Designing a Teaching Programme for Student Teachers of Mathematics: A Trigonometry Case Study

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Designing a teaching programme for student teachers of mathematics: A trigonometry case study

Abstract

This research investigated whether mathematics teacher education sessions on trigonometry which are planned, using the Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005), develop the confidence of prospective teachers in their own ability to teach trigonometry.

The research was carried out in a Teaching School, which has a unique Schools Direct Mathematics programme, where the subject specific sessions are planned and delivered by two mathematics teachers working in the school. Through the use of an action research method it was found that mathematics teacher education sessions on trigonometry planned around the Knowledge Quartet improved the confidence of prospective teachers to teach trigonometry. This research goes onto to suggest that longitudinal research into the use of the Knowledge Quartet in planning teacher educational sessions would be the next intelligent step.

Contents

1. Introduction

$$cos(x + 30) = cosx + cos30$$

This statement in a student's work is one which fills a mathematics teacher with dismay and leads to the question – how did the way I taught trigonometry lead to such a glaring misconception?

This research project arose from a desire to investigate what role a mathematics teacher educator (MTE) plays in ensuring good teaching of trigonometry. The nature of this research project has risen from my teaching experience and role as a school-based mathematics teacher educator. Until three years ago I was a Head of Mathematics with many years of mathematics teaching experience. I have encountered the trigonometric misconception above, and worse, on many occasions. Over the years I have introduced trigonometry in several different ways with varying results. Becoming an MTE three years ago has meant I have had to think about how a prospective teacher learns to teach and what knowledge they need in order to address such misconceptions. The rationale for choosing trigonometry, despite the numerous misconceptions that exist within other topics, is in part due to my professional experience and my role as a MTE. Trigonometry is a fundamental topic in the A-level qualification and in my professional experience students struggle with this particular topic due to their largely procedural understanding. Additionally, I have observed many lessons where I have seen teachers teach a set of rules for students to follow and it is my belief that this then leads to the difficulties faced when studying A-Level mathematics. Watson, Jones and Pratt (2013) discuss the variety of situations where trigonometry becomes important in post-16 mathematics in their book

“Trigonometric functions are pervasive in many parts of pure and applied mathematics” (p.202). Hence, it seems justifiable to focus this research project on how prospective teachers learn to teach trigonometry.

The context of this small-scale research project is a UK based Schools Direct Mathematics Programme where, uniquely, the mathematics specific training is carried out by a colleague and me in our school, as opposed to a university carrying out this element of the training. The prospective teachers study for a Post-Graduate Certificate in Education (PGCE) accredited by a local university. During the pilot year of our course we observed a decrease in confidence in our prospective teachers in their knowledge of mathematics for teaching as the course went on. In their feedback at the end of the course they suggested we had more sessions where they had the opportunity to work on their own mathematics. My colleague and I took on board their evaluations and adapted the course to include “mathematics focus” sessions. This project has arisen from a desire to evaluate if these sessions have improved student teachers’ confidence to teach in particular trigonometry which is a notoriously hard and fundamental aspect of secondary curriculums across the world (Pritchard and Simpson, 1999; Ross, Bruce and Sibbald, 2011; Blackett and Tall, 1991).

The setting for this project is one where the UK government is intent on creating a fully school-led system. This includes moving Initial Teacher Training (ITT) away from Higher Education Institutes (HEI) and into schools so that prospective teachers ‘learn on the job’. This is not a new initiative but has reappeared under various guises with the different governments since the 1980s. Ellis and McNicholl (2015) argue that its implementation has not been as successful as may be claimed, as evidenced by the majority of schools opting to continue to work with HEIs to deliver ITT. Although Brookwood Academy (pseudonym for the school which is the context of this project) is one such school which works in partnership with a local HEI, the course is unique, such that the subject specific sessions are delivered by my colleague and me. However, this has meant we have had to develop a new set of skills and develop our own knowledge for teaching. There has been no formal training and me studying this MSC Teacher Education is the result of a journey to evolve from a teacher of mathematics to a teacher of mathematics education as described by Prestage and Perks (2007).

This project will therefore attempt to answer the following questions:

1. What does the research say about the best approach to teaching trigonometry?
2. What is the confidence of prospective teachers, in their own knowledge, to teach trigonometry?
3. Would mathematics sessions on trigonometry focused on aspects of the Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005) develop prospective teachers' confidence in their own trigonometry teaching?

The first research question will be addressed through the literature review covering the areas of:

- The learning and teaching of trigonometry;
- What pedagogical content knowledge do prospective mathematics teachers need to teach trigonometry?
- Mathematics teacher education sessions;
- Confidence of prospective teachers during their teacher education year.

In order to investigate the further research questions, I will be using an action research methodology, analysing several sessions on trigonometry which I facilitated with a colleague for our six prospective mathematics teachers. Following the first session I used a questionnaire to gather information about the confidence of the prospective teachers and ran a second session as a result of this feedback. There follows a discussion about how four of the prospective teachers went about introducing trigonometry with students for the first time, what was revealed about their knowledge, confidence and the part played by the sessions they attended.

2. Literature Review

The literature review for this research fundamentally covers prospective teachers' confidence to teach mathematics, and in particular trigonometry. My focus is firstly on what the research demonstrates about learning and teaching trigonometry. Secondly, I review the literature relating to the knowledge needed by prospective teachers in order to teach trigonometry. In particular, I draw on the Knowledge Quartet (Rowland et al, 2005) to look in more depth at the different types of knowledge which are revealed during teaching. Thirdly, I look at how confident student teachers appear to be to teach trigonometry during their training year. Finally, I review the research surrounding mathematics teacher education sessions, and the role of the mathematics teacher educator in these, in developing knowledge and confidence of student teachers in the context of trigonometry.

The rationale for the structure of this literature review is because it is acknowledged students perceive and find trigonometry to be a difficult topic (Pritchard and Simpson, 1999). This raises the question about what teachers need to know in order to address these difficulties, and finally what is the role of the MTE in developing the confidence of prospective teachers to teach trigonometry. Ross, Bruce and Sibbald (2011) state:

> *"Deep understanding of trigonometry requires the ability to flip between abstract, visual and concrete representations of mathematical objects. In addition, the subject is confounded by inter-relationship between functions"* (p.121).

Whereas Pritchard and Simpson (1999) describe trigonometry as *"the confluence of a number of streams of mathematical difficulty"* (p.1246).

2.1 Learning and teaching trigonometry

It has become clear through my search for literature about how to approach the learning and teaching of trigonometry there is a paucity of literature; particularly with regards to how prospective mathematics teachers learn to teach trigonometry. I have identified some key themes in the research surrounding learning and teaching trigonometry. These themes are:

- the difficulties concerning learning trigonometry;
- the use of technology in teaching trigonometry;
- developing conceptual understanding of trigonometry.

2.1.1 The difficulties concerning learning trigonometry

In this section I discuss some of the main difficulties students have with learning trigonometry – the connections which need to be made with earlier experiences, trigonometric relationships as functions, relating trigonometry to diagrams and numerical relationships.

In my own teaching experience, I have found that students both find trigonometry difficult and have the perception that it *is* difficult. In my professional experience these difficulties are usually laid bare when students study for A-Level qualifications and their lack of relational understanding (Skemp, 1976) becomes apparent through errors such as that demonstrated in the introduction. The importance of this transitional point from GSCE to A-Level is made by Watson, Jones and Pratt (2013); students need to bring together all their learning up until this point to tackle new ideas about trigonometric functions and calculus. Watson (2009) makes this point about trigonometry robustly:

> "learning trigonometry involves understanding: the definition of triangle; right-angles; recognising them in different orientations; what angle means and how it is measured; typical units for measuring lines; what ratio means; similarity of triangles; how ratio is written as a fraction; how to manipulate a multiplicative relationship; what sin (etc.) means as a symbolic function and so on." (p.5)

It is clear that to make this transition to A-Level students need a sound grasp of a variety of conceptual ideas. Watson (2009) also points out that if a child's only experience of ratio is concerning recipe problems this won't help with developing understanding of trigonometry.

Pritchard and Simpson (1999) argue that trigonometry is the first point at which students come across the concept of a function which is not a numerical manipulation. This could explain why some students are tempted to divide by cos when solving an equation such as $cosx = 0.8$. This student misconception was identified by Pritchard and Simpson (1999) during their small-scale investigation into the role pictorial images played in the solving of trigonometry problems. Despite the fact that Weber's research (2005) was concerning college students, his findings are relevant to school students as he found that when students were asked to estimate and justify the value of sin20° they were unable to do so. He argued that it is important for students to be able to have this type of understanding in order to justify why trigonometric functions have certain properties. However, one might argue that students aged 13 – 16 might not need develop this type of understanding unless they were going to continue to a higher level of mathematics.

Ross et al (2011) state "Deep understanding of trigonometry requires the ability to flip between abstract, visual and concrete representations of mathematical concepts." (p.121).

In contrast Prichard and Simpson (1999) found that while students were able to 'flip' between these representations, the problems arose when they had to move between visual and symbolic ways of working. Ross et al (2011) found many students rely on memorising the mnemonic SOHCAHTOA (Sine Opposite Hypotenuse, Cosine Adjacent Hypotenuse, Tangent Opposite Adjacent). It could be argued this could lead to difficulties when students are faced with more unfamiliar concepts such as angles measured in radians, negative angles and angles greater than 360°. Martinez-Sierra (2008), in a Mexican educational context, found that 'conceptual ruptures' existed around these errors because of a lack of mathematical meaning in school mathematics.

In summary, the research shows that students find, and perceive, trigonometry to be a hard topic to learn which leads to a review of the research around how one should approach the teaching of trigonometry in order to meet these challenges and difficulties. This section has therefore shown that there are difficulties surrounding the learning of trigonometry and is supported in the literature (Watson (2013 and 2009); Blackett and Tall, 1991; Pritchard and Simpson, 1999; Ross, Bruce and Sibbald, 2011; Weber, 2005).

2.1.2 The use of technology in the teaching of trigonometry

Some of the research reviewed has centred on the use of technology in the teaching of trigonometry, for example, calculators and computer software because scientific calculators only became commonplace in the 1980s. One such piece of research Delice and Monaghan (2005) compared and contrasted the different approaches to tool use in Turkey and the UK. Although the research was small-scale and did not compare the participation rates of post-16 mathematics in both countries it is salient that the use of calculators in the UK is widespread compared to Turkey where calculators are not used by students until later in their education. The researchers do not make any assumptions about which approach is better but make the point that different 'tool use' leads to different types of understanding. Blackett and Tall (1991) found when computer software was used the experimental group improved more than a control group who did not. It is perhaps interesting to note that the girls in the experimental group improved more significantly than the boys. I was unable to find any other research which compared trigonometry learning between the genders. In Ross et al (2011) they found, in a Canadian education context, that a blend of computer-assisted learning and whole-class teaching seemed preferable. This is supported by Cavanagh (2008) who, in a reflection about his own teaching experience, did not introduce the use of a calculator until the students needed to calculate the tangent of an angle which was not a multiple of 10°. It might be interesting to investigate the approaches which more mature teachers take, as in those who were teaching or learning prior to the introduction of calculators, to teaching trigonometry. However, this is beyond the scope of this project and so I move on to the research around the different approaches teachers do take to teach trigonometry

2.1.3 Developing conceptual understanding of trigonometry

There is a consensus across much of the research into teaching trigonometry that development of conceptual understanding is the ultimate aim for teachers. In addition, the agreement extends to the understanding that this does not occur as a result of being taught the mnemonic SOHCAHTOA. In my own experience, there is a contrast between what the research says about conceptual understanding and what I observe in classrooms. I have found experienced teachers teaching a set of procedures that allows students to find missing sides and angles. Using the mnemonic and teaching a set of procedures is defined by Skemp (1976) as instrumental understanding "rules without reasons" (p.2) as opposed to relational understanding described as "both knowing what to do and why" (p.2). I use the terms conceptual understanding and 'relational understanding' interchangeably. It appears to me that there may be a disconnect between what is happening in classrooms in practice and what the academics advocate in their research but this is again beyond the scope of this research.

There are two pieces of useful research which discuss the development of conceptual understanding in detail, and which are much quoted in this area – Kendal and Stacey (1996) and Weber (2005). In his article, Weber (2005) expresses the view that despite the difficulties associated with learning trigonometry there is little research in this area. He subsequently points out that the majority of research which does exist is usually centred around comparisons between two groups who are taught using different approaches such as mentioned earlier (Blackett and Tall, 1991) and indeed Kendal and Stacey (1996). Weber (2005), though working with college students, attempts to help them develop their understanding of trigonometric functions as 'procepts' (Gray and Tall, 1994). A 'procept' refers to an "amalgam of concept and process represented by the same symbol" (Gray and Tall, 1994, p.121). Consequently, Weber (2005), found that although at the end of his research the college students could estimate the value of trigonometric functions better in the post-test, he was unable to draw any conclusions about how this approach might be adopted in secondary classrooms.

In contrast to Weber (2005), Kendal and Stacey (1996) compared the unit circle approach to the ratio teaching method with eight Australian year 10 classes. They found that the classes who had been taught using the ratio method were better equipped to master the skills required than those who had experienced the unit circle approach. Many articles reviewed were simply descriptions of mathematics teachers' approaches to teaching trigonometry in their own classrooms (Cavanagh, 2008; Collins, 1973; Steer, de Vila and Eaton, 2009). Prichard and Simpson (1999) argue that when learning trigonometry, students are usually given a mathematical definition (e.g. sine is opposite divided by hypotenuse) before they have developed any concept image (Tall and Vinner, 1981) of the trigonometric functions.

