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A note on the variation in shape of linear rogue waves in the ocean

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Abstract

The average shape of large waves in the open ocean made up of linear waves with random phase is given by the ‘NewWave’ defined by the auto-correlation function. In this note we confirm this result for realistic directionally spread sea-states but show that there is a considerable variation in shape of rogue waves. The practical implications of this are discussed.

Introduction

In many locations around the world ocean waves are the dominant load on ships and offshore structures. Understanding the dynamics of extreme waves is therefore vital for sound engineering design. The simplest model of extreme waves is given by linear potential flow theory, where an extreme wave is assumed to form when individual wave components with random phase, come into phase at a particular point. There is no consensus as to whether, for realistic ocean waves, there is a need to go beyond linear theory for modelling the short-term underlying wave-dynamics (it is theoretically straightforward to correct linear waves to second order as this does not modify the propagation of the free waves). However, even if linear theory is not sufficient for understanding the dynamics of the largest waves in steep sea-states, it remains a useful starting point for examining the dynamics and formation of extreme events – indeed this analysis is necessary to determine whether non-linear physics may be important to wave-group dynamics of the largest waves (Adcock and Taylor, 2014).

There is a large body of work analysing the expected shape of a large wave in a random linear wavefield. This started with the theoretical work of Lindgren (1970) and Boccotti (1983). This was developed for offshore engineering practice by Tromans *et al.* (1991) where it has been called ‘NewWave’ and found to be in good agreement with field measurements (Phillips *et al.*, 1993; Jonathan and Taylor, 1997). NewWave has been shown to be a good model for waves right up the point where waves start to break Haniffah (2013). The NewWave model has been shown to be in agreement with simulations of random linear waves by Tucker (1999). Presumably because of computational demands, Tucker only considered uni-directional waves in his analysis.

This paper is a straightforward extension of Tucker’s work to realistic directional spread waves. The first objective of the paper was to show that the most probable shape of a directionally spread extreme wave is approximately a NewWave since this has only been considered briefly by Forristall (2006). The second objective was to examine how much variation there is from the the expected profile for the size of extreme waves which are of practical interest to scientists and engineers (roughly where the linear crest height is greater than the significant waveheight). Whilst Lindgren (1970) does give an analytical result for variance in shape around a NewWave we investigate the variation in shape using simulations of random waves.

We should perhaps comment on why the variations in the shape of extreme waves are important as this may not be obvious. The most studied form of non-linearity which has been suggested may influence the dynamics of extreme waves is the Benjamin-Feir instability (Benjamin and Feir, 1967). This is an instability which occurs to regular waves in deep water which makes them unstable to small perturbations and causes energy to build up at one location in the wave-train. This instability is inhibited by increasing spectral bandwidth in both mean wave direction, and lateral directions (Janssen and Bidlot, 2009; Gibson and Swan, 2007). However, what is crucial is not the underlying bandwidth of the sea-state, but the localised bandwidth around the extreme wave-group. To put this in a different, slightly crude way, suppose we examine, say, 20 rogue waves which linear theory would predict would have crest height as H_s . All these waves will have different crest widths and different waves

preceding and following it. If it is probable that just one of these will have the local properties which will lead to a strong non-linear change to the dynamics of the wave then this is worth considering in our analysis of extreme sea-states and how we investigate these. Equally, if it is extremely improbable that an extreme wave can form with localised properties that will trigger an instability, this should be appreciated. Thus it is useful to know how much variation in the shape of an extreme wave-group we might expect in waves over a given threshold.

Method

Numerical simulations were carried out using the same general approach as Tucker *et al.* (1984); Tucker (1999) using a method adapted slightly from that appearing in Goda (2010). Care was taken that crest statistics at a point followed the theoretical Rayleigh distribution and that the most probable wave time-history at a point was given by NewWave (consistent with Tucker (1999) and presented in Adcock and Taylor (2014)).

