

The Only Dance in Town: Unique Equilibrium in a Generalized Model of Price Competition*

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Abstract

We study a canonical model of simultaneous price competition between firms that sell a homogeneous good to consumers who are characterized by the number of prices they are exogenously aware of. Our setting subsumes many employed in the literature over the last several decades. We show there is a unique equilibrium if and only if there exist *some* consumers who are aware of *exactly two* prices. The equilibrium we derive is in symmetric mixed strategies. Furthermore, when there are no consumers aware of exactly two prices, we show there is an uncountable-infinity of asymmetric equilibria in addition to the symmetric equilibrium. Our results show the paradigm generically produces a unique equilibrium. We also show that the commonly-sought symmetric equilibrium (which also nests the textbook Bertrand pure strategy equilibrium as a special case) is robust to perturbations in consumer behavior, while the asymmetric equilibria are not. (*JEL: D43, L11*)

Keywords: price competition; price dispersion; unique equilibrium

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1 Introduction

Prices for seemingly homogeneous goods are typically dispersed (see e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; Gorodnichenko, Sheremirov, and Talavera, 2018; Kaplan and Menzio, 2015; Lach and Moraga-González, 2017). The theoretical industrial organization literature offers an elegant rationalization of this phenomenon via games in which firms simultaneously compete in prices for consumers who differ in the number of prices they compare.

We study the elementary and oft-studied such setting in which $n \geq 2$ symmetric firms each sell a homogeneous good at a common marginal cost, $c \geq 0$, and simultaneously set prices in a one-shot game. Consumers demand $q(p)$ units of the good and exogenously differ in the number of prices they know: $I_m \geq 0$ are aware of $m \in \{1, \dots, n\}$ random prices, where we assume $I_1 > 0$ and $I_m > 0$ for at least some $m > 1$.¹ In this setting, it is known that equilibrium price dispersion is produced via mixed-strategies: opposing forces lead firms to “tango” (to use the term of Baye, Kovenock, and De Vries, 1992; henceforth BKV).

There is exactly one symmetric equilibrium. Researchers have almost exclusively relied upon this equilibrium in their analyses. However, a potentially-uncomfortable fact about these popular models is that they can produce very many equilibria. For example, under the assumptions that $I_1, I_n > 0$ and $I_2, \dots, I_{n-1} = 0$, BKV show there is an uncountable infinity of asymmetric equilibria in addition to the symmetric equilibrium.

We contribute by pin-pointing the source of this multiplicity and characterizing when the symmetric equilibrium is in fact the only equilibrium. Specifically, we show that there is a unique equilibrium if and only if $I_2 > 0$. The result is stark: if $I_2 > 0$ the symmetric equilibrium is the unique equilibrium, but if $I_2 = 0$ we show there is a continuum of asymmetric equilibria. In contrast, the literature has mostly unearthed uniqueness in special cases of our setting, perhaps the most well known of which is that of $n = 2$ (BKV). Our result sheds a new light on this finding: uniqueness is not due to duopoly per se; it follows because duopoly implies that some consumers know precisely two prices ($I_2 > 0$).

In the absence of consumers who make the minimal number of comparisons ($I_2 = 0$) each of the infinitely-many asymmetric equilibria feature at least one firm that charges the monopoly price, \bar{p} , with positive probability. When these firms charge \bar{p} , they sell only to their share of “captive” consumers (I_1/n) instead of competing for “contested” (non-captive) consumers by setting lower prices. In contrast, if $I_2 > 0$ each firm competes head-to-head with each other firm for some consumers. This gives firms an incentive to compete over all undominated prices, preventing mass points on any one price (including \bar{p}), dismantling the asymmetric equilibria.

¹It is well known that if $I_1 = 0$ then at least two firms price at marginal cost and all earn zero profit (we discuss the classic Bertrand equilibrium later), and if $I_1 > 0$ but $I_m = 0$ for all $m > 1$ then all firms set the monopoly price \bar{p} .

We highlight four main implications of our result. First, the key determinant of equilibrium uniqueness is not the number of firms, as implied by some studies, but the configuration of consumers' consideration sets. Second, by nesting the vast majority of settings found in the literature, we reconcile existing findings and pinpoint conditions for uniqueness of the commonly-studied symmetric equilibrium. Third, and more generally, the framework may become more attractive to researchers because multiplicity only surfaces in the special case of $I_2 = 0$. Fourth, even if a researcher adopts $I_2 = 0$ and hence faces multiplicity, we provide a novel "stability" rationale in terms of consumer behavior for selecting the symmetric equilibrium: the symmetric equilibrium strategy when $I_2 = 0$ is equal to the unique equilibrium strategy as $I_2 \downarrow 0$.

We next discuss related literature, then provide the model and derive our results. We then detail the equilibrium in several applied settings and discuss the wider implications of our findings.

2 Literature

Models of price competition with heterogeneously-informed consumers offered an early rationalization of price dispersion in homogeneous-goods markets (foundational studies include Rosenthal, 1980; Narasimhan, 1988; Shilony, 1977; Varian, 1980). Since then, the framework has been applied to, or featured in, a wide range of settings including: consumer search (both theoretical, e.g., Atayev, 2019; Burdett and Judd, 1983; Janssen and Moraga-González, 2004; Stahl, 1989, and empirical studies, e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; De los Santos, 2018; Honka, 2014; Honka and Chintagunta, 2016; Pires, 2016); price discrimination (Armstrong and Vickers, 2019; Fabra and Reguant, 2018); product substitutability (Inderst, 2002); strategic clearing-houses such as comparison websites (Arnold and Zhang, 2020; Baye and Morgan, 2001; Moraga-González and Wildenbeest, 2012; Ronayne, 2020; Ronayne and Taylor, 2020; Shelegia and Wilson, 2020); competition with boundedly-rational consumers (e.g., Carlin, 2009; Chioveanu and Zhou, 2013; Gu and Wenzel, 2014; Heidhues, Johnen, and Kőszegi, forthcoming; Inderst and Obradovits, 2018; Johnen, 2020; Piccione and Spiegel, 2012; Spiegel, 2006, 2016); and switching costs (for a review, see Farrell and Klemperer, 2007).

