Essays in the Economics of Higher Education

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A thesis submitted for the degree of

Doctor of Philosophy in Economics

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Abstract

This DPhil thesis consists of three related but independent chapters discussing the question of admission and access to higher education. Chapter 1 explores the extent to which the underrepresentation of students from certain population groups at highly selective universities, can be explained by poor information and high non-monetary application costs, and how the universities' admission policies may affect outcomes. This chapter takes a positive approach and proposes a theoretical model to explore the implications of implementing alternative admission policies. Motivated by the results that arise from this exploration, Chapter 2 proceeds with a normative approach, proposing a general framework to study the optimal selection policy from a pool of applicants, taking into account that the pool of applicants is endogenous. This, it is argued, allows a characterisation of the optimal form of discrimination in university admissions. Chapter 3 studies the relationship between tuition fees and academic selectivity, by developing a different, although somewhat related model of monopolistic competition, where universities compete for students by simultaneously selecting prices and admission standards.

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Over the last couple of years, my work has also benefited greatly from valuable discussions with a number of different people. I should particularly thank the contributions from Debopam Bhattacharya and Clare Leaver, as well as the many interesting and insightful conversations with colleagues and friends —among whom I should especially mention Stefano Caria, Max Rosser, Claudia Herresthal, Bassel Tarbush and Francis Dennig.

I must also thank my parents for making this DPhil possible, by helping me in every possible way.

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Thesis Contents

Foreword

This DPhil thesis consists of three related but independent chapters discussing the question of admission and access to higher education. The three essays in this thesis, one per chapter, share a similar methodological approach and follow a natural order. Nonetheless, each essay has been written as a self-contained paper, so it is possible to read the different chapters in any order.

Chapter 1, based on my MPhil thesis, explores the extent to which the underrepresentation of students from certain population groups at highly selective universities (e.g. students from state schools at Oxford), can be explained by poor information and high non-monetary application costs, and how the universities' admission policies may affect outcomes. This first paper begins with a brief and simple empirical exploration of the nature of the much-remarked underrepresentation of state-school students in 'Oxbridge' admissions. In the light of the evidence, suggesting that the underrepresentation seems to stem more from failure to apply than low probability of acceptance conditional on application, the paper then proceeds to develop an equilibrium model based on the premise that applications are endogenous to the admission policies. In the model, a selective university admits students on the basis of test scores and public information, while students, on the other hand, use private information about their quality to inform their application decisions. The proposed model is, therefore, a framework of selection with two sided asymmetries in information.

In this first essay, the model is solved numerically and simulations are used to explore three particular admission policies. First, an admission policy by quotas, where students from each background are assigned a fixed proportion of the university's capacity; second, a 'blind' rule, where all students are held to the same standard and compete for all the available vacancies; and finally, a differentiated admission policy, where the university treats students differently depending on their background, in order to achieve a measure of parity in the expected quality of entrants from different groups. The main insight from the exercise is that, although all the studied policies may generate the same observable outcomes in terms of representation and admission likelihood of the different population groups −a fact which, in turn, implies that such statistics cannot be used as conclusive evidence of discrimination−, these alternative admission practices do imply very different expected qualities of entrants. Bearing this in mind, the results from Chapter 1 motivate a normative analysis of optimal admission practices under endogenous applications, as developed in the subsequent chapter.

Chapter 2, jointly authored with my supervisor, Ian Jewitt, builds on the motivation from the previous chapter to develop a normative framework for studying admissions as a mechanism design problem. The proposed model allows us to characterise the optimal form of admissions, by assuming that the objective of the university is to maximise the expected quality of entrants. Owing to the fact that efficiency in selection requires using observable and unobservable candidate attributes, the normative approach from this second paper provides a characterisation of the optimal form of discrimination in university admissions. To be more specific, we show that (under sufficient conditions) the optimal form of admissions is consistent with a simple cut-off rule where the university commits to select any applicant who scores above a minimum threshold in the admissions test; and we then use this result to show that the common principle of "choosing the best who apply" (a practice that we call the *benchmark rule*) is not generally optimal, because such principle would typically fail to take into account the feedback that selection has on the composition of the pool of applicants. As shall be clear from the analysis, this result implies that under optimal admissions, candidates belonging to groups that are particularly discouraged from applying should be held to a lower standard, in the sense that the worst entrant from such groups should have lower expected quality. That being the case, our results provide a justification for positive *economic* discrimination.

Chapters 1 and 2 of this thesis, as described above, tackle the question of access to higher education without taking prices into account. Given that prices are a common mechanism to allocate university places, Chapter 3 turns to study the relationship between tuition fees and academic selectivity. For this purpose, the third paper sets up a different, although somewhat related model of monopolistic competition, where profit-maximising universities compete for students by simultaneously selecting prices and admission standards.

The model of monopolistic competition developed in Chapter 3 explores price formation when education is differentiated by quality, and quality is defined by an academic admission requirement (i.e. a minimum ability threshold). In the main result of the paper I show that there are two different configurations that may arise in equilibrium, corresponding to alternative market structures: one in which high-quality colleges serve an elite portion of the market by catering to the high-income/high-ability students, and another in which the high-quality colleges are cheaper than the low-quality colleges which serve low-ability/high-income students. The analysis in this essay indicates that the distinction between these two equilibria is relevant to understand the welfare implications of market reforms, particularly those aimed at deregulating prices and shifting the funding structure of universities towards demand-side financing.

The structure of the thesis is straightforward. Each chapter begins with an abstract and a table of contents, and follows with motivation, related literature, main analysis and conclusions, and finally, its corresponding appendices. All sections, equations and figures are numbered within each chapter.

Chapter 1.

Understanding Access to Elite Universities:

Information, Application Costs and Admission Policies

Chapter abstract

Admissions to leading research universities in England are dominated by private schools, despite the fact that only a small fraction of all students in the country attend such independent institutions. In this paper I explore the extent to which this notorious underrepresentation of maintained schools can be explained by poor information and high non-monetary application costs, and how the universities' admission policies affect outcomes. For this purpose I develop an equilibrium model based on the premise that applications are endogenous to the admissions policies. The model is solved numerically and simulations are used to explore different parametric specifications under three particular admission policies. The analysis points out some interesting dynamics that are involved in the equilibrium, such as the non-monotonic relationship between expected entrant quality and application costs. Using the model to replicate some characteristic features of the admissions statistics from Oxford and Cambridge, it is shown that it is not possible to identify the admission policy from such statistics, so it is not possible to use them as conclusive evidence of discrimination.

Chapter contents

1 Introduction

Top universities in England are under constant scrutiny because of the low representation of certain groups (e.g. students from maintained schools). In this paper I explore the extent to which this underrepresentation can be explained by poor information and high non-monetary application costs, and how the universities' admission policies affect outcomes. Given that students self-select into applications taking into account application costs, as well as university admission procedures, I propose a model of endogenous applications. To be specific, I develop an equilibrium model where students decide whether to submit costly applications to a single university, while the university selects entrants according to an admission test, in the expectation of filling its capacity.

In the proposed framework students are characterised by their school background and their underlying unobservable talent (quality henceforth), so application choices are driven by school-specific application costs and uncertainty. On the university's side, the analysis concentrates on admission decisions, as fully determined by one of three possible admission policies. First, the analysis considers an admission policy by quotas, where students from each background are assigned a fixed proportion of the university's capacity. Second, a 'blind' rule is studied, where all students are treated equally regardless of their background and compete for all the available vacancies. Finally, a differentiated admission policy is explored, where the university treats students differently depending on their background, in order to equate the expected quality of entrants from each group. The approach of the paper is positive, in the sense that it seeks to explain the implications of using different admission rules, rather than to derive *the* optimal admissions policy from a normative objective. The characterisation of optimal admissions with endogenous applications is studied in Chapter 2 of this thesis.[1]

After setting up the model, the analysis begins with an exploration of a benchmark case, where there are no difference across schools, in order to study the main forces underlying the endogeneity of application choices. After this, the analysis

[1] See *DPhil Chapter 2. Selection by Selection: The Case of University Admissions*

proceeds to consider the case of heterogeneity in costs. The model is solved numerically and simulations are used to explore different parametric specifications under the three aforementioned admission policies. Using the model to replicate some characteristic features of the admissions statistics from Oxford and Cambridge, it is shown that it is not possible to identify the admission policy from such statistics.

The rest of the paper is structured as follows. Section 2 presents an illustration of the problem of underrepresentation in admissions using descriptive statistics from administrative data, and then discusses related literature. Section 3 introduces the model, providing a brief discussion of the assumptions and main considerations. Section 4 explores the benchmark case where schools are homogeneous. Section 5 introduces heterogeneity in the application cost, and explores the characteristics of the equilibrium outcomes under each admission policy. Section 6 calibrates the model to reproduce admissions statistics and describes the policy implications of the findings. Section 7 concludes.

2 Background

2.1 A glance at the problem of underrepresentation of maintained schools at *Oxbridge*

In order to illustrate the problem of access to elite universities in England, consider the following descriptive statistic using administrative data from the Universities and Colleges Admissions Service (UCAS). Table 1 presents the school background composition of applications and admissions to study Law at Oxford and Cambridge (*Oxbridge* henceforth), and compares it to the school background composition of the overall population of UK-domiciled students who submitted an application to study Law at any higher education institution in England.[2] As it can be seen, students from private schools constitute only 5.77% of the population who submits applica-

[2]To be specific, the dataset is a 25% random sample from this population pooled over the period 2007-2011. For each applicant in the random sample, in addition to individual characteristics such as school type, gender and qualifications, there is a flag variable indicating whether they applied specifically to the University of Oxford or Cambridge, and if they were offered a place in at least one of the two. See Appendix A.3 for more details.

tions to study Law at any university, yet they account for nearly a quarter of the applications to study Law at Oxbridge. At the admissions stage, such independent schools take almost a third of all available places.

School	**Population**	**Applications**	**Entrants**
Private	5.77%	24.26%	33.04%
Maintained	94.23%	75.74%	66.96%

Table 1: Application and admission shares to Law at Oxbridge by school type

The admission statistics above show that admissions are dominated by candidates from the private sector. This, evidently, has to do at least partly with underlying differences between candidates from different school types, in particular their academic qualifications at the point of application. Given the academic selectivity of Oxbridge, it is natural to investigate the differences in admissions by academic achievement. Using the same dataset, Figure 1 presents the likelihood of admission conditional on application −with the corresponding 95% confidence intervals−, distinguishing candidates by school background and qualifications.

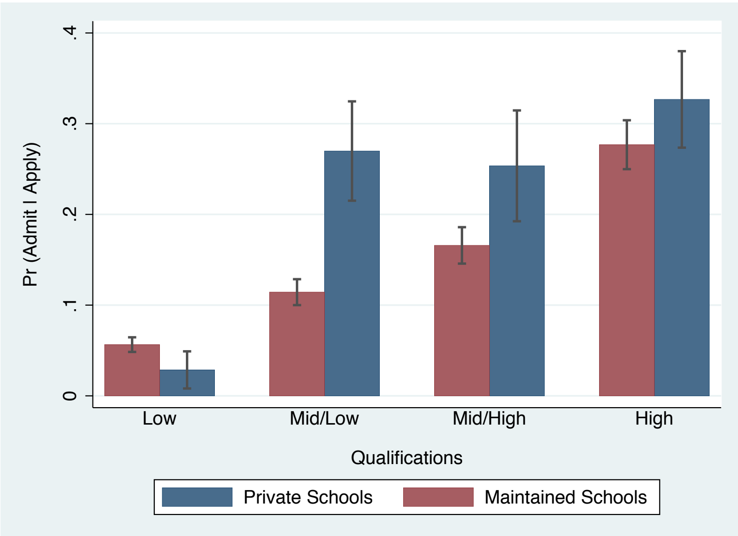

Figure 1: Admission likelihood to study Law at Oxbridge by school type and qualifications

As one would expect, there is a clear gradient in admissions due to qualifications, so students with higher academic achievement are more likely to be admitted regardless of their school background. Interestingly, despite the fact that such gradient is specific to the school type (improvements in qualifications at the higher en of the distribution seem to be associated with changes in admissions probabilities only for maintained schools), the probabilities of admission are statistically similar for candidates from both types of schools across the top qualification levels.[3]

These results suggest that an important part of the issue of underrepresentation of maintained schools stems from application behaviour. Figure 2 presents the likelihood of application by school background and qualifications, using the same categories above. As it can be appreciated, although students with higher academic achievement tend to be more likely to apply to Oxbridge regardless of school background, for most levels of achievement candidates from private schools are more likely to apply than their counterpart from maintained schools −the only exception being the top qualification level, for which the likelihood is higher, but statistically similar at the presented 95% confidence.

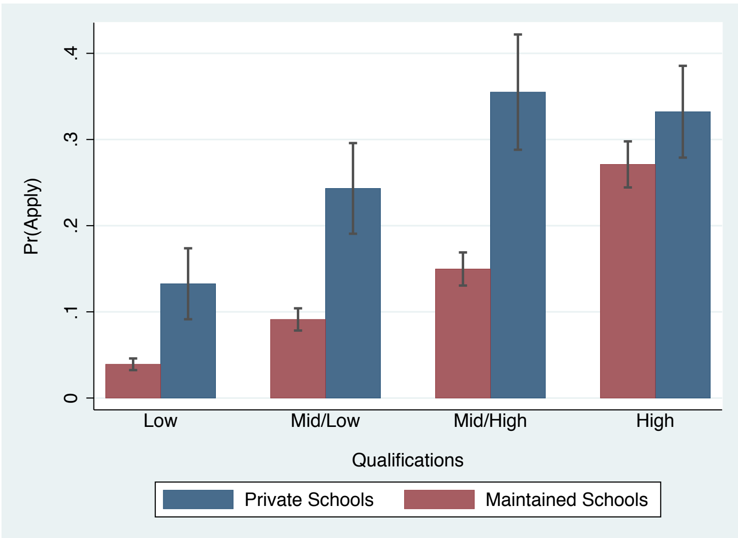

Figure 2: Application likelihood to study Law at Oxbridge by school type and qualifications

The conclusions above can be enhanced by contemplating further candidate characteristics in a simple regression model. The results (available in Appendix A.3) show, in line with what has been discussed already, that attending an independent

[3]The four levels of qualifications used for the analysis, here labeled from low to high for convenience, correspond to the four highest bands in "tariff points", as defined by UCAS. Tariff points are awarded according to results in academic exams such as the International Baccalaureate or the A-levels. It should be noted that below the fourth band there are no admissions in the dataset, in accordance with the minimum A-level requirements stated in conditional offers issued by Oxbridge. For more information see the Appendix A.3.

school is strongly correlated with the probability of applying to Oxbridge, even after controlling for other variables such as qualifications, socioeconomic background, gender and ethnicity.[4] Yet, at the same time, the probability of admission conditional on applying is not statistically related to school type after all characteristics are taken into account. This corroborates claims asserting that there is underrepresentation in applications, in spite of the fact that applicants from state schools are likely to be as successful as similar students from independent schools if they applied.[5]

2.2 Related literature

A number of recent empirical papers, exploring application behaviour in the context of access to selective universities, confirm the importance of understanding uncertainty and application costs as fundamental frictions in the admissions process (Pallais 2015; Carrell and Sacerdote 2013; Hoxby and Avery 2012; Avery and Kane 2004). In particular, Hoxby and Avery (2012) highlight the importance of understanding how these frictions may interact with the background of students. Using a rich array of data from the U.S., Hoxby and Avery (2012) show that the vast majority of very high-achieving students who are low-income do not apply to any selective college or university −and this is despite the fact that selective institutions would often cost them less, owing to generous financial aid, than the resource-poor two-year and non-selective four-year institutions to which they actually apply. The authors document that, because of this, admissions officers underestimate the number of low-income, high-achieving students that in reality exist. Moreover, Hoxby and Avery (2012) also provide evidence that (i) high-achieving, low-income students who do apply to selective institutions are admitted and graduate at high rates; and (ii) the low-income students' application behavior differs greatly from that of their

[4] This is consistent with other sources. In a report using figures from the Higher Education Funding Council, the Sutton Trust (2004) points out that application rates to leading universities are much higher for independent schools, than non-selective state schools with similar A-level results.

[5] As an example of such claims, the BBC reported in 2008 that the Director of Admissions for the Cambridge Colleges declared: "We are disappointed by the figures on admissions. Our research shows that the main cause is a decline in applications from state sector schools and colleges." (http://news.bbc.co.uk/1/hi/ education/7437696.stm)

high-income counterparts who have similar achievement.[6]

In the economics theoretical literature there is little research exploring admissions as part of an equilibrium process with frictions due to application costs and uncertainty. One exception is Chade, Lewis and Smith (2014), CLS hereafter, who depart from the traditional approach of the centralised college matching literature (Galey and Shapley, 1962). In their paper, there is a population of students of heterogeneous ability and two colleges of different quality. The colleges only observe a noisy signal of each applicant's calibre and are interested in filling their capacity with the best available candidates, while the students, on the other side of the market, perfectly observe their own ability and make costly application choices based on common preferences over the two colleges. Since CLS propose a framework where there are several colleges but ability is the only source of heterogeneity across the student population, the authors use the model to explore whether sorting of students and colleges occurs in equilibrium (i.e. whether students of higher ability use in equilibrium more 'aggressive' application portfolios). My approach departs from CLS by considering only one college, but allowing student heterogeneity in observable and unobservable characteristics −specifically, in addition to ability, differences by background in application costs and information. I thus use the model to explore the background composition of applications *within* a college, rather than across colleges. The focus on further student heterogeneity allows me to explore the implications of using background-specific admission policies.

The model that I propose also differs from CLS by allowing a different structure of uncertainty, where ability is fully unobservable and choices are made on the basis of student and college private information.[7] A number of papers on the economics

[6] In a related paper, Pallais (2015) uses confidential micro data from standardised tests in the U.S. to show that very small changes in application costs can account for large differences in application behaviour. She finds that when the ACT increased from three to four the number of free score reports that ACT-takers could send (maintaining a $6 marginal cost for each additional report), the fraction of test-takers sending four reports rose substantially while the fraction sending three fell by an offsetting amount.

[7] Although CLS assume that ability is perfectly observable by students, the authors acknowledge that a general framework placing uncertainty at the heart of the analysis, should consider the case in which ability is unobservable (and both the students and colleges have private information). CLS provide at the appendix a discussion of why, given their assumption that ability is the only source of student heterogeneity, the main results of their analysis are unaffected by the assumption that students perfectly observe ability. Since I introduce further heterogeneity, such argument does

of higher education have highlighted the importance of considering two-sided asymmetric information as part of the admissions problem. Gary-Bobo and Trannoy (2008), for example, consider a population of students endowed with unobservable ability, who must decide whether to buy education from a single college with market power (where the college sets tuition fees and admission standards to maximise an objective that depends on the number and quality of entrants). They show that, in the presence of student and college private signals, a mix of pricing and pre-entry selection induces the optimal amount of student screening (and in contrast, if all signals about ability are public information, screening through prices may be sufficient to induce student screening optimally).[8] The relevance of private information in this context is also confirmed by Fu (2014), who proposes an empirically-driven model to structurally estimate tuition fees, admission policies and enrollment, under the assumption that student ability is unobservable but students and colleges receive private (as well as pubic) signals.

3 The Model

3.1 Definitions and set-up

Consider a continuum of students deciding whether to apply to a single university. There are two relevant student characteristics: quality and school type. Unobservable quality θ is a random variable, and school type is a binary parameter indicating background. Type $i = m$ denotes students attending a maintained school, while type $i = p$ denotes those attending a private school.[9] The proportion of type-p students is noted by α.

The unobservable quality of students, conditional on school type, is denoted $\theta_i \in \mathbb{R}$. While school type is public information, students only receive a private

not apply here.

[8]On a similar vein Epple et al. (2006) model college admissions as a bargaining game between the college and the potential students under asymmetric information. They provide evidence of what they call 'student profiling' practices, which arises when applicant attributes are given weight in funding decisions made by the college, because they are correlated with unobservable student characteristics that the college values.

[9]The term *private* schools is used throughout as the sector complementing the *maintained* schools. In England these schools are classified as independent schools.

and noisy signal S_i of their own quality, which they must use to decide whether to apply to the university. If a student chooses to apply, she pays a cost $\kappa_i \in \mathbb{R}_+$. Once applications are submitted, the university receives a noisy signal T_i about the quality of every applicant, and uses it in accordance with an admission policy to select a group of entrants, with the aim of filling τ vacancies. Entrants receive a reward $\eta_i \in \mathbb{R}_+$, and no payments are made to rejected applicants.

The university's decision to admit an applicant is embodied in the admission policy that it uses. Throughout this paper admissions are assumed to take the form a cut-off rule, so that the university accepts those students with a realised signal $T_i = t_i$ greater than some critical value t_i^*:

$$admit_i = \begin{cases} 1 & \text{if} \quad t_i \geq t_i^* \\ 0 & otherwise \end{cases}$$

The actual value of t_i^* is chosen according to the respective admission policy. While there are many admission protocols that may be employed by the university to choose the entry cut-offs, I will focus here on three specific cases of interest.

(I) **Quotas.** The first admission policy that will be considered, treats applications from each school type separately, and sets two independent admission standards t_p^* and t_m^*. Under this policy, a student from school i is admitted if his or her corresponding signal $T_i = t_i$ is weakly greater than the admission standard t_i^*. Since the admissions are assessed independently, the university chooses t_p^* and t_m^* trying to achieve admission quotas τ_p and τ_m respectively.

(II) **School-blind.** The second admission policy that will be considered gives all students the same treatment regardless of their background by setting a single admission standard t^*, and making all students compete for all vacancies. In this case, an applicant is admitted if his or her corresponding signal $T_i = t_i$ is weakly greater than the unique admission standard t^*. The admission standard is hence chosen in the expectation of achieving τ admissions regardless of the actual composition.

(III) **Quality-matching.** The last admission policy that will be analysed allows the university to use a different threshold for each group, while making students compete for the whole set of vacancies. In this case, the university sets two admission standards t_p^* and t_m^* in order to guarantee that the quality of the average entrant from both school types is the same, and that the expected number of entrants is τ.[10] Once again, an applicant from school i is admitted if he or she provides a signal $T_i = t_i$ weakly greater than the admission standard t_i^*.

Considering the above, a student from school i chooses to apply to the university if he or she observes a signal $S_i = s_i$ which induces a sufficiently high probability of being accepted, given the admission policy. Accordingly, applications are here assumed to follow a cut-off rule, where students choose to apply to the university if they receive a signal weakly greater than some critical value s_i^*:

$$apply_i = \begin{cases} 1 & \text{if} \quad s_i \geq s_i^* \\ 0 & otherwise \end{cases}$$

3.2 Equilibrium

(I) Quotas. An equilibrium under the Quotas policy is a pair $\{\{t_p^*, t_m^*\}, \{s_p^*, s_m^*\}\}$ such that:

(i) Given that the university is choosing $\{t_p^*, t_m^*\}$, then

$$\begin{aligned} \Pr[T_i &\geq t_i^* | S_i = s] \geq \frac{\kappa_i}{\eta_i} \qquad \forall s \geq s_i^*; \quad i \in \{m, p\} \\ \Pr[T_i &\geq t_i^* | S_i = s] < \frac{\kappa_i}{\eta_i} \qquad \forall s < s_i^*; \quad i \in \{m, p\} \end{aligned}$$

(ii) Given that the students are choosing $\{s_p^*, s_m^*\}$ then

$$\begin{aligned} \tau_p &= \alpha \Pr[T_p \geq t_p^*, S_p \geq s_p^*] \\ \tau_m &= (1 - \alpha) \Pr[T_m \geq t_m^*, S_m \geq s_m^*] \end{aligned}$$

[10] There is another policy, also of interest, in which the university sets two different admission thresholds in order to ensure that the quality of the *marginal* entrant from different groups is equated. Such policy is studied in detail in Chapter 2 of this thesis.

(II) School-blind. An equilibrium under the school-blind policy is a pair $\{t^*, \{s_p^*, s_m^*\}\}$ such that:

(i) Given that the university is choosing $\{t^*\}$ then:

$$\begin{aligned} \Pr[T_i &\geq t^*|S_i = s] \geq \frac{\kappa_i}{\eta_i} \quad \forall s \geq s_i^*; \quad i \in \{m,p\} \\ \Pr[T_i &\geq t^*|S_i = s] < \frac{\kappa_i}{\eta_i} \quad \forall s < s_i^*; \quad i \in \{m,p\} \end{aligned}$$

(ii) Given that the students are choosing $\{s_p^*, s_m^*\}$ then:

$$\tau = \alpha \Pr[T_p \geq t^*, S_p \geq s_p^*] + (1-\alpha) \Pr[T_m \geq t^*, S_m \geq s_m^*]$$

(III) Quality-matching. Under the quality-matching admission policy the equilibrium is a pair $\{\{t_p^*, t_m^*\}, \{s_p^*, s_m^*\}\}$ such that:

(i) Given that the university is choosing $\{t_p^*, t_m^*\}$ then:

$$\begin{aligned} \Pr[T_i &\geq t_i^*|S_i = s] \geq \frac{\kappa_i}{\eta_i} \quad \forall s \geq s_i^*; \quad i \in \{m,p\} \\ \Pr[T_i &\geq t_i^*|S_i = s] < \frac{\kappa_i}{\eta_i} \quad \forall s < s_i^*; \quad i \in \{m,p\} \end{aligned}$$

(ii) Given that the students are choosing $\{s_p^*, s_m^*\}$ then:

$$\begin{aligned} E[\theta_p|T_p \geq t_p^*, S_p \geq s_p^*] &= E[\theta_m|T_m \geq t_m^*, S_m \geq s_m^*] \\ \tau = \alpha \Pr[T_p \geq t_p^*, S_p \geq s_p^*] &+ (1-\alpha) \Pr[T_m \geq t_m^*, S_m \geq s_m^*] \end{aligned}$$

3.3 Simplifying assumptions

Assumption 1 Unobservable student quality θ_i is drawn from a standard normal distribution, for both school types.

Assumption 2a The students' signal about unobservable quality is given by:

$$S_i = \theta_i + u_i \quad \text{where} \quad u_i \sim N(0, \sigma_i^2) \quad \text{for} \quad i \in \{m,p\}$$

Assumption 2b The university's signal about unobservable quality is given by:

$$T_i = \theta_i + e \quad \text{where} \quad e \sim N(0,1) \quad \text{for} \quad i \in \{m, p\}$$

Assumption 3 The application cost relative to the benefit of being admitted is given by:

$$c_i \equiv \frac{\kappa_i}{\eta_i} \in (0,1)$$

3.4 Discussion of the model

Unobservable quality. Unobservable student quality, in the sense of the model, can be interpreted as an outcome variable which is unknown at the point of applications and admissions, but which can be (imperfectly) predicted with observable information; for example, university results at final degree exams. It is assumed that quality is equally distributed across the entire population (Simplifying Assumption 1), in order to explore the issue of access to elite universities *in spite* of the correlation of quality with background.

The cost of applications and the admission reward. The model is constructed with deliberately abstract payoffs in mind. The cost parameter captures a variety of aspects such as deadlines, admissions tests and interviews.[11] The reward of admission, on the other hand, captures all the positive dimensions that may be associated with entering elite universities, e.g. higher productivity, signalling effects, social networks, etc. The analysis will focus on the situation in which the costs, relative to the benefits, differ by background. As mentioned before, recent empirical evidence bolsters the relevance of this approach (see, for instance, Carrell and Sacerdote 2013; Sutton Trust 2004).

[11] A more complex (and perhaps realistic) representation of the admissions problem would explicitly consider the fact that students may apply to many universities, but the number of applications is capped, so there is an implicit application cost stemming from this. Since schools may differ in their objectives (e.g., safe vs. top placement of students), school differences in application costs may reflect this, and hence it should be taken into account in a welfare evaluation of policies. Although this issue is outside the scope of the paper, it may provide an interesting avenue of research.

The informational assumptions. In order to explore the claim that underrepresentation in admissions stems from the fact that students from maintained schools have worse information about their quality, the analysis will concentrate on the case in which the noise in the students' signal depends on background (Simplifying Assumption 2a). This will allow an exploration of another channel, different to application costs, through which students from maintained schools are affected in university admissions. In this sense, it provides a link between school background and notions of aspirations (see Avery and Kane 2004 for evidence supporting this relationship in the context of university applications). To reduce the dimensionality of the problem, this paper assumes that the university's signal is equally informative for both school types (Simplifying Assumption 2b).

The admission policies. The three postulated admission policies address, respectively, three different questions:

- Are universities deliberately choosing a fixed share of entrants from private schools? (*Quotas* admission policy)
- Are universities treating all students equally regardless of their background, despite the fact that they may be different? (*School-blind* admission policy)
- Are universities treating students differently depending on their background in order to guarantee a measure of parity in outcomes across sectors? (*Quality-matching* admission policy).

An assessment of the desirability of these policies requires the definition of a normative objective function. I leave such normative analysis for Chapter 2, where I also provide a discussion of what these policies may imply in the context of common notions of discrimination in the economics literature.

4 Homogeneous schools: a benchmark

To understand the main forces interacting in the model, consider initially a simple case in which all students face the same application cost and all student signals are equally informative. In this context, the only relevant admission policy is "blind" and all subscripts can be dropped.

It should be mentioned that due to the endogeneity of application choices, the solution to this simpler admission problem without school heterogeneity is not trivial. This is because, since applications are costly, there is screening at the application stage where candidates use their private information to predict the likelihood of being successful if they apply, given the admission requirement. Hence, selection through admissions begets selection through applications. This section explores the comparative statics of the equilibrium in the simplest case, in order to understand the key underlying dynamics of the model.

4.1 Solving the model

Applications. The students' choice of application, in equilibrium, requires expected utility to be nonnegative after updating beliefs with all available information. The application cut-off can thus be found as the signal level which makes a student indifferent about applying; that is, finding the realisation $S = s$ which equates the updated probability of success with the application cost, namely:

$$\Pr[T \geq t | S = s] = c \tag{1}$$

Given the distributional assumptions it follows that T conditional on $S = s$ is normally distributed with mean and variance:

$$\mu_{T|S} = \frac{s}{1+\sigma^2} \quad \text{and} \quad \sigma^2_{T|S} = \left(2 - \frac{1}{1+\sigma^2}\right)$$

Noting the standard normal cumulative distribution function by $\Phi(\cdot)$, for the marginal applicant it follows that:

$$\Pr[T \geq t | S = s] = 1 - \Phi\left(\frac{t - \frac{1}{1+\sigma^2}s}{\sqrt{2 - \frac{1}{1+\sigma^2}}}\right) = c \tag{2}$$

Using this, the students' cut-off choice can be expressed as a function $s(t;c,\sigma)$ defined by the level of the signal S which solves equation (2).

Given that (2) is an implicit equation associating the application cut-off to the university's admission cut-off, it is possible to use implicit differentiation in order to explore the comparative statics of $s(t;c,\sigma)$. In particular, note that the distributional assumptions imply:

$$\frac{ds(t;c,\sigma)}{dt} = -\frac{\frac{\partial}{\partial t}\Phi\left(\frac{t-\frac{1}{1+\sigma^2}s}{\sqrt{2-\frac{1}{1+\sigma^2}}}\right)}{\frac{\partial}{\partial s}\Phi\left(\frac{t-\frac{1}{1+\sigma^2}s}{\sqrt{2-\frac{1}{1+\sigma^2}}}\right)} > 0 \quad (3)$$

$$\frac{ds(t;c,\sigma)}{dc} = -\frac{1}{\frac{\partial}{\partial s}\Phi\left(\frac{t-\frac{1}{1+\sigma^2}s}{\sqrt{2-\frac{1}{1+\sigma^2}}}\right)} > 0 \quad (4)$$

so it follows, as one would expect, that the application threshold is increasing in the application cost and the entry requirement.

