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INFORMED PRINCIPAL WITH CORRELATION

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INFORMED PRINCIPAL WITH CORRELATION.[*]

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Abstract

In this paper we analyze a simple two-sided adverse selection model with one principal and one agent. They are both risk neutral and have private information about their type. We also assume that the private information of the principal is correlated with the one of the agent. The main result of the paper is that the principal can extract a larger share of the surplus from the agent than in the case where her information is public. The principal can design such a contract because she exploits the fact that her type is an informative signal on the agent's one. We fully characterize the equilibrium of the principal agent game in which different types of principal offer the same menu of contracts that leave the agent uninformed about the principal's type. This gives more freedom to the principal when setting the transfers because the agent's constraints need to hold only at an interim stage. The principal gains from a peculiarity of the correlated environment: different types of agent have different beliefs about the probability distribution over the states of the world.

Keywords: informed principal, private values, correlation.

JEL Classification: D82, D20, L21.

1 Introduction.

Most of the mechanism design theory is built around the hypothesis that there is an uninformed party (the principal) that contracts with an informed party (the agent). The principal offers a contract that the agent accepts or rejects, therefore the main problem for the principal is to find the optimal contractual way to elicit the agent's private information. Although this paradigm has been applied to many different contexts and has proved itself to be quite powerful

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in explaining many economic interactions the assumption of one-sided private information is sometimes restrictive.

Often both principal and agent are endowed with private information which is important for the efficiency of the economic trnsaction between them. One example is the provision of a public good, where usually the lack of information rests with the government (principal) regarding citizens' (agents') private evaluations. However, it is likely that the government possesses superior information about the cost of supplying the good. Also in a regulated market the authority (principal) may have private information about the market demand for the regulated good even if it does not know the costs incurred by the firms (agents). Finally, a discriminating monopolist may have private information about the quality of the good it provides. In all these situations we face what is known in the literature as an "informed-principal" problem.

Once we assume double sided private information the problem that an informed principal has to solve is considerably more complicated. When designing a contract, a principal needs to worry about rent extraction and what to do with her information. She has to decide whether to make it public, and, if so, she has to do it in a credible way. Interestingly, a principal might be worse off by having this private information; in fact in some circumstances she may not be able to use it to her benefit. The main problem in this respect comes from the signalling content that any action taken by the principal may have for the agent.

In this work we analyze a simple two sided adverse selection model with one principal and one agent. They both have private information about their types and a type-dependent quasilinear utility function. We also assume that the private information of the principal is correlated with the one of the agent. In other words, after observing her private information the principal updates non-trivially her beliefs on the agent characteristics. The same is true for the agent.

The main result of the paper is that the principal can extract a larger share of the surplus from the agent than in the case where her information is public. Intuitively, the principal can design such a contract because she exploits the fact that her type is an informative signal of the agent's one.

The existing literature, in particular Maskin and Tirole (1990), has explored this problem in details focusing on the risk sharing benefits to the principal of hiding her private information. They propose the interpretation of this principal-agent game as a fictitious exchange economy where the different types of principal are consumers that trade the slackness on the agent's constraints. They show that there exists a Walrasian equilibrium of this economy and the main reason driving this exchange is risk sharing.

Different types of principal have different attitudes with respect to the risk of facing a "bad" type agent, therefore some arbitrage is mutually beneficial. Hence when the principal is risk averse there exists a strict gain from trade among the different types of principal. If the principal reveals her type when offering the contract, then the incentive compatibility and individual rationality constraints have to be satisfied for each type and hence no trade is possible and no risk sharing possibility arises. If, instead, a risk averse principal hides her private information then she can capture the gains of sharing risk with other types of principal. Clearly, if the principal is risk neutral those gains are not present and hence there is no advantage from hiding her type. It is for this reason that, in their framework, when the utility functions of both parties are assumed to be quasilinear there is no possibility of obtaining a higher payoff: if the different types of principal are risk neutral, they do not benefit from any risk-sharing activity.

In this paper we depart from this analysis by assuming that the principal is risk neutral and that her private information is correlated with the one of the agent. Obviously, in this case, there exist no gains from risk sharing. However, we show the principal can still gain from hiding her private information.

If the contract offer does not reveal any information then the agent remains uninformed about the principal's type and his constraints will have to hold only in expectation. This gives more freedom to the principal who can profit by relaxing these constraints in some states of the world. The peculiarity of this correlated environment is that even if the principal is risk neutral the relative costs of satisfying the agent's constraints is different across types of principal. The presence of correlation, even when principals of every type are risk neutral, allows the marginal rate of substitution between different states of the world to be different across types, so there exists a relative price that makes the exchange beneficial. Obviously the principals do not trade to share risk, they trade because one values relatively more the slack on a constraint than the other does. In other words there exist market prices such that the principals buy something on the market that is cheaper than their private evaluation and sell something that they value relatively less than the market. In this way they can save on the transfers given to the agent.

This can be best seen in the characterization of the equilibrium of the principal agent game when we show that if both types of principal make the same contract offer, they achieve a higher payoff since they manage to reduce the transfers, while the physical allocation remains the same. As a consequence the informational rent of the agent is reduced in favor of an increase in the expected payoff of the principal, who is able to extract more surplus.

We are certainly not the first to suggest that a principal that exploits the informativeness of a private information of individuals, other that the agent, can reduce the rent transferred to the agent (Crémer and McLean (1985,1988), McAfee and Reny (1992)). In particular these papers show that an uninformed principal facing many privately informed agents, whose types are correlated, can design a mechanism that extracts all the surplus from the agents. The principal can then achieve the first best because there is no more need to trade off rents with efficiency. The mechanism uses a sort of yardstick competition among the agents created by constructing lotteries that consist of payments conditional of the announcements of the other agents. The key difference between these papers and ours is that the signal that the principal wishes to exploit is her private information and this implies that any feasible contract must involve incentives for the principal to reveal truthfully. Satisfying these incentive compatibility constraints for the contract designer introduces an extra degree of complexity in our analysis and prevents the principal form extracting all the surplus form the agent as in the above mentioned works.

Another related paper is Riordan and Sappington (1988) where the authors show that a principal can achieve the first best outcome if she can condition a contract on a signal correlated to the type of the agent and that will be made public ex-post. In our case the type of the principal (which is also her signal) remains private information.

The structure of the paper is as follows. In Section 2 we present the setup, we discuss the structure of the game and we study the benchmark of full information. In Section 3 we show that in our fictitious exchange economy there exists a set of equilibria that Pareto dominates the full-information outcome. In Section 4 we characterize the equilibrium of the principal-agent game. In Section 5 concludes. The Appendix contains some proofs and a simple example where it is possible to appreciate the main results in a concise and transparent way.

2 The model.

We are going to compare our analysis to the results of Maskin and Tirole (1990) and also point to some of their results, therefore the set-up and the notation will be kept as close as possible to the ones in their contribution.

2.1 Objective functions and information.

There are two players, a principal and an agent. The principal has a quasi-linear utility function $V^i = \phi^i(y) - t$, where y is an observable and verifiable action, t is a monetary transfer from the principal to the agent, and i is a parameter that represents the principal private information or type. $\phi^i(\cdot)$ is continuous, increasing and concave in y.

The agent has a quasi-linear utility function $U_j = t - \psi_j(y)$, where j represents the agent's type. It is worth noting that U does not depend from j, the principal's private information, this assumption is important and places our model in the private values framework. $\psi_j(\cdot)$ is increasing and convex in y. Moreover we assume that U_j decreases with j, this means that:

$$\psi_1(y) < \psi_2(y) \text{ for all } y.$$

We further make the assumption that $\frac{\partial \psi_1(y)}{\partial y} < \frac{\partial \psi_2(y)}{\partial y}$ for all y.

The agent's reservation utility is normalized to zero.

This abstract set-up fits well the following real world situation: a buyer (the principal), with private information about her preferences for a good, offers a contract to a seller (the agent), that produces the good and has private information about his production costs. In that case y will be the quantity sold and t the price paid by the buyer.

In what follows we may use μ^i_j to indicate the pair (y, t) selected by the mechanism when the principal is of type i and the agent of type j.

To guarantee the existence of equilibrium, we assume that the feasible actions and transfers lie in compact and convex sets[1].

The parameters i and j are drawn from a joint discrete distribution which is common knowledge. We suppose that each parameter can assume only two values, therefore there are only four possible states of the world. The Principal's prior beliefs about the joint distribution of types are:

$$\begin{aligned} p_{11} &= \Pr(i = 1, j = 1) \\ p_{12} &= \Pr(i = 1, j = 2) \\ p_{21} &= \Pr(i = 2, j = 1) \\ p_{22} &= \Pr(i = 2, j = 2) \end{aligned}$$

The agent's prior beliefs are identical to those of the principal, posterior beliefs are denoted by $\dot{p}_{ij}$.

We then define $\rho = p_{11}p_{22} - p_{12}p_{21} \neq 0$ as the correlation coefficient between the two player's information, when it is positive it means that it is relatively more likely that they are of the same type, when negative "mixed" pairs are more likely.

