

ABANDONING FOSSIL FUEL: HOW FAST AND HOW MUCH

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Abstract

Keeping climate change within limits requires that most of the available carbon-based energy sources need to be abandoned underground. We study how fast and how much this transition to carbon-free energy needs to occur within a welfare-maximizing Ramsey growth model of climate change. Our model also addresses the market failure in the development of clean energy which leads to an under-provision of renewable energy, delays the transition time to the carbon-free era, and reduces the amount of dirty fuels locked up *in situ*. Optimal policy requires an aggressive renewables subsidy in the near term and a gradually rising carbon tax which falls in long run. We also study the transition timing and the performance of recently proposed policy rules for the carbon tax.

Keywords: climate change, integrated assessment, Ramsey growth, carbon tax, renewables subsidy, learning by doing, directed technical change, multiplicative damages, additive damages

JEL codes: H21, Q51, Q54

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1. Introduction

Climate policy requires the transition from carbon-based to clean energy. In order to implement this transition in a welfare-maximizing manner, policymakers face two important market failures: the failure for markets to price dirty carbon-based energy to fully internalize all future damages arising from burning another unit of carbon and the failure of markets to internalize the full benefits of learning by doing in using clean, renewable energy (Goulder and Mathai, 2000; de Zwaan et al., 2002; Popp, 2004; Edenhofer et al., 2005). Ideally, climate policy succeeds in locking up most of the available carbon deposits by making their use economically obsolete, thereby leaving these assets abandoned. Climate scientists calculate the amount of hydrocarbon resources to be abandoned through the amount of permissible cumulative emissions. Recent studies estimate such a cumulative ‘carbon budget’ of 1,100 Gt CO₂ or 300 GtC (Allen et al., 2009; Meinshausen et al., 2009; Clarke et al., 2014) that must be kept to in order to ensure that the target of limiting temperature increase to 2°C is not overshoot. We propose a Ramsey growth model of climate change and technological progress which captures both challenges to policymakers and derives the carbon budget in welfare-maximizing way. We also study the performance of policy rules for the carbon tax which have been put forward recently (Golosov et al., 2014).

Theoretically, the carbon tax has to be set equal to the social cost of carbon (*SCC*), which corresponds to the present value of all future marginal global warming damages from burning one extra unit carbon,¹ and the renewable subsidy must be set to the social benefit of learning by doing (*SBL*), which corresponds to the present value of all future reductions in the cost of renewables from using one unit of renewable energy now. Using a calibrated integrated assessment framework which takes account of exhaustible fossil fuel, endogenous efficiency of carbon-free alternatives, gradual and abrupt transitions from fossil fuel to renewables, a temporary population boom, and ongoing technical progress and structural change, we show that dealing with the climate challenge requires an aggressive and temporary renewable subsidy as well as a gradually rising carbon tax to price out

¹ Instead, the optimal price of carbon can be found on an efficient emissions market or can correspond to the shadow price of direct control legislation.

fossil fuel. This combination of policies cuts fossil fuel use and encourages substitution towards renewables, ensures that fossil fuel producers leave more fossil fuel below ground and brings forward end of the fossil-fuel era.

We would like to highlight the following three features of our analysis.

First, we analyse not only the optimal carbon tax but also the optimal transition times for introducing renewables alongside fossil fuel and abandoning fossil fuel altogether *and* the amount of fossil fuel to be left in the ground.² This extends therefore the analysis of Nordhaus (2008, 2014), Rezai et al. (2012) and Golosov et al. (2014). Our model is solved in time steps of one year instead of in time steps of a full decade as earlier integrated assessment models or in time steps of five years as in Nordhaus (2014). This increases granularity allows us to pinpoint much more precisely the transition times towards the carbon-free era and thus avoids significant numerical biases (Cai et. al, 2012). Using IEA data for calibration purposes, we furthermore suppose that fossil-fuel extraction costs rise as less hydrocarbon reserves are left in the ground and less accessible fields are explored. This allows us to quantitatively assess the important role of untapped fossil fuel and stranded assets in the fight against global warming. We thus offer an estimate of the maximum cumulative carbon emissions, i.e. the carbon budget.

In our analysis fossil fuel is exhaustible and extraction costs are stock dependent, hence the price of fossil fuel contains two forward-looking components: the scarcity rent (the present discounted value of all future increases in extraction costs resulting from an extracting an extra unit of fossil fuel) and the *SCC*. The optimal transition times consequently depend on expectations about future developments such as learning by doing in using renewable energy. In contrast, most integrated assessment models assume that fossil fuel is abundant so its demand does not depend on expectations about the price of future renewables and the optimal distribution of finite resources across time and thus the transition simply occurs when the price of fossil fuel inclusive of the carbon tax reaches the price of the renewable energy source.

² We derive our results based on a calibrated and richer version of the theoretical growth and climate model analyzed in van der Ploeg and Withagen (2014).

Second, we suppose that the cost of renewable energy falls as experience increases (Arrow, 1962). Although technological progress, diffusion, and adoption are studied in the economics of climate change, these studies typically do not allow for exhaustible fossil fuel and stock-dependent fossil fuel extraction costs, and do not offer a fully calibrated integrated assessment model.³ If cost reductions from learning-by-doing externalities are not fully internalized, a subsidy is required alongside the carbon tax. We compare “laissez faire” with a global first-best scenario where both the climate and learning-by-doing externalities are fully internalized. Since international climate negotiations have failed miserably and there is room for national renewable energy policies, we also consider a scenario where only learning-by-doing externalities are internalized and the climate externality is not. This leads to faster extraction of fossil fuel and accelerated global warming, since fossil fuel owners fear that their resources will be worth less in the future. This is known as the Green Paradox (Sinn, 2008).

Third, we show that with CES production and utility the optimal carbon tax first rises and then declines with world GDP. The result of Golosov et al. (2014) that the optimal carbon tax is proportional to world GDP is thus not robust;⁴ it depends on logarithmic utility, Cobb-Douglas production, full depreciation of capital in each period, no capital need in fossil fuel extraction, and multiplicative, exponential climate damages in production. The proportional carbon tax performs poorly if policy needs to address multiple market failures and intergenerational inequality aversion differs from unity. The proportional tax of Golosov et al. (2014) rises too slowly relative to the first best and is, thus, insufficient in pricing fossil fuels out of the market. Like the renewable subsidy in isolation, it causes more fossil fuel to be extracted in the near term - a Green Paradox effect.

Section 2 discusses a model of carbon accumulation in the atmosphere with permanent and transient components to explain global mean temperature and presents our benchmark specification of climate damages which exceed those of Nordhaus (2008, 2014) at higher temperatures following Stern (2013) and Dietz and Stern (2015). We do not adopt the approximation to damages used in Golosov et al. (2014), since this leads to unrealistic low damages at higher temperatures. Section 3 formulates

³ See Rezai and van der Ploeg (2016b) for a review of the studies and models of climate change including endogenous technical change and a discussion of second-best issues.

⁴ This formula is also used in Hassler and Krusell (2012) and Gerlagh and Liski (2016).

our general equilibrium model of climate change and Ramsey growth. Section 4 uses our calibrated integrated assessment model to highlight the different outcomes for the optimal carbon tax, the renewables subsidy, untapped fossil fuel, the time it takes to phase in renewable energy and to reach the carbon-free era, and welfare under the optimal, the partial externality and the “laissez faire” scenarios. It also discusses the performance of the simple formula for the carbon tax put forward by Golosov et al. (2014). Section 5 discusses the sensitivity of climate policy. Section 6 concludes.

2. Carbon emissions, temperature and damages

Following Rezai and van der Ploeg (2016a,b), we use a year-by-year version of the decade-by-decade model of the carbon cycle due to Golosov et al. (2014):

$$(1) \quad E_{t+1}^P = E_t^P + \varphi_L F_t, \quad \varphi_L = 0.2, \quad E_0^P = 103 \text{ GtC},$$

$$(2) \quad E_{t+1}^T = (1 - \varphi)E_t^T + \varphi_0(1 - \varphi_L)F_t, \quad \varphi = 0.002304, \quad \varphi_0 = 0.393, \quad E_0^T = 699 \text{ GtC},$$

where E_t^P is permanent component of the stock of carbon (GtC), E_t^T is the remaining transient part of the stock of atmospheric carbon (GtC) that decays at an annual rate φ , and F_t is the rate of fossil fuel (coal, oil and gas) use (GtC/decade). About 20% of carbon emissions is permanent. The remainder has a mean life of about 300 years, so $\varphi = 1 - (1 - 0.0228)^{1/10} = 0.002304$, where 0.0228 is the parameter proposed for the decade-by-decade model in Golosov et al. (2014). The parameter $\varphi_0 = 0.393$, is calibrated to correspond to the stylized fact that about half the carbon impulse is removed after 30 years.

The equilibrium climate sensitivity (*ECS*) is the rise in global mean temperature that eventually results from a doubling of the total carbon stock in the atmosphere, $E_t = E_t^P + E_t^T$. A typical estimate for the *ECS* is 3 (IPCC, 2007). To facilitate comparison with Golosov et al. (2014), we ignore lags between the stock of atmospheric carbon and global mean temperature:⁵

⁵ This lag lowers the *SCC* (see section 5.3 and appendix B).

$$(3) \quad T_t = \omega \ln(E_t / 596.4) / \ln(2), \quad \omega = 3, \quad E_t \equiv E_t^P + E_t^T,$$

where 596.4 GtC (280 ppmv CO₂) is the IPCC figure for the pre-industrial stock of carbon.⁶ The evolution of hydrocarbon reserves in the ground is determined by the depletion equation:

$$(4) \quad S_{t+1} = S_t - F_t, \quad S_0 = 4000 \text{ GtC},$$

where S_t denotes the stock of hydrocarbon reserves at the start of period t .

