DEPARTMENT OF ECONOMICS
OxCarre
Oxford Centre for the Analysis of Resource Rich Economies

Manor Road Building, Manor Road, Oxford OX1 3UQ
Tel: +44(0)1865 281281 Fax: +44(0)1865 271094
oxcarre@economics.ox.ac.uk www.oxcarre.ox.ac.uk

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International Capital Markets, Oil Producers and the Green Paradox

Gerard van der Meijden
VU University
Frederick van der Ploeg
OxCarre
&
Cees Withagen*
VU University

***OxCarre External Research Associate**

Direct tel: +44(0) 1865 281281 E-mail: oxcarre@economics.ox.ac.uk

International Capital Markets, Oil Producers and the Green Paradox

Gerard van der Meijden*, Frederick van der Ploeg** and Cees Withagen***

Abstract

A rapidly rising carbon tax leads to faster extraction of fossil fuels and accelerates global warming. We analyze how general equilibrium effects operating through the international capital market affect this Green Paradox. In a two-region, two-period world with identical homothetic preferences and without investment, the global interest rate falls and the Green Paradox weakens. With investment or a relatively more impatient oil-importing region, the Green Paradox may be strengthened because the future oil demand function shifts downward or because the interest rate rises. If the oil-importing region is very much more patient than the oil-exporting region, the Green Paradox may be reversed but in our calibrated model the effects are tiny. With exploration and endogenous initial oil reserves, a future carbon tax lowers cumulative oil extraction in partial equilibrium. If the boost to current oil extraction is weakened, strengthened or reversed in general equilibrium, so is the fall in cumulative extraction. A partial and general equilibrium welfare analysis of a future carbon tax, both for full and partial exhaustion, is given. The effects of stock-dependent extraction costs are separately discussed in an appendix.

Keywords: Global warming, Green Paradox, Hotelling rule, oil importers, oil producers, investment, capital markets, carbon tax, exploration investment, general equilibrium

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* Corresponding author: Department of Economics, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. E-mail: g.c.vander.meijden@vu.nl. Phone: +31-20-598-2840.

** Oxford Centre for the Analysis of Resource Rich Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK and VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

*** Department of Economics, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

1. Introduction

The idea that a rapidly rising carbon tax or a subsidy for renewable energy encourages oil producers to extract oil more quickly[1] and accelerates global warming has gained traction and is known as the Green Paradox (cf. Sinn, 2008, 2012; Gerlagh, 2011). The underlying mechanism is that a future carbon tax forces oil producers to supply less oil in the future due to lower future demand, which implies that current oil supply goes up. This pushes the current oil price down and therefore boosts today's demand for oil. This is true when a given stock of oil is fully exhausted, but the effect is also present when stock-dependent extraction costs lead to partial exhaustion of reserves albeit that more of reserves are left abandoned.

It is well understood under which conditions the Green Paradox manifests itself in partial equilibrium settings, where the level of investment and the interest rate are taken as given (cf. Sinn, 2008, 2012; Gerlagh, 2011).[2] However, front-loading of oil extraction also influences the global supply of savings and the demand for capital, so that the interest rate must adjust to clear bonds and capital markets. Although integrated assessment models of climate change like DICE and RICE typically do allow for endogenous changes in investment and the interest rate (cf. Nordhaus, 1992; Nordhaus and Zhang, 1996; van der Ploeg and Withagen, 2014; Golosov et al., 2014; Rezai and van der Ploeg, 2014), they do not provide a decomposition analysis to assess the importance of effects operating through the international capital market for the Green Paradox.

Intuition suggests that a rapidly rising carbon tax increases current output relative to future output, which boosts global savings and depresses the global interest rate. The lower interest rate flattens the equilibrium oil price path and induces less current oil demand relative to future oil demand, which leads to less current extraction relative to future extraction. However, in general equilibrium changes in the wealth positions of oil importers and oil exporters due to climate policy will affect the interest rate as well. Moreover, changes in investment will impinge on future oil demand. Our objective is to gain a better understanding of these general equilibrium effects and to show that the benchmark general equilibrium result of mitigation of the Green Paradox is not robust when we allow for investment or when we allow for asymmetric preferences between oil importers and oil exporters. It is then possible that the Green Paradox is reinforced or reversed in general equilibrium.

[1] We refer throughout to 'oil' as short-hand for 'exhaustible fossil fuel resources' such as oil, natural gas and coal.

[2] Studying climate policy in a partial equilibrium framework gives rise to an inconsistency. On the one hand, single-country models with a given interest rate such as Sinn (2008) should be interpreted as models of the entire world economy, because climate change is a global problem. On the other hand, the justification for assuming the interest as given is that we deal with an open economy that is so small compared to the rest of the world that we can neglect its influence on the interest rate. Therefore, it seems a serious shortcoming to abstract from general equilibrium effects of climate policy. We thank an anonymous referee for pointing this out.

We use a two-country, two-period model of oil importers and oil producers, where the interest rate and current and future real oil prices are determined from the conditions for clearing the markets for financial assets and oil, and the Hotelling rule governs optimal oil extraction (cf. Dixit, 1981; Marion and Svensson, 1984; van Wijnbergen, 1985; Djajić, 1988). We suppose that all markets operate under perfect competition and that only the oil-importing countries produce final goods which can be used for consumption and investment. We analyze the effects of changes in future carbon taxes on current oil extraction and the interest rate. In addition, we are interested in the amount of oil that is left unexploited in the crust of the earth as this affects the ultimate degree of global warming. We focus on changes in future taxes instead of optimal climate policy (typically requiring immediate action), because in reality gradually greening policies are observed and strong current action is lacking.

In general equilibrium the interest rate plays a role on both the production and consumption side of the economy. Demand for oil depends on oil prices, which are intertemporally related via the Hotelling rule and feature the interest rate as the opportunity cost of conserving oil. Furthermore, the level of investment depends on the interest rate, because it determines the marginal cost of renting a unit of capital. On the demand side the interest rate determines the relative price of current and future consumption. Moreover, each region's wealth is affected by the interest rate. The interplay between these aspects drives the direction of the change in the interest rate induced by climate policy. The importance of investment for the Green Paradox arises from the imperfect substitutability of capital and oil, which implies that changes in investment shift the future oil demand function. The effects on the interest rate and investment together determine the general equilibrium repercussions for the Green Paradox.

Starting with the case of full exhaustion without extraction and exploration cost we demonstrate that in general equilibrium the Green Paradox induced by a future carbon tax is mitigated if oil importers and oil exporters have identical homothetic preferences over current and future consumption and there are no investment possibilities. Although our results suggest that mitigation is still the most likely outcome under more general conditions with investment possibilities and asymmetric preferences between regions, we are also able to construct cases under which strengthening or reversal of the Green Paradox occurs. In particular, we show that a reversal can occur if oil producers are very much more impatient than oil importers, but in our calibrated model the effects are tiny. Strengthening of the Green Paradox can occur if oil producers are relatively patient or if there is investment in physical capital. When investment in physical capital is possible, it is less likely that the Green Paradox will be reversed. Accounting for partial exhaustion of the stock of oil reserves by imposing exploration costs, we show that in partial equilibrium a future carbon tax ensures that more oil will be locked in the crust of the earth. We demonstrate that general equilibrium effects have similar repercussions for cumulative extraction as they have for current extraction. Our welfare analysis shows that 'green welfare' goes down in the calibrated model with full exhaustion upon an increase in the future carbon tax, but goes up in the specification with partial exhaustion. Furthermore, the welfare analysis suggests that the difference in green welfare effects between the partial and general equilibrium effects are small.

Besides general equilibrium effects, there are at least five other factors driving in the direction of a mitigated Green Paradox that we obtain in our case with identical homothetic preferences without investment. First, Hart and Spiro (2011, p. 7834) report that "scarcity rents seem to have been marginal or non-existent empirically", so that Green Paradox effects will not be large. Potential explanations for this observation are a finite planning horizon of resource owners (Spiro, 2014) and endogenous field openings (Venables, 2015). Moreover, the emerging abundance of shale gas and other forms of unconventional fossil energy reserves might curb existing Hotelling rents even further. Second, a heavily polluting backstop alongside oil and clean renewable resources tends to mitigate the Green Paradox (van der Ploeg and Withagen, 2012b; Michielsen, 2014). Third, if extraction costs of fossil fuel increase with subsoil reserves, climate policy potentially decreases cumulative extraction and cumulative carbon emissions, the beneficial welfare effects of which mitigate the adverse welfare effects of Green Paradox (e.g., Hoel, 2012; van der Ploeg and Withagen, 2012a; van der Ploeg, 2014). Fourth, learning by doing in the renewables sector can mitigate the Green Paradox (Nachtigall and Rübbelke, 2014). Finally, the Green Paradox that occurs after subsidizing renewables might be mitigated if fossil and renewable energy are used simultaneously due to increasing marginal production costs of renewables (cf. van der Ploeg and Withagen, 2012a; Grafton et al., 2013) or imperfect substitution between fossil and renewable resources (Michielsen, 2014).

The model that we use in our analysis is closely related to the models in Eichner and Pethig (2011) and Ritter and Schopf (2014). Eichner and Pethig (2011) study unilateral climate policies in a two-period, three-country model with a finite endowment of oil. Next to the oil-exporting region and the oil-importing region that imposes or strengthens climate policies, they also take into account a second, 'non-abating' oil-importing region. In this setting, they explore conditions under which carbon leakage and the Green Paradox occur. One particular result of Eichner and Pethig (2011) that is relevant for our analysis is their finding that upon an increase in the future carbon tax in the abating region, the Green Paradox will be attenuated and may be reversed even with identical homothetic preferences.[3] There are two reasons for this result. First, the interest rate falls which lowers the return on extracting a marginal barrel of oil and

[3] Eichner and Pethig (2011) actually use emissions caps instead of carbon taxes. In their model, however, these two climate policy instruments yield equivalent outcomes.

thus induces the oil-producers to slow down current extraction.[4] Second, part of the reduced future emissions in the abating country leaks away to the non-abating country in the future, instead of to the present. Reversal of the Green Paradox requires a low intertemporal elasticity of substitution (so that the fall in the interest rate is large enough), and a high price elasticity of oil demand in the non-abating region in the second period (so that enough emissions leak away to the future non-abating region instead of to the present). See Propositions 1 and 6-7 in Eichner and Pethig (2011). Our study is complementary to Eichner and Pethig (2011). Whereas they show that the Green Paradox is attenuated or may even be reversed in general equilibrium with identical homothetic preferences if a third 'non-abating' region is taken into account, we show that in a world without a non-abating country, the Green Paradox may be amplified due to investment in physical capital or reversed due to asymmetric preferences between the two regions.[5] Ritter and Schopf (2014) extend the model of Eichner and Pethig with climate damages and stock-dependent extraction costs. They derive conditions under which a strong Green Paradox (i.e., an increase in the present value of climate damages, cf. Gerlagh, 2011) or even an increase in cumulative extraction occurs upon an increase of the future carbon tax in the abating country. In the current paper, we allow for changes in cumulative exhaustion by introducing exploration costs.[6]

Other related studies are Eichner and Pethig (2013), Long and Stähler (2013), and Sen (2015). Eichner and Pethig (2013) compare cooperative and non-cooperative cost-effective climate policies in a two-period, two-country general equilibrium model. They show that the cooperative outcome is characterized by a uniform carbon tax in the first period only, whereas a cost-effective non-cooperative policy requires emission regulation in both periods. Long and Stähler (2013) consider the general equilibrium effects of green technological progress on the speed of oil extraction, and note that this may lead to a rise in the interest rate thereby offsetting and possibly reversing the initial increase in the rate of oil extraction. Sen (2015) investigates the implications of unilateral climate policy in a two-period, North-South model in which a 'clean' and a 'dirty' intermediate good are produced, which are traded internationally. He finds that if Hotelling rents are positive and the interest rate is exogenously fixed, a unilateral cut of future emissions by the North expands the dirty industry in the South in both periods. Current emissions increase, implying that the Green Paradox occurs.

The structure of the paper is as follows. Section 2 sets up the general two-country model without extraction and exploration costs. Section 3 derives the key partial equilibrium Green Paradox result.

[4] Because Eichner and Pethig (2011) use the consumption good in period 1 as the numéraire, an increase in the price of the second-period good corresponds to a fall in the interest rate.

[5] The variant of our model specification with identical homothetic preferences and without physical capital is a special case of the model of Eichner and Pethig (2011), without a non-abating country. Appendix A4 discusses the inclusion of a non-abating country in our framework.

[6] We discuss stock-dependent extraction costs in Appendix A2.

Section 4 shows that in general equilibrium Green Paradox effects are weakened if there is no investment and if preferences are identical and homothetic, and demonstrates that amplification or reversal of the Green Paradox can occur if there is investment and or preferences differ very much between oil importers and exporters. Section 5 deals with exploration costs and partial exhaustion. Section 6 performs a welfare analysis. Section 7 concludes. We have included three appendices that contain technical details, as well as discussions about the effects of a non-abating country and stock-dependent extraction costs.

2. A two-country, general equilibrium model

We consider a two-period model with two blocks of countries (or regions), one block of homogeneous countries being the oil importers and the other block the oil exporters (denoted by an asterisk). Oil importers produce final goods, which are demanded by both blocks. They use capital and oil (as well as fixed factors such as land and labour) to produce final goods. Their assets consist of capital and bonds. We assume that oil extraction is costless, and discuss exploration costs in Section 5 and stock-dependent extraction costs in Appendix A2. Oil exporters have given initial oil reserves, which will be fully exhausted. Their other assets are capital and bonds. All markets operate under perfect competition and clear in each period. Oil producers have rational foresight and oil prices obey the Hotelling rule. The government of the oil-importing block might levy a specific carbon tax on the use of oil by final goods producers. Tax revenues are distributed in lump-sum fashion to households in the oil-importing region. Our aim is to investigate the effects of the carbon tax on the real price of oil, the world interest rate, investment, and the intertemporal pattern of oil depletion.

