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RISKY ALLOCATIONS FROM A RISK-NEUTRAL INFORMED PRINCIPAL

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Risky Allocations from a Risk-Neutral Informed Principal.*

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Abstract

We study a model of informed principal with private values where the principal is risk neutral and the agent is risk averse. We show that the principal, regardless of her type, gains by not revealing her type to the agent through the contract offer. The equilibrium allocation transfers some ex-ante risk from one type of agent to the other. Despite the increase in the principal's surplus, allocative efficiency does not necessarily improve.

KEYWORDS: Contract, Adverse Selection, Informed Principal, Risk Aversion.

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1 Introduction

We are facing an informed principal problem when, in a principal agent relationship, the player that offers the contract possesses private information, and this is regardless of the agent having private information or not. When dealing with this kind of models the main question is to understand if the principal can gain from having private information and what is the optimal contract that has to be offered to the agent to maximise the principal surplus.

The standard contract theory literature, and the adverse selection one in particular, has mainly focused on the standard principal-agent paradigm where the uninformed party offers a contract to the informed one. This framework has proved very efficient in explaining many economic interactions but in many situations it is realistic to assume that asymmetry of information is double-sided. Moreover studying informed principal problems allows to understand how robust the established results are, how sensitive they are to the uninformed principal assumption.

We study a model with a risk neutral informed principal who deals with a risk averse agent who has private information about his type. We make the further assumptions that types are independently distributed and that the utility function of the agent does not depend from the type of the principal, the latter assumption implies that the model is one of *private values*. We show that the principal benefits from having private information in comparison to the payoff she would have received had her type been public. The optimal contract will involve a pooling offer that will leave the agent uninformed when he has to accept the contract.

Maskin and Tirole [1990] have shown that the principal cannot be worse off when she has private information compared to the payoff she obtains when her type is common knowledge. This is due entirely to the private value assumption, there is no rivalry among different types of principal so that no type of principal will loose out from being pooled with another type and signalling is not an issue[1]. In other words, concealing one's information has no costs because no type of principal can gain by separating herself from other types.

It can happen though that the principal is indifferent between revealing or not, that is there may not be benefits from hiding one's type. Maskin and Tirole [1990] have proved that for a generic subset of utility functions and independently distributed types there are strictly positive gains for the principal from not revealing information.

The benefits stem from the fact that the different types of principals have different relative costs of satisfying the agent's constraints. If the principal makes a not-revealing contract offer then the agent remains uninformed and his constraints need to hold only at an interim stage and this gives more freedom to the principal. What will happen is that each type of principal will be able to relax the relatively more costly constraint while tightening the other. In this equilibrium ex-post constraints may not be satisfied in every state of the world.

Maskin and Tirole [1990] show that the principal cannot gain from a pooling contract offer when both players have quasilinear utility function. Although non

[1]See Maskin and Tirole [1992] for an analysis of common values model where sginalling causes more problems when looking for the optimal contract.

generic, it is a setup frequently analysed in applications. In this case the principal cannot improve upon the payoffs she would get if her type was public information. Cella [2002] shows that gains are positive in that same setup when types are not independently distributed, correlation causes the relative costs of constraints to be different across types of principal.

In this work we further extend the Maskin and Tirole result and show that even when types are independent and the principal is risk neutral there are still benefits for the principal from concealing her information if the agent is risk averse.

Risk aversion on the part of the agent causes the relative costs of satisfying his incentive and participation constraints different for the two types of principal allowing the principal to make some profit from tightening and relaxing the ex-post constraints as long as the interim ones are satisfied.

We first show that the full-information allocation (i.e. the equilibrium when the type of principal is known) is dominated, fro every type of principal, and therefore it cannot be an equilibrium of the game. Then we characterise the optimal contract that each type of principal will offer to obtain a higher utility.

The equilibrium allocation prescribes a reduction in the spread of the ex-post payoff of the efficient agent and an increase in the spread for the inefficient one. In equilibrium then the risk-neutral principal does not eliminate the ex-ante risk that has to be born by the risk-averse agent, what the principal does is somehow "reallocating" some risk from the efficient to the inefficient agent.

Moreover only one type of principal reduces the downward quantity distortion for the inefficient agent while the other type increases it. Total allocative inefficiency is not necessarily reduced in equilibrium despite the increase in the principal's surplus and the reduction in the agent's one.

