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Logarithmically-accurate and positive-definite NLO shower matching

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ABSTRACT: We present methods to achieve NLL+NLO accurate parton showering for processes with two coloured legs: neutral- and charged-current Drell-Yan, and Higgs production in pp collisions, as well as DIS and e^+e^- to jets. The methods include adaptations of existing approaches, as well as a new NLO matching scheme, ESME, that is positive-definite by construction. Our implementations of the methods within the PanScales framework yield highly competitive NLO event generation speeds. We validate the fixed-order and combined resummation accuracy with tests in the limit of small QCD coupling and briefly touch on phenomenological comparisons to standard NLO results and to Drell-Yan data. The progress reported here is an essential step towards showers with logarithmic accuracy beyond NLL for processes with incoming hadrons.

KEYWORDS: Higher-Order Perturbative Calculations, Parton Shower, Resummation

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1 Introduction

As the Large Hadron Collider (LHC) explores the electroweak scale and the energy frontier with continually increasing luminosity, the demands on the accuracy of QCD predictions grow ever more challenging. The only predictive method that approaches physical realism while also being accurate is to match parton showers with fixed order calculations. Next-to-leading order (NLO) matching is widely considered to be a solved problem [1–6], and today most research on matching explores the question at next-to-next-to-leading order (NNLO), see e.g. [7–13].

With the advent of logarithmically-accurate parton showers [14–29], it becomes necessary to revisit NLO matching and examine how to combine fixed-order matching and logarithmic accuracy in a consistent manner. First steps at NLO were explored recently by some of us in ref. [30] for the simple case of unoriented two-body decays. One of the main purposes of this paper is to implement logarithmically-consistent NLO matching for a wider variety of processes, specifically pp scattering producing a Z/γ^* , W , or a Higgs boson; deep-inelastic scattering (DIS); and oriented two-body decays. Our adaptations of standard methods for achieving this are discussed in section 2. That section also addresses our treatment of real radiation, using multiplicative matching [31, 32] (cf. also POWHEG [2], KrkNLO [33] and MAcNLOPS [34]). Furthermore it discusses how to generate the real radiation phase space efficiently in the presence of substantial lepton asymmetries and highlights subtleties in the treatment of flavour in projection-to-Born methods in the presence of parity-violating contributions.

In revisiting NLO matching, the opportunity arises to ask whether it is possible to resolve a long-standing issue, namely the presence of negative-weight events. These are a characteristic of all main modern NLO and NNLO matching approaches, as well as NLO-merging methods [35, 36]. Depending on the process, their fraction may range from about a percent to tens of percent (see e.g. [37]). This causes problems both for statistical convergence and for machine-learning applications. It also results in an effectively unphysical event sample. There are, broadly speaking, two key sources of negative weights. One is connected with the generation of real radiation, and is present only in the MC@NLO method [1], which adds and subtracts contributions to a given shower’s real radiation. This source of negative weights is eliminated in the broad family of multiplicative matching [31, 32] methods, and is embodied in the POWHEG approach [2], as well as KrkNLO [33] and MAcNLOPS [34]. Section 3 presents a family of algorithms that resolve the other part of the problem, namely guaranteeing a positive-definite event-by-event NLO normalisation. It also introduces a method to convert a slicing calculation into the form of a subtraction.

An important part of the general PanScales approach is conclusive numerical testing of the quoted accuracy. It is to be kept in mind that NLO parton-shower matching brings extra terms beyond strict NLO accuracy starting at relative order α_s^2 , in particular, all-order logarithmic contributions. One important exercise will therefore be, in section 4, to compare our NLO matching directly to NLO calculations with physical settings for the coupling. We will additionally verify NLO accuracy using the same type of $\alpha_s \rightarrow 0$ approach that we originally introduced for checking logarithmic accuracy [14]. To the best of our knowledge, this is the first time such a test has been carried out for NLO parton-shower matching. We will also show next-to-next-to-double logarithmic (NNDL, $\alpha_s^n L^{2n-2}$) accuracy tests for event-shape like observables. As was the case for e^+e^- matching [30], the NNDL tests are a crucial step on the way towards full next-to-next-to-leading-logarithmic accuracy (NNLL, $\alpha_s^n L^{n-1}$) [25, 29].

Given that this is the first time that a parton shower demonstrably reaches general NLL+NLO as well as event-shape NNDL accuracy, in section 5.1 we will include a brief comparison to Drell-Yan data and then in section 5.2 discuss event-generation speed, before concluding in section 6.

Finally, some further technical details are discussed in appendices A–D.

2 Adaptations of existing NLO approaches

In this section we start with an overview of NLO matching and associated notation. We then briefly examine how we handle the matching of real radiation, using a multiplicative method that all other parts of this paper rely on. We then consider two alternative ways for the generation of the Born event with the NLO normalisation: one adapts the numerical approach of the POWHEG-BOX [2–4] framework to the PanScales showers, the other, specific to DIS, uses an analytic Projection-to-Born (P2B) methodology [38, 39].

2.1 Overview and notation

The strategies we follow in this work all belong to the multiplicative matching category in which the cross section for the event, starting from a given Born phase-space point Φ_B , can be written as [2]

$$d\sigma_{\text{mult}} = \bar{B}(\Phi_B) d\Phi_B \left[S(v^{\text{PS}}, \Phi_B) \times \frac{R(\Phi_B, \Phi_{\text{rad}})}{B_0(\Phi_B)} d\Phi_{\text{rad}} \right] \times I_{\text{PS}}(v^{\text{PS}}, \Phi_B, \Phi_{\text{rad}}) + \bar{B}(\Phi_B) d\Phi_B S(v_{\text{min}}, \Phi_B). \quad (2.1)$$

Let us focus for now on the first line. Schematically, it depends on three main ingredients. The term in square brackets in eq. (2.1) describes the generation of the first emission parameterised by Φ_{rad} , which is associated with a value of the parton shower (PS) ordering variable v^{PS} . This depends on the Sudakov form factor, $S(v^{\text{PS}}, \Phi_B)$, given by

$$S(v^{\text{PS}}, \Phi_B) = \exp \left[- \int_{v > v^{\text{PS}}} \frac{R(\Phi_B, \Phi_{\text{rad}})}{B_0(\Phi_B)} d\Phi_{\text{rad}} \right], \quad (2.2)$$

which we note is computed using the full matrix element $R(\Phi)$ (with $\Phi \equiv \{\Phi_B, \Phi_{\text{rad}}\}$), as opposed to the shower’s approximate matrix element. We elaborate more on this term in section 2.2. Another ingredient is $I_{\text{PS}}(v^{\text{PS}}, \Phi)$ which denotes the subsequent iterations of the parton shower evolution, starting from v^{PS} . Lastly, the normalisation factor $\bar{B}(\Phi_B)$ is given at NLO by

$$\bar{B}(\Phi_B) = B_0(\Phi_B) + \underbrace{V(\Phi_B) + \int R(\Phi_B, \Phi_{\text{rad}}) d\Phi_{\text{rad}}}_{\text{relative order } \alpha_s}, \quad (2.3)$$

with $B_0(\Phi_B)$ the Born matrix element, $V(\Phi_B)$ the 1-loop contribution and $R(\Phi_B, \Phi_{\text{rad}})$ the real matrix element. The correct NLO cross section is therefore obtained upon integration over the Born phase space

$$\sigma = \int \bar{B}(\Phi_B) d\Phi_B. \quad (2.4)$$

Finally, we comment on the second line of eq. (2.1). It accounts for the no-radiation probability, with v_{min} the transverse momentum scale of the non-perturbative shower cutoff. Since it is a fairly standard term, in what follows we leave it out from our explicit expressions, so as to lighten the notation. However, in our practical implementations such a term is always automatically included.

2.2 Treatment of the real radiation

The shower branching kinematic variables are the dimensionful ordering variable v (e.g. a transverse momentum), and two auxiliary angular variables $\bar{\eta}$ and ϕ (cf. appendix A) and the core equation that we use for the matched branching probability for any given partition of a dipole is

$$\frac{dP}{d \ln v d\bar{\eta} d\phi} = \frac{1}{d\Phi_B} \frac{d\Phi}{d \ln v d\bar{\eta} d\phi} \frac{R_p(\Phi)}{B_0(\Phi_B)}. \quad (2.5)$$

Here $R_p(\Phi)$ is a positive *partition* of the full matrix element for the given final state, designed such that it has the full singularities of the corresponding partition of the dipole, and only those singularities. This ensures that in the infrared the branching probability tends to the shower branching probability, as is required notably for NNDL accuracy [30]. The expressions for $\frac{d\Phi}{d \ln v d\bar{\eta} d\phi}$ together with our partitioning of the matrix elements are outlined in appendix B. For eq. (2.5) to be sufficient for generating the full real radiation, it is necessary that the shower branching map covers phase space. We have verified that this is the case for the PanGlobal and PanLocal showers for up to a total of 3 initial and final-state partons for all processes that we consider here. For the PanLocal showers, this required modifications outlined in appendix A.2.2.

One practical consideration is the boundedness of eq. (2.5). In many of the cases that we examined, the bound from the parton shower approximation $\frac{dP}{d \ln v d\bar{\eta} d\phi}$ was sufficient also with the full matrix element. One situation where it was not was in Drell-Yan production, somewhat away from the Z pole, e.g. around $m_{\ell\ell} = 130\text{--}140$ GeV and more generally also for W production and decay. There, a substantial forward-backward asymmetry arises in the Born matrix element, but not always in the corresponding real matrix element, e.g. for transverse momenta of the order of $m_Z/2$. In phase-space points where the asymmetry causes $B_0(\Phi_B)$ to be particularly small, this enhances the apparent branching probability. Potential solutions include choosing a large overhead factor, which would slow down event generation; or trying to adaptively determine a grid of overhead factors, which might require a warm-up phase in the event generation.

Instead we found it more convenient to simultaneously consider the matrix elements for a given $\ell^+\ell^-$ final state and that where the ℓ^+ and ℓ^- momenta are swapped. Starting from a given Born configuration (say $\ell^+\ell^-$), we first make sure that we reproduce the correct total rate for $\ell^+\ell^-$ or $\ell^-\ell^+$ in the real configuration. Then, where necessary, we adjust the relative rates for $\ell^+\ell^-$ and $\ell^-\ell^+$ by swapping the momenta of the ℓ^+ and ℓ^- with an appropriate probability. This approach, analogous to eq. (4) of ref. [25], avoids a large overhead without the use of an adaptive grid, allowing us to obtain efficient real event generation without a substantial warm-up phase. The concrete algorithm is described in appendix D.¹

2.3 dBNLO: an adaptation of the POWHEG-BOX method

For generic processes, the function \bar{B} as defined by eq. (2.3) is not known analytically. To numerically evaluate \bar{B} , one approach that we adopt is similar to that used in the POWHEG-BOX

¹One open question is whether the swap algorithm could conceivably be generalised and made fully differential rather than discrete, e.g. to sample massive vector boson decays after the NLO generation has been performed.

framework [3]. We dub this approach dBNLO because, as we will see below, a correction to the \bar{B} function is sometimes needed to reproduce NLO accuracy, owing to the fact that the shower map may not agree with the parameterisation used in the POWHEG-BOX.

As in the original approach, the \bar{B} function is trivially rewritten, bringing all contributions in eq. (2.3) (see also eq. (3.2) below) under the same integral sign:

$$\bar{B}(\Phi_B) = \int dX_1 dX_2 dX_3 \tilde{B}(\Phi_B, X_1, X_2, X_3), \quad (2.6)$$

where the equality between eqs. (2.3) and (2.6) effectively defines \tilde{B} . Here, the X_i are the phase-space variables of the real radiation, which are typically taken to be in a unit hypercube (through simple transformation of the integration variables Φ_{rad}).

In order to evaluate eq. (2.6), a specific map for the real radiation phase space Φ_{rad} needs to be chosen. In ref. [3], expressions were derived both within the Frixione-Kunszt-Signer (FKS) [40, 41], and Catani-Seymour (CS) [42] subtraction schemes. On the other hand, in our approach the hardest emission is generated using the PanScales showers. This is done so as to facilitate retaining the logarithmic accuracy, as was discussed in ref. [30]. The kinematic maps associated with the FKS (or CS) schemes are not guaranteed to coincide with those implemented in the PanScales showers in the hard region (which is the one relevant for matching). A mismatch in the mapping from the Born to Born+1 phase space between \bar{B} and the parton shower will induce a spurious $\mathcal{O}(\alpha_s)$, spoiling the NLO accuracy.

There are at least two possibilities to solve this issue. One could calculate the counterterms associated with the PanScales showers so as to correctly sample \bar{B} (these counterterms would need to be computed in $4 - 2\epsilon$ dimensions for each shower variant). Alternatively, one can introduce a correction term to account for the $\mathcal{O}(\alpha_s)$ difference, directly in four dimensions. We opt for the latter and correct \bar{B} as computed with the FKS parameterisation,² as this is the one that is implemented in the POWHEG-BOX [4]. This solution has the advantage that, going forward, we also have the option of using \tilde{B} functions for different processes as implemented in the POWHEG-BOX. The correction term, which we denote by $\Delta\tilde{B}$, can be calculated automatically. It is defined through

$$d\Phi_{\text{rad}}^{\text{FKS}} \tilde{B}^{\text{corr.}}(\Phi_B, X) = d\Phi_{\text{rad}}^{\text{FKS}} \tilde{B}^{\text{FKS}}(\Phi_B, X) + d\Phi_{\text{rad}}^{\text{PS}} R(\Phi^{\text{PS}}) - d\Phi_{\text{rad}}^{\text{FKS}} R(\Phi^{\text{FKS}}) \quad (2.7a)$$

$$= d\Phi_{\text{rad}}^{\text{FKS}} \left[\tilde{B}^{\text{FKS}}(\Phi_B, X) + \Delta\tilde{B}(\Phi_B, X) \right], \quad (2.7b)$$

with $\Delta\tilde{B}$ defined as

$$\Delta\tilde{B}(\Phi_B, X) \equiv \left| \frac{d\Phi_{\text{rad}}^{\text{PS}}}{d\Phi_{\text{rad}}^{\text{FKS}}} \right| R(\Phi^{\text{PS}}) - R(\Phi^{\text{FKS}}). \quad (2.8)$$

The phase-space measures

$$d\Phi^{\text{PS/FKS}} = d\Phi_B d\Phi_{\text{rad}}^{\text{PS/FKS}}, \quad (2.9)$$

implicitly define a map between the Born phase space, Φ_B , and the Born+1 phase space. The $\Delta\tilde{B}(\Phi_B, X)$ term is the main conceptual novelty of this matching method, hence our choice of the dBNLO name.

²Using radiation variables $\xi = \frac{2E_k}{\sqrt{s}}$, $y = \cos\theta_{ik}$, ϕ , where k is the emitted parton, i is the emitter, and ϕ is an azimuthal angle.

Regarding the technical implementation within the PanScales framework, we generate (unweighted) Born events according to $\bar{B}(\Phi_B)$ following the MINT approach [43]: in a first phase, integration grids are generated using the adaptive importance-sampling VEGAS algorithm [44]. In a second phase, upper bounds are found for the integrand \tilde{B} . In MINT these upper bounds can be estimated with the possibility of “folding” the radiation variables multiple times over the integration range, so as to minimise the risk of the integrand being negative-valued. We have implemented the folding procedure in our PanScales framework, though in the following we typically show results without folding, i.e. including negative weights.³

Once upper bounds have been found, the $\tilde{B}(\Phi_B, X)$ function can be sampled randomly, and Born variables are generated with an accept-reject algorithm, where we simply discard the radiation coordinates X . This ensures that the Born phase space is sampled according to the $\bar{B}(\Phi_B)$ distribution.

The correction term $\Delta\tilde{B}(\Phi_B, X)$ is calculated at the same time as the \tilde{B}^{FKS} function, in a semi-automated numerical way. When evaluating the contribution to the integrand for a given (Φ_B, X) , one translates the FKS variables defining the real emission, $\Phi_{\text{rad}}^{\text{FKS}}$, to the PanScales variables, $\Phi_{\text{rad}}^{\text{PS}}$ (associated with the Jacobian $|d\Phi_{\text{rad}}^{\text{PS}}/d\Phi_{\text{rad}}^{\text{FKS}}|$). One can then perform that emission with full kinematics from the Born state Φ_B with any of the PanScales showers as well as our implementation of the FKS map. One then evaluates eq. (2.8) at that phase space point. Note that infrared and collinear divergences cancel in $\Delta\tilde{B}$ and the correction is hence finite.

We have implemented the dBNLO method for $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$, $pp \rightarrow Z$ for both the PanGlobal and PanLocal showers. In these cases the \tilde{B}^{FKS} function is known analytically from ref. [1]. The $\Delta\tilde{B}$ is computed as explained above for $e^+e^- \rightarrow q\bar{q}$ and is simply zero for $pp \rightarrow Z$, because the FKS, PanGlobal and PanLocal shower maps all act equivalently for the first emission.

We have also considered $pp \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ for the PanGlobal shower. For this process, we interfaced the Fortran code from the POWHEG-BOX associated with ref. [45], in order to evaluate the \bar{B} . Here too $\Delta\tilde{B} \equiv 0$ for PanGlobal, whereas the PanLocal map acts differently on γ^*/Z decay products and so would require a non-zero $\Delta\tilde{B}$, which we have yet to implement. Note that when the lepton swaps of appendix D are being used to optimise the generation of the Drell-Yan real matrix element, we employ a suitably adapted version of the POWHEG-BOX Fortran code, to account for the fact that the real contribution in the integral in eq. (2.3) should involve the two lepton permutations. This modification is described in appendix D.

Thinking forward to future work, one potential advantage of the dBNLO method for generating the Born event is that it opens up the possibility of reading in \tilde{B}^{FKS} from the POWHEG-BOX for matching generic processes. The function $\Delta\tilde{B}$ would then be computed separately and automatically. Note however that for general processes the generation of the real emission remains non-trivial and to maintain logarithmic accuracy it is important for it to be generated in a way that is consistent with the shower map and the shower’s specific pattern of higher-order corrections in various infrared limits.

A final comment is that for PanGlobal with $\beta_{\text{ps}} = 0$, for the first emission in any colour singlet process, the POWHEG-BOX kinematic map and ordering variable are identical

³In all processes we have investigated, the fraction of negative weights was below 5 per mille.

to the corresponding shower map and ordering variables. This means that in principle it is also possible to shower events in the Les Houches Event [46] (LHE) format produced from the POWHEG-BOX and retain NLL accuracy. However, for such an interface, some practical aspects remain to be implemented concerning the correct processing of the LHE files and the setup of the corresponding PanScales event for subsequent showering. Furthermore, with the information that is available in LHE files it would not be possible to support the spin-correlation component [16–18] of NLL accuracy.

2.4 Projection-to-Born

Generically, the P2B approach [38] exploits the fact that eq. (2.3) can be computed analytically for certain processes, as a function of the Born kinematics, specifically where the real branching leaves key Born invariants unchanged. In a shower context, ref. [39] used this at NNLO+PS for DIS.⁴ In this paper, we apply it at NLO to DIS and eq. (2.3) is known analytically in terms the proton structure functions [48],

$$\bar{B}(\Phi_B) = \frac{4\pi\alpha^2}{x_{\text{DIS}}Q_{\text{DIS}}^4} \left[\frac{1}{2}(1 + (1 - y_{\text{DIS}})^2)F_2 - \frac{1}{2}y_{\text{DIS}}^2F_L + x_{\text{DIS}}y_{\text{DIS}} \left(1 - \frac{1}{2}y_{\text{DIS}}\right)F_3 \right], \quad (2.10)$$

where the Born variables are x_{DIS} , Q_{DIS}^2 and $y_{\text{DIS}} = Q_{\text{DIS}}^2/(x_{\text{DIS}}s)$, and s is the collider centre-of-mass energy squared. When writing eq. (2.10) we have assumed a shower mapping that preserves the DIS invariants Q_{DIS}^2 and x_{DIS} , as indeed happens for the PanScales showers. We use `Hoppet` [49, 50] for the evaluation of the structure functions, $F_{2,3,L}$.

The expression in eq. (2.10) contains an implicit sum over all possible flavour channels. In the context of parton shower exclusive simulations, for a given event, we generate one specific Born flavour channel and \bar{B} needs to be known for that specific flavour. At LO, the Born flavour label is trivial, but this is not the case for NLO, where real diagrams might originate from several underlying Born configurations. This is the case for gluon-initiated real corrections $g\ell \rightarrow \ell'\bar{q}q'$, where the possible underlying Born channels are $q\ell \rightarrow q'\ell'$ and $\bar{q}\ell \rightarrow \bar{q}'\ell'$. The `Hoppet` structure functions can be decomposed by flavour, but for the gluon-induced axial (F_3) component there is an intrinsic ambiguity in the assignment of a $g\ell \rightarrow X$ contribution to Born flavour and anti-flavour structure functions, with only their difference contributing to the cross section. In practice that F_3 contribution to $g\ell \rightarrow X$ is effectively set to zero.⁵ However, a given matched shower’s backward evolution from quark or anti-quark to gluons will in general not yield an ensemble of final states with equivalent cross sections. This implies that there can be a mismatch between the NLO flavour versus anti-flavour assignment

⁴It is arguable whether a constant (N)NLO normalisation factor, as used e.g. in [27, 30, 47], also counts as P2B. In practice, our PanScales $H \rightarrow gg$ decay process has an inclusive NLO normalisation matching option that is classified as “P2B”, but our nomenclature may evolve.

