building blocks of architecture, have more justification because of their correspondence with the human body.

In the theoretical aspects of *De architectura* Vitruvius seems to be advocating that the architect pay special attention to numbers. It should be remembered, however, that Vitruvius wanted to justify architecture as a science as worthy as mathematics. His emphasis on numbers in the design of orders makes the practice of architecture seem very mathematical and hence scientific. But columns and volutes applied as ornamental parts of buildings are predesigned and are not necessarily the underlying structure of buildings. (This applies especially to Roman architecture where the arch is generally the structural form, as opposed to the post and lintel underpinning of the Greek tradition.) There are passages where Vitruvius details geometrical procedures and in these instances we see the architect as a builder and designer of structure. With the geometrical tools of compass and straightedge the architect can compose the form of a room, the shape of a theater or the foundations of a city.

In describing the layout of theaters, (Book V, chs. 6-7), Vitruvius instructs the reader to draw a circle and inscribe four equilateral triangles at

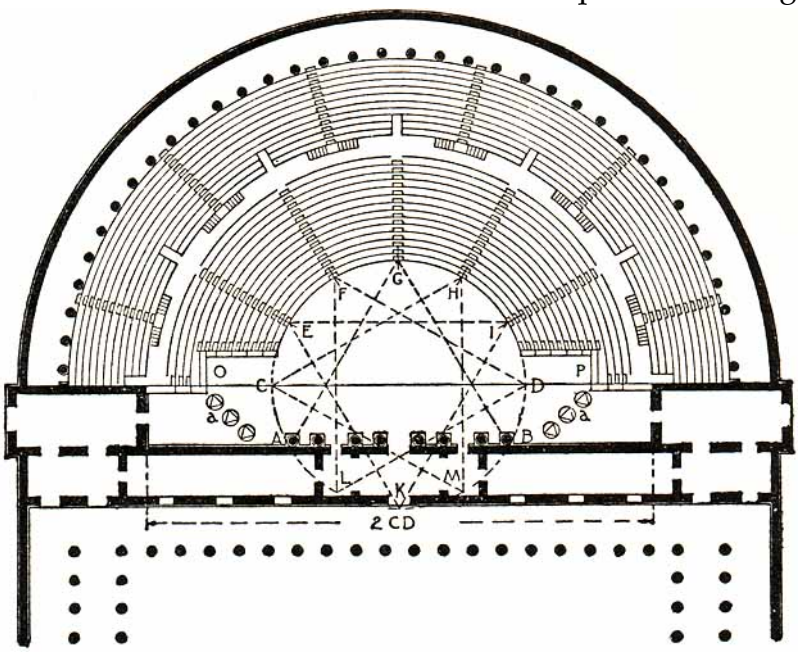


FIGURE 3 ROMAN THEATER PLAN

equal distances apart, touching the circumference (fig. 3, central portion). Already the design process is much more active than the numerical prescriptions for ornaments. It is interesting that the triangles which locate the positions of the scaena and the seat sections are not visible in the finished plan of the theater. They are like scaffolding. The triangles serve to locate twelve equidistant points around the circumference of the circle.

Because of the differences between the theatrical traditions of the Greeks and Romans, the geometrical patterns for the theaters of Greece and Rome also differ. Greek theaters use three squares within a circle instead of triangles within a circle. The three squares result in the same twelve points but the internal guide lines generate different sizes for parts of the theater. Vitruvius could have said, "Start with a circle with twelve equidistant points about its circumference," and the finished plan would have had the same appearance. But the mention of the shapes not only gives a clue as to how one should go about drawing such a circle, it also gives a structural justification, from the point of view of a geometric design process, for the positions which the triangles locate. The scaena, stage, orchestra and seating all relate to each other because they fit within a stable matrix that originated from the use of constructive geometry. The visual unity of

the construction is in contrast to Vitruvius' numerical systems. The numerical proportions for the ornamental elements do not appear to have any justification by reason of a design system.

In the first book of *De architectura* Vitruvius explains how to lay out the streets of a city. Historically, the founding of a city is an activity that begins with rituals. The description in Book I, Chapter 6, of the procedure for locating the axes of streets, has the feel of a ritual. The procedure requires geometer's tools that establish the points and lines needed as reference for all future buildings and streets. Vitruvius stresses exactness. He prescribes using a level to establish a horizontal plane and a rod to cast shadows from the sun at different times of the day. The rest of the calculation is done with constructive geometry. With a circle drawn at the base of the rod and the two shadow marks on the circumference, a compass bisects the two marks and, with a straightedge, creates two lines which intersect at right angles. The lines correspond exactly to the compass points.

Traditionally Roman cities were founded on two axial streets, the Cardo and Decumanus, which align with the compass points.[41] In countless drawings examined for this thesis, the Renaissance artists began their designs with the same procedure that Vitruvius described for the founding of a city. I call it the crossed arcs method and is used for locating a horizontal and vertical line which meet at right angles. The procedure is described in the notebooks of Villard de Honnecourt, *Il libro del arte* of Cennini and the treatises of Filarete, Alberti, Francesco di Giorgio, Pietro Cataneo and Sebastiano Serlio. Chapter 2 discusses further the significance of this method to the design processes in the Renaissance.

Designing with numbers is an exclusively cerebral activity because at architectural scale proportional relationships are all but impossible to perceive. Not only is the eye not equipped to measure distances with the necessary accuracy, vision also carries with it the problems of point-of-view, occlusion and of perspectival diminution. However, these are problems only if one is

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[41]Friedman, David, *Florentine New Towns : Urban Design in the Late Middle Ages*, (New York, 1988), and Macartney, Eugene, 'The Military Indebtedness of Early Rome to Etruria,' *Memoirs of the American Academy of Rome*, (Rome, 1917), 121-155: "The Etruscan surveyor's art, with its fastidious rules for orientation and its complex system for founding cities, is manifestly ritualistic." Also John Onians, 'Architecture, Metaphor and the Mind,' *Architectural History*, vol. 35, (1992), 192-207: [The Romans] inherited from the Etruscans a standard template of architectural organization around a cardo and decumanus crossing each other at right angles."

dependent on the visual perception of the work. If one were truly a proponent of numerical proportion as a design scheme in architecture then it is the *idea* of the building that is the truest expression of the design. If one knows that the measurements of the building follow the proportions of the design, only then can one conceptually appreciate the actual building.

Vitruvius speaks of proportion in his theoretical exposition. However, as with geometry, when it comes to the practical aspects of architecture, numerical consistency becomes secondary. Apparently from experience with actual building situations and from studying extant Greek buildings, Vitruvius found that it was necessary to alter proportions to accommodate for optical distortions. For example in Book III, Ch. 3 he advises that the columns at the corners of a temple be made thicker to counteract the diminution in appearance caused by the different backgrounds for these columns. When one looks at a row of columns that have the temple wall as their background then turns to see the last column of the row, that column has only air as its background and consequently looks narrower. An increase of a fifth of a diameter will achieve the visual consistency. Taller columns have different rules than shorter columns because of the perspective one has looking up at them. The platform upon which a temple sits must be raised a small amount in the middle to keep the horizontal row of columns from appearing to sag midway. The gap between columns in a row is greater for the pair in the middle than for the others in order keep the middle pair from looking too close. With these optical modifications Vitruvius is revealing that compromising the numerical system is sometimes necessary to satisfy the eye even at the expense of conceptual consistency.

There is little known for certain about the interest in Vitruvius' *De architectura* in the centuries between its creation and the Renaissance re-discovery. The canon of proportions of the human figure were occasionally quoted but we know of no architectural writings which respond to Vitruvius' design ideas. There are Vitruvian manuscripts that are known to have been in libraries near where architectural edifices were erected, but it is difficult to prove that a given design was influenced by knowledge of Vitruvian principles. Studies have shown coincidences between Vitruvius' advocation of root-two rectangles and

the application of such figures in Early Christian and Romanesque churches,[42] but the link is speculation. For example, it is possible that a copy of Vitruvius could have influenced the building of St. Michael's, Hildesheim in the Eleventh Century.[43]

By far, however, the importance of Vitruvius' book lies in the influence it had on Renaissance artists and architects. One Fourteenth Century manuscript has marginal notations by Petrarch who may have consulted *De architectura* in the rebuilding of the Pope's palace in Avignon.[44] Boccaccio had a Vitruvian manuscript and there seems to have been Vitruvian influence on Cennino Cennini's handbook for artists in the area of human proportions. The early Italian Humanists were aware of Vitruvius' work and its value as an antique text from the middle of the Fourteenth Century. From these scholars, interest spread to others in the following century, who consulted the text for information on building.[45] The Fifteenth and Sixteenth Centuries saw the direct confrontation between the Ancient Roman architect and the Renaissance designers. The result of that confrontation is discussed in the next chapter on Renaissance sources.

The Use of Design Systems in the Middle Ages

Although Vitruvius' treatise may not have enjoyed a great deal of currency throughout the Middle Ages the theme of geometry and number continued to dominate the design concepts of the period. The written sources that pertain to designing with numbers and geometry come not from artists or architects, but from Christian exegetical writers, such as Augustine, Boethius and their followers. Over the centuries these theologians strove to create an all encompassing, unified theory of God and creation. This thesis concerns the area where the theology intersects with the design process. This area centers on the

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[42]Conant, Kenneth, 'The After-life of Vitruvius in the Middle Ages,' *Journal of the Society of Architectural Historians*, xxvii, (1968), p. 33-38.

[43]Hartwig, Beseler, and Hans Roggenkamp, *Die Michaeliskirche in Hildesheim*, (Berlin, 1954), 112ff. Also Kruft, *Architectural Theory*, (1994), 31.

[44]Oxford, Bodleian Library, Auct. F. 5. 7; cf. Lucia a Ciapponi, "Il *De Architectura* di Vitruvio nel primo umanesimo (dal ms. Bodl. Auct.. F. 5. 7), *Italia medioevale e umanistica* III, 1960, pp. 73ff.

[45]Kruft, *Architectural Theory*, (1994), 39-40.

idea of the architect as a metaphor for God. The exegesis begins with a verse from the Wisdom of Solomon (11:20) where Solomon addresses the Lord as a designer, "Thou hast ordered all things in measure and number and weight."[46] St. Augustine combined this idea of God as an orderly composer with Plato's creator from the *Timaeus* and the Pythagorean numerical harmonies into a thesis where number and geometry were integral parts of God's creative process.[47] The concept of architect was a metaphor for God because architects used geometry and number in their creations in the same way that God had done in His creation. We do not know how much St. Augustine and other Medieval theologians knew about the design procedures of actual architects, but there are a number of rare instances throughout the centuries of the Middle Ages where the methods of actual designers can be surmised.

There is not a great deal of evidence to tell us exactly how the artists and architects of the Middle Ages designed their works. Surviving drawings are very scarce. Paper and parchment were too rare and expensive to scribble out the first ideas of a design.[48] We know of no treatises written on architecture or painting which would tell us the theories, if there were in fact any, behind the design process. An example of the opacity that limits our view of the period is a literary document from c. 1170 by Abbot Suger, of Saint-Denis in the Ile de France just north of Paris. Suger wrote about his efforts to rebuild the narthex and choir of the church of Saint-Denis. In that text Suger mentions that the height of the old nave with the new one were equalized "by means of geometrical and

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[46]"Sed omnia in mensura, et numero, et pondere diposuisti."

[47]Simson, Otto von, *The Gothic Cathedral, Origins of Gothic Architecture and the Medieval Concept of Order,* (New York, 1956), xix. See von Simson's references on *De musica* and *De ordine,* of Augustine and *De musica* of Boethius. Also E. R. Curtius, *European Literature and the Latin Middle Ages*, (New York, 1953), V. F. Hopper, *Mediaeval Number Symbolism*, (New York, 1938), M. Bukofzer, 'Speculative Thinking in Medieval Music,' *Speculum*, XVII, (1942), Johan Huizinga, *The Waning of the Middle Ages*, (New York, 1949).

[48]Branner, Robert, 'Villard de Honnecort, Reims and the Origin of Gothic Architectural Drawing,' *Gazette des Beaux-arts*, lxi, (1963), 129-146. "The contrast between Romanesque and Gothic reflects differences in the procedure of design. The advantage of geometrical schemes is that they can be drawn full scale on the ground. The reason there are no working drawings is the architect worked out the ground plan in his head then drew it on the ground." See also Bucher, François, 'Design in Gothic Architecture: A Preliminary Assessment,' *Journal of the Society of Architectural Historians*, xxvii, (1968), 49-71, and 'Drawings from the 13th Century Architects Shop: The Reims Palimpsest,' *Journal of the Society of Architectural Historians*, xvii, (1958), 9-22.

arithmetical instruments."[49] There is no further explanation of the means.[50]

In the case of architecture, the designers had many practical considerations to contend with, without the restrictions of applying a regulated design scheme. Designers were essentially itinerant builders who went from town to town to create the great monuments of this period.[51] Churches were seldom built on a pristine piece of ground but on the site of a previous church for which there would be foundations, crypts, and sometimes standing walls that had to be incorporated into the new church. There were countless practical limitations such as money, the seasonal building periods and the availability of stone. Building projects often lasted decades or centuries and successive master builders would often disregard what their predecessors had planned. They would start anew with their own ways of building using the current fashion. Master builders were essentially foremen of construction crews. They had risen through the ranks of stone cutters and were not educated in design theory in the way that modern architects are. Without an architect educated in the classical manner with Vitruvian rules of proportion and the rationale behind all of the architectural orders, one might expect to find nothing but a lot of builders who used only standard rules of thumb and were only interested in making the buildings stand up.

On the contrary, the builders of the Middle Ages designed by adapting ideal geometric and arithmetic formulae to the practical considerations of mortar and stone.[52] Through generations of experience in building edifices of increasing complexity, architects created systems with which they designed structures that appear to be miracles of engineering. The real miracle was that they achieved these feats without the benefit of a science of statics. Today engineers can

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[49]'Suger, Libellus Alter de Consecratione Ecclesiae Sancti Dionysii, (The Other Little Book on The Consecration of the Church of St.-Denis), '*The High Middle Ages, Sources in Western Civilization*, Bryce D. Lyon ed., (New York, 1964), 213. Also see, Erwin Panofsky, *Abbot Suger. On the Abbey and Church of St.-Denis and its Art Treasures*, (Princeton, 1946).

[50]For an analysis of the design system at Saint-Denis see Sumner McKnight Crosby, 'Crypt and Choir Plans at Saint-Denis,' *Gesta*, vol. 5, (1966), 4-8.

[51]James, John, *The Template Makers of the Paris Basin*, (Leura, Australia, 1989).

[52]Bucher, François, 'Design in Gothic Architecture, A Preliminary Assessment,' *Journal of the Society of Architectural Historians*, 1968, xxvii, 49-71, 50, and Paul Frankl, 'The Secret of the Mediaeval Master Masons,' *Art Bulletin*, xxvii, 1945, 46-60. Also, Lon R. Shelby, *Gothic Design Techniques: The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schmuttermayer,* (Carbondale, IL, 1977); Franklin Toker, 'Gothic Architecture by Remote Control: An Illustrated Building Contract,' *Art Bulletin*, LXVII, (1985), 67-95.

determine the stability of a design based on scientific formulae, taking into consideration tension and compression and the strengths of building materials to resist these forces. However, what the Medieval builder called *scientia* was a design theory based on long experience of what worked. Without a science of calculating just why a building remained standing, they were left with attributing soundness of structure to their design procedure. While drawing plans on parchment or scratching them on the ground their procedure was to systematically interconnect the drawn shapes using constructive geometry.

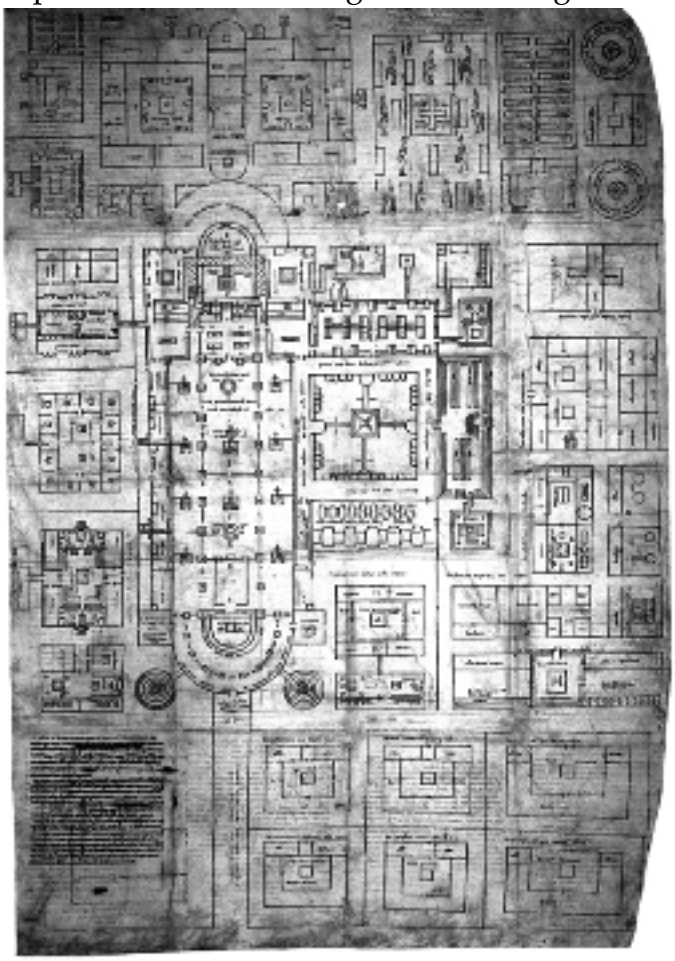


FIGURE 4 PLAN OF ST. GALL

One example of a surviving architectural design is the plan of St. Gall (fig. 4).[53] It is a drawing in ink on parchment of the ground plan of an exemplary monastery c. 820 AD. The circumstances behind the drawing reveal that the St. Gall plan is a rare example where a designer had free reign to design the ideal of a monastery and church without regard for topographical obstructions or preexisting structures. The monastery represented in the plan was to be the archetype for future monasteries. The analysis of the plan comes from the three volume monograph by Walter Horn and Ernest Born, who thoroughly studied the original.

There is evidence that the work is a tracing of a lost prototype. The construction marks which would have easily revealed the design procedure were

[53]Horn, Walter and Ernest Born, *The Plan of St. Gall, A study of the architecture and economy of, & life in a paradigmatic Carolingian Monastery*, (Berkeley, 1979).

lost with the prototype. However, the tracing does provide some clues as to the nature of the prototype. The monastery was designed according to a modular grid system. The square crossing of the church provides the module, encompassing 40 feet, which is repeated to make the apse, transepts and nave. The module is subdivided into sixteen sub-modules and a grid of these squares aligns somewhat with the walls and furnishings of the monastic buildings. There appears to be allowances for wall thicknesses even though the walls are represented with single lines. For the absence of alignment one must take into consideration the fact that the drawing has been traced without the aid of drawing instruments through parchment which is only semi-transparent. Even so, the consistencies of line spacing, angles, straight lines and circles, too precise to have been drawn freehand, indicates that the original must have been created with precise tools.

According to the authors of the 1979 study, Walter Horn and Ernest Born, the procedure of drawing the prototype plan that would have been necessary before the tracing was made is as follows: First, the grid of large modules was drawn to determine the position of the church axis and the main features. Then the larger squares were subdivided into a grid of smaller squares in a manner similar to the parallel lines visible under the text in contemporary illuminated manuscripts. Perhaps the lines were incised with a stylus or drawn with light metalpoint. The ground plan was then laid out according to a system whereby large shapes aligned to the main module lines and the smaller internal shapes aligned with the sub-module lines. Apparently every consideration was given to the appropriate space needed for each of the given buildings. This is a procedure of trial-judgment-modification where the grid has the power to limit the choices. The designer knows to follow a grid line as a trial. If the line needs to be moved then it can be moved to the next grid line. What Egyptian mural designers did with figures, the Medieval draughtsmen did with building elements. The designer of the Plan of St. Gall, however, may also have kept in mind sacred numbers, as is attested by the repetition of patterns of numbers including 3, 4, 10, 12 and 40.

In the area of manuscript illumination we have more direct evidence of design procedures. The Lindisfarne Gospels is a manuscript from c. 800, that contains full page designs for which marks of their construction still exist. These

carpet pages represent the cross in an abstract manner surrounded by intricate patterns that appear to be the result of a profound and almost inhuman degree of concentration. The designs were analyzed by Robert Stevick who, as the result of direct viewing of the original and measuring on a facsimile, was able to unravel the design procedures.[54] Stevick relates, step by step, each successive compass and straightedge operation revealing how the designs of seemingly unthinkable complexity have completely rational skeletons. The conclusion is that these Gospels, as well as other insular manuscripts of the early middle ages, were designed by means of constructive geometry. The fascinating thing about the design process for carpet pages, such as those in the Lindisfarne Gospels, is that the process appears to be an end in itself. Perhaps the monks used geometric operations as a form of ritualistic meditation. "Trying not only the patience but equally to the eyesight of the draughtsmen, the work of decorating a manuscript parallels the austere self-discipline which characterized the ascetic practices of the Irish anchorites. It was another way to attain communion with God."[55]

Another analysis which reveals the importance of geometry in late medieval design is that of the history of Milan cathedral by James Ackerman.[56] The cathedral was founded in 1386. The plan was for a large scale version of the local traditional churches which were broad and low and had triangular elevations. The local engineering skills had not advanced much because of their conservative style and were primitive in comparison with the Northern masters who had built the grand cathedrals. After the foundations were laid and construction begun the building committee summoned French and German architects as consultants because of concerns that the local builders might not be up to the monumental task. Ackerman reconstructs the narrative from the council's records of how the exchanges between the council and outside consultants became adversarial. The council wanted engineering advice while maintaining their traditional style. On the other hand, the advisers insisted that sound engineering was one with the Northern style. We benefit from the

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[54]Stevick, Robert, 'The Design of Lindisfarne Gospels folio 138^{v},' *Gesta*, xxii, (1983), 3-12

[55]Nordenfalk, Carl, *Celtic and Anglo-Saxon Painting, Book Illumination in the British Isles, 600-800*, (New York), 1977, 19.

[56]Ackerman, James, '"*Ars sine scientia nihil est*"' Gothic Theory of Architecture at Milan Cathedral,' *The Art Bulletin*, xxxi, (1949), 84-111.

controversy because the participants made explicit the rationales behind their design processes.

The church's ground plan was unchangeable because the foundation was already laid. The series of consulting architects advised the council to design the elevation of the five aisle nave according the systems *ad triangulum* or *ad quadratum*. The former is a system where the major forms of the design are governed by an equilateral triangle, while in the latter the forms are based on the square. The piers were already being built following a system of heights generated by repetition of a ten unit module, evident in a drawing by Antonio di Vicenzo. Because of the incommensurability of the height to width of the resulting triangle, and the need for builders to use simple measurements, a mathematician, Gabriele Stornaloco was brought in from Piacenza to resolve the conflict. After rounding the numbers, Stornaloco designed an elevation where the heights of the vaults were governed by a system of triangles and modules. Ackerman summarizes:

> The principal function of the geometrical formulae is to establish ideal figures for the whole which are reflected in consistent relationships throughout the parts of the cathedral. Our 'scientia' may be called a theory of consistent relationships. The interrelation of parts provided by geometrical formulae compensates for the Gothic engineer's ignorance of the laws of mechanics. In a sense, this interrelation is a symbol of structural stability. Whatever knowledge the French architect has of stress and thrusts is attributed to geometry. He consequently feels that a structure which does not conform to a simple geometrical figure is unsound.

In this chapter I have attempted to characterize two design systems and to briefly relate some of their Pre-Renaissance occurrences. The intent was to establish a pattern of behavior, that the systematic use of numbers and constructive geometry in design was among the designers' long held traditions. The reason for becoming traditions must have been, to a large extent, the practical benefit the systems provided to the users. This benefit is the effect the use of a system has upon the designer in a creative endeavor. "An understanding of these geometrical and proportional principles lend assurance and a sense of conviction to the designer in that most difficult area – decision-making."[57]

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[57]Coates, Kevin, *Geometry, Proportion and the Art of the Lutherie*, (Oxford, 1985), 167. Coates interprets the evidence from European musical instruments from the 16th to 18th Centuries and

"Knowledge of a basic law gives a feeling of sureness which enables the artist to put into realization dreams which otherwise would have been dissipated in uncertainty."[58]

Knowing the Antique and Medieval traditions behind the use of numbers and geometry brings into clearer focus the design procedures of the Renaissance. We will see in Chapter 2 that the treatise writers of the Renaissance were following in the tradition with their theories. The observation chapters of Part II of this thesis present the physical evidence of how some individual artists and architects applied the traditional procedures to specific works.

argues that the makers of these instruments employed geometry and numerical proportion in the design.

[58]Hambidge, Jay, *The Elements of Dynamic Symmetry*, (Yale, 1926), preface.

Chapter 2
Italian Renaissance Sources

The foregoing outline of historic precedents makes it is reasonable to expect that the artists and architects of the Italian Renaissance had available to them knowledge of the traditional design systems of constructive geometry and numerical proportions. It is also reasonable to expect that the designers of this new period would accept these time honored traditions, modify them for their own purposes and use them in different ways or to different degrees. Those who wrote treatises revealed their thoughts on these systems, though not always in an explicit way. The texts mention elements of the design process such as the crossed arcs method of locating reference axes and what I call the process of trial, judgment and modification. This chapter addresses these aspects of the design process and the two design systems as interpreted through the contemporary written sources.

It is necessary to analyze what contemporary writers said about these methods in order to recognize and interpret the physical evidence from draughting tool marks visible in contemporary drawings. Since the intent of the

treatise writers was not necessarily to make artists or designers out of their readers, many aspects of the design process were left unexplained, or insufficiently explained. Some writers did occasionally reveal clues as to the ideas behind their design processes. By sifting through these clues and analyzing the drawings it is possible to retrace the steps of the designers and sometimes even to reveal the motivation behind the steps.

Trial, Judgment and Modification

The process of trial, judgment and modification is fundamental to the design process of all ages. Designs do not spring fully formed from a designer's head. There must be a period where the designer does not know what is going to happen and has to struggle to make something come into being. I call this struggle the interactive process of trial, judgment and modification. If there is no struggle and the artist knows in advance what the end product will be, then the activity is more accurately designated a craft and the maker a craftsman.[59]

Much of the art of the Middle Ages was dependent upon pattern books. While it is true that copying images straight from pattern books or earlier works is not what we would consider a design process, the process of trial, judgment and modification did occur, only slowly. The formulae for creating a Gospel book carpet page or a Madonna altarpiece may have restricted many of a designer's choices, but there was always room to try something new, even within a strictly dictated program. Over the centuries there was a long-term, collective design process among a large number of designers. When one examines a chronological sequence of carpet page designs, for example, the evolution is clear. The designers were aware of previous works and modified the pattern in their own works according to their own judgment.

Since the rate of change for designs in the Middle Ages was relatively slow, the evidence necessary to reconstruct the individual steps in artists' designs processes is rare. It is not until the Renaissance that we have a few design drawings on which are recorded the telling changes-of-mind. Design drawings record the process of designing in the form of *pentimenti*, the lines and forms that are rejected in favor of something new. When studying the chronological sequence of Italian Renaissance drawings it appears that the

[59]See Janson, H. W., *History of Art*, 3rd ed., (New York, 1986), Introduction, 9-18.

occurrence of *pentimenti* increases significantly toward the end of the Quattrocento. E. H. Gombrich argues that it was Leonardo who began and popularized the process of allowing the struggle to show and letting the struggle ferment into new inventions.[60] Leonardo recommends, "Sketch subject pictures quickly and do not give the limbs too much finish: indicate their position, which you can then work out at your leisure."[61] Sketching quickly with minimal detail allows the artist to delay the moment when he commits himself, and to explore other possibilities that might be better than the initial guess. The resulting figures appear more dynamic and the design more inventive.

Leonardo describes, in his treatise on painting, another technique to enhance the inventiveness of the artist. He relates his experience using this technique: "I have even seen shapes in clouds and on patchy walls which have roused me to beautiful inventions of various things, and even though such shapes totally lack finish in any single part they were yet not devoid of perfection in their gestures or other movements."[62] To Leonardo the creative spark was a response to something in nature.[63] The process of sketching quickly and without worrying about detail gives the artist the opportunity to respond to his own drawing and add that response to the drawing.

Before Leonardo, drawings appear to be records of the draughtsman's skill at creating things fully formed. That skill had to be learned. Cennino Cennini (c1370-1440) describes the process of learning to draw in his handbook, *Il Libro dell'arte*.[64] It was written around the late fourteenth century and focused on the craft of painting. Cennini's handbook contains a great deal of information regarding the materials of the trade and the techniques required of a Trecento artist working on panels or walls. Unfortunately, Cennini does not describe in the same detail a theory of design or procedures for designing. Cennini recommends drawing with charcoal then rubbing the drawing with the barbs of a feather to erase the parts that do not come out correctly. Cennini says to keep starting over from the beginning until the figure agrees in proportion to the

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[60]Gombrich, E. H., *Norm and Form*, (London, 1966), 58-63.
[61]Leonardo, *Treatise on Painting, Codex Urbinas Latinus, 1270*, (ed. A. P. McMahon, Princeton, 1956), fol. 34r.
[62]Leonardo, *Treatise*, fol. 62r.
[63]See Chapter 4 in Part II of this thesis for specific examples of Leonardo's design process.
[64]Cennini, Cennino, *Il Libro dell'arte*, c1400, (ed. *The Craftsman's Handbook*, trans. by D. Thompson, 1932), 17.

model. When the lines are perfect they should be reinforced with a more permanent medium.

The idea of designing by trial, judgment and modification is also expressed by Alberti (1404-1472), in his architectural treatise entitled in Latin, *De re aedificatoria*:

> But I can say this of myself: I have often conceived of projects in the mind that seemed quite commendable at the time; but when I translated them into drawings, I found several errors in the very parts that delighted me most, and quite serious ones; again, when I return to drawings, and measure the dimensions, I recognize and lament my carelessness; finally, when I pass from the drawings to the model, I sometimes notice further mistakes in the individual parts, even over the numbers.[65]

Alberti's design method proceeds in four stages: the design is first imagined and if it gives "delight" it is then reduced into lines. These lines are then measured using numbers and whatever errors are found are then corrected. Finally, a model is made and presumably measured again with numbers to look for errors. Alberti also advocates that the design be reviewed many times by the designer as well as by others. The process, then, is a continuous pattern of trial, review, correction, review.

Crossed Arcs Construction

Another idea that I have come to consider significant is the process of establishing the horizontal and vertical axes of a design. The procedure for establishing these axes I call the crossed arcs method and is based on a simple geometric principle: When two circles are drawn close enough to one another that they overlap, the circles intersect at two points. If a straight line is drawn through these points, that line is at right angles to another line drawn through the centers of the two circles (fig. 5).[66] Since the circles would be drawn with a compass, their centers would be visible as the point-like indentations made by the pivoting foot of the compass.

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[65]Alberti, Leon Battista, *De re aedificatoria*, (Florence, 1486), Bk. 9, ch. 10, (ed. *On the Art of Building in Ten Books*, trans. of by Joseph Rykwert, Neil Leach, and Robert Tavernor, Cambridge, Mass., 1988), 317.

[66]The perpendicular is also exactly halfway between the two centers. Therefore the procedure can also be used to locate the midpoint between any two given points.

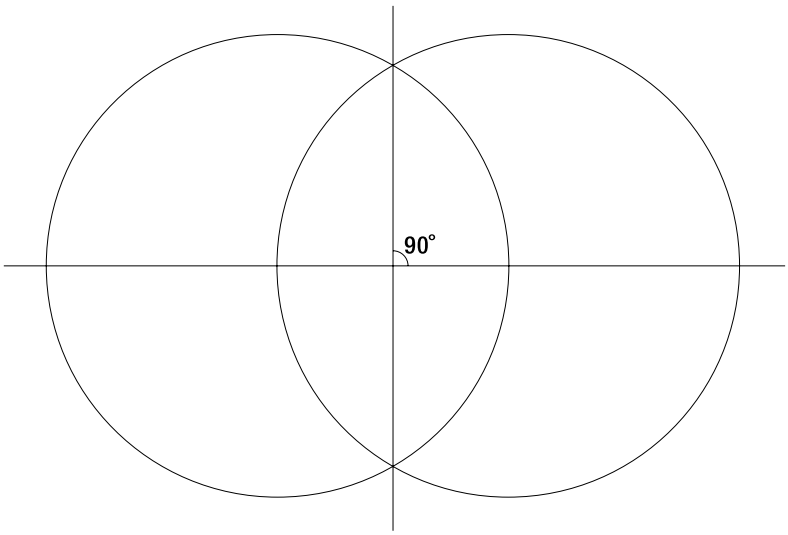


FIGURE 6

The usefulness of this geometric principle is that one can find the ninety degree intersecting line for any given line in two simple steps. First, by drawing two overlapping circles with centers on the given line and then by connecting the two intersecting points where the circles overlap. In practice, the circles are generally not drawn completely. Only enough of the arcs are drawn to locate the points where complete circles would intersect. The obvious sign of this method's having been used is the presence of crossed arcs with ruled lines through their intersections (fig. 6).

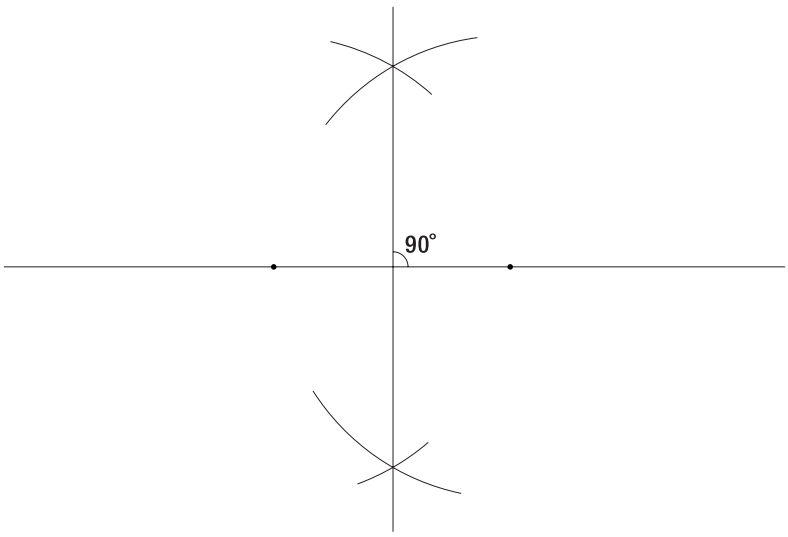


FIGURE 5

Why this simple procedure is here considered significant is a matter requiring some explanation. Evidence of the crossed arcs method is very

frequently present in design drawings where geometer's tools are part of the design process. Yet the procedure has shortcomings. Although the operation is simple in principle, performing it requires patience and some degree of effort. The more care and effort that one invests, the more accurate are the resulting right angles. The procedure can be easily superseded by using a set square or by making a reusable template out of paper with holes pricked in the appropriate locations. Given that the procedure is tedious, that the results vary with the amount of care, and that there are alternate methods that give similar results and require less effort, it is surprising that anyone would go to the trouble of performing the operation. When Andrea Palladio (1508-1580), for example, was designing an architectural plan that required multiple draughts, he began each with the crossed arc procedure. It would have been much easier, quicker and more consistent to have transferred the axes from the first draught through the paper to the second, with pin pricks (see Chapter 7, on Palladio).[67]

Other evidence for the significance of the crossed arcs method is the frequency of its mention in written sources in the context of design instruction. Although a description of the procedure would most expectedly be found in elementary geometry textbooks, the writers of art and architectural treatises felt the need to remind their readers that this elemental procedure was necessary in the process of designing. The method is described by Vitruvius and is illustrated as early as Villard de Honnecourt's sketchbook (2nd quarter of the 13th Century) and as late as Sebastiano Serlio's treatise on architecture (1566).[68]

The ubiquity of the method in theory and practice raises the question as to whether there is some more important purpose for the procedure than just producing a right angle. Perhaps the act of performing the operation is a type of ritual. This possibility will be explored below.

Let us examine now the literary references to the crossed arcs method. A veiled reference to the procedure is in *De re aedificatoria*. There, Alberti advises that when laying out the plan of the foundation on the ground, before starting excavation, one should "mark out all the corners . . . to the correct size and in the

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[67]The evidence for crossed arcs in specific drawings will be discussed in the observations chapters in Part II of this thesis.

[68]Honnecourt, Villard de, *Architector: The Lodgebooks and Sketchbooks of Medieval Architects*, (ed. François Bucher, New York, 1979). Sebastiano, Serlio, *L'Architettura,* (Venice, 1537, Book IV), (1540), Book III, (1545), Book I and II on Geometry and Perspective, (1547), Book V, (ed. *On Architecture*, vol. 1, trans. by Vaughan Hart and Peter Hicks, Yale, 1996).

right place several times, with great care."[69] The procedure of laying out the design, "would not be easy to explain using words alone: the method by which they are drawn is derived from mathematics and would require graphic illustration." But he goes on to explain it anyway:

> Our usual method of defining the foundations is to trace out several lines known as baselines, in the following manner. From a midpoint at the front, we extend a line right to the back of the work; halfway along it we fix a stake into the ground, and through this, following the rules of geometry, we extend the perpendicular. We then relate all the measurements to these two lines. This works wonderfully in every way: the parallel lines are easily drawn, the angles can be defined accurately, and the parts conform and correspond exactly to one another.

The "rules of geometry" must be the crossed arcs method of extending the perpendicular. It is the only way to derive the parallel lines accurately. Alberti's concern, in this example, is to establish self consistency in terms of geometrical accuracy. Marking in the right place, several times with care, while employing the rules of geometry will result in the exact correspondence of parts.

In Francesco di Giorgio's (1439-1502), treatise on architecture, engineering and military arts, he describes and illustrates the crossed arcs method in the context of useful measuring tricks one can do with geometry.[70] There, the method is used to make a perfect square (*perfetta squadra).*

Cennini gives a step-by-step description of the crossed arcs method in his procedures for fresco painting.[71] The focus of the description, as is consistent with the rest of his book, is on the technique and practical considerations associated with the materials of the procedure. The first step is to snap lines at the point when the wall is ready for the underdrawing. (That is, snapping a chalked string, stretched taut over the wall, so as to leave a straight chalk line.) Snapped diagonal lines are used in the first step to locate the center of the space. Finding the center establishes the first point of reference in constructing a composition. The second step is locating the vertical and horizontal references. In the case of building a composition on a wall, Cennini says to first locate the

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[69]Alberti, (ed. *Art of Building*), 62, quotations follow.