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyses the contract when the type of the principal is known, which we use as benchmark. Section 4 studies the optimal contract when the principal is privately informed. Section 5 concludes.

2 The model.

2.1 Objective functions and information.

There are two players, one principal and one agent and both have a type-dependent utility function. Each agent has private information about his or her type but their utility function does not directly depend on the other player's type. This assumption sets our model in what the literature calls a *private values* framework. We are going to denote the type of the principal and the one of the agent with two parameters i and j (with $i = 1, 2$ and $j = 1, 2$)[2] respectively.

The principal is risk neutral with respect to transfers and her utility function takes the following quasi-linear form:

[2] As in Maskin and Tirole [1990, 1992] the limitation on the possible types for the players is not essential but simplifies the analysis and favors the intuition of the results.

$$V^i = S^i(y) - t,$$

where y is an observable and verifiable action, t is a monetary transfer from the principal to the agent. $S^i(\cdot)$ is continuous, increasing and concave in y for every i.

The agent is risk averse and his utility function has the following functional form[3]:

$$U_j = U(t - \theta_j y),$$

where $U(\cdot)$ is continuous, increasing and concave in y and, to simplify, independent from j. We assume that U_j is decreasing in j, implying therefore that: $\theta_1 < \theta_2$.

The agent's reservation utility is normalized to zero.

To guarantee the existence of equilibrium, we assume that the feasible actions and transfers lie in compact and convex sets.

We assume that the parameters i and j are drawn from independent common knowledge distributions. The parameters indicate the type of each player, i is known only to the principal and j to the agent. The prior beliefs of the agent on the type i of the principal are denoted by ϕ_i, type i can assume value 1 (resp. 2) with probability ϕ_1(resp.ϕ_2) such that $\phi_1 + \phi_2 = 1$. We denote by p_j the beliefs of the principal on the possible values of θ; $\theta_j = \theta_1$ and θ_2 with probabilities p_1 and p_2 (with $p_1 + p_2 = 1$).

2.2 The principal-agent game.

The timing of the principal-agent game is as follows:

1. The principal proposes a mechanism in the feasible set M to the agent. A mechanism m in M will specify i) a set of possible messages for each party and ii) for each pair of messages chosen simultaneously an allocation (y, t). Note that the set M includes the set of direct revelation mechanisms in which parties simultaneously announce their types, by invoking the revelation principle for Bayesian game we can restrict the attention to direct truthful mechanisms.[4]

2. The agent updates his prior (if he has learned something from the offer), accepts or refuses the contract offered. If he refuses both players get zero utility and the game ends. If the agent accepts then parties move to the last stage of the game.

3. Both parties announce their types and the proposed mechanism is implemented.

We will study the perfect Bayesian equilibria of the overall game.

[3]The choice of this particular form of utility function is without loss of generality, we just need the agent to be risk-averse.

[4]In this framework (as in Maskin and Tirole [1990]) the principle states that for any mechanism and for given beliefs any equilibrium of the mechanism is equivalent to an equilibrium of a direct revelation mechanism in which types are truthfully announced.

3 Benchmark: the full information allocation.

As a benchmark we study the equilibrium when the principal's information is common knowledge. Maskin and Tirole call it the *full information* case (even if the principal does not know the agent's type) and it is nothing more than the standard screening model.

We know from the revelation principle that every equilibrium allocation of this game can be obtained as an equilibrium of a direct truthful mechanism. The outcome $\left(y_j^i, t_j^i\right)$ that will be implemented in equilibrium will have to satisfy two types of constraints individual rationality and incentive compatibility.

For every i the participation constraints are: $U\left(t_j^i - \theta_j y_j^i\right) \geq 0$ for $j = 1, 2$. While the truthtelling constraints are: $U\left(t_j^i - \theta_j y_j^i\right) \geq U\left(t_k^i - \theta_j y_k^i\right)$ for all j,k.

Standard arguments apply, and in this context only two constraints are binding, the participation constraint of type 2 and the incentive compatibility of type 1.

