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Higher education funding, welfare and inequality in equilibrium

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Abstract

This paper analyses theoretically and quantitatively the effect that different higher education funding policies have on welfare (on aggregate and at the individual level) and wealth inequality. A heterogeneous agent model in continuous time, which has uninsurable income risk and endogenous educational choice is used to evaluate five different higher education financing schemes. Educational investments can be self financed, supported by government guaranteed student loans - that may come with or without income contingent support - or be covered by the public sector. When educational costs are small, differences in outcomes amongst systems are negligible. On the other hand, when these costs rise to realistic levels we see that there can be large gains in welfare and significant drops in inequality by moving to a system with more public sector support. This support can come in the form of tuition subsidies and/or income contingent student loans. However, as the cost of education and the share of debtors in society gets larger, it is preferable to increase public support in the form of tuition subsidies. The reason is that there is a pecuniary externality of debt that gets magnified when student loans become excessive. While I identify large steady state welfare gains from more public sector financing, I show that transition costs can be large enough to justify the status quo.

JEL codes: D52, D58, E24, I22, I23

Keywords: Incomplete markets, Higher education funding, Human capital

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Introduction

Student debt is now the second largest type of household debt in the United States, recently surpassing 1.6 trillion dollars. As shown in Figure (1), the average student at an American university is graduating with over \$34,000 of debt and the stock of student debt, which continues to grow, recently reached 8% of all personal disposable income. While the United States is usually held as a basket case, the United Kingdom is not fairing any better. According to the Institute of Fiscal Studies and the Sutton Trust, the average UK student graduates with over £44,000 worth of debt - Kirby (2016). The rising costs of higher education, student debt defaults in the US and recent modifications of the UK government accounting of student loans have continued to exacerbate calls from the left in favour of either student loan debt forgiveness and/or free tuition at public universities. Those opposing such policies argue that they are regressive. Since the benefits of higher education accrue to the individual pursuing a college degree, while the costs are shared amongst tax payers, many of whom who do not enjoy such benefit, these policies might actually make matters worse (for instance, by reinforcing inequality).

Income contingent student loans have been proposed as an efficient solution for financing tertiary education. They increase access to higher education for low income households by reducing the capital market imperfection in educational investments and lessening income uncertainty with protections covering for bad shocks. The leading proponents for financing higher education with income contingent student loans argue that such a system is the best suited at balancing equity and efficiency trade-offs, is the ‘most efficient’ and that ‘tax funding (of higher education) is unfair’ - Barr and Crawford (2000)¹.

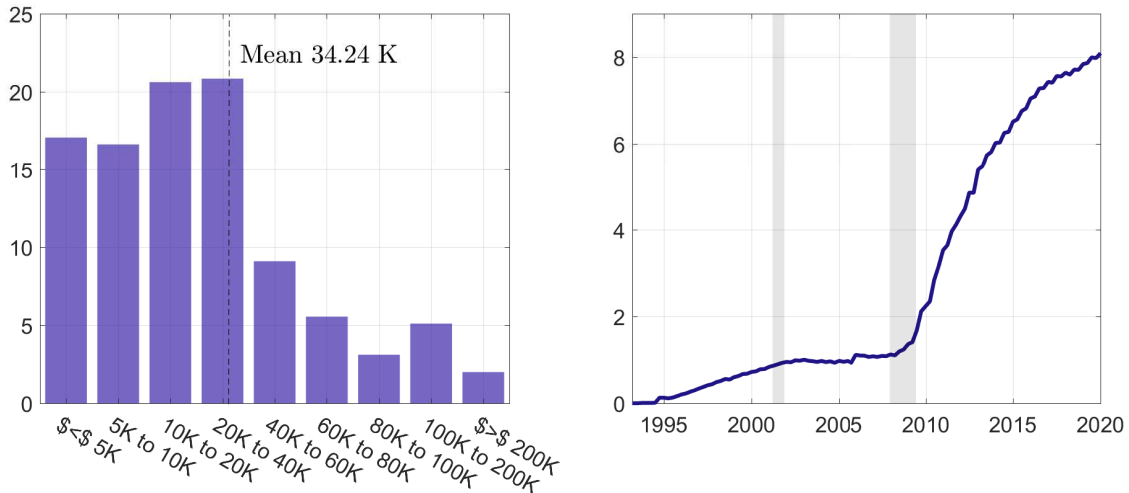


Figure 1: Left: % of borrowers and student loan balances at the 2nd quarter of 2020. Source: U.S. Department of Education. Right: Federal student debt as a percentage of disposable personal income. Source: BEA and Board of Governors

There are considerable reasons to ask if this should be the preferred way to finance tertiary education. First, while there seems to be a consensus, undisputed in some policy circles, on financing higher education with income contingent loans, there is no unique and preferred policy for financing higher education in the OECD². In fact there is plenty of variability, as depicted

¹ ‘(income contingent student debt) is efficient, in that it addresses the major capital market imperfection... It is fair, because people with low earnings make low repayments and people with low lifetime earnings do not repay their loan in full... tax funding (of higher education) is unfair’.

² The same can be said of economists working on this field of research. There is no consensus on which is the best way to finance higher education. Even in the small subset of the literature cited further below we may find that either graduate taxes, tuition subsidies, merit grants or income contingent student loans can be found to be

in Figure (2). The bars represent the share of GDP allocated to tertiary education. The red (dark blue) part captures the share of that expenditure coming from the public (private) sector. South Korea, Japan and anglophone countries fund tertiary education with a relatively higher participation of the private sector. Continental European countries, especially Nordic ones, have the state playing a larger role in financing higher education. Second, contrary to popular perceptions of generous tax financed tertiary education, it appears that larger public spending in higher education, relative to GDP, is associated with lower income inequality in the OECD (see Figure (27) in the appendix). Finally, a set of papers in heterogeneous agent macroeconomic models have shown that agents' savings behaviour may generate pecuniary externalities that can steer away the economy from efficiency - [Aiyagari \(1994\)](#), [Obiols-Homs \(2011\)](#), [Dávila et al. \(2012\)](#), [Nuño and Moll \(2017\)](#) and [Angelopoulos et al. \(2017\)](#). It is not clear a priori if a system of higher education relying on student loans, tuition subsidies or on private self-financing may exacerbate the aforementioned externalities by pushing society to under/over accumulate human and physical capital.

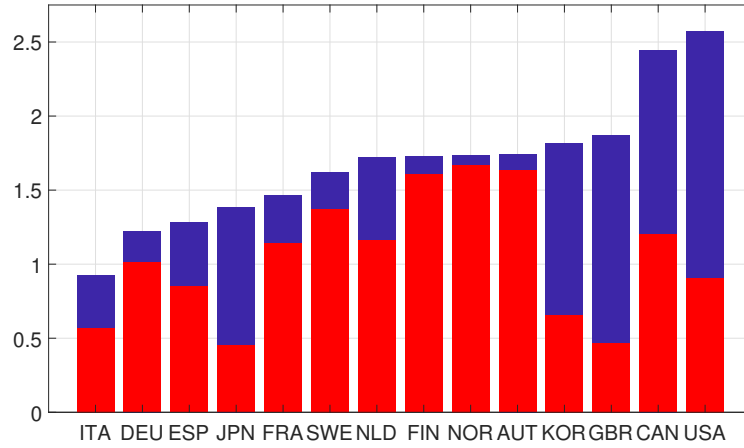


Figure 2: Public and private expenditure on tertiary education relative to GDP in 2015. Source: OECD

In this paper I propose a framework to evaluate the welfare and wealth inequality outcomes of five different higher education financing schemes. I use a heterogeneous agent production economy in continuous time, following [Nuño and Moll \(2017\)](#), extended to allow endogenous educational choices. In the first scheme, called self financing, there is no access to student loans nor tuition subsidies from the government. Only agents with sufficient wealth can afford education. In the next regime the government provides a student loan facility without income contingency features, i.e. agents must pay back their student loans regardless of their income. I then introduce two variants that offer income contingency protections; this is done to highlight how small tweaks in the design of the income contingent student loan program can generate significantly different outcomes. Finally, in a fifth regime the government provides support in the form of tuition subsidies.

This paper highlights the importance of assessing the macroeconomic impact of higher education financing under the light of the price effects of debt described in [Obiols-Homs \(2011\)](#)³; thus making the link between borrowing limits and welfare with higher education financing.

the preferred policy recommendation.

³[Obiols-Homs \(2011\)](#), shows that too lax borrowing constraints may drag down aggregate welfare. When society has a large fraction of net debtors, the beneficial *quantity effect* of large debt limits (because individuals can continue to optimise and smooth consumption with debt), can be overwhelmed by the *price effect* of more debtors putting upward pressure on the interest rate.

The main finding of this paper is that there is a pecuniary externality of debt that manifests itself through the student loan system and becomes more patent as the cost of education rises. When education is relatively easy to achieve, the capital market failures associated with educational investments do not matter enough to warrant government intervention. When the costs of education are calibrated to realistic values, government guaranteed income contingent loans and tuition subsidies provide the best alternatives to finance tertiary education, with the latter yielding the largest welfare gains and drops in wealth inequality. However, as the cost of education and the share of debtors in society gets larger, it becomes much more preferable to increase public support in the form of tuition subsidies. This is particularly important, since tuition costs have been rising in many countries. For instance, these costs have grown consistently faster than CPI, housing and healthcare in the United States - see Figure (28) in the appendix.

By using partial/general equilibrium comparisons, aggregate and individual measures of welfare and a large sensitivity analysis I show that results are affected by two forces: 1) *the shape of the endogenous distribution of income and wealth* and 2) *the price and quantity effects of debt* described in [Obiols-Homs \(2011\)](#). With regards to the former, I show that subsidies, as opposed to loans, generate wealth distributions with smaller amounts of the population as net debtors. Additionally, the equilibrium interest rate ends up being higher, which rewards a society with relatively more lenders. Moreover, equilibria with higher net debtor shares tend to be associated with larger wealth inequality. These distributional impacts have an influence on the public cost of higher education. For instance, I show that depending on the design of the student loan system, the fiscal burden generated by the loan program may turn out to be higher than that of tuition subsidies. While the price and quantity effects of debt are intricately linked to the distributional outcomes of each higher education financing scheme, I isolate the effect of prices by evaluating aggregate and individual welfare of each regime before and after markets clear. Welfare gains of policy changes in higher education financing can either double or vanish completely depending on whether we let markets clear. This stresses the need to evaluate policy changes in general equilibrium.

This article emphasises the importance of evaluating the transitional dynamics of policy changes. While I show substantial steady state gains in terms of consumption equivalent loss of different higher education systems, large transition costs from one regime to another may justify the status-quo. Moving from self financing to a system of public funding (the system yielding the largest welfare gains in the baseline calibration) or to one relying on income contingent loans can be costly enough to eat up more than 70 % of the steady state welfare gains. As a consequence, just comparing steady states may be misleading for policy.

Related literature: There is a large literature at the cross-roads of macroeconomics, education financing and its distributional impact - [García-Peñalosa and Wälde \(2000\)](#), [Bénabou \(2002\)](#), [Hanushek et al. \(2003\)](#), [Bovenberg and Jacobs \(2005\)](#), [Dearden et al. \(2008\)](#), [Johnson \(2013\)](#), [Herrington \(2015\)](#), [Cai and Heathcote \(2018\)](#) and [Luo and Mongey \(2019\)](#). The closest studies to the one presented here are [Ionescu and Simpson \(2016\)](#), [Krueger and Ludwig \(2016\)](#), [Abbott et al. \(2013\)](#) and [Hanushek et al. \(2014\)](#). In the first article the authors arrive at similar findings as in this paper using a life-cycle environment: tax financed grants can have a larger impact in improving welfare than increasing student loan limits, especially if these are too lax. The present study seeks to expand on their results in two ways: endogenising the risk free equilibrium interest rate and factoring transition costs. As shown in this paper, a fixed interest rate dampens one of the major forces driving welfare. Hence, welfare and inequality are computed before and after general equilibrium effects kick in. While the model presented here fails to capture important aspects of lifetime earnings by abstracting from age, it allows us to go beyond steady state comparisons and consider transitional dynamics at a relatively lower computational cost. As will be shown, it is not enough to demonstrate that one regime is better

than another, the costs of transition must also be taken into account as they can be large enough to significantly lessen the desirability of changing to another higher education system.

The paper by [Krueger and Ludwig \(2016\)](#) considers transitions, amongst concerns of optimal taxation and education finance. The paper, however, did not introduce income contingent student loans. This paper abstracts from optimal taxation, on purpose, so that we can see how the results go through even with a flat tax and no public externality in education. One of the most popular arguments against tax financing of higher education is that it is regressive and that in turn, it may reinforce inequality. In this paper I show that even with a tax schedule that is not progressive, we may still find that public financing can be welfare improving for all segments, or at least the vast majority, of society. [Abbott et al. \(2013\)](#) cast similar questions as in this study with a detail-rich life-cycle environment. They find that merit based grants and the current student loan system in the U.S. provides substantial increases in welfare. As the study focused on aspects of the U.S. student loan programs it did not expand on income contingent schemes. [Hanushek et al. \(2014\)](#) compare different higher education funding schemes, as in this paper, with an overlapping generations model. Their findings are somewhat similar to those herein and I contribute to their results by looking at disaggregated measures of welfare and a large sensitivity analysis of the effects of borrowing constraints. Whilst the papers mentioned above focus on the U.S. (controlling for variables such as ability, college quality, gender and elasticity of substitution between educated and non-educated workers), I propose a simpler framework that is general enough to allow for comparisons of different higher education systems. This allows us to evaluate the impact that the most salient features of each educational system (American, British and Continental European) have on welfare and inequality.

Finally, the model developed herein contributes to the literature on debt limits and welfare, confirming the presence of price and quantity effects in environments with two types of debt and the simultaneous presence of physical and human capital. This article also expands on [Angelopoulos et al. \(2017\)](#), who show the pecuniary externalities arising from agents' different savings policies, which vary by education and income profiles. Whereas [Angelopoulos et al. \(2017\)](#) fix exogenously the agent types and restricts flows between groups, this paper endogenises such flows through optimal education choice and evaluates how different higher education systems affect the composition of types in society.

This paper is structured as follows. In the first section I describe the model. In the next section I show steady state comparisons of the different higher education regimes with a large sensitivity analysis on various parameters and perform partial/general equilibrium welfare comparisons. In the third section I analyse if it is worth transitioning from one higher education system to another, specifically from a benchmark towards either of the two top alternatives. The fourth section concludes.

1 Model

The framework developed herein is based on [Aiyagari \(1994\)](#) and [Achdou et al. \(2017\)](#). Time is continuous. There is a continuum of unit measure of agents that are ex-ante identical but ex-post heterogeneous in their wealth, education status and employment state. The main difference is that there is now an endogenous choice of attending university. Getting a college degree increases the labour efficiency of agents. The production side of the economy barely changes; a representative firm hires labour and rents capital to produce output. The labour input is in efficiency units and its distribution is determined endogenously. I will use this framework to rank five different higher education (HE) systems. The first regime, called 'self financing (SF)', depicts a system where there is no government funding of tertiary education nor government guaranteed student loans. Only agents that can cover P , the cost of a college degree, are allowed

to go to university. The second regime then introduces government guaranteed student loans, referred to as NICL (non income contingent loan). The next system makes student loans income-contingent. Only those above a certain earnings threshold repay their student loans, and after 30 years the remaining balance of student debt gets cancelled. The ICL variant has two versions; one is closer to the NICL (ICL1) while the other relies on the repayment scheme that is in place in Britain (ICL2). This will shed light on how the design of loan repayments can affect outcomes. Finally, a fifth regime introduces a government subsidy for tuition fees, reducing the cost agents face to $P(1 - s)$. This system does not have government guaranteed student loans and is called ‘TS’ for tuition subsidies⁴. I will first give a brief overview of agents in the economy. Each type of agent will broadly face the same problem regardless of the HE system. Nonetheless, each regime will have peculiarities affecting agents’ budget constraints. Finally, I will then go into more detail of how the objective and constraints of each type of agent is mathematically formalised for each HE system.

1.1 Agents

Besides students, there are two broad groups of agents in the economy: those without a college degree and those with one. Each of these groups is subdivided into two categories: employed and unemployed. Figure (3) illustrates how agents move between the five types, denoted by θ_i and $i = 1, 2, 3, 4, 5$. The usual flows into and out of employment are written with subscripts denoting the origin and destination (λ_{12} is the flow of non college grads from the unemployed to employed state).

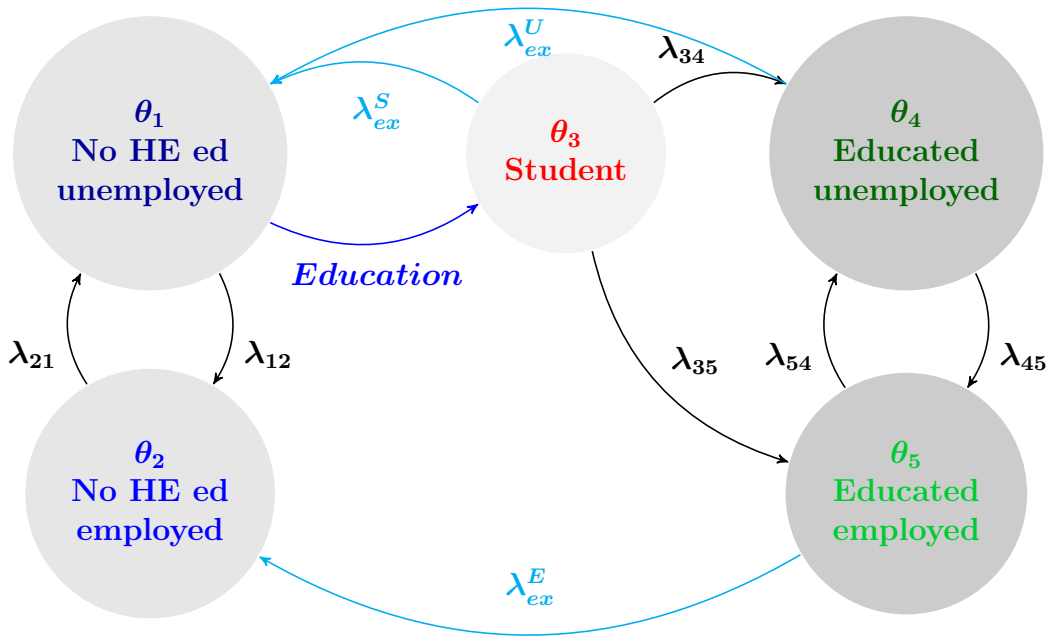


Figure 3: Agent flows

The novelty is the endogenous flow from no HE education to students to HE education⁵.

⁴The model formulation is meant to capture the most salient features of the American (NICL), British (ICL) and Continental/European (TS) systems. In reality, these countries have a mix of the ingredients presented in this model.

