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ORIGINALARBEIT/ORIGINAL ARTICLE

# Negotiating Knowledge Through Mathematical Activities in Classroom Interactions

**Jenni Ingram**

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**Abstract** What students are learning in mathematics classrooms can be analysed from a wide range of perspectives, focusing on the mathematical content, the mathematical processes or the mathematical activities. There is also a social dimension intertwined with this cognitive dimension that focuses on what mathematical activities in particular are established as socio-mathematical norms within a classroom. It is through these norms that students learn what it is to both learn mathematics and what it means to do mathematics. In this article I add a further epistemic dimension to illustrate how these socio-mathematical norms are co-constructed through interactions in a dynamic way. The analysis reported in this paper draws on video data of 65 lesson segments where students made their thinking explicit, taken from an original sample of 133 naturally occurring mathematics lessons with 81 teachers identified as including cognitively demanding mathematics. Using ethnomethodological conversation analysis and the epistemic dimensions of interaction initially outlined by Heritage and colleagues, a fine-grained sequential analysis of the interactions including students' detailed contributions around cognitively demanding activities within these lessons illustrates how epistemic access, primacy and responsibilities are continually negotiated in interaction. Students' rights to make knowledge claims or obligations to demonstrate their knowledge are negotiated in ways that influence students' and teachers' epistemic authority and stance. The analysis shows how the cognitive and social dimensions are intertwined both at an individual level but also at the classroom level that establish and reinforce the norms around the access and primacy of ideas and knowledge in interaction. These epistemic negotiations have consequences on the nature of mathematical activities that students experience and participate in, and subsequently their opportunities to learn mathematics.

✉ Jenni Ingram
University of Oxford, Oxford, UK
E-Mail: Jenni.Ingram@education.ox.ac.uk

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## Wissensvermittlung durch mathematische Aktivitäten im Unterricht

**Zusammenfassung** Was Schülerinnen und Schüler im Mathematikunterricht lernen, kann aus vielfältigen Perspektiven analysiert werden. Beispielsweise können Schwerpunkte gelegt werden auf den mathematischen Inhalt, auf mathematische Prozesse oder auf mathematische Aktivitäten. Diese eher kognitiven Dimensionen sind mit einer sozialen Dimension verflochten, die sich insbesondere darauf bezieht, welche mathematischen Aktivitäten als sozio-mathematische Normen in einem Klassenzimmer etabliert werden. Durch diese Normen lernen die Schüler:innen sowohl was es heißt, Mathematik zu lernen und als auch was es bedeutet, Mathematik zu betreiben. In diesem Artikel ergänze ich eine weitere epistemische Dimension, um zu veranschaulichen, wie diese sozio-mathematischen Normen durch Interaktionen auf dynamische Weise ko-konstruiert werden. Die Analysen in diesem Artikel basieren auf Videodaten von 65 Unterrichtsausschnitten, in denen Schüler:innen ihr Denken explizit zum Ausdruck brachten. Die Videodaten stammen aus einer Originalstichprobe aus 133 regulären Mathematikstunden von 81 Lehrkräften, die sich dadurch auszeichnen, dass sie kognitiv aktivierende Mathematik in ihren Unterricht integrieren. Mittels ethnomethodologischer Gesprächsanalyse und der epistemischen Dimensionen der Interaktion, die ursprünglich von Heritage und Kollegen skizziert wurden, veranschaulicht eine detaillierte sequentielle Analyse der Interaktionen, einschließlich der detaillierten Beiträge der Schüler:innen zu kognitiv aktivierenden Aktivitäten in diesen Unterrichtsstunden, wie epistemischer Zugang, Vorrang und Verantwortlichkeiten kontinuierlich interaktiv ausgehandelt werden. Das Recht der Schüler:innen, Wissensansprüche geltend zu machen, oder die Pflicht, ihr Wissen darzulegen, wird auf Weisen ausgehandelt, die die epistemische Autorität und Haltung von Schüler:innen und Lehrkräften beeinflusst. Die Analyse zeigt, wie die kognitiven und sozialen Dimensionen sowohl auf individueller Ebene als auch auf Klassenebene miteinander verflochten sind und wie sich die Normen rund um den Zugang und den Vorrang von Ideen und Wissen in der Interaktion etablieren und stärken. Diese epistemischen Aushandlungen haben Auswirkungen auf die Art der mathematischen Aktivitäten, die Schüler:innen erleben und an denen sie partizipieren, und in der Folge auf ihre Möglichkeiten, Mathematik zu lernen.

**Schlüsselwörter** Epistemische Aushandlung · Konversationsanalyse · Kognitiv anspruchsvolle Mathematik · Erklärungen

Classroom interaction is complex. It is multidimensional and involves a range of participants. These participants have institutional roles, rights and responsibilities; for example, teachers are responsible for their students' learning. Norms of classroom interaction structure these interactions in ways that enable the interactions to run smoothly most of the time and to support students' learning (Cazden 2001; Mehan

1979; Stivers and Robinson 2006). There has been extensive research focused on classroom interactions that have identified different roles, rights and responsibilities through patterns and norms within classrooms. One of the most well-known norms is the IRE or IRF sequence, where a teacher initiates a question; a student responds before the teacher evaluates or gives feedback on this response (Mehan 1979; Sinclair and Coulthard 1975). More recently, revoicing has become widely used as a sequence where a student shares their thinking and then the teacher 'revoices' this thinking in such a way that the responsibility for the ideas rests with the student (Ingram 2022; C. O'Connor and Michaels 2019; M.C. O'Connor and Michaels 1993). Ethnomethodological Conversation Analysis (EMCA) approaches to analysing classroom interactions have further identified norms around turn-taking (McHoul 1978), the handling of repairs, errors and corrections (Ingram et al. 2015) and student claims of not knowing or remembering (Ingram 2020; Sert and Walsh 2013). In particular, this research has explained and emphasised the important role of the IRE sequence in these teaching and learning contexts, where teachers indicated which actions are acceptable in complex ways alongside what is relevant to these interactions, i.e., what counts as a mathematical explanation.