Nordhaus (2008) has combined micro estimates of the costs of global warming to get aggregate macro costs of global warming of 1.7% of world GDP when global warming is 2.5° C. This figure is used to calibrate the fraction of production output that is left after damages from global warming:

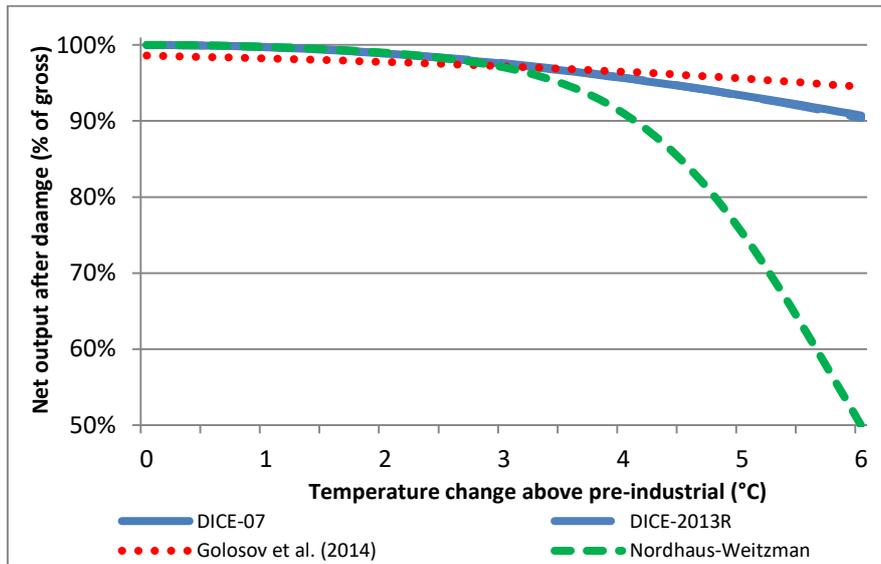
$$(5) \quad \tilde{Z}(T_t) = \frac{1}{1 + \zeta_1 T_t^{\zeta_2} + \zeta_3 T_t^{\zeta_4}}, \quad \text{so } Z(E_t) \equiv \tilde{Z}(\omega \ln(E_t / 596.4) / \ln(2)),$$

with $\zeta_1 = 0.00284$, $\zeta_2 = 2$, and $\zeta_3 = \zeta_4 = 0$ and plotted as the solid line in figure 1. Golosov et al. (2014) approximate this with the exponential net output function $Z(E_t) \equiv \exp[\tilde{\zeta}(E_t - 581)]$ with $\tilde{\zeta} = -2.379 \times 10^{-5}$, which gives rise to the dotted line in figure 1. The fit to the Nordhaus (2008) damages is good for small degrees of global warming from the present, but at higher degrees the fit is much worse and too optimistic about the size of global warming damages.⁷ Weitzman (2010) and Dietz and Stern (2015) argue that damages rise more rapidly at higher levels of temperature than suggested by (5), but empirical studies on the costs of global warming at higher temperatures are not available. Assuming that damages are 50% of world GDP at 6° C and 99% at 12.5° C, Ackerman and Stanton (2012) recalibrate (5) with $\zeta_1 = 0.00245$, $\zeta_2 = 2$, $\zeta_3 = 5.021 \times 10^{-6}$, and $\zeta_4 = 6.76$. The extra term in the denominator captures potentially catastrophic losses at high temperatures.

⁶ To convert to GtCO₂, one should multiply by 44/12.

⁷ The fit deviates by less than 1%-point only between 2° C and 4° C warming. $Z(E_t) = 1.0435 - 0.0001 E_t$ is a much better fit, also at higher temperature and carbon stocks ($R^2 = 0.9994$).

Figure 1: What is left of output after damages from global warming



The dashed line in figure 1 plots this recalibrated net output function. Although the calibration of Nordhaus (2008, 2014) has been derived through a broad consultation process among climate change scientists and economists and can be empirically justified for moderate degrees of global warming, our revised calibration seems more appropriate for higher temperatures while matching damages of Nordhaus (2008, 2014) up to 3°C of warming very closely with deviations of less than 0.5%-point.

3. An integrated assessment model of Ramsey growth and climate change

Since carbon mixed quickly throughout the whole atmosphere and it thus does not matter for global warming where carbon emissions have taken place, we take a global perspective on the climate challenge. Underlying our analysis is thus the assumption that all countries of the global economy cooperate to ensure a globally optimal climate policy which will be sustained with transfers from rich to poor countries to make sure the globally optimal climate policy is incentive compatible for poor countries. To make matters simple, we will in fact assume for each year a representative world citizen. The objective of world policy makers is to maximize a utilitarian social welfare function which adds utilities of all citizens of the world. This welfare function has the form

$$(6) \quad \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t U_t(C_t / L_t) = \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t \left[\frac{(C_t / L_t)^{1-1/\eta} - 1}{1-1/\eta} \right].$$

Here L_t is the exogenous size of the world population, C_t is aggregate consumption, U is the instantaneous utility function which we assume to have a constant elasticity of substitution $\eta > 1$, and $\rho > 0$ is the rate of pure time preference or measure of impatience. The ambition of climate policy thus depends on ethical considerations such as how little weight is given to future generations (small ρ) and on how small intergenerational inequality aversion is (small $1/\eta$) is or how easy it is to substitute current for future consumption per head. We abstract for purposes of our analysis from intra-generational inequality.

Physical capital K_t , labour, L_t , and energy are inputs in production where energy consists of renewable energies R_t , (e.g., solar or wind energy) or fossil fuels (oil, natural gas and coal), F_t . We assume that the production function $H(\cdot)$ has constant returns to scale, is concave and satisfies the Inada conditions. Following Rezai and van der Ploeg (2016a,b), the production of renewable energies is subject to learning by doing and lock-in effects, so that their unit production cost $b(B_t)$ declines with cumulated past production of renewables, denoted by B_t , and thus we have $b' < 0$. Apart from carbon taxes, technological progress is thus an important factor in determining the optimal combination of fossil and renewable energy sources (cf. Acemoglu et al., 2012; Mattauch et al., 2012). We assume instantaneous and perfect spill-over of learning.⁸

Fossil fuel extraction cost is assumed to be more expensive as less accessible fields have to be explored, hence extraction cost of one unit of fossil energy is $G(S_t)$ where S_t denotes remaining reserves in the ground and we thus suppose $G' < 0$. Consumption C_t , is what is left of production after covering the cost of hydrocarbon resource use investments $K_{t+1} - K_t$, and depreciation δK_t , with a constant depreciation rate δ . Hence, accumulation of capital is governed by the following equation:

⁸ We prefer to model endogenous technical progress by learning-by-doing rather than by investment in R&D as done by Bovenberg and Smulders (1996) and Acemoglu et al. (2012), due to the availability of empirically validated learning curves.

$$(7) \quad K_{t+1} = (1 - \delta)K_t + Z(E_t)H(K_t, L_t, F_t + R_t) - G(S_t)F_t - b(B_t)R_t - C_t,$$

where global warming damages are captured by the term $Z(E_t)$ which is given in (5). The initial capital stock K_0 is given. The development of atmospheric carbon follows from (1) and (2), that of fossil fuel stock reserves from (4), and that of learning-by-doing knowledge in the production of renewable energies from

$$(8) \quad B_{t+1} = B_t + R_t, \quad B_0 = 0 \quad \text{or} \quad B_t = \sum_{s=0}^t R_s.$$

Current costs of the various technological options that are available favour fossil energy; complete decarbonization requires substantial cuts in the cost of renewable energies relative to that of fossil fuel.

Proposition 1: *The globally efficient outcome maximizes the social welfare function (6) subject to the constraints (1)-(5) describing the carbon cycle, temperature, damages and hydrocarbon depletion, the accumulation equations (7)-(8). This yields the Euler equation for consumption growth*

$$(9) \quad \frac{C_{t+1}/L_{t+1}}{C_t/L_t} = \left(\frac{1+r_{t+1}}{1+\rho} \right)^\eta, \quad r_{t+1} \equiv Z_{t+1}H_{K_{t+1}} - \delta,$$

and the efficiency conditions for energy use

$$(10a) \quad Z(E_t)H_{F_t+R_t}(K_t, L_t, F_t + R_t) \leq G(S_t) + \theta_t^S + \theta_t^E, \quad F_t \geq 0, \quad \text{c.s.},$$

$$(10b) \quad Z(E_t)H_{F_t+R_t}(K_t, L_t, F_t + R_t) \leq b(B_t) - \theta_t^B, \quad R_t \geq 0, \quad \text{c.s.},$$

where the scarcity rent, the social cost of carbon (SCC) and the social benefit of learning by doing in final goods units (the SBL) are, respectively, given by

$$(11) \quad \theta_t^S = -\sum_{s=0}^{\infty} [G'(S_{t+1+s})F_{t+1+s}\Delta_{t+s}],$$

$$(12) \quad \theta_t^B = -\sum_{s=0}^{\infty} [b'(B_{t+1+s})R_{t+1+s}\Delta_{t+s}] \text{ and}$$

$$(13) \quad \theta_t^E = -\sum_{s=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s \right\} \Delta_{t+s} Z'(E_{t+1+s}) H(K_{t+1+s}, L_{t+1+s}, F_{t+1+s} + R_{t+1+s}) \right],$$

with the compound discount factors given by $\Delta_{t+s} \equiv \prod_{s'=0}^s (1+r_{t+1+s'})^{-1}$, $s \geq 0$.

Proof: see appendix A.

In this globally efficient outcome, the marginal rate of intertemporal substitution has to equal the marginal rate of technical transformation. The Euler equation (9) implies that the growth in consumption per capita rises with the social return on capital (r_{t+1}) and falls with the rate of time preference, especially if intergenerational inequality aversion ($1/\eta$) is small.