In summary, there is little empirical research which I could find despite searching in the major mathematics education journals, that advocates a particular approach to learning and teaching trigonometry. The majority of the research suggests that students find it difficult to learn trigonometry and indeed perceive it to be difficult. In the absence of much research I have discussed the little I have found and will complement it by using wider research about prospective teachers. This leads to further questions such as:

- What do prospective teachers need to know in order to make decisions about teaching trigonometry?
- How can MTEs prepare them to make informed choices that will benefit student achievement?

2.2 What pedagogical content knowledge do prospective mathematics teachers need to teach trigonometry?

As one might expect there is far more research relating to the knowledge that is required for teaching, than about learning and teaching trigonometry which is one branch of mathematics. The first question one might ask is 'what do we mean by knowledge?' It is argued by Rowland et al (2005) that perhaps the seminal piece of research was by Shulman (1987) who categorised the knowledge base for teaching into seven categories. This categorisation was developed by Shulman in the US over a period of three years when he followed the progress of prospective teachers. Since then researchers have particularly focused on 'pedagogical content knowledge' (PCK) as this has been found hard to define and characterise exactly (Rowland et al, 2005). Following on from Shulman, Ball, Thames and Phelps (2008) attempted to develop a useful theory of content knowledge for teaching, and specifically discussing mathematics. They concluded by maintaining teachers need to know the subject they teach but more specifically PCK should not be taken as given but carefully planned and measured. In contrast to Shulman, Ball et al (2008) felt that this aspect of PCK was largely underdeveloped and needed further work.

In order to review the literature surrounding knowledge for teaching I have structured this next section of the literature review using the Knowledge Quartet (Rowland et al, 2005); they argued that PCK is

> "particularly difficult to define and characterise, conceptualising both the link and the distinction between knowing something for oneself and being able to enable others to know it." (p.255).

The Knowledge Quartet offers a way for PCK to be observed i.e. knowledge in action rather than the more static models mentioned above (Shulman, 1987; Ball et al, 2008). The Knowledge Quartet consists of four dimensions – foundation, transformation, connection and contingency. It was developed with primary school settings in mind; however other researchers have successfully applied the quartet to other contexts including secondary settings and the teaching of teacher educators themselves (Rowland, 2012; Rowland and Zazkis, 2013; Watson and Harel, 2013; Chick and Stacey, 2013). The Knowledge Quartet is now reviewed in its separate dimensions below.

2.2.1 Foundation

The foundation dimension of the quartet is defined as a teacher's "mathematics-related knowledge, beliefs and understanding" (Rowland, 2012, p.17). Forgasz and Leder (2008) reviewed literature from 1997 to 2006 and found considerable research around primary teachers' beliefs about pedagogy and learning and less in the secondary context. They found that there was evidence that beliefs about how to teach mathematics were tightly interwoven with beliefs about the nature of mathematics. This is in direct contrast to Beswick (2011) who looked in depth at the relationship between beliefs on the nature of mathematics, and teaching mathematics, of eight case study teachers in Australia. Beswick found that a less experienced teacher experienced a conflict between her own beliefs about mathematics and her beliefs about how to teach it. In addition, she found that a more experienced teacher took on board the latest developments in mathematics pedagogy but this did little to change her beliefs about the nature of mathematics. This raises questions for the MTE in terms of considering the beliefs of prospective teachers and how these align themselves to the guiding principles of the course (to be discussed later in this review).

Additionally, the MTE will need to consider the beliefs of prospective teachers about trigonometry and how their own preconceptions might impact on their subsequent practice in the classroom.

In comparison to the conflict the less experienced teacher encountered in Beswick's (2011) study, Skott (2009) found in a Danish case study of a new teacher that they were also experiencing a conflict between their views of teaching mathematics and the priorities of the school. This conflict is described in various other research studies which review the gap between the theory endorsed in teacher education programmes and the practice of school teaching experience (Burn and Mutton, 2015; Hayward, 1997; Davis et al, 2013; Korthagen and Kessels, 1998). It does raise questions around the design of mathematics teacher education courses, concerning beliefs about mathematics teaching and how these align themselves to the experience the prospective teachers then get in their teaching practice schools. In relation to trigonometry, this conflict could arise where a mathematics department advocates a procedural approach to teaching trigonometry, which is contrary to the prospective teacher's views and beliefs or experience from university sessions.

The foundation knowledge of trigonometry that a prospective teacher needs in order to teach is interesting to consider. Ball and Bass (2002) found that mathematics teachers need to use mathematics in very different ways to that of professional mathematicians. Indeed, Chick and Stacey (2013) referred to mathematics teachers as 'problem-solving applied mathematicians'. Moreover, the mathematics the teachers need to know is largely different to that needed by the students they teach (Davis and Simmt, 2006). For example, drawing on the previous section on learning and teaching trigonometry, the mathematics teacher needs to understand what would lead a child to make the mistake of dividing by cos and therefore have an understanding of the concept of a function. Whereas a professional mathematician and to some extent the child who made the mistake, need to be able to *do* the mathematics and might not understand where or why such a misconception might occur.

With particular reference to trigonometry Akkoc (2008) analysed the understanding of radian measure of six prospective mathematics teachers. The study found that the prospective teachers had a stronger understanding of measurement of angles in degrees rather than in radians which might therefore impact on their PCK later on. Interestingly, Cavey and Berenson (2005), using a lesson study methodology, found that prospective teachers' understanding of trigonometry was largely procedural at the start of teaching. Though not trigonometry-specific, Ball (1990) also found that the knowledge prospective teachers bring to teacher education is largely procedural and "inadequate for teaching mathematics for understanding" (p.464). The next aspect of the Knowledge Quartet to be considered is Transformation.

2.2.2 Transformation

The transformation dimension of the quartet refers to how knowledge transfers to planning and teaching. Ball and Bass (2002) describe the mathematics that mathematicians do as needing to be 'compressed' whereas mathematics teachers need to be able to 'decompress' the knowledge in order to teach it. This act of decompressing or unpacking is what occurs in order to able to teach trigonometry and consider how best to introduce it as a concept. This links to the earlier section where I discussed how Watson (2009) described all the skills that students need in order to be successful at problem and solving trigonometric problems. This means the mathematics teacher must be able to 'unpack' their own knowledge in order to identify these skills.

How prospective teachers learn to plan and the means which teacher education programmes use to teach planning creates a conflict for the prospective teachers because these means are not those practised by experienced teachers (Maroney and Searcy, 1996). Despite the fact that this research is 20 years old the teacher education programmes that we work with have changed little with regards to lesson planning over that period. The decisions made about lessons, choices of examples and tasks are surely informed by the prospective teacher's beliefs and experiences up until that moment? A number of papers described how prospective teachers resort to the ways they have been taught mathematics and lean towards more procedural approaches (Cavey and Berenson, 2005; Hill, 2000; Bowers and Doerr, 2001; Leikin, 2008; Artz, 1999), possibly leading to the development of instrumental understanding rather than relational understanding (Skemp, 1976). Leikin (2008) considers one of the challenges faced by MTEs is "the importance of challenging mathematics and prospective mathematics teachers' limited experience in challenging mathematics" (p.63). I now move on to consider the third aspect of the quartet - Connection.

2.2.3 Connection

The connection dimension refers to not only connections between concepts but the connections that prevail between a series of lessons. Askew, Brown, Wiliam and Johnson (1997) found that of six highly effective teachers, five were shown to have a 'connectionist orientation'. Askew et al (1997) define connectionist orientated teachers as those who "emphasise the links between aspects of the mathematics curriculum" (p.343). They also characterise these types of teachers as those who believe all students can learn mathematics and tend to favour teaching that is "based on dialogue between teacher and pupils to explore each other's understandings" (p.343). Connection is particularly important in the topic of trigonometry as there are many different links that can be made, for example similarity, ratio, algebraic manipulation, measuring, circular motion, series and so on. Finally, I look at Contingency.

2.2.4 Contingency

Perhaps the most interesting dimension of the quartet to observe in the teaching of mathematics is contingency which refers to the actions in the lesson that a teacher takes in response to events that are not anticipated. Rowland, Turner, Thwaites and Huckstep (2009: p.137) counted three scenarios in this dimension:

1. a child's response to a question from the teacher;
2. a child's response to an activity or discussion;
3. a child's incorrect answer to a question or during the course of a discussion.

Rowland and Zazkis (2013) found that mathematical knowledge of the three teachers involved in their research was essential in the responses to the contingent moments they encountered. This dimension of the Knowledge Quartet links with the work of Mason (1998) on awareness and 'shifts of attention'. He argues one of the aims of teacher education is educate prospective teachers' 'awareness-in-discipline' – that is "the sensitivities which enable us to be distanced from the doing sufficiently to instruct others, to give orders, literally for doing things" (p.260). Moreover, Mason (1998) talks about three types of awareness – awareness-in-action, awareness-in-discipline and awareness-in-counsel. It could be argued that awareness cannot be observed per se but it is something that the 'expert' teacher has which makes them look effortless in their job. Mason and Davis (2013) also discussed contingent moments but they referred to these as 'in-the-moment' pedagogical choices. They argue that these 'in-the-moment' choices are informed by their 'mathematical awareness'. How the prospective teacher might respond to the child who divided by cos or expanded $cos(x + 30°)$ reveals something about their PCK thereby particularly showing their confidence in their own knowledge. The knowledge that prospective teachers need is complex and it is the role of the MTE to provide them with opportunities where they can develop their experiences and perhaps encounter contingent moments in a safe and supportive environment.

In summary, prospective teachers need to know the subject they are teaching and then be able to transform this knowledge into pedagogical content knowledge; how they do this is affected by their beliefs about the nature and teaching of mathematics. It is the role of the MTE to design a programme and sessions that challenge these preconceptions and beliefs. The literature and my professional experience suggest that the Knowledge Quartet (Rowland et al, 2005) framework might be a useful way to analyse the knowledge of prospective teachers in action.

2.3 Teacher education sessions

Since this project looks at whether the design of teacher education sessions around the Knowledge Quartet (Rowland et al, 2005) improves the confidence of prospective teachers in their knowledge, it is important to look at the research around the design of mathematics teacher education sessions. In this section I discuss briefly some of the research concerning mathematics teacher education sessions.

The value of prospective teachers *doing* mathematics in sessions has been identified in a review of 111 research papers by Watson and Mason (2007) and Ball and Bass (2002). They argue that it is important for prospective teachers to work both independently and collaboratively on mathematics. This provides opportunity for the prospective teacher to 'unpack' concepts that they are familiar with (Ball and Bass, 2002) as discussed earlier. Burn, Hagger, Mutton and Everton (2003) analysed their own teacher education course and found that from the start the prospective teachers were focused on the learning of the students during the sessions. This principle of experiencing learning from the point of view of the learner was a common theme found when I reviewed literature (Burn et al, 2003; Crowe and Berry, 2007; Loughran, 2006; Korthagen, Loughran and Russell, 2006) about guiding principles of teacher education programmes for a previous assignment. Llinares (2000) describes an activity which could be used in sessions which involved comparing and characterising textbook questions and analysing student responses to questions. This might be useful in relation to trigonometry as the topic is vast when one considers key stages three to five. Characterising trigonometry questions would help to develop the transformation and connection aspects of the Knowledge Quartet (Rowland et al, 2005).

Watson (2008), in a piece about the teacher education course she taught on, described how she asked the prospective teachers to 'collect' misconceptions from their teaching practice so that they could be discussed. Perhaps this is a way to support the growth of knowledge in the contingency dimension as the prospective teachers will encounter more misconceptions in this way and help their response 'in the moment'. Indeed, in their quest to help their prospective teachers respond in the moment, Zazkis, Liljedahl and Sinclair (2009) developed the concept of 'lesson plays'. This approach involved prospective teachers writing a play for their lesson rather than producing a traditional lesson plan. There was a disadvantage to this approach in that the 'lesson play' did not recreate the dynamic nature of a classroom and instead became monologues by the teacher. However, this approach could possibly be adapted to help prospective teachers imagine what they might do when faced with contingent moments in trigonometry lessons.

In summary, there are a number of tasks and activities that can be used in teacher education sessions. The common themes in the literature appear to suggest that it is important for the prospective teachers to engage in mathematics themselves and experience learning from the viewpoint of the students. In this case, it would seem to show the importance of prospective teachers engaging in trigonometry themselves and experiencing learning trigonometry from the point of view of the learner. Crowe and Berry (2007) sum this up well:

> "Beginning teachers need more than a set of activities, ideas and techniques to help them become deliberate, thoughtful teachers who understand the relationship between their teaching and the quality of their students' learning." (p.31)

2.4 Confidence of prospective teachers during their teacher education year

Whilst I use the term 'confidence' regularly in this project when I searched for literature it became apparent that the term 'self-efficacy' was a more commonly used phrase in the literature. Self-efficacy theory was developed by Bandura in the 1970s. Hoy and Spero (2005) describe it thus: "Teachers' sense of efficacy – teachers' judgments about their abilities to promote students' learning" (p.343) which seems to explain self-efficacy in an educational context well. Consequently, I will use 'confidence' and 'self-efficacy' interchangeably. Bandura (1977) found that changes to self-efficacy are more likely to occur at the early stages of teaching. Bandura's theory was predominantly used in discussing psychological changes in behaviour - particularly, what makes an individual persevere with something which could be perceived as threatening. This seems to link to the behaviour of prospective teachers and how they behave when faced with the challenge of teaching for the first time. Corcoran (1981) described this as the 'transition shock', that is, the challenge faced by the prospective teacher as they move from what they know in theory to putting it into practice.

Perhaps unsurprisingly, Ball (1990) found prospective secondary school teachers were more confident in mathematics than prospective elementary school teachers possibly due to the fact the former have studied more mathematics. Interestingly, they found that if the prospective teachers got stuck they felt they were just 'rusty'. Brady and Bowd (2005) found a strong correlation between confidence and 'mathematics anxiety' and whether the participants enjoyed their own experience of formal education.

