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THE TAXING ISSUE OF QUEUES

Terry O'Shaughnessy

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Manor Road Building, Oxford OX1 3UQ

The Taxing Issue of Queues

Terry O'Shaughnessy[1]

St Anne's College
Oxford

Abstract: This paper examines the redistributional function of queues. A system in which subsidies and queues are used to allocate goods may appear attractive to policy makers who are concerned about equity since the resource used in queuing (time) is generally allocated more equally than, say, human or physical capital. Thus a subsidy-queue mechanism performs a role similar to that performed by a tax-benefit system. Both mechanisms, however, bring with them efficiency losses, which the paper compares. It is shown that the subsidy-queue mechanism may appear superior if the trade-off between efficiency and equity is viewed in terms of consumption and the distribution of consumption. On the other hand, if the equity-efficiency trade-off is properly formulated in terms of utility and the distribution of utility the tax-based redistributional mechanism is superior.

Keywords: Redistribution, queues, subsidies, tax-benefit system.

JEL classification: D60, H20

[1]Dr T.J. O'Shaughnessy, St Anne's College, Oxford OX2 6HS, United Kingdom. Telephone +44 (0)1865 274863. Fax +44 (0)1865 274899. Email terry.oshaughnessy@economics.ox.ac.uk.

Introduction[2]

Despite the well-known fact that the market allocation of goods has important efficiency properties, other allocation mechanisms are sometimes observed. In this paper the focus is on allocation by queuing. Situations in which goods are sold at administered prices set below their market-clearing level (possibly at zero price) and allocation is by queues are common in extreme situations such as wartime and the aftermath of natural disasters (when relief is often distributed in this way). But even in normal times allocation by queuing is frequently observed. Thus certain goods (for example health care, public housing, tickets for concerts and sporting events, access to congestible facilities such as roads) are often allocated via queues rather than by price adjustments.

What is common to all these examples of allocation by queuing is the perception that, in some sense, this allocation mechanism is fairer than allocation by price. Queuing reallocates resources because the resource used - time - is more equally distributed[3] than other resources, such as physical and human capital. This is not to deny that arbitrage may take place in which some individuals queue and then resell the good on a secondary market, or that other individuals avoid queuing by buying the rationed good at higher than the administered prices in these secondary markets or by purchasing dearer substitutes. Arbitrage and substitution of this sort complement the redistributional function of queuing itself by giving an individual with little human or physical capital a resource - the ability to queue - which will have a strictly positive price in a queuing equilibrium. To move away from a queuing system it is not sufficient to establish an alternative or secondary market in which the good is available at a higher, market-clearing price. It requires raising the price in the primary market until that market clears without queuing. Doing so will produce efficiency gains, but in the process the price of the ability to queue will fall to zero.

The purpose of this paper is to investigate the redistributional aspect of queuing by comparing this mechanism with an alternative redistributional mechanism: that based on a tax-benefit

[2]An earlier version of this paper appeared as O'Shaughnessy (1991), where the results were obtained by assuming a particular utility function (the "Illustration" described below). This version generalises and provides proofs of these results; see Propositions 1 and 2, below.

[3]Time to queue is not perfectly equally distributed, however. The old, the ill or the disabled may find it difficult or impossible to queue.

system. As will be shown, both the subsidy-queue mechanism and the tax-benefit mechanism have the effect of redistributing income towards those less able to compete in the labour market.[4] The use of both mechanisms, however, involve costs. The relationship between the attractive redistributional features of the two mechanisms and the costs in terms of foregone output and time lost in queuing confronts the public and policy-makers with a choice. The fact than one mechanism (say, queuing) rather than the other (say, allocation by price) is preferred in a particular situation may stem from different values being placed on equity and efficiency, but it may also derive from the fact that the relationship between equity and efficiency in the two redistributional mechanisms is not properly understood. This paper proposes a framework for assessing the equity and efficiency properties of the subsidy-queue and tax-benefit mechanisms which both demonstrates the superiority of the latter, while providing an insight into the popular appeal of the former.

Modelling Strategy

I compare two economies, one that uses a tax-benefit mechanism to redistribute income and the other that employs a subsidy-queue system. Individuals in each economy derive utility from consumption of a single good, c and from leisure, 1-h, where h is the time spent working or queuing[5], measured as a fraction of available time:

$$u(c, 1-h)$$

I also assume that individuals in both economies are characterised by differences in their abilities to produce output. Thus the output of an individual of ability a who chooses to work for

[4]Throughout this paper I am assuming that inequalities in income and consumption stem from differential abilities of wage earners. This leaves out non-wage income. In market economies incomes derived from the ownership of capital are more unequally distributed than wage and salary incomes. Leaving them out might suggest that the problem of inequality is less serious than it is. On the other hand, for well known reasons practical tax-benefit systems focus on wage incomes and the participation decisions of those whose only income opportunities are wages and benefits. Non-wage incomes in the form of privileged access to certain facilities were also important in economies which make extensive use of subsidies and queues (such as the former centrally-planned economies) but are equally difficult to deal with in a formal, comparative way.

