Report no. 01/12

Nonnormal energy transient growth in the Taylor-Couette problem

Á. Meseguer
Oxford University
Numerical Analysis Group

This work is devoted to the study of transient growth of perturbations in the Taylor-Couette problem due to nonnormal mechanisms. The study is carried out for a particular small gap case and is mostly focused on the linearly stable regime of counter-rotation. The exploration covers a wide range of inner and outer angular speeds as well as axial and azimuthal modes. Clear evidence of transient growth is found as long as the counter-rotation is increased. The numerical results are in agreement with former analyses based on energy methods. Similarities with transient growth mechanisms in plane Couette flow and in Hagen-Poiseuille flow are found. This is reflected in the modulation of the basic circular Couette flow by the presence of azimuthal streaks as a result of the nonmodal growth of initial axisymmetric perturbations. This study might shed some light on the subcritical transition to turbulence which is found experimentally in Taylor-Couette flow when the cylinders rotate in opposite directions.

Key words and phrases: Taylor-Couette problem, transient growth, nonnormality, nonmodal analysis

This work was supported by the Engineering and Physical Sciences Research Council of the United Kingdom under Grant GR/M30890.

Oxford University Computing Laboratory
Numerical Analysis Group
Wolfson Building
Parks Road
Oxford, England OX1 3QD
E-mail: alvaro@comlab.oxford.ac.uk

July, 2001

1 Introduction.

Taylor-Couette flow of a viscous fluid confined between independently rotating coaxial cylinders has been one of the most studied problems of fluid dynamics in the last eighty years. Starting with the celebrated work of G. I. Taylor [Taylor, 1923], the Taylor-Couette problem has been an experimental, theoretical and numerical benchmark problem for bifurcation theory and hydrodynamic stability. This flow may become turbulent by means of many different mechanisms which usually involve successive steady or unsteady linear instabilities. We refer the reader to the standard monographs [Chossat & Iooss, 1991] or [Tagg, 1994] for details. Below the critical values predicted by linear stability theory, azimuthal Couette flow is stable with respect to infinitesimal perturbations. Nevertheless, experiments carried out by Coles and Van Atta in the 1960's [Coles, 1965],[Van Atta, 1966] reported striking new phenomena of sudden transition to transient turbulent regimes in the region where the linear theory predicted stability of the basic azimuthal Couette flow. This kind of instability, which Coles termed *catastrophic transition*, cannot be explained by means of eigenvalue analysis of the linearized Navier-Stokes operator, because its spectrum always lies on the stable region of the complex plane. Instead, this subcritical transition seems to be associated with the considerable amplification or transient growth that even very small amplitude perturbations may suffer due to the nonnormality of the linearized operator, i.e. non-orthogonality of its eigenvectors [Kato, 1976]. It has long been known that nonnormality of linearized operators arising in stability analysis of shear flows is responsible for the considerable nonmodal growth of small perturbations. A comprehensive theoretical study of nonmodal analysis for shear flows can be found in the recently published monograph [Schmid & Henningson, 2001] or in [Trefethen *et al.*, 1993]. Shear-dominated flows such as plane Couette or Hagen-Poiseuille (pipe) flows are linearly stable for all Reynolds numbers although they actually become turbulent due to finite amplitude perturbations which are transiently amplified by nonnormal mechanisms. In [Gebhardt & Grossmann, 1993], a comprehensive exploration of the spectra of the Taylor-Couette eigenvalue problem was provided and the nonnormality of the operator was pointed out.

The experiments of Coles and Van Atta were carried out with a small gap apparatus and subcritical transition to turbulence was found in the regime of counter-rotation or when the inner cylinder was at rest. The purpose of this work is to provide evidence of a remarkable energy transient growth of perturbations based on the linear, but nonmodal, analysis of the azimuthal Couette flow under those circumstances. The author does not claim that this mechanism is the only one responsible for the eventual transition to turbulence. A fully nonlinear study based on the analysis of the topological features of the basin of attraction of the azimuthal Couette flow would be needed for a complete understanding of the transition mechanism.

The paper is structured as follows. In §1, we formulate the stability problem and we define the quantities which measure the transient growth of the perturbations. In §2, we provide a comprehensive exploration of the optimal transient growth in the counter-rotation regime for different azimuthal and axial modes and we compare our numerical results with the experimental data available. Finally, in §3, we reveal the existence of streaks as a result of axisymmetric toroidal perturbations and how this mechanism modulates the basic azimuthal flow.