[70]Francesco di Giorgio Martini, *Trattati di architettura ingegneria e arte militare*, ms. Ex-Magliabecchianus II.I.141, fol. 41r, c. 1490-1492, Biblioteca Nazionale Centrale, Florence. (ed. Corrado Maltese, Milano, 1967), 129, t. 30v, tav. 56.

[71]Cennini, (ed. *Handbook*), 42-43.

vertical reference with a plumb line, that is, a string with a lead weight at the end. With a compass placed on the resulting vertical line and opened as large as possible, two short arcs should be drawn on either side of the axis. The foot of the compass is then moved a little distance away along the vertical line and the arcs are repeated, this time crossing the first two arcs. A line can then be drawn between the two resulting intersections forming a horizontal that is at a true right angle to the vertical line and consequently level. This is the same procedure as Figure 5, rotated 90 degrees.

Cennini's method for beginning a painting on a wall is essentially the same as Vitruvius' procedure for founding a city, (see Chapter 1, Vitruvius). Vitruvius and Cennini each use as their foundational anchor a consistent physical phenomenon. In the case of city layouts the sun's shadows provide the first references. In the case of Cennini's frescoes it is the weight of a plumb bob pulling a string straight down. The subsequent steps of the geometric operations could then be accurately made with confidence. The resulting right angles are not only accurate within themselves but they also orient the design with axes of the earth.

My idea that there may be a ritualistic aspect to performing the crossed arcs procedure begins with Vitruvius' origination for the layout of a city. Since the founding of a city is an event that occasions an actual ritual, the initial determination of the orientation of the streets would be an event of significance. The Roman city, as well as the Roman military camp began with two axial streets known by the surveying terms *cardo*, running north and south, and *decumanus*, running east and west.[72] This layout, according to Lewis Mumford, was "oriented to harmonize with cosmic order."[73] Christian churches, which are analogous to the "City of God," also have their streets, the nave and transepts, oriented to the compass points. Because the axes are oriented by means of bisecting the sun's shadows, the hamonizing with cosmic order is, to some extent, literally true. Because the founding a city is an event of great moment, it stands to reason that a ritual is in order at the beginning. The Romans inherited

[72]Polybius, *Histories*, 2nd Century B. C., Book VI. See also, Pliny, *Natural History*, 1st Century AD, XVIII, lines 326-337.
[73]Mumford, Lewis, *The City in History*, (New York, 1961).

the axial city plan from the Etruscans, who founded cities with great ritual.[74]

In the case of making a drawing on a piece of paper, the activity of establishing axial references is perhaps of less moment than the founding of a city, but there may be a similar element of ritual inherent in the invocation. The ritual aspect of the procedure is in the careful repetition of an act for the purpose of making sure that the undertaking is done right and well started. Unlike endeavors that have a predictable outcome, designing requires taking a chance. It is not unusual for those who frequently take chances to indulge in a ritual of some kind.

Constructive Geometry and Numerical Proportions

The motivating idea behind the system of constructive geometry is not expressed in the written sources. That is, no one felt the need to explain why the system was used at all. The quote from Pietro Cataneo, at the beginning of this thesis, extolling the value of geometry to the architect does not specify the reason that geometry is of value in designing. In the case of the Milan cathedral controversy, the council members argued between different geometric principles as to which was better to guide the design, but there was no mention of why geometry is necessary. James Ackerman explains that the *scientia* of the Milan cathedral council was the application of geometry for the purpose of composition and consequently structural stability.[75]

Within the art and architectural treatises of the Renaissance the two themes of geometry and number exist but there is no expression of awareness that geometry needs an explanation. Numbers, however, are a common subject of design theory. Cennini (ch. 30) makes use of a prime measurement (read module) as a means of aiding the accuracy of drawing. The face, he says, following Vitruvius, has three of the same measure: the forehead, the nose and the chin. The designer should use one of these measures as a standard for the whole when copying a painting which is high on a wall. Although Cennini does not describe his method in great detail, he is probably referring to the technique

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[74]Onians, John, 'Architecture, Metaphor and the Mind,' *Architectural History*, vol. 35, (1992), 192-20. Also see, Macartney, (Rome, 1917)), 121-155.

[75]Ackerman, James, '"*Ars sine scientia nihil est*" Gothic Theory of Architecture at Milan Cathedral,' *The Art Bulletin*, xxxi, (1949), 84-111, 105.

of sighting along one's drawing instrument for a measure in the model, and counting that measure a number of times to find the corresponding locations in one's drawing.

Alberti devotes some attention to numbers in Book 9.5, *De re aedificatoria*.[76] He says nature is the justification of using some numbers over others. For example, just as all animals stand on an even number of legs, buildings should stand on an even number of supports. Openings, however, should be odd in number, just as the number of openings of the face. Alberti then extols the virtues of specific numbers, (3, 4, 5, 6, 7, 8, 9, 10), giving instances in nature where each number is significant. Discussing music, he draws a connection between how numbers which make good sounds also affect appearances. "The very same numbers that cause sounds to have that *concinitas*, pleasing to the ears, can also fill the eyes and mind with wondrous delight." He then describes in detail the numerical proportions of musical harmonies, and says that architects should use these numbers. Alberti's numerical system is thoroughly worked out as to what relationships are significant but it is very abstract and not tied specifically to a usable design procedure. Apart from the geometrical layout for a building site, (mentioned above), there is no prescription for a geometrical design process. Alberti's goal was to create a treatise in the Classical sense, rather than provide a handbook for artists.

Filarete's treatise on architecture (c1400-1469) makes a distinction between the use of numbers and geometry.[77] In Book VIII folio 56r, he describes using a number system for the classical orders with their modular proportions similar to Vitruvius and Alberti. When he describes how doors should be proportioned, however, he uses a system of geometrical figures.[78] These are the square, one and one half square and a square and its diagonal. Note that the sizes are given as shapes and not numbers. Similarly, Alberti employs numerical proportions in respect to classical ornament but switches to geometry for the shapes of doors. For shorter doors he specifically mentions the diagonal of a square.[79] Andrea Palladio provides detailed, step-by-step instructions for his readers in the ideal

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[76]Alberti, (ed. *Art of Building*), 305.
[77]Filarete, (Antonio di Piero Averlino), *Dell'Architettura*, (ed. *Treatise on Architecture*, trans. with notes by John R. Spencer, New Haven and London, 1965).
[78]Filarete, (ed. *Treatise*), vol. 1, Book VIII, fol. 60r, 104.
[79]Alberti, (ed. *Art of Building*), for classical proportions: Book 7, 7, 202-205, for proportions of doors: Book 1,12, 29.

relationships of room heights to room shapes. Palladio gives equal attention to both systems. The instructions are given first by geometrical demonstration then by mathematical demonstration.[80]

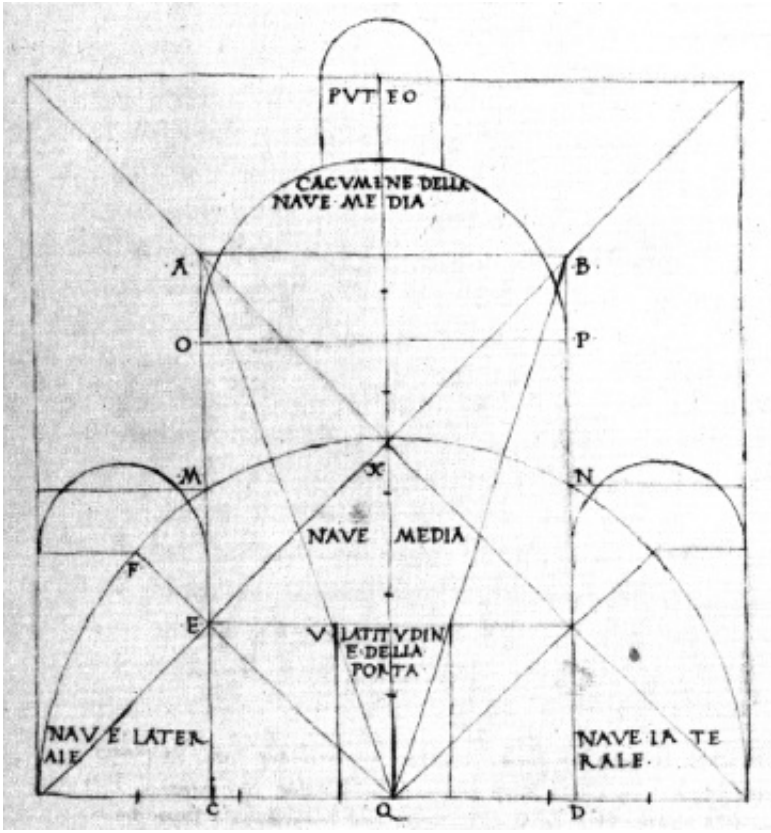


FIGURE 7 SECTIONAL ELEVATION OF A CHURCH

Francesco di Giorgio's use of the two systems is different from the other treatise writers. He advocates a system based on modules but he derives the modules from geometric constructions. On folio 41 of his *Trattati* he draws the sectional elevation of a church with nave, aisles and cupola (fig. 7).[81] The elevation is constructed within a square and Francesco explains the steps of the construction:

> In order to demonstrate other geometric proportions, [by showing] commensurations of temples with oblong naves, first make a square with equal sides and draw inside two diagonal lines from corner to corner. Divide the base of the square into four equal parts. From the points C and D draw two straight lines terminating at the diagonals and connect them with the transversal A, B. Now draw a semicircle from the lower corners, passing through the intersection of the diagonals. Where the circle

[80]Palladio, Andrea, *Il quattro libri dell'architettura*, (Venezia, 1570), ch. xxiii.
[81]Francesco di Giorgio, *Trattati*, ms. Ex-Magliabecchianus II.I.141, fol. 41r, c. 1490-1492, Biblioteca Nazionale Centrale, Florence.

> intersects the two vertical lines at M and N, draw transverse lines: these will be the just height and width of the side aisles. Now draw a line passing through the larger and smaller rectangles, and two others from the mid point of the base through the intersections of the straight lines and the diagonals. The portion which lies within the semicircle, namely E, F will be the module for the whole temple. [Draw] two lines from the said point Q to the points [A] and B and where they intersect [the transversal drawn at point E] this will be the height and width of the door. The same width gives the size of the *puteo* or lantern. Because the diameter of the base is seven times the module E, F, and the height of the smaller rectangle ABCD will be five and one half parts, and the height [of the nave exclusive of the cupola] will be four and one half, draw the line OP, in the middle of which place the compass point with width at P and draw a semicircle OP. This will be the height of the temple.[82]

This passage is very unusual. Unlike Vitruvius' commentary upon theaters that were descriptions of pre-designed works, this is an example of a designer recommending the use of constructive geometry, in which points are located by the intersections of lines and arcs, as a means of designing. It is perhaps the first time that constructive geometry was ever described in such a context. The passage also combines this system, in an original way, with a modular system. The first step of drawing diagonal lines within a square is a significant indication of a geometric design system because neither the square nor the diagonals appear in the finished elevation. They are construction elements for the design in the same way that scaffolding is for a building. The square is the governing shape in the same sense as the *ad triangulum* or *ad quadratum* systems were for the Milan cathedral designers. The diagonal lines,

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[82]Francesco di Giorgio, *Trattati*, 399-400. "E per volere dimostrare alcune altre geometriche proporzioni, commensurazioni de' tempi navali oblonghi, fatto prima uno quadrato d'equali lati nel quale da angulo ad angulo si tirino due linee diagonee, e la basa del quadrato divisa in quattro equali parti, e dal partimento C D si tiri due rette linee terminanti alle linee diagonee con una linea transversa A B. Di poi si tiri un semicirculo dalle estremitá delli anguli della basa, passante la sua altezza all'intersecazione delle linee di[a]gonee X dove la linea del circulo passante interseca per M N, tirate in quel loco le transverse linee, sará iusta altezza alla larghezza della maggiore e minor quadrato, e due altre dal ponto medio della basa o passanti l'intersecazione delle rette linee e diagonee, e vadino a trovare la estremitá della porzione del semicirculo, e quella parte che resta dentro alla porzione, cioé E F, sará modulo a tutto il tempio; e due altre linee dal ditto ponto Q, e vadi insino alla quadrata altezza della B intersecando per lu S: questa sará la larghezza e altezza della porta. La qual medesima larghezza si dia al summo puteo overo lanterna del tolo. Perché il diamitro della basa overo latitudine di tutto il tempio si trovi parti sette del modulo E F, e l'altezza del minore quadrato A B C D saria parti 5 1/2, e a l'altezza di parti 4 1/2, si tiri la linea O P, in mezzo della quale si ponga il centro pigliando la circunferenzia dallo P. E questa sará la somma altezza di tutto il tempio."

from corner to corner, begin a matrix within the square that is stable and upon which one can hang, or against which one can prop, other elements which take part in that stability. The diagonals also find the center and divide the whole into halves, an operation which is basic to geometry.

The halves are again divided into halves and the quarter divisions establish the width of the nave. To this point, the shape of the church could have been expressed as the result of a numerical system. The width of the whole is four units and the two middle units are given to the nave. However, the geometric matrix has a purpose, not only in providing a visualization for the structure of the building, but also in locating points which could not be found using numbers. The width of the door, for example, is not related numerically to any other part of the design but only to the intersections of diagonal and horizontal lines.

The quarter lines rise from the base and stop at the intersections with the diagonals. These intersections form the top of a central square which is one quarter the area of the format square and its base locates the height of the door. The width of the door (*lattidudine della porta* written on the drawing), is determined by two diagonal lines running from Q to A and B, intersecting at the base of the small inner square. Francesco's module, EF, he says is one seventh the width of the whole. In fact, it is slightly more than one seventh. This discrepancy did not seem to bother him since he used it to shorten the height of the springing point of the nave from the line AB to OP.

There appears to be, within Francesco's design procedures, a desire to unify the two conflicting systems of design. First, there is the Vitruvian modular system where a single measure is repeated throughout and numerical proportion is the unifying principle. Francesco's awareness of Vitruvius is apparent because he attempted to translate a text of Vitruvius' *De architectura libri decem* into Italian. Second, there is constructive geometry which might have been received through Francesco's Sienese training. The nature of this synthesis of two systems was analyzed by Richard Betts, who concluded that Francesco used geometry as the structural component of his design process.

His methods for designing churches are based on grids and quadrature, and they usually involve some combination of the two. He used grids to achieve correct proportions in the spaces, which he validated with reference to his theory

of the Human Analogy, and he used variations of quadrature to design the structural forms, among other things. His knowledge of quadrature was probably part of the theory that had been transmitted orally from generation to generation of architects throughout the Middle Ages.[83]

The diagrams in Francesco's treatise appear to be conceptual renderings of plans and elevations because the thin lines cannot possibly represent the thicknesses of walls. However, Betts' analysis of Francesco's manuscripts has determined that the diagrams not only take into account wall thicknesses, but that one of the main purposes of the constructions was to determine the wall thicknesses. The churches that Francesco theorized about and built were vaulted with barrel vaults. The structural requirements of barrel vaults was an experimental science in the Renaissance and Francesco was one of the first practitioners. The diagrams represent experiments in structural innovation. In the absence of a science of statics a designer turns to geometry, a tool he believes will give him structural soundness.

Francesco took from Vitruvius the instruction that a module should be used throughout the design for the purpose of achieving unity. He dispensed with the notion of using the column width as the module and searched for a geometrically derived module that could simultaneously unify the design and provide structure to hold the building up. In an analysis of one of Francesco's ground plans, Richard Betts explains that the module, derived in the same manner as in folio 41, was used to determine the thickness of the supporting walls. For a given span Francesco could plot the necessary wall thickness to carry the load of the vault. Since engineering walls to overcome the enormous thrusts of barrel vaults is serious business, it is an indication of how much faith he had in his systems that he would rely on geometry to determine structural dimensions. Francesco's work was his own unique synthesis. Although his manuscripts were not published, they did influence later architects. Leonardo, for instance, owned and annotated a copy.

Francesco's system of constructing a form within a square, using geometric and numerical divisions of the square, has a parallel in the design of alphabets. In the latter third of the1400s, antiquarians interested in Roman

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[83]Betts, Richard, 'Structural Innovation and Structural Design in Renaissance Architecture,' *Journal of the Society of Architectural Historians*, (March, 1993), 5-25, p.10.

inscriptions developed a geometric means of constructing the letters of the alphabet in the style of epigraphs.[84] In 1467, Felice Feliciano (1433-1479), of Verona wrote the first treatise on the design and construction of capital letters, *Alphabetum romanum*.[85] In it, he describes the construction of letters within a square and circle such that the form of the letters is determined either by a geometric armature within the square or the modular divisions of the square.

In the fourth book of *De divina proporzione* (1509) Luca Pacioli (1445-1517), demonstrates how to construct the forms of capital letters.[86] He advocates using a construction process that combines geometry and numbers. Pacioli says to first draw a circle, then construct a square from the circle with its diagonals. The various letters fit inside the square and relate to a modular system of divisions by ninths. The centers for circles that are used to make the curves of the serifs, for example, are located by the nine part modules. Robert Tavernor argues that because the geometric section of Pacioli's treatise was somewhat plagiarized from Piero della Francesca's treatise *Libellus de quinque corporibus regularibus*, the alphabet could have been lifted as well.[87] Tavernor believes that Alberti could have been the originator of Pacioli's alphabet but that Alberti was influenced by Feliciano.

From this targeted survey of Renaissance treatises we have seen that some writers are more forthcoming than others on the subject of artists' methods and design procedures. It is disappointing that Cennini, with his focus on practical artistic matters, did not delve further into the design process. Perhaps, in the early part of the century, there was no language to describe the process. A century later, Leonardo failed to make explicit much that we would like to know of his design process. Francesco di Giorgio is the most explicit with his advocation of a mixture of constructive geometry and modular ratios, derived from that geometry. The source for such constructions was from the tradition of

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[84]Sperling, Christine M., 'Leon Battista Alberti's Inscriptions on the Holy Sepulchre in the Cappella Rucellai, San Pancrazio, Florence,' *Journal of the Warburg and Courtauld Institutes*, Vol. 52, (1989), p. 221-228. Sperling identifies the group as Alberti, Feliciano, Mantegna, Fra Giocanda da Verona, Damianus Moyllus, and Andrea Bregno, all with ties to Northern Italy, especially Padua.

[85]Feliciano, Felice, *Alphabetum romanum*, Bibl. Vaticana, Vat. Lat. 6852, 1463, (ed. Giovanni Mardersteig, Verona, 1960).

[86]Pacioli, Luca, *De divina proportione*, (Venice, 1509).

[87]Tavernor, Robert, *On Alberti and the Art of Building*, (Yale, 1998), 116.

designing *ad quadratum*, confirmed by Vitruvius' blessing of the square and circle and combined with Vitruvius' modular systems. Alberti, Filarete and Palladio all approach the subject of geometry and numbers with the same structure versus ornament division that characterized Vitruvius' design theory. These architects had absorbed the traditional design processes and had recast them through the medium of the rediscovered Roman architect.

Chapter 3
The Investigation

I began this investigation into draughting tool marks in Renaissance drawings in an attempt to understand the working methods of Renaissance artists. The hypothesis that artists' methods could be discovered was based on the assumption that the artists of the Renaissance sometimes employed methods other than those which were completely intuitive, i.e. they used design systems. The analyses of paintings that originally sparked my interest in the subject had assumed that artists used these design systems and that they could be revealed by a geometrical analysis of the finished works. I found the unexpected degree of correspondence between the elements in the paintings and the superimposed geometry fascinating. But after becoming proficient at discovering the "secret geometry" behind master paintings I discovered the folly of overlaying geometrical grids on reproductions: it was simply too easy to find correspondences. They could be found in works that were obviously not designed by any geometric process. They could even be found in fish.[88] In order

[88]Doczi, György, *The Power of Limits, Proportional Harmonies in Nature, Art & Architecture*, (Boston and London, 1981). Doczi's book provides proportional and geometrical analyses of a wide variety of animals and plants as well as man-made structures. "If we look closely at a flower, and likewise at other natural and man-made creations, we find a unity and an order common to all of them," 1. See also Matila Ghyka, *The Geometry of Art and Life*, (New York, 1946). Ghyka

to discover Renaissance artists' working methods in a manner that could hold up under scrutiny, I needed a research methodology that was less susceptible to error. The decision to forgo spurious, *a priori* theoretical reconstructions and to emphasize observable draughting tool marks was influenced by two investigators who studied artists' working methods, James Elkins and Robert Stevick.

The subject of Elkins' article, "The Case Against Surface Geometry,"[89] is an exposition of the methodological problems associated with theoretical reconstructions of paintings. Such reconstructions are attempts to discover the geometrical plan of paintings by superimposing theoretical grids and looking for correspondences within the paintings. Elkins criticism of a specific reconstruction makes his point:

> The *Baptism* [by Piero della Francesca], it is said, contains a perfect pentagon and an inscribed starred pentagon, perfectly matching the painting at key points. The proponent of the theory, B. A. R. Carter, argues for it on the basis of its accuracy.[90] In the absence of documentation, and without being able to imagine the pentagon without seeing it in place, what criteria would allow us to judge such theses? How is it possible to dispute a perfect fit? Would it be necessary to find something that fits better? . . . [O]ne can argue that the fit is artistically unlikely, since it doesn't correspond to a reasonable working procedure, that it is historically unlikely, since there are no parallels for this kind of organization, and that the fit may indeed be partly fortuitous. On practical, historical and probabilistic grounds, therefore, Piero did not use a pentagon to organize his painting.[91]

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says, "[T]he Platonic – or neo-Platonic – view of Art conceived the Artist as planning his work of Art according to a preexisting system of proportions, as a "symphonic" composition, ruled by a "dynamic symmetry" corresponding in space to musical eurhythmy in time. . . [W]e shall see this point of view transmitted all through the Middle Ages to the Renaissance and beyond with the study, and the application to artistic composition, of the same proportions," introduction ix, x. Ghyka provides geometrical analyses of works of art and architecture ancient and modern.

[89]Elkins, James, 'The Case Against Surface Geometry,' *Art History*, (June, 1991), 143-174.

[90]Carter, B. A. R., 'A Mathematical Interpretation of Piero della Francesca's *Baptism of Christ*,' in M. A. Lavin, *Piero della Francesca's Baptism of Christ*, (New Haven and London, 1981), 149-63.

[91]Elkins, 'Surface Geometry,' 145.

Becoming more specific, Elkins adds that the constructions are inefficient in relation to the forms that they purport to explain; the reconstructions' allowances for error are historically inaccurate; there are no rules to determine how lines should correspond to forms; and there are salient features in the paintings that are not accounted for in the reconstruction.

Elkins' critical arguments made it clear to me that such theoretical reconstructions did not constitute reliable evidence and were to be avoided. More solid evidence would be required in order to discover what geometrical systems, if any, were used by artists of the Renaissance.

Robert Stevick's work also affected my research method because his reconstructions were based on the kind of evidence that could, when found in Renaissance designs, more conclusively demonstrate Renaissance artists' working methods.[92] Stevick's articles on carpet pages from the Lindisfarne Gospels and other Medieval manuscripts demonstrated the possibilities inherent in analyzing the physical evidence of geometric design systems. Since the use of such geometric procedures requires the systematic manipulation of geometric drawing tools, it is sometimes possible to discover the steps of the process. This is because when an artist employs a geometrical design system, each step in the procedure is determined by preceding steps. The marks in the paper are evidence of what operation was performed. Consequently, Stevick was able to give step-by-step instructions for redrawing the geometrical framework and allowing the reader to execute for themselves the same procedures exactly as the original artists had performed them.

Stevick's demonstrations of geometrical design procedures communicated a degree of certainty that was lacking in the kind of theoretical reconstructions criticized by Elkins. Consequently, Stevick's demonstrations were to become the model for portions of my own analyses of Renaissance drawings.

Of course, paintings from the Renaissance differ from completely abstract and geometrical works from the Middle Ages. Had Renaissance artists

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[92]Stevick, Robert, 'The Design of Lindisfarne Gospels folio 138^{v},' *Gesta*, xxii, 1983, 3-12; 'The 4 X 3 Crosses in the Lindisfarne and Lichfield Gospels,' *Gesta*, xxv, 1986, 171-184; see also, 'The Shapes of the Book of Durrow Evangelist Symbol Pages,' *The Art Bulletin*, lxvii, 1986, 182-194; 'The Echternach Gospels' Evangelist Symbol Pages: Forms from the "Two True Measures of Geometry,"' *Peritia*, 5, (for 1986) [pub. 1989], 284-308; 'A Geometer's Art: The Full-page Illuminations in St. Gallen Stiftsbibliothek Cod. Sang. 51, An Insular Gospels Book of the VIIIth Century,' *Scriptorium*, vol. 44, 1990, 161-192.

employed such design systems, the effect on the resulting compositions, I believe, would be much more subtle. This is because the typical Renaissance painting, unlike a carpet page, is a scene with figures in it. When distributing the forms of a figural composition a simple framework may be all that is required of the geometrical construction. Figures and other natural elements are organic in form and generally do not lend themselves to geometricizing, at least, not down to every detail. The central figure of Botticelli's *Birth of Venus*, for example, is anything but geometric. Yet Venus' navel, a geometric point or small circle, lies on the horizon line which is certainly geometric. This, I would suggest, is an intentional geometric correspondence that has the visual effect of fixing the figure in the composition much as a thumb tack would. I am not suggesting that we know, from this correspondence, that Botticelli used a geometrical system to locate the figure (that would require more evidence than I am aware exists for this painting), but, that the employment of a geometrical system may result in images other than carpet pages or architectural elevations.

Linear perspective is another matter. I consider the systems of linear perspective to be a subset of constructive geometry. They are more structured and rule driven than other forms of constructive geometry in the same way that sonnets are more structured and rule driven than other forms of poetry. When using a perspective system, once the vanishing point (or points) and the viewing distance are chosen then everything else is determined by the system. One may think that the essential difference between linear perspective and other forms of constructive geometry is that the former is concerned with the illusionistic disposition of forms in depth and the latter is concerned with the composition of forms on the surface.[93] But experienced users of linear perspective could use that system as a means of composition on the surface as well.[94]

The goal of my research was to lay the foundations for an analytical methodology that could further our understanding of the working methods used by Renaissance artists and architects when designing their works. If we were able to reconstruct, using evidence from draughting tools, exactly how such a

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[93]Elkins, 'Surface Geometry,' footnote 1, 170.

[94]See Martin Kemp, *Visualizations, The Nature Book of Art and Science*, (Univ. of California Press, 2000), 34. See also White, *Birth and Rebirth,* (1987), 19. White writes of the interaction of the artists desire to represent space and his feel for the flatness of the painting's surface.

designer had employed a systematic design procedure, then we would have firmer ground upon which to theorize.

The Investigatory Method

My research method has been to closely examine as many original Italian drawings as possible from the 1400s and 1500s.[95] Although evidence of construction marks exists in panel paintings and frescos, these will have to wait for another investigation. Drawings are much more accessible than painting for these close examinations. Listed below are the repositories of drawings that I visited in my research.

England: The Royal Institute of British Architects, London; The British Museum, London; The Royal Collection at Windsor Castle, Windsor; The Ashmoleon Museum of Art and Archaeology, Oxford.

Italy: Galleria degli Uffizi, Gabinetto Disegni e Stampe, Florence; Casa Buonarroti, Florence; Gallerie dell'Accademia, Venice.

United States: The National Gallery of Art, Washington; The Metropolitan Museum of Art, New York; The Pierpont Morgan Library, New York; The Cooper Hewitt National Design Museum, New York.

When examining drawings, I looked for those specimens that might reveal, with analysis, the use of a design system. Candidates included those with construction marks made by a compass or stylus or anything else that might suggest the use of a design system. Drawings with evidence for the use of systems were very much in the minority because much of the process of making studies for paintings does not require the use of a system. Artists spent a large portion of their efforts drawing studies of heads and figures, either learning to draw or as part of the process that would end in finished paintings. The only systematic aspect of this process might have been Cenini's method for sighting along a thumb or piece of charcoal to measure relative proportions in the model and drawing. Apart from the mental exercise of seeing and depicting, drawing is mostly a series of intuitive guesses and responses to those guesses. The overwhelming majority of Italian drawings from this period are of such figure

[95]Trecento drawings from Siena, at the Museo dell'Opera del Duomo, such as Giotto's tower c1339, did not reveal any construction marks that appeared to be related to design.

studies.[96] Only very occasionally do we see drawings of whole compositions and then, in almost all cases, they are free sketches where no system was employed. In the case of some unknown number of Renaissance works there may have been no need to use a system at any point in the process. Consequently, a very high percentage of the drawings that I examined, numbering in the thousands, had no draughting tool marks and, as a result, were not considered as candidates.

In the case of architectural drawings, there were different kinds to consider and only a few could be candidates for analysis. Some specimens were presentation drawings. With these, it was apparent that the design process had occurred elsewhere and the finished design transferred to another sheet to make an idealized copy. Without the construction marks it would be impossible to recreate the steps of the design process in such presentation drawings. Some specimens were measured drawings of actual buildings, such as Roman ruins. These drawings contained interesting construction marks, but since the buildings depicted were already designed, and the draughtsman was simply illustrating them, then the marks revealed only the process of drawing.

Architectural drawings that revealed evidence of changes, such as one would expect when a draughtsman was engaged in a design process, were the most promising as potential candidates. Erasures would obviously indicate a change of mind but, more commonly, un-inked incised lines would stand out as evidence that a design process had occurred. This is because the design process for architects was typically to incise lines into the paper first, then ink them afterwards. Incisions were made with a stylus or the sharpened foot of a compass. When a change of mind occurred during the incising stage, alternate forms could be drawn and considered before a commitment was made with ink.[97]

The process of examining a prospective candidate for analysis involved spending as much time as possible examining, measuring and recording every mark. Analyzing the marks involved searching for those that indicated locations

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[96]For a discussion of the survival problem of Renaissance drawings, see Francis Ames-Lewis, *Drawing in Early Renaissance Italy*, (New Haven, 1981), 7-12.

[97]This method of drawing the initial idea in colorless incised lines is not limited to architecture. Leonardo and Raphael both used it in non-architectural contexts: see Ashmoleon drawing collection, Leonardo 14 and Raphael 512.

that were geometrically determined. Since lines are never located arbitrarily when using a geometrical design process, their locations must have been part of a series of geometrical steps. The presence of crossed arcs indicated the beginning of a geometrical process since the positions of subsequent lines were determined by it. Untangling the drawing order of the subsequent lines involved many hypotheses that had to be tested. I would suppose that a particular line or point had to have been drawn next, then test if that line or point were dependent on some other, possibly prior line or point. Sometimes I would draw upon my own experience using constructive geometry as a design process in order to initiate hypotheses. In the case of Leonardo's *Madonna and Child with St. Ann and the Infant John the Baptist* (Chapter 4), the sequence of measurements, beginning with a casual operation of the dividers and gradually increasing in precision, appeared logical to me from my own experience with such problems.

Because of limited time and resources it was not possible to catalogue every instance of draughting tool marks that appeared to be related to design, geometry or perspective. Each drawing chosen for extended analysis and inclusion in this thesis was one that presented clear and sufficient evidence for a problem to be solved.[98] They all presented opportunities to interpret the marks and reveal some use of geometrical or numerical design systems. The case studies of Chapter 8 provide additional examples of how this investigatory method can reveal the use of systematic design.

Drawing Instruments

Because of the importance of tool marks in the analyses of the drawings in Part II, some information on the actual draughting tools is in order. One relevant aspect of period instruments, for our purposes, is their precision. Unfortunately it has not been possible to test the precision with the actual tools. We can only surmise their function and limitations from the marks that they left.

The most common type of drawing instruments still in use today, such as the compass, rule and marking devices, have been in existence since Antiquity. Unfortunately, few specimens, if any, before the Roman period have come down to us. A set of bronze drawing instruments from Pompeii (AD 79) has survived,

[98]In the case of the drawings by Andrea Palladio, it was the entire group of examined drawings that presented the problem to be solved.

that includes many familiar tools: a set square, compasses, proportional dividers, and calipers.[99] These appear to be relatively crude but it does not require a great deal of technology to make a compass work. In the Middle Ages, we know from the tool marks that the compasses could describe accurate circles as small as .75 mm.[100]

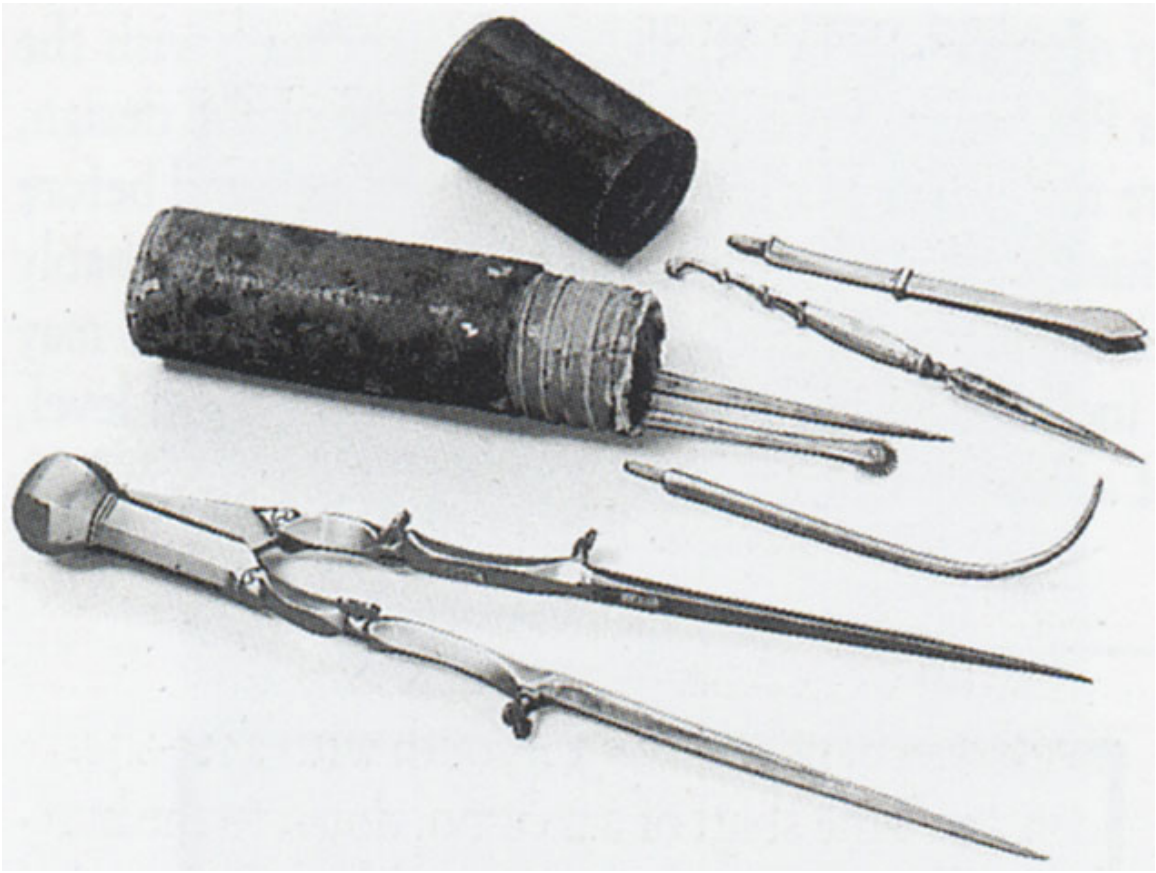


FIGURE 8 DIVIDERS WITH ACCESSORIES, ITALIAN, 16TH

Some tools survive from the Renaissance. I examined one period set of instruments at the exhibition, *The Renaissance from Brunelleschi to Michelangelo, The Representation of Architecture* in Venice. The catalogue entry says:

> The drafting compass exhibited here is made of two brass arms, hinged on a slightly polygonal, spherical head. The arms are retractable and can be fixed in the required position by two steel screws. At the ends of the arms themselves two steel extension points are held in place by steel screws. The dividers (disassembled) are housed in a cylindrical, black cardboard case, which also holds the various standard accessories for compasses of this type: a drawing pen, a pencil attachment, a toothed wheel, a knife,

[99]Hambly, Maya, *Drawing Instruments 1580 – 1980*, (Sotheby's Publications, 1988), fig. 10.
[100]Bruce-Mitford, Rubert, *Evangelia Quattuor Codex Lindisfarnensis*, facsimile ed. Olten and Lausanne, (3 vols. 1956, 1960) vol. II, 222, 226.

and two curved points, all made of steel with brass finish. This compass is said to be that of Michelangelo Buonarroti since it was found among the objects belonging to the great artist (fig. 8).[101]

The drawing pen is the attachable instrument that looks like a forceps. When dipped in ink, a small portion of the fluid is held between the blades by surface tension. A metal ring slides along the shaft to adjust the spacing of the blades and, consequently, the thickness of the drawn line. The instrument immediately below the drawing pen is a stylus.

For comparison purposes I illustrate a second set of instruments in a more formal presentation case, dated in the early 16th Century (fig. 9).[102] It includes a greater variety of devices, including proportional dividers, a pair of compasses with a crayon holder on one leg, a pair of compasses with a knifelike point for scoring and a casket containing ink-wells and a pounce box. This set displays a high level of technological sophistication for the period.

