

LOGICAL CONCEPTS

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A

‘a’. The indefinite article in English. (1) At least one. (2) One and only one. (3) Something or other.

abstract noun. Noun used to denote what cannot be counted: ‘butter’, ‘hydrogen’, ‘endurance’. Noun lacking a plural. *See* count noun, sortal.

actual. Anything is actual in a possible world if and only if it is in that world. Any world, w , is itself actual at w . To say that our world is the actual world is not to say that it exists but to say that we are in it. *See* modal logic, modal realism, possible worlds.

actualism. The doctrine that only the actual world exists. The doctrine that either only the actual world is real or the actual world is more real than other worlds.

ad absurdum. Latin for ‘to absurdity’. *See* *reductio ad absurdum*.

adjective. Word used predicatively. Word used to ascribe a property.

adverb. Word used to denote the manner or time of an action or event.

affirmation. The claiming that a proposition is true. The expressing of a putative truth. The asserting of what is putatively the case.

affirmative. Appropriate for expressing a truth.

affirming the consequent, fallacy of. Wrongly taking as valid the invalid inference from $p \rightarrow q$ and q to p .

algorithm. Purely mechanical proof procedure or method of computation. Proof or computation that a machine could perform.

‘all’. *See* quantifier.

ambiguity, lexical. Ambiguity caused by an ambiguous word.

Lexical ambiguity is not structural. *See* ambiguous, structural ambiguity.

ambiguity, structural. Ambiguity caused by grammatical construction. Structural ambiguity is not lexical. *See* ambiguous, lexical ambiguity.

ambiguous. (1) With two and only two meanings. (2) With at least two meanings.

ampersand. The name of a symbol for conjunction: ‘&’.

analysis. The breaking into logical or semantic parts.

analytic. Definitions of 'analytic' and criteria for being analytic: (1) Any analytic proposition is true, but not all true propositions are analytic. (2) The meaning of an analytic proposition is sufficient for its truth. (3) The truth of an analytic proposition is necessary for its meaning. (4) An analytic proposition is true by definition. (5) An analytic proposition is true in virtue of the meaning of its constituent terms. (6) The negation of an analytic proposition is a contradiction. (7) In an analytic proposition the predicate is 'contained' in the subject. (8) Analytic propositions provide no information about non-linguistic reality.

analyticity. The property of being analytic.

and. ‘*p* and *q*’ is the proposition that *p* is true and *q* is true. The meaning of ‘and’ (‘&’, ‘.’, ‘^’) is defined in the propositional calculus by this truth table:

<u><i>p</i> & <i>q</i></u>			
		<i>q</i>	<i>q</i>
		T	F
<i>p</i>	T	T	F
<i>p</i>	F	F	F

**So the whole proposition ‘*p* & *q*’ is true when *p* is true and when *q* is true but otherwise false. That information may be displayed like this:
p & *q*: TFFF**

&-introduction (&I). The rule of the propositional calculus that legitimates the inference: $p, q \vdash p \& q$. Given p and given q , $p \& q$ follows as conclusion.

&-elimination (&-E). The rule of the propositional calculus which legitimates the inference: $p \& q \vdash p$. Given $p \& q$ as an assumption, p follows or q follows as conclusion.

antecedent. In a conditional proposition, the component proposition which precedes the implication sign: ' $\rightarrow$ '. For example, in ' $p \rightarrow q$ ', ' p ' is the antecedent. *See* consequent.

anti-realism. The thesis that the possibility of a proposition's being true or false does not exceed the possibility of demonstrating its truth or falsity. If p is true or false p is provable or refutable. ' p is true' entails p is provable. ' p is false' entails p is refutable. Anti-realism is inconsistent with realism.

antisymmetric. A relation R between any distinct objects x and y is antisymmetric if and only if x is related by R to y then y is not related by R to x . 'antisymmetric' (*def.*): R is antisymmetric *iff* $(\forall x) (\forall y) \neg(x = y) \& (Rxy \rightarrow \neg Ryx)$.

asymmetric. A relation, ' R ', is asymmetric if and only if, if a is related by R to b , then b is not related by R to a . 'asymmetric' (*def.*): R is asymmetric *iff* $(\forall x) (\forall y) (Rxy \rightarrow \neg Ryx)$.

'any'. *See* quantifier.

a posteriori. (1) p is *a posteriori* if and only if p 's truth value may be decided only through empirical observation. (2) p is *a posteriori* if and only if p 's truth value may not be decided purely intellectually (but requires experience). (3) p is *a posteriori* if and only if p 's truth value is decidable only with more experience than that required to understand p .

a priori. (1) p is *a priori* if and only if p 's truth value may be decided independently of empirical observation. (2) p is *a priori* if and only if p 's truth value may be decided purely intellectually. (3) p is *a priori* if and only if p 's truth value is decidable with no more experience than that required to understand p .

arrow. The name of a sign for implication: ' $\rightarrow$ '.

arrow, double. The name of a sign for logical equivalence: ‘ $\leftrightarrow$ ’.

assertible. Appropriate for advocating as true.

asserting. Advocating as true (whether sincerely or insincerely).

assertion. Proposition advocated as true (whether sincerely or insincerely). A token sentence expressing a proposition, especially as advocated as true..

attribute. Property. That which is reported by the predicate part of an indicative sentence or denoted by an adjective or adjectival expression.

attributive. (1) The grammatical place of an adjective when it qualifies a noun directly rather than predicatively. For example, in ‘Genghis Khan was a tyrannous ruler’ ‘tyrannous’ is used attributively. In ‘The ruler Genghis Khan was tyrannous’ ‘tyrannous’ is not used attributively but predicatively. (2) Use of expression which refers essentially.

arbitrary name. Letter symbolising an arbitrarily selected object in a universe of discourse: $a, b, c, \dots$. No arbitrary name is a proper name. *See* arbitrarily selected object.

arbitrarily selected object. Any object selected from a domain or universe. If an arbitrarily selected object is F then it follows that all objects in the domain are F . In this respect the logical use of ‘arbitrarily selected object’ contrasts sharply with ordinary English. *See* arbitrary name.

argument. A set of propositions such that some, the premises, are putative evidence for another, the conclusion. It makes no sense to talk of arguments as true or false. Arguments are valid or invalid, sound or unsound. Any argument is either a deductive argument or an inductive argument but not both. *See* conclusion, invalid, premise, sound, unsound, valid,

argument-frame (or argument-form). Pattern of argument. Logical form of argument. For example: $p \rightarrow q, p \vdash q$.

ascription. The asserting of possession. The ascription of a property to an object is the claiming that that object has that property.

assertion sign. The name of the sign ' $\vdash$ ' which means 'from...it may be proven that...' or 'from...it follows logically that...' or '...therefore...'. For example in: $p \ \& \ q \vdash p$. 'Assertion sign' is a misleading name because the assertion sign does not signify assertion. It may be reversed to show provability in the other direction: ' $\dashv$ '. It may be placed back to back with a reverse assertion sign to show two-way provability: ' $\dashv \vdash$ '. The assertion sign is also called 'the provability sign' and 'turnstile'.

associative laws. Logical equivalences exhibiting properties of '&': $p \ \& \ (q \ \& \ r) \Leftrightarrow (p \ \& \ q) \ \& \ r$, and 'v': $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$.

assumption. Proposition taken for granted in an argument, rather than proven by it. Typically, an assumption is used as a premise. Nevertheless 'assumption' does not mean 'premise'.

asymmetric. A relation, ' R ', is asymmetric if and only if, if a is related by R to b , then b is not related by R to a . 'asymmetric' (*def.*): R is asymmetric iff $(\forall x) (\forall y) (Rxy \rightarrow \neg Ryx)$

'at least n '. *See* quantifier.

'at least one'. One and possibly more than one. *See* quantifier.

'at least two'. Two and possibly more than two. *See* quantifier.

'at most one'. One and no more than one. *See* quantifier.

atomic proposition. Proposition with no proposition except itself as a part. ' p ' is atomic but ' $p \ \& \ q$ ' is not atomic. $\neg p$ is atomic.

atomic sentence. Sentence with no sentence except itself as a part. Sentence expressing one and only one atomic proposition. *See* atomic proposition.

aut. Latin for 'or' in the exclusive sense: p or q but not both. This is not the sense of 'or' depicted in ordinary propositional calculus. (*See vel.*)

axiom. Proposition for which no proof is required. Axioms typically feature as premises but not as conclusions of arguments. Axioms are thought to be necessary truths or logical truths and are arguably self-evident. *See* logical truth, necessary, tautology, theorem.

axiomatic development of a calculus. The generation of proofs by axioms rather than by rules of inference.

B

‘because’. ‘ q because p ’ is true if and only if $p \rightarrow q$. ‘ q because p ’ means ‘ p therefore q ’.

belief. (1) Psychological attitude entailing a disposition to allocate the truth-value ‘true’ towards a truth-valued content. The believing of what is believed. (2) The truth-valued content of a propositional attitude. That which is believed. Doxastic logic is the logic of belief. *See* proposition, propositional attitude.

both. Both p and q are true if and only if $p \& q$. ‘Both p and q ’ is true if and only if p is true and q is true, so false if p is false or (*vel*) q is false. Conjunction defined by the truth table defining ‘&’:

<u>$p \& q$</u>			
q		q	
	T	F	
p T	T	F	
p F	F	F	

$p \& q$: TFFF

biconditional. The logical operator ‘ $\leftrightarrow$ ’. ‘ $p \leftrightarrow q$ ’ is true if and only if if p is true then q is true and if p is false then q is false, so p and q have the same truth-value. *See* logical equivalence.

Def. ‘ $\leftrightarrow$ ’: $p \leftrightarrow q = (p \rightarrow q) \& (q \rightarrow p)$

$p \leftrightarrow q$

$q \quad q$

	T	F
p	T	T F
p	F	F T

$p \leftrightarrow q$: TFFT

box. The name of the necessary operator: ‘ $\Box$ ’. ‘ p ’ is read ‘necessarily p ’ or ‘box p ’.

box arrow. The name of the necessary implication operator: ‘ $\Box \rightarrow$ ’. ‘ $p \Box \rightarrow q$ ’ is read ‘ p necessarily implies q ’ or ‘ p box arrow q ’.

bivalence. The property of being either true or false.

brackets. Arc-shaped signs deployed in pairs to show the scope of logical connectives: ‘(’, ‘)’’. Curly brackets, ‘{’, ‘}’, show set membership. *See* scope.

C

calculus. Formal language, containing constants and variables, for the determination of the validity of patterns of arguments.