2 Mathematical formulation: linear stability and energy norm

We consider an incompressible fluid of kinematic viscosity ν which is contained between two concentric rotating cylinders whose inner and outer radii and angular velocities are r_i^*, r_o^* and Ω_i, Ω_o respectively. The independent dimensionless parameters appearing in this problem are the radius ratio $\eta = r_i^*/r_o^*$ which fixes the geometry of the annulus, and the Couette flow Reynolds numbers $Ri = dr_i\Omega_i/\nu$ and $Ro = dr_o\Omega_o/\nu$ of the rotating cylinders. Henceforth, all variables will be rendered dimensionless using d, d^2/ν, ν^2/d^2 as units for space, time and the reduced pressure (p^*/ρ^*), respectively. The Navier–Stokes equation and the incompressibility condition for this scaling take the form

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v}, \quad \nabla\cdot\mathbf{v} = 0. \tag{2.1}$$

Let $\mathbf{v} = v_r\,\mathbf{e}_r + v_\theta\,\mathbf{e}_\theta + v_z\,\mathbf{e}_z = (v_r, v_\theta, v_z)$ be the velocity vector $\mathbf{v}$ in cylindrical coordinates (r, θ, z). The basic azimuthal Couette flow $\mathbf{v}^{\mathrm{B}} = (v_r^{\mathrm{B}}, v_\theta^{\mathrm{B}}, v_z^{\mathrm{B}})$ is obtained by assuming independence with respect to t, θ and z:

$$v_r^{\mathrm{B}} = 0, \quad v_\theta^{\mathrm{B}} = Ar + \frac{B}{r}, \quad v_z^{\mathrm{B}} = 0, \quad (r_i \le r \le r_o), \tag{2.2}$$

where $A = (Ro - \eta Ri)/(1+\eta)$, $B = \eta(Ri - \eta Ro)/(1-\eta)(1-\eta^2)$, $r_i = \eta/(1-\eta)$ and $r_o = 1/(1-\eta)$.

For our analysis, the basic flow is perturbed by a small disturbance which is assumed to be periodic in the azimuthal and axial coordinates:

$$\mathbf{v}(r,\theta,z,t) = \mathbf{v}^{\mathrm{B}} + \mathbf{u}(r)\mathrm{e}^{\mathrm{i}(n\theta+kz)+\lambda t}, \tag{2.3}$$

$$p(r,\theta,z,t) = p^{\mathrm{B}} + q(r)\mathrm{e}^{\mathrm{i}(n\theta+kz)+\lambda t}, \tag{2.4}$$

where $n \in \mathbb{Z}$, $k \in \mathbb{R}$ and $\lambda \in \mathbb{C}$. In addition, the perturbation of the velocity field, $\mathbf{u} = (u_r, u_\theta, u_z)$, must cancel at the radial boundaries

$$\mathbf{u}(r_i) = \mathbf{u}(r_o) = \mathbf{0}, \tag{2.5}$$

and satisfy the solenoidal condition

$$\nabla\cdot[\mathrm{e}^{\mathrm{i}(n\theta+kz)}\mathbf{u}(r)] = 0. \tag{2.6}$$

By introducing the perturbed fields (2.3) and (2.4) in the Navier-Stokes equations (2.1) and neglecting nonlinear terms, we obtain the solenoidal eigenvalue problem for the

(n,k) azimuthal-axial mode of the perturbation

$$\begin{aligned}
\lambda u_r &= \mathrm{D}q + \left[\mathrm{D}_+\mathrm{D} - \frac{n^2+1}{r^2} - k^2 - \frac{\mathrm{i}n}{r}v_\theta^{\mathrm{B}}\right] u_r + \left[\frac{2}{r}v_\theta^{\mathrm{B}} - \frac{2\mathrm{i}n}{r^2}\right] u_\theta \ , && (2.7)\\
\lambda u_\theta &= \frac{\mathrm{i}n}{r}q + \left[\mathrm{D}_+\mathrm{D} - \frac{n^2+1}{r^2} - k^2 - \frac{\mathrm{i}n}{r}v_\theta^{\mathrm{B}}\right] u_\theta + \left[\frac{2\mathrm{i}n}{r^2} - (\mathrm{D}_+ v_\theta^{\mathrm{B}})\right] u_r \ , && (2.8)\\
\lambda u_z &= \mathrm{i}kq + \left[\mathrm{D}_+\mathrm{D} - \frac{n^2}{r^2} - k^2 - \frac{\mathrm{i}n}{r}v_\theta^{\mathrm{B}}\right] u_z \ , && (2.9)\\
\mathrm{D}_+ u_r &= -\frac{\mathrm{i}n}{r}u_\theta - \mathrm{i}k u_z \ , && (2.10)
\end{aligned}$$

where $\mathrm{D} = \dfrac{\mathrm{d}}{\mathrm{d}r}$ and $\mathrm{D}_+ = \mathrm{D} + \dfrac{1}{r}$.

