

From image to language: Teachers' collective articulation of spatial reasoning

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Spatial reasoning, the ability to visualise, manipulate, interpret and solve problems using images, is not only key to success in mathematics and other STEM subjects, but also essential in everyday life. Teachers' competence and confidence with spatial reasoning may shape both the extent and the effectiveness of the support they provide to develop these skills in their learners. Using a task-based clinical interview and the Discipline of Noticing, three pairs of secondary mathematics teachers were asked to visualise, describe, and explain a dynamic geometry animation, providing insight into their spatial reasoning. We report three emerging themes in how teachers adopted terminology, interpreted the dynamic nature of the animation, and moved between descriptive accounts and explanatory reasoning. This paper presents the first stage of a study aimed at advancing understanding of spatial reasoning by examining how it becomes visible through language.

Keywords: *Spatial reasoning, visualisation, teacher education, dynamic geometry*

Introduction

Spatial reasoning, the process of imagining, interpreting, and mentally manipulating spatial information, is a key skill for everyday life, underpinning activities such as interpreting maps, giving directions, describing relative position (e.g., under, next to), and using such information to navigate environments, park a car, or carry out multi-step tasks (e.g., loading a dishwasher) (Farran et al., 2024). It is also “at the heart of what it means to discover, learn, and communicate mathematics” (Gilligan-Lee et al., 2022 p.1). In mathematics, spatial reasoning is perhaps most readily associated with geometry, but it is a crucial tool for problem solving across the discipline, where visual representations such as graphs, diagrams, and counters are ubiquitous (Farran et al., 2024). This study investigates how spatial reasoning becomes observable as mathematics teachers collaboratively describe, interpret, and negotiate their understandings of a dynamic geometry animation.

Improving spatial reasoning should be a priority (Wai et al., 2009). Not only key to mathematics, spatial reasoning plays a critical role in developing expertise across STEM subjects, with good spatial skills strongly predicting performance in STEM fields (Uttal et al., 2013). Despite this, there is limited representation of spatial reasoning in school curricula (Farran et al., 2024). With international comparisons such as PISA and TIMSS highlighting UK students' relative weaknesses in shape and space domains (Ingram et al., 2023) and shape and space being given less emphasis in the latest iteration of the English Early Years Statutory Framework (Department for Education, 2025), research into spatial reasoning has never been more timely.

Spatial reasoning has been shown to be teachable, with almost all interventions to develop spatial skills being successful (Uttal et al., 2013). However, weaknesses in teachers' own spatial reasoning may affect whether and how effectively they encourage these skills in their students (Möhring &

Newcombe, 2025). Furthermore, “practitioners receive little instruction during pre-service training on the importance of spatial reasoning and how best to embed spatial activities into their practice” (Bates et al., 2023, p. 572). There is a notable lack of research on teachers’ spatial reasoning skills, with the limited existing work concentrating mainly on early childhood contexts (e.g. Farran et al., 2024). This study focuses on STEM specialists, mathematics teachers, who work with 11-18 year olds and are therefore likely to have stronger than average spatial reasoning skills (Wai et al., 2009), adding a new perspective to the literature which is focused on non-specialists who work with younger children. Investigating how mathematics practitioners talk about mathematical images can reveal ways of reasoning spatially, and subsequently, how to support students to develop these skills. This may also support understanding of reasoning in other domains, for example algebraic reasoning.

The aim of this study is threefold: to develop a tool for observing spatial reasoning through participants’ verbal accounts; to gain insight into different ways of reasoning spatially; and to make suggestions about how to address spatial reasoning in initial teacher education and continued professional development settings. We report here on the pilot stage of the study, presenting emerging themes in the ways secondary mathematics teachers describe and reason about a short animation made using dynamic geometry software.

Spatial reasoning

The National Research Council (2006) describes spatial *thinking* as being a “constructive amalgam of three elements: concepts of space, tools of representation, and processes of reasoning” (p. 3). Most authors, however, discuss spatial reasoning specifically in the context of geometry, relating it to shapes and their transformations. For example, Newcombe (2010) describes spatial thinking as “remembering the locations of objects and their shapes, and being able to mentally manipulate those shapes and track their paths as they move” (p. 33). This focus on manipulating mental images is echoed by Linn and Peterson (1985) who discuss spatial *abilities*, dividing these into spatial perception, mental rotation and spatial visualisation, Lowrie and Logan (2018) who used divisions of spatial orientation, mental rotation and spatial visualisation, and Gilligan-Lee et al. (2022) whose four categories are mental rotation, mental transformation, spatial scaling and perspective taking. Bates et al. (2023) present ten categories of spatial abilities, dividing these into two types: intrinsic (within object) and extrinsic (between objects). Uttal et al. (2013) discuss this same division, adding a second distinction between static and dynamic tasks.

