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High schoolers excel at Oxford quantum course using pictorial mathematics

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Abstract. We are at the dawn of the second quantum revolution, where our ability to create and control individual quantum systems is poised to drive transformative advancements in basic science, computation, and everyday life. However, quantum theory has long been conceived as notoriously hard to learn, creating a significant barrier to workforce development, informed decision-making by stakeholders and policymakers, and broader public understanding.

This paper is concerned with *Quantum Picturalism* [15], a novel visual mathematical language for quantum physics. Originally developed over two decades ago to explore the foundational structure of quantum theory [2], this rigorous diagrammatic framework has since been adopted in both academia and industry as a powerful tool for quantum computing research and software development. Here, we demonstrate its potential as a transformative educational methodology.

We report the findings from a pilot study involving 54 UK high school students [27], randomly selected from a pool of 734 volunteers across the UK. Despite the absence of advanced mathematical prerequisites, these students demonstrated a strong conceptual grasp of key quantum principles and operations. On an assessment comprising university graduate-level exam questions, participants achieved an 82% pass rate, with 48% obtaining a distinction-level grade.

These results pave the way for making quantum more inclusive, lowering traditional cognitive and demographic barriers to quantum learning. This approach has the potential to broaden participation in the field and provide a promising new entry point for stakeholders, future experts, and the general public.

This paper is a contribution to a volume in memory of Basil Hiley, who was one of the 1st fans of QPic, and likely the first person trying to teach it to their very young granddaughter, which happened around 2005. Basil could see well beyond the current standards of physics, in a way very few can do.

1 Introduction

Quantum theory is perhaps the greatest success story of 20th century physics, with our understanding of quantum effects playing a crucial role in everything from the atomic clocks used in GPS navigation to the transistors that make up every computer in the world. The emergence of quantum computation, wherein quantum systems themselves are used to store and process information, is expected to have a dramatic impact in the coming years in fields such as chemistry [11], microbiology [54], medicine [40], material science [3], communications [62, 53], and artificial intelligence (AI) [29]. Governments have already declared quantum technologies to be of the highest geopolitical importance [32], and quantum technologies have been predicted to have a huge economic impact in several major industries in the coming years [31, 23].

At the dawn of this transformative technological revolution, it is imperative to cultivate a skilled workforce, empower stakeholders in business and governance to make informed and responsible decisions, and foster a society that is broadly quantum-ready across different age groups and different educational and socioeconomic backgrounds.

Unfortunately, quantum theory is often regarded as only comprehensible for those with an exceptional intellect and who have had the opportunity to complete one or more advanced science degrees. The language in which quantum theory is traditionally formulated, following von Neumann [65], is that of complex Hilbert spaces and linear maps. Throughout this paper, we will refer to this traditional language as the Hilbert space formalism. The foundations of this language are usually taught in advanced undergraduate-level mathematics and physics courses, with advanced topics and applications, *e.g.* in quantum technologies, often appearing only at postgraduate level.

Note that by 'language' here we mean how quantum systems are described, how one reasons about them, and how calculations are done. Interestingly, von Neumann himself denounced the Hilbert space formalism merely three years after publishing his book on it [66] and dedicated much time thereafter to finding an alternative language [56].

This article is concerned with a new language for quantum theory and its applications, which we refer to as *Quantum Picturalism* (QPic) [15]. It is the subject of two books written by some of the authors: *Picturing Quantum Processes* [20], a textbook which has been used as the basis of a postgraduate course at Oxford University for about a decade; and *Quantum in Pictures* [19], a new accessible book introducing QPic to a broad audience, including young people and readers without any formal mathematical background.

Notably, *Quantum in Pictures* has no mathematical prerequisites beyond what is already taught to 7-11 year olds in the UK, namely the ability to express angles in degrees and add them together [60]. Still, the book covers advanced quantum topics, including some that have been discovered only in the past few decades, or even in just the past few years.

Initially, QPic was not intended to be an educational tool, but rather an abstract language for exploring the fundamental principles of quantum theory [2] and quantum computing [16]. However, it has since found many applications in solving concrete problems in quantum technologies, such as optimising quantum computations to use less time and space [26, 44, 24, 25, 63, 6, 35, 38], modelling and simulating quantum processes on a classical (super)computer [47, 67, 13, 61, 10, 48], producing effective techniques to manage and correct errors in quantum computations [33, 51, 45, 43, 46, 5, 59, 68, 39, 8, 7, 9, 57], and designing interpretable classical and quantum AI systems [52, 41, 49, 28].