We discretize the boundary value problem (2.5)-(2.10) by a solenoidal Petrov-Galerkin spectral method which was already used in [Meseguer & Marques, 2000] for the stability analysis of spiral Couette problem. For a fixed (n,k)-mode, the M-th order spectral approximation of the velocity field is accomplished by means of a linear combination of divergence-free vector fields (trial functions) which identically satisfy (2.10) and (2.5):

$$\mathbf{u}_{\mathrm{M}}(r) = \sum_{m=0}^{\mathrm{M}} a_m \Phi_m(r), \quad \mathrm{D}_+(\Phi_m)_r + \frac{\mathrm{i}n}{r}(\Phi_m)_\theta + \mathrm{i}k(\Phi_m)_z = 0. \qquad (2.11)$$

After substitution of the spectral approximation $\mathbf{u}_{\mathrm{M}}$ in (2.7)-(2.9), the projection is carried out via an inner product over another set of solenoidal fields $\Psi_l(r)$ (test basis)

$$\lambda \sum_{m=0}^{\mathrm{M}} (\Psi_l, \Phi_m) a_m = \sum_{m=0}^{\mathrm{M}} (\Psi_l, \Delta^{\mathrm{B}}\Phi_m) a_m, \quad (l = 0, \ldots, \mathrm{M}), \qquad (2.12)$$

where $(\Psi_l, \Phi_m) = \int_{r_i}^{r_o} \Psi_l^* \cdot \Phi_m r \,\mathrm{d}r$ is the volume integral restricted to the radial domain. In equation (2.12), Δ^{B} stands for the linear operator acting over the velocity perturbation field on the right-hand side of equations (2.7)-(2.9) and the pressure term has been canceled in the projection. The discretization scheme finally leads to an eigenvalue problem for the amplitudes $\mathbf{a} = (a_0, \ldots, a_{\mathrm{M}})^T$ of the spectral representation of the velocity field:

$$\mathbb{L}(Ri, Ro, \eta, n, k)\mathbf{a} = \lambda \mathbf{a}, \qquad (2.13)$$

where the matrix $\mathbb{L}$ implicitly depends on the set of parameters of the boundary value problem. The linear stability problem is then reduced to the computation of the spectrum of $\mathbb{L}$ for each pair of (n,k) azimuthal-axial modes. If, for a fixed set of values Ri, Ro and η, the (n,k)-spectra always lie in the left-hand side of the complex plane, then the basic flow will be stable with respect to infinitesimal perturbations. On the other hand, if one of the eigenvalues has positive real part, then the basic Couette flow will be linearly unstable.

We focus our attention in the transient evolution of perturbations in the regime of linear stability, following the same methodology used in [Schmid & Henningson, 1994] for the study of nonnormal transient growth in Hagen-Poiseuille flow. For a given (n,k) azimuthal-axial mode, consider the linear subspace $\mathrm{S_N}$ spanned by the eigenvectors of the N rightmost eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_\mathrm{N}\}$ of the spectrum of $\mathbb{L}$,

$$\mathrm{S_N} = \langle \tilde{\mathbf{q}}_1, \tilde{\mathbf{q}}_2, \ldots, \tilde{\mathbf{q}}_\mathrm{N} \rangle. \tag{2.14}$$

Any perturbation $\mathbf{q}$ can be expressed as a linear combination of the eigenvectors $\tilde{\mathbf{q}}_i$,

$$\mathbf{q} = \sum_{n=1}^{\mathrm{N}} \kappa_n \tilde{\mathbf{q}}_n = (\kappa_1, \kappa_2, \ldots, \kappa_\mathrm{N})^T, \tag{2.15}$$

and its time evolution is dictated by the diagonal system

$$\frac{\mathrm{d}\kappa_i}{\mathrm{d}t} = \sum_{j=1}^{\mathrm{N}} \Lambda_{ij}\kappa_j, \quad (i = 1, \ldots, \mathrm{N}), \tag{2.16}$$

where $\Lambda_{ij} = \lambda_i \delta_{ij}$, δ_{ij} being the Kronecker symbol.