⁵In the neutral current F_2 and F_L structure functions, quarks and anti-quarks contribute equally, while in the F_3 structure function, quark and anti-quark contributions appear with opposite signs, cf. eq. (18.18) of the 2024 edition of the Structure Functions review by the Particle Data Group [48]. A potential gluon-induced F_3 contribution would come from a convolution of a $C_{qg}^{(3)}$ coefficient function with the gluon distribution. However since the $C_{qg}^{(3)}$ convolution contributes equally to quarks and anti-quarks, its net contribution to the cross section differentially in x and Q^2 will always be zero. Because of this, the $C_{qg}^{(3)}$ coefficient function term is conventionally simply set to be zero (note, for example, its absence in [51]). Consequently, in a standard structure-function based P2B approach, there is no NLO gluon-induced contribution that is attributed to F_3 .

in the structure functions and the true \bar{B} that is actually needed for the given shower mapping. To be able to use these structure functions, we can either partition the gluon-induced real correction consistently, which might not always be possible, or calculate the mismatch.

For photon-induced DIS, where F_3 is zero, any democratic partitioning of the gluon-induced matrix element, such as the one we have implemented in eq. (B.20), yields a result consistent with the structure function. This is due to the fact that the matrix elements for $q\ell \rightarrow q\ell$ and $\bar{q}\ell \rightarrow \bar{q}\ell$ are identical. This is no longer the case if we consider Z or W as mediators, due to the axial component of the coupling. This flavour mismatch will induce a correction to the flavour-decomposed \bar{B} . For the time being, therefore, with the P2B method we focus only on the photon-mediated DIS process and leave for future work the treatment of Z and W mediated process. However, we note that for any infrared safe observable that does not depend on flavour, we would still obtain NLO accuracy once we sum over all the flavours if we use this partitioning for all the DIS processes.⁶

3 Positive-definite NLO event generation

When NLO matching methods for parton showers were first developed [1, 2], the advance was sufficiently revolutionary that a small fraction of negative weights was considered a price well worth paying. However, as NLO parton-shower matching has evolved to become the default accuracy for essentially all studies, and a foundation for first NNLO shower matching methods, the question of negative-weight events is taking on greater importance. Firstly, for a fraction f of negative-weight events, the statistics required for a given accuracy scale as $1/(1 - 2f)^2$. For example, already for $f = 0.15$ this doubles the required statistics. There are key LHC studies where this is a limiting factor [52] and nowadays this is widely considered to be a problem [53–56]. The issue of negative weights turns out to be challenging also with modern machine-learning (ML) approaches (see e.g. [57–62]), which typically assume a physical, i.e. positive-definite event stream. Indeed, one could argue that the core goal of Monte Carlo event simulation, which is to provide a physically realistic simulation of high-energy collisions, is in some way not being met if there is even a single negative-weight event.

There are several strategies in the literature to address the question of negative weights. Some are intended to be used as an intrinsic part of the NLO generation code, for example folding [37, 43] and Born spreading [63] and related methods [64, 65] (see below for further discussion). Methods to reduce the fraction of negative weights have also been explored within the **Sherpa** framework [66]. Other methods effectively modify a sample after it has been generated, notably cell resampling [67–69] and machine-learning based neural resampler methods [70]. In general these methods reduce the fraction of negative weights, but do not completely eliminate them.⁷ In almost all cases, the reduction comes at the cost of a speed

⁶Ref. [39] describes the implementation of a NNLO+PS generator for DIS starting from the NNLO structure functions. The parton shower branching history is used to determine the underlying Born flavour assignment for the configurations containing one (and two) extra emissions, which does not match the partitioning that is effectively present in the (N)NLO structure functions on the final integrated result. We stress that the mismatch cannot be seen for any flavour-summed observable, hence the NNLO accuracy of the generator presented in ref. [39] is not impacted for such observables.

⁷The neural resampler method promises to eliminate negative event weights, as long as the cross section is positive in a given phase space region. As discussed below, this is not always the case.

penalty, a potentially hard-to-quantify NLO bias, a sample that no longer has uniform weights and/or an after-burner stage that complicates the overall event-generation workflow.

The purpose of this section is to introduce a new method that ensures the absence of negative weights, intrinsically as part of the event generation, while maintaining speed and guaranteed NLO accuracy. In section 3.1 we discuss the various potential sources of negative weights (see also the discussion of ref. [37]). One main source is addressed by treatments of real radiation that involve just multiplicative or (positive-definite) additive matching [2, 31–34], cf. our choices in section 2.2. Section 3.2 then introduces a generic method to address the other non-trivial source, connected with the Monte Carlo evaluation of the NLO \bar{B} normalisation. It exploits a Sudakov exponentiation, and we will refer to the resulting generic class of algorithms as “Exponentiated Subtraction for Matching Events,” ESME. It can be seen as a generalisation of a Poisson process, and we note that Poisson processes have been explored for matching also in past work [71, 72]. Section 3.3 then provides a specific implementation that combines real and NLO normalisation into a single algorithm. Finally, section 3.4 highlights a translation that we have used between slicing and subtraction that facilitates the use of our algorithm with the PanScales parton showers.

3.1 The origins of negative weights in standard matching approaches

As discussed in section 2.1, the weight of a Born event, at NLO accuracy, should be generated according to

$$d\sigma = \bar{B}(\Phi_B)d\Phi_B, \tag{3.1}$$

where $\bar{B}(\Phi_B)$ is given in eq. (2.3). In the most common NLO matching approaches, MC@NLO and POWHEG, equations like eq. (2.3)⁸ are evaluated with the help of FKS [40] or dipole [42, 73] subtraction counterterms

$$\bar{B}(\Phi_B) = B_0(\Phi_B) + \underbrace{V(\Phi_B) + C_{\text{int}}(\Phi_B) + \int [R(\Phi) - C(\Phi)] d\Phi_{\text{rad}}}_{\text{relative order } \alpha_s}. \tag{3.2}$$

Generically, $C(\Phi)$ is a counterterm that satisfies $R - C \rightarrow 0$ in the soft and/or collinear limits for Φ_{rad} , and that is sufficiently simple to be integrated analytically

$$C_{\text{int}}(\Phi_B) = \int C(\Phi) d\Phi_{\text{rad}}. \tag{3.3}$$

If we assume that we have positive-definite parton distribution functions (PDFs), as in recent work from the NNPDF group [74], there are three sources of negative weights in common matching procedures.

The *first source of negative weights* lies in the fact that the contents of the underbrace in eq. (3.2) may genuinely be large and negative. For example if considering a process such as $Z + \text{jet}$ production (as the Born process), then in the limit of small- p_t for the jet the underbrace will go as $-B_0(\Phi_B) \times 2\alpha_s C_F / \pi \ln^2 M_Z / p_t$, and the overall \bar{B} as $B_0(\Phi_B)(1 - 2\alpha_s C_F / \pi \ln^2 M_Z / p_t)$. For sufficiently small p_t , this will go negative. In this case the physical

⁸Note that in MC@NLO $R(\Phi)$ is the shower approximation to the real matrix element.

origin is clear.⁹ However in general there may be a range of situations where the NLO coefficient is large and negative and the physical origin will not always be obvious.

A *second source of negative weights* is connected with the way the integral in eq. (3.2) is evaluated. In general, it requires a Monte Carlo evaluation, and this is often done with just a single Φ_{rad} sample for a given Φ_{B} . Even if the underbrace is positive when carrying out the full integration, in a Monte Carlo evaluation with a limited number of Φ_{rad} points, for a given Φ_{B} one may end up sampling a set of Φ_{rad} phase space points such that the underbrace appears large and negative. The main mitigation measure that is used for this is folding [43], which splits the real phase space into distinct regions and samples each of them for any given Φ_{B} . This can improve the situation quite substantially, albeit at a speed cost. Other techniques [63–65] seek to reorganise the integrand. This can reduce the fraction of negative weights without any impact on speed, but it arguably adds complexity to the formulation of the method. As they stand, none of these methods provide a guarantee of positivity.

In purely additive matching schemes, notably the MC@NLO approach, one has a *third source of negative weights*.¹⁰ In such an approach the $\bar{B}_s(\Phi_{\text{B}})$ function reads

$$\bar{B}_s(\Phi_{\text{B}}) = B_0(\Phi_{\text{B}}) + \underbrace{V(\Phi_{\text{B}}) + \int R_s(\Phi) d\Phi_{\text{rad}}}_{\text{relative order } \alpha_s}, \quad (3.4)$$

where $R_s(\Phi)$ is the shower’s approximation of the real matrix element. The Born event generation (with its subsequent showering) is then to be supplemented with an additional stream of events, which generates

$$d\Phi(R - R_s), \quad (3.5)$$

leading to negative weights when $R < R_s$.

In the rest of this section, we will show how to eliminate all sources of negative weights and so guarantee positive-weight events. In the simple cases that we have implemented, this is achieved without any speed penalty relative to the public NLO matching codes that we have tried.

3.2 Exponentiated subtraction for \bar{B}

3.2.1 The general algorithm

Here we present an algorithm that converts any subtraction integral of the form eq. (3.2) into an event-by-event integer, with the option to bound the integer and to control higher-order terms in the Monte Carlo average to some given order. The underlying principles of this algorithm can serve as a basis for a wide range of variants.

⁹And as a result there is an obvious physically-motivated solution in the MiNLO approach [75], which generates the Born event with a Sudakov, whose expansion cancels the negative $\alpha_s \ln^2 M_Z/p_t$ term. Alternatively, if one nests NLO Z and NLO $Z + \text{jet}$ showering then one may use formulas such as those present in refs. [11, 76–78] which cancel the negative $\alpha_s \ln^2 M_Z/p_t$ through the structure of the nested NLO terms.

¹⁰Called \mathbb{H} in e.g. ref. [37]; there it is further split into **N.1** and **N.2**. The first and second sources that we discussed above correspond, together, to \mathbb{S} in ref. [37], or equivalently **N.3**.

As a starting point, we assume a phase-space generation in which one can factorise the radiation phase space $d\Phi_{\text{rad}}$ into an ordering variable v and a 2-dimensional remainder,

$$d\Phi_{\text{rad}} = d\ln v d\Phi_2 J, \tag{3.6}$$

where J is a Jacobian. Standard FKS [40] and Catani-Seymour [73] phase-space generation lend themselves to this organisation, as reflected in their use for parton-shower style real-emission generation in POWHEG-BOX [3] and Sherpa [79]. For the purposes of the discussion below, it may be useful to think of v as being equivalent to a transverse momentum. As with a standard shower, we define a Sudakov factor

$$\begin{aligned} \Delta(v) &= \exp \left[- \int_v^{v_{\text{max}}} \frac{dv'}{v'} \rho(v') \right] \quad \text{with} \\ \rho(v) &= \int d\Phi_2 J \frac{M(\Phi)}{B_0(\Phi_{\text{B}})}, \quad M(\Phi) \geq \max[R(\Phi), C(\Phi)] \end{aligned} \tag{3.7}$$

where $M(\Phi)$ is a generic overestimate function that is always at least as large as the maximum of $R(\Phi)$ and $C(\Phi)$ (this requirement could be loosened here, but will be needed later in section 3.3). In the discussion below we take it to always be of the same order in α_s as the real matrix element, and to share the same infrared and collinear scaling properties. By definition, $R(\Phi)$ is positive definite and we assume a subtraction scheme in which $C(\Phi)$ is also positive definite, possibly after a suitable sum over partitions.¹¹ Furthermore, the integration eq. (3.7) covers the full real phase space, and we have $\Delta(v_{\text{max}}) = 1$ and $\Delta(0) = 0$. With this we can introduce our core procedure, Algorithm 1.

Algorithm 1 calculates an event-by-event normalisation factor n_{B} that multiplies $B_0(\Phi_{\text{B}})$ and whose average across many events with the same Φ_{B} is intended to satisfy

$$\langle n_{\text{B}} \rangle = 1 + r, \quad r \equiv \int \frac{R(\Phi) - C(\Phi)}{B_0(\Phi_{\text{B}})} d\Phi_{\text{rad}}. \tag{3.8}$$

This means that Algorithm 1 can be used in the evaluation of \bar{B} in eq. (3.2), apart from the $V + C_{\text{int}}$ contribution, which we will discuss explicitly below in section 3.3. It achieves this by using the standard Sudakov veto algorithm to choose a phase space point to sample. However, rather than generating an emission as in the normal Sudakov veto algorithm (with probability R/M), it increments or decrements n_{B} (according to the value of $(R - C)/M$). No real emission is ever generated. A second difference relative to the normal Sudakov veto algorithm is that after a change in n_{B} , the algorithm continues down in v , allowing subsequent further changes in n_{B} . It only stops when it reaches v_{min} . If using a fixed coupling inside Algorithm 1, v_{min} does not necessarily have to be connected with the shower cutoff, rather it is akin to a technical cutoff in standard subtraction procedures.

¹¹The $C(\Phi) \geq 0$ restriction can, we believe, be lifted simply by replacing $\max[R(\Phi), C(\Phi)]$, below, with $\max[R(\Phi), C(\Phi), R(\Phi) - C(\Phi)]$. $M(\Phi)$ is generally trivial to find in the infrared. It may be more complicated in the hard region if $R(\Phi)/B_0(\Phi_{\text{B}})$ grows large, however in that case it is conceptually straightforward to add a separate stream of events that accounts for any regions where $M(\Phi)$ is not sufficiently large, using standard unweighting methods. Typically we would expect $C(\Phi)/B_0(\Phi_{\text{B}})$ to remain under good control insofar as the counterterm is constructed from the Born multiplied by a factorised emission.

```

1 Set  $n_B \leftarrow 1$  and  $v \leftarrow v_{\max}$ ;
2 while  $v > v_{\min}$  do
3   generate next  $v$  and  $\Phi_2$  according to Sudakov with density  $\rho(v) d\ln v$ , eq. (3.7);
4   generate random number  $0 < r < 1$ ;
5   if  $r < \frac{|R(\Phi) - C(\Phi)|}{M(\Phi)}$  then
6     if  $R(\Phi) > C(\Phi)$  then
7        $n_B \leftarrow n_B + 1$ ;
8     end
9     else
10       $n_B \leftarrow n_B - 1$ ;
11    end
12  end
13 end
14 return  $n_B$ ;

```

Algorithm 1. General algorithm to convert NLO subtraction integral to integer.

We can demonstrate eq. (3.8) as follows. The probability that the algorithm will have triggered step 3 in a specific $d\ln v$ window is given by $\rho(v)d\ln v$. Given the $\ln v$ value and Φ_2 phase-space point, the algorithm will increment or decrement n_B with conditional probabilities P_+ or P_- respectively

$$\text{if } R(\Phi) > C(\Phi), \text{ increment } n_B \text{ with probability } P_+ = \frac{R(\Phi) - C(\Phi)}{M(\Phi)}, \quad (3.9a)$$

$$\text{if } R(\Phi) < C(\Phi), \text{ decrement } n_B \text{ with probability } P_- = \frac{C(\Phi) - R(\Phi)}{M(\Phi)}, \quad (3.9b)$$

or otherwise leave n_B unchanged. Writing out the integrals for $\rho(v)$, this then gives the following result for the average of n_B ,

$$\langle n_B \rangle = 1 + \int \frac{dv}{v} d\Phi_2 J \frac{M(\Phi)}{B_0(\Phi_B)} (P_+ \Theta(P_+) - P_- \Theta(P_-)), \quad (3.10)$$

which simplifies exactly to eq. (3.8).

Algorithm 1 always gives an integer as its output. It is particularly simple to analyse if $R - C$ always has the same sign. For example if we always have $R - C > 0$, then the probability distribution for n_B is exactly given by a Poisson distribution, i.e. $P(n_B) = e^{-r} r^{n_B-1} / (n_B - 1)!$ for $n_B \geq 1$, from which it is clear to see again that $\langle n_B \rangle = 1 + r$. We note that the use of a Poisson process to obtain a matching weight has been discussed before, notably in the context of NLO merging [71, 72].¹²

In the general case, $R - C$ may sometimes be positive, sometimes negative. Then, all but a fraction $\mathcal{O}(\alpha_s)$ of the time, the integer that is returned is $n_B = 1$, a consequence of the fact that r in eq. (3.8) is of order α_s . A fraction $\mathcal{O}(\alpha_s)$ of the time, the integer will be $n_B = 0$ or $n_B = 2$. A fraction $\mathcal{O}(\alpha_s^2)$ of the time, the integer will be $n_B = -1$ or $n_B = 3$, and so forth.

¹²We thank a referee for bringing this to our attention.

Thus, if we are interested just in NLO accuracy, we can discard any events with $n_B < 0$. Similarly, we are free to replace $n_B \rightarrow \min(n_B, p)$, where p is some integer $p \geq 2$. Assuming we know how to generate unweighted Born events, (positive-definite) unweighted NLO events can then simply be obtained by enhancing the Born event generation cross section by a factor p and then accepting any given Born event with probability n_B/p .

There is considerable freedom in adapting Algorithm 1 according to one’s needs. Below, in section 3.3, we will present a variant that incorporates the real event generation into the same loop, and is NLO accurate and relatively fast. Here we comment briefly on the scope for designing an algorithm that is positive-definite, bounded and that reproduces eq. (3.8) up to and including relative order α_s^m for any choice of positive integer m . The adaptation is remarkably simple: one simply multiplies the density $\rho(v)$ in the Sudakov by m , and increments or decrements n_B in steps 7 and 10 by $1/m$ rather than 1. Without any bounds on n_B , one still reproduces eq. (3.8) exactly. With a positivity bound $n_B \geq 0$, at least $m + 1$ decrement steps are needed to trigger the bound, i.e. the bound affects the results starting only at order α_s^{m+1} . Analogously with an upper bound p (with p at least 2). A detailed mathematical analysis confirming these points is given below in section 3.2.2.

There are also various potential adaptations concerning the speed of the algorithm, i.e. essentially the number of times one must evaluate $R(\Phi)$. In particular, to reproduce eq. (3.8) up to and including order α_s^m , one must allow the algorithm to go through steps 3–10 at least m times. However, after m steps have taken place, there is freedom to simply exit the algorithm even if $v > v_{\min}$. In general we expect a (modest) speed penalty in going to higher m , due to the higher Sudakov density and the larger number of steps that must be carried out before one is allowed to exit the algorithm.

A final comment concerns v_{\min} . One option is to take v_{\min} to be the same as in eq. (2.1), i.e. the scale of the non-perturbative shower cutoff. Another possibility is take the limit $v_{\min} \rightarrow 0$, which is an option as long as the $R(\Phi)$ and $C(\Phi)$ use a fixed-order expansion of the coupling (and PDF), not an all-order form with its Landau pole. If one wishes to obtain exact agreement with a standard NLO calculation, it is the $v_{\min} \rightarrow 0$ option that must be used, with both the coupling and any PDF factors being evaluated at the renormalisation and factorisation scales of the hard process. The difference between the two options will generally involve a non-perturbative power correction associated with the scale $v_{\min} \sim \Lambda_{\text{QCD}}$, as well as beyond-NLO contributions associated with the different possible choices for the running of the coupling and PDF.

We refer to procedures in the family of Algorithm 1 as “Exponentiated Subtraction” and their use for matching showers with fixed-order calculations as “Exponentiated Subtraction for Matching Events” (ESME).

3.2.2 Mathematical analysis of exponentiated subtraction

Let us define

$$r_+ \equiv + \int \frac{R(\Phi) - C(\Phi)}{B_0(\Phi_B)} \Theta(R(\Phi) > C(\Phi)) d\Phi_{\text{rad}}, \tag{3.11a}$$

$$r_- \equiv - \int \frac{R(\Phi) - C(\Phi)}{B_0(\Phi_B)} \Theta(R(\Phi) < C(\Phi)) d\Phi_{\text{rad}}, \tag{3.11b}$$

such that r in eq. (3.8) is given by $r = r_+ - r_-$, with both r_+ and r_- of order α_s , by construction. Let us separately count the number of increments that occur, n_+ and the number of decrements, n_- . The final n_B will be given by $n_B = 1 + n_+ - n_-$. The probability of a given number of increments is given by a Poisson distribution

$$P_+(n_+) = e^{-r_+} \frac{(r_+)^{n_+}}{n_+!}, \quad (3.12)$$

and similarly for the number of decrements,

$$P_-(n_-) = e^{-r_-} \frac{(r_-)^{n_-}}{n_-!}. \quad (3.13)$$

With no bounds on the number of increments or decrements, we have

$$\langle n_{\pm} \rangle = r_{\pm}, \quad \langle n_B \rangle = 1 + \langle n_+ \rangle - \langle n_- \rangle = 1 + r, \quad (3.14)$$

as expected. Next let us consider the probability $P(N)$ to obtain a given value of $N \equiv n_+ - n_-$ (with $n_B = 1 + N$),

$$P(N) = \sum_{n_+=+N}^{\infty} P_+(n_+)P_-(n_+ - N), \quad N \geq 0, \quad (3.15a)$$

$$P(N) = \sum_{n_-=-N}^{\infty} P_-(n_-)P_+(n_- + N), \quad N \leq 0. \quad (3.15b)$$

The sums can be evaluated to yield

$$P(N) = e^{-(r_++r_-)} \left(\frac{r_+}{r_-} \right)^{N/2} I_{|N|}(2\sqrt{r_+r_-}), \quad (3.16)$$

where $I_n(x)$ is the modified Bessel function of the first kind, whose series expansion is $I_n(2x) = \sum_{k=0}^{\infty} \frac{x^{n+2k}}{k!(n+k)!}$. From the first term in that series it is evident that eq. (3.16) is consistent with the expectation that $P(N) \sim \alpha_s^{|N|}$, recalling that the r_{\pm} are of order α_s . Furthermore, one sees that $P(N) = P_{\pm}(N) + \mathcal{O}(\alpha_s^{|N+1|})$ for $N \gtrless 0$, which is as expected given that the lowest order route to achieve a given value of N involves only increments ($N > 0$), or only decrements ($N < 0$).