Firms

Output of final goods is given by $F(K_t, R_t)$, where K_t denotes employed capital and R_t the oil extraction rate in period t ($t = 1, 2$). Taking account of other fixed factors such as land and labour, this production function has strictly decreasing returns to scale and is strictly increasing for positive inputs and strictly concave in each input. With the net rate of return on capital indicated by r_t, the world market oil price by q_t, the specific carbon tax levied on the producers of final goods by τ_t, and the constant rate of depreciation by μ, profits by firms in the oil-importing region in each period are $\Pi_t \equiv F(K_t, R_t) - (r_t + \mu)K_t - (q_t + \tau_t)R_t$. Profit maximization under perfect competition gives:

(1) $$F_K(K_1, R_1) = r_1 + \mu,$$

(2) $$F_R(K_1, R_1) = q_1 + \tau_1,$$

(3) $$F_K(K_2, R_2) = r_2 + \mu,$$

(4) $F_R(K_2, R_2) = q_2 + \tau_2$.

This yields conditional oil demand $R_1(r_1, q_1 + \tau_1)$, capital demand $K_1(r_1, q_1 + \tau_1)$, and the profit function $\Pi_1(r_1, q_1 + \tau_1)$. For period two this yields the factor demands $K_2(r_2, q_2 + \tau_2)$ and $R_2(r_2, q_2 + \tau_2)$, which gives the profit function $\Pi_2(r_2, q_2 + \tau_2)$. Factor demands decrease in own factor prices. If capital and oil are cooperant factors, $F_{KR} > 0$ which we assume, factor demands decrease in the price of the other factor too.

Profit maximization by oil exporters, facing the real interest rate r_2, yields the Hotelling rule:

(5) $q_2 = (1 + r_2)q_1$.

Hence, the return on taking a marginal barrel of oil out of the earth, $r_2 q_1$, must equal the return on keeping a marginal barrel of oil in the earth, i.e., the expected capital gains, $q_2 - q_1$.

Households

Households in the oil-importing region derive utility from present and future consumption and disutility from carbon dioxide in the atmosphere. Preferences of the representative household in the oil-importing region can be represented by:

$$\Lambda(C_1, C_2, E_1, E_2) = U(C_1, C_2) - D(E_1, E_2),$$

where C_t and E_t denote consumption in the oil-importing region and the concentration of carbon dioxide in the atmosphere, respectively, in period t. Carbon emissions are proportional to oil use. By appropriate choice of units we get $E_1 = E_0 + R_1$ and $E_2 = E_0 + R_1 + R_2$.[7] We assume that the utility function $U(C_1, C_2)$ is continuously differentiable, increasing and strictly concave. The atmospheric carbon stock causes damages $D(E_1, E_2)$. Damages are increasing in the atmospheric carbon stock. Households ignore the consequences of their consumption decisions on carbon emissions. The representative household in the oil-exporting region is not affected by climate change or, equivalently, oil exporters do not conduct climate policy. Its preferences are represented by the continuously differentiable, increasing, strictly concave utility function $U^*(C_1^*, C_2^*)$, where C_t^* denotes consumption in period t. The budget restrictions for both regions read:

$$C_1 + A_2 = (1 + r_1)A_1 + \Pi_1 + \tau_1 R_1,$$

$$C_2 = (1 + r_2)A_2 + \Pi_2 + \tau_2 R_2,$$

[7] We abstract from carbon depreciation. For a more detailed modeling of the carbon cycle, see Golosov et al. (2014).

$$C_1^* + A_2^* = (1+r_1)A_1^* + q_1 R_1 ,$$

$$C_2^* = (1+r_2)A_2^* + q_2 R_2 ,$$

where A_t and A_t^* denote asset holdings at the start of period t. The initial asset endowments A_1 and A_1^* are given. It follows that the present-value budget constraints for both regions are

(6) $$C_1 + \frac{C_2}{1+r_2} = (1+r_1)A_1 + \Pi_1 + \tau_1 R_1 + \frac{\Pi_2 + \tau_2 R_2}{1+r_2} \equiv M ,$$

(7) $$C_1^* + \frac{C_2^*}{1+r_2} = (1+r_1)A_1^* + q_1 R_1 + \frac{q_2 R_2}{1+r_2} \equiv M^* ,$$

where M and M^* denote wealth of the oil-importing and oil-exporting region, respectively. Wealth of the oil-importing region is the sum of the net return on assets and the present discounted value of profits and carbon tax refunds. Wealth of the oil-exporting block consists of the return on assets and the present discounted value of oil revenues. Ideally, the carbon taxes in the oil-importing region are optimal (from the perspective of this region). However, in the present paper we consider carbon taxes as exogenous. In Section 6 we perform a welfare analysis taking damages into account.

Equilibrium conditions

Equilibrium on the asset markets requires that capital must be held by one of the two regions:

(8) $$K_1 = A_1 + A_1^*,$$

(9) $$K_2 = A_2 + A_2^*.$$

The initial oil stock is S_1, so that oil market equilibrium (OME) requires

(10) $$S_1 = R_1 + R_2.$$

The goods market equilibrium conditions (GME) for periods one and two are:

(11) $$F(K_1, R_1) + (1-\mu)K_1 = C_1 + C_1^* + K_2 ,$$

(12) $$F(K_2, R_2) + (1-\mu)K_2 = C_2 + C_2^*.$$

This completes the description of the model with full exhaustion of oil reserves, which we analyze in Sections 3-4. In Section 5 we extend the model to allow for partial exhaustion by introducing exploration costs so that the initial oil stock becomes endogenous.

3. Benchmark case: Partial equilibrium

In order to examine the direction of general equilibrium effects on the Green Paradox in Sections 4-6, we first describe the partial equilibrium setting as a benchmark case (cf. Sinn, 2008, 2012; Gerlagh, 2011). Therefore, in this section we study the resource market in isolation thus taking the interest rate r_2 and the capital stock K_2 as fixed. Consider an increase in the future carbon tax τ_2, keeping τ_1 constant. To see that current oil extraction increases, substitute oil demand from (2), (4), and the Hotelling rule (5) in condition (10) for OME:

$$(13) \qquad R_1(q_1+\tau_1)+R_2\left((1+r_2)q_1+\tau_2\right)=S_1,$$

where we use that K_1 and K_2 are given and oil demand R_1 and R_2 are functions of the tax-inclusive oil price only.[8] If the future carbon tax goes up, equilibrium on the oil market does not allow for a higher current world market oil price q_1. From (13) it is immediately apparent that this would reduce oil demand in both periods so that there would be excess supply. Hence, a future carbon tax increases the future cost of oil and curbs future oil use, but depresses the current oil price and thus boosts current oil demand and current carbon emissions. Hence, a future carbon tax induces oil suppliers to supply less oil in the future and thus to supply more today. This is the essence of the Green Paradox for the partial equilibrium context with full exhaustion of reserves.

Next we change carbon taxes in proportion, i.e., $\tau_2=(1+\psi)\tau_1$. The second period resource price is then given by $(1+r_2)q_1+(1+\psi)\tau_1=(1+r_2)[q_1+(1+\psi)/(1+r_2)\tau_1]$, implying that we can rewrite the OME condition (13) as

$$(14) \qquad R_1(q_1+\tau_1)+R_2\left((1+r_2)\left[q_1+\tau_1+\frac{\psi-r_2}{1+r_2}\tau_1\right]\right)=S_1.$$

So, if the growth rate of the carbon tax, ψ, equals the return on capital for the oil-exporting region, r_2, an increase in the first-period carbon tax does not affect $q_1+\tau_1$ or the intertemporal pattern of oil extraction rates. If the growth rate of the carbon tax is bigger than r_2, the future cost of oil rises while the current cost of oil falls so that there is a Green Paradox. However, if the growth rate of the carbon tax is lower than r_2, we deduce immediately from (14) that the future cost of oil falls so that more oil is extracted in the future and less today (no Green Paradox). We summarize the results so far as follows.

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[8] The reason is that $K_1(r_1,q_1+\tau_1)=A_1+A_1^*$ so r_1 is a function of $q_1+\tau_1$ and thus R_1 is a function of $q_1+\tau_1$ only.

Proposition 1 (Green Paradox in partial equilibrium): *With a given interest rate and a fixed resource demand function a higher future carbon tax accelerates oil extraction and global warming whilst a carbon tax that rises faster (slower) than the interest rate accelerates (decelerates) oil extraction and global warming.*

We now extend our analysis towards general equilibrium and focus on a higher future carbon tax (setting the current carbon tax to zero), which captures a carbon tax that rises faster than the interest rate.

4. General equilibrium

In general equilibrium, both the interest rate and the resource demand function are affected by climate policy. As a result, we show that – compared to our benchmark case – the Green Paradox can be attenuated, reversed or amplified.

Definition 1: *Assume the future carbon tax is increased***.**

(i) 'Attenuation' occurs if current oil extraction expands by less than in partial equilibrium.
(ii) 'Reversal' occurs if current oil extraction decreases.
(iii) 'Amplification' occurs if current oil extraction expands by more than in partial equilibrium.

In Section 4.1 we present the case where there is no investment and regions have identical, homothetic preferences. In Sections 4.2-4.4 we use CES utility functions,[9] given by

$$(15) \qquad U(C_1,C_2)=\begin{cases}\dfrac{C_1^{1-\eta}-1}{1-\eta}+\dfrac{1}{1+\rho}\dfrac{C_2^{1-\eta}-1}{1-\eta} & if \quad \eta\neq 1\\[2ex] \ln C_1+\dfrac{1}{1+\rho}\ln C_2 & if \quad \eta=1\end{cases},$$

$$(16) \qquad U^*(C_1^*,C_2^*)=\begin{cases}\dfrac{C_1^{1-\eta^*}-1}{1-\eta^*}+\dfrac{1}{1+\rho^*}\dfrac{C_2^{1-\eta^*}-1}{1-\eta^*} & if \quad \eta^*\neq 1\\[2ex] \ln C_1^*+\dfrac{1}{1+\rho^*}\ln C_2^* & if \quad \eta^*=1\end{cases},$$

where $\eta>0$ and $\eta^*>0$ denote the elasticities of marginal utility (the inverses of the intertemporal elasticities of substitution), and $\rho\geq 0$ and $\rho^*\geq 0$ are the rates of pure time preference in both regions. In Sections 4.2-4.4, we also use the following CES production function

[9] Bergson's theorem states that preferences are time separable and homothetic if and only if they are of the CES type in (15)-(16).

$$(17)\qquad F(R,K)=\left[\beta R^{\frac{\sigma-1}{\sigma}}+\lambda K^{\frac{\sigma-1}{\sigma}}+(1-\beta-\lambda)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with L an exogenously given other input, and σ the elasticity of factor substitution. In Section 4.2 there is no investment, so $\lambda=0$, and the instantaneous utility functions are identical (logarithmic) across regions, that may, however differ in pure rates of time preference. Hence, $\eta=\eta^{*}=1$ and $\rho\neq\rho^{*}$. Section 4.3 introduces capital accumulation in a model with identical (logarithmic) preferences. Hence, $\lambda>0$, $\eta=\eta^{*}=1$ and $\rho=\rho^{*}$. Finally, Section 4.4 goes into the case of capital accumulation, $\lambda>0$ again, and allows for identical instantaneous utilities $\eta=\eta^{*}$ potentially different from unity, and different rates of pure time preference, $\rho\neq\rho^{*}$.

4.1. Identical homothetic preferences, no capital

Without capital as factor of production the oil market equilibrium (OME) condition reads:

$$(18)\qquad R_1(q_1)+R_2\left((1+r_2)q_1+\tau_2\right)=S_1.$$

For a given τ_2, equation (18) gives a negative relationship between r_2 and q_1 and, for given r_2, it gives a negative relationship between τ_2 and q_1. The reason is that a higher interest rate or future carbon tax curbs future oil demand, thus requiring a fall in the current oil price to clear the oil market. Hence, as shown in Figure 1 below, equation (18) corresponds to the downward-sloping OME locus in (r_2,q_1)-space which shifts inwards as the future carbon tax is increased.

To characterize the goods market equilibrium (GME), we assume without loss of generality zero initial asset endowments. Then the present-value budget constraints (6) and (7) become

$$(19)\qquad \begin{aligned} C_1+\frac{C_2}{1+r_2}&=F(R_1(q_1))-q_1R_1(q_1)+\frac{F(R_2((1+r_2)q_1+\tau_2))-(1+r_2)q_1R_2((1+r_2)q_1+\tau_2))}{1+r_2}\\ &\equiv M(q_1,r_2;\tau_2), \end{aligned}$$

$$(20)\qquad C_1^{*}+\frac{C_2^{*}}{1+r_2}=q_1R_1+\frac{q_2R_2}{1+r_2}=q_1S_1\equiv M^{*}(q_1),$$

where profit maximization, the Hotelling rule (5), and (18) have been used. We have explicitly indicated that oil importers' wealth depends on the carbon tax. Let the tax rate be given. For any triplet of prices (q_1,q_2,r_2) such that the Hotelling rule is satisfied, the oil-exporting region derives demand for the final good as functions of this triplet. The Hotelling rule makes sure that (perceived) discounted income of this region equals $q_1 S_1$. For this set of prices (and the given tax rate) the oil-importing region determines profit-maximizing demand for oil, and thereby discounted total income M. Then follows demand for final goods for the two periods. So, we can write $C_t(r_2, M(r_2, q_1; \tau_2))$ and $C_t^*(r_2, M^*(q_1)),\ t = 1, 2$. Using (11)-(12), the GME locus, for which there is equilibrium on the final goods market, is defined by

$$(21) \qquad \frac{C_2(r_2, M(r_2, q_1; \tau_2)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2, q_1; \tau_2)) + C_1^*(r_2, M^*(q_1))} = \frac{F(R_2((1+r_2)q_1 + \tau_2))}{F(R_1(q_1))}.$$

The left-hand-side gives the aggregate demand for future goods relative to the aggregate demand for current goods. In equilibrium, this should equal the production of future goods relative to current goods on the right-hand-side. The oil demand functions on the right hand side follow from (2) and (4), while abstracting from physical capital. A general equilibrium exists where the GME and OME locus intersect (due to Walras' Law).[10] In general, little is known about the shape of GME and the shift that occurs in GME following a change in the tax. A higher carbon tax will affect the current oil price which will lead to a reallocation of wealth, as can be seen from (19) and (20), which in turn affects relative aggregate final goods demand.