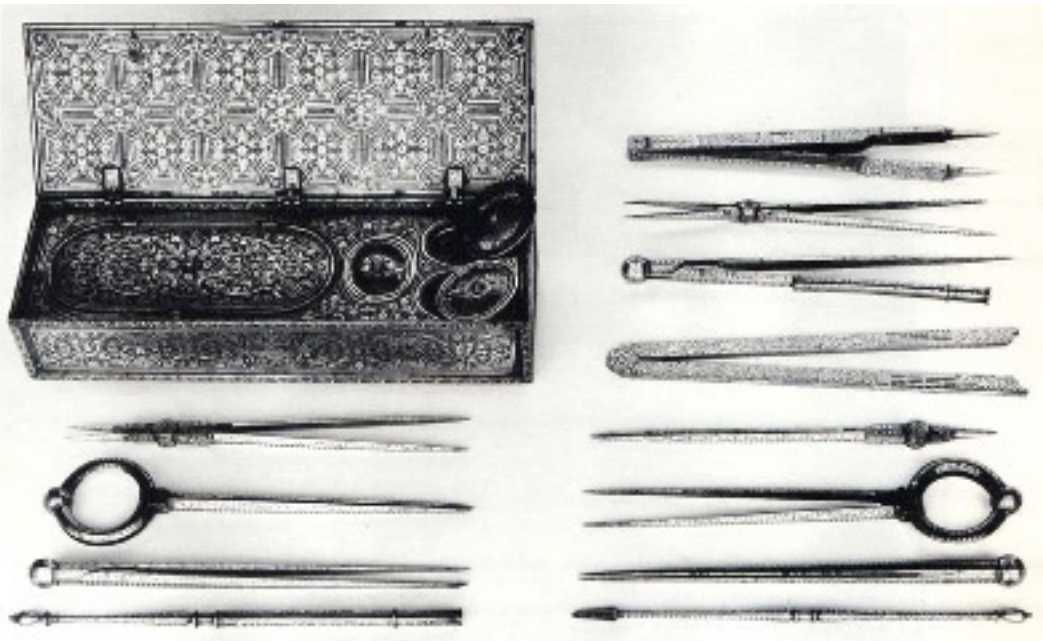


FIGURE 9 INSTRUMENT CASE, MILANESE, EARLY 16TH CENTURY

[101] *The Renaissance from Brunelleschi to Michelangelo*, Henry A. Millon (ed.) and Vittorio Magnago Lampugnani (ed.), (London, 1994), entry 85, 478.
[102] Hambly, *Drawing Instruments*, 157.

Summary of Part I

The tradition of numerical design systems consists of modules, standard units that are organized, fractionated and duplicated according to numerical relationships of simple ratios. The geometrical tradition of design consists of creating an armature of structure out of lines and arcs, drawn with straight edge and compass, to create unity by constructing interlocking geometrical forms.

I argue that there is a consistent theme of geometrical and numerical systems, in the context of design, from ancient times through to the Renaissance. To support this argument, I give examples of implementations of these systems. These examples include Egyptian grids for designing relief murals, Greek geometrical designs for architecture, the gridded plan of St. Gall, constructive geometry in manuscript illumination, and the design *ad triangulum* of Milan Cathedral. Also in support of the argument are the philosophical backgrounds for the systems. Plato and Pythagoras created a mystical/cosmological world view that gave significance to numbers, especially numerical relationships. Vitruvius described his theory of the design process which used numbers for ornamental design and constructive geometry for structural design. Medieval Christian scholars combined the Greek numbers with biblical concepts to create a cosmology that centered on numbers and geometry.

The treatise writers of the Renaissance adopted these traditions, along with the newly rediscovered treatise by Vitruvius, and adapted them to their own situations. The process of trial, judgment and modification, repeated changes involving attempts and responses to those attempts, is expressed by some of the Renaissance writers. The crossed arcs method of drawing a perpendicular lines also occurs in most of the treatises. There is possibly a ritual aspect to its use. Francesco di Giorgio expressed, most explicitly, the design process involving the systems of geometry and numbers. In Francesco's example the large forms of a church façade are determined by constructive geometry and within that geometry is the module that informs the rest of the structure. The interaction of these two systems, geometry and numbers, is the consistent theme that is revealed in the following analyses.

Part II
Analyses of Renaissance Drawings

Chapter 4
Leonardo's Design Process

The two drawings analyzed in this chapter, Leonardo da Vinci's *Perspective Study for the Adoration of the Magi,* (1481) and *The Madonna and Child with St. Ann and the Infant John the Baptist*, (c. 1508), are each part of a sequence of drawings that record part of the process that Leonardo (1452-1519) employed in arriving at compositions. Unfortunately, neither sequence is complete; certainly, there were many other drawings in the sequences that are now lost. Yet, some aspects of Leonardo's design process can be learned by comparing these drawings with others from their respective sequences and considering the evolutionary steps of the processes.[103] But there is additional evidence hidden in the drawings themselves that, when revealed, can shed light on another aspect of Leonardo's design process, i.e. the sequence of steps used within each drawing. The evidence from draughting tool marks in these two drawings allows the sequences to be reconstructed with a great deal of precision.

Many drawings by Leonardo contain draughting tool marks. It is

[103]Ames-Lewis, *Drawing*, 162-165.

common to see compass marks and ruled lines in areas where he was working out geometrical figures, for example, or certain aspects of the theory of perspective. The marks in the two drawings examined here are relatively unusual because they are directly involved in different aspects of the design process.

The quote from Leonardo mentioned in Chapter 2, "Sketch subject pictures quickly and do not give the limbs too much finish: indicate their position, which you can then work out at your leisure," suggests that for

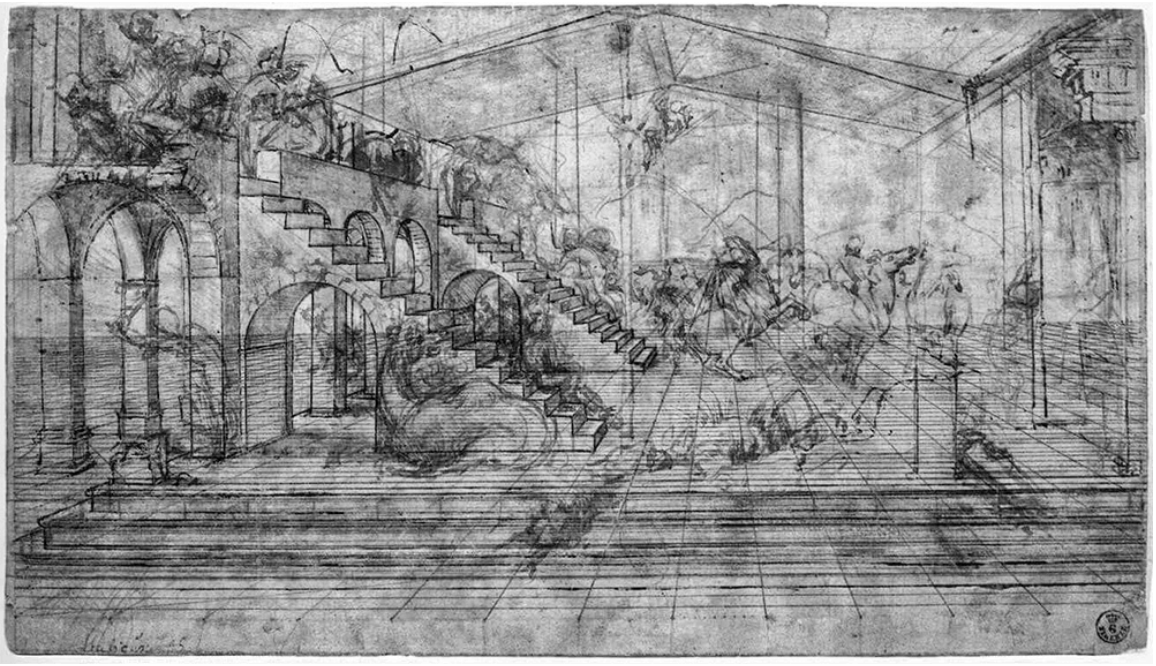


FIGURE 10 *PERSPECTIVE STUDY FOR THE ADORATION OF THE MAGI*

Leonardo there were at least two approaches to the process of arriving at a design.[104] One approach is the quick sketch. Leonardo's opus of extant drawings abounds with examples of sketches of this type. The other approach is that part of the process that is done at one's leisure, when there is time to contemplate and deliberate over the many options. Two of Leonardo's drawings are analyzed here that illustrate these two different approaches. The sequences of approaches he undertook within the drawings are reciprocal to one another. For the first drawing, *Perspective Study for the Adoration of the Magi*, Leonardo began the process slowly and mechanically with calculated steps, then gradually began to

[104]"Il bozzar delle storie sia pronto, e 'l membrificare no' sia troppo finito, sta contento solamente a' siti d' esse membra, i quali poi a' bel' aggio piacendoti potrai finire," Leonardo, *Treatise on Painting, Codex Urbinas Latinus, 1270*, fol. 34r, Leonardo, *Treatise*, (ed. A. P. McMahon).

introduce informalities and finally ended the process with the quick sketch approach (fig. 10). With each step he moved in the direction of a technique characterized by complete freedom and spontaneity. In the later drawing, *The Madonna and Child with St. Ann and the Infant John the Baptist,* Leonardo reversed this pattern (fig. 15). He began the drawing with completely free marks inspired only by intuition and the desire to "attend to the movements appropriate to the mental state of the creatures that make up the picture," and ended the process with precise calculated steps.[105] The steps of trial and modification are here analyzed to allow this interpretation of what Leonardo's judgments were regarding these drawings.

Perspective Study for the Adoration

This drawing is in pen and ink and has some metalpoint lines underneath the ink. It is an unfinished compositional study. Leonardo worked on it as a step in the process of composing the panel painting *The Adoration of the Magi* that was also never finished. The drawing is a horizontal rectangle; the original compositional boundary, referred to here as the drawing's format, is twice as long as it is high.[106] The image relates, in many respects, to the upper half of the panel painting which is nearly square. The drawing depicts an exterior scene occurring on a gridded ground plane rendered meticulously in one-point perspective. Figures of men and animals move about on the plane and on stairs leading to the top of a ruined loggia which recedes from the left side of the composition to the vanishing point in the center right. There are also some loose indications of rocks and plants in the foreground, mountains in the background and a wide roof, held up by some narrow supports, covering the middle ground.

The execution of the drawing required a mixture of two different technical approaches. First, there is the process of rendering a perspective construction

[105]"attendi prima alli movimenti apropriati alli accidenti mentali de li animali componitori della storia," fol. 62r, Leonardo, *Treatise*, (ed. A. P. McMahon).

[106]Uffizi 436E. The paper is 165 x 290 mm. There is a blank margin at the bottom of the page below the lower edge of the drawing's format. The drawing spills over the upper edge of the format which is not clearly visible in the illustration, and extends to the upper edge of the paper. The specific locations of the drawing's format are discussed below.

with mechanical precision. The lines of the grid are ruled and inked, the orthogonals (lines perpendicular to the picture plane, going away from the viewer) are measured and recede regularly to a vanishing point. The narrowly spaced transversals (lines parallel to the picture plane), march back into space creating a smooth stage floor upon which the drama unfolds. The architecture is composed of geometric solids that are hard-edged and obediently follow the checkerboard grid of the floor. The figures, however, were created by means of a completely different technique. Drawn with fluid pen strokes, they appear to move in and around the architectural elements like spirits moving too fast to be captured by the artist. They are drawn so loose as to become turbulent clouds of creative energy. The unresolved character of these strokes contrasts sharply with the cold regularity of the ruled geometry. One can think of these two drawing approaches as two different states of mind that Leonardo underwent during the design process. The first can be analyzed because it is the result of a geometrical system, one-point perspective, and Leonardo's draughting tools left behind sufficient marks for a reconstruction of how he used this system. The second quick and intuitive process cannot be reconstructed because it resulted from the magic of Leonardo's unique creativity.

In order to decipher the procedures for the perspective construction, it was necessary to look through all the subsequent layers of drawing and recover the skeletal framework of the construction. In some places the important locations had been obscured but could be surmised by extrapolating from the marks that were visible. Investigating this drawing required two separate methods of analysis. First, there was a thorough examination of the original where measurements were carefully obtained and all of the draughting tool marks were recorded. The second method of analysis was performed on a large color reproduction. This reproduction was enlarged 165% of the original and was uncropped. I laid a sheet of clear acetate over the reproduction and etched lines with a straightedge and sharp stylus corresponding to the lines and points in the drawing. Because the etched lines were as thin, or thinner, as those in the reproduction, a very precise reconstruction could be obtained for those lines or points that had become obscured by overdrawing.

Interpreting the tool marks for the initial phase of the perspective grid is best considered in the context of Alberti's advice for such constructions.

Alberti's treatise, *On Painting,*[107] gives step-by-step instructions for drawing a perspective construction very similar to the one in the present drawing. Leonardo's theory of art was much influenced by Alberti's treatise and he followed Alberti in some aspects of this perspective grid.[108] However, he modified the system in areas that were more suited to this situation.

Alberti's first step in making a perspective construction is to draw a rectangle, of any size, that he regards as an open window through which the scene is viewed. This, too, was Leonardo's starting point. We know this because all the subsequent geometrical construction marks are dependent on the positions of the sides of this rectangle.

The first problem to solve in the investigation of this drawing was the location of the four corners of the drawing's original format. Only one of the corners, the lower left, remains clearly visible. The other three corners are obscured by overdrawing or faded lines. Although there were difficulties reconstructing these three corners of the original rectangle, sufficient evidence from lines that are clearly visible made it possible to located them accurately. Listed below are the details of the evidence concerning the borders and corners of the original rectangle.

- The base line is incised and uninked; it is, none the less, visible for most of its length and its position is reinforced because many of the orthogonals (drawn later in the construction, see below) terminate on it. Only at the extreme right are the details of the base line lost.
- The left side of the rectangle is drawn in metalpoint very near the left edge of the paper. The upper end of this line is lost under the confusion of subsequent marks. Fortunately, the left border is at a true right angle to the base line and extends to the bottom of the paper leaving a clear intersection with the base. Thus the lower left corner is securely located.
- The right border is confused by three vertical lines very near the edge of the paper. The line closest to the edge is lost in places because of the irregularities of the edge. Paper damage also obscures the intersections of these lines with the base line. In order to locate the lower right corner it is necessary to follow the orthogonal that leads part of the way to that corner. Extending this orthogonal

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[107]Alberti, Leon Battista, *De pictura*, 1435, Bk. I, 19, first published (Basel, 1540), Latin, (Venice, 1547), Italian, (ed. *On Painting*, Penguin Classics, trans. by Cecil Grayson, 1991), 54.
[108]Kemp, Martin, *Leonardo da Vinci, The Marvellous works of Nature and Man*, (London, 1981), 32.

until it intersects with the base line locates the lost intersection. This position can also be confirmed by continuing the regular divisions of the baseline.

• The upper border is the least visible because at some later point in the process of drawing it ceased to contain the composition and became covered. It is a pale metalpoint line most clearly evident in the upper right area between the sloping roof and the entablature. Following the upper line to the left, it appears to fade in and out through the roof area and is last seen just to the upper right of the face of the leaning figure standing at the top of the near stairs. The upper border of the rectangle can be recovered with a straightedge aligned with those parts of that line that are still visible. Once the upper border is recovered it is then possible to locate the positions of the upper left and right corners. The height of the top of the rectangle, where visible, is the same as half the length of the base line. With this information it is possible to reconstruct the positions of the upper left and right corners and to determine that the whole rectangle had a ratio of one to two.

The next step in Alberti's instructions is to decide the height of a man and divide that measure into three equal parts, each corresponding to a *braccio*.[109] Alberti advises dividing the baseline into as many *braccia*, as it will hold. Leonardo divided his baseline into twelve equal parts. The figures in Leonardo's drawing are all drawn much too late in the process to obtain this early measurement. Even after-the-fact it is difficult to determine if the figures are of the proper size because none of them is standing straight enough to obtain an accurate measurement.

Locating the "centric point" or vanishing point is next. Alberti explains that this is the point where the centric ray strikes. It can be anywhere in the painting but most suitably no higher than a man standing on the baseline. This allows the viewers to be on the same plane as the figures in the painting. The figures in the present drawing are from a much later stage and were not used to locate the vanishing point. However, the heads of the men standing on the plane, if they were standing straight, would be on or near the level of the vanishing point.

Leonardo's choice for the location of the vanishing point raises two

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[109]The Florentine *braccio*, is approximately 58 cm. Alberti (ed. *On Painting)*, footnote 13.

questions: What is the rationale behind the decision? How was it carried out? Answering the first question is partly dependent on the second. Answering the second question would be easier if there were a ruled line passing through the vanishing point, either horizontally, vertically or diagonally. But, there are no such lines. In order to plot a specific point within a rectangle one needs either to measure a distance along an existing line or use crossed arcs, measured with a compass, from two other positions. There do not appear to be any crossed arcs on the vanishing point. Since it is unlikely that the location of such an important feature would have been chosen at random, and there are no marks to indicate how it was chosen, then the position of the vanishing point presents a puzzle.[110] There are some sketchy lines near the vanishing point to indicate a horizon but there does not appear to be any ruled lines extending to either edge.[111] The vanishing point is also not directly over any of the divisions of the base line. As a consequence, there is no perfectly vertical orthogonal.

One clue to the rationale behind the vanishing point's position is that a continuation of the orthogonal from the lower left corner to the vanishing point intersects with the reconstructed upper right corner. This alignment seems more than coincidental. Also, the distance from the vanishing point to the upper right corner is equal to five units, as counted along the baseline. These criteria, although they are not in themselves a reason, seem to indicate that deliberate choices were made. How these choices were made is not clear because the hypothetical diagonal from the vanishing point to the upper right corner was not drawn or incised. The orthogonal, too, since it depends on the vanishing point for its angle must have been preceded by the vanishing point. This means that the position of the vanishing point was chosen before there were lines within the rectangle to locate it. The only way, then, to locate the vanishing point would have been to lay a straightedge down, from corner to corner, and mark the point from a pre-measured spot on the straightedge, without drawing a line. Although

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[110]Martin Kemp suggests that the location of the vanishing point is close to the golden section between the left and right margins. (Kemp, *Leonardo)*, 73. It is possible to locate the golden section at this stage in the drawing with a simple geometrical construction. There are no tool marks to support this theory, however. Also, at this stage in the drawing Leonardo was being very precise. The vanishing point would determine the positions of most of the lines that follow. If Leonardo had tried to put the vanishing point at the golden section by some means that left no trace, then he missed it by almost a full percentage point.

[111]The ruled line visible between the arches on the left side of the composition is actually an orthogonal positioned below the horizon.

not entirely natural, this is not altogether implausible since the evidence suggests that, in the initial steps of the construction, Leonardo began the composition with the greatest care and with an interest in precision. Avoiding extraneous lines might have been a priority. Also, the technique of locating a point with a straightedge was repeated later to draw the orthogonals, described below.

The question of why Leonardo chose to put the vanishing point where he did is one for which there is insufficient evidence to answer. It is interesting to note, however, that the point was not chosen at random and that its position is justified both geometrically and numerically. Geometrically, because it lies on the diagonal of the whole rectangle and numerically, because its placement on that diagonal is derived from a number of modules.

Once the vanishing point was located, lines could be drawn to each of the

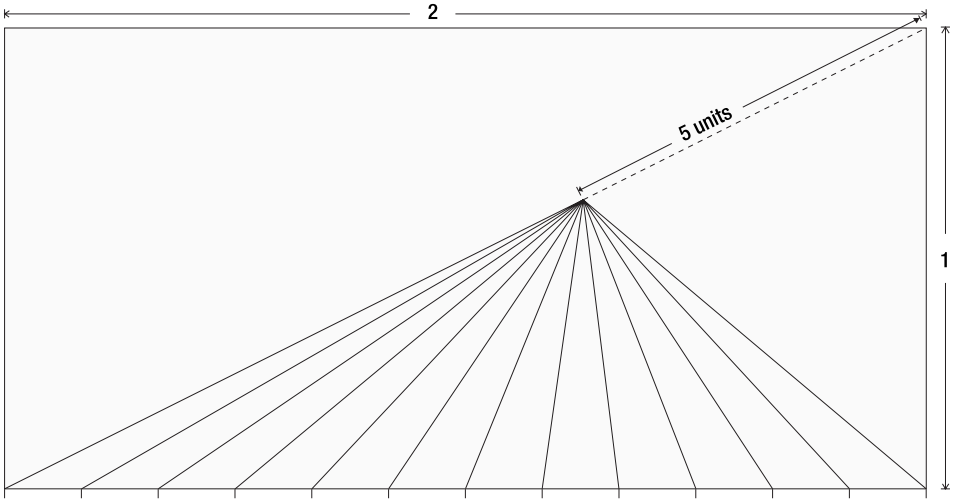


FIGURE 11 1ST STAGE

divisions of the base line. Figure 11 illustrates the initial state of the composition with the first thirteen orthogonals. There is, in this illustration, a reconstructed diagonal from the vanishing point to the upper right corner.[112] Although the orthogonals converge on what is now a mass of ink, precise reconstruction of each line shows that Leonardo took care that they converged exactly on the point. Later in the course of constructing the perspectival framework, some of the lines miss the vanishing point. This carelessness suggests that the artist was growing impatient with the tedious, repetitive task.

[112]The portion of this diagonal above the vanishing point is for the purpose of illustrating the continuation of the orthogonal. It was not drawn or incised on the original.

Departing from Alberti's method at this stage in the procedure, Leonardo set out to draw more orthogonals to the left and right of those already drawn. These additional lines would provide a wider surface for the pavement. He first drew a horizontal line through the middle of the rectangle that was to become a distant limit to the recession of the pavement. The remaining orthogonals all stop on this line. One reason for limiting the pavement is to avoid having too many lines converge on the vanishing point. The growing network of radiating lines not only causes physical stress on the paper but their number and closeness would limit Leonardo's ability to draw over them after the construction was completed. This horizontal line is not the horizon line that would technically be on a level with the vanishing point. The horizon line, in fact, was never drawn.

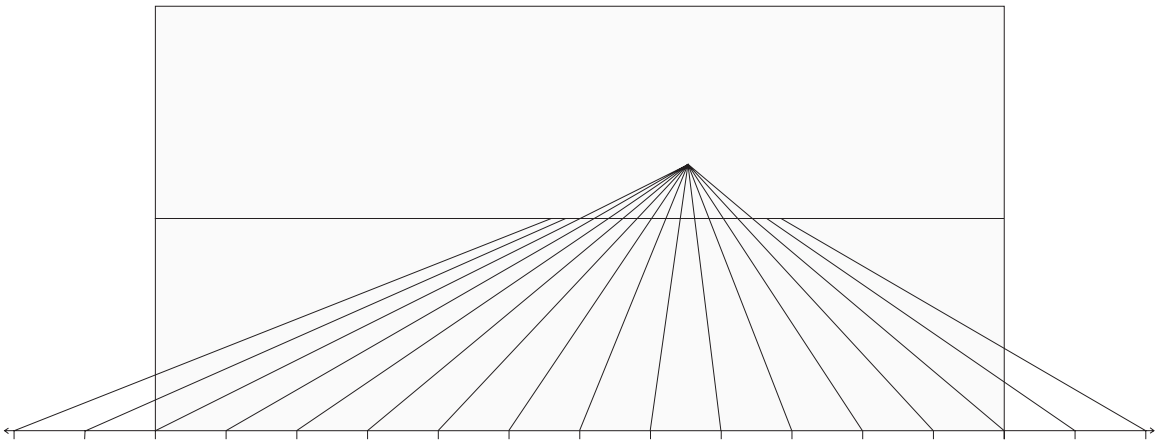


FIGURE 12 2ND STAGE

The position of this horizontal line is not quite level. The intersection with the left side of the format is near the exact center, while on the right side it intersects a little above the center. This was probably the result of a careless error. The slant is not visible and Leonardo did not notice it at first. Later in the process, when he did notice it, he strove to hide the error.

In order to plot the base positions of the orthogonals to the right and left of the baseline, it was necessary to extend the baseline, outside the paper on either side. These extensions could then be divided into units equal to those already established (figure 12). Lines drawn from each of these new units to the vanishing point filled out another twelve or so orthogonals on each side.[113] For

[113]Only the first two of the new orthogonals, on either side, are illustrated in figure 12. Although the lines extend in the direction of the vanishing point they stop at the horizontal line.

the remaining orthogonals, it appears from the irregularities of the spacings that Leonardo guessed at the positions of the lines. In order to have plotted each of these remaining orthogonals, he would have had to extend the base line to an unmanageable distance on either side.[114] The added effort of laying down a straightedge, nearly a meter in length, to each of nearly twenty additional base line units would have gained him very little in the way of illusory effect. The finished construction appears to be as accurate as one made by a machine.

Although Leonardo seems to have begun the perspective grid with the intention of being as accurate as possible, at the stage in the process where he decides to guess at the positions of the orthogonals, he changes his approach. From this point in his procedure, he settles for merely giving the impression of accuracy.

There are marks near the left edge of the format that require an explanation. They form an irregular column of indentations a few millimeters from the left border. Each indentation has one of eleven orthogonals passing through it. These orthogonals are the ones that required an extension of the base line on the left side of the drawing. The indentations were used to plot the positions of the eleven orthogonals. It appears that Leonardo laid a straightedge down from the vanishing point to each of the extended baseline units and pricked the paper near the edge of the rectangle to mark the position of each line. He then presumably traced the orthogonals from these indentations in a second step. His procedure greatly increased his work, because the time-consuming part of perspective constructions is the repeated aligning of the straightedge to points. Why not draw the line when the straightedge was already in place the first time? The reason for the added step could have been his tools. Perhaps his longer straightedge, which would have been used for these orthogonals as well as the main diagonal, was not good for incising lines, only plotting them. This method of laying down a straightedge, used to plot the orthogonals, gives weight to the theory expressed above regarding the plotting of the vanishing point. There, the vanishing point was located with the straightedge laid down corner to corner. This procedure is consistent with Leonardo's initial approach to the drawing

[114]The distance would be about two and a half times the width of the drawing on the left side.

where he took great care to maintain accuracy and avoid extraneous marks. He did not repeat this step on the right side of the drawing.

With the orthogonals in place, Leonardo next turned his attention to drawing the transversals, the lines parallel to the baseline. The transversals represent lines that, when seen from above, would be equidistant. The angle of the ground plane to the viewer, however, causes the transversals to become progressively closer together the further away they are from the viewer. Plotting this diminution of spacing is a matter of some controversy according to Alberti's instructions. Apparently, there were those who used a method that Alberti considered to be wrong. This method is to place each successive line, back into space, in such a way that the ratio between the more distant space to the one immediately prior is always two units to three. Alberti did not consider this numerical system to be a sufficiently accurate approximation of the intersection of the visual pyramid with the picture plane. Alberti's preferred method amounts to drawing a side view of the picture plane, along with the viewer's eye and the pavement floor marked with equidistant points representing the distances of each transversal to the viewer's eye. Lines drawn from each of these points to the viewer's eye intersect the picture plane line in a manner that represents the desired diminution of spacing. [115]

Leonardo did not use either of these methods, but devised one of his own.

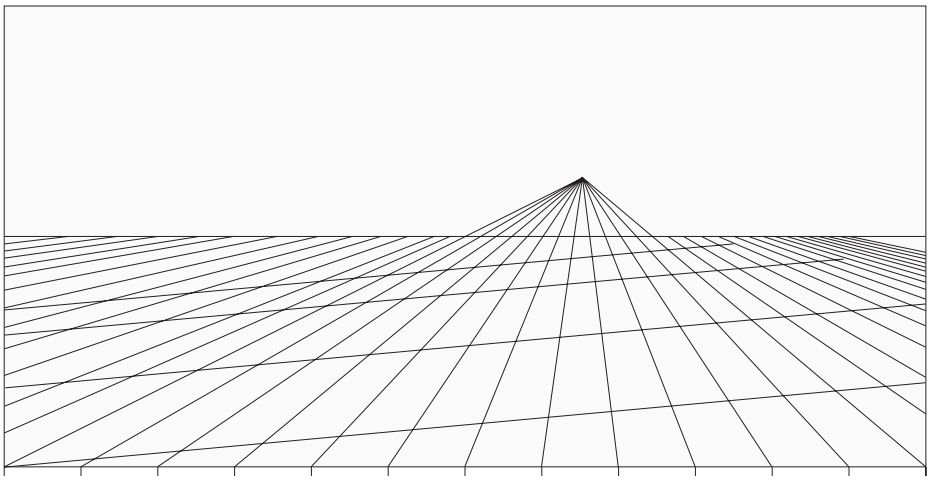


FIGURE 13 DIAGONAL LINES TO PLOT THE

The theory behind Alberti's method was that the transversals and orthogonals

[115]Alberti, *De pictura*, Bk. I, 20, (ed. *On Painting*), 57, fig. 10.

delineated square tiles. Leonardo's new method disposed of square tiles in favor of a large number of transversals delineating long rectangles. Leonardo used a series of four shallow diagonals to determine the spacing of each of the horizontal lines(fig. 13). The faintly drawn diagonals are parallel to each other but their angle to the base line appears to be arbitrary. The lowest line begins at the lower left corner and intersects the right side at approximately the same distance from the lower right corner as one baseline unit. The shallowness of the angle was probably chosen so as to result in a large number of closely spaced transversals.

At each intersection of these diagonals and orthogonals, an indentation marked the location for a transversal. The shallowness of the diagonals resulted in the very short intervals between the transversals. The plotting of these lines, however, turned out to be a source of frustration. He followed this procedure of diminution about halfway to the end of the pavement then settled for intervals that were approximately uniform in width. There are several errors in the drawing of the transversals that, perhaps, indicate Leonardo's impatience with the task. At about three quarters of the way up the plane, working from bottom to top, he must have realized that the horizontal he was approaching with his transversals was not parallel to the lines he was drawing. To hide this error, he gradually diminished the intervals on the left side. His success is measured by the fact that the whole perspective scheme appears to be perfectly accurate.



FIGURE 14 PARIS *ANNUNCIATION* (1481)

Francesco di Giorgio's method for locating the transversals shares one common characteristics with Leonardo's. It begins with a similar diagonal line at the lower left corner, continuing out of the picture a distance to the right. But Francesco's diagonal quickly intersects with the horizon line and is consequently much steeper. The resulting transversals would be spaced wider and would more accurately describe square tiles.[116] Although Leonardo's method for locating the transversals is closer to Francesco's than it is to Alberti's, it does not appear to be an orthodox scheme.

With the floor in place, Leonardo could then begin constructing the architecture. He must have had the basic layout in mind since the arrangement of elements matches closely those in an earlier sketch (in reverse) of the same subject (fig. 14, Paris, Louvre): a pair of stairs abutting a wall pierced with arched openings, columns supporting a ruined entablature, and a widely pitched roof bridging these two elements. If the Paris drawing represents the "first thoughts" for the background Roman ruins, then the present drawing is the solidification and regularization of those forms. The overlapping of sketched forms in the Paris drawing provides clues to the order of those first thoughts. With the foreground figures in place he appears to have first conceived the architecture as a rectangular frame above the Virgin. The square corner column on the left, the primitive post on the right and the lintel under the gable are the three sides of this frame. The stairs and loggia, with moving figures, were then sketched in and around the frame without any perspectival guidelines.

In the Uffizi drawing, however, the stairs and loggia were constructed first with the roof and columnar elements added afterwards. The desire to maintain precision is apparent in the overall construction lines, although the inking of details, such as the steps, was executed with less care. The widths of the two stairs were determined by counting four transversals back into space, each at the base. This was done without taking into account the uncontrolled intervals of the transversals and resulted in awkward foreshortening. There is an indication of shading with a subtle ink wash on the stairs and loggia that suggests a light source from the left and gives the architecture some solidity on the wire mesh of the floor.

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[116]Francesco di Giorgio, *Trattati*, folio 33, tav. 61.

Leonardo follows Alberti in the initial construction of architectural solids.[117] It is a convenience of such wire mesh constructions that as long as the forms are rectilinear and are oriented on the same axes as both the picture plane and its perpendicular, then the lines on the floor become the plans for the buildings. The draughtsman needs only count the number of tiles he wishes the architectural forms to cover. Leonardo reinforced the lines of the grid with wider, darker lines in the areas where they were to describe the outlines of the architectural bases. The upward extrusions of these plans were easily constructed because the outlines are either horizontal, vertical or are projected to the vanishing point.

The arches under the two sets of stairs were drawn with a compass because they are parallel to the picture plane and consequently undistorted. The arches that recede into space, however, were drawn freehand. Although Alberti's method contains instructions for foreshortening such arches, it is a tedious process. Leonardo was apparently finished with the accurate approach that he had demonstrated at the beginning of the drawing and was now well on his way to unburdening himself of technical rules. The two relatively unsuccessful arches visible on the wall between the stairs are testimony to Leonardo's impatience with the system. After the stairs and loggia of the left side of the composition were constructed with ever increasing haste, there is a marked turn from systematic drawing. The pitched roof area apparently offered some trouble and extended above the carefully drawn format.

At some point after the relatively careful construction of this architecture, the technique of careful measuring and ruling gave way to improvisation. This may have been occasioned by Leonardo's inability to fit the roof into his one-by-two rectangular format. It is easy to see how fascination with the tool of perspective can carry one through the tedious process. But with each error, that fascination slips away. The careful planning of all the elements may have been more than Leonardo's patience would have allowed. With the roof breaking the upper border and thus breaking the order he had established, the technique shifted from perspectival rule-following to trial-judgment-modification. The roof was tried with different pitches, widths and sizes. The recession lines on the right side of the roof show that Leonardo had lost the patience to place the

[117]Alberti, *De pictura*, Bk. 2, 33-34, (ed. *On Painting)*, 67-71.

straightedge on the vanishing point. Some of the architectural lines are drawn without a straightedge. But, without the constraints of the geometric system and its tools, a landscape burst to life. The quill pen moved freely and fluidly over the mechanical perspective grid without concern for the hours of



FIGURE 15 *ADORATION OF THE MAGI*, UFFIZI

preparation.

To summarize, the design process for the Uffizi *Adoration* drawing followed a path that began with many self-imposed constraints. Alberti's one-point perspective is a system of constraints, and once certain choices are made there is only one outcome. Leonardo followed the system, as one follows a road, until the time came to turn off in a new direction. The constraints were dropped, one by one, first, because of the tedious nature of the task and second, because once error creeps in and the pristine quality is gone, there is no reason to

continue being precise. Then, after the foundations were sufficient, he began to lead the system rather than follow it. The priorities of the geometric system gave way to the urge to experiment and respond.

The Adoration *Sequence*

The perspectival scheme within the *Perspective Study for the Adoration* helps to illuminate another aspect of Leonardo's design process. The change of point-of-view within the images in the *Adoration* sequence reveals a change in compositional approach. In the *Adoration* sequence, Leonardo's perspective choices are the result of a change of intention regarding the illusion of space versus surface composition from preliminary drawings to painting. Using a perspective system typically involves making a few initial choices and letting the system do the rest. That is why the initial choices are so important. For Leonardo, the initial choices for the points-of-view changed from image to image revealing different intensions regarding the viewer's interpretation of space.

Leonardo has also affected the spatial cues in the last stage, also in order to diminish the illusion of space. Spatial cues are the parts of the image that allow the viewer to interpret the patterns of the image as representing three-dimensional objects in space. For example, when one image overlays another, the upper image is interpreted as being in front of the other in space. The pattern of light and dark is also a spatial cue that helps the viewer interpret colors of paint as objects lit by a light source in a fictive space. The whole system of linear perspective is a means of providing consistent spatial cues that can allow the viewer to interpret the whole scene as being viewed from the same vantage point.

The sequence I am considering includes two drawings and a painting: the Paris drawing (fig. 14), the Uffizi drawing 436E (fig. 10) and the unfinished painting *The Adoration of the Magi* in the Uffizi (fig. 15). The architectural elements in the two drawings contain most of the same features, only reversed left to right. The painting has most of the elements as the Uffizi drawing, on the left side, but is missing those of the right side and the roof in the center. The comparison presented here concerns the depiction of the loggia and stairs in each image and the point-of-view that determines how each of these architectural images relates to its fictive space and the surface composition.

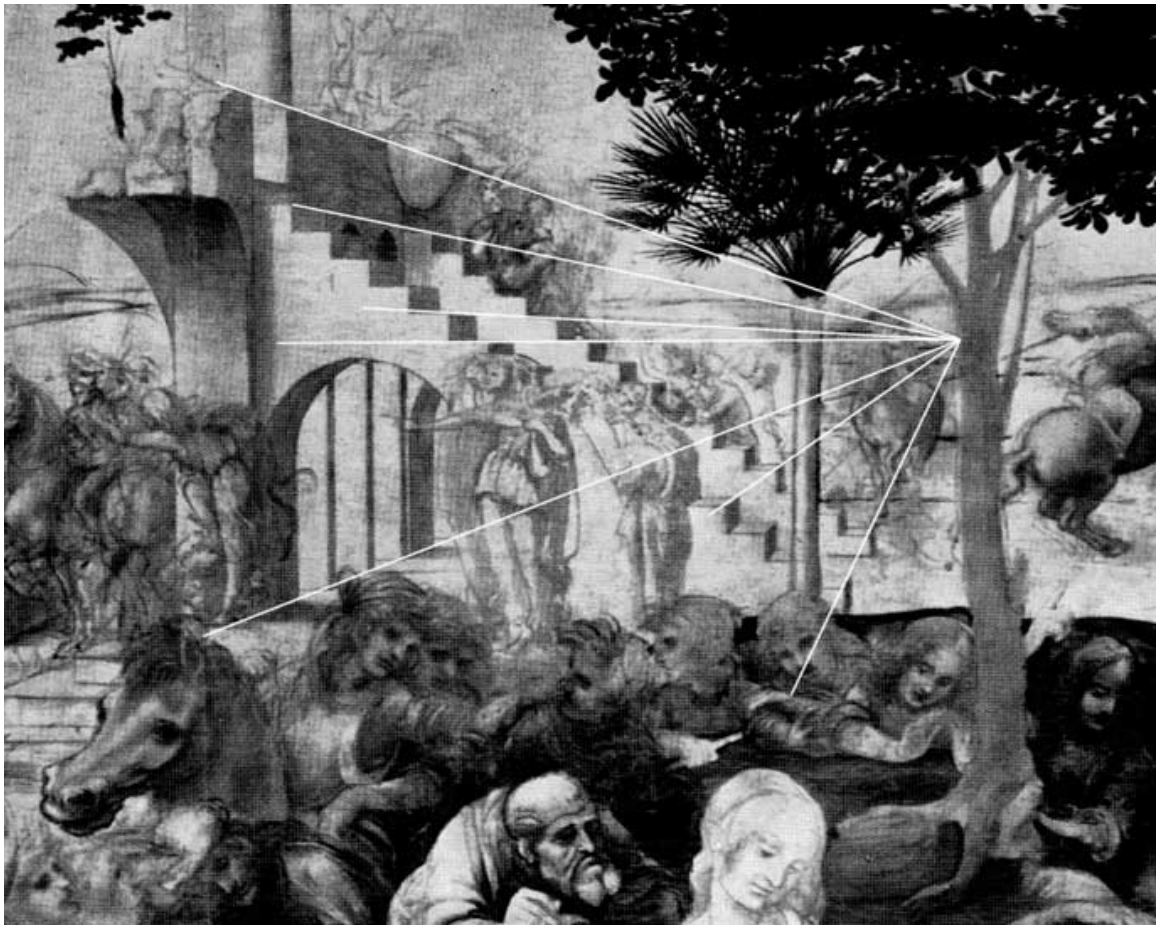


FIGURE 15A *ADORATION* PAINTING (DETAIL) WITH IMPLIED ORTHOGONALS

The Paris drawing appears to be very early if not the first compositional idea for the subject. The architecture seems to have developed without a plan. It is rendered completely in the intuition mode; the perspective of the architecture is created without a system. There are no measurements or ruled lines. The characteristic of the perspective, to contrast with the other works in the sequence, is the point-of-view. The viewer's eye is positioned well above the architecture looking down on the top of the roof and loggia. This point-of-view is a little disjointed compared to that of the foreground figures, who appear more on the viewer's eye level. The character of the drawing styles between the figural group and the background architecture are also different, giving the impression that the two areas represented different problems to solve. The foreground figures are like a frieze; all are on the same plane, parallel to the picture plane. The architecture, however, is drawn to give maximum spatial impact. We see everything laid out clearly, as if the intention for this point-of-view were to show the viewer how much space the architecture was taking up and how of a much of

a gulf of space the distant figures had in which to move around.