We define the *energy norm* of the perturbation $\mathbf{q}$ by means of the inner product

$$\varepsilon(\mathbf{q}) = (\mathbf{q}, \mathbf{q})_\mathrm{E} = \frac{1}{2}\int_{r_i}^{r_o} \mathbf{q}^* \cdot \mathbf{q}\, r \mathrm{d}r, \tag{2.17}$$

where $*$ stands for complex conjugation. For practical purposes, it is convenient to work with the standard 2-norm in the space $\mathrm{S_N}$,

$$||\mathbf{q}||_2^2 = \sum_{j=1}^{\mathrm{N}} \kappa_j^* \kappa_j, \quad \forall \mathbf{q} \in \mathrm{S_N}. \tag{2.18}$$

We consider the matrix of inner products between the eigenvectors

$$\mathbb{M}_{ij} = (\tilde{\mathbf{q}}_i, \tilde{\mathbf{q}}_j)_\mathrm{E}. \tag{2.19}$$

This matrix is positive definite and it admits a decomposition of the form $\mathbb{M} = \mathbb{F}^\dagger \mathbb{F}$, where $\dagger$ stands for the complex conjugate transpose. This decomposition can be accomplished by means of the standard QR algorithm [Trefethen & Bau, 1997]). The energy norm of the perturbation $\mathbf{q}$ in (2.17) can be expressed in the standard 2-norm in $\mathrm{S_N}$ by means of the components $\mathbb{F}$ and $\mathbb{F}^\dagger$:

$$\varepsilon(\mathbf{q}) = \kappa^\dagger \mathbb{M} \kappa = (\mathbb{F}\kappa, \mathbb{F}\kappa)_2 = (\kappa, \kappa)_\mathrm{E} = ||\kappa||_\mathrm{E}^2 = ||\mathbb{F}\kappa||_2^2,$$

where $\kappa = (\kappa_1, \kappa_2, \ldots, \kappa_\mathrm{N})^T$.

We are interested in the measurement of the energy growth of an initial condition κ_0 as a function of time. More specifically, we define the *energy amplification factor*, $\mathrm{g}(t)$, as the ratio between the energy norm of the perturbation at time t and its initial norm,

$$\mathrm{g}(t) = \frac{||\kappa(t)||_\mathrm{E}^2}{||\kappa_0||_\mathrm{E}^2} = \frac{||\mathrm{e}^{\Lambda t}\kappa_0||_\mathrm{E}^2}{||\kappa_0||_\mathrm{E}^2}. \tag{2.20}$$

For a fixed time t, we want to maximize $\mathrm{g}(t)$ in (2.20) over the set of *all* possible initial conditions κ_0. Maximization of the ratio appearing in (2.20) leads to the quantity $\mathrm{G}(t)$, the *optimal energy amplification factor*

$$\mathrm{G}(t) = \max_{||\kappa_0||\neq 0} \mathrm{g}(t) = \max_{||\kappa_0||\neq 0} \frac{||\mathrm{e}^{\Lambda t}\kappa_0||_{\mathrm{E}}^2}{||\kappa_0||_{\mathrm{E}}^2} = \max_{||\kappa_0||\neq 0} \frac{||\mathbb{F}\mathrm{e}^{\Lambda t}\kappa_0||_2^2}{||\mathbb{F}\kappa_0||_2^2} = ||\mathbb{F}\mathrm{e}^{\Lambda t}\mathbb{F}^{-1}||_2^2. \tag{2.21}$$

The quantity $||\mathbb{F}\mathrm{e}^{\Lambda t}\mathbb{F}^{-1}||_2$ is the principal singular value σ_1 of the operator $\mathbb{F}\mathrm{e}^{\Lambda t}\mathbb{F}^{-1}$ and its computation is straightforward via the SVD algorithm [Trefethen & Bau, 1997],

$$\mathrm{G}(t) = \sigma_1^2(\mathbb{F}\mathrm{e}^{\Lambda t}\mathbb{F}^{-1}). \tag{2.22}$$

This is equivalent to solve the variational problem of maximizing the factor $\mathrm{g}(t)$ for a prescribed time t and considering the initial conditions as the degrees of freedom of the problem [Butler & Farrell, 1992]. The optimal growth $\mathrm{G}(t)$ in (2.22) has been obtained from the linear operator Λ associated with the (n, k) azimuthal-axial mode and for a prescribed positive time t. Therefore, for a fixed set of values Ri, Ro and η, the *maximum energy amplification factor*, $\mathrm{G}_{\mathrm{max}}$, is obtained by maximizing $\mathrm{G}(t)$ in (2.22) for all the pairs $(n, k) \in \mathbb{Z} \times \mathbb{R}$ and for $t \in \mathbb{R}^+$

$$\mathrm{G}_{\mathrm{max}}(Ri, Ro, \eta) = \sup_{(n,k,t)} \mathrm{G}(t). \tag{2.23}$$