Efficient resource use requires that, if a type of fuel is used, its marginal product should equal total marginal cost. For fossil fuel total cost in Equation (10a) is the sum of current extraction cost $G(S_t)$ the scarcity rent θ_t^S and the SCC θ_t^E . The complimentary slackness condition ensures that if fossil fuel cannot cover its marginal cost, it is not used. Similarly for the use of renewable energies, equation (10b) states that for renewable energies to be used, its marginal product must be greater than or equal its current cost $b(B_t)$ minus the SBL θ_t^B .

Equation (11) states that the scarcity rent of keeping an extra unit of fossil fuel unexploited is the present discounted value of all future reductions in fossil fuel extraction costs. It follows from the Hotelling rule which states that the return on extracting an extra unit of fossil, selling it and getting a return on it, the rate of interest ($r_{t+1}\theta_t^S$) must equal the expected capital gain from keeping an extra unit of fossil fuel in the earth ($\theta_{t+1}^S - \theta_t^S$). The presence of extraction costs lowers the return of extracting today by shifting up all future extraction costs. This effect is captured by the term $(-G'(S_{t+1})F_{t+1})$ in Equation (11). The presence of a perfect substitute, a so-called back-stop technology, implies that depending on extraction costs some of the resource stock can become economically obsolete and be abandoned *in situ*. In this case, the scarcity rent only captures the cost of extracting an extra unit today on all future extraction costs as less accessible deposits have to be mined. Learning by doing operates like the mirror image of the extraction costs. Past use of

renewable energies lowers current and future production costs. Optimal policy captures this positive aggregate effect by reducing the marginal cost of renewable energies by the *SBL* which in equation (12) equals the present discounted value of all future learning-by-doing reductions in the cost of renewable energy.

Equation (13) states that the *SCC* is given by the usual expression, namely the present discounted value of all future marginal global warming damages from burning one unit of carbon today, taking due account of that part stays in the atmosphere for ever and the rest gradually decays at a rate corresponding to roughly 1/300 per year.

Golosov et al. (2014) use a decadal Cobb-Douglas production function with negative exponential damages such that $Z(E_t)H(K_t, F_t + R_t) = \exp[-\gamma(2.13E_t - 581)]AK_t^\alpha(F_t + R_t)^\beta$, where K_t is the capital stock at the start of period t , A is the calibrated total factor productivity (including the contribution of fixed factors such as labour and land), α is the share of capital in value added, and β is the share of energy in value added. Under the further assumptions that the carbon cycle follows the two-box formulation of equations (1) and (2), assume logarithmic utility, 100% depreciation of manmade capital each period, zero fossil fuel extraction costs and thus full exhaustion of initial fossil fuel reserves, Golosov et al. (2014) show that the optimal *SCC* is proportional to world GDP:

$$(13') \quad \tau_t^{GHKT} = \tilde{\zeta} [\varphi_L(1+\rho)/\rho + (1-\varphi_L)\varphi_0(1+\rho)/(\rho+\varphi)] Z(E_t)H(K_t, L_t, F_t + R_t).$$

With $\gamma = 2.379 \times 10^{-5}$, decadal world GDP of 630 US\$ trillion in 2010, a discount rate of 10% per decade (or 0.96% per year), the *SCC* equalled 75 US\$/tC (or roughly 20 \$/tCO₂) and continues to grow at the rate of output thereafter. The beauty of this expression is that no detailed integrated assessment model of growth and climate change is needed. The carbon tax follows directly from the rate of time preference, world GDP and some technical damage and carbon cycle parameters. The formula breaks down if intergenerational inequality aversion or factor substitution differs from unity, extraction costs are non-zero (especially if they are stock dependent) and the bad fit of the exponential approximation of (5) becomes important.

Decentralized market outcome

It is easy to see how to sustain the efficient outcome of Proposition 1 in a decentralized competitive market economy with atomistic agents, where firms maximize profits and households maximize utility taking the wage rate w_t , the market interest rate r_{t+1} , the market price for fossil fuel p_t , the specific tax τ_t on carbon emissions, the market price for renewable energy q_t , the specific subsidy v_t on the use of renewable energy, and the stock of atmospheric carbon E_t as given. Fossil fuel owners also operate under perfect competition and maximize the present value of their profits taking the market price of fossil fuel p_t as given and internalizing the effect of depletion on future extraction costs. Producers of renewable energy also operate under perfect competition and maximize the present value of their profits taking the market price of renewable energy q_t and the stock of accumulated knowledge about using renewable energy B_t as given. The government balances its books and hands back revenue from carbon taxes minus cost of renewable subsidies as lump-sum transfers.

Proposition 2: *The decentralized market outcome satisfies (1)-(8), the Euler equation (9), the energy producers' optimality conditions*

$$(10a') \quad p_t \leq G(S_t) + \theta_t^S, \quad F_t \geq 0, \quad \text{c.s.},$$

$$(10b') \quad q_t \leq b(B_t), \quad R_t \geq 0, \quad \text{c.s.},$$

where the scarcity rent θ_t^S is (11), and the final good firms' optimality conditions $w_t = \tilde{Z}(T_t)H_L$,

$$r_{t+1} + \delta = \tilde{Z}(T_t)H_K, \quad p_t + \tau_t = q_t - v_t = \tilde{Z}(T_t)H_{F_t+R_t}.$$

Proof: Follows from the optimality conditions for firms, fossil fuel producers and households. \square

Proposition 3: *The globally efficient outcome presented in proposition 1 is replicated in the decentralized market economy if $\tau_t = \theta_t^E$ and $v_t = \theta_t^B$, $\forall t \geq 0$, where these follow from (12) and (13).*

Proof: Comparing conditions of proposition 1 with those of proposition 2. \square

The first best is thus sustained in the market economy if the specific carbon tax is set to the *SCC*, the specific subsidy on renewable subsidy is set to the *SBL*, and net revenue is rebated in lump sums. Based on the decentralized equilibrium defined in proposition 2 we define the following policy scenarios:

Definition 1: A decentralized market economy has the following outcomes.

- I. “First best” or globally efficient outcome with $\tau_t = \theta_t^E$ and $\nu_t = \theta_t^B, \forall t \geq 0$.
- II. “Laissez faire” or business as usual with $\tau_t = \nu_t = 0, \forall t \geq 0$.
- III. “Rule-based tax” with $\tau_t = \tau_t^{GHKT}$ from (13’) and no subsidy $\nu_t = 0, \forall t \geq 0$.
- IV. “Rule-based tax and optimal subsidy” with $\tau_t = \tau_t^{GHKT}$ from (13’) and $\nu_t = \theta_t^B, \forall t \geq 0$. \square

Policy scenario I leads to the social optimum and sets the specific carbon tax τ_t to the *SCC* (13) and the renewable subsidy ν_t to the *SBL* (12). Scenario II to “laissez faire” and corrects neither for climate nor for learning by doing, so $\tau_t = \nu_t = 0$. Scenarios III and IV test the proportionality rule for the carbon tax (13’) by implementing it in the decentralized economy with either no renewable subsidy, $\nu_t = 0$, in Scenario III or optimal renewable subsidy, $\nu_t = \theta_t^B$, as in Scenario IV.

Table 1: Comparing Integrated Assessment Models of Climate Change

	DICE	FUND	GHKT	GL	AABH	RP
Unit of time (in years)	10 or 5	1	10	10	5	1
Scarce fossil resources			✓	✓		✓
Stranded reserves						✓
Explicit energy market			✓	✓		✓
Endogenous technical progress in renewables		✓			✓	✓
Population growth	✓	✓				✓
Productivity growth	✓	✓		✓	✓	✓
Optimal growth	✓		✓	✓	✓	✓
Aggregate production function (incl. energy)	Leontief	N. A.	C.-D.	C.-D.	CES	CES
CES (not log) utility	✓	N. A.			✓	✓
100% depreciation			✓	✓		
Climate damages (at high temperature)	low	N. A.	very low	very low	catastrophic at 6°C	high
Type of damage	Prod.	Health	Prod.	Prod.&Utility	Utility	Prod.

Table 1 compares the key benchmark properties of our model with that of various prominent IAMs: the DICE-2013R model as discussed in the decadal version of Nordhaus (2008) and the semi-decadal version of Nordhaus (2014), the FUND model as presented in Tol (2002), the GHKT model as put forward in Golosov et al. (2014), the GL model as developed by Gerlagh and Liski (2016), and the AABH model as set out in Acemoglu et al. (2012), and finally the RP model as put forward in the present paper. While most models conduct sensitivity analysis for some of their assumption, table 1 reports the baseline version of each model.

4. Policy simulation and optimization

We simulate our model from 2010 to 2600 on a yearly time grid. The functional forms and calibration of the carbon cycle, temperature module and global warming damages have been discussed in section 2. The functional forms and benchmark parameter values for the economic part of our model put forward in section 3 are discussed in appendix C. We choose standard macroeconomic parameter values for capital depreciation and intertemporal preferences and adopt assumptions on near-term productivity and population growth from Nordhaus (2014). Current production possibilities imply low fossil fuel extraction costs and an initially high cost for renewable energy generation due to past biases in innovation towards fossil energy production. The calibration of our benchmark scenario reflect this cost structure. The reported simulations use the CES production function (with elasticity of substitution between energy and the capital-labour aggregate equal to $\vartheta = 0.5$). We refer the reader to appendix C for a detailed description of the calibration. Table 2 restates scenarios I-IV and the results for these are reported in figure 2 and tables 3 and 4.

