

Abstract

The Mobility and Stability of the Human Knee Joint

Jennifer Dorothy Feikes
Balliol College

Doctor of Philosophy
Trinity Term 1999

Separating the study of kinematic geometry of the human knee from the study of its behaviour under load provides insight into the complex relationship between form and function at the joint. The development of a three-dimensional mathematical model which examines the mobility and stability of the joint in sequence is described in this thesis.

A previously proposed model of knee mobility, in which the ligaments and articular surfaces act as rigid constraints between the bones in a single degree-of-freedom spatial mechanism, was re-examined and its limitations addressed. A new geometric-numerical approach to solving the model kinematics, capable of handling both idealised and more anatomical representations of the articular surfaces, was developed.

A database of specimen-specific motion and geometry was established, based on cadaver studies. Articular contact kinematics and ligament length patterns were also quantified. In experiment, all components of passive knee movement were found to be coupled to the flexion angle, providing justification for the underlying concept of the model of knee mobility.

Specimen-specific models of mobility were successful in predicting the main features of passive knee motion through a full range of flexion. Incorporation of second order tibial articular surfaces permitted the prediction of physiological motion compatible with more realistic contact point movement.

Through incorporation of continuous three-dimensional arrays of extensible ligament fibres, a preliminary model of knee stability was formulated. Although in need of further refinement, sample predictions of joint behaviour during a/p drawer and axial rotation have demonstrated the potential of the model in highlighting the subtleties of ligament mechanics.

It was concluded that the sequential approach is appropriate for the study of joint behaviour in three dimensions and that, based on the success of the analogous two-dimensional theory, it provides an invaluable tool in the study of joint mechanics in activity and in the design and assessment of surgical procedures for treating knee injury and disease.

THE MOBILITY AND STABILITY
OF
THE HUMAN KNEE JOINT

A DISSERTATION SUBMITTED TO THE
UNIVERSITY OF OXFORD

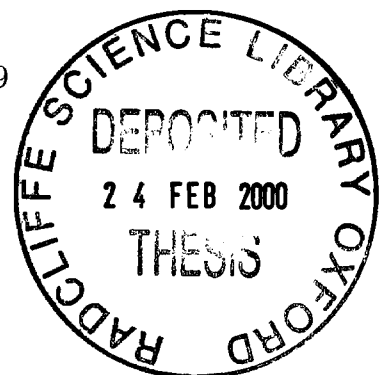


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JENNIFER DOROTHY FEIKES

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¹The material described in this section has been submitted as a Technical Note to the Journal of Biomechanics and is currently under review, February 1999.

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³The material presented in this section is based on the two- and 3-D ligament force calculations put forth by Zavatsky [148, 146]

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Chapter 1

Introduction

1.1 Overview

The knee is the largest and most complex joint in the human body and its functional subtleties are not yet fully understood. The structure of the knee is a compromise aiming to provide both the necessary *mobility* and *stability* to the leg [64].

Disease or injury at the joint is particularly debilitating due its central role in all locomotor activities. Meniscal tears, ligament ruptures, lesions in the articular surfaces, and/or fracture of the bones surrounding the joint are most often caused by trauma due to sporting mishaps and automobile accidents. The incidence of knee ligament injury in Western Society has been estimated at over 98 per 100,000 population per year (San Diego Kaiser study [54]). Osteoarthritis, a widespread disease among the growing elderly population involves the degeneration of the articular cartilage. It is thought to represent a physiological imbalance between the stresses placed on the joint and the tissue's ability to withstand them [107]. Patients suffer varying degrees of debilitation and pain at the joint. Rheumatoid arthritis, a congenital form of the disease, also affects relatively large numbers of the population.

Experimental observation of and the ability to mathematically model how the structures at the joint act together to provide its natural mobility and stability is invaluable in the assessment, optimal treatment and rehabilitation of the widespread range of disorders occurring at the joint. Such an understanding is particularly vital for the successful design and implantation of artificial surfaces and ligaments in the treatment of knee injury and disease. Separating the study of knee *mobility*, describing the range and pattern of movement of the unloaded joint, from the study of joint *stability*, describing the stiffness of the joint under load, provides a simple, yet logical, approach to understanding the complex nature of knee mechanics.

This chapter serves to provide a brief description of the anatomy of the joint, an overview of existing knowledge and previous studies of knee joint kinematics and

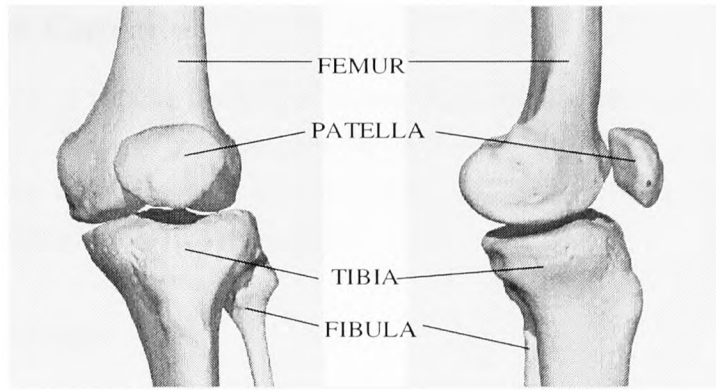


Figure 1.1: The bones at the knee joint: frontal view (left) and medial sagittal view (right)

mechanics, and an outline of the aims and scope of this thesis.

1.2 Anatomy of Human Knee Joint

The knee is a *diarthrodial* joint, a joint at which the connecting bones articulate with one another with limited constraints to movement within a certain range. A brief description of the mechanical structures at the joint follows. The clinical terminology used is summarised in Appendix A.

1.2.1 Bones

The knee is the junction between the long bones of the lower limb: the femur and the tibia (Figure 1.1). A third long bone, the fibula, is aligned alongside the tibia but does not form part of the joint. The patella, an oval-shaped, sesamoid bone, is embedded in the tendon of the quadriceps muscle and articulates with the femur. There are three separate compartments of the joint: the medial and lateral tibiofemoral compartments and the patellofemoral compartment.

The distal femur and the proximal tibia both widen at the knee and separate into distinct sections called condyles. The posterior femoral condyles are convex in both the sagittal and frontal planes and are nearly spherical [67]. The two femoral condyles are separated by the intercondylar fossa or notch. The notch extends anteriorly to form the patellofemoral groove, the articular surface in contact with the patella. The protrusions on the outer medial and lateral condyles are called epicondyles. The tibial condyles are both slightly concave in the frontal plane. In the sagittal plane, the medial condyle is essentially flat, whereas the lateral condyle is slightly convex [64]. The resulting tibiofemoral articulation is between incongruent surfaces. Both tibial condyles slope posteriorly. In the central region of the plateau lie the medial and lateral tibial eminences or spines (protrusions of bone).

1.2.2 Joint Capsule

The joint capsule, a tough, protective tissue lined with a synovial membrane, surrounds the joint. The membrane secretes a viscous fluid called synovium, which not only provides the joint with nutrients but also introduces a form of boundary lubrication between the articular surfaces.

1.2.3 Cartilage

The articular surface in each joint compartment is covered with a layer of articular cartilage. This thin, fibrous, connective tissue made up of a matrix of mostly water, in conjunction with the synovium, provides the surface with a coefficient of friction as low as $\mu = 0.001$ [108]. Under compressive loads, a film of fluid is squeezed out of the cartilage, providing a form of hydrostatic lubrication. In addition to providing a smooth bearing surface, deformation of the cartilage promotes a more even load distribution on the bones.

1.2.4 Menisci

Reducing the incongruence at the tibiofemoral articulation, especially in the sagittal plane, a *meniscus* is located between the bones in both the medial and lateral compartments of the knee (Figure 1.2). Made up of tough, fibrous tissue, the menisci are semi-lunar in a transverse view and wedge-shaped in cross-section. By filling the space between the bones around the outer edges of the articular compartments, they serve to increase contact area and lower contact stresses across the condyles. With attachments in the region of the tibial spines, they are free to shift backwards and forwards with the femur as it rolls back and forth to maintain congruence throughout the range of flexion.

1.2.5 Ligaments

A ligament is by definition a soft tissue structure which connects one articulating bone to another. Ligaments are *passive* structures, *i.e.* there is no neural input, although they may play a sensory role [63].

Histologically, the basic unit of a ligament is the collagen microfibril, groups of which form fibrils. Collagen fibres (diameter approx. $10\ \mu\text{m}$) consist of groups of fibrils and lie oriented approximately parallel to the length of the ligament. The hierarchy is extended to subfascicular units and then *fasciculi*, which are the bundles of fibres visible to the naked eye (diameters larger than $0.25\ \text{mm}$). These fascicles are commonly, and throughout this thesis, referred to as the ‘fibres’ of the ligament.

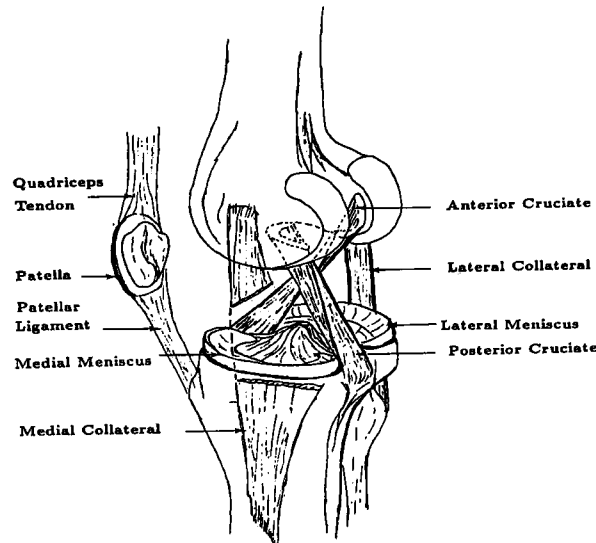


Figure 1.2: The major soft tissue structures surrounding the knee joint (after FitzPatrick [38])

At the knee, the ligaments guide joint motion and, by transmitting forces across the joint, also provide stability. The four major ligaments crossing the knee are shown in Figure 1.2: the Anterior Cruciate Ligament (ACL), the Posterior Cruciate Ligament (PCL), the Medial Collateral Ligament (MCL), and the Lateral Collateral Ligament (LCL).

The cruciate ligaments are cord-like, intra-capsular but extra-synovial ligaments. Lying near the centre of joint, they appear crossed (hence ‘cruciate’) when viewed in the sagittal and the frontal planes. In full extension, the ACL femoral attachment lies posteriorly on the lateral wall of the intercondylar notch, in an approximately sagittal plane. The ACL courses anteriorly, distally and medially to its insertion on the proximal tibia, a virtually transverse planar attachment. The PCL originates anteriorly on the medial wall of the intercondylar notch and inserts posteriorly on the proximal tibia between the two condyles and distal to the joint surface. The collateral ligaments, both extra-capsular and extra-synovial, span the medial and lateral sides of the joint. The most significant portion of the MCL is its superficial layer, a flat, belt-like structure, which originates on the medial epicondyle of the femur and inserts distally on the medial side of the tibia. The deep layer of the MCL is a weaker ligament also known as the capsular ligament. The LCL is cord-like in morphology and originates on the lateral femoral epicondyle and inserts on the proximal fibula.

1.2.6 Muscles

Skeletal muscles actuate the bones of the body to produce movement at the joints and provide stability by resisting movement across joints. Muscles are *active* structures, *i.e.* they are activated by the central nervous system. In concentric contractions, muscle produces force through contraction and shortening of its bundles of muscle fibres. During eccentric contractions, the muscle fibres contract and lengthen. An isometric contraction produces force with constant muscle length.

At the knee, the muscles can be divided into flexors and extensors. The main flexors include the hamstrings group (semimembranosus, semitendinosus and biceps femoris) and the medial and lateral gastrocnemius muscles. Smaller muscles (sartorius, gracilis and popliteus) also play minor roles in knee flexion. The principal extensors make up the quadriceps group (rectus femoris; vastii lateralis, medialis, and intermedius) which inserts at the tibia via the quadriceps tendon, patella and patellar tendon in series (Figure 1.2).

1.3 Mechanics of the Knee Joint

1.3.1 Kinematics

The knee is a highly mobile joint. The kinematics of the joint involve not only the rigid body motion of the adjacent bones, but also the details of changes in the orientation of surrounding soft tissue structures and in the location of the contact areas on the bones. Any changes in the joint's geometric configuration alter the distribution of load between the bones, muscles, ligaments and other soft tissues. The motion directly affects the mechanics at the joint.

Injury and disease affect the kinematics and subsequently the mechanics of the joint with potentially damaging consequences. The movement pattern and range of motion of normal knees provide a kinematic baseline for studying the effects of joint disorders and their treatment. The design of artificial ligaments and articular surface replacement relies not only on an accurate description of normal knee movement but also requires an understanding of the relative importance of the structures at the joint in controlling this movement.

1.3.1.1 Passive vs Active Motion

The kinematics of the knee result from the geometric arrangement and the mechanical properties of the structures at the joint. Patterns of muscle activation and external loads also affect the motion. To begin to understand such a complex system, it is useful to start with the simplest situation: the unloaded knee. *Passive*

motion describes that which occurs in the absence of muscle action. Joint *mobility*, more specifically, refers to the range and pattern of motion in the unloaded knee, *i.e.* in the absence of both muscle and external loads. Knee *mobility* is determined by the geometry of the articular surfaces and soft tissues of the joint alone. In contrast, *active* kinematics describe the movement of the loaded joint in the presence of muscle activity and external forces.

1.3.1.2 Movement Studies

Quantifying both passive and active motion at the knee joint has been the focus of much work over the past century. Both *in vivo* (living subject) and *in vitro* (post-mortem specimen) investigations have been undertaken.

In vivo experiments Clearly, it is most desirable to measure active motion at the knee *in vivo* during activity. Murray *et al.* [93] were among the first to quantify joint angulation patterns in three planes of motion during level-walking. Since then others, including Sutherland *et al.* [122] and Chao *et al.* [27], have reported normative data on knee motion, useful in diagnosing pathological gait.

More than a purely diagnostic tool, gait analysis has contributed to the study of joint function in injured patients, pre- and post- surgical intervention. Andriacchi *et al.* [5], for example, have explained different gait patterns observed after total knee arthroplasty (TKA) through changes in the lever arms of muscles and contact point locations. Comparing the motion at the knee between small groups of normal and ACL-deficient subjects, Shiavi *et al.* [116] found increased adduction and external rotation of the femur during certain periods of stride. Shiavi *et al.* were also the first to attempt to quantify the true 3-D kinematics of the knee *in vivo* [117], *i.e.* the spatial rigid body displacement of the tibia relative to the femur, versus the previously adopted projected planar angulations. They established that the normal knee does not act as a hinge, nor does it move in a single plane.

Contemporary techniques used in measuring 3-D movement *in vivo*, reviewed by Chao *et al.* [28], include stereometry (*e.g.* [25, 75]), which relies on spatial tracking of points with cameras, video, or electro-optical sensing, followed by point reconstruction in 3-D; goniometry (*e.g.* [27, 117]), or exo-skeletal linkage methods, which can provide a more direct method of measurement; and magnetic tracking systems (*e.g.* [4, 94]), which have gained popularity recently due their simple set-up and use. The drawback of the majority of *in vivo* measurement techniques lies in the fact that measurement devices are fixed to outer soft tissue and not to the bones themselves. Relative motion between the skin and underlying bone and problems in locating reference axes fixed to the bones both represent sources of significant error. The limited available resolution also prevents the detection of relatively small joint

translations.

These problems have been addressed by Lafortune *et al.* [69] with the implantation of bone pins into the femur and tibia of five subjects. The external ends of the pins were tracked with cameras, and radiographs were used to relate the segment motion to that of the joint's internal structures. Due to the invasive nature of the technique, however, only a small number of subjects could be studied, making it difficult to draw firm conclusions. Magnetic resonance imaging (MRI) of passive motion [95] and fluoroscopic techniques for recording active motion [43] are currently being used and may soon provide more accurate quantification of 3-D knee kinematics *in vivo*.

In vitro experiments Although only an approximation to knee kinematics *in vivo*, *in vitro* testing has numerous advantages. Exposure of the bones avoids errors due to soft tissue motion; hence, joint movement can be measured with much greater accuracy. Direct measurement of patellar motion and tracking of the ligaments and menisci are also possible. Injury to the soft tissue may be simulated by means of simple cutting experiments and the effects of surgical intervention may also be assessed. The characteristics of both passive motion in the unloaded knee and active motion under simulated physiological loading have been studied. *In vitro* experiments allow one to distinguish between the two, providing further insight into the complex nature of knee movement.

As early as 1836, Weber and Weber [134] qualitatively described the coupled posterior translation of the contact points on the tibia with flexion as observed in cadaver studies. Meyer [85], fifteen years later, described the internal rotation of the tibia accompanying flexion, now often referred to as the 'screw-home' motion.

In the 1970's researchers strove to locate the rotational axes of the knee. Shaw and Murray [115], among others, looked at active and passive kinematics to determine the location of the long axis of rotation of the knee and the role of the cruciates in controlling the rotation. Their testing rig, however, along with others of the time [101], constrained the motion at the joint, *i.e.* there were less than six degrees-of-freedom (dof) available between the femur and tibia. The early tendency was to isolate components of the motion in planar analyses rather than to examine the spatial motion as a whole. Using an optical technique, Blacharski and Somers [12] were the first to measure the 3-D movement of the knee *in vitro*. However, the method of Rouleaux, an inherently two-dimensional (2-D) method, was used to locate instantaneous centres of rotation in two planes.

More recent studies have aimed to quantify completely the motion in terms of spatial displacements of the bones. For this purpose, Selvik [114] and Van Dijk *et al.* [35] developed a roentgen stereophotogrammetric (RSP) system using biplanar

radiographs and small metallic beads implanted into the bones. Kurosawa *et al.* [67] employed this method to study the excursion of the centres of the femoral condyles and the external rotation of the femoral axis over the tibia with flexion. Biden [10], FitzPatrick [38], and Shoemaker *et al.* [118] used six dof testing rigs which simulated flexed-knee stance to measure knee position in 3-D during passive and active flexion. These rigs did not constrain the knee motion in any way. All three report a coupling of internal tibial rotation to flexion in both the passive and active case. The motion path during knee extension was not measured, however, and the specific path of movement and the magnitude of the internal rotation varied widely between the studies. Whether this was due to differences in the conventions used in calculating the components of movement or in the loading itself is not known.

Blankevoort *et al.* [15], using the RSP system and a six dof motion rig, measured the passive motion characteristics of the knee. ‘Envelopes’ of passive knee motion for four knee specimens were defined by the limits of internal and external tibial rotation at a series of flexion angles when the tibia was subjected to 3 Nm of internal and external torque respectively. The motion was presented as an ‘envelope’ because no consistent path was found in the completely unloaded state. Although described as passive in the sense that muscle forces were not applied to the joint, since torque was applied to the joint, the movement recorded does not describe the *mobility* of the joint as defined in Section 1.3.1.1. The ‘envelopes’ of motion could be considered descriptions of passive stability rather than passive motion (Section 1.3.2.1).

In vitro studies have also examined the effects of joint disorders on the overall kinematics of the joint. Shoemaker *et al.* [118], for example, recorded changes in passive and active motion patterns of the joint with the cutting of the ACL. Under simulated quadriceps loading, reconstruction of the ACL with a patellar tendon graft restored the kinematics to that of the normal knee.

In vitro experiments allow direct control over the conditions of the experiment. This freedom is also one of the reasons why there is still confusion with regard to the details of knee kinematics. Different rig constraints and loading configurations result in different patterns of movement at the joint. Although the need for six dof testing rigs in the assessment of total knee motion has been recognised, and a standardisation proposal for the description of joint motion has recently been put forth [144], variations in the simulation of physiological loading, differences in specimen alignment, remaining rig constraints, and the description of the motion itself still cause disagreement as to the detailed kinematics of the joint, even with respect to the intact, unloaded knee.

1.3.2 Stability

The knee joint must transmit large loads and provide great stability by maintaining the union between the articulating bones. The coordinated action of the articular surfaces, ligaments, menisci, muscles, capsule and other surrounding soft tissues ensures the overall stability of the joint. While *mobility* described the movement of the unloaded joint, *stability* describes the movement of the joint under load. Stability is quantified by the stiffness, a force per unit displacement, or the laxity of the joint, the resulting displacement under a specified load.

Joint stability is a second criterion for normal joint function. It is clear that mobility and stability are inversely proportional. The less stiff the joint, the more mobile it becomes. The knee, however, reconciles these apparently mutually exclusive criteria [53]. Its complex, composite structure guarantees mobility in certain configurations, *e.g.* full flexion, and stability in others, *e.g.* full extension. Stability, too, affects the overall mechanics of the joint. Changes in the mechanical properties of the ligaments or articular surfaces affect the deformation of these structures under load, the resulting configuration of the joint, and hence, the force distribution between the various structures.

Stiffness and laxity measurements in normal knees also provide a baseline in examining the effects of disease, injury and subsequent treatment. By isolating the effects of load from the effects of movement, the relative importance of the structures at the joint in restraining, rather than controlling movement, can be assessed. Although numerous studies have provided insight into the functional stabilisers at the knee, a unified concept of knee stability does not yet exist [110].

1.3.2.1 Active vs Passive Stability

During normal motion in the unloaded joint, the ligaments and articular surfaces provide guidance in maintaining joint kinematics and deform minimally. In the absence of muscle activity, the stiffness due to the passive soft tissue structures spanning the joint and the articular surfaces alone is termed *passive* stability. *Active* or dynamic stability is defined as stability with the added contribution of muscle activity and external load.

1.3.2.2 Stability Studies

The stability of the knee has been studied extensively both in the clinic (*in vivo*) and experimentally (*in vitro*). The contributions of the various structures at the joint have been examined.

Articular Surfaces The constraints provided by the joint surfaces alone appear to be inadequate for providing functional stability to the joint due to the low conformity of the tibiofemoral articulation. In the virtually unloaded joint, Hsieh and Walker [56] found that the ligaments and other soft tissues were responsible for providing a/p and torsional stability to the joint. They claimed, however, that under compressive loads, geometric conformity of the condyles was an important factor in reducing joint laxity. Further stiffness studies of cadaver knees have revealed that the geometry of articular surfaces (in particular the interaction between the tibial eminences and the femoral intercondylar notch) may play a small but significant role in resisting mediolateral translation [103] and valgus/varus rotation [113] at the joint.

Menisci The menisci increase the conformity at the tibiofemoral articulation. There is, however, no firm evidence that they provide functional stability to the joint. Biden *et al.* [10] demonstrated that meniscectomy (removal of the meniscus) had no effect on the axial rotational stiffness of the joint in both the unloaded and loaded cadaver knee. *In vitro*, Levy *et al.* [72] showed that a/p knee laxity was not affected by medial meniscectomy. Similarly, Hsieh and Walker [56] confirmed this observation in the case of bicompartmental meniscectomy. In resisting mediolateral translation and varus/valgus rotation, however, a minor contribution of the menisci has been reported [103, 113].

Ligaments Ligamentous structures provide stability by limiting knee motion through their ability to develop tensile stresses along the orientation of their fibres.

Historically, ligamentous injury has been diagnosed by displacement of the joint under a specified load *in vivo*, *i.e.* through laxity tests. Because the resulting movements are functions of the flexion angle, it is still debated at which flexion angle such injuries are best detected. Current clinical tests for ACL injury, for example, include the anterior drawer test at 90° knee flexion and the Lachman test at 25 – 30° flexion. Markolf *et al.* [80], among others, presented a quantitative method using an instrumented clinical testing machine. They measured the a/p force versus displacement and varus/valgus moment versus angulation of the knee. The results compared well with measurements made on cadavers. Decreases in laxity of up to 50% were found when patients were asked to tense their muscles, highlighting the importance of differentiating between *active* and *passive* stability.

In vitro studies have permitted a more direct evaluation of the contributions of the ligaments to laxity through sequential cutting tests. The role of a sectioned ligament in stabilising the joint has been determined either in terms of the increase in displacement under applied load (*e.g.* [42, 81]) or in terms of the decrease in the load transmission across the joint in response to an applied displacement (*e.g.*

[24, 48]). The inherent limitation of the former method is that the increase in laxity depends on the cutting order of the structures. A disadvantage of the latter stiffness approach is that the removal of a structure may indirectly influence the loading of the remaining structures. All of these studies have confirmed that the ACL and PCL are the primary stabilisers in anterior and posterior drawer respectively. Both ligaments were also found to resist axial rotation. In a more precise application of the latter method, Piziali *et al.* [102], described a system for completely describing the coupled, non-linear stiffness matrix of the knee. By applying controlled displacements of the bones relative to one another in any direction and measuring the complete system of restraining forces and moments required to achieve the displacement for normal and sectioned knees, they were able to quantify the contributions of individual ligaments to the stability of the joint with respect to any of the six dof of the joint [103, 113].

A direct relationship between the forces in the ligaments and the *normal* displacement of the tibia or femur, for example during passive or active flexion, has been more difficult to determine. Lewis *et al.* [74] and Ahmed *et al.* [1], for example, have measured the forces in the ligaments directly, using buckle transducers on selected fibres. Direct measurements of strain in the ligaments have also been undertaken [36, 65] where strain-gauges are sutured to selected fibres of the ligaments. Miniature Hall-effect transducers have been fixed to certain fibres [7, 109] not only *in vitro*, but recently also *in vivo* [9]. Unfortunately, localised strain measurements cannot be correlated with total ligament force. Furthermore, since the fibre tensioning patterns vary within each ligament, these techniques provide a limited description of overall ligament function. Implantation or attachment of such measurement devices may also affect the normal functioning of the ligament. Markolf *et al.* [79] developed a method of measuring the total force in a ligament directly by mounting a load cell to a bone plug to which the ligament is attached. The system, however, has been prone to error due to movement of the bone plug relative to the rest of the bone. More recently, Fujie *et al.* [41] implemented a variation of the stiffness method described above by using a robotic manipulator to reproduce the movement of the intact joint in a knee with only the ACL left unsectioned. A six degree-of-freedom load cell measured the system of forces and moments acting on the tibia during the motion. This allowed not only the magnitude but also the direction and point of application of the ACL force to be determined.

1.3.3 Mathematical Models of the Knee

1.3.3.1 Why Model ?

In vitro and *in vivo* experiments have contributed substantially to today's understanding of knee joint function. Nevertheless, there are still limits to the number

and types of experiments that can be done. Both technical and ethical reasons prevent adequately detailed measurements of kinematics and stability *in vivo*. While *in vitro* tests allow a more accurate quantification of motion and loading at the joint, it remains difficult to simulate and compare *multiple* conditions resulting from injury and the numerous possible surgical interventions on a single knee.

For these reasons, a mathematical model of the knee presents a useful tool in biomechanical analysis. Not only do models serve to predict mechanical quantities which are difficult to measure experimentally, particularly *in vivo*, but they aid in planning appropriate experiments and interpreting results. A model simplifies the reality by making a number of assumptions and must be validated by experiment in the context of its application. Development of a model is thus often an iterative procedure. Because representations of the anatomical structures can be easily modified, removed or added, models can also be used to study the effect of injury, disease or surgery on joint mechanics.

Anatomically-based models of the knee can be divided into models which predict joint stability, those which predict joint kinematics, and those which are capable of predicting both.

1.3.3.2 Models of Knee Stability

Modelling of the knee joint thus far has largely been devoted to the study of joint stability. The approach involves representation of the anatomical structures as either springs or rigid constraints.

Crowninshield [33] developed a model in which the ligaments at the knee were represented by 13 straight line elements with specific load-deflection characteristics. Motion known *a priori* prescribed the position of the femoral attachments of the ligaments relative to their attachments on the tibia. Surface interaction was not considered. The lengths and tension patterns of the ligaments were calculated and the stiffness of the model knee determined at a number of flexion angles. Only limited model validation was undertaken due to the variability in stiffness measurements taken from a small number of cadaver knees.

Wismans *et al.* [139] presented a model which included non-linear spring elements representing the ligaments and capsule and rigid constraints representing the tibial and femoral articular surfaces. The geometrical data, including the polynomial surface shapes, were based on anatomical measurements. The material properties for the ligaments were chosen based on the literature. For a given 3-D loading, the model predicted joint position and ligament, capsule and contact forces by simultaneously solving the equations of mechanical equilibrium and the equations of constraint satisfying articular contact. Friction at the articulation, deformation of the surface and the menisci were not taken into account. A sensitivity study of

the model parameters showed that the results were highly dependent on the chosen initial ligament strains at full extension, parameters which are not well documented in the literature.

A 3-D model was put forth by Andriacchi *et al.* [6] to describe the mechanical response of the knee to various loading conditions. The influence of the ligaments on the kinematics of the knee was incorporated into the model by using a direct stiffness approach. The ligaments and capsule were represented by 21 tension-only linear springs and the menisci were modelled as two shear beam elements. The model bones were rigid except for the contact regions which were represented by 10 hydrostatic elements. With a solution method based on the minimisation of total energy, the model could maintain equilibrium from 0° to 90° flexion in a chosen polycentric, planar movement of the joint. The model predicted joint response to various loads. The influence of the surface shapes was not considered. The model was validated with results published in the literature and *in vitro* data from a single knee.

Models similar, but more advanced than that of Wismans *et al.*, were developed by Essinger *et al.* [37] and Blankevoort *et al.* [18]. The former group included a simplified patellofemoral joint and used the model to study the mechanics of TKA. The tibia was represented by a deformable surface and the femur, patella and patellar tendon were considered inelastic. Using the principle of minimisation of total energy, the model predicted the relative movements of the ligaments and contact areas, ligament tensions, and the articular surface pressures. Motion patterns were compared for the simulated surfaces of a normal knee and three types of TKA. Initial strains in the ligaments again posed a problem and were adjusted so that model predictions compared well with experimental tension patterns. The stiffness parameters of the ligaments were also modified to avoid ligament fibre tensions over 400 N. Model predictions were restricted to the simulation of a particular experimental set-up [67], and unfortunately the model was only validated with data from normal knees.

Blankevoort *et al.* [18] examined the effect of rigid and deformable articular contact on the passive motion characteristics of their model knee. The deformable contact was mathematically described by a thin, either inelastic or elastic layer on a rigid foundation. The incorporation of the deformable surface did not alter the kinematic behaviour in a qualitative sense. Quantitative changes were small for moderate loading conditions and negligible between linear and non-linear elastic descriptions. Blankevoort also extended the Wismans model by adding a further six ligament line elements for more realistic fibre recruitment and joint stiffness. Again the reference ligament strains were adjusted in such a way that the model predictions approximated the motion data from specific experiments, in this case from the same

knee specimen from which the geometric data was taken. A validation of the model was also described by comparing the predictions of four specimen-specific models to knee motions recorded in experiment and laxity data reported in the literature [14].

Recently, Mommersteeg *et al.* [88, 87, 90, 89] have advanced Blankevoort's model with respect to the representations of the ligaments. Using a numerical-experimental approach, the minimum number of fibre bundles required to represent each ligament was deduced and the mechanical properties of each bundle, both the stiffness and reference length, were determined by comparing the results of a series of tensile tests of bone-ligament-bone complexes with the behaviour of the model ligaments [88, 87]. The method, although useful, has only been applied to a single specimen and is, by the authors' own admission, extremely time-consuming. To validate the improved multi-bundle ligament representation, it was incorporated into the original knee model with input from the single set of specimen-specific parameters and validated for specific a/p and varus/valgus loading cases. The restraining functions of the various ligament bundles under these loading conditions were also examined [89]. The knee model, however, was found to be unstable when axial torque was applied to the joint, in particular in external rotation [90]. This loading case could therefore not be examined. The complexity of the model and parameter derivation may also limit its practical use.

While modelling the knee as a structure is often useful, these models have been prone to difficulties, particularly when light loads were considered. Flexion or extension moments were required to flex or extend the model knee or to hold it in a fixed position [37, 139]. Despite their intention to model the *passive* characteristics of the joint, Blankevoort *et al.* required a 150 N compressive force to avoid 'contact loosening' [18]. The models were also critically sensitive to the initial tensions of the ligament spring elements [18, 139]. Adjustments were necessary to compensate for the single or multiple line element representations of the ligaments. By only examining motion under load, such models fail to predict well-known features of knee kinematics, including the internal and external rotation of the tibia accompanying flexion and extension, the 'screw-home' motion, as quantified in *in vitro* experiments on the unloaded knee (*e.g.* [34, 118]).

Because most of these models are based on simultaneously solving equations of mechanical equilibrium together with the constitutive equations and compatibility conditions, the forces and moments must be known first in order to determine the geometry of the joint structures. This is a potential limitation in their application to studying ligament and contact forces *in vivo*. In gait analysis, for example, the geometry of the structures at the joint needs to be prescribed *a priori* before the structural analysis can be performed (*e.g.* [76]).

1.3.3.3 Models of Knee Mobility

Articulating bones at a joint and the ligaments which hold the bones together can be modelled as a mechanical linkage. Such *geometric* or *kinematic* models predict joint motion based solely on the geometry of the joint.

A number of authors have explained the functional anatomy of the knee in the sagittal plane using a four-bar linkage [45, 64, 84, 92, 121]. In such a model, it is assumed that one fibre within the ACL and one fibre within the PCL remain isometric throughout the range of flexion (approximately 0° to 120° flexion). The two isometric ligament fibres, the femur and the tibia make up the four rigid links of the planar mechanism. The concept highlights the importance of the cruciates in guiding knee motion. The instantaneous centre of the linkage, or the flexion axis of the knee, is located at the intersection of the isometric fibres of the ACL and PCL. The geometry of the single dof linkage in a starting position dictates the kinematics of the joint in the sagittal plane. It predicts the coupled posterior translation of the contact points with flexion and the combination of rolling and sliding at the articular contact [64].

The four-bar linkage model has provided the basis for a number of biomechanical analyses. Bradley *et al.* [19] used the 2-D model to study knee kinematics in the sagittal plane and to predict changes in ligament lengths with flexion. O'Connor *et al.* [99] described the interactions between the articular surfaces and the cruciate ligaments during flexion and extension. They noted that complementary articular surfaces are required to keep the ligaments just taut so they may guide knee movement. For a particular ligament configuration, the complementary femoral surface to a flat tibial surface was constructed based on the principle that, to avoid penetration and separation, the common normal to the two surfaces must pass through the instantaneous centre. The resulting surface shape closely resembled the anatomy of the posterior femoral condyles in the sagittal plane, *i.e.* almost circular.

Huson *et al.* [58] also described the kinematics of the knee in the sagittal plane with the four-bar linkage model. They introduced a second dof which allowed rotation about a vertical axis lying in the plane of the four-bar linkage and passing through the instantaneous centre. Lengsfeld [71] attempted to describe 3-D knee motion by a set of two planar four-bar linkages based on the crossing of the cruciate ligaments in both the sagittal and transverse planes. The two planes of movement and their instantaneous centre pathways were considered independently. The guidance of the movement was attributed to the geometry of the cruciate ligaments alone, ignoring the contribution of the surface shapes necessary to accommodate the motion. Both Huson's and Lengsfeld's models are purely descriptive and are unable to predict fully the 3-D motion of the knee.

Applications of Geometric Models of Mobility A true geometric model, such as the aforementioned four-bar linkage model used by Bradley *et al.* and O'Connor *et al.*, promotes an understanding of the mobility of the joint before addressing its more complex behaviour under load. It allows the role of the different anatomic structures in controlling joint motion to be examined.

Load transmission across the joint is of great interest in the understanding of injury mechanisms, prosthesis design and implantation, and the development of safe rehabilitation regimes. In a typical mechanical analysis, a known resultant force and moment at the knee due to external loading must be balanced by muscle, ligament and contact forces at the joint. Due to the number of load-bearing structures at the knee, the problem is a statically indeterminate one. The problem is simplified somewhat by including only the primary structures at the knee and combining groups of flexor and extensor muscles. At this point, a geometric model is useful in prescribing the position of the femur relative to the tibia at any flexion angle. By providing the lines of action of the muscles, ligaments and contact normals and the location of the instantaneous axis of rotation, such a model can further reduce the number of unknowns.

Morrison [91] modelled the knee geometrically as a hinge in an analysis of forces at the knee during normal walking. The mechanical model included three muscle groups, the cruciate and collateral ligaments, and the tibiofemoral contact. The active muscle groups were chosen from electromyographic (EMG) recordings of muscle activity. The lines of action of the hamstrings, gastrocnemius and ligaments were based on the positions of their origins and insertions and the position of the femur relative to the tibia as predicted by the geometric model. The line of action of the patellar tendon was deduced from radiographs. Numerous experimental studies have since demonstrated, however, that the knee does not move about a fixed axis (*e.g.* [117]).

O'Connor *et al.* [98] added muscle tendons to the four-bar linkage model of the knee and determined their directions and moment arms over the flexion range. The model was then incorporated into a lower limb model to study muscle-ligament interactions at the knee during level walking (Collins and O'Connor [30]). Besides Morrison's analysis, this was one of the first studies to calculate ligament forces during an activity such as gait. The parameters for the model were derived from anthropometrical studies [19]. A 'limiting solution' approach was taken in solving the remaining indeterminate system. All possible solutions involving tensile muscle and ligament forces and compressive contact forces were considered. EMG data was consistent with the solutions over most of the gait cycle. Because the model ligament forces were found to be quite high, it was concluded that assuming inextensible ligaments provided an upper bound for ligament forces in gait. Lu [76] advanced

the 2-D model of the pelvis-leg apparatus and validated its predictions with femoral forces telemetred from two subjects during isometric exercise and level walking [78]. The kinematic geometry of the knee joint was again prescribed by the inextensible four-bar linkage model.

Another application of the four-bar linkage model has been in prosthesis design. Goodfellow and O'Connor [45] designed a meniscal bearing knee replacement which restores sagittal plane kinematics of the knee as predicted by the model. Both cruciate ligaments are retained in the surgery and, through the thickness of the meniscal component, they are set to be just taut to maintain their critical role in guiding the motion. For a set ligament configuration, the model predicted that the ideal shape for a tibial component complementary to a spherical femoral component was flat.

1.3.3.4 Models of Mobility & Stability: The Sequential Approach

While certain models of joint stability [15, 37, 139] appear to predict both kinematics and stability of the knee, this is true only in a limited sense. The models are tailored and parameters adjusted to predict motion as observed under certain experimental loading conditions.

A *sequential*, versus *simultaneous*, approach provides an alternative to the combined study of knee mobility and stability. First, a geometric model is used to predict the kinematics of the joint, providing the relative positions and orientations of the bones, ligaments, muscle attachments and articular contact areas at each flexion angle. In this step the structures of the linkage are assumed to be rigid. This neutral configuration of the joint is then used as a starting point for a subsequent mechanical analysis in which the structures are allowed to deform. Their elasticity is accounted for by incorporating the appropriate stress-strain relationships. A displacement analysis from the undeformed state follows.

FitzPatrick and O'Connor [39] used the four bar linkage model to analyse tibial displacement and to calculate the relative contributions of the knee ligaments to stability in an anterior drawer test. The model ACL and PCL were allowed to deform. The strain in the ligaments and the force transmitted by each of the ligaments as a result of tibial displacement was calculated based on anthropometric data and material properties taken from the literature. Relationships between tibial displacement, ligament force, applied a/p force, and tibiofemoral contact force were calculated. The study highlighted that a force analysis which ignores the stretching of ligaments overestimates cruciate ligament forces.

Zavatsky and O'Connor [147] incorporated arrays of extensible fibres into the sagittal plane model of FitzPatrick and O'Connor. In addition to the chosen isometric fibre, a 2-D linear mapping of cruciate ligament fibres between approximated

tibial and femoral attachment lines was defined. These arrays included the isometric fibres of the ACL and PCL which defined the four-bar linkage. Arrays of collateral ligament fibres were similarly mapped. The model ligament attachment lines were based on measurements by Van Dijk *et al.* [35]. Coordinates of the isometric fibres and their zero-tension positions were deduced from observations reported in the literature. First, the length and shape changes of the ligaments were studied during passive flexion/extension. In a second step, they studied fibre recruitment under load [148]. The procedure was similar to that used by FitzPatrick and O'Connor for single fibres. In this case, the force in a ligament was calculated by summing the individual contributions of each of the fibres. In addition to the effects of passive flexion and a-p translation, ligament length changes, load sharing between the ligaments, and the interaction between muscle and ligament during isometric quadriceps contractions were also studied [149]. The incorporation of arrays of ligament fibres provided a more realistic model of joint stiffness versus single or two- or three-line element models of the ligaments, and highlighted some of the subtleties of ligament function.

To assess the effect of assuming inextensible ligaments in 2-D modelling of the lower limb, Lu [76] proposed a method of incorporating the ligament model of Zavatsky and O'Connor [147, 148]. By taking the original inextensible solution as a starting point, the tibial a/p displacement could be varied until the force generated in the extensible ligament (ACL or PCL) matched that predicted by the original lower limb model with the tibia shifted accordingly. Lu suggested that incorporation of extensible ligaments, while important for the study of the stability of the joint and for the accurate estimation of ligament forces *in vivo*, would not affect the estimation of muscle forces. The utility of the sequential approach described by Lu has been demonstrated by Toutoungi [126] who used the lower limb model with extensible ligaments to study forces in the ACL and PCL arising during rehabilitation exercises *in vivo*. She confirmed Lu's suggestions and showed that the inextensible theory does indeed provide upper bounds on the ligament forces.

The sensitivity to component placement of meniscal cruciate-retaining arthroplasty has recently been studied by Imran and O'Connor [61]. They employed the Zavatsky model to quantify the effects of distal or proximal placement of the tibial component on joint stability and cruciate ligament forces. They demonstrated that the sensitivity of component malplacement can be modelled with ease by applying the sequential modelling approach.

Huss *et al.* [59] have recently examined the effect of cartilage deformation on the predictions of Zavatsky's model and have shown that it is secondary to the effect of ligament extension in determining the passive laxity of the joint; however, in the case of dynamic laxity, *i.e.* with application of muscle force in the loaded joint, the

effects have been shown to be more significant [60]. The results suggest that, for the estimation of ligament forces, surface deformation should also be incorporated into the geometric knee models used in the study of gait or other activities in which the joint experiences high compressive loads.

1.4 Aims and Scope of This Thesis

Geometric modelling of the knee joint as a four-bar linkage in two dimensions has proved successful in a number of applications. By describing the relationship between the shape of the articular surfaces and the geometric arrangement of the ligaments, it has proved valuable in the design of surface replacement [45]. By prescribing the relative movement of the tibia and femur and the lines of action of the ligaments and contact forces throughout the flexion range, the four-bar linkage is an essential component in a 2-D musculoskeletal model of the locomotor system [78, 76]. Such a model can be used to study muscle, ligament and contact forces during activity [126]. Finally, the 2-D geometric model provides a starting point for the study of joint movement under load, in a sequential vs simultaneous approach to examining knee stability. In the 2-D knee model, representing the ligaments by continuous arrays of extensible fibres has provided a means of examining patterns of fibre recruitment and joint laxity in a/p drawer [147, 148]. The elastic ligament model allows tissue deformation to be taken into account in the distribution of forces at the joint in a model of the lower limb [76, 126]. It has also allowed the study of the effects of component malplacement on stability after cruciate-retaining arthroplasty [61]. The 2-D theory has recently examined the effect of surface deformation on joint stability and muscle-ligament interaction [59, 60].

The 2-D theory is limited in that it cannot account for potentially significant out-of-plane effects, *i.e.* in that both the geometry and motion of the joint are restricted to the sagittal plane. It can take no account of, for example, the internal rotation of the tibia which is coupled to flexion. A geometric model capable of predicting the 3-D movement of the unloaded knee throughout the range of flexion is required. A 3-D model of mobility analogous to the four-bar linkage model would also allow the spatial relationships between surface and ligament geometry to be examined anew, with possible applications to surface replacement design. A 3-D musculoskeletal model of the lower limb also depends on an accurate prescription of the spatial kinematic geometry of the knee during flexion [76, 77].

With respect to the elastic ligament model, Zavatsky and O'Connor [148] also recognised the need for a 3-D description of the movement at the joint as well as a 3-D representation of the ligament fibres. They extended the representation of the cruciate ligaments to three dimensions but the results were not entirely satisfactory.

The limitations were considered a combination of inappropriate attachment site and fibre mapping definitions and the fact that the model knee motion was still restricted to the sagittal plane [146, 150]. Thus a 3-D model of knee mobility is also required, in conjunction with a more appropriate spatial representation of the extensible ligaments, to apply the sequential approach in the development of a 3-D model of knee stability. Ultimately, by incorporating the effects of surface deformation, the 3-D model of stability would be capable of predicting the complete stiffness matrix of the joint at any flexion angle for any given displacement. The work presented in this thesis focuses on the first necessary steps in achieving this final goal.

The first iteration of a spatial geometric model of the knee was recently developed by Wilson [135]. The original model predicted knee motion only through a limited range of flexion and was based on average parameters taken from the literature. The formulation of the model was also limited in that only spherical and planar articular surfaces could be considered. Despite this, the model has already been incorporated into a 3-D model of the locomotor system [76, 77]. Its formulation and limitations will be described in detail in Chapter 5. For purposes of parameter derivation and model validation, some experimental measurements of knee geometry and motion also accompanied Wilson's original work (five specimens); however, the model was not successfully validated for specimen-specific parameter sets. The experimental and theoretical studies of Wilson represent the starting point for the current work.

Hence, the aims of this thesis are:

- To enlarge the experimental database of specimen-specific joint geometry and mobility. Specifically:
 - To quantify fully the passive movement of the knee joint *in vitro*.
 - To derive more physiological geometrical representations of the articular surfaces and ligaments of the joint.
 - To estimate articular contact movement and ligament fibre length changes during passive knee motion.
 - To assess the associated experimental error and accuracy of the experimental results.
- To develop further a 3-D model of knee mobility. Specifically:
 - To review the original spatial mechanism model concept, its underlying assumptions and solution method (Wilson).
 - To examine the limitations of the original mechanism-based approach to solving the model kinematics.

- To develop a numerical approach to the model solution capable of predicting the full range of flexion and handling more complex representation of surface geometry.
 - To examine and validate the predictions of specimen-specific models with both idealised and anatomical surface representation.
 - To study the sensitivity of the model.
- To develop a preliminary model of 3-D knee stability. Specifically:
 - To incorporate 3-D arrays of ligament fibres into the model of knee mobility.
 - To define zero-strain reference lengths and mechanical properties of the model ligament fibres.
 - To study fibre recruitment in passive flexion, anterior and posterior drawer and tibial rotation.
 - To develop a method of determining ligament force for a given a/p translational or axial rotational displacement of the joint.
 - To study a/p and torsional passive joint laxity.
- To outline the clinical and biomechanical applications of modelling the mobility and stability of the knee.
- To recommend directions for future work.

Chapter 2

Experimental Methods

A prerequisite for the successful formulation of a model of knee mobility and stability is the observation and study of the behaviour of the real system. Although there has been extensive work done describing the kinematics of the knee joint, a full description of truly passive knee motion is not yet available in the literature. Such a description is essential in the development of a model of knee mobility. Observed knee motion also provides a means of validating the predictions of the model on either a specimen-specific basis. Furthermore, from measurements made on the cadaver knees, physiological, specimen-specific model parameters can be derived. For these reasons a series of *in vitro* experiments on post-mortem human knee specimens were undertaken. The aims were:

- To measure and describe the passive flexion and extension of the knee
- To measure and describe the geometry of the femoral and tibial condyles
- To estimate contact point movement during the measured passive motion
- To measure and describe the attachment sites and the fibre architecture of the cruciate (ACL and PCL) and collateral (MCL and LCL) ligaments
- To determine ligament fibre length changes during the measured motion and to identify the most isometric fibre in each ligament

These experiments thus sought to provide a conceptual basis, sets of specimen-specific parameters and a means of validation for a model of knee mobility.

A previously established experimental protocol was evaluated and adopted with some modifications. Similar experiments had already been performed on five knee specimens before the start of the current project (Knees J, D, M, L and X) [135]. In addition to testing a further ten specimens, data from all fifteen knees were evaluated using the current methods of analysis.

This chapter describes the experimental apparatus and the methods used to measure knee motion and geometry. The data analysis performed to describe the motion and geometry of the joint and to derive model parameters is also presented.

2.1 Specimens

Fifteen¹ post-mortem human knee specimens (Table 2.1), each including approximately 15 cm of bone above and below the joint line, were dissected down to the capsule, leaving all ligaments intact. The quadriceps tendon was transected 3-5 cm above the patella, eliminating the functional role of the extensor mechanism. A visual check of the collateral ligaments was made, and each joint was examined manually before the experiment to detect any abnormal laxity. The integrity of the cruciate ligaments and articular surfaces was checked after the experiments. Knee specimens were obtained at autopsy and kept frozen at -18°C prior to testing.

On the day of the experiment, the specimens were immersed in a cold water bath for three hours to thaw. In the case of the first eight specimens (Knees J, D, L, M, X, I, E and G) a minimum of four small nails were driven into both the tibia and femur. These were used as reference markers. The markers were located so that they were evenly distributed around the shafts of the bones. The measurement system used, described in Section 2.3, is sensitive to the presence of metal in the vicinity. For this reason, with the final seven specimens (Knees F, C, B, A, N, P and H), the nails were replaced by 2 cm long sections of 3 mm diameter nylon studding which were fixed into pre-drilled holes in the bone with a cyanoacrylate adhesive ('super glue'). Once secure, the plastic markers protruded approximately 1 cm from the bony surface which allowed for easier access with the digitising stylus of the measurement system (described in Section 2.4.1). A small hole was drilled into the tip of each plastic marker beforehand with a 1 mm diameter drill to permit more consistent placement of the stylus.

With the first eleven specimens (see Table 2.1), the tibia was potted with wax or plaster into a section of plastic tubing (3.5 inch diameter) which could then be secured to the testing rig. A perspex rod was fixed in the femoral intermedullary canal through a press-fit, and if necessary secured with wax or plaster. With the final four specimens (Knees A, N, P and H), the preparation for fixation to the rig was simplified. A length of nylon studding (12 mm diameter) was fixed in the tibial intermedullary canal using polymethylmethacrylate (bone cement). The tibia could then be fixed directly and rigidly to the rig by screwing it into a mounting block fixed to the rig. A similar piece of nylon studding was also cemented into the femoral canal. One end of the original femoral perspex rod was threaded and it was then fixed to the studding via a connector.

¹Fifteen including the five tested previously and the ten new specimens.

Specimen	Test Date	Lab	Mounting	Markers
J (Left)	April 4 1994	1	potting (wax)	nails
D (Right)	June 26 1995	1	potting (wax)	nails
L (Left)	July 14 1995	1	potting (wax)	nails
M (Right)	July 25 1995	1	potting (wax)	nails
X (Right)	Sept 15 1995	1	potting (wax)	nails
I (Right)	Nov 14 1995	1	potting (wax)	nails
E (Right)	Feb 8 1996	1	potting (wax)	nails
G (Left)	Feb 16 1996	1	potting (wax)	nails
F (Left)	Nov 17 996	2	potting (plaster)	plastic
C (Left)	Dec 12 1996	2	potting (plaster)	plastic
B (Left)	Mar 20 1997	2	potting (plaster)	plastic
A (Left)	June 12 1997	2	nylon studding	plastic
N (Left)	June 13 1997	2	nylon studding	plastic
P (Right)	July 22 1997	2	nylon studding	plastic
H (Right)	Aug 18 1998	2	nylon studding	plastic

Table 2.1: The fifteen knee specimens and their corresponding test dates. The list includes the lab in which the specimen was tested, the type of fixation used to mount the specimens to the rig and the type of reference marker implanted. It is also noted whether the specimen was a right or a left knee.

2.2 The Testing Rig

Each specimen was flexed and extended in a rig designed to move the knee through its range of motion while applying minimal load to the joint. There were no constraints to the natural motion of the joint. The set-up of the testing rig is shown in Figure 2.1. The workbench and the flexing rig were built almost entirely out of plastic and wood. The tibia was positioned approximately vertically and secured firmly to the workbench. The perspex rod fixed in the femoral canal was supported by a second horizontal rod held by a frame which rotated about two hinges on the workbench. The frame was lowered and raised by a nylon cord wound upon a lightweight reel mounted at a height of approximately 2 m at a distance of approximately 1.5 m from the workbench. The knee was flexed by unwinding the cord and allowing the frame to fall slowly through its arc under its own weight and that of the femur. Extension of the joint was brought about by winding the cord back onto the reel, thus pulling the frame back up through its arc.

2.3 Motion Tracking System

A 3-D electromagnetic tracking system, the Isotrak II (Polhemus Inc., Colchester, Vermont, U.S.A.), was used to track the location of the femur relative to the tibia.

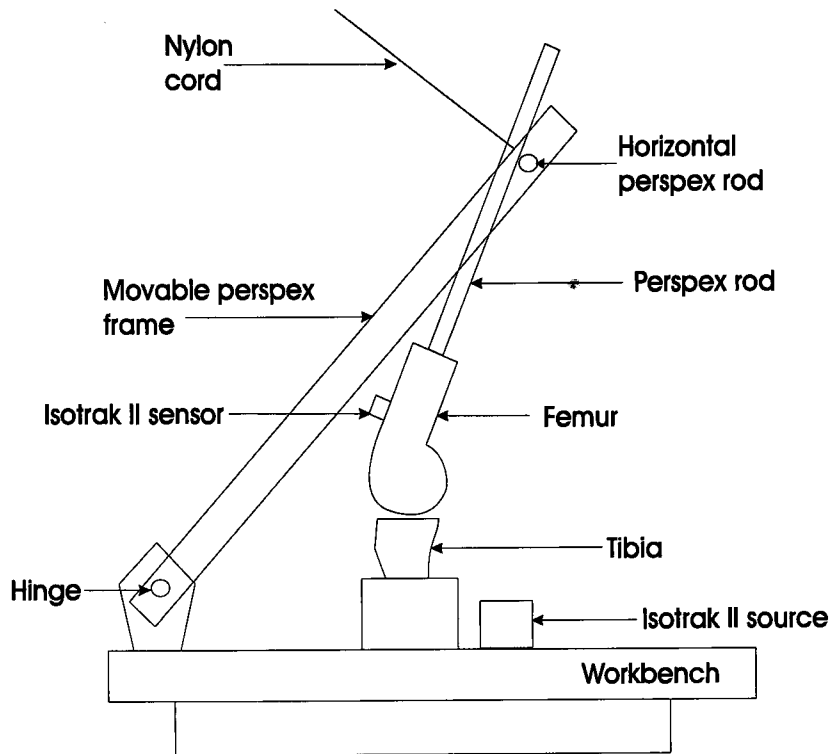


Figure 2.1: A schematic of the testing rig. Sagittal view.

The Isotrak II system consists of a power supply, a systems electronic unit (SEU), a transmitter, a sensor, and a digitising stylus (pen).

2.3.1 Operation

The system uses electromagnetic principles to measure the position and orientation of the sensor with respect to the reference frame of the transmitter. Three orthogonal coils in the transmitter emit low-frequency, quasi-static magnetic fields. These magnetic fields induce currents in three orthogonal coils in the sensor. The measurement of these currents determines the position and orientation of the sensor coordinate system relative to that of the transmitter. The digitising stylus also has three orthogonal coils embedded in it and works in the same way. Given the offset between the centre of the coordinate system defined by the coils and the tip of the stylus, the output data describes the position of the tip. As it relies on magnetic principles for operation, the system is sensitive to metal in the test area. The manufacturer therefore suggests that large metal objects be kept at least 2 m away from the transmitter.

2.3.2 Data Logging

The SEU was connected to a personal computer (IBM compatible 486, Opus Technology, U.K.) by a standard RS-232 connection. Software provided with the Isotrak II system was used for data acquisition. The program allows the configuration of the system as a ‘tracker’ or a ‘digitiser’. In the ‘digitiser’ mode, the system is set to collect a data point every time a button is pressed on the stylus. When configured as a ‘tracker’, the system, by default, collects data at 60 Hz. With each sampled position, the system sends a data record to a file made up of three coordinates (x, y, z), describing the origin of the sensor coordinate system (or the tip of the stylus) and three Euler angles (roll, elevation, azimuth), describing the orientation of the sensor (or stylus) coordinate system.

2.3.3 System Set-Up

In the current tests, the tracker sensor was fixed rigidly to the femur via a perspex clamp, while the transmitter was fixed rigidly to the workbench to which the tibia was fixed. The knee was aligned in the rig so that the X , Y and Z axes of the transmitter coordinate system approximately corresponded to the anteroposterior, mediolateral, and proximodistal axes of the tibia, respectively. Data averaging was implemented so that each data record represented the average of three data samples, effectively reducing the sampling rate to 20 Hz. In ‘tracker’ mode, the system was also set to collect a data record only when the sensor had moved a minimum of 1 mm from its previously recorded position, thereby reducing the amount of data collected. An adaptive, single-pole, low-pass filter, provided with the manufacturer’s software, was used to remove extraneous noise, due to vibrations and electrical pickup, from the position and orientation data.

2.4 Experimental Protocol

Each of the fifteen knee specimens was tested using the experimental protocol described in this section. A number of specimens were used to collect additional data. The experimental protocols involved in these additional tests are described in the relevant sections.

2.4.1 Reference Position

Full extension of the knee was defined by extending the femur until significant resistance was felt. This was comparable to the limit felt in a clinical laxity test, a solid end point. With the knee held at full extension by securing the reel, the position of

each reference marker on each bone was measured three times using the digitising stylus. The position and orientation of the sensor was also recorded.

2.4.2 Passive Motion and ‘Deviation’ Tests

Three cycles of passive flexion and extension were recorded from full extension. A full flexion/extension cycle took approximately 25 seconds. To test qualitatively the resistance of the knee to an applied load, ‘deviation’ tests were carried out during one flexion/extension cycle for each specimen. At each of three flexion positions, the femur was displaced manually along the horizontal rod in both the medial and lateral directions. After each deviation, the femur was quickly released and the flexion/extension cycle resumed.

2.4.3 Loading at the Joint

During the passive motion tests, the joints were virtually unloaded. At any flexion angle, the only load applied to the distal femur was that due to its weight and the reaction force of the perspex rod from the supporting frame. The weight of the distal femur was a maximum of 5 N. The magnitude of the resultant force at the joint was therefore always less than 5 N. The flexing/extending moment applied to the knee during a motion cycle, determined during one experiment by measuring the geometry of the test set-up and the tension in the cord using an in-line force transducer (Transducer World, Aylesbury, U.K.), was less than 0.4 Nm. Based on a simple mechanical analysis, the magnitude of the moment is consistent with the maximum moment required to raise or lower the weight of the distal femur and the frame. Friction between the perspex rods was estimated, using the maximum weight supported by the rods and the coefficient of friction ($\mu \approx 0.25$), to apply no more than 0.2 N of force and 0.4 Nm of moment to the joint in any direction.

For the ‘deviation’ tests, loads, applied approximately 25 cm from the tibial plateau, varied from 1 to 3 N as measured by a hand-held force gauge (Mecmesin Systems, Southwater, U.K.). These correspond therefore to a maximum deviating moment of 0.75 Nm.

2.4.4 Joint Digitisation

To relate knee movement to knee anatomy and to generate model parameters, further data was collected. After the motion tests, the knee was disarticulated. First the tibia and then the femur were fixed to the workbench, dissected further and their geometry digitised. Upon removal of the perspex rod, the femur could be fixed to the workbench in the same way as the tibia (either through potting or nylon

studding). On each bone, the menisci were removed and the attachment areas of the ligaments exposed. The following features were digitised (surface points were as equally spaced as possible):

- Positions of the reference markers on each disarticulated bone (3 times)
- Points on the femoral and tibial attachment areas of the ACL, PCL, MCL and LCL (approx. 200 each)²
- Points on the medial and lateral tibial condyles (approx. 400 each)
- Points on the medial and lateral femoral condyles (approx. 400 each)
- Points on the external bony surfaces of the femur and tibia (approx. 500 each)

2.5 Reconstruction of Intact Knee Geometry

In the experiments, the positions of the anatomical features were measured in the disarticulated tibia and femur, and not in the intact, fully-extended knee. To relate the joint motion to the anatomy and to derive useful parameters for the model, the positions of these features in the intact joint were required.

The displacement of a rigid body from location A to location B can be represented by a rotation, $\mathbf{R}$, followed by a translation, $\mathbf{v}$ (see Appendix B.1 for notes on matrix notation). The bones, in this case, were assumed to be rigid bodies. The mean position vectors of the four or more reference markers in the disarticulated knee (*i.e.* Location A) and the fully extended knee (Location B), together with a least-squares algorithm developed by Veldpaus *et al.* [131], were used to estimate the unknown rotation matrices $\hat{\mathbf{R}}_{fem}$ and $\hat{\mathbf{R}}_{tib}$ and translation vectors $\hat{\mathbf{v}}_{fem}$ and $\hat{\mathbf{v}}_{tib}$.

These rotations and translations describe the displacement from the disarticulated knee to the intact knee of the femur and tibia respectively. The coordinates of the anatomical features measured in the disarticulated femur, $\mathbf{p}_{fdis}$, and tibia, $\mathbf{p}_{tdis}$, were then mapped to their positions in the intact knee at full extension, to $\mathbf{p}_{fint}$ and $\mathbf{p}_{tint}$:

$$\begin{aligned}\mathbf{p}_{fint} &= \hat{\mathbf{R}}_{fem}\mathbf{p}_{fdis} + \hat{\mathbf{v}}_{fem} \\ \mathbf{p}_{tint} &= \hat{\mathbf{R}}_{tib}\mathbf{p}_{tdis} + \hat{\mathbf{v}}_{tib}\end{aligned}\tag{2.1}$$

These coordinates are described with respect to the coordinate system of the tracking system transmitter, which will be referred to from this point on as the *global* coordinate system, $\{g\}$. Points whose coordinates are measured relative to this system may be noted either without or with a superscript g , *e.g.* $\mathbf{p}_{tint}^g$ and $\mathbf{p}_{fint}^g$.

²For the first five specimens (Knees J, D, L, M and X), only points on the periphery of the attachment site were digitised.

2.6 Articular Surface Geometry

Geometrical descriptions of the femoral and tibial condyles provide a means of defining consistent anatomically-based coordinate systems in each bone. Joint motion may then also be described in a consistent and well-defined manner. A description of surface geometry also allows an estimation of contact point movement during the motion. Finally, the articular surface geometry constitutes an essential parameter in a model of knee mobility.

2.6.1 Femoral Condyles

The femoral condyles were first fitted with spheres. The line joining the two fitted sphere centres provided a consistent definition of a medial-lateral axis of motion. Then, for a more physiological geometrical representation, the femoral condyles were fitted with polynomial surfaces.

2.6.1.1 Femoral Spheres

Spheres were fitted to the points measured on each femoral condyle by minimising the sum of the squares of the distances from each data point to the surface of the sphere. If (x_c, y_c, z_c) are the coordinates of the centre of the sphere and R is the radius, the sum of the squares of the distances from n data points (x_i, y_i, z_i) to the surface of the sphere is represented by the function F :

$$F = \sum_{i=1}^n |(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 - R^2| \quad (2.2)$$

The centre of the sphere (x_c, y_c, z_c) and the radius R were determined for each condyle by minimising F using Matlab's 'constr' function (Appendix C). The mean error of the fit was then expressed as the standard deviation³ over all n points of:

$$\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2} - R \quad (2.3)$$

Spheres were fitted to two sets of measured surface points for each condyle. First, a sphere was fitted to points measured on both the posterior and distal aspects of the condyle ('whole' sphere). Another sphere was fitted to points on the posterior aspect of the condyle only ('posterior' sphere) (Figure 2.2).

2.6.1.2 Femoral Polynomial Surfaces

To fit polynomial surfaces to the femoral condyles it was first necessary to transform the points on each condyle from global cartesian coordinates to local spherical coordinates. A local coordinate system, $\{f\}$, with axes parallel to those of the global

³Equivalent to the root mean squared error.

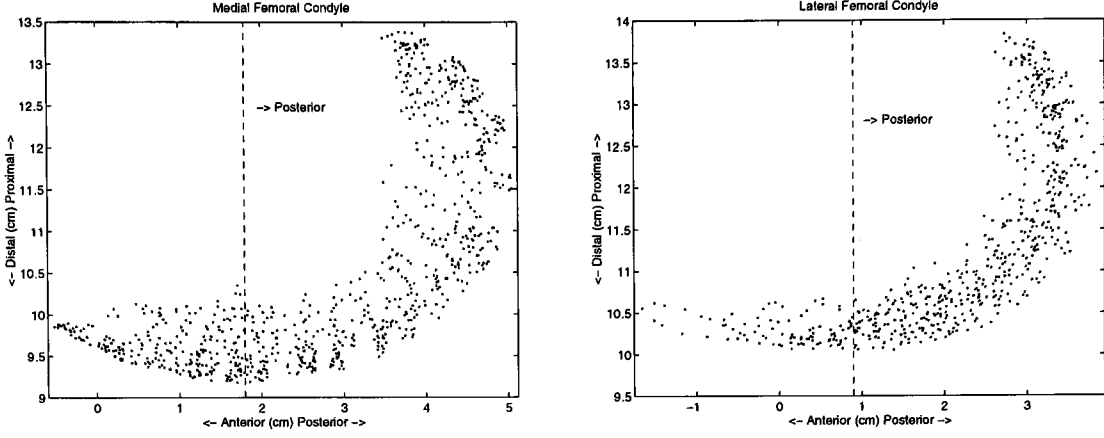


Figure 2.2: Points measured on the femoral condyles. ‘Whole’ spheres were fitted to all the points while ‘posterior’ spheres were fitted to points lying posterior to the most distal point on the condyle.

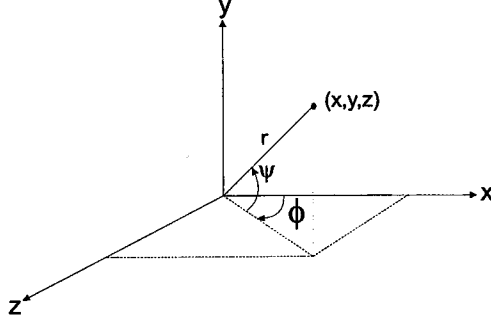


Figure 2.3: Definition of the spherical coordinates used to describe points on each femoral condyle. The origin is taken to be the centre of the best-fit sphere.

system in the fully extended knee was defined in each femoral condyle. The origin of each local frame was located at the centre of the fitted ‘whole’ sphere (x_c, y_c, z_c) . The transformation from the global to each local femoral coordinate system consisted of a pure translation so that a point on the surface in global coordinates, $\mathbf{p}_i^g$, was given by $\mathbf{p}_i^f$ in local coordinates:

$$\mathbf{p}_i^f = \mathbf{p}_i^g - \begin{bmatrix} x_c & y_c & z_c \end{bmatrix}^T \quad (2.4)$$

The local $\mathbf{p}_i^f$ were then converted from cartesian coordinates (x_i, y_i, z_i) to the spherical coordinates (ψ_i, ϕ_i, r_i) (Figure 2.3) such that:

$$\begin{aligned} r &= \sqrt{x_i^2 + y_i^2 + z_i^2} \\ h &= \sqrt{x_i^2 + z_i^2} \\ \psi &= \arctan(y_i/h) \\ \phi &= \arctan(z_i/x_i) \end{aligned} \quad (2.5)$$

where $\arctan$ represents the four-quadrant inverse tangent. Polynomial surfaces were then fitted to the spherical coordinates. The radial coordinate, r , of a point on the

surface was defined by a polynomial function of the ψ and ϕ coordinates:

$$r = f(\psi, \phi) = \sum_{i=0}^k \sum_{j=0}^m p_{ij} \psi^i \phi^j \quad (2.6)$$

The best-fit coefficients p_{ij} , a total of $(k+1)(m+1)$, were determined in a least-squares sense using Matlab's 'polyfit2d' function (Appendix C). The mean error of the fit was then expressed as the standard deviation over the n measured points of $r_i - f(\psi_i, \phi_i)$.

2.6.2 Tibial Condyles

To provide a second axis of motion, a longitudinal axis of the tibia was defined based on the measured geometry of the condyles. The tibial condyles were then each fitted with a plane, as a first approximation, and then, for a more physiological representation, with a polynomial surface.

2.6.2.1 Tibial Longitudinal Axis

The orientation of the longitudinal axis of the tibia was defined using a bi-planar analysis of the tibial condyle geometry. The definition was based on the 'functional' axis described by Yoshioka *et al.* [145] in their study of tibial geometry. Points on the tibial articular surfaces were examined in both the sagittal and frontal planes (Figure 2.4). In each plane, lines approximating the slopes of the medial and lateral condyles were related to the orientation of the tibial axis. The orientation of this proximodistal axis was defined by the non-unit vector:

$$\mathbf{v}_{pd} = \begin{bmatrix} 1/m_s & 1/m_f & 1.0 \end{bmatrix}^T \quad (2.7)$$

where m_s is the slope of the line in the sagittal plane perpendicular to an antero-posterior axis, inclined at an angle of 7° (average λ_m reported by Yoshioka *et al.*) to the line joining the most anterior and posterior points measured on the medial tibial plateau (Figure 2.4). m_f is the slope of the line in the frontal plane perpendicular to a transverse axis, inclined at an angle of 2° (average θ reported by Yoshioka *et al.*) to the line joining the most medial and lateral points measured on the medial and lateral tibial condyles respectively (Figure 2.4).

2.6.2.2 Tibial Planes

For the planar fits, the sum of the squared distances of the data points to the fitted plane was minimised. The equation for a plane is:

$$ax + by + cz + d = 0. \quad (2.8)$$

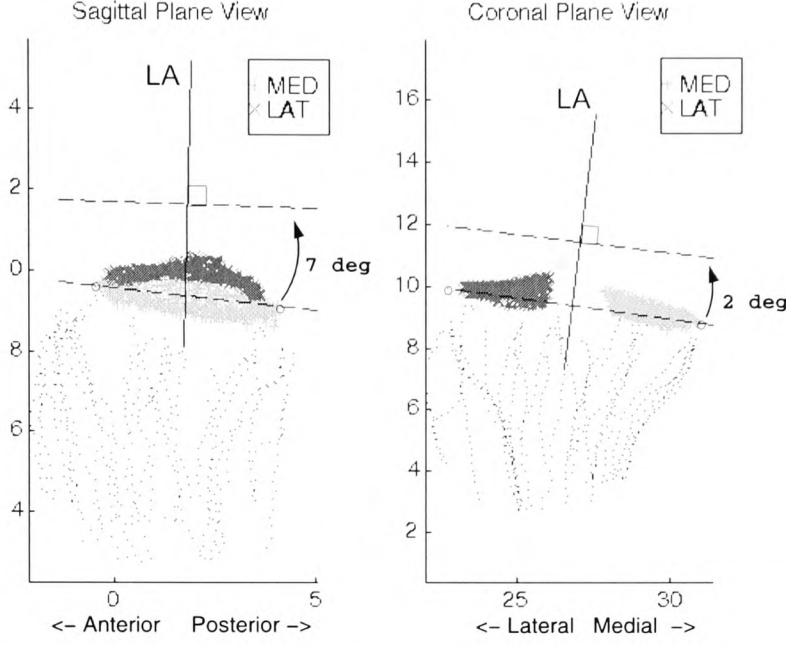


Figure 2.4: The digitised tibia viewed in the sagittal (left) and frontal (right) planes.

$\mathbf{v}_n = [a \ b \ c]^T$ represents a vector normal to the plane, and $\mathbf{v}_p = [0 \ 0 \ -d/c]^T$ a point on the plane. The distance, D_i , from a data point (x_i, y_i, z_i) to the plane is:

$$D_i = ([x_i \ y_i \ z_i]^T - \mathbf{v}_p) \cdot \hat{\mathbf{v}}_n \quad (2.9)$$

where $\hat{\mathbf{v}}_n$ is the unit normal vector to the plane. The function:

$$F = \sum_{i=1}^n (D_i)^2 \quad (2.10)$$

where n is the number of measured points, was minimised using Matlab's 'constr' function (Appendix C) to determine the parameters a , b , c and d of the best-fit plane. The mean error associated with the each fitted plane was given by the standard deviation of D_i over the n measured points.

A plane whose normal vector was parallel to the defined longitudinal axis of the tibia, was also fitted to each tibial condyle. In this case, the distance from each point to the 'transverse' plane is given by Equation 2.9 where $\hat{\mathbf{v}}_n$ is known and equal to the unit vector $\hat{\mathbf{v}}_{pd}$. Again, Matlab's 'constr' algorithm was used to minimise the sum of the squared distances D_i and to find the best-fit z intercept of the plane, $-d/c$. A measure of the error was also given by the standard deviation of D_i over all n measured points.

2.6.2.3 Tibial Polynomial Surfaces

Polynomial surfaces were fitted to the measured surface points. The z coordinate of a point on the surface was defined by a polynomial function of the x and y

coordinates:

$$z = f(x, y) = \sum_{i=0}^k \sum_{j=0}^m p_{ij} x^i y^j \quad (2.11)$$

The best-fit coefficients p_{ij} , a total of $(k+1)(m+1)$, were determined in a least-squares sense using Matlab's 'polyfit2d' function (Appendix C). The mean error of the fit was then expressed as the standard deviation of $z_i - f(x_i, y_i)$ over the n measured points.

2.7 Ligament Geometry

To track the lengths of individual ligament fibres during the experimental motion and to incorporate arrays of 3-D fibres into a model of knee mobility and stability, a description of the ligament fibre architecture was required. Geometric representation of the architecture involves a description of the ligament attachment sites on the bones and of the relationships between the femoral and tibial attachment sites of individual fibres.

2.7.1 Attachment Sites

To describe the ligament attachment sites, planes and polynomial surfaces were fitted to the measured data points.

2.7.1.1 Planes of Attachment

A plane was fitted to the points measured on each attachment as described in Section 2.6.2.2 for the tibial condyles. The fitted plane was again defined by a vector normal to the plane, $\mathbf{v}_n = [a \ b \ c]^T$, and the z intercept of the plane, $-d/c$.

2.7.1.2 Local Coordinate Systems

A local coordinate system, $\{l\}$, was defined at each ligament attachment site. The Z axis, described by the unit vector $\mathbf{u}_z$, was defined to lie along the normal to the fitted plane. The X axis ($\mathbf{u}_x$) was defined to lie in the plane such that it was directed approximately posteriorly. The Y axis ($\mathbf{u}_y$) was then defined by the cross product of $\mathbf{u}_z$ and $\mathbf{u}_x$.

The transformation matrix describing the position and orientation of the local coordinate system at the ligament attachment site $\{l\}$ relative to the global coordinate system $\{g\}$,

$$\mathbf{T}_l^g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \mathbf{p}_{lp} & \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \end{bmatrix} \quad (2.12)$$

where $\mathbf{p}_{1p}$ is the projection of one of the measured points to the plane. The matrix was used to describe the points measured on the attachment site, $\mathbf{p}_i^g$, with respect to the corresponding local attachment frame:

$$\mathbf{p}_i^l = (\mathbf{T}_l^g)^{-1} \mathbf{p}_i^g \quad (2.13)$$

2.7.1.3 Polynomial Surfaces of Attachment

A polynomial surface was fitted to the local coordinates, $\mathbf{p}_i^l$, of the measured points at each attachment. The z coordinate of a point on the surface was defined by a polynomial function of the x and y coordinates:

$$z = f(x, y) = \sum_{i=0}^k \sum_{j=0}^m p_{ij} x^i y^j \quad (2.14)$$

The best-fit coefficients p_{ij} , a total of $(k+1)(m+1)$, were determined in a least-squares sense, as in the case of the tibial polynomial surface fitting, using the Matlab function 'polyfit2d' (Appendix C). Again, the mean error of the fit was then expressed as the standard deviation of $z_i - f(x_i, y_i)$ over the n measured points.

2.7.1.4 Measured Attachment Site Boundaries

The boundary of each measured attachment site was described by a series of bounding rectangles in the fitted plane (local X - Y plane). The area in the plane covered by the local (x, y) coordinates of the measured points was divided into a series of rectangular strips parallel to the local x axis. Each rectangular strip i was defined by the y coordinate of its lower edge, y_i , and the minimum and maximum x coordinate within each strip, $x_{i,min}$ and $x_{i,max}$. These coordinates defined a series of n bounding rectangles in the plane:

$$\mathbf{B}_m = \begin{bmatrix} y_1 & x_{1,min} & x_{1,max} \\ y_2 & y_{2,min} & x_{2,max} \\ & \dots & \\ y_n & x_{n,min} & x_{n,max} \end{bmatrix} \quad (2.15)$$

The joined outer edges of these rectangles defined the boundary of the *measured* attachment site in the plane (Figure 2.5). The total area covered by the rectangles provided an estimate of the area of the *measured* attachment site.

