Uncertainty, Investment and Capital Accumulation: A Structural Econometric Approach

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To my parents, who seed the belief in truth and beauty.

To my husband, Qu Feng, who completes me on the path to be "a couple of economists".

I declare that this thesis represents my own work, except where otherwise specified, and that it has not been previously included in a thesis, dissertation or report submitted to this University or to any institution for a degree, diploma or other qualifications.

Signature: **Date:**

ABSTRACT

This thesis contributes to the empirical literature about how uncertainty affects firm-level investment behavior and capital accumulation using a structural econometric approach. Chapter 2 surveys the literature and highlights that there are two key channels through which uncertainty may affect investment decisions. One reflects the non-linearity of operating profits in stochastic demand or productivity parameters, summarized as the Hartman-Abel-Caballero (HAC) effect. Another reflects frictions in capital adjustment, summarized by different forms of capital adjustment costs: partial irreversibility, a fixed cost of undertaking any investment and quadratic adjustment costs. Chapter 3 presents simulation evidence about the effects of uncertainty on investment dynamics and capital accumulation through different forms of adjustment costs. Using the Method of Simulated Moments, Chapters 4 and 5 estimate fully parametric structural investment models, for panels of Brazilian and UK manufacturing firms, respectively. Chapter 4 investigates the effects of reducing capital adjustment costs. Counterfactual simulations indicate that investment would be much more responsive to new information about profitability if firms in Brazil faced a lower level of adjustment costs. A lower level of adjustment costs would also induce firms to operate with substantially higher capital stocks. Both these effects are mainly due to the importance of the estimated quadratic adjustment costs. Chapter 5 then investigates the effects of changing the level of uncertainty. The estimated investment models predict a small effect of uncertainty on investment dynamics in the short-run, and a negative and potentially large effect of uncertainty on capital accumulation in the long-run. The long-run effect of uncertainty operates through the negative effect of quadratic adjustment costs in the baseline model, or through a richer combination of effects in an extended model that allows discount rates to vary with the level of uncertainty.

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CONTENT

Chapter 1

Introduction

Investment is one of the most important and volatile components of macroeconomic activity. In the short-run, the relationship between uncertainty and investment is central to understanding the business cycle. In the long-run, the effect of uncertainty on capital accumulation has significant implications for economic growth and development.

The main goal of this thesis is to contribute to the empirical literature about how uncertainty affects firm-level investment behavior and capital accumulation.

Achieving such a goal requires three methodological ingredients: first, firm-level investment data; second, a microeconomic model of investment that is consistent with the data; and third, an econometric approach that allows these questions to be addressed. This thesis incorporates these ingredients in a unified structural framework.

Under the assumptions of risk-neutrality, perfect capital markets, and no adjustment costs, within the framework of static neoclassical producer theory, a firm's optimal investment policy would equalize the marginal revenue product of capital and the user cost of capital, as derived by Jorgenson (1963). Two practical features of investment change the static problem into a dynamic one: uncertainty about future profitability and costly adjustment of the capital stock. Optimal investment then needs to take into account the intertemporal linkage between current investment and future returns to capital.

Chapter 2 surveys consensus, controversies and open questions in the literature on investment and uncertainty in such a dynamic framework. It illustrates that there are two key channels through which uncertainty may affect investment dynamics and capital accumulation. One reflects non-linearity of the operating profits with respect to the variables that characterize uncertainty, summarized as the Hartman-Abel-Caballero (HAC) effect. Another reflects the frictions in capital adjustment, summarized by different forms of capital adjustment costs. This implies that reliable evidence about the effects of uncertainty requires the specification and estimation of an investment model, which incorporates these two potential channels. This is achieved in a sequential order in Chapters 3 to 5.

The theoretical analysis in Chapter 3 focuses exclusively on the second channel. It extends the investment model of Abel and Eberly (1999), by generalizing the adjustment cost function from complete irreversibility only, to one which includes partial irreversibility, a fixed cost of undertaking any investment, and quadratic adjustment costs. In order to abstract the HAC effect, some commonly-used functional forms are chosen so that by construction, in the absence of any adjustment costs, both investment dynamics and capital accumulation are invariant to the level of uncertainty. This provides a useful benchmark that allows me to investigate two questions: first, what are the effects of different forms of adjustment costs, compared with the frictionless benchmark; and second, what are the effects of uncertainty in the presence of each of these forms of adjustment costs?

Chapter 3 finds that even such a simple investment model generates rich implications for investment behavior, depending on the precise form of adjustment costs. The impact effects of positive shocks to profitability on capital adjustment differ dramatically, according to the form of adjustment costs. These effects may be dampened, amplified or even reversed when translated into the expected level of the capital stock. Other parameters in the model also affect the quantities of interest in a substantial fashion. These findings highlight the importance of the empirical work undertaken in Chapters 4 and 5.

Using firm-level investment data for a panel of Brazilian manufacturing firms, Chapter 4 estimates a fully parametric structural investment model similar to that studied in Chapter 3, using the Method of Simulated Moments. Within this model, investment and capital accumulation are determined by five factors: the Jorgensonian user cost of capital, the production technology, the demand schedule, the stochastic process characterizing uncertainty in the profitability, and different forms of adjustment costs. For a given Jorgensonian user cost of capital, structural parameters for the other four factors are estimated by matching simulated model moments to empirical moments.

Distinctive features of the model estimated here are that, I allow for unobserved heterogeneity across firms in long-run profitability growth rate, and for measurement errors in the firm-level data. The empirical results suggest an important role for quadratic adjustment costs, although at least one type of non-convex adjustment costs is also needed to match features of this firm-level dataset.

The estimated model is used to investigate how these firms' investment dynamics and capital accumulation would differ if they faced different levels of adjustment costs at the estimated level of uncertainty. Counterfactual simulations indicate that investment would be much more responsive to new information about profitability if firms in Brazil faced a lower level of adjustment costs. In the long run, a lower level of adjustment costs would induce firms to operate with substantially higher capital stocks.

Both the theoretical analysis in Chapter 3 and the empirical findings in Chapter 4 therefore highlight the importance of the adjustment costs. Meanwhile they also raise the question studied in Chapter 5: given these capital adjustment costs, what would be the effects on investment dynamics and capital accumulation if firms faced a difference level of uncertainty? To address this, the model is extended to allow for the HAC effect. Identification requires variation across firms in the level of uncertainty, and a firm-level proxy for uncertainty to be available in the data. This chapter uses panel data for UK manufacturing firms, which is observed over a longer time period (1972-1991).

Chapter 5 captures the HAC effect by generalizing the investment model considered in Chapters 3 and 4. By modelling uncertainty in both productivity and demand, the investment model in this chapter has two important implications, based upon which I design a 'two-step' identification procedure.

First, in the absence of adjustment costs, the expected capital stock level could increase, decrease or be invariant to the level of uncertainty, depending on the relative weight of productivity and demand uncertainty in overall uncertainty. With this property, the identification of the HAC effect is transformed into identifying the relative weight of these different sources of uncertainty.

Second, by construction the linear homogeneity of the investment model implies the HAC effect affects the level of variables from this model, but not ratios or growth rates. Therefore despite the presence of the HAC effect, the irrelevance of the HAC effect on variables in ratio or growth rate allows me to estimate the four factors mentioned above, i.e. the production technology, the demand schedule, the stochastic process, and different forms of adjustment costs, by using information for variables in ratio or growth rate only, as achieved in Chapter 4. This is implemented in step 1 of the 'two-step' procedure.

In step 2 of the 'two-step' procedure, holding the parameters estimated from step 1 fixed, the model estimates the parameter that characterizes the HAC effect by matching information for variables in level. In particular, this requires matching the correlation between a measure of capital stock and a proxy of uncertainty.

Finally, as a robustness check, I consider a more general specification, which further allows for a risk-premium component in the discount rate. The correlation between the capital-to-sales ratio and uncertainty provides the identification for the additional parameter describing this discount rate effect.

With this empirical strategy, estimating the effects through each channel is transformed into estimating a set of structural parameters of the model. Using a simulated minimum distance estimator, these parameters are then estimated by matching simulated model moments with empirical data moments from a panel of UK manufacturing firms. The empirical results imply that a mix of partial irreversibility and quadratic adjustment costs fits the features of investment behavior in this dataset best; while the estimates for the HAC effect depend on whether the model allows for risk premium component in the discount rate or not.

Counterfactual simulations find that, in the short-run, the estimated investment model predicts a small effect of uncertainty on investment dynamics. This is mainly due to the importance of quadratic adjustment costs in the estimated model. In the long-run, this chapter estimates a negative and potentially large effect of uncertainty on capital accumulation, which operates through the negative effect of quadratic adjustment costs in the baseline model without discount rate effect, or through a richer combination of factors in the extended model with a discount rate effect.

Chapter 6 summarizes the findings of the thesis, discusses potential limitations of this methodology, and outlines some important and interesting extensions for future research.

Chapter 2

Uncertainty, Investment and Capital Accumulation:

What Do We Know and What Do We Not Know?

1 Introduction

What are the effects of uncertainty on the investment decision and capital accumulation of an ongoing firm? Since Hartman (1972), this research question has been interesting economists for a long time. This chapter surveys areas of consensus, controversies and open questions in the literature within a general dynamic programming framework. It shows that the effects of uncertainty depend on the forms of adjustment costs, the properties of the operating profit function and the quantities of interest. In particular, two special combinations of adjustment costs and operating profits are investigated to see why some well-known controversies have emerged; and whether each of them could provide unambiguous predictions on all the quantities of our interest. The final section reviews some empirical findings. It also highlights how a structural econometric approach can be applied to answer the part that we do not know yet.

2 Theoretical Framework

Under the assumptions of risk-neutrality and perfect capital markets, and within the framework of standard static neoclassical producer theory, a firm's optimal investment is to equalize the marginal revenue product of capital to the user cost of capital, as derived by Jorgenson (1963). Two practical features of investment change the static problem into dynamic: uncertainty about future economic environment and costly adjustment of capital stock. Optimal investment then needs to take into account the intertemporal linkage between current investment and future returns to capital. By combining different structures of capital adjustment costs that have been highlighted in the investment literature, Abel and Eberly (1994) unify an investment model under uncertainty in a dynamic programming problem.

2.1 A Dynamic Programming Problem

Consider that a risk-neutral firm chooses investment I_t to maximize the expected present value of current and future operating profit $\pi(Z_t, K_t)$ less the total investment cost $C(I_t, K_t)$, subject to the law of motion for capital stock K_t and an exogenous stochastic variable Z_t. This optimization problem can be represented by the Bellman equation

$$\begin{aligned} V(Z_t, K_t) &= \max_{I_t}\{\pi(Z_t, K_t) - C(I_t, K_t) + \frac{1}{1+r}E_t\left[V(Z_{t+1}, K_{t+1})\right]\} \quad (1) \\ s.t. \quad K_{t+1} &= K\left(I_t, K_t, \delta\right) \\ Z_{t+1} &= Z\left(Z_t, \rho, \mu, \sigma\right) \end{aligned}$$

where r is a constant discount rate; δ is a constant rate of capital depreciation; Z_t encompasses all the stochastic from factor prices, technology and demand, and represents the profitability of the investment project. Z_t is usually assumed to evolve according to a first-order Markov process with serial correlation ρ and growth rate μ. The parameter σ is the standard deviation of the exogenous innovations that drive Z_t, hence is the measure of uncertainty in this model.

2.2 Capital Adjustment Costs

The cost function $C(I_t, K_t)$ in (1) includes both a direct purchase cost of capital goods and the associated costs of capital adjustment. The adjustment costs are introduced into the investment literature as an convenient way to model those frictions that dampen the instantaneous response of investment to new information about profitability. In the early literature, as surveyed in Chirinko (1993), the adjustment costs merely refer to what is called convex adjustment costs in more recent literature. Irreversibility stands as another type of friction that affects firm's investment decision. $C(I_t, K_t)$ is augmented in Abel and Eberly (1994) to include both traditional convex adjustment costs and irreversibility, together with a fixed component of capital adjustment costs. Hamermesh and Pfann (1996) have provided an extensive discussion about the economic rationale of these adjustment costs and the appropriateness of the specification.

2.2.1 Convex Adjustment Costs

Two specifications are widely adopted in the literature assuming convex adjustment costs. Convexity is in the sense of the common assumption $C_{II}\left(I_t, K_t\right) > 0$.

The first specification further assumes $C_K(I_t, K_t) = 0$, for example, Hartman (1972), Abel (1983, 1984, 1985) and Caballero (1991); while the second assumes $C(I_t, K_t)$ is linearly homogenous in I_t and K_t, as in many macro studies on investment or growth, for example, Lucas and Prescott (1971), and Abel (2002).

2.2.2 Irreversibility, Partial Irreversibility or Costly Reversibility

The fact that investment sometimes appears irreversible was first modeled as a non-negativity constraint on investment, for example Lucas and Prescott (1971) and Pindyck (1988), or infinite convex costs for downward adjustment in Caballero (1991). However, within the framework of (1), a zero resale price of capital goods is sufficient to guarantee non-negative optimal investment. And more generally, a resale price that is lower than purchase price implies costly reversibility as studied in Abel and Eberly (1996) and Abel *et al.* (1996), which is also known as partial irreversibility in more recent literature, for example, Cooper and Haltiwanger (2006) and Bloom (2009). With this modelling strategy, the cost function has the properties that $C_K(I_t, K_t) = 0$, and $C(I_t, K_t)$ is piecewise linear in I_t.

2.2.3 Fixed Adjustment Costs

Two types of fixed costs are studied in the literature. The first assumes a lump-sum cost that has to be paid to set up a project, for example, Lee and Shin (2000). This is in fact a fixed operating cost often modelled in industry organization research. More generally, the second assumes fixed costs per unit time that are independent of the level of investment and are incurred at each point in time when investment is non-zero. Under this assumption, the fixed costs are modelled either proportional to operating profit (Abel and Eberly, 1994 and Bloom, 2009) or proportional to capital stock (Caballero and Leahy, 1996 and Cooper and Haltiwanger, 2006). If the operating profit is linear in K_t, both specifications of the second type imply that $C_I(I_t, K_t) = 0$, and $C(I_t, K_t)$ is linear in K_t.

2.3 Properties of Operating Profit

The instantaneous operating profit $\pi(Z_t, K_t)$ is the maximized short-run profit for a given capital stock, technology and demand, obtained by choosing optimal variable inputs or utilization of capacity. In order to obtain closed-form solutions, different combinations for technology and demand have been employed under different forms of adjustment costs, which determine two key properties of $\pi(Z_t, K_t)$: First, whether the operating profit $\pi(Z_t, K_t)$ is linear or concave in K_t, or equivalently, whether the marginal revenue product of capital $\pi_K(Z_t, K_t)$ is independent of K_t, i.e. whether $\pi_{KK}(Z_t, K_t) = 0$ or $\pi_{KK}(Z_t, K_t) < 0$; Second, whether the marginal revenue product of capital $\pi_K(Z_t, K_t)$ is convex, linear or concave in Z_t., i.e. whether $\pi_{KZZ}(Z_t, K_t) > 0$, $\pi_{KZZ}(Z_t, K_t) = 0$ or $\pi_{KZZ}(Z_t, K_t) < 0$.

For studying convex adjustment costs, given the convexity of $C(I_t, K_t)$, a linear operating profit function $\pi(Z_t, K_t)$ is sufficient to guarantee the concavity of the net revenue $\pi(Z_t, K_t) - C(I_t, K_t)$, hence the existence of a unique solution to (1). A combination of linearly homogenous technology and perfect competition in both factor and product markets generate the linearity of $\pi(Z_t, K_t)$ in K_t.

For example, Hartman (1972) assumes a constant return to scale Cobb-Douglas production function, $Q_t = K_t^{\beta} L_t^{1-\beta}$, where $0 < \beta < 1$. Suppose capital is quasi-fixed due to adjustment costs and there is a variable input, say labour L_t. The unit price of labour is w_t and unit price of output is p_t. Denote $Z_t = (p_t, w_t)$. By choosing optimal variable input L_t, the operating profit takes the form of $\pi(Z_t, K_t) = \max_{L_t}\{p_t Q_t - w_t L_t\} = f(Z_t)K_t$, where $\pi_{KK}(Z_t, K_t) = 0$, and $\pi_{KPP}(Z_t, K_t) = f''(Z_t) > 0$. The convexity of $f(Z_t)$ in this case, or of $\pi_K(Z_t, K_t)$ in more general cases, comes from the fact that for a given capital stock, after the uncertainty is resolved, the firm can partially compensate for what, *ex post*, appears to be a poor decision regarding the capital input by choosing an appropriate level of labour. The optimal choice must make $\pi_K(Z_t, K_t)$ increase faster when Z_t increases, and decreases slower when Z_t falls, which leads to the convexity of $\pi_K(Z_t, K_t)$ in Z_t. Figure 2.1 is based on Varian (1992). It provides an illustration for the convexity of the profit function in factor or output price under perfect competition and constant return to scale. Given $\pi(Z_t, K_t) = f(Z_t)K_t$ under these assumptions, this implies that $\pi_K(Z_t, K_t)$ is convex in Z_t. Hartman (1976) highlights the importance of substitutability between capital and labour in generating this convexity property, by generalizing the analysis for a Cobb-Douglas production function to a CES production function.

As proved in Abel (1983, 1984, 1985), the convexity of the operating profit function with respect to the stochastic variable that characterizes uncertainty, predicts a positive relationship between uncertainty and the expected investment expenditure due to the Jensen's inequality effect. This result is generalized in Caballero (1991) to different degree of homogeneity of the production technology and the market power of the firm.

In the investment literature, the important consequence regarding the effects of uncertainty on investment through the curvature of the marginal revenue product of capital with respect to the stochastic variable, is known as the '**Hartman-Abel-Caballero effect**'. Throughout this thesis, I denote it as the 'HAC effect' in short notation.

For studying non-convex adjustment costs, given the linearity of $C(I_t, K_t)$, a concave operating profit function $\pi(Z_t, K_t)$ is necessary for the existence of a unique solution to (1). This has been modelled in two alternative ways in the literature. First, as in Pindyck (1988), assume that each unit of capital provides the capacity to produce one unit of output Q_t per time period. Then a downward sloping demand curve such as $p_t = Z_t - Q_t$ is necessary to make $\pi_{KK}(Z_t, K_t) < 0$. Now if the firm has to utilize all the capacity that has been invested, $\pi_{KPP}(Z_t, K_t) = 0$. In contrast, if the firm could choose the optimal utilization of capacity according to the realization of Z_t, $\pi_{KZZ}(Z_t, K_t) > 0$. This convexity is also due to the possibility of instantaneous variability in operations, as if there was a variable input in production.

The second alternative is to use the same technology as that in the convex adjustment costs literature but assume imperfect competition. Together with the constant return to scale assumption, this would generate $\pi_{KK}(Z_t, K_t) < 0$ unambiguously. However, by assuming isoelastic demand function, two different specifications of demand uncertainty generate different curvatures of the marginal revenue product of capital. The first specification, for example, in Abel and Eberly (1996), interprets demand shocks Z_t as changes in the quantity demanded Q_t for any given price p_t (horizontal demand shocks), i.e. $Q_t = Z_t p_t^{-\varepsilon}$, where ε is the demand elasticity. $\pi_K(Z_t, K_t)$ is unambiguously concave in Z_t under this specification. The second one, for example, in Caballero (1991), interprets demand shocks Z_t as changes in the price p_t associated with any quantity of output demanded Q_t (vertical demand shocks), i.e. $p_t = Z_t Q_t^{(1-\psi)/\psi}$, where ψ is a markup coefficient and $\psi = \frac{\varepsilon}{\varepsilon - 1}$. In contrast, $\pi_K(Z_t, K_t)$ is unambiguously convex in Z_t under this specification. Which specification is a more realistic characterization of monopoly behavior remains an open question even in the theory of industrial organization (Klemperer and Meyer, 1989).

3 The Effects of Uncertainty

To study the effects of uncertainty, it is standard to apply a mean-preserving spread for the stochastic process Z_t. Operationally, that is to keep all the parameters in (1) constant and increase the measure of uncertainty σ.

3.1 Quantities of Interest

3.1.1 Short-Run Adjustment

Most studies in the literature have focused on the effects of uncertainty on the level (or rate) of investment undertaken by a firm. This is an instantaneous uncertainty investment relationship in the sense that it asks for two otherwise identical firms, whether the firm facing larger σ would have a higher or lower level (or rate) of investment than the firm facing smaller σ, if they get the same realization of Z_t. Denote this quantity of interest as $\frac{\partial I_t}{\partial \sigma}$ (or $\frac{\partial (I_t/K_t)}{\partial \sigma}$).

This is different from another quantity of interest—the effects of uncertainty on the expected level (or rate) of investment. This is an expected uncertainty investment relationship in the sense that it asks for the same firm, whether we would expect it to have higher or lower rate of investment on average when it faces a larger σ than when it faces a smaller σ. As pointed out by Dixit and Pindyck (1994), $\frac{\partial I_t}{\partial \sigma}$ (or $\frac{\partial (I_t/K_t)}{\partial \sigma}$) is different from $\frac{\partial E[I_t]}{\partial \sigma}$ (or $\frac{\partial E[I_t/K_t]}{\partial \sigma}$). This is because with a larger σ, which means a more volatile stochastic process, it is more likely for this firm to get more extreme realizations of Z_t than with a smaller σ. In other words, the support of Z_t would be "spreaded" with higher σ.

To taking into account this support effect, the impact effect of uncertainty has been studied in the more recent literature, for example, Bloom, Bond and Van Reenen (2007) and Bloom (2009). This quantity of interest can be denoted as $\frac{\partial I_t/\partial Z_t}{\partial \sigma}$(or $\frac{\partial (I_t/K_t)/\partial Z_t}{\partial \sigma}$), which asks how different level of uncertainty would affect the responsiveness of the level (or rate) of investment to changes in Z_t.

3.1.2 Long-Run Accumulation

Despite the importance of short-run capital adjustment to the business cycle, the stock of capital is what ultimately produces output and creates firm value. Hence it is important and interesting to investigate the effects of uncertainty on capital stock. Abel (1984) is the first paper that explicitly studies the effects of uncertainty on expected long-run capital stock in the presence of convex adjustment costs, in which the quantity of interest is $\frac{\partial \lim_{s\to\infty} E_t[K_s]}{\partial \sigma}$. A related quantity of interest is $\frac{\partial \lim_{s\to\infty} E_t[K_s^I]/E_t[K_s^R]}{\partial \sigma}$ investigated in Abel and Eberly (1999), where $E_t[K_s^I]$ and $E_t[K_s^R]$ are expected long-run capital stock under complete irreversibility and reversibility, respectively.

The fact that the numerator is normalized by the frictionless capital stock in Abel and Eberly (1999) but not in Abel (1984) is due to different assumptions about the stochastic process Z_t. In Abel and Eberly (1999), Z_t evolves according to a random walk ($\rho = 1$), which makes analytical solution possible, but meanwhile makes the variance of Z_t hence K_t grow without bound as the forecast horizon grows. Therefore a normalization by the frictionless capital stock is necessary to stabilize the quantity of interest. In contrast, Abel (1984) assumes a mean-reverting process of Z_t ($\rho < 1$) so that there would be a steady state capital stock in the long run.

3.2 What Do We Know?

It is well-known that in general there is no closed-form solution to the optimization problem (1) unless some particular functional form is assumed and/or the problem is simplified into a two-period horizon context. Except some interesting special cases, such as under a particular form of fixed adjustment costs studied in Caballero and Leahy (1996), it is also well known that in general, the investment incentive in the dynamic programming (1) can be summarized by the marginal valuation of installed capital, i.e. $q_t = \frac{\partial V(P_t, K_t)}{\partial K_t}$, a quantity known as marginal q.

3.2.1 Finding I: Unambiguously Positive HAC Effect

Suppose the operating profit function is linear in K_t and the marginal revenue product of capital is convex in Z_t, so that it takes the form as $\pi(Z_t, K_t) = f(Z_t)K_t$ with $f''(Z_t) > 0$. Suppose the augmented adjustment costs function $C(I_t, K_t)$ does not depend on K_t, i.e. $C_K(I_t, K_t) \equiv 0$. As proved in their Appendix B by Abel and Eberly (1994), the value of the firm is a linear function of the capital stock, i.e. $V(Z_t, K_t) = q_t(Z_t)K_t + J(Z_t)$, so that the marginal q is independent of K_t. Furthermore, the convexity of $f(Z_t)$ guarantees that q_t increases with σ for a given Z_t. Since investment I_t is a strictly increasing function of q_t if $C(I_t, K_t)$ only includes convex adjustment costs which do not depend on K_t, it follows directly that $\frac{\partial I_t}{\partial \sigma} > 0$. This is a generalization of the findings in Hartman (1972) and Abel (1983, 1984, 1985). In case of irreversibility so that $C(I_t, K_t)$ does not depend on K_t neither, I_t is a non-decreasing function of q_t, it then follows $\frac{\partial I_t}{\partial \sigma} \geq 0$. This is a generalization of the special case for perfect competition and irreversibility studied in Caballero (1991). One obvious reason for the strong and unambiguous result under this specification is the HAC effect that has been discussed above. A less obvious reason is due to the irrelevance of capital stock in the investment decision that will be discussed next.

3.2.2 Finding II: Unambiguously Negative Marginal Call Option Effect

Suppose the operating profit function is concave in K_t and the augmented adjustment costs function $C(I_t, K_t)$ includes partial irreversibility. In a two-period horizon problem, Abel *et al.* (1996) show that the marginal q in period 1 includes three components, i.e. $q = N(Z_2, K_1) + \frac{1}{1+r}P'(Z_2, K_1) - \frac{1}{1+r}C'(Z_2, K_1)$. The first term $N(Z_2, K_1)$ is the expected present value of the marginal returns to capital evaluated at the first-period capital stock K_1, assuming capital stock remains constant at this level in the future, which is called naive net-present-value (NPV). The second component $\frac{1}{1+r}P'(Z_2, K_1)$ is the present value of the marginal put option, i.e. the option to sell capital in period 2 at a price lower than purchase price, which the firm will choose to exercise if Z_2 is lower than a critical value. Finally, the third component $\frac{1}{1+r}C'(Z_2, K_1)$ is the present value of the marginal call option, i.e. the option to buy capital in period 2 at the purchase price, which the firm will choose to exercise if Z_2 is higher than a critical value. This call option is subtracted from NPV because investing in period 1 extinguishes the option. In the special case of complete irreversibility, the put option disappears. It has been emphasized by Bertola (1988), Pindyck (1988), Dixit (1989), Dixit and Pindyck (1994) and Abel *et al.* (1996), that the marginal call option increases unambiguously as σ increases. This means all else being equal, through the channel of marginal call option, marginal q, the investment incentive would decreases with σ. Or equivalently, the critical value of Z_2 at or above which it is optimal to invest is higher with a higher σ.

3.2.3 Reconcile: The Relevance of Capital Stock in Marginal q

The strikingly different conclusions from the above two cases got reconciled in 1990's. When investment is irreversible and the future economic environment is uncertain, the firm cannot disinvest should the economic conditions change adversely. Hence an investment decision involves the exercising of the call option, the option to productively invest at any time in the future. However, as first pointed out by Caballero (1991) and further explored in Pindyck (1993), under finding I, the assumption of linearly homogeneous technology and perfect competition simplifies the operating profit to be linear in capital stock, but also makes the marginal q be independent of capital stock. Since the adjustment costs are a function of the level of investment and not the stock of capital, investment in each period is independent of investment or capital stock in any other period. Hence the firm does not "kill an option" when it invests. This effectively turns off the negative marginal call option effect under irreversibility while the positive HAC effect remains.

This comparison highlights the importance of intertemporal linkage between investment for studying the effects of uncertainty. A sufficient condition to achieve the intertemporal linkage is through the relevance of capital stock in marginal q, which will be true as long as the operating profit is strictly concave in capital stock, i.e. $\pi_{KK}(P_t, K_t) < 0$.

3.3 What Do We Not Know?

3.3.1 HAC Effect v.s. Marginal Call Option Effect

In the presence of irreversibility, Abel *et al.* (1996) provide important insights to the effects of uncertainty on the marginal q through three different components. An increase in σ increases (decreases) the NPV if the marginal revenue product of capital is a convex (concave) function of Z_t through the HAC effect. The marginal call option decreases with σ hence decreases the incentive to investment; while the marginal put option increases with σ hence increases the incentive to investment. Therefore, the net effect on the incentive to invest will be ambiguous.

Similarly, in presence of fixed adjustment costs, Lee and Shin (2000) show that uncertainty affects investment through two channels. The first is a positive HAC effect, due to the convexity of marginal revenue product of capital. The second is a negative call option value effect, due to the option value of waiting. Since the former comes from firm's ability to vary the optimal input of labour in face of uncertainty, the share of labour in the production function determines the convexity of the marginal revenue product of capital, so that the importance of the HAC effect. For a given option value of investment due to the fixed costs, the net effect of uncertainty therefore depends on the share of labour. Of course, how large the labour share is in a production function is an empirical question.

3.3.2 HAC Effect v.s. User Cost Effect

It is now clear that the specification with $\pi_{KK}(Z_t, K_t) = 0$, $\pi_{KZZ}(Z_t, K_t) = f''(Z_t) > 0$ and $C_K(I_t, K_t) = 0$ is not a sufficient framework to study the role of irreversibility. However, it provides analytical convenience to study the role of convex adjustment costs, i.e. $C_{II}(I_t, K_t) > 0$. Together with the assumption of mean-reverting stochastic process, Abel (1984) shows that the expected long-run capital stock $\lim_{s\to\infty} E_t[K_s] = \frac{\lim_{s\to\infty} E_t[I_s]}{\delta}$, while $\frac{\partial \lim_{s\to\infty} E_t[I_s]}{\partial\sigma} = \eta_f - F(\rho, \delta, r)\,\eta_{c_I}$, where $F(\rho, \delta, r)$ is a function of the Jorgensonian user cost of capital; η_f and η_{c_I} are the curvatures of the marginal revenue product of capital $f(Z_t)$ and the marginal adjustment costs $C_I(I_t, K_t)$. Hence the first term η_f measures the magnitude of the HAC effect, and the second term $F(\rho, \delta, r)\,\eta_{c_I}$ reflects the importance of the user cost effect appropriately defined to take account of the convex adjustment costs. Observe that with $f(Z_t)$ being linear and with $C(I_t, K_t)$ being quadratic, the firm's optimization problem is a linear-quadratic problem for which the certainty equivalence principle applies. Thus the expected long-run capital stock would be invariant to σ. This is because in this case, neither the HAC effect nor the amplified user cost effect is in operation. With $f(Z_t)$ being convex and $C(I_t, K_t)$ being quadratic, the expected long-run capital stock must increase with σ due to the positive HAC effect.

This provides some intuition to the more general case when $\pi_{KK}(Z_t, K_t) < 0$, $C_{II}(I_t, K_t) > 0$ and $C_K(I_t, K_t) \neq 0$. Although no closed-form relationship between $\lim_{s\to\infty} E_t[K_s]$ and σ can be derived, the trade-off would still between the positive HAC effect and the negative user cost effect due to capital adjustment costs. However, neither the sign nor magnitude are clear unless all the functional forms and structural parameters are imposed.

3.3.3 User Cost Effect v.s. Hangover Effect

Although in the presence of irreversibility and $\pi_{KK}(Z_t, K_t) < 0$, uncertainty has an unambiguous negative effect on the incentive to invest in current period through the marginal call option effect, it does not necessarily imply a negative effect on the expected long-run capital stock even in the absence of positive HAC effect. This is because under irreversibility, the firm cannot sell capital even if its marginal revenue product of capital is lower than the Jorgensonian user cost of capital, and it would be constrained by its own past investment behavior to have a capital stock that is higher than it would choose if it could start as fresh. Abel and Eberly (1999) call these two countervailing effects due to irreversibility as the 'user cost effect' and the 'hangover effect'.

Under the assumption of complete irreversibility and no physical depreciation, they prove that the user cost and hangover effects have opposite implications for $E_t[K_s^I]$ relative to $E_t[K_s^R]$, i.e. the expected long-run capital stock under irreversibility relative to that under reversibility. The two effects also give opposite answers to the sign of $\frac{\partial \lim_{s\to\infty} E_t[K_s^I]/E_t[K_s^R]}{\partial\sigma}$, i.e. regarding the effect of increased uncertainty on the expected long-run capital stock under irreversibility relative to that under reversibility, since higher uncertainty strengthens both the user cost and hangover effects.

Therefore it is not clear which effect would dominate in the special case Abel and Eberly(1999) have studied, since these effects depend on characteristics of the firm and its environment. It is not clear either which effect would dominate in more general cases, such as if complete irreversibility is relaxed and replaced by partial irreversibility, or if capital depreciates at a positive rate, since both user cost effect and hangover effect would become weaker.

4 A Structural Econometric Approach

This literature review highlights the difficulties in gauging the qualitative and quantitative importance of uncertainty on investment and capital accumulation. Relatively simple cases provide closed-form solutions and unambiguous predictions. But as a by-product of analytical convenience, a lack of intertemporal linkage of investment also invalidates the importance of adjustment costs involving option value. More interesting and realistic models need particular specifications for adjustment costs and operating profit, together with some other restrictions, like two-period horizon or no physical depreciation. Therefore in general no unambiguous conclusion can be reached, in some cases about the sign, in other cases about both the sign and the magnitude. All these ambiguities highlights the importance of empirical work.

4.1 Some Empirical Evidence

Two lines of empirical research in the investment literature are particularly relevant to the research question of our interest.

Since the effects of uncertainty depend on the form of adjustment costs, the first line asks what is the nature of the capital adjustment costs. Highlighted in the work of Doms and Dunne (1994), Cooper, Haltiwanger and Power (1999), and Caballero, Engel and Haltiwanger (1995), extensive empirical evidence of discrete and lumpy adjustments has been found in firm-level or plant-level data. Cooper and Haltiwanger (2007) take further step by providing structural estimates of adjustment costs, and find an investment model which mixes both convex, irreversibility, and a fixed adjustment cost which entails the disruption of production fits the data best.

The second line has aimed at signing the effect of uncertainty on investment directly. The first attempt comes from Leahy and Whited (1996). They use yearly volatility of daily returns of stock as a measure of uncertainty and estimate the relationship between investment rate and uncertainty. Various sample splitting tests are performed in order to test comparative static implications of effects through different channels. The main finding of the paper, appears to be at variance with the HAC effect and leaves irreversibility as the most likely explanation of the negative uncertainty-investment rate relationship. Guiso and Parigi (1999) measure uncertainty based on a survey of Italian firms' distribution of demand growth expectations. When planned investment rate is regressed over the measure of subjective uncertainty, they find the effect of an increase in uncertainty is to reduce the responsiveness of investment rate to demand shocks. This is consistent with the prediction from an investment model with the presence of irreversibility. The significant role of irreversibility has also been found in Bond, Bloom and Van

Reenen (2006), both numerically for a model with a rich mix of adjustment costs, and also empirically for a panel of manufacturing firms. In particular, when they allow for both a positive and a negative HAC effect (convex and concave marginal revenue product of capital in uncertainty), these effects have little impact on how uncertainty would affect investment dynamics in the simulated investment data. More recently, Bloom (2009) provides a structural framework to estimate the effects of uncertainty in an investment model with rich specifications of both capital and labour adjustment costs. His findings further emphasize the negative impact effect of uncertainty on investment dynamics.

4.2 The Remaining Puzzle

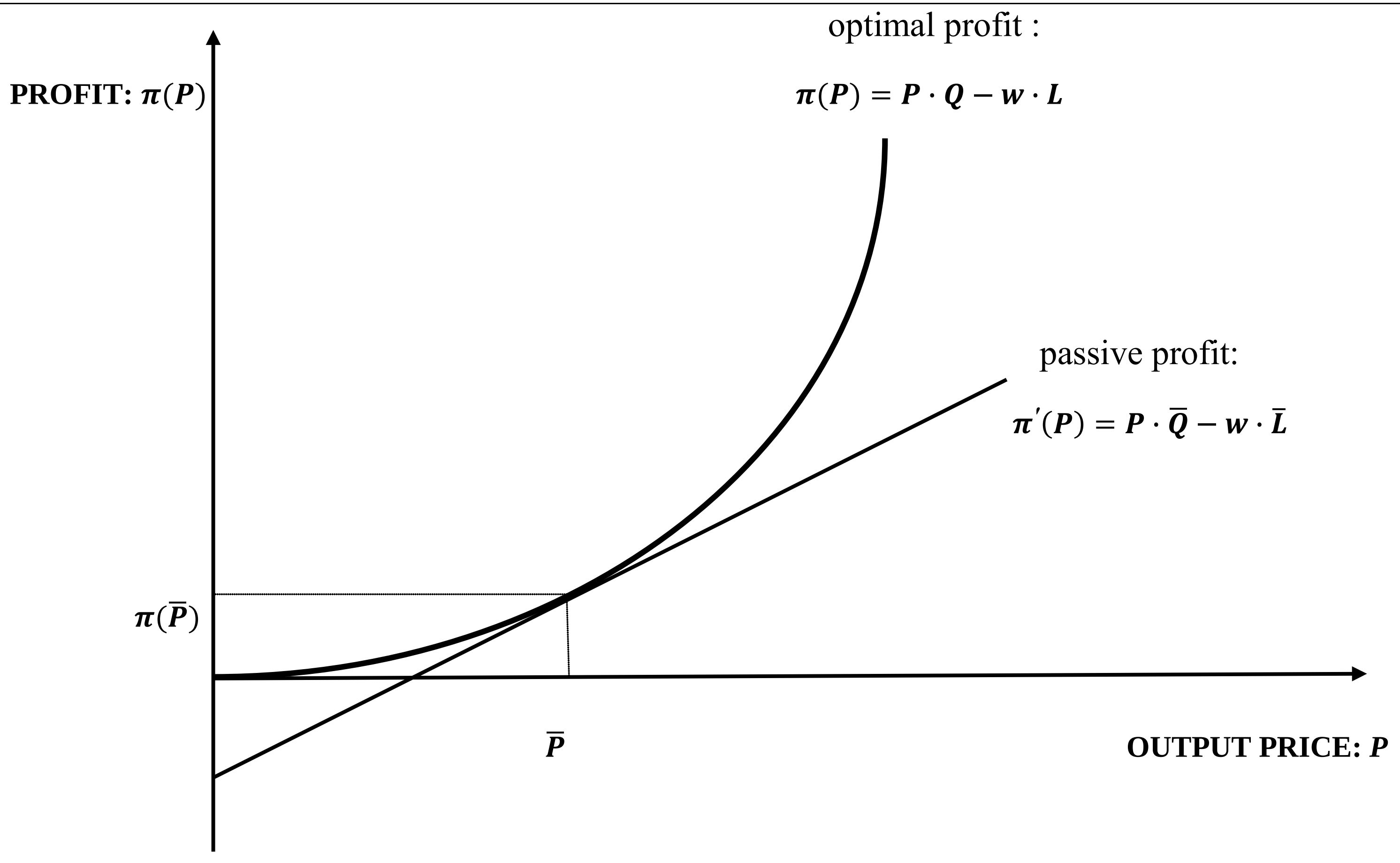
One interesting puzzle from these empirical evidence is that, why compared with the importance of irreversibility, there has been very little empirical evidence for the HAC effect. One possible explanation is that the HAC effect is a level effect but not a ratio effect. That is when the expected investment expenditure increases/decrease due to the HAC effect, the expected capital stock also increases/decrease proportionally. This explanation is consistent with the prediction from an investment model, in which both the operating profit function and the adjustment cost function are linear homogeneous in capital stock and the variables characterizing uncertainty. This explanation is also consistent with the empirical evidence surveyed above, in which investment rate instead of the level of investment expenditure is studied.

If this explanation is valid, it has two important implications. First, even if the HAC effect may be irrelevant in studying the short-run relationship between uncertainty and investment dynamics, it might be important in studying the long-run relationship between uncertainty and capital accumulation, in addition to the capital adjustment cost effect. Ignoring one of the two main channels suggested by the theoretical literature seems potentially dangerous. That is why this thesis has tried to take the HAC effect into account in estimating the long-run uncertainty-capital accumulation relationship.

Second, the limited existing empirical evidence about the sign and magnitude of the HAC effect also motivates further empirical research. The above explanation indicates the possibility of identifying the HAC effect by using information in level instead of in ratio. One major contribution of this thesis is to use variables in level and design an identification strategy for the HAC effect through a structural econometric approach.

The advantage of this approach is to provide a structural framework in which countervailing effects could be estimated to quantify the effects of uncertainty on the quantities of our interest unambiguously. The disadvantage of this approach is that the choice of particular functional forms is potentially important, which could rule out some of the effects discussed in the theoretical literature. However, as Dixit and Pindyck (1994) predict, a promising strategy is from a more ambitious attempt to estimate the optimality conditions of the fully stochastic dynamic programming problem using structural approach. This thesis is one of those attempts undergoing.

Figure 2.1: Convexity of Profit Function under Perfect Competition



Note: from *Varian* (1992), page 42.

Chapter 3

The Effects of Adjustment Costs and Uncertainty on Investment Dynamics and Capital Accumulation

1 Introduction

As surveyed in Chapter 2, one important wisdom draw from the literature is that, even in a risk-neutral Modigliani-Miller world, the neoclassical producer theory has generated very rich predictions about the effects of uncertainty on investment dynamics and capital accumulation. The two key channels are the curvature of the operating profit function, summarized as the HAC effect, and the frictions in the capital adjustment, summarized by different forms of capital adjustment costs.

This theoretical chapter focuses exclusively on the second channel, and focuses simultaneously on both short-run investment dynamics and long-run capital accumulation. In order to isolate the HAC effect, some commonly-used functional forms are chosen so that by construction in the absence of any adjustment costs, both investment dynamics and capital accumulation are invariant to the level of uncertainty. This provides an useful benchmark that allows me to investigate two sequential questions: first, what are the effects of different forms of adjustment costs, compared with the frictionless benchmark; and second, what are the effects of uncertainty through each form of these adjustment costs?

Compared with fixed and quadratic adjustment costs, economists have paid much more attention to irreversibility and achieved important insights about the effects of irreversibility on firm's investment behavior.

When investment is irreversible, the optimal investment policy is to purchase capital only as needed to prevent the marginal revenue product of capital from rising above an optimally derived hurdle. The hurdle, which is the user cost of capital appropriately defined to take account of irreversibility and uncertainty, is higher than the Jorgensonian frictionless user cost. This result is known as the 'marginal call option' effect in the option literature, or the 'user cost' effect in the investment literature. Firm anticipates that the irreversibility constraint may bind in the future and thus is more reluctant to invest today. This finding has been emphasized by Bertola (1988), Pindyck (1988), Dixit (1989), and Dixit and Pindyck (1994).

A related result is that an increase in uncertainty facing the firm tends to increase the user cost under irreversibility without affecting the user cost in the frictionless case. The increase in the user cost due to increased uncertainty tends to further reduce the optimal investment under irreversibility. This negative relationship is formalized as a negative effect of uncertainty on the impact effect of positive profitability shocks on investment dynamics, and has been extensively analyzed in Bloom, Bond and Van Reenen (2007) and Bloom (2009).

Nevertheless the relationship between uncertainty and capital stock level is much less clear in this class of model. This is because if demand for the firm's output is unusually low at date t, the firm would like to sell some of its capital at a positive price. However, under irreversibility, the firm cannot sell capital, and it would be constrained by its own past investment behavior to have a capital stock that is higher than it would choose if it could start fresh at date t. Abel and Eberly (1999) refer to this effect as the 'hangover' effect to indicate the dependence of the current capital stock on past behavior.

In the special case of complete irreversibility, no depreciation, a Brownian motion demand process and an infinite time horizon, Abel and Eberly (1999) demonstrate analytically that the user cost and hangover effects have opposite implications for the expected long-run capital stock. Neither the user cost effect nor the hangover effect dominates globally, so that irreversibility may increase or decrease capital accumulation relative to the frictionless benchmark. Furthermore, the two effects also give opposite answers regarding the effect of uncertainty on the expected long-run capital stock under irreversibility. Therefore an increase in uncertainty can either increase or decrease the long-run capital stock under irreversibility relative to that under frictionless case.

This chapter generalizes the model of Abel and Eberly (1999) to allow for other forms of capital adjustment costs, namely partial irreversibility, fixed adjustment costs and quadratic adjustment costs and solves it using numerical dynamic programming methods. Following Bloom (2009), I define the impact effect of positive demand shocks on capital adjustment as the quantity of interest for the short-run dynamics; and following Abel and Eberly (1999), I define the ratio of the expected capital stock level with adjustment costs to the expected capital stock level without adjustment costs as the quantity of interest for the long-run accumulation.

Concerning the short-run effects, I find the presence of complete irreversibility, partial irreversibility and quadratic adjustment costs all dampen the responsiveness of investment to new information about demand shocks. Furthermore, in the presence of complete irreversibility, partial irreversibility and fixed adjustment costs, the impact effect of positive demand shocks on capital stock adjustment is a non-increasing function of the level of uncertainty. This confirms the findings in Bloom (2009) for partial irreversibility and also highlights the importance of other forms of non-convex adjustment costs.

Concerning the long-run effects, my numerical solution replicates the analytical finding in Abel and Eberly (1999) for the case of complete irreversibility. I also find that the same conclusion applies to partial irreversibility, in the sense that the presence of partial irreversibility could either increase or decrease the expected long-run capital stock relative to that under frictionless case; and uncertainty does not ease the ambiguity regarding the long-effect of partial irreversibility, but rather deepens it. In contrast, in the presence of both fixed and quadratic adjustment costs, the expected long-run capital stock would be unambiguously lower than the frictionless level, due to an user cost of capital that is higher than the Jorgensonian user cost of capital. Furthermore, in the case of quadratic adjustment costs, this user cost effect gets stronger with an increase in uncertainty hence further decreases the level of the expected long-run capital stock.

This chapter also investigates the effects of other model parameters on these findings, in particular, the demand growth rate, the discount rate, the capital share in the production function and the demand elasticity. As another generalization for the framework in Abel and Eberly (1999), the numerical methods also allow me to study the effects of positive physical capital depreciation and a trend stationary demand process. Among these six parameters, most of them have clear predictions on how capital accumulation would change with these parameters under different forms of adjustment costs.

The rest of the Chapter is organized as follows. Section 2 constructs an investment model under uncertainty and characterizes the optimal investment decision with adjustment costs and uncertainty. Section 3 presents the main results, for the effects of adjustment costs and uncertainty on both short-run capital adjustment and long-run capital accumulation. Section 4 examines the effects of other model parameters. And Section 5 offers concluding remarks.

2 An Investment Model under Uncertainty

This section sets up a simple model of investment for a firm operating under uncertainty. The functional forms are chosen strictly following Abel and Eberly (1999), except for two variations. First, a discrete rather than continuous time, which allows me to solve this model numerically. Second, depreciation of capital stock and more general forms of capital adjustment costs, which confirm the analytical results in Abel and Eberly (1999) and generalize the analysis to other interesting cases.

2.1 Short-run Profit Optimization

Time is discrete and horizon is infinite. By paying capital adjustment costs, new investment I_t contributes to productive capital $\widehat{K}_t$ immediately in period t, which depreciates at the end of each period.[1]

Assumption 1 ***Timing:***

The law of motion for capital stock is

$$K_{t+1} = (1-\delta)\left(K_t + I_t\right) \equiv (1-\delta)\widehat{K}_t \tag{1}$$

where δ is the constant depreciation rate.

[1] Compared with alternative lagged timing assumption, such as $K_{t+1} = (1-\delta)K_t + I_t$, Assumption 1 does not affect the qualitative implication of the model, but allows for a closed-form solution to the investment problem in the frictionless case, which does not involve any expectation term. This provides a convenient benchmark for studying the effets of captial adjustment costs. In the special case of no depreciation as assumed in Abel and Eberly (1999), this timing difference even vanishes.

Consider a firm that uses capital stock $\widehat{K}_t$ and labour L_t to produce nonstorable output Q_t, according to a nonstochastic constant returns to scale Cobb-Douglas technology.

Assumption 2 ***Production:***

The production function is

$$Q_t = L_t^{1-\beta} \widehat{K}_t^{\beta} \tag{2}$$

where the capital share β *satisfies* $0 < \beta < 1$.

The firm faces an isoelastic, downward-sloping, stochastic demand curve. Denote X_t as the random component in demand, which can be interpreted as changes in the quantity demanded Q_t for any given price of output P_t, and is called 'horizontal demand shocks' in Abel and Eberly (1999). As discussed in Chapter 2, compared with the alternative 'vertical demand shocks', within this class of model, the specification for the horizontal demand shocks effectively turns off the HAC effect.

Assumption 3 ***Demand:***

The demand schedule is

$$Q_t = X_t P_t^{-\varepsilon} \tag{3}$$

where $-\varepsilon < -1$ *is the demand elasticity with respect to price.*

The demand shift parameter X_t is the only source of uncertainty in this model. Abel and Eberly (1999) assume X_t evolves exogenously according to a geometric Brownian motion with mean μt and variance $\sigma^2 t$. By Ito's Lemma, this implies the log of X_t follows a Brownian motion with mean $(\mu - 0.5\sigma^2)\, t$ and variance $\sigma^2 t$. The discrete time analogue of this process is described in the following assumption:

Assumption 4 ***Demand Stochastic:***

The law of motion for X_t *is*

$$x_t \equiv \ln X_t = x_{t-1} + \widetilde{\mu} + e_t \tag{4}$$

where $\widetilde{\mu} = \mu - 0.5\sigma^2 > 0$, $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$, *and* $x_0 = 0$.

Firms making decisions in period t know X_t and the parameter values of X_0, μ and σ, but are uncertain about future levels of demand which depend on future realizations of the demand shocks e_t. The standard deviation of these demand shocks σ therefore measures the level of uncertainty faced by the firm in our model. The condition $\mu > 0.5\sigma^2$ guarantees that the marginal revenue product of capital has a non-degenerate ergodic distribution, as restricted in the Eq. (7) of Abel and Eberly (1999).

Labour is a variable input and can be adjusted instantaneously and costlessly. At each point of time, for given capital stock and demand realization, the firm chooses labour L_t to maximize its instantaneous operating profit $P_tQ_t - wL_t$, where w is a constant wage rate.

Lemma 1 ***Operating Profit:***

The maximized value of operating profit is given by

$$\pi(X_t, \widehat{K}_t) = const0 \cdot X_t^{\gamma} \widehat{K}_t^{1-\gamma} \tag{5}$$

where

$$0 < \frac{1}{\varepsilon} < \gamma = \frac{1}{1 + \beta(\varepsilon - 1)} < 1 \tag{6}$$

and

$$const0 = \left(\frac{\gamma\varepsilon - 1}{w}\right)^{\gamma\varepsilon - 1} (\gamma\varepsilon)^{-\gamma\varepsilon} \tag{7}$$

Proof: See the Eq. (3) in Abel and Eberly (1999).

2.2 Adjustment Cost Function

Besides production technology and demand conditions, the firm's investment behavior also depends on capital adjustment costs. I depart from the assumption in Abel and Eberly (1999) of complete irreversibility by allowing for more general forms of adjustment costs, as considered in Abel and Eberly (1994).

2.2.1 Partial Irreversibility

Partial irreversibility allows a gap between the purchase price of capital p^I and the sale price of capital p^S, as a result of capital specificity, or more generally, the adverse selection in the market for used capital goods. Normalize the purchase price

p^I to one and denote $b_i = 1 - p^S \geqslant 0$, so that the parameter b_i can be interpreted as the difference between the purchase price and the sale price expressed as a percentage of the purchase price. For example, $p^S = 0.8$ gives $b_i = 0.2$, indicating that the sale price is 20% lower than the purchase price.

Assumption 5 ***Partial Irreversibility:***

The functional form of partial irreversibility is

$$G(I_t) = -b_i I_t \mathbf{1}_{[I_t<0]}$$

where $\mathbf{1}_{[I_t<0]}$ *is an indicator equal to one if investment is strictly negative.*

Letting $p^S = 0$, or letting $b_i = 1$, ensures that the firm never chooses to sell capital, and mimics the investment behavior under a complete irreversibility constraint, which is the case studied in Abel and Eberly (1999).

2.2.2 Fixed Adjustment Costs

Fixed adjustment costs reflect those costs that are independent of the level of investment or disinvestment and are paid at each point of time if any investment or disinvestment is undertaken. I model the level of these costs to be proportional to the operating profit. Under this specification, first, these costs can be rationalized as profit loss due to the interruption in production during periods of large investment or disinvestment; second, these costs do not become irrelevant as the firm grows larger; third, they can be avoided by choosing zero investment.

Assumption 6 ***Fixed Adjustment Costs:***

The functional form of fixed adjustment costs is

$$G(X_t, K_t; I_t) = b_f \mathbf{1}_{[I_t \neq 0]} \pi_t$$

where $\mathbf{1}_{[I_t \neq 0]}$ *is an indicator equal to one if investment is non-zero.* π_t *is defined in equation (5). The parameter* b_f *is interpreted as the fraction of operating profit loss due to any non-zero investment.*

2.2.3 Quadratic Adjustment Costs

Quadratic adjustment costs reflect those costs that increase convexly in the level of investment or disinvestment. I consider a specification that includes three features. First, the costs are quadratic in investment rate, to reflect higher costs due to more rapid changes and to allow for analytical tractability. Second, the costs attain their minimum value of zero at zero investment, so that the firm can avoid these costs by setting investment equal to zero. Third, the level of these costs is proportional to capital stock, so that a given investment rate imposes costs that increase with the size of the firm, and do not become irrelevant as the firm grows larger.

Assumption 7 ***Quadratic Adjustment Costs:***

The functional form of quadratic adjustment costs is

$$G(K_t; I_t) = \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t$$

where b_q measures the magnitude of quadratic adjustment costs.