Plummer and Peterson (2009) found in their case study of a prospective secondary teacher she found a means by which to protect herself as a mathematics expert. They found her comments reflected a belief about teaching mathematics in a conceptual way and that she said she was confident in her knowledge of mathematics. However, when she was placed in a situation which challenged her own conceptual understanding she sought to protect herself using a variety of strategies. This could be evidence of a 'fixed mind-set' (Dweck, 2006) and a fear about admitting she did not understand the mathematics.

Hoy and Spero (2005) mentioned that the impact of different types of input received by prospective teachers in their ITT year was significant on the development of a strong sense of efficacy. This thus supports the case made earlier of the tension that is created between the theory advocated by MTEs and what is actually experienced during school experience. It appears that where there is a disconnect which means prospective teachers' confidence is likely to be affected. They went on to find that efficacy "rose during teacher preparation and student teaching, but fell with actual experience as a teacher" (p.352). This could be why the teaching schools in my local area have put so many resources in to supporting those teachers in their first, second and third years of teaching, as it would appear that the withdrawal of support after ITT has a negative effect on confidence.

In summary, it appears that at the start of training prospective secondary teachers arrive full of confidence in their mathematical knowledge. When this is challenged there is some evidence that those with a 'fixed mind-set' (Dweck, 2006) might avoid challenging situations where their conceptual understanding is challenged.

2.5 Literature review summary

As can be seen above, there is a greater body of literature relating to the knowledge needed by teachers than there is regarding specific literature surrounding the learning and teaching of trigonometry. What is interesting is the dearth of literature which advocates a particular approach to teaching trigonometry especially in a world where resources and ideas are shared on the worldwide web with a click of a button.

As the title of this project implies "Designing a mathematics teacher education programme: A trigonometry case study" it made sense to structure the literature review so that it mirrors what happens in the design of our particular course. It started with what we need to know about learning and teaching trigonometry, to what knowledge a prospective teacher needs, to how one develops a mathematics specific session for prospective teachers and finally how all of these influence the confidence of these teachers to carry out the act of teaching.

In summary this chapter highlighted:

- The research does not show that one particular approach to teaching trigonometry is better than another;
- There is a paucity of literature about how student teachers learn to teach trigonometry;
- Prospective secondary teachers arrive at the start of their ITT with high levels of confidence in their mathematical knowledge. Their confidence in their teaching diminishes as they experience what it is like to be a 'real' teacher;
- The knowledge of prospective teachers tends to be predominantly procedural at the start of their training;
- PCK can be seen in action by using the Knowledge Quartet (Rowland et al, 2005).

This project, therefore, seeks to contribute to the area around how prospective teachers learn to teach trigonometry whilst reflecting on the changes we made to our mathematics teacher education programme in order to support the professional growth of our prospective teachers.

3. Methodology

The review of the literature on learning and teaching trigonometry, in particular that of prospective teachers, highlighted that there is an absence of research in the area of prospective teachers learning how to teach trigonometry. This small-scale project aims to contribute to this area of research. The motivation for this project has emerged as a consequence of my own experience of working as a mathematics teacher educator in a school. This chapter specifies the rationale for the methodological approaches used in the research.

3.1 Context of the research

This research took place in an inner city secondary school (ages 11 – 19) which is a designated Teaching School. Brookwood Academy is a standalone academy which means the school receives its funding directly from government rather than through a local authority. The school has been a training provider since 2000 meaning it can award Qualified Teacher Status (QTS) in its own right. Brookwood Academy has a 16-year history of successfully training teachers through the Graduate Trainee Programme (GTP) now known as the Schools Direct salaried programme.

Two years ago the school went into partnership with a local university to set up a SCITT (School Centred Initial Teacher Training) course specifically for mathematics. This means that the school is responsible for awarding QTS and the university awards the Post Graduate Certificate of Education (PGCE). This course is quite unique as the mathematics specific content is delivered by my colleague, Jenny (a pseudonym), and me. In the majority of

SCITT courses the university deliver the mathematics content or the provider has to buy in some additional services.

In the first year the size of the cohort was five and this year (2015 – 2016) there were six, who constitute the participants of this research. The six prospective teachers were a mix of ages with two having had long careers in the city before opting for a career change. The educational background of the participants was varied – two with a mathematics degree, two with engineering, one with a business/economics related degree and one with a science related degree.

The format of the course follows a typical university-based PGCE model – front loaded with time spent in school in workshops, a short teaching experience lasting until Christmas, time spent back at Brookwood Academy in sessions, a second longer teaching experience in a different school, followed by a final two weeks back at Brookwood Academy. During the year the prospective teachers are required to collate evidence to show that they meet the Teaching Standards (Department for Education, 2011) in addition to completing three PGCE assignments, two of which have associated masters' credits. The prospective teachers attend mathematics workshops planned and run by Jenny and me and General Professional Studies (GPS) sessions run by various 'experts'.

Any teacher education course will need to be informed by some guiding principles which are necessary to be explicit about what the course seeks to achieve. In order to provide the reader with a feel for the course that we have developed, here are the guiding principles of our course, developed as a result of my work for assignment four of this course.

1. Mathematics teaching should give students the opportunity to develop their conceptual understanding – the programme helps develop the student teachers' conceptual understanding (Skemp, 1976) and to engage critically with mathematics (Watson, 2008).
2. Being a teacher requires a growth mind-set (Dweck, 2006) and the desire to promote mathematics growth mind-sets in our students.
3. Learning to teach involves an understanding of the theory of teaching and learning and the opportunity to learn how this occurs in the classroom with the support of practising teachers.
4. Becoming a teacher requires the ability to be able to understand and evaluate theories such as assessment for learning, differentiation (not just mathematics and teaching theories but those of wider education including child development).

(Goodden, 2016, p.11)

This research project arose through the work that I am doing to develop our unique course and in particular respond to the needs and feedback of the prospective teachers. At the end of the first year of our course the teachers said they would have liked more sessions on their own conceptual understanding of mathematical concepts. This project is an in-depth look at how we developed the course the following year to respond to this feedback, and to develop conceptual understanding of trigonometry and confidence to teach it. It is longitudinal as it follows the six prospective teachers throughout their year-long teacher training course.

3.2 Research questions and information required

The research questions that were drawn up are as follows:

1. What does the research say about the best approach to teaching trigonometry?
2. What is the confidence of prospective teachers, in their own knowledge, to teach trigonometry?
3. Would mathematics sessions on trigonometry focused on aspects of the Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005) develop prospective teachers' confidence in their own trigonometry teaching?

The first requirement was a literature search which would enable me to consider the various approaches that have been taken in the learning and teaching of trigonometry. I needed literature and research studies which would tell me about the confidence the prospective teachers had in their knowledge of trigonometry pre and post trigonometry teacher education sessions. Using the search facilities on Solo and Google Scholar I found that there was a paucity of research in the area of prospective teachers' learning of how to teach trigonometry. Consequently, the literature review covered approaches to teaching trigonometry, the knowledge that prospective teachers bring and need with regards to trigonometry, the role of the MTE and the confidence of prospective teachers at various points in their training.

In order to answer the research questions, as well as literature, I needed information about the six participants' confidence regarding aspects of teaching trigonometry both prior to and post mathematics teacher education sessions. It was also useful to observe the participants teaching trigonometry and interview them after the lessons.

3.3 Action research

Denscombe (2005) stated that action research “is normally associated with ‘hands-on’, small-scale research projects” (p.73) which is exactly the nature of this project. This project is an in-depth look at how we (Jenny and myself) designed sessions to develop the conceptual understanding and confidence of prospective teachers to teach trigonometry. Punch (2009) argues that what is meant by action research is clear in the name – action and research. “Action research brings together the acting (or doing) and the researching (or the inquiring)” (Punch, 2009, p.160). Several researchers identify the cyclical nature of action research (Denscombe, 2005; Punch, 2009; Elliott, 1991):

The figure originally presented here cannot be made freely available via ORA because of copyright. The figure was sourced at Denscombe, M. (2005) ***The Good Research Guide for small-scale social research projects***. 2nd ed. Maidenhead: Open University Press

(Denscombe, 2005, p.76)

The associated advantages with using action research for this project are:

- There are positive benefits to our school linking directly back to developing our mathematics teacher education programme.

- This involved my colleague so not only provided me with a professional development opportunity it also developed her practice.
- It was time efficient – we constantly engage in evaluating, reflecting and changing our programme so it is felt that this is an activity we are doing anyway.

There are, of course, some disadvantages with using action research:

- The research is limited to the school in which I am working so generalisations regarding teacher education on a wider scale are limited.
- There will be an element of bias on my part as it is difficult to detach myself from the programme I have developed in order to be critical. The approaches I took to overcome this potential for bias are discussed within the ethical considerations section of this methodology.

In summary, action research was the most appropriate research method to use for this project as it is something that I do as a practitioner regularly, thereby making it fairly time efficient.

3.4 Specific innovation

This project seeks, through a practitioner research enquiry, to design and implement a specific innovation in teaching and learning. This is to design teacher educations sessions on trigonometry to develop confidence and promote the conceptual understanding of students. Part of this research will include an evaluation of this specific innovation. In particular, do sessions that focus on aspects of the Knowledge Quartet (Rowland et al, 2005) help develop that confidence and knowledge for teaching conceptual understanding of trigonometry?

3.4.1 First trigonometry session

The first trigonometry session was designed with the first three aspects of the Knowledge Quartet (Rowland et al, 2005) in mind – foundation, transformation and connection. The session consisted of three parts.

Table 1: The first trigonometry session and how it was linked to the Knowledge Quartet

Part of session	**Description**	**Aspect of Knowledge Quartet**
Refreshing their own subject knowledge	For example, recognise 'nested' similar triangles. Anne Watson shared these with me during discussions about this project. I then adapted the skills to suit our course and prospective teachers. The prospective teachers were required to understand each statement for themselves and then write a possible question.	Foundation
Constructing a network of trigonometric skills	Using the skills from the first part of the session the prospective teachers had to build a network to show the sequence in which the skills should be taught (see appendix 5).	Transformation and connection
How to introduce trigonometry for the first time?	The prospective teachers experienced and critiqued various approaches including similarity, functions, ratio and procedural (Watson, 2013).	Transformation

3.4.2 Second trigonometry session

Following the questionnaires and collaboration with Jenny a second trigonometry session was planned and delivered.

Table 2: The second trigonometry session and how it was linked to the Knowledge Quartet

Part of session	**Description**	**Aspect of Knowledge Quartet**
Refreshing their own subject knowledge	The session started with the question 'can you prove the sine rule?' Prospective teachers worked together to come up with various proofs.	Foundation
Marking student work on the sine and cosine rules	The prospective teachers were given several questions which had been answered by students – some correctly and some incorrectly. The prospective teachers had to mark the work identifying errors/misconceptions which had occurred.	Contingency
Experience of misconceptions	The prospective teachers were given many misconceptions on trigonometry from across the age ranges 13 – 18. These misconceptions were some that Jenny and I have come across as well as others which other members of the mathematics department thought of. The prospective teachers were asked to think about these questions: What is the misconception or error that has occurred? How might it have occurred? How might this misconception be addressed?	Transformation and contingency

3.5 Choice of methods for information gathering

3.5.1 Questionnaires

There were several ways that I could have collected data about the participants' confidence to teach trigonometry – questionnaires, focus group or interviews. I decided against a focus group because although it would have allowed the participants to interact and given a collective view (Denscombe, 2005), there were two participants who could dominate the others in group situations. I chose not to interview initially to ascertain confidence in teaching trigonometry due to my role in school. As the course director and a senior leader I felt that the participants may have felt pressured to give answers they thought I wanted to hear, especially at earlier stages of the course, before trust and credibility had been established (Denscombe, 2005).

I decided that I would administer questionnaires at two points during the year – in the autumn term following an initial trigonometry session (appendix 1) and in the spring term following a second session (appendix 2). This would provide me with data at different stages in the teachers' development. I used a questionnaire so that the prospective teachers could give their responses anonymously and they were easy to administer at the end of the day following the mathematics sessions.

The questionnaires, which had the advantages of being anonymous and time efficient, were developed over a period of time using the literature reviewed, with guidance from my supervisor and through the use of a pilot questionnaire with some of the mathematics teachers who work in the Mathematics Department at Brookwood Academy. I opted for a questionnaire which had scaled items of Likert-type.

Table 3: Changes made to rating scale in the questionnaires

Initial rating scales	**Rating scales following discussion with my supervisor**
Strongly agree Moderately agree Agree slightly more than disagree Disagree slightly more than agree Moderately disagree Strongly disagree	Strongly agree Moderately agree Neutral Moderately disagree Strongly disagree

The use of the scaled items allowed me to collect data that measures attitudes about set statements (Wilkinson and Birmingham, 2003).

The choice of questions was influenced by the literature review particularly trying to establish whether the prospective teachers had predominantly procedural or conceptual understanding of trigonometry (Ball, 1990), the type of PCK they possess particularly the transformation dimension from the Knowledge Quartet (Rowland et al, 2005) and confidence levels around teaching trigonometry. Wilkinson and Birmingham (2003) advocate the use of a pilot as it allows " 'fresh eyes' to comment on its suitability and clarity" (p.19). So following the small pilot of the questionnaire amongst my colleagues I changed the wording of some of the questions. The changes I had to make were regarding variety as the questions did not have sufficient variety in terms of 'agree' and 'disagree' statements meaning that the respondents could fall into a pattern (Denscombe, 2005) without necessarily having to read the questions. Each questionnaire finished with open questions to "allow for the recording of any response" (Wilkinson and Birmingham, 2003, p.11).

In summary, I used two questionnaires in order to collect data regarding views about set statements. Although there were only six participants the use of the questionnaire negated the conflict of interest regarding my role and allowed them to respond anonymously. The questionnaires were developed in light of the literature, feedback from my supervisor and the result of a pilot study within a small group of colleagues.