2.7.2 Fibre Mapping

The tracing of individual fibres from their attachment on the tibia to their attachment on the femur was not done in the experiment. The tibial and femoral attachment sites of the ligaments were measured after joint disarticulation. Therefore, in

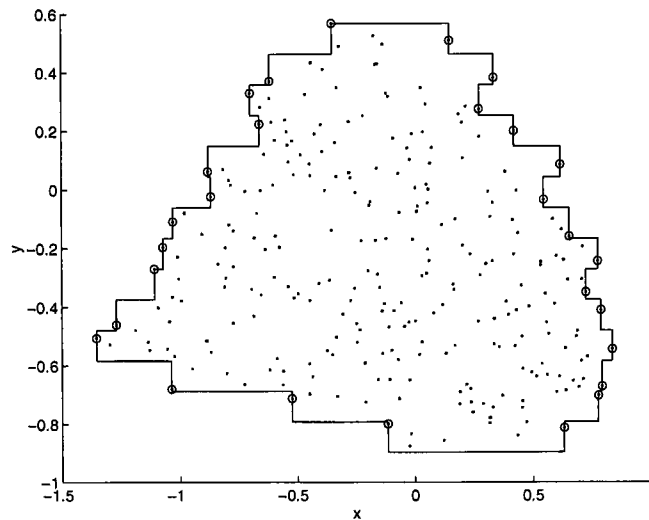


Figure 2.5: The boundary of a ligament attachment site as defined by $\mathbf{B}_m$ (example of an ACL femoral attachment).

order to describe a relationship between the tibial and femoral attachment sites of the individual fibres of a ligament, descriptions of ligament fibre anatomy reported in the literature and the author's own experimental observations were relied upon.

Based on the geometry of the measured attachment sites and general anatomical descriptions, a 3-D mapping function was derived for each ligament. The mapping function relates local coordinates of a fibre's tibial attachment, (x_t, y_t) , to those of its femoral attachment, (x_f, y_f) .

The relationship was derived based on a method described by Van der Helm and Veenbaas [52] which maps muscle fibres from a line attachment to a planar attachment region of arbitrary shape. The method was extended to map planar regions to planar regions. Two separate functions were formulated, one which maps the (x_t, y_t) coordinates to the corresponding x_f coordinate and one which maps the (x_t, y_t) coordinates to the corresponding y_f :

$$x_f = f_x(x_t, y_t) \quad (2.16)$$

$$y_f = f_y(x_t, y_t) \quad (2.17)$$

To establish the functions, f_x and f_y , an interactive computer program was developed in Matlab. First, the measured points on the tibial and femoral attachment sites were projected to and plotted in the local X - Y planes (Figure 2.6). The most posterior and anterior, medial and lateral, and proximal and distal of these points were identified as those with the largest and smallest global x , y and z coordinates respectively.

Descriptions of fibre bundle anatomy shown in Figures 2.7 and 2.8 provided a basis for selecting pairs of physiological fibre attachment sites. The anatomy of

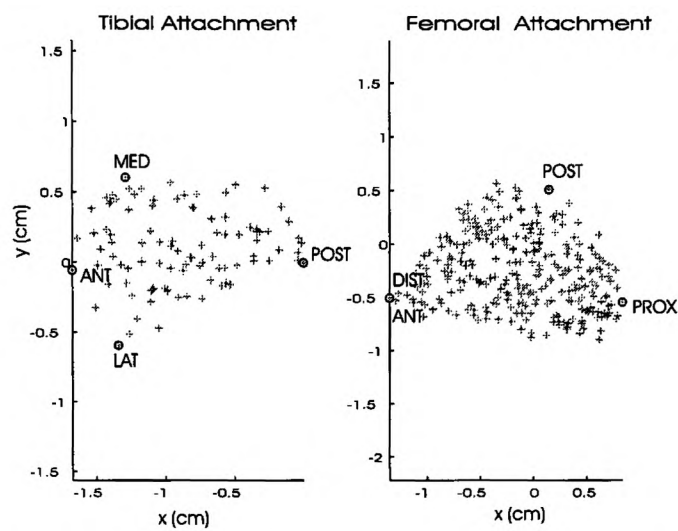


Figure 2.6: The Matlab interface to allow the user to pick off pairs of attachment sites on the tibial and femoral attachment sites corresponding to approximately physiological fibres. ACL attachments of a representative specimen are shown.

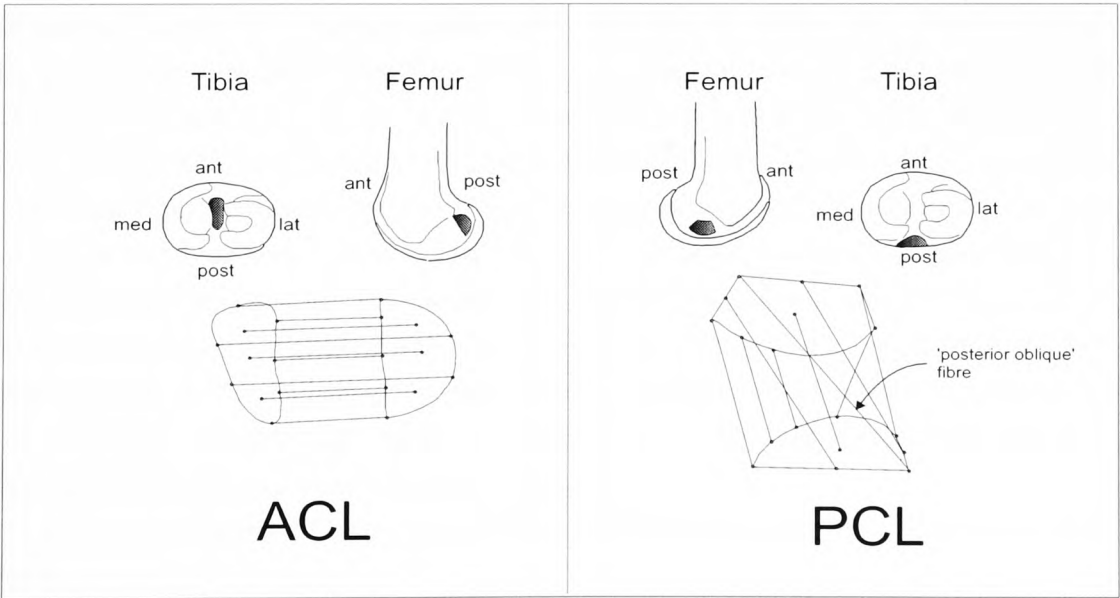


Figure 2.7: Description of ACL and PCL fibre bundle anatomy derived from Friederich *et al.* [40]. Mapping of specific fibres from their tibial to femoral attachment sites is shown. The attachment sites of additional fibres (not described by Friederich *et al.*), approximated through interpolation, have been included to enhance the description.

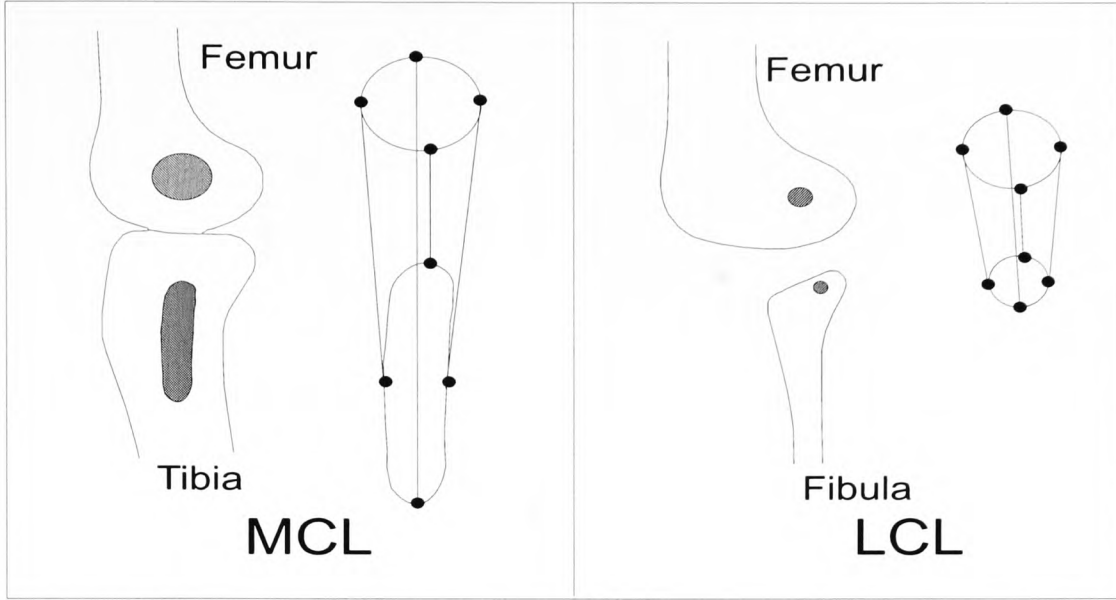


Figure 2.8: Description of MCL and LCL fibre bundle anatomy based on the author’s own observations and general descriptions of these ligaments in the literature. Mapping of specific fibres from their tibial to femoral attachment sites is shown. Diagrams of the bones and the ligament attachment sites were derived from Zavatsky and O’Connor [147].

individual fibre bundle attachments of the cruciate ligaments (ACL and PCL) (Figure 2.7) was derived from observations made by Friederich *et al.* [40] and O’Brien *et al.* [97]. Mommersteeg [86] also measured the specific mapping of fibre bundles of the ACL and PCL on three specimens. However, a more general mapping description was required. The fibre bundle attachments of the collateral ligaments (Figure 2.8) were based on general anatomical descriptions (*e.g.* Burks [21], Wilson *et al.* [137]) and the author’s own experimental observations due to a lack of detailed information in the literature. The program prompted the user to record pairs of points (by selecting points within the plots with the mouse): a point within the tibial attachment, $(\hat{x}_t, \hat{y}_t)$, followed by one within the femoral attachment, $(\hat{x}_f, \hat{y}_f)$, so that each pair represented the approximate attachments of a ‘physiological’ fibre. Referring to the descriptions of fibre bundle anatomy based on the shapes of the attachments, at least eight pairs of points on the tibia and femoral attachments, $(\hat{x}_{ti}, \hat{y}_{ti})$ and $(\hat{x}_{fi}, \hat{y}_{fi})$ were selected (Figure 2.9).

Given these pairs of data points, the best-fit linear functions f_x and f_y of the form:

$$x_f = f_x(x_t, y_t) = a_1 x_t + b_1 y_t + c_1 \quad (2.18)$$

$$y_f = f_y(x_t, y_t) = a_2 x_t + b_2 y_t + c_2 \quad (2.19)$$

were determined in a least-squares sense using Matlab’s ‘constr’ function (Appendix C).

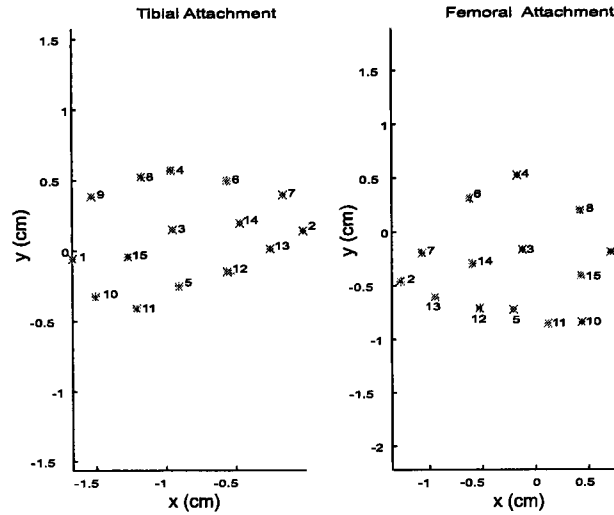


Figure 2.9: Example of the pairs of attachment sites of the ACL of a representative specimen selected to correspond to approximately physiological fibres.

2.7.2.1 Fibre Mapping Evaluation

The mapping function, defined by f_x and f_y , was evaluated in two ways. First, the procedure was assessed by mapping the coordinates of the tibial attachments of the selected physiological fibres, $(\hat{x}_{ti}, \hat{y}_{ti})$, and comparing the coordinates of the mapped femoral attachments, (x_{fi}, y_{fi}) (calculated using Equations 2.18 and 2.19), to those of the selected fibres, $(\hat{x}_{fi}, \hat{y}_{fi})$. A measure of the error involved in the mapping was then given by the mean distance between the two sets of points in the plane of the femoral attachment.

Then, using data derived from fibre bundle tracing experiments of the ACL and PCL reported in the literature, a second evaluation was made. Mommersteeg [86] provided detailed descriptions of the fibre bundle anatomy of the ACL and PCL in the form of connectivity maps based on measurements made on three cadaver specimens. Between 8 and 9 bundles for the ACL and between 6 and 10 bundles for the PCL were described.

The planar views of the tibial and femoral attachments provided for each ligament of each specimen (Figures 4.7 and 4.8 in [86]) were scanned and saved as separate images (*e.g.* Figure 2.10). Using software provided with the Silicon Graphics computer system ('imgview'), it was possible to identify the pixel coordinates of the cursor location within each image. Mapping functions were derived for each of the ACL and PCL using the methods described above. Only the shape of the outer boundary was used to guide the selection of pairs of 'physiological' attachment points. These were described in image pixel coordinates. The derived functions, f_x and f_y , were then evaluated by mapping the coordinates of the approximate centroid of the tibial attachment of each bundle, and comparing the coordinates of

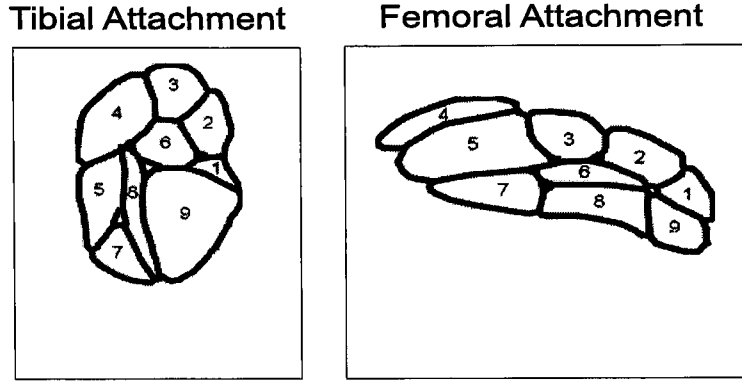


Figure 2.10: Example of the figures provided by Mommersteeg (ACL attachments of Specimen 1) [86].

the mapped femoral attachment with those of the approximate centroid of the corresponding femoral bundle attachment. A measure of the error involved in the mapping was then given by the mean distance between the two sets of centroids in the plane of the femoral attachment.

No similar evaluation was done for the MCL or LCL. Only qualitative descriptions of the fibre bundle anatomy of the MCL and LCL were found in the literature. This may be explained by the difficulty in identifying individual MCL fibre bundle insertions on the tibia and the cord-like nature of the LCL.

2.7.3 Fibre Array Definition and Modified Attachments

For each ligament, an array of fibres was defined based on the mapping functions (f_x and f_y), the fitted polynomial surface functions (f_{polyf} and f_{polyt}), the bounding rectangles ($\mathbf{B}_{mt}$ and $\mathbf{B}_{mf}$), and the transformation matrices ($\mathbf{T}_{tt}^g$ and $\mathbf{T}_{tf}^g$).

Within the measured tibial attachment site boundary, $\mathbf{B}_{mt}$, a grid of (x_t, y_t) coordinates was defined. The density of the grid, chosen to be 0.5 mm for visualisation purposes and for tracking ligament fibre lengths, determined the final number of fibres in the array. For each set of tibial coordinates (x_t, y_t) , the corresponding femoral coordinates (x_f, y_f) were determined based on the mapping functions. A fibre was only defined, however, if the resulting (x_f, y_f) fell within the measured femoral attachment site boundary, $\mathbf{B}_{mf}$. Based on the final array of fibres, new boundaries for the tibial attachment, $\mathbf{B}_t$, and femoral attachment, $\mathbf{B}_f$, and corresponding attachment site areas were defined in the attachment planes. The z coordinates of the tibial and femoral fibre attachment sites were then given by $z = f_{poly}(x, y)$ where f_{poly} represents the fitted polynomial surface of the tibial or femoral attachment. The local coordinates of the points on each attachment site were then converted back to global coordinates using the corresponding transformation matrix $\mathbf{T}_l^g$. Because the attachment areas on the femur and tibia differed, the density of fibres on

the two attachment sites also differed.

The repeatability of the entire process, including the derivation of the mapping function, was assessed by performing the procedure three times for each ligament with one specimen (Knee F).

2.8 Knee Motion

The passive movement of the knee was measured to provide a conceptual basis for the formulation of a model of knee mobility (Chapter 5) and to provide data with which to validate the subsequent predictions of such a model (Chapters 6 and 7). Knee motion was described as the relative movement between the tibia and the femur. A complete description of the 3-D joint movement was provided by three rotations and three components of translation with respect to sets of clinically relevant axes. The relative motion was also expressed by a set of finite helical axes. The uniqueness of the motion path was examined.

2.8.1 Joint Position

To calculate the location of the femur with respect to the tibia during each flexion/extension cycle, the location of the sensor fixed to the femur with respect to the transmitter (fixed to the workbench and hence the tibia) was computed. For a given output $(x, y, z, \psi, \theta, \phi)$ for the Polhemus sensor location, a transformation describing the location of the sensor fixed frame, $\{s\}$, relative to the global frame (*i.e.* of the transmitter), $\{g\}$, was computed. The rotation matrix was calculated as follows:

$$\mathbf{R}_s^g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.20)$$

The complete transformation was then expressed as a 4x4 matrix $\mathbf{T}_s^g$ based on the rotation matrix $\mathbf{R} = \mathbf{R}_s^g$ and the translation vector $\mathbf{v} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ (Appendix B.1). If the location of the sensor at full extension is described by the transformation $\mathbf{T}_{s0}^g$ and at a subsequent location i by the transformation $\mathbf{T}_{si}^g$, the transformation which displaces vectors in the global frame $\{g\}$ from location 0 to location i according to the displacement of the sensor, and hence the femur, is then:

$$\mathbf{T}_{0i} = \mathbf{T}_{si}^g (\mathbf{T}_{s0}^g)^{-1} \quad (2.21)$$

The operator $\mathbf{T}_{0i}$ displaces the vectors describing the coordinates of femoral anatomical features at full extension according to the measured displacement of the femur.

The inverse of this matrix $(\mathbf{T}_{0i})^{-1}$ is the operator which displaces the tibial features relative to a fixed femur.

2.8.2 Tibial and Femoral Body-Fixed Axes

Two clinically relevant axes were defined with respect to the anatomy of the femur and tibia as described in previous sections:

a mediolateral axis defined by the unit vector $\hat{\mathbf{v}}_{ml}$ lying along the line connecting the centres of the spheres fitted to the femoral condyles at full extension ('posterior' spheres, Section 2.6.1.1), directed medially for both right and left knees

a proximodistal axis defined by the unit vector $\hat{\mathbf{v}}_{pd}$ describing the orientation of the functional longitudinal axis of the tibia, directed proximally (Section 2.6.2.1)

Sets of orthogonal, body-fixed axes in the femur and tibia were then defined based on these axis orientations and represented by the unit vectors $(\hat{\mathbf{f}}_f, \hat{\mathbf{l}}_f, \hat{\mathbf{t}}_f)$ and $(\hat{\mathbf{f}}_t, \hat{\mathbf{l}}_t, \hat{\mathbf{t}}_t)$ respectively. The unit vector $\hat{\mathbf{f}}$ refers to a mediolateral (or flexion) axis, $\hat{\mathbf{l}}$ to a proximodistal (or axial rotation) axis and $\hat{\mathbf{t}}$ to an anteroposterior (or ab/adduction) axis. The set of body-fixed axes in each bone was defined as the set of *orthogonal* axes closest to those based on the clinical directions described above. For a right knee, the femoral axes were defined as:

$$\hat{\mathbf{f}}_f = \hat{\mathbf{v}}_{ml} \quad (2.22)$$

$$\hat{\mathbf{t}}_f = \hat{\mathbf{f}}_f \times \hat{\mathbf{v}}_{pd} \quad (2.23)$$

$$\hat{\mathbf{l}}_f = \hat{\mathbf{t}}_f \times \hat{\mathbf{f}}_f \quad (2.24)$$

and the tibial axes as:

$$\hat{\mathbf{l}}_t = \hat{\mathbf{v}}_{pd} \quad (2.25)$$

$$\hat{\mathbf{t}}_t = \hat{\mathbf{v}}_{ml} \times \hat{\mathbf{l}}_t \quad (2.26)$$

$$\hat{\mathbf{f}}_t = \hat{\mathbf{l}}_t \times \hat{\mathbf{t}}_t \quad (2.27)$$

so that $\hat{\mathbf{f}}_f$ and $\hat{\mathbf{f}}_t$ are directed medially, $\hat{\mathbf{l}}_f$ and $\hat{\mathbf{l}}_t$ proximally, and $\hat{\mathbf{t}}_f$ and $\hat{\mathbf{t}}_t$ anteriorly.

For a left knee, the axes are defined in the same way with the following exceptions:

$$\hat{\mathbf{f}}_f = -\hat{\mathbf{v}}_{ml} \quad (2.28)$$

$$\hat{\mathbf{t}}_f = \hat{\mathbf{f}}_f \times -\hat{\mathbf{v}}_{pd} \quad (2.29)$$

and:

$$\hat{\mathbf{l}}_t = -\hat{\mathbf{v}}_{pd} \quad (2.30)$$

$$\hat{\mathbf{t}}_t = -\hat{\mathbf{v}}_{ml} \times \hat{\mathbf{l}}_t \quad (2.31)$$

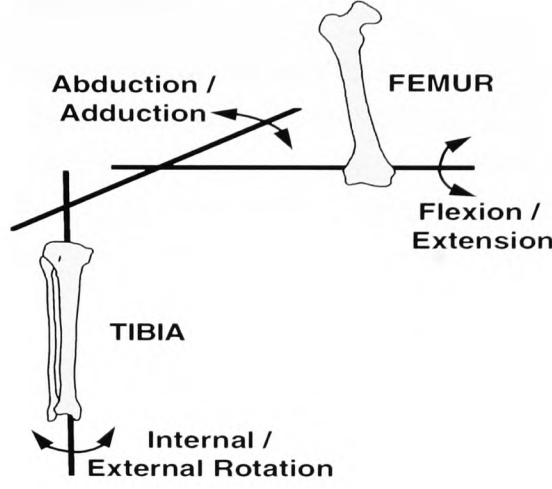


Figure 2.11: The Joint Coordinate System

so that $\hat{\mathbf{f}}_f$ and $\hat{\mathbf{f}}_t$ are directed medially, $\hat{\mathbf{l}}_f$ and $\hat{\mathbf{l}}_t$ distally, and $\hat{\mathbf{t}}_f$ and $\hat{\mathbf{t}}_t$ posteriorly.

2.8.3 Joint Rotations

To describe joint rotations, the tibia was considered to be moving relative to a fixed femur. The unit vectors representing the body-fixed axes of the tibia were displaced by the transformation $(\mathbf{T}_{0i})^{-1}$ for each motion step i . Knee orientation was then described by three rotations about the axes of a Joint Coordinate System (JCS) [29, 49], a standardised, widely-used method of assigning motion axes at a joint (Figure 2.11). The JCS assigns an internal/external rotation axis fixed in the tibia, a flexion/extension axis fixed in the femur and a ‘floating’ abduction/adduction axis defined as the cross product of the other two axes. The axes of the JCS were defined as follows:

- flexion/extension axis: $\hat{\mathbf{e}}_1 = \hat{\mathbf{f}}_f$
- internal/external rotation axis: $\hat{\mathbf{e}}_3 = \hat{\mathbf{l}}_t$
- abduction/adduction axis: $\hat{\mathbf{e}}_2 = \hat{\mathbf{e}}_3 \times \hat{\mathbf{e}}_1$

Next the angles of rotation were computed using an algorithm developed by Cole *et al.* [29]:

- Flexion angle: $\Phi_1 = \arccos(\hat{\mathbf{e}}_2 \cdot \hat{\mathbf{t}}_f) * \text{sign}(\hat{\mathbf{e}}_2 \cdot \hat{\mathbf{l}}_f)$
- Ab/adduction angle: $\Phi_2 = \arccos(\hat{\mathbf{r}} \cdot \hat{\mathbf{l}}_t) * \text{sign}(\hat{\mathbf{f}}_f \cdot \hat{\mathbf{l}}_t)$, where $\hat{\mathbf{r}} = \left(\frac{\hat{\mathbf{f}}_f \times \hat{\mathbf{e}}_2}{|\hat{\mathbf{f}}_f \times \hat{\mathbf{e}}_2|} \right)$
- Internal/external rotation: $\Phi_3 = \arccos(\hat{\mathbf{e}}_2 \cdot \hat{\mathbf{t}}_t) * \text{sign}(\hat{\mathbf{e}}_2 \cdot \hat{\mathbf{f}}_t)$

Internal rotation of the tibia and knee abduction were therefore defined as positive.

2.8.4 Joint Translations

The translation of a reference point on the tibia from its position in the fully extended knee with respect to a fixed femur was also determined. The reference point, $\mathbf{p}_{ref}$, was chosen to be the most posterior point on the digitised tibial attachment of the ACL. The position of the reference point at a subsequent knee position i , $\mathbf{p}_{ref,i}$ was then given by:

$$\mathbf{p}_{ref,i} = (\mathbf{T}_{0i})^{-1} \mathbf{p}_{ref,0} \quad (2.32)$$

where $\mathbf{p}_{ref,0}$ refers to its position in the fully extended knee. The displacement of the point from its original position to its current position i was then described relative to the coordinate system fixed in the femur:

$$\mathbf{d}_i^f = (\mathbf{p}_{ref,i} - \mathbf{p}_{ref,0}) \cdot [\hat{\mathbf{t}}_f \quad \hat{\mathbf{f}}_f \quad \hat{\mathbf{l}}_f]^T \quad (2.33)$$

Whether a right or a left knee, for the translations, the posterior direction was always assigned as positive x , medial positive y , and proximal positive z .

2.8.5 Flexion and Extension Paths

To calculate differences between the flexing and extending paths of movement and to describe average motion, internal/external rotation, ab/adduction and the three components of translation for one cycle were split into flexing and extending subsets for each specimen. These subsets were then fitted with quintic splines [140] which were evaluated every 1° of flexion over the range of motion.

2.8.6 Finite Helical Axes

The motion of the tibia with respect to the femur (and vice versa) was also described by a series of finite helical axes (FHAs). The positions and orientations of the axes were calculated for steps of 20° flexion or extension (0-20, 10-30, 20-40, 30-50, 40-60, 50-70, 60-80, 70-90, 80-100°). For each flexion angle considered, the closest corresponding movement step and associated transformation matrix $\mathbf{T}_s^g$, describing the location of the sensor in the global frame, was found. The transformations $\mathbf{T}_{s1}^g$ and $\mathbf{T}_{s2}^g$ describing the sensor locations at two subsequent locations of the femur, were used to calculate the transformation which displaces the femur in the global frame from location 1 to 2:

$$\mathbf{T}_{12} = \mathbf{T}_{s2}^g (\mathbf{T}_{s1}^g)^{-1} \quad (2.34)$$

The inverse $(\mathbf{T}_{12})^{-1}$ describes the displacement of the tibia relative to the femur. For both the case of a moving tibia relative to a fixed femur and a moving femur relative to a fixed tibia, the parameters of the helical motion:

- the orientation of the axis given by the direction vector, $\mathbf{v}_n$
- a point on the axis given by the position vector, $\mathbf{p}_s$
- the angle of rotation about the axis, ϕ
- the translation along the axis, t

were determined from the corresponding rotation matrix and translation vector, using an algorithm by Spoor and Veldpaus [120].

In each case, a mean helical axis (MHA), the axis closest to all nine FHAs in a least-squares sense, was also computed for both flexing and extending subsets using an algorithm described by Woltring [141].

2.9 Articular Contact Movement

Contact between the tibia and femur occurs in both the medial and lateral compartments of the knee. Because direct measurement in experiment was not possible, an algorithm based on a geometric description of the articular surfaces and a proximity criterion was developed to estimate the movement of the contact areas with flexion. The contact in each compartment was idealised by a point, assumed to represent the geometrical centre of the contact area. Contact points were estimated every 5° of flexion from 0° to 100°.

2.9.1 Surface Geometry

The femoral and tibial condyles were represented by fitted polynomial surfaces (Sections 2.6.2.3, 2.6.1.2).

2.9.2 Proximity Criterion

The experimental error in the reconstruction of the intact knee geometry results in some interpenetration or separation of the measured tibial and femoral surfaces in the intact knee (at any flexion angle). A minimisation scheme capable of handling both occurrences was developed to identify the point which best represented the location of the contact point in each compartment.

For both the medial and lateral compartments, a function was formulated which describes the difference between the radial coordinate of a given point on the tibial polynomial surface, r_t , and that of a corresponding point on the femoral surface, r_f . The difference between these radii describes the intersection of the two surfaces. The function $(r_t - r_f)$ was minimised to find the points on tibial and femoral surfaces, defined by coordinates (x_t, y_t) and (ψ_f, ϕ_f) , which best describe the contact. The function is such that when the surfaces did not intersect, the minimum corresponded

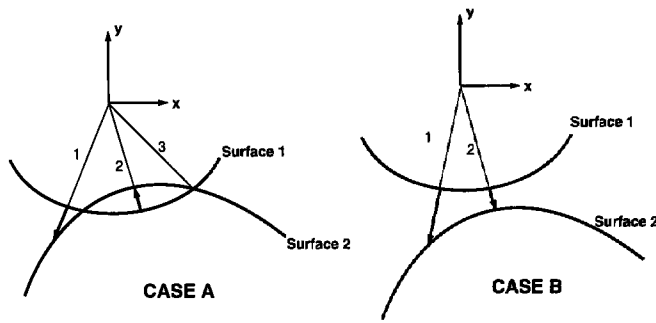


Figure 2.12: 2-D analogy of surface proximity function. If the surfaces intersect in the region of interest (Case A), the function is positive where there is separation (1), negative where there is interpenetration (2) and zero where they intersect (3). If surfaces do not intersect in the region of interest (Case B), the function is always positive (1,2).

to the point on the surface which lies closest to the opposing surface. In the case of interpenetration, the minimum corresponded to the point which penetrates furthest radially into the opposing surface. These cases are illustrated by the 2-D analogy depicted in Figure 2.12.

The radial difference was expressed first as a function of the tibial coordinates (x_t, y_t) . The z coordinate of the point on the tibial surface was found: $z_t = f(x_t, y_t)$, where f describes the fitted polynomial function. The coordinates of this point on the tibia were then displaced according to the measured position of the knee i at the flexion angle of interest:

$$\mathbf{p}_{ti} = (\mathbf{T}_{0i})^{-1} \mathbf{p}_{t0} \quad (2.35)$$

where $\mathbf{p}_{t0} = [x_t \ y_t \ z_t]^T$. This point described with respect to the local femoral frame was determined by:

$$\mathbf{p}_f = \mathbf{p}_{ti} - [x_c \ y_c \ z_c]^T \quad (2.36)$$

where (x_c, y_c, z_c) are the coordinates of the centre of the femoral sphere. The radial coordinate of this point on the tibial surface was then given by: $r_t = \|\mathbf{p}_f\|$.

The spherical coordinates of the point $\mathbf{p}_f$ were calculated using Equations 2.5. For the associated (ψ, ϕ) coordinates, the radial coordinate of the corresponding point on the femoral surface was: $r_f = f(\psi, \phi)$, where f in this case describes the fitted femoral polynomial surface.

The function:

$$f(x_t, y_t) = r_t - r_f \quad (2.37)$$

was then minimised using Matlab's 'constr' function (Appendix C) to determine the (x_t, y_t) best describing the tibial contact point. Lower and upper bounds were set on the variables based on the measured range of points. At 0° of flexion, an initial

guess for the tibial (x_t, y_t) coordinates was supplied by the (x, y) coordinates of the corresponding femoral sphere centre. For subsequent flexion angles, the points found for the previous flexion step provided the initial guess.

A similar procedure, in which the radial difference was expressed as a function of the femoral coordinates (ψ_f, ϕ_f) , was used to estimate the contact points on the femoral condyles. In the case of the femur, $(0, -\pi/2)$ provided an initial guess for the femoral coordinates, (ψ, ϕ) , in each compartment.

2.10 Ligament Fibre Lengths

The lengths of individual ligament fibres were tracked during the recorded passive movement of the knee. For each of the ACL, PCL, MCL and LCL, two sets of fibres were tracked:

Combination Fibres based on all possible combinations of tibial and femoral *measured* attachment points.

Physiological Fibres making up the array defined by the derived mapping functions, the fitted polynomial surface functions, and the bounding rectangles of the ligament's measured attachment sites (Section 2.7.3).

In each case, the femoral attachment points described in global coordinates were displaced according to the measured movement by the transformation $\mathbf{T}_{0i}$, corresponding to the knee position i . The fibre length was then defined as the length of the line segment joining the femoral and tibial attachment points. The lengths of the fibres were tracked over the range of motion in steps of 5° flexion. A length ratio was defined as:

$$LR = L_i / L_o \quad (2.38)$$

where L_i is the length of the ligament at position i and L_o its length in full extension. For each fibre, the maximum change in the length ratio over a 100° flexion range was expressed as a percentage:

$$\Delta LR_{max} = \frac{L_{max} - L_{min}}{L_o} \times 100 \quad (2.39)$$

A search over all possible fibres yielded the fibre with the lowest ΔLR_{max} . This fibre was defined as the most nearly isometric.

2.11 Accuracy and Experimental Error

2.11.1 Manufacturer's Data

The static accuracy of the Polhemus Isotrak II electromagnetic tracking system is quoted by the manufacturer as 2.4 mm root mean square (RMS) for position (x, y, z) and 0.75° RMS for angular position (roll, elevation, azimuth) within a sensor range of 10.2 cm to 71.1 cm from the source. The quoted resolution of the device for the range used in the current experiments was 0.25 - 0.4 mm for position coordinates and 0.03° for angular values.

2.11.2 Digitisation Error

The error associated with the digitisation of spatial coordinates was re-evaluated using a method developed previously [135]. A perspex plate with a reference grid of countersunk holes of 1 mm diameter (6 holes x 4 holes) was mounted approximately 20 cm from the transmitter (the digitisation of the specimens was performed in a sensor range of approximately 15 cm to 30 cm from the transmitter). The stylus was used to digitise each of the holes in turn. Based on the known separation of the holes on the grid (accurate to 0.01 mm) and using the procedure of Veldpaus *et al.* [131] as described in Section 2.5, the spatial transformation between the measured points and the actual positions of the holes was determined. This was done for four orientations of the plate relative to the transmitter. A measure of the static digitisation error was given by the mean distance between the measured points and the points predicted by the calculated transformations.

2.11.3 Reconstruction Error

The reconstruction error was evaluated by calculating the mean distance between the positions of the reference markers measured in the fully extended intact knee and those transformed to the fully extended intact knee from the measured coordinates in the disarticulated bones.

2.11.4 Contact Error

Contact error describes the minimum separation or maximum interpenetration of the measured surfaces. It is a manifestation of the error involved in the reconstruction of the intact knee geometry. It is unlikely to reflect true surface deformation or separation because loading of the joint was virtually zero (< 5 N, Section 2.4.3). Using polynomial surface descriptions and the proximity criterion described in Section 2.9, the contact error in each compartment was defined at 0, 30, 60 and 90°

flexion as the distance between the contact point on the tibial condyle and the contact point on the femoral condyle. It was noted whether the error represented separation or interpenetration of the surfaces.

2.11.5 Movement Error

The error associated with the calculated rotations and translations at the knee was also re-assessed by comparing the simulated and measured motion of sensor mounted on an arm rotating about a fixed hinge inclined at 45° to the workbench [135]. The sensor was positioned approximately 15 cm from the axis. In the test, the arm was swung slowly through its entire range of motion. At each position i , the transformation matrix $\mathbf{T}_{si}^g$ describing the position and orientation of the sensor relative to the transmitter was recorded. The transformations $\mathbf{T}_{s0}^g$ and $\mathbf{T}_{si}^g$ describing the sensor locations at the initial position and position i respectively, were used to calculate the transformation which describes the associated displacement of the arm $\mathbf{T}_{0i}$ as described in Equation 2.21.

Given the rotation matrix component of the $\mathbf{T}_{0i}$ matrix, the movement of the arm relative to the fixed base was described in terms of JCS rotations (Section 2.8.3) with respect to sets of axes fixed in the arm and in the rig base.

The translation of a reference point on the arm located 1.5 cm from the axis was also calculated. This position was chosen to correspond with the approximate location of the reference point used to describe the knee motion. The transformation matrix $\mathbf{T}_{0i}$ between the two extreme positions of the arm was used to calculate the finite helical axis of the motion based on the algorithm by Spoor and Veldpaus [120] (as described in Section 2.8.6). The finite helical axis was defined by its orientation $\mathbf{v}_n$ and a point on the axis $\mathbf{p}_s$. Given the initial position of the sensor origin, $\mathbf{v}_{s0}$, the radius vector from the sensor to the axis is:

$$\mathbf{v}_r = ((\mathbf{v}_{s0} - \mathbf{p}_s) \cdot \mathbf{v}_n)\mathbf{v}_n - \mathbf{v}_{s0} + \mathbf{p}_s \quad (2.40)$$

The position of the reference point in the global coordinate system at arm position i was given by:

$$\mathbf{p}_{ri} = \mathbf{v}_{si} + (\|\mathbf{v}_r\| - 1.5) \frac{\mathbf{v}_r}{\|\mathbf{v}_r\|} \quad (2.41)$$

where $\mathbf{v}_{si}$ describes the position of the sensor origin at arm position i . The displacement of the reference point from its initial position to the position i , $\mathbf{p}_{ri} - \mathbf{p}_{r0}$ was then described in terms of the set of axes fixed in the rig base.

The *true* motion of the arm was simulated by predicting its movement about the calculated finite helical axis. For a rotation θ about a fixed axis whose orientation is described by the unit vector $[n_x \ n_y \ n_z]^T$, the rotation matrix is given by the

equation documented by Kinzel *et al.* [66]:

$$\mathbf{R} = \begin{bmatrix} n_x^2(1 - \cos \theta) + \cos \theta & n_x n_y(1 - \cos \theta) - n_z \sin \theta & n_x n_z(1 - \cos \theta) + n_y \sin \theta \\ n_x n_y(1 - \cos \theta) + n_z \sin \theta & n_y^2(1 - \cos \theta) + \cos \theta & n_y n_z(1 - \cos \theta) - n_x \sin \theta \\ n_x n_z(1 - \cos \theta) - n_y \sin \theta & n_y n_z(1 - \cos \theta) + n_x \sin \theta & n_z^2(1 - \cos \theta) + \cos \theta \end{bmatrix} \quad (2.42)$$

For θ ranging from 0° to 179° in steps of 1° , the rotation matrix was calculated and used to simulate the actual JCS rotations with respect to the same axes used to describe the measured rotations.

The movement of the reference point was also simulated. A coordinate system $\{h\}$ whose z axis is aligned along the axis with an origin at $\mathbf{p}_s$ was defined. The position of a point moving at a fixed radius of $r = 15$ mm about a fixed axis with respect to frame H is:

$$\mathbf{p}_{rsi}^h = \begin{bmatrix} r \sin \theta & r \cos \theta & 0 \end{bmatrix}^T \quad (2.43)$$

In the global coordinate system, the position of the reference point was calculated as:

$$\mathbf{p}_{rsi} = \mathbf{T}_h^g \mathbf{p}_{rsi}^h \quad (2.44)$$

where the columns of the associated rotation matrix $\mathbf{R}_h^g$ were given by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively:

$$\mathbf{c} = \mathbf{v}_n \quad (2.45)$$

$$\mathbf{a} = \frac{-\mathbf{v}_r}{\|\mathbf{v}_r\|} \quad (2.46)$$

$$\mathbf{b} = \mathbf{c} \times \mathbf{a} \quad (2.47)$$

and the translation component of $\mathbf{T}_h^g$ by $\mathbf{p}_s$. The displacement of the reference point from its initial position to the position i , $\mathbf{p}_{rsi} - \mathbf{p}_{rs0}$ was then described in terms of the set of axes fixed in the rig base.

The mean absolute differences between the simulated and measured rotations and translations were computed. The mean distance from the simulated position of a point to the measured position of the reference point was also calculated.

2.11.5.1 Effect of Specimen Alignment and Sensor Placement

Because knee movement is described with respect to axes based on the measured anatomy of the joint and because the joints are virtually unloaded, small differences in specimen alignment were expected to have little effect on the reported motion data. This was tested by repeating the entire motion experiment, including the fixation and alignment of the tibia on the workbench and the mounting of the sensor on the femur, with one specimen (Knee H). The mean absolute differences between the joint rotations and translations and between the helical axis parameters for the two separate experiments were calculated.

	Fixed Bone	Orientation	Direction
1*	Tibia	Vertical	Proximal Up
2	Tibia	Vertical	Distal Up
3	Tibia	Horizontal	Anterior Up
4	Tibia	Horizontal	Posterior Up
5	Tibia	Horizontal	Medial Up
6	Tibia	Horizontal	Lateral Up
7	Femur	Vertical	Distal Up

Table 2.2: The seven different spatial configurations tested to assess the effect of gravitational load (*standard configuration shown in Figure 2.1).

2.11.5.2 Effect of Gravitational Load

To assess the effect of the weight of the moving bone on the measured motion pattern and on the uniqueness of the motion path, one specimen (Knee A) was tested in the seven different spatial configurations listed in Table 2.2.

2.11.5.3 Effect of Speed of Movement

An increase in the speed of the moving segment may affect the measured motion through an increase in inertial effects or a delay in the signal transmission of the tracking system. Variations in speed ranging from 4-12° of flexion per second were examined during the movement error test using the sensor-mounted rotating arm. The weight of the arm corresponded approximately to that of the distal femur so that inertial effects were comparable.

2.11.6 Influence of Presence of Metal

The effect on the tracker measurements of the presence of a number of large metallic objects in the original laboratory (Lab 1) was examined. Initially the metal objects were considered far enough away from the source and sensor, and their influence was assumed negligible. The digitisation, reconstruction and movement error described above was re-evaluated in a second laboratory (Lab 2) where there was far less metal present. The final seven out of the fifteen specimens (Knees F, C, B, A, N, P, and H) were tested in Lab 2.

Chapter 3

Measured Geometry and Movement of the Knee

The measured geometry and passive motion characteristics of the intact human knee, the results of the experiments described in Chapter 2, are presented and discussed in this chapter¹. The articular surface geometry and the ligament architecture of the joint are described. This is followed by a description of the passive joint motion of these specimens both in terms of joint rotations and translations and sets of finite helical axes. The concept of fully coupled passive motion is put forth. The relevance of the experimental results is discussed in light of current understanding of knee kinematics and geometry and in the context of the development of a model of knee mobility and stability.

3.1 Articular Surface Geometry

Upon disarticulation, two of the fifteen specimens (Knees C and P) were found to be arthritic and with a third (Knee N) a damaged MCL was observed; these three knees were excluded from the analyses, leaving twelve for analysis.

3.1.1 Femoral Condyles

Spheres fitted to the medial and lateral posterior femoral condyles of twelve knee specimens were of average radii 22.5 mm and 21.9 mm respectively ('posterior' spheres, Table 3.1). The radii of the fitted spheres correspond well with the range of values reported by Mensch and Amstutz [83] based on a study of 30 cadaver knees and 53 knees of normal subjects. Aiming to provide a guide for knee replacement design, they found circular radii in the sagittal plane ranging from 18 to 24 mm for the medial posterior condyle and from 16 to 27 mm for the lateral posterior condyle.

¹The experimental error and accuracy results are described in the next chapter.

Spec	Medial Condyle				Lateral Condyle			
	Posterior (mm)		Whole (mm)		Posterior (mm)		Whole (mm)	
	R	ME	R	ME	R	ME	R	ME
J	21.4	1.8	24.9	2.2	21.9	2.1	24.1	3.0
D	21.8	0.9	26.9	1.3	23.1	1.0	28.0	1.6
L	21.8	1.2	23.9	2.4	23.1	0.8	26.4	1.6
M	25.5	1.0	27.8	1.5	23.3	0.9	25.2	1.2
X	21.8	0.8	27.6	1.7	22.1	0.6	29.1	2.0
I	24.3	1.1	28.6	1.7	22.0	1.1	27.8	2.0
E	22.7	1.1	28.8	2.0	21.6	0.9	27.0	1.9
G	25.8	1.3	28.9	1.5	23.2	1.2	24.8	1.3
F	21.8	1.1	27.0	1.9	20.8	1.1	24.3	1.6
B	23.0	1.1	29.0	1.9	21.2	1.1	23.6	1.4
A	20.6	0.7	23.7	1.2	21.3	0.8	23.1	1.2
H	19.6	0.6	27.4	1.3	19.2	0.7	23.3	1.8
Mean	22.5	1.1	27.0	1.7	21.9	1.0	25.6	1.7
SD	1.9	0.3	1.9	0.4	1.2	0.4	2.0	0.5

Table 3.1: The radii (R) and mean errors (ME) associated with ‘posterior’ and ‘whole’ spheres fitted to the medial and lateral femoral condyles.

The representation of the posterior femoral condyles as spheres is reasonable with overall mean errors between the fitted spherical surfaces and the measured surface points of less than 1.1 mm for both the medial and lateral condyles across the twelve specimens. This observation supports the findings of Kurosawa *et al.* [67]. They digitised the shapes of sections of human knees and found that “the posterior femoral condyles were shown to closely fit spherical surfaces”.

Spheres fitted to the distal and posterior aspects of the femoral condyles were of average radii of 27.0 mm (medial) and 25.6 mm (lateral), about 20% larger than those fitted to the posterior condyles only (‘whole’ spheres, Table 3.1). The combined distal and posterior aspects of the femoral condyles are not as well described by spherical surfaces as the posterior condyles, with associated mean errors of 1.7 mm for both the medial and lateral condyles, but they still provide an adequate idealised representation of the *whole* condylar surface.

The accuracy of the polynomial surface fits clearly depends on the degree of polynomial functions fitted. Table 3.2 lists the associated error of polynomials of varying degrees for a representative specimen (Knee F).

Polynomial surfaces of degree four in both ψ and ϕ were found to provide a sufficiently accurate physiological representation of the femoral articular surfaces including both the distal and posterior portions of the condyles (Figure 3.1). With

Degree		Mean Error (mm)		Accuracy Increase (%) (relative to previous)	
ψ	ϕ	Medial	Lateral	Medial	Lateral
0	0	1.86	1.55	-	-
1	1	1.75	1.54	5.9	0.7
2	2	1.48	1.31	15.4	14.9
3	3	0.791	0.853	46.6	34.9
4	4	0.750	0.789	5.2	7.5
5	5	0.722	0.784	3.7	0.6
6	6	0.718	0.774	0.6	1.3

Table 3.2: Mean errors associated with the polynomial surfaces fitted to the measured medial (Med) and lateral (Lat) femoral condyles of a representative specimen (Knee F). Polynomials of degree 0 correspond to spheres.

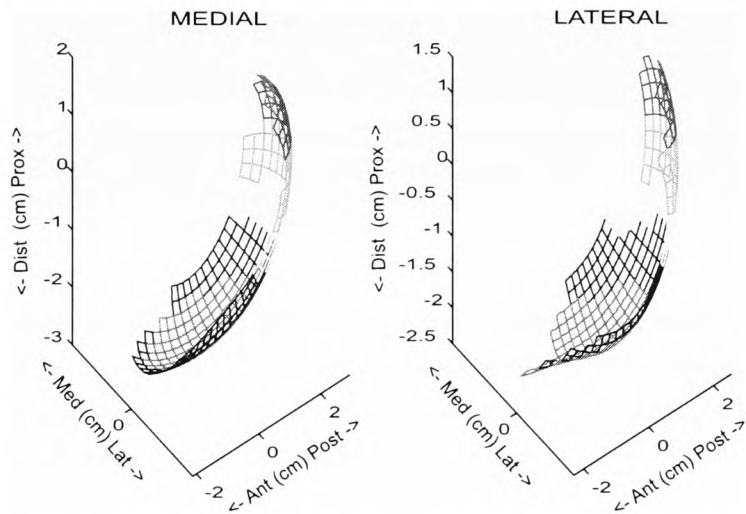


Figure 3.1: The biquartic polynomial surfaces (degree 4 in ψ and ϕ) fitted to the medial and lateral femoral ‘whole’ condyles of a representative specimen (Knee F).

higher degree polynomial surfaces it was possible to further reduce the error; however, the relatively small gain in accuracy (less than 5% beyond degree 4) was not deemed worth the increased complexity of the description (Table 3.2). The overall mean error associated with the biquartic polynomial surface fits was 0.7 mm for both medial and lateral condyles across the twelve specimens, approximately 60% less than that associated with the spherical fits (‘whole’ spheres).

While the fitted spheres provide a consistent means of defining a mediolateral reference axis of motion for the joint (the line joining the sphere centres) and an idealised surface representation of the femoral condyles, the fitted biquartic polynomials constitute a more accurate physiological description of the articular surfaces.

Spec	Medial Plane				Lateral Plane			
	Free			Tran	Free			Tran
	λ ($^{\circ}$)	γ ($^{\circ}$)	ME (mm)	ME (mm)	λ ($^{\circ}$)	γ ($^{\circ}$)	ME (mm)	ME (mm)
J	2.96	-6.50	1.46	1.94	9.83	3.44	2.01	2.54
D	7.80	-9.67	0.73	1.87	5.23	6.98	1.35	1.85
L	7.57	-10.60	0.87	2.18	-2.13	8.60	1.25	1.68
M	2.33	-8.86	1.11	1.86	8.66	15.13	1.57	3.00
X	7.48	-9.59	1.04	2.24	7.82	11.74	1.56	2.53
I	5.02	-7.84	0.84	1.65	0.17	11.23	1.07	1.71
E	5.34	-8.76	1.19	2.18	8.15	15.45	1.34	2.82
G	4.19	-13.26	0.75	1.92	-0.30	6.29	1.15	1.40
F	7.76	-8.97	0.96	2.41	5.19	8.06	1.68	2.15
B	2.27	-9.74	0.64	1.38	2.51	6.80	0.86	1.22
A	5.80	-10.79	0.67	1.79	-0.43	3.22	1.11	1.18
H	6.59	-3.36	0.87	1.64	1.00	16.90	1.61	2.39
Mean	5.43	-8.99	0.93	1.92	3.81	9.49	1.38	2.04
SD	2.11	2.43	0.24	0.29	4.17	4.60	0.32	0.62

Table 3.3: Planar fits of the medial and lateral tibial condyles. The orientations of fitted ‘free’ planes (Free) (see Figure 3.2) and the associated mean errors (ME). Mean errors associated with the fitted ‘transverse’ planes (Tran) are also reported ($\lambda, \gamma=0$).

A geometrical description of the distal femur is critical in the design of femoral components of knee replacement. Models of knee mobility incorporating both spherical and polynomial representations of the femoral articular surfaces have been developed and will be described in detail in Chapter 5.

3.1.2 Tibial Condyles

The orientation of the ‘free’ plane fitted to each tibial condyle is described by the angles between the projections of the normal to the plane and the tibial longitudinal axis²: λ in the sagittal plane and γ in the frontal plane (Figure 3.2).

The average orientation of the medial condyle in the sagittal plane as defined by the fitted ‘free’ planes, was found here to be 5.4° (Table 3.3) and not the average 7° reported by Yoshioka *et al.* [145] in their study of tibial geometry (Section 2.6.2.1). This discrepancy is due to the difference in the definition of condyle orientation. Here the orientation is described by the best-fit plane whereas Yoshioka *et al.* defined the orientation based on the most anterior and posterior points measured on the condyle. Similarly, for the lateral condyle, Yoshioka *et al.* reported an average posterior tilt of 8° in the sagittal plane in contrast to the average 3.8° found here (Table 3.3). These

²The definition of the longitudinal axis is described in Section 2.6.2.1.

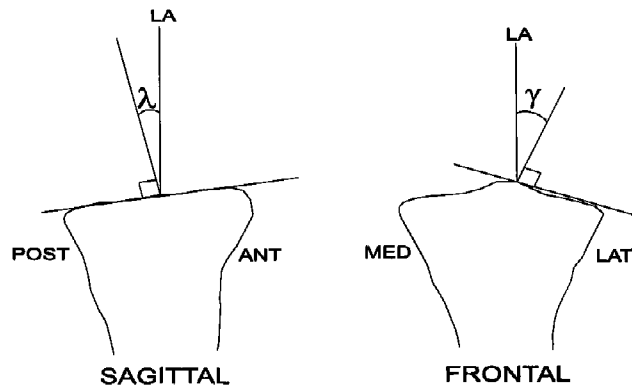


Figure 3.2: The angles in the sagittal (λ) and frontal (γ) planes which describe the orientation of the 'free' planes fitted to the medial and lateral tibial condyles with respect to the tibial longitudinal axis (LA). Positive λ indicates posterior tilt of the plane and positive γ represents a medial tilt for the medial condyle and a lateral tilt for the lateral condyle.

differences have implications for the design and implantation of tibial components of knee replacement systems. One would deduce from the study of Yoshioka *et al.* that the medial and lateral tibial component surfaces should be tilted posteriorly by approximately the same amount with respect to the longitudinal axis of the tibia. The results from the current study, however, suggest that the lateral surface should be oriented more horizontally relative to the medial surface.

In the frontal plane, both medial and lateral tibial 'free' planes were tilted outward at mean angles γ of 9.0° and 9.5° respectively (Table 3.3). The orientations of the medial and lateral condyles in the frontal plane were not described by Yoshioka *et al.*

The overall mean errors associated with the fitted 'free' tibial planes, 0.9 mm and 1.4 mm for the medial and lateral condyles respectively (Table 3.3), are of the same order as those associated with the fitted 'posterior' femoral spheres (Table 3.1).

The overall mean errors associated with the fitted 'transverse' planes, 1.9 mm for the medial and 2.0 mm for the lateral condyle, were approximately twice as high as those for the 'free' planes. Accepting the reduction in accuracy of the description, the 'transverse' planes provide the simplest idealised surface representation of the tibial condyles.

As in the case of the femoral condyles, the accuracy of the tibial polynomial surface fits depends on the degree of polynomial functions fitted. Table 3.4 presents the associated error of polynomials of varying degrees for a representative specimen (Knee F).

Polynomial surfaces of degree 4 in x and 3 in y were found to provide a sufficiently accurate physiological representation of the tibial condyles (Figure 3.3). Again, higher degree polynomial surfaces could reduce the error even further, but

Degree		Mean Error (mm)		Accuracy Increase (%) (relative to previous)	
x	y	Medial	Lateral	Medial	Lateral
0	0	2.41	2.15	-	-
1	1	0.975	1.66	59.5	22.8
2	2	0.617	0.624	36.7	62.4
3	3	0.519	0.463	15.9	25.8
4	3	0.508	0.444	2.1	4.1
4	4	0.501 (RD)	0.443 (RD)	1.4	0.2
5	5	0.494 (RD)	0.440 (RD)	1.4	0.7

Table 3.4: Mean errors associated with polynomial surfaces fitted to the measured medial (Med) and lateral (Lat) tibial condyles of a representative specimen (Knee F). Polynomials of degree 0 correspond to ‘transverse’ planes. RD indicates the occurrence of rank deficiency.

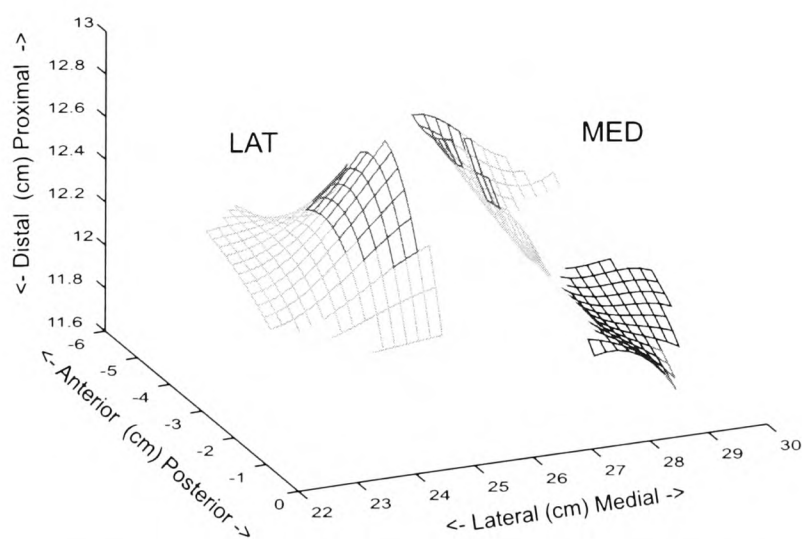


Figure 3.3: The tibial polynomial surfaces (degree 4 in x and degree 3 in y) fitted to the medial and lateral tibial condyles of a representative specimen (Knee F).

the relatively small gain in accuracy was not deemed worth the increased complexity of the description (Table 3.4). Furthermore, in fitting polynomials higher than degree 4 in y , rank deficiency in the matrix used to solve the least-squares problem commonly occurred. The associated mean errors ranged from 0.3 to 0.9 mm for the medial condyle and from 0.2 to 0.8 mm for the lateral condyle across the twelve specimens. The overall mean error, 0.4 mm for both the medial and lateral tibial condyles, is over 50% less than that associated with the ‘free’ planar fits. While the lateral condyle was found to be distinctly less planar than the medial condyle, the condyles are equally well represented by the polynomial surface descriptions with associated errors of the same order.

While the fitted planes provide reasonable idealised geometrical representations of the tibial condyles and a means of describing the overall orientation of each condyle with respect to the longitudinal axis of the tibia (‘free’ planes), the fitted polynomials constitute a more physiological description of the surface. Models of knee mobility incorporating both planar and polynomial tibial articular surfaces have been developed and will be described in detail in Chapter 5.

3.2 Ligament Architecture

In this section, the geometry of the attachment site surfaces, the fibre mapping functions, the definition of fibre arrays and the approximate areas of attachment of the ACL, PCL, MCL and LCL are described based on the data from the seven intact knee specimens for which complete ligament attachment data was available (Knees I, E, G, F, B, A and H). For these specimens, the protocol was altered so that points distributed over the entire ligament attachment sites were measured. In the tests prior, only points lying along the periphery of the attachment were digitised.

3.2.1 Attachment Site Surfaces

Both planes and bicubic polynomial surfaces (degree 3 in x and y) were successfully fitted to the points measured on the attachment sites of the ACL, PCL, MCL and LCL. Mean errors associated with the planar representations of the ligament attachment sites are presented in Table 3.5. The PCL femoral attachment was found to be the least planar, followed by the PCL tibial attachment. The errors associated with the planar fits of the attachment sites of the ACL, MCL and LCL were all under 0.6 mm. The increase in the accuracy (reduction in error) associated with the polynomial versus planar surface representation ranged from 22 to 61%. Higher degree polynomial surfaces again reduce the error further but the increase in the complexity of the description did not merit such a representation.

Lig	Tibial			Femoral		
	Plane ME (mm)	Poly ME (mm)	% Diff	Plane ME (mm)	Poly ME (mm)	% Diff
ACL	0.53	0.28	47.1	0.60	0.37	38.3
PCL	0.65	0.32	50.8	1.03	0.40	61.2
MCL	0.47	0.27	42.6	0.40	0.27	32.5
LCL	0.20	0.15	25.0	0.40	0.31	22.5

Table 3.5: The mean errors (ME) associated with the planar (Plane) and bicubic polynomial (Poly) surface fits to the ligament attachment sites for the seven specimens analysed. The percentage increase in the accuracy (reduction in error) of the polynomial versus planar representation is also given.

The findings are in general agreement with previous observations. Zavatsky and O'Connor [150] concluded, based on measurements made on a single knee specimen, that planar representation of the attachment sites of the ACL was reasonable. Elliptical attachments of the ligament were then incorporated into their geometric model of the knee.

In a study of the isometric femoral attachments of the PCL, Grood *et al.* [46] found that a plane adequately represented the tibial attachment of the ligament but a quadratic surface was required for the femoral attachment. The measurements and observations made by Zavatsky and O'Connor, too, suggest that the PCL femoral attachment is distinctly non-planar [150]. They found, however, that the tibial attachment could be adequately represented by a plane. Although both attachment sites of the PCL were represented by ellipses in their model, they acknowledged that the representation of the femoral attachment of the PCL was inaccurate and constituted a necessary area for improvement.

Previous descriptions of the collateral ligament attachment sites are found only in general anatomical texts. There has been no specific mention of the planarity of the attachment sites of the MCL or LCL. However, the anatomy of the bones at the locations of the attachment sites and the errors found here suggest that planar representation is reasonable.

Planar descriptions of the attachment sites facilitate the definition of local coordinate systems required in deriving the fibre mapping functions. Not all ligament attachment sites, specifically those of the PCL, however, were adequately represented by planar regions. Therefore, bicubic polynomial descriptions were used to represent more accurately the surfaces of the ligament attachment sites.

3.2.2 Attachment Site Shape

The shapes of the measured attachment sites of the ACL, PCL, MCL and LCL were similar in nature between the seven specimens examined. The ACL tibial attachment was close to elliptical while the femoral attachment resembled a semi-circle. The PCL tibial attachment could be described either as rectangular or triangular with rounded corners. The PCL femoral attachment was trapezoidal in shape with a convex distal edge. The femoral attachment of the MCL and both attachments of the LCL may well be approximated by circles or ellipses. The tibial attachment of the MCL, however, was more rectangular in nature, longer proximodistally. The tibial attachments of the MCL were also most variable in size and shape across the specimens due to the ill-defined nature of the site on the bone.

The shapes of the attachment sites measured were in good agreement with previous descriptions found in the literature. Odensten and Gillquist [100], in a study of ACL functional anatomy, described its tibial and femoral attachment sites as oval-shaped. Similarly, the femoral attachment of the ACL has often been described as a semi-circle [8, 40, 44] or an ellipse [62]. The tibial and femoral attachments of the PCL have been depicted by Girgis *et al.* [44] as rectangular and semi-circular respectively. Friederich *et al.* [40] described the tibial attachment of the PCL as semi-circular and found the femoral attachment of the PCL to resemble a trapezoid with a curved distal side. In their development of a 2-D model of the knee ligaments, Zavatsky O'Connor [147] described the femoral attachments of the MCL and LCL and the tibial attachment of the LCL as circular. They too depicted the MCL tibial attachment as a long rectangular shape with rounded ends, similar to that observed here.

3.2.3 Fibre Mapping

The functions, f_x and f_y , describing the mapping of tibial attachment sites to femoral attachment sites of individual fibres (Section 2.7.2) were successfully derived for each of the ACL, PCL, MCL and LCL for the seven specimens analysed. One measure of the mapping error is given by the mean distance between the selected femoral attachment sites of approximately physiological fibres and the femoral attachment sites determined based on the mapping functions and on the selected tibial attachment sites (Section 2.7.2.1). These errors are presented in Table 3.6. The highest mean error was found for the mapping of the PCL most likely due to the more complex spatial arrangement of its fibres.

Based on mapping functions derived for the ACL and PCL of three specimens described by Mommersteeg [86] (Section 2.7.2.1), mean errors between the true and mapped femoral fibre bundle attachments of 1.23 mm, 1.57 mm and 1.89 mm

Spec	Mapping Error (mm)			
	ACL	PCL	MCL	LCL
I	1.79	1.61	0.82	0.92
E	1.72	1.52	1.36	0.63
G	1.43	1.13	1.35	0.70
F	2.48	1.86	1.12	1.09
B	0.74	1.86	1.35	0.38
A	1.13	2.00	0.84	0.97
H	1.10	2.30	1.42	0.71
Mean	1.44	1.77	1.18	0.77
SD	0.57	0.37	0.24	0.22

Table 3.6: A measure of error associated with the mapping of individual fibre attachment sites from the tibia to the femur for the ACL, PCL, MCL, and LCL.

were found for the ACL and of 2.67 mm, 1.35 mm and 3.29 mm for the PCL. By taking into account measured inter-specimen differences, this second measure of error describes the accuracy of the procedure as a whole. With mean errors of 1.6 mm for the ACL and 2.4 mm for the PCL, the procedure was deemed successful in deriving functions which closely describe the true fibre bundle anatomy of the ligaments. Again, the higher error for the PCL is a reflection of the higher degree of inter-specimen variability and its more complex fibre arrangement.

3.2.4 Fibre Array Definition

Figures 3.4 and 3.5 depict the attachment sites of arrays of fibres representing the ACL, PCL, MCL and LCL of a representative specimen (Knee F). In general, the attachment sites of the defined array of fibres corresponded well with the original measured attachment sites. The modified attachment site boundaries for the ACL, corresponding to the defined array of fibres, are shown in Figure 3.6 for the same representative specimen. Similar results were found for the remaining six specimens.

The ligament attachment areas, approximated by the areas enclosed by the modified boundaries in the plane of the attachment (Figure 3.6), are presented in Table 3.7. The average areas for the ACL attachments are on the order of, but slightly lower than, those reported by Odensten and Gillquist [100] based on a study of 33 cadaver knees. If taken to be ellipses, the average oval shaped areas they described were $1.5 \text{ cm}^2 (\pm 0.5 \text{ cm}^2)$ and $1.6 \text{ cm}^2 (\pm 0.5 \text{ cm}^2)$ for the tibial and femoral areas respectively. The average PCL femoral attachment area found concurs with the approximate 2.0 cm^2 area measured by O'Connor *et al.* [98] on 15 specimens. No reports of MCL and LCL attachment areas could be found in the literature. In the

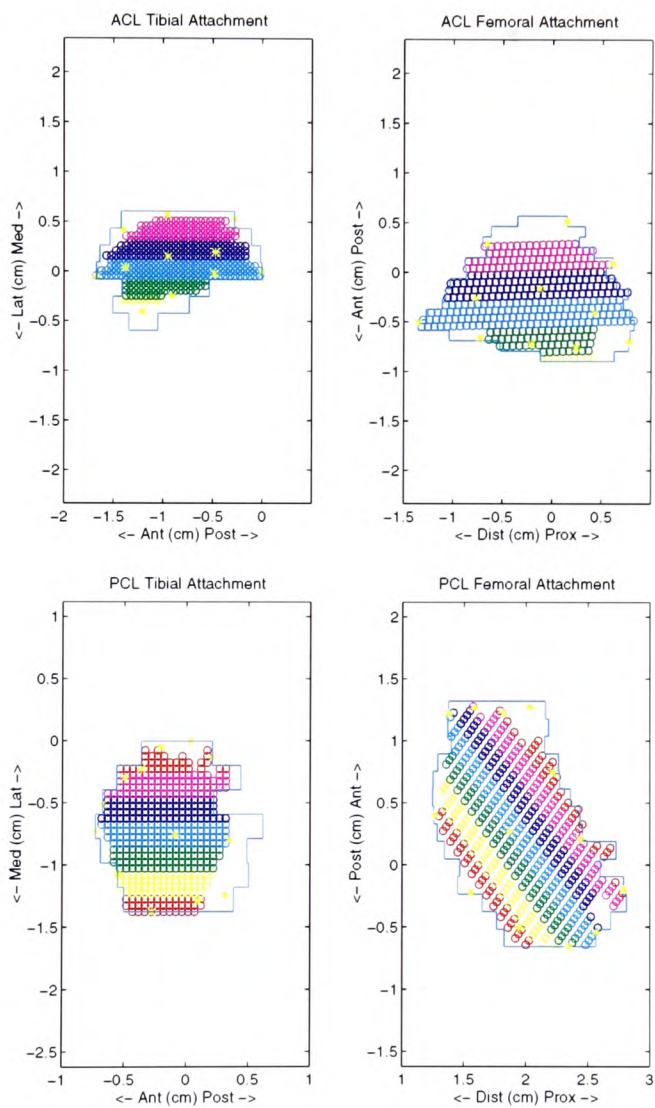


Figure 3.4: The ACL and PCL attachment sites (o) of a defined array of fibres for a representative specimen (Knee F). Colour coding describes the mapping of the fibres from the tibial to the femoral attachment. The selected attachment sites of approximately physiological fibres used to derive the mapping functions (*) and the originally measured attachment site boundaries (solid lines) are also shown.

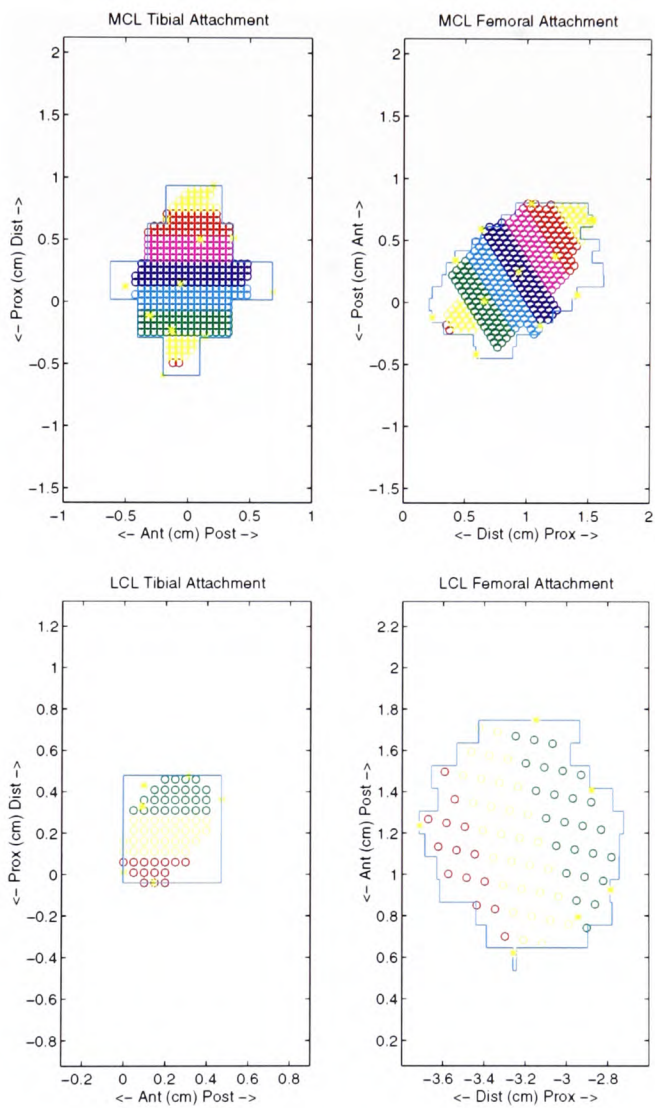


Figure 3.5: The MCL and LCL attachment sites (o) of a defined array of fibres for a representative specimen (Knee F). Colour coding describes the mapping of the fibres from the tibial to the femoral attachment. The selected attachment sites of approximately physiological fibres used to define the mapping functions (*) and the originally measured attachment site boundaries (solid lines) are also shown.

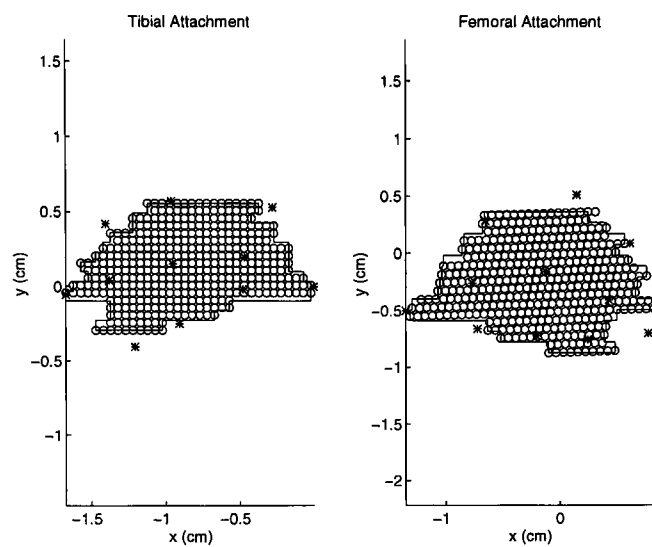


Figure 3.6: The modified boundary of the ACL attachment based on the defined array of fibres shown in Figure 3.4.

Spec	ACL (cm ²)		PCL (cm ²)		MCL (cm ²)		LCL (cm ²)	
	Tib	Fem	Tib	Fem	Tib	Fem	Tib	Fem
I	1.01	1.33	1.18	1.51	0.97	0.74	0.18	0.64
E	0.85	1.12	0.95	0.99	1.54	1.25	0.37	0.84
G	1.01	1.29	1.10	1.92	1.86	1.56	0.40	0.59
F	0.93	1.72	1.05	1.80	0.83	0.87	0.18	0.80
B	1.34	0.92	1.03	1.66	0.89	1.51	0.29	0.29
A	1.18	1.07	0.96	1.53	1.54	1.02	0.14	1.11
H	1.12	0.77	0.65	2.35	2.38	2.68	0.50	0.67
Mean	1.06	1.17	0.99	1.68	1.43	1.38	0.30	0.71
SD	0.15	0.29	0.16	0.39	0.53	0.60	0.13	0.23

Table 3.7: The areas of the tibial (Tib) and femoral (Fem) attachment sites of the ACL, PCL, MCL, and LCL of the seven specimens analysed (approximated by the area enclosed by the modified boundary in the plane of attachment, *e.g.* Figure 3.6).

Spec	ACL	PCL	MCL	LCL
I	0.76	0.78	1.31	0.28
E	0.76	0.96	1.23	0.44
G	0.78	0.57	1.19	0.68
F	0.54	0.58	0.95	0.23
B	1.46	0.62	0.59	1.00
A	1.10	0.63	1.51	0.13
H	1.45	0.28	0.89	0.75
Mean	0.98	0.63	1.10	0.50
SD	0.34	0.19	0.28	0.29

Table 3.8: The ratios of tibial to femoral attachment site areas for the ACL, PCL, MCL and LCL based on the values presented in Table 3.7.

current study, the size of tibial attachment area of the MCL was most variable. This can be explained in part by the difficulty in defining the limits of the attachment on the bone; it is not well-defined. Most of the differences in size and shape found here and reported in the literature can be attributed to inter-specimen variability.

The ratio of the attachment area on the tibia to that on the femur (Table 3.8) gives a measure of the relative density of the fibres on each attachment site, assuming that the total number of fibres is constant over the ligament length. For the ACL and MCL, the ratio was on the order of 1, while for the PCL and LCL, the area on the tibia was close to half that on the femur. The fibres of the PCL and LCL are on average more concentrated at their tibial attachments than at their femoral attachments. This relative density will be of relevance in the development of a model of knee stability (Chapter 8).

3.3 Passive Knee Movement

The recorded *in vitro* movement of the virtually unloaded knee is described in this section, both in terms of joint rotations and translations and sets of finite helical axes. The coupled nature of the movement is highlighted and the uniqueness of the motion path quantified. Differences between normal and abnormal knees are examined. The results are discussed in the light of previous descriptions of passive knee kinematics. The implications of the observations with respect to the development of a model of knee mobility are considered.

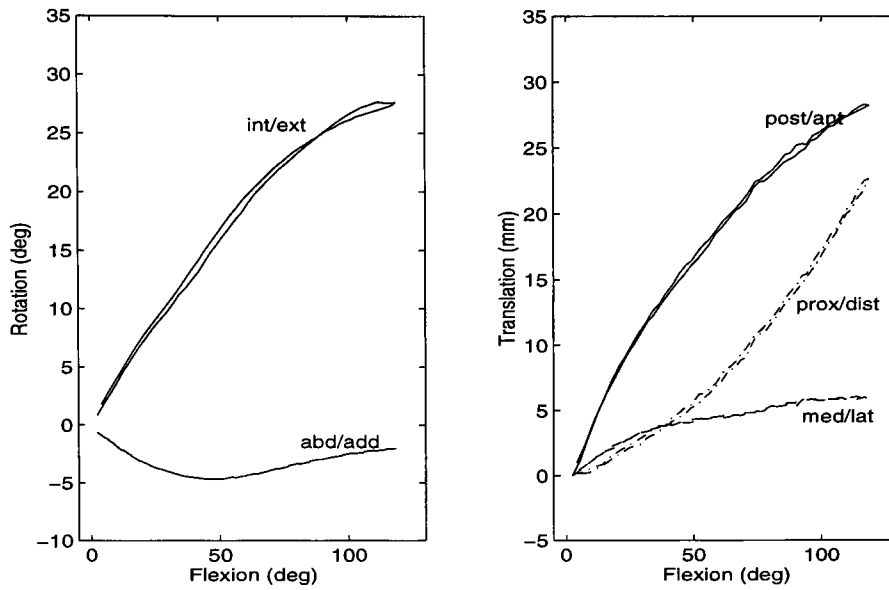


Figure 3.7: Flexing and extending movement paths for a representative knee specimen (Knee F). Internal(+)/external(-) tibial rotation, abduction(+)/adduction(-), posterior(+)/anterior(-), medial(+)/lateral(-) and proximal(+)/distal(-) translation are all coupled to the flexion angle.

3.3.1 Coupled Motion

The path of movement during flexion was virtually the same as that during extension for the twelve normal knee specimens. The paths were repeatable over the three cycles of motion. Internal/external rotation, ab/adduction, and the three components of translation of the reference point were all coupled to the flexion angle, as shown in Figure 3.7 for a representative specimen (Knee F) and in Figure 3.8 for all twelve normal knees. The three remaining knees followed non-unique, yet repeatable paths (Figure 3.9). As mentioned previously, of these three, two were arthritic (Knees C and P) and the third had a MCL with a partial tear (Knee N). The uniqueness of these motion paths is quantified in Section 3.3.3.

3.3.2 Rotations and Translations

All twelve normal knees displayed similar patterns of coupled movement (Figure 3.10 and 3.8). The tibia rotated internally with increasing flexion angle, with the peak rotation ranging from 14° to 36° . Abduction/adduction was less consistent, with maximum values ranging from 9° abduction to 9° adduction. As the knee flexed, the reference point on the tibia, the most posterior point on the ACL attachment, always moved posteriorly, proximally, and medially with respect to a fixed femur. The maximum translations ranged from 20 to 34 mm posteriorly, 2 to 9 mm medially and 6 to 24 mm proximally.