We chose to simulate waves over an area of 800 m by 4000 m. The resolution in the mean wave direction was 5 m and in the lateral direction 15 m. Each simulation lasted for 1 hour. In each simulation we analysed the largest wave in the record. If the maximum of this wave was not within a central square 310 m by 2805 m the simulation was discarded. If the maximum crest did fall in this region the free surface of the surrounding area was recorded. We base our analysis on 3000 simulations, which leaves us approximately 800 large waves to analyse. Of the waves analysed in this study all had crest heights greater than $1.15H_s$ with the average crest height being $\sim 1.4H_s$. As such it is reasonable to class all the waves in this study as rogue waves.

We confine our investigation to waves on deep water (i.e. propagation is governed by the deepwater linear dispersion relation). We analysed two cases both of which have a JONSWAP spectrum but with differing peak enhancement factors of $\gamma = 1$ and $\gamma = 3$. 3 and frequency resolution of $2.777 \times 10^{-4} \text{ s}^{-1}$. We used a simple wrapped normal

directional spreading with $s = 15^\circ$ across all frequencies. The peak frequency chosen is $T_p = 10$ s.

Results

Comparison with NewWave

The expected shape of an extreme wave event is given by the NewWave. In a directional sea-state this is

$$\eta(x, y) = \frac{A \sum_n \sum_m S(k_{x,n}, k_{x,n}) \cos(k_{x,n}x + k_{x,n}y)}{\sum_n \sum_m S(k_{x,n}, k_{x,n})},$$

where S is the spectral density function and A is the desired amplitude at the crest.

Figure 1a and 1b show the contours of the NewWave wavegroup for the two spectra considered in this study. It can be seen that the group with the narrower spectral bandwidth ($\gamma = 3.3$) has deeper troughs, and larger crests, preceding and following it.

The NewWave prediction is compared with the average shape found in our random wave simulations in Figure 1. There are two scenarios that we are interested in: the average shape around a the largest crest at any spatial or temporal point in our simulation and the average shape of the largest crest at a given point in our simulations. Due to the detailed computational method used, in the former case we had approximately 800 extreme waves to analyse, most of which had a crest greater than 1.2Hs. In the latter case, we analysed 5000 extreme wave shapes. These had a rather smaller crest size with a mean crest height of approximately 0.9Hs. In both analyses we introduced thresholds and only analysed waves larger than these, but this produced no significant difference in the results and so the average shape of all the waves in the record is used here to give the smoothest results. The waves were normalised by crest amplitude at $x = 0, y = 0$.

There is good general agreement between the NewWave group and the shape of the extreme waves. As expected, the agreement in shape is rather better when the spatio/temporal extreme shape is considered. When the extreme is around an arbitrary

point, which is obviously not necessarily at the highest crest of the wave, then the average shape is unsurprisingly broader in the lateral direction.

The variation between the average shape of the spatio/ temporal extreme wave and the NewWave is examined further in Figure 2. This Figure shows the variation in the mean wave direction through the crest of the extreme wave. Also shown are the bounds which 95% of the simulated extreme waves fall between.

There is clearly an excellent general agreement between the NewWave shape and the average shape of the waves in our simulations. The spatial location of the trough and peak on either side of the extreme crest is very accurately predicted. For the waves considered in this study, which are of the order of the ones of most practical interest, the NewWave slightly under-predicts the magnitude of the peaks and troughs either side of the extreme crest. These results are consistent with Tucker's study of uni-directional waves.

Variation from NewWave

Figure 2 shows the range of wave shapes within which 95% of our simulations fall between along the mean wave direction through the line of the maximum crest. There is clearly considerable variation in the shape of an extreme wave for both spectra analysed.

It is impossible to find a single parameter which captures the variation in the shape of an extreme wave-group. However, one approach is to examine the local bandwidths of the wave-group envelopes (this follows the work on these bandwidths for isolated wave-groups by Gibbs and Taylor (2005)). As discussed above, this is important for determining whether a rogue wave will occur. Probably the simplest parameter describing the local non-linearity is the amplitude to bandwidth ratio Adcock (2009) which is of the form

$$a^2 k_0^4 / s_x s_y,$$

where a is the amplitude of the crest and s_x and s_y the bandwidths of the wave-group in mean-wave and lateral directions. The characteristic wavenumber k_0 is somewhat

ambiguous to define and here we use a value corresponding to the mean frequency of the spectrum. This parameter is effectively a localised version of the Benjamin-Feir index, which for uni-directional waves was introduced in the seminal paper by Janssen (2003).