Now let us suppose that we truncate N in such a way that when $N > N_{\max}$, we reset it to N_{\max} and when $N < -N_{\max}$ we reset it to $-N_{\max}$. Thus if $N_{\max} = 1$, this is like bounding $0 \leq n_B \leq 2$, i.e. whenever $n_B > 2$ after completing Algorithm 1, n_B is reset to 2, and whenever $n_B < 0$ after completing Algorithm 1 n_B is reset to 0. The average bounded n_B is given by

$$\langle n_B \rangle_{N_{\max}} \equiv 1 + \sum_{N=0}^{N_{\max}} N[P(N) - P(-N)] + \sum_{N=N_{\max}+1}^{\infty} N_{\max}[P(N) - P(-N)]. \quad (3.17)$$

We have not been able to evaluate this in general, however it is possible to establish the structure of the first non-trivial terms,

$$\langle n_B \rangle_{N_{\max}} = 1 + (r_+ - r_-) - \frac{1}{(N_{\max} + 1)!} \left(r_+^{N_{\max}+1} - r_-^{N_{\max}+1} \right) + \mathcal{O} \left(r_{\pm}^{N_{\max}+2} \right). \quad (3.18)$$

Thus if r_+ and r_- are both of order α_s , free of any logarithmic enhancements, the difference between $\langle n_B \rangle_{N_{\max}}$ and the unbounded $\langle n_B \rangle$ becomes of order $\alpha_s^{N_{\max}+1}$, as expected, and the coefficient multiplying that difference is factorially suppressed.

Finally, let us consider the variant of the algorithm that is designed to guarantee positive-definite results with control of terms up to and including α_s^m . As discussed above, one multiplies $\rho(v)$ by a positive integer m , which introduces an extra factor of m in front of every occurrence of r_{\pm} above. Additionally, one defines

$$n_B^{(m)} = 1 + \frac{N}{m}, \tag{3.19}$$

and takes $N_{\max} = m$. That yields

$$\langle n_B^{(m)} \rangle_{N_{\max}=m} = 1 + (r_+ - r_-) - \frac{m^m}{(m+1)!} (r_+^{m+1} - r_-^{m+1}) + \mathcal{O}(r_{\pm}^{m+2}). \tag{3.20}$$

If we consider $m = 3$, which would be sufficient to eliminate spurious terms up to and including N³LO, the coefficient multiplying $(r_+^{m+1} - r_-^{m+1})$ is $27/24$, which is comfortably close to 1.

A final comment is that it is important that r_+ and r_- are separately of order α_s , free of logarithmic enhancements, i.e. one should not rely on large cancellations between r_+ and r_- , or between their difference and C_{int} in eq. (3.2). In general we expect that this condition will be satisfied for single-scale problems and local subtraction schemes that avoid restrictive phase space cuts in the counterterm. For multi-scale problems, instead, the dependence on N_{\max} or m would provide a measure of potential higher order terms that is distinct from, and thus complementary to, scale variation. It would be interesting to further explore this in future work, keeping in mind that multi-scale problems anyway need a dedicated treatment, for example as in the MiNLO approach [8, 75].

3.3 An ESME algorithm with joint reals and subtractions

Here, we adapt the algorithm of section 3.2 not only to ensure NLO accuracy with positive-definite weights, but to organise it such that the effective \bar{B} evaluation and the real-emission generation share evaluations of the real matrix element. This helps reduce the total number of real matrix-element and associated PDF evaluations, and so can contribute to faster NLO event generation. It should be seen as just one among many possible algorithms founded on the principles of section 3.2.

One consideration is that in section 3.2 we left free the details of how to incorporate the $V(\Phi_B) + C_{\text{int}}(\Phi_B)$ contributions in eqs. (3.2) and (3.3), while here we will give specific prescriptions. The starting point of our method is that we will generate Born events with a weight $\bar{B}_C(\Phi_B)$, defined as

$$\bar{B}_C(\Phi_B) = B_0(\Phi_B) + V(\Phi_B) + C_{\text{int}}(\Phi_B). \tag{3.21}$$

Standard approaches instead generate the Born events with weight $\bar{B}(\Phi_B)$. The key difference is that $\bar{B}_C(\Phi_B)$ does not involve the Monte Carlo integral over $d\Phi_{\text{rad}}$ in eq. (3.2). This has a potential practical advantage, namely that to obtain the weight for any given Born configuration, one does not need a separate explicit integration over the real phase space. However, it obviously misses part of the overall \bar{B} normalisation. We will recover the

normalisation through the use of two non-unitary streams of events, which will account for the $\int d\Phi_{\text{rad}}[R(\Phi) - C(\Phi)]$ contribution to \bar{B} . Physically, if $R(\Phi) < C(\Phi)$, then $\bar{B}_C(\Phi_B)$ is too large and we need to eliminate some of the events generated with weight $\bar{B}_C(\Phi_B)$ (stream 1). If instead $R(\Phi) > C(\Phi)$, this implies that $\bar{B}_C(\Phi_B)$ is too small and we need an extra source of events (stream 2).

Each of the two streams will effectively account for specific parts of Algorithm 1, which, we recall, precisely evaluates $\int d\Phi_{\text{rad}}[R(\Phi) - C(\Phi)]$. The algorithm for 2 will address the situations in Algorithm 1 where $n_B = 1$ or $n_B = 0$, i.e. n_B is unchanged or decremented. Conversely, the algorithm for 3 will provide an additional source of events to account for the situations in Algorithm 1 where n_B is incremented. In other words, stream 1 will discard a fraction $\mathcal{O}(\alpha_s)$ of events relative to the Born rate, while stream 2 will add a fraction $\mathcal{O}(\alpha_s)$ of events. The sum of the two streams will also generate the hardest emission in such a way as to produce the correct real matrix element.

Let us first look at the algorithm for 2. Step 11 is a critical part of the algorithm, because it is the only step that is non-unitary. Specifically, it rejects the event with probability $[C(\Phi) - R(\Phi)]/M(\Phi)$. It is the direct analogue of step 10 of Algorithm 1, which decrements n_B . In standard NLO approaches for evaluating eq. (3.2), such regions with $C > R$ would be associated with a risk of negative-weight events. Because we account for that region through a rejection mechanism, that danger does not arise here. Aside from that, the $C > R$ branch is very much the standard Sudakov veto algorithm, accepting the emission with probability $R(\Phi)/M(\Phi)$ in step 14r. In the other branch, $C \leq R$, the stream 1 algorithm deviates from the standard Sudakov algorithm, because the emission is accepted with probability $C(\Phi)/M(\Phi)$ rather than $R(\Phi)/M(\Phi)$. The missing difference $R(\Phi) - C(\Phi)$, which connects with the increment of n_B in step 7 of Algorithm 1, will be accounted for in the algorithm for 3. Specifically, stream 2's step 15 occurs with probability proportional to $R(\Phi) - C(\Phi)$. It also generates a real emission, compensating the missing contribution for real emissions in stream 1's step 23. Stream 2's step 15 is also the only step that leads to an *event* being accepted in that stream. To order α_s , the corresponding probability, $[R(\Phi) - C(\Phi)]/M(\Phi)$ (when $R > C$) exactly matches the probability for incrementing n_B in step 7 of Algorithm 1. The behaviour of the overall algorithm is illustrated also in figure 1.

The combination of the two streams is reminiscent in some ways of the MAcNLOPS method [34], while stream 1 alone is similarly reminiscent of KrKNLO [33, 80]. But, where those references aimed to eliminate negative-weight events when trying to obtain the correct real part of the showering, here our intention is to also address difficulties that arise with the overall normalisation.

A further comment is that each stream exits the main matching loop as soon as a non-trivial action has taken place (i.e. reject event, or accept the emission and continue normal showering of the accepted event). If $M(\Phi)$ is chosen carefully enough, i.e. to be of order α_s , then there is an $\mathcal{O}(1)$ probability of exiting the loop at each stage, leading to an $\mathcal{O}(1)$ total number iterations around the loop. This is to be contrasted with the default formulation of Algorithm 1, which would typically require a number of steps proportional to $\ln^2 v_{\text{max}}/v_{\text{min}}$, most of which would bring no action because $|R - C| \ll M$ for small v .

Overall the above approach addresses the *second* source of negative events discussed in section 3.1, i.e. the one associated with the NLO normalisation, as well as potentially speeding up the generation by not requiring evaluation of the $d\Phi_{\text{rad}}(R - C)$ integral directly in the

```

1 Generate Born event according to  $\bar{B}_C$  distribution and set  $v = v_{\max}$ ;
2 while  $v > v_{\min}$  do
3   generate next  $v$  and  $\Phi_2$  according to Sudakov with density  $\rho(v) d\ln v$ , eq. (3.7);
4   generate random number  $0 < r < 1$ ;
5   if  $C(\Phi) > R(\Phi)$  then
6     if  $r > \frac{C(\Phi)}{M(\Phi)}$  then
7       | veto emission;
8     end
9     else
10      if  $r > \frac{R(\Phi)}{M(\Phi)}$  then
11        | return reject event;
12      end
13      else
14        | accept emission and return continue shower, accept event;
15      end
16    end
17  end
18  else
19    if  $r > \frac{C(\Phi)}{M(\Phi)}$  then
20      | veto emission;
21    end
22    else
23      | accept emission and return continue shower, accept event;
24    end
25  end
26 end
27 return accept event;

```

Algorithm 2. (ESME) Born + NLO rejection.

Born. The *third* source of negative events (associated with the generation of real radiation) was already implicitly addressed by our use of multiplicative matching, cf. section 2.2. There remains a potential for the *first* source of negative events, i.e. there could be Born phase space regions where $\bar{B}_C(\Phi_B) < 0$ because of large negative NLO contributions. Contrary to the other two sources, in this case there is often a clear physical reason, notably due to the breakdown of perturbative convergence for standard fixed-order calculations in the presence of disparate scales. In any region where this occurs, plain NLO cross sections are anyway devoid of physical meaning, and we are therefore free to reorganise the perturbative series so as to make it positive definite. In particular, defining B_0 and $\alpha_s B_{C,1}$ as the LO and NLO contributions to \bar{B}_C , we have the freedom to use

$$\bar{B}_C(\Phi_B) = B_0[1 + f(\alpha_s B_{C,1}/B_0)], \quad (3.22)$$

```

1 Generate Born event according to  $B_0$  (or  $\bar{B}_C$ ) distribution and set  $v = v_{\max}$ ;
2 while  $v > v_{\min}$  do
3   generate next  $v$  and  $\Phi_2$  according to Sudakov with density  $\rho(v) d\ln v$ , eq. (3.7);
4   generate random number  $0 < r < 1$ ;
5   if  $C(\Phi) > R(\Phi)$  then
6     if  $r > \frac{R(\Phi)}{M(\Phi)}$  then
7       | veto emission;
8     else
9       | return reject event;
10    end
11  else
12    if  $r > \frac{R(\Phi)}{M(\Phi)}$  then
13      | veto emission;
14    else if  $r > \frac{C(\Phi)}{M(\Phi)}$  then
15      | accept emsn, return continue shower, accept event;
16    end
17    else
18      | return reject event;
19    end
20  end
21 end
22 return reject event;

```

Algorithm 3. (ESME) NLO addition.

where $f(x)$ is any function that satisfies $f(x) = x + \mathcal{O}(x^2)$. If we additionally choose $f(x)$ such that $f(x) > -1$ for all x , then we will guarantee the total absence of negative-weight events. In practice we take

$$f(x) = \begin{cases} x & \text{for } x \geq 0 \\ \tanh x & \text{for } x < 0, \end{cases} \quad (3.23)$$

so that spurious higher-order contributions start only at relative order α_s^3 and large positive K factors are not modified.¹³

It is important to be aware that some aspects of the algorithm as formulated in this section still induce spurious α_s^2 contributions. Some of these can be mitigated: for example

¹³It is natural to ask whether one could use such an approach at the level of the *integrand* in standard NLO matching. While we cannot rule out that it might work, it is potentially more delicate. For example taking $f(x) = \max(-1, x)$ and integrating $\int_0^1 dv [1 + f(-\alpha_s v^{-2/3})]$ yields $1 - 3\alpha_s + 2\alpha_s^{3/2}$.

This is *not* correct, because of the $\alpha_s^{3/2}$ term, which is parametrically larger than a NNLO contribution. To what extent such an issue would arise in practice depends critically on the adaptive phase space generation for $d\Phi_{\text{rad}}$.

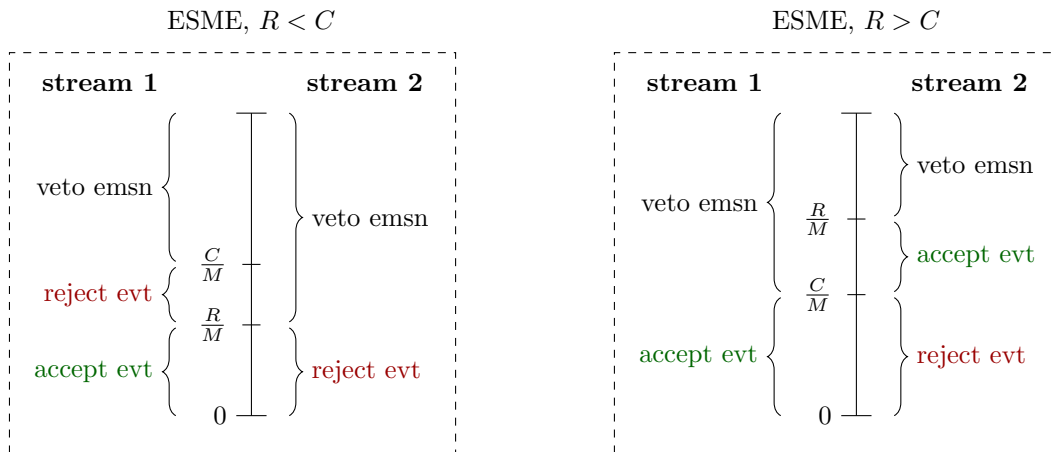


Figure 1. Simple illustration of the different possible actions in the two streams of the ESME algorithm with joint reals and subtractions. The actions are shown separately for the cases $R(\Phi) < C(\Phi)$ (left) and $R(\Phi) > C(\Phi)$ (right). In each case, when summing the two streams, one sees that the “accept evt” action occurs with total weight R/M . One can also verify that the contribution to the total event rate change relative to the \bar{B}_C normalisation is $(R - C)/M$. Recall that the default action in stream 1 (2) is to accept (reject) the event if the shower scale reaches v_{\min} — only when the action is different from the stream’s default is the total event rate affected.

cross terms between the \bar{B}_C normalisation and the stream 2 additions can be eliminated by generating stream 2 with a weight B_0 . In stream 1, there are α_s^2 effects from the product of $\alpha_s B_{C,1}$ and the order α_s event-rejection probability, which can be eliminated by renormalising that probability with a suitable factor.¹⁴ These mitigation strategies are included by default in our implementation, and in the phenomenological results presented in section 5. Yet other spurious second-order contributions are intrinsically associated with the structure of the algorithm. Their elimination would require adaptation of some of the techniques discussed at the end of section 3.2. We will briefly discuss the size of these terms below, in section 4.1.

The algorithms for each of the two streams both contain a v_{\min} cutoff, as in eq. (3.1) and Algorithm 1. Given the interplay with real radiation, here v_{\min} is always taken to be the shower non-perturbative cutoff. Technically that could introduce non-perturbative power corrections, however we have not found any evidence of these being numerically significant.

A final comment is that there is freedom also to replace stream 2 with a direct generation of real radiation events in proportion to $d\Phi(R - C)\Theta(R - C)$. For now we have not explicitly explored this option because of the need for an additional warm-up phase in order to efficiently sample the corresponding phase space.

3.4 Counterterm from slicing

When combining multiplicative matching of section 2.2 and the ESME treatment of section 3.3 for NLO normalisation, it is convenient to have a counterterm $C(\Phi)$ that can be easily

¹⁴Specifically, if $\bar{B}_C/B_0 > 1$, then the event-rejection probability, $[R(\Phi) - C(\Phi)]/M(\Phi)$ that is used in step 11 of 2 is divided by \bar{B}_C/B_0 . Otherwise it is multiplied by $2 - \bar{B}_C/B_0 = 1 - \alpha_s B_{C,1}/B_0$.

represented in the same v , $\bar{\eta}$ and ϕ variables as used for the shower. As in the preceding sections, there is considerable freedom in how to approach this. Here we outline the specific route we have taken, highlighting the ability to relate a slicing calculation to a shower-based subtraction approach, which we dub a slice-to-subtraction approach. In this subsection, to illustrate the approach, we focus on $e^+e^- \rightarrow 2$ jets and the PanGlobal shower, with other 2-leg processes discussed in appendix C.

The starting point will be a slicing calculation of the NLO rate for producing a given Born configuration with a tight constraint on any additional radiation. At NLO, conversion between calculations for different slicing variables is relatively straightforward and we will choose a slicing variable that coincides with the shower ordering variable v everywhere in phase space in the limit of small v . Specifically, for $e^+e^- \rightarrow q\bar{q}$ and the PanGlobal shower with $\beta_{\text{PS}} = 0$, the NLO normalisation with a slicing constraint $v < e^{-|L|}$, is given by

$$\bar{B}_{\text{PG}}(v < e^{-|L|}) = B_0(\Phi_{\text{B}}) \left[1 - \frac{\alpha_s C_F}{2\pi} \left(4L^2 + 6L + \frac{\pi^2}{3} + 2 \right) \right], \quad (3.24)$$

valid for large and negative L . This result was obtained by adapting the calculation presented in [81].

In general, it is possible to convert a slicing calculation into a subtraction calculation by constructing a counterterm $C(\Phi)$ that has the full QCD behaviour in the soft and/or collinear regions and can in practice be integrated over the full phase space above some arbitrary small v . Normally this is done with a counterterm that lives in the actual real-radiation phase space. However, one is free to choose a counterterm that is non-zero even for values of shower generation variables that do not map to valid phase space regions, e.g. if this facilitates the integration of the counterterm.¹⁵ Expressed in terms of the shower phase-space generation variables $\ln v$ and $\bar{\eta}$ (cf. appendix A), we take the counterterm to be

$$\frac{C(\Phi)}{B_0(\Phi_{\text{B}})} d\Phi_{\text{rad}} \rightarrow \frac{dv}{v} d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s}{\pi} z P_{gq}(z), \quad \ln z = \bar{\eta} - \bar{\eta}_{\text{max}}, \quad 0 < \bar{\eta} < \bar{\eta}_{\text{max}} = \ln Q/v, \quad (3.25)$$

where Q is the total centre-of-mass energy, for the quark and similarly for the anti-quark. Here $P_{gq}(z) = C_F(1 + (1 - z)^2)/z$ is the usual LO splitting function. The integration above $Qe^{-|L|}$ is simple and, after a sum over (half) dipoles, gives

$$\frac{C_{\text{int}}(v > Qe^{-|L|})}{B_0(\Phi_{\text{B}})} = 2 \int_{Qe^{-|L|}}^Q \frac{dv}{v} \int_0^{\ln Q/v} d\bar{\eta} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{\alpha_s}{\pi} z P_{gq}(z) = \frac{\alpha_s C_F}{2\pi} (4L^2 + 6L + 7), \quad (3.26)$$

for large $|L|$. From this, we can work out \bar{B}_C as in eq. (3.21) with this specific counterterm,

$$\bar{B}_C = \Sigma_{\text{PG}}^{\text{NLO}}(v < Qe^{-|L|}) + C_{\text{int}}(v > Qe^{-|L|}) = B_0(\Phi_{\text{B}}) \left[1 + \frac{\alpha_s C_F}{2\pi} \left(5 - \frac{\pi^2}{3} \right) \right]. \quad (3.27)$$

4 NLO and NNDL tests

In this section we validate the fixed-order and the logarithmic accuracy of our matched predictions. In particular, in section 4.1 we show that the $\mathcal{O}(\alpha_s)$ expansion of such predictions

¹⁵For the specific case of the PanGlobal final-state shower that we use as an illustration here, the counterterm phase space will actually coincide exactly with the genuine real phase space.

reproduces NLO calculations. In section 4.2 we instead demonstrate that we achieve NNDL accuracy for a series of continuously global observables.

4.1 NLO tests

We start by testing the relative $\mathcal{O}(\alpha_s)$ accuracy of the matched shower algorithms. For each process and observable, we first carry out a comparison to a standard NLO calculation with phenomenological settings for the coupling and shower parameters. For generic observables, NLO-matched showers give predictions that differ from the pure NLO result, because of higher-order differences that can come from e.g. the Sudakov form factor and its intrinsic connection to the momentum mapping during showering. In the ESME method, an additional source of higher-order differences comes from the elimination of negative event weights. Given that, in general, matched showers will not exactly agree with a pure NLO calculation, it is important to check that any differences are genuinely of higher order and not a small mistake in the NLO coefficient. Accordingly, we also explore the $\alpha_s \rightarrow 0$ limit of the NLO matched shower to isolate the pure NLO coefficient in the shower and thus conclusively establish the shower's NLO accuracy in any given matching scheme. This procedure is inspired by the standard PanScales approach for testing logarithmic accuracy, but we believe that it is the first time that it has been applied to tests of NLO shower matching.

