

Processes of students' effort exertion, competence beliefs and motivation: Cyclic and  
dynamic effects of learning experiences within school days and school subjects:

Supplementary Materials

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Three models are presented here: (1) Model 1 is a measurement model for the factor structure of the variables at time T. (2) Model 2 includes the within-day lag, and is applicable to the within-school-subject lag by switching the lagged variables from within-day to within-school-subject. (3) Model 3 includes both lagged variables and the between level factor structure. We follow LISREL notation for SEM specification.

#### (1) Model 1: Two-level confirmatory factor analysis

Eight observed indicators ( $y_1$ - $y_8$ ):  $y_1$  = effort exertion (eff),  $y_2$  = understood the contents (und),  $y_3$  = how well did you do it (well),  $y_4$  = interested (int),  $y_5$  = enjoyment (enj),  $y_6$  = choice (cho),  $y_7$  = I had to do it (had),  $y_8$  = teacher wanted me to do it (want), were decomposed into their within-level indicators ( $y_{W1}$ - $y_{W8}$ ) and between-level indicators ( $y_{B1}$ - $y_{B8}$ ). Four latent constructs at both the within- and between-levels (indexed by "W" and "B" respectively):  $\eta_{1T}$  = effort exertion,  $\eta_{2T}$  = competence belief,  $\eta_{3T}$  = autonomous motivation, and  $\eta_{4T}$  = controlled motivation, were linked with their respective within and between-level indicators, specified as  $Y = \Lambda H + \Theta_\epsilon$  in Equation 1.

$$\begin{bmatrix} y_{B1} \\ y_{B2} \\ y_{B3} \\ y_{B4} \\ y_{B5} \\ y_{B6} \\ y_{B7} \\ y_{B8} \\ \hline y_{W1} \\ y_{W2} \\ y_{W3} \\ y_{W4} \\ y_{W5} \\ y_{W6} \\ y_{W7} \\ y_{W8} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \lambda_{B3,2}(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \lambda_{B5,3}(b) & 0 \\ 0 & 0 & \lambda_{B6,3}(c) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda_{B8,4}(d) \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \lambda_{W3,2}(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \lambda_{W5,3}(b) & 0 \\ 0 & 0 & \lambda_{W6,3}(c) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda_{W8,4}(d) \end{bmatrix} \times \begin{bmatrix} \eta_{B1T} \\ \eta_{B2T} \\ \eta_{B3T} \\ \eta_{B4T} \\ \hline \eta_{W1T} \\ \eta_{W2T} \\ \eta_{W3T} \\ \eta_{W4T} \end{bmatrix} + \begin{bmatrix} \epsilon_{B1,1} \\ \epsilon_{B2,2} \\ \epsilon_{B3,3} \\ \epsilon_{B4,4} \\ \epsilon_{B5,5} \\ \epsilon_{B6,6} \\ \epsilon_{B7,7} \\ \epsilon_{B8,8} \\ \hline \epsilon_{W1,1} \\ \epsilon_{W2,2} \\ \epsilon_{W3,3} \\ \epsilon_{W4,4} \\ \epsilon_{W5,5} \\ \epsilon_{W6,6} \\ \epsilon_{W7,7} \\ \epsilon_{W8,8} \end{bmatrix} \quad (1)$$

in which  $\lambda$ s are factor loadings,  $\eta$ s latent constructs, and  $\epsilon$ s are uniquenesses.

Subscripts, (e.g., "(a)") indicate that factor loadings are equated, for obtaining metric invariance across the two levels. For obtaining configural invariance across the levels, i.e., a model in which the factor loading are freely estimated, these constraints can be relaxed. The structural part of the model is a variance-covariance matrix ( $\Psi$ ) of the latent variables at both levels in Equation 2.

$$\psi = \begin{bmatrix} \psi_{B1,1} \\ \psi_{B2,1} \ \psi_{B2,2} \\ \psi_{B3,1} \ \psi_{B3,2} \ \psi_{B3,3} \\ \psi_{B4,1} \ \psi_{B4,2} \ \psi_{B4,3} \ \psi_{B4,4} \\ \hline \psi_{W1,1} \\ \psi_{W2,1} \ \psi_{W2,2} \\ \psi_{W3,1} \ \psi_{W3,2} \ \psi_{W3,3} \\ \psi_{W4,1} \ \psi_{W4,2} \ \psi_{W4,3} \ \psi_{W4,4} \end{bmatrix} \quad (2)$$

in which diagonal elements of  $\psi$  are variances and off-diagonal elements are covariances. Not shown is the matrix of uniquenesses ( $\Theta_\epsilon$ ) with diagonal elements estimated at both the between and within-levels.

**(2) Model 2: Effects of lagged within-day and lagged within-school-subject constructs**

Model 2 is presented using the within-day lag (i.e., "T-1D" indicators and constructs), and is applicable to the within-school-subject lag by switching the lagged variables to within-school-subject (i.e., "T-1S" indicators and constructs).