‘can’. Modal term expressing possibility. p can be true if and only if: $\Diamond p$. *See* possibility.

canonical. Accepted. Established.

category. Class. Set.

character. The properties of an object, especially as sufficient for the object’s belonging to a type.

characterising. Of properties: pertaining to an object, especially as sufficient for the object’s belonging to a type.

circular argument. Argument in which the conclusion features as a premise.

circular definition. Definition in which the defined concept features as a defining concept.

class. Plurality defined by a common property.

class-variable. class-variable. Letters used in the theory of classes to refer to classes: $\alpha, \beta, \gamma \dots$

coherence. p and q are coherent if and only if the truth of p does not preclude the truth of q and the truth of q does not preclude the truth of p . Consistency. *See* truth, consistency.

coherence theory of truth. The thesis that the truth of a proposition (belief, theory) is its consistency with endorsed propositions (beliefs, theories). The theory cannot be an analysis of truth because consistency with endorsed propositions is neither necessary nor sufficient for truth. As a definition of truth it is circular because the concept of truth is presupposed by consistency. Nevertheless, the theory points towards a pragmatic test for the endorsement or repudiation of beliefs.

comparative. Adjective used to express a relational property: 'greater than', 'less than'.

coimplicant. p and q are coimplicant if and only if p implies q and q implies p . *See* biconditional, logical equivalence.

common noun. Noun admitting of a plural. Noun that is not abstract. *See* abstract noun.

commutative laws. Logical truths concerning '&' and ' $\vee$ ': (1) $p \& q \leftrightarrow q \& p$. (2) $p \vee q \leftrightarrow q \vee p$.

complement. The class of objects that are not members of a given class. For example, the complement of set α is all the things that are not members of α . The sign ' $a \mid$ ' means 'the complement of a ': 'complement' *def*: $a \mid = \{x: \neg(x \in a)\}$. The complement of a set is confined to a relevant universe of discourse. *See* universe of discourse.

completeness of propositional calculus. The property the propositional calculus possesses if and only if any tautologous sequent is provable by its rules.

completeness of predicate calculus. The property the predicate calculus possesses if and only if any logically true sequent is provable by its rules.

complex conjunction. Conjunction with conjunctions other than itself as parts.

complex disjunction. Disjunction with disjunctions other than itself as parts.

conclusion. (1) Proposition proven by an argument's premises. (2) Proposition putatively proven by an argument's premises. Any proposition in an argument is either a conclusion or a premise or both.

conditional. Complex proposition in which the antecedent implies the consequent. Proposition of the form 'if p then q ': $p \rightarrow q$. It is controversial whether conditionals have truth conditions. *See* antecedent, consequent, implication.

Conditional Proof (CP). Rule legitimising inferences with a conditional conclusion. For example: $p \rightarrow q \vdash \neg q \rightarrow \neg p$. The antecedent of the conclusion to be proven is assumed and its consequent is derived as a conclusion. That antecedent and its consequent jointly comprise the proven conclusion. In $p \rightarrow q \vdash \neg q \rightarrow \neg p$, assuming $\neg q$, we may derive $\neg p$ from $\neg q$ and $p \rightarrow q$ by *modus tollens*. $\neg q$ is the antecedent and $\neg p$ the consequent of $\neg q \rightarrow \neg p$.

concept. (1) Discriminatory ability: A has the concept of x if and only if A can discriminate x 's from non x 's. By this criterion, a robot that, say, sorts boxes has the concept of a box. (2) Mental representation.

condition, necessary. *See* necessary condition.

condition, sufficient. *See* sufficient condition.

condition, truth. *See* truth condition.

conjunct. Proposition related to another proposition by '&'. In ' $p \& q$ ', p is a conjunct and q is a conjunct.

conjunction. (1) Word used to relate two propositions which are jointly asserted: ‘and’, ‘both’, ‘but’. (2) The proposition that both p and q are true: $p \ \& \ q$. ‘&’ is defined by this truth-table:

$p \ \& \ q$			
	q	q	
	T	F	
p	T	T	F
p	F	F	F

‘ $p \ \& \ q$ ’ is true when p is true and q is true but false if either conjunct is false or if both conjuncts are false: $p \ \& \ q$: TFFF

connective. Proposition forming operator on propositions. In propositional calculus: ‘ $\neg$ ’, ‘&’, ‘ $\vee$ ’, ‘ $\rightarrow$ ’. ‘ $\leftrightarrow$ ’. *See* negation, conjunction, disjunction, implication, biconditional.

consequent. The implied proposition in a conditional proposition. In ‘ $p \rightarrow q$ ’, q is the consequent. *See* antecedent.

consistency of predicate calculus. The property the predicate calculus possesses if and only if any sequent provable by its rules is a logical truth. *See* completeness.

consistency of propositional calculus. The property the propositional calculus possesses if and only if any sequent provable by its rules is tautologous. *See* completeness.

consistent. p and q are consistent if and only if the truth of p does not preclude the truth of q and the truth of q does not preclude the truth of p : $\neg(p \rightarrow \neg q) \ \& \ \neg(q \rightarrow \neg p)$. *See* contrary.

consistent wff. A wff is consistent if and only if it is tautologous or contingent. That is, either: in every allocation of truth values to its component propositions it takes the truth value ‘true’ or: for at least one assignment it takes the truth value ‘true’ and for at least one assignment it takes the truth value ‘false’. *See* inconsistent wff.

constituent. Logical or semantic part.

contingency. The property of being contingent. *See* contingent.

contingent. If p is contingent then if p is true then p could have been false and if p is false then p could have been true. If p is contingent then p is not necessary. Contingent propositions do not have their truth values essentially. *See* necessary.

contingent wff. wff that is neither a tautology nor a contradiction as determined by the truth tables of the propositional calculus.

contradiction. The conjunction of a proposition and its negation. Proposition of the form $(p \ \& \ \neg p)$ ('both p and not p '). *See* contradiction, law of.

contradiction, law of (or law of non-contradiction). The rule that the conjunction of a proposition and its negation is false. The law that any proposition of the form $(p \ \& \ \neg p)$ ('both p and not p ') is false.

contradictory. p and q are contradictory if and only if: if p is true then q is false and if p is false then q is true and if q is true then p is false and if q is false then p is true. If p and q are contradictory they differ in truth value.

'contradictory' (*def.*): p and q are contradictory iff $\neg(p \leftrightarrow q)$ is a tautology.

contrary. p and q are contrary if and only if: if p is true then q is false and if p is false then q is true. If p and q are contraries they are not both true but may both be false. 'contrary' (*def.*): p and q are contrary iff $\neg(p \ \& \ q)$ is a tautology. *See* inconsistent, subcontrary.

conventionalism. The doctrine that necessary truths are true by linguistic convention. The thesis that the meaning of a necessary truth is sufficient for its truth, or all necessary truths are analytic.

converse. The proposition that results from exchanging the propositions on the left hand and right hand side of a (main) connective. The converse of $p \rightarrow q$ is $q \rightarrow p$.

correspondence. The relation between a proposition and a state of affairs such that if the state of affairs obtains, the proposition is true and if the state of affairs does not obtain, the proposition is false.

correspondence theory of truth. The doctrine that p is true if and only if p . The thesis that the obtaining or non-obtaining of states of affairs makes propositions true or false. *See* correspondence.

counter example. Particular truth falsifying a generalisation. True proposition of the form $(\exists x) \neg(Fx)$ so inconsistent with $(\forall x) (Fx)$, or of the form $(\exists x) (Fx)$ so inconsistent with $(\forall x) \neg(Fx)$.

counterfactual. *adj.* Contrary to fact. Only hypothetical. *n. abbrev.* counterfactual conditional.

counterfactual conditonal. Conditional with hypothetical antecedent. For example: If kangaroos did not have tails they would fall over.

counterpart theory.

D

decidable. Demonstrable with regard to truth value. Provable or refutable. *See* proof, refutation.

decision procedure. Method of proof or refutation. Method for demonstrating truth value.

declarative. Apt for making an assertion. Truth-valued.

deduce. Infer. Logically derive. Prove. *See* deduction.

deduction. Kind of argument in which, putatively, the conclusion logically follows from the premises. Argument type in which, putatively, it is impossible for the premises to be true but the conclusion false. Kind of argument in which, putatively, the conjunction of the premises with the negation of the conclusion entails a contradiction. Logical inference. Logical derivation. Proof. No deduction is induction. *See* induction.

deductive. *See* deduction.

defeasible. Vulnerable to refutation. Open to counter example.

definite article. ‘The’. Word used to refer to anything countable, specifying it as that particular thing but without thereby mentioning what it is. *See* indefinite article.

definite description. Expression with sense used to denote exactly one individual: ‘the tallest man in the room’. Definite descriptions are typically introduced by the definite article ‘the’. Definite descriptions are distinguished from proper names. *See* proper name.

definite reference. Reference to a specific thing. Reference to one and only one individual.

degree. (1) Of application of a predicate: Fully applying or not fully applying but not not applying. For example if a is F to some degree, a is F to some degree more than 0 but less than 1, for example : a is F to degree 0.7 but not F to degree 0.3. (2) Of truth of a proposition: Fully true or not fully true but not not true. If p is to some degree true then p true to some degree between 0 and 1, for example, true to degree 0.6 but false to degree 0.4. Fuzzy logic is the logic of degrees of truth. *See* fuzzy logic.

degrees of truth. *See* fuzzy logic.

demonstration. Valid argument. Sound argument. If p is demonstrated, then p is true. If p is not true, then p is not demonstrated.

demonstrative. Referring pronoun: ‘that’, ‘this’, ‘I’, ‘now’.

denial. The assertion that a proposition is false. The denial that p is the assertion that $\neg p$.

deny. To assert of a proposition that it is false. To deny that p is to assert that $\neg p$.

denying the antecedent. The fallacy of taking as valid the invalid inference: $(p \rightarrow q) \ \& \ \neg p \rightarrow \neg q$.

deontic. Pertaining to obligation.

deontic logic. The logic of obligation.

designator. Referring expression. *See* rigid designator.

discourse. Language. Portion of language characterised by its subject matter.

disjunct. Proposition expressing one alternative in a disjunction. Proposition related to another proposition by ‘ $\vee$ ’. In $p \vee q$, p is a disjunct and q is a disjunct. *See* complex disjunction, disjunction.

disjunction. The proposition that either p is true or q is true or both: $p \vee q$. Logical relation defined by the truth table for ‘ $\vee$ ’:

$p \vee q$	
q	q
T	F
p T	T T
p F	T F

‘ $p \vee q$ ’ is true if p is true, p if q is true, q is both p and q are true but false if both p and q are false: $p \vee q$: TTTF

$\vee$ -introduction (v-I). Rule of the propositional calculus legitimating the inference: $p \vdash p \vee q$. Given p as premise, $p \vee q$ follows as conclusion.

$\vee$ -elimination (v-E). Rule of the propositional calculus legitimating the inference: $p \vee q, p \rightarrow r, q \rightarrow r, \vdash r$. Given $p \vee q$ as assumption, and given a proof of r from p as assumption and a proof of r from q as assumption, r follows as conclusion.

distinct. a and b are distinct if and only if a is not b : $\neg(a = b)$. *See* identity.

distinction. The fact of two things not being the same thing. What their being distinct consists in.

domain. Set of objects referred to. The domain of a relation, R , is the set of objects that stand in relation R to something. *See* range, universe.

double negation. The negation of the negation of a proposition. $\neg\neg p$ is the double negation of p . The double negation of a proposition is logically equivalent to that proposition.

double negation, rule of (DN). Rule of the propositional calculus legitimating inferences of the form: $p \vdash \neg\neg p$ and $\neg\neg p \vdash p$.

dyadic. Of relations: two term. 'is next two' is a two term relation.

E

'E Shriek'. ' $\exists!$ '. Quantifier with the meaning: 'There exists one and only one...', 'There exists exactly one...', or 'There is a unique...'.
'each'. *See* quantifier.