In this article, we provide evidence that QPic enables one to teach advanced quantum concepts at the high school level, and that moreover, the students can use that knowledge to solve sophisticated problems. We demonstrate that, after a short course, a group of 54 high school students, selected randomly from a pool of 734 volunteers from across the UK, could not only pass an exam based on past exam questions from an Oxford postgraduate-level course, but that many of them excelled. Of the students that participated in the study, 82% passed the exam and 48% obtained a distinction, *i.e.*, a first-class mark of 70 or above.[1] These results are especially promising given that this was a completely voluntary course undertaken by UK high school students, many of whom were simultaneously preparing for their A-level examinations. [36]. This supports our argument that QPic is a powerful new tool to make quantum education more inclusive than imagined, in a way that goes hand-in-hand with cutting-edge quantum technology development.

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[1] These results were obtained after pre-filtering students by means of a "canary question", which was designed to measure basic engagement of participants with the course material. That is, a canary question was a question which tested participants' overall understanding; correct answers to the canary question (it had two parts) was a prerequisite for passing the assessment.

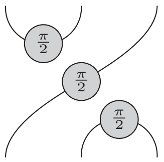
2 Quantum Picturalism

Quantum Picturalism (QPic) employs a new and highly intuitive form of mathematics known as *diagrammatic reasoning*, departing from conventional symbolic methods. This mathematical framework was formalised in the 1990s and early 2000s through the theory of monoidal categories [4, 42, 12, 50]. However, the particular types of diagrams used trace back two decades earlier to the graphical tensor notation introduced by Penrose [55]. QPic shows that this pictorial notation can be extended, capturing all of the essential quantum notions like entanglement, mixed states and processes, and observables and their complementarity [14, 58, 21, 22, 17, 18].

QPic originated as an attempt to provide a high-level language for quantum computation, aimed at helping quantum computing researchers better understand and design algorithms. The basic idea proceeds in analogy with the case of classical software. Whereas the very first classical computers were programmed using very rudimentary machine code, which got translated directly into streams of 1s and 0s, modern sophisticated software would not be possible without *high-level* programming languages like C++ and Python. These languages allow programmers to describe complex behaviours in ways that more closely resemble how they think about problems, rather than focusing on the low-level details of the machine.

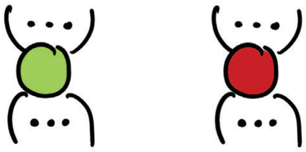
The analogy goes as follows. While the low-level language of classical computing consists of binary numbers, *i.e.* matrices of 0's and 1's, the 'low-level language' of quantum theory comprises matrices of complex numbers. QPic gives us a high-level language for describing quantum theory using pictures, *e.g.* from [20]:

$$\frac{1}{4}\begin{pmatrix} -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \end{pmatrix} \Longrightarrow$$

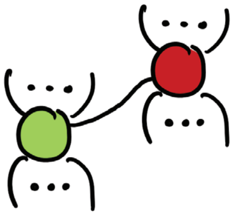


The matrix on the left and the picture on the right describe the same mathematical object. However, even without knowing the details of quantum theory, one can immediately recognise certain features from the picture, *e.g.* which parts are connected to others—that are completely obscured by the matrix.

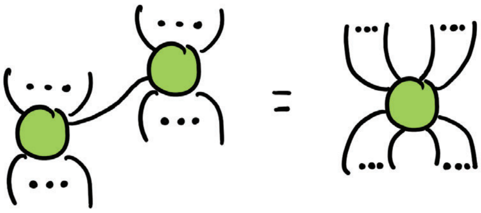
We now briefly give a taste of QPic. The basic building blocks from which we built quantum states and processes are called *spiders*. These spiders are of two colours and depicted as follows:



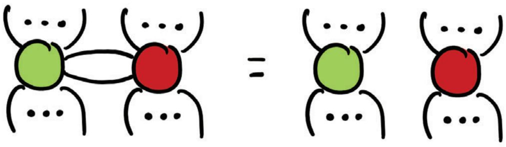
These can then be plugged together by *connecting legs*:



