

A ‘lost’ chapter in the calculation of π : Baron Zach and MS Bodleian 949

Abstract

The Bodleian library holds a manuscript containing mathematical tables and a calculation of π to 154 decimal places, last described (in part) in 1802. This paper provides an outline of the manuscript’s contents and relates it to contemporary developments in the computation of π .

Keywords

History of mathematics, mathematical tables, pi, Baron Zach.

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1. Introduction

In the second edition of Montucla’s *Histoire des mathématiques*, a calculation of π is mentioned:

M. le baron de Zach a vu dans un manuscrit de la bibliothèque de Ratclif, à Oxford, le calcul poussé ... jusqu’à 155 chiffres [Montucla 1802, p. 640].

At the time this was the most ambitious computation of the value of π ever to have been performed. But its author remained unidentified, and the manuscript was never published, eventually slipping into obscurity. It has become something of a lost chapter in the computation of π . This paper describes the manuscript, still held in Oxford, and the little we can glean about its author.

2. MS Bodl. 949

Oxford, Bodleian Library, MS Bodl. 949 contains a calculation of the ratio of a circle’s circumference to its diameter to 154 decimal places; 155 figures including the initial ‘3’ (fols. 114^v–121^v). In the manuscript the calculation is called a ‘Quadrature

of the Circle’, and the term π is not used, but for the sake of brevity I refer to this as a ‘calculation of π ’, and the resulting number a ‘value of π ’.

The manuscript, which measures about 18 inches by 11 inches and contains over 250 pages of text, is the fair copy of various mathematical tables and calculations, done over a period of more than 30 years. It is on good quality paper, but it is not all that ‘fair’: although much of it is quite neatly laid out on pages with ruled margins, it has many corrections and widely variable handwriting. It has no title page or table of contents. Notes interspersed with its contents make it clear when the work was done. The calculation of π was initially carried out in Philadelphia, Pennsylvania ‘around 1721’ (fol. 121^v). There is no other indication of the location(s) where the calculations took place. The calculation of π was ‘worked over’ in 1723–4 with correction of three figures. From the beginning of February 1727 the author then worked on a large table of common logarithms to 30 decimal places, eventually computing the values of 10,000 logarithms. Also calculated during this period were several smaller supplementary tables of logarithms (such as the logarithms of radices from 1.0000 to 1.0009 in increments of 0.0001), and a complete table of sines and logarithmic sines in increments of one minute, to (mostly) 28 decimal places. This work took until 1759.

Initially all of the calculations were apparently done on loose sheets – reference is made to a ‘loose copy’ – but from probably quite early in the period 1727–1759 the author began to copy the results into the large, bound volume that is now MS Bodl. 949 (see Figure 1). Gaps were left in between the different tables at this stage, and the volume was initially fairly neatly copied.

In 1737 the first part of the π calculation was repeated once again, and was copied into the end of the volume; the second part was repeated in 1758–9, and also copied in: apparently just one figure needed to be adjusted compared with the earlier computation. At around the same time the author used the blank pages in between the different tables for quite extensive, and somewhat less tidy, notes on their construction and use. Between March and June 1759 the author checked the logarithm tables and the sine table against the original ‘loose copy’ and calculations; in the logarithm tables no more than 20 figures needed to be corrected. (I have spot-checked

twenty of the logarithms and sines using WolframAlpha; all the logarithms were entirely correct, while five of the sines contained a total of six incorrect digits.) The strong clear figures of the 1730s had now deteriorated into a cramped, crabbed hand, by which these later sections, and the late corrections made to the tables, can be clearly identified. The impulse to calculate was not dead, though, and the author began a new series of ‘tables of arches’: preliminary calculations apparently intended to support the construction of a new table of sines. This ended, or petered out, after just 13 pages in the fair copy; no dates were added to it. Nothing more was copied out, and out of a volume of 280 leaves, over 150 remained blank.