3.5.2 Observations

Questionnaires were not sufficient in evaluating and analysing the confidence the prospective teachers had with regards to teaching trigonometry because what they might say could be different to what they actually do, as was found by Ball (1990). Therefore, I decided I needed to observe as many of the prospective teachers actually teaching trigonometry as possible. Wilkinson and Birmingham (2003) state:

> "Merely asking about or reporting the activities people carry out in different social settings and situations will no doubt give you a flavour of what is involved, but in order to understand fully what these activities mean to people, how they themselves perceive them and what their perspective is on them, it is necessary to see those people in action, to experience what it is they do" (p.116)

In order to be able to analyse the lessons for this project I decided that recording the lesson would be the best course of action. I could have used field notes but there is so much going on in a lesson it is easy to miss subtle actions made by the teacher and responses of the students. Having a recording of the lessons enabled the prospective teacher and me to watch the lesson again and so trigger our memory. However, observing lessons had a number of disadvantages:

- The prospective teachers could be nervous due to my role in the school and the course;
- The responses of the students in the lesson could have been affected by my presence;
- I was only able to observe two lessons (one each of two prospective teachers) due to time constraints in school.

In summary, I chose to observe two of the prospective teachers in action so that I could identify the impact of any of the activities we did in trigonometry sessions and what was revealed about the teacher's PCK. I analysed the lessons I observed using the codes identified in the Knowledge Quartet (Rowland et al, 2005). This involved me immersing myself in the video recordings of the two lessons and looking for the codes identified as characteristic of the four dimensions of the Knowledge Quartet (Rowland et al, 2005).

3.5.3 Interviews

There are broadly three types of interview: structured, unstructured and semi-structured (Denscombe, 2005; Wilkinson and Birmingham, 2003). A structured interview, where the questions are predetermined and described as rigid, was not appropriate as I needed to have some flexibility to pursue issues that might arise and which may help to explain the impact of the sessions on their teaching. An unstructured interview was not appropriate as I had specific questions which needed to be asked about confidence and reasons for choices made about teaching trigonometry. In addition, an unstructured interview would not have been practical in the time constraints of the day-to-day life of a school. It therefore became apparent that a semi-structured interview would be most appropriate. This format has a list of questions that the interviewer follows but "there is sufficient flexibility to allow the interviewee an opportunity to shape the flow of information" (Wilkinson and Birmingham, 2003, p.45). This format also allowed me to react with flexibility if other issues arose during the interview.

For the semi-structured interviews, I created a table (appendix 3) which enabled me to keep notes during the interview and I made an audio recording of each one. This offered me a record of what was said and was “complete in terms of the speech that occurs” (Denscombe, 2005, p.175). However, a disadvantage of audio recording is that it only captures speech and not non-verbal cues. Following the interviews, I listened to the audio recordings in order to summarise the responses to each question whilst transcribing some responses for quotes. A disadvantage of this technique, rather than simply transcribing, was that the summaries were dependent on my interpretation. The major advantage, however, was that it saved me a great amount of time.

The questions for the interview directly related to the research questions of this project and were drawn up as a result of the findings in the literature review:

1. How confident were you about teaching trigonometry at the start of the course?
2. What made you choose this particular approach to introducing trigonometry?
3. What was your confidence like at the end of the course?
4. Was there any particular aspect of the course which influenced your choices about how to teach trigonometry?
5. Is there anything else about teaching trigonometry which you would like to mention?

The questions were carefully planned to collect data and information that could not be gathered from other methods (Wilkinson and Birmingham, 2003). I would not have been able to find out the reasons the participants chose their particular approach through a questionnaire, observation of the lesson or the written lesson plan. There was a risk with the first question that the participants would have forgotten how they felt about their confidence at the start of the course. However, it proved to be that all the participants were able to reflect on their confidence at the start of the course fairly easily.

In summary, I have explained my reasons for the use of a semi-structured interviewing format and how the interviews were conducted. Ethical principles were adhered to in this research ensuring anonymity and all the information was stored securely.

3.6 Data collection

The table below shows how the data collected links to the research questions:

Table 4: Summary of research questions and data collection method

Research question	Data collected
1. What does the research say about the best approach to teaching trigonometry?	Literature search
2. What is the confidence of prospective teachers, in their own knowledge, to teach trigonometry?	Pre-session questionnaires
3. Would mathematics sessions on trigonometry focused on aspects of the Knowledge Quartet (Rowland et al, 2005) develop prospective teachers' confidence in their own trigonometry teaching?	Post-session questionnaires Lesson observations Lesson observation written feedback Post-lesson semi-structured interviews

In essence this research project consisted of the following stages of work:

1. Data was collected about prospective teachers' confidence in their knowledge of trigonometry for teaching following an initial teacher education session on trigonometry.
2. A trigonometry session was designed and carried out as a follow up to the information from the evaluations and questionnaires.

3. A post-session questionnaire was given to the participants.
4. Two lessons were observed of prospective teachers introducing trigonometry to students for the first time.
5. Post-lesson semi-structured interviews were conducted (two where I observed the lesson and two where I had the lesson plans for the lesson but did not observe). In the case of the two lessons which I did not observe I used secondary data (lesson plans) to inform the primary analysis.

3.7 Collaboration with other adults

A significant aspect of this research project was the collaboration with other adults. Zaslavsky and Leikin (2004) argue that mathematics teacher educators develop whilst working in 'communities of practice' (Lave and Wenger, 1991) which contain experienced and senior members as well as newcomers. In this case the 'community of practice' contains the prospective teachers, Jenny (a mathematics colleague), the mathematics department and myself. Jenny and I worked together to develop the trigonometry sessions. We both looked at the results from the first questionnaires and planned how to address some of the areas where the prospective teachers felt their confidence was lacking. Indeed, Jenny has been involved at every stage of the project. Other members of the mathematics department at Brookwood Academy were involved in contributing misconceptions which they had encountered regarding trigonometry in their teaching careers. Following the first trigonometry session I adapted the third part (how to introduce trigonometry) to run a workshop for the mathematics department as the new GCSE foundation curriculum includes trigonometry for the first time. Finally, I will be sharing the findings from this project with

Jenny, my colleagues in the mathematics department and the lecturers with whom we work in partnership from the local university.

3.8 Ethical considerations

This project was approved by the Central University Research Ethics Committee of the University of Oxford and I had permission from the Principal of Brookwood Academy. Following the guidance of the BERA (2011) I took the following steps when conducting this research:

- The process and nature of the research was explained to all the participants.
- The participants voluntarily agreed to take part in this research (see appendix 4 for the consent letter).
- The participants had the right to withdraw at any stage.
- Pseudonyms have been used for the school and all the participants to protect their identities.
- Questionnaires were completed anonymously.
- The recordings of the lessons and interviews were encrypted and will be destroyed on the completion of this project.
- There was a potential conflict of interests as I was also the course director as well as the researcher.

The most significant two ethical issues which are likely to influence this research project are researcher bias and the conflict of interests created due to my roles in school. I needed to be mindful of the possibility that when analysing the findings of this research I identified any researcher bias. Miles and Huberman (1994) made suggestions about how to reduce bias and I have used these to review my actions in terms of ethical considerations.

Table 5: Tactics to reduce bias (Miles and Huberman, 1994) and how these were used in this research

Tactics recommended by Miles and Huberman	How these were used in this research
Checking for representativeness	My colleague Jenny also looked at the findings of the questionnaires, interviews and lessons. The findings will not be used to make wider generalisations.
Checking for researcher effects	The interviews took place in a non-threatening environment (i.e. not my office) and where the participants were comfortable. Ensured the reason for the interviews was clear to all participants. The participants saw a great deal of me throughout the year, I was part of the 'local landscape'. They were comfortable giving open and honest feedback due to the 'growth mind-set' (Dweck, 2006) we promoted. None of the four participants who were interviewed were classed as 'cause for concern'.
Weighting the evidence	I checked understanding with the participants throughout the interviews. I triangulated the data as much as possible.
Checking the meaning of outliers	I took into account the contrast in the final gradings of the two prospective teachers whose lessons I observed.
Follow up surprises	For example, Tom's interview revealed that he had decided in future he would teach trigonometry in a way which went against his beliefs. This was explored until he clarified that it depended on the ability of the students and how likely they were to go on to study A-Level mathematics.
Replicating findings	Collaboration with Jenny about the findings.
Getting feedback from informants	The findings were shared with participants throughout the research project. For example, at the start of the second trigonometry session I shared the findings from the questionnaires and the rationale for the session.

3.9 Limitations of the methodology

There are a number of limitations of this research project:

- The project is small-scale with only six participants.
- It is a ‘work-site’ (Denscombe, 2005) specific project which means it cannot be used to generalise across teacher education on a wider scale.
- Researcher bias could exist due to the nature of my role in the school but as described above I took steps to minimise this.
- There could be conflict of interests between my role as researcher and my role in school but as above I took steps to minimise this.
- Due to organisation constraints I was only able to observe two of the teachers teach trigonometry.

4. Findings

This chapter outlines the findings of this project which are then analysed with specific reference to the literature review in the next chapter. I share the findings from the questionnaires which were given prior to and after the mathematics specific sessions which Jenny and I planned and delivered. I summarise, for the reader, the two lessons (one of Tom's and one of Lucy's) which I observed with specific reference to the Knowledge Quartet (Rowland et al, 2005) followed by a discussion about the responses made during the semi-structured interviews.

4.1 Pre-session questionnaire

Below I summarise the themes that emerged from the questionnaire which was distributed following the initial mathematics specific session on trigonometry. The findings were then used to plan the subsequent session.

4.1.1 Responses to scaled items

The responses to the scaled items in the questionnaire after the first session on trigonometry can be seen in the table below.

Table 6: Responses to scaled items in the first questionnaire

		Number of responses				
Statement		Strongly agree	Moderately agree	Neutral	Moderately disagree	Strongly disagree
1	I can remember my first experience of trigonometry from my school days.	1		2	2	1
2	My own experiences of trigonometry prior to this course were largely procedural.	1	3	2		
3	Prior to the course I had a good conceptual understanding of right-angle trigonometry.	2	2	1	1	
4	Before the trigonometry session I felt confident in my own knowledge of trigonometry and the skills involved.	3		2	1	
5	After the trigonometry session I felt less confident about my own knowledge of trigonometry and the skills involved.				2	4
6	If I have to introduce trigonometry to a class for the first time I know exactly which approach I will take.	1	1	2	2	
7	If I have to introduce trigonometry to a class for the first time I don't know which approaches are possible.			2	2	2
8	If I have to introduce trigonometry to a class for the first time I know which approaches are possible but am undecided which one to take.	1	4	1		
9	When it comes to planning a sequence of lessons on trigonometry I have a clear idea about how to order the teaching.		2	2	2	
10	I am not confident when it comes to planning an individual lesson on trigonometry.		1	4		1
11	I feel confident about knowing the misconceptions which exist when teaching trigonometry.			4	2	
12	I feel confident about planning to address misconceptions which exist around trigonometry.			4	2	

Unsurprisingly, only one of the participants was able to recall their own experience of learning trigonometry from their school days. All the participants disagreed (to a varying degree) with the statement "After the trigonometry session I felt less confident about my own knowledge of trigonometry and the skills involved". However, the majority response for questions about confidence in planning, knowing misconceptions and planning for misconceptions were neutral. It appears that the first trigonometry session on the course helped with the prospective teachers' foundation knowledge (Rowland et al, 2005) but there were still some doubts when it came to the actual planning to teach. Five of the six prospective teachers knew what the different approaches to introducing trigonometry are but only two knew which one they would actually choose.

I will now pick out the common responses to the open ended questions in the questionnaires.

4.1.2 I think the best aspect of the trigonometry session was…

There were two common aspects which all participants commented on:

- building the network of skills and
- going outside to measure the height of the school using a clinometer.

All participants found it useful in the development of their own knowledge of trigonometry to consider all the skills needed from key stage three to key stage five (see appendix 5 for a photo of the completed network). The participants commented that the practical activity of going outside made the session memorable and they were likely to plan a similar activity in the future. One prospective teacher said *"I prefer the ratio method as I believe it can be* *used in any class"*. This particular respondent also mentioned the ability of the class in terms of it making a difference to what approach was decided upon.

4.1.3 I think the aspect which could have most been improved was…

There were no common themes in response to this question. One participant made the comment *"This was a great session but having not taught this topic since I fear some of its value was lost"*. One participant made the comment that they felt less confident introducing trigonometry as a function referring to a discussion had in the session about the Weber (2005) article. One participant wanted more time spent on higher trigonometric concepts *"maybe go through some proofs of identities and then examples"*. This is interesting as trigonometric identities first appear in the A-level curriculum and whilst the course is for teaching ages 11 – 18 it is unlikely the prospective teachers would be 'let loose' on an A-level class immediately.

4.1.4 Which part of teaching trigonometry, if any, would you like more support with?

There were two parts which the prospective teachers identified as requiring more support with:

- how to teach the identities without just giving them to students and
- a deeper understanding of student misconceptions.

To summarise, the findings from the first questionnaire demonstrated to me that while the first session was helpful in developing the prospective teachers' own understanding of trigonometry, it showed they lacked confidence in transforming their knowledge into planning, especially for misconceptions. They also required further support with how to teach the trigonometric identities. As a result of these findings I developed a second session on trigonometry on identities and misconceptions.

4.2 Post session questionnaire

In this section I present the findings of the questionnaire which was distributed following the second session and developed after the first questionnaire. The format of the findings is the same as in the previous session.

