Essays on Asset Pricing and Rational Expectations

Daniel Pesch

Exeter College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Hilary 2024

Abstract

This thesis explores the impact of belief heterogeneity on subjective risk premia within international financial markets, leveraging expert survey forecasts to shed light on price formation mechanisms, strategic market games, and expected excess returns. Taking expectations as observable circumvents the issues of projecting expectations on state variables and results in model-free estimates of agents' subjective perceptions of reality. Leveraging data across individuals, asset classes, and countries allows for the combination of classical asset pricing questions with puzzles from international and behavioural finance.

The first chapter sets the scene by assessing belief heterogeneity in the context of classical strategic market games, demonstrating the superiority of the Shapley Window model over Cournot-type models in stabilising prices and volatilities. Through simulations, this chapter illustrates the moderating effect of market liquidity on the impact of heterogeneous beliefs on prices and volatilities, suggesting novel insights into the stability of financial markets.

By exploiting international survey datasets on interest rates and exchange rates, the second chapter delves into the concept of subjective risk premia, revealing their cyclical nature and predictive power over future realised returns. These findings are underpinned by a structural estimation within a multi-country asset pricing model, highlighting the significance of financial market belief distortions in understanding the classical risk-return trade-off.

In the final chapter, the focus shifts to the expected excess return on the long carry trade, uncovering its non-zero, time-varying nature and its predictive capabilities over future realisations, which starkly contrasts with traditional models that utilize statistical projections. By comparing and contrasting survey-based observations with those derived from such projections, this chapter investigates the relationship between exchange rate risk premia and carry trade premia, calling for an extension of traditional economic models to incorporate these empirical findings.

Collectively, this thesis bridges the gap between subjective risk assessments and market dynamics from both, theoretical and empirical points of view.

Essays on Asset Pricing and Rational Expectations



Daniel Pesch

Exeter College

University of Oxford

A thesis submitted for the degree of

Doctor of Philosophy

Hilary 2024

Acknowledgements

First and foremost I want to thank both of my supervisors, Dimitrios Tsomocos and Ilaria Piatti. Dimitri has been a great mentor who has taught me not only about economics and finance but also about how to think like a researcher. The fact that he is well-read in so many different fields has been a great inspiration to me. The community that Dimitri formed in W1.37 over the years is one-of-a-kind and I am grateful for the regular reading groups, research discussions and dinners which made Oxford even more enjoyable.

Ilaria is one of the most intelligent, rigorous and quantitatively skilled individuals I have ever met. At the same time, she is also one of the kindest people I know. Thank you for continuing to be an excellent supervisor even after leaving Oxford.

My gratitude also goes to all the incredible academics that helped me by assessing my work and providing feedback along the way: Oren Sussman, Alan Morrison, Pedro Bordalo, Jiri Knesl and Han Özsöylev. I also want to thank my co-author Paul Whelan who always pushed me to improve as a writer and researcher, much to the benefit of the second chapter of this thesis.

Special thanks to Christian Andres for encouraging me to consider pursue doctoral studies and for convincing me to undertake a Master's beforehand. I am also indebted to Steve Roberts for introducing me to quantitative finance and machine learning; these skills have proven extremely beneficial.

Over the course of the DPhil, I have taught courses in microeconomics, theoretical asset pricing, advanced asset pricing, fixed income and derivatives, financial crises and risk management, and machine learning at the Department of Economics, Saïd Business School, ESCP Business School and Magdalen College. It has been a great privilege to be entrusted not only with teaching assistance but also with developing my own lectures and courses for students from many different backgrounds and I want to thank my supervisors as well as Thomas Norman, Matthias Holweg, Agni Orfanoudaki and Markus Bick for making this possible. And while the book that I wrote together with Dimitri during the COVID19 pandemic is not at all part of this thesis, I still want to thank Ben and Jean-Guillaume for their support with this project.

I am also grateful to my colleagues, friends and flatmates in Oxford for the many days and nights we shared. To Javier for being a role model of a DPhil

student who motivated me to submit my thesis and for being one of the (three) best flatmates one can wish for; also thanks to you and Diana for your light-heartedness and encouragement. To David for proposing and introducing our elective courses at ESCP Business School and for always being a reliable, pragmatic and kind flatmate and friend. To Xuan for introducing me to the opportunity to teach undergraduates at Magdalen; something that made me value the Oxford system even more. To Ken for always trying to make Oxford, and particularly the business school, a better place and for always having an open door. To all of my other intergenerational peers in Dimitri's reading group: Nikos, Fanis, Tatjana, Aditya, Yixiao, Chris, Jasper and everyone else. And especially to Luisa for enduring the endless nights of working towards conference deadlines and for always supporting me.