The calculation of π was performed using a method that had been published in Henry Sherwin’s *Mathematical Tables* in 1706, ‘communicated by Dr. Halley’, although the lesser-known mathematician Abraham Sharp would claim the method had been stolen from him (see [Willmoth 2011]):

Divide the square Root of 12 continually by 3, and the several Quotients again by all the Odd Numbers successively, viz. the first quote by 3, the second by 5, the third by 7, &c. Then to the $\sqrt{12}$ add the $1/5$ of the second Quote, $1/9$ of the fourth, $1/13$ of the sixth, &c. in *infinitum*, and from the Sum substract $1/3$ of the first Quote, $1/7$ of the third, $1/11$ of the fifth, &c. in *infinitum*, and the Remainder shall be the Circumference sought [Sherwin 1706, p. 54].

The author of MS Bodl. 949 quoted this verbatim and imitated the page layout of the calculations from Sherwin’s book. In modern terms, the method was to sum

$$\sqrt{12} - \sqrt{12}/3.3 + \sqrt{12}/9.5 - \sqrt{12}/27.7 \dots + (-1)^n \cdot \sqrt{12}/3^n \cdot (2n + 1) \dots$$

In this case it was summed for n up to 314, having first calculated $\sqrt{12}$ to 154 decimal places (there is no evident reason for the choice of 154). Like Halley, the author first calculated the positive terms, and summed them, then calculated the negative terms, and summed them, and finally subtracted the one sum from the other (see Figure 2). The result given is

3.141592 653589 793238 462643 383279 502884 197164 399375 105820
974944 592307 816406 286208 998628 034825 342117 067982 148086
513282 306647 093844 609550 582231 725359 408128 4802.

The author remarked (fol. 121^v) that ‘I believe the whole may be depended on as far as one hundred fivety one or fivety two places of figures’, which indeed it can. Digits 153 and 154 are incorrect, apparently not because any mistake was made but simply as a result of the slow convergence of Halley’s series. This result represented an enormous increase over the 72 digits Sharp had calculated using the same method [Sherwin 1706, p. 57]. But despite the labour expended on the calculations, their author did not trouble to stay up-to-date with the subject. John Machin published more rapidly converging series for π in [Jones 1706, p. 263]; he also published 101 decimal places of π in [ibid., p. 243]. The author of MS Bodl. 949 mentioned neither this nor other subsequent work on the subject such as that of de Lagny.

As for the logarithm tables, both Wallis and Newton were cited (fol. 1^r), but a note in the manuscript states that it was reading of Halley’s [1695] work on logarithms that provided the real stimulus:

The Hint given in that Transaction, that a logarithmick Table calculated to more Places, would be acceptable, and the use it would be of, to the Curious, in the managem[en]t of great unweildy numbers; put me on a Designe of makeing, one hundred Chiliads of Logarithms; to 30 Places [fol. 1^r].

I suspect that the reference to ‘use’ here is misleading; indeed, it is the logarithms in these tables that are ‘great unweildy numbers’, more than the radices. For most if not all purposes a 30-place table would actually be of *less* use than the 10-place or 14-place tables already in print. Much the same is true of the calculation of π , the precision of which corresponds to no conceivable situation of ‘use’.

No printed version of these tables is known to the major bibliographical databases for the period, and the manuscript is neither finished nor polished in a way that suggests readiness for printing. Nor do the notes in the manuscript mention printing, either as a plan, an accomplished fact or an abandoned aspiration. Printers were not fond of such

things, and I think it probable that the tables were never published. (One comparison: the ‘Tables du Cadastre’ constructed in France in the 1790s and giving 100,000 logarithms to 19 places and a further 100,000 to 24 places, also remained in manuscript despite serious plans to publish them: see [Roegel 2010] and [Peaucelle 2011, 2012].)