4.2.1 Responses to scaled items

This table shows the responses to the scaled items in the questionnaire:

Table 7: Responses to scaled items in the post session questionnaire

		Number of responses				
Statement		**Strongly agree**	**Moderately agree**	**Neutral**	**Moderately disagree**	**Strongly disagree**
1	I feel confident about knowing the misconceptions which exist when teaching trigonometry.	5	1			
2	I do not feel confident about planning to address misconceptions which exist around trigonometry.				2	4
3	This session was not helpful in terms of developing my own conceptual understanding of trigonometry.					6
4	I know how some teaching approaches can lead to misconceptions within trigonometry.	2	3	1		

It can be seen in the table above that in general the participants indicated a feeling of confidence with respect to knowing and planning for misconceptions in contrast to the questionnaire completed prior to the second session.

4.2.2 I think the best aspect of the trigonometry session was…

Here are some of the responses to this question:

"Having the opportunity to answer some questions on trig, see students work, identify what could be an error or misconception and how to prevent this from happening."

"Perhaps addressing the misconceptions"

"Addressing misconceptions…algebraic misconceptions not trig."

This participant was referring to knowing when the misconception is caused by an inability to perform some kind of algebraic manipulation rather that a misconception associated specifically to trigonometry.

"I thought the identifying mistakes in marking section was useful, as some mistakes were quite difficult to spot. Going through these examples and possible areas for mistakes would help me spot them as a teacher."

From these responses it was clear that the prospective teachers found the second session useful, in particular the focus on errors and misconceptions.

4.2.3 I think the aspect which was least useful was…

Five out of the six respondents had nothing to say in response to this question. This could be because they do not know what they do not know yet and with more time and experience they may have been able to give an answer. One of the respondents said:

"I thought the A3 sheets in groups wasn't a good idea on our behalf as it didn't allow us to think of any possible misconceptions on our own accord."

This prospective teacher was referring to an activity where I had identified questions ranging from introductory trigonometry (key stage three) to more advanced (A-Level) which led to common misconceptions. These were identified by using my professional experience with help from my colleague Jenny. The prospective teacher made a valid point and this could be incorporated in future sessions.

To conclude, the findings of the post session questionnaire indicate the prospective teachers not only enjoyed the session, but reported they felt more confident with respect to misconceptions which might occur when they are teaching trigonometry. The next stage of this project was to discern whether the impact of the sessions could be seen in the act of their teaching and if their comments matched their practice. In the next two sections I discuss in detail the two lessons I observed and the prospective teachers' responses in the post-lesson interviews.

4.3 Tom's lesson on trigonometry and post-lesson interview

4.3.1 Tom's background

Tom is a mature prospective teacher with a mathematics degree who, after a successful career in stocks and shares, decided to make the career change to become a teacher. He generally found the career change difficult in terms of classroom management but his knowledge of mathematics was stronger than the rest of the group. Early on in the course he highlighted his lack of pedagogical content knowledge and how he was having to 'unlearn' what he has learnt over the years in order to teach mathematics.

4.3.2 Tom's lesson

Tom's lesson took place during his second teaching practice in an independent boys' school. The class was an able group of year eights (ages 12 – 13) and this was his first attempt at teaching trigonometry. Tom decided to focus on introducing the tangent ratio in his introductory lesson. His chosen approach was the ratio method as described by Kendal and Stacey (1996). The students spent time in the lesson measuring sides and angles of similar triangles. Tom distributed a table which showed the values of the tangent of angles from between 0° and 89° rather than using a calculator approach.

In the table below I illustrate parts of the lesson under each aspect of the Knowledge Quartet (Rowland et al, 2005).

Table 8: Illustrations of teaching in Tom's lesson in which aspects of the KQ were seen

Foundation Use of terminology – a great deal of time in the lesson was devoted to discussion about the best names for the sides. Tom explained the meaning of the work 'trigonometry' breaking it down into three parts. Tom demonstrated a deep understanding of the mathematics he was teaching in this lesson. His decision to focus on the tangent of an angle only and use the similar triangle approach to introducing trigonometry – linking to the session he had at Brookwood Academy. Students were given the opportunity to explore the triangles themselves – possibly developing 'relational understanding' (Skemp, 1976). Correct use of mathematical language, except at one point where Tom said "isosceles triangles do not have a right-angle" – he later corrected this.	**Transformation** At the start of the lesson Tom asked students to identify the largest angle and longest side – at this point there was only one right-angled triangle. Tom chose a triangle with opposite and adjacent sides of 3cm and 7cm, the other two were then enlargements of this initial triangle. The types of questions Tom used were: is that always the case? Can you think of an example when it might not be true?
Connection Tom made links to previous lessons on Pythagoras – particularly when using the word hypotenuse. The parts of the lessons were linked together – identifying longest sides and angles, naming the opposite and adjacent sides, measuring the sides and angles of similar triangles and finally revealing the tangent function as a way of expressing the relationship between the opposite and adjacent sides.	**Contingency** During the lesson, several times, it was clear that Tom did not quite understand what a student was saying – in general he asked them to try and explain again and listened carefully. He was then able to understand and respond.

This shows that in a single lesson it was possible to observe all four dimensions of the Knowledge Quartet (Rowland et al, 2005).

4.3.2 Interview

Appendix 6 shows the responses during the interviews made by the four participants. With respect to his confidence Tom felt confident about his own knowledge of trigonometry especially having taught several lessons by now. It was clear that Tom felt 'rusty' at the start of the course but knew he could refresh his knowledge in a fairly short amount of time.

> *"It's just a case…you see it's been such long time since I was at school, it's just a case of remembering basic rules. I would be confident after an hour or so of teaching, it's something I think would develop quite quickly."*

Tom chose to focus on introducing the tangent function for reasons that for him felt quite intuitive and because of a discussion which had taken place in one of our mathematics sessions.

> *"It seemed obvious to me to start with tan because it comes up in real life, it's a bit like in our session when we went outside to measure the height of the building. The first two sides you come across really are the opposite and adjacent. I am sure that is what I did when I was at school."*

Perhaps one of the most interesting parts of the interview was when Tom was asked to explain why and how his lesson came about. Shortly after I had asked the prospective teachers if they would be willing to be part of my research Tom had the opportunity to watch an experienced teacher, who he described as *"passionate about how she introduces it"*, introduce trigonometry. He went on to describe how this teacher introduced the sine, cosine and tangent functions using SOHCAHTOA so that the students had to get used to all three at once. He felt at the time that he would not teach it in that manner, as the students would fail to understand the link between ratio and trigonometry, but rather it would become more about pressing buttons on the calculator. Owing to his strong feelings that this was not the right approach, despite a conversation with his mentor to the contrary, he planned the lesson I subsequently observed. However, having taught this lesson and the following series of lessons he changed his mind:

> *"I am wrong though, to get most people through the exam that seems to be the way it is taught. It just feels that it is a good way bulk way to get everyone to be able to answer the questions. It's more efficient to teach all three at the same time. This approach probably gets the better results."*

Tom went on to say that despite this method being *"shallow and lacking the depth of understanding"* he would use this procedural approach in future.

4.4 Lucy's lesson on trigonometry and post-lesson interview

4.4.1 Lucy's background

Lucy is a young teacher, having made the career change after a short period working in marketing. At university she did an economics degree. From the outset of the course Lucy indicated that she wanted to teach mathematics in a way that helped the students understand it but that she lacked the conceptual understanding herself.

4.4.2 Lucy's lesson

Lucy was teaching a second set year nine class (ages 13 – 14) who had not encountered trigonometry before. The lesson started with four questions checking the students had the required skills of manipulating equations which involve fractions. The lesson progressed onto how to label the sides of a right-angled triangle and students were introduced to the terms 'opposite', 'adjacent' and 'theta'. Lucy's chosen approach was also the ratio method.

However, she chose to give the students five nested right-angled triangles with an angle of 30°. Students were asked to measure the three sides and then work out the ratios. The results were shared briefly and the lesson finished with students asked to find missing sides (only the opposite or hypotenuse) of 30° triangles.

Table 9: Illustrations of teaching in Lucy's lesson in which aspects of the KQ were seen

Foundation Use of the term 'inverse operation' during the starter. Lucy encouraged students to say 'length of the opposite' rather than just 'opposite'. Lucy had on the board: "The relationship or ratio between the length of the opposite and the length of the hypotenuse is called sine." Lucy created her own worksheet with nested triangles – no adherence to or use of a textbook. Clear from the planned lesson Lucy was trying to develop understanding – except when explaining the last starter question (9.5 = 10/x) when she said "to make it easier you multiply both sides by x".	**Transformation** The last starter question had the variable as the denominator. Students were required to identify the sides of right-angled triangles using the terms opposite and adjacent. The nested triangles approach meant that some students did not 'see' the individual triangles. Lucy chose to focus on the sine ratio for the last 15 minutes. A mix of questions were used "can you explain", "why do you think that", "can you see a relationship".
Connection Lesson started with some equations to solve where one side is a fraction e.g. x/4 = 4. There were no explicit links made between the starter and the topic of trigonometry, there was opportunity at the end but the link was not made explicit. Made links to Pythagoras and the word hypotenuse. Anticipation of the complexity of using the word 'ratio' in a context which students were not used to was evident. Lucy had clearly though about the wording she would use (see above).	**Contingency** Lucy responded to all the students' comments appropriately. When students gave incorrect responses Lucy's most common response was to ask another student to correct him or her. There was no deviation from the agenda.

As with Tom's lesson, I observed all four dimensions of the Knowledge Quartet (Rowland et al, 2005) in the trigonometry lesson.

4.4.3 Lucy's interview

In contrast to Tom, Lucy did not feel confident about teaching trigonometry and recognised this when it came to planning the lesson *"The fact that I was nervous and worried about how to reveal the concept showed me how unconfident I was"*. Lucy could not articulate why she chose to focus on the sine function beyond an instinctive feeling that this was the first trigonometric ratio that is usually introduced in textbooks and possibly the first one she encountered in her school days. She revealed something about her own knowledge in her decision to focus on sine of 30°:

> *"I wanted to make it as simple as possible. I wanted to make them be able to deal with numbers..umm…with it being a half meant you could talk about it being a half rather than dealing with all the decimals which I knew a lot of them were not confident with anyway."*

This also reveals that Lucy considered the prior learning of her students in choosing examples and their areas of strength and development.

Lucy found that the parts of the trigonometry sessions which most impacted on her decisions in the planning of the lesson were the practical elements of the session and real-life links. She commented that knowing how trigonometry can be applied in real life gave her some confidence in dealing with student responses. During the interview Lucy also revealed how useful she found using trigonometric tables as opposed to calculators. She felt this helped with classroom management as students got tired of using the tables and began to bring their calculators but also she felt they had a better sense of the numbers involved.

4.5 Other interviews

As mentioned earlier I observed two lessons (Tom's and Lucy's) but I carried out four interviews. The final two interviews were that of Brian and Kate who introduced trigonometry to a class during their second teaching practices. Whilst I was not able to observe the lessons I did have access to their lesson plans and resources (appendix 7). It was not possible to use the Knowledge Quartet (Rowland et al, 2005) to analyse Brian's and Kate's lessons as I did not observe the lessons myself.

4.5.1 Brian's interview

Brian was teaching a year nine class (ages 13 – 14) who were the group below that of Lucy's class. He chose to take the nested similar triangle approach to introducing trigonometry. The students measured the sides and were encouraged to find the relationships between the sides. Brian chose to introduce all three ratios in this lesson.

In his interview Brian talked about how his confidence changed from the start of the course when he felt confident in his own knowledge. This confidence dipped when it came to thinking about how to teach trigonometry *"I was more confident until I put myself in the students' shoes and thought how would I approach it"*. He recognised the challenge in thinking about a concept which he knows and transitioning to teaching the same concept. Brain mentioned on several occasions how important it was to link new concepts with concepts which students are already familiar with, in this case proportion. This seemed very important to Brian throughout the interview.

> *"If you were to ask me how to teach it at the start of the course I would have gone straight for sine, cos and tan, here are the formulas, this is how to put the numbers in. It was very procedural. But the way I chose linked to what they already knew."*

When asked about how the sessions impacted on his confidence he was positive about these and mentioned how valuable the variety of experiences was. The school placement and working with other mathematics teachers were vital components for him too.

4.5.1 Kate's interview

Kate was teaching a year nine class (ages 13 – 14) of the most able students. Like Tom's school, Kate's class was single gender (girls as opposed to boys). Kate chose to use similar triangles as an introduction to trigonometry but these were not nested triangles. Students were put into groups with different triangles (the same angle, for example one group had a 30° angle, another 60°). Kate's approach was faster than that of the other prospective teachers as she speedily got on to using SOHCAHTOA and modelling several examples in the lesson.

Kate was more confident with her knowledge of trigonometry than some of the other prospective teachers. She mentioned regularly that where she was not confident it was just a case of refreshing her own knowledge. Interestingly in her interview Kate valued the importance of giving the students the chance to investigate the trigonometric ratios themselves, however in practice this did not happen. The written feedback from the experienced observer witnessed a pressure to cover content rather than really build the students' conceptual understanding. When asked about this Kate said *"I was told that by the end of the lesson they should be able to calculate the length of the side"*. Consequently, this meant Kate gave the students a small amount of time to investigate and quickly moved on to modelling the procedures. Kate was particularly surprised by how difficult the students found it to label the sides which is something she finds intuitive herself:

> *"What surprised me was how much difficulty they had labelling the sides, it seemed to be intuitively a difficult concept for them. I assumed this would be easy for them."*

In conclusion, this section shared the findings from the questionnaires, the lesson observations and the post-lesson interviews. In the next chapter I discuss these findings and link them to the literature reviewed in chapter two.

5. Discussion

In this chapter I discuss the literature reviewed in chapter two in light of the findings of the action research project. This discussion is structured around the literature review headings.