The next work in the sequence is the Uffizi drawing. It was begun, as we have seen, in a more intellectual mode than the previous drawing. The architectural features were positionally reversed and the geometry was thought out and systematized with perspective. The first decision that Leonardo had to make, after the format was drawn, was where to put the vanishing point. The vanishing point would determine the position from which the viewer would see the buildings and pavement. We have seen how Leonardo positioned the vanishing point in relation to the rectangular format, but now consider this decision in relation to the resulting point-of-view. Compared to the Paris drawing, the viewer's eye shifts much further downward and in the direction of the stairs. That is, the stairs become more foreshortened. Foreshortening compresses space and diminishes the illusion of space. The figures still have space to move around in, yet, because of the foreshortening, the space is shallower. The three horses and surrounding figures, along with those figures climbing the far stairs, are all on the same plane. There is also a mound of grassy earth sketched in the foreground to help compress the middle space even more. Even so, without the Paris drawing for comparison the perspective construction is still a rather spacious setting.

The perspective scheme of the painting is different from that of the Uffizi drawing. The transition from the Uffizi drawing to the painting must have involved an intermediary perspective construction, either on another sheet of paper or on the panel itself. The vanishing point has moved, in relation to the stairs and loggia, so as to increase the foreshortening and further compress the space. The viewing distance has also changed so as to compress the space even more, effecting the diminution of the architectural recessions as a telephoto lens would.

The following figures represent the *Adoration* painting with overlays illustrating the perspective schemes of the painting and the Uffizi drawing: Figure 15A is a detail of the loggia and stairs with white lines overlaid to highlight the implied orthogonals leading to the vanishing point. Figure 15B is a detail of the upper area of the painting with an overlay, in black lines, of the



FIGURE 15B *ADORATION* PAINTING (DETAIL) OVERLAY OF PERSPECTIVE STUDY

relevant parts of the linear framework from the Uffizi drawing.[118] Figure 15C combines the two overlays of the previous two figures onto the whole painting, so as to reveal the differences in the architecture that the two vanishing points caused.

The linear representation of the Uffizi drawing does not fit exactly into the upper half of the painting. It is reduced in size (to about 89% of the original[119]) and shifted to the right. I made these adjustments in order to make the overlay match, as closely as possible, the perspective scheme of the painting. This way the differences between the two perspective schemes can become more apparent. I used the length of the facing plane of the near stairs as the initial point of

[118]The perspective construction overlay was created by importing a high resolution scan of the Uffizi drawing into the application Adobe Illustrator as a background. With the computer's draughting tools, I traced the lines in such a way as to achieve a highly accurate representation of the perspective construction. The results were imported into Adobe Photoshop as object art layered above a scan of the painting. With the image editing tools, I manipulated the linear representation of the Uffizi drawing, scaling without distortion, to find the ideal alignment.

[119]This is assuming that the one by two rectangle of the original perspective scheme was to extend from edge to edge of the painting. The Uffizi drawing itself would have to increase about seven and a half times to extend to the size represented in figure 15C.



FIGURE 15C *ADORATION* PAINTING WITH OVERLAYS FROM FIGS. 15A AND B

comparison and adjusted the overlay to match that length. For both images, the steps of the facing plane of the near stairs lined up, indicating that whatever else Leonardo changed in the perspective from drawing to painting, this element was the same.

In figure 15B it is possible to see the degree of change that the movement of the vanishing point and the change in viewing distance caused in the foreshortening of the stairs. The receding faces of the stairs, on the lowest steps, almost disappear. In fact, the steps are as foreshortened as possible while still being readable as steps. The near and far stairs are much closer together, very little of the wall is visible and the arcade is reduced to some vertical lines seen through the archway under the near stairs. In the Paris drawing, the reason for the pair of stairways was to open the gulf of space for the movement of figures.

Leonardo kept the elements of the architecture but changed the point-of-view in the painting so that the stairs lost that gulf of space. As a result, the stairs are now closer to a flat pattern of shapes. The figures have less space in which to move and that compression adds to their dynamism.

All of these changes in the architecture are the result of changes in the vanishing point and viewing distance. Since the effect of these changes is to diminish the power of the spatial illusion, I contend that it was Leonardo's intention to diminish the power of the spatial illusion. The Uffizi perspective drawing and the Paris sketch are ample evidence that Leonardo was capable of creating the illusion of space, both systematically and unsystematically, had that been required.

In addition to the changes in the architecture, Leonardo also effected other spatial cues in an effort to diminish the illusion of space caused by the architecture. Many of the elements of the spatial illusion of the architecture are covered up with figural elements. The palm tree and figures covering the lower part of the stairs affect an important aspect of the illusion. Even with the new foreshortening the lower half of the steps could still be read as objects in space because the near steps aligned with the far steps along receding orthogonals. To locate a receding orthogonal one must match a near step to its mate on the far steps. The palm tree is so positioned as to block these imaginary lines for steps two and three, counting from the bottom. Steps five to nine are blocked by the group of figures. Above that, there is very little recession to worry about because the dark risers against the light forward faces begin to read as a harlequin pattern of diamond shapes. Another example of anti-illusionism is the broken arches on the left side of the loggia. These arches, which are parts of ruined vaulting, appear to align counter to what their illusionistic positions would indicate, to make a partial semicircle that reads as a semicircle on the surface of the composition. Much of the architecture that is still visible around the occluding figures has become flat, geometric patterns. Leonardo has manipulated the organic and geometric forms so as to create as much spatial ambiguity as possible.

Why did Leonardo reject the original perspective scheme that he had gone to such pains to create? Perhaps the scheme of the perspective drawing, in the context of the whole painting, would have caused too much of a distraction,

drawing attention away from the foreground scene. Perhaps Leonardo subscribed to the notion, current among modern designers, that compositional design occurs on the surface and that the strong illusion of space weakens the design. Martin Kemp suggests that, "Those perspectivists who have most assiduously followed the geometrical rules to make linear constructions have also exhibited the greatest awareness of the implications of the surface configuration of projected forms."[120] Although Leonardo might be considered only an honorary member of this group, he does not always follow rules, he does exhibit a hyper awareness of surface design. This is most evident in the painting of the *Last Supper* (Milan, 1495-97). There, the perspective scheme implies a room of much greater depth than it appears. Leonardo controls the spatial cues, not allowing the viewer to see enough of the perspective construction to accurately judge how much space is there. At the same time, the orthogonals that lead into depth, in the ceiling and upper borders of the tapestries, do double duty, pointing across the surface to Christ's face and forming axial diagonals of the rectangle.

Observations of the *Adoration* sequence seem to indicate that Leonardo's design process moved from the illustration of space toward surface composition. What is more, when one considers his two approaches of creation, the quick, intuitive mode and the systematic, intellectual mode, the intellectual decisions are the ones responsible for the space-denying elements.

The Madonna and Child with St. Anne and the Infant John the Baptist

The following analysis is of another Leonardo drawing with much the same design approaches as the *Adoration*, but in reverse. It is on the page numbered 1875-6-12-17 in the British Museum (fig. 16).[121] Among the various drawings on this small page is a labored compositional sketch for the Madonna and Child with St. Anne (?) and the Infant John the Baptist. It is in ink and black chalk with the main forms reinforced by incised lines. This analysis will focus on observations of the marks making up the format of the compositional sketch. Based on this evidence, I will attempt to reconstruct the sequence of drawing procedures that must have occurred in order to create these marks. The se-

[120]Kemp, *Visualizations*, 34.
[121]26.7 x 20.1cm

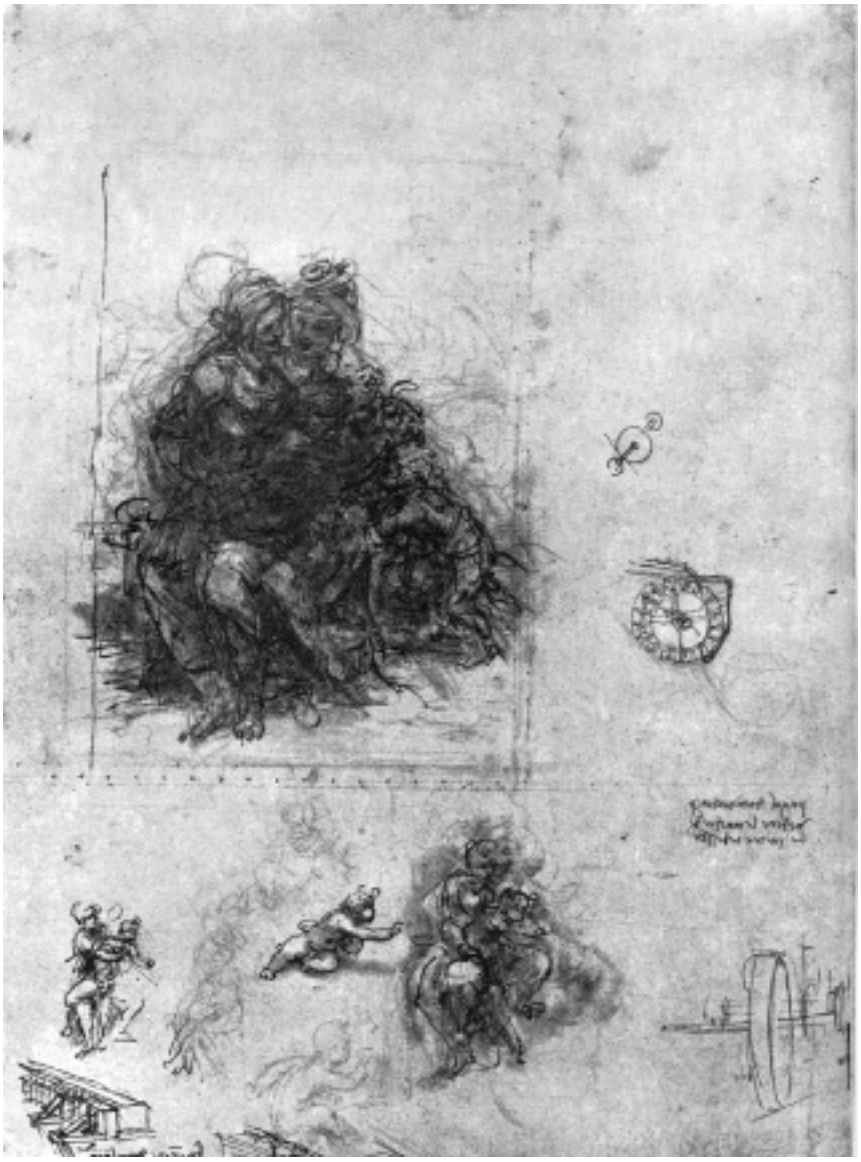


FIGURE 16 PAGE OF STUDIES (BM 1875-6-12-17, C. 1508)

quence of the operations reveal that on this occasion, Leonardo began the work as loosely as possible and then moved toward a more calculated technique.

The composition is one of several smaller studies on the page of similar figures among other miscellaneous sketches. It sits on the page positionally unrelated to the other drawings and without regard for its position on the page. In reverse order to the *Adoration,* Leonardo began the drawing without constraints, completely free to invent and change as intuition led. It is a virtuoso performance of the trial-judgment-modification design process. Beginning with a light touch of black chalk, he created a cloud of lines drawn loosely in circular strokes. The forms evolved on top of each other. When the forms became lost in the haze he bore down harder on the black chalk. When the forms became lost in the black chalk he switched to black ink and raised the highlights with some

white ink. Finally, with the forms almost obliterated by the layers of multiple trials he transferred the image to the verso. He did this by placing the paper over a blackened surface and engraving the lines he wanted to keep with a

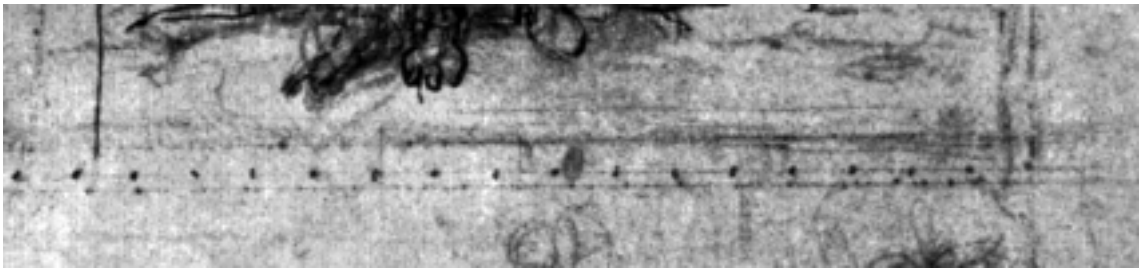


FIGURE 17 DETAIL OF BM 1875-6-12 17

stylus.[122] The rationale behind this design process was not merely to achieve the highest possible number of trials and selecting the best one, but to bring about a situation were the artist could mold the forms on the paper like clay.

Given the enormous variety of Madonna figural groups in Leonardo's sketches and his inexhaustible ability to create ever new combinations, it is interesting that this apparent "first thoughts" sketch is so remarkably similar to the London cartoon. Almost every feature in the cartoon has its precedent in this drawing. It is as if Leonardo's technique of creating layered turbulent clouds of line and tone has resulted in an image with the same perfection as his "clouds and patchy walls." He must have seen something in these ambiguous tones that was exactly what he wanted because when Leonardo was satisfied with the internal elements of the composition he turned his attention to the enclosing frame.

The figural group is enclosed in a rectangular format bordered at the top by a sketchy freehand line and on the other three sides by multiple freehand and ruled lines. There are also evenly spaced "points" (indentations made by dividers) that fall along most of these lines. Some of the points are inked. The multiple lines of the format are a record of various attempts to establish the left, right and lower borders of the composition, while the points record the attempts to establish a numerical ratio of height and width in even units. The subtle changes in the positions of the borders are like that of a photographer cropping an image by a millimeter here and a millimeter there. The effort to bring the

[122]Leonardo did not pursue the composition on the verso. The subsequent measurements with dividers on the recto are visible on the verso but there are no measurements originating on the verso.

rectangle into an even number of units proceeded, as in the drawing it encloses, from a sequence of loose gestural marks in chalk to the more precise incised and inked marks.[123]

The construction of the rectangular format began with a quick sketch to loosely indicate the container of the figural group. The upper border (line 2 of figure 18) is a pale chalk line, drawn with a few loose freehand strokes, which was left unaltered throughout the following process. In the first stage of drawing, the format must have been made of four such lines (with a few alternate lines at the bottom and right side) made when the figural group was in the early state. Some remnants of this early state are still visible in the pentimenti that are not covered by ink in the upper left and right of the figures. The technique of multiple trials was the same for the format as for the figures. One expects that Leonardo kept the borders deliberately vague during the evolution of the figural group in order to leave room for changes.

Among the possible freehand lines that could have served as the lower border, Leonardo chose one (line 2 fig. 18 and detail fig. 17) and incised it with a rule, making it straight in preparation for measurement. After selecting two points as the ends of this line (points A and B in the diagram) he located the centerpoint. His use of trial and error resulted in a cluster of points in the middle area and a few to the left of B. His method was to put one foot of the dividers at A and the other at a guessed-at midpoint. After pressing the foot into the paper to mark this midpoint he swung the other foot around to B only to find he had overshot the end of the line. After some similar guesses he found the center, leaving behind the clusters of points. In the same way, he located the quarter point between A and the center. He then went about dividing the base into seventeen equal units. Since seventeen is not divisible by two or four his center and quarter points were used only as reference. The distance from point A to the first quarter needed to be four-and-one-quarter units of about 6mm each. The cluster of points just to the right of the quarter point record the number of trials it took to arrive at this measure.

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[123]The vertical ink line visible inside the left border and made with jagged strokes, appears to have been made apart from the process described here. It may have been drawn after this process or, if it were made before, it was ignored as a boundary during the subsequent measurements. Its positional relationship to the figures matches closely the left border in the London cartoon, see below.

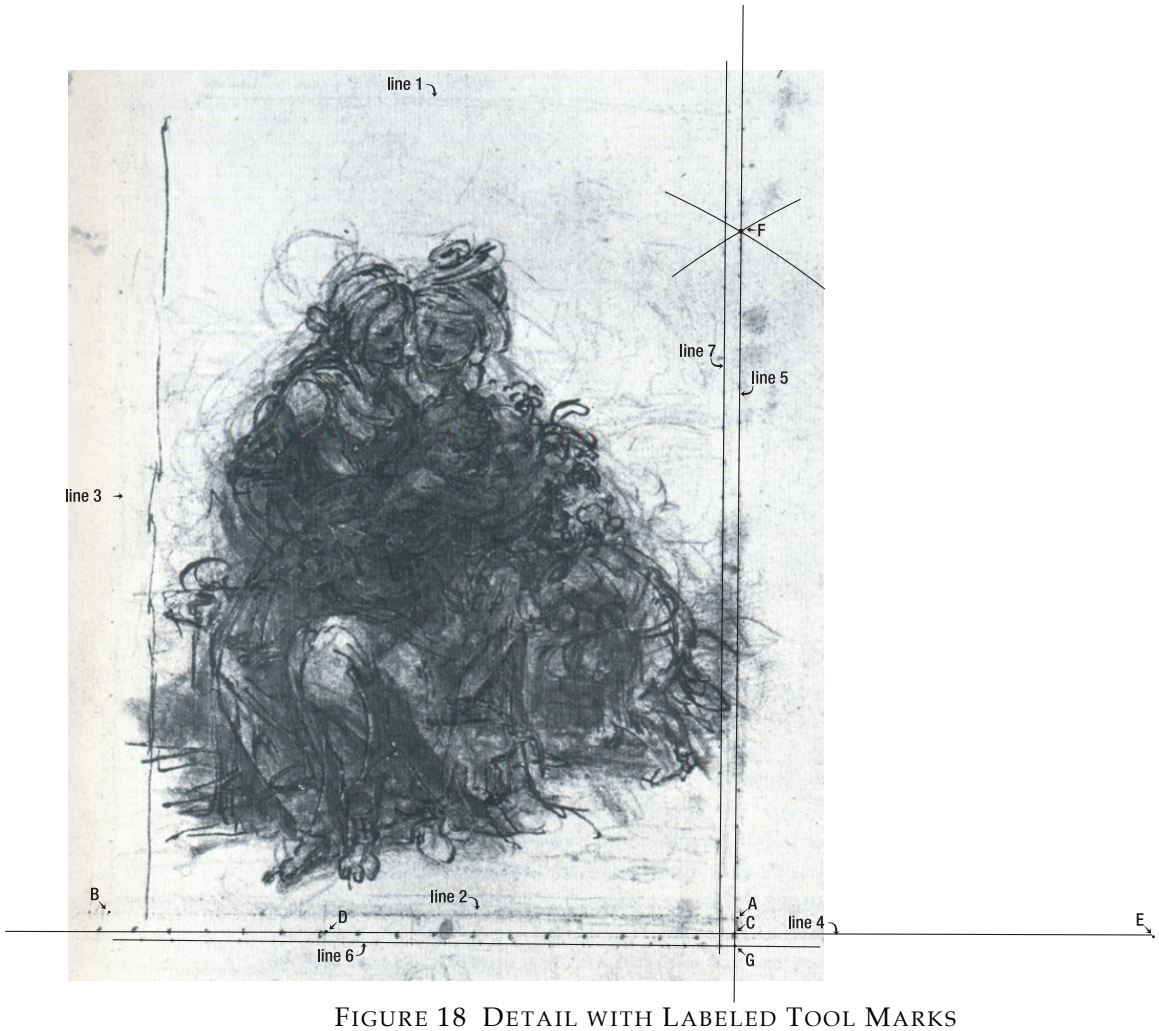


FIGURE 18 DETAIL WITH LABELED TOOL MARKS

Beginning at point B Leonardo then measured vertically twenty-five of these new units along line 3, an unruled chalk line forming the left border. The indentations exceed the height of the format, as defined by line 1, by three units. Line 1 intersects with line 3 approximately at point twenty-two. This operation appears casual as though accuracy were not important. Without a ruled incised line to follow, the dividers step along the irregular path of the chalk line with occasional misses. On three occasions (measures 4, 6 and 18) the foot of the divider was allowed to drag during the step leaving an incised arc. (These arcs are visible in light raking top to bottom across the page.) Also evident here is that Leonardo's dividers had asymmetrical points: one was needle sharp and frequently pierced the paper, while the other was blunt and left shallow depressions. Throughout the measurement process these dividers left trails of alternately sharp and dull indentations.

The circumstance of interpreting the different characteristics of the indentations is a useful example of how different interpretations can be made from the same evidence. Initially, the appearance of shallow impressions alternating with sharp, paper-piercing impressions led me to deduce that Leonardo had measured the distance twice with two different sets of dividers. This interpretation, however, led to puzzling conclusions. No sense could be made of the numbers and the size of the measured spaces. These inconsistencies compelled me to rethink the initial assumption regarding the dividers. The explanation of the use of two sets of dividers seemed odd, even for Leonardo. The realization that one set of dividers could make two different kinds of marks led to the simpler interpretation which allowed the evidence to tell a rational story. As for why a set of dividers would have two different kinds of points there is a possible explanation. The Renaissance drawing instruments illustrated in Chapter 3 included dividers that had removable arms. This feature allows replacement arms to be attached for other uses. If the dividers were used with alternate arms then the pivoting point (that is un-removable) will receive more wear than the point that can be removed.

For some reason that is not apparent, Leonardo was not satisfied with the results of the first attempt to establish and measure the format. It may have been that the number of units he wanted for the left side resulted in a rectangle that was too tall. The number twenty-two, which coincided with the height of the rectangle in this first attempt, must also have been undesirable as a ratio with seventeen. The method of measurement must also have been insufficiently precise since his next attempt in the process is characterized by an apparent desire for more accuracy.

Beginning anew, Leonardo placed a straightedge parallel to line 2 (and about 3mm below) and incised line 4 all the way across the page. The excessive length of the line was in preparation for establishing a perpendicular that would define the right border of the composition. Along line 4 a point (C in the figure just below A) was selected as the new position of the lower right corner of the format. In order to draw a perpendicular line at point C, Leonardo used the crossed-arcs construction: With dividers he measured and indented two points (D and E) on line 4 at equal distances from point C. The distance needed to be as wide as possible within the limitation of the right edge of the paper so as to make

the perpendicular as accurate as possible. Finding points D and E involved incising two short arcs crossing line 4 with one foot of the dividers pivoting at point C. (These two short arcs are visible in light raking left-to-right across the page.) With the dividers opened to the distance from point D to point E the pivoting foot was then moved to point D while the other foot incised a relatively long arc eleven centimeters above point C. This step was repeated with the pivoting foot at point E and a shorter arc incised intersecting the longer arc and locating point F. A straightedge was then used to make the incised line connecting points C and F (line 5). The significance of the different lengths of the two arcs locating point F is that the first arc represented a safe guess as to the future location of the second arc. The second could be shorter because all that was needed of it was the intersection.

Having constructed a precise right angle, Leonardo then remeasured the new, lower border with seventeen units, from right to left, beginning at point C. These units are slightly longer than those in line 2 (the original lower border), and so the seventeen units extend further to the left by 1 mm. There are no other indentations on this line indicating there was no need for trial-and-error. He then reinforced these points with brown ink as if he wanted to make his choice permanent. Twenty-five of these same units were measured up the right border (line 5) but are not inked. The level of line 1, which is the only indication of the height of the format, falls between the units twenty-two and twenty-three

Apparently dissatisfied with the resulting numbers, Leonardo incised a fresh baseline (line 6) 2 mm below and parallel to line 4. He then measured from right to left (beginning at point G) eighteen slightly smaller units ending about 1 mm shy of the left border. He then measured vertically from point G along line 5 with these new units. This resulted in a measure of twenty-four with line 1, the upper border, falling between twenty-three and twenty-four.

After a minuscule reduction in the unit Leonardo then remeasured line 6 from left to right with eighteen units. The measuring begins directly under line 3 which defines the left border and ends on a point 3 mm from line 5. Through this last point on line 6, a vertical (line 7) is drawn with a straightedge parallel to line 5 becoming the new right border. Line 7 is measured with the same units as line 6 to a height of twenty-four aligning with the top border.

To summarize the steps in making the format:

1. The format is sketched loosely in chalk
2. The bottom and left borders are measured, without attention to precision, as 17 x 25, three units higher than the upper border.
3. The lower border is lowered and a precise right angle is constructed for a new right border.
4. The new lower border and the right border are measured with new longer units as 17 x 25 still higher than the upper border.
5. The lower border is again lowered and measured with eighteen smaller units. The right border is measured as twenty-four of these units, but again exceeds the upper border.
6. The lower border is shifted 1 mm to the left and remeasured with 18 smaller units.
7. A new right border is drawn 3mm to the left to align with the eighteenth point of the new lower border and measured with 24 units aligning at the top with the upper border.

The sequence of operations here outlined, reveal some of Leonardo's priorities regarding the format of this drawing. 1) The upper border must have been where he wanted it from the start. He never moved the upper border and the other lines' measurements and positions were altered to accommodate it. 2) The response to the first attempt at measurement shows that it was more important to change the position of the lines than to change the ratio of 17 x 25. 3) When it became evident that the preferred ratio would not work without moving the lower line down further than was desirable, the ratio was changed to 18 x 24.

There were other format changes between this drawing and the London cartoon of the same subject. Although the position of the first lower line (line 2) matches the cartoon quite closely, all the other lines have moved much closer to the figures. This is true, especially, of the upper border. The damaged state of the cartoon makes it impossible to tell the exact ratio it had originally, but it is now approximately 18 x 24.

If my analysis is correct, we now know the sequence of steps that Leonardo followed in his pursuit of measured boundaries for his nascent

composition. We are still in the dark, however, concerning the reasons behind these steps. One possibility is that Leonardo, knowing that he was going to make a painting from this drawing, already had a frame in mind for the painting. But this hypothesis does not account for the change of mind regarding the ratios. If the frame were 17 x 25 then 18 x 24 would not be an acceptable ratio. He may have been concerned only that the rectangle be made of whole number units. The decision to change ratios would then have been based on the suitability of the new ratio to the drawing. But his trials of stepping the dividers up 25 units, well past the upper boundary of the composition, indicates that he wanted 25 to be the height paired with 17 as the width. His increasingly precise efforts were directed toward establishing a numerical ratio for the height and width of the little sketch. We have seen, in the background survey, that numerical systems were a traditional tool at the service of Renaissance designers. In this case, however, we do not know what system that Leonardo was using. One thing is certainly true; numbers were an important part of the process.

Chapter 5
Sangallo's Pilaster

Antonio da Sangallo the Younger (1484-1546), was an architect involved in the project to build the new St. Peters in Rome during the first half of the Sixteenth century. From 1520 to 1546 he was the chief architect for the project.[124] Scores of Sangallo's various architectural designs and sketches are preserved in the Uffizi drawing cabinets. They record the typical interests of Renaissance architects: notes on the measurements of antique monuments, recordings of antique inscriptions, sketches of architectural details, and constructions of spirals for ionic volutes and mechanical contraptions. There are also sketches, designs and presentation drawings of palaces, churches and fortifications.

This analysis will focus on a single drawing, Uff. 7976A, made in 1518-19 while Sangallo was an assistant to Raphael who was then chief of works (fig. 19).[125] This drawing is of special interest here because it reveals much about

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[124]He assumed the position after the death of Raphael who was official architect (1514-1520). Antonio's brother Giuliano da Sangallo was also official architect at St. Peter's (1520-1536), as was Baldassare Peruzzi (1520-1536). Michelangelo became the official architect after Antonio's death (1546-1564).

[125]*Modano for the Base of the Monumental Order of Interior Pilasters (St. Peter's, Vatican)*. See Wolff Metternich, Franz Graf, *Die Erbauung der Peterskirche zu Rom im 16. Jahrhundert I,* (Vienna/Munich, 1972), n.47. Christoph Frommel, 'San Pietro. Storia della sua construzione,' *Raffaello architetto*, n. 2.15.19, (Milan, 1984), 241-310.

Sangallo's process for designing using geometry and numbers. It is a tall, narrow drawing, measuring 171.5 cm by 57.5 cm, and consists of the profile of a pilaster base constructed by means of a grid of ruled lines. The drawing is a *modano*. It represents one step in the process of making an actual size template intended for stone cutters to use in cutting the interior pilaster bases for St. Peter's. The marks and incised lines visible in the paper record the architect's process of designing the form of the pilaster. The physical evidence indicates that the finished profile was partially the result of a program set out from the beginning of the work and partially the result of improvised changes made after the design was under way.[126] The nature of the evidence is such that some fairly certain conclusions can be drawn regarding the sequence of steps Sangallo employed in the construction of the forms. Where evidence is insufficient for certainty, arguments will be made for the most probable scenario.

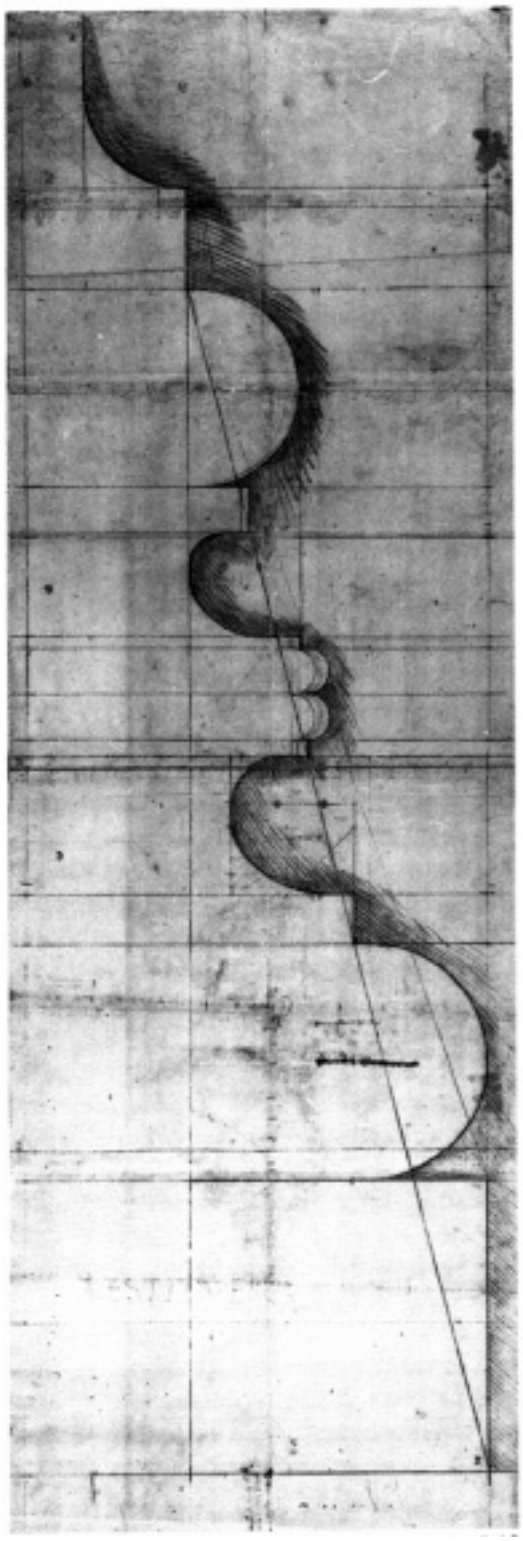


FIGURE 19 PILASTER BASE PROFILE

The paper is made of five sheets glued side to side, joined with four horizontal splices and shows signs of having been folded in many places. Most of the lines were incised first with a sharp instrument then drawn over with black chalk, the point of which was sometimes blunt and sometimes sharp, and applied with varying degrees of pressure. There are also a few marks in red chalk used to make some locations more visible. The

[126]This process is reminiscent of that observed by Haselberger at Didyma, see Chapter 1.

geometry was drawn precisely with dividers and straightedge, although, occasionally measurements were taken with a casual attitude toward precision. Sangallo reinforced the outline with freehand black chalk to help distinguish the form of the final profile from all of the preceding trials. He also sketched rough hatchings along the perimeter, outside the forms, throwing them into relief. The slanting of the hatchings from upper left to lower right indicates that the right-handed Sangallo must have had the drawing positioned horizontally on a table and was standing along one of the sides. The large scale of the paper would have made drawing in the vertical position difficult. There are some ink marks which have bled through from the verso in the area of the lower torus. On the plinth is the word "RESTITVER" in charcoal written in Roman capitals in imitation of an antique inscription. On the verso are sketches of profiles for pedestals and ambulatory details along with inscriptions in Sangallo's hand: 1) *Modano della basa gra*[*n*]*de / di s.to pietro de pilastri* (rotated 180°, above center); 2) *Basa gra*[*n*]*di s.to pietro / traglie gra*[*n*]*di fatte di pietra / per la facciata di s.to pietro* (rotated 270°, right margin).[127]

A pilaster is an architectural feature like a column, except that it is flattened and attached to a wall. It has a capital, shaft and base. The profiles of pilaster bases are the same as those for columns but the bases themselves are rectangular in plan rather than circular. The present pilaster base profile consists of alternating curved and rectilinear forms projecting to the right from the surface of the shaft, indicated by a vertical line in the upper left corner. In this analysis the forms will be characterized by their size, or vertical measure, and projection, which is the distance that they extend to the right. The names of the forms, from bottom to top, are as follows: the plinth, the lower torus, a large fillet, the lower scotia, a small fillet, two identical small tori or astragals, a small fillet, the upper scotia, a large fillet and the upper torus.[128] With the exception of the plinth each form is one of a pair of forms of the same shape. There is a

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[127]"Model of the large base of the pilasters of Saint Peters. Large pilaster base, made of stone, drawn large, for the building of Saint Peters." Tracy E. Cooper, n. 116, *The Renaissance from Brunelleschi to Michelangelo*, Millon ed., 1994, 499. Christoph Frommel, *The Architectural Drawings of Antonio da Sangallo the Younger and his Circle*, (New York, 1994), 4, says that Sangallo identified on the verso the content of the drawings on the recto.

[128]According to the architectural stone cutters at the cathedral workshop in Florence, the astragals are called "bastone," the larger fillet is a "listello," and the small fillet is a "listelliao." The curve which joins the base to the column is the "guscio."

pattern, followed by all of the pairs except the two astragals, where the lower member of each pair is larger and projects further to the right than the upper member.

What makes this drawing interesting from the point of view of the design process is Sangallo's departure from the usual numerical formulae of Classical ornament. Typically, the forms of capitals, entablatures and bases consisted of long-established numerical programs which dictated the measure of each form as a fraction or multiple of a module. Vitruvius described the Attic and Ionic styles of bases for columns with their various modular recipes.[129] The Attic base consisted of plinth-torus-scotia-torus, the Ionic base of plinth-scotia-astragals-scotia-torus. Alberti followed Vitruvius for the Attic but described the Ionic with an extra torus making a base with the same set of forms as those in the present drawing.[130] Alberti described two different modular programs, used by the Ancients, for the ratios of Ionic bases. The first, similar to Vitruvius' instructions, involved a sequence of fractional divisions. With each step of the sequence a space was divided into a number of parts with one part given to a form, then the remainder was divided by a different number.

Once the plinth had been marked off, the rest of the height was divided into sevenths, two of which gave the thickness of the lower torus. The space left over by the torus and the plinth was further divided into thirds, with the top one being given over to the upper torus, and the two remaining thirds given to two scotiae and two rings, sandwiched between the two tori.[131]

The whole sequence of fractional divisions from the columnar module to the fillets was 1/2, 1/4, 1/7, 1/3, 1/7, 1/8. No common denominator was used; each division was a separate procedure further and further removed from the original module. The problem that a designer would face with this literary tradition is that it prescribed the way the forms should be at the end of the process, not the process for arriving at them. It is difficult to take a line segment and divide it evenly into an odd number of units. A trial and error procedure can only approximate it. In order for one to make a pilaster base easily that

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[129]Vitruvius, *Architectura*, Book III, Chapter 5, (ed. *Vitruvius, The Ten Books on Architecture*, trans. by Morris Hicky Morgan, New York, 1960), 90-97.
[130]Alberti, *De re aedificatoria*, Book VII, Chapter 7, (ed. *Art of Building*), 203.
[131]Alberti, (ed. *Art of Building*), 203.

followed Vitruvian and Albertian prescriptions, one would have to begin with the smallest unit, that of the small fillets, and multiply the units according to the changing numbers to arrive at the prescribed ratios. This method, however, would result in a base of an unpredictable size.

Alberti's second method alleviated this problem by giving each form a number of modules:

> [A]part from the plinth, the thickness of the base was divided into sixteen parts, called modules; four of these were taken up by the lower torus, three by the upper torus, three and a half by the lower scotia and, similarly, by the upper one, and two by the intermediate rings.[132]

The advantage of this method is that the consistent module allows the total height to be predicted, if one begins with the smallest form, or if one begins with the whole. Divisions into sixteenths can be made easily. Sangallo's overall approach, as we shall see, contained elements of both methods as well as his own original contribution.

In addition to the literary traditions, Sangallo also had at his disposal examples of bases from surviving antique buildings. In Sangallo's note pages there are sketches of Ionic base profiles with measurements that record the information necessary to make an exact copy of the base.[133] The profile of the present drawing has been linked to the pilasters in the *pronaos* (front porch) of the Pantheon.[134] One would think that the process of adapting a form from the Pantheon to one in St. Peter's would be a mechanical process of taking the measurements and enlarging them to the appropriate scale. However, the evidence in this Uffizi drawing shows that Sangallo's intent was to use a base of the Pantheon type as a point of departure. While designing he must have had in mind the writings of Vitruvius and Alberti, the physical evidence of Roman ruins and the practical necessities of the St. Peter project. He must also have felt free to develop his own process of invention while at the same time deciding how much of the tradition to follow.