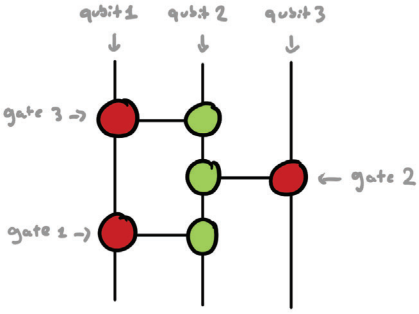
Connected spiders are subject to the following rules. If they are of the same colour, they *fuse*:



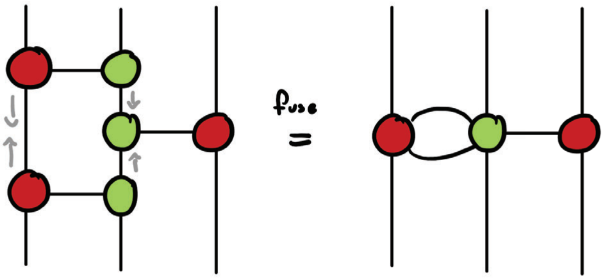
If they are of different colour and share two legs, we *chop* those two legs:



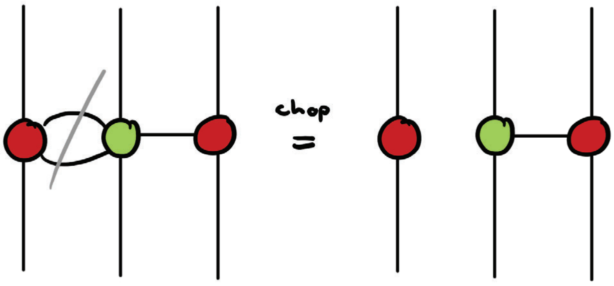
Now consider the following *quantum circuit*, in which the vertical lines are *qubits*—so there are three of those—and the horizontal lines with spiders at each end are *CNOT-gates*—hence there are three of those:



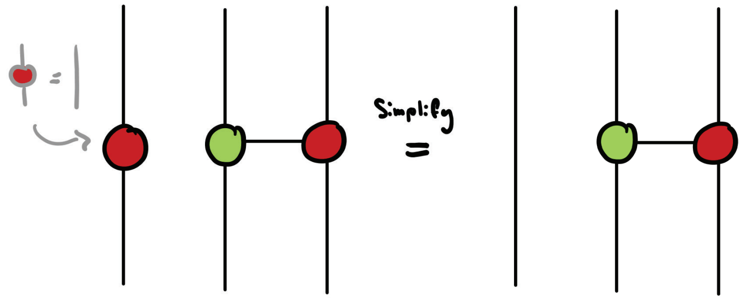
We immediately see that there are connected spiders of the same colour, so we can fuse them:



Next we see that there are two shared legs between two spiders of different colours, so they vanish:



Spiders with two legs are just wires, therefore:



Hence we reduced this quantum circuit from having three CNOT-gates to having only one CNOT-gate. This is of course a very simple example, but the methods based on simple diagram transformations like these have been used in state-of-the-art quantum circuit optimisation procedures [26, 24, 25]. This is very important, as it allows large quantum programs to be turned into much smaller ones that can be executed faster and with few opportunities for errors. Other equally simple methods underpin many other important tools for quantum technologies.

How broadly applicable is QPic? For over 10 years, it has been the subject of a popular post-graduate course at Oxford University, containing all essentials expected from a quantum computing course [20]. In addition to five courses at Oxford University teaching QPic, it has also been taught in courses at the University of Edinburgh and the University of Cambridge in the UK, Indiana University Bloomington in the US, the University of Amsterdam in the Netherlands, Kwame Nkrumah University of Science and Technology in Ghana, and more courses listed in the extended version of this paper [30]. QPic methods have been applied in roughly 300 research papers including publications by 10 quantum computing companies, across 18 areas: quantum error correction, optimisation, classical simulation, variational algorithms, verification, measurement-based quantum computation, natural language processing, photonic circuits, condensed matter, chemistry, higher-dimensional quantum computation, continuous-variable quantum computation, cryptography, entanglement, stabiliser theory, complexity theory, quantum computer architecture, and quantum software. The utility of QPic in research and teaching stems from the following results:

- *Universality* [15]. All states and processes of the Hilbert space formalism can be represented within QPic.
- *Completeness* [37, 64]. Any equation between states and processes that can be derived within the Hilbert space formalism can also be derived within QPic.