We are therefore assuming that each player's type is an informative signal of the other's type. As a consequence conditional distribution of the agent's type are different for the two types of principal, the same is true for the agent.

As in Maskin and Tirole the limitation on the possible types for the players is not essential but simplifies the analysis and favors the intuition of the results.

2.2 The principal-agent game.

The timing of the principal-agent game is as follows:

1. The principal proposes a mechanism in the feasible set M to the agent. A mechanism m in M will specify i) a set of possible messages for each party and ii) for each pair of messages chosen simultaneously an allocation (y, t). Note that the set M includes the set of direct revelation mechanisms in which parties simultaneously announce their types, by invoking the revelation principle for Bayesian game we can restrict the attention to direct truthful mechanisms.[2]

2. The agent updates his prior (if he has learned something from the offer), accepts or refuses the contract offered. If he refuses both players get zero utility and the game ends. If the

[1] This assumption can be seen partly as a sort of limited liability for the agent, the differenc eis that the transfers not only have a lower bound but also an upper bound. This is needed to ensure the compactness of the budget set in the fictitious Walrasian economy in the next section.

[2] In this framework (as in Maskin and Tirole (1990)) the principle states that for any mechanism and for given beliefs any equilibrium of the mechanism is equivalent to an equilibrium of a direct revelation mechanism in which types are truthfully announced.

agent accepts, the principal updates her beliefs, and the parties move to the last stage of the game.

3. Both parties announce their types and the proposed mechanism is implemented.

We will study the perfect Bayesian equilibria of the overall game.

2.3 The case of full information.

As a benchmark we study the equilibrium when the principal's information is common knowledge. Maskin and Tirole call it the *full information* case (even if the principal does not know the agent's type) and it is nothing more than the standard screening model.

We know from the revelation principle that every equilibrium allocation of this game can be obtained as an equilibrium of a direct truthful mechanism. The outcome μ_j^i that will be implemented in equilibrium will have to satisfy two types of constraints individual rationality and incentive compatibility.

For every i the participation constraints are: $U_j\left(\mu_j^i\right) \geq 0$ for $j = 1, 2$. While the truthtelling constraints are: $U_j\left(\mu_j^i\right) \geq U_j\left(\mu_k^i\right)$ for all j,k.

Standard arguments apply, and in this context only two constraints are binding, the participation constraint of type 2 and the incentive compatibility of type 1.

Therefore in the case of full information a principal of type i proposes a contract $\left\{\mu_1^i, \mu_2^i\right\}$ that solves the following program:

$$\left(F^i\right)\begin{cases} \max\limits_{\left\{\mu_j^i\right\}} \sum\limits_{j=1}^{2} p_{ij} V^i\left(\mu_j^i\right) \text{ such that} & \\ \mathrm{IR}^i : U_2\left(\mu_2^i\right) \geq 0 & \left(\rho^i\right) \\ \mathrm{IC}^i : U_1\left(\mu_1^i\right) \geq U_1\left(\mu_2^i\right) & \left(\gamma^i\right), \end{cases}$$

where ρ^i and γ^i are the Lagrange multipliers for the IR and IC constraints.[3]

Given the specific functional forms chosen for the utility functions of the two players we can actually find the precise solution to this problem.

A principal of type i will offer the following decreasing schedule of output and the respective transfers, $(y_{i1}, y_{i2}, t_{i1}, t_{i2})$:

$$\begin{aligned} \phi^{i\prime}\left(y_{i1}\right) &= \psi_1'\left(y_{i1}\right) \text{ and } t_{i1} = \psi_1\left(y_{i1}\right) + \left(\psi_2\left(y_{i2}\right) - \psi_1\left(y_{i2}\right)\right) \\ \phi^{i\prime}\left(y_{i2}\right) &= \psi_2'\left(y_{i2}\right) + \frac{p_{i1}}{p_{i2}}\left(\psi_2'\left(y_{i2}\right) - \psi_1'\left(y_{i2}\right)\right) \text{ and } t_{i2} = \psi_2\left(y_{i2}\right). \end{aligned}$$

As one could expect, the solution preserves standard characteristics like the "no distortion at the top" property and no informational rent for the "bad" agent.

[3] A more detailed solution of the full-information problem can be found in the Appendix.

For future reference denote by $\left(\overline{\mu}^i_\cdot, \overline{\rho}^i, \overline{\gamma}^i\right)$ the solution to the full information program $\left(F^i\right)$ and let $\overline{v}^i \equiv \sum_j p_j V^i\left(\overline{\mu}^i_j\right)$ be the type i principal's payoff.

Moreover at the full-information allocation the ratios of the Lagrange multipliers of the two types of principal are different ($\frac{\overline{\gamma}^1}{\overline{\rho}^1} = \frac{p_{11}}{p_{12}+p_{11}} \neq \frac{\overline{\gamma}^2}{\overline{\rho}^2} = \frac{p_{21}}{p_{21}+p_{22}}$) meaning that the relative cost of fulfilling the individual rationality and incentive compatibility constraint is not the same across principals. This fact is going to be extremely important to prove the results in what follows.[4]

A feature, common to all the private values models, is that, regardless of the agent's information about the principal's type, $\overline{v}^i$ provides a lower bound to the type i principal's equilibrium payoff. This means that the full information contract is incentive compatible[5] for each type of principal. We are going to show that it is possible to find equilibria that improve on this payoff.

3 The fictitious exchange economy and the Walrasian equilibria.

The problem we are studying changes considerably if the type of the principal is private information. From the previous section we know that offering the full information contract is a possibility since it is incentive compatible. Another possibility for the principal is to hide her type until the implementation stage, in this way when the agent accepts the contract he would not know with certainty the state of the world. He would know his type and have a belief on the type of the principal which coincides with the prior probabilities if he has not learned anything from the contract offer. We are going to show that for the different types of principal it is possible to design a mechanism that Pareto dominates the full information allocation. It is going to achieve this by "pooling" the agent's IR and IC constraints over the different types of principal, i.e. by having the constraints hold only in expectation rather than for each single type.

This methodology leads to the study of the Walrasian equilibria of the fictitious pure exchange economy where the traders are the two types of principal that exchange the slack variables of the agent binding constraints. When we are in the full information framework trade is not possible because the constraints have to be satisfied ex-post in every state of the world and slackness on them is not allowed (as if markets were totally absent). As soon as the agent does not know the type of the principal then his constraints need to be satisfied only in expectation, offering the principals (the different types) the possibility of exchanging slackness (as if markets were now open and complete).

The following proposition introduces the idea of existing gains from the trade of slackness in our "economy".

[4] The ratio of the Lagrange multipliers would be the same for the two types of principal if types were independently distributed.

[5] Each type of principal maximises his expected payoff over the same set of constraints, therefore they cannot do better by claiming to be of another type and implement this other type's contract.

Proposition 1 *When utility functions are quasilinear and there exists correlation between the information of the principal and the one of the agent, there exists an allocation that satisfies interim IR and IC constraints for the agent and that Pareto dominates the full-information allocation* $\overline{\mu}$ *(for all the different types of principal).*

Proof. Consider the solution $\left(\overline{\mu}^i_\cdot, \overline{\rho}^i_\cdot, \overline{\gamma}^i_\cdot\right)$ to $\left(F^i\right)$, we have shown already that $\frac{\overline{\gamma}^1}{\overline{\rho}^1} \neq \frac{\overline{\gamma}^2}{\overline{\rho}^2}$.

For an allocation $\mu^i_\cdot$, define now $r^i\left(\mu^i_\cdot\right)$ and $c^i\left(\mu^i_\cdot\right)$ as the negatives of the slack variables associated with the IR^i and IC^i constraints:

$$\begin{aligned} r^i\left(\mu^i_\cdot\right) &\equiv -U_2\left(\mu^i_2\right) \\ c^i\left(\mu^i_\cdot\right) &\equiv U_1\left(\mu^i_2\right) - U_1\left(\mu^i_1\right). \end{aligned}$$

In particular, $r^i\left(\overline{\mu}^i_\cdot\right) = 0$ and $c^i\left(\overline{\mu}^i_\cdot\right) = 0$; in fact in the full information problem the constraints have to be satisfied state by state therefore the slack variables in each of them have to be necessarily equal to zero. In case the offer of the contract is not fully revealing then, as we said before, then the constraints would have to be satisfied only in expectation. In terms of slack variables as we just have defined them the IR and IC constraints can be expressed as:

$$IR : \sum_i \dot{p}_{ij} r^i\left(\mu^i_\cdot\right) \leq 0 \text{ and } IC : \sum_{ij} \dot{p}_{ij} c^i\left(\mu^i_\cdot\right) \leq 0.$$

Which says that the negatives of the slack variables need only be non-positive on average, and not for each type of principal. More precisely, in the case of only two principal's types, the above conditions are equivalent to:

$$r^2 = -\frac{\dot{p}_{21}}{\dot{p}_{22}} r^1 \text{ and } c^2 = -\frac{\dot{p}_{11}}{\dot{p}_{12}} c^1.$$

Consider now the following perturbed version of the full information program:

$$\left(F^i_*\right) \begin{cases} \max\limits_{\{\mu^i_j\}} \sum\limits_{j=1}^{2} p_{ij} V^i\left(\mu^i_j\right) \quad \text{such that} \\ U_2\left(\mu^i_2\right) \geq -r^i \\ U_1\left(\mu^i_1\right) \geq U_1\left(\mu^i_2\right) - c^i \end{cases}$$

It is evident that the only difference from the full-information program is that now there is some slack allowed on each constraint.