To illustrate this process it is necessary to attempt a reconstruction of Sangallo's working sequence. This can be done by analyzing the marks in such a

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[132]Alberti, (ed. *Art of Building*), 203.
[133] Uff. 1140a, 1255a, 1061a.
[134]Frommel, 'San Pietro,' (Milan, 1984), 277.

way as to determine which marks depend on other marks for their position. Using this strategy one can work backwards to the beginning and reconstruct the steps necessary to produce the original design. The locations and sizes of the forms of the pilaster base are governed by an armature of horizontal and vertical lines which meet at right angles. The armature must then have preceded the pilaster. The armature was itself preceded by the first horizontal and vertical lines which established the right angle upon which the rest of the design depended for its accuracy. With one horizontal and one vertical that are known to be perpendicular one can locate any number of parallels with the dividers or ruler by measuring equal distances.[135]

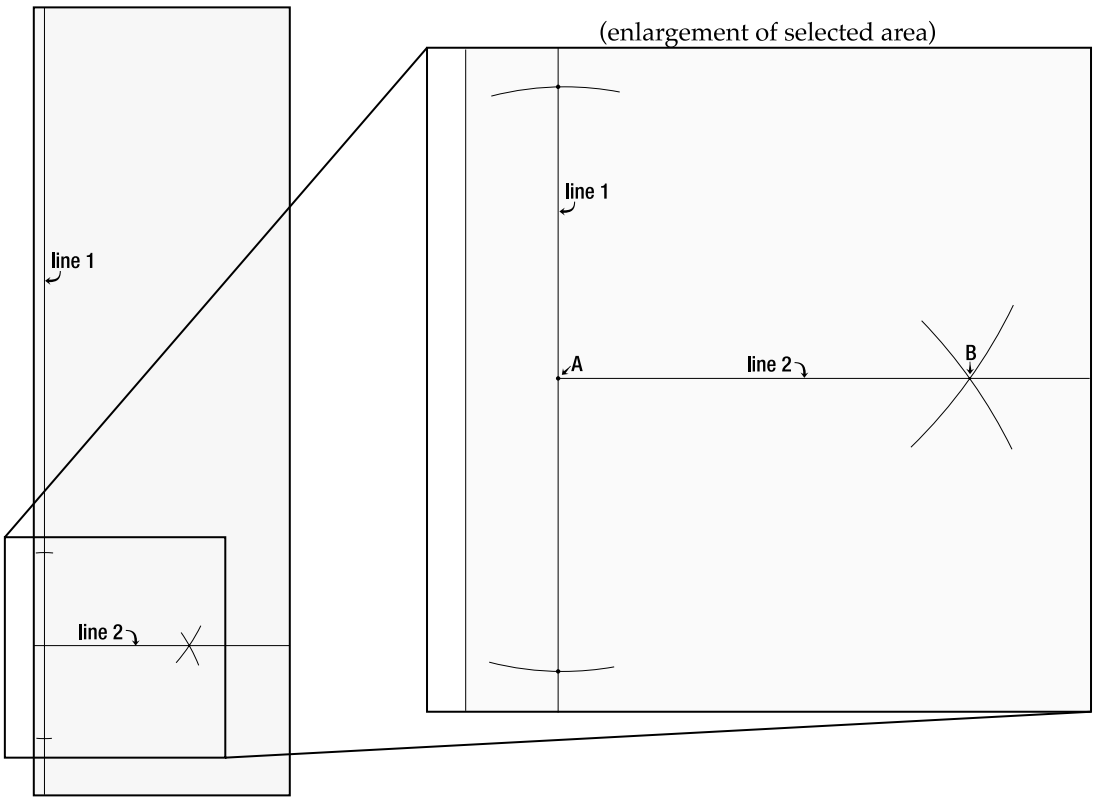


FIGURE 20 ESTABLISHING A RIGHT ANGLE

Figure 20 illustrates the first stage in this process. Sangallo incised the origination line 1 with a long straightedge near the left edge of the paper. This line was needed as a reference because the edge of the paper was not true. He then arbitrarily chose a point (which I have labeled point A) near the lower end

[135]In the diagrams the major lines are numbered in the probable order of execution. When clarity requires, points, arcs and minor lines will be identified by letters. These letters will be used sequentially from diagram to diagram but do not imply the sequence of every mark.

of this line to be the site of the perpendicular. With one foot of a large pair of dividers on point A, he incised two short arcs (visible under raking light) intersecting with line 1 at equal distances from point A. The intersections of these two short arcs with line 1 then became locations for the compass to inscribe two more short arcs which intersect at point B. A straight line (line 2) was then incised across the paper intersecting points A and B and forming a right angle to line 1. The position of the perpendicular so far from the center of the origination line is unusual. Generally, either the origination line or the perpendicular will be along either the horizontal or vertical axis. This would minimize the error of transferring parallel lines across the length of the paper. Perhaps Sangallo wanted to leave the center free, knowing in advance that the mid section was yet to be designed. The perpendicular, which is only used to construct the armature, might have interfered with the yet-to-be-located horizontal transversals.

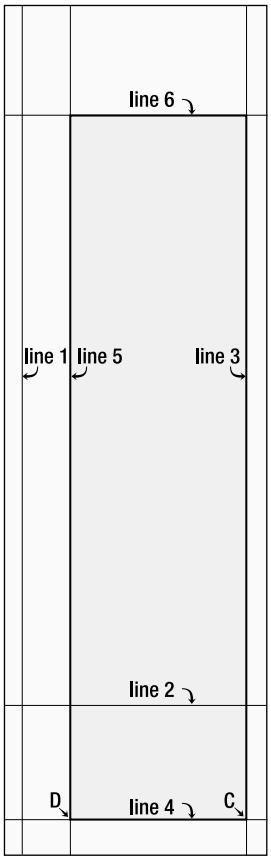


FIGURE 21
DESIGN FORMAT

The evidence of the crossed arcs and the related compass indentations establishes the priority of lines 1 and 2. There are no other crossed arcs of this type on the paper, therefore, all subsequent right angles must have been made with reference to lines 1 and 2. Sangallo then incised an armature of lines parallel to lines 1 and 2 using the dividers and/or a long rule to measure equal distances (figure 21, lines 3, 4, 5, 6). The armature serves as a frame for the pilaster forms. I have numbered the lines in the most likely order of construction which, as a group, must have followed lines 1 and 2 and preceded all subsequent lines. Lines 3 and 4, which make up the right and lower border of the frame, are parallel to lines 1 and 2 respectively. Sangallo chose their positions for their proximity to the edges of the paper and thus allowed maximum room for the design to take place. The positions of lines 5 and 6 will be the result of measurements from lines 3 and 4 based on a preconceived plan.

From the beginning Sangallo had the Roman *palmo* in mind as a module.[136] Measuring from point C (right to left, figure 21) along line 4 he made a short vertical chalk mark at 1 *palmo* and another at 1.5 *palmi* at the point labeled in the diagram as D. This second mark located the position for line 5. The measure was transferred to the top and line 5 was incised. To locate line 6 Sangallo measured with a rule 6 *palmi* vertically from points C and D. Since there are no compass marks at intervals of one *palmo* or 1.5 *palmi*, he must have had a rule with *palmi* already marked.

The rectangle bordered by lines 3 through 6 (shown in bold in figure 21) became the format in which Sangallo designed the pilaster base. In order to draw the base within this format Sangallo needed to determine, 1) the height for each of the base's eleven forms, 2) the distance that each form would project from the left edge of the rectangle, and 3) the location of the center points for the compass to draw the curved forms. Figure 22 illustrates this inner rectangular format, removed from the rest of the drawing, with the finished pilaster profile and scales of *palmi* in one and one and a half units. The boundaries of the format define the limits of the outer projection of the plinth and lower torus, the depth of the upper scotia, and the height of the pilaster base up to the upper torus.

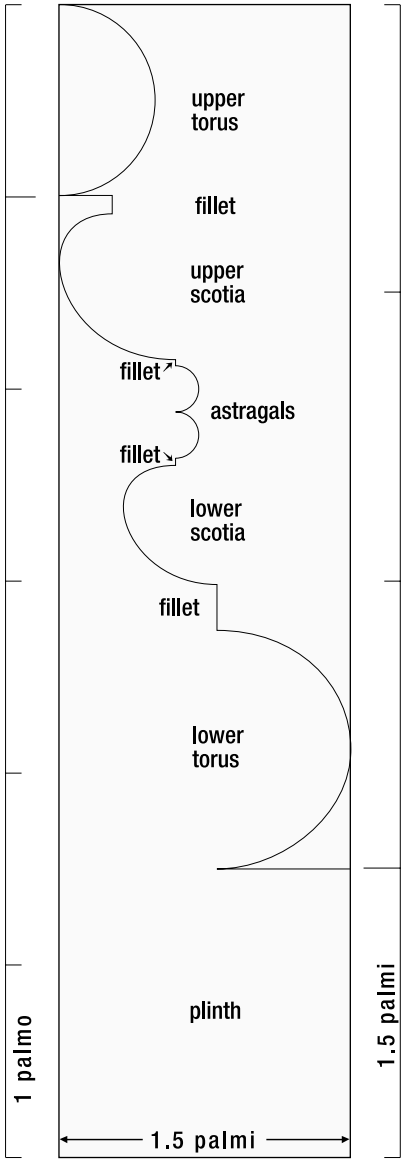


FIGURE 22 RATIOS

The ratio of the format is 1 to 4 or, measured in *palmi romani*, 1.5 to 6 (1 *palmo romano* is 0.2234 m). Two of the forms of the pilaster base relate vertically to the units. The size of the plinth is 1.5 *palmi* and the size of the upper torus is 1 *palmo*. Also, the 1.5 *palmi* width of the format determines, on the left side, the deepest cut into the stone (the upper

[136]Frommel, 'San Pietro,' (Milan, 1984), 277. Frommel uses the presence of the *palmo* measure to date the drawing since it was in 1518-19 that Raphael and Sangallo decided upon the module.

scotia) and, on the right side, the furthest projection (the lower torus and plinth). It is evident, then, from the coincidence of forms with modules, that from the beginning of the design process Sangallo had a program in mind to determine at least part of the base's form. As we shall see, other forms are the result of procedures improvised after the drawing was begun.

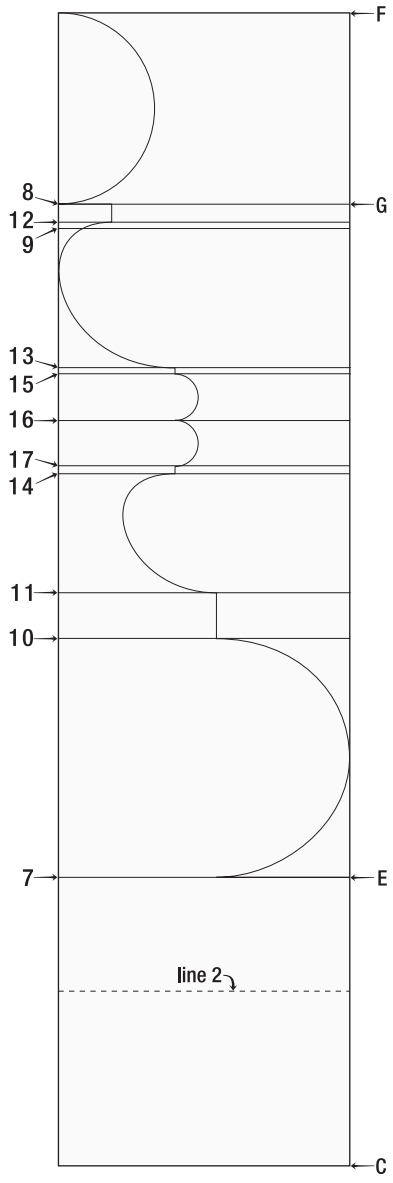


FIGURE 23

With the rectangle drawn, the next step in building the armature was to subdivide the format into sections with a series of incised transverse horizontal lines. The space between each line would become the height of each form. Figure 23 illustrates the format with these transversals, numbered here in the order they were most likely drawn, along with the final form of the pilaster for reference.[137] Sangallo measured along the vertical lines 1 and 3 (as labeled in figure 21) to locate the points from which to draw the transverse lines across the width of the paper. The transverse lines determined the heights at which one form of the base would change to the next. The order of drawing was ascertained in this study by determining, for each line, what other lines were necessary for its position. The first two transversals (lines 7 and 8), for example, defining the heights of the plinth and upper torus, were measured from the bottom and top of the format respectively. Therefore it was necessary for the format to be in position before they could be located. Because all of the other transversals depended on the locations of lines 7 and 8, as we shall see, their priority could be established.

[137]The transversals were reinforced with black chalk and are visible in the reproduction figure 19. The horizontal lines visible above the lines that I have labeled 7 and 11 in figure 23, are splices in the paper and are not part of the design.

It was possible to determine that lines 7 and 8 were measured from the top and bottom of the format by the character of the construction marks. The curvature of a short compass arc at point E, locating the position of line 7, indicates that the opposite foot of the compass had to have been at point C.

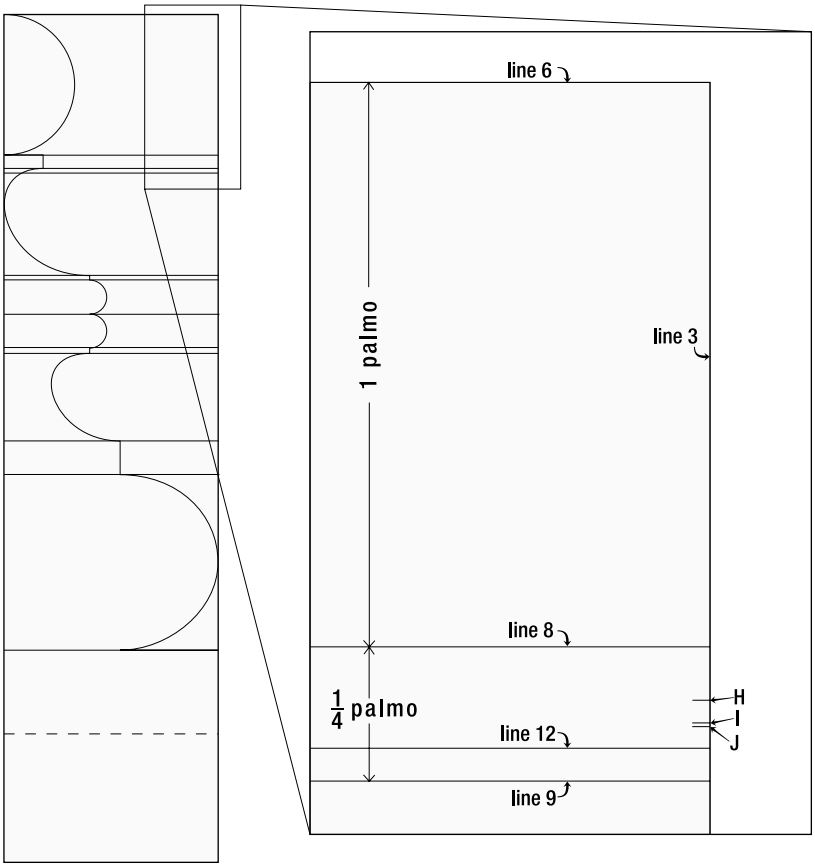


FIGURE 24 SEARCH FOR A MEASUREMENT

Therefore line 7 was measured from the bottom. Similarly, an arc at point G was measured from point F indicating that line 8 was measured from the top. Sangallo made these marks by measuring modules and the marks represent the extent to which he was certain about the positions of transversals. From this point the rest of the design was, to varying degrees, improvised.

The next transversal, line 9, was located, incised, reinforced in black chalk and later rejected and its function replaced by line 12, 3.8 mm higher. The position of line 9 determined the original size of the upper fillet as one quarter *palmo*. Line 9 was the only transversal in the design process to be replaced. The change of mind must have occurred after lines 10 and 11, defining the lower fillet (fig. 26), were located in order to make the upper fillet slightly smaller than the lower.

Because of the large size of the original drawing and the necessity of illustrating relatively small areas, the remaining diagrams will follow the format of reproducing figure 23 in its essentials with an enlarged detail of the relevant area. Figure 24 depicts an enlarged view of the upper right corner of figure 23. The area here under consideration is a small segment of line 3 between the transversals 8 and 9. It is here that Sangallo searched for the vertical measure which was to become the lower torus, the next base form to be located. Sangallo was measuring along the upper part of line 3 to arrive at a vertical measure that he was going to transfer to the lower part of line 3. This may seem odd, but there is a reason for it. He must have intended the lower torus to be larger than the upper torus, as was traditional, but not by a previously specified amount. The distance between lines 6 and 8 represented a visual reminder of the size of the upper torus; so, in order to determine the size of the lower torus, he had to add to this measure. He made a chalk mark at point H guessing at the halfway point between lines 8 and 9. This would enlarge the vertical measure by one eighth *palmo* according to the module. He then enlarged this by one sixteenth *palmo* more by making another chalk mark at point J (after a rejected guess at point I). This last point added three sixteenths to the one *palmo* to become the vertical measure for the lower torus.[138] This measure, obtained at the top of the drawing, was then transferred with a compass to the area above the plinth, locating line 10 and leaving the appropriate compass arc incisions (see fig. 26).

The preceding procedure seems odd considering that the net result, obtaining a measure of 1 and 3/16th *palmi*, could have been taken simply and accurately from a ruler that measured *palmi*, which Sangallo must have had. The act of adding marks, however, to a visible distance must have helped him visualize the increased size of the lower torus in relation to the upper torus. It may be that he first considered 1 and 1/4 *palmi* and found that that appeared too big, then considered 1 and 1/8th *palmi* and found that too small. The fact that the marks were located by eye and not measured possibly reveals that he was just "thinking out loud." The process of making marks and judging visual distances is telling since it is counter to a process of judgment based strictly on

[138]Frommel ('San Pietro,' (Milan, 1984), 277), says that the lower torus is 1 and 1/6 *palmo* which is half a centimeter less than 1 and 3/16 *palmo*. The evidence from the rest of the drawing indicates that the module was divided into sixteen parts instead of sixths.

mathematical or "harmonic" considerations. The fact that his choices for consideration fall into the category of simple fractions indicates that even though he felt himself free to redesign an antique architectural form, he had, nevertheless, self-imposed limitations for his possible choices.

Line 11, which defined the size of the lower fillet, was located next by adding one quarter *palmo* above line 10. This step resulted in the lower fillet being equal to the upper one. This situation ran counter to the pattern, which was to be the final pilaster solution, that the paired forms had a major and minor example, therefore in order to slightly reduce the upper fillet line 12 was then drawn as an adjustment to line 9 (fig. 24).

The remaining forms were then to fit into the remaining space using one sixteenth *palmi* as units and a system of numbers. The transversals were to be measured giving the upper scotia nine units, the lower scotia twelve units and the two astragals four units each.[139] There also had to be room for two small fillets flanking the two astragals. A little over thirty units were needed. Unfortunately the established unit of one-sixteenth *palmo* or 14 mm was too large for the 404 mm between lines 11 and 12. Solving the problem involved some trial and error measuring since Sangallo would allow the size of the units to be changed but not the number of units. Along the right edge of the format between lines 11 and 12 there are over one hundred and fifty indentations at various intervals. One interval is the one-sixteenth *palmo* beginning at the intersection of line three and line 12. These indentations become lost in the confusion of marks which record the trial and error search for a new unit which would measure the proper intervals. Once the new unit was obtained on the right side of the format, Sangallo counted out new units cleanly on the left side. There, (along line 1 in fig. 21), between the transversals 11 and 12, there are evenly spaced indentations of the new unit of about 13 mm. Red chalk marks highlight the points at which the transversals were to be drawn. Lines 13 and 14 were located first because they were known quantities. The scotiae had definite numbers for their measures. The two small fillets were variable and resulted in one and one half units for the lower and a little more than one unit for the upper.

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[139]These numbers do not match Vitruvius or Alberti nor any of the above mentioned examples from the Uffizi. However, the type of modular division is similar to Alberti's second method. The numbers match more closely the proportions of the Pantheon described below.

The preceding procedure of dividing a remaining space by a given number of units relates to the prescriptions of Vitruvius and Alberti. Alberti's first method, mentioned above, was to divide the remaining distance of each step by a different fraction than the last step without regard for a common denominator. Sangallo seems to have done this with the middle section of his pilaster resulting in inconsistently sized units. The extra trouble that he went to resulted from the conditions that he set for himself from the beginning. The decision to make the height of the pilaster six *palmi*, each divided into sixteen units meant that he had ninety-six units to work with. He ended up with ninety-eight and one quarter units with the surplus taken up by reducing the middle twenty-seven and one half units. Therefore the original six *palmi* height and forms comprising ninety-eight plus units must have been more important to Sangallo than having units of consistent size.

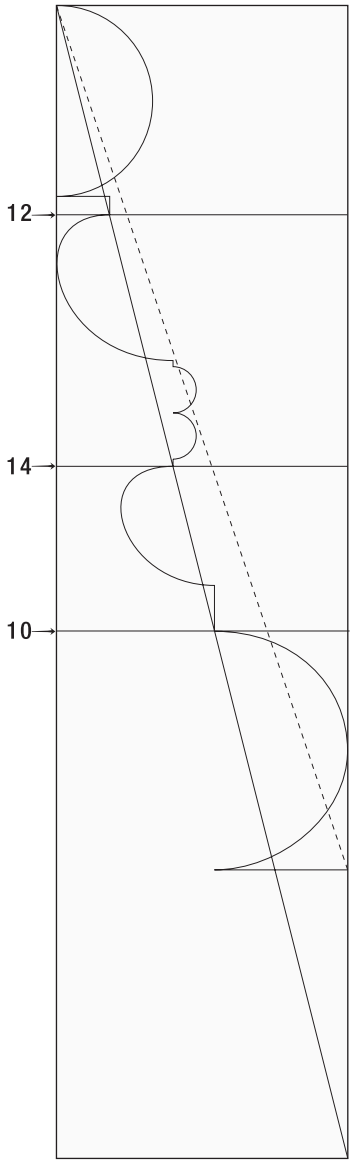


FIGURE 25
DIAGONALS

With the transversals in place, defining the levels at which one form changed to another, the next set of procedures was to determine the projection of the forms. Normally, illustrations of architectural profiles have a plumb line tangent to the farthest projecting point with numbers of modules to indicate the distance to the plumb line from each salient and reentrant form. Had Sangallo been following such a program, he would simply have measured in from the right edge of the rectangle a number of modules from the preset program. For this design, however, Sangallo used a diagonal line running from the upper left of the format to the lower right. This solution is unusual and may be unique. The intersections of this diagonal with the transversals numbered 12, 14 and 10, became the horizontal limits for three projecting forms which govern the overall slope of the base (fig. 25). The positions of all the forms of the pilaster base are governed by a combination of these three intersections,

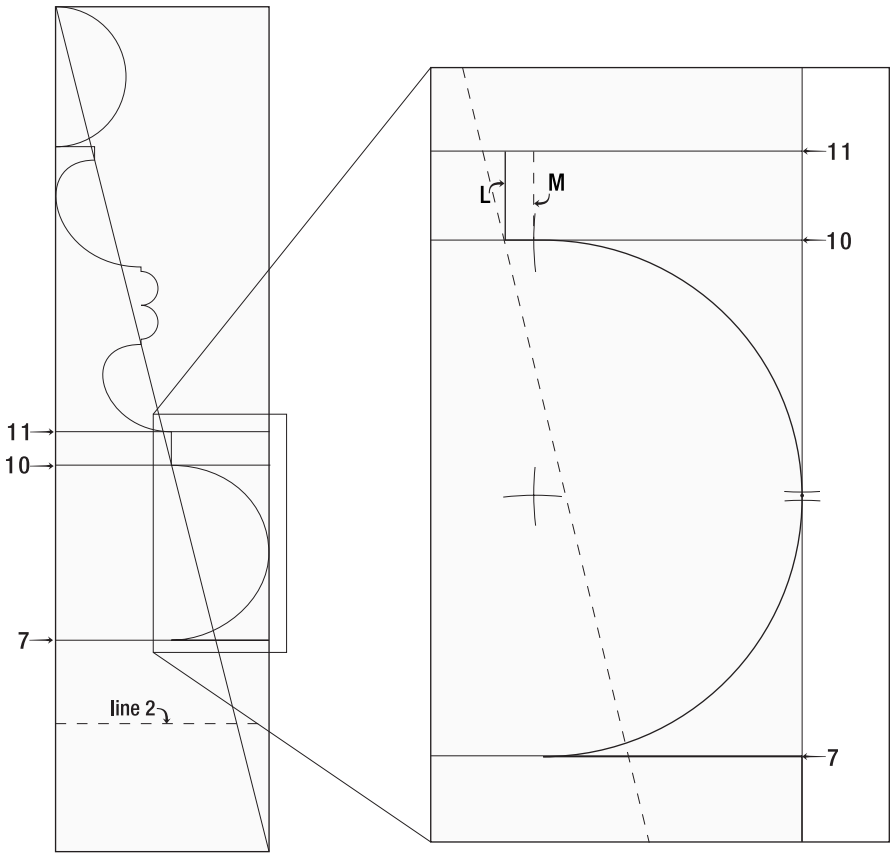


FIGURE 26 LOWER TORUS AND FILLET

directly or indirectly, and the transversals. Another diagonal (shown as a dotted line) was also drawn. It may have been a rejected first attempt which would have given the base a less steep profile. The presence of an alternate diagonal which would drastically change the finished form of the pilaster is further evidence that Sangallo was engaged in a design process and not following a program.

Figure 26 illustrates the area of the lower torus and its construction marks. Since the torus was to be a semicircle, between lines 7 and 10, and tangent to the right side of the format, drawing the form involved simply locating the center point for the compass. The first step was to locate the center between lines 7 and 10 along the right edge. Sangallo's procedure for doing this was to open the dividers an amount approximating half the distance, making a short arc, then transferred the pivot point to the other side and making another mark. The two arcs being 5 mm apart allowed an easy guess at the center. He had used the same method to locate the center of the plinth, although he never used that measure. With the dividers now open to one half the height of the torus, Sangallo could locate the center with crossed arcs and draw the lower torus.

The projection of the lower fillet, which rests upon the lower torus, is expressed by a vertical line between transversals 10 and 11 (labeled L in figure 26). The lower end of line L originates at the intersection of the long

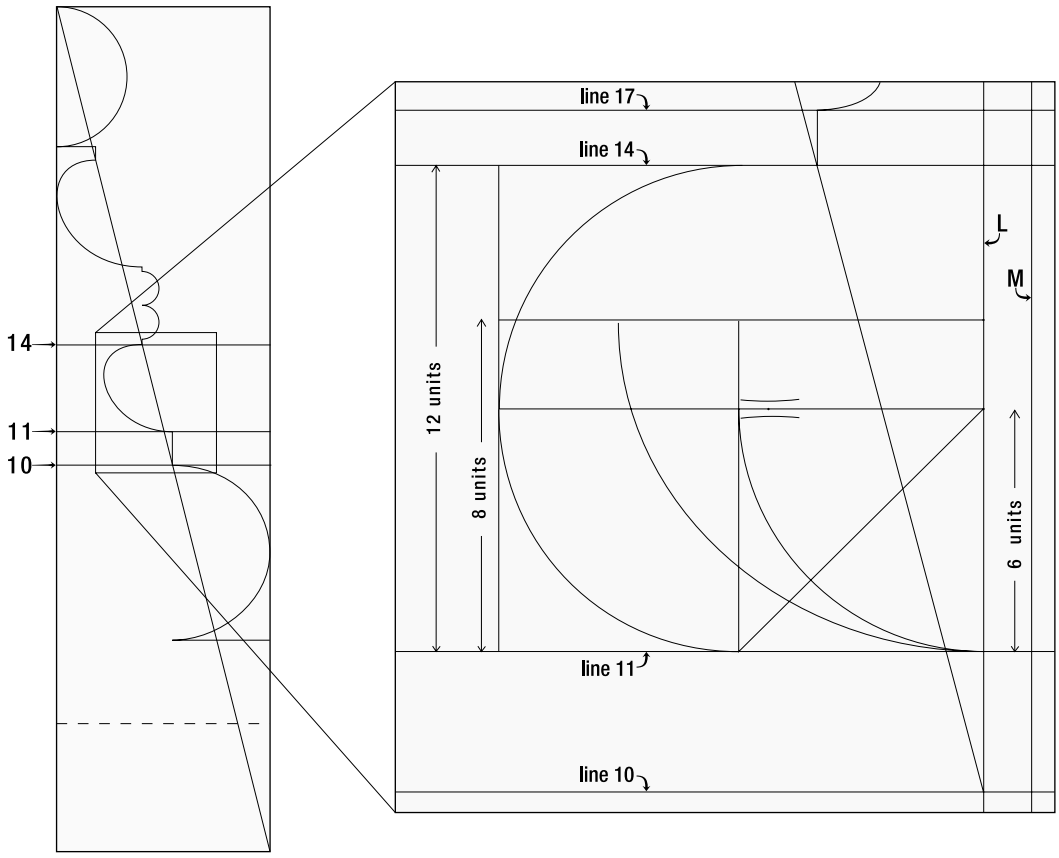


FIGURE 27 LOWER SCOTIA (FIRST ATTEMPTS)

diagonal, indicating that the diagonal determined the position of line L. The lightly drawn line M was an optional position located directly above the center of the torus. Here again, Sangallo was choosing between visible options. This time, each was arrived at geometrically. An extension of the rejected line M was used in subsequent step of the process.

The design of the lower scotia proved to be a complex ordeal. In order to unravel the confusion of marks, three diagrams of the area of the lower torus will illustrate the procedures. Figures 27 and 28 depict procedures which were considered and rejected, and figure 29 depicts the final profile with its construction.

The first step in designing the lower scotia between the transversals 11 and 14 was to locate a square within which to construct the form. This would

not have been necessary if Sangallo had known in advance what the solution was to be. The vertical distance between the two transversals had been measured
as twelve "new" units (which had been reduced from the original 1/16th *palmo*.)

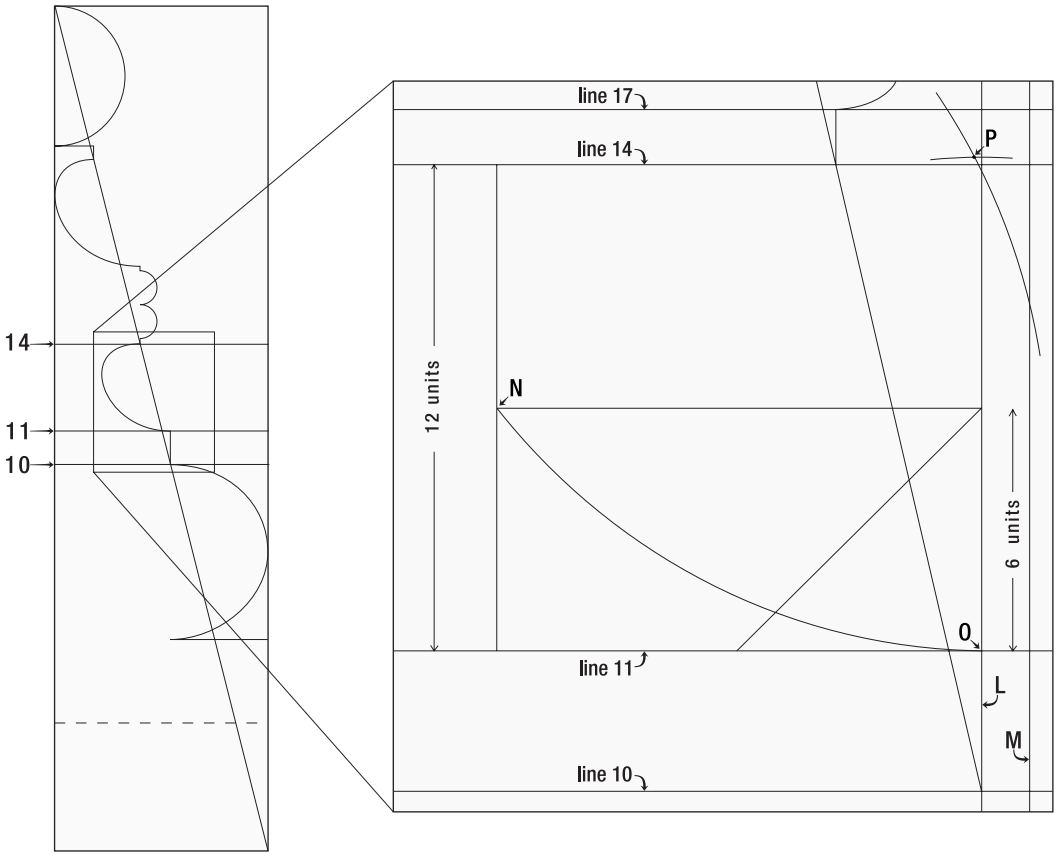


FIGURE 28 LOWER SCOTIA (ANOTHER ATTEMPT)

A vertical extension of line M (from figure 26) became the right side of the square and twelve of the new units were measured to the left of line M to locate the left side. The long diagonal served to locate the position of the upper part of the scotia, defining the projection of the small fillet. Sangallo then located the center, using the same method he had used for the lower torus, and incised a semicircular scotia. Since all of the models for this kind of base have compound curves for the scotiae this semicircle was anomalous. It may have served only as a visual reference and might not even have been intended as a possible solution. Figure 27 also shows how two quarter circles were drawn with centers along an extension of line L at six and eight units. The quarter circle at eight units could have become the lower segment of a compound curve to make a scotia but it would have been much shallower than the one later decided upon.

Although the sequence of the rejected procedures is not certain, the next attempt appears to follow the last two in that it portends a deeper scotia and relies on line L. Figure 28 illustrates this procedure. With the dividers open to the distance from point N to point O, and one foot on point N, Sangallo

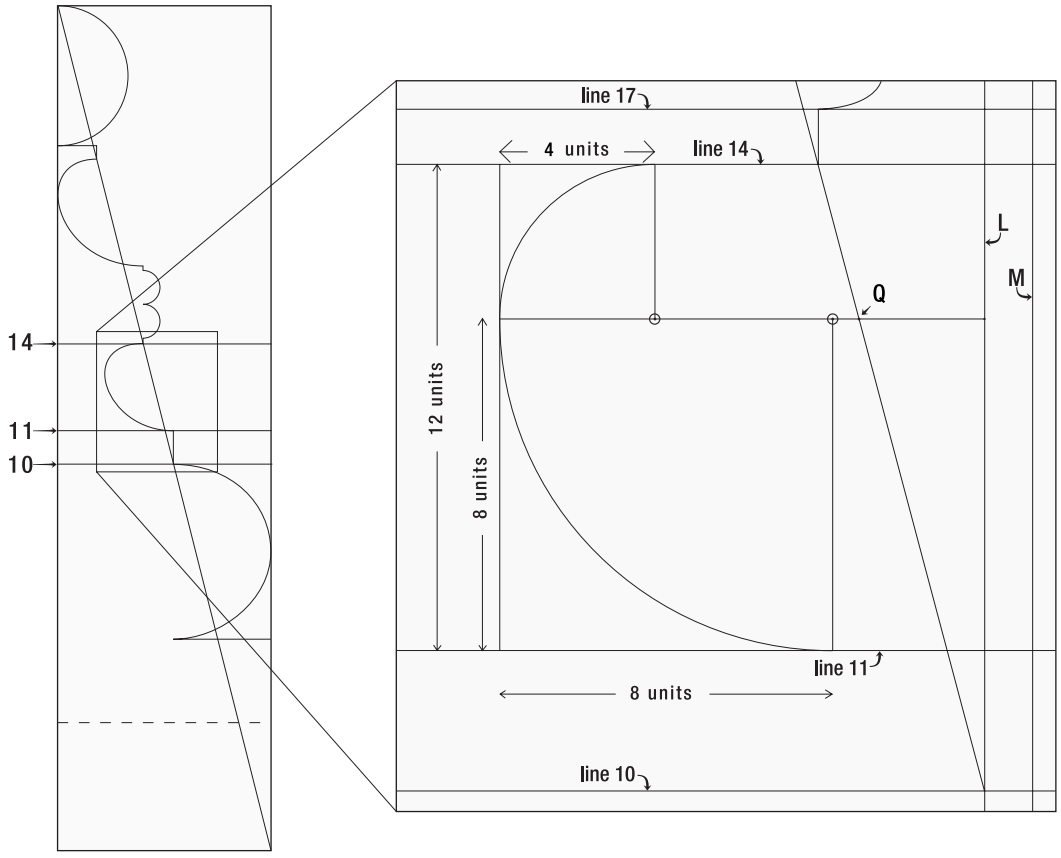


FIGURE 29 LOWER SCOTIA (FINAL)

incised a long arc intersecting the extension of line L. He then made a shorter arc with one foot at O intersecting the first arc at P. Changing the pivot foot to P, he then incised an arc connecting O and N. This arc would have provided a smoother transition of the scotia from the lower fillet than the previous trials, but it would also have required the upper segment of the scotia to cut deeper than the established square provided.

The final solution for the lower scotia is illustrated in figure 29. This solution is oriented to the left side of the square and locates compass centers using the fractions one third and two thirds. The upper arc segment is a quarter circle with its center at four units (or one third of the square) from the top and four units from the left side. The lower segment is a quarter circle with its center at eight units (or two thirds of the square) from the bottom and eight units from

the left side. These two centers, which represent the final solution, were circled to distinguish them from previous trials.[140] The point labeled Q, where the diagonal intersects one of the construction lines, will be of significance in later constructions.