The almost-ubiquitous assumption made is that consumers are aware of symmetrically- and randomly-drawn prices (without replacement), which is the setting we study. As such, each consumer's information is completely characterized by the number of prices they know. Perhaps the most well known is the "Model of Sales" of Varian (1980, 1981), which assumes $I_1, I_n > 0$ and $I_{m \neq 1, n} = 0$. There, BKV show there is an uncountable infinity of equilibria in addition to the symmetric equilibrium when $n > 2$. Most of the literature deals with this multiplicity by focusing on the symmetric equilibrium (for example, and in addition to those cited above: Armstrong, 2015; Armstrong, Vickers, and Zhou, 2009; Lach and Moraga-González, 2017; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer,

2013). Uniqueness has been found in some special cases of our setting e.g., when $n = 2$ (BKV), or when consumers’ awareness of any two prices is independent (Spiegler, 2006)², while Armstrong and Vickers (2020) find uniqueness in a similar setting to ours in which $I_2 > 0$. We connect all these results by proving that $I_2 > 0$ is both a sufficient and necessary condition for equilibrium uniqueness.

In the multiple equilibria identified by BKV, firms earn the same profits, but may set price via very different distributions. Equilibrium price distributions are of key interest in many models of consumer search. They drive consumers’ incentive to become informed (e.g., Armstrong, Vickers, and Zhou, 2009; Baye and Morgan, 2001; Burdett and Judd, 1983; Fershtman and Fishman, 1994; Moraga-González, Sándor, and Wildenbeest, 2017), and their comparative static properties are the central focus of many studies (e.g. Janssen and Moraga-González, 2004; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer, 2013).³ The robustness of results in these papers depends on the existence of asymmetric equilibria. Our finding that asymmetric equilibria are knife-edge phenomena implies that predictions derived with symmetric equilibria are the relevant ones.

Arguments in favor of the symmetric equilibrium have been made in some settings. In an extension of their main analysis, BKV allow those consumers willing to check exactly one firm to choose which firms to buy from. They show that game has a unique solution where firms adopt symmetric pricing strategies. Modifying the “Model of Sales” setting to include a positive fixed-cost of advertising à la Baye and Morgan (2001), Arnold and Zhang (2014) prove the symmetric outcome is the unique outcome. In our setting without advertising but with more general information sets, we provide a distinct argument for the symmetric equilibrium based on continuity, and without extending or otherwise changing the game’s structure.

A related literature is that on all-pay auctions. Perhaps the most relevant paper there is Baye, Kovenock, and De Vries (1996), which documents the equilibria in an all-pay auction with complete information and so does not explore the role of different information sets.⁴

A few have made progress examining equilibria in some particular asymmetric settings, including, e.g., Arnold and Zhang (2020), BKV (Section V), Inderst (2002); Ireland (1993); McAfee (1994); Narasimhan (1988); Szech (2011). Allowing for more general asymmetries in con-

²Szech (2011) extends Spiegler’s result to show uniqueness extends to the asymmetric independent case.

³In contrast, Myatt and Ronayne (2019) allow firms to set price in two stages and predict pure-strategy outcomes.

⁴Baye, Kovenock, and De Vries (1996, Theorem 3) indicates that asymmetries (e.g., in marginal costs) between firms (bidders) in settings where consumers see either one price or all (à la Varian, 1980), would lead to asymmetric equilibria in which some players mix continuously while others demonstrate complete rigidity at the monopoly price via a common pure strategy. In the context of pricing with otherwise similar firms, the coexistence of such polar strategies seems empirically unattractive. These equilibria are also not robust to $I_2 > 0$. In addition, although symmetry is a restrictive assumption, firms having the same marginal cost seems reasonable in some important settings featuring price dispersion, e.g., online platforms who charge the same fees to multiple sellers.

sumers' information sets is challenging and little is known about equilibria there. Armstrong and Vickers (2020) is an exception, offering a rich characterization when $n = 3$. In this paper, we analyze the standard (symmetric and random) configuration of consumers' information.

3 Model and Equilibrium

Model. There are $n \geq 2$ symmetric firms, $i = 1, \dots, n$, that produce a homogeneous good at marginal cost c to sell to consumers who each wish to buy $q(p)$ units of the good at a price p .⁵ We assume that $q(p)$ is such that $(p - c)q(p)$ has a unique maximum $\bar{p} \equiv \arg \max_p (p - c)q(p)$ with $\bar{p} > c$, $q(\bar{p}) > 0$, and that $(p - c)q(p)$ is continuous for all $p \in [c, \bar{p}]$ and strictly increasing for all $p \in [c, \bar{p}]$.^{6,7} We define the monopoly profits per customer $\bar{\pi} \equiv (\bar{p} - c)q(\bar{p})$. This demand function includes the commonly-used special case of unit demand: consumers buy a unit if and only if the price is weakly below a reservation price r , and zero otherwise. Firms simultaneously choose price and firm i 's price is denoted p_i . Consumers differ by the number of prices they are exogenously aware of i.e., the size of their “information” or “consideration” sets.⁸ Consumers buy at the lowest price in their consideration set. Where there is a tie in the lowest price, any interior tie-breaking rule may be assumed. The mass of consumers informed of $m \in \{1, \dots, n\}$ prices is denoted $I_m \geq 0$, with $I_1 > 0$ and $I_m > 0$ for some $m > 1$.⁹ For each type of consumer, consideration sets are symmetrically and randomly distributed across firms. This means, e.g., that I_2 comprises of the same share, $I_2/\binom{n}{2}$, of consumers with each consideration set $\{1, 2\}, \{1, 3\}, \{1, 4\}$, etc. We refer to the I_1 consumers as captive and all others as contested. Before deriving our result we illustrate it through an example.

Example. Consider a triopoly of symmetric firms and consumers all with unit demand and a reservation price r , so that $\bar{p} = r$. Suppose no consumer sees exactly two prices ($I_2 = 0$), but some consumers observe one ($I_1 > 0$) and others all three ($I_3 > 0$).