Admissions. Regarding admission thresholds, the university requires the expected number of admitted students relative to the entire population to be equal to τ. In the benchmark case this requirement translates into:

$$\tau = \Pr[T \geq t, S \geq s]$$

Denoting the joint cumulative distribution of T, S by $F_{T,S}(\cdot)$ (with corresponding joint density $f_{T,S}(\cdot)$), the capacity constraint can be rewritten as:

$$\begin{aligned}\tau &= \int_s^\infty \int_t^\infty f_{T,S}(u,v)\, du\, dv \\ &= 1 - \Phi\left(\frac{t}{\sqrt{2}}\right) - \Phi\left(\frac{s}{\sqrt{1+\sigma^2}}\right) + F_{T,S}(t,s) \quad (5)\end{aligned}$$

From this, the university's choice of admission cut-off can be expressed as a function $t(s;\sigma,\tau)$, defined as the value of t solving (5). As before, even though this function does not have a closed form, implicit differentiation yields:

$$\begin{aligned}
\frac{dt(s;\sigma,\tau)}{ds} &= -\frac{\frac{\partial}{\partial s}\int_s^\infty \int_t^\infty f_{T,S}(u,v)\, du\, dv}{\frac{\partial}{\partial t}\int_s^\infty \int_t^\infty f_{T,S}(u,v)\, du\, dv} \\
&= -\frac{\int_t^\infty f_{T,S}(u,s)\, du}{\int_s^\infty f_{T,S}(t,v)\, dv} < 0 \qquad (6) \\
\frac{dt(s;\sigma,\tau)}{d\tau} &= -\frac{-1}{\frac{\partial}{\partial t}\int_s^\infty \int_t^\infty f_{T,S}(u,v)\, du\, dv} \\
&= -\frac{1}{\int_s^\infty f_{T,S}(t,v)\, dv} < 0 \qquad (7)
\end{aligned}$$

Thus, the university's entry threshold is decreasing in s and τ. This is intuitive; considering that there is a capacity constraint, the more screening takes place at the application stage, the less screening should take place at the admission stage. By implication, if the capacity constraint is more stringent, everything else equal, then selection in admissions should be stricter.

Equilibrium. In this case, following the reasoning above, the equilibrium $\{t^*, s(t^*)\}$ corresponds to the value of t^* solving:

$$\tau = 1 - \Phi\left(\frac{t^*}{\sqrt{2}}\right) - \Phi\left(\frac{s(t^*;c,\sigma)}{\sqrt{1+\sigma^2}}\right) + F_{T,S}(t^*, s(t^*;c,\sigma)) \qquad (8)$$

The results from (3) and (6) imply that the equilibrium is unique, so it can be easily computed numerically. To see how this computation may be implemented, notice that the student application condition from (2) can be written as:

$$\Phi^{-1}(1-c) = \frac{t - s\beta_{TS}}{\sigma_{T|s}}$$

where $\beta_{TS} \equiv \frac{1}{1+\sigma^2}$ is the regression coefficient of T on S, and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal CDF. Since this inverse function can be used directly as an input in many computational software programs, such as *Mathematica*, the application cut-off can implemented with the simple function:

$$s(t;c,\sigma) = \frac{t - \Phi^{-1}(1-c)\sigma_{T|s}}{\beta_{TS}} \qquad (9)$$

Thus, the unique equilibrium can be found through a standard iteration routine that given a starting point, searches numerically for the root of equation (8) with $s(t; c, \sigma)$ defined by (9). All of the simulations throughout the remainder of this section were produced using this procedure.

4.2 Characteristics of the equilibrium at the benchmark

The application cost c determines the students' direct incentives to apply to the university. The application cut-off is positively related to c (recall that $\frac{\partial s(t;c,\sigma)}{\partial c} > 0$) and thus fewer but better students, at least on average, are willing to submit applications when it is more costly. It follows that the application cost serves the purpose of screening students at the application stage. Note that this screening role of c at the application stage is crucial in determining how much information is extracted from the students' signal S. If applications are too costly, then application incentives are low, and the university has to set a low admission standard t to fill all the vacancies (recall that $\frac{\partial s(t;c,\sigma)}{\partial t} > 0$). This will then mean that most of the screening will be achieved through the information contained in the signal S. Conversely, if applications are inexpensive then application incentives are high, and most of the screening takes place at the admissions stage with the information contained in the signal T. This feature has interesting implications for the expected quality of entrants, affecting it in a non-monotonic fashion. The expected quality of entrants is a measure of the degree of information transmitted in the equilibrium, and is consequently maximised at some intermediate point where information is extracted from both signals. Intuitively, this is simply saying that the expected quality of entrants is higher when a large number of students are screened at both the application *and* admission stages.

Assuming that all the signals are equally informative (i.e. setting $\sigma = 1$) and fixing the university capacity at $\tau = 0.01$, it is possible to solve numerically for $\{t^*, s^*\}$ following the procedure already discussed, and then compute $E[\theta | T \geq t^*, S \geq s^*]$ for any value of c. Figure 3 presents the results from this exercise. As it can be verified from the figure, the expected quality of entrants is non-monotonic in the application cost with a maximum at $c = 0.168$. This means that, for the given parametric specification, any application cost higher or lower than 0.168 entails a suboptimal use of the information contained in each of the signals.

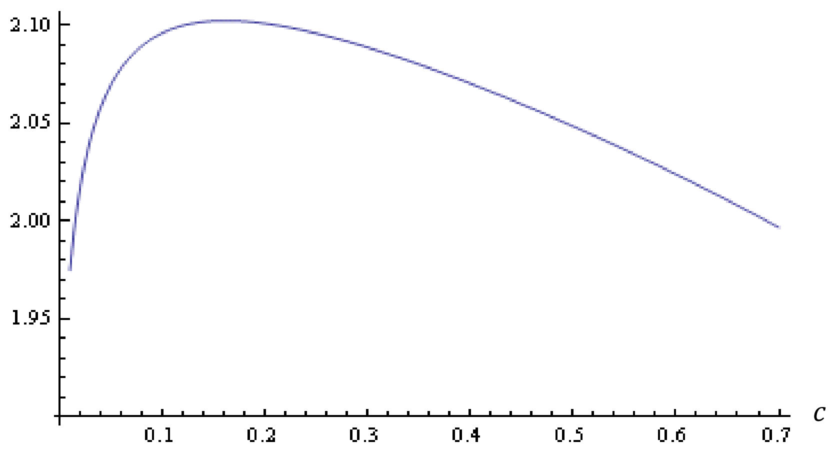

Figure 3: Equilibrium expected quality of entrants ($E[\theta|T \geq t^*, S \geq s^*]$) as a function of c, for $\sigma = 1$ and $\tau = 0.01$

Result 1 (Non-monotonicity of quality) *The expected quality of entrants is non-monotonic quasiconvex in the application cost.*

Another important aspect to discuss is the role of σ. By definition, this parameter captures the informativeness of the students' signals in the model, so the comparative statics are more complicated because σ affects simultaneously $s(t; c, \sigma)$ and $t(s; \sigma, \tau)$ in a non-trivial way.[12] However, it is intuitive that better information about quality improves the expected quality of entrants in equilibrium. When the signal S is highly informative, the distributions of quality among applicants and entrants are highly skewed and have thinner tails as more students are concentrated around the admission cut-off. Furthermore, these distributions become similar as information improves, because applicants become progressively more successful.

The above observations can be appreciated graphically by fixing the parameters and solving numerically for $\{t^*, s^*\}$, and then using these values to plot the probability density function of quality for the pool of applicants (i.e. $f(\theta|S \geq s^*)$) and the pool of entrants (i.e. $f(\theta|T \geq t^*, S \geq s^*)$). Figure 4 compares these distributions for $\sigma = 0.1$, fixing $\tau = 0.01$ and $c = 0.3$. As it can be seen by comparison with

[12] It can be checked, for instance, that $\frac{ds(t;c,\sigma)}{d\sigma} > 0$ only when t is sufficiently large.

the tail of the distribution of quality in the entire population (i.e. the probability density function of the standard normal), the distributions of quality for applicants and entrants are very close and skewed (means marked with dashed lines).

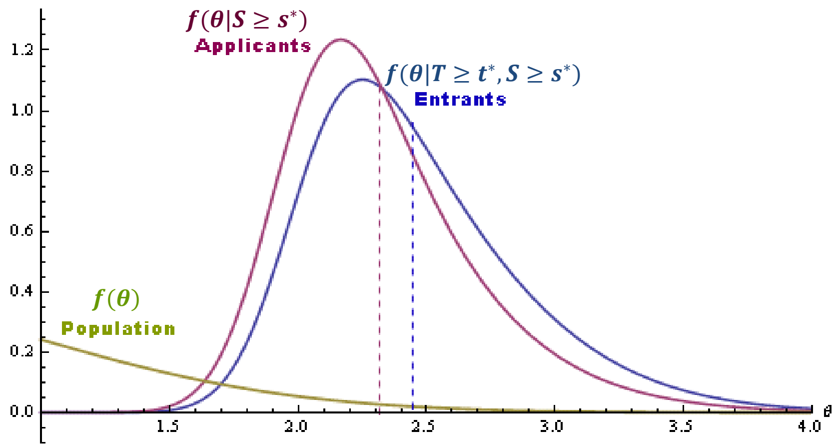

Figure 4: Equilibrium distributions of quality, for $\sigma = 0.1$, $\tau = 0.01$ and $c = 0.3$

Observation 1 *The distributions of quality among applicants and entrants become more similar as the student signal becomes more informative.*

5 Heterogeneity in costs

5.1 Preliminaries

This section explores the implications of school heterogeneity by assuming that maintained schools have a higher application cost. The main objective is to compare outcomes across policies and evaluate the sensitivity of the equilibrium to changes in the application cost. Analogously to the benchmark case already discussed, in this section the equilibrium calculations are obtained by fixing the parameters and finding numerically the solution to the application and admission conditions of each policy, as defined in Section 3. Appendix A.2 presents the equations and routines used for the implementation of the solution.

It should also be mentioned that for the Quotas policy, the analysis in this section assumes that the entry requirements are such that there is no underrepresentation, in the sense that $\tau_p = \alpha\tau$ and $\tau_m = (1-\alpha)\tau$. This assumption is, evidently, in-compatible with underrepresentation of maintained schools; so it will be dropped in Section 6 for the calibration exercise.

5.2 Outcome comparison across policies: an example

The debate on admissions is complex partly because it is not possible for an external observer to compare alternative policies *ceteris paribus*. In this sense, it is insightful to use an equilibrium example to compare the following outcomes across different admission policies:

- Aggregate number of applications: $\alpha Pr[S_p \geq s_p^*] + (1-\alpha)Pr[S_m \geq s_m^*]$
- Share of private-school applications: $\frac{\alpha Pr[S_p \geq s_p^*]}{\alpha Pr[S_p \geq s_p^*]+(1-\alpha)Pr[S_m \geq s_m^*]}$
- Share of private-school admissions: $\frac{\alpha}{\tau} Pr[T_p \geq t_p^*, S_p \geq s_p^*]$
- Admission likelihood conditional on application (sometimes referred to as success rate): $Pr[T_i \geq t_i^* | S_i \geq s_i^*] \quad \text{for} \quad i = \{m, p\}$
- Distributions of quality: $f(\theta_i | S_i \geq s_i^*)$ and $f(\theta_i | T_i \geq t_i^*, S_i \geq s_i^*)$ for $i = \{m, p\}$

Figure 5 presents a comparison of these outcomes, under the parametric assumptions summarised below.

Parameters	**Assumptions**
Application cost	$c_p = 0.3$; $c_m = 0.6$
Informativeness of signals	$\sigma_p = \sigma_m = 1$
University capacity	$\tau = 0.01$
Fraction of type-p candidates	$\alpha = 0.07$
Share of admissions for Quotas policy	$\tau_p = \alpha\tau$; $\tau_m = (1-\alpha)\tau$

Table 2: Parameter values for outcome comparison

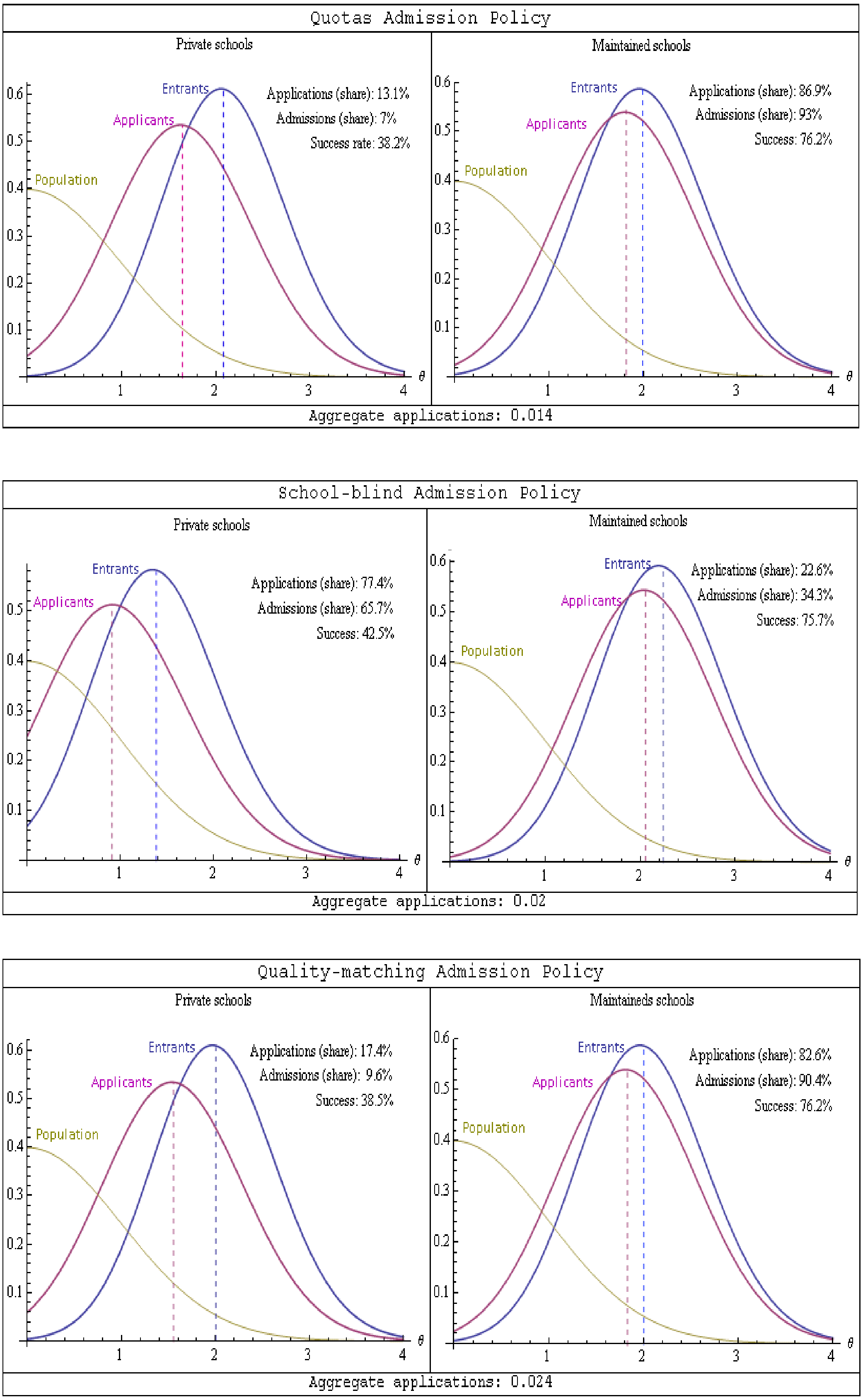

Figure 5: Equilibrium outcomes by policy, for parametric specification in Table 2

As it can be appreciated from Figure 5, maintained schools are highly successful under all admission policies, but application rates are remarkably low in every case. As a consequence, maintained schools turn out to be underrepresented in the School-blind and Quality-matching policies (maintained schools cannot be underrepresented in the Quotas policy, here by assumption; yet application rates are comparably low). It should be noted, however, that there is a significant difference in the order of magnitude of the underrepresentation that arises from the School-blind and Quality-matching policies; the former yields six times as many entrants from private schools as the latter.

This example shows that from all admission procedures, the School-blind policy generates the largest inequality between schools, in terms of applications, admissions and quality. Furthermore, the School-blind admission policy yields the lowest average quality of entrants. The Quality-matching policy, on the other hand, ensures that there is no inequality in terms of the calibre of the average entrant (by definition), but yields the largest number of applications (almost 70% more than the Quotas policy).

5.3 Comparative statics

The previous example explored outcomes across policies for a fixed positive level of cost heterogeneity between schools. The rest of this section explores the comparative statics associated with changes in the extent of such heterogeneity. For simplicity, results are presented for the gap parameter $\tilde{c} = c_m - c_p = c_m - 0.3$, while all other parameters remain fixed as per Table 2.[13]

(I) Quotas. Consider first the quotas admission policy. In this case the university is faced with two independent 'benchmark cases' with different parameter values and corresponding outcomes. Evidently, because of the nature of this admission policy, there is no strategic interaction between schools: an increase in the cost of maintained-school applications only affects equilibrium outcomes through its

[13] Simulations for several different values of c_p suggest that the results for $\tilde{c}$ are qualitatively robust to changes in the location of the gap.

direct effect on maintained schools. This implies that whenever $\tilde{c}$ goes up (i.e. c_m increases), the maintained school incentives to apply are reduced and the university lowers the admission standard only for them. The resulting outcome is fewer but more successful applications from maintained schools (and constant applications from private schools). The comparative statics from the benchmark case apply; in particular, Result 1 implies that the expected quality of type-m entrants is non-monotonic quasiconvex in c_m, while quality of type-p entrants is independent.

Observation 2 *Under admissions by quotas, the university deals with two independent problems by assessing admissions separately. Any change in the parameters of a particular school type will only have an effect on the equilibrium outcomes of that particular type.*

(II) School-blind. Consider now the School-blind policy. Notice that due to the fact that the university must set a unique admission threshold, all students compete for the available vacancies. In this sense, under School-blind admissions the applications of one group impose an externality on the other, so it is no longer true that outcomes are independent. To be specific, applications are subject to two effects related to an increase in $\tilde{c}$. The first and more obvious effect is that maintained schools are discouraged from applying (*direct effect*); however, at the same time the university lowers the admission standard and students are encouraged to apply (*indirect effect*). The aggregate effect on overall applications will be given by the trade-off between these two opposing forces.

To determine the relationship between the *direct* and *indirect* effects, note that the former can be further divided into two different effects: fewer applications from maintained schools are submitted as incentives go down (*reduction effect*), and those who apply are more likely to be successful because only better students will be willing to cover the higher application cost (*quality effect*). Interestingly, for relatively small levels of $\tilde{c}$, the *quality effect* is strong and thus private schools are typically under-encouraged to compensate applications because of the increase in the success rate of maintained schools. However, the *reduction effect* becomes more important as $\tilde{c}$ is sufficiently large, because there are so few type-m applications that even if such applications are highly successful, they can only take few vacancies. Precisely at the point at which the *reduction effect* starts to dominate there is minimum in the total number of applications; a further increase in $\tilde{c}$ means that the *indirect* effect will offset the *direct*. This can be described as a non-monotonicity of aggregate applications. Figure 6 bears out this intuition. The last part of Appendix A.1 presents a further discussion of this result.

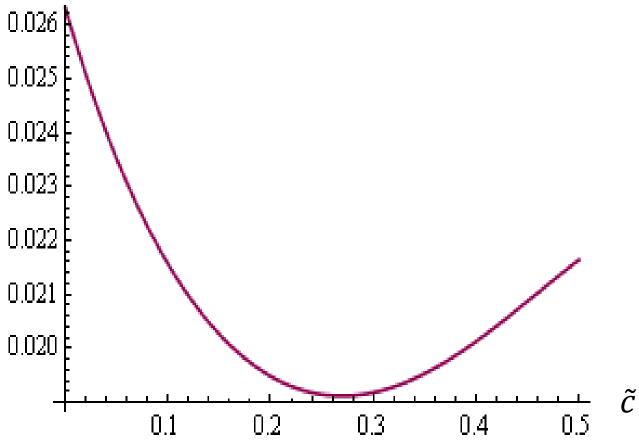

Figure 6: School-blind aggregate applications ($\alpha Pr[S_p \geq s_p^*] + (1-\alpha)Pr[S_m \geq s_m^*]$) as a function of $\tilde{c}$, for $\alpha = 0.07$, $\tau = 0.01$ and $\sigma_p = \sigma_m = 1$.

Result 2 **(Non-monotonicity of applications)** *Under the School-blind admission policy, an increase in the application cost for one school may increase or decrease the total number of all applications (i.e. the sum of applications from both school types).*

The reasoning behind Result 2 also suggests that, under school-blind admissions, the representation of maintained schools amongst entrants should be worsened by the introduction of $\tilde{c} > 0$. This is because the direct effect impacts only the maintained schools (fewer type-m applications) while the indirect effect benefits all candidates irrespective of background (more applications from both types), so introducing $\tilde{c} > 0$ implies a smaller share of type-m applications. But at the same time, because of the indirect effect, the university's reduction of the admission threshold for everybody, means that some worse private-school candidates with cheaper applications can now enter the university. As a result, introducing $\tilde{c} > 0$ also implies proportionally more (and worse) entrants from private schools. Figure 7 presents a simulation illustrating this point; private schools dominate admissions as $\tilde{c}$ increases, yet their expected quality goes down (and hence the overall average quality of entrants goes down).

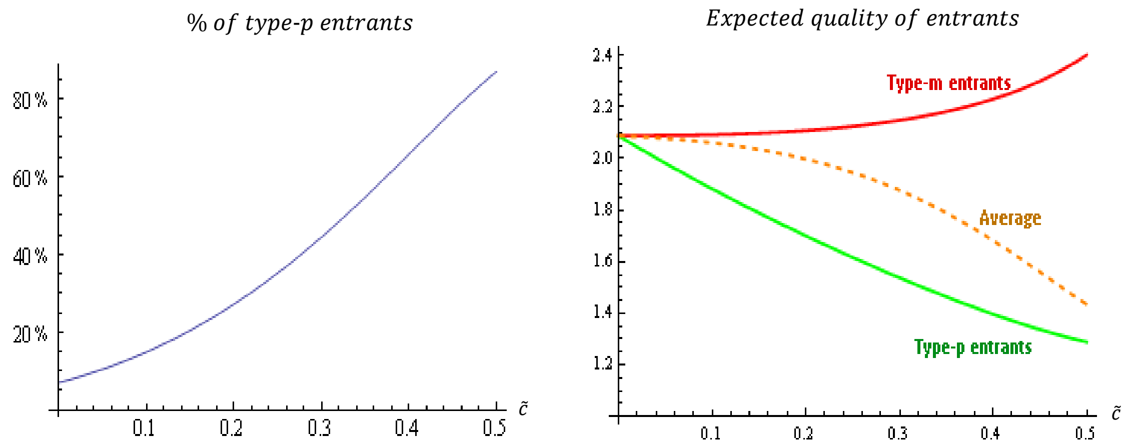

Figure 7: (a) Share of private-school admissions ($\frac{\alpha}{\tau}Pr[T_p \geq t^*, S_p \geq s_p^*]$) and (b) Expected quality of entrants ($E[\theta_i|T_i \geq t^*, S_i \geq s_i^*]$), as a function of $\tilde{c}$ under School-blind admissions, for $\alpha = 0.07$, $\tau = 0.01$ and $\sigma_p = \sigma_m = 1$

Result 3 (Underrepresentation of maintained schools) *Under the School-blind admission policy, introducing $\tilde{c} > 0$ implies that the proportion of type-m entrants is smaller than $\tau(1-\alpha)$*

(III) Quality-matching. The last case to analyse is the quality-matching policy. As it should be clear from the definition, this case is richer because it considers all students simultaneously while allowing the university to treat both school types differently in order to compensate candidates for their underlying heterogeneity in costs. The equilibrium under this admission policy shares most of the qualitative features described above for the school-blind policy. The similarity between these two policies rests on the externalities that applicants from different schools impose on one another; since both policies have students compete for all vacancies, there are conceptually similar direct and indirect effects involved.

As a consequence of the above, it remains true that the proportion of type-m applications is decreasing in $\tilde{c}$ under quality-matching admissions. Similarly, maintained schools are underrepresented in the university whenever $\tilde{c} > 0$. However, the extent to which private schools dominate admissions is significantly reduced because the university compensates the gap by requiring a higher threshold for the private schools. Figure 8 presents a simulation of the equilibrium admissions under this admission policy. A comparison of Figures 8 and 7 reveals the extent to which the quality-matching admission policy improves the participation of maintained schools. Even though the trends are similar, the order of magnitude is drastically reduced.

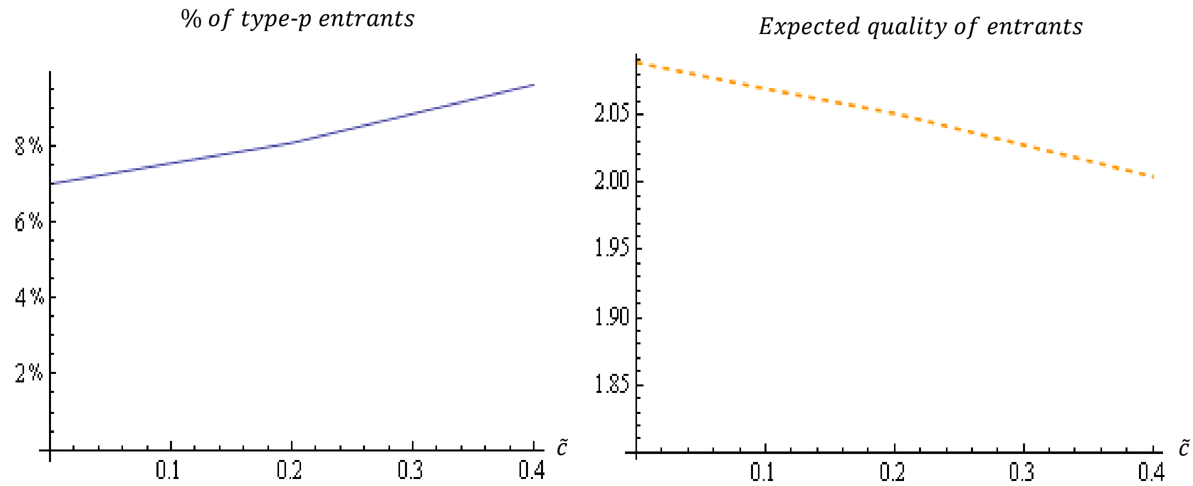

Figure 8: (a) Share of private-school admissions ($\frac{\alpha}{\tau}Pr[T_p \geq t_p^*, S_p \geq s_p^*]$) and (b) Expected quality of entrants ($E[\theta_i|T_i \geq t_i^*, S_i \geq s_i^*]$), as a function of $\tilde{c}$ under Quality-matching admissions, for $\alpha = 0.07$, $\tau = 0.01$ and $\sigma_p = \sigma_m = 1$

The crucial difference between School-blind and Quality-matching admissions is that in the latter the university is allowed to offer different entry requirement to each school type, in order to achieve parity in the quality of the average entrants. The result is that under Quality-matching, an increase in $\tilde{c}$ implies a reduction in the admission standard for both school types, but in a far greater amount for maintained schools. This remark can be corroborated by simulating the admission standard for both school types as a function of $\tilde{c}$. Figure 9 shows that the admission standard for maintained schools is always lower whenever $\tilde{c} > 0$.

Result 4 (Compensation of maintained schools) *Under the Quality-matching admission policy, introducing $\tilde{c} > 0$ implies that maintained schools are compensated by means of a lower entry requirement.*

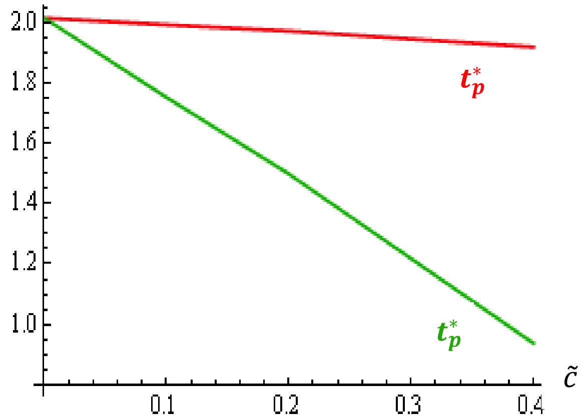

Figure 9: Admission standards (t_p^*, t_m^*) by school type as a function of $\tilde{c}$, under Quality-matching admissions for $\alpha = 0.07$, $\tau = 0.01$ and $\sigma_p = \sigma_m = 1$

6 Replicating observable admission statistics

Preliminaries. As it has been mentioned in Section 2.1, publicly available undergraduate admissions statistics suggest that maintained schools are systematically underrepresented in elite universities, even if private and maintained schools turn out to be almost equally successful conditional on applying. In this section the objective is to explore the extent to which the model, with the different admissions policies, is able to reproduce equilibrium outcomes similar to those observed in practice. To be specific, it shall be assumed that $\tau = 0.1$ and $\alpha = 0.07$, and then for each policy, the objective will be to search for combinations of the other parameters such that the resulting equilibrium exhibits the following two properties:

(a) $\frac{\alpha}{\tau} Pr[T_p \geq t_p^*, S_p \geq s_p^*] = 0.5$

(b) $Pr[T_p \geq t_p^* | S_p \geq s_p^*] = Pr[T_m \geq t_m^* | S_m \geq s_m^*]$

Property (a) captures the fact that private schools take half the places in admissions (and since $\alpha = 0.07$, this is despite the fact that private schools account for a small fraction of the population of candidates). Property (b) captures the fact that maintained schools have the same admission likelihood to private schools conditional on applying.

(I) Quotas. Starting with the policy of admissions by quotas, it is trivial to see that maintained schools will be underrepresented if the university is using a quota that induces underrepresentation. Furthermore, both school types may be equally successful (i.e. exhibit the same conditional admission likelihood) if it is the case that maintained schools are faced with a strictly higher application cost. This is because, if costs are the same, private schools are more successful whenever $\tau_p > \alpha\tau$; but the success of maintained schools is increasing in their application cost (while type-p outcomes are independent of c_m). Thus, increasing c_m by the right amount may increase the admission likelihood of maintained schools just enough to equate it with private schools.

Suppose that $\tau = 0.1$, $\alpha = 0.07$, and the quotas are such that half of the capacity must go to each school type. By fixing $c_p = 0.3$ and $\sigma_p = \sigma_m = 1$, it is possible to search numerically for a value of $c_m > 0.3$ that, in equilibrium, generates the same likelihood of admissions for both schools. Table 3 presents the results of such calibration exercise. As it can be seen, to achieve outcomes similar to those observed in practice (i.e. fulfill properties (a) and (b)) the maintained schools would have to face a much higher application cost, and as a consequence, there would be a remarkable underlying difference in the quality of entrants across school types.

	Parameters	
	Private Schools	*Maintained Schools*
Application Cost	$c_p = 0.3$	$c_m = 0.339$
Signal Noise	$\sigma_p = 1$	$\sigma_m = 1$
Quota	$\tau_p = 0.005$	$\tau_m = 0.005$
	Outcomes	
	Private Schools	*Maintained Schools*
Share of Applications	52.56%	47.44%
Share of Admissions	50%	50%
Admission likelihood	0.42	0.42
Expected quality of entrants	1.49609	2.23738

Table 3: Calibration under admissions by quotas (with $\tau = 0.1$ and $\alpha = 0.07$)

(II) School-blind. In contrast, under blind admissions $c_m > c_p$ is not sufficient to generate an outcome characterised by (a) and (b). This is simply because in this case, whenever $c_m > c_p$, maintained-school students are underrepresented, but *ceteris paribus*, they are also more successful conditional on application. Hence, the calibration exercise with School-blind admissions requires considering an additional source of heterogeneity. Here we explore the case where maintained schools have less accurate signals about their own quality, so $\tilde{\sigma} = \sigma_m - \sigma_p = \sigma_m - 1 > 0$.