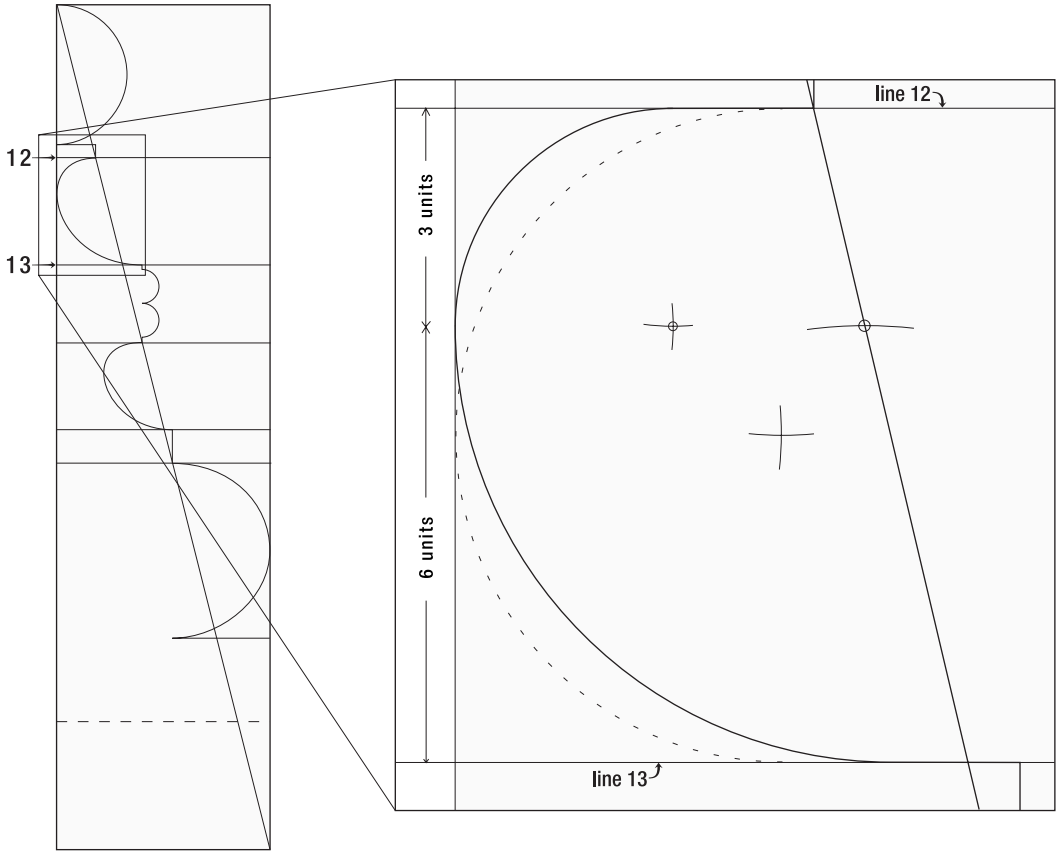


FIGURE 30 UPPER

The form for the upper scotia follows the same program as the last procedure for the lower scotia. The limits for the upper scotia are the transversals 13 and 9, which are nine units apart (fig. 30). The depth of the scotia is limited by the left side of the overall format and the projection of the boundary is determined by the diagonal line. The overall height is nine units so the fractions of one third and two thirds divide evenly as they did in the lower scotia. The center for the lower segment of the compound curve coincidentally aligns with the diagonal line. Again these centers are circled. There is no confusion of

[140]These circles that mark the compass centers were possibly for the purpose of reconstructing the final design elsewhere, once it was finished. Since the paper was never cut out to make a template, the actual template that was used would have to have been made from a copy. The copy could have been made by pricking the important points through to another paper surface (or directly onto the template material) and redrawing the lines and arcs.

rejected construction marks for the upper scotia, only an incised semicircle used as in the lower scotia, perhaps as a visual reference.

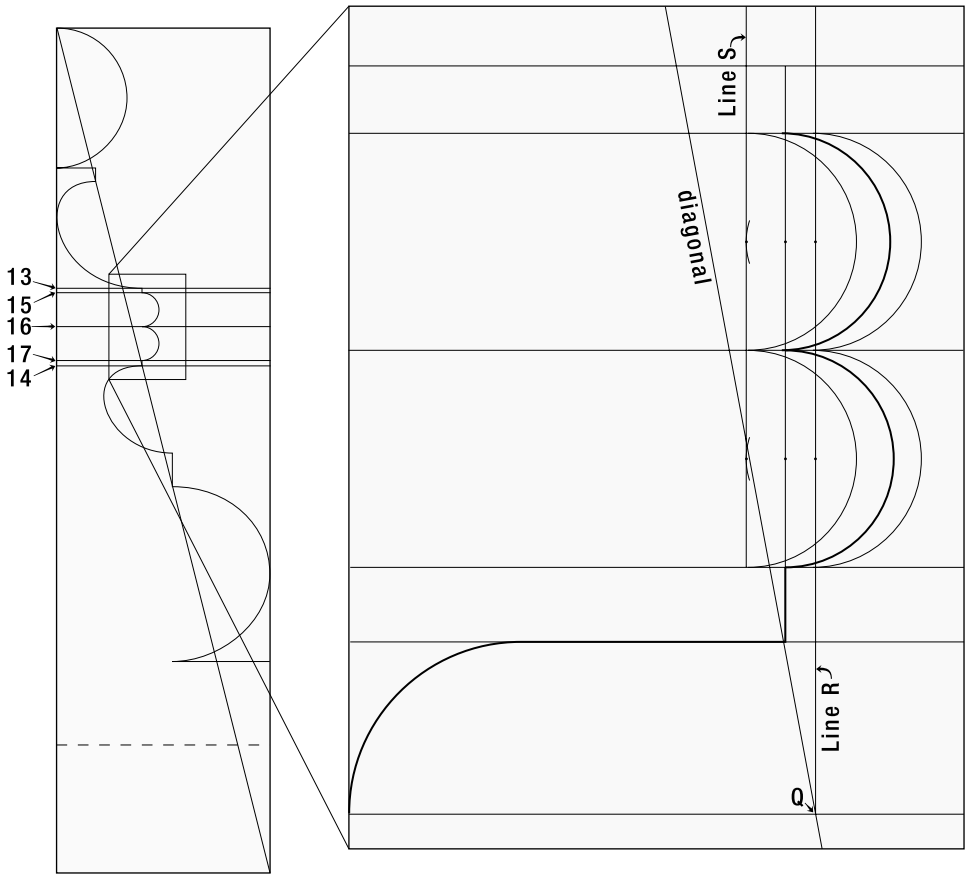


FIGURE 31 ASTRAGALS

Figure 31 illustrates the area of the central projection of the base with its two astragals and flanking fillets. The five transversals 13 through 17 define the vertical limits of the four forms which amount to two, short vertical lines and two semicircles. The final form, outlined in bold, depends, for its projected distance, on the intersection of the diagonal with line 14. Two alternate forms derive from a vertical line, labeled R, which originates at point Q from figure 29. Centers on this line form the lightly drawn semicircle to the right of the final pair of astragals and also locate the centers for another pair to the left of the final form. The line upon which these last centers rest (labeled S) extends upward along with line R to the top of the format where they form part of the construction of the upper torus.

Throughout the preceding constructions, the upper torus may not have been left blank. Among the several pentimenti arcs which vied for the upper torus is a dotted arc. This arc was made with a compass, its center on the left

border and its marking foot skipping over the paper. This arc represented the shortest possible projection for the upper torus and may have been used as a visual reference while the rest of the pilaster was being designed. The final form was not to be located until the lower forms were constructed because lines extending from the lower forms were to be used to determine the projection of the torus (figure 32). The arc was probably dotted in the early stages because it was not considered a candidate for the final form but was needed for Sangallo to help visualize the approximate shape. Three other trial arcs were made with the final one at 1/16 *palmo* from the left edge marked with a circle around the center point. The center was located not by measuring from the left border but by using the extended line S, from figure 31, as the projection limit. Line R was also used in this way but the resulting torus was rejected.

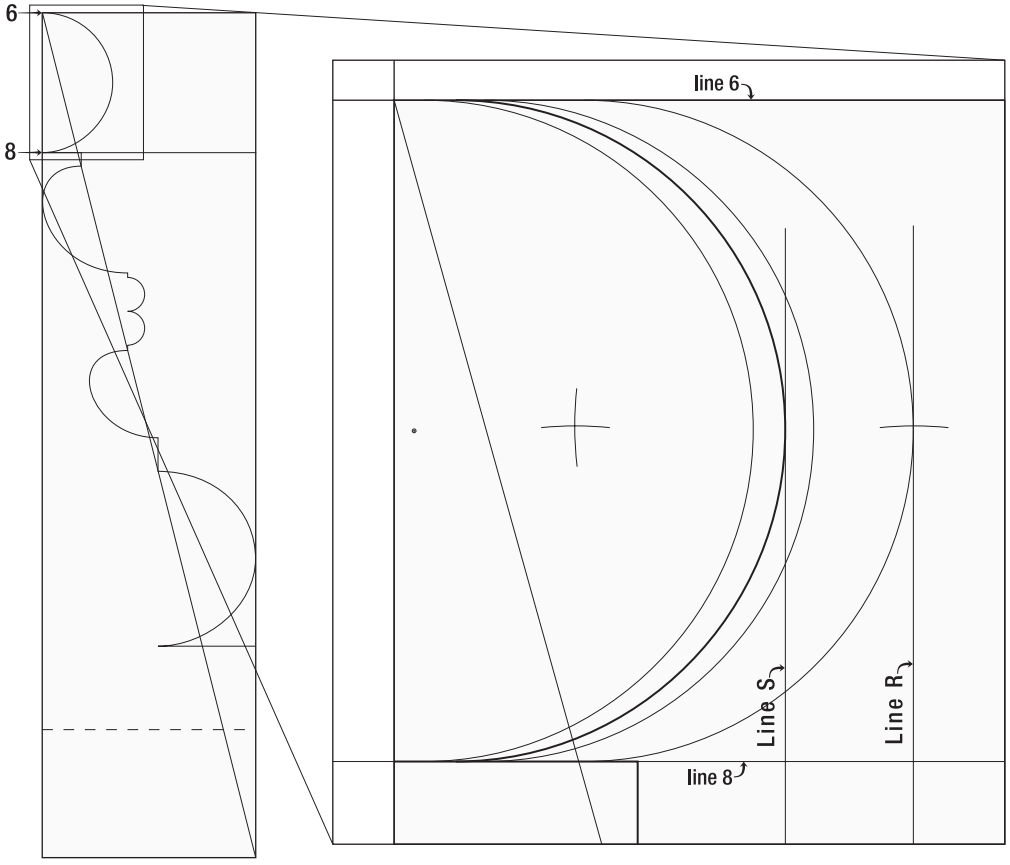


FIGURE 32 UPPER TORUS

Now that the sequence of procedures is reconstructed it is necessary to compare the finished design to the antique prototype. Frommel uses a drawing from the Codex Mellon by Domenico da Varignana to represent the Pantheon

pilaster base.[141] The relative sizes of the forms match Sangallo's design, to some extent, but the projections differ more greatly. The overall slope of Sangallo's base is steeper. The Pantheon base is also reproduced in Palladio's *Quattro libri* with the measurements in modules that correspond almost exactly to Sangallo's sixteenths of *palmi*.[142] The numbers for the vertical measures match exactly the plinth, lower torus, fillets and astragals and within a fraction of a unit for the other forms. The projection measures, however, again show a less steep slope than the Sangallo. If the Pantheon base served as the model for the Sangallo drawing then the purpose of the design procedures that we have reconstructed was to alter that model. Tracy E. Cooper says that "Sangallo's interpretation is a bolder version of the Pantheon base." And further, "Compare the slighter projection and less emphatic concavities of Varignana's copy [of the Pantheon base].

In a building as monumental as St. Peter's, this boldness served to accentuate the individual parts whose articulation contributed to the overall expression of structural grandeur in the vast interior."[143] The Varignana copy actually projects more than the Sangallo, but the concavities are less bold.

The reconstruction of Sangallo's design process showed that his major concern was to fit the base into a 1 by 4 rectangular format with the diagonal of that rectangle governing the slope of the base. A wider rectangle would have resulted in a less steep slope and consequently a further projecting base. (According to the rejected diagonal (fig. 25), Sangallo considered a less steep base.) The desire for a steeper slope and less projecting base may have been less for monumentality than for an accommodation for scale. To a viewer standing in front of an interior pilaster of the gigantic order which was a scaled copy of the Pantheon, the base may have intruded too far into his space.

Whatever the practical reason might have been to alter the model, the method Sangallo chose to execute the alteration reveals a mind bent on solving a problem within constraints imposed by tradition and by the designer himself. The conditions at the beginning of the design must have been:

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[141]Frommel, 'San Pietro,' (Milan, 1984), New York Pierpont Morgan Library, f. 28r.
[142]Plate LVI.
[143]Tracy E. Cooper, n. 116, *The Renaissance from Brunelleschi to Michelangelo*, Millon ed., 1994, p. 499.

1. design a pilaster base on a gigantic scale, similar to the Pantheon, but with a steeper slope and deeper scotiae.
2. The forms from plinth to upper torus cannot exceed six *palmi* and the furthest projection of the plinth to deepest cut of the upper scotia will be one and a half *palmi*.
3. The diagonal line of the resulting rectangle will govern some of the projections and, consequently, the slope of the base.
4. The vertical measure at which one form changes to the next will be governed by a given set of numerical units which has priority over consistency of unit size.
5. Some of the forms, including the scotiae and astragals, will be determined on site by employing constructive geometry, simple fractions and visual judgment.
6. Each consideration must be justified in terms of compositional unity either by geometrical or numerical alignment.

The pilaster drawing is an unusual example among drawings in that it contains a great deal of decipherable evidence of the artist's design process. It is also unusual among *modani*, the template drawings, because it survives. Very few *modani* survive because they were construction guides on the building site. It was a necessary part of the process for the drawing to be destroyed and the only record remaining of the design would be the cut stone molding. Why has this example been preserved? Perhaps because it was a record of Sangallo's inventiveness. Had the profile of the pilaster base been drawn from a program, such as the numerical instructions given by Vitruvius or Alberti, there would not have been anything interesting to preserve. Because the marks and pentimenti on the drawing showed that Sangallo was altering a prototype, perhaps the drawing became a record and justification of the alterations. Many of Michelangelo's *modani* are preserved because they record his unorthodox method of design.[144] Michelangelo did not follow numerical or geometrical prescriptions but drew bold outlines freehand and altered them by eye.

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[144]Cooper, Tracy E., 'I Modani: *Template Drawings*,' *The Renaissance from Brunelleschi to Michelangelo*," by Henry A. Millon and Vittorio Magnago Lampugnani, eds., London, 1994, 494-498.

Even with the excuse that *modani* are seldom preserved, the absence of similar examples by Sangallo or his circle suggests that this type of design was unusual. Most architectural profiles probably did not require such effort. The design approach, however, of using constructive geometry and numerical ratios is fairly common among the Uffizi's collection. An apparently favorite subject was the construction for the volute of an Ionic capital. These, however, are demonstrations and seldom have marks that reveal a design process. One example of a design for a Corinthian capital does depart from prescriptive construction and will be analyzed in the following chapter.

Chapter 6
Sangallo's Capital Design

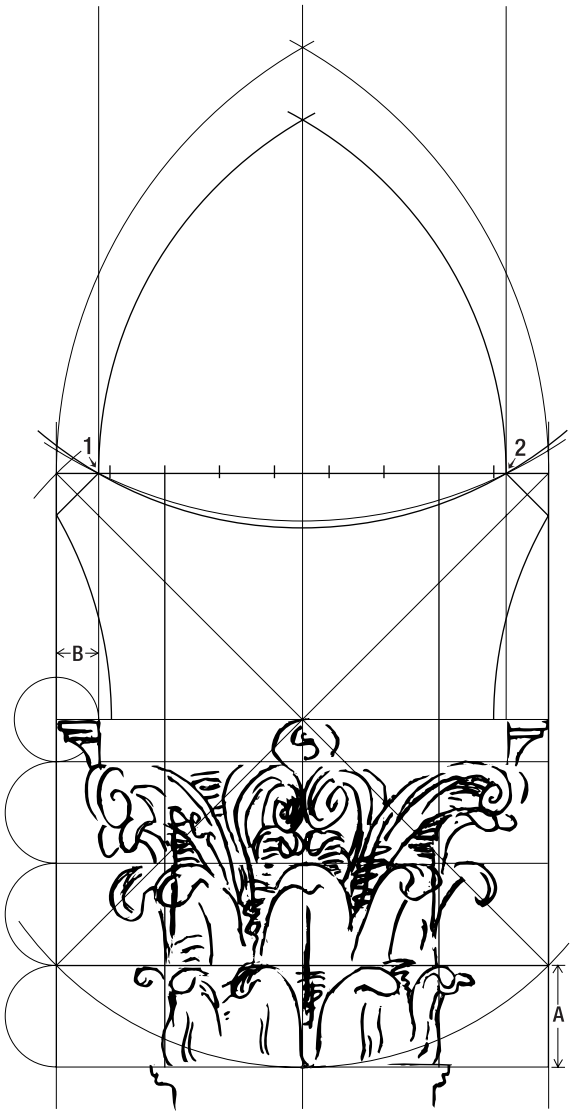


FIGURE 33 UFF. 1121A

Among the Uffizi's collection of architectural studies by Antonio da Sangallo is a small sketch for the plan and elevation of a Corinthian capital (redrawn in figure 33).[145] The procedures that Sangallo employed for this design relate closely to those of the actual size pilaster base (Uff. 7976A): he followed Vitruvius up to a point, then improvised a new, more visually composed design using constructive geometry. The drawing appears odd at first because it combines different views of the capital into one image with apparently extraneous arches. This is because the capital's plan and elevation are constructed together in one plane. The elevation in the lower

[145]Uff.1121A, pen and ink on incised construction lines, 17.5 x 24cm (including restored edges).

portion renders the familiar Corinthian features within a grid of lines: two equal rows of acanthus leaves under a similar row of stalks and volutes supporting a narrower abacus. In the elevation the capital's widths are constrained by vertical lines. Above the elevation is half of the abacus, seen in plan, showing the concave sides and the circles indicating where the column and collar extend. The construction above this plan, which appears to be a Gothic arch, represents the means by which the concavities of the abacus were determined.

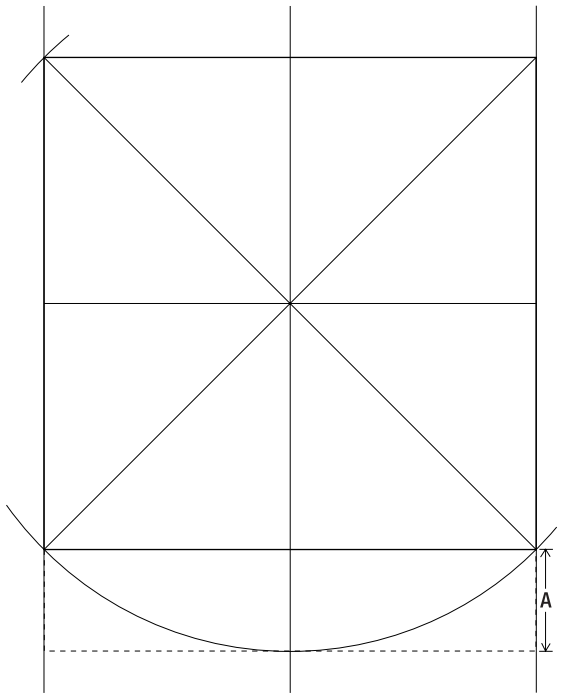


FIGURE 34 CONSTRUCTION SEQUENCE

The construction began with an armature of incised lines around the horizontal and vertical axes. Figure 34 shows the originating square from which the capital was generated. With one foot of the compass at the center of this square an arc was incised extending between the two lower corners. A horizontal line was added tangent to the lower extremity of this arc increasing the height of the square by the amount indicated by the letter A. Here, Sangallo was following the instructions for constructing a Corinthian capital given by Vitruvius:

> The proportions of this capital should be fixed as follows. Let the height of the capital, including its abacus [the uppermost element of the capital], be equivalent to the thickness of the base of a column. Let the breadth of the abacus [in plan] be proportioned so that diagonals drawn from one corner of it to the other shall be twice the height of the capitals, which will give the proper breadth to each face of the abacus.[146]

Vitruvius was using a dimension from the plan to determine the elevation. The corners of the abacus are the corners of the original square. If the diagonal of this square is twice the height of the capital then half of that, the distance from

[146]Vitruvius, *Architectura*, Book IV chapter 1, (ed. Vitruvius, *The Ten Books on Architecture*, trans. by Morris Hicky Morgan, New York, 1960), 106.

the center of the square to one of the corners, is equal to the height of the capital. Using the original square as the plan and the elevation, Sangallo has efficiently illustrated the relationship between the two views of the three dimensional capital just as Vitruvius had done literally.[147] The visible path of the arc makes the relationship visual.

The next step was to take the amount A, copy it with the dividers and transfer it twice onto the vertical axis in the lower half of the original square (fig. 35). Two new horizontal lines from these measures make three horizontal bands of equal widths. The width of the inverted bell area of the capital was determined by dividing the top of the square into nine equal units with the two verticals drawn from units 2 and 7 (fig. 36). The lines below the horizontal axis of the original square now form the complete armature for the elevation of the capital. The upper of the three equally spaced divisions is for the volutes and the other two are given to the acanthus leaves. The narrow space between these bands and the midline of the square determines the height of the abacus. The construction at this point seems to have been made without any revisions which would indicate a change of mind. Vitruvius continues:

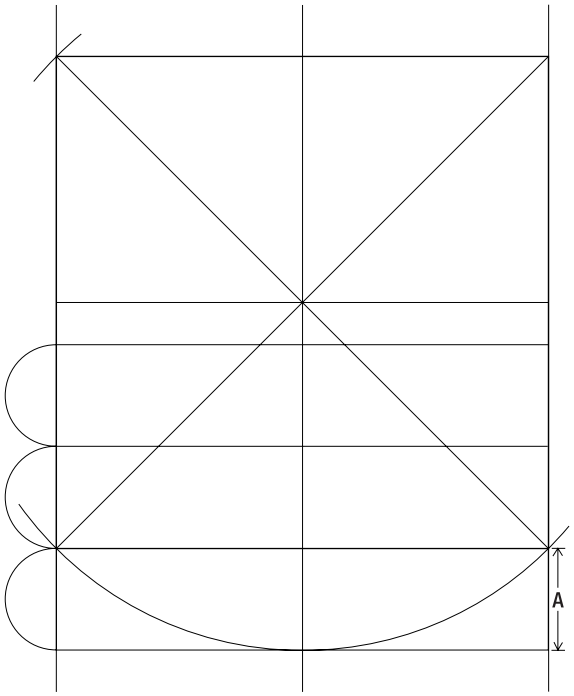


FIGURE 35 CONSTRUCTION SEQUENCE

> The height of the abacus is one seventh of the height of the capital. . . let the rest be divided into three parts, of which one should be given to the lowest leaf. Let the second leaf occupy the middle part of the height. Of the same height should be the stalks, out of which grow leaves projected so as to support the volutes which proceed from the stalks, and run out to the utmost corners of the abacus.[148]

[147]Leonardo does something similar with the *Vitruvian Man* drawing by combining the circle and square with the multi-limbed figure (see Chapter 8).

[148]Alberti's instructions are the same as Vitruvius for the elevation but the plan has ten modules and deeper concavities, *De re aedificatoria*, Book VII ch. 8, (ed. *Art of Building*), 208.

In these instructions the height is divided into seven parts, with one part for the abacus and two parts each for the three lower divisions. Sangallo has followed Vitruvius for the relative sizes of the lower three registers but departed from the recipe with its division into sevenths. He used the line which had been the bottom of the square as the first division, and used the unit A as a close approximation to the required two-sevenths. The difference in size between Sangallo's choice and the prescribed two-sevenths is only two and a half percent.[149] The unit A makes more sense visually than the arbitrary division by seven, because it is a geometrical relationship already present in the construction. It is also not desirable to attempt to divide a given line into seven parts because it requires trial and error which is not exact.

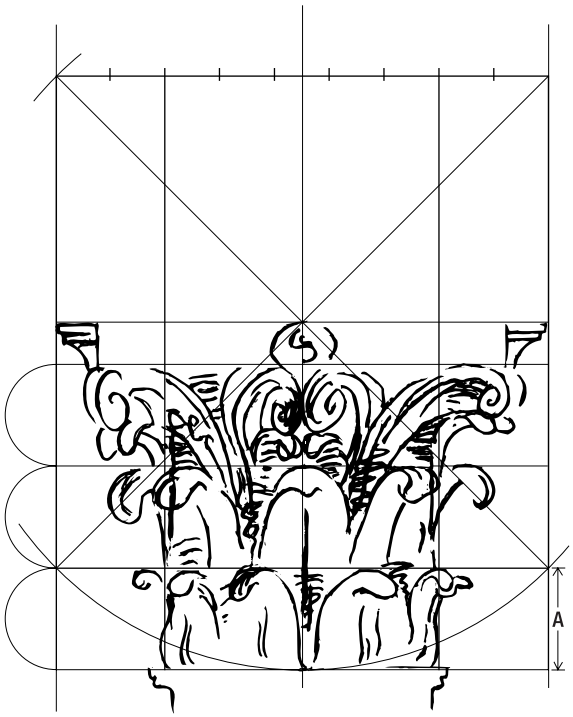


FIGURE 36 CAPITAL ELEVATION

Another construction of a Corinthian capital by Sangallo, Uff.6770 Av, shows that the architect was aware of the Vitruvian requirements (fig. 37). This is a rendering of the section profile of a Corinthian capital showing only the forms which project from the vertical edge. The drawing appears to be an illustration of rules; there are no indications of a design process. Here, the abacus measures one-seventh the total height while the other forms measure two-sevenths each.[150]

The plan of a Corinthian abacus is a square with concave sides and truncated corners. Vitruvius says that "the faces should curve inwards, by one ninth of the breadth of the face, from the outside edge of the corners of the abacus."[151] Alberti says that the faces, "curve inward until the distance between

[149]Using one half of the original square as one unit then the height of the capital is the square root of two, 1.414... because it is equal to the diagonal of a square of one unit. The length of the unit A, then, is .414... compared to two sevenths of the square root of two which is .404....

[150]A small sketch on folio U6341 of the profile of a Corinthian capital has a steep diagonal line from bottom to top that determines the projection of the forms. This diagonal lines appears to have the same function of that in the pilaster base design.

[151]Vitruvius, *Architectura*, Book IV chapter 1, (ed. Vitruvius, *The Ten Books*)106.

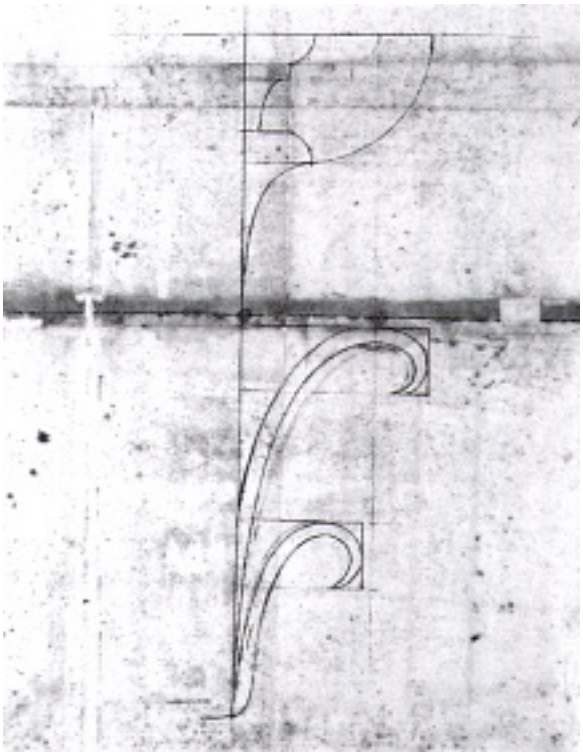


FIGURE 37 UFF.6770 CAPITAL PROFILE

them is the same as the bottom of the [capital]."[152] Sangallo had divided the top of the square into nine units (by trial and error) to determine the width of the capital at the base, but did not follow Vitruvius or Alberti in the construction of the curved faces. For these he experimented with constructive geometry. The curved faces of the abacus were to be made with segments of circles. In order to draw these segments with a compass one needs to know where along the vertical axis to put the pivoting foot. Along the vertical axis are numerous compass holes which represent the different attempts Sangallo made in searching for a new solution more satisfactory than those of Vitruvius or Alberti.

Alberti advocated dividing the width of the abacus into ten modules then clipping off one half module for either corner. Sangallo's solution, however, was to relate the plan to the previous procedures in the elevation (fig. 33). The corners were truncated by the amount B which was the width of the abacus in the elevation. This amount had been one of the results from the first construction. Along the top edge of the square points 1 and 2 were transferred from either end by the amount B. With the compass open between points 1 and 2 arcs were drawn pivoting from each of the two points. The resulting cross at the vertical axis located the new point from which to draw the curving face of the abacus. The arches visible in figure 33 outside those just described were rejected after being inked. These arches originated from above the square on extensions of the sides of the square. The resulting concavity was shallower than the one chosen.

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[152] Alberti, *De re aedificatoria*, Book VII ch. 8, (ed. *Art of Building*), 208.

We have seen that Sangallo has followed the prescriptions of Vitruvius to a point, then altered them based on the application of his own values of three dimensional geometric composition. To Sangallo a geometric solution was better than a numerical one. The reason for Vitruvius' advocating numerical formulae may have been that it was easier to communicate verbally when using numbers than to trying to describe geometrical procedures without illustrations.

Chapter 7
Palladio's Design Process

Andrea Palladio, (1508-1580), was an architect from Vicenza and Venice whose works, including villas, palaces, churches and public buildings, are spread throughout the Veneto. He studied ancient Roman buildings, which he measured to create scaled drawings of their imagined reconstructions. He created his own synthesis of the antique architectural vocabulary and designed and erected buildings that rivaled the antique. He also wrote a practical treatise on architecture. He garnered considerable fame in his lifetime and his legacy of buildings, drawings and literary works have impacted European and American architectural theory and style to such a degree that he is arguably the West's most influential architect.[153]

The following interpretation of Palladio's design process is the result of analysis of his design drawings in the Burlington-Devonshire collection at the Royal Institute of British Architects in London.[154] These drawings contain evidence of Palladio's working procedures for constructing plans and elevations for

[153]For the most recent assessment of Palladio's career see Bruce Boucher, *Andrea Palladio, The Architect in His Time*, (New York, 1998). See also James Ackerman, *Palladio*, (Penguin Books, 1966), revised ed. (Harmondsworth, England), 1977.

[154]My appreciation to the staff at R. I. B. A. for their assistance in allowing me to examine the Palladio collection. For the authenticity of the collection, see Douglas Lewis, *The Drawings of Palladio*, exhib. Cat., (Washington D. C., 1981-1982).

his villas and palaces.[155] The drawings date variously from 1538 to 1547.[156] The goal of the observations was to determine the role of geometrical systems versus that of numerical systems in Palladio's design process by searching for evidence of his techniques for designing plans and elevations. Because the evidence from tool marks indicates Palladio was trying out ideas on the paper and, on many occasions, was changing his mind, it is reasonable to assume that at least part of his design process occurred during the making of the drawings. An analysis, then, of the tracks he left behind should help us understand not only how he created his designs but also the thinking behind his decisions.

When an architect makes a design there are, at the outset, many restrictions affecting the outcome: the nature of the building materials and their costs, the patron's desires, the characteristics of the location, the decorum appropriate to the time and place. There are also the restrictions imposed by tradition on the design process. The two kinds of design systems discussed in this thesis as traditional appear to be present in Palladio's drawings, as was the case with the drawings of Leonardo and Sangallo. The salient issues, with regard to Palladio's drawings, are what was the relative effect of each of these design systems on Palladio's design process and which should be considered central to the architect's design process.

The Conventional View

Before reviewing the evidence from the drawings it is necessary to consider the conventional view of Palladio's design process. Rudolf Wittkower, in his influential book, *Architectural Principles in the Age of Humanism*, created the view that has become the standard model of interpretation of Palladio's work.[157] According to Wittkower, Palladio used a specific numerical system as his central

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[155]The sample, here considered, is restricted to design drawings. The Burlington-Devonshire collection includes numerous examples of drawings after antique monuments. Although these drawings contain draughting tool marks, they cannot be candidates for revealing Palladio's design process.

[156]Lewis, *Drawings* (1981-82). Lewis relates the reasons for the variable survivability of Palladio's drawings, including the consumption during the process of building and the destruction during the process of transferring to woodcut printing blocks for publication, pp. 6-9.

[157]Wittkower, *Architectural Principles*, (1971). Other works that expand upon Wittkower's thesis are Ackerman, *Palladio*, (1966), 160-167; Colin Rowe, *The Mathematics of the Ideal Villa and Other Essays*, MIT Press, (Cambridge, Massachusetts, 1976), 8-16; and Robert Tavernor, *Palladio and Palladianism*, (London, 1991), 37-42.

design process, "Palladio took the greatest care in employing harmonic ratios not only inside each single room, but also in the relation of the rooms to each other, and it is this demand for the right ratio which is at the center of Palladio's architecture."[158] Harmonic ratios are the numerical equivalents of the musical intervals of two-note chords, such as the octave or fifth (1:2 or 3:2). Applied to architecture, these ratios translate into linear dimensions, for example, the ratio of the length to the width in rooms. Three numbers in a sequence can also be considered harmonic, such as when the height of a room is compared to the length and the width.

It is tempting to accept that Palladio was concerned with harmonic ratios. Many numerical relationships occurring in Palladio's design *can* be characterized as harmonic and much weight was given to harmonic ratios by his contemporaries and the architectural treatises available to him.[159] But numbers that are harmonic in the context of a numerical system may also be interpreted as the result of geometric shapes in the context of geometrical systems. Had Palladio used geometrical systems the resulting numbers might also be harmonic by coincidence.

Wittkower builds a case partly on an analysis of the numerical proportions that are published in Palladio's treatise, *I quattro libri dell'architettura*, but much of his argument rests on works by theoreticians in Palladio's circle. Daniele Barbaro, for instance, was a close friend and collaborator of Palladio who published a translation of Vitruvius with extensive commentaries. These commentaries include much on theories of proportion and detailed mathematical explanations for numerical systems of harmony. Palladio himself wrote little about proportional theory and nothing about harmonic systems in his treatise, which is mostly concerned with practical matters of architectural design. There is one statement in *I quattro libr*i however, that Wittkower considers "very important" for his argument:

> But the large rooms ought to be so related (*compartite*) to the middle ones, and these to the small, that, as I have said elsewhere, one part of the building may correspond with the other, so that the whole body of the

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[158]Wittkower, *Architectural Principles*, (1967) 72.

[159]e. g. Vitruvius, *Architectura*, Book V, Chapter IV, Alberti, *De re aedificatoria*, Book IX, Chapters V and VI. The treatises of Vitruvius, Alberti and Filarete treat numerical systems as an ideal, intellectual subject while they considered geometrical systems as being more "hands-on" and practical.

> edifice may have in itself a certain harmony (*convenienza*) of members which may make it entirely beautiful and graceful.[160]

Wittkower translates the Italian word, *convenienza*, as "harmony" perhaps to strengthen his thesis that Palladio employed harmonic ratios. But *convenienza*,

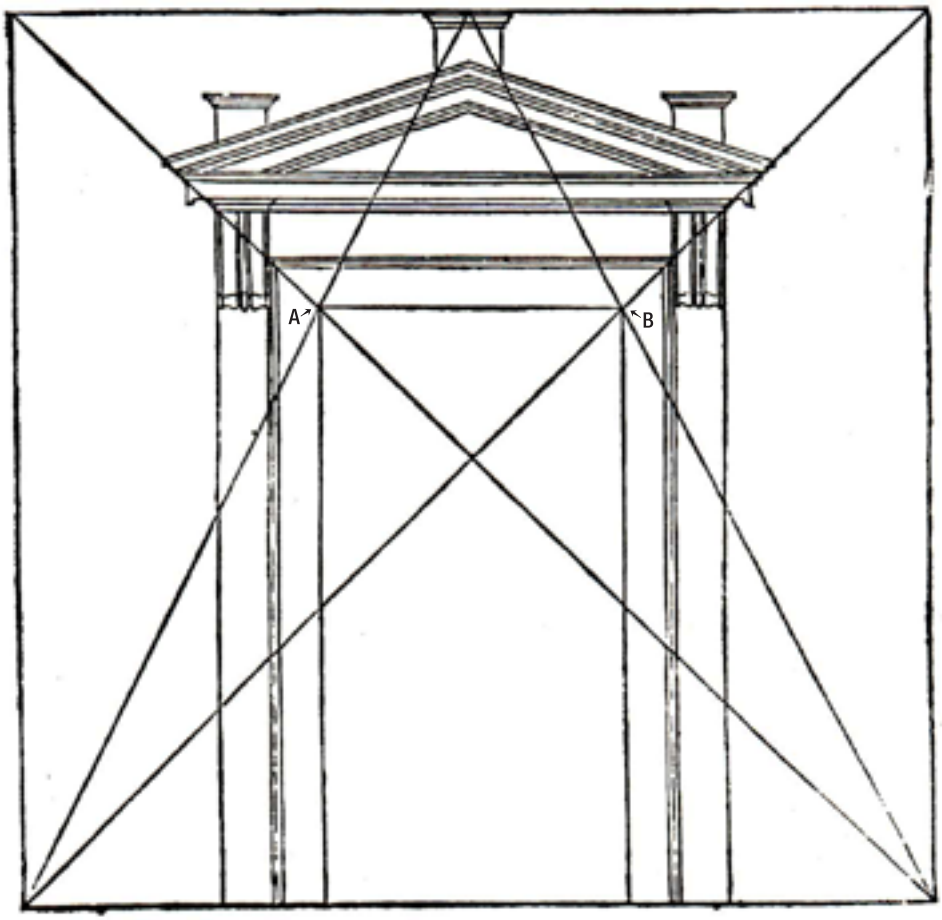


FIGURE 38 CONSTRUCTION OF A DOOR - SERLIO

in this context, has no numerical significance. The word is rendered "an inherently suitable distribution" in the newest translation by Robert Tavernor and Richard Schofield.[161] Wittkower appears to be conflating the meaning of "harmony" in the sense of "beautiful and graceful" with that of harmonic ratios. The quote from Palladio suggests only that architects should make the different sized rooms correspond to one another for the sake of convenience, beauty and grace.

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[160]Wittkower, *Architectural Principles*, (1971), 130. The quote is from Andrea Palladio, *I quattro libri*, Bk. II, chap. 2.

[161]Andrea Palladio, *I quattro libri dell'Architettura*, (Venice, 1570), (ed. *The Four Books on Architecture*, trans. by Robert Tavernor and Richard Schofield, MIT Press, Cambridge, Massachusetts, 1997), 78.

