

Online Appendix for Commodity Cycles and Financial Instability in Emerging Economies

M.Andreev*, M. U. Peiris[†], A. Shirobokov[‡], D. P. Tsomocos[§]

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Abstract

This document presents additional materials not included in the main text of the paper. Section [1](#) presents the historical decomposition of the time series used for model estimation and implied shock series. Section [2](#) presents the optimality conditions of the model.

*Bank of Russia, Russian Federation

[†]Department of Economics, Oberlin College, USA

[‡]ICEF, HSE University, Russian Federation

[§]Saïd Business School and St Edmund Hall, University of Oxford, United Kingdom

1 Quantitative results

1.1 Historical decomposition

Figures 1a to 2h show historical decomposition of the observed data series by shocks for the endogenous and exogenous financial frictions models. Overall the endogenous financial frictions model is able to capture more of the dynamics of the data by the oil shock series, which is especially the case for loans, deposits and non-performing loans to total loans. Oil prices are better captured in the endogenous case with the initial values having a more persistent effect in 1f.

Figures 2e and 2f show that deposits are well matched by the oil price shock in the case of endogenous financial frictions, while in the exogenous financial frictions case the dynamics is matched through the relatively large contributions of different shocks. The superiority of the endogenous frictions model is best seen in Figures 2g and 2h where the endogenous frictions model can explain most of the fluctuations in non-performing loans by oil price shocks while the exogenous frictions model requires measurement errors.

Other studies on Russia provide a more moderate presence of the oil price shock in economic dynamics. For example, Polbin (2014) builds a New Keynesian model with a number of frictions and shows that the oil price shock has the main role in explanation of Great recession in Russia. Kreptsev and Seleznev (2018) build a model with a stochastic oil price trend but no financial sector and with labor as the only production function. They find an insignificant contribution of the oil price shock in the dynamics of GDP. Kreptsev and Seleznev (2017) builds a large-scale DSGE model with the banking sector and the financial frictions along the lines of Bernanke et al. (1999) and show that GDP is explained well by the oil price shocks during Great recession, while during crisis episode of 2015 GDP was affected by oil price shocks less so. In our paper, the oil price shock largely explains both the crisis episodes of 2008-2009 and 2015.

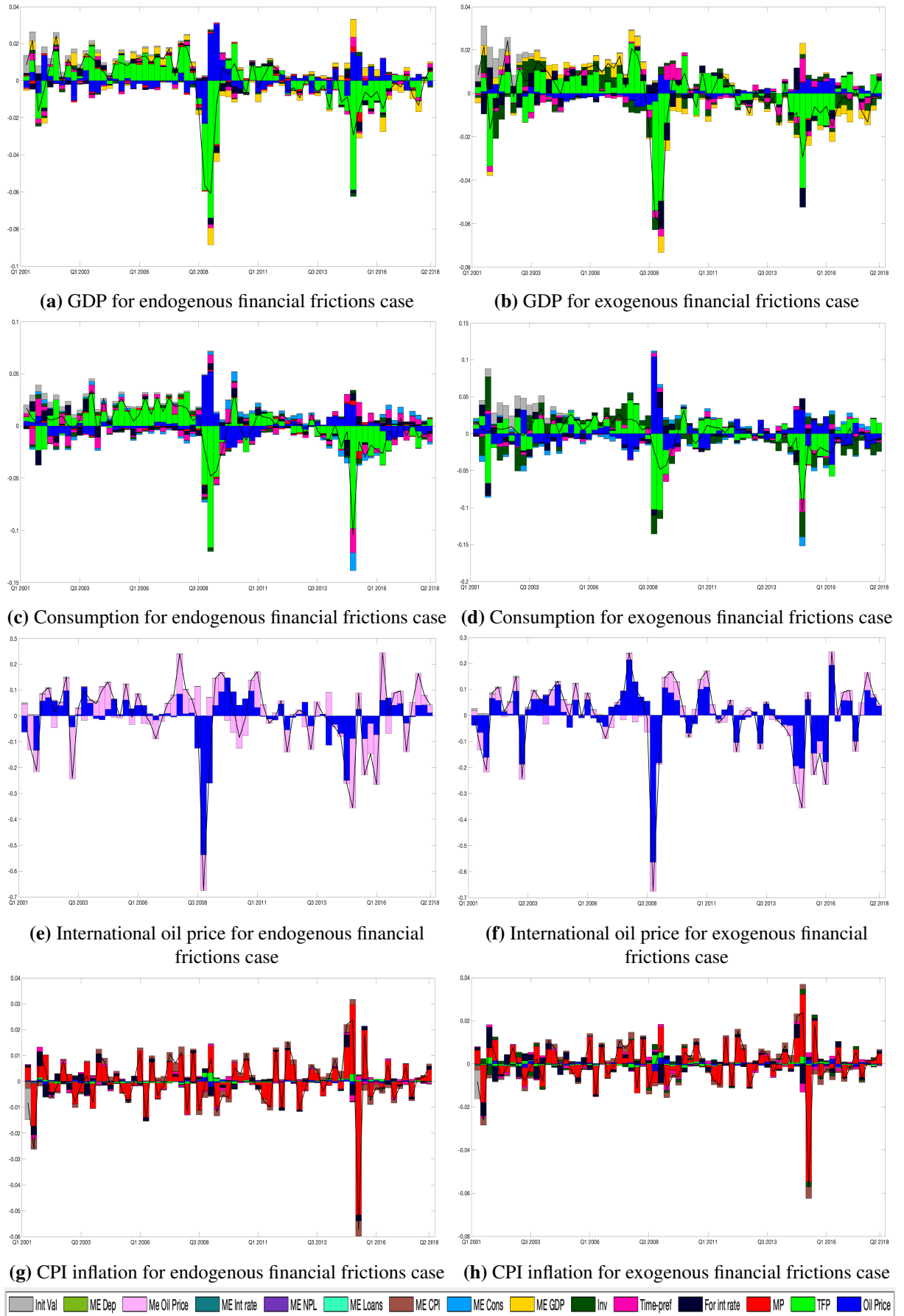
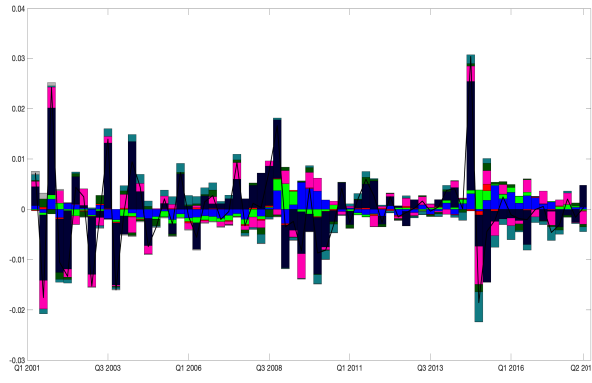
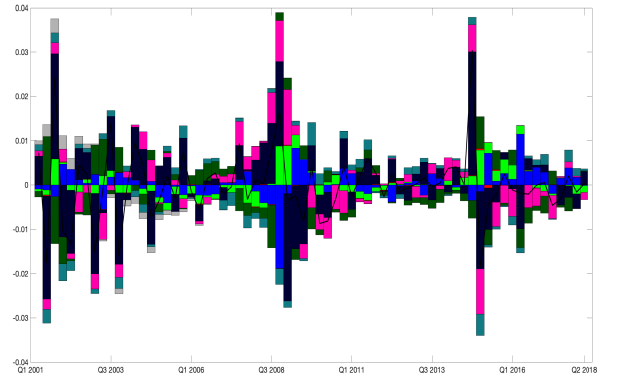