Table 2: Scenarios

		<i>Carbon tax, τ_t</i>		
		<i>Optimal</i>	0	<i>Proportional to GDP (13')</i>
<i>Renewable subsidy, ν_t</i>	<i>Optimal</i>	(I) first-best 		(III) proportional carbon tax and optimal renewable subsidy 
	0		(II) "laissez faire" 	(IV) only proportional carbon tax 

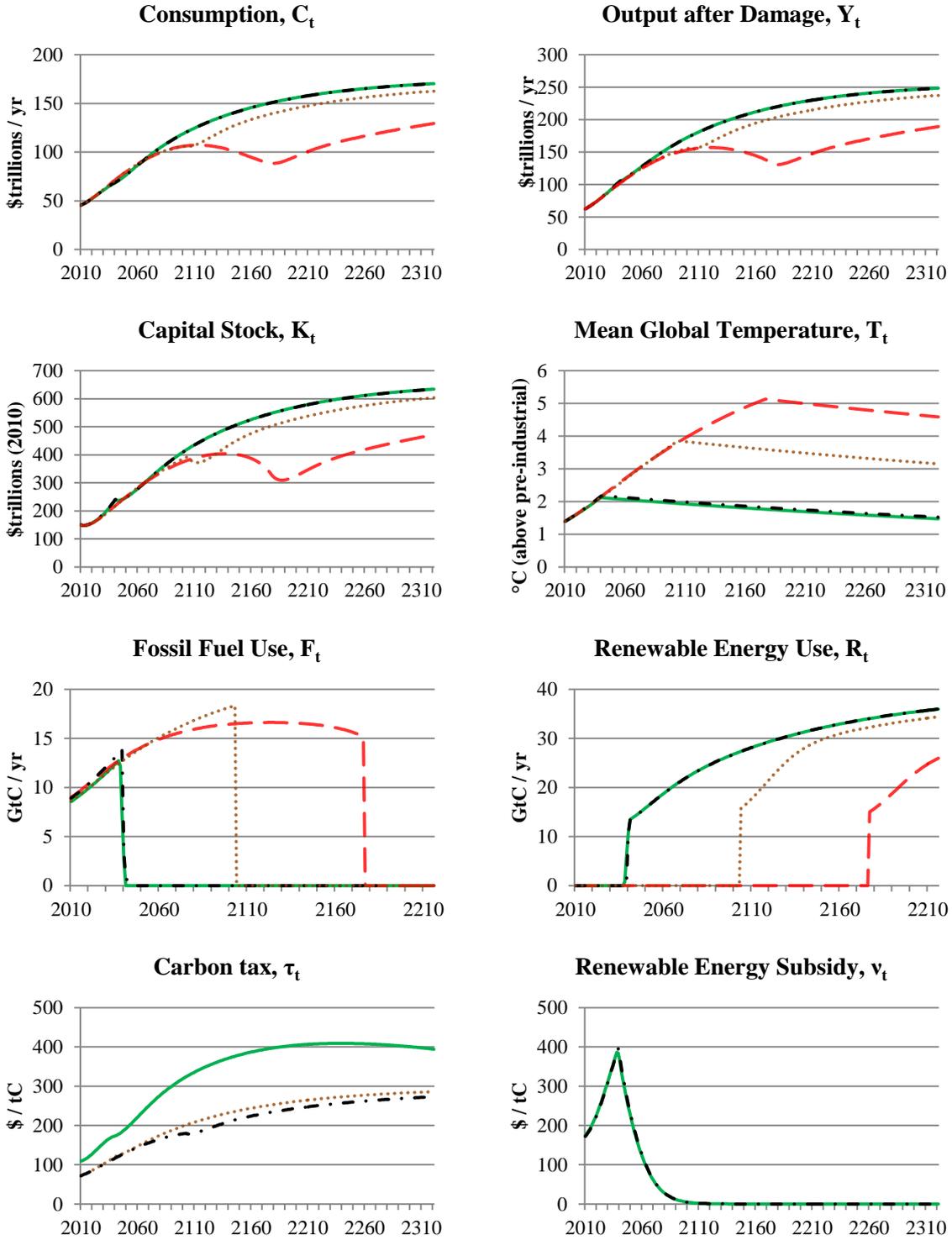
4.1. How to quickly de-carbonize and leave more fossil fuel untapped

As discussed in Rezai and van der Ploeg (2016b) and depicted in figure 2 and table 3, under the first-best scenario I (green, solid) population growth and technological progress allow a smooth transition of consumption, GDP and the capital stock to their steady state value. First-best policy imposes the social cost of carbon emission on the producers and consumers of carbon by levying a carbon tax starting at 109 \$/tC or 30 \$/tCO₂ and reaching a maximum of 174 \$/tC or 47 \$/tCO₂ at the end of the fossil fuel era. As discussed below, the tax incidence of the increased market price of fossil fuel is shared by producers and consumers, as oil producers partly lower their price in light of diminished sales prospects. First-best policy also includes subsidies for the consumption of renewable energy to foster and accelerate learning in this sector. Both policy instruments combine to allow a smooth and early energy transition with renewable energy uptake by 2035; the last drop of fossil energy used in 2040. Cumulative emissions are limited to 320 GtC which implies that the vast majority of the available 4000 GtC of fossil fuel reserves are abandoned. The carbon budget slightly exceeds the Note that in figure 2 the maximum of the social cost of carbon occurs only in 2240, long after the transition to renewable energy. This maximum is only hypothetical, however, as fossil fuel is already priced out of the market, given high extraction costs due to past extraction and low costs of the back-stop, even in the absence of a carbon tax. The important feature of the carbon tax is to depress fossil fuel use while the renewable alternative is not competitive. The renewable subsidy can help start the transition. In the first best scenario it starts at 174 \$/tC or 47\$/tCO₂ and quickly rises to 387 \$/tC or 105 \$/tCO₂ in 2038 and rapidly falls to zero as all learning has occurred by the end of this century.

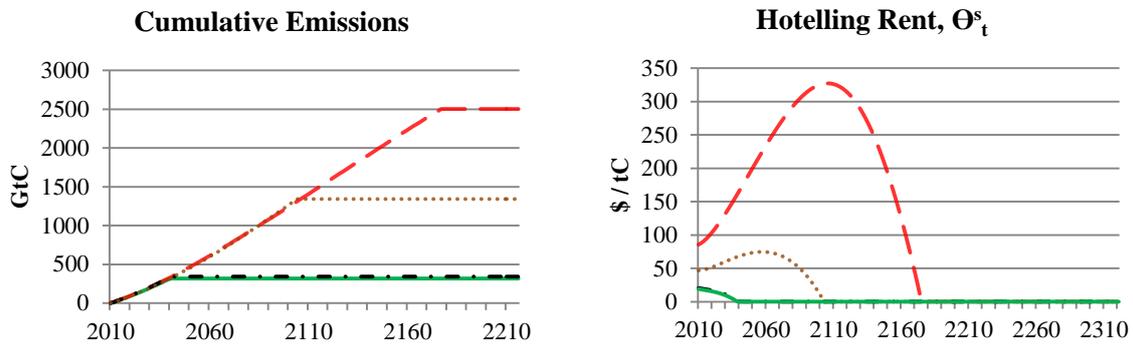
Table 3: Transition times and carbon budget

	Fossil fuel	Simultaneous use	Renewable	Carbon budget
I Globally efficient outcome	2010-2035	2035-2040	2040 –	320 GtC
II “Laissez faire“	2010-2175	x	2175 –	2,500 GtC
III Rule-based tax	2010–2105	x	2105 –	1,340 GtC
IV Rule-based tax, optimal subsidy	2010–2040	2040–2045	2045 –	340 GtC

Figure 2: Benchmark policy simulations



Key: laissez faire (— —), proportional carbon tax only (.....), first best (—), prop. tax and subsidy (— · — ·)

Figure 2: Benchmark policy simulations (continued)

Key: laissez faire (— —), proportional carbon tax only (.....), first best (—), prop. tax and subsidy (— · — ·)

300 GtC proposed by climate scientists so global mean temperature peaks at 2.1°C, slightly overshooting the 2° limit.⁹

Figure 2 and table 2 and 3 illustrate the optimal policy mix combines a renewable subsidy, which has the potential to bring renewable energy into use and facilitate endogenous technological progress early on, with a gradually rising carbon tax which persistently rises the market price of fossil fuel and ensures that fossil fuel becomes economically obsolete once and for all.

Table 4: Social costs of carbon, renewable subsidies, and welfare losses

	Welfare Loss (% of initial GDP)	Maximum carbon tax τ (\$/tC)	Maximum renewable subsidy (\$/tC)	max T (°C)
I Social optimum	0%	170 \$/tC	390 \$/tC	2.1 °C
II Laissez faire	-598%			5.1 °C
III Rule-based tax	-155%	200 \$/tC		3.2 °C
IV Rule-based tax, optimal subsidy	-1%	125 \$/tC	395 \$/tC	2.2 °C

Key: welfare losses expressed in percent of initial GDP; maximum effective carbon tax / renewable subsidy while fossil fuel / renewable energy is in use, rounded to the next multiple of 5; peak warming in °C.