Whilst this parameter is useful for illustrating the properties of wave-groups, it is somewhat difficult to calculate a definitive local bandwidth. For instance Gibbs and Taylor (2005) used a very localised value and calculated this from the gradient of the wave and its Hilbert transform at the wave-crest (see also Gibbs (2004)). An alternative approach is to fit a Gaussian to the spatial wavegroup over a characteristic area around the crest (e.g. Adcock *et al.* (2012)). This latter approach is taken here as it captures the properties of the wave-group over a greater area than simply examining the peak. However, the value is sensitive to the area over which it is fitted. In this study we have chosen to present estimates of the amplitude to bandwidth parameter by fitting a Gaussian in the mean-wave and lateral directions over a distance of 120 m and 180 m respectively (i.e. over roughly 1 characteristic wavelength). Note that whilst the numerical values are different if we fit over different areas the general conclusion we draw from these results is unchanged.

Figure 3 presents the amplitude to bandwidth parameter for each of the wave crests in our study. We also show the amplitude to bandwidth parameter of a NewWave wavegroup for each spectrum. There is clearly considerable scatter in the amplitude to bandwidth parameter.

In this instance, the spectrum with $\gamma = 1$ appears to have more non-linear groups than the narrower banded $\gamma = 3.3$. This is slightly misleading as this difference is due to the difference in k_0 values of the two spectra. Rather more important than the actual numbers on the y-axis is that there is clearly a considerable scatter in the bandwidths of different extreme wave-groups. The data runs through the line predicted by the NewWave wave-group, which is slightly more non-linear than the average wave-group for a crest of a given amplitude.

Figure 4 demonstrates qualitatively the variation in the shape of the wave-group associated with extreme waves. This figure shows surface and contour plots of the

highest and lowest bandwidth groups from the $\gamma = 3.3$ simulations. In this instance the group with the larger bandwidth ratio is a higher wave (amplitude $1.47H_s$).

Discussion

This paper demonstrates that although the NewWave is an accurate representation of the average shape of an extreme wave, there is considerable variation in the shape of extreme waves at the probability levels that are of practical interest to scientists and engineers. It is worth discussing the consequences of this for practical engineering and scientific applications.

One application of NewWave is providing a target focused wave-group for investigating wave-structure interactions in a physical or numerical wave-tank. In general, the extreme loading or structural response at a given probability level is likely to be given by a wave profile which is significantly different from the NewWave. Nevertheless, the NewWave approach remains extremely useful for avoiding the need to run costly random wave simulations and indeed it is usually straightforward to examine the sensitivity in the loading to small changes in the form of the wave-group. Using a constrained NewWave (e.g. Taylor *et al.* (1997)) is a straightforward and efficient way of examining the variations in response due to slightly different shapes of extreme wave without having to simulate fully random waves.

A problem for which the spatial shape of extreme waves is key is the inundation on the decks of offshore platforms. Forristall (2006) argues that even if an extreme wave contacts a deck, the forces are often still acceptable as the area of the deck inundated may due to the shape of the wave. He bases his analysis of this on the average shape of an extreme crest (i.e. NewWave). Whilst this is a good first approximation it is clear that some waves may have a much broader crest than the average, increasing the loading on the deck.

An alternative application of NewWave wave-groups is in investigating the non-linear dynamics of extreme waves (Baldock *et al.*, 1996; Johannessen

and Swan, 2001; Adcock and Taylor, 2009; Yan and Ma, 2010). This remains an excellent way to investigate the physics of such waves but clearly cannot be used to provide quantitative guidance on the non-linear dynamics of an extreme wave-group of a given height as this will always be strongly dependent on the local shape of the wave-group. There is also a large variation in the second order characteristics of extreme waves due to the randomness in their shape which is discussed at length in Adcock and Draper (2015).