This source of heterogeneity affects the admission likelihood of maintained schools in the opposite direction to $\tilde{c}$. Fewer and less successful students from maintained schools will apply as their information deteriorates with respect to private schools. Consequently, it is possible to have highly unequal scenarios under which private schools are overrepresented while their likelihood of admission is the same as maintained schools; in fact, if $\tilde{\sigma}$ is large enough private-school applications may even be more successful. Figure 10 exemplifies this point by showing all the possible combinations of $(\tilde{c}, \tilde{\sigma})$ for which the admission likelihood of private schools is larger under the School-blind policy: any combination of $(\tilde{c}, \tilde{\sigma})$ that lies in the dark area of the parameter space implies that private-school applications are more successful.

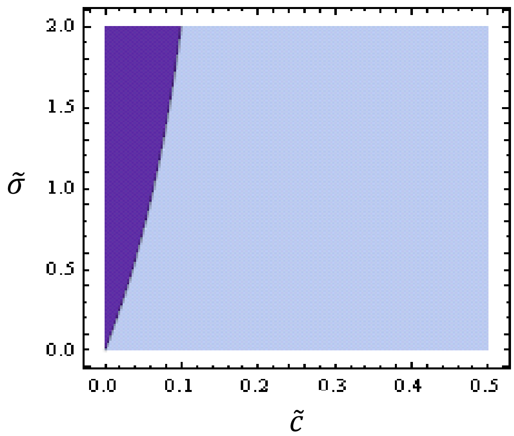

Figure 10: Combinations of $(\tilde{c}, \tilde{\sigma})$ yielding a higher admission likelihood of private schools (darker area) under School-blind admissions (with $\tau = 0.01$ and $\alpha = 0.07$)

In addition to the above, there are other intuitive implications that follow from introducing $\tilde{\sigma} > 0$. The expected quality of entrants is reduced because there is less information to extract from the signals, and the proportion of entrants from private schools is rapidly increased as fewer and less successful type-m students apply. Appendix A.1 presents simulations supporting these results, and exploring the equilibrium interaction between $\tilde{c}$ and $\tilde{\sigma}$ under this policy.

Based on the above remarks, Table 4 presents a calibration for School-blind admission. The result shows that in order to achieve outcomes similar to those observed in practice, there must be a significant underlying difference between schools, both in terms of information and application costs. Interestingly, however, the size of these differences would have to be much larger under School-blind admissions than under Quotas; so it turns out that the quality of entrants from maintained schools under School-blind admissions would be lower than in the Quotas policy. As a consequence, for the same observed outcomes in terms of entrant shares and admission likelihoods, the underlying difference in the quality of the average entrants from different school types would be smaller under School-blind admissions.

	Parameters	
	Private Schools	*Maintained Schools*
Application Cost	$c_p = 0.3$	$c_m = 0.37658$
Signal Noise	$\sigma_p = 1$	$\sigma_m = 2.3092$
	Outcomes	
	Private Schools	*Maintained Schools*
Share of Applications	49.9%	50.1%
Share of Admissions	50%	50%
Admission likelihood	0.42	0.42
Expected quality of entrants	1.4961	1.61434

Table 4: Calibration under School-blind admissions (with $\tau = 0.01$ and $\alpha = 0.07$)

(III) Quality-matching. Turning to the last admission policy, using the same reasoning from above it is possible to search for values of $c_m > 0.3$ and $\sigma_m > 1$ that achieve the target equilibrium properties (a) and (b) under quality-matching. Table 5 presents such calibration. As it can be appreciated, in comparison with the School-blind case, the calibration under Quality-matching requires yet larger underlying gaps in information and costs across schools. So in this case the underlying quality of the average maintained-school entrant would be even lower −in fact as low as the quality of the average private school entrant, so that there was no underlying difference in average entrant quality across schools, as required by the policy.

	Parameters	
	Private Schools	*Maintained Schools*
Application Cost	$c_p = 0.3$	$c_m = 0.382$
Signal Noise	$\sigma_p = 1$	$\sigma_m = 2.718$
	Outcomes	
	Private Schools	*Maintained Schools*
Share of Applications	49.8%	50.2%
Share of Admissions	50%	50%
Admission likelihood	0.42	0.42
Expected quality of entrants	1.4983	1.4983

Table 5: Calibration under Quality-matching admissions (with $\tau = 0.01$ and $\alpha = 0.07$)

Further observations. It is important to highlight that, owing to the fact that any of the three considered policies can generate the same observable outcomes, it is not possible to identify the admissions policy from the publicly available admissions statistics. This implies that, to the extent that there is no publicly available information about the true quality of applicants and entrants from different groups, it is not possible to determine whether the university is holding all candidates to the same standard.

Observation 3 *Publicly available admissions statistics (acceptances and success rates of applications) are not enough information to identify Oxford's admission policy. As a consequence, such admissions statistics cannot be used as conclusive evidence of discrimination.*

Another important remark that follows from these simple calibrations is that, under all the postulated admission policies, the observed admissions statistics are generated by some notion of heterogeneity in the application behaviour of candidates.

Observation 4 *In the model, $\tilde{c} > 0$ and $\tilde{\sigma} > 0$ are necessary conditions for observing underrepresentation of maintained schools and equal likelihood of admission across school types under the School-blind and Quality-matching admission policies. In the case of the Quotas policy, only $\tilde{c} > 0$ is necessary. This* *suggests that reducing the cost of application for maintained schools (ceteris paribus) would improve their representation, independently of the policy.*

Taking the above point further, it should be noted that given the interaction of cost and information disadvantages (see Appendix A.1), an intervention that simultaneously reduced the application cost for maintained schools ($\downarrow \tilde{c}$) and improved the information about their own quality ($\downarrow \tilde{\sigma}$), could have the property of simultaneously improving the participation of maintained schools while crowding out weaker applicants from private schools.[14]

Finally, it is important to emphasise that, although all the studied policies may generate the same observable outcomes in terms of representation and admission likelihood, these alternative admission practices do imply very different expected qualities of entrants. Bearing this in mind, the results from this paper motivate the relevance of a normative analysis of optimal admission practices under endogenous applications.

7 Conclusions

This paper explored the extent to which the notorious underrepresentation of students from maintained schools in elite English universities can be explained by poor information and high application costs, and how the universities' admission policies affect admission outcomes. Here I developed an equilibrium model of two-sided asymmetric information where uncertainty and application costs determine application choices, and acceptances are defined by an admission policy. The model was solved numerically and simulations were used to explore different parametric specifications under three particular admission policies: admissions by quotas (*Quotas*

[14]It is important to highlight, however, that the results discussed so far do not take into account the fact that certain pre-university characteristics, in particular school affiliation, are possibly endogenous. In fact, since in practice school affiliation often follows from an investment decision by the student's family with academic outcomes in mind –a factor not explored in this thesis–, it is possible that differences in application costs by background partly reflect differences in preferences for such an outcome. That being the case, a caveat is in order: any accurate evaluation of the real-world welfare implications of different admission policies (and related policy interventions such as reducing application costs) requires a careful analysis of the mechanisms through which applications respond to changes in admission policies; including, crucially, those that operate through the acquisition of relevant characteristics prior to entering the selection competition.

policy), a school-blind policy that treats all students equally (*School-blind* policy) and a differentiated policy that equates the expected quality of entrants (*Quality-matching* policy).

The analysis started with the case of homogeneous schools, concentrating on the role of information and application costs. It was shown that the expected quality of entrants is non-monotonic in the cost of applications, and that the distributions of quality among applicants and entrants become increasingly similar as the students' signals becomes more informative. After studying the benchmark without school heterogeneity, the analysis turned to the case in which students attending maintained schools face a higher application cost. It was shown that all admission policies may yield underrepresentation of maintained schools, yet with some remarkable differences –notably in terms of the number and quality of applications and acceptances.

Using simulations to explore the comparative statics of the model, the paper exposed different dynamics that are involved in the equilibrium. It was shown that increasing the application cost of maintained schools may affect the outcomes of private schools through an indirect effect. As a consequence, under the School-blind and Quality-matching policies there is a non-monotonic behaviour of aggregate applications. This means that reducing the application cost for maintained schools may yield a reduction in the *total* number of applications.

The model was also used to replicate some stylised features of the admissions statistics at elite English universities. The analysis revealed that it is possible to calibrate the model to fit the characteristic features of this particular stance for the three admission policies. The corollary is that it is not possible to identify the admission policy from the available statistics, and hence, it is not possible to know whether the university is treating all students equally, setting admission quotas, or compensating students to achieve parity in the expected quality of entrants across schools. Considering that the three policies imply a different expected quality of the average entrant, it would be interesting to obtain data on the calibre of university students from different backgrounds −such as final exam results−, in order to calibrate the model. This would be a natural avenue for future work.

Finally, the results from this paper suggest that, while interventions aimed at reducing the cost of application for maintained schools are likely to improve their representation, such interventions also entail non-trivial consequences on the number and quality of applications −and ultimately, the expected caliber of entrants from *all* schools− which crucially depend on the admission policy that is implemented. This motivates the normative analysis of optimal admission practices under endogenous applications, as developed in Chapter 2 of this thesis.

A Appendix for Chapter 1

A.1 Equilibrium implications of introducing $\tilde{\sigma} > 0$

The model can be used to explore what happens when maintained schools have worse information. In this part of the appendix I explore the implications of introducing $\tilde{\sigma} = \sigma_m - \sigma_p = \sigma_m - 1 > 0$. The analysis concentrates on the equilibrium composition of applications and admissions, as well as the expected quality of entrants. All results here correspond to the School-blind admission policy. This is because the simulations for the Quality-matching policy are qualitatively equivalent and significantly more computationally demanding.[15]

Applications. When students have worse information about their own quality they have less incentives to apply because it is less likely that they will be successful. Hence, type-m applications decrease when $\tilde{\sigma}$ increases. Type-p applications, in response, go up. Figure 11 bears out this intuition with a simulation of applications as a function of $\tilde{\sigma}$, fixing $c_p = c_m = 0.3$.

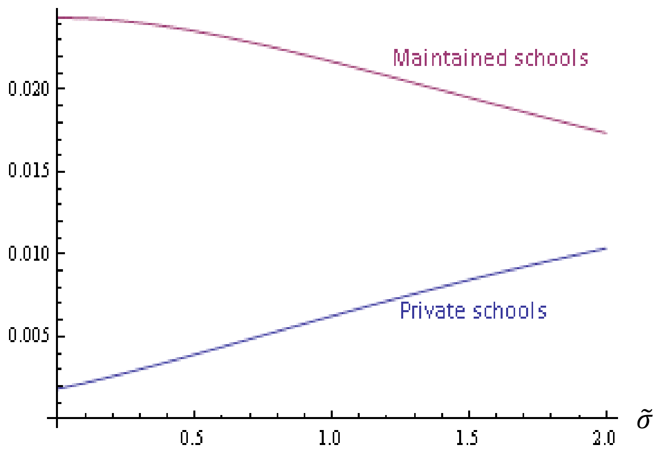

Figure 11: $\alpha Pr[S_p \geq s_p^*]$ and $(1-\alpha)Pr[S_m \geq s_m^*]$ as a function of $\tilde{\sigma}$ under the School-blind policy, for $c_p = c_m = 0.3$, $\tau = 0.1$ and $\alpha = 0.07$

Admissions: As checked above, the result of introducing $\tilde{\sigma} > 0$ is that fewer type-m applications are submitted, and because of poor information, they are less

[15] Notice that the results for the Quotas policy follow the benchmark, so it is not necessary to discuss them separately in this appendix.

successful on average. This in turn means that the share of type-p entrants is increasing in $\tilde{\sigma}$. Figure 12 shows a simulation of the share of private-school admissions, as a function of $\tilde{\sigma} > 0$.

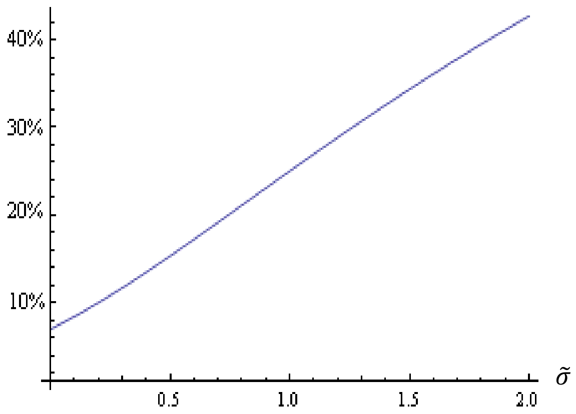

Figure 12: $\frac{\alpha}{\tau} Pr[T_p \geq t_p^*, S_p \geq s_p^*]$ as a function of $\tilde{\sigma}$ under the School-blind policy, for $c_p = c_m = 0.3$, $\tau = 0.1$ and $\alpha = 0.07$

Expected quality of entrants: The expected quality of entrants is an indicator of how much information is transmitted in the equilibrium. Figure 13 shows that expected quality goes down for both school types as maintained schools have worse information.

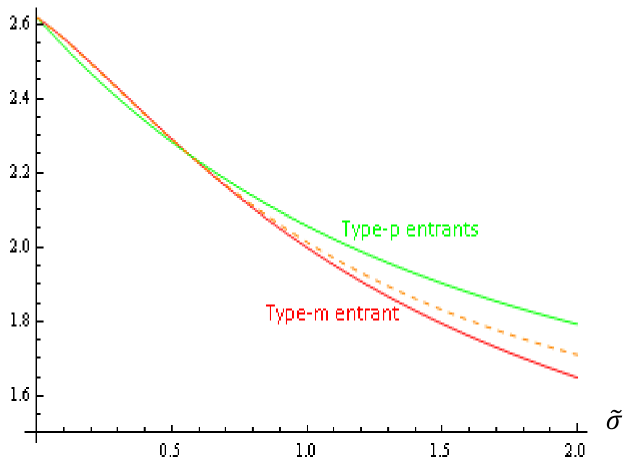

Figure 13: $E[\theta_i | T_i \geq t^*, S_i \geq s_i^*]$ as a function of $\tilde{\sigma}$ under the School-blind policy, for $c_p = c_m = 0.3$, $\tau = 0.1$ and $\alpha = 0.07$

Interaction with $\tilde{c}$: In the analysis of cost heterogeneity it was argued that aggregate applications need not be monotonic in $\tilde{c}$. This was explained by showing that an increase in $\tilde{c}$ yields both fewer and better type-m applications (*reduction* and *quality* effects). Note that in this explanation there is an interaction between $\tilde{\sigma}$ and $\tilde{c}$. The *quality* effect is strong only if students are well informed. Intuitively this means that the extent of this non-monotonicity (of applications with respect to $\tilde{c}$) depends on the actual level of $\tilde{\sigma}$. Figure 14 presents a contour plot of the total number of applications as a function of $\tilde{c}$ and $\tilde{\sigma}$ (lighter colours represent more applications).

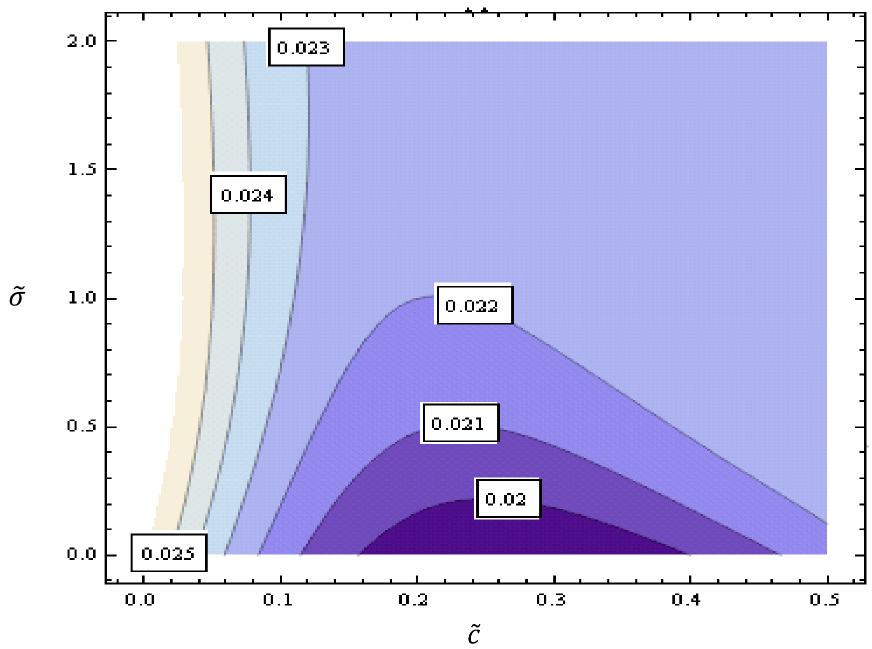

Figure 14: Aggregate applications ($\alpha Pr[S_p \geq s_p^*]+(1-\alpha)Pr[S_m \geq s_m^*]$) as a function of $\tilde{c}$ and $\tilde{\sigma}$ under the School-blind policy, for $\tau = 0.1$ and $\alpha = 0.07$

A.2 Solution and Numerical Implementation

Following the derivations from Section 4, the student application threshold can be expressed as a function

$$s_i(t_i; c_i, \sigma_i) = \frac{t_i - \Phi^{-1}(1 - c_i)\sigma_{T_i|s_i}}{\beta_{T_i S_i}} \qquad i \in \{m, p\} \tag{10}$$

where $\beta_{T_i S_i}$ and $\sigma_{T_i|s_i}$ follow the notation already introduced.

Intuitively, this equation captures the student best response to any admission requirement t_i, so it always takes the same form regardless of the policy used to determine the actual requirement. Hence, analogously to the benchmark case, under heterogeneity across schools the model can be solved by substituting $s_i(t_i; c_i, \sigma_i)$ into the university's policy-specific admission conditions and finding the corresponding roots.

Below are the equations that were solved numerically through standard iteration routines, to find the equilibrium admission requirements in each case.[16]

Quotas admission policy

$$\tau_p = 1 - \Phi\left(\frac{t_p^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_p(t_p^*; c_p, \sigma_p)}{\sqrt{1+\sigma_p^2}}\right) + F_{T,S}(t_p^*, s_p(t_p^*; c_p, \sigma_p)) \tag{Ia}$$

$$\tau_m = 1 - \Phi\left(\frac{t_m^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_m(t_m^*; c_m, \sigma_m)}{\sqrt{1+\sigma_m^2}}\right) + F_{T,S}(t_m^*, s_m(t_m^*; c_m, \sigma_m)) \tag{Ib}$$

School-blind admission policy

$$\begin{aligned}\tau =& \alpha\left(1 - \Phi\left(\frac{t^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_p(t^*; c_p, \sigma_p)}{\sqrt{1+\sigma_p^2}}\right) + F_{T,S}(t^*, s_p(t^*; c_p, \sigma_p))\right)\\ &+ (1-\alpha)\left(1 - \Phi\left(\frac{t^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_m(t^*; c_m, \sigma_m)}{\sqrt{1+\sigma_m^2}}\right) + F_{T,S}(t^*, s_m(t^*; c_m, \sigma_m))\right)\end{aligned} \tag{II}$$

[16] For the solution under the *Quotas* and *School-blind* admission policies, the *FindRoot* routine was implemented. This routine, which is built in the software *Mathematica*, searches for a numerical root using Newton methods given a starting point. The solution under the *Quality-matching* policy was slightly different because it was more difficult to implement using standard routines. In this specific case the solution was found by a combination of numerical integration (to estimate the expected quality of entrants) and manual search (to fill vacancies in expectation).

Quality-matching admission policy

$$\tau = \alpha \left(1 - \Phi\left(\frac{t_p^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_p(t_p^*; c_p, \sigma_p)}{\sqrt{1+\sigma_p^2}}\right) + F_{T,S}(t_p^*, s_p(t_p^*; s_p, \sigma_p)) \right)$$
$$+ (1-\alpha) \left(1 - \Phi\left(\frac{t_m^*}{\sqrt{2}}\right) - \Phi\left(\frac{s_m(t_m^*; c_m, \sigma_m)}{\sqrt{1+\sigma_m^2}}\right) + F_{T,S}(t_m^*, s_m(t_m^*; c_m, \sigma_m)) \right) \tag{IIIa}$$

$$E[\theta_i | T \geq t_p^*, S_p > s_p(t_p^*; c_p, \sigma_p)] = E[\theta_i | T \geq t_m^*, S_m > s_m(t_m^*; c_m, \sigma_m)] \tag{IIIb}$$

A.3 Analysis of admissions statistics

The dataset used was provided by the Universities and Colleges Admissions Service (UCAS). It consists of 29,440 observations corresponding to a 25% random sample of the entire pool of applicants to at least one degree course in the subject line M (Law), during the period 2007-2011. To ensure anonymity, suppressions to the pool were applied before taking the random sample, whenever there were less than 3 students with the same characteristics (except when a student had applied to the University of Oxford). This resulted in the suppression of around 2.5% of the observations in the original pool. To check whether suppression led to bias, the estimated probabilities of admissions in the dataset were contrasted with reported admission likelihoods in administrative records from Oxford; this exercise suggested there was no bias due to suppression.

The following tables present a summary of the variables, as well as some basic descriptive statistics including regressions of the probability of application and admission controlling for student characteristics.

Content	Labels	Description	Remark
Gender	*male*	Dummy variable (1=male, 0=female)	
Ethnic group	*ethnic0, ethnic1, ... ethnic5*	Set of dummy variable for students from each of the following (mutually exclusive) ethnic groups: Unknown/No answer, *Asian, Black, Mixed, Other, White*	In some regressions the variable *ethnic5* is renamed to *white* to make output more explicit
HE background	*polar2*	Discrete variable taking values from 1 to 5, classifying students' postal code by quintiles in the distribution of youth progression to higher education	Missing data for applicants older than 19 years
Independent school	*private*	Dummy variable (1=independent school, 0=otherwise)	-Data available for 6 different school types -Non-independent schools include all state schools and those classified as "other" and "unknown"
UCAS Tariff band	*tariff*	Discrete variable classifying tariff points into 10 successive bands, taking values from 1 to 10 according to the following ranges: 1-79; 80-119; 120-179; 180-239;240-299;300-359 … >540	-Tariff points are allocated according to tariff tables (www.ucas.com). -At A-levels, A* = 140 points, A=120, B=100, and so on. -Tariffs are only supplied by UCAS for verified qualifications. -Tariffs for students with non-verified qualifications are reported as missing.
Application	*apply*	Dummy variable (1=Applied to Oxbridge, 0=otherwise)	
Admission	*admit*	Dummy variable (1=Admitted into Oxbrige, 0=otherwise)	Admits are those who receive and accept an offer. Missing data correspond to non-applicants

Table 6: Summary of variables in the data set

Variable	Obs	Mean	St. Err.	Obs	Mean	St. Err.	Difference	95% Conf. Interval	
	Non-independent schools			Independent schools					
male	27742	0.3644	0.0029	1698	0.4417	0.0121	-0.0773	-0.1009	-0.0536
white	27742	0.6902	0.0028	1698	0.8127	0.0095	-0.1225	-0.1450	-0.1000
polar2	21661	3.3321	.00921	1515	4.2706	.02628	-.93851	-1.0081	-.86886
tariff	22057	5.6537	0.0147	1603	7.3082	0.0495	-1.6545	-1.7641	-1.5448
apply	27742	0.0333	0.0011	1698	0.1743	0.0092	-0.1410	-0.1506	-0.1314
admit	924	0.1623	0.0121	296	0.2500	0.0252	-0.0877	-0.1382	-0.0371

Table 7: Comparison of student characteristics by school type

```
Logistic regression                               Number of obs   =      23660
                                                  LR chi2(13)     =    2531.74
                                                  Prob > chi2     =     0.0000
Log likelihood = -3219.6583                       Pseudo R2       =     0.2822
```

apply	Odds Ratio	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tariff	2.349685	.0548582	36.59	0.000	2.244588	2.459703
private	2.486586	.216621	10.46	0.000	2.096286	2.949556
male	1.25918	.0876293	3.31	0.001	1.098628	1.443195
ethnic1	.1131112	.0432479	-5.70	0.000	.0534626	.2393104
ethnic2	.1704954	.0690168	-4.37	0.000	.0771161	.3769473
ethnic3	.1651739	.0671948	-4.43	0.000	.0744153	.3666238
ethnic4	.346314	.1687682	-2.18	0.030	.1332467	.9000852
ethnic5	.0669802	.024944	-7.26	0.000	.0322813	.1389765
polar1	1.170698	.2594927	0.71	0.477	.7581768	1.80767
polar2	1.374438	.2752435	1.59	0.112	.9282492	2.035101
polar3	1.316675	.255388	1.42	0.156	.9002768	1.925668
polar4	1.319539	.2475747	1.48	0.139	.91352	1.906016
polar5	1.110724	.2033496	0.57	0.566	.7758348	1.590167
_cons	.0007819	.0003362	-16.64	0.000	.0003367	.001816

Table 8: Regression of probability of application (odds ratio)

```
Logistic regression                                   Number of obs   =       1112
                                                      LR chi2(13)     =      94.00
                                                      Prob > chi2     =     0.0000
Log likelihood = -493.27328                           Pseudo R2       =     0.0870
```

admit	Odds Ratio	Std. Err.	z	P>|z|	[95% Conf.	Interval]
tariff	1.644847	.1268451	6.45	0.000	1.414112	1.913231
private	1.276533	.2286409	1.36	0.173	.8986128	1.813391
male	.99068	.162681	-0.06	0.955	.7180528	1.366817
ethnic1	.3903746	.2387293	-1.54	0.124	.117744	1.294269
ethnic2	.7910406	.5478859	-0.34	0.735	.2035376	3.074348
ethnic3	1.149838	.7428517	0.22	0.829	.3241298	4.079008
ethnic4	.7565584	.6710425	-0.31	0.753	.1330009	4.303585
ethnic5	.5200071	.2913379	-1.17	0.243	.1734272	1.559199
polar1	.4188587	.2634098	-1.38	0.166	.1221147	1.436703
polar2	.4307652	.2382384	-1.52	0.128	.1457056	1.273517
polar3	.3285812	.179101	-2.04	0.041	.1128951	.9563355
polar4	.9352751	.4643152	-0.13	0.893	.3534779	2.474665
polar5	1.073818	.5203982	0.15	0.883	.4153535	2.776154
_cons	.0061109	.0058286	-5.34	0.000	.0009424	.0396265

Table 9: Regression of probability of admission (odds ratio)

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Chapter 2.

Selection by Selection: The Case of University Admissions

Chapter abstract

There are many prizes in life for which one needs to apply, in order to win. Here, we focus mainly on the prize for young adults of admission to an elite university. At such universities, as has been much remarked, there is a notable underrepresentation from disadvantaged social groups. This underrepresentation seems to stem more from the failure to apply than low probability of acceptance conditional on application. Since, students self-select into application taking into account university policy and procedures in selecting from the pool of applicants, the pool of applicants from which universities choose is endogenous to their selection policies. Selection begets selection. This paper characterises the optimal selection policy from a pool of applicants taking into account that the pool of applicants is endogenous. Owing to the fact that optimal selection in our framework requires making use of observable and unobservable candidate attributes, our approach allows a characterisation of the optimal form of discrimination in university admissions.

Chapter contents

1 Introduction

Motivation. Top universities in England are under constant scrutiny because of the low representation of certain groups. In particular, the composition of the pool of university entrants is often used to suggest that admissions are biased.[1] Universities, on the other hand, defend themselves by arguing that the observed underrepresentation of specific groups (e.g. students from state schools) stems from the failure of these groups to apply, rather than low probability of acceptance conditional on application. Indeed, a glance at the admissions statistics from Oxford highlights the relevance of the debate.

	All UK Students	Oxford Applicants	Oxford Entrants
Non-state schools	7%	37.2 %	43.2 %
State schools	93%	62.8 %	56.8 %

Source: University of Oxford, 2013

Chapter 1 of this thesis presents evidence supporting the claim that candidate background is relevant to predict applications, but not admissions (conditional on application). Since students self-select into applications taking into account university policies and procedures in selecting from the pool of applicants, the pool of applicants from which universities choose is endogenous to their selection policies: selection through admissions begets selection through applications. On this account, a natural question to ask is whether a selective university interested in maximising the quality of its entrants, should be more lenient in admitting candidates from state schools if they are particularly discouraged from applying. This paper provides a framework to answer this question.

A normative analysis of selection from an endogenous pool is in essence a problem of mechanism design, for the selection rule must induce applicants to self-select

[1] The following quote from Sir Peter Lampl, in a report by the Sutton Trust −a think tank working towards improving social mobility through education in the U.K.−, summarises well the nature of such claims: *"Every year there are some 3,000 well-qualified young people attending state schools and sixth form colleges who are not among the 30,000 students who are admitted to our dozen or so leading universities, despite achieving grades as good as or better than the entry requirements to courses in those universities. This suggests perhaps a lack of ambition or confidence, but certainly a potential waste of talent. And the corollary is that, far from the university entry system discriminating against pupils from the independent sector, it is acting in their favour."*

optimally into the pool on the basis of their private information. In this paper we propose a general framework for studying admissions as a mechanism design problem, and show that under sufficient conditions the optimal general mechanism can be equivalently implemented through a simple and intuitive cut-off rule: the university publishes a minimum entry threshold on an admissions test, candidates decide whether to apply, and if they do, they are admitted only if they score above the entry threshold. Using this result we then characterise the optimal admission thresholds.

In order to view the optimal policy in context, we explore first an intuitive policy benchmark in which students apply without knowing the admission thresholds and the university simply selects the best who apply (i.e. a benchmark with non-committed admissions). We show that the optimal admissions rule is not generally equivalent to the non-committed benchmark, so the optimal way of admitting the candidates with the highest potential in the *population* is not necessarily to admit the candidates with the highest potential in the pool of *applicants*. As shall be clear, this result implies that under optimal admissions, candidates belonging to groups that are particularly discouraged from applying would be held to a lower standard, in the sense that the worst entrant from such groups would have lower predicted quality. That being the case, our results provide a justification for positive economic discrimination on the grounds of efficiency.

Related Literature. A growing strand of empirical literature highlights the relevance of our approach, beyond the case of elite English universities (Pallais 2013; Carrell and Sacerdote 2013; Hoxby and Avery 2012; Avery and Kane 2004). In particular, Hoxby and Avery (2012) use a rich array of data from the U.S.to show that the vast majority of very high-achieving students who are low-income do not apply to any selective college or university (and this is despite the fact that selective institutions would often cost them less, owing to generous financial aid, than the resource-poor and non-selective institutions to which they actually apply). The authors document that, because of this, admissions officers underestimate the number of low-income, high-achieving students that in reality exist. Moreover, Hoxby and Avery (2012) also provide evidence that (i) high-achieving, low-income students who do apply to selective institutions are admitted and graduate at high rates; and (ii) the low-income students' application behavior differs greatly from that of their high-income counterparts who have similar achievement.