The above discussion shows that QPic diagrams are somewhat different in nature from other familiar diagrammatic tools, such as Feynman diagrams. Whereas the latter serve as a visual aid to guide rigorous calculations that must still be carried out using the 'standard' mathematics of quantum theory, QPic is already a fully rigorous language in which calculations can be performed directly.

Importantly, in many cases, the representation and the derivations in QPic turn out to be much simpler than those in the Hilbert space formalism. How much simpler? This brings us to our main result of this paper.

3 The course

We describe the course materials and the content of the exam in this section. Additional background, context, methodology and more details of the study and the results reported in this paper can be found in the pre-experiment paper [27] and the extended and more technical version of this paper [30].

3.1 Course materials

The course materials consisted of a course textbook, which was a slightly abridged version of the book *Quantum in Pictures*, covering the following topics:

- general states, processes, their sequential composition (cf. matrix multiplication) and parallel composition (cf. tensor product), process theories and Choi-Jamiołkowski isomorphism, spacetime diagrams, quantum and classical bits, quantum gates (CNOT, H, Z phase, X phase), quantum circuits,

> unitary processes, inner product, Bell states/effects, observables, non-determinism, measurement disturbance, complementarity, causality and no-signalling, quantum teleportation, entanglement swapping, reversible computation, entanglement, uncertainty principle, gate-based quantum computing, quantum circuit optimisation, measurement-based quantum computing, multi-party quantum communication protocols, mixed state quantum information

In *Quantum in Pictures*, many of these concepts were given alternative (and somewhat less obscure) names that more directly resemble their respective pictorial incarnations and, similarly, the subjects were given names that more directly relate to daily reality. A translation is provided at the end of *Quantum in Pictures*.

Remark 3.1. Two notable omissions here are the Schrödinger equation and any detailed discussion of quantum algorithms. For the former, we follow the conventions of a typical quantum information/computation course and break time evolution down into discrete time steps, thus not requiring manipulation of differential equations. For the latter, producing small, digestible examples of problems that can be solved by a quantum algorithm suitable for education at this level is a topic of future work. We hope that this will build on the progress that has already been made in the analysis of quantum algorithms themselves using QPic [34].

In addition to the textbook, we also filmed blackboard lectures covering all the relevant material. Additional materials also included exercises with filmed solutions. All of these soon will be made publicly available, after post-processing. Here is a pre-production example of a lecture:

https://www.youtube.com/watch?v=uVdWzQwAO48

of an exercise:

https://forms.office.com/pages/responsepage.aspx?id=G96VzPWXk0-
0uv5ouFLPkQY1yz0RM0lIrQkpLDzPs4ZUOFA3N0k3Rlg3MVhQTzNCM0NUSlI0OEFJMy4u

and the solutions:

https://drive.google.com/drive/folders/18T54TfSCNuTOxJ6OHpGo6omGAE-17JEv

3.2 Course structure

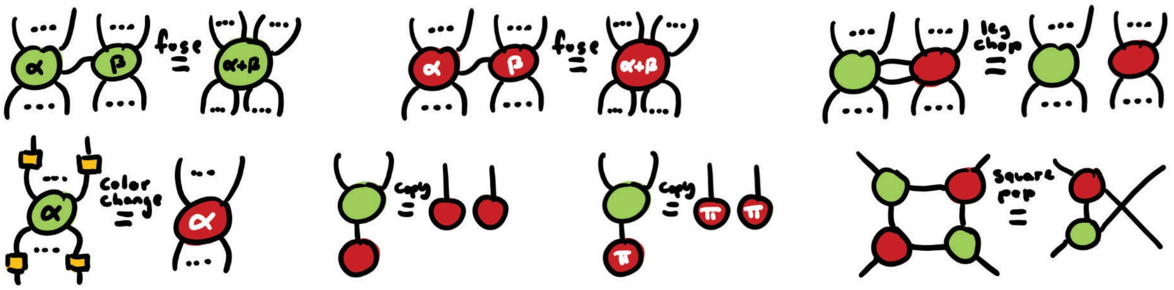
The structure of the course was based roughly on that of an Oxford University postgraduate course. This consisted of an eight-week taught portion, with lectures, tutorials, and weekly exercises, followed by a take-home exam (which in Oxford post-graduate courses is sometimes called a *mini-project*).