We specified four latent constructs at the between-level, the average learning experiences during the week ( $\eta_{B1T}$  to  $\eta_{B4T}$ ). At the within-level we specified eight latent constructs (Equation 3). These were latent constructs at time-point T (i.e., time-point zero):  $\eta_{W1T}$  = effort exertion at time T,  $\eta_{W2T}$  = competence belief at time T,  $\eta_{W3T}$  = autonomous motivation at time T, and  $\eta_{W4T}$  = controlled motivation at time T, and four lagged constructs at time-point T-1D:  $\eta_{W5T-1D}$  = effort exertion at time T-1D,  $\eta_{W6T-1D}$  = competence belief at time T-1D,  $\eta_{W7T-1D}$  = autonomous motivation at time T-1D, and  $\eta_{W8T-1D}$  = controlled motivation at time T-1D. These constructs were linked with eight observed variables at time T ( $y_1$ - $y_8$  as in Model 1) and with eight lagged variables at time T-1D:  $y_9$  = effort exertion at T-1D ( $\text{eff}_{T-1D}$ ),  $y_{10}$  = understood the contents at T-1D ( $\text{und}_{T-1D}$ ),  $y_{11}$  = how well did you do it at T-1D ( $\text{well}_{T-1D}$ ),  $y_{12}$  = interested at T-1D ( $\text{int}_{T-1D}$ ),  $y_{13}$  = enjoyment at T-1D ( $\text{enj}_{T-1D}$ ),  $y_{14}$  = choice at T-1D ( $\text{cho}_{T-1D}$ ),  $y_{15}$  = I had to do it at T-1D ( $\text{had}_{T-1D}$ ),  $y_{16}$  = teacher wanted me to do it at T-1D ( $\text{want}_{T-1D}$ ). The lagged variables are only defined at the within-level of the model.

$$\begin{aligned}
& \begin{bmatrix} y_{B1} \\ y_{B2} \\ y_{B3} \\ y_{B4} \\ y_{B5} \\ y_{B6} \\ y_{B7} \\ y_{B8} \\ y_{W1} \\ y_{W2} \\ y_{W3} \\ y_{W4} \\ y_{W5} \\ y_{W6} \\ y_{W7} \\ y_{W8} \\ y_{W9} \\ y_{W10} \\ y_{W11} \\ y_{W12} \\ y_{W13} \\ y_{W14} \\ y_{W15} \\ y_{W16} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \lambda_{B3,2}(a) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda_{B5,3}(b) & 0 & 0 \\ 0 & 0 & \lambda_{B6,3}(c) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda_{B8,4}(d) & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & \lambda_{W3,2}(a) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \lambda_{W5,3}(b) & 0 & 0 \\ 0 & 0 & \lambda_{W6,3}(c) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \lambda_{W8,4}(d) & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \lambda_{W11,6}(a) & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \lambda_{W13,7}(b) \\ 0 & 0 & 0 & 0 & \lambda_{W14,7}(c) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \lambda_{W16,8}(d) \end{bmatrix} \times \begin{bmatrix} \eta_{B1T} \\ \eta_{B2T} \\ \eta_{B3T} \\ \eta_{B4T} \\ \eta_{W1T} \\ \eta_{W2T} \\ \eta_{W3T} \\ \eta_{W4T} \\ \eta_{W5T-1D} \\ \eta_{W6T-1D} \\ \eta_{W7T-1D} \\ \eta_{W8T-1D} \end{bmatrix} + \begin{bmatrix} \epsilon_{B1,1} \\ \epsilon_{B2,2} \\ \epsilon_{B3,3} \\ \epsilon_{B4,4} \\ \epsilon_{B5,5} \\ \epsilon_{B6,6} \\ \epsilon_{B7,7} \\ \epsilon_{B8,8} \\ \epsilon_{W1,1} \\ \epsilon_{W2,2} \\ \epsilon_{W3,3} \\ \epsilon_{W4,4} \\ \epsilon_{W5,5} \\ \epsilon_{W6,6} \\ \epsilon_{W7,7} \\ \epsilon_{W8,8} \\ \epsilon_{W9,9} \\ \epsilon_{W10,10} \\ \epsilon_{W11,11} \\ \epsilon_{W12,12} \\ \epsilon_{W13,13} \\ \epsilon_{W14,14} \\ \epsilon_{W15,15} \\ \epsilon_{W16,16} \end{bmatrix} \\
& \hspace{10em} (3)
\end{aligned}$$