From a theoretical point of view, the design of optimal admission policies to select candidates from a fixed pool, is a well-studied problem that has been explored extensively, both in the literature on statistical selection (Lehmann 1966; Eaton 1967; Eaton 1982), and the literature on statistical discrimination (Phelps 1972; Arrow 1973; Welch 1976). The problem of devising optimal selection procedures when the pool is *endogenous*, however, has received much less attention. In the case of university admissions, specifically, there is little research exploring optimal selection as part of an equilibrium process with frictions due to application costs and uncertainty. Two exceptions are Gary-Bobo and Trannoy (2008) and Chade, Lewis and Smith (2013). In both cases the admission policy is assumed to take the form of a cut-off rule, and the only source of student heterogeneity is unobservable ability. The contributions of this paper are twofold: we provide conditions under which a cut-off rule is efficient, and characterise optimal cut-offs when there is heterogeneity both in unobservable ability and observable student characteristics (e.g. gender, ethnicity, etc.). This allows us to explore the optimal form of discrimination in university admissions.

The remainder of this paper is organised as follows. Section 2 introduces the general mechanism design problem; Section 3 discusses a benchmark case where admission policies are non-committed; Section 4 characterises the optimal admissions policy; Section 5 discusses our results using a Gaussian parameterisation; Section 6 presents a discussion of the results by relating them to notions of discrimination traditionally used in the economics literature; and Section 7 finishes with the concluding remarks.

2 The Mechanism Design Problem

2.1 The general problem

Consider a set of students deciding whether to attend a single university. These students form a continuum of unit measure and are classified into one of n different groups, or *sectors*, labeled $i \in I$ for $I = \{1, ..., n\}$. Each sector is characterised by a vector of observable student characteristics (e.g. gender, ethnicity, qualifications, school type, etc.), and contains a mass $\alpha_i \in [0, 1]$ of students endowed with unobservable quality $\theta_i \in \mathbb{R}$, for $i \in I$.

Although candidates do not observe their quality, they receive a related real-valued signal S_i. If a candidate applies to the university, she pays a cost $c_i \in [0, 1]$ and submits a report of her private information. The university, interested in maximising the expected quality of entrants (subject to capacity), observes in turn a related real-valued private signal T_i about the quality of each applicant (e.g. admission test, interview, etc.).

Since students have private information, which they use to inform their application choices, the university's optimal choice of admissions from the pool of applicants corresponds to a problem of mechanism design.

Definition 1 (Direct Mechanism) A direct mechanism is a pair of functions $(\hat{q}_i, \hat{p}_i)$ for each $i \in I$, where

(a) $\hat{q}_i : \mathbb{R} \to [0, 1]$

(b) $\hat{p}_i : \mathbb{R}^2 \to [0, 1]$

The first function, $\widehat{q}_i(s_i)$, gives the probability that a candidate from sector $i \in I$ applies when observing $S_i = s_i$. The second function, $\widehat{p}_i(\tilde{s}_i, t_i)$, gives the probability of admission for a candidate who applies from sector $i \in I$ reporting a realisation $\tilde{s}_i$ and producing an admission test score $T_i = t_i$.

A direct mechanism, which successfully implements admissions for the university, must satisfy feasibility and incentive compatibility. The former requires that the university fills capacity (noted hereafter $\tau \in [0,1]$), so a feasible direct mechanism must be such that

$$\tau = \sum_{i=1}^{n} \alpha_i \int\int \hat{p}_i(s_i,t_i)\hat{q}_i(s_i) f_{S_i,T_i}(s_i,t_i) ds_i dt_i \tag{1}$$

where $f_{S_i,T_i}(s_i,t_i)$ denotes the joint distribution of T_i and S_i for $i \in I$. Incentive compatibility, on the other hand, requires that when candidates apply, they are content to report their private information truthfully. Hence, an incentive compatible direct mechanism in this context must be such that for all $\tilde{s}_i$ and s_i in the real line,

$$\hat{q}_i(\tilde{s}_i)\left(\int \widehat{p}_i(\tilde{s}_i,t_i) f_{T_i|S_i}(t_i|s_i)dt_i - c_i\right) \leq \hat{q}_i(s_i)\left(\int \widehat{p}_i(s_i,t_i) f_{T_i|S_i}(t_i|s_i)dt_i - c_i\right) \tag{2}$$

where $f_{T_i|S_i}(\cdot|\cdot)$ denotes the conditional probability distribution of T_i given S_i, for $i \in I$.[2]

In addition to the incentive compatibility constraint above, in the case of university admissions from an endogenous pool of applicants there is an extra condition requiring the university to respect privacy of application decisions. This additional incentive compatibility condition arises because candidates will only apply and report their private information if they want to. Hence, an incentive compatible direct mechanism is also constrained by

$$\int \widehat{p}_i(s_i,t_i) f_{T_i|S_i}(t_i|s_i)dt_i \begin{array}{c} > \\ = \\ < \end{array} c_i \quad \Leftrightarrow \quad \widehat{q}_i(s_i) \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array} \tag{3}$$

Considering this, the general admissions problem corresponds to a class of mechanisms, representable by a *constrained* direct mechanism: the university seeks to maximise the expected quality of entrants by using a direct mechanism $(\widehat{p}_i, \widehat{q}_i)$, $i \in I$, such that conditions (1), (2) and (3) are satisfied.

A natural, albeit more specific, alternative formulation of the admissions problem is through *indirect* mechanisms admitting probabilistic acceptance rules, conditional

[2] In what follows, all distributions will use the obvious corresponding notation. For example, the marginal distributions of S_i and T_i will be respectively noted $f_{S_i}(\cdot)$ and $f_{T_i}(\cdot)$ for $i \in I$.

on test scores. In contrast to the general constrained direct mechanism already introduced, in the indirect counterpart candidates from sector $i \in I$ are admitted with probability $p_i(t_i)$ if they produces test score $T_i = t_i$.

Definition 2. (Probabilistic Acceptance Mechanism) A probabilistic acceptance mechanism is a function of the form $p = (p_1, ..., p_n)$ where $p_i : \mathbb{R} \to [0, 1]$ for each sector $i \in I$.

Similarly to the direct mechanisms, a feasible probabilistic admissions rule must fill capacity, and hence satisfy

$$\tau = \sum_{i=1}^{n} \alpha_i \int\int p_i(t_i) q_i(s_i) f_{S_i,T_i}(s_i, t_i) ds_i dt_i \tag{4}$$

where $q_i(s_i)$ denotes the best response application choice associated with the probabilistic admission rule $p_i(t_i)$, and where by definition, for a candidate observing $S_i = s_i$,

$$\int p_i(t) f_{T_i|S_i}(t_i|s_i) dt_i \begin{array}{c} > \\ = \\ < \end{array} c_i \Leftrightarrow q_i(s_i) \begin{array}{c} = 1 \\ \in [0, 1] \\ = 0 \end{array} \tag{5}$$

2.2 Optimality of cut-off rules

The following two propositions justify the optimality of cut-off rules in two different scenarios. In Proposition 1, we establish optimality from mechanisms representable by constrained direct mechanisms. In Proposition 2 we establish optimality from mechanisms representable by probabilistic acceptance rules.

Proposition 1. *Suppose*

1. *For each sector $i \in I$, the random vector (T_i, S_i, θ_i) is affiliated.*

2. *For each sector $i \in I$, $\theta_i \perp\!\!\!\perp T_i \mid S_i$*

Then, for any feasible constrained direct revelation mechanism there is a feasible cut-off rule (of the form "for each sector $i \in I$ admit those applicants who score at *least $T_i \geq t_i$ for some t_i") which implements the same mass of students from each sector but with higher average quality.*

Proof. *See Appendix B.2.*

Proposition 2. *Suppose*

1. *For each sector $i \in I$, the random vector (T_i, S_i, θ_i) is affiliated.*

2. *For each sector $i \in I$, $E[\theta_i \mid S_i = s_i, T_i = t_i]$ is an additive function.*

Then for any feasible probabilistic acceptance mechanism, there is a feasible acceptance cut-off rule (of the form "for each sector $i \in I$ admit with probability one for those applicants who score at least $T_i \geq t_i$ for some t_i") which implements the same mass of students from each sector but with higher average quality.

Proof. *See Appendix B.3.*

Notice that, when implementing a cut-off rule $(t_1^*, ..., t_n^*)$ which optimally induces application thresholds $(s_1^*, ..., s_n^*)$, the candidates who are accepted are above two margins, as illustrated by the shaded rectangle in Figure 1.

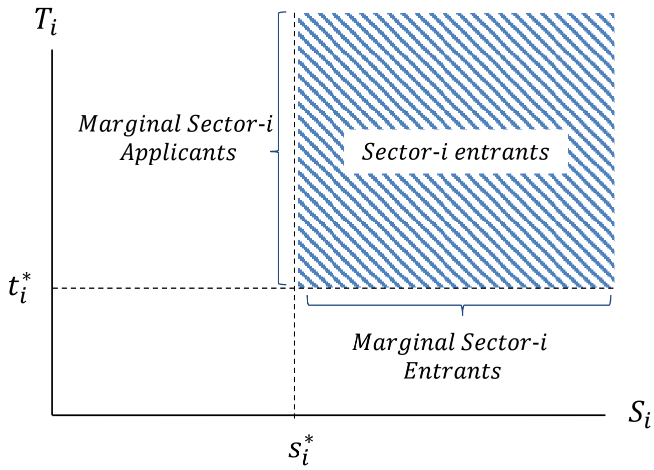

Figure 1: Selection on T_i and S_i

The first margin consists of those candidates who were indifferent about applying (the vertical dashed line in Figure 1), while the second margin consists of those applicants whom the university was indifferent about admitting (the horizontal dashed line). A cut-off admissions problem, therefore, consists in setting the entry requirements that produce the appropriate shape and size of these rectangles across sectors, such that the expected number of entrants equals the capacity, and the average quality of entrants is maximised. Propositions 1 and 2 provide conditions under which it is possible to study optimal admissions by concentrating on the solution to the cut-off admissions problem. This will be the approach for the remainder of this paper.

Before continuing it should be noted that cut-off rules, as we have defined them, assume commitment power by the university, in the sense that the university commits the thresholds and admits every candidate who applies and scores above them. There is an alternative selection procedure, also of interest, where the university does not commit to a minimum standard, but rather selects the best who apply to fill capacity. We will refer to such selection procedure as the *no-commitment benchmark*. An intuitive way to distinguish between these alternative selection procedures is to think in terms of sequential interaction. In the case of committed admissions the university promises the thresholds before observing the applicants, hence taking into account the indirect effect that the thresholds have on the composition of the pool of applicants. In contrast, in the no-commitment benchmark the university sets the thresholds only after observing the applicants, so it does not account for the indirect effect. Interestingly, in both cases admissions and applications are defined in equilibrium —after all, with or without commitment students only apply if they think that they will be admitted. Because of this, the benchmark is relevant from a normative point of view: it captures the equilibrium dynamics in a setting where the university is constrained to choose the best applicants.

3 The No-commitment Benchmark

Suppose admission thresholds are not committed. In this case the university takes the application choices given and seeks to choose admissions in order to maximise the expected quality of entrants, subject to filling capacity in expectation. It is easy to see that, given any application behaviour by students (i.e. ex-post the submission of applications), efficiency in admissions entails selecting those applicants with the highest test score (i.e. admit students according to a cut-off rule). That being the case, the best response application policy by students, it can be checked, is also to apply according to a cut-off rule (i.e. apply if and only if the student signal is greater than some critical value $S_i = s_i$, for $i \in I$).

Considering the above, in this case the university's problem is to choose the thresholds $t_1, t_2, ..., t_n$ to maximise

$$\begin{aligned}
&\sum_{i=1}^{n} \alpha_i \mathbb{E}[\theta_i | T_i \geq t_i, S_i \geq s_i] \Pr[T_i \geq t_i, S_i \geq s_i] \\
&= \sum_{i=1}^{I} \alpha_i \mathbb{E}[\theta_i \cdot \mathbb{I}_{\{T_i \geq t_i, S_i \geq s_i\}}] \\
&= \sum_{i=1}^{n} \alpha_i \int_{-\infty}^{\infty} \int_{s_i}^{\infty} \int_{t_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(T_i, S_i, \theta_i) \; dT_i \, dS_i \, d\theta \qquad (6)
\end{aligned}$$

subject to filling capacity

$$\tau = \sum_{i=1}^{n} \alpha_i \int_{s_i}^{\infty} \int_{t_i}^{\infty} f_{T_i,S_i}(T_i, S_i) \; dT_i \, dS_i \qquad (7)$$

And hence the Lagrangean for this problem, with multiplier λ for the capacity constraint, is

$$\begin{aligned}
\mathcal{L}(t_1, ..., t_n) =& \sum_{i=1}^{n} \alpha_i \int_{-\infty}^{\infty} \int_{s_i}^{\infty} \int_{t_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(T_i, S_i, \theta_i) \; dT_i \, dS_i \, d\theta \\
&+ \lambda \left(\tau - \sum_{i=1}^{n} \alpha_i \int_{s_i}^{\infty} \int_{t_i}^{\infty} f_{T_i,S_i}(T_i, S_i) \; dT_i \, dS_i \right) \qquad (8)
\end{aligned}$$

Partially differentiating (8) gives the following first order conditions for each $i \in I$ (henceforth FOCs):

$$\begin{aligned}
\frac{\partial \mathcal{L}(t_1, ..., t_n)}{\partial t_i} =& \alpha_i \lambda \int_{s_i}^{\infty} f_{T_i,S_i}(t_i, S_i) \; dS_i \\
&- \alpha_i \int_{-\infty}^{\infty} \int_{s_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(t_i, S_i, \theta_i) \; dS_i d\theta = 0 \qquad (9)
\end{aligned}$$

Noting that

$$E[\theta_i|T_i = t_i, S_i \geq s_i] = \frac{\int_{-\infty}^{\infty}\int_{s_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(t_i, S_i, \theta_i)\, dS_i\, d\theta}{\int_{s_i}^{\infty} f_{T_i,S_i}(t_i, S_i)\, dS_i}$$

we can simplify (9) to obtain

$$E[\theta_i|T_i = t_i, S_i \geq s_i] = \lambda \qquad \text{for } i \in I \tag{10}$$

The no-commitment benchmark rule above states: *choose cut-offs* $t_1, t_2, ...t_n$ *to equate the expected quality of marginal entrants across sectors.* This follows the familiar economic intuition, since selecting the best applicants requires implementing entry thresholds such that the worst entrant from each sector has the same predicted quality. Otherwise, if two marginal entrants failed to have the same expected quality, there would be a positive mass of applicants with expected quality in between, whom the university would prefer over the marginal entrant of lesser predicted quality. The value of t_i that solves (10) for any value of s_i, henceforth noted $t_i(s_i)$, is the university's best response in the no-commitment benchmark.

To pin down the equilibrium notice that a candidates observing t_i is indifferent about applying if and only if

$$\Pr[T_i \geq t_i|S_i = s_i] = c_i \tag{11}$$

so the candidates' best response, henceforth noted $s_i(t_i; c_i)$, is given by the realisation of S_i that solves (11). Consequently, the benchmark equilibrium is a set of threshold pairs $(t_1^e, s_1^e), ..., (t_n^e, s_n^e)$ such that:

$$s_i^e = s_i(t_i^e; c_i) \quad \text{and} \quad t_i^e = t_i(s_i^e) \qquad \text{for } i \in I$$

Proposition 3. *Suppose the random vector* (T_i, S_i, θ_i) *is affiliated for each sector* $i \in I$*. It follows that*

1. $s_i(t_i; c_i)$ *is non-decreasing in* t_i *and* $t_i(s_i)$ *is non-increasing in* s_i*, so the no-commitment benchmark equilibrium is unique when it exists*

2. *$s_i(t_i; c_i)$ is non-increasing in c_i, so the no-commitment benchmark equilibrium is characterised by $t_i^e \geq t_j^e$ and $s_i^e \leq s_j^e$ when $c_i \leq c_j$ (everything else constant across sectors)*

Proof. Affiliation of the random vector (T_i, S_i, θ_i) implies that $\mathbb{E}[\theta_i | T_i = t_i, S_i \geq s_i]$ is non-decreasing in t_i and s_i. From implicit differentiation this yields

$$\frac{\partial t_i(s_i)}{\partial s_i} \leq 0$$

Affiliation also implies that $\Pr[T_i \leq t_i | S_i = s_i]$ is non-increasing in s_i, so we get that

$$\frac{\partial s_i(t_i; c_i)}{\partial t_i} \geq 0 \quad \text{and} \quad \frac{\partial s_i(t_i; c_i)}{\partial c_i} \geq 0$$

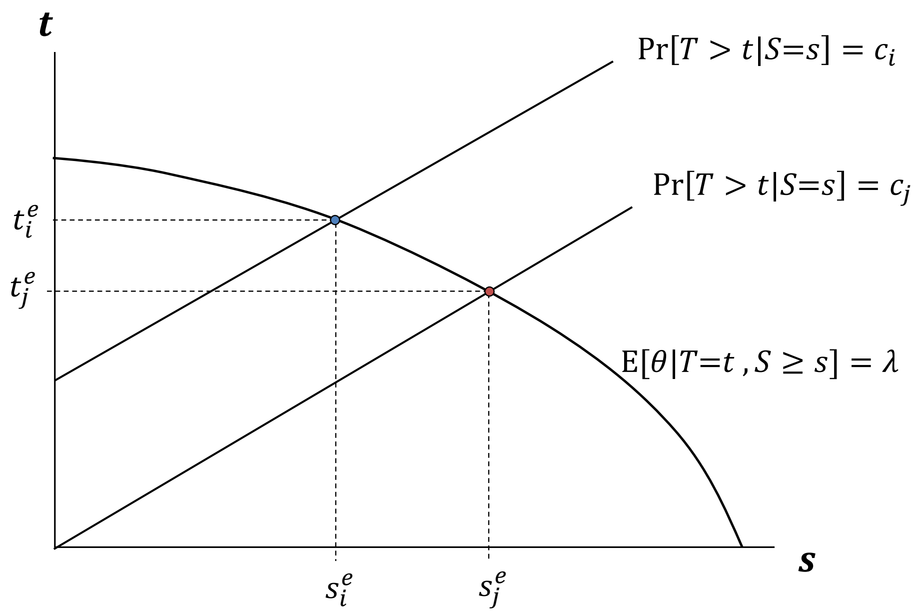

Figure 2: No-commitment benchmark equilibrium when sectors are identical except for $c_i < c_j$

□

Recall that the objective of the university depends on quality along two margins, as already introduced in Figure 1: the calibre of those entrants who were indifferent about applying (Marginal Applicants), and the calibre of those applicants whom the university was indifferent about admitting (Marginal Entrants). Let us label now these margins to facilitate the subsequent analysis.

Definition 3a (Marginal Entrant Quality)

$$MEQ_i(s_i, t_i) \equiv E[\theta_i | T_i = t_i, S_i \geq s_i]$$

Definition 3b (Marginal Applicant Quality)

$$MAQ_i(s_i, t_i) \equiv E[\theta_i | T_i \geq t_i, S_i = s_i]$$

We have already shown that when the university selects the best applicants, it sets admission thresholds to ensure that $MEQ_i(s_i, t_i)$ is constant across sectors (equation (10)). Since the expected quality of entrants depends also on $MAQ_i(s_i, t_i)$, it is natural to investigate how the 'iso-quality' levels corresponding to $MAQ(\cdot)$ compare to those of $MEQ(\cdot)$. Consider the following intuition.

- Suppose $f_{S_i}(\cdot)$ and $f_{T_i}(\cdot)$ have finite support, with respective upper bounds denoted $\overline{s}_i$ and $\overline{t}_i$ for $i \in I$
- Let $\hat{\theta}_i(s_i, t_i) \equiv \mathbb{E}[\theta_i | T_i = t_i, S_i = s_i]$
- At the boundary of the distribution of T_i we know that $MAQ_i(s_i, \overline{t}_i)$ takes the same value as $\hat{\theta}_i(s_i, \overline{t}_i)$, because by definition, T_i cannot take higher values than $\overline{t}_i$. For any other threshold $t_i < \overline{t}_i$, however, under sufficient monotonicity it must be the case that

  $$MAQ_i(s_i, t_i) > \hat{\theta}_i(s_i, t_i)$$

  so the levels of $MAQ_i(\cdot)$ cut the levels of $\hat{\theta}_i(\cdot)$ once from above
- Similarly, at the other boundary $MEQ_i(\overline{s}_i, t_i) = \hat{\theta}_i(\overline{s}_i, t_i)$, but for $s_i < \overline{s}_i$ we know that

  $$MEQ_i(s_i, t_i) > \hat{\theta}_i(s_i, t_i)$$

so the level sets of $MEQ_i(\cdot)$ cut the level sets of $\hat{\theta}_i(\cdot)$ at least once from below.

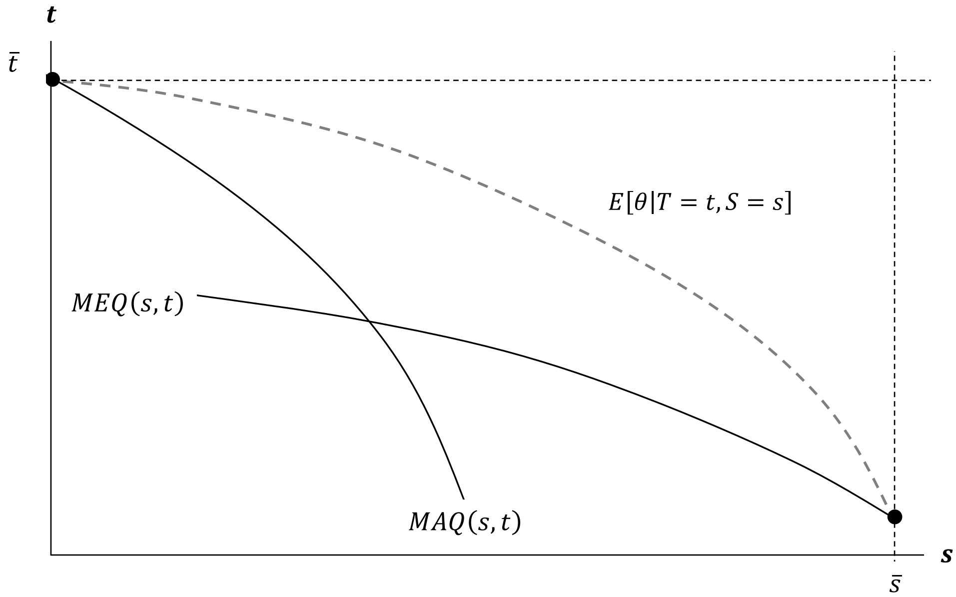

Figure 3: Intuition for single-crossing relationship between $MEQ(\cdot)$ and $MAQ(\cdot)$ in the finite support case

These facts, illustrated in Figure 3, imply that there is a level set of $MEQ_i(\cdot)$ which crosses the level set of $MAQ_i(\cdot)$ (at least once) from below. The following proposition strengthens this claim, albeit with additional conditions (all satisfied in the Gaussian case studied in the next section).

Proposition 4. *For each sector $i \in I$, suppose:*

(1) S_i, T_i are affiliated.

(2a) Conditional on $T_i = t_i$, for each t_i, S_i has decreasing mean remaining life; and

(2b) conditional on $S_i = s_i$, for each s_i, T_i has decreasing mean remaining life.

(3) $\mathbb{E}[\theta_i|S_i = s_i, T_i = t_i]$ is supermodular.

(4a) $s_i \to \mathbb{E}[\theta_i|T_i = t_i, S_i = s_i]$ is concave for each t_i; (4b) $t_i \to \mathbb{E}[\theta_i|T_i = t_i, S_i = s_i]$ is concave for each s_i.

It follows that

$$\frac{\frac{\partial}{\partial s_i} MEQ_i(s_i, t_i)}{\frac{\partial}{\partial t_i} MEQ_i(s_i, t_i)} \leq \frac{\frac{\partial}{\partial s_i}\mathbb{E}[\theta_i|T_i = t_i, S_i = s_i]}{\frac{\partial}{\partial t_i}\mathbb{E}[\theta_i|T_i = t_i, S_i = s_i]} \leq \frac{\frac{\partial}{\partial t} MAQ_i(s_i, t_i)}{\frac{\partial}{\partial s} MAQ_i(s_i, t_i)}.$$

Proof: See Appendix B.4

The single-crossing property established in Proposition 4, together with the second part of Proposition 3, imply that $MAQ_i(s_i^e, t_i^e) \leq MAQ_j(s_j^e, t_j^e)$ for $c_i \leq c_j$. This means that the no-commitment benchmark induces an unbalanced pool of applicants across sectors, in the sense that the worst low-cost applicant is of lesser quality. This is illustrated in Figure 4.

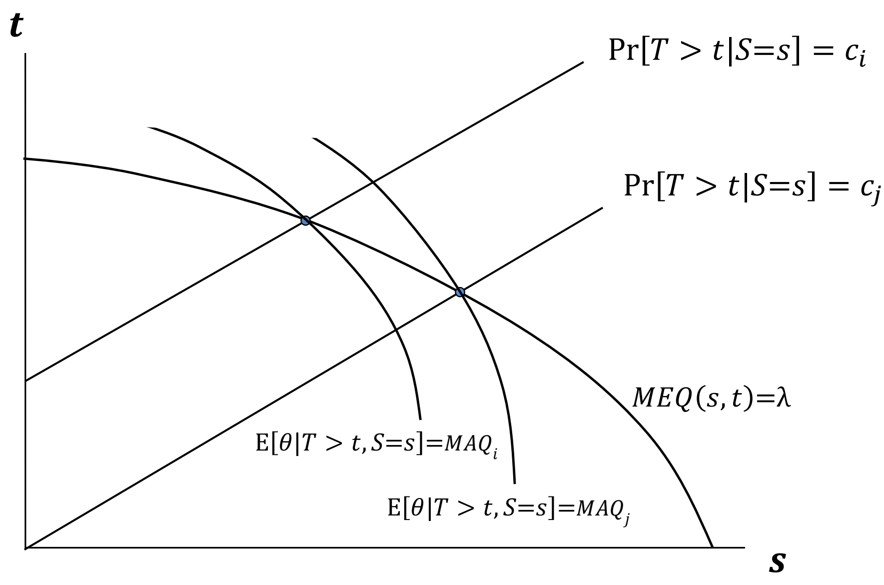

Figure 4: Levels of $MAQ_i(\cdot)$ and $MAQ_j(\cdot)$ for $c_i < c_j$ in the no-commitment benchmark (everything else equal across sectors)

To see how the university could benefit from commitment, suppose that there are only two sectors, and one has a very high cost of application. In such case, the benchmark rule would imply that the university should offer a lower threshold to the high-cost sector, yet there would be in equilibrium fewer but better high-cost applicants. Suppose that the university decided to promise instead an even lower admission requirement to the high-cost sector, together with a corresponding higher threshold to the low-cost counterpart, ensuring that capacity was still filled in expectation. Provided that applications were sufficiently responsive to changes in the admission policy, such lower threshold would lead to more but worse applications from the high-cost sector, and simultaneously, fewer but better applications from the low-cost sector. If the original gap in the equilibrium quality of applicants was sufficiently large, such intervention could mean that on average the university is better-off, because the improved quality of low-cost applications and entrants more than compensates the reduction in the quality of the high-cost counterpart. This intuition motivates the following formal characterisation of optimal admissions.

4 Optimal Admissions

Recall that Propositions 1 and 2 justify the optimality of *committed* cut-off rules. When a candidate observes an admission requirement t_i, which has been committed, she is indifferent about applying if and only if

$$\Pr[T_i \geq t_i | S_i = s_i] = c_i$$

As before, the signal level that solves this equation gives the candidates' best response $s_i(t_i; c_i)$. Accordingly, the university incorporates this in its optimisation problem to solve

$$\begin{aligned} \underset{t_1,...,t_n}{\text{MAX}} \quad & \textstyle\sum_{i=1}^n \alpha_i \mathbb{E}[\theta_i | T_i \geq t_i, S_i \geq s_i(t_i)] \Pr[T_i \geq t_i, S_i \geq s_i(t_i)] \\ \text{s.t.} \quad & \tau = \sum_{i=1}^n \alpha_i \Pr[T_i \geq t_i, S_i \geq s_i(t_i)] \end{aligned}$$

As before, rewritting the problem in terms of the joint distributions yields the following Lagrangean:

$$\begin{aligned} \mathcal{L}(t_1, ..., t_n) = & \sum_{i=1}^n \alpha_i \int_{-\infty}^{\infty} \int_{s_i(t_i)}^{\infty} \int_{t_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(T_i, S_i, \theta_i) \, dT_i \, dS_i \, d\theta \\ & + \lambda \left(\tau - \sum_{i=1}^n \alpha_i \int_{s_i(t_i)}^{\infty} \int_{t_i}^{\infty} f_{T_i,S_i}(T_i, S_i) \, dT_i \, dS_i \right) \end{aligned} \tag{12}$$

with FOCs given by

$$\begin{aligned}\frac{\partial \mathcal{L}(t_1, t_2, ..., t_n)}{\partial t_i} &= -\left(\int_{-\infty}^{\infty}\int_{s_i(t_i)}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(t_i, S_i, \theta_i)\; dS_i\, d\theta\right) \\ &- \left(\frac{\partial s_i(t_i)}{\partial t_i}\int_{-\infty}^{\infty}\int_{t_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(T_i, s_i(t_i), \theta_i)\; dT_i\, d\theta\right) \\ &+ \mu\left(\int_{s_i(t_i)}^{\infty} f_{T_i,S_i}(t_i, S_i)\; dS_i + \frac{\partial s_i(t_i)}{\partial t_i}\int_{t_i}^{\infty} f_{T_i,S_i}(T_i, s_i(t_i)) +\; dT_i\right) \\ &= 0 \end{aligned} \tag{13}$$

Proposition 5. *The optimal admissions policy equates weighted average marginal applicant and entrant qualities. Specifically, for each sector* $i \in I$*,*

$$\rho_i\, MEQ_i(s_i(t_i), t_i) + (1 - \rho_i)\, MAQ_i(s_i(t_i), t_i) = \lambda$$

where

$$\rho_i = \frac{\Pr[T_i = t_i, S_i \geq s_i(t_i)]}{\Pr[T_i = t_i, S_i \geq s_i(t_i)] + \frac{\partial s_i(t_i)}{\partial t_i}\Pr[T_i \geq t_i, S_i = s_i(t_i)]}$$

Proof. From Definition 3 we have

$$MAQ_i(s_i(t_i), t_i) = \frac{\int_{-\infty}^{\infty}\int_{t_i}^{\infty} \theta_i f_{T_i,S_i,\theta_i}(T_i, s_i(t_i), \theta_i)\; dT_i\, d\theta}{\int_{s_i(t_i)}^{\infty} f_{T_i,S_i}(t_i, S_i)\; dS_i}$$

Using this in equation (13) and rearranging terms completes the proof. □

Proposition 5 offers an intuitive characterisation of the optimal admissions: *choose* $t_1, t_2, ...t_n$ *to equate weighted average marginal applicant and entrant qualities, with the weight determined by a parameter measuring the behavioural response of students to admissions requirements.* The direct implication of this rules is that selection must consider the feedback on the quality of the marginal applicants across sectors, as it is not generally efficient to simply equate the quality of the marginal entrants. As we would expect, whenever students are unresponsive to admissions, it follows that $\frac{\partial s_i(t_i)}{\partial t_i} = 0$ and $\rho_i = 1$, so Proposition 5 reduces to equation (10). In general, however, the optimal mechanism is different to the benchmark case where the university selects the best who apply.

Corollary. *For any sectors $i, j \in I$, the optimal admissions policy satisfies*

$$min\{MEQ_i(s_i(t_i), t_i), MAQ_i(s_i(t_i), t_i)\} \leq max\{MEQ_j(s_j(t_j), t_j), MAQ_j(s_j(t_j), t_j)\}$$

This corollary is useful from an empirical point of view. If the quality of the marginal applicants and entrants could be identified from the data, it would be possible to reject efficiency in admissions by testing the condition above. In the next section we explore the Gaussian case and provide some intuition regarding further testable predictions.

