



Generalised adaptive optics method for high-NA aberration-free refocusing in refractive-index-mismatched media

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Abstract: Phase aberrations are introduced when focusing by a high-numerical aperture (NA) objective lens into refractive-index-mismatched (RIM) media. The axial focus position in these media can be adjusted through either optical remote-focusing or mechanical stage translation. Despite the wide interest in remote-focusing, no generalised control algorithm using Zernike polynomials has been presented that performs independent remote-focusing and RIM correction in combination with mechanical stage translation. In this work, we thoroughly review derivations that model high-NA defocus and RIM aberration. We show through both numerical simulation and experimental results that optical remote-focusing using an adaptive device and mechanical stage translation are not optically equivalent processes, such that one cannot fully compensate for the other without additional aberration compensation. We further establish new orthogonal modes formulated using conventional Zernike modes and discuss its device programming to control high-NA remote-focusing and RIM correction as independent primary modes in combination with mechanical stage translation for aberration-free refocusing. Numerical simulations are performed, and control algorithms are validated experimentally by fabricating graphitic features in diamond using direct laser writing.

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1. Introduction

Precise and aberration-free focusing with high numerical aperture (NA) is essential in many applications such as microscopy [1,2] and laser fabrication [3,4]. Aberrations commonly lead to a distorted focal spot when focusing through refractive-index-mismatched (RIM) media with planar interfaces [5,6]. The correction of RIM is important not only for different microscope modalities [7–10], but also for emerging applications that generate larger amounts of RIM, typically when using air immersion objectives to provide longer working distances [2], fabricating in high-RI material [11], and imaging in optically-cleared biological samples [12]. Using an adaptive device, such as a deformable mirror (DM) or spatial light modulator (SLM), one can not only correct for RIM using adaptive optics (AO) methods [13,14], but also perform remote-focusing [15,16], i.e., fast optical refocusing without mechanical movement of the objective lens or sample translation stage. Remote-focusing can also be achieved using focus-tunable lenses [17–19] and enable fast axial scanning up to kilo-hertz frames rates, but the degradation in axial resolution due to RIM and focus tuning remains a topic for further research. Here we focus upon the use of common multi-actuator adaptive devices due to its higher versatility for aberration correction.

Both defocus and spherical aberration can be modelled as functions over a unit disk representing the optical pupil [20]. With a low-NA objective lens, these two functions coincide well with two of the well-known Zernike polynomials [21], which are a set of orthogonal basis functions commonly employed to decompose an arbitrary aberration function. However, when high-NA lenses are used, defocus takes the form of a spherical wavefront as opposed to the quadratic function describing Zernike defocus [15], and RIM aberration no longer coincides with the individual Zernike spherical terms [7,8]. Practically, this means that Zernike defocus and Zernike spherical terms of first and higher orders must be both manipulated for remote-focusing and RIM correction, such that the two operations are coupled in terms of Zernike representation.

The correction of high-NA RIM aberration using common multi-actuator adaptive devices has been well addressed [4,9,22–25]. However, there is not yet a generalised predictive method that provides simultaneous support for aberration-free remote-focusing with consideration of mechanical stage translation in a high-NA system. Ideally, a new set of mathematical modes analogous to Zernike modes should be derived, such that the new modes maintain orthogonality while allowing high-NA defocus to independently adjust the axial focus position without inducing aberrations; and the new high-NA spherical mode to independently correct for RIM aberration without shifting the focus position.

In this paper, we derive modified models of the conventional Zernike defocus and Zernike spherical for high-NA systems by means of QR decomposition. We first consider the representation of high-NA defocus before discussing high-NA remote-focusing in a homogeneous medium and independent remote-focusing and RIM correction in stratified media. Two distinct control strategies are provided and verified through numerical simulations. Experiments of remote-focusing in laser fabrication are further performed. Finally, we discuss how the proposed theory can be extended to achieve aberration-free refocusing in stratified media with multiple interfaces.

2. Representation of high-NA defocus Φ_d

The illumination field at the focus of a high-NA lens is accurately modelled by the Debye–Wolf diffraction integral [26], the geometry of which is illustrated in Fig. 1(a). This integral is formulated with respect to the maximum semi-aperture angle α of the Gaussian reference sphere G , which relates to the numerical aperture and image space refractive index n by $\text{NA} = n \sin \alpha$. θ represents the incidence angle of an arbitrary meridional ray that converges to the Gaussian focus O . Note that in this work, we only consider on-axis points in the sample for telecentric systems where the adaptive device is placed at an infinity-focused exit pupil imaged by $4f$ telescope systems from the back aperture of an infinity-corrected objective lens. By mapping the Gaussian reference sphere onto a unit aperture disk A for an objective lens obeying Abbe's sine condition [15], the radial coordinate ρ can be expressed by $\rho = \sin \theta / \sin \alpha$.

The phase representation of high-NA defocus Φ_d can be derived by referring to the dependency of the diffraction integral upon axial coordinate z . Assume the presence of an axial focal displacement Δ expressed in micrometres, that causes the Gaussian focus to shift to O' , as illustrated in Fig. 1(b). The corresponding phase in the unit aperture disk can then be obtained as

$$\Phi_d = k\Delta \cos \theta, \quad (1)$$

where $k = 2\pi n / \lambda$ and λ is the incident wavelength expressed also in micrometres. For convenience of representation in the unit pupil disk, using the geometrical relations given in Fig. 1(a), one can also express Φ_d as a function of the radial coordinate ρ ,

$$\Phi_d = k\Delta \sqrt{1 - (\rho \sin \alpha)^2}. \quad (2)$$

The operation of inducing an axial focal displacement of Δ using a DM or SLM (remote-focusing), without mechanical movement of the objective lens or sample stage, can then be achieved by correcting for Φ_d , equivalent to applying a phase of $-\Phi_d$ at the pupil plane of the optical system.