In this article I build on existing research using an EMCA approach that focuses on three epistemic dimensions that apply to all interactions: epistemic access, epistemic primacy, and epistemic responsibility. This approach to the analysis of mathematics classroom interactions describes patterns of interactions that are normative, drawing on both the social and cognitive dimensions of interaction. Teachers and students have different access, primacy, authority, entitlements, rights, obligations, and responsibilities in relation to what they know, but also in terms of how they share or display this knowledge as a consequence of their institutional roles. Furthermore, participants in classroom interactions also have different interactional roles (expert-novice) and communicative roles (speaker-listener). These too are accompanied by differences in epistemic access and responsibilities. Classroom interaction is organised and structured by these relative epistemic dimensions (Enfield 2011), and these structures are norms for what it means to learn and do mathematics. The specific contribution of this paper to the existing work on classroom interaction is the identification of the role of epistemic management around cognitively demanding mathematics in ordinary classrooms.

I begin by introducing the theoretical principles of EMCA before outlining the 'epistemic engine' of interaction (Heritage 2012b) that distinguishes between the different epistemic roles, rights and responsibilities of the participants. I then briefly describe the conversation analytic notions of alignment and affiliation used in the qualitative analysis within the study, before outlining the data that were used in the analysis for this article. Findings are then presented and discussed. In particular, in this article I detail how teachers and students navigate the complex epistemic negotiations involved in interactions around cognitively demanding activities. Specifically, how epistemic access, epistemic primacy and epistemic responsibilities are negotiated and co-constructed in interaction and the consequences this has in relation to the role of epistemics in mathematics classroom interactions.

## 1 Ethnomethodological Conversation Analysis

EMCA is both a theoretical perspective and a methodology (Drew 2005; Ingram 2018, 2021; Sidnell and Stivers 2012). EMCA works with the premise that all social interaction is ordered and through participants' reciprocal and reflexive interactional work, a shared understanding in negotiated. This interactional order is commonly referred to as norms or social norms, though all norms are social (Sekiguchi 2006), normative patterns (Herbel-Eisenmann 2003) or simply patterns (Ingram 2021). The interactional work is observable in participants' turns as they display, repair and ratify the unfolding interpretations (Abrahamson et al. 2019). EMCA focuses on describing the fine details of these processes that are visible in interaction.

EMCA focuses on the joint construction or co-construction of intersubjectivity in interaction. Co-construction in EMCA focuses on in-the-moment interactional processes such as collaboration, cooperation and coordination that are visible in conversational interaction rather than a focus on the role of social interaction in overall cognitive development often found in sociocultural research (Jacoby and Ochs 1995). In classroom interaction research this means that the analysis focuses on the complex interactional work of both students and teachers, rather than focusing specifically on the actions of the teacher for example. Focusing on the actions of the teacher can overemphasise their role in structuring the activity at the expense of recognising the active and competent roles of students within these interactions, through which tasks are done and goals are achieved (Elbers 2003; Gardner and Forrester 2010). The analysis of norms in EMCA research makes a similar temporal distinction as norms are considered visible in moment-to-moment interactions through turn design and what is treated as acceptable, aligned and affiliative or not by the teachers and students in interaction. In contrast, a socio-cultural approach usually considers the establishment and impact of norms over time and at the classroom level rather than at the interactional level.

The ethnomethodological principles within EMCA also mean a focus on what students and teachers orient to as important or relevant through what they make visible in their interactions, rather than the more usual researcher interests. This principle has important implications for how context is treated in the analysis. The fact that an interaction occurs in a mathematics lesson does not necessarily make this context relevant to the interaction. Many interactions within mathematics classrooms could be seen as occurring in other classrooms, other teaching and learning contexts, or in everyday contexts. Rather than the researcher identifying the context as a mathematics lesson, in EMCA the analysis needs to show how the context is consequential to the interaction taking place (Schegloff 1997). Similarly, mathematical content is made relevant through how the teacher and student use it and acknowledge it in interaction.

Research focusing on classroom interactions have revealed a range of structures and patterns within interaction. One of the most well-known of these is the IRE (Initiation-response-evaluation) sequence (Mehan 1979; Sinclair and Coulthard 1975). The prevalence of this sequence in teaching-learning interactions, such as those occurring in classrooms illustrate one of the institutional norms. However, the use (and critique) of IRE overlooks the complexity and multiplicity of the work of each term.

This also extends to other research that focuses on categorising the functions of turns. EMCA instead focuses on the variety of actions being performed in each turn. Coding reduces this complexity, yet in doing so the multiplicity of functions as well as the reflexive nature and sequential context of the turn is lost. EMCA aims to provide a detailed analysis that reveals this complexity and how teachers and students recognise particular norms of interaction as relevant (Garfinkel 1967). These norms include those patterns identified by Yackel and Cobb (1996) as sociomathematical norms that focus on what counts as "mathematically different, mathematically sophisticated, mathematically efficient and mathematically elegant" (p. 461). In particular, using EMCA focuses not on the establishment of classroom norms such as students are obliged to explain and justify (Yackel and Cobb 1996), but on how these norms are visible and oriented to in the moment-to-moment interactions between teachers and students.

There has been considerable research using EMCA focusing on knowledge in interaction, with the work of Heritage (Heritage 2011, 2012b, 2013; Heritage and Raymond 2012) in particular examining the systematic ways in which knowledge is co-constructed in both every day and institutional contexts. From this perspective, knowledge is dynamic, negotiated and changed in interaction (van Dijk 2014). More recently this research has considered mathematics teaching and learning, including a focus on mathematics in early years interactions (e.g., Bateman 2015) and multilingual classrooms (e.g., Barwell 2013). In addition, EMCA research has also considered the consequences that knowledge asymmetries commonly encountered in mathematics classrooms have on social relations. Stivers et al. (2011) introduced the terms alignment and affiliation to distinguish between the structural and social roles of turns in interactions involving these knowledge asymmetries. In the following sections, Heritage's 'epistemic engine' is described, followed by a more detailed explanation of affiliation and alignment.