Our model allows for these three forms of adjustment costs, specifying the adjustment cost function to be

$$G(X_t, K_t; I_t) = -b_i I_t \mathbf{1}_{[I_t<0]} + b_f \mathbf{1}_{[I_t \neq 0]} \pi_t + \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t \tag{8}$$

2.3 Dynamic Optimization

Denote $\Pi(X_t, K_t; I_t)$ as the net revenue of the firm in each period t. That is

$$\Pi(X_t, K_t; I_t) = \pi(X_t, K_t; I_t) - G(X_t, K_t; I_t) - I_t \tag{9}$$

Or equivalently,

$$\begin{aligned} &\Pi(X_t, K_t; I_t) \\ = &\begin{cases} \pi(X_t, K_t; I_t) & \text{if } I_t = 0 \\ (1-b_f)\,\pi(X_t, K_t; I_t) - \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t - I_t & \text{if } I_t > 0 \\ (1-b_f)\,\pi(X_t, K_t; I_t) - \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t + (1-b_i)\,|I_t| & \text{if } I_t < 0 \end{cases} \end{aligned}$$

Assumption 8 *The firm is risk-neutral and discounts future net revenue at a constant rate r, where $r > \exp(\mu) - 1$.*

As explained in Appendix B.1, the condition $r > \exp(\mu) - 1$ guarantees a finite firm value hence is one of those necessary conditions for the existence of a solution to firm's optimization problem.

In each period investment is chosen to maximize the discounted present value of current and expected future net revenues, where expectations are taken over the distribution of future demand conditions.

$$V(X_t, K_t) = \max_{I_t} E_t \left\{ \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \Pi(X_{t+s}, K_{t+s}; I_{t+s}) \right\}$$

According to the Principle of Optimality (Theorem 9.2, Stokey and Lucas, 1989), this investment decision can be represented as the solution to a dynamic optimization problem defined by the stochastic Bellman equation

$$V(X_t, K_t) = \max_{I_t} \{\Pi(X_t, K_t; I_t) + \frac{1}{1+r} E_t [V(X_{t+1}, K_{t+1})]\} \tag{10}$$

together with the law of motion (1) and (4) for K_t and X_t. Here $V(X_t, K_t)$ is the value of the firm in period t; $E_t [V(X_{t+1}, K_{t+1})]$ is the expected value of the firm in period $t+1$ conditional on information available in period t.

2.4 Investment Policy

In the special case of no capital adjustment costs, there is a closed-form solution that describes the optimal investment policy analytically.

2.4.1 Frictionless Case

Proposition 1 ***Investment Policy in the Frictionless Case:***

If $G(X_t, K_t; I_t) \equiv 0$, the Euler equation for the optimization problem (10) is

$$const0 \cdot (1-\gamma) \cdot \left[\frac{X_t}{\widehat{K}_t} \right]^{\gamma} = \frac{r+\delta}{1+r} \tag{11}$$

Hence the optimal investment rate can be derived as

$$\frac{I_t^*}{K_t} = const1 \cdot \frac{X_t}{K_t} - 1 \tag{12}$$

Or equivalently expressed in levels, the optimal productive capital stock is

$$\widehat{K}_t^* = I_t^* + K_t = const1 \cdot X_t \tag{13}$$

where

$$const1 = \left[const0 \cdot (1-\gamma) \,/ \left(\frac{r+\delta}{1+r} \right) \right]^{\frac{1}{\gamma}} \tag{14}$$

Furthermore, if $\sigma = 0$, the optimal investment rate under certainty would be

$$\frac{I_t^{**}}{K_t} = \frac{\exp(\mu)}{1-\delta} - 1 \tag{15}$$

Proof: See Appendix A.1.

The left hand side of equation (11) is simply the marginal revenue product of capital, while the right side is known as the Jorgensonian user cost of capital. Hence in spite of the uncertainty about future demand, this intertemporal optimality condition is equivalent to the first order condition in a static decision problem of the neoclassical producer theory. This is solely the result of the firm being able to adjust its capital stock instantaneously and costlessly in this case.

Equations (12) and (13) imply that without any friction, the optimal investment rate is a linear function of demand relative to inherited capital stock to meet the imbalance between the optimal productive capital stock and the level of demand in each period, where the slope term ($const1$) reflects production technology (β), demand elasticity (ε), factor price (w), and the Jorgensonian user cost of capital $\left(\frac{r+\delta}{1+r}\right)$.

2.4.2 Friction Cases

In the presence of adjustment costs, uncertainty about future demand affects current investment since current and future adjustment of capital stock incurs costs. Optimal investment then needs to take into account the intertemporal linkage between current investment and future returns to capital and becomes indeed an interesting dynamic problem. However, only in very special case, such as the one studied in Abel and Eberly (1999), there is in general no analytical solution with the more general capital adjustment costs that I consider in equation (8). Appendix B.1 explains how I solve the dynamic optimization problem (10) numerically.

The investment model outlined above is fully parametric. Once I specify values for the parameters I could illustrate investment policy derived from the numerical solutions. Throughout Sections 2 and 3 of this chapter, I impose common parameter values as those in Fig. 1 of Abel and Eberly (1999). That is, depreciation rate $\delta = 0$, discount rate $r = 0.05$, capital share $\beta = 0.33$, demand elasticity $\varepsilon = 10$, demand growth rate $\mu = 0.029$. Given the restriction $\mu > \frac{1}{2}\sigma^2$, the highest level of uncertainty that could be considered is $\boldsymbol{\sigma} = 0.2405$. So I denote 0.2405 as the reference level of uncertainty in this model.

Figures 3.1-3.3 present the investment policy derived from the numerical solutions under different forms of capital adjustment costs and at half of the reference level of uncertainty ($\sigma = 0.5\boldsymbol{\sigma}$).

I plot the optimal investment rate (I_t/K_t) against the scaled demand ($const1 \cdot X_t/K_t - 1$), so that the investment policy in the frictionless case (12) would be a 45^o line and is plotted as a benchmark in each of these figures. Since this scaled demand is a monotonic increasing transformation of the marginal revenue product of capital, this 45^o line highlights the proposition that in the absence of any adjustment cost, investment rate is a continuous and strictly increasing function of the marginal revenue product of capital.

Figure 3.1a illustrates the investment policy with complete irreversibility only ($b_i = 1.0$, $b_q = b_f = 0$) that has been studied in Abel and Eberly (1999), and Figure 3.1b with partial irreversibility only ($b_i = 0.10$, $b_q = b_f = 0$) . In both these two figures, there is a region of inaction in the investment policy. Positive investment is triggered only when the marginal revenue product of capital reaches a right critical level; and for further higher levels of the marginal revenue product of capital the investment rate continues to be lower than what would be chosen in the frictionless case. Since the investment rate on the 45^o line would equalize the Jorgensonian user cost of capital and the marginal revenue product of capital, this implies the introduction of irreversibility increases the user cost of capital relative to the Jorgensonian user cost of capital, as highlighted in Abel and Eberly (1999). Under partial irreversibility, no disinvestment occurs unless the marginal revenue product of capital falls below a left critical level; and for further lower levels of the marginal revenue product of capital the disinvestment rate is much smaller than what would be chosen in the frictionless case. Under complete irreversibility, no disinvestment would ever happen, no matter how low the marginal revenue product of capital is. To summarize, the optimal investment policy under irreversibility is a 'barrier control' policy and a non-decreasing function of the marginal revenue product of capital.

Figure 3.2 illustrates both a region of inaction and discontinuities in the investment policy with fixed adjustment costs only ($b_f = 0.05$, $b_i = b_q = 0$). Similar to partial irreversibility, investment and disinvestment occur only when the marginal revenue product of capital exceeds the right and left critical levels that determine a region of inaction. Outside this region, the optimal investment decisions are quite different from those under partial irreversibility. Small adjustments to the capital stock do not generate benefits that are sufficiently high to warrant paying a fixed cost to implement them. Therefore capital stock adjusts to new information about demand through infrequent but large adjustments. When the marginal revenue product of capital exceeds the critical levels, optimal investment rate jumps discontinuously to an investment policy, in which the absolute magnitude is close to or even larger than that in the frictionless case, as the result of two countervailing effects. On the one hand, similar to irreversibility, the introduction of fixed costs increases the user cost of capital relative to the Jorgensonian user cost of capital. Hence the investment rate that equalizes the user cost of capital and the marginal revenue product of capital would be lower than the 45^o line. On the other hand, as illustrated in Cooper, Haltiwanger and Power (1999), with deterministic positive demand growth in this model (and/or physical capital depreciation more generally) the fixed costs of adjustment provides an incentive for the firm to 'overshoot its target', that is whenever investment is implemented, it is optimal to overinvest to make the marginal revenue product of capital lower than the user cost of capital. To summarize, the optimal investment policy under fixed adjustment costs is a 'jump control' policy and a non-decreasing function of the marginal revenue product of capital.

Figure 3.3 illustrates the optimal investment policy with quadratic adjustment costs only ($b_q = 0.50$, $b_i = b_f = 0$). Investment and disinvestment still occur at all levels of the marginal revenue product of capital. However, with quadratic adjustment costs, the increasing marginal adjustment costs penalize high rates of investment and disinvestment, capital stock adjusts to new information about demand through a series of continuous but small adjustments. To summarize, the optimal investment policy under quadratic adjustment costs is also a continuous and strictly increasing function of the marginal revenue product of capital, but much dampened compared with the 45^o line.

3 The Effects of Adjustment Costs and Uncertainty

This section examines the effects of uncertainty on the capital stock adjustment and the expected capital stock level under different forms of adjustment costs. In order to isolate the effect of uncertainty, I analyze changes in the distribution of demand shocks that preserve the mean level of demand $E[X_t]$.

Lemma 2 ***Mean-preserving Spread:***

Keeping μ constant and increasing σ is a mean-preserving spread for X_t, i.e. conditioning on $x_0 = 0$,

$$\begin{aligned} E[X_t] &= \exp(\mu t) \\ Var[X_t] &= [\exp(2\mu t)] \left[\exp\left(\sigma^2 t\right) - 1\right] \end{aligned}$$

Proof: See Appendix A.3.

3.1 Investment Policy at Different Level of Uncertainty

In order to study the effects of uncertainty, it is useful to illustrate how investment policy under different forms of adjustment costs would vary with uncertainty. In addition to the investment policy plotted at half of the reference level of uncertainty ($\sigma = 0.5\boldsymbol{\sigma}$) as those in Figures 3.1-3.3, Figures 3.4-3.6 also plot the investment policy at the reference level of uncertainty ($\sigma = \boldsymbol{\sigma}$) on the same figure, keeping all other parameter values constant. The comparison between the dark and light lines in Figures 3.4-3.6 therefore show the effects of uncertainty on investment policy under each form of adjustment costs.

Figure 3.4a and 3.4b consider these effects under complete irreversibility ($b_i = 1.0$, $b_q = b_f = 0$) and partial irreversibility ($b_i = 0.10$, $b_q = b_f = 0$). Both in the case of complete and partial irreversibility, higher level of uncertainty has two effects: first, enlarge the region of inaction; and second, lower the rate of positive investment, if positive investment would take place under both level of uncertainty. This implies the user cost of capital in the presence of irreversibility is an increasing function of uncertainty, a proposition emphasized in Abel and Eberly (1999). Similar effects are found in Figure 3.5 for fixed adjustment costs ($b_f = 0.05$, $b_i = b_q = 0$) as well. However, these effects are very different in Figure 3.6 for quadratic adjustment costs ($b_q = 0.50$, $b_i = b_f = 0$), where the shape of investment policy does not vary with the level of uncertainty, but a lower level of uncertainty implies a higher rate of investment for any given level of marginal revenue product of capital.

3.2 Short-run Capital Stock Adjustment

Following Bloom (2009), I illustrate the effects of adjustment costs and uncertainty on short-run investment dynamics by considering the impact effect of demand shocks (e_t) on the adjustment of the capital stock in the same period. One measure for how much capital stock is adjusted in period t is the change in the log of capital stock level in this period. Denote this measure as $\iota(t)$. That is

$$\iota(t) \equiv \Delta \ln \widehat{K}_t = \ln \widehat{K}_t - \ln \widehat{K}_{t-1} \tag{16}$$

Together with the capital accumulation formula (1), this quantity is approximately equal to investment rate net of depreciation rate, i.e. $\iota(t) = \ln\left[\left(1 + \frac{I_t}{K_t}\right)(1-\delta)\right] \simeq \frac{I_t}{K_t} - \delta$. A weaker impact effect indicates a smaller response of capital stock to new information about demand, hence slower investment dynamics.

Lemma 3 ***Capital Stock Adjustment in the Frictionless Case:***

If $G(X_t, K_t; I_t) \equiv 0$, the capital stock adjusts to demand shocks instantaneously *and fully according to a one-to-one linear relationship*

$$\iota^* (t) = \widetilde{\mu} + e_t \tag{17}$$

Proof: By equation (13) and (4).

Figures 3.7-3.9 illustrate how the level of uncertainty affects this impact effect under different forms of adjustment costs, at the reference level of uncertainty ($\sigma = \boldsymbol{\sigma}$) and at half of that level ($\sigma = 0.5\boldsymbol{\sigma}$).

I plot the capital stock adjustment ($\iota (t)$) against the demand shocks (e_t), so that the relationship in the frictionless case (17) would be a 45^o line and is plotted as a benchmark in each of these figures. Since $\widetilde{\mu} = \mu - \frac{1}{2}\sigma^2$, keeping μ constant and varying σ implies that $\widetilde{\mu}$ would vary with the level of uncertainty, which is reflected in the difference between the dash and solid straight lines.[2] However, uncertainty only makes the difference in the intercept but not in the shape and slope of how capital stock would adjust to demand shocks. Therefore in the absence of adjustment costs, the impact effect of demand shocks on capital stock adjustment is insensitive to the level of uncertainty.

With adjustment costs, I construct a simulated sample of 1000 hypothetical firms as the empirical analogue to answer the question of our interest. Under each form of adjustment costs, and at different level of uncertainty, firms apply the specific investment policies solved out as those illustrated in Figures 3.4-3.6, and draw independent innovations for demand shocks from the distribution $\epsilon_{i,t} \overset{i.i.d.}{\sim} N(0, 1)$. Each firm then adjusts its capital stock according to the investment policy and its realization of demand shocks. Same random numbers for the innovations are used in each simulation so as to isolate the effect of adjustment costs and uncertainty. The impact effect is then estimated by fitting a non-parametric regression[3] to data on capital stock adjustment ($\iota_{i,t}$) over demand shocks ($e_{i,t} = \sigma\epsilon_{i,t}$), where $i = 1, 2, \cdots, 1000$, and is plotted in circle ($\sigma = \boldsymbol{\sigma}$) and asterisk ($\sigma = 0.5\boldsymbol{\sigma}$) lines, respectively. Therefore, comparison between the circle/asterisk line and

[2] This is a natural result of the unit root process defined in equation (4). In order to keep the mean of X_t equal to μ, the mean of $\Delta \ln X_t$ and hence of $\Delta \ln \widehat{K}_t$ varies with the level of σ.

[3] These non-parametric regressions are obtained by malowess function in Matlab, with robust line fit and span=0.5.

the dash/solid 45^o line illustrates the effect of adjustment costs; and comparison between the circle line and asterisk line illustrates the effect of uncertainty.

Figure 3.7a and 3.7b consider these effects under complete irreversibility and partial irreversibility. As expected from the investment policy shown in Figure 3.1a and 3.1b, the impact effect of positive demand shocks on capital stock growth is much weaker under irreversibility than in the frictionless case. Whereas all firms adjust instantaneously and fully to new information about demand in the frictionless case, for those firms whose demand shocks leave them within their region of inaction, capital stock does not adjust at all in the current period under irreversibility. And for those firms that do some adjustment in the current period, the magnitude of the adjustment is much smaller than that in the absence of adjustment costs. Also as expected, the impact effect of negative demand shocks on capital stock adjustment is much weaker under partial irreversibility, reflecting the greater reluctance of firms to undertake disinvestment. And this impact effect is exactly zero under complete irreversibility, reflecting the no disinvestment constraint.

Consistent with the investment policies at different level of uncertainty illustrated in Figure 3.4a and 3.4b, the asterisk lines shown in Figure 3.7a and 3.7b illustrate that the impact effect of positive demand shocks on capital stock growth is noticeably stronger when firms subject to irreversibility operate in a less uncertain environment, although how capital stock responses to negative demand shocks is almost indistinguishable.

Figure 3.8 illustrates these effects under fixed adjustment costs only. Similar to the effect of irreversibility, for those firms whose demand shocks leave them within their region of inaction, capital stock does not adjust at all in the current period under fixed adjustment costs. Different from the effect of irreversibility, for those firms that do some adjustment in the current period, the magnitude of the adjustment is close to or even larger than that in the absence of adjustment costs. Similar to that under irreversibility, the impact effect of positive demand shocks on capital stock growth is also much stronger when firms subject to fixed adjustment costs operate in a less uncertain environment.

Figure 3.9 shows these effects under quadratic adjustment costs only. As expected from the investment policy shown in Figure 3.3, the impact effect of both positive and negative demand shocks on capital stock adjustment is much weaker under quadratic adjustment costs than in the frictionless case. Furthermore, consistent with the investment policies at different level of uncertainty illustrated in Figure 3.6, the impact effect is insensitive to the level of uncertainty over the whole range of demand shocks. Similar to that in the frictionless case, uncertainty only makes the difference in the intercept but not in the shape and slope of how capital stock adjusts to demand shocks under quadratic adjustment costs.

Properties illustrated in Figure 3. 4-6 are summarized in Proposition 2.

Proposition 2 ***The Short-run Effect of Adjustment Costs:***

if $b_i > 0$ *or* $b_q > 0$, $\partial\iota(t)/\partial e_t < \partial\iota^*(t)/\partial e_t$, $\forall e_t$;

if $b_f > 0$, *the effect is ambiguous.*

The Short-run Effect of Uncertainty:

if $b_i > 0$ *or* $b_f > 0$, $\partial^2\iota(t)/\partial e_t \partial\sigma \leq 0$, $\forall e_t > 0$;

if $b_q > 0$, $\partial^2\iota(t)/\partial e_t \partial\sigma = 0$, $\forall e_t$.

3.3 Long-run Capital Stock Accumulation

I illustrate the effects of adjustment costs and uncertainty on capital stock accumulation by considering the expected capital stock level $\left(E\left[\widehat{K}_t\right]\right)$ at different levels of uncertainty (σ).

Lemma 4 ***Expected Capital Stock Level in the Frictionless Case:***

If $G(X_t, K_t; I_t) \equiv 0$, *the expected capital stock level is given by*

$$E\left[\widehat{K}_t^*\right] = const1 \cdot \exp(\mu t) \tag{18}$$

Proof: By equation (13) and Lemma 2.

Following the Eq. (14a) in Abel and Eberly (1999), define $\kappa(t)$ as the ratio of the expected capital stock level at date t under different forms of adjustment costs to the expected capital stock level at date t in the frictionless case. That is

$$\kappa(t) = \frac{E\left[\widehat{K}_t\right]}{E\left[\widehat{K}_t^*\right]} \tag{19}$$

Lemma 4 implies the denominator in $\kappa(t)$ is invariant to the level of uncertainty and is a constant for given parameter values and date t. Therefore how $\kappa(t)$ is different from 1 reflects the effect of adjustment costs and how $\kappa(t)$ varies with σ reflects the effect of uncertainty.

Figures 3.10-3.13 illustrate how $\kappa(t)$ varies with σ under different forms of adjustment costs, over a range from a low level of uncertainty ($\sigma = 0.0485$) to the reference level of uncertainty ($\sigma = \boldsymbol{\sigma}$).

Figure 3.10 is an analytical replicate for the Fig. 1. in Abel and Eberly (1999) and is plotted according to their analytical solution derived in the particular case: complete irreversibility only ($b_i = 1.0$, $b_q = b_f = 0$), no depreciation ($\delta = 0$), infinite time horizon ($t = \infty$) and continuous time.

In Figures 3.11-3.13, I construct a simulated panel of 1,000,000 hypothetical firms for 100 years as the empirical analogue of the answer to the question of our interest. Under each form of adjustment costs, and at each level of uncertainty, same random numbers for the innovations are used in each simulation so as to isolate the effect of adjustment costs and uncertainty. I track the optimal capital stock $\left(\widehat{K}_{i,t}\right)$ that is chosen by each firm at each year and calculate the mean of capital stock levels across these firms in the same reference year, i.e. $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$, where $N = 1,000,000$, and $t = 100$. The large number of firms ($N = 1,000,000$) is chosen for two purposes: first, make the sample analogue $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ to be a good approximation to the population quantity $E\left[\widehat{K}_t\right]$ according to law of large number; and second, to average out the cyclical pattern between investment inaction and investment spike for individual firms in the presence of fixed adjustment costs. The reference year ($t = 100$) is chosen to be long enough that any effect of the initialization for the simulations has become negligible, and I check that similar results are found for later reference years. The denominator $E\left[\widehat{K}_t^*\right]$ is calculated according to the analytical solution (18). I also check that the difference between $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}^{*}$ and $E\left[\widehat{K}_{t}^{*}\right]$ is less than 0.5% and does not systematically vary with the level of uncertainty.

Figure 3.11a plots the ratio $\kappa(t)$ against σ with complete irreversibility only ($b_i = 1.0$, $b_q = b_f = 0$). Therefore it is a numerical replicate for the Fig 1. in Abel and Eberly (1999) and for Figure 3.10. The dashed line shows the actual estimates of $\kappa(t)$ at different levels of σ, which fluctuate somewhat as the result of numerical discretization. The solid line fits a simple 3-order polynomial regression through these points to illustrate the general pattern. This reproduces the main features of Figure 3.10, which confirms the analytical results in Abel and Eberly (1999) and suggests that our numerical results are in the right ballpark.

There are two key features of $\kappa(t)$ in this special case, which highlight the two important findings from Abel and Eberly (1999). First, $\kappa(t)$ may be greater than, less than , or equal to 1. Second, the behavior of $\kappa(t)$ is not monotonic in uncertainty. To be more specific, at very low levels of uncertainty, the presence of complete irreversibility has almost no effect on the expected level of the capital stock. Indeed as $\sigma \to 0$, complete irreversibility becomes irrelevant for firms that are experiencing certain, positive growth in demand. As σ increases, $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ initially increases relative to $E\left[\widehat{K}_{t}^{*}\right]$. Over this range the 'hangover' effect described in Abel and Eberly (1999) dominates the 'user cost' effect, so that on average $\kappa(t) > 1$ and $\partial\kappa(t)/\partial\sigma > 0$. This effect peaks at values of σ around 0.17, where $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 1.3 per cent higher than $E\left[\widehat{K}_{t}^{*}\right]$. After this peak, $\partial\kappa(t)/\partial\sigma < 0$. For higher values of σ, the 'user cost' effect dominates the 'hangover' effect so that $\kappa(t) < 1$. At the reference level of uncertainty, $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 0.3 per cent lower than $E\left[\widehat{K}_{t}^{*}\right]$.

Figure 3.11b considers a specification with partial irreversibility only. In this case, the relationship between $\kappa(t)$ and σ has a similar pattern to that shown under complete irreversibility in Figure 3.11a, but the magnitudes are different. At low levels of uncertainty, $\kappa(t)$ is again increasing in σ. At the peak $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 0.5 per cent higher than $E\left[\widehat{K}_{t}^{*}\right]$, and this peak occurs at lower values of σ around 0.13. At higher levels of uncertainty, $\kappa(t)$ is again decreasing in σ. At the reference level of uncertainty, the effect of uncertainty under partial irreversibility is to reduce $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ by about 7 per cent of $E\left[\widehat{K}_t^*\right]$. The 'hangover' effect appears to be less important in the case of partial irreversibility, where firms can choose to adjust capital stocks downwards, than it is under complete irreversibility.

Figure 3.12 presents the relationship with fixed adjustment costs only. First, different from that under irreversibility, $\kappa(t)$ is less than 1 at all levels of uncertainty in the presence of fixed adjustment costs. Firms with deterministic positive demand growth in this model (and/or with physical capital depreciation more generally) will want to have growing capital stocks, which requires positive investment on average. Under fixed adjustment costs, this adjustment will take the form of infrequent, large investments, implying a user cost of capital associated with fixed adjustment costs that is higher than the Jorgensonian user cost of capital. This user cost effect reduces $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ relative to $E\left[\widehat{K}_t^*\right]$ even in the case of complete certainty ($\sigma \to 0$). For example, at $\sigma = 0.0485$, $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 3.5 per cent lower than $E\left[\widehat{K}_t^*\right]$.

Second, under fixed adjustment costs, as illustrated in Figure 3.5, a higher level of uncertainty will first, enlarge the region of investment inaction. This will reduce both investment and disinvestment relative to that under a lower level of uncertainty, hence has an ambiguous effect on the expected capital stock level; Second, outside the region of inaction, a higher level of uncertainty will decrease the magnitude of investment and increase the magnitude of disinvestment relative to that under a lower level of uncertainty. This will unambiguously reduce the expected capital stock level. Finally, as illustrated in Figure 3.8, a higher level of uncertainty will enlarge the support of demand shocks, so that some larger capital adjustment which would not occur under a lower level of uncertainty will take place under a higher level of uncertainty. However, since this implies larger adjustment both upwards and downwards, it also has an ambiguous effect on the expected capital stock level. Taking into account all these effects, how $\kappa(t)$ would vary with σ under fixed adjustment costs is therefore ambiguous. For the case under illustration, there is an inverse U-shape relationship. At low levels of uncertainty, $\partial\kappa(t)/\partial\sigma > 0$ and $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ peaks at value of σ around 0.08. For higher levels of uncertainty, $\partial\kappa(t)/\partial\sigma < 0$. At the reference level of uncertainty, the effect of uncertainty in the presence of these fixed adjustment costs is to reduce $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ by about 5 per cent of $E\left[\widehat{K}_t^*\right]$. This is similar to the effect found in the specification with partial irreversibility, and considerably larger than the effect under complete irreversibility.

Figure 3.13 studies a case with quadratic adjustment costs only. First, similar to that under fixed adjustment costs, the presence of quadratic adjustment costs makes $\kappa(t) < 1$ even in an environment with complete certainty. For example, at $\sigma = 0.0485$, $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 5 per cent lower than $E\left[\widehat{K}_t^*\right]$. This is because at $\sigma = 0$, firms with deterministic positive demand growth in this model (and/or physical capital depreciation more generally) will require positive investment. With the functional form of the quadratic adjustment costs considered here, positive investment implies that some adjustment costs must be paid, hence a user cost of capital associated with quadratic adjustment costs that is higher than the Jorgensonian user cost of capital. This user cost effect unambiguously reduces $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ relative to $E\left[\widehat{K}_t^*\right]$ in the case of complete certainty. Appendix A.2 provides a formal illustration for this property by comparing the investment Euler equations with quadratic adjustment costs and in the frictionless case.

Furthermore, different from fixed adjustment costs, under quadratic adjustment costs, $\kappa(t)$ falls monotonically with σ. The magnitude of this effect is also much greater than what has been found with partial irreversibility or with fixed adjustment costs. For $\sigma = 0.15$, $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ is about 10 per cent lower than $E\left[\widehat{K}_t^*\right]$. At the reference level of uncertainty, the effect of uncertainty in the presence of these quadratic adjustment costs is to reduce $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ by about 35 per cent of $E\left[\widehat{K}_t^*\right]$. The intuition for this negative monotonic effect of uncertainty on the expected capital stock level is from the short-run capital adjustment behavior. As illustrated in Figure 3.9, a higher level of uncertainty will enlarge the support of demand shocks, which implies larger upwards and downwards adjustment will take place relative to that under a lower level of uncertainty. Different from fixed adjustment costs, under which the adjustment cost incurred is independent of the rate of investment, the cost incurred under quadratic adjustment costs increases monotonically with the rate of investment. Therefore the user cost effect associated with quadratic adjustment costs increases monotonically with the level of uncertainty and lead to an unambiguous negative relationship between $\kappa(t)$ and σ.

3.4 The Cost-to-Profit Ratio

Compared with $b_i = 0.1$, which can be interpreted as capital being sold at a price 10% lower than the purchase price if disinvestment occurs, it is less clear how costly capital adjustment is due to a fixed adjustment cost at the magnitude of $b_f = 0.05$ and a quadratic adjustment cost at the magnitude of $b_q = 0.50$. In order to have an indication of the relative magnitude of these costs, I calculate the sample average for the actual adjustment costs incurred as a ratio of the operating profit across these firms at the reference year.

When σ increases from 0.0485 to 0.2405, at $b_f = 0.05$, this cost-to-profit ratio increases monotonically from 0.31% to 0.65%, with an average ratio 0.38%; at $b_q = 0.50$, the cost-to-profit ratio increases monotonically from 0.36% to 1.56%, with an average ratio 0.79%. This implies the actual adjustment costs incurred in the presence of fixed and quadratic adjustment costs both increase with the level of uncertainty.

A related question is why the effect of uncertainty on $\kappa(t)$ appears to be much larger in the presence of quadratic adjustment costs than that in the presence of fixed adjustment costs, at least in the cases illustrated in Figure 3.13 and 3.12. Is it simply because that $b_q = 0.50$ implies a higher average cost-to-profit ratio than that implied by $b_f = 0.05$, or is it because the effects of uncertainty in the presence of these two forms adjustment costs are fundamentally different, even if they would incur the same cost-to-profit ratio on average?

In order to control for the first possibility, I gradually increase the value of b_f and decrease the value of b_q and redo this exercise. At $b_f = 0.10$, the cost-to-profit ratio increases monotonically from 0.48% to 0.90%, with an average ratio

0.6%; at $b_q = 0.30$, the cost-to-profit ratio increases monotonically from 0.23% to 1.25%, with an average ratio 0.6% too. At the median/mean level of uncertainty $\sigma = 0.1445$, the cost-to-profit ratio is about 0.55% for both $b_f = 0.10$ and $b_q = 0.30$.

Figure 3.14 plots how $\kappa(t)$ varies with σ at $b_f = 0.10$ and $b_q = 0.30$ on the same scale. The line for $\kappa(t)$ associated with $b_f = 0.10$ starts with 0.9433 at $\sigma = 0.0485$ and decreases to 0.9133 at $\sigma = 0.2405$; while the line for $\kappa(t)$ associated with $b_q = 0.30$ decreases from 0.9686 to 0.7941. And it is also around the median/mean level of uncertainty that these two lines intersect. Finally, if I add the lines for $\kappa(t)$ at $b_f = 0.05$ and $b_q = 0.50$ on the same figure, the line for $b_f = 0.10$ is below the one for $b_f = 0.05$ at any level of uncertainty; and the line for $b_q = 0.30$ is above the one for $b_q = 0.50$ at any level of uncertainty.

This exercise implies first: in the presence of both fixed and quadratic adjustment costs, the cost-to-profit ratio is an informative indicator for how much the adjustment costs would reduce the $\frac{1}{N}\sum_{i=1}^{N}\widehat{K}_{i,t}$ relative to $E\left[\widehat{K}_t^*\right]$. Second, $\kappa(t)$ responses to σ in a stronger pattern in the presence of quadratic adjustment costs than that in the presence of fixed adjustment costs, even if the costs incurred are similar on average over the range of uncertainty. Third, $\kappa(t)$ is a decreasing function of b_f and b_q. In other words, all else being equal, higher fixed and quadratic adjustment costs imply lower expected capital stock level.

Properties discussed in this section are summarized in Proposition 3.

Proposition 3 ***The Long-run Effect of Adjustment Costs:***

if $b_f > 0$ *or* $b_q > 0$, $\kappa(t) < 1$; *and* $\partial\kappa(t)/\partial b_f < 0$, $\partial\kappa(t)/\partial b_q < 0$;

if $b_i > 0$, *the effect is ambiguous.*

The Long-run Effect of Uncertainty:

if $b_q > 0$, $\partial\kappa(t)/\partial\sigma < 0$;

if $b_i > 0$ *or* $b_f > 0$, *the effect is ambiguous.*

4 The Effects of Other Model Parameters

These results are obtained by using particular parameter values imposed in Abel and Eberly (1999). This section studies for given level of uncertainty and capital adjustment costs considered in this model, whether and how these findings would vary with the value of other model parameters, which determines firm's characteristics and economic environment.

Following the section 5 of Abel and Eberly (1999), I consider the effects of demand growth rate (μ), the discount rate (r), the capital share in the production function (β) and the price elasticity of demand (ε). In addition, I also consider the effects of physical capital depreciation (δ) as a generalization.

I study how the investment policy, short-run capital adjustment and long-run capital accumulation would vary with each of these parameters, under each form of capital adjustment cost. Since for most of these parameters, the variation is most informative in the long-run capital accumulation, I plot these effects in Figures 3.15-3.17. The lines labelled as 'AE parameters' in these figures are plotted at those parameter values imposed in Abel and Eberly (1999) and employed in Figures 3.11-3.13, namely $\mu = 0.029$, $r = 0.05$, $\beta = 0.33$, $\varepsilon = 10$ and $\delta = 0$. I use these values as benchmark, and vary each of them individually in plotting each of the other five lines in these figures for comparative static analysis. The alternative values I consider are $\mu = 0.04$, $r = 0.10$, $\beta = 0.13$, $\varepsilon = 20$ and $\delta = 0.05$.

Abel and Eberly (1999) find that in the special case of complete irreversibility only, changing in the first four parameters leads to clear changes in capital accumulation. To be more specific, first, $\partial\kappa(t)/\partial\mu < 0$. This is because although the irreversibility constraint become less important in a higher growth environment, the hangover effect is weakened even more than the user cost effect as μ increases. Second, $\partial\kappa(t)/\partial r > 0$. While the user cost under both irreversibility and frictionless case rises with r, which tends to reduce the capital stock level in both cases, this effect is weaker under irreversibility than in the frictionless case. Finally, $\partial\kappa(t)/\partial\beta < 0$ and $\partial\kappa(t)/\partial\varepsilon < 0$. Since $\gamma = \frac{1}{1+\beta(\varepsilon-1)}$, as derived in equation (5), together, the capital share and the demand elasticity determine the concavity of the profit function, measured by γ. As γ rises, the profit function becomes more concave and thus deviations from the optimal frictionless capital stock are more costly to the firm, so that $\partial\kappa(t)/\partial\gamma > 0$, or equivalently $\partial\kappa(t)/\partial\beta < 0$ and

$\partial\kappa(t)/\partial\varepsilon < 0$. Figure 3.15a presents how $\kappa(t)$ varies with these four parameters in the presence of complete irreversibility only, and confirms these predictions from Abel and Eberly (1999).

Different from the complete irreversibility case, in the presence of other forms of adjustment costs, since there are no analytical results to draw on, the results illustrated in Figures 3.15b-3.17 are based on simulation. I attempt to provide some interpretations.

First, when there is partial irreversibility only, similar patterns for the effects of these four parameters that are discussed in Figure 3.15a, are found in Figure 3.15b as well.

Second, in the presence of fixed and quadratic adjustment costs, as illustrated in Figure 3.16 and 3.17, an increase in r will decrease $\kappa(t)$ if $b_f > 0$, since the future benefits of paying a fixed cost to implement investment are discounted by more; an increase in μ will decrease $\kappa(t)$ if $b_q > 0$, since the part of user cost associated with quadratic adjustment costs is an increasing function of μ. The effect of μ when $b_f > 0$ and the effect of r when $b_q > 0$, depend on the level of uncertainty and are therefore ambiguous. In contrast, similar to that under irreversibility, with relatively low capital share and inelastic demand, it is more costly for the firm to deviate from the optimal frictionless capital stock, hence an increase in β and ε will also decrease $\kappa(t)$ in the presence of fixed and quadratic adjustment costs.

Concerning the effect of depreciation rate, an increase in δ makes the investment policy under partial irreversibility more similar to that under complete irreversibility. For example, keeping all other parameters constant, at $\delta = 0.05$, even the partial irreversibility is at the magnitude of $b_i = 0.1$, no disinvestment would ever happen as if $b_i = 1$. This is because with a higher depreciation rate, it is optimal for firms to sell capital much less often, given any resale incurs a loss. This 'less necessary to disinvest' weakens the 'hangover' effect, thus also firm's reaction to it, that is the 'user cost' effect, hence the overall effect of δ on $\kappa(t)$ is ambiguous if $b_i > 0$, as illustrated in Figure 3.15a and 3.15b.

In the presence of fixed adjustment costs, with a higher δ, it is optimal for the firm to adjust capital stock more often than otherwise, therefore paying more adjustment cost on average. This user cost effect implies $\partial\kappa(t)/\partial\delta < 0$ if $b_f > 0$.

In the presence of quadratic adjustment costs, instead of simply affecting the level of $\kappa(t)$, a higher δ will also dampen the effect of uncertainty on $\kappa(t)$. This is because the user cost associated with quadratic adjustment increases with the rate of investment, while there are two determinants for the optimal investment rate in the presence of quadratic adjustment costs: one is the non-stochastic target level, which is an increasing function of δ as demonstrated in Proposition 1; another is the stochastic optimal response to the actual realization of demand shocks. Therefore an increase in δ increases the user cost by increasing the target investment rate. Meanwhile a higher δ also reduces the relative weight of the stochastic part in determining the optimal investment rate and thus the user cost, therefore reduce the sensitivity of $\kappa(t)$ to the level of uncertainty.

Properties discussed in this section are summarized in Proposition 4.

Proposition 4 ***The Effects of Model Parameters on*** $\kappa(t)$***:***

For $b_i > 0$, $b_f > 0$ *or* $b_q > 0$, $\partial\kappa(t)/\partial\beta < 0$ *and* $\partial\kappa(t)/\partial\varepsilon < 0$*;*

if $b_i > 0$, $\partial\kappa(t)/\partial\mu < 0$, $\partial\kappa(t)/\partial r > 0$*;*

if $b_f > 0$, $\partial\kappa(t)/\partial r < 0$, $\partial\kappa(t)/\partial\delta < 0$*;*

if $b_q > 0$, $\partial\kappa(t)/\partial\mu < 0$, $\partial^2\kappa(t)/\partial\sigma\partial\delta < 0$.

Finally, in order to allow for the demand shocks to have a persistent but not permanent effect on investment behavior, I consider an alternative specification for the stochastic process as a generalization for Abel and Eberly (1999).

Assumption 9 ***Alternative Demand Stochastic:***

The law of motion for X_t *is*

$$\begin{aligned} x_t &= \log X_t \\ x_t &= c + \mu t + \zeta_t \qquad (20) \\ \zeta_t &= \overline{\varsigma} + \rho\zeta_{t-1} + e_t = \overline{\varsigma} + \sum_{s=0}^{t-1} \rho^s e_{t-s} \end{aligned}$$

where $0 < \rho < 1$, $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$ *and* $\zeta_0 = 0$.

Under this trend-stationary specification, by adjusting the constant term c properly, keeping μ constant and increasing σ will be a mean-preserving spread for X_t as well.

Lemma 5 ***Alternative Mean-preserving Spread:***

For any $0 < \rho < 1$, *by choosing* $c = -0.5\sigma^2/(1-\rho^2)$, *keeping* μ *constant and increasing* σ *is a mean-preserving spread for* X_t, *i.e. conditioning on* $\zeta_0 = 0$,

$$\begin{aligned} E[X_t] &= \exp(\overline{\varsigma} + \mu t) \\ Var[X_t] &= [\exp(2\overline{\varsigma} + 2\mu t)]\left[\exp\left(\sigma^2/\left(1-\rho^2\right)\right) - 1\right] \end{aligned}$$

Proof: See Appendix A.3.

By setting the intercept term $\overline{\varsigma} = 0$, the stationary process (20) then implies the same expected value for X_t as predicted in Lemma 2 for the non-stationary process (4). Although (4) can be regarded as the limit case of (20) when $\rho \to 1$, the effects of uncertainty generated from (4) and (20) are not completely comparable in general. This is because for a given $E[X_t]$, $Var[X_t]$ is time-invariant under Assumption 9, but will increase with t under Assumption 4.

Appendix B.2 explains how I solve the dynamic optimization problem (10) numerically under this alternative specification. When $0 < \rho < 1$, similar patterns for the investment policy, short-run capital adjustment and long-run capital accumulation are found as those illustrated in previous sections of this chapter. Finally, I add the relationship between $\kappa(t)$ and σ derived at $\rho = 0.9$ and $t = 100$ in Figures 3.15-3.17 to illustrate the effect of uncertainty on capital accumulation under a trend-stationary stochastic process.

5 Conclusions

This chapter extends the framework of Abel and Eberly (1999) to illustrate how capital adjustment costs and uncertainty affect investment dynamics and capital accumulation. Even such a simple investment model could generate very rich implications for investment behavior according to different forms of adjustment costs. The impact effects of demand shocks on capital adjustment also dramatically differ depending on the form of adjustment costs; however, these effects may be dampened, amplified or even reversed when translated into the expected long-run capital stock level. Other parameters in the model would also affect the quantities of our interest in a substantial fashion.

These findings therefore highlight the importance of empirical work. In order to obtain the right sign and quantify the magnitude for the effects of uncertainty, it is therefore important to study which form of capital adjustment costs could explain the investment behavior best, conditioning on a good control for the firm's characteristics and economic environment.

This motivates the structural estimation in Chapters 4 and 5.

Appendix A: Proof

A.1 Proof for Proposition 1

In the frictionless case $G(K_t, X_t; I_t) = 0$, so that the Bellman equation is

$$V(K_t, X_t) = \max_{I_t}\{const0 \cdot X_t^\gamma (K_t + I_t)^{1-\gamma} - I_t + \frac{1}{1+r}E_t [V(X_{t+1}, K_{t+1})]\}$$

with the law of motion for capital stock

$$K_{t+1} = (1-\delta)(K_t + I_t)$$

Taking derivative with respect to K_t on both sides of the Bellman equation generates the quantity known as marginal q

$$\frac{\partial V_t}{\partial K_t} = 1 \quad \text{(A1)}$$

which is a constant due to our timing assumption.

The first order condition with respect to K_{t+1} is

$$\frac{\partial V_t}{\partial K_{t+1}} = const0 \cdot \frac{1-\gamma}{1-\delta}\left[\frac{X_t}{K_t + I_t}\right]^\gamma - \frac{1}{1-\delta} + \frac{1}{1+r}E_t\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = 0 \quad \text{(A2)}$$

Replacing $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ in (A2) with (A1) leads to the investment Euler equation

$$const0 \cdot (1-\gamma) \cdot \left[\frac{X_t}{\widehat{K}_t}\right]^\gamma = \frac{r+\delta}{1+r} \quad \text{(A3)}$$

Rearranging this equation gives the optimal investment rate in equation (12)

$$\frac{I_t^*}{K_t} = const1 \cdot \frac{X_t}{K_t} - 1$$

and optimal capital stock in equation (13)

$$\widehat{K}_t^* = I_t^* + K_t = const1 \cdot X_t$$

If $\sigma = 0$, the law of motion for X_t implies that $X_t = X_{t-1}\exp(\mu)$. Then $K_{t+1}^* = K_t^* \exp(\mu)$. This implies

$$\widehat{K}_t^* \equiv \frac{K_{t+1}^*}{(1-\delta)} = \frac{K_t^* \exp(\mu)}{(1-\delta)} = const1 \cdot X_t$$

so that as claimed in equation (15), in the absence of adjustment costs and uncertainty, the optimal investment rate would be

$$\frac{I_t^{**}}{K_t} = \frac{\exp(\mu)}{1-\delta} - 1$$

A.2 Investment Euler Equation if $b_q > 0$, and $b_i = b_f = 0$

If there are **quadratic adjustment costs** only, $V(K_t, X_t)$ is differentiable so that we could also derive a closed-form investment Euler equation. Now the Bellman equation can be written as

$$\begin{aligned} V(K_t, X_t) &= \max_{I_t}\{const0 \cdot X_t^{\gamma}\,(K_t + I_t)^{1-\gamma} - \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t - I_t \\ &\quad + \frac{1}{1+r}E_t\left[V(X_{t+1}, K_{t+1})\right]\} \end{aligned}$$

Taking derivative with respect to K_t on both sides of this equation, marginal q now becomes

$$\frac{\partial V_t}{\partial K_t} = 1 + \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 \tag{A1'}$$

The first order condition with respect to K_{t+1} is

$$\begin{aligned} \frac{\partial V_t}{\partial K_{t+1}} &= const0 \cdot \frac{1-\gamma}{1-\delta}\left[\frac{X_t}{K_t + I_t}\right]^{\gamma} - \frac{1}{1-\delta} - \frac{b_q}{1-\delta}\left(\frac{I_t}{K_t}\right) \\ &\quad + \frac{1}{1+r}E_t\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] = 0 \end{aligned} \tag{A2'}$$

Replacing $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ in (A2') with (A1') and moving one-period forward leads to the Euler equation

$$const0 \cdot (1-\gamma) \cdot \left[\frac{X_t}{\widehat{K}_t}\right]^{\gamma} = \left(\frac{r+\delta}{1+r}\right) + b_q\left(\frac{I_t}{K_t}\right) - \frac{b_q}{2}\left(\frac{1-\delta}{1+r}\right)E_t\left[\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right] \tag{A3'}$$

Both in equation (A3) and (A3'), the left hand side is simply the marginal revenue product of capital, the right hand side is the user cost of capital. In addition to the Jorgensonian user cost of capital $\left(\frac{r+\delta}{1+r}\right)$ in the frictionless case, the presence of quadratic adjustment costs introduces an additional term

$$c = b_q\left(\frac{I_t}{K_t}\right) - \frac{b_q}{2}\left(\frac{1-\delta}{1+r}\right)E_t\left[\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right]$$

in the user cost of capital. Since the quadratic term in c has a second-order magnitude compared with the linear term, for moderate investment rate that would happen in the presence of quadratic adjustment costs as those illustrated in Figure 3.3, the overall value of c is determined by the first term. In an environment positive demand growth ($\mu > 0$) and/or physical capital depreciation ($\delta > 0$), firms will need positive investment rate on average, i.e. $\frac{I_t}{K_t} > 0$. This implies $c > 0$, that is the user cost of capital under quadratic adjustment costs must be higher than the Jorgensonian user cost of capital. Therefore the expected capital stock level under quadratic adjustment costs must be lower than the frictionless level.

A.3 Proof for Lemma 2 and Lemma 5

It is well-known that for any random variable $p_t \overset{i.i.d}{\sim} N(a,\, b)$, if $P_t = \exp(p_t)$, the first moment for P_t is

$$\begin{aligned} E[P_t] &= \exp\left(E[p_t] + 0.5V[p_t]\right) \\ &= \exp(a + 0.5b) \end{aligned} \tag{A4}$$

And the second moment is

$$\begin{aligned} V[P_t] &= E\left[P_t^2\right] - \left(E[P_t]\right)^2 \\ &= E\left[\exp(2p_t)\right] - \left[\exp(a + 0.5b)\right]^2 \\ &= \exp\left[2a + 0.5 \cdot 4b\right] - \exp(2a + b) \\ &= \exp(2a + 2b) - \exp(2a + b) \\ &= \left[\exp(2a + b)\right]\left[\exp(b) - 1\right] \end{aligned} \tag{A5}$$

Under Assumption 4, the law of motion for X_t is

$$x_t \equiv \ln X_t = x_{t-1} + \widetilde{\mu} + e_t$$

where $\widetilde{\mu} = \mu - 0.5\sigma^2 > 0$, $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$, and $x_0 = 0$.

This implies

$$\begin{aligned} x_t &= x_0 + \widetilde{\mu}t + \sum_{s=0}^{t-1} e_{t-s} \\ E[x_t] &= \widetilde{\mu}t \text{ if } x_0 = 0 \\ Var[x_t] &= \sigma^2 t \end{aligned}$$

Therefore $x_t \overset{i.i.d}{\sim} N(\widetilde{\mu}t,\, \sigma^2 t)$. Apply the formula derived in (A4) and (A5),

$$\begin{aligned} E[X_t] &= \exp\left(\widetilde{\mu}t + 0.5\sigma^2 t\right) \\ &= \exp\left[\left(\mu - 0.5\sigma^2\right)t + 0.5\sigma^2 t\right] \\ &= \exp(\mu t) \end{aligned}$$

and

$$\begin{aligned}Var\left[X_t\right] &= \left[\exp\left(2\widetilde{\mu}t+\sigma^2 t\right)\right]\left[\exp\left(\sigma^2 t\right)-1\right] \\ &\quad \left[\exp\left(2\mu t\right)\right]\left[\exp\left(\sigma^2 t\right)-1\right]\end{aligned}$$

This proves Lemma 2.

Under Assumption 9, the law of motion for X_t is

$$\begin{aligned}x_t &= \log X_t \\ x_t &= c+\mu t+\zeta_t \\ \zeta_t &= \overline{\zeta}+\rho\zeta_{t-1}+e_t=\overline{\zeta}+\sum_{s=0}^{t-1}\rho^s e_{t-s}\end{aligned}$$

where $0<\rho<1$, $e_t=\sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$ and $\zeta_0=0$.

This implies

$$\begin{aligned}x_t &= c+\mu t+\overline{\zeta}+\sum_{s=0}^{t-1}\rho^s e_{t-s} \\ E[x_t] &= E[c+\mu t+\overline{\zeta}+\sum_{s=0}^{t-1}\rho^s e_{t-s}] \\ &= c+\mu t+\overline{\zeta} \\ Var\left[x_t\right] &= Var[c+\mu t+\zeta_t] \\ &= V[\sum_{s=0}^{t-1}\rho^s e_{t-s}] \\ &= \sigma^2/\left(1-\rho^2\right) \text{ when } 0<\rho<1 \text{ and } t\to\infty\end{aligned}$$

Therefore $x_t \overset{i.i.d}{\sim} N\left(c+\mu t+\overline{\zeta},\, \sigma^2/\left(1-\rho^2\right)\right)$. Apply the formula derived in (A4) and (A5), if $c=-0.5\sigma^2/\left(1-\rho^2\right)$,

$$\begin{aligned}E\left[X_t\right] &= \exp\left(c+\mu t+\overline{\zeta}+0.5\sigma^2/\left(1-\rho^2\right)\right) \\ &= \exp\left(\mu t+\overline{\zeta}\right)\end{aligned}$$

and

$$\begin{aligned}Var\left[X_t\right] &= \left[\exp\left(2(c+\mu t+\overline{\zeta})+\sigma^2/\left(1-\rho^2\right)\right)\right]\left[\exp\left(\sigma^2/\left(1-\rho^2\right)\right)-1\right] \\ &= \left[\exp\left(2\mu t+2\overline{\zeta}\right)\right]\left[\exp\left(\sigma^2/\left(1-\rho^2\right)\right)-1\right]\end{aligned}$$

This proves Lemma 5.

Appendix B: Numerical Methods

B.1 Solve (10) Numerically with Non-Stationary Variables

The problem I need to solve is

$$\begin{aligned} V(X_t, K_t) &= \max_{I_t}\{\Pi(X_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(X_{t+1}, K_{t+1})\right]\} \\ s.t.\ K_{t+1} &= (1-\delta)(K_t + I_t) \\ X_t &= \exp(x_t) \\ x_t &= x_{t-1} + \widetilde{\mu} + e_t \end{aligned} \tag{B1}$$

where $\widetilde{\mu} = \mu - 0.5\sigma^2 > 0$, $e_t = \sigma\epsilon_t, \epsilon_t \overset{i.i.d}{\sim} N(0, 1)$ and $x_0 = 0$.

Suppose there exists a unique solution to this problem. In order to find the solution it by standard numerical methods, for example, value function iteration, the state and control variables must be stationary.

This is a stochastic dynamic program with two state variables and one control variable. The demand shift parameter X_t and the beginning-of-period capital stock K_t are the state variables. Investment I_t, or equivalently the end-of-period capital stock $K_t + I_t$, is the control variable. The exogenous state variable X_t follows an $I(1)$ process; the state variable K_t endogenously follows X_t hence is also an $I(1)$ process; the linear capital accumulation formula then implies I_t is an $I(1)$ process as well. A useful strategy in such situation is to normalize the value function as suggested by Bloom (2009).

According to Proposition 1, in the absence of adjustment costs, the analytical solution for optimal capital stock is

$$K^*_{t+1} \equiv (1-\delta)\cdot \widehat{K}^*_t = (1-\delta)\cdot const1 \cdot X_t$$

The law of motion for X_t implies for given information at $t-1$, $E_{t-1}[X_t] = X_{t-1}\exp(\mu)$. Then on the balanced-growth-path, $K^*_{t+1} = K^*_t \exp(\mu)$, so that in the absence of adjustment costs

$$K^*_t = steady \cdot X_t \tag{B2}$$

where

$$steady = (1-\delta)\cdot const1/\exp(\mu)$$

This indicates that the ratio X_t/K_t^* is a constant. Bloom (2000) shows that with any finite adjustment costs, the ratio K_t^*/K_t is stationary or, equivalently, that $\ln K_t^*$ and $\ln K_t$ are cointegrated. Therefore the ratio X_t/K_t must also be stationary.