Economic and environmental prospects are much depressed in the “laissez faire” scenario II (red, dashed) where carbon tax and renewable subsidy are fixed at zero and none of the externalities is internalized. In the absence of any tax on carbon, the market price for fossil fuel is lower and as a

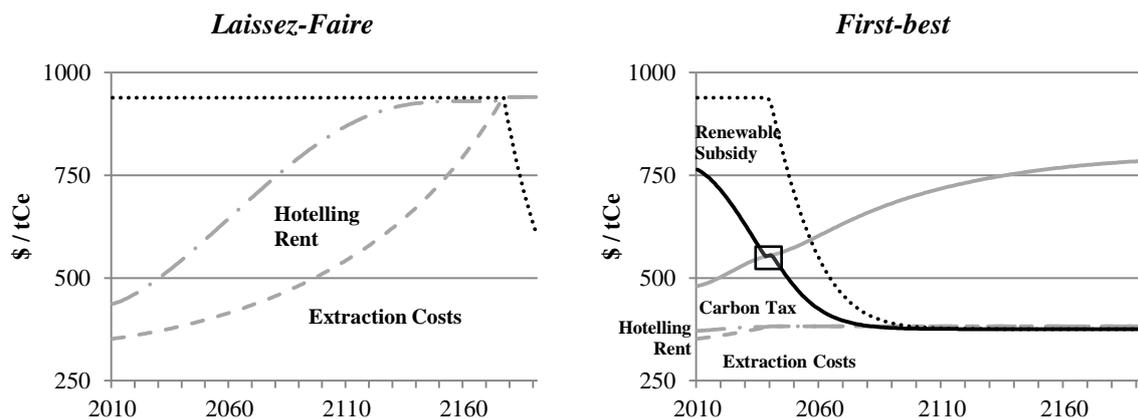
⁹ Recent estimates by the IPCC (2014) state that cumulative emissions have to be limited to 790 GtC (with an uncertainty range of 700-860 GtC) if global warming is to remain below 2°C. By 2011 520 GtC had been emitted, giving a remaining carbon budget of only 270 GtC.

result cumulative use of more fossil fuel increases to 2500 GtC. Peak warming increases to 5.1°C matching recent IPCC and IEA estimates for business as usual. Given that no subsidy is available to the users of renewable energy, the transition to renewable energy is severely delayed to 2175. Table 4 indicates that these inefficiencies (see section 5.2 for more detail) cause a substantial welfare loss relative to the first best of almost 6 times today's world GDP.¹⁰

4.2. Time paths for the market price of fossil fuel and the renewable

Market prices for both types of fossil and renewable energy are depicted in figure 3. Grey lines depict components of the market price of fossil energy, black lines components of the market price of renewable energy. Equation (10a) shows that the market price of fossil energy consists of the sum of marginal extraction cost and the Hotelling rent plus any carbon tax. The market price of renewable energy is set to its production cost minus any learning subsidy (see equation (10b)). Initially prices are rising in all scenarios and only on these rising sections are fossil fuels used. A renewable subsidy enables a period of simultaneous use (indicated by boxed sections) and smooths the transition to the carbon-free era.

Figure 3: The market price of fossil and renewable energy (\$/tC)



Key: Fossil energy (grey) and its cost components: Extraction costs (dashed), Hotelling (dot-dashed) and carbon tax (solid). Renewable energy (black) and its cost components: Generation costs (dotted) and renewable subsidy (solid).

¹⁰ Stern (2007) expresses cost of inaction in annuity terms; Nordhaus (2008) in terms of today's consumption. We calculate the difference in the total welfare, evaluated at initial prices, and express it as a share of initial GDP. Our welfare measure equals the loss due to inaction as a share of initial GDP.

The dynamics of the market prices of energy are best understood by starting with the “laissez faire” scenario II (left panel) where the market price of fossil energy comprises only extraction costs and the forward-looking Hotelling rent. No carbon tax is imposed. The market cost of renewable energy comprises only the costs of generation as the subsidy also absent. Our calibration assumes that initially extraction costs of fossil fuel are lower than the generation cost of renewable energy, otherwise fossil fuel would never be used and global warming always limited to current levels. Consequently, fossil fuel is in use initially and, during this use, its price rises along with increasing extraction costs and the increasing Hotelling rent. Energy procurement switches from fossil to renewable energy once the market price of fossil energy exceeds that of renewable energy. However, as this switch point nears, the scarcity rent of keeping fossil fuel under the ground for later use falls and the Hotelling rent falls. The cost of producing renewable energy falls quickly and approaches its lower floor due to learning by doing. This scenario is clearly sub-optimal, since renewable energy is too expensive for much too long relative to fossil fuel.

To implement the first best, scenario I (right panel), a carbon tax and a renewable subsidy need to be implemented. Adding the *SCC* to the price of fossil energy increases it beyond the “laissez faire” level and its rate of growth significantly.¹¹ The subsidy lowers the market price of renewable energy initially and brings forward the switch points to simultaneous use (indicated by boxed sections) and to full renewable energy use. Once renewable energy is brought into use, its price stops falling and rises with that of fossil energy during the period of simultaneous use. The price resumes its decline only as the transition to carbon-free energy is complete.

The optimal policy mix locks up nearly 90% of the fossil resources burnt under “laissez-faire”. This significant reduction in the carbon budget lowers the scarcity rent component of the fossil fuel price

¹¹ If the *SCC* is taken account of, the price of fossil energy continues to rise even as no fossil energy is produced (see the grey solid lines). The *SCC* rises initially, since decay is limited and consumption is increasing. This yields a smaller marginal utility of consumption and thus a higher *SCC* (see (13)). However, after some point of time decay of atmospheric carbon dominates the decrease in marginal utility of consumption and the *SCC* – and with it the market price of fossil energy – start to fall. If the fall is sufficiently large, fossil energy becomes competitive again. As figure 3 indicates, the time horizon that we consider is too short and the learning-based reduction in renewable costs too large to make such re-switching optimal. A carbon tax proportional to GDP or consumption does not allow for re-switching.

as soon as the policy is implemented (or anticipated). This reduction in the market price could lead to a positive demand response, pushing short-run fossil fuel use beyond its “laissez-faire” level. The optimal carbon tax, however, is sufficiently large to compensate the fall in the Hotelling rent and increase the optimal fossil fuel price above “laissez faire”, precluding the Green Paradox effect.

4.3. “Laissez faire” overinvests in dirty and underinvests in clean capital

How does “laissez faire”, scenario II, differ from the first best, scenario I? Initially, output, consumption and capital accumulation take place at very similar levels. However, the climate damage and the learning externality drastically impede the accumulation of capital as climate change worsens. The slowing of accumulation and short period of decumulation of capital is clearly visible in the “laissez faire” paths in figure 2. The corresponding disruption in consumption translates into a substantial welfare loss of about 6 times initial GDP. “Laissez faire” leads to inefficient allocation of resources, since the economy overvalues the returns to conventional capital accumulation and undervalues investments in green energy sources. Failure to cooperate induces excessive fossil fuel extraction and capital accumulation leading to high global warming damages.

This inefficient use of resources lowers welfare because it keeps consumption low in early periods of the program to allow for capital accumulation and consumption low in future periods due to high global warming damages. Damages under “laissez faire” are large enough to lower factor returns sufficiently to induce decumulation of capital and a fall in consumption. From 2140-2190 the capital stock falls by 25% from a peak of \$410 trillion to a trough \$310 trillion, consumption drops by 17% from a peak of \$107 trillion to a trough of \$88 trillion. This climate crisis is ended as extraction costs rise above the cost of renewable energy. At this point all of energy use is sourced from renewables and the climate and economy recover from previous excessive use of fossil fuels. The inefficiency of this scenario is also reflected in the high share of GDP expended on energy which rises from around 7% in 2010 to 10% in 2110, compared to 6.5% and 6%, respectively, under first best. Once the economy switches to the renewable and stocks of atmospheric carbon recede, the return to capital and the interest rate increase. This leads to a resumption of global growth. As only a fraction of carbon

dissipates, welfare remains below the social optimum even in steady state. Close inspection demonstrates that consumption in the social optimum is the lowest for some initial periods. This implies that internalization of the climate externality has to be phased in more slowly which is the case with higher intergenerational inequality aversion (lower η).¹²

4.4. The optimal carbon tax versus aggregate consumption or world GDP

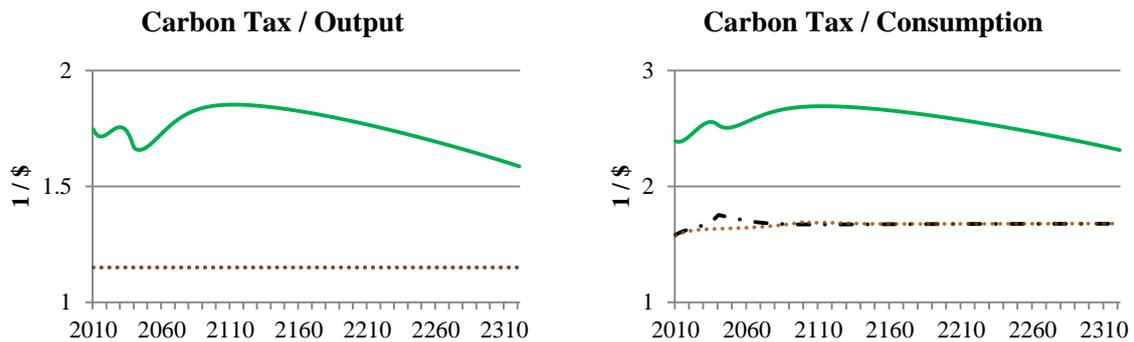
To examine whether the linear formula for the optimal carbon tax (13') put forward by Golosov et al. (2014) holds up in our more general and detailed integrated assessment model of Ramsey growth and climate change, figure 4 plots the ratio of the optimal carbon tax to both world GDP and aggregate consumption. We immediately see that the optimal carbon tax (green line) is not well described by a constant proportion of world GDP or aggregate consumption. The general pattern is that during the initial phases of fossil fuel use the social cost of carbon rises as a proportion of world GDP as more carbon emissions raise marginal damages of global warming and during the carbon-free phases the *SCC* falls as a proportion of world GDP as a significant part of the stock of carbon in the atmosphere is gradually returned to the surface of the oceans and the earth. The required rise and fall of the *SCC* is substantial and non-linear due to rapid rises in productivity and population.