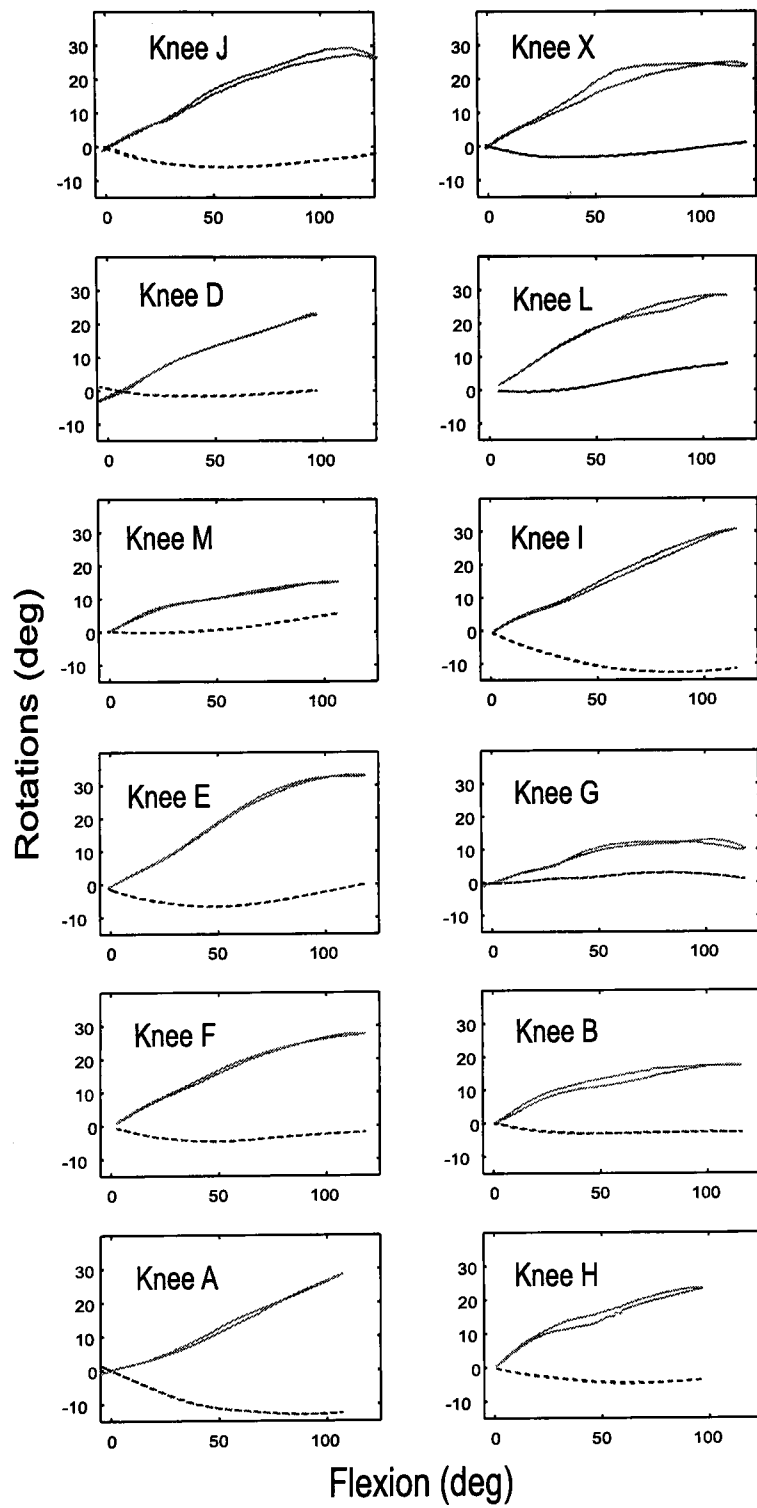


Figure 3.8: Flexing and extending movement paths for all twelve normal knee specimens. Internal(+)/external(-) tibial rotation (solid), abduction(+)/adduction(-) (dashed) are coupled to the flexion angle.

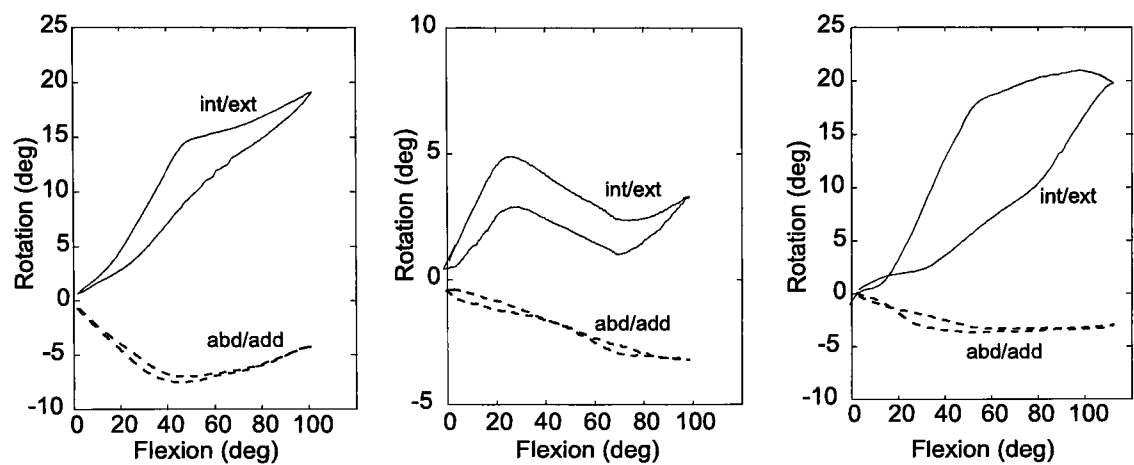


Figure 3.9: Flexing and extending movement paths for the three abnormal knee specimens (Knees P, C and N from left to right).

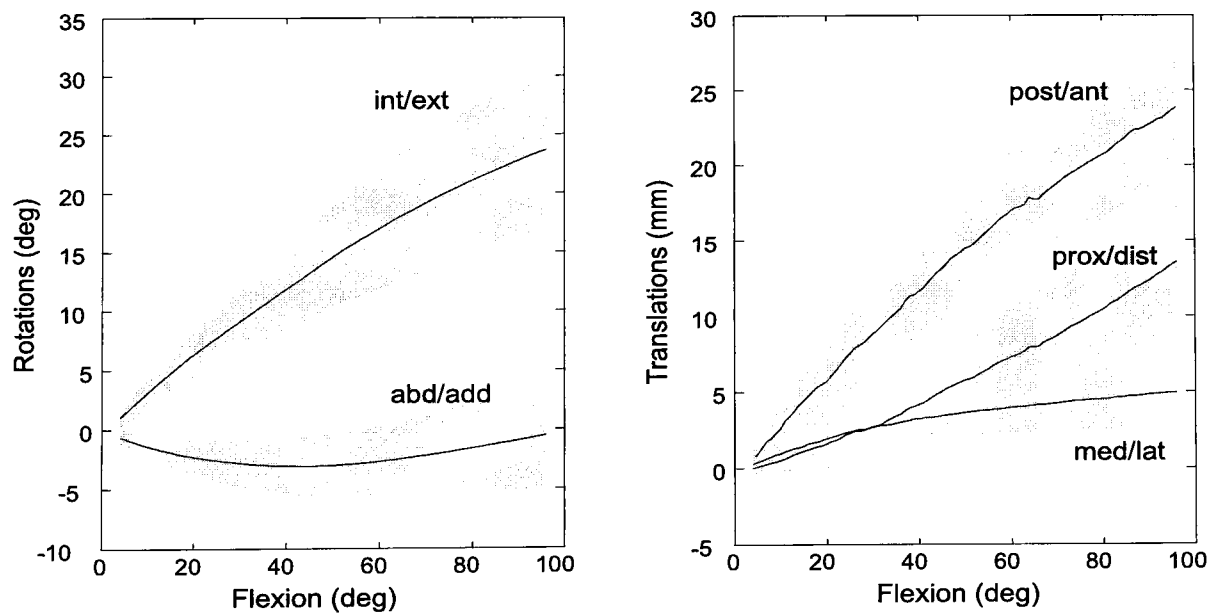


Figure 3.10: Average passive motion of the twelve normal knee specimens (flexing paths). The shading indicates one standard deviation above and below the mean.

	Int/Ext (°)	Abd/Add (°)	Post/Ant (mm)	Med/Lat (mm)	Prox/Dist (mm)
Normal (n=12)	0.93 (0.28)	0.14 (0.03)	1.18 (0.34)	0.26 (0.05)	0.58 (0.17)
Abnormal (n=3)	3.8* (3.1)	0.32* (0.14)	0.88 (0.06)	0.44 (0.44)	0.35 (0.14)

Table 3.9: The mean absolute differences between flexing and extending paths for each component of movement (plotted in Figure 3.11 for the twelve normal specimens) averaged over the flexion range. Associated standard deviations are shown in brackets. Significantly greater differences in the flexing and extending rotation paths were found for the abnormal knees (*, Wilcoxon rank sum test, $p < 0.05$).

Sagittal Plane	Flexion		Extension	
	Post (mm)	Dist (mm)	Ant (mm)	Prox (mm)
Mid	8.2 (3.0)	7.5 (2.3)	8.6 (4.4)	6.9 (2.2)
Medial	3.6 (3.8)	11.5 (3.4)	4.1 (5.4)	13.0 (2.9)
Lateral	12.7 (3.0)	3.5 (2.8)	13.1 (4.0)	0.9 (3.9)

Table 3.10: The mean shifts of the FHAs in the mid-sagittal (Mid) and para-sagittal planes passing through the medial (Med) and lateral (Lat) femoral sphere centres at full extension. Standard deviations are shown in brackets.

3.3.3 Flexion and Extension Paths

As already observed qualitatively, the path of movement during flexion was virtually the same as that during extension for the twelve normal specimens. The mean absolute difference between flexing and extending subsets for each component of movement, shown in Figure 3.11, quantifies the uniqueness of the path as a function of flexion angle. The mean absolute differences averaged over the flexion range are presented for each component of movement in Table 3.9. For the normal knees, these differences are on the order of the measurement uncertainty described later in Section 4.3.4.

The differences between flexing and extending paths corresponding to the three abnormal specimens are also included in Table 3.9. The abnormal knees displayed significantly greater differences between flexing and extending joint rotation paths compared to the normal knees (Wilcoxon rank sum test, $p < 0.05$).

3.3.4 Finite Helical Axes

The finite helical axes (FHAs) describing the movement of the femur relative to a fixed tibia, initially inclined in the frontal plane and tilted towards the lateral

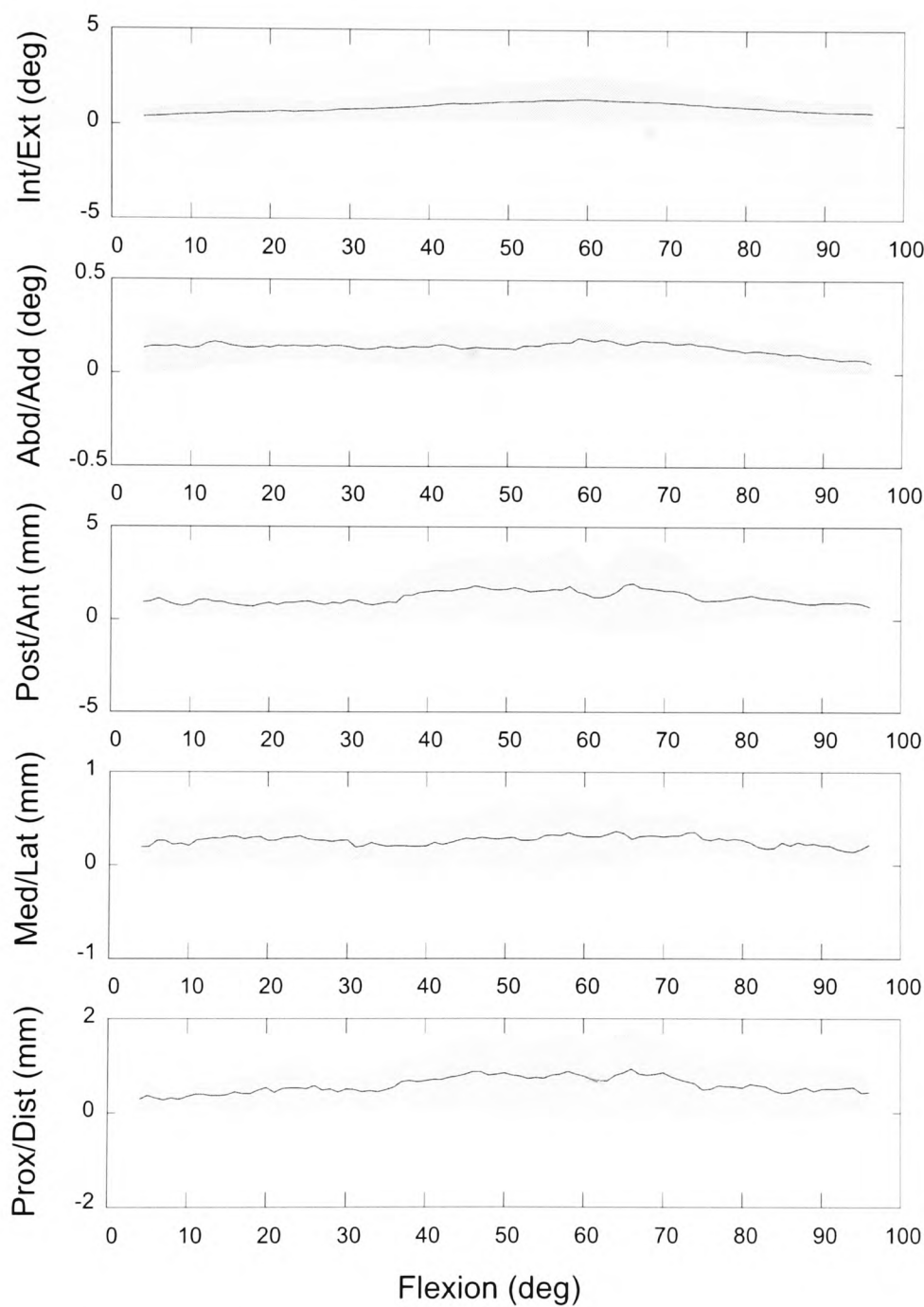


Figure 3.11: Mean absolute differences between flexing and extending joint rotation and translation paths as a function of flexion angle for the twelve normal knees ($n=12$). The shading indicates one standard deviation above and below the mean. NB the difference in scale between plots.

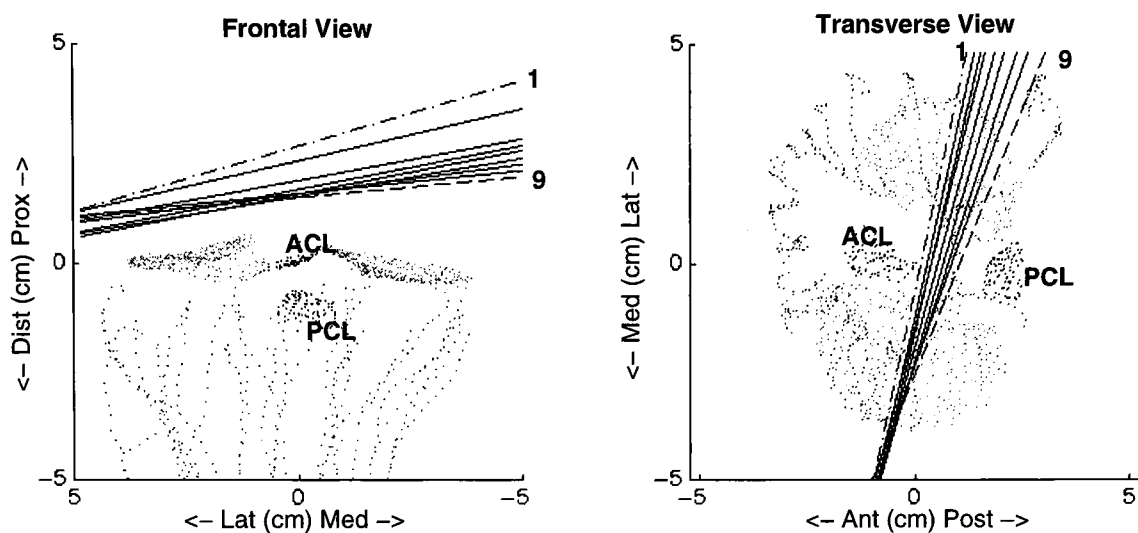


Figure 3.12: Finite helical axes describing the motion of the femur relative to the tibia for a representative specimen (Knee F). Frontal and transverse views of the nine FHAs describing flexion with respect to the tibial coordinate system ($\hat{t}_t, \hat{l}_t, \hat{p}_t$). The tibial anatomy, including the attachment sites of the anterior (ACL) and posterior (PCL) cruciate ligaments, is shown.

plateau, became nearly horizontal with increasing flexion (Figure 3.12). This reflects the decreasing contribution of axial rotation which is also observed in the decreasing slope of the internal rotation versus flexion curve at higher flexion angles (*e.g.* Figure 3.7). Simultaneously, the axes shifted posteriorly in the transverse plane (Figure 3.12). The reverse was observed during extension.

The intersections of the FHAs with a mid-sagittal and medial and lateral para-sagittal (parallel to sagittal) planes are shown in Figure 3.13 for a representative specimen (Knee F). The medial and lateral para-sagittal planes are defined as those passing through the femoral 'posterior' sphere centres in the fully extended knee. The FHAs for flexion were virtually the same as those for extension. The shifts of these intersections in the sagittal planes are quantified in Table 3.10. For the twelve normal knees, the shift of the FHAs in the mid-sagittal plane during flexion was posteriorly and distally directed. During extension, anterior and proximal shifts were recorded. In the para-sagittal planes passing through the medial and lateral femoral sphere centres, the shifts of the FHAs differed in magnitude but were similar in direction.

The orientations of the mean helical axes (MHAs) are presented in Table 3.11. A flexing and an extending MHA is reported for each of the twelve normal specimens, describing the motion of the femur relative to the tibia during flexion and extension respectively. In the frontal plane, α ranges from 2 to 15° and in the transverse plane, β ranges from 6 to 21°. The mean absolute differences between the orientations of the flexing and extending MHAs over the twelve specimens, 0.45° (α) and 0.86° (β)

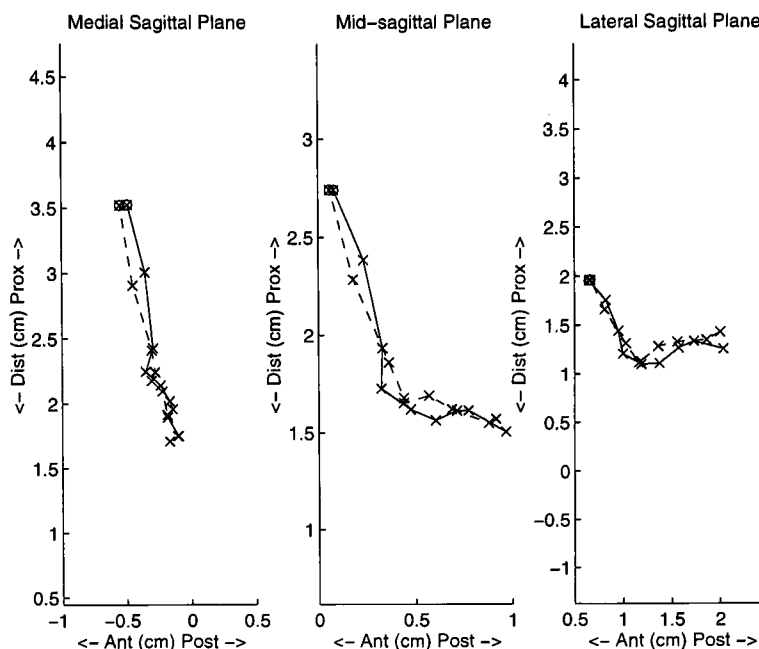


Figure 3.13: Intersections of the nine FHAs of a representative specimen (Knee F) with a mid-sagittal and medial and lateral para-sagittal planes. The solid line corresponds to the flexing and the dashed line to the extending path. The circle (o) represents the FHA of the 0-20° flexion or 20-0° extension step.

(Table 3.11), are less than the associated error described in Section 4.3.4 (Table 4.8).

The FHAs describing tibial motion relative to a fixed femur displayed similar patterns of movement during knee flexion and extension. The shift of the axes in the mid-sagittal plane ranged from 5.2 to 11.2 mm (mean=8.4 mm) posteriorly during flexion and from 4.5 to 10.0 mm (mean=7.0 mm) anteriorly during extension.

3.3.5 ‘Deviation’ Tests

With the application of a small load, the femur was displaced from its preferred path of motion. Upon release of the deviating force, however, the femur of each normal intact joint sprang back to its preferred position on the original motion path (Figure 3.14). It continued to follow that path when flexion or extension was resumed. The deviation consisted predominantly of internal/external tibial rotation, revealed as a spike in this component on the graph.

3.3.6 Discussion

Unconstrained passive movement of the knee was measured in cadaver specimens to show that all components of movement are coupled to flexion. In previous studies of passive knee movement, the method for representing motion is not described

Specimen	Alpha (deg)			Beta (deg)		
	Flexion	Extension	Δ	Flexion	Extension	Δ
J	15.23	13.77	1.45	17.60	16.24	1.36
D	4.20	4.06	0.14	12.61	12.55	0.06
L	14.20	15.12	0.92	12.42	12.89	0.47
M	8.33	7.96	0.37	6.79	7.26	0.47
X	10.64	11.03	0.38	14.56	16.56	2.00
I	12.03	12.45	0.45	20.17	21.16	0.99
E	11.50	11.72	0.23	17.68	18.26	0.58
G	11.75	11.28	0.47	6.43	6.91	0.49
F	10.35	9.90	0.45	16.20	16.83	0.63
B	7.19	7.12	0.07	11.90	13.30	1.39
A	2.10	1.76	0.35	19.22	19.72	0.50
H	14.85	14.70	0.15	16.26	17.64	1.38
Mean	10.20	10.07	0.45	14.32	14.94	0.86
SD	4.10	4.16	0.39	4.46	4.51	0.56

Table 3.11: The orientations of the MHAs for flexion and extension of the femur relative to a fixed tibia. Alpha (α) and Beta (β) are the angles between the projections of the MHA and the tibial mediolateral axis, f_t in the frontal and transverse planes respectively. Positive α describes an axis which is tilted towards the lateral plateau in the frontal plane and positive β describes an axis which is inclined posterolaterally in the transverse plane. The absolute differences (Δ) between the angles corresponding to the flexing and extending axes are also reported.

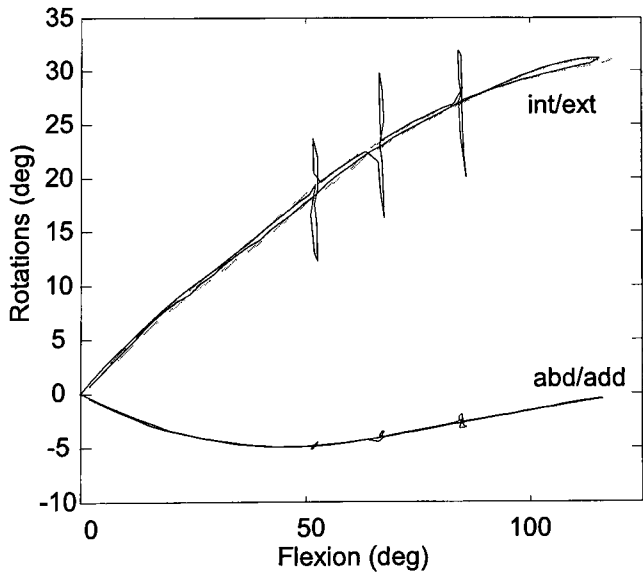


Figure 3.14: Deviations from the neutral passive motion path under a small applied load for a representative specimen (Knee F) (solid curve). The specimen always returned to and continued to follow the original neutral path of motion (dashed curve) after each deviation.

completely, some components of motion are not reported, or passive flexion is described by an envelope rather than a path. A complete and accurate database of knee kinematics of the virtually unloaded knee has now been compiled to provide a conceptual basis for and a means of validation for a 3-D model of knee mobility (Chapter 5).

The movement observed confirms that internal rotation, ab/adduction and the three components of translation are coupled to flexion in the virtually unloaded knee. There are two clear pieces of evidence for coupling. First, for each normal specimen, the flexing and extending paths of motion were similar, with differences between the paths on the order of the uncertainty in the measurements. The similarity between the flexing and extending paths was observed both in the components of joint rotation and translation and in the mean finite helical axes representing the motion. Second, in each specimen, the femur sprang back to its position on the preferred path of movement after loads were applied to rotate the knee internally and externally. If internal/external rotation were not coupled to flexion, the femur would have remained in the position to which it had been rotated. Both the unique path of movement and the consistent elastic 'spring-back' to it suggest that, when virtually unloaded in passive flexion, the knee behaves as a one degree of freedom (dof) joint. All other components of movement can be fully specified by the flexion angle alone. It can also be inferred that the passive flexion path represents a series of zero- or minimum-energy positions. In other words the results suggest that zero or minimal deformation of the soft tissues of the joint is associated with movement of the bones along the path while deviations of the bones from the path are resisted by soft tissue deformation. Hence, the application of external loads to the joint would clearly alter the observed motion path.

The results of the present study are consistent with the concept of an 'envelope' of passive knee motion, but provide a more precise description of the kinematics of the unloaded knee. Blankevoort *et al.* [15] acknowledged that the regions of motion in flexion/extension and internal/external tibial rotation can be treated either as regions of low stiffness or as regions of freedom of motion. They chose the latter approach, treating the knee as a two dof joint, linking all other movements to the flexion angle and tibial rotation. Unable to observe a stable, consistent path in the unloaded knee, they examined the movement under applied internal and external tibial torques of 3 Nm and described an envelope of passive motion. The envelope of passive knee flexion defined in Blankevoort *et al.*'s study had a total width from the 'internal' to the 'external' pathway of about 40°, within which the position of the knee could not be specified for a given flexion angle. In contrast, the present study shows that internal/external tibial rotation can be specified within an average of one degree at a given flexion angle. Since, as Blankevoort *et al.* note: "The basis

for the understanding of the kinematics of the knee joint lies in the description of its passive motion characteristics", it seems important that these characteristics be described under the simplest possible loading conditions and with as much precision as possible. The current database is therefore a valuable addition to the experimental results of Blankevoort *et al.* [15].

The patterns of coupled rotation and translation and the finite helical axes measured in this study are consistent with results reported in the literature. Internal tibial rotation with increasing flexion was measured in all specimens in the present study. The magnitude of the rotation observed was higher than that reported in most previous studies, although a wide range of peak rotations has been reported [8.5° (Markolf *et al.* [81]), 10° (Shoemaker *et al.* [118]), 14.5° (Biden *et al.* [11]), 20° (FitzPatrick [38]), and 37° (Trent *et al.* [128])]. The large variation is likely due to different test rigs, different conventions for representing knee rotations and definitions of coordinate systems. Both Woltring [141] and Cole *et al.* [29] have demonstrated that the magnitudes of calculated joint angles are sensitive to the convention chosen to represent them. The measurements of internal rotation in the present study do lie within the passive motion envelope specified by Blankevoort *et al.* [15]. They are closest to the 'internal pathway' and quite far from the 'external pathway', suggesting that the knee is more compliant in external rotation than in internal rotation. In contrast to their findings, however, automatic internal rotation of the tibia with flexion, often described as the screw-home mechanism, has been shown here to be a passive characteristic of the knee.

No consistency in the direction of knee ab/adduction was observed. Coupled ab/adduction of the knee with flexion has not been documented extensively. The current results are consistent with FitzPatrick [38].

The translations of a point at the knee cannot easily be compared to other published results because they depend critically on the chosen point. Because the same reference point was used in this study and in Blankevoort *et al.*'s study [15], these results can be compared. Blankevoort measured consistent posterior translation of up to 19.5 mm, while measurements of 20 to 34 mm were made in the present study. Blankevoort measured slight medial translation of the reference point of up to 4.5 mm, while in the current study medial translations of 2 to 9 mm were measured. Blankevoort measured proximal displacements of up to 15.0 mm, consistent with the current results of 6 to 24 mm.

Few studies have examined the axodes of the finite screw axes of the knee in passive motion. The unpopularity is partially due to the poor consistency and reproducibility reported in early studies, such as that of Blacharski and Somerset [12]. The position and direction parameters of the axes are highly susceptible to measurement errors [142]. In a more recent study, however, Blankevoort *et al.* [16],

successfully described knee movement by a series of finite screw axes with respect to both the femur and the tibia. The positions and orientations of the finite helical axes for the passive motion found here are reasonably consistent with those reported by his group, although direct comparison is difficult because the loading conditions were not identical in the two studies. For both the internal and external pathways of motion, Blankevoort *et al.* observed that the FHA was oblique in the frontal plane near extension, tilted towards the lateral plateau, and moved to a horizontal orientation near full flexion, which is consistent with our observations. The posterior shift of the FHAs describing a moving tibia relative to a fixed femur in the mid-sagittal plane, averaging 8.4 mm for flexion, was also on the order of the average 8 mm reported by Blankevoort *et al.* for both pathways.

A primary function of the human knee joint is to permit mobility between the tibia and femur. Hence, a primary objective of knee replacement and ligament reconstruction is the restoration of joint mobility, both the range and pattern of motion of the unloaded joint. Intra-operatively, the surgeon assesses the mobility of the knee to confirm the correct sizing and placement of the components. The mobility of the normal knee can be described by a unique and repeatable path of motion. Any changes to the geometry of the normal intact joint, for example by the erosion of the articular surfaces (Knees C and P) or the defunctioning of a ligament (Knee N), can change the number of dof and alter the passive kinematics of the joint. The unique track of motion followed by the virtually unloaded knee may provide a valuable kinematic baseline in assessing the restoration of normal kinematics.

To conclude, a unique path of passive motion is associated with the intact, unloaded knee. Internal/external rotation, ab/adduction, and the three components of translation are all coupled to knee flexion. This preferred path of movement was also well described by a set of FHAs. These experimental results support the hypothesis that the intact unloaded knee behaves as a one dof joint in passive flexion and extension. The results justify modelling the *unloaded* knee as a single dof mechanism, *i.e.* of mobility one. The kinematic database, together with the geometric articular surface and ligament data, provides a means for specimen-specific validation of a model of knee mobility.

Chapter 4

Further Experimental Data Analysis

Based on the measured geometry and passive motion characteristics of the intact human knee described in Chapter 3, estimates of articular contact movement and ligament fibre length patterns accompanying the measured motion are reported in this chapter. The locations of the most nearly isometric ligament fibres are described. The results are compared to previous findings described in the literature and discussed in the context of the development of a model of knee mobility and stability. Measures of experimental error and accuracy are also given.

4.1 Articular Contact Movement

The movement of the contact area in the medial and lateral compartments of the knee during passive flexion, estimated by examining the proximity of the moving surfaces representing the femoral and tibial condyles, is reported in this section. The femoral condyles were described by polynomials of degree 4 in ψ and ϕ and the tibial polynomials of degree 4 in x and 3 in y . The anteroposterior contact point excursions on the tibia in the medial and lateral compartment and the associated slip ratios based on data from twelve knee specimens are examined. Conclusions are drawn with respect to observed trends in the estimated contact kinematics and the validity of the estimations.

4.1.1 Contact Point Excursion

Although large variations in the magnitude of the displacements were found, in particular in the medial compartment, some trends in the estimated contact point movement were observed. The contact point on the lateral tibial condyle always moved posteriorly with flexion in the twelve specimens examined (Table 4.1), by an

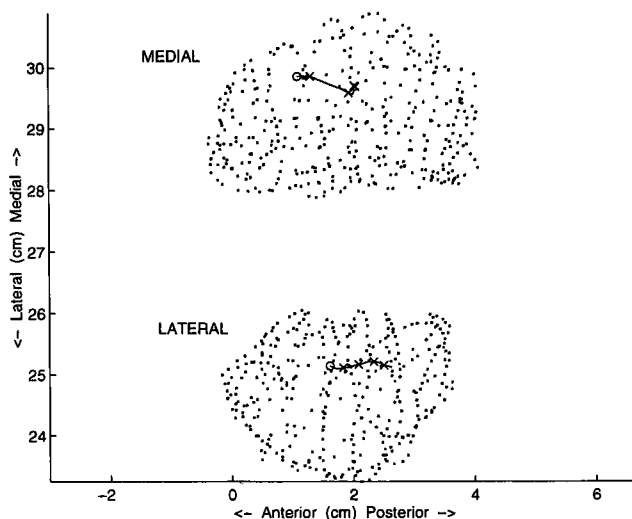


Figure 4.1: Estimated tibial contact point movement. Results for a single specimen (Knee F). Transverse view. The circle (o) represents the contact point at full extension (0°) and the crosses (x) correspond to 20° , 40° , 60° and 80° flexion.

average of 14.7 mm (net) for 100° of flexion. In all but one knee (Knee M), this was accompanied by net posterior movement of the contact point in the medial compartment, averaging 7.9 mm. In the medial compartment, any differences between net and maximum posterior movement of the contacts reflect a degree of anteriorly directed displacement in early flexion due to early internal rotation.

Figure 4.1 shows the estimated contact point movement on the tibia for a single specimen (Knee F). The location of the contact on the tibial condyles at full extension and the rate of the excursion of the contact points were variable across the twelve specimens. In the medial compartment, the location of the 'contact' was often found to lie on the edges of the condyles, either the anterior edge or towards the tibial eminence. Such extremes are most likely a manifestation of the error in the procedure as will be discussed in Section 4.1.2.

The slip ratio is defined as the displacement of the contact point on the tibia divided by that on the femur for steps of 5° flexion. A value of one corresponds to pure rolling, zero to rotational sliding, and ∞ to translational sliding of the femur on the tibia. The slip ratio associated with the same single specimen (Knee F) is shown in Figure 4.2. In the medial compartment, the ratio remains below 0.6 for the entire flexion range, indicating a combination of rolling and sliding of the femur on the tibia. The fact that the ratio was found for the most part to lie between 0 and 0.5 suggests a greater proportion of sliding versus rolling of the femur on the tibia. In the lateral compartment, the ratio is on the whole somewhat higher, with values on the order of 0.5 for a large portion of the flexion range. The results were again variable over the twelve specimens (Figure 4.3), particularly at certain flexion angles. The results do suggest, however, that the proportion of rolling in the lateral

Specimen	Side	Medial		Lateral	
		Net (mm)	Max (mm)	Net (mm)	Max (mm)
J	L	11.4	13.1	9.2	9.2
D	R	6.3	6.3	13.1	13.1
L	L	0.7	0.7	16.5	16.5
M	R	-4.5	-7.3	9.0	9.0
X	R	7.4	7.4	18.0	18.0
I	R	3.3	3.5	24.6	24.6
E	R	1.9	1.9	22.3	22.3
G	L	19.6	19.8	12.9	14.5
F	L	9.1	9.8	10.2	10.2
B	L	18.0	18.1	15.2	15.2
A	L	8.8	10.5	11.6	11.6
H	R	12.4	12.4	13.5	13.5
Mean		7.9	8.0	14.7	14.8
SD		7.0	7.7	5.0	4.9

Table 4.1: Estimated posterior movement of the tibial contact points with flexion. Negative values indicate anterior movement. Net movement corresponds to that recorded from full extension (0°) to 100° flexion while maximum (Max) movement is the maximum occurring within the 0-100° range.

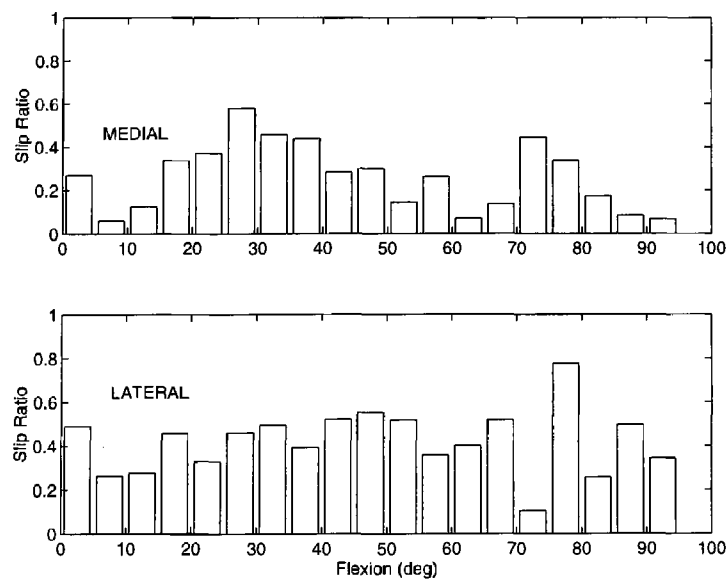


Figure 4.2: Estimated slip ratio as a function of flexion angle for a single specimen (Knee F).

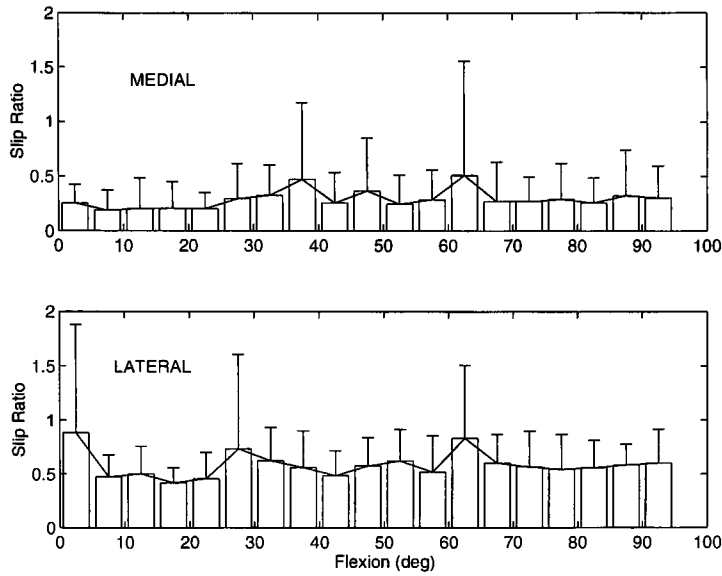


Figure 4.3: Average slip ratio as a function of flexion angle ($n=12$ specimens).

compartment is generally higher than that in the medial compartment. They also imply that pure rolling of the femur on the tibia is not a distinct feature of passive knee motion. It must be noted that where anterior movement of the tibia contact point was observed in the medial compartment in early flexion, the femur is still described as ‘sliding’ on the tibia and the slip ratio still defined as positive in the current analysis. The ratio is never greater than one in early flexion in the medial compartment and therefore could not be mistaken with pure rolling.

4.1.2 Discussion

Despite variations between specimens, some trends in contact kinematics during passive motion were observed. In all cases, the contact point moved posteriorly in the lateral compartment of the knee with flexion. In this compartment, the femoral motion was a combination of rolling and rotational sliding on the tibia. The average slip ratio remained below unity. In the medial compartment, the net average movement was also directed posteriorly; however, the magnitude was less than that in the lateral compartment. The proportion of rolling in the medial compartment was less than that in the lateral compartment (ratio closer to zero). The average slip ratios in both compartments suggest that pure rolling is not a consistent feature of passive knee kinematics.

The average displacement of the contact points was on the order of previously reported meniscal movement during flexion. Kapandji [64] described how the menisci follow the movements of the contact points between the femoral and tibial condyles. He further quantified the posterior movement of the medial meniscus to be roughly

half that of the lateral, reporting values of 6 mm and 12 mm of movement respectively, compared with the average 8 mm and 15 mm found here. Thompson *et al.* [125], measuring the excursion of the menisci in the unloaded cadaver knee using magnetic resonance imaging, found average values of 5.1 mm and 11.2 mm for the medial and lateral compartments respectively.

The trends observed in the slip ratio are in agreement with observations made by O'Connor *et al.* [98]. They examined the geometry and movement of the knee in two dimensions by modelling the joint as a four-bar linkage based on the cruciate mechanism. According to their calculations, the femur rolls as well as slides on the tibia and the proportions of rolling and sliding depend on the articular surface shape and the flexion angle. Nowhere did they find the slip ratio to be one, corresponding to pure rolling, consistent with the present results.

While due in part to inter-specimen differences, some of the variation in contact kinematics found across the specimens may also be explained by the error involved in the reconstruction of the intact knee geometry and to a lesser extent error associated with the polynomial surface representation. Contact error, quantified specifically as the amount of separation or interpenetration of the polynomial surfaces at the contact, is presented and discussed in detail in Section 4.3.3; however, it must be noted here that, with values of up to 1 cm at higher flexion angles with some specimens, the contact error is a major limitation in the quantification of contact point movement based on the current data. The large inter-specimen variability in the contact error partially explains the large variability in estimated contact point movement.

Further variability can be attributed to the nature of the proximity criterion used. The solution for the contact point location is not unique and can not be determined analytically. A minimisation algorithm was implemented and the solution therefore depended on the initial guess and the bounds placed on the variables. As a result, the path followed by the contact points was often discontinuous in nature. Jumps in the contact movement at certain flexion angles account for some of the variability. The slip ratio, for example, at full extension in the lateral compartment, reflects a large degree of variability (Figure 4.3). In this case, one specimen exhibited a large jump in the contact point location on the tibia from 0° to 5° flexion, while another showed virtually no movement (remained at edge of condyle). A similar explanation applies to the high standard deviations found at other flexion angles.

To conclude, despite the variability in the results, general patterns with respect to contact point movement were noted. Knowledge about contact excursion in the normal knee is critical in the design and development (*e.g.* wear testing) of surface replacements. The specimen-specific contact kinematics estimated here also provide additional data for the validation of a model of knee mobility.

4.2 Ligament Fibre Isometry and Length Patterns

The length change patterns of fibres within the ACL, PCL, MCL and LCL are examined in this section. Tracking sets of both ‘combination’ fibres and ‘physiological’ fibres (as defined in Section 2.10) during the measured passive motion yielded those which were most nearly isometric in each of the four ligaments. The locations of these fibres and their corresponding length changes are described. Patterns of fibre lengthening and shortening during passive motion within the arrays of ‘physiological’ fibres are also examined. The findings are discussed in the light of previous work on ligament length patterns and isometry and with respect to the development of a model of knee mobility and stability.

4.2.1 Ligament Fibre Isometry

4.2.1.1 Isometric Fibre Location

The locations of the most nearly isometric fibres found in the ACL, PCL and MCL are illustrated in Figure 4.4. To observe trends across the twelve specimens, approximate locations are shown with respect to idealised tibial and femoral attachment areas. Results for the LCL are not presented. The attachment sites of this ligament are relatively small; hence, there is little difference in the isometry of the fibres with respect to attachment site. The locations of both the most isometric ‘combination’ and ‘physiological’ fibres of the ACL, PCL and MCL are discussed below. Anatomical directions refer to the fully extended knee.

‘Combination’ Fibres The insertion site of the most isometric ACL fibre on the tibia was least consistent. With six specimens, the fibre inserted within the posterior third of the attachment, with four, within the anterior third, and with the remaining two, it was found to lie within the central third. The femoral origin of the ACL fibre was more consistent. In almost all cases, the site was proximal and lay close to the anterior edge of the attachment area. Hence, the ACL isometric fibres for which a posterior tibial attachment site was found may not correspond well to physiological fibres (Chapter 2, Figure 2.7).

The location of the origin and insertion of the most isometric PCL fibre was fairly consistent across specimens. On the tibia, the fibre always inserted in the distal-lateral region of the attachment. The origin on the femur was consistently in the proximal, posterior region of the attachment area. In this case, this combination of attachment sites corresponds well to the location of a physiological fibre bundle, often referred to as the ‘posterior oblique’ fibre (Chapter 2, Figure 2.7).

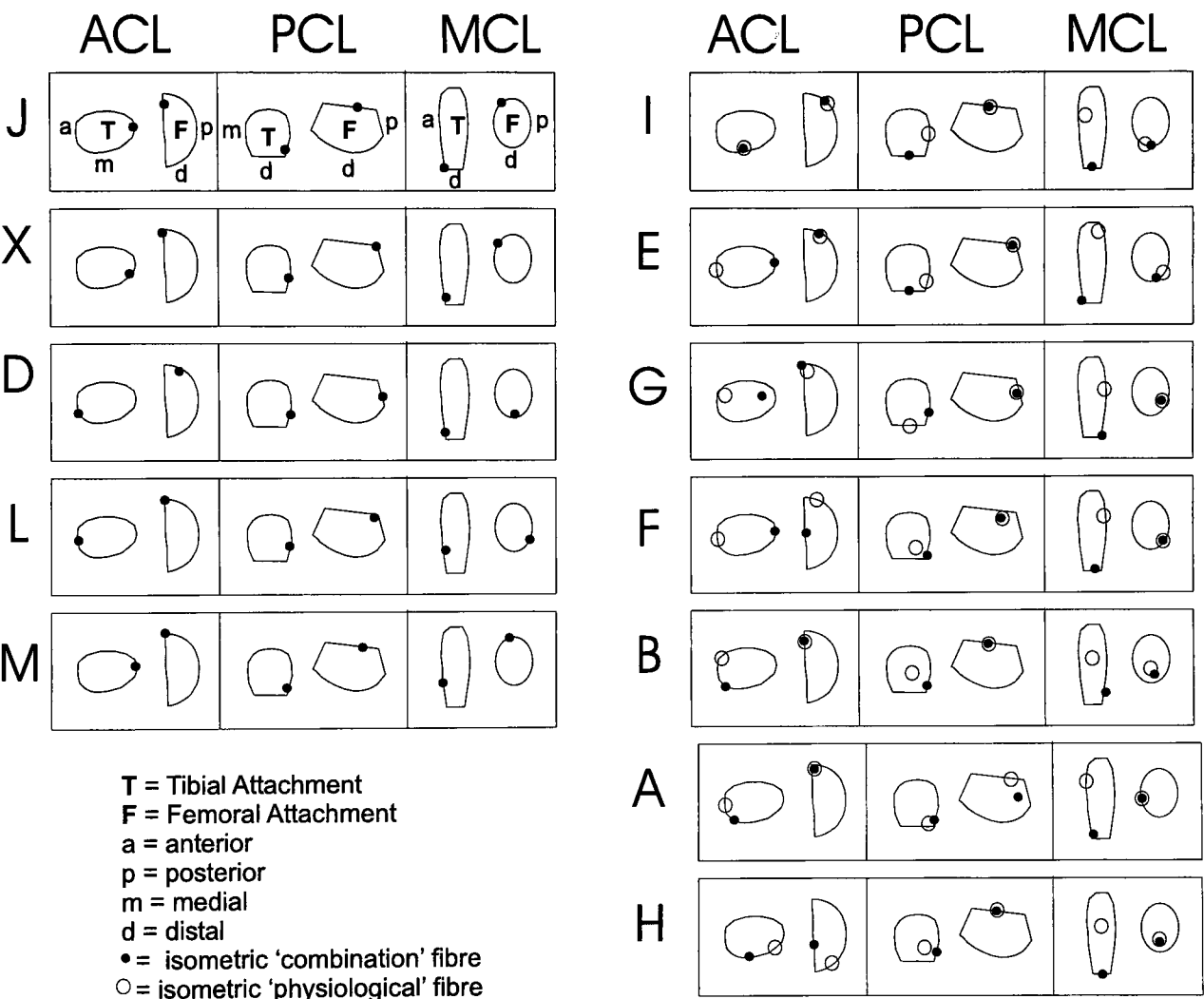


Figure 4.4: A schematic showing the approximate locations of the origins and insertions of the most nearly isometric fibres found in the ACL, PCL and MCL with respect to idealised tibial (T) and femoral (F) attachment sites. For all twelve specimens, the origins and insertions of the most isometric 'combination' fibre are depicted (solid dot). For seven specimens, the locations of the most isometric 'physiological' fibre attachments are also shown (open circle). The ACL tibial attachment is viewed in the transverse plane, the PCL tibial attachment in the frontal plane, and the ACL and PCL femoral attachments and both attachments of the MCL in the sagittal plane.

In the case of the MCL fibre, the tibial insertion site was found to lie distally and in seven knees, on or close to the anterior edge of the attachment. The location of the femoral origin was variable. With an anterior insertion on the tibia, a fibre is generally expected to originate anteriorly on the femur (Chapter 2, Figure 2.8). In this case, fibres with posterior and central femoral origins may not correspond well to physiological fibres.

‘Physiological’ Fibres By searching through a defined array of ‘physiological’ fibres versus a set of ‘combination’ fibres, the location of the most isometric fibre found in each ligament did not change drastically. For the seven specimens for which ‘physiological’ fibres were defined, the femoral origins of the ACL, PCL and MCL isometric fibres were virtually the same as those of the most isometric ‘combination’ fibres found. There were, however, notable differences in the locations of the tibial insertion sites of the ‘physiological’ and ‘combination’ fibres.

In five of the seven knees, the most isometric ACL fibre, originating anteriorly and proximally on the femur, inserted anteriorly on the tibia (all except Knees I and H). The tibial insertion sites of the PCL fibre were close to those found by searching through the sets of ‘combination’ fibres. All fibres inserted within the distal-lateral region of the tibial attachment. The location of the tibial insertion site of the most isometric MCL fibre was variable. It was, on average, more proximal compared to those of the ‘combination’ fibres. The ACL, PCL and MCL fibres identified correspond, by definition, more closely to physiological fibres.

4.2.1.2 Isometric Fibre Length Change

The maximum changes in the length ratio over the flexion range (0-100°), ΔLR_{max} , of the most nearly isometric fibres are presented in Table 4.2 for the ‘combination’ fibres and in Table 4.3 for the ‘physiological’ fibres. These changes reflect the difference between the longest and shortest calculated lengths.

Among the most isometric ‘combination’ fibres (Table 4.2), the ACL and MCL fibres experienced length changes of under 6% and 5% respectively. The largest changes in length were seen by the PCL fibre, with some ΔLR_{max} as high as 15% and 18% (Knees J and M). The results for the PCL were most variable. With the exception of one knee (Knee I), the LCL fibre experienced length changes of under 7%.

The change in length experienced by the most isometric ‘physiological’ fibre was of the same order of magnitude but somewhat higher than that experienced by the ‘combination’ fibre found in the same ligament of the same knee (Table 4.3). With the exception of the ACL fibre of Knee E and the LCL fibre of Knee I, the length changes of the most isometric ACL, PCL, MCL and LCL fibres for the seven

Specimen	ACL (%)	PCL (%)	MCL (%)	LCL (%)
J	3.03	14.57	3.95	6.75
D	2.89	6.60	3.74	5.79
L	5.37	7.97	4.68	4.78
M	5.91	18.28	3.21	4.24
X	4.81	8.92	3.72	3.72
I	1.84	3.17	1.46	14.15
E	3.00	1.41	1.20	2.60
G	2.98	2.58	1.67	1.65
F	1.82	1.73	1.40	1.03
B	3.62	4.13	1.67	2.14
A	2.34	6.02	2.53	3.21
H	3.58	2.93	1.20	2.29
Mean	3.43	6.53	2.54	4.36
SD	1.32	5.27	1.26	3.52
Mean (Knees I-H)	2.74	3.14	1.59	3.87
SD	0.76	1.56	0.46	4.58

Table 4.2: The maximum changes in the length ratio, ΔLR_{max} , of the most nearly isometric ‘combination’ fibres.

specimens were all under 8%.

Figure 4.5 shows the length ratios of the most isometric ‘combination’ and ‘physiological’ fibres of the ACL, PCL and MCL throughout the entire flexion-extension cycle for two specimens (Knees F and B). The locations of the most isometric ‘physiological’ and ‘combination’ fibres for Knees F and B were similar (Figure 4.4), except for the ACL fibre of Knee F. Hence, the associated length change patterns were similar in all but this case. The ‘physiological’ fibres always experienced, even if only slightly, more deformation than the ‘combination’ fibres over the range of flexion.

Repeatability of Fibre Array Definition The assessment of the repeatability of the definition of arrays of physiological fibres consisted of searching for the most nearly isometric ‘physiological’ fibre within each of the three sets of defined fibre arrays for the ACL, PCL and MCL of a single specimen (Knee F) (Section 2.7.3). For each ligament, the location of the most isometric fibre found based on the three array definitions was virtually identical. This is demonstrated by the changing length ratios of these fibres which exhibited virtually no differences between the sets of data (Figure 4.6).

Specimen	ACL (%)	PCL (%)	MCL (%)	LCL (%)
I	1.93	3.70	1.41	17.23
E	14.82	2.43	1.19	2.86
G	3.31	3.02	2.16	1.73
F	2.63	1.94	1.60	1.42
B	3.43	4.64	1.69	2.22
A	2.39	7.23	3.47	3.92
H	7.33	3.66	1.44	2.49
Mean	5.12	3.80	1.85	4.55
SD	4.64	1.75	0.78	5.65

Table 4.3: The maximum change in length ratio, ΔLR_{max} , of the most nearly isometric ‘physiological’ fibres.

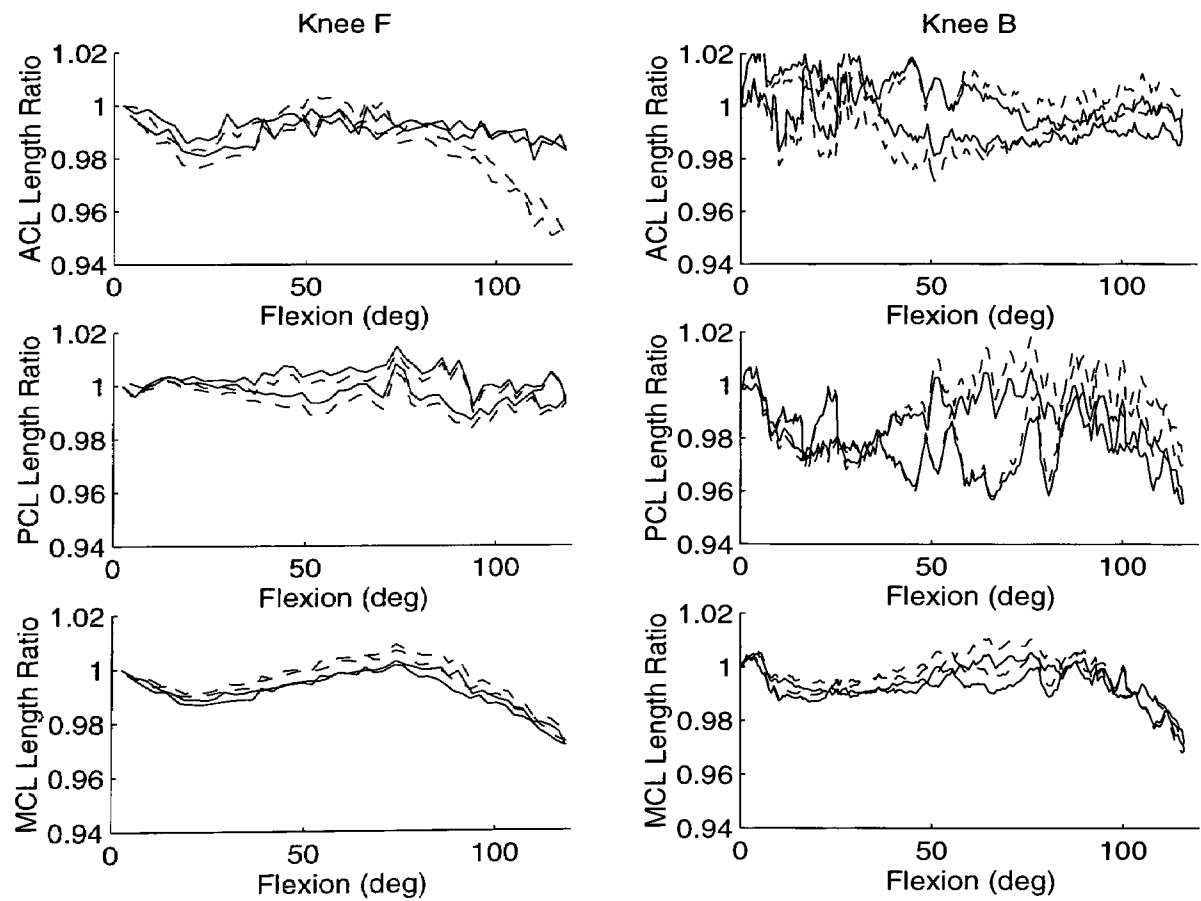


Figure 4.5: The length ratios (LR) of the most isometric ‘combination’ (solid) and ‘physiological’ (dashed) fibres of the ACL, PCL and MCL for the entire flexion-extension cycle for two specimens (Knees F and B).

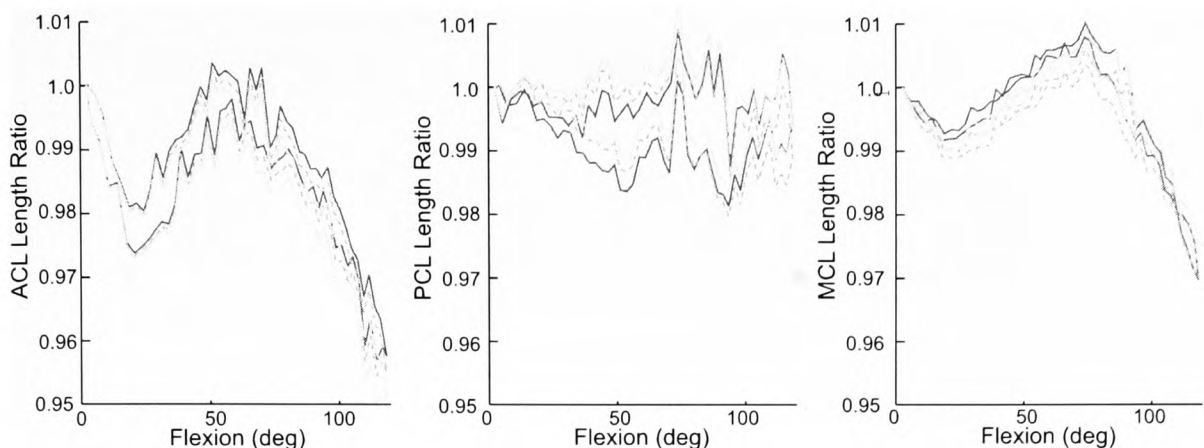


Figure 4.6: Assessment of the repeatability of the fibre mapping procedure and array definition. The length ratios of the most isometric ‘physiological’ fibres of the ACL, PCL and MCL of a representative specimen (Knee F). The three curves represent the length ratios of fibres found based on three subsequent definitions of the fibre arrays.

4.2.2 Ligament Fibre Length Patterns

The contour maps shown in Figures 4.7 and 4.9 illustrate the length changes of the arrays of ‘physiological’ fibres representing the ACL and PCL of two specimens (Knees B and F) during passive knee flexion. For 30°, 60° and 90° flexion, contour lines representing different levels of the fibre length ratio (LR) were plotted in the plane of the tibial ligament attachment. The maximum percentage changes in the length ratio over the 100° flexion range (ΔLR_{max}) of the ligament fibres for the same two specimens (Knees F and B) are described by a second series of contour maps in Figures 4.8 and 4.10.

Significant differences in the magnitude and the pattern of ACL fibre length changes were noted between the two specimens (Figure 4.7). With Knee F, fibres in the anteromedial region of the ligament shortened slightly with flexion while the posterolateral fibres lengthened, some by as much as 40% within 90° of passive flexion. Examining Knee B, however, almost the opposite was observed. Nearly all the ACL fibres shortened with flexion. Only a small number of anterior fibres lengthened slightly up to 30° flexion. The posterolateral fibres shortened with flexion, some by up to 14%, and then began to lengthen again as full flexion was approached. Despite these differences, the anterior region on the tibial attachment site corresponded to the most isometric fibres of the ACL for both specimens with ΔLR_{max} on the order of 5% (Figure 4.8). The posterior region for both knees was least isometric but the posterior fibres of Knee F experienced almost twice the percentage change in length compared to those of Knee B. The ACL fibres of Knees I, G, A and H exhibited length change patterns similar to that of Knee B, while the ACL fibres of Knee E behaved more like those of Knee F. The magnitude of the length changes

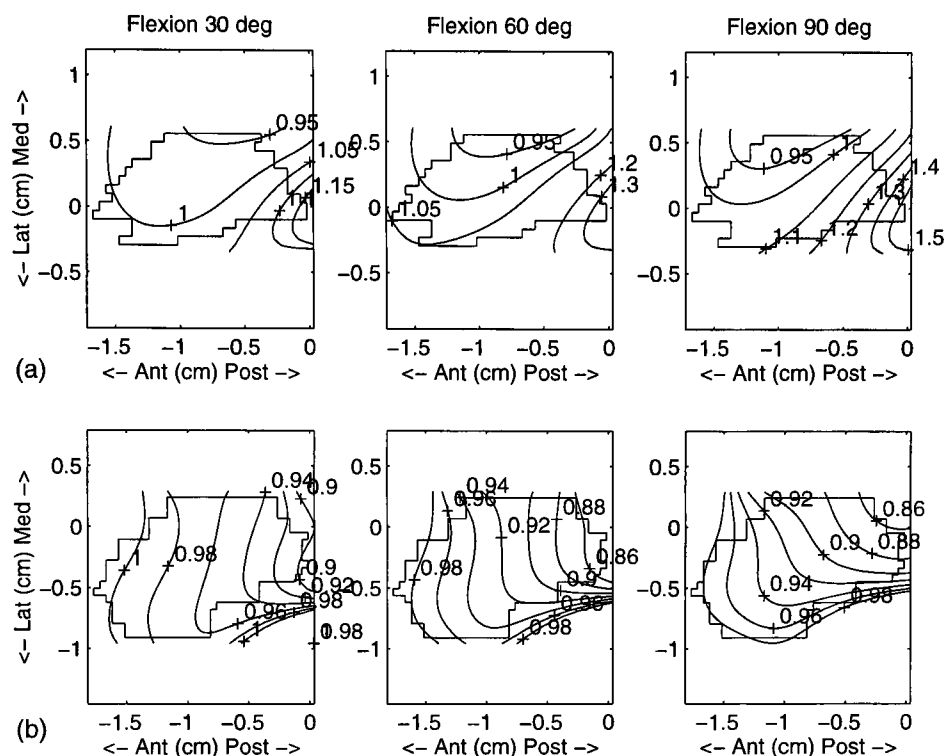


Figure 4.7: The length change patterns of arrays of ‘physiological’ fibres of the ACL for two specimens (a) Knee F and (b) Knee B. Contour maps of the length ratio are shown with respect to the plane of the tibial ligament attachment (transverse view).

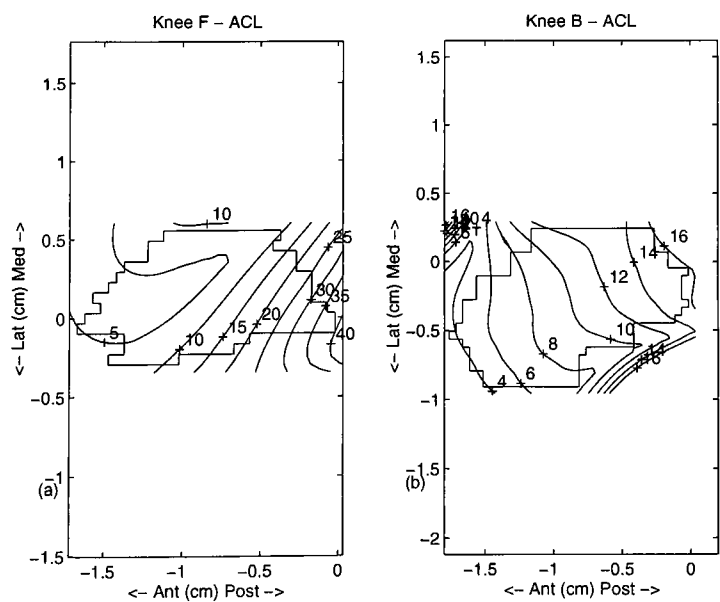


Figure 4.8: The maximum percentage changes in the length ratio (ΔLR_{max}) over the flexion range for arrays of ‘physiological’ fibres of the ACL. Contour maps for the two specimens (Knees F and B), are shown with respect to the plane of the tibial ligament attachment (transverse view).

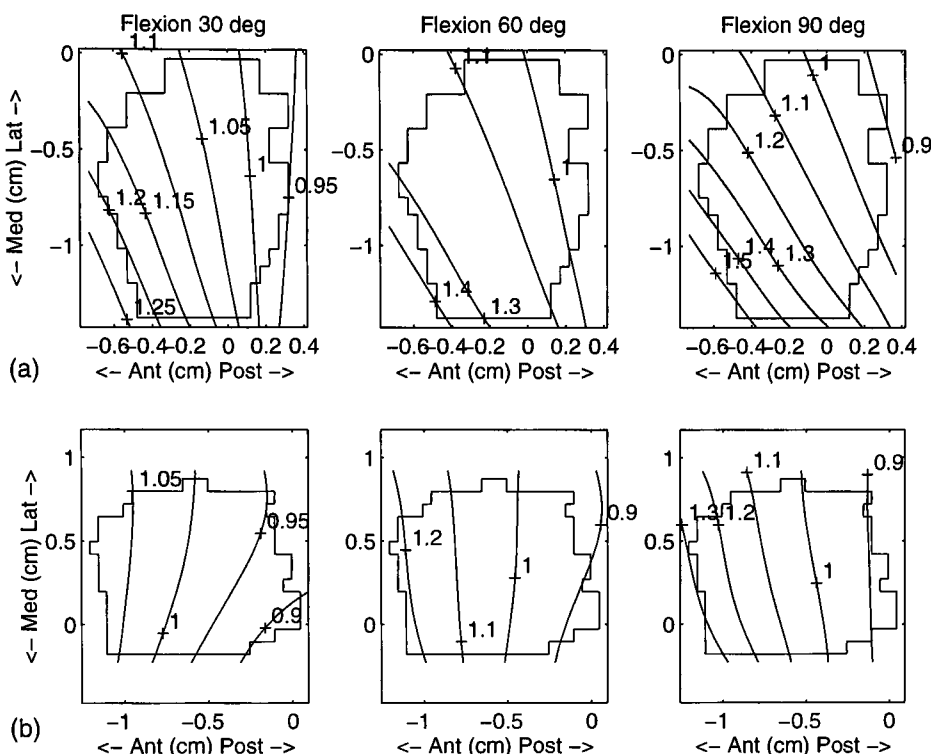


Figure 4.9: The length change patterns of arrays of ‘physiological’ fibres of the PCL for two specimens, (a) Knee F and (b) Knee B. Contour maps of the length ratio are shown with respect to the plane of the tibial ligament attachment (approx. transverse view).

varied somewhat across the specimens with shortening of up to 20% (Knee A) and lengthening up to 40% (Knee F) experienced by some fibres .

For the PCL fibres, the length change patterns observed were more consistent (Figure 4.9). For Knees F and B, the anterior (proximal) fibres tended to lengthen, by up to 50%, with flexion while the posterior (distal) fibres shortened, by approximately 10%, with respect to their length at full extension. The posterolateral region on the tibial attachment site corresponded to the most isometric fibres of the PCL for both specimens with ΔLR_{max} on the order of 5% (Figure 4.10). The PCL fibres of the remaining five knee specimens displayed similar magnitudes and patterns of length change during passive flexion.

The degree of lengthening and shortening of the MCL and LCL fibres varied somewhat (no figure), but the patterns were similar for the seven specimens. In all knees, the anterior fibres of the MCL and LCL remained close to constant or lengthened slightly during flexion. The posterior fibres of both ligaments shortened with flexion. Fibre shortening and lengthening of up to 20% was found within the range of motion examined.

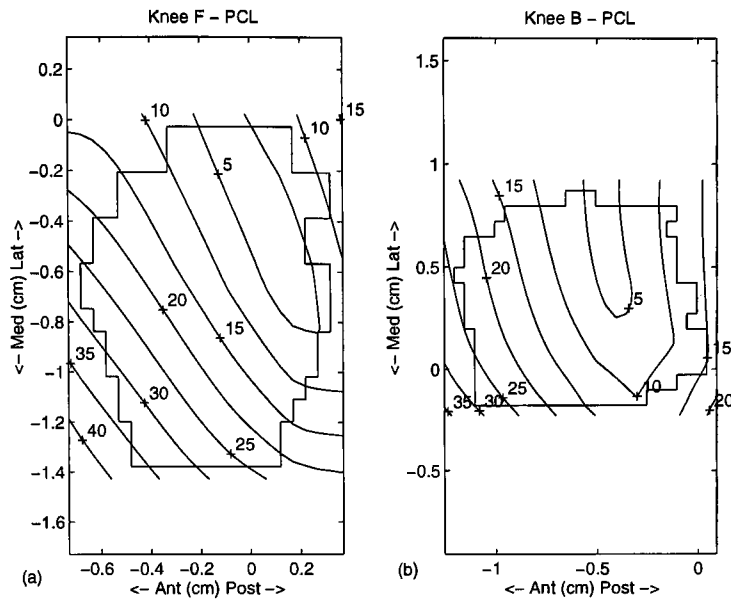


Figure 4.10: The maximum percentage changes in the length ratio (ΔLR_{max}) over the flexion range for arrays of ‘physiological’ fibres of the PCL. Contour maps for the two specimens (Knees F and B) are shown with respect to the plane of the tibial ligament attachment (approx. transverse view).

4.2.3 Discussion

Studies of ligament fibre lengths during passive flexion were undertaken to determine the locations and associated length changes of the most nearly isometric ‘combination’ and ‘physiological’ fibres in each of the ACL, PCL, MCL and LCL and to provide a complete description of ‘physiological’ fibre length patterns during passive motion.

4.2.3.1 Ligament Fibre Isometry

The locations and length changes associated with the most isometric ‘physiological’ fibres in the ACL and PCL were generally consistent with previous descriptions of ligament isometry in the literature. The locations of the most isometric ‘physiological’ fibres in the MCL and LCL, however, were variable although associated length changes were consistently small.

ACL In a review of isometry studies of the ACL, Amis and Zavras [3] concluded that, although there is general agreement that no *truly* isometric ACL fibre exists, nearly isometric fibres have been identified with length changes of less than 2 mm, corresponding approximately to a 6% maximum length change. It is generally agreed that as the anterior fibre is approached, the fibres become more isometric and that the anteroproximal corner of the femoral attachment is most isometric [3]. Here too, for five out of the seven specimens examined, the most isometric ‘physiological’

fibre was identified as the anterior fibre. In a sixth specimen, a fibre inserting anteromedially on the tibia was found. The mean maximum length change of the 'physiological' ACL fibre in the present study was 5%, on the order of previously reported values.

PCL With respect to the PCL there has been less agreement as to the location of the most isometric fibre. Friederich *et al.* [40] and Covey *et al.* [31], however, have suggested that the posterior oblique band of the PCL, which runs from posterosuperior on the femur to posterolateral on the tibia is most isometric, similar to the fibres found in the current study. Although some anteroposterior forces were applied in their study, Grood *et al.* [46], for a central tibial insertion site, defined an anteroposteriorly directed isometric line on the femoral attachment, lying parallel and close to its proximal edge. The origins of the PCL fibres found here all lie in the vicinity of this line.

MCL For the MCL, maximum length changes were consistently under 4%; however, the location of the fibre varied across the seven specimens. To the author's knowledge, there have been no reported attempts to identify the most isometric fibres in the MCL. Among the few who have previously examined length changes or strains in the MCL, there has been little agreement. Warren *et al.* [133] have suggested that fibres lying both along the anterior and posterior edges experience length changes of under 2 mm, the anterior fibre lengthening and the posterior one shortening, suggesting a most isometric central fibre. Blankevoort *et al.* [17] measured length changes of specific bundles of the MCL in four cadaver knees under light loading during passive motion. From their data, the anterior fibre appears to be most isometric, with corresponding length changes of approximately 5%. In the case of the MCL, some of the variability across the seven specimens and between different studies can be explained by the ill-defined nature of the ligament and locations of the fibre insertion sites on the tibia.

LCL For the LCL, both the location and maximum length change of the most isometric 'physiological' fibre were found to be variable. The average maximum length change of 5% is much lower than the few previously reported observations. Blankevoort *et al.* [17] in study of ligament recruitment under light loading also reported variable length changes for a number of LCL fibre bundles but found length changes of under 10% for the anterior or superior bundle. Rovick *et al.* [111] found a central fibre of the LCL to shorten by approximately 20%.

‘Combination’ vs ‘Physiological’ Fibres The search through sets of ‘combination’ fibres often yielded most nearly isometric fibres which were close to ‘physiological’ in nature. For the PCL, a posterior oblique fibre was found in all cases. In the case of the ACL and MCL, anterior fibres were found in a majority of specimens. The locations of the most isometric ‘combination’ fibres provides useful information for the theory of ligament reconstruction as graft placement is not restricted to the locations of physiological fibres.

The length changes experienced by fibres within all four ligaments were more sensitive to the location of the femoral origin than to that of the tibial insertion site. Comparing the most isometric ‘physiological’ and ‘combination’ fibre for the same ligament of the same knee, the location of the femoral origin was often virtually the same. Despite notable differences in the location of the tibial insertion site, however, the variations in the overall length changes were small. This has been noted by Hefzy *et al.* [51] and Grood *et al.* [46] to hold true for both the ACL and PCL and was explained by the proximity of the femoral attachments to the flexion axis [46]. The present results suggest that it also holds true for the collateral ligaments.

4.2.3.2 Ligament Fibre Length Patterns

To the author’s knowledge the current data on ligament length patterns represents the first description of the length change patterns for entire arrays of ‘physiological’ ligament fibres in the ACL, PCL, MCL and LCL during measured passive knee motion. Most previous studies have focused on measuring length changes or strain in specific fibres or bundles of the ligaments (*e.g.* ACL [2, 40, 68, 112]). Only Sidles *et al.* [119] in a similar study presented the isometry of ACL and PCL fibres corresponding to all possible insertions sites on the tibial attachment for selected femoral origins. Because their study was aimed at finding the most isometric combinations of femoral origin and tibial insertion sites to guide graft placement during ligament reconstruction, no data was reported for the length change patterns corresponding to fibres of the intact ligaments.

Despite some notable inter-specimen variations, the observed length change patterns of the cruciate and collateral ligament fibres are generally consistent with descriptions of individual fibre or bundle length change patterns previously reported in the literature.

The pattern of ACL fibre lengthening and shortening recorded in five of the seven knees (*e.g.* Knee B) is in agreement with measurements made by Sapega *et al.* [112]. They found the anteromedial region of the ligament to be most isometric and observed slackening followed by lengthening of the fibres in the central and posterior regions of the ligament during passive flexion. Blankevoort *et al.*, too, found a posterior bundle to shorten and an anterior bundle to remain close to constant

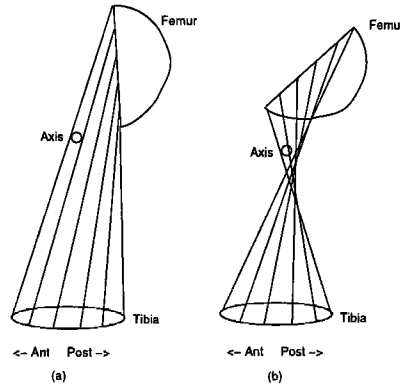


Figure 4.11: Explanation for differences observed in ACL fibre length change patterns. The ligament configurations depicted in the sagittal plane in the fully extended knee.

length or shorten with flexion [17]. A reversed pattern of ACL fibre lengthening and shortening, however, was found for two specimens, (*e.g.* Knee F). Such differences can be explained by small variations in the positions and orientations of the tibial and femoral attachments of the ligament relative to the axis of motion. Configuration (a) in Figure 4.11 is that most commonly described: the ACL fibres which lie anterior to the axis of motion will lengthen and the fibres which lie posterior to the axis will shorten with flexion (*e.g.* Knee B). For a slightly different orientation of the femoral attachment in configuration (Figure 4.11(b)), however, the anterior fibres will shorten and the posterior fibres lengthen with flexion (*e.g.* Knee F).

The pattern of PCL fibre lengthening and shortening described for the seven specimens is consistent with the previous studies of the ligament. The anterior part of the ligament has been described as being slack in full extension and becoming taut or lengthening with increasing flexion [20, 40, 44]. Covey *et al.* [31], in testing for PCL isometry, found the anterior and central fibres of the PCL to lengthen by equal amounts and the posterior fibres to slacken and then re-tighten. Rovick *et al.* [111] found the posterior fibres to remain close to constant in length and the anterior fibres to lengthen by up to 30%. Blankevoort *et al.* [17] reported specific anterior bundles to lengthen by up to 40%, compared to the maximum of 50% found here; their results for the posterior bundle were variable.

The lengthening and shortening patterns of MCL and LCL fibres for the seven specimens are also in accordance with previous observations. Kapandji [64] reported that both the MCL and LCL are taut in extension and slacken with flexion. Warren *et al.* [133] found the anterior border of the parallel fibres of the MCL to lengthen

with flexion while the fibres posterior to the leading edge slackened. Rovick *et al.* [111] confirmed these observations with reported shortening of up to 20% for posterior fibres of the MCL and central fibres of the LCL. Blankevoort *et al.* [17] found quantitative differences in the length changes of the LCL. They found that most fibres of the LCL shortened, the posterior bundle by up to 30%. In one specimen, however, the anterior fibre of the LCL lengthened by over 30%. Similar variability was found in the current study.