Let v^i_* be the maximized value of the maximand, by definition of the shadow prices $\overline{\rho}^i$ and $\overline{\gamma}^i$ it approximately equals $\overline{v}^i + \overline{\rho}^i r^i + \overline{\gamma}^i c^i$ for small values of r^i and c^i. Let μ^i_* be a solution to F^i_*.

Choose negative slack variables $\left(r^1, c^1\right)$ for the type 1 principal; then the slack variables for type 2 are defined according to the above conditions. Therefore we can now write:

$$
\begin{aligned}
v_*^1 - \overline{v}^1 &\simeq \overline{\rho}^1 r^1 + \overline{\gamma}^1 c^1 \\
v_*^2 - \overline{v}^2 &\simeq \overline{\rho}^2 \left(-\frac{\dot{p}_{21}}{\dot{p}_{22}} r^1 \right) + \overline{\gamma}^2 \left(-\frac{\dot{p}_{11}}{\dot{p}_{12}} c_1 \right)
\end{aligned}
$$

The left hand sides of the above equalities are both positive if:

$$
\begin{cases}
\frac{\overline{\rho}^1}{\overline{\gamma}^1} > -\frac{c^1}{r^1} \\
-\frac{c^1}{r^1} > \frac{\overline{\rho}^2}{\overline{\gamma}^2} \frac{\dot{p}_{12}\dot{p}_{21}}{\dot{p}_{11}\dot{p}_{22}}
\end{cases}
\text{when } r^1 > 0 \ ,
$$

and:

$$
\begin{cases}
\frac{\overline{\rho}^1}{\overline{\gamma}^1} < -\frac{c^1}{r^1} \\
-\frac{c^1}{r^1} < \frac{\overline{\rho}^2}{\overline{\gamma}^2} \frac{\dot{p}_{12}\dot{p}_{21}}{\dot{p}_{11}\dot{p}_{22}}
\end{cases}
\text{when } r^1 < 0 \ ,
$$

We know, from the solution to the full information case, that $\frac{\overline{\gamma}^1}{\overline{\rho}^1} = \frac{p_{11}}{p_{12}+p_{11}}$ and $\frac{\overline{\gamma}^2}{\overline{\rho}^2} = \frac{p_{21}}{p_{21}+p_{22}}$ if we substitute these values in the above conditions and the compute the beliefs using Bayes' rule[6] we get that both systems are satisfied if:

$$\rho > 0 \text{ (for } r^1 > 0) \text{ and } \rho < 0$$

which hold by assumption.

We have therefore shown that μ_*^i solution to $\left(F_*^i\right)$ Pareto dominates the full information allocation $\overline{\mu}^i$ from the perspective of both types of principal, when the correlation is non zero. ■

The intuition of Proposition 1 is relatively simple. In the full information case the agent's constraints have to be satisfied for each type i of principal; if we introduce a small amount of slack $-r^i$ and $-c^i$ on these constraints then the principal can obtain the payoff: $\overline{v}^i + \overline{\rho}^i r^i + \overline{\gamma}^i c^i$.

As long as $\sum_i \dot{p}_{ij} r^i \left(\mu_\cdot^i\right) \leq 0$ and $\sum_{ij} \dot{p}_{ij} c^i \left(\mu_\cdot^i\right) \leq 0$, then the agent's constraints hold in expectation. We can choose $\left(r^1, c^1, r^2, c^2\right)$ in such a way that $v^i - \overline{v}^i$ is strictly positive for $i = 1, 2$. The allocation $\mu_{\cdot *}^{\cdot}$ corresponding to this choice then Pareto dominates $\overline{\mu}_\cdot^\cdot$.

A useful and fruitful interpretation is thinking of $\mu_{\cdot *}^{\cdot}$ as being generated by the different types of principal "trading" slack variables. In that case the full information allocation corresponds to autarchy.

The idea is that if different types of principal have different shadow values for the constraints that means that they value differently the relative slackness on the constraints and they can gain from trading it. This means that the two types of principal have different marginal rates of substitution between different state of the world, because given their type they attach dif-

[6] To compute beliefs we have to consider all the possible types of contract offer by the different types of principal. The contract acts as a signal. Note that $\frac{\dot{p}_{12}\dot{p}_{21}}{\dot{p}_{11}\dot{p}_{22}} = \frac{p_{12}p_{21}}{p_{11}p_{22}}$ in case of pooling or semi-separating offer by the principal. We therefore obtain that our conditions cannot be satisfied if the offer is separating, i.e. the full information allocation.

ferent probability to each state of the world. They are allowed to trade slackness only if the constraints for the agent have to hold in expectation which can happen when the principal does not reveal her type at the contract offer stage. This amounts to having different marginal rates of substitution between two states of the world and not disclosing information allows to exploit advantageous trading opportunity, which would be unavailable otherwise.[7]

This Pareto improvement is available to the types of principal also when their utility function is quasilinear and it is due to the correlation between the private information of the principal and the one of the agent.

In the quasilinear case when types are independently distributed, as in Maskin and Tirole (1990), the ratio of the shadow values of the full information case is the same for both types of principal therefore no gain from trade exists in that exchange economy. But the equality of these ratio is not due only to the specific functional form of the utility function but also to the independence hypothesis. These ratios represent the probability of the agent being of type 1 given the type of the principal, because of correlation these ratios have to be different for different types of principal.

Therefore correlation allows principals of different types with a quasilinear utility function to benefit from the trade of slackness across different states of the world.

In the work of Maskin and Tirole (1990) the quasilinear case represented a subset of the more general framework (with generic utility functions) in which the possibility for the principal of gaining from concealing her private information did not hold. As mentioned in the introduction, they trade for risk sharing reasons, so when the principal is risk neutral there are no gains to be made from risk sharing. Adding correlated types to the picture puts the quasilinear case back in line with their main results, even if the motivation for trading is different. The principals trade because they have a higher valuation then the market for the slackness they buy, and lower for the one they sell.

Let now $V_I^i\left(r^i, c^i\right)$ be the principals' indirect utility when there is slack $-r^i$ and $-c^i$ in the agent's participation and incentive compatibility constraints, respectively. Thus $V_I^i\left(r^i, c^i\right)$ is the value function of the perturbed full information problem $\left(F_*^i\right)$ we introduced in the Proof of Proposition 1 and that we can rewrite with our specific functional form as:

$$\left(F_*^i\right)\begin{cases}\max\limits_{\{y_{ij},t_{ij}\}}\sum\limits_{j=1}^{2}p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right)\ \text{ such that}\\ t_{i2}-\psi_2\left(y_{i2}\right)=-r^i\\ t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)=-c^i\end{cases}$$

[7]In the literature this fact is also known as "Hirshleifer" effect (see Hirshleifer (1971)).

The solution to this problem entails the same quantities as the full information case[8] but different transfers which depend on r^i and c^i, namely:

$$\begin{aligned} t_{i1} &= \psi_1(y_{i1}) + (\psi_2(y_{i2}) - \psi_1(y_{i2})) - r^i - c^i \\ t_{i2} &= \psi_2(y_{i2}) - r^i. \end{aligned}$$

By implementing a contract with the quantities and different transfers the principal does not reduce the productive inefficiency of the full-information allocation (which is nothing but the "usual" second best solution), she just manages to improve her expected payoff by reducing the transfers given to the agent. By trading slackness they succeed in extracting more surplus from the agent.

The indirect utility function $V_I^i(r^i, c^i)$ already incorporates a maximization over quantities and transfers, we can obtain its specific form by substituting the argmax of problem (F_*^i). Since we are interested only in the effect of the slack variables we can consider as constant everything in the value function which does not depend on r^i and c^i. We can therefore write:

$$V_I^i(r^i, c^i) = p_{i1}(K_{i1} + r^i + c^i) + p_{i2}(K_{i2} + r^i).$$

Suppose now that the type i principal can "buy" and "sell" slack in the agent's constraints at prices ρ and γ subject to the "budget" constraint that the value of the negative slack purchased be non-positive.[9] The principal would then take her trading decision through the solution of the following problem:

$$(D^i) = \begin{cases} \max_{\{r^i, c^i\}} p_{i1}(K_{i1} + r^i + c^i) + p_{i2}(K_{i2} + r^i) \text{ subject to} \\ \rho r^i + \gamma c^i \leq 0 \end{cases}$$

We now have to check whether the conditions for the existence of a solution to this "consumer" problem are satisfied. The utility functions are linear, therefore also concave. Let $B^i(\rho, \gamma) \equiv \{r^i, c^i \text{ s.t. } \rho r^i + \gamma c^i \leq 0\}$ be the budget set of principal i, the following Lemma proves its compactness.

Lemma 2 *B^i is a compact set.*

Proof. By assumption feasible actions and transfers lie in a compact set. The function that maps action and transfers, y_{ij} and t_{ij}, into the space of "feasible" slack variables is linear.