'either...or...'. *See* disjunction.

elementary. Without parts.

elementary conjunction. Conjunction which has no conjunction except itself as a part.

elementary disjunction. Disjunction which has no disjunction except itself as a part.

elimination, rules of. *See* $\&$ -elimination, $\vee$ -elimination, $\forall$ -elimination, $\exists$ -elimination, identity-elimination.

empirical. Knowable through experience.

empty class. Class with no members. The null set.

empty field. The field of a relation with no relata. A relation that holds between nothing and nothing has an empty field. *See* field.

empty universe. Universe containing no objects.

entailment. (1) Implication. The relation between p and q expressed by ' $\rightarrow$ '. p entails q if and only if: $p \rightarrow q$. ' p entails q ' is true unless p is

true and q is false. (2) Strict implication. The relation between p and q expressed by ' $\rightarrow$ '. p entails q if and only if: $p \rightarrow q$. 'p entails q' if and only if p necessarily implies q . This relationship is sometimes called 'strict implication. *See* material implication.

entails. *See* entailment.

equivalence. The relation between p and q if and only if: if p is true then q is true and if q is true then p is true, that is, if they have the same truth value. The relation between p and q if and only if their biconditional is true: $p \leftrightarrow q$. Sometimes equivalence is called 'logical equivalence'. *See* biconditional.

equivalence relation. A relation, ' R ', is an equivalence relation if and only if R is transitive, symmetric and reflexive. *See* reflexive, relation, symmetric, transitive.

'equivalence relation'. A relation, ' R ', is an equivalence relation if and only if: R is transitive, symmetric and reflexive. 'equivalence relation' (*def.*): R is an equivalence relation iff $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow Rxz) \ \& \ (Rxy \rightarrow Ryx) \ \& \ (\exists x) Rxx$

equivalent. p and q are logically equivalent if and only if: if p is true q is true and if q is true p is true and if p is false q is false and if q is false p is false that is, if they have the same truth value. Two propositions, p , q , are logically equivalent if and only if their biconditional is true: $p \leftrightarrow q$. The meaning of ' $\leftrightarrow$ ' is given by this truth table:

$p \leftrightarrow q$

	q	q
	T	F
p	T	T F
p	F	F T

' $p \leftrightarrow q$ ' is true if p is true and q is true and if p is false and q is false. 'p $\leftrightarrow$ q' is false if p is true and q is false and if q is true and p is false:
 $p \leftrightarrow q$: TFFT

essential property. A property something could not lack and either: (1) remain the thing it is, or, (2) remain the sort of thing it is, or, (3) exist. *See* property, inessential property.

‘every’. *See* quantifier.

‘everything’. *See* quantifier.

evidence. p is evidence that q (or for q) if and only if p ’s truth would make the truth of q more likely than p ’s not being true.

‘exactly one’. No more than one and no fewer than one. *See* numerically definite quantifier.

‘except if’. And not if. ‘ p is true except if q is true’ means ‘ $q \rightarrow \neg p$ ’ which is logically equivalent to ‘ $p \rightarrow \neg q$ ’.

exception. *See* counter example.

excluded middle, law of. Rule that any proposition of the form ‘ p or not p ’ is a necessary truth. The claim that the disjunction of a proposition and its negation, $p \vee \neg p$, is a tautology. The law of excluded middle is not admitted by intuitionism. Fuzzy logic admits degrees of truth and falsity. Free logics admit truth values other than ‘true’ and ‘false’. *See* free logics, fuzzy logics, intuitionism, proposition.

exclusive. (1) Of a distinction: admitting of no other term or kind. (2) A sense of ‘or’. ‘ p or q ’ is true in the exclusive sense of ‘or’ if and only if at least one and at most one of p and q is true. The meaning of ‘or’ in the exclusive sense is defined by this truth table:

p or q (exclusive)

	q	q
	T	F
p	T	F
p	F	T

‘ p or q ’ is true if p is true but q is false and if p is false but q is true but false if p and q are both false and false if p and q are both true: p or q : FTTF (exclusive). The exclusive sense of ‘or’ is *not* the one depicted by ‘ $\vee$ ’ in ordinary propositional calculus. *See* disjunction, ‘or’.

existence. (1) What there is. (2) The being of what there is.

existential. Pertaining to existence.

existential assumption. Proposition governed by the existential quantifier, featuring in an argument typically as a premise but never as a conclusion.

existential generalisation. Proposition expressing the claim that all members of a class have a certain property. Proposition of the form: $\forall x (Fx)$.

existential ‘is’, the. *See* ‘is’.

existential proposition. Proposition apt for asserting existence. Proposition governed by the existential quantifier.

existential quantifier. The symbol ‘ $\exists$ ’ for ‘there is a’ or ‘there exists at least one’. A sentence governed by the existential quantifier is logically equivalent to a complex disjunction. For example, in the universe of discourse m, n, o , $(\exists x Fx)$ means $(Fm \vee Fn \vee Fo)$.

Existential Quantifier Introduction ($\exists I$). Rule of inference of the predicate calculus legitimating inferences of the form: $Fm \vdash (\exists x)(Fx)$. Intuitively, if a particular object is F then there exists something or other that is F .

Existential Quantifier Elimination ($\exists E$). Rule of inference of the predicate calculus legitimating inferences of the form: $(\exists x)(Fx), Fa \rightarrow p, \vdash p$. Intuitively, If something specific is F then p , and something is F , then p .

extension. The set of objects to which an expression applies. The reference of a proposition. The set of objects falling under a concept. *See* set theoretic extension.

F

false. p is false if and only if not p . (no italics) p is false if and only if the truth-conditions sufficient for p ’s falsity obtain. If a proposition is not true, and not neither true nor false, it is false.

falsity. The property of being false. The truth of $\neg p$ is necessary and sufficient for the falsity of p .

fallacies. Invalid patterns of inference commonly taken to be valid.

field. The field of a relation R is the set of objects either in the domain of R or in the converse domain of R . *See* converse domain, domain, empty field, relation.

finite. A set is finite if and only if for some natural number, 0, 1, 2..., the number of its members is that number. Equivalently, a set is finite if and only if a one-one mapping obtains between its members and a subset of the natural numbers. A set is finite if and only if it is not infinite. *See* infinite.

fixed. Unchanging, in particular with regard to referent.

fixed universe of discourse. Set of objects, or type of objects, referred to that does not gain or lose members.

follows from. q follows from p if and only if p entails q . *See* entailment.

‘for any x ’. *See* Universal Quantifier.

form of argument. If one argument is a substitution instance of another they have the same logical form.

formal implication. Conditional generalisation. Universal quantification over material implication. Proposition of the form: $(\forall x) (Fx \rightarrow Gx)$.

formal language. Symbolic system comprising rules, operators and variables, for the development of proofs. Language with its content subtracted. Abstract system in which meaning, truth and validity depend upon logical form. *See* logical form.

formation rules. The rules governing what counts as a *wff*. in a formal language.

formula. A sentence or putative sentence of a formal language. Formulae are not well-formed (that is, ill-formed) or well-formed. ‘*wff*’ abbreviates ‘well formed formula’. *See* *wff*.

formulated. Translated into symbolic notation. *See* formula, *wff*.

foundation. (1) Necessary condition. (2) Ultimate necessary condition, that is: necessary condition with no necessary condition. *See* necessary condition.

free logic. Logic in which truth values other than ‘true’ and ‘false’ are admitted. For example: ‘neither true nor false’, ‘both true and false’, ‘undecidable’.

free variable. Variable not bound by a quantifier.

function. *See* propositional function, truth function.

fuzzy logic. logic in which degrees of truth and falsity are admitted. For example, if p is true to degree 0.6 then p is false to degree 0.4. Classical logic is the limiting case in which p is true to degree 1 or true to degree 0, and so false.

G

gap, truth value. *See* truth value gap.

given. (1) Unargued. For example, assumptions are given because they are justified by no stretch of argument. (2) Uninterpreted.

grammar. The set of implicit or explicit rules governing what may be expressed in a language.

H

I

idempotence, laws of. $p \ \& \ p \leftrightarrow p$, $p \ \vee \ p \leftrightarrow p$.

identity. (1) The relationship between a and b which holds if and only if a is b : $a = b$. (2) The relationship between a and b which holds if and only if a is like b : $Fa \leftrightarrow Fb$. *See* identical.

identity and class. Two classes are identical if and only if they have the same members. Any two empty classes are identical.

identity elimination (=E). Rule legitimating inferences of the form:
 $a = b, Fa \vdash Fb$.

identity introduction (=I). Rule legitimating the introduction of a proposition the form $x = x$ at any stage of a proof. Because $x = x$ is a logical truth, it rests on no assumptions.

identity statements. *See* 'is'.

identity, law of. *See* law of identity.

identical. The same. 'Same' is ambiguous between 'same one' and 'same sort'. a is the same one as b if and only if a is numerically identical to b . a is of the same sort as b if and only if a is qualitatively similar to b . a is of only the same sort as b if and only if a is qualitatively identical to b . (1) a and b are numerically identical if and only if a is b : $a = b$. (2) a and b are qualitatively identical if and only if a is (exactly) like b : $Fa \leftrightarrow Fb$.

a is numerically identical to b if and only if a is b . a is numerically identical to b if and only if a is the same one as b . a is qualitatively similar to b if and only if a is like b in at least one respect. a is qualitatively identical to b if and only if a is like b in every respect. a is qualitatively similar to b if and only if a and b are of the same sort (kind, type).

In predicate calculus, 'a is numerically identical to b' is symbolised ' $a = b$ '. Numerical identity is a transitive relation: $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z)$ If a is numerically identical to b and b is numerically identical to c , then a is numerically identical to c .

Numerical identity is a symmetrical relation: $(\forall x) (\forall y) (x = y) \leftrightarrow (y = x)$ If a is numerically identical to b then b is numerically identical to a .

Numerical identity is a reflexive relation: $(\forall x) (x = x)$ a is numerically identical to a .

Numerical identity is an equivalence relation because it is transitive, symmetrical and reflexive : $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z) \ \& \ (\forall x) (\forall y) (x = y) \leftrightarrow (y = x) \ \& \ (\forall x) (x = x)$.

‘if’. ‘ q if p ’ means ‘ p is a sufficient condition for q ’: $p \rightarrow q$. *See* implication, necessary condition, sufficient condition.