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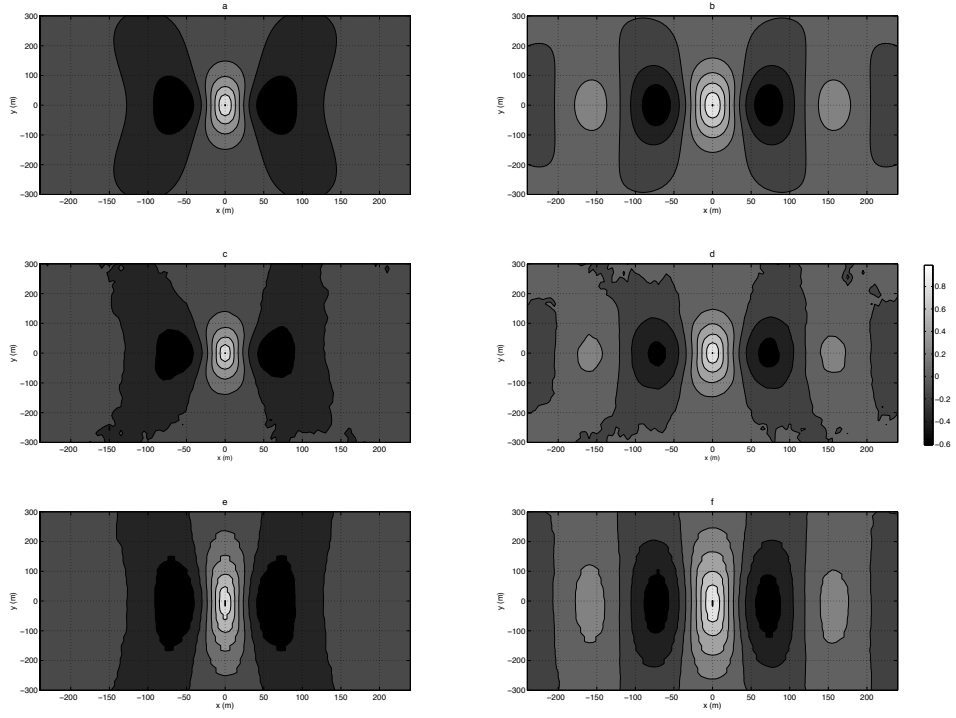


Figure 1 The shape of extreme wave-groups. Left (a, c, e) — $\gamma = 1$; right (b, d, f) — $\gamma = 3.3$ Top row (a, b) — NewWave prediction; Middle row (c, d) — average shape from simulations around a maximum in space and time; Bottom row (e, f) — average shape from simulations around a temporal maximum at an arbitrary fixed point.