[5]I assume that the disutility of an hour of queuing is equal to the disutility of an hour of work, though other weights could be used.

period h is

$$y(a) = ha$$

The distribution of abilities in the population is described by a density function $f(a)$ defined on a support (0,1) and a corresponding distribution function

$$F(a) = \int_0^a f(t)dt$$

so that $F(a)$ is the proportion of the population with ability up to and including a.

In a competitive economy with no taxes or benefits individuals of different abilities would be paid at different wages rates per time period w:

$$w = ap$$

where p is the price at which output is sold (which we will fix at 1.0). In such an economy individuals would choose to supply h^* units of labour where h^* is chosen to maximize

$$u(ah, 1 - h)$$

The resulting distribution of income (and consumption) is unequal for two reasons: individuals with different abilities choose to supply different amounts of labour and, when they do so, they are paid different amounts for what they do supply.[6]

A Tax-Benefit System

Income and consumption will become more equal if a tax-benefit system is introduced. Here we will consider a very simple system: the income of those who work is taxed at a flat rate t and the taxes raised in this way are used to finance a (non-taxed) benefit b which is paid to each (voluntarily) unemployed person. For individuals who choose to work we have a consumption level of

$$c = wh(1 - t)$$

and utility of

[6] If h^* is increasing as a function of ability these two effects will reinforce one another: the more able will work longer hours and will receive more for each hour they work. This is what occurs in the example developed below, but it is not the only possibility. If the more able strongly prefer leisure h^* will fall as a function of ability so that income (and consumption) will be more equally distributed than ability.

$$u(ah(1-t),1-h)$$

Again, let $h^*(a)$ be the value of $h(a)$ which maximizes this.

For some low ability individuals their utility when unemployed will be greater than $u(h^*(a))$. They will obtain a utility level of

$$u(b,1)$$

from the benefit system. There will be a cut-off ability level a^* such that those with ability $a > a^*$ choose to work and those with $a < a^*$ choose to receive benefit instead, defined by

$$u(a^* h^*(a^*)(1-t),1-h^*(a^*)) = u(b,1)$$

Since each employed person pays twh in tax, total tax receipts are

$$T = \int_{a^*}^{1} tah^*(a)f(a)da$$

For the tax-benefit system to be self-financing, total tax receipts T must be equal to the total paid out in benefits, B :

$$B = b\int_{0}^{a^*} f(a)da = T = \int_{a^*}^{1} tah^*(a)f(a)da$$

Illustration

To illustrate this mechanism, let us specify the utility function as

$$u(c,1-h) = \sqrt{c} + (1-h)$$

and assume that the density function $f(a)$ is uniform on (0,1).

It follows that

$$u(h) = \sqrt{ah(1-t)} + 1 - h$$

Choosing h^* to maximize $u(h)$ yields:

$$h^* = \frac{a(1-t)}{4}$$

in which case

$$u^* = 1 + \frac{a(1-t)}{4}$$

If individuals choose not to supply labour they will obtain

$$u^* = \sqrt{b} + 1$$

Equating these two expressions defines a^*, the cut-off level of ability which divides the employed from the unemployed:

$$a^* = 4\frac{\sqrt{b}}{(1-t)}$$

It follows that the relationship between the cut-off ability level a^* and the level of benefits is

$$b = \frac{(1-t)^2 a^{*2}}{16}$$

The condition for the tax-benefit system to be self-financing becomes

$$T = \int_{a^*}^{1} tah^* da = B = b\int_{0}^{a^*} da = ba^*$$

Thus

$$T = \int_{a^*}^{1} ta^2(1-t)da = B = \frac{ba^3}{4} = \frac{(1-t)a^{*3}t(1-t)}{16} \quad 12$$

from which we obtain

$$a^* = \sqrt[3]{\frac{4t}{3+t}}$$

Figure 1 illustrates the situation facing individuals with different abilities for a tax rate of t = 0.1. Three budget constraints are drawn for $w = a$ = 0.4, 0.6 and 0.8. Someone with a = 0.4 could consume at point A or anywhere along the lower of the three budget constraints. Clearly he or she would prefer to be at A and so chooses not to work. On the other hand, someone for whom a = 0.6 will choose point B and so provide some labour, but not as much as the person with a = 0.8 who chooses point C.

Raising the tax rate t has two effects on these choices. Higher tax rates cause the budget constraints to rotate anticlockwise so that those who choose to work offer fewer hours. Higher tax rates also enable a more generous level of benefit to be funded, at least up to a point. This means that a larger proportion of less able individuals will choose not to participate in the labour force. A consequence is that total consumption is lower the higher the tax rate but that consumption (and utility) are more equally distributed. Note that utility is distributed more equally than consumption, whatever the tax rate. There are two reasons for this. One is that the marginal utility of consumption falls with higher consumption so that the able obtain less satisfaction from each unit of their higher level of consumption than do the less able. The other is that the utility calculation takes into account the utility of leisure, of which the less able consume more.

How Much Redistribution is Justified?

There are a number of approaches to answering this question, three of which will be considered in this paper. One is to posit egalitarianism as an argument in individuals' (or policy-makers') utility functions[7] and to examine the trade-off between equity and efficiency that arises when utility functions of this sort are maximized. As we will see, this requires careful specification of the way in which "equity" and "efficiency" are perceived by those interested in such a trade-off. Before doing so, however, it may be useful to mention two other answers to the question of how far redistribution should proceed.