‘if and only if’. *See* biconditional.

iff. abbrev. ‘if and only if’. *See* biconditional.

imperative. Sentence appropriate for expressing a command or suggestion. Imperatives are not capable of being true or false, only obeyed, disobeyed, ignored.

imperative logic. The logic of imperatives. Even though imperatives are not truth-valued, logical relations may obtain between them, For example: ‘All A ’s do x ’, ‘This is an A ’, ‘ A , do x ’.

implication. The relation between two propositions expressed by ‘ $\rightarrow$ ’. ‘ $p \rightarrow q$ ’ is true unless p is true and q false.

implication, formal. *See* formal implication.

implication, material. *See* material implication.

implicature, conversational. Unstated or non literal meaning assumed in ordinary conversation. For example: ‘Oh, you are awful’.

implies. p implies q if and only if $p \rightarrow q$. $p \rightarrow q$ is true unless p is false and q is true. *See* material implication.

included in. Set α is included in set β if and only if every member of α is a member of β , that is, if and only if $\alpha \subseteq \beta$ is true. The inclusion sign, ‘ $\subseteq$ ’, is defined as follows: ‘ $\subseteq$ ’ *def*: $\alpha \subseteq \beta \leftrightarrow (\forall x) (x \in \alpha \rightarrow x \in \beta)$

(Inclusion is not membership. It is a category mistake to say set α is a member of set β if β includes α , except if β is a set of sets. Even then, the inclusion relation is not the membership relation.)

inclusion. The relation which holds between two sets α and β if and only if ' $\alpha \subseteq \beta$ ' is true. *See* included in.

inconsistency. The relation which holds between p and q if and only if $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$. *See* inconsistent.

inconsistent. p and q are inconsistent if and only if: if p then not q and if q then not p , that is: $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$.

inconsistent wff. wff that for all possible allocations of truth values to its component propositions takes the truth value 'false'. wff that is neither tautologous nor contingent. *See* consistent wff, truth table.

indefinite article. 'a', 'an'. Word used to refer to anything countable without specifying it as that particular thing. *See* definite article, indefinite reference.

indefinite reference. Reference to anything countable without specifying what it is. Reference using, or equivalent to using, the indefinite article: 'a', 'an'. *See* indefinite article.

independent. *See* logically independent.

indeterminacy of translation. The thesis that any knowledge of behaviour is insufficient for the correct interpretation of utterances.

indexical. Expression requiring knowledge of the context of utterance for the determination of its reference. For example, demonstrative and personal pronouns: 'I', 'here' 'this', 'that', 'now', 'yesterday', 'today', 'tomorrow', and all tensed verbs. To understand who 'I' refers to' it is necessary to know who is saying 'I'. To know which time 'now' refers to, it is necessary to know when 'now' is uttered. To know where 'here' refers to, it is necessary to know where 'here' is uttered. Indexicals are sometimes called 'token reflexives'.

indexicality. The property of being indexical. *See* indexical.

indicative. Capable of being true or false.

indirect proof. *See* *reductio ad absurdum*.

individual. Particular thing, as opposed to type, sort or universal.

individual variable. Symbolic place holder for an individual. In predicate calculus: 'x' stands for any individual, 'y' stands for any individual distinct from x and so on. *See* individual, predicate calculus, variable.

individual variable substitution. The replacement of all the occurrences of one variable in a *wff.* by another variable.

induction. The putative derivation of a conclusion from premises by probabilistic rather than deductive means. ' The drawing of conclusions of the form ' $(\forall x) (Fx)$ ' from premises of the form ' $Fa, Fb...$ ' All inductive arguments are formally invalid.

inductive. Of an inference: probabilistic rather than deductive. *See* induction.

intransitive. A relation, 'R', is intransitive if and only if: if *a* is related by *R* to *b*, and if *b* is related by *R* to *c*, then *a* is not related by *R* to *c*. 'intransitive' (*def.*): *R* is intransitive *iff* $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow \neg Rxz)$

inessential property. A property something could lack and nevertheless, either: (1) remain the thing it is, or, (2) remain the sort of thing it is, or, (3) exist.

inference. (1) The relation between premises and conclusion in a valid argument. (2) The relation between premises and conclusion in a putatively valid argument.

infinite. A set is infinite if and only if there is a one-one mapping between its members and a proper subset of that set. Intuitively: a set is infinite if and only if, for any natural number 0, 1, 2,..., the number of its members is greater than that number. A set is infinite if and only if it is not finite. *See* finite, one-one mapping.

infinite conjunction. Conjunction with infinitely many conjunctions as parts: $p \ \& \ q \ \& \ r...n$.

infinite disjunction. Disjunction with infinitely many disjunctions as parts: $p \vee q \vee r...n$.

infinity. *See* infinite.

informative. (1) synthetic. (2) non-tautological.

integer. Number.

integer, non-negative. Number in the series 0, 1, 2, 3... n .

integer, positive. Number in the series 1, 2, 3... n .

intension. (1) Sense, especially the sense of a sentence or proposition. (2) Property in virtue of which the members of a class belong to that class.

interderivability. If q is provable from p and p is provable from q then p and q are interderivable. If $q \vdash p$ and $p \vdash q$ then $p \dashv \vdash q$. See proof, assertion sign.

internal structure. The parts of a proposition related in ways that affect meaning, truth and validity. In propositional calculus, the parts of propositions are only propositions. For example, ' p ' and ' q ' are parts of ' $p \& q$ '. An atomic proposition, p , has no parts (except itself). In predicate calculus the parts of propositions are names, predicates and quantifiers. For example, the parts of ' $\exists a (Fa)$ ' are ' $\exists a$ ', ' F ' and ' a '. What could only be treated as an atomic proposition in propositional calculus may be 'opened up' in predicate calculus and inferences that depend upon relations between parts of the proposition depicted.

interpretation. The assignment of truth values to the component propositions of a proposition in propositional calculus. The assignment of objects, properties and relations to propositions of the predicate calculus.

intersection. The set of objects common to two sets. The intersection of α and β is the set of objects that are members of both α and β . The sign for intersection, ' $\cap$ ', is defined in this way: ' $\cap$ ' def. $\alpha \cap \beta = \{x: x \in \alpha \& x \in \beta\}$.

introduction rules. See $\&$ -introduction ($\&I$), $\vee$ -introduction ($\vee I$), $\exists$ -introduction ($\exists I$), $\forall$ -introduction ($\forall I$), = introduction ($=I$).

intransitive. The kind of relation ' R ' such that if x is related by R to y and y is related by R to z then x is not related by R to z : $(\forall x) (\forall y) (\forall z) (xRy \& yRz) \rightarrow \neg(xRz)$.

intuitionism. (1) Logic not admitting the law of excluded middle, the law of double negation and *reductio ad absurdum*. (2) The anti-realist thesis that truth entails provability. (3) The meta-mathematical thesis that proofs are not discovered but invented or ‘constructed’. *See* anti-realism, double negation, excluded middle, *reductio ad absurdum*.

invalid. Of a deductive argument: the truth of the conclusion is consistent with the falsity of the premises. If an argument is invalid, then even if the premises are true, the conclusion could still be false. If an argument is invalid it cannot be sound. If an argument is not invalid then it is valid.

invalidity. The property a deductive argument possesses if and only if the truth of the conclusion is consistent with the falsity of the premises. Invalidity is inconsistent with soundness.

irreflexive. A relation is irreflexive if it does not hold between anything and itself. ‘*R*’, is irreflexive if and only if, *a* is not related by *R* to *a*. ‘irreflexive’ (*def.*): *R* is irreflexive *iff* $(\forall x) \neg Rxx$

‘is’. (1) ‘is the same thing as’. ‘is identical to’ The ‘is’ of identity: ‘*a* is *b*’ is true if and only if ‘*a* = *b*’ is true. (2) ‘is’ in ‘is *F*’, ‘is *G*’ (where ‘*F*’ and ‘*G*’ denote properties). The ‘is of predication’: ‘*a* is *F*’ is true if and only if ‘*Fa*’ is true. (3) ‘exists’. The existential ‘is’: ‘*x* exists’ is true if and only if ‘ $\exists x Fx$ ’ is true.

‘is a member of’. Some object *x* is a member of some set α if and only if ‘ $x \in \alpha$ ’ is true.

is implied by. *q* is implied by *p* if and only if $p \rightarrow q$. *See* implication, material implication.

‘is the same object as’. *See* identical.

‘is’ of identity, the. *See* identical, ‘is’.

‘is’ of predication, the. *See* ‘is’.

J

jointly exhaustive. Two propositions are jointly exhaustive if and only if: if p is not true then q is true and if q is not true then p is true: $(\neg p \rightarrow q) \ \& \ (\neg q \rightarrow p)$.

K

L

law. *See* associative, commutative, conversion, de Morgan's, distributive, double negation, excluded middle, idempotence, obversion.

law of excluded middle. Rule that any proposition of the form p or not p is a necessary truth. Rule that at least and at most one of the disjunction of a proposition and its negation is true. Claim that $p \vee \neg p$ is a tautology. The law of excluded middle is not admitted by intuitionism.

law of identity. Rule that if a proposition is true then it is true. The entailment of a proposition by that proposition. The claim that $p \rightarrow p$ is a tautology.

law of non-contradiction. The rule that the conjunction of a proposition and its negation is necessarily false. The claim that $(p \ \& \ \neg p)$ is a contradiction.

least. The smallest. If n is the least number in some domain then there is no number smaller than n in that domain.

lemma. Auxiliary result used in a proof.

logic. The distinguishing of valid from invalid patterns of argument.

logic, many valued. *See* free logic.

logical connective. Proposition forming operator on propositions that logically relates propositions by its application. For example, ‘v’ but not ‘ $\neg$ ’.

logical constant. Logical operator that is either a connective or not a connective. For example ‘ $\neg$ ’ is a logical constant but not a connective, ‘&’ both a logical constant and a logical connective. ‘Constant’ contrasts with ‘variable’. *See* logical connective, variable.

logical equivalence. The relationship that holds between p and q if and only if p and q have the same truth value.

$p \leftrightarrow q$

	q	q
	T	F
p	T	T F
p	F	F T

$p \leftrightarrow q$: TFFT

logical form. What remains of a proposition if its content is subtracted. The properties of propositions bestowed only by relations between variables and connectives, in virtue of which meaning, truth and validity obtain. If one argument is a substitution instance of another they have the same logical form.

logical independence. p and q are logically independent if and only if: p and q are not contrary, not subcontrary, p does not imply q , q is not implied by p , p and q are not equivalent and p and q are not contradictory. *See* contradictory, contrary, subcontrary.

logically follows from. *See* implication, proof.

logically true. *See* logical truth.

logical truth. Proposition true only for logical reasons. Proposition for whose truth the principles of logic are sufficient. Truth made true by logic. No logical truth is synthetic or contingent.

M

main column. In a truth table for a *wff*, the list of possible truth values of the whole *wff* which shows whether the *wff* is a tautology, a contradiction or contingent. *See* connective, main connective, truth table, *wff*.

main connective. That connective in a *wff* to which all others are subordinate in scope. The connective in a *wff* with the widest scope. In any *wff* there is one and only one main connective. In a truth table for a *wff*, the possible truth values of the *wff* as a whole appear in the column under the main connective. *See* connective, main column, truth table, *wff*.

many-valued logics. *See* free logic.

material implication. The relation between *p* and *q* that holds if and only if $p \rightarrow q$ is true. The conditional $p \rightarrow q$ is true unless *p* is true and *q* is false. Material implication falls short of entailment, or strict implication, because if *p* entails *q* then necessarily $p \rightarrow q$, ie: $(p \rightarrow q)$. The material implication of *q* by *p* differs from a formally valid inference from *p* to *q* because that requires the impossibility of *p*'s falsity given *p*. If *q* is validly derivable from *p* then '*p* but not *q*' entails a contradiction. If *p* materially implies *q* '*p* but not *q*' does not entail a contradiction even though $p \rightarrow q$ is in fact true. The meaning of the logical connective ' $\rightarrow$ ' is given by this truth table:

$p \rightarrow q$	
<i>q</i>	
<i>T</i>	
<i>F</i>	
<i>p</i>	<i>T</i>
<i>p</i>	<i>F</i>
<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>
<i>T</i>	<i>F</i>
<i>F</i>	<i>F</i>

So $p \rightarrow q$ is true under all allocations of truth values except if the antecedent, p , is true and the consequent, q , is false: $p \rightarrow q$: TFFT.

material implication, paradoxes of. If p is false then $p \rightarrow q$, so a false proposition implies any proposition, which is counter-intuitive. Paradoxically, if p implies any proposition, p implies q and p implies $\neg q$. If q is true then $p \rightarrow q$, so a true proposition is implied by any proposition, which is counter-intuitive. Paradoxically, if q is true, q is implied by p and q is implied by $\neg p$. *See* material implication.