⁵I make the simplifying assumption that only unemployed agents have time to go to university. While this assumption does not drastically alter the results, it reduces a significant amount of computational cost. Additionally, it reinforces the opportunity cost of going to university (missing out on the possibility of gaining a

Additionally, a distinctive feature are the flows in the opposite direction λ_{ex}^U and λ_{ex}^E . They represent the rate at which a college degree depreciates. There are three reasons why I introduced such flows. The first reason is that in an infinitely lived agents environment, education becomes an absorbing state if we shut off λ_{ex}^U and λ_{ex}^E . The second reason is that these flows can capture how technological advances make redundant some careers that required tertiary education qualifications. This opens the door to study policy in an environment of increasing automation. The approach is not different from Ben-Porath models, where skills can depreciate through time - Ben-Porath (1967) and Manuelli et al. (2012). Furthermore, given that $\lambda_{ex}^U > \lambda_{ex}^E$ we may capture how unemployment spells can have an impact on skills and labour market outcomes - Arrazola et al. (2005) and Hugonnier et al. (2019). Third, while I consider natural to account for HE degrees depreciating in an infinitely lived environment, as in Hugonnier et al. (2019), these flows can be re-interpreted as mortality rates⁶.

The transitions between unemployment and employment will be calibrated so as to capture that people with a HE degree tend to face a better job market (higher transition rate into employment and a lower one into unemployment, relative to those without a college degree). Finally, in order to capture uncertainty at the student stage I introduce a college drop out rate λ_{ex}^S . All agents face a standard consumption-savings problem with a debt limit on b , the amount on money they have in a checking account. The debt limit $\underline{b}$ is tighter than the natural borrowing limit for the lowest income type. When we introduce government guaranteed student loans a , agents will be able to finance the cost of higher education with both a and b . There is also a finite debt limit on student loans $\underline{a}$. Common to all agents, preferences are determined by a strictly increasing and strictly concave utility function $u(c)$ and the subjective discount rate ρ . Throughout this paper I assume that all agents (regardless of type) have CRRA preferences described by $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. In the following subsection I formalise the different type of agents' problem.

1.1.1 θ_1 Unemployed and no higher education

As in Achdou et al. (2017) agents maximise utility subject to a flow budget constraint. The only idiosyncratic shock affects income z_i , $i = 1, 2$, which is a two point jump process, where λ_{12} and λ_{21} are the Poisson rates of jumps from unemployed to employed and vice-versa, respectively. Besides choosing consumption, the agent can now choose a time T where, if it has sufficient funds to cover the cost of education, it enrolls in university and becomes a student. Since the problem will be solved in the state domain, we will essentially be looking for a free boundary in b (or in b and a in the systems with student loans). Let such boundaries in b and a be denoted by a $\dagger$ superscript and let b^* and a^* represent the target points where agents end up at after covering education costs. The general problem of a type 1 agent in any higher education regime is shown next.

$$V_1(b_t, a_t) = \max_{c, T} E_t \left[\int_t^T e^{-\rho(s-t)} u(c) ds + e^{-\rho(T-t)} V_3(b^*, a^*) \right] \quad (1)$$

$$\text{s.t. } \frac{db}{dt} = z_i + r_b b - c + \phi_b(b, a, \theta), \quad b \geq \underline{b} > -\infty, \quad (2)$$

$$\frac{da}{dt} = \phi_a(b, a, \theta) \quad \text{and} \quad 0 \geq a \geq \underline{a} > -\infty \quad (3)$$

A type 1 agent receives unemployment benefits $\mu w z_L = z_1$, where μ , w and z_L are the replacement rate, aggregate wage and uneducated labour efficiency, respectively. The Poisson jumps in z are

full-time job and earning a higher income relative to the unemployed and students)

⁶I introduce a perpetual youth extension to this paper (in a separate article) where these rates capture stochastic lifetimes. The results are qualitatively similar to those described herein.

not made explicit in the drift⁷. The agent pays (receives) interest in b if it is a net debtor (saver). If the agent has student loans, it pays them back according to $\phi_b(b, a, \theta)$ and $\phi_a(b, a, \theta)$. These functions depend on the peculiarities of each higher education system and they will be described further below. Following Moll (2016) we can show that the solution to this problem satisfies the Hamilton-Jacobi-Bellman (HJB) equation⁸

$$\rho V_1 = \max_c u(c) + \frac{\partial V_1}{\partial b} S_b + \frac{\partial V_1}{\partial a} S_a + \lambda_{12} [V_2 - V_1], \quad (4)$$

and satisfies the constraint (5) in the region where higher education is not chosen

$$V_1(b, a) > V_3(b, a). \quad (5)$$

We can express the problem as a variational inequality

$$\begin{aligned} \min \{ \rho V_1 - u(c) - \mathbf{A}V_1, V_1 - V_3(b^*, a^*) \} &= 0, \\ \text{where } \mathbf{A}V_1 &= \frac{\partial V_1}{\partial b} S_b + \frac{\partial V_1}{\partial a} S_a + \lambda_{12} [V_2 - V_1]. \end{aligned} \quad (6)$$

When agents have access to student loans they will face a portfolio type problem⁹, where they choose the combination of a and b that lets them achieve the highest $V_3(b, a)$ after covering the cost of education P . As mentioned earlier, instead of looking for the optimal stopping time T , we will be solving for the threshold values $b^\dagger$ and $a^\dagger$ where the agent optimally chooses to pay for education (if the agent does not have enough funds to pay, it cannot jump to type 3 and become a student). In systems such as SF and TS, we encounter single asset problems, e.g. there is no dependence on a , and as a consequence the third term on the right hand side of equation (4) drops out. Additionally, there is no portfolio problem in the single asset case and as a consequence $V_3(b^*) = V_3(b^\dagger - (1-s)P)$. Equation (6) can be conveniently solved as a linear complementarity problem (LCP) - See Moll (2016) and Huang and Pang (1998). The Poisson jumps due to debt cancellation are made explicit in the Kolmogorov Forward Equations (KFE) - more on this below - and not in the student loan drift¹⁰.

Finally, in the no schooling region we have the standard first order condition in consumption given by

$$u'(c_1) = \frac{\partial V_1}{\partial b}. \quad (7)$$

1.1.2 θ_2 Employed and no higher education

Agents that are employed and do not have a college degree cannot go to university unless they become unemployed again, which occurs at rate λ_{21} . Hence, they face the standard consumption savings problem. They receive after tax income $z_2 = (1-\tau)wz_L$ and smooth consumption with b ; they pay (receive) interest on b and must pay back student loans a (if they have any). Employed agents without a college degree lose their jobs at a rate that is much higher than those employed with a college degree (i.e. $\lambda_{21} > \lambda_{54}$). Workers supply labour inelastically. Hence, the HJB equation of a type θ_2 agent will be analogous to (4).

⁷It appears to be a convention to follow this approach with notation.

⁸For notational convenience I will be denoting the drift as S_b (S_a) instead of $\frac{db}{dt}$ ($\frac{da}{dt}$).

⁹An earlier version of this model allowed for a fully fledged portfolio type problem, where the agent chooses the optimal combination of a^* and b^* . The results are equivalent to a less computationally demanding method akin to a so called finance pecking order model - for more on this see section (5.2) in the appendix.

¹⁰Following the convention in Achdou et al. (2017).

$$\rho V_2 = \max_c u(c) + \frac{\partial V_2}{\partial b} S_b + \frac{\partial V_2}{\partial a} S_a + \lambda_{21} [V_1 - V_2] \quad (8)$$

As mentioned above, in the single asset case the third term on the right hand side drops out. The expression in (8) can be solved as in Achdou et al. (2017) or in an LCP setting where the cost of going to university is large enough to never make the option desirable. The first order condition is analogous to that of (7).

1.1.3 θ_3 Students

Students are allowed to work a reduced number of hours¹¹. Given that we have inelastic labour supply, I scale their labour efficiency accordingly. After spending, on average, $\frac{1}{\Delta_{ed}}$ years as a student, the agent may graduate with (without) a job at rate λ_{35} (λ_{34}). There is a risk that the agent will not graduate, captured as λ_{ex}^S . Students do not pay income taxes. The HJB equation of students is shown next.

$$\rho V_3 = \max_c u(c) + \frac{\partial V_3}{\partial b} S_b + \frac{\partial V_3}{\partial a} S_a + \lambda_{34} V_4 + \lambda_{35} V_5 + \lambda_{ex}^S V_1 - (\lambda_{34} + \lambda_{35} + \lambda_{ex}^S) V_3 \quad (9)$$

1.1.4 θ_4 and θ_5 Unemployed and employed with higher education

Agents with a college degree face the standard consumption savings problem as in Huggett (1993) and Achdou et al. (2017). They have a higher labour efficiency $z_H > z_L$ and thus receive higher after tax income (or unemployment benefits, if unemployed). Agents gain (lose) jobs at a higher (lower) rate, when compared to agents without a university education. The two HJB equations for those with a college degree are given by

$$\rho V_i = \max_c u(c) + \frac{\partial V_i}{\partial b} S_b + \frac{\partial V_i}{\partial a} S_a + \lambda_{ij} [V_j - V_i] + \lambda_{ex}^k [V_{i-3} - V_i] \quad (10)$$

where $i = 4, 5$, $i \neq j$ and $k = E, U$. $\lambda_{ex}^U > \lambda_{ex}^E$ captures that skills gained by a college degree ‘depreciate’ faster when the agent is unemployed. The first order condition is analogous to that of (7). The next subsection elaborates on the peculiarities of each higher education system and specially on the student loan repayment function $\phi(b, a, \theta)$.

1.2 Higher education financing and agents’ budget constraints

Self financing (SF) and tuition subsidies (TS): The main defining feature of self financing and tax financed systems is that they are single asset models, i.e. there are no student loans. The cost of education that the agent faces is $P(1 - s)$, where P and s are the price and subsidy rate from the state, respectively. Self financing is captured by setting $s = 0$ and not having access to student loans. In the SF and TS systems, if the agent decides to go to university the agent subtracts $P(1 - s)$ from its wealth stock and migrates to θ_3 . As shown further below, the government covers the cost of tuition subsidies by adjusting the income tax rate.

Non income contingent student loans (NICL): Agents are now allowed to pay for higher education with student loans a (or combinations of b and a if the student loan debt limit is binding). The $\phi(b, a, \theta)$ functions describe the student loan repayment scheme.

¹¹According to Sonnet (2010) the share of ‘working students’ may vary substantially by country, from the low single digits to well over a third of students. According to Carnevale et al. (2015) around two thirds of tertiary students in the US are engaged in work. Of those that work, 30 hours per week is the average.

$$\phi_b(b, a, \theta) = \begin{cases} (r_A + \delta)a & \text{for } \theta_i = 1, 2, 4, 5 \\ 0 & \text{for } \theta_3 \end{cases} \quad \phi_a(b, a, \theta) = \begin{cases} -\delta a & \text{for } \theta_i = 1, 2, 4, 5 \\ r_A a & \text{for } \theta_3 \end{cases}$$

If the agent holds student loans, it pays $(r_A + \delta)a$, the interest and amortisation rates on student debt, regardless of its income state. The exception is for students, who accrue debt while at university. Debt forgiveness is not allowed, so the debt cancellation premium $\lambda_{np} = 0$ and thus $r_A = r_B$. This follows closely federal unsubsidised student loans in the U.S.

Income contingent loan with repayment subsidies (ICL1): The $\phi(b, a, \theta)$ functions describe the student loan repayment scheme.

$$\phi_b(b, a, \theta) = \begin{cases} (r_A + \delta)a & \text{for } \theta_5 \\ 0 & \text{otherwise} \end{cases} \quad \phi_a(b, a, \theta) = -\delta a \quad \forall \theta$$

In ICL1 the income contingency protection kicks in. Agents pay their student loans only when they reach a high enough income (they reach type 5, i.e. they become employed and educated). The government covers interest and amortisation otherwise. Agents are now allowed to receive debt forgiveness; loans are cancelled, on average, after $1/\lambda_{np}$ years. The government recovers such losses by charging a premium on student loans and thus $r_A = r_B + \lambda_{np}$.

Income contingent loan without repayment subsidies (ICL2): Agents pay a tax r_p on earnings above the threshold z_T . Earnings encompass labour and capital income, so any agent with earnings above the threshold will be subject to the tax as long as their student loan balance is not zero. That is, an uneducated agent carrying student loans (say because it suffered a college dropout or skill depreciation shock) that is wealthy in b can still be liable for student loan repayments. This charge draws down the student loan balance. High labour income earners pay an extra interest on their student debt, set to r_A to keep some comparability with ICL1. Students accumulate debt at rate r_A . This system follows closely that of the UK¹².

$$\phi_b(b, a, \theta) = \begin{cases} -r_p \mathbb{1}_{\{a < 0\}} \max\{z_i + r_b \max\{b, 0\} - z_T, 0\} & \text{for } \theta_i = 1, 2, 4 \\ 0 & \text{for } \theta_3 \\ -r_p \mathbb{1}_{\{a < 0\}} \max\{z_i + r_b \max\{b, 0\} - z_T, 0\} & \text{for } \theta_5 \end{cases}$$

$$\phi_a(b, a, \theta) = \begin{cases} r_p \mathbb{1}_{\{a < 0\}} \max\{z_i + r_b \max\{b, 0\} - z_T, 0\} & \text{for } \theta_i = 1, 2, 4 \\ r_A a & \text{for } \theta_3 \\ r_A a + r_p \mathbb{1}_{\{a < 0\}} \max\{z_i + r_b \max\{b, 0\} - z_T, 0\} & \text{for } \theta_5 \end{cases}$$

The remaining student loan balance is cancelled after a certain period (as in ICL1, after $1/\lambda_{np}$ years). Besides the repayment scheme, the main difference with ICL1 is that in ICL2 the government does not provide debt repayment subsidies for those receiving income contingency protections. In ICL1 the student loan balance is always decreasing regardless of the income state of the individual; in ICL2 the balance can increase if tax payments on earnings over the threshold z_T are not large enough to cover interest.

In NICL and ICL2 there is one additional subsidy from the state in the student loan program. Any agent with a negative drift at $\underline{a}$, will have interest payments on student debt covered by the government. This is done to prevent mass escaping the state space¹³. These costs are covered

¹²As mentioned earlier, in the United Kingdom student loan interest rates are charged during studies and vary depending on income later in life. The rate charged to students and high income earners tends to be larger than the risk free rate.

¹³This rarely affects results for the calibrations considered in this paper.

through the tax revenue raised from labour income. Additional variants of the ICL system are left as potential extensions of this paper. In the next subsection I describe how agents interact with the other sectors of the economy.

1.3 Firms, government, education and asset market

The rest of the economy is composed of a representative firm (as in [Aiyagari \(1994\)](#)), asset market and government. Figure (4) depicts the flows between the different players in the economy. Agents supply labour to a representative firm and receive wages net of taxes in return. Taxes go to fund unemployment insurance and education costs (if there is such support). Agents supply capital to the representative firm, through a financial market that is omitted from the figure since it acts as an invisible intermediary. In return, agents receive interest income. The simplified diagram in Figure (4) represents such flows as agents supplying labour and capital and receiving consumption goods and education in return. Figure (5) represents the additional flows in two asset economies (NICL, ICL1 and ICL2), mainly how the government acts as a financial intermediary by supporting the student loan program. For this purpose the channelling of funds through financial markets is made explicit.

Higher education has a fixed resource cost. This is an explicit modelling choice; this assumption is made so that we can evaluate the impact of P in the capital market imperfection of educational investments, and in turn, on the rankings between the different higher education regimes. It is important to highlight the role of P , especially when tuition costs have risen so dramatically in many countries. An alternative interpretation is to treat education as an import, which is not a far fetched assumption for small countries that educate their workforce abroad¹⁴. The economy invests $P \int_a \int_b g_1(b, a) \mathbb{1}_{\{V_3(b^*, a^*) > V_1(b, a)\}} db da$ in education.

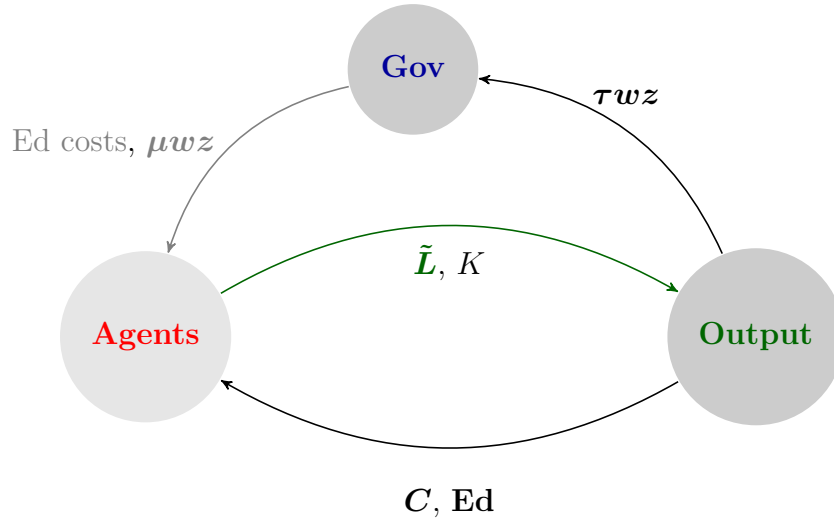


Figure 4: Common flows in all HE systems

1.3.1 Representative firm

As in [Aiyagari \(1994\)](#), there is a representative firm with Cobb-Douglas technology. The firm rents capital, which depreciates at rate δ , from agents and hires labour. Labour differs in

¹⁴The perpetual youth extension of this model endogenises P as in [Hanushek et al. \(2003\)](#). The conclusions are broadly similar to the ones presented here under plausible parametrisations, although in that setting we cannot evaluate how P magnifies differences between systems directly.

productivity; agents with a university degree have a higher efficiency. The production function is given by

$$Y = AK^\alpha \tilde{L}^{1-\alpha}, \quad (11)$$

where A is a positive constant and α is the capital share. The effective labour supply is given by adding the efficiencies of the employed with and without college degree and students. This embeds an assumption of perfect labour substitutability between educated and non-educated workers and of production externalities¹⁵. Remark that students' effective labour supply scales z_L by z_s to capture their working hours.

$$\tilde{L} = z_L(\theta_2 + z_s\theta_3) + z_H\theta_5 \quad (12)$$

Factor prices are given by the next two expressions.

$$r = \alpha \frac{Y}{K} - \delta, \quad (13)$$

$$w = (1 - \alpha) \frac{Y}{\tilde{L}}. \quad (14)$$

Equation (13) gives us capital demand.