The present results highlight the potential effect of inter-specimen anatomical variability on recorded ligament length patterns. This effect may explain some of the differences in the length changes of specific ligament fibres or bundles reported in previous studies. The description of the location of the studied fibres or bundles is most often qualitative. The current results have demonstrated the sensitivity of the length change patterns to the specific fibre origins and insertions. Such variability must also be considered in the context of ligament reconstruction and may be examined theoretically by means of mathematical model of knee mobility and stability described in Chapters 5 through 8.

4.2.3.3 Limitations of the Analysis

The fibre length changes presented here can not be used to draw conclusions about the *strain* experienced by the ligament fibres. To determine strain, a zero-strain or zero-load reference length for each fibre must be known. Hence, the present data provides no information as to the slackness or tautness of the fibres. The minimal load required to passively flex and extend the joint (Section 2.4.3) suggests that there is minimal tissue deformation, *i.e.* stretching of the ligament fibres. This in turn would suggest that most fibres are slack or just taut during passive motion. This explains some of the large changes in fibre length observed. Given the failure strain of ligament tissue to be on the order of 20% (*e.g.* [96]), such changes could only occur if the particular fibres are slack for most of the range of motion.

A further limitation of the current description of fibre length patterns is that the length of a fibre has been defined as the straight line distance from the tibial insertion site to the femoral origin. Fibres do not always run a straight course from tibia to femur. The MCL for example wraps slightly around the edge of the medial tibial plateau. In the fully extended knee, impingement of the ACL with the femoral intercondylar notch is possible and a degree of interaction between the cruciates may also occur during flexion. It is assumed that these interactions play a minimal role in the length patterns observed in passive flexion; however, the effects must be reconsidered when examining length changes in the loaded joint, *e.g.* in a study of knee stability (Chapter 8).

While due in part to inter-specimen differences, some of the variation in the

length change patterns of ligament fibres found across the specimens may be explained by the error involved in the reconstruction of the intact knee geometry, in the measured movement and in the establishment of fibre mapping functions used to define the arrays of ‘physiological’ fibres. Estimates of reconstruction error and movement error will be presented in Sections 4.3.2 and 4.3.4. The mean reconstruction errors, on the order of 2 mm (for knees tested in Lab 1: J through G) and 0.5 mm (for knees tested in Lab 2: F through H) may explain some of the larger length changes associated with specimens tested in Lab 2. Differences in associated reconstruction error between Labs 1 and 2 were not, however, reflected in the locations of the most isometric fibres or in the qualitative description of fibre length change patterns. Both Knees F and B, for which ligament length patterns were presented, were tested in Lab 2.

4.2.3.4 Conclusions

The current observations confirm much of what has been previously reported in the literature with respect to overall ligament fibre length patterns and the locations and length changes of the most nearly isometric fibres. The present results do however fill some gaps in the existing knowledge of ligament isometry and length change patterns. Complete specimen-specific descriptions of fibre length changes of the ACL, PCL, MCL and LCL as a function of the tibial insertion site and flexion angle are reported for passive flexion. Similar descriptions relative to the femoral origins of the fibres may also be generated. The degree of inter-specimen variability has been highlighted and provides an explanation for some of the disagreement reported in the literature. The additional information is of relevance to the design of and technique associated with ligament reconstruction.

The present results also provide evidence that nearly isometric fibres exist in each of ACL, PCL, MCL and LCL during passive flexion, important in justifying the conceptual basis of a model of knee mobility (Chapter 5). The specimen-specific fibre length change patterns also provide a means of validating theoretical predictions of a 3-D extensible ligament model (Chapter 8) which may be used to estimate fibre strain patterns in the loaded joint.

4.3 Experimental Error and Accuracy

4.3.1 Digitisation Error

The effect of the metal objects in the original lab was found to be significant. The error corresponding to every grid point measured is plotted in Figure 4.12 against the distance of the point from the transmitter. Lab 1 refers to the measurements

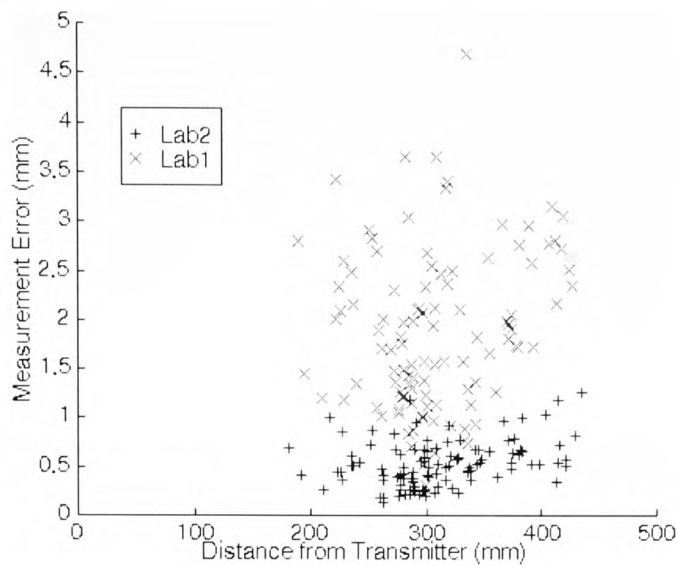


Figure 4.12: Digitisation error in reference grid points

performed in the original lab and Lab 2 is the second lab in which was virtually metal-free. The mean error in the original laboratory (Lab1) was 1.92 mm (S.D. 0.87 mm). In the second lab (Lab 2), the mean error fell to 0.5 mm (S.D. 0.2 mm). The presence of metal in the original lab (Lab 1) reduced the accuracy of the tracking system in the digitisation of 3-D coordinates.

A limitation of the method used to assess the digitisation error is that it cannot account for any systematic error or bias in the digitisation of the points. An optimised transformation was used to provide the actual positions of the points on the reference grid relative to the measured positions. This was done because it was not possible to identify physically the origin of the transmitter coordinate system. The positions of the points therefore could not be measured absolutely relative to the transmitter using an external device. If there is a bias present, the method may underestimate the error; however, because the measurements on the knees are all relative, a consistent bias would have a limited effect on the accuracy of the results.

4.3.2 Reconstruction Error

The distances between the reconstructed positions of the tibial and femoral reference markers and their positions measured in the intact knee were calculated for each knee. The mean and maximum distances (errors) over the total number of markers are presented in Table 4.4.

The reduction in reconstruction error in Lab 2 confirms that the presence of metal in the original lab (Lab 1) influenced the accuracy of the tracking system

Specimen	Femur (mm)		Tibia (mm)	
	mean error	max error	mean error	max error
J	1.91	2.52	0.46	0.73
D	2.87	3.97	1.37	2.39
L	3.01	3.75	0.3	0.42
M	1.00	1.76	1.40	2.00
X	2.57	3.65	1.16	1.51
I	2.30	3.13	0.97	1.19
E	2.56	5.03	0.53	0.85
G	3.22	5.18	2.01	4.22
Mean (Lab 1)	2.43	3.62	1.03	1.66
SD	0.66	1.09	0.54	1.15
F	0.95	1.25	0.37	0.72
C	0.51	0.77	0.56	0.91
B	0.67	1.24	0.41	0.84
A	0.55	0.68	0.52	1.00
N	0.67	0.94	0.33	0.41
P	0.55	0.91	0.26	0.34
H	0.46	0.77	0.34	0.75
Mean (Lab 2)	0.62	0.94	0.40	0.71
SD	0.15	0.21	0.10	0.23

Table 4.4: The reconstruction error in the tibial and femoral reference markers.

Specimen	Lab	Medial (mm)				Lateral (mm)			
		0°	30°	60°	90°	0°	30°	60°	90°
J	1	1.24	1.17	2.33	4.48	0.84	0.86	2.64	4.63
X	1	-1.64	0.85	3.17	5.06	-0.57	4.31	7.76	9.40
D	1	0.53	1.28	1.29	0.03	3.06	5.29	6.66	6.87
L	1	3.02	5.90	11.00	13.88	5.66	8.56	11.14	10.44
M	1	-2.59	-4.67	-1.17	-0.71	1.84	4.93	8.10	7.88
I	1	-6.04	-6.45	-6.58	-5.97	-1.09	4.74	8.16	9.89
E	1	-3.11	-3.22	-2.43	-1.43	7.64	10.74	13.00	13.62
G	1	0.41	4.20	8.50	11.38	1.10	1.98	3.17	2.63
F	2	1.80	1.58	0.23	-0.26	1.34	2.16	1.63	1.48
B	2	0.73	1.72	1.77	1.61	1.70	2.29	3.18	3.01
A	2	1.85	-0.74	-3.13	-5.96	2.16	4.40	4.55	4.40
H	2	1.06	1.52	1.53	1.50	1.81	2.18	2.88	3.45

Table 4.5: Contact errors in the medial and lateral tibiofemoral compartments at a series of flexion angles. Positive values indicate separation and negative values interpenetration of the surfaces.

significantly. Specimens F, C, B, A, N, P, and H, tested in Lab 2, the virtually metal-free laboratory, had much lower errors (all < 1 mm). The femoral reconstruction error associated with Lab 2 was 75% less than that associated with Lab 1 (the original lab). The tibial reconstruction error was 60% less.

The reconstruction errors are of the same order of magnitude as the digitisation errors. This suggests that the optimised transformation used is successful in reconstructing the intact geometry without introducing further error.

The error in the reconstruction of the femoral markers was found to be greater than that for the tibial markers. This is due to the fact that the tibia is displaced minimally between the motion experiment and the digitisation procedure, whereas the femur undergoes a rotation of approximately 180° in the frontal plane.

4.3.3 Contact Error

The contact error between the femoral and tibial articular surfaces in the reconstructed intact knee was quantified by the degree of separation or interpenetration of their polynomial surface representations (Table 4.5).

Contact error is a manifestation of the error involved in the reconstruction of the intact joint geometry from that measured in the disarticulated knee. The absolute value of the error is therefore on average higher for the knees tested in the original lab (Lab 1) than those tested in Lab 2 (Table 4.6). The average contact errors are larger than the associated reconstruction errors. This can be explained by the fact that the error in the reconstructed surface geometry of both the tibia and the femur

	Lab 1 (mm)				Lab 2 (mm)			
	Medial		Lateral		Medial		Lateral	
	0°	90°	0°	90°	0°	90°	0°	90°
Mean	2.32	5.37	2.73	8.17	1.36	2.33	1.75	3.09
SD	1.83	5.01	2.60	3.47	0.55	2.50	0.34	1.21

Table 4.6: Mean contact errors for knees tested in Lab 1 and Lab 2. The absolute values of the contact errors were averaged.

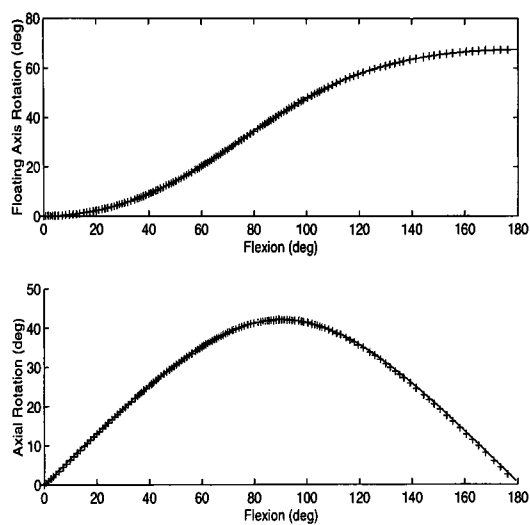


Figure 4.13: Assessment of movement error. Measured (+) and simulated (-) rotations.

and the error associated with the polynomial fits all contribute to the amount of separation or interpenetration.

4.3.4 Movement Error

Based on tests performed in the second lab (Lab 2), measured rotations and translations are compared to simulated rotations and translations in Figures 4.13 and 4.14. Through the range of motion of the test rig, the RMS difference between the simulated and measured angles was 0.21° for axial rotation and 0.12° for floating axis rotation. The mean distance from the simulated position of a point to the measured position of the point was 1.73 mm.

The same test done was also performed in the original lab (Lab 1), resulting in differences between measured and simulated rotations and translations that were twice as large. The RMS difference between the simulated and measured angles was 0.45° for axial rotation and 0.32° for floating axis rotation. The mean distance from the simulated position of a point to the measured position of the point was 2.84 mm.

The larger movement error associated with Lab 1 did not affect the similarity

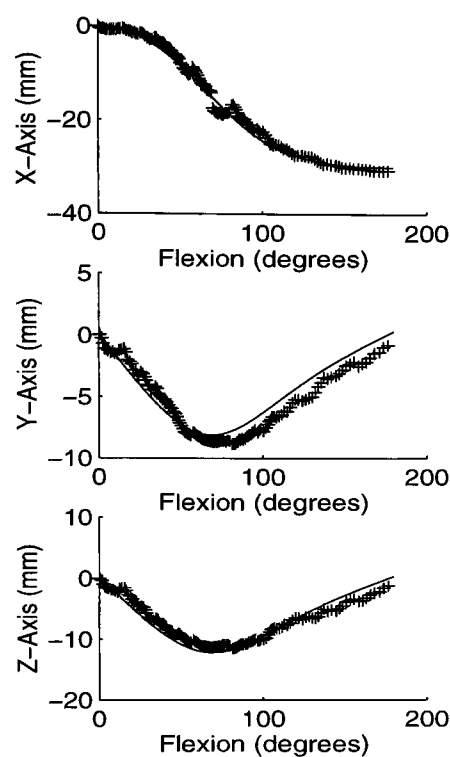


Figure 4.14: Assessment of movement error. Measured (+) and simulated (-) translations. NB the irregularity in the position coordinates due to the speed of movement not being exactly constant and the delay in the position updating of the tracking system.

of the flexing and extending motion paths of knees tested in the original lab. The mean absolute differences in each component of movement described in Table 3.9 for all twelve knees were similar for knees tested in Lab 1 and those tested in Lab 2 (Table 4.7).

4.3.4.1 Effect of Specimen Alignment and Sensor Placement

The results based on two repeated experiments in Lab 2 with the same specimen are virtually the same. The small differences in the joint rotation and translation curves

	Int/Ext (°)	Abd/Add (°)	Post/Ant (mm)	Med/Lat (mm)	Prox/Dist (mm)
Normal (Lab 1) (n=8)	0.86 (0.25)	0.14 (0.03)	1.33 (0.48)	0.31 (0.06)	0.62 (0.21)
Normal (Lab 2) (n=4)	1.05 (0.44)	0.14 (0.03)	0.78 (0.25)	0.17 (0.07)	1.51 (0.17)

Table 4.7: The mean absolute differences between flexing and extending paths for each component of movement averaged over the flexion range as reported in Table 3.9.

	Quantity	Mean	Std Dev
Rotation	Ab/Adduction (deg)	0.15	0.08
	Int/External (deg)	0.53	0.47
Translation	Posterior/Anterior (mm)	0.48	0.30
	Medial/Lateral (mm)	0.34	0.15
	Proximal/Distal (mm)	0.21	0.18
Finite Helical Axis	α (deg)	1.51	1.31
	β (deg)	1.53	1.16
	Intersection, a_x (mm)	0.85	0.62
	Intersection, a_y (mm)	2.05	1.41

Table 4.8: The differences between the movement results from the repeated testing of one specimen. Mean differences in the joint rotations and translations are averaged over the flexion range (0-100°). Mean differences in the FHA parameters are averaged over the nine axes used to describe the motion. α and β are the orientations of the FHA in the frontal and transverse planes. a_x and a_y correspond to the coordinates of the intersection of the axis with the mid-sagittal plane.

and in the helical axis parameters for the two separate tests, shown in Table 4.8, demonstrate the repeatability of the experimental measurements. Small variations in the alignment of the knee in the rig and the placement of the sensor have little effect on the reported results.

4.3.4.2 Effect of Gravitational Load

Although gravity does affect the motion path slightly, it does not affect the coupling of joint translations and rotations to flexion. For the single specimen tested in seven different spatial configurations, the different motion paths are ‘unique’ in each orientation (Figure 4.15) but different between the orientations. They do, however, lie within the standard deviation curves for the twelve normal specimens. The results suggest that a truly unloaded joint would most likely move along a single unique track that lies in the vicinity of the mean of the seven individual tracks.

4.3.4.3 Effect of Speed of Movement

Variations of speed within the testing range had virtually no effect on the measured motion (Figure 4.16).

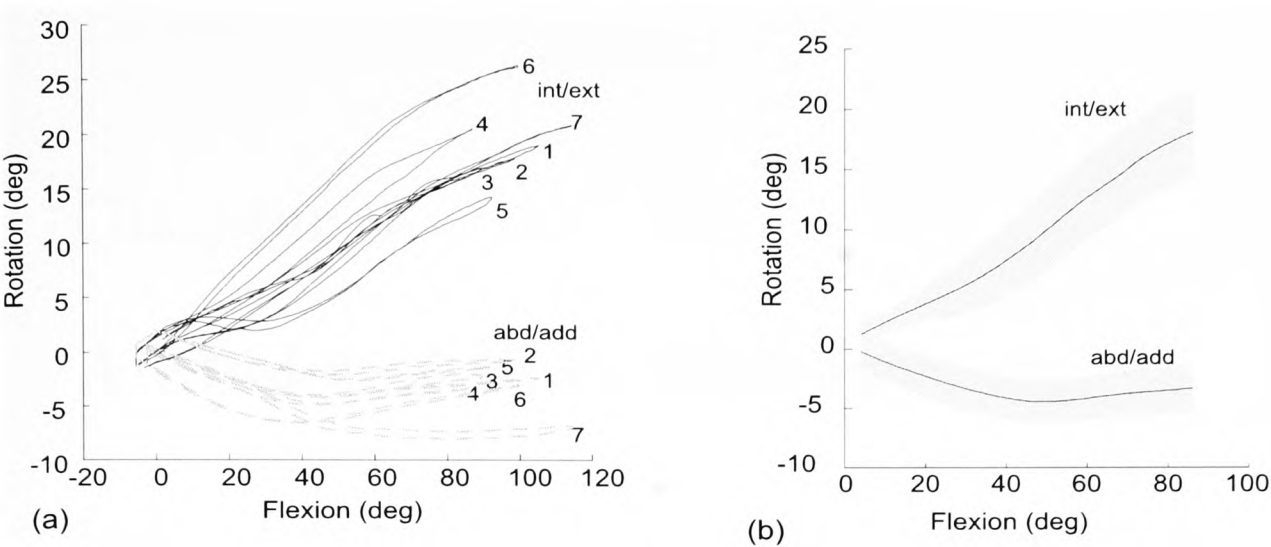


Figure 4.15: Passive joint rotations for specimen (Knee A) tested in seven different spatial orientations. (a) Individual tracks shown for each configuration listed in Table 2.2. (b) Mean and standard deviation of the curves in (a).

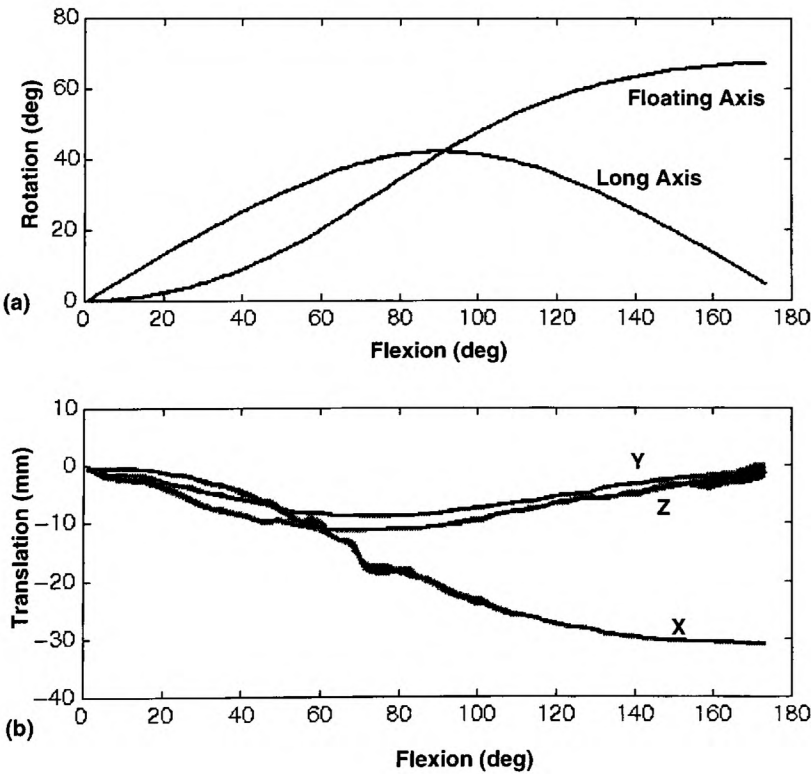


Figure 4.16: The effect of speed of movement. The mean and standard deviation curves for two JCS rotations (long axis and floating axis) and three components of translation (X , Y and Z) of a sensor rotating about a fixed axis with speeds of 4, 6, 8, 10, and 12 degrees of flexion per second. Standard deviations of the rotations were virtually zero.

Chapter 5

A Model of Knee Mobility

The need for an anatomical 3-D model of knee mobility was discussed in Chapter 1. Such a model must be capable of predicting the range and pattern of 3-D movement of the unloaded knee, given, as input, the spatial geometry of the relevant passive joint structures. It must predict the coupling of axial rotation to flexion, unaccounted for by 2-D models such as the four-bar linkage.

In this chapter, the development of such a model is described. First, the underlying model concept and assumptions are presented. The formulation of the model as a parallel spatial mechanism and the mechanism-based approach to solving the model knee displacement originally developed by Wilson *et al.* [136] are reviewed. The limitations of the original solution method are examined. The subsequent development and implementation of a more robust and computationally efficient solution method is described. Facilitated by the new geometric-numerical approach, formulations of both an idealised and a general surface model of knee mobility are put forth. Finally, the derivation of specimen-specific model parameters and the quantification of model knee motion is described.

5.1 Model Concept and Assumptions

In vitro experiments have shown that the knee follows a virtually unique path of movement in passive flexion and extension and that external forces are needed to deviate the joint from this preferred path (Chapter 3). This observation provides a basis for modelling the unloaded knee as a single degree-of-freedom (dof), 3-D mechanism or system.

The first step in formulating the model of knee mobility was therefore the specification of the anatomical structures which act as constraints in reducing the dof of the joint from six to one and, hence, act together to guide the bones along the observed movement path.

The passive structures at the knee are not rigid and, in isolation, can only provide one-sided constraints. The ligaments and capsule can only transmit tensile loads and are compliant in tension. The articular surfaces can only resist interpenetration but not separation and are compliant in compression. The unique geometric arrangement of the joint, however, is such that during passive motion the ligaments at the joint prevent the surfaces from separating and the surfaces keep some parts of the ligaments just taut. It was assumed that the unloaded knee can be flexed and extended without resistance, as suggested by the experiments described in Chapter 3, and hence, that deformation of the ligaments and articular surfaces during passive motion is negligible. Under these conditions the deformable one-sided constraints can then be modelled as rigid two-sided constraints. In the sections to follow assumptions made regarding the identification of the individual anatomical constraints and justification for these assumptions based on evidence in the literature and *in vitro* experiments will be described.

5.1.1 Ligaments

Within a ligament, each individual fibre experiences a different pattern of lengthening and shortening during passive motion. Some fibres remain slack throughout the motion while others are ‘just taut’, *i.e.* not slack or stretched, for part of the motion. For a fibre to represent a constraint to the motion it must remain ‘just taut’ throughout the range of motion. Such a fibre is *isometric* as its length does not change during the motion.

Based on evidence in the literature and the experimental results presented in Chapter 4 (Section 4.2), it was assumed that, in passive flexion, there is an isometric fascicle in each of the ACL, PCL, and MCL. Due to the inconsistency in the LCL results between current findings and evidence in the literature, the ligament was not assumed to act as a constraint to passive knee motion. The isometric fascicle in each of the ACL, PCL and MCL was defined as a straight line from its tibial attachment to its femoral attachment. It was assumed to be rigid and in the first instance, wrapping of the isometric fascicles around the bones was ignored.

5.1.2 Articular Contact

Bone-to-bone contact between the femur and tibia occurs in both the medial and lateral compartments of the joint. It was assumed that contact was maintained in both compartments throughout the range of flexion.

The deformation of the layers of articular cartilage covering the femoral and tibial condyles, under conditions of passive flexion, was considered negligible. The femoral and tibial condyles were considered rigid. As a result, a single point of

contact is specified in each compartment. With an associated coefficient of friction on the order of 0.001 [108], articular contact was assumed to be frictionless.

As a first approximation, the femoral condyles were assumed to be spherical and the medial and lateral tibial condyles were assumed to be planar. The mean error associated with fitting spheres to the femoral condyles and planes to the tibial condyles based on current measurements was found to be on the order of 1-2 mm (Section 3.1), providing satisfactory justification for the idealised articular surface representation.

5.1.3 Other Passive Structures

When the knee is passively flexed, the menisci slide freely in the anteroposterior direction, pulled along by the femoral condyles [64]. Observations made by Thompson *et al.* [125] based on a magnetic resonance imaging study of meniscal movement in cadavers support the assumption that the menisci offer no resistance to passive flexion.

Based on current experimental observation, the posterior portion of the joint capsule only tightens within 10-15° of full extension and therefore was considered incapable of constraining knee motion through the range of flexion. It only provides a constraint to hyperextension.

5.2 Original Model Formulation and Solution Method (Wilson)

This section describes the original formulation of the model as a parallel spatial mechanism developed by Wilson [135]. The mechanism-based approach originally used to solve the model knee kinematics and the predictions reported by Wilson based on a preliminary parameter set are reviewed. The limitations associated with the original solution method and the preliminary model parameter set are then examined and discussed.

5.2.1 A Parallel Spatial Mechanism

Acting as geometric constraints to knee motion, the isometric fascicles in the ACL, PCL and MCL and the contact in the medial and lateral compartments were represented by connections made up of standard kinematic pairs linking one rigid bone to the other. The problem was to find the movement path of the joint which satisfies these constraints.

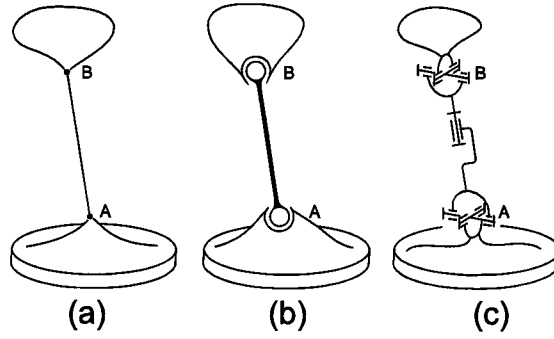


Figure 5.1: Kinematically equivalent mechanical connections: isometric ligament fibre (after Hunt [57]).

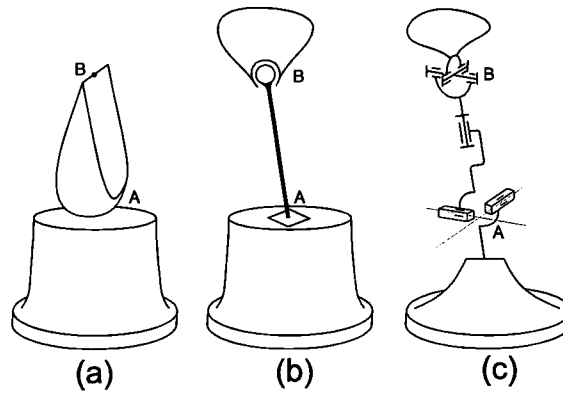


Figure 5.2: Kinematically equivalent mechanical connections: articular contact (after Hunt [57]).

Wilson represented each isometric fascicle by a solid rod connecting a spherical pair on the tibia to a spherical pair on the femur (Figure 5.1a,b). This constraint corresponds to the isometricity of the ligament fascicle.

Medial and lateral tibiofemoral contacts were each represented by a spherical pair at the centre of the femoral condyle connected by a rod to a planar pair at the point of contact on the tibial condyle (Figure 5.2a,b). This connection allows rolling and sliding at the contact point.

The resulting configuration of mechanically connected links is a parallel spatial mechanism (PSM) (Figure 5.3). It is so named because the output link, the femur in this case, is connected to the input link, the tibia, by a parallel arrangement of constraints in three dimensions.

5.2.2 Mechanism-Based Kinematic Analysis

In the first step of the original kinematic analysis, Wilson determined the mobility of the mechanism. The mobility of a spatial mechanism, M , with u linearly independent constraints is defined by Hunt as [57]:

$$M = 6 - u \quad (5.1)$$

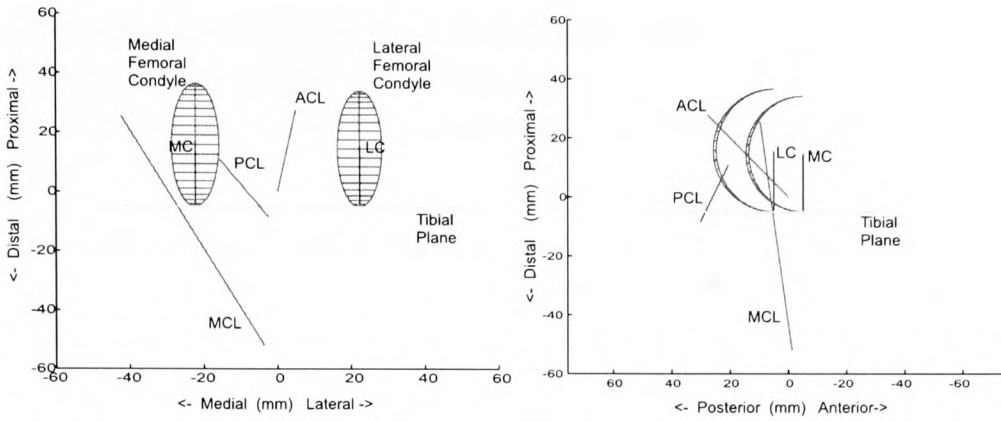


Figure 5.3: Representation of the model of knee mobility as a parallel spatial mechanism (preliminary parameters). Frontal (left) and sagittal (right) views. MC=Medial Contact Normal, LC=Lateral Contact Normal.

Because the three ligament connections and the two articular connections each represent one independent constraint, the PSM is left with a single dof. In a one dof mechanism, one input parameter is sufficient to specify the location of the output link and all connecting links. In the model, the position of the femur relative to the tibia can therefore be specified fully by one quantity alone, such as the flexion angle.

The path of the femur relative to the tibia as a function of the flexion angle was then found by solving the forward kinematics of the mechanism from a starting position in full extension. Because no closed-form solution to the forward kinematics of the PSM had been found [70], an iterative method developed by Uicker *et al.* [129] was originally used:

1. The spherical and planar pairs making up the ligament and articular connections were replaced by kinematically equivalent chains of one dof revolute or prismatic pairs (Figures 5.1c and 5.2c).
2. Each one dof pair was assigned a coordinate system using an established convention (Uicker *et al.* [129]). Four pair quantities, a_i , s_i , θ_i and α_i define the transformation from the coordinate system in one pair to the coordinate system in an adjacent pair. These four quantities were calculated for each of the 25 one dof pairs using simple geometry, yielding 100 pair quantities describing the starting position of the mechanism. In a revolute pair, only θ_i changes as the pair rotates, and in a prismatic pair, only s_i changes as the pair slides. Thus, as the mechanism moves, 75 pair parameters remain constant and 25 pair variables change.
3. The transformation across every pair was written as a 4×4 transformation matrix, $A_{i,j}$, whose elements are functions of a_i , s_i , θ_i and α_i [129].

4. Four independent closed loops were defined, each starting and ending at the tibial insertion of the ACL: ACL-PCL, ACL-MCL, ACL-Medial Contact Normal, ACL-Lateral Contact Normal, each loop yielding six independent equations.
5. The mediolaterally oriented revolute pair fixed to the tibia in the ACL connection was chosen as the input and named pair 1. A small change in the input angle θ_1 was applied to move the mechanism from the starting position. Values of the 24 other pair variables were determined using the iterative computations described by Uicker *et al.* [129].
6. At every increment of the input angle, the transformation matrix between a coordinate system fixed in the femur and a coordinate system in the tibia was calculated.

The model knee motion was defined with respect to a moving tibia and a fixed femur, as was done for the experiments. The joint coordinate system (JCS) described in Chapter 2 (Section 2.8) was used to calculate flexion about a mediolateral axis fixed in the femur passing through both femoral sphere centres, internal/external rotation about a longitudinal axis in the tibia and ab/adduction about a third floating axis defined as the cross-product of the other two axes. Translation of the tibial insertion of the ACL link was described with respect to a set of axes fixed in the femur parallel to the JCS axes at full extension ($\hat{\mathbf{t}}_f, \hat{\mathbf{f}}_f, \hat{\mathbf{l}}_f$).

5.2.3 Preliminary Parameter Derivation

A preliminary set of parameters derived from studies reported in the literature was originally used by Wilson [135]. The anterior fascicle of the ACL, the posterior fascicle of the PCL and the anterior fascicle of the MCL measured in a single knee by Blankevoort *et al.* [17] were taken to be the isometric fascicles. The coordinates of the femoral and tibial attachments at full extension were derived from the published data. The tibial condyles were represented by a transverse plane passing through the tibial attachment of the isometric fascicle of the ACL. The radii of the spherical femoral condyles and the distance separating their centres were taken from the mean values measured by Kurosawa *et al.* [67]. At full extension the medial contact point was located 5 mm posterior to the ACL tibial attachment and the lateral contact point 5 mm anterior.

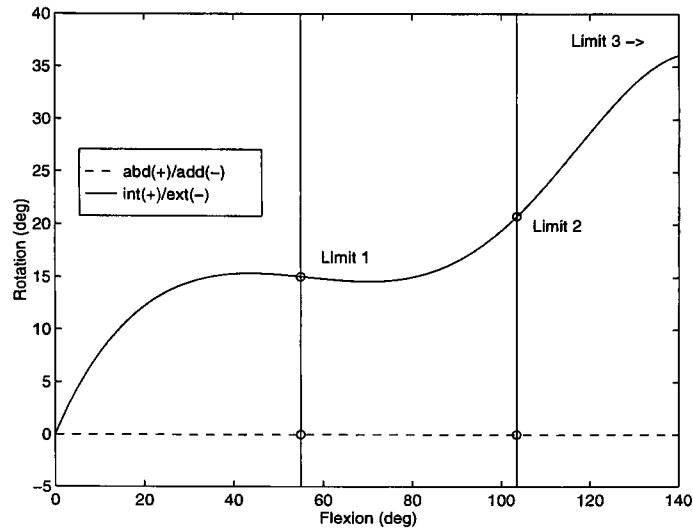


Figure 5.4: Model knee rotations based on preliminary parameter set. Motion up to Limit 1 originally predicted by Wilson [135] using a mechanism-based solution procedure. Extended range of motion (Limit 2) predicted using an alternate definition of lower pair orientations to avoid gimbal-lock. Motion beyond 140° (Limit 3) flexion predicted by new geometric-numerical approach.

5.2.4 Model Predictions

The mechanism model defined by the preliminary parameter set predicted a range of motion from -18° hyperextension to 54° flexion [135] (Figure 5.4, Limit 1), considerably smaller than the 100° or more of passive flexion observed in the natural knee. At the extremes of the range, the solution no longer converged. In the absence of theorems which define the occurrence of locking of parallel spatial mechanisms, Wilson originally assumed that the femur had reached its physical limit of movement due to the geometric configuration of the ligament and articular constraints. Any further movement, it was thought, would result in the stretching of the isometric fascicles or the separation or interpenetration of the articular surfaces.

Within the limited range of motion, the model successfully predicted the coupling of internal rotation to flexion with a peak internal rotation of 15° . Zero ab/adduction was predicted as the tibial condyles were represented by a single transverse plane.

5.2.5 Limitations of the Original Approach

5.2.5.1 Underestimation of Range of Motion

To determine whether the limited range of motion predicted by the original mechanism-based approach was a manifestation of the violation of the conditions of ligament isometricity and articular contact or simply a result of the chosen mathematical representation or solution method, explanations for the ‘locking’ of the mechanism were examined as part of the current study.

The displacement solution no longer converges when the mechanism is at or nearing a *stationary* configuration. A *stationary* configuration is one at which one of the pair freedoms of the mechanism becomes transitorily inactive. For a mechanism made up of n links and m pairs where each pair i possesses f_i dof, the expanded form of Grubler's criterion for the mobility, M , of the mechanism [57]:

$$M = 6(n - m - 1) + \sum_{i=1}^m f_i \quad (5.2)$$

can also be used to verify that, when every pair freedom is active, the mobility of the mechanism made up of the equivalent chains of one dof pairs is indeed one ($n=22$, $m=25$ and $\sum_i^m f_i = 25$). It is also clear based on Equation 5.2, how, if any one of the 25 lower pairs becomes transitorily inactive, the loss of that single pair freedom reduces the mobility to zero. The mechanism 'locks' and the solution no longer converges.

The freedom at a lower pair is transitorily lost when either the special alignment of a set of lower pairs renders a single pair inactive or when the excursion about or along the pair reaches its limit before the reversal of the direction of motion. Both types of stationary configurations, discussed below, provide explanations for the premature 'locking' of the original mechanism and resulting underestimation of the flexion range.

'Gimbal-lock' The chains of one dof pairs representing the ligament and articular connections proved prone to 'gimbal-lock'. Upon examining the positions and orientations of the 25 individual lower pairs during the propagation of the original mechanism, it was found that at the limiting angle of 54° , the three revolute pair axes at the femoral attachment of the PCL link, together representing a spherical pair, approached a coplanar or 'gimbal lock' configuration in which one of the lower pairs was rendered inactive. The loss of the single degree of freedom in the PCL connection caused the overall mobility to fall to zero and the mechanism to 'lock'. By redefining the lower pair orientations at the starting position of the mechanism, however, it was possible to avoid 'gimbal lock', enabling the model to predict up to 103° flexion (Figure 5.4, Limit 2). At this locking point, no special configurations in the alignment of the lower pairs were noted.

Choice of input pair Driving the flexion motion by incrementing displacement about an input pair can also lead to an underestimation of the range of motion of the mechanism. A mediolaterally oriented revolute pair at the ACL tibial attachment was originally chosen as the input pair. The point at which the ACL connection reaches its maximal excursion about this axis constitutes a stationary configuration as the pair transitorily loses its freedom, causing the mechanism to lock. Although

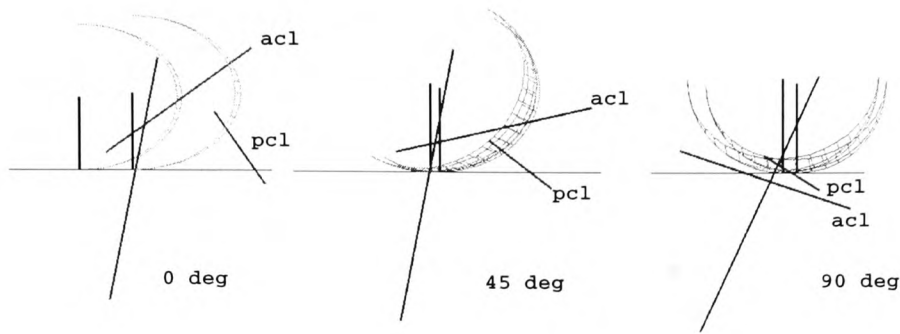


Figure 5.5: The preliminary model at 0°, 45° and 90° flexion in the sagittal plane view

no further movement can occur with associated rotation about the input pair, further knee flexion may still be possible. This is demonstrated to be the case using a new geometrical-numerical approach (Section 5.3). In other words, the use of a driving pair assumes that the input displacement increases monotonically with increasing flexion; the range of flexion is underestimated when this assumption is violated.

5.2.5.2 Limitations of Articular Surface Representation

The mechanism-based formulation of the model is limited in that it is only capable of representing idealised articular surface shapes. Only articulations between spheres and planes can be represented by connections made up of standard kinematic pairs. A sphere-on-sphere articulation, for example, can be modelled as a rod connecting a spherical pair at the centre of one sphere to a spherical pair at the centre of the second sphere. A plane-on-plane articulation can be modelled simply by a planar pair. To incorporate more physiological articular surface shapes into the model, however, and to study the effects of different surface replacement designs on the kinematics and mechanics of the joint, an approach capable of handling more complex surfaces was required.

5.2.5.3 Unphysiological Motion - Preliminary Parameter Set

The set of preliminary model parameters lead to the prediction of unphysiological ligament orientations beyond the initially predicted 54° flexion. The locations of the ligament links at full extension (0°), 45°, and 90° flexion are shown in Figure 5.5. By 90° flexion, the ACL has rotated through the tibial plateau. A more physiological set of parameters was therefore required.

5.3 A Geometric-Numerical Model Formulation and Solution Method¹

In the previous section it was shown that the original mechanism-based approach to solving for the model knee kinematics leads to an underestimation of the range of motion. Furthermore, the need for a model formulation and solution algorithm capable of handling more complex surface representations was recognised. A purely numerical model formulation and solution method was therefore developed to solve for the path of joint motion which satisfies the given ligament and contact constraints. By expressing the model ligament and surface constraints as functions of the knee position, the resulting system of equations can then be solved for a given flexion angle to determine the overall position of the joint. In this section, the new approach is described, first, in its application to the idealised sphere-on-plane surface model. The approach is then extended to the formulation and solution method of a general articular surface model are then described.

5.3.1 Idealised Surface Model Formulation

5.3.1.1 Description of Knee Position

Reference coordinate frames were defined in the tibia and femur (Figure 5.6). In a left knee, the tibial X axis was oriented posteriorly, the Y axis medially and the Z axis proximally, aligned with its longitudinal axis. The origin of the tibial frame was located at the tibial attachment of the isometric ACL fibre. The femoral X axis was oriented laterally and was defined to lie along the line joining the medial and lateral sphere centres. The femoral Y axis was directed posteriorly and the Z axis proximally. The femoral frame origin was located at the centre of the medial femoral sphere. The tibial and femoral axes were not coincident nor were they necessarily aligned in the starting position (full extension).

The location of the femur relative to the tibia can be fully described by six quantities: the three position coordinates, $\mathbf{s} = (s_x, s_y, s_z)$, describing the position of the femoral frame origin relative to the tibial frame; and three Euler angles, $\theta_1, \theta_2, \theta_3$, describing rotations about the femoral Z , Y and X axes, respectively. The rotation θ_3 of the femur about the mediolateral axis X_f passing through the femoral sphere centres is therefore equivalent to the flexion angle as defined in previous modelling work and in reporting observed experimental motion (Section 2.8). The rotation matrix $\mathbf{R}_f^t$ describes the orientation of the femoral frame relative to the

¹The material described in this section has been submitted as a Technical Note to the Journal of Biomechanics and is currently under review, February 1999.

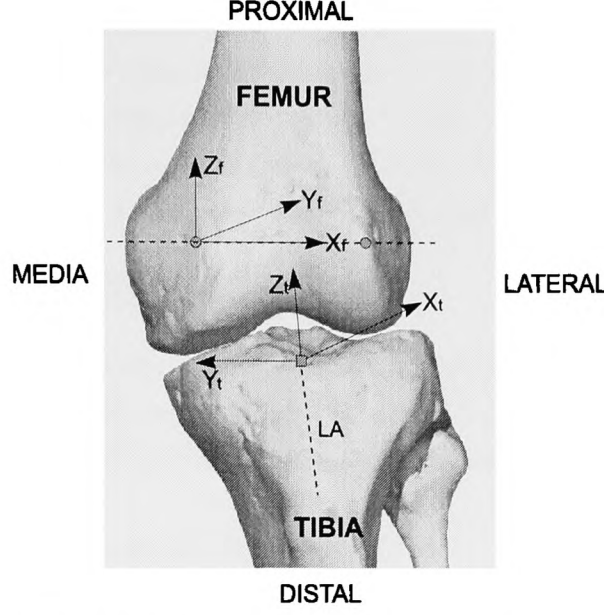


Figure 5.6: Model reference coordinate frames defined in the tibia and femur (left knee). The tibial Z axis (Z_t) lies parallel to the tibial longitudinal axis (LA). The tibial origin ($\square$) lies at the tibial insertion of the model ACL isometric fibre. The femoral X axis (X_f) lies along the line joining the medial and lateral fitted sphere centres ($\circ$).

tibial frame:

$$\mathbf{R}_f^t = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_3 & -\sin \theta_3 \\ 0 & \sin \theta_3 & \cos \theta_3 \end{bmatrix} \quad (5.3)$$

The transformation matrix $\mathbf{T}_f^t$ incorporates the rotation ($\mathbf{R} = \mathbf{R}_f^t$) and position ($\mathbf{v} = \mathbf{s}$) information.

5.3.1.2 Constraint Equations

If completely unconnected, the bones have six dof relative to each other. By constraining the bones to move without violating the conditions of ligament isometricity and continuity of contact, the number of dof are reduced to one. The three equations governing the conditions of ligament isometry for the ACL ($i = 1$), PCL ($i = 2$) and MCL ($i = 3$) are:

$$\| \mathbf{T}_f^t \mathbf{b}_i^f - \mathbf{a}_i^t \| - L_i = 0 \quad i = 1, 2, 3 \quad (5.4)$$

where the vector $\mathbf{b}_i^f$ represents the coordinates of the femoral attachment site of a ligament fascicle with respect to the femoral frame, the vector $\mathbf{a}_i^t$ the position of the corresponding tibial attachment site described in the tibial frame, and L_i the length of the fascicle measured at full extension.

Continuity of contact is ensured first, when the contact points on the two articulating surfaces are coincident, and second, when the outward normal vectors of the

two surfaces at the point of contact are opposite in direction. In the case of a sphere-on-plane articulation, these two conditions may be expressed by a single equation stating that the shortest distance from the tibial plane to the corresponding femoral sphere centre is equal to the sphere radius. Hence, the conditions of continuity of contact in the medial ($i = 1$) and lateral ($i = 2$) compartments are expressed as:

$$(\mathbf{T}_f^t \mathbf{c}_i^f - \mathbf{d}_i^t) \cdot \hat{\mathbf{n}}_i^t - R_i = 0 \quad i = 1, 2 \quad (5.5)$$

where the vector $\mathbf{c}_i^f$ represents the coordinates of the femoral sphere centre in the femoral frame, the vector $\mathbf{d}_i^t$ any point on the corresponding tibial plane, the vector $\hat{\mathbf{n}}_i^t$ the unit outward normal to the plane, and R_i the radius of the femoral sphere.

5.3.1.3 Determination of Model Kinematics

Equations 5.4 and 5.5 provide five independent equations in six unknowns, appropriate to a mechanism with a single dof. For a specified flexion angle, θ_3 , the five other variables, s_x , s_y , s_z , θ_1 and θ_2 , can be determined by solving simultaneously the system of five non-linear equations. Together, the six variables describe the position of the femur relative to the tibia. By incrementing θ_3 from 0° (full extension) in steps of 1° , the full range and pattern of movement of the femur relative to the tibia in the unloaded knee can be determined.

A non-linear least-squares root-finding algorithm, Matlab's 'fsolve' (Appendix C), was used to find the values of the five variables (s_x , s_y , s_z , θ_1 and θ_2) which satisfied the five non-linear equations at each flexion step. For the first step of movement, an initial guess for the five variables was provided by the position of the femur relative to the tibia at the starting position. In subsequent steps, the initial guess was supplied by the solution for the previous flexion angle. The maximum range of movement was defined by the flexion angle at which the solution no longer converged to a root.

5.3.2 General Surface Model Formulation

The geometric-numerical approach can be used to examine the kinematics of a system governed by any number of constraint equations involving an equal number of unknowns. The method therefore allows the formulation of a model of knee mobility incorporating more general surface descriptions of the tibial and femoral condyles.

5.3.2.1 Constraint Equations

The three equations governing the conditions of ligament isometricity are the same as those described for the idealised surface model (Equation 5.4).

The equations governing the conditions of contact must now be formulated in a more general way. Constraint equations enforcing coincidence of contact in the medial ($i = 1$) and lateral ($i = 2$) compartments are expressed as:

$$\mathbf{T}_f^t \mathbf{c}_i^f - \mathbf{d}_i^t = 0 \quad i = 1, 2 \quad (5.6)$$

where $\mathbf{c}_i^f$ and $\mathbf{d}_i^t$ represent the coordinates of the contact points on the femoral and tibial surfaces respectively, defined with respect to the corresponding femoral or tibial frame. The coincidence of contact condition provides a total of six equations for the medial and lateral compartments.

The second contact condition requires that the femoral and tibial outward surface normals at each contact point be opposite in direction. This provides four additional equations:

$$\mathbf{R}_f^t \hat{\mathbf{n}}_i^f + \hat{\mathbf{n}}_i^t = 0 \quad i = 1, 2 \quad (5.7)$$

where $\hat{\mathbf{n}}_i^f$ and $\hat{\mathbf{n}}_i^t$ represent the unit outward normals of the femoral and tibial surfaces respectively. Again these are defined relative to the corresponding femoral or tibial frame. Only the rotation portion of the transformation matrix $\mathbf{T}_f^t$ was required to describe the direction of the femoral unit surface normal in the tibial frame.

5.3.2.2 Additional System Unknowns

In this general formulation further unknowns have been introduced. The positions of the contact points on the femoral and tibial surfaces, $\mathbf{c}_i^f$ and $\mathbf{d}_i^t$, and the corresponding unit surface normals, $\hat{\mathbf{n}}_i^f$ and $\hat{\mathbf{n}}_i^t$, may not be deduced directly from the position variables as in the case of the sphere-on-plane articulations. The only unknowns associated with each contact, however, are the (x, y) coordinates of the tibial contact point and the (ψ, ϕ) coordinates of the femoral contact point. Given the surface descriptions, the (x, y) coordinates define fully $\mathbf{d}_i^t$ and $\hat{\mathbf{n}}_i^t$ and the (ψ, ϕ) coordinates define fully $\mathbf{c}_i^f$ and $\hat{\mathbf{n}}_i^f$.

Four unknowns, (x, y) and (ψ, ϕ) , have been introduced in each compartment (medial and lateral), increasing the total number of unknowns from six to fourteen.

5.3.2.3 Determination of Model Kinematics

This increase in the number of unknowns is accompanied by an increase in the number of constraint equations. In this case, the model displacement is governed by 13 independent equations in 14 unknowns, again appropriate to a single dof system. By fixing the flexion angle, θ_3 , the remaining thirteen variables can be determined by solving simultaneously the system of thirteen non-linear equations. The six variables ($s_x, s_y, s_z, \theta_1, \theta_2$ and θ_3) once again define fully the position of

the femur relative to the tibia. The remaining variables (the (x, y) and (ψ, ϕ) in the medial and lateral compartments) specify the location of the contact points on the two bones. As before, by incrementing θ_3 from 0° (full extension) in steps of 1° , the full range and pattern of passive movement of the femur relative to the tibia may be determined.

As in the case of the idealised surface model, a non-linear least-squares root-finding algorithm (Matlab's 'fsolve', Appendix C) was used to find the values of the 13 variables which satisfied the 13 constraint equations at each flexion step. For the first step of movement, an initial guess for the 13 variables was provided by the position of the femur relative to the tibia at the starting position and the geometry of the surfaces (described in more detail in Section 5.4.4). In subsequent steps, the initial guess was supplied by the solution for the previous flexion angle. The maximum range of movement was again defined by the flexion angle at which the solution no longer converged to a root.

5.3.2.4 Additional Geometric Constraints

Surface Interpenetration In the case of idealised surface model, the nature of the sphere-on-plane articulation is such that the contact conditions are met at only one point; the surfaces do not interpenetrate at any other point.

In the case of the general surface description, satisfying the contact conditions does not necessarily guarantee that the surfaces do not interpenetrate at any other point within the region of interest. In the first instance, a verification was performed after the determination of the model knee kinematics. Contact error was evaluated as described in Section 2.11.4 every 10 degrees of predicted flexion. Negative errors indicated that interpenetration of surfaces within the measured boundaries of the condyles occurred and the solution to the model kinematics was considered invalid.

5.3.3 Combination Surface Model Formulation

The geometric-numerical approach can be applied to any combination of idealised and general surface models. For example, a model consisting of a sphere-on-plane articulation in the medial compartment of the joint and a general surface articulation in the lateral compartment reduces to a system of ten unknowns: six variables describing the position of the femur relative to the tibia, $(s_x, s_y, s_z, \theta_1, \theta_2, \theta_3)$ and four describing the tibial and femoral contact points in the lateral compartment: (x, y) and (ψ, ϕ) . Nine equations can be written to describe the mobility of the system: three corresponding to ligament isometricity and six to the conditions of contact.

5.4 Specimen-Specific Parameter Derivation

The prediction of unphysiological motion with a set of preliminary parameters based on the literature was described in Section 5.2.5.3. An improved derivation of specimen-specific ligament and surface parameters based on the measurements of cadaver knees was developed.

5.4.1 Global to Model Coordinate Systems

The coordinates of points measured on the articular surfaces and ligament attachment areas in the intact joint were described relative to the global coordinate system of the experiment $\{g\}$. The transformation matrices describing the location of the femoral and tibial coordinate systems of the model (Section 5.3.1.1) relative to the global experimental frame in the fully extended knee were thus required. The location of the tibial frame relative to the global frame was defined by:

$$\mathbf{T}_t^g = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ - & - & - & - \\ \mathbf{v}_{torg} & & \mathbf{R}_t^g & \end{array} \right] \quad (5.8)$$

where $\mathbf{v}_{torg}$ describes the tibial attachment of the isometric ACL fibre in global coordinates and the three columns of $\mathbf{R}_t^g$ are defined by the experimental axes unit vectors $\hat{\mathbf{t}}_t$, $-\hat{\mathbf{f}}_t$ and $\hat{\mathbf{l}}_t$ for a right knee and $-\hat{\mathbf{t}}_t$, $-\hat{\mathbf{f}}_t$ and $-\hat{\mathbf{l}}_t$ for a left knee.

If in the starting position of the model, *i.e.* the fully extended knee, the transformation matrix $\mathbf{T}_f^t$ describes the location of the model's femoral frame relative to its tibial frame, the transformation matrix describing the location of the model's femoral frame relative to the global frame in the fully extended knee is given by:

$$\mathbf{T}_f^g = \mathbf{T}_t^g \mathbf{T}_f^t \quad (5.9)$$

5.4.2 Surface Parameters

In Section 3.1, geometrical descriptions of the articular surfaces of the measured cadaver knees were presented. The surface representations consisted of different least-squares surface fits to the 3-D coordinate data measured on the condylar surfaces. Model parameters were similarly derived. Before fitting the surfaces, however, the measured points on the tibial and femoral condyles were described relative to the tibial or femoral frame of the model, using $(\mathbf{T}_t^g)^{-1}$ and $(\mathbf{T}_f^g)^{-1}$ respectively.

5.4.2.1 Tibial Condyles

The tibial condyles were described either by best-fit planes (‘free’ or ‘transverse’) or polynomial surfaces of degree k and m in x and y respectively. The planar

and polynomial surface functions were both described with respect to the global coordinate system $\{t\}$.

If the fitted plane was given by Equation 2.8, the unit normal to the plane, $\hat{\mathbf{n}}^t$, was defined by normalising the vector: $\mathbf{n} = [a \ b \ c]^T$.

If $\mathcal{S} = f(x, y) - z$ defined the fitted polynomial surface, the outward surface normal at a point (x, y) was then defined as:

$$\mathbf{n}(x, y) = \nabla \mathcal{S} = \frac{\partial \mathcal{S}}{\partial x} \hat{\mathbf{i}} + \frac{\partial \mathcal{S}}{\partial y} \hat{\mathbf{j}} + \frac{\partial \mathcal{S}}{\partial z} \hat{\mathbf{k}} \quad (5.10)$$

Hence, the components of $\mathbf{n}(x, y)$ are simply: $\left[\frac{\partial f}{\partial x} \ \frac{\partial f}{\partial y} \ -1 \right]$ where the partial derivatives are evaluated at (x, y) . This vector was then normalised to provide the unit outward surface normal, $\hat{\mathbf{n}}^t$.

5.4.2.2 Femoral Condyles

The femoral condyles were represented by fitted spheres ('whole' or 'posterior') or polynomial surfaces (functions of the spherical coordinates ψ and ϕ).

For the description of polynomial surfaces, additional local femoral frames, $\{f_1\}$ and $\{f_2\}$, were established such that their axes were parallel to those of the model tibial frame in full extension and their origins were located at the centres of the medial and lateral fitted 'whole' spheres respectively. The transformation matrices $\mathbf{T}_{f_1}^f$ and $\mathbf{T}_{f_2}^f$ were thereby defined. The measured surface points described in the model femoral frame, $\{f\}$, were then described with respect to the corresponding local femoral frame, $\{f_i\}$, using the appropriate $\mathbf{T}_{f_i}^f$ transformation and the cartesian coordinates were converted to spherical coordinates using Equations 2.5.

If $\mathcal{S} = f(\psi, \phi) - r$ defined the fitted polynomial surface, the components of the outward surface normal in cartesian coordinates relative to the local femoral frame, $\mathbf{n}^{f_i}$, were determined from:

$$\begin{aligned} n_x &= \frac{\partial \mathcal{S}}{\partial x} = \frac{\partial \mathcal{S}}{\partial \psi} \cdot \frac{\partial \psi}{\partial x} + \frac{\partial \mathcal{S}}{\partial \phi} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial \mathcal{S}}{\partial r} \cdot \frac{\partial r}{\partial x} \\ n_y &= \frac{\partial \mathcal{S}}{\partial y} = \frac{\partial \mathcal{S}}{\partial \psi} \cdot \frac{\partial \psi}{\partial y} + \frac{\partial \mathcal{S}}{\partial \phi} \cdot \frac{\partial \phi}{\partial y} + \frac{\partial \mathcal{S}}{\partial r} \cdot \frac{\partial r}{\partial y} \\ n_z &= \frac{\partial \mathcal{S}}{\partial z} = \frac{\partial \mathcal{S}}{\partial \psi} \cdot \frac{\partial \psi}{\partial z} + \frac{\partial \mathcal{S}}{\partial \phi} \cdot \frac{\partial \phi}{\partial z} + \frac{\partial \mathcal{S}}{\partial r} \cdot \frac{\partial r}{\partial z} \end{aligned} \quad (5.11)$$

and the relationships between the spherical (ψ, ϕ, r) and cartesian (x, y, z) coordinates (Equations 2.5). In the femoral frame $\{f\}$, the unit normal was then described by:

$$\hat{\mathbf{n}}^f = \mathbf{R}_{f_i}^f \hat{\mathbf{n}}^{f_i} \quad (5.12)$$

where $\mathbf{R}_{f_i}^f$ is the rotation component of $\mathbf{T}_{f_i}^f$ and $\hat{\mathbf{n}}^{f_i}$ is the normalised vector $\mathbf{n}^{f_i}$.

5.4.3 Ligament Parameters

In Section 4.2, the isometry of fibres in the knee ligaments during passive motion was examined. The model isometric fibres in the ACL, PCL and MCL were defined as either the most nearly isometric ‘combination’ fibres, or, where additional ligament geometry data was available, the most nearly isometric ‘physiological’ fibres. Because the attachment sites were originally defined in global coordinates, $\mathbf{p}^g$, the coordinates were transformed to the model tibial and femoral frames using $(\mathbf{T}_t^g)^{-1}$ and $(\mathbf{T}_f^g)^{-1}$ respectively.

5.4.4 Contact Error Correction

As described in Section 4.3.3, the error associated with reconstruction of the positions of the bones in the intact knee from the digitised data in the disarticulated joint, results in ‘contact error’, referring to a measured separation or interpenetration of the femoral and tibial surfaces.

Because the model surfaces are based on the experimental measurements, they exhibit a similar degree of contact error in the starting position (full extension). If one attempts to solve the governing equations without correcting for the separation or interpenetration of the surfaces, the first flexion step will reflect the change in femoral position which, not only must bring the tibial and femoral surfaces into together, but must also satisfy the contact conditions and ligament isometry. The first step of motion would then involve a pronounced shift in the orientation and position of the femur relative to the tibia.

To avoid such discontinuity in the predicted motion, the contact error was corrected by manipulation of the specimen-specific parameters so that the tibial and femoral surfaces were brought into contact in the starting position (full extension).

5.4.4.1 Idealised Surface Model

In the case of the spheres-on-planes model, the femoral and tibial surfaces were brought into contact at the starting position by extending or decreasing the radius of each femoral sphere until contact was made with the corresponding tibial plane. The position of the femur relative to the tibia and hence, all other parameters, remained unchanged in the starting position. It must be noted that extending or decreasing the sphere radius is kinematically equivalent to raising or lowering the height of the tibial plane to meet the spherical surface (in a direction perpendicular to the plane).

5.4.4.2 General Surface Model

In the case of polynomial surface articulations, the contact error problem required a more sophisticated correction of the specimen-specific model parameters.

The location of the model femur at the starting position (full extension) was first optimised so that the contact conditions were best satisfied. The problem involved 10 objective functions to be minimised (Equations 5.6 and 5.7) and the 13 variables describing the location of the femur and the tibial and femoral contact points (Section 5.3.2.3). The optimal solution for the 13 variables was found using Matlab's multi-objective optimisation algorithm, 'attgoal' (Appendix C). The ten objective functions were weighted equally. An initial guess for the first six variables, x , y , z , θ_1 , θ_2 and θ_3 , was provided by the original measured position of the femur in the starting position (full extension). An initial guess for the tibial and femoral contact points was given by the experimental estimates based on the tibial and femoral surface geometry and the measured knee position at 0° flexion (Section 4.1).

The measured motion was then described relative to this new starting position. The experimental analysis of movement and the search for the most nearly isometric fibres was performed again based on this modified description of the motion.

5.5 Description of Model Knee Motion

Within the predicted range, model knee motion was described in a manner similar to that used to describe the experimentally measured movement (Section 2.8).

5.5.1 Joint Position

For each step of flexion θ_3 , the model solves for the five remaining variables: x , y , z , θ_1 and θ_2 , which together describe the location of the femoral frame with respect to the tibial frame expressed by the transformation matrix $\mathbf{T}_f^t$. If at full extension the transformation matrix is described by $\mathbf{T}_{fo}^t$ and at a flexion angle i by $\mathbf{T}_{fi}^t$, then the operator which describes the displacement of femur (and points on the femur) with respect to the tibial frame is:

$$\mathbf{T}_{0i} = \mathbf{T}_{fi}^t (\mathbf{T}_{fo}^t)^{-1} \quad (5.13)$$

5.5.2 Tibial and Femoral Body-Fixed Axes

Body-fixed axes, $\hat{\mathbf{f}}_f, \hat{\mathbf{l}}_f, \hat{\mathbf{t}}_f$ in the femur, and $\hat{\mathbf{f}}_t, \hat{\mathbf{l}}_t, \hat{\mathbf{t}}_t$ in the tibia, were defined as described in Section 2.8.2 in the experimental analysis of joint motion. The unit vectors were now, however, defined relative to the model tibial frame. The model

tibial axes, X_t , Y_t and Z_t , correspond exactly to the tibial body-fixed axes of experiment. The model femoral axes, however, are defined differently.

5.5.3 Joint Rotations and Translations

Knee orientation was then quantified by three rotations about the axes of a Joint Coordinate System (JCS) [29, 49] as was done in the analysis of experimental motion (Section 2.8.3). The movement of the tibia was again described relative to a fixed femur.

The translation of a reference point on the tibia from its position in the fully extended knee with respect to a fixed femur was also determined as in the experimental analysis (Section 2.8.4). The same reference point tracked during the experimental motion was chosen, *i.e.* the most posterior point measured on the ACL tibial attachment.

5.5.4 Instantaneous Helical Axes²

The relative motion of the model bones was also quantified by a series of instantaneous helical axes (IHA). The IHA was calculated at each flexion step using the principle of reciprocity. Each ligament and contact constraint i was represented as a wrench. Because the constraints are capable of transmitting force but no moment, the pitch of each wrench is zero. The screw coordinates are in this case equivalent to the line coordinates $(\mathcal{L}_i \mathcal{M}_i \mathcal{N}_i; \mathcal{P}_i \mathcal{Q}_i \mathcal{R}_i)$. Provided these five wrenches are linearly independent, the instantaneous motion of the femur is described by the unique screw which is reciprocal to these five wrenches.

The Plücker line coordinates of any line can be determined from the positions of two points on the line (Equations B.2 and B.3). For each ligament constraint, the line coordinates were derived from the positions of the tibial and femoral attachments described in the model tibial frame. For each contact constraint, in the case of the idealised surface model, the line coordinates were derived from the position of the femoral sphere centre and a point lying a distance from the centre along the normal to the plane described in the tibial frame. In the case of the general surface model, the line coordinates of each contact wrench were found from the position of the contact point on the tibia and a point lying a distance along the outward surface normal at that point, both described in the tibial frame.

The screw which is reciprocal to the five wrenches was determined by solving five equations provided by the condition of reciprocity and an additional equation

²Theoretical background to this section is described in Appendix B.2.

$\mathcal{P}^* = 1$ to assign an arbitrary amplitude to the screw:

$$\begin{bmatrix} \mathcal{L}_1 & \mathcal{M}_1 & \mathcal{N}_1 & \mathcal{P}_1 & \mathcal{Q}_1 & \mathcal{R}_1 \\ \mathcal{L}_2 & \mathcal{M}_2 & \mathcal{N}_2 & \mathcal{P}_2 & \mathcal{Q}_2 & \mathcal{R}_2 \\ \mathcal{L}_3 & \mathcal{M}_3 & \mathcal{N}_3 & \mathcal{P}_3 & \mathcal{Q}_3 & \mathcal{R}_3 \\ \mathcal{L}_4 & \mathcal{M}_4 & \mathcal{N}_4 & \mathcal{P}_4 & \mathcal{Q}_4 & \mathcal{R}_4 \\ \mathcal{L}_5 & \mathcal{M}_5 & \mathcal{N}_5 & \mathcal{P}_5 & \mathcal{Q}_5 & \mathcal{R}_5 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathcal{P}^* \\ \mathcal{Q}^* \\ \mathcal{R}^* \\ \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (5.14)$$

The screw coordinates were normalised and the pitch of the screw calculated (Equations B.7 and B.8). The Plücker line coordinates of the screw (Equation B.6) and two points on the screw axis (Equation B.5), $\mathbf{s}_1^t$ and $\mathbf{s}_2^t$, were then determined to describe the location of the IHA relative to the tibial coordinate system. The points on the IHA were also described in the femoral frame at each step of motion by: $\mathbf{s}_i^f = \mathbf{T}_t^f \mathbf{s}_i^t$. The series of IHAs described relative to the fixed tibial frame define the tibial or fixed axode while their locations relative to the moving femoral frame define the femoral or moving axode.

5.5.5 Contact Point Movement

In the case of the idealised surface model, at each motion step, the contact point on the tibia in compartment i was calculated as:

$$\mathbf{p}_i^t = \mathbf{T}_f^t \mathbf{c}_i^f - R_i \hat{\mathbf{n}}_i^t \quad (5.15)$$

where, once again, $\mathbf{c}_i^f$ represents the coordinates of the femoral sphere centre in the femoral frame, the vector $\hat{\mathbf{n}}_i^t$ the unit outward normal to the tibial plane, and R_i the radius of the femoral sphere. The contact point on the femur described in the femoral frame at the same motion step was given by:

$$\mathbf{p}_i^f = \mathbf{T}_t^f \mathbf{p}_i^t \quad (5.16)$$

In the case of the general surface model, the locations of the contact points on the tibia and femur were system unknowns and were solved for directly.

5.5.6 Computer Animation

The model knee movement of both the idealised and general surface models of mobility was computer animated on a Silicon Graphics Indy workstation using Matlab and Inventor (Silicon Graphics Software).

Chapter 6

Knee Mobility: Idealised Surface Model Predictions and Validation

The predictions of an idealised surface model of mobility (Section 5.3.1) are presented and discussed in this chapter. First, variations both in the chosen spherical and planar representations of the femoral and tibial condyles and in the definition of the isometric ligament fibres are investigated. Using the most appropriate spherical and planar articular surface representation and isometric ligament fibre definition, the predictions of a number of specimen-specific models are presented. The relative movement between the model bones is quantified both by joint rotations and translations and the tibial axodes of the instantaneous helical axis (IHA). Movement of the contact points on the two bones is also examined. Model predictions of joint movement are compared to experimental observations of cadaver knee motion. The model solution behaviour and the required computational effort are reviewed. The sensitivity of the model to uncertainties in the model parameters and to the solution method and associated convergence criteria is reported. The utility and the limitations of the idealised surface model of knee mobility are discussed in the light of the results.

6.1 Representation of Surface and Ligament Constraints

In this section, two significant aspects of knee mobility: the range of predicted motion and the degree of coupled internal tibial rotation, are examined to establish which representations, given the idealised nature of the model articular surfaces and ligaments, best describe the contact and ligament constraints and result in the most realistic predictions of knee mobility.

Specimen	WTC Model		WFC Model	
	ROF (deg)	Max Rot (deg)	ROF (deg)	Max Rot (deg)
J	15	31.3	11	-30.7
D	113	24.1	96	23.4
L	>120	5.0	>120	2.3
M	> 120	18.6	>120	16.7
X	>120	36.0	114	160
I	9	34.2	5	-13.1
E	>120	-57.0	94	105
G	>120	-16.9	>120	63.5
F	>120	33.6	4	-11.4
B	>120	15.9	>120	61.7
A	32	11.4	2	-
H	115	66.6	74	-7.7

Table 6.1: Model predictions for sets of specimen-specific WTC and WFC parameters. Range of flexion (ROF) and maximum internal rotation of the tibia (Max Rot) are reported (negative values indicate external rotation).

6.1.1 Spherical and Planar Surface Representation

The effect of the chosen spherical and planar representations of the femoral and tibial condyles in an idealised surface model is investigated by evaluating four variations of the specimen-specific parameters: WTC, WFC, PTC, and PFC where the following notation applies:

- ‘W’ or ‘P’ Femoral condyles represented by spheres fitted to whole femoral condyles (‘W’), including both the posterior and distal aspects, *or* to the posterior femoral condyles only (‘P’).
- ‘T’ or ‘F’ Transverse (‘T’) *or* free (‘F’) planes fitted to the tibial condyles.
- ‘C’ or ‘P’ The most nearly isometric ‘combination’ (‘C’) fibres *or* ‘physiological’ (‘P’) fibres selected to model the isometric fibres in the ACL, PCL and MCL.

The range of flexion and the maximum coupled internal tibial rotation predicted by twelve specimen-specific models are presented in Tables 6.1 and 6.2.

The results suggest that spheres fitted to distal and posterior regions of the femoral condyles (‘W’) and transverse planes fitted to the tibial condyles (‘T’) together provide the most appropriate idealised representation of the constraint provided by the articular contact in a model of knee mobility. Despite some variability in the results, nine out of the twelve specimen-specific WTC models predicted a range of flexion of over 100°. Among those nine, seven predicted coupled internal

Specimen	PTC Model		PFC Model	
	ROF (deg)	Max Rot (deg)	ROF (deg)	Max Rot (deg)
J	1	-	1	-
D	>120	25.5	21	15.2
L	>120	1.9	>120	-25.5
M	>120	20.5	> 120	23.8
X	>120	44.6	0	-
I	2	-	1	-
E	3	-	0	-
G	>120	-8.4	>120	45.3
F	1	-	0	-
B	40	0.5	0	-
A	14	2.7	0	-
H	1	-	0	-

Table 6.2: Model predictions for sets of specimen-specific PTC and PFC parameters. Range of flexion (ROF) and maximum internal rotation of the tibia (Max Rot) are reported (negative values indicate external rotation).

rotation of the tibia with flexion with peaks ranging from 5° to 66°. The remaining two predicted coupled external rotation. Three models (J, I and A) exhibited limited mobility with ranges of flexion of under 40°.

Although they provide a better mathematical fit to the measured tibial condylar surfaces, the data presented in Table 6.1 suggests that the best-fit ‘free’ planes provide a less reliable representation of the contact constraints to knee mobility. Overall, the WFC model predictions were more variable than those of the WTC models. Only five models predicted a range of knee flexion of over 100°. Out of those five, four predicted coupled internal rotation with peak values ranging from 2° to 64° and one predicted a maximum internal rotation of 160°. The remaining seven models predicted limited flexion, ranging from 2° to 96°, coupled with, in all cases but one, high internal rotation or external rotation.

Representing the femoral articular surfaces by spheres fitted to the posterior condyles alone (‘P’) tended to reduce the range of predicted knee flexion. Although five PTC models predicted flexion of over 100°, of the remaining seven, five predicted virtually zero mobility in flexion. Where flexion was predicted, it was accompanied by peak internal rotations ranging from 21° to 45° in three cases; the remaining two predicted virtually zero internal rotation.

Modelling the tibial condyles as best-fit free planes (‘F’) and the femoral articular surfaces as spheres fitted to the posterior condyles (‘P’) appeared to provide the least successful representation of the contact constraints¹. Eight of the PFC models were

¹The PTC model parameter set of Model M is that described in [136]

Specimen	WTC Model		WTP Model	
	ROF (deg)	Max Rot (deg)	ROF (deg)	Max Rot (deg)
I	9	34.2	5	-6.1
E	>120	-57.0	>120	99.7
G	>120	-16.9	>120	-17.4
F	>120	33.6	>120	91.1
B	>120	15.9	>120	12.6
A	32	11.4	30	15.1
H	115	66.6	22	31.1

Table 6.3: Model predictions for sets of specimen-specific WTC and WTP parameters. Range of flexion (ROF) and maximum internal rotation of the tibia (Max Rot) are reported (negative values indicate external rotation).

virtually immobile in flexion. Out of the three models which predicted a full range of flexion, one predicted coupled external tibial rotation and another high internal rotation.

6.1.2 Isometric Ligament Fibre Definition

In this section, the effect of the representation of the ligament constraints is examined by comparing the predictions of models based on sets of WTC versus WTP parameters (as defined in Section 6.1.1)². Table 6.3 shows the range of predicted flexion and maximum coupled tibial rotation for each definition of the model ligament parameters for the seven models.

The range of motion was virtually unaffected in all cases but one (Model H) but the maximum predicted tibial rotation was significantly influenced by the choice of isometric ligament fibre definition. Using the WTC parameters, two models were identified which predicted coupled internal rotation within the physiological range (Models F and B). Among the WTP parameter sets, only Model B provided a reasonable prediction of tibial rotation through a full range of flexion. For this reason and because isometric ‘physiological’ fibres were only defined for seven of the twelve specimens, it was decided that the most nearly isometric ‘combination’ fibres provide the most appropriate representation of the constraints supplied by each ligament, even though they did not always correspond closely to the location of a physiological fibre.

²For seven specimens both the most nearly isometric ‘physiological’ fibre and the most nearly isometric ‘combination’ fibre during the measured passive motion were identified in each of the ACL, PCL and MCL.

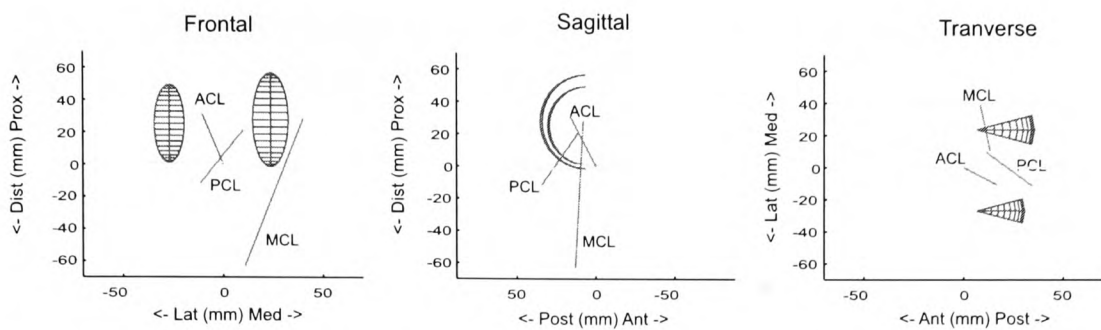


Figure 6.1: The configuration of a specimen-specific idealised surface model of knee mobility (Model B). The geometry is shown relative to the tibial coordinate system (Figure 5.6) in the frontal, sagittal and transverse views.

Model	Medial (mm)	Lateral (mm)
D	-1.5	3.8
F	-0.1	1.2
B	0.03	0.55

Table 6.4: The differences between the original fitted radii and the those of the model, modified to correct for the initial contact error. Positive differences indicate an increase in the model radius.

6.2 Specimen-Specific Model Predictions and Validation

The results presented in the previous sections suggest that the WTC parameters provide the most appropriate representation of the articular surface and ligament constraints in a idealised surface model of knee mobility. In this section, predictions of three of the most successful specimen-specific WTC models, Models D, F and B, are presented. The theoretical results are compared to the experimentally measured motion of Knees D, F and B and, for aspects of the motion for which direct measurements were not available, to observations reported in the literature.

6.2.1 Model Parameters

The parameters of the three models are listed in Appendix D. The differences between the original fitted radii of the femoral spheres and those modified to correct for the initial contact error (Section 5.4.4) are presented in Table 6.4. Figure 6.1 shows the configuration of one of the models (Model B) in the starting position (full extension).