materially implies. p materially implies q if and only if $p \rightarrow q$. *See* material implication.

mathematics. The study of reality through the medium of number.

matrix. Truth table for the definition of connectives or determination of sequents as tautologous, contradictory or contingent:

$\neg p$

	p	$\neg p$
p T	T	F
p F	F	T

$\neg p$: FT

$p \rightarrow q$

	p	q	$p \rightarrow q$
		T	F
p T	T	T	F
p F	F	T	T

$p \rightarrow q$: TFFT

$p \& q$

	p	q	$p \& q$
		T	F
p T	T	T	F
p F	F	F	F

$p \& q$: TFFF

$p \vee q$

$q \quad q$

	T	F
p	T	T
p	F	T

$p \vee q$: TTTF

$p \leftrightarrow q$

	q	q
	T	F
p	T	T
p	F	F

$p \leftrightarrow q$: TFFT

meaning. (1) Sense. An expression has a sense if and only if it has a verbal definition. (2) Reference. An expression has reference if and only if it has a referent.

mechanical proof check. See: algorithm.

mechanical proof-discovery. Proof discovery that could be conducted by a machine.

mechanical test. Test that could be conducted by a machine. For example: the truth table test for the logical status of *wffs* of the propositional calculus.

member. Item possessing a property necessary and sufficient for set membership.

membership-relation. Property possession sufficient for set membership. *See* member.

metatheorem. Theorem about a theorem. *See* *theorem*.

metalogical variable. Variable used to describe variables.

modal. Pertaining to necessity, possibility or probability.

modal idioms. Informal terms expressing necessity, possibility or probability: 'it is necessary that', 'it cannot be false that', 'probably'.

modal logic. The logic of necessity, possibility and probability. Modal logic includes propositional and predicate calculus as parts.

modal realism. The view that all possible worlds exist. The actual world does not differ from other worlds by existing rather than not existing but by our being in it. Modal realism entails the existence of an infinite number of possible worlds. *See* actual, ersatz world, ersatzism, possible world,.

***modus ponens* (MP).** Rule of inference of the propositional calculus legitimating inferences of the form: $p \rightarrow q, p \vdash q$.

***modus tollens* (MT).** Rule of inference of the propositional calculus legitimating inferences of the form: $p \rightarrow q, \neg q, \vdash \neg p$.

monotonic. Property a valid deductive argument possesses just in case it remains valid no matter what is added to it.

‘most’. More than half.

mutually exclusive. Two propositions are mutually exclusive if and only if: if one is true then the other is false: $(p \rightarrow \neg q) \ \& \ (q \rightarrow \neg p)$. *See* jointly exhaustive.

N

naïve. Pre theoretical.

name. Term used to refer to an individual. Substantive. Noun. *See* arbitrary name, proper name.

name-letter. Letter that replaces a proper name: ‘m’, ‘n’... .

name substitution. The replacement of all the occurrences of one name in a *wff.* by another name.

natural deduction. The discovery of proofs using rules of inference, rather than truth tables.

natural kind. Type of thing that exists, and is the way it is, independently of any classificatory system.

natural number. Member of the set of numbers: $\{0, 1, 2, 3 \dots n\}$.
Non-negative integer.

natural structure. That which is indicated by the relations of scope and subordination in a *wff*. See rank, scope, subordination.

necessary. If p is necessary, then if p is true p could not have been false and if p is false then p could not have been true. If p is necessary then p has its truth value essentially, or could not have had a different truth value. See analytic, logical truth, theorem, truth value.

necessary condition. If the truth of p is a necessary condition for the truth of q then if not p then not q (ie if p is not true then q is not true). If p is a necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p . If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . Everything entails its own prerequisites. See sufficient condition.

necessary falsehood. False proposition that could not have been true.
Proposition that entails any proposition.

necessary truth. True proposition that could not have been false.
Proposition that follows from any proposition.

negation. The result of prefacing a proposition by the negation sign ' $\neg$ '. Contradictory of a proposition. Proposition, including a negation sign, used to assert that a proposition is false. 'not p ' is true if and only if p is false. The meaning of the negation sign ' $\neg$ ' is given by the following truth table:

$\neg p$	
p	$\neg p$
T	F
F	T

' $\neg p$ ' is true if p is false, false if p is true: $\neg p$: FT

Negation, Double, See: Double Negation (DN).

negative sentence. Sentence including a negative expression.

negative proposition. Proposition including a negative expression.

‘neither...nor...’. ‘Neither p nor q ’ is true if and only if $\neg p \ \& \ \neg q$.

‘no’. Not any. *See* quantifier.

non-contradiction, law of. The rule that the conjunction of a proposition and its negation is false: $\neg (p \ \& \ \neg p)$.

non-empty term. A term ‘ F ’ is empty if and only if there is nothing that has the property of being F : $\neg(\exists x) (Fx)$.

non-empty universe of discourse. A universe which can be referred to, in which at least one thing exists.

non-negative integer. Member of the set: $\{0, 1, 2, 3...n\}$. Natural number.

non-particular. General or abstract.

non-symmetric. ‘non symmetric’ (*def.*): R is non symmetric *iff* $(\exists x) (\exists y) (Rxy \ \& \ \neg Ryx)$

non-tautologous. (1) In propositional calculus: contingent or contradictory. (2) Not analytic. A non-tautological proposition’s meaning is not sufficient for its truth.

non-transitive. A relation, R , is non-transitive if and only if R is not transitive and R is not intransitive. *See* intransitive, relation, transitive.

‘none’. Not any. *See* quantifier.

‘not’. *See* negation.

‘not any’. None. *See* quantifier.

‘not both...and...’. ‘Not both p and q ’ is true if and only if $\neg(p \ \& \ q)$ is true, that is, if either p is false or q is false.

‘nothing’. Not anything. *See* quantifier.

noun. Word used to refer to an object rather than ascribe a property. Substantive. *See* name, proper name.

null class. The class with no members. There is only one null class because all classes with no members are identical.

number. Abstract object that has mathematical relations essentially. (Unity or) plurality. Set. *See* natural number, rational number, real number.

numeral. Symbol used to refer to a number. For example, '1' refers to the number one.

numerically definite quantifier. Quantifier used to assert the existence of some exact number of objects. For example ' $(\exists x^3) (Fx)$ ' is true if and only if there are exactly three things that have the property of being *F*.

numerical identity. The relationship between *a* and *b* if and only if *a* is *b*: $a = b$. Numerical identity is a transitive relation: $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z)$. If *a* is numerically identical to *b* and *b* is numerically identical to *c*, then *a* is numerically identical to *c*. Numerical identity is a symmetrical relation: $(\forall x) (\forall y) (x = y) \leftrightarrow (y = x)$ If *a* is numerically identical to *b* then *b* is numerically identical to *a*. Numerical identity is a reflexive relation: $(\forall x) (x = x)$ *a* is numerically identical to *a*. Numerical identity is an equivalence relation: The relation 'is numerically identical to' is transitive, symmetrical and reflexive. Therefore it is an equivalence relation. $(\forall x) (\forall y) (\forall z) (x = y \ \& \ y = z \rightarrow x = z) \ \& \ (\forall x) (\forall y) (x = y) \leftrightarrow (y = x) \ \& \ (\forall x) (x = x)$. *See* identity, identical, qualitative identity.

O

O-form proposition. Particular negative proposition, of the form: $(\exists x) (Fx \ \& \ \neg Gx)$.

one-one mapping. Relation holding between two sets, α and β , if and only if there is a relation, R , such that each member of α bears R to exactly one member of β and each member of β bears R to exactly one member of α .

one-many mapping. Relation, ' R ', between two sets, α and β , such that each member of α bears R to at least one member of β and each member of β bears R to at least one member of α .

'only'. 'Only if p then q ' means ' p is necessary for q '. If p is necessary for q then q is sufficient for p , so 'Only if p then q ' is true if and only if 'if q then p ' is true. 'Only if p then q ' therefore means: $q \rightarrow p$.

'Only things that are F are G ' means being F is necessary for being G . If being F is necessary for being G then being G is sufficient for being F , so 'Only things that are F are G ' is true if and only if 'Anything that is G is F ' is true. 'Only things that are F are G ' therefore means: $(\forall x) (Gx \rightarrow Fx)$

'only, the'. 'The only things that are F are G ' means 'Anything that is F is G ': $(\forall x) (Fx \rightarrow Gx)$

'only if...then...'. 'Only if p then q ' means ' p is necessary for q '. If p is necessary for q then q is sufficient for p , so 'Only if p then q ' means if q then p : $q \rightarrow p$.

ontological. Pertaining to existence.

ontological commitment. The existence entailed by the truth of an existential proposition. The existence entailed by the use of the existential quantifier.

opaque. A context in a sentence is referentially opaque if and only if the truth value of that sentence is not necessarily retained through substitution of propositions in that context. For example, 'believes that' is referentially opaque because in 'A believes that p ', q cannot be substituted for p without the possibility of changing truth value.

operator. Logical constant. Logical connective. *See* Logical constant. Logical connective.

'or'. *See* disjunction.

order of proper names. Changing the order of proper names in a sentence changes its meaning and might change its truth value. For example: Rmn (m bears relation R to n) might be true but Rnm (n bears R to m) might be false.

order of quantifiers. Changing the order of quantifiers in a sentence containing both existential and universal quantifiers might change its truth-value. For example: $(\forall y) (\exists x) (Rxy)$ (For any y , there exists an x such that x bears relation R to y) might be true but $(\exists y) (\forall x) (Rxy)$ (There exists a y such that for all x , x bears relation R to y) might be false. Changing the order of quantifiers in a sentence containing only existential or only universal quantifiers does not change its truth values. *See* quantifier shift, quantifier shift principles.

ordered n-tuple. Sequence essentially ordered such that its members are the same members as another sequence. For example, $\langle 1, 2, 3, 4 \rangle$ is an ordered sequence in relation to $\langle 1, 2, 3, 4 \rangle$ but not in relation to $\langle 1, 4, 3, 2 \rangle$. *See* ordered pair.

ordered pair. A pair essentially ordered such that its members are the same members as another pair. For example, $\langle 1, 2 \rangle$ is an ordered pair in relation to $\langle 1, 2 \rangle$ but not in relation to $\langle 2, 1 \rangle$. *See* ordered sequence.

ostensive definition. Defining an expression by pointing to an object it refers to and naming it using that expression.