1.3.2 Government and tertiary education

The government has a balanced budget constraint and raises revenue from labour income with a flat tax applied to workers τ . In all versions of the model we have unemployment insurance. Additional tax revenue may be raised to cover subsidies to P (in TS), to interest or contingency of student loans (in ICL1 and ICL2). Hence, the income tax rate τ is shown next.

$$\tau = \frac{\overbrace{\mu[z_L\theta_1 + z_H\theta_4]}^{\tau_{UI}}}{z_L\theta_2 + z_H\theta_5} + \frac{\text{Education costs}}{w[z_L\theta_2 + z_H\theta_5]} \quad (15)$$

The first term, τ_{UI} , is the tax rate needed to cover unemployment benefits. The unemployment benefit system is common in all the five regimes being considered. The second term captures the public cost of financing the higher education system. As mentioned previously, in the ICL regimes, the government raises extra revenue with premiums on student loans, so as to cover debt cancellation.

$$r_A = r_B + \lambda_{np} \quad (16)$$

The student loan balance of an individual gets cancelled after a period of length $\frac{1}{\lambda_{np}}$ has elapsed, hence this cost is covered by the risk premium λ_{np} . If we denote the total amount of newly issued student loans as A^{new} , the aggregate stock of student debt as A and agents' aggregated net savings as B , we can represent the government's role as an intermediary in student loan programs as follows.

¹⁵In the perpetual youth extension of this article I consider a more general framework, using a CES aggregator of educated and non educated workers as in [Hanushek et al. \(2003\)](#) and [Abbott et al. \(2013\)](#). Once again, the results are broadly similar to those presented here under calibrations commonly found in the literature.

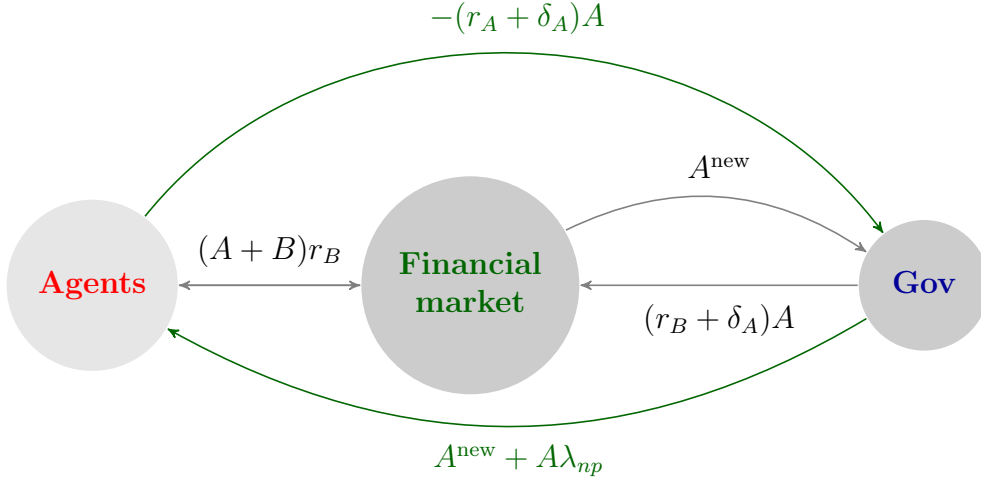


Figure 5: Government intermediation in student loans

The government acts as an intermediary, raising funds in the financial market, issuing student loans to agents and acting as guarantor in case of debt forgiveness. Any loses in the student loan program are covered by the state either through tax revenue and/or risk premiums. In the next subsection I define what is an equilibrium in the economy and the welfare ranking.

1.4 General equilibrium

The stationary equilibrium in this model is defined by a set of policy functions in consumption and educational investment (given by the HJB equations shown above) for each agent type, a joint income and wealth distribution that is ergodic, a government balanced budget and a risk free rate that clears the asset market. During transitions, the asset market clears at every instant. The income and wealth distribution is governed by the following Kolmogorov Forward Equations (KFE). Let g represent the density, ∂_k denote the partial derivative w.r.t. k and the subscripts in g depict the agent type. The KFEs are shown below

$$\partial_t g_1 = -\partial_a[\dot{a}_1 g_1] - \partial_b[\dot{b}_1 g_1] + \lambda_{21} g_2 - \lambda_{12} g_1 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4 - g_1 \delta(a - a^\dagger)(b - b^\dagger), \quad (17)$$

$$\partial_t g_2 = -\partial_a[\dot{a}_2 g_2] - \partial_b[\dot{b}_2 g_2] + \lambda_{12} g_1 - \lambda_{21} g_2 + \lambda_{ex}^E g_5, \quad (18)$$

$$\partial_t g_3 = -\partial_a[\dot{a}_3 g_3] - \partial_b[\dot{b}_3 g_3] - [\lambda_{ex}^S + \lambda_{34} + \lambda_{35}] g_3 + g_1 \delta(a - a^*)(b - b^*), \quad (19)$$

$$\partial_t g_4 = -\partial_a[\dot{a}_4 g_4] - \partial_b[\dot{b}_4 g_4] + \lambda_{54} g_5 - [\lambda_{ex}^U + \lambda_{45}] g_4 + \lambda_{34} g_3, \quad (20)$$

$$\partial_t g_5 = -\partial_a[\dot{a}_5 g_5] - \partial_b[\dot{b}_5 g_5] + \lambda_{35} g_3 + \lambda_{45} g_4 - [\lambda_{ex}^E + \lambda_{54}] g_5, \quad (21)$$

where the Dirac delta function $\delta(\cdot)$ captures the flow¹⁶, at $(a^\dagger, b^\dagger)$, of no HE unemployed agents into students and where a^*, b^* are the targets. In single asset regimes we drop the dependence on a . In steady state $\dot{g} = 0 \forall i, a, b$. Market clearing requires $K_S - K_D = 0$, where

$$K_S = \sum_{i=1}^5 \int_a^0 \int_b^\infty (b + a) g_i db da.$$

Capital demand being equal to capital supply implies the national accounting identity¹⁷ $Y = C + I + \text{Education costs}$. There is no proof of existence and uniqueness of equilibrium for

¹⁶While notation is a bit loose here, note that $(a^\dagger, b^\dagger)$ will be a function of states, generating a region/combination of a, b pairs where becoming a student is optimal and feasible.

¹⁷A heuristic proof is left in the appendix.

the model with educational choice. The downward sloping and continuous demand of capital remains the same as in [Aiyagari \(1994\)](#). Nevertheless, capital supply is affected by the different education types - [Angelopoulos et al. \(2017\)](#) - and by the educational choice. Quantitative evaluations for a large parameter space show that it is the case that the aggregate capital supply K_S is monotonically upward sloping, approaching ρ from below, continuous and that there is a single crossing of capital demand and supply. I evaluate aggregate and individual welfare via consumption equivalent loss (CEL), as shown next. Let V_0 and V_c denote the steady state value function in the benchmark and alternative regimes, respectively.

$$\tilde{c} = \left[\left(\frac{V_c + \frac{1}{\rho(1-\sigma)}}{V_o + \frac{1}{\rho(1-\sigma)}} \right)^{\frac{1}{1-\sigma}} - 1 \right] * 100 \quad (22)$$

Remark that CEL will be presented in percentage terms. V_c and V_o are computed as follows.

$$\sum_{i=1}^5 \int_{\underline{a}}^0 \int_{\underline{b}}^{\infty} V_i(b, a) g_i(b, a) db da. \quad (23)$$

This measure will be computed as an average for the whole economy (as in (23) above) and also for each point in the state space (using $V_i(b, a)$ only), giving us a disaggregated view of which groups in society favour/are against policy changes relative to a common benchmark. Given that each regime yields a different distribution of income and wealth, the disaggregate CEL comparisons will be unweighed comparison of raw value functions. Hence, this analysis will be complemented by comparing the income and wealth distributions of HE systems. The benchmark V_o will be set to welfare in the self financing regime. Positive values of $\tilde{c}$ mean that agents in the SF system would be as well off as in the alternative system if their lifetime consumption is increased by $\tilde{c}$ per cent. Negative values mean that we would have to subtract $\tilde{c}$ per cent of the life time consumption of agents in the SF regime, in order to make them as worse off as in the alternative higher education system. The numerical method used is the finite differences approach presented in [Achdou et al. \(2017\)](#). The agent's decision to become a student is computed with an LCP solver as in [Moll \(2016\)](#) on non-uniform grids.

1.5 Calibration

The baseline calibration of the model is shown in Table (1). The model economy has 24 parameters. These are discussed below in separate categories. The baseline is set based on studies focused in the U.S.¹⁸. The goal is not construct a model that matches U.S. data, but to show how different higher education schemes have an impact on welfare and inequality while using a reasonable calibration.

Preferences: Preferences are described by a constant relative risk aversion utility function with risk aversion coefficient σ and a subjective discount rate ρ . These are set to standard values found in the literature. All parameters are calibrated so that everything is understood in annual terms.

Labour market transitions: The labour market transition rates from unemployment to employment, and vice-versa, are taken from [Lamadon et al. \(2013\)](#). One can see in Table (1) how labour market outcomes are more favourable for graduates as they face a higher probability of being employed and lower probability of falling into unemployment. The transition rates from student to educated is set to $\Delta_{ed} = 0.25$, reflecting that on average it takes four years to

¹⁸Forthcoming work expands the model presented in this paper in a life-cycle environment based on a UK calibration.

complete a bachelor's degree in the U.S.¹⁹. The flow from student to educated Δ_{ed} is split into transitions to unemployed and educated (λ_{34}), and employed and educated (λ_{35}). According to the National Center for Educational Statistics (NCES), roughly two thirds of students find employment within the first 9 months after graduation - [Staklis and Bentz \(2016\)](#). This figure is roughly constant despite fluctuations over the business cycle. The skills depreciation rate is taken from [Manuelli et al. \(2012\)](#). The magnitude seems to be more or less the same among other papers using Ben-Porath type models, for instance [Ionescu \(2009\)](#). The doubling of this rate for those that are unemployed is inspired from evidence highlighted in [Arrazola et al. \(2005\)](#) and [Hugonnier et al. \(2019\)](#). The dropout rate for students λ_{ex}^S is taken from NCES (2019) while the replacement rate μ is taken from 2019 estimates from the U.S. Department of Labor.

Education and skills premium: The cost of education is difficult to pin down since it is not clear if we should include living expenses. For instance, what fraction of students stay at home or rent elsewhere during their studies? It is not clear what percentage of students move out of home when they enrol at university²⁰. This is important since it gives us a better idea of whether student accommodation counts as an extra expense accounted by c or P . The benchmark P will be set to lie between %95 (including living expenses) and %129 (just tuition) of costs in the U.S., according to data from [The College Board \(2019\)](#). These percentages are found by matching the ratio of higher education costs to GDP per capita, with the latter rescaled to reflect only those in the labour force (this model only has employed and unemployed people, we exclude those not in the labour force). This ratio is then multiplied by the mean income in the economy. Whilst mean income is endogenous and varies with each higher education regime, it is broadly stable for the vast majority of cases considered here, lying in a range between 0.5 and 0.59. Hence, the baseline P is set to 1.1. This is a conservative calibration of education costs, nonetheless a sensitivity analysis with $P \in [0.3, 2.1]$ is performed. This is crucial, since as Figure (28) shows, tuition inflation has outstripped healthcare and housing costs and, as will be shown next, as P rises we magnify welfare differences amongst the different HE systems. The appendix goes into more detail on how the benchmark calibration of P is obtained. The higher education wage premium ψ is set to 1.7, following²¹ evidence from [James \(2012\)](#) and [Valletta \(2018\)](#).

Student loans and debt limits: The amortisation rate in NICL and ICL1 is set to 1/30. This corresponds to the maximum maturity in the *Standard Repayment* schedule for American student loans. The reason I picked this number is twofold. First, high amortisation rates reduce the state space where consumption can remain positive when indebted and second, a 30 year loan allows some degree of comparability with other regimes where loans are forgiven after 30 years. Using the re-scaled GDP per capita method outlined in the appendix, we obtain the values for $\underline{b}$ and $\underline{a}$. The student debt limit (4 year degree cumulative) for Stafford loans is \$23000 while the average unsecured debt amount is at \$17000 according to the Survey of Consumer Finances (2016). Thus, following [Athreya et al. \(2019\)](#) this allows us to set the debt limits $\underline{b}$ and $\underline{a}$ to -0.174 and -0.24, respectively. Aggregate welfare is sensitive to debt limits, as pointed out by [Obiols-Homs \(2011\)](#). Therefore, a large sensitivity analysis on debt limits is carried out to illustrate how they affect the rankings of the higher education funding schemes discussed herein. The graduate tax in ICL1 is described by two parameters: z_T and r_p . The threshold z_T is set

¹⁹According to NCES, in the U.S., the most common is to graduate in 4 years. Results with further sensitivity analysis on Δ_{ed} can be reproduced upon request.

²⁰According to NCES (2016) about a quarter of university students in the U.S. move out of state. This is not enough information to pin down the fraction of students that incur extra accommodation costs due to tertiary education.

²¹I emphasize a fixed premium as the last decades have seen a stable, if not rising, college wage premium in both the US and UK despite large increases in the supply of college educated workers. [Belfield et al. \(2018a\)](#) and [Belfield et al. \(2018b\)](#) have shown that the premium is driven by substantial heterogeneity. Introducing heterogeneity in ψ is left for an extension.

	Values	Description	Source
σ	2	CRRA	Common
λ_{12}	1.368	Poisson rate $z_1 \rightarrow z_2$	Lamadon et al. (2013)
λ_{21}	0.36	Poisson rate $z_2 \rightarrow z_1$	Lamadon et al. (2013)
λ_{45}	1.5	Poisson rate $z_4 \rightarrow z_5$	Lamadon et al. (2013)
λ_{54}	0.072	Poisson rate $z_5 \rightarrow z_4$	Lamadon et al. (2013)
λ_{34}	$\Delta_{ed} \frac{1}{3}$	Poisson rate $z_3 \rightarrow z_4$	Staklis and Bentz (2016)
λ_{35}	$\Delta_{ed} \frac{2}{3}$	Poisson rate $z_3 \rightarrow z_5$	Staklis and Bentz (2016)
ρ	0.05	Discount rate	Common
λ_{ex}^E	0.024	Poisson obsolescence	Manuelli et al. (2012)
λ_{ex}^U	0.048	Poisson obsolescence	Manuelli et al. (2012) and Arrazola et al. (2005)
λ_{ex}^S	0.148	Poisson dropout	USDE-NCES (2019)
s	[0,1]	Subsidy	-
μ	0.382	Replacement rate	USDL (2019)
Δ_{ed}	{1, 0.25}	Inverse years until grad	-
A	0.45	Productivity	Convenient
λ_{np}	1/30	Premium on student loans	UK
r_p	0.09	ICL2 graduate tax	UK
z_T	0.2811	ICL2 income thresh	UK
$\underline{b}$	[0, -1.5]	Exogenous b limit	0-268% avg US income
$\underline{a}$	[0, -2.35]	Exogenous a limit	0-150% avg ed costs US*
P	[0.3, 2.1]	Education cost	30-130% avg ed costs US*
ψ	1.7	HE premium	James (2012), Valletta (2018)
δ	0.05	Capital depreciation	Common
δ_A	1/30	Amortisation in NICL and ICL1	**

Table 1: *Average cost of a private non-profit university. The % may vary depending on including board.** Maximum maturity for *Standard Repayment* in the US. Values in red denote parameters where sensitivity analysis is performed.

to mimic the UK's student loan system, matching the ratio of the taxable threshold to GDP per capita while the tax rate r_p on earnings above the threshold is set to 9 %. Following the methodology in the calibration for P , we translate z_T to a comparable figure in the US.

Production: Three parameters describe the productive technology in the economy. These are the capital elasticity α , depreciation δ and TFP A . The first two are set to commonly used values while the third is adjusted for convenience to keep the state space in wealth of a reasonable size.

2 Steady state results

All CEL computations set SF as the benchmark regime. Table (2) shows²² that self financing is the worst out of all systems. It has the lowest CEL, share of employed workers with a college degree, capital stock, GDP, expected earnings, the highest wealth inequality and the highest net debtor population share. Debtors face an interest rate that is higher than most systems, so net borrowers (net lenders) suffer more (gain more). The income tax rate is higher than in the student loan systems and that of TS 50%. As we see more involvement of the public sector, be it with student loans or tuition subsidies, wealth inequality decreases, relative to self financing. Besides this, there are two striking results. First, while the system with income contingent loans brings substantial CEL gains, it can vary substantially depending on how it is designed. Second, tuition subsidies at 100 % of education costs bring the largest CEL gains vis-à-vis self financing; it attains the highest share of workers with a college degree - more than doubling the share in self financing. Moreover it reaches the largest interest rate *with the lowest share of the population in debt*. This last result points to a powerful force driving aggregate welfare: *the price and quantity*

²²In Table (2), the TS system columns have a percentage attached to denote what fraction of education costs are covered by the state.

effects of debt described in Obiols-Homs (2011).

	SF	NICL	ICL1	ICL2	TS 50 %	TS 75 %	TS 100 %
CEL %	0.0000	17.3155	30.9271	21.5248	51.1487	54.2979	55.2659
K	2.1642	2.6490	2.9138	2.7448	3.1385	3.0208	2.8903
$\mathbb{E}[z]$	0.4254	0.4967	0.5378	0.5103	0.5800	0.5758	0.5684
r	0.0416	0.0390	0.0384	0.0385	0.0393	0.0421	0.0451
Y	0.6006	0.7144	0.7803	0.7360	0.8489	0.8434	0.8329
θ_1	0.1303	0.0758	0.0430	0.0656	0.0054	0.0014	0.0000
θ_2	0.5155	0.3224	0.2064	0.2864	0.0736	0.0592	0.0544
θ_3	0.0324	0.0551	0.0687	0.0593	0.0843	0.0860	0.0866
θ_4	0.0160	0.0271	0.0338	0.0292	0.0415	0.0424	0.0426
θ_5	0.3058	0.5196	0.6480	0.5595	0.7952	0.8110	0.8164
τ	0.0581	0.0397	0.0362	0.0380	0.0538	0.0710	0.0892
τ_{UI}	0.0581	0.0386	0.0294	0.0356	0.0204	0.0195	0.0192
CV	1.2797	0.8862	0.7144	0.8371	0.4605	0.4337	0.4698
Gini _w	0.6350	0.4976	0.4100	0.4744	0.2622	0.2470	0.2671
Debtors	0.0389	0.0340	0.0294	0.0368	0.0166	0.0093	0.0073

Table 2: $\Delta_{ed} = 1/4 - P = 1.1$

Welfare rankings coincide almost perfectly with the share of the population in net debt and with measures of wealth inequality. Systems that generate more net debtors and more inequality have lower aggregate CEL. If a relatively high net debtor share is compounded with a larger interest rate welfare will be depressed even further. Systems with student loans have a larger fraction of the population as net debtors. That is, student loan systems are more prone to the negative impact on welfare coming through the price effect described in Obiols-Homs (2011). Whilst the income contingent loan systems shields agents from the effect of interest rates, debt balances may potentially accumulate at a faster rate and for longer periods. This is because of the debt cancellation premium that is added on top of r and because agents accrue debt when they don't have a high enough income²³. Even if the amount of debt is notional and may not affect the individual, the government still has to raise revenue to cover interest payments and cancellations. This will become quantitatively more patent in the next subsection.