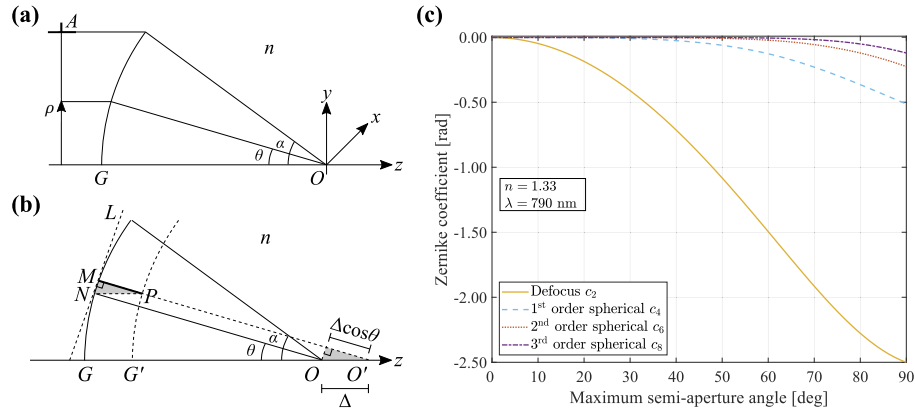


Fig. 1. Representation of high-NA defocus Φ_d . (a) Frame of reference for the Debye–Wolf diffraction integral. (b) Geometrical derivation of Φ_d for focal displacement Δ . (c) The first four Zernike coefficients c_{2p} in Eq. (3) (excl. Zernike piston) evaluated as a function of α for 1 μm focal displacement in a homogeneous medium of refractive index $n = 1.33$ at wavelength $\lambda = 790 \text{ nm}$.

2.1. Zernike expansion of Φ_d

Phase Φ_d can be accurately expanded into a series of radially symmetric Zernike polynomials $\mathcal{Z}_{2p}^0(\rho, \phi)$ by [7,27],

$$\Phi_d = k\Delta\sqrt{1 - (\rho \sin \alpha)^2} = k\Delta \sum_{p=0}^{\infty} c_{2p}^0 \cdot \mathcal{Z}_{2p}^0(\rho, \phi), \quad (3)$$

where $\mathcal{Z}_{2p}^0(\rho, \phi)$ is normalised and ordered as outlined in [20,28], and c_{2p}^0 is a Zernike coefficient that represents the root-mean-square (RMS) phase value of its corresponding Zernike mode. Both single $(\cdot)_{\#i}$ and double $(\cdot)_n^m$ index notations can be used, and the latter $(\cdot)_{n=2p}^{m=0}$ is adopted for derivations in this work using radially symmetric Zernike polynomials with radial order $n = 2p$ [20,28]. Explicit mention of the azimuthal order $m = 0$ and azimuthal coordinate ϕ will also be dropped for the remainder of this work. It should be noted that except for interferometric techniques [29], Zernike piston coefficients c_0 or $c_{\#1}$ can be safely ignored during the evaluation of phase, as it only introduces a constant offset to the phase but no variations within. However, we retain the Zernike piston term in mathematical derivations throughout this work for completeness.

To appreciate the difference between low-NA and high-NA defocus, the first four Zernike coefficients c_{2p} in Eq. (3) (excl. Zernike piston) are evaluated in radians as a function of α for 1 μm focal displacement in a homogeneous medium of refractive index $n = 1.33$ at a wavelength of $\lambda = 790 \text{ nm}$. c_{2p} can be calculated as [27],

$$c_{2p} = \left[\frac{2p-1}{2p+3} \tan^4\left(\frac{\alpha}{2}\right) - 1 \right] \frac{\sin(\alpha) \tan^{2p-1}(\alpha/2)}{2(2p-1)\sqrt{2p+1}}. \quad (4)$$

Results in Fig. 1(c) show that only Zernike defocus is significantly non-zero when α is small ($\alpha \approx 22^\circ$ for $\text{NA} = 0.5$ and $n = 1.33$). Therefore, refocusing using a low-NA lens in a homogeneous medium generally results in a phase contribution that is essentially equivalent to Zernike defocus $c_2 \mathcal{Z}_2(\rho)$. On the other hand, Zernike spherical terms become more significant at larger angles, such that the additional terms in Eq. (3) cannot be neglected. We also note that when the focal displacement Δ is large in a low-NA system, the absolute value of Zernike spherical terms may

also become non-negligible, in which case the aberration-free refocusing algorithm proposed in this work would also be relevant.

2.2. Representation using a vector of Zernike coefficients

When considering a finite sum, the Zernike coefficients in Eq. (3) can be collected into a vector $\mathbf{s}_l \in \mathbb{R}^{K+1}$ expressed in the following form,

$$\mathbf{s}_l = \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ \vdots \\ c_{2K} \end{bmatrix} = \begin{bmatrix} c_{\#1} \\ c_{\#4} \\ c_{\#11} \\ \vdots \\ c_{\#J} \end{bmatrix}, \quad (5)$$

where the double and single index notations are used for the first and second vectors, respectively. We refer to $\mathbf{s}_l$ as the vector of low-NA spherical coefficients, where the maximum radial order is $2K$, and relates to the maximum Noll's index J by $J = 1 + 2K(2K + 1)/2$. When controlling an adaptive device in practice, it is also necessary to consider the complete set of Zernike coefficients $\mathbf{z}_l \in \mathbb{R}^J$, where zeros take the place of non-radially symmetric elements that appear in the Noll's single-index enumeration. High-NA defocus Φ_d can then be represented by $k\Delta \cdot \mathbf{s}_l$ or $k\Delta \cdot \mathbf{z}_l$ for a focal displacement of Δ . In practice, this means that in order to refocus from point O to O' in Fig. 1(b) using an adaptive device, the vector $-k\Delta \cdot \mathbf{z}_l$ should be added to the vector of Zernike coefficients representing its current state of control.

3. High-NA remote-focusing in a homogeneous medium

As detailed in Section 2.1, due to the focussing geometry through high-NA lenses, defocus is no longer equivalent to Zernike defocus, and consists also of Zernike spherical terms of first and higher orders (see Eq. (3)). In practice, this means for multi-actuator adaptive devices that are conveniently controlled using Zernike polynomials, such as SLMs and membrane DMs, adjusting Zernike defocus only for refocusing would inherently induce aberrations and adjusting any Zernike spherical term of first and higher order would shift the focus position. This is highly inconvenient, as ideally one would like to control one element only for remote-focusing and correct for other aberrations without affecting the focus position.

3.1. Independent remote-focusing and aberration control

In this section, we derive a modal decomposition method for independent remote-focusing and aberration correction in a homogeneous medium. A new set of orthogonal modes will be defined, which we call high-NA modes, where high-NA defocus is an independent degree of freedom (DoF) equivalent to the role of Zernike defocus in the low-NA case for aberration-free remote-focusing.