The structural part of the model (Equation 4) includes lagged, directional relationships between the within-level constructs. The constructs at time T ( $\eta_T$ ) are regressed on the constructs at time T-1D ( $\eta_{T-1D}$ ) in the form ( $\eta_t = B \times \eta_{T-1D} + \zeta$ ). The diagonal elements of the B-matrix ( $\beta_{W1,5}, \beta_{W2,6}, \beta_{W3,7}, \beta_{W4,8}$ ) are auto-regressive parameters, indicating rank-order stability of the experiences. The off-diagonal elements indicate the cross-lags, for example  $\beta_{3,6}$ , the effect of autonomous motivation at time T regressed on competence belief at time T-1D.

$$\begin{bmatrix} \eta_{B1T} \\ \eta_{B2T} \\ \eta_{B3T} \\ \eta_{B4T} \\ \dots \\ \eta_{W1T} \\ \eta_{W2T} \\ \eta_{W3T} \\ \eta_{W4T} \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ \dots & & & & \\ \beta_{W1,5} & \beta_{W1,6} & \beta_{W1,7} & \beta_{W1,8} \\ \beta_{W2,5} & \beta_{W2,6} & \beta_{W2,7} & \beta_{W2,8} \\ \beta_{W3,5} & \beta_{W3,6} & \beta_{W3,7} & \beta_{W3,8} \\ \beta_{W4,5} & \beta_{W4,6} & \beta_{W4,7} & \beta_{W4,8} \end{bmatrix} \times \begin{bmatrix} \\ \\ \\ \\ \dots \\ \eta_{1T-1D} \\ \eta_{2T-1D} \\ \eta_{3T-1D} \\ \eta_{4T-1D} \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \dots \\ \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix} \quad (4)$$

with the  $\Psi$ -matrix (Equation 5):

$$\psi = \begin{bmatrix} \psi_{B1,1} \\ \psi_{B2,1} & \psi_{B2,2} \\ \psi_{B3,1} & \psi_{B3,2} & \psi_{B3,3} \\ \psi_{B4,1} & \psi_{B4,2} & \psi_{B4,3} & \psi_{B4,4} \\ \dots & & & \\ \zeta_{W1,1} \\ \psi_{W2,1} & \zeta_{W2,2} \\ \psi_{W3,1} & \psi_{W3,2} & \zeta_{W3,3} \\ \psi_{W4,1} & \psi_{W4,2} & \psi_{W4,3} & \zeta_{W4,4} \\ & & & \psi_{W5,5} \\ & & & \psi_{W6,5} & \psi_{W6,6} \\ & & & \psi_{W7,5} & \psi_{W7,6} & \psi_{W7,7} \\ & & & \psi_{W8,5} & \psi_{W8,6} & \psi_{W8,7} & \psi_{W8,8} \end{bmatrix} \quad (5)$$

The matrix for the residuals at the between-level is a diagonal matrix. At the within level we include correlated uniquenesses between indicators at time T and T-1D (Equation 6).



**(3) Model 3: Effects of lagged within-day and within-school-subject constructs at the within-level and teacher-perceptions at the between-level**

The first eight indicators  $y_1$ - $y_8$  have variance partitioned into both the between and within parts, as in Models 1 and 2. The next 15 indicators  $y_9$ - $y_{23}$  are estimated at the between-level only. The lagged within-day indicators  $y_{24}$ - $y_{31}$  and lagged within-school-subject indicators  $y_{32}$ - $y_{39}$  are estimated only at the within level. Note that we assigned these indicators other numeric values than in Model 2.

We specified seven substantive latent constructs at the between-level (Equation 7). In addition to the time-point T between-level constructs ( $\text{eff}_i$ ,  $\text{comp}_i$ ,  $\text{auto}_i$  and  $\text{cntr}_i$ ) we added teacher-reported academic performance (3 indicators;  $y_{B9}$ - $y_{B11}$ ), teacher-perceived student task-focus (6 indicators;  $y_{B12}$ - $y_{B17}$ ) with an additional method construct for negative wording ( $y_{B15}$ - $y_{B17}$ ), and teacher involvement with each student (6 indicators;  $y_{B18}$ - $y_{B23}$ ). The method construct was fixed to be uncorrelated with any other construct. This is represented as factor correlations fixed at zero in the  $\Psi$ -matrix in Equation 10.

In the within-part of the model the constructs at time T ( $\eta_{W1T-1D}$  to  $\eta_{W4T-1D}$ ) were specified by ( $\lambda_{w1,1}$  to  $\lambda_{w8,4}$ ). The lagged within-day constructs ( $\eta_{W9T-1D}$  to  $\eta_{W12T-1D}$ ) were specified by ( $\lambda_{w26,9}$  to  $\lambda_{w31,12}$ ), and the lagged within-school-subject constructs ( $\eta_{W13T-1D}$  to  $\eta_{W16T-1D}$ ) were specified by ( $\lambda_{w32,13}$  to  $\lambda_{w39,16}$ ), expressed in Equation 8.



[illegible]

(2)

[illegible]





(10)