By substituting $S_1 - R_1(q_1)$ for $R_2((1+r_2)q_1 + \tau_2)$ in (21), we obtain a 'modified GME' or MGME locus:

$$(22) \qquad \frac{C_2(r_2, M(r_2, q_1)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2, q_1)) + C_1^*(r_2, M^*(q_1))} = \frac{F\left(S_1 - R_1(q_1)\right)}{F(R_1(q_1))}.$$

The advantage of working with the MGME-locus instead of with the GME-locus, is that the former does not feature the future carbon tax τ_2 and therefore does not shift upon a change in the future carbon tax. To determine the general equilibrium effect of a higher carbon tax, we still need to establish whether the MGME locus is upward-sloping or downward-sloping in (r_2, q_1)-space.

Figure 1 shows the MGME and OME loci in (r_2, q_1)-space, for three different cases, in the neighbourhood of the initial general equilibrium. Because the OME line shifts down upon an increase in the future carbon tax whereas the MGME line remains unaffected, the comparative statics results depend on the relative slopes of the MGME and OME loci with a crucial role played by the interest rate. In partial equilibrium the interest rate is exogenous and the equilibrium oil price jumps down from point E to the

[10] In total, 9 equilibrium conditions are incorporated in (18) and (21): (2), (4), (10), (11), (12), $C_1 = C_1(r_2, M(r_2, q_1; \tau_2))$, $C_2 = C_2(r_2, M(r_2, q_1; \tau_2))$, $C_1^* = C_1^*(r_2, M^*(q_1))$, and $C_2^* = C_2^*(r_2, M^*(q_1))$, to find 8 unknowns: C_1, C_2, C_1^*, C_2^*, R_1, R_2, q_1, and r_2. Instead of using Walras' law to disregard one equilibrium condition (as we do in Online Appendix A4), we have used the ratio of the two equilibrium conditions (9)-(10). If the ratio holds, (9) and (10) will automatically be satisfied if the other 7 equilibrium conditions are satisfied.

level corresponding with point PE (partial equilibrium) in each panel, which implies shifting oil extraction from the future to the present.[11] In general equilibrium, however, the interest rate adjusts to the level corresponding with point GE (general equilibrium) to clear the asset market.

Attenuation and reversal of the Green Paradox feature a fall in the interest rate. This induces oil exporters to pump more slowly, so that the partial equilibrium Green Paradox is attenuated (panel (a)) or reversed (panel (b)). Amplification of the Green Paradox (panel (c)) is associated with a higher interest rate, which further boosts current oil extraction.

Which of the cases prevails, depends on preferences, and in particular on the wealth effects of a future carbon tax. With homothetic preferences, relative aggregate consumption of future and current final goods in each country is independent of wealth. It follows from the optimality conditions for households in each region that demand for future relative to current goods depends solely on the interest rate, i.e. $C_2 / C_1 = \Phi(r_2)$ and $C_2^* / C_1^* = \Phi^*(r_2)$, with $\Phi' > 0$ and $\Phi^{*}{}' > 0$. Hence, with identical preferences the MGME locus reduces to

$$\Phi(r_2) = \frac{F\left(S_1 - R_1(q_1)\right)}{F(R_1(q_1))}.$$

We thus get an upward-sloping MGME locus, which implies weakening of the Green Paradox relative to the partial equilibrium outcome.

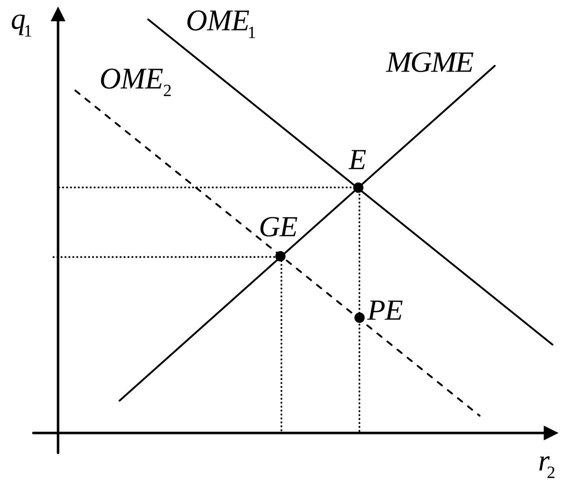
Proposition 2 (Attenuation of the Green Paradox): *Suppose the two regions have identical and homothetic preferences, that there is no investment, and that the future carbon tax is increased. Then, in general equilibrium acceleration of oil extraction occurs but is always less than in partial equilibrium.*

The intuition is as follows. The partial equilibrium effect of a future carbon tax lowers q_1 so that current oil extraction and current production of final goods increase and future oil extraction and final goods production decrease. Because of identical homothetic preferences, wealth effects do not affect the aggregate relative demand for future goods. As a result, an excess demand for future goods materializes at the prevailing interest rate. Hence, the interest rate must decrease to reduce relative demand for future goods, thus eliminating excess demand. The lower interest rate encourages oil-exporters to extract oil less quickly, thus attenuating the Green Paradox.

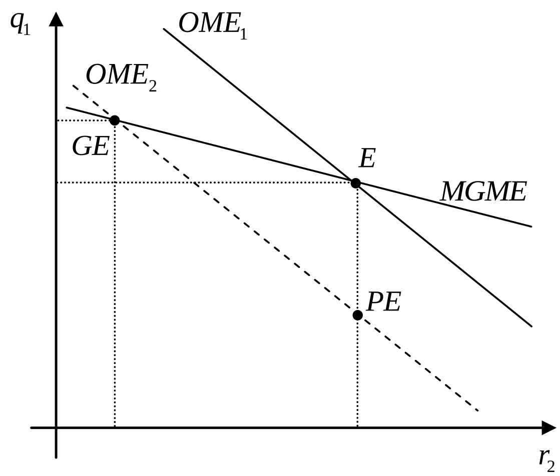
[11] We assume that the economy is in a general equilibrium initially and that the initial interest rate is known (observed) by agents and policy makers.

Figure 1: Three different general equilibrium scenarios

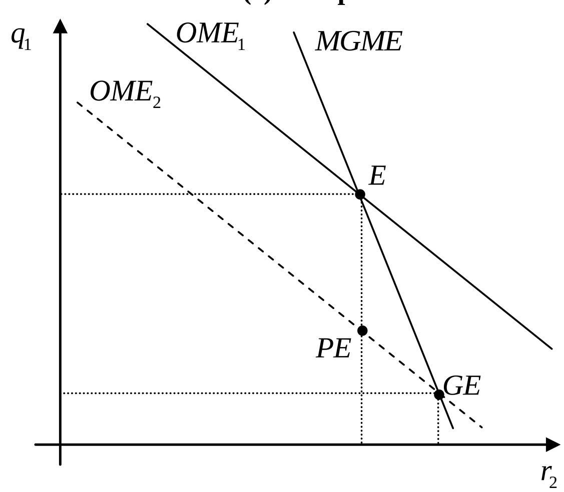
Panel (a) - Attenuation



Panel (b) - Reversal



Panel (c) - Amplification



Notes: The intersection of the solid lines (point E) gives the initial equilibrium. The intersection at point GE gives the equilibrium after the increase in the future carbon tax. The movement from point E to point PE gives the partial equilibrium effect of an increase in the future carbon tax.

4.2. Asymmetric preferences

To see whether Proposition 2 also holds more generally, we allow for different, but homothetic, preferences across regions. The aggregate relative consumption of future and current goods is then

$$\frac{C_2 + C_2^*}{C_1 + C_1^*} = \Phi(r_2) + \left[\Phi^*(r_2) - \Phi(r_2)\right]\frac{C_1^*}{C_1 + C_1^*}\,. \tag{23}$$

The extra term in square brackets in (23) captures a *wealth reallocation effect*: Even with homothetic preferences, a reallocation of wealth between countries affects aggregate relative demand for future goods at an unchanged interest rate. If relative current consumption of the region with the highest equilibrium future-to-current consumption ratio increases, then aggregate future-to-current consumption increases and *vice versa*.

In this section we restrict ourselves to the case of logarithmic utility, $\eta = \eta^* = 1$, in which case the income and substitution effects of changes in the interest rate cancel out, resulting in the fixed expenditure shares

$$\begin{aligned} &C_1 = \alpha M, \\ &\frac{C_2}{1+r_2} = (1-\alpha)M, \\ &C_1^* = \alpha^* M^*, \\ &\frac{C_2^*}{1+r_2} = (1-\alpha^*)M^*. \end{aligned} \tag{24}$$

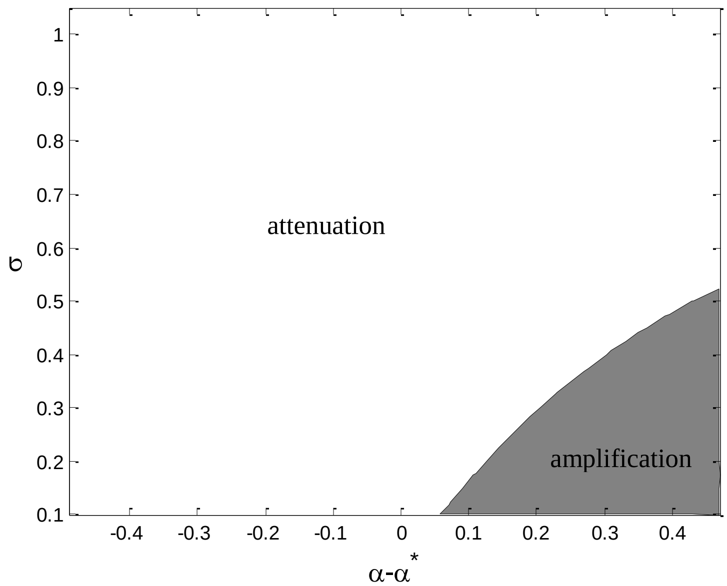
where $\alpha \equiv (1+\rho)/(2+\rho)$ and $\alpha^* \equiv (1+\rho^*)/(2+\rho^*)$. Note that $\alpha, \alpha^* \in [0.5,1)$ due to non-negative discounting. The case of non-unitary elasticities of marginal utility is explored in Section 4.4.

Proposition 3 (Logarithmic utility and general equilibrium): *Suppose* $\eta = \eta^* = 1$, *and that there is no investment. With an increase in the future carbon tax, the Green Paradox is attenuated if oil importers are more patient than oil exporters* $(\rho \le \rho^*)$, *and may be amplified if oil importers are more impatient than oil exporters* $(\rho > \rho^*)$. *Reversal of the Green Paradox cannot occur.*

Proof: See Appendix A1. □

If the oil-importing region is more patient or not too much more impatient than the oil-exporting region, Green Paradox effects are attenuated. However, Figure 2 shows that if the oil-importing region is much more impatient than the oil-exporting region, the Green Paradox may be amplified.[12] In the figure we have $\beta = 0.1$, $\lambda = 0$, $\eta = \eta^* = 1$, $L = 1$, and $S_1 = 2$. The carbon tax τ_2 is increased from zero to 0.01.

Figure 2: Attenuation versus amplification of the Green Paradox



Notes: Parameters are set to $\beta = 0.1$, $\lambda = 0$, $\eta = \eta^* = 1$, $L = 1$, and $S_1 = 2$. The carbon tax τ_2 is increased from 0 to 0.01. The horizontal axis features a mean-preserving spread with $\alpha = 1/2 + x$, $\alpha^* = 1 - x$, $x \in (0,1/2)$; the shaded area indicates amplification of the Green Paradox.

The intuition for amplification of the Green Paradox is as follows. At a given interest rate, the future carbon tax induces excess supply of current goods (as in Proposition 1), but also increases the relative wealth of the oil-importing region due to the decline in the oil price. If the oil-importing region has a relatively high current-to-future consumption rate (which is the case if $\rho > \rho^*$), the wealth reallocation effect positively affects aggregate current-to-future consumption, which diminishes the excess supply of current final goods that is caused by the partial equilibrium Green Paradox effect. If strong enough, this wealth reallocation effect can even lead to excess demand for current goods. The interest rate then needs to rise instead of fall to restore general equilibrium so that the Green Paradox is amplified.

[12] In this example and all examples to come, existence of an equilibrium poses no problem. Moreover, local stability of the usual Walrasian tâtonnement process is easily verified (i.e., Assumption A1 in Online Appendix A4 is satisfied in all the reported cases). Furthermore, one can formally show that in our CES utility and production specification of the model with $\sigma < 1$ and without physical capital, the equilibrium is always locally stable in the Walrasian sense.

The strength of the wealth reallocation effect depends on the difference between current expenditures shares, $\alpha - \alpha^*$ (or, equivalently on how much more impatient the oil-importing region is relative to the oil-exporting region, as measured by $\rho - \rho^*$). The strength of the supply effect (the change in current output relative to future output due to the lower current world price of oil) depends crucially on the elasticity of factor substitution, σ, through its effect on the price elasticity of oil demand ε_1:[13]

$$dF[R_1(q_1)] = F'\left(R_1(q_1)\right)\left(\frac{\partial R_1}{\partial q_1}\right)dq_1 = -\varepsilon_1 R_1 dq_1 \text{ with } \varepsilon_1 \equiv -\left(\frac{q_1 \partial R_1}{R_1 \partial q_1}\right) = \sigma\left[1+\left(\frac{\beta}{1-\beta}\right)\left(\frac{R_1}{L}\right)^{\frac{\sigma-1}{\sigma}}\right] > 0.$$

The grey area in Figure 2 gives combinations of $\alpha - \alpha^*$ and σ for which the Green Paradox is amplified.