Define a basis matrix $\mathbf{A} \in \mathbb{R}^{(K+1) \times (K+1)}$, where each column is a basis mode represented by a vector of low-NA spherical coefficients. When using low-NA Zernike modes only, $\mathbf{A}$ will be an identity matrix. Now substitute the second column of $\mathbf{A}$ with the Zernike decomposition of high-NA defocus expressed in radians for 1 μm focal displacement in a homogeneous medium

with refractive index n (see Eq. (4)), then basis matrix $\mathbf{A}$ would take the following form,

$$\mathbf{A} = \begin{bmatrix} 1 & kc_0 & 0 & 0 & \dots & 0 \\ 0 & kc_2 & 0 & 0 & \dots & 0 \\ 0 & kc_4 & 1 & 0 & \dots & 0 \\ 0 & kc_6 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & kc_{2K} & 0 & 0 & \dots & 1 \end{bmatrix}. \quad (6)$$

To make all subsequent spherical modes orthogonal to high-NA defocus while being mutually orthogonal, a QR decomposition process can be applied to matrix $\mathbf{A}$ [30],

$$\mathbf{A} = \mathbf{QR} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \dots & \mathbf{q}_{K+1} \end{bmatrix} \mathbf{R}, \quad (7)$$

where $\mathbf{Q} \in \mathbb{R}^{(K+1) \times (K+1)}$ is an orthonormal matrix whose j -th column $\mathbf{q}_j$ represents the j -th new high-NA mode, and $\mathbf{R} \in \mathbb{R}^{(K+1) \times (K+1)}$ is an upper triangular matrix whose j -th column represents combinations of the basis modes in $\mathbf{A}$ that are required to create the j -th new high-NA mode. As a result, each high-NA mode is a unique linear combination of radially symmetric Zernike modes and represent the high-NA analogues of Zernike piston, defocus, 1st order spherical, and higher order spherical terms, after removal of preceding high-NA components.

We can now introduce a vector of high-NA spherical coefficients $\mathbf{s}_h \in \mathbb{R}^{K+1}$, of which the second element controls high-NA defocus, analogous to that in $\mathbf{s}_l$ in the low-NA case for device control. The remaining elements are orthogonal to the second element, as well as being mutually orthogonal. In order to find the relation between $\mathbf{s}_h$ and $\mathbf{s}_l$, conversion matrix $\mathbf{T}_{l \leftarrow h} \in \mathbb{R}^{(K+1) \times (K+1)}$ can be defined to convert from $\mathbf{s}_h$ to $\mathbf{s}_l$, expressed as

$$\mathbf{s}_l = \mathbf{T}_{l \leftarrow h} \mathbf{s}_h. \quad (8)$$

As matrix $\mathbf{Q}$ is orthonormal, each column represents a high-NA mode whose unit magnitude corresponds to a phase of 1 radians RMS. In practice, however, one may wish to relate high-NA defocus to the amount of focal displacement in micrometres. For this purpose, the conversion matrices can be obtained by rescaling the columns of $\mathbf{Q}$ by the diagonal elements of $\mathbf{R}$, such that

$$\begin{aligned} \mathbf{T}_{l \leftarrow h} &= \mathbf{Q} \cdot \text{diag}(\mathbf{R}) \\ &= \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \dots & \mathbf{q}_{K+1} \end{bmatrix} \begin{bmatrix} r_{11} & 0 & \dots & 0 \\ 0 & r_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & r_{(K+1)(K+1)} \end{bmatrix}. \end{aligned} \quad (9)$$

The above process allows aberration-free remote-focusing to be achieved in a high-NA system by solely adjusting the second element of vector $\mathbf{s}_h$. For example, to refocus an offset Δ away from the Gaussian focus, a vector $\mathbf{s}_h$ with zeros assigned to each element is first taken. Then, $-\Delta$ is assigned to the second element of $\mathbf{s}_h$ to set high-NA defocus. Finally, a vector of low-NA spherical coefficients $\mathbf{s}_l$ can be computed using Eq. (8) to reconfigure an adaptive device controlled using Zernike polynomials. It should be mentioned that as all elements in $\mathbf{s}_h$ are now mutually orthogonal, adjusting the remaining elements of $\mathbf{s}_h$ would not affect the focus position.

Simulations of the proposed control algorithm were performed for a refocusing offset of $\Delta = 20 \mu\text{m}$ with parameters $\text{NA} = 1.25$, $n = 1.33$, and wavelength $\lambda = 790 \text{ nm}$. Figure 2(a) simulates a diffraction-limited focal spot at the Gaussian focus, the maximum intensity of which is used to normalise all subsequent simulation plots. In Fig. 2(b), a low-NA approximation of $-\Phi_d$ consisting of pure Zernike defocus was applied to the pupil disk. This results in the focal spot being shifted towards the desired depth albeit with significant distortions and a Strehl ratio (SR) of 0.16, since Φ_d and its low-NA approximation do not perfectly cancel. Figure 2(c) uses the proposed control algorithm by setting the second element of a zero vector $\mathbf{s}_h = \mathbf{0}$ to -20 and obtaining $\mathbf{s}_l$ by computing $\mathbf{s}_l = \mathbf{T}_{l \leftarrow h}$. Aberration-free remote-focusing was thus achieved at the desired offset of $20 \mu\text{m}$ with an SR of 1, as indicated in the colorbar. To further validate that adjusting other elements of $\mathbf{s}_h$ would not affect the focus position, in Fig. 2(d), the second element was set to zero while all other elements were set to normally distributed random numbers after normalising by the coefficient vector length. This led to an aberrated focal spot centred at the Gaussian focus, confirming that elements of $\mathbf{s}_h$ excluding the second could indeed control other aberration modes without shifting the focus. Finally, Fig. 2(e) also assigns -20 to the second element of $\mathbf{s}_h$ and shows independent control of remote-focusing and aberration correction.

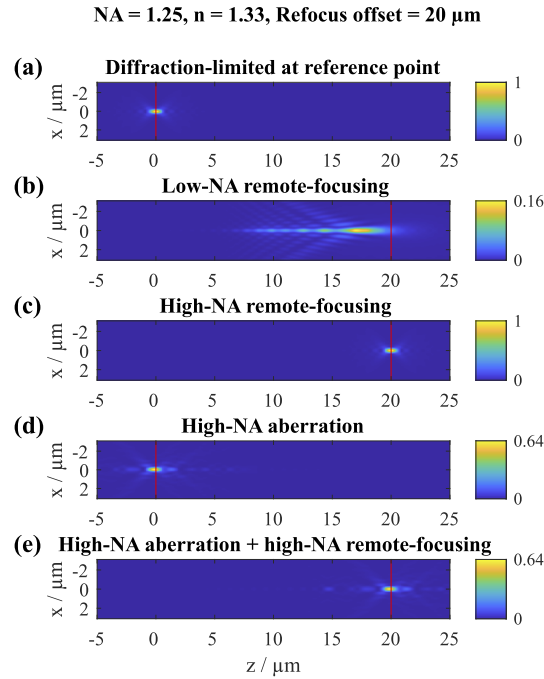


Fig. 2. Simulation of the proposed algorithm for refocusing offset $\Delta = 20 \mu\text{m}$ with parameters $\text{NA} = 1.25$, $n = 1.33$, and wavelength $\lambda = 790 \text{ nm}$. Plots show focal intensity distributions in the cases of: (a) applying zero phase; (b) applying a low-NA approximation to $-\Phi_d$; (c) applying high-NA defocus by assigning -20 to the second element of zero vector $\mathbf{s}_h = \mathbf{0}$; (d) assigning normalised random values to all elements of $\mathbf{s}_h$ excluding the second; (e) further assigning -20 to the second element of $\mathbf{s}_h$.