Model	Original Method (Wilson)		New Method
	Pair Defn 1	Pair Defn 2	
D	33.6°	68.6°	113°
F	37.8°	113.1°	>120°
B	32.5°	102.9°	>120°

Table 6.5: The maximum range of flexion predicted by three specimen-specific models using both the original mechanism-based approach (two different pair definitions) developed by Wilson [135] and the new geometric-numerical method.

6.2.2 Range of Motion

Each of the three models predicted over 100° of flexion (Table 6.1). As already demonstrated in Chapter 5 (Section 5.2.5), use of the original mechanism-based solution method of Wilson [135] led to a significant underestimation of the range of model knee motion (Table 6.5). Each model predicted under 40° of flexion using the original method. A redefinition of the lower pair orientations allowed further flexion in each case but the range of motion was still lower than that predicted using the new method.

In extension, Model F predicted only 2° of hyperextension while Models D and B predicted over 25°. It is important to note that the posterior capsule does tighten near full extension, providing an additional constraint to extension of the knee which would cause the joint to lock before reaching 25° of hyperextension.

6.2.3 Joint Rotations and Translations

Models D, F and B successfully predicted coupled internal rotation with flexion with peak rotations of 24.1°, 33.6° and 15.9° respectively (Figure 6.2). Predicted internal tibial rotation was within the range of the measured rotations³. With Model D, a fair match between predicted rotations and experiment was found within the range of measured motion, *i.e.* up to 100° flexion. Although of the same order, peak internal rotations somewhat higher than those recorded in experiment were predicted by Models F and B. The pattern of predicted rotation of Model F differed from the corresponding experimental curve in that model peak rotation occurred in mid-flexion rather than approaching full flexion. Model B’s internal rotation curve, however, was qualitatively similar to the measured pattern.

Zero ab/adduction of the knee was predicted by each model due to the nature of the surface representation and the definition of the motion axes (Figure 6.2). The

³Both the measured and predicted joint rotations are described here with the flexion axis defined to lie along the line passing through the centres of the *whole* femoral spheres.

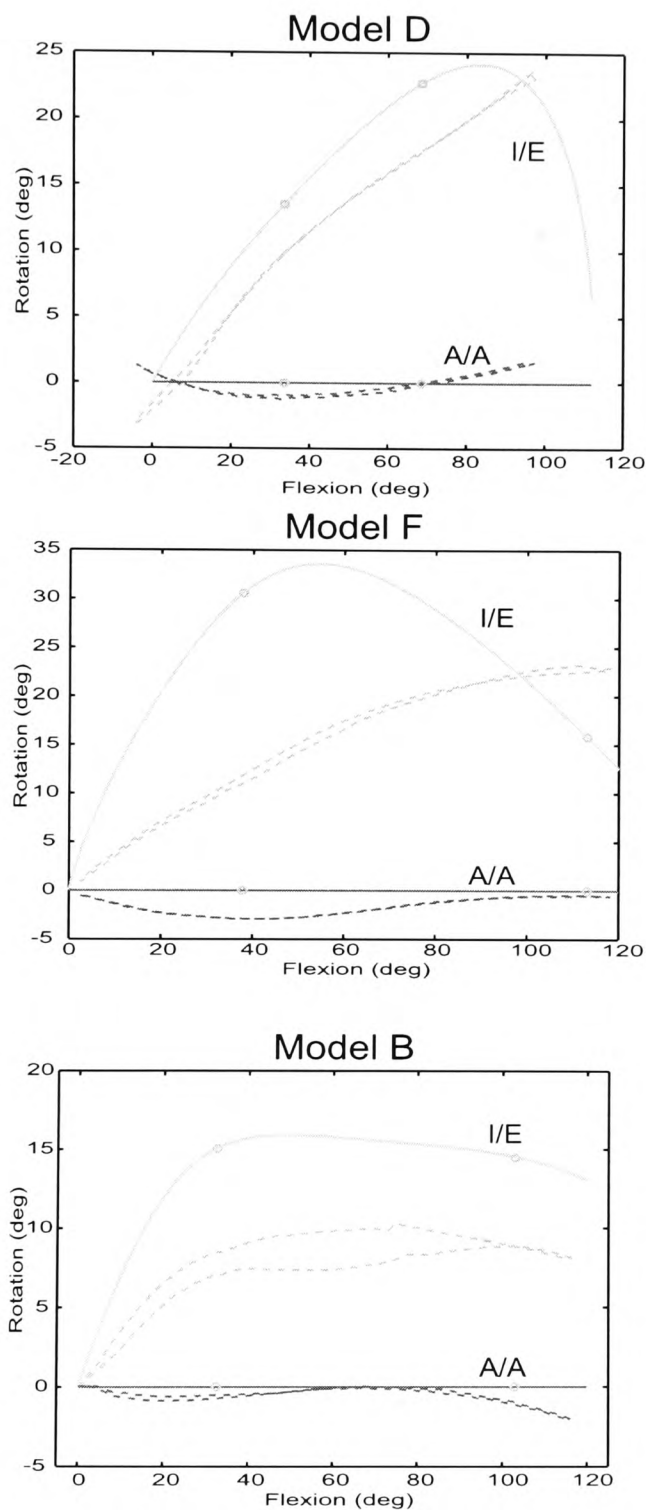


Figure 6.2: Joint rotations predicted by three specimen-specific models (solid lines). Internal(+)/external(-) tibial rotation (I/E) and abduction(+)/adduction(-) (A/A) are plotted versus flexion. The circles (o) on the model curves depict the limits of the range of flexion predicted using the original approach (Table 6.5). The measured rotations of the corresponding knee specimen, *i.e.* the specimen used to derive the model parameters, are also shown (dashed lines).

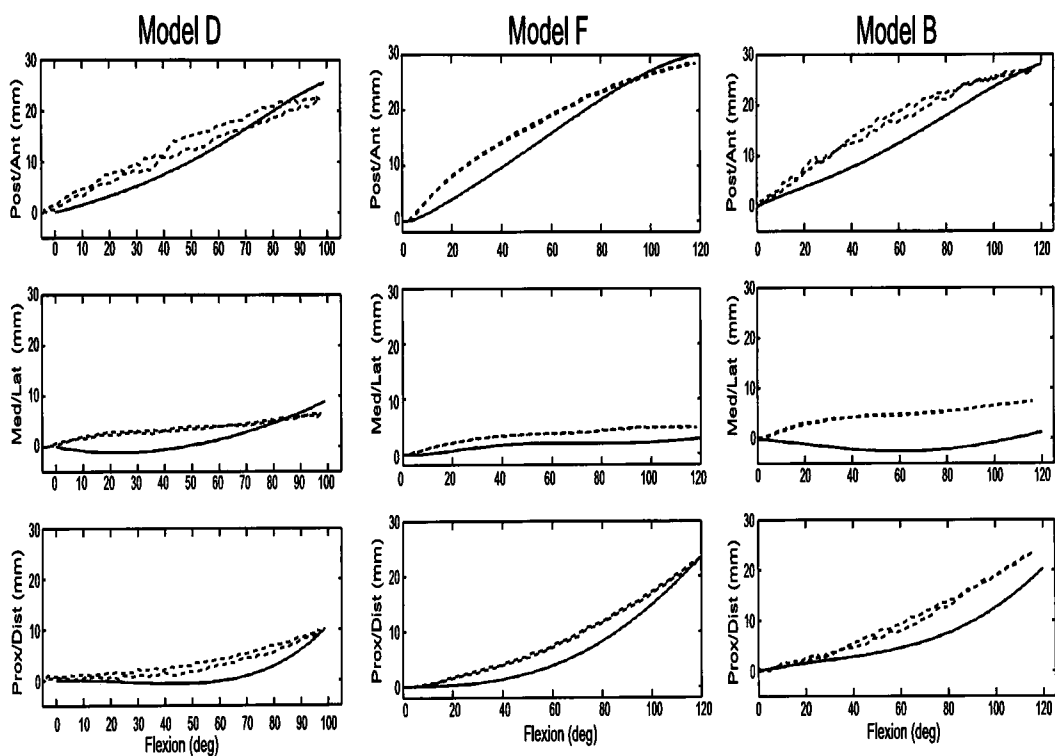


Figure 6.3: Joint translations predicted by three specimen-specific models (solid lines). Posterior(+)/anterior(-), medial(+)/lateral(-) and proximal(+)/distal(-) components of the translation of the reference point are plotted versus flexion. The translations of the same reference point measured during flexion and extension of the corresponding knee specimen are also shown (dashed lines).

tibial condyles were represented by two parallel transverse planes. By enforcing continuity of contact between the femoral spheres and the tibial planes, the orientation of the flexion axes in the frontal plane can not change relative to the mediolateral tibial axis during the motion. The result is, in any case, consistent with the little ab/adduction observed in experiment.

Posterior and proximal translation of the tibial reference point was predicted by all three models (Figure 6.3). Models D, F and B exhibited maximum posterior translations of 26.7, 30.2 and 28.1 mm and maximum proximal translations of 19.6, 23.6 and 20.4 mm respectively. Medial translation was predicted by Model F (maximum 2.8 mm) while Models D and B predicted a small amount of lateral translation followed by medial translation (maximums 9.2 mm and 1.3 mm respectively). On the whole, the theoretical predictions corresponded well with experimental measurements.

6.2.4 Instantaneous Helical Axes

The instantaneous helical axes (IHAs) describing model knee movement are represented with respect to the fixed tibial coordinate system in Figure 6.4. The nine axes, corresponding to flexion increments of 10° from 10° to 90° flexion, describe the tibial axode of each model knee. The nine IHAs are analogous to the nine finite helical axes (FHAs) describing the experimental motion for the flexion steps of 0-20, 10-30, ..., 80-100° (Section 3.3.4).

In the frontal plane, the predicted IHAs, initially oblique and inclined towards the lateral tibial plateau, shift to lie more parallel to the plateau with increasing flexion (Figure 6.4). The axes tend to pivot about a point central to the joint. While Model B's IHAs remain close to horizontal, the IHAs of Models D and F continue to tilt towards the medial plateau as 90° flexion is approached. The changing orientations in the frontal plane reflect the degree of axial rotation accompanying flexion. Up to the point where the models predicted internal rotation with increasing flexion (Figure 6.2), the corresponding IHAs are tilted laterally. As the IHA comes to lie parallel to the flexion axis, the contribution of axial rotation approaches zero. Because Models D and F exhibit a decrease in internal tibial rotation (relative external rotation) as the knee nears full flexion, the corresponding IHAs are seen to tilt medially.

The changing orientations of the IHAs in the frontal plane are generally consistent with those of the FHAs describing the experimentally observed movement (*e.g.* Figure 3.12) with the exception of the final tilting towards the medial plateau (Models D and F) and a more central location of the pivot point⁴.

⁴The experimental FHA results were susceptible to noise and hence quite variable across the

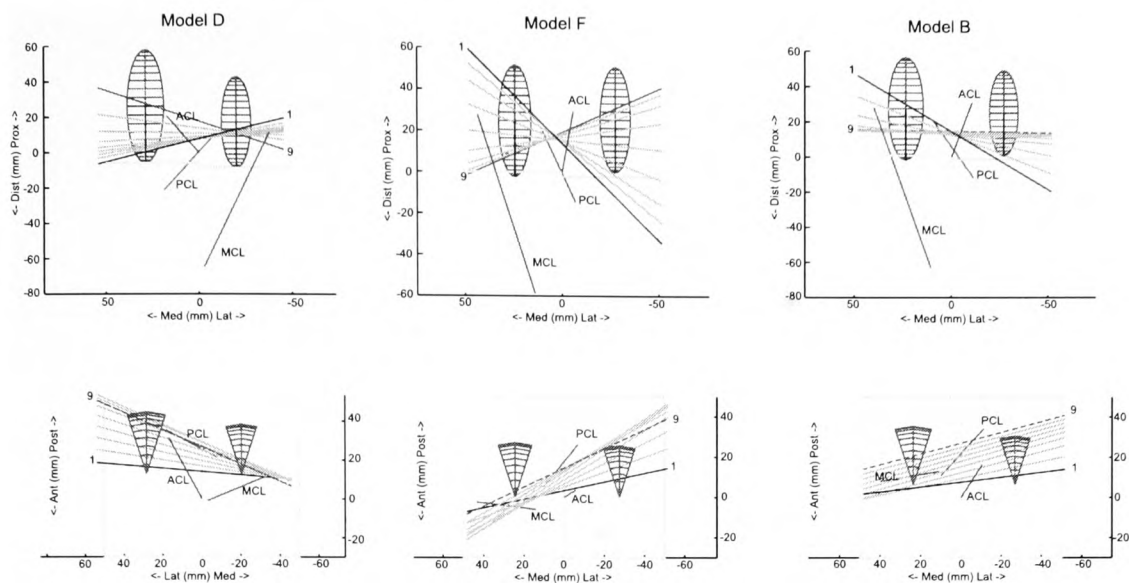


Figure 6.4: Model IHAs describing the motion of the femur relative to a fixed tibia (tibial axodes). Nine axes are shown with respect to the tibial coordinate system both in the frontal and transverse planes. The first axis (1) corresponds to 10° flexion and the final axis (9) to 90° flexion. The model configuration is depicted in its starting position (full extension).

In the transverse plane, the model IHAs, initially nearly parallel to the mediolateral axis of the tibia, move posteriorly with increasing flexion while rotating about a pivot point medial to the joint (Figure 6.4). The changing orientation of the IHAs in the transverse plane reflects the changing orientation of the flexion axis during the motion. The changing pattern of the model IHA orientation in the transverse plane is similar to that of the experimental FHAs (*e.g.* Figure 3.12) despite a somewhat more medial pivot point.

The intersections of the model IHAs with a mid-sagittal and medial and lateral para-sagittal planes are shown in Figure 6.5. The para-sagittal planes are analogous to those used to describe the movements of the corresponding experimental FHAs (Section 3.3.4)⁵.

The net posterior and proximodistal shifts of the IHAs in the three planes are summarised in Table 6.6. The IHAs moved posteriorly in each of the sagittal planes, with the largest excursions occurring in the lateral sagittal plane. Models D and F predicted posterior shifts in the medial and mid-sagittal planes on the order of the mean values recorded in experiment: 3.6 and 8.2 respectively (flexion path) (Table 3.10). Model B predicted larger posterior shifts in all three planes. The model IHAs moved proximally in the mid and lateral sagittal planes, however, in

twelve specimens tested (Section 3.3.4). Specimen-specific comparisons were not considered particularly meaningful. Trends observed across the twelve specimens and the orientations of the mean helical axes were considered.

⁵For the model results, the planes are defined to pass through the centres of the ‘whole’ spheres. The mediolateral positions of these centres are virtually identical to those of the ‘posterior’ spheres used to define the para-sagittal planes in the experimental FHA analysis.

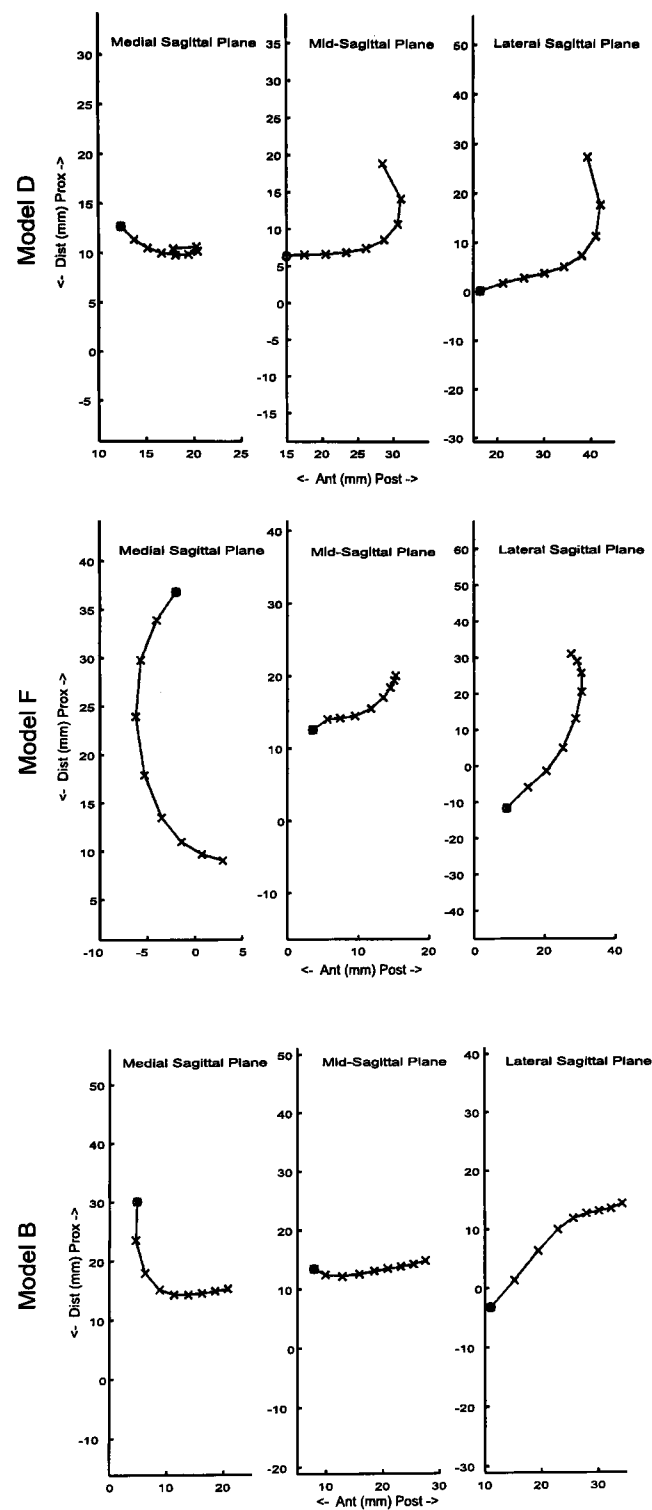


Figure 6.5: Intersections of the model IHAs with mid-sagittal and medial and lateral para-sagittal planes. The para-sagittal planes pass through the femoral sphere centres in the fully extended knee. The circle (o) represents the IHA corresponding to 10° flexion and the crosses (x) indicate the axis locations for 10° flexion increments.

Model	Posterior Shift (mm)			Distal Shift (mm)		
	Med	Mid	Lat	Med	Mid	Lat
D	5.5	14.2	22.8	2.3	-12.5	-27.3
F	4.9	11.7	18.4	27.78	-7.5	-42.7
B	15.9	19.6	23.3	14.9	-1.3	-17.5

Table 6.6: Shifts of the IHAs in the mid-sagittal (Mid) and para-sagittal planes passing through the medial (Med) and lateral (Lat) femoral sphere centres. Posterior and distal movement is defined as positive.

Model	Mean Helical Axis Orientation	
	Alpha (deg)	Beta (deg)
D	3.3	16.1
F	8.4	27.2
B	10.2	13.8

Table 6.7: Orientations of the mean helical axes (MHAs). Alpha (α) and Beta (β) are the angles between the projections of the MHA and the tibial mediolateral axis, Y_t , in the frontal and transverse planes respectively. A positive α describes an axis which is tilted towards the lateral plateau in the frontal plane and positive β an axis which is inclined posterolaterally in the transverse plane.

contrast to experimental observations. This is a result of the axes rotating about a more central pivot point in the frontal plane. The distal shift of the model IHAs in the medial plane was variable but was on average close to the mean 11.5 mm recorded in experiment (Table 3.10).

For each model, the set of IHAs describing knee motion from 0° to 100° flexion in increments of 1° flexion was quantified by a mean helical axis (MHA), the axis closest to all IHAs in a least-squares sense. Orientations of the model MHAs are presented in Table 6.7. The orientations are generally on the same order as those of the MHAs describing the passive flexion of the twelve specimens: α =10.2 and β =14.3 (Table 3.11). Only the MHAs of Model F in the transverse plane and of Model D in the frontal plane are considerably more oblique than the experimental axes.

6.2.5 Contact Movement

Each of the three models successfully predicted coupled posterior translation of the contact points on the tibia with flexion (Figure 6.6). Within 100° flexion⁶, net

⁶Range chosen to allow comparison with experimental contact movement which was reported for 100° of flexion.

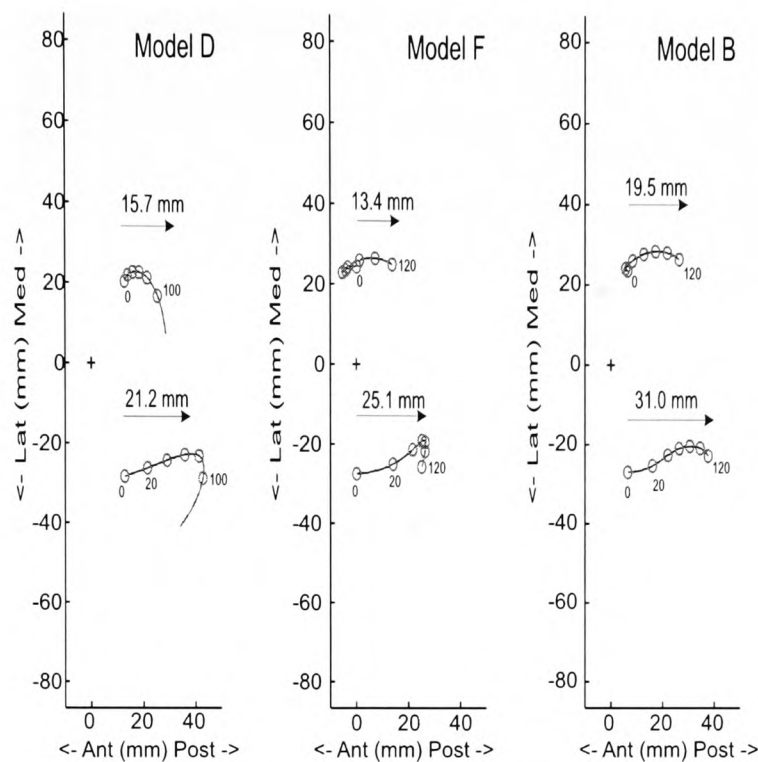


Figure 6.6: Tibial contact point displacement in the medial and lateral compartments predicted by three specimen-specific models. The circles (o) on the contact paths represent steps of 20° flexion. The site of the ACL tibial attachment is also shown (+).

posterior excursions of 12.3, 6.9 and 14.9 mm in the medial compartment and 30.1, 26.2 and 27.7 mm in the lateral compartment were predicted by Models D, F and B respectively.

Translation in the lateral compartment was approximately twice that in the medial compartment reflecting the internal rotation of the tibia with flexion. Models F and B exhibited some initial anterior displacement of the contact in the medial compartment. This is a reflection of the sharp increase in internal rotation predicted by the two models in early flexion (Figure 6.2). The slope of Model F's axial rotation curve is initially greater than one. The larger proportion of internal tibial rotation versus flexion explains the initial anterior displacement of the tibial contact point exhibited in the medial compartment and the large of posterior movement in the lateral compartment. Similarly, the anterior excursion of the contact points of Models D and F in the lateral compartment near full flexion reflects the sharp decrease in internal rotation (relative external rotation) predicted as full flexion was approached. Minimal mediolateral displacement was predicted by the three models within the 100° flexion range.

The excursions of the contact points on the tibia are generally larger than values estimated based on measurements of surface geometry and motion (Section 4.1) and those reported previously in the literature. Based on experiment, mean estimates

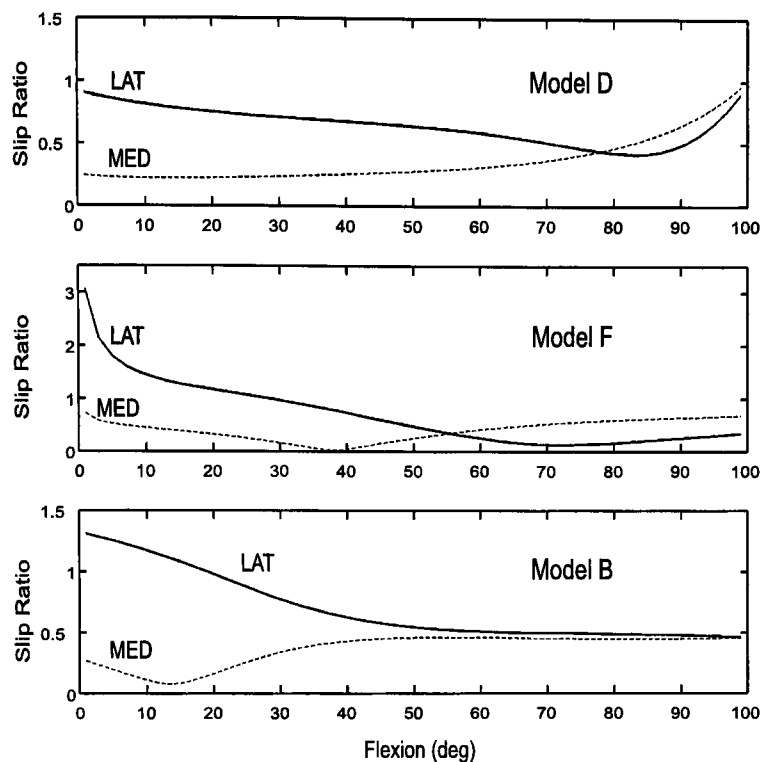


Figure 6.7: The slip ratio as a function of flexion angle predicted by three specimen-specific models. The ratio is defined as the displacement of the contact point on the tibia divided by that on the femur for increments of 2° flexion. The ratios in the medial (MED) and lateral (LAT) compartment are shown.

of posterior contact point movement on the tibia were 7.9 mm (S.D. 7.0 mm) in the medial compartment and 14.7 mm (S.D. 5.0 mm) in the lateral compartment (Table 4.1). These experimental estimates were found to be on the order of previous reports in the literature [64, 125] (Section 4.1). The theoretical values reported here are up to twice as large. In the lateral compartment, the contact points near full flexion are located near the posterior edge of the articular surface of the tibial condyle.

Relative motion at the contact was characterised by the slip ratio, shown in Figure 6.7 for the three models. As in the experimental analysis, the ratio was defined as the distance travelled by the contact point on the tibia divided by that travelled on the femur over a specified increment of movement. A slip ratio of zero indicates pure rotational sliding while a ratio of one indicates pure rolling of the femur on the tibia. A slip ratio between zero and one describes a combination of rolling and sliding. A ratio greater than one implies a degree of translational sliding of the femur on the tibia. The magnitude of the slip ratio at a particular flexion angle must be considered in conjunction with the relative directions of the tibial and femoral contact point movement. In the medial compartment some of the early sliding of the femur on the tibia occurs with anteriorly directed movement of the

tibial contact point (Models F and B); the simple definition of the slip ratio used in the current analysis, however, was always positive.

Although there were inter-model differences, each model predicted a greater proportion of sliding in the medial compartment compared to the lateral compartment for most of the flexion range. This is consistent with mean estimates of the slip ratio based on the measured motion (Section 4.1). At higher flexion angles, the model slip ratios in the medial and lateral compartments approach one another, and both lie between zero and one. Models F and B predicted slip ratios in the lateral compartment greater than one near full extension, implying some proportion of translational sliding of the femur.

6.3 Computational Effort

The behaviour of the solution process and the computational effort associated with determining the kinematics of an idealised surface model of knee mobility were examined using the parameter set of Model B.

Using the non-linear least-squares root-finding optimisation algorithm programmed in Matlab (Matlab's 'fsolve' routine, Appendix C), the determination of the average model kinematics through 120° of flexion required a total of 3641 function evaluations. Each of the five constraint functions were evaluated 3641 times. The evaluation time was 71 seconds, corresponding to a solution rate of approximately 0.6 seconds per degree of flexion running on an Indy workstation (Silicon Graphics, USA).

Using the original mechanism-based approach to solving the model kinematics was less computationally efficient than the new method of solution. Coded in Matlab, the algorithm exhibited an approximate solution rate of 1.5 seconds per degree of flexion.

6.4 Discussion

An idealised surface model of knee mobility defined by select sets of specimen-specific parameters was successful in predicting the main features of passive knee motion: coupled internal rotation of the tibia and posterior translation of the contact points on the tibia with flexion. Only few models, Models D, F and B, predicted peak internal tibial rotation and knee abduction on the order of that recorded *in vitro* in the cadaver knees used to derive the model parameters. Joint translations, too, were consistent with experimental measurements. The orientations of the model IHAs were similar to those of the experimental FHAs, despite the differences in the paths of intersections in the sagittal planes. Although nearly twice too large in magnitude,

posterior movement of the tibial contact points was predicted in both the medial and lateral compartments of the knee, with the excursion in the lateral compartment nearly double that in the medial, a pattern consistent with experimental estimates and previous reports in the literature.

The examination of a number of specimen-specific models has confirmed the superiority of the new geometric-numerical approach in determining the kinematics of a model of mobility. Using the original mechanism-based approach of Wilson [135] led to a significant underestimation of the range of motion of Models D, F and B. Furthermore, the computational solution rate of the new method is higher than that of the original. It provides a more robust and computationally efficient approach to solving for the unique path of model knee motion.

The chosen representation of the idealised surface constraints had a significant effect on both the predicted range of motion and the degree of coupled internal rotation (Tables 6.1 and 6.2). Examining model predictions based on twelve sets of specimen-specific parameters, it was found that, overall, the constraints were most appropriately represented by modelling each femoral condyle as a sphere fitted to its distal and posterior portions and each tibial condyle as the best-fit transverse plane (WTC parameters). With this surface definition, the majority of the specimen-specific models predicted a full range of flexion with coupled internal tibial rotation. In contrast, by fitting spheres to the posterior femoral condyles alone and ‘free’ planes to the tibial condyles (PFC parameters), the majority of the specimen-specific models were virtually immobile. The effects of differences in the representation of the articular surfaces on knee mobility may now be examined and interpreted in a more systematic fashion with a focus on implications for knee replacement design.

The representation of each ligament constraint by either the most nearly isometric ‘physiological’ or ‘combination’ fibre was also found to influence the mobility of the model joint. The pattern, rather than the range of motion, appeared to be most affected by the choice of ligament fibre. The effect of the isometric fibre locations on the range and pattern of knee motion may now be systematically examined with a focus on implications for ligament reconstruction.

A principal limitation of the idealised surface model is the geometrical simplification of the anatomical structures which act as constraints to the joint motion. The differences between the experimental joint and contact kinematics and the predictions of the three specimen-specific idealised surface models and the complete failure of others in predicting joint mobility can be attributed in part to such simplifications.

In the model, the articular surface shapes are idealised. Although the posterior condyles are close to spherical, the combined distal and posterior regions of the condyle are less so (Section 3.1). The tibial condyles are curved in the frontal and sagittal planes, in particular in the case of the lateral plateau. The large contact

point displacement on the tibia may be explained by these simplifications. The four-bar linkage model of the knee showed that the slip ratio (and by extension the contact point movement) is sensitive to the assumed shape of the tibial plateau [99]. Inclusion of more anatomical shapes for the condyles may yield contact point excursions closer to physiological values. The geometric-numerical approach readily allows the incorporation of further geometric detail, such as more complex surface shapes, into the model of mobility as was described in Chapter 5 (Section 5.4.4.2). The predictions of such a model will be presented in Chapter 7.

Differences between the model and natural knee ligaments may also be of particular importance. The ligament fibres were modelled as straight line segments joining the femoral and tibial attachments, while the natural ligament fibres can wrap around each other or neighbouring bony surfaces. The natural MCL wraps around the tibial plateau. Kinematically, the constraint provided by an isometric fascicle ‘bent’ around the plateau is identical to that provided by a straight line segment connecting the attachments. The problem then consists of identifying the attachment sites of such an isometric fascicle during the search. Tibial and femoral attachments may be identified which define a straight-line segment between them which is most nearly isometric during the recorded motion, however, the corresponding ligament fibre may well be anisometric because the point of wrapping changes with knee flexion. Furthermore, the most isometric ‘combination’ fibre found in experiment does not necessarily correspond to the location of a real fibre. Specimen-specific models incorporating ‘physiological’ fibres to represent the ligamentous constraints, however, did not fare any better than those using ‘combination’ fibres.

Error in the experimental measurement of the geometry of the joint used to derive the model parameters may also explain some of the differences between theory and experiment. Significant differences were found in the reconstruction and movement error between specimens tested in Lab 1 and those tested in Lab 2 (Section 4.3). Lower error, however, did not necessarily ensure a more successful model formulation. It is a combination of the deviations due to measurement error and those due to the simplified geometrical representation of the surfaces which most likely dictates whether a particular set of model parameters leads to a successful model or not. The effects on model mobility of deviations in the parameters on the order of the experimental error are examined in a separate sensitivity study (Section 6.5).

Well-defined protocols were used to derive the model parameters from measurements on cadaver knees. It is possible that with additional modifications to the protocol, the potential of each of the specimen-specific models to predict a unique path of passive knee motion similar to that observed in experiment would be realised. It has already been demonstrated what effect differences in the definition of surface representation and isometric ligament fibres have on joint model mobility

(Section 6.1.1 and 6.1.2). Other representations of the ligament and contact constraints, still based on the measured geometry of the joint, may exist which together provide a more successful formulation of the model of mobility. The difficulty lies in establishing the ‘best’ definition of the model parameters. The parameter derivations examined (WTC, WFC, PTC, PFC, WTP) were considered the most likely candidates.

The model of mobility presented here is the first 3-D model to examine the *full* range and path of knee movement in the unloaded state. A number of knee models have been developed to predict 3-D joint motion in response to an applied load [6, 18, 37, 139]. These models are unable to predict the fully coupled nature of the motion of the unloaded joint because load is required to hold the knee at any flexion angle [37] or to maintain articular contact [18]. Displacement of the model knee in passive flexion involves zero tissue deformation and hence does not require the application of external forces or moments. The fact that the features of passive knee kinematics can be reproduced by such a model implies that passive flexion of the real knee, too, involves minimal tissue deformation and that a unique path of unresisted passive motion exists (Section 3.3.6).

The spatial model of mobility adds to the insights gained from previous 2-D geometric knee modelling work (Section 1.3.3.3). In the sagittal plane, the four-bar linkage model has successfully predicted and provided an explanation for the posterior movement of the instantaneous centre of rotation, located at the point where the two cruciate links cross, and the combination of rolling and sliding of the femur on the tibia during passive flexion [45, 98]. The 2-D model, however, is limited because it can only predict knee motion in the sagittal plane. The model of mobility presented here represents the 3-D analogue to the 2-D four-bar linkage model, predicting the spatial movement of the bones in the unloaded joint, the location of the instantaneous helical axis and the excursion of the articular contacts in both the medial and lateral compartments at any flexion angle. It thus provides insight into the spatial relationships between surface and ligament geometry which control the mobility, factors of interest in the design and implantation of knee prostheses and in ligament reconstruction.

Examining the kinematic geometry of the unloaded joint not only allows the study of the influence of joint geometry on knee mobility, but also provides a practical starting point for the mechanical analysis of the joint under load as described in Chapter 1. 3-D force analysis of the musculoskeletal system (*e.g.* [76, 77]), for example, depends on an accurate prescription of the spatial kinematic geometry of the knee joint. By providing the relative positions of the bones and hence the lines of action of the contact normals, ligaments and muscle tendons throughout the range of flexion, a 3-D geometric model, such as that described in this chapter, reduces

the number of unknowns in the mechanically indeterminate problem of distributing the forces at the joint. Furthermore, by providing the knee's *neutral* configuration at any given flexion angle, the 3-D model of knee mobility is also the essential first component in a sequential vs simultaneous approach to the theoretical study of passive knee stability (Chapter 8).

6.5 Model Sensitivity

In the previous sections, the predictions of an idealised surface model of knee mobility have been examined for selected sets of specimen-specific parameters. While certain parameter sets yielded models which were successful in predicting a path of passive knee motion close to that observed in experiment, others described models which were virtually immobile. Measurement error and geometrical simplifications underlying the parameter derivation were given as possible reasons for the diverse results. To investigate the effect of such differences, a parameter sensitivity study was performed by examining examples of both the 'best' and 'worst' specimen-specific models. The aim was to establish whether deviations in the model parameters on the order of the aforementioned measurement and geometrical approximations provide an explanation for the success of certain specimen-specific models versus the failure of others in predicting the mobility of the joint. A similar analysis had been previously performed by Wilson [135] based on the original model formulation and a preliminary set of parameters (Section 5.2.3). Because the original approach led to an underestimation of the range of motion and the preliminary parameter set proved unphysiological, it was necessary to reassess the model sensitivity. The sensitivity of the model predictions to the method used to solve the sets of non-linear constraint equations and the effect of relaxing the tolerances involved in satisfying these equations were also investigated. The analyses and results are presented and discussed in this section.

6.5.1 Parameter Sensitivity Study

To study the sensitivity of the idealised surface model of mobility to uncertainty in the input parameters, two specimen-specific WTC parameter sets were considered: Models D and A. The predictions of Model D were generally consistent with experimental measurements of joint movement. A full range of flexion of over 100° and coupled internal rotation was predicted. In contrast, Model A predicted only a limited range of flexion (32°) (Table 6.1). The effect of uncertainty in the definition of the parameters on the main features of the model kinematics was examined.

6.5.1.1 Methods

A procedure similar to that used by Wilson [135] in studying the parameter sensitivity of the original model formulation was implemented. Starting with the basic parameter set of each specimen-specific model, each of the thirty independent parameters was perturbed in sequence: the x , y and z components of the tibial and femoral attachments of the ACL, PCL and MCL (18), the x , y and z components of the medial and lateral femoral sphere centres (6), the radii of the spheres (2) and the orientations of the tibial planes (4). The orientation of each tibial plane was described by two angles, α and β . α describes the angle between the projection of the normal and the z axis in the sagittal plane and β the angle between the projection of the normal to the plane and the z axis in the frontal plane. The thirty parameters are listed in Table 6.8. The remaining parameters, such as the contact points on the tibia, are dependent on the locations of the sphere centres, the radii of the spheres and the orientations of the planes.

The effect of uncertainties of ± 5 mm in the position and length parameters and $\pm 5^\circ$ in the angular parameters was investigated. Parameter sets were generated identical to the basic set except that one of the independent parameters P_i was replaced by $P_i \pm u_{P_i}$, where u_{P_i} represents the uncertainty in the parameter P_i . For the resulting 60 parameter sets, the model knee displacement was determined using the method described in Section 5.3.1.3. The range of flexion predicted and the internal tibial rotation and posterior movement of the contact points on the tibia were noted in each case. These represent the significant features of passive knee motion which the model should reproduce.

The sensitivity of the range of flexion to each individual parameter j was quantified by the difference between the range of flexion for a change of $+5$ mm or $+5^\circ$ in the particular parameter and that for a change of -5 mm or -5° . The maximum internal rotation and posterior movement of the tibial contact points predicted by the model are clearly dependent on the model's range of motion. By examining the sensitivity of these kinematic quantities at a specific flexion angle, this dependence on range of motion is removed. The sensitivity of the maximum internal rotation and the net posterior contact point movement at a chosen flexion angle was determined for each parameter.

In a text on error analysis, Taylor presented the theoretical background to the propagation of uncertainties [124]. The total uncertainty in an output Q due to the uncertainties in individual parameters, P_i , can be approximated by:

$$u_Q \approx \pm \sqrt{(\|\frac{\partial Q}{\partial P_1}\| u_{P_1})^2 + (\|\frac{\partial Q}{\partial P_2}\| u_{P_2})^2 + \dots + (\|\frac{\partial Q}{\partial P_n}\| u_{P_n})^2} \quad (6.1)$$

where $u_{P_1}, u_{P_2}, \dots, u_{P_n}$, the individual parameter uncertainties, are independent

Constraint	Symbol	Model Parameter
ACL	xt	x coordinate of the ACL tibial attachment
	yt	y coordinate of the ACL tibial attachment
	zt	z coordinate of the ACL tibial attachment
	xf	x coordinate of the ACL femoral attachment
	yf	y coordinate of the ACL femoral attachment
	zf	z coordinate of the ACL femoral attachment
PCL	xt	x coordinate of the PCL tibial attachment
	yt	y coordinate of the PCL tibial attachment
	zt	z coordinate of the PCL tibial attachment
	xf	x coordinate of the PCL femoral attachment
	yf	y coordinate of the PCL femoral attachment
	zf	z coordinate of the PCL femoral attachment
MCL	xt	x coordinate of the MCL tibial attachment
	yt	y coordinate of the MCL tibial attachment
	zt	z coordinate of the MCL tibial attachment
	xf	x coordinate of the MCL femoral attachment
	yf	y coordinate of the MCL femoral attachment
	zf	z coordinate of the MCL femoral attachment
Mcon	a	α , sagittal plane orientation of the medial tibial plane
	b	β , frontal plane orientation of the medial tibial plane
	xf	x coordinate of the medial femoral sphere centre
	yf	y coordinate of the medial femoral sphere centre
	zf	z coordinate of the medial femoral sphere centre
	R	radius of the medial femoral sphere centre
Lcon	a	α , sagittal plane orientation of the lateral tibial plane
	b	β , frontal plane orientation of the lateral tibial plane
	xf	x coordinate of the lateral femoral sphere centre
	yf	y coordinate of the lateral femoral sphere centre
	zf	z coordinate of the lateral femoral sphere centre
	R	radius of the lateral femoral sphere centre

Table 6.8: The thirty independent parameters of the idealised surface model of knee mobility.

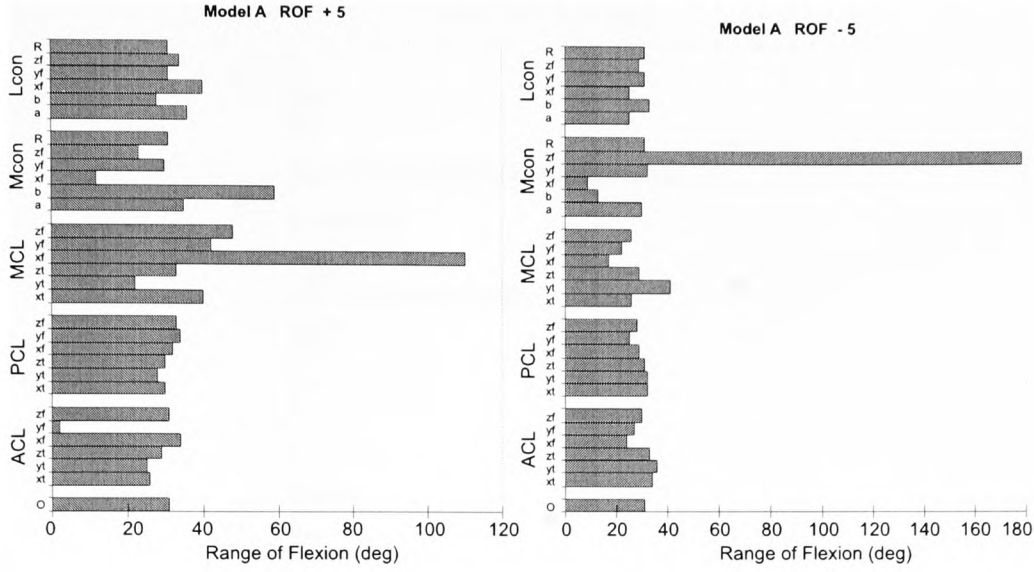


Figure 6.8: Range of flexion predicted by Model A. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters. ‘O’ refers the original parameter set.

and random. Because the range of flexion, the maximum internal rotation, and the net posterior contact point movement in the medial and lateral compartments predicted by the model are not analytical functions of the parameters, each unsquared term in the summation under the square root of Equation 6.1 was approximated by:

$$\left\| \frac{\partial Q}{\partial P_j} \right\| u_{P_j} \approx Q(P_1, P_2, \dots, P_j + u_{P_j}, \dots, P_n) - Q(P_1, P_2, \dots, P_n) \quad (6.2)$$

Using the above formulae, the overall uncertainty in the predicted range of flexion, $u_{Q_{rof}}$, was determined. The overall uncertainty in the maximum internal rotation, $u_{Q_{rot}}$, and the net medial and lateral posterior tibial contact point movement, $u_{Q_{ctm}}$ and $u_{Q_{ctl}}$, were also computed for a specific flexion range.

6.5.1.2 Results

The predicted range of motion showed little sensitivity to uncertainty in some parameters while changes in others greatly influenced the result.

Figure 6.8 shows the predicted range of motion for changes of ± 5 mm or $\pm 5^\circ$ in each parameter for Model A. While it originally predicted only 32° of flexion, a range of over 100° flexion with either a $+5$ mm change in the x coordinate of the MCL femoral attachment or a -5 mm change in the z coordinate of the medial femoral sphere centre was predicted. The smallest range of motion was found for uncertainty of $+5$ mm in the y coordinate of the ACL femoral attachment (9°). The overall sensitivity of Model A’s range of motion to uncertainties of ± 5 mm or $\pm 5^\circ$ in each parameter is shown in Figure 6.9. The predicted range of motion was most sensitive to the x coordinate of the MCL femoral attachment and the z coordinate

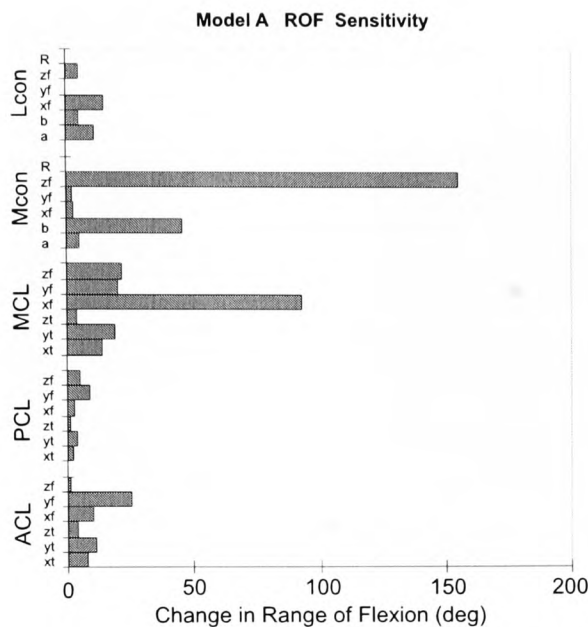


Figure 6.9: Sensitivity of range of flexion. Model A. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters.

of the medial sphere centre. Uncertainty in the orientation of the medial tibial plane in the frontal plane (β) also had a pronounced effect. Changes in the parameters relating to the PCL and lateral contact constraints were found to have the least effect. Changes to the radii of the femoral spheres made no difference to the model range of motion. The reason for this is that changing the femoral sphere radius in the model is equivalent to moving the height of the corresponding tibial plane in the direction of the contact normal. Kinematically, the resulting mechanism is equivalent and the prediction of model kinematics identical. The overall uncertainty in the model range of motion was found to be $\pm 95.4^\circ$.

Similarly, with Model D, a model deemed successful based on its original set of parameters, the predicted range of motion was highly sensitive to uncertainty in specific parameters and virtually insensitive to changes in others. Figure 6.10 shows the predicted range of motion for changes of ± 5 mm or $\pm 5^\circ$ in each parameter. The largest range of motion corresponded to a change of -5 mm in the z coordinate of the medial femoral sphere centre (176°). The smallest range of motion was predicted with a $+5$ mm change in its x coordinate (65°). The overall sensitivity of Model D's range of motion to uncertainties of ± 5 mm or $\pm 5^\circ$ in each parameter is shown in Figure 6.11. As in the case of Model A, uncertainty in the z coordinate of the medial femoral sphere centre had the largest effect on the model range of motion. Changes in the x coordinate of the medial femoral sphere centre and the z coordinate of the PCL femoral attachment were also highly influential. Again, changes in the parameters relating to the lateral contact constraints seemed to have little effect on

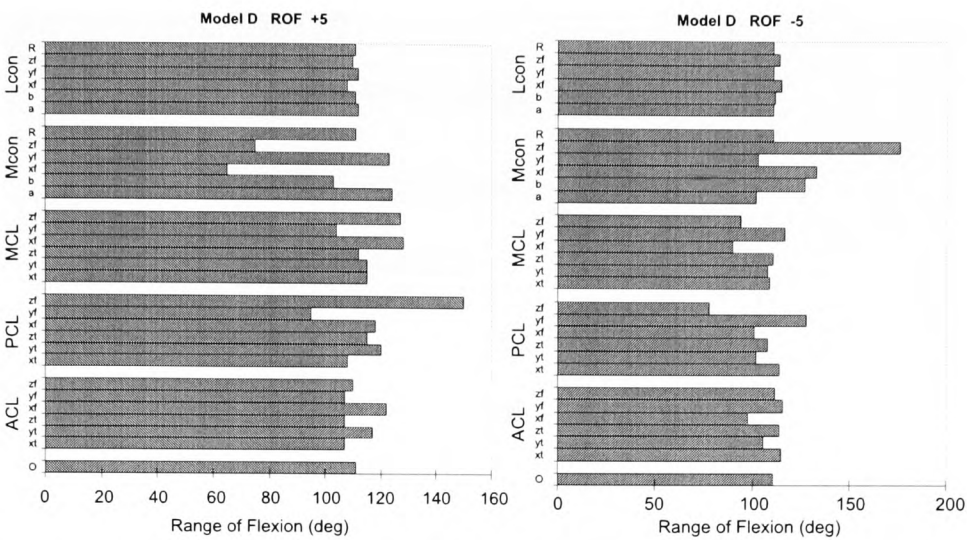


Figure 6.10: Range of flexion predicted by Model D. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters. ‘O’ refers the original parameter set.

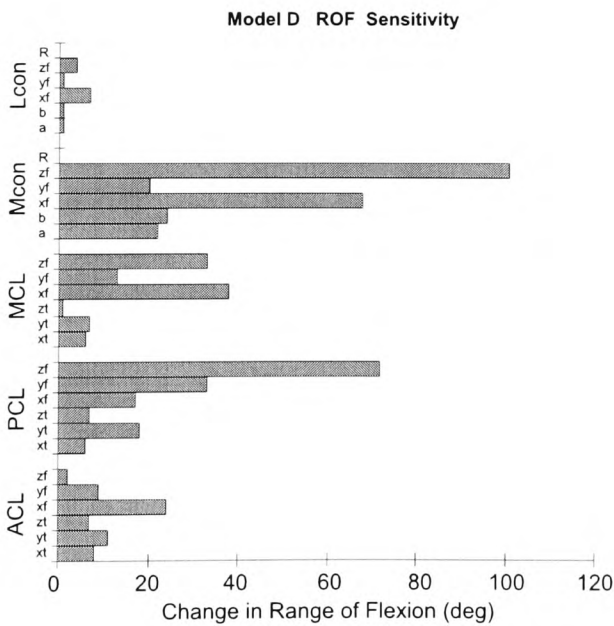


Figure 6.11: Sensitivity of range of flexion. Model D. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters.

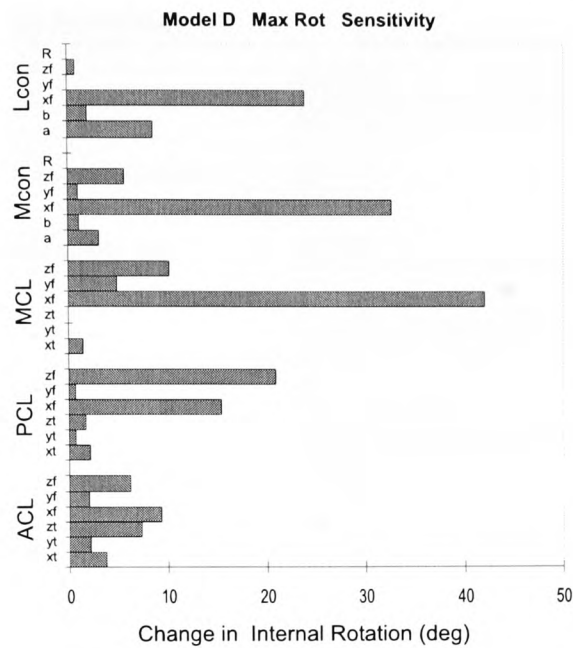


Figure 6.12: Sensitivity of maximum internal tibial rotation at 65° flexion (smallest range of motion predicted). Model D. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters.

the predicted range of motion and changes to the radii of the femoral spheres made no difference. The overall uncertainty in the model range of motion was found, in this case, to be $\pm 81.0^\circ$.

Because the smallest range of flexion predicted by Model A was virtually zero, the sensitivity of the model’s maximum internal rotation and net posterior contact excursions to parameter uncertainty was not examined.

The sensitivity of the maximum internal rotation and net posterior translation of the tibial contact points were examined in the case of Model D for the 65° flexion range (Figures 6.12 and 6.13). This was the smallest range of flexion predicted by any one of the 60 parameter sets. Both aspects of the movement were highly sensitive to uncertainties in some parameters and virtually insensitive to changes in others.

The maximum internal rotation (Figure 6.12) was most sensitive to changes in the x coordinate of the femoral MCL attachment and the x coordinates of the medial and lateral femoral sphere centres. The coupled rotation was insensitive to uncertainty in the length of the femoral radii (for the same reason described above), in the y and z coordinates of the MCL tibial attachment and the y coordinate of the lateral femoral sphere centre. The overall uncertainty in the maximum internal rotation (within the 65° flexion range) was calculated to be $\pm 30.4^\circ$.

The net posterior translation of the medial tibial contact point (Figure 6.13) was highly sensitive to changes in the x coordinates of the medial femoral condyle centre and the x coordinate of the MCL femoral attachment. The posterior excursion

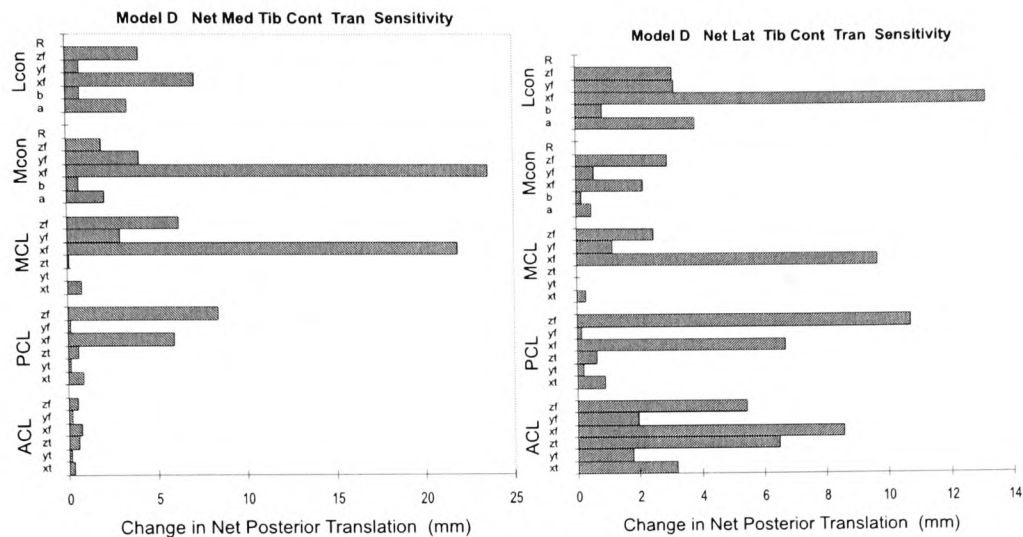


Figure 6.13: Sensitivity of net posterior translation of the tibial contact points within the 65° flexion range (smallest range of motion predicted). Medial compartment (left) and lateral compartment (right). Model D. Uncertainty of ± 5 mm or $\pm 5^\circ$ in the model parameters.

of the medial contact point was, however, virtually insensitive to changes in the tibial attachments of the ACL, PCL and MCL and again in the radii of the femoral spheres.

In the lateral compartment, the posterior excursion of the tibial contact showed the greatest sensitivity to uncertainty in the x coordinate of the lateral femoral condyle centre, the z coordinate of the PCL femoral attachment and the x coordinates of the MCL and ACL femoral attachments. The contact point movement laterally was virtually insensitive to changes in the coordinates of the MCL tibial attachment and, again, to the radii of the femoral spheres.

The overall uncertainty in the net posterior translation (within the 65° flexion range) was calculated to be ± 17.8 mm in the medial compartment and ± 10.9 mm in the lateral compartment.

6.5.1.3 Discussion

The results of the parameter study reveal that uncertainty in the input parameters can have a significant effect on the model range of motion. In the case of Model D, a model which predicts a full range of flexion based on its original parameters, a small change in one of the parameters caused the flexion range to be halved. Similarly, in the case of Model A, a model which predicted only limited flexion, a small parameter change led to the prediction of over 120° of flexion. The overall uncertainty in the range of motion was found to be 95° and 81° for the two models examined, suggesting that other specimen-specific models which were virtually immobile could achieve almost a full range of flexion with a combination of changes to the parameters

within the chosen levels of uncertainty.

The predicted pattern of model knee motion within a given range of flexion was also sensitive to uncertainties in the model input parameters. Both the coupled internal tibial rotation and the posterior excursion of the contact points on the tibia with flexion exhibit high sensitivity to certain input parameters. For Model D, the overall uncertainty in the predicted internal rotation (at 65°) of $\pm 30.4^\circ$ suggests that, for specimen-specific models which predicted coupled internal rotation which was either too high or too low, parameter sets within the range of uncertainty could be found which would lead to the prediction of a reasonable degree of axial rotation. Similarly, overall uncertainties in the net posterior contact point movement, ± 17.8 mm in the medial and ± 10.9 mm in the lateral compartment, suggest that parameter sets exist within the level of uncertainty which would yield more physiological contact point movement.

The results suggest that differences between the specimen-specific model predictions and measurements of knee movement reported earlier in this chapter can be attributed to uncertainty in the definition of the model parameters. The level of uncertainty chosen in the study, ± 5 mm or $\pm 5^\circ$, was on the order of the uncertainty in the definition of the model parameters, taking into account the influence of measurement error in the digitisation and reconstruction of the joint geometry and error associated with the simplified geometrical representation of both the ligaments and articular surfaces.

The study suggests that for each specimen-specific model an ideal set of parameters within the given level of uncertainty can be found; however, the problem lies in identifying this set. Wilson concluded, based in his original parameter sensitivity study, that with 30 independent parameters, considering only two possible values of each parameter, 2^{30} different parameter sets would have to be analysed [135]. Even if the solution time for the model kinematics were reduced to one second per parameter set, to examine all parameter sets would take over 33 years. Although searching for such an ideal set considering all possible sets within the range of uncertainty is not a realistic option, formulating the problem as a numerical optimisation may prove fruitful.

Earlier in the chapter, in an attempt to identify such an ideal set of parameters, a number of different representations of the surface and ligament constraints were considered in defining sets of specimen-specific models. The differences between the parameters corresponding to the WFC, WTC, PFC, PTC and WTP sets were on the order of the sample uncertainty. This permitted the identification of the most successful and consistent derivation of the model parameters from experimental measurements given the idealised surface and ligament representation.

One possible approach to improving the identification of the best-suited model

parameters is to identify those parameters to which the model outputs are particularly sensitive and try to reduce the uncertainty involved in their derivation. The parameter study has revealed certain parameters to which the model kinematics are particularly sensitive and others which have no effect on the predicted motion. The anteroposterior and proximodistal positions of the femoral condyle centres were seen to play a decisive role in the prediction of the range and pattern of knee motion. The range of motion is particularly sensitive to the proximodistal position of the medial condyle centre. This sensitivity to femoral condyle centre position may be explained by the fact that not only do the sphere centres define the location of the model contact constraints, but together they define the axis of flexion, the axis about which motion is driven in the model solution. The overall greater sensitivity of the model kinematics to the MCL parameters suggests that the difficulty in defining the tibial attachment of the MCL and hence in identifying the most isometric MCL fibre is of significant concern. In future, better identification and representation of the MCL isometric fibre must be a priority. Given the locations of the sphere centres, changes to the radii of the femoral spheres make no difference to the knee mobility. To accommodate a larger femoral radius, for example, the tibial plane is simply lowered so that contact may be maintained in the starting position. The resulting knee mechanism is kinematically equivalent.

The limitations of the methods used in studying the parameter sensitivity of the model must be considered. Only specific features of the model kinematics were examined. Investigating the sensitivity of every aspect of the model knee motion would serve only to complicate the interpretation of the results. The main features of passive knee movement which the model aimed to reproduce were considered sufficient in arriving at conclusions regarding the model sensitivity and improvement of the parameter derivation. The sensitivity of the pattern of knee motion was only studied using the basic parameter set of one model (Model D) and then only within a 65° range of flexion. Despite the limited flexion range, the results still serve to indicate which parameters were most critical in the prediction of coupled rotation and posterior contact point movement. Finally, it must be remembered that the overall uncertainties in the model outputs are only estimates. Whether the approximations made in formulating Equation 6.1 are valid is not certain due to the non-linear nature of the model analyses.

The results of the parameter study also have a number of clinical applications. Where the model mobility is particularly sensitive to changes in the ACL ligament parameters is of significance in designing and implanting a successful reconstruction of the ligament. Similarly, the fact that the range and pattern of knee movement is highly sensitive to the location of the medial femoral condyle centre suggests that the correct placement of the femoral component in knee replacement is critical if joint

mobility is to be restored. It is the proximodistal and anteroposterior positioning of the component in the fully extended knee which is of particular importance. The insensitivity of model knee motion to the length of the radii of the condyles of the femoral implies that when all other joint constraints have been retained or restored, *i.e.* by proper soft tissue balancing, kinematically, the size of the femoral component is irrelevant (in the case of sphere-on-plane geometry).

6.5.2 Constraint Violation Sensitivity Study

The aim of this study was to examine the sensitivity of the model to the method used to solve the set of non-linear constraint equations and to the permitted levels of constraint violation. The determination of model knee kinematics of a specimen-specific model which predicted only a limited range of flexion (Model A) was examined in greater detail.

6.5.2.1 Methods

Two different algorithms were implemented to solve the set of five non-linear constraint equations (Equations 5.4 and 5.5), *i.e.* to solve for the relative positions of the model bones, at each flexion angle.

Non-linear least squares method The first algorithm is the original non-linear least-squares method (Matlab’s ‘fsolve’, Appendix C) used to generate results presented thus far in this chapter. The algorithm aims to minimise the sum of the squares of the five constraint functions. Irrespective of whether the solution is considered a ‘root’, the algorithm always yields the solution to the least-squares minimisation problem. These minima were determined for a full range of flexion (120°). The associated model knee motion and the violation of the individual constraint functions was quantified.

Goal-attainment method A second algorithm, a goal-attainment method (Matlab’s ‘attgoal’, Appendix C), was also implemented. The multi-objective minimisation procedure aims to bring simultaneously each of the five individual constraint functions as close to zero as possible. Equal weighting of the individual constraint functions was chosen so that the minimum found was associated with the same level of violation of each constraint. This is appropriate because each constraint function represents the length of the kinematic link representing the ligament or contact constraint. These lengths are of the same order. As in the case of the least-squares minimisation described above, the local minima were determined for a full range of

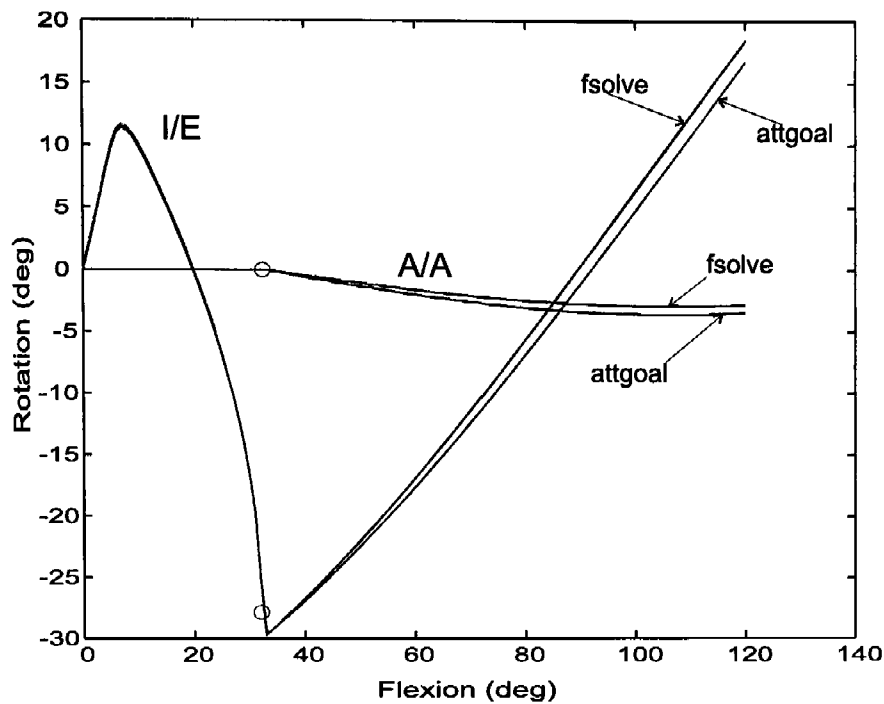


Figure 6.14: Predicted model joint rotations versus flexion angle using the non-linear least-squares method ('fsolve') or a goal-attainment method ('attgoal') to solve for the minimum of the governing constraint equations at each flexion angle. Up to the original 32° flexion (o), the solution is considered the exact root to the model equations.

flexion (120°). Again, the associated model knee motion and the violation of the constraint functions was quantified.

6.5.2.2 Results

Non-Linear Least-Squares Method At each flexion angle up to 120°, where a 'root' did not exist, the non-linear least-squares method successfully yielded a local minimum to the constraint equations. These local minima described model knee motion as shown in Figure 6.14. With the convergence tolerance set to its default value (10^{-4}), these minima constitute roots to the equations only up to 32° flexion. Just beyond 32° flexion, the slope of the motion path was discontinuous. The tibia, which had been externally rotating, suddenly began to rotate internally. In Figure 6.15, the corresponding levels of constraint violation are depicted for a series of flexion angles within the 120° range. With increasing flexion angle, the level of constraint violation increased up to 100°. Upon reaching 120°, however, the levels dropped off somewhat. The violation of the MCL isometry and the medial contact conditions was highest at each flexion angle. To achieve 120° of flexion, the maximum the MCL had to stretch was 1.4 mm and the maximum interpenetration of the medial femoral and tibial surfaces was 2.0 mm. The ACL and PCL never stretched more than 0.2 mm and the lateral articular surfaces never separated more

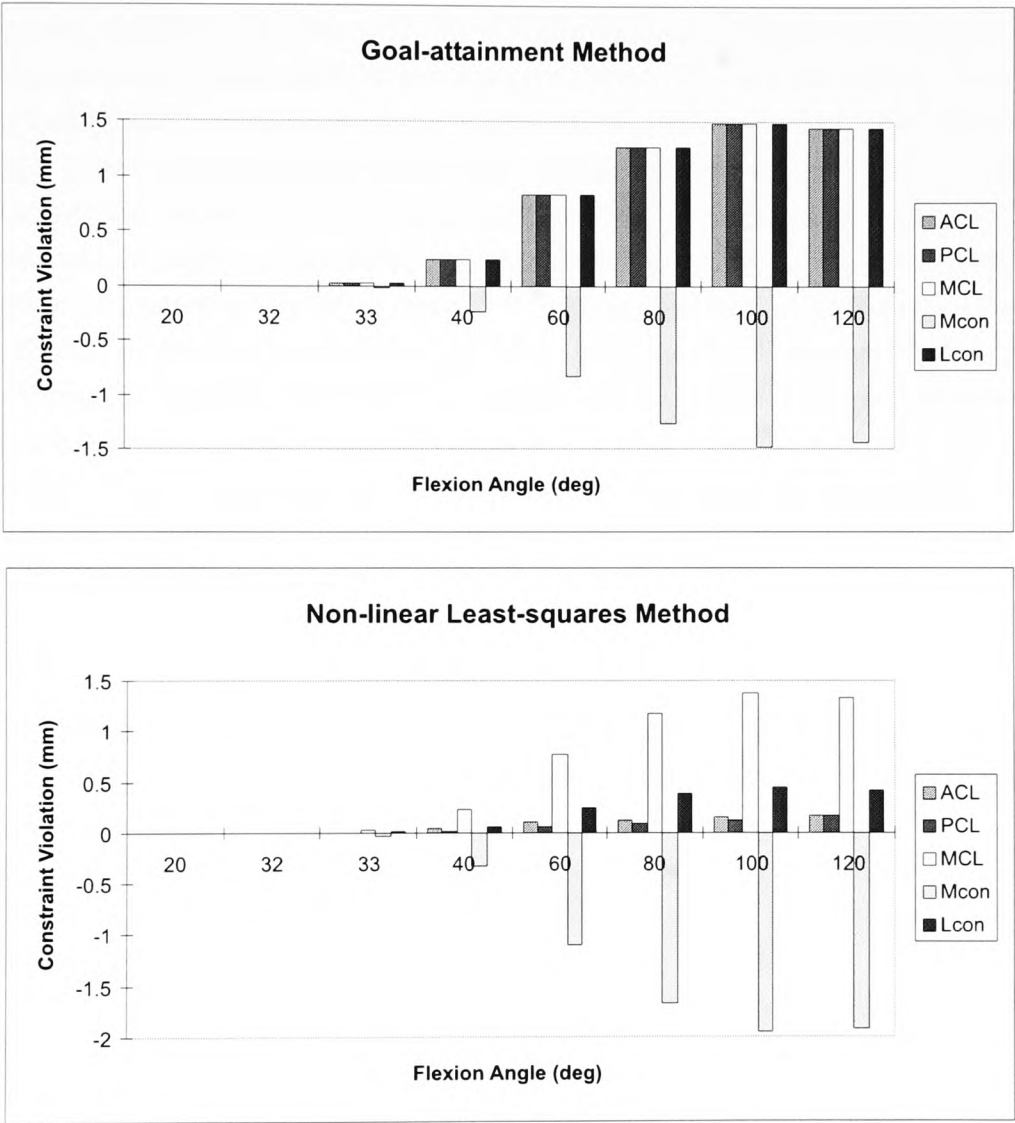


Figure 6.15: Levels of violation of each of the five constraint equations governing the idealised surface model of knee mobility. Positive and negative violation of the three ligament isometry conditions (ACL, PCL and MCL) represent stretching or slackening of the ligament respectively. Positive and negative violation of the contact conditions in the medial and lateral compartments (MCon and LCon) represent separation or interpenetration of the articulating surfaces respectively. Results for both the goal-attainment method and the non-linear least-squares method are shown.

than 0.5 mm.

Goal-Attainment Method Figure 6.14 also shows the joint rotations versus flexion described by the minima found using the multi-objective minimisation procedure. Model knee motion up to 120° flexion was only possible provided a level of constraint violation was tolerated. Up to 32°, the path of motion was identical to that found using the non-linear least-squares method. Beyond 32°, the motion paths described by the two different sets of minima were almost identical. In Figure 6.15, the associated levels of constraint violation are depicted for a series of flexion angles. The algorithm succeeded in determining a minimum at each flexion angle such that the amount of under- or over-achievement of each constraint equation was identical. To describe motion up to 60° of flexion, for example, a level of constraint violation of 0.83 mm must be tolerated. The ACL, PCL and MCL isometric fibres must be allowed to stretch by 0.83 mm, the medial surfaces allowed to interpenetrate by 0.83 mm and the lateral surfaces to separate by 0.83 mm. To achieve a full range of motion (120°) up to 1.5 mm of constraint violation must be permitted. At 32° flexion, the violation of constraints was virtually zero ($< 1 \times 10^{-11}$) while at 33° flexion, the level of violation was up by an order of 1×10^9 .

6.5.2.3 Discussion

Two different optimisation methods were implemented to solve the governing constraint equations of an idealised surface model of knee mobility. The aim was to clarify the definition of an ‘exact’ root to the equations and to examine the levels of constraint violation associated with predicted knee mobility. The study was performed to provide insight into the factors limiting the range of motion. The minimisation algorithms were also used to investigate the possibility of describing knee motion associated with minimal tissue deformation where an exact solution did not exist.

The range of motion for which an exact solution to the model equations exists is distinct from the range within which local minima to the constraint equations may be successfully identified. Using either the least-squares or the goal-attainment method, the level of constraint violation up to 32° flexion was less than 1×10^{-7} , virtually zero. Beyond that point, solutions representing minima were successfully identified but involved distinctly higher levels of constraint violation. With the goal-attainment method, for example, constraint violation associated with the minimum at 33° flexion, although still considered low at 0.02 mm, was on the order of 1×10^9 higher than that for the solution corresponding to 32° flexion.

Where an exact solution does not exist, minimisation of the governing constraint equations provides insight into which individual constraints are active in limiting

further motion. Using a non-linear least-squares to find solutions representing local minimums beyond the limiting flexion angle, the levels of violation of the MCL isometry and medial contact conditions were always highest. These results indicate that it is further stretching of the MCL isometric fibre or further interpenetration of the femoral and tibial articular surfaces in the medial compartment which are most likely limiting further flexion. This information indicates how one might alter the parameter set within the range of uncertainty to allow the prediction of further motion.

The study introduces the concept of a minimum-violation path versus a zero-violation path of motion along which the model bones move in passive flexion. If the stiffnesses of the constraints were incorporated into the objective functions these would be equivalent to minimum-energy and zero-energy paths. The zero-violation path is defined as that along which violation of the constraint equations is virtually zero ($< 1 \times 10^{-7}$), along which the bones can move with virtually zero deformation of the ligaments and articular surfaces. A minimum-violation path describes a path along which the bones can move while keeping stretching of the ligaments and interpenetration or separation of the articular surfaces to a minimum. The zero-violation path is also by definition a minimum-violation path. The difference between the two paths clearly depends on the definition of what level of deformation can be considered 'zero'. While it is extremely encouraging that in theory model knees have been successfully formulated which predict a full range of motion with virtually zero deformation (*e.g.* Models D, F and B), in reality, passive motion of the knee involves some minimal deformation of the ligaments and articular surfaces. The most isometric fibres, identified in experiment, for example, underwent strains on the order of 2-4%. The strict definition of knee mobility may explain why some models are incapable of predicting a full range of knee motion.

By solving for a minimum-violation path versus a zero-violation path, a description of model knee motion through a full range of flexion (120°) is possible, although not necessarily unique. Multi-dimensional minimisation is not a trivial problem [104]. In this case, using two different minimisation algorithms, slightly different motion paths were determined beyond the limiting flexion angle (Figure 6.14). By defining a maximum allowable level of constraint violation, a minimum-violation path can still provide a description of passive model knee motion beyond an original limited range of flexion. Although in this case the minima identified using the two methods were virtually the same, as the solution is not unique, the effects of using different minimisation criteria should always be examined. This approach will be discussed in more detail in the context of the predictions of a general surface model of knee mobility (Chapter 7).

Chapter 7

Knee Mobility: General Surface Model Predictions and Validation

The idealised surface model of knee mobility proved successful in predicting the main features of passive motion for selected sets of model parameters (Chapter 6). A principal limitation of the simplified surface model, however, was the overestimation of posterior contact point excursion on the tibial condyles during flexion. It was suggested that inclusion of more anatomical surface shapes would result in more physiological predictions of contact kinematics. In this chapter, predictions of a general surface model of mobility in which the femoral and tibial condyles are represented by polynomial surfaces are examined.

First, to determine the optimal level of surface complexity, the effect of increasing the degree of the polynomial surfaces representing the tibial and femoral condyles is examined. The predictions of a number of specimen-specific models are then reviewed. The associated computational effort and the nature of the solution involved in determining the model kinematics are presented. To demonstrate the utility of the model in the context of surface replacement and ligament reconstruction design, the effect of specific ligament and surface parameter changes on knee mobility are investigated. Finally, the applications and limitations of the general surface model are discussed.

7.1 Polynomial Surface Representation

A general surface model of mobility permits the incorporation of articular surface shapes of varying mathematical complexity. While in the experimental analysis of surface geometry (Section 3.1), polynomials of degree 4 and 3 (in x and y) were chosen to describe most accurately the tibial condyles and polynomials of degree 4 (in ψ and ϕ) were selected to describe most accurately the femoral condyles, these

higher order surfaces do not necessarily provide the most appropriate representations of the kinematic constraints supplied by the tibiofemoral articulation. The effect of increasing the degree of the polynomials from zero (spheres or planes) is studied in this section. The aim is to establish the minimum level of surface complexity required to improve the theoretical prediction of physiological knee motion, in particular with respect to contact point excursions.

7.1.1 Model Analyses

In this analysis, parameters are derived from the ligament and surface geometry of a single specimen (Knee A). The most nearly isometric ‘combination’ fibres were used to represent the ligamentous constraints provided by the ACL, PCL and MCL. The notation $[abcd]$ is used to describe a model in which the polynomials representing the tibial condyles are of degree a in x and degree b in y and those representing the femoral condyles are of degree c in ψ and d in ϕ . The surface model $[0000]$ corresponds to the case of femoral spheres articulating on transverse tibial planes¹. Tibial polynomials of degree 0 to 3 in x and y and femoral polynomials of degree 0 to 2 in ψ and ϕ were considered.

In this preliminary analysis, the total of over 20,000 possible tibial and femoral surface combinations was reduced to 12 by considering, in the first instance, only combinations in which the medial and lateral condyles were represented by polynomials of the same degree and in which each polynomial was of the same degree in the two orthogonal directions (x and y or ψ and ϕ). Errors involved in fitting the range of polynomial surfaces to the measured tibial and femoral condyles of a representative specimen are reported in Section 3.1.

7.1.2 Results

The predictions of a specimen-specific model of mobility in which the femoral and tibial condyles are represented by different combinations of polynomial surfaces are summarised in Table 7.1. It is also noted in the table whether any surface interpenetration of the tibial and femoral condyles occurred during the motion.