BKV show that the following equilibria exist. Two firms, say 1 and 2, randomize continuously over an interval $[\underline{p}, r]$. The remaining firm, 3, randomizes continuously over some $[\underline{p}, x) \cup r$ with $x \in (\underline{p}, r]$, placing a mass point at r whenever $x < r$.¹⁰ This is an equilibrium for all $x \in [\underline{p}, r]$,

⁵Alternatively, one could assume heterogeneous consumer demands and let $q(p)$ denote aggregate demand with the added assumption that per consumer demand is independent of which prices they know.

⁶We do not need $q(p)$ to be weakly decreasing, only that $(p - c)q(p)$ increases continuously between c and \bar{p} .

⁷Under an alternative (unrealistic) assumption that demand is unbounded, mixed-strategy equilibria can exist, as shown by Kaplan and Wettstein (2000) in the case that $I_n > 0$ and $I_m = 0$ for $m < n$.

⁸Some evidence indicates that such “fixed-sample” or “simultaneous” search often describes consumer behavior well. De los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2016) examine data from markets for books and auto insurance, respectively. Both studies fail to find a relationship between the prices consumers have seen and their decision to search on, consistent with the premise of simultaneous search.

⁹As we also detail later, information on how many offers consumers consider is available to firms in at least some markets. For example, the Consumer Financial Protection Bureau (2015) reported that approximately 45% of mortgage borrowers only seriously consider one lender, 40% two, and 10% three.

¹⁰ \underline{p} is the lowest undominated price; see (2). Equilibrium strategies for this example are in the proof of Lemma 2.

hence there is an uncountable infinity of equilibria.¹¹ When $x = r$, equilibrium strategies are symmetric. The asymmetric equilibria require firm 3 to have a mass point on r . In all equilibria, each firm's profit is determined by what they can earn from charging the monopoly price to their captive consumers, their minmax payoff, $\bar{\pi}I_1/3 = (r - c)I_1/3$.

In each equilibrium, 1 and 2 trade-off exploiting captive consumers and competing for the I_3 contested consumers. But the asymmetric equilibria “sideline” 3: 1 and 2 compete head-to-head for I_3 by mixing over a common interval such that 3 does not gain by joining the competition, and can therefore focus more on exploiting its captive consumers by placing a mass point on r .

In contrast, when $I_2 > 0$ each firm competes head-to-head with each of its rivals for some consumers. This incentivizes each firm to compete for contested consumers: no firm can sit on the sideline in equilibrium so the asymmetric equilibria no longer exist. To see this more precisely, add some consumers to this example triopoly setting who compare two prices so that $I_2 > 0$, and suppose asymmetric strategies are played in which firm 3 places a mass point at r .

When charging r , firm 3 loses all contested consumers and earns $(r - c)I_1/3$. In contrast, because $I_2 > 0$ and firm 3 has a mass point on r , firms 1 and 2 each sell to $I_2/3$ contested consumers with positive probability, even at prices arbitrarily close to r . Thus, firms 1 and 2 earn strictly more than $(r - c)I_1/3$. But then firm 3 can increase its profit by competing for the $2I_2/3$ contested consumers who see firm 3 and exactly one other firm by charging prices below r (it has a strict incentive to shift the probability mass from r to lower prices).

Our result generalizes this intuition to $n \geq 2$, while allowing for a more general demand function. If $I_2 > 0$, each firm competes head-to-head with each rival for at least some contested consumers. This rules out the possibility of sidelined firms in equilibrium and leads to the uniqueness of the symmetric equilibrium.

Analysis. We prove our result (Proposition 1) via the construction of two primary lemmas (Lemmas 1 and 2). Lemma 1 provides the unique equilibrium when $I_2 > 0$, and Lemma 2 says there are uncountably-many equilibria when $I_2 = 0$. We construct our proof of Lemma 1 through a sequence of intermediate results. We provide an overview of those results next and refer the interested reader to the Appendix for the technical details. The initial parts do not require $I_2 > 0$ and are somewhat similar to proofs in the literature.

We first confirm the range of permissible equilibrium prices. Firms are guaranteed a profit of at least $\pi_i = \bar{\pi} \cdot I_1/n > 0$ by setting a price of \bar{p} which sells $q(\bar{p})$ units to their I_1/n captive consumers regardless of other prices. Per customer profit has a unique maximum at \bar{p} so every price $p_i > \bar{p}$ is strictly dominated by \bar{p} . The highest profit a price p_i below \bar{p} can generate is

¹¹More generally for $n \geq 2$ (and $I_1, I_n > 0$ and $I_2, \dots, I_{n-1} = 0$), at least two firms continuously mix over $[p, r]$ in any equilibrium, while all other firms may have mass points on r .

found when the firm sells with certainty to all consumers who are aware of its price:

$$(p_i - c)q(p_i) \sum_{m=1}^n I_m \frac{\binom{n-1}{m-1}}{\binom{n}{m}} = (p_i - c)q(p_i) \sum_{m=1}^n I_m m/n, \quad (1)$$

hence only prices in $[\underline{p}, \bar{p}]$ arise in equilibrium, where \underline{p} is the lowest undominated price:¹²

$$(\underline{p} - c)q(\underline{p}) \sum_{m=1}^n I_m m/n = \bar{\pi} \cdot I_1/n. \quad (2)$$

As $(p - c)q(p)$ is continuous and strictly increasing for $p \in [c, \bar{p})$, (2) yields a unique $\underline{p} \in (c, \bar{p})$.

To continue characterizing equilibrium pricing behavior we denote i 's price distribution by G_i , with \underline{s}_i and \bar{s}_i as the minimum and maximum of its support. The next steps (corresponding to Lemmas A1 to A5 in the Appendix) establish that: (i) at least two firms have $\bar{s}_i = \bar{p}$; (ii) all firms earn the same profit; and (iii) no firm places any mass point at any $p \in [\underline{p}, \bar{p})$ in equilibrium.