5.1 Learning and teaching trigonometry

5.1.1 The difficulties concerning learning trigonometry

It is accepted that there are difficulties associated with children learning trigonometry both in school and then when they go on to further study (Watson et al, 2013; Watson, 2009; Pritchard and Simpson, 1999; Weber, 2005; Ross et al, 2011; Blackett and Tall, 1991). The prospective teachers in this research project recognised the challenges faced by children when learning trigonometry, particularly using the term ratio in the form of a fraction, and initially did not feel confident about the approaches which could be used to address these difficulties. Lucy thought extremely carefully about how she would use the word ratio as she was concerned that the students would be uncomfortable without the use of the ratio notation. She used this definition: *"the relationship or ratio between the length of the opposite and the length of the hypotenuse is called sine"*. This links to the research by Watson (2009) about the difficulties in learning trigonometry being linked to the number of different mathematical understandings students need. All the prospective teachers in this project decided to approach these difficulties by using the ratio approach in the classroom in an effort to help the children make links with areas of mathematics which were more familiar to them. This was despite the fact our first teacher education session demonstrated several ways to introduce trigonometry. My perceptions around why they each chose the ratio method is because this was the one they felt most comfortable with and felt the students would be able to access more easily. Tom particularly talked about in the future, matching his approach to the ability of the students he would be teaching and whether they would be going to continue with the study of mathematics post-16. The use of the ratio approach was also supported by the research conducted by Kendal and Stacey (1996) in Australia when they compared the unit circle and ratio approaches.

In terms of suggestions for future practice from the discussion above the prospective teachers had some understanding of the difficulties that face children when they learn trigonometry. Thus it might be intriguing to look through a longitudinal study at how prospective teachers change their approach, as they learn themselves more about the difficulties with learning trigonometry, as they move from novices to experts. Both trigonometry teacher education sessions attempted to give the prospective teachers a sense of the complexity involved in teaching trigonometry by building the trigonometry skills network and then analysis of student misconceptions and errors. This helped them justify the decisions they took in the post-lesson interviews.

5.1.2 The use of technology in the teaching of trigonometry

It is interesting that three of the four teachers observed teaching took the decision not to involve any technology (scientific calculator or dynamic computer software) in their lessons. They reported feeling that using a calculator would not be helpful for supporting the development of conceptual understanding. This differs from the research reviewed which found technology to be a useful tool in the teaching of trigonometry (Blackett and Tall, 1991; Cavanagh, 2008). Indeed, it was Delice and Monaghan (2005) who found that whilst no system is better than another you do 'get what you teach'. This was found to be the case in Lucy's perception of her class's learning. She found that using the trigonometric tables rather than calculators gave the children a greater appreciation of the relationships. Lucy felt this was evident when three children who joined her class later in the term used the keys far more erroneously than the children who were in her class from the start.

Worthy of note also is that in the first teacher education session Jenny and I shared a copy of the trigonometric table as a tool for the prospective teachers to use if they chose to. It is difficult to know whether this influenced their decision or whether they made a pedagogical decision. Interestingly, none of the prospective teachers used any dynamic geometry software, which again we did not use in either trigonometry teacher education session. It is possible that the phrase 'you get what you teach' (Delice and Monaghan, 2005) refers to teacher education too, for example had we used dynamic software in our session the prospective teachers may have introduced trigonometry using it.

This project has not looked at the impact of the prospective teachers' practice on student learning (which would have been too big for a small-scale project). Therefore, it is difficult to comment on student learning of trigonometry with regards to the use of technology. However, the three prospective teachers' perceptions (who were observed) was that the avoidance of calculators was an approach they would adopt again in order to give children a 'sense' of the numbers involved. This is another interesting point as in the comparison between the Turkish and UK systems carried out by Delice and Monaghan (2005) the Turkish system avoid these long decimal numbers as they do not use calculators until later. This leads to Turkish students being more comfortable with surds at an earlier stage than students from the UK. Another avenue for future investigation would be to compare and contrast the approaches taken with regard to technology and student learning of trigonometry. Perhaps, in this project I could have looked in more detail about the impact of technology on teaching mathematics more generally to support and contrast with my findings.

5.1.3 Developing conceptual understanding of trigonometry

In accordance with the literature (Kendal and Stacey, 1996; Skemp, 1976; Weber, 2005) where researchers looked at approaches that help children understand trigonometry, my research showed that this was also salient to the prospective teachers. It was clear from the interviews that helping students to understand the concept of trigonometry and in particular the relationship between the sides was important to all the prospective teachers. Although they all approached this using a ratio approach there were differences with one choosing to focus on sine, one selecting tangent and two introducing all three ratios in one lesson. Consistent with 'usual' practice reported by Pritchard and Simpson, in each case the prospective teacher did not give a mathematical definition of sine, cosine or tangent before giving a concept image (Tall and Vinner, 1981).

Interestingly, in her interview Kate articulated her beliefs about the significance of children's conceptual understanding of trigonometry, and talked about her lesson in a way that suggested she planned to teach in that way. However, it became swiftly apparent from the lesson feedback that while her lesson started using a guided discovery style, she quickly resorted to teaching a set of procedures because of the pressure she felt to cover the content. In point of fact the experienced teacher who observed her wrote in his feedback "Could you try to cover less content and try to develop the children's conceptual understanding?". This conflict between Kate's views and the priorities of the school is in line with the research by Skott (2009) reviewed earlier. This conflict was also identified by Plummer and Peterson (2009) who described the case of the prospective teacher who advocated teaching conceptual understanding which was then not observed in her teaching. The teacher in this research felt confident in her conceptual understanding but then took moves to protect herself when it became revealed she did not have this understanding. A deeper investigation into Kate's attitudes and teaching might have revealed a similar picture but in this small scale project it is difficult to say whether the similarities are strong.

5.2 What pedagogical content knowledge do prospective mathematics teachers need to teach trigonometry?

In the findings I showed that using the Knowledge Quartet (Rowland et al, 2005) was helpful when looking at PCK in action in the lessons but also in the planning of the teacher education sessions. In the sections below I discuss this, making links to the literature reviewed in chapter two.

5.2.1 Foundation

As discussed in the previous section there was a conflict of beliefs and the priorities of the school in Kate's case which links to the literature reviewed earlier (Skott, 2009; Beswick, 2011). This was found to be the case with Tom too who started with firm beliefs about his views of how children should learn trigonometry having watched an experienced teacher introduce trigonometry. Following his experience Tom decided that he would teach trigonometry using a more procedural approach in future. It could be argued that this has occurred because of the gap between the theory espoused on our teacher education programme and his school experience. This supports the literature reviewed regarding the gap (Burn and Mutton, 2015; Hayward, 1997; Davis et al, 2013; Korthagen and Kessels, 1998). In his interview Tom wondered whether this conceptual approach was only possible in particular schools such as Brookwood Academy. Again, it is difficult to know the impact of the teacher education sessions on Tom's beliefs and how he came to change his mind. Once more, it would be interesting to follow Tom in a longer study to analyse his beliefs and how they are enacted in the classroom.

It was clear in the findings that the foundation dimension of the Knowledge Quartet (Rowland et al, 2005) could be observed in the prospective teachers' teaching. From the interviews and observed lessons, it was apparent that all the prospective teachers found the trigonometry sessions helpful in developing their own knowledge of the subject and particularly progressing themselves as 'problem-solving applied mathematicians' (Chick and Stacey, 2013). The findings from the post-session questionnaire showed that they all found the time spent analysing the misconceptions and errors contributed to their foundation knowledge which was also evident in the lessons I observed.

In summary, my findings seem to support the literature reviewed but going forward it might be pertinent to look at how the beliefs of our mathematics teachers change - or not, over time as an indicator of the success of the guiding principles of our teacher education programme.

5.2.2 Transformation

Predictability, most of the prospective teachers were unable to recall how they were taught trigonometry, though Tom reported that he was possibly introduced to the tangent function first. As discussed in the literature review the research showed that prospective teachers full back on the ways they were taught (Cavey and Berenson, 2005; Hill, 2000; Bowers and Doerr, 2001; Leikin, 3008; Artz, 1999). My research does not suggest this to be the case unless of course it has a subconscious impact on the approaches they choose. The fact that Kate ended up taking a procedural approach by the end of the lesson could suggest she had resorted to the same methods that she was taught. Lucy also reported that later on in the unit of trigonometry she did teach SOHCAHTOA but felt the children had a better understanding because of the groundwork she had done.

There was little time spent in the trigonometry sessions spent on the actual planning of lessons. There is a definite conflict between the means with which we expect the prospective teachers to plan and what 'real' teachers do on our course as supported by literature (Maroney and Searcy, 1996). Consequently, it was apparent that the prospective teachers spent an inordinate amount of time planning their lessons. Although in the first session we spent some time sequencing the variety of skills associated with trigonometry it might be necessary in the future to think about how we develop the transformation dimension of Knowledge Quartet (Rowland et al, 2005) to aid planning.

5.2.3 Connection

The first trigonometry session certainly helped the prospective teachers with regards to the connection dimension. They all commented on the importance of making links with other areas of mathematics with which the children might be familiar – in this case ratio and similarity. In the lessons I observed the prospective teachers made explicit links with other areas of mathematics and Kate commented that when she teaches trigonometry again she will spend time teaching similarity beforehand. If the research by Askew et al (1997) about highly effective teachers being ‘connectionist orientation’ is correct then it is possible that the teacher education sessions have been successful. Time and observation will tell whether the prospective teachers become highly effective; however, at this stage they have all passed the course with a grading of either good or outstanding. Again an implication from this research is that a more longitudinal study looking at teacher learning and growth over time much as the Developing Expertise as a Beginning Teacher (DEBT) did (for example Burn et al, 2003; Burn et al, 2008). This would be beneficial and allow the researcher to better evaluate the effectiveness of the sessions and their impact beyond the short term reported in this study.

5.2.4 Contingency

In the lessons I observed I did not see any contingent moments of note. It is possible these occurred in subsequent lessons which means that it would have been pertinent to have observed a whole series of lessons. However, this was not logistically an option for this project. It is perhaps notable that when asked a question which Tom was not sure about, initially, he took the time to think about an answer before responding. When asked if anything happened in the lesson which they did not expect Kate and Brian mentioned that they were surprised with how long it took the students to get used to labelling the sides of a right-angled triangle. In the lessons I saw there was no ’deviation from agenda’ (Rowland et al, 2009). In conclusion, this project did not find anything significant with regards to how our teacher education sessions affected the prospective teachers’ ability to respond in the moment. However, by observing the teachers over a series of lessons this may have been evident as in my professional experience the misconceptions usually begin to arise after several lessons. As I only observed an introductory lesson on trigonometry by Lucy and Tom this is a limitation of this research project.

5.3 Teacher education sessions

The findings showed that the prospective teachers felt more confident, following the sessions, about their ability to teach trigonometry in terms of their own knowledge and the different approaches that could be taken. Since our sessions provided the prospective teachers with a chance to 'unpack' the mathematics (Ball and Bass, 2002) and work on mathematics collaboratively this supports the literature reviewed. The nature of the collaboration of prospective teachers is a complex one and different to that of practising teachers (Leikin, 2008). The prospective teachers not only find themselves in a 'community of practice' (Lave and Wenger, 1991) with the others on their course but also within a 'community of practice' within their school placement. Leikin (2008) argues that knowledge is developed in these social groups but that they may not be able to integrate fully into their school group; it could be that this leads to a greater importance placed on the community of prospective teachers. All the questionnaires after the second session were positive about the session and pointed to an increased confidence.

The second session presented the prospective teachers with a collection of misconceptions. However, as one of the participants said in their questionnaire it might have been more useful to encourage them to think about possible misconceptions themselves as described in the literature review (Watson, 2008). It was clear in the post-lesson interviews that the sessions led the prospective teachers to be reflective in terms of the choices they took and were focused on how the children would learn about trigonometry in line with the quote used in the literature review from Crowe and Berry (2007).

In summary, the findings and literature show that teacher education sessions planned to directly take account of the Knowledge Quartet (Rowland et al, 2005) result in improved confidence in knowledge to teach trigonometry. However, in the future the sessions would need to have a greater focus on medium term planning and we need to think more carefully around how we can create situations in our sessions to help the prospective teachers handle contingent moments.

5.4 Confidence of prospective teachers during their teacher education year

In answer to research question three it was clear from the findings, in particular the questionnaire and post-lesson interviews, that the prospective teachers felt more confident as a result of the teacher education sessions. It could be argued that in the interview they said what they thought I wanted to hear. However, I observed a difference in their confidence in the classroom and in their own knowledge from the start of the course to the end. In line with the findings of Hoy and Spero (2005) at the start of the course it was evident that the prospective teachers were confident in their own knowledge, though some reported to being 'rusty', but they were more worried about aspects of teaching such as classroom management. This reporting of 'rustiness' was also found to be common in secondary mathematics teachers by Ball (1990). The sessions led them to the realisation that they may not understand the concept of trigonometry as much as they thought they did. However, after the second session they all reported increased confidence, which I saw in the lessons and the interviews. As reviewed earlier there is evidence of a dip in confidence when the prospective teachers enter the profession (Hoy and Spero, 2005; Corcoran, 1981), however this research project was not able to look at this due to time restrictions of this course.

6. Conclusion and future implications

This project set out to investigate whether the design of teacher education session planned around the Knowledge Quartet (Rowland et al, 2005), specifically on trigonometry, had an impact on prospective teachers' confidence in their knowledge to teach using an action research. In this chapter I will return to the research questions to see how far I have been able to answer them. I will outline implications for future research taking account of the limitations of this project.

6.1 Research questions

What does the research say about the best approach to teaching trigonometry?

The literature showed that there was a paucity of research in this specific area and the most commonly referred to research was conducted by Kendal and Stacey (1996) who found the ratio approach to be the most effective in the teaching of trigonometry

What is the confidence of prospective teachers, in their own knowledge, to teach trigonometry?