I want to thank Exeter College, the Oxford-Man Institute of Quantitative Finance and the Saïd Business School Foundation for their generous support of my work.

Most importantly, I want to thank my entire family and for their support and especially my father for encouraging me to stay at Oxford and to take the challenge.

The past years have been humbling, eye-opening, challenging and rewarding and I would not want to miss them.

Abstract

This thesis explores the impact of belief heterogeneity on subjective risk premia within international financial markets, leveraging expert survey forecasts to shed light on price formation mechanisms, strategic market games, and expected excess returns. Taking expectations as observable circumvents the issues of projecting expectations on state variables and results in model-free estimates of agents' subjective perceptions of reality. Leveraging data across individuals, asset classes, and countries allows for the combination of classical asset pricing questions with puzzles from international and behavioural finance.

The first chapter sets the scene by assessing belief heterogeneity in the context of classical strategic market games, demonstrating the superiority of the Shapley Window model over Cournot-type models in stabilising prices and volatilities. Through simulations, this chapter illustrates the moderating effect of market liquidity on the impact of heterogeneous beliefs on prices and volatilities, suggesting novel insights into the stability of financial markets.

By exploiting international survey datasets on interest rates and exchange rates, the second chapter delves into the concept of subjective risk premia, revealing their cyclical nature and predictive power over future realised returns. These findings are underpinned by a structural estimation within a multi-country asset pricing model, highlighting the significance of financial market belief distortions in understanding the classical risk-return trade-off.

In the final chapter, the focus shifts to the expected excess return on the long carry trade, uncovering its non-zero, time-varying nature and its predictive capabilities over future realisations, which starkly contrasts with traditional models that utilize statistical projections. By comparing and contrasting survey-based observations with those derived from such projections, this chapter investigates the relationship between exchange rate risk premia and carry trade premia, calling for an extension of traditional economic models to incorporate these empirical findings.

Collectively, this thesis bridges the gap between subjective risk assessments and market dynamics from both, theoretical and empirical points of view.

Contents

List of Figures xi

List of Tables xiii

Introduction 1

1 The Impact of Beliefs on Price Formation 9
- 1.1 Introduction . . . 10
- 1.2 Five Price Formation Mechanisms . . . 14
  - 1.2.1 Sell-All model . . . 16
  - 1.2.2 Bid-Offer mechanism . . . 19
  - 1.2.3 Complete markets mechanism . . . 20
  - 1.2.4 Shapley's "Window model" mechanism . . . 22
  - 1.2.5 Double Auction model . . . 24
- 1.3 Properties of Price Formation Mechanisms . . . 27
- 1.4 Simulations . . . 31
  - 1.4.1 More Players . . . 33
  - 1.4.2 More Commodities . . . 34
  - 1.4.3 Wealth Distributions . . . 36
  - 1.4.4 Matching Stylised Facts . . . 40
- 1.5 Conclusion . . . 41

2 Subjective Risk Premia in Bond and FX Markets 43
- 2.1 Introduction . . . 44
- 2.2 Data . . . 48
- 2.3 Framework and Notation . . . 50
  - 2.3.1 Risk premia in the fixed income market . . . 50
  - 2.3.2 Risk premia in the foreign exchange market . . . 51
- 2.4 Subjective Risk Premia . . . 52
  - 2.4.1 Subjective Bond Risk Premia . . . 52
  - 2.4.2 Subjective Exchange Rate Risk Premia . . . 55
  - 2.4.3 Comparison with statistical risk premia . . . 56