3 Parametric study of $\mathrm{G}_{\mathrm{max}}$

In this section we describe the global features of the growth factor G_{max} defined in equation (2.23). The exploration is carried out for the particular case $\eta = 0.881$ and for inner and outer Reynolds numbers in the domain $(Ri, Ro) \in [0, 900] \times [-4000, 500]$, following the specifications of the experimental study provided in [Coles, 1965]. Our attention is mainly focused in the counter-rotating regime, where the flow exhibited subcritical transitions in the laboratory. Nevertheless, for completeness we enhanced our exploration to a small region in the co-rotating regime. In order to simplify the exploration, we take advantage of the $\mathsf{O}(2)$-symmetry of the problem, i.e., invariance of the system (2.5)-(2.10) under axial translations and specular reflections with respect to orthogonal planes to the common axis of the cylinders. The system also obeys $\mathsf{SO}(2)$-symmetry, i.e., invariance with respect to azimuthal rotations around the center axis [Chossat & Iooss, 1991]. Therefore, we have restricted our computations to the case when both n and k are positive or zero. In this particular exploration we have maximized the factor G in (2.22) for positive times, for azimuthal modes in the range $0 \leq n \leq 15$, and for axial wavenumbers in the range $0 \leq k \leq 10$. Our results are summarized in figure 1. The shaded zone represents the region of the (Ro, Ri)-plane where the circular Couette flow is linearly unstable. This region has a lower boundary which is the critical curve where the first linear instability appears. This critical curve has been computed by solving the eigenvalue problem (2.13) and imposing the condition that the real part of the rightmost eigenvalue of $\mathbb{L}$ is zero. Below the critical boundary prescribed by

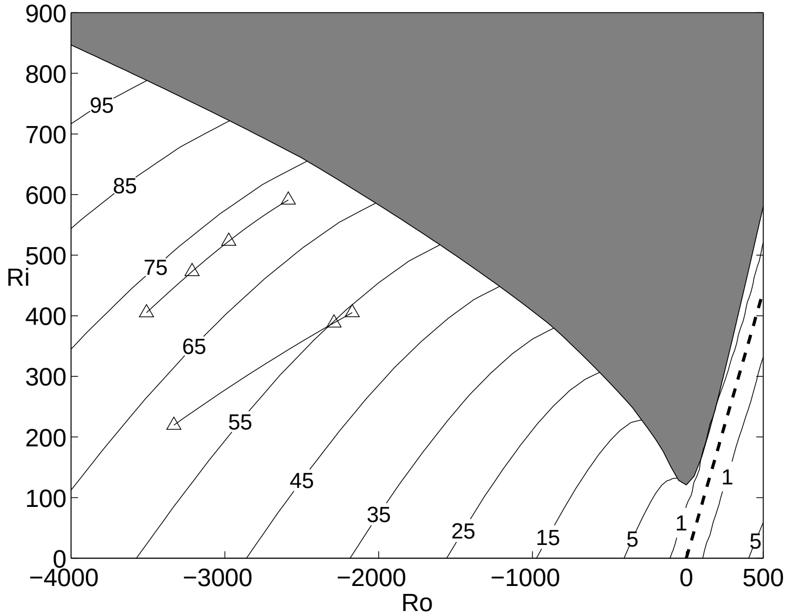

Figure 1: Maximum transient growth factor $G_{\max}$ in the $(Ro, Ri)-$plane. The dashed line represents the rigid body rotation curve $Ri = \eta Ro$. The lines with white triangles represent the experimental boundaries of transition to turbulence provided in [Coles, 1965].

the modal analysis, the figure shows isovalues of the function $G_{max}(Ro, Ri)$. Different features can be pointed out. First, at the bottom right of figure 1 we have represented the *rigid rotation curve*, $Ri = \eta Ro$, by a dashed line representing the region where both cylinders rotate with the same angular speeds, $\Omega_i = \Omega_o$. We can observe that, close to that region, the Couette flow does not exhibit transient growth. This is clearly visualized in the figure by a narrow stripe containing the rigid rotation curve within which $G_{\max} = 1$. This result is in agreement with previous analyses based on energy methods which concluded that near the rigid rotation region, circular Couette flow is absolutely, monotonically and globally stable [Joseph, 1976]. Second, in the counter-rotation region, we can observe a monotonic growth of G_{max}, which ranges between 1 and 100. This would imply that the energy of any small perturbation would be potentially amplified by almost two orders of magnitude in the counter-rotation region explored in this case. Nevertheless, this growth mechanism, due to nonnormal linear effects, is only transient and the perturbation will eventually decay for long times, as predicted by the linear stability analysis. Second, the contours of G_{max} are *not* tangent to the shaded region over the linear instability boundary. In fact, the intersection in transversal, implying that nonmodal transient growth may still be found slightly above the linear critical values. Finally, the figure includes the experimental data from [Coles, 1965]. The lines with white triangles represent the experimental boundaries of catastrophic transition to turbulence reported by Coles for two experiments carried out with different fluids. In [Coles, 1965], the discrepancy between the two experimental boundaries was not completely understood. Nevertheless, the upper experimental boundary from figure 1 is clearly aligned with the contour curves of G_{max}, revealing a dependence between the catastrophic transition and the energy amplification factor.