Therefore in the case of full information a principal of type i proposes a contract $\left\{\left(y_1^i, t_1^i\right), \left(y_2^i, t_2^i\right)\right\}$ that solves the following program:

$$\left(F^i\right) = \begin{cases} \max\limits_{\left(y_j^i, t_j^i\right)} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y_j^i\right) - t_j^i\right) \text{ such that} & \\ \text{IR}^i : U\left(t_2^i - \theta_2 y_2^i\right) = 0 & \left(\rho^i\right) \\ \text{IC}^i : U\left(t_1^i - \theta_1 y_1^i\right) = U\left(t_2^i - \theta_1 y_2^i\right) & \left(\gamma^i\right), \end{cases}$$

where ρ^i and γ^i are the Lagrange multipliers for the IR and IC constraints.

Given the specific functional forms chosen for the utility functions of the two players we can actually find the precise solution to this problem.

A principal of type i will offer the following decreasing schedule of output and the respective transfers, $\left\{\left(y_1^i, t_1^i\right), \left(y_2^i, t_2^i\right)\right\}$:

$$\begin{aligned} \text{type 1} \quad &: \quad S^{i\prime}\left(y_1^i\right) = \theta_1 \text{ and } t_1^i = \theta_1 y_1^i + \Delta\theta y_2^i \\ \text{type 2} \quad &: \quad S^{i\prime}\left(y_1^i\right) = \theta_2 + \frac{p_1}{p_2}\Delta\theta \text{ and } t_2^i = \theta_2 y_2^i. \end{aligned}$$

The solution has standard characteristics like the "no distortion at the top" property and no informational rent for the "low-type" agent. It preserves also the feature that the risk aversion on the part of the agent does not influence the solution In particular, the downward distortion in the production requested to a type 2 agent is the same that one would observe if the agent was risk neutral.

For future reference denote by $\left(\overline{y}_{\cdot}^i, \overline{t}_{\cdot}^i, \overline{\rho}^i, \overline{\gamma}^i\right)$ the solution to the full information program $\left(F^i\right)$. Let $\overline{v}^i \equiv \sum_j p_j V^i\left(\overline{y}_j^i, \overline{t}_j^i\right)$ be the type i principal's expected payoff.

At this stage it is essential to look at the ratio of the Lagrange multipliers of the full information optimization problem; they are different across types of principal and have the following expression:

$$\frac{\rho^i}{\gamma^i} = \frac{U'\left(t_1^i - \theta_1 y_1^i\right)}{p_1 U'\left(t_2^i - \theta_2 y_2^i\right)} = \frac{U'\left(\Delta\theta y_2^i\right)}{p_1 U'(0)}.$$

This implies that the relative cost of fulfilling the individual rationality and incentive compatibility constraint is different for each type of principal[5].

As Maskin and Tirole [1990] have shown, as long as the relative costs of the incentive and participation constraints is not the same for both types of principal there are some gains coming from the fact one type of principal can relax the constraint that is relatively more costly for her, while enforcing the one that is relatively less costly. This result can be achieved only if the constraints of the agent can be satisfied at an interim stage (and not ex-post as in the full information case) and they need to hold only on average exactly when the agent does not know the type of the principal, in other words when the offer is not revealing.

4 The pooling offer.

As anticipated in the previous section we now return to the case in which the type of the principal is not known to the agent, and we are going to show that not revealing her type to the agent until the third stage of the game will allow any type of principal to obtain a payoff higher than the full-information one.

First of all it is important to bear in mind that the full information allocation can still be implemented when the type of the principal is not known to the agent. This is true because we are in the private values framework, as Maskin and Tirole [1990] explain, there is no rivalry between different types of principal and therefore revealing the type through the contract offer at the first stage is incentive compatible for the principal.

An informed principal has the alternative choice of not revealing her type, and we are going to show that indeed the full-information allocation can never be an equilibrium when the type of the principal is private information to her.

First of all we show that it is dominated and then we characterize the new equilibrium.

Proposition 1 *The full information allocation* $\left(\overline{y}^i_{\cdot}, \overline{t}^i_{\cdot}\right)$ *is dominated from the point of view of every type of principal* $i = 1, 2$ *by the allocation* $\left(\overline{y}^i_{\cdot *}, \overline{t}^i_{\cdot *}\right)$ *, which is the solution to the following program:*

$$\left(F^i_*\right) = \begin{cases} \max\limits_{\left(y^i_j, t^i_j\right)} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y^i_j\right) - t^i_j\right) \text{ such that} \\ IR^i : U\left(t^i_2 - \theta_2 y^i_2\right) = -r^i \\ IC^i : U\left(t^i_1 - \theta_1 y^i_1\right) - U\left(t^i_2 - \theta_1 y^i_2\right) = -c^i. \end{cases}$$

Proof. It is evident that the only difference from the full-information program is that now there is some slack allowed on each constraint.