## 2 Epistemics in Classroom Interaction

Mathematics classroom interaction is fundamentally characterised by the negotiation of knowledge and meaning. Both teachers and students design their turns in ways that take into account what those listening already know, or what is assumed that they know. There is a cognitive relationship between participants in classroom interaction "without which relationship our activity, behavioural and verbal, could not be meaningfully organized" (Goffman 1983, p. 4).

Mathematics classroom norms are established and negotiated through classroom interactions. In this article I use a sociological and emergent perspective on norms (Herbel-Eisenmann 2003), focusing on "patterns of interaction" (Bauersfeld 1988; Voigt 1985) that are usually implicit but are observable through a microanalysis of interaction. Within these interactions, the normative and moral aspects of knowledge can be examined. In contrast to many perspectives on classroom norms where these norms can be viewed as constraints on what both teachers and students do, these norms instead are viewed as resources that enable participants to make sense of each other, make sense of the mathematics in focus, and achieve the institutional goals underpinning these classroom interactions.

Analysing the interactional resources used by both teachers and students as they interact about knowledge makes visible what is treated by the teacher as something their students should know, such as teacher requests for students to display this knowledge, student claims of knowing or not knowing, and student demonstrations of their knowledge. The normative aspect of this is highlighted in the prolific IRE or IRF sequence (Ingram 2021; McHoul 1978; Mehan 1979).

In classrooms, teachers generally have a good command over what knowledge students should, could, or actually have access to as a consequence of the curriculum and previous lessons, tasks or activities that involved this knowledge. This is evident in both how teachers often structure their questions and explanations to include references to previous lessons, tasks or topics, or by phrasing the question as a request to remember, and also in how students' claims or not having access to this knowledge is subsequently handled by teachers in these interactions (Ingram 2020). The normative asymmetry in epistemic access between teachers and students is further evident in the frequent use of lexical certainty markers such as "I think" by students when they respond to teachers' questions (Rowland 1995) and in the ways that teachers handle mistakes or errors (Ingram et al. 2015; Seedhouse 1997).

The negotiation, demonstration and claiming of knowledge within classroom interactions are ubiquitous and the epistemic access, rights and responsibilities of the different participants are made visible through the way teachers and students structure their turns around the context of the interaction. The use of different epistemic resources are normatively organised (Stivers et al. 2011) and this has enabled researchers to examine how teachers and students use these epistemic resources in similar ways, but also in different ways.

In classrooms there are asymmetrical positions in relation to content knowledge, and students and teachers "orient to the normatively organized social distributions of authoritative access to bodies or types of knowledge" (Drew 1991, p. 45). Mathematics teachers normally behave and are treated as experts on the mathematical content of a lesson. Teachers generally have epistemic access to the mathematical content, and the right to evaluate students' contributions as appropriate (or not) (Barwell 2013).

In this article Heritage's epistemic engine of interaction (Heritage 2012b, a, 2013) which focuses on the three primary dimensions of knowledge that are visible in interaction and are governed by norms. These dimensions are epistemic access, epistemic primacy and epistemic responsibility. Through analysing extracts of interaction around cognitively demanding mathematical activity, these three dimensions illustrate both the similarities and the differences between classrooms in terms of what it means to do and learn mathematics.

Different participants have different access to knowledge, and these differences in epistemic access result in participants having different epistemic statuses in interactions. There can be a range of relative states of knowledge which Heritage (2012a) describes as an epistemic gradient where different participants occupy different positions between being more knowledgeable or less knowledgeable. This epistemic status varies over time and across epistemic domains, and evolves through interaction. Learning is a change in epistemic access and status. In mathematics classrooms, the teacher has access to a wide range of mathematical knowledge, as well as curricula knowledge and pedagogical knowledge that they draw on when they interact. Students' access to this range of knowledge is far more limited. On the other hand, students will have primary access to their own ideas, experiences and ways of thinking.

Epistemic stance refers to the moment-by-moment expression of the relationship between the epistemic statuses of the participants. For example, a teacher could ask a student "Are you stuck?" Here the teacher indicates that they have recognised something as needing attention that might be a consequence of the student being stuck. The student has more information about whether they are stuck or not, even though they may not be sure about whether they are stuck. There is an epistemic gradient between the teacher and the student, with the student being more knowledgeable here than the teacher. However, a teacher could have said "You're stuck aren't you?". Here the teacher has taken the epistemic stance of knowing that the student is stuck, which may or may not be challenged by the student in their response. Epistemic stance can be used by people to appear more, or less, knowledgeable than they actually are. For example, teachers may use the pedagogical technique of pretending not to know something in order to give a student a need to explain their reasoning.

The second dimension is epistemic primacy which describes the relative rights to tell, inform, assert or assess what is being said, and also differences in the depth and completeness of knowledge. In classrooms many of these rights are institutional and teachers and students orient to these asymmetries in these rights, but they also vary between classrooms. Teachers consider what their students already know and what is new as well as who might be able to answer their questions. Where teachers and students have the same epistemic access to mathematical knowledge, there is still a qualitative difference in the depth and completeness of that mathematical knowledge—that is teachers usually also have more epistemic authority. Within the structure of interactions, students often downgrade their answers or assertions by using mitigations such as "I think". While in classrooms epistemic primacy is derivable from the institutional roles of teacher and student, it is also derived from the interactional roles, e.g. questioner, answer, teller, assessor (Heritage and Raymond 2005).

The final dimension is epistemic responsibilities. Knowledge is a resource, it is also a position and this imposes responsibilities (Enfield 2011). Teachers and students have particular responsibilities, such as around asking and answering questions or around knowing what is treated as shared knowledge. Teachers take this shared knowledge into account when they design their questions and tasks and in doing so treat students as being responsible for knowing this shared knowledge (Ingram 2020).