Note that although refocusing is controlled by the second element of $\mathbf{s}_h$, large values in the other elements may also cause distortions that shift the position of maximum intensity along the optical axis, similar to the case of Zernike spherical aberration [31].

4. High-NA refocusing in stratified media

In this section, we consider the case of refocusing in stratified media, where the index of refraction is uniform within each stratum, but is different between consecutive strata. The interface between each stratum is assumed to be planar and orthogonal to the optical axis, and cause additional spherical terms that are absent in the case of focusing in a homogeneous medium.

4.1. Effective numerical aperture

When focusing into stratified media, it is important to verify whether the full NA of the objective lens can be exploited. Assume a medium with N strata, and let n_i denote the refractive index of the i th stratum, due to possible total internal reflection at an interface i where $n_i > n_{i+1}$, the effective numerical aperture NA_e is always limited to the minimum refractive index of the stratified media, and should be used in place of the nominal numerical aperture NA_n after finding the index i for which the ratio NA_n/n_i is maximum and no larger than 1.

4.2. Stratified media with a single interface

The geometry for deriving the phase contribution due to RIM is given in Fig. 3, where θ_1 represents the angle of propagation of a ray in stratum 1 and θ_2 is the angle of this ray refracted into stratum 2. The distance h travelled by the sample translation stage (or objective lens relative to the RIM interface) will be referred to as the stage depth, or Gaussian depth, which is also equivalent to the corresponding focal shift in a homogeneous medium without the presence of RIM. We define the phase contribution due to RIM as Φ_i , which can be obtained by computing the optical path difference (OPD) between ray 1 and ray 2. Φ_i can thus be obtained using geometrical optics and Snell's Law as [7]

$$\Phi_i = -k_1 h \cos \theta_1 + k_2 h \cos \theta_2, \quad (10)$$

where $k_1 = 2\pi n_1/\lambda$ and $k_2 = 2\pi n_2/\lambda$. In order to implement remote-focusing in stratum 2, the phase contribution of high-NA defocus Φ_d should be expressed according to Eq. (1),

$$\Phi_d = k_2 \Delta \cos \theta_2, \quad (11)$$

Thus the total phase considering both RIM aberration and remote-focusing is expressed as [32]

$$\Phi_t = \Phi_i + \Phi_d = -k_1 h \cos \theta_1 + k_2 (h + \Delta) \cos \theta_2. \quad (12)$$

If Φ_i is cancelled by an adaptive device, an aberration-free focus will form at Gaussian depth h . A subsequent adjustment of Φ_d will then allow remote-focusing to an offset Δ with respect to O . Therefore, the total phase for simultaneous RIM correction and remote-focusing to desired depth $h + \Delta$ is $-\Phi_t$. It is important to realise that Φ_i is a function of both n_1, α_1 and n_2, α_2 . On the contrary, Φ_d is only a function of n_2, α_2 . Consequently, if Φ_i is cancelled by the adaptive device, the problem of remote-focusing within stratum 2 would be identical to that outlined for the homogeneous medium in Section 2. We remark that for the derivations in this work, we always assume remote-focusing within the last stratum without total internal reflection.

According to Eq. (3), Eq. (12) can be rewritten as a function of ρ in the unit pupil disk,

$$\Phi_t = -k_1 h \sqrt{1 - (\rho \sin \alpha_1)^2} + k_2 (h + \Delta) \sqrt{1 - (\rho \sin \alpha_2)^2}, \quad (13)$$

and be further expanded into a series of radially symmetric Zernike polynomials $\mathcal{Z}_{2p}(\rho)$,

$$\Phi_t = -k_1 h \sum_{p=0}^{\infty} c_{2p} \cdot \mathcal{Z}_{2p}(\rho) + k_2 (h + \Delta) \sum_{p=0}^{\infty} d_{2p} \cdot \mathcal{Z}_{2p}(\rho), \quad (14)$$

where c_{2p} and d_{2p} are computed from Eq. (4) using α_1 and α_2 , respectively. Following the same reasoning in Section 2.2, c_{2p} and d_{2p} can be collected into two vectors $\mathbf{s}_{l1}$ and $\mathbf{s}_{l2}$ (see Eq. (5)),

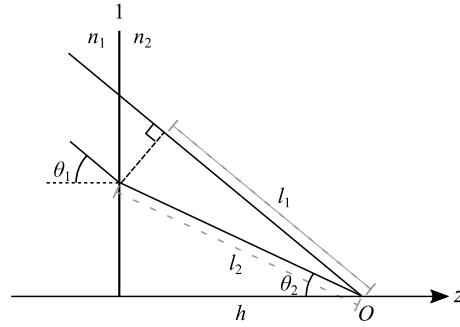


Fig. 3. Geometry for deriving the phase contribution due to RIM when focusing through stratified media with a single interface. The refractive index on either side of the interface is denoted by n_1 and n_2 . The illustration shows the case where $n_1 < n_2$. Ray 1 depicts the case of focusing at O in a homogeneous medium without RIM aberration; and ray 2 depicts the case of focusing at the same position, albeit in a stratified media such that the ray is refracted at the RIM interface before arriving at O . The two geometries have been superimposed.

and Φ_l can be represented with a vector of low-NA spherical coefficients $\mathbf{s}_l$ as,

$$\mathbf{s}_l = -k_1 h \mathbf{s}_{l1} + k_2 (h + \Delta) \mathbf{s}_{l2}. \quad (15)$$

Applying $-\mathbf{s}_l$ to the adaptive device will simultaneously correct for the RIM aberration induced at stage depth h and remote-focus to the desired depth $h + \Delta$.

In practice, it is beneficial to control high-NA defocus and RIM spherical aberration directly as independent primary modes, rather than indirectly as combinations of Zernike basis modes. This would enable the operation of remote-focusing, which is equivalent to controlling high-NA defocus, and correcting for high-NA RIM spherical aberration, to be achieved by independently adjusting one mode only. As these two operations are associated with the same Zernike spherical terms for high-NA lenses, decoupling is essential for independent adjustment.