[8] In fact the quantities are inplicitely defined by:

- $\phi^{i\prime}(y_{i1}) = \psi_1'(y_{i1})$
- $\phi^{i\prime}(y_{i2}) = \psi_2'(y_{i2}) + \frac{p_{i1}}{p_{i2}}(\psi_2'(y_{i2}) - \psi_1'(y_{i2}))$

[9] This means that the value of the final slack "consumption" bundle should not exceed the value of her endowment, which for both types is zero since they start from the full information allocation.

Therefore it is continuous and has a continuous inverse therefore also the space of r^i and c^i is compact. Then also $B^i(\rho,\gamma)$ is compact. ■

Therefore each principal maximizes a linear (and concave) function over a compact set, then the solution to the program $\left(D^i\right)$ is a correspondence $D^i(\rho,\gamma)$ which we can interpret as the Walrasian demand of slackness of principal i.

It can be shown (the Proof is in the Appendix) that a Walrasian Equilibrium of this economy exists and it is a pair of positive prices (ρ,γ) and a choice of negative slack variables $\left(r^i,c^i\right)$ for each type i such that:

$$\begin{aligned} \sum_i \dot{p}_{ij} r^i\left(\mu_\cdot^i\right) &= 0 \text{ and } \sum_i \dot{p}_{ij} c^i\left(\mu_\cdot^i\right) = 0 \\ \left(r^i,c^i\right) &\in D^i(\rho,\gamma) \end{aligned}$$

The first set of conditions are "market clearing" requirements, which ensure that the average amount of slack demanded is equal to the average supply, i.e. zero.[10]

In Figure 1 we represent our exchange economy in a Edgeworth-like diagram. First note that the economy is not represented by a box, this is because there are no fixed endowments of slackness. In principle, as long as the market clearing conditions are satisfied we can have a very large amount of slackness (e.g. if c^1 is very big and positive then c^2 will be very big and negative). The indifference curves of the two types of principal are straight lines with different slopes, they would have the same slope if the types were independently distributed.[11]

The origin of the axis is the endowment point, in fact the types of principal start trading having zero slackness on the constraints. The area in the bottom right between the two thick indifference curves that go through the origin represents the possible gains from trade. In this area both types of principal would be on a higher indifference curve.

From the picture it is clear that we will have infinitely many possible equilibria, all in the bottom right region.[12] To put it simply, we have two degrees of freedom, by choosing a value for r_1 and c_1 (in that region) then we univocally define the values for r_2 and c_2 while the terms of the trade are going to be given by the slope of the line that goes through the equilibrium and the endowment[13].

It can be proven that these Walrasian equilibria possess a whole set of properties which carry

[10] This exactly amounts to satisfying the agent's constraints in expectation.

[11] The indifference curves of type 1, in the r^1,c^1 space are of the following type:

$$c^1 = -\left(1+\frac{p_{12}}{p_{11}}\right)r^1,$$

while those of type 2 are:

$$c^1 = -\left(1+\frac{p_{12}}{p_{11}} - \frac{\rho}{p_{11}p_{22}}\right)r^1.$$

If $\rho = 0$ then they would exactly coincide.

[12] Since r and c belong to a compact set, the equilibria will be on the boundary of this set.

[13] The equilbirum prices will be $\frac{\widehat{\gamma}}{\widehat{\rho}} \in \left[\frac{p_{21}}{p_{21}+p_{22}}, \frac{p_{11}}{p_{11}+p_{12}}\right]$

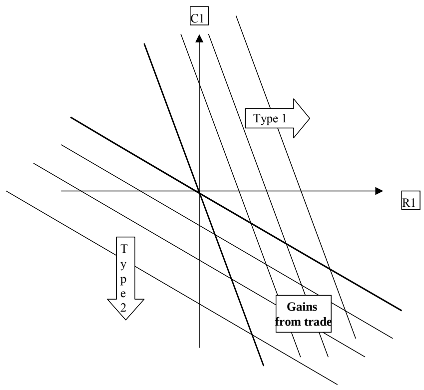

Figure 1: Gains from trade in the fictitious exchange ecnomy.

the flavor of the First and Second welfare theorems, but for all these results refer to Maskin and Tirole (1990).

4 The equilibrium in the principal-agent game.

In the previous section we have shown that there exists many possible equilibria that secure to both types of principal a higher payoff when she is able to satisfy the agent's constraints in expectation. At this stage it is worth discussing the relationship between Walrasian equilibria of the fictitious exchange economy and the Bayesian equilibria of the principal-agent game. This is the goal of the following proposition.

Proposition 3 *For any Walrasian allocation of the fictitious exchange economy there exists a perfect Bayesian equilibrium where both types of principal propose the same contract and where the equilibrium outcome is this Walrasian allocation.*

Proof. Consider a Walrasian equilibrium $\left\{(\widehat{\rho},\widehat{\gamma}),\left(\widehat{r}^i,\widehat{c}^i\right)_{i=1,2}\right\}$ and let $\widehat{\mu}_\cdot^\cdot$ be the corresponding allocation. The equilibrium path is going to be the following: both principals propose the direct revelation mechanism $\widehat{\mu}_\cdot^\cdot$, the agent does not learn anything from this offer about the principal's type so his belief do not change. All types of agent accept the contract and both parties announce their types truthfully at the third stage.

To show that this is an equilibrium we proceed backward. At the third stage the agent will reveal his type truthfully because is interim IC constraints are satisfied by the Walrasian allocation. Because of the property of "no-envy" of the Walrasian allocation also the IC constraint of both types of principal are satisfied.

At the second stage the agent will accept the contract because at the third stage he will obtain at least his reservation utility.

It remains to show that at the first stage the principal does not want to offer a contract other than $\widehat{\mu}_{\cdot}^{\cdot}$. This can be done by choosing the appropriate off-equilibrium path strategies and beliefs. These are arbitrary, as cannot be derived with Bayes rule. These belief need to be chosen in such a way that if the principal proposes another mechanism all types of principal are no better off than with $\widehat{\mu}_{\cdot}^{\cdot}$. Suppose, as argued in Maskin and Tirole (1990) that a mechanism m is offered, and suppose that the agent has out of equilibrium beliefs such that $(\dot{p}_{1j} = 1, \dot{p}_{2j} = 0)$, then a type 1 principal will receive at most the full information payoff $\overline{v}^1$. Similarly if beliefs are $(\dot{p}_{1j} = 1, \dot{p}_{2j} = 0)$ then type 2 will at most obtain $\overline{v}^2$. From continuity and because $\widehat{\mu}_{\cdot}^{\cdot}$ is strongly pareto optimal for the prior beliefs, then there exist intermediate beliefs for which both types do not prefer of deviate and offer m. ■

In synthesis, on the equilibrium path the principal proposes a Walrasian allocation as a direct revelation mechanism, the proposal is accepted and at the final stage both parties announce their types truthfully.

In this section our goal is to characterize such equilibria in the more standard principal-agent framework. More precisely we are going to study the contract offer that is going to lead to one of those equilibria.

First of all we need to ensure that the agent does not learn the type of the principal after she has offered the contract. That is why we are going to look for a "pooling offer" by the two types of principal. Pooling offer does not mean that in equilibrium different types of principal will implement the same allocation it just means the different types of principal will offer the same menu of contracts, knowing already that some of them (the ones contingent on types other that their true one) will never be implemented. In our particular setup each principal is going to offer four pairs, an action and a transfer, one for each possible state of the world knowing already that two of them will not be implemented. Offering a whole menu of contracts turns out to be a useful device for keeping the agent uninformed.

Secondarily we are going to make sure that the contract will be accepted by both types of agents and that it is incentive compatible. It has to be true in fact that, at the third stage when the players announce simultaneously their type, the agent reveals it truthfully.

Third, we are going to require that the contract is incentive compatible for the principal. That is, the contract offer has to be such that each principal prefers her possible allocations to the one of the other type. This will ensure a truthful announcement by the principal at the third stage.

Finally we want the different types of principal to profit from this game with respect to the full information case, that, in a private values framework, constitutes the lower bound to each principal's payoff.

Then, in a pooling offer equilibrium the contract proposed by the principal, regardless of her type, will be $(y_{ij}, t_{ij})_{i,j=1,2}$ such that it is a solution, for $i, k = 1, 2$ $(i \neq k)$, of:

$$\left(P^i\right)\begin{cases} \max\limits_{\{y_{ij},t_{ij}\}} \sum\limits_{j=1}^{2} p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right) \text{ such that} \\ IR_2 : \sum\limits_{i=1}^{2} \dot{p}_{i2}\left(t_{i2}-\psi_2\left(y_{i2}\right)\right)\geq 0 \\ IC_1 : \sum\limits_{i=1}^{2} \dot{p}_{i1}\left(t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)\right)\geq 0 \\ ICP^i : \sum\limits_{j=1}^{2} p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right)\geq \sum\limits_{j=1}^{2} p_{ij}\left(\phi^i\left(y_{kj}\right)-t_{kj}\right) \end{cases}$$

A solution to this problem is then incentive compatible for the principal and the agent and will be accepted by both types of agent.[14] In addition it can be shown that the incentive compatibility constraint of an agent of type 2 is not binding at the optimum (a proof can be found in the Appendix).