4.1.1 e^+e^- collisions

A non-trivial observable with which to test the NLO accuracy for the $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$ process is the polar angle of the thrust axis, $\cos\theta_T$. At NLO its differential distribution is given analytically by [82, 83]

$$\frac{d\sigma_{\text{an}}}{d\cos\theta_T} = \sigma_0 \left[\frac{3}{8} (1 + \cos^2\theta_T) R_U + (1 - 3\cos^2\theta_T) R_L \right], \quad (4.1)$$

$$R_U = 1 + \frac{\alpha_s}{\pi} \frac{3C_F}{4} + \mathcal{O}(\alpha_s^2), \quad R_L = \frac{\alpha_s}{\pi} \frac{3C_F}{8} \left(8 \ln \frac{3}{2} - 3 \right) + \mathcal{O}(\alpha_s^2), \quad (4.2)$$

with σ_0 the inclusive Born cross section. Note that showers (including the PanScales showers) typically do not preserve the thrust axis, even in the presence of just one emission. Therefore the tests will verify both the effective \bar{B} for the Born configuration, as well as the structure of the real radiation.

In figure 2, we show fixed-order tests of the $\cos\theta_T$ distribution. In the top panel, we show NLO results with the PanGlobal $\beta_{\text{ps}} = 0$ shower, for a phenomenological setup with $\sqrt{s} = 91.1876$ GeV, with the dBNLO and ESME methods, using only γ^* exchange. We compare it to the analytical LO and NLO results from eq. (4.1). The middle panel shows the ratio to the total NLO result. Qualitatively the shower is similar to the NLO result, both in normalisation and shape, though there is about a 1% offset in the ESME method. The dBNLO method shows essentially no statistically significant offset. The difference between them is one measure of the size of the higher-order corrections associated with the elimination of negative weights and the observed 1% effect is consistent with the expected order of magnitude of an order α_s^2 term.

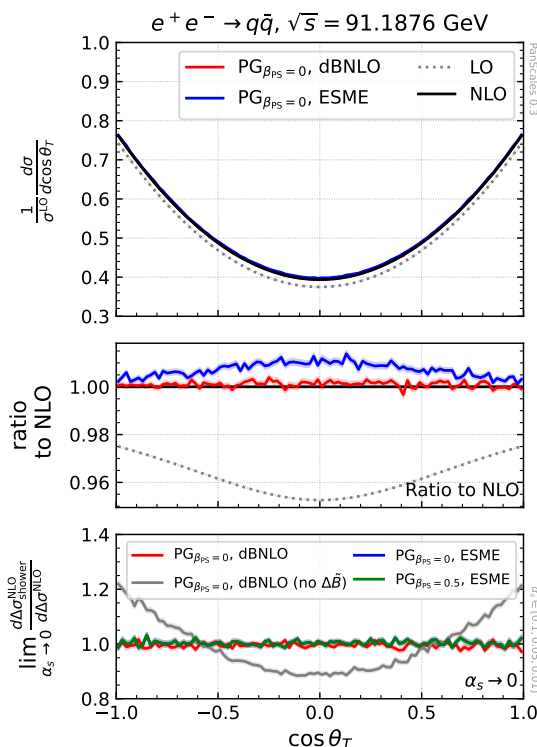


Figure 2. Tests of NLO-matched showers, showing the oriented thrust axis distribution in $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$ collisions. The top panel shows the ratio to the total Born cross section for a phenomenological setup with $\sqrt{s} = 91.1876$ GeV, $\alpha_s(\sqrt{s}) = 0.118$ and a showering cutoff of 0.5 GeV. The middle panel shows the ratio to the differential NLO cross section with the same settings for \sqrt{s} and α_s . The bottom panel shows the ratio of the pure NLO coefficient in the matched shower to the known exact NLO coefficient, i.e. the ratio in eq. (4.3).

To verify the correctness of the pure NLO coefficient we examine

$$\lim_{\alpha_s \rightarrow 0} \frac{d\Delta\sigma_{\text{shower}}^{\text{NLO}}}{d\Delta\sigma_{\text{an.}}^{\text{NLO}}}, \quad d\Delta\sigma^{\text{NLO}} \equiv d\sigma^{\text{NLO}} - d\sigma^{\text{LO}}, \quad (4.3)$$

where $d\sigma^{\text{LO}}$ ($d\sigma^{\text{NLO}}$) is the leading-order (next-to-leading-order) differential cross section for a specific observable. Note that for the analytic result, $\Delta\sigma_{\text{an.}}^{\text{NLO}}$ is a pure (relative) $\mathcal{O}(\alpha_s)$ correction. In the shower case, any higher-order corrections will be eliminated by taking the $\alpha_s \rightarrow 0$ limit. The $\alpha_s \rightarrow 0$ extrapolation is performed from runs at three values of $\alpha_s \in [0.1, 0.05, 0.01]$. The bottom panel of figure 2 shows eq. (4.3). The result is consistent with 1, to within statistical errors, confirming the NLO correctness of the ESME implementation. That same panel shows the NLO test for other NLO matching choices: ESME with the PanGlobal shower and $\beta_{\text{PS}} = 0.5$ and dBNLO with PanGlobal $\beta_{\text{PS}} = 0.0$, confirming their NLO correctness as well.

The bottom panel of figure 2 also shows the $\alpha_s \rightarrow 0$ result in the dBNLO method with the $\Delta\tilde{B}$ term artificially set to zero (grey curve). As expected, this does not agree with the true NLO correction, confirming the necessity of the $\Delta\tilde{B}$ term. Note that distributions that are insensitive to — or averaged over — the orientation of the event would be unaffected by

the absence of $\Delta\tilde{B}$ correction. Equivalently, for the case of oriented e^+e^- events, the effect of the $\Delta\tilde{B}$ correction is visible only in the longitudinal component of the NLO coefficient in eq. (4.1). This is because for one single final-state emission, the PanGlobal and the FKS maps are identical up to an overall event rotation, and the PanLocal map is formally identical to the FKS map up to the partitioning of singular regions.

4.1.2 pp tests

In this section we examine colour-singlet production in pp collisions. We consider three processes, namely (i) neutral-current Drell–Yan ($pp \rightarrow Z/\gamma^* \rightarrow e^+e^-$), (ii) charged-current Drell–Yan ($pp \rightarrow W^+ \rightarrow e^+\nu_e$), and (iii) Higgs production in gluon fusion $pp \rightarrow H$. For our tests, we use the positive-definite NNPDF40MC_nlo_as_01180 PDF set [74], with $\alpha_s(m_Z) = 0.118$. We use a centre-of-mass energy of 13.6 TeV. We work in the G_μ electroweak scheme [84] and take the following input parameters [85]

$$\begin{aligned} m_Z &= 91.1876 \text{ GeV}, & m_W &= 80.377 \text{ GeV}, & G_F &= 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}, \\ m_H &= 125 \text{ GeV}, & \Gamma_Z &= 2.4952 \text{ GeV}, & \Gamma_W &= 2.085 \text{ GeV}. \end{aligned} \tag{4.4}$$

In the Higgs case, we also use the infinite top mass limit [86, 87]. To obtain the LO and NLO baselines, we use MCFM v10.3 [88–92]. For phenomenological results we take the event-by-event di-lepton (or Higgs) invariant mass as our central renormalisation scale and carry out 7-point scale variation to show uncertainty bands. For the extraction of the pure NLO coefficient, we instead use a fixed renormalisation and factorisation scale equal to the on-shell mass of the produced boson, as given in eq. (4.4).

Figure 3 shows results for $pp \rightarrow Z/\gamma^* \rightarrow e^+e^-$ with the following lepton cuts: $p_{t\ell} > 27$ GeV and $|\eta_\ell| < 2.5$, $66 < m_{\ell\ell} < 116$ GeV. The left-hand panel is for the distribution of $m_{\ell\ell}$, the lepton-pair invariant mass; the right-hand panel is for the rapidity of the boson (or equivalently, the lepton pair). As in figure 2, the upper panels show the differential cross section, while the middle panels show the ratio to NLO. One observes agreement to within about a percent for both ESME and dBNLO. Note that with our specific lepton cuts, the NLO K -factor is quite close to 1 near the Z mass and for central rapidities. This comes from an interplay between a positive NLO effect in the total cross section and negative NLO effect due to the cuts. That interplay is also responsible for much of the kinematic dependence of the NLO K -factor. The lower panels show the $\alpha_s \rightarrow 0$ test of NLO accuracy. Given that the NLO coefficient is close to zero in parts of the phase space, we show a ratio to α_s times the LO result,

$$\lim_{\alpha_s \rightarrow 0} \frac{d\Delta\sigma_{\text{shower}}^{\text{NLO}} - d\Delta\sigma^{\text{NLO}}}{\alpha_s d\sigma^{\text{LO}}}. \tag{4.5}$$

Note that for these NLO accuracy tests (i.e. the bottom panel of each plot), we limit the shower to the first emission, since higher numbers of emissions can only modify α_s^2 terms and beyond. We also freeze the PDF at the factorisation scale used in the fixed-order calculation (the Z on-shell mass in this case) and we use a fixed renormalisation scale, independent of $m_{\ell\ell}$ (again, m_Z). We see agreement to within statistical uncertainties, shown as a band. Figure 4 shows analogous tests for W^+ production, without lepton cuts, and the conclusions

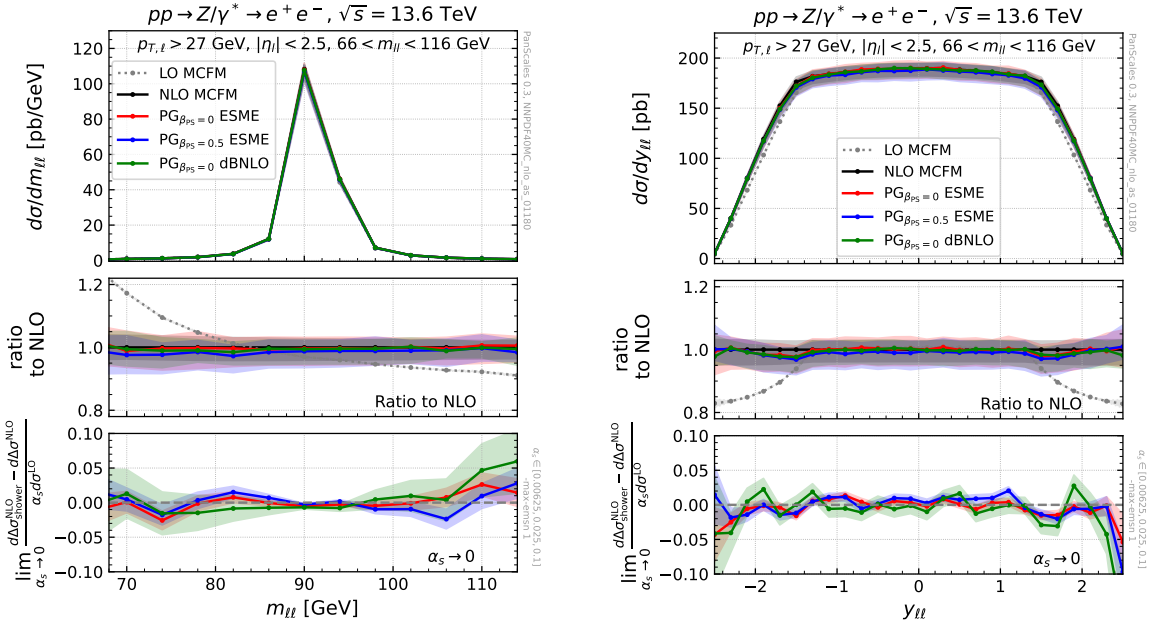


Figure 3. NLO tests for the $pp \rightarrow Z/\gamma^* \rightarrow e^+e^-$ process with cuts on the lepton transverse momentum and rapidity. Left (right): the invariant mass (rapidity) of the colour singlet. The top and middle panels show results with phenomenological settings, compared to NLO predictions from MCFM. Bands correspond to 7-scale uncertainty, $m_{\ell\ell}/2 \leq \mu_R, \mu_F \leq 2m_{\ell\ell}$ with $1/2 \leq \mu_R/\mu_F \leq 2$. The bottom panel shows the ratio of the shower NLO coefficient (extracted in an $\alpha_s \rightarrow 0$ limit) to the NLO coefficient from MCFM. The bands represent the combined statistical uncertainty on the ratio.

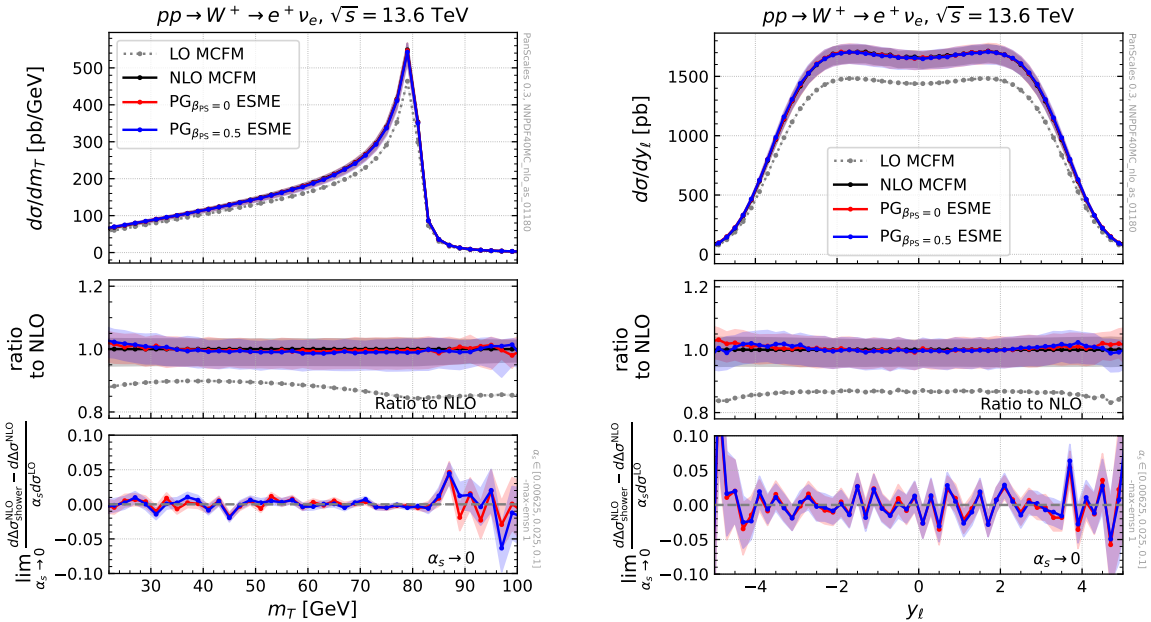


Figure 4. Analogue of figure 3, showing the W transverse mass and the charged lepton rapidity, without lepton or missing momentum cuts.

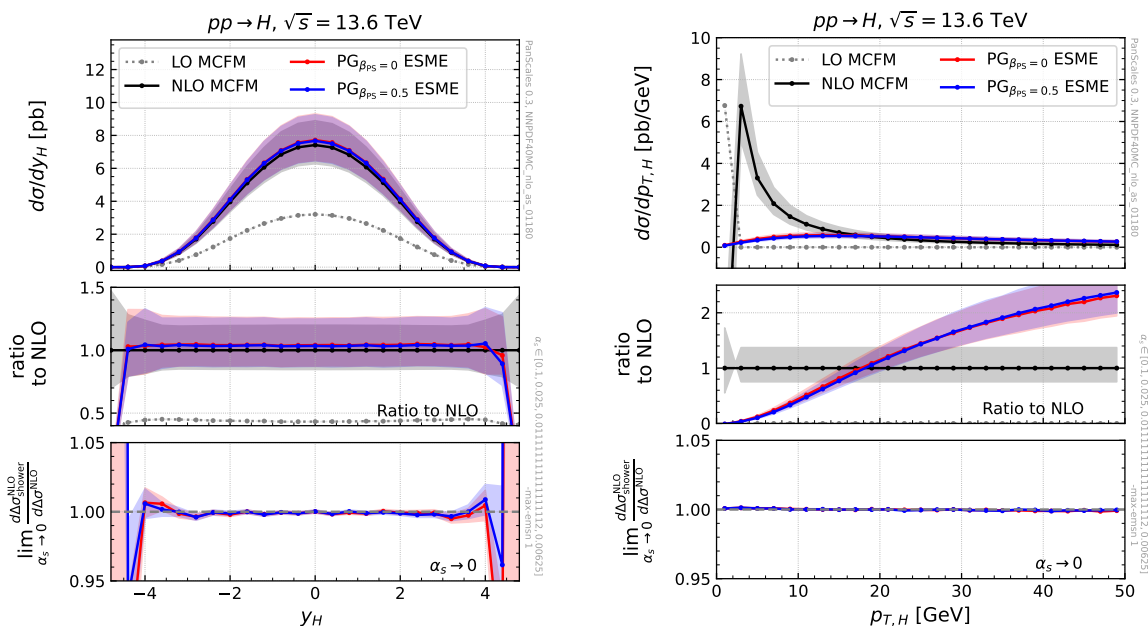


Figure 5. Analogue of figure 3 for Higgs production, showing the Higgs rapidity distribution (left) and the transverse momentum distribution (right).

are similar. Since the lepton rapidity distribution is sensitive to the $V - A$ structure of the W interaction, it provides a direct check of the lepton swap procedure discussed in section 2.2.

In figure 5 we provide results for Higgs production. In the left panel, we illustrate the H rapidity: here too the conclusions are similar, though in the middle panel the ratio to NLO (middle panel) deviates from 1 by about 3–4% rather than the 1% seen for Z and W production. Note, however, that this deviation is numerically small compared to the size of scale uncertainties. The NLO coefficient itself is in excellent agreement with the MCFM result. The right-hand panel illustrates the H transverse momentum. As this observable is highly sensitive to all-order corrections, it is no surprise that a parton shower differs significantly from a NLO result, and the $\alpha_s \rightarrow 0$ limit is necessary to verify that differences are indeed purely due to higher-order effects.

4.1.3 DIS tests

For our first set of DIS tests, in figure 6, we consider photon-mediated e^-p collisions at $\sqrt{s} = 300$ GeV. The left and right-hand plots show the distributions in the $x_{\text{DIS}} = \frac{Q_{\text{DIS}}^2}{2p \cdot q}$ and $y_{\text{DIS}} = \frac{p \cdot q}{p \cdot k}$ variables respectively. Here p and k are the incoming proton and electron momenta and q is the photon momentum. The plots include a constraint $Q_{\text{DIS}}^2 > 25$ GeV². We obtain our reference NLO results from the `disorder` code [93] which itself relies on `Hoppet` [49] and `DISENT` [73]. For the phenomenological predictions, we use as central renormalisation and factorisation scale Q_{DIS} , while for the extraction of the pure α_s correction we use a fixed scale equal to the on-shell Z mass.

The two NLO matching methods that we have explored in DIS are ESME, which we have implemented with the PanGlobal shower, and P2B, which works with both PanGlobal

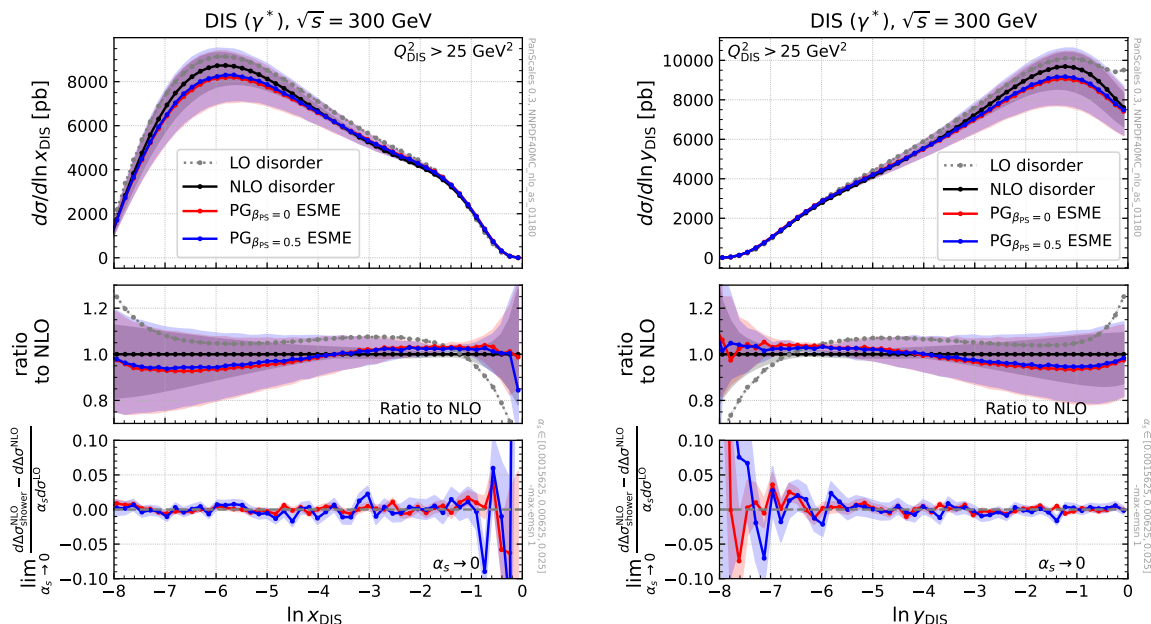


Figure 6. Analogue of figure 3 for DIS. The left-hand plot shows $\ln x_{\text{DIS}}$, the right-hand one $\ln y_{\text{DIS}}$. The reference LO and NLO results have been obtained with the `disorder` code.

and PanLocal showers. For the P2B method, since the PanScales showers conserve x_{DIS} and y_{DIS} , we expect the NLO shower results to be identical to pure NLO predictions. We have verified that this is the case. In figure 6 we therefore focus on the ESME method. The phenomenological predictions from the ESME method, in the upper two panels, are in agreement with the exact NLO to within about 5%, and well within the scale uncertainty bands.¹⁶ The $\alpha_s \rightarrow 0$ NLO accuracy test, in the bottom panel, demonstrates the correctness of the shower’s pure NLO contribution, to within statistical uncertainties.