4.2.1. Independent remote-focusing and RIM aberration correction

Following the derivations made in Section 3.1, we use the same QR decomposition method to achieve independent remote-focusing and RIM correction in a stratified media, with slight differences only in the representation of basis matrix $\mathbf{A}$. The second column should now consist of the Zernike coefficients associated with high-NA defocus in n_2 (last stratum). In addition, those associated with high-NA RIM spherical aberration should now be used as the primary high-NA spherical mode in the third column, such that

$$\mathbf{A} = \begin{bmatrix} 1 & k_2 d_0 & k_2 d_0 - k_1 c_0 & 0 & \dots & 0 \\ 0 & k_2 d_2 & k_2 d_2 - k_1 c_2 & 0 & \dots & 0 \\ 0 & k_2 d_4 & k_2 d_4 - k_1 c_4 & 0 & \dots & 0 \\ 0 & k_2 d_6 & k_2 d_6 - k_1 c_6 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & k_2 d_{2K} & k_2 d_{2K} - k_1 c_{2K} & 0 & \dots & 1 \end{bmatrix}. \quad (16)$$

QR decomposition will then create a high-NA spherical aberration mode that is orthogonal to high-NA defocus. Specifically, the second and third columns of orthonormal matrix $\mathbf{Q}$ now

represent normalised versions of independent high-NA defocus Φ_d and RIM spherical aberration Φ_i , respectively, and conversion matrix $\mathbf{T}_{l \leftarrow h}$ can be obtained in the same way as Eq. (9). Now we can achieve independent remote-focusing and RIM aberration correction by setting the second element of the newly-defined vector of high-NA spherical coefficients $\mathbf{s}_h$ to $-\Delta$ to refocus to an offset Δ away from the Gaussian focus, and the third element of $\mathbf{s}_h$ to $-h$ to correct for the RIM aberration induced at stage depth h . We note that in column three, using only the latter vector would also give the same results. Here we retain both terms for its physical RIM presentation.

In order to validate the proposed control algorithm for focusing in high-RI medium with a large RIM between the immersion media, Fig. 4 demonstrates simulation results for independent remote focusing and RIM aberration correction focusing from oil ($n_1 = 1.52$) to diamond ($n_2 = 2.42$) using an oil immersion lens of NA = 1.45 with incident wavelength $\lambda = 790$ nm. Figure 4(a) first simulates a diffraction-limited focal spot at stage depth $h_1 = 20$ μm below a conceptual interface between $n_1 = n_2 = 2.42$ for an NA = 1.45 objective lens designed for perfect operation at this refractive index. The maximum focal intensity is again used to normalise all consecutive plots. Figure 4(b) illustrates the RIM aberration in the case of $n_1 = 1.52$ and $n_2 = 2.42$ when zero phase is applied to the adaptive device at the same stage depth. In Fig. 4(c), the RIM aberration is corrected for independently of its high-NA defocus component, i.e., with the defocus component removed, by assigning -20 to the third element and zero to the remaining elements of $\mathbf{s}_h$. A diffraction-limited focal spot (SR = 1) is thus obtained at the original focus position [4]. Note that in the presence of severe spherical aberrations, this is different from the position of maximum intensity in Fig. 4(b), which is determined by the innermost rays of the illumination cone [33]. To demonstrate independent remote-focusing after correction of the RIM aberration for stage depth $h_1 = 20$ μm , a refocusing offset of $\Delta = -10$ μm is introduced in Fig. 4(d) by assigning 10 to the second element of $\mathbf{s}_h$, as well as -20 to the third. As expected, this operation causes the focus to shift 10 μm towards the negative z -axis without introducing additional aberrations. Next, we set the second element of $\mathbf{s}_h$ back to zero, and translate the sample stage axially by 5 μm to a stage depth of $h_2 = 15$ μm without updating the RIM correction accordingly. Figure 4(e) demonstrates how failing to apply the correct RIM correction for each stage shift leads to non-optimal results. Finally, the diffraction-limited focal spot is restored by assigning -15 to the third element of $\mathbf{s}_h$, as shown in Fig. 4(f).

Here, we see how the RIM aberration is only related to stage depth h , and not affected by the magnitude of Δ . As a result, adjustments made to the remote-focusing term Φ_d alone cannot cancel the additional RIM aberration incurred by a stage shift, as Φ_d has no terms in $k_1 \cos \theta_1$. Therefore, every stage movement must be followed by appropriate compensation of the additional RIM aberration. This is a different concept to remote-focusing, where only the adaptive device is used for refocusing such that Φ_d changes while Φ_i remains constant.

4.2.2. Minimal root-mean-square balancing for a desired focal depth

Axial refocusing in an adaptive optical microscope can normally be achieved in two ways, either by translating the sample stage (or objective lens), or remote-focusing using an adaptive device. The former changes stage depth h , thus the amount of RIM aberration, while the latter changes refocusing offset Δ . The QR decomposition method discussed in previous sections allows these two operations to be performed independently. However, it does not consider the total demanded stroke of the adaptive device, which could be a limiting factor in the case of large RIM aberrations or desired refocusing offsets. As we always assume refocusing in the last stratum, the first and second terms in Eq. (12) have opposite signs. This raises the possibility of balancing the phase term required for correction of RIM aberration $-\Phi_i$, which is a function of stage depth h , with that for remote-focusing $-\Phi_d$, which is a function of refocusing offset Δ . In many practical cases, one may only be concerned about the desired focal depth at which to perform imaging or direct laser writing, and are happy for the values of h and Δ to be adjusted accordingly. In such a

