

# Irradiation pressure effects in close binary systems

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## ABSTRACT

We present a method for the calculation of the effects of external irradiation on the geometrical shape of the secondary in a close binary containing a compact star, the source of the radiation, and a normal companion star, where we include the possibility of shadowing by an accretion disc. The model is based on a simple modification of the standard Roche binary potential in which the radiation pressure force is parametrized using the ratio of the radiation to the gravitational force. We have constructed numerical solutions of approximate three-dimensional irradiated equipotential surfaces to demonstrate the main geometrical effects of external radiation pressure. For systems in which the inner Lagrangian point is irradiated directly and for sufficiently high irradiation fluxes, the critical condition for which the secondary fills its tidal lobe changes from an inner to an outer critical configuration, where the critical equipotential surface connects to one of the outer rather than inner Lagrangian points. Such a situation may apply to evaporating binary pulsar systems (e.g. PSR 1957 + 20), stars orbiting supermassive black holes in AGN and some high-mass X-ray binaries (HMXBs, including Centaurus X-3). For systems containing an accretion disc, which shadows the inner Lagrangian point from the external irradiation, the presence of significant radiation pressure causes a non-axisymmetric deformation of the stellar surface. This has particularly important consequences for low-mass X-ray binaries, for which the X-ray luminosities can be close to the Eddington limit. We have calculated modified Roche potentials to determine the main effects on the optical light curves and radial velocity curves for typical binary parameters. Compared to previous studies, the inclusion of irradiation pressure effects results in changes in the derived system parameters (e.g. component masses, radial velocities) that may be as high as  $\sim 30$  per cent. We conclude that the proper inclusion of irradiation pressure effects is essential for a reliable analysis of close binary systems in which the secondaries are strongly irradiated.

**Key words:** radiative transfer – binaries: close – stars: individual: Centaurus X-3 – stars: neutron – pulsars: individual: PSR 1957 + 20 – X-rays: stars.

## 1 INTRODUCTION

Irradiation of the secondaries in close binary systems can significantly change their appearance and even their internal structure. These are particularly dramatic in the case of low-mass X-ray binaries (LMXBs), in which the luminosity of the compact object can be as high as  $10^{38} \text{ erg s}^{-1}$  (i.e. close to the Eddington luminosity of a neutron star), and where the luminosity intercepted by the secondary may be several orders of magnitude larger than the internal luminosity. Depending on the structure and chemical composition of the stellar atmosphere, a fraction of the incident radiation will be absorbed, thereby heating material in the region of penetration. Some of the luminosity deposited in the secondary will be re-radiated at

lower wavelengths, increasing the outward flux and raising the surface temperature. The rest will move to cooler regions of the surface via circulation currents. This leads to a change in the temperature and entropy profiles of the envelope. The former will affect the emitted flux, altering the phase-dependent light curve. It will also cause a variation in the temperature gradient, which can modify the spectral line profiles and, in turn, the radial velocity curves determined from spectral lines. The change in the entropy in the layer where the irradiation flux is thermalized may influence the long-term equilibrium structure of the star and can, under certain conditions, lead to significant expansion (Podsiadlowski 1991).

Many of these effects have been addressed by a number of authors in the past; see, for example, reviews by Peraiah (1982), Kopal (1988) and van Paradijs (1998), and also Davey & Smith (1992) and Martin & Davey (1995). The purpose of the present study is to investigate an associated, but often neglected effect of irradiation: the

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pressure force as a result of the momentum transfer of the incident photons. This is particularly important for X-ray binaries in which the radiation pressure force acting on the secondary may be a substantial fraction of the gravitational force. The consequences of this can be quite dramatic, sometimes involving a large deformation of the secondary's surface and a possible shift in the position of the effective inner Lagrangian point  $L_1$ . In extreme cases, mass loss from the secondary can occur through one of the outer Lagrangian points,  $L_2$  or  $L_3$ , rather than the inner one. Irradiation can then drastically change the dynamics of mass transfer and the nature of the binary interaction.

Standard procedures for the modelling of photometric light curves of close binary systems and most models for binary interactions are based on the Roche model for the equipotential surfaces of the binary components. This assumes that both components are centrally concentrated and that their gravitational effects can be approximated as those of point masses. The Roche model has been extended by various authors to overcome some of the intrinsic approximations of circular orbits and synchronous rotation; see, for example, Plavec (1958) and Kruszewski (1966). In addition, tidal and rotational distortions, the reflection effect, limb- and gravity-darkening can all be treated in a reasonably physical manner; see, for example, Kopal (1959) and Al Naimiy (1978). However, the classical Roche model allows only for gravitational and centrifugal forces, and no provision is made for systems in which external radiation pressure acting on either component is important.

Podsiadlowski & Rees (1994) were the first to derive the condition when external irradiation causes a shift from an inner to an outer critical configuration and Drechsel et al. (1995) developed a computational method to calculate modified equipotential surfaces that approximately take into account the effects of external irradiation. Drechsel et al. (1995) applied their method mainly to early-type stars and showed how external irradiation can shift the position of the inner Lagrangian point, significantly changing the shape of an irradiated star and as a consequence the light curve of the system.

In this paper, we present a systematic investigation of the effects of external irradiation pressure on the standard model of binary stellar surfaces. Our approach is based on and extends the method developed by Drechsel et al. (1995). In Section 2 we justify the approach and in Section 3 we show how the Roche-lobe model can be modified to include approximately the effects of external irradiation. In Section 4 we describe the numerical procedure used to construct these approximate irradiated equipotential surfaces. Binary systems with and without accretion discs are considered, and the validity of our solutions for given parameters is examined. In Sections 5 and 6, we apply these solutions to a number of physical phenomena and systems, with and without accretion discs, respectively. In particular, we discuss applications to binary pulsars, systems with extreme mass ratios and LMXBs and HMXBs.

## 2 THE RADIATION PRESSURE FORCE TERM

Radiation pressure is caused by the interaction between electromagnetic radiation and stellar matter. Its strength depends on the momentum transfer per photon absorbed or scattered in the irradiated photosphere and is generally a very complicated function of local opacity and optical depth.

In the presence of a radiation source, the equation of hydrostatic equilibrium, relating the pressure  $P$ , the density  $\rho$ , and the potential  $\Phi$  for any point  $\mathbf{r}(r, \theta, \phi)$  in the secondary, will acquire an additional term corresponding to the force resulting from this *external* radiation pressure:

$$\nabla P = -\rho \nabla \Phi + \frac{\kappa \rho}{c} f(\tau) \hat{\mathbf{n}}_1. \quad (1)$$

Here  $\hat{\mathbf{n}}_1$  is a unit vector in the direction of the external radiation,  $\kappa$  is the local opacity,  $c$  is the speed of light,  $f$  is the irradiation flux and  $\tau$  is the local optical depth in the secondary envelope with respect to the external radiation. (Here we have neglected inertial terms resulting from the rotation of the system, i.e. the Coriolis force and circulation terms.) For a given source luminosity,  $f(\tau)$  at a point below the atmosphere will depend both on the local physical and chemical conditions in the secondary's envelope and on the geometry of the system, i.e. the distance from the radiation source and the angle of incidence of the flux vector. The complicated functional dependence of this term renders a precise analytical representation impossible. To obtain an explicit expression for this term, we generally make the simplifying assumption that the X-ray flux decreases exponentially [see, for example, Podsiadlowski (1991, 1992)], i.e.

$$f(\tau) \sim \frac{L_x}{4\pi|\mathbf{r} - \mathbf{r}_1|^2} e^{-\tau}, \quad (2)$$

where  $|\mathbf{r} - \mathbf{r}_1|$  represents the distance from the radiation source, and  $L_x$  denotes its X-ray luminosity.

However, in general the irradiation term in equation (1) does not represent a conservative force, i.e. a force that can be written as the gradient of a scalar potential, since it depends on the local conditions in the region of flux absorption. Therefore, the inclusion of radiation pressure in the force equation precludes the existence of an equipotential surface. This also implies that there exists no surface for which the tangential force component is identical to zero everywhere. An accurate determination of the irradiated secondary surface would therefore require a full treatment of the surface motion, including the meridional circulation of stellar material as a result of rotation (Von Zeipel 1924a) and the additional circulatory currents driven by the external radiation. A full solution is beyond the scope of this paper, and the correct circulation patterns even for non-irradiated single stars are still subject to debate (Osaki 1982; Charbonneau 1992; Maeder & Zahn 1998).

The purpose of this paper is to investigate the effects of the external irradiation in the absence of circulatory currents, i.e. we consider the extreme case in which tangential forces on the irradiated surface are small. In effect, we seek a solution for which the irradiated surface locally approximates an equipotential surface. Compared to a solution that fully accounts for irradiation-driven circulation, we may expect that the true surface of an irradiated star will lie somewhere between this extreme case and the standard Roche potential.

## 3 A MODIFIED ROCHE POTENTIAL

Even though the surface of an irradiated star cannot be represented by an exact equipotential surface, we can still modify the classical Roche potential to account for the radiation pressure term in an approximate way.

To do this, we first rewrite equation (1), expressing the balance of forces per unit volume at the secondary surface, using the approximation of equation (2):

$$\nabla P + \rho \nabla \Phi_1 = -\rho \frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|^2} \hat{\mathbf{n}}_1 + \frac{\kappa \rho}{c} \frac{L_x}{4\pi|\mathbf{r} - \mathbf{r}_1|^2} e^{-\tau} \hat{\mathbf{n}}_1. \quad (3)$$

Here the effective potential  $\Phi_1$  combines the gravitational potential of the secondary, treated as a point mass, with the non-inertial potential terms resulting from the rotation of the system (i.e. as a result of the centrifugal force).

Depending on the irradiation spectrum and the local physical and chemical conditions in the secondary, the external flux will penetrate to a certain depth below the photosphere. In our treatment of the irradiated surface, we consider a layer of sufficient depth that most external radiation has been absorbed. However, this depth should be small compared to the secondary's radius, in order that  $|\mathbf{r} - \mathbf{r}_1|$  can be considered constant in this region and  $\kappa$  and  $\rho$  can be well represented by average values  $\bar{\kappa}$  and  $\bar{\rho}$ . Momentum transfer can then be regarded as an 'on the spot' event.

We now integrate equation (3) in the direction of the flux vector  $\hat{\mathbf{n}}_1$ , from the top of the secondary's atmosphere to some fixed surface layer of vertical optical depth  $\tau_{0,n} \gg 1$ . We denote the path length of penetration as  $s_0$  with a corresponding optical depth to X-rays as  $\tau_0 = \tau_{0,n} \sec \gamma$ , where  $\gamma$  is the angle between the surface normal and the flux direction  $\hat{\mathbf{n}}_1$ . With the assumption that only the exponential term changes significantly over this distance, the right-hand side of equation (3) becomes

$$-\bar{\rho} \frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|^2} \int_0^{s_0} ds + \frac{\bar{\kappa} \bar{\rho}}{c} \frac{L_x}{4\pi |\mathbf{r} - \mathbf{r}_1|^2} \int_0^{s_0} e^{-\tau} ds. \quad (4)$$

The physical depth of penetration and the optical depth are related by  $d\tau = \bar{\kappa} \bar{\rho} ds$  and therefore  $\tau_0 = \bar{\kappa} \bar{\rho} s_0$ . So for  $\tau_0 \gg 1$ , the right-hand side approximates to

$$\begin{aligned} -\bar{\rho} \frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|^2} s_0 + \frac{L_x}{4\pi c |\mathbf{r} - \mathbf{r}_1|^2} \\ = -\bar{\rho} \frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|^2} s_0 \left\{ 1 - \frac{L_x \bar{\kappa}}{4\pi GM_1 c \tau_0} \right\} \\ = -\rho \frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|^2} s_0 \left\{ 1 - \frac{f_i \bar{\kappa} \cos \gamma}{c g \tau_{0,n}} \right\}, \end{aligned} \quad (5)$$

where  $f_i$  is the total integrated flux and  $g$  is the gravitational acceleration resulting from the compact radiation source. Since both of these quantities have the same  $|\mathbf{r} - \mathbf{r}_1|$  dependence, the term in brackets in equation (5) is effectively constant for a given flux direction. Thus, at a given depth parallel to the top of the atmosphere, we may combine the forces resulting from gravity and radiation pressure from the compact object as a 'reduced' gravitational force

$$\mathbf{F}_{\text{grav}}^{\text{eff}} = \mathbf{F}_{\text{grav}} - \mathbf{F}_{\text{rad}} = (1 - \delta) \mathbf{F}_{\text{grav}}, \quad (6)$$

where  $\delta = \text{constant} \times \cos \gamma$ . The factor  $\delta$ , expressing the ratio of radiation to gravitational forces, therefore depends only on the cosine of the angle between the surface normal and the flux vector. However, it is this dependence that renders the reduced force non-central.