Furthermore, note that the operating profit and the adjustment cost functions are both homogenous of degree one in $(X_t, K_t; I_t)$. This implies the revenue function $\Pi(X_t, K_t; I_t)$ and hence the value function $V(X_t, K_t; I_t)$ must be homogenous of degree one in $(X_t, K_t; I_t)$, so that (B1) can be arranged as

$$K_t V(g_t) = \max_{i_t}\{K_t \Pi(g_t; i_t) + \frac{1}{1+r} K_{t+1} E_t\left[V(g_{t+1})\right]\}$$

where $g_t = X_t/K_t$ and $i_t = I_t/K_t$. Dividing by K_t on both sides, and using the capital accumulation formula,

$$V(g_t) = \max_{i_t}\{\Pi(g_t; i_t) + \frac{(1+i_i)(1-\delta)}{1+r} E_t\left[V(g_{t+1})\right]\}$$

Now define $\widetilde{g}_t = X_t/K_{t+1}$. Then g_t, i_t and $\widetilde{g}_t$ are linked by the law of motion

$$i_t = \frac{1}{(1-\delta)} \cdot \frac{g_t}{\widetilde{g}_t} - 1 \tag{B3}$$

Therefore by normalization I have transformed the problem (B1) into the following problem

$$V_t(g_t) = \max_{\widetilde{g}_t} \Pi(g_t; \widetilde{g}_t) + \frac{(g_t/\widetilde{g}_t)}{1+r} E_t\left[V_{t+1}(g_{t+1})\right] \tag{B4}$$

Now in this formulation, g_t is the state variable and $\widetilde{g}_t$ is the control variable, which are both stationary. Since on average $g_t/\widetilde{g}_t = K_{t+1}/K_t = \exp(\mu)$, the effective discount rate is $\frac{\exp(\mu)}{1+r}$, which is less than 1, as assumed in Assumption 8.

Using (B2) as a benchmark, in order to take into account the effect of uncertainty and capital adjustment costs, the support of g_t is defined be

$$\left[\exp\left(-\ln(steady) - 10\sigma\right),\ \ \exp\left(-\ln(steady) + 5\sigma\right)\right]$$

and this state space is discretized using 500 grid points. Conditional expectations are then calculated based on the transition matrix according to the normal CDF.

Since $\Pi(g_t; \widetilde{g}_t)$ is real valued, continuous, concave and bounded for any $0 < \gamma < 1$; and the constraint set $\Omega \equiv \{g_t(i)\}_{(i=1,2,\cdots 500)}$ is compact and convex; together with $\frac{\exp(\mu)}{1+r} < 1$, by the Contraction Mapping Theorem (Theorem 9.8 in Stokey and Lucas, 1989), a unique investment policy $i_t = h(g_t)$ must be obtained by using value function iteration.

The value function iteration starts with an arbitrary initial guess $V(g_t)_{[0]}$. For each g_t, the program searches along the state space $g0$ for the optimal policy rule $\widetilde{g}_t$, or equivalently i_t, which would maximize the value of the firm $V(g_t)_{[1]}$. $V(g_t)_{[1]}$ then updates $V(g_t)_{[0]}$. This procedure is repeated until the difference between $V(g_t)_{[n-1]}$ and $V(g_t)_{[n]}$ is within the predefined tolerance $1e-8$. At this point, there is convergence and the optimal solution $\widetilde{g}_t = g(g_t)$, or equivalently $i_t = h(g_t)$ are obtained.

This numerical solution is applied to generate a simulated panel data with N firms. All the simulated firms are endowed with the initial condition $X_{i,0} = 1$ and $K_{i,1} = steady$, i.e. $\widetilde{g}_{i,0} = X_{i,0}/K_{i,1} = 1/steady$, where $i = 1, \cdots, N$. According to the law of motion for demand, $g_{i,1} = X_{i,1}/K_{i,1} = \widetilde{g}_{i,0} \exp(\widetilde{\mu} + \sigma\epsilon_{i,1})$. The optimal investment rate $i_{i,1}$ are then pinned down using the policy rule derived above for each firm i in the first period 1. Then according to the law of motion for capital stock, $K_{i,2} = K_{i,1}(1 + i_{i,1})(1 - \delta)$, which updates $g_{i,1}$ into $\widetilde{g}_{i,1}$. In all subsequent periods, $\widetilde{g}_{i,t-1}$ becomes $g_{i,t}$ when $X_{i,t}$ evolves exogenously according to its law of motion. For given $g_{i,t}$, optimal investment rates $i_{i,t}$ are found from the numerical solution for each firm i in each period t. Then $g_{i,t}$ becomes $\widetilde{g}_{i,t}$ when $K_{i,t+1}$ evolves endogenously according to its law of motion.

The actual level of investment is easily recovered as $I_{i,t} = i_{i,t}K_{i,t}$. And the actual level of capital stock for each period is tracked by $K_{i,t}$. With the simulated data for $I_{i,t}$ and $K_{i,t}$, and given model parameter values, other variables of interest such as adjustment costs and operating profit can be easily calculated.

B.2 Solve (10) Numerically with Trend-Stationary Variables

The problem I need to solve is

$$\begin{aligned}
V(X_t, K_t) &= \max_{I_t}\{\Pi(X_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(X_{t+1}, K_{t+1})\right]\} \qquad \text{(B5)}\\
s.t.\ K_{t+1} &= (1-\delta)(K_t + I_t)\\
X_t &= \exp(x_t)\\
x_t &= c + \mu t + \zeta_t\\
\zeta_t &= \overline{\zeta} + \rho\zeta_{t-1} + e_t = \overline{\zeta} + \sum_{s=0}^{t-1}\rho^s e_{t-s}
\end{aligned}$$

where $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d}{\sim} N(0,\, 1)$ and $\zeta_0 = 0$.

This is a stochastic dynamic program with two state variables and one control variable. The demand shift parameter X_t and the beginning-of-period capital stock K_t are the state variables. Investment I_t, or equivalently the end-of-period capital stock $K_t + I_t$, is the control variable. The exogenous state variable X_t follows a trend stationary process; the state variable K_t endogenously follows X_t hence also follows a trend stationary process; the linear capital accumulation formula then implies I_t follows a trend stationary process as well. In this situation, I adopt a normalization strategy different from that in the problem (B1).

Define $\Psi_t \equiv \exp\left(c + \overline{\zeta} + \mu t\right)$. Denote $\widetilde{X}_t = \frac{X_t}{\Psi_t}$, $\widetilde{K}_t = \frac{K_t}{\Psi_t}$, $\widetilde{I}_t = \frac{I_t}{\Psi_t}$, $\widetilde{X}_{t+1} = \frac{X_{t+1}}{\Psi_{t+1}}$, and $\widetilde{K}_{t+1} = \frac{K_{t+1}}{\Psi_{t+1}}$. Then due to the linear homogeneity of V in $(X_t, K_t; I_t)$,

$$\begin{aligned}
V(\widetilde{X}_t, \widetilde{K}_t) &= V(\frac{X_t}{\Psi_t}, \frac{K_t}{\Psi_t})\\
&= \frac{1}{\Psi_t}V(X_t, K_t)\\
&= \max_{I_t/\Psi_t}\{\frac{1}{\Psi_t}\Pi(X_t, K_t; I_t) + \frac{\exp(\mu)}{1+r}\frac{1}{\Psi_{t+1}}E_t\left[V(X_{t+1}, K_{t+1})\right]\}\\
&= \max_{I_t/\Psi_t}\{\Pi\left[\frac{X_t}{\Psi_t}, \frac{K_t}{\Psi_t}; \frac{I_t}{\Psi_t}\right] + \frac{\exp(\mu)}{1+r}E_t\left[V(\frac{X_{t+1}}{\Psi_{t+1}}, \frac{K_{t+1}}{\Psi_{t+1}})\right]\}\\
&= \max_{\widetilde{I}_t}\{\Pi(\widetilde{X}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp(\mu)}{1+r}E_t\left[V(\widetilde{X}_{t+1}, \widetilde{K}_{t+1})\right]\}
\end{aligned}$$

where I have used the fact that $\Psi_{t+1}/\Psi_t = \exp(\mu)$. Therefore by normalization I have transformed the problem (B5) into the following problem

$$\begin{aligned} V(\widetilde{X}_t, \widetilde{K}_t) &= \max_{\widetilde{I}_t}\{\Pi(\widetilde{X}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp(\mu)}{1+r} E_t \left[V(\widetilde{X}_{t+1}, \widetilde{K}_{t+1})\right]\} \qquad \text{(B6)} \\ s.t.\ \widetilde{K}_{t+1} &= \exp(-\mu)(1-\delta)\left(\widetilde{I}_t + \widetilde{K}_t\right) \\ \widetilde{X}_t &= \exp\left(\widetilde{\zeta}_t\right) \\ \widetilde{\zeta}_t &= \rho\widetilde{\zeta}_{t-1} + e_t = \sum_{s=0}^{t-1} \rho^s e_{t-s} \end{aligned}$$

where $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d}{\sim} N(0, 1)$ and

$$\widetilde{\zeta}_t = \zeta_t - \overline{\zeta}$$

That is $\widetilde{\zeta}_t$ evolves according to the same law of motion as ζ_t but net of the intercept term $\overline{\zeta}$. When $\overline{\zeta} = 0$, the difference between $\widetilde{\zeta}_t$ and ζ_t is trivial, but the algorithm provided here is applicable to more general case when $\overline{\zeta} \neq 0$.

Now in this formulation, $\widetilde{X}_t$ and $\widetilde{K}_t$ are the two state variables, and $\widetilde{I}_t$ is the control variable, which are all stationary. The investment rate is not affected by this normalization, i.e. $I_t/K_t = \widetilde{I}_t/\widetilde{K}_t$, which is convenient. The effective discount factor is $\frac{\exp(\mu)}{1+r}$, which is less than 1, as assumed in Assumption 8.

Since conditional expectations need to be formed based on $\widetilde{X}_t$, I adopt the approximation method in Tauchen (1986) to discretize the continuous $AR(1)$ process for $\widetilde{\zeta}_t$ into a 9-state Markov process for given parameters ρ and σ. Then I get the support for $\widetilde{X}_t$ to be $\left[\exp(\widetilde{\zeta}_t(1)),\ \exp(\widetilde{\zeta}_t(9))\right]$ with 9 grid points.

In the absence of adjustment costs, on the balanced-growth-path, $K_t^* = steady \cdot X_t$, this implies $\widetilde{K}_t^* = steady \cdot \widetilde{X}_t$. Using this relationship as a benchmark, in order to take into account the effect of uncertainty and capital adjustment costs, the support of $\widetilde{K}_t$ is defined to be

$$\exp\left[\ln\left(steady \cdot \widetilde{X}_t(1)\right),\ \ln\left(steady \cdot \widetilde{X}_t(9)\right)\right]$$

and this state space is discretized using 500 grid points.

Since $\Pi(\widetilde{X}_t, \widetilde{K}_t; \widetilde{I}_t)$ is real valued, continuous, concave and bounded for any $0 < \gamma < 1$; and the constraint set $\Omega \equiv \{(\widetilde{X}_t(i), \widetilde{K}_t(j))\}_{(i=1,2,\cdots 9; j=1,2,\cdots 200)}$ is compact and convex; together with $\frac{\exp(\mu)}{1+r} < 1$, by the Contraction Mapping Theorem (Theorem

9.8 in Stokey and Lucas, 1989), a unique investment policy $\widetilde{I}_t = h(\widetilde{X}_t, \widetilde{K}_t)$ can be obtained by using value function iteration.

In practice, within each value function iteration, I apply a policy improvement algorithm (Chapter 20, Ljungqvist and Sargent, 2000). This costs more time for each value function iteration but substantially saves overall numbers of iterations. In total it saves about two thirds of the time compared with simple value function iteration in solving (B6). The termination condition is set as the difference in the value function between two consecutive iterations to be smaller than the predefined tolerance $1e-8$.

After getting the optimal solution $\widetilde{I}_t = h(\widetilde{X}_t, \widetilde{K}_t)$, linear interpolation is implemented, so that the final state space for $\widetilde{X}_t$ has 100 grid points, $\widetilde{K}_t$ has 2000 grid points, and the final policy space for $\widetilde{I}_t$ is of the dimension (100×2000).

This numerical solution is applied to generate a simulated panel data with N firms. When $t = 0$, all the simulated firms are endowed with the initial condition $\widetilde{\zeta}_{i,0} = 0$ and the corresponding initial capital stock $\widetilde{K}_{i,0} = steady \cdot \widetilde{X}_{i,0} = steady \cdot \exp(\widetilde{\zeta}_{i,0})$. For all subsequent periods, random demand shocks $e_{i,t} = \sigma\epsilon_{i,t}$ are drawn for each firm i and in each period t. Given the realization of $\widetilde{X}_{i,t}$ and the inherited $\widetilde{K}_{i,t}$, the optimal investment $\widetilde{I}_{i,t}$ are then pinned down using the policy rule derived above. Then $\widetilde{X}_{i,t}$ or equivalently $\widetilde{\zeta}_{i,t}$ evolves exogenously according to its law of motion, $\widetilde{K}_{i,t}$ evolves endogenously according to capital accumulation formula. The control variable in this period then becomes the state variable in next period. Finally the intercept and time trend $\exp\left(c + \overline{\zeta} + \mu t\right)$ are recovered for the level variables of the original model. For firm i in period t, the actual investment is therefore $I_{i,t} = \widetilde{I}_{i,t} \exp\left(c + \overline{\zeta} + \mu t\right)$; and the actual capital stock is $K_{i,t} = \widetilde{K}_{i,t} \exp\left(c + \overline{\zeta} + \mu t\right)$. In this chapter, I set $c = -0.5\sigma^2 / \left(1 - \rho^2\right)$ and $\overline{\zeta} = 0$, so that according to Lemma 5, $E\left[X_t\right] = \exp\left(\mu t\right)$. With the simulated data for $I_{i,t}$ and $K_{i,t}$, and given model parameter values, other variables of interest such as adjustment costs and operating profit can be easily calculated.

Figure 3.1a: Investment Policy for Complete Irreversibility only

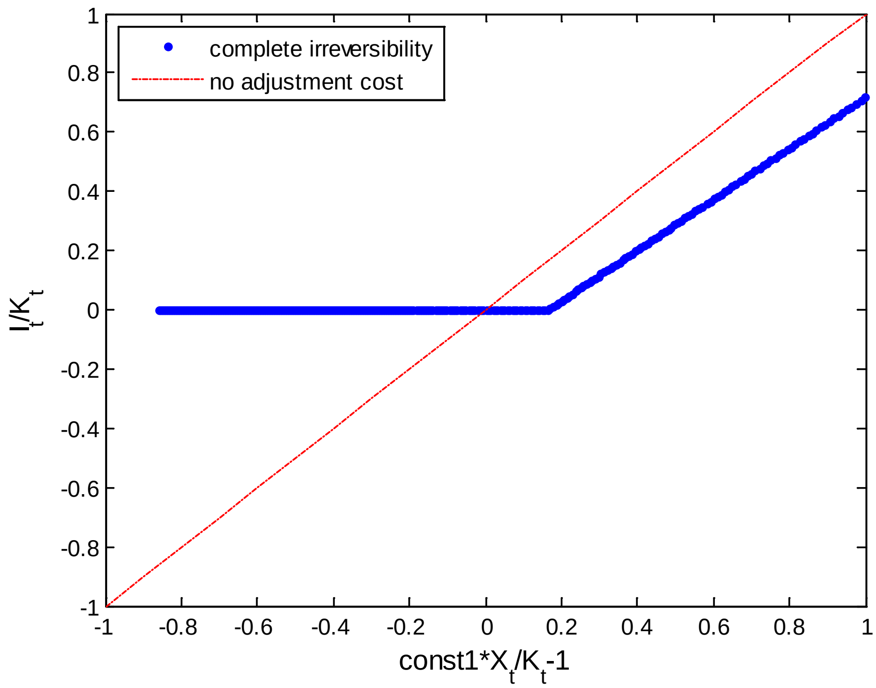


Figure 3.1b: Investment Policy for Partial Irreversibility only

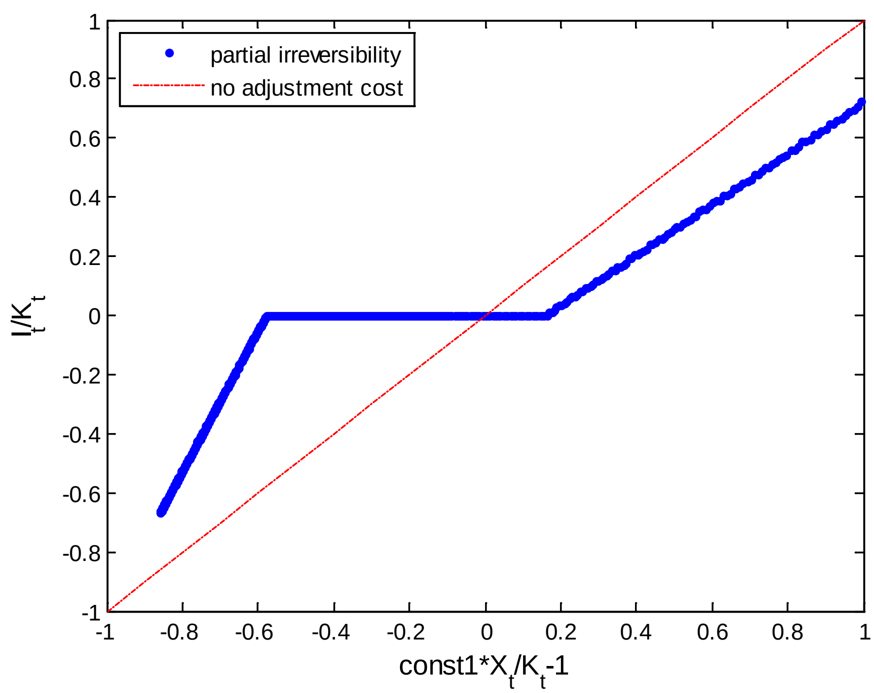


Figure 3.2: Investment Policy for Fixed Adjustment Cost only

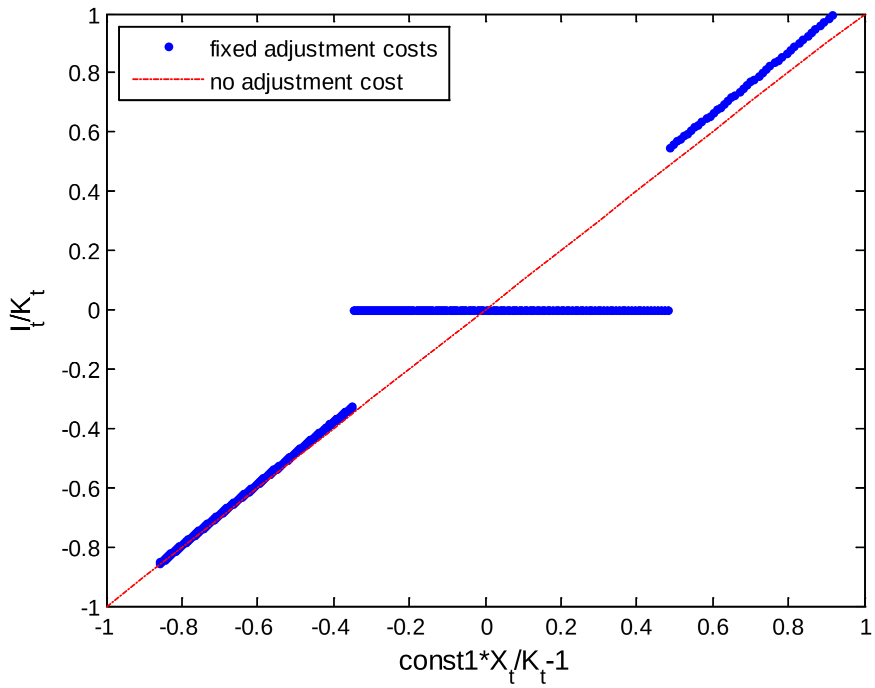


Figure 3.3: Investment Policy for Quadratic Adjustment Cost only

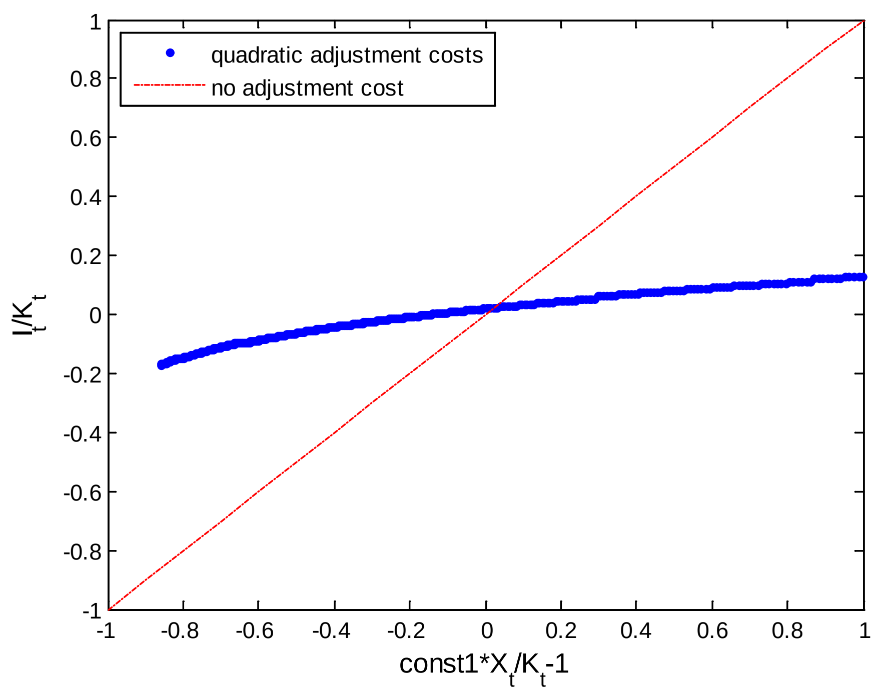


Figure 3.4a: Investment Policy at Different level of Uncertainty: Complete Irreversibility

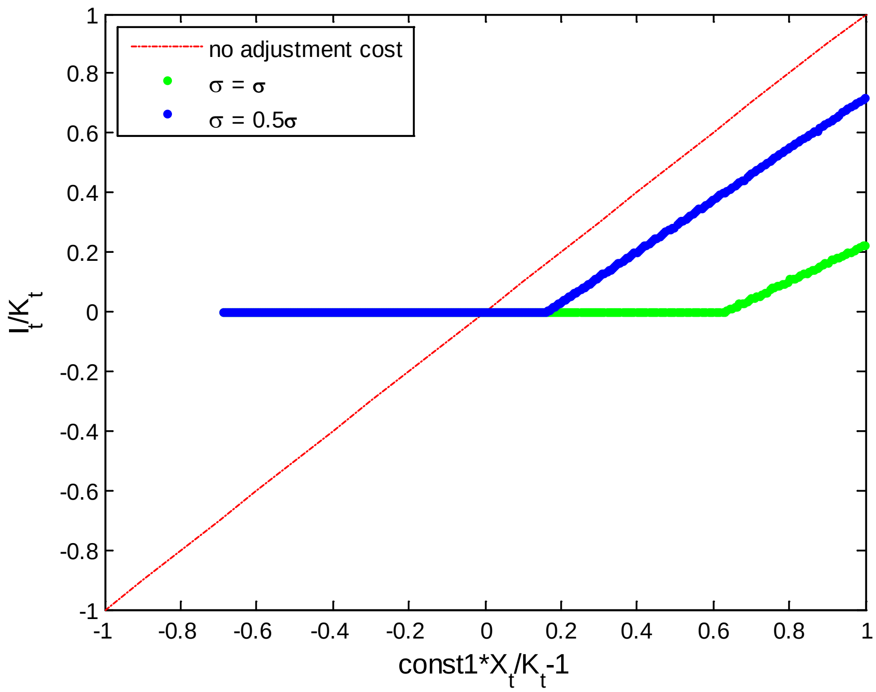


Figure 3.4b: Investment Policy at Different level of Uncertainty: Partial Irreversibility

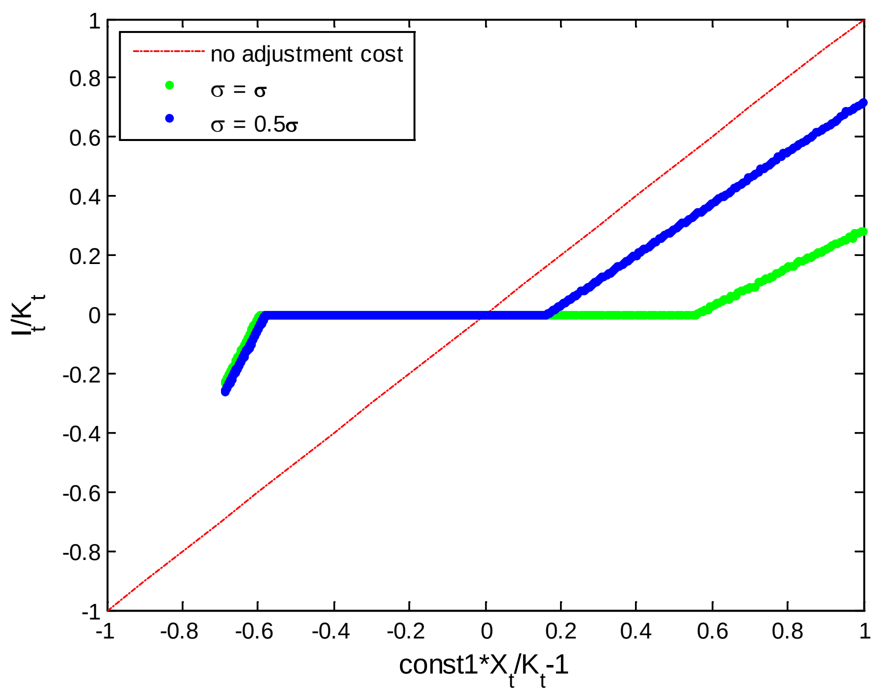


Figure 3.5: Investment Policy at Different level of Uncertainty: Fixed Adjustment Cost

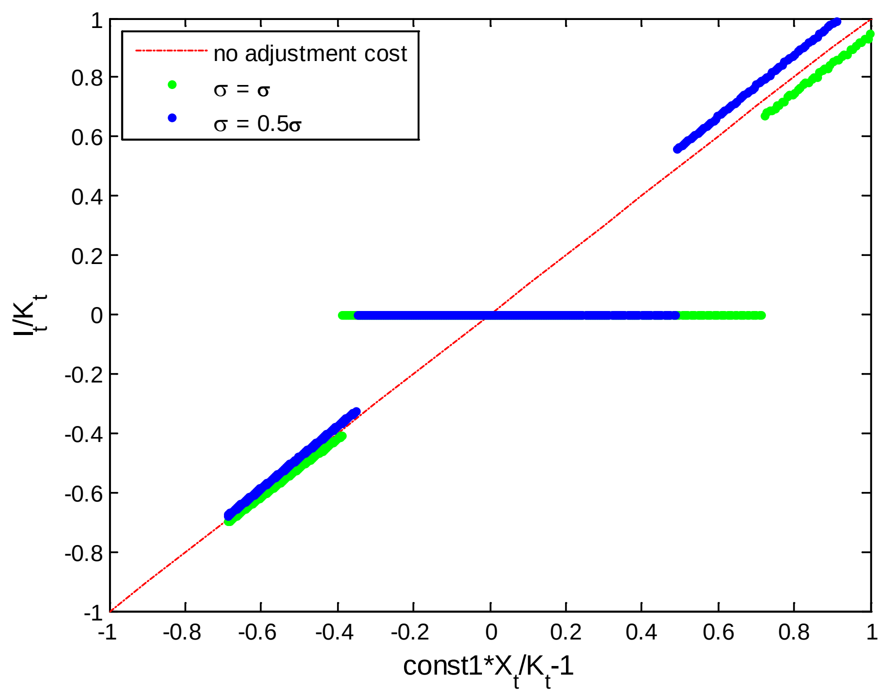


Figure 3.6: Investment Policy at Different level of Uncertainty: Quadratic Adjustment Cost

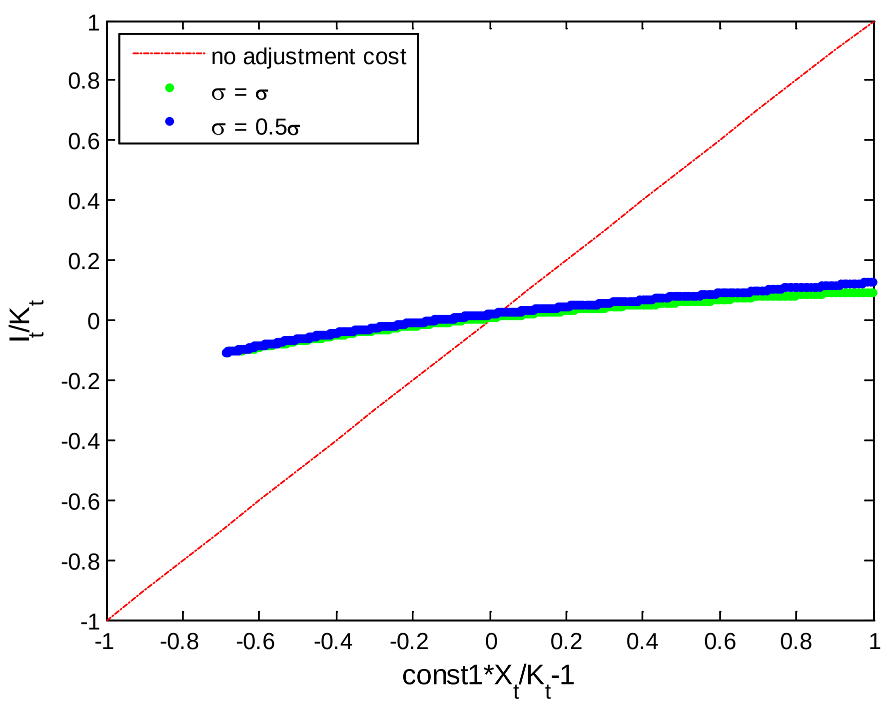


Figure 3.7a: Short-run Effects for Complete Irreversibility only

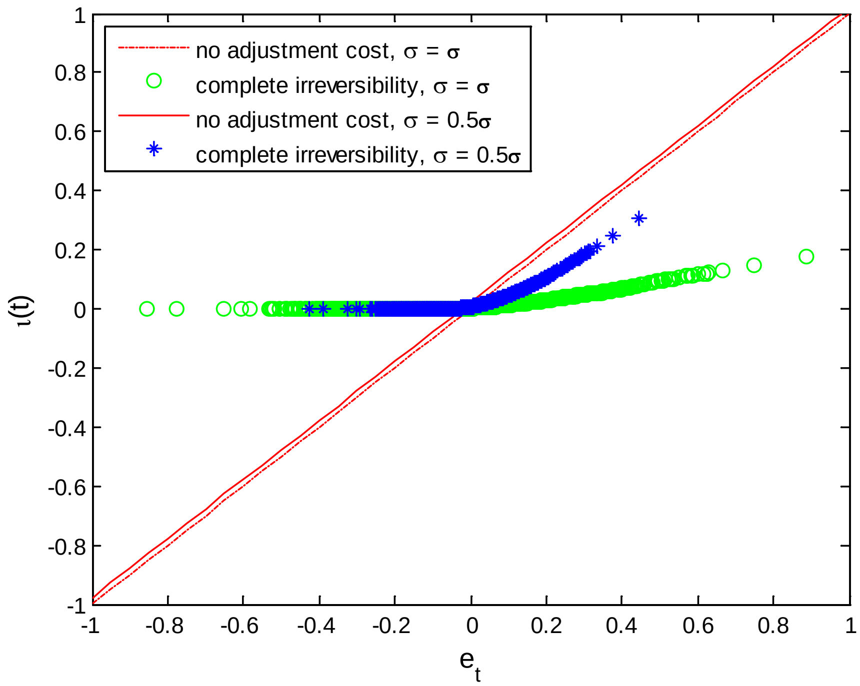


Figure 3.7b: Short-run Effects for Partial Irreversibility only

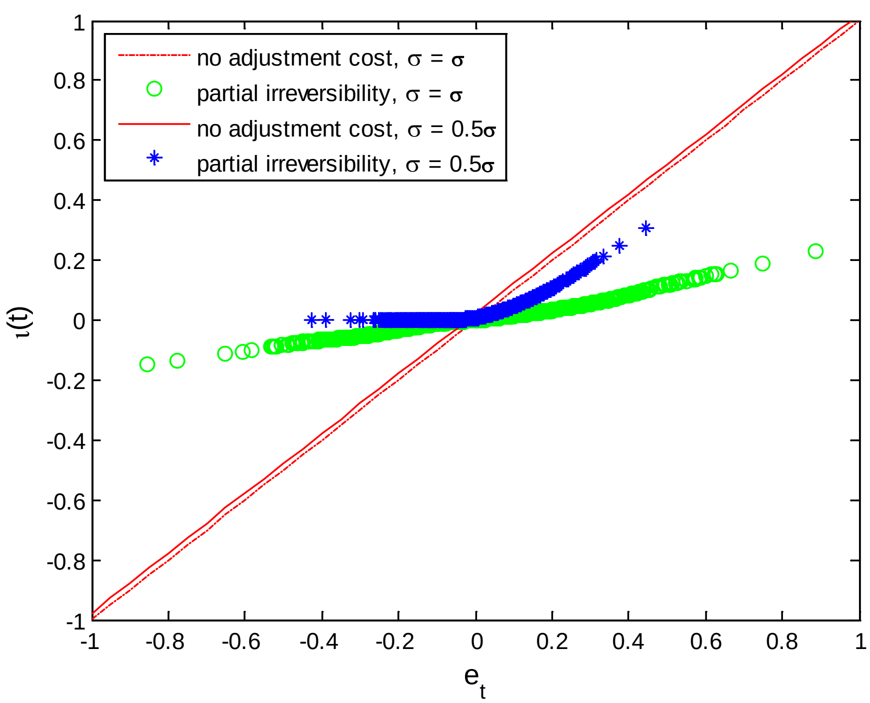


Figure 3.8: Short-run Effects for Fixed Adjustment Cost only

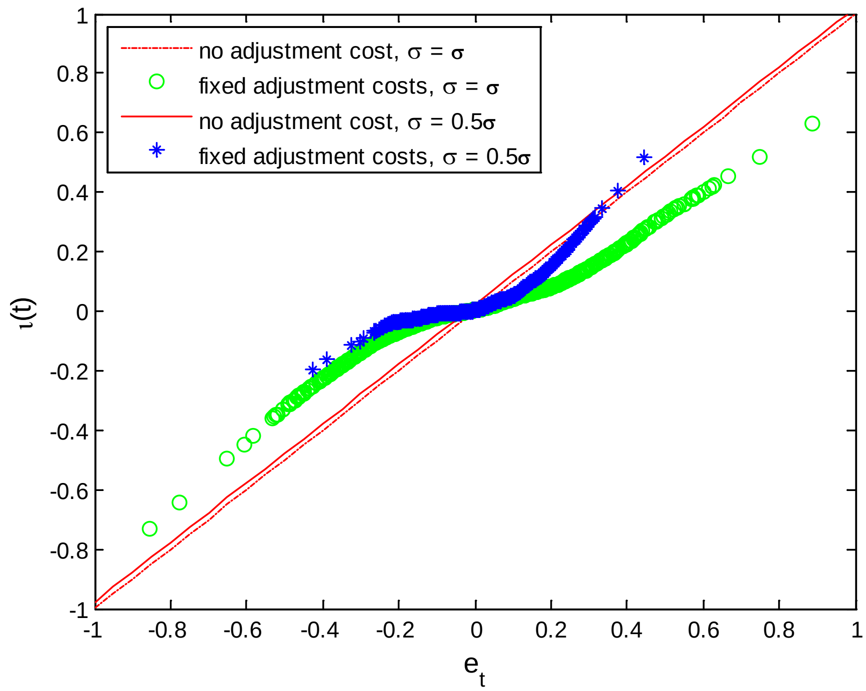


Figure 3.9: Short-run Effects for Quadratic Adjustment Cost only

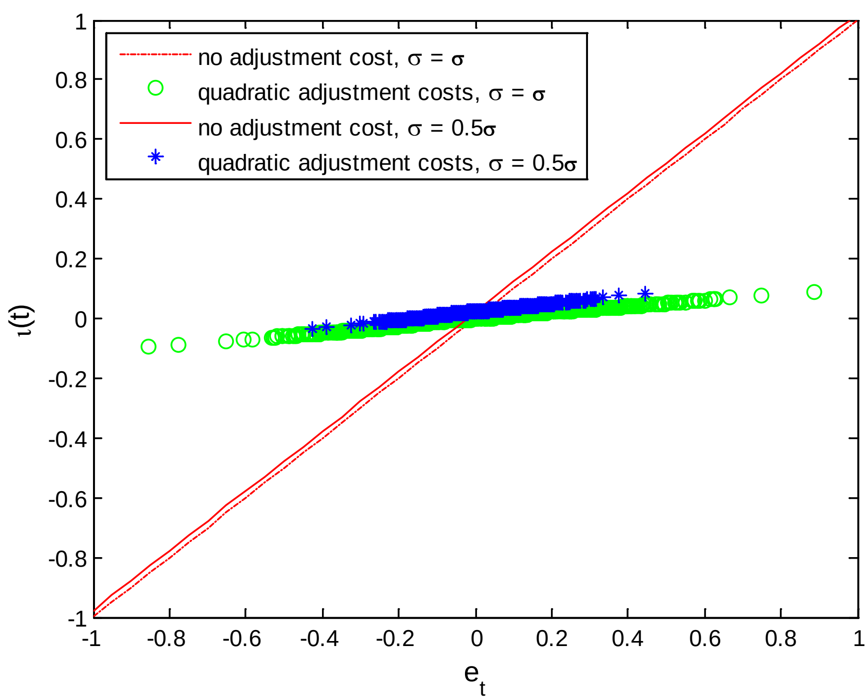


Figure 3.10: Analytical Replication for the Fig. 1 in Abel and Eberly (1999)

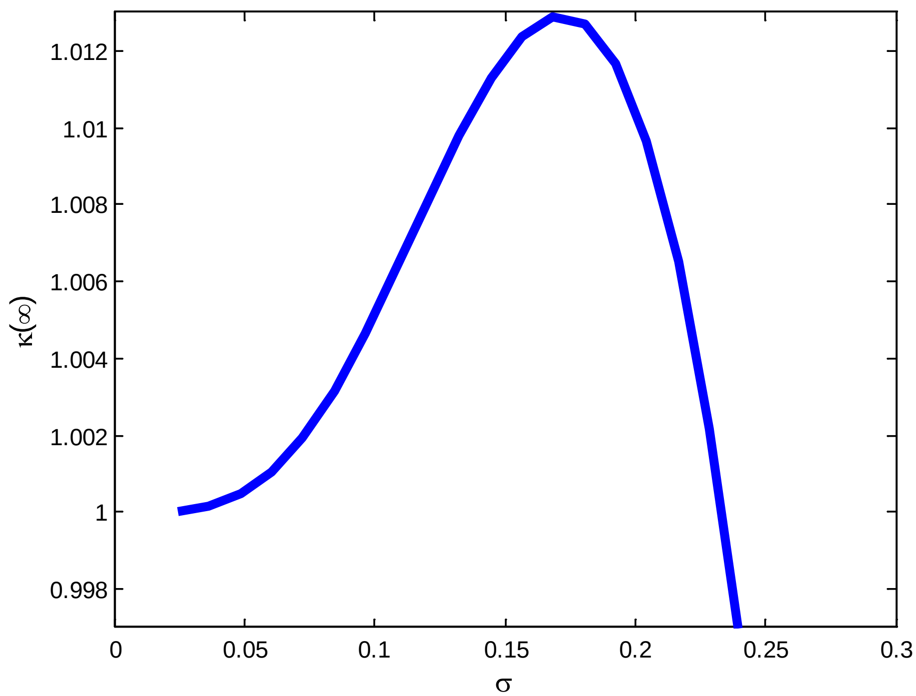


Figure 3.11a: Long-run Effects for Complete Irreversibility only

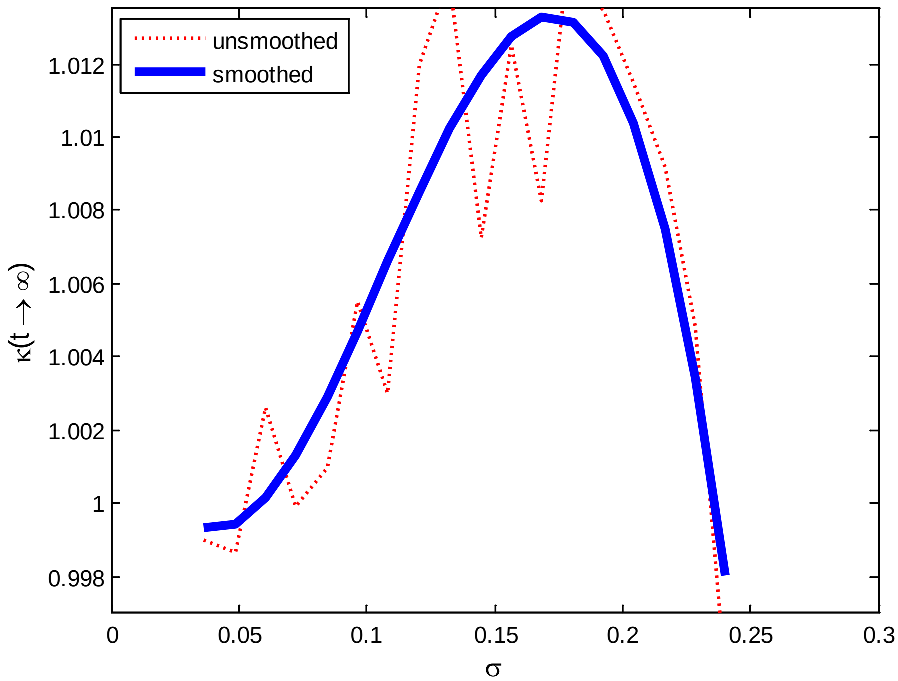


Figure 3.11b: Long-run Effects for Partial Irreversibility only

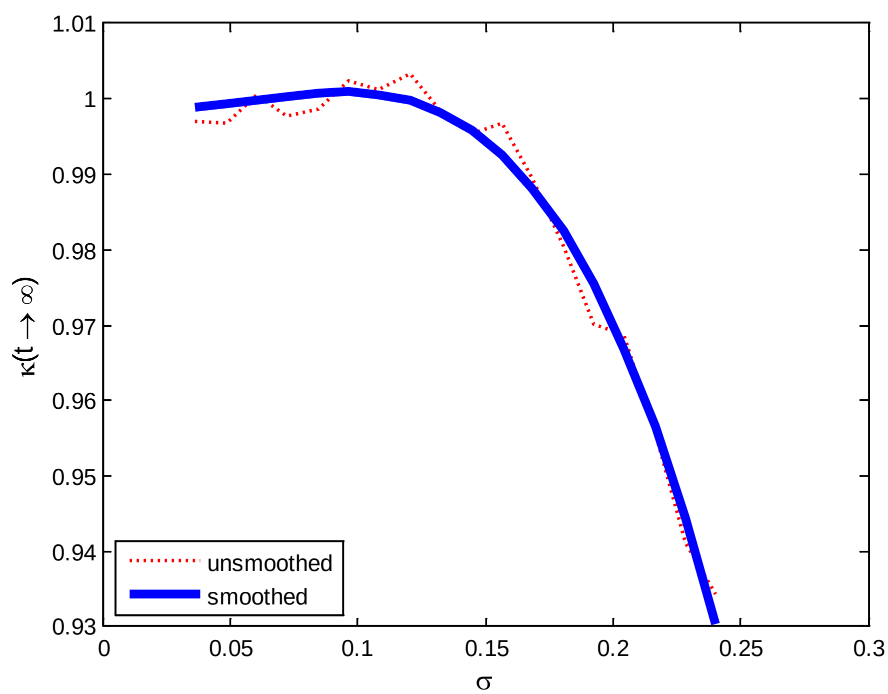


Figure 3.12: Long-run Effects for Fixed Adjustment Cost only

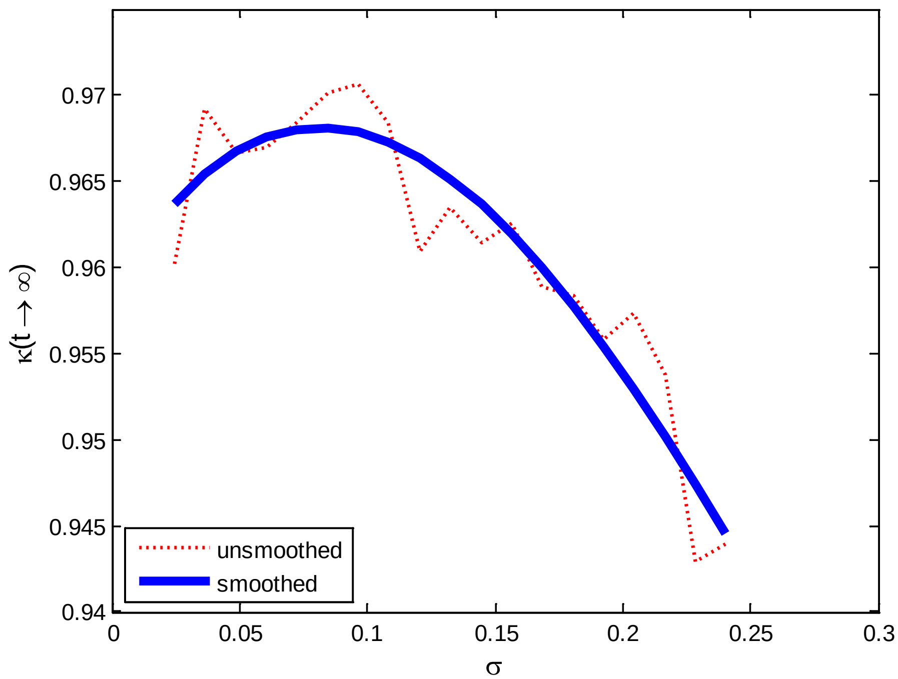


Figure 3.13: Long-run Effects for Quadratic Adjustment Cost only

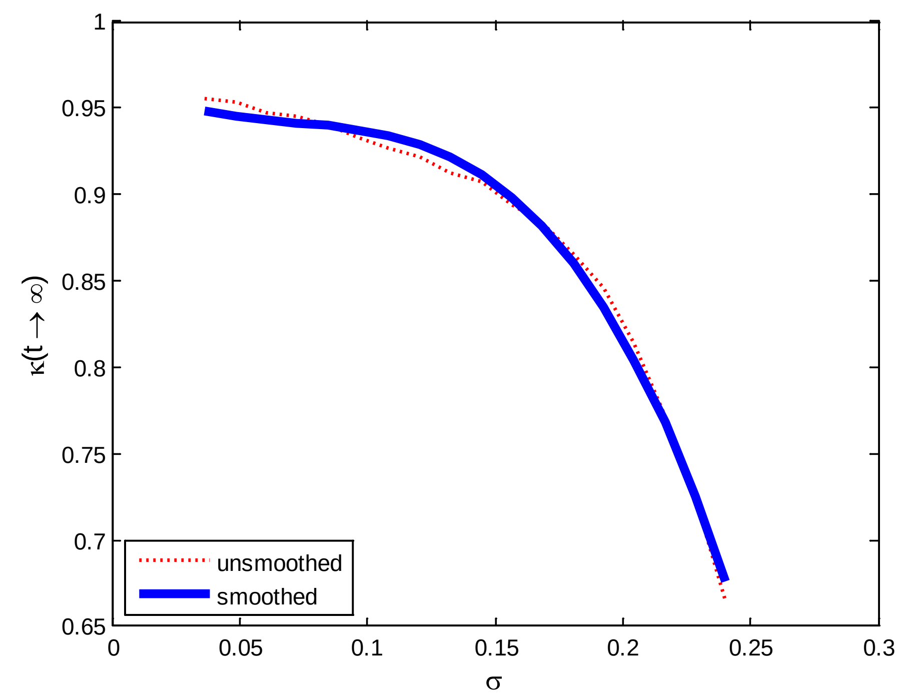


Figure 3.14: Comparison between Fixed and Quadratic Adjustment Costs

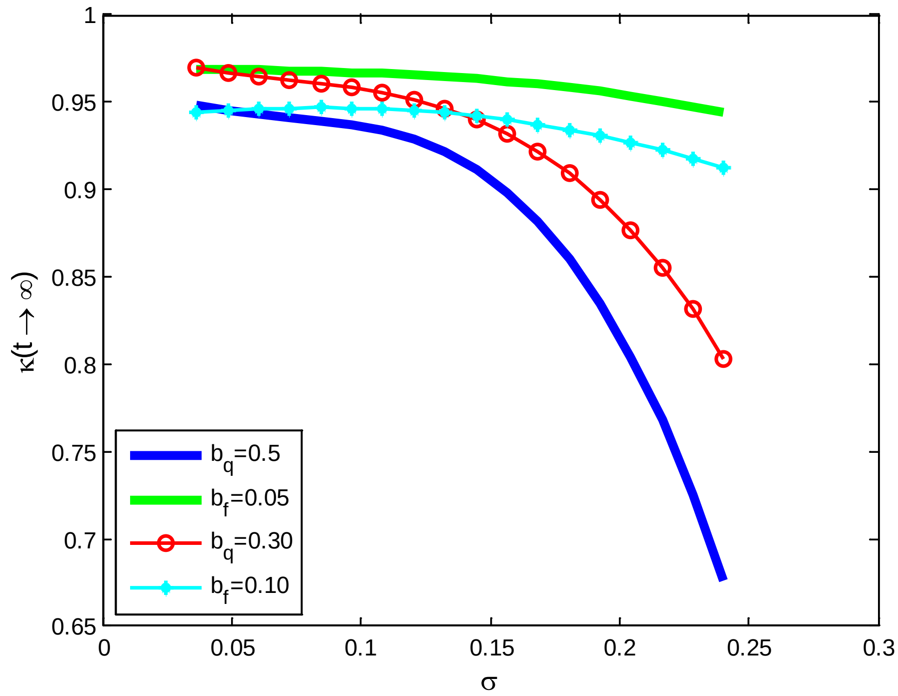


Figure 3.15a: The Effects of Model Parameters with Complete Irreversibility only

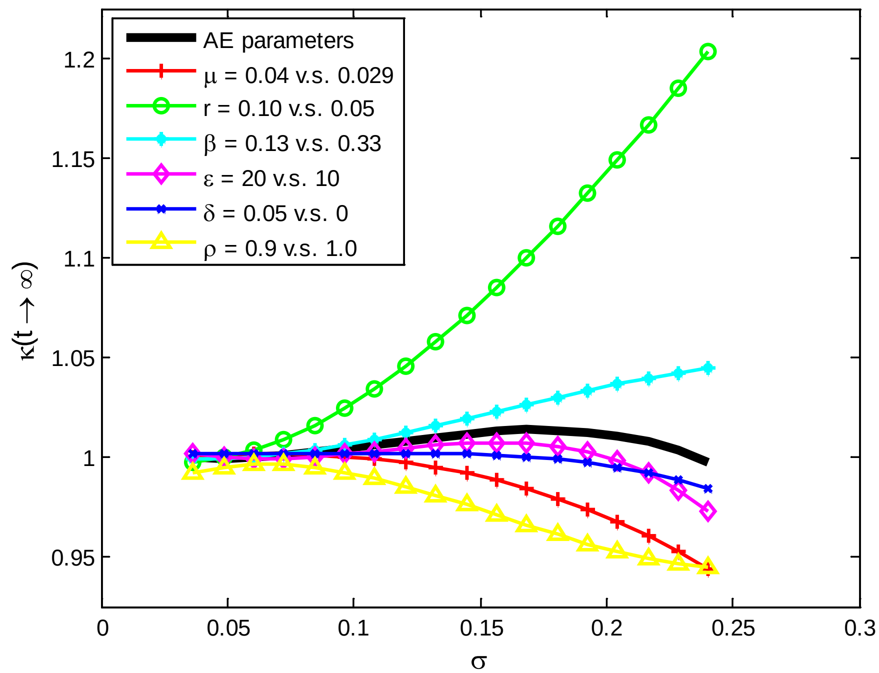


Figure 3.15b: The Effects of Model Parameters with Partial Irreversibility only

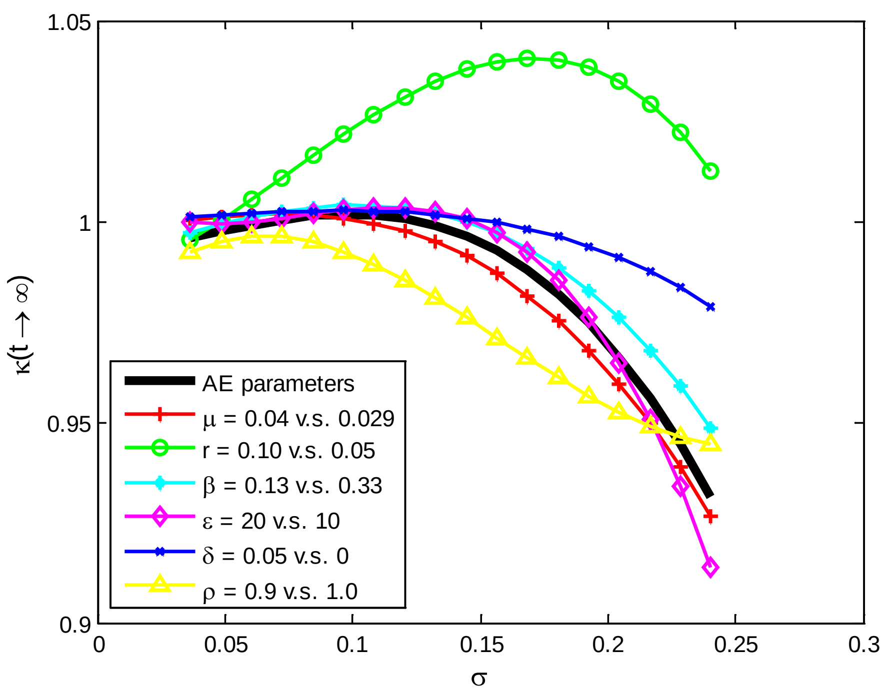


Figure 3.16: The Effects of Model Parameters with Fixed Adjustment Cost only

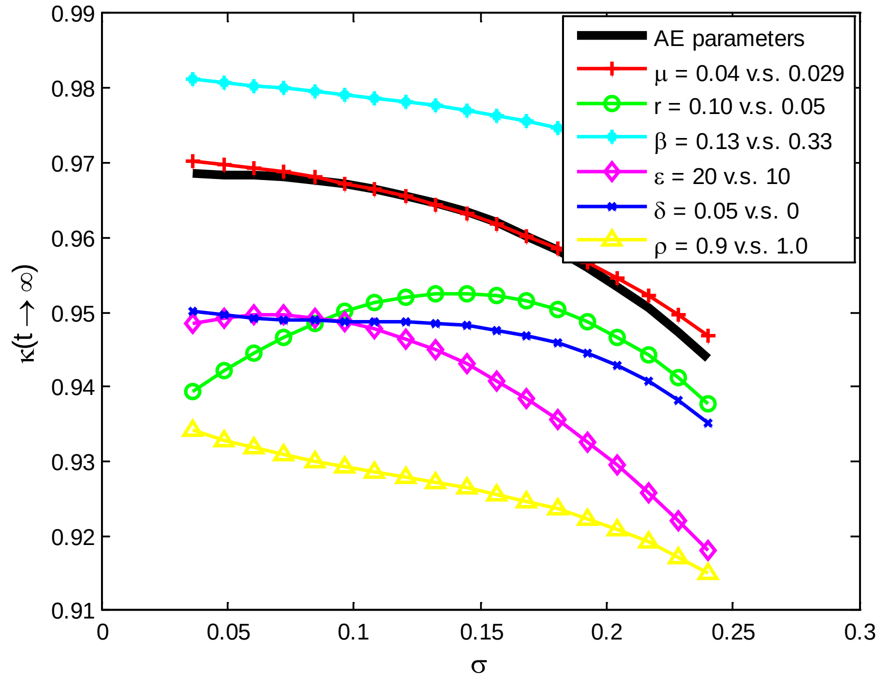
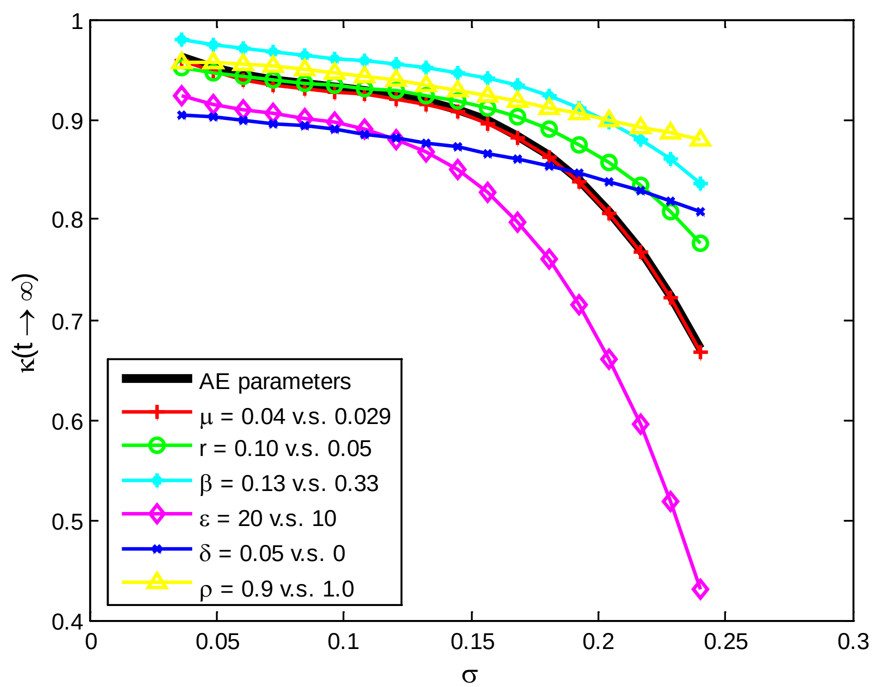


Figure 3.17: The Effects of Model Parameters with Quadratic Adjustment Cost only



Chapter 4

How Costly is Capital Adjustment?