2.5 Predictability Regressions . . . 57
2.5.1 Bond Risk Premia . . . 58
2.5.2 Exchange Rate Risk Premia . . . 59
2.6 Expectation Errors . . . 61
2.6.1 Forecast Error Predictability . . . 62
2.7 An Equilibrium Model with Subjective Beliefs . . . 63
2.7.1 Stylised Facts . . . 64
2.7.2 Fundamentals and Preferences . . . 65
2.7.3 Bond Pricing . . . 65
2.7.4 Subjective Measure and Belief Distortion . . . 67
2.7.5 Subjective Bond Risk Premia and Yield Forecast Errors . . . 69
2.7.6 Foreign Countries . . . 70
2.7.7 Estimation Results . . . 72
2.7.8 Introducing Stochastic Volatility . . . 75
2.8 Conclusion . . . 79
2.9 Figures . . . 81
2.10 Tables . . . 87
A.1 Data Appendix . . . 94
A.1.1 Interest Rate Expectations . . . 95
A.1.2 Macro Economic Expectations . . . 96
A.2 Supplementary material to Section 3.4 . . . 96
A.2.1 Are bonds risky bets or hedges? . . . 97
A.2.2 FX Factors . . . 97
A.3 Supplementary material to Section 2.6 . . . 98
A.3.1 How Rational are Survey Forecasts? . . . 98
A.3.2 Over-reaction (under-reaction) to news . . . 99
A.3.3 Forecast Error Predictability . . . 100
A.3.4 Economic Significance of Deviation from Rational Expectations 100
A.3.5 Note on the Driscoll and Kraay (1998) estimator . . . 102
A.4 Asset Pricing Derivations . . . 103
A.4.1 Economic Dynamics . . . 103
A.4.2 Preferences . . . 103
A.4.3 Bond Pricing . . . 105
A.4.4 The Subjective Measure . . . 109
A.4.5 CRRA Solution . . . 110
A.4.6 Recursive HJB equation . . . 111
A.4.7 Exact Recursive Solution $\psi \to 1$ . . . 111
A.4.8 Log-linear solution . . . 114
A.4.9 Risk Free Rate . . . 118

A.5 Simulated Method of Moments 120
A.6 Tables 123
A.7 Figures 145

3 Subjective Risk Premia and the Long Carry Trade 159
3.1 Introduction 160
3.2 Data 163
3.3 Framework and Notation 164
3.3.1 Risk premia in the fixed income market 165
3.3.2 Risk premia in the foreign exchange market 165
3.3.3 Risk premia on dollar-denominated foreign bonds 166
3.4 Subjective Risk Premia 168
3.4.1 Foreign Bond Dollar Risk Premia 168
3.4.2 Comparison with Statistical Models 172
3.5 Forecast Errors and Predictability 174
3.5.1 Predicting Foreign Bond Dollar Risk Premia 174
3.5.2 Errors in Forecasting Excess Returns 176
3.5.3 Trading Strategies 178
3.6 Risk Premia Implied by an Equilibrium Model 180
3.6.1 Model 182
3.6.2 Model-Implied Foreign Bond Dollar Risk Premium 184
3.7 Conclusion 186
3.8 Tables 187
3.9 Figures 194
A.8 Figures 211

Conclusion 219

Bibliography 221

List of Figures

1.1 A two-period economy with state-uncertainty 15
1.2 Interior Bid Offer Constellations 25
1.3 Corner Solution Bid Offer Constellations 26
1.4 Visualisation of the two-period economy with state-uncertainty . . . 27
1.5 Impact of number of players on volatility 33
1.6 Impact of number of commodities on volatility 34
1.7 Wealth Distributions (no aggregate uncertainty) 36
1.8 Impact of Wealth Distributions on Prices 37
1.9 Impact of Wealth Distributions on Aggregate Wealth 38
1.10 Impact of Disagreement jointly on Welfare and Price Volatility . . . 39

2.11 Subjective Bond Risk Premia 82
2.12 Subjective Exchange Rate Risk Premia 83
2.13 Survey vs Projected Risk Premia 84
2.14 Forecast Errors 85
2.15 Model Implied Exchange Rate Risk Premia 86
A.16 Number of long-term bond yield forecasters by country 145
A.17 Comparison of H15 and BB yields 146
A.18 Comparison of compounding frequencies 147
A.19 Constant Maturity Macro Expectations 147
A.20 Stock-Bond Correlation 148
A.21 Subjective Macro Expectations 149
A.22 Volatility of realized bond returns and realized and risk-neutral spot exchange rates 150
A.23 PCAs of Subjective Exchange Rate Risk Premia 151
A.24 Interest Rate Spreads 152
A.25 Projected Bond Risk Premia 153
A.26 Projected Exchange Rate Risk Premia 154
A.27 Correcting Errors: Δ RMSE in the Fixed Income market 155
A.28 Correcting Errors: Δ RMSE in the Foreign Exchange market 156
A.29 Forecast errors of inflation expectations 157