After having found a solution to the above problem we will have to check that the expected payoff for each type of principal is higher with this allocation than at the full information equilibrium, that is the following conditions must hold:

$$\begin{aligned} \pi_1 &: \sum_{j=1}^{2} p_{1j}V^1\left(\widetilde{\mu}_j^1\right)\geq \sum_{j=1}^{2} p_{1j}V^1\left(\overline{\mu}_j^1\right) \\ \pi_2 &: \sum_{j=1}^{2} p_{2j}V^2\left(\widetilde{\mu}_j^2\right)\geq \sum_{j=1}^{2} p_{2j}V^2\left(\overline{\mu}_j^2\right), \end{aligned}$$

where $\widetilde{\mu}_j^i$ is solution to P^i while $\overline{\mu}_j^i$ is the equilibrium allocation of the full information case.

We are now ready to state the following result, which characterizes the equilibrium of the principal-agent game.

Proposition 4 *The following strategies form a perfect Bayesian equilibrium of the principal agent game:*

- *At date 1 both types of principal offer the same contract* $\widetilde{m}=\left(y_{ij},t_{ij}\right)_{i,j=1,2}$ *that satisfies the following conditions for* $i=1,2$*:*
  - $\phi^{i\prime}\left(y_{i1}\right)=\psi_1'\left(y_{i1}\right)$
  - $\phi^{i\prime}\left(y_{i2}\right)=\psi_2'\left(y_{i2}\right)+\frac{p_{i1}}{p_{i2}}\left(\psi_2'\left(y_{i2}\right)-\psi_1'\left(y_{i2}\right)\right)$
  - $\sum\limits_{i=1}^{2} p_{i2}\left(t_{i2}-\psi_2\left(y_{i2}\right)\right)=0$
  - $\sum\limits_{i=1}^{2} p_{i1}\left(t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)\right)=0$
- *At date 2 the posterior beliefs of the agent are unchanged and equal to the priors, and all types of agent accept the proposed mechanism* $\widetilde{m}$*.*

[14] Standard considerations ensure the satisfaction of the participation constraint of a type 1 agent.

- *At date 3 both parties announce their type truthfully and implement the mechanism.*
- *The equilibriumj Pareto dominates the Full Information allocation.*

Proof. First note that any contract m which is a solution to problem $\left(P^i\right)$ is incentive compatible for the principal and agent and is individually rational for the agent. We need therefore to show that $\widetilde{m}$ is indeed a solution to $\left(P^i\right)$.

To begin with, note that $\widetilde{m}$ is a solution to the less constrained problem $\left(P_*^i\right)$ which is defined as:

$$\left(P_*^i\right)\begin{cases}\max\limits_{\{y_{ij},t_{ij}\}}\sum\limits_{j=1}^{2}p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right)\text{ such that}\\ IR_2:\sum\limits_{i=1}^{2}\dot{p}_{i2}\left(t_{i2}-\psi_2\left(y_{i2}\right)\right)\geq 0\quad\left(\widetilde{\rho}_i\right)\\ IC_1:\sum\limits_{i=1}^{2}\dot{p}_{i1}\left(t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)\right)\geq 0\quad\left(\widetilde{\gamma}_i\right).\end{cases}$$

Program $\left(P_*^i\right)$ is the same as $\left(P^i\right)$ except that the incentive compatibility constraints for the two types of principal have been omitted.

The first order conditions for this problem are:

$$\begin{aligned}\frac{\partial L}{\partial y_{i1}}&=p_{i1}\phi^{i\prime}\left(y_{i1}\right)+\dot{p}_{i1}\widetilde{\gamma}_i\psi_1'\left(y_{i1}\right)=0\\ \frac{\partial L}{\partial y_{i2}}&=p_{i2}\phi^{i\prime}\left(y_{i2}\right)-\dot{p}_{i1}\widetilde{\gamma}_i\psi_1'\left(y_{i2}\right)+\dot{p}_{i2}\widetilde{\gamma}_i\widetilde{\rho}_i\psi_2'\left(y_{i2}\right)=0\\ \frac{\partial L}{\partial t_{i1}}&=-p_{i1}-\widetilde{\gamma}_i\dot{p}_{i1}=0\\ \frac{\partial L}{\partial t_{i2}}&=-p_{i2}+\widetilde{\gamma}_i\dot{p}_{i1}-\widetilde{\rho}_i\dot{p}_{i2}=0\\ \frac{\partial L}{\partial\widetilde{\rho}_i}&=\sum_{i=1}^{2}\dot{p}_{i2}\left(t_{i2}-\psi_2\left(y_{i2}\right)\right)=0\\ \frac{\partial L}{\partial\widetilde{\gamma}_i}&=\sum_{i=1}^{2}\dot{p}_{i1}\left(t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)\right)=0\end{aligned}$$

From $\left(\frac{\partial L}{\partial y_{i1}}\right)$ and $\left(\frac{\partial L}{\partial t_{i1}}\right)$ we obtain the first condition which implicitly defines y_{i1}, while from $\left(\frac{\partial L}{\partial y_{i2}}\right)$ and $\left(\frac{\partial L}{\partial t_{i2}}\right)$ we get the definition of y_{i2}. Then $\left(\frac{\partial L}{\partial\widetilde{\rho}_i}\right)$ and $\left(\frac{\partial L}{\partial\widetilde{\gamma}_i}\right)$ give the last two conditions on the transfer. So $\widetilde{m}$ satisfies the first order conditions of problem $\left(P_*^i\right)$.

Now $\widetilde{m}$ is also a solution to $\left(P^i\right)$ if it is incentive compatible for both types of principal. To show this first note that each principal maximizes her expected utility over the same set of constraints. Therefore the value at the optimum of the utility function cannot be higher if the principal then lies and chooses the optimal allocation chosen by the other type. More precisely, call $h\left(y_{ij},t_{ij}\right)=0$ the set of constraints of problem $\left(P_*^i\right)$ and let $\left(\widehat{y}_{1j},\widehat{t}_{1j}\right)$ be the allocations that maximize $\sum_{j=1}^{2} p_{1j}\left(\phi^1(y_{ij}) - t_{ij}\right)$ over $h(y_{ij}, t_{ij}) = 0$ and $\left(\widehat{y}_{2j}, \widehat{t}_{2j}\right)$ the equivalent for a type 2 principal. It is evident that the following holds:

$$\sum_{j=1}^{2} p_{1j}\left(\phi^1(\widehat{y}_{1j}) - \widehat{t}_{1j}\right) \geq \sum_{j=1}^{2} p_{1j}\left(\phi^1(\widehat{y}_{2j}) - \widehat{t}_{2j}\right),$$

that is: each principal prefers her optimal allocations to the ones of the other type. If they were preferred, they would have been chosen in the first place because the set of constraints is the same.

So $\widetilde{m}$ is incentive compatible for both types of principal and is therefore a solution to $\left(P^i\right)$.

Going back to the Walrasian analysis, the principals have the same endowment, they exchange slackness, the equilibrium will have the property of being "envy-free".

Now we need to check that conditions π_1 and π_2 are satisfied by the new allocation, and that indeed the pooling offer guarantees a higher expected payoff for the two types of principal. Both $\widetilde{\mu}_j^i$ and $\overline{\mu}_j^i$ prescribe the same actions, therefore the satisfaction of the above conditions will depend exclusively on the transfers chosen. The transfers of the full information case are: $t_{i1} = \psi_1(y_{i1}) + (\psi_2(y_{i2}) - \psi_1(y_{i2}))$ and $t_{i2} = \psi_2(y_{i2})$. While the ones of the pooling offer have to satisfy the following conditions:

$\sum_{i=1}^{2} \dot{p}_{i2}\left(t_{i2} - \psi_2(y_{i2})\right) = 0$

$\sum_{i=1}^{2} \dot{p}_{i1}\left(t_{i1} - t_{i2} - \left(\psi_1(y_{i1}) - \psi_1(y_{i2})\right)\right) = 0.$

Let $\widetilde{t}_{i1}$ and $\widetilde{t}_{i2}$ be the transfers in the pooling offer $\widetilde{m}$, the conditions π_1 and π_2 then become respectively:

$$\begin{aligned} p_{11}t_{11} + p_{12}t_{12} &\geq p_{11}\widetilde{t}_{11} + p_{12}\widetilde{t}_{12} \\ p_{21}t_{21} + p_{22}t_{22} &\geq p_{21}\widetilde{t}_{21} + p_{22}\widetilde{t}_{22}. \end{aligned}$$