Empirical Evidence from a Structural Estimation

1 Introduction

There are three important conclusions in Chapter 3. First, a standard neoclassical investment model augmented with capital adjustment costs, can generate very different patterns of investment depending on the form of adjustment costs. Second, the effects of uncertainty on capital adjustment and on the expected long-run capital stock differ dramatically depending on the form of adjustment costs. And finally, other parameters in this model, such as those determining production technology and demand schedule, can also impact significantly on these effects.

This raises several important questions. What are the patterns of investment at the micro level? Do these patterns support the presence of irreversibility, fixed or quadratic adjustment costs? What are the magnitudes of such adjustment costs? What are the estimates of other model parameters that are consistent with firm's characteristics and economic environment? And what are the effects of adjustment costs on short-run investment dynamics and long-run capital accumulation of the estimated investment model? Using firm-level data from Brazil, this chapter reports structural estimates for these effects, by estimating a reasonably flexible investment model using a simulated minimal distance estimator.

In their pioneering study, Cooper and Haltiwanger (2006) adopt a method of simulated moments (MSM) to recover structural parameters of capital adjustment costs, by matching plant-level moments from the Longitudinal Research Database (LRD) with simulated moments generated from an investment model with different forms of adjustment costs. Applying the same method to a firm-level panel dataset from the US Compustat, Bloom (2009) shows it is possible to distinguish the stochastic process and adjustment costs simultaneously by using moments of investment rate and sales growth series jointly. Furthermore, this paper also highlights the importance of distinguishing persistent differences across firms in their level of investment and sales growth rates from adjustment costs in order to obtain consistent estimates of adjustment costs.

This chapter employs the MSM to estimate the investment model specified in Chapter 3, using a panel of Brazilian manufacturing firms collected by the World

Bank. This dataset has two important advantages for this study. First, one unique feature of this dataset relative to other studies that measure investment is that information on both investment expenditure and value of disinvestment is explicitly asked in the survey for each firm and over each year. As discussed in Chapter 3, models with non-convex adjustment costs (partial irreversibility or fixed adjustment costs) predict an important region of investment inaction, in which firms prefer to undertake no investment rather than costly upward or downward adjustments. Consistent with this, 36.74% of observations in this sample report zero investment. Observations with zero investment expenditure are much fewer in LRD and extremely rare in annual data for publicly traded US or UK firms, such as Compustat or Datastream. Second, using firm-level data for one of the largest newly-emerging economies, allows us to consider investment behavior in an environment that has both stronger growth option and more investment frictions, compared with richer countries like the USA or the UK. This implies it might be more likely to capture the effects of adjustment costs on investment dynamics and capital accumulation from such an interesting sample.

There are several important findings in this chapter.

First, in line with Cooper and Haltiwanger (2006) and Bloom (2009), I find that at least one form of non-convex adjustment costs is required to fit these firm-level investment data. Different from these two previous studies, the estimated quadratic adjustment cost is much larger. In particular, a mix of quadratic adjustment costs and fixed adjustment costs fits the data best. Counterfactual simulations indicate that, these capital adjustment costs substantially dampen the instantaneous investment response to new information about demand in the short-run, and substantially reduce the average capital stock level in the long-run. Both these effects are mainly driven by the presence of significant quadratic adjustment costs.

Second, consistent with Bloom (2009), I find the stochastic process and adjustment costs are jointly identified by using moments based on investment rates and sales growth series simultaneously. Taking one step further than Bloom (2009), I explicitly model and estimate unobserved heterogeneity in the demand growth rate to distinguish permanent differences across firms in their level of investment and sale growth rates from adjustment costs. I find that ignoring this form of heterogeneity would lead to an upward bias in the estimates of adjustment costs and persistence of the stochastic process.

Third, both Cooper and Haltiwanger (2006) and Bloom (2009) estimate the investment model by imposing values for parameters in the production technology and demand schedule. I note that for a given Jorgensonian user cost of capital, and conditional on capital adjustment costs being identified separately, the short-run profit maximization properties provide identification for the capital share in the production function and the demand elasticity, by using information about profit-to-sales ratio and capital-to-sales ratio jointly.

Finally, a less attractive feature of survey data for firms in developing countries is the possibility of significant measurement error in recorded variables. To take this into account, I allow for an unusually rich structure of measurement errors in the empirical specification and find that this helps to fit some important features of the data.

The rest of the paper is organized as follows. Section 2 outlines the investment model for estimation. Section 3 presents the data and empirical specification. Section 4 discusses identification. Section 5 reports the empirical results. Section 6 provides the counterfactual simulations for the effects of adjustment costs. Section 7 concludes.

2 The Investment Model

This section sets up an investment model for estimation, building on the framework outlined in Chapter 3. Compared to the model in Chapter 3, I make two modifications here. First, a production function with three factors instead of two is adopted. This is because I will use data on sales, not value-added, as the proxy for 'output' in the empirical analysis. Second, I model a trend-stationary stochastic process instead of a process with a unit root. This allows demand shocks to have persistent but not permanent effect on the level of demand.

2.1 Model

Time is discrete and the time horizon is infinite. By paying capital adjustment costs, new investment I_t contributes to productive capital $\widehat{K}_t$ immediately in period t, which depreciates at the end of each period.

Assumption 1 ***Timing:***

The law of motion for capital stock is

$$K_{t+1} = (1-\delta)\,(K_t + I_t) \equiv (1-\delta)\widehat{K}_t \tag{1}$$

where δ is the constant depreciation rate.

Consider a firm that uses capital stock $\widehat{K}_t$, labour L_t and material M_t to produce nonstorable output Q_t, according to a nonstochastic constant returns to scale Cobb-Douglas technology.

Assumption 2 ***Production:***

The production function is

$$Q_t = \widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta} \tag{2}$$

where the capital share β satisfies $0 < \beta < 1$, labour share α satisfies $0 < \alpha < 1$, and $0 < \alpha + \beta < 1$.

The firm faces an isoelastic, downward-sloping, stochastic demand curve.

Assumption 3 ***Demand:***

The demand schedule is

$$Q_t = X_t P_t^{-\varepsilon} \tag{3}$$

where $-\varepsilon < -1$ is the demand elasticity with respect to price.

The demand shift parameter X_t is the only source of uncertainty in this model and follows a trend-stationary process.

Assumption 4 ***Demand Stochastic:***

The law of motion for X_t is

$$
\begin{aligned}
x_t &= \ln X_t \\
x_t &= c + \mu t + \zeta_t \\
\zeta_t &= \overline{\zeta} + \rho\zeta_{t-1} + e_t = \overline{\zeta} + \sum_{s=0}^{t-1} \rho^s e_{t-s}
\end{aligned} \tag{4}
$$

where $0 < \rho < 1$, $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$, $c = -0.5\sigma^2 / \left(1 - \rho^2\right)$ *and* $\zeta_0 = 0$.

Denote sales as $Y_t = P_t Q_t$. Labour and material are two variable inputs and can be adjusted instantaneously and costlessly. At each point of time, for given capital stock and demand realization, the firm chooses labour L_t and material M_t to maximize its instantaneous operating profit $\pi_t = \max\limits_{L_t,\, M_t} \{Y_t - wL_t - mM_t\}$, where w is a constant wage rate and m is a constant price for material.

Lemma 1 ***Operating Profit and Sales:***

The maximized value of operating profit and sales are given by

$$\pi_t = const0 \cdot X_t^{\gamma} \widehat{K}_t^{1-\gamma} \tag{5}$$

and

$$Y_t = \gamma\varepsilon \cdot \pi_t \tag{6}$$

where

$$0 < \frac{1}{\varepsilon} < \gamma = \frac{1}{1 + \beta(\varepsilon - 1)} < 1 \tag{7}$$

and

$$const0 = \frac{1}{\gamma\varepsilon} \left[\left(\frac{\alpha}{w}\right)^{\alpha} \left(\frac{1-\alpha-\beta}{m}\right)^{1-\alpha-\beta} \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{1-\beta} \right]^{\gamma(\varepsilon-1)} \tag{8}$$

Proof: See Appendix A.1.

In contrast with labour and material, frictions prevent instantaneous and costless adjustment of capital stock, which are modelled through three forms of capital adjustment costs: complete/partial irreversibility, fixed adjustment costs and quadratic adjustment costs.

Assumption 5 ***Capital Adjustment Costs:***

The functional form for capital adjustment costs is

$$G(X_t, K_t; I_t) = -b_i I_t \mathbf{1}_{[I_t<0]} + b_f \mathbf{1}_{[I_t\neq 0]}\pi_t + \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t \tag{9}$$

where $\mathbf{1}_{[I_t<0]}$ *and* $\mathbf{1}_{[I_t\neq 0]}$ *are indicators for negative and non-zero investment;* b_i *can be interpreted as the difference between the purchase price and the sale price expressed as a percentage of the purchase price of capital goods;* b_f *is interpreted as the fraction of operating profit loss due to any non-zero investment; and* b_q *measures the magnitude of quadratic adjustment costs.*

The net revenue of the firm in each period t is therefore

$$\Pi(X_t, K_t; I_t) = \pi(X_t, K_t; I_t) - G(X_t, K_t; I_t) - I_t \tag{10}$$

Assumption 6 *The firm is risk-neutral and discounts future net revenue at a constant rate* r*, where* $r > \exp(\mu) - 1$*.*

In each period investment is chosen to maximize the discounted present value of current and expected future net revenues, which can be represented as the solution to a dynamic optimization problem defined by the stochastic Bellman equation

$$V(X_t, K_t) = \max_{I_t}\{\Pi(X_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(X_{t+1}, K_{t+1})\right]\} \tag{11}$$

together with the law of motion (1) and (4) for K_t and X_t. Here $V(X_t, K_t)$ is the value of the firm in period t; $E_t\left[V(X_{t+1}, K_{t+1})\right]$ is the expected value of the firm in period $t+1$ conditional on information available in period t.

2.2 Key Variables

Structural estimation involves using variables, that are generated from the model, and also measurable and observable from the data. The identification strategy adopted in this chapter requires information on four such variables: Investment (I_t); Capital stock (K_t); Sales (Y_t) and Operating Profit net of fixed and quadratic adjustment costs, defined as

$$PI_t = \pi_t - b_f \mathbf{1}_{[I_t\neq 0]}\pi_t - \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t$$

Based on these four variables, five ratio or growth rate variables are generated.[1]

Definition 1 ***Key Variables for Identification:***

Investment Rate:

$$i_t \equiv I_t / K_t$$

Sales Growth Rate:

$$dy_t \equiv \ln (Y_t / Y_{t-1})$$

log of Sales-to-(beginning-of-period)Capital Ratio:

$$yk_t \equiv \ln (Y_t / K_t)$$

log of (productive) Capital-to-Sales Ratio:

$$ky_t \equiv \ln \left(\widehat{K}_t / Y_t \right)$$

Profit-to-Sales Ratio:

$$PIY_t \equiv PI_t / Y_t$$

2.3 Properties

This section presents the model predictions for these five variables.

The investment rate i_t is the most central variable. In the special case of no capital adjustment cost, Proposition 1 describes the optimal investment policy analytically.

Proposition 1 ***Optimal Investment Rate in the Frictionless Case:***

If $G(X_t, K_t; I_t) \equiv 0$, the optimal investment rate is

$$i_t^* = const1 \cdot (X_t / K_t) - 1 \tag{12}$$

where

$$const1 = \left[const0 \cdot (1 - \gamma) / \left(\frac{r + \delta}{1 + r} \right) \right]^{\frac{1}{\gamma}} \tag{13}$$

[1] Although I define these five variables separately, this does not mean that they contain information exclusively to each other. In particular, the fourth variable is approximately a linear combination of the first and the third variables. Therefore, defining five variables is for the purpose of convenience in constructing the variables from the data, and in discussing model predictions. In the stage of estimation and identification, I use proper moments for these variables to avoid the potential 'linear-dependency' problem. See Section 4.3 for detailed discussion.

Proof: See Chapter 3.

In the presence of adjustment costs, optimal investment policy is solved out using numerical dynamic programming method explained in the Appendix B.2 of Chapter 3. Figures 3.1-3.3 illustrate these policy under different forms of adjustment costs. Here is a short summary: first, irrespective to the form of adjustment costs, the optimal investment policy is always a non-decreasing function of the marginal revenue product of capital. Second, the optimal investment policy is a 'barrier control' policy in the presence of complete/partial irreversibility only and a 'jump control' policy in the presence fixed adjustment costs only; Finally, the optimal investment policy in the presence of quadratic adjustment costs only is a continuous function of the marginal revenue product of capital but with a dampened slope.

Since the demand X_t is not observable, if sales Y_t is used as a proxy, the log of sales-to-capital ratio yk_t would be a proxy for the marginal revenue product of capital.

As a corollary of Proposition 1, the frictionless optimal productive capital stock must be a linear function of the level of demand.

Lemma 2 ***Optimal Capital Stock in the Frictionless Case:***

$$\widehat{K}_t^* \equiv I_t^* + K_t = const1 \cdot X_t \tag{14}$$

The log of capital-to-sales ratio and profit-to-sales ratio measure the capital intensity and profitability of the firm, respectively. Given the definition for sales Y_t, operating profit π_t, net profit PI_t, together with Lemma 1 and 2, it is straightforward to derive the optimality for these two ratio variables in the absence of adjustment costs.

Lemma 3 ***Optimal Capital-to-Sales Ratio and Profit-to-Sales Ratio in the Frictionless Case:***

If $G(X_t, K_t; I_t) \equiv 0$, the optimal log of capital-to-sales ratio is

$$ky_t^* = \ln\left[\beta\left(1 - \frac{1}{\varepsilon}\right) / \left(\frac{r+\delta}{1+r}\right)\right] \tag{15}$$

and the optimal profit-to-sales ratio is

$$PIY_t^* = \frac{1+\beta(\varepsilon-1)}{\varepsilon} \tag{16}$$

Proof: By Lemma 1 and 2.

Finally, by definition, the sales growth rate is a weighted average of growth rate in demand and in capital stock, i.e.

$$dy_t \equiv \gamma \ln(X_t/X_{t-1}) + (1-\gamma)\ln\left(\widehat{K}_t/\widehat{K}_{t-1}\right) \tag{17}$$

Proposition 2 provides the long-run property for the growth rate in capital stock and in sales.

Proposition 2 ***Long-run Growth Rate:***

In the long-run, both sales and capital stock grows at rate μ on average, i.e.

$$dy_T \equiv \lim_{T\to\infty}\frac{1}{T}\ln(Y_{T+t}/Y_t) = \lim_{T\to\infty}\frac{1}{T}\ln\left(\widehat{K}_{T+t}/\widehat{K}_t\right) = \mu \tag{18}$$

Proof: See Appendix A.2.

The discussion of estimation and identification in Section 4.3 are based on these theoretical properties.

3 Data and Empirical Specification

3.1 Data

The dataset used for estimation comes from the World Bank's Investment Climate Surveys, conducted in 2003, providing annual observations for up to 3 years in the period 2000-2002 for 729 private-owned Brazilian manufacturing firms, distributed across 9 industries and 13 cities.

Four key variables are collected from the data:

Investment ($I_{j,t}$): value of investment expenditure minus value of disinvestment in machinery, equipment and plant; Capital stock ($K_{j,t}$): net book value of machinery, equipment and plant; Sales ($Y_{j,t}$): total value of sales; Profit ($PI_{j,t}$): sales minus direct costs of raw materials and inputs, minus total energy costs, minus total labour costs and minus other administrative costs, where j denotes firm and t denotes year. Using this information, the five key variables in ratios or growth rates are calculated according to Definition 1 above. Data Appendix C explains how these variables are deflated and cleaned.

Figure 4.1a plots the empirical distribution for investment rate $i_{j,t}$. The most distinctive feature is a considerable mass at zero. Indeed 36.74% firm-year observation reports exactly zero investment. The second feature is the large dispersion of the investment rate, varying from -58.16% to 100%. Finally, there is a striking asymmetry between investment and disinvestment and the distribution is highly skewed to the right. Figure 4.1b illustrates the empirical distribution for sales growth rate $dy_{j,t}$. Similar to that of investment rate, there is also a large dispersion in the sales growth rate, varying from -41.61% to 62.51%. Different from that of investment rate, sales growth rates are distributed symmetrically around 5% with standard tests for normality not being rejected.

3.2 Three Empirical Specification

In estimating the investment model using this firm-level dataset, I allow for three empirical specifications.

3.2.1 Growth Rate Heterogeneity

There are two reasons why modelling heterogeneity in the growth rates is important. First, both Cooper and Haltiwanger (2006) and Bloom (2007) illustrate the possibility of identifying different forms of capital adjustment costs from different features in the investment rate. However, as recognized in these studies, a key challenge in estimating adjustment costs is to distinguish the permanent differences in the stochastic process from the adjustment costs. For example, differences across firms in the demand growth rate, as well as high quadratic adjustment costs, can both lead to persistent differences across firms in their investment rates. Second, a key challenge in estimating the stochastic process itself is to distinguish the permanent differences in the growth rate from a highly serially correlated driving process. For example, both differences across firms in the demand growth rate and a highly serially correlated driving process can lead to persistent differences across firms in their sales growth rates. In order to distinguish between unobserved heterogeneity and state dependence, I therefore explicitly model heterogeneity in the growth rate.

Assumption 7 ***Growth Rate Heterogeneity:***

The demand growth rate for firm j is μ_j, where $\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$.

That is each firm j has a firm-specific demand growth rate μ_j, where μ_j is drawn independently from an identical normal distribution with mean μ_μ and standard deviation σ_μ. According to Proposition 2, the sales growth rate for firm j, dy_j is equal to the demand growth rate of firm j, μ_j. This normality assumption is consistent with the empirical distribution for the sales growth rate as illustrated in Figure 4.1b.

The investment policy under different growth rate is different. Hence the dynamic programming problem described in (11) must be solved for each firm j with value μ_j, which is infeasible even for a small sample. Therefore I adopt a standard approach used in the literature, for example, Eckstein and Wolpin (1999), to allow for a finite type of firms.

Assumption 8 ***A Finite Type of Firms:***

There are a finite, say V types of firms, each comprising a fixed proportion $1/V$ of the population, where the type set is defined as $\digamma = \{(\mu_v) : v = 1, \cdots, V\}$.

Under this assumption, I discretize the continuous distributions for μ_j by Tauchen (1989) method. Due to 'the curse of dimensionality', I have to be conservative about the number of grids and set it to be 3. This implies in the simulated data, there are 3 type of firms, each comprising a fixed proportion 1/3 of the population.

3.2.2 Measurement Errors

Given the important role of investment rate and sales in estimation, I allow for a rich structure of measurement errors in the empirical specification for these two variables. This is motivated by two considerations. First, measurement errors are common in firm-level survey data. Second and more fundamentally, allowing for permanent components of measurement errors in the investment rate is a computationally tractable way, to control for persistent differences between firms in investment rate, for example, due to persistent differences in depreciation rate, which might not have been fully controlled for through modelling heterogeneities in demand growth rate. Similarly, allowing for permanent components of measurement errors in the sales is a computationally tractable way, to control for persistent differences between firms in capital-to-sales ratio, since the model imposes a common production technology and demand schedule across firms.

Assumption 9 ***Measurement Errors in Investment Rate:***

$i_{j,t} = i^*_{j,t}\exp(e^I_{j,t})$, *where* $e^I_{j,t} = e^{IT}_{j,t} + e^{IP}_j$, *and* $e^{IP}_j \overset{i.i.d}{\sim} N(0,\sigma^2_{IP})$, $e^{IT}_{j,t} \overset{i.i.d}{\sim} N(0,\sigma^2_{IT})$.

That is there is a multiplicative structure for measurement errors in the investment rate, where $i_{j,t}$ denotes the observed investment rate, $i^*_{j,t}$ denotes the true underlying investment rate which is not measured accurately in the data, and the measurement error $e^I_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{IT} and σ_{IP}, respectively. This specification has the property that the sign of recorded investment rate is not affected by measurement error, and treats observations with zero investment in the data as true zeros.

Assumption 10 ***Measurement Errors in Sales:***

$Y_{j,t} = Y^*_{j,t}\exp(e^Y_{j,t})$, *where* $e^Y_{j,t} = e^{YT}_{j,t} + e^{YP}_j$, *and* $e^{YP}_j \overset{i.i.d}{\sim} N(0,\sigma^2_{YP})$, $e^{YT}_{j,t} \overset{i.i.d}{\sim} N(0,\sigma^2_{YT})$.

That is there is a multiplicative structure for measurement errors in sales, where $Y_{j,t}$ denotes the observed level of sales, $Y^*_{j,t}$ denotes the true underlying level of sales which is not measured accurately in the data, and the measurement error $e^Y_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{YT} and σ_{YP}, respectively.

3.2.3 Aggregation at the Firm-Level

Another feature for firm-level data is that these data might be consolidated across several plants within the firm. As pointed out by Bloom (2009), investment rate is featured by spikes and zeros at the plant-level but by much smoother serials at the firm-level. Without accounting for possible aggregation in the firm-level data could thus lead to an overestimate for the quadratic adjustment costs and an underestimate for the non-convex adjustment costs. Since the sample used in this chapter is a firm-level dataset, it is important to take this into account. The actual relevance of aggregation depends on the features of the data, hence has to be determined empirically.

Assumption 11 ***Aggregation:***

Each firm is made of p *plants, where* $p \geq 1$. *For plant* i *of firm* j *in period* t, *the law of motion for* $X_{i,j,t}$ *is given by*

$$\begin{aligned} x_{i,j,t} &= \log X_{i,j,t} \\ x_{i,j,t} &= c + \mu_j t + \zeta_{i,j,t} \\ \zeta_{i,j,t} &= \overline{\zeta} + \rho \zeta_{i,j,t-1} + e_{i,j,t} = \overline{\zeta} + \sum_{s=0}^{t-1} \rho^s e_{i,j,t-s} \end{aligned} \tag{19}$$

where $0 < \rho < 1$, $e_{i,j,t} = \sigma \epsilon_{i,j,t}$, $\epsilon_{i,j,t} \overset{i.i.d.}{\sim} N(0,1)$, $c = -0.5\sigma^2/\left(1-\rho^2\right)$ *and* $\zeta_0 = 0$.

That is there is heterogeneity across firms in the demand growth rate; which determine the 'type' of a firm; however, plants within the same firm are of the same type but have plant-level idiosyncratic innovations $\epsilon_{i,j,t}$.

Appendix B.1 explains how I solve the optimization problem numerically and simulate the data under these three assumptions.

4 Structural Estimation

The ultimate goal of this section is to provide estimates for the structural parameters in the model. It is achieved using the Method of Simulated Moments (MSM), which has been widely employed in the recent empirical investment literature.

For example, Cooper and Haltiwanger (2006) and Bloom (2009) estimate capital adjustment costs; Cooper and Ejarque (2003) and Eberly, Rebelo and Vincent (2008) evaluate the Q-model; Henessy and Whited (2007) and Bond, Söderbom and Wu (2007) study the cost of equity and debt finance, all through this structural econometric approach.

4.1 Method of Simulated Moments

The MSM aims at estimating a vector of unknown parameters by solving a minimum quadratic distance problem. Figure 4.2 provides an intuitive illustration for the basic idea of this method. Formally, following Gouriéroux and Monfort (1996), the **MSM estimator** Θ^* solves

$$\widehat{\Theta}^* = \arg\min_{\Theta} \left(\widehat{\Phi}^D - \frac{1}{H}\sum_{h=1}^{H} \widehat{\Phi}_h^S(\Theta) \right)' \Omega \left(\widehat{\Phi}^D - \frac{1}{H}\sum_{h=1}^{H} \widehat{\Phi}_h^S(\Theta) \right) \tag{20}$$

where Θ is the vector of parameters of our interest; $\widehat{\Phi}^D$is a set of empirical moments estimated from an empirical dataset; $\widehat{\Phi}^S(\Theta)$ is the same set of simulated moments estimated from a simulated dataset based on the structural model; H is the number of simulation paths; Ω is a positive definite weighting matrix.

Suppose the empirical dataset is a panel with N firms and T years. Given we model unobserved heterogeneity across firms, the asymptotics is for fixed T and $N \to \infty$. At the efficient choice for the weighting matrix Ω^*, the MSM procedure provides a global specification test of the overidentifying restrictions of the model, i.e. if the model is well-specified, the test statistics OI follows a chi-square distribution with degree of freedom equal to the difference between the number of moments and the number of parameters:

$$\begin{aligned} OI &= \frac{NH}{1+H} \left(\widehat{\Phi}^D - \frac{1}{H}\sum_{h=1}^{H} \widehat{\Phi}_h^S(\Theta) \right)' \Omega^* \left(\widehat{\Phi}^D - \frac{1}{H}\sum_{h=1}^{H} \widehat{\Phi}_h^S(\Theta) \right) \\ &\sim \chi^2 \left[\dim\left(\widehat{\Phi}\right) - \dim(\Theta) \right]. \end{aligned} \tag{21}$$

If the optimal weighting matrix Ω^* is used in solving (20), the MSM estimator is asymptotically normal for fixed H and T, and $N \to \infty$, i.e.

$$\sqrt{N}\left(\widehat{\Theta} - \Theta^*\right) \xrightarrow{D} N\left(0, W\left(H, \Omega^*\right)\right) \tag{22}$$

where

$$W(H,\Omega^*) = \left(1+\frac{1}{H}\right)\left(E\left[\partial\widehat{\Phi}^S\left(\widehat{\Theta}\right)'/\partial\Theta\right]\Omega^* E\left[\partial\widehat{\Phi}^S\left(\widehat{\Theta}\right)/\partial\Theta'\right]\right)^{-1}$$

Define binding function as $\widehat{\Phi}^S = \widehat{\Phi}^S(\Theta)$, that is how the simulated moments change with the structural parameters. Define the Jacobian matrix for the binding functions as $J = \left[\partial\widehat{\Phi}^S\left(\widehat{\Theta}\right)'/\partial\Theta\right]$. The crucial point of MSM is that the simulated moments $\widehat{\Phi}^S\left(\widehat{\Theta}\right)'$ depend on the structural parameters $\widehat{\Theta}$ used in that particular round of simulation. Therefore identification requires the variation in the simulated moments being informative about the changes in the underlying structural parameters. The sufficient condition for local identification is that the Jacobian matrix J has full row rank.

Appendix B.2 reports how I estimate the efficient weighting matrix, solve the minimum quadratic distance problem, calculate the numerical derivatives and check if the local identification appears to be achieved.

4.2 Structural Parameters

Table 4.1 lists the set of parameters Θ that I aim to estimate, which can be divided into 4 categories. First, parameters measuring the magnitude of capital adjustment costs, i.e. quadratic adjustment costs b_q, partial irreversibility b_i and fixed adjustment costs b_f. Denote $b = (b_q, b_i, b_f)$. Second, parameters characterizing technology and demand, i.e. the capital share in production function β and the demand elasticity with respect to price ε. Third, parameters characterizing the stochastic process, i.e. the serial correlation of demand shocks ρ; the mean and standard deviation of the growth rate μ_μ and σ_μ; and the standard deviation of demand shocks or measure of uncertainty σ. Fourth, parameters measuring the magnitude of measurement errors in the data, i.e. the standard deviation of transitory and permanent measurement errors in investment rate σ_{IT} and σ_{IP}, and in sales σ_{YT} and σ_{YP}.

In addition to these 13 structural parameters, there are 2 parameters in the investment model that are imposed but would also affect the investment policy and capital accumulation through the Jorgensonian user cost of capital. First, the depreciation rate δ. As Proposition 2 states, in the long-run, capital stock and sales grow at the same rate, since the growth rate of capital stock is approximately equal to the investment rate net of depreciation rate, i.e. $\ln\left(\widehat{K}_t/\widehat{K}_{t-1}\right) \simeq i_{j,t} - \delta$, I therefore impose $\delta = 0.05$, the difference between the mean of investment rate and of sales growth rate in the data. Second, the discount rate r. I impose $r = 0.15$. This is mainly driven by a highly dispersed sales growth rate observed in the data, which implies a certain fraction of firms have a high demand growth rate. According to Assumption 6, a high value of discount rate has to be imposed in order to guarantee a finite firm value. I consider the sensitivity of the estimates to imposing different values for both δ and r in later section.

4.3 Moments and Identification

Table 4.2 lists the set of moments $\widehat{\Phi}^D$ that I aim to match. The second column reports the value of these empirical moments estimated from the data and the standard errors are provided in the third column. Overall four means, threes standard deviations, three serial correlations, three cross correlations and four sample proportions of the five key variables defined in Definition 1 are utilized.

The selection of moments is guided by the properties of the model discussed in Section 2.3 and the empirical specifications discussed in Section 3.2. I have chosen not to use the basic moments for all the five key variables. This is because for $PIY_{j,t}$, in the absence of adjustment costs, it is determined by β, ε, r and δ according to (16). With homogeneity in these four parameters, there is very little variation in $PIY_{j,t}$. That is why I only use mean information for this variable. As for $ky_{j,t}$, since it is approximately equal to $i_{j,t} - yk_{j,t}$, I only use two out of three variables from $(ky_{j,t}, i_{j,t}, yk_{j,t})$ in calculating any set of moments.

This section discusses how the moments listed in Table 4.2 provide the possibility of identifying the parameters listed in Table 4.1.

The mean of profit-to-sales ratio $mean\,(PIY_{j,t})$ and log of capital-to-sales ratio $mean\,(ky_{j,t})$ are informative for the capital share in production function β and the demand elasticity with respect to price ε. As Lemma 3 claims, in the absence of adjustment costs, $PIY_{j,t}$ only depends on β and ε; and $ky_{j,t}$ only depends on β, ε,

r and δ. So in the absence of any adjustment costs, for a given Jorgensonian user cost of capital r and δ, identifying β and ε is equivalent to solving two equations (15) and (16) for two unknowns simultaneously. Table 4.2 reports a sample average of 23.35% profit-to-sales ratio and a value of -1.4280 for log of capital-to-sales ratio.

According to Proposition 2, in the long-run, both sales and capital stock grow at rate μ. Hence the mean of investment rate $mean\,(i_{j,t})$ and sales growth rate $mean\,(dy_{j,t})$ are informative about the mean of demand growth rate μ_μ, while the difference between these two moments depends on the depreciation rate δ. On average firms in this sample have an investment rate 10.68% and sales growth rate 4.97%.

The standard deviation of investment rate $std\,(i_{j,t})$ and sales growth rate $std\,(dy_{j,t})$ both depend on how volatile and heterogeneous the stochastic process is, therefore are both informative about the level of uncertainty σ and the heterogeneity in the growth rate σ_μ. In addition, both type of measurement errors in investment rate $(\sigma_{IT}, \sigma_{IP})$ will increase $std\,(i_{j,t})$ but only transitory measurement error in sales (σ_{IT}) will increase $std\,(dy_{j,t})$. Finally, since adjustment costs have a direct effect of investment and indirect effect on sales, the difference between these two moments reflects the importance of the adjustment costs. In the data, $std\,(i_{j,t})$ is indeed smaller than $std\,(dy_{j,t})$, which indicates the possible presence of adjustment costs. In contrast, if there are no adjustment costs, as proved in Lemma 3, $ky_{j,t}$ is a constant, for which the model imposes common values across firms, hence $std\,(ky_{j,t})$ would be zero. Therefore the non-zero value reflects the importance of adjustment costs, together with measurement errors in sales $(\sigma_{YT}, \sigma_{YP})$.

The serial correlation in investment rates $corr\,(i_{j,t}, i_{j,t-1})$, sales growth rates $corr\,(dy_{j,t}, dy_{j,t-1})$ and log of sales-to-capital ratio $corr\,(yk_{j,t}, yk_{j,t-1})$ all increase with the persistence of the stochastic process, which is determined by ρ. In addition, permanent difference in the demand growth rate across firms will lead to persistent difference in the investment rate and sales growth rate, hence $corr\,(i_{j,t}, i_{j,t-1})$ and $corr\,(dy_{j,t}, dy_{j,t-1})$ are also informative for the magnitude of σ_μ. In addition, transitory and permanent measurement errors affect these serial correlation coefficients in the opposite direction. Finally, quadratic adjustment costs will generate a smooth investment series, therefore a higher b_q will increase both $corr\left(i_{j,t}, i_{j,t-1}\right)$ and $corr\left(dy_{j,t}, dy_{j,t-1}\right)$, but with a direct effect on the former and indirect effect on the latter. Hence using these two moments simultaneously distinguish high adjustment costs from persistent or heterogeneous stochastic process. In the data, both investment rates and sales growth rates are positively serially correlated, which implies a high ρ or a large σ_μ. Furthermore, the serial correlation is indeed much stronger for investment rates than for sales growth rates, which might reflect the importance of b_q.

The cross correlation coefficient $corr\left(i_{j,t}, yk_{j,t}\right)$ and $corr\left(dy_{j,t}, yk_{j,t}\right)$ reflect the responsiveness of investment rate and sales growth rate to the proxy for the marginal revenue product of capital. The investment policies illustrated in Figures 3.1-3.3 indicate that high values of b_q and b_i dampen this responsiveness, with a stronger effect on investment rate than on sales growth rate. Therefore, both a small correlation $corr\left(i_{j,t}, dy_{j,t}\right)$ between these two variables, and the difference between $corr\left(i_{j,t}, yk_{j,t}\right)$ and $corr\left(dy_{j,t}, yk_{j,t}\right)$ reflects the importance of the adjustment costs. Finally, any measurement errors in investment rate $(\sigma_{IT}, \sigma_{IP})$ or sales $(\sigma_{YT}, \sigma_{YP})$ can cause attenuation in these coefficients, which may partly account for the low correlations observed from the data.

Following Cooper and Haltiwanger (2006), I use four proportion moments to capture the distribution of investment rate: $prop\left(0.2 < i_{j,t} < 1\right)$, $prop\left(0 < i_{j,t} < 0.2\right)$, $prop\left(i_{j,t} = 0\right)$ and $prop\left(-0.2 < i_{j,t} < 0\right)$ report the proportions of investment spikes, moderate investment, zero investment and moderate disinvestment, with the proportion of disinvestment spikes being one minus these four reported values. Recall that the investment model predicts that both partial irreversibility b_i and fixed adjustment costs b_f would generate a range of inaction and hence a mass point at zero investment. However, partial irreversibility would lead to less disinvestment, while fixed adjustment costs more investment spikes. Furthermore, quadratic adjustment costs b_q implies large investment rates are costly, hence predicts less investment spikes. Very few disinvestment and a large proportion of zero investment are recorded in the empirical sample. This implies the importance of at least one form of the non-convex adjustment costs.

Finally, I have investigated that applying the MSM procedure to this model using generated data, for which I know the true parameter values, this set moments indeed recovers reasonable parameter estimates.

5 Empirical Results

5.1 Estimates

Table 4.3 presents the estimation results for the full model, imposing $p = 1$ plant across each firm, a depreciation rate $\delta = 0.05$, and a discount rate $r = 0.15$. The first column in the right panel reports the optimal estimates of the structural parameters and the second column lists the numerical standard errors of these estimates.

The estimates for two out of three forms of capital adjustment costs are found to be significantly different from zero. In particular, $\widehat{b_q} = 2.4156$, this implies a quadratic adjustment cost, which is about 1.25% of the operating profit, evaluated at the sample average. $\widehat{b_f} = 0.0105$, implies any investment or disinvestment would result in a 1.05% loss of operating profit. Since about 36% firms are at the inaction region of investment, this implies on average firms are paying a fixed adjustment cost equal to 0.67% of the operating profit.

The estimated $\widehat{\beta} = 0.0576$, a reasonable value for the capital share in a gross output production function. The estimate for the demand elasticity with respect to price is $\widehat{\varepsilon} = 5.2148$, which implies a mark-up coefficient $\frac{\widehat{\varepsilon}}{\widehat{\varepsilon}-1} = 1.2373$. These two estimates together determine the estimate for the capital coefficient in the operating profit $1 - \widehat{\gamma} = 0.20$, an indication of strong concavity.

The estimated serial correlation parameter for the stochastic process is 0.8515 and significantly different from 1, which implies the effect of demand shocks are persistent but not permanent. The estimates for the mean and standard deviation of the growth rate are 0.0276 and 0.0735, respectively, and are both significantly different from zero. This implies on average the demand grows at 2.76% per year, meanwhile there is large heterogeneity in the growth rate across firms in this sample. The estimated standard deviation for demand shocks σ is 0.2197, which measures the level of uncertainty in this model. A value of σ at 0.2 has been considered as 'typical' for simulation purposes in theoretical research, for example, Pindyck (1988).

Three out of four types of measurement errors under our specification are significant. In particular, measurement errors in investment rate have a large transitory component while measurement errors in sales have a large permanent component. In contrast with transitory measurement errors, which mainly reflect the pure recording errors in the firm-level data, permanent component may reflect the existence of other 'unobserved heterogeneity' across the firms that I have not modelled 'structurally', for example, the Jorgensonian user cost of capital, the production technology and the demand schedule.

The left panel of Table 4.3 compares the empirical moments from the data and the simulated moments from the model evaluated at the optimal estimates.

Among the four mean moments, the model could fit the profit-to-sales ratio and capital intensity very well; but the model slightly over-estimates the mean for investment rate but under-estimates the mean for sales growth rate. All the three standard deviation moments are closely fitted. The model could generate high serial correlation for investment rate and sales-to-capital ratio, but not for the sales growth rate. This is because the specification for the stochastic process implies $dy^*_{j,t} = \mu_j + \zeta_{j,t} - \zeta_{j,t-1}$ and $dy^*_{j,t-1} = \mu_j + \zeta_{j,t-1} - \zeta_{j,t-2}$, hence if there was neither adjustment costs nor heterogeneity in μ, the serial correlation for $dy_{j,t}$ would be negative. In the estimated model, the non-zero adjustment costs and heterogeneity in μ indeed generate a positive serial correlation, but not as large as in the data. This suggests there might be other factors which can lead to a more persistent sales growth rate, that have not been included in the model. Similarly, the simulated cross correlations between investment rate, sales growth rate and sales-to-capital ratio all have the same sign as the empirical moments, but the magnitudes are larger than the empirical counterparts. This indicates that the investment model under our specification can successful generate a positive relationship between investment rate, sales growth rate and the marginal revenue product of capital; the presence of the adjustment costs indeed dampens the response of investment and sales to new information about demand, but there might be other frictions which would further dampen the responsiveness. Finally, all the four moments characterizing the empirical distribution of investment rates are closely matched.

Overall, the overidentifying restriction test statistics is 93 with 4 degrees of freedom under this specification.

5.2 Specification Tests

Table 4.4 reports specification tests for several restricted models, where the preferred full model is listed in Column (1) as benchmark.

Column (2) shows the result of imposing $b_q = 0$, i.e. imposing no quadratic adjustment costs. Compared with our preferred full model, the criterion value increases a lot, mainly because the restricted model cannot fit the positive serial correlation in sales growth rate; it also predicts too many investment spikes.

Column (3) shows the result of imposing $b_i = 0$, i.e. imposing no irreversibility. This restricted model generates very similar parameter estimates and simulated moments to those in the benchmark model and the same criterion value as for the benchmark model. This implies the null hypothesis of no irreversibility cannot be rejected.

This raises an interesting puzzle: does it mean that the resale of capital goods does not result in a loss for these Brazilian firms? Is this consistent with the fact that only 4% disinvestment are reported in this sample? This puzzle can be solved by studying the role of other forms of adjustment costs. At the estimated value for b_q and b_f, even if there is no discrepancy between the sales price and the purchase price of capital goods, any disinvestment would incur a quadratic adjustment cost and a fixed adjustment cost which are both about 1% of the operating profit. Given a 5% depreciation rate and a 3% demand growth rate, on average these firms would need an investment rate around 8%, therefore a disinvestment which would cost 2% of the operating profit will very seldom be an optimal choice, even if $b_i = 0$.

To study the role of other forms of adjustment costs, Column (4) shows the result of imposing $b_f = 0$, i.e. imposing no fixed adjustment costs. Under this restriction, the estimated b_i increases to 0.2133. Similarly, in Column (2) where b_q is imposed to be 0, the model also estimates a value for b_i at 0.1569. This implies in the absence of quadratic or fixed adjustment costs, the model would predict a non trivial difference between the sales price and the purchase price of capital goods in order to fit the data.

In other words, under the current specification of the capital adjustment cost function, the model could match the limited extent of disinvestment observed in the data through two alternative ways: one is $b_i > 0$, that is a direct loss of disinvestment since the resale price is lower than the purchase price; another is $b_q > 0$ and $b_f > 0$, that is an indirect loss of disinvestment since any non-zero investment incurs capital adjustment costs. For this sample, clearly the second specification is preferred.

Column (5) shows the result of imposing $b_i = b_f = 0$, i.e. imposing no non-convex adjustment costs. This specification is massively rejected, mainly because it fails to generate a significant proportion of zero investment but instead predicts too many moderate investment. This implies at least one form of non-convex adjustment costs is required in order to match this distinctive feature of the data. To choose between fixed adjustment costs and irreversibility, comparison between Column (3) and (4) indicates that a model with a combination of fixed adjustment costs and quadratic adjustment costs would fit the data better than a model with a combination of irreversibility and quadratic adjustment costs.

Column (6) shows the result of imposing $\sigma_\mu = 0$, i.e. imposing no heterogeneity in the growth rate. As we may expect, under this restriction, the model first, cannot fit the positive serial correlation of the sales growth rate, even if ρ, i.e. the serial correlation of the stochastic process, is over-estimated; and second, it estimates higher values for all three forms of adjustment costs, in particular the quadratic and fixed adjustment costs. This highlights the importance of allowing for heterogeneity in growth rate in order to get consistent estimates for the adjustment costs and the stochastic process.

Column (7) shows the result of imposing $\sigma_{IT} = \sigma_{IP} = \sigma_{YT} = \sigma_{YP} = 0$, i.e. imposing no measurement errors in investment rate and sales. Since with the full model, I have found three out of four types of measurement errors to be significant, this restricted model mainly tests the effect of not allowing for these measurement errors. Not surprisingly, this restricted specification is massively rejected, mainly because the simulated cross correlations are too large compared with the empirical counterparts. In order to compensate for these effects, the model generates much higher estimates for the adjustment costs, which in turn makes investment being too persistent and not dispersed enough compared with the real data. This implies that an investment model with adjustment costs only would generate investment rates and sales growth rates, that are too responsive to new information about demand, compared to the real data. Measurement errors in the data could partly account for this discrepancy.

5.3 Robustness Tests

Table 4.5 presents results for three robustness tests. Column (1) is the benchmark model, imposing a depreciation rate $\delta = 0.05$, a discount rate $r = 0.15$ and the number of plants within each firm to be $p = 1$.

Column (2) and (3) show the results for the same model but imposing the depreciation rate to be 0.04 and 0.06 respectively. The model with δ =0.04 fits the data slightly better than our benchmark model, because both the mean of investment rate and of the sales growth rate are closer to the empirical moments. Nevertheless, except the estimates for the mean of demand growth rate μ_μ, the estimates for adjustment costs, stochastic process and measurement errors are relatively robust to the choice of depreciation rate.

Column (4) and (5) present the results for the same model but imposing the discount rate to be 0.13 and 0.17, respectively. The model with r =0.13 fits the data slightly better and with $r = 0.17$ slightly worse, than the benchmark model. However, r cannot be further reduced below 0.13 in order to meet the requirement of finite firm values. Comparison between Column (1), (4) and (5) implies that the estimates for adjustment costs, stochastic process and measurement errors are robust to the choice of discount rate within this range.

Recall that a higher depreciation rate and a higher discount rate will both increase the Jorgensonian user cost of capital $\left(\frac{r+\delta}{1+r}\right)$. For given a Jorgensonian user cost of capital, in the absence of adjustment costs, the capital share β and the demand elasticity ε determine the capital-to-sales ratio, as claimed in Lemma 3. Therefore in order to match the empirical mean of capital-to-sales ratio, when δ and r varies across Column (1) to (5), the estimates for β and ε also vary across these columns; and the direction of variation is consistent with what Lemma 3 would predict. Hence, despite the presence of adjustment costs, the mechanism of how capital intensity depends on the Jorgensonian user cost of capital, the capital share and the demand elasticity as described in Lemma 3 still works.

Column (6) lists the results for estimating the same model but imposing the number of plants within each firm to be three, i.e. $p = 3$. As we may expect, since aggregation is one of the important sources of smoothing, a higher number of plants per firm results in a smaller estimate for b_q, higher estimates for b_i and b_f, and higher estimates for parameters measuring dispersion σ_μ and σ. The overall fit deteriorates compared with $p = 1$. If I further increase p, say $p = 5$, the fit gets even worse. In contrast with Bloom (2009), where moments from very large firms (median number of employees 3,450) are employed so that aggregation across plants improves the fit, in this Brazilian sample, where moments from relatively small firms (median number of employees 48) are employed, the model prefers the specification with a single firm to that with aggregation.

5.4 Comparison with Cooper and Haltiwanger(2006)

Table 4.6 compares the estimates on capital adjustment costs from this chapter to that from Cooper and Haltiwanger (2006), together with some common moments reported in both samples. The estimates from these two studies are more comparable to each other, relative to other studies in the literature. This is because, first, the functional form for capital adjustment costs are the same. Second, estimates listed in Table 4.6 are both for single decision unit without aggregation. Finally, the adjustment cost parameters are both obtained from structural estimation. However, the results should not be compared too literally, since there is also difference in the choice of functional form for the operating profit and the stochastic process.

Interestingly, both these two studies find a mix of quadratic and fixed adjustment costs fit the data best. However, the estimated fixed adjustment cost is much larger in Cooper and Haltiwanger (2006), while the estimate for the quadratic adjustment costs is much higher in this chapter. This is consistent with how the moments in these two samples differ. The highly dispersed investment rates and a large proportion of disinvestment spikes in LRD imply that fixed adjustment costs will be important. In contrast, the high serial correlation of investment rates is consistent with the non-trivial quadratic adjustment costs estimated for the Brazilian sample.

According to the investment policy illustrated in Figure 3.2, fixed adjustment costs is one way to generate investment inaction. One interesting question is why a fixed adjustment cost of 20% operating profit in the LRD sample generates about 8% zero investment, but a fixed adjustment cost of 1% operating profit in Brazilian sample generates 37% zero investment. There are two reasons.

First, Cooper and Haltiwanger (2006) impose a production technology and demand schedule which imply a markup about 16% in their sample; while this chapter estimates β and ε which imply a markup about 24%. This means firms in the Brazilian sample are more profitable than those in the LRD sample. Therefore a cost of same proportion to that of operating profit would imply different actual loss.

Second, this chapter allows for heterogeneous demand growth rates. According to the estimates in Table 4.3, about one-third firms in this sample have a growth rate around -5%. All else being equal, these firms should disinvest on average. However, given there is a 5% depreciation rate, capital stock physically reduces at the same rate of demand. Since any investment or disinvestment implies non-zero quadratic and fixed adjustment costs, it would be optimal for these firms to stay inactive. Indeed, if the model imposes a common demand growth rate and estimates the rate to be 2.8% as that in Column (5) of Table 4.4, so that on average firms need an investment rate around 7.8% to follow demand growth and compensate physical depreciation, the estimated fixed adjustment costs immediately increase to 4% in order to match the large proportion of zero investment in the data. Once again, this highlights the importance of estimating adjustment costs and stochastic process simultaneously in order to get consistent estimates.

6 Counterfactual Simulations

This section answers the question 'How costly is capital adjustment' through counterfactual simulations. Similar to the analysis in Chapter 3, I study the effects of adjustment costs on both short-run investment dynamics and long-run capital accumulation.

6.1 Short-Run Effects

Following Chapter 3, I illustrate the effects of adjustment costs by considering the effect of demand shocks e_t on the adjustment of the capital stock $\iota(t)$ in the same period, where $\iota(t) \equiv \Delta \ln \widehat{K}_t = \ln \widehat{K}_t - \ln \widehat{K}_{t-1}$. Recall this quantity is approximately equal to investment rate net of depreciation rate, i.e. $\iota(t) \simeq \frac{I_t}{K_t} - \delta$. A weaker impact effect indicates a smaller response of capital stock to new information about demand, hence slower investment dynamics.

Figure 4.3a plots how $\iota(t)$ varies with demand shocks e_t, using the estimated parameter values reported in Table 4.3. Given there is heterogeneity in growth rate within this sample, the effects presented here are for the type of firms with the median level of growth rate, i.e. 0.0276. Although the lines we see in Figure 4.3a would shift downwards and upwards for firms with lower and higher growth rates, similar patterns in the shape and the slope of these lines are found for firms with different levels of growth rate.

The dashed 45^o line is plotted as a benchmark to illustrate the relationship between $\iota(t)$ and e_t if there were no adjustment costs. The circle line is plotted at the actual level of uncertainty while the asterisk line is plotted at half of that value. Comparison between the circle line and the 45^o line highlights the significant effect of capital adjustment costs in dampening investment response to new information about demand. Comparing the circle line and the asterisk line, it is clear that the effect of uncertainty on investment dynamics is virtually zero under the estimated value of adjustment costs. Consider a realization of demand shock $e_t = 0.2$, which is about the same value as the standard deviation of these shocks σ. It would induce a 20% growth rate in capital stock in the absence of adjustment costs, but only stimulates a 5% growth rate in capital stock, at both levels of uncertainty, due to the presence of these adjustment costs.