Rankings between systems change depending on the cost of education, which is affected by P , the price of a college degree, and the time it takes to graduate ($1/\Delta_{ed}$). When P is low and Δ_{ed} is high, the capital market friction in educational investments and riskiness of becoming a student diminishes and the results go in the opposite direction, as shown in Table (4) and Figure (25) in the appendix. Even though it is highly implausible that we could keep education quality constant with high Δ_{ed} and low P , it is worth considering these results. They illustrate under which conditions we may get that government intervention in education financing can reinforce inequality, a commonly repeated link by detractors of public financing. Thus, there seems to be a little bit of truth in two popular perceptions higher education: financing tertiary education with too much debt or with too much government support can foster inequality and reduce the social gains from educational attainment. Nonetheless, as Tables (2) and (4) illustrate, results are sensitive to P and Δ_{ed} .

We know that it is not empirically plausible to have low P and high Δ_{ed} . It is also worth noting that larger public expenditure in tertiary education relative to GDP is associated with lower income inequality²⁴ in the OECD - see Figure (27) in the appendix. Table (2) shows a

²³This is one of the main distinctions between ICL1 and ICL2. In the former the government covers student loan repayments when agents do not make contributions whereas in the latter debt continues to accrue. Changing z_T and r_p may accentuate or soften these effects.

²⁴Given the lack of available data that is consistent for cross country comparisons of wealth inequality in the OECD, I could only estimate this relationship for measures of income inequality.

similar relationship between wealth inequality and the amount of financial support in tertiary education coming from the public sector. The gap between τ and τ_{UI} is the amount of tax that needs to be raised to fund the higher education system. The larger the gap the bigger the public sector involvement in tertiary education is. The size of the gap coincides with the CEL and inequality rankings.

Table (2) illustrates how the welfare ranking moves in lockstep with the net debtor population share and wealth inequality. This points to the welfare effects of borrowing limits described earlier. This is developed further in the next subsection. In order to verify that it is indeed the price and quantity effects outlined by [Obiols-Homs \(2011\)](#), I repeat the exercise with 2304 combinations of $\underline{b}$ and $\underline{a}$, and various values of P ranging from 27% to 190% of its benchmark calibration value.

2.1 Borrowing limits and welfare

A natural question to ask is how much are the aggregate CEL results driven by the choice of debt limits $\underline{b}$ and $\underline{a}$. In a first round of experiments, we keep the SF benchmark at the baseline calibration and compute steady state results for all other higher education financing schemes with 2304 combinations of $\underline{b}$ and s ($\underline{b}$ and $\underline{a}$ in systems with student loans). Figure (6) depicts the case of SF and TS.

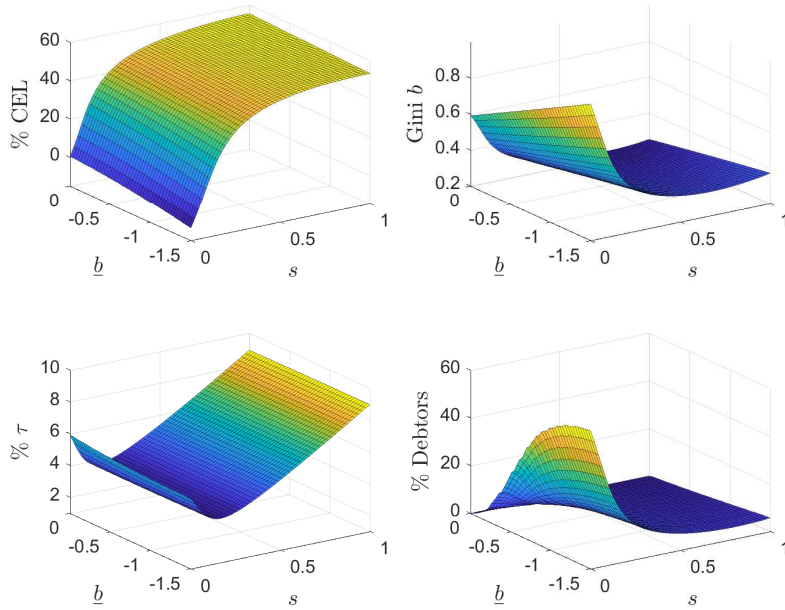


Figure 6: CEL, wealth Gini, tax rate and net debtor share in SF and TS

The results look robust to the findings shown above. First, welfare is sensitive to debt limits. In the SF and TS cases the optimal debt limit $\underline{b}$ is located at zero, regardless of the subsidy rate. This is a natural result in an Aiyagari economy - [Obiols-Homs \(2011\)](#) and [Nuño and Moll \(2017\)](#). The negative impact of laxer debt limits is diminished by increasing the subsidy rate. Second, wealth inequality and the net debtor population share appear to be closely related to the CEL welfare ranking. This is not the case with the tax rate, which has a minimum at strictly positive values of the subsidy rate. Financing tuition subsidies with higher tax rates are not necessarily associated with lower welfare, even when we have a flat tax rate. In order to repeat the same exercise with the regimes that have student loans, steady states are computed for 2304 combinations of $\underline{b}$ and $\underline{a}$ and compared to the benchmark SF. The cases for ICL1 and NICL are

displayed here, those of ICL2 are similar to those of ICL1 and are thus left in the appendix - see Figure (29). The choice of focusing on ICL1 is also motivated by the fact that it tends to outperform ICL2.

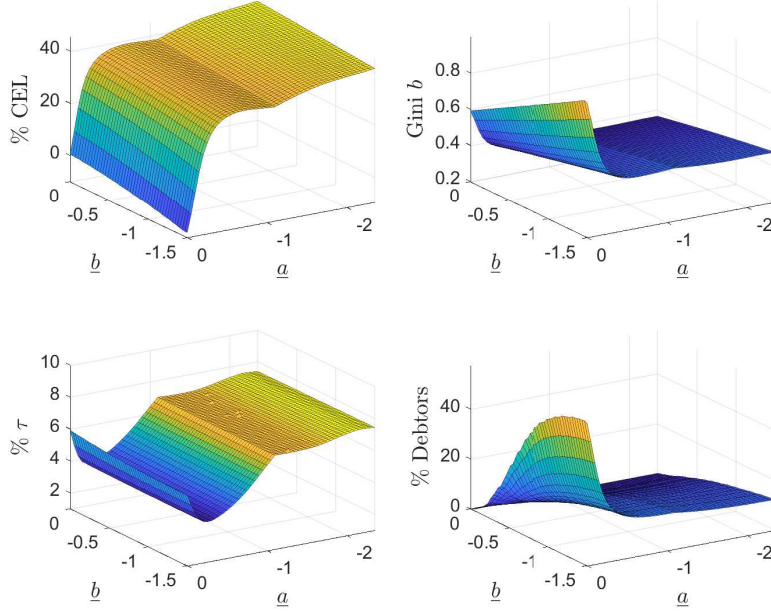


Figure 7: CEL, wealth Gini, tax rate and net debtor share in ICL1

Figure (7) above displays the outcomes in ICL1. The direction of results go in the same way as in the previous experiment with tuition subsidies; mainly that there is lower aggregate welfare when the economy generates a larger mass of debtors. The debt limit on student loans has a similar effect to that of the subsidy rate with the twist that the relationship between aggregate welfare and $\underline{a}$ in ICL1 and ICL2 is not monotonic. The % CEL over SF initially peaks around average income, then dips and then starts to increase again when $\underline{a}$ reaches P , the monetary cost of education. Similar results have been found in the literature, [Johnson \(2013\)](#), [Ionescu and Simpson \(2016\)](#) and [Abbott et al. \(2013\)](#) identify a similar effect of student debt limits on welfare: laxer limits on a can provide diminishing gains or even drag down welfare - while more generous subsidies can generate stronger gains. The novelty of this paper is making the connection with Obiol-Homs' price and quantity effects of debt and comparing systems under this light. Another remarkable result is the scale of the vertical axes on the top left panels of Figures (6) and (7): welfare ranks consistently higher in TS for a large area of the parameter space.

Another robust outcome of the experiment is that tax rates can be higher in economies with income contingent loans, relative to that of a system relying on tuition subsidies - see the results for ICL2 in Figure (29) in the appendix. It is surprising that this is more patent in ICL2, despite the fact that in ICL1 the government covers the interest *and amortisation* of agents that do not earn a high enough income. This happens for two reasons. First, remember that in ICL2 agents accumulate student loan interest when they do not earn enough (when the income contingency protects low earners). Furthermore, for a vast area of the state space, the tax r_p of earnings over the threshold z_T contributes little, if at all, to pay down the student loan balance. This is probably putting a larger burden on the public sector. Higher taxes and lower income contingency protections in ICL2 explains why it delivers a smaller share of the population with a college degree. This takes us to the second point. Notice how ICL1 spends less in unemployment insurance, relative to ICL2 and NICL, in Table (2) and how the overall income tax rate is lower,

especially when the student debt limit is lax, relative to ICL2 in Figures (7) and (29). As ICL1 delivers a larger share of college graduates, who have face a better labour market (less unemployment) and earn more than non-graduates, the cost of unemployment insurance drops, lowering the income tax rate. It seems that, on aggregate, it is a better deal for the government to help bring down student loan balances of those that do not earn enough. The case with student loans that do not have an income contingency protections is shown next.

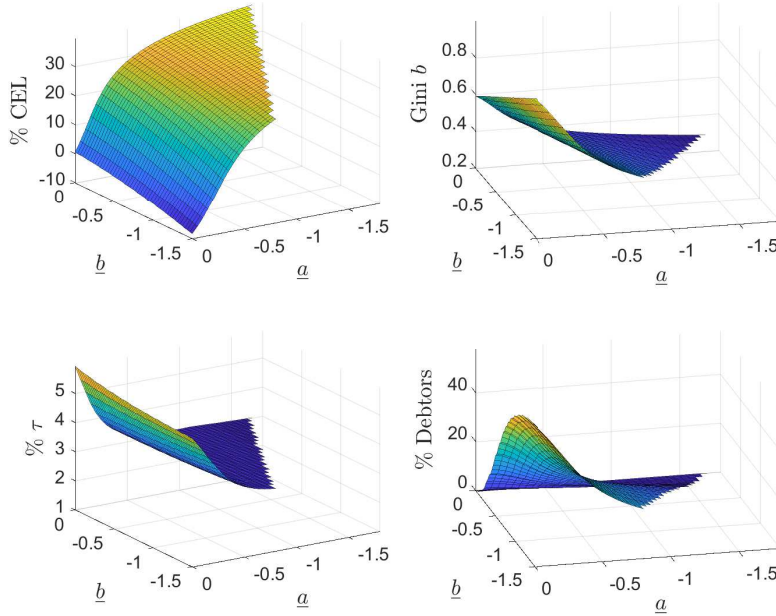


Figure 8: CEL, wealth Gini, tax rate and net debtor share in NICL

Figure (8) reveals two defining aspects of the NICL system. First, income taxes are almost always lower than in any other system with government support, either via income contingent student loans or tuition subsidies. Second, the regions where a steady state equilibrium can be achieved are reduced. This is because agents must pay amortisation and interest in student loans regardless of their income state. When a is large enough (in absolute value), interest and amortisation overtake all earnings, pushing towards zero consumption. These cases are ruled out since bankruptcy is not possible. CEL gains relative to self financing are much smaller than those of the income contingent loan programs, despite having lower income tax rates. As will be shown further below, NICL does outperform ICL2 when the student loan limit is large, around the same region where ICL2 starts to generate a large mass of net debtors.

It is interesting to revisit the link between CEL, subsidy rate and debt limits. In the TS case, as the subsidy rate approaches zero, aggregate CEL decreases more sharply as the debt limit is relaxed. As the subsidy rate becomes positive we see that this relationship is weakened. This relates to the pecuniary externalities described in [Nuño and Moll \(2017\)](#), where higher wealth inequality was found to be associated with higher welfare²⁵. The twist is that now the configuration yielding the highest CEL in TS is associated with the lowest wealth inequality. The same can be said of all student loan programs. While more lax student debt limits increase welfare relative to SF, the welfare gains drop as $\underline{b}$ is loosened.

When the monetary and time costs in educational investments are low, loosening $\underline{a}$ or providing tuition subsidies will decrease welfare, regardless of $\underline{b}$ - see Figures (25) and (26) in the appendix. I repeat this exercise with different values of P for each system. Given that this adds

²⁵This opens an interesting extension for constrained efficiency considerations in [Nuño and Moll \(2017\)](#).

an extra dimension to the analysis, these results are illustrated as animations²⁶. These show how CEL, tax rate, the wealth Gini coefficient and the net debtor share change with P . The animations illustrate how the results shown above are robust to sensible values of P . Larger values of P magnify the capital market frictions and riskiness in educational investments and welfare gains from government intervention. As P rises the marginal benefit from laxer $\underline{a}$ flattens out more markedly. As for the benchmark calibration of P (and higher values) and in most of the parameter configurations considered here, it is safe to say that TS often yields higher steady state CEL gains (and lower wealth inequality) than the student loan systems. I confirm this with a new experiment.

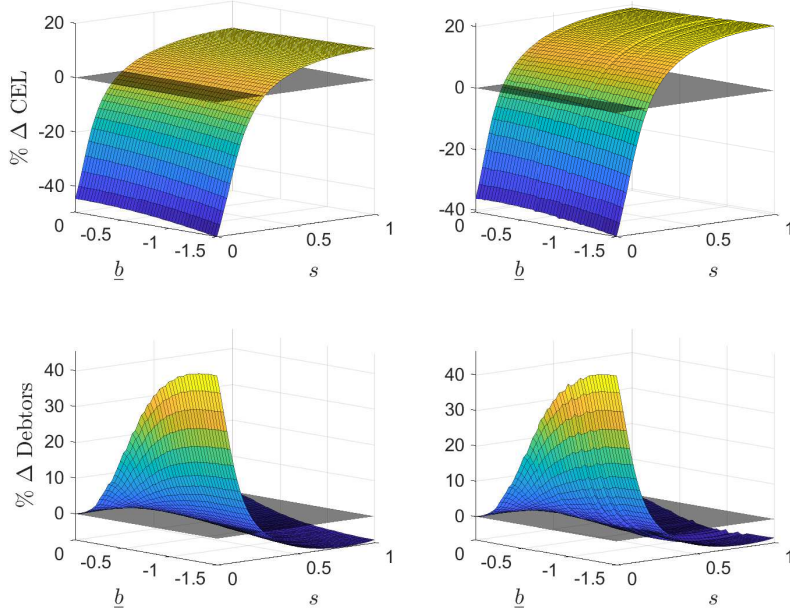


Figure 9: Left to right: $CEL_{TS} - CEL_{ICL1}$ and $CEL_{TS} - CEL_{ICL2}$ and difference in net debtor share

Figure (9) computes the difference between the outcomes in²⁷ TS and ICL1 and TS with ICL2. Since the student loan systems have an extra dimension I compare single and two asset regimes by fixing $\underline{b}$ and picking the student loan debt limit that yields the highest CEL relative to SF. This is then used to compare with the results in TS under the same $\underline{b}$. I then vary the tuition subsidy rate s . When the subsidy rate is low, the student loan systems ICL1 and ICL2 tend to outperform TS. Figure (9) clearly indicates that this coincides with whichever system has the larger share of the population as net debtors. TS starts to outperform ICLs at subsidy rates above %30. Finally, comparing the student loan systems only, for each $(\underline{b}, \underline{a})$ combination, yields Figure (10) below. We can see once again how CEL is tightly linked to the debtor share. At laxer debt limits ICL2 and NICL are not that different in terms of aggregate welfare (and sometimes ICL2 generates more borrowers and lower CEL). As NICL forces agents to pay down student debt regardless of income state, ICL2 lets agents accumulate interest in student debt when earnings are low (this becomes less of an attractive outcome when $\underline{a}$ is too loose).

Having conducted a sensitivity analysis to illustrate how aggregate welfare is affected by debt limits and thus how higher education funding policies interact with the price and quantity effects of debt, the next section looks at disaggregated measures of welfare of each system under the baseline calibration described in Table (1).

²⁶These animations can be found at <http://gustavomellior.com/animations/>.

²⁷Comparisons between TS and NICL are omitted for brevity but can be reproduced upon request.

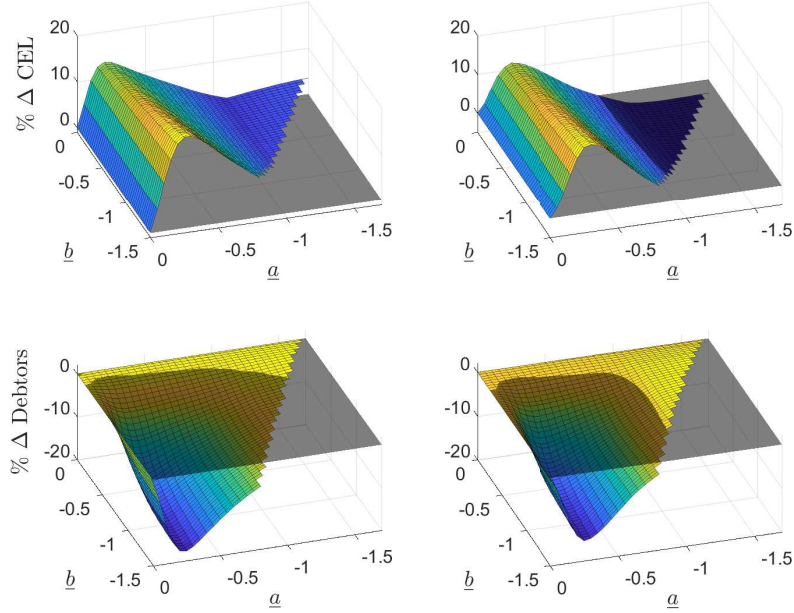


Figure 10: Left to right: $CEL_{ICL1} - CEL_{NICL}$ and $CEL_{ICL2} - CEL_{NICL}$ and difference in net debtor share

2.2 Disaggregated CEL

A benefit of working with a heterogeneous agent model is that we can disaggregate the CEL and see how it fares at each point of the state space²⁸ using equation (22). Due to the fact that the state space is not the same between systems without student loans (SF and TS) and those that do have them (NICL, ICL1 and ICL2), I compare single asset and student loan systems in two different ways. The first method, presented further below, sets SF as a benchmark V_o for each grid point. In an separate appendix that can be given upon request, disaggregated comparisons are performed between systems with the same state space only²⁹. In this section I concentrate on the two systems delivering the largest welfare gains - TS and ICL1 - and results for ICL2 and NICL are left in the appendix

Self financing and tuition subsidies: Figure (11) illustrates the unweighted and disaggregated CEL of the single asset regimes considered back in Table (2). The student type is omitted to save space and since it generally has small mass relative to the other types. Figure (11) reveals that the disaggregated CEL is not monotone in assets. Virtually everybody in the state space favours the policy change from self financing towards high partial or full public financing of tertiary education. The exception are the asset rich agents in TS % 50. This is partly due to the lower interest rate ($r_{SF} > r_{TS50}$), as will be shown in the partial equilibrium exercises in the next subsection. Nevertheless, once we weigh who finds themselves in this region post-reform it turns out that it is less than 0.01% of the population. It is clear that comparing raw value functions is not enough as each higher education financing scheme yields a different distribution. Hence, I now compare distributions by taking the mass in each TS regime and then subtracting that of SF. This allows us to see how tuition subsidies changes the distribution, reduces wealth inequality and increases the capital stock. Values below (above) zero tell us that SF places more

²⁸Remark that each higher education system will generate a distinct density. In order to abstract from different general equilibrium densities, which may place more or less weight in different parts of the state space, I compare raw value functions following (22). I elaborate on densities further below.