[5] Lagrange multipliers represent the shadow cost of the constraints in the maximization problem. Their ratio would be the same for the two types of principal if also the agent was risk neutral (see Maskin and Tirole [1990]).

Let v_*^i be the maximized value of the maximand, by definition of the shadow prices $\overline{\rho}^i$ and $\overline{\gamma}^i$ it approximately equals $\overline{v}^i + \overline{\rho}^i r^i + \overline{\gamma}^i c^i$ for small values of r^i and c^i. Let μ_*^i be a solution to F_*^i.

Choose negative slack variables (r^1, c^1) for the type 1 principal; then the slack variables for type 2 are defined as: $r^2 = -\frac{\phi_1}{\phi_2} r^1$ and $c^2 = -\frac{\phi_1}{\phi_2} c^1$. We can then write:

$v_*^1 - \overline{v}^1 \simeq \overline{\rho}^1 r^1 + \overline{\gamma}^1 c^1$

$v_*^2 - \overline{v}^2 \simeq -\frac{\phi_1}{\phi_2} \left(\overline{\rho}^2 r^1 + \overline{\gamma}^2 c^1\right)$.

Both the above left hand sides can be positive for (r^1, c^1) if and only if $\frac{\overline{\rho}^1}{\overline{\gamma}^1} \neq \frac{\overline{\rho}^2}{\overline{\gamma}^2}$, as it is in our case. ■

We have therefore shown that when each type of principal is allowed some slackness on the agent's constraints then they can achieve a higher payoff than in the full information case.

The principal, whatever her type, will offer the same contract which will consist of a menu of four allocations, one for each possible state of the world. If she does so then the agent will not be able to infer anything about her type and accept the contract still on the basis of the prior probability distribution. What really happens is that the principal offers a contract that contains allocations that will never be implemented (e.g. a type1-1principal offers also those that are designed for a type-2 principal), but she does so because this will keep the agent ignorant about her type. This means that agent's constraints, individual rationality and incentive compatibility, will have to hold only in expectation leaving the principal the freedom of not satisfying some of the ex-post ones. We have already shown that each type of principal gains from being able to not satisfy the constraint that is relatively more expensive for her, in what follows we are going to characterize the equilibrium contract.

In a pooling offer each type of principal will offer a contract $\left(y_j^i, t_j^i\right)_{i,j=1,2}$ such that it is a solution, for $i = 1, 2$, of:

$$\left(P^i\right) = \begin{cases} \max\limits_{\left\{y_j^i, t_j^i\right\}} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y_j^i\right) - t_j^i\right) \text{ such that} \\ IR_2 : \sum\limits_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0 \\ IC_1 : \sum\limits_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0 \\ ICP^1 : \sum\limits_{j=1}^{2} p_j \left(S^1\left(y_j^1\right) - t_j^1\right) \geq \sum\limits_{j=1}^{2} p_j \left(S^1\left(y_j^2\right) - t_j^2\right) \\ ICP^2 : \sum\limits_{j=1}^{2} p_j \left(S^2\left(y_j^2\right) - t_j^2\right) \geq \sum\limits_{j=1}^{2} p_j \left(S^2\left(y_j^1\right) - t_j^1\right) \end{cases}$$

A solution to this problem is then incentive compatible for the principal and the agent and will be accepted by both types of agent.[6]

We can now state the following result, which characterizes the equilibrium of the principal-agent game.

[6]Standard considerations ensure the satisfaction of the participation constraint of a type 1 agent and of the incentive compatibility constraints of a type 2 agent.

Proposition 2 *In a perfect Bayesian equilibrium of the principal agent game any type of principal will offer the same contract* $\widetilde{m} = \left(y_j^i, t_j^i\right)_{i,j=1,2}$. *The agent's beliefs are unchanged and accepts the contract. At the last stage both parties announce their type truthfully and the contract is implemented.*

Contract $\widetilde{m} = \left(y_j^i, t_j^i\right)_{i,j=1,2}$ *that satisfies the following conditions for* $i = 1, 2$*:*

1. $S^{i\prime}\left(y_1^i\right) = \theta_1$

2. $S^{i\prime}\left(y_2^i\right) = \theta_2 + \frac{p_1}{p_2}\Delta\theta\frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}$

3. $\sum_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0$

4. $\sum_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0$

5. $-\frac{U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)}{U\left(t_2^i - \theta_2 y_2^i\right)} = \frac{p_1}{p_2}\frac{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)} + \frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)}.$

Proof. First note that any contract m which is a solution to problem (P^i) is incentive compatible for the principal and agent and is individually rational for the agent. We need therefore to show that $\widetilde{m}$ is indeed a solution to (P^i).