This research project found initially the prospective teachers felt confident in their own knowledge of trigonometry but not their knowledge *to teach it*. Later in the course this confidence in their knowledge to teach increased as a result of the teacher education programme. This was research in action as education lessons were developed to address questions of confidence which arose from the questionnaires.

Would mathematics sessions on trigonometry focused on aspects of the Knowledge Quartet (Rowland, Huckstep and Thwaites, 2005) develop prospective teachers' confidence in their own knowledge?

The research project found that mathematics sessions on trigonometry planned around the Knowledge Quartet (Rowland et al, 2005) had a positive impact on the confidence of the prospective teachers. This was experienced by themselves as well as in observations of their teaching by me.

6.2 Evaluation of this project

This project is limited because the numbers of prospective teachers involved was very small. Moreover, it took place in a particularly unique context where the subject specific component of a Schools Direct programme is delivered by working mathematics teachers as opposed to university lecturers. In hindsight the project would have benefitted from more depth in the literature review and a narrower focus on confidence. Due to the word limit and short nature of this MSc course the research project was limited to one year. However, far more could have been learned had the study taken place over a longer period.

6.3 Implications for the future

I now consider the implications of my research project for researchers. The analysis revealed that it is useful for mathematics educators to use the Knowledge Quartet (Rowland et al, 2005) when planning sessions. There are some further questions which are of interest to researchers:

- Can the Knowledge Quartet be used to plan sessions that are focused on other topics?

- To what extent does confidence change as prospective teachers enter the profession following a teacher education programme designed around the Knowledge Quartet?
- What is the most effective approach to introduce trigonometry to children? What is the long term impact of using technology in the teaching of trigonometry?
- To what extent do teachers use research to inform the choices they make with regards to introducing trigonometry?

6.4 Implications for my professional development

This project has had a significant impact on my own development. Through this study I have learned how pedagogical content knowledge can be looked at through the four dimensions of the Knowledge Quartet (Rowland et al, 2005). A major part of my role is running mathematics teacher education sessions for prospective teachers and experienced teachers. In the future I will plan these sessions with a clear idea of which dimension I am looking to develop - whether it be improving the knowledge of mathematics for newer teachers or helping more experienced teachers respond to contingent moments. Equally important to my own professional development is future collaboration. As the next academic year begins I have an increased responsibility for continuing professional development of teachers across the whole school, not just in mathematics. I think there is great value the Knowledge Quartet can play in the development of teaching across the school in our 'community of practice' (Lave and Wenger, 1991).

I believe this project has made a valid contribution to the research surrounding mathematics teacher education. In the current climate, where mathematics teacher educators are increasingly carrying out the role alongside their duties in school, the

Knowledge Quartet could be the framework for developing a national teacher education programme for mathematics.

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Appendix 1

Trigonometry Efficacy Questionnaire

A number of statements related to your own learning of trigonometry, your current knowledge of trigonometry, your knowledge of how to teach trigonometry and the trigonometry session are presented below. There are no correct/incorrect answers, I am interested in your opinions. This is completely anonymous.

Read each statement and circle the number which relates to the following key.

1 = strongly agree **2 = moderately agree** **3 = neutral**

4 = moderately disagree **5 = strongly disagree**

Statement		Scale				
1	I can remember my first experience of trigonometry from my school days.	1	2	3	4	5
2	My own experiences of trigonometry prior to this course were largely procedural.	1	2	3	4	5
3	Prior to the course I had a good conceptual understanding of right-angle trigonometry.	1	2	3	4	5
4	Before the trigonometry session I felt confident in my own knowledge of trigonometry and the skills involved.	1	2	3	4	5
5	After the trigonometry session I felt less confident about my own knowledge of trigonometry and the skills involved.	1	2	3	4	5
6	If I have to introduce trigonometry to a class for the first time I know exactly which approach I will take.	1	2	3	4	5
7	If I have to introduce trigonometry to a class for the first time I don't know which approaches are possible.	1	2	3	4	5
8	If I have to introduce trigonometry to a class for the first time I know which approaches are possible but am undecided which one to take.	1	2	3	4	5
9	When it comes to planning a sequence of lessons on trigonometry I have a clear idea about how to order the teaching.	1	2	3	4	5
10	I am not confident when it comes to planning an individual lesson on trigonometry.	1	2	3	4	5
11	I feel confident about knowing the misconceptions which exist when teaching trigonometry.	1	2	3	4	5
12	I feel confident about planning to address misconceptions which exist around trigonometry.	1	2	3	4	5

For this section I would like to hear your views in your own words.

1. **I think the best aspect of the trigonometry session was**

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2. **I think the aspect which could have most been improved was**

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3. **Which part of teaching trigonometry, if any, would you like more support with?**

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Appendix 2

Trigonometry Efficacy Questionnaire 2

Please complete the questionnaire below. There are no correct/incorrect answers, I am interested in your opinions. This is completely anonymous.
Read each statement and circle the number which relates to the following key.
1 = strongly agree **2 = moderately agree** **3 = neutral**

4 = moderately disagree **5 = strongly disagree**

Statement		Scale				
1	I feel confident about knowing the misconceptions which exist when teaching trigonometry.	**1**	**2**	**3**	**4**	**5**
2	I do <u>not</u> feel confident about planning to address misconceptions which exist around trigonometry.	**1**	**2**	**3**	**4**	**5**
3	This session was <u>not</u> helpful in terms of developing my own conceptual understanding of trigonometry.	**1**	**2**	**3**	**4**	**5**
4	I know how some teaching approaches can lead to misconceptions within trigonometry.	**1**	**2**	**3**	**4**	**5**

For this section I would like to hear your views in your own words.

1. **I think the best aspect of the trigonometry session (part 2) was**

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2. **I think the aspect which was least useful was**

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Appendix 3

Post-Lesson Interview Questions (semi-structured questionnaire)

Research questions: a) What does the research say about the best approach to teaching trigonometry? b) What is the confidence of prospective teachers to teach trigonometry? c) How do the mathematics sessions on trigonometry delivered on our teaching programme develop prospective teachers' confidence to teach? What is the role of the Mathematics Teacher Educator (MTE) in ensuring effective teaching of trigonometry?
How confident were you about teaching trigonometry at the start of the course?
What made you choose this particular approach to introducing trigonometry?
What was your confidence like at the end of the course? If it changes, what do you put that down to?
Was there any particular aspect of the course which influenced your choices about how to teach trigonometry?

Is there anything else about teaching trigonometry which you would like to mention?
Supplementary Questions

Appendix 4

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Director Professor Jo-Anne Baird

Dear XXXX

As a mathematics teacher and teacher educator at Southfields Academy I have recognised what a challenge it is to learn to teach and how confidence to teach fluctuates throughout the training year. I am therefore planning a research project to investigate how mathematics sessions (on trigonometry) impact on your confidence to teach trigonometry. I have already tentatively approached you to ask if you would be willing to participate in this project and you have kindly agreed. However, it is important that I receive your written permission to go ahead with this research.

Your involvement would entail: completion of questionnaires at various points, analysis of your subject knowledge audits, a lesson observation (trigonometry) followed by a post-lesson interview. All this data will be used to analyse your confidence to teach trigonometry and how the time spent in mathematics sessions is enacted in practice in the classroom. This will allow us to improve our understanding of how mathematics sessions can be planned to improve prospective teachers' confidence to teach trigonometry.

In order to carry out this project I am required to complete the full University ethics procedure and for this I have to ensure that I have secured informed consent from everyone involved. I will change the name of the schools and teachers when I write about the research ensuring confidentially and anonymity. The video files from the lesson observation and post lesson interviews will be completely anonymous and password protected.

If you are still happy to be involved in this exciting project with me please complete the consent form below. However, if you require further information and/or clarification then do not hesitate to contact me.

Yours truly,

Sinead Goodden
sinead.goodden@education.ox.ac.uk

I have read the above information and understand what my participation will involve and I am willing to participate in this research project.

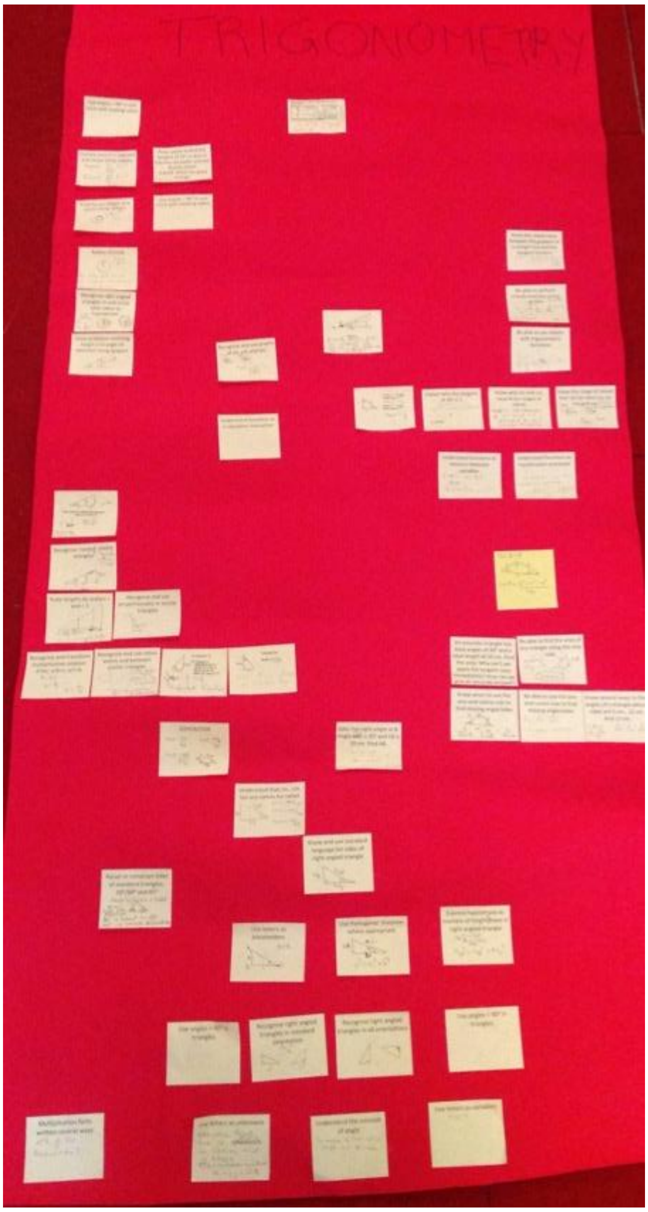
Participant's Name: ________________________

Signature: ________________________ **Date:**

Researcher's Name: ________________________

Signature: ________________________ **Date:**

Appendix 5



Appendix 6 Responses to Interview Questions

Question	**Tom**	**Lucy**	**Brian**	**Kate**
How confident were you about teaching trigonometry at the start of the course?	Moderately confident. “It’s just a case…you see it’s been such long time since I was at school, it’s just a case of remembering basic rules” “I would be confident after an hour or so of teaching, it’s something I think would develop quite quickly”	Not very confident “very unconfident”. This became obvious to Lucy when she came to teach it for the first time. “The fact that I was nervous and worried about how to reveal the concept showed me how unconfident I was”. What was your own confidence in your own ability at the time? Since teaching it she has felt more confident indicating that her own knowledge was not 100%. Do you think that your own understanding of trigonometry has changed now you have taught it? “Yes and no. Because when I first learnt it I can’t remember if I understood it. I feel like I was very good at applying it…but going over it again has meant that I have deepened that understanding.”	Felt he was ok. “More centred about what he knew about trigonometry but not how to teach it”. “I was more confident until I put myself in the students’ shoes and thought how would I approach it”. Started with high confidence until he realised there were some things he needed to brush up on himself. “How it can transition from what I know to how to teach it. For example, I had never thought about proportions with trigonometry.” Recognised how using something students had come across before [proportion] could help them with understanding trig. What role did the subject knowledge enhancement course play? Helped him better understand trig and put him in the shoes, experienced some activities. A lot more to trig rather than just solving problems.	Subject knowledge was sketchy at the start so was not confident.

What made you choose this particular approach to introducing trigonometry?	"The first section about what is an angle seemed to me to be fundamental to discuss and get the kids involved rather than assume everyone knows what an angle is." Moving away from Pythagoras to look at non-right-angled triangles. With trig you need to start naming the sides so it was important to get them thinking about that. "Trying to get them to intuitively think about what the sides might be called so that if they start to think about it on their own they will be able to remember it more easily."	"I wanted to make it as simple as possible. I wanted to make them to be able to deal with numbers..umm…with it being a half meant you could talk about it being a half rather than dealing with all the decimals which I knew a lot of them were not confident with anyway." Avoiding the long decimal numbers was important to Lucy to make it accessible to the students. Why sine and not cos 60° or tan 45°? Because it is the one that is introduced first. It was the one that Lucy remembered being introduced to first and in the textbooks that she looked at. She described an order in her head and wasn't sure where it had come from. Lucy said that learning trigonometry in the order sine, cosine and tangent helped the students later when she introduced the mnemonic SOHCAHTOA. The nested triangles leads itself to sine more easily as you can see the opposite getting bigger.	"If you were to ask me how to teach it at the start of the course I would have gone straight for sine, cos and tan, here are the formulas, this is how to put the numbers in. It was very procedural. But the way I chose linked to what they already knew." Talked about bridging the gap with previous topics.	"I wanted them to have a more investigative approach to what the trig ratios meant. I didn't want to give them a formula at the start which I had seen other teachers do. I wanted them to have some experience of getting there by themselves." She looked at what others had done and what we had talked about in sessions and felt the students would be able to deal with all three ratios in the lesson – contrary to others advice. "I tried to do too much in the lesson and I didn't totally leverage the investigative part of the lesson. I didn't have the time to do what I had planned. I was too demanding."