Fig. 1. Historical decomposition (1)
ME: Measurement Error

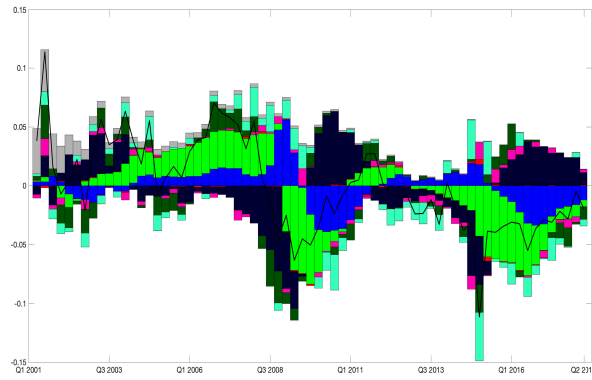
1.2 Implied Shock Series



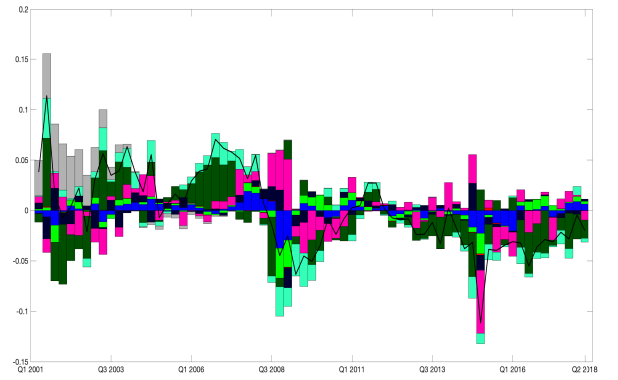
(a) CB interest rate for endogenous financial frictions case



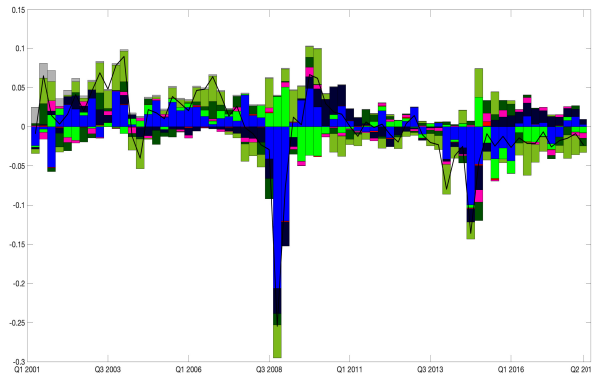
(b) CB interest rate for exogenous financial frictions case



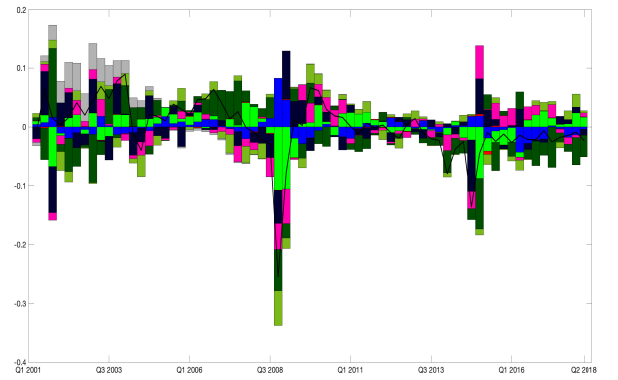
(c) Total loans for endogenous financial frictions case



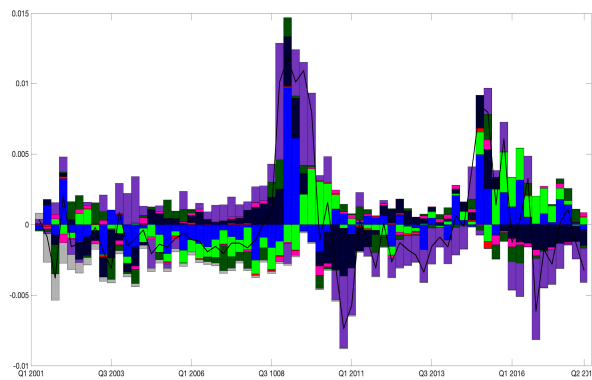
(d) Total loans for exogenous financial frictions case