4.3. Physical capital

Changes in investment affect the future capital stock and will thus shift future resource demand. We show that, as a consequence, amplification of the Green Paradox no longer requires an increase in the interest rate. With investment, the OME condition reads[14]

$$(25) \qquad R_1(q_1) + R_2(r_2, (1+r_2)q_1 + \tau_2) = S_1.$$

With the use of (11)-(12), the GME locus is now:

$$\frac{C_2(r_2, M(r_2, q_1; \tau_2)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2, q_1; \tau_2)) + C_1^*(r_2, M^*(q_1))} = \frac{(1-\mu)K_2(r_2, (1+r_2)q_1 + \tau_2)) + F(K_2(r_2, (1+r_2)q_1 + \tau_2), R_2(r_2, (1+r_2)q_1 + \tau_2))}{(1-\mu)K_1 + F(R_1(q_1)) - K_2(r_2, (1+r_2)q_1 + \tau_2)},$$

where we have used that the first-period interest rate can be written as a function of q_1 only, which follows from $F_K(K_1, R_1) = r_1 + \mu$, $F_R(K_1, R_1) = q_1$, and $K_1 = A_1 + A_1^*$. The left-hand-side features the relative consumption of future and current goods, which should equal the relative supply of future and current goods on the right-hand-side in equilibrium. The factor demand equations on the right-hand-side follow from (1)-(4). As in the previous section, the GME locus can be turned into an MGME locus without the carbon tax appearing, but this is a bit more complicated now. First, as before, we replace $R_2(r_2, (1+r_2)q_1 + \tau_2)$ by $S_1 - R_1(q_1)$. Second, for every q_1 we know $S_1 - R_1(q_1)$, so that (with slight abuse of notation), future demand for capital is $K_2(r_2, S_1 - R_1(q_1))$. So, we arrive at

[13] In the simulation, we have $R_1 > L$ for all combinations of σ and $\alpha - \alpha^*$ shown in Figure 2. Note that the relationship between σ and ε_1 may become non-monotonic if $R_1 < L$.

[14] The difference with OME condition (18) is that second-period oil demand now also depends (negatively) on the interest rate r_2 (through the effect of r_2 on the demand for capital), instead of only on the second-period oil price $(1+r_2)q_1 + \tau_2$.

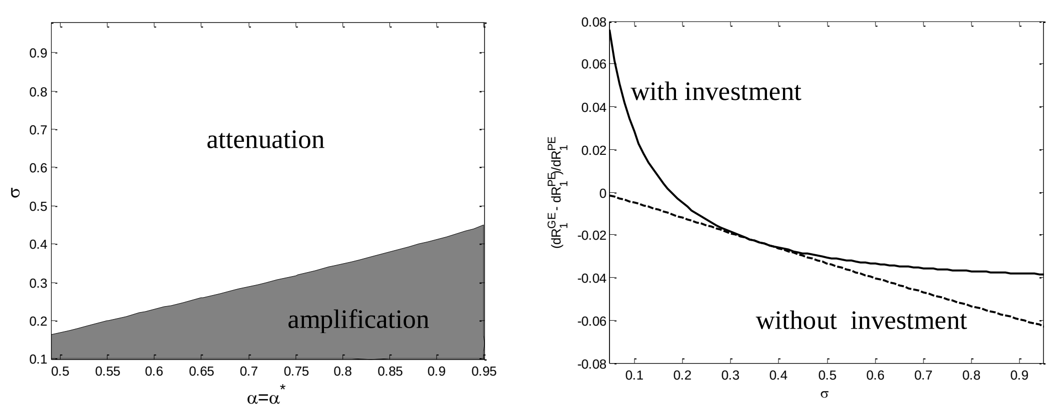
$$(26)\quad \frac{C_2(r_2, M(r_2,q_1)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2,q_1)) + C_1^*(r_2, M^*(q_1))} = \frac{(1-\mu)K_2(r_2, S_1 - R_1(q_1)) + F(K_2(r_2, S_1 - R_1(q_1)), S_1 - R_1(q_1))}{(1-\mu)K_1 + F(R_1(q_1)) - K_2(r_2, S_1 - R_1(q_1))}.$$

In terms of the direction of changes in the equilibrium prices, the different possibilities are still described by the three panels of Figure 1. But there is a key difference with Sections 4.1-4.2: a change in the future capital stock shifts future oil demand, so the movement from E to PE no longer corresponds with the partial equilibrium effect. Hence, the Green Paradox can be amplified even without a higher interest rate. We illustrate this using identical preferences, so $(C_2 + C_2^*)/(C_1 + C_1^*) = \Phi(r_2)$.

It can be seen from (26) that the interest rate then always goes down after an increase in the future carbon tax. Figure 3 shows the outcome when τ_2 is increased from zero to 0.01. We use $A_1 = 1$, $A_1^* = 0$, $\beta = 0.1$, $\lambda = 1/3$, $\eta = \eta^* = 1$, $\mu = 0.1$, $L = 1$, and $S_1 = 2$.

Figure 3: Attenuation versus amplification under identical homothetic preferences

Panel (a): attenuation vs. amplification **Panel (b): strength of general equilibrium effect**



Notes: $A_1 = 1$, $A_1^* = 0$, $\beta = 0.1$, $\lambda = 1/3$, $\eta = \eta^* = 1$, $\mu = 0.1$, $L = 1$, and $S_1 = 2$. In panel (a), the shaded area indicates amplification and the white area attenuation of the Green Paradox. Panel (b) shows the difference in the change in current extraction between the general and the partial equilibrium model, as a share of the change in the partial equilibrium model, for the specification with physical capital (solid line) and without physical capital (dashed line). In panel (b), $\rho = \rho^* = 0.05$, implying $\alpha = \alpha^* = 1.05/2.05$. The carbon tax τ_2 is raised from zero to 0.01.

The shaded area in panel (a) indicates the combinations of the elasticity of factor substitution σ and the common current expenditure share α for which the Green Paradox is amplified despite the decrease in the interest rate, due to a downward shift in future oil demand. Panel (b) shows the strength of the general equilibrium effect as a function of the elasticity of factor substitution, with $\rho = \rho^* = 0.05$ imposed (so $\alpha = \alpha^* = 1.05/2.05$). More specifically, the graph in panel (b) shows the difference in the change in first-period extraction between general equilibrium and partial equilibrium as a share of the change in first-period extraction in partial equilibrium when there is investment (solid line) and when there is no investment (dashed line), i.e. the lines in panel (b) represent $(dR_1^{GE} - dR_1^{PE}) / dR_1^{PE}$ for the models with and without investment, where the superscript GE indicates 'general equilibrium', and PE 'partial equilibrium'. This measure equals zero if the future carbon tax has exactly the same effect in general equilibrium as it has in partial equilibrium. It is positive if the increase in current extraction in general equilibrium is larger than in partial equilibrium (amplification) and negative if the change in general equilibrium is smaller than in partial equilibrium (attenuation). For the plotted range of elasticities of factor substitution (from 0.05 to 0.95), the general equilibrium effect on first-period extraction varies from an amplification of 7.5 percent in the model with investment to an attenuation of 6.5 percent without investment. Allowing for investment does not change the result of Proposition 3 that reversal of the Green Paradox is impossible if $\eta = \eta^* = 1$. We summarize the results of this section as follows.

Proposition 4 (Investment and the Green Paradox): Suppose $\eta = \eta^* = 1$ *and investment in physical capital is possible. With an increase in the future carbon tax, amplification of the Green Paradox can occur even if preferences are identical (* $\rho = \rho^*$ *) but reversal of the Green Paradox cannot occur.*

Proof: See Appendix A1. □

4.4. Non-unitary elasticity of marginal utility

Propositions 3 and 4 establish that reversal of the Green Paradox cannot occur with $\eta = \eta^* = 1$. Unitary elasticities of marginal utility give rise to constant current expenditure shares as income and substitution effects of changes in the interest rate cancel out against each other. However, if the income effect dominates the substitution effect and current expenditure shares depend positively on the interest rate, reversal of the Green Paradox may occur. The intuition behind this result is as follows. Recall that if the oil-importing region has a relatively low current-to-future consumption ratio, the wealth reallocation effect amplifies the excess supply of current goods resulting from an increase in the future carbon tax. Hence, the interest rate must fall to equilibrate relative demand for and relative supply of current goods. However, if current expenditure shares fall together with the interest rate, this constitutes a counteracting effect on the excess supply of current goods. Therefore, the required decrease in the interest rate to clear the intertemporal goods market is larger than in the case of constant expenditure shares. If the negative effect on current expenditure shares is strong enough, the Green Paradox can thus be reversed in general equilibrium.

As an example, take $\eta = \eta^* > 1$. The current expenditure shares resulting from utility maximization are given by

$$\frac{C_1}{M} = \left[1 + (1+r_2)^{\frac{1-\eta}{\eta}} (1+\rho)^{-\frac{1}{\eta}}\right]^{-1}, \quad \frac{C_1^*}{M^*} = \left[1 + (1+r_2)^{\frac{1-\eta^*}{\eta^*}} (1+\rho)^{-\frac{1}{\eta^*}}\right]^{-1},$$

which depend positively on the interest rate if $\eta = \eta^* > 1$. Figure 4 shows simulation results for different combinations of σ, $\eta = \eta^*$, and $\alpha - \alpha^*$. The two upper (lower) panels give the outcome for the model without (with) investment. We set $A_1 = 0$, $A_1^* = 1$, $\beta = 0.1$, $\lambda = 1/3$, $\mu = 0.1$, $L = 1$, and $S_1 = 1$.[15] [16] The carbon tax τ_2 is increased from zero to 0.01. Attenuation occurs in the shaded areas of the four panels, whereas reversal of the Green Paradox occurs in the white areas.

The figure illustrates that a combination of relatively patient oil importers, a sufficiently high elasticity of marginal utility, together with a sufficiently low elasticity of factor substitution can give rise to reversal of the Green Paradox. However, it is clear from Figure 4 that we need to impose rather extreme conditions to obtain this outcome.[17]

A comparison of panels (a)-(b) and (c)-(d) suggests that with investment it is even more difficult to find conditions under which strong reversal occurs. To understand this, consider a strong reversal in the model without capital, so that R_2 goes up and r_2 goes down. In the model with capital, this would induce an increase in investment. The resulting increase in the future capital stock has two opposing effects on the Green Paradox. First, relative supply of future consumption goods goes up, which increases the equilibrium rate of interest and thus boosts current extraction through the Hotelling rule, thereby enhancing the Green Paradox. Second, the increase in the future capital stock induces an upward shift in future oil demand, working against the Green Paradox. The first effect dominates the second one, so that on balance, strong reversal of the Green Paradox is less likely if the possibility of investment in physical capital is taken into account (cf. Online Appendix A4).

Proposition 5: Suppose $\eta = \eta^* > 1$ *and there is investment in physical capital. An increase in the future carbon tax can reverse the Green Paradox in general equilibrium.*

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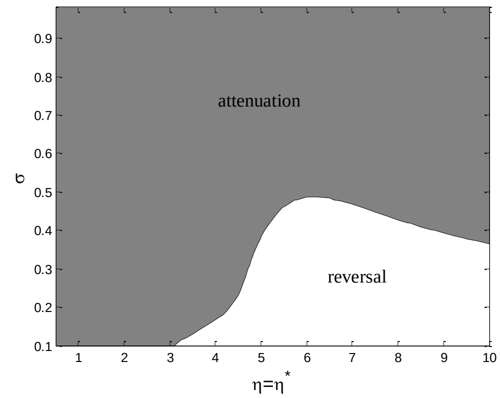
[15] Note that the initial asset endowment and the initial oil stock differ from the other simulations. We need a relatively high initial asset endowment for the oil exporter and a low initial oil stock to obtain examples of reversal.

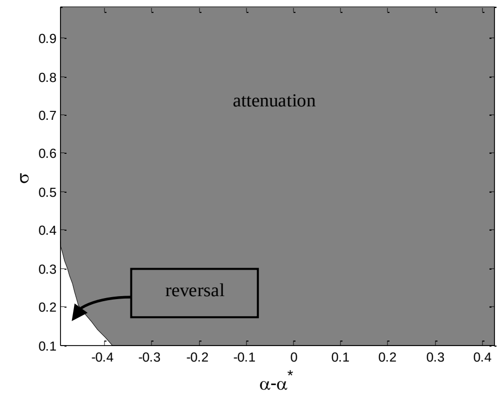
[16] To facilitate the comparison between the models with and without capital, we take the pre-tax equilibrium values of K_1, K_2, and r_1 from the model with capital and use those as exogenous variables in the model without.

[17] The reversal obtained in our numerical example is small in magnitude: the largest decrease in first-period extraction that we find is a factor 10 (10^3) smaller than the increase in extraction in the example with identical preferences underlying Figure 3 with $\rho = \rho^* = 0.05$ and $\sigma = 0.5$ in the model without (with) investment.

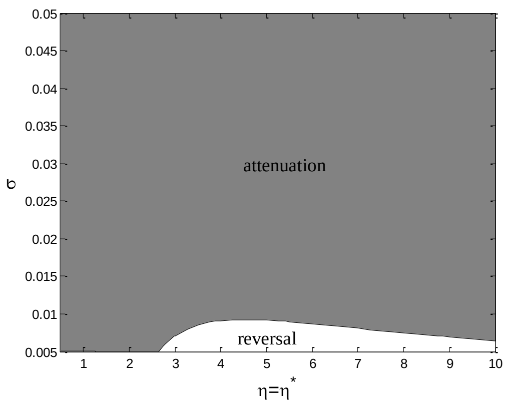
Figure 4: Attenuation versus reversal of the Green Paradox with a future carbon tax

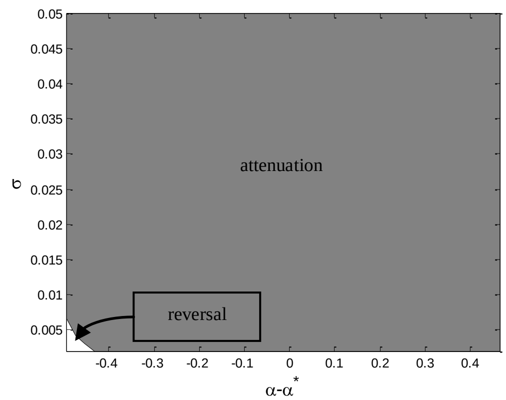
Panel (a): without capital, $\alpha = 0.5, \alpha^* = 0.995$ **Panel (b): without capital,** $\eta = \eta^* = 10$





Panel (c): with capital, $\alpha = 0.5, \alpha^* = 0.995$ **Panel (d): with capital,** $\eta = \eta^* = 10$





Notes: $A_1 = 0$, $A_1^* = 1$, $\beta = 0.1, \lambda = 1/3$, $\mu = 0.1$, $L = 1$, and $S_1 = 1$. The carbon tax τ_2 is increased from zero to 0.01. The white (shaded) area gives the region in which the Green Paradox is reversed (attenuated).

Proposition 5 summarizes the result of this section.