To begin with, note that $\widetilde{m}$ is a solution to the less constrained problem (P_*^i) which is defined as:

$$\left(P_*^i\right) = \begin{cases} \max_{\left\{y_j^i, t_j^i\right\}} \sum_{j=1}^{2} p_j \left(S^i\left(y_j^i\right) - t_j^i\right) \text{ such that} & \\ IR_2 : \sum_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0 & \left(\widetilde{\rho}^i\right) \\ IC_1 : \sum_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0 & \left(\widetilde{\gamma}^i\right) \end{cases}$$

Program (P_*^i) is the same as (P^i) except that the incentive compatibility constraints for the two types of principal have been omitted.

The first order conditions for this problem are:

$$\frac{\partial L}{\partial y_1^i} = p_1 S^{i\prime}\left(y_1^i\right) + \phi_1 \widetilde{\gamma}^i \theta_1 U'\left(t_1^i - \theta_1 y_1^i\right) = 0$$
$$\frac{\partial L}{\partial y_2^i} = p_2 S^{i\prime}\left(y_2^i\right) - \phi_1 \widetilde{\rho}^i \theta_2 U'\left(t_2^i - \theta_2 y_2^i\right) + \phi_1 \widetilde{\gamma}^i \theta_1 U'\left(t_2^i - \theta_1 y_2^i\right) = 0$$
$$\frac{\partial L}{\partial t_1^i} = -p_1 - \phi_1 \widetilde{\gamma}^i U'\left(t_1^i - \theta_1 y_1^i\right) = 0$$
$$\frac{\partial L}{\partial t_2^i} = -p_2 + \phi_1 \widetilde{\rho}^i U'\left(t_2^i - \theta_2 y_2^i\right) - \phi_1 \widetilde{\gamma}^i U'\left(t_2^i - \theta_1 y_2^i\right) = 0$$
$$\frac{\partial L}{\partial \widetilde{\rho}^i} = \sum_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0$$
$$\frac{\partial L}{\partial \widetilde{\gamma}^i} = \sum_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0$$

From $\left(\frac{\partial L}{\partial y_1^i}\right)$ and $\left(\frac{\partial L}{\partial t_1^i}\right)$ we obtain the first condition which implicitly defines y_1^i, while from $\left(\frac{\partial L}{\partial y_2^i}\right)$ and $\left(\frac{\partial L}{\partial t_2^i}\right)$ we get the definition of y_2^i. Then $\left(\frac{\partial L}{\partial \widetilde{\rho}^i}\right)$ and $\left(\frac{\partial L}{\partial \widetilde{\gamma}^i}\right)$ give the agent's constrains (IR and IC). So $\widetilde{m}$ satisfies the first order conditions of problem (P_*^i).

Now $\widetilde{m}$ is also a solution to (P^i) if it is incentive compatible for both types of principal.

To show this first note that the ratio of the Lagrange multipliers is:

$$\frac{\widetilde{\rho}^i}{\widetilde{\gamma}^i} = \frac{p_1}{p_2}\frac{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)} + \frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)}.$$

This can be used to rewrite condition 5 as:

$$\widetilde{\rho}^i r^i + \widetilde{\gamma}^i c^i = 0^7$$

where $r^i = -U\left(t_2^i - \theta_2 y_2^i\right)$ and $c^i = -\left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right]$.