The dotted lines in figure 2 provide further details on the time profiles of key variables under scenarios III and IV which set climate policy according to (13'). This proportional tax internalizes part of the *SCC* and improves welfare relative to no policy. A carbon tax proportional to GDP increases the price of fossil fuel and encourages a transition to renewable energy earlier on. However, the proportional tax starts at a too low level and rises too slowly (and even falls in scenario IV) compared to first best. In the case where the only instrument is a proportional carbon tax, the transition is delayed until the end of the 21st century, leading to excess fossil fuel of 1 TtC and an increase in peak temperature increase of 1 °C. The welfare loss of such an inefficient policy is equivalent to 155% of current GDP. Once the proportional carbon tax is supplemented with an

¹² Alternatively, the social optimum might have to be abandoned and instead one has to devise second-best intergenerational compensation schemes for policy to increase consumption in all periods. Karp and Rezai (2014) and Karp (2016) discuss the identification of generations in Ramsey models, intergenerational discounting, and the various effects of environmental policy in an OLG setting.

optimal renewable subsidy (scenario IV), however, the social optimum is matched quite closely and the welfare loss is less than 1% of initial GDP. Fossil fuels are used slightly longer and the cumulative budget slightly higher than under first-best. The carbon tax necessary to achieve this, however, is lower with a maximum of 125 \$/tC.

Figure 4: The carbon tax as fraction of aggregate world GDP and consumption



Key: first best (—), proportional carbon tax only (.....), proportional tax and optimal subsidy (- · - ·)

Still, in these sub-optimal scenarios a Green Paradox effect emerges as depicted in the panel on fossil fuel use in figure 2. Under scenario IV, and more so scenario III, the carbon tax is set too low to prevent fossil fuel use from rising above the “laissez-faire” level. The anticipation of a future obsolescence of part of the “laissez-faire” carbon budget, reduces the scarcity rent component of the market price for fossil fuel. If carbon is not taxed sufficiently high to offset this price decrease, demand for fossil fuel increases and climate change is temporarily worse than if no policy was implemented at all.

5. Robustness of the optimal climate policy

Figure 5 shows the sensitivity of the social optimum to some other key parameters and to the proportional tax rule proposed by Golosov et al. (2014) and table 5 gives the corresponding transition times and carbon budgets. Our general finding of an inverted-U time profile for the social cost of carbon is robust. The exact timing and magnitude depends on specific parameters. Introducing a time

lag in the temperature response to carbon increases as in Gerlagh and Liski (2016), lowers the *SCC* and slows the transition to carbon-free sources of energy.

Table 5: Transition times and carbon budget (extended)

<i>Scenario</i>	<i>Fossil fuel</i>	<i>Simultaneous use</i>	<i>Renewable Only</i>	<i>Carbon used</i>
<i>Baseline</i>	2010–2035	2035–2040	2040 –	320 GtC
<i>Temperature lag</i>	2010–2055	2055–2060	2060 –	600 GtC
<i>Nordhaus damage</i>	2010–2105	2105–2110	2110 –	845 GtC
<i>$\vartheta = 0$ (Leontief)</i>	2010–2035	2035–2040	2040 –	325 GtC

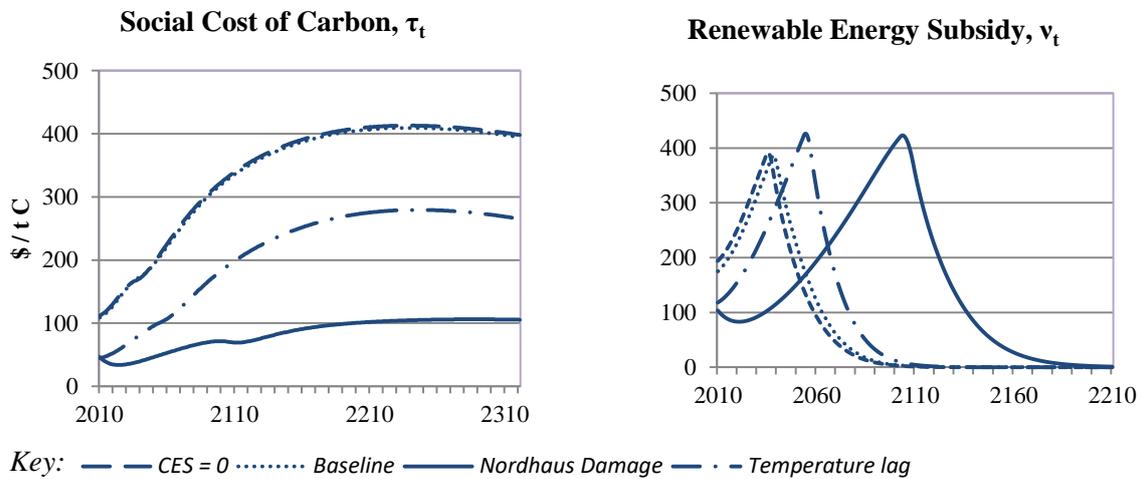
Climate policy is also less ambitious if damages are lower and less convex than in Nordhaus (2008), which implies that more carbon is burnt and less fossil fuel is left abandoned in the crust of the earth.

Lowering the substitutability between the capital-labour aggregate and energy in the CES production function ($\vartheta = 0$) leads to higher (fossil and renewable) energy use, which leads to more atmospheric carbon and a higher *SCC* as the destructive and costly energy production cannot be substituted.

Climate policy is less ambitious with a proportional carbon tax set according to (13') where the presence of an optimal subsidy reduces the amount of carbon burnt to mimic the first best closely.

The subsidy for renewable energy use and the optimal carbon tax are complements. The subsidy policy increases in cases in which the carbon tax (and with it the market price of fossil energy) falls:

Figure 5: Sensitivity analysis for the time paths of the optimal carbon tax



lower damages and less responsive temperature. We also conducted further robustness checks. A much lower social rate of discount ($\rho = 0$) leads to a much more ambitious climate policy with a much higher social cost of carbon, earlier phasing in of renewables and more fossil fuel left in situ. A higher elasticity of intertemporal substitution implies less intergenerational inequality aversion. For example, with zero intergenerational inequality aversion ($\eta \rightarrow \infty$ instead of $\eta = 1/2$), this implies in a growing economy that the carbon tax hurts earlier generations much more than later generations. Policy makers in the globally optimal outcome are relatively more concerned with fighting global warming than with avoiding big differences in consumption of different generations. Climate policy is also more aggressive with a higher equilibrium climate sensitivity ($\omega = 6$). Starting with half the initial capital stock ($K_0 = 100$) or increasing population and productivity growth ($A(\infty) = 6$ and $L(\infty) = 10.6$) hardly affect the *SCC*. The increase in the *SCC* under lower discounting and intergenerational inequality aversion and a more sensitive climate allows for less aggressive renewable subsidies because of instrument complementarity.

6. Conclusion

Climate policy can be derived from an integrated assessment model, which contains an economic block describing Ramsey growth and convergence and a climate block describing heat exchange, the carbon cycle and temperature. In our integrated assessment model we have highlighted the importance of two externalities: the climate externality which is a truly global one and a learning-by-doing externalities arising from the free (and unpatented) flow of knowledge in using renewables. The globally efficient policy response is to price carbon and subsidize renewable use. Such a two-handed policy is able to limit fossil fuel use and carbon emissions and at the same time promote substitution away from fossil fuel towards renewable energies, leaving more hydrocarbon reserves untapped in the ground, and bringing forward the end of the fossil-fuel era and the beginning of the carbon-free era.

Our benchmark calibration of our integrated assessment model suggests a combination of global carbon tax which should rise from about 110 \$/tC (or 30 \$/tCO₂) now to 174 \$/tC (47 \$/tCO₂) and a renewable subsidy which should rise from 174 \$/tC (47 \$/tCO₂) now to 380 \$/tC (104 \$/tCO₂) in 2040 and then falls quickly to zero. Our analysis thus suggests an optimal policy mix, which consists of an aggressive subsidy making renewable energy competitive early on and a gradually rising carbon tax eventually pricing fossil energy out of the market. The total amount of carbon burnt in this efficient policy scenario is 320 GtC which is much less than the 2,500 GtC under “laissez faire”. Consequently, the social optimum manages to limit the maximum temperature to 2.1 °C instead of the much higher 5.1 °C under “laissez faire”. Remarkably, the welfare loss without policy under “laissez faire” is almost 6 times today’s world GDP.

We have also performed some robustness exercises, which suggest that climate policy becomes less ambitious as the social cost of carbon is lower, fossil fuel is abandoned less quickly and more carbon is used in total if the discount rate is higher, intergenerational inequality aversion is stronger, and the equilibrium climate sensitivity is lower. Furthermore, less substitutability between energy and the capital-labour aggregate leads to less energy use especially in capital-scarce economies and less global warming. Hence, the required global carbon tax or permit price is lower.

A recent interagency working group (2010) suggests that US institutions use a social cost of carbon of initially \$80/tC (22 \$/tCO₂) rising to 165 \$/tC (45 \$/tCO₂) in 2050 in project appraisal based on a discount rate of 3% per annum. A discount rate of 2.5% per annum would imply an initial social cost of carbon of 129 \$/tC (35 \$/tCO₂) rising to 238 \$/tC (65 \$/tCO₂) in 2050, which is in line with our estimates.¹³ These figures are typically based on existing integrated assessment models which are often very elaborate and in only one of which consumers maximize utility as in the Ramsey model. Golosov et al. (2014) offer a tractable fully consistent general equilibrium model of climate change and Ramsey growth, but ignore the temperature lag in the carbon cycle and employ unrealistically

¹³ Such models of climate change yield estimates of the social cost of carbon starting from 5 to 35 \$ per ton of carbon in 2010 and rising to \$16 to \$50 per ton in 2050 (e.g., the DICE, PAGE and FUND models of Nordhaus (2008), Alberth and Hope (2007) and Tol (2002), respectively), but the Stern Review obtains much higher estimates of the social cost of carbon with a much lower discount rate (Stern, 2007).

low damages at higher temperatures and need to make some very bold assumptions to ensure that both aggregate consumption and the carbon tax are a fixed proportion of world GDP. Much too much carbon is burnt and the welfare losses are substantial with this proportional carbon tax, especially if there is no policy in place for encouraging learning by doing in renewable production. In the absence of an optimal renewable subsidy, the proportional tax starts at a too low level and rises too slowly and causes extraction rates to rise, as predicted by the Green Paradox.