Each of these epistemic dimensions is evident in classroom interactions through how teachers and students construct what they say and how they say it. Solem (2016) illustrated how students making assertions can challenge the epistemic norms within classrooms, but also highlight the teacher's epistemic primacy, their right and responsibility to ratify what knowledge counts. For example, students often phrase their initiations as questions which maintains the epistemic order, or they can "upgrade their epistemic stance by repeating or backing their claims with accounts and providing evidence of them" (2016, p. 737) (i.e., providing an explanation for their answer).

The negotiation of knowledge within mathematics classrooms is not only influenced by these epistemic dimensions. Teachers and students also structure their turns around the broader sequential organisation within the interaction and affective dimensions. These two further dimensions are described as alignment and affiliation which I describe next.

## 3 Affiliation and Alignment

When participants interact, they structure what they say and how they say it in ways that not only enable the interaction to run smoothly but also in ways that take into account the social relationships between themselves and the other participants. These two aspects are referred to as alignment and affiliation in EMCA research (Stivers et al. 2011). Alignment is the structural features of interaction that support the progression of the interaction, such as accepting presuppositions being made by other speakers. Affiliative responses are more affective and include actions such as matching the other speakers' evaluative stance.

Interactional turns can be congruently aligning and affiliative, but they can also align but be disaffiliative, and vice versa. Teachers may not acknowledge the knowledge being expressed by a student if it does not contribute to the trajectory of the interaction for example, by not being directly relevant to the topic the teacher is focusing on, or because the student spoke out of turn (Clayman and Raymond 2021; Weatherall and Keevallik 2016). This action enables the interaction to continue smoothly, but is not affiliative in the handling of the student's turn.

Conversation analysis and other interactionist approaches focus on a range of actions in turns at talk, including how speakers align or affiliate themselves with each other. The focus is not just what is said, but also how it is said. Conversation analysis in particular also stresses the importance of the sequential organisation of knowledge management in interactions and the norms that are established through this sequential organisation. Norms, including sociomathematical norms, are co-constructed by teachers and students, and this co-construction is often unconscious, implicit and locally specific (Voigt 1995). Voigt illustrated the indirect and often unconscious influence mathematics teachers can have on students' mathematical thinking and learning through their turns including through their evaluations, what they draw attention to when they respond to students, as well as their affective reactions to students' turns. Norms can also be consciously developed by mathematics teachers (Makar et al. 2015).

## 4 Methods

This article reports on a secondary analysis of the data collected and analysed for the TALIS Video Study (OECD 2020). This study analysed mathematics teaching in eight countries and economies, using videos of lessons focused on the teaching of quadratic equations to students aged between 13 and 17 years old, the accompanying lesson materials such as worksheets and slides, alongside pre- and post-student tests and teacher and student questionnaires. The analysis in this article focuses on the 167 videos of 85 teachers collected from one of the participating countries, England, as these videos were accessible to the author and had been partially transcribed as part of the national analysis (Ingram and Gorgen 2020). Some videos from the study from other country contexts are available through the study website (https://www.globalteachinginsights.org/) in the cases where consent was given for this sharing.

In the original study, the videos were divided into 16-minute segments and coded by two trained raters across six domains of practice. In this article, the focus is on two of the higher inference codes (called components) used in the study; *Engagement in Cognitively Demanding Subject Matter* and *Eliciting Student Thinking* (Bell et al. 2020a). In the study, "tasks that required thoughtful analysis, creation or evaluation were considered more cognitively demanding" (p. 129). The average ratings for teachers in each of the country contexts for each of these component codes can be found in the chapter from the final project report focusing on teaching and instruction (Bell et al. 2020b). In England, only 8% of teachers used cognitively demanding tasks across the two lessons recorded for the study (Bell et al. 2020b, p. 290).

The analysis for this article focuses on lesson segments coded as students *engaging in cognitively demanding subject matter*, whether this was occasionally (a rating of 2), sometimes (a rating of 3) or frequently (a rating of 4), and lesson segments coded as having a moderate (a rating of 3) or a lot of (a rating of 4) student thinking present (see Bell et al. (2020a) for further details of the observation system used in the study). The first stage involved identifying lesson segments around cognitively demanding mathematics where students were actively involved in the interactions around this mathematics. Using the existing coding, 302 lesson segments from 133 lessons from 81 teachers that were coded with a rating of 3 or more for *engaging in cognitively demanding subject matter* were identified and initially transcribed verbatim as part of the original data analysis in England. The interactions from lesson segments where the student turns were extended and focused on the cognitively demanding subject matter identified in the coding (extracts from 65 lesson segments) were then transcribed in more detail using the Jefferson transcription system (Hepburn and Bolden 2013) to enable the micro-analysis involved in EMCA. Simplified transcripts have been used for the dissemination of the original qualitative findings from the England study to teachers (Ingram and Gorgen 2020) and in this article for ease of reading.

The interactions within these 65 lesson segments that focused on cognitively demanding mathematical activities were then analysed using EMCA. This is similar to the approach taken by Yackel and Cobb in identifying sociomathematical norms where norms are inferred by "identifying regularities in patterns of social interaction" (Yackel and Cobb 1996, p. 460). However, EMCA focuses on systematically identifying these regularities across a range of classroom contexts to identify sociomathematical norms in mathematics classrooms rather than a mathematics classroom over time (Sidnell 2010). It also focuses on the co-construction of these norms rather than the focus on individual sense-making within interactions in other interactionist approaches (Krummheuer 2011; Voigt 1995; Yackel and Cobb 1996). In particular, the analysis focused on the negotiation of epistemic access, primacy and responsibilities to identify norms of interaction in these cognitively demanding interactions.