We now extend the concept of a reduced force to define a reduced potential. Both gravitational and radiative forces resulting from the X-ray source can then be characterized as  $-(1 - \delta) \int \mathbf{F} \cdot d\mathbf{r} = (1 - \delta) \phi_{\text{grav}}$ , and the standard Roche expression for the binary potential may be modified directly by the inclusion of this  $(1 - \delta)$  factor. This formulation is strictly only valid in the case where gravitation and radiation pressure are acting in exactly opposite directions, and the resulting equipotential surface lies normal to both. The  $\cos \gamma$  dependence of  $\delta$  implies that this condition is not fulfilled, i.e. that a fictitious force results from the inclusion of a variable  $\delta$  factor in the total potential (see Section 4.1).

A number of authors have previously attempted to derive equipotential surfaces based on a reduced gravitational potential; see, for example, Schuerman (1972), Kondo & McCluskey (1976), Vanbeveren (1977) and Zhou & Leung (1988). However, these attempts did not fully account for the true physical and geometric conditions of irradiated systems, resulting in unrealistic solutions. We base our approach on the method developed by Drechsel et al.

(1995), who treated the geometry in a far more sophisticated way, although we use a different treatment for the inner Lagrangian point and extend their model to include the effects of an accretion disc, if present.

#### 4 DESCRIPTION OF THE MODEL

Following Drechsel et al. (1995), we use a modified Roche potential that approximately accounts for the effects of external irradiation to calculate the surface of the secondary, assumed to coincide with an equipotential surface. Since the external pressure force cannot be represented analytically, the equipotential surface must be calculated numerically. In the case of an accreting binary system, the model must also take into account the presence of an accretion disc, which may shadow a significant portion of the secondary and most importantly the inner Lagrangian ( $L_1$ ) point. In addition, when applying these modified surfaces to the calculations of photometric light curves or radial velocity curves, the effects of gravity- and limb-darkening, which cause significant perturbations to the surface temperature distribution, must also be included. This also requires a precise numerical determination of the gradients of the potential on the stellar surface.

We consider the compact object as an isotropic point source of radiation, a fraction of which is incident on the surface of the secondary. The dimensionless ratio of radiation pressure force to the gravitational force as a result of the compact companion at a given position on the secondary is defined as

$$\delta(r, \theta, \phi) = \frac{|\mathbf{F}_{\text{rad}}|}{|\mathbf{F}_{\text{grav}}|}. \quad (7)$$

In the case where the irradiation flux is totally absorbed in the secondary's surface layer, which will be largely justified in the case of irradiation by hard X-rays (see Section 4.4), these forces will be co-directional. We may then use the approximations of a reduced gravitational force and a reduced potential introduced in the previous section. We take the centre of the secondary star as the origin of spherical polar coordinates, with the compact object along the polar axis of the coordinate system (i.e. in the direction  $\theta = 0$ ) and the pole of the secondary in the direction  $\theta = 90^\circ$ ,  $\phi = 0$ . The value of  $\delta$  varies as the angle  $\gamma$  between the surface normal and flux vector increases from its minimum value, obtained when the flux vector lies along the line of centres, up to  $90^\circ$  at the horizon, causing a respective decrease in the radiation pressure force proportional to the  $\cos \gamma$  term in equation (5).

In the Roche model, the stellar surfaces are obtained as closed equipotential surfaces for given values of the potential and mass ratios (Kopal 1959). When radiation pressure is included as a reduced gravitational potential, the modified total potential for a surface element of the secondary star, assuming synchronous rotation, takes the dimensionless form

$$-\Omega(r, \theta, \phi) = \frac{1}{r} + q \frac{1 - \delta(r, \theta, \phi)}{\sqrt{1 - 2\lambda r + r^2}} - q\lambda r + \frac{(q+1)}{2} r^2 (1 - v^2), \quad (8)$$

where  $\lambda = \sin \theta \cos \phi$  and  $v = \cos \theta$ .

The variation of  $\delta$  across the surface of the secondary has to be evaluated numerically. Once specified, the stellar surfaces can be determined point by point using a standard Newton–Raphson scheme; see, for example, Press et al. (1992). For given values of  $q$  and the critical potential  $\Omega_0$ , the radius  $r_0(\theta, \phi)$  for any direction  $(\theta, \phi)$  is obtained iteratively from the condition that the function  $g$ , defined as

$$g(\theta, \phi, r, \Omega_0, q) = \Omega_0 - \Omega(\theta, \phi, r, q), \quad (9)$$

is identical to zero.

Writing  $\delta = \delta_{\max} \cos \gamma$ , we may relate  $\delta_{\max}$  to the stellar mass and luminosity of the companion, and to the mean absorption coefficient  $\bar{\kappa}$  (see equation 5). This will generally be a complicated function, varying significantly across the surface of the secondary. In the present, exploratory investigation, we therefore make the simplifying assumption that  $\delta_{\max}$  can be treated as an independent parameter. This has the added advantage of only introducing one new parameter into the expression for the modified potential. This is desirable given the already large number of parameters necessary to specify the physical and geometrical structure of close binaries even for the classical Roche model. In Section 4.4, we make some rough estimates of the magnitude of  $\delta_{\max}$  for typical parameters found in X-ray binaries.

#### 4.1 Validity of the model

Before we proceed to apply this model, we need to discuss its limitations. In the model we treat the radiation source as a point source. Since our main application is to systems containing neutron stars which have a radius many orders of magnitudes smaller than the orbital separation, this is well justified as long as the radiation is emitted from the neighbourhood of the neutron star. In systems where a significant fraction of the X-rays originates from an extended accretion disc corona, this would no longer be strictly correct. This could particularly affect systems containing accretion discs, since radiation from an extended source may then be able to irradiate regions near the  $L_1$  point that would otherwise be shadowed by the disc, effectively modifying the equipotential surfaces.

As already stressed in Section 2, the external radiation pressure force is not a conservative force and thus cannot be expressed precisely as the gradient of a potential. By assuming that the surface of the secondary coincides with one of these modified equipotential surfaces, we therefore make an error in the force equation (1) by effectively introducing a fictitious force. In addition, the assumption of hydrostatic equilibrium ignores the effects of irradiation-driven circulation.

In our formulation, the external radiation pressure force is introduced as a reduction in the gravity term,  $(1 - \delta)\mathbf{F}_{\text{grav}}$ , as in equation (6). The modified potential resulting from the combined forces of radiation and gravity from the compact object is then taken to be  $-(1 - \delta) \int \mathbf{F} \cdot d\mathbf{r}$ , as in the second term of equation (8). However, this clearly introduces a fictitious and unphysical force as a result of the variation of  $\delta$  given by

$$\mathbf{F}_\delta = -\frac{\partial \Omega}{\partial \delta} \nabla \delta. \quad (10)$$

In order to see the effect of this term on the determination of the surface geometry, it is helpful to visualize that the construction of the equipotential surface is equivalent to piecing together surface elements that are perpendicular to the net local force. This provides a unique solution as long as there are a sufficient number of boundary conditions that allow a complete covering of the surface by these surface elements. In the present problem, this is easily fulfilled since many surface points are already fixed, irrespective of irradiation – for example, those in the shadowed region, once the potential at the inner Lagrangian point is specified. The forces derived from the modified potential then provide a good representation of the actual forces if, for all points, the fictitious force component does not significantly perturb the surface normal vector parallel to the actual force,  $-(\nabla \Omega)_\delta$ , given by the gradient of the potential with  $\delta$  kept constant.

To quantify this condition, we define the ‘true’ irradiated surface by the constraint

$$-(\nabla \Omega)_\delta \wedge \hat{\mathbf{n}}_0 = 0, \quad (11)$$

where  $\hat{\mathbf{n}}_0$  is the unit vector normal to the surface. By using a reduced potential, as given by equation (8), the surface actually calculated is given by

$$[-(\nabla \Omega)_\delta + \mathbf{F}_\delta] \wedge [\hat{\mathbf{n}}_0 + \delta \hat{\mathbf{n}}_0] = 0. \quad (12)$$

The quantity  $\delta \hat{\mathbf{n}}_0$  represents the deviation from the true surface normal and hence should be small ( $\ll 1$ ) for the model to represent the surface geometry faithfully. Expanding this expression and using equation (11), we obtain

$$[-(\nabla \Omega)_\delta \wedge \delta \hat{\mathbf{n}}_0] + [\mathbf{F}_\delta \wedge \hat{\mathbf{n}}_0] = 0, \quad (13)$$

where  $\hat{\mathbf{n}}_1 = \hat{\mathbf{n}}_0 + \delta \hat{\mathbf{n}}_0$  is the calculated surface normal in the model. But  $-(\nabla \Omega)_\delta \equiv |(\nabla \Omega)_\delta| \hat{\mathbf{n}}_0$ , and hence we may write

$$|\hat{\mathbf{n}}_0 \wedge \delta \hat{\mathbf{n}}_0| = \frac{|\mathbf{F}_\delta \wedge \hat{\mathbf{n}}_0|}{|(\nabla \Omega)_\delta|}. \quad (14)$$

It follows from the definition of a unit vector that  $|\hat{\mathbf{n}}_0 \wedge \delta \hat{\mathbf{n}}_0| \approx |\delta \hat{\mathbf{n}}_0|$  (correct to second order in  $\delta \hat{\mathbf{n}}_0$ ). Thus, the requirement that  $|\delta \hat{\mathbf{n}}_0| \ll 1$  may be expressed as

$$\eta \equiv \frac{|\mathbf{F}_\delta \wedge \hat{\mathbf{n}}_0|}{|(\nabla \Omega)_\delta|} \ll 1. \quad (15)$$

When this inequality is satisfied, the inclusion of the effects of external radiation pressure in the form of a modified potential only introduces a small error in the determination of the surface geometry.

In our present model, we do not consider the effects of irradiation-induced circulation in the outer layers of the secondary; see, for example, Kippenhahn & Thomas (1979). Indeed, in general there is no physical reason why the surface of the secondary should be an equipotential surface, even approximately, if circulation terms are important. If the surface were not close to an equipotential, this would imply large, often supersonic circulation velocities. Since in a steady state, we would expect subsonic velocities, the surface geometry has to readjust itself in such a way that the circulation becomes subsonic everywhere. This illustrates that a realistic treatment of the circulation problems requires that the surface geometry of an irradiated secondary is determined simultaneously with the circulation. The inclusion of circulation is beyond the scope of the present paper, but we have already initiated a detailed study of its consequences (Beer & Podsiadlowski 2002, in preparation). By assuming that the surface is given by a modified equipotential, we effectively minimize the effects of circulation. In this sense, these surfaces represent an extreme limit, and we may expect that the true surface of an irradiated secondary lies somewhere between this limit and the other extreme, the unperturbed Roche potential.

#### 4.2 Evaluation of $\delta(r, \theta, \phi)$

In our calculations we divide the surface of the secondary into an array of grid points spaced equally in  $\theta$  and  $\phi$ . The value of  $\delta(r, \theta, \phi)$  is then determined for all grid points, where we need to take into account shadowing by an accretion disc, if present. In all cases, we assume that the compact object is a point source of radiation.