P

pair class. The class whose members are either a or b : $\{a, b\}$. If x is a member of the pair class $\{a, b\}$ then either x is a or x is b .

paradigm. Example.

paradigm case. Typical example.

paradox. Apparent contradiction or inconsistency generated by otherwise uncontroversial logical means.

paradoxes of formal implication. Given universal quantification over material implication: $(\forall x) (Fx \rightarrow Gx)$, it follows from $(\forall x) (Fx)$ that $(\forall x) (Fx \rightarrow Gx)$ and from $(\forall x) \neg(Fx)$ that $(\forall x) (Fx \rightarrow Gx)$. But if $(\forall x) (Fx) \vdash (\forall x) (Fx \rightarrow Gx)$ then if anything has a certain property it has a certain distinct property, which is counter intuitive and if $(\forall x) \neg(Fx) \vdash (\forall x) (Fx \rightarrow Gx)$ then if anything lacks a certain property it follows that it has a certain property, which is counter intuitive. If anything either possesses or lacks a certain property then it has a certain distinct property, which is paradoxical.

paradoxes of material implication. If p is false then $p \rightarrow q$, so a false proposition implies any proposition, which is counter-intuitive. Paradoxically, if p implies any proposition, p implies q and p implies $\neg q$. If q is true then $p \rightarrow q$, so a true proposition is implied by any proposition, which is counter-intuitive. Paradoxically, if q is true, q is implied by p and q is implied by $\neg p$. *See material implication.*

particle. Grammatical connective, typically a preposition: ‘if’, ‘on’, ‘in’, or a conjunction: ‘and’, ‘but’.

particular. (1) *n.* Individual as opposed to universal. Token rather than type. (2) *adj.* Referring to one and only one thing. Appropriate for referring to only one thing. *See individual, universal.*

particular affirmative. Proposition of the form: $(\exists x) (Fx \ \& \ Gx)$.

particular negative. Proposition of the form: $(\exists x) (Fx \ \& \ \neg Gx)$.

positive term. Term definable without the use of negative terms.

positive integer. Member of the set of numbers 1, 2, 3... n .

possessive. Expressive of ownership or belonging. Expressible by the genitive case.

pattern of argument. *See form of argument, logical form.*

polyadic. Many term. A polyadic relation is a relation that holds between more than two relata. *See dyadic, relation.*

possibility. That which is not contradictory to describe. The modality expressed by '◇'.

possible. Not contradictory to describe. In modal logic possibility and necessity are interdefinable: possibly p if and only if not necessarily not p : $\Diamond p \leftrightarrow \neg \Box \neg p$ and necessarily p if and only if not possibly not p : $\Box p \leftrightarrow \neg \Diamond \neg p$.

predicate. The result of subtracting the proper name from an indicative sentence. A word or string of words used to ascribe a property. The sense of a word, or senses of a string of words, used to ascribe a property.

predicate calculus. Formal language consisting in individual variables ($x, y, \dots$), predicate letters ($F, G, \dots$) individual constants ($m, n, \dots$), propositional connectives ($\&, \leftrightarrow, \rightarrow, \neg, \vee$) and either the universal quantifier ($\forall$) or the existential quantifier ($\exists$) or both and governed by both propositional calculus rules and quantifier rules.

predicate calculus with identity. *See* identical.

predicate calculus, intuitionist. Predicate calculus in which the law of excluded middle is not admitted.

predicate calculus and propositional calculus. The predicate calculus includes the propositional calculus as a part.

predicate, grammatical. That which remains of an indicative sentence if a proper name is deleted. A word or set of words used to ascribe a property. Adjectival expression. *See* proper name, subject.

predicate letter. In the predicate calculus, an upper case letter: ' F ', ' G '..., which replaces and stands for a predicate.

predicate-letter substitution. The replacement of all the occurrences of one predicate letter in a *wff*. by another predicate letter.

predicate logic. Predicate calculus. *See* predicate calculus.

premise. Any proposition that is putative evidence for the conclusion of an argument. In a valid argument it is not possible for the premises to be true but the conclusion false because the conjunction of the premises and the conclusion entails a contradiction. Any

proposition in an argument is either a premise or a conclusion or both. *See* argument, conclusion, sound, valid.

presupposition. Necessary condition. If q is a presupposition of p then if not q then not p , so p entails q . If p presupposes q then: $(\neg q \rightarrow \neg p)$ but $(\neg q \rightarrow \neg p) \leftrightarrow (p \rightarrow q)$ so if p presupposes q then: $(p \rightarrow q)$. Everything entails its own presuppositions. *See* necessary condition, sufficient condition.

primitive rule. Rule of inference that is not derived. Axiomatic rule. *See* derived rules.

principle of identity. If anything is the very same thing as anything then any property of one is a property of the other and if any property of one thing is a property of the other then they are the very same thing: $(\forall x) (\forall y) (x = y) \leftrightarrow (Fx \leftrightarrow Fy)$.

principle of quantifier shift. *See* quantifier-shift principles.

progressive. The application of a proven result in further proofs.

pronoun. Word that may replace a noun in a sentence without changing its truth value.

proof. Demonstration of truth. Valid argument. Sound argument. If p is proven, then p is true. If p is not true, then p is not proven. *See* completeness proof, conditional proof, consistency proof, indirect proof.

proof by induction. From Fx it follows that $(\forall x) (Fx)$ on the assumption that any successor of x is F . For example, if it can be shown that 0 is F and that the successor to any number has F , it may be validly concluded that all numbers are (or any number is) F .

proof, mechanical check of. *See* mechanical check of proof.

proof procedure. Any method of exhibiting the validity of valid arguments by deploying rules of inference. *See* argument, rule of inference, valid.

proof, stage of. Any of: the making of assumptions; the deploying of rules of inference; the drawing of a conclusion; the use of the

conclusion of one application of a rule of inference as a premise for the next.

provability sign. The assertion sign: ‘ $\vdash$ ’ meaning ‘given...it is provable that...’. *See* assertion sign.

propositional variable. Lower case letter, ‘ p ’, ‘ q ’, ‘ r ’... used to replace and stand for a proposition. ‘ p ’ stands for a proposition. ‘ q ’ stands for a proposition distinct from ‘ p ’ *etc.*

proper name. Referring expression that is a name. Name used to refer to just one thing or kind of thing: ‘London’, ‘New York’, ‘Jesus’, ‘oxygen’. In predicate calculus, the lower-case letters ‘ m ’, ‘ n ’..., replace and symbolise proper names.

property. Characteristic. Anything true of anything. (There are arguable exceptions: It is true of any object that exists that it exists, but existing might not be a property.) Property possession determines set membership.

property-expression. Predicate. Adjectival term. *See* predicate, property.

proposition. (1) What is expressed by an indicative (declarative) sentence. (2) The meaning of an indicative (declarative) sentence. (3) What is asserted when something is asserted. (4) What is expressed by an indicative sentence. A proposition is capable of being true or false. The letter ‘ p ’ stands for any proposition. ‘ q ’ stands for any proposition distinct from p ‘ r ’ for any proposition distinct from both p and q , *etc.* If p is true then p is not false. If p is false then p is not true. It makes no sense to talk of propositions as valid or invalid, sound or unsound but true or false. *See* affirmative, conditional, existential proposition, internal structure of proposition, negative, particular, sentence, statement, universal proposition.

propositional attitude. Psychological relation a subject bears to some propositional content. For example, as reported in: ‘ A hopes that p ’, ‘ A doubts that p ’, ‘ A wonders whether p ’, ‘ A desires that p ’, ‘ A believes that p ’. In propositional attitude sentences it is possible to vary the subject, the attitude or the proposition. For example: ‘ A desires that p ’, ‘ B hopes that q ’, ‘ C doubts that r ’.

propositional calculus. Formal language for the proof of sequents whose validity depends on relations between propositions rather than on the internal structure of propositions. *See* completeness of the propositional calculus, consistency of the propositional calculus, intuitionism, intuitionist propositional calculus propositional calculus connectives, propositional calculus rules, propositional calculus variables.

propositional calculus rules. *See* Rule of Assumptions (A), *modus ponens* (MP), *modus tollens* (MT), Double Negation (DN), Conditional Proof (CP), &-Introduction (&-I), &-Elimination (&-E), $\vee$ -Introduction ($\vee$ -I), $\vee$ -Elimination ($\vee$ -E), *reductio ad absurdum* (RAA) and biconditional.

propositional calculus connectives. The proposition-forming operators on propositions: ' $\neg$ ', ' $\vee$ ', '&', ' $\rightarrow$ ', ' $\leftrightarrow$ '.

propositional calculus variables. Individual letters used to stand for propositions: ' p ', ' q ', ' r '... n . p is any proposition. q is any proposition distinct from p . r is any proposition distinct from p and q . The number of variables available in the propositional calculus is assumed to be infinite.

propositional function. Formula such as: ' Fx ', or ' $Fx \rightarrow Gx$ ', which falls short of being well-formed, and so does not express a proposition, either because it contains no names or its variables are not bound by quantifiers. A propositional function may be turned into a *wff*. by the addition of names or quantifiers. For example: Fx might become Fm or $(\exists x) (Fx)$, or $Fx \rightarrow Gx$ might become $Fm \rightarrow Gn$ or $(\forall x) (Fx \rightarrow Gx)$. *See* function, well-formed formula, *wff*.

propositional logic. Propositional calculus. *See* propositional calculus.

proved. (1) Demonstrated to validly follow from premises. (2) Demonstrated to soundly follow from premises and so true.

proven. (1) Demonstrated to validly follow from premises. (2) Demonstrated to soundly follow from premises and so true.

proven sequent. Sequent either: (1) Demonstrated to validly follow from premises, or: (2) Demonstrated to soundly follow from premises and so true. *See* derivable sequent, sequent, valid sequent.

Q

qualitative identity. *a* is qualitatively identical to *b* if and only if *a* is like *b* in every respect: $(Fa \leftrightarrow Fb)$. Qualitative identity logically follows from numerical identity: $(\forall x) (\forall y) (x = y) \rightarrow (Fx \leftrightarrow Fy)$ and (more controversially) numerical identity follows from qualitative identity: $(\forall x) (\forall y) (Fx \leftrightarrow Fy) \rightarrow (x = y)$. *See* identical.

quality. Property. That which is ascribed by the use of a true sentence containing a predicate.

quantification theory. *See* predicate calculus.

quantification. The application of quantifiers (for example: ‘all’, ‘some’, ‘a few’ ‘more than half’). In predicate logic, the prefacing of a sentence by either the universal quantifier $(\forall x)$, ‘For any *x*’, or the existential quantifier $(\exists x)$, ‘There exists at least one *x* such that’. Quantification turns a sentence with free variables into a sentence with bound variables, for example: *Fx*, ‘Something is *F*’, into $(\exists x) Fx$, ‘There exists at least one *x* that is *F*’. The propriety of the universal and existential quantifiers is not beyond philosophical question. The use of the universal quantifier assimilates ‘all’, ‘any’, and ‘every’ which are arguably distinct concepts. The use of the existential quantifier enshrines formally the doctrine that ‘exists’ is not a first-order predicate, but ‘exists’ might be some kind of first-order predicate. Quantification theory is predicate logic. *See* predicate calculus.

quantifier. Term used to express a quantity, such as: ‘all’, ‘each’, ‘every’, ‘any’, ‘some’, ‘most’, ‘many’, ‘fewer than half’, ‘none’, ‘at least one’, ‘nothing’. *See* existential quantifier, universal quantifier.

quantifier, existential. *See* existential quantifier.

quantifier, numerically definite. *See* numerically definite quantifier.

quantifier phrase. Expression employing quantifiers short of a sentence: ‘most of them’, ‘not all’, ‘many’. *See* existential quantifier, quantifier, universal quantifier.

quantifier rules. *See* $\forall$ -introduction, $\forall$ -elimination, $\exists$ -introduction, $\exists$ -elimination.

quantifier, scope of. *See* scope.

quantifier, universal. *See* universal quantifier.

quantifier, vacuous. *See* vacuous quantifier.

quantifier-shift. The changing of the order of quantifiers in a sequent. For example: changing; ‘ $(\forall x) (\forall y) (Fxy)$ ’ to ‘ $(\forall y) (\forall x) (Fxy)$ ’ or changing; ‘ $(\forall x) (\exists y) (Fxy)$ ’ to ‘ $(\exists y) (\forall x) (Fxy)$ ’. *See* order of quantifiers, quantifier shift principles.

quantifier-shift principles. The rules that the order of only universal quantifiers in a sequent and the order of only existential quantifiers in a sequent do not affect meaning, validity and truth. The rules depend upon two interderivability results: $(\forall x) (\forall y) (Fxy) \dashv \vdash (\forall y) (\forall x) (Fxy)$ and $(\exists x) (\exists y) (Fxy) \dashv \vdash (\exists y) (\exists x) (Fxy)$. $(\forall x) (\forall y) (Fxy)$ and $(\forall y) (\forall x) (Fxy)$ mean the same, are true or false under the same conditions and are mutually entailing. $(\exists x) (\exists y) (Fxy)$ and $(\exists y) (\exists x) (Fxy)$ mean the same, are true or false under the same circumstances and are mutually entailing.