For a utilitarian this question has a straightforward answer: redistribution should proceed until total (or average) utility is maximized. As the upper function in Figure 2 shows, such a maximum occurs in the illustrative tax-benefit model at about $t = 13.0\%$. Up to this point higher levels of consumption (and more leisure) on the part of the less able increases their utility by more than the amount that the utility of the more able is reduced.

Another straightforward answer to the question of how much redistribution is justified is provided by those who would seek to maximize the utility of the worst off person in the economy. In this model the worst off person is unemployed for all $t > 0$ and so his or her utility depends only on how generous a benefit level may be funded. Here the benefit level increases in the tax rate until $t = 26.0\%$ and then declines again as the combined effect of fewer people working and fewer hours being worked by those who do work starts to reduce total tax receipts. This model therefore illustrates the point made by Rawls (1972, pp. 77-8) that utilitarianism allows larger inequalities than does maximizing the utility of the worst-off person.[8]

A Subsidy-Queue System

An alternative method of achieving a more equal distribution of consumption and utility is to set up a system within which people queue for goods which are distributed at subsidised (perhaps even zero) prices. To fix ideas, we will imagine such a scheme being implemented by a (beneficent) social planner. As before, there is a single good. The planner purchases all the output produced by (profit maximizing) enterprises at a price $p_0 = 1.0$. Enterprises pay each

[7] See Breit (1974), and Lambert (1985a, 1985b, 1990).

[8] Here I am positing the notion of a "Rawlsian" policy-maker who seeks to maximize the utility of the worst-off person. This does not correspond exactly with Rawls's own argument in Rawls (1972) which is couched in terms of "primary social goods" (Rawls's list includes "income and wealth") rather than utility.

worker a wage, w, equal to his or her ability, a. Although there is only one good there are two markets. In the first "official" market a low price is set and supply is rationed by queuing.[9] In the other, which may be an "unofficial" or "black" market, supply is rationed by price. Let the equilibrium value of this "unofficial" price by p_1. The planner sets the "official" price of the good, p_2, and allocates output to the two markets.

Individuals face the following decision problem: choose a value of labour supply (h) in order to obtain an income wh. Then decide whether to purchase the good at p_1, without queuing, in which case

$$wh = c_1 p_1$$

and

$$u(c,h) = u(\frac{wh}{p_1}, 1 - h)$$

Denote the value of h which maximizes utility in this situation as h_1*. Alternatively, individuals may decide to purchase the good at p_2 in the official or subsidized market, but if they do so they will have to queue for a time[10] q for each unit of the commodity that they purchase.[11] In this case

[9]Stahl and Alexeev (1985) investigate the properties of a queue-rationed centrally planned economy and consider the effects of arbitrage between "official" and "unofficial" markets. One difference between their model and the one employed here is that they take money income as given and do not model the labour supply decision.

[10]We abstract from uncertainty. In reality, individuals join queues without knowing how long they will have to wait. They also face uncertainty about whether the supplier will run out of stocks before they reach the head of the queue. Both these considerations have implications for equity and efficiency. For example, risk averse individuals may wish to avoid these sources of uncertainty by purchasing at the market price - though such transactions may be risky for other reasons if, for example, the authorities, in trying to prevent arbitrage on "black" markets, make such transactions illegal. Also, the fact that supply is uncertain after queuing means that individuals may join queues for "precautionary" motives, even if they do not need the commodity on offer at the time. (In the former Soviet Union it was suggested that people would do this, even if they did not know what the commodity on offer happened to be.) These implications of uncertainty, while of interest, are beyond the scope of this paper.

[11]Stahl and Alexeev (1985, p. 235) choose a similar specification, rejecting the alternative of treating queuing times as "transactions costs" independent of the amount purchased. They note that, in reality in centrally planned economies, government stores usually limited the amount that could be purchased per transaction, so that if a consumer wanted more he or she would

$$wh = c_2 p_2$$

and

$$u(c,h) = u(\frac{wh}{p_2}, 1 - h - qc_2) = u(\frac{wh}{p_2}, 1 - h - \frac{qwh}{p_2})$$

Denote the value of h which maximizes utility in this situation as h_2*. Now consider an individual who happens to be indifferent between these two courses of action. For this individual

$$u(\frac{wh_1^*}{p_1}, 1 - h_1^*) = u(\frac{wh_2^*}{p_2}, 1 - h_2^* - \frac{qwh_2^*}{p_2})$$

Denote this person's wage as w* (corresponding to ability level a*). Individuals for whom a is less than a* will choose to queue. For those with very low values of a the labour supply decision will be mainly influenced by p_2 (the subsidized price) relative to a. They will spend little time queuing, since their consumption is so low, so the value of q will not matter very much to them. But as ability (and w) increase the proportion of time spent queuing rises. Supplying labour now seems less attractive if time must also be spent queuing.[12] Naturally, an effect of reduced labour supply on the part of such workers will be lower aggregate output.

At higher levels of ability still, individuals will abandon the subsidized market and will purchase at the higher, market price. However, such highly skilled individuals are still subject to the influence of the queuing system, since the prices they face are higher than they would be in

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have to stand in line again. In fact, the case for this specification is stronger than they grant: if queuing was costly but arbitrage was permitted and unlimited purchases were allowed, there would have been no queues since all consumers would have employed a single individual - the one of their number with the lowest queuing cost - to purchase on their behalf.