5 The Gaussian case

Suppose that (T_i, S_i, θ_i) are jointly normally distributed with mean μ_i and covariance matrix

$$\Sigma_i = \begin{pmatrix} \sigma^2_{T_i} & \sigma_{S_iT_i} & \sigma_{\theta_iT_i} \\ \sigma_{S_iT_i} & \sigma^2_{S_i} & \sigma_{\theta_iS_i} \\ \sigma_{\theta_iT_i} & \sigma_{\theta_iS_i} & \sigma^2_{\theta_i} \end{pmatrix}$$

Given this, $\Pr[T_i \geq t_i | S_i = s_i] = c_i$ can be expressed as $F^{-1}(1 - c_i) = \frac{t_i - \mu_{T_i|s_i}}{\sigma_{T_i|s_i}}$, where $\mu_{T_i|s_i}$ and $\sigma_{T_i|s_i}$ denote the mean and variance of the conditional distribution of T_i given $S_i = s_i$, and $F(\cdot)$ is the standard normal CDF. Solving for s_i thus obtains

$$s_i(t_i; c_i) = \frac{t_i + \mu_{S_i} - \sigma_{T_i|s_i} F^{-1}(1 - c_i)}{\beta_{T_iS_i}} \tag{14}$$

where β_{T_i,S_i} is the regression coefficient of T_i on S_i.

From (14) it is easy to see that the responsiveness of applications is simply given by the inverse of the regression coefficient of the private signals,

$$\frac{\partial s_i(t_i; c_i)}{\partial t_i} = \frac{1}{\beta_{T_iS_i}} = \frac{\sigma^2_{S_i}}{\sigma_{S_iT_i}} \tag{15}$$

which means that, in the Gaussian case, the responsiveness of applications to changes in the admission policy –for any given sector– will be larger when (i) the dispersion in the admissions test is small compared to the dispersion in the candidates' private information, and (ii) when the admissions test co-varies less with the candidates' private information.

The Gaussian case also allows a convenient parameterisation of $MAQ_i(s_i, t_i)$ and $MEQ_i(s_i, t_i)$ in terms of the partial regression coefficients $\beta_{\theta T_i;S_i}$ and $\beta_{\theta S_i;T_i}$:

$$\begin{aligned} MEQ_i(s_i,t_i) &= E[\theta_i | T_i = t_i, S_i \geq s_i] \\ &= \beta_{\theta_i T_i;S_i} t_i + \beta_{\theta_i S_i;T_i} \mu_{S_i|t_i} + h\left(\frac{s_i - \mu_{S_i|t_i}}{\mu_{S_i|t_i}}\right) \beta_{\theta_i S_i;T_i} \sigma_{S_i|t_i} \qquad (16) \\ MAQ_i(s_i,t_i) &= E[\theta_i | T_i \geq t_i, S_i = s_i] \\ &= \beta_{\theta_i S_i;T_i} s_i + \beta_{\theta_i T_i;S_i} \mu_{T_i|s_i} + h\left(\frac{t_i - \mu_{T_i|s_i}}{\mu_{T_i|s_i}}\right) \beta_{\theta_i T_i;S_i} \sigma_{T_i|s_i} \qquad (17) \end{aligned}$$

where $h(\cdot)$ denotes the standard Gaussian hazard rate function.

A Numerical Example. Suppose there are only two sectors $i \in \{1, 2\}$ with $c_1 < c_2$. In order to explore the solution numerically, Figure (5) presents the best response functions —and the corresponding equilibrium— for a particular parametric configuration.

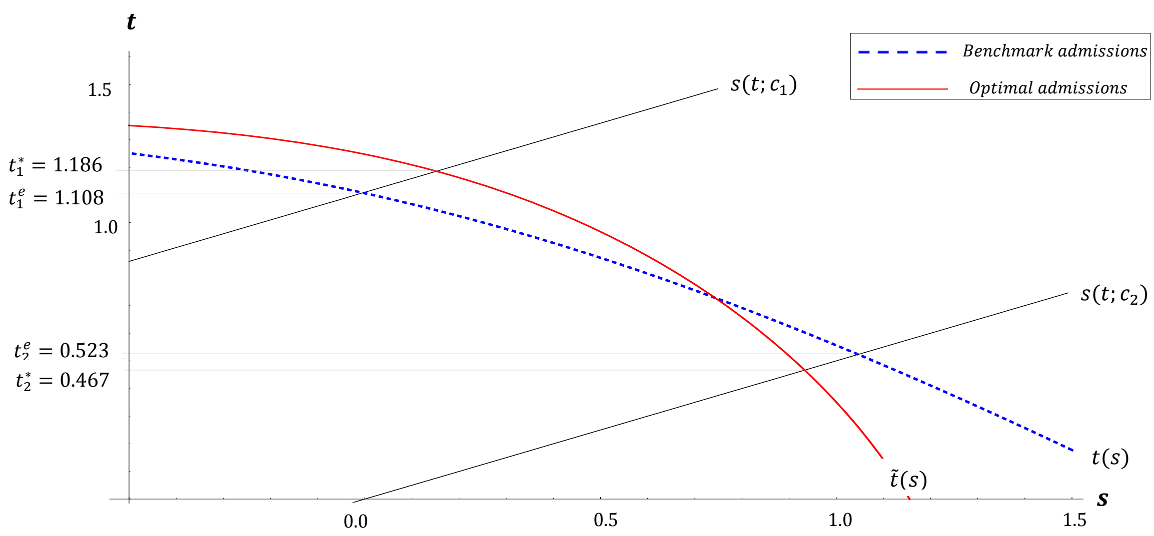

Figure 5: Equilibrium admissions for a Gaussian example with $\alpha_1 = \alpha_2 = 0.5$; $c_1 = 0.1$, $c_2 = 0.5$; $\tau = 0.1$; $\mu_1 = \mu_2 = (0, 0, 0)$; $\sigma_{S_i T_i} = \sigma_{\theta_i T_i} = \sigma_{\theta_i S_i} = 0.5$; $\sigma^2_{\theta_i} = \sigma^2_{T_i} = \sigma^2_{S_i} = 1$

As it can be appreciated, the curves marking the university's best response functions do not depend on the sector. Although this is not generally the case, in the considered example it is true because we have assumed that the only source of heterogeneity across sectors is the application cost. Also, notice that the red line depicting the university's best response for the optimal policy is steeper and cuts from above the dashed blue line marking the best response under the no-commitment benchmark. This is because, as we showed in the previous section, the iso-quality curves corresponding to $MAQ(\cdot)$ are steeper than those corresponding to $MEQ(\cdot)$ (Proposition 4), and optimal admissions equate weighted average marginal applicant and entrant qualities (Proposition 5).

To visualise the comparative impact of selection in applications and admissions, Figure 6 plots the conditional probability density functions of θ_i that results from truncating T_i and S_i below the equilibrium thresholds.[3] To complete the numerical analysis, Table 1 summarises the equilibrium outcomes under the benchmark and optimal policies.

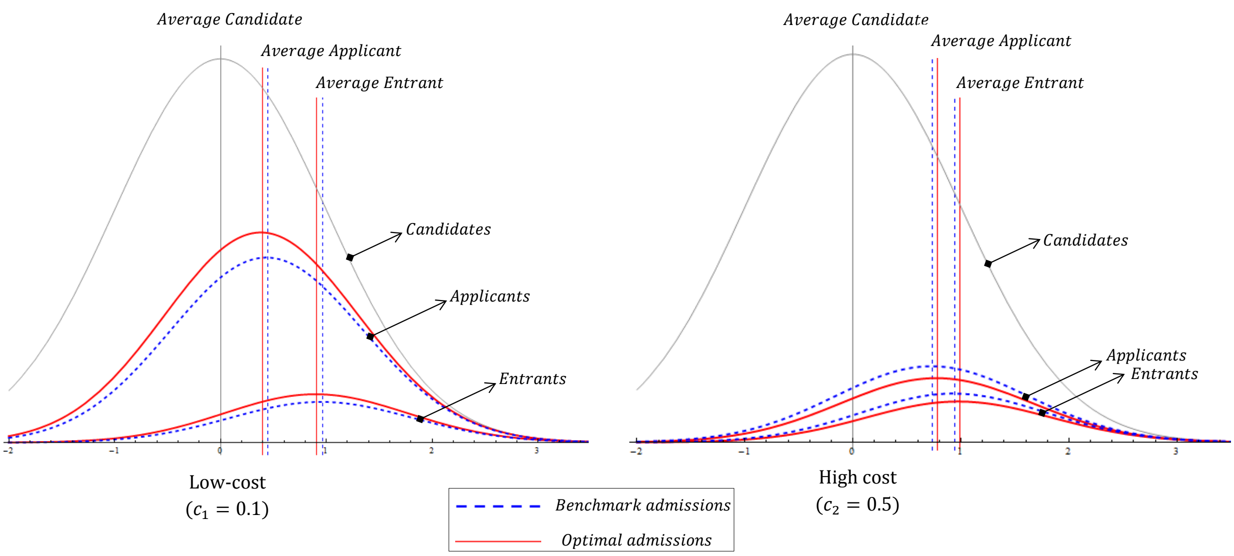

Figure 6: Unscaled equilibrium distribution of quality for a numerical Gaussian example with $\alpha_1 = \alpha_2 = 0.5$; $c_1 = 0.1$, $c_2 = 0.5$; $\tau = 0.1$; $\mu_1 = \mu_2 = (0, 0, 0)$; $\sigma_{S_i T_i} = \sigma_{\theta_i T_i} = \sigma_{\theta_i S_i} = 0.5$; $\sigma^2_{\theta_i} = \sigma^2_{T_i} = \sigma^2_{S_i} = 1$

[3]Note that the truncated distributions in Figure 6 have not been scaled, so the area under each curve reflects the size of the relevant population. To be precise, Figure 6 plots

$$f(\theta_i | S_i > s_i) = \int_{s_i}^{\infty} f_{T_i, S_i, \theta_i}(T_i, S_i, \theta_i)\, dS_i$$

$$f(\theta_i | S_i > s_i, T > t_i) = \int_{s_i}^{\infty} \int_{t_i}^{\infty} f_{T_i, S_i, \theta_i}(T_i, S_i, \theta_i)\, dT_i\, dS_i$$

where (s_i, t_i) is set to take the values of (s_i^e, t_i^e) and (s_i^*, t_i^*).

Benchmark equilibrium							
Cost	Pop shares	Application shares	Entry Thresholds	Entrant shares	Quality of applicants	Quality of entrants	Total Quality
0.1	50%	77.26%	1.10794	54.78%	0.3977	0.9117	0.946395
0.5	50%	22.74%	0.523273	45.22%	0.7813	0.9885	

Optimal equilibrium							
Cost	Pop shares	Application shares	Entry Thresholds	Entrant shares	Quality of applicants	Quality of entrants	Total Quality
0.1	50%	71.47%	1.18602	45.88%	0.4487	0.9622	0.951561
0.5	50%	28.53%	0.466524	54.12%	0.7359	0.9426	

Table 1: Equilibrium outcome summary for a numerical Gaussian example with $\alpha_1 = \alpha_2 = 0.5$; $c_1 = 0.1$, $c_2 = 0.5$; $\tau = 0.1$; $\mu_1 = \mu_2 = (0,0,0)$; $\sigma_{S_iT_i} = \sigma_{\theta_iT_i} = \sigma_{\theta_iS_i} = 0.5$; $\sigma^2_{\theta_i} = \sigma^2_{T_i} = \sigma^2_{S_i} = 1$

As it can be checked, the benchmark entails a lower entry requirement for the high-cost sector vis-a-vis the low-cost sector, yet the quality of high-cost applicants and entrant is significantly higher (almost twice for the average applicant). In terms of representation in admissions, the high-cost candidates are underrepresented, in the sense that they take less places at the university than their population share. Interestingly, however, here the optimal admissions also entail a lower requirement for the high-cost (vis-a-vis the low-cost) sector, but the optimal threshold for them is lower than the benchmark procedure would suggest. In fact Table 1 shows that moving from the benchmark to the optimal procedure implies that the gap in the quality of applicants across sectors is narrowed and the high-cost sector becomes *over* represented in admissions. This substantiates the intuition described in the previous section: commitment allows the decision-maker to promise the high-cost (low-cost) students a lower (higher) entry requirement than would be necessary to select the best applicants.

Testable Predictions: Special Cases. The Gaussian case can also be used to illustrate how to produce testable predictions by introducing further structure to the problem. Suppose that all relevant public information about the background of a candidates is summarised in the kind of school she attends, so that $I = \{m, p\}$, with m and p denoting *maintained* and *private* school sectors, respectively. Define the students' private information S_i as 'encouragement', and suppose that this encouragement is not directly informative about quality. To be specific, let (T_i, S_i, θ_i) be jointly normally distributed and suppose that S_i is orthogonal to θ_i given T_i.

Suppose now that private-school candidates are coached by tutors who know very well the admissions process and specifically encourage those candidates who are more likely to score high on the admissions tests. In contrast, suppose that maintained-school students are encouraged across the board (or equivalently, that their tutors cannot encourage them differently because of lack of familiarity with the admissions process).[4] For the sake of simplicity, take the extreme case in which the regression coefficients of T_i on S_i for $i \in \{m, p\}$ are:

$$\beta_{T_p S_p} = +\infty \quad \text{and} \quad \beta_{T_m S_m} = 0 \tag{18}$$

The assumption above implies that private-school tutors are perfectly able to induce encouragement proportionally to the candidates' potential admissions test scores if they apply, yet are not able to observe quality directly −not any better than the university could with the same available information. By contrast, encouragement in maintained schools is not related at all to the candidates' calibre.

Recall that we showed in equation (15) that the responsiveness of candidates for the Gaussian model corresponds to $\frac{1}{\beta_{T_i S_i}}$. Hence, encouragement as we have defined it in equation (18) implies that t_p has no effect on private-school applications, while t_m has an infinitely large effect on candidates from maintained schools. In other words, because of coaching, students from private schools would be irresponsive to changes in the admission requirements, in stark contrast to students from maintained schools, who would be extremely sensitive to the treatment they would receive at the admissions stage if they applied.

Taking the above into account, Proposition 5 for this special case reduces to:

$$\mathbb{E}[\theta | T_p = t_p] = \mathbb{E}[\theta | T_m \geq t_m] \tag{19}$$

[4] The report by the National Council for Education Excellence in England from 2008 makes references to issues similar to these.

This condition states: *choose t_1 and t_2 to equate the expected quality of* ***marginal*** *private-school entrants, with the expected quality of the* ***average*** *maintained-school entrants.* It is an efficiency prediction which can be tested with admissions data, since the expected quality of the worst admitted applicant from private schools and the mean expected quality of entrants from maintained schools can be estimated by looking at the outcomes of university entrants (where the outcomes of interest are suitably defined by the university's notion of quality).

6 Discussion of results

The analysis that has been presented so far provides a characterisation of the optimal form of discrimination on the basis of observable group characteristics. This section relates our results to classic notions of discrimination in economics. In what follows we build on the concepts and definitions presented in Fang and Moro (2010).

A voluminous strand of literature on the economics of discrimination studies situations where inequality in treatment arises because a decision-maker, who receives information about a fixed pool of candidates with different group identity, uses observable characteristics of these individuals as a proxy for unobservable, but outcome-relevant characteristics. In the classic conceptualisation of the problem by Phelps (1972), and the literature that originated from it, exogenous differences between groups of candidates can account for the fact that two workers who give the decision-maker the same signal about quality, end up receiving in equilibrium a different treatment depending on their group identity. This notion of discrimination is traditionally referred to as *statistical* discrimination. Different studies in the literature concentrate on variations in the character of the groups' intrinsic differences, consequently finding different results regarding the nature of inequality in treatment. Yet the key feature of statistical discrimination, as it is traditionally studied in the literature, is that in spite of it, in equilibrium candidates with the same expected quality (conditional on all available information) receive the same treatment. So even though same-signal candidates may receive different treatment, under statistical discrimination all candidates are paid their expected productivity.

Our analysis of the benchmark admissions problem relates directly to the aforementioned notion of discrimination. As stated in Proposition 5, if candidates are irresponsive to changes in the admissions policy, the optimal selection procedure is equivalent to implementing the benchmark rule, and as shown in Proposition 3, in such case individuals with higher cost of application would receive in equilibrium a lower selection cut-off. Evidently, this constitutes a standard form of positive statistical discrimination from exogenous differences, as discussed above: a decision-maker implementing the benchmark rule uses observable background characteristics of individuals as a proxy for relevant unobservable characteristics, so in equilibrium, exogenous differences between candidates, such background-specific costs of application, imply that two candidates with the same test score in the selection test may receive a different treatment depending on their background −yet two candidates with the same conditional expected quality would, by definition of the benchmark rule, receive the same treatment regardless of background. Consequently, implementing the benchmark rule implies that all candidates are paid their expected productivity, so there is no *economic* discrimination.

The importance of our approach becomes clear once we relax the assumption that applicants are irresponsive to the selection procedure. Proposition 5 shows that the optimal selection mechanism is not generally equivalent to the benchmark rule of selecting the applicants with the highest expected quality, so in equilibrium, exogenous background-specific differences can explain differential treatment for candidates with the same predicted quality conditional on all available information. This is a novel mechanisms through which economic discrimination can be explained from exogenous group differences, without assuming that the decision-maker has a preference bias towards individuals from certain backgrounds.

At the heart of our analysis is the fact that different groups of candidates (sectors) are competing for a fixed number of vacancies. Recall the numerical example: the only reason why high-cost candidates are being held to a lower standard in equilibrium is the fact that a large number of low-cost candidates finds it too easy to apply, so they do not screen themselves sufficiently on the basis of their private information. In this sense, we show that economic discrimination may efficiently arise in equilibrium due to the externality that the applications of one group impose on the other groups.

Other studies have explored alternative mechanisms leading to economic discrimination through inter-group externalities. Mailath, Samuelson and Shaked (2000) show that economic discrimination in the labour market may arise from search frictions. In their conceptualisation of the problem, discrimination is sustained in equilibrium as a "self-fulfilling stereotype". They argue that, if firms focus their search in one type of candidates, this depresses the incentives of the other candidates to acquire skills, which in equilibrium justifies the firms' decision to target search in the initial group to begin with. Our result highlights a different channel, one which can explain positive discrimination in favour of a group simply because it is disadvantaged in an exogenous way which correlates with an observable characteristic.

Our framework proposes a structured and transparent analysis of (i) the objective of selection, (ii) the domain of feasible selection policies that can be implemented, and (iii) the sources that lead to differential treatment. Stating the problem in these terms, and concentrating on *economic* discrimination, seems especially useful in the current context of polarised opinions regarding the implications of affirmative action in university admissions; particularly in the U.S., where according to the current standards of legal scrutiny established by the Supreme Court, a preferential policy can be permitted if it constitutes a *narrowly tailored* means of furthering a *compelling state interest* (Holzer and Neumark 2010).

7 Conclusions

This paper characterised the optimal selection policy from a pool of applicants taking into account the fact that the pool of applicants is endogenous. We focused on admissions to selective universities, but our framework speaks to a broad class of problems were scarce resources (e.g. jobs, credits, public services) are allocated through selection procedures that consider outcome-relevant characteristics of applicants.

Based on the premise that the choice of applying to take part in a costly and uncertain selection process depends on the probability of being selected, we developed a general mechanism design problem and showed that under sufficient conditions the optimal mechanism can be equivalently implemented through a simple and intuitive cut-off rule, where the decision-maker commits to select any applicant scoring above a minimum threshold in a relevant selection test. Using this result, we turned to study a simpler equilibrium model where cut-off admissions and applications best-respond to each other. This allowed us to characterise the optimal use of public information about candidates in the design of the selection rule. We showed that the common principle of "choosing the best who apply" (a practice that we called the *benchmark rule*) is not generally optimal, because such principle fails to take into account the feedback that selection has on the composition of the pool of applicants.

Specifically, we showed that the optimal selection procedure requires equating weighted average marginal applicant and entrant qualities, with weights determined by the behavioural response of candidates to changes in the admission requirements. This, it was argued, allows exogenous background-specific differences across candidates (e.g. application costs) to explain differential treatment for applicants with the same predicted quality conditional on all available information. That being the case, our results provide a justification for positive *economic* discrimination.

A natural extension of our framework is to consider endogenous application costs, for example through monetary transfers. Although a general form of the mechanism design problem with monetary transfers is beyond the scope of this paper, the Appendix B1 provides a motivating analysis where the university can choose application costs directly, in order to maximise the expected quality of entrants. We show that the expected quality of the entrants is non-monotonic in the application cost, reaching the maximum at some intermediate level that induces sufficient screening both in applications and admissions. To be specific, the appendix shows that under the optimal cost policy, the university equates marginal applicant qualities.

Although concrete policy prescriptions will require a detailed assessment of issues not included in our framework —such as the endogenous choice of candidates to acquire characteristics prior to entering the selection competition—, our results do suggest that the policy debate should incorporate an empirical analysis aimed at (i) identifying the quality of marginal applicants and how they compare to marginal entrants, and (ii) estimating the responsiveness of different groups of applicants to changes in the admission requirements.

B Appendix for Chapter 2

B.1 Costs as choice variables

Suppose that the university can choose the application costs $\boldsymbol{c} = \{c_1, c_2, ..., c_n\}$ in addition to the admission requirements $\boldsymbol{t} = \{t_1, t_2, ..., t_n\}$. As before, in this case candidates are indifferent about applying iff $\Pr[T_i \leq t_i | S_i = s_i] = 1 - c_i$, and thus the candidates' best response $s_i(c_i, t_i)$ is given by the realisation of S_i which makes a candidate indifferent.

In this case, the university's problem is

$$\underset{\boldsymbol{c}, \boldsymbol{t}}{\text{MAX}} \quad \sum_{i=1}^{n} \frac{\alpha_i}{\tau} \mathbb{E}[\theta | T_i \geq t_i, S_i \geq s_i(c_i, t_i)] \Pr[T_i \geq t_i, S_i \geq s_i(c_i, t_i)]$$

$$\text{s.t.} \qquad \tau = \sum_{i=1}^{n} \alpha_i \Pr[T_i \geq t_i, S_i \geq s_i(c_i, t_i)]$$

and the corresponding Lagrangean is given by

$$\begin{aligned}\mathcal{L}(\boldsymbol{c}, \boldsymbol{t}) = & \sum_{i=1}^{n} \alpha_i \int_{-\infty}^{\infty} \int_{s_i(c_i,t_i)}^{\infty} \int_{t_i}^{\infty} \theta f_{T_i,S_i,\theta}(T_i, S_i, \theta)\; dT_i\, dS_i\, d\theta \\ & + \lambda \left(\tau - \sum_{i=1}^{n} \alpha_i \int_{s_i(c_i,t_i)}^{\infty} \int_{t_i}^{\infty} f_{T_i,S_i}(T_i, S_i)\; dT_i\, dS_i \right)\end{aligned}$$

Taking derivatives with respect to c_i, the FOCs for costs are

$$\begin{aligned}\frac{\partial \mathcal{L}(\boldsymbol{c}, \boldsymbol{t})}{\partial c_i} = & - \alpha_i \frac{\partial s_i(c_i, t_i)}{\partial c_i} \int_{-\infty}^{\infty} \int_{t_i}^{\infty} \theta f_{T_i,S_i,\theta}(T_i, s_i(c_i, t_i), \theta)\; d\theta\; dT_i + \\ & \alpha_i \lambda \frac{\partial s_i(c_i, t_i)}{\partial c_i} \int_{t_i}^{\infty} f_{T_i,S_i}(T_i, s_i(c_i, t_i))\; dT_i = 0\end{aligned}$$

As noted in the proof of Proposition 5, we know that

$$\begin{aligned}MAQ_i(s_i(c_i, t_i), t_i) \equiv & E[\theta | T_i \geq t_i, s_i(c_i, t_i)] \\ = & \frac{\int_{-\infty}^{\infty} \int_{t_i}^{\infty} \theta f_{T_i,S_i,\theta}(T_i, s_i(c_i, t_i), \theta)\; dT_i\, d\theta}{\int_{t_i}^{\infty} f_{T_i,S_i,\theta}(T_i, s_i(c_i, t_i))\; dT_i}\end{aligned}$$

so the FOCs for $\boldsymbol{c}$ are

$$MAQ_i(s_i(c_i, t_i), t_i) = \lambda \qquad \text{for} \quad i = 1, 2, ..n \tag{B.1.1}$$

It follows that optimal costs are interior because, as stated in the proof of Proposition 3, affiliation of the random vector (θ_i, T_i, S_i) implies that $\frac{\partial s_i(c_i,t_i)}{\partial c_i} \geq 0$. Therefore, noting that the FOCs for $\boldsymbol{t}$ are the same from Proposition 5, condition B.1.1 implies that the optimal combination of costs and admissions requirements must be such that

$$MAQ_i(s_i(c_i, t_i), t_i) = MEQ_i(s_i(c_i, t_i), t_i) = \lambda \qquad \text{for} \quad i = 1, 2, ..n \tag{B.1.2}$$

Using the same parameterisation from the Gaussian example in Section 5, but allowing costs to be endogenous, the efficiency condition B.1.2 implies that the university would choose $t_1^* = t_2^* = 0.772$ and $c_1^* = c_2^* = 0.328$. By comparison with the results from Table 1, where we assumed that costs were fixed at $c_1 = 0.1$ and $c_2 = 0.5$, we can check that the university would like to transfer almost one third of the cost of application from the high to the low cost candidates; clearly, this would be coupled with an increase (decrease) in the admissions thresholds for the high (low) cost candidates.

B.2 Proof for Proposition 1.

Proposition 1:

Suppose

1. *For each sector $i \in I$, the random vector (T_i, S_i, θ_i) is affiliated.*

2. *For each sector $i \in I$, $\theta_i \perp\!\!\!\perp T_i \mid S_i$.*

Then, for any feasible constrained direct revelation mechanism there is a feasible cut-off rule (of the form for each sector $i \in I$ admit those applicants who score at least $T_i \geq t_i$ for some t_i) which implements the same mass of students from each sector but with higher average quality.

Proof. Given a cut-off mechanism $p = (p_1, ..., p_n)$, applicants observing s_i in sector $i \in I$ apply with probability $q_i(s_i)$ where

$$\int p_i(t_i) f_{T_i|S_i}(t_i|s_i)dt_i \begin{array}{c} > \\ = \\ < \end{array} c_i \Leftrightarrow q_i(s_i) \begin{array}{c} = 1 \\ \in [0,1] \\ = 0. \end{array}$$

For the direct mechanism $(\widehat{p}, \widehat{q})$, by incentive compatibility for applications,

$$\int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i \begin{array}{c} > \\ = \\ < \end{array} c_i \quad \Leftrightarrow \quad \widehat{q}_i(s_i) \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array}$$

The real line can be partitioned into intervals where $\int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i - c_i$ is respectively positive, negative or zero. Hence, if negative, incentive compatibility implies $\hat{q}_i(s_i) = 0$. If zero in an interval, then $\frac{\partial}{\partial s_i} \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i = 0$ on that interval. And if positive, $\hat{q}_i(s_i) = 1$. Incentive compatibility requires that for all $\tilde{s}_i$ in a neighborhood in which $\hat{q}_i(s_i) = 1$, that

$$\int \widehat{p}_i(\tilde{s}_i, t_i)) f_{T_i|S_i}(t_i|s_i)dt_i - c_i \leq \int \widehat{p}_i(s_i, t_i)) f_{T_i|S_i}(t_i|s_i)dt_i - c_i.$$

Hence, in such a neighborhood, the following first-order condition obtains

$$\frac{\partial \int \widehat{p}_i(\tilde{s}_i, t_i) f_{T_i|S_i}(t_i|s_i) dt_i}{\partial \tilde{s}_i} |_{\tilde{s}_i = s_i} = 0.$$

Lemma: Let p_i be an acceptance cut-off rule for sector $i \in I$, then for any direct mechanism $(\widehat{p}, \widehat{q})$,

$$q_i(s_i) \int p_i(t_i) f_{T_i|S_i}(t|s_i) dt_i - \widehat{q}_i(s_i) \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i) dt_i$$

has a single change of sign from negative to positive.

Proof of Lemma. We can restrict attention to the region where $q_i(s_i) = 1$ since there is evidently no crossings when $q_i(s_i) = 0$ and given the strict affiliation assumption, $0 < q_i(s_i) < 1$ occurs with probability zero ($\int p_i(t_i) f_{T_i|S_i}(t_i|s_i) dt_i$ is strictly increasing).

Given this, we can also restrict attention to the case where $\hat{q}_i(s_i) = 1$.

- If $\hat{q}_i(s_i) = 0$, then $q_i(s_i) \int p_i(t_i) f_{T_i|S_i}(t_i|s_i) dt_i \geq \widehat{q}_i(s_i) \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i) dt$ as required.
- If $0 < \hat{q}_i(s_i) < 1$, then

$$\widehat{q}_i(s_i) \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i) dt = \widehat{q}_i(s_i) c_i < c_i \leq \int p_i(t_i) f_{T_i|S_i}(t|s_i) dt_i$$

Hence, it suffices to show that $\int p_i(t_i) f_{T_i|S_i}(t_i|s_i) dt_i - \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i) dt_i$ has a single change of sign in the region where

$$\frac{\partial \int \widehat{p}_i(\tilde{s}_i, t_i) f_{T_i|S_i}(t_i|s_i) dt_i}{\partial \tilde{s}_i} |_{\tilde{s}_i = s_i} = 0$$

Suppose for some $s_i = s_i', \int p_i(t_i) f_{T_i|S_i}(t_i|s_i') dt_i = \int \widehat{p}_i(s_i', t_i) f_{T_i|S_i}(t_i|s_i') dt$. Since $p_i(t_i) - \widehat{p}_i(s_i', t_i)$ has a single sign change from negative to positive as t_i traverses the real line from minus to plus infinity, it follows from affiliation (Karlin (1967), Theorem 3.1 variation diminishing property) that for $s_i'' > s_i'$,

$$\int p_i(t_i) f_{T_i|S_i}(t_i|s_i'')dt_i > \int \widehat{p}_i(s_i', t_i) f_{T_i|S_i}(t_i|s_i'')dt_i$$

Therefore,

$$\frac{d \int p_i(t_i) f_{T_i|S_i}(t_i|s_i')dt_i}{ds_i} - \frac{\partial \int \widehat{p}_i(s_i', t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{\partial s_i} |_{s_i'=s_i} > 0.$$

Which, together with the incentive compatibility condition

$$\frac{\partial \int \widehat{p}_i(\tilde{s}_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{\partial \tilde{s}_i} |_{\tilde{s}_i=s_i} = 0$$

implies that at $s_i = s_i'$,

$$\frac{d \int p_i(t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{ds_i} - \frac{d \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{ds_i} > 0.$$

Hence, $\int p_i(t_i) f_{T_i|S_i}(t_i|s_i)dt_i = \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i$ implies

$$\frac{d \int p_i(t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{ds_i} > \frac{d \int \widehat{p}_i(s_i, t_i) f_{T_i|S_i}(t_i|s_i)dt_i}{ds_i}$$

This in turn implies the required single crossing condition. □

To complete the proof of Proposition 1, we show that for any constrained direct mechanism $(\widehat{p}, \widehat{q})$ there is a cut-off rule admission policy p and associated best response q such that

$$\int \int p_i(t_i) q_i(s_i) f_{S_i,T_i}(s_i, t_i) ds_i dt_i = \int \int \hat{p}_i(s_i, t_i) \hat{q}_i(s) f_{S_i,T_i}(s_i, t_i) ds_i dt_i \qquad \text{(B2.1)}$$

$$\begin{aligned} &\int \int E[\theta_i | T_i = t_i, S_i = s_i] p_i(t_i) q_i(s_i) f_{T_i,S_i}(t_i, s_i) ds_i dt_i \\ &\geq \int \int E[\theta_i | T_i = t_i, S_i = s_i] \hat{p}_i(s_i, t_i) \hat{q}_i(s_i) f_{S_i,T_i}(s_i, t_i) ds_i dt_i \qquad \text{(B2.2)} \end{aligned}$$

First, note that more lenient cut-off rules attract more applicants, so more lenient acceptance cut-off rules increase the mass of students admitted, continuously so given the assumption that (S_i, T_i) has no atoms. Hence, there is a cut-off pair p_i, q_i satisfying (B2.1). By Assumption 1, we may write

$$E[\theta|T_i = t, S_i = s] = \varphi(s).$$

Hence,

$$\begin{aligned}
&\int E[\theta_i|T_i = t_i, S_i = s_i]p_i(t_i)q_i(s)f_{T_i|S_i}(t_i|s_i)dt_i - \\
&\int E[\theta_i|T_i = t_i, S_i = s_i]\hat{p}_i(s_i, t_i)\widehat{q}_i(s_i)f_{T_i|S_i}(t_i|s_i)dt_i \\
&= \int \varphi(s_i)\left(q_i(s_i)\int p_i(t_i)f_{T_i|S_i}(t_i|s_i)dt_i - \hat{q}_i(s_i)\int \hat{p}_i(s_i, t_i)f_{T_i|S_i}(t_i|s_i)dt_i\right)ds_i.
\end{aligned}$$

Recall,

$$q_i(s_i)\int p_i(t)f_{T_i|S_i}(t_i|s_i)dt_i - \hat{q}_i(s_i)\int \hat{p}_i(s_i, t_i)f_{T_i|S_i}(t_i|s_i)dt_i$$

has the single crossing property from negative to positive. Also, by affiliation, $\varphi(s)$ is increasing in s, therefore for any $k \in \mathbb{R}$, $\varphi(s) - k$ has a single crossing property, from negative to positive. Therefore, for a suitably chosen value of $k \in \mathbb{R}$,

$$\int (\varphi(s_i) - k)\left(q_i(s_i)\int p_i(t_i)f_{T_i|S_i}(t_i|s_i)dt_i - \hat{q}_i(s_i)\int \hat{p}_i(s_i, t_i)f_{T_i|S_i}(t_i|s_i)dt_i\right)f_{T_i|S_i}(s_i)ds_i \geq 0.$$

Rearranging and using (B2.1) gives

$$\int\int \varphi(s_i)q_i(s_i)p_i(t_i)f_{S_i,T_i}(s_i, t_i)dt_i ds_i \geq \int\int \varphi(s_i)\hat{q}_i(s_i)\hat{p}_i(s_i, t_i)f_{S_i,T_i}(s_i, t_i)dt_i ds_i,$$

which is the desired result. □

B.3 Proof for Proposition 2.