4 Growth mechanism and azimuthal streaks

In this section we study how the nonnormal growth mechanism affects the basic azimuthal Couette flow. It has long been known that shear flows such as plane Couette or pipe Poiseuille flow exhibit transition to secondary transient flows usually termed *streaks*. These flows are particularly easy to trigger when perturbing the basic field by means of *streamwise vortices*, i.e., vortical structures which are uniform along the direction of the basic flow. Initially, the streamwise vortices only perturb the spanwise and normal components of the flow. The *lift-up effect* is eventually responsible for the formation of the streaks by transferring the spanwise-normal contribution of the energy to the streamwise direction [Schmid & Henningson, 2001]. Streaks are regions of the fluid where the modulated flow attains high and low relative speeds. The modulated flow results in a profile which is, in a transient sense, linearly unstable with respect to three-dimensional perturbations. This last instability is usually termed *streak breakdown* and is one possible route of transition to turbulence in shear flows.

In the Taylor-Couette narrow gap geometry, where the curvature is considerably reduced, the *azimuthal* coordinate plays the role of the *streamwise* direction and axisymmetric *toroidal* vector fields are suitable candidates to be streamwise vortices. Two factors are essential in order to study the time evolution of the perturbations and the modulation of the Couette flow. The first is the energy of the initial perturbation with respect to the energy of the basic flow, $\mathrm{E_B}$. This quantity is given by the expression

$$\mathrm{E_B} = \frac{1}{2}\int_{r_i}^{r_o} \mathbf{v}^{\mathrm{B}} \cdot \mathbf{v}^{\mathrm{B}} r \,\mathrm{d}r = \frac{A^2}{8}(r_o^4 - r_i^4) - \frac{B^2}{2}\ln\eta + \frac{AB}{2}(r_o^2 - r_i^2). \tag{4.1}$$

The second is the time scale during which the transient streaks achieve their maximum amplitude. In our nondimensionalization, the time scale was given by the *viscous time*, $t = d^2/\nu$. We are interested in the characteristic time that a perturbation needs to reach its maximum amplitude and how this time is related to the driving dynamics of the cylinders. In counter-rotation situations, a suitable advective time scale is given by the *outer rotation period*, τ_o, which is the time that the outer cylinder needs to complete one rotation

$$\tau_o = \frac{2\pi}{Ro(\eta - 1)}, \quad Ro < 0. \tag{4.2}$$

In figure 2, we have plotted $\mathrm{G}(t)$ for $Ro = -4000$, $Ri = 0$ and $k = \pi/2$. The plot provides the optimal growth for different azimuthal modes ranging from $n = 0$ to $n = 7$. We observe that the axisymmetric mode does not exhibit a remarkable growth in comparison with other non-axisymmetric (oblique) modes. Nevertheless, we have studied the effect of streamwise perturbations and its implications in the formation of streaks. In figure 3,

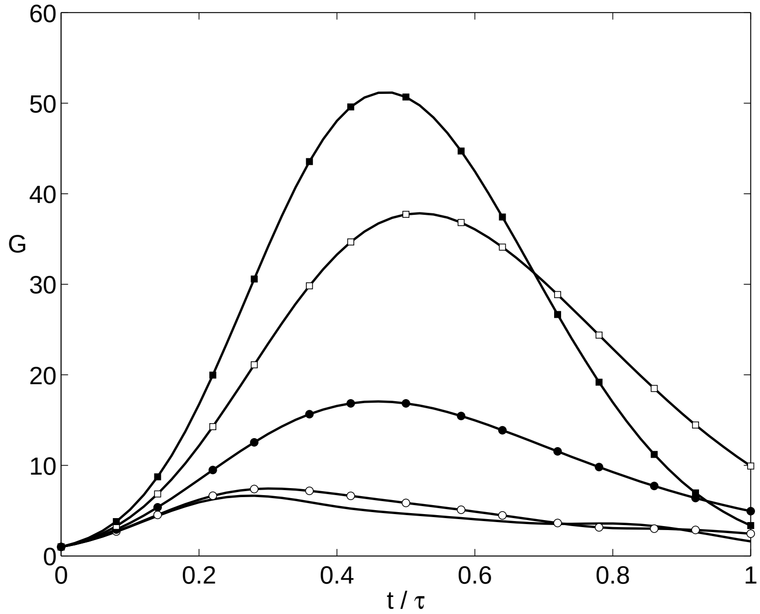

Figure 2: Transient growth for different azimuthal modes for $Ro = -4000$, $Ri = 0$ and $k = \pi/2$: $n = 0$ (solid line),$n = 1$ (open circles), $n = 3$ (filled circles), $n = 5$ (open squares), $n = 7$ (filled squares).