Our second set of tests in DIS considers the net flavour of the leading jet, in order to study the effects discussed in section 2.4, specifically the impact of the gluon-induced axial (F_3) component in the P2B method. To probe the issue, we identify jets with the DIS version [20] of the Cambridge/Aachen algorithm [94, 95], implemented as a FastJet [96] plugin, used in the Breit frame, and consider the jet with the largest light-cone component in the current direction. As has been extensively discussed in the literature [97–100], the jet flavour is not an infrared safe quantity for standard jet algorithms, because of configurations associated with a pair of soft quarks, starting at $\mathcal{O}(\alpha_s^2)$.¹⁷ We therefore limit our study to events where we generate just the first shower emission, which ensures that the infrared unsafe configuration is not present. As a reference, we use the NLO+PS event generator of ref. [101], which

¹⁶We also explored a modification of 3 without steps 9 and 18 and it brought the ESME NLO results closer to pure NLO. We note however (not shown) that the ESME NLO result is remarkably close to the actual NNLO result. Indeed, in several instances in our investigations, we have observed that higher-order freedom in the formulation of ESME brings shape differences that are similar to those seen in NLO→NNLO, and not hinted at by normal NLO scale variation. We believe that further study on this question may be of interest in any future work that addresses higher orders and their uncertainties more comprehensively.

¹⁷The algorithms proposed in those articles have yet to be extended to DIS.

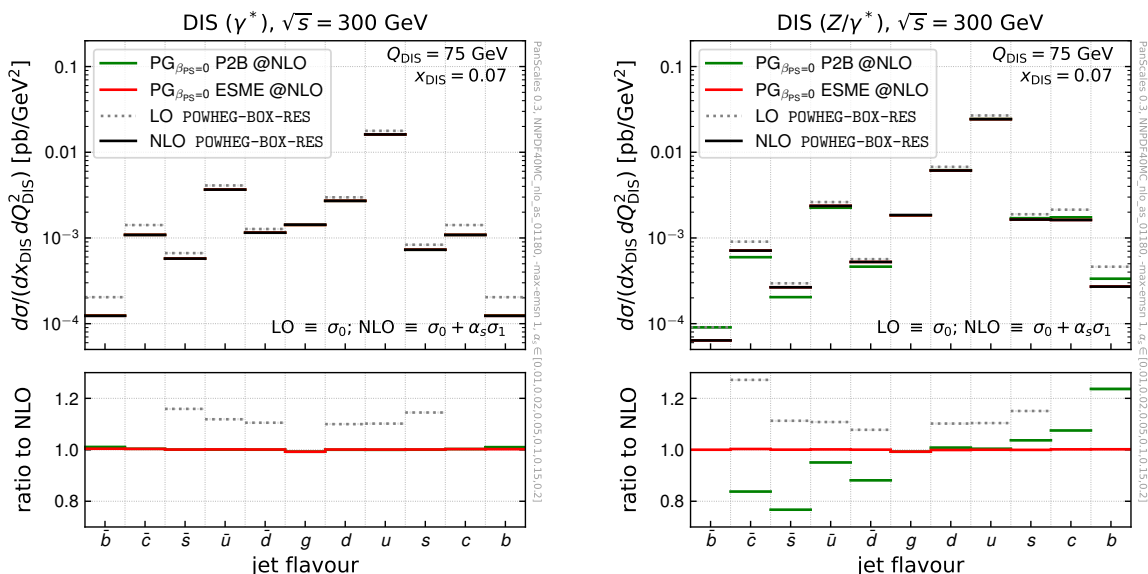


Figure 7. NLO flavour tests for the DIS process, showing the distribution of net flavour of the jet with largest light-cone component along the current direction. The left-hand plot is for photon exchange where both the P2B and ESME methods are expected (and observed) to agree with NLO. The right-hand plot is for the full γ^*/Z contribution where one sees that the P2B approach differs from NLO, due to the missing gluon-induced axial contribution. The reference LO and NLO results have been produced running the POWHEG-BOX-RES DIS generator. The ESME and P2B predictions are shown at pure NLO, i.e. the LO and NLO coefficients have been extracted from an $\alpha_s \rightarrow 0$ extrapolation and then the physical pure NLO result has been obtained by adding them with the actual $\alpha_s(Q)$ multiplying the NLO coefficient. This choice provides a phenomenologically relevant indication of the size of NLO effects, while avoiding any potential confusion associated with differing treatments of orders beyond NLO.

can perform fixed-order (“stage 2”) differential calculations with explicit flavour dependence thanks to the `detailedNLO` feature of the POWHEG-BOX-RES framework [102]. We still consider e^-p collisions at $\sqrt{s} = 300$ GeV, but we fix $x_{\text{DIS}} = 0.07$ and $Q_{\text{DIS}} = 75$ GeV, a combination that helps enhance the relative size of the gluon-induced axial contribution. For this test, we also use $\mu_F = \mu_R = Q_{\text{DIS}}$. Figure 7 shows the cross section in bins of the leading jet’s net flavour, with just photon exchange (left) and full γ^*/Z exchange (right). At LO, the results are driven entirely by the flavour distribution of the proton PDF and the associated quark charges. At NLO, with just photon exchange, the P2B and ESME methods both agree with the predictions from POWHEG-BOX. With γ^*/Z exchange, while the ESME method is correct at NLO, one sees that the P2B method is not. The differences relative to NLO are generally larger for anti-quark flavours and for flavours where the ratio of gluon-induced to Born contributions is enhanced. One can imagine various resolutions of this issue, for example supplementing P2B with a flavour-related $\Delta\bar{B}$ contribution or, perhaps, through an adaptation of the swap technique of appendix D, applied between outgoing quark and anti-quark flavours in the gluon-induced channel. We leave their investigation to future work.

4.2 NN DL tests

In this section we provide a numerical demonstration that the matching algorithms that we have introduced bring NN DL accuracy for event-shape like observables, i.e. control of terms $\alpha_s^n L^{2n-p}$ with $p \leq 2$. This is specifically in the cumulative cross section, $\Sigma_{\text{PS}}(\alpha_s, L)$, for a given dimensionless observable to have a value v less than e^L (with L large and negative). If done properly, NLO matching together with NLL-accurate parton showers should automatically provide NN DL event-shape accuracy in $\Sigma_{\text{PS}}(\alpha_s, L)$. Testing the NN DL accuracy provides a key validation of one of the necessary ingredients towards general NN LL accuracy in $\ln \Sigma_{\text{PS}}(\alpha_s, L)$. For a matched shower to achieve NN DL precision, the cumulative cross section must satisfy [30]

$$\lim_{\substack{\alpha_s \rightarrow 0 \\ \xi \text{ fixed}}} \frac{\Sigma_{\text{PS}}(\alpha_s, L) - \Sigma_{\text{NN DL}}(\alpha_s, L)}{\alpha_s \Sigma_{\text{DL}}(\alpha_s, L)} = 0. \quad (4.6)$$

Note that $\alpha_s = \alpha_s(Q)$ and that $\xi = \alpha_s L^2$ is kept fixed when evaluating eq. (4.6), so as to isolate the pure NN DL contribution to v . In practice, we numerically evaluate eq. (4.6) by first running the showers with fixed values of $\alpha_s = 0.1/N^2$ with $N \in \{6, 7, 12, 24\}$ and then performing a polynomial extrapolation in powers of $\sqrt{\alpha_s}$ so as to obtain the $\alpha_s \rightarrow 0$ limit. Typically the extrapolation uses a subset of the points. We estimate a systematic uncertainty associated with this procedure by considering a second extrapolation with different α_s values. For all processes, we use $\xi = 1.296$.

To test the showers, we use observables based on the Lund-plane [103, 104] picture, as introduced for PanScales NLL testing in e^+e^- [14], pp [18, 19] and DIS [20] collisions. These Lund-based event shapes are defined as

$$M_\beta = \max_{j \in \text{decl.}} \left\{ \frac{|p_{t,j}|}{Q} e^{-\beta y_j} \right\}, \quad S_\beta = \sum_{j \in \text{decl.}} \frac{|p_{t,j}|}{Q} e^{-\beta |\eta_j|}, \quad (4.7)$$

where β is a free parameter and we explore three values, $\beta \in \{0, 0.5, 1\}$. In eq. (4.7), the max and sum run over primary declusterings, which are suitably defined depending on the hard-scattering process.

In e^+e^- collisions the whole event is clustered into two jets using the Cambridge [94] algorithm. For each of these jets we undo the last clustering, and define $p_{t,j} = E_j |\sin \theta_{ij}|$ and $\eta_j = -\ln \tan \frac{\theta_{ij}}{2}$, with E_j the energy of the softer particle in the branching, and θ_{ij} the angle between the two particles. We repeat the declustering following the harder subjet, such that only the set of primary declusterings is considered [14, 104, 105]. In proton-proton collisions we cluster the full event with the Cambridge/Aachen (C/A) algorithm [94, 95] into jets with radius $R = 1$. We then calculate the transverse momentum and rapidity of each of the jets with respect to the beam, which defines $p_{t,j}$ and η_j [19]. For deep-inelastic scattering we use the algorithm defined in appendix C of ref. [20], and specifically eqs. (C.4), (C.5) of that reference to define the transverse momentum and rapidity that enter into eq. (4.7).

Resummed predictions for M_β and S_β at NN LL accuracy will be presented in ref. [106] for all processes considered in this work. We have used the NN DL expansion of those results to test the showers.

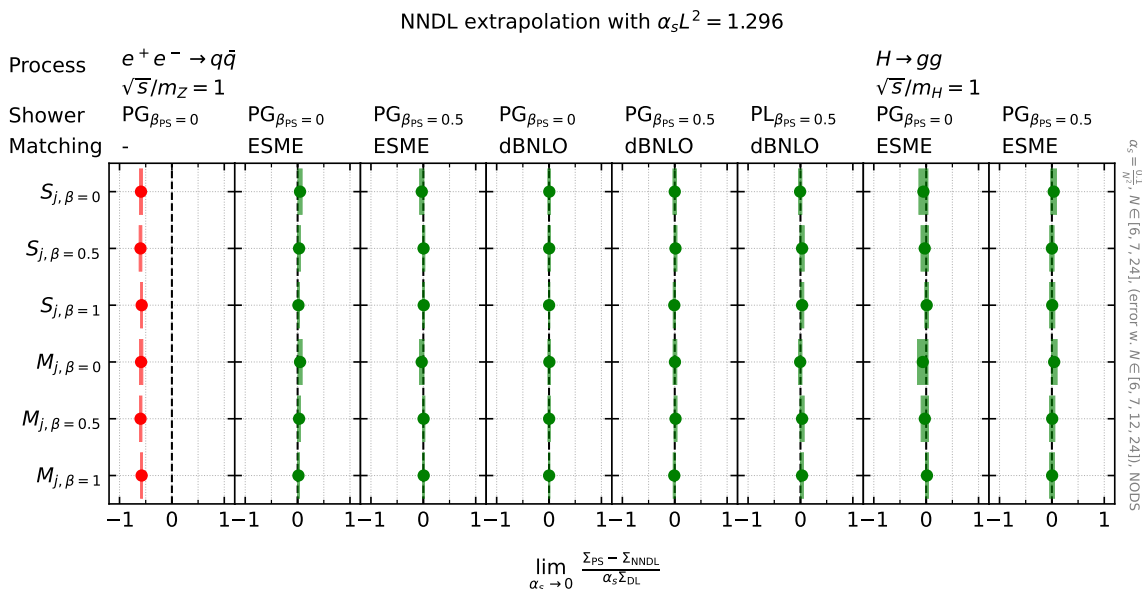


Figure 8. Results of the NNDL accuracy tests, eq. (4.6), at fixed $\xi = \alpha_s L^2$ for $e^+e^- \rightarrow q\bar{q}$ and $H \rightarrow gg$ with $\sqrt{s}/m_X = 1$. The tests are carried out for a fixed Born configuration (for $e^+e^- \rightarrow q\bar{q}$ we use $\cos\theta_{q,\text{beam}} = 0.5$ and for $H \rightarrow gg$ we align the gluons along the z -axis).

For processes with incoming protons, we use the toy PDF set described in appendix A.3 of ref. [19], designed specifically for logarithmic accuracy tests. For quark-initiated processes, we assume an initial d quark. The runs are carried out using the NODS colour scheme [15, 18], and turning off spin correlations [16–18] as they do not affect the NNDL accuracy. In all plots, the α_s values used to obtain the central value of the extrapolation and its error are quoted on the side of each figure in grey. Finally, all NNDL tests are performed for fixed Born kinematics and flavour.

We begin by examining the e^+e^- results. NNDL tests have already been carried out for event shape observables in ref. [30]. There, the tests used a generator without (oriented) NLO normalisation, by comparing to an NNDL calculation that is divided by the total NLO cross section. In figure 8, instead, we have the explicit NLO normalisation in the generator, with both ESME and dBNLO methods and the NNDL calculation is correspondingly normalised to the total LO cross section. With NLO matching, all combinations of observable, matching scheme and process are consistent with NNDL accuracy.

We test for NNDL accuracy in pp collisions in figure 9, which shows ESME and dBNLO results for Z production and ESME results for Higgs production, including one case of matching with the PanLocal shower. For these tests, we fix the colour-singlet rapidity to be 0. The left-most column of the plot shows that without matching there is a large discrepancy, illustrating the power of the test to diagnose potential issues. All the matched results are in agreement with NNDL prediction. Figure 10 shows corresponding tests for DIS, with ESME and P2B matching (the latter also with the PanLocal shower). The pp and DIS tests represent the first time that NNDL event-shape accuracy has been demonstrated for a matched parton shower with incoming hadron beams.

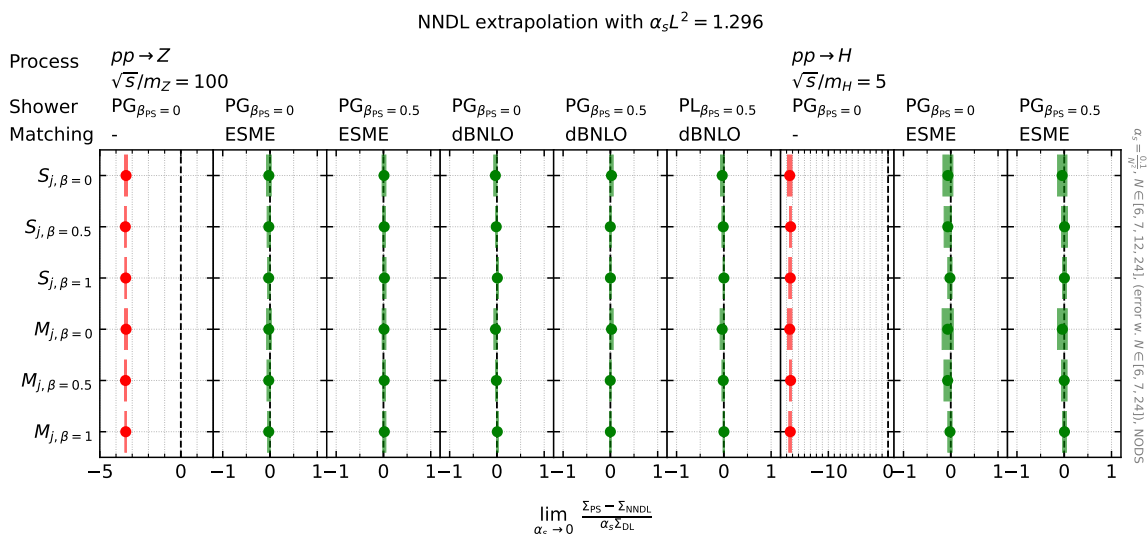


Figure 9. Results of the NNDL accuracy tests at fixed $\xi = \alpha_s L^2$ for $pp \rightarrow Z$ with $\sqrt{s}/m_Z = 100$ and $pp \rightarrow H$ with $\sqrt{s}/m_H = 5$. The rapidity of the colour singlet is set to 0. For the case of Z production, we consider $d\bar{d} \rightarrow Z$ as the Born flavour configuration.

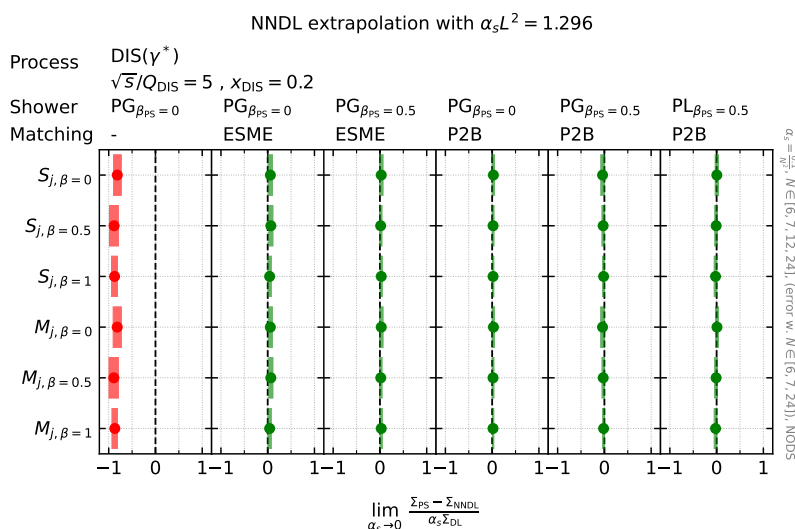


Figure 10. Results of the NNDL accuracy tests at fixed $\xi = \alpha_s L^2$ and fixed Born flavour configuration $\gamma^* d \rightarrow d$ with $\sqrt{s}/Q_{\text{DIS}} = 5$ and $x_{\text{DIS}} = 0.2$.

5 Brief comparison to data and performance studies

5.1 Comparison to data

Several features are still missing from PanScales and its Pythia interface in order to carry out a full phenomenological comparison to data with incoming hadrons. These include QED effects and, in pp collisions, multi-parton interactions. Therefore in this section we consider only a very first basic comparison, with the intention of elaborating on the results shown here

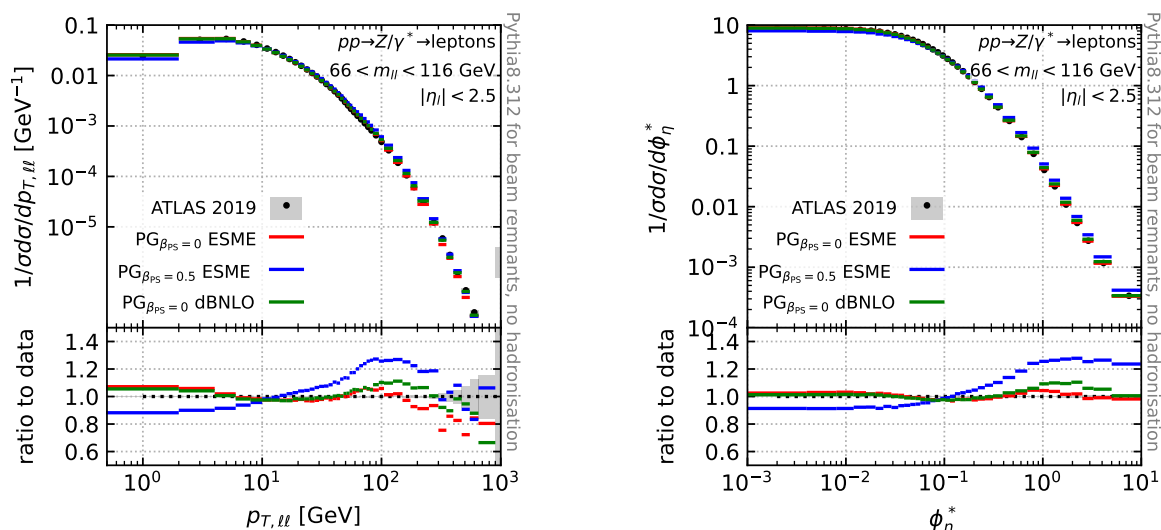


Figure 11. PanScales NLL+NLO matched showers, interfaced with Pythia [107], as compared to 13 TeV QED-Born di-lepton data from the ATLAS collaboration [108]. The left-hand plot is for the di-lepton transverse momentum distribution, while the right-hand plot is for the ϕ_η^* variable [109], cf. eq. (5.1). In the Pythia interface, we include Pythia’s primordial transverse momentum but not hadronisation, QED effects or multi-parton interactions.

in future work. The showers that we show here are the first to have demonstrated general NLL accuracy combined with NLO together with NN DL accuracy for event-shape like observables.