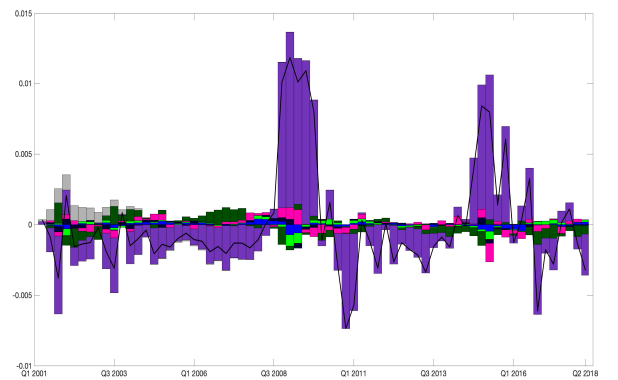
(e) Deposits for endogenous financial frictions case



(f) Deposits for exogenous financial frictions case



(g) Non-performing loans as a share of total loans for endogenous financial frictions case



(h) Non-performing loans as a share of total loans for exogenous financial frictions case



Fig. 2. Historical decomposition (2)
ME: Measurement Error

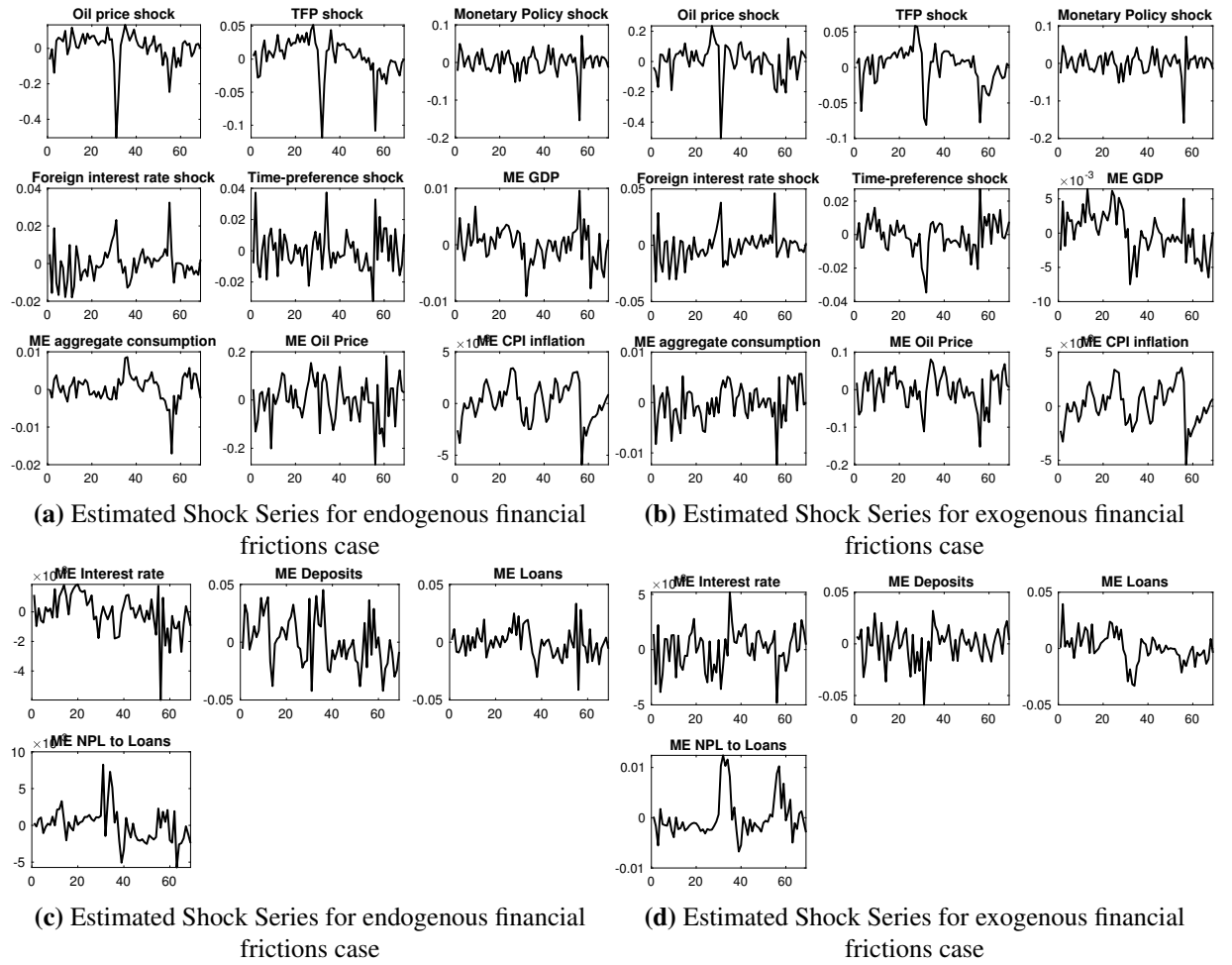


Fig. 3

2 Optimality conditions

Household optimality conditions

F.O.C. for consumption of domestic goods $c_{N,t}$:

$$c_{N,t} = \phi^h (\lambda_t^h)^{-\nu_c} c_t^{1-\nu_c\sigma} (A^c)^{\nu_c-1} \quad (1)$$

F.O.C. for consumption of imported goods $c_{T,t}$:

$$c_{N,t} = (1 - \phi^h) (p_t^{imp} \lambda_t^h)^{-\nu_c} c_t^{1-\nu_c\sigma} (A^c)^{\nu_c-1} \quad (2)$$

F.O.C. for deposits d_{t+1}^h :

$$\lambda_t^h (1 + a^{h,d} (d_{t+1}^h - d_{ss}^h)) = \beta_t^h (1 + r_{t+1}^d) \lambda_{t+1}^{sav} \quad (3)$$

F.O.C. for equity of a bank e_t^{bank} :

$$e_t^{bank} (1 + a^{h,b,e} (e_t^{bank} - e_{ss}^{bank})) = \frac{\lambda_{t+1}^h \beta_t^h}{\lambda_t^h} (\Pi_t^{bank}) \quad (4)$$