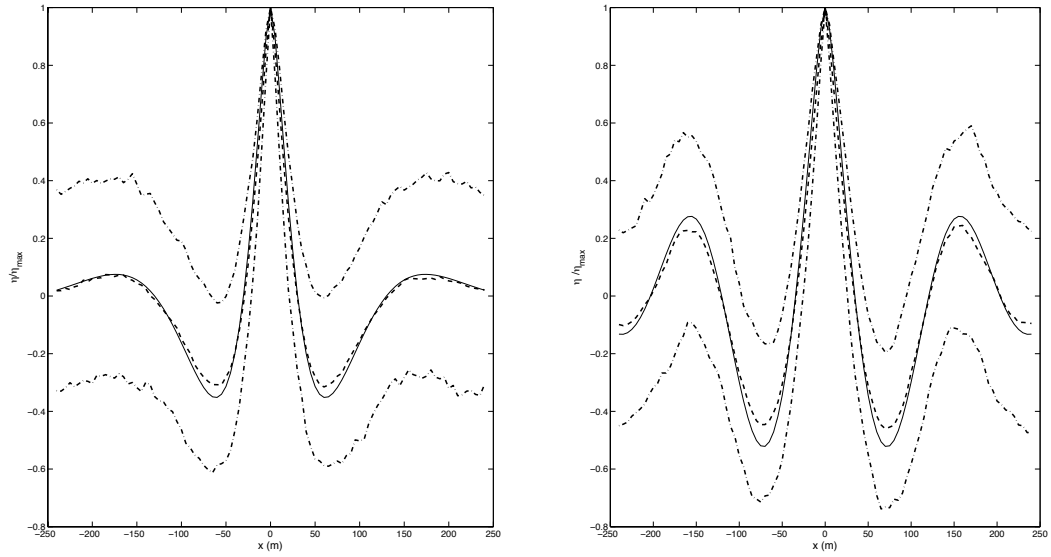


Figure 2 The shape of extreme wave-groups in the mean wave direction. Left — $\gamma = 1$; right — $\gamma = 3.3$. Thin continuous line NewWave profile. Thick dashed line — average wave shape of simulations; thick dot/dash lines within which 95% of the simulations fall between.

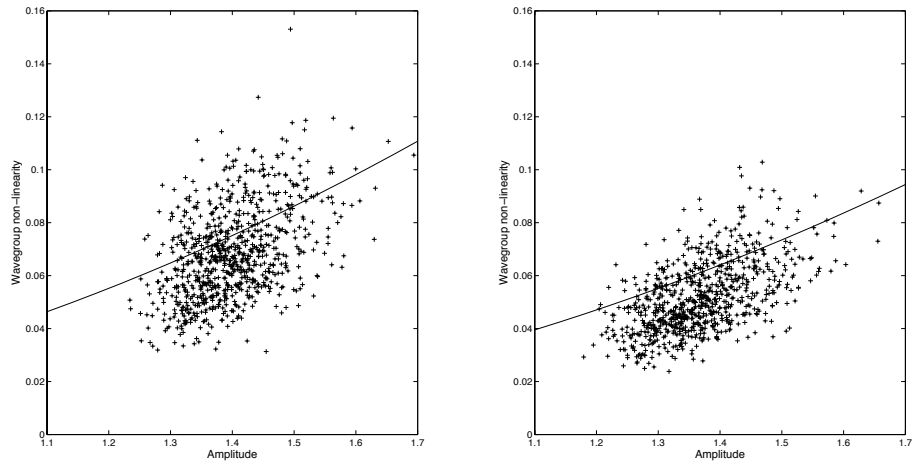


Figure 3 The amplitude to bandwidth ratio of each wave in simulations shown with cross. Left $\gamma = 1$; right $\gamma = 3.3$. Amplitude to bandwidth ratio of equivalent amplitude NewWave is shown with a continuous line. Amplitude normalised by H_s .

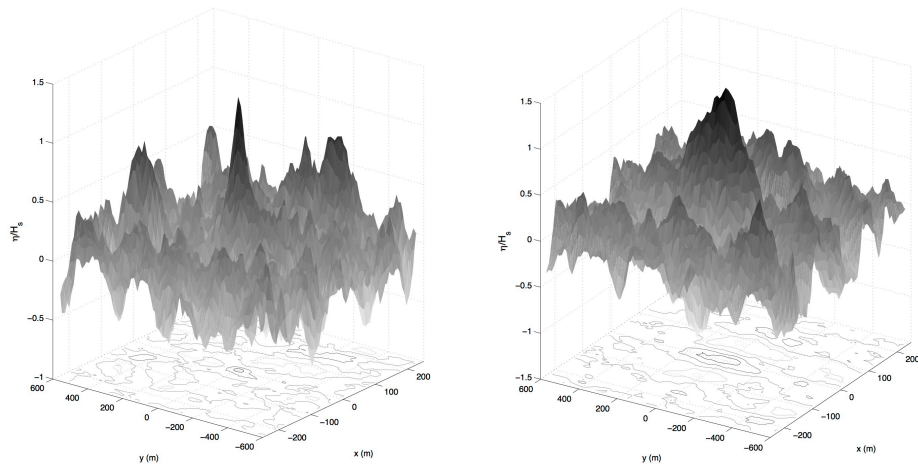


Figure 4 Examples of wave-groups with low (left) and high (right) amplitude to bandwidth ratios. Examples are extreme cases from $\gamma = 3.3$ simulations.