The wide range of categories and descriptions of spatial reasoning demonstrates the complexity of the concept. A commonality amongst the categorisations is visualisation; all authors agree that picturing something in one’s mind is central to spatial reasoning. Spatial reasoning research is predominantly situated within cognitive psychology and typically assesses participants’ ability to mentally manipulate visualisations, asking them, for example, to mentally rotate objects or imagine them from different perspectives (Farran & Atkinson, 2016; Gilligan-Lee et al., 2022). This study takes a different approach: instead of asking participants to perform actions on visualisations, they were asked to remember, describe and interpret a geometry animation, articulating the mental images and spatial relationships they drew upon. Rather than testing a set of predefined spatial

reasoning skills, a data-driven approach was adopted to see whether different ways of reasoning spatially could be observed in participants' verbal accounts. Grounding the methods and analysis in Mason's (2002) Discipline of Noticing, participants were asked to give an account *of* the animation (describe what they saw), and account *for* it (attempt to make sense of it). We conceptualise spatial reasoning as occurring when participants draw on their visualisations to make sense of (Schoenfeld, 2016), interpret (Steenpaß & Steinbring, 2014), or make logical inferences about (Demiray & Koştur, 2026), the animation or its element. We conjecture that spatial reasoning is observable when participants attempt to articulate this sense-making (Macmillan et al., 2026).

Task based interview

Observing reasoning, something that happens within one's head, is challenging. This study employed a task-based interview drawing on the clinical interview technique, originally developed by Piaget and later adapted for research into mathematical thinking (Ginsburg, 1981). Ginsburg argues that this technique aims to "facilitate rich verbalization" by using open-ended questions, enabling the researcher to "discover" features of thought whilst shedding light on underlying processes (p. 5). It allows investigation into participants' spatial reasoning, with "a degree of insight that is virtually impossible to obtain through standard tests" (p. 5). Participants were interviewed in pairs. Working in pairs allowed them to discuss their thoughts, which, as Ginsburg notes, is a more natural forum for verbalising their thinking than monologuing.

The pairs watched a dynamic geometry animation developed by Dave Hewitt, demonstrating a construction for a square with the same area as a rectangle ("squaring the rectangle"). A stationary rectangle, with a point that moved left and right along its base edge, was central to the animation. As the point moved, shapes were revealed, removed, or emphasised. For example, in Figure 1, congruent shapes are emphasised with the use of colour, shading and bold lines. The participants were not permitted to draw or take notes, and the animation was removed before asking them to discuss it, requiring them to visualise then represent their visualisations with verbalisations.

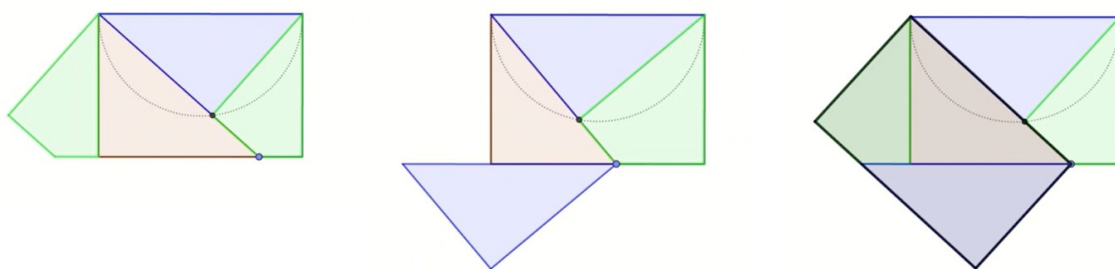


Figure 1: Example frames from the animation

The animation was broken into two parts, each around three minutes long. Drawing on Mason's (2002) Discipline of Noticing, participants were asked three questions after watching each part of the animation. First, they were asked to give an *account of* the image, describing what they saw. The question aimed to make visible what the participants were picturing: their visualisation skills. Second, they were asked to give an *account for* the image, to encourage them to draw conclusions, pose questions, explain and justify their thinking, providing insight into their spatial reasoning

skills. Finally, they were asked to reflect on the pedagogical decisions made in the construction of the dynamic image, encouraging them to notice what decisions were made, and conjecture why they may have been made, revealing how they connected their own spatial reasoning skills to teaching.