²⁹That is, SF vs. TS, NICL vs. ICLs, ICL1 vs ICL2. A disaggregated CEL of NICL vs. ICL1, the worst and best student loan programs, is nonetheless shown in Figure (30) in the appendix.

(less) mass, relative to TS, in that region.

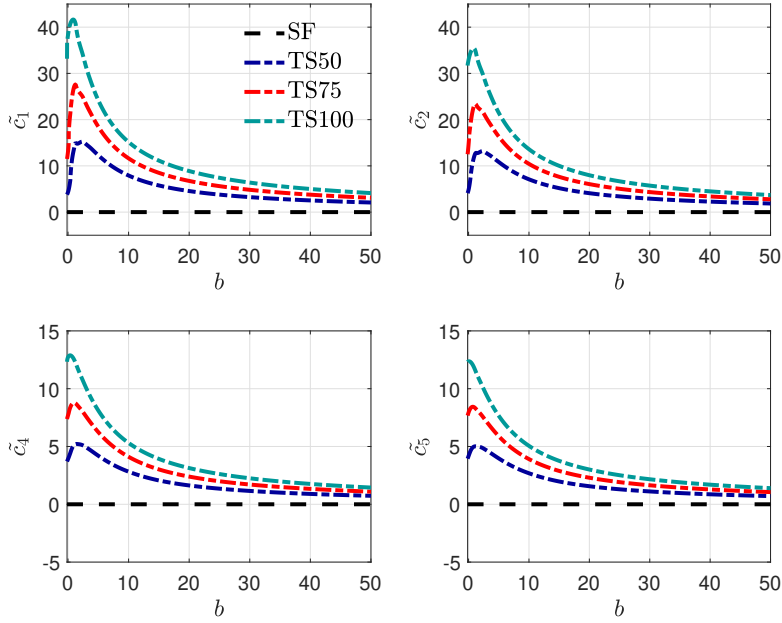


Figure 11: Disaggregated CEL in baseline calibration

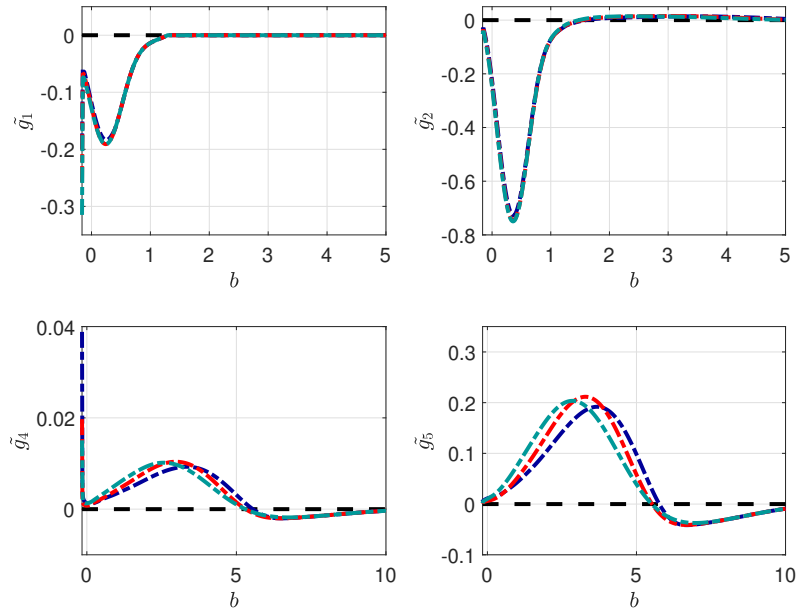


Figure 12: Density difference $\tilde{g} = g_{TS} - g_{SF}$

Figure (12) shows how partial and full subsidisation of tuition puts more mass in moderately high values of b and less on the high-low extremes³⁰. As tuition subsidies increase, precautionary savings fall and r goes up, as can be seen in Table (2). This would be even more patent if employment and skill depreciation transition rates were more favourable for educated agents (lower risk of falling in bad states). The disaggregated results for NICL vs ICLs are shown next.

³⁰Angelopoulos et al. (2017) reach similar findings in an environment where education types are exogenously defined.

Government guaranteed student loans ICL1: The disaggregated CEL of ICL1 reveals the same forces outlined earlier. Relative to SF, ICL1 is more favourable for lower income and lower wealth agents. Besides the income contingency protections, ICL1 has a lower equilibrium interest rate. This benefits borrowers and not lenders. This is reflected in Figure (13); the point where SF performs better is when asset income dominates labour income. As the student loan balance increases, ICL1 becomes less desirable, and this is most patent in the lower right panel of the figure, depicting educated and employed agents. This is precisely where agents have to pay back student loans. We will see how these effects accentuate when we repeat this exercise in a partial equilibrium setting.

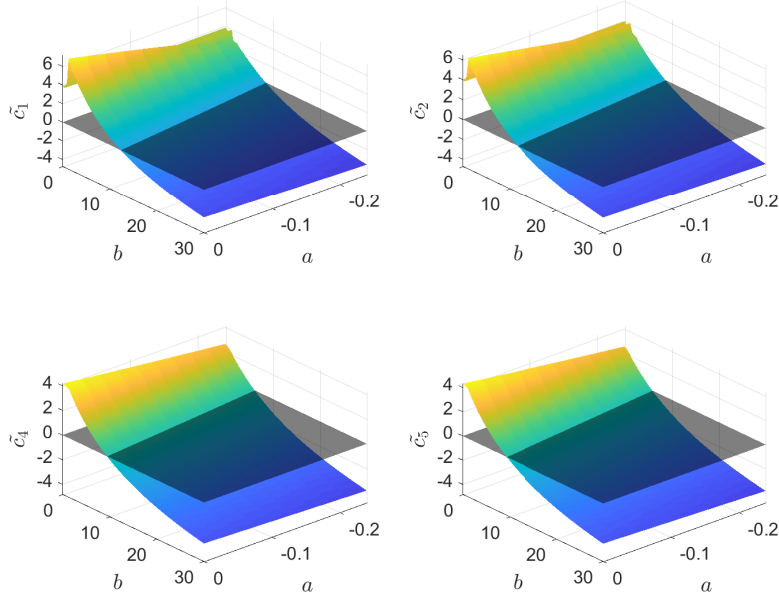


Figure 13: General equilibrium disaggregated CEL in ICL1

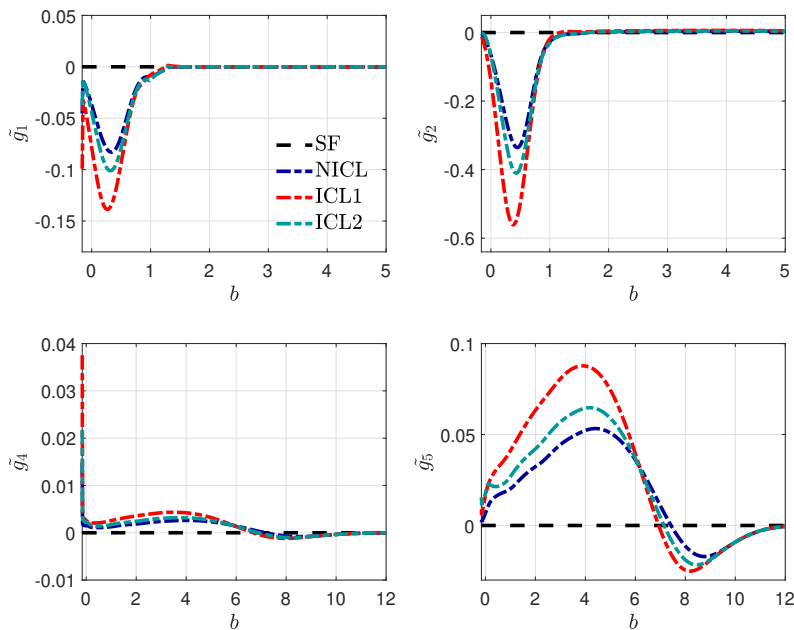


Figure 14: Density difference $\tilde{g} = g_{ICL1} - g_{SF}$

The density difference between student loan programs and SF is shown in³¹ Figure (14). The density of NICL is the closest to that of SF and has a larger spread than those of the income contingent loan systems. As the student loan program becomes more generous, precautionary savings decrease and so does the spread of the distribution. Furthermore, as the loan program becomes more generous, more mass is placed in the educated group (the lower row in the figure). Given the debt cancellation offered in the income contingent loan programs, more people will have no student loans in the ICLs than in NICL; the distribution will place more mass in the regions where welfare gains are strongest.

2.3 Partial and general equilibrium comparisons

This section demonstrates how results in partial, as opposed to general equilibrium, can double or even eliminate welfare gains. Changes in welfare are once again tightly linked to the price effects of debt mentioned earlier. This is illustrated by evaluating the CEL (relative to SF) of changes in HE financing before market forces respond, i.e. by holding the prices of labour and capital fixed³². I will first report aggregate results, shown in Figure (15) and Table (3). I then repeat the disaggregated exercise, comparing raw value functions and densities.

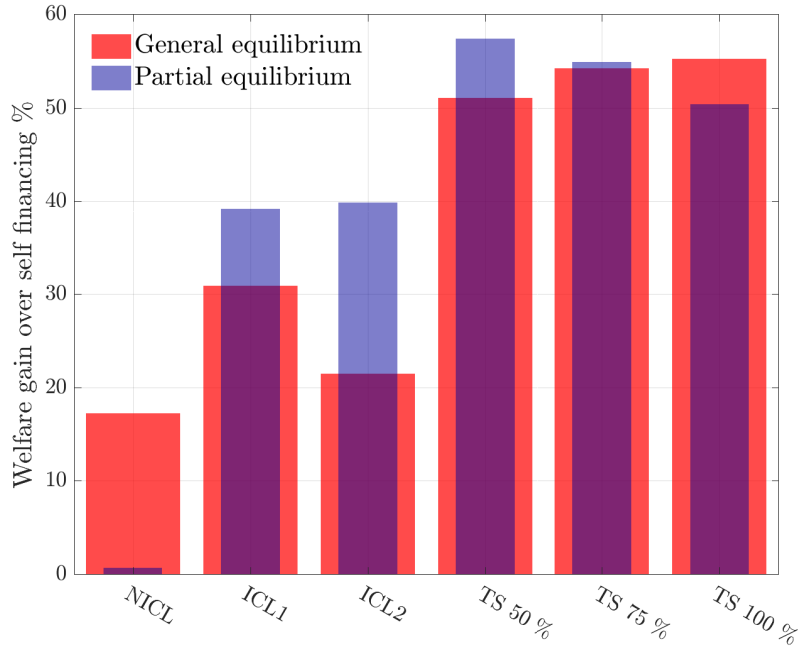


Figure 15: Partial vs general equilibrium steady state welfare gains

Figure (15) and Table (3) reveal that imposing SF prices and tax rate leads substantial welfare improvements (relative to when we let markets clear) in the ICLs, moderate improvement in TS 50%, small loss of welfare gains in TS 100 % and a complete loss of all welfare gains NICL. The change in rankings and in welfare gains broadly follow the price effects of debt and the relative change in the exposure to idiosyncratic shocks. With regards to the former it must be noted that the general equilibrium interest rate in SF is higher than in the student loan programs and TS 50 %, about the same in TS 75 % and lower than in TS 100 %. A higher r benefits lenders and harms borrowers so the direction of additional welfare gains in partial equilibrium

³¹Welfare and density comparison between NICL and ICL1 and NICL and ICL2 can be found in the appendix in Figures (30) - (31).

³²The income tax rate is also held fixed at its equilibrium value prior to the enactment of the new HE financing policy. Prices and taxes are fixed at the SF equilibrium values.

will ultimately depend on where does the endogenous distribution of income and wealth places more mass. This takes us to the second point, how prices interact with the relative change to exposure to idiosyncratic shocks. Systems with income contingency protections such as ICL1 and ICL2 shield agents from movements in the interest rate during bad times (when unemployed or with low income), effectively reducing the risk of investing in education. A higher interest rate increases student loan payments; in NICL this happens in both good and bad states, elevating the riskiness of educational investments. In the next few paragraphs I elaborate on how these forces (price effects of debt) and exposure to risk compound in each system.

	SF	NICL	ICL1	ICL2	TS 50 %	TS 75 %	TS 100 %
CEL %	0.0000	0.0072	39.2704	39.9776	57.4633	54.9671	50.4247
$\mathbb{E}[z]$	0.4254	0.4254	0.5395	0.5372	0.5744	0.5774	0.5791
θ_1	0.1303	0.1303	0.0336	0.0355	0.0040	0.0014	0.0000
θ_2	0.5155	0.5154	0.1733	0.1800	0.0687	0.0596	0.0544
θ_3	0.0324	0.0324	0.0726	0.0718	0.0849	0.0860	0.0866
θ_4	0.0160	0.0160	0.0358	0.0354	0.0418	0.0423	0.0426
θ_5	0.3058	0.3059	0.6848	0.6773	0.8006	0.8107	0.8164
CV	1.2797	1.2795	0.6694	0.6623	0.4516	0.4324	0.4567
Gini _w	0.6350	0.6350	0.3843	0.3806	0.2571	0.2462	0.2606
Debtors	0.0389	0.0389	0.0206	0.0203	0.0122	0.0099	0.0130

Table 3: $\Delta_{ed} = 1/4 - P = 1.1$

The largest gains relative to general equilibrium are in ICL1 and ICL2. In these two cases the composition of the distribution improves; there are more students and more college educated agents (employed and unemployed). These systems now have a larger share of the population facing a better labour market (lower transitions to unemployment and higher ones to employment) and diminished uncertainty. Given that we are holding prices fixed at the SF level, wages do not respond to the influx of newly educated workers - so average pay is higher. Remark that w remains fixed, but the composition of the labour force affects $\mathbb{E}[z]$. The interest rate is higher than when we let markets clear, which benefits a distribution that generates less debtors. The net debtor share and inequality fall. Despite higher income taxes, aggregate welfare is higher when holding prices fixed. The results in TS 50 % follow the same pattern.

Focusing on TS 100% reveals an interesting feature. Almost all distributional effects are isolated as the composition of types remains the same as when we let markets clear. Thus we have something close to a pure price effect since the only differences lie in labour income, interest rates and in the income tax. Nonetheless, while the composition of types is close to identical we can still have differences in asset holdings; the debtor share rises in the partial equilibrium exercise. The interest rate is lower than when markets clear, so lenders receive less asset income. As shown further below (see Figure 17), the distribution of wealth will shift to lower values of b . So despite more debtors paying less interest, the shifts in the distribution are both bad for lenders and borrowers, dragging down aggregate welfare. Despite a lower income tax and higher expected labour earnings, aggregate welfare gains over SF are about four percentage points smaller than when markets clear. As mentioned above, TS 50% goes in the opposite direction, benefiting the most (out of the three TS variants) in the partial equilibrium exercise. It has a relatively higher interest rate, improvements in educational attainment and unambiguously lower inequality. The distribution of wealth places more mass to the right of the general equilibrium result. So we now have less debtors, more lenders and higher capital income. These effects dominate a higher income tax rate and slightly lower labour earnings than when markets clear.

In NICL, expected earnings are substantially lower and interest rates are much higher than when markets clear. These two results combined with no income contingency protections lower the attractiveness of earning a college degree. The distribution worsens relative to when markets

clear and inequality rises, fairing marginally better than SF. Almost all welfare gains vanish when we do not let markets clear in NICL, whereas they double in ICL2. This highlights the importance of evaluating higher education policy changes in general equilibrium.

Finally, I repeat the exercise of section 2.1, on borrowing limits and welfare, but this time in partial equilibrium. Figures (34) and (35) in the appendix show how solving the model in partial equilibrium can over (under) estimate welfare gains at low (high) subsidy rates and student loan debt limits. The over/under estimation follows closely the net debtor share and wealth inequality, illustrating once again how the distributional and price effects of debt are key determinants of aggregate welfare. In the next subsections I evaluate disaggregated comparisons as done earlier but in a partial equilibrium setting.

2.3.1 Disaggregated partial equilibrium results

Disaggregated partial equilibrium in TS: Figure (16) reveals two results that stand out. First, all TS variants are superior to SF, everywhere in the state space, in the partial equilibrium exercise. Second, when prices are fixed at SF the welfare gains of TS 100% (which has a higher interest rate in general equilibrium) decrease for wealthy agents. The opposite happens in TS 50% (asset poor agents have smaller welfare gains relative to general equilibrium but asset rich agents gain more). Note that r in SF is not that different of that of TS 75 % when markets clear so there is not much change between the partial and general equilibrium CELs across all of the state space.

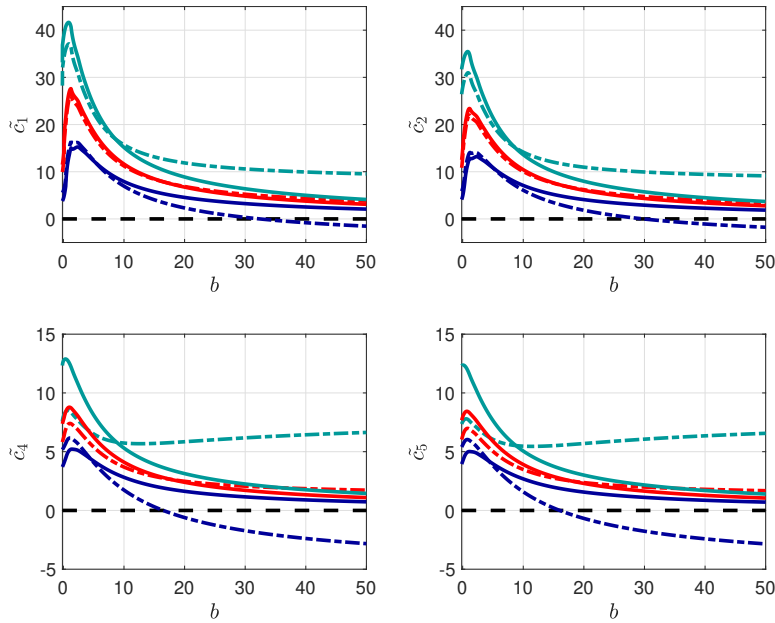


Figure 16: Dash/dotted lines are the same as in Figure (11), solid lines introduce the new educational policy but hold all SF prices and tax rate fixed.