Planar or bilinear polynomial surface representation of the tibial condyles resulted in unphysiologically large excursions of the tibial contact point in the lateral compartment. With over 30 mm of translation, the lateral tibial contact point moved beyond the posterior edge of the condyle. Overestimation of contact point

¹In this analysis, rather than extending or decreasing the radii of the femoral spheres to correct for any initial contact error, the method implemented for the general surface model described in Section 5.4.4.2 was used. The predictions are therefore different from those of the corresponding WTC model (Section 6.1.1).

Surfaces [abcd]	ROF (deg)	Max Rot (deg)	NPM (mm)	NPL (mm)	IP (mm)
[0000]	>100	30.9	6.6	32.0	NO
[0011]	24	27.6	-6.9	16.2	YES (<0.2)
[0022]	>100	26.2	9.7	32.3	YES (<0.5)
[1100]	>100	42.8	1.0	33.0	NO
[1111]	>100	52.1	-0.7	34.5	YES (<0.2)
[1122]	>100	57.7	-1.9	34.9	YES (<0.6)
[2200]	>100	37.3	2.9	16.4	NO
[2211]	>100	43.7	1.1	15.7	YES (<0.3)
[2222]	8	8.0	-1.5	2.3	YES (<0.2)
[3300]	9	-5.8	3.4	0.2	NO
[3311]	>100	29.8	25.1	9.7	YES (<5.0)
[3322]	>100	49.2	4.6	18.3	YES (<7.0)

Table 7.1: Predictions of a specimen-specific general surface model of knee mobility (Model A) for 12 different polynomial tibial and femoral surface combinations [abcd]. Range of flexion (ROF), maximum internal tibial rotation (Max Rot) and net posterior tibial contact point excursions in the medial (NPM) and lateral (NPL) compartments are reported. IP=surface interpenetration

excursion was previously described as a limitation of the even the most successful idealised surface models (Section 6.2.5).

Retaining the spherical representation of the femoral condyles and increasing the order of the tibial polynomial surfaces, model predictions were generally closer to experimentally observed joint mobility. Incorporation of tibial surfaces of degree 2 in x and y ([2200]), resulted in predictions of coupled internal rotation on the order of that observed in *in vitro* experiments (Section 3.3.2) and contact point movement on the order of estimates based on experiment (Section 4.1)². Increasing the degree of the tibial polynomials further ([3300]), however, resulted in a reduced range of flexion. No surface interpenetration was found for any of the models in which the femoral condyles were represented by spheres (degree zero).

Increasing the degree of the polynomial surfaces representing the femoral condyles did not result in models which predicted motion any closer to that observed in experiment. The range of flexion was often reduced ([2222]) or internal rotation considered somewhat high ([2211]). Although in all but two cases ([3311] and [3322]) it was less than 0.6 mm, some degree of surface interpenetration accompanied the predicted joint movement of models in which the femoral condyles were represented by polynomials of degree greater than zero. These model predictions were considered invalid.

²The predictions of this model will be presented in detail in Section 7.2.

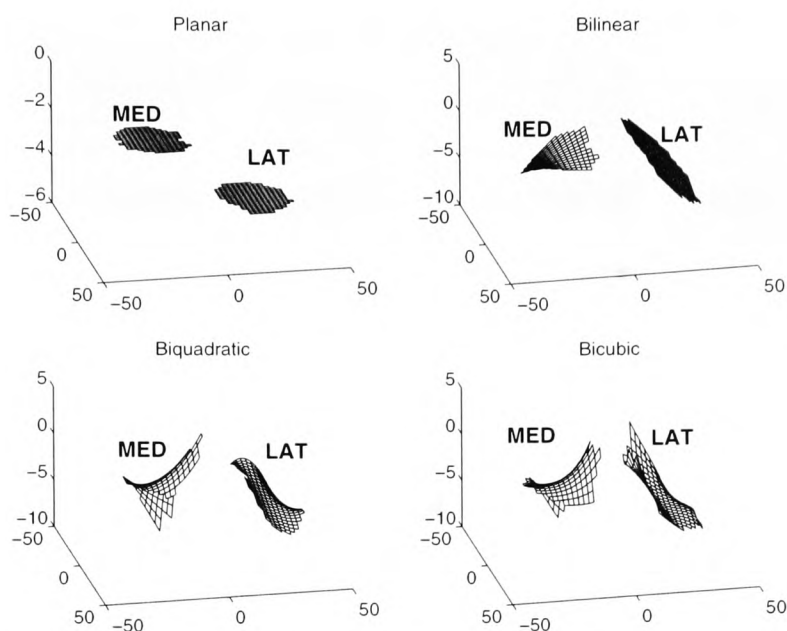


Figure 7.1: Polynomial representations of the tibial condyles in a specimen-specific model of mobility (Model A). Planar (degree 0 in x and y), bilinear (degree 1), biquadratic (degree 2) and bicubic (degree 3).

7.1.3 Discussion

The preliminary study suggests that spherical representation of the femoral condyles and biquadratic polynomial representation of the tibial condyles ([2200]) provides the best compromise between idealised and anatomical surface shape in a model of knee mobility. By incorporating any further complexity into the description of the femoral condyles (*e.g.* [2211] and [2222]), problems with surface interpenetration and limited ranges of motion are encountered. Describing the tibial condyles by polynomials of degree 3 ([3300]), in this case, resulted too in a model with only limited mobility. On the other hand, further idealisation of the tibial condylar surfaces (*e.g.* [0000]) results in predictions of knee motion, specifically contact point excursions, which are less physiological and closer to those of the idealised surface models (Chapter 6).

A limitation of the preliminary study is that the model analyses were based on surface and ligament parameters derived from a single specimen. The configuration of the ligament parameters may influence the trends observed in the effect of surface shape representation on model mobility. To address these issues, the predictions of three additional specimen-specific models with surface representation [2200] or [3300] will be examined in Section 7.3.

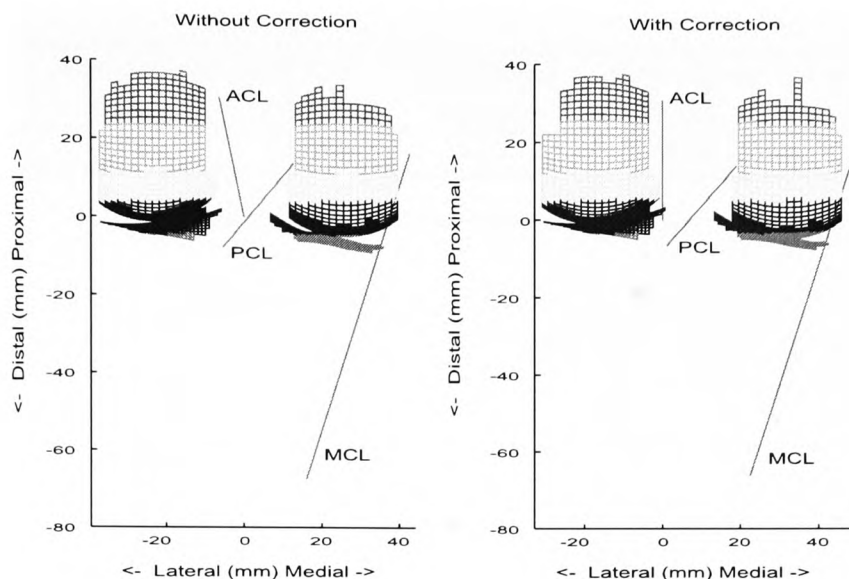


Figure 7.2: The starting configuration of a specimen-specific polynomial surface model (Model A). The relative positions of the bones and isometric ligament fibres without and with the associated contact error correction are shown. NB. The isometric ligament fibres in the corrected configuration, based on the adjusted experimental motion and a new search, are different and not simply shifted.

7.2 Specimen-Specific Model Predictions and Validation

In this section, all aspects of the predicted movement of the specimen-specific polynomial surface model (Model A) are examined in further detail. Based on the preliminary analysis described above, the tibial condyles were represented by biquadratic polynomials (degree 2 in x and y) and the femoral condyles by spherical surfaces (degree 0 in ψ and ϕ). Again, the most isometric ‘combination’ fibres provide the ligamentous constraints. The theoretical results are compared to the experimentally measured motion of Knee A and to observations reported in the literature. The predictions are also compared to those of the most successful idealised surface models.

7.2.1 Model Parameters

The parameters of the specimen-specific model (Model A) are given in Appendix D. The geometric configuration of the model in the starting position without and with the associated initial position correction (Section 5.4.4) is shown in Figure 7.2. The correction ensures that the contact conditions are met in the starting position of full extension. The joint rotations (under 0.2°) and translations (under 4 mm) describing the femoral displacement from its original measured position in full extension to the corrected position, calculated as described in Section 5.5, are shown in Table 7.2).

Flexion (deg)	Abduction (deg)	Int Rot (deg)	Post Trans (mm)	Med Trans (mm)	Prox Trans (mm)
0.00	-0.18	-0.06	3.73	0.76	0.24

Table 7.2: The initial position correction.

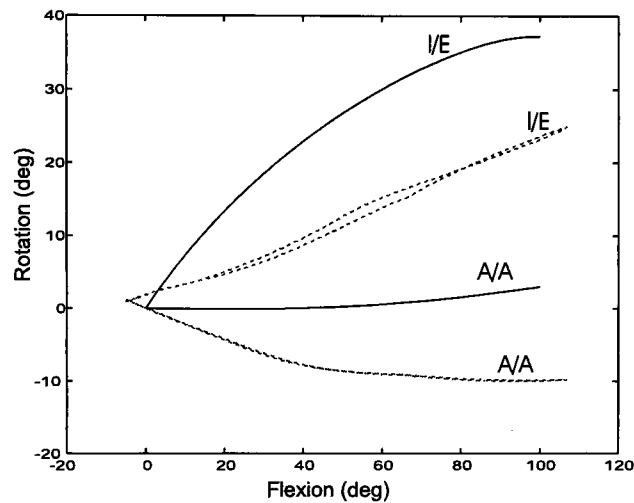


Figure 7.3: Predicted joint rotations (solid lines). Internal(+)/external(-) tibial rotation (I/E) and abduction(+)/adduction(-) (A/A) are plotted versus flexion. The measured rotations of the knee specimen used to derive the model parameters are also shown (Knee A) (dashed lines).

7.2.2 Joint Rotations and Translations

The specimen-specific polynomial surface model successfully predicted coupled internal rotation with flexion. The peak rotation of 37.3° is somewhat higher than the maximum of 24.9° recorded in experiment (Figure 7.3)³. The maximum predicted internal tibial rotation, however, does lie near the high end of the range of measured peak rotations of twelve knee specimens (14°-36°, Section 3.3.2).

The pattern of coupled internal/external rotation predicted by the polynomial surface model is closer to that of the experimentally recorded movement than that predicted by the most successful specimen-specific idealised surface models (Section 6.2.3). Although current peak internal tibial rotation is somewhat higher than that predicted by the idealised surface models (24.1, 33.6 and 15.9° for Models D, F and B), the tibia continues to rotate internally with increasing flexion as observed in experiment. In contrast, with the idealised surface models, internal rotation peaks near mid-flexion and then the tibia begins to externally rotate.

While the model predicted higher internal rotation than observed in experiment, it exhibited less adduction, a maximum of 3.0° abduction occurring at 100° flexion.

³Both the measured and predicted joint rotations are described here with the flexion axis defined to lie along the line passing through the centres of the *whole* femoral spheres.

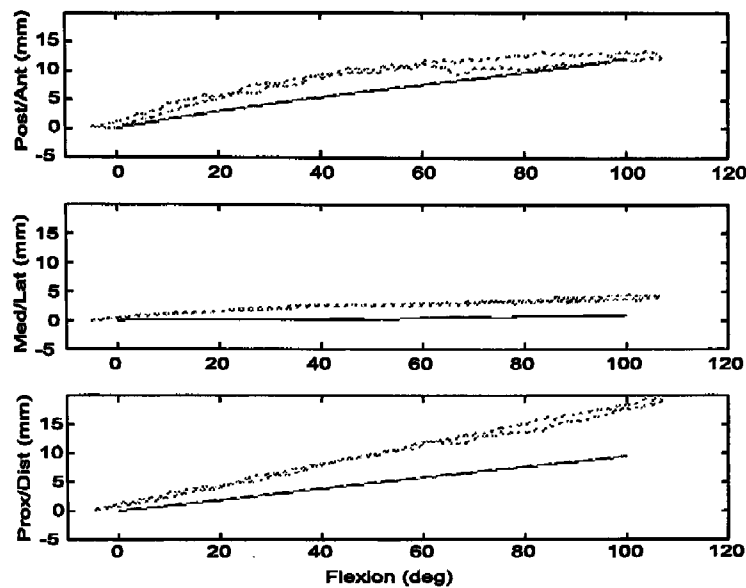


Figure 7.4: Predicted joint translations (solid lines). Posterior(+)/anterior(-), medial(+)/lateral(-) and proximal(+)/distal(-) components of the translation of the reference point are plotted versus flexion. The translations of the same reference point measured during flexion and extension of the knee specimen used to derive the model parameters are also shown (Knee A) (dashed lines).

Knee adduction reaching a peak of 10.0° , however, was recorded in experiment. Again, the predicted value lay within the range of measured peak rotations of twelve knee specimens (9° adduction to 9° abduction, Section 3.3.2).

Although the match between theoretical and experimentally recorded ab/adduction is not excellent, a polynomial surface model is capable of predicting this component of movement. The idealised surface models were not able to account for any ab/adduction due to the relative orientations of the tibial planes and the axes of motion.

Posterior and proximal translation of the tibial reference point was predicted by the model (Figure 7.4). The maximum posterior translation of 12.1 mm corresponds exactly with the 12.1 mm of posterior translation recorded in experiment at 100° flexion. The peak proximal translation of 9.5 mm, however, is somewhat lower than the 17.5 mm recorded in experiment. Almost no medial translation was predicted by the model (maximum 0.9 mm), consistent with the minimal medial translation observed in experiment (maximum 3.7 mm).

7.2.3 Instantaneous Helical Axes

The instantaneous helical axes (IHAs) describing model knee movement are represented with respect to the fixed tibial coordinate system in Figure 7.5. The nine axes, corresponding to flexion increments of 10° from 10° to 90° flexion, describe the tibial axode of the model knee.

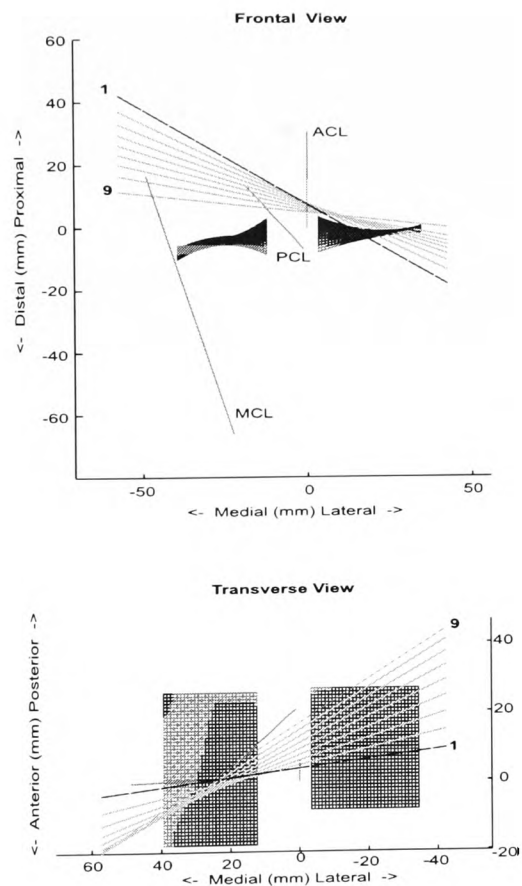


Figure 7.5: The model IHAs describing the motion of the femur relative to a fixed tibia (tibial axode). The first axis (1) corresponds to 10° flexion and the ninth axis (9) to 90° flexion. The model configuration is depicted in its starting position (full extension).

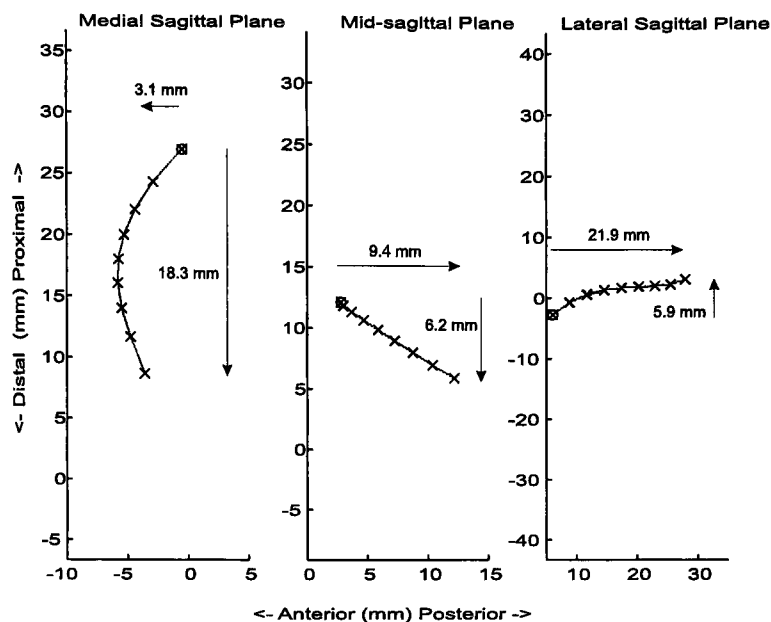


Figure 7.6: Intersections of the model IHAs with mid-sagittal and medial and lateral para-sagittal planes. The circle (o) represents the IHA corresponding to 10° flexion and the crosses (x) indicate the axis locations for 10° flexion increments. The net posterior and distal shifts of the IHA in each plane are also noted.

In the frontal plane, the predicted IHAs, initially oblique and inclined towards the lateral tibial plateau, shift to lie more parallel to the plateau with increasing flexion, reflecting the coupling of internal rotation to flexion (Figure 7.5). The changing orientation of the model IHAs in the frontal plane is consistent with that of sets of finite helical axes (FHAs) describing the motion of cadaver knees (*e.g.* Figure 3.12(a)). The approximate pivot point of the model IHAs in the frontal plane did, however, lie somewhat more central to the joint.

In the transverse plane, the model IHAs, initially nearly parallel to the medio-lateral axis of the tibia, move posteriorly and become more oblique with increasing flexion (Figure 7.5). The changing orientation of the model IHAs in the transverse plane is also similar to that of sets of experimental FHAs (*e.g.* Figure 3.12(b)). The IHAs tend to pivot approximately about a point located over the medial compartment of the joint whereas the approximate pivot point of the experimental FHAs lay more medial to the joint.

The intersections of the model IHAs with a mid-sagittal and para-sagittal planes passing through the medial and lateral femoral sphere centres in the fully extended knee are shown in Figure 7.6. The para-sagittal planes are analogous to those used to describe the movements of the corresponding experimental FHAs (Section 3.3.4)⁴.

The posterior and distal shifts of the IHAs in the mid-sagittal plane, 9.4 mm and

⁴Again, for the model results, the planes are defined to pass through the centres of the 'whole' spheres. The mediolateral positions of these centres are virtually identical to those of the 'posterior' spheres used to define the para-sagittal planes in the experimental FHA analysis.

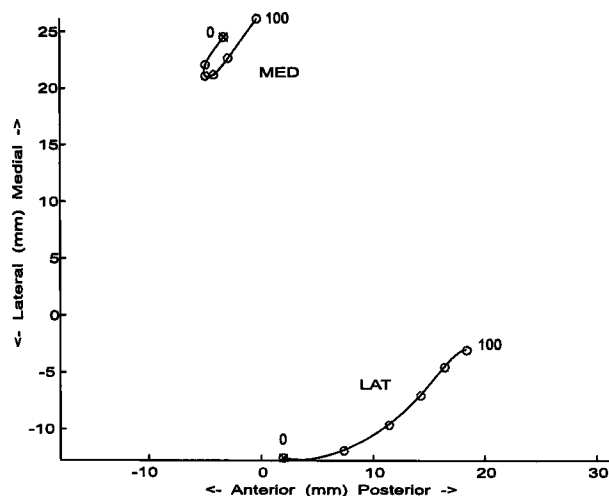


Figure 7.7: Transverse view of tibial contact point displacement in the medial and lateral compartments. The circles (o) on the contact paths represent steps of 20° flexion.

6.2 mm respectively, were similar to the corresponding mean shifts of the experimental FHAs for twelve specimens (8.2 mm and 7.5 mm, Table 3.10). The match between theory and experiment is better than that found for the idealised surface models which predicted generally larger posterior shifts and *proximal* shifts of the IHA in the mid-sagittal plane (Section 6.2.4).

The IHA shifts in the medial and lateral para-sagittal planes were not as consistent with the mean pattern of FHA movement as those in the mid-sagittal plane; however, the match between theory and experiment was generally better compared to that reported for the idealised surface models (Section 6.2.4).

The mean helical axis (MHA), representing the average of the set of model IHAs describing knee motion from 0° to 100° flexion in increments of 1° flexion, is oriented at 19.0° from the tibial mediolateral axis in the frontal plane (α) and at 23.2° relative to the same axis in the transverse plane (β). The orientation of the MHA is generally consistent but somewhat more oblique in both planes compared to the mean values describing the flexing motion of twelve cadaver knees ($\alpha=10.2^\circ$, $\beta=14.3^\circ$, Table 3.11).

7.2.4 Contact Movement

The polynomial surface model predicted net posterior translation of the tibial contact points of 2.9 mm and 16.4 mm in the medial and lateral compartments respectively (Figure 7.7). In the medial compartment, anterior translation of the contact point up to 40° flexion was exhibited. Some net medial translation of the contact points was also predicted with contact in the lateral compartment occurring in the region of the tibial eminence at higher flexion angles (Figure 7.8).

The magnitude and pattern of contact point excursion is in good agreement with

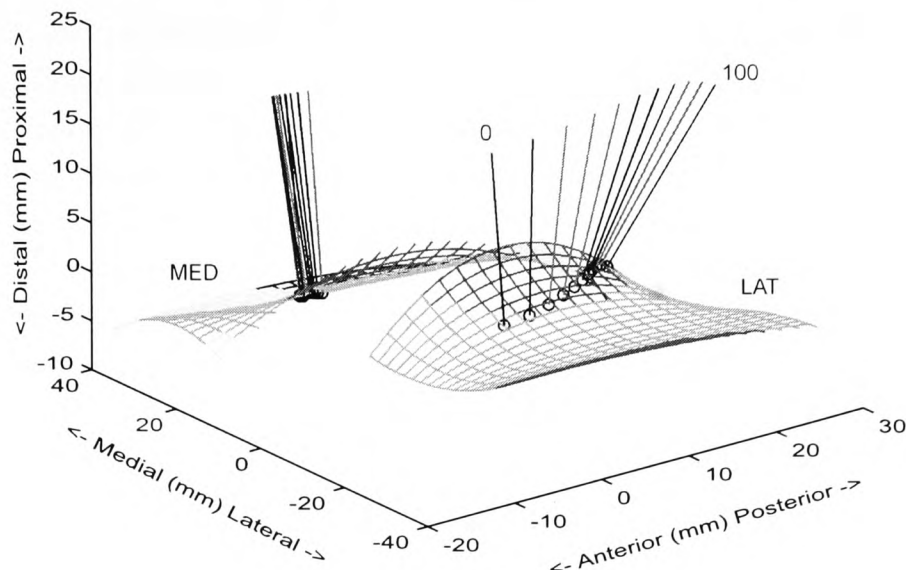


Figure 7.8: Contact point displacement on the medial and lateral tibial condyles. The circles (o) on the contact paths represent steps of 10° flexion and the lines depict the direction of the contact normals.

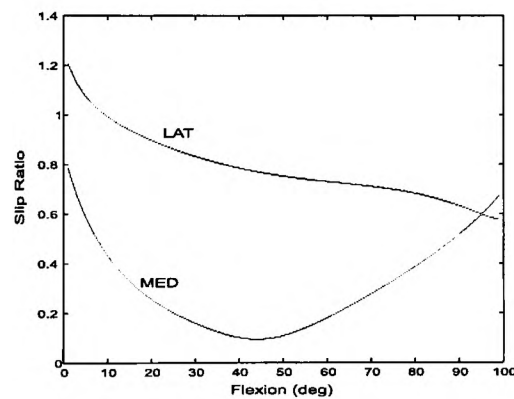


Figure 7.9: The slip ratio as a function of flexion angle.

estimates based on measurements of surface geometry and motion (Section 4.1). In the lateral compartment, the net posterior translation of 16.4 mm lies close to the mean estimate of 14.7 mm for twelve knee specimens (Table 4.1). The net posterior movement of the medial contact point, 2.6 mm, is smaller than the mean experimental estimate of 7.9 mm but lies within the range of one standard deviation (± 7 mm). The pattern of tibial contact point movement, with greater excursion in the lateral compartment than in the medial compartment, is also consistent with previous reports in the literature [64, 125] (Section 4.1). The polynomial surface model predicts more physiological contact point movement compared to that exhibited by idealised surface models, in particular with respect to the lateral compartment of the joint.

Relative motion at the medial and lateral contact points was once again characterised by the slip ratio as described in Section 6.2.5 in the context of the idealised

surface model predictions. In the lateral compartment the ratio starts off above one, implying a proportion of translational sliding of the femur on the tibia in early flexion (Figure 7.9). Near 10° flexion, the femoral motion consists transitionally of pure rolling. In the medial compartment, the ratio remains below one throughout the flexion range, reflecting a combination of rolling and sliding of the femur on the tibia. Once again, the magnitude of the slip ratio must be considered in conjunction with the relative directions of the tibial and femoral contact point movement. In the medial compartment some of the early sliding of the femur on the tibia occurs with anteriorly directed movement of the tibial contact point; the simple definition of the slip ratio used in the current analysis, however, was always positive. At higher flexion angles, the slip ratio in the lateral compartment approaches that in the medial compartment. Consistent with estimates of experimental contact kinematics (Section 4.1.1), the polynomial surface model predicts a greater proportion of sliding in the medial compartment than in the lateral compartment for almost the entire flexion range. Less translational sliding of the femur in the lateral compartment is predicted in early flexion compared to the idealised surface models. Such sliding was also not a predominant characteristic of the experimental contact kinematics.

7.3 Summary of Additional Specimen-Specific Model Predictions

The predictions of three additional specimen-specific polynomial surface models are summarised in this section. The parameters of the three models examined were derived from measurements made on Knees F, B and H, the remaining intact specimens tested in Lab 2. The contact error associated with these specimens was much lower than that for knees tested in Lab 1 (Section 4.3.3). The initial position correction required was therefore smaller and the model parameters deemed more accurate. Based on the success found with a single specimen-specific model (Model A), the model femoral condyles were represented in each case by polynomials of degree 0 in ψ and ϕ (spheres) and the model tibial condyles by polynomials of degree 2 or 3 in x and y respectively (biquadratic or bicubic). The ligamentous constraints were again represented by the most isometric ‘combination’ fibres in the ACL, PCL and MCL.

7.3.1 Range of Motion

In the case of Model A described in the previous section, the solution to the constraint equations defining the model knee kinematics was ‘exact’ for the reported range of flexion. The model bones moved along the unique zero-violation path for

Model	Surfaces	ROF (deg)	Max Rot (deg)	NPM (mm)	NPL (mm)
A	[2200]	100 (0-100)	37.3	2.9	16.5
F	[2200]	100 (0-6)	36.5	2.9	16.3
B	[2200]	100 (0-23, 42-100)	27.6	7.0	19.7
H	[3300]	100 (0-56)	38.3	2.4	15.7

Table 7.3: Polynomial surface model predictions for four sets of specimen-specific parameters. For the range of flexion, the range indicated in brackets refers to that for which an ‘exact’ root was found.

over 100° of flexion. The three additional specimen-specific models exhibited more limited ranges of flexion: 6, 23 and 56° flexion (Table 7.3). In this analysis, where the solution method no longer converged to an ‘exact’ root, the minimum determined by the non-linear least-squares root-finding procedure (Matlab’s ‘fsolve’, Appendix C) is considered to provide an approximation to the passive joint movement for the 100° range of flexion. Associated levels of constraint violation are reported and the sensitivity of the predictions to the optimisation method used discussed in Section 7.4.

7.3.2 Pattern of Motion

The three additional specimen-specific polynomial surface models exhibit coupled internal rotation and posterior translation of the tibial contact points with flexion on the order of that predicted by Model A (Table 7.3). Despite the higher peak internal tibial rotation, the pattern of the model knee movement was in general agreement with measured rotations and translations of the corresponding cadaver knees (Figures 7.10 and 7.11). Contact point excursions in the medial and lateral compartments were close to the physiological levels predicted by Model A.

Although it provides an estimate of the model knee rotations, translations and contact point excursions, the description of knee movement provided by the minimum-violation path is limited by its inexact nature. At the point where the model knee motion switches from being described by the unique zero-violation path to being approximated by a minimum-violation path, discontinuities arise. These discontinuities, not always apparent in the joint rotation and translation curves, prohibit a meaningful description of the contact kinematics in terms of the slip ratio. Moreover, where the solution to the model kinematics is not exact, the joint system can no longer be described by a true mechanism and the direct calculation of the IHA using the principle of reciprocity is not applicable.

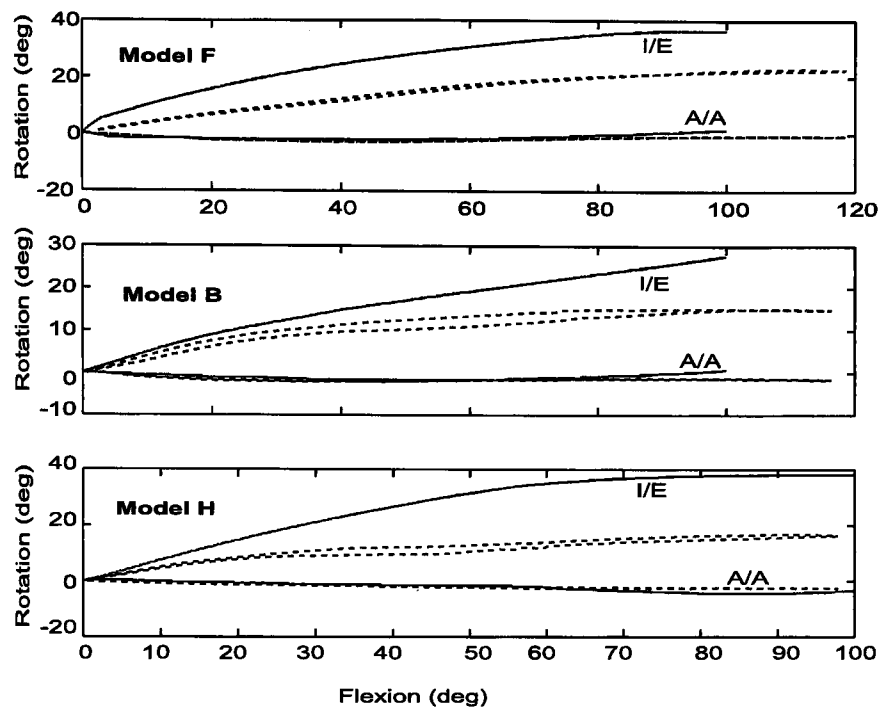


Figure 7.10: Predicted joint rotations of Models F, B and H (solid lines). The measured rotations of the knee specimens used to derive the model parameters are also shown (Knees F, B and H) (dashed lines).

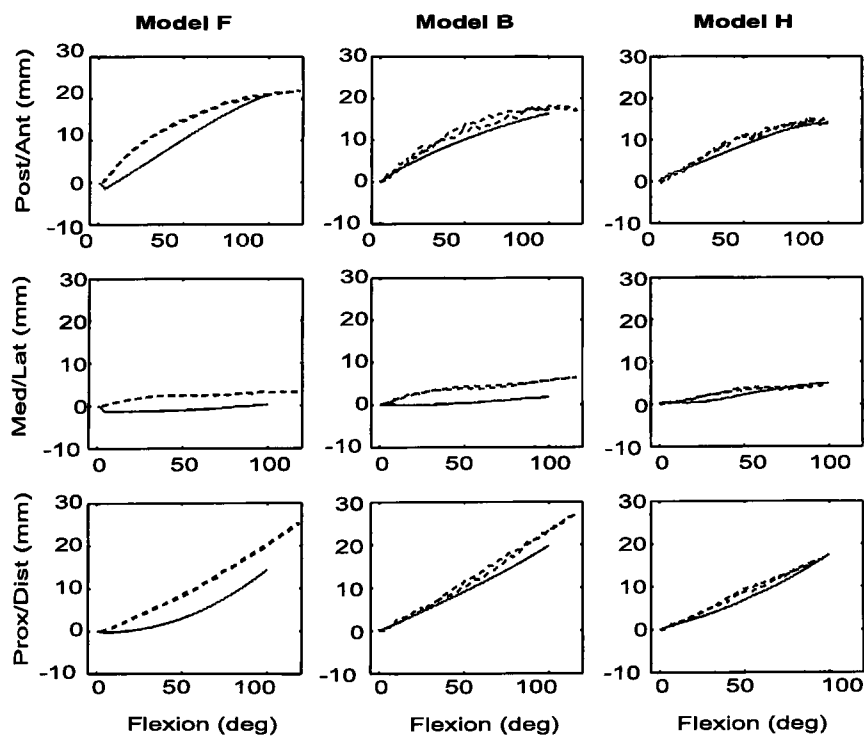


Figure 7.11: Predicted joint translations of Models F, B and H (solid lines). The translations of the same reference point measured during flexion and extension of the knee specimens used to derive the model parameters are also shown (Knees F, B and H) (dashed lines).

7.4 Nature of the Model Solution and Constraint Violation

In this section, the nature of the solution to the model kinematics of the polynomial surface model of knee mobility is examined in more detail. The validity of the approximation of passive knee movement provided by the minimum, rather than the ‘exact’ root, to the constraint equations is assessed. The levels of constraint violation associated with the description of a minimum-violation versus a zero-violation path, and the sensitivity of the results to the minimisation procedure used, are investigated.

7.4.1 ‘Exact’ Solution

‘Exact’ solutions define the unique zero-violation path of the one-degree of freedom joint system. As described in Section 6.5.2.3 in the context of the idealised surface model, an ‘exact’ solution corresponded to constraint function violations on the order of 10^{-7} . As in the case of the idealised surface model, the range of motion for which an ‘exact’ solution exists was clearly distinct from that for which a minimum approximating a root was determinable. The range of flexion for which an ‘exact’ solution existed varied across the four specimen-specific polynomial surface models (Table 7.3). Only in the case of Model A did an ‘exact’ solution exist for over 100° of flexion. In contrast, for Model F, an ‘exact’ solution was found for only 6° of flexion.

7.4.2 ‘Minimum’ Solution

In the case of Models F, B and H, where an exact solution did not exist, the non-linear least-squares optimisation was successful in determining a minimum to the constraint equations throughout the 100° flexion range examined. The procedure minimises the sum of the squares of the 13 constraint equations (Section 5.3.2.1).

7.4.2.1 Constraint Violation

The minima provide a description of passive knee movement which is associated with a minimal level of constraint violation. Constraint violation represents stretching or slackening of the ‘isometric’ ligament fibres or interpenetration or separation of the tibial and femoral articular surfaces. Figures 7.12, 7.13, and 7.14 depict the levels of constraint violation associated with the solution to the model knee motion of Models F, B and H. In the case of Model F, for example, the ‘isometric’ fibres of the ACL and MCL must stretch by up to 0.02 mm and 1.38 mm, respectively, and the

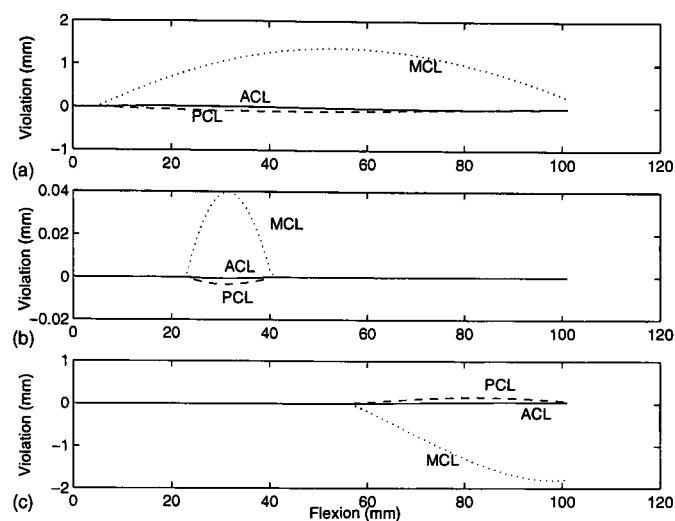


Figure 7.12: The ligament constraint violations associated with the reported kinematics of (a) Model F, (b) Model B, and (c) Model H. The violations correspond to the stretching or slackening of the ACL, PCL and MCL ‘isometric’ fibres with respect to their lengths in the fully extended knee.

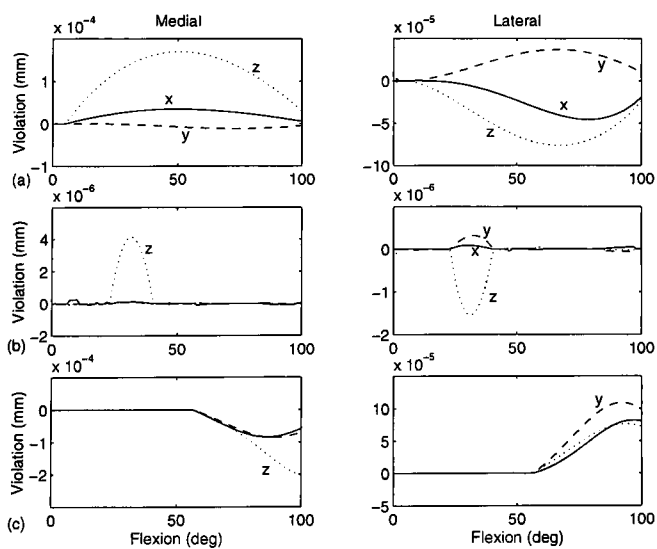


Figure 7.13: Coincidence of contact constraint violations associated with the reported kinematics of (a) Model F, (b) Model B, and (c) Model H. The violations correspond to the differences in the x (posterior), y (medial) and z (proximal) coordinates of the tibial and femoral contact points in the medial and lateral compartments.

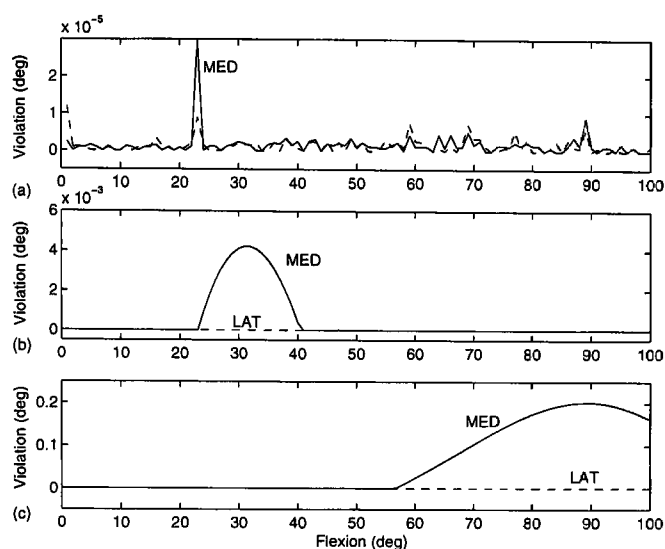


Figure 7.14: Contact normal constraint violations associated with the reported kinematics of (a) Model F, (b) Model B, and (c) Model H. The violations correspond to the angular deviation between the tibial and femoral contact normals in the medial (MED) and lateral (LAT) compartments.

PCL fibre must slacken by 0.1 mm to achieve the reported 100° range and pattern of passive flexion. Violation of the conditions enforcing coincidence of contact was less than 10^{-3} mm and the tibial and femoral contact normals at the point of contact deviated from each other by less than 10^{-4} degrees.

Some differences in the associated constraint violation were found between specimen-specific models. The ligament constraint violations accompanying the predicted motion of Model B, for example, were much lower than those of Model F. Only up to 0.001 mm and 0.01 mm slackening of the ACL and PCL fibres, respectively, and up to 0.4 mm stretching of the MCL fibre accompanied the motion. With Model H, the ACL and PCL fibres stretched by no more than 0.04 mm and 0.2 mm, respectively, but the MCL fibre slackened rather than stretched by up to 1.9 mm.

Despite these differences, there were some trends observed in the levels and patterns of constraint violations across the three models. In all three cases, the misalignment of the contact normals was always greater in the medial compartment than in the lateral compartment. The MCL fibre was always required to stretch or slacken by significantly more than the ACL and PCL fibres. As was suggested in a similar study of an idealised surface model (Section 6.5.2.3), it is the isometry of the MCL fibre and the inability of the femoral and tibial articular surfaces to penetrate or separate in the medial compartment which tend to limit the flexion range. Again, such information indicates how one might improve the model formulation to allow the ‘exact’ prediction of a full range of physiological motion.

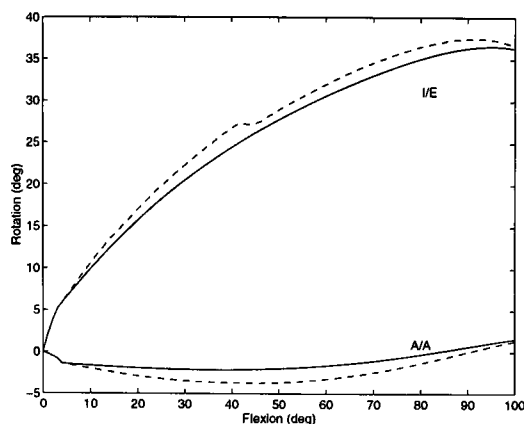


Figure 7.15: The joint rotations exhibited by Model F using both a non-linear least-squares method (solid lines) and a goal-attainment method (dashed lines).

7.4.2.2 Sensitivity of Minimum-Violation Path

As was mentioned in the context of the idealised surface model (Section 6.5.2), by solving for a minimum-violation path versus a zero-violation path, a description of model knee motion through a full range of flexion is possible, although not necessarily unique. The minimum-violation path found is, for example, sensitive to the minimisation procedure used.

A multi-objective, goal-attainment method (Matlab's 'attgoal', Appendix C) was also used to solve for the kinematics of Model F. In the case of the polynomial surface model formulation, the goal-attainment method aims simultaneously to bring each of the 13 individual constraint functions as close to zero as possible. The weighting of the individual constraint functions was defined such that the levels of violation associated with each minimum are equal for each of the 13 ligament and contact constraint functions.

Despite significant differences in the levels of constraint violation, the model joint motion is remarkably similar. The minimum-violation path determined using the goal-attainment optimisation method differs only slightly from that found using the non-linear least-squares method (Figure 7.15). In this case, the maximum violation of each of the 13 constraint functions, however, was 0.12 mm. With respect to the contact normals, this level corresponded to deviations of 10.0° in the medial compartment and 0.53° in the lateral compartment.

7.5 Computational Effort

Using the non-linear, least-squares root-finding optimisation algorithm programmed in Matlab (Matlab's 'fsolve' routine, Appendix C), the determination of the model kinematics of Model A through 100° of flexion required a total of 12 493 function

Model	Function Evaluations	Solution Rate (sec/deg)
A	12 493	9.7
F	46 913	31.7
B	30 053	20.8
H	30 677	26.3

Table 7.4: Computational effort associated with four specimen-specific polynomial surface models.

evaluations, *i.e.* each of the thirteen constraint functions was evaluated 12 493 times. The evaluation time was 16.3 minutes, corresponding to a solution rate of approximately 9.7 seconds per degree of flexion running on an Indy workstation (Silicon Graphics, USA).

Where an ‘exact’ solution did not exist and the method solved for a minimum to the constraint equations, the number of function evaluations and the computation time required was significantly increased (Table 7.4).

7.6 Parameter Studies

The preliminary studies of surface constraint representation (Section 7.1) provided some insight into the parameter sensitivity of the polynomial surface model of knee mobility. Due to the substantial computational effort and the more complex description of the model surface parameters, the effects of uncertainties in each of the independent parameters were not studied as was done in the case of the idealised surface model. Instead, the effect of specific, clinically relevant ligament and surface parameter variations are examined. The parameter studies, focusing on aspects of ligament reconstruction and surface replacement, serve to demonstrate the utility of the model of mobility.

7.6.1 Ligament Parameters

A primary objective of ligament reconstruction is the restoration of the joint’s range of motion and the pattern of movement within that range. In designing and implanting a successful ACL reconstruction, the effect of variations in the geometric parameters, *i.e.* the attachment sites of the graft, on the resulting joint mobility is therefore of utmost relevance. Reviewing the variety of techniques employed during ACL reconstruction, geometrically, they differ for the most part with respect to the location of the femoral attachment site. Insight into the effects of such differences on the restoration of knee mobility is critical in determining the optimal placement

Variation	ROF (deg)	Max Rot (deg)	NPM (mm)	NPL (mm)
Post 5 mm	100	39.0	12.8	14.0
Ant 5 mm	130	29.1	4.3	15.7
Med 5 mm	113	37.3	4.0	14.9
Lat 5 mm	116	35.2	3.4	15.0
Prox 5 mm	112	36.7	3.8	14.9
Dist 5 mm	120	35.7	3.4	15.0

Table 7.5: Predictions of a polynomial surface model of knee mobility (Model A) for variations in the femoral attachment of the ACL. The maximum internal tibial rotation (Max Rot) and tibial contact point excursions (NPM and NPL) correspond to the 100° flexion range.

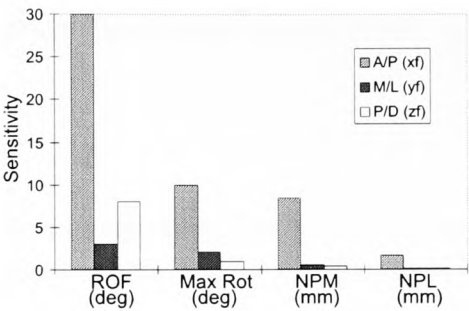


Figure 7.16: The sensitivity of the main features of the predicted motion to variations in the location of the femoral attachment of the ACL.

of a graft.

7.6.1.1 Methods

The effect of variations in the location of the femoral attachment of the ACL on knee mobility were examined. The attachment site was varied by ± 5 mm in the antero-posterior (x), mediolateral (y) and proximodistal (z) directions in the fully extended knee. The analyses are based on the specimen-specific parameters of Model A (Section 7.2).

7.6.1.2 Results

The predicted joint motion was most affected by changes in the anteroposterior location of the femoral attachment of the ACL fibre (Table 7.5). The sensitivity of the range of flexion and maximum internal rotation and net posterior tibial contact point excursions, are highest for deviations in the x coordinate (Figure 7.16). The net translation of the tibial contact point in the medial compartment was also more sensitive to changes in the ACL femoral parameters than that in the lateral compartment.

The relative sensitivity of the polynomial surface model to uncertainties in the location of the ACL femoral attachment is comparable to that of an idealised surface model. In the case of an idealised surface model (Model D), the main features of the model knee motion were also most sensitive to variations in the x coordinate of the ACL femoral attachment compared to its y and z coordinates (Figures 6.11, 6.12 and 6.13). The sensitivities of the range of motion and of the maximum internal rotation to the anteroposterior location of the attachment, 30° and 10° , respectively, are of the same order as those corresponding to an idealised surface model (25° and 10° respectively). With the idealised surface model, the sensitivity of the net contact point excursions also varied between medial and lateral compartments; however, movement in the lateral compartment tended to be more sensitive to changes in the ACL parameters than that in the medial compartment, the opposite result to that found here.

7.6.1.3 Discussion

The results suggest that when performing an ACL reconstruction it is the anteroposterior positioning of the femoral attachment which is most critical to the restoration of normal range and pattern of knee mobility. If, for example, the graft is placed too far posteriorly on the femur, the model knee's range of motion is somewhat reduced. The graft in this case would have to stretch or slacken to allow the joint a full range of motion. In either case, the guiding role of the ligament is either lost and, in the case of stretch, there is a risk of graft damage. Determining a minimum-violation path beyond the predicted range of flexion, such as that described in Section 7.4.2, the level of stretch or slackening may be estimated.

The current findings are in agreement with previous studies of ligament isometry and reconstruction reported in the literature but provide additional insight into the effects of graft placement on post-reconstruction joint kinematics. In an isometry study of the ACL, Hefzy and Grood [50] found that the anteroposterior position of the femoral attachment was the primary determinant of whether the distance between tibial and femoral attachment sites changes with flexion. In a theoretical study of ACL reconstruction, Blankevoort [13] concluded that isometry and the success of a graft in restoring normal laxity to the joint depends mainly on the anteroposterior location of the femoral attachment. It was also pointed out in both studies that any change in the motion characteristics of the knee after ligament reconstruction may affect the isometry of a ligament. In the current approach, a number of isometric ACL fibre locations are considered; how these fibres then control the range and pattern of knee movement is studied. The approach is novel because it allows the study of the effect of graft placement on the range and pattern of coupled knee movement, rather than on ligament/graft isometry alone which is most likely

affected by the change in knee motion after reconstruction.

7.6.2 Surface Parameters

As in the case of ligament reconstruction, a main aim of joint replacement is restoration of joint mobility. Again, the kinematic geometry of the joint depends not on the mechanical properties of the implants but on the shapes of the components, on their positions on the bones and the geometric arrangement of the residual soft tissues.

In knee replacement, the tibial and femoral anatomical surfaces are often replaced by idealised surfaces. With the Oxford Knee, for example, a meniscal unicompartamental replacement, all ligaments are retained and the tibiofemoral articulation is replaced by that of a sphere on a plane. Knowledge of the effect of such surface shape changes on knee mobility is critical in the design and implantation of such components. In this chapter it has already been demonstrated that spherical surfaces adequately represent the femoral condyles in controlling knee mobility. Second order polynomial surfaces, however, were found to provide a more appropriate representation of the tibial condyles. In this section, the effect on knee mobility of replacing the medial or lateral tibial polynomial surface with a planar surface is examined.

7.6.2.1 Methods

The analyses are based on the specimen-specific parameters of Model A (Section 7.2). The planes corresponded to the best-fit ‘free’ planes based on the measured surface data of Knee A (Section 3.1.2). A combination model solution method involving nine equations and nine unknowns was used to determine the model kinematics (Section 5.3.3). In the affected compartment, the radius of the corresponding femoral sphere was adjusted so that the contact conditions were met in the starting position.

7.6.2.2 Results and Discussion

By replacing the medial condyle with a planar surface all aspects of normal joint mobility may be restored; however, in the lateral compartment this was not the case. The range and pattern of joint rotation was virtually unaffected by replacing either the medial or lateral anatomical polynomial tibial articular surface by a planar surface (Figure 7.17). By replacing the lateral tibial biquadratic polynomial surface by a planar surface, however, the posterior excursion of the tibial contact point in the lateral compartment becomes unphysiologically high (>30 mm) (Figure 7.18).

The preliminary results provide some explanation for the success of the Oxford Knee in medial unicompartamental replacement [26] and the unreliability of the arthroplasty in the lateral compartment because of dislocation [82]. The model

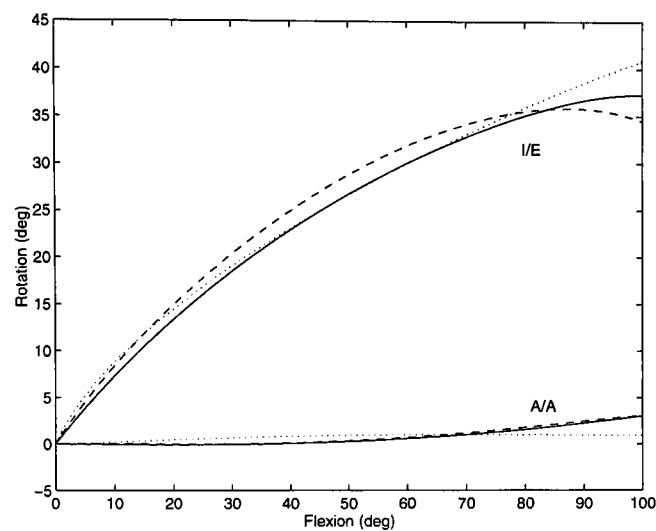


Figure 7.17: Predicted joint rotations for the original specimen-specific polynomial surface model (Model A) (solid lines) and the original model with either the medial (dashed lines) or the lateral (dotted lines) tibial condyle replaced by a planar surface.

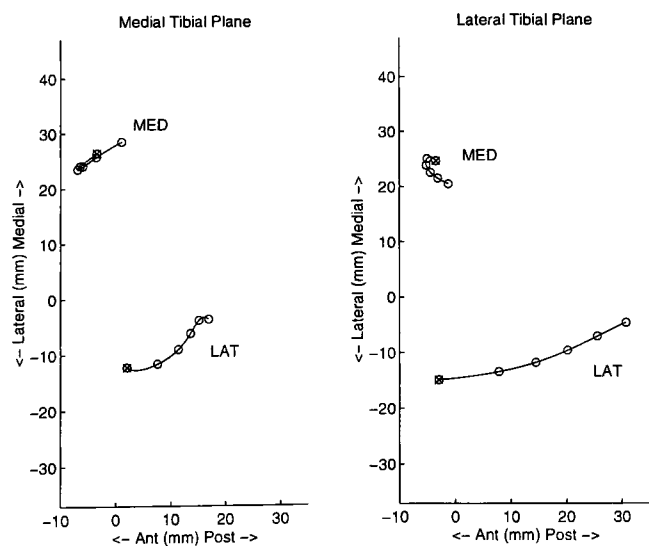


Figure 7.18: Predicted tibial contact point excursions for the original specimen-specific polynomial surface model (Model A) with the medial (left) or lateral (right) tibial condyle replaced by a planar surface.

may now be used to devise not only a more appropriate surface shape, but also the optimal placement on the bone, of the lateral tibial component of the Oxford Knee.

7.6.3 Discussion

Incorporation of more anatomical tibial surfaces into a model of knee mobility results in closer to physiological predictions of joint motion, in particular with respect to contact point movement. A specimen-specific model in which the tibial condyles are represented by biquadratic polynomial surfaces and the femoral condyles by spherical surfaces predicts a range and pattern of passive knee movement more similar to that observed in experiment. The main features of knee mobility are exhibited, increasing internal tibial rotation and net posterior translation of the tibial contact points coupled to flexion. The magnitude of posterior excursions of the tibial contact points and the pattern of coupled rotation were both closer to physiological levels than that predicted by idealised surface models in which the tibial condyles are represented by planes.

Spherical representation of the femoral condyles in a model of knee mobility is satisfactory in describing their role in controlling the passive movement of the joint. Although spheres do not constitute the most accurate representation (Section 3.1.1), the results suggest that they provide an appropriate kinematic representation of the condyles. By modelling the femoral condyles by polynomial surfaces, difficulties in satisfying the conditions of contact result in a reduced range of flexion and problems with tibial and femoral surface interpenetration arise.

Incorporation of second order tibial surface shape information into a model of knee mobility appears to be critical in the prediction of physiological joint motion and contact point movement. Such higher order surfaces are better able to take into account features such as the convexity of the lateral plateau in the sagittal plane and the medial and lateral tibial eminences (Figure 7.1). The results suggest that the inclusion of these features may be of particular importance in restoring the normal range and pattern of knee mobility during surface replacement.

The geometric-numerical approach and the polynomial surface model brings us one step closer to understanding the spatial relationships between surface and ligament geometry which control the mobility of the joint. These relationships are critical to the optimal design and implantation of surface and ligament prostheses at the knee. The preliminary study of polynomial surface representation (Section 7.1) served to give an idea as to the effect of surface shape on the kinematics of the unloaded knee. The utility of the model in the study of surface replacement design and of ligament reconstruction was demonstrated in Sections 7.6.2 and 7.6.1.

As put forth in Chapter 1 and discussed in the context of the idealised surface

model (Section 6.4), a model of mobility provides the starting point for the development of a model of knee stability in a sequential approach to the study of knee mechanics. The first iteration of such a model of stability, based on the simpler kinematic geometry of an idealised surface model of mobility, is described in Chapter 8. The next iteration will incorporate the kinematic geometry of a model of mobility capable of predicting more realistic contact point excursions such as that described in this chapter in which the tibial condyles are represented by biquadratic polynomial surfaces.

The polynomial surface model of mobility represents the next generation of a geometric model of the knee for incorporation into a 3-D model of the locomotor system [76, 77]. A model of the whole lower limb including muscles is used to study the joint mechanics during gait and other activities. An idealised surface model represents the first iteration of such a knee model (Section 6.4). The more accurate description of the kinematic geometry of the knee provided by a polynomial surface model of mobility, *i.e.* with respect to contact point movement, will allow a more accurate description of knee mechanics in activity.

While the incorporation of more anatomical descriptions of the tibial articular surface shapes has improved the prediction of physiological knee mobility, some of the remaining differences between experimental and theoretical joint kinematics and the limited range of movement of some specimen-specific models may be explained by the remaining simplifications in the geometrical representation of the ligament constraints and by the error in the experimental measurements used to derive the model parameters. The implications of such simplifications and of the associated error have already been discussed in the context of the idealised surface model (Section 6.4).

A limitation of the model remains its inability to prevent interpenetration of higher order tibial and femoral articulating surfaces. The contact constraints, both those ensuring coincidence of contact and those ensuring alignment of the contact surface normals, are not sufficient in preventing such interpenetration. Additional inequality constraints would have to be incorporated into the model solution formulation, *e.g.* constraints ensuring that the contact error in each compartment is greater or equal to zero. The non-linear least-squares method used would no longer be capable of solving such a problem. A multi-objective goal attainment method would allow incorporation of such additional constraints, however, only at the expense of significantly increased computational effort. The results of the preliminary study of polynomial surface representation (Section 7.1) suggest, in any case, that by prohibiting surface interpenetration, the resulting models would tend to be limited in their range of movement, in particular in the case of higher order surfaces. The contact conditions were only satisfied by allowing some surface interpenetration.

Where an 'exact' solution to the model kinematics does not exist, a minimum solution path may prove useful, for example, in the study of the effects of surface shape on some aspects of passive knee motion, *e.g.* joint rotations and translations. Although mildly sensitive to the method used to solve the minimisation problem, the pattern of knee movement described by a minimum-violation path for three specimen-specific models was similar to that of the unique zero-violation path predicted by Model A. An accurate description of contact kinematics and the direct calculation of the IHA was, however, not possible. In terms of prescribing the true kinematic geometry of the joint as a basis for a model of knee stability or as an integral component of a musculoskeletal model of the lower limb, the unique, 'exact' path is required; the prescription of motion provided by a series of minima to the model constraint equations therefore remains limited.

Chapter 8

Towards a Model of Knee Stability

A model of knee mobility such as that described in Chapters 5, 6 and 7 permits the study of the range and pattern of motion in the unloaded joint. In this chapter, the effect of load on joint movement is considered. In a sequential approach to studying knee mechanics (Section 1.3.3.4), the kinematic geometry of the joint as prescribed by a 3-D model of knee mobility, together with a description of the 3-D ligament fibre architecture, provide the starting point for the study of behaviour of the joint under load.

The detailed formulation and predictions of a preliminary model of 3-D passive knee stability are presented in this chapter. The formulation of the current model follows along the lines of the previous 2-D elastic ligament model developed by Zavatsky and O'Connor [147, 148, 150]. In the sections to follow, the relative displacement of the bones from their neutral unloaded configuration, the determination of generated fibre strain and stress, and the governing equations of mechanical equilibrium at the joint are described. Sample theoretical predictions of fibre recruitment and joint stiffness in both anteroposterior (a/p) and torsional stability of the joint are reported, partially validated with experimental results in the literature, and compared to the analogous 2-D theory. Finally, the applications and current limitations of the model and suggestions for further work are discussed.

8.1 Geometry

The first step in the sequential approach to modelling knee mechanics is the prescription of the geometry of the passive structures in the unloaded state at any given flexion angle. Then, by displacing the bones relative to one another from the neutral configuration, the geometry of the joint structures in the deformed state can be described. In the current analysis, only the ligaments are considered deformable and able to provide passive stability to the joint. The articular surfaces are assumed

to be rigid.

8.1.1 Kinematics

8.1.1.1 Passive Flexion

In the current study, an idealised surface model of mobility based on a set of specimen-specific parameters (WTC) was used to predict the unique path of motion followed by the unloaded joint during passive flexion. The tibial condyles are represented by the fitted ‘transverse’ planes and the femoral articular surfaces by spheres fitted to the ‘whole’ condyles (Section 6.1.1). The ligamentous constraints were represented by the most isometric ‘combination’ fibres (Section 6.1.2).

The model knee motion is described by a series of transformation matrices, $\mathbf{T}_i$, each corresponding to a knee flexion angle i , ranging from 0° to 120° . The transformation describes the displacement of the femur relative to the tibia from its location in the starting position, *i.e.* the fully extended knee. The displacement is described relative to the model tibial coordinate system (Section 5.3.1.1). The locations of the contact points on the medial and lateral tibial condyles are also prescribed by the model for any flexion angle (Section 5.5.5).

8.1.1.2 A/P tibial translation

An a/p tibial translation other than that prescribed by the model of mobility can be described by a displacement of the femur in the direction of the tibial X_t axis. The transformation matrix which describes the additional a/p femoral displacement may be written as:

$$\mathbf{T}_{ap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \pm\delta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8.1)$$

where positive δ corresponds to a posterior and negative δ to an anterior *femoral* translation. Only the x coordinates of points on the femur are affected. The deformation of the articular cartilage and of the menisci which may accompany a/p translation of the bones and which may affect the z coordinates of points on the femur is not taken into account in this analysis of knee stability. The implications of this assumption will be discussed in Section 8.5.2.3.

With the neutral configuration of the femur relative to the tibia at a given flexion angle n specified by $\mathbf{T}_n$, and an applied femoral a/p translation defined by $\mathbf{T}_{ap}$, the total displacement of the femur is prescribed by:

$$\mathbf{T}_{tot} = \mathbf{T}_{ap} \mathbf{T}_n \quad (8.2)$$

In each compartment, the location of the tibial contact point is that prescribed by the model of mobility for the specified flexion angle n displaced anteriorly or posteriorly by δ .

8.1.1.3 Tibial rotation

A tibial internal or external rotation other than that prescribed by the model of mobility can be described by a rotation of the femur about the tibial Z_t axis. The transformation matrix which describes the additional femoral displacement may be written as:

$$\mathbf{T}_{ie} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8.3)$$

where a positive θ corresponds to an external and a negative θ to an internal *femoral* rotation for a left knee. Again, the deformation of the articular cartilage and of the menisci is not taken into account; hence, only the x and y coordinates of points on the femur are affected.

With the neutral configuration of the femur relative to the tibia at a given flexion angle specified by $\mathbf{T}_n$, and an applied femoral internal/external rotation defined by $\mathbf{T}_{ie}$, the total displacement of the femur is prescribed by:

$$\mathbf{T}_{tot} = \mathbf{T}_{ie} \mathbf{T}_n \quad (8.4)$$

In each compartment, the location of the contact point on the tibia is that prescribed by the model of mobility for the specified flexion angle displaced by $\mathbf{T}_{ie}$.

The above describes a rotation about the Z_t axis passing through the tibial origin. The axial rotation may be prescribed about an axis parallel to the Z_t axis passing through any other point (x, y) in the transverse plane. The associated femoral rotation is then described by:

$$\mathbf{T}_{ie} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ x \cos \theta - y \sin \theta & \cos \theta & -\sin \theta & 0 \\ x \sin \theta + y \cos \theta & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8.5)$$

8.1.2 Ligament Fibre Architecture

In this study, the four major ligaments at the knee, the ACL, PCL, MCL, and LCL, were considered to provide the passive stability to the joint. Each of the model ligaments was described by a 3-D array of straight-line fibres connecting points on the tibia and femur.

Specimen-specific tibial and femoral attachment site geometry and fibre mapping functions derived from measurements made on a cadaver knee were used to define the fibre arrays. The arrays were then incorporated into the idealised surface model of mobility whose isometric ligament fibre and articular surface parameters were derived from the same knee specimen. It must be noted that in this case, because the ligamentous constraints in the model of mobility were represented by the most nearly isometric ‘combination’ fibres of the ACL, PCL and MCL, these did not correspond exactly to fibres within the defined array of ‘physiological’ fibres.

The following ligament parameters were obtained from the experimental analysis (Section 3.2):

$\mathbf{T}_{lt}^g, \mathbf{T}_{lf}^g$ The transformation matrices describing the location of the local coordinate systems at the tibial ($\{lt\}$) and femoral ($\{lf\}$) ligament attachment areas relative to the global coordinate system used in experiment, $\{g\}$.

f_{polyt}, f_{polyf} The bicubic polynomial functions describing the surfaces of the tibial and femoral attachment areas.

$\mathbf{B}_t, \mathbf{B}_f$ The boundaries of the tibial and femoral attachment sites of the defined array of fibres.

f_x, f_y The mapping functions which, for a given insertion site on the tibial attachment determine the coordinates of the fibre origin on the femoral attachment area.

The transformation matrix describing the location of the tibial coordinate system of the model relative to the global system of the experiment, $\mathbf{T}_t^g$, was defined as part of the derivation of the specimen-specific parameters for the model of mobility (Section 5.4). The transformation matrices describing the location of the local ligament attachment coordinate systems relative to the tibial frame were then given by:

$$\mathbf{T}_{lt}^t = (\mathbf{T}_t^g)^{-1} \mathbf{T}_{lt}^g \quad (8.6)$$

$$\mathbf{T}_{lf}^t = (\mathbf{T}_t^g)^{-1} \mathbf{T}_{lf}^g \quad (8.7)$$

8.1.3 Fibre Lengths in Passive Flexion

For each of the ACL, PCL, MCL and LCL, the lengths of fibres were first tracked during the prescribed passive motion of the unloaded model knee as follows.

The length of a fibre inserting at (x_t, y_t) on the tibial attachment was calculated by determining the corresponding (x_f, y_f) coordinates of the fibre origin on the femur using the mapping functions f_x and f_y :

$$x_f = f_x(x_t, y_t) \quad (8.8)$$

$$y_f = f_y(x_t, y_t) \quad (8.9)$$

The local z coordinates of the fibre insertion and origin were then determined based on the polynomial functions f_{polyt} and f_{polyf} , describing the tibial and femoral attachment site surfaces respectively, so that:

$$\mathbf{p}_t^{lt} = \begin{bmatrix} x_t & y_t & f_{polyt}(x_t, y_t) \end{bmatrix} \quad (8.10)$$

$$\mathbf{p}_f^{lf} = \begin{bmatrix} x_f & y_f & f_{polyf}(x_f, y_f) \end{bmatrix} \quad (8.11)$$

In the model tibial coordinate system, the position of the insertion, $\mathbf{p}_{to}$, and of the origin, $\mathbf{p}_{fo}$, in the fully extended knee were then given by:

$$\mathbf{p}_{to}^t = \mathbf{T}_{lt}^t \mathbf{p}_t^{lt} \quad (8.12)$$

$$\mathbf{p}_{fo}^t = \mathbf{T}_{lf}^t \mathbf{p}_f^{lf} \quad (8.13)$$

The femoral coordinates were then displaced according to the prescribed passive movement of the model femur for a given flexion angle n :

$$\mathbf{p}_f^t = \mathbf{T}_n \mathbf{p}_{fo}^t \quad (8.14)$$

Finally, the corresponding length of the fibre was given by:

$$L = \| \mathbf{p}_f^t - \mathbf{p}_{to}^t \| \quad (8.15)$$

A length ratio, $LR = L/L_o$, was defined where L_o corresponds to the length of the fibre in the fully extended knee.

Contour Plots Contour plots of the fibre length ratio LR as a function of the local (x_t, y_t) coordinates of the insertion on the tibial attachment site were generated for a series of flexion angles as follows.

Fibre insertions sites on the tibia, equally spaced by 0.05 mm, were defined over a rectangular grid including the entire tibial attachment area as defined by the bounding rectangle $\mathbf{B}_t$. The coordinates of the tibial origins of these fibres were described by (x_{ti}, y_{tj}) in the local ligament attachment coordinate system $\{lt\}$, where $i = 1 \dots n$ and $j = 1 \dots m$; $n \times m$ is the size of the rectangular grid of insertion sites, *i.e.* the total number of fibres considered. For a given flexion angle, the length ratios of these fibres were calculated as described above. Using Matlab's 'contour' algorithm, contour lines were determined and plotted as a function of the insertion site (x_t, y_t) in the plane of the tibial attachment. The boundary of the physiological array of fibres, $\mathbf{B}_t$, was then superimposed.

8.1.3.1 Fibre Zero-Strain Reference Lengths

To determine the strain in a particular ligament fibre, the length of the fibre which corresponds to zero strain, the zero-strain reference length, must be known. The

results of the *in vitro* motion tests suggested that movement of the unloaded bones along the virtually unique path of motion required minimal tissue deformation (Section 3.3.6). It was therefore assumed that no tissue deformation occurs during passive flexion. For this to hold true, no single ligament fibre may stretch; all fibres must remain either just taut or slack. The maximum length exhibited by a fibre during passive flexion was therefore taken to represent the zero-strain reference length of the particular fibre.

The maximum length, and hence the zero-strain reference length, of each fibre defined by the tibial insertion site (x_{ti}, y_{tj}) within the array was determined for the 0-100° passive flexion range by computing its length every 1° of flexion. An $n \times m$ array of reference lengths, **Lref**, was stored for each ligament.

The reference length for a fibre inserting at *any* (x_t, y_t) on the tibial attachment was determined, first, by identifying the coordinates (x_{ti}, y_{tj}) of the four surrounding pre-defined fibres (x_{ti}, y_{tj}) and then linearly interpolating between the corresponding reference lengths stored in **Lref**.

8.2 Mechanics

8.2.1 Ligament Fibre Strain and Stress

The length of a fibre can be compared to its zero-strain reference length by the ratio:

$$r = \frac{L}{L_{ref}} \quad (8.16)$$

where L is the length of the fibre at the current knee position and L_{ref} refers to the zero-strain reference length of the fibre (Section 8.1.3.1). If $r > 1$ then the ligament fibre is stretched. The strain in the fibre was then defined as:

$$\epsilon = \ln(r) \quad (8.17)$$

The tensile stress arising in a stretched fibre was then related to the strain in the fibre. A non-linear constitutive relationship was idealised as shown in Figure 8.1. Stress, σ , was taken to be a quadratic function of the strain for strains less than $\epsilon = \epsilon_o$ and a linear function of strain for $\epsilon \geq \epsilon_o$ [139]. If the slope of the curve at $\epsilon = \epsilon_o$ is taken to be E_o , then the stress-strain relationship is defined by the equations:

$$\sigma = \begin{cases} (\frac{E_o}{2\epsilon_o})\epsilon^2, & \epsilon < \epsilon_o \\ E_o(\epsilon - \frac{\epsilon_o}{2}), & \epsilon \geq \epsilon_o \end{cases} \quad (8.18)$$

The above formulation models the unfolding of the crimp in the ligament fibre which is exhibited in the initial toe region of load-elongation and stress-strain curves for ligamentous tissue.

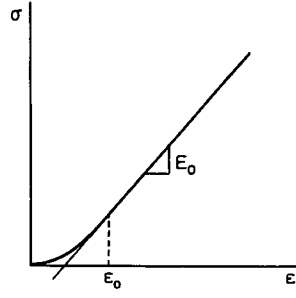


Figure 8.1: Idealised non-linear stress-strain curve for ligament fibres. ϵ_o is the strain at which fibres become uncrimped; E_o is the slope of the linear part of the stress-strain curve.

8.2.1.1 Strain Adjustment

To account for the variable fibre densities on the tibial and femoral attachments, each fibre was idealised by a conical section where the ratio of the cross-sectional area at the femoral end to that at the tibial end, b , was set equal to the specimen-specific ratio of the ligament attachment area on the femur to that on the tibia (Table 3.8). In the first instance, the strain at the tibial attachment is assumed to be equal to the overall fibre strain, ϵ , as calculated in Equation 8.17. For the tension in the fibre to be constant along the fibre, the strain at the femoral end must then be taken to be $\frac{\epsilon}{b}$.

8.2.2 Ligament Force: Magnitude and Direction¹

The total force in a ligament $\mathbf{F}_L$ is the sum of the force contributions $d\mathbf{F}$ from individual tensed fibres, *i.e.*:

$$\mathbf{F}_L = \int d\mathbf{F} \quad (8.19)$$

where the integral is carried out over only that part of the ligament which is taut, *i.e.* over those fibres stretched beyond their zero-strain reference lengths ($r > 1$). The force in a tensed fibre, $d\mathbf{F}$, is the product of the stress σ in the fibre and its cross-sectional area dA , perpendicular to the fibre direction.

To sum up these individual fibre contributions, however, the integration was carried out over the area of the tibial attachment of the ligament. Therefore, the stresses acting on that surface were required. As a first approximation, the stresses acting on the plane estimating the tibial attachment area were considered, rather than the stresses acting on the corresponding polynomial surface. Considering σ the principal tensile stress acting along the ligament fibre, the transformed stresses acting on the plane of the tibial attachment area were determined as follows.

¹The material presented in this section is based on the two- and 3-D ligament force calculations put forth by Zavatsky [148, 146]

A system of stress can be written as a second-order tensor $\mathbf{T}$:

$$\mathbf{T} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \quad (8.20)$$

where the σ s are direct stresses and the τ s shear stresses. The second subscript refers to the direction of the stress. The stress acts on a plane perpendicular to the direction given by the first subscript. Moment equilibrium requires that $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{zy}$.

A local coordinate system, $\{fib\}$, was defined in the fibre so that its Z axis was aligned with the direction of the fibre. The X and Y axes lay in a plane perpendicular to the fibre. In the local fibre coordinate system, the state of stress can then be described by:

$$\mathbf{T}^{fib} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma \end{bmatrix} \quad (8.21)$$

where σ is the tensile stress generated along the fibre.

In the coordinate system defined in the plane of the tibial attachment, $\{lt\}$, the state of stress was described by $\mathbf{T}^{lt}$ and determined by transforming the stress tensor, $\mathbf{T}^{fib}$:

$$\mathbf{T}^{lt} = \mathbf{A} \mathbf{T}^{fib} \mathbf{A}^{-1} \quad (8.22)$$

where $\mathbf{A} = \mathbf{A}_{fib}^{lt}$ is a rotation matrix describing the orientation of the local fibre coordinate system $\{fib\}$ relative to the tibial attachment coordinate system $\{lt\}$. Assigning elements to the matrix $\mathbf{A}$, the transformation equation becomes:

$$\mathbf{T}^{lt} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \quad (8.23)$$

$$\mathbf{T}^{lt} = \begin{bmatrix} (a_{13})^2 \sigma & a_{13} a_{23} \sigma & a_{13} a_{33} \sigma \\ a_{23} a_{13} \sigma & (a_{23})^2 \sigma & a_{23} a_{33} \sigma \\ a_{33} a_{13} \sigma & a_{23} a_{33} \sigma & (a_{33})^2 \sigma \end{bmatrix} \quad (8.24)$$

The stresses acting on the plane of the tibial attachment, *i.e.* on the $X - Y$ plane of the tibial attachment coordinate system $\{lt\}$, are described by the components of the stress tensor: σ_{zz} , τ_{zx} and τ_{zy} . From Equation 8.24, these were expressed as:

$$\sigma_{zz} = (a_{33})^2 \sigma \quad (8.25)$$

$$\tau_{zx} = a_{13} a_{33} \sigma \quad (8.26)$$

$$\tau_{zy} = a_{23} a_{33} \sigma \quad (8.27)$$

In the above equations, only three elements of the $\mathbf{A}$ matrix are required. The direction of the fibre in the fibre coordinate system is described by the unit vector $\hat{\mathbf{n}}^{fib} = [0 \ 0 \ 1]^T$. The components of the unit vector in the tibial attachment coordinate system are then described by:

$$\hat{\mathbf{n}}^{lt} = \mathbf{A}_{fib}^{lt} \hat{\mathbf{n}}^{fib} = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \quad (8.28)$$

In the model tibial coordinate system, the direction of the fibre is given by:

$$\hat{\mathbf{n}}^t = \frac{\mathbf{p}_f^t - \mathbf{p}_{to}^t}{\|\mathbf{p}_f^t - \mathbf{p}_{to}^t\|} \quad (8.29)$$

The components of $\hat{\mathbf{n}}^{lt}$ were then determined from:

$$\hat{\mathbf{n}}^{lt} = \mathbf{R}_t^{lt} \hat{\mathbf{n}}^t \quad (8.30)$$

where $\mathbf{R}_t^{lt}$ is the rotation component of the transformation matrix $(\mathbf{T}_{lt}^t)^{-1}$. The elements of the matrix $\mathbf{A}$ were defined by substituting the expression for $\hat{\mathbf{n}}^{lt}$ of Equation 8.30 into Equation 8.28.