To see the relative importance of quadratic and fixed adjustment costs in generating this low responsiveness, Figure 4.3b plots three lines: the diamond line uses the actual b_q and b_f estimated from the model, together with other model parameters. Hence it illustrates the actual capital adjustment in the data. The asterisk line keeps all else equal but imposes $b_q = 0$, i.e. as if there was no quadratic adjustment cost; the circle line keeps all else equal but imposes $b_f = 0$, i.e. as if there was no fixed adjustment cost. The striking similarity between the diamond and circle lines, and the obvious difference between the diamond and asterisk lines, indicates that it is the quadratic adjustment costs that substantially dampens investment dynamics.

6.2 Long-Run Effects

Although the short-run investment dynamics has important implications for the business cycle, it is the level of capital stock that determines the long-run economic performance. In Chapter 4, I have studied how $\kappa(t) \equiv E\left[\widehat{K}_t\right]/E\left[\widehat{K}_t^*\right]$ varies with adjustment costs. Since the denominator is invariant to the adjustment cost parameters, so any variation in the ratio $\kappa(t)$ must reflect variation in the numerator, which is what I choose to report directly in this section.

Figures 4.4a illustrates how the level of quadratic adjustment costs affects the average capital stock in the estimated model, using the estimated parameter values reported in Table 4.3 and the same random numbers employed during estimation. It takes the estimated b_q as the reference level and decreases and increases this value by one-third on the horizontal axis. The average capital stock level is scaled by the level of average capital stock in the simulation using the reference level of b_q, so that the values on the vertical axis can be read as percentage changes in the average capital stock as we change b_q. The downward sloping line in this Figure highlights the negative effect of quadratic adjustment costs on capital accumulation. All else being equal, a permanent reduction in the level of b_q by one-third is estimated to increase average capital stock levels by about 9%.

Figure 4.4b reports the actual adjustment costs associated with b_q as a ratio to operating profit. When b_q increases from 1.6 to 3.2, the adjustment costs associated with b_q increases from 0.95% to 1.55% of the operating profit. If the cost-to-profit ratio is scaled by the ratio in the simulation using the reference level of b_q, Figure 4.4c indicates that all else being equal, a permanent reduction in the level of b_q by one-third is estimated to decrease the cost-to-profit ratio by about 25%.

Figure 4.5a to 4.5c are the counterparts of Figure 4.4a to 4.4c, but focuses on the effects of fixed adjustment costs. As illustrated in Figure 4.5a, although there is also a negative relationship between average capital stock and fixed adjustment costs, the effect of varying b_f is much smaller than that for b_q. All else being equal, a permanent reduction in the level of b_f by one-third is estimated to increase average capital stock levels by only about 0.5%.

Figure 4.5b and 4.5c plot the actual and scaled cost-to-profit ratio associated with these different values of b_f. All else being equal, when b_f increases from 0.007 to 0.014, the adjustment costs associated with b_f increases from 0.48% to 0.85% of the operating profit. Translated into percentage change, a permanent reduction in the level of b_f by one-third is estimated to decrease the cost-to-profit ratio by about 30%.

This implies that, changes in b_q and b_f, which cause similar variation in the cost-to-profit ratio, would lead to very different magnitude for the effects on average capital stock. This highlights the importance of quadratic adjustment costs on long-run capital accumulation.

Finally, to check whether the findings in Figure 4.3, 4.4 and 4.5 are robust to the choice of depreciation rate, discount rate and the number of plants, I also implement the same counterfactual simulations based on the estimates in Column (2) to (6) of Table 4.5. In spite of some trivial quantitative differences, the effects of adjustment costs on investment dynamics and capital accumulation we find in this section are qualitatively and quantitatively robust.

7 Conclusion

This chapter presents estimates of the effects of capital adjustment costs on both short run investment behavior and long run capital accumulation for a panel of Brazilian manufacturing firms.

I estimate a structural investment model, in which investment and capital accumulation is determined by five factors: the Jorgensonian user cost of capital, the production technology, the demand schedule, the stochastic process characterizing uncertainty in the economic environment, and different forms of adjustment costs: partial irreversibility, a fixed cost of undertaking any investment at all, and quadratic adjustment costs. For a given Jorgensonian user cost of capital, structural parameters for the other four factors are estimated by matching simulated model moments to empirical moments, using a simulated minimum distance estimator. In particular, under the specification of the functional form, the profit-to-sales ratio and capital-to-sales ratio identify the production technology and demand schedule; the difference between investment rate and sales growth rate in their distribution, dynamics and response to new information about demand, jointly identifies the adjustment costs and stochastic process. The empirical results imply that a mix of fixed and quadratic adjustment costs fits the data best. In the estimated model, the adjustment costs associated with these fixed and quadratic adjustment costs are about 0.67% and 1.25% of the operating profit, respectively.

The estimated model is used to investigate how these firms' investment dynamics and capital accumulation would differ if they faced different levels of adjustment costs. Counterfactual simulations indicate that investment would be much more responsive to new information about demand if firms in Brazil faced a lower level of adjustment costs. In the long run, a lower level of adjustment costs would induce firms to operate with substantially higher capital stocks. While much of the recent literature on investment and adjustment costs has emphasized the role of non-convex forms of adjustment costs, this chapter finds that it is the presence of a significant convex component in our estimated adjustment cost functions that generates these potentially large effects .

Appendix A: Proof

A.1 Proof for Lemma 1

For given production techonology $Q_t = \widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta}$, demand schedule $Q_t = X_t P_t^{-\varepsilon}$, and variable factor price w and m, the short-run profit maximization problem is

$$\pi_t = \max_{L_t,\, M_t} \{Y_t - wL_t - mM_t\} \tag{A1}$$

Since $Y_t = P_t Q_t$, it would be useful to derive a relationship between Y_t and π_t first.

$$\begin{aligned}
Y_t &= P_t \times Q_t \\
Y_t &= \left(\frac{Q_t}{X_t}\right)^{-\frac{1}{\varepsilon}} \times Q_t \\
Y_t &= X_t^{\frac{1}{\varepsilon}} \left(Q_t\right)^{1-\frac{1}{\varepsilon}} \\
Y_t &= X_t^{\frac{1}{\varepsilon}} Q_t^{\frac{\varepsilon-1}{\varepsilon}} \\
Y_t &= X_t^{\frac{1}{\varepsilon}} \left(\widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta}\right)^{\frac{\varepsilon-1}{\varepsilon}} \\
Y_t &= X_t^{\frac{1}{\varepsilon}} \widehat{K}_t^{b} L_t^{a} M_t^{c} \qquad \text{(A2)}
\end{aligned}$$

where $b = \beta\frac{\varepsilon-1}{\varepsilon}$, $a = \alpha\frac{\varepsilon-1}{\varepsilon}$, $c = (1-\alpha-\beta)\frac{\varepsilon-1}{\varepsilon}$. Thus $0 < b + a + c = \frac{\varepsilon-1}{\varepsilon} < 1$. Define $\gamma = \frac{1}{1+\beta(\varepsilon-1)}$. Then

$$1 - a - c = \frac{1}{\varepsilon} + b = \frac{1}{\varepsilon} + \beta\frac{\varepsilon-1}{\varepsilon} \equiv \frac{1}{\gamma\varepsilon} \tag{A3}$$

The two first-order conditions for (A1) with respect to L_t and M_t are

$$a\frac{Y_t}{L_t} = w \tag{A4}$$

$$c\frac{Y_t}{M_t} = m \tag{A5}$$

which implies $wL_t = aY_t$ and $mM_t = cY_t$. Substituting them into (A1), the maximized π_t can then be expressed as

$$\pi_t = Y_t\left(1 - a - c\right)$$

Together with (A3), this implies the optimal choice of L_t and M_t would make the maximized π_t and Y_t always proportional to each other, where the proportion only depends on the capital share in the production function β, and the demand elasticity ε. That is

$$\frac{\pi_t}{Y_t} = \frac{1}{\gamma\varepsilon} \tag{A6}$$

In order to derive the value of sales (and operating profit) as a function of X_t and $\widehat{K}_t$ only, substitute (A4) and (A5) into (A2) and simplify

$$\begin{aligned}
Y_t &= \left(\frac{a}{w}Y_t\right)^a \left(\frac{c}{m}Y_t\right)^c X_t^{\frac{1}{\varepsilon}} \widehat{K}_t^b \\
Y_t &= \left(\frac{a}{w}\right)^a \left(\frac{c}{m}\right)^c Y_t^{(a+c)} X_t^{\frac{1}{\varepsilon}} \widehat{K}_t^b \\
Y_t^{(1-a-c)} &= \left(\frac{a}{w}\right)^a \left(\frac{c}{m}\right)^c X_t^{\frac{1}{\varepsilon}} \widehat{K}_t^b \\
Y_t &= \left[\left(\frac{a}{w}\right)^a \left(\frac{c}{m}\right)^c\right]^{\gamma\varepsilon} X_t^{\gamma} \widehat{K}_t^{1-\gamma}
\end{aligned}$$

where we use the fact

$$b\gamma\varepsilon = \beta\frac{\varepsilon-1}{\varepsilon}\cdot\frac{1}{1+\beta(\varepsilon-1)}\cdot\varepsilon = 1-\gamma$$

Hence

$$\pi_t = \frac{1}{\gamma\varepsilon}Y_t = const0 \cdot X_t^{\gamma}\widehat{K}_t^{1-\gamma}$$

where

$$\begin{aligned}
const0 &= \frac{1}{\gamma\varepsilon}\left[\left(\frac{a}{w}\right)^a \left(\frac{c}{m}\right)^c\right]^{\gamma\varepsilon} \\
&= \frac{1}{\gamma\varepsilon}\left[\left(\frac{\alpha}{w}\right)^{\alpha}\left(\frac{1-\alpha-\beta}{m}\right)^{1-\alpha-\beta}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{1-\beta}\right]^{\gamma(\varepsilon-1)}
\end{aligned}$$

A.2 Proof for Proposition 2

By definition, the sales growth rate is a weighted average of growth rate in demand and in captial stock, i.e.

$$dy_t \equiv \gamma\ln\left(X_t/X_{t-1}\right) + (1-\gamma)\ln\left(\widehat{K}_t/\widehat{K}_{t-1}\right) \tag{A7}$$

In the abence of of adjustment cost, $\widehat{K}_t^* = const1 \cdot X_t$, hence

$$\ln\left(\widehat{K}_t^*/\widehat{K}_{t-1}^*\right) = \ln\left(X_t/X_{t-1}\right) \tag{A8}$$

Bloom (2000) shows that the actual capital stock series chosen by a firm in the friction case has a long run growth rate equal to that of the hypothetical capital stock series that the same firm would choose in the frictionless case, essentially because the gap between these two series is bounded, i.e.

$$\lim_{T\to\infty}\frac{1}{T}\ln\left(\widehat{K}_{T+t}/\widehat{K}_t\right)=\lim_{T\to\infty}\frac{1}{T}\ln\left(\widehat{K}^*_{T+t}/\widehat{K}^*_t\right) \tag{A9}$$

According to (A8), the frictionless long-run growth rate of capital stock is equal to that of the demand,

$$\begin{aligned}\lim_{T\to\infty}\frac{1}{T}\ln\left(\widehat{K}^*_{T+t}/\widehat{K}^*_t\right) &= \lim_{T\to\infty}\frac{1}{T}\ln\left(X_{T+t}/X_t\right)\\ &= \lim_{T\to\infty}\frac{1}{T}\left(\mu T+\zeta_T-\zeta_t\right)\\ &= \mu \end{aligned} \tag{A10}$$

where we use the fact

$$\begin{aligned}\zeta_t &= \zeta_0+\sum_{s=0}^{t-1}\rho^s e_{t-s}\\ \zeta_T &= \zeta_0+\sum_{s=0}^{T-1}\rho^s e_{T-s}\end{aligned}$$

Thus for any $\rho<1$

$$\lim_{T\to\infty}\frac{1}{T}\left(\zeta_T-\zeta_t\right)=\lim_{T\to\infty}\frac{1}{T}\sum_{s=t}^{T-1}\rho^s e_{T-s}=0$$

(A7)-(A10) therefore lead to the long-run sales growth rate as described in Proposition 2.

$$\begin{aligned}dy_T &\equiv \lim_{T\to\infty}\frac{1}{T}\ln\left(Y_{T+t}/Y_t\right)\\ &= \gamma\lim_{T\to\infty}\frac{1}{T}\ln\left(X_{T+t}/X_t\right)+(1-\gamma)\lim_{T\to\infty}\frac{1}{T}\ln\left(\widehat{K}_{T+t}/\widehat{K}_t\right)\\ &= \lim_{T\to\infty}\frac{1}{T}\ln\left(X_{T+t}/X_t\right)\\ &= \mu\end{aligned}$$

Appendix B: Numerical Methods

B.1 Solve the model and simulate the data

The problem I need to solve is

$$\begin{aligned}
V(X_t, K_t) &= \max_{I_t}\{\Pi(X_t, K_t; I_t) + \frac{1}{1+r} E_t\left[V(X_{t+1}, K_{t+1})\right]\} \qquad \text{(B1)}\\
s.t.\ K_{t+1} &= (1-\delta)(K_t + I_t)\\
X_t &= \exp(x_t)\\
x_t &= c + \mu t + \zeta_t\\
\zeta_t &= \overline{\zeta} + \rho\zeta_{t-1} + e_t = \overline{\zeta} + \sum_{s=0}^{t-1}\rho^s e_{t-s}
\end{aligned}$$

where $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d}{\sim} N(0,\, 1)$, $c = -0.5\sigma^2/(1-\rho^2)$, $\zeta_0 = 0$ and

$$\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$$

Following the same trick as in Chapter 3, define $\Psi_t \equiv \exp\left(c + \overline{\zeta} + \mu t\right)$; denote $\widetilde{X}_t = \frac{X_t}{\Psi_t}$, $\widetilde{K}_t = \frac{K_t}{\Psi_t}$, $\widetilde{I}_t = \frac{I_t}{\Psi_t}$, and $\widetilde{\zeta}_t = \zeta_t - \overline{\zeta}$, then the problem (B1) is transformed into

$$\begin{aligned}
V(\widetilde{X}_t, \widetilde{K}_t) &= \max_{\widetilde{I}_t}\{\Pi(\widetilde{X}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp(\mu)}{1+r} E_t\left[V(\widetilde{X}_{t+1}, \widetilde{K}_{t+1})\right]\} \qquad \text{(B2)}\\
s.t.\ \widetilde{K}_{t+1} &= \exp(-\mu)(1-\delta)\left(\widetilde{I}_t + \widetilde{K}_t\right)\\
\widetilde{X}_t &= \exp\left(\widetilde{\zeta}_t\right)\\
\widetilde{\zeta}_t &= \rho\widetilde{\zeta}_{t-1} + e_t = \sum_{s=0}^{t-1}\rho^s e_{t-s}
\end{aligned}$$

where $e_t = \sigma\epsilon_t$ and $\epsilon_t \overset{i.i.d}{\sim} N(0,\, 1)$.

The level of demand growth rate μ would affect the investment policy. Hence the dynamic program (B2) must be solved for each type of μ. In other words, μ is an additional state variable besides $\widetilde{X}_t$ and $\widetilde{K}_t$. Given $\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$, I discretize this continuous distribution by Tauchen (1989) method. Due to 'the curse of dimensionality', I have to be conservative about the number of grids and set it to be 3 for this state variable. I experiment with higher number of grids and find the simulated moments are not very sensitive. The procedure of how to solve the model can be described as:

for $\mu = (\mu_1, \mu_2, \mu_3)$

for value function iteration converges

policy improvement algorithm

(See Appendix B.2 in Chapter 3)

end

interpolate for $\widetilde{I}_t = h(\widetilde{X}_t, \widetilde{K}_t)$

end

In other words, the dynamic program (B2) is solved for each type of firms from the type set $F = \{(\mu_v) : v = 1, \cdots, 3\}$. Under Assumption 8, this implies in the simulated data, there are 3 type of firms, each comprising a fixed proportion 1/3 of the population.

This numerical solution is applied to generate a simulated panel data with N firms and T years, where each firm is made of m independent plants. The empirical sample I use is a balanced panel with $N = 729$ firms and $T = 3$ years of observation for each firm. In order to simulate a panel for firms which are on their balanced-growth-path so that Propostion 2 applies, I allow for a burning period of Υ years in simulating the data. In application to this sample, I find $\Upsilon = 50$ is long enough in the sense that after living for Υ years, in the next T years, the simulated mean of sales growth rate is equal to the mean of demand growth rate imposed in that particular round of simulation, as predicted in Proposition 2.

For given N, T, p, and Υ, the procedure to simulate the data includes four steps, where $j = 1, \cdots, N$, $i = 1, \cdots, p$ and $t = 1, \cdots, (\Upsilon + T)$.

Step 1: Simulate data for each plant i of firm j in period t. When $t = 0$, all the simulated plants of firm j are endowed with the initial condition $\widetilde{\zeta}_{i,j,0} = 0$ and the corresponding initial capital stock $\widetilde{K}_{i,j,0} = steady_j \cdot \widetilde{X}_{i,j,0} = steady_j \cdot \exp(\widetilde{\zeta}_{i,j,0})$. Here $steady_j = (1 - \delta) \cdot const1/ \exp\left(\mu_j\right)$, (see Appendix B.1 of Chapter 3 for definition), which reflects the effect of heterogeneity in growth rate on the steady-state capital stock. For all subsequent periods, random demand shocks $e_{i,j,t} = \sigma\epsilon_{i,j,t}$ are draw for each each plant i of firm i and in each period t. Given the realization of $\widetilde{X}_{i,j,t}$ and the inherited $\widetilde{K}_{i,j,t}$, the optimal investment $\widetilde{I}_{i,j,t}$ are then pinned down using the policy rule derived above. Then $\widetilde{X}_{i,j,t}$ or equivalently $\widetilde{\zeta}_{i,j,t}$ evolves exogenously according to its law of motion, $\widetilde{K}_{i,j,t}$ evolves endogenously according to capital accumulation formula. The control variable in this period then becomes the state variable in next period.

Step 2: Plant-level data are aggregated into firm-level data. For firm j in period t, the normalized investment data is $\widetilde{I}_{j,t} = \sum_{i=1}^{p} \widetilde{I}_{i,j,t}$; capital stock is $\widetilde{K}_{j,t} = \sum_{i=1}^{p} \widetilde{K}_{i,j,t}$; operating profit is $\widetilde{\pi}_{j,t} = \sum_{i=1}^{p} \widetilde{\pi}_{i,j,t}$.

Step 3: The intercept and time trend $\exp\left(c + \overline{\zeta} + \mu_j t\right)$ are recovered for the level variables of the original model. For firm j in period t, the actual investment is therefore $I_{j,t} = \widetilde{I}_{j,t} \exp\left(c + \overline{\zeta} + \mu_j t\right)$; the actual capital stock is $K_{j,t} = \widetilde{K}_{j,t} \exp\left(c + \overline{\zeta} + \mu_j t\right)$; the actual operating profit is $\pi_{j,t} = \widetilde{\pi}_{j,t} \exp(c + \overline{\zeta} + \mu_j t)$; the sales is $Y_{j,t} = \gamma\varepsilon \cdot \pi_{j,t}$ according to equation (6). The investment rate is $i_{j,t} = I_{j,t}/K_{j,t}$.

Since only variables in ratio or growth rate are utilized in this chapter, while the value of $\overline{\zeta}$ does not affect any variables in ratio or growth rate (see Appedix A.5 of Chapter 5 for proof), I impose $\overline{\zeta} = 0$ in this chapter. The value of c is irrelevant for variables in ratio or growth rate, neither. But by setting $c = -0.5\sigma^2/\left(1 - \rho^2\right)$, $E\left[X_t\right] = \exp\left(\mu t\right)$, which is a property derived in Lemma 5 of Chapter 3 and hence can be used to check the accuracy of the simulation.

Step 4: Add measurement errors, so that the observed investment rate is $i_{j,t} = i^*_{j,t} \exp(e^I_{j,t})$, and the observed level of sales is $Y_{j,t} = Y^*_{j,t} \exp(e^Y_{j,t})$.

With the simulated data for $I_{j,t}$, $K_{j,t}$, $\pi_{j,t}$, $Y_{j,t}$, and given model parameter values, other variables of interest such as adjustment costs can be easily calculated by definition. The simulated moments are calculated only using the last $T = 3$ years observation to match those in the empirical sample.

B.2 Method of Simulated Moments

The empirical sample I use is a balanced panel with $N = 729$ firms and $T = 3$ years of observation for each firm. As suggested by Michaelides and Ng (2000), for MSM, simulating H path of $(N \times T)$ is equivalent to simulate $(HN \times T)$. In addition, each firm has p plants, and I allow for $\Upsilon = 50$ years to start from the ergodic distribution and discard them in calculating the moments. Therefore the simulated data panel is of size $[HNp \times (\Upsilon + T)]$ where I use $H = 10$ in this application.

I use a bootstrapping method (*bstrap* command in Stata) to estimate the inverse of the variance-covariance matrix of the empirical moments. This provides a candidate for the efficient choice of the weighting matrix in solving the minimum quadratic distance problem (20), i.e.

$$\Omega^* = \left[N \cdot var\left(\widehat{\Phi}^D\right)\right]^{-1}$$

In the extreme case of $var\left(\widehat{\Phi}^D\right)$ being singular, it is invertible. However, in practice, even if $var\left(\widehat{\Phi}^D\right)$ is not singular, due to the large dimension of $\widehat{\Phi}^D$, $var\left(\widehat{\Phi}^D\right)$ could be ill-conditioned. Then the inverse of $var\left(\widehat{\Phi}^D\right)$ gets inaccurate, so that sometimes Ω^* could even be not positive definite. To avoid this potential concern, I use a weighting matrix that is the inverse of the matrix, which is made of the diagonal elements of $var\left(\widehat{\Phi}^D\right)$ only. Denote this weighting matrix as $\widehat{\Omega}$, i.e.

$$\widehat{\Omega} = \left[N \cdot diag\left(var\left(\widehat{\Phi}^D\right)\right)\right]^{-1}$$

By using $\widehat{\Omega}$ instead of Ω^*, I effectively neglect information from the data about correlation between moments in measuring the quadratic distance between simulated moments and empirical moments. However, $\widehat{\Omega}$ does contain the information from the data about how relatively important each moment is in the set of moments we have chosen to match. All the empirical results I report in this thesis is based on $\widehat{\Omega}$.

To see the effect of using different weighting matrices, I re-estimate the benchmark model in Table 4.3. by using Ω^*. The estimates for b_q, b_f and ρ increase about 10%, but estimates for all other parameters stay relatively stable. In the counterfactual simulations, the 10% increase in b_q results in a 8% larger effect of b_q on capital accumulation, and has very little effect on capital adjustment.

Due to the discretization of the state spaces and the discontinuities of the investment policy in the presence of non-convex adjustment costs, I adopt a simulated annealing algorithm described in Goffe, Ferrier and Rogers (1994) to avoid local minima in solving the minimization problem (20).

There are 17 moments and 13 structural parameters in this application so that the Jacobian matrix for the binding functions is

$$J = \frac{\partial \widehat{\Phi}^S\left(\widehat{\Theta}\right)'}{\partial \Theta} = \begin{bmatrix} \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_1'}{\partial \Theta_1} & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_2'}{\partial \Theta_1} & \cdots & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_{17}'}{\partial \Theta_1} \\ \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_1'}{\partial \Theta_2} & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_2'}{\partial \Theta_2} & \cdots & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_{17}'}{\partial \Theta_2} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_1'}{\partial \Theta_{13}} & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_2'}{\partial \Theta_{13}} & \cdots & \frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_{17}'}{\partial \Theta_{13}} \end{bmatrix}$$

When calculating the numerical derivatives, in order to smooth the possible wiggles in the binding function, I use a simple regression techniques as follows. Given parameter set Θ_i $(i = 1, 2, \cdots 13)$, moment set Φ_j $(j = 1, 2, \cdots, 17)$, for each parameter Θ_i, define a range of values $\left[0.8\widehat{\Theta}_i, 1.2\widehat{\Theta}_i\right]$[1] around the optimal estimate $\widehat{\Theta}_i$ and discretize this range into 20 grids $\left[\widehat{\Theta}_i^1, \widehat{\Theta}_i^2, \cdots, \widehat{\Theta}_i^{20}\right]$, meanwhile fix all other parameters at their optimal estimates $\widehat{\Theta}_{-i}$, for each moment Φ_j, I calculate its value at each $\widehat{\Theta}_i^q$ $(q = 1, 2, \cdots 20)$, which will produce 20 values $\widehat{\Phi}_j^q$ $(q = 1, 2, \cdots 20)$. Plotting a figure of $\widehat{\Phi}_j^q$ against $\widehat{\Theta}_i^q$ illustrates the shape of the binding function. A flat line implies the moment $\widehat{\Phi}_j$ does not vary with the parameter $\widehat{\Theta}_i$; while a steep line implies $\widehat{\Phi}_j$ is informative about the variation in $\widehat{\Theta}_i$, at least at the local area of the optimal estimates. In most cases, these binding function has a linear shape, hence I run the regression using OLS, e.g.

$$\widehat{\Phi}_j^q = \alpha_{0j,i} + \alpha_{j,i}\widehat{\Theta}_i^q + \vartheta_{j,i}$$

As an illustration, Figure 4.6a plots how the proportion of zero investment $p\left(i_{j,t} = 0\right)$ in the simulated data would vary with the value of b_f, keeping all other parameters at their optimal estimates; and Figure 4.6b presents the relationship between mean profit-to-sales ratio $mean\left(PIY_{j,t}\right)$ and demand elasticity ε. The dots in these figures are the actual 20 grids evaluated and the straight lines fit these dots using OLS.

[1] For the serial correlation parameter, I only consider the range $[0.9\widehat{\rho}, 1.1\widehat{\rho}]$, given the estimated $\widehat{\rho}$ is around 0.85 and due to the restriction of $0 < \rho < 1$ in using Tauchen method. For estimates such as $\widehat{b_i}$ and $\widehat{\sigma}_{meYT}$, which are already at their lower limits, I only consider the range on one side.

Then the slope coefficient from the regression $\widehat{\alpha}_{j,i}$ fills the element $\frac{\partial \widehat{\Phi}^S(\widehat{\Theta})_j'}{\partial \Theta_i}$ in the Jacobian matrix J, which turns out to be indeed of full row rank. This implies that first, no column has all zeros—no redundant moment; second, no row has all zeros—all parameters have the possibility to be identified; and third, no rows are linear dependant—none of any two parameters lead to same variation in all moments.

With this Jacobian matrix, the asymptotic variance-covariance matrix of the optimal estimates is calculated according to (22), which produces the standard errors reported in Table 4.3.

Appendix C: Data

C.1 Sampling

After registration the original dataset and questionnaire is downloadable from: http://www.enterprisesurveys.org/

The dataset for Brazil used in this chapter was collected by the World Bank's Investment Climate Surveys in 2003, providing annual observations for up to 3 years in the period 2000-2002 for 1642 Brazilian firms, distributed across 9 industries and 13 cities.

C.2 Definition of Variables (number of observations with missing value in brackets)

- Define I = investment-disinvestment in machinery, equipment and plants (336).
- Define K= net book value of machinery, equipment and plants (441).
- Define Y = total sales (280).
- Define PI = sales minus direct costs of raw materials and inputs, minus total energy costs, minus total labour costs and minus other administrative costs (514).

C.3 Cleaning Rules (reduction in number of observations after each rule in brackets)

- Drop non-manufacturing firms (0).
- Drop state-owned firms--government ownership more than 50% (3).
- Drop firms with employment less than 10 or larger than 1000 (99).
- Rule out Investment Rate less than -1 or larger than 1 (229).
- Rule out top and bottom 5% observations for other ratio or growth rate variables (1286).
- Deflate sales growth rate using GDP deflator for Brazil [6.8% in 2001, 8.4% in 2002].
- Drop firms with less than two consecutive observations for investment rate or sales growth rate (1249).

Table 4.1 Set of Parameters

Category	**Symbol**	**Definition**
Adjustment Costs	b_q	quadratic adjustment costs
	b_i	partial irreversibility
	b_f	fixed adjustment costs
Technology and Demand	β	capital share in production function
	ε	demand elasticity with respect to price
Stochastic Process	ρ	serial correlation of demand shocks
	μ_μ	mean of μ, where μ is the growth rate of demand
	σ_μ	standard deviation of μ
	σ	standard deviation of demand shocks, level of uncertainty
Measurement Errors	σ_{IT}	s.d. of transitory measurement errors in investment rates
	σ_{IP}	s.d. of permanent measurement errors in investment rates
	σ_{YT}	s.d. of transitory measurement errors in sales
	σ_{YP}	s.d. of permanent measurement errors in sales

Table 4.2 Set of Moments

Symbol	Moments	s.e.	Definition	Informativeness
$mean(PIY_{j,t})$	0.2335	0.0054	mean of profit-to-sales ratio	β, ε
$mean(i_{j,t})$	0.1068	0.0049	mean of investment rate	$\mu_\mu, (\delta)$
$mean(dy_{j,t})$	0.0497	0.0055	mean of sales growth rate	μ_μ
$mean(ky_{j,t})$	-1.4280	0.0297	mean of log capital-to-sales ratio	$\beta, \varepsilon, (r, \delta)$
$sd(i_{j,t})$	0.1821	0.0065	standard deviation of investment rate	$b, \sigma_\mu, \sigma, \sigma_{IT}, \sigma_{IP}$
$sd(dy_{j,t})$	0.1914	0.0038	standard deviation of sales growth rate	$\sigma_\mu, \sigma, \sigma_{YT}$
$sd(ky_{j,t})$	0.8607	0.0176	standard deviation of log capital-to-sales ratio	$b, \sigma_{YT}, \sigma_{YP}$
$corr(i_{j,t}, i_{j,t-1})$	0.4244	0.0415	serial correlation of investment rate	$b_q, \rho, \sigma_\mu, \sigma_{IT}, \sigma_{IP}$
$corr(dy_{j,t}, dy_{j,t-1})$	0.2169	0.0377	serial correlation of sales growth rate	$\rho, \sigma_\mu, \sigma_{YT}$
$corr(yk_{j,t}, yk_{j,t-1})$	0.9412	0.0064	serial correlation of log sales-to-capital ratio	$\rho, \sigma_{YT}, \sigma_{YP}$
$corr(i_{j,t}, dy_{j,t})$	0.1987	0.0330	corr. btw. investment rate & sales growth rate	$b, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}$
$corr(i_{j,t}, yk_{j,t})$	0.1243	0.0271	corr. btw. investment rate & log sales-to-capital ratio	$b, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}, \sigma_{YP}$
$corr(dy_{j,t}, yk_{j,t})$	0.1179	0.0272	corr. btw. sales growth rate & log sales-to-capital ratio	$b, \sigma_{YT}, \sigma_{YP}$
$p(0.2 < i_{j,t} < 1)$	0.1909	0.0101	proportion of investment spikes	b_f, σ_μ
$p(0 < i_{j,t} < 0.2)$	0.4019	0.0132	proportion of moderate investment	b_f, b_q
$p(i_{j,t} = 0)$	0.3674	0.0151	proportion of zero investment	b_i, b_f
$p(-0.2 < i_{j,t} < 0)$	0.0345	0.0047	proportion of moderate disinvestment	b_i, σ_μ

Table 4.3 Empirical Results

Symbol	Empirical	Simulated	Parameter	Estimates	s.e.
$mean(PIY_{j,t})$	0.2335	0.2337	b_q	2.4156	0.0645
$mean(i_{j,t})$	0.1068	0.1163	b_i	0.0020	0.0075
$mean(dy_{j,t})$	0.0497	0.0292	b_f	0.0105	0.0008
$mean(ky_{j,t})$	-1.4280	-1.4324	β	0.0576	0.0004
$sd(i_{j,t})$	0.1821	0.1860	ε	5.2148	0.0064
$sd(dy_{j,t})$	0.1914	0.1925	ρ	0.8515	0.0063
$sd(ky_{j,t})$	0.8607	0.8580	μ_μ	0.0276	0.0001
$corr(i_{j,t}, i_{j,t-1})$	0.4244	0.4303	σ_μ	0.0735	0.0007
$corr(dy_{j,t}, dy_{j,t-1})$	0.2169	0.0470	σ	0.2197	0.0008
$corr(yk_{j,t}, yk_{j,t-1})$	0.9412	0.9763	σ_{IT}	0.6914	0.0024
$corr(i_{j,t}, dy_{j,t})$	0.1987	0.3486	σ_{IP}	0.3716	0.0045
$corr(i_{j,t}, yk_{j,t})$	0.1243	0.1805	σ_{YT}	0.0000	0.0130
$corr(dy_{j,t}, yk_{j,t})$	0.1179	0.1982	σ_{YP}	0.8287	0.0011
$p(0.2 < i_{j,t} < 1)$	0.1909	0.1908			
$p(0 < i_{j,t} < 0.2)$	0.4019	0.4092			
$p(i_{j,t} = 0)$	0.3674	0.3608	**OI**		**93**
$p(-0.2 < i_{j,t} < 0)$	0.0345	0.0322	**Degree of Freedom**		**4**

Table 4.4 Specification Tests

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Restriction	none	$b_q = 0$	$b_i = 0$	$b_f = 0$	$b_{i,f} = 0$	$\sigma_\mu = 0$	$m.e.$=0
Estimates							
b_q	2.4156	**0.0000**	2.5380	1.9086	3.9220	3.4468	3.9898
b_i	0.0020	0.1569	**0.0000**	0.2133	**0.0000**	0.0376	0.2783
b_f	0.0105	0.0001	0.0116	**0.0000**	**0.0000**	0.0435	0.0325
β	0.0576	0.0494	0.0580	0.0519	0.0620	0.0719	0.0470
ε	5.2148	5.1673	5.2235	5.1426	5.3346	5.3624	4.8399
ρ	0.8515	0.9161	0.8592	0.6832	0.8803	0.9469	0.9472
μ_μ	0.0276	0.0343	0.0278	0.0342	0.0342	0.0275	0.0208
σ_μ	0.0735	0.0839	0.0736	0.0828	0.0427	**0.0000**	0.0901
σ	0.2197	0.1012	0.2209	0.2023	0.2398	0.2397	0.2196
σ_{IT}	0.6914	0.4007	0.6912	0.7030	0.6617	0.4342	**0.0000**
σ_{IP}	0.3716	0.4614	0.3648	0.2977	0.5225	0.6300	**0.0000**
σ_{YT}	0.0000	0.1024	0.0000	0.0002	0.0000	0.0000	**0.0000**
σ_{YP}	0.8287	0.8495	0.8277	0.8314	0.8195	0.8355	**0.0000**
Moments							
$mean(PIY_{j,t})$	0.2337	0.2333	0.2334	0.2338	0.2338	0.2339	0.2354
$mean(i_{j,t})$	0.1163	0.1157	0.1166	0.1249	0.1303	0.1157	0.0880
$mean(dy_{j,t})$	0.0292	0.0338	0.0295	0.0359	0.0359	0.0292	0.0235
$mean(ky_{j,t})$	-1.4324	-1.4268	-1.4321	-1.4308	-1.4249	-1.4312	-1.4056
$sd(i_{j,t})$	0.1860	0.1909	0.1853	0.1828	0.1736	0.1902	0.1017
$sd(dy_{j,t})$	0.1925	0.1873	0.1930	0.1893	0.1958	0.1926	0.2039
$sd(ky_{j,t})$	0.8580	0.8556	0.8590	0.8655	0.8532	0.8554	0.8240
$corr(i_{j,t}, i_{j,t-1})$	0.4303	0.4307	0.4296	0.4217	0.4251	0.4411	0.7763
$corr(dy_{j,t}, dy_{j,t-1})$	0.0470	-0.1491	0.0504	0.0059	-0.0238	-0.0031	0.1294
$corr(yk_{j,t}, yk_{j,t-1})$	0.9763	0.9765	0.9763	0.9790	0.9733	0.9710	0.9747
$corr(i_{j,t}, dy_{j,t})$	0.3486	0.4396	0.3480	0.3192	0.2352	0.4136	0.6071
$corr(i_{j,t}, yk_{j,t})$	0.1805	0.1509	0.1836	0.2052	0.1526	0.1554	0.6979
$corr(dy_{j,t}, yk_{j,t})$	0.1982	0.1786	0.1998	0.2204	0.1815	0.1929	0.4346
$p(0.2 < i_{j,t} < 1)$	0.1908	0.1999	0.1919	0.2034	0.1832	0.1879	0.1903
$p(0 < i_{j,t} < 0.2)$	0.4092	0.3967	0.4090	0.4580	0.7680	0.4276	0.3207
$p(i_{j,t} = 0)$	0.3608	0.3587	0.3616	0.3018	0.0091	0.3775	0.4646
$p(-0.2 < i_{j,t} < 0)$	0.0322	0.0353	0.0309	0.0311	0.0336	0.0000	0.0243
OI	**93**	**181**	**93**	**147**	**1306**	**167**	**1027**
Degree of Freedom	**4**	**5**	**5**	**5**	**6**	**5**	**8**

Table 4.5 Robustness Tests

Column	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Variation	benchmark	$\delta = 0.04$	$\delta = 0.06$	$r = 0.13$	$r = 0.17$	$p = 3$
Estimates						
b_q	2.4156	3.0072	2.5212	2.5738	2.9318	2.0428
b_i	0.0020	0.0002	0.0000	0.0008	0.0044	0.0042
b_f	0.0105	0.0127	0.0100	0.0111	0.0120	0.0364
β	0.0576	0.0563	0.0613	0.0533	0.0645	0.0620
ε	5.2148	5.1955	5.2872	5.1154	5.3572	5.1849
ρ	0.8515	0.8837	0.8314	0.8585	0.8597	0.7991
μ_μ	0.0276	0.0352	0.0210	0.0286	0.0275	0.0319
σ_μ	0.0735	0.0704	0.0770	0.0745	0.0730	0.0773
σ	0.2197	0.2206	0.2206	0.2155	0.2273	0.3560
σ_{IT}	0.6914	0.6943	0.6835	0.6913	0.6973	0.6660
σ_{IP}	0.3716	0.4059	0.3420	0.3604	0.3626	0.3656
σ_{YT}	0.0000	0.0000	0.0005	0.0001	0.0000	0.0038
σ_{YP}	0.8287	0.8229	0.8285	0.8263	0.8264	0.8316
Moments						
$mean(PIY_{j,t})$	0.2337	0.2326	0.2339	0.2334	0.2337	0.2346
$mean(i_{j,t})$	0.1163	0.1137	0.1204	0.1175	0.1165	0.1190
$mean(dy_{j,t})$	0.0292	0.0368	0.0227	0.0302	0.0292	0.0318
$mean(ky_{j,t})$	-1.4324	-1.4284	-1.4283	-1.4268	-1.4238	-1.4323
$sd(i_{j,t})$	0.1860	0.1872	0.1845	0.1849	0.1855	0.1873
$sd(dy_{j,t})$	0.1925	0.1921	0.1923	0.1921	0.1931	0.1915
$sd(ky_{j,t})$	0.8580	0.8602	0.8568	0.8578	0.8612	0.8565
$corr(i_{j,t}, i_{j,t-1})$	0.4303	0.4345	0.4315	0.4289	0.4252	0.4315
$corr(dy_{j,t}, dy_{j,t-1})$	0.0470	0.0516	0.0494	0.0526	0.0492	0.0275
$corr(yk_{j,t}, yk_{j,t-1})$	0.9763	0.9765	0.9767	0.9766	0.9765	0.9761
$corr(i_{j,t}, dy_{j,t})$	0.3486	0.3329	0.3527	0.3427	0.3423	0.3925
$corr(i_{j,t}, yk_{j,t})$	0.1805	0.1875	0.1842	0.1791	0.1931	0.1582
$corr(dy_{j,t}, yk_{j,t})$	0.1982	0.2010	0.2015	0.1969	0.2053	0.1941
$p(0.2 < i_{j,t} < 1)$	0.1908	0.1839	0.2004	0.1936	0.1912	0.1974
$p(0 < i_{j,t} < 0.2)$	0.4092	0.4112	0.4117	0.4087	0.4087	0.4250
$p(i_{j,t} = 0)$	0.3608	0.3661	0.3530	0.3611	0.3644	0.3389
$p(-0.2 < i_{j,t} < 0)$	0.0322	0.0318	0.0288	0.0301	0.0291	0.0324
OI	**93**	**82**	**111**	**91**	**96**	**112**
Degree of Freedom	**4**	**4**	**4**	**4**	**4**	**4**

Table 4.6 Comparison with Cooper & Haltiwanger (2006)

	This Chapter	Cooper & Haltiwanger (2006)
Estimates		
b_q	2.416	0.153
b_i	0.004	0.019
b_f	0.011	0.204
Sample		
	World Bank Survey Brazil	**LRD USA**
Data		
$mean(i_{j,t})$	0.107	0.122
$sd(i_{j,t})$	0.182	0.337
$corr(i_{j,t}, yk_{j,t})$	0.124	0.143
$corr(i_{j,t}, i_{j,t-1})$	0.424	0.058
$p(0.2 < i_{j,t} < 1)$	0.191	0.186
$p(0 < i_{j,t} < 0.2)$	0.402	..
$p(i_{j,t} = 0)$	0.367	0.081
$p(-0.2 < i_{j,t} < 0)$	0.035	..
$p(-1 < i_{j,t} < -0.2)$	0.005	0.018