Students were asked to watch one video lecture per week in preparation for the weekly live tutorials. During the hour-long tutorial sessions, conducted in groups of up to 15 students, a tutor facilitated discussions, allowing participants to ask questions about the material and work through exercises under the tutor's guidance. Twelve additional homework exercises, with the filmed solutions, were also released during these 8 weeks, and the students were given about a week to try to do them, before solutions were released.

3.3 The exam

The structure of the exam closely mirrored that of an Oxford University postgraduate examination. It was double-marked, and in cases where the marks deviated by more than nine points, a third marker was involved. The exam followed a take-home assignment format, allowing participants three weeks to complete their assignments. With the exception of a two-part canary question, all questions were adapted from previous Oxford postgraduate exams, modified only slightly to align with the terminology and notation taught during the eight-week course. The exam consisted of three multi-part questions, each progressively increasing in difficulty.

For example, participants were given the following table of rules to use to calculate their solutions:



Below is an outline of the exam questions, an indication of their expected difficulty, and the QPic rules that needed to be used:

- The first question concerned quantum circuit simplification. Question 1a required the application of a four-qubit circuit to a state. It can be solved fairly straightforwardly both in QPic (using fusion and copy) and in Hilbert space formalism (applying gates one-by-one). Question 1b required showing that a three-qubit circuit reduces to the identity. In QPic it requires some more rules (leg-chop and colour-change), and in Hilbert space formalism multiplying 8x8 matrices for verification which is a lot more tedious, and easily leads to mistakes. Question 1c asks what a three-qubit circuit simplifies to. In QPic it requires one more rule (square-pop), but in Hilbert space formalism this question would again require a sizeable matrix calculation.

- The second question concerned multi-party protocols, more specifically, how a number of parties can together prepare a joint state using local operations and classical communication. Questions 2a and 2b are much more intuitive to solve with QPic than in the Hilbert space formalism. Question 2c is very hard in the Hilbert space formalism, as it requires an impossibility argument.

- The third question concerns computing classical correlations when measuring quantum states. The first two parts of Question 3, 3a.1 and 3a.2 are extremely straightforward in QPic, and essentially required them to reproduce two definitions given in the course. We deliberately included this two-part ‘canary’ question to identify papers where students made very little effort to read the material and answer the questions. Questions 3a.3 and 3a.4, on the other hand, were more difficult. To solve them properly, they require mixed states in the Hilbert space formalism, as they involve “tracing out” quantum systems, while in QPic they only require the much easier *doubling* of diagrams, which then allows one to just stick to the same rules. Question 3b is difficult even in QPic, so gives us an indication of which students gained an exceptionally strong grasp of the material. We refer to this as the ‘sentinel’ question.

4 Results

Discarding those exams from students who failed to solve the two-part canary question, we found that 82% of the students passed, of which 48% had a distinction.[2]

There were several limitations in the course format during the pilot study, which, if addressed, could lead to even better student performance and experience. The first was that all course components were delivered online, with lectures and exercise solutions provided as pre-recorded materials. Due to institutional restrictions on experiments involving minors, participants had no opportunity to interact with each other outside tutorial sessions, such as through online platforms. An in-person course or an online format with enhanced peer interaction could foster collaboration, enabling participants to support one another and engage more naturally with tutors. Finally, the course timing posed a challenge, as it coincided with a busy exam season followed by family holiday period in the UK. A different schedule may have allowed participants to have fewer competing commitments.

Students completed a questionnaire at the end of the course. Figure 1 presents a composite view of some of the data collected. Panel (a) shows that almost 80% students reported limited to minimal prior knowledge of quantum. Panel (b) provides a breakdown of the number of hours participants

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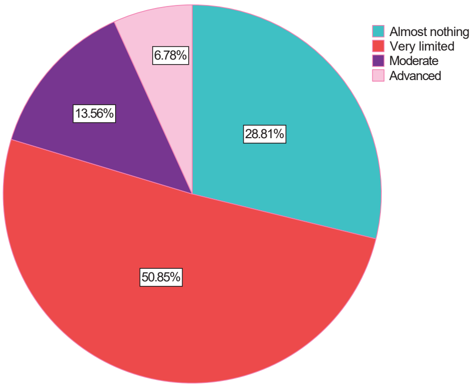
[2] Readers interested in a comprehensive analysis of the experimental results are referred to the longer, more detailed paper [30].

reported studying per week, with almost two-thirds reporting study hours within the bounds of course expectations. Panel (c) displays all the other responsibilities the students were balancing during the course. As shown in the panel, the sample reflects a broad cross-section of students, indicating that it included varying levels of academic background, effort and engagement. Among the 54 students, more than half (55.6%) attended state schools, which are funded by the government and usually do not require entrance exams. 25.9% attended grammar schools, 5.6% attended state boarding schools, 3.7% attended private boarding schools, 3.7% attended private independent schools, and 5.5% (3 participants) opted not to respond. Further tables and detailed discussion of this data can be found in the extended version of this paper [30].