4.5 Summing up

The results so far have shown that the Green Paradox effect associated with the announcement of a future carbon tax is attenuated in general equilibrium if the oil-importing and the oil-exporting region have identical, homothetic preferences and if the analysis abstracts from investment. However, if oil-importers are relatively impatient or if the possibility of investment in capital is taken into account, the Green Paradox may be amplified in general equilibrium. Finally, we have shown that with very patient oil importers, a low elasticity of factor substitution, and an elasticity of marginal utility larger than unity, the Green Paradox can be reversed, but in our calibrated model the effects are tiny.

5. Exploration costs and partial exhaustion

For climate policy not only the speed of extraction matters, but also how much reserves to lock in the crust of the earth forever (e.g., Hoel, 2012; van der Ploeg and Withagen, 2012a; van der Ploeg, 2013). Instead of having stock-dependent extraction costs as this literature does, we choose to have endogenous exploration investment (Cairns, 1990; Gaudet and Laserre, 1988). We thus assume that the total recoverable stock of oil S_1 depends on initial exploration investment

(27) $S_1 = H(I)$,

with $H' > 0$, $H'' < 0$, where I denotes exploration investment.[18]

Hence, the return on oil exploration falls as less accessible fields have to be explored. Profit maximization gives the Hotelling rule (5) and exploration investment and initial reserves as increasing function of the initial oil price:

(28) $q_1 H'(I) = 1$,

Implying that we can write $I = H^{-1}(1/q_1) \equiv I(q_1)$, with $I' > 0$, and $S_1 = S_1(q_1)$ with $S_1^{'}(q_1) > 0$. With exploration costs, the OME condition (21) becomes

(29) $R_1(q_1) + R_2(r_2, (1+r_2)q_1 + \tau_2) = S_1(q_1)$.

In partial equilibrium with given interest rate r_2 and capital stocks K_1 and K_2 an increase in the future carbon tax τ_2 boosts current oil extraction, since the return to conserving oil drops (as in the case without exploration costs). In addition, however, cumulative extraction goes down as the current oil price falls so that the return to exploration becomes lower. We have thus the following proposition.

Proposition 6 (Abandoning oil reserves – partial equilibrium): *In partial equilibrium a higher future carbon tax τ_2 boosts current oil extraction and curbs cumulative oil extraction.*

Proof: Given $d\tau_2 > 0$, suppose that $dq_1 > 0$. This would imply that the left-hand-side of (29) goes down, whereas the right-hand-side goes up. Therefore, we need $dq_1 < 0$ so that $dR_1 > 0$ and $dS_1 < 0$. □

Hence, the effect of a future carbon tax on current oil extraction and cumulative oil extraction works in opposite direction. As a result, moving our focus to global warming, there is a trade-off between the speed of emissions and cumulative emissions (cf. van der Ploeg and Withagen, 2012a; van der Ploeg, 2013).

[18] Assuming that total oil reserves in the crust of the earth are given by $S_0 \geq S_1$, it follows that $S_0 - S_1$ units of oil remain untapped in the market equilibrium, because they are not worthwhile to exploit.

Using the Hotelling rule (5), the OME condition (29), and taking investment in exploration and the dependence of initial oil reserves on the oil price into account, the MGME condition is

$$(30)\quad \frac{C_2(r_2, M(r_2,q_1)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2,q_1)) + C_1^*(r_2, M^*(q_1))} = \frac{(1-\mu)K_2(r_2, S_1(q_1) - R_1(q_1)) + F(K_2(r_2, S_1(q_1) - R_1(q_1)), S_1(q_1) - R_1(q_1))}{(1-\mu)K_1 + F(R_1(q_1)) - K_2(r_2, S_1(q_1) - R_1(q_1)) - I(q_1)},$$

where the wealth levels for the oil-importing and exporting-region are, respectively,

$$(31)\quad \begin{aligned} M &= (1+r_1)A_1 + F(K_1, R_1(q_1)) - (r_1+\mu)K_1 + \frac{F(K_2(r_2,q_1), S_1(q_1) - R_1(r_2,q_1))}{1+r_2} - q_1 S_1(q_1), \\ M^* &= (1+r_1)A_1^* + q_1 S_1(q_1) - I(q_1). \end{aligned}$$

The MGME condition (30) requires that the relative demand for future and current consumption (left-hand side) equals the relative supply of future and current final goods (right-hand side) in equilibrium. Exploration costs $I(q_1)$ are subtracted from the denominator on the right-hand side. The factor demand equations are again derived from (1)-(4). Both the MGME condition (30) and the expressions for the wealth levels in (31) take into account that the initial stock depends on the oil price: $S_1 = S_1(q_1)$.

Depending on the relative slopes of the OME and MGME loci, we get as before attenuation, reversal or amplification of the Green Paradox in general equilibrium. The various possible directions of changes in the equilibrium prices are once more described by the panels of Figure 1. Nevertheless, there is an important difference with the analysis in Section 4: the general equilibrium consequences for the equilibrium price of oil also affect cumulative resource extraction, so that we get the following result.

Proposition 7 (Abandoning oil reserves – general equilibrium): *If the partial equilibrium effect on current oil extraction of a higher future carbon tax is attenuated, reversed or amplified in general equilibrium, then so will be the effect on cumulative oil extraction.*

Proof: With attenuation, q_1 changes in the same direction, but by less than in partial equilibrium. With reversal, q_1 changes in the opposite direction, compared to the partial equilibrium outcome. With amplification, q_1 changes in the same direction, but by more than in partial equilibrium. Because S_1 depends positively on q_1, the same qualifications hold for cumulative extraction. □

For example, consider the case of weakening of the Green Paradox in panel (a) of Figure 1. The current oil price drops by less than in partial equilibrium. As a result, cumulative oil extraction drops by less than in partial equilibrium, so general equilibrium effects weaken both the increase in current oil extraction and the decrease in cumulative oil extraction. Similarly, if the Green Paradox is strongly reversed, as in panel (b) of Figure 1, cumulative oil extraction goes up upon the announcement of a future carbon tax.

Since cumulative extraction directly affects the stock of atmospheric carbon, Proposition 7 implies that the general equilibrium consequences of future carbon taxes on global warming are (partly) offset by the general equilibrium consequences of future carbon taxes on cumulative extraction.

In Appendix A2, we discuss a specification of the model in which partial exhaustion is achieved through stock-dependent extraction costs, instead of through exploration costs. We show that if the Green Paradox occurs in a partial equilibrium setting, the general equilibrium effects on current extraction are comparable to the specification of the model discussed here and in Section 4: amplification of the Green Paradox can occur if the elasticity of factor substitution is low and if the importer is relatively impatient, whereas reversal is possible if the elasticity of marginal utility is high and the importer is relatively patient. The effects on cumulative extraction may be different from the model with exploration costs, depending on the price elasticity of oil demand in the pre-tax equilibrium. If the second-period price elasticity of demand is low enough, the change in current extraction depends positively on the change in the interest rate. Details can be found in the appendix.

6. Welfare analysis

Here we quantify the general equilibrium welfare effects of a future carbon tax for the oil-importing region, both for full and partial exhaustion. We also compare the effects in general equilibrium with those in partial equilibrium. We present results for the case of $\rho = \rho^* = 0.05$ and $\eta = \eta^* = 1$. We vary the elasticity of factor substitution from 0.05 to 0.95.[19] Starting from no taxation at all, we consider the introduction of a future carbon tax of 0.1, which roughly corresponds to a tax of 100 US dollars per ton carbon.[20] Damages from carbon emissions in period 1 are ψE_1 and in period 2 are $\psi(E_1 + E_2)$, where we set $\psi = 0.1$ to get damages of about 100 US dollars per ton carbon (the implied marginal utility of consumption is close to 1 in the pre-tax equilibrium). Initial asset endowments are $A_1 = 1$ and $A_1^* = 0$. The exploration function is $H(I) = \chi\sqrt{I}$, where $\chi = 7.1747$.[21] Other parameters are $\beta = 0.1$, $\lambda = 1/3$, $\mu = 0.1$, and $L = 1$. Figure 5 shows the welfare effects in percentage changes of current consumption.

[19] Van der Werf (2008) reports estimates for the elasticity of factor substitution varying from 0.17 to 0.61.

[20] Take a world output of 75 trillion US2013$ (World Bank, 2014), oil reserves amounting to 150 billion ton carbon (OPEC, 2013). If $S_1 = 2$ and the implied value of output $Y_1 = 1$, we get a tax of 100$/ton carbon.

[21] We have chosen $\chi = 7.1747$ to get $S_1 = 2$ in the pre-tax equilibrium with $\sigma = 0.5$ (the middle of the ranges on the horizontal axes Figure 5). Furthermore, we have endowed the oil-exporting region with an additional amount of initial assets equal to the exploration cost that they incur in the pre-tax equilibrium with $\sigma = 0.5$. As a result, for $\sigma = 0.5$ the pre-tax equilibria of the models with and without exploration costs are identical in terms of equilibrium prices q_1 and r_2. In the initial equilibrium with $\sigma = 0.5$ the oil income share $q_1 R_1 / Y_1$ equals 8.8 percent, which

The total welfare effect (solid line) is decomposed in a ‘green’ effect (dashed line) and ‘grey’ effect (dotted line). Panels (a) and (b) show the results for full exhaustion. The welfare effects for partial exhaustion are given in panels (c) and (d). The left panels show the welfare changes in general equilibrium. With full exhaustion, grey welfare increases as a result of the fall in the oil price. Green welfare, however, falls because of the Green Paradox: extraction is brought forward so that discounted damages rise. The effect on total welfare varies from an increase of over 3 percent of current consumption to a decrease of about 1 percent for low and high values of the elasticity of factor substitution, respectively.

With partial exhaustion green welfare goes up instead of down due to the decrease in cumulative extraction: the adverse green welfare effect of more rapid extraction of a given stock of reserves (the Green Paradox) is swamped by the positive welfare effect of locking up more fossil fuel in the ground. The fall in cumulative extraction at the same time dampens the positive effect on grey welfare. Interestingly, the effect on total welfare now remains positive and varies from a 0.8 to a 1.8 percent increase in current consumption.

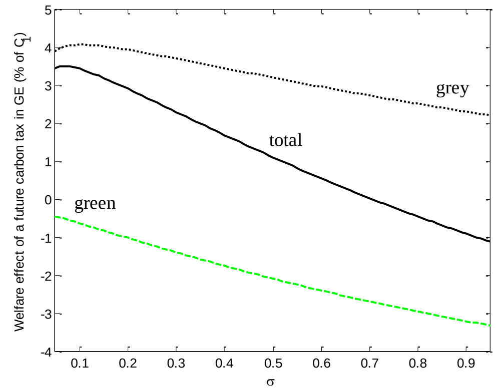
The two panels on the right of Figure 5 show the difference in welfare changes between general and partial equilibrium. Recall from Figure 3 that the Green Paradox is mitigated (amplified) in general equilibrium if the elasticity of factor substitution is high (low) enough. Accordingly, panel (b) shows that with full exhaustion, climate damages go down by less in general equilibrium if σ is high (where the dashed line is positive) and by more if σ is low (where the dashed line is negative). Panel (d) shows that with partial exhaustion the effect on cumulative oil extraction dominates: at high values, the effect on cumulative oil extraction is mitigated, leading to a smaller green welfare gain in general equilibrium. At low values of σ the decrease in cumulative oil extraction is amplified, so that the increase in green welfare is larger in general equilibrium.

Both for the specifications with full and partial exhaustion, the difference in green welfare effects between partial and general equilibrium ranges from about minus 0.05 percent to plus 0.1 percent of current consumption. Total welfare is not much affected either. For example, with $\sigma = 0.5$, consumption rises by approximately 1 percent in general equilibrium and by 0.9 percent in partial equilibrium (with full exhaustion). Hence, the ‘mistake’ made by assuming that the interest rate is unaffected upon an increase in the carbon tax is not detrimental to welfare. The welfare effects in the model with stock-dependent extraction costs in Appendix A2 are comparable to those reported here.

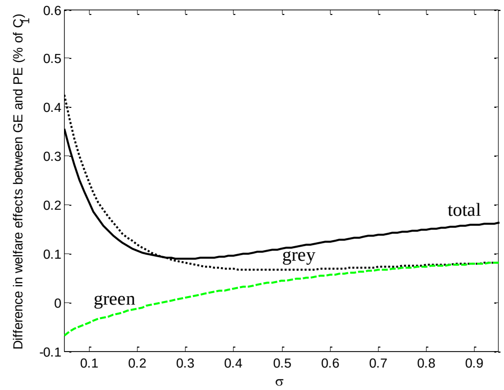
matches the average US energy expenditure share in GDP over the period 1970-2009 (U.S. Energy Information Administration, 2012).

Figure 5: Welfare effects of a future carbon tax

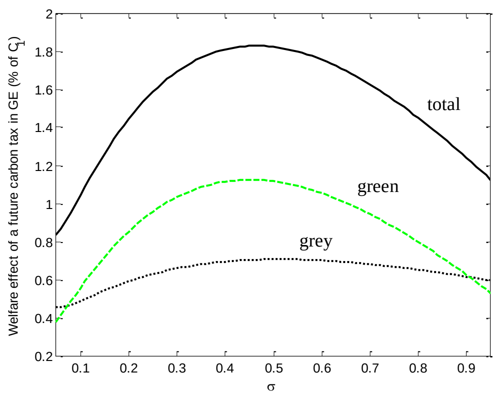
Panel (a): Full exhaustion - GE



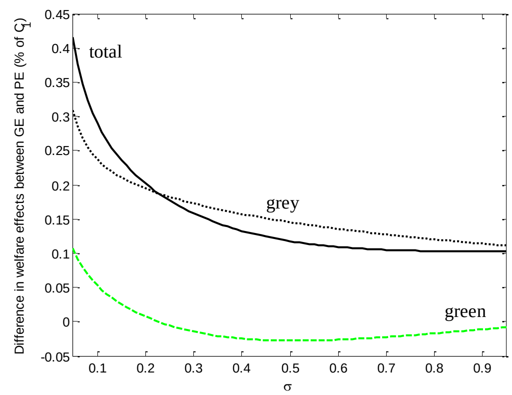
Panel (b): Full exhaustion - GE vs. PE



Panel (c): Partial exhaustion - GE



Panel (d): Partial exhaustion - GE vs. PE



Notes: $A_1 = 1$, $A_1^* = 0$, $\beta = 0.1$, $\lambda = 1/3$, $\eta = \eta^* = 1$, $\mu = 0.1$, $\psi = 0.1$, $\rho = \rho^* = 0.05$, $L = 1$, and $\chi = 7.1747$. The carbon tax τ_2 is raised from zero to 0.1. The solid black line gives the total welfare effect, the dashed and dotted lines represent the 'green' and 'grey' welfare effects, respectively. Panels (a) and (c) show the welfare effect under general equilibrium (GE). Panels (b) and (d) report the difference in welfare changes between general and partial equilibrium (PE).