First, if fewer than two firms have $\bar{s}_i = \bar{p}$, then a firm with the highest $\bar{s}_i < \bar{p}$ can profit by shifting probability mass from (or just below) \bar{s}_i to just below \bar{p} ; thus at least two firms have $\bar{s}_i = \bar{p}$. To see that all firms receive the same equilibrium profit, note that if $\pi_i < \pi_j$ for some i and j , i has a profitable deviation to price slightly below \underline{s}_j which would bring π_i arbitrarily close to π_j . Last, \bar{p} is the only place any firm can place mass in equilibrium. If i placed mass elsewhere in $[\underline{p}, \bar{p})$ there would be a “gap” just above the mass in which no firm would ever price, meaning that i could profitably shift this mass onto some higher price within the gap.

The next steps are crucial in establishing the sufficiency of $I_2 > 0$ for equilibrium uniqueness (Lemma 1). They show that if $I_2 > 0$, no firm has a mass point at \bar{p} (Lemmas A6 and A7).

If two firms placed mass at \bar{p} , there would be a positive probability that they would tie for the consumers who compare the prices of only those two firms ($I_2 > 0$ ensures there is a positive measure of those consumers). But then either firm could profit by slightly undercutting the other (gaining a discrete boost in sales for a continuous loss in price). In the case that exactly one firm places mass at \bar{p} , there is a rival firm that sets prices slightly below \bar{p} (because at least two firms have $\bar{s}_i = \bar{p}$). Because these two firms compete for some consumers that compare only their prices (because $I_2 > 0$), the rival earns strictly more than the firm with the mass point, contradicting that firms all make the same equilibrium profit.

Next we derive payoffs when $I_2 > 0$ (Lemma A8). We know two firms have $\bar{s}_i = \bar{p}$ and that they do not have mass points there. As such, when these firms set prices approaching \bar{p} from below, there is a vanishing chance of (i) a tie in price, or (ii) any other firm setting a higher price; so

¹²We follow convention to ignore the possibility that firms choose suboptimal prices with probability zero.

their profits are $\bar{\pi}I_1/n$. Because payoffs are symmetric, this is every firm's payoff.

We can now speak to pricing behavior at lower prices (Lemma A9). First, no firm will ever set a price outright lower than those that any other firm ever sets (because it would do better to raise any such price to match the lowest price that any other would ever set). In addition, the lowest prices ever set by any firm cannot exceed \underline{p} : if they did, then the firms setting those prices would (by definition of \underline{p} ; see (2)) earn strictly more than $\bar{\pi}I_1/n$, a contradiction.

Taking stock, we know equilibrium profits and (a fair amount) about pricing behavior at \underline{p} , \bar{p} , and in the points in-between, when $I_2 > 0$. The remaining steps in the Appendix (Lemmas A10 to A14) build on this knowledge and are more technical in nature. They show that all firms mix via the same, continuous, and strictly increasing CDF, G , over the entire interval $[\underline{p}, \bar{p}]$.

Because in a mixed-strategy equilibrium firms must be indifferent between all elements of their support, the profit garnered from any price, $p \in [\underline{p}, \bar{p}]$, must be equal to $\bar{\pi}I_1/n$:

$$(p - c)q(p) \sum_{m=1}^n \frac{m}{n} I_m (1 - G(p))^{m-1} = \bar{\pi} \cdot \frac{I_1}{n}. \quad (3)$$

Because $(p - c)q(p)$ is strictly increasing and continuous in p , (3) pins down a unique solution for $G(p)$. This expression can be understood simply: each summand on the left hand side is a firm's expected quantity of sales from consumers aware of both its price and that of $m - 1 \in \{0, \dots, n - 1\}$ rivals. The independence of information sets permits many simplifications and so allows (3) to be quite a concise expression.

The analysis so far combines to provide the first of our two primary lemmas.

Lemma 1 (Sufficiency). *If $I_2 > 0$, there is a unique equilibrium which is symmetric. The CDF, G , solves (3) and has support $[\underline{p}, \bar{p}]$.*

To show that $I_2 > 0$ is also necessary for equilibrium uniqueness, we show that multiple equilibria exist if $I_2 = 0$. In fact, we find the multiplicity to be extreme in the sense that there are uncountably many equilibria. The result extends the intuition provided in our earlier triopoly example: when $I_2 = 0$, “sidelined” firms are accommodated in equilibrium. Sidelined firms have mass points at \bar{p} . When they price at \bar{p} they only sell to their captive consumers. As such, no other firm can grab customers by slightly undercutting \bar{p} , and, if there are many sidelined firms, those sidelined have no incentive to undercut each other.

The multiplicity arises because when $I_2 > 0$ there is no discipline on how much mass sidelined firms allocate to \bar{p} ; it can be anywhere between 0 (not sidelined at all) to 1 (fully sidelined). If $I_2 = 0$, the symmetric equilibrium defined by (3) is still an equilibrium of the game, and corresponds to the case in which every firm is not sidelined at all. The issue though, is that

infinitely many other equilibria also exist, corresponding to all the intermediate degrees of how sidelined firms are. We summarize this with Lemma 2.

Lemma 2 (Necessity). *If $I_2 = 0$, infinitely many equilibria exist.*

Exactly how many sidelined firms are permitted in equilibrium when $I_2 = 0$ is a function of how many prices consumers are (not) aware of. If $I_2 = 0$ but $I_3 > 0$, then there can be no more than one sidelined firm, as in our triopoly example. More generally, if we define $k \geq 2$ such that $I_{m \leq k} = 0$ (for all $m \geq 2$) and $I_{k+1} > 0$, then there can be up to $k - 1$ sidelined firms in equilibrium.

Putting together Lemmas 1 and 2 we get Proposition 1, the main result of this article.

Proposition 1. *There is a unique equilibrium if and only if $I_2 > 0$. The equilibrium is symmetric: firms continuously mix over the support $[\underline{p}, \bar{p}]$ via a common CDF that solves (3). When $I_2 = 0$, there are uncountably-many equilibria.*

In each equilibrium at least two firms put no mass at \bar{p} and compete fully for contested consumers. If $I_2 = 0$, remaining firms may be sidelined. If $I_2 > 0$, that is not possible and the equilibrium is unique: each firm goes head-to-head with each rival for at least some consumers, giving each an incentive to compete for contested consumers.