F.O.C. for holding of domestic bonds $B_{t+1}^{g,h}$:

$$\lambda_t^h (1 - a^{h,b,g} (B_{t+1}^{g,h} - B_{ss}^{g,h})) = \beta_t^h \lambda_{t+1}^h (1 + r_{t+1}^b) \quad (5)$$

F.O.C. for holding of foreign bonds B_{t+1}^{for} :

$$\lambda_t^{sav} (1 - a^{h,b,f} (B_{t+1}^f - B_{ss}^f)) = \beta_t^h \lambda_{t+1}^h (1 + r_{t+1}^f) \quad (6)$$

F.O.C. for firm equity $e_t^{w,total}$:

$$\lambda_t^h (1 + a^{h,w,e} (e_t^{w,total} - e_{ss}^{w,total})) = \lambda_{t+1}^h \beta_t^h \frac{\Pi_{t+1}^w \theta^w + \bar{\Pi}_{t+1}^w (1 - \theta^w)}{e_t^{w,total}} \quad (7)$$

Wage setting problem derivation

In the wage rigidity set up demand for individual labor takes the form similar to the demand for individual firm output in the case of price stickiness. And so, demand for individual labor becomes a function of total labor demand, aggregate wage and individual wage. In particular, it takes the form:

$$l_t^h(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} l_t^{sav} \quad (8)$$

Then the part of the saver's Lagrangian that is associated with the choice of labor can be represented as (note that for the time being nominal BC is used):

$$\tilde{L} = -\theta^h \frac{(l_0^h(j))^{1+\gamma^h}}{1+\gamma^h} + \lambda_0(W_0(j)l_0^h(j)) + E_0 \sum_{t=1}^{\infty} (\beta_{t-1}^h)^t \left(-\theta^h \frac{(l_t^h(j))^{1+\gamma^h}}{1+\gamma^h} + \lambda_t(W_t(j)l_t^h(j)) \right) \quad (9)$$

Given the demand for individual labor the previous expression can be written as:

$$\begin{aligned} \tilde{L} = & -\theta^h \frac{\left(\left(\frac{W_0(j)}{W_0}\right)^{-\epsilon_w} l_0^h\right)^{1+\gamma^h}}{1+\gamma^h} + \lambda_0(W_0(j) \left(\frac{W_0(j)}{W_0}\right)^{-\epsilon_w} l_0^h) + \\ & + E_0 \sum_{t=1}^{\infty} (\beta_{t-1}^h)^t \left(-\theta^h \frac{\left(\left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} l_t^h\right)^{1+\gamma^h}}{1+\gamma^h} + \lambda_t(W_t(j) \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} l_t^h) \right) \end{aligned} \quad (10)$$

Individual real wage can be expressed as:

$$w_t(j) = \frac{W_t(j)}{P_t} \quad (11)$$

Aggregate real wage can be expressed as:

$$w_t = \frac{W_t}{P_t} \quad (12)$$

Given that an individual can reset his nominal wage next period with probability $1 - \theta^{pw}$, real wage that individual gets at period $t + s$ if he is stuck with the wage he chose at time t can be represented as:

$$w_{t+s}(j) = \frac{W_t(j)}{P_{t+s}} = \frac{W_t(j)}{P_t} \frac{P_t}{P_{t+s}} = w_t(j) \Pi_{t,t+s}^{-1}, \quad (13)$$

where $\Pi_{t,t+s} = \prod_{m=1}^s \Pi_{t+m} = \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \dots = \frac{P_{t+s}}{P_t}$

Then, for the choice of real wage rate at time t corresponding part of the Lagrangian will be:

$$\begin{aligned} \tilde{L}_t = & -\theta^h \frac{\left(\left(\frac{w_t(j) \Pi_{t,t}^{-1}}{w_t}\right)^{-\epsilon_w} l_t^h\right)^{1+\gamma^h}}{1+\gamma^h} + \lambda_t(w_t(j) \Pi_{t,t}^{-1} \left(\frac{w_t(j) \Pi_{t,t}^{-1}}{w_t}\right)^{-\epsilon_w} l_t^h) + \\ & + E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \left(-\theta^h \frac{\left(\left(\frac{w_t(j) \Pi_{t,t+s}^{-1}}{w_{t+s}}\right)^{-\epsilon_w} l_{t+s}^h\right)^{1+\gamma^h}}{1+\gamma^h} + \lambda_t(w_t(j) \Pi_{t,t+s}^{-1} \left(\frac{w_t(j) \Pi_{t,t+s}^{-1}}{w_{t+s}}\right)^{-\epsilon_w} l_{t+s}^h) \right) \end{aligned} \quad (14)$$

The FOC for $w_t(j)$ becomes:

$$\begin{aligned}
& \epsilon_w w_t(j)^{-\epsilon_w(1+\gamma^h)-1} (\theta^h(w_t^{ed})^{\epsilon_w(1+\gamma^h)} \Pi_{t,t}^{\epsilon_w(1+\gamma^h)} (L_t^{sav})^{(1+\gamma^h)} + \\
& + E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \theta^h(w_{t+s}^{ed})^{\epsilon_w(1+\gamma^h)} \Pi_{t,t+s}^{\epsilon_w(1+\gamma^h)} (L_{t+s}^h)^{(1+\gamma^h)}) = \\
& (\epsilon_w - 1) w_t(j)^{-\epsilon} (\lambda_t(w_t^{ed})^{\epsilon_w} \Pi_{t,t}^{\epsilon_w-1} (L_t^h) + E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \lambda_{t+s}(w_{t+s}^{ed})^{\epsilon_w} \Pi_{t,t+s}^{\epsilon_w-1} (L_{t+s}^h))
\end{aligned} \tag{15}$$