We use a pre-release PanScales version 0.3 with its interface [110] to Pythia 8.312 [107] as well as Pythia’s HepMC3 [111] interface to the RIVET tool [112] in order to carry out the ATLAS di-lepton analysis of ref. [108]. The analysis considers events with two oppositely charged leptons (in a “QED-Born” definition), each with $p_{t\ell} > 27$ GeV and $|\eta_\ell| < 2.5$. The left-hand plot of figure 11 shows the di-lepton p_t distribution, normalised to the total cross section, while the right-hand plot shows the ϕ_η^* [109] distribution with

$$\phi_\eta^* \equiv \tan\left(\frac{\pi - \Delta\phi_{\ell\ell}}{2}\right) \sin\theta_\eta^*, \quad \cos\theta_\eta^* = \tanh\frac{\eta_{\ell^-} - \eta_{\ell^+}}{2}. \quad (5.1)$$

Both observables are in the $\beta_{\text{obs}} = 0$ class but they have substantially different NLL resummation structures. The figures show curves from the ESME NLO-matching method with the PanGlobal shower, with $\beta_{\text{PS}} = 0$ and 0.5, and the dBNLO method with PanGlobal $\beta_{\text{PS}} = 0$. The differences between various shower and matching choices provide an indication of the size of uncertainties that are associated with missing higher orders (we defer a more extensive study of uncertainties to future work). Those differences are of the order of 20% and this is consistent with the size of NNLL corrections observed for the PanGlobal shower in ref. [29] for $e^+e^- \rightarrow q\bar{q}$ event shapes, though we note that in that case the two β_{PS} values gave very similar results even at NLL accuracy. Within the 20% NLL+NLO uncertainties, as well as the expected size of higher-order matching uncertainties at large $p_{t,\ell\ell}$ and ϕ_η^* , there is good agreement with the data. While we do not have MPI, we find that in a plain Pythia run with the Monash 2013 tune [113], adding MPI effects reduces the lowest $p_{t,\ell\ell}$ bin by about

10% and has a negligible effect elsewhere. We would expect a broadly similar impact with our shower. *Pythia*'s primordial transverse momentum *is* included in our simulation and its impact was a $\mathcal{O}(20\%)$ reduction in the smallest $p_{t,\ell\ell}$ bin and $\mathcal{O}(5\text{--}8\%)$ reduction in the few smallest ϕ_η^* bins. Note that these effects are likely to depend on the tune and one might ultimately want to develop updated tunes with the PanScales showers.

5.2 Performance studies

Of the matching methods that we have discussed in this article, dBNLO is based on a well-established underlying methodology and P2B is an intrinsically simple and efficient method. On the other hand the ESME method is qualitatively new and, given that it offers the prospect of general positive-definite matching, it is especially important to determine whether its speed performance is at least comparable to other methods.

Comparisons of speed bring many aspects into play: the efficiency of the underlying code for matrix-elements, which may be hard-coded or automatically generated; trade-offs between warm-up time and event-generation efficiency; the efficiency of unweighting; etc. There are also delicate technical aspects of speed measurement, especially as not all codes produce equivalent outputs (e.g. full showered events versus just a first emission). Some caution is therefore needed in interpreting any timing results.

For our first performance test we considered the $pp \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ process at NLO at $\sqrt{s} = 13.6$ TeV. We examined this process with the POWHEG-BOX-V2 revision 3985 [4, 45], Herwig version 7.3.0 [114], Sherpa version 3.0.1 [115] and MG5_aMC version 3.6.1 [6], starting from default settings for each. We focused on the time to produce the NLO event up and to and including first emission, either explicitly where this was possible, or deducing it from the difference between NLO+PS and LO+PS runs. We found the POWHEG-BOX to be both the fastest and the one with the smallest fraction of negative-weighted events, and so took it as our baseline. We used it with the NNPDF40MC_nlo_as_01180 [74] PDF, which is positive-definite, thus alleviating the one potential extraneous source of negative-weight events. We also considered γ^* -mediated DIS at $\sqrt{s} = 317$ GeV and $Q_{\text{DIS}}^2 > 100$ GeV². We used as baseline the POWHEG-BOX-RES [102] (revision 4057) generator developed by some of us [101],¹⁸ and we employed the same NNPDF40MC_nlo_as_01180 PDF set.

The POWHEG-BOX has so-called *ifold* parameters for each of the three phase space variables, (ξ, y, ϕ) , in the \bar{B} integration (cf. section 2.3). The left panel of figure 12 shows the fraction of negative-weight events in the POWHEG-BOX for Drell-Yan production versus the timing per event, with the three blue points corresponding to folding choices of 5,1,1 (left), 2,1,1 (middle) and 1,1,1 (right).¹⁹ The fraction of negative weights is low for this process, below 1%. For many practical purposes, a user could even set out to verify that they do not cluster in any specific phase space regions and then arguably just discard them. Still this process helps illustrate the trade-off between folding, event-generation time and negative-weight fraction. For DIS, we found that the optimal folding is the one over the

¹⁸For this comparison, we modified the POWHEG-BOX-RES to use analytic matrix elements, which are not enabled by default because they are only valid for the γ^* , non-polarised process. This leads to a 20% speed gain compared to the default.

¹⁹We also examined 5,5,1 and 2,2,1 and found the same fractions of negative weights as 5,1,1 and 2,1,1, and slower timings.

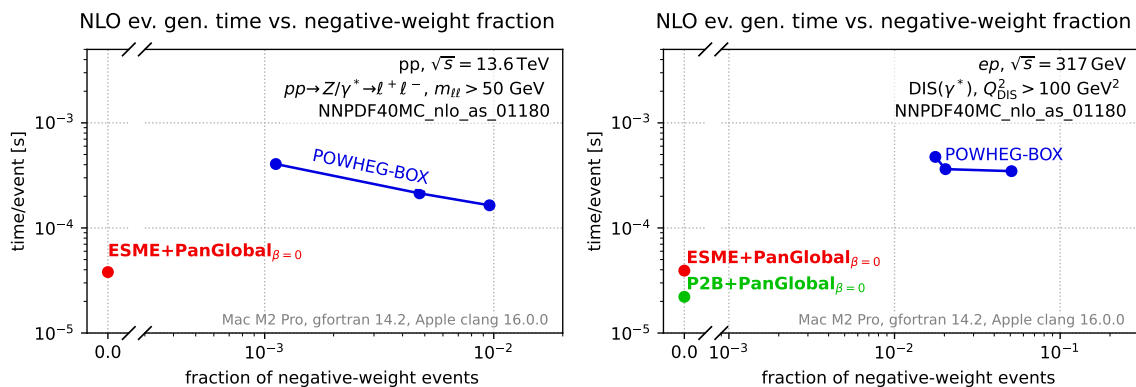


Figure 12. Illustration of the performance of our ESME implementation as compared to POWHEG-BOX-V2, for $pp \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ (left panel), and of our ESME and P2B implementations for the γ^* -mediated DIS as compared to the POWHEG-BOX-RES (right panel). The plot shows the time per event versus the fraction of negative weights. The three POWHEG-BOX points for Drell-Yan production correspond (from right to left) to folding choices for ξ , y and ϕ of 1,1,1, 2,1,1 and 5,1,1. For DIS we instead considered the folding choices: 1,1,1, 1,1,2 and 1,1,5 (from right to left). For all cases, the event generation time is for NLO accuracy with just the hardest emission, and is evaluated by running on a single core of an Apple M2 Pro processor.

ϕ -variable, and we considered 1, 2 and 5 foldings (from right to left). The fraction of negative weight is slightly higher than in Drell-Yan production, but always below 5%, and in particular it is equal to 2% with 2 foldings over the ϕ -variable, which does not seem to induce an appreciable speed penalty. Increasing to 5 foldings leads only to a marginal reduction of negative-weighted events, but a substantial increase of the run time. In general, the higher fraction of negative weights compared to the Drell-Yan case, is due to the lower scale of the process under consideration.²⁰

Figure 12 also shows the timing of the ESME method of section 3.3 (red point) for both Drell-Yan (left) and DIS (right), as well as P2B of section 2.4 (green point) for DIS only. By construction, neither the ESME nor the P2B methods have any negative-weight events. ESME is about four times faster than the fastest of the POWHEG-BOX configurations for Drell-Yan, and ten times for DIS, taking about $40 \mu\text{s}$ per event regardless of the process considered. It is also interesting to note that the time per ESME event is only roughly double the P2B one, despite the more complicated rejection algorithm involved in ESME. This high speed should be put into context: we devoted some effort to understand the generation of the phase space, which led, e.g. to the lepton-swap technique mentioned in section 2.2 and also enabled us to limit the warm-up phase. We hard-coded our own matrix elements, which allowed the matrix element for the two lepton-swap configurations to be evaluated in almost the same time as a single configuration. We also used a pre-release version of Hoppet 2.0.0 [49, 50] to evaluate the PDFs at each x, μ_F point, rather than direct LHAPDF [116] evaluation. Finally, focusing on Drell-Yan production, one should keep in mind that parton showering with PanScales (without the Pythia [107] interface) would add a further $40\text{--}70 \mu\text{s}$

²⁰Increasing the lower cut on Q_{DIS} to 50 GeV, and using 2 folds on ϕ , the fraction of negative weights is 0.2%, i.e. 10 times smaller, and the time per event is 0.6 ms.

per event. However, for Drell-Yan production with today’s tools, it would ultimately be `Pythia`’s generation of hadronisation and, especially, multi-parton interactions that would dominate, at about 2 ms per event.

We also briefly compared Higgs production against the `POWHEG-BOX` and, in that case too, found that `ESME` was faster.²¹ A final test that we carried out was of our `dBNLO` implementation versus `ESME`, for Z production without lepton decays (our `dBNLO` implementation with lepton decays relies on the `POWHEG-BOX` \tilde{B} and so is not an independent speed test). With folding turned off, the `dBNLO` negative-event rate was similar to that of `POWHEG-BOX`, and the speed was about 30% faster than `ESME`. It remains to be seen, however, how this observation would generalise to other processes.

6 Conclusions

In this article we have explored a range of methods for NLO matching such that shower logarithmic accuracy is preserved and even augmented to NNDL for event-shape like observables. We focused on colour-singlet production in pp collisions, oriented $e^+e^- \rightarrow 2$ jets and DIS. From a theoretical point of view, such matching is a milestone in its own right. It is also a crucial step towards higher general logarithmic accuracy in parton showers, specifically NNLL accuracy. In prior work [29] in an e^+e^- context, NNLL shower accuracy has been shown to have up to 20% effects, and so is expected to have broad phenomenological importance, given that many measurements have accuracies at the few percent level or better. Thus our work here is an essential step on the way towards bringing this progress to processes with incoming hadrons.

Some of the methods that we used are adaptations of existing approaches. That was the case for the `dBNLO` method (section 2.3), which adapts the widely used `POWHEG` method. It brings a generalisation of the ordering variable, uses a real-matched shower instead of the FKS map, and a corresponding $\Delta\bar{B}$ added to standard FKS \bar{B} to account for the different relation between the Born and real kinematics. The projection to Born (P2B) method of section 2.4 is most suited to DIS type processes, where it has been used in the past for NNLO matching [39]. One subtlety that we encountered concerns its use with parity-violating interactions and the correct separation of flavour and anti-flavour Born channels.

We also explored a qualitatively new approach. In particular in section 3.2 (algorithm 1) we introduced a new core approach — Exponentiated Subtraction for Matching Events (`ESME`) — that reformulates the standard Monte Carlo evaluation of \bar{B} as a problem of generating a Sudakov distribution. It can be straightforwardly made positive definite, while retaining accuracy up to any specified order in α_s . We believe that it has the potential to serve as a foundation that accommodates many variations and that it should be feasible also

²¹There is one caveat here, namely that at low $\mu_F \sim 1$ GeV the gluon distribution in the `NNPDF40MC_nlo_as_01180` set approaches zero at small x , but not at moderate x . This leads to large ratios of $g(x_{\text{large}}, \mu_F)/g(x_{\text{small}}, \mu_F)$ in R/B at low μ_F . The `PanScales` code currently uses an x -dependent but μ_F -independent overhead factor. This ends up being set according to the very large overhead required at low μ_F , with a corresponding speed impact. If we simply freeze the PDF set below $\mu_F = 1.3$ GeV, which has limited phenomenological impact, the large overhead is no longer required and we find speeds that are only 20% slower than for di-lepton production.

for other codes to adopt and/or adapt it. The specific variant that we chose was described in section 3.3. We combined it with a shower-based NLO subtraction method in which we started from a slicing calculation and used an approximation of the shower to promote the slicing calculation into a subtraction method. We envisage that this approach may also have wider applications.

To validate the methods, we carried out tests of both NLO and event-shape NNDL accuracy. In particular, given that shower NLO matching typically introduces terms also beyond NLO, we highlighted the value of studying the $\alpha_s \rightarrow 0$ limit of the matched result, explicitly extracting the pure NLO coefficient. The use of the $\alpha_s \rightarrow 0$ limit is already widespread in logarithmic accuracy tests and we applied it also for NNDL validation, verifying that our matching correctly achieves this milestone, as needed for future work on high logarithmic accuracy, notably for processes with incoming hadrons.

A final consideration concerns event-generation efficiency. Existing methods for reducing the fraction of negative-weight events bring penalties in speed and/or complexity. We compared the ESME method with the POWHEG-BOX with various degrees of folding, cf. figure 12. Our implementation of the ESME method is not only positive definite, but turned out to be several times faster per event than (unfolded) POWHEG-BOX, which was the fastest of the public tools we examined and the one with fewest negative weights. It also sidestepped the need for a substantial warm-up phase, making for instance NLO Drell-Yan showered event generation as easy and fast as LO Pythia showered event generation.

Taken together, our results represent a key step on the path to higher logarithmic accuracy in parton showers and also suggest that there may be significant value in further exploring new matching methods at NLO and beyond.

The developments presented in this work are available from <https://gitlab.com/panscales/panscales-0.X>, as part of the 0.3.0 release of the PanScales code.

Acknowledgments

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A Overview of the PanScales showers

Here we summarise the kinematic maps for the showers considered in this work and provide the shower emission probability in the absence of matching. The modifications to the latter when considering NLO matching are discussed in appendix B. A common ingredient entering the kinematic map of the PanScales showers and their emission kernel is a process-dependent reference four-vector Q^μ , which defines a reference frame for measuring angular distances. For the processes considered in this paper we set its four-momentum (p_x, p_y, p_z, E) to:

- **e^+e^- collisions:** for the decay of a colour singlet X with momentum p_X^μ , we use $Q^\mu = p_X^\mu$, i.e., we operate in the rest frame of X .
- **pp collisions:** when we consider the production of a colour singlet X with mass m_X and rapidity y_X in hadron-hadron collisions, we set

$$Q^\mu = m_X (0, 0, \sinh y_X, \cosh y_X). \quad (\text{A.1})$$

This corresponds to the rest frame of the colour singlet before showering.

- **Deep inelastic scattering:** when we consider lepton-hadron scattering $\ell(p_1)h(P) \rightarrow \ell'(p_2)X$, with the standard DIS variables

$$q_{\text{DIS}}^\mu = p_1^\mu - p_2^\mu, \quad x_{\text{DIS}} = \frac{-q_{\text{DIS}}^2}{2q_{\text{DIS}} \cdot P}, \quad (\text{A.2})$$

we define the reference vectors

$$n_{\text{in}}^\mu = x_{\text{DIS}} P^\mu, \quad n_{\text{out}}^\mu = q_{\text{DIS}}^\mu + n_{\text{in}}^\mu, \quad (\text{A.3})$$

which we employ to define

$$Q^\mu = n_{\text{in}}^\mu + n_{\text{out}}^\mu = q_{\text{DIS}}^\mu + 2x_{\text{DIS}} P^\mu. \quad (\text{A.4})$$

A.1 Shower emission probability

Let us now consider the emission of a parton k from a dipole i, j , with pre-branching momentum \tilde{p}_i, \tilde{p}_j . We define the invariants

$$\tilde{s}_{ij} = 2\tilde{p}_i \cdot \tilde{p}_j, \quad \tilde{s}_i = 2\tilde{p}_i \cdot Q, \quad \tilde{s}_j = 2\tilde{p}_j \cdot Q, \quad (\text{A.5})$$

where Q^μ is the process-dependent reference four-vector introduced earlier. The emission probability is expressed as a function of three variables: v , the ordering variable, which carries

the dimension of a transverse momentum, a rapidity-like variable $\bar{\eta}$ and an azimuthal angle ϕ . From such variables we can build the effective transverse momentum

$$\kappa_t = \kappa_t(v, \bar{\eta}, \tilde{s}_{ij}, \tilde{s}_i, \tilde{s}_j, Q^2; \beta_{\text{PS}}), \quad (\text{A.6})$$

where the specific relation depends on the shower under consideration and is detailed in appendix A.2, and $0 \leq \beta_{\text{PS}} < 1$. Given κ_t and $\bar{\eta}$, we can compute two auxiliary variables,

$$\alpha_k = \sqrt{\frac{\kappa_t^2 \tilde{s}_j}{s_{ij} \tilde{s}_i}} e^{+\bar{\eta}}, \quad \beta_k = \sqrt{\frac{\kappa_t^2 \tilde{s}_i}{s_{ij} \tilde{s}_j}} e^{-\bar{\eta}}, \quad (\text{A.7})$$

which we use to build the momentum fractions z_i and z_j that enter into the emission probability. In particular we have for final-state branchings

$$z_i = \alpha_k, \quad z_j = \beta_k, \quad (\text{final-state}), \quad (\text{A.8})$$

while for initial-state branchings,

$$z_i = \frac{\alpha_k}{1 + \alpha_k}, \quad z_j = \frac{\beta_k}{1 + \beta_k} \quad (\text{initial-state}). \quad (\text{A.9})$$

The emission probability then reads

$$d\mathcal{P}_{i\bar{j} \rightarrow ij\bar{k}} = \sum_{\ell=i,j} \frac{\alpha_s^{\text{eff}}(\mu_R)}{\pi} d \ln v d\bar{\eta} \frac{d\phi}{2\pi} \frac{\partial \ln \kappa_t}{\partial \ln v} \mathcal{L}_{\ell\bar{\ell}}(\tilde{x}_\ell, z_\ell, \mu_F) z_\ell P_{\ell k}^{\text{IS/FS}}(z_\ell) f(\bar{\eta}_\ell), \quad (\text{A.10})$$

where

- $\alpha_s^{\text{eff}}(\mu_R)$ is the QCD coupling constant evaluated at the scale $\mu_R = \kappa_t$ according to the Catani-Marchesini-Webber prescription [118];
- The luminosity factor is given by

$$\mathcal{L}_{\ell\bar{\ell}}(\tilde{x}_\ell, z_\ell, \mu_F) = \begin{cases} \frac{f_\ell\left(\frac{\tilde{x}_\ell}{1-z_\ell}, \mu_F\right)}{(1-z_\ell)f_{\bar{\ell}}(\tilde{x}_\ell, \mu_F)} & \text{for initial state (IS)}, \\ 1 & \text{for final-state (FS)}, \end{cases} \quad (\text{A.11})$$

where $f_a(x, \mu_F)$ is the parton distribution function (PDF) for a parton with flavour a and longitudinal momentum fraction x , evaluated at the factorisation scale μ_F , which we take to be equal to

$$\mu_F = v \left(\frac{Q}{v} \right)^{\frac{\beta_{\text{PS}}}{1+\beta_{\text{PS}}}}. \quad (\text{A.12})$$

- The splitting functions $P_{\ell k}^{\text{IS/FS}}(z_\ell)$ are given by

$$P_{\ell\bar{\ell}}^{\text{FS}}(z_\ell) = P_{\bar{\ell} \rightarrow \ell, k}^{\text{FS}}(z_\ell), \quad P_{\ell\bar{\ell}}^{\text{IS}}(z_\ell) = (1-z_\ell) P_{\ell \rightarrow \bar{\ell}, k}^{\text{IS}}(z_\ell), \quad (\text{A.13})$$

where $P_{\ell \rightarrow \bar{\ell}, k}^{\text{IS}}(z_\ell)$ are opportunely symmetrised DGLAP splitting functions, and are reported in appendix A of ref. [18].

- $f(\bar{\eta}_\ell)$ is a function used to partition the emission probability between the leg \tilde{i} and \tilde{j} . For an antenna shower, if \tilde{i} carries a colour index, and \tilde{j} carries the anti-colour one, we use $\bar{\eta}_i = +\bar{\eta}$, $\bar{\eta}_j = -\bar{\eta}$ and

$$f(\bar{\eta}) = f_{\text{ant}}(\bar{\eta}) = \frac{1}{1 + e^{-2\bar{\eta}}}. \quad (\text{A.14})$$

For a dipole shower, the two contributions in eq. (A.10) are handled separately, so that we can imagine \tilde{i} always being the emitter and $\bar{\eta}_\ell = \bar{\eta}_i = \bar{\eta}$, and

$$f(\bar{\eta}) = g_{\text{dip}}(\bar{\eta}) = \begin{cases} 0 & \text{if } \bar{\eta} < -1 \\ \frac{15}{16} \left(\frac{\bar{\eta}^5}{5} - \frac{2\bar{\eta}^3}{3} + \bar{\eta} + \frac{8}{15} \right) & \text{if } -1 \leq \bar{\eta} \leq 1. \\ 1 & \text{if } \bar{\eta} > 1 \end{cases} \quad (\text{A.15})$$

In the following we specify how to build κ_t , and the new momenta given the shower variables v , $\bar{\eta}$ and ϕ for two variants of the PanScales showers, namely PanGlobal and PanLocal.

A.2 Kinematic maps

We now discuss the emission generation for two variants of the PanScales showers, namely PanGlobal A.2.1 and PanLocal A.2.2. For the latter, we introduce a new interpretation of the relation between the ordering variable $\ln v$ and the actual Sudakov decomposition used to write the new momenta, which enables better phase space coverage in the presence of initial-state radiation, as compared to the maps used in refs. [18–20].