The analysis uses the EMCA methods to demonstrate how students and teachers orient to the sequential organisation of classroom talk, with each turn responding to both the immediately prior turn and the prior sequence of turns. By making prior turns relevant, teachers and students are demonstrating and understanding and developing knowledge-in-interaction. In constructing their turns, teachers and students display an understanding of previous turns at a multiplicity of levels, including process and content. These displays are then responded to in the turns that follow, with explicit or implicit evaluations or repairs (Schegloff 1993). These EMCA methods include analysis of turn design, the sequential organisation of turns including preference organisation, turn-taking and turn allocation, and trouble in interaction and the subsequent repair or this trouble (see Sidnell and Stivers (2012) for further details of each of these methods). The extracts below have been chosen for their clarity in illustrating the two identified norms of this epistemic negotiation in mathematically cognitively demanding interactions. The unit of the analysis is the interaction rather than the teacher or the students. EMCA is also agnostic towards what is going on inside teachers' and students' heads (Potter and Edwards 2013).

The EMCA analysis began with an unmotivated viewing of the 302 lesson segments coded as including cognitively demanding subject matter that is common to EMCA research. These viewings revealed that there were differences in the quantity and quality of student contributions to the interactions around this cognitively demanding mathematics. Beginning with the 302 lesson segments that included evidence of students engaging in cognitively demanding subject matter, watching the videos of these lesson segments revealed that 217 of these included students making this thinking public in some way. The rest of the lesson segments included students working on tasks individually or in pairs in ways that means that the reasoning is not visible in the interactions captured by the videos. Through multiple viewings, these 217 segments were then analysed to look at the norms around the explanations involved in the cognitively demanding subject matter and 65 segments were identified as including student turns that were extended and focused on the cognitively demanding subject matter identified in the coding. These 65 lesson segments were then used as the basis for the subsequent analysis. Repeated viewings of these lesson segments led to the identification of sequential patterns with some variations pertaining to the interaction and epistemic work around explanations when considering cognitively demanding mathematics.

The process of building a collection of cases led to the organisation of these segments into collections of cases based on whether the students were involved in producing these explanations or responding to these explanations (see Sidnell (2013) for a more detailed description of this process of case collection in EMCA).

This approach to case collection allows for individual interactions to be included in multiple collections, for example where a student initiates an explanation of an idea which the teacher then builds on adding more information or connections to other ideas would appear in both the collection of cases where the teacher produced an explanation and the collection of cases where a student produced an explanation as the explanation is co-constructed through the interaction. The fuzzy boundaries of the case collections developed through EMCA underly the absence of frequencies of cases in the reporting of any analysis with a preference for terms such as most, several or few (Stivers 2015).

This process of identifying case collections addresses the following questions: What are the epistemic and interactional resources that teachers and students use when explaining ideas and processes involved in working with cognitively demanding mathematics?

## 5 Findings

In this section, I present two extracts that illustrate the two collections of cases that focus on the types of norms of interaction that were evident in all of the lesson segments involving engaging students in cognitively demanding subject matter. These common norms are both social and mathematical in nature in that they both consider explanations focused on meanings, but in the first type of norm presented, these explanations were given by teachers and only by teachers. This norm, which is illustrated in Extract 1, was visible in all of the lesson segments where the students' thinking around the cognitively demanding subject matter was audible in the video. The second collection focuses on cases where these explanations were also given by students. In total 78 lesson segments from 36 teachers included students giving explanations, in 65 of these lesson segments this student explanation was audible in the video recording, and in 36 of these lesson segments with 25 teachers these explanations focused on the mathematical meanings, and it is these 36 lesson segments that are included in the second collection. Extract 2 illustrates the different ways in which these explanations occurred. As mentioned in the methods sections, these lesson segments could include multiple explanations which could be included in either or both collections.

### 5.1 Collection 1—Teachers Are Responsible for Explaining Meanings

The first extract is taken from a lesson segment that had the maximum rating by both raters for students engaging in cognitively demanding subject matter, as the question posed required student to analyse the relationship between the coordinates of the turning point (minimum or maximum) of a quadratic curve and the quadratic equation represented in completed square form.

### *5.1.1 Extract 1*

Teacher: okay, keep your hand up if you've got an idea, how I can find the turning point. (.) some of us have got it, go on Simon

Simon: completing the square.

Teacher: (2.2) you're on the right lines, so something to do with go- that's- this is why we complete the square.

Simon: errr (2.1) I- (.) (I dunno, I'm not sure) really

Teacher: it's something to do with co- when we complete the square, Sarah

Sarah: is it like (.) the number that you square but the opposite of it.

Teacher: yes. Because, think about that minimum value, I know that (.) the lowest number here, I need that bracket to be zero. Because anything- anything squared, it- zero squared is going to give me zero. it's going to give me the smallest number.

Sam: I was going to say it's just the-, it's always the first digit in the minimum turning point.

Teacher: so, in the brackets. Next to the x, that's my x value, so think about what must x be to make this the smallest number possible. and then that will be what I'm left with for the y coordinate.

In this first extract we see the common norm of classroom interaction that is well documented in decades of research. The whole interaction can be described in terms of a sequence of IRE sequences. This illustrates the institutional epistemic roles and responsibilities of teachers and students. The teacher initiation illustrates the epistemic access the teacher has to the mathematical content but also positions the students as having this access. Notably, by asking students to raise their hands the teacher is requesting multiple students to make a claim of knowing how to find the turning point. Students raising their hands are making this claim and are taking an epistemic stance that is both affiliative and aligns with the teacher's epistemic stance visible in lines 1 to 3. The teacher's nomination of Simon requires them to demonstrate this knowledge.