[12] This may lead to a *fall* in the amount of labour supplied as a function of ability. This occurs in the illustrative economy described below when $p_2 < 0.5\ p_1$.

the absence of queuing. One particular effect is that such individuals may supply less labour than they would if market clearing prices were lower; this too will affect aggregate output.

To investigate the consequences of these individual decisions it is necessary to derive conditions for equilibrium in the subsidy-queue economy. Goods market equilibrium requires that total output, which is made up of the output of those for whom $a < a^*$ and those for whom $a > a^*$, must equal total demand. Thus

$$\int_0^{a^*} ah_2^* f(a)da + \int_{a^*}^1 ah_1^* f(a)da = \int_0^{a^*} \frac{ah_2^*}{p_2} f(a)da + \int_{a^*}^1 \frac{ah_1^*}{p_1} f(a)da$$

or:

$$(1 - p_2)\int_0^{a^*} ah_2^* f(a)da = (p_1 - 1)\int_{a^*}^1 ah_1^* f(a)da$$

Illustration

As an illustration, consider a subsidy-queue system in the economy described above where agents have utility

$$u(c, 1 - h) = \sqrt{c} + (1 - h)$$

Those who *choose not to queue* obtain utility

$$u = \sqrt{\frac{wh}{p_1}} + 1.0 - h$$

and supply labour according to

$$h_1^* = \frac{w}{4p_1}$$

so that

$$u = \frac{w}{4p_2} + 1$$

Individuals who *choose to queue* obtain utility

$$u = \sqrt{\frac{wh}{p_2}} + 1.0 - h - qc_2 = \sqrt{\frac{wh}{p_2}} + 1.0 - h - \frac{qwh}{p_2}$$

and therefore supply labour according to

$$h_2^* = \frac{w}{4p_2(1+\frac{qw}{p_2})^2}$$

so that

$$u = \frac{w}{4p_2(1+\frac{qw}{p_2})} + 1$$

This means that someone will choose not to queue if

$$w = a > \frac{p_1 - p_2}{q}$$

The goods market equilibrium condition is therefore

$$\int_0^{a^*} \frac{a^2}{4p_2(1+\frac{qa}{p_2})^2}da + \int_{a^*}^{1} \frac{a^2}{4p_1}da = \int_0^{a^*} \frac{a^2}{4p_2^2(1+\frac{qa}{p_2})^2}da + \int_{a^*}^{1} \frac{a^2}{4p_1^2}da$$

with

$$a^* = \frac{p_1 - p_2}{q}$$

so that

$$(1-p_2)\int_0^{a^*} \frac{a^2}{4p_2^2(1+\frac{qa}{p_2})^2}da = (p_1-1)\int_{a^*}^{1} \frac{a^2}{4p_1^2}da$$

The integral on the left hand side of this equation may be evaluated using the following change of variable:

$$u = 1+\frac{qa}{p_2}; \quad a = \frac{p_2 u - p_2}{q}; \quad da = \frac{p_2}{q}du$$

so that the condition becomes

$$(1-p_2)\int_1^{1+\frac{qa^*}{p_2}} \frac{p_2(u-1)^2}{q^3u^2}du = (p_1-1)\int_{a^*}^{1} \frac{a^2}{4p_1^2}da$$

Thus

$$\frac{(1-p_2)p_2}{q^3}\left(\frac{qa^*}{p_2} - 2\ln(1+\frac{qa^*}{p_2}) + \frac{\frac{qa^*}{p_2}}{1+\frac{qa^*}{p_2}}\right) = \frac{(p_1-1)(1-a^{*3})}{3p_1^2}$$

Using the fact that

$$\frac{p_1}{p_2} = 1 + \frac{qa^*}{p_2}$$

allows us to express the equilibrium queuing time, q^*, in terms of p_1 and p_2:

$$q^* = \left((p_1 - p_2)^3 + \frac{3(1 - p_2)p_1^2 p_2}{(p_1 - 1)} \left(\frac{p_1}{p_2} - 2\ln\frac{p_1}{p_2} - \frac{p_2}{p_1} \right) \right)^{1/3}$$

Thus the more p_1 and p_2 diverge the more attractive it is to buy at p_2, so the longer queuing times have to be. This will, in turn, influence the labour supply decisions of individuals with different abilities. The overall effect will be to reduce aggregate output. There is also an impact on distribution. The more p_1 diverges from p_2 the more equally consumption and utility are distributed across individuals with different abilities. A consequence is that a price reform which raised p_2 towards p_1 would raise total output and improve the position of those whose ability lies in the middle of the distribution of abilities - but at the cost of worsening the position of the less able. (The highly skilled would continue to avoid queues by buying on the unsubsidized market.)

Comparing the Two Mechanisms - Illustrative Case

It is now possible to compare the tax-benefit and queue-subsidy mechanisms in terms of their impacts on equity and efficiency. More severe versions of each will redistribute consumption and utility towards the less able, but at the expense of the performance of the economy as a whole. In order to compare the two mechanisms, a range of tax rates (for the tax-benefit system) and price combinations (for the subsidy-queue system) were investigated for the economy we are using as an illustration. Two equity-efficiency "trade-offs" were constructed. In the first of these trade-offs, "equity" is measured by the Gini coefficient of concentration of consumption - call this Gini(C) for convenience - while "efficiency" is measured by total output.