In addition from the agent's constrains we know that: $r^2 = -\frac{\phi_1}{\phi_2} r^1$ and $c^2 = -\frac{\phi_1}{\phi_2} c^1$, which together with the above implies that also the following must hold:

$$\widetilde{\rho}^k r^i + \widetilde{\gamma}^k c^i = 0, \text{ with } k \neq i.$$

Since in general, r^i and c^i are different from zero, this implies that in equilibrium:

$$\frac{\widetilde{\rho}^1}{\widetilde{\gamma}^1} = \frac{\widetilde{\rho}^2}{\widetilde{\gamma}^2}.$$

The above means that at the equilibrium the relative cost of satisfying the constraints is the same for the two types of principal, there are therefore no more gains to be obtained from relaxing further some constraints. Moreover $\frac{\rho}{\gamma}$ is the slope of

[7] This ensures that "the value" of the slackness (positive and negative) for each type of principal is the same. Morevoer it is equal to zero, the value of the slackness of the full-information allocation.

the value function at the new optimum, the fact that it is the same for each type of principal means that we have indeed found a Pareto optimal allocation.

Finally note that no type of principal would like to implement the other type's allocation since each of them has relaxed the constraint that was more costly to her. This indeed shows that the allocation is incentive compatible for both types of principal.

It remains to show that no type of principal can gain by deviating by offering a contract different from $\widetilde{m}$. This can be done by choosing the appropriate off-equilibrium path beliefs, that are arbitrary. As Maskin and Tirole [1990] argue these belief need to be chosen in such a way that if the principal proposes another mechanism all types of principal are no better off than with $\widetilde{m}$. Suppose that a mechanism m is offered, and suppose that the agent has out of equilibrium beliefs such that $\left(\dot{\phi}_1 = 1, \dot{\phi}_2 = 0\right)$, the a type 1 principal will receive at most the full information payoff $\overline{v}^1$. Similarly if beliefs are $\left(\dot{\phi}_1 = 1, \dot{\phi}_2 = 0\right)$ then type 2 will at most obtain $\overline{v}^2$. From continuity and because $\widetilde{m}$ is strongly Pareto optimal for the prior beliefs, then there exist intermediate beliefs for which both types do not whish to deviate and offer m. ■

We have therefore characterized the allocation that a risk neutral informed principal, regardless of her type, will offer to a risk averse agent. The main feature of this allocation is that it does not satisfy all the agent's ex-post constraints, but only the interim ones. As a consequence the risk aversion of the agent plays a role in determining the optimal downward distortion in the quantity of a low-type agent. Efficiency for the high-type is preserved. Finally, in equilibrium, the ratios of the lagrange multipliers of the two types of principal are the same. This implies that no more gains can be reaped by tightening and relaxing the constraints.

To understand more clearly what goes on in the equilibrium we need to specify which principal gains from relaxing which constraint. Let's assume that $S^1(\cdot) > S^2(\cdot)$ for every y (and that $S^{1\prime}(\cdot) > S^{2\prime}(\cdot)$). This enables us to say that the full information quantities can be ordered as follows:

$$\overline{y}_1^1 > \overline{y}_1^2 \text{ and } \overline{y}_2^1 > \overline{y}_2^2.$$

Which in turn tells us that the informational rent enjoyed by an efficient agent is greater when the principal is of type 1 (the informational rent is $\Delta\theta\overline{y}_2^i$, for every i). This means that, when the type of principal is not known, if she offers the full information contract to the agent then the expected payoff of the two types of agent would be agent would be:

$$\phi_1 U\left(\Delta\theta\overline{y}_2^1\right) + \phi_2 U\left(\Delta\theta\overline{y}_2^2\right), \text{ for a type 1 agent,}$$

and

$$\phi_1 U(0) + \phi_2 U(0), \text{ for a type 2 agent.}$$

As we have shown above the full-information allocation is not optimal for the principal. In particular, from the concavity of the agent's utility function we get:

$$\frac{U'\left(\Delta\theta\overline{y}_2^1\right)}{p_1 U'(0)} = \frac{\overline{\rho}^1}{\overline{\gamma}^1} < \frac{\overline{\rho}^2}{\overline{\gamma}^2} = \frac{U'\left(\Delta\theta\overline{y}_2^2\right)}{p_1 U'(0)},$$

meaning that a type 2 principal finds relatively more costly to satisfy the individual rationality constraint than a type 1. In the pooling offer type 2 principal is going to relax the individual rationality constraint, i.e.:

$$U\left(t_2^2 - \theta_2 y_2^2\right) < 0,$$

but because the constraints have to hold in expectation the former inequality implies the following:

$$U\left(t_2^2 - \theta_2 y_2^2\right) > 0.$$

In other words a type 1 principal is giving rent also to a low type agent.