we have plotted the isovalues of the modulated azimuthal Couette flow for $Ro = -4000$, $Ri = 0$, $t = 0, \tau_o/10, \tau_o/5, 2\tau_o$. In this computation, the initial perturbation was an axisymmetric vortex ($n = 0$) with $k = \pi/2$, zero azimuthal component and initial energy 1.5% of $\mathrm{E_B}$. It can be clearly observed the formation of azimuthal streaks near the inner and outer cylinders. Finally, figure 4 shows the time evolution of the perturbation field for the same computation. We observe that the radial-axial components of the perturbation decay monotonically, transferring the energy to the azimuthal component.

5 Conclusions

A comprehensive exploration of the nonnormal transient growth in counter-rotating Taylor-Couette flow has been provided. Remarkable energy transient growth is found in the counter-rotating regime. The numerical computations reveal a dependence between the experimental transition to turbulence and the maximum amplification factor obtained by nonmodal analysis. Oblique modes seem to be more effective in the transient mechanism and azimuthal streaks may be observed as well although they exhibit a weaker amplification. Fully nonlinear analysis is required to understand completely the transition mechanism to turbulent regimes.

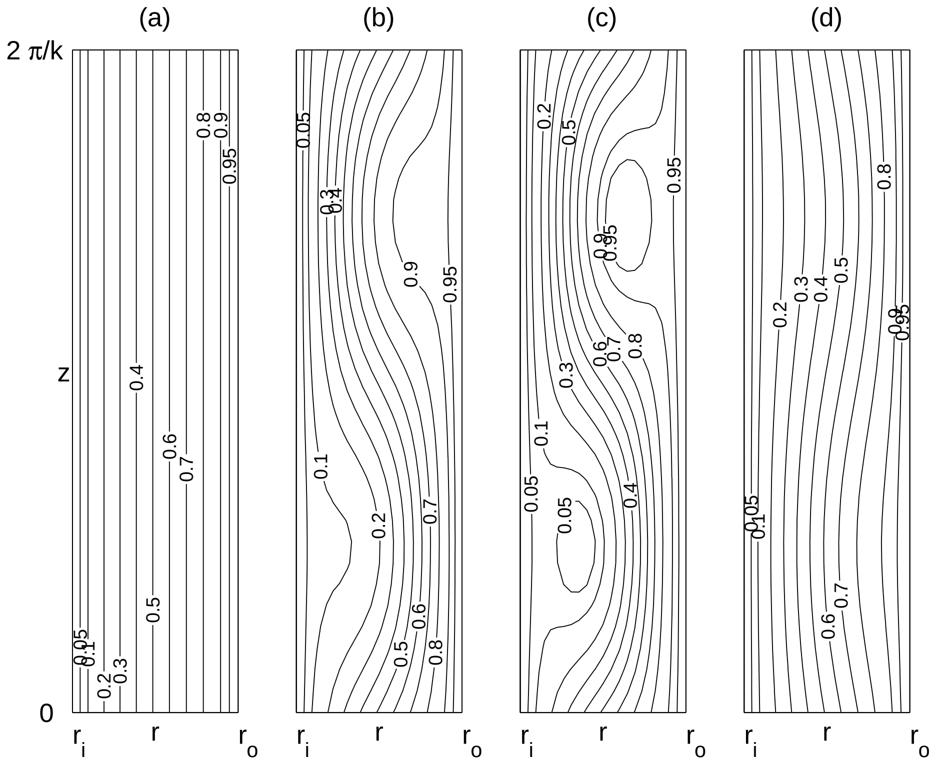

Figure 3: Isovalues of the modulated azimuthal Couette flow for $Ri = 0$, $Ro = -4000$ at different times: (a) $t = 0$, (b) $t = \tau_o/10$, (c) $t = \tau_o/5$, (d) $t = 2\tau_o$. The flow has been renormalized with respect to Ro.