A.2.1 The PanGlobal shower

The PanGlobal shower is an antenna shower with local longitudinal momentum conservation, but global transverse momentum conservation. The first step is the calculation of the effective transverse momentum κ_t of eq. (A.6), which is given by

$$\kappa_t \equiv \left(\frac{\tilde{s}_i \tilde{s}_j}{\tilde{s}_{ij} Q^2} \right)^{\beta_{\text{ps}}} v e^{\beta_{\text{ps}} |\bar{\eta}|}, \quad (\text{A.16})$$

with $0 \leq \beta_{\text{ps}} < 1$. The Jacobian $\partial \ln \kappa_t / \partial \ln v$ appearing in eq. (A.10) is thus always 1. One then builds the variables α_k and β_k from eq. (A.7) that are used to define the intermediate momenta

$$\vec{p}_k^\mu = r_L (\alpha_k \vec{p}_i^\mu + \beta_k \vec{p}_j^\mu + k_\perp^\mu), \quad (\text{A.17a})$$

$$\vec{p}_i^\mu = r_L (1 \mp \alpha_k) \vec{p}_i^\mu, \quad (\text{A.17b})$$

$$\vec{p}_j^\mu = r_L (1 \mp \beta_k) \vec{p}_j^\mu, \quad (\text{A.17c})$$

where k_\perp^μ is a four-vector orthogonal to \vec{p}_i^μ and \vec{p}_j^μ with $k_\perp^2 = -\alpha_k \beta_k \tilde{s}_{ij}$, and the sign \mp takes into account if the leg is a final-state one ($-$) or an initial-state ($+$) one. The factor r_L is equal to 1 for initial-initial (II) and initial-final (IF) dipoles, while it is different from

1 for final-final (FF) dipoles. In particular, when considering the decay of a colour singlet in e^+e^- we define [25]

$$r_L = \frac{-\tilde{p}_m \cdot \bar{p}_{ijk} + \sqrt{(\tilde{p}_m \cdot \bar{p}_{ijk})^2 + \tilde{p}_{ijk}^2(Q^2 - \tilde{p}_m^2)}}{\bar{p}_{ijk}}, \quad (e^+e^-) \quad (\text{A.18})$$

where $\tilde{p}_m = Q - \tilde{p}_i - \tilde{p}_j$ and $\bar{p}_{ijk} = \bar{p}_i + \bar{p}_j + \bar{p}_k$. For processes involving at least one initial-state hadron we use [20]

$$r_L = \frac{\tilde{s}_i + \tilde{s}_j}{\tilde{s}_i + \tilde{s}_j + 2k_\perp \cdot Q} \quad (pp \text{ and DIS}). \quad (\text{A.19})$$

The choice in eq. (A.18) ensures that the mass of the colour-singlet is left unchanged, while the choice in eq. (A.19) ensures that the energy of the dipole in the rest frame of Q^μ does not change. This factor was absent in the original formulation of the PanGlobal showers [14, 18] and it is necessary [25] to avoid an issue of long-distance correlations that otherwise arises with triple-collinear configurations.

The relation between the final momenta p_l^μ and \tilde{p}_l^μ is process dependent.

- e^+e^- : for the decay of the colour singlet, one defines $\bar{Q}^\mu = \tilde{p}_i^\mu + \tilde{p}_j^\mu + \tilde{p}_k^\mu + \tilde{p}_m^\mu$, that corresponds to the new total final-state momentum, and applies the following boost to all final-state particles:

$$\Lambda^{\mu\nu} = g^{\mu\nu} + \frac{2Q^\mu \bar{Q}^\nu}{Q^2} - \frac{2(Q + \bar{Q})^\mu (Q + \bar{Q})^\nu}{(Q + \bar{Q})^2}. \quad (\text{A.20})$$

- pp : when considering the production of a colour singlet X in hadron collisions,²² we use the colour singlet to absorb the transverse-momentum imbalance, and then rescale the beams to ensure longitudinal momentum conservation. In practice, for each final-state particle, excluding the colour singlet, we define

$$p_l^\mu = \bar{p}_l^\mu \text{ if } l \in i, j, k, \quad p_l^\mu = \tilde{p}_l^\mu \text{ otherwise}. \quad (\text{A.21})$$

We then calculate p_m , i.e., the momentum of all final-state particles excluding X , and we decompose it along the directions of the hadron beams P_a^μ and P_b^μ , to get

$$\bar{p}_m^\mu = a_m P_a^\mu + b_m P_b^\mu + q_\perp^\mu, \quad (\text{A.22})$$

where q_\perp is the transverse momentum component. The momentum of the colour-singlet is modified to be

$$p_X^\mu = \sqrt{\frac{|q_\perp^2| + m_X^2}{\sqrt{S}}} (e^{yx} P_a^\mu + e^{-yx} P_b^\mu) - q_\perp^\mu, \quad (\text{A.23})$$

with $S = (P_a + P_b)^2$, and we reset the momenta of the incoming partons to be

$$p_a^\mu = \left(a_m + e^{yx} \sqrt{\frac{|q_\perp^2| + m_X^2}{\sqrt{S}}} \right) P_a^\mu, \quad p_b^\mu = \left(b_m + e^{-yx} \sqrt{\frac{|q_\perp^2| + m_X^2}{\sqrt{S}}} \right) P_b^\mu. \quad (\text{A.24})$$

²²A discussion on the generalisation of our map for generic hadron-hadron collider processes can be found in ref. [18].

- **DIS:** for lepton-hadron collisions, we first calculate the total final-state momentum, excluding the final state lepton, \bar{p}_X , and we decompose it along the directions n_{in}^μ and n_{out}^μ introduced in eq. (A.3)

$$\bar{p}_m^\mu = \frac{|q_\perp^2| + \bar{p}_m^2}{b_m} n_{\text{in}}^\mu + b_m n_{\text{out}}^\mu + q_\perp^\mu. \quad (\text{A.25})$$

The incoming parton momentum is reset to (by construction $Q^2 = -q_{\text{DIS}}^2$, see eq. (A.4))

$$p_a^\mu = \frac{\bar{p}_m^2 + Q^2}{Q^2} n_{\text{in}}^\mu, \quad (\text{A.26})$$

while all the final state partons are boosted according to

$$\Lambda^{\mu\nu} = g^{\mu\nu} + \frac{2n_{\text{in}}^\mu}{Q^2} \left[(b_m - 1)n_{\text{out}}^\nu + \frac{|q_\perp^2|}{b_m Q^2} n_{\text{in}}^\nu + q_\perp^\nu \right] + \frac{2n_{\text{out}}^\mu n_{\text{in}}^\nu}{Q^2} \frac{1 - b_m}{b_m} - \frac{2q_\perp^\mu n_{\text{in}}^\nu}{b_m Q^2}. \quad (\text{A.27})$$

It is easy to see that

$$p_m \equiv \Lambda^\mu{}_\nu \bar{p}_m^\nu = \frac{\bar{p}_m^2}{Q^2} n_{\text{in}}^\mu + n_{\text{out}}^\mu, \quad (\text{A.28})$$

so that $p_m^\mu - p_a^\mu = q_{\text{DIS}}^\mu$, as required for momentum conservation. Notice that the boost in eq. (A.27) was specifically designed to avoid assigning a large transverse-momentum component to partons aligned along the incoming-beam direction n_{in}^μ [20].

A.2.2 The (new) PanLocal shower

We now present a refined version for handling initial-state radiation in the PanLocal shower, as compared to that presented in refs. [18, 20], that is well-suited for generic processes and has an improved phase-space coverage. Again, the first step consists in calculating the effective transverse momentum κ_t

$$\kappa_t \equiv \min(\tilde{\kappa}_t, \mu_F), \quad (\text{A.29})$$

where $\tilde{\kappa}_t$ coincides with the PanGlobal definition in eq. (A.16), while μ_F is defined in eq. (A.12). This ensures we switch to transverse-momentum ordering when, in the rest frame of Q^μ , the emission is hard-collinear with energy larger than $\sqrt{Q^2}$. This is necessary to avoid long-distance correlations in the presence of very energetic collinear emissions, as pointed out in ref. [18]. Notice that now the Jacobian in eq. (A.10) will be

$$\frac{\partial \ln \kappa_t}{\partial \ln v} = \begin{cases} 1 & \text{for } \kappa_t = \tilde{\kappa}_t \\ \frac{1}{1 + \beta_{\text{ps}}} & \text{for } \kappa_t = \mu_F. \end{cases} \quad (\text{A.30})$$

Given κ_t and $\bar{\eta}$, we then build the auxiliary variables α_k and β_k as in eq. (A.7). These variables are then employed to build the coefficients a_k and b_k , which are used to construct the momenta of the new emission \bar{p}_k

$$\bar{p}_k^\mu = a_k \tilde{p}_i^\mu + b_k \tilde{p}_j^\mu + k_\perp^\mu, \quad (\text{A.31})$$

with $k_\perp^2 = -a_k b_k \tilde{s}_{ij}$. How a_k and b_k are related to the auxiliary variables α_k and β_k , and how the recoiled momenta $\bar{p}_{i,j}$ are defined, depends on the type of the dipole. In the following we

consider only the dipole variant, and not the antenna one, so we need to distinguish between IF (initial-final) and FI (final-initial) dipoles, where the first label is used to denote the emitter. The transverse momentum is conserved locally within the dipole, and in particular it is absorbed only by the emitter.

- For FF and FI dipoles, we choose

$$a_k = \alpha_k, \quad b_k = \beta_k \tag{A.32}$$

and

$$\bar{p}_i = (1 - a_k)\tilde{p}_i + \frac{a_k b_k}{1 - a_k}\tilde{p}_j - k_\perp, \tag{A.33a}$$

$$\bar{p}_j = \frac{1 - a_k \mp b_k}{1 - a_k}\tilde{p}_j, \tag{A.33b}$$

where the sign $-$ is used when j is a final-state spectator, and the sign $+$ is used for a FF dipole.

- For an II map, we choose

$$a_k = \frac{\alpha_k}{\sqrt{1 + \alpha_k \beta_k}}, \quad b_k = \beta_k \frac{(\alpha_k + \sqrt{1 + \alpha_k \beta_k})^2}{\sqrt{1 + \alpha_k \beta_k}}, \tag{A.34}$$

and the recoiled momenta read

$$\bar{p}_i = (1 + a_k)\tilde{p}_i + \frac{a_k b_k}{1 + a_k}\tilde{p}_j + k_\perp, \tag{A.35a}$$

$$\bar{p}_j = \frac{1 + a_k + b_k}{1 + a_k}\tilde{p}_j, \tag{A.35b}$$

$$\bar{p}_m = \tilde{p}_i + \tilde{p}_j, \tag{A.35c}$$

where \bar{p}_m denotes the collection of all other final state particles, except k . This choice ensures that the transverse momentum of \bar{p}_k with respect to the new beams \bar{p}_i and \bar{p}_j is equal to κ_i^2 , and that the difference in rapidity between \bar{p}_k and \bar{p}_m is equal to

$$y_k - y_m = \frac{1}{2} \ln \frac{(\bar{p}_k \cdot \bar{p}_j)(\bar{p}_m \cdot \bar{p}_i)}{(\bar{p}_k \cdot \bar{p}_i)(\bar{p}_m \cdot \bar{p}_j)} = \bar{\eta} + \frac{1}{2} \ln \frac{\tilde{s}_i}{\tilde{s}_j}. \tag{A.36}$$

- For an IF map we instead adopt

$$a_k = \frac{\alpha_k}{(1 - \beta_k)^2 - \alpha_k \beta_k}, \quad b_k = \frac{\beta_k(1 + \alpha_k - \beta_k)^2}{(1 - \beta_k)^2 - \alpha_k \beta_k}, \tag{A.37}$$

and

$$\bar{p}_i = (1 + a_k)\tilde{p}_i + \frac{a_k b_k}{1 + a_k}\tilde{p}_j + k_\perp, \tag{A.38a}$$

$$\bar{p}_j = \frac{1 + a_k - b_k}{1 + a_k}\tilde{p}_j. \tag{A.38b}$$

The expressions in eq. (A.37) are derived such that the invariants obtained with the IF map exactly match those of the corresponding FI map. From eq. (A.37) it is also easy to notice that there is a non-singular region corresponding to large β_k , where $(1 - \beta_k)^2 < \alpha_k \beta_k$, such that a_k and b_k can become negative. If we look at the corresponding FI map in the dipole frame, this would correspond to a case where the original final-state leg develops a longitudinal component along the incoming parton \tilde{p}_i which is larger than the residual component along \tilde{p}_j (i.e., if we think about the first emission in DIS, the original final-state leg recoils in the remnant hemisphere). Thus, to populate this region, when $(1 - \beta_k)^2 < \alpha_k \beta_k$ we instead use

$$b_k = -\frac{\alpha_k}{(1 - \beta_k)^2 - \alpha_k \beta_k}, \quad a_k = -\frac{\beta_k(1 + \alpha_k - \beta_k)^2}{(1 - \beta_k)^2 - \alpha_k \beta_k}, \quad (\text{A.39})$$

and

$$\bar{p}_i = \frac{a_k b_k}{b_k - 1} \tilde{p}_i + (b_k - 1) \tilde{p}_j + k_\perp, \quad (\text{A.40a})$$

$$\bar{p}_j = \frac{1 + a_k - b_k}{b_k - 1} \tilde{p}_i. \quad (\text{A.40b})$$

Notice that in the soft and in the collinear limit ($\beta_k \ll 1$), both for the IF and II maps we obtain $a_k \sim \alpha_k$ and $b_k \sim \beta_k(1 + \alpha_k)^2$, as originally implemented in the old map of ref. [18]. All the FF, FI, II and IF maps are fully local, in the sense $\pm \tilde{p}_i \pm \tilde{p}_j = \pm p_i \pm p_j + p_k$ (+ for outgoing partons, - for incoming ones). For FF and FI, we can use $p_l = \bar{p}_l$, where p_l are the final momenta used to update the event record. For IF and II maps, the incoming momentum \bar{p}_i is no longer aligned with the beam, so we need to apply a Lorentz transformation to the momenta in the event to realign the beams. The Lorentz transformation depends upon the process under consideration.

- **pp**: we apply a boost and a rotation so that \bar{p}_i (the emitting initial-state parton) and \bar{p}_b (the other initial-state parton, which coincided with \bar{p}_j for an II map) are back-to-back and aligned along the z axis. For the case of colour-singlet production, we then apply a longitudinal boost so that the rapidity of the colour singlet is preserved.²³
- **DIS**: we first apply a rotation to the initial-state and to the final-state partons (i.e. all the particles but the leptons) so that \bar{p}_i is aligned along the direction of the incoming proton. Then we decompose the total partonic final-state momentum as in eq. (A.25), and we apply the boost in eq. (A.27) to all the partons (including \bar{p}_i). This boost simply acts as a rescaling for everything parallel to n_{in}^μ , so that the final expression for p_i^μ corresponds to the one in eq. (A.26).

²³An alternative option, suited for generic processes, is to apply a longitudinal boost so that $p_b = \bar{p}_b$, i.e. the other initial-state beam is preserved.

B Hardest emission matrix elements in the PanScales showers

When considering NLO matching, the effective parton-shower matrix element (see eq. (A.10)) is replaced with

$$\sum_{\ell=i,j} \mathcal{L}_{\ell\bar{\ell}}(\tilde{x}_\ell, z_\ell, \mu_F) z_\ell P_{\ell k}^{\text{IS/FS}}(z_\ell) f(\bar{\eta}_\ell) \rightarrow \frac{d\Phi}{d\Phi_B d \ln \kappa_t d\bar{\eta} d\phi} \frac{R_{ijk}(\mu_R, \mu_F; \Phi)}{B_{ij}(\mu_R, \mu_F; \Phi_B)}, \quad (\text{B.1})$$

for the first emission only. There are three different ingredients in the NLO shower weight given by eq. (B.1). First, we have introduced a process-dependent Jacobian associated with the transformation from the radiative phase-space to the shower variables. More concretely, the phase-space factor is equal to

$$\frac{d\Phi}{d\Phi_B d \ln \kappa_t d\bar{\eta} d\phi} = \frac{k_t^2}{16\pi^2} \frac{d\phi}{2\pi} \times \begin{cases} 1 & \text{for colour-singlet production,} \\ \frac{s_{ij} + s_{ik}}{s_{ij}} & \text{for DIS (with } i \text{ IS parton, } k \text{ the emitted one).} \end{cases} \quad (\text{B.2})$$

where k_t is the physical transverse momentum of the emission. The corresponding phase-space factor for e^+e^- is given in appendix C.1 of ref. [30]. Another ingredient in eq. (B.1) is the Born squared matrix-element

$$B_{ij}(\mu_R, \mu_F; \Phi_B) = f_i(\tilde{x}_i, \mu_F) f_j(\tilde{x}_j, \mu_F) \frac{|M^{(0)}(\Phi_B)|^2}{2\tilde{s}}, \quad (\text{B.3})$$

evaluated at the underlying Born phase-space point Φ_B . This includes the tree-level Born matrix element divided by the flux factor, $|M^{(0)}(\Phi_B)|^2/(2\tilde{s})$, and the product of PDFs evaluated at scale μ_F as given in eq. (A.12). Similarly, $R_{ijk}(\mu_R, \mu_F; \Phi)$ is the real matrix element, stripped of a factor α_s/π

$$R_{ijk}(\mu_R, \mu_F; \Phi) = \frac{\pi}{\alpha_s(\mu_R)} f_i(x_i, \mu_F) f_j(x_j, \mu_F) \frac{|M^{(0)}(\Phi)|^2}{2s}. \quad (\text{B.4})$$

As discussed in the main text, the real-matrix element needs to be partitioned between all possible emitting dipoles. Denoting with $R(\Phi)$ the total real matrix element (including PDF and π/α_s factors) we write

$$R_{ijk}(\mu_R, \mu_F; \Phi) = \sum_p R_p(\Phi) = \sum_p R(\Phi) w_p(\Phi), \quad (\text{B.5})$$

where the $w_p(\Phi)$ are the partitioning functions (satisfying $0 \leq w_p(\Phi) \leq 1$) that we use to separate our total real cross section into several blocks, each of them to be interpreted as a dipole emission probability. The e^+e^- case was worked out in appendix C of ref. [30]. Here, we discuss how to build the partitioning for all other processes.

- **Drell Yan.** Let us consider the LO process $q\bar{q} \rightarrow Z$, which is characterised by the following oriented dipoles: (q, \bar{q}) and (\bar{q}, q) , where the first element denotes the emitter. If we assume the quark moves along the positive z direction, we have

1. gluon emission from the dipole (q, \bar{q})

$$R_1(\Phi) = R_{q\bar{q} \rightarrow Zg}(\Phi) \times \begin{cases} g_{\text{dip}}(y - y_Z) & \text{for PanLocal} \\ f_{\text{ant}}(y - y_Z) & \text{for PanGlobal,} \end{cases} \quad (\text{B.6})$$

where y is the rapidity of the emission, and y_Z is the rapidity of the Z boson;

2. gluon emission from the dipole (\bar{q}, q)

$$R_2(\Phi) = R_{q\bar{q} \rightarrow Zg}(\Phi) - R_1(\Phi); \quad (\text{B.7})$$

3. anti-quark emission from the dipole (q, \bar{q})

$$R_3(\Phi) = R_{g\bar{q} \rightarrow Z\bar{q}}(\Phi); \quad (\text{B.8})$$

4. quark emission from the dipole (\bar{q}, q)

$$R_4(\Phi) = R_{qg \rightarrow Zq}(\Phi). \quad (\text{B.9})$$

- **Gluon fusion.** Let us consider the LO process $g_1 g_2 \rightarrow H$, which is characterised by the following oriented dipoles: (g_1^C, g_2^A) , (g_1^A, g_2^C) , (g_2^C, g_1^A) and (g_2^A, g_1^C) , where again the first element is the emitter. The prefix C and A denote if the gluon carries a colour or an anti-colour index. If we assume g_1 moves along the positive z direction, we have

1. gluon emission from the dipole (g_1^C, g_2^A)

$$R_1(\Phi) = \frac{1}{2} R_{g\bar{g} \rightarrow Hg}(\Phi) \times \begin{cases} g_{\text{dip}}(y - y_H) & \text{for PanLocal} \\ f_{\text{ant}}(y - y_H) & \text{for PanGlobal,} \end{cases} \quad (\text{B.10})$$

where y is the rapidity of the emission, and y_H is the rapidity of the H boson;

2. gluon emission from the dipole (g_1^A, g_2^C)

$$R_2(\Phi) = R_1(\Phi); \quad (\text{B.11})$$

3. gluon emission from the dipole (g_2^C, g_1^A)

$$R_3(\Phi) = \frac{1}{2} R_{g\bar{g} \rightarrow Hg}(\Phi) - R_2(\Phi) \quad (\text{B.12})$$

4. gluon emission from the dipole (g_2^A, g_1^C)

$$R_4(\Phi) = R_3(\Phi); \quad (\text{B.13})$$

5. anti-quark emission from the dipole (g_1^C, g_2^A)

$$R_5(\Phi) = R_{\bar{q}g \rightarrow H\bar{q}}(\Phi); \quad (\text{B.14})$$

6. anti-quark emission from the dipole (g_2^C, g_1^A)

$$R_6(\Phi) = R_{g\bar{q} \rightarrow H\bar{q}}(\Phi); \quad (\text{B.15})$$

7. quark emission from the dipole (g_1^A, g_2^C)

$$R_7(\Phi) = R_{qg \rightarrow Hq}(\Phi) \tag{B.16}$$

8. quark emission from the dipole (g_2^A, g_1^C)

$$R_8(\Phi) = R_{gq \rightarrow Hq}(\Phi). \tag{B.17}$$

• **Deep inelastic scattering.** Let us consider the LO process $\ell q_I \rightarrow \ell q_F$, and assume the momenta are in the Breit frame and q_F moves along the positive z direction. We have three regions:

1. gluon emission from the dipole (q_F, q_I)

$$R_1(\Phi) = R_{\ell q \rightarrow \ell qg}(\Phi) \times \begin{cases} g_{\text{dip}}(y) & \text{for PanLocal} \\ f_{\text{ant}}(y) & \text{for PanGlobal,} \end{cases} \tag{B.18}$$

where y is the rapidity of the emission in the Breit frame;

2. gluon emission from the dipole (q_I, q_F)

$$R_2(\Phi) = R_{\ell q \rightarrow \ell qg}(\Phi) - R_1(\Phi); \tag{B.19}$$

3. anti-quark emission from the dipole (q_I, q_F)

$$R_3(\Phi) = R_{\ell g \rightarrow \ell q\bar{q}}(\Phi) \Theta(p_{\bar{q}} \cdot p_g < p_q \cdot p_g), \tag{B.20}$$

where, as discussed before the Θ prevents double counting this contribution since it can also be reached starting from the LO process $\ell \bar{q}_I \rightarrow \ell \bar{q}_F$.