(a) Prior knowledge of quantum

(b) Hours spent to study per week.

Hours/week	*Percent*
1-2	64.4
3-4	30.5
4-5	1.7
Total	*96.6*

(c) Other commitments over the course period.

	Exam preparation	*Part-time job*	*Internship*	*Sport activity*	*Camps/leisure*
%	100	20.3	11.9	10.2	57.6

Figure 1: Panel combining (a) A pie chart describing students' prior knowledge of quantum before attending training. The students were asked: "How much did you know about quantum theory before participating in this program?" 4 in 5 students responded either 'Very limited' or 'Almost nothing'. (b) A table reporting study hours per week, and (c) A table reporting other commitments over the course period.

The students' achievement was assessed through an end-of-course exam, which is described in Section 3.3. The exams were marked by University of Oxford faculty members according to standard procedures of the university.

Importantly, the students also enjoyed the course: 86.4% of students agreed with the statement, "The course motivated me to pursue a STEM career," with 54.2% strongly agreeing.

5 Impact

The students' mastery of the key quantum theory concepts we tested exceeded our expectations, demonstrating promise that it is possible to teach a subject which has historically had a high barrier to entry. The significance of these results warrants further studies, particularly those featuring in-person teaching, which may further enhance comprehension and engagement. Preparations for these forthcoming studies are underway in the UK and internationally. International interest has been substantial since the very beginning, with many applications received from outside the UK to participate in this first trial, which were excluded in order to comply with Oxford University's ethical approval requirements. At the time of writing, some follow-up studies are being planned with US funding agencies in collaboration with the Colorado high school system, while others are being discussed in developing countries, such as South Africa, Ghana, Morocco, and Pakistan. These further studies are expected to provide deeper insights into the demographic and age groups that may benefit most, from utilising QPic to teach quantum concepts more accurately and more accessibly.

On a broader scale, QPic is being considered for systematic integration in educational curricula, both at the high school and university level, with plans with the Colorado high school system to examine the quantum concepts learnable by even younger age groups. In Greece, the "KYMA (Quantum Computing for Students)" [1] initiative was launched in 2021 with the goal of bringing high school students in contact with the world of Quantum Information. Their 2024-2025 program – now engaging schools nationwide in Greece – has incorporated elements of QPic and at the time of writing, plans are underway to further expand its integration in future realizations. The KYMA initiative follows up on Hellenic Governmental Authorities' official approval and Hellenic Mathematical Society's support, reflecting national interest in expanding quantum computing education at the high school level, with QPic now playing a significant role in this effort. Notably, Greece is not the only country where QPic has supported national quantum education strategies. The inclusive nature of QPic has also been acknowledged by Elevate Quantum, the largest US quantum consortium prioritising quantum workforce development in the Mountain West. Under their mission to secure the region as a global epicenter for quantum, they have endorsed QPic as a key strategy for advancing equitable access to quantum science education.

As evidenced by the worldwide interest outlined above, the QPic experiment not only challenges assumptions about the prerequisites for Quantum Information Science and Technology, but, in doing so, highlights new pathways to quantum readiness. The findings open up new possibilities for early exposure to quantum, particularly at critical stages of education and career decision-making. As the next quantum revolution rapidly approaches, this raises important considerations for how to best prepare a skilled and adaptable workforce.

There also is the more general goal of making STEM research more broadly attractive. One notable piece of feedback we received from students was that "the course was so much fun". With QPic-alike languages being developed for several other STEM areas, there is now a very promising prospect indeed to change the face of STEM subjects.

6 Additional Information

Additional background, context, methodology and more details on the results reported in this paper can be found in the pre-experiment report [27] and the extended and more technical version of this paper [30].

7 Acknowledgements

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