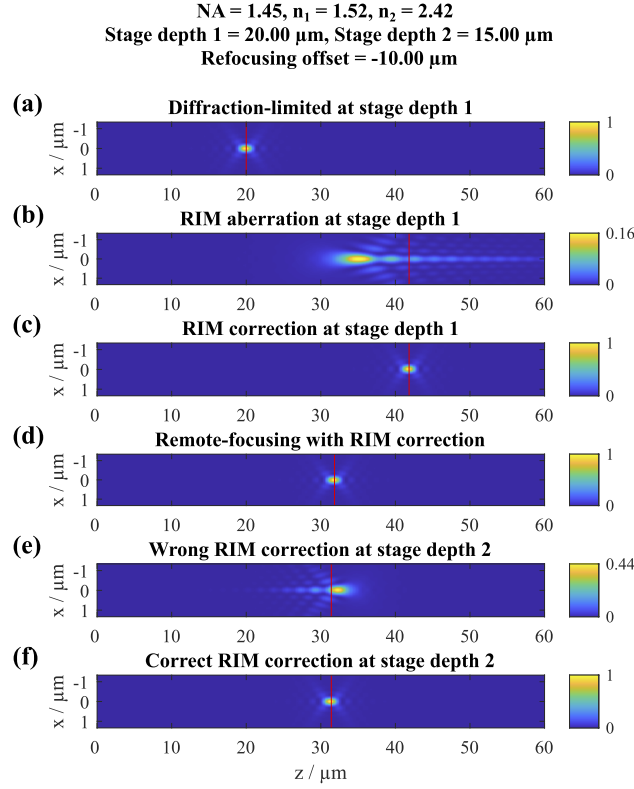


Fig. 4. Simulation of the proposed control algorithm for independent remote-focusing and RIM aberration correction focusing from oil ($n_1 = 1.52$) to diamond ($n_2 = 2.42$) using an oil immersion lens of NA = 1.45 with incident wavelength $\lambda = 790$ nm. Plots show focal intensity distributions in the cases of: (a) diffraction-limited in stratum 2; (b) RIM aberration induced at $h_1 = 20 \mu\text{m}$; (c) independent RIM correction for $h_1 = 20 \mu\text{m}$; (d) independent remote-focusing by an offset of $\Delta = -10 \mu\text{m}$; (e) RIM correction for $h_1 = 20 \mu\text{m}$ when at $h_2 = 15 \mu\text{m}$ without remote-focusing; (f) correct RIM correction for $h_2 = 15 \mu\text{m}$.

scenario, the desired focal depth, which we denote as τ , can be considered a fixed parameter in the relation $\tau = h + \Delta$, and a criterion can be established such that h and Δ are functions of τ . One such criterion is to balance h and Δ such that the RMS of the total phase Φ_t is minimised. This may be an important consideration, if using an adaptive device with limited stroke. Finally, aberration-free refocusing to the desired focal depth τ can be achieved by adjusting both the sample translation stage and adaptive device.

Taking advantage of Eq. (15) and the orthogonality of Zernike polynomials, the RMS of Φ_t can be expressed as

$$\begin{aligned} \text{RMS}(\Phi_t)^2 &= |-k_1 h \mathbf{s}_{l1} + k_2 (h + \Delta) \mathbf{s}_{l2}|^2 \\ &= |k_1 \mathbf{s}_{l1} - k_2 \mathbf{s}_{l2}|^2 \tau^2 + 2(k_1 k_2 \mathbf{s}_{l1}^T \mathbf{s}_{l2} - |k_1 \mathbf{s}_{l1}|^2) \tau \Delta + |k_1 \mathbf{s}_{l1}|^2 \Delta^2, \end{aligned} \quad (17)$$

which is a quadratic function if one of the two variables τ and Δ is fixed and the other is free. By computing the derivative of Eq. (17) with respect to Δ and assign a value of zero to yield

$$\Delta^*(\tau) = \underset{\Delta}{\text{argmin}} [\text{RMS}(\Phi_t)^2] = \left(1 - \frac{k_2 \mathbf{s}_{l1}^T \mathbf{s}_{l2}}{k_1 |\mathbf{s}_{l1}|^2} \right) \tau, \quad (18)$$

and

$$h^*(\tau) = \tau - \Delta^*(\tau) = \frac{k_2 \mathbf{s}_{l1}^T \mathbf{s}_{l2}}{k_1 |\mathbf{s}_{l1}|^2} \tau. \quad (19)$$

Note that the first element of $\mathbf{s}_{l1}$ and $\mathbf{s}_{l2}$, which correspond to Zernike piston, should be removed before calculation. Using the same high-NA modes as those obtained by QR decomposition of basis matrix $\mathbf{A}$ given in Eq. (16), the second element of $\mathbf{s}_h$ can be set to Δ^* for high-NA remote-focusing, while h^* is used to shift the translation stage, as well as to set the third element of $\mathbf{s}_h$ to correct for RIM aberration at this stage depth. As a result, aberration-free focusing will be achieved at desired depth τ below the RIM interface with minimal RMS stroke demanded from the adaptive device.

We note that other criteria for optimisation may also be applied, dependent on the scenario. For example, when using an SLM one may want to minimise the maximum derivative of Φ_l for minimal phase-wrapping [4]; when using membrane DMs, one might need to minimise the maximum curvature of Φ_l . Alternatively, a different normalisation procedure can be applied to the Zernike polynomials to evaluate the peak-to-valley magnitude of the phase, if the absolute stroke of the adaptive device is limited.

4.2.3. Balanced remote-focusing and RIM aberration correction

So far, we have treated Δ and h as two independent degrees of freedom (DoFs), i.e., both values need to be explicitly assigned to vector $\mathbf{s}_h$ for focus control. Alternatively, a simpler and more efficient strategy would be to consider τ as a single DoF. This can be accomplished by rewriting Eq. (15) as

$$\mathbf{s}_l = -k_1 h(\tau) \mathbf{s}_{l1} + k_2 \tau \mathbf{s}_{l2}, \quad (20)$$

and expressing $h(\tau)$ using Eq. (19), which leads to

$$\mathbf{s}_l = \left(-k_2 \frac{\mathbf{s}_{l1}^T \mathbf{s}_{l2}}{|\mathbf{s}_{l1}|^2} \mathbf{s}_{l1} + k_2 \mathbf{s}_{l2} \right) \tau. \quad (21)$$

QR decomposition can now be performed on basis matrix $\mathbf{A}$ after substituting the third column of Eq. (16) with the first component of Eq. (21), which represents the high-NA RIM spherical aberration that exhibits minimal demanded stroke for a desired focal depth $\tau = 1 \mu\text{m}$, such that

$$\mathbf{A} = \begin{bmatrix} 1 & k_2 d_0 & k_2 d_0 - k_2 \Omega \cdot c_0 & 0 & \dots & 0 \\ 0 & k_2 d_2 & k_2 d_2 - k_2 \Omega \cdot c_2 & 0 & \dots & 0 \\ 0 & k_2 d_4 & k_2 d_4 - k_2 \Omega \cdot c_4 & 0 & \dots & 0 \\ 0 & k_2 d_6 & k_2 d_6 - k_2 \Omega \cdot c_6 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & k_2 d_{2K} & k_2 d_{2K} - k_2 \Omega \cdot c_{2K} & 0 & \dots & 1 \end{bmatrix}, \quad (22)$$

where $\Omega = \sum_{p=1}^K c_{2p} \cdot d_{2p} / \sum_{p=1}^K c_{2p}^2$. An aberration-free focal spot can thus be obtained at τ after shifting the translation stage to h^* , assigning $-\tau$ to the third element of $\mathbf{s}_h$, and applying the vector of low-NA spherical coefficients $\mathbf{s}_l$ computed by Eq. (8) to the adaptive device. Note that the Zernike coefficients for high-NA defocus are retained in the second column of $\mathbf{A}$, such that fine remote-focusing could still be achieved without shifting the sample stage.