##### 4.2.1 Systems without accretion discs

In systems without an accretion disc, there is no shielding of the secondary, and  $\delta$  has its maximum value along the line of centres,

at the point  $\theta = \phi = 0$ . At all other points,  $\delta$  is calculated according to

$$\delta(r, \theta, \phi) = \begin{cases} \delta_{\max} \cos \gamma(r, \theta, \phi) & \text{if } \cos \gamma > 0 \\ 0 & \text{otherwise} \end{cases}, \quad (16)$$

where  $\gamma(r, \theta, \phi)$  is the angle subtended by the surface normal and the vector directed from the point  $(r, \theta, \phi)$  to the source of the radiation. The constant  $\delta_{\max}$  is an input parameter, which defines the maximum value of  $\delta$  obtained in the absence of an accretion disc when the flux vector is incident normally on the irradiated surface. (Note, however, that there may not be any region of the surface on which the flux vector is incident normally; see Section 5.1.1.)

#### 4.2.2 Systems with accretion discs

LMXBs in general and many HMXBs are believed to contain some form of an accretion disc. A disc will cast a shadow on a significant portion of the secondary's surface and will generally shield the  $L_1$  point from external irradiation.

To model the effect of this shadowing, we assume that the accretion disc is axially symmetric and has an inner opaque region and an outer region which transforms smoothly from opaque to transparent at the outermost edge.

We define the opening (half) angle of the disc as  $\alpha$  and assume that the disc is completely opaque within an angle  $\beta_1$  of the equatorial plane ( $\beta_1 < \alpha$ ), where these angles are defined with respect to the compact object.

In Cartesian coordinates, where the compact object lies along the  $x$ -axis and where the axis of orbital rotation and the axis of symmetry of the accretion disc are parallel to the  $z$ -axis,  $\beta$  is given by

$$\sin \beta = |z|/d, \quad (17)$$

where  $d$  denotes the distance from a point on the secondary  $(x, y, z)$  to the compact object, given by

$$d = ((a - x)^2 + y^2 + z^2)^{1/2} \quad (18)$$

where  $a$  is the orbital separation.

To model the partially transparent region of the disc, we define a composite function  $T(r, \theta, \phi)$ :

$$T = \begin{cases} 0 & \text{if } \beta \leq \beta_1 \\ \frac{1}{2} \left\{ 1 - \cos \left[ \left( \frac{\beta - \beta_1}{\alpha - \beta_1} \right) \pi \right] \right\} & \text{if } \beta_1 \leq \beta \leq \alpha \\ 1 & \text{if } \beta \geq \alpha \end{cases} \quad (19)$$

We have also tested a linear transparency function in the partially transparent region, but we found that a smooth sinusoidal dependence improves the rate of convergence in the numerical iteration process.

Using this penetration factor, values of  $\delta(r, \theta, \phi)$  are determined over the surface of the secondary in a similar manner to equation (16)

$$\delta(r, \theta, \phi) = \begin{cases} \delta_{\max} T \cos \gamma(r, \theta, \phi) & \text{if } \cos \gamma > 0 \\ 0 & \text{otherwise} \end{cases}, \quad (20)$$

where the constant  $\delta_{\max}$  is defined as before.

#### 4.3 The convergence procedure

We now have a recipe for calculating  $\delta(r, \theta, \phi)$  in terms of  $\gamma(r, \theta, \phi)$  over the surface of the secondary. The angle  $\gamma(r, \theta, \phi)$  is the angle subtended by the surface normal and the vector from the point

$(r, \theta, \phi)$  to the radiation source. Its evaluation requires the knowledge of the three-dimensional surface of the secondary under the influence of radiation pressure, which is a priori unknown and can only be determined iteratively.

Following Drechsel et al. (1995), we model the surface of the secondary in the first iteration as a triaxial ellipsoid, defined by

$$\frac{x^2}{r_{\text{point}}^2} + \frac{y^2}{r_{\text{side}}^2} + \frac{z^2}{r_{\text{pole}}^2} = 1, \quad (21)$$

with semi-axes  $r_{\text{point}}$ ,  $r_{\text{side}}$  and  $r_{\text{pole}}$  identical with the Roche radii in the directions  $(\theta = 0)$ ,  $(\theta = 90^\circ, \phi = 90^\circ)$  and  $(\theta = 90^\circ, \phi = 0)$ , respectively. The radii  $r_{\text{side}}$  and  $r_{\text{pole}}$  are taken to be equal to the classical Roche radii since these points lie beyond the radiation horizon. In the presence of an accretion disc, we assume that the inner Lagrangian point is shielded from radiation – hence  $r_{\text{point}}$  will also be equal to the classical Roche radius. In systems which do not contain a disc, the position of the  $L_1$  point will be modified. The value of  $\delta$  at this point must simultaneously satisfy the requirement that the total force along the  $x$ -axis vanishes, i.e.  $\partial \Omega / \partial x = 0$  where  $\Omega$  is given by equation (8), and must also satisfy equation (16), which relates  $\delta$  to the surface normal. Note that, as in the case of the unperturbed Roche lobe, the surface normal at the  $L_1$  point is strictly undefined. We therefore consider a point infinitesimally close to the  $L_1$  point and obtain an approximate surface normal by extrapolation. The value of  $r_{\text{point}}$  is then fixed and forms one boundary condition for the iteration scheme.

In each iteration, we need the vector from each surface grid point to the radiation source and the vector defining the surface normal. The former is readily calculated. However, a correct calculation of the surface normal is a crucial factor for a numerically stable iteration process, since this affects the variation of  $\delta$  with  $\theta$  over the irradiated region. For a correctly converging solution,  $\delta$  must fall to zero *continuously* (though not necessarily smoothly) at the interface points, i.e. those points which separate the irradiated and shadowed regions. This forms a second boundary condition. We also require a continuous variation of  $\delta$  over the region shadowed by the accretion disc; this is ensured by our choice of the functional form of the transparency function in equation (19).

Thus, for any point on the irradiated ellipsoid, the surface normal and flux vectors are determined. A value for  $\gamma$  can then be calculated and hence the local value of  $\delta$ . Substituting these  $\delta$  values into the modified potential equation (8) then allows the determination of improved radius values for all points on the surface by application of the Newton–Raphson scheme.

#### 4.4 Physical estimates of $\delta_{\max}$

So far, we have considered  $\delta_{\max}$  as a free parameter. However, it is straightforward to estimate the value of  $\delta_{\max}$  for actual binary systems.

Consider an irradiated secondary. The radiation pressure acting on a unit surface element of the secondary is given by

$$P_{\text{rad}} = \frac{1}{c} \int_{\omega} \int_{\nu} I_{\nu} \cos^2 \gamma \, d\nu \, d\omega, \quad (22)$$

where  $c$  is the velocity of light,  $\gamma$  is the angle between the surface normal and the incident radiation,  $d\omega$  is the opening solid angle of the radiation cone, and  $I_{\nu}$  is the intensity in the frequency interval  $d\nu$  around  $\nu$ . Assuming a plane parallel atmosphere and negligible scattering of the incident radiation, we obtain for the radiation force per unit mass

$$F_{\text{rad}} = -\frac{1}{\rho} \frac{dP_{\text{rad}}}{dr} = \frac{1}{c} \int_{\omega} \int_{\nu} \kappa_{\nu} I_{\nu} \cos \gamma \, d\nu \, d\omega, \quad (23)$$

where  $\rho$  is the mass density and  $\kappa_{\nu}$  is the monochromatic mass absorption coefficient for radiation absorbed by the secondary. By taking an integrated mean opacity,  $\bar{\kappa}$ , we may take this quantity out of the integral and obtain  $F_{\text{rad}} = f_i \bar{\kappa} / c$ , where  $f_i$  is the total integrated flux. The gravitational force as a result of the compact object per unit mass of the secondary is simply  $F_{\text{grav}} = GM/d^2 = g$ , where  $M$  and  $d$  are the mass of the compact object and its distance from a point on the irradiated component. The maximum ratio of pressure to gravitational forces, assuming normally incident radiation ( $\gamma = 0$ ), is therefore given by

$$\delta_{\text{max}} = \frac{F_{\text{rad}}}{F_{\text{grav}}} = \frac{f_i \bar{\kappa}}{cg}, \quad (24)$$

which has the same functional dependence as in equation (5). We may also relate  $\delta_{\text{max}}$  to the Eddington luminosity of the secondary,  $L_{\text{Edd}}$ . Writing the flux as  $f_i = L/4\pi d^2$ , where  $L$  is the irradiating luminosity,  $\delta_{\text{max}}$  becomes

$$\delta_{\text{max}} = \frac{L\bar{\kappa}}{4\pi d^2 cg} = \frac{L\bar{\kappa}}{4\pi cGM} = \frac{1}{q} \frac{L}{L_{\text{Edd}}}, \quad (25)$$

where  $m$  is the mass of the secondary, and  $q = M/m$  is the mass ratio.

In general, for a given frequency of radiation, the opacity will be a very sensitive function of the local temperature, density and chemical composition, and can vary by orders of magnitude across the surface of the secondary. If the material on the surface of the secondary consisted of fully ionized hydrogen, electron scattering would be the dominant opacity source, i.e.  $\bar{\kappa} = \sigma_T/m_p \sim 0.4 \text{ cm}^2 \text{ g}^{-1}$ , where  $\sigma_T = 6.7 \times 10^{-25} \text{ cm}^2$  is the Thomson cross-section.

For systems where the luminosity in X-rays is close to the Eddington luminosity or where  $\bar{\kappa}$  is much larger than the Thomson opacity, the ratio of radiative to gravitational forces may easily be

of the order of unity, implying a significant deformation of the classical Roche lobe. Indeed, it is possible that  $\delta_{\text{max}}$  exceeds unity. In this case, there would be no inner Lagrangian point. However, this is unlikely in practice as such systems are very likely to contain a thick accretion disc which will shield much of the inner face of the secondary.

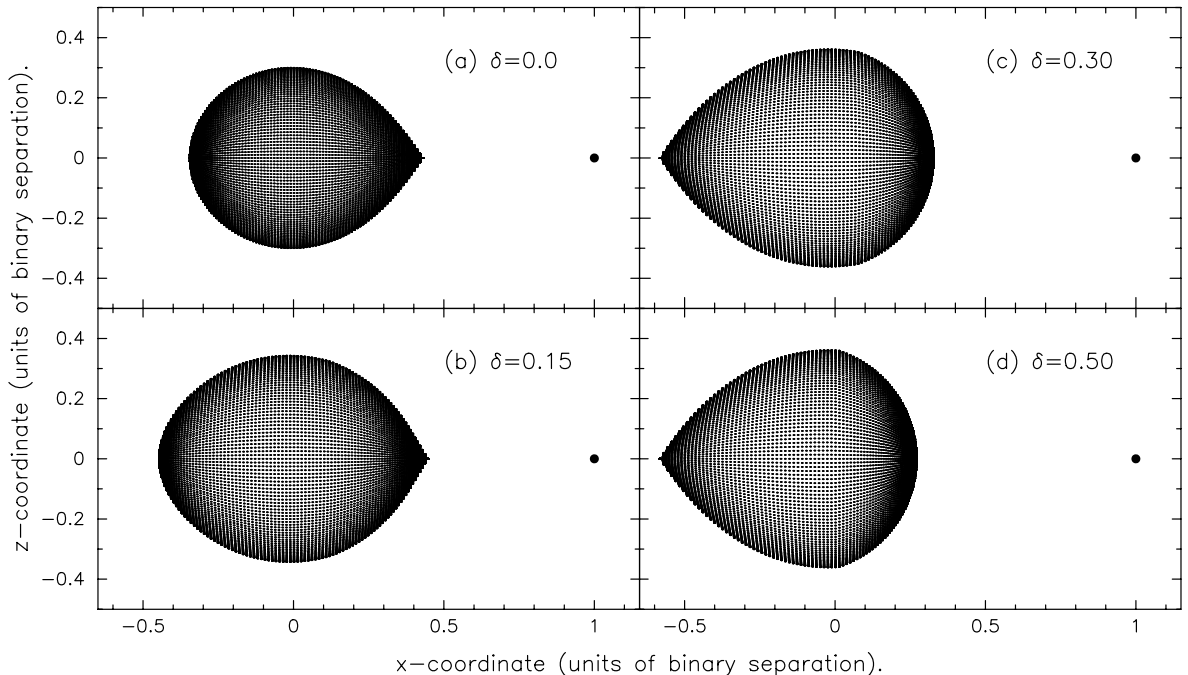
## 5 THE EFFECTS OF EXTERNAL RADIATION PRESSURE ON THE BINARY STRUCTURE

External radiation pressure changes the structure and appearance of the secondary compared to the case of Roche-lobe overflow without irradiation. The most important effects are: (1) the distortion of the secondary's surface, and the associated change in the surface temperature distribution which affects the modelling of light curves and radial velocity curves (see Section 6); and (2) a possible change of the critical configuration for systems that do not contain accretion discs (or contain discs that are tilted with respect to the binary plane), where the  $L_1$  point is directly irradiated, which affects mass transfer and mass loss from the system. We discuss these various possibilities in detail in this and the subsequent section.