When these two measures are plotted against one another we obtain a version of Breit's (1974) "geometrical artifact" by which he proposed representing the policy-maker's preferences by an indifference map in output-Gini(C) space. It is plausible, he argued, that most people would be prepared to forego some real output for greater equality; this means that indifference curves in output-Gini(C) space would slope down to the left.[13] In Figure 3, which is reproduced from

[13]Breit also suggests that such indifference curves will be convex to the origin.

Breit's paper, the curve through W, P and H shows available combinations of output and Gin(C) while the indifference curves are labelled I and II.

Combinations of output and Gini(C) for the tax-benefit economy and for the subsidy-queue economy are shown in Figure 4. Both mechanisms display the equity-efficiency trade-off, in the sense that the curves slope down and to the left. However, three aspects of Figure 4 are worthy of note. The first is that the output-Gini(C) relationship for the tax-benefit system is not concave. The possibility that this could occur has been noted by Lambert (1990). He constructs an example which looks rather similar to the relationship shown in Figure 4, even though it is based on a different distribution of initial income and a different utility function from the one I have used. Lambert points out that non-concavity could mean that Breit's construction failed to prescribe the degree of redistributive taxation and consequent efficiency loss appropriate to maximize social welfare.[14]

A second feature of Figure 4 is that the subsidy-queue system with $p_2 = 0$ dominates less extreme versions of the mechanism.

A third feature of Figure 4 is that the various subsidy-queue trade-offs dominate the tax-benefit trade-off.[15] In other words, more equality in consumption may be bought at the cost of less output loss by the use of the subsidy-queue mechanism. This is clearly what makes the mechanism attractive in situations where some weight is given to equity in consumption. A policy-maker, contemplating a version of Figure 4 based on the actual distribution of individuals' abilities, individuals' preferences between consumption and leisure and his or her own views about the merits of equity and efficiency, may well be tempted to employ a subsidy-queue mechanism.

However, Figure 4 does not tell the whole story. What it leaves out is the welfare loss of queuing. By using output as a measure of efficiency and the degree of inequality in consumption as a measure of equity, crucial features of the two redistributional mechanisms are neglected. A more soundly based approach is to measure efficiency in terms of total (or average) *utility* and equity in terms of the *distribution of utility*. Figure 5 shows such a representation of the effects

[14]What matters, as Lambert's own example of a social welfare function involving output and Gini(C) shows, is not whether the trade-off is concave, but whether the indifference curves describing the social welfare function are *more* concave. If they are, as they are in Lambert's Figure 3, a redistributive maximum may still be found.

[15]These trade-offs, unlike that derived from the tax-benefit mechanism, are concave.

of the tax-benefit and subsidy-queue mechanisms. In the same way as before, the tax rate (in the tax-benefit system) and the subsidised price (in the subsidy-queue system) are varied and the effect on total utility and its distribution observed.[16]

Figure 5 shows some important contrasts with Figure 4. First, it should be noted that both the subsidy-queue and the tax-benefit systems yield concave trade-offs when the relationships are drawn in total utility-Gini(U) space. Since it is in such a space that our judgements about efficiency and equity should properly take form, this may provide a way in which the difficulties Lambert has with Breit's "geometrical artifact" could be resolved.[17]

The second feature of Figure 5 is that it shows the tax-benefit mechanism in a much better light. Redistribution via that mechanism actually increases both total utility and the degree of equality (as measured by Gini(U)), at least up to a point. On the other hand, the subsidy-queue mechanism does promote equality, but only at a high cost in terms of foregone utility. Since the tax-benefit trade-off dominates the subsidy-queue trade-off, choosing on the basis of Figure 5 would lead the policy-maker to prefer a tax-benefit mechanism.[18]

Comparing the Two Mechanisms - General Case

Figures 4 and 5 were drawn for a particular utility function and a particular distribution of abilities. The question arises as to whether these ways of ranking the two mechanisms survive in

[16]The horizontal axis shows the Gini coefficient for the distribution of utilities (Gini(U)). Note that utility will be more equally distributed than ability or consumption in the illustrative economy because of the way that the utility function is defined. Changing the specification of the utility function will also change Gini(U). This does not undermine the usefulness of Gini(U). All that we have in mind is performing a comparison between two redistributional mechanisms. Gini(U) may be used so long a utility is specified in the same way in both economies.

[17]Breit (1974, footnote 26) points out that, since valued items like leisure are not counted in total output, the dimensions along both axes in his diagram should "for purposes of logical rigour" be measured in terms of "some utility dimension". However, "for purposes of objective measurement" he prefers to use the Gini coefficient of money income distribution and the money value of output.

[18]Note that, in doing so, the policy-maker need not stop at the point at which the tax-benefit trade-off peaks, although this would be the position selected by a utilitarian. If the policy-maker's welfare function included both total utility and the degree of equality, redistribution past this point would be carried through.

a more general setting.[19] The answer is that they do.

Proposition 1. In an economy in which individuals' labour supply is not decreasing in the real consumption wage, a subsidy-queue mechanism dominates a tax-benefit mechanism when viewed in Total Consumption *x* Gini(Consumption) space.