4 Discussion

Proposition 1 tells us there is a unique equilibrium when $I_2 > 0$. The unique equilibrium CDF (the solution to the polynomial (3)) does not generally have an analytic solution except in some cases. We now report the unique equilibrium for some such settings that feature $I_2 > 0$. To do so we consider again the commonly-studied case of unit demand with a reservation price r , normalize (without loss of generality) $\sum_{m=1}^n I_m = 1$, and express (3) more parsimoniously as a probability generating function associated with the number of rivals faced by each firm:¹³

$$\phi(x) = \sum_{m=1}^n a_m x^{m-1}, \quad (4)$$

where $a_m \equiv I_m m / n$ and the equilibrium CDF is the solution to

$$\frac{\phi(1 - G(p))}{\phi(0)} = \frac{r - c}{p - c}, \quad \text{where} \quad \underline{p} \equiv \frac{r\phi(0) + c(\phi(1) - \phi(0))}{\phi(1)}. \quad (5)$$

Example 1. Suppose consumers only check one or two prices i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$. Then $\phi(x) = a_1 + a_2 x$. This setting is particularly relevant. First, it is the model of Varian (1980), i.e., $I_1, I_n > 0$ and $I_1 + I_n = 1$, with $n = 2$. The unique equilibrium is reported by BKV in

¹³We are grateful to Mark Armstrong for the suggestion to utilize probability generating functions.

their full characterization of equilibria in Varian's model. Our Proposition 1 shows uniqueness is obtained not because $n = 2$, but because $I_2 > 0$. Second, our analysis treated consumers' information as exogenous. In their canonical consumer search model, Burdett and Judd (1983) show ex-ante symmetric consumers endogenously choose to search either once or twice (i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$) and derive a symmetric equilibrium.¹⁴ Our Proposition 1 shows this is in fact the unique equilibrium in such a setting.

Example 2. Extending the first example, suppose that consumers are again aware of few prices, but either one, two, or three (i.e., $I_1, I_2, I_3 > 0$ and $I_{m>3} = 0$). Then $\phi(x) = a_1 + a_2x + a_3x^2$, for which one of the two solutions to (5) is valid, which by our result gives the unique equilibrium. More generally, the solution to the setting with $I_1, \dots, I_k > 0$ and $I_{m>k} = 0$ quickly becomes intractable as k grows. However, some approaches simplify (4), as the next example shows.

Example 3. Consider a setting in which each consumer is independently aware of each firm's price with probability $\alpha \in (0, 1)$. Then $I_m = \binom{n}{m} \alpha^m (1-\alpha)^{n-m} > 0$ for $m = 1, \dots, n$. Application of the binomial theorem to (4) yields $\phi(x) = \alpha(1 - \alpha(1-x))^{n-1}$. This is the symmetric version of the models of awareness and advertising by Ireland (1993) and McAfee (1994), the equilibrium of which is shown to be unique by Spiegler (2006). Our more general result shows that uniqueness follows in this special setting because $I_2 > 0$, which is implied by $\alpha \in (0, 1)$.

Textbook Bertrand. Consider perhaps the simplest version of Bertrand competition where all consumers compare prices ($I_1 = 0$). With $n > 2$ it is well known that there are infinitely-many equilibria (two charge marginal cost while others charge any prices). Our results offer an intuitive rationale to focus on the symmetric (price-equals-marginal-cost) equilibrium: When $I_1, I_2 > 0$ (and any $I_m \geq 0$ for $m > 2$) Proposition 1 shows the only equilibrium is symmetric; and as $I_1 \downarrow 0$ ceteris paribus, profits go to zero and the equilibrium pricing distribution converges to the degenerate distribution with all its probability mass on marginal cost.

Other Implications. More generally, our result may make this classic price competition framework more attractive to researchers. For those adopting it, we argue that our result makes the symmetric equilibrium the practically- and theoretically-relevant one.

Firstly, the assumption that some consumers check two prices ($I_2 > 0$) appears quite reasonable. It seems intuitive that a wide range of search-cost distributions would lead some consumers to compare two prices. Indeed, Burdett and Judd (1983) find that even with homogeneous (and linear) search costs, a positive mass of consumers gather two price quotations in equilibrium.¹⁵ Empirical evidence also broadly supports this. The Consumer Financial Protection Bureau (2015) find that around 90% of home-buyers consider either one or two mortgage lenders/brokers, consistent with the estimate of Woodward and Hall (2012). De los Santos,

¹⁴Burdett and Judd (1983) assume a continuum of firms, but the equilibrium strategy is the same for any $n \geq 2$.

¹⁵Such search behavior is also found in the "high-intensity" equilibria of Janssen and Moraga-González (2004).

Hortaçsu, and Wildenbeest (2012) find that around 70% of consumers visited one or two online book stores before purchasing, while De los Santos (2018) finds the average number to be 1.27. In the US auto-insurance market, Honka (2014) finds 35% obtain two quotes.

From a theoretical perspective, the framework as a whole may be more attractive to researchers because we now know that multiple equilibria only arise in the special case of $I_2 = 0$. Furthermore, even in that special case, the symmetric equilibrium is robust in the sense that the strategy is right-continuous at $I_2 = 0$: as $I_2 \downarrow 0$, the equilibrium strategy is equal to the symmetric equilibrium strategy when $I_2 = 0$. Conversely, suppose one predicts an asymmetric equilibrium when $I_2 = 0$ and consider a perturbation in consumer behavior $I_2 = \epsilon > 0$, ceteris paribus. Following the shock, the unique and symmetric equilibrium strategies of each firm jumps, and can be qualitatively very different from firms' supposed asymmetric strategies before the change. This offers a stability justification for the restriction to symmetric equilibria made in prior work.

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Appendix

This appendix houses proofs and results omitted from, but referred to, in the main text. We first provide intermediate results, then the proofs of Lemmas 1 and 2.

Intermediate Results

Lemma A1. *If some firm i has a mass point at $\bar{s} \equiv \max_j \{\bar{s}_j\}$, i sells only to its I_1/n captive consumers when it sets $p_i = \bar{s}$.*

Proof. Suppose instead there was a positive probability that i sells to some contested consumers when setting $p_i = \bar{s}$. Then some other firm, $j \neq i$, also has a mass point at \bar{s} , implying $\lim_{p \uparrow \bar{s}} \pi_i(p) > \pi_i(\bar{s})$, a contradiction.¹⁶

Lemma A2. $\exists i: \bar{s}_i = \bar{p}$.