By denoting the optimal choice of $w_t(j)$ at time t by $w_t^\#$ we get the following expression:

$$\begin{aligned}
w_t^{\#,1+\epsilon_w\gamma^h} &= \frac{\epsilon_w}{\epsilon_w - 1} \frac{\theta^{sav}(w_t)^{\epsilon_w(1+\gamma^h)} \Pi_{t,t}^{\epsilon_w(1+\gamma^h)} (l_t^h)^{(1+\gamma^h)}}{\lambda_t(w_t)^{\epsilon_w} \Pi_{t,t}^{\epsilon_w-1} (l_t^h) + E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \lambda_{t+s}(w_{t+s})^{\epsilon_w} \Pi_{t,t+s}^{\epsilon_w-1} (l_{t+s}^h)} + \\
& + \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \theta^h(w_{t+s})^{\epsilon_w(1+\gamma^h)} \Pi_{t,t+s}^{\epsilon_w(1+\gamma^h)} (l_{t+s}^h)^{(1+\gamma^h)}}{\lambda_t(w_t)^{\epsilon_w} \Pi_{t,t}^{\epsilon_w-1} (l_t^h) + E_t \sum_{s=1}^{\infty} (\beta_{t+s-1}^h \theta^{pw})^s \lambda_{t+s}(w_{t+s})^{\epsilon_w} \Pi_{t,t+s}^{\epsilon_w-1} (l_{t+s}^h)}
\end{aligned} \tag{16}$$

Then expression for $w_t^\#$ can be represented as:

$$w_t^{\#,1+\epsilon_w\gamma^{sav}} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{H_1}{H_2}, \tag{17}$$

where ϵ_w - elasticity of labor substitution.

$$H_{1,t} = \theta^h w_t^{\epsilon_w(1+\gamma^{sav})} l_t^{h,1+\gamma^h} + \beta_t^h \theta^{pw} \Pi_{t+1}^{\epsilon_w(1+\gamma^h)} H_{1,t+1}, \tag{18}$$

where θ^{pw} – probability of saver household not to be able to adjust his wage rate next period.

$$H_{2,t} = \lambda_t^h w_t^{\epsilon_w} l_t^h + \beta_t^h \theta^{pw} \Pi_{t+1}^{\epsilon_w-1} H_{2,t+1} \tag{19}$$

And labor wage rate dynamics follows the following equation (similar to the dynamics of inflation in case of price stickiness):

$$w_t^{1-\epsilon_w} = (1 - \theta^{pw}) w_t^{\#,1-\epsilon_w} + \theta^{pw} \Pi_t^{\epsilon_w-1} w_{t-1}^{1-\epsilon_w} \tag{20}$$

Wholesale producer's optimality conditions

F.O.C. for labour in high state

$$w_t = \frac{(1 - \alpha) p_t^w y_t^{w,high}}{l_t^{w,high}} \tag{21}$$

F.O.C. for labour in low state

$$w_t = \frac{(1 - \alpha) p_t^w y_t^{w,low}}{l_t^{w,low}} \tag{22}$$

F.O.C. for secured borrowing in endogenous financial frictions case:

$$\lambda_{t+1}^{sav} \beta_t^{sav} (1 + r_{t+1}^{w,s}) = \lambda_t^w (1 - a^{w,s} (\mu_{t+1}^{w,s} - \mu_{ss}^{w,s})) - \psi_t^w (1 + r_{t+1}^{w,s}) \quad (23)$$

F.O.C. for secured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,s}) = \lambda_t^w (1 - a^{w,s} (\mu_{t+1}^{w,s} - \mu_{ss}^{w,s})) - \psi_{ss}^w (1 + r_{ss}^{w,s}) \quad (24)$$

F.O.C. for unsecured borrowing in endogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,u}) = \lambda_t^w (1 - a^{w,u} (\mu_{t+1}^{w,u} - \mu_{ss}^{w,u})) \quad (25)$$

F.O.C. for unsecured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h ((1 + r_{t+1}^{w,u})(1 - \delta_{ss}^w) + \delta_{ss}^w (1 + r_{ss}^{w,u})) = \lambda_t^w (1 - a^{w,u} (\mu_{t+1}^{w,u} - \mu_{ss}^{w,u})) \quad (26)$$

F.O.C. for capital in endogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (\alpha p_{t+1}^w A_{t+1}^w (k_{t+1}^w)^{\alpha-1} (l_{t+1}^w)^{1-\alpha} + (1-\tau) p_{t+1}^K) = \lambda_t^w p_t^K (1 + a^{w,k} (k_{t+1}^w - k_{ss}^w)) - \psi_t^w coll (1-\tau) p_{t+1}^K, \quad (27)$$

F.O.C. for capital in exogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (\alpha p_{t+1}^w A_{t+1}^w (k_{t+1}^w)^{\alpha-1} (l_{t+1}^w)^{1-\alpha} + (1-\tau) p_{t+1}^K) = \lambda_t^w p_t^K (1 + a^{w,k} (k_{t+1}^w - k_{ss}^w)) - \psi_{ss}^w coll (1-\tau) p_{ss}^K, \quad (28)$$

F.O.C. for default rate:

$$\Omega_t \frac{cost_t^{def}}{\delta_t^w} = \mu_t^{w,u} (1 + r_t^{w,u}), \quad (29)$$

$$\text{where } cost_t^{def} = \left(\delta_t^w \mu_t^{w,u} (1 + r_t^{w,u}) \right)^{1+\psi}$$