The following equations describe the forces acting on the plane of the tibial attachment area of the ligament:

$$\begin{aligned} F_x &= \iint \tau_{zx} dx dy \\ &= \iint a_{13} a_{33} \sigma dx dy \end{aligned} \quad (8.31)$$

$$\begin{aligned} F_y &= \iint \tau_{zy} dx dy \\ &= \iint a_{23} a_{33} \sigma dx dy \end{aligned} \quad (8.32)$$

$$\begin{aligned} F_z &= \iint \sigma_{zz} dx dy \\ &= \iint (a_{33})^2 \sigma dx dy \end{aligned} \quad (8.33)$$

where the x , y and z components of the force refer to the attachment coordinate system, $\{lt\}$. The computation of these integrals over the area in the plane of the tibial attachment corresponding to the insertions of stretched fibres will be described in Section 8.2.3.1.

The components of force in the tibial coordinate system were then given by:

$$\mathbf{F}_L^t = \mathbf{R}_{lt}^t \mathbf{F}_L^{lt} \quad (8.34)$$

where

$$\mathbf{F}_L^{lt} = \begin{bmatrix} F_x & F_y & F_z \end{bmatrix}^T \quad (8.35)$$

8.2.3 Ligament Force: Point of Application

In addition to the magnitude and direction, the point of application of the ligament force on the tibia is required for subsequent mechanical analysis of the joint.

The ligament fibre force system described in the previous section was composed of a set of non-parallel forces acting on the plane estimating the tibial attachment. This force system was used to approximate the set of forces acting on the real surface of the attachment. The force system can be reduced to the resultant ligament force $\mathbf{F}_L$ ($\mathbf{F}_L^t$ described in the previous section) and the total moment $\mathbf{M}_{Lo}$ of the fibre forces with respect to the origin of the tibial coordinate system. The total moment can be defined by:

$$\mathbf{M}_{Lo} = \int \mathbf{r} \times d\mathbf{F} \quad (8.36)$$

where, again, the integral is carried out over only that part of the ligament which is taut, *i.e.* over those fibres stretched beyond their zero-strain reference lengths ($r > 1$).

It was assumed that for each stretched fibre, the fibre force was applied at its insertion site on the polynomial surface of the tibial attachment area. In the tibial attachment frame, $\{lt\}$, the vector from the origin to the point of application of the fibre force is $\mathbf{r}^{lt} = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix}^T$ where z_t is defined by $f_{polyt}(x_t, y_t)$. In the tibial coordinate system, the vector $\mathbf{r}$ is then given by:

$$\mathbf{r}^t = \mathbf{T}_{lt}^t \mathbf{r}^{lt} \quad (8.37)$$

The fibre force was still, however, approximated by integrating the stress acting on the plane of the attachment so that the three components of the force $d\mathbf{F}^{lt}$ were defined as before:

$$dF_x = a_{13}a_{33}\sigma dxdy \quad (8.38)$$

$$dF_y = a_{23}a_{33}\sigma dxdy \quad (8.39)$$

$$dF_z = (a_{33})^2\sigma dxdy \quad (8.40)$$

The components of the force relative to the tibial frame are:

$$d\mathbf{F}^t = \mathbf{R}_{lt}^t d\mathbf{F}^{lt} \quad (8.41)$$

By substituting the expressions for $d\mathbf{F}^t$ and $\mathbf{r}^t$ into Equation 8.36, each component of $\mathbf{M}_{Lo}$ reduces to a linear combination of the integrals: $\int dF_x$, $\int dF_y$, $\int dF_z$, $\int x_t dF_y$, $\int x_t dF_z$, $\int y_t dF_x$, $\int y_t dF_z$, $\int z_t dF_x$ and $\int z_t dF_y$. The x , y and z in the integrals once again refer to the attachment coordinate system, $\{lt\}$. The scalar constants describing the linear combination for each component are derived from the elements of the transformation matrix $\mathbf{T}_{lt}^t$. The computation of these integrals over the portion of

the planar tibial attachment area for which the ligament is taut will be described in Section 8.2.3.1.

The general force system of $\mathbf{F}_L$ and $\mathbf{M}_{Lo}$ can also be represented as a wrench (Appendix B.2). A wrench consists of a force $\mathbf{R}$ and a couple $\mathbf{G}$ whose directions are parallel such that:

$$\mathbf{G} = p\mathbf{R} \quad (8.42)$$

where p is the pitch of the wrench. In this case, the force system has been reduced to a single force $\mathbf{R} = \mathbf{F}_L$ at the origin of the tibial coordinate system o , and a couple $\mathbf{G} = \mathbf{M}_{Lo}$. The condition of $\mathbf{R}$ and $\mathbf{G}$ being parallel is not necessarily fulfilled. By changing the base point to a point q , where $\mathbf{q}$ describes the vector from the origin o to q , the new force $\mathbf{R}_q$ and couple $\mathbf{G}_q$ are described by:

$$\mathbf{R}_q = \mathbf{R} \quad (8.43)$$

$$\mathbf{G}_q = \mathbf{G} - \mathbf{q} \times \mathbf{R} \quad (8.44)$$

These constitute a wrench if:

$$\mathbf{G}_q = p\mathbf{R}_q \quad (8.45)$$

Substituting Equations 8.43 and 8.44 into Equation 8.45 gives a vector equation for $\mathbf{q}$ and p :

$$\mathbf{G} - \mathbf{q} \times \mathbf{R} = p\mathbf{R} \quad (8.46)$$

which in turn provides the scalar equations for a line, namely:

$$\frac{G_1 - yR_3 + zR_2}{R_1} = \frac{G_2 - zR_1 + xR_3}{R_2} = \frac{G_3 - xR_2 + yR_1}{R_3} = p \quad (8.47)$$

where G_i and R_i represent the components of $\mathbf{G}$ and $\mathbf{R}$ respectively, and (x, y, z) are the components of $\mathbf{q}$. The pitch of the wrench is calculated by taking the dot product of $\mathbf{R}$ with each side of Equation 8.46:

$$p = \frac{\mathbf{R} \cdot \mathbf{G}}{\|\mathbf{R}\|^2} \quad (8.48)$$

The above equations demonstrate that $\mathbf{R}_q$ and $\mathbf{G}_q$ constitute a wrench provided that $\mathbf{q}$ lies on the line described by Equations 8.47. The point where the line intersects the surface of the ligament attachment site can be taken as the point of application of the resultant ligament force. As a first approximation, the point of application is taken to be the point where the line intersects the plane of the attachment.

8.2.3.1 Evaluation of Integrals

The integrals required in the calculation of the force and moment contributions of the ligaments were evaluated over the area of the tibial attachment corresponding

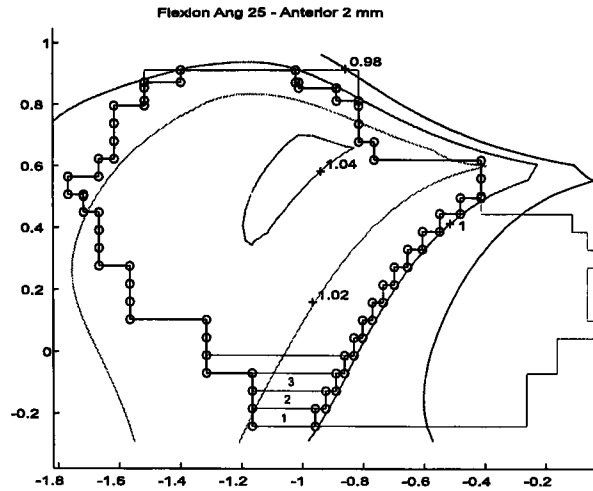


Figure 8.2: Example of definition of modified rectangular areas of integration (ACL).

to the insertions of taut fibres ($r > 1$). The whole attachment area was divided up into the series of bounding rectangles defined by B_t . Each rectangular area i was examined in sequence. If the contour line corresponding to $r = 1$ did not intersect the rectangular area, the fibres with attachments within the area were either all taut or all slack. If slack, the contribution of those fibres was zero. If all taut, the bounds on the 2-D integral:

$$\int_{y_{low}}^{y_{high}} \int_{x_{low}}^{x_{high}} I(x, y) dx dy \quad (8.49)$$

were taken to be the limits of the rectangular area i such that $x_{low} = x_{i,min}$, $x_{high} = x_{i,max}$, $y_{low} = y_i$ and $y_{high} = y_{i+1}$. If the contour line $r = 1$ did intersect the area, then an algorithm capable of handling every possible type of intersection was used to define one or more modified rectangular areas enclosing only the taut fibres ($r > 1$) and to re-define the bounds on the corresponding integrals accordingly. Figure 8.2 shows an example of the modified rectangular areas of integration on the ACL tibial attachment where only part of the ligament was taut. The defined areas of integration represent an approximation (underestimate) of the true area of integration; the contribution of a small number of taut fibres was ignored.

Once the bounds on a rectangular area of integration were defined, the integral was evaluated using a modified 2-D version of Matlab's 'quad' function, 'quad2dcgen', which uses a Gauss-Chebyshev quadrature integration scheme (Appendix C).

8.2.4 Joint Stability

It has already been described how, given as input the configuration of the unloaded joint at any flexion angle, and an additional relative displacement of the bones, the resultant force generated in each ligament acting on the tibia can be calculated

using geometric and mechanical analyses of the deformed fibres. Analysis of static equilibrium of the tibia now provides the magnitude of external forces and moments that must be applied to the tibia to bring about the additional displacement.

8.2.4.1 Joint Loading

The forces acting on the tibia include the ligament forces applied by the ACL, PCL, MCL and LCL, the contact forces in the medial and lateral compartments and any external forces required to bring about the prescribed displacement.

Ligament Forces and Moments The sums of the x , y and z components of the resultant ligament forces generated in the ACL, PCL, MCL and LCL as a result of the prescribed tibial displacement are given by $\sum F_{Lx}$, $\sum F_{Ly}$ and $\sum F_{Lz}$ as described in Section 8.2.2. The sums of the resultant moments about the X , Y and Z axes of the ligament forces are given by $\sum M_{Lxo}$, $\sum M_{Ly0}$ and $\sum M_{Lzo}$ as described in Section 8.2.3.

Contact Force and Moments In the case of the idealised surface model of mobility, where the tibial condyles are modelled as transverse planes, the contact forces in the medial and lateral compartments both act in the same direction, *i.e.* normal to the planes in the z direction. As a result, they contribute only to the balance of forces in the z direction and the balance of moments about the x and y axes. The magnitudes of the medial and lateral contact forces, F_{MC} and F_{LC} respectively, are unknown. Their points of application, however, are known: each contact force acts at the prescribed contact point on the tibial plane. The corresponding moments about the X and Y axes, M_{MCx} , M_{MCy} , M_{LCx} and M_{LCy} , can then be determined by the magnitudes of the contact forces and the x and y coordinates of the contact points.

External Forces and Moments External loads must be applied to the tibia to bring about and maintain the prescribed displacement relative to the fixed femur. In general, a 3-D force system, described by three components of force and three components of moment, is required to produce the desired movement. For example, in the study of anterior drawer, not only must an a/p directed force, F_{ap} , be applied to the tibia to overcome and balance the resistance provided by the ligaments in this direction but additional external forces and moments must also be applied to prevent any tibial movement with respect to any of the other freedoms. To prevent mediolateral tibial displacement an external mediolateral force, F_{ml} , must be applied to balance any resultant ligament forces in this direction. Normally, a

similar proximodistal force would also be required; however, in this analysis, deformation of the articular surfaces is not taken into account. Hence, proximodistal movement of the tibia relative to the femur is not possible and the z components of the ligament and contact forces simply balance each other. Furthermore, any resultant moments about the Y and Z axes due to the generated ligament and contact forces must be balanced by an externally applied flexing/extending moment, M_{fe} , and an internal/external rotary moment, M_{ie} . Because the bones are taken to be rigid, ab/adduction about the tibial origin is also not feasible and an external ab/adducting moment is accounted for by the contact forces. The moments about the X axis due to the ligament and contact forces must balance each other.

Solution of the Equilibrium Equations Considering the tibia as a free body, the equations describing static equilibrium can be written as follows :

$$\begin{aligned}
 \sum F_x &= \sum F_{Lx} + F_{ap} &= 0 \\
 \sum F_y &= \sum F_{Ly} + F_{ml} &= 0 \\
 \sum F_z &= \sum F_{Lz} + F_{MC} + F_{LC} &= 0 \\
 \sum M_{xo} &= \sum M_{Lxo} + M_{MCx} + M_{LCx} &= 0 \\
 \sum M_{yo} &= \sum M_{Lyo} + M_{MCy} + M_{LCy} + M_{fe} &= 0 \\
 \sum M_{zo} &= \sum M_{Lzo} + M_{ie} &= 0
 \end{aligned} \tag{8.50}$$

where the forces F_{ap} and F_{ml} act at the origin of the tibial coordinate system.

The resulting system involves six unknowns: F_{ap} , F_{ml} , M_{fe} , M_{ie} , F_{MC} and F_{LC} . The applied a/p force and internal/external moment are simply equal and opposite in direction to $\sum F_{Lx}$ and $\sum M_{Lzo}$ respectively. From the remaining equilibrium equations, F_{MC} , F_{LC} , F_{ml} , M_{fe} and M_{ie} can also be determined.

8.3 Model Parameters

In the current analyses, the kinematic geometry of the joint was prescribed by a specimen-specific idealised surface model, Model B, described in Section 6.2. The parameters describing the ligament architecture listed in Section 8.1.2 were derived from the same specimen, Knee B.

The values of E_o for the model ligament fibres, are listed in Table 8.1 for the ACL, PCL, MCL and LCL. The values for the ACL and PCL fibres were chosen such that the model a/p force-displacement curve at zero degrees flexion (shown later in Figure 8.7) matched the experimental results of Piziali *et al.* [103]. The E_o values for the MCL and LCL fibres were defined to lie approximately half way between that of the ACL and PCL fibres. The magnitudes of these fibre moduli are discussed in Section 8.5.3.1. The parameter ϵ_o was taken to be 0.03, based on the measurements of Butler *et al.* [23].

Ligament	E_o (MPa)
ACL	40
PCL	450
MCL	250
LCL	250

Table 8.1: Values of parameter, E_o , describing the elastic moduli of the model ligament fibres.

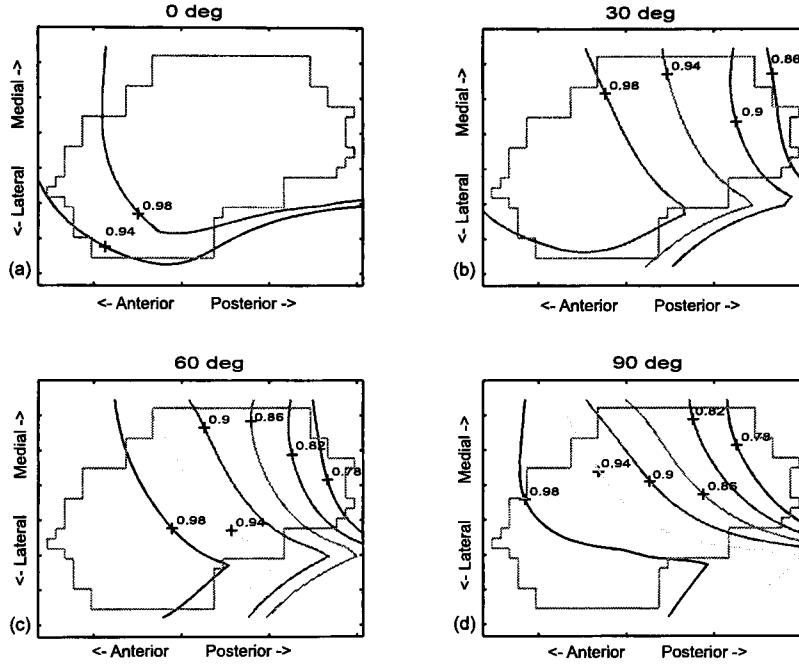


Figure 8.3: Strain pattern in the the model ACL fibres during flexion of the unloaded knee (0° , 30° , 60° and 90° flexion). Contour maps of the length ratio r (Equation 8.16) are shown with respect to the plane of the tibial attachment (transverse view). For location of isometric 'combination' fibre tibial attachment, refer to Figure 4.4.

8.4 Results

8.4.1 Fibre Strain Patterns During Passive Flexion

The fibre strain patterns of the model ACL and PCL during passive flexion are described as a function of the fibre tibial insertion sites in Figures 8.3 and 8.4. The anterior fibres of the ACL remain close to just taut throughout the range of motion while the posterior fibres slacken. The anterior fibres of the PCL start off slack in full extension and only become taut near full flexion. The fibres of the posterior PCL start off just taut and tend to slacken with flexion. For both the MCL and the LCL (no figure), the anterior fibres start off slack and become taut near full flexion. The posterior fibres see the reverse pattern of lengthening and shortening and the central fibres remain closest to constant in length.

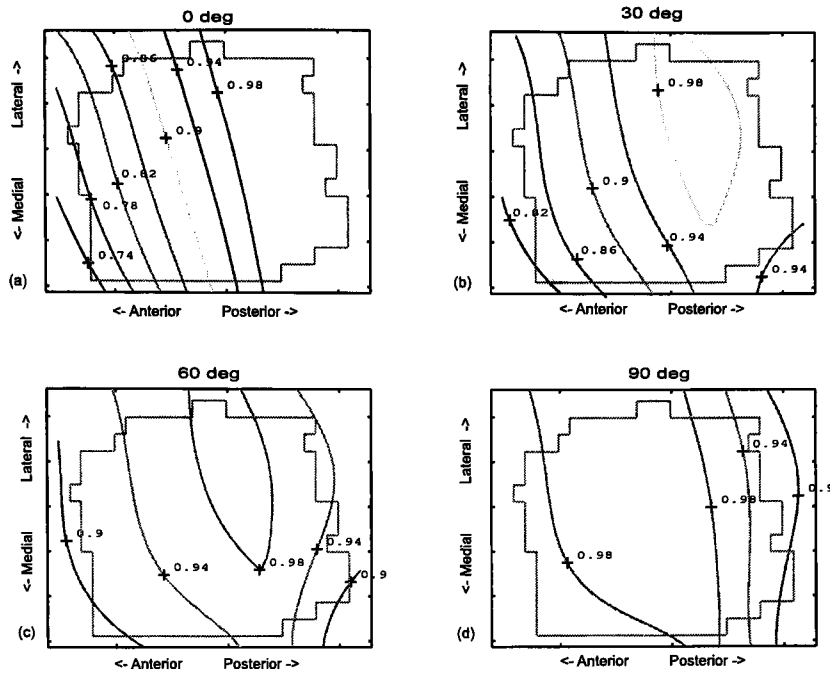


Figure 8.4: Strain pattern in the the model PCL fibres during flexion of the unloaded knee (0° , 30° , 60° and 90° flexion). Contour maps of the length ratio r (Equation 8.16) are shown with respect to the plane of the tibial attachment (transverse view). For location of isometric 'combination' fibre tibial attachment, refer to Figure 4.4.

8.4.1.1 Comparison to Experimental Results

The patterns and magnitudes of the fibre length changes predicted for all four model ligaments are similar to those estimated based on the measured passive motion of the specimen from which the model parameters were derived (Knee B) (Section 4.2.2). Similarity between the theoretical and experimental ligament length patterns is expected as the predicted model knee motion is on the order of that recorded in experiment (Figure 6.2) and the definition of the ligament fibre arrays is identical.

The experimental observations of ligament length patterns were compared to previous experimental results described in the literature in the context of the *in vitro* work (Section 4.2.3.2). Despite notable variation between the specimens tested, the current experimental length change patterns of the fibres of the ACL, PCL, MCL and LCL were generally consistent with descriptions of individual fibre or bundle length change patterns reported by a number of authors. As the current theoretical patterns of fibre strain are in good agreement with the length patterns based on the experimental motion for the same specimen, similar comparisons are not repeated here.

8.4.1.2 Comparison to 2-D Model Predictions

The predictions of the current 3-D model of passive knee stability confirm much of what the analogous 2-D theory of Zavatsky and O'Connor has taught us with regard to fibre length patterns within the ACL, PCL, MCL and LCL during passive flexion [147]; however, some differences are noted.

The ACL and PCL fibre strain patterns during passive flexion exhibited by the current model are consistent with the predictions of the 2-D model, despite the difference in the definition of the zero-strain reference lengths. In the 2-D model, fibres of the ACL and the posterior fibres of the PCL were taken to be just taut in the fully extended knee while the anterior fibres of the PCL were just taut at 120° flexion. Fibre length changes were of the same order for the PCL but not for the ACL; in the 2-D model, at 90° flexion, the central and posterior ACL fibres slackened by up to 20% and 40%, respectively, twice as much as the levels exhibited by similar fibres in the current model. The current predictions are closer to those estimated based on experiment.

The strain pattern within the collateral ligaments during passive flexion differs somewhat between the 2-D and current 3-D models. The current definition of the zero-strain reference lengths ensures that no fibres in the MCL or LCL stretch during the prescribed passive motion. In the 2-D model, however, full extension was defined as the zero-strain reference position for both ligaments. As a result, some fibres of the MCL experienced strain up to 3% and some force was required to flex the knee passively. For the LCL, the 2-D model predicts that all fibres slacken with flexion, while the anterior fibres of the LCL in the current model start off slack and become just taut with flexion.

The differences described above can be explained in part by the inability of the 2-D model to prescribe the axial rotation of the joint which accompanies flexion and by the differences in the definition of the zero-strain reference length.

Zavatsky and O'Connor [150] furthered their ligament model work by extending the representation of the cruciate ligaments to three dimensions. The kinematic geometry of the knee was, however, still prescribed by the sagittal-plane four-bar linkage model and the zero-strain reference positions defined as in the original model. In analysing passive flexion, the general pattern of lengthening and shortening of the ACL fibres was similar to that found here with only 25% slackening of the posterior fibres at 90° flexion, compared to the current 23%. However, they found that some fibres within the ACL stretched during passive flexion, a prediction inconsistent with the supposition that passive flexion involves zero tissue deformation. This demonstrated that not only was the prescription of motion restricted to the sagittal plane a limitation, but also the chosen 3-D representation of the ligaments was not entirely satisfactory.

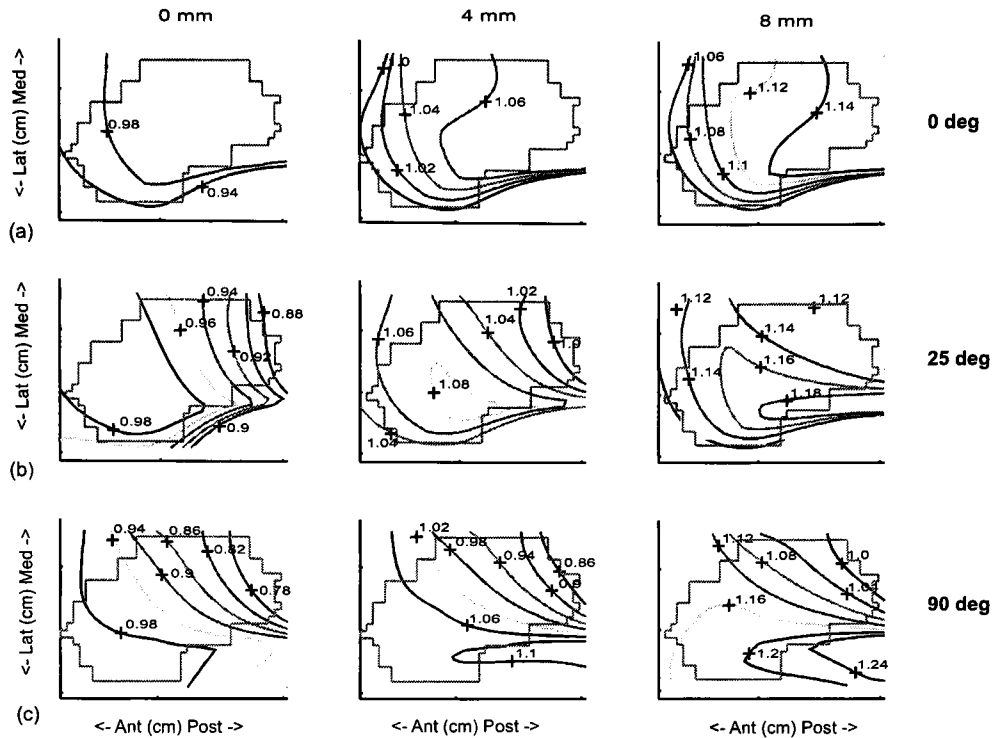


Figure 8.5: Strain pattern in the the model ACL fibres for increasing anterior displacement of the tibia relative to the femur at (a) 0°, (b) 25° and (c) 90° knee flexion.

8.4.2 A/P Stability

8.4.2.1 Geometry

Ligament fibre recruitment resulting from a/p displacement of the tibia is described by the resulting strain patterns in the model ACL, PCL, MCL and LCL². For anterior tibial translation, fibres in the ACL, MCL and LCL and for posterior translation, fibres in the PCL, MCL and LCL, are recruited to bear load. Figures 8.5 and 8.6 show the strain patterns for the ACL and PCL for tibial translations of 0, 4 and 8 mm at 0°, 25° and 90° flexion. The corresponding contour plots for the MCL and LCL are not included here. Tables 8.2 and 8.3, however, summarise the maximum and minimum strain levels experienced by fibres within each of the model ligaments.

During anterior drawer in the fully extended knee, the posterior fibres of the ACL are recruited first and reach the highest strain levels within the ligament. As the flexion angle increases, the pattern of recruitment shifts: at 25° and 90° flexion, the central and lateral fibres are recruited first and exhibit the highest strains. The maximum strains experienced by certain fibres of the ACL (up to 25%) are potentially damaging to the tissue. This will be discussed further in Section 8.4.2.3.

With posterior tibial translation of up to 8 mm at 0° flexion, the posterolateral

²The results are presented in terms of *tibial* translation, commonly described in the clinic as anterior or posterior drawer.

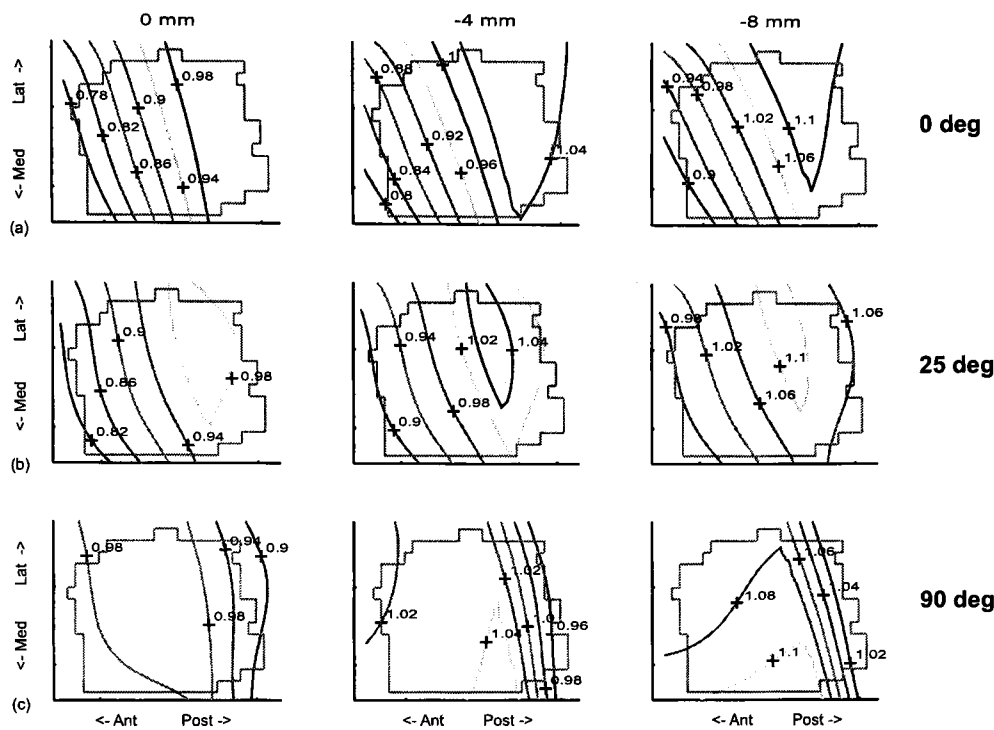


Figure 8.6: Strain pattern in the the model PCL fibres for increasing posterior displacement of the tibia relative to the femur at (a) 0°, (b) 25° and (c) 90° knee flexion.

fibres of the PCL are recruited first and experience the highest strains while the anterior fibres remain slack. Again the pattern shifts so that at 90° flexion, these are replaced by central and medial fibres.

The posterior fibres of the MCL are recruited to resist anterior tibial translation in the fully extended knee while the anterior fibres remain slack. With increasing flexion, there is a shift from posterior to anterior in the location of the fibres which are recruited to bear load. During posterior drawer, the same pattern in MCL fibre strain is observed, although at 90° flexion, the MCL fibres are no longer recruited to bear load (Table 8.2), even at the 8 mm level.

During anterior drawer, the anterior fibres of the LCL are recruited to bear load when the knee is in 90° flexion. Only in the fully extended knee do the posterior fibres of the LCL act to resist posterior tibial translations (Table 8.3).

The shifting patterns of fibre recruitment with flexion, observed with all the ligaments, is a reflection of the location of the fibres which are just taut and ready to bear load.

Ligament	Flexion	4 mm		8 mm	
		max	min	max	min
ACL	0	1.08	0.98	1.18	1.04
	25	1.09	0.97	1.21	1.10
	90	1.11	0.83	1.25	0.96
MCL	0	1.00	0.86	1.01	0.86
	25	1.00	0.89	1.00	0.89
	90	1.01	0.93	1.01	0.94
LCL	0	0.99	0.90	0.97	0.89
	25	0.98	0.9	0.99	0.90
	90	1.02	0.96	1.05	0.99

Table 8.2: The maximum and minimum strain levels (r) experienced by fibres of the ACL, MCL and LCL with anterior tibial translations. Values of $r > 1$ in bold indicate fibre recruitment. NB. All the PCL fibres go slack.

Ligament	Flexion	-4 mm		-8 mm	
		max	min	max	min
PCL	0	1.06	0.80	1.13	0.87
	25	1.05	0.88	1.12	0.95
	90	1.05	0.93	1.11	0.98
MCL	0	1.00	0.87	1.01	0.88
	25	1.00	0.90	1.01	0.91
	90	1.00	0.93	1.00	0.93
LCL	0	1.02	0.93	1.05	0.96
	25	0.98	0.90	0.98	0.91
	90	0.98	0.92	0.97	0.91

Table 8.3: The maximum and minimum strain levels (r) experienced by fibres of the PCL, MCL and LCL with posterior tibial translations. Values of $r > 1$ in bold indicate fibre recruitment. NB. All the ACL fibres go slack.

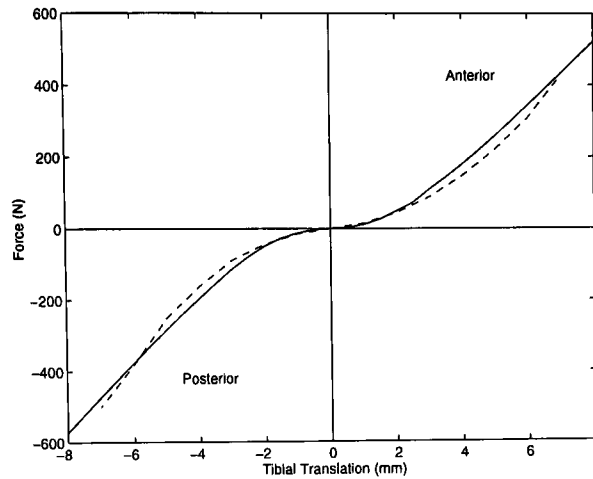


Figure 8.7: The applied a/p force versus tibial translation for the model knee in full extension (cruciates only). The dashed lines are derived from the experimental results of Piziali *et al.* [103].

8.4.2.2 Mechanics

An anterior or posterior force, F_{ap} , must be applied to the tibia to produce and maintain the tibial translations described above (Section 8.2.4.1)³. Figure 8.7 shows the applied a/p force plotted versus anterior (+) or posterior (-) tibial displacement for the model knee in full extension (0° degrees flexion) with only the cruciates to resist the applied load. ACL fibres are recruited to bear load with anterior tibial displacement and PCL fibres with posterior displacement. The curve derived from the experimental results of Piziali *et al.* [103] for a fully extended knee (cruciates only) is also shown in the figure. As noted in Section 8.3, the elastic moduli, E_o , of the ACL and PCL fibres were defined such that these theoretical and experimental curves nearly matched.

With both the cruciates and the collateral ligaments to resist the load, the resulting a/p force-displacement curve is virtually the same as that shown in Figure 8.7 for the fully extended knee. With only the collateral ligaments acting to resist the applied load, the force required to displace the tibia by 8 mm anteriorly at 0° flexion is less than 0.1 N (MCL fibres recruited). Less than 2 N is required to bring about 8 mm of posterior tibial translation (LCL and MCL fibres recruited).

The a/p force required depends on both the magnitude of the tibial translation and on the flexion angle. Figures 8.8 and 8.9 show the levels of a/p force corresponding to a series of anterior and posterior tibial translations for the intact knee (cruciates and collaterals) at 0° , 25° and 90° flexion.

At each flexion angle examined, the force required to displace the tibia increases with the amount of tibial displacement as fibres are stretched further and more

³In this preliminary analysis of passive a/p stability, the values of the additional flexing/extending and axial moments and mediolateral force required to prescribe the pure a/p displacement are not reported.

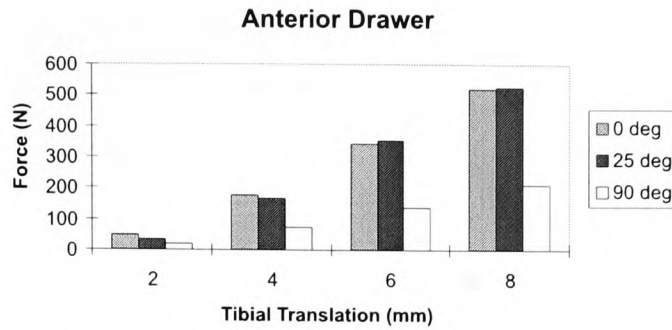


Figure 8.8: The a/p force versus anterior tibial translation for the model knee at 0°, 25° and 90° flexion.

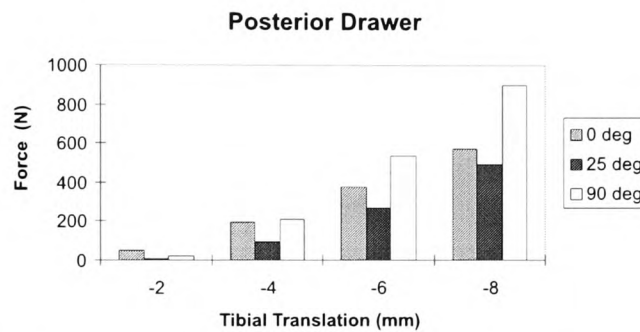


Figure 8.9: The a/p force versus posterior tibial translation for the model knee at 0°, 25° and 90° flexion.

fibres are recruited to bear load. For anterior drawer (Figure 8.8), the level of force required at 90° knee flexion is approximately half that required near full extension (0° or 25° flexion); this is because the fibres of the model ACL are more slack in flexion than near extension (Figure 8.3). In posterior drawer (Figure 8.9), the opposite is seen as more PCL fibres are closer to just taut in flexion compared to extension (Figure 8.4).

The model MCL and LCL make only small contributions to a/p knee stability throughout the flexion range. At 25° and 90° flexion, the contribution of the collateral ligaments in resisting anterior or posterior tibial translations (Figures 8.8 and 8.9) is also minimal. With anterior drawer at 25° flexion and posterior drawer at 90° flexion, the collaterals provide no resistance. To bring about 8 mm of anterior tibial translation at 90° flexion in a knee with only the collateral ligaments to resist the load, an applied force of only 12.3 N is required (MCL and LCL recruited). Posterior translation of 8 mm requires 0.28 N at 25° flexion (MCL recruited).

Figure 8.10 shows the model ACL force versus flexion angle for 5 mm of anterior tibial translation. The force in the ligament decreases with increasing flexion, again a reflection of the higher degree of fibre slackness at the higher flexion angles. The model results fall within the wide range of values derived from the experimental studies of Butler *et al.* [24] and Vahey and Draganich [130]. These comparisons will

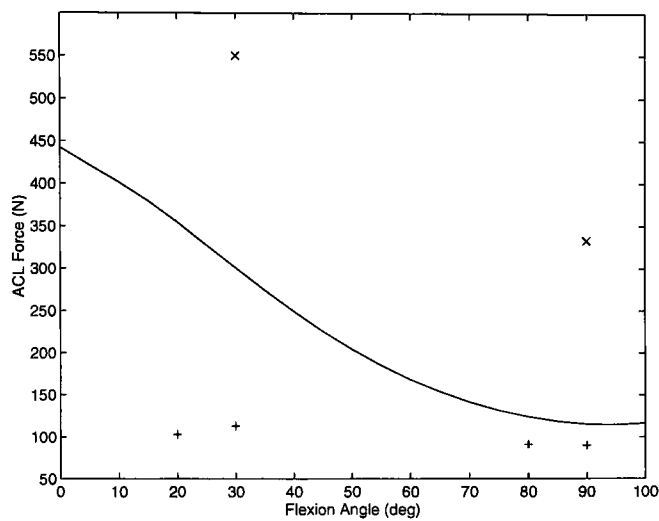


Figure 8.10: The magnitude of the resultant force in the ACL versus flexion angle for 5 mm of anterior tibial translation. Experimental results derived from Butler *et al.* [24] (x) and Vahey and Draganich [130] (+) are also shown.

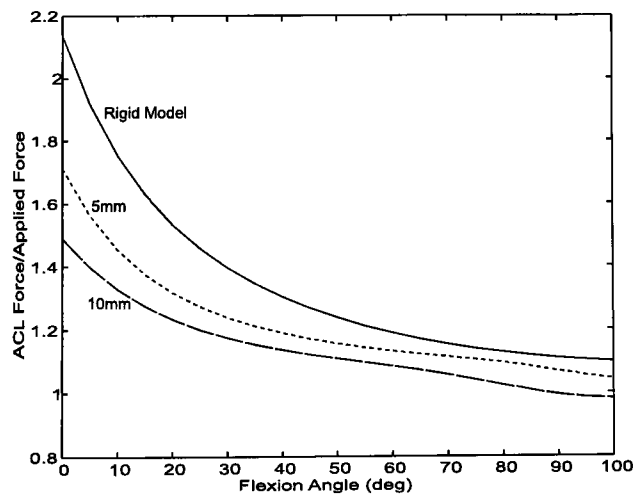


Figure 8.11: The force in the ACL as a proportion of the applied anterior force versus flexion angle. The curves for 5 mm and 10 mm of tibial translation and for the rigid ligament model are shown.

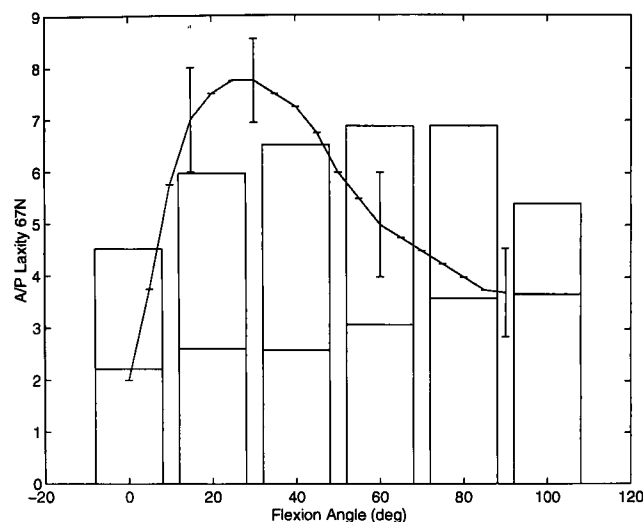


Figure 8.12: The total a/p laxity of the joint (mm) at 0, 20, 40, 60, 80 and 100° flexion for a force level of 67 N. The bars are divided into anterior (bottom section) and posterior (top section) laxities. The average experimental results of Grood and Noyes [47] are also shown (solid line with standard deviations).

be discussed further in Section 8.4.2.3.

Figure 8.11 shows the force in the ACL as a proportion of the applied anterior force versus flexion angle. The ratios corresponding to 5 mm and 10 mm of anterior tibial translation are shown. The figure also includes the curve showing the ligament force per unit applied load based on the orientation of the corresponding isometric fibre of the model of mobility (rigid model). In all three cases, the force ratio decreases with increasing flexion as the ACL becomes more horizontal and hence better oriented to resist the applied load. This effect is heightened when ligament fibres are allowed to stretch.

In applying a force first anteriorly to the tibia and then posteriorly, the resulting total tibial translation, *i.e.* the total a/p laxity, depends on the flexion angle. Figure 8.12 shows the total a/p laxity corresponding to a force level of 67 N for a series of flexion angles (cruciates and collaterals). The model results are on the same order of the experimental measurements of Grood and Noyes [47]; the comparison between theory and experiment will be discussed further in Section 8.4.2.3. From 20° to 80° flexion, the model a/p laxity increases but remains close to constant. As the laxity in anterior drawer increases, that in the posterior direction decreases.

The direction and the point of application of the resultant ligament forces change with the amount of tibial translation and the flexion angle. To illustrate such changes, Figure 8.13 shows the resultant ACL ligament force with respect to its tibial attachment site for a series of anterior displacements with the knee in full extension, at 25° and 90° flexion. At each flexion angle, the force becomes more horizontally oriented with increasing tibial translation. At 25° and 90° flexion, the

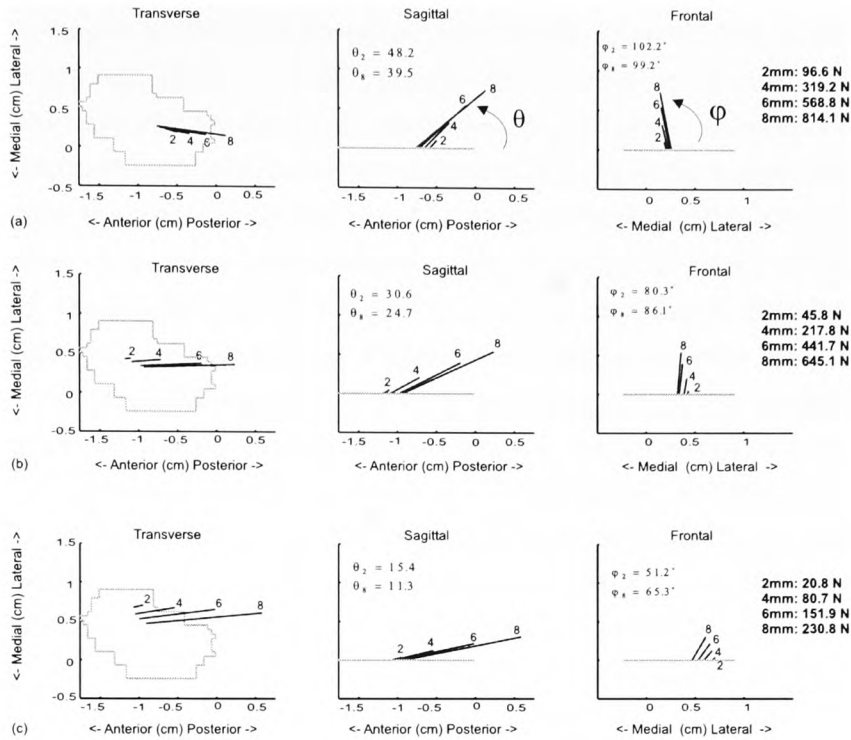


Figure 8.13: The direction and point of application of the resultant force in the ACL for a series of anterior tibial translations (a) in the fully extended knee (b) at 25° and (c) at 90° flexion. NB. Different scales for the three flexion angles.

point of application moves posteriorly while at 0° flexion, it moves anteriorly with increasing tibial translation.

8.4.2.3 Comparisons with Experimental Results

Fibre Recruitment Despite differences in loading conditions, the fibre recruitment patterns of the model ligaments during a/p drawer are consistent with previous experimental results reported in the literature.

The role of the ACL and PCL as primary stabilisers in anterior and posterior drawer respectively has been established by a number of authors, either by studying the effect of ligament sectioning on the joint laxity (*e.g.* [42, 81]) or on the restraining force (*e.g.* [24, 103]). There are, however, fewer detailed descriptions of fibre recruitment within the cruciate and collateral ligaments.

Hollis *et al.* [55] studied length changes of three parts of the ACL in cadaver knee specimens. They found that during anterior drawer at fixed flexion angles, all ACL fibres increased in length, the central fibres by somewhat more than the anteromedial and posterolateral fibres, suggesting a pattern of fibre recruitment similar to that predicted by the model (Figure 8.5).

Blankevoort *et al.* [17] determined the probability that certain ligament fibres were recruited to resist a/p displacement of the tibia. They defined a minimal

recruitment length which was based on (a) bundle lengths measured in cadaver knee specimens undergoing motion brought about by relatively low applied forces and moments and (b) an assumed maximum strain level. Blankevoort observed greater recruitment probability for the anterior ACL fibres at higher flexion angles. This supports the current prediction of a shift occurring with increasing flexion from posterior to anterior with respect to the location of fibres within the ACL experiencing the highest strains (Figure 8.5). They also found that the MCL was recruited during anterior drawer; at higher flexion angles, however, only the anterior part of the superficial ligament tended to be recruited. Here, the model MCL fibres were also recruited to bear load in anterior drawer, the anterior fibres at 90° flexion (Table 8.2), consistent with Blankevoort *et al.*'s reported probabilities.

Blankevoort *et al.* found too, however, that with posterior force applied to the tibia, the PCL was not recruited at lower flexion angles. Only at 90° flexion was the recruitment of the anterior bundle of the PCL highly probable. The current results are in agreement with the observations at higher flexion angles: at 90° flexion, the anterior fibres of the model PCL are recruited (Figure 8.6). The current results, however, do not support the results for lower flexion angles: the model posterior PCL fibres are recruited to bear load at both 0° and 25° (Figure 8.6) which is in agreement with the widely accepted view that the PCL acts as a primary stabiliser in posterior drawer. Such differences may be explained by the fact that Blankevoort *et al.* measured the length changes of only two fibre bundles in the PCL.

For posterior forces at low flexion angles, Blankevoort *et al.* also found the MCL and LCL fibres were recruited. This is in good agreement with the model results for 0° flexion (Table 8.3).

The method used by Blankevoort to describe fibre recruitment in experiment is similar to the approach used to define the fibre zero-strain reference lengths in the current model. Here, a maximum fibre strain level of 0% was assumed for passive flexion of the unloaded knee (*i.e.* no deformation) while Blankevoort *et al.* assumed levels of maximum strain of 5% and 10% corresponding to knee flexion under moderate load.

Tibial Translation Piziali *et al.* [103] measured the applied forces and moments required to achieve pure a/p tibial translation in cadaver knee specimens. The amount of model tibial translation brought about by an applied a/p force in the fully extended knee (cruciates only) is compared to their experimental results in Figure 8.7. The agreement between theory and experiment is good; however, to achieve such a match, the elastic modulus, E_o , for the ACL fibres was taken to be 40 MPa, approximately one tenth of that of the model PCL fibres (450 MPa). These values will be discussed in Section 8.5.3.1.

The model MCL and LCL played little role in resisting a/p drawer. The contribution of the MCL and LCL to the a/p force measured by Piziali *et al.* [103] was also small, but slightly higher than that predicted by the model. For the specimen corresponding to the curve shown in Figure 8.7, they found the a/p forces required to overcome the resistance offered by the MCL and LCL during 7.5 mm of anterior drawer to be 22.4 N and 0 N respectively. For posterior tibial translation of 7.7 mm, they found the forces carried by the MCL and LCL to be 0 N and 5.6 N respectively. An increase in the elastic moduli, E_o , of the MCL and LCL fibres would lead to a better match with Piziali's data.

A/P Laxity The dependence of laxity on flexion angle as shown in Figure 8.12 has been previously reported by others. This dependence is not simply explained by the changing primary directions of the ligaments with flexion. For example, the model ACL is better oriented to resist anterior loads in full flexion than in full extension; however, the model knee is more lax in anterior drawer at 90° flexion than at 0° or 25° flexion (Figure 8.8). This can be explained by the fact that the laxity reflects both the overall orientation of the ligament fibres and the degree of fibre slackness. The calculated total a/p laxity, although on the order of values measured by Grood and Noyes [47] (Figure 8.12), differs somewhat in pattern; the a/p laxity of the model knee increases with increasing flexion up to 80° flexion, in contrast to the peak laxity of Grood and Noyes observed near 30° flexion. Peak laxity at this flexion angle has also been reported by others [42, 55]. Beyond 60°, the model a/p laxity is higher than that reported by Grood and Noyes. A better match in this flexion range would most likely be possible by incorporating further structures into the model such as the joint capsule or the deep fibres of the MCL or, as described previously, by increasing the moduli of the MCL and LCL fibres. The a/p laxity is also overestimated at 0° flexion compared to the results of Grood and Noyes. This may be explained by the fact the model does not include the posterior capsule which tightens and may be recruited to bear load near full extension.

Ligament Force Magnitude The calculated fibre strains and resultant forces are not inconsistent with experimental estimates of ultimate strain and load for the knee ligaments. A few model ACL fibres exhibit strains up to 25% with 8 mm of anterior drawer at 90° flexion (Table 8.2). Assuming a strain limit of approximately 20% [96], these fibres would fail with this level of translation. The corresponding resultant ACL force, however, is relatively low (Figure 8.13(c)) and only comes close to the ultimate load levels reported in the literature for older specimens [143]. These predictions are not inconsistent since the reported ultimate loads and strains are based on tests done on bone-ligament-bone complexes aligned approximately

along the ACL axis. Thus, almost all the ACL fibres are stretched, requiring much higher ligament loads for the same strain. The example also demonstrates how the model may be useful in explaining the phenomenon of partial tears of the ligament.

The resultant ACL ligament forces are consistent with previously reported force levels derived from experiment. Vahey and Draganich [130] measured the difference in applied force required to produce a given displacement with the ligaments intact and transected at a series of flexion angles. By measuring the orientations of the ligaments radiographically, they calculated the total ligament force using trigonometric relationships. Butler *et al.* [24] used the same principle to measure the anterior component of the restraining force provided by the ACL with 5 mm of tibial translation. Toutoungi *et al.* [127], in a parameter study of the 3-D extension of the ligament model of Zavatsky and O'Connor [150], calculated the ACL forces corresponding to Butler's data using the orientation of the ACL in the sagittal plane as predicted by the four-bar linkage model. The current model ACL force corresponding to 5 mm anterior tibial translation falls between the two sets of experimental data (Figure 8.10). The two experimental studies were the only ones found, outside that of Piziali *et al.*, which reported ACL force or a/p force in terms of tibial displacement rather than of applied load. Although both studies used similar methods, their results differ considerably in magnitude.

Lewis *et al.* [73], using buckle transducers to monitor loads in both the anterior and posterior portions of the ligament *in vitro* during anterior drawer, reported that the ACL force is nearly always smaller than the applied force. However, as Zavatsky and O'Connor [148] previously demonstrated, these observations disagree with the widely accepted view that the ACL is the primary restraint to anterior drawer at all flexion angles. If the ACL provides 85% of the restraining load and is oriented at an angle of 45° to the plateau in the sagittal plane, the resulting ACL force should be 1.2 times the applied load. These simple geometric calculations put into question the use of buckle transducers to measure ligament forces.

Markolf *et al.* [79] measured the ACL force by isolating its tibial attachment and fixing a load transducer to the bone plug containing the attachment. In applying an anteriorly directed force of 200 N to the tibia, ACL forces of 207 N and 210 N were measured at 0° and 20° flexion respectively. The current model predicts that the ACL force is always greater than applied load, although the ratios are much higher than the 1-9% reported by Markolf *et al.*. More recently, Takai *et al.* [123], using a non-contact, kinematic method to measure ACL forces *in vitro*, found that with an anterior load of 100 N, the ACL force reached levels of up to 30% greater than the applied load near full extension. For 5 mm of tibial translation, the model ACL force, however, is still up to 1.7 times the applied load at full extension. This may be explained by the fact that the ACL is virtually the model knee's only restraint

to anterior drawer. Again, by increasing the contribution of the MCL in resisting anterior drawer or by including representations of other passive structures, the ACL force as a percentage of the applied load would be lower and closer to experimental findings.

Only with the recent application by Fujie *et al.* [41] of a universal force-moment sensor in a non-contact method to study *in vitro* joint mechanics, has the determination of not only the magnitude of a resultant ligament force *in situ*, but also that of the direction and the point of application of the force, become possible. Fujie *et al.* demonstrated the utility of the approach in a study of ACL force during anterior drawer at 30° flexion. They found the ACL force to be oriented at an average of 47.7° to the tibial plateau in the sagittal plane and at 12.9° lateral to the vertical in the frontal plane under 20 N of applied load. These angles shifted with increasing load to reach 41.7° and 12.5° respectively with 100 N. The model results for the knee at 25° flexion shown in Figure 8.13(b) are in fair agreement with these experimental results. Fujie *et al.* also determined the point of application of the ACL force to lie, with respect to an average-sized specimen, 4.0 mm medial and 2.0 mm posterior of the geometric centre of the attachment. The points of application of the model ACL force at 25° flexion for the range of translations also lie close to the geometric centre of the attachment (Figure 8.13(b)).

8.4.2.4 Comparison with 2-D Model Predictions

In this section, the current 3-D model predictions are compared to the analogous 2-D analysis of passive a/p stability presented by Zavatsky and O'Connor [148] which focused on the behaviour of the ACL during anterior drawer.

Both 2-D and 3-D models describe the progressive recruitment of appropriately oriented fibres to resist a/p load, the increasing 'effective' area corresponding to taut fibres with increasing translation, the changing position of the resultant forces within each ligament and the non-linear relationship between a/p force and displacement. The non-linearity of the joint system stems from the non-linear relationship between a/p tibial translation and fibre length, the non-linear fibre stress-strain relationship, and the sequential nature of the recruitment of fibres within the ligaments [148].

While both models predict the recruitment of the ACL in anterior drawer and that of the PCL in posterior drawer, the recruitment of the collateral ligaments differs somewhat. In the 2-D model, with the knee in 25° flexion, the ACL and MCL fibres are stretched to resist anterior tibial translation and the fibres of the PCL and LCL become slack; the opposite is predicted for posterior tibial translations. In the current model, however, the ACL provides the only resistance to anterior drawer at 25° flexion (Table 8.2). With posterior tibial translations at 25° knee flexion, the current model PCL and MCL fibres are recruited. In any case, they are of little

importance since, in both two- and 3-D models, the MCL and LCL play only a small role in resisting anterior or posterior drawer.

While the maximum levels of fibre strain are similar, some differences are noted in the pattern of ACL fibre recruitment during anterior drawer between the two- and 3-D models. The sagittal plane theory predicts that, in the fully extended knee, all fibres of the ACL tighten during anterior drawer with the posterior fibres experiencing the highest strains of up to 20% with 8 mm of translation. At 90° flexion, only the anterior fibres stretch and then by up to 20%. Beyond 35° flexion, even with 10 mm of tibial displacement the posterior fibres remain slack. Maximum strain levels predicted by the current model are also on the order of 20% with 8 mm of tibial translation (0°, 25° and 90° flexion); however, no fibres remain slack at this level of translation.

Despite the differences in recruitment patterns, the applied force versus a/p displacement is similar between the two- and 3-D theory. The match between theory and the experimental results of Piziali *et al.* [103] is good for both the 2-D and current 3-D models, however, due to the model parameter variations made. In case of the sagittal-plane model, Zavatsky and O'Connor [148] defined the elastic moduli of the ligament fibres based on values reported in the literature for bone-ligament-bone complexes [23] and then chose the mediolateral widths of the ACL and PCL tibial attachments to achieve the best match, while in the current model, the attachment areas were based on specimen-specific measurements and the elastic moduli E_o of the ACL and PCL fibres were defined so as to closely match the results of Piziali *et al.*.

Both models predict that, for small anterior tibial translations, the knee is stiffest in anterior drawer in the fully extended knee (Figure 8.8). However, while the current model predicts similar levels of a/p force for a prescribed tibial translation at 0° and 25° flexion, the 2-D theory predicts that the greatest differences between 0° and 30° flexion.

The differences in a/p force levels described in the previous section are also reflected in the results for total a/p laxity. The 2-D model predicts total a/p laxity which is somewhat closer in pattern to that measured by Grood and Noyes [47] than the current model (Figure 8.12). The peak total laxity predicted by the 2-D model occurs close to 35° flexion, similar to the data of Grood and Noyes. Both 2-D and 3-D models, however, predict laxity which is larger than the experimental values beyond 35°.

The changing direction and point of application of the ACL force in the sagittal plane during anterior drawer predicted by the current model were similar to 2-D predictions. The magnitude of the force was also of the same order but somewhat higher at each level of tibial translation in the current model. With 8 mm of anterior

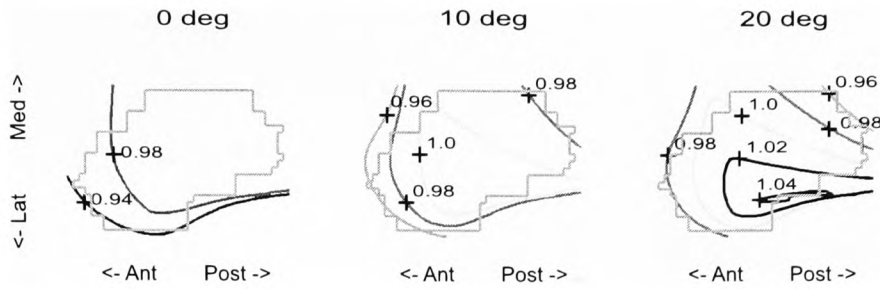


Figure 8.14: Strain pattern in the the model ACL fibres for increasing internal rotation of the tibia relative to the femur at 0° flexion.

tibial translation at 25° flexion, the 2-D ACL force was 536 N compared to the 645 N reported here.

The noted differences between the two and 3-D theory of a/p drawer stem from the fact that the latter takes into account the tibial rotation coupled to flexion and relies on a definition of the zero-strain reference lengths which ensures that no tissue deformation accompanies the motion of the unloaded knee. These two factors dictate the degree of fibre slackness within the ligaments at any given flexion angle before the application of load (Section 8.4.1.2).

8.4.3 Torsional Stability

8.4.3.1 Geometry

Ligament fibre recruitment resulting from tibial internal or external rotational displacements will be illustrated by the resulting strain patterns in the model ACL, PCL, MCL and LCL for rotations about a single, fixed axis. In this case, the axis was chosen to pass through the origin of the model tibial coordinate system which lies central to the joint (Figure 8.18). The results depend on the location of the axis and in Section 8.4.3.2 the effect of the axis location on the ligamentous response will be examined briefly.

For internal tibial rotation, fibres in the ACL, PCL, MCL and LCL and for external rotation, fibres in the ACL and LCL, are recruited to bear load. Figures 8.14 and 8.15 show the resulting fibre strain patterns for the ACL and PCL during internal tibial rotation of 10° and 20° in the fully extended knee. Corresponding plots for external rotation and for the collateral ligaments are not shown but Tables 8.4 and 8.5 summarise the maximum and minimum strain levels experienced by the fibres within the model ACL, PCL, MCL and LCL during internal and external rotations of 10 and 20 degrees.

Internal Rotation In the fully extended knee, the central and posterolateral fibres of the ACL are recruited first to bear load but experience strains of up to only

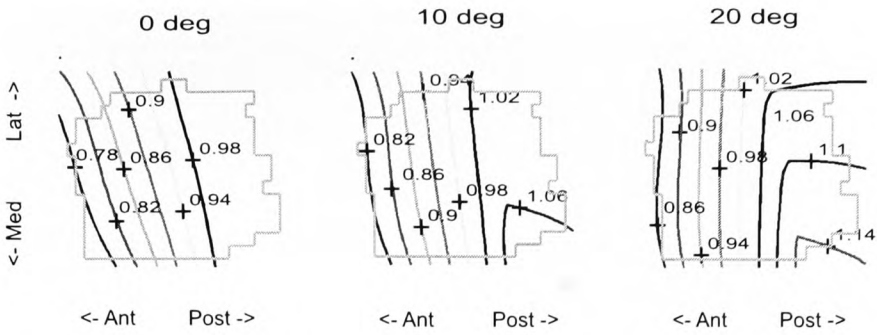


Figure 8.15: Strain pattern in the the model PCL fibres for increasing internal rotation of the tibia relative to the femur at 0° flexion.

Ligament	Flexion	10 deg		20 deg	
		max	min	max	min
ACL	0	1.02	0.95	1.04	0.95
	25	1.03	0.85	1.06	0.86
	90	1.03	0.69	1.08	0.70
PCL	0	1.07	0.81	1.15	0.85
	25	1.06	0.86	1.14	0.88
	90	1.07	0.96	1.15	1.05
MCL	0	1.01	0.87	1.02	0.88
	25	1.00	0.91	1.02	0.92
	90	1.01	0.94	1.02	0.95
LCL	0	0.98	0.89	0.99	0.89
	25	1.00	0.93	1.06	0.97
	90	1.08	1.02	1.18	1.12

Table 8.4: The maximum and minimum strain levels (r) experienced by fibres of the ACL, PCL, MCL and LCL with internal tibial rotation. Values of $r > 1$ in bold indicate fibre recruitment.

4% with 20° of internal rotation (Figure 8.14). The anterior and posteromedial ACL fibres remain slack. With the knee in 25° or 90° flexion, the pattern of ACL fibre recruitment is similar but the anterolateral rather than the posterolateral fibres are recruited first.

With internal tibial rotation at 0° flexion, the posterior fibres of the PCL are recruited with the posteromedial fibres experiencing strain levels of up to 15% (Figure 8.15). The anterior fibres remain slack. At 25° and 90° flexion, the pattern is similar. At 90° flexion, all the PCL fibres are recruited with 20° rotation (Table 8.4).

In full extension, the posterior fibres of the MCL are recruited to resist internal tibial rotation but strain levels remain below 3%. At 25° and 90° flexion, the central and anterior fibres of the MCL are recruited, respectively. At 0° flexion, the LCL offers no resistance to internal rotation. At 25° flexion, the posterior fibres of the LCL are recruited while at 90° flexion, all the LCL fibres stretch to resist internal

Ligament	Flexion	-10 deg		-20 deg	
		max	min	max	min
ACL	0	1.04	0.91	1.09	0.9
	25	0.99	0.87	0.99	0.89
	90	0.97	0.73	0.95	0.77
LCL	0	1.04	0.95	1.11	1.01
	25	0.97	0.89	0.99	0.92
	90	0.95	0.88	0.92	0.85

Table 8.5: The maximum and minimum strain levels (r) experienced by fibres of the ACL and LCL with external tibial rotation. Values of $r > 1$ in bold indicate fibre recruitment. All PCL and MCL remain just taut or slack.

rotation with the central fibres seeing strains as high as 18% with 20° rotation.

External Rotation With external rotation of up to 20° in the fully extended knee, the posterior fibres of the ACL and most LCL fibres are recruited to bear load. In the current model, at both 25° and 90° flexion, no resistance is offered by the ligaments to axial rotations of up to 20°.

The patterns of fibre recruitment in axial rotation described above can be explained in part by the positions and orientations of the different ligament fibres with respect to the chosen axis. The pattern of recruitment also depends on the degree of fibre slackness at the given flexion angle. For example, while the MCL is oriented close to vertical in the sagittal plane and has some fibres which are just taut throughout the flexion range, there is minimal or no fibre recruitment predicted for internal or external rotations of the tibia. This can be explained by considering the overall orientation of the ligament in the frontal plane and the proximity of the tibial attachment to the axis of rotation (Figure 8.18).

8.4.3.2 Mechanics

Where resistance is offered by the ligaments, an internal or external moment, M_{ie} , must be applied to the tibia to produce and maintain the axial rotations described above (Section 8.2.4.1)⁴.

Figure 8.16 shows the internal or external moments required to produce 10° and 20° of internal or external tibial rotation respectively. The applied moment is

⁴In this preliminary analysis of passive torsional stability, the values of the additional flexing/extending moment and mediolateral and a/p forces required to prescribe the pure rotation about the fixed axis are not reported.

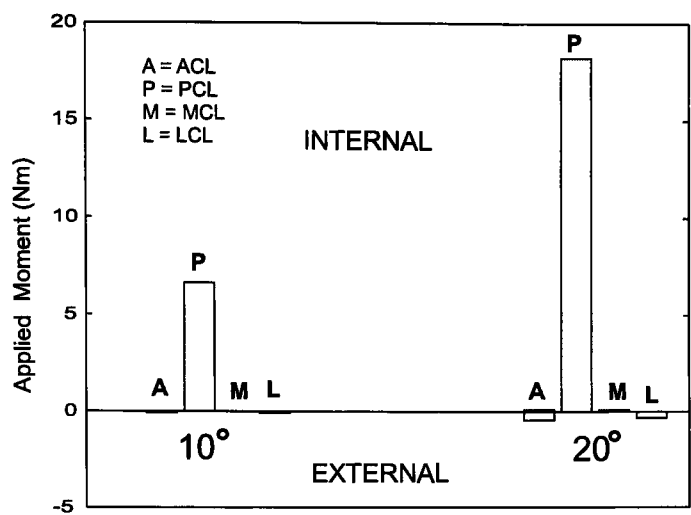


Figure 8.16: The applied moment required to overcome the resistance provided by each of the model ligaments in bringing about 10° and 20° of internal or external rotation in the fully extended knee.

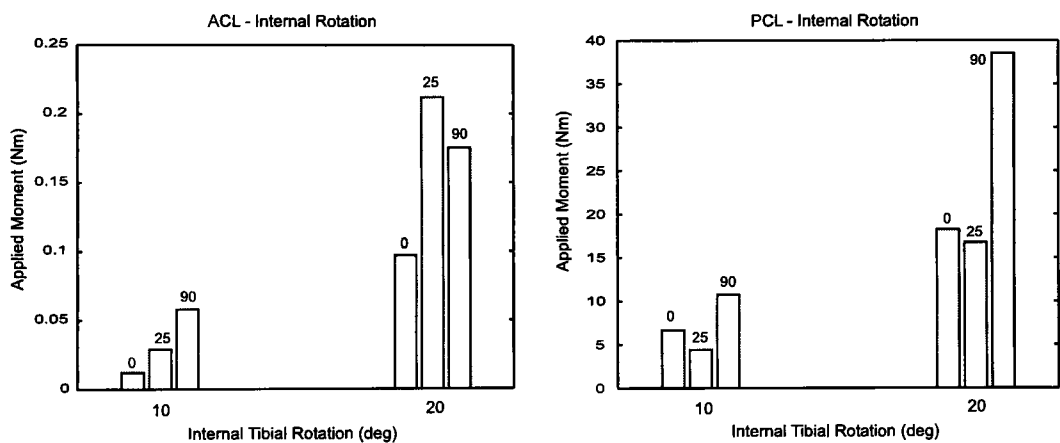


Figure 8.17: The applied moment required to overcome the resistance provided by each of the ACL and PCL in bringing about 10° and 20° of internal rotation at 0°, 25° and 90° flexion. NB the different scaling between the graphs.

broken down into the proportions required to balance the resistance offered by each of the ACL, PCL, MCL and LCL. The resistance of the PCL dominates in internal rotation, requiring an applied moment as high as 18.2 Nm to bring about 20° of rotation. In external rotation, the total moment required to overcome the resistance offered by both the ACL and LCL is less than 1 Nm.

As the knee is flexed, the resistance offered by the PCL to internal rotation about the chosen axis first decreases and then almost doubles with the knee in 90° flexion (Figure 8.17). With 10° of internal tibial rotation, the ACL provides more resistance with the knee at 90° flexion than at 25° flexion; however, the opposite trend is noted at the higher level of rotation (Figure 8.17). For both ligaments, the resistance to internal rotation jumps significantly from the 10° level to the 20°

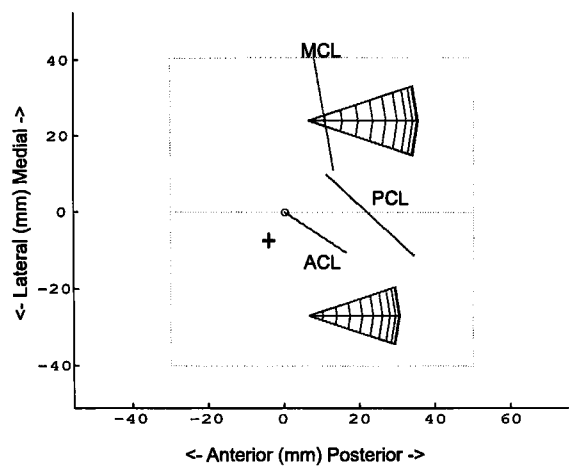


Figure 8.18: The locations of the original axis (o) and an ‘optimised’ axis (+) relative to the model tibia (transverse view). The isometric fibres of the model of mobility are shown.

Axis Centre (mm)		Applied Internal Moment (Nm)				
x	y	ACL	PCL	MCL	LCL	Total
0	0	0.1	18.2	0.1	-	18.4
-4.0	-7.4	-0.1*	1.7	-	-	1.6

Table 8.6: The applied moment required to overcome the resistance offered by the ligaments in resisting internal rotation of 20° in the fully extended knee. The results for the original axis (0,0) and an ‘optimised’ axis are presented. *In this case, the resultant moment generated by the ACL is directed internally.

level, at any flexion angle. This is due not only to an increase in the number of fibres recruited but also to the higher levels of strain in the already stretched fibres. As the strain levels surpass 3% and the fibres uncrimp, an additional increase in the overall resistance may be explained by the transition from the non-linear to the linear region of the stress-strain curve.

Location of the Axis of Rotation To demonstrate the effect of shifting the axis, an axis (x_c, y_c) was determined using an optimisation procedure (Matlab’s ‘attgoal’, Appendix C) such that the coupled a/p and mediolateral forces, generated by the ligaments during internal rotation of 20° in the fully extended knee, were essentially brought to zero. The resultant internal moment was thereby also decreased. The resulting axis location is shown in Figure 8.18 and the axial moments generated by the ligaments summarised in Table 8.6. It must be noted that the location of such an ‘optimised’ axis will clearly depend both on the flexion angle of the knee and the direction and amount of rotation.

8.4.3.3 Comparison with Experimental Results

The difficulty in comparing the predictions of the current model with previously reported experimental results lies in the dependence of the results, both theoretical and experimental, on the choice of axis of rotation. *In vivo* or in the unconstrained cadaver knee, the joint will rotate about a moving axis. The position of this axis will depend on the action of external and internal loads. If a fixed axis is chosen, the restraining function of the ligaments will depend on the location of this axis.

Fibre Recruitment Previous descriptions of ligament fibre recruitment or strain during axial rotation based on experiment are somewhat variable. Although there are certain features which have been consistently observed, others remain controversial. This can be explained in part by the fact that most studies focus only on length changes or tension measurements of a number of specific fibres or fibre bundles within each ligament. A complete picture of the ligamentous response such as that predicted by the model has not been previously described based on experiment. Furthermore, in the case of passive torsional stability, the comparison between the theory and experimental studies is complicated further, as already mentioned, by differences in the definition of the axis of rotation. Despite these difficulties, the model predictions do generally agree with previously observed features of ligament recruitment during axial rotation.

The role of the cruciates in restraining internal rotation has been established by a number of authors based on early *in vitro* laxity studies [44, 128] and qualitative anatomical analyses [64]. Their role has been confirmed in a precise study of *in vitro* torsional stability about a single fixed axis [113]. The model, too, predicts that, with internal rotation of the tibia about a centrally located axis, fibres in both the model ACL and PCL are recruited. Other authors, although not ruling out recruitment of fibres within the cruciates, have attributed little importance to the restraining function of the ACL in axial rotation [81, 132]. Some authors have found it difficult to draw firm conclusions due to the examination of only a limited number of fibre bundles and inter-specimen variability [1, 17].

Using buckle transducers, Ahmed *et al.* [1] measured the tensions generated in single, selected bands in each of the ACL and PCL respectively during passive axial rotation (with the tibia otherwise unconstrained) and found the resulting tension pattern to exhibit considerable variation between specimens. Recruitment of the ACL was not observed in some knees, while in others, recruitment of the ligament in both internal and external rotation was reported. In the current analysis, model ACL fibres were also recruited in external rotation with the knee in full extension. Ahmed *et al.*'s results for the PCL were similarly variable.

In addition to studying a/p drawer (Section 8.4.2.3), Blankevoort *et al.* [17]

also examined the fibre recruitment probability of specific ligament bundles under the application of internal and external moments (again with the tibia otherwise unconstrained). They found that, at low flexion angles, the recruitment level of the ACL with internal rotation was relatively low; however, they considered only an anterior and a posterior bundle of the ligament. At 90° flexion, Blankevoort *et al.* found that only the anterior bundle of the ACL was recruited, consistent with the predicted anterior shift in the ACL fibre recruitment pattern with increasing flexion (Figure 8.14). The posterior bundle of the PCL was also recruited in internal rotation, as was the case here. At 90° flexion, however, the anterior bundle of the PCL, too, was recruited in external rotation, an observation which has not been reported elsewhere.

The role of the collateral ligaments in restraining axial rotation has been documented in a number of studies [1, 17, 20, 81, 113, 133]. While some report that the MCL plays a significant role in restraining external rotation [133], others have found that the ligament is functional only in resisting internal rotation [81, 113]. Ahmed *et al.* [1] found that the MCL played a restraining role in both internal and external rotation but that its role in external rotation was greater than in internal rotation. The results of Blankevoort *et al.* [17] support this finding. They, too, found that, in external rotation, the anterior portion of the deep MCL, the portion of the ligament which is not represented in the current model, had the highest recruitment probability. Seering *et al.* [113], on the other hand, found the MCL to be functional only in internal rotation. This is consistent with the recruitment of the model MCL in internal rotation.

Previous experimental studies have shown that the LCL either plays virtually no role in restraining axial rotation [20] or if so, only in external rotation [113]. Again, there have also been reports of variable results. Ahmed *et al.* [1], for example, found wide variability between specimens, more so than in any of the other ligaments, more likely reflecting a true variation in LCL function between specimens. They found that the LCL could contribute to both internal and external rotational stability. Where it did play a significant role, it offered more resistance to external than internal rotation. This observation is supported also by Blankevoort *et al.* [17] who found that the LCL fibres tended to be recruited more in external rotation and at the lower flexion angles. This finding is consistent with the model prediction of LCL fibre recruitment during external rotation in the fully extended knee; at the higher flexion angles, however, the model LCL offers no resistance.

Torque versus Rotation Although the patterns of fibre recruitment are not inconsistent with previous descriptions of ligamentous response to internal and external axial rotations, differences between the calculated and measured torques required

to bring about the rotations highlight the need for further development of the model with regard to torsional stability.

The model predictions of the predicted torque versus displacement are compared here to the experimental results of Seering *et al.* [113]. They measured the required forces and moments necessary to achieve a pure tibial rotation about a single, centrally-located, fixed axis. The testing protocol used also permitted the determination of the coupled load-displacement characteristics of the individual passive structures of the joint. The axis of rotation was perpendicular to the tibial plateaus and passed through a point midway between the centres of the plateaus.

Seering *et al.* quantified the torque required to rotate a fully extended knee internally by 20° , with only the cruciates and collaterals able to resist the rotation, to be 19.7 Nm and 30.6 Nm for two specimens. For an external rotation, the applied torques were 27.3 Nm and 25.4 Nm, respectively. The levels of torque for internal rotation are on the order of the predicted 18.4 Nm about a similar, centrally-located axes; however, the torque required to externally rotate the fully extended model knee was less than 2 Nm. Furthermore, despite some inter-specimen variation, Seering *et al.* found that the cruciates and the MCL contributed equally in resisting internal rotation. In contrast, the resistance of the model PCL dominates while the contribution of the MCL is virtually zero. This difference can be explained partly by the fact that Seering *et al.* include in the contribution both the superficial and deep portions of the MCL, while the model MCL represents only the superficial portion. An increase in the modulus of the model MCL fibres would, in this case, alter the model predictions only slightly as the associated strain levels are so low.

At 30° flexion, Seering *et al.* reported a decrease in the levels of applied torque required: 15.4 Nm and 11.7 Nm for internal rotation and 18.5 Nm and 8.0 Nm for external, for the two specimens. The model also reflects a decrease in the torque associated with the resistance offered by the PCL at 25° versus 0° flexion with internal rotation (Figure 8.17); however, this torque is almost doubled when the knee is in 90° flexion. Unfortunately, Seering *et al.* did not report results for flexion angles greater than 30° . The contribution of the ACL, on the other hand, increases from 0° to 25° flexion. Because its contribution is so small, this change is of little consequence.

Although they found the principal resistance to torsional rotation was provided by the cruciate and collateral ligaments, the contributions of these ligaments in full extension made up 58% and 65% of the total torque with 20° of internal rotation, and 63% and 78% with 20° of external rotation, for the two specimens. The remaining resistive torque was attributed to the capsular structures, the menisci, and other structures. The capsular structures on the lateral side were judged most important in restricting external rotation. In the current model, only the ACL, PCL, MCL

and LCL are represented and the MCL includes only the superficial portion of the ligament.

8.5 Discussion

8.5.1 From Two to Three Dimensions

While the current 3-D model confirms much of what the analogous 2-D model [147, 148, 150] predicted with regard to ligament mechanics during a/p drawer (Section 8.4.2.4), it provides a more complete picture and, by taking into account the spatial geometry and motion of the joint, has made the theory more powerful.