The distributional effects go in the same direction. Figure (17) depicts the density difference of TS with self financing. TS 50 % benefits the most from the relatively higher pre-reform interest rate and moves the distribution to larger values of b . The opposite happens for TS 100%, the distribution shifts towards lower values of b . As a consequence rankings change between these two variants of tuition subsidies. TS 75 % has very similar prices to those of SF, except for the income tax rate, which is slightly higher in general equilibrium, and sees no major changes in aggregate and individual welfare measures; the distribution is also about the same.

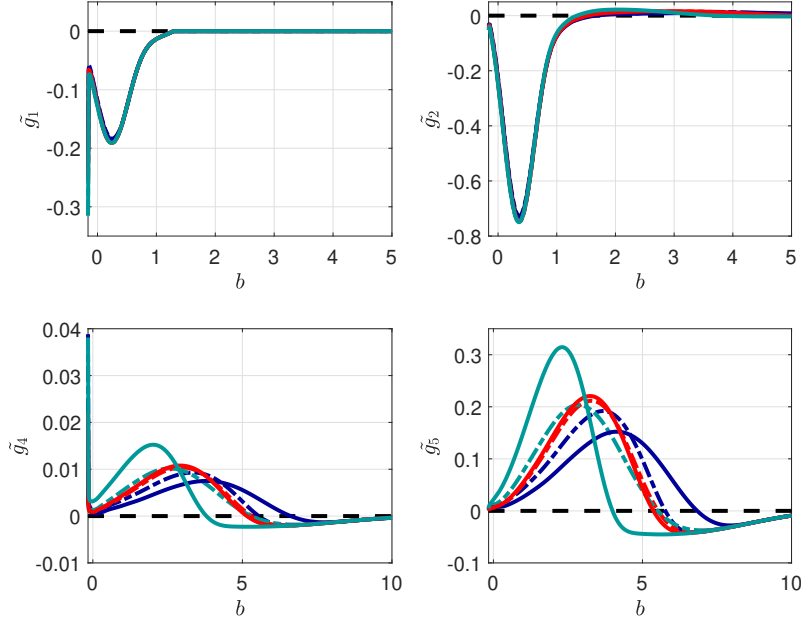


Figure 17: Dash/dotted lines are the same as in Figure (11), solid lines introduce the new educational policy but hold all SF prices and tax rate fixed.

Disaggregated partial equilibrium in ICL1: As mentioned previously, given the difference in state space sizes, I will show the partial equilibrium comparison of every grid point in ICL1 against the benchmark calibration of self financing. Figure (18) depicts the difference in CEL, when we hold the SF prices fixed and then switch HE funding to that of ICL1.

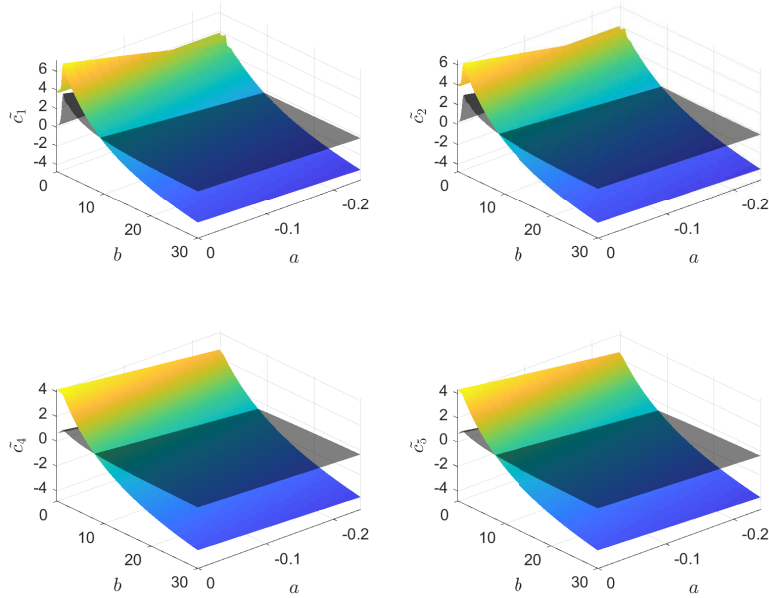


Figure 18: General equilibrium (green-blue) and partial equilibrium (black) disaggregated CEL in ICL1

The interest rate at SF is higher than when we let markets clear at ICL1. Thus, the partial equilibrium CEL surfaces are slightly lower (higher) for low (high) wealth agents relative to those in general equilibrium. The density difference plots are displayed in Figure (33) in the appendix.

The figures reveal a shift in the distribution towards more educated agents and towards larger values of b . The influx of educated workers does not depress wages, as they are fixed at the higher value in SF. The price effects and the endogenous change in the distribution illustrate how ICLs improve under the partial equilibrium experiment (with ICL2 almost doubling aggregate gains, relative to when markets clear). Those most vulnerable (those with high debt in b and a) benefit the most from the income contingent protections in ICL1. Nonetheless, when repayment of student loan starts, the student loan interest rate is much higher than in other systems since $r_A = r + \lambda_{np}$. As a consequence, it should not surprise us that the group of educated and employed agents benefits the least from having the policy change with prices fixed at SF. Hence, while reductions in income uncertainty, thanks to the endogenous change in the composition of types, contribute to welfare gains, these are dampened (increased) by less (more) favourable prices for borrowers (lenders) in partial equilibrium.

It is evident that the price effect described in [Obiols-Homs \(2011\)](#) and the endogenous make up of agent types play an important role in shaping the welfare gains of higher education reforms, especially through the link between the share of the population in net debt and wealth inequality.

3 Transitions

Since there are large steady state welfare differences amongst the five HE systems considered in this paper, is it worth making the transition to the system yielding the largest welfare gain? This section seeks to answer that question. The results shown in Table (2) and Figure (6) indicate that the biggest steady state welfare gain is between SF and TS 100%. Thus the next experiment considers an unexpected, immediate³³ and permanent change of the subsidy rate from %0 to 100%. Figures (19) and (20) show the transitional dynamics of key aggregates.

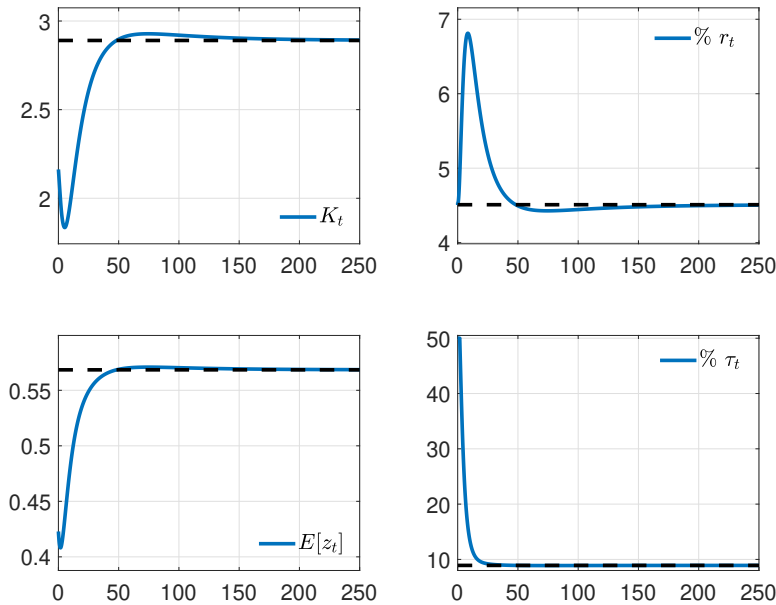


Figure 19: SF to TS %100

³³Similar experiments where policy changes are announced in advance yield lower gains since agents postpone enrolling in university until the subsidy is in place. This has negative aggregate effects given that it initially lowers θ_5 . A perpetual youth or life-cycle formulation would most likely dampen such an effect since the education choice will probably be made once in a single life time and sooner rather than later.

After the policy change the share of workers with a university degree increases, roughly doubling in about 12 years. Wealth inequality and the net debtor population share decrease after an initial spike. During the spike, interest rates remain substantially higher than in the initial and terminal equilibrium. Furthermore, the early years of the transition are accompanied by a large jump in the income tax rate (almost a tenfold increase from the initial SF steady state τ value of 5.81% to 49 %). As seen earlier, higher net debtor shares and interest rates drag down welfare. Given that agents discount future gains, more weight is placed on the immediate sacrifices incurred during the transition.

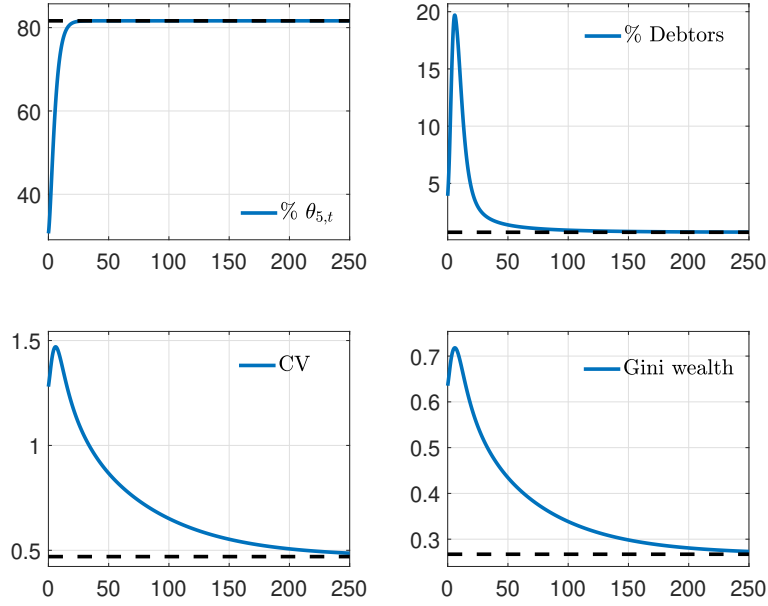


Figure 20: SF to TS %100

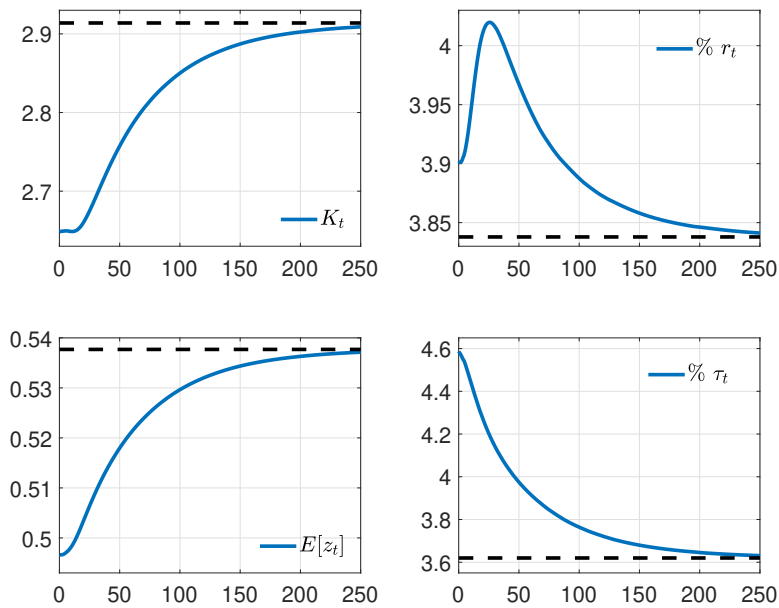


Figure 21: NICL to ICL1

Hence, despite the substantial steady state gains the transition costs make the policy change less appealing than when we compare just steady states. The aggregate CEL of the transition amounts to a gain of 12 %, that is forty three percentage points lower than in the steady state comparisons. This indicates that steady state comparisons alone may be misleading. I repeat the exercise, but this time for a smaller reform, transitioning among two different student loan programs, computing the transition from NICL to ICL1. Could it be that smaller reforms that are less costly make policy changes more viable? The transition paths are shown in Figures (21) and (22).

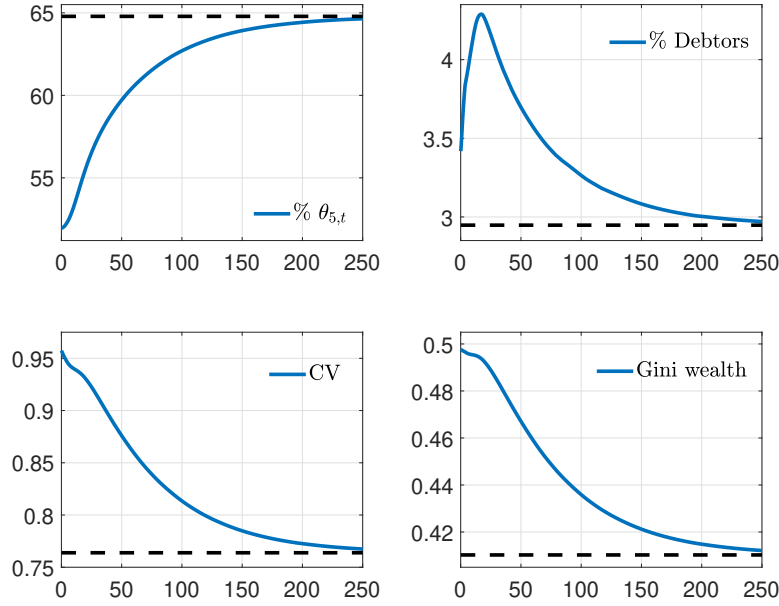


Figure 22: NICL to ICL1

As before, the initial periods are marked by higher taxes, interest rates and indebtedness while gains accrue much later. While the rise in the tax, net debtor population share and interest rate is more muted, the reform yields a positive but small CEL of 0.6223 %. Now that we have an idea of what makes transitions, big or small, costly, the same exercise is repeated for transitions from 1) SF to all possible subsidy rates between 30 and 100 % (say transitioning from SF to TS 30 % and from SF to TS 40 % and so on) and 2) from SF to ICL1 with student debt limits covering 40 to 200% of the costs of education (for instance, from SF to ICL1 with $\underline{a} = -1$ and then from SF to ICL1 with $\underline{a} = -1.05$ and so on), keeping the rest of the parameters in their baseline calibration.

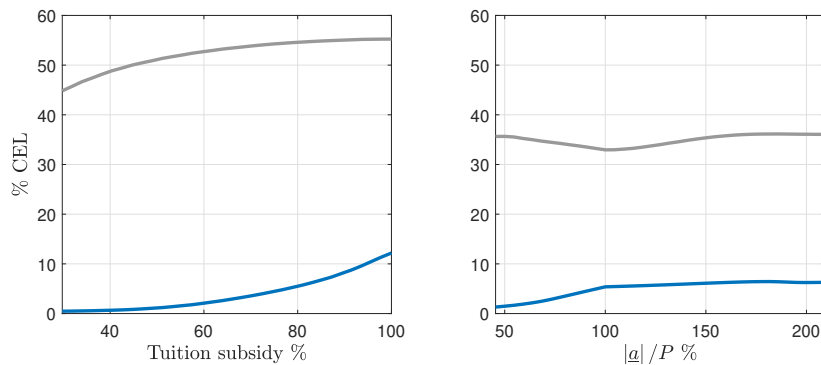


Figure 23: Steady state CEL gain (grey) and dynamic CEL (in blue) - SF to TS - right - SF to ICL1

Figure (23) reveals that when we compute the transition between regimes, the welfare gains can be four to ten times smaller than when just comparing the steady state CELs. Figure (36) in the appendix does not overlay the steady state gains to those factoring the transition costs, so that one can zoom in on the scale of gains. This last experiment reveals that moving towards moderate partial tuition subsidies ($s < 50\%$) is dominated by ICL1. However, generous tuition subsidies (closer to 100 %) outperform ICL1, regardless of the student loan limit. It is clear that policy changes in higher education financing should factor the costs of transitions.

4 Conclusion

In this paper I evaluate the welfare and wealth inequality outcomes of five different higher education financing schemes, with the help of a heterogeneous agent production economy in continuous time, extended to allow for endogenous educational choices. The main contribution of this study is to evaluate the financing of tertiary education under the light of *the price and quantity effects of debt*. When we ignore the pecuniary externalities described in [Obiols-Homs \(2011\)](#), [Nuño and Moll \(2017\)](#) and [Angelopoulos et al. \(2017\)](#) we miss general equilibrium effects that are powerful enough to tilt the balance on which higher education system yields the largest aggregate and individual welfare gains. This article also contributes in identifying when government intervention in tertiary education can increase welfare, reduce inequality and at what cost. This contribution can be broken into four findings.

First, the ranking between systems depend on the cost of education. If the price of education is low and the time length of study is short, then tuition subsidies or student loans may drag down welfare relative to self financing. That is, when education is relatively easy to achieve, the capital market failures associated with educational investments do not matter enough to warrant government intervention. When the costs of education are calibrated to realistic values, government guaranteed income contingent loans and tuition subsidies are found to be the best choice, out of the five systems considered herein, to finance higher education, with the latter yielding the largest steady state gains.

Second, while I show significant steady state welfare differences between various higher education systems, large transition costs from one regime to another diminish the desirability of policy changes. That is, comparing steady states alone may be misleading for policy, transition costs must be factored in.

Third, balanced budget tax rates can be higher (relative to regimes with tuition subsidies) in systems relying on income contingent student loans. Fourth, public financing of higher education only increases inequality when the cost of education is extremely low; if this cost rises to realistic levels inequality falls as public sector support increases. This is particularly true with tuition subsidies; they yield the lowest inequality outcomes in all the systems considered in this paper.

This paper will be extended along three directions. The first extension addresses the fact that this study has abstracted from important dimensions such as labour skill substitutability, age and ability. A future extension of this work will include these dimensions, emphasise educational investments as a once-in-a-lifetime decision and explore the interactions between higher education funding and intergenerational inequality.

A separate extension will disaggregate the college wage premium, as recent UK based studies - [Belfield et al. \(2018a\)](#) and [Belfield et al. \(2018b\)](#) - have shown that while the average premium has remained stable, if not rising, it is driven by substantial heterogeneity and large outliers, where the returns to higher education vary by subject, institution and agents' gender. Should we fund higher education generously when the distribution of returns is highly skewed? The third avenue that will be pursued seeks to understand how increased longevity and increasing exposure

to automation risk may warrant repeated educational investments and additional government support in higher education. Would longer lives lead us to retrain and go back to university at older ages? What role can the public sector play to mitigate the riskiness and capital market imperfections affecting tertiary education?