Proof. Denote $i: \bar{s}_i = \max_j \{\bar{s}_j\}$ and suppose $\bar{s}_i < \bar{p}$. Suppose some firm, j , has a mass point at \bar{s}_j . By Lemma A1, $\pi_j(\bar{s}_i) = (\bar{s}_i - c)q(\bar{s}_i)I_1/n < \bar{\pi} \cdot I_1/n = \pi_j(\bar{p})$. If no firm has a mass point at \bar{s}_i , $\lim_{p \uparrow \bar{s}_i} \pi_i(p) < \pi_i(\bar{p})$.

Lemma A3. $\exists i, j: i \neq j \ \& \ \bar{s}_i = \bar{s}_j = \bar{p}$.

Proof. From Lemma A2 we know one firm, say i , has $\bar{s}_i = \bar{p}$. Denote $j \neq i$ as a firm with the second-highest support-maximum and suppose $\bar{s}_j < \bar{p}$. Note that as a result, firm i places no mass on prices in (\bar{s}_i, \bar{p}) . Suppose some firm, k , has a mass point at \bar{s}_j . By the same argument as in Lemma A1, firm k only ever sells to two types of consumers when it sets $p_k = \bar{s}_j$: its captive consumers, and contested consumers who only see the price of i and k . But then $\pi_k(\bar{s}_j) < \lim_{p \uparrow \bar{p}} \pi_k(p)$. If no firm has a mass point at \bar{s}_j , $\lim_{p \uparrow \bar{s}_j} \pi_j(p) < \lim_{p \uparrow \bar{p}} \pi_j(p)$.

Lemma A4. $\pi_i = \pi_j \ \forall i, j$.

Proof. Suppose $\pi_i < \pi_j$ for some i and j . Then $\lim_{p \uparrow \bar{s}_j} \pi_i(p) = \pi_j > \pi_i$.

Lemma A5. *No firm places a mass point at any $p \in [p, \bar{p})$.*

Proof. Suppose that firm i has a mass point at $p_i \in [p, \bar{p})$. There exists some interval $(p_i, p_i + \epsilon)$ in which no other firm puts probability mass (suppose some firms did, and let $j \neq i$ be a firm with $p_j > p_i$ in its support such that no other firm $k \neq i, j$ has (p_i, p_j) in its support: because $I_m > 0$ for

¹⁶The limit argument means firm i can increase profits by shifting probability mass from the mass point at \bar{s} to just below \bar{s} . We use similar limit arguments throughout.

some $m > 1$, there are consumers informed of i 's and j 's price, hence $\lim_{p \uparrow p_i} \pi_j(p) > \pi_j(p_i + \delta)$ for $\delta \in (0, p_j - p_i)$. But then $\pi_i(p_i + \delta) > \pi_i(p_i)$ for $\delta \in (0, \epsilon)$.

Lemma A6. *If $I_2 > 0$, at most one firm places a mass point at \bar{p} .*

Proof. Suppose i and j place mass points at \bar{p} . Because $I_2 > 0$, there are consumers who are informed of i 's and j 's price and no other price, so $\pi_i(\bar{p}) < \lim_{p \uparrow \bar{p}} \pi_i(p)$.

Lemma A7. *If $I_2 > 0$, no firm has a mass point at \bar{p} .*

Proof. By Lemma A6, at most one firm has a mass point at \bar{p} . Suppose exactly one firm, i , has a mass point at \bar{p} . By Lemma A1, $\pi_i = \bar{\pi} \cdot I_1/n$. By Lemma A3 there is some $j \neq i$ with $\bar{s}_j = \bar{p}$. Because $I_2 > 0$ there are consumers who are informed of the prices from firm i and j and no other price, hence $\lim_{p \uparrow \bar{p}} \pi_j(p) > \pi_i$, contradicting Lemma A4.

Lemma A8. *If $I_2 > 0$, $\pi_i = \bar{\pi} \cdot I_1/n \forall i$.*

Proof. From Lemma A3, $\exists i, j: i \neq j$ and $\bar{s}_i = \bar{s}_j = \bar{p}$. By Lemma A7, no firm has a mass point at \bar{p} , hence $\pi_i = \pi_j = \bar{\pi} \cdot I_1/n$. By Lemma A4, all firms make this profit.

Lemma A9. *If $I_2 > 0$, $\exists i, j: \underline{s}_i = \underline{s}_j = \underline{p}$.*

Proof. Index firms such that $\underline{s}_1 \leq \underline{s}_2 \leq \dots \leq \underline{s}_n$ and suppose $\underline{s}_1 < \underline{s}_2$. Firm 1 strictly increases profit by shifting the mass it places on prices in $[\underline{s}_1, \underline{s}_2)$, to prices slightly below \underline{s}_2 , hence $\underline{s}_2 = \underline{s}_1$. Next, suppose $\underline{s}_1 = \underline{s}_2 > \underline{p}$. By Lemma A5 no firm places a mass point on \underline{s}_1 . But then $\lim_{p \uparrow \underline{s}_1} \pi_i(p) > \bar{\pi} \cdot I_1/n$ for any i , contradicting Lemma A8.

We now introduce notation to characterize i 's expected profits. First, D_i^m , which is the expected share of consumers i sells to among those who see m prices by setting a price $p_i < \bar{p}$. To write D_i^m we use $S_{m-1}(i)$, the set of all vectors of length $m-1$ of distinct firms that do not include i :

$$S_{m-1}(i) \equiv \{a | a = (j_1, \dots, j_{m-1}) \not\ni i, \text{ and } j_k \neq j_l \forall j_k, j_l \in a\}, \quad (6)$$

$$D_i^m \equiv \begin{cases} n^{-1} & m = 1, \\ \binom{n}{m}^{-1} \sum_{a \in S_{m-1}(i)} \prod_{k \in a} (1 - G_k) & m = 2, \dots, n. \end{cases} \quad (7)$$

Term D_i^1 is the share of consumers who see one price and buy from i . More generally, let a be a consideration set of a consumer of length m that includes firm i and $m-1$ other firms. Consumers with this consideration set buy from i if $p_i \leq p_k$ for all $k \in a$.¹⁷ The probability of this is $\prod_{k \in a} (1 - G_k)$. Summing over all sets of firms $a \in S_{m-1}(i)$ that are in a consideration set

¹⁷By Lemma A5, ties are zero-probability events so we do not need to specify a tie-breaking rule.

with i , and dividing by the number of consumers who see m prices, $\binom{n}{m}$, gives the expected share of consumers that firm i attracts among consumers who see m prices.