### 5.1 The binary configuration

Let us first consider the effects of radiation pressure in systems without accretion discs. This case has possible applications to binary pulsar systems, stars orbiting supermassive black holes and, in some cases, HMXBs.

To illustrate how external irradiation can change the binary configuration, we have considered a binary with a fixed mass ratio  $q = M_1/M_2 = 2$ , but with different values for  $\delta_{\text{max}} = 0.0, 0.15, 0.30$  and  $0.50$ . We calculated the critical modified potential, as described above, such that the secondary fills its maximum possible volume. In Fig. 1 we present meridional sections in the  $x$ - $z$  plane of the



**Figure 1.** A sequence of modified ‘Roche’ lobes including the effects of external irradiation for a binary system with mass ratio  $q = M_1/M_2 = 2$ . The compact companion, of mass  $M_1$ , is represented as a single point at coordinates (1, 0). The values of  $\delta_{\text{max}}$ , the maximum ratio of the external radiation force to the gravitational force as a result of the compact star at the secondary surface, are set to (a) 0.0 (the standard Roche lobe), (b) 0.15, (c) 0.30 and (d) 0.50, respectively.

modified critical potential surfaces for this sequence of models, where the same scale has been used in all panels; see also section 3 in Drechsel et al. (1995) for a very similar analysis.

Fig. 1(a) shows the unperturbed Roche lobe without external irradiation pressure. In Fig. 1(b) the external irradiation force has been increased to 15 per cent of the gravitational force. As a consequence, the modified inner Lagrangian point  $L_1$  moves towards the compact object (since its effective gravity has been reduced), and the volume of the critical potential lobe increases. The critical potential, as defined by equation (8), decreases in magnitude as  $\delta$  is increased, until it reaches the same value as the potential of the outer Lagrangian point,  $L_2$ , which is unaffected by irradiation as it always lies in the shadowed region of the secondary. Once the outer Lagrangian point has a higher potential than the inner Lagrangian point, it takes over the role of defining the potential of the critical lobe, and the critical potential then passes through  $L_2$  rather than  $L_1$ . This change from an inner to an outer critical configuration implies that mass loss now takes place through the outer rather than the inner critical point, most likely leading to the formation of a circumbinary disc and mass loss from the system rather than mass transfer to the compact object. For  $q = 2$ , this transition occurs at  $\delta_{\max} = 0.202$ . In Fig. 1(c), where  $\delta_{\max} = 0.30$ , the switch of configuration has already occurred. Increasing  $\delta$  further no longer changes the position of  $L_2$  (since its potential is unaffected by external radiation), but the shape of the critical potential is further deformed and its enclosed volume decreases (see Fig. 1d).

### 5.1.1 The conical inner Lagrangian point

As can be seen in Fig. 1, for  $\delta_{\max} < \delta_{\text{crit}}$ , where  $\delta_{\text{crit}}$  is the critical value at which the system configuration change occurs, the region around the irradiated  $L_1$  point has a conical geometry (as for the standard Roche lobe). Hence the normal vector at the  $L_1$  point is undefined, and its position must be determined from the asymptotic value obtained in its neighbourhood (i.e. in the limit of small  $\theta$ , the angle measured from the line of centre, or  $x$ -axis). Because of the centrifugal term in the total potential [equation (8)] there is no axial symmetry about the  $x$ -axis. The asymptotic value of the  $L_1$  normal vector therefore depends on the plane containing the  $x$ -axis being considered, i.e. it depends on the angle  $\phi$ . The critical equipotential surface is defined as the surface with the lowest potential for which any matter can leave the secondary's gravitational field. It is easy to show that this is the case for  $\phi = 0$ ; this implies that an expanding irradiated secondary star will first lose mass through the inner Lagrangian point in the  $x$ - $z$  plane. (In practice however, if mass loss takes place, the overflowing material is likely to be optically thick and will therefore shield the  $L_1$  point from radiation, thus obscuring the subtle geometrical details described above.)

The conical nature of the irradiated  $L_1$  point also implies that the maximum value of the ratio  $\delta$ , obtained by extrapolation, is less than  $\delta_{\max}$ . At no point on the irradiated stellar surface is the flux vector at normal incidence. This is in contrast to the model described by Drechsel et al. (1995) in which the  $L_1$  point is apparently calculated assuming perpendicular incidence of the external radiation flux, such that  $\delta = \delta_{\max}$  at this point. Such a 'flat'  $L_1$  point also affects the critical value,  $\delta_{\text{crit}}$ , at which the configuration change occurs. In the Drechsel model it is lower for a given mass ratio  $q$ . For example, for  $q = 2$  it would occur at  $\delta_{\text{crit}} = 0.167$  for the assumption of a flat  $L_1$  point instead of  $\delta_{\text{crit}} = 0.202$ , as obtained above.

To show that a flat geometry near the  $L_1$  point is not appropriate, we can examine the factor  $\eta$  defined in Section 4.1.1 [see

equation (15)], which compares the fictitious force acting within the equipotential surface (resulting from the equipotential approximation) to the 'true' force directed approximately normally to this surface. We find that  $\eta$  is at least an order of magnitude less than  $\delta_{\max}$  at all points; this confirms that this modified equipotential surface provides a good representation of the 'true' forces. On the other hand, for a flat  $L_1$  geometry,  $\eta$  is always of the order of unity in the neighbourhood of the  $L_1$  point. This implies that the true and fictitious forces are of comparable magnitude and proves that the corresponding equipotential surface provides a poor approximation for an irradiated surface.

(Note that both models are consistent for outer critical configurations set by the  $L_2$  potential. In this case, the irradiated surface at  $\theta = 0$  is flat and  $\delta$  acquires its maximum value at this point,  $\delta_{\max}$ .)

## 5.2 Implications for binary pulsar systems

As shown in Section 5.1, if the irradiation force exceeds a critical value of the gravitational force,  $\delta_{\max}$ , the critical potential configuration changes from an inner to an outer configuration. However, this can only apply to systems in which the inner Lagrangian point is directly irradiated and hence this can never be the case for systems, such as LMXBs, where the accretion disc effectively screens the  $L_1$  point from the X-ray irradiation. In such systems, the main consequence of irradiation will be a deformation of the surface (see Section 6).

On the other hand, binary systems containing millisecond pulsars probably do not contain accretion discs. Hence pulsar radiation (in the form of electromagnetic waves, particle flux, etc.) can play the role of X-rays and can potentially cause a configuration change. Mass will then be lost from the outer Lagrangian point, which may then leave the system entirely. Continued irradiation may eventually lead to the total evaporation of the secondary. This is the main mechanism, originally proposed by Ruderman, Shaham & Tavani (1989), by which a pulsar may be able to destroy its companion leaving a single millisecond pulsar. The 'black-widow' pulsar, PSR 1557+20 (Fruchter, Stinebring & Taylor 1988), and the binary pulsar PSR 1718+19 (Lyne et al. 1990; McCormick & Frank 1993) are believed to be examples where the pulsar radiation is in the process of evaporating the companion.

In the case of PSR 1557+20, the mass function and the radio eclipse suggest a secondary mass near  $0.022 M_{\odot}$  [for an assumed pulsar mass of  $1.4 M_{\odot}$ ; see, for example, Fruchter et al. (1988)]. This implies a mass ratio of around 60 and a corresponding value for  $\delta_{\text{crit}}$  of approximately 0.01 (see Table 1). We can also estimate the pulsar luminosity from the observed spin-down rate to be  $\sim 40 L_{\odot}$ ; see, for example, Fruchter et al. (1990). For the irradiated component we have  $L_{\text{Edd}} \simeq (10^{36} \text{ erg s}^{-1})$ , and therefore  $\delta_{\max} = L/qL_{\text{Edd}} = 0.0025\bar{\kappa}$ . So  $\delta_{\max}$  will exceed  $\delta_{\text{crit}}$  if the mean photospheric opacity on the irradiated side is greater than  $\sim 4 \text{ cm}^2 \text{ g}^{-1}$ , a value that may not be unreasonable (it depends on the uncertain thermodynamic parameters in the irradiated photosphere of the secondary). In this case, we may expect a configuration change where mass loss occurs from the outer Lagrangian point, leading to the formation of an excretion disc rather than an accretion disc.

## 5.3 Application to binary systems with extreme mass ratios

Table 1 shows the values of  $\delta_{\text{crit}}$ , the product  $q\delta_{\text{crit}}$ , and the critical potential  $\Omega_{\text{crit}}$  of the secondary, as defined by equation (8), for a sequence of mass ratios. For higher mass ratios, the critical value of  $\delta_{\max}$  decreases, and the magnitude of the critical potential increases,

**Table 1.** Values of  $\delta_{\text{crit}}$ , the product  $q\delta_{\text{crit}}$ , and  $\Omega_{\text{crit}}$  for a sequence of mass ratios.

Mass ratio, $q$	$\delta_{\text{crit}}$	$q\delta_{\text{crit}}$	$-\Omega_{\text{crit}}$
0.5	0.459	0.230	2.408
1.0	0.318	0.318	3.207
1.5	0.246	0.369	3.949
2.0	0.202	0.405	4.655
2.5	0.172	0.431	5.335
10	0.0564	0.564	14.44
$10^2$	$6.89 \times 10^{-3}$	0.589	109.7
$10^3$	$7.48 \times 10^{-4}$	0.748	$1.02 \times 10^3$
$10^4$	$7.76 \times 10^{-5}$	0.776	$1.00 \times 10^4$
$10^5$	$7.89 \times 10^{-6}$	0.789	$1.00 \times 10^5$
$10^6$	$7.94 \times 10^{-7}$	0.794	$1.00 \times 10^6$

corresponding to a smaller Roche volume. Note that the product  $q\delta_{\text{crit}}$  converges towards a constant value for increasing  $q$ . In conjunction with equation (25), this implies that, in the asymptotic limit of high mass ratio, the critical luminosity scales directly with the Eddington luminosity of the irradiated object:

$$L_{\text{crit}} = q\delta_{\text{crit}}L_{\text{Edd}} \propto L_{\text{Edd}}, \quad \text{for } q \gg 1. \quad (26)$$

A related formula was derived by Podsiadlowski & Rees (1994), where they expressed the difference in potential between the inner and outer Lagrangian points in terms of the ratio of irradiating to Eddington luminosities. Their equation (7) (without a spurious factor of  $(m/3M)^{1/3}$ ) reads

$$\Delta U \equiv U_2 - U_1 \simeq \frac{Gm}{a} \left[ \frac{2}{3} - \gamma \right], \quad (27)$$

where  $\gamma = L/L_{\text{Edd}}$ ,  $a$  is the binary separation, and  $m$  is the mass of the irradiated star. However, this equation contains the assumption that the irradiating flux is perpendicular to the irradiated surface at  $L_1$ , i.e.  $\delta_{\text{max}}/\delta_{L_1} = 1$ . As we have shown above, this is not a good approximation. Based on our calculations, performed for a range of values of  $q$  and  $\delta_{\text{max}}$ , we find this ratio to be approximately 1.20. Hence the Lagrangian points have the same potential, i.e.  $\Delta U = 0$ , for  $\gamma = 1.20 \times \frac{2}{3}$ . Equation (26) therefore becomes

$$L_{\text{crit}} \rightarrow 0.80L_{\text{Edd}}, \quad \text{as } q \rightarrow \infty, \quad (28)$$

in agreement with Table 1.