Capital producer's optimality conditions

with respect to domestic investment component:

$$\begin{aligned} 1 = & p_t^K \left(1 - \frac{\varkappa}{2} \left(\frac{i_{t+1}}{i_t} - 1 \right)^2 - \varkappa \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right) \right) (A^i)^{1-\frac{1}{\nu_i}} (\phi^i)^{\frac{1}{\nu_i}} i_{N,t+1}^{\frac{-1}{\nu_i}} i_{t+1}^{\frac{1}{\nu_i}} + \\ & + \beta_t^h \left(\frac{\lambda_{t+1}^h}{\lambda_t^h} \right) (p_{t+1}^K \varkappa \left(\frac{i_{t+2}}{i_{t+1}} - 1 \right) \left(\left(\frac{i_{t+2}}{i_{t+1}} \right)^2 \right)) (A^i)^{1-\frac{1}{\nu_i}} (\phi^i)^{\frac{1}{\nu_i}} i_{N,t+1}^{\frac{-1}{\nu_i}} i_{t+1}^{\frac{1}{\nu_i}} \end{aligned} \quad (30)$$

with respect to imported investment component:

$$p_t^{imp} = p_t^K \left(1 - \frac{\varkappa}{2} \left(\frac{i_{t+1}}{i_t} - 1\right)^2 - \varkappa \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right) (A^i)^{1-\frac{1}{\nu_i}} (1 - \phi^i)^{\frac{1}{\nu_i}} i_{T,t+1}^{\frac{-1}{\nu_i}} i_{t+1}^{\frac{1}{\nu_i}} + \right. \\ \left. + \beta_t^h \left(\frac{\lambda_{t+1}^h}{\lambda_t^h}\right) (p_{t+1}^K \varkappa \left(\frac{i_{t+2}}{i_{t+1}} - 1\right) \left(\frac{i_{t+2}}{i_{t+1}}\right)^2) (A^i)^{1-\frac{1}{\nu_i}} (1 - \phi^i)^{\frac{1}{\nu_i}} i_{T,t}^{\frac{-1}{\nu_i}} i_{t+1}^{\frac{1}{\nu_i}} \right) \quad (31)$$

Bank's optimality conditions

with respect to deposits:

$$\mathbb{E} \frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} \left((1 + r_{t+1}^d) \right) = \lambda_t^{bank} (1 - a^{b,d} (d_{t+1}^{bank} - d_{ss}^{bank})) \quad (32)$$

with respect to secured loans to firms:

$$\mathbb{E} \frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} (1 + r_{t+1}^{w,s}) + (k_t^{bank} - k_t^{\bar{bank}}) r \bar{w} \frac{e_t^{bank}}{r w a_t^{bank}} = \lambda_t^{bank} (1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s})) \quad (33)$$

with respect to unsecured loans to firms:

$$\mathbb{E} \frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} [(1 + r_{t+1}^{w,u}) (1 - \theta^w \delta_{t+1}^w)] + (k_t^{bank} - k_t^{\bar{bank}}) r \bar{w} \frac{e_t^{bank}}{r w a_t^{bank}} = \lambda_t^{bank} (1 + a^{b,u} (\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u})) \quad (34)$$

Log-linearized equations

F.O.C. for secured borrowing in endogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,s}) = \lambda_t^w (1 - a(\mu_t^{w,s} - \mu_{ss}^{w,s})) - \psi_t^w (1 + r_{t+1}^{w,s}) \quad (35)$$

$$\Rightarrow \log(\lambda_{t+1}^h) + \log(\beta_t^h) + \log((1 + r_{t+1}^{w,s})) = \log(\lambda_t^w (1 - a(\mu_t^{w,s} - \mu_{ss}^{w,s})) - \psi_t^w (1 + r_{t+1}^{w,s})) \quad (36)$$

$$\Rightarrow \log(\lambda^{h,ss}) + \frac{\lambda_{t+1}^h - \lambda^{h,ss}}{\lambda^{h,ss}} + \frac{\beta_t^h - \beta^h}{\beta^h} + \log((1 + r^{w,s,ss})) + \frac{r_{t+1}^{w,s} - r^{w,s,ss}}{1 + r^{w,s,ss}} = \\ = \log(\lambda^{w,ss} (1 - a(\mu_{ss}^{w,s} - \mu_{ss}^{w,s})) - \psi^{w,ss} (1 + r^{w,s,ss})) + \frac{1}{\lambda^{w,ss} - \psi^{w,ss} (1 + r^{w,s,ss})} (\lambda_t^w - \lambda^{w,ss}) - \\ - \frac{a \lambda^{w,ss}}{\lambda^{w,ss} - \psi^{w,ss} (1 + r^{w,s,ss})} (\mu_t^{w,s} - \mu_{ss}^{w,s}) - \frac{1 + r^{w,s,ss}}{\lambda^{w,ss} - \psi^{w,ss} (1 + r^{w,s,ss})} (\psi_t^w - \psi^{w,ss}) - \\ - \frac{\psi^{w,ss}}{\lambda^{w,ss} - \psi^{w,ss} (1 + r^{w,s,ss})} (r_{t+1}^{w,s} - r^{w,s,ss}) \quad (37)$$

$$\begin{aligned} \Rightarrow \frac{\lambda_{t+1}^h - \lambda_{t+1}^{h,ss}}{\lambda_{t+1}^{h,ss}} + \frac{\beta_t^h - \beta_t^h}{\beta_t^h} + \frac{r_{t+1}^{w,s} - r_{t+1}^{w,s,ss}}{1 + r_{t+1}^{w,s,ss}} &= \frac{1}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (\lambda_t^w - \lambda_{t+1}^{w,ss}) - \\ &- \frac{a\lambda_{t+1}^{w,ss}}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (\mu_t^{w,s} - \mu_{t+1}^{w,s}) - \frac{1 + r_{t+1}^{w,s,ss}}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (\psi_t^w - \psi_{t+1}^{w,ss}) - \\ &- \frac{\psi_{t+1}^{w,ss}}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (r_{t+1}^{w,s} - r_{t+1}^{w,s,ss}) \end{aligned} \quad (38)$$