Similarly, we define D_{ij}^m using the set of vectors of non-equal firms excluding i and $j \neq i$:

$$S_{m-1}(ij) \equiv \{a | a = (j_1, \dots, j_{m-1}) \not\ni i, j: i \neq j, \text{ and } j_k \neq j_l, \forall j_k, j_l \in a\}, \quad (8)$$

$$D_{ij}^m \equiv \begin{cases} n^{-1} & m = 1 \\ \binom{n}{m}^{-1} \sum_{a \in S_{m-1}(ij)} \prod_{k \in a} (1 - G_k) & m = 2, \dots, n-1 \\ 0 & m = n. \end{cases} \quad (9)$$

Relating D_i^m and D_{ij}^m , we find

$$D_i^m = D_{ij}^m + \frac{\binom{n}{m-1}}{\binom{n}{m}} (1 - G_j) D_{ij}^{m-1}, \quad m = 2, \dots, n \text{ and } i \neq j. \quad (10)$$

This decomposition will be useful for the proceeding results. Note that $D_i^n = \prod_{j=1; j \neq i}^n (1 - G_j)$, $D_{ij}^{n-1} = \frac{1}{n} \prod_{k=1; k \neq i, j}^{n-1} (1 - G_k)$, and $D_i^n = (1 - G_j) n D_{ij}^{n-1}$. The total profits at a price p from consumers who see m prices is $K^m(p) \equiv (p - c)q(p)I_m$ for $m = 1, \dots, n$, so that the expected profit of firm i is

$$B_i(p_i) \equiv \sum_{m=1}^n K^m(p_i) D_i^m(p_i), \quad i = 1, \dots, n. \quad (11)$$

$K^m(p_i)$ is the total profit from consumers who see m prices when they pay p_i . $D_i^m(p_i)$ is the expected share of consumers of firm i when charging p_i among consumers who see m prices. Thus, $K^m(p_i) D_i^m(p_i)$ is the expected profit of firm i from consumers who see m prices, and $B_i(p_i)$ is the expected profit of firm i given that each rival j plays G_j .

Lemma A10. *If $I_2 > 0$, $B_i(p_i)$ is constant and equal to $\bar{\pi} \cdot I_1/n$ at the points of increase of G_i on $[p, \bar{p})$ for all i .*

Proof. If p_i is a point of increase of G_i , then firm i must earn the equilibrium profit at p_i , which by Lemma A8 is $\bar{\pi} \cdot I_1/n$.

Lemma A11. *If p is a point of increase of G_i and G_j , then $G_i = G_j$ at p . $B_i(p)$ is continuous at every point of increase of G_i .¹⁸*

Proof. First, we can use (10) to rewrite

$$B_i(p) = \frac{1}{n} K^1(p) + \sum_{m=2}^{n-1} D_{ij}^m(p) K^m(p) + (1 - G_j(p)) \sum_{m=2}^n \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p) K^m(p). \quad (12)$$

¹⁸A “point of increase of G_i ” is p such that i plays prices in $(p - \epsilon, p + \epsilon)$ with strictly positive probability $\forall \epsilon > 0$.

Because p is a point of increase of G_i and G_j , Lemma A4 implies that $B_i(p) = B_j(p)$. Using $D_{ij}^m = D_{ji}^m$ and $D_{ij}^{m-1} = D_{ji}^{m-1}$, it follows that $B_i(p) = B_j(p)$:

$$\begin{aligned} \Rightarrow (1 - G_j(p)) \sum_{m=2}^n \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p) K^m(p) &= (1 - G_i(p)) \sum_{m=2}^n \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p) K^m(p) \\ \Rightarrow G_i(p) &= G_j(p). \end{aligned} \quad (13)$$

Because $D_{ij}^1 = n^{-1}$, the summations are strictly positive, allowing the last implication. To show that $B_i(p)$ is continuous at every point of increase p of G_i , we use that all firms j who also have a point of increase at p set $G_j(p) = G_i(p)$. For all other firms k , $G_k(p)$ is constant around p . Additionally, observe that $K^m(p)$ is continuous for all m and $p \in [c, \bar{p}]$, and $\bar{\pi} \cdot I_1/n$ is continuous. Thus, $B_i(p) = \bar{\pi} \cdot I_1/n$ pins down a continuous function $G_i(p)$ around p . We conclude that $B_i(p)$ is continuous at p .

Lemma A12. *If $I_2 > 0$, for every i and every point of increase p of G_i in $[p, \bar{p})$, there is at least one G_j with $j \neq i$ such that G_j increases at p .*

Proof. Suppose instead there is no such firm $j \neq i$. Because p is a point of increase of G_i , $\frac{dB_i(p)}{dp} = 0$ by Lemma A10. By Lemma A11, we can differentiate $B_i(p)$ with respect to p , leading to

$$\sum_{m=1}^n D_i^m(p) \frac{dK^m}{dp} = 0. \quad (14)$$

Note that because no firm $j \neq i$ has a point of increase at p , $\frac{dD_i^m}{dp} = 0$ for all $m > 1$; hence $\frac{dD_i^m}{dp}$ terms are absent from (14). Because $D_i^m(p) \frac{dK^m}{dp} \geq 0$ for all m and $D_i^1 \frac{dK^1}{dp} = \frac{1}{n} \frac{dK^1}{dp} > 0$ for all $p \in [p, \bar{p})$, the left-hand-side of (14) is strictly positive, but the right-hand-side is zero, a contradiction. We conclude that $\frac{dD_i^m}{dp} < 0$ for some m , implying that at least one G_j has to increase at p .