Thirty-two years is a long time, and only a few logarithms, sines, or digits of π need have been computed each day over that period in order to produce the results in this manuscript. This was a project that was steadily picked at, not rapidly polished off: the fact that in 1759 it took two months to check about 100 pages suggests that perhaps half an hour to an hour was being spent on the project each day, a level of activity I think is consistent with the calculation of the tables steadily over a period of 30 years. I say a little more about what this suggests in the conclusion to this note: the blunt fact is that we do not know why the tables were made, nor by whom.

The manuscript itself contains no indication of authorship. The notes it contains mention Antony Thacker (d. 1743; see [Taylor 1966, p. 166]) as a ‘young man’ ‘to whom I was greatly obliged to for some instructions in the compiling these tables’, and also ‘Henry Beaton’ (i.e. Beighton: 1686/7–1743). Both were provincial English mathematicians or ‘mathematical practitioners’, and both were editors of the *Ladies’ Diary*, that annual compendium of verse puzzles and problems that had become a major hub of amateur mathematical communication in the English-speaking world (see Albree and Brown 2009; also Despeaux 2014). Some answers to mathematical problems appeared in the *Diary* over the signature ‘Philadelphia’, and it is possible that this was the same – unknown – person who wrote this manuscript.

Other clues are too scanty to justify speculation: the manuscript contains nothing that implies a really high degree of mathematical ability, a grammar school education or an unusual amount of leisure. My perusal of material on the American Philosophical Society, and its precursor the ‘Junto’, has yielded no name that can plausibly be associated with this manuscript. Unless there is a serendipitous find – some reference to ‘so-and-so the calculator’ in Pennsylvanian correspondence or a document with handwriting that matches this one – the identity of the author seems completely open.

3. The manuscript in Oxford

The manuscript itself gives no indication of how or when it reached Oxford; although it is in its original binding and shows no sign of having lost any leaves, there is no trace of a covering letter or notes on its provenance. (There is likewise no sign of the ‘loose’ copy of the tables, or of the original calculations.) Knowing neither the author’s location, date of death, nor intentions, it seems useless to speculate: the road to Oxford could have been direct or indirect, intended or unintended by the author.

There is no evidence that the manuscript circulated among mathematicians, or that it was read at all before Baron Zach saw it in the 1780s. British publications on logarithms, including those of Gardiner and Hutton, remained unaware of its existence. Editions of *Sherwin’s Tables* which appeared during this period continued to give Sharp’s calculation of π as the best (for example [Sherwin 1771, pp. 110–11]).

Franz Xaver, Baron Zach (1754–1832), Hungarian astronomer and scientific intelligencer, was in England working as a tutor from November 1783 (or perhaps slightly earlier) for about two and a half years, minus a European sojourn. He often visited Oxford during this period [Vargha 2005, p. 21]. His phrase in the report quoted above was ‘la bibliothèque de Ratclif’, and probably means the manuscript was in the Radcliffe Camera in central Oxford, which housed scientific books at this time: or, less likely, the Radcliffe Observatory, north of the city centre.

Both institutions were in at least the nominal charge of the same man, Thomas Hornsby, the Savilian Professor of Astronomy (1733–1810: see [Wallis 2011]) and at this date also Observer at the Radcliffe Observatory and Radcliffe Librarian. He became a correspondent of Zach [Vargha 2005, p. 21; McConnell 2004, pp. 35–6], and it seems very likely that it was he who showed the π manuscript to him.

Since Hornsby possibly also had charge of the papers of James Bradley (1692–1762), the late Astronomer Royal, at this time (see [Madan 1895, pp. 604–5]) no more can really be said except that the manuscript had somehow been passed into the network(s) of people who worked at the observatories of Oxford or Greenwich. Searches in the published works of Bradley [1798, 1832–3] and his manuscript

correspondence [Bodleian Library, MSS Bradley 44–45], and the printed catalogue of the Radcliffe library ([Kidd 1835]), find no relevant reference, while the papers of the the Radcliffe Trustees are much too voluminous (85 boxes: see [Clapinson and Rogers, 1991, p. 1393]) for a search to be feasible. Falconer Madan, the Victorian cataloguer of Bodleian manuscripts, describes this item and certain others in the same shelfmark sequence as ‘waifs and strays of which the date of acquisition is not certainly known’ ([1895, pp. 801–2]).