Our model of Ramsey growth and climate change has more realistic damages and finds that the optimal carbon tax is a hump-shaped function of world GDP. Our analysis also pays careful attention to how fast and how much fossil fuel should be abandoned and how quickly and how much renewables should be phased in. Our results suggest a ‘third way’ in climate policy which consists of a quick and aggressive path of upfront renewable subsidies to stimulate use of renewables and enjoy the fruits of learning by doing, combining the logic of direct technical change and kick-starting green innovation developed in Acemoglu et al. (2012) and Mattauch et al. (2012) with a gradually rising carbon tax as advocated in most integrated assessment studies including Nordhaus (2008, 2014), Stern (2007) and Dietz and Stern (2015).

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Appendix A: Proof of proposition 1

The adjoined Lagrangian for our model reads:

$$\begin{aligned} L \equiv & \sum_{t=0}^{\infty} (1+\rho)^{-t} \left[L_t U_t(C_t / L_t) - \mu_t^S (S_{t+1} - S_t + F_t) - \mu_t^B (B_{t+1} - B_t - R_t) \right] \\ & + \sum_{t=0}^{\infty} (1+\rho)^{-t} \left[\mu_t^{PE} (E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE} \left\{ E_{t+1}^T - (1-\varphi) E_t^T - \varphi_0 (1-\varphi_L) F_t \right\} \right] \\ & - \sum_{t=0}^{\infty} (1+\rho)^{-t} \lambda_t \left[K_{t+1} - (1-\delta) K_t - Z(E_t^P + E_t^T) H(K_t, L_t, F_t + R_t) + G(S_t) F_t + b(B_t) R_t + C_t \right], \end{aligned}$$

where μ_t^S denotes the shadow value of *in-situ* fossil fuel, μ_t^B the shadow value of learning by doing, μ_t^{PE} and μ_t^{TE} the shadow disvalue of the permanent and transient stocks of atmospheric carbon, and λ_t the shadow value of manmade capital. Necessary conditions for a social optimum are:

$$(A2a) \quad U'(C_t / L_t) = (C_t / L_t)^{-1/\eta} = \lambda_t,$$

$$(A2b) \quad Z_t H_{F_t+R_t} \leq G(S_t) + \left[\mu_t^S + \varphi_L \mu_t^{PE} + \varphi_0 (1-\varphi_L) \mu_t^{TE} \right] / \lambda_t, \quad F_t \geq 0, \quad \text{c.s.},$$

$$(A2c) \quad Z_t H_{F_t+R_t} \leq b(B_t) - \mu_t^B / \lambda_t, \quad R_t \geq 0, \quad \text{c.s.},$$

$$(A2d) \quad (1-\delta + Z_{t+1} H_{K_{t+1}}) \lambda_{t+1} = (1+\rho) \lambda_t,$$

$$(A2e) \quad \mu_{t+1}^S = (1+\rho) \mu_t^S + G'(S_{t+1}) F_{t+1} \lambda_{t+1},$$

$$(A2f) \quad \mu_{t+1}^B = (1+\rho) \mu_t^B - b'(B_{t+1}) R_{t+1} \lambda_{t+1},$$

$$(A2g) \quad \mu_{t+1}^{PE} = (1+\rho) \mu_t^{PE} + Z'(E_{t+1}^P + E_{t+1}^T) H_{t+1} \lambda_{t+1},$$

$$(A2h) \quad (1-\varphi) \mu_{t+1}^{TE} = (1+\rho) \mu_t^{TE} + Z'(E_{t+1}^P + E_{t+1}^T) H_{t+1} \lambda_{t+1}.$$

Equations (A2a) and (A2d) yield the Euler equation (9). The Kuhn-Tucker conditions (A2b) and

(A2c) give (10a) and (10b) after defining $\theta_t^S \equiv \mu_t^S / \lambda_t$, $\theta_t^B \equiv \mu_t^B / \lambda_t$ and

$\theta_t^E \equiv \left[\varphi_L \mu_t^{PE} + \varphi_0 (1-\varphi_L) \mu_t^{TE} \right] / \lambda_t$ in final good units. (A2e) and (A2d) yield the Hotelling rule

$$(A3) \quad \theta_{t+1}^S = (1+r_{t+1}) \theta_t^S + G'(S_{t+1}) F_{t+1}.$$

Forward summation over time of (A3) and using the transversality conditions gives (11). Equations

(A2f) and (A2d) yield

$$(A4) \quad \theta_{t+1}^B = (1+r_{t+1}) \theta_t^B + b'(B_{t+1}) R_{t+1}.$$

Forward summation over time of (A4) and using the transversality conditions yields (12). Defining

$\theta_t^{PE} \equiv \mu_t^{PE} / \lambda_t$ and $\theta_t^{TE} \equiv \mu_t^{TE} / \lambda_t$ in final good units we use (A2g), (A2h) and (A2a) to get:

$$(A5a) \quad \theta_{t+1}^{PE} = (1+r_{t+1}) \theta_t^{PE} + Z'(E_{t+1}^P + E_{t+1}^T) H_{t+1},$$

$$(A5b) \quad (1 - \varphi)\theta_{t+1}^{TE} = (1 + r_{t+1})\theta_t^{TE} + Z'(E_{t+1}^P + E_{t+1}^T)H_{t+1}.$$

Solving (A5a) and (A5b), using the transversality conditions and $\theta_t^E \equiv \varphi_L \theta_t^{PE} + \varphi_0(1 - \varphi_L)\theta_t^{TE}$, we obtain equation (13).

Appendix B: A temperature lag

Suppose a lag between atmospheric stock of carbon and temperature:

$$(3') \quad T_{t+1} = (1 - \varphi_T)T_t + \varphi_T \omega \ln\left(\frac{E_t^P + E_t^T}{581}\right),$$

where φ_T is the speed at which a higher stock of atmospheric carbon gets translated into a higher global mean temperature. Since it takes on average about 70 years, we set $\varphi_T = 1.70$. With this additional equation the Lagrangian function becomes:

$$\begin{aligned} L \equiv & \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[L_t U_t (C_t / L_t) - \mu_t^S (S_{t+1} - S_t + F_t) - \mu_t^B (B_{t+1} - B_t - R_t) \right] \\ & - \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[\mu_t^T \left\{ T_{t+1} - (1 - \varphi_T)T_t - \varphi_T \omega \ln\left(\frac{E_t^P + E_t^T}{581}\right) \right\} \right] \\ & + \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[\mu_t^{PE} (E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE} \left\{ E_{t+1}^T - (1 - \varphi)E_t^T - \varphi_0(1 - \varphi_L)F_t \right\} \right] \\ & - \sum_{t=0}^{\infty} (1 + \rho)^{-t} \lambda_t \left[K_{t+1} - (1 - \delta)K_t - Z(T_t)H(K_t, L_t, F_t + R_t) + G(S_t)F_t + b(B_t)R_t + C_t \right]. \end{aligned}$$

where μ_t^T is the marginal disvalue of global warming. Necessary conditions for a social optimum are (11a)-(11f) as before and:

$$(11g') \quad \mu_{t+1}^{PE} = (1 + \rho)\mu_t^{PE} + \varphi_T \omega (E_{t+1}^P + E_{t+1}^T)^{-1} \mu_{t+1}^T,$$

$$(11h') \quad (1 - \varphi)\mu_{t+1}^{TE} = (1 + \rho)\mu_t^{TE} + \varphi_T \omega (E_{t+1}^P + E_{t+1}^T)^{-1} \mu_{t+1}^T, \text{ and}$$

$$(11i') \quad (1 - \varphi_T)\mu_{t+1}^T = (1 + \rho)\mu_t^T + Z'(T_{t+1})H_{t+1}\lambda_{t+1}.$$

We now get as before (12)-(15). The dynamics of the permanent and transient components of the SCC become:

$$(16') \quad \begin{aligned} \theta_{t+1}^{PE} &= (1 + r_{t+1})\theta_t^{PE} + \omega(E_{t+1}^P + E_{t+1}^T)^{-1} \theta_{t+1}^T, \\ (1 - \varphi)\theta_{t+1}^{TE} &= (1 + r_{t+1})\theta_t^{TE} + \omega(E_{t+1}^P + E_{t+1}^T)^{-1} \theta_{t+1}^T, \end{aligned}$$

where $\theta_t^T \equiv \mu_t^T / \lambda_t$. Solving (16') and using the transversality conditions yields the SCC as the present discounted value of all future marginal damages from global warming:

$$(A6) \quad \theta_t^C = \sum_{s=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0(1 - \varphi_L)(1 - \varphi)^s \right\} \Delta_{t+s} \omega(E_{t+s+1}^P + E_{t+s+1}^T)^{-1} \theta_{t+s+1}^T \right].$$

Solving (11i) together with (11d) yields:

$$(A7) \quad (1 - \varphi_T)\theta_{t+1}^T = (1 + r_{t+1})\theta_t^T + Z'(T_{t+1})H_{t+1}.$$

Using the transversality conditions, this equation yields the marginal cost of global warming:

$$(A8) \quad \theta_t^T = -\sum_{s=0}^{\infty} \left[(1 - \varphi_T)^s \Delta_{t+s} Z'(T_{t+s+1}) H_{t+s+1} \right].$$

Upon substituting (A8) into (A6) we get the SCC.