Lemma A13. *If $I_2 > 0$ and G_i is strictly increasing on some open interval (x, y) , $p < x < y < \bar{p}$, then G_i is strictly increasing on the whole interval $[p, y)$.*

Proof. Suppose instead that G_i is, without loss of generality, constant on (z, x) for some $z \in [p, x)$. By Lemma A5, there are no mass points at z or x , hence $G_i(z) = G_i(x)$. At least two firms, k and l , must charge prices with positive probability in $(x - \epsilon, x)$ for some $\epsilon > 0$. (If no firm charges a price in $(x - \epsilon, x)$, take $\bar{\epsilon}$ to be the supremum of the set of $\epsilon > 0$ such that this is true. Firms charging prices in some neighborhood around $x - \bar{\epsilon}$ strictly increase profits by moving probability mass into $(x - \bar{\epsilon}, x)$. Thus, at least one firm charges prices with positive probability in $(x - \epsilon, x) \forall \epsilon > 0$. By Lemma A12, this extends to two firms.)

By Lemma A10, for every $p \in (x - \epsilon, x)$, $B_k(p) = B_l(p) = \bar{\pi} \cdot I_1/n$. And because there are no

mass points on $[\underline{p}, \bar{p}]$ (Lemmas A5 and A7), $B_i(x) = B_k(x) = B_l(x) = \bar{\pi} \cdot I_1/n$. Because $x < \bar{p}$ and i charges prices in (x, y) with positive probability, Lemma A11 implies that

$$G_k(x) = G_l(x) = G_i(x) < 1.$$

But with $B_i(x) = B_l(x) = B_l(p)$ for all $p \in (x - \epsilon, x)$, it must be that $B_i(p) \leq B_l(p)$ for all $p \in (x - \epsilon, x)$, because these p are not in i 's support. But then by the same arguments used in Lemma A11, $B_i(p) \leq B_l(p)$ implies $G_i(p) \leq G_l(p)$. But this contradicts $G_i(x) = G_l(x)$, because $G_l(p)$ is increasing on $(x - \epsilon, x)$, but $G_i(p)$ is constant.

Lemma A14. *If $I_2 > 0$, all firms mix continuously via some symmetric $G(p)$ over the support $[\underline{p}, \bar{p}]$, in any equilibrium.*

Proof. We first show that $\bar{s}_i = \bar{p} \forall i$. Suppose instead there exists a firm i with $\bar{s}_i < \bar{p}$. Then by Lemmas A5 and A13, this firm charges prices on $[\underline{p}, \bar{s}_i]$. By Lemma A11, all firms that charge prices $p \in [\underline{p}, \bar{s}_i]$ play symmetric CDFs $G(\cdot)$ for these prices. Because some firms have \bar{p} in their support (Lemma A3) and charge prices in $[\underline{p}, \bar{p}]$ (Lemmas A7 and A13), we have $G(\bar{s}_i) < 1$, a contradiction. We conclude that $\bar{s}_i = \bar{p} \forall i$.

Because $\bar{s}_i = \bar{p} \forall i$ and no firm has a mass point at \bar{p} (Lemma A7), Lemma A13 implies that all firms charge prices on every open subinterval of $[\underline{p}, \bar{p}]$ with strictly positive probability. Lemma A11 then implies that all firms play a symmetric price distribution over the whole support.

Proofs of Lemmas 1 and 2

Proof of Lemma 1. By Lemma A14 all firms play symmetric price distributions, G over the support $[\underline{p}, \bar{p}]$, in any equilibrium. By Lemma A10, G must solve $B(p) = \bar{\pi} \cdot I_1/n$ (3) for all $p \in [\underline{p}, \bar{p}]$. Equation (3) is satisfied at $G(\underline{p}) = 0$ and $G(\bar{p}) = 1$. Because $(p - c)q(p)$ is strictly increasing and continuous in p for $p \in [c, \bar{p})$, the equation pins down a unique strictly increasing and continuous $G(p)$ for all $p \in [\underline{p}, \bar{p}]$. To complete the specification, let $G(p) = 0$ for $p < \underline{p}$, and $G(p) = 1$ for $p \geq \bar{p}$.

Proof of Lemma 2. We show there are uncountably-many asymmetric equilibria in which one firm has a mass point at \bar{p} when $I_2 = 0$. Specifically, the following strategies constitute Nash Equilibria, parameterized by $x \in (\underline{p}, \bar{p})$. Firms $i = 1, \dots, n - 1$ have support $[\underline{p}, \bar{p}]$, while one firm, n , has support $[\underline{p}, x] \cup \{\bar{p}\}$. Over $[\underline{p}, x]$, all firms play the symmetric mixed strategy $G(\cdot)$ that solves (3). Firm n places its remaining probability mass $1 - G(x)$ at \bar{p} . Over $[x, \bar{p})$, firms $i = 1, \dots, n - 1$ play the symmetric mixed strategy $\hat{G}(\cdot)$. Adjusting (12) appropriately, we see

that $\hat{G}(\cdot)$ solves

$$\frac{1}{n}K^1(p) + \sum_{m=2}^{n-1} \frac{\binom{n-2}{m-1}}{\binom{n}{m}} (1 - \hat{G}(p))^{m-1} K^m(p) + (1 - G(x)) \sum_{m=2}^n \frac{\binom{n-2}{m-2}}{\binom{n}{m}} (1 - \hat{G}(p))^{m-2} K^m(p) = \bar{\pi} \cdot \frac{I_1}{n}. \quad (15)$$

As $p \uparrow \bar{p}$, the left-hand-side converges to $K^1(\bar{p})/n = \bar{\pi} \cdot I_1/n$. Because $I_2 = 0$, firms that charge prices arbitrarily close to \bar{p} cannot attract any contested consumers from firm n when n charges \bar{p} . Thus, even arbitrarily close to \bar{p} , firms $i \neq n$ still only earn $(p - c)q(p)I_1/n$ (in contrast to the proof of Lemma A7 when $I_2 > 0$). These equilibria exist for $I_2 = 0$ and any $I_3, I_4, \dots, I_m \geq 0$. If $I_2 = 0$ and $I_3 = 0$, there are equilibria where two firms have a mass point at \bar{p} , and so on.