4. Zach’s report and its aftermath

In an 1818 article discussing recent great calculators, Zach mentioned no English authors, nor the Oxford manuscript [Zach 1818, pp. 221–2]. But he passed his information about the manuscript to either Montucla or Jérôme Lalande (the latter was among Zach’s friends: [Vargha 2005, p. 187]), and it appeared in the posthumous volume 4 of Montucla’s history which Lalande edited, in 1802.

This appears to be the first and, in the nineteenth century, the only report of anyone having actually seen MS Bodl. 949. It also included the final digits of the anonymous calculation of π , and thus at this point the Oxford manuscript became the largest calculation of π to have been published, overtaking the 140 figures given by Vega in [1794, p. 633]. Montucla or Lalande did not remark on the discrepancy between figures 137–140 in the two values.

Zach described the manuscript accurately; the sequence of digits printed by Montucla is identical to that found in the manuscript. As mentioned above, those digits were in fact correct up to the 152nd place.

Although the learned world was tired of circle-squarers – both the Paris Academy and the Royal Society resolved during the later eighteenth century to examine no more attempts at the problem (see [Hobson 1913: 3–4]) – the related activity of accurately computing π retained a good deal of prestige. Jurij Vega’s calculation of π to 144 places in the 1780s, indeed, was done with the aim of election to the Petersburg Academy, although he died before the reward could be conferred (see [Sandifer 2006]). One result was that the ‘Oxford’ value for π enjoyed a reasonable circulation

in the first half of the nineteenth century. In 1822 it was printed, with some inaccuracies, by the German mathematician and teacher Bernhard Friedrich Thibaut in the fourth edition of his *Grundriss der reinen Mathematik* [p. 314] but without, as far as I can see, any indication of where it came from (see also [Razpet 2013, pp. 133–4]). The errors were corrected in the following edition (1831), but not the attribution. It is notable that Thibaut chose to trust the anonymous manuscript rather than Vega’s published version for the value of π .

In 1831 a new edition of Montucla’s book about the quadrature of the circle appeared, prepared by an anonymous editor; this included the 127 digits published by de Lagny in 1719, and noted in a footnote that the 114th should be corrected by reference to Vega and that the sequence could be extended to 154 places using the Oxford manuscript [Montucla 1831, p. 157; repeated in Paris 1834, p. 677]: thus Vega’s final four digits were once again disregarded, this time without explicit comment. An ‘addition’ at the end of the volume set out the case in more detail, drawing attention to the fact that for digits 137–140 the Oxford value differed from Vega’s, and suggesting that Montucla in the *Histoire* may have made an error of transcription [Montucla 1831, p. 282]. The implication is that this editor was inclined to let Vega trump the anonymous author of the Oxford manuscript, which was in any case known only at second- or third-hand.

Augustus De Morgan, writing anonymously in the English *Penny Cyclopædia* in 1841 [Long 1841, p. 187; see De Morgan 1882, p. 412], once again repeated the account from Montucla’s *Histoire*, giving also Vega’s value and pointing out that from the 137th place onwards there was as yet no consensus. In the same year the English mathematician William Rutherford published his computation of π to 208 places in the *Philosophical Transactions* [1841, p. 281], mentioning the Oxford manuscript as giving ‘152 places’. Despite his insistence that ‘the computations have been very carefully conducted’ [p. 282], Rutherford’s calculation was by a curious coincidence also incorrect from the 153rd place onwards. The ‘Oxford’ value of π was therefore not in fact improved upon until a few years later, when Leopold von Schulz Strasznický in the *Journal für die reine und angewandte Mathematik* for [1844] reported a value to 205 decimal places (of which 200 were in fact correct) computed by the German calculating phenomenon Zacharias Dase. Strasznický remarked on

Vega's error, on Thibaut's value and its probable source in the Oxford manuscript, and on the correctness of the Oxford value up to the 152nd place; he did not mention Rutherford's computation.