Proof. Consider a tax benefit system with a proportionate rate of tax t which is used to fund an unemployment benefit b. Corresponding to $t = b = 0$ there will be a level of aggregate consumption C and a value of Gini(C) which may be plotted against one another in Figure 6, as point A. (This point also corresponds to a subsidy-queue system with no subsidy; that is, $p_1 = p_2 = 1$.) As the tax rate t increases, total consumption falls but consumption becomes more equally distributed, so we move down a locus like that shown, to point B, say. At this point we will suppose that everyone with ability $a > a_t^*$ will choose to supply labour while those with ability $a = a_t^*$ are indifferent between working and not working:

$$u(a_t^* h^*(a_t^*)(1-t), 1-h^*(a_t^*)) = u(b,1)$$

Compare this with a subsidy-queue system with $p_2 = 0$, with the same "cutoff" value of $a = a_q^* = a_t^*$ and with $1 - t = 1/p_1$. With these parameters, everyone with ability $a > a_q^*$ will choose to supply the same amount of labour and will consume the same amount as they would in the tax-benefit economy. It follows that the aggregate of the difference between the output and the consumption of those with ability $a > a_q^*$, that is

$$\int_{a_q^*}^{1} (1 - \frac{1}{p_1}) a h^*(a) f(a) da$$

would be just sufficient to provide those with $a < a_q^*$ with a level of consumption equal to b, the benefit level in the tax-benefit economy. However, if consumption goods amounting to b per person were distributed via a queuing mechanism to each person with ability $a < a_q^*$ each would obtain a utility level of

[19] In this section we consider a general distribution of abilities, $F(a)$, and a general utility function, $u(c,1-h)$. However, we retain the simple proportional tax/ flat rate unemployment benefit mechanism used previously. We also retain a simple version of the subsidy-queue system, with $p_2 = 0$. Thus further generalisation (to incorporate, say, a more complicated tax system) would be possible.

$$u(b,1-qb) < u(b,1)$$

Moreover,

$$u(\frac{a_q^* h^*(a_q^*)}{p_1}, 1-h^*(a_q^*)) _ u(b,1-qb)$$

so that the point B (characterised by b, a_q*, p_1) does *not* constitute an equilibrium of a subsidy-queue economy. Now define a level of b (say b') such that

$$u(\frac{a_q^* h^*(a_q^*)}{p_1}, 1-h^*(a_q^*)) = u(b', 1-q'b)$$

Consider the point D (characterised by b', a_q*, p_1). Since at D consumption of everyone with $a > a_q$* is the same as at B while everyone with $a < a_q$* consumes an amount $b' - b$ more than at B, total consumption at D is greater than at B by an amount

$$(b'-b)\int_0^{a_q^*} f(a)da$$

(This is represented by the distance CD on Figure 6.) Also, since, when moving from B to D, the consumption of everyone with the lowest level of consumption at B has increased while the consumption of those consuming more than this minimum at B remains unchanged, Gini(C) must be lower at D than at B (by an amount represented by BC on Figure 6.) Note, however, that D (characterised by b', a_q*, p_1) is not an equilibrium of the subsidy-queue economy either, since total consumption is greater than total output.

Now consider an increase in p_1, say to p_1'. Increasing p_1 has the effect of reducing both the output[20] and the consumption of everyone who is supplying labour (that is, with $a > a_q$*) but it reduces their consumption more quickly, since

[20]This assumes that labour supply is *increasing* in the real consumption wage. If it is decreasing in the real consumption wage (the "backward bending" labour supply case) a rise in p_1 will more quickly close the gap between aggregate consumption and output, since the former will fall and the latter will rise with p_1 .

$$\frac{dy(a)}{dp_1} = a\frac{dh(a)}{dp_1}; \qquad \frac{dc(a)}{dp_1} = \frac{a}{p_1}\frac{dh(a)}{dp_1} - \frac{ah(a)}{p_1^2}$$

As well as closing the gap between aggregate consumption and aggregate output, this increase in p_1 also has a positive impact on the attractiveness of queuing for the subsidised good. In order for this not to have an impact on a_q* it must be the case that more subsided goods (say, b" rather than b') are made available for each person with $a < a_q$*, so reducing queuing time. Let p_1' be defined as the price which equilibrates aggregate consumption and aggregate output, taking into account the extra output that needs to be diverted from those with $a > a_q$* to those with $a < a_q$* in order to raise b' to b".

It is now clear that the point characterised by (b", a_q*, p_1') - point F in Figure 6 - is an equilibrium in the subsidy-queue economy. It is also clear that both the decrease in the consumption of those with $a > a_q$* and the increase in the consumption of those with $a < a_q$* serve to decrease Gini(C); the impact of these two effects together is represented by the distance DE.