An analysis from *Architectural Principles* of a drawing by Sebastiano Serlio (1475-1554), gives some indication of Wittkower's overall approach by focusing on what he calls a, "fugal system of proportions (fig. 38).[162] The drawing is of a doorway; its frame and pediment are placed inside a square that has diagonals drawn from each corner to the opposite corner and from the lower corners to the top center. The intersections of these diagonals (I have labeled A and B), locate the upper corners of the doorway while the corner-to-corner diagonals coincide with the outer edges of the pediment. The drawing, in its apparent application of a geometrical design system, recalls the elevation section of a church by Francesco di Giorgio, (fig. 7).

Wittkower rejects the interpretation that the design of the doorway is the result of an *ad quadratum* geometrical design system practiced in the Middle Ages. He correctly identifies the irrational relationships between the two kinds of diagonals and the side of the square, but his view of Italian Renaissance design theory does not include the use of irrational relationships. Wittkower reconciles the appearance of the geometrical scheme with the numerical system that he argues for, by stating that the geometrical scheme was arrived at after the ratios were chosen for the door. The ratio of the rectangular doorway, it turns out, is one to two. This fortuitous commensurability occurs because the intersections of the diagonals at points A and B divide the square into three equal parts. The doorway has the width of one of these parts and the height of two parts. "Thus," Wittkower concludes, "an interrelated series of ratios of small integral numbers is really at the basis of Serlio's design. 'Mediaeval' geometry here is no more than the veneer that enables practitioners to achieve commensurable ratios without much ado."

In my opinion, Serlio's door is an illustration of exactly what it appears to be: the application of a geometrical system. It is true that the doorway has a ratio that, within a numerical system, would be considered harmonic. However, in the light of the drawing's context, the numerical interpretation appears anomalous. Serlio's design for a door is the last item in Book I of his architectural treatise. Book I deals exclusively with geometry and the practical application of geometry

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[162]Wittkower, *Architectural Principles*, (1971), 126-127. The illustration is from Sebastiano Serlio, (ed. *On Architecture*, vol. 1, trans. by Vaughan Hart and Peter Hicks, Yale, 1996), p. 33. Book 1, *On Geometry*, from Serlio's treatise, was first published in 1545, twenty-five years before Palladio's treatise was first published.

for the architect. It begins with the definition of a point, as Euclid's *Elements* does, and continues, in steps, to define and illustrate basic geometrical problems that an architect might encounter. The item prior to the door design, in Book I, is a demonstration of how to locate the centerpoint of a circle within an arched pediment, so that an oculus can be drawn. The arched pediment is drawn enclosed by a rectangle, subdivided by the same diagonals as the door design. The intersection of the corner-to-corner diagonals locates the centerpoint of the oculus while the other intersections determine the width of the oculus. The door design appears to be the result of the same rule-of-thumb geometrical procedure for determining the size and position of an opening within a given rectangle. Nowhere in the book is there any mention of the numerical implications of the geometrical figures.

To account for Wittkower's conclusions, it appears that he is holding at least three assumptions regarding the evidence he considers both in the case of the Serlio drawing and with Palladio's works. These assumptions are that geometrical systems deal primarily with incommensurable ratios; the appearance of a ratio of small integers necessarily implies a numerical system; and the use of geometrical systems would be anachronistic for Renaissance architects. I believe that Wittkower's analysis suffers on account of these assumptions because they prevent him from seeing what, to me, appears to be obvious. Firstly, the idea of commensurability does not apply within a geometrical system. For example, the procedure for doubling the length of a square is almost identical to that for increasing length of a square by the square root of two. The former involves commensurable ratios, the latter does not. As far as the geometrical system is concerned, the fact that the diagonal of a square is incommensurable with the side is not relevant. Secondly, ratios of small numbers are a feature of both numerical and geometrical systems so that the presence of ratios does not necessarily imply numerical systems. And lastly, as we have seen, some Renaissance designers did use and write about geometrical systems.

There is not universal agreement concerning Wittkower's interpretation. Bruce Boucher believes that Wittkower has invested Palladio's treatment of proportion with "excessive theoretical ballast more worthy of Alberti or Danielo

Barbaro."[163] Howard Burns comments, "Wittkower's reservations and qualifications. . .are probably nearer the mark than his interpretation itself."[164] Burns believes that Palladio, like every other educated person of the time, knew about proportions and harmonic numbers and tried to use them whenever practically possible:

> Palladio's desire to create a unified assemblage of parts, and his arrangement of rooms in symmetrical bands on either side of the sala meant inevitably that a short side of a larger room would end up as the long side of a smaller one. And, with this as a starting point, his preference for certain basic geometrical forms inevitably generated proportional interrelations throughout the building.[165]

Wittkower inferred the existence of a design process based on harmonic numbers because in the context of musical ratios, many of the numbers in Palladio's published designs can be interpreted as harmonic. This inference was questioned by Deborah Howard and Malcolm Longair in an analysis of Palladio's published designs.[166] They demonstrated that in the context of musical ratios, Palladio's numbers are not consistently harmonic. Their analysis was much more scientific and complete than Wittkower's, whose argument rests on a limited selection of works. Howard and Longair took into account all of the plans from Book II of Palladio's treatise, *I quattro libri*, and even used a far more generous definition of harmonic numbers. They combined different systems that Palladio might have used, the JUST scale and the Pythagorean, to arrive at a list of whole numbers that could individually be combined in any way to make harmonic relationships. Although it is unlikely that Palladio would have used more than one system at a time, the list of numbers was designed to give the architect the widest possible margin to see if he had used harmonic numbers. Even though the possibilities included thirty-four numbers out of those from one to 100, still only two thirds of the numbers that Palladio used in his printed plans were harmonic. On occasions, some works were left un-harmonic when only insignificant changes would bring them into harmony, implying that the numbers were not part of a

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[163]Boucher, Bruce, *Andrea Palladio*, (New York, 1998), 211.
[164]Burns, Howard, with Lynda Fairbairn and Bruce Boucher, *Andrea Palladio 1508-1580 The portico and the farmyard*, (The Arts Council of Great Britain, 1975), 224.
[165]Burns, *Andrea Palladio*, (1975), 224.
[166]Howard, Deborah and Malcolm Longair, 'Harmonic Proportion and Palladio's *Quattro Libri*,' *Journal of the Society of Architectural Historians*, (May 1982), 116-143.

harmonic system in the first place. Further, the presence of harmonic numbers could have resulted from the coincidence that most of Palladio's favorite room shapes, in addition to being harmonic, also happen to be equivalent to ratios of small whole numbers. These simple ratios would have been desirable for their ease of use, irrespective of their musical interpretation. Howard and Longair did not consider a non-numerical explanation for Palladio's design process.

The modern idea that Palladio could have used harmonic numbers as the center of his design process may also have been encouraged by the kind of drafting tools used in our time, which circumvent the need for constructive geometry. With a parallel bar and right triangle a modern designer can draw a square that is mechanically "true." This encourages the designer to think of a square as the ratio of 1 to 1 instead of a geometrically constructed object. When one thinks of a square as simply a numerical ratio then the process of creating the square becomes irrelevant. My interpretation of Palladio's design process hinges on how the geometrical objects were conceived.

In my view, the evidence from Palladio's training, writing and drawings suggests that numbers may not have been the controlling method as Wittkower states, especially when it came to the actual decisions made while designing. As we saw in the case of Sangallo's pilaster, numerical and geometric systems were both used in the same design while neither could definitely be said to be the controlling method in that process.

As detailed in Chapter 1, constructive geometry involves the manipulation of a compass and straightedge to locate points by making intersections with ruled lines and arcs. These located points then become the footings for geometric shapes whose parts interconnect and reinforce each other, forming a stable fabric. Tools are an essential part of a design process based on constructive geometry. Harmonic numbers, however, are an intellectual construct with no need for tools. Only numerical relationships matter in such systems. Composing with musical ratios may result in works with complex numerical relationships, but as a design process it is more akin to creating an arithmetical puzzle, such as a magic square, than to constructing a physical object.[167]

The evidence from Palladio's writings, drawings as well as from his stone

[167] A magic square is a square containing a number of integers so arranged that the sum of the numbers in each row, column and diagonal is always the same.

mason background indicates that the architect used tools to construct the initial shapes, and afterwards considered the numbers. Palladio was a stone mason and sculptor until he was thirty. He was trained in the same kind of environment as Mediaeval builders and designers, working with tools to craft physical objects and employing techniques valued for their simplicity and utility. He handled his draughting tools as a skilled craftsman, and even for sketches he would deftly undertake what would seem to be laborious procedures. This suggests that his work habits carried over into his design practices.

In his writing Palladio does not support the theory that harmonic numbers were at the center of his architecture. On the contrary, the design instructions in his book, *I quattro libri*, reveal Palladio's geometrical thinking. In fact, Palladio never refers to numbers as harmonic or explicitly advocates designing with musical ratios. Instead, he speaks in terms of geometric shapes. In Book I, for example, he recommends the ideal shapes for rooms:

> There are seven types of room that are the most beautiful and well proportioned and turn out better: they can be made circular, though these are rare; or square; or their length will equal the diagonal of the square of the breadth; or a square and a third; or a square and a half; or a square and two thirds; or two squares.[168]

Although all but one of these room shapes can be expressed, in numerical terms, as musical ratios, Palladio chose to describe them, instead, as geometric shapes. He describes five of the room shapes in terms of their construction from squares. The idea of making rooms geometrically out of squares is again expressed in his recommendation for the length of halls: "Usually I do not make halls longer than two squares, which are derived from the breadth, but the closer they are to being square, the more praiseworthy and practical they will be."[169] The presence, then, of ratios that can be termed "harmonic" is the coincidental effect of designing with room shapes derived from squares. In fact, the third room, "the diagonal of the square," can only be described as a constructed

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[168]Palladio, *I quattro libri,* Bk. I, ch. 21. "Le piú belle e proporzionate maniere di stanze, e che riescono meglio, sono sette, perchioché o si faranno ritonde, e queste di rado, o quadrate o la lunghezza loro sará per la linea diagonale del quadrato della larghezza, o d'un quadro et un terzo, o d'un quadro e mezo, o d'un quadro e due terzi, o di due quadri." (ed. *The Four Books,* Tavernor, 1997), 57.

[169]Palladio, *I quattro libri,* "Io sono solito non eccedere nella lunghezza delle sale due quadri. I quali si faccianno dalla larghezza. . ." (ed. *The Four Books,* Tavernor, 1997), 57.

geometric shape because of its ratio of one to the square root of two.
This ratio cannot be expressed in terms of whole numbers, which is a necessary condition for musical ratios

Another example of description in geometric rather than numeric terms occurs in Chapter 23 of the first book of *I quattro libri* where Palladio considers the heights of vaults of ground floor rooms. In order for the floor above the ground floor to be level it is necessary that the vaults underneath rise to the same height. The problem is how can the same height be proportional to all the different horizontal dimensions of the rooms. The solution is to use different procedures to arrive at the heights of the different rooms. Palladio gives three procedures, the first being higher than the second and the second higher than the third. He illustrates them with three geometric diagrams and explains each, first by describing the geometric procedure then by giving the equivalent numerical operation.[170] Since the calculations for each room must arrive at the same answer, the purpose of the calculations is to justify the heights for each room. Even with three different procedures there would be occasions where a height could not be justified and in these cases Palladio says that the architect should use his judgment according to necessity.

This last statement sums up Palladio's approach to the business of architecture. It is practical. The picture that Wittkower paints of Palladio's intellectual environment, including scholars devoted to theory, may be entirely correct. But, Palladio himself eschewed theoretical considerations in favor of those which were useful to the builder. When compared to previous architectural treatises *I quattro libri* has very little to say about theory. The prose is pared down to the minimum and the pictures carry much of the explanations. Although Wittkower's argument is partly supported by the numbers in Book II, where there are plans for Palladio's own designs, the text does not mention the harmonic implications of these numbers.

The case made here against Wittkower's interpretation of Palladio's design theory is not meant to expunge all vestiges of harmonic numbers from the architect's work. Palladio, certainly, must have been aware of all of the numerical systems that were discussed by his peers. He probably could not help but feel

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[170]The same three operations were given by Alberti, *De re aedificatoria*, Book XI/6, (ed. *On the Art of Building*), 308-9. Alberti comments that the geometrical mean is very difficult to ascertain numerically, although it may be found very easily using lines.

added justification whenever he used almost any ratio. But the issue here is what is at the center of his architecture. I contend that if harmonic numbers were at the center of Palladio's architecture, then he would have used them consistently, advised his readers to do so and told them why. He would also have avoided advice that was incompatible with harmonic systems and avoided using incompatible measurements in his own work. Since he did not, I conclude that harmonic numbers were not central.

Observations of Palladio's Drawings

From theory we turn now to practice. The large number of fairly uniform drawings under consideration reveals a similar dialogue between numerical and geometrical systems as was evident in the previous chapters on Leonardo and Sangallo. The following analysis is based on close examinations of seventy-six drawings from volumes XIV, XVI and XVII of the Palladio collection at RIBA. The drawings are architectural plans and elevations of villas and palaces and consist, for the most part, of various arrangements of rectangles. Unlike Palladio's more numerous studies of antique monuments, the drawings are original designs, not renderings of existing buildings, and therefore provide a unique opportunity for studying his design methods.

The drawings are in pen and brown ink with occasional metal point or chalk. The common elements of the ground plans are straight lines meeting at right angles to form rectangles of various sizes. Circles and semicircles also occur, though less frequently. The elevations, in addition to the straight lines and right angles, also include renderings of ornamental elements such as columns, pediments and the patterns of stone courses. All the drawings are bilaterally symmetrical, and variations of the size and shape of elements occur within a small range of possibilities.

In most of the drawings, raking light reveals numerous incised lines, arcs and point-like indentations in the surface of the paper. The incised lines were made with a stylus[171] drawn lightly against a straightedge or, in the case of arcs, drawn with dividers. The shallow, point-like indentations were sometimes made

[171]Burns, *Andrea Palladio*, (1975), 225. Burns remarks that a sharpened stylus would scratch the paper and cause the ink to bleed. A soft metal such as silver, used as a stylus, would leave no mark on unprepared paper while insuring as little violence to the paper as possible.

by the pivoting foot of the dividers or compass while other points were pressed into the paper with a stylus to mark locations. Some indentations pierce the paper while others are very shallow and can only be seen under light directed from certain angles.

The evidence from these tool marks indicates Palladio's drawing technique was first to incise lines, points and arcs and then to use these indented marks as guides for the inked lines. For example, when he needed several evenly spaced parallel lines to indicate stairs or stone courses, he would first make two rows of indentations by stepping the dividers; then he would connect the indentations with lines incised against a straightedge. If the position and spacing of this incised armature were acceptable he would then redraw the lines with ink. Sometimes incised lines appear that are not inked, indicating either that a change of mind about the position of the lines, or that the incised lines were used as reference in the process of construction, as described below.

The evidence of changes of mind is not due to later corrections following clients' comments. Most of the pentimenti are from the incision stage of the drawing, before ink was applied. At this stage, a drawing that is entirely incised is very difficult to see and wouldn't be shown to a patron in that state for judgment. Often, walls, stairwells and vaulting shapes were incised in one place and re-incised and inked in another place. The only explanation for this is that the designer changed his mind while designing.

The drawings considered here consist of forty-six ground plans and thirty elevations. Typical of Palladio's work, each of the ground plans has a main structure that is bilaterally symmetrical, and all but one have an incised axis, which I term the "origination line." It appears that this line is the reference from which the rest of the design is measured. The elevations are also bilaterally symmetrical, with the base lines of the elevations generally used as the origination line. The points that became the corners of the rectangular rooms were located by stepping the dividers a number of units from the origination line or its perpendicular. Equidistant indentations along a line reveal where the individual modules were counted. Once a scale was made, the dividers could be opened to a given numerical measure and this measure could be transferred to any location requiring that distance. The numbers would then be written within the rectangle to show the dimensions of the length and width.

In a typical design for the plan of a building, Palladio began with an incised origination line (line O) longitudinally bisecting the paper and then from this line constructed the first perpendicular using one of two crossed arc constructions (figs. 39 and 40).

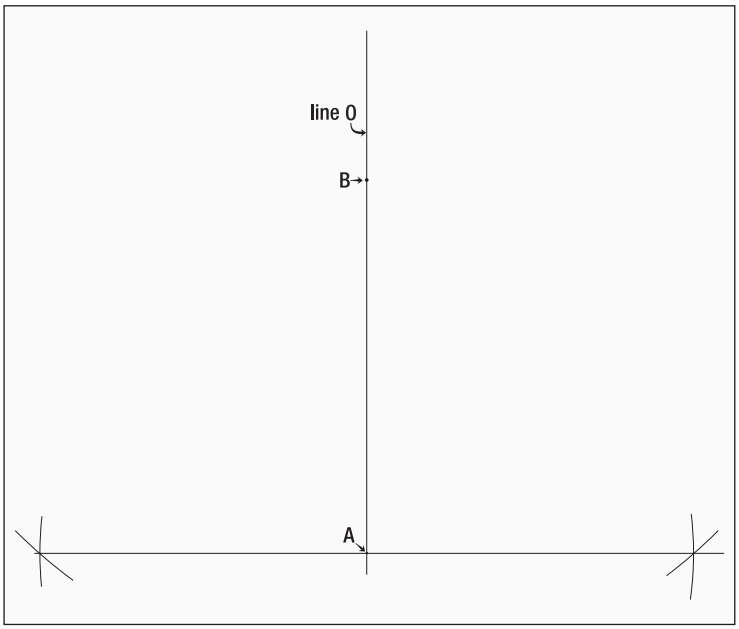


FIGURE 39

Near the lower end of line O Palladio chose a point and marked it with an indentation [point A]. This would locate on the plan the future position of the façade wall. Then, using dividers open to a width approximating (but not necessarily coincident with) half the anticipated width of the façade, he placed one foot at point A and, with the other foot, incised two short arcs to the left and right of Line O in the general area of ninety degrees. Then, opening the dividers wider and moving the foot to a random location [point B] further towards the center of the paper on line O, he inscribed two more arcs crossing the first two. He then drew an incised line through the two intersections of crossed arcs passing through line O at exactly ninety degrees.

A similar crossed arc construction was used mostly for façade elevations (fig. 40). First, Palladio incised line O near and parallel to the bottom edge of the paper. This was to become the base line of the elevation. An indentation [point A] marked the center of this line. The dividers, open to a random width, measured and marked with indentations two points (B) along Line O equidistant from point

A. From points B the dividers, opened wider now, measured and located with crossed arcs point C above point A. The perpendicular line through points A and C became the vertical axis of the elevation.

Crossed arc constructions are evident on eight of the forty-six ground plans and eleven of the thirty elevations.[172] Half of the elevations were begun

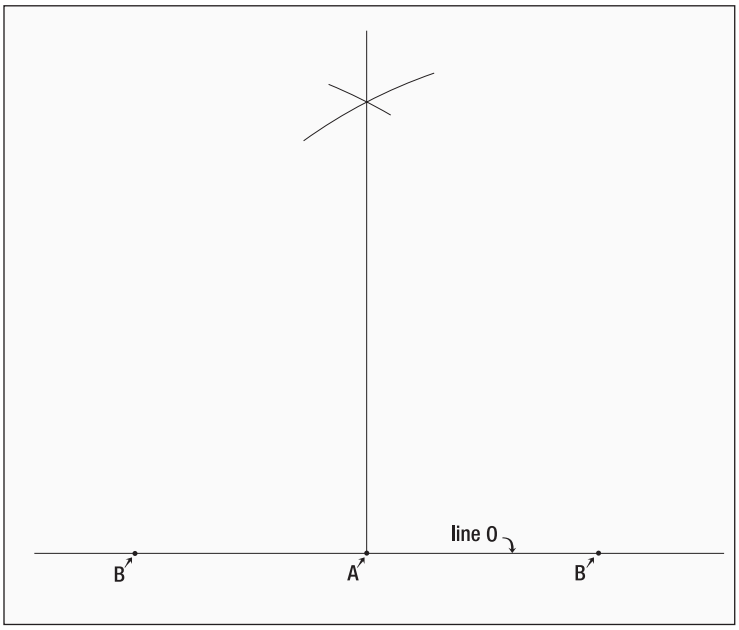


FIGURE 40

with the first crossed arc method[173] while two of the ground plans used the second crossed arc method.[174] On seven drawings a plan and elevation of the same building occur on the same folio and share a common origination line.[175] There are also no occasions where two crossed arc constructions occur in the same drawing. Once an axis and a perpendicular are created the rest of the lines and angles are measured from them.

The incidence of crossed arcs is not 100% because the drawings represent a variety of stages in the design process. Once the axes are found in a drawing they can easily be transferred to another drawing by pricking. This method is, in fact, easier than the original crossed arc construction, to the extent that one might

[172]Plans: Volume XIV numbers 13, 16; Volume XVI numbers 1, 6, 8, 9, 12, 14B, 15, Volume XVII numbers 15, 18, 27. Elevations: Volume XIV number 14; Volume XVII numbers 2, 9, 11, 12, 17, 18, 19, 20, 21, 23, 26, 27.
[173]Volume XVII numbers 2, 17, 19, 23, 26.
[174]Volume XVI/12 and Volume XVII/27.
[175]Volume XVII numbers 1, 2, 14, 16, 17, 18, 27.

wonder why an architect would bother repeating the construction more than once in his lifetime. The possibility that there is a ritual aspect of the construction is explained in Chapter 2 on the crossed arcs method.

There is no evidence that Palladio used a set square or other template to make right angles. A template in the shape of a right triangle, such as we use today, would leave a broken line at the intersection with the origination line. All lines cross the origination line unbroken. Using a triangle would also have been out of character, because of the imprecision involved in aligning the tool to the incised line. Imprecision would not be accepted in a central structural member of the drawing. It would be like a construction worker guessing that his wall was vertical rather than using a plumb line.

Incised lines also reveal Palladio's method for drawing voussoirs over arched doorways and windows (fig. 41). A rectangular opening was first incised with a semicircular arched top dotted along its circumference with ten equally spaced indentations to make eleven voussoirs. Trial and error was necessary to

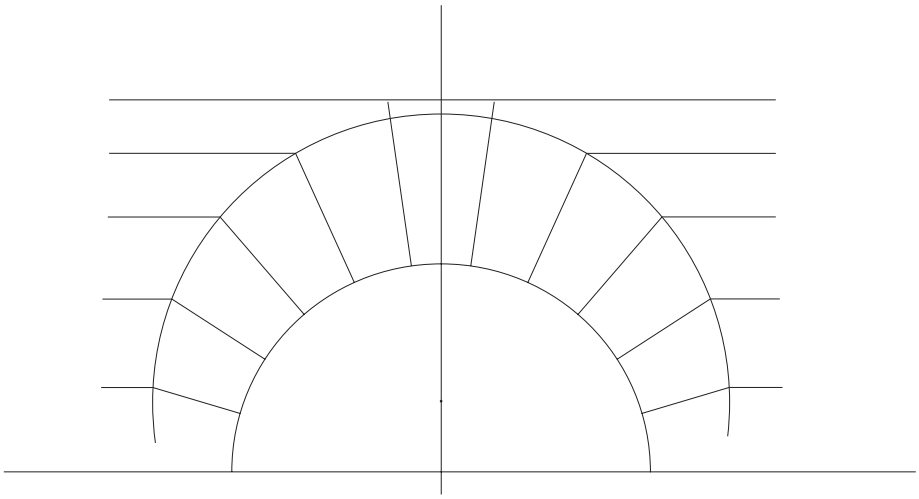


FIGURE 41

determine the divisions of the semicircle because Palladio had no geometric method for dividing a semicircle into eleven equal parts. One foot of the dividers was then placed a measured distance above the center of the arch and another larger arch was incised. Lines radiating from the center of the inner arch were then incised passing through the ten indentations. The intersections of these radiating lines with the outer arch became the corners of the voussoirs and determined widths of the horizontal stone courses. The larger circle remained uninked in the finished drawing leaving the corners of the voussoirs to express the

vestige of the outer arch.

For one elevation, Palladio employed this technique in part of the design and pricked the intersections through to the other side with the paper folded. Folio XVII/11 is a two-story elevation with two arched windows on the ground floor, flanking a central arched doorway. Several indentations along the inner arches reveal that the two left windows were constructed as in figure 38. Since the number of trials is greater for the far left window than the inner window one can suppose that the trial and error process began on the far left and the results transferred to the right. Even with all the experimentation, however, neither of the windows have exactly uniform voussoirs. Indentations also occur on the left half of the arch of the central door. The two windows on the right and the right half of the center door have no incised lines and the paper shows evidence of having been folded in half along the vertical axis. There are also no experimental indentations on the right side. The voussoirs of the right two windows have indentations, which were pierced from the back of the paper through to the front, marking the intersections of the voussoir divisions and the masonry courses. Apparently, in order to save time, Palladio made all the necessary construction marks on the left side then folded the right half around the back of the left half and pricked the intersections with a stylus.

Another construction involving indentations was used to draw five octagons on three ground plans.[176] Palladio created his equilateral octagons by truncating squares (fig. 42 steps 1 and 2). First, a square was incised with its diagonals (fig. 42 step 1). Then, a measure was taken with the dividers, between the center [A] and the side of the square [B]. This measure was then transferred along each diagonal with one foot of the dividers on the center and the other foot making indentations on the four diagonals radiating from the center [C]. These indentations located the centers of small circles that became the corner niches of the octagon. The measure from A to B also was used to make short arcs with centers at D on the diagonals. The points marked D were located by an unmeasured distance from the center. These short arcs located the truncated sides of the square. The bold lines in figure 42 (step 2) indicate the final positions of the walls[177]

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[176]Volume XVI/8 and 19, Volume XVII/16.

[177]There is a much simpler method for determining the side of and octagon inside a square illustrated in Book 1 of Serlio's treatise, folio 19r.

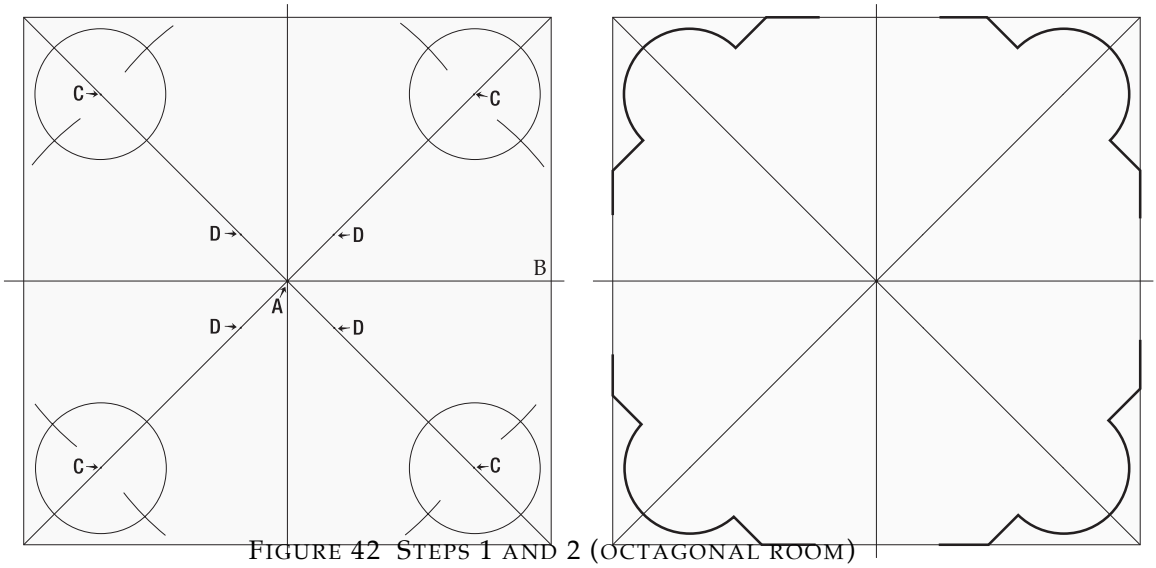


FIGURE 42 STEPS 1 AND 2 (OCTAGONAL ROOM)

As with the favorite room shapes, the octagonal rooms are derived from squares. In a numerical system, the units used to measure the sides of the squares could not be used to locate the sides of the octagons. This is because their relationship to the sides of the square is irrational. The octagons could only have been constructed geometrically.

But, what is at the center of Palladio's design process? To summarize, the question is whether Palladio chose a given dimension for its harmonic qualities or for its geometric shape. We have seen that Wittkower's thesis may have been founded on erroneous assumptions. If harmonic numbers governed Palladio's decisions, then he was not very adept at using them. Even when harmonic numbers are defined so generously that thirty four numbers out of one hundred are harmonic in any combination, Palladio's actual numbers are only two thirds harmonic. He used them inconsistently; he failed to make minor adjustments that would have made buildings harmonic, he repeatedly used numbers that were not harmonic; and he advocated as an ideal room shape a rectangle which could not be expressed as a simple ratio. We have seen that the physical evidence visible in the original drawings indicates that Palladio used tools to make shapes using constructive geometry; that constructive geometry implies a practical design process where the use of tools affects the thinking behind the process; that making things with constructive geometry is analogous to building on the paper, which is appropriate to architecture; and that the geometric design process, here described, is also wholly consistent with Palladio's training as well as his writing.

Chapter 8

Case Studies

This last chapter gathers together some briefer case studies, which, on the surface are very heterogeneous, but serve to demonstrate that different artists faced with apparently different design tasks can resort to the same or similar design resources. These case studies include Leonardo da Vinci's, *Study of Human Proportions in the Manner of Vitruvius*; Paolo Uccello's (attributed) *Perspectival Study of a Chalice*; an anonymous Italian architectural drawing, from the 16th or 17th Century; Michelangelo's *Project for San Giovanni dei Fiorentini*; and Baldassarre Peruzzi's *Central-Plan Project for San Giovanni dei Fiorentini*. All these case studies provide comparative examples and data that show how tool mark analysis can potentially be extended across a wider range of drawings than might be expected. Not all drawings are as extensively marked by draughting tools as those in the preceding chapters, yet some informative interpretations can be gleaned from limited evidence. For some drawings the problems of interpretation comes not from a dearth of evidence but from a surplus. The design process, in these heavily marked works, is such that there may be a tangle that is too complex to unravel.

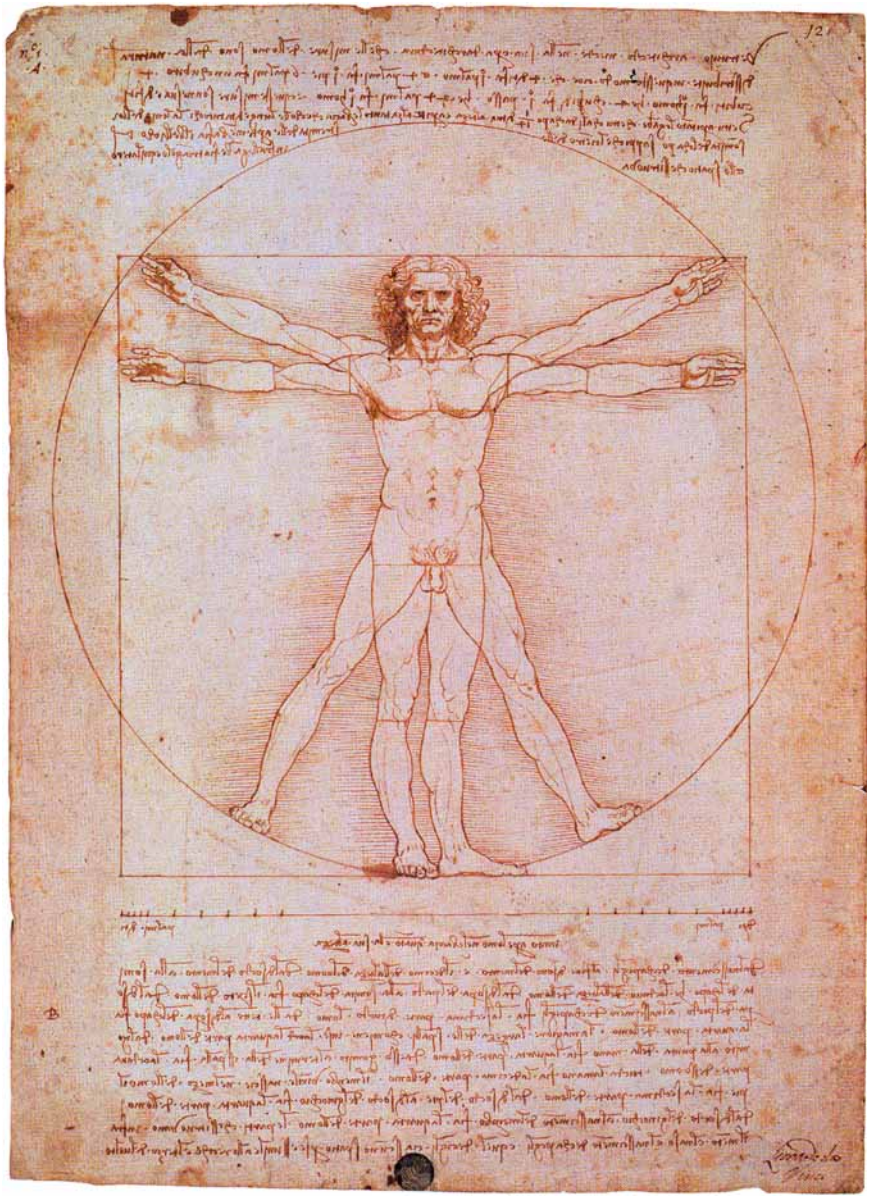


FIGURE 43

Leonardo da Vinci
(1452-1519)
Study of Human Proportions in the Manner of Vitruvius
c. 1490
pen, brown ink, wash, and red chalk on paper
34.3 x 24.5
Gallerie dell'Accademia, inv. 228, Venice.

The examination of Leonardo's famous *Vitruvian Man* drawing did not yield sufficient evidence for an independent chapter (fig. 43). This is because the figure and geometry were not designed or constructed on this sheet but were part of a process that began on another sheet or sheets, now lost, and mechanically transferred. This might then be considered a "clean copy" taken from a source that contained the evidence of Leonardo's design process. Nevertheless, there are important points to be made regarding the marks on the drawing within the context of our theme of geometrical and numerical systems of design.

The transfer was made by placing the original on top of this rather thick sheet and incising the lines of the figure with a stylus.[178] The impression from the top sheet left a deep groove along the contours of the figure (except for the top of the head) and shallower grooves for some of the figure's inner details. The geometry was transferred by means of tiny pricked holes through the necessary points from the top drawing: the corners of the square, the center of the circle and the intersections of the circle with the square. The geometry was then inked with a ruling pen and the figure, with its hatched shading, with a very fine quill.

There are countless point-like impressions in the paper from dividers at various locations that are not part of the construction process. The left side of the square is pockmarked with at least a hundred of these points indicating that dividers walked this length numerous times with various measures. Other points cluster along the vertical axis of the figure: the navel, the xiphoid process, around the calf and on the face, especially the nose. There are points on the left horizontal forearm and hand and under each spread foot are two points in line with the circle along the axes of the legs at a distance of 7 mm. All of these points record the process of measurements taken *from* the drawing, as if Leonardo, after making the figure, regarded it as a standard from which ratios could be obtained.[179] The process of determining ratios by walking the dividers along the edge and changing the measures is reminiscent of the process used in the later drawing, the *Madonna and Child with St. Anne*, analyzed in Chapter 4.

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[178]Bambach, Carmen, *Drawing and Painting in the Italian Renaissance Workshop,* (Cambridge, 1999), 308. Bambach suggests that the transfer process involved *spolvero*, pounced dots, as indicated by infrared reflectography. These dots were not visible in normal viewing conditions.

[179]Kemp, Martin, *Circa 1492, Art in the Age of Exploration*, Jay A. Levenson, ed., (National Gallery of Art, Washington, D.C., 1991), 277.

Illustrations of Vitruvius' prescription regarding human proportions had been made before, most notably by Francesco di Giorgio. But Leonardo's illustration was new in that it combined the two elemental shapes, the square and the circle, into one diagram. In order to show the different positions of the two figures, Leonardo combined them into one with transparent overlapping at the intersections. Although the multi-limbed figure looks natural enough in its proportions, it must have taken numerous trials to construct a figure that so perfectly matched Vitruvian proportions. Once the Vitruvian figure was drawn with its enclosing geometry, Leonardo set about exploring further the numerical consonances within the human body.

This thesis argues that the design systems using numbers and geometry were a tradition among designers from ancient times to the Renaissance. This drawing is a testimony to the validity of that claim. Here, a Renaissance artist not only creates a definitive demonstration of an ancient Roman's precepts but also takes it further. He acknowledges the validity of Vitruvius' system but also explores the possibility that the system could be expanded to include even more numerical ratios.