Remembering that when $\widetilde{m}$ is offered the posterior beliefs of the agent are unchanged and that the priors of principal and agent were identical, from the above equality constraints we can solve for:

$$\widetilde{t}_{22} = \psi_2(y_{22}) + \frac{p_{12}}{p_{22}}\psi_2(y_{12}) - \frac{p_{12}}{p_{22}}\widetilde{t}_{12}$$

and then using what we just found:

$$\begin{aligned} \widetilde{t}_{11} = \psi_1(y_{11}) + \widetilde{t}_{12} - \psi_1(y_{12}) - \frac{p_{21}}{p_{11}}\widetilde{t}_{21} + \frac{p_{21}}{p_{11}}\psi_1(y_{21}) \\ + \frac{p_{12}}{p_{11}}\left(\psi_2(y_{22}) + \frac{p_{12}}{p_{22}}\psi_2(y_{12}) - \frac{p_{12}}{p_{22}}\widetilde{t}_{12}\right) - \frac{p_{21}}{p_{11}}\psi_1(y_{21}). \end{aligned}$$

Substitute the values of full information transfers and those of $\widetilde{t}_{22}$ and $\widetilde{t}_{11}$ in the two inequalities, what we obtain are two linear inequalities in two unknowns:

$$\begin{aligned}\widetilde{t}_{21}-\widetilde{t}_{12} &\geq \psi_1(y_{21})-\psi_2(y_{12})+\psi_2(y_{22})-\psi_1(y_{22})\\ p_{21}\widetilde{t}_{21}-\frac{\rho}{p_{22}}\widetilde{t}_{12} &\geq p_{21}(\psi_1(y_{21})-\psi_2(y_{12})+\psi_2(y_{22})-\psi_1(y_{22}))-\frac{\rho}{p_{22}}\psi_2(y_{12})\end{aligned}$$

There exists infinite solutions to this system of inequalities, therefore it is possible to choose four transfers such that the conditions π_1 and π_2 are satisfied. ■

The last proposition has characterized the strategies which can constitute an equilibrium of our principal-agent game. The contract offer is pooling, in the sense that both principals offer the same menu of four allocations, this does not allow the agent to learn anything about the type of the principal he is facing. The actions prescribed are the same as in the full information case while the transfers are potentially different. We are in fact left with two degrees of freedom in choosing the transfers (four unknowns and two equations), one possibility are the transfers of the full information case.[15] Our claim is though that both types of principal can do better than in the full information case so we can exploit these degrees of freedom in the constraints to make sure that the conditions for a higher equilibrium payoff are satisfied by the chosen transfers.

This proposition proves that for both types of principal it is possible to do better than in the full information case. They can achieve a higher payoff making a contract offer that does not reveal anything about their type to the agent. Moreover there are infinitely many contracts that allow a higher expected utility.[16]

In the case of no correlation all these equilibria would bear a payoff equal to the full information one, therefore the principal would be completely indifferent between revealing her information to the agent at the contract offer stage or keeping it secret until the third stage. Correlation allows to break this indifference.

5 Concluding remarks.

In this work we have shown how an informed principal with a quasilinear utility function and whose type is correlated with the one of the agent can improve his expected payoff with respect to the one he would obtain if he had no private information. The increase in payoff comes from pure redistribution of surplus that she manages to extract from the agent. It does not come, as elsewhere in the literature, from the elimination of risk. In this sense the efficiency of the economy as a whole is not improved, the principal, however, uses in a more efficient way the tools in her hands: the possibility of designing the contract.

[15] The fact that the full information contract is still a solution when the information is not public is a typical feature of the private values case.

[16] Note that this is perfectly consistent with the result found in the previous section, also here we keep two degress of freedom.

Hence we have shown that the assumption of correlation between the information of the two parties is of great consequence in a world of private values. We believe it would be interesting to extend this analysis to the case of common values for which the literature offers less general and clear cut results, since in the common values framework issues of rent extraction mix with signaling problems.

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6 Appendix

6.1 The full information case.

In the full information framework each type of principal i solves the following problem:

$$\max_{\{y_{ij},t_{ij}\}} \sum_{j=1}^{2} p_{ij} \left(\phi^{i} \left(y_{ij} \right) - t_{ij} \right)$$

subject to:

$$\begin{aligned} t_{i1} - \psi_1 (y_{i1}) &= t_{i2} - \psi_1 (y_{i2}) \\ t_{i2} - \psi_2 (y_{i2}) &= 0 \end{aligned}$$

Each Lagrangian would then be:

$$L^i = \sum_{j=1}^{2} p_{ij} \left(\phi^i (y_{ij}) - t_{ij} \right) - \gamma_i (t_{i1} - \psi_1 (y_{i1}) - t_{i2} + \psi_1 (y_{i2})) - \rho_i (t_{i2} - \psi_2 (y_{i2}))$$

and maximizing it with respect to y_{ij}, t_{ij}, γ_i and ρ_i we obtain the following solution:

- $\phi^{i\prime} (y_{i1}) = \psi_1' (y_{i1})$ and $t_{i1} = \psi_1 (y_{i1}) + (\psi_2 (y_{i2}) - \psi_1 (y_{i2}))$
- $\phi^{i\prime} (y_{i2}) = \psi_2' (y_{i2}) + \frac{p_{i1}}{p_{i2}} \left(\psi_2' (y_{i2}) - \psi_1' (y_{i2}) \right)$ and $t_{i2} = \psi_2 (y_{i2})$.

It is important to stress that the ratio of the Lagrange multipliers (i.e. the shadow value of the constraints) at the optimum is different across principals, more precisely:

$$\frac{\gamma_1}{\rho_1} = \frac{p_{11}}{p_{12} + p_{11}} \neq \frac{\gamma_2}{\rho_2} = \frac{p_{21}}{p_{21} + p_{22}}.$$

Had we been in a framework of independently distributed information these ratios would be the same because of the quasilinearity of the utility functions.

6.2 Existence of Walrasian Equilibrium in the fictitious exchange economy.

Textbook microeconomics tells us that to prove with "standard" theorems the existence of a Walrasian equilibrium in an exchange economy where the agents have strongly monotone utility functions we need the aggregate excess demand correspondence $z(p)$, defined for all price vectors $p \gg 0$, to satisfy the following properties:

1. $z(\cdot)$ is upper hemi-continuous;
2. $z(\cdot)$ is homogeneous of degree zero;
3. $pz(p) = 0$ for all p (Walras' law);
4. There is an $s > 0$ such that $z_l(p) > -s$ for every commodity l and all p;
5. $\lim_{p \to \partial\Delta} \inf \|z(p)\| \to \infty$.

One can easily check that the first four conditions are satisfied in our framework.

We have problems with the fifth property because of the assumption of compact choice sets (r^i and c^i belong to a compact set for $i = 1, 2$). In our case excess demands for both goods do not tend to infinity when their price tends to the boundary of the simplex because the choice set is bounded.

This inconvenience can be solved by removing the assumption of compact choice set for one principal[17]. We have therefore obtained that the aggregate excess demand for a commodity will go to infinity if the price of that commodity is zero. This allows to apply standard existence theorems to our framework.

The removal of the assumption of compactness does not cause any further problem because the market clearing conditions (which are the constraints of the agent in the principal agent game) will hold and together with the compactness assumption for the other principal will ensure that the equilibrium allocation will belong to a compact set.

6.3 IC2 is not binding at an optimum.

The argument of this proof is very similar to the one adopted in the proof of Lemma 1 in Maskin and Tirole (1990).

We need to show that IC constraint for type 2 agent is not binding at the optimum of program P_*^i, in other words that a solution of such program satisfies $IC2$.

If $\mu_\cdot^\cdot$ is a solution to P_*^i then:

$$V^i\left(\mu_1^i\right) \geq V^i\left(\mu_2^i\right) \qquad (*)$$

must hold because it if was violated then the pooling allocation $\widetilde{\mu}$, defined so that for every i:

$$\widetilde{\mu}_1^i = \widetilde{\mu}_2^i = \mu_2^i$$

that also satisfies the constraints of P_*^i would generate higher values of the maximand.

If $\mu_\cdot^\cdot$ violates $IC2$ (that is type 2 strictly prefers $\mu_1^\cdot$ to $\mu_2^\cdot$), then define $\widehat{\mu}_\cdot^\cdot$ so that $\widehat{\mu}_1^i = \widehat{\mu}_2^i = \mu_1^i$ for all i. The allocation $\widehat{\mu}_\cdot^\cdot$ satisfies all the constraints of P_*^i and, from (*), generates at least as high a value of the maximand as $\mu_\cdot^\cdot$. But, because the type 2 agent strictly prefers $\mu_1^\cdot$ to $\mu_2^\cdot$, we can slightly reduce the transfer from the principal to the agent in $\widehat{\mu}_\cdot^\cdot$ without violating the constraints. But then $\widehat{\mu}_\cdot^\cdot$ generates a higher values of the maximand than $\mu_\cdot^\cdot$, a violation of $\mu_\cdot^\cdot$'s optimality.

6.4 A simple example.

In what follows we are going to apply our propositions in an extremely simplified framework and we are going to find a numerical solution so that it is going to be more evident that an informed principal can profit from having and concealing private information.

One principal wants to sell one unit of a good which he can produce at costs c_1 or c_2, with $c_1 < c_2$. The cost of producing the good is private information.

[17] In general equilibrium theory with incomplete markets this procedure is known as the "Cass-trick".

One agent wants to buy one unit of the same good and he values that unit v_1 or v_2, with $v_1 < v_2$. The valuation for the good is private information.