References

- ABBOTT, B., G. GALLIPOLI, C. MEGHIR, AND G. L. VIOLANTE (2013): “Education policy and intergenerational transfers in equilibrium,” NBER Working Papers 18782, National Bureau of Economic Research.
- ACHDOU, Y., J. HAN, J.-M. LASRY, P.-L. LIONS, AND B. MOLL (2017): “Income and wealth distribution in macroeconomics: A continuous-time approach,” NBER Working Papers 23732, National Bureau of Economic Research.
- AIYAGARI, S. R. (1994): “Uninsured Idiosyncratic Risk and Aggregate Saving,” *The Quarterly Journal of Economics*, 109, 659–684.
- ANGELOPOULOS, K., S. LAZARAKIS, AND J. R. MALLEY (2017): “Wealth inequality and externalities from ex ante skill heterogeneity,” .
- ARRAZOLA, M., J. DE HEVIA, M. RISUEÑO, J. F. SANZ, ET AL. (2005): “A proposal to estimate human capital depreciation: Some evidence for Spain,” *Hacienda Publica Espanola-Revista de Economia Publica*, 172, 9–22.
- ATHREYA, K., F. IONESCU, U. NEELAKANTAN, AND I. VIDANGOS (2019): “Who Values Access to College?” Finance and Economics Discussion Series 2019-015, Washington: Board of Governors of the Federal Reserve System.
- BARR, N. AND I. CRAWFORD (2000): “Student Loans: A Hungarian Proposal,” Discussion paper.
- BELFIELD, C., J. BRITTON, F. BUSCHA, L. DEARDEN, M. DICKSON, L. VAN DER ERVE, A. SIBIETA, A. VIGNOLES, I. WALKER, AND Y. ZHU (2018a): “The impact of undergraduate degrees on early-career earnings,” Institute for Fiscal Studies and Department of Education.
- BELFIELD, C., J. BRITTON, F. BUSCHA, L. DEARDEN, M. DICKSON, L. VAN DER ERVE, L. SIBIETA, A. VIGNOLES, I. WALKER, AND Y. ZHU (2018b): “The relative labour market returns to different degrees: Research report: June 2018,” Institute for Fiscal Studies and Department of Education.
- BEN-PORATH, Y. (1967): “The production of human capital and the life cycle of earnings,” *Journal of Political Economy*, 75, 352–365.
- BÉNABOU, R. (2002): “Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?” *Econometrica*, 70, 481–517.
- BOVENBERG, A. L. AND B. JACOBS (2005): “Redistribution and education subsidies are Siamese twins,” *Journal of Public Economics*, 89.
- CAI, Z. AND J. HEATHCOTE (2018): “College tuition and income inequality,” CEPR Discussion Paper No. DP13101, CEPR.
- CARNEVALE, A. P., N. SMITH, M. MELTON, E. PRICE, ET AL. (2015): “Learning While Earning: The New Normal.” *Georgetown University Center on Education and the Workforce*.
- DÁVILA, J., J. HONG, P. KRUSELL, AND J. V. RÍOS RULL (2012): “Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks,” *Econometrica*, 80, 2431–2467.

- DEARDEN, L., E. FITZSIMONS, A. GOODMAN, AND G. KAPLAN (2008): “Higher education funding reforms in England: The distributional effects and the shifting balance of costs,” *The Economic Journal*, 118, F100–F125.
- GARCÍA-PENALOSA, C. AND K. WÄLDE (2000): “Efficiency and equity effects of subsidies to higher education,” *Oxford Economic Papers*, 52, 702–722.
- HANUSHEK, E. A., C. K. Y. LEUNG, AND K. YILMAZ (2003): “Redistribution through education and other transfer mechanisms,” *Journal of Monetary Economics*, 50, 1719–1750.
- (2014): “Borrowing constraints, college aid, and intergenerational mobility,” *Journal of Human Capital*, 8, 1–41.
- HERRINGTON, C. M. (2015): “Public education financing, earnings inequality, and intergenerational mobility,” *Review of Economic Dynamics*, 18, 822–842.
- HUANG, J. AND J.-S. PANG (1998): “Option pricing and linear complementarity,” *Journal of Computational Finance*, 2, 30–61.
- HUGGETT, M. (1993): “The risk-free rate in heterogeneous-agent incomplete-insurance economies,” *Journal of Economic Dynamics and Control*, 17, 953–969.
- HUGONNIER, J., F. PELGRIN, AND P. ST-AMOUR (2019): “Self-Inflicted Unemployment Scarring and Stigma,” .
- IONESCU, F. (2009): “The federal student loan program: Quantitative implications for college enrollment and default rates,” *Review of Economic dynamics*, 12, 205–231.
- IONESCU, F. AND N. SIMPSON (2016): “Default risk and private student loans: Implications for higher education policies,” *Journal of Economic Dynamics and Control*, 64, 119–147.
- JAMES, J. (2012): “The college wage premium,” Economic Commentary 2012-10, Federal Reserve Bank of Cleveland.
- JOHNSON, M. T. (2013): “Borrowing constraints, college enrollment, and delayed entry,” *Journal of Labor Economics*, 31, 669–725.
- KIRBY, P. (2016): “Degrees of Debt: Funding and Finance for Undergraduates in Anglophone Countries.” Tech. rep., Sutton Trust.
- KRUEGER, D. AND A. LUDWIG (2016): “On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium,” *Journal of Monetary Economics*, 77, 72–98.
- LAMADON, T., J. LISE, C. MEGHIR, AND J.-M. ROBIN (2013): “Matching, Sorting, and Wages,” Discussion paper.
- LUO, M. AND S. MONGEY (2019): “Assets and Job Choice: Student Debt, Wages and Amenities,” Discussion paper.
- MANUELLI, R., A. SESHADRI, AND Y. SHIN (2012): “Lifetime labor supply and human capital investment,” Discussion paper.
- MOLL, B. (2016): “Exercising an Option: Numerical Solution,” Technical note.
- NUÑO, G. AND B. MOLL (2017): “Social optima in economies with heterogeneous agents,” *Review of Economic Dynamics*.

- OBIOLS-HOMS, F. (2011): “On borrowing limits and welfare,” *Review of Economic Dynamics*, 14, 279–294.
- SONNET, A. (2010): *Off to a good start? Jobs for youth*, OECD.
- STAKLIS, S. AND A. BENTZ (2016): “Employment and Enrollment Status of Baccalaureate Degree Recipients 1 Year after Graduation: 1994, 2001, and 2009. Web Tables. NCES 2017-407.” *National Center for Education Statistics*.
- THE COLLEGE BOARD (2019): “Trends in College Pricing,” Trends in Higher Education Series.
- VALLETTA, R. G. (2018): “Recent flattening in the higher education wage premium: Polarization, skill downgrading, or both?” in *Education, skills, and technical change: Implications for future US GDP growth*, University of Chicago Press, 313–342.

5 Appendix

5.1 Market clearing

In this subsection I show a heuristic proof of how to show that $K_S = K_D$ implies $Y = C + I + \text{Ed costs}$. The same steps can be applied to any regime and will lead to the same conclusion. For the sake of brevity, I illustrate this with the TS regime³⁴. Following [Nuño and Moll \(2017\)](#) we start by the aggregate law of motion of capital.

$$\begin{aligned}
\frac{d}{dt} [b_i g_i] &= S_i g_i + b_i \dot{g}_i \\
\sum_{i=1}^5 \int_b \frac{d}{dt} [b_i g_i] db &= \sum_{i=1}^5 \int_b S_i g_i db + \sum_{i=1}^5 \int_b b \dot{g}_i db \\
\frac{d}{dt} \sum_{i=1}^5 \int_b [b_i g_i] db &= \sum_{i=1}^5 \int_b S_i g_i db + \sum_{i=1}^5 \int_b b \dot{g}_i db \\
\frac{d}{dt} K &= \sum_{i=1}^5 \int_b S_i g_i db + \sum_{i=1}^5 \int_b b \dot{g}_i db \\
\dot{K} &= \sum_{i=1}^5 \int_b S_i g_i db + \sum_{i=1}^5 \int_b b \dot{g}_i db \\
0 &= \sum_{i=1}^5 \int_b S_i g_i db + \sum_{i=1}^5 \int_b b \dot{g}_i db
\end{aligned} \tag{24}$$

Expanding the first term gives us the following result.

$$\begin{aligned}
&w [(1 - \tau)[z_L \theta_2 + z_H \theta_5] + \mu[z_L \theta_1 + z_H \theta_4] + z_L z_s \theta_3] + r \sum_{i=1}^5 \int_{\underline{b}}^{\bar{b}} b g_i db - C \\
&\frac{(1 - \alpha)Y}{\tilde{L}} [z_L \theta_2 + z_H \theta_5 + z_L z_s \theta_3] - sP \int_{b^\dagger}^{\bar{b}} g_1 db + \left[\frac{\alpha Y}{K_D} - \delta \right] K_S - C \\
&(1 - \alpha)Y + \alpha Y - I - C - sP \int_{b^\dagger}^{\bar{b}} g_1 db
\end{aligned} \tag{25}$$

The last step requires that $K_S = K_D$, which is what was intended to be shown. Expanding the second term in (24) yields aggregate education costs. The KFEs of the single asset economy are given by the following five expressions.

$$\partial_t g_1 = -\partial_b [\dot{b}_1 g_1] + \lambda_{21} g_2 - \lambda_{12} g_1 + \lambda_{ex}^S g_3 + \lambda_{ex}^U g_4 - g_1 \delta (b - b^\dagger) \tag{26}$$

$$\partial_t g_2 = -\partial_b [\dot{b}_2 g_2] + \lambda_{12} g_1 - \lambda_{21} g_2 + \lambda_{ex}^E g_5 \tag{27}$$

$$\partial_t g_3 = -\partial_b [\dot{b}_3 g_3] - [\lambda_{ex}^S + \lambda_{34} + \lambda_{35}] g_3 + g_1 \delta (b - b^*) \tag{28}$$

$$\partial_t g_4 = -\partial_b [\dot{b}_4 g_4] + \lambda_{54} g_5 - [\lambda_{ex}^U + \lambda_{45}] g_4 + \lambda_{34} g_3 \tag{29}$$

$$\partial_t g_5 = -\partial_b [\dot{b}_5 g_5] + \lambda_{35} g_3 + \lambda_{45} g_4 - [\lambda_{ex}^E + \lambda_{54}] g_5 \tag{30}$$

³⁴With two asset models we have to perform multivariate integration parts. I omit the derivation in this paper, but can be reproduced upon request.

Adding the KFEs, multiplying by b and integrating, as in (24), yields the next result.

$$\sum_{i=1}^5 \int_b b g_i db = - \sum_{i=1}^5 \int_b \partial_b [S_i g_i] db + \int_b b g_1 [\delta(b - b^*) - \delta(b - b^\dagger)] db \quad (31)$$

Using integration by parts one can show that the first term on the right hand side is equal to zero. The second term captures education costs covered by agents, the difference between b^* and $b^\dagger$ is $P(1 - s)$. Putting everything together gives us the national accounting identity, augmented with aggregate flow educational expenditure. Since we are solving for the steady state, these educational costs will be equal to the depreciation of the stock of those with higher education.

$$\begin{aligned} 0 &= Y - I - C - P \int_{b^\dagger}^{\bar{b}} g_1 db \\ Y &= I + C + \text{Ed costs} \end{aligned} \quad (32)$$

The proof for models with student loans, although a bit more tedious due to two types of assets, can be shown following the same steps.

5.2 Portfolio problem and pecking order

An earlier version of this model allowed agents to maximise V_3 by choosing how to pay P with the best feasible combination of b and a in the NICL, ICL1 and ICL2 regimes. The results are identical to the ones presented here. Except for unrealistic calibrations, agents rarely pay for university using exclusively b . This motivated the use of a so-called ‘pecking order’ mechanism to model the decision of how to cover P . This is computationally less expensive. An example of how this works is shown in Figure (24).

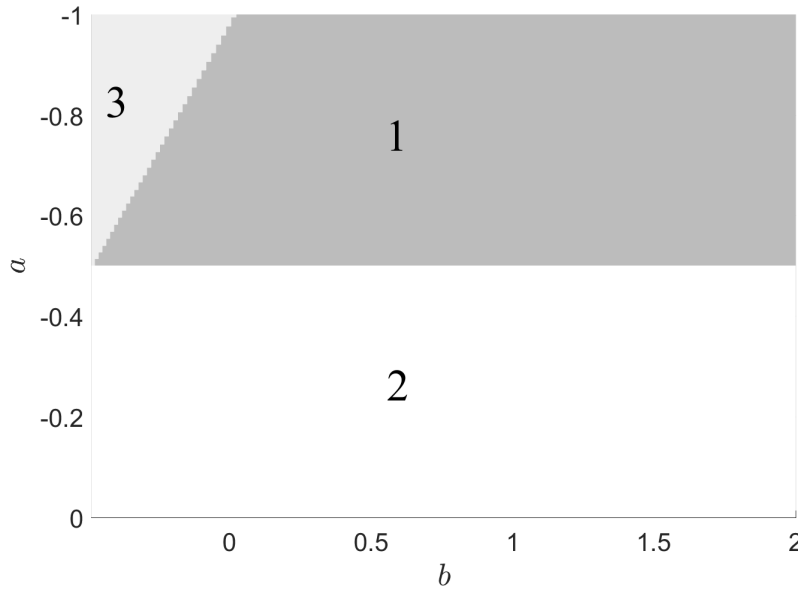


Figure 24: 1 - Cover tuition with mix of b and a . 2 - Cover tuition with a only. 3 - Cannot afford to go to university.

Suppose that $P = 0.5$, $\underline{a} = -1$ and $\underline{b} = -0.5$. The agent will first try to cover P exclusively with student loans, a situation depicted in region 2. If the agent has more than 0.5 in student loans, it will only be able to cover the difference between $\underline{a}$ and -0.5 in new student loans, and cover the rest of P with b . This case is that of region 1. Finally, if the agent has little b and a large stock of a , then it will not be able to go to university, the case of region 3.

5.3 Benchmark calibration of P

In 2018 U.S. GDP per capita stood at \$54541 according to the World Bank. The U.S. Bureau of Economic Analysis reports that U.S. population reached 327.436 million while the OECD estimates that the working age population in the U.S. stood at 206.513 million in 2018. Given that in the ‘NICL’ regime I set $s = 0$, I calibrate P to the cost of attending a private non-profit university. According to the College Board, the average annual cost of an American private non-profit university in 2018 was \$37430 (tuition) and \$48510 (tuition and board). The benchmark calibration of P can be thus set to:

$$\begin{aligned} P_B &= \frac{37430 * 4}{\frac{327.436}{206.513} * 54541} * w\mathbb{E}[z] && \text{Lower bound} \\ \bar{P}_B &= \frac{48510 * 4}{\frac{327.436}{206.513} * 54541} * w\mathbb{E}[z] && \text{Upper bound} \end{aligned}$$

When equilibria deliver a positive amount of college graduates, $w\mathbb{E}[z]$ is fairly stable between 0.5 and 0.59, with an average of 0.56. So for the benchmark calibration I choose 0.56. Given that $w\mathbb{E}[z]$ is an endogenous result and the uncertainty around to what extent do we include student accommodation expenses in the costs of education, the reader is asked to consider P as an arbitrary yet reasonable pick for a benchmark. The sensitivity analysis in P gives further indication of how each regime fares with a large range of educational prices. As mentioned earlier, a perpetual youth extension of this paper endogenises P and matches it’s relationship to average income in the United States. The results are fairly consistent with what is shown here.

5.4 Results when P is low and Δ_{ed} is high

When the price of education is low and it takes less time to become educated, the capital market imperfection in educational investment is no longer large enough to justify government support, either with student loans or tuition subsidies. As shown in Table (4) the CEL over self financing is now negative for all regimes. All HE systems give more or less the same share of educated workers. Nonetheless, income taxes are lower in self financing than in any other system. As mentioned earlier, the relationship between inequality and government support is now reversed; more government support fosters inequality. Additionally, the rankings have mostly reversed. Tuition subsidies yield the lowest CEL. The relationship between CEL and the population net debtor share seems to have weakened.

	SF	NICL	ICL1	ICL2	TS50%	TS75%	TS100%
CEL %	0.0000	-0.1659	-0.1981	-0.1596	-0.1480	-0.2691	-0.4061
K	2.9693	2.9529	2.9420	2.9505	2.9047	2.8821	2.8648
$\mathbb{E}[z]$	0.5909	0.5899	0.5892	0.5897	0.5869	0.5854	0.5843
r	0.0462	0.0465	0.0468	0.0466	0.0476	0.0482	0.0486
Y	0.8654	0.8639	0.8629	0.8637	0.8595	0.8574	0.8557
θ_1	0.0004	0.0003	0.0003	0.0003	0.0001	0.0001	0.0000
θ_2	0.0595	0.0593	0.0593	0.0592	0.0586	0.0583	0.0582
θ_3	0.0231	0.0231	0.0231	0.0231	0.0231	0.0231	0.0231
θ_4	0.0455	0.0455	0.0455	0.0455	0.0456	0.0456	0.0456
θ_5	0.8715	0.8717	0.8717	0.8718	0.8726	0.8729	0.8730
τ	0.0193	0.0196	0.0212	0.0202	0.0308	0.0367	0.0425
τ_{UI}	0.0193	0.0193	0.0193	0.0193	0.0192	0.0192	0.0192
CV	0.4085	0.4175	0.4363	0.4399	0.4659	0.5022	0.5401
Gini _w	0.2306	0.2350	0.2451	0.2478	0.2603	0.2788	0.2984
Debtors	0.0017	0.0014	0.0016	0.0022	0.0012	0.0012	0.0019

Table 4: $\Delta_{ed} = 1 - P = 0.5$

Figure (25) illustrates that steady state CEL gains are robust to different debt limits in b and a . For sake of brevity I only show the cases for NICL (in red) and ICL1 (in yellow/blue)³⁵.