Figure 4.1a: Empirical Distribution of Investment Rate

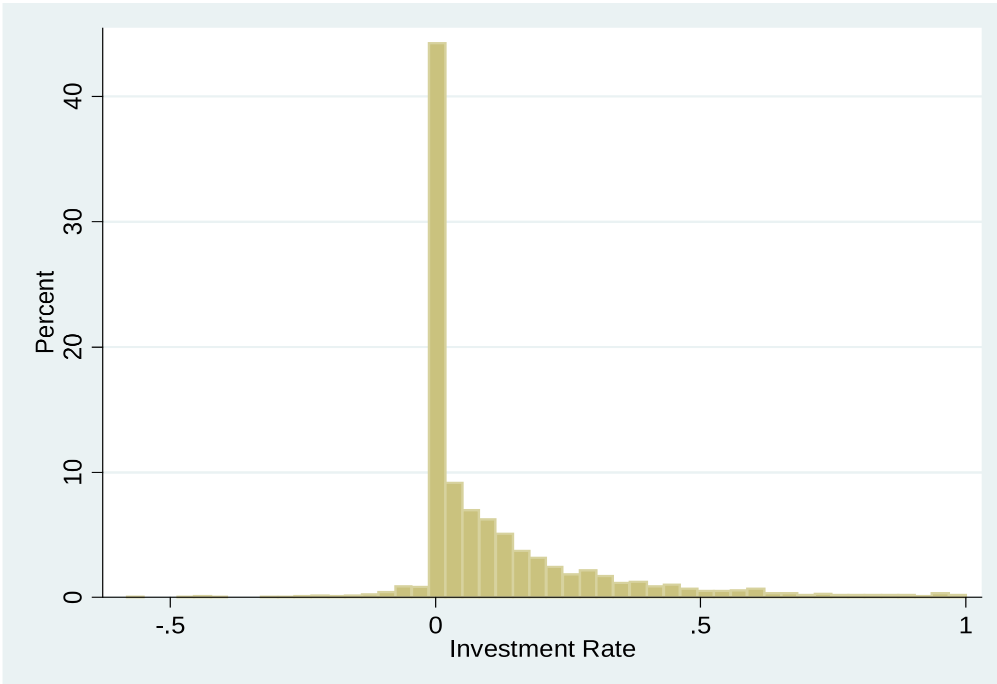


Figure 4.1b: Empirical Distribution of Sales Growth Rate

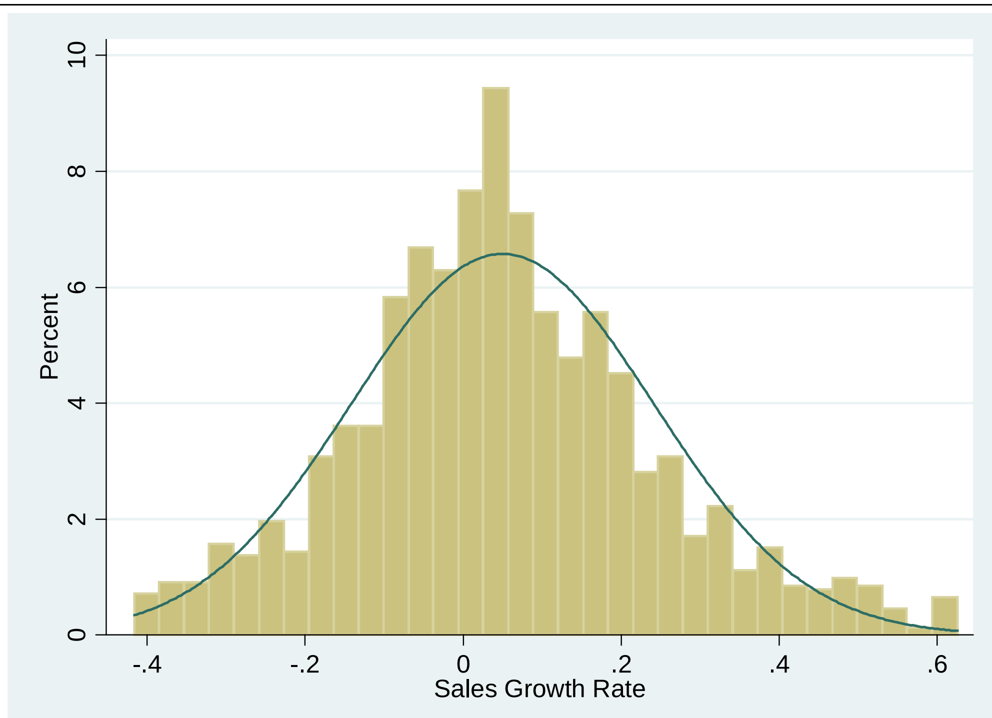


Figure 4.2: Illustration of Method of Simulated Moments

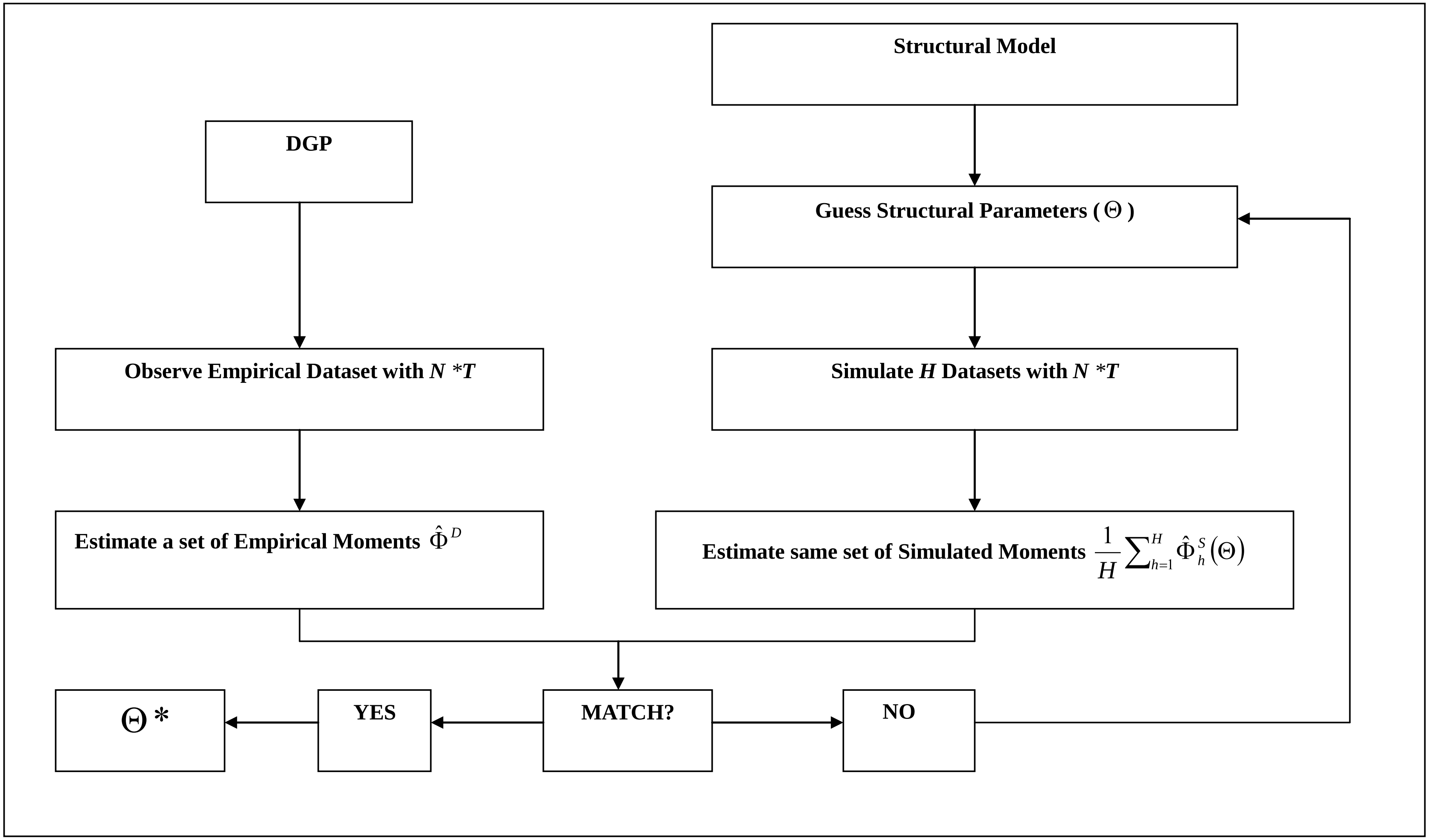


Figure 4.3a: Capital Adjustment at Different Level of Uncertainty

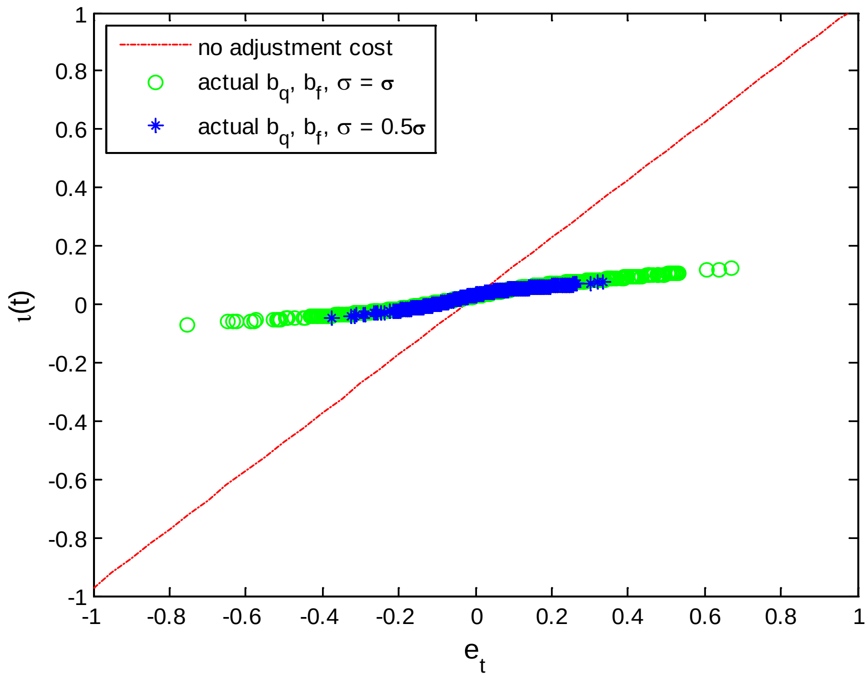


Figure 4.3b: Capital Adjustment at Different Level of Adjustment Costs

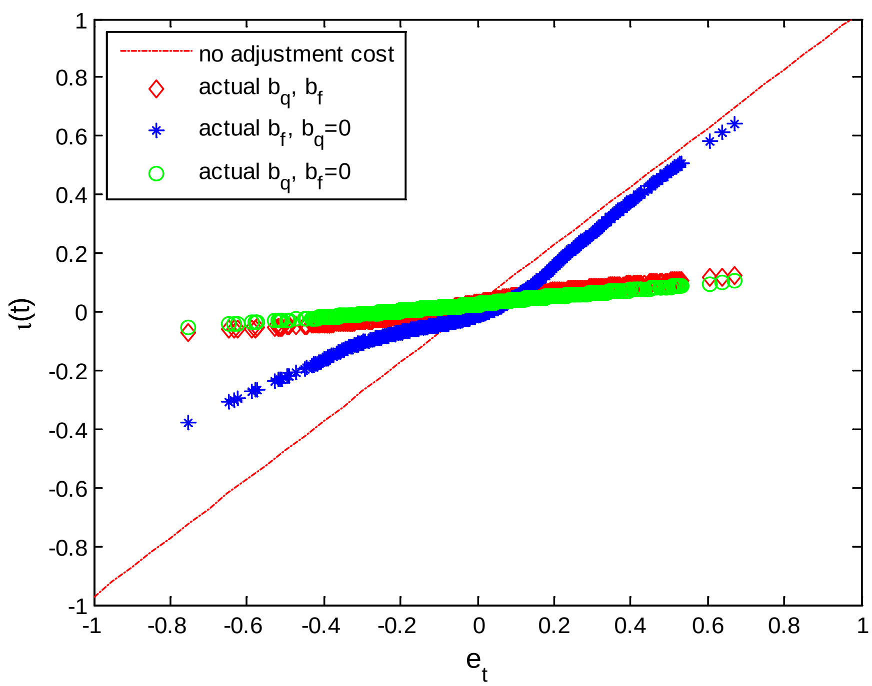


Figure 4.4a: Normalized Capital Stock at Different Level of b_q

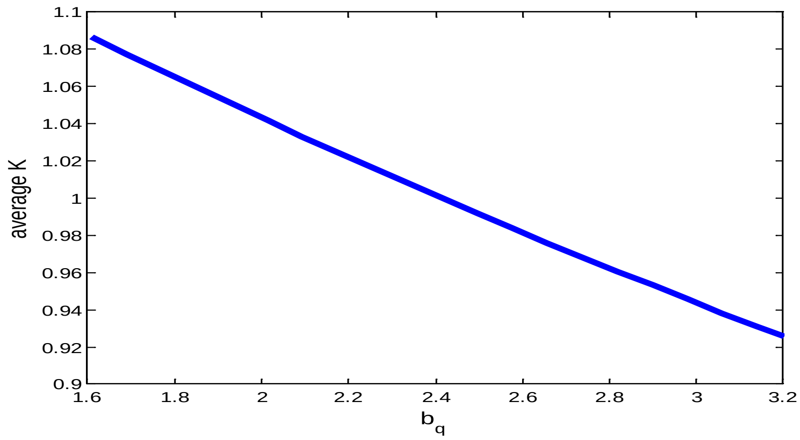


Figure 4.4b: Actual Cost-to-Profit Ratio at Different Level of b_q

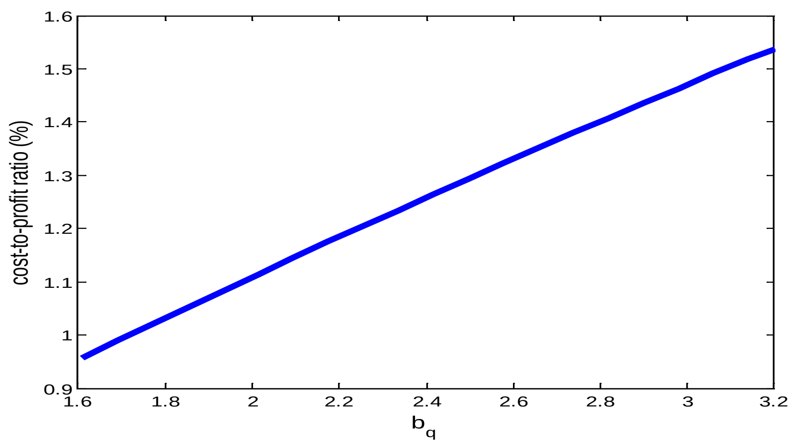


Figure 4.4c: Normalized Cost-to-Profit Ratio at Different Level of b_q

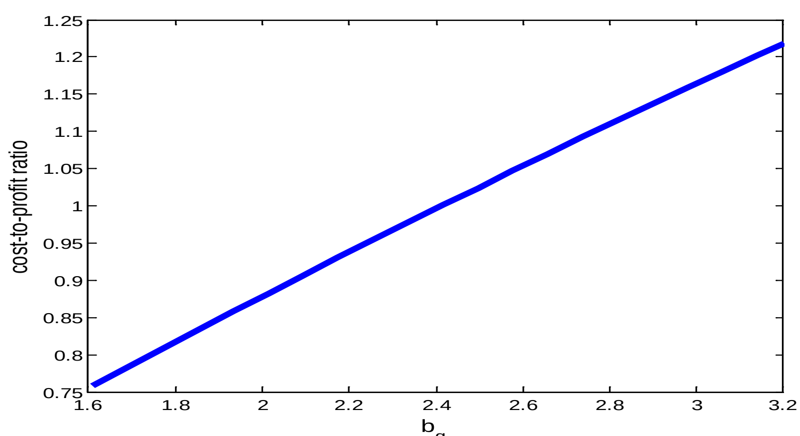


Figure 4.5a: Normalized Capital Stock at Different Level of b_f

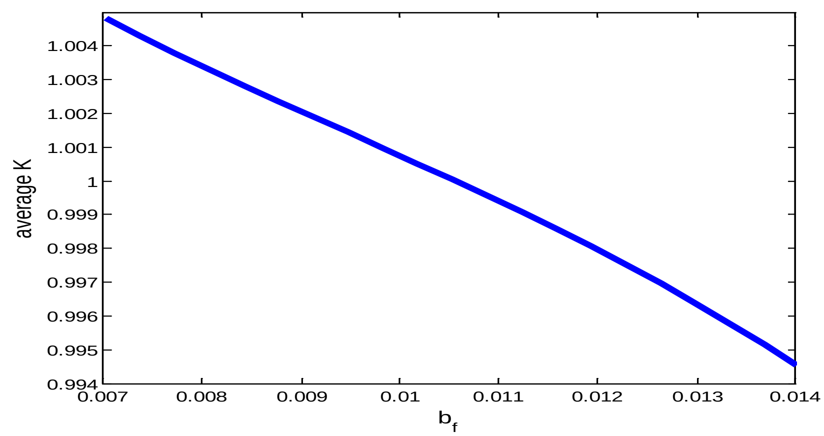


Figure 4.5b: Actual Cost-to-Profit Ratio at Different Level of b_f

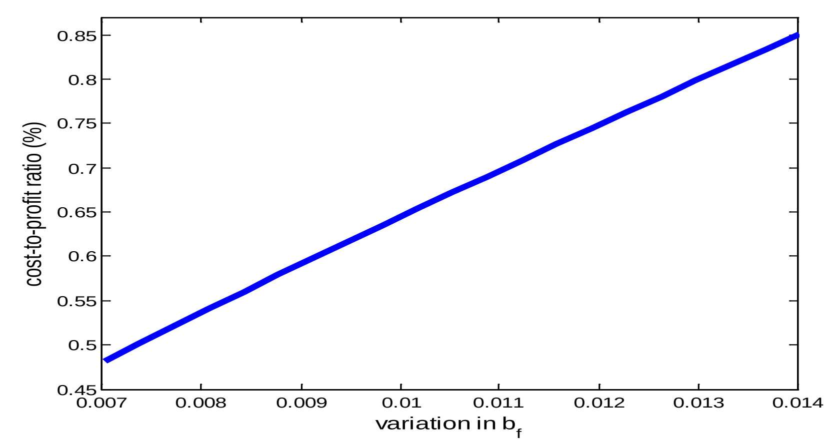


Figure 4.5c: Normalized Cost-to-Profit Ratio at Different Level of b_f

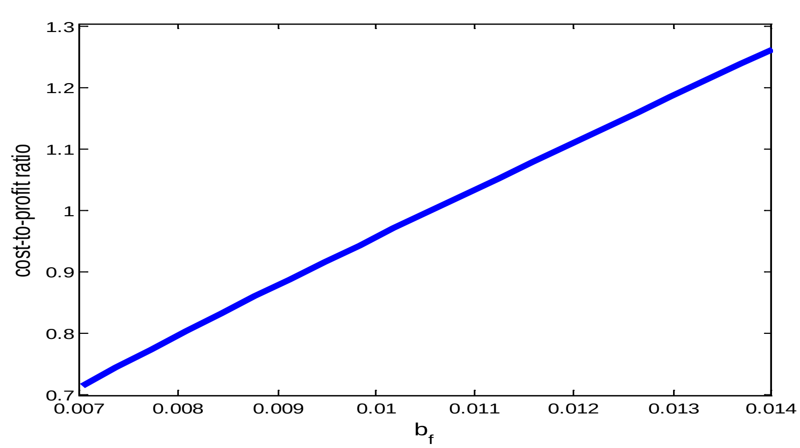


Figure 4.6a: Binding Function--How Proportion of Zero Investment Varies with b_f

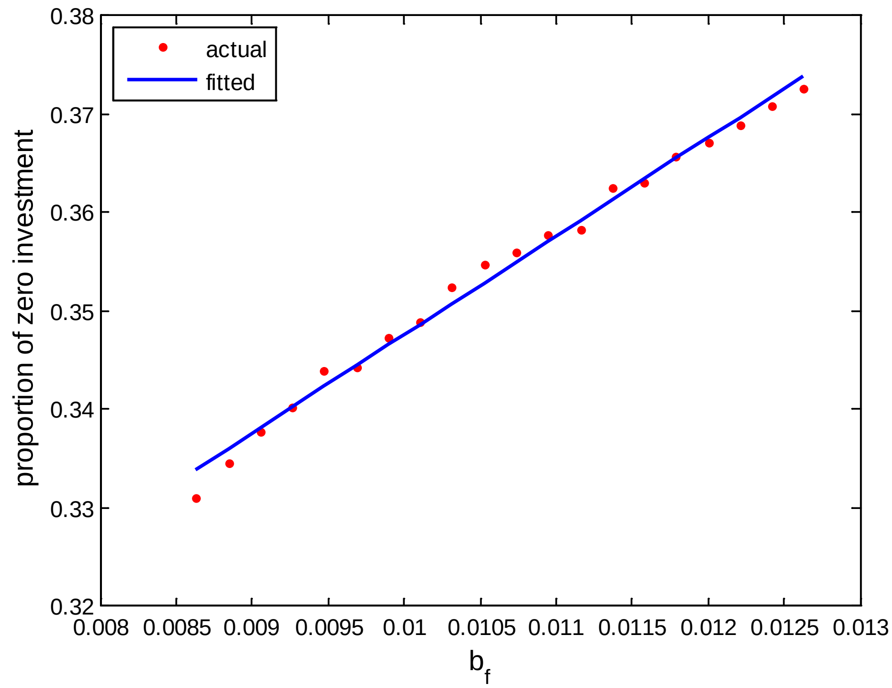
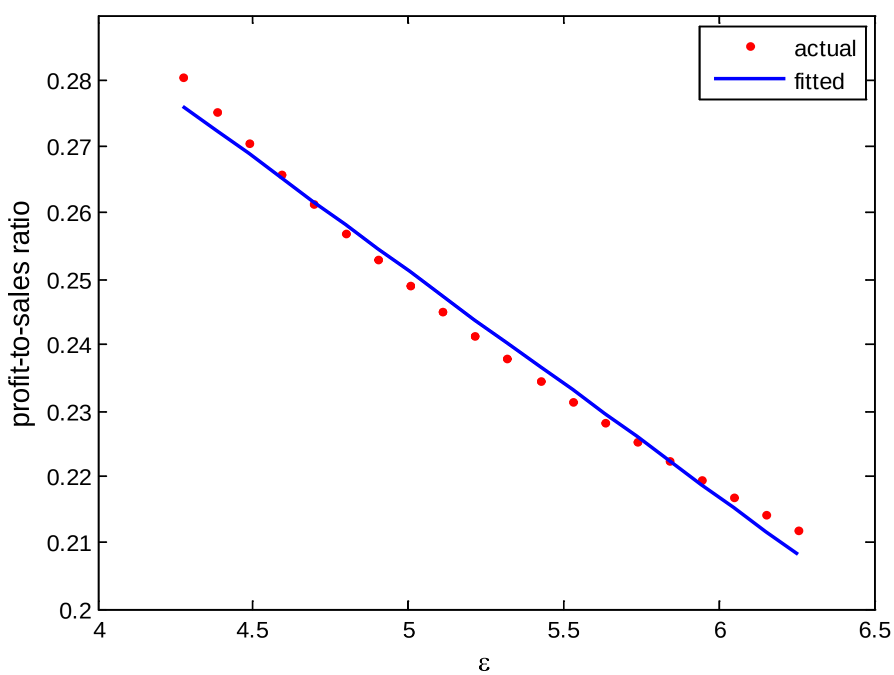


Figure 4.6b: Binding Function--How Profit-to-Sales Ratio Varies with ε



Chapter 5

A Structural Estimation for the Effects of Uncertainty on Capital Accumulation with Heterogeneous Firms

1 Introduction

This chapter develops a structural framework to estimate the effects of uncertainty on investment behavior and capital accumulation at the firm level. As surveyed in Chapter 2, within the framework of risk-neutrality and perfect capital markets, the neoclassical producer theory predicts two possible channels for how uncertainty might affect investment decision and capital accumulation: first, through the curvature of the operating profit, which is called the Hartman-Abel-Caballero (HAC) effect in the literature; and second, through the frictions associated with capital adjustment, modelled through different forms of capital adjustment costs.

Chapter 3 focuses exclusively on the second channel and studies the effects of uncertainty in the absence of a HAC effect from a theoretical point of view. In that chapter, by construction in the absence of adjustment costs, both the investment dynamics and capital accumulation are invariant to the level of uncertainty. So in this class of investment models, uncertainty affects these two quantities of interest only through different forms of adjustment costs. To see whether the investment behavior at the micro level is consistent with the presence of adjustment costs, Chapter 4 provides structural estimates of the adjustment cost parameters by matching simulated moments from the investment model with empirical moments from a panel of Brazilian manufacturing firms. The results indicate that the model would fit the data best with the mix of a quadratic adjustment cost, and at least one form of non-convex adjustment costs.

The theoretical analysis in Chapter 3 and the empirical findings in Chapter 4 therefore highlight the importance of adjustment costs. Meanwhile they also raise an interesting question: what would be the net effect of uncertainty if the HAC effect is also in operation? This chapter goes further and investigates the overall effects of uncertainty on short-run investment dynamics and, particularly, on long-run capital accumulation, allowing for the HAC effect, in addition to the capital adjustment cost effect.

Recall that the HAC effect affects the investment decision through the curvature of the operating profit with respect to variables that characterize uncertainty.

This chapter captures this effect by modelling uncertainty from both productivity and demand, together with commonly-used functional forms for production technology and demand schedule. The investment model under this specification has two important implications, based on which I design a 'two-step' identification procedure.

First, in the absence of adjustment costs, the expected capital stock level could increase, decrease or be invariant to the level of uncertainty, depending on the relative importance of productivity and demand uncertainty in the overall uncertainty. Therefore, the identification of the HAC effect requires identifying the relative weight of different sources of uncertainty. In the presence of adjustment costs, the overall effect of uncertainty on the expected capital stock then depends on the net effect of the HAC effect and the adjustment cost effect.

Second, by construction both the operating profit and the adjustment cost function are linear homogenous in the stochastic variable and capital stock. This implies the HAC effect affects the level of key variables such as capital stock, sales and operating profit, but not their growth rates or ratios, such as investment rate, sales growth rate, capital-to-sales ratio and profit-to-sales ratio. That is, when the expected investment expenditure increases/decreases due to the HAC effect, the expected capital stock, sales, and operating profit all increase/decrease proportionally. The irrelevance of the HAC effect on variables in ratio or growth rate is consistent with the empirical evidence, such as Leahy and Whited (1996) and Bond, Bloom and Van Reenen (2006), where uncertainty is found to have little impact on the variation in investment rate through the HAC effect.

For a given Jorgensonian user cost of capital, the expected capital stock level is determined by five factors in the model: production technology, demand schedule, stochastic process, adjustment costs and the HAC effect. As illustrated in Chapter 4, using moments for variables in ratio or growth rate only, variation in the data can identify the parameters of the production technology, the demand schedule, the stochastic process and the adjustment costs. Therefore in this chapter, despite the presence of the HAC effect, the irrelevance of the HAC effect on moments in ratio or growth rate would allow me to estimate the other four factors by using moments in ratio or growth rate only. This is implemented in step 1 of the 'two-step' procedure.

In step 2 of the 'two-step' procedure, for given four factors estimated from step 1, I estimate the HAC effect by matching moments in level. In particular, identification of the HAC effect comes from matching the correlation between a measure of capital stock and a proxy for the level of uncertainty, for a given effect of uncertainty on capital accumulation through the adjustment cost effects estimating in step 1.

This implies that a necessary condition for identification in step 2 is to allow for some variation in the level of uncertainty. This is achieved by modelling heterogeneity in the level of uncertainty across firms. In addition, I also allow for heterogeneity in the growth rate of the stochastic process and heterogeneity in the intercept of the stochastic process, in order to control for unobserved factors that may lead to permanent differences in firm size.

With this empirical strategy, estimating the effects through each channel separately involves estimating a set of structural parameters of the model. Using a simulated minimum distance estimator, these parameters are estimated by matching simulated model moments with empirical data moments from a panel of UK manufacturing firms in Datastream. The panel structure of this dataset allows me to proxy the measure of uncertainty for each firm using the within-group standard deviation of sales growth rate, while the between-group standard deviation of this proxy measures the variation in the level of uncertainty across firms in this sample.

Finally, I generalize the specification by allowing for a possible omitted variable bias through a risk-premium component in the discount rate. If risk-neutrality and full diversification do not hold, this extension should lead to a better fit. The negative correlation between the capital-to-sales ratio and the proxy for uncertainty is key for the identification of the discount rate effect, through which uncertainty may have a further negative effect on capital accumulation, in addition to the adjustment cost effect and the HAC effect.

Counterfactual simulations find that, in the short-run, the estimated investment model predicts a small effect of uncertainty on capital adjustment. This is mainly due to the importance of quadratic adjustment costs and hence is robust to whether the model allows for a discount rate effect or not. In the long-run, the estimates indicate a negative and potentially large effect of uncertainty on capital accumulation. This operates mainly through the negative effect of quadratic adjustment costs in the baseline model without discount rate effect, and through a richer combination of factors in the extended model with a discount rate effect.

The rest of the chapter is organized as follows. Section 2 outlines the investment model for estimation and examines the model predictions about the effects of uncertainty on capital accumulation. Section 3 presents the features of the data and the empirical specifications. Section 4 discusses identification. Section 5 reports the empirical results. Section 6 considers an extended model with a simple link between uncertainty and firm-specific discount rates. Section 7 contains the counterfactual simulations for the effects of uncertainty. Section 8 concludes.

2 The Effects of Uncertainty

This section sets up an investment model for estimation. The model is the same as the one in Chapter 4 with one exception: randomness in productivity as well as demand is introduced. With this generalization, under a commonly-used functional form, uncertainty from the different sources implies different curvature of the operating profit, therefore has different implications for the sign and the magnitude of the effects of uncertainty on capital accumulation.

2.1 The Investment Model

2.1.1 Production and Technology

Time is discrete and the horizon is infinite. By paying capital adjustment costs, new investment I_t contributes to productive capital $\widehat{K}_t$ immediately in period t, which depreciates at the end of each period.

Assumption 1 ***Timing:***

The law of motion for capital stock is

$$K_{t+1} = (1-\delta)(K_t + I_t) \equiv (1-\delta)\widehat{K}_t \tag{1}$$

where δ is the constant depreciation rate.

Consider a firm that uses capital stock $\widehat{K}_t$, labour L_t and material M_t to produce nonstorable output Q_t, according to a stochastic constant returns to scale Cobb-Douglas technology.

Assumption 2 ***Production:***

The production function is

$$Q_t = A_t \widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta} \tag{2}$$

where A_t represents the randomness in productivity; the capital share β satisfies $0 < \beta < 1$, labour share α satisfies $0 < \alpha < 1$, and $0 < \alpha + \beta < 1$.

The firm faces an isoelastic, downward-sloping, stochastic demand curve.

Assumption 3 ***Demand:***

The demand schedule is

$$Q_t = X_t P_t^{-\varepsilon} \tag{3}$$

where X_t represents the randomness in demand[1]*; $-\varepsilon < -1$ is the demand elasticity with respect to price.*

Denote sales as $Y_t = P_t Q_t$. Labour and material are two variable inputs and can be adjusted instantaneously and costlessly. At each point of time, for a given capital stock, productivity and demand realization, the firm chooses labour L_t and material M_t to maximize its instantaneous operating profit $\pi_t = \max_{L_t,\, M_t} \{Y_t - wL_t - mM_t\}$, where w is a constant wage rate and m is a constant price for material.

[1] This is called "horizontal demand shocks" in Abel and Eberly (1999). Alternatively, if we model "vertical demand shocks", such as $P_t = X_t Q_t^{-1/\varepsilon}$ in Caballero (1991), the operating profit can be derived as $\pi(X_t, A_t, \widehat{K}_t) = const0 \cdot (X_t^{\gamma})^{\varepsilon} (A_t^{\gamma})^{\varepsilon-1} \widehat{K}_t^{1-\gamma}$. As it will become clear in Section 2.2, this specification does not allow us to estimate the relative importance of the HAC effect. On the other hand, both horizontal and vertical demand shocks could be justified to model demand uncertainty faced by a monopoly (Klemperer and Meyer, 1986).

Lemma 1 ***Operating Profit and Sales:***

The maximized value of operating profit and sales are given by

$$\pi_t = const0 \cdot X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma} \tag{4}$$

and

$$Y_t = \gamma\varepsilon \cdot \pi_t \tag{5}$$

where

$$0 < \frac{1}{\varepsilon} < \gamma = \frac{1}{1+\beta(\varepsilon-1)} < 1 \tag{6}$$

and

$$const0 = \frac{1}{\gamma\varepsilon}\left[\left(\frac{\alpha}{w}\right)^{\alpha}\left(\frac{1-\alpha-\beta}{m}\right)^{1-\alpha-\beta}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{1-\beta}\right]^{\gamma(\varepsilon-1)} \tag{7}$$

Proof: See Appendix A.1.

2.1.2 Stochastic Process

The demand shift parameter X_t and the level of productivity A_t are the two possible sources of uncertainty in this model.

Assumption 4 ***Demand Stochastic:***

The law of motion for X_t is

$$\begin{aligned} x_t &= \ln X_t \\ x_t &= c_x + \mu_x t + \zeta_t^x \\ \zeta_t^x &= \overline{\zeta}^x + \rho_x \zeta_{t-1}^x + e_t^x = \overline{\zeta}^x + \sum_{s=0}^{t-1} \rho^s e_{t-s}^x \end{aligned} \tag{8}$$

where $0 < \rho_x < 1$, $e_t^x = \sigma_x \epsilon_t^x$, $\epsilon_t^x \overset{i.i.d.}{\sim} N(0,1)$, $c_x = -0.5\sigma_x^2/\left(1-\rho_x^2\right)$ *and* $\zeta_0^x = 0$.

Assumption 5 ***Productivity Stochastic:***

The law of motion for A_t is

$$\begin{aligned} a_t &= \ln A_t \\ a_t &= c_a + \mu_a t + \zeta_t^a \\ \zeta_t^a &= \overline{\zeta}^a + \rho_a \zeta_{t-1}^a + e_t^a = \overline{\zeta}^a + \sum_{s=0}^{t-1} \rho_a^s e_{t-s}^a \end{aligned} \tag{9}$$

where $0 < \rho_a < 1$, $e_t^a = \sigma_a \epsilon_t^a$, $\epsilon_t^a \overset{i.i.d.}{\sim} N(0,1)$, $c_a = -0.5\sigma_a^2/\left(1-\rho_a^2\right)$ *and* $\zeta_0^a = 0$.

As (4) indicates, it is $X_t^{\gamma}\left(A_t^{\gamma}\right)^{\varepsilon-1}$ that jointly determines the marginal revenue product of capital and hence the investment decision in this model.

Lemma 2 ***Reparameterization:***

By imposing $\rho_x = \rho_a = \rho$, and assuming that ϵ_t^x and ϵ_t^a are independent, X_t and A_t can be combined into one single random variable, i.e.

$$Z_t = X_t \left(A_t\right)^{\varepsilon-1} \tag{10}$$

The law of motion for Z_t is given by

$$\begin{aligned} z_t &= \log Z_t \\ z_t &= c + \mu t + \zeta_t \\ \zeta_t &= \overline{\zeta} + \rho\zeta_{t-1} + e_t = \overline{\zeta} + \sum_{s=0}^{t-1} \rho^s e_{t-s} \end{aligned} \tag{11}$$

where $0 < \rho < 1$, $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$, and $\zeta_0 = 0$. In particular,

$$\begin{aligned} \overline{\zeta} &= \overline{\zeta}^x + (\varepsilon-1)\,\overline{\zeta}^a \\ c &= c_x + (\varepsilon-1)\,c_a \\ \mu &= \mu_x + (\varepsilon-1)\,\mu_a \\ \sigma^2 &= \sigma_x^2 + (\varepsilon-1)^2\,\sigma_a^2 \end{aligned} \tag{12}$$

Proof: See Appendix A.2.

With this reparameterization, the operating profit can be written as

$$\pi(Z_t, \widehat{K}_t) = const0 \cdot Z_t^{\gamma}\widehat{K}_t^{1-\gamma} \tag{13}$$

where Z_t incorporates uncertainty from both demand and productivity. Henceforth, I call Z_t 'profitability'. If $\sigma_a^2 = 0$, it is equivalent to a model where all the uncertainty is from demand; this corresponds to the model studied in Chapter 3 and 4. If $\sigma_x^2 = 0$, it is equivalent to a model where all the uncertainty is from productivity. When uncertainty comes from both demand and productivity, σ^2 is a measure of the overall uncertainty faced by the firm.

Assumption 6 ***Constant Proportion of Demand Uncertainty:***

Among the overall uncertainty, there is a constant proportion τ of uncertainty coming from demand, i.e.

$$\sigma_x^2 = \tau\sigma^2 \tag{14}$$

Since $\sigma^2 = \sigma_x^2 + (\varepsilon - 1)^2 \sigma_a^2$, this assumption also implies a constant proportion of uncertainty from productivity, i.e. $\sigma_a^2 = \frac{(1-\tau)}{(\varepsilon-1)^2}\sigma^2$. Hence $\tau = 1$ is equivalent to a model where all the uncertainty is from demand; and $\tau = 0$ is equivalent to a model where all the uncertainty is from productivity.

2.1.3 Dynamic Optimization

In contrast with labour and material, frictions prevent instantaneous and costless adjustment of capital stock, which are modelled through three forms of capital adjustment costs: partial irreversibility, fixed adjustment costs and quadratic adjustment costs.

Assumption 7 ***Capital Adjustment Costs:***

The functional form for capital adjustment costs is

$$G(Z_t, K_t; I_t) = -b_i I_t \mathbf{1}_{[I_t<0]} + b_f \mathbf{1}_{[I_t \neq 0]} \pi_t + \frac{b_q}{2}\left(\frac{I_t}{K_t}\right)^2 K_t \tag{15}$$

where $\mathbf{1}_{[I_t<0]}$ and $\mathbf{1}_{[I_t \neq 0]}$ are indicators for negative and non-zero investment; b_i can be interpreted as the difference between the purchase price and the sale price expressed as a percentage of the purchase price of capital goods; b_f is interpreted as the fraction of operating profit loss due to any non-zero investment; and b_q measures the magnitude of quadratic adjustment costs.

The net revenue of the firm in each period t is therefore

$$\Pi(Z_t, K_t; I_t) = \pi(Z_t, K_t; I_t) - G(Z_t, K_t; I_t) - I_t \tag{16}$$

Assumption 8 *The firm is risk-neutral and discounts future net revenue at a constant rate r, where $r > \exp(\mu) - 1$.*

In each period investment is chosen to maximize the discounted present value of current and expected future net revenues, which can be represented as the solution to a dynamic optimization problem defined by the stochastic Bellman equation

$$V(Z_t, K_t) = \max_{I_t}\{\Pi(Z_t, K_t; I_t) + \frac{1}{1+r} E_t \left[V(Z_{t+1}, K_{t+1})\right]\} \tag{17}$$

together with the law of motion (1) and (11) for K_t and Z_t. Here $V(Z_t, K_t)$ is the value of the firm in period t; $E_t \left[V(Z_{t+1}, K_{t+1})\right]$ is the expected value of the firm in period $t+1$ conditional on information available in period t.

2.1.4 Properties

Given the role Z_t plays in equation (17) of this chapter is the same as X_t plays in equation (11) of Chapter 3, and the law of motion for K_t and Z_t in this chapter is the same as those for K_t and X_t in Chapter 3, the optimal investment decision derived in Chapter 3 also applies to the dynamic program in this chapter, by replacing X_t with Z_t in the notation.

Proposition 1 ***Optimal Investment Rate and Capital Stock in the Frictionless Case:***

If $G(Z_t, K_t; I_t) \equiv 0$, the optimal investment rate can be derived as

$$\frac{I_t^*}{K_t} = const1 \cdot \frac{Z_t}{K_t} - 1 \tag{18}$$

Or equivalently expressed in levels, the optimal productive capital stock is

$$\widehat{K}_t^* \equiv I_t^* + K_t = const1 \cdot Z_t \tag{19}$$

where

$$const1 = \left[const0 \cdot (1-\gamma) \,/ \left(\frac{r+\delta}{1+r}\right)\right]^{\frac{1}{\gamma}} \tag{20}$$

Furthermore, given the same specification for the investment models, all the properties about variables in ratio or growth rate derived in the Section 2.3 of Chapter 4 also applies to the same variables in ratio or growth rate in this chapter.

2.2 The Effects of Uncertainty on Capital Accumulation

This section analyzes the effects of uncertainty on the expected level of capital stock through two possible channels predicted by this investment model.

2.2.1 Uncertainty and the HAC Effect

In order to abstract from any effects of uncertainty through capital adjustment costs, this subsection imposes $G(Z_t, K_t; I_t) \equiv 0$.

Following Chapter 3, to study the effects of uncertainty, I apply **a mean-preserving spread** for the underlying stochastic process. Equation (8) and (9) imply that keeping $\mu_x$and μ_a constant while increasing σ_x^2 and σ_a^2, is a mean-preserving spread for the logarithms x_t and a_t, respectively. Since the demand shift parameter X_t and the level of productivity A_t are the two stochastic variables that expectation is taken over, I therefore focus on increases in uncertainty that preserve the mean of X_t and A_t.

Lemma 3 ***Mean-Preserving Spread for*** X_t ***and*** A_t***:***

By setting $c_x = -\frac{\sigma_x^2}{2(1-\rho^2)}$ *and* $c_a = -\frac{\sigma_a^2}{2(1-\rho^2)}$, $E[X_t] = \exp\left(\overline{\zeta}^x + \mu_x t\right)$ *and* $E[A_t] = \exp\left(\overline{\zeta}^a + \mu_a t\right)$, *i.e. keeping* μ_x *and* μ_a *constant while increasing* σ_x^2 *and* σ_a^2, *is a mean-preserving spread for* X_t *and* A_t, *respectively.*

Proof: See Chapter 3.

Recall that $Z_t = X_t (A_t)^{\varepsilon-1}$, where $\mu = \mu_x + (\varepsilon-1)\mu_a$ and $\sigma^2 = \sigma_x^2 + (\varepsilon-1)^2 \sigma_a^2$. Hence keeping μ_x and μ_a constant while increasing σ_x^2 and σ_a^2 also implies keeping μ constant while increasing σ^2. However, this is in general not a mean-preserving spread for Z_t.

Lemma 4 ***Expectation of*** Z_t***:***

Keeping μ *constant while increasing* σ^2 *is in general not a mean-preserving spread for* Z_t*. In particular,*

$$E[Z_t] = \exp\left[\overline{\zeta} + \mu t + \frac{(\varepsilon-2)(1-\tau)}{2(\varepsilon-1)(1-\rho^2)}\sigma^2\right] \tag{21}$$

Proof: See Appendix A.3.

Lemma 4 implies that the effect of keeping μ constant while increasing σ^2 on $E[Z_t]$ includes three cases. First, either when all uncertainty is from demand so that $\sigma_x^2 = \sigma^2$ or equivalently $\tau = 1$, or when the demand elasticity $\varepsilon = 2$, then $\partial E[Z_t] / \partial\sigma = 0$. Second, if there is any uncertainty from productivity so that $\sigma_a^2 >$

0 or equivalently $\tau < 1$, and the demand elasticity $\varepsilon > 2$, then $\partial E[Z_t]/\partial\sigma > 0$. Finally, if there is any uncertainty from productivity so that $\sigma_a^2 > 0$ or equivalently $\tau < 1$, and the demand elasticity ε satisfies $1 < \varepsilon < 2$, then $\partial E[Z_t]/\partial\sigma < 0$.

According to Proposition 1, $\widehat{K}_t = const1 \cdot Z_t$ in the absence of adjustment costs, Lemma 4 implies that the effects of uncertainty on the expected capital stock depend on the value of τ and ε.

Proposition 2 ***Uncertainty and the HAC Effect:***

$$\textit{If } G(Z_t, K_t; I_t) \equiv 0,\ \partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma \begin{cases} = 0 & \textit{if } \tau = 1 \textit{ or } \varepsilon = 2 \\ > 0 & \textit{if } \tau < 1 \textit{ and } \varepsilon > 2 \\ < 0 & \textit{if } \tau < 1 \textit{ and } 1 < \varepsilon < 2 \end{cases}$$

Proof: By Proposition 1 and Lemma 4.

Proposition 2 implies that this investment model allows for the uncertainty to affect the expected capital stock through the curvature of the operating profit, and this effect could be positive, negative or zero, depending on the source of uncertainty and the demand elasticity.

In the case of $\tau = 1$, i.e. when all the uncertainty is from demand, this investment model is equivalent to the ones studied in Chapters 3 and 4, where we have established the property that $\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma = 0$ in the absence of adjustment costs.

The cases studied in the literature that lead to a positive HAC effect, for example, uncertainty in output price (Hartman, 1972, 1976; Abel, 1983), and in the price of variable input (Abel, 1985; Lee and Shin, 2000), or uncertainty from horizontal demand shocks (Caballero, 1991; Pindyck, 1993) can be represented by the case of $\tau < 1$ and $\varepsilon > 2$. Furthermore, within this case, equation (21) implies that $\partial\left(\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma\right)/\partial\varepsilon > 0$. This verifies the insight in Caballero (1991) about the role of degree of competition in determining the importance of the HAC effect. In the limiting case of perfection competition, i.e. $\varepsilon = \infty$, the magnitude of the HAC effect is infinitely large and dominates the effects of uncertainty through any other channel, which is consistent with the prediction from Abel and Eberly (1994).

2.2.2 Uncertainty and the Adjustment Cost Effect

In order to abstract from any effect of uncertainty through the HAC effect, this subsection assumes all the uncertainty is from demand, i.e. $\tau = 1$. Under this scenario, the investment model is the same as that studied Chapter 3. Hence it is straightforward to derive the relationship between uncertainty and the expected capital stock under different forms of adjustment costs.

Proposition 3 ***Uncertainty and the Adjustment Cost Effect:***

$$If\ \tau \equiv 1,\ \partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma \begin{cases} < 0 & if\ b_q > 0 \\ \lesseqgtr 0 & if\ b_i > 0\ or\ b_f > 0 \end{cases}$$

Proof: Recall the Proposition 3 in Chapter 3 claims that the effect of uncertainty on capital accumulation depends on the form of adjustment costs. In particular, if $b_q > 0$, $\partial\kappa(t)/\partial\sigma < 0$; and if $b_i > 0$ or $b_f > 0$, the effect is ambiguous, where $\kappa(t) = E\left[\widehat{K}_t(\sigma)\right]/E\left[\widehat{K}^*_t(\sigma)\right]$. Given $E\left[\widehat{K}^*_t(\sigma)\right] = const1 \cdot \exp(\overline{\zeta} + \mu t)$ when $\tau = 1$, which is invariant to σ, the sign of $\partial E\left[\widehat{K}_t(\sigma)\right]/\partial\sigma$ must be the same as the sign of $\partial\kappa(t)/\partial\sigma$.

The analysis in this section illustrates the rich implications about the effects of uncertainty in our investment model. With an increase in the level of uncertainty, the HAC effect would increase/decrease/leave unchanged the expected level of capital stock, depending on the value of τ and ε (Proposition 2); capital adjustment costs would affect the expected capital stock, depending on the exact form of the adjustment costs (Proposition 3). This implies the effects of uncertainty on capital accumulation is fundamentally an empirical question. Within the framework of this investment model, quantifying the effects of uncertainty requires estimating the structural parameters of the model.

3 Data and Empirical Specification

3.1 Data and Variables

This chapter uses a sample for UK manufacturing firms, analyzed previously by Bloom, Bond and Van Reenen (2007), for the investment dynamics under uncertainty and partial irreversibility. This sample contains firm-level data for an unbalanced panel of 629 firms between 1972 and 1991. On average there are 10 annual observations per firm. These company data are taken from the consolidated accounts of manufacturing firms listed on the U.K. stock market and are obtained from the Datastream on-line service. Four key variables are collected from the data: Investment ($I_{j,t}$); Capital stock ($K_{j,t}$); Sales ($Y_{j,t}$); and Profit ($PI_{j,t}$), where j denotes firm and t denotes year. Data Appendix C explains how these variables are constructed, deflated and cleaned.

Using this information, five key variables in ratio or growth rate are calculated as those defined in Chapter 4.

Definition 1 ***Variables in Ratio or Growth Rate:***

Investment Rate:

$$i_{j,t} \equiv I_{j,t}/K_{j,t}$$

Sales Growth Rate:

$$dy_{j,t} \equiv \ln\left(Y_{j,t}/Y_{j,t-1}\right)$$

log of Sales-to-(beginning-of-period)Capital Ratio:

$$yk_{j,t} \equiv \ln\left(Y_{j,t}/K_{j,t}\right)$$

log of (productive) Capital-to-Sales Ratio:

$$ky_{j,t} \equiv \ln\left(\widehat{K}_{j,t}/Y_{j,t}\right)$$

Profit-to-Sales Ratio:

$$PIY_{j,t} \equiv PI_{j,t}/Y_{j,t}$$

Identifying the parameter τ and hence the HAC effect, a measure of uncertainty and expected capital stock have to be constructed from the data. Identification could be based on the variation of these two variables either across time or across firms. Since the empirical sample used in this chapter is a short panel, and a main feature in firm-level investment data is the importance of 'fixed-effects' (Bond and Van Reenen, 2003), I model this variation as cross-sectional variation.

Definition 2 ***Measure of Uncertainty:***

The within-group standard deviation of sales growth rate for firm j across time proxies the level of uncertainty of firm j:

$$SDdy_j \equiv sd_t\left(dy_{j,t}\right) \simeq \sigma_j$$

Definition 3 ***Variables in Level:***

The within-group means of the capital stock and sales for firm j across time proxy the size of firm j:

Expected Capital Stock:

$$E\widehat{K}_j \equiv mean_t\left(\widehat{K}_{j,t}\right)$$

Expected Sales:

$$EY_j \equiv mean_t\left(Y_{j,t}\right)$$

3.2 Empirical Specification

3.2.1 Uncertainty Heterogeneity

In order to identify the effect of uncertainty through the HAC effect, a necessary condition is that there is some variation in the level of uncertainty. Since the identification is based on cross-sectional variation, the fundamental empirical specification is the heterogeneity in the level of uncertainty across firms.

Assumption 9 ***Uncertainty Heterogeneity:***

The measure of uncertainty for firm j is σ_j, where $\log\sigma_j \overset{i.i.d}{\sim} N\left(\mu_{l\sigma},\, \sigma_{l\sigma}^2\right)$.

That is each firm j faces a firm-specific standard deviation of profitability shocks σ_j, where $\log\sigma_j$ is drawn independently from an identical normal distribution with mean $\mu_{l\sigma}$ and standard deviation $\sigma_{l\sigma}$.

3.2.2 Growth Rate Heterogeneity

Heterogeneity in the growth rate is not a necessary condition for identification, but there are two reasons why allowing for this heterogeneity may be important. First, as explained in Chapter 4, I allow for the heterogeneity in the growth rate in order to get robust estimates for the adjustment cost parameters. Second, heterogeneity in the growth rate will lead to permanent differences in firm size. All else being equal, on average firms with higher growth rates grow larger than firms with lower growth rate. Since the cross-sectional differences in firm size is key for identifying the HAC effect, it is therefore important to control for this type of heterogeneity.

Assumption 10 ***Growth Rate Heterogeneity:***

The profitability growth rate for firm j is μ_j, where $\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$ and $cov\left(\mu_j, \sigma_j\right) = 0$..

That is each firm j has a firm-specific profitability growth rate μ_j, where μ_j is drawn independently from an identical normal distribution with mean μ_μ and standard deviation σ_μ. With heterogeneities in both σ and μ, I further assume that they are uncorrelated with each other so that the effects of uncertainty can be separated from the effects of growth rate.

Figure 5.1a plots the empirical distribution of the within-group standard deviation of sales growth rate ($SDdy_j$) for the sample, which displays a log-normal pattern. Figure 5.1b presents the distribution of the within-group mean of sales growth rate (Edy_j), which are better approximated by a normal distribution. Given $SDdy_j \equiv sd_t\left(dy_{j,t}\right) \simeq \sigma_j$ and $Edy_j \equiv mean_t\left(dy_{j,t}\right) = \mu_j$, this explains the specification for the distributions of σ_j and μ_j in Assumptions 9 and 10.

Both the level of uncertainty and the growth rate would affect the investment policy. Hence the dynamic program described in (17) must be solved for each firm j with value σ_j and μ_j, which is infeasible even for a small sample. Therefore I adopt a standard approach used in the literature, for example, Eckstein and Wolpin (1999), to allow for a finite mixture of types.

Assumption 11 ***A Finite Mixture of Types:***

There are a finite mixture of types, say $U \times V$ types of firms, each comprising a fixed proportion $1/(U \times V)$ of the population, where the type set is defined as $\digamma = \{(\sigma_u, \mu_v) : u = 1, \cdots, U; v = 1, \cdots, V\}$.

Under this assumption, I discretize the continuous distributions for $\log \sigma_j$ and μ_j individually by Tauchen (1989) method. Due to 'the curse of dimensionality', I have to be conservative about the number of grids and set them to be 3 for both of these two variables. This implies in the simulated data, there are 9 type of firms, each comprising a fixed proportion 1/9 of the population.

3.2.3 Intercept Heterogeneity

Similar to the heterogeneity in the growth rate, heterogeneity in the intercept of the stochastic process is not a necessary condition for identification, but it is important to control for this potential heterogeneity which would lead to permanent differences in firm size. In addition to the HAC effect and the capital adjustment costs effect that I have explicitly modelled, Proposition 1 indicates that the expected capital stock depends on the Jorgensonian user cost of capital (r, δ), the production technology (β), the demand elasticity (ε), the relative price of variable input (w, m), the time period a firm has been in operation (t), the unit in measuring variables in level (£ or £1000), and finally the intercept in the stochastic process $\left(\overline{\zeta}\right)$. Differences in these factors across firms will lead to permanent differences in the expected capital stock and sales across firms. Modelling heterogeneity in these aspects structurally is desirable but not feasible due to the 'curse of dimensionality'. My empirical strategy is to impose common values for r, δ, β and ε, choose arbitrary values for w, m, t, and the unit of measurement, and to model and estimate the distribution of $\overline{\zeta}$.

Assumption 12 ***Intercept Heterogeneity:***
The intercept in the stochastic process for firm j is $\overline{\zeta}_j$, where $\overline{\zeta}_j \overset{i.i.d}{\sim} N\left(\mu_{\overline{\zeta}}, \sigma_{\overline{\zeta}}\right)$, $cov\left(\overline{\zeta}_j, \sigma_j\right) = 0$, and $cov\left(\overline{\zeta}_j, \mu_j\right) = 0$.

That is each firm j has a firm-specific intercept $\overline{\zeta}_j$ in the stochastic process, where $\overline{\zeta}_j$ is drawn independently from an identical normal distribution with mean $\mu_{\overline{\zeta}}$ and standard deviation $\sigma_{\overline{\zeta}}$. With heterogeneities in σ, μ and $\overline{\zeta}$, I further assume that they are uncorrelated with each other so that the factors that lead to permanent differences in the expected capital stock through $\overline{\zeta}_j$ are uncorrelated with the level of uncertainty and the growth rate of the firms. This technical devise is based on the important property summarized by Lemma 5.

Lemma 5 ***Level and Dispersion of Firm Size:***

In the absence of capital adjustment costs, for given HAC effect and heterogeneity in growth rate, the effect of imposing common value for $(r, \delta, \beta, \varepsilon, w, m, t)$ on the dispersion of the expected capital stock and sales can be accounted for by adjusting $\sigma_{\overline{\zeta}}$; the effect of choosing arbitrary value for (w, m, t) and the unit of measurement on the mean of the expected capital stock and sales can be accounted for by adjusting $\mu_{\overline{\zeta}}$.

Proof: See Appendix A.4.

Different from the level of uncertainty and the growth rate, the value of $\overline{\zeta}$ does not affect the investment policy due to the linear homogeneity property of the investment model. Hence there could be 'infinite' type for the intercept in the stochastic process.

3.2.4 Measurement Errors

Given the important role of investment rate and sales in estimating adjustment costs, I allow for a rich structure of measurement errors in the empirical specification for these two variables as those specified in Chapter 4.

Assumption 13 ***Measurement Errors in Investment Rate:***

$i_{j,t} = i^*_{j,t} \exp(e^I_{j,t})$, *where* $e^I_{j,t} = e^{IT}_{j,t} + e^{IP}_j$, *and* $e^{IP}_j \overset{i.i.d}{\sim} N(0, \sigma^2_{IP})$, $e^{IT}_{j,t} \overset{i.i.d}{\sim} N(0, \sigma^2_{IT})$.

That is there is a multiplicative structure for measurement error in the investment rate, where $i_{j,t}$ denotes the observed investment rate, $i^*_{j,t}$ denotes the true underlying investment rate which is not measured accurately in the data, and the measurement error $e^I_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{IT} and σ_{IP}, respectively. This specification has the property that the sign of recorded investment rate is not affected by measurement error, and treats observations with zero investment in the data as true zeros.

Assumption 14 ***Measurement Errors in Sales:***

$Y_{j,t} = Y^*_{j,t}\exp(e^Y_{j,t})$, where $e^Y_{j,t} = e^{YT}_{j,t} + e^{YP}_j$, and $e^{YP}_j \overset{i.i.d}{\sim} N(0, \sigma^2_{YP})$, $e^{YT}_{j,t} \overset{i.i.d}{\sim} N(0, \sigma^2_{YT})$.

That is there is a multiplicative structure for measurement error in sales, where $Y_{j,t}$ denotes the observed level of sales, $Y^*_{j,t}$ denotes the true underlying level of sales which is not measured accurately in the data, and the measurement error $e^Y_{j,t}$ has both transitory and permanent components with mean zero and standard deviation σ_{YT} and σ_{YP}, respectively.

3.2.5 Aggregation at the Firm-Level

Different from that in Chapter 4, where about one-third observations report zero investment in the Brazilian sample, in the Datastream sample studied in this chapter, only about 2% observations record an investment rate less than 1% in absolute value, which is called 'inaction rate' in Cooper and Haltiwanger (2006). This might indicate the insignificance of non-convex adjustment costs or the importance of aggregation in this sample. As discussed in Chapter 4, not accounting for the possible aggregation in the firm-level data could lead to an over-estimate for the quadratic adjustment costs and an under-estimate for the non-convex adjustment costs. Therefore it is important to consider the possible aggregation at the firm-level. Comparison between the empirical distribution of investment rate for the Brazilian sample plotted in Figure 4.1a, and for the Datastream sample plotted in Figure 5.1c, indicates aggregation might be more relevant in the later sample.

Assumption 15 ***Aggregation:***

Each firm is made of p plants, where $p \geq 1$. For plant i of firm j in period t, the law of motion for $Z_{i,j,t}$ is given by

$$\begin{aligned} z_{i,j,t} &= \ln Z_{i,j,t} \\ z_{i,j,t} &= c_j + \mu_j t + \zeta_{i,j,t} \qquad (22) \\ \zeta_{i,j,t} &= \overline{\zeta}_{i,j,t} + \rho\zeta_{i,j,t-1} + e_{i,j,t} = \overline{\zeta}_{i,j,t} + \sum_{s=0}^{t-1} \rho^s e_{i,j,t-s} \end{aligned}$$

where $0 < \rho < 1$, $e_{i,j,t} = \sigma_j \epsilon_{i,j,t}$, $\epsilon_{i,j,t} \overset{i.i.d.}{\sim} N(0,1)$, *and* $\zeta_{i,j,0} = 0$.

That is there are heterogeneities across firms in the level of uncertainty, profitability growth rate and intercept of the stochastic process, which determine the 'type' of a firm; however, plants within the same firm are of the same type but have plant-level idiosyncratic innovations $\epsilon_{i,j,t}$.

Appendix B.1 explains how I solve the optimization problem numerically and simulate the data under the empirical specifications in this section.

4 A Structural Estimation

Under the theoretical specification in Section 2 and the empirical specification in Section 3, estimating the effects of uncertainty on investment dynamics and capital accumulation is translated into estimating a set of structural parameters in the model. Using the same method as in Chapter 4, these parameters are estimated by matching the moments from the model with the moments from the data.

4.1 Structural Parameters

Table 5.1 lists the set of parameters that I aim to estimate. Overall there are 17 parameters, 12 of which are common across this chapter and chapter 4. They are, 3 parameters measuring the magnitude of capital adjustment costs, i.e. quadratic adjustment costs b_q, partial irreversibility b_i and fixed adjustment costs b_f. Denote $\mathbf{b} = (b_q, b_i, b_f)$; 2 parameters characterizing technology and demand, i.e. the capital share in production function β and the demand elasticity with respect to price ε; 3 parameters characterizing the stochastic process, i.e. the serial correlation of demand shocks ρ; the mean and standard deviation of the growth rate μ_μ and σ_μ; and 4 parameters measuring the magnitude of measurement errors in the data, i.e. the standard deviation of transitory and permanent measurement errors in investment rate σ_{IT} and σ_{IP}, and in sales σ_{YT} and σ_{YP}.

The additional 5 parameters introduced in this chapter are, the mean and standard deviation of the level of uncertainty μ_σ and σ_σ, which replace the homogeneous σ in chapter 4; the parameter that determines the sign and magnitude of the HAC effect τ; and the mean and standard deviation of intercept $\mu_{\bar{\zeta}}$ and $\sigma_{\bar{\zeta}}$.

These 17 parameters can be divided into two subsets. Define

$$\Theta_1 \equiv \left(\mathbf{b},\ \beta,\ \varepsilon,\ \rho,\ \mu_\mu,\ \sigma_\mu,\ \mu_\sigma,\ \sigma_\sigma,\ \sigma_{IT},\ \sigma_{IP},\ \sigma_{YT},\ \sigma_{YP}\right)$$

and

$$\Theta_2 \equiv \left(\tau,\ \mu_{\bar{\zeta}},\ \sigma_{\bar{\zeta}}\right)$$

The difference between Θ_1 and Θ_2 is that, parameters in both sets would affect variables in level, but the 3 parameters in Θ_2 do not affect any variable in ratio or growth rate.

Proposition 4 ***Irrelevance of*** Θ_2 ***for Variables in Ratio or Growth Rate:***

The value of Θ_2 *does not affect any variable in ratio or growth rate.*

Proof: By the linear homogeneity property of the model and see Appendix A.4 for detail.

Similar to Chapter 4, the two parameters in the Jorgensonian user cost of capital are imposed. First, the depreciation rate δ, which is imposed at 0.08, the rate that was used in constructing the capital stock series with perpetual inventory method in the empirical sample. Second, the discount rate r. I impose $r = 0.09$ to guarantee a finite firm value.

4.2 Identification: A Two-Step Procedure

Table 5.2 defines the set of moments that I aim to match. The second column reports the value of these empirical moments estimated from the data and the standard errors are provided in the third column. Moments with subscripts j and t are measured using all the firm-year observations; and with j only are measured using firm-averages. Overall there are 26 moments, which can also be divided into two subsets. Define Φ_1 to be the subset made of 19 moments, which are only for variables in ratio or growth rate. Define Φ_2 to be the subset made of the remaining 7 moments, which involve variables in level.

In estimating the 17 parameters by matching these 26 moments, I adopt a **two-step procedure**. In step 1, estimate Θ_1 using the moments in Φ_1 only. In step 2, impose $\Theta_1 = \widehat{\Theta}_1$, i.e. at the estimates from step 1, estimate Θ_2 using the moments in Φ_2 only. These two steps are separable by Proposition 4.

Within the 19 moments in Φ_1, there are 13 common moments that are employed both in this chapter and in Chapter 4. In addition, there are 4 proportion moments capturing the distribution of investment in Chapter 4. For the sample used in this chapter, no disinvestment spike is observed. So I use the proportion of investment spike $prop\,(0.2 < i_{j,t} < 1)$, investment inaction $prop\,(|i_{j,t}| < 0.01)$ and disinvestment $prop\,(i_{j,t} < -0.01)$ instead, with the proportion of moderate investment being one minus these three values. Chapter 4 has provided a detailed discussion about how these 13 common moments together with the proportion moments are used to identify the 12 parameters $\left(\mathbf{b},\ \beta,\ \varepsilon,\ \rho,\ \mu_\mu,\ \sigma_\mu,\ \sigma_{IT},\ \sigma_{IP},\ \sigma_{YT},\ \sigma_{YP}\right)$, which are common in both these two chapters. The remaining two parameters in Θ_1 are μ_σ and σ_σ. They are identified by the mean and standard deviation across firms for the within-group standard deviation of sales growth rate $mean\,(SDdy_j)$ and $sd\,(SDdy_j)$, which are also moments for variable in growth rate. This adds up to 18 moments. The cross correlation between the mean capital-to-sales ratio and the measure of uncertainty, i.e. $corr\,(Eky_j, SDdy_j)$ is another moment for variables in ratio or growth rate. According to Lemma 4 in Chapter 4, in the absence of adjustment costs, $ky^*_{j,t} = \ln\left[\beta\left(1-\frac{1}{\varepsilon}\right)/\left(\frac{r+\delta}{1+r}\right)\right]$. Hence within this model, any non zero correlation can only be attributed to adjustment costs. This completes the discussion of identification in step 1.

In step 2, by imposing $\Theta_1 = \widehat{\Theta}_1$, factors from adjustment costs, technology and demand, stochastic process and measurement errors that will affect moments for variables in level have been fixed, leaving Θ_2, i.e. the HAC effect and the intercept heterogeneity as the remaining factors to match moments in Φ_2.