On the other side a type 1 principal will relax the incentive compatibility constraint:

$$U\left(t_1^1 - \theta_1 y_1^1\right) < U\left(t_2^1 - \theta_1 y_2^1\right).$$

The fact that a type 1 principal is saving on informational rent implies that a type 2 principal will grant higher informational rent to a high type agent and the following must hold:

$$U\left(t_1^2 - \theta_1 y_1^2\right) > U\left(t_2^2 - \theta_1 y_2^2\right).$$

This constraints relaxing and tightening have, obviously, an effect on the marginal utility of the agent and the term which defines the downward distortion of an inefficient agent will be different for each type of principal. In particular:

$$\frac{U'\left(\left(t_2^1 - \theta_1 y_2^1\right)\right)}{U'\left(\left(t_1^1 - \theta_1 y_1^1\right)\right)} < 1 \text{ and } \frac{U'\left(\left(t_2^2 - \theta_1 y_2^2\right)\right)}{U'\left(\left(t_1^2 - \theta_1 y_1^2\right)\right)} > 1,$$

while in the full information allocation they were both equal to 1. As a consequence y_2^1 is now less distorted that in the full information allocation ($y_2^1 > \overline{y}_2^1$)and y_2^2 is more distorted ($y_2^2 < \overline{y}_2^2$), therefore widening the gap between the quantities produced under the two types of principal.

In other words the type 1 principal who is saving on informational rent, by relaxing her ex-post incentive compatibility constraint, will reduce the distortion while the type 2 principal will increase the inefficiency compared to the full-information allocation.

The quantities in question will then be ranked in the following way:

$$y_2^1 > \overline{y}_2^1 > \overline{y}_2^2 > y_2^2.$$

The above does not mean that the ex-post payoffs for an efficient agent will have more spread than in the full-information allocation. To the contrary the principal can bring them closed and save up on risk premium. In fact while it's true that $\Delta\theta y_2^1 > \Delta\theta\overline{y}_2^1$, it is also true that now $t_1^1 - \theta_1 y_1^1 < \Delta\theta y_2^1$. Similarly even if$\Delta\theta y_2^2 < \Delta\theta\overline{y}_2^2$ we now have that in equilibrium $t_1^2 - \theta_1 y_1^2 > \Delta\theta y_2^2$.

While the principal can reduce the ex-ante risk born by a type 1 agent, a type 2 agent, who in the full information allocation does not face any ex-ante risk (getting his reservation utility with any type of principal), will get a positive rent in one state of the world and negative in the other:

$$\phi_1 U\big(\underset{-}{t_2^1 - \theta_2 y_2^1}\big) + \phi_2 U\big(\underset{+}{t_2^2 - \theta_2 y_2^2}\big).$$

What we observe then is an increase in the spread of the payoffs of an inefficient agent and a reduction in the spread for an efficient agent. The principal therefore does not eliminate risk from the equilibrium allocation because she benefits from it, with a gain that exceeds the total risk premia that she pays.

What is peculiar about this allocation though is that only one type of principal reduces the allocative inefficiency, while the other increases it. This is because the total gain derived through a pooling offer by an informed principal does not come only from saving on informational rent.

The "traditional theory" results which may come to mind and that can be used as a comparison are those of ex-ante contracting between a risk-neutral (uninformed) principal and a privately informed risk-averse agent.

The theory tells us that when the agent accepts a contract before knowing his type the principal gains by relaxing the participation constraint. This way she is able to offer the wedge in ex-post payoffs needed for truth-telling with a lower expected cost (i.e. informational rent) and will consequently reduce the distortion of the quantity for the lower type agent.

When studying an informed principal problem, the one just told is only a little part of the story. An informed principal behaves in equilibrium in a way that is not present at all in traditional incentive theory.

5 Concluding remarks.

We have shown that in a model with a risk neutral informed principal and a risk averse agent the principal gains by making a not revealing contract offer that will keep the agent uninformed about her type until the implementation stage.

The optimal contract will result in an equilibrium allocation that involves a larger spread in the ex-post payoffs of the inefficient agent, forcing him to take up more risk when he accepts the contract. In addition the principal will not eliminate completely ex-ante risk for an efficient agent. Moreover only one type of principal reduces the downward distortion in quantities, while the other increases.

We believe that this highlights the interest of informed principal problem as some of the usual features of well-known one sided screening models no longer hold. In some circumstances therefore the usual assumption of an uninformed principal is not without loss of generality.

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