8.5.1.1 Ligament Representation

In the 2-D model, the ligaments are represented by arrays of fibres all parallel to the sagittal plane with rectangular areas of attachment on the two bones. The 2-D theory is, as a result, unable to describe the twisting of fibres about themselves with flexion. With the preliminary extension of the cruciate ligament representation to three dimensions, Zavatsky and O'Connor [150] were able to describe such twisting; however, the results were found to be not entirely satisfactory. Fibres in the ACL stretched during passive flexion [150] and unrealistically high levels of force were generated by the ACL during anterior drawer [146]. Possible reasons for these inconsistencies included the inappropriate elliptical, planar representations of the ligament attachment areas.

The current 3-D model, in contrast, incorporates detailed, specimen-specific geometrical representations of both the cruciate and collateral ligaments. The attachment site geometry and fibre mapping functions are derived from measurements made on a single cadaver knee specimen. The positions and orientations of the attachments areas with respect to the instantaneous helical axis are hence more realistic. The fibre mapping is also able to take into account some of the complexity present, for example, in the mapping of the PCL fibres (Figure 2.7). Although in the calculation of the resultant ligament force, the stress is still integrated over the best-fit plane of the tibial attachment, polynomial surface representations of the tibial and femoral attachments are used in the determination of fibre length.

Most previous 3-D models of the knee that include representations of the ligaments use only a limited number of line elements to represent each ligament [6, 18, 37, 90, 139]. In contrast, the more realistic representation of the ligaments by a continuous mapping of fibres between the femur and tibia permits the examination of the functional subtleties of ligament mechanics. The changing position of the resultant force within a ligament, for example, can also be calculated (Figure 8.13).

8.5.1.2 Prescription of Passive Movement

In the 2-D analysis of a/p stability, the four-bar linkage model prescribes the passive movement of the bones in the sagittal plane, and hence ignores the axial rotation of the knee coupled to flexion. Zavatsky and O'Connor [150] suggested another possible reason for the inconsistencies in their 3-D ligament model: the four-bar linkage may not be applicable to passive flexion in three dimensions. In the current model formulation, the 3-D movement of the unloaded knee is prescribed by a spatial specimen-specific model of mobility (Chapter 6). Such a model takes into account the coupled internal rotation of the tibia with flexion. The nature of the unloaded motion affects the degree of fibre slackness or tautness within the ligaments during passive motion, and hence, subsequent fibre recruitment under load.

8.5.1.3 Parameter Derivation

In the 2-D model, the parameters of the four-bar linkage and of the ligament geometry were based approximately on the attachment sites of anterior and posterior fibre bundles of each ligament measured by Van Dijk *et al.* [35]. With the preliminary 3-D representation of the ACL and PCL [150], the sizes and orientation of elliptical attachments were based on radiographic measurements made on a single cadaver knee, while the four-bar linkage parameters were defined as in the 2-D model. The current predictions are made more powerful still by the fact that the geometry of the model bones, ligament fibre arrays and underlying model of mobility is derived from the same specimen.

8.5.1.4 Spatial Displacements

The current 3-D model formulation has the potential to examine fibre recruitment patterns compatible with motions other than a/p drawer and flexion/extension.

In this chapter, preliminary analyses of both a/p translation and axial rotation are presented. The reasons for examining these two motions first are as follows. First, the ligaments, as opposed to the bones and menisci, constitute the primary structures acting to limit these motions [47]. Thus, the simplified geometry and rigidity of the articular surfaces and ignoring the effects of bone-ligament interaction were not thought to significantly affect the results. Second, because of the strong ligamentous contribution to resisting these motions, combinations of a/p drawer and axial rotation are commonly used in the clinic to diagnose knee injury. As a result, much data has been reported in the literature with respect to the behaviour of the knee during these motions. Comparison of the current model predictions with such experimental results has provided a means of verifying the theory.

While studying the joint response to mediolateral translation, ab/adduction

about an axis passing through either the medial or lateral contact point, or joint separation is also possible (given the rigidity of the surfaces), the effect of ligament-bone interaction may play a more significant role. Blankevoort *et al.* [17], for example, examined the effect of MCL-bone interaction on model predictions of rotational laxity. Although negligible in the case of internal/external laxity, the wrapping of the ligament around the edge of the tibial condyle provided more effective resistance during knee abduction. Ligament-bone interaction may also play an important role with respect to adduction (LCL) and mediolateral translations (MCL or LCL).

8.5.2 Current Limitations

The predicted patterns of fibre recruitment during a/p drawer and axial rotation are generally consistent with previous observations reported in the literature (Section 8.4.2.3). The remaining differences between theory and experiment with respect to a/p and torsional joint stability can be attributed to a number of limitations of the current model formulation.

8.5.2.1 Ligament Representation

Despite the improvements in ligament representation with respect to specimen-specific attachment areas and ‘physiological’ fibre mapping, each ligament is still idealised by an array of straight-line fibres running from femur to tibia. This representation can only approximate the true nature of the ligaments which are complex, collagenous structures. There is no account taken of the interaction between ligament fascicles which may arise through the presence of interfascicular connective tissue or cross-linking, nor of fibres twisting about their long axes or wrapping around each other. Such interactions and twisting could affect both fibre tension and fibre orientation.

8.5.2.2 Articular Surface Geometry

As a first approximation, an idealised surface model of knee mobility (femoral spheres acting on transverse parallel tibial planes) provides a reasonable description of the movement of the unloaded joint during flexion. Given the rigidity of the surfaces, the model geometry also permits the prescription of pure tibial a/p translation and axial rotation. The model knee motion, however, although reasonably close to that observed in experiment, is associated with unphysiologically large contact point excursions (Chapter 6). Although incorporating a polynomial surface model of mobility (*e.g.* Model A, Chapter 7) would allow a prescription of passive movement compatible with more realistic contact point excursions, the inclusion of non-planar rigid articular surfaces would make the analysis of pure a/p translation

or axial rotation impossible. Even the study of a/p displacement with some degree of coupled rotation and translation would most likely be limited. The incorporation of more anatomical surfaces is therefore only deemed appropriate in a model which also takes surface deformation into account.

8.5.2.3 Articular Surface Deformation

In this first iteration of a 3-D model of passive knee stability, the effect of surface deformation was not taken into account. Huss *et al.* [59] found that inclusion of deformable cartilage layers in the analogous 2-D model caused a reduction between 10 and 35% in the force required to produce a given tibial a/p displacement. So, although an important modifier of the loading response, it was considered secondary to the effect of ligament stretching. The flexion angle dependence of the passive laxity was also much more strongly influenced by the ligament fibre recruitment than by deformation of the cartilage. Similar effects are expected with the incorporation of surface deformation in the current 3-D model.

Although passive stability is minimally affected, with the eventual application of muscle forces and the study of dynamic laxity, these effects will most likely be much more significant due to higher contact forces [60]. At that stage, it will also be important to consider the inclusion of more anatomical surface shapes and the load-bearing function the menisci.

8.5.2.4 Torsional Stability

Comparison with previous experimental results reported in the literature has revealed that, although the model is generally successful in predicting the behaviour of the joint in a/p drawer and the fibre recruitment associated with axial rotation, its utility in predicting realistic load-displacement characteristics in axial rotation is not satisfactory. While the deficiency may be explained in part by inter-specimen variation, an inexact specification of the location of the axis of rotation, or poor choice of ligament mechanical properties, the observations of others suggest that representations of the deep medial collateral ligament and the medial and lateral capsular structures may lead to a more realistic prediction of torsional stability, in particular during external rotation. Inclusion of more anatomical articular surfaces may also improve the predictions of the model in this respect.

8.5.3 Model Parameters

All the input parameters of the 3-D model presented in this chapter, except those describing the mechanical properties of the ligament fibres, are based on measurements made on a single cadaver knee specimen. These include the parameters of the

model of mobility and the parameters describing the geometry of the surface and the ligament fibre arrays. The zero-strain reference lengths of the fibres are then defined based on the supposition that no fibre stretches during passive motion from full extension to 100° flexion. The ligament forces and level of applied forces and moments required can then be calculated once the mechanical properties of the ligaments have been specified.

8.5.3.1 Ligament Mechanical Properties

Although the differences between model fibre elastic moduli of the ligaments are large (Table 8.1), all values fall within the extremely wide range of experimental and theoretical values described or adopted previously in the literature.

ACL The model ACL fibre modulus is significantly lower than the average elastic moduli reported by Butler *et al.* [22] for anterior and posterior bone-ACL-bone sub-units (284 MPa and 155 MPa respectively); however, reported standard deviations were on the order of 100 MPa and a modulus as low as 40 MPa was found for the posterior sub-unit of one specimen. Mommersteeg *et al.* [88], in examining the effect of relative insertion orientations on the tensile stiffness of the cruciate and collateral ligaments, found an average stiffness per unit strain of 7 136 N (determined by multiplying the reported average linear stiffness by the average zero-load length of the ligament). The E_o defined for the model ACL fibres is also equivalent to a stiffness per unit strain of 5 600 N (calculated by multiplying E_o by the area of the tibial attachment of Knee B) which lies close to the levels adopted by Blankevoort *et al.* [18] (5 000 N) and Andriacchi *et al.* [6] (7 200 N as computed in [18]) in their knee models. It is also on the order of the stiffness corresponding to the moduli and area of attachment of the array of ACL fibres in the analogous 2-D model (8 505 N) [148].

PCL The modulus of the model PCL fibres is within one standard deviation of the average modulus determined experimentally by Butler *et al.* [23] (345 ± 107 MPa) and within two standard deviations of the average modulus reported by Race and Amis [105] for the anterior bundle of the ligament (248 ± 119 MPa). The corresponding stiffness per unit strain of 45 000 N, however, is approximately four times larger than the experimental average found by Mommersteeg *et al.* [88] of 10 114 N and the values used to model the ligament in the aforementioned theoretical models of Blankevoort *et al.* and Andriacchi *et al.* (9 000 N and 14 300 N). The value is also about three times as large as the stiffness corresponding to fibre moduli and attachment area of the PCL (14 900 N) used in the 2-D model [148].

MCL and LCL The elastic moduli of the model MCL and LCL fibres correspond to stiffnesses per unit strain of 22 500 N and 7 500 N respectively. While similar for the LCL (7 706 N), the model MCL stiffness is greater than the value determined by Mommersteeg *et al.* [88] (13 936 N). In the case of the LCL, the stiffness is on the order of that used by Andriacchi *et al.* (7 300 N) but more than double that adopted by Blankevoort *et al.* (2 000 N). For the MCL, however, the stiffness is much larger than previous theoretical estimates (2 750 N [18] and 8 200 N [6]). In the 2-D model [148], the corresponding stiffness values, 12 112 N and 2 905 N for the MCL and LCL respectively, are about half those used here.

The variation in the experimental and theoretical estimates of the fibre stiffness and moduli again highlight the need for further specimen-specific experimental data in order to best define these parameters. Experiments like those of Piziali *et al.* [103] have proved useful in defining the moduli of the ACL and PCL fibres in the current model. Ultimately, it would be desirable to obtain such data for the same specimen used to derive the geometrical parameters of the model and with respect to all six degrees of freedom. Then applying a numerical approach similar to that of Mommersteeg *et al.* [87], the most appropriate definition of the ligament fibre moduli could be determined. Different moduli may need to be considered for different portions of a ligament as suggested by experimental studies [22, 106].

8.5.3.2 Parameter Sensitivity

The model predictions presented thus far are based on a single set of specimen-specific parameters. The utility of such a model must now also be assessed by examining the sensitivity of the model predictions to inter-specimen parameter variations. The model must be robust enough to predict the established features of joint stability based on a set of normal joint parameters but also sensitive enough to describe true inter-specimen variations and to highlight which parameters are most critical in restoring aspects of normal joint stability, for example, in the context of ligament reconstruction. A complete parameter sensitivity study was considered outside the scope of the current thesis, and may only be warranted once the model response in torsional stability is considered satisfactory.

8.5.4 Applications of the Model

8.5.4.1 Calculation of Ligament Forces *In Vivo*

Even in its current state, the 3-D model of knee stability has a significant application in the study of ligament forces *in vivo*. The importance of taking tissue deformation into account in the estimation of ligament forces using a 2-D model of the lower

limb [76, 126] was described in Chapter 1 (Section 1.3.3.4). A 3-D musculoskeletal model of the lower limb in which the kinematic geometry of the knee is prescribed by an idealised surface model of knee mobility such as that described in Chapter 6 is currently limited in that tissue deformation is not taken into account [76, 77].

8.5.4.2 Characterisation of the Stiffness Matrix of the Joint

With further refinement, a 3-D model of passive knee stability such as that formulated in this chapter will be able to provide a complete description of the stiffness matrix of the joint. Due to the non-linear, directional dependence of the load-displacement characteristics, each coefficient of this matrix would depend on the flexion angle and the amount and direction of the corresponding displacement, as observed in experiments [102, 103]. This would require that the model be able to calculate the system of forces and moments required to bring about the pure displacement or rotation about any of the three axes.

8.5.4.3 Clinical Studies

The model of stability, in isolation, will also provide a useful tool in the study of ligament reconstruction. Parametric analyses focusing on the effects of altering graft placement and the mechanical properties of the graft will allow numerous clinical questions to be addressed. Furthermore, with the incorporation more complex surface geometry and surface deformation, the effect of component shape and placement on joint laxity may then also be examined in the context of knee replacement design. Studies similar to those performed by Imran and O'Connor [61] in two dimensions may be undertaken.

Chapter 9

Conclusions and Recommendations

In Chapter 1, the need for a mathematical model of the human knee capable of predicting both the 3-D mobility and stability of the joint was put forth. Studying the movement of the unloaded joint (mobility) before examining the joint response to load (stability), in a sequential versus simultaneous approach, elucidates the functional anatomy of the joint and simplifies the complex nature of knee mechanics.

The overall aim of this thesis was to develop further and to validate a 3-D model of knee mobility and to formulate a preliminary, 3-D model of knee stability. In this context, experimental studies were also undertaken to justify the underlying concept of the model, derive input parameters and provide a means of validation.

In this final chapter, the extent to which these goals have been met is discussed. In the light of the experimental and theoretical work presented in the previous chapters, general conclusions are drawn and recommendations for further work are outlined. The chapter ends with a general summary of the thesis.

9.1 Experimental Work

9.1.1 Measurements of Knee Geometry and Movement

Essential to the successful formulation of a model of knee mobility was the observation of the range and pattern of passive movement in the real joint. Based on the *in vitro* testing of twelve normal knee specimens described in detail in Chapter 3, several conclusions can be drawn:

- All components of passive movement: internal/external rotation, ab/adduction and three components of translation are coupled to the flexion angle in the intact, unloaded joint.

- In passive flexion, the tibia rotates internally as the knee is flexed.
- In the unloaded joint, minimal deformation of the soft tissues is associated with passive knee flexion.
- In passive flexion, the unloaded knee behaves as, and hence may be modelled as, a single degree-of-freedom joint.
- The coupled paths of joint movement are consistent with the concept of an 'envelope' of passive knee motion, but provide a more precise description of the kinematics of the unloaded knee.
- The mobility of the joint may be compromised by injury or disease. The unique track of motion followed by the virtually unloaded knee may provide a valuable kinematic baseline in assessing the restoration of normal kinematics.

The development of the current model of knee mobility and stability also required, as input, descriptions of the 3-D geometry of the articular surfaces and ligaments of the joint. While the measured geometry of cadaver knee specimens was described in detail in Chapter 3, the following general points can be made:

- The line joining the centres of spheres fitted to the medial and lateral femoral condyles provides a consistent definition of a mediolateral reference axis for describing joint motion.
- The longitudinal axis of the tibia, as determined using a bi-planar analysis of the measured geometry of the condyles, provides a consistent definition of a second reference axis for the description of motion.
- Spheres provide an idealised and fairly accurate representation of the articular surface shape of the femoral condyles.
- Planar surfaces provide an idealised and reasonably accurate representation of the geometry of the tibial condyles. The lateral condyle was found to be less planar than the medial condyle.
- Fitted polynomial surfaces provide more accurate, physiological descriptions of both the tibial and femoral condyles with mean errors of over 50% less than those associated with the spherical and planar surfaces.
- Fibre mappings, based on the measured geometry of the tibial and femoral attachment sites and qualitative descriptions of fibre bundle anatomy, provide a means of representing each of the ACL, PCL, MCL and LCL by continuous 3-D arrays of 'physiological' fibres.

In conjunction with the measured motion, the 'physiological' geometrical representations of the articular surfaces and ligaments have permitted the quantification of

contact kinematics and ligament length patterns associated with passive knee flexion. Despite some inter-specimen variability, certain conclusions can be drawn based on the observations made in Chapter 4:

- The contact point (approximating the true contact area) moves posteriorly on the tibia in the lateral compartment of the joint during passive flexion (0-100°), by an average of 15 mm.
- In the medial compartment, although anterior translation of the tibial contact point is possible in early flexion, the average excursion during passive flexion (0-100°) of 6 mm is posteriorly directed.
- The proportion of rolling in the medial compartment is less than that in the lateral compartment. Pure rolling is not a consistent feature of passive knee kinematics in either compartment.
- Nearly isometric fibres exist within the ACL, PCL, MCL and LCL. Average maximum length changes during passive flexion of these fibres are 5% or less.
- Combinations of origin and insertion sites within the measured femoral and tibial attachments may define ‘fibres’ which are closer to isometric than the most nearly isometric ‘physiological’ fibres.
- Sensitivity of the length change patterns to the chosen origin and insertion of a ligament fibre and inter-specimen variability both suggest that it may be misleading to describe the function of a ligament or its length change pattern based on the tracking of a limited number of fibres or fibre bundles.

The specimen-specific data describing contact point excursions and ligament length patterns fills in some gaps in the existing knowledge with respect to the distributions of length changes within the ligaments and the sensitivity of the resulting patterns to fibre location and with respect to the quantification of contact kinematics during passive flexion in the unloaded knee. It also provides an additional means of validating predictions of a model of knee mobility.

9.2 Theoretical Work

9.2.1 A Model of Knee Mobility

In Chapter 5, a previously proposed model of knee mobility [135], in which the ligaments and articular surfaces act as rigid constraints between the bones in a single degree-of-freedom spatial mechanism, was re-examined and its limitations addressed. A new geometric-numerical approach to solving the model kinematics, capable of handling both idealised and more anatomical representations of the articular surfaces, was developed. In Chapters 6 and 7, predictions of idealised and polynomial

surface models were presented and validated. During the course of the model development and validation, a number of conclusions were reached. Specifically:

- A geometric-numerical approach represents a computationally efficient, robust means of determining the kinematics of a model of mobility. It is also flexible enough to handle the incorporation of more complex surface and ligament geometries.
- An idealised surface model of knee mobility defined by selected sets of specimen-specific parameters is successful in predicting the main features of passive knee motion: coupled internal rotation of the tibia and posterior translation of the contact points on the tibia with flexion. The principal limitation of the simplified surface model, however, is the overestimation of contact point excursion.
- Incorporation of second order tibial surface shape information into a model of knee mobility is critical in the prediction of physiological motion compatible with realistic contact point movement.
- Spherical representation of the femoral condyles in a model of mobility is satisfactory in describing the constraint offered by the surfaces and their role in controlling passive knee movement.
- The coupled movements of the unloaded knee are largely a consequence of geometrical constraints offered by the ligaments and articular surfaces, implying that passive flexion of the real knee, too, involves minimal tissue deformation and that a unique path of unresisted passive motion exists.

Parameter sensitivity of the idealised surface model of mobility and the sensitivity of the predictions of both idealised and polynomial surface models to the tolerated levels of constraint violation were examined. Based on these studies, the following conclusions were drawn:

- Differences between the specimen-specific model predictions and measurements of knee movement may be attributed to uncertainty in the definition of the model parameters.
- The range of motion for which an exact solution to the model equations exists is distinct from the range within which local minima to the constraint equations may be successfully identified.
- Where an exact solution does not exist, minimisation of the governing constraint equations provides insight into the individual constraints which are active in limiting further motion. By solving for a minimum-violation path versus a zero-violation path, a description of model knee motion through a full range of flexion is possible, although not necessarily unique.

9.2.2 A Model of Knee Stability

With the accurate prescription of the spatial kinematic geometry of the joint by a model of knee mobility and the incorporation of 3-D arrays of extensible ligament fibres, the sequential approach to studying knee mechanics has been successfully applied to three dimensions. The detailed formulation and predictions of a 3-D model of passive knee stability were presented in Chapter 8. Based on the preliminary theoretical analyses of a/p and torsional stability, the following general points can be made:

- The supposition that passive flexion of the unloaded joint involves no tissue deformation facilitates the definition of zero-strain reference lengths of the ligament fibres.
- 3-D geometrical analysis of the extensible ligament fibres permits the determination of the spatial distribution of strain and, by assuming mechanical properties of the fibres, stress within the ligaments resulting from relative displacement of the bones. Load sharing between the ligaments may then also be deduced.
- The current 3-D model of stability permits the examination of fibre recruitment and generated forces in the ACL, PCL, MCL and LCL during a/p translation and axial rotation about any fixed axis. The study of ligament response to displacements with respect to the other freedoms, however, will most likely require further refinement of the model to take into account the effect of bone-ligament contact interactions.
- The preliminary 3-D model of knee stability confirms much of what the analogous 2-D theory has taught us with respect to ligament mechanics during a/p drawer. However, the theoretical predictions of a/p stability have been made more powerful by taking into account specimen-specific, spatial geometry and motion of the joint.
- The cruciate and collateral ligaments are not sufficient in offering resistance to external rotation at the joint. Incorporating representations of the deep MCL and the capsular structures into the model may improve the model torsional stability.
- The pattern of fibre recruitment during axial rotation and the torsional stiffness are highly sensitive to the choice of axis of rotation.
- An unresolved difficulty, also encountered by others, in reproducing the resistance offered by the cruciates in the real knee during a/p drawer, is that it was necessary to assume the fibres of the model PCL to be up to ten times stiffer than those of the ACL.

9.3 Recommendations for Future Work

9.3.1 Experimental Studies

During the course of the passive motion experiments, it was observed that when the geometry of the joint had been altered in some way through disease or injury, the path of movement in the unloaded joint was no longer fully coupled. A more systematic examination of the effects of ligament sacrifice and subsequent reconstruction, or of surface replacement, on the mobility of the joint is now required. In conjunction with the model of knee mobility, such experimental data will be invaluable, in the theoretical study of knee replacement and ligament reconstruction.

With respect to the derivation of geometrical model parameters, the acquisition of specimen-specific fibre bundle anatomy would permit a more accurate representation of the ligaments. The current method of deriving fibre mapping functions, although adequate, may not take sufficient account of inter-specimen differences. Fibre tracing may prove more difficult in the case of the MCL; however, it is the MCL which appears to be the ligament to which the mobility of the joint is most sensitive.

Testing of specimens using a magnetic tracking device has proved satisfactory, when experiments were conducted in a virtually metal-free lab. A more accurate measurement system, however, would allow for more reliable quantification of contact movement and ligament length patterns during experimental motion.

9.3.2 Model of Mobility

By implementing a new geometric-numerical approach and by incorporating more anatomical tibial articular surface shapes, the model of knee mobility now prescribes both a physiological range and pattern of movement. The polynomial surface model presented in Chapter 7 is ready for incorporation into a 3-D model of the locomotor system [76, 77]. By providing a more realistic prescription of the spatial kinematic geometry of the joint, it more accurately prescribes the changing lines of action of the ligaments, muscle tendons and contact normals at the joint during flexion. The polynomial surface model also provides an invaluable tool in the study of knee replacement design as was demonstrated in Chapter 7. Questions related to the optimal surface shape of the lateral component of the Oxford Knee, for example, may be now addressed.

The idealised surface model of mobility, despite the somewhat high contact point excursions, still constitutes a valuable tool in studying the relationship between form and function at the knee joint. Parameter studies are more readily performed due to the less computationally intensive nature of the model solution. Questions related

to the functional anatomy of the joint and to the effects on knee mobility of graft placement during ligament reconstruction, for example, may now be addressed.

There remain various limitations and areas for improvement in the formulation of the model of knee mobility. These have been discussed in detail in Chapters 6 and 7 but will be briefly reviewed here. First, taking into account the effect of the MCL wrapping around the edge of the tibial condyle may improve the identification of the most isometric fibre within the ligament, and hence permit a more realistic representation of the kinematic constraint. The rigidity of the surfaces still limits the degree of surface complexity which may be incorporated into the model. A model solution algorithm which is capable of not only satisfying the ligament and contact constraints but of prohibiting surface interpenetration may allow the examination of further surface complexity, albeit most likely through a more limited range of motion. The potential of the determination of a minimum versus zero-energy path must be examined further. Finally, it may be possible to define additional specimen-specific models which successfully predict a full range of physiological motion by optimising the parameters within the range of uncertainties.

9.3.3 Model of Stability

Specific limitations of the model of knee stability and ways in which they should be addressed were reviewed in detail in Chapter 8, and so will not be repeated here. It is important to reiterate, though, the need for specimen-specific data with respect to the load-displacement behaviour of the joints. The definition of the mechanical properties of the ligament fibres represents one of the weakest aspects of the model formulation and would be much improved by such data. A next vital step also consists of the assessment of the parameter sensitivity of the model in its current state. Predictions of additional specimen-specific models must be examined. Only once the elastic ligament response to a/p drawer and torsional rotation is satisfactory, should the effects of surface deformation and of different surface shapes on joint laxity be examined. With further refinement, the model may then be used in the context of surface replacement or ligament reconstruction to study the effects of component or graft placement on the 3-D stability of the joint. Following along the lines of the 2-D work, the 3-D model of stability will also prove useful in taking account of tissue deformation at the knee in a 3-D model of the locomotor system [76, 77], allowing for more realistic estimates of ligament forces during activity.

9.4 General Summary

In this thesis, the study of knee mobility was distinguished from the study of knee stability in a sequential approach to modelling the joint. The main focus of the thesis was the further development of a 3-D model of knee mobility. The first iteration of the model, in which the ligaments and articular surfaces act as rigid constraints between the bones in a single degree-of-freedom mechanism, was re-examined and its limitations addressed. A new geometric-numerical approach to solving the model kinematics was put forth. Detailed formulations of both an idealised and a general surface model of mobility were presented.

An experimental database of 3-D joint motion and geometry was compiled and served to justify the underlying concept of modelling the unloaded knee as a single degree-of-freedom system, to provide model input parameters and a means of validating the theory. Based on the measured motion and geometry, 3-D articular contact kinematics and ligament length patterns accompanying passive flexion were also examined.

An idealised surface model of knee mobility defined by selected sets of specimen-specific parameters was successful in predicting the main features of passive knee motion: coupled internal rotation of the tibia and posterior translation of the contact points on the tibia with flexion. Incorporation of second order tibial surfaces, however, led to the prediction of physiological motion compatible with more realistic contact point movement. A number of parameter studies were undertaken.

By incorporating continuous 3-D arrays of extensible ligament fibres into an idealised surface model of mobility, a preliminary model of knee stability was put forth. Fibre recruitment within the ligaments during a/p drawer and axial rotation was examined and sample calculations of a/p and torsional stability were presented. Despite the current limitations, the potential of the model of stability and the sequential approach in highlighting the subtleties of functional ligament mechanics was demonstrated.

Finally, general conclusions were drawn and recommendations for further development suggested. Although still in need of further refinement, the 3-D theory has now been advanced far enough such that the model is in a state to provide an essential component in the study of joint mechanics in activity and a valuable tool in the design and assessment of surgical procedures, such as knee replacement and ligament reconstruction, for treating knee injury and disease. The sequential approach as put forth in this thesis may now also be applied to the study of other joints.

Appendix A

Clinical Terminology

A.1 Reference Planes

The reference planes in the body are shown in Figure A.1 and are described as follows [138]:

sagittal plane divides the body vertically into left and right sections

frontal or coronal plane divides the body vertically into front and rear sections, parallel to the sagittal plane

transverse plane divides the body horizontally into upper and lower sections, perpendicular to the sagittal and frontal planes

The neutral position of the body with respect to these planes is defined when the body is upright, with feet together and arms at the sides with the palms facing forward.

A.2 Anatomical Directions

When describing the structures and the motion at a joint with respect to anatomical directions, the following terminology has been adopted:

anterior towards the front of the body or in front

posterior towards the rear of the body or in the rear

medial towards the midline of the body

lateral away from the midline of the body

proximal towards the upper extremity of the body

distal towards the lower extremity of the body

A.3 Clinical Description of Motion

In describing motion at a joint, different conventions have been adopted for different joints. At the knee, rotations in three reference planes are usually considered:

flexion/extension Rotation of a moving body segment relative to an adjacent one in the sagittal plane. Flexion bends the knee while extension straightens it.

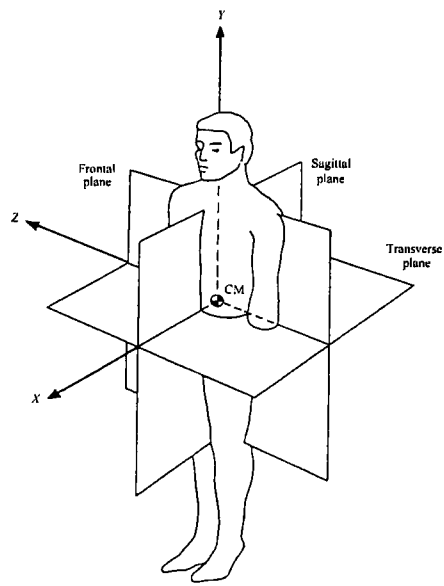


Figure A.1: Anatomic reference planes

abduction/adduction Rotation in the frontal plane. Abduction moves the foot away from the mid-line of the body and adduction moves the foot towards the mid-line.

internal/external rotation Rotation in the transverse plane about the long axis of the moving body segment. With internal rotation of the shank, the toes point in towards the mid-line and in external rotation, the toes point outward

Appendix B

Mathematical Notation and Screw Theory

B.1 Spatial Transformations

The notation described by Craig [32] is used in this thesis. The spatial transformation $\mathbf{T}_b^a$ describes the position and orientation of Cartesian frame $\{b\}$ relative to a frame $\{a\}$ in space. The 4×4 homogeneous transformation matrix may be written as:

$$\mathbf{T}_b^a = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 \\ v_x & R_{11} & R_{12} & R_{13} \\ v_y & R_{21} & R_{22} & R_{23} \\ v_z & R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (\text{B.1})$$

where $\mathbf{v} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}$ is the position of the origin of $\{b\}$ relative to the $\{a\}$ frame and $\mathbf{R} = \mathbf{R}_b^a$ is the rotation matrix whose column vectors are the unit vectors of $\{b\}$ expressed with respect to $\{a\}$ (direction cosines).

The transformation $\mathbf{T}_b^a$ may be used to map vectors from one coordinate system or frame to another. For example, a point described relative to the frame $\{b\}$ by the vector, $\mathbf{v}_p^b$, may be expressed with respect to the frame $\{a\}$ as: $\mathbf{v}_p^a = \mathbf{T}_b^a \cdot \mathbf{v}_p^b$. It is implied that $\mathbf{v}_p^a$ and $\mathbf{v}_p^b$ have been extended to 4×1 vectors (1 in the first row) for this calculation.

The transformation $\mathbf{T}_b^a$ may also be thought of as an operator with notation $\mathbf{T}$ or $\mathbf{T}_{12}$. In the case where $\mathbf{T} = \mathbf{T}_b^a$, $\mathbf{T}$ operates on vectors to displace them according to the displacement from the location of frame $\{a\}$ (location 1) to the location of frame $\{b\}$ (location 2). A position vector described with respect to a global reference frame $\{g\}$ by $\mathbf{v}_p^g$ is displaced by $\mathbf{T}$ or $\mathbf{T}_{12}$ to its new position with respect to $\{g\}$ by: $\mathbf{v}_{p2}^g = \mathbf{T} \cdot \mathbf{v}_{p1}^g$.

Compound transformations involving multiple frames $\{a\}, \{b\}, \{c\}$ are such that $\mathbf{T}_c^a = \mathbf{T}_b^a \mathbf{T}_c^b$. The inverse of a transform $\mathbf{T}_b^a$ is simply $\mathbf{T}_a^b$ based on the rotation matrix $\mathbf{R} = \mathbf{R}_b^{a^T}$, and the translation vector $\mathbf{v} = -\mathbf{R}_b^{a^T} \mathbf{v}_{b \text{ org}}^a$.

B.2 Screw Theory

Any displacement of a rigid body in space can be effected by a single rotation about a unique axis combined with a translation along that axis. The displacement is referred to as a *screw displacement* or *twist*. The unique axis is the *finite screw axis* (FSA). The ratio of translation, t , to rotation, ϕ , is called the pitch of the screw, h . If the pitch is zero, the displacement is a pure rotation. A pitch of ∞ corresponds to a pure translation where the FSA lies at infinity and is oriented perpendicular to the direction of the translation.

As the displacement of a body becomes infinitesimally small, the uniquely defined screw axis is referred to as the *instantaneous screw axis* (ISA). It is then convenient to express the rotation and translation as vectors of angular velocity, ω , and translational velocity, τ , respectively. The expression for the pitch becomes $h = \tau/\omega$. The surface generated by a series of sequential ISAs is called an *axode* of movement.

B.2.1 Line Geometry and Screw Coordinates

B.2.1.1 Plücker Line Coordinates

In three dimensions Plücker line coordinates provide a convenient way of describing a line in space. Given the line segment L_{12} passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) , the first three coordinates consist of direction ratios:

$$\begin{aligned}\mathcal{L} &= x_2 - x_1 \\ \mathcal{M} &= y_2 - y_1 \\ \mathcal{N} &= z_2 - z_1\end{aligned}\tag{B.2}$$

The second set of three coordinates represent the moments of the line about the x , y and z axes respectively:

$$\begin{aligned}\mathcal{P} &= y_1 z_2 - y_2 z_1 \\ \mathcal{Q} &= z_1 x_2 - z_2 x_1 \\ \mathcal{R} &= x_1 y_2 - x_2 y_1\end{aligned}\tag{B.3}$$

The six coordinates are written as $(\mathcal{L}, \mathcal{M}, \mathcal{N}; \mathcal{P}, \mathcal{Q}, \mathcal{R})$ and the following relationship always holds:

$$\mathcal{L}\mathcal{P} + \mathcal{M}\mathcal{Q} + \mathcal{N}\mathcal{R} \equiv 0\tag{B.4}$$

For a finite line, $\mathcal{L}, \mathcal{M}$ or $\mathcal{N}$ must be non-zero. If $\mathcal{P}, \mathcal{Q}$ and $\mathcal{R}$ are all zero, the line passes through the origin.

From the line coordinates, the position of a point on the line may be given by:

$$\mathbf{v} = \left(\begin{bmatrix} \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \end{bmatrix} \times \begin{bmatrix} \mathcal{P} \\ \mathcal{Q} \\ \mathcal{R} \end{bmatrix} \right) + \Delta \begin{bmatrix} \mathcal{L} \\ \mathcal{M} \\ \mathcal{N} \end{bmatrix}\tag{B.5}$$

B.2.1.2 Screw Coordinates

The axis of a screw may be described by its Plücker line coordinates $\mathcal{L}, \mathcal{M}, \mathcal{N}$. Three additional coordinates related to the Plücker line coordinates and the pitch of the screw, h , are introduced:

$$\begin{aligned}\mathcal{P}^* &= \mathcal{P} + h\mathcal{L} \\ \mathcal{Q}^* &= \mathcal{Q} + h\mathcal{M} \\ \mathcal{R}^* &= \mathcal{R} + h\mathcal{N}\end{aligned}\tag{B.6}$$

The *screw* coordinates are then written as $(\mathcal{L}, \mathcal{M}, \mathcal{N}; \mathcal{P}^*, \mathcal{Q}^*, \mathcal{R}^*)$ and may be normalised as follows:

$$(\mathcal{L}_N \mathcal{M}_N \mathcal{N}_N; \mathcal{P}_N^* \mathcal{Q}_N^* \mathcal{R}_N^*) = \frac{(\mathcal{L} \mathcal{M} \mathcal{N}; \mathcal{P}^* \mathcal{Q}^* \mathcal{R}^*)}{\sqrt{\mathcal{L}^2 + \mathcal{M}^2 + \mathcal{N}^2}}\tag{B.7}$$

The pitch of the screw, h_N , is given by:

$$h_N = \mathcal{L}_N \mathcal{P}_N^* + \mathcal{M}_N \mathcal{Q}_N^* + \mathcal{N}_N \mathcal{R}_N^*\tag{B.8}$$

B.2.2 Wrenches and Twists - Reciprocal Screws

There is a direct analogy between statics and instantaneous kinematics. A *wrench* is defined as a force-couple of resultant force, $\mathbf{F}$, acting along a line w and of resultant couple, $\mathbf{C}$, parallel to w . A wrench has a scalar pitch h equal to C/F . A wrench on a screw can be expressed by its screw coordinates: $(\mathcal{L}_w, \mathcal{M}_w, \mathcal{N}_w; \mathcal{P}_w^*, \mathcal{Q}_w^*, \mathcal{R}_w^*)$.

The *wrench* on a screw (statics) is analogous to the *twist* on a screw (instantaneous kinematics). The couple, $\mathbf{C}$, and the translational velocity, τ , are both free vectors, whereas the force, $\mathbf{F}$, and angular velocity, ω , are line vectors. The intensity of a wrench is F , analogous to the amplitude of a twist, ϕ .

If a body is constrained to move about an ISA $\mathcal{S}$, and a wrench acting on a screw $\mathcal{S}'$ contributes nothing to the rate at which work is being done on the body, the screws $\mathcal{S}$ and $\mathcal{S}'$ are said to be *reciprocal*. The condition for reciprocity states that the location and pitches of the screws are such that:

$$(h + h') = r \tan \lambda \quad (\text{B.9})$$

where λ is the angle between the axes and r is the shortest distance between them. Equation B.9 defines the general relationship between constraint and freedom in mechanisms and may also be written in terms of the screw coordinates as:

$$\begin{bmatrix} \mathcal{L} & \mathcal{M} & \mathcal{N} & \mathcal{P}^* & \mathcal{Q}^* & \mathcal{R}^* \end{bmatrix} \cdot \begin{bmatrix} \mathcal{P}_w & \mathcal{Q}_w & \mathcal{R}_w & \mathcal{L}_w^* & \mathcal{M}_w^* & \mathcal{N}_w^* \end{bmatrix}^T = 0 \quad (\text{B.10})$$

In an ideal mechanism no force is required to displace the mechanism along its freedoms and no work is done. The screws representing freedoms must then be reciprocal to those representing constraint. For a mechanism with a single freedom, there must be five independent constraints.

Appendix C

Numerical Methods

Brief descriptions of the numerical methods used are given below. For further details the reader is referred to the *Optimization Toolbox User's Guide for Use with Matlab* and the *Reference Guide* for Matlab both published by The MathWorks Inc. (1994).

Constrained Non-Linear Optimization The function 'constr' was used to find the constrained or unconstrained minimum of a scalar function of several variables, starting with an initial estimate. The algorithm uses a Sequential Quadratic Programming (SQP) method. Gradients were computed by finite differences.

Solution of System of Non-Linear Equations The function 'fsolve' was used to find the root of a set of non-linear equations. For the current analyses, a Levenberg-Marquardt method was implemented. The line search algorithm consisted of a mixed quadratic and cubic polynomial interpolation and extrapolation method. Gradients were computed by finite differences. The algorithm aims to minimise the sum of the squares of the constraint functions. The solution vector at the minimum constitutes a 'root' to the equations when the final function value is below a tolerance level. The default value, 1×10^{-4} , was used. When the termination criterion is not met, i.e. when a 'root' is not found, the solution simply represents the local minimum.

Multi-Objective Goal Attainment The function 'attgoal' was used to minimise a set of objective functions simultaneously. The function implements a Sequential Quadratic Programming (SQP) method. The user assigns each objective function a goal value and a weighting factor which controls the under or over attainment of these goal values. There is also the option to attempt to bring the objective functions as close to the goal values as possible. The procedure terminates if the identification of a minimum is achieved, even if the objective functions have not been successfully brought to 'zero'. It is not strictly a root-finding procedure. Using this algorithm, however, the minimum can be considered a root to the equations when the sum of the squares of the constraint functions is below a set level, e.g. 1×10^{-4} .

Polynomial Surface Fitting The function 'polyfit2d', a two-dimensional version of Matlab's 'polyfit' contributed by Perry W. Stout (1995), was used to find the coefficients of a polynomial function of 2 variables in a least-squares sense.

Two-Dimensional Numerical Integration The function 'quad2dcgen', a 2-D version of Matlab's 'quad' function contributed by Bryce Gardner of Purdue University (1993), was used for 2-D integration over a general 2-D region. The algorithm uses a Gauss-Chebyshev quadrature integration scheme.

Appendix D

Model Parameters

D.1 Idealised and Polynomial Surface Models

The specimen-specific parameters of the polynomial (Model A) and idealised (Models D, F and B) surface models of mobility presented in Chapters 6 and 7 are given below. The parameters are described in the starting configuration of the model, *i.e.* in full extension, with respect to the model tibial coordinate system (Figure 5.6). For the polynomial surface model, the contact error correction is taken into account.

Model A (Poly [2200])	x (mm)	y (mm)	z (mm)
ACL tibial attachment	0	0	0
PCL tibial attachment	20.9	1.0	-6.6
MCL tibial attachment	0.7	22.5	-66.2
ACL femoral attachment	6.2	-0.1	30.9
PCL femoral attachment	4.6	19.0	14.0
MCL femoral attachment	-0.4	48.9	16.7
Medial femoral condyle centre	-2.0	32.1	19.0
Lateral femoral condyle centre	-2.0	-17.3	22.1

Medial Femoral Poly Coeff (radius)	23.7		
Lateral Femoral Poly Coeff (radius)	23.1		
Medial Tibial Poly Coeff	0.00	0.00	-0.06
	0.00	-0.02	0.25
	0.01	-1.05	13.68
Lateral Tibial Poly Coeff	0.00	0.00	-0.04
	0.00	0.02	0.50
	0.01	0.46	3.49

Model D (WTC)	x (mm)	y (mm)	z (mm)
ACL tibial attachment	0.0	0.0	0.0
PCL tibial attachment	35.5	18.3	-21.3
MCL tibial attachment	-1.0	-3.0	-64.6
Medial tibial contact point	12.8	-20.4	-8.7
Lateral tibial contact point	12.8	28.3	-6.0
ACL femoral attachment	31.4	17.0	20.4
PCL femoral attachment	21.3	-7.9	8.8
MCL femoral attachment	13.8	-38.0	11.2
Medial femoral condyle centre	12.8	-20.4	16.8
Lateral femoral condyle centre	12.8	28.3	25.8
Medial tibial plane (normal)	0.0	0.0	1.0
Lateral tibial plane (normal)	0.0	0.0	1.0

Model F (WTC)	x (mm)	y (mm)	z (mm)
ACL tibial attachment	0.0	0.0	0.0
PCL tibial attachment	26.1	-6.9	-14.3
MCL tibial attachment	-6.0	14.0	-58.8
Medial tibial contact point	0.1	24.3	-2.1
Lateral tibial contact point	0.1	-27.5	0.1
ACL femoral attachment	3.6	-5.7	29.7
PCL femoral attachment	5.4	10.2	20.2
MCL femoral attachment	-2.4	43.5	27.7
Medial femoral condyle centre	0.1	24.3	24.8
Lateral femoral condyle centre	0.1	-27.5	25.6
Medial tibial plane (normal)	0.0	0.0	1.0
Lateral tibial plane (normal)	0.0	0.0	1.0

Model B (WTC)	x (mm)	y (mm)	z (mm)
ACL tibial attachment	0.0	0.0	0.0
PCL tibial attachment	34.5	-11.4	-11.6
MCL tibial attachment	13.1	10.6	-63.1
Medial tibial contact point	6.5	23.6	-1.4
Lateral tibial contact point	6.5	-26.9	1.0
ACL femoral attachment	16.6	-10.6	31.3
PCL femoral attachment	11.1	9.8	21.1
MCL femoral attachment	8.1	39.9	28.2
Medial femoral condyle centre	6.5	23.6	27.6
Lateral femoral condyle centre	6.5	-26.9	25.1
Medial tibial plane (normal)	0.0	0.0	1.0
Lateral tibial plane (normal)	0.0	0.0	1.0

Appendix E

Related Publications

E.1 Refereed Conference Proceedings

1. **European Society of Biomechanics Meeting** Toulouse, France, 8-11 July 1998. *Articular surface representation in a 3-D model of knee mobility.* Feikes, J.D. (Podium - J.D. Feikes) *The knee in passive flexion has one degree of unresisted freedom.* Feikes, J.D., Wilson, D.R., Zavatsky, A.B., O'Connor, J.J. (Poster)
2. **Orthopaedic Research Society Meeting** New Orleans, U.S.A., 16-19 March 1998. *The unique track of intact passive knee motion as a kinematic baseline.* Feikes, J.D., Wilson, D.R., O'Connor, J.J. (Podium presentation - J.D. Feikes)
3. **European Orthopaedic Research Society Meeting** Barcelona, Spain, 21-23 March 1997. *Knee replacements and mathematical models.* Feikes, J.D., Wilson, D.R., O'Connor, J.J. (Podium - J.J. O'Connor)
4. **British Orthopaedic Research Society Meeting** Sheffield, U.K., 3-4 March 1997. *Knee replacements and mathematical models.* Feikes, J.D., Wilson, D.R. and O'Connor, J.J. (Poster)
5. **International Symposium on the 3-D Analysis of Human Movement** Grenoble, France, 30 June-3 July, 1996. *The One Degree-of-Freedom Nature of the Human Knee Joint - Basis for a Kinematic Model.* Wilson, D.R., Feikes, J.D., Zavatsky, A.B., Bayona, F., O'Connor, J.J. (Podium - J.D. Feikes)
6. **Canadian Society of Biomechanics IX Biennial Conference** Vancouver, Canada, 21-24 August 1996. *The One Degree-of-Freedom Nature of the Human Knee Joint - Basis for a Kinematic Model.* Wilson, D.R., Feikes, J.D., Zavatsky, A.B., Bayona, F., O'Connor, J.J. (Podium - J.D. Feikes)
7. **British Orthopaedic Research Society Meeting** Oswestry, U.K., 28-29 March 1996. *The Human Knee: A One Degree-of-Freedom Joint?* Wilson, D.R., Feikes, J.D., Zavatsky, A.B., Bayona, F., O'Connor, J.J. (Podium - J.D. Feikes)

E.2 Published Articles

1. **Ligaments and articular contact guide passive knee flexion** Wilson, D.R., Feikes, J.D., O'Connor, J.J. *Journal of Biomechanics*, **31** 1127-1136, 1998.
2. **The components of passive knee movement are coupled to flexion angle** Wilson, D.R., Feikes, J.D., Zavatsky, A.B., O'Connor, J.J. *Journal of Biomechanics*, in press, 1999.
3. **A geometric-numerical approach to modelling the mobility of the human knee joint** Feikes, J.D., O'Connor, J.J. *Journal of Biomechanics*, under review, 1999.

Bibliography

- [1] A.M. Ahmed, A. Hyder, D.L. Burke, and K.H. Chan. In-vitro ligament tension pattern in the flexed knee in passive loading. *J Orthop Res*, 5:217–30, 1987.
- [2] A.A. Amis and P.C. Dawkins. Functional anatomy of the anterior cruciate ligament. *J Bone Jt Surg [Br]*, 73-B:260–267, 1991.
- [3] A.A. Amis and T.D. Zavras. Isometricity and graft placement during anterior cruciate ligament reconstruction. *The Knee*, 2:5–17, 1995.
- [4] K.N. An, M.C. Jacobsen, L.J. Berglund, and Chao E.Y.S. Application of a magnetic tracking device to kinesiological studies. *J Biomech*, 21:613–20, 1988.
- [5] T.P. Andriacchi, J.O. Galante, and R.W. Fermier. The influence of total knee replacement design during walking and stairclimbing. *J Bone Jt Surg [Am]*, 64-A:1328–1335, 1982.
- [6] T.P. Andriacchi, R.P. Mikosz, S.J. Hampton, and J.O. Galante. Model studies of the stiffness characteristics of the human knee joint. *J Biomech*, 16, 1983.
- [7] S.W. Arms, M.H. Pope, R.J. Johnson, R.A. Fischer, I. Arvidsson, and E. Eriksson. The biomechanics of anterior cruciate ligament rehabilitation and reconstruction. *Am J Sports Med*, 12:8–18, 1984.
- [8] S.P. Arnoczky. Anatomy of the anterior cruciate ligament. *Clin Orthop*, 172:19–25, 1983.
- [9] B.D. Beynon, B.C. Fleming, R.J. Johnson, C.E. Nichols, P.A. Renstrom, and M.H. Pope. Anterior cruciate ligament strain behavior during rehabilitation exercises in vivo. *Am J Sports Med*, 23:24–34, 1995.
- [10] E. Biden. *Mechanics of Synovial Joints*. PhD thesis, University of Oxford, 1981.
- [11] E. Biden, J.J. O'Connor, and J. Goodfellow. Tibial rotation in the cadaver knee. In *Trans Orthop Res Soc*, page 30, 1984. Atlanta.
- [12] P.A. Blacharski and J.H. Somerset. A three-dimensional study of the kinematics of the human knee. *J Biomech*, 8:375–384, 1975.
- [13] L. Blankevoort. *Passive motion characteristics of the human knee joint. Experiments and computer simulations*. PhD thesis, Catholic University, 1991.
- [14] L. Blankevoort and R. Huiskes. Validation of a three-dimensional model of the knee. *J Biomech*, 29:955–961, 1996.
- [15] L. Blankevoort, R. Huiskes, and A. de Lange. The envelope of passive knee joint motion. *J Biomech*, 21:705–20, 1988.
- [16] L. Blankevoort, R. Huiskes, and A. de Lange. Helical axes of passive knee joint motions. *J Biomech*, 23:1219–1229, 1990.
- [17] L. Blankevoort, R. Huiskes, and A. de Lange. Recruitment of knee joint ligaments. *Trans ASME - J Biomech Eng*, 13:94–103, 1991.
- [18] L. Blankevoort, J.H. Kuiper, R. Huiskes, and H.J. Grootenboer. Articular contact in a three-dimensional model of the knee. *J Biomech*, 24:1019–1031, 1991.

- [19] J. Bradley, D. FitzPatrick, D. Daniel, T. Shercliff, and J. O'Connor. Orientation of the cruciate ligaments in the sagittal plane. *J Bone Jt Surg [Br]*, 70-B:94–9, 1988.
- [20] O.C. Brantigan and A.F. Voshell. The mechanics of the ligaments and menisci of the knee joint. *J Bone Jt Surg [Am]*, 23:44–66, 1941.
- [21] R.T. Burks. Gross anatomy. In D.M. Daniel, W.H. Akeson, and J.J. O'Connor, editors, *Knee Ligaments: Structure, Function, Injury, and Repair*, chapter 4, pages 59–76. Raven Press, New York, 1990.
- [22] D.L. Butler, Y. Guan, M.D. Kay, J.F. Cummings, S.M. Feder, and M.S. Levy. Location-dependent variations in the material properties of the anterior cruciate ligament. *J Biomech*, 25:511–8, 1992.
- [23] D.L. Butler, M.D. Kay, and D.C. Stouffer. Comparison of material properties in fascicle-bone units from human patellar tendon and knee ligaments. *J Biomech*, 19:425–32, 1986.
- [24] D.L. Butler, F.R. Noyes, and E.S. Grood. Ligamentous restraints to anterior-posterior drawer in the human knee. *J Bone Jt Surg [Am]*, 62-A:259–270, 1980.
- [25] A. Cappozzo, F. Catani, A. Leardini, M.G. Benedetti, and U. Della Croce. Position and orientation in space of bones during movement: experimental artefacts. *Clin Biomech*, 11:90–100, 1996.
- [26] A.J. Carr, G. Keyes, R. Miller, J.J. O'Connor, and J. Goodfellow. Medial unicompartmental arthroplasty: a survival study of the Oxford Meniscal Knee. *Clin Orthop*, 295:205–13, 1993.
- [27] E.Y. Chao, R.K. Laughman, E. Schneider, and R.N. Stauffer. Normative data of knee joint motion and ground reaction forces in adult level walking. *J Biomech*, 16:219–233, 1983.
- [28] E.Y.S. Chao. Goniometry, accelerometry, and other methods. In N. Berme and A. Cappozzo, editors, *Biomechanics of human movement. Applications in rehabilitation, sports and ergonomics*, chapter 7.3, pages 130–139. Bertec Corporation, 1990.
- [29] G.K. Cole, B.M. Nigg, J.L. Ronsky, and M.R. Yeadon. Application of the three-dimensional joint attitude and movement representation: a standardization proposal. *Trans ASME - J Biomech Eng*, 115:344–349, 1993.
- [30] J.J. Collins and J.J. O'Connor. Muscle-ligament interactions at the knee during walking. *Proc Inst Mech Eng Part H, J Engng Med*, 205:11–18, 1991.
- [31] D.C. Covey, A.A. Sapega, G.M. Sherman, and J.S. Torg. Testing for “isometry” during posterior cruciate ligament reconstruction. In *Trans Orthop Res Soc*, page 665, 1992.
- [32] J.J. Craig. *Introduction to Robotics. Mechanics and Control*. Addison-Wesley Publishing Co., 1989.
- [33] R. Crowninshield, M.H. Pope, and R.J. Johnson. An analytical model of the knee. *J Biomech*, 9:397–405, 1976.
- [34] R. van Dijk. *The Behaviour of the Cruciate Ligaments of the Knee*. PhD thesis, Catholic University, 1983.
- [35] R. van Dijk, R. Huiskes, and G. Selvik. Roentgen stereophotogrammetric methods for the evaluation of the three dimensional kinematic behavior and cruciate ligament length patterns of the human knee joint. *J Biomech*, 12:727–31, 1979.
- [36] R.G. Edwards, J.F. Lafferty, and K.O. Lange. Ligament strain in the human knee joint. *Trans ASME - J Biomech Eng*, pages 131–136, March 1970.
- [37] J.R. Essinger, P.F. Leyvraz, J.H. Heegard, and D.D. Robertson. A mathematical model for the evaluation of the behaviour during flexion of condylar-type knee prostheses. *J Biomech*, 22:1229–1241, 1989.
- [38] D.P. FitzPatrick. *Mechanics of The Knee Joint*. PhD thesis, University of Oxford, 1989.

- [39] D.P. FitzPatrick and J.J. O'Connor. Theoretical modelling of the knee applied to the anterior drawer test. In *Proc Inst Mech Eng - Changing Role of Engineering in Orthopaedics*, pages 79–83, 1989.
- [40] N.F. Friederich, W. Muller, and W.R. O'Brien. Klinische anwendung biomechanischer und funktionell anatomischer daten am kniegelenk. *Orthopade*, 21:41–50, 1992.
- [41] H. Fujie, G.A. Livesay, S.L.-Y. Woo, S. Kashiwaguchi, and G. Blomstrom. The use of a universal force-moment sensor to determine in-situ forces in ligaments: a new methodology. *Trans ASME - J Biomech Eng*, 117:1–7, Feb 1995.
- [42] T. Fukubayashi, P.A. Torzilli, M.F. Sherman, and R.F. Warren. An in-vitro biomechanical evaluation of anterior-posterior motion of the knee: Tibial displacement, rotation and torque. *J Bone Jt Surg [Am]*, 64-A:258–64, 1982.
- [43] S.M. Gabriel, D.A. Dennis, R.D. Komistek, W.A. Hoff, and J.B. Stiehl. In vivo TKA kinematics with consequences for system stresses and strains. In *Trans Orthop Res Soc*, page 212, 1996.
- [44] F.G. Girgis, J.L. Marshall, and A.R.S. Al Monajem. The cruciate ligaments of the knee joint - anatomical, functional and experimental analysis. *Clin Orthop*, 106:216–31, 1975.
- [45] J.W. Goodfellow and J.J. O'Connor. The mechanics of the knee and prosthesis design. *J Bone Jt Surg [Br]*, 60-B:358–369, 1978.
- [46] E.S. Grood, M.S. Hefzy, and T.L. Lindenfield. Factors affecting the region of most isometric femoral attachments. Part I: The posterior cruciate ligament. *Am J Sports Med*, 63:1257–1269, 1989.
- [47] E.S. Grood and F.R. Noyes. Diagnosis of knee ligament injuries: Biomechanical precepts. In *The Crucial Ligaments: Diagnosis and Treatment of Ligamentous Injuries About the Knee*, chapter 9, pages 245–85. Churchill Livingstone, New York, 1988.
- [48] E.S. Grood, F.R. Noyes, D.L. Butler, and W.J. Suntay. Ligamentous and capsular restraints preventing straight medial and lateral laxity in intact human cadaver knees. *J Bone Jt Surg [Am]*, 63-A:1257–1269, 1981.
- [49] E.S. Grood and W.J. Suntay. A joint coordinate system for the clinical description of three-dimensional motions: application to the knee. *Trans ASME - J Biomech Eng*, 105:136–144, 1983.
- [50] M.S. Hefzy and E.S. Grood. Sensitivity of insertion locations on length patterns of anterior cruciate ligament fibers. *Trans ASME - J Biomech Eng*, 108:119–26, 1986.
- [51] M.S. Hefzy, E.S. Grood, and F.R. Noyes. Factors affecting the region of most isometric femoral attachments, Part II: The anterior cruciate ligament. *Am J Sports Med*, 17:208–16, 1989.
- [52] F.C.T. Van der Helm and R. Veenbaas. Modelling the mechanical effect of muscles with large attachment sites: Application to the shoulder mechanism. *J Biomech*, 24:1151–1163, 1991.
- [53] S. Hirokawa. Biomechanics of the knee joint: A critical review. *Critical Reviews in Biomedical Engineering*, 21:79–135, 1993.
- [54] H.P. Hirshman, D.M. Daniel, and K. Miyasaka. The fate of unoperated knee ligament injuries. In D.M. Daniel, W.H. Akeson, and J.J. O'Connor, editors, *Knee Ligaments: Structure, Function, Injury, and Repair*, chapter 27, pages 481–504. Raven Press, New York, 1990.
- [55] J.M. Hollis, S. Takai, D.J. Adams, S. Horibe, and S.L.-Y. Woo. The effects of knee motion and external loading on the length of the anterior cruciate ligament (ACL): A kinematic study. *Trans ASME - J Biomech Eng*, 113:208–214, 1991.
- [56] H.-H. Hsieh and P.S. Walker. Stabilizing mechanisms of the loaded and unloaded knee joint. *J Bone Jt Surg [Am]*, 54-A:87–93, 1976.

- [57] K.H. Hunt. *Kinematic Geometry of Mechanisms*. Oxford University Press, Oxford, 1978.
- [58] A. Huson, C.W. Spoor, and A.J. Verbout. A model of the human knee, derived from kinematic principles and its relevance for endoprosthesis design. *Acta Morphol Neerl Scan*, 27:45–62, 1989.
- [59] R.A. Huss, H. Holstein, and J.J. O'Connor. The effects of cartilage deformation on the laxity of the knee joint. *Proc Inst Mech Eng Part H, J Engng Med*, 213:19–32, 1999.
- [60] R.A. Huss, H. Holstein, and J.J. O'Connor. A mathematical model of forces in the knee under isometric quadriceps contractions. *Proc Inst Mech Eng Part H, J Engng Med*, submitted, 1999.
- [61] A. Imran and J.J. O'Connor. The effect of tibial surface curvature on cruciate ligament loading. *J Bone Jt Surg [Br]*, 78-B(S1):34, 1996.
- [62] A. Inderster, K.P. Benedetto, K.H. Künzel, O. Gaber, and R.A. Balyk. Fiber orientation of anterior cruciate ligament: An experimental morphological and functional study, Part I. *Clin Anat*, 6:26–32, 1993.
- [63] H. Johansson, P. Sjolander, and P. Sojka. A sensory role for the cruciate ligaments. *Clin Orthop*, 268:161–78, 1991.
- [64] I.A. Kapandji. *The Physiology of the Joints, Vol. 2 The Lower Limb*. Churchill Livingstone, London, 5th edition, 1987. (Translated by L.H. Honore).
- [65] J.C. Kennedy, R.J. Hawkins, and R.B. Willis. Strain gauge analysis of knee ligaments. *Clin Orthop*, 129:225–9, 1977.
- [66] G.L. Kinzel, A.S. Hall, and B.M. Hillberry. Measurement of the total motion between two body segments - I. Analytical development. *J Biomech*, 5:93–105, 1972.
- [67] H. Kurosawa, P.S. Walker, S. Abe, A. Garg, and T. Hunter. Geometry and motion of the knee for implant and orthotic design. *J Biomech*, 18(7):487–499, 1985.
- [68] H. Kurosawa, K.-I. Yamakoshi, K. Yasuda, and T. Sasaki. Simultaneous measurement of changes in length of the cruciate ligaments during knee motion. *Clin Orthop*, 265:233–240, 1991.
- [69] M.A. Lafortune, P.R. Cavanagh, H.J. Sommer III, and A. Kalenak. Three-dimensional kinematics of the human knee during walking. *J Biomech*, 25(4):347–357, 1992.
- [70] H.Y. Lee, W. Lin, and J. Duffy. A method for forward displacement analysis of in-parallel platform mechanisms. *Mechatronics*, 3:659–69, 1993.
- [71] M. Lengsfeld. Rotationsbewegungen im kniegelenk, ein mathematisch-kinematisches modell. *Z Orthop*, 127:243–247, 1989.
- [72] M. Levy, P.A. Torzilli, and R.F. Warren. The effect of medial menisectomy on anterior-posterior motion of the knee. *J Bone Jt Surg [Am]*, 71-A:401–406, 1978.
- [73] J.L. Lewis, W.D. Lew, J.A. Hill, P. Hanley, K. Ohland, S. Kirstukas, and R.E. Hunter. Knee joint motion and ligament forces before and after ACL reconstruction. *Trans ASME - J Biomech Eng*, 111:97–106, 1989.
- [74] J.L. Lewis, W.D. Lew, and J. Schmidt. A note on the application and evaluation of the buckle transducer for knee ligament force measurement. *Trans ASME - J Biomech Eng*, 104:125, 1982.
- [75] J. Li, U.P. Wyss, P.A. Costigan, and K.J. Deluzio. An integrated procedure to assess knee-joint kinematics and kinetics during gait using an optoelectric system and standardized X-rays. *J Biomed Eng*, 15:392–400, 1993.
- [76] T.-W. Lu. *Geometric and Mechanical Modelling of the Human Locomotor System*. PhD thesis, University of Oxford, 1997.

- [77] T.-W. Lu and J.J. O'Connor. A three-dimensional computer graphics-based animated model of the human locomotor system with anatomical joint constraints. In *Trans Orthop Res Soc*, 1998.
- [78] T.-W. Lu, J.J. O'Connor, S.J.G. Taylor, and P.S. Walker. Validation of a lower limb model with *in vivo* femoral forces telemetered from two subjects. *J Biomech*, 31:63–9, 1997.
- [79] K.L. Markolf, J.F. Gorek, M. Kabo, and M.S. Shapiro. Direct measurement of resultant forces in the anterior cruciate ligament. *J Bone Jt Surg [Am]*, 72-A:557–67, 1990.
- [80] K.L. Markolf, A. Graff-Radford, and H.C. Amstutz. In-vivo knee stability - a quantitative assessment using an instrumented clinical testing apparatus. *J Bone Jt Surg [Am]*, 60-A:664–9, 1978.
- [81] K.L. Markolf, J.S. Mensch, and H.C. Amstutz. Stiffness and laxity of the knee - The contributions of the supporting structures. A quantitative in-vitro study. *J Bone Jt Surg [Am]*, 58-A:583–93, 1976.
- [82] G.D. Markovich, S.A. Banks, and W.A. Hodge. Functional stability in total knee replacements. In *Trans Orthop Res Soc*, page 730, 1996.
- [83] J.S. Mensch and H.C. Amstutz. Knee morphology as a guide to knee replacement. *Clin Orthop*, 112:231–41, 1975.
- [84] A. Menshik. Mechanik des kniegelenks - Teil 1. *Z Orthop*, 112:481–95, 1974.
- [85] H. Meyer. Die Mechanik des Kniegelenks. *Archiv fur Anatomie und Physiologie*, pages 497–547, 1853.
- [86] T.J.A. Mommersteeg. *Human knee ligaments: structure, function and mechanical properties*. PhD thesis, Catholic University of Nijmegen, 1994.
- [87] T.J.A. Mommersteeg, L. Blankevoort, R. Huiskes, J.G.M. Kooloos, and J.M.G. Kauer. Characterization of the mechanical behaviour of human knee ligaments: a numerical-experimental approach. *J Biomech*, 29:151–160, 1996.
- [88] T.J.A. Mommersteeg, L. Blankevoort, R. Huiskes, J.G.M. Kooloos, J.M.G. Kauer, and J.C.M. Hendriks. The effect of variable relative insertion orientation of the human knee bone-ligament-bone complexes on the tensile stiffness. *J Biomech*, 28:745–752, 1995.
- [89] T.J.A. Mommersteeg, R. Huiskes, L. Blankevoort, J.G.M. Kooloos, and J.M.G. Kauer. An inverse dynamics modeling approach to determine the restraining function of human knee ligament bundles. *J Biomech*, 30:139–146, 1997.
- [90] T.J.A. Mommersteeg, R. Huiskes, L. Blankevoort, J.G.M. Kooloos, J.M.G. Kauer, and P.G.M. Maathuis. A global verification study of a quasi-static knee model with multi-bundle ligaments. *J Biomech*, 29:1659–1664, 1996.
- [91] J.B. Morrison. The mechanics of the knee joint in relation to normal walking. *J Biomech*, 3:51–61, 1970.
- [92] W. Müller. *The Knee: Form, Function, and Reconstruction*. Springer-Verlag, Berlin, 1983.
- [93] M. P. Murray, A. B. Drought, and R. C. Kory. Walking patterns of normal men. *J Bone Jt Surg [Am]*, 46-A:335–60, 1964.
- [94] B.E.L. Nahass, M.M. Madson, and P.S. Walker. Motion of the knee after condylar resurfacing - An *in vivo* study. *J Biomech*, 24:1107–1117, 1991.
- [95] M. Niitsu, H. Akisada, I. Anno, and S. Miyakawa. Moving knee joint: technique for kinematic mr imaging. *Radiology*, 174:569–570, 1990.
- [96] F.R. Noyes and E.S. Grood. The strength of the anterior cruciate ligament in humans and rhesus monkeys. *J Bone Jt Surg [Am]*, 58-A:1074–82, 1976.
- [97] W.R. O'Brien, N.F. Friederich, W. Muller, and C.E. Henning. Functional anatomy of the cruciate ligaments. Unpublished, 1989.

- [98] J.J. O'Connor, T. Shercliff, D. FitzPatrick, J. Bradley, D. Daniel, E. Biden, and J. Goodfellow. Geometry of the knee. In D.M. Daniel, W.H. Akeson, and J.J. O'Connor, editors, *Knee Ligaments: Structure, Function, Injury, and Repair*, chapter 10, pages 163–200. Raven Press, New York, 1990.
- [99] J.J. O'Connor, T.L. Shercliff, E. Biden, and J.W. Goodfellow. The geometry of the knee in the sagittal plane. *Proc Inst Mech Eng Part H, J Engng Med*, 203:223–33, 1989.
- [100] M. Odensten and J. Gillquist. Functional anatomy of the anterior cruciate ligament and a rationale for reconstruction. *J Bone Jt Surg [Am]*, 67-A:257–62, 1985.
- [101] J. Perry, D. Antonelli, and W. Ford. Analysis of knee-joint forces during flexed-knee stance. *J Bone Jt Surg [Am]*, 57-A:961–967, 1975.
- [102] R.L. Piziali, J.C. Rastegar, and D.A. Nagel. Measurement of the nonlinear, coupled stiffness characteristics of the human knee. *J Biomech*, 10:45–51, 1977.
- [103] R.L. Piziali, W.P. Seering, D.A. Nagel, and D.J. Schurman. The function of the primary ligaments of the knee in anterior-posterior and medial-lateral motions. *J Biomech*, 13:777–84, 1980.
- [104] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C. The Art of Scientific Computing. Second Edition*. Cambridge University Press, 1992.
- [105] A. Race and A.A. Amis. The mechanical properties of the two bundles of the human posterior cruciate ligament. *J Biomech*, 27:13–24, 1994.
- [106] A. Race and A.A. Amis. Loading of the two bundles of the posterior cruciate ligament: an analysis of bundle function in a-p drawer. *J Biomech*, 29:873–879, 1996.
- [107] E.L. Radin. Orthopaedic surgery in the ageing population. In V. Wright, editor, *Bone and Joint Disease in the Elderly*, chapter 15, pages 275–297. Churchill Livingstone, Edinburgh, 1983.
- [108] E.L. Radin and I.L. Paul. Response of joints to impact loading. I, *In vitro* wear. *Arth Rheum*, 14:356–362, 1971.
- [109] P. Renstrom, S.W. Arms, T.S. Stanwyck, R.J. Johnson, and M.H. Pope. Strain within the anterior cruciate ligament during hamstring and quadriceps activity. *Am J Sports Med*, 14:83–7, 1986.
- [110] A. Rosenberg, R.P. Mikosz, and C.G. Mohler. Basic knee biomechanics. In W.N. Scott, editor, *The Knee*, chapter 4, pages 75–95. Mosby-Year Book, Inc., St.Louis, 1994.
- [111] J.S. Rovick, J.D. Reuben, R.J. Schragar, and P.S. Walker. Relation between knee motion and ligament length patterns. *Clin Biomech*, 6:213–220, 1991.
- [112] A.A. Sapega, R.A. Moyer, C. Schneck, and N. Komalahiranya. Testing for isometry during reconstruction of the anterior cruciate ligament. *J Bone Jt Surg [Am]*, 72-A:259–267, 1990.
- [113] W.P. Seering, R.L. Piziali, D.A. Nagel, and D.J. Schurman. The function of the primary ligaments of the knee in varus-valgus and axial rotation. *J Biomech*, 13:785–94, 1980.
- [114] G. Selvik. *A roentgen stereophotogrammetric method for the study of the kinematics of the skeletal system*. PhD thesis, Univeristy of Lund, 1974.
- [115] J.A. Shaw and D.G. Murray. The longitudinal axis of the knee and the role of the cruciate ligaments in controlling transverse rotation. *J Bone Jt Surg [Am]*, 56-A:1603–7, 1974.
- [116] R. Shiavi, T. Limbird, M. Frazer, K. Stivers, A. Strauss, and J. Abramovitz. Helical motion analysis of the knee - II. Kinematics of uninjured and injured knee during walking and pivoting. *J Biomech*, 20:653–665, 1987.
- [117] R. Shiavi, T. Limbird, M. Frazer, K. Stivers, A. Strauss, and J. Abramovitz. Helical motion analysis of the knee - I. Methodology for studying kinematics during locomotion. *J Biomech*, 20, 1987.

- [118] S. C. Shoemaker, D. Adams, D. M. Daniel, and S. L. Woo. Quadriceps anterior cruciate graft interaction: An in vitro study of joint kinematics and ACL graft tension. *Clin Orthop*, 294:379–390, 1993.
- [119] J.A. Sidles, R.V. Larson, J.L. Garbini, D.J. Downey, and F.A. Matsen III. Ligament length relationships in the moving knee. *J Orthop Res*, 6:593–610, 1988.
- [120] C.W. Spoor and F.E. Veldpaus. Rigid body motion calculated from spatial coordinates of markers. *J Biomech*, 13:391–393, 1980.
- [121] H. Strasser. *Lehrbuch der Muskel- und Gelenkmechanik*. Springer, Berlin, 1917.
- [122] D.H. Sutherland, R. Olshen, L. Cooper, and S. Woo. The development of mature gait. *J Bone Jt Surg [Am]*, 62-A:336–53, 1980.
- [123] S. Takai, S.L.-Y. Woo, G.A. Livesay, D.J. Adams, and F.H. Fu. Determination of the in situ loads on the anterior cruciate ligament. *J Orthop Res*, 11:686–695, 1993.
- [124] J.R. Taylor. *An Introduction to Error Analysis. The Study of Uncertainties in Physical Measurements*. University Science Books, Mill Valley, CA, USA, 1982.
- [125] W.O. Thompson, F.H. Fu, F.L. Thaete, and S. Dye. Assessment of tibial meniscal kinematics by 3-D magnetic resonance imaging. *Trans Orthop Res Soc*, page 245, 1990.
- [126] D.E. Toutoungi. *The mechanics of rehabilitation exercises for the cruciate-ligament injured knee*. PhD thesis, University of Oxford, 1997.
- [127] D.E. Toutoungi, A.B. Zavatsky, and J.J. O'Connor. Parameter sensitivity of a mathematical model of the anterior cruciate ligament. *Proc Inst Mech Eng Part H, J Engng Med*, 211:235–246, 1997.
- [128] P.S. Trent, P.S. Walker, and B. Wolf. Ligament length patterns, strength and rotational axes of the knee joint. *Clin Orthop*, 117:263–70, 1976.
- [129] J.J. Uicker, J. Denavit, and R.S. Hartenberg. An iterative method for the displacement analysis of spatial mechanisms. *Trans ASME - J App Mech*, pages 309–14, June 1964.
- [130] J.W. Vahey and L.F. Draganich. Tensions in the anterior and posterior cruciate ligaments of the knee during passive loading: Predicting ligament loads from in situ measurements. *J Orthop Res*, 9:529–38, 1991.
- [131] F.E. Veldpaus, H.J. Woltring, and Dortmans L.J.M.G. A least-squares algorithm for the equiform transformation from spatial marker co-ordinates. *J Biomech*, 21:45–54, 1988.
- [132] C-J. Wang and P.S. Walker. Rotatory laxity of the human knee joint. *J Bone Jt Surg [Am]*, 56-A:161–70, 1974.
- [133] L.F. Warren, J.L. Marshall, and F. Girgis. The prime static stabilizer of the medial side of the knee. *J Bone Jt Surg [Am]*, 56-A:665–74, 1974.
- [134] W.E. Weber and E.F.W. Weber. *Mechanik der menschlichen Gehwerkzeuge*. Dietrichschen Buchhandlung, Gottingen, 1836.
- [135] D.R. Wilson. *Three-dimensional kinematics of the knee*. PhD thesis, University of Oxford, 1996.
- [136] D.R. Wilson, J.D. Feikes, and J.J. O'Connor. Ligaments and articular contact guide passive knee flexion. *J Biomech*, 31:1127–1136, 1998.
- [137] S.A. Wilson, V.J. Vigorita, and W.N. Scott. Anatomy. In W.N. Scott, editor, *The Knee*, chapter 2, pages 15–54. Mosby-Year Book, Inc., St. Louis, 1994.
- [138] D.A. Winter. *Biomechanics and Motor Control of Human Movement*. John Wiley and Sons, New York, 2nd edition, 1990.
- [139] J. Wismans, F. Veldpaus, and J. Janssen. A three-dimensional mathematical model of the knee-joint. *J Biomech*, 13:677–685, 1980.

- [140] H. J. Woltring. A fortran package for generalized, cross-validatory spline smoothing and differentiation. *Adv. Engng Software*, 8:104–113, 1986.
- [141] H.J. Woltring. Model and measurement error influences in data processing. In N. Berme and A. Cappozzo, editors, *Biomechanics of human movement. Applications in rehabilitation, sports and ergonomics*, chapter 10.1, pages 203–237. Bertec Corporation, 1990.
- [142] H.J. Woltring, R. Huiskes, and A. De Lange. Finite centroid and helical axis estimation from noisy landmark measurements in the study of human joint kinematics. *J Biomech*, 18:379–389, 1985.
- [143] S.L-Y. Woo and D.J. Adams. The tensile properties of human anterior cruciate ligament (ACL) and ACL graft tissues. In D.M. Daniel, W.H. Akeson, and J.J. O'Connor, editors, *Knee Ligaments: Structure, Function, Injury and Repair*, chapter 13, pages 279–90. Raven Press, New York, 1990.
- [144] G. Wu and P.R. Cavanagh. ISB recommendations for standardization in the reporting of kinematic data. *J Biomech*, 28:1257–1261, 1995.
- [145] Y. Yoshioka, D.W. Siu, R.A. Scudamore, and T.D.V. Cooke. Tibial anatomy and functional axes. *J Orthop Res*, 7:132–7, 1989.
- [146] A.B. Zavatsky. *The Functional Architecture of Human Knee Ligaments*. PhD thesis, University of Oxford, 1993.
- [147] A.B. Zavatsky and J.J. O'Connor. A model of human knee ligaments in the sagittal plane: Part I. Response to passive flexion. *Proc Inst Mech Eng Part H, J Engng Med*, 206:125–34, 1992.
- [148] A.B. Zavatsky and J.J. O'Connor. A model of human knee ligaments in the sagittal plane: Part II. Fibre recruitment under load. *Proc Inst Mech Eng Part H, J Engng Med*, 206:135–45, 1992.
- [149] A.B. Zavatsky and J.J. O'Connor. Muscle-ligament interaction at the knee during isometric quadriceps contractions. *Proc Inst Mech Eng Part H, J Engng Med*, 207:7–18, 1993.
- [150] A.B. Zavatsky and J.J. O'Connor. Three-dimensional geometrical models of human knee ligaments. *Proc Inst Mech Eng Part H, J Engng Med*, 208:229–40, 1994.