7. Conclusion

The Green Paradox states that a carbon tax that increases at a rate larger than the interest rate induces oil producers to extract their reserves more quickly, so that the problem of global warming is exacerbated in a world in which the entire initial stock of oil is extracted eventually. The intuition is that a future carbon tax forces oil producers to supply less oil in the future due to lower future demand, which implies that current oil supply goes up. This pushes the current oil price down and therefore increases today's demand for oil. Most of the discussion of the Green Paradox has been cast in a partial equilibrium framework and has taken the interest rate as given. Since the interest rate is the key intertemporal price driving saving and investment decisions as well as oil depletion decisions and since the future oil demand function shifts upon changes in the future capital stock, this seems a serious shortcoming.

We show that the Green Paradox is mitigated in general equilibrium if oil exporters and oil importers have identical, homothetic preferences and if there are no investment possibilities. The mechanism behind this result is simple: the increase in the future carbon tax decreases the current oil price and hence increases current oil demand. This induces a rise in relative current output. Consequently, there will be excess supply of current output at the going interest rate. To restore equilibrium, the interest rate needs to fall, which implies that oil exporters will slow down their current extraction so that the Green Paradox is mitigated.

However, under less restrictive conditions, the Green Paradox might be amplified instead of attenuated in general equilibrium. First, if investment in physical capital is taken into account, the decrease of future extraction may cause a fall in investment given that oil and capital are cooperative in production. As a result, the future resource demand function shifts inwards, so that oil extraction (supply) is brought forward, thereby amplifying the partial equilibrium Green Paradox outcome. Second, the Green Paradox may be amplified if the oil-importing region is relatively more impatient than the oil-exporting region. The reason is that environmental policy induces a wealth reallocation effect between oil producers and oil importers. If oil importers are relatively impatient, this wealth reallocation effect diminishes the excess supply of savings resulting from the increase in the future carbon tax. If this effect is strong enough, excess supply may turn into excess demand for current goods at the prevailing interest rate. In that case, the interest rate needs to rise to restore equilibrium, so that oil producers respond by increasing current extraction and the partial equilibrium Green Paradox outcome is amplified.

If oil importers are very much more patient than oil exporters, the Green Paradox can even be reversed. The wealth reallocation effect then amplifies the excess supply of current goods resulting from the increase in the future carbon tax. Hence, the interest rate must decrease further than under identical preferences to restore equilibrium. We have shown that under certain conditions (an elasticity of factor substitution close to zero together with an extreme difference in the pure rate of time preference between the two regions, and an elasticity of marginal utility exceeding unity) the decline in the interest rate can be large enough to reverse the partial equilibrium Green Paradox outcome but effects will be tiny. Our numerical examples with factor income shares and elasticities of factor substitution in line with the empirics and with only moderate differences in patience, suggest that mitigation of the Green Paradox is the most likely outcome in a world with CES utility and CES production.

We have also studied the case of partial exhaustion in the presence of exploration costs. In practice, exploration costs rise as less accessible fields are explored and then it is seldom optimal to fully exhaust all oil reserves. An effective climate policy must thus focus on the supply side of the carbon market as well as on the demand side because it is crucial that not all oil that is in the crust of the earth is burned. We show that in partial equilibrium, a future carbon tax will reduce cumulative emissions. In general equilibrium, however, this effect may be mitigated, amplified or even reversed. In particular, we show that if the change in current extraction is mitigated, amplified or reversed in general equilibrium, so will be the change in cumulative extraction. We find that the adverse welfare effects of the Green Paradox can easily be swamped by the beneficial welfare effects of locking up more fossil fuel in the crust of the earth.

Our analysis has demonstrated that general equilibrium effects may have important consequences for the Green Paradox. In future research it would be interesting to assess these effects empirically, to evaluate how asset taxes as suggested by Sinn (2008, 2012) and Jaakkola (2012) fare in general equilibrium and might avoid the Green Paradox altogether, and to explore how these affect the strategic analysis of climate policies.

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Appendix A1: Proofs

For purposes of the proofs in this appendix, we distinguish between *weak* and *strong* reversal.

Definition A1: *Assume the future carbon tax is increased.*

(i) Weak reversal occurs if current oil extraction is unaffected.

(ii) Strong reversal occurs if current oil extraction decreases.

Proof of Proposition 3: Strong reversal implies a higher current world market oil price q_1, to have less demand for current oil, and a lower future world market oil price $(1+r_2)q_1$, to have more demand for future oil (note that the oil price for the final good producer is $(1+r_2)q_1+\tau_2$, which has to fall in spite of a higher tax). Hence, it is immediate from (20) and (24) that demand for current final goods by oil exporters goes up and its demand for future final goods goes down. With lower present production and higher future production, the demand response of oil importers must be the other way around. But this is excluded by (24).

Weak reversal occurs if q_1 and $(1+r_2)q_1+\tau_2$ remain unaffected. Hence the wealth of oil exporters is unaffected as well as its first-period consumption. Its second-period consumption goes down because $(1+r_2)q_1$ goes down. So, current consumption in the oil-importing region stays unaffected and future consumption increases. This is incompatible with (24).

Equality between first-period consumption and production requires $\alpha M+\alpha^* M^* = F(R_1)$. Using $M^* = q_1 S$ and $M = F(R_1)+F(R_2)/(1+r_2)-q_1 S_1$, we get

$$(\alpha^* - \alpha)q_1 S_1 = (1-\alpha)F(R_1(q_1)) - \alpha F(R_2((1+r_2)q_1+\tau_2))/(1+r_2)\,.$$

From this expression, we see that $\alpha > \alpha^*$ (note that $\alpha > \alpha^* \Leftrightarrow \rho > \rho^*$) is a necessary condition for amplification: occurrence of the Green Paradox implies an increase in R_1 and a decrease in R_2. Hence, q_1 must go down. Furthermore, amplification of the Green Paradox requires an increase in r_2. Therefore, the right hand side of the expression increases. Given that q_1 goes down, the left hand side can only increase if $\alpha > \alpha^*$. To prove that $\alpha \le \alpha^*$ implies attenuation, we observe that under this condition there is no amplification and, as reversal is excluded too, there must be attenuation. □

Proof of Proposition 4: The existence of the amplification region in panel (a) of Figure 4 proves the first part of the proposition. To prove the second part, we first show that weak reversal cannot occur. Weak reversal of the Green Paradox requires $dR_1 = 0$ and $dR_2 = 0$ (from (25)), therefore $dr_1 = 0$ and $dq_1 = 0$ (from (1)), and $dC_1^* = \alpha^* dM^* = \alpha^* dq_1 S_1 = 0$. Furthermore, note that $R_2(r_2, q_1(1+r_2)+\tau_2)$ decreases if r_2 and $q_1(1+r_2)+\tau_2$ increase. Hence, the constancy of R_2 and q_1 imply that $dr_2 < 0$, so that $dK_2 > 0$ (from

(3)). Consequently, (11) implies that $d(C_1 + C_1^*) < 0$. Given that $dC_1^* = 0$, this requires $dC_1 < 0$. But, we also have $C_1 = \alpha M = \alpha\big((1+r_1)A_1 + F(K_1, R_1) - (r_1 + \mu)K_1 + [F(K_2, R_2) - (r_2 + \mu)K_2]/(1+r_2) - q_1 S_1\big)$. The change in the bracketed term is given by $[F_K(K_2, R_2) - (r_2 + \mu)]dK_2 - K_2 dr_2 = -K_2 dr_2$, where the equality uses (3). Therefore, $dC_1 > 0$, so that we get a contradiction.

A consequence of the absence of weak reversal is that the use of first-period oil is monotonic in the second-period tax rate. To exclude strong reversal we thus need to show that for the tax large enough there exists an equilibrium with second-period oil use close to zero, implying first-period oil use close to the total available stock of oil. Hence, first-period oil use is increasing in the tax rate, and no strong reversal occurs. Define $\bar{q}_1$ by $F_R(A_1 + A_1^*, S_1) = \bar{q}_1$. Hence, it is the first-period world market oil price such that the entire stock of oil is demanded in the first period, with full employment of all capital. Define $\bar{r}_1$ by $F_K(A_1 + A_1^*, S_1) = \bar{r}_1 + \mu$ Suppose $r_2 + \mu = 0$. Then, with $K_1 = A_1 + A_1^*$, the equilibrium in this economy can be characterized as follows:

$$C_1 + C_1^* = (1-\mu)K_1 + F(K_1, S_1) - K_2,$$

$$C_2 + C_2^* = (1-\mu)K_2.$$

Moreover

$$M = \{(1+\bar{r}_1)A_1 + F(K_1, S_1) - (\bar{r}_1 + \mu)K_1 - \bar{q}_1 S_1\},$$

$$M^* = \{(1+\bar{r}_1)A_1^* + \bar{q}_1 S_1\}.$$

We also have

$$\text{(A.1)} \quad \begin{aligned} &C_2 + C_2^* = (1-\mu)K_2 = (1-\mu)[(1-\alpha)M + (1-\alpha^*)M^*] = \\ &(1-\alpha)(1-\mu)\{(1+\bar{r}_1)A_1 + F(K_1, S_1) - (\bar{r}_1 + \mu)K_1 - \bar{q}_1 S_1\} + (1-\alpha^*)\{(1+\bar{r}_1)A_1 + \bar{q}_1 S_1\}. \end{aligned}$$

From this final condition we can solve for K_2. We now have a set of prices and allocations. It is claimed that this set constitutes the limit of an equilibrium of an economy for the second-period carbon tax going to infinity. Given these prices consumers maximize their utility, subject to their budget constraints. First-period profits are maximized. Demand for final goods as well as for oil equal supply in both periods. The difficulty lies in second-period profit maximization and demand for second-period capital in production. Profits can be written as $F(K_2, R_2) - (r_2 + \mu)K_2 - ((1+r_2)\bar{q}_1 + \tau_2)R_2$. Given the solution K_2 of (A.1) the optimal oil input goes to zero as τ_2 goes to infinity. Moreover, given zero oil input, the capital stock K_2 maximizes profits. This establishes the absence of a strong reversal. □

Appendix A2: Extraction costs

In modelling extraction costs we follow Ritter and Schopf (2014) by assuming that the marginal extraction costs depend on the existing stock at the *outset* of each period.[22] The oil exporter takes period 1 and period 2 world market oil prices q_1 and q_2 as given. The maximization problem of the oil exporter reads

$$\max_{R_1,R_2} \quad \left[q_1 - G(S_1)\right]R_1 + \frac{1}{1+r}\left[q_2 - G(S_1 - R_1)\right]R_2$$

subject to $R_1 + R_2 \leq S_1$ and the nonnegativity conditions on the extraction rates, which we will ignore, assuming there is demand for oil in both periods. The Lagrangian reads

$$\left[q_1 - G(S_1)\right]R_1 + \frac{1}{1+r}\left[q_2 - G(S_1 - R_1)\right]R_2 + \omega(S_1 - R_1 - R_2)\,.$$

Necessary conditions are

$$\text{(A.2)} \qquad q_1 - G(S_1) + \frac{1}{1+r}G'(S_1 - R_1)R_2 = \omega\,,$$

$$\text{(A.3)} \qquad \frac{1}{1+r}\left[q_2 - G(S_1 - R_1)\right] = \omega\,.$$

Equilibrium on the oil market requires[23]

$$\text{(A.4)} \qquad q_1 = F'(R_1)\,,$$

$$\text{(A.5)} \qquad q_2 = F'(R_2) - \tau_2\,.$$

In general, two cases should be distinguished. One where there is full exhaustion in the two periods, the other where some oil is left at the end of the second period. Here we will restrict attention to the case of partial exhaustion, implying that $\omega = 0$.

Partial equilibrium

Linearizing (A.2)-(A.3) with $\omega = 0$ and (A.4)-(A.5) imposed around a zero-tax equilibrium gives the following system[24]

$$\text{(A.6)} \qquad \begin{pmatrix} 1 + \dfrac{p\varepsilon_1 R_1 R_2 G''}{q_1} & -\dfrac{p\varepsilon_2 R_2 G'}{G} \\ -\dfrac{G'\varepsilon_1 R_1}{q_1} & 1 \end{pmatrix} \begin{pmatrix} dq_1 \\ dq_2 \end{pmatrix} = \begin{pmatrix} -G'R_2 & \dfrac{p\varepsilon_2 R_2 G'}{G} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} dp \\ d\tau_2 \end{pmatrix}\,,$$

---

[22] In contrast to Ritter and Schopf (2014), we assume that in each period the marginal extraction cost is independent of that period's extraction rate.

[23] We focus on the specification of the model without physical capital (see Sections 4.1-4.2).