5. Modified control matrix for adaptive device

In general, an adaptive device is configured to create not only radially symmetric but also non-radially symmetric Zernike modes. In this section, we discuss how in practice the control matrix of common multi-actuator adaptive devices can be updated to account for the newly derived radially symmetric high-NA modes without affecting its previous functionalities.

Let $\mathbf{a}$ represent a vector of Zernike coefficients commonly used for device control, and $\mathbf{e}_{\#i}$ the i -th standard unit vector, i.e., a vector whose elements are all zero except for the i -th element which is set to one. We define a permutation matrix $\mathbf{P}_f$ as

$$\mathbf{P}_f = \begin{bmatrix} \mathbf{e}_{\#1} & \mathbf{e}_{\#4} & \mathbf{e}_{\#11} & \dots & \mathbf{e}_{\#J} & \mathbf{e}_{\#2} & \mathbf{e}_{\#3} & \mathbf{e}_{\#5} & \dots \end{bmatrix}^T, \quad (23)$$

where the first part corresponds to the indices of radially symmetric Zernike modes under the single index notation. Then define a new vector of coefficients $\mathbf{a}'$ which replaces the radially symmetric Zernike coefficients in $\mathbf{a}$ with the new high-NA spherical coefficients. $\mathbf{P}_f$ can then be used to permute vector $\mathbf{a}'$ by

$$\begin{bmatrix} \mathbf{s}_h \\ \mathbf{s}_o \end{bmatrix} = \mathbf{P}_f \cdot \mathbf{a}', \quad (24)$$

where $\mathbf{s}_h$ is the vector of high-NA spherical coefficients discussed in previous sections, and $\mathbf{s}_o$ is a vector of all non-radially symmetric Zernike coefficients. According to Eq. (8), we have

$$\begin{bmatrix} \mathbf{s}_l \\ \mathbf{s}_o \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{l \leftarrow h} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{s}_h \\ \mathbf{s}_o \end{bmatrix}, \quad (25)$$

which is now a vector of pure Zernike coefficients. Finally, all coefficients need to be permuted back to their original order as defined in [20,28], which results in

$$\mathbf{a} = \mathbf{P}_f^T \begin{bmatrix} \mathbf{s}_l \\ \mathbf{s}_o \end{bmatrix}. \quad (26)$$

For the case of using an SLM, no further modifications are required assuming a linear relationship between drive signal and phase. For a membrane DM that uses a control matrix $\mathbf{B}$ to calculate the desired actuator control signals $\mathbf{c}$ from Zernike coefficients $\mathbf{a}$ according to

$$\mathbf{c} = \mathbf{B}\mathbf{a}, \quad (27)$$

a new control matrix $\mathbf{B}'$ that relates between $\mathbf{a}'$ and $\mathbf{c}$ can be given by

$$\mathbf{B}' = \mathbf{B}\mathbf{P}_f^T \begin{bmatrix} \mathbf{T}_{l \leftarrow h} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{P}_f \quad (28)$$

Thus the only modification to the experimental DM control program is to replace $\mathbf{B}$ with $\mathbf{B}'$, and the new high-NA modes discussed in previous sections can be used without further changes.

6. Experimental validation

Validation experiments were performed by ultrafast laser fabrication of graphitic marks in diamond ($n_2 = 2.42$) [34] using a 1.45 NA oil objective lens through a layer of immersion oil ($n_1 = 1.52$). RIM correction for the oil–diamond interface allows for well localised graphitic

state changes and even single vacancy creation at the focal spot [11]. This breakdown of diamond is highly non-linear and therefore sensitive to optical aberrations [35]. The experiment consisted of two parts which used the fabrication and confocal imaging systems illustrated in Fig. 5(a), respectively. During the fabrication step, ultrafast pulses were produced by a Ti:Sapphire laser (790 nm, 250 fs, Spectra Physics Solstice), and different phase patterns were applied to an SLM (Hamamatsu, X10468) placed conjugate to the objective pupil plane. Experiment 1 was designed to validate the independent high-NA remote-focusing performance without shifting the xyz -translation stage. The optimal RIM correction was applied to the SLM for a constant stage depth of $h = 20\ \mu\text{m}$ by setting the third element of $\mathbf{s}_h$ to -20 . During experiments, refocusing offsets with $1\ \mu\text{m}$ increments within a $\pm 10\ \mu\text{m}$ depth range were applied to the SLM by continuously updating the second element of $\mathbf{s}_h$. Graphitic features at each depth (21 in total) were fabricated using one pulse of 80.5 nJ while translating the stage along the x -axis, as indicated by "RF+/SS-/RIM+" in Fig. 5(b). Experiment 2 was designed to demonstrate the effects of axially shifting the xyz -translation stage alone without remote-focusing to fabricate at the same depths as in experiment 1, but without updating the RIM correction for each varying stage depth, such that a constant value of -20 was retained for the third element of $\mathbf{s}_h$. Additional points were then fabricated along the y -axis at each depth while increasing the pulse energy from that used in experiment 1, as indicated by "RF-/SS+/RIM-" in Fig. 5(b). Experiment 3 was designed to validate the independent RIM correction performance without remote-focusing by continuously updating the third element of $\mathbf{s}_h$ to apply the optimal RIM correction when shifting the stage to the same depths as in experiment 2. The same pulse energy as that in experiment 1 was used, results are as indicated by "RF-/SS+/RIM+" in Fig. 5(b).