Simon's response makes visible their access to the method of completing the square, and makes this method relevant to the process of finding the turning point. Simon has given evidence that at least some of the students are able to make this connection between the task of finding a turning point and the method of completing the square—i.e., that they have access to the knowledge needed to answer the teacher's question. In line 5, the teacher's primacy over this knowledge is evident through the assessment of Simon's response, which supports the relevance of completing the square to the process but assesses this as not answering the question. The teacher's turn is affiliative in how they construct this assessment with no explicit negative assessment and illustrating the preference for teacher-initiated self-repair in classroom interactions by returning the turn to Simon. This partial acceptance of Simon's turn being "on the right lines" also emphasises the students' epistemic access to the knowledge needed to explain how to find the turning point and gives them the epistemic right and responsibility to use this knowledge in this interaction. However, this implicit negative evaluation also shows that the teacher treats Simon's response as not aligning with the trajectory of interaction, emphasising the epistemic primacy of the teacher in this interaction to assess the students' explanations.

Simon then demonstrates their understanding of the interaction so far, accepting that the turn has returned to them and that their original response in line 4 did not answer the teacher's question in a way that the teacher was anticipating (i.e., it did not align). In this turn, Simon takes responsibility for the lack of knowledge needed to answer the question by displaying uncertainty over the answer needed, claiming that they do not have access to the knowledge needed. The sequential position of this turn, however, suggests that the difficulty is with addressing the teacher's initiation in lines 1–3, rather than accessing the mathematical knowledge needed, which they have demonstrated in line 4. This turn is marked by hesitations, a repeated start and upgrading the claim of not being able to answer with the ending "really?". This marking illustrates the social norms in classroom interactions of the epistemic rights and responsibilities around responding to teacher initiations, demonstrating Simon's affiliative stance towards these rights and responsibilities whilst also not giving the response the teacher is expecting that would enable the interaction to continue smoothly.

The teacher responds to this turn by emphasising the connection of Simon's first response in line 4 to the answer they are looking for, accepting and reinforcing the affiliative nature of Simon's second response. This positive assessment of Simon's first response is affiliative and indicates that completing the square is relevant to the interaction. It also supports the ongoing interaction by rephrasing the question to be about how we can find the turning point using something connected to completing the square, and then by nominating a different student to take the next turn. Sarah takes this turn, describing a particular number, a description that applies to a quadratic equation in completed square form. This description treats as shared that the interaction is about an equation in completed square form, which the teacher ratifies in their immediate, bald, positive assessment in line 12.

Sarah's turn is phrased as a question, a common student turn structure that highlights the teacher's epistemic primacy (Bateman 2015). Sarah's turn demonstrates that they have epistemic access to the process of identifying the $x$-coordinate of a turning point from the completed square form, though Sarah has not demonstrated that this process only identifies the $x$-coordinate. This focus on identifying the $x$-coordinate is continued in the teacher's turn. Here the teacher treats knowledge of the connection between the minimum value of a function and the turning point as shared. The teacher also treats the knowledge that to achieve this minimum value the expression within the bracket needs to be zero, as something students should have access to, at least after thinking about it. This treats the students as capable of making this deduction. The teacher's explanation explains why "the number that you square but the opposite of it" gives the x-coordinate using knowledge and deductive reasoning that has not been referred to previously in this interaction. By offering this explanation for Sarah's response the teacher is taking epistemic responsibility for ensuring that this reasoning is made explicit for all students to have access to, whilst not challenging at least some students existing access.

Sam begins their turn marking that their idea is one that they were willing to articulate earlier, which also marks what they are about to say as a change in topic. By positioning the idea in the past, Sam also mitigates any potential negative evaluation. Sam then focuses on the first digit (the x-coordinate) of the turning point but uses a dietic reference ('it's') to indicate the relevance of this digit. The teacher picks up on the potential ambiguity associated with using 'its' and focuses on the specific location of the digit Sam is referring to in the completed square form, before continuing to explain the connection by focusing on the value of x needed to minimise the expression in the bracket before also connecting the y-coordinate to the final term in the expression. The acceptance of and expansion of Sam's response also demonstrates that this answer is an acceptable response to the teacher's initiation, but here the responsibility for the procedure that Sam describes for identifying the minimum point (or specifically the x-coordinate of the minimum point) to have meaning is reinforced through the teacher making the connection to their previous explanation by thinking about the value of x needed to make the bracket in completed square form zero.

Both Sarah's and Sam's responses focus on the location of particular numbers, with Sarah focusing on the number inside the bracket in the completed square form representation and Sam focusing on the $x$-coordinate of the turning point. These two representations are treated as shared knowledge by both the students and the teacher. Both Sarah and Sam identify how to find the $x$-coordinate of the turning point, but it is the teacher who explains the meaning behind why that particular number in the completed square from will be the same as the x-coordinate of the turning point.

The interaction in this first extract illustrates a norm that was common across many of the lesson segments including students engaging with cognitively demanding mathematics around explaining meanings and will be familiar to many readers. In these interactions the teacher offers the explanations that focused on the meaning or on why particular processes were appropriate, as in Extract 1. These explanations are given in response to students' ideas and responses to the previous initiation. In terms of epistemic access, throughout the extract the teacher and the students orient to all parties in the interaction having access to the mathematics needed to respond to the teacher's initiation asking students to describe or explain how to find the turning points. In terms of epistemic rights, both the students and the teacher have the right to share this knowledge they have access to. The students are given this right by the teacher through the initiations and invitations to speak. This is reflective of the usual social norms of classroom interaction (Cazden 2001; Mehan 1979). In terms of epistemic responsibilities, the teacher's explanation in lines 12–16 and again in lines 19–22 illustrate the teacher's epistemic responsibilities to not only engage students in cognitively demanding subject matter, but also to give meaning to approaches and processes that students engage in when doing this. These explanations are responsive to the responses given by the students. The turn design and sequential location of this explanation maintain the positioning of the students' access to the knowledge needed to find the turning point, but the explanation explains why completing the square leads to finding this minimum value by modelling the thought process involved. The teacher's and the students' turns do not suggest that the students have a responsibility to explain the meaning behind 'how'. The original initiation focuses on how, and the teacher accepts or partially accepts answers that describe how, without prompting for whys. There are no attempts, either successful or unsuccessful (Ingram et al. 2019), by the students to offer an explanation for why their procedures find the minimum points. It is in these epistemic responsibilities to explain meanings where the two collections of cases differ.