[24] All variables and functions in (A.6) take their pre-tax equilibrium values, e.g. $G = G(S_1 - R_1(q_1))$, where q_1 takes its pre-tax equilibrium value.

where $\varepsilon_t \equiv -(\partial R_t / R_t)/(\partial q_t / q_t) > 0$ is the price elasticity of oil demand at time $t = 1, 2$ and $p \equiv 1/(1+r)$. Using Cramer's rule, we obtain

$$\text{(A.7)} \qquad \frac{dq_1}{d\tau_2} = \frac{p\varepsilon_2 R_2 G'}{|\Gamma| G}, \; \frac{dq_1}{dp} = -\frac{R_2 G'}{|\Gamma|}, \; \frac{dq_2}{d\tau_2} = \frac{p\varepsilon_1\varepsilon_2 R_1 R_2 (G')^2}{|\Gamma| q_1 G}, \text{ and } \frac{dq_2}{dp} = -\frac{\varepsilon_1 R_1 R_2 (G')^2}{q_1 |\Gamma|},$$

where $|\Gamma|$ is the determinant of the Jacobian matrix in (A.6). We take the extraction cost function to be $G(S) = \gamma / S$, which is decreasing and strictly convex in the stock size: $G' < 0$, $G'' > 0$. Consequently, the determinant $|\Gamma|$ can be written as

$$|\Gamma| \equiv 1 + \frac{p\varepsilon_1 R_1 R_2}{q_1}\left(G'' - \frac{(G')^2}{G}\varepsilon_2 \right) = 1 + \frac{p\varepsilon_1 R_1 R_2 \gamma}{q_1 (S_1 - R_1)^3}\left(2 - \varepsilon_2\right).$$

If $\varepsilon_2 < 2$ we have $|\Gamma| > 0$, implying $dq_1 / d\tau_2 < 0,\ dq_1 / dp > 0,\ dq_2 / d\tau_2 > 0$, and $dq_2 / dp < 0$. Hence, if $\varepsilon_2 < 2$ current extraction goes up upon an increase in the future carbon tax at an unchanged interest rate, implying that a Green Paradox occurs in partial equilibrium. Furthermore, current extraction depends positively on the interest rate ($dq_1 / dp > 0 \Leftrightarrow dq_1 / dr < 0$). Hence, an increase (decrease) in the interest rate will amplify (attenuate or reverse) the Green Paradox in general equilibrium, as shown in Figures A.1 and A.2 in the next section. By using (A.4)-(A.5) and (A.7), we obtain the following comparative statics for cumulative extraction:

$$\frac{d(R_1 + R_2)}{d\tau_2} = \frac{\varepsilon_1 \varepsilon_2 R_1 R_2 p}{(S_1 - R_1)|\Gamma| q_1}\left(1 - \varepsilon_2 \frac{R_2}{S_1 - R_1}\right) - \frac{\varepsilon_2 R_2}{q_2},$$

$$\frac{d(R_1 + R_2)}{dp} = -\frac{\varepsilon_1 R_1 R_2 q_2}{(S_1 - R_1)|\Gamma| q_1}\left(1 - \varepsilon_2 \frac{R_2}{S_1 - R_1}\right).$$

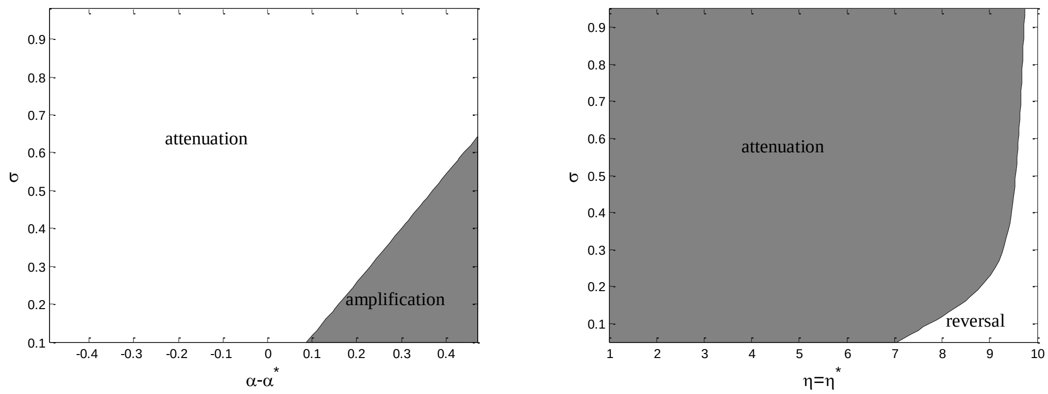
Hence, the effects of changes in the future carbon tax and in the interest rate on cumulative extraction are ambiguous. An increase (decrease) in the interest rate, i.e, $dp < 0$, increases (lowers) cumulative extraction if $\varepsilon_2 < \min\{2, (S_1 - R_1)/R_2\}$. In other words, if general equilibrium effects attenuate or reverse the Green Paradox, they will also lower cumulative extraction if $\varepsilon_2 < \min\{2, (S_1 - R_1)/R_2\}$. However, if $\varepsilon_2 \in ((S_1 - R_1)/R_2, 2)$, general equilibrium effects increase cumulative extraction if they attenuate or reverse the Green Paradox.

General equilibrium

Figure A.1 gives simulation results for the general equilibrium model described in Section 2 of the paper, with (3) and (8) replaced by (A.2)-(A.3), with the CES production function (20), and without physical capital. The results for current extraction are comparable to the full exhaustion model: amplification might occur if σ is low and $\alpha-\alpha^*$ is high, whereas reversal is possible if $\eta=\eta^*$ is high and $\alpha-\alpha^*$ is low. See also Figures 2 and 4(a) in the paper.

Figure A.1: Attenuation, amplification, and reversal of the Green Paradox with extraction costs

Panel (a): $\eta=\eta^*=1$ **Panel (b):** $\alpha=0.5$, $\alpha^*=0.995$



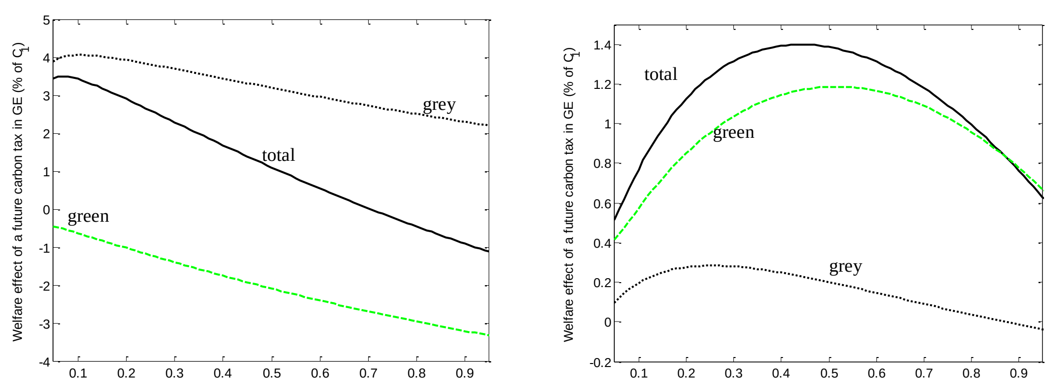
Notes: $A_1=0$, $A_1^*=1$, $\beta=0.1$, $\gamma=0.15$, $\lambda=1/3$, $L=1$, and $S_1=1.5$. The carbon tax τ_2 is increased from zero to 0.01.

Welfare

For the welfare analysis, we move on the full model with physical capital described in Section 2, with (3) and (8) replaced by (A.2)-(A.3), and with the CES production function (23). Figure A.2 shows the welfare effects in percentage changes of current consumption. For ease of comparison with the case of full exhaustion, we have copied panel (a) of Figure 5 from the paper.

Figure A.2: Welfare effects of a future carbon tax

Panel (a): Full exhaustion **Panel (b): Partial exhaustion**



Notes: $A_1=1$, $A_1^*=0$, $\beta=1/3$, $\gamma=0.15$, $\eta=\eta^*=1$, $\lambda=0.1$, $\mu=0.1$, $\rho=\rho^*=0.05$, $L=1$, $S_1=2$. τ_2 is raised from zero to 0.1. The solid black line gives the total welfare effect, the dashed and dotted lines represent the 'green' and 'grey' welfare effects, respectively.

The total welfare effect (solid line) is decomposed in a 'green' effect (dashed line) and 'grey' effect (dotted line). Panel (a) gives the result for full exhaustion, whereas panel (b) shows the results for the case of partial exhaustion due to stock-dependent extraction costs. The results are comparable to those of the model with partial exhaustion due to exploration costs that we have discussed in the paper (see panel (c) of Figure 5). Although green welfare goes down under full exhaustion, with partial exhaustion, it goes up, due to a decrease in cumulative extraction: the negative green welfare effect of more rapid extraction of a given stock of fossil fuels is dominated by the positive green welfare effect of leaving more oil untapped. The fall in cumulative extraction at the same time lowers the grey welfare effect of a future carbon tax.

Appendix A3: Including a non-abating country

Here we study the effect of including a non-abating country in our analysis (cf. Eichner and Pethig, 2011). Apart from the oil exporter (denoted by *) and the oil importer that conducts climate policy (denoted by ^), we introduce another oil-importing region, which does not conduct climate policy (denoted by ~).[25]

The oil producers maximize utility subject to their budget constraints:

$$\max_{C_1^*, C_2^*} U(C_1^*, C_2^*)\text{, subject to } C_1^* + \frac{1}{1+r}C_2^* = q_1 R_1^* + \frac{1}{1+r}q_2 R_2^*\text{, and } R_1^* + R_2^* = S_1 .$$

In equilibrium, the oil price is governed by the Hotelling rule:

$$\text{(A.8)} \qquad q_1 = \frac{1}{1+r}q_2 .$$

The maximization problem of the households in the oil-importing region that conducts climate policy is given by:

$$\max_{\hat{C}_1, \hat{C}_2} U(\hat{C}_1, \hat{C}_2)\text{, subject to } \hat{C}_1 + \frac{1}{1+r}\hat{C}_2 = \hat{\Pi} + \tau_1 \hat{R}_1 + \frac{1}{1+r}\tau_2 \hat{R}_2 ,$$

where profits $\hat{\Pi}$ follow from the maximization problem of the firms:

$$\text{(A.9)} \qquad \hat{\Pi} = \max_{\hat{R}_1, \hat{R}_2} \hat{F}(\hat{R}_1) - (q_1 + \tau_1)\hat{R}_1 + \frac{1}{1+r}(\hat{F}(\hat{R}_2) - (q_2 + \tau_2)\hat{R}_2) .$$

The maximization problem of the households in the oil-importing region that does not conduct climate policy reads

[25] We use the consumption good as the numéraire, whereas Eichner and Pethig (2011) use the consumption good *in period 1* as the numéraire. As a result, their second-period price p_{x2} equals the inverse of the gross interest rate in our model: $p_{x2} = 1/(1+r_2)$.

$$\max_{\tilde{C}_1, \tilde{C}_2} U(\tilde{C}_1, \tilde{C}_2) \text{, subject to } \tilde{C}_1 + \frac{1}{1+r}\tilde{C}_2 = \tilde{\Pi} \text{,}$$

where profits $\tilde{\Pi}$ are obtained from

$$\text{(A.10)} \quad \tilde{\Pi} = \max_{\tilde{R}_1, \tilde{R}_2} \tilde{F}(\tilde{R}_1) - q_1 \tilde{R}_1 + \frac{1}{1+r}(\tilde{F}(\tilde{R}_2) - q_2 \tilde{R}_2) \text{.}$$

Goods market equilibrium

Following Eichner and Pethig (2011), we assume that the utility functions are identical and homothetic. Hence, in equilibrium we have

$$\text{(A.11)} \quad \frac{C_2^* + \hat{C}_2 + \tilde{C}_2}{C_1^* + \hat{C}_1 + \tilde{C}_1} = \Phi(r) = \frac{\hat{F}(\hat{R}_2) + \tilde{F}(\tilde{R}_2)}{\hat{F}(\hat{R}_1) + \tilde{F}(\tilde{R}_1)} \text{,}$$

with $\varphi'(r) > 0$. Let us define

$$C_1 \equiv C_1^* + \hat{C}_1 + \tilde{C}_1 \text{, } C_2 \equiv C_2^* + \hat{C}_2 + \tilde{C}_2 \text{, } F_1 \equiv \hat{F}(\hat{R}_1) + \tilde{F}(\tilde{R}_1) \text{, and } F_2 \equiv \hat{F}(\hat{R}_2) + \tilde{F}(\tilde{R}_2) \text{.}$$

Then

$$\text{(A.12)} \quad \frac{C_2}{C_1} = \Phi(r) = \frac{F_2}{F_1} \text{.}$$

Carbon tax

We focus on the effects of an increase in the future carbon tax. So, we have $d\tau_2 > 0,\ d\tau_1 = 0\ (\tau_1 = 0)$.

Proposition A1: *Suppose the three regions have identical and homothetic preferences and that the future carbon tax is increased. Then,* $dr < 0$ *and* $dq_2 < 0$.

Proof:

Step 1. Suppose $dq_1 > 0,\ dq_2 > 0$. Then $d\hat{R}_1 < 0,\ d\tilde{R}_1 < 0,\ d\hat{R}_2 < 0,\ d\tilde{R}_2 < 0.$ Hence excess supply of oil. Ruled out.

Step 2. Suppose $dq_2 > 0$. Then $dq_1 < 0$ from step 1. Moreover, $dr > 0$ since $q_1 = \frac{1}{1+r} q_2$ (Hotelling). Hence, $d\hat{R}_2 < 0$, $d\tilde{R}_2 < 0$, $d\hat{R}_1 > 0$, and $d\tilde{R}_1 > 0$. So, $d(F_2 / F_1) < 0$. But also $d\varphi(r) > 0$. A contradiction. So, we have $dq_2 < 0$.

Step 3. Suppose $dr > 0$. We then have $dq_1 = d\frac{1}{1+r} q_2 < 0$ and $d(\hat{F}(\hat{R}_1) + \tilde{F}(\tilde{R}_1)) = q_1 d(\hat{R}_1 + \tilde{R}_1)$ $= -q_1 d(\hat{R}_2 + \tilde{R}_2) > 0$ since first-period oil gets cheaper. Furthermore, we have $d(\hat{F}(\hat{R}_2) + \tilde{F}(\tilde{R}_2))$ $= (q_2 + \tau_2) d\hat{R}_2 + q_2 d\tilde{R}_2 = (1+r) q_1 d(\hat{R}_2 + \tilde{R}_2) + \tau_2 d\hat{R}_2 < 0$ since second-period oil use must get smaller (in view of more first-period demand) and $d(\hat{F}'(\hat{R}_2)/\tilde{F}'(\tilde{R}_2)) = d(1+\tau_2/q_2) > 0$ implying that $d(\hat{R}_2/\tilde{R}_2) < 0$ and hence $d\hat{R}_2 < 0$. So, we have $d(F_2/F_1) < 0$ and $d\Phi(r) > 0$. Therefore, we get a contradiction again. So, we have $dr < 0$. □

In contrast to the result of Proposition 2 in Section 4.1, the first- period price $q_1 = \frac{1}{1+r} q_2$ may go up, in which case a reversal of the Green Paradox occurs, even with identical and homothetic preferences. The reason is that the MGME locus, which is obtained by substitution of $\hat{R}_2 = S_1 - \hat{R}_1 - \tilde{R}_1 - \tilde{R}_2$ and the factor demand equations derived from (A.9) and (A.10) into (A.12)),

$$\text{(A.13)} \quad \Phi(r_2) = \frac{\hat{F}\left(S_1 - \hat{R}_1(q_1) - \tilde{R}_1(q_1) - \tilde{R}_2\left((1+r_2)q_1\right)\right) + \tilde{F}\left(\tilde{R}_2\left((1+r_2)q_1\right)\right)}{\hat{F}(\hat{R}_1(q_1)) + \tilde{F}(\tilde{R}_1(q_1))}$$

is not necessarily upward sloping, due to the last term in the numerator, which depends negatively on q_1. As a result, the general equilibrium can be described by panel (a) or panel (b) of Figure 1.[26] The potential occurrence of reversal can be explained intuitively by the existence of the non-abating region: part of the future emission reduction in region (^) will leak away to region (~) instead of to the present. Without the non-abating region, the last term in the numerator drops out, so that the MGME reduces to (16) in the main text.