Proposition 2: *Suppose*

1. *For each sector $i \in I$, the random vector (T_i, S_i, θ_i) is affiliated.*

2. *For each sector $i \in I$, $E[\theta_i \mid S_i = s_i, T_i = t_i]$ is an additive function.*

Then for any feasible probabilistic acceptance mechanism, there is a feasible acceptance cut-off rule (of the form for each sector $i \in I$ admit with probability one for those applicants who score at least $T_i \geq t_i$ for some t_i) which implements the same mass of students from each sector but with higher average quality.

Proof. Let $\widehat{p} : \mathbb{R}^{\mathrm{n}} \rightarrow [0,1]^{\mathrm{n}}$ be a feasible probabilistic acceptance mechanism and $p : R^{\mathrm{n}} \rightarrow \{0,1\}^{\mathrm{n}}$ denote a cut-off rule.

Given p_i in sector i, applicants observing s_i apply with probability $q_i(s_i)$ where

$$\int p_i(t_i) f_{T_i|S_i}(t_i|s_i) dt_i \begin{array}{c} > \\ = \\ < \end{array} c_i \Leftrightarrow q_i(s_i) \begin{array}{c} = 1 \\ \in [0,1] \\ = 0 \end{array}$$

Similarly, for $\hat{p}$.

The following are easy consequences of affiliation

1. If p_i is an acceptance cut-off rule for sector $i \in I$, then $\int p_i(t) f_{T_i|S_i}(t|s) dt$ is increasing. Hence, the best response q_i is an application cut-off rule.

2. Let p_i be an acceptance cut-off rule for sector $i \in I$, then for any probabilistic acceptance mechanism $\widehat{p}_i$, $\int p_i(t) f_{T_i|S_i}(t|s_i) dt - \int \widehat{p}_i(t) f_{T_i|S_i}(t|s_i) dt$ has a single change of sign from negative to positive (variation diminishing property).

3. Let p_i be an acceptance cut-off rule for sector $i \in I$, then for any probabilistic acceptance mechanism $\widehat{p}_i$, $\int q_i(s) f_{S_i|T_i}(s|t_i) ds - \int \widehat{q}_i(t) f_{S_i|T_i}(s|t_i) ds$ has at most a single change of sign, if a sign change takes place it is from negative to positive. This follows from 2. and the variation diminishing property.

4. Let p_i be an acceptance cut-off rule for sector $i \in I$, then for any acceptance policy $\widehat{p}_i$, $q_i(s_i)\int p_i(t)f_{T_i|S_i}(t|s_i)dt - \widehat{q}_i(s_i)\int \widehat{p}_i(t)f_{T_i|S_i}(t|s_i)dt$ has at most a single change of sign, if a sign change takes place it is from negative to positive. This follows from the fact that if $\int p_i(t)f_{T_i|S_i}(t|s_i)dt \geq \int \widehat{p}_i(t)f_{T_i|S_i}(t|s_i)dt$, then $q_i(s_i) \geq \widehat{q}_i(s_i)$, so the single crossing property is preserved.

5. Let p_i be an acceptance cut-off rule for sector $i \in I$, then for any acceptance policy $\widehat{p}_i$, $p_i(t_i)\int q_i(s)f_i(s|t_i)ds - \widehat{p}_i(t_i)\int \widehat{q}_i(t_i)f_i(s|t_i)ds$ has at most a single change of sign, if a sign change takes place it is from negative to positive. Similar to 4.

To complete the proof, we show that for any $\widehat{p}_i, \widehat{q}_i$ pair there is a p_i, q_i pair of cut-off rules such that

$$\int\int p_i(t)q_i(s)f_{S_i,T_i}(s,t)dsdt = \int\int \widehat{p}_i(t)\widehat{q}_i(s)f_{S_i,T_i}(s,t)dsdt \tag{B3.2}$$

$$\begin{aligned}&\int\int E\left[\theta|T_i = t, S_i = s\right]p(t)q_i(s)f_{S_i,T_i}(s,t)dsdt \\ &\geq \int\int E[\theta|T_i = t, S_i = s]\widehat{p}_i(t)\widehat{q}_i(s)f_{S_i,T_i}(s,t)dsdt\end{aligned} \tag{B3.2}$$

First, note that more lenient cut-off rules attract more applicants, so more lenient acceptance cut-off rules increase the mass of students admitted. Continuously so given the assumption that S_i, T_i has no atoms. Hence, there is a cut-off pair p_i, q_i satisfying (B3.1). Assumption 1 (additivity), we may write $E[\theta_i|T_i = t, S_i = s] = \varphi(s) + \phi(t)$, hence,

$$\begin{aligned}&\int\int E\left[\theta|T_i = t, S_i = s\right]p_i(t)q_i(s)f_{S_i,T_i}(s,t)dsdt \\ &= \int \varphi(s)\left(q_i(s)\int p_i(t)f_{T_i|S_i}(t|s)dt\right)f_{S_i}(s)ds \\ &+ \int \phi(t)\left(p_i(t)\int q_i(s)f_{S_i|T_i}(s|t)ds\right)f_{T_i}(t)dt\end{aligned}$$

By 3. above, $q_i(s)\int p_i(t)f_{T_i|S_i}(t|s)dt - \widehat{q}_i(s)\int \widehat{p}_i(t)f_{T_i|S_i}(t|s)dt$ has the single crossing property. Also, by affiliation, $\varphi(s)$ is increasing in s, therefore for any $k \in \mathbb{R}$, $\varphi(s) - k$

also has at most a single crossing property, also from negative to positive if it occurs. Therefore, for a suitably chosen value of k,

$$\int (\varphi(s) - k) \left(q_i(s) \int p_i(t) f_{S_i,T_i}(s,t)dt - \widehat{q}_i(s) \int \widehat{p}_i(t) f_{S_i,T_i}(s,t)dt \right) ds \geq 0$$

That is, upon rearranging and using (B3.1), inequality

$$\int \varphi(s) \left(q_i(s) \int p_i(t) f_{T_i|S_i}(t|s)dt \right) f_{S_i}(s)ds \geq \int \varphi(s) \left(\widehat{q}_i(s) \int \widehat{p}_i(t) f_{T_i|S_i}(t|s)dt \right) f_{S_i}(s)ds$$

is established. An identical argument establishes

$$\int \phi(t) \left(p_i(t_i) \int q_i(s) f_{S_i|T_i}(s|t_i)ds \right) f_{T_i}(t)dt \geq \int \phi(t) \left(\widehat{p}_i(t_i) \int \widehat{q}_i(s) f_{S_i|T_i}(s|t_i)ds \right) f_{T_i}(t)dt$$

Summing the two inequalities gives the result. □

B.4 Proof for Proposition 4.

Proposition 4. *For each sector $i \in I$, suppose:*

(1) S_i, T_i are affiliated.

(2a) Conditional on $T_i = t_i$, for each t_i, S_i has decreasing mean remaining life; and

(2b) conditional on $S_i = s_i$, for each s_i, T_i has decreasing mean remaining life.

(3) $\mathbb{E}[\theta_i | S_i = s_i, T_i = t_i]$ is supermodular.

(4a) $s_i \to \mathbb{E}[\theta_i | T_i = t_i, S_i = s_i]$ is concave for each t_i; (4b) $t_i \to \mathbb{E}[\theta_i | T_i = t_i, S_i = s_i]$ is concave for each s_i.

It follows that

$$\frac{\frac{\partial}{\partial s_i} MEQ_i(s_i, t_i)}{\frac{\partial}{\partial t_i} MEQ_i(s_i, t_i)} \leq \frac{\frac{\partial}{\partial s_i} \mathbb{E}[\theta_i | T_i = t_i, S_i = s_i]}{\frac{\partial}{\partial t_i} \mathbb{E}[\theta_i | T_i = t_i, S_i = s_i]} \leq \frac{\frac{\partial}{\partial t} MAQ_i(s_i, t_i)}{\frac{\partial}{\partial s} MAQ_i(s_i, t_i)}.$$

Proof. We provide below a detailed proof for the second inequality; the second follows from a symmetrical argument. Since the entire argument applies to $i \in I$, we drop the subscripts without loss of generality (N.B. distributions remain with the consistent notation, so for example $f(s|t)$ denotes the conditional distribution of S_i given T_i).

Let $\hat{\theta}(s,t) \equiv \mathbb{E}[\theta | S = s, T = t]$ and express the quality of the marginal entrant as:

$$MEQ(s,t) \equiv \mathbb{E}[\theta | S \geq s, T = t] = \frac{\int_s^\infty \hat{\theta}(\eta, t) f(\eta|t) d\eta}{1 - F(s|t)} \tag{B.4.1}$$

Taking partial derivatives with respect to s yields:

$$\begin{aligned} \frac{\partial}{\partial s} \frac{\int_s^\infty \hat{\theta}(\eta, t) f(\eta|t) d\eta}{1 - F(s|t)} &= -\hat{\theta}(s,t) \frac{f(s|t)}{1 - F(s|t)} + \frac{f(s|t)}{1 - F(s|t)} \frac{\int_s^\infty \hat{\theta}(\eta, t) f(\eta|t) d\eta}{1 - F(s|t)} \\ &= \frac{f(s|t)}{1 - F(s|t)} \frac{\int_s^\infty \left(\hat{\theta}(\eta, t) - \hat{\theta}(s,t)\right) f(\eta|t) d\eta}{1 - F(s|t)} \end{aligned}$$

Supposing $s \to \hat{\theta}(s,t)$ to be concave, it follows that:

$$\frac{\partial MEQ(s,t)}{\partial s} \leq \frac{f(s|t)}{1-F(s|t)} \frac{\int_s^\infty \frac{\partial \hat{\theta}(s,t)}{\partial s} (\eta - s) f(\eta|t) d\eta}{1-F(s|t)} = \frac{\partial \hat{\theta}(s,t)}{\partial s} \frac{\frac{\int_s^\infty (\eta-s) f(\eta|t) d\eta}{1-F(s|t)}}{\frac{1-F(s|t)}{f(s|t)}} \tag{B.4.2}$$

The term $\frac{\int_s^\infty (\eta-s) f(\eta|t) d\eta}{1-F(s|t)}$ above is the *mean remaining life* of S, which we denote as a function $m(s;t)$. It is easy to check that

$$\frac{\partial m(s;t)}{\partial s} = \frac{f(s|t)}{1-F(s|t)} m(s;t) - 1$$

so we can express the inequality (B.4.2) as:

$$\frac{\partial MEQ(s,t)}{\partial s} \leq \frac{\partial \hat{\theta}(s,t)}{\partial s} \left(1 + \frac{\partial m(s;t)}{\partial s} \right)$$

And, if $m(s;t)$ is decreasing in s, it follows that[5]

$$\frac{\partial MEQ(s,t)}{\partial s} \leq \frac{\partial \hat{\theta}(s,t)}{\partial s} \tag{B.4.3}$$

On the other hand, taking derivatives of (B.4.1) with respect to t gives:

$$\frac{\partial MEQ(s,t)}{\partial t} = \frac{\int_s^\infty \frac{\partial \hat{\theta}(\eta,t)}{\partial t} f(\eta|t) d\eta}{1-F(s|t)} + \int_s^\infty e(\eta,t) \frac{\partial \frac{f(\eta|t)}{1-F(s|t)}}{\partial t} d\eta$$

and supposing that S and T are affiliated, it follows that

$$\int_s^\infty \hat{\theta}(\eta,t) \frac{\partial \frac{f(\eta|t)}{1-F(s|t)}}{\partial t} d\eta \geq 0$$

[5]This can be interpreted by noting that $m(s;t) = \frac{\int_s^\infty (\eta-s) f(\eta|t) d\eta}{1-F(s|t)}$ is the expected lifetime remaining of the random variable S, supposing it to have survived to time s. If there were an exponential distribution with (constant) hazard rate $\frac{f(s|t)}{1-F(s|t)}$, then it would have mean duration $\frac{1-F(s|t)}{f(s|t)}$. Evidently, any distribution with hazard rate always greater than $\frac{f(s|t)}{1-F(s|t)}$ will have mean duration less than $\frac{1-F(s|t)}{f(s|t)}$. Therefore, if f has increasing hazard rate, it also has expected lifetime remaining less than $\frac{1-F(s|t)}{f(s|t)}$, in which case, $\frac{\int_s^\infty (\eta-s) f(\eta|t) d\eta}{1-F(s|t)} \leq \frac{1-F(s|t)}{f(s|t)}$

which in turn means that

$$\begin{aligned}\frac{\partial MEQ(s,t)}{\partial t} &= \frac{\int_s^\infty \frac{\partial}{\partial t}\hat{\theta}(\eta,t)f(\eta|t)d\eta}{1-F(s|t)} + \int_s^\infty \hat{\theta}(\eta,t)\frac{\partial \frac{f(\eta|t)}{1-F(s|t)}}{\partial t}d\eta \\ &\geq \frac{\int_s^\infty \frac{\partial}{\partial t}\hat{\theta}(\eta,t)f(\eta|t)d\eta}{1-F(s|t)}\end{aligned} \tag{B.4.4}$$

From this, it is clear that supermodularity of $\hat{\theta}(s,t)$ implies:

$$\frac{\partial MEQ(s,t)}{\partial t} \geq \frac{\int_s^\infty \frac{\partial}{\partial t}\hat{\theta}(\eta,t)f(\eta|t)d\eta}{1-F(s|t)} \geq \frac{\int_s^\infty \frac{\partial}{\partial t}\hat{\theta}(s,t)f(\eta|t)d\eta}{1-F(s|t)} = \frac{\partial\hat{\theta}(s,t)}{\partial t}$$

Thus:

$$\frac{\frac{\partial}{\partial t}MEQ(s,t)}{\frac{\partial}{\partial s}MEQ(s,t)} \geq \frac{\frac{\partial}{\partial t}\hat{\theta}(s,t)}{\frac{\partial}{\partial s}\hat{\theta}(s,t)}$$

□

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Chapter 3.

Monopolistic Competition and Exclusive Education

Chapter abstract

This paper explores price formation when education is differentiated by quality, and higher quality yields higher returns only to competent students who are able to cope with the teaching. I propose a model of monopolistic competition with product differentiation to show that there are two relevant equilibrium market segmentations: one in which high-quality colleges serve an elite portion of the market by catering to the high-income/high-ability students, and another in which the high-quality colleges are cheaper than the low-quality colleges which serve low-ability/high-income students. The analysis indicates that the distinction between these two equilibria is relevant to understand the welfare implications of market reforms.

Chapter contents

1 Introduction

Background and motivation. Tuition fees at publicly funded higher education institutions are at the centre of policy debates around the world. The discussions typically revolve around the structure of funding; state transfers, graduate taxes, student loans, scholarships and vouchers are at the core of the analysis. Interestingly, however, the debate often omits issues related to price formation and market dynamics, despite the indisputable fact that the market for higher education is extremely particular due to the complex nature of the product in question. Here, I focus on an often omitted feature of demand and supply analysis in education markets, namely that higher tuition quality is a product characteristic that may be valuable only to sufficiently competent students.

Understanding the interaction of prices and tuition quality is very timely in the light of recent policy reforms aiming to shift the funding structure towards demand-side financing, while deregulating prices. In England, for example, the Higher Education White Paper *Students at the Heart of the System* raised in 2011 the cap on undergraduate tuition fees from 3,375 to 9,000 GBP per year, while enabling for-profit companies and foreign providers to enter the market more easily, and introducing measures to foster competition for top-achieving students across providers. Soon after the publication of the White Paper, the English media was filled with pronouncements expressing fears that the measures could compromise quality through downward pressure on fees, and that the induced competition for top-achieving students could lead to the polarisation of the system. The underlying concern amongst critics of the reform was that there would be unintended consequences.[1]

The theoretical economics literature on price formation in higher education is very limited, and there is no clear consensus as to how we should think about Industrial Organisation in this specific context. This is because generic models of

[1]Kim Katcheside, a journalist writing for the newspaper The Guardian, expressed this concern very graphically in an article published 29 June 2011: *"My worry is that this attempt to rig the market will lead to yet more unintended consequences. Images of American bullfrogs invading Sussex ponds, the triffid-like advance of Japanese knotweed and rats murdering endangered birds on distant islands, come to mind. A delicate eco-system is at risk and I fear we can have little confidence that policy makers have much idea how their plans will play out in the real world".*

competition fail to take into account the relevant particularities of demand and supply for higher education. As noted by Winston (1999) in a paper entitled *"Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education"*, one key factor that is obscured by a traditional analysis of price formation is that, in the case of higher education, the 'firms' depend on the quality of their customers to provide their services. As a consequence firms trying to meet a certain quality standard care very much about who their customers are and how well-equipped they are with the necessary characteristics. Quality, in this context, is desirable and has an important element of exclusivity.

An important feature of the market for higher education is that institutions compete not only in prices, but also in quality. Winston (1999), for example, provides extensive detail on how different providers of higher education differ in dimensions closely related to quality, such as the selectivity of their admission procedures. In this paper I will focus on tuition quality as defined by the academic level at which colleges target their teaching, with the inevitable consequence of excluding certain students who are not sufficiently competent to cope with the chosen level of tuition quality. There are, of course, many other important dimensions of tuition quality that do not require excluding certain students (e.g. the size of classes, the calibre of the teachers and their teaching load, or the menu of complimentary inputs and resources available to students). These other dimensions of quality will not be considered in this paper.

A natural instrument that embodies quality, in the sense that will be discussed in this paper, is the admissions policy: a minimum academic standard below which candidates are not admitted, as they would not be able to cope with the complexity of the teaching. Considering this, I develop a simple model of monopolistic competition with product differentiation, where profit-maximising colleges choose prices and tuition quality, the latter by setting a minimum ability threshold. Students, on the other side of the market, are characterised by income and ability and derive utility from disposable income and education quality. The model is first explored in a benchmark case with only one college and a fixed outside option, and then strategic competition across colleges is introduced. The analysis of the benchmark is important, because the problem of a single college maximising profits with respect to its own price and quality is not trivial; as shall be clear, quality not only affects demand through its effect on prices, but also directly affects the *quantity* that can be produced, by excluding potential customer. This will play a central role in the dynamics of the problem with strategic competition.

The main insight from this paper is that there are two relevant market segmentations that may be sustained in equilibrium: one in which high-quality colleges serve an elite portion of the market by selling their services to the high-income/high-ability students, and another in which the high-quality colleges are cheaper than the low-quality colleges which serve low-ability/high-income students. Given that these two equilibria are characterised by a different allocation of students into colleges, they entail a different aggregate student welfare; thus, distinguishing between them is relevant to understand the welfare implications of market reforms. Consider the following example. Suppose that the market is regulated, such that prices are fixed and colleges compete only along the quality dimension. If the tuition fees are fixed at a price level such that the *status-quo* regulated outcome (i.e. the equilibrium outcome of competition only in quality) yields a welfare level in between the two associated *laissez-faire* equilibria, then it is not possible to know whether liberalising the market to allow competition in prices is beneficial or detrimental to students, because it will depend on the type of deregulated equilibrium that arises. This example will be formally explored as part of the equilibrium analysis in Section 5.

Related literature. The model that I propose extends the work on monopolistic competition, as developed in Gabszewicz and Thisse (1979) and Shaked and Sutton (1982), by introducing ability as an additional relevant dimension, so that consumers' preferences for quality-price combinations (or 'menus') depend on income and ability. This paper hence contributes to the literature on monopolistic competition by studying the scope for competition in quality, when high quality is exclusive because some customers do not have the required ability to perceive the benefits. Although I concentrate throughout the paper on the market for higher education, the proposed approach could be applied in other contexts; for example, in the market for computer software, where the profitability of more sophisticated products may depend on the 'marginal consumer ability' (i.e. the ability of the consumer who is just skilled enough to be able to use the product).

As mentioned before, I propose a model based on the premise that educational quality is defined by academic standards, and those standards can be implemented through an admission policy. Consequently, since admission policies reflect the quality of teaching, the proposed framework provides a mechanism through which students have preferences over admission policies. In this sense, my analysis complements the traditional approach taken in the literature on the economics of education, where peer-effects are the main mechanism linking admissions policies and tuition fees (see, for example, Rothschild and White 1995, Epple and Romano 1998 and Epple, Romano and Sieg 2006). The key difference between the approach taken in this paper and the approach taken in the peer-effect literature, is that I explore the case in which students have preferences over the ability of the *marginal* entrant (i.e. willingness to pay as a function of the ability of the least able student attending college) rather than the *average* entrant. This simple distinction between preferences over *marginal* and *average* entrant ability leads to different equilibrium results. Epple and Romano (1998), for example, propose a model where tuition-financed profit-maximising selective schools compete for students (characterised by ability and income) who seek to maximise academic achievement defined by average peer-group ability. They show that in equilibrium the schools are selective in ability and exhibit within-college price discrimination (charging a lower fees to high-ability / low-income students). As it shall be explained in Section 5, this is in contrast to an analysis of price formation like the one I propose, where price discrimination becomes irrelevant because willingness to pay depends on the ability of the worst entrant, so colleges do not benefit from subsidising high-ability / low-income students.

The rest of this paper is structured as follows. Section 2 introduces the model, Section 3 discusses a benchmark without strategic interaction, Section 4 solves the model with monopolistic competition, Section 5 analyses the equilibrium solutions, and Section 6 finishes with the concluding remarks.

2 Set-up and definitions

Consider a market for higher education consisting of two ex-ante identical profit-maximising colleges, indexed $i = 1, 2$. The two colleges choose a quality standard a_i and tuition fees p_i. By assumption, all students attending a given college pay the same fees and perceive the same quality. The colleges' technologies are such that the cost of teaching an extra student is c, regardless of the choice of quality or the total number of students. As mentioned before, quality can be interpreted as the minimum ability that is required for students to cope with the teaching. The objective of the colleges is to maximise profits.

On the other side of the market, consider a continuum of students deciding whether to acquire higher education by making indivisible and mutually exclusive purchases from the colleges. A student either makes no purchase and takes a fixed outside option, or else buys exactly one unit from the most preferred college. Students are characerised by ability $\theta \in [0, 1]$ and income $\omega \in [0, 1]$. The student ability-income type (θ, ω) is publicly observable and drawn with equal probability in the unit square.

Students have preferences over educational quality and disposable income. The utility of consuming education is simply the product of quality and disposable income; however, since consuming education is only available to students of sufficient ability, the utility achieved by a student of type $t = (\theta, \omega)$ attending college $i \in \{1, 2\}$ is given by:

$$U^t(p_i, a_i) = \begin{cases} (\omega - p_i) \cdot a_i & \text{if} \quad \theta \geq a_i \\ 0 & otherwise \end{cases}$$

These preferences capture the fact that educational quality and income are complementary for students of sufficient calibre. Interpreting quality as the admission standard, these preferences assume that conditional on being admitted, wealthier students are willing to pay more for selectivity than poorer students of comparable ability. In the (a, p)-plane, this implies 'single-crossing'; that is, for students of the same (sufficiently high) ability, any indifference curve of a higher-income individual cuts any indifference curve of a lower-income individual from below. This property is consistent with other studies in the literature on the economics of education, such as Epple and Romano (1998), and is also familiar in other models of price competition through quality product differentiation such as Shaked and Sutton (1982).[2]

The outside option will be denoted as a college that offers quality q_0 to any student (regardless of ability) at no cost. Thus, the utility of taking the outside option is simply:

$$U^t(0, q_0) = \omega \cdot q_0$$

Students and colleges interact in three successive stages:

I) *Tuition Quality:* In the first stage, colleges simultaneously choose quality $a_i \in [0, 1]$

II) *Tuition fees:* In the second stage, after quality standards are chosen and observed, colleges simultaneously set the price $p_i \in [0, 1]$ that they will charge to admitted students

III) *Student placement:* In the third and last stage, students observe prices and qualities and select their most preferred alternative. If the two colleges offer the same quality and prices, students are indifferent and attend either college with equal probability.

It should be noted that in the model introduced so far, if competition across colleges is left aside, the problem is reduced to a single college that maximises profits with respect to its own price and quality. This is a two-dimensional monopoly problem: the college must choose p and a to maximise profits. Despite the simplicity of the model, this two-dimensional monopoly problem is not trivial because a affects both the price that students are willing to pay, and the number of potential students who can attend college. This trade-off between tuition quality and fees makes the problem different from a standard one-dimensional monopoly, and will be crucial to understand the dynamics of the more complicated model with strategic interaction.

[2] As pointed out by Epple and Romano (1998), this 'single-crossing' property is consistent with available −yet unfortunately limited− empirical estimates of income elasticities of demand for educational quality as measured by educational spending.

3 Benchmark with a single college

3.1 Student demand

If there is only one college, students with ability below a have no choice and take the outside option. Sufficiently skilled students, on the other hand, must decide between attending college or taking the outside option. Let $y(p, a; q_0)$ be the threshold income for which a student of ability $\theta \geq a$ is indifferent between attending the college with quality a at price p, and taking the outside option. It then follows that:

$$\begin{aligned} U(p, a) &= U(0, q_0) \\ (y - p)(a) &= y \cdot q_0 \\ y(p, a; q_0) &= \frac{p \cdot a}{a - q_0} \end{aligned} \tag{1}$$

From the assumed utility function it is clear that, among all students of ability $\theta \geq a$, those with income $\omega \geq y(p, a; q_0)$ will prefer the college over the outside option (recall the 'single-crossing' property of the assumed preferences). As a result, the college demand corresponds to a segment of the student population, as depicted in Figure 1.

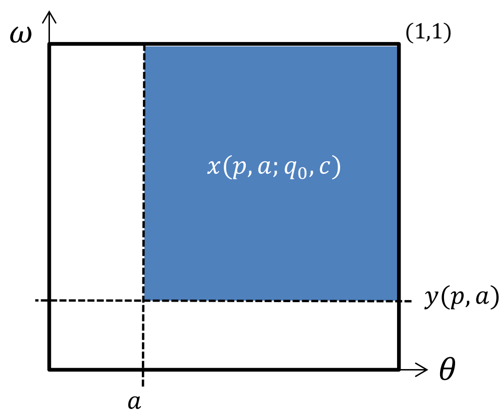


Figure 1: Demand in the monopolist benchmark

Using equation (1), the demand function is thus given by:

$$x(p, a; q_0, c) = \begin{cases} (1 - y(p, a; q_0))(1 - a) & \text{if} \quad 0 \leq y(p, a; q_0) \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

3.2 Tuition fees and quality

The problem faced by the monopolist, considering the above, is a standard constrained maximisation problem:

$$\underset{p,a}{\text{MAX}} \quad \pi(p,a;q_0,c) = (p-c)(1-a)(1-y(p,a;q_0))$$

$$\text{s.t.}$$
$$0 \leq y(p,a;q_0) \leq 1$$
$$c \leq p \leq 1$$
$$q_0 \leq a \leq 1$$

The necessary first order conditions for an interior solution (subscripts noting partial derivatives) are:

$$\pi_p(p,a;q_0,c) = (1-a)(1-y(p,a;q_0)-(p-c)y_p(p,a;q_0)) = 0$$
$$\pi_a(p,a;q_0,c) = (p-c)(1-y(p,a;q_0)+(1-a)y_a(p,a;q_0)) = 0$$

Using (1) above yields two equations in two unknowns:

$$p(a;q_0,c) = \frac{a(1+c)-q_0}{2a} \tag{3}$$
$$a(p;q_0) = q_0 + \frac{\sqrt{(1-p)(1-q_0)pq_0}}{1-p} \tag{4}$$

The solution to the benchmark problem is the menu (p^*, a^*) solving the simulatenous equations above. This solution is interior as long as marginal cost c and the value of the outside option q_0 are sufficiently small.

Proposition 1. *The benchmark problem has an interior solution only if*

$$0 < q_0 + c < 1$$

Proof. Substituting (3) into (1) gives

$$y(p, a; q_0) = \frac{p(a; q_0, c) \cdot a}{a - q_0} = \frac{a(1 + c) - q_0}{2(a - q_0)}$$

but $a^* \in (q_0, 1)$ and $p^* \in (c, 1)$ imply that $0 < \frac{a^*(1+c)-q_0}{2(a^*-q_0)} < 1$ only if $0 < c+q_0 < 1$ □

If costs are low, such that $0 < c + q_0 < 1$, fixing $\overline{a}$ makes the problem of choosing prices equivalent to a standard one-dimensional monopolist: the optimal pricing rule is to equate marginal revenue with marginal cost, as required by $p(\overline{a}; q_0, c)$ in (3). However, if quality is not fixed, the two-dimensional problem is generally different to the traditional monopolist problem because marginal revenue is a function of quality. Hence, finding a pricing rule requires solving a pair of simultaneous equations. Figure 2 presents profits as a function of p and a for $c = q_0 = 0.1$ (higher profit levels are depicted in lighter shades). The dotted black line marks the constraint (i.e. the boundary where demand becomes zero). It follows that the profit function $\pi(p, a; q_0, c)$ has a unique maximum for intermediate values of p and a, so the monopolist would profit from serving the segment of high-income/high-ability students.