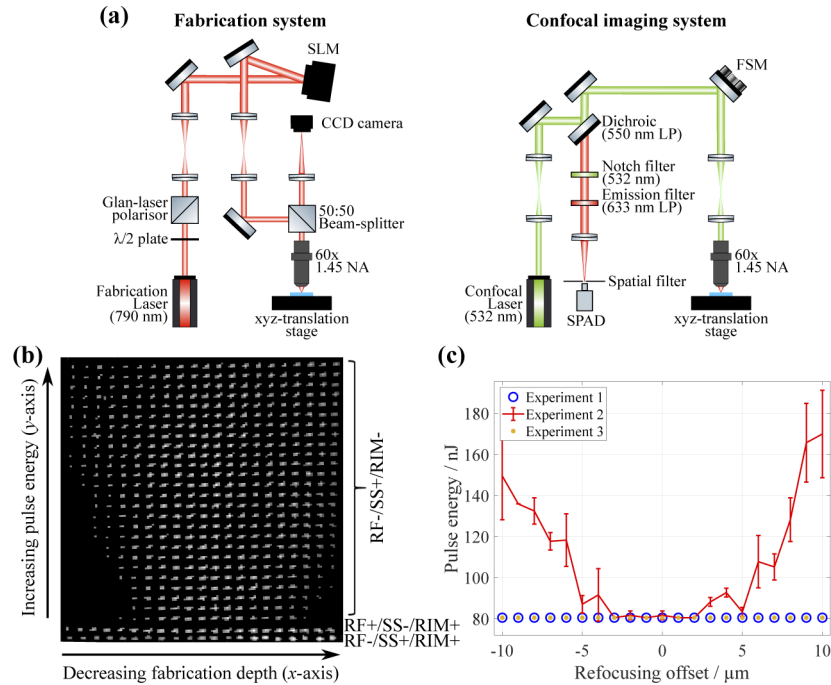


Fig. 5. Fabrication of graphitic marks when focusing from oil ($n_1 = 1.52$) to diamond ($n_2 = 2.42$) using a 1.45 NA oil immersion lens. (a) Fabrication and confocal imaging system diagram. (b) Summed 3D stack image of fabricated graphitic marks at all depths. RF: remote-focusing; SS: stage shift; RIM: refractive index mismatch correction. "+": applied; "-": not applied. (c) Comparison of pulse energy vs. refocusing offset between all three experiments.

To assess the fabrication performance, the graphitic marks were imaged by confocal laser scanning microscopy, and we assume that the measured fluorescence intensity is proportional to the amount of induced graphite. A continuous wave 532 nm laser beam (Cobalt Samba) was scanned by a dual-axis fast scanning mirror (Newport, FSM-300-001) and coupled into the same 1.45 NA objective lens used in the fabrication system. The fluorescence signal was then descanned and detected by a SPAD (Excelitas, SPCM-AQRH-14-FC) via a single-mode fibre acting as a spatial filter for optical sectioning. A 633 nm long pass emission filter was used in combination with a 532 nm notch filter to block both the induced Raman lines and residual excitation light. A full lateral scan of the fabrication area was acquired at each fabrication depth, and the summed 3D stack image of all scans generated Fig. 5(b).

Analysis was performed by comparing the fabrication laser pulse energies used to create graphitic marks of similar fluorescence intensity at each refocusing offset for all three experiments, as shown in Fig. 5(c). Specifically, using the same minimal laser pulse energy of 80.5 nJ for fabrication at all refocusing offsets in experiments 1 and 3, we were able to obtain graphitic marks of the same quality at all depths, as shown in Fig. 5(b) and labelled in blue and orange in Fig. 5(c). This result most critically showed that by adjusting the second element of $\mathbf{s}_h$ for independent remote-focusing and updating the third element of $\mathbf{s}_h$ following each mechanical stage shift, we were able to retain the fabrication performance at each depth without coupling between the two operations, confirming the validity of our proposed algorithm. We then analysed the fabrication performance in experiment 2, i.e., when a non-optimal RIM correction was applied at each depth except for zero refocusing offset. The fluorescence intensity from graphitic marks in each column of experiment 2 was measured to find the lowest pulse energy which yielded a similar fluorescence intensity to that at the same depth in experiment 1. Corresponding pulse energies are labelled in red in Fig. 5(c) with error bars indicating a 15 % tolerance range. It is shown that with a non-optimal RIM correction, aberrations reduced the focal intensity such that a higher laser fabrication pulse energy was needed to generate a graphitic mark with the same fluorescence intensity as the aberration-free focus, and this phenomenon was more detrimental with larger refocusing offsets. This further showed how optical remote-focusing and mechanical stage translation were not equivalent processes, and cannot be used interchangeably. Slight asymmetry and fluctuation in fluorescence intensity are likely due to non-radially symmetric residual system aberrations, or may be related to some non-linearity during laser fabrication.

7. Discussion

We present in this work a generalised method by means of QR decomposition for common multi-actuator adaptive devices (DM or SLM) to perform predictive axial refocusing considering the mechanical stage position with simultaneous RIM aberration correction when imaging using high-NA lenses. In particular, we show that when focusing in stratified media, both the adaptive device and translation stage should be operated simultaneously to achieve optimal refocusing. This implies in the first instance that each axial stage movement should be followed by a subsequent adjustment of the adaptive device. Secondly, one can focus at a desired depth below the RIM interface with minimal RMS stroke by balancing the phase required for correction of RIM aberration with that for remote-focusing. An important distinction should thus be made between focusing in a homogeneous and stratified medium. In the former case, the position of the translation stage is irrelevant, and one can use a single strategy to control the adaptive device for aberration-free refocusing, as outlined in Section 3. In the latter case, the current state of the translation stage must be considered to maintain an aberration-free focus. Thus two distinct control strategies were thoroughly discussed. The first strategy allows for independent remote-focusing and RIM correction without any hardware or focal depth restrictions, as outlined in Section 4.2.1, and can be further employed for closed-loop adaptive optics control. The second strategy described in Section 4.2.3 allows the user to precisely place an aberration-free focus at a

desired depth below the RIM interface with minimal stroke demanded of the adaptive device. Both strategies may prove useful in practice.

We also note that the proposed QR decomposition method is easily adaptable towards stratified media with multiple interfaces, e.g., when a cover glass is used, for both independent and balanced control of remote-focusing and RIM aberration correction. In such cases, refocusing is normally performed in the last stratum only, such that the distances between intermediate strata is fixed and their contribution to RIM aberration can be removed as a constant prior to dynamic control of the device. Thus, only the RIM aberration between the first (immersion medium) and last (sample) stratum needs to be considered. Future work should focus not only on the position and quality of the focal spot in the object space, but also how this would affect the imaging process [36] in the detection space with careful consideration of the microscope configuration, e.g., epi-/trans-illumination, the number of beam-passes on the adaptive device between the light source and detector, the type of image detector used, the effect of aberrations in the conjugate space between the adaptive device and objective lens [19], as well as how the design of different objective lenses behaves internally for rays passing through various lens elements when the model is not considered ideal. It would also be interesting to examine off-axis points in the sample and expand the approach towards non-telecentric systems.

8. Conclusion

In summary, we demonstrated that remote-focusing and RIM correction are coupled in terms of Zernike representation in a high-NA system, and that remote-focusing and stage translation are not equivalent processes. To address mode coupling between remote-focusing and RIM correction, we established new orthogonal modes using QR decomposition to control high-NA remote-focusing and high-NA RIM correction as independent primary modes for aberration-free refocusing in stratified media. Two distinct control strategies were proposed for scenarios with or without restrictions of device stroke and focusing depth. Numerical simulations validated the feasibility of the proposed algorithm, which was further confirmed by laser fabrication experiments using remote-focusing inside refractive-index-mismatched media.

Funding. European Research Council (695140, 812998); Engineering and Physical Sciences Research Council (R004803/01).

Acknowledgments. The authors thank Brian Patton for valuable discussions.

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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