F.O.C. for secured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,s}) = \lambda_t^w (1 - a^{w,s} (\mu_{t+1}^{w,s} - \mu_{ss}^{w,s})) - \psi_{ss}^w (1 + r_{ss}^{w,s}) \quad (39)$$

$$\begin{aligned} \Rightarrow \frac{\lambda_{t+1}^h - \lambda_{t+1}^{h,ss}}{\lambda_{t+1}^{h,ss}} + \frac{\beta_t^h - \beta_t^h}{\beta_t^h} + \frac{r_{t+1}^{w,s} - r_{t+1}^{w,s,ss}}{1 + r_{t+1}^{w,s,ss}} &= \frac{1}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (\lambda_t^w - \lambda_{t+1}^{w,ss}) - \\ &- \frac{a\lambda_{t+1}^{w,ss}}{\lambda_{t+1}^{w,ss} - \psi_{t+1}^{w,ss}(1 + r_{t+1}^{w,s,ss})} (\mu_t^{w,s} - \mu_{t+1}^{w,s}) \end{aligned} \quad (40)$$

F.O.C. for unsecured borrowing in endogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h (1 + r_{t+1}^{w,u}) = \lambda_t^w (1 - a(\mu_t^{w,u} - \mu_{ss}^{w,u})) \quad (41)$$

$$\Rightarrow \frac{\lambda_{t+1}^h - \lambda_{t+1}^{h,ss}}{\lambda_{t+1}^{h,ss}} + \frac{\beta_t^h - \beta_t^h}{\beta_t^h} + \frac{r_{t+1}^{w,u} - r_{t+1}^{w,u,ss}}{1 + r_{t+1}^{w,u,ss}} = \frac{(\lambda_t^w - \lambda_{t+1}^{w,ss})}{\lambda_{t+1}^{w,ss}} - a(\mu_t^{w,s} - \mu_{ss}^{w,s}) \quad (42)$$

F.O.C. for unsecured borrowing in exogenous financial frictions case:

$$\lambda_{t+1}^h \beta_t^h ((1 + r_{t+1}^{w,u})(1 - \delta_{ss}^w) + \delta_{ss}^w (1 + r_{ss}^{w,u})) = \lambda_t^w (1 - a^{w,u} (\mu_{t+1}^{w,u} - \mu_{ss}^{w,u})) \quad (43)$$

$$\Rightarrow \frac{\lambda_{t+1}^h - \lambda_{t+1}^{h,ss}}{\lambda_{t+1}^{h,ss}} + \frac{\beta_t^h - \beta_t^h}{\beta_t^h} + \frac{(1 - \delta_{ss}^w)(r_{t+1}^{w,u} - r_{t+1}^{w,u,ss})}{1 + r_{t+1}^{w,u,ss}} = \frac{(\lambda_t^w - \lambda_{t+1}^{w,ss})}{\lambda_{t+1}^{w,ss}} - a(\mu_t^{w,s} - \mu_{ss}^{w,s}) \quad (44)$$

Collateral constraint of a firm in endogenous financial frictions case::

$$\mathbb{E}(1 + r_{t+1}^{w,s}) \mu_{t+1}^{w,s} \leq coll(1 - \tau) k_{t+1}^w \mathbb{E} p_{t+1}^K \quad (45)$$

$$\Rightarrow \frac{r_{t+1}^{w,s} - r_{t+1}^{w,s,ss}}{1 + r_{t+1}^{w,s,ss}} + \frac{\mu_{t+1}^{w,s} - \mu_{t+1}^{w,s,ss}}{\mu_{t+1}^{w,s,ss}} = \frac{k_{t+1}^w - k_{t+1}^{w,ss}}{k_{t+1}^{w,ss}} + \frac{p_{t+1}^K - p_{t+1}^{K,ss}}{p_{t+1}^{K,ss}} \quad (46)$$

F.O.C. for δ_t^w :

$$\Omega_t \frac{cost_t^{def}}{\delta_t^w} = \mu_{t-1}^{w,u} (1 + r_t^{w,u}) \quad (47)$$

$$\Rightarrow \log(\Omega_t) + \log(cost_t^{def}) - \log(\delta_t^w) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u}) \quad (48)$$

$$\Rightarrow \log(\Omega_t) + \log((\delta_t^w \mu_{t-1}^{w,u} (1 + r_t^{w,u}))^{1+\psi}) - \log(\delta_t^w) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u}) \quad (49)$$

$$\Rightarrow \log(\Omega_t) + (1+\psi)(\log(\delta_t^w) + \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u})) - \log(\delta_t^w) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u}) \quad (50)$$

$$\Rightarrow \log(\Omega_t) + (1+\psi)(\log(\delta_t^w) + \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u})) - \log(\delta_t^w) = \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u}) \quad (51)$$

$$\Rightarrow \log(\Omega_t) + \psi(\log(\delta_t^w) + \log(\mu_{t-1}^{w,u}) + \log(1 + r_t^{w,u})) = 0 \quad (52)$$

$$\Rightarrow \frac{\Omega_t - \Omega_{ss}}{\Omega_{ss}} + \psi \frac{\delta_t^w - \delta_{ss}^w}{\delta_{ss}^w} + \psi \frac{\mu_{t-1}^{w,u} - \mu_{ss}^{w,u}}{\mu_{ss}^{w,u}} + \psi \frac{r_t^{w,u} - r_{ss}^{w,u}}{1 + r_{ss}^{w,u}} = 0 \quad (53)$$

F.O.C. for secured loans:

$$\frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} (1 + r_{t+1}^{w,s}) + (k_t^{bank} - k^{\bar{bank}}) r \bar{w} \frac{e_t^{bank}}{r w a_t^{bank}} = \lambda_t^{bank} (1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s})) \quad (54)$$

$$\Rightarrow \frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} (1 + r_{t+1}^{w,s}) + \left(\frac{e_t^{bank}}{r w a_t^{bank}} - k^{\bar{bank}} \right) r \bar{w} \frac{e_t^{bank}}{r \bar{w} (\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u})} = \lambda_t^{bank} (1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s})) \quad (55)$$

$$\Rightarrow \frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} (1 + r_{t+1}^{w,s}) + \left(\frac{e_t^{bank}}{r \bar{w} (\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u})} - k^{\bar{bank}} \right) \frac{e_t^{bank}}{(\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u})} = \lambda_t^{bank} (1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s})) \quad (56)$$