One interesting application of this case is low-mass stars orbiting supermassive black holes in active galactic nuclei (AGN) which are being irradiated by the central black hole. Such systems can be considered binaries with extreme mass ratios, typically of the order of  $10^5$  to  $10^8$ , and some highly unusual properties; see, for example, Podsiadlowski & Rees (1994). The X-ray luminosity in these systems will generally be far in excess of the Eddington luminosity of the stellar companion,  $\sim 10^{38} \text{ erg s}^{-1}$  for a star of the order of  $1 M_{\odot}$ . (In general, for luminous supermassive black holes, we may expect  $L/L_{\text{Edd}}^{\text{star}} \sim 10^4$ – $10^7$ .) So it is clear from equation (28) that only a tiny fraction of the central source's X-ray luminosity has to be emitted in the direction of the companion and reach the inner Lagrangian point for  $L$  to exceed  $L_{\text{crit}}$ . Hence, any Roche-lobe overflow from the stellar companion must occur via the outer Lagrangian ( $L_2$ ) point, leading to the formation of an excretion disc rather than an accretion disc. The tidal coupling between this excretion disc and the secondary may then provide a tidal barrier and actually prevent accretion of disc material on to the black hole; see, for example, Artymowicz et al. (1991).

## 5.4 Self-sustaining mass loss through the outer Lagrangian point

In the previous sections, we have shown how external irradiation pressure can change the configuration of the critical tidal lobe for the mass-losing component in a binary system from an inner to an outer critical configuration where mass loss occurs through the outer rather than the inner Lagrangian point. Since this matter will carry off an associated amount of angular momentum that is larger than the systemic specific angular momentum, this leads to a net decrease in the specific angular momentum of the system. Such mass loss can dramatically increase the mass-loss rate from the mass loser and sometimes even destabilize such systems completely. To estimate this more quantitatively, we derived the predicted orbital period change following a simple analysis similar to Rappaport, Verbunt & Joss (1983; see also Ritter 1996).

Assuming that all the mass is lost from the outer Lagrangian point and neglecting tidal interaction terms, the change of the orbital period can be written in its differentiated form as

$$\frac{\dot{P}_{\text{orb}}}{P_{\text{orb}}} = - \left\{ \frac{2 + 3q(1 - \alpha)}{1 + q} \right\} \frac{\dot{M}_2}{M_2}, \quad (29)$$

where  $q = M_1/M_2$  is the mass ratio,  $\dot{M}_2$  is the mass-loss rate from the secondary (and the system), and  $\alpha$  is the specific angular momentum of the matter lost from the system in units of the specific angular momentum of the secondary.

In recent years, a number of systems have been observed to exhibit orbital period decays, for example, the massive X-ray binary SMC X-1 with  $\dot{P}_{\text{orb}}/P_{\text{orb}} = (-3.36 \pm 0.02) \times 10^{-6} \text{ yr}^{-1}$  (Levine et al. 1993) and the system Centaurus X-3 (Cen X-3) with  $\dot{P}_{\text{orb}}/P_{\text{orb}} = (-1.8 \pm 0.1) \times 10^{-6} \text{ yr}^{-1}$  (Kelley et al. 1983). These period changes have traditionally been explained by a tidal instability, the Darwin instability, which causes the orbit to decay in the process of spinning up the secondary (Darwin 1879; Pringle 1974).

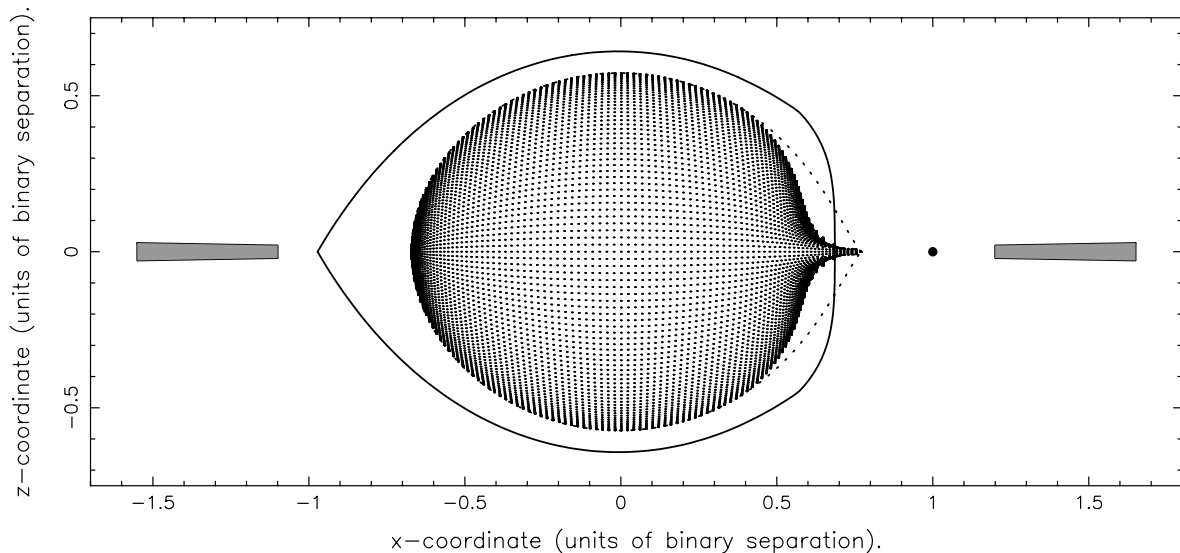
However, as we demonstrate below, using Cen X-3 as an example, a relatively moderate outflow through  $L_2$  could also account for the observed change and may help to explain some other observations that are not well understood at the present time. Note that here we do not consider the tidal coupling of the circumbinary disc with the binary. This leads to a transfer of orbital angular momentum from the binary to the circumbinary disc which can also drive the evolution of the orbit; see, for example, Meyer & Meyer-Hofmeister (2001) and Taam & Spruit (2001).

Cen X-3 is a massive X-ray binary with an orbital period of 2.87 d, observed to radiate near the Eddington luminosity for a neutron star,  $\sim 2 \times 10^{38}$  (White, Swank & Holt 1983). For a mass ratio of 0.05 (Rappaport & Joss 1983), this is about 50 per cent greater than the flux required to produce a configuration change provided that the inner Lagrangian point is directly irradiated. Assuming that mass loss occurs through the outer Lagrangian point, we can estimate  $\alpha$  to be  $\sim 460$ . Equation (29) then becomes

$$\frac{\dot{P}_{\text{orb}}}{P_{\text{orb}}} \sim 60 \frac{\dot{M}_2}{M_2}. \quad (30)$$

This should be compared to the case where mass transfer is conservative through  $L_2$ , where the factor in front of  $\dot{M}_2/M_2$  would be  $\sim 20$  instead of 60. Thus, to reproduce the observed rate of orbital decay ( $\dot{P}_{\text{orb}}/P_{\text{orb}} \sim -1.8 \times 10^{-6} \text{ yr}^{-1}$ ), a mass-loss rate of around  $6.0 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$  through the outer Lagrangian point would be required. This is consistent with the initial rates expected for overflow through the  $L_1$  point in HMXBs during the early phase





**Figure 2.** Approximate equipotential surfaces for the HMXB system Cen X-3, assuming a mass ratio of 0.05. The dotted curve represents the unperturbed Roche lobe, with the compact X-ray source at coordinates (1, 0). The dotted inner region shows the deformed geometry of the critical lobe when the effects of X-ray irradiation are included (for an Eddington-limited X-ray source), but the  $L_1$  point is shadowed by a thin accretion disc (not shown). The thick solid curve shows the critical potential surface when the  $L_1$  is irradiated directly with a luminosity of  $2 \times 10^{38} \text{ erg s}^{-1}$ . Matter will then preferentially escape from the outer Lagrangian point, forming an *excretion* disc, as indicated schematically by the shaded regions.

of mass transfer; see, for example, Lamers, van den Heuvel & Petterson (1976). In this case, however, we expect the formation of an *excretion* disc emanating from the  $L_2$  point [see, for example, Shu, Lubow & Anderson (1979)], from which emission or absorption lines may be observed depending on the orientation of the observer. Indeed, strong evidence for material extending beyond the limiting Roche-lobe surface of the secondary has been reported by Mauder (1975) and Clark, Minato & Guozhu (1988), although the true origin of this material remains unclear. If indeed the secondary loses mass through the outer Lagrangian point, an efficient mechanism would be required to fuel the accretion-powered X-ray source, most likely in the form of a stellar wind, quite likely enhanced by X-ray irradiation (Day & Stevens 1993).

Fig. 2 shows approximate equipotential surfaces for Cen X-3 for a mass ratio  $q = 0.05$ . The dotted curve represents the unperturbed Roche lobe, with the compact X-ray source at coordinates (1, 0). The dotted inner region represents the deformed secondary when X-ray irradiation (assumed to be Eddington limited) is included, and the  $L_1$  point is shadowed by a thin accretion disc (not shown). The cross-section of the irradiated Roche lobe is significantly reduced by the incident radiation, causing a bottleneck near the  $L_1$  point and strongly reducing the effective cross-section for a stream emanating from  $L_1$ ; see Lubow & Shu (1975). We caution, however, that in reality this bottleneck may be widened by other effects not included in our model (e.g. small-scale turbulent motion, large-scale circulation currents, Coriolis-force effects, X-ray emission from an extended source).

If the accretion disc is either tilted or absent, allowing direct irradiation of the  $L_1$  point, the secondary can expand to fill the outer equipotential surface (solid curve). Matter will then preferentially escape from the outer Lagrangian point forming an *excretion* disc (indicated schematically as shaded regions).

If Cen X-3 is losing mass through the outer Lagrangian point, this has several important consequences for both the evolutionary stage of the system and the modelling of the system. Although we use Cen X-3 as an example, the model could apply equally well to

similar high-luminosity HMXBs, e.g. SMC X-1. We require only that the X-ray flux at the  $L_1$  point is greater than the critical flux; for systems radiating close to the Eddington limit, this will generally be the case.

#### 5.4.1 The lifetime of the X-ray phase

One consequence of mass loss through the outer Lagrangian point is that it may increase the lifetime of the X-ray-active phase of the system. High-luminosity HMXBs are commonly assumed to be powered by Roche-lobe overflow through the  $L_1$  point (Lamers et al. 1976). However, this will rapidly lead to runaway mass transfer on a thermal time-scale, which would result in accretion rates of the order of  $10^{-4}$  to  $10^{-3} M_{\odot} \text{ yr}^{-1}$  (van den Heuvel & De Loore 1973). Such large rates would quickly and completely extinguish the X-ray source by absorption, suggesting a maximum X-ray lifetime of  $\leq 3 \times 10^3 \text{ yr}$  (Terman, Taam & Savage 1998). In the case of outflow through  $L_2$ , all of this mass loss from the secondary will feed a circumbinary disc. The X-ray active lifetime will be determined by the rate at which the wind accretion rate on the neutron star increases. It could be comparable to the lifetime estimated for low-luminosity HMXBs,  $\sim 10^4$ – $10^5 \text{ yr}$ , which are thought to be predominantly wind-fed (Meurs & van den Heuvel 1989).