It is now necessary to check the relative distances EF and DC. EF is made up of two components: the decrease in total consumption of the more skilled as a consequence of p_1 rising to p_1', and the increase in consumption of the less skilled. Thus

$$EF = \int_{a_q^*}^{1} \frac{ah^*(a, p_1)}{p_1} f(a)da - \int_{a_q^*}^{1} \frac{ah^*(a, p_{1'})}{p_{1'}} f(a)da + (b' - b'') \int_{a_q^*}^{1} f(a)da$$

Hence

$$EF - CD = \int_{a_q^*}^{1} \frac{ah^*(a, p_1)}{p_1} f(a)da - \int_{a_q^*}^{1} \frac{ah^*(a, p_{1'})}{p_{1'}} f(a)da -$$

$$\int_{a_q^*}^{1} (1 - \frac{1}{p_{1'}}) ah^*(a, p_{1'}) f(a)da + \int_{a_q^*}^{1} (1 - \frac{1}{p_1}) ah^*(a, p_1) f(a)da$$

which is non-negative so long as

$$\int_{a_q^*}^{1} ah^*(a, p_1) f(a)da \geq \int_{a_q^*}^{1} ah^*(a, p_{1'}) f(a)da$$

Thus will be the case if individuals' labour supply is not increasing in the real consumption wage, as we are assuming here.

Since F lies to the left of B and not below C (and B), the subsidy-queue locus in Figure 6 (which passes through A and F) dominates the tax-benefit locus (through A and B).

This completes the proof of Proposition 1.

Proposition 2. In an economy in which individuals' labour supply is not decreasing in the real consumption wage, a tax-benefit mechanism dominates a subsidy-queue mechanism when viewed in Total Utility *x* Gini(Utility) space.

Proof. Consider a subsidy-queue system in a single good economy in which the authorities purchase all the economy's output from competitive firms and distribute a portion of this output free to agents who queue for it. The authorities sell the rest to agents who choose not to queue at price p_1.

Lemma 1. The locus of subsidy-queue equilibria slopes downwards to the left in Total Utility *x* Gini(Utility) space.

Proof of Lemma 1. Consider a subsidy-queue equilibrium in which those with abilities less than or equal to *a** choose to queue. Now consider an increase in p_1 which is used to purchase a greater quantity of the good for free distribution. This will raise the utility of everyone with ability less than or equal to *a** and reduce the utility of those with ability greater than less than *a**.[21] Hence Gini(U) will decrease.

To investigate what happens to total utility we may calculate

[21]Note that *a**, the cut-off between those who choose to queue and those who choose not to, will rise as a result of the increase in p_1.

$$\frac{d}{dp_1}[\int_0^{a^*(p_1)} u(s(p_1),1-s(p_1)q(p_1))f(a)da+\int_{a^*(p_1)}^1 u(\frac{ah(p_1)}{p_1},1-h(p_1))f(a)da]$$

subject to the constraint

$$(1-\frac{1}{p_1})\int_{a^*}^1 ah(a,p_1)f(a)da = s\int_0^{a^*} f(a)da$$

To simplify the derivative we apply Leibniz's formula so that it becomes

$$u(s(p_1),1-s(p_1)q(p_1))f(a^*)\frac{da^*}{dp_1}+\int_0^{a^*}\frac{du(s(p_1),1-s(p_1)q(p_1))}{dp_1}f(a)da$$

$$-u(\frac{a^*h(a^*)}{p_1},1-h(a^*))f(a^*)\frac{da^*}{dp_1}+\int_{a^*}^1\frac{du(\frac{ah(p_1)}{p_1},1-h(p_1)}{dp_1}f(a)da$$

The first and third terms cancel, leaving

$$\int_0^{a^*}\frac{du(s(p_1),1-s(p_1)q(p_1))}{dp_1}f(a)da+\int_{a^*}^1\frac{du(\frac{ah(p_1)}{p_1},1-h(p_1))}{dp_1}f(a)da$$

Differentiating the utility functions under the two integrals yields

$$\int_0^{a^*}[u_1\frac{ds}{dp_1}-u_2s\frac{dq}{dp_1}-u_2q\frac{ds}{dp_1}]f(a)da+\int_{a^*}^1[\frac{u_1a}{p_1}\frac{dh}{dp_1}-\frac{u_1ah}{p_1^2}-u_2]f(a)da$$

where u_j (j =1,2) is the derivative of the utility function with respect to its jth argument.

Since

$$u_1 = -qu_2$$

for those choosing to queue, this simplifies to

$$\int_0^{a^*}[-u_2 s\frac{dq}{dp_1}]f(a)da + \int_{a^*}^{1}[\frac{u_1 a}{p_1}\frac{dh}{dp_1} - \frac{u_1 ah}{p_1^2} - u_2]f(a)da$$

Since we are assuming that dh/dp_1 is less than or equal to zero, this expression is negative. Thus total utility is decreasing in p_1, which proves the lemma.