3.30 Motivation . . . 194
3.31 Projected Foreign Bond Dollar Bond Risk Premia . . . 195
3.32 Realised Foreign Bond Dollar Bond Risk Premia . . . 196
3.33 Subjective Foreign Bond Dollar Bond Risk Premia . . . 197
3.34 Subjective Long Term Dollar Bond Risk Premia Decomposed . . . 198
3.35 Comparison of subjective, projected and realised mean expected excess returns . . . 199
3.36 $\Delta BRP^{\$}$ Decomposition: Average (AVG) . . . 200
3.37 $\Delta BRP^{\$}$ Decomposition: Australian Dollar (AUD) . . . 201
3.38 $\Delta BRP^{\$}$ Decomposition: Swiss Franc (CHF) . . . 202
3.39 PCAs of Subjective Foreign Bond Dollar Risk Premia . . . 203
3.40 Time Series of Forecast Errors of subjective risk premia . . . 204
3.41 Time Series of Forecast Errors of projected risk premia . . . 205
3.42 Correcting Errors: Δ RMSE in the long carry trade . . . 206
3.43 Cross-Sectional Trading Strategies using subjective expectations . . . 207
3.44 Time Series Trading Strategies using subjective expectations . . . 208
3.45 Model-implied correlations for CHF and AUD . . . 209
3.46 Realised correlations for AUD and CHF . . . 210
A.47 $\Delta BRP^{\$}$ Decomposition: Canadian Dollar (CAD) . . . 212
A.48 $\Delta BRP^{\$}$ Decomposition: Euro (EUR) . . . 213
A.49 $\Delta BRP^{\$}$ Decomposition: British Pound (GBP) . . . 214
A.50 $\Delta BRP^{\$}$ Decomposition: Japanese Yen (JPY) . . . 215
A.51 $\Delta BRP^{\$}$ Decomposition: Norwegian Krona (NOK) . . . 216
A.52 $\Delta BRP^{\$}$ Decomposition: New Zealand Dollar (NZD) . . . 217
A.53 $\Delta BRP^{\$}$ Decomposition: Swedish Krona (SEK) . . . 218

List of Tables

2.1 Descriptive Statistics for Subjective Risk Premia 87
2.2 Cyclicality Regressions: Subjective Bond & FX Risk Premia 88
2.3 Bond & FX Return Panel Predictability Regressions 89
2.4 Summary Statistics: Expectation Errors 90
2.5 Forecast Error Predictability 91
2.6 Term Structure Estimation 92
2.7 Belief Distortion 93
A.8 All countries included in the Consensus Economics datasets 123
A.9 Summary Statistics: Par Yields and Zero Yields 124
A.10 Sub Sample Summary Statistics BRP 124
A.11 Descriptive Statistics for Projected Risk Premia 125
A.12 Cross Country Correlations of Survey Premia 126
A.13 Cross Country Correlations of Projected Premia 127
A.14 Cross Country Correlations of realized Premia 128
A.15 PCA Analysis: Variance Explained (%) 128
A.16 Descriptive Statistics Predictive Regressions 129
A.17 Forecast Error Predictability 130
A.18 Cyclicality Regressions: Subjective Bond & FX Risk Premia 131
A.19 Bond & FX Return Panel Predictability Regressions 132
A.20 Information Frictions: Coibion and Gorodnichenko (2015) 133
A.21 Forecast Error Predictability: Individual Country Regressions . . . 134
A.22 Cyclicality: Subjective FX Risk Premia (Macro Spreads) 135
A.23 Cyclicality Regressions: Subjective Bond Risk Premia 136
A.24 Cyclicality Regressions: Subjective Currency Risk Premia 137
A.25 Individual Predictability Regressions 138
A.26 Summary Statistics: Term Structure Moments 139
A.27 Summary Statistics: Inflation Moments 140
A.28 Correlation of realized inflation 140
A.29 Summary Statistics: Growth Moments 141
A.30 Correlation of realized consumption growth 141
A.31 US Consumption Growth Estimates 142

A.32 Swiss Consumption Growth Estimates 143
A.33 Australian Consumption Growth Estimates 144

3.34 Descriptive Statistics for Subjective Risk Premia 187
3.35 Descriptive Statistics for Projected Risk Premia 188
3.36 Summary Statistics Forecast Errors 189
3.37 Summary Statistics Projected Forecast Errors 190
3.38 PCA Analysis: LRP Variance Explained (%) 191
3.39 Predictability Regressions 191
3.40 Cross Country Correlations of Subjective Foreign Bond Dollar Risk Premia 192
3.41 Cross Country Correlations of Projected Foreign Bond Dollar Risk Premia 192
3.42 Forecast Error Predictability 193
3.43 Cyclicality Regressions: Subjective LRPs 193

Introduction

Modern financial markets resemble pure complexity, with countless market participants from varied backgrounds engaging in the buying and selling of assets. Some of these agents are large institutional investors, such as pension funds or insurance companies, while others are small retail investors looking to invest their savings. Each of these agents holds their own set of beliefs, preferences, and strategies, contributing to the market's complexity.