Within the 7 moments in Φ_2, first, 4 of them measure the level and dispersion of firm size, i.e. for capital stock $mean\left(\widehat{K}_{j,t}\right)$ and $sd\left(\widehat{K}_{j,t}\right)$ and for sales $mean\,(Y_{j,t})$ and $sd\,(Y_{j,t})$. For given τ, the mean and standard deviation of the intercept $\mu_{\overline{\zeta}}$ and $\sigma_{\overline{\zeta}}$ determine the value of these four moments. Second, $corr\left(\widehat{K}_{j,t}, Y_{j,t}\right)$ measures the cross correlation between capital stock and sales. Since without adjustment costs, these two variables would be linearly proportional to each other, a correlation coefficient less than 1 is the sign for the presence of adjustment costs, while the magnitude of this moment depends on the dispersion of the two level variables, hence depends on $\mu_{\overline{\zeta}}$ and $\sigma_{\overline{\zeta}}$. Finally, according to Proposition 2 and 3, the cross correlation between firm size and measure of uncertainty, i.e. $corr\left(E\widehat{K}_j, SDdy_j\right)$ and $corr\left(EY_j, SDdy_j\right)$ depend on the adjustment cost effect and the HAC effect. For given adjustment costs, these two key moments therefore identify the HAC effect, i.e. parameter τ. Since these two correlations are negative in the data, this implies either the importance of quadratic adjustment costs, or a case with $\tau < 1$ and $1 < \varepsilon < 2$, or both.

This two-step procedure is motivated by three considerations.

First, both heterogeneity in growth rate $(\sigma_\mu > 0)$ and in intercept $\left(\sigma_{\overline{\zeta}} > 0\right)$ will generate firm size dispersion. Dispersion generated from $\left(\sigma_{\overline{\zeta}} > 0\right)$ is time-invariant, but dispersion generated from $(\sigma_\mu > 0)$ increases with the time horizon in simulation. So how long should the time horizon be when we simulate the data? There is a trade-off. In order to make the simulated firms on their balanced-growth-path so that the effects we estimate are stable over time, a sufficiently long period should be simulated to allow firms to adjust capital stock to follow the balanced-growth-path. One necessary property for the balanced-growth-path is that sales are growing at the rate μ (Proposition 2 in Chapter 4). Suppose the minimal period after which sales are growing at the rate μ is $\underline{t}$. If the 17 parameters are estimated in one-step simultaneously, in case $\underline{t}$ is very large, heterogeneity in growth rate $(\sigma_\mu > 0)$ will generate a firm size dispersion, which is so large that even if there is no heterogeneity in intercept $\left(\sigma_{\overline{\zeta}} = 0\right)$ the simulated dispersion is still too large compared with the empirical one. In order to match the empirical dispersion, the model would then lead to a downward bias in the estimate for σ_μ and probably an upward bias in $\sigma_{\overline{\zeta}}$. In contrast, by estimating σ_μ and $\sigma_{\overline{\zeta}}$ in two steps, σ_μ can be consistently estimated from step 1 for any choice of $t \geqslant \underline{t}$. In step 2, for a given $\underline{t}$, if the estimated $\sigma_{\overline{\zeta}}$ is larger than zero, it indicates the potential constraint that '$\underline{t}$ is too large' is not binding so that $\sigma_{\overline{\zeta}}$ can be consistently estimated from step 2.

Second, the existing investment literature using structural estimation only matches moments in ratio or growth rate, for example Cooper and Haltiwanger (2006) and Bloom (2009). Therefore the results from the step 1 are comparable to the literature and to those in Chapter 4. It also highlights the role of additional moments for variables in level in identifying the HAC effect in step 2.

Finally, this two-step procedure also provides a check for identification. If the 19 moments used in step 1, i.e. Φ_1 is sufficient to identify the 12 parameters in step 1, i.e. Θ_1, $\widehat{\Theta}_1$ estimated by using Φ_1 together with any other additional moments should be similar to the $\widehat{\Theta}_1$ estimated by using Φ_1 only. Given the additional moments could be any moments generated from the investment model, the 7 moments used in step 2, i.e. Φ_2 could serve the purpose of checking. This is equivalent to say that, if Φ_1 is sufficient to identify Θ_1, when the 17 parameters (Θ_1, Θ_2) are estimated simultaneously in one-step by matching all the 26 moments (Φ_1, Φ_2), estimates from the two-step and the one-step procedures should be the same.[2]

5 Empirical Results

5.1 Estimates

Table 5.3 presents the estimation results for the full model, imposing p=10 plants across each firm, a depreciation rate $\delta = 0.08$, and a discount rate $r = 0.09$. The upper panel reports the estimated values of the structural parameters and the lower panel compares the empirical moments from the data and the simulated moments from the model, evaluated at the estimates listed in the corresponding columns in the upper panel.

[2] This would not be true, if the efficient weighting matrix, i.e. the inverse of the variance-covariance matrix of the moments are used in estimation. In this case, estimates from the one-step procedure should be more effient, since it uses full information in the weighting matrix. Instead doing it in two-step (even if the efficient weighting matrix for each subset of moments in each step is used) effectively neglects information about correlation bewteen moments in ratio and moments in level. In order to make the estimates from the two-step and the one-step procedure indeed comparable, I only use the diagonal elements from the inverse of the variance-covariance matrix of the moments as the weighting matrix.

The first two columns describe the empirical results from the two-step procedure.

In step 1, the model estimates $\widehat{b_q} = 1.3844$, this implies a quadratic adjustment cost, which is about 4.5% of the operating profit, evaluated at the sample average. $\widehat{b_i} = 0.1716$ implies that resale of a capital goods would incur a sell-loss, which is about 17.2% of its original purchase price. $\widehat{b_f} = 0.009$ implies any investment or disinvestment would result in a loss that is 0.9% of operating profit.

The estimated $\widehat{\beta} = 0.0810$, a capital share in the production function that is higher than the estimates for the same parameter in Chapter 4. The estimate for the demand elasticity with respect to price is $\widehat{\varepsilon} = 24.6869$, which implies these firms have less market power than firms in the Brazilian sample. These two estimates together determine the estimate for the capital coefficient in the operating profit $1 - \widehat{\gamma} = 0.6574$.

The estimated serial correlation parameter for the stochastic process is $\widehat{\rho}$ =0.9496, which implies the effect of profitability shocks are highly persistent in this sample. Since at such a high value of ρ, the process discretized by the Tauchen (1989) method becomes a poorer approximation for the original continuous distribution, I check the robustness of the results by re-estimating the model and imposing $\rho = 0.9$. Both the estimates for the parameters and the counterfactual simulations are similar at these two different values of ρ. The estimates for the mean and standard deviation of the growth rate are $\widehat{\mu_\mu}$ =0.0192 and $\widehat{\sigma_\mu}$ =0.0381, respectively. So on average the profitability grows at 1.92% per year, meanwhile there is large heterogeneity in the growth rate across firms in this sample. The estimated mean and standard deviation for the log of σ can be transformed into the mean and standard deviation for σ itself, which are 0.6627 and 0.2103, respectively. A mean value of σ at 0.6627 seems to be much larger than the estimates for the σ in Brazilian sample in Chapter 4, which is about 0.2. However, since 0.6627 measures the standard deviation of the idiosyncratic profitability shocks at each of 10 plants, it is not comparable with a measure for the standard deviation of shocks at the firm-level. Instead, the value of $mean\,(SDdy_j)$ proxies the average level of uncertainty across firms in this sample, which is about 0.12. The standard deviation for the σ being 0.2103 implies there is indeed heterogeneity in the level of uncertainty across firms. Recall this is a necessary condition for identifying the HAC effect within this model.

Similar to the Brazilian sample, three out of four types of measurement errors under our specification are significant. However, the magnitude found here is smaller.

Overall the model fits most of the 19 moments in the subset Φ_1 relatively well, although the model cannot even match the sign for one moment $corr\,(Eky_j, SDdy_j)$.

Imposing these parameters at their estimates from step 1, step 2 estimates $\widehat{\tau} = 1$. According to Proposition 2, this rejects the existence of any HAC effect in this sample. The estimated $\widehat{\sigma_{\overline{\zeta}}} = 1.1147$, which is far away from zero. This implies two things. First, in addition to the heterogeneity in growth rate, there is heterogeneity in the intercept which would generate permanent difference in firm size. Second, the concern that the simulation period being too long is not binding. The estimate $\widehat{\mu_{\overline{\zeta}}}$ does not have a direct economic meaning. But as Lemma 5 claims, it accounts for the effect of choosing arbitrary value for (w, m, t) and the unit of measurement on the mean of the expected capital stock and sales.

Overall the model fits the 7 moments in the subset Φ_2 also relatively well, in particular, the model successfully generates the negative correlation between measures of firm size and uncertainty. However, the simulated magnitude is still too small, compared with the empirical one. Given $\widehat{\varepsilon} > 2$, any value of τ that is less than 1 would generate a positive correlation and hence make the simulated moments further away from the empirical ones. This explains why the model estimates a value for τ that would not generate any positive HAC effect.

The last column in Table 5.3 reports the estimated parameters and simulated moments when the model is estimated in one-step. The similarity between the results from the two-step procedure and the one-step procedure provides a robustness check for the identification.

5.2 Specification Tests

Table 5.4 reports specification tests for several restricted models. The preferred full model is listed in Column (1) as benchmark. Columns (2) to (8) test specifications that only involve restricting parameters in subset Θ_1, hence I only report step 1 estimates. Column (9) tests a specification that only involves restricting a parameter in subset Θ_2, hence I impose Θ_1 at the estimates of the preferred full model and re-estimate step 2.

Columns (2) to (5) test the importance of different forms of adjustment costs. A model without quadratic adjustment costs is massively rejected and a model without any form of non-convex adjustment costs is also strongly rejected. This implies, similar to what we find in Chapter 4, that a mix of quadratic adjustment costs and at least one form of non-convex adjustment costs are required to fit the data in this sample, too. Different from the results in Chapter 4, where a mix of quadratic and fixed adjustment costs fits the data best, here a mix of quadratic and partial irreversibility fits the data best.

Columns (6) and (7) test the importance of heterogeneity in the growth rate and level of uncertainty, respectively. As we may expect, a model imposing a homogeneous growth rate cannot match the high serial correlation in investment rates and sales growth rates, while a model imposing a homogeneous level of uncertainty cannot match the large dispersion in the within-group standard deviation of sales growth rates across firms.

Column (8) lists the results obtained by imposing no measurement errors in investment rate and sales. This restricted specification is massively rejected, mainly because the simulated cross correlations are too large compared with their empirical counterparts; in order to compensate for these effects, the model generates much higher estimates for the adjustment costs, which in turn leads to too much serial correlation in investment rate, and too little dispersion in investment rate and capital-to-sales ratio. Another interesting finding is that, at these estimated values of adjustment costs, the positive correlation between capital-to-sales ratio and measure of uncertainty, i.e. $corr\,(Eky_j, SDdy_j)$ gets even stronger, hence further away from the empirical one. This implies within this model, the capital adjustment costs cannot generate the negative correlation between measure of capital intensity and uncertainty.[3]

Column (9) presents the results of imposing $\sigma_{\overline{\zeta}} = 0$ in step 2. Without any heterogeneity in the intercept, even if the model estimates heterogeneity in the growth rate, the simulated dispersion for the firm size is still too low compared with the empirical one. For given negative covariance between firm size and level of uncertainty, a smaller dispersion in firm size generates a larger magnitude in the correlation coefficient $corr\left(E\widehat{K}_j, SDdy_j\right)$ and $corr\left(EY_j, SDdy_j\right)$, which would be too negative compared with their empirical counterparts. In this case, the model estimates $\widehat{\tau} = 0.7199$ to offset the 'over' negative effect generated by the quadratic adjustment costs. Together with an estimate $\widehat{\varepsilon} > 2$, this implies the presence of a positive HAC effect. However, the overall fit is worse under this restriction compared with the benchmark model.

5.3 Robustness Tests

Table 5.5 presents results for two robustness tests. Column (1) is the benchmark full model, imposing a depreciation rate δ =0.08, a discount rate r =0.09, and the number of plants within each firm to be $p = 10$. Since the capital stock series is constructed with perpetual inventory method by imposing δ =0.08 in the empirical sample, I only test the robustness to the choice of r and p. Results reported here are all obtained by the two-step procedure and I also check the similarity to the results from the one-step procedure under each specification.

[3]The effect of uncertainty on the expected capital intensity–measured as expected capital-to-sales ratio or expected capital-to-labour ratio–is an open question in the class of this model. Abel and Eberly (1999) prove that in the presence of complete irreversibility only, an increase in uncertainty will lead to higher expected capital intensity, see their footnote 18.

The intuition is that, conditional on positive profitability shocks, the firm uses more variable inputs than capital since it is free to adjust variable inputs but costly to adjust capital; conditional on negative profitability shocks, the firm uses less variable inputs but cannot disinvest under the complete irreversibility constraint. Since the expectation is taken over both positive and negative shocks, this leads to a positive relationship between uncertainty and the expected capital intensity. Similar intuition applies in the presence of partial irreversibility, fixed adjustment costs or quadratic adjustment costs. But without the complete irreversibility, the firm could also adjust capital stock downwards conditional on negative profitability shocks. Therefore in theory the effect of uncertainty on the expected capital intensity under these forms of adjustment costs is ambiguous.

Columns (2) and (3) estimate the same model but imposing the discount rate to be $r = 0.075$ and $r = 0.11$, respectively. Comparison between the first three columns implies that the estimates for the model are robust to the choice of discount rate within this range.

Columns (4) to (6) list the results for estimating the same full model but imposing the number of plants within each firm to be 1, 5, and 15, respectively. Since aggregation is one of the important sources of smoothing, assuming fewer number of plants for aggregation, i.e. when p decreases from 15 to 1, results in a higher estimate for b_q, lower estimates for b_i and b_f, lower estimate for average level of uncertainty μ_σ at the plant level, and lower estimates for parameters measuring dispersion in the data σ_μ and $\sigma_{l\sigma}$. The estimate for the mean of the intercept $\mu_{\bar{\zeta}}$ increases, since with fewer number of plants for aggregation, each plant has to be 'larger' to match the empirical mean of the firm size. According to the overall fit, a model imposing $p = 10$ would be preferred to those imposing $p = 1$, 5 or $p = 15$. The fact that I allow for $p = 10$ plants to take into account the effect of aggregation while Bloom (2009) allows for $p = 250$ plants, is partly due to the difference in firm size across these two samples: median number of employees is 3,450 in Bloom (2009) and is 1,100 in this chapter; but is mainly because the profitability shocks in this model are idiosyncratic across plants, while the shocks in Bloom(2009) have both an idiosyncratic component at the plant level and a common component at the firm level.

5.4 Comparison with the Literature

Since this is the first study that offers structural estimates of the HAC effect, there is no existing literature that can be used to compare the finding in this line. However, the pioneering work of Cooper and Haltiwanger (2006), Bloom (2009) and the estimates from Chapter 4 provide the possibility to compare the estimates on capital adjustment costs.

Table 5.6 lists the estimates on capital adjustment costs from Cooper and Haltiwanger (2006), Bloom (2009), the Brazilian sample in Chapter 4 and the Datastream sample in this chapter, together with median number of employees at each firm, assumed number of plants aggregated within each firm, and common moments reported in each of these studies. The estimates are comparable to each other, in the sense that the functional form for capital adjustment costs are the same and the adjustment cost parameters are all obtained from structural estimation. However, the estimates should not be compared too literally since different studies have different assumptions about the functional form for the operating profit and the stochastic process, and the number of plants for aggregation.

Across these four studies, the estimates in Cooper and Haltiwanger (2006) are highlighted by a large fixed adjustment cost while Bloom (2009) estimates a substantial partial irreversibility parameter. In contrast, Chapter 4 finds the importance of quadratic and fixed adjustment costs and Chapter 5 estimates a moderate quadratic adjustment cost, partial irreversibility and a relatively small fixed adjustment cost. For quadratic adjustment costs, the estimates from Chapter 4 and 5 are larger compared with those in Cooper and Haltiwanger (2006) and Bloom (2009), which is consistent with the higher serial correlation of investment rate in samples used in these two chapters. Recall these estimates for the quadratic adjustment costs would be even larger, had I not controlled for heterogeneity in growth rate and not allowed for potential measurement error in investment rate. Nevertheless, compared with empirical research inferring quadratic adjustment costs from the 'Q-model', for example, Hayashi (1982), the structural estimates listed in Table 5.6 are more closer to each other and significantly lower than those traditional findings.

6 A More General Specification

The empirical results from Tables 5.3, 5.4 and 5.5 indicate that, under the specification assumed in Sections 2 and 3, an investment model with adjustment costs and no HAC effect fits the data best. Here the negative correlation between measure of firm size and uncertainty, i.e. $corr\left(E\widehat{K}_j, SDdy_j\right)$ and $corr\left(EY_j, SDdy_j\right)$, is due to the significant quadratic adjustment cost estimated from the data.

However, even at our preferred specification, the model cannot match the negative correlation between measures of capital intensity and uncertainty, i.e. $corr\left(Eky_j, SDdy_j\right)$. As I discussed previously, within this model, any non zero correlation between Eky_j and $SDdy_j$ can only be attributed to adjustment costs. But the estimated adjustment costs, which fit the overall features of the data best, cannot generate this negative correlation observed from the data.

This highlights the importance of potential mis-specification. One potential mis-specification that I explore in this section is the assumption that the discount rate r is common to all firms, whether they face a high or low level of uncertainty. A more general specification allows the discount rate to vary with the level of uncertainty, reflecting one version of a 'risk premium' in the required rate of return or the cost of capital.

This section studies the effect of relaxing this assumption by allowing for one possible form of omitted variable bias through a discount rate effect.

Assumption 16 ***Discount Rate Effect:***

The firm j discounts future net revenue at a constant rate r_j, where

$$r_j = \overline{r} + \theta \cdot \sigma_j^2 \tag{23}$$

Assumption 16 allows the discount rate of firm j to be the sum of a constant term that is same across firms and a firm-specific term that depends on the firm-specific level of uncertainty. Since the profitability shocks I model in this chapter are idiosyncratic, under the assumption that the owners of firms are risk-neutral or even if are risk-averse but fully-diversified, θ must be zero. In contrast, if the owners are risk-averse and not fully-diversified, either because of incomplete markets, as analyzed in Angeletos and Calvet (2006), or as the result of an optimal incentive scheme due to agency conflict, as modelled in Himmelberg, Hubbard and Love (2002), idiosyncratic risks would also affect the required rate of return, so that θ could be positive and $\theta \cdot \sigma_j^2$ is then the 'risk premium component' in the discount rate of the firm.

Since in the absence of adjustment costs, $E\left[\widehat{K}_t^*\right] = const1 \cdot E\left[Z_t\right]$, where $const1$ depends negatively on r, and $ky_{j,t}^* = \ln\left[\beta\left(1 - \frac{1}{\varepsilon}\right) / \left(\frac{r+\delta}{1+r}\right)\right]$, all else being equal, a positive θ would imply a negative correlation for both firm size and uncertainty, and capital intensity and uncertainty. I call this effect the 'discount rate effect'.

The right panel in Table 5.7 reports the empirical results for estimating the full model with this additional specification, assuming $p = 10$ plants within each firm and $\delta = 0.08$ as those in Table 5.3. I impose $\overline{r} = 0.03$ and estimate θ together with the other 17 structural parameters, both in the two-step procedure and one-step procedure. The results of imposing $r = 0.09$ and $\theta = 0$ are listed for comparison on the left panel.

The model allowing for the discount rate effect estimates $\widehat{\theta} = 0.1345$ in step 1 and produces a better overall fit than a model imposing $\theta = 0$, mainly because it generates a negative value for $corr\,(Eky_j, SDdy_j)$ that is close to the empirical value. The estimates for the other structural parameters are broadly similar across the two models with and without discount rate effect.

In step 2, given that the model without discount rate effect fits $corr\left(E\widehat{K}_j, SDdy_j\right)$ and $corr\,(EY_j, SDdy_j)$ reasonably well only through the adjustment cost effect, and $\theta > 0$ would imply a negative effect on firm size through the discount rate effect, it is not surprising that the more general model with discount rate effect estimates a positive HAC effect, i.e. $\widehat{\tau} = 0.8843 < 1$. In addition, this model also produces a better overall fit than the model without discount rate effect in this step.

A similar pattern is found when the models are estimated by the one-step procedure. To confirm that the improvement in the fit by allowing for the discount rate effect is indeed due to $\theta > 0$ instead of a different value for the average discount rate, I calculate the sample average of the discount rates r_j according to equation (23) at the estimated parameter values, which are 0.0907 and 0.0926 in the two-step and one-step procedures, respectively, and are very close to 0.09, the value of discount rate imposed in the baseline model without discount rate effect.

7 Counterfactual Simulations

This section examines the effects of uncertainty through counterfactual simulations. Similar to those exercises in Chapter 3, I study the effects of uncertainty on both short-run investment dynamics and long-run capital accumulation.

7.1 Short-Run Effects

In Chapter 3, I illustrate the effects of uncertainty by considering the impact effect of profitability shocks e_t on the adjustment of capital stock $\iota(t)$ in the same period. Different from those in Chapter 3 or 4, where each firm has single plant so that there is a one-to-one correspondence between e_t and $\iota(t)$, with aggregation across plants in this chapter, what we are interested in is how capital adjustment responds to new information about profitability at the firm-level.

Figure 5.2a and 5.2b plot how the growth rate of the capital stock $\iota(t)$, where $\iota(t) \equiv \Delta \ln \widehat{K}_t = \ln \widehat{K}_t - \ln \widehat{K}_{t-1}$, varies with the sales growth rate dy_t across firms in the same period, using the estimated parameter values for adjustment costs, production technology and demand schedule reported in Table 5.3.

Given there is heterogeneity in growth rate within this sample, both figures plot the relationship for the type of firms with the median level of growth rate, i.e. 0.0192. The heterogeneity in the level of uncertainty provides the possibility to study the effect of uncertainty on this representation of the impact effect of new information about profitability on investment. Figure 5.2a plots the impact effect for firms with different level of uncertainty by discretizing the estimated distribution for σ_j into three grids, so that the low, median and high σ lines in the figure correspond to values of 0.5179 (circle line), 0.6627 (diamond line) and 0.8479 (asterisk line), respectively. Comparison between these three lines and the 45^o line highlights the significant effect of capital adjustment costs in dampening the response of investment to new information about profitability. However, the similarity between these three lines implies that the effect of uncertainty on short-run investment dynamics is very small across firms with different levels of uncertainty within this sample.

Figure 5.2b plots an out-of-sample counterfactual simulation. That is, for firms with median growth rate and level of uncertainty, what would be the effect if the level of uncertainty was decreased or increased by 50%? Therefore the circle, diamond and asterisk lines correspond to σ at values of 0.3313, 0.6627 and 0.9940, respectively. The effect of uncertainty on investment dynamics is more evident than in Figure 5.2a, however the effect estimated here is still substantially lower than those reported in Bloom (2009). This is because this chapter has estimated a substantial quadratic adjustment cost, which is almost zero in Bloom(2009).

As a robustness check, I also simulate the short-run effects using estimates of the more general specification, allowing for the discount rate effect, as those reported in Table 5.7. Since both $\iota(t)$ and dy_t are variables in growth rate, the estimated positive HAC effect does not influence these short-run effects. The positive risk-premium in the discount rate further dampens the effect of uncertainty on investment dynamics. However, as illustrated in Figure 5.3a and 5.3b, at the estimated value of adjustment costs, both the pattern and magnitude of the short-run effects from a model with discount rate effect are similar to those from a model without discount rate effect.

7.2 Long-Run Effects

Figure 5.4a illustrates how the level of uncertainty affects the average level of the capital stock in the estimated model, using the estimated parameter values reported in Table 5.3 and the same random numbers employed during estimation. Therefore these effects are aggregated across plants, averaged over firms with heterogeneous growth rate and level of uncertainty. The counterfactual simulation takes the estimated median level of uncertainty, i.e. σ =0.6627 as the reference level and decreases and increases this value by 10% along the horizontal axis, maintaining the heterogeneity in the level of uncertainty at the estimated value. The average capital stock level is scaled by the level in the simulation using the reference level of σ, so that the values on the vertical axis can be read as percentage changes in the average capital stock as we vary σ. The downward sloping line in this figure highlights the negative effect of uncertainty on capital accumulation.

All else being equal, a permanent increase in the level of uncertainty by 10% at the plant-level is estimated to decrease the average capital stock levels by about 20% at the firm-level. Since the model in Table 5.3 does not estimate any HAC effect and does not allow for any discount rate effect, according to Proposition 3, the negative effect estimated here is fully due to the presence of capital adjustment costs, and in particular due to the substantial estimated quadratic adjustment costs.

To check whether this finding is robust to the inclusion of a discount rate effect, Figure 5.4b does the same exercise using the estimated parameter values reported in Table 5.7. Under this specification, uncertainty also generates a negative effect on capital accumulation, and the magnitude is even larger than that found in Figure 5.4a.

To see how uncertainty affects capital accumulation through different channels in this model, Figures 5.5a, 5.5b and 5.5c decompose the overall effect estimated in Figure 5.4b into adjustment cost effect only, discount rate effect only and the HAC effect only. In other words, in Figure 5.5a, I impose $\theta = 0$ (no discount rate effect) and $\tau = 1$ (no HAC effect) and plot the effect using estimates for other parameters as reported in Table 5.7. Therefore, a permanent increase in the level of uncertainty by 10% is estimated to decrease the average capital stock levels by about 21%, through the adjustment cost effect only, which is indeed similar to the finding in Figure 5.4a. In Figure 5.5b, I plot the discount rate effect by calculate how the $const1$ in equation (20) varies with σ through the discount rate, keeping all else constant. The model estimates the average capital stock levels would decrease about 22% with a permanent 10% increase in the level of uncertainty, through the discount rate effect only. Finally, in Figure 5.5c, I plot the HAC effect by calculating how $E\left[Z_t\right]$ in equation (21) varies with σ through the HAC effect. All else being equal, the model predicts that a permanent increase in the level of uncertainty by 10% would increase the average capital stock levels by about 6%, through the HAC effect only.

Since uncertainty affects capital accumulation through these effects simultaneously and nonlinearly, the overall effects are different from the simple additive or multiplicative result of the three individual effects. Instead, a simulation which allows for the three effects to operate simultaneously is reported in Figure 5.4b.

Comparison between Figure 5.4a and 5.4b implies, although imposing the discount rate to be common for all firms in our main specification may under-estimate the HAC effect, the overall effect of uncertainty on capital accumulation is estimated to be broadly similar in both specifications of the model.

8 Conclusion

This chapter presents estimates for the effects of uncertainty on both short run investment behavior and long run capital accumulation for a panel of UK manufacturing firms. I estimate a structural investment model, in which uncertainty would affect capital accumulation through two possible channels: the capital adjustment cost effect and the HAC effect. Information about variables in ratio or growth rate provides the identification for the capital adjustment costs, together with production technology, demand schedule, stochastic process and measurement errors in the data; while additional information about variables in level provides the identification for the HAC effect. I also allow for a more general specification in which discount rates may vary with the level of uncertainty.

The estimated model is used to investigate how these firms' investment dynamics and capital accumulation would differ if they faced different levels of uncertainty. Counterfactual simulations indicate that the impact effect of new information about profitability on investment is not sensitive to variation in the level of uncertainty, mainly because the presence of quadratic adjustment costs substantially dampens the responsiveness of capital adjustment to new information about profitability. In the long run, a lower level of uncertainty would induce firms to operate with substantially higher levels of capital, which operates through the negative effect of quadratic adjustment costs in a baseline model without discount rate effect, or through a negative adjustment cost effect, a negative discount rate effect and a positive HAC effect in the more general specification.

Appendix A: Proof

A.1 Proof for Lemma 1

For given production techonology $Q_t = A_t \widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta}$, and demand schedule $Q_t = X_t P_t^{-\varepsilon}$, rewrite sales as that in Chapter 4

$$
\begin{aligned}
Y_t &= P_t \times Q_t \\
Y_t &= \left(\frac{Q_t}{X_t}\right)^{-\frac{1}{\varepsilon}} \times Q_t \\
Y_t &= X_t^{\frac{1}{\varepsilon}} \left(Q_t\right)^{1-\frac{1}{\varepsilon}} \\
Y_t &= X_t^{\frac{1}{\varepsilon}} Q_t^{\frac{\varepsilon-1}{\varepsilon}} \\
Y_t &= X_t^{\frac{1}{\varepsilon}} A_t^{\frac{\varepsilon-1}{\varepsilon}} \left(\widehat{K}_t^{\beta} L_t^{\alpha} M_t^{1-\alpha-\beta}\right)^{\frac{\varepsilon-1}{\varepsilon}}
\end{aligned}
$$

Note that $A_t^{\frac{1}{\varepsilon}}$ is playing as the same role as $X_t^{\frac{1}{\varepsilon}}$ in the sales, but have an exponential term $(\varepsilon - 1)$. Since the short-run profit optimization is solved for given X_t and A_t, following the solution in Appendix A.1 in Chapter 4, it is straightforward to derive

$$\pi_t = const0 \cdot X_t^{\gamma} \left(A_t^{\gamma}\right)^{\varepsilon-1} \widehat{K}_t^{1-\gamma}$$

A.2 Proof for Lemma 2

Taking the log on both sides of equation (4) leads to

$$
\begin{aligned}
\ln \pi_t &= \ln const0 + \gamma \ln X_t + \gamma \left(\varepsilon - 1\right) \ln A_t + \left(1-\gamma\right) \ln \widehat{K}_t \\
&= \ln const0 + \gamma \left[\ln X_t + \left(\varepsilon - 1\right) \ln A_t\right] + \left(1-\gamma\right) \ln \widehat{K}_t \qquad \text{(A1)}
\end{aligned}
$$

Under Assumption 4 and 5, the stochastic part of (A1) can be written as

$$
\begin{aligned}
& \gamma \left\{\left[c_x + \mu_x t + \zeta_t^x\right] + \left(\varepsilon - 1\right)\left[c_a + \mu_a t + \zeta_t^a\right]\right\} \\
= \; & \gamma \left\{ \begin{array}{c} \left[\overline{\zeta}^x + \left(\varepsilon - 1\right)\overline{\zeta}^a\right] + \left[c_x + \left(\varepsilon - 1\right) c_a\right] + \left[\mu_x + \left(\varepsilon - 1\right)\mu_a\right] t \\ + \left[\sum_{s=0}^{t-1} \rho_x^s e_{t-s}^x + \left(\varepsilon - 1\right) \sum_{s=0}^{t-1} \rho_a^s e_{t-s}^a\right] \end{array} \right\}
\end{aligned}
$$

By imposing $\rho_x = \rho_a = \rho$, since e_t^x and e_t^a are independently normally distributed, a linear combination of these two random variables is still normally distributed with proper mean and variance, i.e.

$$\sum_{s=0}^{t-1} \rho_x^s e_{t-s}^x + \left(\varepsilon - 1\right) \sum_{s=0}^{t-1} \rho_a^s e_{t-s}^a = \sum_{s=0}^{t-1} \rho^s \left[e_{t-s}^x + \left(\varepsilon - 1\right) e_{t-s}^a\right] = \sum_{s=0}^{t-1} \rho^s e_{t-s}$$

where

$$e_t \overset{i.i.d}{\sim} N\left(0,\, \sigma_x^2 + (\varepsilon - 1)^2 \sigma_a^2\right)$$

This implies that the log of operating profit (A1) can be rewritten as

$$\log \pi_t = \log const0 + \gamma \log Z_t + (1 - \gamma) \log \widehat{K}_t$$

or equivalently the operating profit can be rewritten as

$$\pi(Z_t, \widehat{K}_t) = const0 \cdot Z_t^{\gamma} \widehat{K}_t^{1-\gamma}$$

The law of motion for the combined random variable Z_t is given by (11), with parameters in this stochastic process defined in (12).

A.3 Proof for Lemma 4

Given $c = c_x + (\varepsilon - 1) c_a$, $c_x = -0.5\sigma_x^2/\left(1 - \rho^2\right)$, $c_a = -0.5\sigma_a^2/\left(1 - \rho^2\right)$, $\sigma^2 = \sigma_x^2 + (\varepsilon - 1)^2 \sigma_a^2$, and $\sigma_a^2 = (1 - \tau)\, \sigma^2/\left(\varepsilon - 1\right)^2$, we can derive that

$$\begin{aligned}
E\left[Z_t\right] &= \exp\left[E\left(z_t\right) + 0.5V\left(z_t\right)\right] \\
&= \exp\left[\overline{\zeta} + c + \mu t + 0.5\sigma^2/\left(1 - \rho^2\right)\right] \\
&= \exp\left[\overline{\zeta} + \mu t + \left(c_x + \frac{\sigma_x^2}{2\left(1 - \rho^2\right)}\right) + (\varepsilon - 1)\left(c_a + \frac{(\varepsilon - 1)\,\sigma_a^2}{2\left(1 - \rho^2\right)}\right)\right] \\
&= \exp\left[\overline{\zeta} + \mu t + (0) + (\varepsilon - 1)\left(\frac{(\varepsilon - 2)\,\sigma_a^2}{2\left(1 - \rho^2\right)}\right)\right] \\
&= \exp\left[\overline{\zeta} + \mu t + \frac{(\varepsilon - 1)(\varepsilon - 2)}{2\left(1 - \rho^2\right)}\sigma_a^2\right] \\
&= \exp\left[\overline{\zeta} + \mu t + \frac{(\varepsilon - 2)(1 - \tau)}{2(\varepsilon - 1)\left(1 - \rho^2\right)}\sigma^2\right] \qquad \text{(A2)}
\end{aligned}$$

A.4 Proof for Lemma 5

As derived in the Appendix A.3 of Chapter 3, for any random variable $x_i \overset{i.i.d}{\sim} N\left(\mu,\, \sigma^2\right)$, if $X_i = \exp\left(x_i\right)$, the first moment for X_i is

$$\begin{aligned}
E\left[X_i\right] &= \exp\left(E\left[x_i\right] + 0.5V\left[x_i\right]\right) \\
&= \exp\left(\mu + 0.5\sigma^2\right) \qquad \text{(A3)}
\end{aligned}$$

And the second moment is

$$\begin{aligned}
V\left[X_i\right] &= E\left[X_i^2\right] - \left(E\left[X_i\right]\right)^2 \\
&= \left[\exp\left(2\mu + \sigma^2\right)\right]\left[\exp\left(\sigma^2\right) - 1\right]
\end{aligned}$$

or equivalently, the standard deviation is given be

$$sd\left[X_i\right] = \exp\left(\mu + 0.5\sigma^2\right)\sqrt{\left[\exp\left(\sigma^2\right) - 1\right]} \tag{A4}$$

Assume $\tau = 1$ (no HAC effect) and $\sigma_u = 0$ (homogeneous growth rate) for now. Denote operation taking over firm and time with subscript j and t, respectively. Suppose the p plants within the same firms have similar size. Then the within-group capital stock for firm j is

$$E\widehat{K}_j = E_t\left[\widehat{K}_{j,t}\right] = E_t\left[\sum\nolimits_{i=1}^{p}\widehat{K}_{i,j,t}\right] \simeq E_t\left[p \cdot \widehat{K}_{i,j,t}\right] = pE_t\left[\widehat{K}_{i,j,t}\right]$$

In the absence of adjustment costs,

$$\begin{aligned} E_j\left[E\widehat{K}_j\right] &= E_j\left[pE_t\left[\widehat{K}_{i,j,t}\right]\right] = p \cdot const1 \cdot E_j\left[E_t\left[Z_{i,j,t}\right]\right] \\ sd_j\left[E\widehat{K}_j\right] &= sd_j\left[pE_t\left[\widehat{K}_{i,j,t}\right]\right] = p \cdot const1 \cdot sd_j\left[E_t\left[Z_{i,j,t}\right]\right] \end{aligned}$$

The law of motion for $Z_{i,j,t}$ is given by equation (26), by which we get

$$\begin{aligned} E_t[z_{i,j,t}] &= c_j + \overline{\zeta}_j + \mu t \\ Var_t[z_{i,j,t}] &= \frac{\sigma_j^2}{1-\rho^2} \end{aligned}$$

Using the result in (A3)

$$\begin{aligned} E_t\left[Z_{i,j,t}\right] &= \exp\left(E_t[z_{i,j,t}] + 0.5Var_t[z_{i,j,t}]\right) \\ &= \exp\left(c_j + \overline{\zeta}_j + \mu t + 0.5\sigma_j^2/\left(1-\rho^2\right)\right) \\ &= \exp\left(\overline{\zeta}_j + \mu t\right) \\ &= \exp\left(\mu t\right)\exp\left(\overline{\zeta}_j\right) \end{aligned}$$

Calculating the between-group mean and standard deviation leads to

$$\begin{aligned} E_j\left[E_t\left[Z_{i,j,t}\right]\right] &= \exp\left(\mu t\right)E_j\left[\exp\left(\overline{\zeta}_j\right)\right] \\ sd_j\left[E_t\left[Z_{i,j,t}\right]\right] &= \exp\left(\mu t\right)sd_j\left[\exp\left(\overline{\zeta}_j\right)\right] \end{aligned}$$

Since $\overline{\zeta}_j \overset{i.i.d}{\sim} N\left(\mu_{\overline{\zeta}}, \sigma_{\overline{\zeta}}^2\right)$, applying the the result in (A3) and (A4), we have

$$\begin{aligned} E_j\left[\exp\left(\overline{\zeta}_j\right)\right] &= \exp\left(\mu_{\overline{\zeta}} + 0.5\sigma_{\overline{\zeta}}^2\right) \\ sd_j\left[\exp\left(\overline{\zeta}_j\right)\right] &= \exp\left(\mu_{\overline{\zeta}} + 0.5\sigma_{\overline{\zeta}}^2\right)\sqrt{\left[\exp\left(\sigma_{\overline{\zeta}}^2\right) - 1\right]} \end{aligned}$$

Suppose the capital stock is recorded in £ and I normalize it with £M. Empirically, I calculate the first two moments of $\widehat{K}_{j,t}$ being 0.0862 and 0.3780. This implies conditional on τ and $\mathbf{b}$ being identified by other moments, the identification for $\mu_{\overline{\zeta}}$ and $\sigma^2_{\overline{\zeta}}$ can be achieved by solving the simultaneous equations

$$\Gamma \cdot \exp\left(\mu_{\overline{\zeta}} + 0.5\sigma^2_{\overline{\zeta}}\right) = 0.0862 \tag{A5}$$

$$\Gamma \cdot \exp\left(\mu_{\overline{\zeta}} + 0.5\sigma^2_{\overline{\zeta}}\right)\sqrt{\left[\exp\left(\sigma^2_{\overline{\zeta}}\right) - 1\right]} = 0.3780 \tag{A6}$$

where $\Gamma = p \cdot const1 \cdot \exp(\mu t)/M$, and $const1$ is a function of $(r, \delta, \beta, \varepsilon, w, m)$.

Divide equation (A6) by (A5), we get the solution to $\sigma_{\overline{\zeta}}$, which is independent of Γ and only depends on the coefficient of variation of the capital stock. Substitute this solution into (A5), $\mu_{\overline{\zeta}}$ can be solved out for any Γ. Hence given $(\mu, r, \delta, \beta, \varepsilon)$ are pinned down by other moments or predefined, the effect of any arbitrary choice of w, m, t, the unit of measurement M, and the number of plants p, on the level of expected capital stock can be accounted for by adjusting $\mu_{\overline{\zeta}}$.

A.5 Proof for Proposition 4

Both the operating profit function and the capital adjustment cost function are linear homogeneous in $(Z_t, K_t; I_t)$. This implies the linear homogeneity of $\Pi(Z_t, K_t; I_t)$, hence the linear homogeneity of firm value in $(Z_t, K_t; I_t)$. Therefore if I_t is the optimal solution to the following problem

$$V(Z_t, K_t) = \max_{I_t}\{\Pi(Z_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(Z_{t+1}, K_{t+1})\right]\}$$

then $\widetilde{I}_t \equiv I_t/\Omega$, where Ω is any positive constant, must be the optimal solution to the normalized problem

$$V(\widetilde{Z}_t, \widetilde{K}_t) = \max_{\widetilde{I}_t}\{\Pi(\widetilde{Z}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{1}{1+r}E_t\left[V(\widetilde{Z}_{t+1}, \widetilde{K}_{t+1})\right]\}$$

where $\widetilde{Z}_t = Z_t/\Omega$, $\widetilde{K}_t = K_t/\Omega$, $\widetilde{Z}_{t+1} = Z_{t+1}/\Omega$, $\widetilde{K}_{t+1} = K_{t+1}/\Omega$.

Given Ω could be any positive constant, following (A2), choosing

$$\Omega = \exp\left(\overline{\zeta} + \frac{(\varepsilon-2)(1-\tau)}{2(\varepsilon-1)(1-\rho^2)}\sigma^2\right)$$

leads to Proposition 4. This is because $\Theta_2 \equiv \left(\tau,\ \mu_{\overline{\zeta}},\ \sigma_{\overline{\zeta}}\right)$, different value of τ, $\mu_{\overline{\zeta}}$ and $\sigma_{\overline{\zeta}}$ will generate different value for Ω, hence different firm value and variables in level, however, they would not affect the value of any variable in ratio or growth rate. That is for firm j, by choosing

$$\Omega_j = \exp\left(\overline{\zeta}_j + \frac{(\varepsilon - 2)(1-\tau)}{2(\varepsilon - 1)(1-\rho^2)}\sigma_j^2\right)$$

Investment Rate:

$$i_{j,t} \equiv I_{j,t}/K_{j,t} = \widetilde{I}_{j,t}/\widetilde{K}_{j,t}$$

Sales Growth Rate:

$$dy_{j,t} \equiv \ln\left(Y_{j,t}/Y_{j,t-1}\right) = \ln\left(\widetilde{Y}_{j,t}/\widetilde{Y}_{j,t-1}\right)$$

log of Sales-to-(beginning-of-period)Capital Ratio:

$$yk_{j,t} \equiv \ln\left(Y_{j,t}/K_{j,t}\right) = \ln\left(\widetilde{Y}_{j,t}/\widetilde{K}_{j,t}\right)$$

log of (productive) Capital-to-Sales Ratio:

$$ky_{j,t} \equiv \ln\left(\widehat{K}_{j,t}/Y_{j,t}\right) = \ln\left(\left(\widetilde{K}_{j,t} + \widetilde{I}_{j,t}\right)/Y_{j,t}\right)$$

Profit-to-Sales Ratio:

$$PIY_{j,t} \equiv PI_{j,t}/Y_{j,t} = \widetilde{PI}_{j,t}/\widetilde{Y}_{j,t}$$

Appendix B: Numerical Methods

B.1 Solve the model and simulate the data

The problem I need to solve is

$$\begin{aligned}
V(Z_t, K_t) &= \max_{I_t}\{\Pi(Z_t, K_t; I_t) + \frac{1}{1+r}E_t\left[V(Z_{t+1}, K_{t+1})\right]\} \qquad \text{(B1)}\\
s.t.\ K_{t+1} &= (1-\delta)\left(K_t + I_t\right)\\
Z_t &= \exp\left(z_t\right)\\
z_t &= c + \mu t + \zeta_t\\
\zeta_t &= \overline{\zeta} + \rho\zeta_{t-1} + e_t = \overline{\zeta} + \sum_{s=0}^{t-1}\rho^s e_{t-s}
\end{aligned}$$

where $e_t = \sigma\epsilon_t$, $\epsilon_t \overset{i.i.d}{\sim} N\left(0,\, 1\right)$, $\zeta_0 = 0$.

Assumption 6, Lemma 2 and 3 imply that

$$\begin{aligned}
c &= c_x + (\varepsilon - 1)\, c_a\\
&= -\frac{\sigma^2}{2\left(1-\rho^2\right)}\left[\tau + \frac{(1-\tau)}{(\varepsilon-1)}\right]
\end{aligned}$$

and Assumption 9, 10 and 13 assume that for firm j,

$$\begin{aligned}
&\log\sigma_j \overset{i.i.d}{\sim} N\left(\mu_{l\sigma},\, \sigma^2_{l\sigma}\right)\\
&\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma^2_\mu\right)\\
&\overline{\zeta}_j \overset{i.i.d}{\sim} N\left(\mu_{\overline{\zeta}}, \sigma^2_{\overline{\zeta}}\right)
\end{aligned}$$

Following the same approach as in Chapter 3, define $\Psi_t \equiv \exp\left(c + \overline{\zeta} + \mu t\right)$; denote $\widetilde{Z}_t = \frac{Z_t}{\Psi_t}$, $\widetilde{K}_t = \frac{K_t}{\Psi_t}$, $\widetilde{I}_t = \frac{I_t}{\Psi_t}$, and $\widetilde{\zeta}_t = \zeta_t - \overline{\zeta}$, then the problem (B1) is transformed into

$$\begin{aligned}
V(\widetilde{Z}_t, \widetilde{K}_t) &= \max_{\widetilde{I}_t}\{\Pi(\widetilde{Z}_t, \widetilde{K}_t; \widetilde{I}_t) + \frac{\exp\left(\mu\right)}{1+r}E_t\left[V(\widetilde{Z}_{t+1}, \widetilde{K}_{t+1})\right]\} \qquad \text{(B2)}\\
s.t.\ \widetilde{K}_{t+1} &= \exp(-\mu)\left(1-\delta\right)\left(\widetilde{I}_t + \widetilde{K}_t\right)\\
\widetilde{Z}_t &= \exp\left(\widetilde{\zeta}_t\right)\\
\widetilde{\zeta}_t &= \rho\widetilde{\zeta}_{t-1} + e_t = \sum_{s=0}^{t-1}\rho^s e_{t-s}
\end{aligned}$$

where $e_t = \sigma\epsilon_t$ and $\epsilon_t \overset{i.i.d}{\sim} N\left(0,\, 1\right)$.

Both the level of uncertainty σ and the growth rate μ would affect the investment policy. Hence the dynamic program (B2) must be solved for each type of σ and μ. In other words, σ and μ are two additional state variables besides $\widetilde{Z}_t$ and $\widetilde{K}_t$. Given $\log \sigma_j \overset{i.i.d}{\sim} N\left(\mu_{l\sigma}, \sigma_{l\sigma}^2\right)$, and $\mu_j \overset{i.i.d}{\sim} N\left(\mu_\mu, \sigma_\mu^2\right)$, I discretize these continuous distributions individually by Tauchen (1989) method. Due to 'the curse of dimensionality', I have to be conservative about the number of grids and set them to be 3 for both of these two state variables. I experiment with higher number of grids for these two state variables and find the simulated moments are not very sensitive. The procedure of how to solve the model can be described as:

for $\sigma = \exp\left(\log \sigma_1, \log \sigma_2, \log \sigma_3\right)$

  for $\mu = (\mu_1, \mu_2, \mu_3)$

    for value function iteration converges

      policy improvement algorithm

      (See Appendix B.2 in Chapter 3)

    end

    interpolate for $\widetilde{I}_t = h(\widetilde{Z}_t, \widetilde{K}_t)$

  end

end

In other words, the dynamic program (B2) is solved for each type of firms from the type set $\digamma = \{(\sigma_u, \mu_v) : u = 1, \cdots, 3;\ v = 1, \cdots, 3\}$. Under Assumption 8, this implies in the simulated data, there are 9 type of firms, each comprising a fixed proportion 1/9 of the population.

This numerical solution is applied to generate a simulated panel data with N firms and T years, where each firm is made of p independent plants. The empirical sample I use is an unbalanced panel with $N = 629$ firms and $T = 10$ years of observation for each firm on average. In order to simulate a panel for firms which are on their balanced-growth-path so that Propostion 2 applies, I allow for a burning period of Υ years in simulating the data. In application to this sample, I find $\Upsilon = 50$ is long enough in the sense that after living for Υ years, in the next T years, the simulated mean of sales growth rate is equal to the mean of profitability growth rate imposed in that particular round of simulation, as predicted in Proposition 2.

For given N, T, p, and Υ, the procedure to simulate the data includes four steps, where $j = 1, \cdots, N$, $i = 1, \cdots, p$ and $t = 1, \cdots, (\Upsilon + T)$.

Step 1: Simulate data for each plant i of firm j in period t. When $t = 0$, all the simulated plants of firm j are endowed with the initial condition $\widetilde{\zeta}_{i,j,0} = 0$ and the corresponding initial capital stock $\widetilde{K}_{i,j,0} = steady_j \cdot \widetilde{Z}_{i,j,0} = steady_j \cdot \exp(\widetilde{\zeta}_{i,j,0})$. Here $steady_j = (1 - \delta) \cdot const1/ \exp\left(\mu_j\right)$, (see Appendix B.1 of Chapter 3 for definition), which reflects the effect of heterogeneity in growth rate on the steady-state capital stock. For all subsequent periods, random profitability shocks $e_{i,j,t} = \sigma_j \epsilon_{i,j,t}$ are draw for each each plant i of firm j and in each period t. Given the realization of $\widetilde{Z}_{i,j,t}$ and the inherited $\widetilde{K}_{i,j,t}$, the optimal investment $\widetilde{I}_{i,j,t}$ are then pinned down using the policy rule derived above. Then $\widetilde{Z}_{i,j,t}$ or equivalently $\widetilde{\zeta}_{i,j,t}$ evolves exogenously according to its law of motion, $\widetilde{K}_{i,j,t}$ evolves endogenously according to capital accumulation formula. The control variable in this period then becomes the state variable in next period.

Step 2: Plant-level data are aggregated into firm-level data. For firm j in period t, the normalized investment data is $\widetilde{I}_{j,t} = \sum_{i=1}^{p} \widetilde{I}_{i,j,t}$; capital stock is $\widetilde{K}_{j,t} = \sum_{i=1}^{p} \widetilde{K}_{i,j,t}$; operating profit is $\widetilde{\pi}_{j,t} = \sum_{i=1}^{p} \widetilde{\pi}_{i,j,t}$.

Step 3: The intercept and time trend $\exp\left(c_j + \overline{\zeta}_j + \mu_j t\right)$ are recovered for the level variables of the original model. For firm j in period t, the actual investment is therefore $I_{j,t} = \widetilde{I}_{j,t} \exp\left(c_j + \overline{\zeta}_j + \mu_j t\right)$; the actual capital stock is $K_{j,t} = \widetilde{K}_{j,t} \exp\left(c_j + \overline{\zeta}_j + \mu_j t\right)$; the actual operating profit is $\pi_{j,t} = \widetilde{\pi}_{j,t} \exp(c_j + \overline{\zeta}_j + \mu_j t)$; the sales is $Y_{j,t} = \gamma\varepsilon \cdot \pi_{j,t}$ according to equation (6). The investment rate is $i_{j,t} = I_{j,t}/K_{j,t}$.

Different from those in Chapters 3 and 4, in this chapter c_j reflects the importance of HAC effect on variables in level and is defined as

$$c_j = -\frac{\sigma_j^2}{2\left(1 - \rho^2\right)} \left[\tau + \frac{(1 - \tau)}{(\varepsilon - 1)}\right]$$

$\overline{\zeta}_j$ is simulated according to

$$\overline{\zeta}_j \overset{i.i.d}{\sim} N\left(\mu_{\overline{\zeta}}, \sigma_{\overline{\zeta}}^2\right)$$

for each firm by drawing random numbers from a normal distribution generated by the computing software.

Step 4: Add measurement errors, so that the observed investment rate is $i_{j,t} = i^{*}_{j,t} \exp(e^{I}_{j,t})$, and the observed level of sales is $Y_{j,t} = Y^{*}_{j,t} \exp(e^{Y}_{j,t})$.

With the simulated data for $I_{j,t}$, $K_{j,t}$, $\pi_{j,t}$, $Y_{j,t}$, and given model parameter values, other variables of interest such as adjustment costs can be easily calculated by definition. The simulated moments are calculated only using the last $T = 10$ years observation to match those in the empirical sample.

Appendix C: Data

C.1 Sampling

The dataset used in this chapter is from Bloom, Bond and Van Reenen (2007). The original sample contains firm-level data for an unbalanced panel of 672 firms between 1972 and 1991. These company data are taken from the consolidated accounts of manufacturing firms listed on the U.K. stock market and are obtained from the Datastream on-line service.

C.2 Definition of Variables

- Define I = total new fixed assets (DS435) less sales of fixed assets (DS423).
- Define K=constructed by applying a perpetual inventory procedure with a depreciation rate of 8%. The starting value was based on the net book value of tangible fixed capital assets (DS339) in the first observation within sample period.
- Define Y = total sales (DS104).
- Define PI = profit before interest, taxation and depreciation (DS137+DS136). This information is missing for 43 firms in the original sample so that I finally use a sample consisted of 629 firms.

C.3 Cleaning Rules

- Bloom, Bond and Van Reenen (2007) have deleted firms with less than three consecutive observations, broke the series for firms where accounting periods fell outside the range 300 to 400 days (due to changes in year ends), and excluded observations for firms where there are jumps of greater than 150% in any of the basic variables. In addition, I further clean the sample in following steps.
- Deflate the capital stock series by the aggregate series for investment goods prices (INJJ/INKL from the Office of National Statistics)
- Deflate the sales series by the aggregate GDP deflator (YBMA/ABMI from the Office of National Statistics)
- Convert the values of capital stock and sales into constant price of year 1982 and divide these two series by 1000000.
- Rule out investment rate less than -1 or larger than 1. This deletes 18 observations from the original sample.
- Rule out top and bottom 1% observations for sales growth rate. This further deletes 120 observations from the original sample.