The conversation transcripts were analysed collaboratively by the research team to identify emerging themes that could inform the subsequent development of a coding framework for observing spatial reasoning. Consistent with Mason's (2002) Discipline of Noticing, the researchers read and discussed the transcripts multiple times, initially focusing on giving an account *of* each conversation before attempting to account *for* what was noticed. Through describing what we each noticed, drawing attention to specific excerpts or individual words, themes and patterns began to emerge, as did deviations from those themes and patterns. Whilst attempting to account *for* these observations, connections to spatial reasoning were conjectured and explored.

In this pilot stage, three pairs of teachers were interviewed, and their conversations were analysed, to explore how visualisations and reasoning were articulated, negotiated, and co-constructed through language in collaborative dialogue. We report here on three emerging themes in the participants' discussions: adoption of shared colloquial and invented language; interpretation of the dynamic nature of the animation; and interplay between description and interpretation.

Shared language

Relative position was a key feature in all descriptions with participants using a combination of formal and informal terms, for example "top left," "outside," and "touching." Similarly, participants used a range of formal and colloquial terms for geometrical features, for example using "vertex" and "corner," and "base edge" and "bottom line of the rectangle," interchangeably. Terms were invented and adopted with shared meaning, for example "the golden point" which one pair used to describe the moment in the animation where the square and rectangle had equal areas. Likewise, participants established shared meanings for geometrical terms, even when these departed from conventional mathematical usage. For example, one pair used the terms "similar" and "reflection" throughout the interview, whilst the context indicated they meant "congruent", and "translation":

- 2B Yeah. So for for the left side of the rectangle, then the rest of the left vertical line became reflection line for the green and brown shapes...and then the horizontal line at the bottom of the rectangle became the mirror line for the blue shape.
- 2A Yeah, yeah.
- 2B Which kept changing because the line kept changing. The line made the shapes.
- 2A So we ended up with that bottom dot moving and changing the proportion of the brown shaded, the blue shaded and the green shaded, and eventually we had that reflection line that you're saying with the brown, green and blue outside at different points.

Rather than treating mathematical vocabulary as a prerequisite for communicating spatial ideas, these interactions suggest that meaning was negotiated through dialogue, with participants establishing shared referents and locally stable interpretations that supported reasoning about the animation.

Dynamic image

Whilst all participants paid attention to shifts in position, size, and spatial configuration which they reported sequentially, they attended differently to the dynamic nature of the animation. For some, the continual movement was central to their descriptions, for example:

3A: Then when it came back to the left, a second point was introduced that followed the line as it was moving from left to right, but now followed it along the arc of the semicircle. It did the same iteration left-vertex right-vertex with that second line now. Then after that there are two more iterations where another layer was adding on.

For others, changes were reported chronologically as a series of static images, or described as if they were explaining the construction of a single stationary image:

1A: So I saw a rectangle first and then a semicircle inside of that ... I saw that the diameter of the circle is equal to the width of the box, and then the radius of the semicircle was the height of the rectangle rather than the box, um, and then I think they took a point on the base of the rectangle?

Where movement was foregrounded, participants used predominantly visual rather than technical language talking about points moving “back and forth,” lines “stretching,” squares getting “bigger” and “smaller,” and shapes “swinging.” In one case where movement was not a central feature of participants’ descriptions, they expressed confusion about the purpose of the movement, for example:

2A: I don’t understand why we had to, why it kept going back and forth, why it didn’t reveal to start with. Because if it’s just if the end result is just the the reflection shape, but the shapes are the same.

The different ways in which participants attended to movement suggests that the dynamic nature of the animation was not interpreted uniformly. For some, movement appeared to be an integral aspect of the mathematical object under consideration, with attention focused on the changing relationships between points, lines, and shapes over time. For others, the animation was treated primarily as a sequence of states leading to a final configuration, with movement functioning only as a means of revealing that configuration. This distinction is potentially significant in the context of dynamic geometry, where movement can be used not merely to display a figure but to represent generality through a range of possible cases and draw attention to invariant relationships. The findings therefore raise questions about the extent to which learners and teachers perceive movement itself as mathematically meaningful, rather than as a purely presentational feature of a visualisation.