Relationship to Golosov et al. (2014)

Golosov et al. (2014) assume that there is no temperature lag. Following Gerlagh and Liski (2016) we do allow for such a lag. Under this set of assumptions, (A8) shows that the marginal cost of global warming at the social optimum is proportional to global GDP (cf. equation (6)):

$$(A8') \quad \theta_t^T = 2.379 \times 10^{-5} \left(\frac{1 + \rho}{\rho + \varphi_T} \right) Z(T_t) H(K_t, L_t, F_t + R_t).$$

Upon substitution of (A8') into (A6), we get:

$$\theta_t^C = 2.379 \times 10^{-5} \left(\frac{1 + \rho}{\rho + \varphi_T} \right) \sum_{s=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0(1 - \varphi_L)(1 - \varphi)^s \right\} \Delta_{t+s} \omega(E_{t+s+1}^P + E_{t+s+1}^T)^{-1} Z(T_{t+s+1}) H_{t+s+1} \right].$$
 This

expression does not simplify to a simple expression depending only on current global GDP. However, if we use a dynamic reduced-form temperature module with a distributed lag between carbon stock and damages,

$$(A9) \quad Z_t = Z(E_t), \quad E_{t+1} = (1 - \varphi_T)E_t + \varphi_T(E_{t+1}^P + E_{t+1}^T),$$

we have the Lagrangian function

$$\begin{aligned} L \equiv & \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[L_t U_t(C_t / L_t) - \mu_t^S (S_{t+1} - S_t + F_t) - \mu_t^B (B_{t+1} - B_t - R_t) \right] \\ & - \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[\mu_t^E \left\{ E_{t+1} - (1 - \varphi_T)E_t - \varphi_T(E_{t+1}^P + E_{t+1}^T) \right\} \right] \\ & + \sum_{t=0}^{\infty} (1 + \rho)^{-t} \left[\mu_t^{PE} (E_{t+1}^P - E_t^P - \varphi_L F_t) + \mu_t^{TE} \left\{ E_{t+1}^T - (1 - \varphi)E_t^T - \varphi_0(1 - \varphi_L)F_t \right\} \right] \\ & - \sum_{t=0}^{\infty} (1 + \rho)^{-t} \lambda_t \left[K_{t+1} - (1 - \delta)K_t - Z(E_t)H(K_t, L_t, F_t + R_t) + G(S_t)F_t + b(B_t)R_t + C_t \right]. \end{aligned}$$

and thus we get:

$$(11g'') \quad \mu_{t+1}^{PE} = (1 + \rho)(\mu_t^{PE} + \varphi_T \mu_t^E),$$

$$(11h'') \quad (1 - \varphi)\mu_{t+1}^{TE} = (1 + \rho)(\mu_t^{TE} + \varphi_T \mu_t^E),$$

$$(11i') \quad (1 - \varphi_T)\mu_{t+1}^E = (1 + \rho)\mu_t^E + Z'(E_{t+1})H_{t+1}\lambda_{t+1}.$$

It thus follows that

$$(A10) \quad \theta_t^C = \sum_{s=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^s \right\} \Delta_{t+s} \theta_{t+s+1}^E \right],$$

$$(A11) \quad \theta_t^E = - \sum_{s=0}^{\infty} \left[(1-\varphi_T)^s \Delta_{t+s} Z'(E_{t+s+1}) H_{t+s+1} \right].$$

Under the assumptions made by Golosov et al. (2014), we get

$$(A11') \quad \theta_t^E = 2.379 \times 10^{-5} \left(\frac{1+\rho}{\rho+\varphi_T} \right) Z(E_t) H_t$$

and thus the following simple expression for the *SCC*:

$$(A10') \quad \theta_t^C = 2.379 \times 10^{-5} \left(\frac{1+\rho}{\rho+\varphi_T} \right) \left[\left(\frac{1+\rho}{\rho} \right) \varphi_L + \left(\frac{1+\rho}{\rho+\varphi} \right) \varphi_0(1-\varphi_L) \right] Z(E_t) H(K_t, L_t, F_t + R_t).$$

A lag between atmospheric carbon and damages ($\varphi_T > 0$) thus pushes down the *SCC* so our estimates of the optimal global carbon tax will be biased upwards.

Appendix C: Functional forms and calibration

Preferences

As stated in (8), we suppose a CES utility function. We set the elasticity of intertemporal substitution to $\eta = 1/2$ and thus intergenerational inequality aversion to 2. The rate of pure time preference ρ is 1% per year.

Cost of energy

We employ an extraction technology of the form $G(S) = \gamma_1 (S_0 / S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 6%-7% depending on the policy scenario. This translates to fossil production costs of \$350/tC (\$35/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon, giving $G(S_0) = \gamma_1 = 0.35$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume only a doubling of production costs if a total 2000 GtC is extracted. With

$s_0 = 4000$ GtC,¹ this gives $\gamma_2 = 1$.² This implies that we assume very low extraction costs and a high initial stock of reserves which biases our findings toward using more fossil fuel longer.

Initial capital stock and depreciation rate

The initial capital stock is set to 200 (US\$ trillion), which is taken from Rezai et al. (2012). We set the depreciation rate δ to be 0.1 per year.

Global production and global warming damages

Output before damages is $H_t = \left[(1 - \beta) \left(AK_t^\alpha (A_t^L L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta \left(\frac{F_t + R_t}{\sigma} \right)^{1-1/\vartheta} \right]^{1/(1-1/\vartheta)}$,

$\vartheta \geq 0$, $0 < \alpha < 1$ and $0 < \beta < 1$. This is a constant-returns-to scale CES production function in energy and a capital-labour composite with θ the elasticity of substitution and β the share the parameter for energy. The capital-labour composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital, A total factor productivity and A_t^L the efficiency of labour. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same level of global warming damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as

$H_t = H_0 \left[(1 - \beta) \left(\frac{AK_t^\alpha (A_t^L L_t)^{1-\alpha}}{H_0} \right)^{1-1/\vartheta} + \beta \left(\frac{F_t + R_t}{\sigma H_0} \right)^{1-1/\vartheta} \right]^{1/(1-1/\vartheta)}$. We set the share of capital to $\alpha =$

0.35, the energy share parameter to $\beta = 0.06$, and the elasticity of factor substitution $\vartheta = 0.5$ (as in the WITCH 2008 model) which we will refer to as the CES run. Initial world GDP in 2010 is \$63 trillion. Given $A_t^L = 1$, we calibrate $A = 3.78$ to yield initial output under “laissez faire”. The energy intensity of output σ is calibrated to an initial energy use of 9 GtC under “laissez faire”, $\sigma = 0.15$.

Population growth and labour-augmenting technical progress

Population in 2010 (L_1) is 6.5 billion people. Following Nordhaus (2008) and UN projections population growth is given by $L_t = 8.6 - 2.1e^{-0.35t}$. Population growth starts at 1% per year and falls below 0.1% percent within six decades and flattens out at 8.6 billion people. In the sensitivity analysis in section 4.3 we assume faster growth and a higher plateau to reflect more recent forecasts. Without loss of generality the efficiency of labour $A_t^L = 3 - 2e^{-0.2t}$ starts out with $A_1^L = 1$ and an initial

¹ Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2-scenario that 7000 GtCO₂ (with 3.66 tCO₂ per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

² Since $G(2000) / G(4000) = (4000 / 2000)^{\gamma_2} = 2^{\gamma_2}$ and $2^{\gamma_2} = 2$.

Harrod-neutral rate of technical progress of 2% per year. The efficiency of labour stabilizes at 3 times its current level.

Cost of the renewable and learning by doing

We model learning by doing with initial cost reductions and a lower limit for the cost of the renewable, i.e., $b(B_t) = \chi_1 + \chi_2 e^{-\chi_3 B_t}$, $\chi_1, \chi_2, \chi_3 \geq 0$. This formulation differs from the usual power law definition of learning curves (Manne and Richels, 2004) but allows us to better calibrate initial learning rates (which can reach infinity for power law) and specify a lower limit for unit cost. We calibrate unit cost of renewable energy to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\rho \rightarrow 0$, energy demand is $\sigma Z_t H_t$. The cost of generating all energy carbon free is $\sigma Z_t H_t b / Z_t H_t = \sigma b_t$. Nordhaus (2008) states that it costs 5.6% of GDP to decarbonise today's economy in a model of back-stop mitigation. We increase this cost to 7-8% of GDP in order to reach the learning potential given below. To calibrate the production cost of renewable energy, this cost estimate needs to be combined with the cost of producing conventional energy which ranges between 6%-7%. This gives $\sigma b_1 = 0.14$ or with $\sigma = 0.15$ we get $b_1 = b(0) = \chi_1 + \chi_2 = 0.94$ or \$940/tC. Through learning by doing this cost can be reduced by 60% to a lower limit of 9% of GDP, so that $b(\infty) = \chi_1 = 0.563$ and thus $\chi_2 = 0.375$. In our simulations with $\rho > 0$, this lower limit falls to about 6% of GDP due to substitution of energy through capital, i.e. in the long run the share of energy falls back to its current level. We assume that experience in renewable production lowers unit cost at a falling rate and the parameter χ_3 measures this speed of learning. We calibrate learning such that costs would decrease slowly. We suppose a mere 1.25% reduction in cost if all of world energy use would be supplied by renewable sources in the initial period, so that $\chi_3 = 0.00375$. These assumptions imply at most a 14% reduction for a doubling of experience at the peak of learning-by-doing and fall within the broad range reported for learning rates in renewable generation (McDonald and Schrattenholzer, 2001). Manne and Richels (2004) use a 20% cost reduction for a doubling of experience (and thus need to impose unrealistic constraints on renewable use due to the strong curvature of the power-law learning curve). Alberth and Hope (2007) assume a cost reduction of 5%-25% for a doubling of experience so our calibrated 14% falls in this range. Goulder and Mathai (2000) calibrate their learning-by-doing technology to a 0.5%-4% of GDP cost of a 25% emission reduction in 2020. This translates into a cost of renewable energy which is a factor 3-8 times current fossil energy prices. In our calibration current renewable production prices are little less than 3 times those of fossil energy. These numbers illustrate the large scientific uncertainties surrounding the possible trajectories of renewable energy prices. Our calibration falls within the ballpark figures presented above, giving high costs relative to Nordhaus (2008) who also includes CCS technologies in his assessment.