Order *does* matter to validity and truth in sequents mixing types of quantifier. For example, ‘ $(\forall x) (\exists y) (Fxy)$ ’ and ‘ $(\exists y) (\forall x) (Fxy)$ ’ have different meanings, are true or false under different conditions and are not mutually entailing. *See* order of quantifiers, quantifier-shift.

R

radical reference failure. Putative reference in which nothing is referred to.

range. The converse domain of a relation. The set of entities such that something stands to them in the relation R .

rank of connectives. The order of logical connectives according to their scope: ' $\neg$ ' has a narrower scope than '&' and ' $\vee$ '. '&' and ' $\vee$ ' have a narrower scope than ' $\rightarrow$ ', and ' $\rightarrow$ ' has a narrower scope than ' $\leftrightarrow$ '. The order ' $\neg$ ', '&', ' $\vee$ ', ' $\rightarrow$ ', ' $\leftrightarrow$ ' makes brackets redundant from, for example: $\neg p \vee q, p \rightarrow p \& q$ or $p \& q \rightarrow p$.

realism. The thesis that something exists whether or not it is: (1) believed to exist, (2) perceived to exist, (3) proven to exist, or (4) could be proven to exist, or any combination of these. Realism about truth is this thesis that any proposition is true or false, that is, that the principle of excluded middle holds. *See* anti-realism, excluded middle. *See also* modal realism.

recursive. Pertaining to the first and subsequent members of a series.

recursive definition. Application of a property to the first term of a series and the inference that any successor in the series has that property.

***reductio ad absurdum*, rule of.** Rule legitimising inferences of the form $p \rightarrow q, p \rightarrow \neg q \vdash \neg p$. If a contradiction may be derived from p then it may be concluded that not p . If q follows from p and if $\neg q$ follows from p then $q \& \neg q$ follows from p . $q \& \neg q$ is a contradiction, so p is false.

redundancy theory of truth. The thesis that ' p is true' means nothing more or less than ' p '.

reference. Semantically picking out in the world. Relation between a referring expression and its referent.

reference failure. (1) Reference, but not by the use of a term to refer to what it conventionally refers to. For example: 'That man drinking black currant juice' might be used successfully to refer to that man, even though he is drinking red wine and 'man drinking blackcurrant juice' is not conventionally used to refer to a man drinking red wine. (2) Failure to refer to anything. Radical reference failure. *See* radical reference failure.

referent. What is referred to when a referring expression is used correctly or when a sentence including a referring expression is true.

referential. Pertaining to reference.

referentially opaque. A context in a sentence is referentially opaque if and only if substitution of one referring expression by another might change the sentence's truth value.

referring. Semantically picking out.

referring expression. (1) Part of language correctly used to pick something out. (2) Term whose sense is insufficient for its reference.

reflexive. A relation, ' R ', is reflexive if and only if, a is related by R to a : 'reflexive' (*def.*): R is reflexive *iff* $(\exists x) Rxx$. See irreflexive, relation,

reflexivity. The property of being reflexive. *See* irreflexive, reflexive relation, relation.

result. Conclusion validly derived.

relation. That kind of property holding between two or more objects or an object and itself.

relation of identity. a is numerically identical to b if and only if a is b . a is numerically identical to b if and only if a is the same one as b . a is qualitatively similar to b if and only if a is like b in at least one respect. a is qualitatively identical to b if and only if a is like b in every respect. a is qualitatively similar to b if and only if a and b are of the same sort (kind, type).

relational expression. Expression correctly used to refer to a relation: 'next to', 'larger than', 'on top of', 'is a brother of'.

relational sentence. (1) Sentence used to report or describe a relation. Sentence containing at least one relational predicate.

relationship, logical. Two or more propositions being logically dependent or logically independent. *See* logically dependent, logically independent.

‘rests on’. A conclusion rests on some assumption used as a premise if and only if the truth of that assumption is necessary for the truth of that conclusion. If q rests on p then p is necessary for q . In a valid argument, the premises are singularly necessary but jointly sufficient for the conclusion so the conclusion is necessary for the set of its premises. ‘rests on’ means ‘depends on’ or ‘based on’. See argument, conclusion, entailment, premise, presupposition.

‘reverse E’. A name for ‘ $\exists$ ’, the existential quantifier. *See* existential quantifier.

rigid designator. Expression which picks out the same object in all possible worlds. Expression which changes its meaning if it changes its referent.

robust. Difficult to refute.

rule recorded in proof. The citation on the right hand side of a proven line of argument of the rule of inference used to prove it, usually in abbreviated form.

rule, derived. A rule that is not primitive but logically derived from a primitive rule. *See* primitive.

rule, primitive. *See* primitive rule.

rule, quantifier. *See* quantifier rule.

rule, safe. Rule of inference that ensures validity, that is, from true premises yields only true conclusions.

rule of assumptions (A). Rule of inference legitimising the introduction of any proposition at any stage of an argument as an assumption. Such an assumption typically features as a premise. The conclusion of the argument rests on any such assumptions. *See* argument, assumption, premise, ‘rests on’, conclusion, valid.

rule of conditional proof (CP). *See* conditional proof.

rule of inference. Rule legitimising the valid drawing of a conclusion from premises. *See* double negation (DN), existential quantifier elimination ($\exists E$), existential quantifier introduction ($\exists I$), identity elimination ($=E$), identity introduction ($=I$), *modus ponens* (MP),

modus tollens (MT), *reductio ad absurdum* (RAA), sequent introduction (SI), theorem introduction, universal quantifier elimination ($\forall$ E), rule of universal quantifier introduction ($\forall$ I), &-elimination (&-E), &-introduction (&-I), $\vee$ -elimination ($\vee$ -E), $\vee$ -introduction ($\vee$ -I).

S

safe rule. Rule of inference that never yields a false conclusion from true premises. Rule that legitimises only valid inferences.

salva veritate. Latin for: saving (the) truth, truth preserving. See *substitution salva veritate*.

same. (1) a is the same as b if and only if a is b : $a = b$. (2) a is the same as b if and only if a is like b : $Fa \leftrightarrow Fb$. See identity.

schematic letter. Letter replacing and standing for an indicative sentence ' p ', ' q '..., name, ' m ', ' n '... or property ' F ', ' G '... . Variable. See variable.

scope. The scope of a logical connective is the *wff*. it operates upon. This is always the shortest *wff*. in which it features. For example, in ' $(p \vee q) \rightarrow r$ ', the scope of ' $\vee$ ' is ' $p \vee q$ '. In ' $p \leftrightarrow \neg p$ ' the scope of ' $\leftrightarrow$ ' is that entire *wff*. The scope of one connective may be contained within the scope of another. For example in ' $(p \& q) \vee r$ ' the scope of '&' is included in the scope of ' $\vee$ '. The connective with the contained scope is said to be 'subordinate' to the connective with the containing scope. The scope of a quantifier is always the shortest function in which it occurs. For example, in ' $(\forall x) (Fx) \rightarrow (\exists y) (Gy)$ ' the scope of ' $(\exists y)$ ' is ' $(\exists y) (Gy)$ '. In ' $(\forall x) (Fx)$ ' the scope of ' $(\forall x)$ ' is the entire function ' $(\forall x) (Fx)$ '. See rank.

second order. Applying to a language that applies to the world. If a language applies to the world it is first order.

self-contradictory. Contradictory. Entailing a contradiction. See contradiction.

self-evident. *p* is self evident if and only if perceiving the truth of *p* is a necessary condition for understanding *p*.

self-membered class. A class that is a member of itself. The class of classes is a member of itself because it is a class. The existence of a class of classes that are not members of themselves gives rise to a paradox: That class is and is not a member of itself, which is contradictory. It is nevertheless intuitively plausible that there is a class of classes that are not members of themselves because the most classes are not members of themselves. *See* paradox.

semantic. Pertaining to meaning.

semantic equivalence. Synonymy. *See* synonymous.

semantic theory of truth. Thesis that truth is the truth of true sentences.

sentence. A grammatically well-formed portion of language appropriate for making assertions, asking questions or giving orders, so indicative, interrogative or imperative in mood. *See* atomic sentence, sentence connective. binary, unary, declarative sentence, indicative sentence, proposition, relational sentence, subject-predicate sentence, subjunctive.

sentence, analysis of. (1) Grammatically, the exhibition of the parts of speech in a sentence: nouns, pronouns, adjectives, verbs, adverbs, prepositions, articles and the distinguishing of subject from predicate. (2) Logically, the translation of a sentence into the language of propositional or predicate calculus, exhibiting its logical form in order to determine its logical status or exhibit its role in argument.

sentence-forming operator. *See* logical connective.

sentential calculus. *See* propositional calculus.

sequence. Set of things ordered by some principle.

sequent. An argument form consisting in the provability sign ‘ $\vdash$ ’ flanked on the left hand side by assumptions and on the right hand side by a conclusion which putatively follows from them.

sequent, derivable. *See* derivable.

sequent expression. Any series of *wffs.* that separated by an occurrence of the assertion sign. *See* assertion sign, proof.

sequent-expression substitution. The replacement of one sequent expression by another with the same logical form. Any substitution instance of a proven sequent can be proven.

sequent introduction (SI). Derived rule permitting the introduction of a proven sequent, or substitution instance of a proven sequent, at any stage of a proof.

set. Plurality of objects with a common property. Class.

set theoretic extension. The members of a set.

singular. Concerning one and only one item. Not plural.

singular term. Expression correctly used to refer to one and only one thing. Proper name. Definite description. *See* definite description, proper name.

‘so’. Conclusion marker in ordinary English. Therefore. ‘ p so q ’ means ‘ q because p ’.