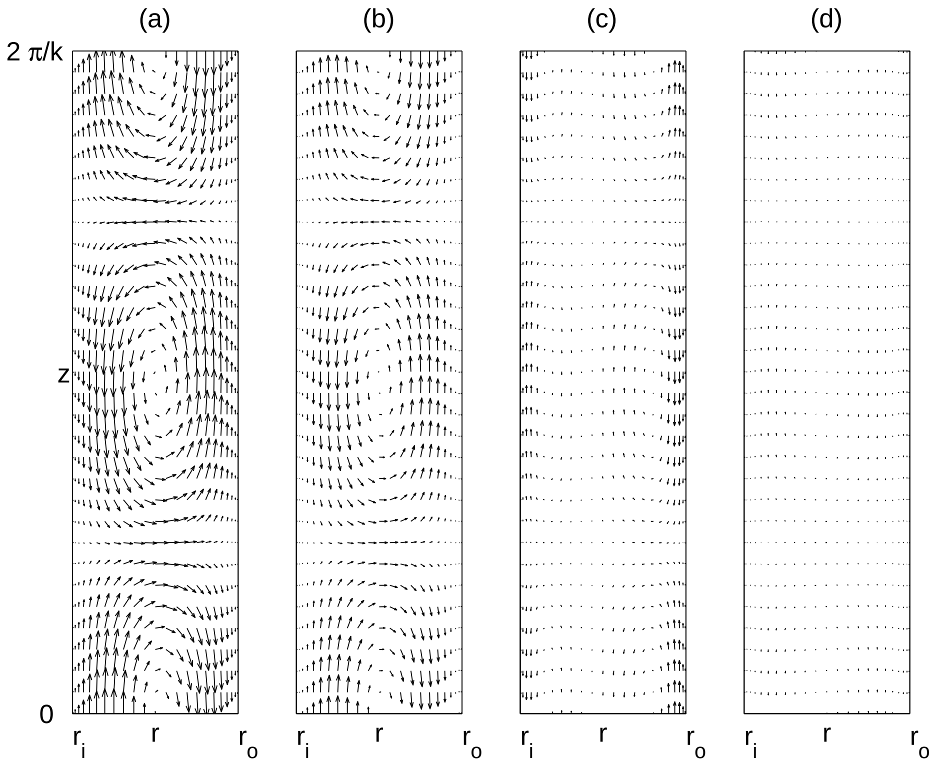

Figure 4: Radial-axial components of the perturbation field for $Ri = 0$, $Ro = -4000$ at different times: (a) $t = 0$, (b) $t = \tau_o/10$, (c) $t = \tau_o/5$, (d) $t = 2\tau_o$.

Acknowledgments

I thank Nick Trefethen for his fruitful discussions. This work was supported by the Engineering and Physical Sciences Research Council of the United Kingdom under Grant GR/M30890.

References

[Butler & Farrell, 1992] Butler, K. M., Farrell, B. F., 1992, *Three-dimensional optimal perturbations in viscous sheaar flow.*, Phys. Fluids A **48**.

[Chossat & Iooss, 1991] Chossat, P., Iooss, G., 1991, *The Couette-Taylor Problem*, App. Math. Sci. **102**, Springer-Verlag.

[Coles, 1965] Coles, D., 1965, *Transition in circular Couette flow*, J. Fluid Mech. **21**: 385-425.

[Gebhardt & Grossmann, 1993] Gebhardt, T., Grossmann, S., 1993 C., *The Taylor-Couette eigenvalue problem with independently rotating cylinders*, Z. Phys. B **90**: 475-490.

[Joseph, 1976] Joseph, D. D., 1976, *Stability of Fluid Motions* vol. I and II. Springer Tracts in Natural Philosophy, 27-28. Springer-Verlag, Berlin.

[Kato, 1976] Kato, T., 1976, *Perturbation Theory for Linear Operators*, Springer-Verlag, Berlin.

[Meseguer & Marques, 2000] Meseguer, A., Marques, F., 2000, *On the competition between centrifugal and shear instability in spiral Couette flow.*, J. Fluid Mech. **402**: 33-56.

[Schmid & Henningson, 1994] Schmid, P. J., Henningson, D. S., 1994, *Optimal energy growth in Hagen-Poiseuille flow*, J. Fluid Mech. **277**.

[Schmid & Henningson, 2001] Schmid, P. J., Henningson, D. S., 2001, *Stability and Transition in Shear Flows*, Applied Mathematical Sciences **142**, Springer-Verlag, New York.

[Tagg, 1994] Tagg, R., 1994, *The Couette-Taylor Problem*, Nonlinear Science **4**(3).

[Taylor, 1923] Taylor, G. I., 1923, *Stability of a viscous fluid contained between two rotating cylinders*, Phil. Trans. Roy. Soc. London Ser A **223**.

[Trefethen & Bau, 1997] Trefethen, L. N., Bau, D., 1997 *Numerical Linear Algebra*, SIAM.

[Trefethen *et al.*, 1993] Trefethen, L. N. *et al.*, 1993, *Hydrodynamic Stability Without Eigenvalues*, Science, **261**.

[Van Atta, 1966] Van Atta, C., 1966, *Exploratory measurements in spiral turbulence*, J. Fluid Mech. **25**: 495-512.