Despite all of this complexity, the price of an asset stands out as a fundamental element that holds universal significance across all groups. It acts as common ground, reflecting the collective interplay of supply and demand, informed by the diversity of information available to the market's participants.

Understanding how asset prices are determined in such a complex environment is crucial, as it sheds light on the inner workings of financial markets and the factors that drive the decision-making process of its various actors. This thesis contributes to this understanding by exploring the formation of asset prices in financial markets and the role of subjective beliefs in shaping these prices.

Traditional economics research, e.g., Hicks (1939) and Lucas (1972), attempts to build simple, static Walrasian models that are able to match empirical observations of asset prices. Such models often try to rationalise production, unemployment, or consumption but rarely characterise the precise mechanisms of exchange. In doing so, these models often fail to capture the complexity of real-world financial markets, where agents are not price-takers but (partial) price-makers, actively influencing the prices of assets through their trading strategies.

One way to account for this strategic formation of asset prices in financial markets is to model them as strategic market games (SMGs). In such games, the

formal rules of exchange are defined explicitly and are formulated in a playable fashion. In theory, an SMG can be physically played out by a group of individuals in a lab with predictable outcomes. While these predictions, as anything in economics, heavily depend on assumptions around rationality, information, and preferences, SMGs provide deep insights on how asset prices originate from first principles.

Many of the seminal papers in the discipline such as Shubik (1973), Shapley and Shubik (1977), Dubey and Shubik (1977a), Dubey and Shubik (1977b), or Radner (1968) were written in a time characterised by severely limited computing power. While the theory of SMGs is inherently interesting, the potential for agent-based simulations of entire markets only developed in the last few years. Papers like Zheng et al. (2022) show that SMGs are a useful vehicle for such simulations. Harnessing SMGs, which by definition are predictable and interpretable, and using them in combination with machine learning and big data is a promising path for economics in the digital age.

A particularly challenging avenue is to study how market dynamics change as financial frictions are introduced. Such frictions can be related to the accessibility of information, the introduction of credit and default, or a heterogeneous perception of reality altogether. All three chapters of this thesis incorporate subjective beliefs, which we understand as personalised opinions. Such differing perceptions can be linked to behavioural biases that induce optimism or pessimism, or be the result of a completely rational informational advantage. Traditionally, beliefs have been neglected by economists because of their unstable nature and the difficulty of quantifying them. A seminal paper in this literature is Muth (1961) who articulates the rational expectations hypothesis which suggests that markets, in aggregate, do not waste information. This theory suggests that expectations about future events are essentially the same as theoretical predictions. His testing of this hypothesis in a simplified, one-commodity economy, however, has been challenged extensively. Fostel and Geanakoplos (2008) show that when market sentiment is dominated by pessimists, a flight to quality can lead to a reduced availability of credit and economic contraction. Simsek (2013) studies how spirals of optimism and pessimism

can explain the subprime mortgage crisis. Papers like Bordalo et al. (2023), Bordalo et al. (2020) and Buraschi et al. (2022) show that beliefs can help explain market outcomes that are otherwise hard to rationalise, e.g., the price volatility puzzle of Shiller (1981), which posits that observed price volatility cannot be explained by volatility of dividends alone. In this particular case, the authors show that this puzzle can be explained by considering the excess volatility in agents' subjective beliefs. Chapters Two and Three will use the same methodological vehicle as many of the aforementioned papers and use survey data to alleviate this measurement issue.

Chapter One of this thesis addresses the impact of heterogeneous beliefs on Strategic Market Games (SMGs) and discusses what we learn about the stability of markets. It provides an overview of a wide range of price formation mechanisms that have been proposed to explain the formation of asset prices in financial markets. Moreover, it is the first paper to extend the framework of Dubey and Shubik (1977b) to a range of SMGs, forming a standardised theoretical framework to compare a variety of price formation mechanisms. Chapter One is aimed at two distinct audiences. Firstly, it caters to theoretical economists who are interested in exploring the properties and dynamics of price formation mechanisms. Secondly, it is designed for applied economists and machine learning researchers who are keen on conducting agent-based market simulations. The chapter provides a concise overview of the most suitable game-theoretic formulations and assesses their respective strengths and weaknesses.