#### 5.4.2 The size of the X-ray eclipse

Cen X-3 is an eclipsing system where the total X-ray eclipse lasts 0.488 d (Schreier et al. 1972), i.e. a fraction of 0.17 of the total period. Defining the eclipse half-angle  $\theta_e$  as the orbital phase angle at which the occultation of the X-ray source is just complete, the data imply  $\theta_e \approx 42^\circ$ . This angle may be directly related to the radius of the secondary  $R_2$  according to

$$\frac{R_2}{a} = (1 - \sin^2 i \cos^2 \theta_e)^{1/2}, \quad (31)$$

(Rappaport & Joss 1983) where  $i$  is the system inclination and  $a$  is the component separation. If we assume that the secondary fills its

standard Roche lobe, we can then use the expression of Eggleton (1983) for the Roche-lobe radius to uniquely determine the mass ratio  $q$ . Because the mass function is approximately  $15 M_{\odot}$  (Schreier et al. 1972), this would yield a mass of the compact object of less than  $0.7 M_{\odot}$  (Davidson & Ostriker 1973), which is not consistent with current estimates of  $\sim 1.4 M_{\odot}$  [see, for example, Thorsett et al. (1993)] for a typical neutron star. Clark et al. (1988) attempted to fit the Cen X-3 data using a spherically symmetric coronal wind model, which took into account the gradual eclipse ingresses and egresses. Their model predicted a more reasonable neutron star mass of  $1.23 \pm 0.60 M_{\odot}$ , but required a much lower (total) eclipse half-angle of  $32.9^{\circ}$ .

A much simpler solution of this discrepancy is to increase the size of the eclipsing region, as occurs naturally if the secondary fills its outer critical lobe (see Fig. 2). To demonstrate this, we consider an inclination angle in the probable range  $70\text{--}80^{\circ}$  (Nagase 1989) and we assume that neither the star itself nor any material associated with it can exceed the dimensions of the irradiated Roche lobe. Following the analysis of Schreier et al. (1972), we adopt  $\theta_c \approx 45^{\circ}$ ; although Wilson (1972) has suggested values as high as  $53^{\circ}$ . Equation (31) then implies that the irradiated Roche-lobe radius is in the range  $0.72\text{--}0.75$ , in units of the orbital separation. Using our model, rather than Eggleton's formula, we relate  $R_2$  to  $q$  by calculating the total volume of the irradiated lobe for a given mass ratio and equating this with  $4/3\pi R_2^3$ . We obtain values for  $q$  in the range of  $0.101\text{--}0.062$ , which corresponds to a far more plausible range for the neutron star mass of  $1.26\text{--}1.92 M_{\odot}$ . We can expect that a more thorough investigation will yield tighter constraints on the possible masses, the inclination and the eclipse angle.

#### 5.4.3 The optical light curve

A second observational problem associated with Cen X-3 is related to the modelling of the optical light curve of the system; see, for example, Mauder (1975) and Tjemkes, Zuiderwijk & van Paradijs (1986). Using a standard model for the ellipsoidal variations, Tjemkes et al. (1986) found that the depths of the minima at phases 0.0 and 0.5 were much greater than the typical model parameters predicted. They discounted the possibility of X-ray heating as this would further decrease the depth at phase 0.5. However, Hutchings et al. (1979) presented conflicting evidence. They found the spectral type of the companion star to vary with orbital phase, ranging from O9 near phase 0.0 to O6 near phase 0.5, which strongly suggested appreciable X-ray heating. Our model would appear to broadly reconcile these results if we allow irradiation of the  $L_1$  point, because the initial increase in the companion Roche lobe actually causes an *increase* in this depth as a result of the change in geometry. Clearly, this effect should be included in accurate light-curve modelling of the system. We should also note that strong irradiation need not necessarily result in a strong 'reflection' effect, since only a fraction of the incident X-ray flux has to be thermalized, while circulation currents will further distribute the heated material around the secondary (Kippenhahn & Thomas 1979; Schandl, Meyer-Hofmeister & Meyer 1997).

#### 5.4.4 Caveats

In our analysis we have made two important assumptions: (1) that the secondary is in (almost) synchronous rotation, such that the Roche-lobe approximation may be applicable; and (2) that the  $L_1$

point is directly irradiated, with an associated change in the value of the critical potential.

It is generally believed that any initial non-circularity in the binary orbit or frequency difference in the rates of orbital and axial rotation would have been rapidly removed by tides exerted by the compact star on the secondary. The synchronization time-scale in X-ray binaries is generally of the same order of (or shorter than) the circularization time-scale (Lecar, Wheeler & McKee 1976; Zahn 1975; Hut 1981). As Fabbiano & Schreier (1977) have shown that the secondary in Cen X-3, also known as Krzeminski's star, is in a highly circular orbit ( $e = 0.0008 \pm 0.0001$ ), it is safe to assume that the secondary has synchronized its spin with the orbit.

The second of these caveats is somewhat more problematic. In the simplest model of an X-ray binary, we assume that matter is accreted on to the compact object through an accretion disc which forms in the orbital plane. The  $L_1$  point is then shadowed from a point-like X-ray source positioned at the compact object. However, there are several possible alternatives.

First, even if an accretion disc is present, the  $L_1$  point of the secondary may be directly irradiated by X-ray radiation coming from an extended Compton-heated thick accretion disc corona for which evidence has been found in the partial X-ray eclipses of several non-pulsating accretion-powered binary X-ray sources by White & Holt (1982).

Secondly, the disc plane may be tilted with respect to the binary plane and allow direct irradiation of the secondary (depending on the precession phase). Such tilted discs have variously been proposed to explain precessing discs in systems such as Her X-1, LMC X-4 and SS 433 [see, for example, Roberts (1974), Petterson (1977) and Priedhorsky & Holt (1987)] and even Cen X-3 (Iping & Petterson 1990). As Pringle (1996) has shown, even initially planar discs are unstable to warping as a result of the external radiation pressure, implying that warping is a generic phenomenon in centrally illuminated accretion discs; see also Maloney & Begelman (1997).

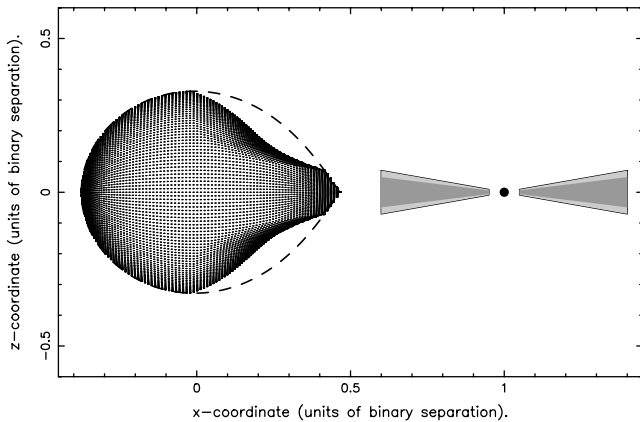
Thirdly, it is by no means clear that an accretion disc forms if mass accretion on to the compact object occurs through a stellar wind, because very little of the orbital angular momentum is actually accreted with the wind material; see, for example, the studies by Ruffert (1994, 1997). No disc is expected to form if the circularization radius, which depends on the specific angular momentum of the accreted material, is less than the Alfvén radius, which defines the radius below which the neutron star's magnetic field will determine the dynamics of the accretion flow.

## 6 BINARY SYSTEMS CONTAINING AN ACCRETION DISC

In general, LMXBs and some HMXBs contain substantial accretion discs. These shield the inner Lagrangian point from X-ray irradiation, thereby maintaining the relative positions and values of the Lagrangian points. However, radiation pressure may still cause significant deformation of the irradiated stellar surfaces, particularly in systems with high accretion rates. This can change the appearance of the optical light curves and radial velocity curves, thereby altering the derived system parameters.

#### 6.1 Observational effects of surface deformations

To illustrate the effects of radiation pressure, we consider a binary system with physical parameters that may be appropriate for the LMXB Scorpius X-1 with an orbital period of 18.9 h (Kallman,



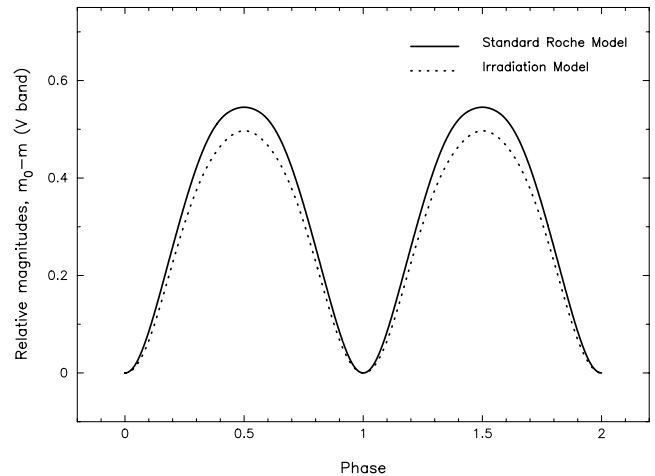
**Figure 3.** Deformation of the Roche lobe for  $\delta_{\max} = 0.99$ ,  $q = 1.4$ . The outline of the standard Roche lobe is shown as a dashed curve, and the X-ray source is represented as a point with coordinates (1, 0). The cross-section of the accretion disc is shown containing a heavily shaded opaque region and a more lightly shaded transition region.

Borison & Vrtilek 1998), where we take the masses of the neutron star and the secondary to be 1.4 and 1  $M_{\odot}$ , respectively. The system then has an orbital separation of 4.8  $R_{\odot}$ . The irradiating X-ray luminosity is assumed to be  $2 \times 10^{38} \text{ erg s}^{-1}$ , which is approximately the Eddington luminosity of the neutron star. The optical companion in this system is thought to be a Roche-lobe filling star near the end of or just beyond its main-sequence phase (Cowley & Crampton 1975). We take an effective (polar) temperature of 16000 K,<sup>1</sup> a gravity darkening coefficient of 0.08, and a constant limb-darkening coefficient of 0.35, appropriate to V-band observations (see appendix). For the accretion disc, we assume a conservative opening (half) angle of  $10^{\circ}$  with a fully opaque region of  $7^{\circ}$ . These parameters imply a mass ratio of  $q = 1.4$ , and therefore a ratio of radiation to pressure forces  $\delta_{\max} = L/q L_{\text{Edd}} \sim 2.5\bar{\kappa}$ . Using the approximation described in Section 4.4, we assume  $\bar{\kappa} = 0.4$ , implying  $\delta_{\max} = 0.99$ . We emphasize that this model should not be considered to be very realistic, because it makes a number of simplifications, e.g. it assumes constant opacity across the irradiated surface and that all the energy is thermalized below the photosphere of the secondary (i.e. it takes an X-ray albedo of 1). The main purpose of the model is to demonstrate the magnitude of the effects of irradiation pressure on the derived system parameters.

The results of the model using these parameters are shown in Fig. 3, together with the outline of the Roche lobe due to the standard model. The compact object is again represented as a single point with coordinates (1, 0). The cross-section of the accretion disc is shown, containing a heavily shaded opaque region and a more lightly shaded transition region. Its effect as an X-ray shield may be readily inferred from the lack of any distortion around the  $L_1$  point.

The deformation of the Roche surface is very clear. The effect is greatest in the region just beyond the accretion disc shadow and decreases as the Roche-lobe horizon is approached. It should also be noted that the surface equipotential ‘cuts’ the interior equipotentials to a depth determined by the extent of the deformation (about 1/5

<sup>1</sup>This temperature is significantly higher than the effective temperature of a 1  $M_{\odot}$  star, but could be appropriate if the secondary were in a similar evolutionary phase as the secondary in Cyg X-2 (Casares, Charles & Kuulkers 1998) or if surface circulation were very efficient in re-distributing the irradiation flux around the secondary.



**Figure 4.** Comparison of V-band optical light curves due to standard and modified Roche lobes, for the parameters given in the text. Relative magnitudes are calculated by subtracting the theoretical magnitude at each phase,  $m$ , from the magnitude at phase 0,  $m_0$ .