$$\begin{aligned} \Rightarrow \log\left(\frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})_{s_{bank}}} (1 + r_{t+1}^{w,s}) + \left(\frac{(e_t^{bank})^2}{r \bar{w} (\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u})^2} - \frac{k^{\bar{bank}} e_t^{bank}}{(\mu_{t+1}^{bank,s} + \mu_{t+1}^{bank,u})} \right) \right) = \\ = \log(\lambda_t^{bank} (1 + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}))) \end{aligned} \quad (57)$$

$$\begin{aligned}
\Rightarrow & -\frac{\beta^{bank}(1+r_{ss}^{w,s})}{((\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}})^2} \frac{(\hat{\Pi}_{t+1}^{bank})^{\varsigma_{bank}} - (\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} + \frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}} \frac{r_{t+1}^{w,s} - r_{ss}^{w,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} + \\
& + (2 \frac{(e_{ss}^{bank})}{r\bar{w}(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})^2} - \frac{k_t^{bank}}{(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})}) \frac{e_t^{bank} - e_{ss}^{bank}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} \\
& + (-2 \frac{(e_{ss}^{bank})^2}{r\bar{w}(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})^3} + \frac{k_t^{bank} e_{ss}^{bank}}{(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})^2}) \frac{\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} \\
& + (-2 \frac{(e_{ss}^{bank})^2}{r\bar{w}(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})^3} + \frac{k_t^{bank} e_{ss}^{bank}}{(\mu_{ss}^{bank,s} + \mu_{ss}^{bank,u})^2}) \frac{\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} = \\
& = \frac{\lambda_t^{bank} - \lambda_{ss}^{bank}}{\lambda_{ss}^{bank}} + a^{b,s}(\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \quad (58)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow & -\frac{\beta^{bank}(1+r_{ss}^{w,s})}{((\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}})^2} \frac{(\hat{\Pi}_{t+1}^{bank})^{\varsigma_{bank}} - (\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} + \frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}} \frac{r_{t+1}^{w,s} - r_{ss}^{w,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} + \\
& + \frac{e_t^{bank} - e_{ss}^{bank}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} - \frac{\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} - \frac{\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} = \\
& = \frac{\lambda_t^{bank} - \lambda_{ss}^{bank}}{\lambda_{ss}^{bank}} + a^{b,s}(\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \quad (59)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow & -\frac{(\hat{\Pi}_{t+1}^{bank})^{\varsigma_{bank}} - (\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}} + \frac{r_{t+1}^{w,s} - r_{ss}^{w,s}}{(1+r_{ss}^{w,s})} + \\
& + \frac{e_t^{bank} - e_{ss}^{bank}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} - \frac{\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} - \frac{\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})^{\varsigma_{bank}}}(1+r_{ss}^{w,s})} = \\
& = \frac{\lambda_t^{bank} - \lambda_{ss}^{bank}}{\lambda_{ss}^{bank}} + a^{b,s}(\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \quad (60)
\end{aligned}$$

F.O.C. for unsecured loans:

$$\frac{\beta^{bank}}{(\hat{\Pi}_{t+1}^{bank})^{\varsigma_{bank}}} [(1+r_{t+1}^{w,u})(1-\theta^w \delta_{t+1}^w)] + (k_t^{bank} - k_t^{\bar{bank}}) r\bar{w} \frac{e_t^{bank}}{r w a_t^{bank}} = \lambda_t^{bank} (1 + a^{b,u}(\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u})) \quad (61)$$

$$\begin{aligned}
\Rightarrow & -\frac{(\hat{\Pi}_{t+1}^{bank})_{\varsigma_{bank}} - (\hat{\Pi}_{ss}^{bank})_{\varsigma_{bank}}}{(\hat{\Pi}_{ss}^{bank})_{\varsigma_{bank}}} + \frac{r_{t+1}^{w,u} - r_{ss}^{w,u}}{(1 + r_{ss}^{w,u})} - \theta^w \frac{\delta_{t+1}^w - \delta_{ss}^w}{(1 - \theta^w \delta_{ss}^w)} + \frac{e_t^{bank} - e_{ss}^{bank}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})_{\varsigma_{bank}}} (1 + r_{ss}^{w,s}) (1 - \theta^w \delta_{ss}^w)} \\
& - \frac{\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})_{\varsigma_{bank}}} (1 + r_{ss}^{w,s}) (1 - \theta^w \delta_{ss}^w)} - \frac{\mu_{t+1}^{bank,u} - \mu_{ss}^{bank,u}}{\frac{\beta^{bank}}{(\hat{\Pi}_{ss}^{bank})_{\varsigma_{bank}}} (1 + r_{ss}^{w,s}) (1 - \theta^w \delta_{ss}^w)} = \\
& = \frac{\lambda_t^{bank} - \lambda_{ss}^{bank}}{\lambda_{ss}^{bank}} + a^{b,s} (\mu_{t+1}^{bank,s} - \mu_{ss}^{bank,s}) \quad (62)
\end{aligned}$$

Taylor rule:

$$\frac{1 + i_t^b}{1 + i_{ss}^b} = \left(\frac{1 + i_{t-1}^b}{1 + i_{ss}^b} \right)^{r_R} \left(\frac{1 + \pi_t^{cpi}}{1 + \pi_{ss}^{cpi}} \right)^{1+r_\pi} \varepsilon_t^R, \quad (63)$$

$$\Rightarrow \log(1 + i_t^b) - \log(1 + i_{ss}^b) = r_R (\log(1 + i_{t-1}^b) - \log(1 + i_{ss}^b)) + (1 + r_\pi) (\log(1 + \pi_t^{cpi}) - \log(1 + \pi_{ss}^{cpi})) \quad (64)$$

$$\Rightarrow \frac{i_t^b - i_{ss}^b}{1 + i_{ss}^b} = r_R \frac{i_{t-1}^b - i_{ss}^b}{1 + i_{ss}^b} + (1 + r_\pi) \frac{\pi_t^{cpi} - \pi_{ss}^{cpi}}{1 + \pi_{ss}^{cpi}} \quad (65)$$

3 References

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