## 5.2 Collection 2—Students are also Responsible for Explaining Meanings

Extract 2 focuses on a different norm around explaining meanings that occurred more rarely in these interactions around cognitively demanding subject matter. Here the interaction also includes explaining meanings but with the students contributing to this meaning. This lesson segment focuses on finding the area of an L shape where some of the side lengths are given using linear expressions.

### *5.2.1 Extract 2*

Polly: could you (.) work out the area of the s:quare, and
then work out the area of the rectangle? Ah >°it's
not a square it's a rectangle°< but [an y way .]
Teacher: [well corrected]
nice. so you're going to split it
Polly: yeah
Teacher: are you going across there?
Polly: yeah
Teacher: alright. so, Polly is going to split the shape,
because it's a compound shape. We don't know a formula
to do area of like an L-shape. There isn't a standard
one. So we're going to split it into shapes we know. so
we got a little rectangle at the top which has an area
of (1.2)
Peter: twenty centimetre[s ]
Teacher: [twenty cen]timetres squared. right.
I'm going to label them A and B just because it's
easier for (.) the way I may write it down, (.) so
area A (1.1) we're saying is four times five it's
twenty. okay! Please make sure you put your units
on! Keep those consistent! How am I going to do <the
area of part B>, Phil
Phil: it's (.) you do the brackets x minus three bracket x
plus four. [t-]
Teacher: [wh]y

Phil: because (.) that’s (.) just (.) what you’re (.) trying
to answer
Teacher: why did you times those two things together.
Phil: because that’s two different like lengths=
Teacher: =yeah thank you, I was just making sure that everyone
knew why you’d used x take three and not x plus two.
so I’m using this rectangle at the bottom, which has a
height of x take three and a width of x plus four. So
they’re my two measurements.

The norms around turn-taking and the use of the IRE sequence are still evident in this extract. In contrast to Extract 1, there is a difference in the epistemic access at the start of this interaction. Polly has epistemic primacy at the beginning of this extract as they have suggested splitting the L shape into two rectangles, which the teacher builds on by asking where the dividing line is in line 7 and which differs from how the teacher had imagined splitting the L as stated by the teacher later in the interaction (not included). This is further supported in line 9 by the teacher who revoices Polly’s suggestion, attributing the suggestion to her. In this turn we also see the same patterns in epistemic roles and responsibilities as in Extract 1 in that it is the teacher who is explaining the meaning behind Polly’s actions on the shape. This explanation returns the epistemic primacy to the teacher and makes explicit that the teacher shares the same epistemic access as Polly once the specific splitting of the L shape has been completed. It also implicitly positively evaluates Polly’s suggestion by shifting from attributing the actions specifically to Polly to attributing the knowledge and the actions to ‘we’. The teacher treats this epistemic access as shared by the class as a whole in their shift to using the pronoun ‘we’, and by passing the next turn to Peter, and the following one to Phil, through the addition of units in the expansion of Peter’s turn without immediately indicating that there is a problem with Peter’s turn. The epistemic primacy of the teacher is also evident in the teacher’s response to Peter in line 16 through the references to introducing notation to make the writing easier and through the attention to units.

In both extracts, the teacher’s responsibility to ensure meanings are addressed is visible, but in this second extract, the teacher also gives the responsibility to Phil to explain the meaning behind his action through the question ‘why’ following his description of how. Phil’s first response in lines 23–24 describes the area of B using words that describe how it is written but without any indication of the meaning behind this action. Phil has trouble responding to the teacher’s ‘why’, recognising that an explanation is needed by beginning their turn with ‘because’, but the turn is given hesitantly with several pauses, indicating that the trouble is with the question the teacher has asked, not with the mathematics involved. The teacher clarifies the question in line 28. Phil’s next response is aligned and draws on the mathematical meaning behind his original action. This trouble with the teacher’s original question is also accepted by the teacher, who accepts Phil’s explanation in line 29 quickly before accounting for why they had asked for an explanation even though from Phil’s perspective, “That’s just what you’re trying to answer”. This account also makes clear that the teacher also has epistemic access to this meaning, but also makes visible the epistemic responsibility the teacher has to make these explanations of meaning public. However, while the teacher positions Phil as having the responsibility to offer an explanation in response to the why question, the first response of Phil plus the teacher explicitly requesting this explanation indicates that the norm focuses on the responsibility of students to respond to a request for an explanation of meaning rather than to provide these explanations. In this extract and across the collection of cases, the teachers treat the students as having epistemic access to the knowledge needed to engage in the cognitively demanding mathematics through the design of both their initiation and their responses to the students. This is the same as in the first collection of cases. In both collections, the teacher also has epistemic primacy, which is interactionally managed through turn-taking and the evaluation of students' responses. What is distinctive between the two collections is that in the second collection, the teacher also prompts students to take responsibility for explaining meanings.