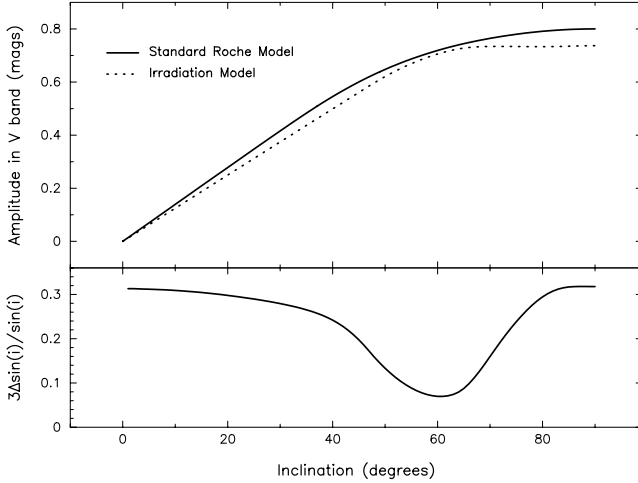
of the radius for the illustrated case). This may have implications for the circulation of energy along equipotentials.

#### 6.1.1 Effects of radiation pressure on optical light curves

To demonstrate the effect of X-ray radiation pressure on optical light curves, we have calculated these (using a standard light-curve code whose main ingredients are listed in the Appendix) for both a standard Roche-lobe model and a model with a distorted Roche lobe (Fig. 3) for the parameters given above. In order to avoid complications as a result of eclipses of the secondary or the accretion disc, we have chosen an inclination of  $40^{\circ}$ . Fig. 4 shows the resulting optical V-band light curves calculated by summing the flux over the surface of the modified Roche lobe and the standard Roche lobe, respectively. In both cases, the accretion disc is assumed to contribute a fixed amount of flux. For each curve, the relative magnitude is plotted against phase, where the former is calculated by subtracting the theoretical magnitude at each phase,  $m$ , from the magnitude at phase 0,  $m_0$ . This vertical shift ensures that both curves pass through the origin, thereby highlighting their differences at phase 0.5.

The greatest difference between the two models occurs at phase 0.5, for which the irradiated side of the secondary is most visible and the observed flux is a maximum. The flux resulting from the pressure-modified lobe is lower, since the distortion results in a decrease in the heated area perpendicular to the line of sight. Around phase 0, the curves differ much less, since the back side of the secondary is visible, and its geometry is not affected by radiation pressure.

The amplitude of a light curve is defined as the difference in magnitudes between phases 0 and 0.5 and is generally a sensitive function of the inclination of the system. It provides an empirical correlation between the observations and the system parameters; see, for example, Bochkarev, Karitskaya & Shakura (1979). If the mass ratio  $q$  is known for a system, for example from radial velocity curves determined spectroscopically, it is common to use the light-curve amplitude to deduce the orbital inclination and with it the component masses, which are proportional to  $\sin i$  to the third power. Thus, a fractional error in the value of  $\sin i$  propagates to three times this error in the component masses:



**Figure 5.** Top panel: theoretical V-band amplitudes obtained for the standard Roche model and irradiation model as a function of inclination. Bottom panel: the fractional error in  $(\sin i)^3$  as a function of inclination. For a given amplitude, the irradiation model is assumed to provide the true inclination.

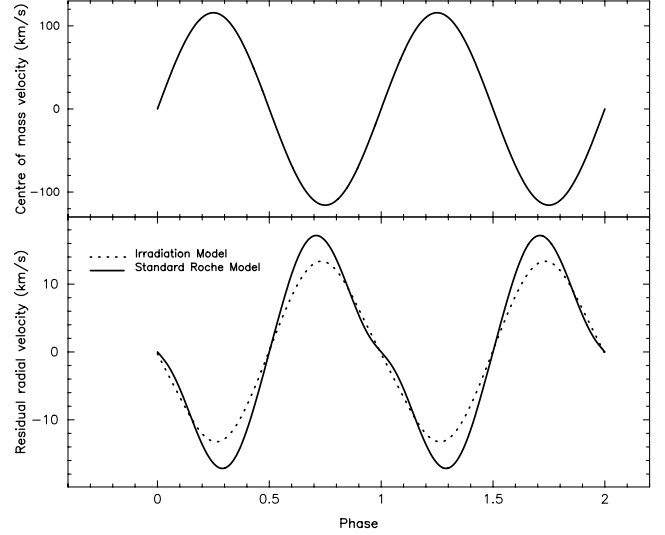
$$\frac{\Delta m}{m} \simeq \frac{3\Delta(\sin i)}{\sin i}. \quad (32)$$

The top panel of Fig. 5 shows the theoretical amplitudes expected for this system as a function of inclination, calculated for both the standard Roche model and the irradiated model. At an inclination of zero, the observer views the system face-on and no variation in the light curve is observed. The amplitude has a maximum at an inclination of  $90^\circ$  when the line of sight is parallel to the orbital plane (of course, at large inclination eclipse effects will also strongly affect the light curve). As described earlier, the theoretical amplitude is systematically smaller for the irradiated model than in the standard model. Thus, for a given observed amplitude, different models will correspond to different inclinations and hence implied masses [through equation (32)]. In the lower panel of Fig. 5, we have plotted the quantity  $3\Delta \sin i / \sin i$  against inclination, where we assume that the true value of  $\sin i$  refers to the irradiation model. This correction is significant ( $\geq 7$  per cent) at all inclinations and rises beyond 30 per cent at the extremes. At an inclination of  $40^\circ$  to the line of sight, the value of  $3\Delta \sin i / \sin i$  is approximately 0.26, implying a  $\sim 25$  per cent error in the component masses. At lower inclinations, the error is greater still.

Systems where external radiation pressure can be expected to be important are systems such as Scorpius X-1, Cygnus X-2 or the black-hole candidate GRO J1655-40, in which the secondary is sufficiently evolved so as to allow optical observations during outburst (Orosz & Bailyn 1997). The effect will also be of importance in HMXBs, whose optical output is dominated by the high-mass secondary star; for example, the super-Eddington X-ray binary, LMC X-4 – see Heemskerck & van Paradijs (1989).

### 6.1.2 Effects of radiation pressure on radial velocity curves

The distortion of the secondary also affects the interpretation of radial velocity curves obtained from the phase-dependent Doppler shifts of spectral lines. In the presence of irradiation, the temperature of the heated side of the secondary may be increased by a factor of 2 or more, depending on the type of system and X-ray luminosity. This causes phase-dependent asymmetries in the shapes of individual spectral lines and shifts the effective velocity centre of the lines.



**Figure 6.** Top panel: the radial velocity curve of the centre of mass as a function of phase. Bottom panel: the residual radial velocity curves as a function of phase for the standard Roche lobe and radiation pressure model. The relevant parameters are given in the text.

If these effects are not properly taken into account, this leads to incorrect determinations of the radial velocity amplitude; see Wade & Horne (1988), Shahbaz & Wood (1996), and Phillips, Shahbaz & Podsiadlowski (1999). The deformation of the Roche lobe introduces a further higher-order error. In order to illustrate the magnitude of these effects, we consider flux-weighted *residual* radial velocity curves calculated using the same system parameters as above (with  $M_1 = 1.4 M_\odot$ ,  $M_2 = 1 M_\odot$ ,  $P_{\text{orb}} = 18.9$  h,  $L_x = 2 \times 10^{38}$  erg s $^{-1}$ ).

The lower panel of Fig. 6 shows the residual radial velocity curves calculated with these parameters and an inclination of  $40^\circ$  for a model that includes only the variation of the surface temperature (including irradiation) and a model that includes the effects resulting from the temperature variation and the surface distortion. Here, the residual velocity was calculated according to

$$V_{\text{resid}}(\Phi, i) = \frac{\sum_{r, \theta, \phi} V_{\text{rad}}(r, \theta, \phi, \Phi, i) \Delta F(r, \theta, \phi, \Phi, i)}{\sum_{r, \theta, \phi} \Delta F(r, \theta, \phi, \Phi, i)} - V_{\text{rad}}^{\text{CM}}(\Phi, i), \quad (33)$$

where  $V_{\text{rad}}(r, \theta, \phi, \Phi, i)$  is the radial velocity for a given orbital phase angle  $\Phi$  and inclination  $i$  at a grid point  $(r, \theta, \phi)$ ,  $\Delta f(r, \theta, \phi, \Phi, i)$  is the observed flux for a given surface element, and  $V_{\text{rad}}^{\text{CM}}(\Phi, i)$  is the centre-of-mass radial velocity of the secondary. (The summation is only performed over the visible area of the star.)

At phase 0, the observer sees the back side of the secondary, and the radial velocity (and residual radial velocity) is zero. The radial velocity increases and becomes positive as the secondary moves away from the observer. However, the heated face of the secondary is now visible, and this shifts the ‘effective centre’ of the velocity towards the centre of mass of the binary orbit. Hence, the radial velocity is reduced and the residual radial velocity is negative. It falls to zero once more at phase 0.5, when the heated face of the secondary is directly in the observer’s line of sight. After phase 0.5, the secondary moves back towards the observer and so the reverse effect is seen. The amplitude of these effects is substantially smaller (by about 30 per cent or  $\sim 4$  km s $^{-1}$ ) for the pressure-modified surface compared with the standard Roche lobe. Again, this is a consequence of the decrease in the projected heated area

perpendicular to the observer's line of sight, as a result of the deformation of the surface. This demonstrates that the distortion of the surface reduces the  $K$  correction that needs to be applied to obtain the true underlying radial velocity curve. This will be particularly important for systems such as Sco X-1, where the measured X-ray luminosity [ $\sim 1.8 \times 10^{38}$  erg s $^{-1}$ ; Kallman et al. (1998)] indicates strong heating at phase 0.5.

## 7 CONCLUSIONS

We have shown that irradiation of the secondary in a close binary can have a significant influence on the shape of irradiated stellar surfaces and on system configurations. We have constructed approximate numerical solutions of the equipotential surfaces of secondaries where external irradiation effects are important to demonstrate the main physical effects. These generally scale with the luminosity of the radiation source and become dramatic whenever the ratio of radiation to gravitational forces approaches the critical value. The inclusion of the radiation pressure term in the total effective potential leads to considerable deviations from the standard Roche geometry. Depending on the luminosity of the radiation source and the relative efficiency of absorption in the irradiated photosphere, the most important effects are:

- (1) The geometry of the equipotential surface in the vicinity of the irradiated region is altered, in particular for high X-ray luminosities.
- (2) For systems without accretion discs, the position of the irradiated inner Lagrangian ( $L_1$ ) point is shifted. This leads to an increase in the size of the critical Roche lobe.
- (3) When the irradiating luminosity increases above a critical value, the (irradiated) inner Lagrangian point ceases to be the Lagrangian point with the lowest potential. Then the critical potential will be set by the  $L_2$  point (or the  $L_3$  point in HMXBs) and an outer critical configuration will be obtained.
- (4) In such outer critical configurations, mass loss occurs through the outer Lagrangian point. This may influence the evolutionary processes of systems such as binary pulsars or stars orbiting supermassive black holes in AGN.
- (5) In the case of mass loss purely from the  $L_2$  point, the associated change in angular momentum can lead to self-sustaining mass loss. This may be relevant for binary systems which show a steady decay in orbital period, and may lengthen the stable lifetimes of some HMXBs. Cen X-3 would appear to be a possible candidate.
- (6) Because of the deformation of the stellar surface, light curves and radial velocity curves in LMXBs and HMXBs are modified. These effects need to be included for a reliable determination of system parameters.

Finally, we note that, while it has yet to be demonstrated conclusively that radiation pressure can affect the global structure of a star to the extent proposed in this paper, it is clear that the pressure force resulting from external irradiation can be of a considerable magnitude. No analysis of high-luminosity X-ray binary systems can be complete without an adequate treatment of these effects.