We also assume that $c_1 < v_1 < c_2 < v_2$, therefore a type 1 principal has always gains from trade while for type 2 gains from trade are conditional on the agent having a high valuation for the good.

The utility for the principal is:

$$V = t - c,$$

and the one of the agent is:

$$U = v - t,$$

where t is the price paid for the good (i.e. a transfer form the agent to the principal).

We are therefore in a world of bilateral asymmetric information and our informed principal problem falls in the realm of the private values case because the agent does not care directly about the cost of production of the good (i.e. the type of the principal).

Each player knows only the *a priori* distribution of the other player's type. The two types of the players are equally likely with $\Pr(v_i) = \Pr(c_i) = \frac{1}{2}$ with $i = 1, 2$. However they are not independently distributed with the conditional distributions being:

$$\begin{aligned} \Pr(v_i \mid c_i) &= \Pr(c_i \mid v_i) = \frac{3}{4} \\ \Pr(v_i \mid c_j) &= \Pr(c_j \mid v_i) = \frac{1}{4} \end{aligned}$$

with $i, j = 1, 2$ and $i \neq j$.

We are assuming therefore that there is higher probability of the two player's being of the same type with one's type acting like an informative signal on the other party's type.

After learning his cost of production, the principal offers a contract to the agent that specifies a price to be paid for the good in each state of the world. We are going to show that the principal will gain from not revealing her type at the contract offer stage. This means that both types of principal will offer the same menu of four prices (one for each state of the world). At that stage the principal already knows her type and so knows that two of those prices will never be implemented but by making this pooling contract-offer she does not allow the agent to learn anything new about her type. The agent will therefore accept the contract on the basis of the a priori distribution which he has not been able to update because no information has come from the offer. This therefore means that the participation and incentive constraints will have to hold only in expectation, leaving therefore more freedom to the principal when setting the prices.

At the third stage both principal and agent make an announcement about their type, we are going to show that the optimal contract which is also incentive compatible for the principal, in the sense that the principal will report truthfully his type.

At the final stage the contract is implemented and the transaction takes place at the chosen price.

We will also show that the principal is better off when concealing her information than in the case she reveals it from the very beginning. Finally we are going to show that correlation of information plays a big role in all this by showing that when types are independently distributed the principal cannot improve upon the full information payoff.

6.4.1 The full information case.

As we did before we are now going to study the contracts when the type of the principal is common knowledge, we are going to use it as a benchmark for evaluating the gains for the principal.

If the principal has cost of production c_1 then she offers a contract which consists of two prices, one for an agent that has valuation v_1 and one for an agent that has valuation v_2.

In order to have a lighter notation, simplify the analysis by making it more clear, we are going to assign the following specific values to valuations and costs: $v_2 = 3$, $v_1 = 1$, $c_2 = \frac{3}{2}$, $c_1 = 0$.[18]

The prices are determined through the following optimization problem:

$$\max_{t(v_1,c_1),t(v_2,c_1)} E(V_1) = \frac{3}{4}t(v_1,c_1) + \frac{1}{4}t(v_2,c_1)$$

subject to the following participation and incentive constraints:

$$\begin{aligned} IR_1 &: 1 - t(v_1,c_1) \geq 0 \\ IR_2 &: 3 - t(v_2,c_1) \geq 0 \\ IC_1 &: 1 - t(v_1,c_1) \geq 1 - t(v_2,c_1) \\ IC_2 &: 3 - t(v_2,c_1) \geq 3 - t(v_1,c_1). \end{aligned}$$

It is clear that the only way to satisfy the incentive constraints is setting equal transfers for both types of agent. If the principal sets transfers:

$$t(v_1,c_1) = t(v_2,c_1) = 1,$$

then she will sell to both types of agent and his expect payoff will be:

$$E(V_1) = 1,$$

which is higher than what she would obtain by setting the transfers equal to 3 because that

[18] We chose these numbers in order that the assumed ranking was manatained, in fact it is still true that: $c_1 < v_1 < c_2 < v_2$. Moreover we wanted: $v_1 - c_1 > \frac{1}{4}(v_2 - c_1)$ so that it is optimal for a type 1 principal to sell to both types of agent in the full-information case. This assumption ensures also efficiency.

would ensure her a payoff of $\frac{3}{4}$[19]

Therefore both types of agent consume the good, and the agent with high valuation enjoys some rent (he pays a price which is well below his valuation and $E(U_2) = \frac{1}{2}$).

When the principal has high cost of production, then the two prices (always contingent on the type of the agent) will solve the following problem:

$$\max_{t(v_2,c_2),t(v_1,c_2)} E(V) = \frac{3}{4}\left(t(v_2,c_2) - \frac{3}{2}\right) + \frac{1}{4}\left(t(v_1,c_2) - \frac{3}{2}\right)$$

subject to the following participation and incentive constraints:

$$\begin{aligned} IR_1 &: \quad 1 - t(v_1,c_2) \geq 0 \\ IR_2 &: \quad 3 - t(v_2,c_2) \geq 0 \\ IC_1 &: \quad 1 - t(v_1,c_2) \geq 1 - t(v_2,c_2) \\ IC_2 &: \quad 3 - t(v_2,c_2) \geq 3 - t(v_1,c_2). \end{aligned}$$

Again both transfers have to be equal and since setting $t(v_1,c_2) = t(v_2,c_2) = 1$ would gain the principal a negative expected payoff, then this time the transfers will be:

$$t(v_1,c_2) = t(v_2,c_2) = 3.$$

The principal will sell (and produce) the good only to a high valuation agent because the price is to high for a low valuation agent that prefers to enjoy his reservation utility. In this case the principal's equilibrium payoff is going to be:

$$E(V_2) = \frac{9}{8}.$$

Now both type of agents receive the same equilibrium payoff, even if one agent consumes and the other not.

Note also that, with the assumption that we made on the parameters, we obtain efficiency: the principal sells the good all the times that the valuation of the good by the agent is higher than her cost of production.

6.4.2 The pooling offer.

In this section we return to the case of privately informed principal and we show that the optimal contract is a menu of prices to be offered by both types of principal such that:

- it satisfies individual rationality and incentive compatibility constraints for both types of agent;

[19] In that case a type 1 agent would refuse the contract.

- it satisfies incentive compatibility for the principal;
- it secures a higher equilibrium expected payoff to both types of principal;
- it maintains efficiency (i.e. when the valuation is v_1 and the cost c_2 the good is not exchanged or produced).

This menu will be offered by both types of principal so that the agent does not learn any information from the contract offer so that his beliefs on the type of the principal coincides with the priors. This also means that his constraints will have to hold only in expectations (at an *interim* stage) leaving more freedom to the principal in setting the transfers.

At this stage we introduce four new variables $q_{ij} \in \{0,1\}$ with $i,j = 1,2$ which indicates whether a principal of type j will sell or not the good when she is paired with an agent of type i.

Then the expected utility for the different types of principal is going to be:

$$\begin{aligned} E(V_1) &= \frac{3}{4}t(v_1,c_1) + \frac{1}{4}t(v_2,c_1) \\ E(V_2) &= \frac{3}{4}\left(t(v_2,c_2) - \frac{3}{2}q_{22}\right) + \frac{1}{4}\left(t(v_1,c_2) - \frac{3}{2}q_{12}\right), \end{aligned}$$

The optimal contract will be the solution to the maximization of these objective function with respect to $t(v_i,c_j)$ and q_{ij} provided that a set of constraints is satisfied. This set includes participation constraints for the agent, incentive compatibility constraints for both agent and principal. Finally we are going to show also that the expected payoff when both types of principal offer this optimal contract will be higher than the one of the full-information case.

The individual rationality constraints that have to be satisfied are:

$$\begin{aligned} IR_1 &: \frac{3}{4}(q_{11} - t(v_1,c_1)) + \frac{1}{4}(q_{12} - t(v_1,c_2)) \geq 0 \\ IR_2 &: \frac{3}{4}(3q_{22} - t(v_2,c_2)) + \frac{1}{4}(3q_{21} - t(v_2,c_1)) \geq 0, \end{aligned}$$

The incentive compatibility constraints for the agent are:

$$\begin{aligned} IC_1 &: \frac{3}{4}(q_{11} - t(v_1,c_1)) + \frac{1}{4}(q_{12} - t(v_1,c_2)) \geq \frac{3}{4}(q_{21} - t(v_2,c_1)) + \frac{1}{4}(q_{22} - t(v_2,c_2)) \\ IC_2 &: \frac{3}{4}(3q_{22} - t(v_2,c_2)) + \frac{1}{4}(3q_{21} - t(v_2,c_1)) \geq \frac{3}{4}(3q_{12} - t(v_1,c_2)) + \frac{1}{4}(3q_{11} - t(v_1,c_1)). \end{aligned}$$

This time though we are going to require that the principal reveals truthfully her type at the third stage, therefore it has to be that, for each given type, she prefers "her prices" to the ones of other type. More precisely, incentive compatibility for the principal requires that the following two constraints are satisfied:

$ICP^1 : \frac{3}{4}t(v_1,c_1) + \frac{1}{4}t(v_2,c_1) \geq \frac{3}{4}t(v_1,c_2) + \frac{1}{4}t(v_2,c_2)$

$$ICP^2 : \tfrac{3}{4}\left(t\left(v_2,c_2\right)-\tfrac{3}{2}q_{22}\right)+\tfrac{1}{4}\left(t\left(v_1,c_2\right)-\tfrac{3}{2}q_{12}\right) \geq \tfrac{3}{4}\left(t\left(v_2,c_1\right)-\tfrac{3}{2}q_{21}\right)+\tfrac{1}{4}\left(t\left(v_1,c_1\right)-\tfrac{3}{2}q_{11}\right)$$

In addition, since we want to show that the principal benefit from concealing her type, then we require that the solution satisfies also the following inequalities are satisfied:

$$\begin{aligned} \pi_1 &: \frac{3}{4}t\left(v_1,c_1\right)+\frac{1}{4}t\left(v_2,c_1\right) \geq 1 \\ \pi_2 &: \frac{3}{4}\left(t\left(v_2,c_2\right)-\frac{3}{2}q_{22}\right)+\frac{1}{4}\left(t\left(v_1,c_2\right)-\frac{3}{2}q_{12}\right) \geq \frac{9}{8}. \end{aligned}$$

In other words the expected payoff for each type of principal have to be greater than what she could achieved by revealing her information, namely the full information payoff we computed in the previous section.