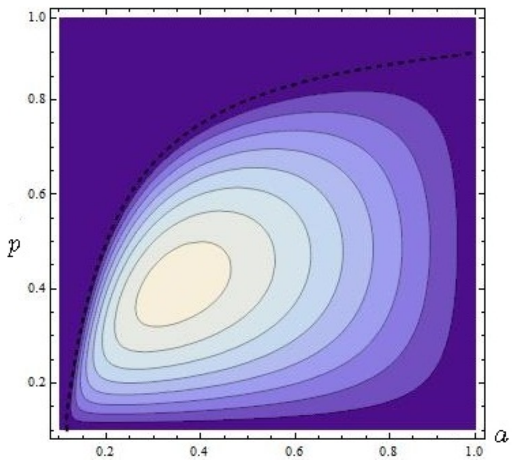


Figure 2: Iso-profit contours as a function of p and a (for $c = q_0 = 0.1$)

Proposition 2. *Suppose* $0 < q_0 + c < 1$. *Then the necessary first order conditions* (3) *and* (4) *have at most one solution satisfying* $a \in (q_0, 1)$ *and* $p \in (c, 1)$. *Hence, the interior solution* (p^*, a^*) *that maximises* $\pi(p, a; q_0, c)$ *is unique, whenever it exists.*

Proof: See Appendix C1.

3.3 Comparative statics

Changes in the outside option. An interesting way of exploring outcomes in this set-up is to fix the marginal cost at some arbitrarily low level, and then trace the optimal tuition fee and quality that correspond to different values of the outside option. Figure 3 plots the set of optimal price-quality combinations as a function of q_0, setting $c = 0.1$.

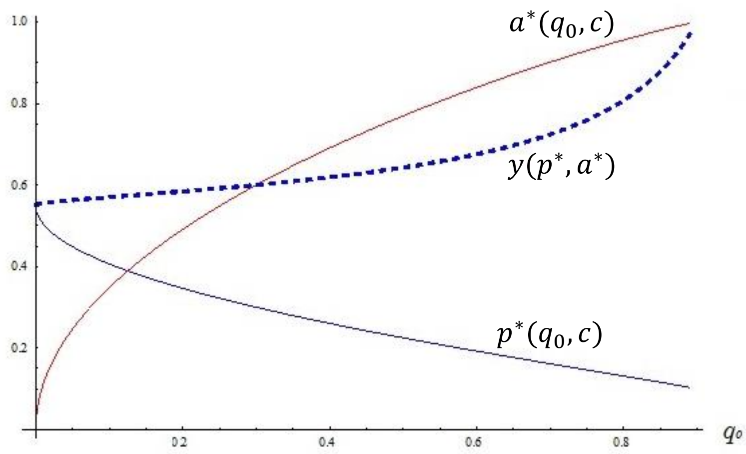


Figure 3: Optimal fees and quality as a function of the outside option, for $c = 0.1$

Notice that for low levels of q_0 fees are high and quality is low, but as q_0 grows, the college raises the quality and lowers fees. Moreover, when q_0 goes up there is an increase in the threshold income at the optimal fees and quality.

The total effect of q_0 on the threshold income can be decomposed in two effects, interacting in opposite directions. When the outside option becomes more valuable attending college becomes less attractive, so there is a positive direct effect on the income of the marginal student attending college (note that $\frac{\partial y(p,a;q_0)}{\partial q_0} > 0$). At the same time, however, when the outside option becomes more valuable the college responds by lowering prices and raising quality, so there is a negative indirect effect on threshold income. It can be checked that the negative indirect effect only offsets the positive direct effect partly, so the threshold income is increasing in q_0.

Proposition 3. *Provided that an interior solution exists, the optimal price-quality* *menu is such that:*

$$(i) \quad \frac{dp^*(q_0, c)}{dq_0} < 0 \qquad (ii) \quad \frac{da^*(q_0, c)}{dq_0} > 0 \qquad (iii) \quad \frac{dy(p^*, a^*; q_0)}{dq_0} > 0$$

Proof: See Appendix C2.

This result is intuitive given that marginal utility of quality is increasing in income (by assumption). As a consequence, whenever the value of the outside option increases, the college responds by making education more attractive (lower prices and higher quality), but attracts proportionally more students from the upper end of the income distribution. The implication, hence, is that an increase in the value of the outside option translates into a reduction in the representation of low-income students in the college −even after taking into account that tuition fees go down.

Changes in the marginal cost. To explore the impact of changes in the marginal cost, Figure 4 plots the optimal menu as a function of c, setting $q_0 = 0.1$.

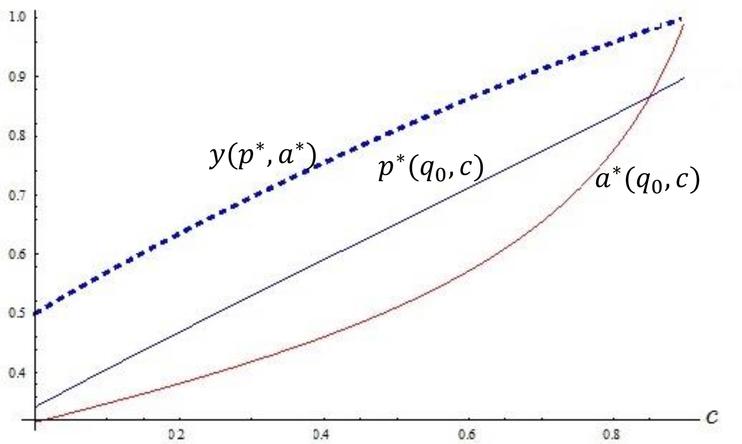


Figure 4: Optimal fees and quality as a function of the outside option, for $q_0 = 0.1$

As it can be seen, in this case both $p^*(q_0, c)$ and $a^*(q_0, c)$ are monotonically increasing in c. An increase in marginal cost reduces the profit margin and the college responds by raising prices. The choice of quality, on the other hand, does not depend on c directly —recall from equation (4) that the first order condition of quality is independent of c—, so an increase in the marginal cost rises quality only indirectly, through the increase in prices. Once again, it can be checked that these two effects imply that the threshold income is increasing in c for any level of q_0 for which an interior solution exists.

Proposition 4. *Provided that an interior solution exists, the optimal price-quality menu is such that:*

$$(i) \quad \frac{dp^*(q_0, c)}{dc} > 0 \qquad (ii) \quad \frac{da^*(q_0, c)}{dc} > 0 \qquad (iii) \quad \frac{dy(p^*, a^*; q_0)}{dc} > 0$$

Proof: See Appendix C2.

This result, as before, is consistent with the intuition of a two-dimensional monopolist: higher marginal costs imply that the college must operate at a smaller scale with higher marginal benefits, so it increases prices and quality, hence catering for a more selective segment of brighter and richer students, who are willing to pay more. In a traditional one-dimensional monopolist model, a reduction (increase) in the marginal cost would also lead to a reduction (increase) in fees. Yet, such a model would fail to take into account the additional indirect impact of costs on quality, via prices.

3.4 Welfare analysis

For any price-quality menu (p, a) it is possible to classify students in three mutually exclusive groups, as illustrated in Figure 5. One group consists of those individuals who are sufficiently rich and skilled to attend college; a second group consists of individuals who have sufficient ability to attend college but decide to take the outside option because of low income; and finally, there is a third group of individuals who must take the outside option, regardless of income, because of low ability. Student welfare in the two-dimensional monopoly can be expressed as the sum of welfare contributions across these three groups.

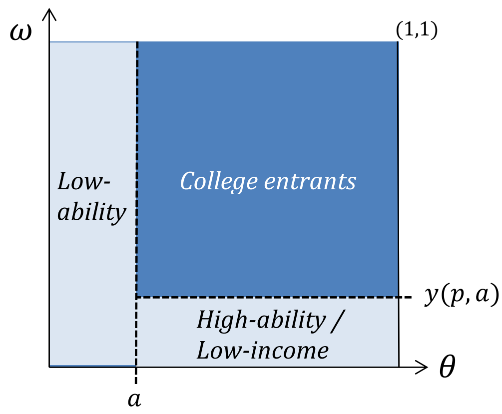


Figure 5: Groups for calculating welfare in the 2-dimensional monopoly

For the group of college entrants, the welfare contribution is given by:

$$E[(\omega - p)a|\theta \geq a, \omega \geq y(p,a)] \; \Pr[\theta \geq a, \omega \geq y(p,a)]$$
$$= \left(\frac{y(p,a;q_0)+1}{2} - p\right) a(1-a)(1-y(p,a;q_0))$$

Similarly, the welfare contribution for the high-ability/low-income group is:

$$E[\omega q_0|\theta \geq a, \omega < y(p,a)] \Pr[\theta \geq a, \omega < y(p,a)]$$
$$= \frac{y(p,a;q_0)}{2} q_0(1-a)y(p,a;q_0)$$

and for the low-ability group:

$$E[\omega q_0|\theta < a] \Pr[\theta < a] = \frac{q_0}{2}a$$

Bringing these together, student welfare is simply the sum of the above contributions:

$$SW(p,a;q_0) = \left(\frac{y(p,a)+1}{2} - p\right) a(1-a)(1-y(p,a;q_0)) + \frac{y^2(p,a;q_0)}{2} q_0(1-a) + \frac{q_0}{2}a \quad (5)$$

Since $SW(p,a;q_0)$ is non-increasing in prices, aggregate student welfare is maximised when prices equal marginal cost. This is intuitive given that utility is non-increasing in tuition fees, regardless of whether a student attends the college or not. Tuition quality, in contrast, implies a trade-off because an increase in a is associated with higher welfare via higher utility of college entrants, but at the same time lower welfare via a reduction in the number of college entrants. This means that a welfare-maximising college would face a tension between raising quality standards and making college more accessible. To be specific, the necessary first order conditions for maximising student welfare are $p^{sw}(c) = c$ and

$$a^{sw}(q_0, c) = \frac{1}{4}\left(1 + 3q_0 + \sqrt{\frac{(1-q_0)(1-c-q_0+9cq_0)}{1-c}}\right)$$

Figure 6 plots student welfare under (p^{sw}, a^{sw}) and (p^*, a^*) as a function of q_0. It can be checked that (i) a monopolist would offer a menu with higher prices and lower quality than desirable for aggregate student welfare, and (ii) the resulting loss in student welfare would be decreasing in q_0.

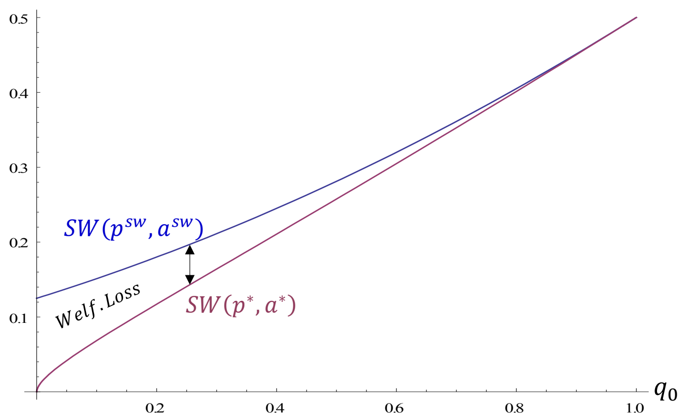


Figure 6: Student welfare loss as a function of q_0 (for $c = 0$)

An important point to highlight is that loss in student welfare due to monopoly, in this two-dimensional framework, requires an analysis of the *allocation* of demand −rather than just the size of served demand− because not all students benefit equally from attending college. Consider the following example where the monopolist has complete power over the market because there is no outside option and the marginal cost of education is zero (i.e. suppose $c = q_0 = 0$). Since there is no outside option, assume that students always prefer to attend the college if they can afford it, so the threshold income in this particular case is given by $y(p) = p$.

In this scenario, student welfare would be maximised at:

$$p^{sw} = c = 0$$
$$a^{sw} = \underset{a}{\text{argmax}} \left[SW(0,a) \right] = \underset{a}{\text{argmax}} \left[\frac{a(1-a)}{2} \right] = \frac{1}{2}$$

which means that only the most able half of the students should go to college (irrespective of income). In this same example, however, profits are $\pi(p, a; 0, 0) = p(1-a)(1-p)$, so a monopolist would choose $a^* = 0$ and $p^* = \frac{1}{2}$. Thus, at the menu offered by the monopolist the student welfare would be entirely lost, but the size of served demand would be at the welfare-maximising level −only the richest half of the student population would attend college.

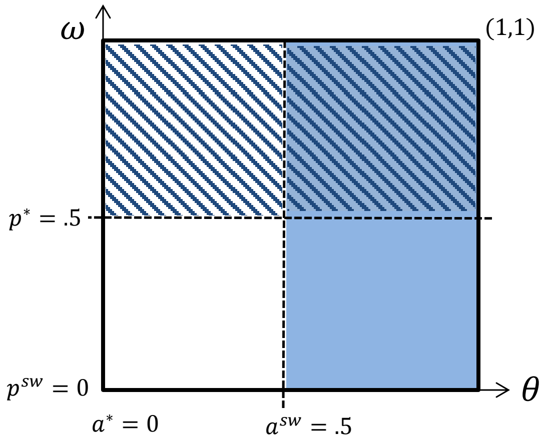


Figure 7: Profit-maximising student allocation (dashed region) vs. Welfare-maximising allocation (shaded region), for $q_0 = c = 0$.

4 Monopolistic competition

4.1 Solution approach

The monopolistic competition problem features strategic choices in stages; as it is standard in this type of framework, the solution follows backward-induction. The remainder of this section proceeds accordingly by finding a demand curve (third stage), which in turn is used to derive equilibrium prices (second stage) and then, finally, equilibrium qualities (first stage).

Before continuing, note that because of backward induction the solution must be such that the pricing choice at the second stage constitutes a best response for all possible values of quality from the first stage. If colleges were to set $a_1 = a_2$ at the first stage, they would move to the pricing stage facing the same demand curve. In this case, the scenario that results (i.e. the sub-game following $a_1 = a_2$) corresponds to a Bertrand competition game, in which equilibrium prices are driven down to marginal cost.

Based on the above remark, any choice of quality $q_0 < a_1 < a_2$ that yields positive profit would be strictly preferred by the colleges. In what follows it will be assumed that the equilibrium qualities are $a_1 < a_2$, and then it will be shown that the solution would, indeed, yield positive profit; so the Bertrand subgame would not be reached in equilibrium.

4.2 Demand curves

Starting at the final stage, students make decision about which college to attend knowing that $a_1 < a_2$. Students with ability $\theta < a_1$ trivially opt for the outside option, while students with ability $a_1 \leq \theta < a_2$ compare the outside option with the low-quality college, since for them the high-quality alternative is out of question. Sufficiently skilled students with ability $a_2 \leq \theta$, on the other hand, have all doors open and compare both colleges against each other and the outside option.

To illustrate the problem faced by students with ability $a_2 \leq \theta$, consider the following scenarios. If the price of the the high-quality option is weakly lower than the low-quality alternative, bright students only have to compare the top college with the outside option, as the bottom college offering worse quality at a higher price is clearly inferior. However, if the low-quality college is sufficiently cheaper than the high-quality alternative, there could be some bright students who prefer the low-quality college, some who prefer the high-quality alternative, and some who still prefer the outside option. More formally, demand in this case is determined by three income margins.

- When a student is comparing the **low-quality** college with the **outside**

option, the threshold income for which she is indifferent is:

$$\begin{aligned}(y_{1,0} - p_1)(a_1) &= y_{1,0} q_0 \\ y_{1,0} &= \frac{p_1 a_1}{a_1 - q_0}\end{aligned} \tag{6}$$

- Similarly, when a student is comparing the **high-quality** college and the **outside option**, the threshold income is:

$$\begin{aligned}(y_{2,0} - p_2)(a_2) &= y_{2,0} q_0 \\ y_{2,0} &= \frac{p_2 a_2}{a_2 - q_0}\end{aligned} \tag{7}$$

- Finally, when a student is comparing the **high** and **low** quality college with each other, the threshold income for which she is indifferent is:

$$\begin{aligned}(y_{2,1} - p_2)(a_2) &= (y_{2,1} - p_1)(a_1) \\ y_{2,1} &= \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}\end{aligned} \tag{8}$$

These thresholds are clearly functions of prices and quality (as well as the underlying parameters), so they are endogenous variables of the model. Since they will be used recurrently throughout the rest of the paper, they will be simply noted as variables $y_{1,0}$, $y_{2,0}$ and $y_{2,1}$ and the arguments shall be omitted for notational brevity.

To reiterate, income affects the student choices differently depending on whether θ is below a_1, between a_1 and a_2, or above a_2. Any student with ability below a_1 will choose the outside option regardless of income, while students with ability between a_1 and a_2 will prefer the low-quality college if and only if $\omega \geq y_{1,0}$. For students with ability above a_2 there are two options. If $y_{2,0} \geq y_{2,1}$, those students with income $\omega \geq y_{2,0}$ will choose the high-quality option while the rest choose the outside option; if on the contrary $y_{2,0} < y_{2,1}$, then students with income $\omega \geq y_{2,1}$ will choose the high-quality option, students with income $y_{2,1} > \omega \geq y_{1,0}$ will choose the low-quality option, and the rest will choose the outside option.

Graphically, the resulting segmentations of the market can be characterised in two relevant cases, as depicted in Figure 8.

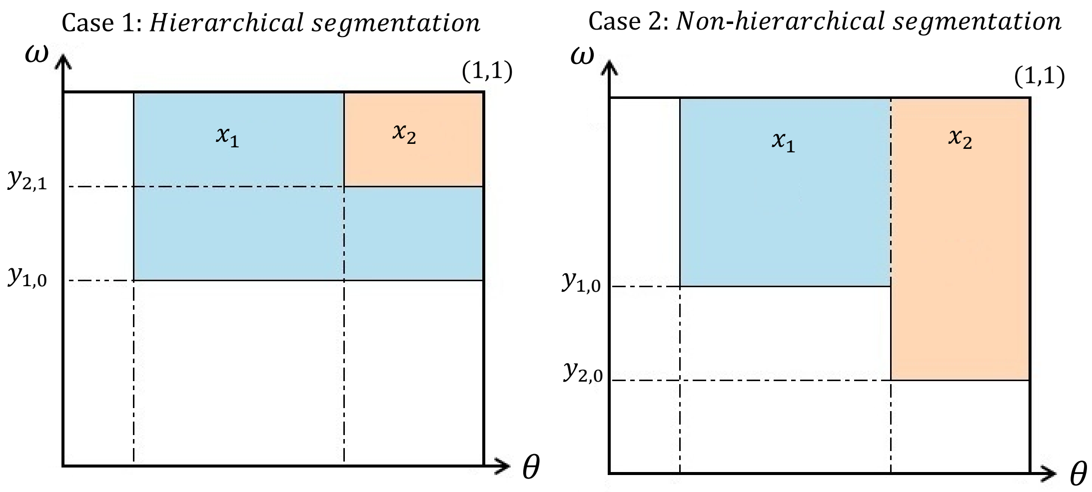


Figure 8: Characterisation of market segmentation by cases

To understand why there are only two relevant cases despite the fact that there are three income margins, note that there are six different permutations for $\{y_{2,1}; y_{2,0}; y_{1,0}\}$. All permutations in which $y_{2,0}$ is the highest or the lowest cannot arise if $a_2 > a_1$. This leaves only two relevant permutations, namely $y_{2,1} > y_{2,0} > y_{1,0}$ and $y_{1,0} > y_{2,0} > y_{2,1}$. The former leads to a hierarchical segmentation where more selective colleges are more expensive, while in the latter the segmentation is not hierarchical and more selective colleges are cheaper.

Using the remarks above, the demand curves can be derived by cases, depending on whether $y_{2,0}$ is larger or smaller than $y_{2,1}$.

CASE 1. Hierarchical segmentation. The low-quality college is significantly cheaper than the high-quality college, so that:

$$\frac{p_2 a_2}{a_2 - q_0} < \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}$$

or equivalently, the low-quality college is so cheap that:

$$p_1 < \frac{a_2 p_2 (a_1 - q_0)}{a_1 (a_2 - q_0)} \tag{9}$$

and demand (denoted with superscript h to label the *hierarchical* case) is given by:

$$\begin{aligned} x_1^h &= (a_2 - a_1)(1 - y_{1,0}) + (1 - a_2)(y_{2,1} - y_{1,0}) \\ &= (a_2 - a_1)\left(1 - \frac{p_1 a_1}{a_1 - q_0}\right) \\ &+ (1 - a_2)\left(\frac{p_2 a_2 - p_1 a_1}{a_2 - a_1} - \frac{p_1 a_1}{a_1 - q_0}\right) \end{aligned} \tag{10}$$

$$\begin{aligned} x_2^h &= (1 - a_2)(1 - y_{2,1}) \\ &= (1 - a_2)\left(1 - \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}\right) \end{aligned} \tag{11}$$

CASE 2. Non-hierarchical segmentation. The low-quality college is more expensive, or offers a very similar price given its relative quality, so that:

$$\frac{p_2 a_2}{a_2 - q_0} > \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}$$

or equivalently, prices are such that:

$$\frac{a_2 p_2 (a_1 - q_0)}{a_1 (a_2 - q_0)} < p_1 \tag{12}$$

and demand (denoted with superscript nh to label the *non-hierarchical* case) is given by:

$$\begin{aligned} x_1^{nh} &= (a_2 - a_1)(1 - y_{1,0}) \\ &= (a_2 - a_1)\left(1 - \frac{p_1 a_1}{a_1 - q_0}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} x_2^{nh} &= (1 - a_2)(1 - y_{2,0}) \\ &= (1 - a_2)\left(1 - \frac{p_2 a_2}{a_2 - q_0}\right) \end{aligned} \tag{14}$$

The remainder of this section will solve the respective price and quality choice problems for each of the cases above. The equilibrium solutions will be noted by $\boldsymbol{p^h} = (p_1^h, p_2^h)$, $\boldsymbol{a^h} = (a_1^h, a_2^h)$; and $\boldsymbol{p^{nh}} = (p_1^{nh}, p_2^{nh})$, $\boldsymbol{a^{nh}} = (a_1^{nh}, a_2^{nh})$.

4.3 Solution for Case 1: Hierarchical segmentation

4.3.1 Prices

Using demand from equations (10) and (11), and threshold incomes from equations (6)-(8), the problems for the low and high-quality colleges at the pricing stage are:

$$\underset{p_1 \in [c,1]}{\text{MAX}} \quad (p_1 - c)\Big((a_2 - a_1)(1 - y_{1,0}) + (1 - a_2)(y_{2,1} - y_{1,0})\Big)$$
$$\text{s.t.} \quad 0 \leq y_{1,0} \leq y_{2,1} \leq 1$$

$$\underset{p_2 \in [c,1]}{\text{MAX}} \quad (p_2 - c)(1 - a_2)(1 - y_{2,1})$$
$$\text{s.t.} \quad 0 \leq y_{1,0} \leq y_{2,1} \leq 1$$

In this case it is easy to check that all corner solutions are uninteresting from the perspective of strategic interaction because at least one college would prefer not to operate.

- If condition $0 \leq y_{1,0}$ binds, then:
  - It follows that $y_{1,0} = 0$, and hence $p_1^h = 0$, so the low-quality college makes no profit
  - This solution would lead to only the high-quality college operating, so the analysis would follow the benchmark case and hence: $p_2^h = \frac{a_2(1+c)-a_1}{2a_2}$
- If condition $0 \leq y_{2,1}$ binds, then:
  - It follows that $y_{2,1} = 0$, and hence $p_1^h = p_2^h = 0$, so both colleges make no profit
- If condition $y_{2,1} \leq 1$ binds, then:
  - It follows that $y_{2,1} = 1$, and hence $p_2^h = 1$ so the high-quality college has no demand
  - This solution would again lead to only one college operating, so the analysis would follow the benchmark case and hence $p_1^h = \frac{a_1(1+c)-q_0}{2a_1}$
- If condition $y_{1,0} \leq y_{2,1}$ binds:
  - It follows that $y_{2,1} = y_{1,0} = \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}$
  - In such scenario, the two colleges solve symmetric problems

- The solution of equilibrium prices for which the two threshold incomes are equated is:

$$p_i^h = \frac{(1+2c)a_i - (1-c)a_i}{3a_i}$$

- Substituting this back into the threshold income gives $y_{2,1} = y_{1,0} = \frac{2c}{3}$ which implies negative profits for the low-quality college.

Regarding interior solutions, the necessary first-order conditions for the price-maximisation problems yield two linear equations that can be expressed as price reaction functions for the two colleges:

$$p_2(p_1, a_1, a_2; q_0, c) = A_0 + A_1 p_1 \tag{15}$$

$$p_1(p_2, a_1, a_2; q_0, c) = B_0 + B_1 p_2 \tag{16}$$

where the slope and intercept of the reaction functions is given by

$$B_0 = \frac{a_1^3(1+c) - a_2^2 q_0 - a_1^2(2a_2(1+c)+q_0) + a_1\left(a_2^2 - cq_0 + a_2(c+2q_0+cq_0)\right)}{2a_1\left(a_1^2 + a_2 - 2a_1a_2 - q_0 + a_2q_0\right)}$$

$$B_1 = \frac{a_2(1-a_2)(a_1-q_0)}{2a_1\left(a_1^2 + a_2 - 2a_1a_2 - q_0 + a_2q_0\right)}$$

$$A_0 = \frac{a_2(1+c) - a_1}{2a_2}$$

$$A_1 = \frac{a_1}{2a_2}$$

The equilibrium prices for the hierarchical segmentation correspond to the intersection of the reaction functions given by equations (15) and (16), namely $p_1^h = p_1(p_2^h)$ and $p_2^h = p_2(p_1^h)$. Figure 9 illustrates this in a diagram.

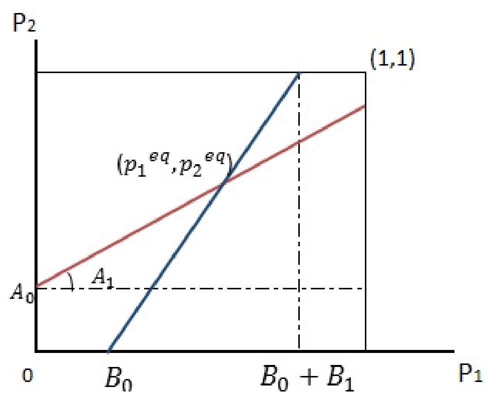


Figure 9: Equilibrium prices for the hierarchical segmentation

Proposition 5. *The reaction functions given by equations* (15) *and* (16) *intersect exactly once for* $q_0 < a_1 < a_2 < 1$

Proof. Through algebraic manipulation it can be shown that A_0, A_1, B_0, B_1 are strictly positive and smaller than 1 whenever $q_0 < a_1 < a_2 < 1$. From Figure 9 it can be checked that due to linearity, this implies that the reaction functions must intersect exactly once. □

Before continuing to the stage of competition in tuition quality, it is important to discuss the relationship between equilibrium prices and quality. Figure 10 explores this by plotting p_1^h and p_2^h as a function of quality.

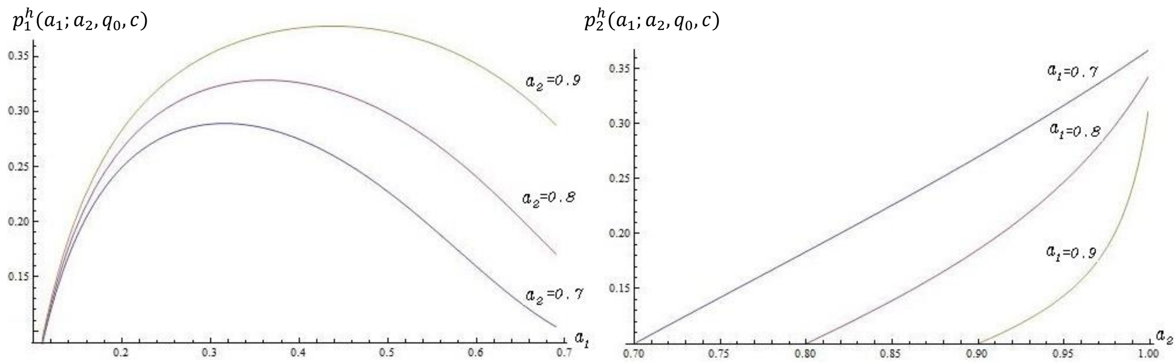


Figure 10: Hierarchical equilibrium prices as a function of quality, for $c = q_0 = 0.1$

For the high-quality college, p_2^h is increasing in a_2 and decreasing in a_1 and q_0 (i.e. prices at the high-quality college are increasing in own quality and decreasing in the quality of the other options). This is because when a_2 goes up (or equivalently, when a_1 or q_0 go down) the high-quality college becomes comparatively more attractive than the alternatives, so it can consequently set a higher price in equilibrium. This follows from the same reasoning that was discussed for the benchmark problem with only one college (Proposition 3). For the low-quality college, on the other hand, p_1^h is increasing in a_2 because whenever the high-quality college increases its quality, the low-quality college becomes less similar to its competitor and can hence increases its price. Interestingly, however, p_1^h is non-monotonic in a_1 because whenever the low-quality college increases its own quality, everything else equal, it simultaneously becomes less similar to the outside option but more similar to the high-quality college; so fees reach a maximum at some intermediate point between q_0 and a_2 where the differentiation is as large as possible.

4.3.2 Tuition quality

The equilibrium prices $\boldsymbol{p^h} = (p_1^h, p_2^h)$ depend on quality choices, so they can be substituted into profits to obtain a function only in terms of qualities. Replacing an explicit form of $\boldsymbol{p^h}$ into profits to derive first order conditions would unfortunately yield extremely long expressions which would not be easy to manipulate or interpret analytically (recall that the reaction functions in prices −equations (15) and (16)− are third degree polynomials in a_1 and a_2). On account of this, it is convenient to fix the underlying parameters and solve the problem numerically.

Figure 11 superimposes the iso-profit contours for both colleges as a function of quality standards for $c = q_0 = 0.1$. Since the solution under a hierarchical segmentation is only relevant if the equilibrium is indeed consistent with the case-specific condition (9) −which is what guarantees a hierarchical segmentation in the first place− the second panel in Figure 11 presents again the same plot, but cropping the region to include only combinations of quality where the threshold incomes are consistent with

$$0 < y_{1,0}(p_1^h, a_1) < y_{2,0}(p_2^h, a_2) < y_{2,1}(\boldsymbol{p}^h, \boldsymbol{a}) < 1$$

(where $y_{1,0}(p_1^h, a_1)$, $y_{2,0}(p_2^h, a_2)$ and $y_{2,1}(\boldsymbol{p}^h, \boldsymbol{a})$ denote the threshold income after substituting the equilibrium prices).

As it can be seen from this example, the iso-profit contours are convex. The reaction functions corresponding to the level of tuition quality that maximises profits for a given choice of quality by the competitor intersect only once (red and pink lines in Figure 11) and the interior solution is consistent with a hierarchical equilibrium segmentation.