‘some’. (1) In logic: at least one. If ‘one a is F ’ is true then ‘some a ’s are F ’ is true. If ‘more than one a is F ’ is true then ‘some a ’s are F ’ is true. If ‘all a ’s are F ’ is true then ‘some a ’s are F ’ is true. If ‘no a ’s are F ’ is true, then ‘some a ’s are F ’ is false. (2) In ordinary English: at least two but not all. *See* quantifier.

sorites (σοριτες). Greek for ‘heap’.

sorites paradox. *See* vague.

sortal. Term correctly used to pick out things that may be counted: ‘book’, ‘house’, ‘molecule’ but not ‘butter’, ‘wine’, ‘oxygen’. *See* universal.

sound. Of an argument: valid with true premises and so with a true conclusion. If an argument is sound then it is valid but a valid

argument is not sound if any of its premises is false. All the propositions in a sound argument are true. If an argument is not sound it is unsound. (It is a category mistake to speak of sentences or propositions as sound.) *See* unsound.

soundness. *See* sound, unsound.

statement. (1) Sentence expressing a proposition. (2) Token assertion of a sentence expressing a proposition. *See* proposition.

stipulative definition. Verbal explanation of meaning that is invented or decided rather than discovered.

strategy. Technique for proof discovery.

subalternate. Subimplicant. *See* subimplicant.

subcontrary. p and q are subcontrary if and only if: if p is false then q is true and if q is false then p is true. If p and q are subcontraries: (1) p and q are not both false (2) p and q may both be true and (3) at least one of p and q is true. ‘subcontrary’ (*def.*): p and q are subcontrary iff $p \vee q$ is a tautology.

subimplicant. If q is implied by p then q is subimplicant to p . *See* implication.

subject. Grammatical subject. Expression within a sentence denoting an object to which a property is ascribed by a predicate. What remains of an indicative sentence when its predicate is deleted. A subject is typically a noun or pronoun or nominal or pronominal phrase. (Confusingly, the term ‘subject’ is also used to refer to the object the grammatical subject denotes. The grammatical subject is a word or string of words. The subject in this second sense is (typically) a non-linguistic item. No parallel confusion exists between ‘predicate’ and ‘property’.) *See* predicate.

subject-predicate sentence. Sentence in which a property is ascribed to an object. *See* predicate, subject.

subordinate. *See* scope.

substitution. The replacement of one expression by another of the same logical form, especially to provide an assumption for an argument.

substitution-instance. One sequent is a substitution-instance of another if they have the same logical form. Any substitution-instance of a tautology is a tautology. Any substitution instance of an inconsistent proposition is an inconsistent proposition. However, a substitution instance of a contingent proposition might be a tautologous, contingent or inconsistent proposition. Any substitution instance of a proven theorem can be proven.

substitutivity *salve veritate*. The retention of truth value despite substitution. *See salva veritate*, substitution.

sufficient. If p is sufficient for q then if p then q : $p \rightarrow q$. If p is sufficient for q then q is necessary for p : $(\neg q \rightarrow \neg p)$, so $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$. *See* sufficient condition.

sufficient condition. If the truth of p is a sufficient condition for the truth of q then if p then q (ie if p is true then q is true). If the truth of p is both necessary and sufficient for the truth of q then q if and only if p . If p is a necessary condition for q then q is a sufficient condition for p . If p is a sufficient condition for q then q is a necessary condition for p . *See* necessary condition

superalternate. Superimplicant. *See* superimplicant.

superimplicant. If $p \rightarrow q$ is tautologous then p is superimplicant to q .

syllogism. Argument comprising two premises and one conclusion usually relying on quantifiers.

symmetric. A relation, ' R ', is symmetric if and only if: if a is related by R to b , then b is related by R to a . 'symmetric' (*def.*): R is symmetric iff $(\forall x) (\forall y) (Rxy \rightarrow Ryx)$. *See* not symmetric, asymmetric.

synonymous. With the same meaning.

syntax. Grammar.

synthetic. (1) The meaning of a synthetic proposition is insufficient for its truth. (2) The truth of a synthetic proposition is not necessary

for its meaning. (3) A synthetic proposition is true or false but not by definition. (4) A synthetic proposition is true or false but not in virtue of the meaning of its constituent terms. (5) The negation of a synthetic proposition is internally consistent. (6) In a synthetic proposition the predicate is not 'contained' in the subject. (7) Synthetic propositions might provide information about non-linguistic reality. It is a mistake to define 'synthetic' as 'contingent' even if controversial thesis that all and only synthetic propositions are contingent is true. *See* analytic.

T

tautologous sequent. A sequent is tautologous if and only if, for every allocation of truth values to its component propositions, it takes the truth value 'true'.

tautologous wff. wff that for every possible allocation of truth values to its component propositions, takes the truth value 'true'. Every theorem of the propositional calculus is a tautology. *See* consistent, contingent, truth table, wff.

tautology. (1) Proposition taking the truth value 'true' under all possible allocations of truth values to its component propositions. (2) Logical truth. (3) Substitution instance of a logical law. *See* consistent, inconsistent, logical law, necessary.

tense. A grammatical form of a verb, determining whether it refers to past, present or future. Not all natural languages include tensed verbs.

tense logic. The study of the validity and invalidity of arguments depending on the tenses of verbs in their component propositions.

'the'. The definite article in English. Word used to refer to some specific thing without thereby specifying what it is.

theorem. Logical truth. The conclusion of a provable sequent in which the number of assumptions is zero. Substitution instance of a logical law. For example: $p \rightarrow p$, $\neg(p \ \& \ \neg p)$.

theorem introduction (TI.) Rule legitimising the introduction of a proven theorem, or substitution instance of a proven theorem, as an assumption at any stage of a proof.

theory of (definite) descriptions. The depiction of the logical form of sentences such as ‘The king of France is bald’ as $(\exists x) (\text{bald } x)$ to do justice to the fact that there are two ways in which ‘The king of France is bald’ can be false: either because there is no king of France or because there is a king of France but he is not bald. ‘ $(\exists x) (\text{bald } x)$ ’ is false if $\neg(\exists x) (\text{bald } x)$ or if $(\exists x) \neg(\text{bald } x)$.

theory of presupposition. Thesis that a subject-predicate sentence does not express a proposition unless the subject refers. ie: A subject-predicate sentence is not true or false unless what the subject refers to exists.

theory of types. Putative solution to Russell’s paradox which prohibits claims that a set is a member of itself as ill-formed.

‘there is an x such that...’. See existential quantifier.

‘therefore’. ‘ p therefore q ’ means ‘ q is provable from p ’: $p \vdash q$.
See assertion sign, proof.

tilde. A name of the negation sign: ‘ $\sim$ ’, pronounced ‘not’.

token. Particular. Individual. See type.

token-reflexive. Expression overtly or covertly referring to the sentence in which it occurs. For example: ‘Now’ refers to when the sentence it occurs in is uttered. ‘Here’ refers to where the sentence it occurs in is uttered. Indexical. See indexical.

transitive. A relation, ‘ R ’, is transitive if and only if: if a is related by R to b , and if b is related by R to c , then a is related by R to c .

‘transitive’ (*def.*): R is transitive *iff* $(\forall x) (\forall y) (\forall z) (Rxy \ \& \ Ryz \rightarrow Rxz)$

true. p is true if and only if p (with no italics). p is true if and only if the truth conditions sufficient for p ’s truth obtain. If a proposition is not false, and not neither true nor false, nor both true and false, it is true.

true, logically. p is logically true if and only if p is: (1) a tautology (2) a theorem (3) a logical law or a substitution instance of a logical law, or if (4) the meaning of the logical constants in p are sufficient for p 's truth.

truth conditions. The conditions under which a proposition is true or false.

truth-function. A proposition whose truth value is wholly determined by the truth value of its constituent propositions. For example, if p is true and if q is true, then $p \& q$ is true. If either or both of p and q is false then $p \& q$ is false. The truth value of a truth function's constituent propositions is necessary and sufficient for its truth value.

truth table. Matrix for the definition of connectives or proof of sequents.

$\neg p$

	$\neg p$
p T	F
p F	T

$\neg p$: FT

$p \rightarrow q$

	q	q
	T	F
p T	T	F
p F	T	T

$p \rightarrow q$: TFFT

$p \& q$

	q	q
	T	F
p T	T	F
p F	F	F

$p \& q$: TFFF

$p \vee q$

q	q
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	T	F
p	T	T
p	F	T

$p \vee q$: TTTF

$p \leftrightarrow q$

	q	q
	T	F
p	T	T
p	F	F

$p \leftrightarrow q$: TFFT

truth-value. The property of being true or false. If p is true then p has the truth value ‘true’. If p is false then p has the truth value ‘false’. Intuitionism admits truth values other than ‘true’ and ‘false’, for example: ‘neither true nor false’, ‘indeterminate’. Fuzzy logic admits ‘to some degree true and to some degree false’. *See* fuzzy logic, intuitionism.

turnstile. A name of the provability symbol or assertion sign: ‘ $\vdash$ ’. *See* assertion sign.

type. Some objects are of the same type if and only if they share a property. Category. Sort. Set. Class.

U

union. The union of two sets α and β is set of objects which are either members of α or members of β .

unique. If x is unique one and only one x exists. Nothing like x exists except x .

unit class. Class with only one member.

universal. n . Type. Sort. Kind. *adj.* Without exception.

universal affirmative. Proposition of the form: $(\forall x) (Fx \ \& \ Gx)$.

universal negative. Proposition of the form: $(\forall x) (Fx \ \& \ \neg Gx)$.

universal proposition. Proposition either of the form $(\forall x) (Fx \ \& \ Gx)$ or of the form $(\forall x) (Fx \ \& \ \neg Gx)$.

universal quantifier. The sign: $(\forall x)$ which is read ‘for all x ’ or ‘for every x ’, or ‘everything’ or ‘anything’. *See* predicate calculus.

***Universal Quantifier Elimination* ($\forall E$).** Rule legitimising the inference: $(\forall x) (Fx) \vdash Fm$. If everything is F , then a particular thing is F . If each object in a universe of discourse has a property, then a specific object in that universe has that property.

***Universal Quantifier Introduction* ($\forall I$).** Rule legitimising the inference: $Fa \vdash (\forall x) (Fx)$. If an arbitrarily selected object, a , is F then everything is F . If any arbitrarily selected object in a domain is F then any object in that domain is F .

universals. Types. Sorts. Kinds.

universe. Set of objects.

universe class. The class of which everything is a member.

universe of discourse. Set of objects referred to.

universe, empty. Set not containing any objects.

universe, non-empty. Set containing objects.

‘unless’. ‘ q unless p ’ means ‘if not p then q ’: $\neg p \rightarrow q$. *See* necessary condition, only, unless...then... .

‘unless...then...’. ‘unless p then q ’ means ‘if not p then q ’: $\neg p \rightarrow q$.
See necessary condition, only, unless.

unsound. Argument that is either invalid or possesses at least one false premise or both. If an argument is valid then it is either sound or unsound. If an argument is invalid then it is unsound. *See* sound, valid, invalid.

V

vacuous. Without referent.

vacuous quantifier. Quantifier governing no variable.

vague. Indefinite, especially in reference. For example, it might be thought that there is no exact number of items enough to comprise a heap, so ‘heap’ is vague in its application. It is not clear under exactly which conditions it should be applied and under which conditions withheld. *See* sorites.

valid. If an argument is valid, then the truth of the premises is inconsistent with the falsity of the conclusion. If an argument is valid, then if the premises are true then the conclusion is true. If an argument is not valid it is invalid.

vel. Latin for ‘or’ in the inclusive sense: either p or q or both: $p \vee q$.
(*See aut.*)

variable. Symbol used to replace an expression. Propositional variables $p, q, r...$, replace propositions. In predicate calculus, $x, y...$, replace names.

vel. Latin for ‘or’ in the inclusive sense of: ‘either...or...or both’.

wff. *abbrev.* well-formed formula.

verb. Word used to denote an action or event.

verb, intransitive. Verb not taking a direct object. *See* verb.

verb, transitive. Verb taking a direct object. *See* verb.

W

well-formed formula (*wff*). Member of the class of formulae obeying the syntactic formation rules of a calculus.

wff. abbrev. well-formed formula.

‘when’. If.

‘when and only when’. If and only if.

word. The smallest unit of meaning in a natural language.

XYZ