Arguments standard in this literature allow us to solve the programs with a binding participation constraint of a low valuation agent (IR_1) and a binding incentive compatibility constraint of a high valuation agent (IC_2).

As a result of optimization we find that $q_{11} = q_{21} = q_{22} = 1$ and $q_{12} = 0$, this means that we are going to observe trade in all case but when the valuation of the agent is lower than the cost of production for the principal.

Now that we know the optimal values for the q's we can substitute them in the constraint and from the binding IR_1 we can derive the following:

$$t\left(v_1,c_1\right) = 1-\frac{1}{3}t\left(v_1,c_2\right);$$

while from IC_2, after having plugged in $t\left(v_1,c_1\right)$, we obtain:

$$t\left(v_2,c_2\right) = \frac{10}{3}-\frac{1}{3}t\left(v_2,c_1\right)+\frac{8}{9}t\left(v_1,c_2\right).$$

We can now use these two expressions to simplify the remaining inequalities and obtain a system of linear inequalities in only two variables, $t\left(v_1,c_2\right)$ and $t\left(v_2,c_1\right)$. After the simplifications the constraints become:

$$\begin{cases} IR_2 : t\left(v_1,c_2\right) \leq \frac{3}{4} \\ IC_2 : t\left(v_2,c_1\right) \geq \frac{1}{4}-\frac{1}{3}t\left(v_1,c_2\right) \\ ICP^1 : t\left(v_2,c_1\right) \geq \frac{1}{4}+\frac{11}{3}t\left(v_1,c_2\right) \\ ICP^2 : t\left(v_2,c_1\right) \leq \frac{21}{8}+t\left(v_1,c_2\right) \\ \pi_1 : t\left(v_2,c_1\right) \geq 1+t\left(v_1,c_2\right) \\ \pi_2 : t\left(v_2,c_1\right) \leq 1+\frac{11}{3}t\left(v_1,c_2\right) \end{cases}$$

We can graph the corresponding equations (Fig.2) and look for a solution.

We can see that the shaded area in the graph satisfies all inequalities, that means that there exists infinite number of solutions, there are two degrees of freedom when choosing two of the

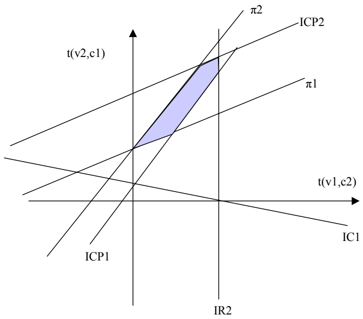

Figure 2: Equilibria that ensure both types of principal a higher payoff than in the full-information case.

prices, namely $t(v_2, c_1)$ and $t(v_1, c_2)$. The set of the solutions is also a Pareto set from the point of view of the types of principal, in fact the expected payoff of type 1 principal increase with $t(v_2, c_1)$ and decrease with $t(v_1, c_2)$ while the reverse holds for the expected payoff of a type 2 principal.

We can now pick a pair of prices inside the shaded area and verify that all the constraints are satisfied, for example: $t(v_1, c_2) = 0.6$ and $t(v_2, c_1) = 2.7$. From the binding constraints we derive that $t(v_1, c_1) = 0.8$ and $t(v_2, c_2) \simeq 2.966$.

Table 1	FULL-INFO	POOLING
$t(v_1, c_1)$	1	0.8
$t(v_1, c_2)$	0	0.6
$t(v_2, c_1)$	1	2.5
$t(v_2, c_2)$	3	2.96
rent A_1	0	0
rent A_2	0.5	0.10
π_1	1	1.275
π_2	$\frac{9}{8}$	1.249

We can then verify in table 1 that an agent who has low valuation obtains his reservation utility as in the full information case, while a high type agent enjoys a smaller expected rent of 0.100. We also get that IC_2 is binding while IC_1 is not. None of the incentive compatibility constraints for the principal is binding and she also enjoys higher expected profits, with respect to the full information case, whatever her type. In fact a type one principal gets 1.275 while in the former case she would get only 1; while a type two principal receives an expected payoff of 1.249 while it was only 1.125 when her information was known to the agent. It is therefore evident that by making the pooling offer the types of principal manages to extract some surplus from the agent. Some Crémer-McLean flavor emerges from the fact that with the new mechanism the principal raises the prices in the less likely states of the world therefore reducing the informational rent of the agent in those states.

6.4.3 The case of independent types.

To stress the importance of correlation in getting our result we are going to analyze the case in which the private information is independently distributed. We will show that even if the principals make a pooling offer she is not able to improve upon the full information outcome.

We maintain the assumption previously made that both types are equally likely with $\Pr(v_i) = \Pr(c_i) = \frac{1}{2}$ with $i = 1, 2$, but this time the conditionals distribution are going to be equal to the marginal ones:

$$\begin{aligned} \Pr(v_i \mid c_i) &= \Pr(c_i \mid v_i) = \frac{1}{2} \\ \Pr(v_i \mid c_j) &= \Pr(c_j \mid v_i) = \frac{1}{2}, \end{aligned}$$

this means precisely that one's type is non-informative signal of the other party's type.

The full information optimal contracts are the following pairs of prices and trade possibilities[20]:

$$\begin{aligned} t(v_2, c_j) &= 3 \text{ with } q_{2j} = 1 \\ t(v_1, c_j) &= 0 \text{ with } q_{1j} = 0, \end{aligned}$$

with $j = 1, 2$. This time both types of principal do not sell the good to a low valuation agent, therefore also leaving some gains from trade unexploited. The expected payoff for these contracts are respectively:

$$\begin{aligned} E(V_1) &= \frac{3}{2} \\ E(V_2) &= \frac{3}{4}. \end{aligned}$$

We now study the optimal contract when the different types of principal conceal their type at the offer stage. Optimal trade possibilities remain unchanged, with the low valuation agent not able to consume the good even when paired with a low cost principal.

At this stage the constraints have to be satisfied in expectation and if we let the individual rationality constraint of a low valuation agent and incentive compatibility of a high type be

[20] These are the solution to the following problems, with $i = 1, 2$:
$\max \frac{1}{2}(t(v_1, c_j) - q_{1j}c_j) + \frac{1}{2}(t(v_2, c_j) - q_{2j}c_j)$
subject to usual IC and IR constraints for the two types of agent.

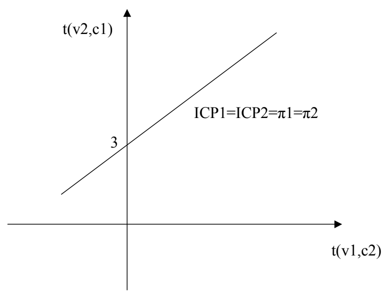

Figure 3: Equilibria of the pooling contract offer when types are independent.

binding we obtain:

$$\begin{aligned} t(v_1, c_1) &= -t(v_1, c_2) \\ t(v_2, c_2) &= 6 - t(v_2, c_1). \end{aligned}$$

If we then substitute this values inside the other constraints we obtain the other constraints for the agent are always satisfied while those of the principal become:

$$\begin{aligned} ICP^1 &: \ t(v_2, c_1) \geq 3 + t(v_1, c_2) \\ ICP^2 &: \ t(v_2, c_1) \leq 3 + t(v_1, c_2) \\ \pi_1 &: \ t(v_2, c_1) \geq 3 + t(v_1, c_2) \\ \pi_2 &: \ t(v_2, c_1) \leq 3 + t(v_1, c_2). \end{aligned}$$

This time the shaded area where the solution to the system of constraints where lying collapses to a single straight line with all constraints strictly binding.

In Fig.3 we can see that there is a continuum of equilibria of the game with pooling offer. All of them belong to the same line which is the locus of the transfers which leave the two types of principal at the same expected payoff level of the full information payoff. We have therefore shown that when types are independent the different types of principal are indifferent between revealing and concealing their private information because they neither gain nor lose from it.