All of these later reports of the Oxford value of π depended on the version in Montucla's *Histoire*, and thus on Zach's (fortunately accurate) transmission of the value; none of the authors had seen the manuscript in person. As far as I am aware the first and last person to identify MS Bodl. 949 as the one Zach had seen was the anonymous archivist who supplied the relevant entry for the *Summary Catalogue* of Bodleian manuscripts around 1895. There, after a description of the manuscript, it was recorded that 'it is stated that baron F. X. von Zach (*d.* 1832) saw this MS. in the Radcliffe Library at Oxford' [Madan 1895, p. 806]. Evidently by this date the manuscript had been transferred to the Bodleian Library, and bore its modern shelfmark. Conceivably some documentation was associated with the manuscript, subsequently lost, to make the identification secure; perhaps the author of the description simply compared its value of π with that in Montucla's *Histoire*.

Despite the existence of this identification in (admittedly rather obscure) print, despite the accuracy of the 'Oxford' value compared with those of Vega and Rutherford, and despite its reasonably frequent appearances in print in the early nineteenth century, subsequent histories of the computation of π have generally given little or no space to Zach's report and the 'Oxford' value of π . The few that did, like the later nineteenth-century references and that of Montucla himself, did not say which of the Bodleian's innumerable manuscripts Zach saw (for example, [Agarwal 2013, p. 29–30]). It has thus, paradoxically, taken on something of the status of a 'lost text' since 1895. With the recent digitization of the *Summary Catalogue* it has now become easily feasible to reconnect Zach's report with MS Bodl. 949.

5. Conclusion

On the evidence we have, the Oxford manuscript represented the world's most accurate value of π for well over a century. It was the most accurate published value for more than 40 years. For these reasons alone it is more than a footnote, and deserves a somewhat more prominent place in histories of π than it has generally

received. It also tells an interesting story about mathematics as a hobby in the early modern period.

When historians think of mathematics as a hobby in this period they are apt to think of its more sociable manifestations: local talents running mathematical schools, working men educating themselves and one another at mathematical clubs, ladies and gentlemen of the middling sort submitting problems and solutions to mathematical periodicals. Mathematics could also be a very private hobby: a daily discipline perhaps comparable to the exercises in prose composition or translation undertaken by many an early modern self-improver, or to daily musical practice. This was mathematics as a form of virtuous leisure, but also as a private ritual that might have stood alongside (or even replaced) the other private rituals of physical exercise, prayer or self-examination. Diaries and autobiographies from the period occasionally provide glimpses of such things: Babbage rising at dawn to study arithmetic books [1864, p. 19], or Anne Lister steadily working through Euclid's *Elements* as a private, and consciously a demanding, discipline (for example [2010, p. 14]). There is no evidence that MS Bodl. 949 was intended to serve a sociable purpose: although like much about this manuscript, we do not know for certain.

The evidence suggests, in fact, that the impulse to lengthy, repetitious calculation was widespread among those with some (or more than some) mathematical ability in the early modern world. The many published tables of logarithms, of trigonometrical values, and – from the later eighteenth century – of lunar positions for navigators, bear witness to the fact that there were plenty of individuals prepared to devote hundreds or even thousands of hours to computation. What is exceptional about MS Bodl. 949, I believe, is not the impulse that created it but the chance that it has survived. It is an unfortunate fact for the historian of mathematics that pages filled with unintelligible numbers and symbols tend sooner or later to be thrown away. We will never know how many amateurs there were who computed neither for pay nor for glory but simply as a pastime or an exercise: how many others calculated π to many digits in the eighteenth century and did not publish their results.

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Captions

Figure 1. MS Bodl. 949, fol. 5v. The first page of the ‘great table’ of logarithms.

Figure 2. MS Bodl. 949, fol. 5v. 121v. The end of the calculation of π , showing the final positive terms, the eventual result and a note on the dates of the calculations.