What was your confidence like at the end of the course? If it changes, what do you put that down to?	Pretty confident now. "Going over it a couple of times, lesson planning first of all and then having taught it a couple of times I am pretty confident"	Teaching trigonometry has improved her confidence. Importance of applying trig to real world problems gave her extra confidence – she recalled this from the trig sessions and others.	More aware of what to expect from students. Speaking to teachers and the sessions. "Here is a way to transition, for the kids, something that could sound like gibberish to them to something that they are more familiar with." "relay information"	"Totally confident to deliver trigonometry lessons up to GCSE" Not as confident up to A-Level. Lack of experience.
Was there any particular aspect of the course which influenced your choices about how to teach trigonometry?	"From the sessions I did a section on similar triangles" Introduction of tan from the session	Lucy mentioned measuring buildings and gradients as being memorable aspects of the sessions. Lucy talked about how important discovery was as influencing her choices. "My choice to tell them to divide was definitely influenced by the course because although you emphasises discovery and getting the kids to work it out for themselves you emphasised that they are not reinventing the wheel they do need to be told how to do things and if this is going to help them channel the hour of the lesson that is important."	Personally prefers to learn visually. He would choose something he feels confident with. "It was nice to be given many different options" Gave him confidence at the front of the class. Talked about how these ideas might help him in future and in order to help him be flexible.	"My starting point was so weak in terms of needing to refresh my knowledge. To be given so many ways to think about what I had learnt about trig historically. I felt that what I had learnt about trig was quite dry and almost remembering rules. This was the first time in my education I had sat down and thought about what trig actually is." This gave her more confidence to plan the lessons. Felt equipped herself rather than explicitly thinking about the sessions. Found the engagement in literature really helpful.

Is there anything else about teaching trigonometry which you would like to mention?	Tom did not have anything to say for this question.	Lucy talked about using the trig tables. It was helpful for students who did not have calculators and was a motivator to bring in calculators. She felt that the tables helped them understand the numbers and the nature of the relationship between the sides. Talked about her concern that the students' algebra is not strong. She found that students did not use calculator keys erroneously. Three students who joined the class late and missed out on using the tables made more errors than others.	Appreciates all the experiences he has been given. Would like to explore how other ways could be used to introduce trig. Brain did read the trig articles. He did some further reading about how to program a calculator to work out sine and cosine.	Not confident on A-Level trig. She would like to see how other teachers would tackle it. "If I could do it again I would make more of the similar triangles." Talked about the starter questions about finding a missing side, students used Pythagoras but it could have been worked out using similar triangles. She felt in hindsight she should have addressed this.
	How did you use a textbook or SOW to plan? Used his brain and collaborated with his mentor. At this point Tom wanted to share how the lesson came about… At the start of the placement Tom's year 9 class was starting trig but the class teacher was so passionate about the way she taught trig that Tom was not going to be able to teach it. He felt put out. The way she		How did your lesson go? Students were enthusiastic. "Something I need to improve on is finding a way to bridge the gap the task and what I want them to find out." He mentioned about the pace being affected by him talking too much. How will you teach it next time? "I will do lessons on similarity and congruence prior to	The written feedback stated that it appeared you were under pressure to cover content. Where did that pressure come from? Self-imposed. "I was told that by the end of the lesson they should be able to calculate the length of the side." She recognised that there was a lot to cover to be able to get to that point. Made the point

	taught it made him think 'he wouldn't teach it like this' and discussed it with his mentor. The mentor was in agreement about the approach to teaching trig in order to get the majority of boys to pass the exams. All three functions are introduced at the same time using SOHCAHTOA so that the get used to deciding which ratio to use, everything was mixed in. Tom felt they didn't get an understanding of what is going on with the angle, they don't get the idea that they are functions and just see them as buttons on the calculator. The idea of ratios seems to be left behind and Tom felt it was a shame. "It seemed obvious to me to start with tan because it comes up in real life, it's a bit like in our session when we went outside to measure the height of the building. The first two sides you come across really are the opposite and adjacent. I am sure that is what I did when I was at school. To introduce them all at once doesn't give them the depth of knowledge to go on with the		introducing trigonometry so they hopefully see the relationship quicker." Mentioned the clinometer task!	that the ultimate decision was hers alone. Lucy felt that 2/3 of the class could calculate the length of a side by the end of the lesson. "What surprised me was how much difficulty they had labelling the sides, it seemed to be intuitively a difficult concept for them. I assumed this would be easy for them." Students kept asking what the formula was. "If I were to do that lesson again there are two things I would do differently: 1. More time on the investigation and made more of it. 2. Spend longer on labelling sides." She would still introduce sine, cosine and tangent in one lesson. Kate talked about how she found the lesson stressful because it was being observed.

	study of maths. It might help them pass their GCSE but it doesn't give you a depth of understanding, it's quite shallow." "I am wrong though, to get most people through the exam that seems to be the way it is taught. It just feels that is a good way bulk way to get everyone to be able to answer the questions. It's more efficient to teach all three at the same time. This approach probably gets the better results." Tom talked about the fact he might use a more investigative approach with a top group rather than a middle group. He talked about the use of a calculator versus the table – he felt you got a better sense of the values from a table. Was there anything in the lesson that took you by surprise? He thought that the initial discussion about the angle would be quicker. He expected that the boys would have been able to see that the triangles			

	could be nested in each other quicker.			

Appendix 7

Brian and Kate's Lesson Plans

Topic/Unit:	Trigonometry	Lesson	Introduction to trigonometry

Learning Objectives (using learning words)

All will be able to:

Name and label the sides of a right angled triangle with relation to the angle indicated

Identify the relationship between the different sides of a right angled triangle with the same angle

Some will be able to:

Use the knowledge of the relationship between sides to identify the unknown length on a right angle triangle with an angle of 30°

Prior Knowledge

How to identify what a right angle triangle is; How to identify which length is the hypotenuse; How to use the Pythagoras equation "$a^2 + b^2 = c^2$", Rounding to decimal places and significant values, how to measure distances with a ruler

Key Words

Hypotenuse, adjacent, opposite, right angle, rearrange, make subject, decimal places, trigonometry, relationship

Misconceptions and Common Errors

Students may assume or think:

- That the 2 short sides can be added together and then square rooted to give the hypotenuse ($2^2 + 3^2 = c^2 = 5^2 = c^2 = 2.24$)
- When finding 1 of the shorter sides, they don't rearrange properly ($a^2 \neq c^2 + b^2$)
- The relationship between the opposite and hypotenuse applies to the adjacent and hypotenuse
- Students might think they're measuring the hypotenuse but are measuring the distance between the opposite lengths on the worksheet rather than the hypotenuse

Assessment for Learning Tools

- Using questions to find out who has grasped the concepts of the lesson and altering the task to better guide the students to discovering what the relationship between different sides are

Key Questions

- What can you notice about the relationship between the hypotenuse and the opposite distances?
- What if you drew another line on this sheet and measured the distances? Is the relationship still the same?
- Is there a relationship between the opposite and adjacent? Or the adjacent and hypotenuse?
- What would happen to the relationship if I used a different angle?
- Do you think the relationship applies to all triangles?
- If the relationship is always the same, what would the hypotenuse equal to if the opposite was 12cm?

Differentiation

Support/Less Able	**Progression/More Able**
During the activity, if students are struggling to see the relationship, a sheet that allows them to compare the relationship between the sides is provided. This sheet allows them to find the relationship between the different sides.	Students, if they find a relationship between the lengths, are asked to see if the relationship is always true. They are then tasked to draw their own lines on the sheet and see if the relationship is still there.

Targeted use of other adults	• Within the class, there are 2 SLCN students. The LSA will assist with checking those students understand the instructions given for the task. • The LSA will assist by asking key questions in case students are abler or need more support • The LSA will be watchful of potential behaviour that limits students
SEND	• [redacted] have the lowest reading ages so they are checked first to see if they understand the task • [redacted] has a visual impairment so the activity and slides are a good size so that she can see. She is sat at the front to better see the questions on the board • [redacted] have been showing poor effort so they are checked by other adults in the class to ensure they are on task

Timings (mins)	Activity	Instruction	Learning gained and AfL
7 mins	Starter: Find the missing lengths on right angle triangles	Students refresh their knowledge by finding the missing lengths on the triangles on the board.	As students are attempting the questions, the adults in the class check to see which students are on the right track and help individuals where need be. A hint is put on the board if there are a few students that are still unsure how to find the missing lengths and a sheet is provided to practice the skill
7 mins	Identifying name of lengths on a right angle triangle	Students are shown how to identify the adjacent and opposite lengths and they note it down in their books	Students learn how to identify the lengths of a right angle triangle where an angle is indicated. MWB to check if students can find the different names on a right angle triangle. Students that are still unsure how to name the lengths are spoken to individually by myself or the other adults and guided to finding the name of distances.
25 mins	Measuring the distances of the right angle triangles	Students attempt the main task and see if they can identify a relationship between the hypotenuse and the opposite distance of multiple triangles with the same angles. They record their results in a table.	Students identify the relationship between the hypotenuse and the opposite lengths of the triangle and see the common pattern. Adults in the room ask the students questions to get them thinking about the relationship between the distances. If students are still struggling, a sheet is handed out to compare the distances to make it more apparent what is happening
10 mins	Plenary: The question at the start	Students attempt to answer the question at the start that had only 1 distance and 1 angle indicated	Students, using what they have discovered, can find the missing distance on the shape. Students explain to each other and to the class what they think the answer will be. If there are students that are exceptional at finding the missing length, a mini intro to trig tables is implemented and extension questions are placed on the board.

Lesson Plan & Evaluation Form

Class:	9X1	Date	9/5/ 16	Period	1	Time	8:55
Topic/Unit:	Trigonometry	Lesson		Finding unknown side of right angled triangle			

Learning Objectives (using learning words)

Objectives:
To be able to find the missing side of a right angled triangle when we know one side and one angle.

Prior Knowledge

Can recognise and draw right angle triangle
Angles of a triangle sum to 180 degrees
Can calculate the side of a right angle using Pythagoras theorem. Can rearrange formula to find missing side
Can re-arrange formula involving fractions. Can calculate to 3sf. Spot similar triangles

Key Words

Opposite, hypotenuse, adjacent, included angle.
Sin, Cos, Tan
Significant figures
Similar triangles

Misconceptions and Common Errors

Confusion on what to substitute into pythag formula
If given longest side, failing to rearrange the formula for missing side
Not correctly label triangle: O, H, A
Confusion on which sides you need to use and which trig formula to apply.
Incorrect use of calculator

Assessment for Learning Tools and Key Questions

Starter – Prior knowledge of Pythagoras theorem. Students use $a^2 + b^2 = c^2$ (c is longest side). Able students should recognise 2nd Q requires rearranging of formula
Exam Q to illustrate what LO is – revisit as plenary to show students met LO
Model Qs on labelling- give 3 triangles and ask class to label - testing students understanding of basic concept of labelling triangles
AfL using in class assessment by walking around room during independent work, random student choice based on independent task assessment to ensure all students engaged in class discussion. At each stage this will determine when to introduce next level/pace of lesson
Students to refer to LO at end of lesson as to what they have learnt and identify any areas of weakness

Differentiation

Support/Less Able	*Progression/More Able*
Class discussion to include all to ensure basic concept has been grasped Model all independent tasks on board which support the less able students collectively. Utilise AS to support class on independent tasks	Discovery of key trig relationships-Students encouraged to share understanding in class context. Independent task encourage student to work out which sides are h/o/a and work out which trig function needed. Ask to 3sf to test prior knowledge Extension Qs in Exercise – can students find x if it's the denominator
Targeted use of other adults	No SEN requirements Use AS to be my 'positive' spy. Whos on task, whos not and also to help me model practical task

Homework/Marking Due:	Students to finish any unanswered Qs - eg plenary/ main task

Timings	Task/Teaching Point	Instruction – What to do, where	Effective Learning for All Students (Differentiation)

8.55 09:05	Welcome and settle class. Starter on board. Take register Discuss starter task + show class **what they should be able to do by end of lesson**	Write title, LO and have a go at starter. Check answers and mark	Starter allows for prior knowledge assessment- PYTHAG and how to rearrange formulae when given longest side. I can further assess when students re-group to discuss Creates a calm and learning conducive atmosphere.
09:10 09:15	Labelling sides-Ask class how to label sides then give definitions. Discover key trig ratios. Sin, Cos Tan and 18degree	Hand out -Class to label triangles and stick in books Paired work-Hand out Groups- different size angle triangles. Measure sides and record results.	3 types of triangles test understanding of o/h/a ME to check students work- are class discovering that the relationship is the same for SIMILAR triangles?
09:20	Stop class -ME to ask each group if their ratio, 0.5 1 etc Mention trig tables	Discuss conclusions- relationship of sides determined by angle	Introduces key concept of trig, Group discussions tests understanding and misconceptions
09:25	Give ratios	Students to copy in books	Behaviour management ensuring all students on task
09:30	Using your Calculator	Check you can use your calculator- clear instructions	More able spend less time and weaker students can help when doing Ex 1
09:35	ME TO MODEL 3 EXAMPLES 1 on own, 2x students	Pens down and focus	Involve class to ensure students learn from others
09:40 09:45	Exercise 1 Stop class and give answers	Rough diagrams in books Mark work- green pen	ME/AS to assist where required. e and f test application-extension
09:50	Plenary- original LO	Do Q on whiteboard using students	Have they met their LO ?

Evaluation	
www: Made really good progress in lesson; good pace and students engaged. LO achieved	***ebi***: Should have spent more time on discovery task.

Were the learning objectives achieved?
Yes – all students able to calculate missing side using trig

What evidence was there of students' progress? Plenary demonstrated understanding

Targets	Standard/s	Were they met?
AfL TECHNIQUES		Good class circulation, marking. Perhaps could have asked class if happy to move to next stage
Encourage independent thinking and differentiation		Yes, but always need more extension work for a few