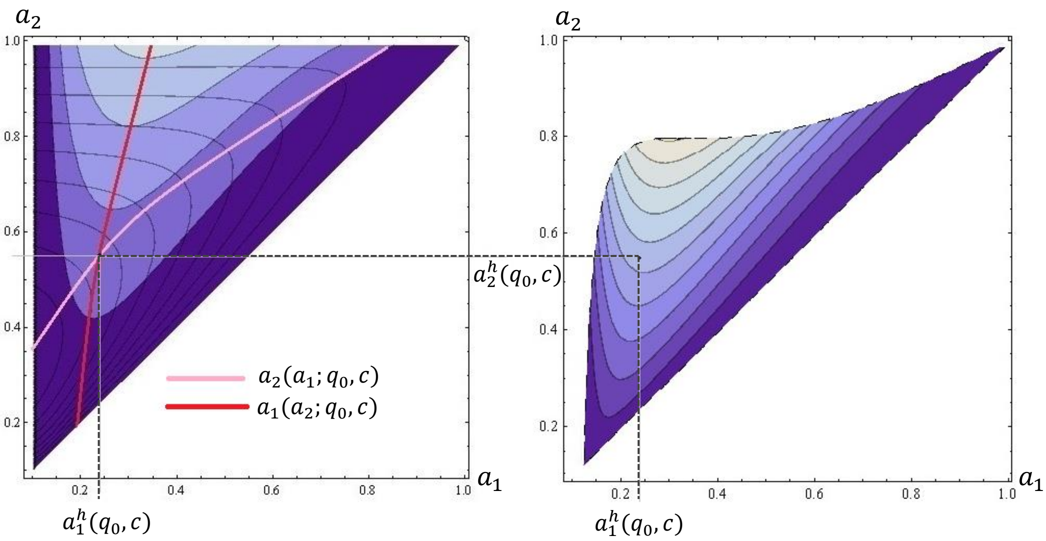


Figure 11: Superimposed iso-profit contours and optimal quality choices for $c = q_0 = 0.1$. Right panel restricted to case-consistent combinations of (a_1, a_2)

4.4 Solution for Case 2: Expensive low-quality College

4.4.1 Prices

Analogously to the previous case, the respective problems at the pricing stage under a non-hierarchical segmentation are:

$$\begin{array}{cc} \underset{p_1 \in [0,1]}{\text{MAX}} & (p_1 - c)(a_2 - a_1)(1 - y_{1,0}) \\ \text{s.t.} & 0 \leq y_{2,0} \leq y_{1,0} \leq 1 \end{array}$$

$$\begin{array}{cc} \underset{p_2 \in [0,1]}{\text{MAX}} & (p_2 - c)(1 - a_2)(1 - y_{2,0}) \\ \text{s.t.} & 0 \leq y_{2,0} \leq y_{1,0} \leq 1 \end{array}$$

Once again corner solutions are uninteresting because they would lead to at least one college not making positive profits. Concentrating on interior solutions, from inspection of the profit functions above it follows that demand only depends on own prices. In this case, the first order conditions at the pricing stage yield two independent equations:

$$p_i^{nh} = \frac{a_i(1+c) - q_0}{2a_i} \quad \text{for} \quad i = 1, 2 \tag{17}$$

This means that there is no strategic interaction at this stage and the pricing reaction functions are flat.

4.4.2 Tuition quality

The optimal prices given by equations (17) depend only on own quality. Moreover, since the profits of the high-quality college are not affected by the choice of quality of the other college, the maximisation problem that college 2 faces is independent from the choice of quality of college 1.

Making use of this, the optimal tuition quality can be found by solving for $a_2^{nh}(q_0, c)$ first, and then substituting into the problem of the low-quality college to solve for $a_1^{nh}(q_0, c) = a_1(a_2^{nh}(q_0, c))$. Figure 12 presents an example of the solution.[3]

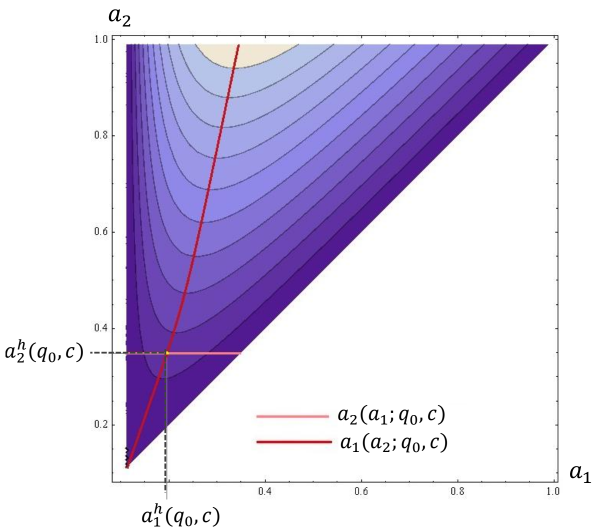


Figure 12: Equilibrium qualities derived graphically from superimposing iso-profit contours of the high-quality college (for $c = q_0 = 0.1$)

[3]Here it is not necessary to present an additional panel cropping the plot because all combinations of a_1 and a_2 (for which $q_0 < a_1 < a_2 < 1$) are consistent with $0 < y_{2,1}(\boldsymbol{p}^h, \boldsymbol{a}) < y_{1,0}(p_1^h, a_1) < y_{2,0}(p_2^h, a_2) < 1$

5 Equilibrium analysis

A market outcome is a combination of hierarchical and non-hierachical equilibrium qualities and prices, for a given combination of the parameters q_0 and c. As should be clear from the analysis in the preceding section, an analytic characterisation of market outcomes is not convenient given the algebraic complexity of the solutions; so it is necessary to rely on numerical implementations.

The simulations presented in more detail in the Appendix C3 indicate that the hierarchical and non-hierarchical equilibrium prices are increasing in c but decreasing in q_0 (for both colleges), while equilibrium qualities are increasing in c and q_0 (for both colleges). This is in line with the comparative statics from the benchmark problem: regardless of whether the equilibrium segmentation is hierarchical or not, a more valuable outside option implies that both colleges set lower prices and offer higher quality, while a higher marginal cost implies that both colleges operate at a smaller scale with higher prices and higher quality.[4]

The numerical results also show that for all combinations of q_0 and c the hierarchical equilibrium yields lower prices, higher quality and larger served demand than the non-hierarchical equilibrium; as a result, the hierarchical equilibrium leads to higher student welfare.

To illustrate this, Table 1 presents the market outcomes and the associated welfare when $q_0 = c = 0.1$. Figure 13 shows the corresponding outcome segmentations graphically. The following points are worth emphasising:

- The hierarchical segmentation leads to lower prices, higher quality, lower threshold incomes and larger aggregate student welfare

- The total number of students attending college is higher under the hierarchical segmentation, and this is mostly due to the demand served by the low-quality college taking students who are either not rich or skilled enough.

- In the non-hierarchical case, the low-quality college (which is by definition

[4]This does not mean that the effect on demand associated with changes in q_0 is the same across cases. Under the hierarchical case, increasing q_0 narrows the gap in the marginal income of entrants between colleges, while under a non-hierarchical segmentation the gap is widened. This is because under the hierarchy only the low-quality college is directly affected by q_0.

expensive for the relative quality it offers) is still cheaper than the high-quality alternative. This makes the average income of students attending college very similar across colleges in the non-hierarchical case.

- Students attending the high-quality college under the non-hierarchical case are, on average, richer and brighter than students attending the high-quality college under the hierarchy. This is despite the fact that under a hierarchical segmentation the high-quality college concentrates on the segment of high-income/high-ability students.

Case 1: Hierarchical equilibrium						
College	p_i^h	a_i^h	$y_i(p^h, a^h)$	$x_i(p^h, a^h)$	$SW_i(p^h, a^h)$	$SW(p^h, a^h)$
1	0.21	0.23	0.37	0.21	0.019	
2	0.35	0.46	0.49	0.27	0.050	0.086
Outside	-	-	-	0.52	0.017	
Case 2: Non-hierarchical equilibrium						
College	p^{nh}	a^{nh}	$y_i(p^{nh}, a^{nh})$	$x_i(p^{nh}, a^{nh})$	$SW_i(p^{nh}, a^{nh})$	$SW(p^{nh}, a^{nh})$
1	0.31	0.20	0.60	0.06	0.006	
2	0.41	0.35	0.57	0.28	0.037	0.066
Outside	-	-	-	0.56	0.023	

Table 1: Equilibrium outcome for $q_0 = c = 0.1$. Total student welfare in last column corresponds to the sum of contributions in the penultimate column. Row "Out" presents information about students at the outside option.

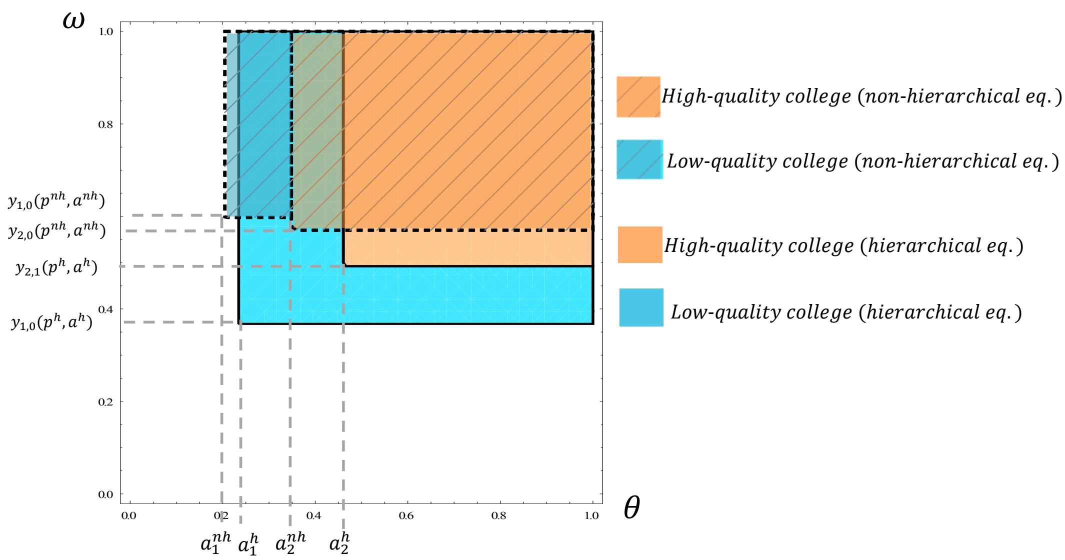


Figure 13: Equilibrium demand for $q_0 = c = 0.1$

The above remarks are important to understand the possible welfare implications associated with policy interventions aimed at deregulating the market for higher education. Consider the following example. Suppose that the market is regulated, such that prices are fixed at some level $\overline{p} > c$ and colleges compete only along the quality dimension. By definition, such scenario would necessarily fall under the non-hierarchical case because at the same price, the low-quality college would only be attractive to students who are not skilled enough to attend the high-quality college. Since the high-quality college does not face strategic interaction in quality under this non-hierarchical segmentation, fixed prices would imply that college 2 can choose quality according to the benchmark, namely:

$$a_2^{nh}(\overline{p}, q_0, c) = q_0 + \frac{\sqrt{(1-\overline{p})(1-q_0)\overline{p}q_0}}{1-\overline{p}}$$

The low-quality college, in turn, would take this as given and solve:

$$a_1^{nh}(\overline{p}, q_0, c) = \underset{a_1}{\text{argmax}} \left[(\overline{p} - c)\left(a_2^{nh} - a_1\right)\left(1 - \frac{\overline{p}a_1}{a_1 - q_0}\right)\right]$$

$$= q_0 + \frac{\sqrt{(1-\overline{p})(a_2^{nh} - q_0)\overline{p}q_0}}{1-\overline{p}}$$

Table 2 presents a summary of the market outcome for $q_0 = c = 0.1$ assuming that fees are fixed at $\overline{p} = 0.2$.

Fixed prices						
College	$\overline{p}$	a^{nh}	$y_i(\overline{p}, a^{nh})$	$x_i(\overline{p}, a^{nh})$	$SW_i(\overline{p}, a^{nh})$	$SW(\overline{p}, a^{nh})$
1	0.2	0.16	0.53	0.4	0.004	
2	0.2	0.25	0.33	0.5	0.058	0.076
Outside	-	-	-	0.1	0.013	

Table 2: Equilibrium outcome summary for $q_0 = c = 0.1$ under fixed prices

As it can be checked by comparing these results with those in Table 1, the regulated outcome would have a higher student welfare than the non-hierarchical *laissez-faire* equilibrium, but lower than the hierarchical alternative. This means that if the regulated market was suddenly deregulated to allow competition in prices, the resulting segmentation of the market could become either hierarchical or non-hierarchical, which could in turn imply either an improvement or a deterioration in student welfare due to changes in the allocation of students into colleges.

It should be noted that, although the hierarchical equilibrium under monopolistic competition leads to higher student welfare, the allocation of students into colleges that maximises student welfare is not hierarchical. The Appendix C4 discusses this point in more detail. Intuitively, since the utility of individuals attending college is decreasing in prices −regardless of which college they attend− a social planner aiming to maximise aggregate student welfare would choose $p_1 = p_2 = c$; and as has been pointed out, this can only be consistent with a non-hierarchical allocation of students into colleges. Appendix C4 shows that for $q_0 = c = 0.1$ student welfare is maximised where $p_1^{SW} = p_2^{SW} = 0.1$ and $\left(a_1^{SW}, a_2^{SW}\right) = (0.41, 0.7)$

Before moving on to the concluding remarks, it should also be noted that all the results discussed so far would follow under an alternative interpretation of the model, where student preferences over a_i capture peer-effects rather than preferences over the intrinsic quality of the teaching. Under such alternative interpretation, sufficiently talented students would prefer a more exclusive admission policy (i.e. a higher a_i) because it provides them with better peers, rather than better tutoring *per se*. Since a_i would still determine the ability of the marginal entrant, this alternative interpretation allows an insightful comparison of the results with those in the literature on peer-effects, where utility is assumed to depend on the *average* entrant ability.[5]

When preferences depend on *average* entrant quality, colleges have an incentive to offer a lower price to poor students of high ability, because this allows them to charge a suitably higher price to richer students of low ability. This, in turn, implies that it is not be possible to sustain in equilibrium a non-hirarchical segmentation where colleges with low admission standards charge higher fees than the more selective alternatives. Indeed, as it was mentioned in the review of related literature at the

[5] It is important to keep in mind that under this peer-effect interpretation of the model, low quality students are not excluded because of their lack of ability to benefit from the teaching, but because they deteriorate the educational experience of higher-ability students attending the college. Given the assumption of perfect information about student quality, this distinction is irrelevant. Clearly, this interpretation does not allow a trivial comparison with the results from the recent literature on peer effects and pricing under imperfect information (see Fu 2013).

introductory section, Epple and Romano (1998) study a model where students have preferences over disposable income and average entrant quality (with otherwise very similar assumptions, for example, regarding the complementarity of income and quality), and find that there is a unique equilibrium exhibiting a hierarchy of college qualities and within-college price discrimination.

By contrast, when student preferences depend on *marginal* entrant quality, colleges have no incentive to subsidise certain students, because for any level of college quality, willingness to pay is independent of the composition of the pool of entrants. Thus, this simple distinction between preferences over marginal and average peer ability −stemming from a conceptual difference in the definition of educational quality as valued by students−, underlies the equilibrium analysis of the paper and contextualises the relevance of its contribution.

6 Conclusions

This paper developed a model of monopolistic competition with product differentiation, where profit-maximising colleges choose prices and tuition quality, the latter by setting a minimum ability threshold. The model was first explored in a benchmark case with only one college and a fixed outside option, in order to show that the tension between making standards high and excluding potential customers, makes it optimal for a monopolist to use a combination of prices and admission requirements to maximise profits. Strategic interaction between ex ante identical colleges was then introduced, and it was shown that there are two possible market segmentations that are sustainable in equilibrium: a hierarchical allocation of students to colleges, in which high-quality colleges serve an elite portion of the market by selling their services to the high-income/high-ability students, and another, non-hierarchical allocation, in which the high-quality colleges are cheaper than the low-quality colleges which serve low-ability/high-income students.

The presented analysis suggests that the distinction between these two equilibria is relevant to understand the welfare implications of market reforms, such as the deregulation of prices. Numerical examples showed that if a market with fixed tuition fees is liberalised to allow competition in prices, the resulting equilibrium may imply either an improvement or a deterioration in student welfare depending on the type of equilibrium that arises. This point is important because, as was discussed in the introduction, demand and supply analysis in education markets often omit the role of quality as a variable that colleges can choose strategically, and which crucially affects the segmentation of the market and the associated student welfare.

C Appendix for Chapter 3

C.1 Quasi-concavity of profits for the monopoly benchmark

Proposition 1 . *Suppose* $0 < q_0 + c < 1$. *Then the necessary first order conditions* (3) *and* (4) *have at most one solution satisfying* $a \in (q_0, 1)$ *and* $p \in (c, 1)$. *Hence, the interior solution* (p^*, a^*) *that maximises* $\pi(p, a; q_0, c)$ *is unique, whenever it exists.*

Proof. Note that equations (3) and (4) can be solved for p to obtain:

$$p - \frac{a(1+c) - q_0}{2a} = 0$$
$$p - \frac{(a - q_0)^2}{a^2 + q_0 - 2aq_0} = 0$$

So the optimal value of quality corresponds to a solving

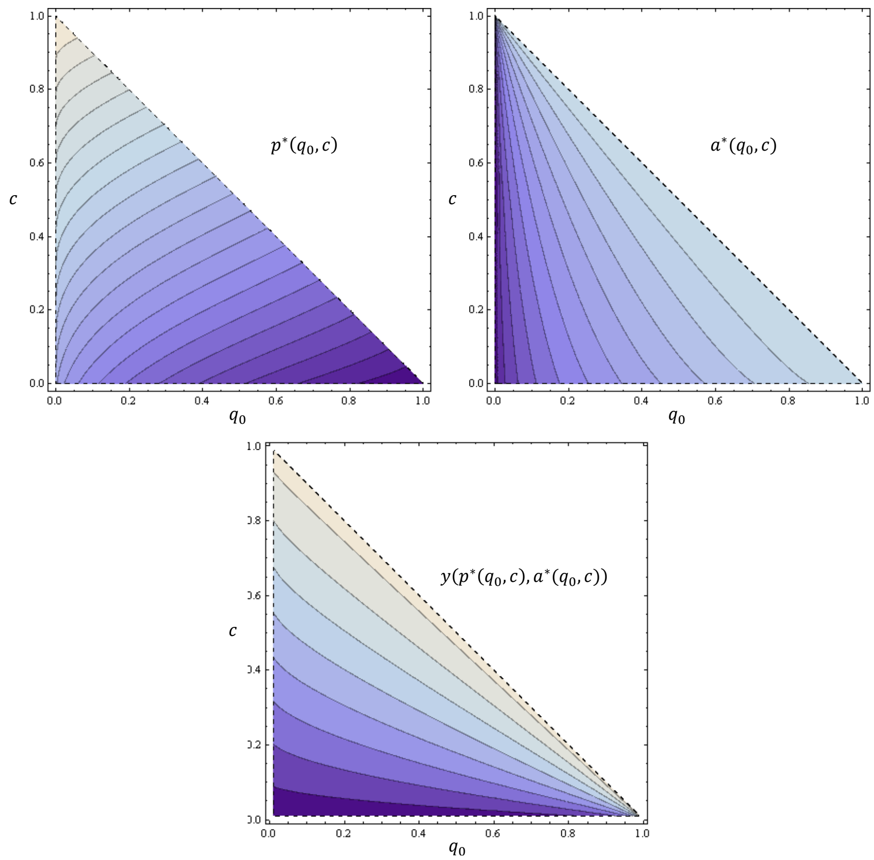
$$\frac{a^3(-1+c) + a(1+c)q_0 - q_0^2 + a^2(q_0 - 2cq_0)}{2a\left(a^2 + q_0 - 2aq_0\right)} = 0 \tag{C1}$$

The denominator in equation C1 is strictly positive for $0 < q_0 < a < 1$. So the solution must correspond to the roots of the cubic polynomial in the numerator. It can be checked that, from the three possible roots of this polynomial, whenever $0 < q_0 + c < 1$ there is only one real root $a^* \in (q_0, 1)$. □

C.2 Comparative statics for the monopoly benchmark

As it should be clear from inspection of equation C1, the comparative statics of the optimal menu (p^*, a^*) are not easy to derive analytically. Nevertheless, it is straightforward to calculate the solutions numerically for any parametric combination (q_0, c); in fact, it is possible to inspect the solutions graphically for the entire parameter space, by depicting level sets. This allows for an exhaustive test of regularity to substantiate Propositions 3 and 4.

The following are the contour plots corresponding to $p^*(q_0, c)$, $a^*(q_0, c)$ and $y(p^*(q_0, c), a^*(q_0, c))$ for $q \in [0, 1]$ and $c \in [0, 1]$, such that $0 < c + q_0 < 1$. Lighter shades represent higher levels. It is clear that a^* and $y(p^*, a^*)$ are both monotonically increasing in c and q_0 −but p^* is monotonically increasing in c and decreasing in q_0.



C.3 Monopolistic competition simulations

The following tables present a summary of market outcomes for different combinations of the underlying parameters. A selection of these outcomes is presented graphically in Figure 14 to allow a visual comparison.

Case 1. Hierarchical eq.							
q_0	a_1^h	a_2^h	p_1^h	p_2^h	$y_{1,0}(p^h, a^h)$	$y_{2,1}(p^h, a^h)$	$SW(p^h, a^h)$
0.01	0.05	0.20	0.27	0.45	0.33	0.52	0.028
0.1	0.23	0.46	0.21	0.35	0.37	0.49	0.086
0.3	0.49	0.69	0.16	0.26	0.43	0.47	0.174
Case 2. Non-hierarchical eq.							
q_0	a_1^{nh}	a_2^{nh}	p_1^{nh}	p_2^{nh}	$y_{1,0}(p^{nh}, a^{nh})$	$y_{2,0}(p^{nh}, a^{nh})$	$SW(p^{nh}, a^{nh})$
0.01	0.04	0.11	0.41	0.50	0.57	0.55	0.014
0.1	0.20	0.35	0.31	0.41	0.60	0.57	0.066
0.3	0.46	0.60	0.23	0.30	0.64	0.60	0.161

Table 3: Market outcomes for different values of q_0, holding $c = 0.1$

Case 1. Hierarchical eq.							
c	a_1^h	a_2^h	p_1^h	p_2^h	$y_{1,0}(p^h, a^h)$	$y_{2,1}(p^h, a^h)$	$SW(p^h, a^h)$
0.01	0.20	0.42	0.14	0.30	0.27	0.45	0.091
0.10	0.23	0.46	0.21	0.35	0.37	0.49	0.086
0.30	0.31	0.54	0.38	0.47	0.56	0.60	0.073
Case 2. Non-hierarchical eq.							
c	a_1^{nh}	a_2^{nh}	p_1^{nh}	p_2^{nh}	$y_{1,0}(p^{nh}, a^{nh})$	$y_{2,0}(p^{nh}, a^{nh})$	$SW(p^{nh}, a^{nh})$
0.01	0.18	0.32	0.23	0.35	0.51	0.51	0.069
0.10	0.20	0.35	0.31	0.41	0.60	0.57	0.066
0.30	0.27	0.42	0.46	0.53	0.74	0.70	0.059

Table 4: Market outcomes for different values of c, holding $q_0 = 0.1$

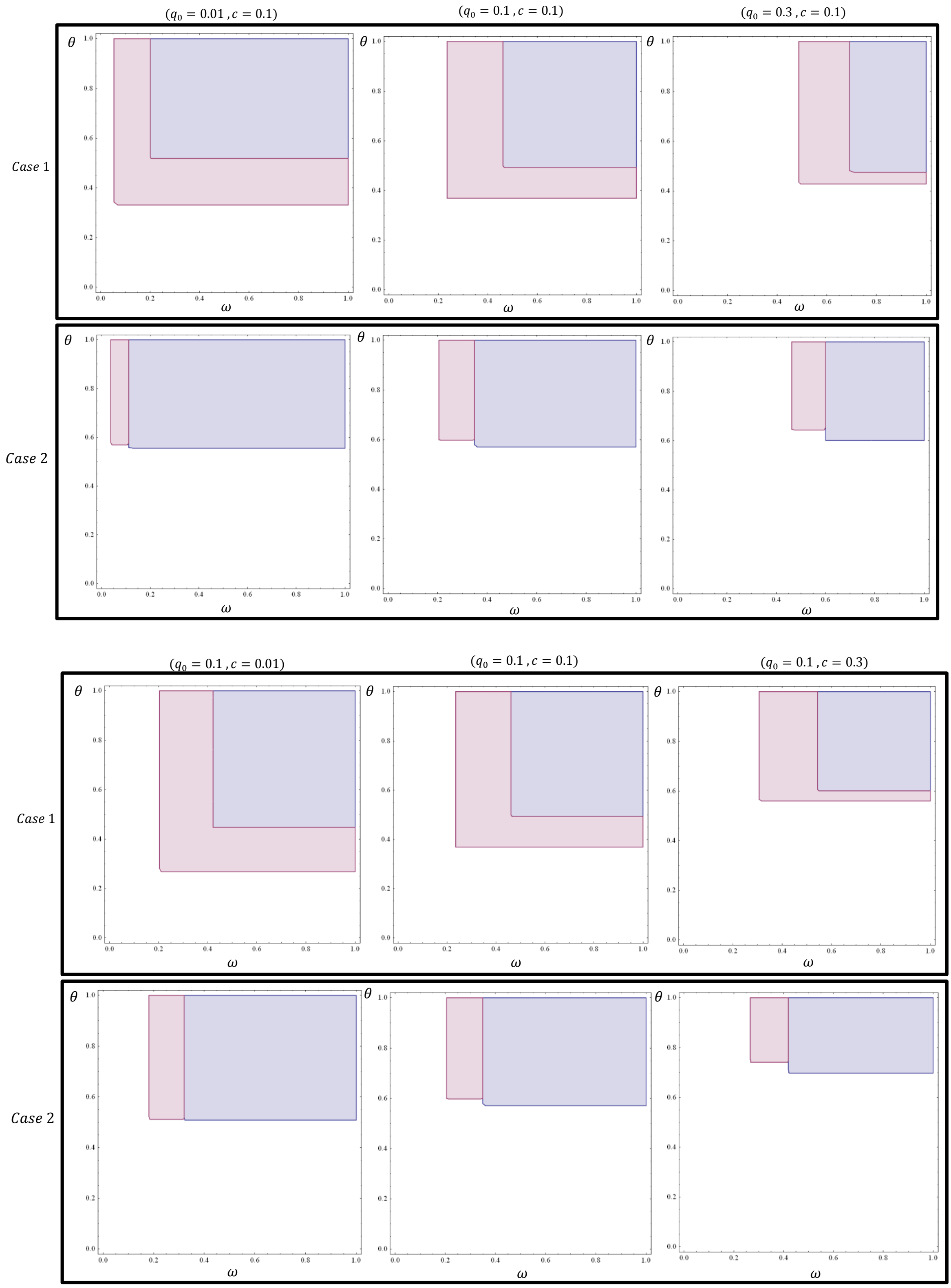


Figure 14: Market outcome by cases for different parameter specifications

C.4 Student welfare under monopolistic competition

Analogously to the welfare analysis from the benchmark case, aggregate student welfare under monopolistic competition can be calculated as the expected utility associated with a menu (a_1, a_2, p_1, p_2). For the hierarchical and non-hierarchical equilibria, student welfare is given respectively by

$$\begin{aligned} SW^{nh}(a_1, a_2, p_1, p_2) =& a_1 (a_2 - a_1) (1 - y_{1,0}) \left(\frac{1 + y_{1,0}}{2} - p_1 \right) \\ &+ a_2 (1 - a_2) (1 - y_{2,0}) \left(\frac{1 + y_{2,0}}{2} - p_2 \right) \\ &+ q_0 \frac{y_{1,0}}{2} (a_2 - a_1) y_{1,0} + q_0 \frac{y_{2,0}}{2} (1 - a_2) y_{2,0} + \frac{q_0}{2} a_1 \end{aligned}$$

and

$$\begin{aligned} SW^{h}(a_1, a_2, p_1, p_2) =& a_2(1 - a_2)(1 - y_{2,1}) \left(\frac{1 + y_{2,1}}{2} - p_2 \right) \\ &+ a_1(1 - a_2)(y_{2,1} - y_{1,0}) \left(\frac{y_{2,1} + y_{1,0}}{2} - p_1 \right) \\ &+ a_1(a_2 - a_1)(1 - y_{1,0}) \left(\frac{1 + y_{1,0}}{2} - p_1 \right) \\ &+ q_0 \frac{y_{1,0}}{2} (1 - a_1) y_{1,0} + q_0 \frac{a_1}{2} \end{aligned}$$

where $y_{1,0}$, $y_{2,0}$ and $y_{2,1}$ are defined by equations (6)-(8).

Both of these expressions for student welfare are decreasing in p_1 and p_2. So a social planner interested in student welfare would choose $\left(p_1^{SW}, p_2^{SW}\right) = (c, c)$, which would in turn mean that the resulting allocation would have to be non-hierarchical (any student who is sufficiently skilled would take the high-quality college instead of low-quality alternative for the same price). This also means that $y_{1,0} = c \frac{a_1}{a_1 - q_0}$

and $y_{2,0} = c\frac{a_2}{a_2 - q_0}$, so the planner would choose (a_1, a_2) to maximise

$$\begin{aligned} SW(a_1,a_2) = & a_1(a_2-a_1)\left(1-\frac{a_1c}{a_1-q_0}\right)\left(\frac{1}{2}\left(1+\frac{a_1c}{a_1-q_0}\right)-c\right) \\ & + a_2(1-a_2)\left(1-\frac{a_2c}{a_2-q_0}\right)\left(\frac{1}{2}\left(1+\frac{a_2c}{a_2-q_0}\right)-c\right) \\ & + \frac{q_0\frac{a_1c}{a_1-q_0}}{2}(a_2-a_1)\frac{a_1c}{a_1-q_0} + \frac{q_0\frac{a_2c}{a_2-q_0}}{2}(1-a_2)\frac{a_2c}{a_2-q_0} + \frac{q_0}{2}a_1 \end{aligned}$$

Figure 15 presents a plot of iso-welfare levels for $q_0 = c = 0.1$ (higher contours depicted in lighter shades). As it can be seen, aggregate student welfare is maximised for $\left(a_1^{SW}, a_2^{SW}\right) = (0.41, 0.71)$.

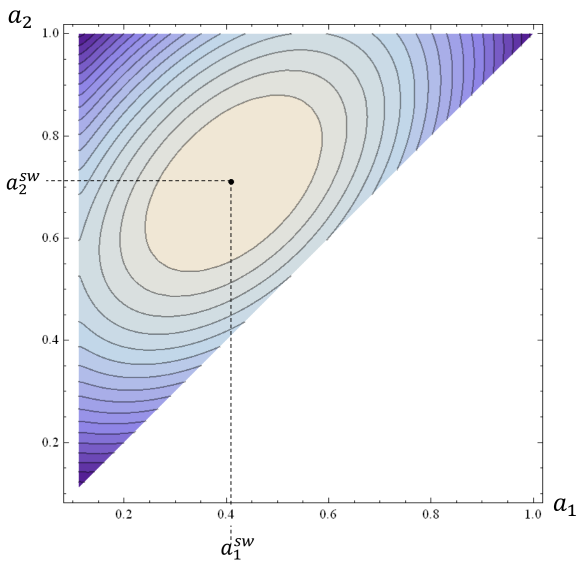


Figure 15: Contour plot of student welfare as a function of quality standards, for $p_1 = p_2 = c$ and $q_0 = c = 0.1$

This exercise shows that a social planner would choose a segmentation where students are divided into segments of similar size along the ability dimension. In fact, if there was no outside option and education had no cost, such that $q_0 = c = 0$, then a planner would choose $\left(p_1^{SW}, p_2^{SW}\right) = (0, 0)$ and

$$\left(a_1^{SW}, a_2^{SW}\right) = \underset{(a_1,a_2)}{\text{argmax}} \left[\frac{1}{2}(1-a_2)a_2 + \frac{1}{2}a_1(a_2-a_1)\right] = \left(\frac{1}{3}, \frac{2}{3}\right)$$

so quality would split the student space in equidistant segments along the ability dimension.

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