Description and interpretation

We conceptualised participants to be reasoning spatially when they were drawing on their visualisations to attempt to make sense of elements of the animation, or the animation as a whole (Macmillan et al., in press). One way in which spatial reasoning became visible was when participants moved beyond description to infer properties or relationships, for example a triangle having a right-angle, or a pair of shapes being congruent. Whilst we anticipated the accounts *for* would provide most opportunity to observe spatial reasoning, moments of interpretation were more

prevalent whilst participants were being asked to give an account *of* the image, where they moved fluidly between description and inference, particularly when discussing congruence. Participants conjectured relationships between elements visible at a particular moment in the animation, and across different moments of the animation, for example:

3A: ...there was a point at which the new highlighted inscribed rectangle within the outer rectangle from animation one became a square. And there was, towards the end of the animation it showed when that becomes a square that the area inside the remaining shape also becomes a square, similar to how in animation one they showed in a bold black line when the two shapes were identical.

We anticipated that mathematics teachers would have stronger than average spatial reasoning skills (Wai et al., 2009). Although all pairs gave detailed accounts of the first part of the animation, the amount of detail, the features they attended to (Mason, 2002), and the accuracy of their descriptions varied both between participants and between the first and second accounts. In the second account, all groups foregrounded more structural (geometrical) features, placing greater emphasis on shapes, their properties, and relationships than on, for example, colour or the number of iterations of a movement. However, the second account *of* contained less descriptive detail. One explanation for this reduction in descriptive detail is that participants assumed a shared understanding, an interpretation supported by the development of shared language. We conjecture, however, that this difference may also be related to the extent of participants' sense making. Where participants' accounts suggested that they had made limited sense of the animation, their second accounts *of* were more tentative, and contained inaccuracies. In contrast, where participants' discussions indicated that they had developed a coherent interpretation, description gave way to inference. In these cases, rather than providing an account from which an observer could visualise the animation, participants foregrounded the spatial and mathematical relationships they perceived, as illustrated in the following excerpt:

3A: By the end of it, we saw some congruent triangles that related to the areas of the shapes, both inside the original shape and now being constructed outside by the lines that were moving. ... And it indicated that that the new areas created the same rectangle as the original one

More substantial changes occurred when participants moved from giving an account *of* the image to accounting *for* it. The accounts *for* were significantly shorter than the accounts *of*, both in terms of time and number of words uttered, and less developed in that each participant spoke fewer times, meaning utterances functioned as statements rather than chains of an argument. All three pairs framed the animation as a theorem-seeking exercise, assuming there was a geometrical generalisation to be made. However, in their accounts *for*, instead of attempting to build on their accounts *of*, all participants initially focused on one of the triangles, asserting that it had a right angle, a property no participants had previously mentioned. They all conjectured that the animation may have been related to Pythagoras' Theorem, an area of school curricula where right-triangles are commonly seen. This triangle was inscribed inside the semicircle, as can be seen in Figure 1, and the right-angle was either assumed from its appearance, or inferred from their prior knowledge of circle theorems. Participants stopped referring to specific features of the animation and began discussing right-triangles generically, drawing on prior mathematical knowledge to conjecture what

the intention of the animation might have been. All pairs demonstrated epistemic vulnerability in their accounts *for*, expressing confusion or frustration when they made comments such as “I feel like I should know,” “Is this university level?”, “How smart do I need to be?” and “Without context it’s hard.”

Our findings that spatial reasoning emerged during the accounting *of* phase and appeared to be curtailed during the accounting *for* phase raises further questions. First, might learners respond differently to mathematics teachers because they are accustomed to encountering unfamiliar mathematics? Second, might epistemic vulnerability have inhibited participants’ spatial reasoning? These questions have implications for classroom practice and teacher education.

Conclusion

Observing spatial reasoning is challenging as it happens in one’s head. Communicating that reasoning to others, even when the participants have had a shared experience of a dynamic image, and, as mathematics teachers, have experience and expertise in mathematics and communication, is also challenging. Using a task-based clinical interview where secondary mathematics teachers were invited to give *accounts of* and *accounts for* a geometry animation, we were able to observe how participants co-constructed language, interpreted the movement of the animation differently, and moved between description and interpretation. Spatial reasoning became visible when participants moved from descriptive to interpretive statements, inferring properties of geometric components of the animation, and articulating underlying relationships both between geometric components, and between sections of the animation. Moments of inference were more common during the descriptive phase of the interviews and when asked to account *for* it, participants focused on a particular component, attempting to match it to school level mathematics they were familiar with, rather than continue to draw on their visualisations of the whole animation. When this happened, conversations were less developed and ended quickly, with participants expressing confusion and frustration.

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