## 6 Discussion

Each of these social actions is normative in the majority of classroom interactions, as widely discussed in decades of research. However, interactions focused on cognitively demanding mathematics highlight the complexity of intersubjectivity and epistemic management. The epistemic primacy of the teacher is evident in all these interactions; it is the teacher who controls the topic and the direction of the topic, who makes assessments, and who gives students the right to share their thinking, knowledge and understanding. In cognitively demanding activities, the epistemic access to the concepts or knowledge was treated as being available to the students through the teachers' turn design and the sequential organisation of the interaction, but was not treated as necessarily shared. Teachers concluded these interactions through summaries and explanations of the accepted response to the original initiation. This sequential organisation (closing) makes visible the teacher's responsibility to ensure that the knowledge or ideas that are taken as shared are in fact shared by the class, including when an explanation has been given by a student. In each of the cases of lesson segments that focused on cognitively demanding mathematics these explanations were given and this responsibility is also visible through the construction of these explanations (turn design) and how they reflexively build on and respond to students' turns. This treatment of students' epistemic access to the knowledge needed and the responsibility to ensure that there are explanations of meaning made visible during this work were structured in ways that were affiliative with the students' responses. Negative assessments of students' turns were mitigated and treated not as a problem of epistemic access, but as a problem of alignment. Explanations were almost always constructed in ways that built on and responded to students' descriptions, in ways that expanded on these descriptions rather than corrected these descriptions. These interactional norms that use affiliative actions are one of the ways that teachers implicitly create a safe environment for students to engage in cognitively demanding mathematics. There are also occasions where multiple solutions strategies are possible, as illustrated in Extract 2. This can result in shifts in epistemic access and primacy temporarily away from the teacher, where a student can suggest a method that differs from the one the teacher was anticipating or would use themselves. These interactions adapt, extend and build on the normative patterns of interaction found across classrooms to take into account these shifts in epistemic access both at the individual student level and for the class as a whole.

Where the two collections differed was in the management of the responsibility of students to explain meanings. In the first collection of cases the responsibility lies with the teacher. In the second collection of cases the responsibility is given to and recognised by the students. However, these student explanations are not given spontaneously, suggesting that the norm of interaction is that these explanations need to be given when asked for, but not as possible, answers to the questions and tasks that the teachers initiate.

A key contribution of this research is that the findings result from video recordings of everyday mathematics lessons, with teachers selected using a random sampling method, and without any intervention or instructions. These are not classrooms selected to evaluate an intervention, to identify differences between different teaching approaches, or to illustrate 'good' teaching. The quantitative analysis in the original TALIS video study demonstrates the variety of teaching practices captured in these videos. The analysis in this paper focused specifically on lesson segments where there were interactions identified as including cognitively demanding mathematics. What is interesting is the relative consistency of the treatment of epistemic access and primacy by both students and teachers across those classrooms where this activity occurred. These institutional norms carry across different teaching approaches, tasks and groups of students. It was the epistemic responsibility that varied across these classrooms.

## 7 Conclusion

This article has identified two types of norm around explanations of meaning when engaging with cognitively demanding mathematics. These explanations are demanding by their very nature and teachers have a responsibility to support individual students and the class as a whole with this cognitive demand. This responsibility is visible in teachers' turn design and the sequential organisation of the interaction around this cognitively demanding mathematics. In some cases, the responsibility for these explanations was taken only by the teacher. In other cases, this responsibility was shared with the students. In Extract 2 the teacher explicitly invited the student to give this explanation. There were also cases where the student gave explanations without the teacher explicitly asking for one, though only within interactions where teachers had previously explicitly asked students for an explanation. In these situations the sequential organisation of the interaction still positions the teacher as requesting this explanation.

In each of the extracts, the norms of interaction are structured around the teachers' epistemic primacy of the cognitively demanding content. The teacher also uses these interactional structures to give epistemic access to the wider audience of students beyond the student contributing their thinking, to ensure this knowledge and understanding is shared. The institutional role of teacher is accompanied by epistemic rights and responsibilities that are visible in the well-established structures of classroom interaction across the range of classrooms considered in this study.

The analysis also shows how the epistemic and social dimensions are intertwined both at an individual level but also at the classroom level that establish and reinforce the norms around what is valued in the learning of mathematics in different classrooms in relation to explanations of meaning. The analysis only focused on extracts from students engaging in cognitively demanding mathematics. It might be that these differences in norms around explanations extend beyond this particular type of interactional context. Further research is needed to explore this in naturally occurring classroom contexts, particularly given the relatively few instances of students offering these explanations around cognitively demanding mathematics.

These norms of interaction around cognitively demanding mathematics are what enable classroom interaction to run smoothly. While a great deal of research has been conducted to look at ways of changing these norms, any changes have wider implications for classroom interactions. While it has been widely established that students explaining meanings supports their learning, it is also often considered desirable for students to give these explanations of meaning without teacher prompting (Prediger and Erath 2014). That is, for students to have the responsibility for explaining meanings not just in response to a teacher's request for this explanation. In everyday classrooms these unprompted explanations are rare and often result from specific task or lesson designs rather than as being established norms. In most cases in this study and in many of the cases documented in the literature where students are explaining meanings, these explanations are prompted by the teacher. This institutional norm where the teacher has the epistemic responsibility to ensure that students have epistemic access to these meanings is the core to the aim of teaching mathematics. What this study shows is that within these norms teachers can handle this responsibility in ways that involve the students, builds and expands on their ideas, in affiliative ways during work that is cognitively demanding whilst still ensuring that the students have access to the meanings involved. Teachers can work within these norms rather than working to change these norms (e.g. by supporting students to give explanations of meaning without prompting) to achieve these goals.

## 8 Transcript Notation

**Table 1** Transcript Notation. (Adapted from Jefferson 2004)

| Convention | Name | Use |
|---|---|---|
| [text] | Brackets | Indicates the start and end points of overlapping speech |
| = | Equal signs | Indicates no break or gap |
| (0.5) | Timed silence | Indicates the length, in seconds, of a silence |
| (.) | Micropause | A hearable pause, usually less than 0.2 s |
| . | Period | Indicates falling pitch or intonation |
| ? or ↑ | Question mark or Up arrow | Indicates rising pitch or intonation |
| , | Comma | Indicates a temporary rise or fall in intonation |
| – | Hyphen | Indicates an abrupt halt or interruption in utterance |
| >< | Right/left carats | Bracketed material is spoken more quickly than surrounding speech |
| <> | Left/right carats | Bracketed material is spoken more slowly than surrounding speech |
| ° | Degree symbol | Indicates quiet speech |
| underline | Underlined text | Indicates the speaker is emphasising or stressing the speech |
| ::: | Colon(s) | Indicates prolongation of a sound |
| (text) | Parentheses | Speech which is unclear in the transcript |

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