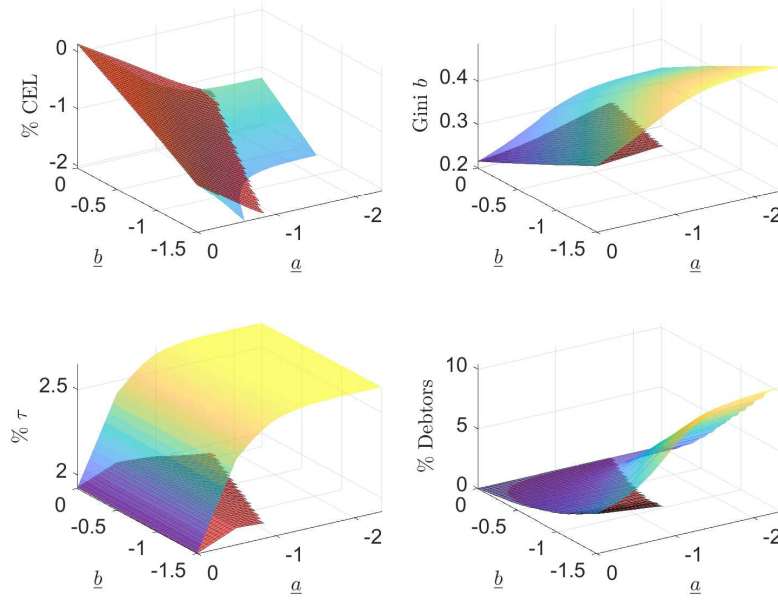


Figure 25: CEL, wealth Gini, tax rate and net debtor share in NICL and ICL1 with $P = 0.5$ and $\Delta_{ed} = 1$. The red (blue/yellow) surface depicts NICL (ICL1).

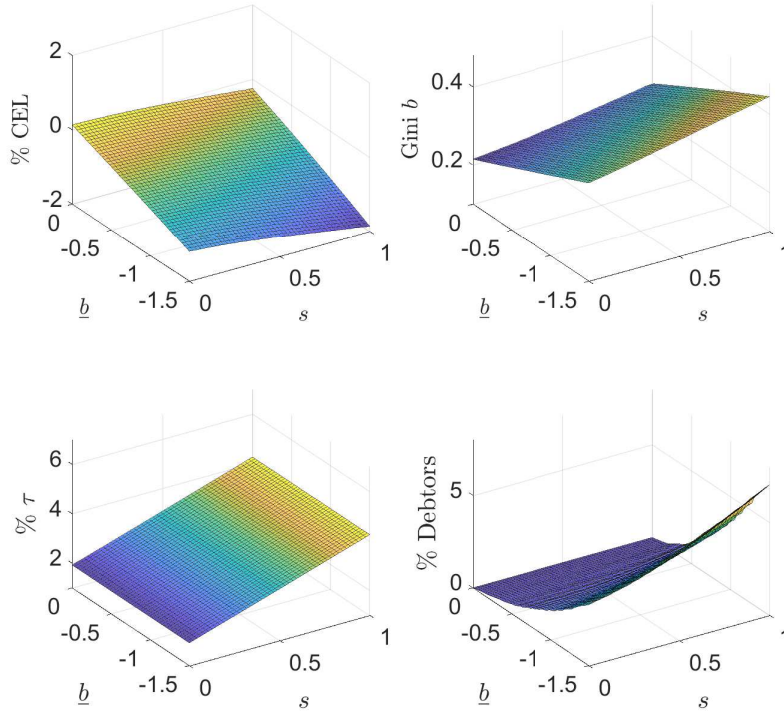


Figure 26: CEL, wealth Gini, tax rate and net debtor share in TS with $P = 0.5$ and $\Delta_{ed} = 1$

³⁵Results for TS and ICL2 can be produced upon request.

5.5 Additional tables and figures

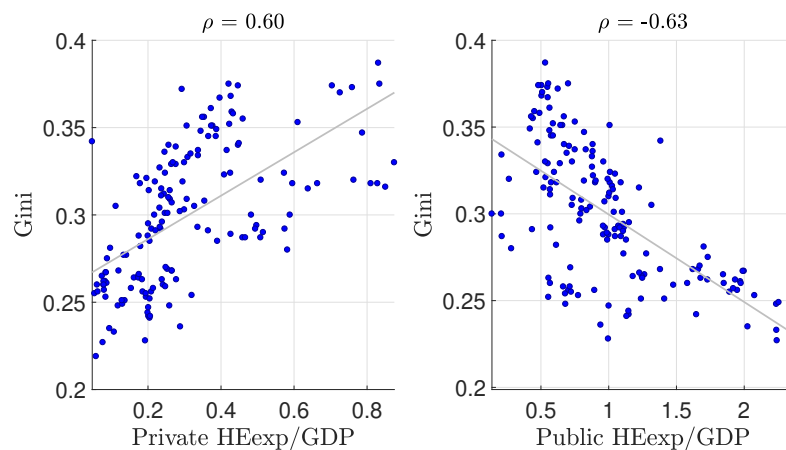


Figure 27: Income Gini and HE expenditure relative to GDP in OECD countries* 2000-2016. Source: OECD. *Mexico and Chile are excluded. Note that, surprisingly, their exclusion does not affect the correlation, even though they are outlier cases.

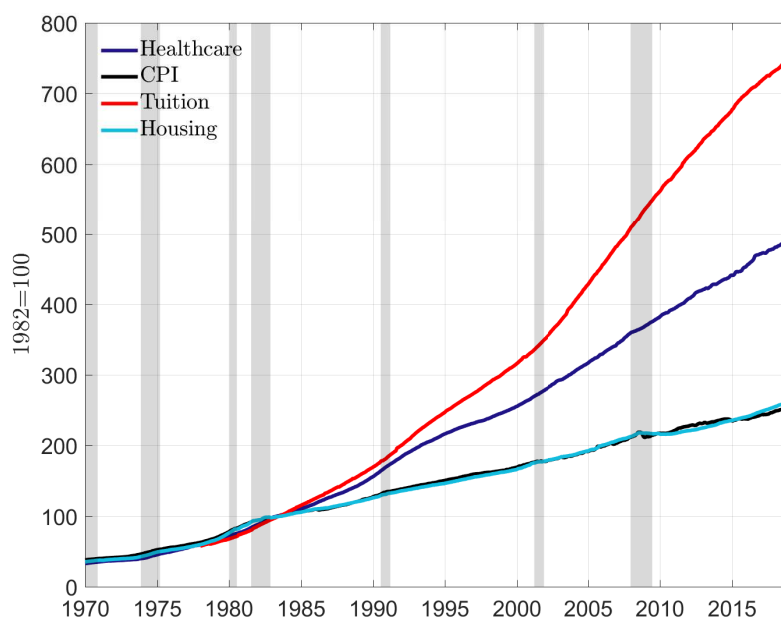


Figure 28: Tuition inflation. Source: US Bureau of Labor Statistics (2019)

5.5.1 Borrowing limits and welfare in ICL2

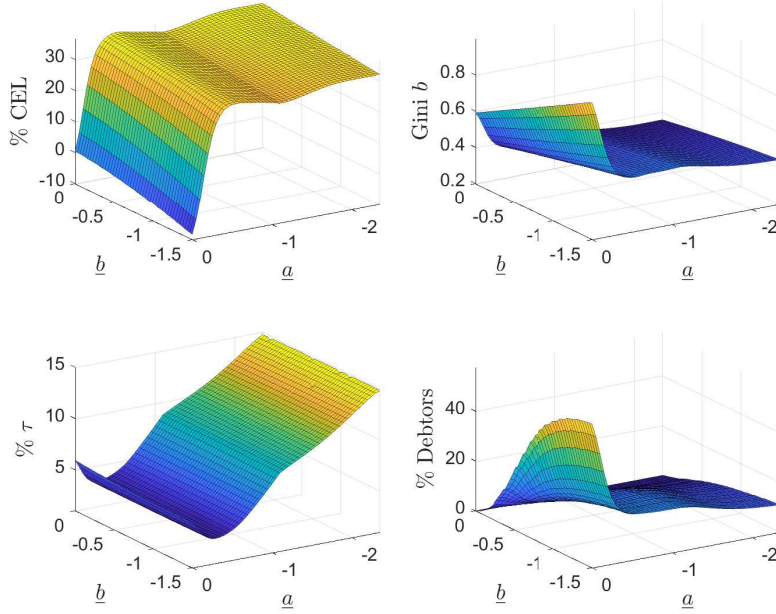


Figure 29: CEL, wealth Gini, tax rate and net debtor share in ICL2

5.5.2 NICL vs. ICL1 at benchmark calibration

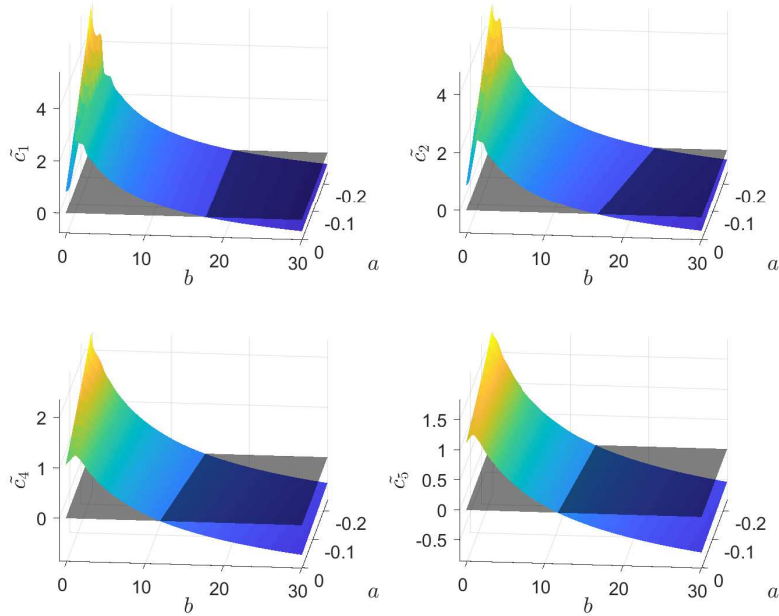


Figure 30: NICL (in black) vs. ICL1 in baseline calibration

5.5.3 Density difference amongst student loan programs

The most salient fact is that ICL1 places more mass in zero student debt workers with a college degree. This is probably due the interest and amortisation subsidies and especially due to debt cancellation. Additionally, NICL places more mass, relative to ICL1, on groups 1 and 2; this is

reflected on a smaller share of workers with a college degree in Table (2). Comparisons between NICL and ICL2 are broadly similar, except that the latter places less mass in groups 4 and 5 (and thus has a larger share of non college-educated workers).

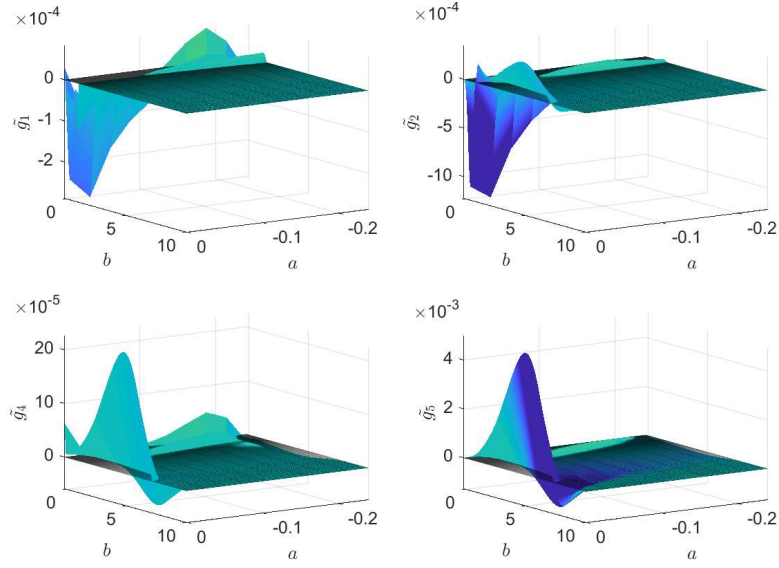


Figure 31: Density difference - $g_{ICL1} - g_{NICL}$ in baseline calibration

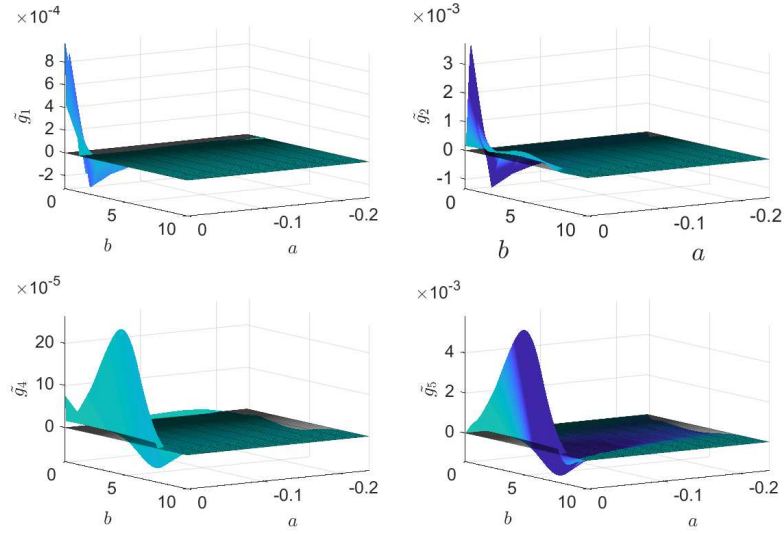


Figure 32: Density difference $g_{ICL2} - g_{NICL}$ in baseline calibration

5.5.4 Density difference of student loan programs in partial and general equilibrium

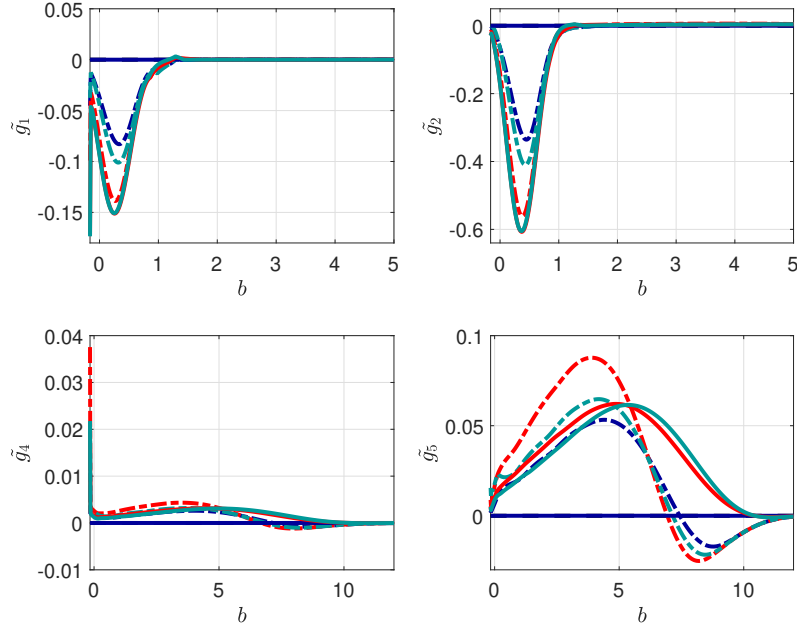


Figure 33: Density difference $g_{ICL1} - g_{SF}$ in partial and general equilibrium. As in Figure(14), dash dotted (solid) lines represent general (partial) equilibrium results and NICL, ICL1 and ICL2 are represented in blue, red and turquoise, respectively.

5.5.5 Borrowing limits and welfare - GE vs PE

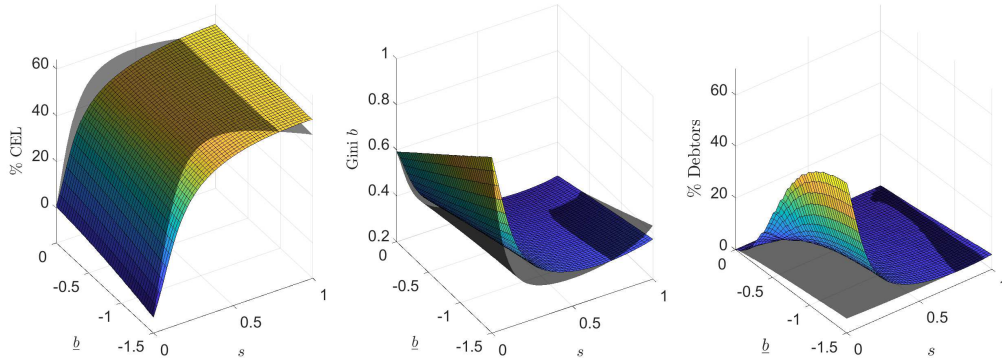


Figure 34: CEL, wealth Gini, tax rate and net debtor share in SF and TS - PE (black). $P = 1.1$ and $\Delta_{ed} = 0.25$

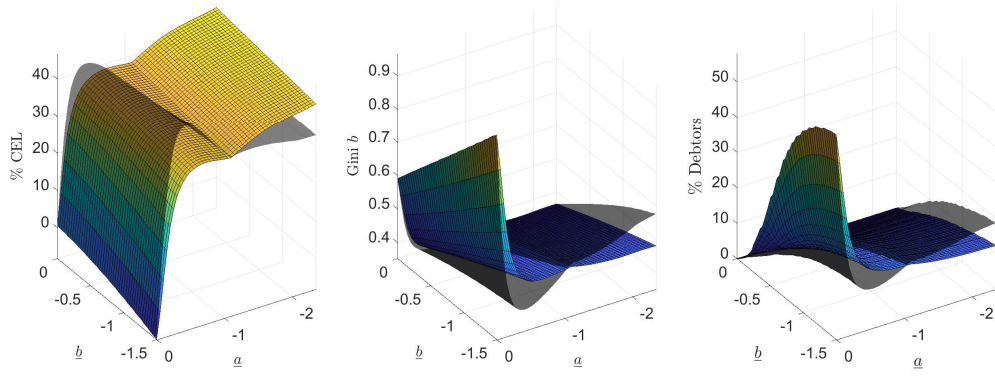


Figure 35: CEL, wealth Gini, tax rate and net debtor share in ICL1 - PE (black). $P = 1.1$ and $\Delta_{ed} = 0.25$

5.5.6 Zooming in transition CEL

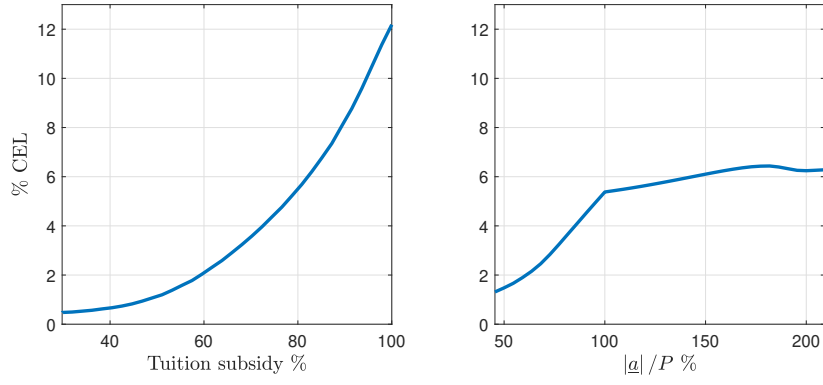


Figure 36: Left - SF to TS for $s \in [0.3, 1]$ - right - SF to ICL1 $a \in [-2.35, 0.5]$