C Slice to subtraction expressions for processes with two coloured legs

In this appendix we give the expressions for the $\bar{B}_C(\Phi_B)$ weights, and the corresponding differential counterterms, for the PanGlobal pp , DIS and e^+e^- shower variants for all processes considered in this paper.

C.1 $e^+e^- \rightarrow q\bar{q}$ and $H \rightarrow gg$

For the e^+e^- PanGlobal shower we use a counterterm real-radiation probability given by (cf. section 3.4)

$$\frac{C(\Phi)}{B_0(\Phi_B)} d\Phi_{\text{rad}} \rightarrow \frac{dv}{v} d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s}{\pi} z P_{ij}(z), \quad \ln z = \bar{\eta} - \bar{\eta}_{\text{max}}, \quad 0 < \bar{\eta} < \bar{\eta}_{\text{max}} = \ln Q/v, \tag{C.1}$$

where the splitting functions that enter into the $e^+e^- \rightarrow q\bar{q}$ and $H \rightarrow gg$ processes are given by

$$P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z}, \tag{C.2a}$$

$$P_{gg}(z) = C_A \left[\frac{1 + (1-z)^3}{z} + w_{gg}(1-2z) \right], \tag{C.2b}$$

$$P_{qq}(z) = 2T_R \left[(1-z)^2 - w_{qq}(1/2-z) \right]. \tag{C.2c}$$

The factors w_{qq} and w_{gg} govern the partitioning of the P_{qq} and P_{gg} splitting functions as they enter in the PanGlobal shower [14], and are by default set to 0. These counterterms are simple enough to be integrated analytically, and following the procedure of section 3.4 we arrive at the following weight for $e^+e^- \rightarrow q\bar{q}$

$$\bar{B}_C^{\text{PG},q\bar{q}}(\Phi_B) = B_0^{e^+e^- \rightarrow q\bar{q}}(\Phi_B) \left(1 + \frac{\alpha_s C_F}{2\pi} \left[5 - \frac{\pi^2}{3} \right] \right). \quad (\text{C.3})$$

Similarly, we can derive the effective NLO weight for $H \rightarrow gg$, which enters into the decay width

$$\bar{B}_C^{\text{PG},gg}(\Phi_B) = B_0^{H \rightarrow gg}(\Phi_B) \left[1 + \frac{\alpha_s(\mu_R)}{2\pi} \left(C_A \left(\frac{167}{9} - \frac{\pi^2}{3} \right) - T_R n_f \frac{46}{9} + 8\pi b_0 \ln \frac{\mu_R}{Q} \right) \right]. \quad (\text{C.4})$$

C.2 Worked example for Drell-Yan production

Before giving the equations that enter into processes with initial-state radiation, it is instructive to look at a detailed example. For this purpose we will look at $pp \rightarrow V$. The results presented here are independent of whether or not the vector boson is allowed to decay. The first step is to formulate a shower counterterm where the ordering variable resembles that of the actual shower in the infrared limits, and the emission probability reproduces the correct singularity structure of the full matrix element. This ‘‘approximate shower’’ has to be simple enough that it can be integrated at order α_s above some slicing cutoff. It will have approximate phase-space bounds and will not need an explicit kinematic mapping other than in the IR. For our pp showers, we will define a shower that has a single (unregularised) leading-order splitting function $p_{ij}(z)$ for each side of the event, i.e. $p_{ij}(z_1)\Theta(\bar{\eta} > 0)$ and $p_{ij}(z_2)\Theta(\bar{\eta} < 0)$, where z_1 and z_2 are momentum fractions for the forward and backward going beams respectively. We consider the $\beta_{\text{PS}} = 0$ PanGlobal shower in the $\bar{\eta} > 0$ hemisphere, and define (see appendix A)

$$\bar{z} \equiv 1 - z = \frac{1}{1 + \alpha_k}, \quad \alpha_k = \frac{\kappa_t}{Q} e^{\bar{\eta}}, \quad (\text{C.5})$$

where Q is the invariant mass of the colour singlet. The requirement $\bar{\eta} > 0$ translates to

$$\bar{z} < \frac{Q}{Q + \kappa_t}. \quad (\text{C.6})$$

The counterterm real-radiation probability is then given by

$$\frac{C(\Phi)}{B_0(\Phi_B)} d\Phi_{\text{rad}} \rightarrow \frac{d\kappa_t}{\kappa_t} d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s}{\pi} (1 - \bar{z}) p_{qk}(\bar{z}) \frac{f_k(x/\bar{z})}{f_q(x)} \Theta \left(x < \bar{z} < \frac{Q}{Q + \kappa_t} \right), \quad (\text{C.7})$$

where x is the momentum fraction of the incoming parton. For $q\bar{q} \rightarrow Z$ there are two channels to consider: one where the flavour is preserved, and one where the (anti-)quark backwards evolves into a gluon. The corresponding splitting functions are given by

$$p_{qq}(\bar{z}) = C_F \left[\frac{1 + \bar{z}^2}{1 - \bar{z}} \right], \quad (\text{C.8a})$$

$$p_{qg}(\bar{z}) = T_R [\bar{z}^2 + (1 - \bar{z})^2]. \quad (\text{C.8b})$$

Above a slicing cutoff $\kappa_t = Qe^L$, the approximate shower will give a cross section of

$$\frac{C_{\text{int}}(v > Qe^L)}{B_0(\Phi_B)} = 2 \frac{\alpha_s}{2\pi} \sum_{i=1,2} \sum_{k \in q,g} \int \frac{d\kappa_t}{\kappa_t} \frac{d\bar{z}}{\bar{z}} p_{qk}(\bar{z}) \frac{f_k(x_i/\bar{z})}{f_q(x_i)} \Theta\left(x_i < \bar{z} < \frac{Q}{Q+\kappa_t}\right) \Theta(Qe^L < \kappa_t < Q), \quad (\text{C.9})$$

where $f_i(x) \equiv f_i(x, \mu_F = \kappa_t)$ denote the PDFs for flavour i and momentum fraction x . This may be written in a form that separates the PDF dependence as

$$\begin{aligned} \frac{C_{\text{int}}(v > Qe^L)}{B_0(\Phi_B)} &= 2 \frac{\alpha_s}{2\pi} \int_{Qe^L}^Q \frac{d\kappa_t}{\kappa_t} \left[2 \int_0^{Q/(Q+\kappa_t)} d\bar{z} p_{qq}(\bar{z}) \right. \\ &\quad + \sum_{i=1,2} \int_{x_i}^1 \frac{d\bar{z}}{\bar{z}} \left(p_{qq}(\bar{z}) \Theta\left(\bar{z} < \frac{Q}{Q+\kappa_t}\right) \right)_+ \frac{f_q(x_i/\bar{z})}{f_q(x_i)} \\ &\quad \left. + \sum_{i=1,2} \int_{x_i}^{Q/(Q+\kappa_t)} \frac{d\bar{z}}{\bar{z}} p_{qg}(\bar{z}) \frac{f_g(x_i/\bar{z})}{f_q(x_i)} \right]. \end{aligned} \quad (\text{C.10})$$

In the limit of large negative L , the first line gives us

$$4 \int_{Qe^L}^Q \frac{d\kappa_t}{\kappa_t} \int_0^{Q/(Q+\kappa_t)} d\bar{z} p_{qq}(\bar{z}) = 4C_F L^2 + 6C_F L + \bar{H}_{qq}^{(1)} + \mathcal{O}(e^L), \quad (\text{C.11})$$

with

$$\bar{H}_{qq}^{(1)} = C_F \left(1 + 6 \ln 2 + \frac{2\pi^2}{3} \right). \quad (\text{C.12})$$

For the last two lines of eq. (C.10) we replace $\Theta\left(\bar{z} < \frac{Q}{Q+\kappa_t}\right)$ with $1 - \Theta\left(\bar{z} > \frac{Q}{Q+\kappa_t}\right)$ and interchange the κ_t and \bar{z} integration orders. We get for these last two lines

$$= -2L \frac{(P_{qk} \otimes f_k)(x_i)}{f_q(x_i)} + \frac{(\bar{C}_{qk}^{(1)} \otimes f_k)(x_i)}{f_q(x_i)} + i \leftrightarrow j, \quad (\text{C.13})$$

where P_{qk} is now the regularised splitting function and

$$\bar{C}_{qq}^{(1)}(\bar{z}) = -2 \left(p_{qq}(\bar{z}) \ln \frac{\bar{z}}{1-\bar{z}} \Theta(\bar{z} > 1/2) \right)_+, \quad (\text{C.14a})$$

$$\bar{C}_{qg}^{(1)}(\bar{z}) = -2 p_{qg}(\bar{z}) \ln \frac{\bar{z}}{1-\bar{z}} \Theta(\bar{z} > 1/2). \quad (\text{C.14b})$$

The complete result for the counterterm thus becomes

$$\frac{C_{\text{int}}(v > Qe^L)}{B_0(\Phi_B)} = \frac{\alpha_s}{2\pi} \left[4C_F L^2 + 6C_F L + \bar{H}_{qq}^{(1)} + \frac{((-2LP_{qk} + \bar{C}_{qk}^{(1)}) \otimes f_k)(x_i)}{f_q(x_i)} + i \leftrightarrow j \right]. \quad (\text{C.15})$$

We tabulate and then interpolate the results of the convolutions with **Hoppet**, which we adapted in order to accurately handle the $\Theta(\bar{z} > 1/2)$ in eq. (C.14).

To work out \bar{B}_C we also need a slicing calculation in the shower variable, $\Sigma_{\text{PG}}^{\text{NLO}}(v < Qe^L)$. The first emission of the PanGlobal shower coincides with the transverse momentum of the leading jet. This means we may use the results of ref. [119] (cf. the supplemental material therein) directly to obtain $\Sigma_{\text{PG}}^{\text{NLO}}(v < Qe^L)$. At $\mathcal{O}(\alpha_s)$, at fixed Born flavours with momentum fractions x_i and x_j , we can write those results as

$$\frac{\Sigma(v < Qe^L)}{B_0(\Phi_B)} = 1 + \frac{\alpha_s}{2\pi} \left[-2A_q^{(1)}L^2 + 2B_q^{(1)}L + H_{q\bar{q}}^{(1)} + \frac{\left((2LP_{ik} + C_{ik}^{(1)}) \otimes f_k \right) (x_i)}{f_i(x_i)} + i \leftrightarrow j \right], \quad (\text{C.16})$$

with

$$A_q^{(1)} = 2C_F, \quad B_q^{(1)} = -3C_F, \quad H_{q\bar{q}}^{(1)} = C_F \left(-8 + \frac{7\pi^2}{6} \right), \quad (\text{C.17a})$$

$$C_{ij}^{(1)}(z) = -P_{ij}^{(0),\epsilon}(z) - \delta_{ij}\delta(1-z)C_F\frac{\pi^2}{12} + 2P_{ij}(z)\ln\frac{Q}{\mu_F}. \quad (\text{C.17b})$$

Here $P_{ij}^{(0),\epsilon}$ denote the ϵ -dependent part of the leading-order splitting functions in $D = 4 - 2\epsilon$ dimensions

$$P_{q\bar{q}}^{(0),\epsilon}(z) = -C_F(1-z), \quad (\text{C.18a})$$

$$P_{gq}^{(0),\epsilon}(z) = -C_F z, \quad (\text{C.18b})$$

$$P_{qg}^{(0),\epsilon}(z) = -2T_R z(1-z), \quad (\text{C.18c})$$

$$P_{gg}^{(0),\epsilon}(z) = 0. \quad (\text{C.18d})$$

where $P_{q\bar{q}}^{(0),\epsilon}(z)$ is for a single flavour or anti-flavour of quark (not the sum of flavour and anti-flavour). Adding together eqs. (C.15) and (C.16) gives us

$$\bar{B}_C^{\text{PG},pp \rightarrow V} = B_0^{pp \rightarrow V}(\Phi_B) \left[1 + \frac{\alpha_s(\mu_R)}{2\pi} \left(H_{q\bar{q}}^{(1)} + \bar{H}_{q\bar{q}}^{(1)} + \frac{\left((C_{ik}^{(1)} + \bar{C}_{ik}^{(1)}) \otimes f_k \right) (x_i, \mu_F)}{f_i(x_i, \mu_F)} + i \leftrightarrow j \right) \right]. \quad (\text{C.19})$$

C.3 Results for gluon fusion Higgs production and DIS

In order to state an equivalent result to eq. (C.19) for gluon fusion Higgs production, we need first to give the splitting functions that enter into eqs. (C.7) and (C.9)

$$p_{gq}(\bar{z}) = C_F \left[\frac{1 + (1 - \bar{z})^2}{\bar{z}} \right], \quad (\text{C.20a})$$

$$\tilde{p}_{gg}(\bar{z}) = 2C_A \left[\frac{\bar{z}}{1 - \bar{z}} + \frac{\bar{z}(1 - \bar{z})}{2} \right], \quad (\text{C.20b})$$

$$p_{gg}(\bar{z}) = \tilde{p}_{gg}(\bar{z}) + \tilde{p}_{gg}(1 - \bar{z}). \quad (\text{C.20c})$$

It is worth noting that the regularised P_{gg} splitting function can be written as

$$P_{gg} = (\tilde{p}_{gg}(\bar{z}))_+ + \tilde{p}_{gg}(1 - \bar{z}) - \frac{4n_f T_R}{6} \delta(1 - \bar{z}). \quad (\text{C.21})$$

For Higgs production in the limit of a large top-quark mass and vanishing bottom-quark mass, the counterterm real-radiation probability is then given by

$$\frac{C(\Phi)}{B_0(\Phi_B)} d\Phi_{\text{rad}} \rightarrow \frac{d\kappa_t}{\kappa_t} d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s}{\pi} (1-\bar{z}) p_{gk}(\bar{z}) \frac{f_k(x/\bar{z})}{f_g(x)} \Theta\left(x < \bar{z} < \frac{Q}{Q+\kappa_t}\right), \quad (\text{C.22})$$

while the NLO weight is

$$\bar{B}_C^{\text{PG}, pp \rightarrow H} = B_0^{pp \rightarrow H}(\Phi_B) \left[1 + \frac{\alpha_s(\mu_R)}{2\pi} \left(H_{gg}^{(1)} + \bar{H}_{gg}^{(1)} + \frac{\left((C_{ik}^{(1)} + \bar{C}_{ik}^{(1)}) \otimes f_k \right) (x_i, \mu_F)}{f_i(x_i, \mu_F)} + i \leftrightarrow j \right) \right], \quad (\text{C.23})$$

where

$$H_{gg}^{(1)} = C_A \left(5 + \frac{7}{6} \pi^2 \right) - 3C_F + 8\pi b_0 \ln \frac{\mu_R}{Q}, \quad \bar{H}_{gg}^{(1)} = C_A \left(\frac{1}{6} + \frac{2\pi^2}{3} + \frac{22}{3} \ln 2 \right), \quad (\text{C.24})$$

and

$$\bar{C}_{gq}^{(1)} = -2p_{gq}(\bar{z}) \ln \frac{\bar{z}}{1-\bar{z}} \Theta(\bar{z} > 1/2), \quad (\text{C.25a})$$

$$\bar{C}_{gg}^{(1)} = -2 \left[\tilde{p}_{gg}(\bar{z}) \ln \frac{\bar{z}}{1-\bar{z}} \Theta(\bar{z} > 1/2) \right]_+ - 2\tilde{p}_{gg}(1-\bar{z}) \ln \frac{\bar{z}}{1-\bar{z}} \Theta(\bar{z} > 1/2). \quad (\text{C.25b})$$

The $C_{ik}^{(1)}$ is given as in eq. (C.17b) with the replacement $C_F \rightarrow C_A$. Finally, for DIS we need to combine elements of the e^+e^- and pp analyses above. Specifically, the differential counterterms are given by eq. (C.1) in the current hemisphere and eq. (C.7) in the remnant hemisphere, and the sum of the integrated counterterm and virtual correction is

$$\bar{B}_C^{\text{PG}, \text{DIS}}(\Phi_B) = B_0^{\text{DIS}}(\Phi_B) \left[1 + \frac{\alpha_s(\mu_R)}{2\pi} \left(H_{\text{DIS}}^{(1)} + \bar{H}_{\text{DIS}}^{(1)} + \frac{\left((C_{ik}^{(1)} + \bar{C}_{ik}^{(1)}) \otimes f_k \right) (x_i, \mu_F)}{f_i(x_i, \mu_F)} \right) \right], \quad (\text{C.26})$$

with

$$H_{\text{DIS}}^{(1)} = -8C_F, \quad \bar{H}_{\text{DIS}}^{(1)} = C_F \left(7 - \frac{\pi^2}{4} + 3 \ln 2 \right). \quad (\text{C.27})$$

D Lepton-swap algorithm for the Drell-Yan process

In this section we describe the swap algorithm that we have implemented to efficiently generate the hardest emission in Drell-Yan processes, where the lepton-anti lepton asymmetry can lead to a nearly (exactly) vanishing Born squared matrix element $B_0(\Phi_B)$ in neutral-current (charged-current) production, causing the ratio $R(\Phi_B, \Phi_{\text{rad}})/B_0(\Phi_B)$ to become very large away from a singular configuration.

Hardest radiation generation. We aim to generate an emission given (a) the underlying Born phase space Φ_B and (b) radiation variables Φ_{rad} . For standard multiplicative matching, we generate the hardest emission with the following branching probability,

$$\frac{dP}{d\Phi_{\text{rad}}} = \frac{R(\Phi_B, \Phi_{\text{rad}})}{B_0(\Phi_B)}, \quad (\text{D.1})$$

cf. eq. (2.5). When we activate the swap algorithm we instead use

$$\frac{dP^{(\text{swap})}(\Phi_B, \Phi_{\text{rad}})}{d\Phi_{\text{rad}}} = \frac{R(\Phi_B, \Phi_{\text{rad}}) + R(\Phi'_B, \Phi_{\text{rad}})}{B_0(\Phi) + B_0(\Phi'_B)}, \quad (\text{D.2})$$

where Φ'_B is the underlying Born phase space in which the lepton momenta have been swapped. It is important to notice that eq. (D.1) and eq. (D.2) become identical in the limit where the emission is either soft or collinear.

If an emission is accepted, one computes

$$f_B = \frac{B_0(\Phi_B)}{B_0(\Phi_B) + B_0(\Phi'_B)}, \quad f_R = \frac{R(\Phi_B, \Phi_{\text{rad}})}{R(\Phi_B, \Phi_{\text{rad}}) + R(\Phi'_B, \Phi_{\text{rad}})}, \quad (\text{D.3})$$

and swaps the lepton kinematics with probability

$$p^{(\text{swap})} = \max\left(0, 1 - \frac{f_R}{f_B}\right). \quad (\text{D.4})$$

The swap ensures that the lepton kinematics in the presence of a resolved real emission is distributed according to $R(\Phi_B, \Phi_{\text{rad}})$. This effectively corrects for eq. (D.2), bringing the real generation probability back to eq. (D.1).

Virtual corrections in dBNLO. Any change in the generation of real radiation implies also a corresponding change in the \bar{B} NLO normalisation factor for the Born configuration, which now reads

$$\bar{B}^{(\text{swap})}(\Phi_B) = B_0(\Phi_B) + V(\Phi_B) + C_{\text{int}}(\Phi_B) + \int \left[B_0(\Phi_B) \frac{dP^{(\text{swap})}(\Phi_B, \Phi_{\text{rad}})}{d\Phi_{\text{rad}}} - C(\Phi) \right] d\Phi_{\text{rad}}, \quad (\text{D.5})$$

where we rely on the property that $V(\Phi_B)/V(\Phi'_B) = B_0(\Phi_B)/B_0(\Phi'_B)$ and similarly for the counterterm, both differential and integrated.

Virtual corrections in ESME. For the ESME algorithm, the normalisation of Born-like events is given by \bar{B}_C of eq. (3.21), which is a function of the Born amplitude squared B_0 , the virtual one V and the integrated real counterterm C_{int} . Recall the ratios $V(\Phi_B)/B_0(\Phi_B)$ and $C_{\text{int}}(\Phi_B)/B_0(\Phi_B)$ are independent from the decay angles of the vector boson. Thus \bar{B}_C stays the same for the lepton-swapped configuration. The real radiation part of ESME uses $dP^{(\text{swap})}(\Phi_B, \Phi_{\text{rad}})$ of eq. (D.2). This ensures that eq. (D.5) is automatically reproduced, without the need to modify the Born event normalisation.

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has associated code in a code repository.

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