We now return to the proof of Proposition 2. Consider a subsidy-queue equilibrium (s_1,a_{q1}*, p_1) corresponding to point B in Figure 7b. (Point A corresponds to the no-subsidy/no-queue case, (0,1,1); it also corresponds to a tax-benefit equilibrium with $t = b = 0$.)
The total volume of the subsidised good distributed to those who choose to queue at B is

$$s_1 F(a_{q1}^*)$$

Consider distributing this same volume of goods to the less skilled but without requiring them to queue. In the absence of queues consumption of the subsidised good becomes more attractive so that more people - that is, those with ability level a up to and including, say, a_{q2}* - choose not to supply labour. This reduces the volume of the subsidised good available to each person to

$$s_2 = \frac{s_1 F(a_{q1}^*)}{F(a_{q2}^*)}$$

where a_{q2}* is defined by the condition that

$$u(\frac{a_{q2}^* h(a_{q2}^*, p_1)}{p_1}, 1 - h(a_{q2}^*, p_1)) = u(s_2, 1)$$

This situation corresponds to point C on Figure 7a. However C is not an equilibrium since at C the volume of the subsidised good available for distribution is not equal to $s_1F(a_{q1}$*) but falls short of this by an amount

$$\int_{a_{q1}^*}^{a_{q2}^*}(1 - \frac{1}{p_1})ah(a, p_1)f(a)da$$

owing to the fact that those in the ability range $a_{q1}^* \leq a \leq a_{q2}^*$ are no longer contributing to the production of output. The amount that is actually available for distribution to those who do not supply labour will therefore be an amount less than s_2 - say, s_3. This will, in turn, effect the attractiveness of supplying labour rather than receiving s_3 so that only those with $a \leq a_{q3}^*$ choose not to supply labour where s_3 and a_{q2}^* are defined by the conditions

$$u(\frac{a_{q3}^* h(a_{q3}^*, p_1)}{p_1}, 1 - h(a_{q3}^*, p_1)) = u(s_3, 1)$$

and

$$\int_{a_{q3}^*}^{1} (1 - \frac{1}{p_1}) a h(a, p_1) f(a) da = s_3 \int_{0}^{a_{q3}^*} f(a) da = s_3 F(a_{q3}^*)$$

This corresponds to the point D in both Figures 7a and 7b. It also corresponds to the equilibrium in a tax-benefit economy with benefit level $b = s_3$ and tax rate $t = 1 - 1/p_1$.

Comparing points B and D in Figure 7a, it is clear that (i) total utility has risen and (ii) the whole of the increase in utility has accrued to those whose utility was lowest at B so that Gini(U) must be lower at D than at B. Hence in Figure 7b D must lie to the left and above B and the locus through A and D (corresponding to tax-benefit economies) must dominate the subsidy-queue locus through A and B.

This completes the proof of Proposition 2.

Conclusion

The aim of this paper was to shed some light on the allocation of goods by queues. Despite the obvious inefficiencies associated with queuing, it occurs and survives in situations where considerations of fairness are thought to be particularly important. This is because allocation by queuing has a redistributional role if the ability to queue is allocated more equally than, say, human or physical capital. In this sense, a queuing mechanism plays a role similar to that performed by a tax-benefit system. In both cases, however, we need to pay careful attention to the efficiency-equity trade-offs involved.

The paper shows how a queuing mechanism may look rather attractive if the focus is on the

level of consumption and its distribution. However, this attraction is superficial. When the choice between these two mechanisms is properly formulated in terms of utility and the distribution of utility the advantages of a tax-based redistributional mechanism are clear.

While the main purpose of the modelling strategy employed in this paper was to compare the equity and efficiency properties of the subsidy-queue and tax-benefit mechanisms, the models developed above also provide some insight into the comparative properties of capitalist and Soviet-type economies (where the subsidy-queue mechanism was used extensively). Thus, in the Soviet-type economies labour force participation was high but labour supplied per worker was low, especially on the part of highly skilled workers who, faced with queues or high "unofficial" prices, chose leisure. The tax-benefit system used in market economies has the advantage of getting more output out of the highly skilled, but a consequence is that this group consumes a lot since their incomes - even after tax - are high. The disincentives of the tax-benefit system are well known and lead to less labour being supplied by the least skilled. In one sense this is a more sensible distribution of labour effort than that which occurs under the subsidy-queue system since those best able to produce output do more of the work.[22] On the other hand, it may have negative consequences. These will arise if labour force participation confers non-monetary benefits by, for example, contributing to an individual's sense of worth. Another consequence is that pressure may develop among taxpayers to limit the extent of tax-financed redistribution. If a (rational) decision on the part of a less skilled individual not to participate in the labour force is represented as shirking, this pressure will be harder to resist. Such political factors may limit how far redistributional tax-benefit policies are pushed. In turn, this may make the use of a subsidy-queue system seem attractive, especially if the equity-efficiency trade-offs are not properly perceived.

Nevertheless, it should be stressed that the subsidy-queue mechanism is not the best way of overcoming inequalities in the ability to earn income. This has implications both in former Soviet-type economies and elsewhere. In a number of former centrally-planned economies price liberalisation has occurred without the development of properly functioning tax-benefit systems. Many people are worse off than under the previous subsidy-queue regime and have expressed their discontent politically, for example by voting for political parties pledged to restore the old system. If the development of a workable tax-benefit system continues to lag behind price

[22]This feature of the model depends on the specification of the utility function. If leisure were valued more highly at high income levels labour supply might not be increasing in the post-tax wage over the whole range of the latter. Such a "backward bending" labour supply function would modify this result.

reforms designed to displace the subsidy-queue mechanism, such opposition could grow and could even undermine the whole reform process.

Elsewhere, policy-makers should move towards dispensing with allocation by queuing, whether in health-care or housing markets or in allocation to congestible facilities. Defenders of queuing as an allocation mechanism are often influenced by a concern with equity, but this concern would be better expressed by supporting a more thorough-going redistribution of income through the tax mechanism.

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