Table 5.1 Set of Parameters

Subset	Category	Symbol	Definition
Θ_1			
	Adjustment Costs	b_q	quadratic adjustment costs
		b_i	partial irreversibility
		b_f	fixed adjustment costs
	Technology and Demand	β	capital share in production function
		ε	demand elasticity with respect to price
	Stochastic Process	ρ	serial correlation of profitability shocks
		μ_μ	mean of μ, where μ is the growth rate of profitability
		σ_μ	standard deviation of μ
		$\mu_{l\sigma}$	mean of $\ln(\sigma)$, where σ measures the level of uncertainty
		$\sigma_{l\sigma}$	standard deviation of $\ln(\sigma)$
	Measurement Errors	σ_{IT}	sd of transitory measurement errors in investment rates
		σ_{IP}	sd of permanent measurement errors in investment rates
		σ_{YT}	sd of transitory measurement errors in sales
		σ_{YP}	sd of permanent measurement errors in sales
Θ_2			
	HAC Effect	τ	$\sigma_x^2 = \tau\sigma^2$
	Intercept	$\mu_{\bar{\zeta}}$	mean of $\bar{\zeta}$, where $\bar{\zeta}$ is the intercept
		$\sigma_{\bar{\zeta}}$	standard deviation of $\bar{\zeta}$

Table 5.2 Set of Moments

Subset	Symbol	Moments	s.e.	Definition	Informativeness
Φ_1					
	$mean(PIY_{j,t})$	0.1108	0.0022	mean of profit-to-sales ratio	β, ε
	$mean(i_{j,t})$	0.1246	0.0027	mean of investment rate	$\mu_\mu, (\delta)$
	$mean(dy_{j,t})$	0.0292	0.0022	mean of sales growth rate	μ_μ
	$mean(ky_{j,t})$	-0.7886	0.0223	mean of log capital-to-sales ratio	$\beta, \varepsilon, (r, \delta)$
	$mean(SDdy_j)$	0.1198	0.0019	BG mean of WG s.d. of sales growth rates	$\mu_{l\sigma}, \sigma_{YT}$
	$sd(i_{j,t})$	0.1260	0.0036	standard deviation of investment rate	$b, \sigma_\mu, \sigma, \sigma_{IT}, \sigma_{IP}$
	$sd(dy_{j,t})$	0.1337	0.0020	standard deviation of sales growth rate	$\sigma_\mu, \sigma, \sigma_{YT}$
	$sd(ky_{j,t})$	0.5672	0.0191	standard deviation of log capital-to-sales ratio	$b, \sigma_{YT}, \sigma_{YP}$
	$sd(SDdy_j)$	0.0467	0.0014	BG s.d. of WG s.d. of sales growth rates	$\sigma_{l\sigma}, \sigma_{YT}$
	$corr(i_{j,t}, i_{j,t-1})$	0.3916	0.0236	serial correlation of investment rate	$b_q, \rho, \sigma_\mu, \sigma_{IT}, \sigma_{IP}$
	$corr(dy_{j,t}, dy_{j,t-1})$	0.1958	0.0155	serial correlation of sales growth rate	$\rho, \sigma_\mu, \sigma_{YT}$
	$corr(yk_{j,t}, yk_{j,t-1})$	0.9679	0.0027	serial correlation of log sales-to-capital ratio	$\rho, \sigma_{YT}, \sigma_{YP}$
	$corr(i_{j,t}, dy_{j,t})$	0.3971	0.0155	corr. btw. investment rate & sales growth rate	$b, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}$
	$corr(i_{j,t}, yk_{j,t})$	0.1356	0.0212	corr. btw. investment rate & log sales-to-capital ratio	$b, \sigma_{IT}, \sigma_{IP}, \sigma_{YT}, \sigma_{YP}$
	$corr(dy_{j,t}, yk_{j,t})$	0.1438	0.0167	corr. btw. sales growth rate & log sales-to-capital ratio	$b, \sigma_{YT}, \sigma_{YP}$
	$corr(Eky_j, SDdy_j)$	-0.1021	0.0459	corr. btw. BG capital-to-sales ratio & measure of uncertainty	$b, \mu_{l\sigma}, \sigma_{l\sigma}$
	$p(i_{j,t} > 0.2)$	0.1533	0.0071	proportion of investment spikes	b_f, σ_μ
	$p(\|i_{j,t}\| < 0.01)$	0.0266	0.0024	proportion of investment inaction	b_i, b_f
	$p(i_{j,t} < -0.01)$	0.0239	0.0022	proportion of disinvestment	b_i, σ_μ
Φ_2					
	$mean(\widehat{K}_{j,t})$	0.0862	0.0151	mean of capital stock	$\mu_{\bar{\zeta}}, \tau, b_q, (r, \delta)$
	$mean(Y_{j,t})$	0.1591	0.0200	mean of sales	$\mu_{\bar{\zeta}}, \tau, (r, \delta)$
	$sd(\widehat{K}_{j,t})$	0.3780	0.1037	standard deviation of capital stock	$\sigma_{\bar{\zeta}}, \tau, b_q$
	$sd(Y_{j,t})$	0.5423	0.0972	standard deviation of sales	$\sigma_{\bar{\zeta}}, \tau$
	$corr(\widehat{K}_{j,t}, Y_{j,t})$	0.8760	0.0243	corr. btw. capital stock & sales	$\sigma_{\bar{\zeta}}, b_q$
	$corr(E\widehat{K}_j, SDdy_j)$	-0.1031	0.0325	corr. btw. BG capital stock & measure of uncertainty	$\sigma_{\bar{\zeta}}, \tau, b_q, \mu_{l\sigma}, \sigma_{l\sigma}$
	$corr(EY_j, SDdy_j)$	-0.1153	0.0332	corr. btw. BG sales & measure of uncertainty	$\sigma_{\bar{\zeta}}, \tau, \mu_{l\sigma}, \sigma_{l\sigma}$

Table 5.3 Empirical Results

Procedure		two-step		one-step
		step 1	step 2	
Estimates				
b_q		1.3844		1.2659
b_i		0.1716		0.1632
b_f		0.0093		0.0053
β		0.0810		0.0787
ε		24.6869		23.3530
ρ		0.9496		0.9377
μ_μ		0.0192		0.0199
σ_μ		0.0381		0.0419
$\mu_{l\sigma}$		-0.4115		-0.4199
$\sigma_{l\sigma}$		0.2260		0.2429
σ_{IT}		0.5289		0.5242
σ_{IP}		0.2129		0.1993
σ_{YT}		0.0003		0.0009
σ_{YP}		0.5646		0.5625
τ			1.0000	0.9954
$\mu_{\bar{\zeta}}$			-16.8570	-16.7542
$\sigma_{\bar{\zeta}}$			1.1147	1.1062
Moments	**Empirical**	**Simulated**	**Simulated**	**Simulated**
$mean(PIY_{j,t})$	0.1108	0.1097		0.1106
$mean(i_{j,t})$	0.1246	0.1288		0.1293
$mean(dy_{j,t})$	0.0292	0.0192		0.0200
$mean(ky_{j,t})$	-0.7886	-0.7756		-0.7872
$mean(SDdy_j)$	0.1198	0.1183		0.1182
$sd(i_{j,t})$	0.1260	0.1183		0.1174
$sd(dy_{j,t})$	0.1337	0.1323		0.1327
$sd(ky_{j,t})$	0.5672	0.5863		0.5850
$sd(SDdy_j)$	0.0467	0.0467		0.0473
$corr(i_{j,t}, i_{j,t-1})$	0.3916	0.3913		0.3955
$corr(dy_{j,t}, dy_{j,t-1})$	0.1958	0.1136		0.1098
$corr(yk_{j,t}, yk_{j,t-1})$	0.9679	0.9753		0.9748
$corr(i_{j,t}, dy_{j,t})$	0.3971	0.4095		0.4172
$corr(i_{j,t}, yk_{j,t})$	0.1356	0.1906		0.1842
$corr(dy_{j,t}, yk_{j,t})$	0.1438	0.2173		0.2136
$corr(Eky_j, SDdy_j)$	-0.1021	0.0254		0.0297
$p(i_{j,t} > 0.2)$	0.1533	0.1860		0.1889
$p(\lvert i_{j,t}\rvert < 0.01)$	0.0266	0.0284		0.0287
$p(i_{j,t} < -0.01)$	0.0239	0.0230		0.0240
$mean(\widehat{K}_{j,t})$	0.0862		0.0658	0.0664
$mean(Y_{j,t})$	0.1591		0.1727	0.1735
$sd(\widehat{K}_{j,t})$	0.3780		0.2298	0.2267
$sd(Y_{j,t})$	0.5423		0.5925	0.5834
$corr(\widehat{K}_{j,t}, Y_{j,t})$	0.8760		0.8759	0.8663
$corr(E\widehat{K}_j, SDdy_j)$	-0.1031		-0.1003	-0.0911
$corr(EY_j, SDdy_j)$	-0.1153		-0.0977	-0.0885
OI		**111.3**	**4.4**	**117.9**
Degree of Freedom		**5**	**4**	**9**

Table 5.4 Specification Tests

Column	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Restriction	none	$b_q = 0$	$b_i = 0$	$b_f = 0$	$b_{i,f} = 0$
Estimates					
b_q	1.3844	**0.0000**	1.4240	1.2033	1.5010
b_i	0.1716	0.2478	**0.0000**	0.1953	**0.0000**
b_f	0.0093	0.0001	0.0530	**0.0000**	**0.0000**
β	0.0810	0.0701	0.0853	0.0782	0.0825
ε	24.6869	22.6478	24.3083	23.7270	24.8537
ρ	0.9496	0.7766	0.9497	0.9386	0.9493
μ_μ	0.0192	0.0207	0.0193	0.0197	0.0203
σ_μ	0.0381	0.0207	0.0289	0.0421	0.0259
$\mu_{l\sigma}$	-0.4115	-0.6058	-0.4168	-0.4174	-0.4092
$\sigma_{l\sigma}$	0.2260	0.3562	0.2319	0.2348	0.2243
σ_{IT}	0.5289	0.2409	0.5161	0.5307	0.5086
σ_{IP}	0.2129	0.4955	0.2571	0.1900	0.2525
σ_{YT}	0.0003	0.0001	0.0001	0.0003	0.0000
σ_{YP}	0.5646	0.5769	0.5624	0.5545	0.5640
Moments					
$mean(PIY_{j,t})$	0.1097	0.1111	0.1092	0.1104	0.1108
$mean(i_{j,t})$	0.1288	0.1315	0.1296	0.1291	0.1301
$mean(dy_{j,t})$	0.0192	0.0204	0.0194	0.0198	0.0204
$mean(ky_{j,t})$	-0.7756	-0.8117	-0.7903	-0.7898	-0.7752
$mean(SDdy_j)$	0.1183	0.1174	0.1187	0.1178	0.1180
$sd(i_{j,t})$	0.1183	0.1401	0.1199	0.1177	0.1154
$sd(dy_{j,t})$	0.1323	0.1248	0.1311	0.1322	0.1300
$sd(ky_{j,t})$	0.5863	0.5818	0.5814	0.5745	0.5826
$sd(SDdy_j)$	0.0467	0.0488	0.0466	0.0465	0.0466
$corr(i_{j,t}, i_{j,t-1})$	0.3913	0.3631	0.3897	0.3880	0.3981
$corr(dy_{j,t}, dy_{j,t-1})$	0.1136	-0.0609	0.1004	0.1125	0.0912
$corr(yk_{j,t}, yk_{j,t-1})$	0.9753	0.9685	0.9743	0.9745	0.9747
$corr(i_{j,t}, dy_{j,t})$	0.4095	0.5850	0.4195	0.4162	0.4026
$corr(i_{j,t}, yk_{j,t})$	0.1906	0.1347	0.1851	0.1873	0.1825
$corr(dy_{j,t}, yk_{j,t})$	0.2173	0.1818	0.2156	0.2170	0.2114
$corr(Eky_j, SDdy_j)$	0.0254	0.0296	0.0320	0.0305	0.0284
$p(i_{j,t} > 0.2)$	0.0230	0.1943	0.1888	0.1890	0.1877
$p(\lvert i_{j,t}\rvert < 0.01)$	0.0284	0.0445	0.0281	0.0292	0.0256
$p(i_{j,t} < -0.01)$	0.1860	0.0206	0.0266	0.0229	0.0290
OI	**111.3**	**538.7**	**122.3**	**113.6**	**131.8**
Degree of Freedom	**5**	**6**	**6**	**6**	**7**

Table 5.4 Specification Tests--Continued

Column	**(1)**	**(6)**	**(7)**	**(8)**	**(1)**	**(9)**
Restriction	none step 1	$\sigma_{\mu}=0$	$\sigma_{l\sigma}=0$	*m.e.*=0	none step 2	$\sigma_{\zeta_0}=0$
Estimates						
b_q	1.3844	0.8199	0.9599	3.9498		
b_i	0.1716	0.1670	0.1962	0.6770		
b_f	0.0093	0.0102	0.0002	0.0999		
β	0.0810	0.0795	0.0772	0.1037		
ε	24.6869	26.2638	24.0627	26.1052		
ρ	0.9496	0.9498	0.9496	0.9499		
μ_{μ}	0.0192	0.0188	0.0192	0.0299		
σ_{μ}	0.0381	**0.0000**	0.0330	0.0500		
$\mu_{l\sigma}$	-0.4115	-0.4216	-0.4014	-0.2924		
$\sigma_{l\sigma}$	0.2260	0.2198	**0.0000**	0.4207		
σ_{IT}	0.5289	0.5180	0.5172	**0.0000**		
σ_{IP}	0.2129	0.2829	0.2407	**0.0000**		
σ_{YT}	0.0003	0.0002	0.0000	**0.0000**		
σ_{YP}	0.5646	0.5683	0.5686	**0.0000**		
τ					1.0000	0.7199
$\mu_{\bar{\zeta}}$					-16.8570	-16.7349
$\sigma_{\bar{\zeta}}$					1.1147	**0.0000**
Moments						
$mean(PIY_{j,t})$	0.1097	0.1083	0.1098	0.1085		
$mean(i_{j,t})$	0.1288	0.1296	0.1292	0.1149		
$mean(dy_{j,t})$	0.0192	0.0189	0.0193	0.0268		
$mean(ky_{j,t})$	-0.7756	-0.7555	-0.7744	-0.7411		
$mean(SDdy_j)$	0.1183	0.1202	0.1242	0.1138		
$sd(i_{j,t})$	0.1183	0.1191	0.1192	0.0689		
$sd(dy_{j,t})$	0.1323	0.1305	0.1331	0.1385		
$sd(ky_{j,t})$	0.5863	0.5851	0.5874	0.3562		
$sd(SDdy_j)$	0.0467	0.0468	0.0344	0.0635		
$corr(i_{j,t}, i_{j,t-1})$	0.3913	0.3767	0.3998	0.9152		
$corr(dy_{j,t}, dy_{j,t-1})$	0.1136	0.0825	0.1047	0.1315		
$corr(yk_{j,t}, yk_{j,t-1})$	0.9753	0.9738	0.9745	0.9427		
$corr(i_{j,t}, dy_{j,t})$	0.4095	0.4228	0.4251	0.5719		
$corr(i_{j,t}, yk_{j,t})$	0.1906	0.1781	0.1878	0.6878		
$corr(dy_{j,t}, yk_{j,t})$	0.2173	0.2103	0.2161	0.4179		
$corr(Eky_j, SDdy_j)$	0.0254	0.0238	0.0104	0.3662		
$p(i_{j,t} > 0.2)$	0.1860	0.1839	0.1877	0.1238		
$p(\lvert i_{j,t}\rvert < 0.01)$	0.0284	0.0249	0.0299	0.0452		
$p(i_{j,t} < -0.01)$	0.0230	0.0247	0.0230	0.0021		
$mean(\widehat{K}_{j,t})$					0.0658	0.0701
$mean(Y_{j,t})$					0.1727	0.1901
$sd(\widehat{K}_{j,t})$					0.2298	0.0982
$sd(Y_{j,t})$					0.5925	0.3394
$corr(\widehat{K}_{j,t}, Y_{j,t})$					0.8759	0.8036
$corr(E\widehat{K}_j, SDdy_j)$					-0.1003	-0.1282
$corr(EY_j, SDdy_j)$					-0.0977	-0.1015
OI	**111.3**	**130.9**	**190.0**	**2275.6**	**4.4**	**22.6**
Degree of Freedom	**5**	**6**	**6**	**9**	**4**	**5**

Table 5.5 Robustness Tests

Column Variation	(1) benchmark	(2) *r* =0.075	(3) *r* =0.11	(4) *p*=1	(5) *p*=5	(6) *p*=15
Estimates						
b_q	1.3844	1.5119	1.2796	2.5945	1.3979	0.8516
b_i	0.1716	0.0975	0.2023	0.0001	0.1059	0.2856
b_f	0.0093	0.0325	0.0067	0.0000	0.0004	0.0007
β	0.0810	0.0797	0.0847	0.0859	0.0807	0.0757
ε	24.6869	24.2363	25.8412	29.8106	25.3781	22.4906
ρ	0.9496	0.9483	0.9497	0.9300	0.9478	0.9490
μ_μ	0.0192	0.0201	0.0193	0.0204	0.0194	0.0202
σ_μ	0.0381	0.0399	0.0348	0.0343	0.0274	0.0436
$\mu_{l\sigma}$	-0.4115	-0.4254	-0.3806	-1.0397	-0.6264	-0.3676
$\sigma_{l\sigma}$	0.2260	0.2306	0.2232	0.3489	0.2460	0.2196
σ_{IT}	0.5289	0.5244	0.5264	0.5355	0.5241	0.5398
σ_{IP}	0.2129	0.2120	0.2272	0.2056	0.2570	0.1985
σ_{YT}	0.0003	0.0000	0.0000	0.0011	0.0001	0.0001
σ_{YP}	0.5646	0.5629	0.5665	0.5544	0.5775	0.5657
τ	1.0000	1.0000	1.0000	0.9973	1.0000	1.0000
$\mu_{\bar{\zeta}}$	-16.8570	-16.9581	-16.8900	-17.2303	-17.1539	-16.7137
$\sigma_{\bar{\zeta}}$	1.1147	1.1062	1.1291	0.8991	0.8887	0.9947
Moments						
$mean(PIY_{j,t})$	0.1097	0.1058	0.1124	0.1091	0.1105	0.1110
$mean(i_{j,t})$	0.1288	0.1298	0.1292	0.1303	0.1298	0.1293
$mean(dy_{j,t})$	0.0192	0.0201	0.0193	0.0202	0.0201	0.0195
$mean(ky_{j,t})$	-0.7756	-0.7559	-0.8116	-0.7754	-0.7859	-0.7930
$mean(SDdy_j)$	0.1183	0.1182	0.1179	0.1172	0.1185	0.1182
$sd(i_{j,t})$	0.1183	0.1187	0.1193	0.1174	0.1199	0.1202
$sd(dy_{j,t})$	0.1323	0.1323	0.1315	0.1325	0.1318	0.1323
$sd(ky_{j,t})$	0.5863	0.5840	0.5874	0.5869	0.5916	0.5874
$sd(SDdy_j)$	0.0467	0.0465	0.0464	0.0474	0.0469	0.0466
$corr(i_{j,t}, i_{j,t-1})$	0.3913	0.3925	0.3929	0.3943	0.4022	0.3862
$corr(dy_{j,t}, dy_{j,t-1})$	0.1136	0.1147	0.1162	0.1204	0.1136	0.1129
$corr(yk_{j,t}, yk_{j,t-1})$	0.9753	0.9751	0.9754	0.9757	0.9754	0.9755
$corr(i_{j,t}, dy_{j,t})$	0.4095	0.4156	0.4157	0.4022	0.4177	0.4236
$corr(i_{j,t}, yk_{j,t})$	0.1906	0.1857	0.1918	0.2233	0.1939	0.1920
$corr(dy_{j,t}, yk_{j,t})$	0.2173	0.2148	0.2186	0.2412	0.2278	0.2181
$corr(Eky_j, SDdy_j)$	0.0254	0.0267	0.0255	0.0180	0.0361	0.0346
$p(i_{j,t} > 0.2)$	0.1860	0.1901	0.1866	0.1877	0.1869	0.1865
$p(\lvert i_{j,t}\rvert < 0.01)$	0.0284	0.0284	0.0283	0.0295	0.0285	0.0293
$p(i_{j,t} < -0.01)$	0.0230	0.0237	0.0238	0.0245	0.0232	0.0232
$mean(\widehat{K}_{j,t})$	0.0658	0.0675	0.0638	0.0646	0.0674	0.0630
$mean(Y_{j,t})$	0.1727	0.1724	0.1733	0.1807	0.1831	0.1747
$sd(\widehat{K}_{j,t})$	0.2298	0.2336	0.2248	0.1501	0.1398	0.1789
$sd(Y_{j,t})$	0.5925	0.5892	0.5941	0.5554	0.4968	0.5887
$corr(\widehat{K}_{j,t}, Y_{j,t})$	0.8759	0.8747	0.8784	0.8379	0.8102	0.8627
$corr(E\widehat{K}_j, SDdy_j)$	-0.1003	-0.0982	-0.1033	-0.0410	-0.0889	-0.1073
$corr(EY_j, SDdy_j)$	-0.0977	-0.0948	-0.1020	-0.0295	-0.0619	-0.0943
OI	**115.7**	**121.6**	**118.2**	**152.4**	**135.2**	**121.6**
Degree of Freedom	**9**	**9**	**9**	**9**	**9**	**9**

Table 5.6 Comparison with the Literature

	Cooper&Haltiwanger (2006)	Bloom(2009) (a)	Bloom(2009) (b)	Chapter 4	Chapter 5
Estimates					
b_q	0.153	0.000	0.616	2.416	1.384
b_i	0.019	0.330	0.303	0.004	0.172
b_f	0.204	0.015	0.009	0.011	0.009
Sample	**LRD USA**	**Compustat USA**	**Compustat USA**	**World Bank Survey Brazil**	**Datastream UK**
No. of Employees (sample median)	..	3450	3450	48	1100
No. of Plants (assumed)	1	250	25	1	10
Data					
$mean(i_{j,t})$	0.122	..	..	0.107	0.129
$sd(i_{j,t})$	0.337	0.139	0.139	0.182	0.118
$corr(i_{j,t}, yk_{j,t})$	0.143	0.260	0.260	0.124	0.136
$corr(i_{j,t}, i_{j,t-1})$	0.058	0.328	0.328	0.424	0.392
$p(0.2 < i_{j,t} < 1)$	0.186	..	..	0.191	0.153
$p(0 < i_{j,t} < 0.2)$	..	..	..	0.402	0.796
$p(i_{j,t} = 0)$	0.081	..	..	0.367	0.027
$p(-0.2 < i_{j,t} < 0)$	..	..	..	0.035	0.024
$p(-1 < i_{j,t} < -0.2)$	0.018	..	..	0.005	0.000

Note: $\mathrm{p}(\mathrm{i}_{\mathrm{j,t}} = 0)$ is defined as zero investment precisely in Chapter 4, and as investment rate less than 1% in absolute value in Cooper and Haltiwanger (2006) and in Chapter 5.

Table 5.7 A More General Specification

		No Discount Rate Effect			Discount Rate Effect		
Procedure		**two-step step 1**	**two-step step 2**	**one-step**	**two-step step 1**	**two-step step 2**	**one-step**
Estimates							
θ		**0.0000**		**0.0000**	0.1345		0.1311
b_q		1.3844		1.2659	1.3113		1.3712
b_i		0.1716		0.1632	0.2120		0.1730
b_f		0.0093		0.0053	0.0127		0.0158
β		0.0810		0.0787	0.0820		0.0816
ε		24.6869		23.3530	25.0966		24.6686
ρ		0.9496		0.9377	0.9493		0.9470
μ_μ		0.0192		0.0199	0.0188		0.0193
σ_μ		0.0381		0.0419	0.0368		0.0380
$\mu_{l\sigma}$		-0.4115		-0.4199	-0.3987		-0.4068
$\sigma_{l\sigma}$		0.2260		0.2429	0.2153		0.2208
σ_{IT}		0.5289		0.5242	0.5293		0.5324
σ_{IP}		0.2129		0.1993	0.2173		0.2125
σ_{YT}		0.0003		0.0009	0.0000		0.0000
σ_{YP}		0.5646		0.5625	0.5561		0.5547
τ			1.0000	0.9954		0.8843	0.8786
$\mu_{\bar{\zeta}}$			-16.8570	-16.7542		-17.2775	-17.2351
$\sigma_{\bar{\zeta}}$			1.1147	1.1062		1.1422	1.1482
Moments	**Empirical**	**Simulated**			**Simulated**		
$mean(PIY_{j,t})$	0.1108	0.1097		0.1106	0.1101		0.1097
$mean(i_{j,t})$	0.1246	0.1288		0.1293	0.1284		0.1292
$mean(dy_{j,t})$	0.0292	0.0192		0.0200	0.0188		0.0193
$mean(ky_{j,t})$	-0.7886	-0.7756		-0.7872	-0.7727		-0.7821
$mean(SDdy_j)$	0.1198	0.1183		0.1182	0.1186		0.1178
$sd(i_{j,t})$	0.1260	0.1183		0.1174	0.1191		0.1199
$sd(dy_{j,t})$	0.1337	0.1323		0.1327	0.1325		0.1319
$sd(ky_{j,t})$	0.5672	0.5863		0.5850	0.5876		0.5858
$sd(SDdy_j)$	0.0467	0.0467		0.0473	0.0469		0.0469
$corr(i_{j,t}, i_{j,t-1})$	0.3916	0.3913		0.3955	0.3898		0.3857
$corr(dy_{j,t}, dy_{j,t-1})$	0.1958	0.1136		0.1098	0.1146		0.1147
$corr(yk_{j,t}, yk_{j,t-1})$	0.9679	0.9753		0.9748	0.9752		0.9752
$corr(i_{j,t}, dy_{j,t})$	0.3971	0.4095		0.4172	0.4163		0.4168
$corr(i_{j,t}, yk_{j,t})$	0.1356	0.1906		0.1842	0.1917		0.1890
$corr(dy_{j,t}, yk_{j,t})$	0.1438	0.2173		0.2136	0.2189		0.2185
$corr(Eky_j, SDdy_j)$	-0.1021	0.0254		0.0297	-0.1169		-0.1194
$p(i_{j,t} > 0.2)$	0.1533	0.1860		0.1889	0.1845		0.1873
$p(\lvert i_{j,t}\rvert < 0.01)$	0.0266	0.0284		0.0287	0.0275		0.0276
$p(i_{j,t} < -0.01)$	0.0239	0.0230		0.0240	0.0230		0.0243
$mean(\widehat{K}_{j,t})$	0.0862		0.0658	0.0664		0.0693	0.0688
$mean(Y_{j,t})$	0.1591		0.1727	0.1735		0.1677	0.1675
$sd(\widehat{K}_{j,t})$	0.3780		0.2298	0.2267		0.2740	0.2737
$sd(Y_{j,t})$	0.5423		0.5925	0.5834		0.6074	0.6085
$corr(\widehat{K}_{j,t}, Y_{j,t})$	0.8760		0.8759	0.8663		0.8806	0.8796
$corr(E\widehat{K}_j, SDdy_j)$	-0.1031		-0.1003	-0.0911		-0.1175	-0.1163
$corr(EY_j, SDdy_j)$	-0.1153		-0.0977	-0.0885		-0.1086	-0.1071
OI		**111.3**	**4.4**	**117.9**	**103.4**	**2.9**	**107.7**
Degree of Freedom		**5**	**4**	**9**	**5**	**4**	**9**

Figure 5.1a: Empirical Distribution of Within-Group Standard Deviation of Sales Growth Rate

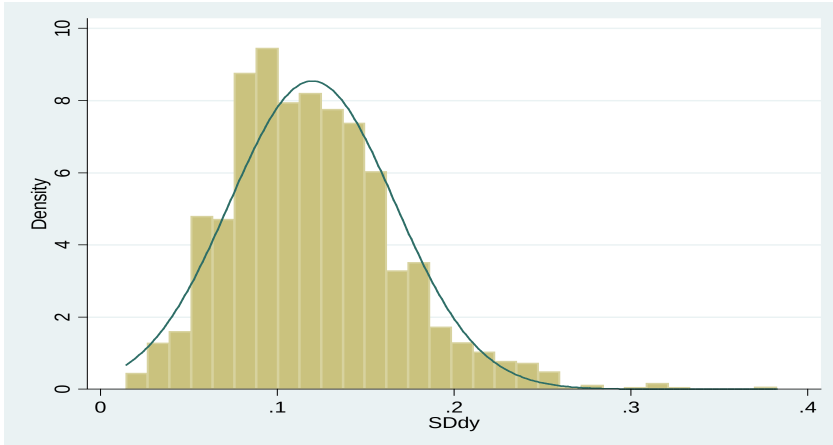


Figure 5.1b: Empirical Distribution of Within-Group Mean of Sales Growth Rate

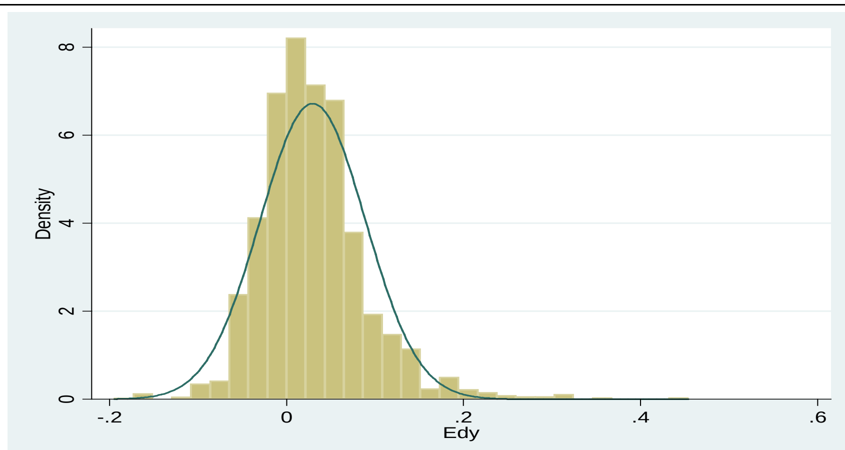


Figure 5.1c: Empirical Distribution of Investment Rate

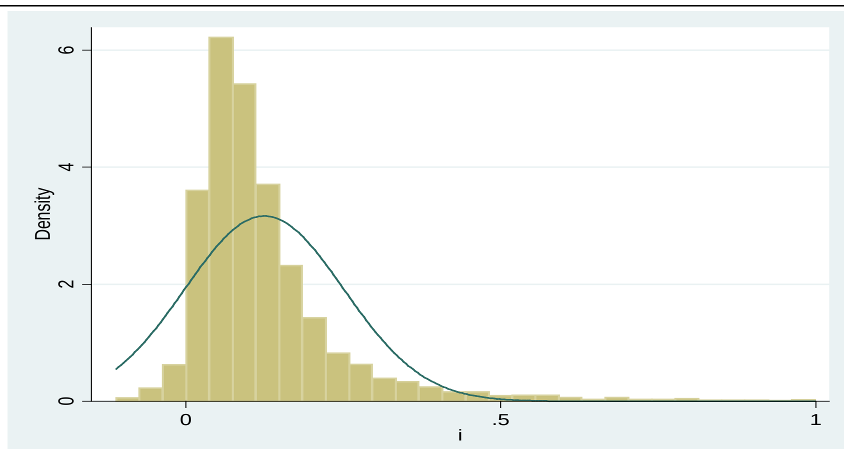


Figure 5.2a: Uncertainty-Capital Adjustment (θ = 0): Within-Sample

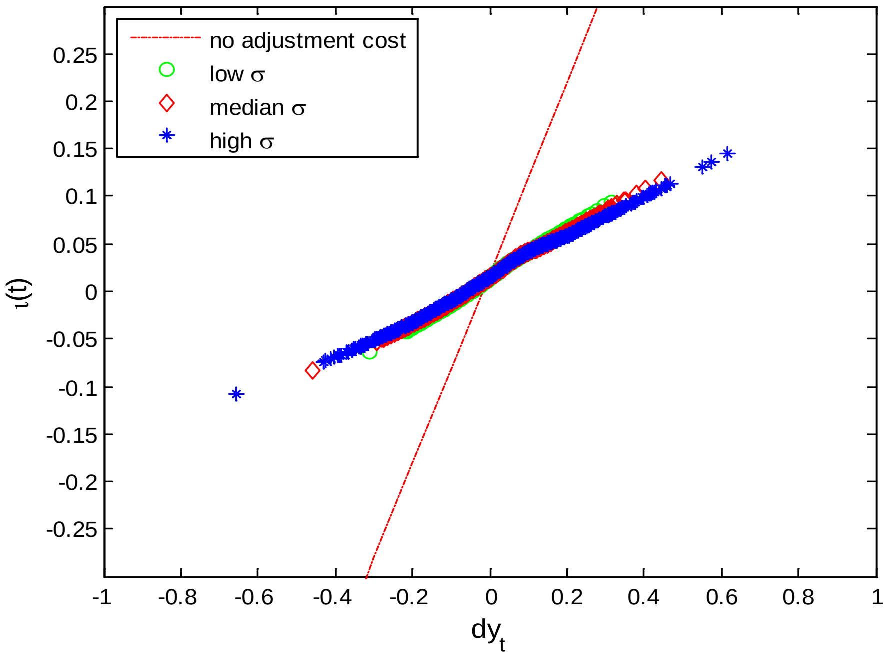


Figure 5.2b: Uncertainty-Capital Adjustment (θ = 0): Out-of-Sample

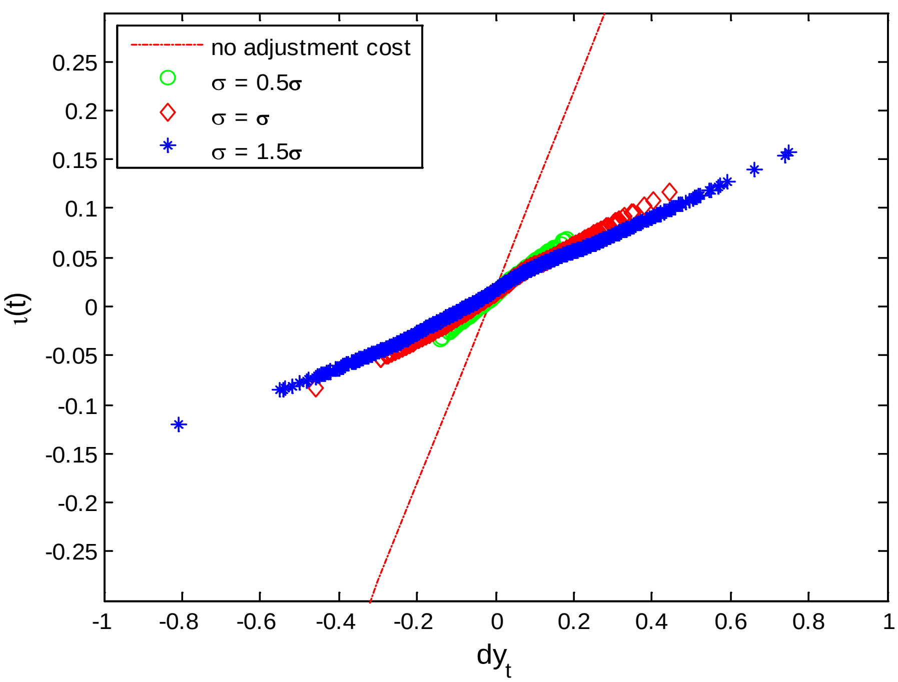


Figure 5.3a: Uncertainty-Capital Adjustment (θ > 0): Within-Sample

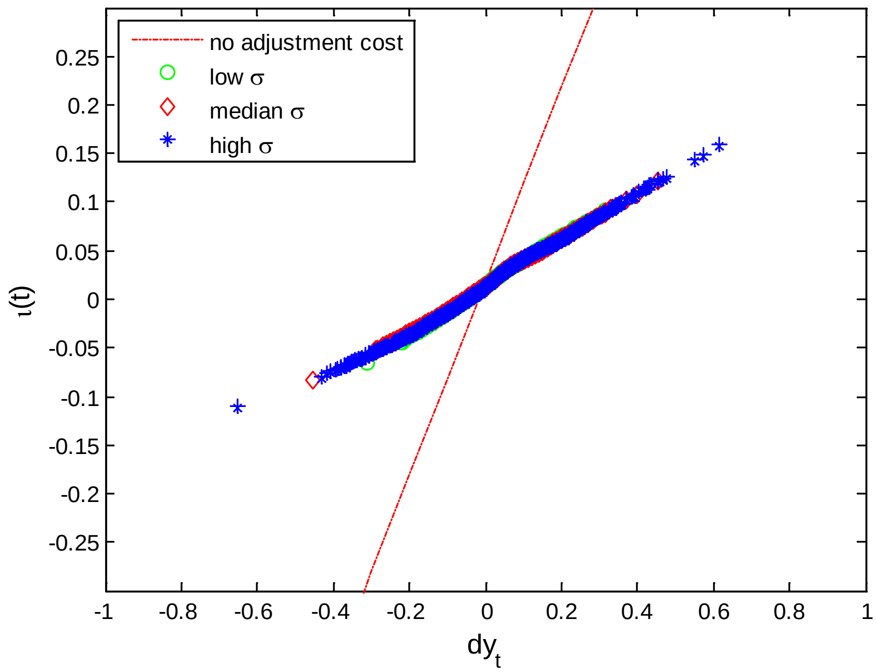


Figure 5.3b: Uncertainty-Capital Adjustment (θ > 0): Out-of-Sample

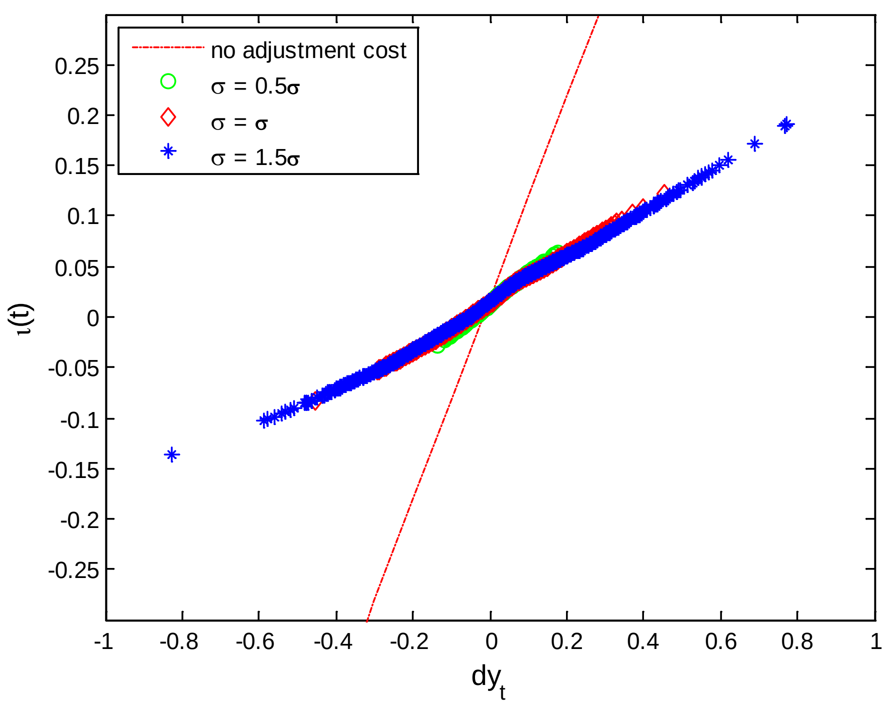


Figure 5.4a: Uncertainty-Capital Accumulation (θ = 0)

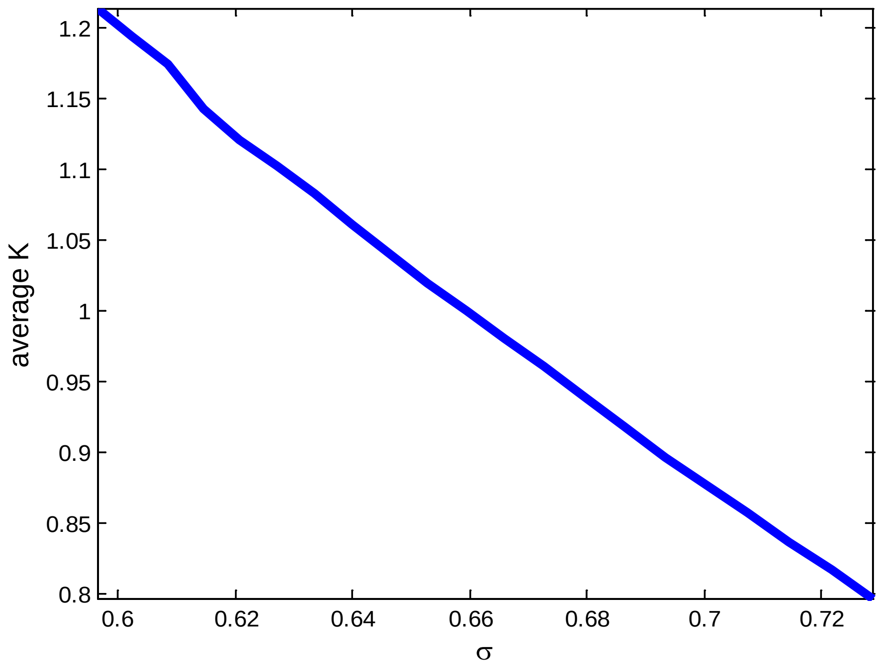


Figure 5.4b: Uncertainty-Capital Accumulation (θ >0)

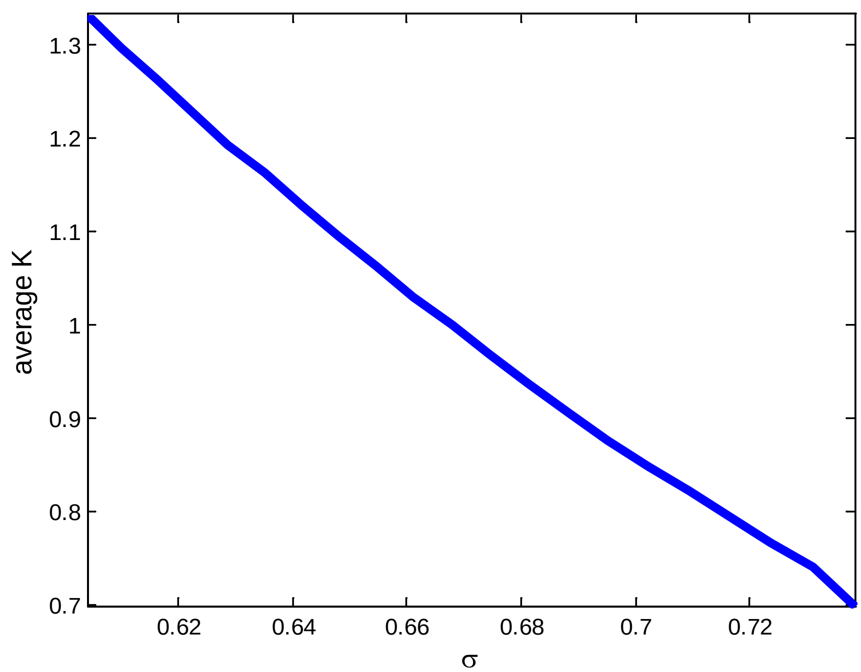


Figure 5.5a: Decomposition--Adjustment Cost Effect

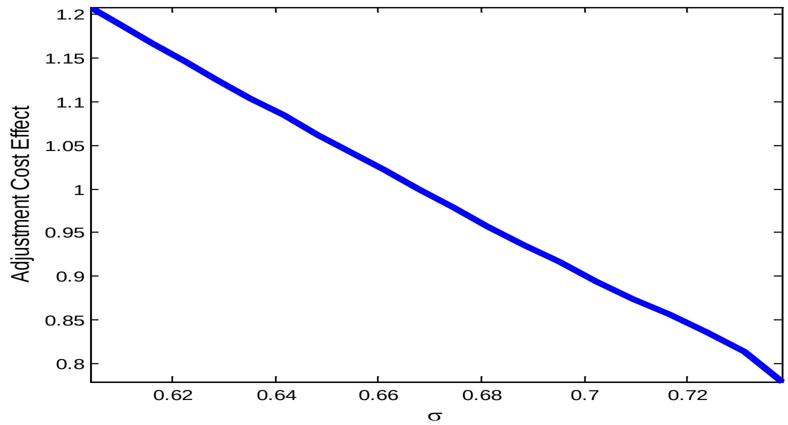


Figure 5.5b: Decomposition--Discount Rate Effect

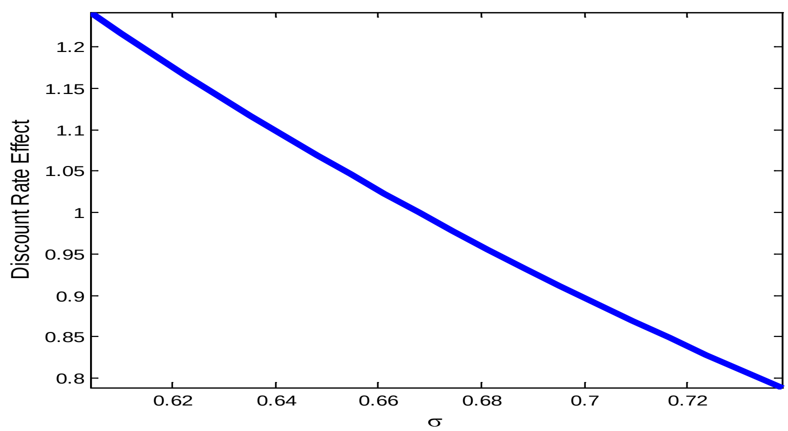
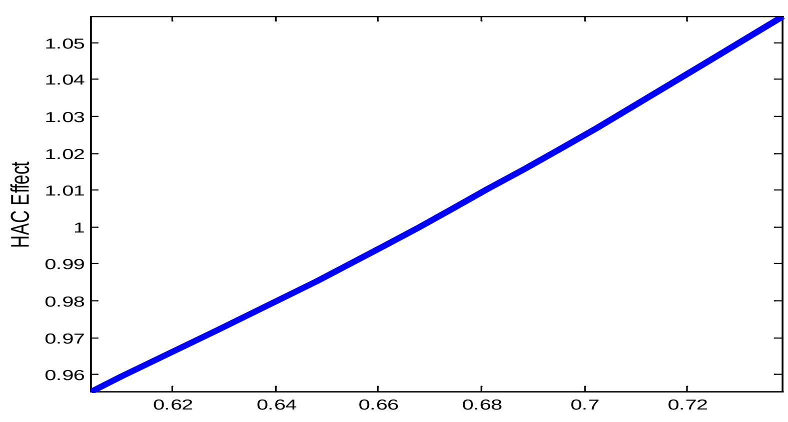


Figure 5.5c: Decomposition--HAC Effect



Chapter 6

Conclusion and Extension

1 Findings

As Abel and Eberly (1994) conclude after presenting a unified model of investment under uncertainty, 'The ultimate goal of this line of research is to derive an econometric specification to apply these models to aggregated and disaggregated data on investment'. This thesis adopts a structural econometric approach to estimate the effects of uncertainty on both short-run investment behavior and long-run capital accumulation at the firm-level.

Under the assumptions of risk-neutrality and perfect capital markets, the investment models specified in this thesis allow uncertainty to affect investment dynamics and capital accumulation through two main channels. First, the curvature of the operating profit function with respect to the variables that characterize uncertainty, summarized by the Hartman-Abel-Caballero (HAC) effect; and second, frictions in capital adjustment, summarized by different forms of capital adjustment costs.

Method of Simulated Moments is utilized to estimate the parameters of these investment models, by matching simulated moments from the model and empirical moments from panels of Brazilian and UK manufacturing firms. Compared to earlier models estimated in this literature, notably, Cooper and Haltiwanger (2006) and Bloom (2009), these specifications allow for considerable unobserved heterogeneities across firms. In Chapter 5, the specification estimated is the first in this literature to allow for the possibility of a HAC effect.

The empirical findings of this thesis can be summarized as below.

First, in both samples I find that an investment model with quadratic adjustment costs together with at least one form of non-convex adjustment costs (partial irreversibility or fixed adjustment costs), provides a reasonably good fit to the features of investment data at the firm level.

Second, regarding the effects of adjustment costs, the estimated investment models predict that these costs substantially dampen the response of investment to new information about profitability in the short-run, and substantially reduce the average capital stock level in the long-run. Both these effects are mainly driven by the presence of significant quadratic adjustment costs.

Third, regarding the effects of uncertainty, in the short-run, the estimated investment model in Chapter 5 predicts a small effect of uncertainty on investment dynamics. This is mainly due to the presence of quadratic adjustment costs, and is robust to whether the specification allows for an effect of uncertainty on discount rate or not. In the long-run, the mode suggests a negative and potentially large effect of uncertainty on capital accumulation. This operates through the negative effect of quadratic adjustment costs in a model without discount rate effect, or through a richer combination of effects in a model with discount rate effect.

Therefore, while much of the recent literature on investment and adjustment costs has emphasized the role of non-convex forms of adjustment costs, it is the presence of a significant convex component in the estimated adjustment cost functions that accounts for these main findings.

2 Limitations

The findings in this thesis are subject to several limitations.

First, the advantage of the structural econometric approach is to provide framework in which the relative importance of countervailing effects can be estimated. Since the effects of uncertainty work through different channels simultaneously and nonlinearly, it would be difficult to sign the effects of uncertainty and sort the relative importance of various channels in a reduced form econometric approach. The disadvantage of the structural econometric approach is that the choice of particular functional forms is potentially important, which could rule out some of the effects discussed in the theoretical literature. For example, although compared with similar studies in this line of research, I have allowed for the most general functional forms for production and technology, the investment model under estimation implies that any HAC effect on capital stock levels must be positive if the price elasticity of demand is smaller than 2 in absolute value.

Second, one stylized fact of firm-level investment data is the importance of unobserved heterogeneity across firms. The empirical specification proposed in this thesis allows for unobserved heterogeneities in several dimensions. First, cross-sectional heterogeneity in the level of uncertainty, which is a necessary condition for identifying the HAC effect; second, cross-sectional heterogeneity in the long-run growth rate of profitability, which will lead to permanent differences in both investment rates and capital stock levels; and finally, cross-sectional heterogeneity in firm size, given if long-run growth rates are common. Despite these richer specifications compared with the existing literature in this line of research, a crucial assumption is that the unobserved heterogeneity in growth rate or firm size is uncorrelated with differences in the level of uncertainty. The estimated effects of uncertainty are subject to the validity of this assumption.

Third, like most empirical research in the investment literature, the investment model in this thesis is explicitly partial equilibrium in nature. This allows me to model a rich structure for the stochastic process facing an ongoing firm and to study its investment behavior at given preference, technology and factor prices. In a general equilibrium framework, the analysis of uncertainty on capital accumulation should focus not only through the stochastic process, but also through preference, technology, relative factor prices and the entry and exit decision. The quantitative and possibly qualitative effects of uncertainty estimated in this thesis are therefore subject to these considerations.

3 Extension

Despite the general success of the investment model in matching firm-level investment data, within the framework of risk-neutrality and perfect capital markets, there are two features in the data that the neoclassical investment model does not fit or does not fit well. This motivates the discussion for two extensions in future research.

First, one important assumption of the thesis is that the discount rate of the firm r_j, remains unchanged when the level of uncertainty of the firm σ_j varies. As pointed out in Abel (1985), risk-neutrality *per se* is not required for the invariance of r_j with respect to σ_j. This is because in the traditional capital asset pricing model (CAPM), the discount rate of a firm is independent of the standard deviation of idiosyncratic risk if the rate of return on the firm is uncorrelated with the rate of return on the market portfolio. In the context of the more recent consumption-CAPM, the discount rate of the firm will be independent of the standard deviation of idiosyncratic risk if the rate of return on the firm is uncorrelated with the marginal utility of consumption. If the owner of the firm can fully diversify the idiosyncratic risk, his marginal utility of consumption will be uncorrelated with the return on the firm. Therefore, either risk-neutrality or the possibility of full diversification would imply the independence of r_j with respect to σ_j.

Under this assumption, as illustrated in Chapter 5, the benchmark investment model, in which uncertainty affects capital accumulation only through the HAC effect and the adjustment cost effect, cannot match the negative correlation between capital-to-sales ratio and a measure of uncertainty. In order to match this feature in the data, I allow for a possible effect of uncertainty on discount rates, and estimate a significant positive risk premium that varies across firms. Given that the productivity and demand shocks I model in this thesis are idiosyncratic, and that firms in Datastream are large publicly-traded firms in the UK stock market, this raises the interesting puzzle of why this risk-adjusted discount rate effect appears to be important in this sample.

This highlights the importance of relaxing the assumptions of risk-neutrality or full diversification, and estimating the model in a more generalized framework. However, in order to study the role of the discount rate effect, we either need a logically coherent theory which predicts a specific relationship between r_j with respect to σ_j, or need to consider other measures of risk in this context, such as those suggested by the asset pricing theories. For the former, the incomplete market literature, such as Angeletos and Calvet (2006), and the corporate finance literature, such as Himmelberg, Hubbard and Love (2002), offer the potential structural for such a model. For the later, the consumption-CAPM is the working horse model.

Second, the empirical results in both Chapters 4 and 5 indicate that the simulated cross correlations between investment rates, sales growth rates and a proxy for the marginal revenue product of capital all have the same sign as those in the empirical moments, but the magnitude is much larger than their empirical counterpart. Allowing for measurement errors in investment rate and sales attenuate these correlations, but cannot fully eliminate the discrepancy between the simulated and empirical moments. This indicates that the presence of the adjustment costs indeed dampens the responses of investment and sales to new information about profitability, but there may be other frictions which may further dampen their responses.

As highlighted elsewhere by the economic literature on investment, one of the possible frictions comes from the cost or difficulty of financing investment. Therefore, it is important to relax the perfect capital market assumption and study investment behavior in the presence of financing constraints. Recent papers by Hennessy and Whited (2007) and Bond, Soderbom and Wu (2007), have made a start in this direction, but impose comparatively simple forms of adjustment costs. A challenge for future work will be to develop a tractable specification that combines a rich specification of adjustment costs with financing frictions.

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