Paolo Uccello (attributed)
1397-1475
Perspectival Study of a Chalice
c. 1450-1470
pen and dark brown ink, Uff.1758A (29 x 24.5)
Galleria degli Uffizi, Gabinetto Disegni e Stampe Florence

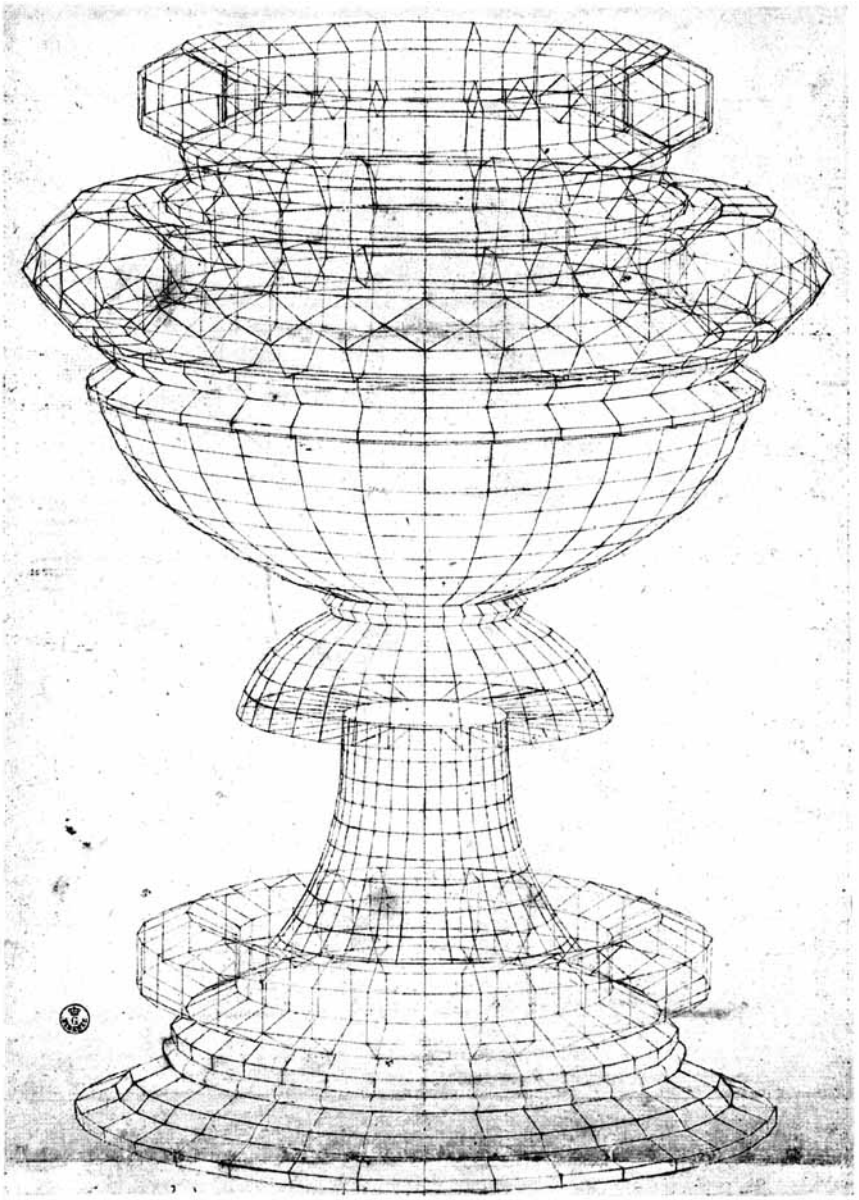


FIGURE 44

Although this sheet is not a design drawing or even a study for a composition, it illustrates, at the extreme end of the scale, the possibilities of constructive geometry when applied to the depiction of three dimensional

objects on a flat surface. To Twenty-first Century observers it appears to be a familiar image of a wire frame model created by a 3-d computer program. In modern computer graphics, 3-d space is conceived as an organization of three intersecting, mutually perpendicular grids, where every point can be expressed in terms of coordinates in relation to the axes of those grids, *x*, *y* and *z*. This conception of space, however, did not come about until centuries after the chalice drawing was made. The chalice was conceived not as an object in space but as an object composed of an arrangement of intersections, lines from which stretch from the chalice to the viewer's eye. The image represents the intersections of these lines with the picture plane. The locations of these intersections on the picture plane are plotted using a system of constructive geometry.

To the enthusiastic perspectivists of the mid-Fifteenth century the magic of perspective was that, with enough patience, all the points of a three dimensional object could be geometrically plotted. Without relying on the eye's judgment, which is full of distortions, a perspectivist could turn a few parameters of a real or imaginary object into an apparently accurate image on a flat surface with mathematical precision. This aspect of perspective would have been more on the order of a magician's trick than a useful tool for the typical artist. For most artists, everyday perspective systems were simpler and were carried out using rules-of-thumb - more for compositional organization than for fooling the eye. One telling illustration of the limited appeal of overly complex constructions can be found in the construction marks in the *Trinity Fresco* by Masaccio. On one of a pair of Ionic capitals flanking the main figural group, Masaccio used a simplified plotting technique to accurately depict the curving form of the capital seen from below.[180] On the opposite side, he did away with the complex construction and drew the capital freehand. For the practical job of painting, the time consuming ordeal of plotting even a few points was not worth the trouble, especially when a serviceable illusion could be obtained by eye.

There were artists, however, who were fascinated by the space-plotting possibilities of the system and who were undaunted by painstaking geometrical procedures. Paolo Uccello and Piero della Francesca were among these. The drawing of a chalice is a product of this kind of fascination. It is a masterful

[180]Schlegel, Ursula, 'Observations on Masaccio's Holy Trinity Fresco in Santa Maria Novella,' *Art Bulletin*, (March, 1963), 23.

demonstration of what perspective could do. The image, rendered in inked lines, appears to be that of an elaborate chalice turned on a lathe, except that short, straight lines replace curved lines and planar facets replace curved forms. It is transparent, in areas, as if it were made of straight lengths of wire, with the backside visible through the front. Elsewhere, the surface appears solid and occludes the backside.

We view the chalice with very little foreshortening, as through a telephoto lens and from an angle just above the top rim. The greatest difference in length between a form on the front of the chalice and the same form on the back is just seven percent, almost unnoticeable. The torus shapes at the top and near the base are the same width and represent the same shape. Because of the position of the viewer's eye in relation to the chalice, the upper ring is more foreshortened than the lower one. The difference in eccentricity of the ovals (that is the vertical measure), which determines the amount of foreshortening of the ovals, is about thirty-seven percent. I mention these details to counter the claim that the chalice is depicted in non-convergent or orthographic perspective.[181] Such a method of depiction would mean that the near and far pairs of facets would be identical in size, and that the ovals would all have the same eccentricity. Since this is not the case the chalice was not depicted in orthographic perspective.

The particular type of linear perspective used in this drawing is related, but not identical to, that of Piero della Francesca's "other method."[182] In that method, points are plotted from a schematization of the plan and elevation of the object and then located on the picture plane, point by point, by means of lines and intersections. While Piero's second method did not involve a vanishing point, the chalice method of construction does require a vanishing point and was more complicated than Piero's method, if that can be imagined.

In constructing the chalice, the inked lines represent just the tip of the iceberg. The lines that made it possible to plot the position of each intersecting point of the chalice were incised and pricked into the paper in an unbelievably intricate network. These incised lines, too, are only part of the story, since even more construction must have occurred elsewhere to obtain the information that was encoded in the paper's surface.

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[181]Kemp, *Circa 1492*, 242.
[182]Evans, Robin, *The Projective Cast*, (MIT, 1995), 131-142.

The chalice is made as a column of horizontally positioned polygons of different sizes, rather like a stack of coins of various diameters. Each polygon has thirty-two sides and corners. The essential program for building this object was to locate the points representing the corners of each of these polygons and then to draw straight lines from point to adjacent point, so as to form an interconnected network. Although each step of the process is relatively simple, the difficulty lies in executing the tasks countless times with minor variations each time. Even with hundreds of points and innumerable possibilities for mistaken connections, the drawing is without error.

The following is my theory regarding the procedures that the artist employed to draw the chalice. The explanation, I believe, accounts for the hundreds of incised lines and points visible in the original. The construction of the chalice began elsewhere, on paper or other surface, which is now lost. It is possible, however, to reconstruct this part of the process by inferring from the marks on the paper what the perspectivist must have done to obtain the information encoded in these marks.

The layers of polygons that make up the chalice are distorted by perspective but they maintain most of the qualities of the same polygons rendered flat and undistorted. In order to create the distorted view one must first draw the polygon in plan, that is, undistorted. One does this by drawing a circle, with a compass, inside a square. Since each corner of a thirty-two sided polygon lines up neatly on a perfect circle, all that is required is to find the thirty-two points along the circumference of the circle. In geometry, points are located, or plotted, by the intersections of lines, arcs or circles. The intersections of the diagonals of the square with the circle locate the first four points. With the compass on each of these points it is possible, using crossed arcs, to locate the halfway points between each. Repeating this process of halving two more times arrives at thirty-two equally spaced points. Since it is the points and not the faces of the polygons that will be plotted on the chalice it is not necessary, here, to draw the faces (fig. 45 A).

It is much easier to distort a square in perspective than a polygon. Therefore, it is desirable to plot the polygon within a square, distort the square and then replot the polygon. We have already located the polygon within a square but a problem will arise when we attempt to replot the same polygon

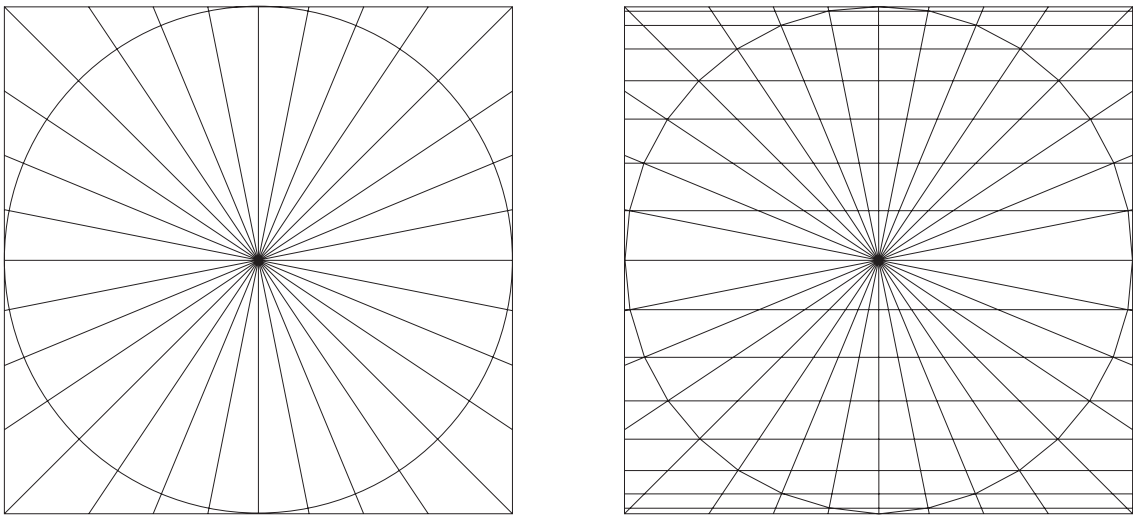


FIGURE 45 A,B

within a distorted square because our compass cannot draw a distorted circle needed to plot the polygon. This is the problem that Masaccio ran into with the Ionic capitals. Masaccio's solution was to draw the curves by eye, but the artist of the chalice drawing did not resort to guesswork.

Suppose we wanted to replot the thirty-two intersections in Figure 45 A onto another square without using the compass. We could draw parallel lines within the first square that would intersect on the polygon's corners. Along the perimeter of the square would then be the endpoints of these lines, as well as those of the radiating lines. Were we to reproduce these endpoints along the perimeter of another square and draw lines connecting the points in the same pattern as the first square, then the resulting intersections within the new square would be a reproduction of the original polygon corners (fig. 45 B).

The artist of the chalice drawing used the intersections of horizontal and radiating lines to locate his points. Armed with this information, his next problem to solve was how to draw the two sets of lines within a number of distorted squares. Let us return to our undistorted polygon within a square. Along the perimeter of the square are the intersections of the radiating lines and horizontal lines which pass through each of the corners of the polygon. These points of intersection with the square are the information we need to transfer to our distorted squares. When a square is distorted by perspective, some or all of the lines change in length and the corners change their angles. In the chalice drawing, the imaginary squares which surround each polygon are distorted, that is shortened, on their sides and back but not on the leading edge because the

leading edge is parallel to the picture plane. The information from our undistorted polygon in square can be transferred undistorted onto the leading edges of the squares of the chalice drawing. The artist might have used a straight strip of paper or other surface, laid along the leading edge of the square to transfer the points. He then incised vertical lines at the same intervals as the points on the leading edge of the square. Each line would then intersect at the same place on each square.

The problem of distortion for the back side of the square could be solved by taking the strip of paper with the transfer points and proportionally reducing its length to equal that of the sides of the distorted square. This can be done by drawing a long right triangle with the transfer points plotted along the short side of the triangle. From each of the transfer points a straight line would be drawn to the opposite corner of the triangle. Now, any line within this triangle drawn parallel to the short side, such as line A, will pass through all the lines and the resulting intersections will be proportionally reduced versions of the transfer points (fig. 46).

Since each of the polygons in the chalice drawing is seen from a slightly

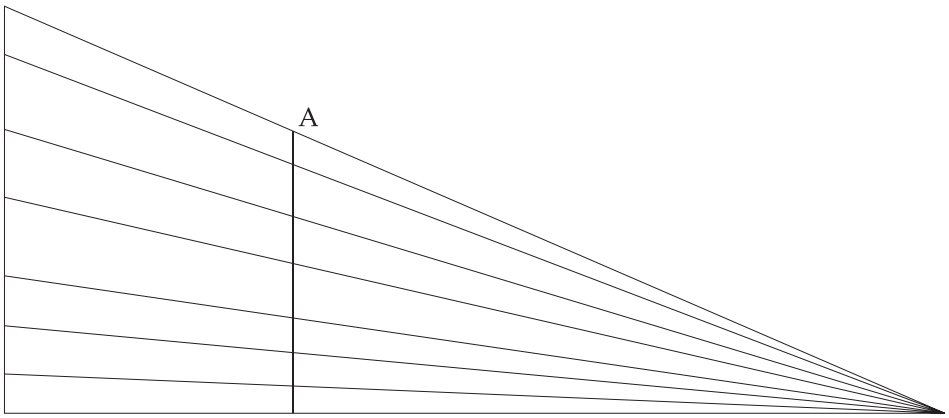


Figure 46

different angle, each of the imaginary squares surrounding the polygons would have been distorted differently as well. These squares were not incised on the chalice drawing so it was necessary to draw them somewhere else. This other drawing would have been of a tall, vertical, rectangular solid like a box, (fig. 47, left). This box would have been large enough to contain the chalice and would have been divided on its face by sixty-three horizontal lines, each the leading edge of a square receding back in perspective.

Information on the points needed to plot the polygon was transferred from the preliminary drawing to the chalice drawing by means of a template shaped as a trapezoid (fig 47, right). The points for the radiating lines and horizontals were marked on the left and right edges of the template and transferred along incised lines up the left and right sides of the chalice drawing. The purpose of the angled sides of the template was to avoid overlapping the points onto one another.

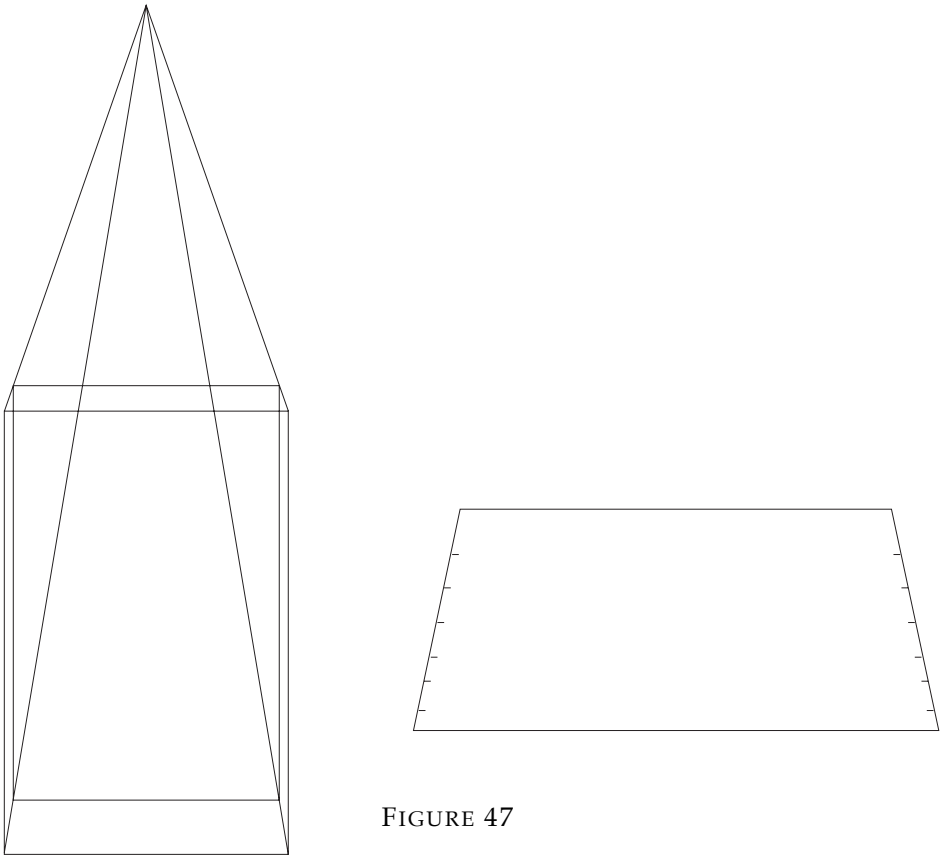


FIGURE 47

Anonymous architectural drawing,
16th/17th Century Italian
ink, metalpoint, wash and paper
38.5 x 26.1 cm
Folio 31 of the Talman Album
Ashmolean Museum, Oxford

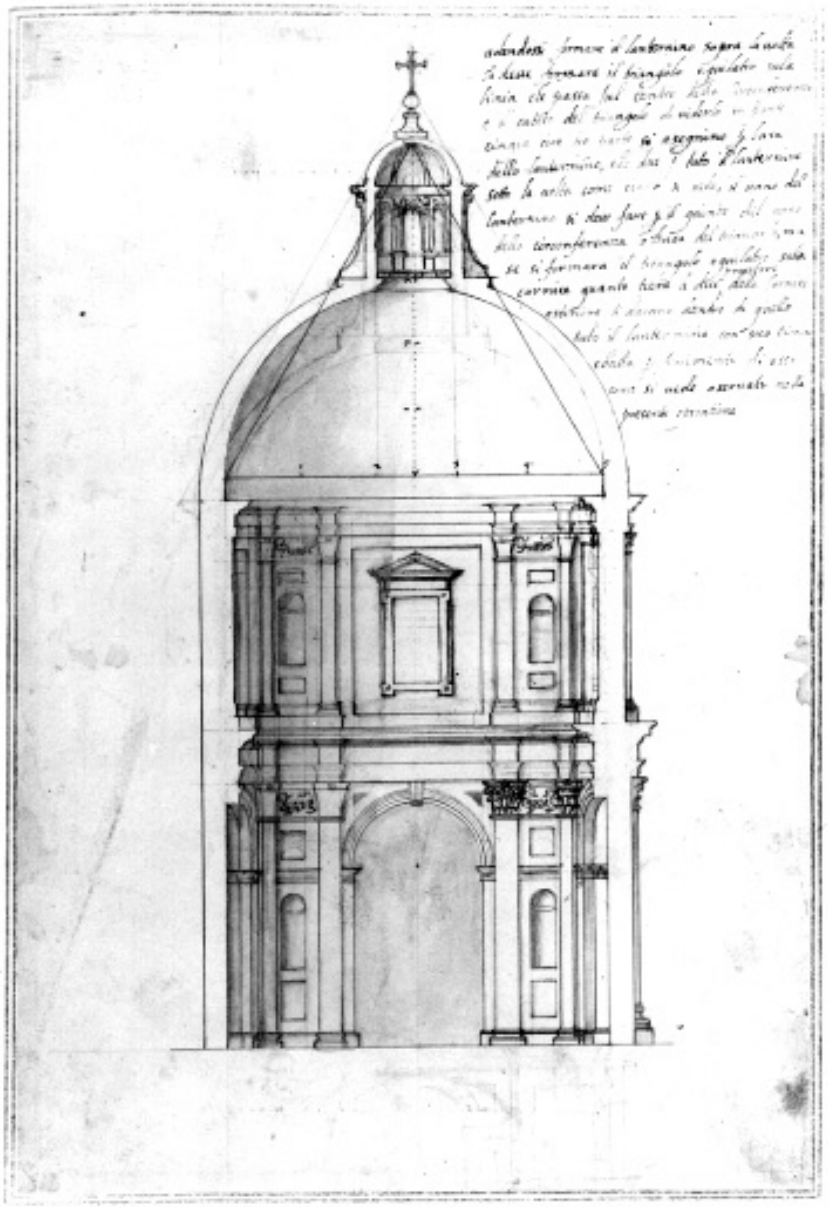


FIGURE 48 ASHMOLEAN

This architectural drawing illustrates, in an unusually clear manner, how a designer might use a geometrical form as a tool in the design process (fig. 48). The design process exhibited in the Ashmolean drawing is an unusual

implementation of the method advocated by Francesco di Giorgio of perhaps a century preceding. Unfortunately, little is known of the origin of the drawing, which is part of the Talman collection, apart from the fact that it is Italian and from the Sixteenth or perhaps the Seventeenth Century. The architect is unknown, as is the building represented. The unusual aspect of the work is not only that the geometrical aspect of the design process is clearly depicted, but that the process is annotated as well.

The drawing is in ink on ruled metalpoint lines and there is a faint wash to indicate shadows. The subject is an elevation cross section of an octagonal chapel with a hemispherical dome. The dome is crowned with an octagonal lantern upon which rests a smaller hemispherical dome, an orb and a cross. The interior walls are decorated with two levels of Corinthian pilasters and entablatures in a typical Italian Renaissance style. On the lower level, the pilasters flank a large arched opening while above they flank a gabled window. Foreshortened elements on the left and right indicate the angled walls of the octagon that were truncated by the section view. There is a slight asymmetry in the building: the left exterior wall is indicated by a straight vertical line while the corresponding line on the right follows the contour of an upper and lower pilaster with cornice moldings. The departure from strict symmetry may have been to accommodate the details of an actual site.

Filling the upper right margin of the paper is the annotation which refers to two triangles, with measurements, that relate to the dome and lantern (fig. 49):

> If you want to make a little lantern upon a [hemispherical] vault, you must form an equilateral triangle on the line that passes through the center of the circumference [of the vault]. The height of the triangle [should be] divided into five parts, that is, three parts up to the base of the lantern and two [parts] for the lantern itself, above the vault, as you see clearly. The [internal width] of the lantern you must make [the same as] one fifth of the circumference of the base of the triangle. But, if you make the equilateral triangle on the cornice for as far as the two projections of the exterior cornices extend, the inside of that triangle will give [the height of] the lantern with its summit-ball, as you see observed in the present work.[183]

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[183]"Volendossi formare il lanternino sopra la volta si deve formare il triangolo equilatro sula linia che passa sul centro della circonferenza e il cateto del triangolo dividerlo in parte cinque cioe tre parte si asegnino p[er] basa dello lanternino, elle due p[er] tuto il lanternino sotto la volta come ciaro si vede, il vano del lanternino si deve fare p[er] il quinto dil vano della circonferenza o basa del triangolo, ma se si formara il triangolo equilatro sula cornice quanto tiene le due

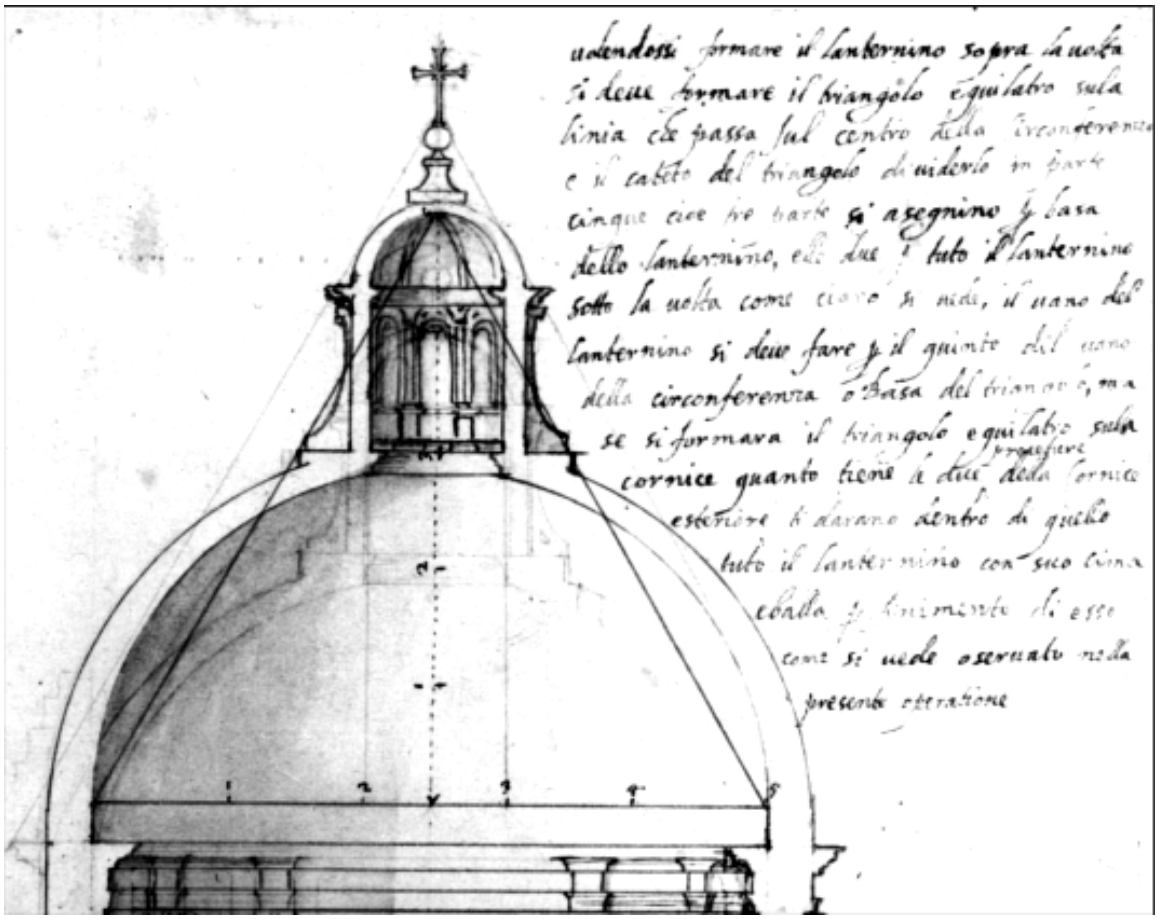


FIGURE 49 DETAIL OF ASHMOLEAN DRAWING

The dome is drawn with an outer and an inner semicircle, the centers of which are aligned on the central axis about three millimeters apart. The first triangle mentioned in the annotation is in ink and its base sits on the compass indentation, which is the center of the inner semicircle of the dome. This is the "center of the circumference" of the vault. A vertical dotted line through the center of this triangle is divided into five numbered marks. The third mark corresponds to the base of the lantern, which is coincident with the top of the outer arch. The fifth mark and top of the triangle correspond to the inner arch of the lantern's dome. The second triangle mentioned in the annotation is in metalpoint and is larger than the first triangle. It rests on the cornice three millimeters below the center of the outer arch. The base of the smaller triangle is also divided into five units and numbered. Two metalpoint vertical lines rise

proieture della cornice esteriore li darano dentro di quello tuto il lanternino con sua cima eballa p[er] finimento di essocome si vede oservato nella presente operatione."

from the marks numbered two and three and align with the inner walls of the lantern.

There are some ruled construction lines in metalpoint visible under the wash, most of which are horizontal and vertical reference lines that assisted in the layout of the pilasters and cornices. There is also a second complete drawing of the dome and lantern outlines displaced nineteen millimeters below the inked one. The width of this ghost image is the same as the final version but the vertical dimension is stretched a little as though the regulating triangles were positioned differently. Along the right inner wall on the upper level are some ghost cornice moldings which must have belonged to the lower dome stage. The presence of the alternate images appears to contradict the definitive tone of the annotation.

The annotation also leaves out some necessary information regarding the relationship of the small triangle to the large one. The distance between the base lines is approximately one-third the vertically measured units but may have been chosen arbitrarily. The size of the larger triangle seems to also have been chosen without a rule. Even without all of the necessary information, it is an extraordinary example of the use of a geometric system to aid in the design process.

Michelangelo
1475-1564
Project for San Giovanni dei Fiorentini
1559
Pen and ink, wash, sanguine and black pencil on paper
417 x 376 mm, CB124r
Casa Buonarroti, Florence

Baldassarre Peruzzi
1481-1536
Central-Plan Project for San Giovanni dei Fiorentini
1519
Pen and Ink, on paper, red chalk
456 x 427mm, Uff. 505A
Galleria degli Uffizi, Gabinetto Disegni e Stampe Florence

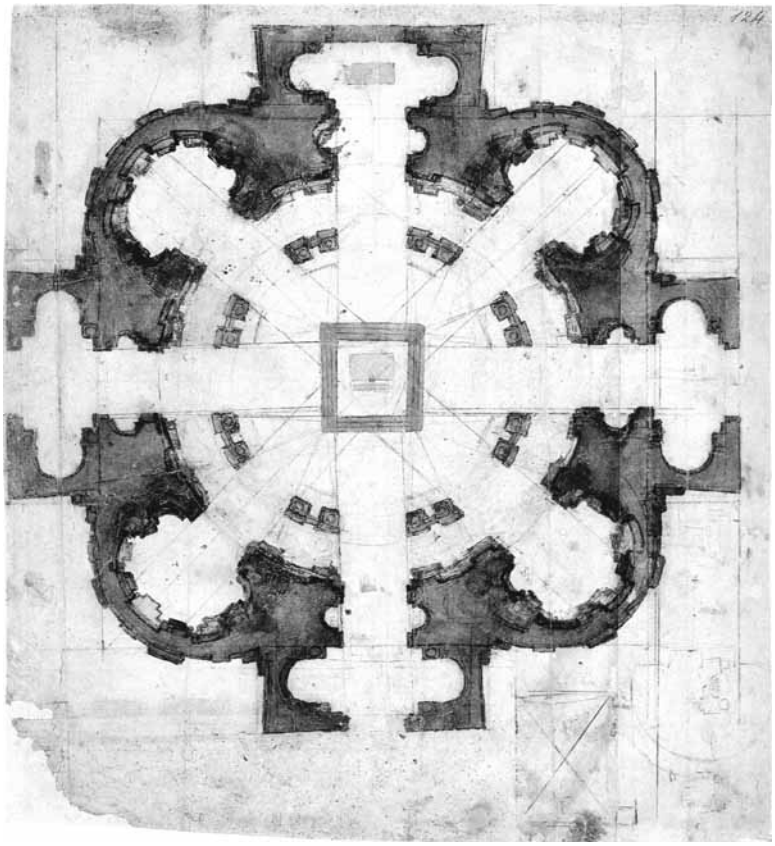


FIGURE 50 MICHELANGELO

These two drawings are here compared in order to show the different approaches used by two different designers working on the same project. They

are both design drawings for the church San Giovanni dei Fiorentini, a project with a long, complex history that involved many Italian architects throughout the 16th Century, including Bramante, Antonio da Sangallo, and Sansovino. In both drawings, chapels located on the horizontal, vertical and diagonal axes fill up the space between a circular central area and a square perimeter. The Peruzzi drawing employs the same kind of systematic design procedures that are treated in this thesis. Michelangelo, however, used no such systems.

Instead of systems we have considered throughout this thesis, in Michelangelo's drawing (fig. 50), we have the intuitive approach to design. This is similar to what we saw in the figures of Leonardo's Madonna and Child with St. Anne. Multiple lines appear around many of the forms, especially around the chapels, recording the trials that were considered in the process of searching for the right form. There are places where the number of pentimenti become so confusing that Michelangelo had to reinforce the desired contours with ink wash to distinguish them from the rejected forms. Although there is evidence of the use of draughting tools, the tools were not applied according to a geometric system. A straightedge was used to draw many of the pencil lines for the underdrawing but these are not at right angles to one another. Apparently Michelangelo just laid the rule down, relying on the judgment of his eye for squareness. There are incised arcs made with a compass, but these were located with the same casual attitude toward precision. There are no crossed arcs and those arcs and circles that were used as reference for the shapes of chapels, niches and the ambulatory were not closely followed. Instead of a systematic application of constructive geometry, we have a bold, expressive record of Michelangelo's intuitive design process.

The Peruzzi drawing (fig. 51), on the other hand, records the application of the geometrical and numerical systems that we saw in the Sangallo pilaster drawing. Crossed arcs were used to obtain true horizontal and vertical axes. The multiple lines and arcs that are visible reminders of the trial, judgment, modification process all conform to the geometrical or numerical system. Various chapels and niches were considered and rejected but all were consistent with the grammar of the system. A scale is visible below the horizontal axis and numerical measures were taken from it so as to maintain the proper ratios within

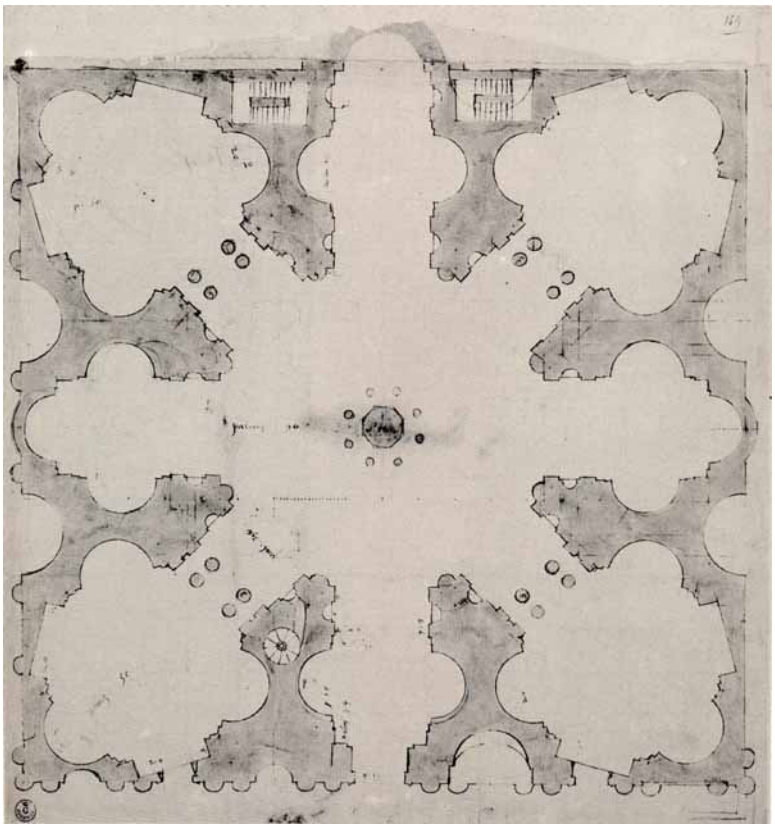


FIGURE 51 PERUZZI

a numerical system. Because of the complex aspect of this drawing and my own inability to untangle all of the steps of the design process, there will be no step-by-step reconstruction. I include this drawing here to illustrate: that there is evidence that another designer used the systems this thesis treats, that the systems are not limited to ornamental fixtures and that the same subject that is treated by Michelangelo in an intuitive approach can also be treated in a systematic approach.

Looking through the Uffizi's collection of Peruzzi drawing it appears that Peruzzi had the same geometrical interests as Sangallo. There are sketches of pilaster base profiles with measurements, along with other architectural moldings. There are also a number of geometrical constructions for Ionic volutes and ovals. Examples of drawings with complex tangles of geometrical lines and arcs that are part of design processes, such as in the present drawing, include, Uff. 494Ar., Uff. 581Ar., Uff. 513Ar. and Uff. 510Ar. Although these drawings may not be decipherable in terms of every step in their design processes, their existence adds weight to the assertion that the few drawings that are thoroughly analyzed in this thesis are not exceptional cases.

Looking through the drawing collection at Casa Buonarroti reinforces the above characterization of Michelangelo's drawing process. Almost all of Michelangelo's architectural drawings were rendered with the same bold, intuitive approach as the project for San Giovanni dei Fiorentini, without relying on any system. The lines from a straightedge and the arcs from a compass were merely suggestions because Michelangelo would not be limited by any system or accept any authority other than his own eye.

Conclusion

The overall conclusion of the thesis is that draughting tool marks are worth investigating. The tiny and sometimes almost invisible deformations of the work surface form an archaeological record of the designers' operations while engaged in a design process. Analyses of these marks can reveal information about the working methods of specific artists and architects that can usually be found nowhere else. Because the marks are the result of implementations of systematic design processes, the information that is found can be considered with a great deal of certainty. The results of the foregoing investigations support this conclusion.

In the case of Leonardo, we have seen his methods up close. His (mostly) systematic use of perspective in the *Adoration* drawing allows us to see every decision and consider why he made it. The revelation of Leonardo's care, his mistakes, his attempts to cover up mistakes, and his improvising alternatives to the system that he was using are new. In the case of the *Madonna and Child with St. Anne*, we saw Leonardo intently trying to reconcile a ratio with a composition. Even though the ultimate reason is still unknown, it is fascinating to see how the process was carried out.

In the case of Sangallo's pilaster base, we could watch as a designer

struggled to reconcile traditional prescriptions with the needs of the moment and with the demonstration of his own creative invention. His solutions were certainly inventive, but the means of changing the module in mid design seem, to the modern eye, strange. Nevertheless, it is instructive of how his priorities differ from ours. Long established numerical programs were more important than consistently sized measuring units. The analysis of his little capital design is also instructive of what a Renaissance architect considered important. Unification of the plan and elevation of the capital allowed both dimensions to be designed according to the same constructive geometry and to make previously numerical relationships geometrical ones.

In the case of Palladio, the myth of harmonic numbers needed to be excised. The implication of Wittkower's efforts is that geometric design processes are somehow inferior to numerical ones. This may be true for Wittkower, but Palladio and other Renaissance architects held the two systems as complementary and both necessary tools in their toolboxes.

The usefulness of this investigatory methodology is not limited to the drawings analyzed in this thesis. There are numerous drawings from the Italian Renaissance, as well as prior and subsequent periods, that contain draughting tool marks. Moreover, tool marks can be found in finished works as well as in drawings. The use of numerical and geometrical systems is an ancient tradition that has continued through to the 20th Century.

However, not all works with draughting tool marks will yield results. The types of works that are not analyzable with this method are those where the artists used tools without using a system. Also unfruitful will be those works where the artists' steps are so numerous and convoluted that they cannot be reliably untangled.

The types of works to yield best results will more likely be architectural or geometrical than figural. Leonardo's *St. Anne* drawing would not have been an apparent candidate had it not been for the frame. Perspective constructions are particularly well suited to analysis because the greater limitations of the system makes the puzzles easier to solve. The limitless possibilities of compositional constructive geometry, on the other hand, offer more challenging investigations. The ideal candidates would be those that include pentimenti, where decisions

concerning different geometrical possibilities are recorded, allowing the opportunities to see the artists at work.

The methodology of examining and recording the tool marks and determining the sequences of operations by reference to the design traditions is essential to the successful interpretations of the evidence. It is my hope that others who share the appreciation of design and the processes of its creation will investigate further and that they can benefit from the methodology here employed and the investigatory results of this thesis.

Bibliography

Note: The potential bibliography of this topic is vast, and I have here concentrated on sources used most directly in this thesis, and those (such as Ames-Lewis and Cappel) which provide the reader with a broader introduction to the forms and functions of Renaissance drawings.

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