The MGME locus (A.13) clearly shows the complementarity between the analysis of Eichner and Pethig (2011) and our paper. Eichner and Pethig (2011) obtain interesting effects because they have a more general specification on the right-hand-side (i.e., production in a non-abating country). We obtain interesting effects because of a more general specification on the left-hand-side (i.e., non-identical preferences), but also on the right-hand-side (by introducing physical capital). The generalization of Eichner and Pethig (2011) opens up the possibility of obtaining a reversal of the Green Paradox. Our generalization with physical capital leads to the possibility of amplification of the Green Paradox with identical homothetic preferences, and our generalization to non-identical homothetic preferences opens up the possibilities of amplification and reversal of the Green Paradox.

[26] The constellation in panel (c) of Figure 1 can be excluded, because of the result in Proposition A1 that the interest rate falls.

Online Appendix A4: Comparative statics tâtonnement around an equilibrium with zero taxes

This appendix discusses the local Walrasian stability of the different equilibria and derives the comparative statics around an equilibrium outcome with zero taxes. The tâtonnement versions of the OME and GME conditions, respectively,

$$\dot{q}_1 = \lambda_1 \left[R_1(q_1) + R_2\left(r_2,(1+r_2)q_1+\tau_2\right) - S_1 \right],$$

$$\dot{r}_2 = \lambda_2 \left[C_1(r_2, M(r_2,q_2)) + C_1^*(r_2, M^*(q_1)) + K_2(r_2, R_2\left(r_2,(1+r_2)q_1+\tau_2\right)) - F(K_1, R_1(q_1)) - (1-\mu)K_1 \right],$$ where

dots above variables denote changes and the wealth levels for both regions are given by

$$\begin{aligned} M &= \left(1+r_1[R_1(q_1)]\right)A_1 + F[R_1(q_1)] - \left(r_1[R_1(q_1)]+\mu\right)K_1 - q_1 S_1 \\ &+ \frac{F\left(K_2[r_2, R_2\left(r_2,(1+r_2)q_1+\tau_2\right)], R_2\left(r_2,(1+r_2)q_1+\tau_2\right)\right)}{1+r_2} \\ &- \frac{(r_2+\mu)K_2(r_2, R_2\left(r_2,(1+r_2)q_1+\tau_2\right)) - \nu_2\left(A_2^*(r_2,q_1) + q_1 R_2\left(r_2,(1+r_2)q_1+\tau_2\right)\right)}{1+r_2}, \end{aligned}$$

$$M^* = \left(1+r_1[R_1(q_1)]\right)A_1^* + q_1 R_1 + \frac{q_2}{1+r_2}R_2 = \left(1+r_1[R_1(q_1)]\right)A_1^* + q_1 S_1.$$

Standard Walrasian tâtonnement implies that the auctioneer would raise oil prices if demand for oil exceeds the supply of oil, hence $\lambda_1 > 0$. It also implies that the auctioneer would raise the current final goods price which corresponds to a fall in the future final goods price and a rise in r_2 if current demand for final goods exceeds current supply of final goods, hence $\lambda_2 > 0$.

Linearizing this system around a steady state with $\tau_2 = 0$ we get the following system:

$$\begin{pmatrix} \dot{q}_1 \\ \dot{r}_2 \end{pmatrix} = \begin{pmatrix} -\lambda_1\omega_q & -\lambda_1\omega_r \\ \lambda_2\gamma_q & -\lambda_2\gamma_r \end{pmatrix} \begin{pmatrix} dq_1 \\ dr_2 \end{pmatrix} + \begin{pmatrix} -\lambda_1\omega_\tau \\ -\lambda_2\gamma_\tau \end{pmatrix} d\tau_2,$$

with the stationary state of this system giving the following steady-state solution

$$dq_1 = -\frac{\Gamma_{q\tau}}{|\Delta|}d\tau_2, \; dr_2 = -\frac{\Gamma_{r\tau}}{|\Delta|}d\tau_2,$$

where $|\Delta| \equiv \omega_q\gamma_r + \omega_r\gamma_q$, and we have defined the following coefficients and elasticities:

$$\gamma_r \equiv \frac{\partial C_1}{\partial M}\left(\frac{Y_2-(r_2+\mu)K_2}{(1+r_2)^2} + \frac{K_2}{1+r_2} + \frac{\varepsilon_2 q_2 R_2}{(1+r_2)^2} \right) - \left[\frac{\partial C_1}{\partial r_2} + \frac{\partial C_1^*}{\partial r_2} \right] + \left\{ \frac{\partial C_1}{\partial M}\frac{\varepsilon_2^{Rr}}{1+r_2}\frac{q_2 R_2}{r_2+\mu} + \frac{\eta_2 K_2}{r_2+\mu} + \eta_2^{KR}K_2\left(\frac{\varepsilon_2}{1+r_2} + \frac{\varepsilon_2^{Rr}}{r_2+\mu} \right) \right\},$$

$$\gamma_q \equiv \left(\frac{\partial C_1^*}{\partial M^*} - \frac{\partial C_1}{\partial M} \right) S_1 + \varepsilon_1 R_1 - \frac{\partial C_1}{\partial M}\left(\varepsilon_1 R_1 + \varepsilon_2 R_2\right) + \left\{ \left(\frac{\partial C_1}{\partial M} - \frac{\partial C_1^*}{\partial M^*} \right) A_1^* \eta_1^{rR} \varepsilon_1 \frac{r_1+\mu}{q_1} - \eta_2^{KR}\frac{\theta_K}{\theta_R}\frac{1+r_2}{r_2+\mu}\varepsilon_2 R_2 \right\},$$

$$\gamma_\tau \equiv \frac{\partial C_1}{\partial M}\frac{\varepsilon_2 R_2}{1+r_2} + \left\{ \frac{\varepsilon_2 \eta_2^{KR} K_2}{q_2} \right\} > 0, \quad \omega_q \equiv \varepsilon_1 R_1 + \varepsilon_2 R_2 > 0, \; \omega_r \equiv \left(\left\{ \frac{\varepsilon_2^{Rr}}{r_2+\mu} \right\} + \frac{\varepsilon_2}{1+r_2} \right) q_1 R_2 > 0, \quad \omega_\tau \equiv \frac{\varepsilon_2 R_2}{1+r_2} > 0,$$

$$\Gamma_{q\tau} \equiv \gamma_r\omega_\tau - \gamma_\tau\omega_r = \frac{\varepsilon_2 R_2}{1+r_2}\left(\frac{\partial C_1}{\partial M}\left(\frac{Y_2-(r_2+\mu)K_2}{(1+r_2)^2} + \frac{K_2}{1+r_2} \right) - \left[\frac{\partial C_1}{\partial r_2} + \frac{\partial C_1^*}{\partial r_2} \right] + \left\{ \frac{\eta_2 K_2}{r_2+\mu} \right\} \right),$$

$$\Gamma_{r\tau} \equiv \gamma_\tau\omega_q + \gamma_q\omega_\tau = \frac{\varepsilon_2 R_2}{1+r_2}\left(\left(\frac{\partial C_1^*}{\partial M^*} - \frac{\partial C_1}{\partial M} \right) S_1 + \varepsilon_1 R_1 + \left\{ \left(\frac{\partial C_1}{\partial M} - \frac{\partial C_1^*}{\partial M^*} \right) A_1^* \eta_1^{rR} \varepsilon_1 \frac{r_1+\mu}{q_1} + \eta_2^{KR}\varepsilon_1 R_1 \frac{\theta_K}{\theta_R}\frac{1+r_2}{r_2+\mu} \right\} \right),$$

$$|\Delta| \equiv \omega_q \gamma_r + \omega_r \gamma_q, \quad \theta_K \equiv \frac{(r_2+\mu)K_2}{Y} \in (0,1), \quad \theta_R \equiv \frac{q_2 R_2}{Y} \in (0,1), \quad \varepsilon_1 \equiv -\frac{\partial R_1(q_1)}{\partial q_1}\frac{q_1}{R_1} > 0, \quad \varepsilon_2 \equiv -\frac{\partial R_2(r_2,q_2)}{\partial q_2}\frac{q_2}{R_2} > 0,$$

$$\varepsilon_2^{Rr} \equiv -\frac{\partial R_2(r_2,q_2)}{\partial r_2}\frac{r_2+\mu}{R_2} > 0, \quad \eta_1^{rR} \equiv \frac{\partial r_1(K_1,R_1)}{\partial R_1}\frac{R_1}{r_1+\mu} > 0, \quad \eta_2 \equiv -\frac{\partial K_2(r_2,R_2)}{\partial r_2}\frac{r_2+\mu}{K_2} > 0, \quad \eta_2^{KR} \equiv \frac{\partial K_2(r_2,R_2)}{\partial R_2}\frac{R_2}{K_2} > 0.$$

The terms in curly brackets drop out if there is no investment, the terms in square brackets drop out if expenditure shares are constant (i.e., if the elasticity of intertemporal substitution (EIS) equals unity in both regions), and the terms in double square brackets drop out if preferences are identical and homothetic. We discuss comparative statics induced by a future carbon tax in four different cases. Stability of the tâtonnement mechanism requires that the Jacobian matrix of the dynamic system for the tâtonnement process, i.e.,

$$J = \begin{pmatrix} -\lambda_1 \omega_q & -\lambda_1 \omega_r \\ \lambda_2 \gamma_q & -\lambda_2 \gamma_r \end{pmatrix},$$

has two eigenvalues with negative real parts. This requires $\det(J) = \lambda_1 \lambda_2 \left[\gamma_r \omega_q + \gamma_q \omega_r\right] > 0$ or $|\Delta| > 0$ and $\mathrm{trace}(J) = -\lambda_1 \omega_q - \lambda_2 \gamma_r < 0$.

Assumption A1: $|\Delta| > 0$ **and** $\lambda_1 \omega_q + \lambda_2 \gamma_r > 0$, $\forall \lambda_1 > 0, \lambda_2 > 0$.

In the comparative statics below, we assume that Assumption A1 is satisfied.[27]

Identical homothetic preferences, EIS=1 and no investment: Terms in curly, square and double square brackets drop out, so $\Gamma_{q\tau} = \frac{\varepsilon_2 R_2}{1+r_2}\frac{\partial C_1}{\partial M}\left(\frac{Y_2 - (r_2+\mu)K_2}{(1+r_2)^2} + \frac{K_2}{1+r_2}\right) > 0$, $\Gamma_{r\tau} = \frac{\varepsilon_1 R_1 \varepsilon_2 R_2}{1+r_2} > 0$. Therefore, a future carbon tax pushes down the interest rate and the Green Paradox is attenuated.

Heterogeneous homothetic preferences, IES=1 and no investment: Terms in curly and square brackets drop out, so $\Gamma_{q\tau} = \frac{\varepsilon_2 R_2}{1+r_2}\frac{\partial C_1}{\partial M}\left(\frac{Y_2 - (r_2+\mu)K_2}{(1+r_2)^2} + \frac{K_2}{1+r_2}\right) > 0$, $\Gamma_{r\tau} = \frac{\varepsilon_2 R_2}{1+r_2}\left(\left[\!\left[\frac{\partial C_1^*}{\partial M^*} - \frac{\partial C_1}{\partial M}\right]\!\right] S_1 + \varepsilon_1 R_1\right)$. We thus see that $\Gamma_{r\tau}$ can become negative if oil exporters are much more patient than oil importers.[28] In that case, a future carbon tax pushes up the interest rate and the Green Paradox is amplified.

Heterogeneous homothetic preferences and no investment: Terms in curly brackets drop out, so

$$\Gamma_{q\tau} = \frac{\varepsilon_2 R_2}{1+r_2}\left(\frac{\partial C_1}{\partial M}\left(\frac{Y_2 - (r_2+\mu)K_2}{(1+r_2)^2} + \frac{K_2}{1+r_2}\right) - \left[\frac{\partial C_1}{\partial r_2} + \frac{\partial C_1^*}{\partial r_2}\right]\right), \quad \Gamma_{r\tau} = \frac{\varepsilon_2 R_2}{1+r_2}\left(\left[\!\left[\frac{\partial C_1^*}{\partial M^*} - \frac{\partial C_1}{\partial M}\right]\!\right] S_1 + \varepsilon_1 R_1\right).$$

Hence, $\Gamma_{q\tau}$ can become negative if the IES < 1 and importers are not too impatient. In that case, the Green Paradox is strongly reversed.[29]

Investment: The terms in curly brackets appear. Investment makes it more likely for $\Gamma_{q\tau}$ to be positive. Hence, strong reversal is less likely with investment.

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[27] One can show that Assumption A1 is always satisfied in the equilibrium of the model with CES utility and production with $\sigma < 1$ and without physical capital.

[28] As in the main text, we use the first period expenditure share, $\partial C_1 / \partial M$, as a measure of impatience.

[29] We interpret IES here as a weighted average of the intertemporal elasticities of substitution in both regions, such that IES<1 if $\partial C_1 / \partial r_2 + \partial C_1^* / \partial r_2 > 0$.