## REFERENCES

Al Naimiy H. M., 1978, *Ap&SS*, 53, 181  
 Artymowicz P., Clarke C. J., Lubow S. H., Pringle J. E., 1991, *ApJ*, 370, L35  
 Avni Y., 1978, in Giacconi R., Ruffini R., eds, *Physics and Astrophysics of Neutron Stars and Black Holes*. North-Holland, Amsterdam, p. 42  
 Bochkarev N. G., Karitskaya E. A., Shakura N. I., 1979, *Astron. Zh.*, 56, 16  
 Casares J., Charles P., Kuulkers E., 1998, *ApJ*, 493, L39

Charbonneau P., 1992, *A&A*, 259, 134  
 Clark G. W., Minato J. R., Guozhu, M., 1988, *ApJ*, 324, 974  
 Cowley A. P., Crampton D., 1975, *ApJ*, 201, L65  
 Darwin G. H., 1879, *Proc. R. Soc.*, 29, 168  
 Davey S., Smith R. C., 1992, *MNRAS*, 257, 476  
 Davidson K., Ostriker J. P., 1973, *ApJ*, 179, 585  
 Day C. S. R., Stevens I. R., 1993, *ApJ*, 403, 322  
 Drechsel H., Haas S., Lorenz R., Gayler S., 1995, *A&A*, 294, 723  
 Eggleton P. P., 1983, *ApJ*, 268, 368  
 Fabbiano G., Schreier E. J., 1977, *ApJ*, 214, 235  
 Fruchter A. S., Stinebring D. R., Taylor, J. H., 1988, *Nat*, 333, 237  
 Fruchter A. S. et al., 1990, *ApJ*, 351, 642  
 Heemskerk M. H. M., van Paradijs J., 1989, *A&A*, 223, 154  
 Hut P., 1981, *A&A*, 99, 126  
 Hutchings J. B., Cowley A. P., Crampton D., van Paradijs J., White N. E., 1979, *ApJ*, 229, 1079  
 Iping R. C., Petterson J. A., 1990, *A&A*, 239, 221  
 Kallman T., Boroson B., Vrtilik S. D., 1998, *ApJ*, 502, 441  
 Kelley R. L., Rappaport S., Clark G. W., Petro L. D., 1983, *ApJ*, 268, 790  
 Kippenhahn R., Thomas H. C., 1979, *A&A*, 75, 281  
 Kondo Y., McCluskey G. E., 1976, in Eggleton P. P., Mitton S., Whelan J., eds, *IAU Symp. 73. Structure and Evolution of Close Binary Systems*. Reidel, Dordrecht, p. 277  
 Kopal Z., 1959, *Close Binary Systems*, Wiley, New York.  
 Kopal Z., 1988, *Ap&SS*, 144, 557  
 Kopal Z., Kitamura K., 1968, *Adv. Astron. Astrophys.*, 6, 125  
 Kruszewski A., 1966, *Adv. Astron. Astrophys.*, 4, 233  
 Lamers H. J. G. L. M., van den Heuvel E. P. J., Petterson J. A., 1976, *A&A*, 49, 327  
 Lecar M., Wheeler J. C., McKee C. F., 1976, *ApJ*, 205, 556  
 Levine A., Rappaport S., Deeter J. E., Boynton P. E., Nagase F., 1993, *ApJ*, 410, 328  
 Lubow S. H., Shu F. H., 1975, *ApJ*, 198, 383  
 Lucy L. B., 1967, *Z. Ap.*, 65, 89  
 Lyne A. G., Johnston S., Manchester R. N., Staveley-Smith L., D'Amico N., 1990, *Nat*, 347, 650  
 Nagase F., 1989, *PASJ*, 41, 1  
 Maeder A., Zahn J. P., 1998, *A&A*, 334, 1000  
 Maloney P. R., Begelman M. C., 1997, *ApJ*, 491, L43  
 Martin T. J., Davey S. C., 1995, *MNRAS*, 275, 31  
 Mauder H., 1975, *ApJ*, 195, L27  
 McCormick P., Frank J., 1993, *BAAS*, 182, 6409  
 Meurs E. J., van den Heuvel E. P. J., 1989, *A&A*, 226, 88  
 Meyer F., Meyer-Hofmeister E., 2001, in Podsiadlowski Ph., Rappaport S., King A. R., D'Antona F., Burderi L., eds, *ASP Conf. Ser. Vol. 229, Evolution of Binary and Multiple Star Systems*. Astron. Soc. Pac., San Francisco, p. 167  
 Orosz J. A., Bailyn C. D., 1997, *ApJ*, 477, 876  
 Osaki Y., 1982, *PASJ*, 34, 257  
 Peraiah A., 1982, *JApA*, 3, 485  
 Petterson J. A., 1977, *ApJ*, 216, 827  
 Phillips S. N., Shahbaz T., Podsiadlowski Ph., 1999, *MNRAS*, 304, 839  
 Plavec M. J., 1958, *Mem. Soc. R. Sci. Liège*, 20, 411  
 Podsiadlowski Ph., 1991, *Nat*, 350, 136  
 Podsiadlowski Ph., 1992, in Kondo, Y., Sisteró R. F., Polidan R. S., eds, *IAU Symp. No. 151, Evolutionary Processes in Interacting Binaries*. Kluwer, Dordrecht, p. 457  
 Podsiadlowski Ph., Rees M. J., 1994, in Holt S. S., Day C. S., eds, *AIP Conf., Evolution of X-Ray Binaries*. AIP, New York, p. 71  
 Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, *Numerical Recipes in Fortran: the Art of Scientific Computing*. Cambridge University Press, Cambridge, Chapter 9.4  
 Priedhorsky W. C., Holt S. S., 1987, *Space Sci. Rev.*, 45, 291  
 Pringle J. E., 1974, *MNRAS*, 168, 13  
 Pringle J. E., 1996, *MNRAS*, 281, 357  
 Rappaport S., Joss P. C., 1983, in Lewin W. H. G., van den Heuvel E. P. J., eds, *Accretion-driven Stellar X-Ray Sources*. Cambridge University Press, Cambridge, p. 1

- Rappaport S., Verbunt F., Joss P. C., 1983, *ApJ*, 275, 713  
 Ritter H., 1996, in Wijers M. J., Davies M. B., Tout C. A., eds, *Evolutionary Processes in Binary Stars*. Kluwer, Dordrecht. p. 223  
 Roberts W. J., 1974, *ApJ*, 187, 575  
 Ruderman M., Shaham J., Tavani M., 1989, *ApJ*, 336, 507  
 Ruffert M., 1994, *A&AS*, 106, 505  
 Ruffert M., 1997, *A&A*, 317, 793  
 Schandl S., Meyer-Hofmeister E., Meyer F., 1997, *A&A*, 318, 73  
 Schreier E., Levinson R., Gursky H., Kellogg E., Tananbaum H., Giacconi R., 1972, *ApJ*, 172, L79  
 Schuerman D. W., 1972, *Ap&SS*, 19, 351  
 Shahbaz T., Wood J. H., 1996, *MNRAS*, 282, 362  
 Shu F. H., Lubow S. H., Anderson L., 1979, *ApJ*, 229, 223  
 Taam R. E., Spruit H. C., 2001, *ApJ*, 561, 329  
 Terman J. L., Taam R. E., Savage C. O., 1998, *MNRAS*, 293, 113  
 Thorsett S. E., Arzoumanian Z., McKinnon M. M., Taylor J. H., 1993, *ApJ*, 405, L29  
 Tjemkes S. A., Zuiderwijk E. J., van Paradijs J., 1986, *A&A*, 154, 77  
 van den Heuvel E. P. J., De Loore C., 1973, *A&A*, 25, 387  
 van Paradijs J., 1998, in Buccheri R., van Paradijs J., Alpar M. A., eds, *The Many Faces of Neutron Stars*. Kluwer, Dordrecht, p. 279  
 Vanbeveren D., 1977, *A&A*, 54, 877  
 Von Zeipel H., 1924a, in Kienle H., ed., *Probleme der Astronomie. Festschrift für H. v. Seeliger*, Springer, Berlin, p. 144  
 Von Zeipel H., 1924b, *MNRAS*, 84, 665  
 Wade R. A., Horne K., 1998, *ApJ*, 324, 411  
 Wilson R. E., 1972, *ApJ*, 174, L27  
 White N. E., Holt S. S., 1982, *ApJ*, 257, 318  
 White N. E., Swank J. H., Holt S. S., 1983, *ApJ*, 270, 711  
 Zahn J. P., 1975, *A&A*, 41, 329  
 Zhou H. N., Leung K. C., 1988, *Ap&SS*, 141, 257

## APPENDIX A: THE OPTICAL LIGHT-CURVE CODE

### A1 Surface geometry

The secondary is modelled using a surface modified by irradiation pressure with the assumptions of synchronous rotation and a circular orbit. The surface is then divided into grid points spaced equally in  $\cos \theta$  and  $\phi$ . The accretion disc is modelled as described in Section 4.2.2.

### A2 Gravity darkening

The theorem of Von Zeipel (1924b) provides a relationship between the local potential gradient and the local emergent flux  $f$  in a tidally or rotationally distorted star in radiative equilibrium,  $f_{\text{rad}} = \sigma T^4 \propto |\nabla \Omega|$ . Consequently, the temperature at any point on the star is given by

$$\frac{T(x, y, z)}{T_{\text{pole}}} = \left[ \frac{|\nabla \Omega|(x, y, z)}{|\nabla \Omega|_{\text{pole}}} \right]^{\beta}, \quad (\text{A1})$$

where  $T_{\text{pole}}$  and  $|\nabla \Omega|_{\text{pole}}$  are the temperature and potential gradient at the pole of the star. The gravity-darkening exponent  $\beta$  has two values: 0.25 for stars with radiative atmospheres (Von Zeipel 1924b), and 0.08 for stars with fully convective envelopes (Lucy 1967).

### A3 Limb darkening

For each grid point on the secondary surface, the temperature obtained from gravitational darkening is used to compute the monochromatic intensity according to the Planck relation:

$$I(\lambda, T) \propto [\exp(hc/k\lambda T) - 1]^{-1}. \quad (\text{A2})$$

This intensity is modified using a standard linear limb-darkening law

$$I(\mu) = I(1)\{1 - u + u \cos \gamma\}, \quad (\text{A3})$$

where  $\gamma$  is the angle of foreshortening, and  $\mu = \cos \gamma$ .  $I(\mu)$  represents the distribution of the emergent intensity which varies with the angle of foreshortening. The limb-darkening coefficients  $u$  are taken from standard tables computed from model atmospheres; see, for example, Al Naimiy (1978). See Kopal & Kitamura (1968) and Avni (1978) for more discussions on these approximations.

### A4 X-ray heating

The X-ray flux that each surface element intercepts is given by

$$f_{\text{X-ray}}(x, y, z) = \frac{L_{\text{x}}}{4\pi d^2(x, y, z)} \cos \epsilon(x, y, z), \quad (\text{A4})$$

where  $L_{\text{x}}$  is the luminosity of the source,  $d$  is the distance from the element to the source, and  $\epsilon$  is the angle of foreshortening. Assuming total thermalization of the incident radiation, we can determine the modified temperature of each surface element visible from the source (i.e. for which  $\cos \epsilon > 0$ ) from

$$T_{\text{X-ray}}^4(x, y, z) = T_{\text{pole}}^4 \left( \frac{|\nabla \Omega|(x, y, z)}{|\nabla \Omega|_{\text{pole}}} \right)^{4\beta} + \frac{(1 - W)f_{\text{X-ray}}(x, y, z)}{\sigma}, \quad (\text{A5})$$

where  $W$  is the albedo, and  $\sigma$  is the Stefan constant.

We now have a grid of temperature values over the secondary surface from which we can calculate the total luminosity and flux. This calculation is repeated in steps of the phase angle. Optical light curves and flux-weighted radial velocity curves may then be calculated.

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