The main theoretical finding of Chapter One is the demonstration that price formation mechanisms that show more interconnectedness of markets, e.g., Shapley's window model (Sahi and Yao, 1989), are more resilient to heterogeneous beliefs than those that are not. We document that markets that force agents to sell all of their endowments in the style of Shubik (1973) are the least interconnected, therefore showing higher volatility in the presence of heterogeneous beliefs. This differential in sensitivity to heterogeneous beliefs exists because more tightly coupled markets are able to better distribute "friction-induced" shocks to relative prices across multiple commodities, softening the blow to any one price. This logic translates well to

liquidity introduced by adding more players, as they help reduce the marginal impact of agents holding a contrarian view of the market. Another key aspect of the simulations that are presented in Chapter One is varying the initial endowments of the participating agents. The benchmark case is a uniform wealth distribution to highlight the features of the studied mechanisms without adding another layer of complexity. Moving to a less equitable distribution of wealth turns out to further exacerbate the impact of heterogeneous beliefs on relative prices and volatility.

Since the paper uses SMGs and simulations, some of its theoretical predictions can be tested in lab experiments. While this is beyond the scope of the chapter, Huber et al. (2010) run such experiments and support some main findings. Furthermore, the chapter links theoretical findings to empirical evidence in the form of stylised facts in commodity markets. While the first chapter culminates in this consideration of empirical facts, it is first and foremost a theoretical piece. However, at least as interesting as confirming new theory by empirics is challenging old theory on the basis of empirics. This is why the two latter chapters focus on two puzzles in international finance, their empirical manifestations, and implications for model building. At the heart of both puzzles is the use of subjective beliefs to gain insights into how market participants form expectations about the future.

Chapter Two presents an empirical study of subjective beliefs in fixed income and foreign exchange markets. Both markets are characterised by enormous volumes and plenty of hypotheses about their drivers, including two testable hypotheses: the Uncovered Interest Rate Parity (UIP) and the Expectation Hypothesis (EH).

On the foreign exchange side, the UIP is derived from the idea that a trader could borrow a low-interest-rate currency, use the proceeds to buy a high-interest-rate currency, and then, after some holding period, convert their proceeds back into the low-interest-rate currency to pay back their loan. The UIP predicts that the gains this trader will earn should be zero on average since the high-interest-rate currency must depreciate proportionally to the interest rate differential. Papers such as Fama (1984), Hansen and Hodrick (1980), and Froot and Frankel (1989) discuss deviations from this hypothesis, often referred to as the *UIP puzzle*.

Similarly, the EH suggests that an agent should not expect to earn a premium from buying a long-term bond, selling it after some holding period, and financing the trade by selling short-term bonds. This concept goes back to Fisher (1896).

In both cases, we are interested in the expected excess returns, also called the risk premia, that investors expect to gain from entering into the UIP and EH trades. While there is a body of literature on both of these trades, Chapter Two brings survey data to the table. The dataset we use is obtained from Consensus Economics and includes month-by-month forecasts of international finance professionals. These professionals forecast interest rates, exchange rates, and many macroeconomic variables. While these forecasts are not perfect, as shown by Piazzesi et al. (2015), Cieslak (2017), Frankel and Froot (1987), and this chapter, they do allow for a model-free, real-time estimate of traders' thoughts. Buraschi et al. (2022) even demonstrate that one can use the persistence in forecast accuracy to construct survey-based measures that are surprisingly good at predicting future returns, offering a new perspective on Greenwood and Schleifer (2014).

Using these survey estimates allows us to reconstruct what agents must have thought when entering trades and what they must have expected to gain from them. Comparing these subjective expected excess returns to the excess returns that were realised ex-post allows us to study their predictability, cyclicality, and their implications for asset pricing models. We show that the subjective risk premia that agents demand in the bond and currency markets are non-zero and time-varying. We also find that subjective bond risk premia are counter-cyclical with respect to subjective expectations of growth, challenging asset pricing models that use long run risks (Bansal and Yaron, 2004), habit preferences (Campbell and Cochrane, 1999), and rare disasters (Weil, 1989) to argue for the pro-cyclicality of bond risk premia. In currency markets, however, we document that risk premia, from the view of a US investor, vary pro-cyclically with the business cycle since investors tend to flee to the US Dollar as a safe haven in bad time; leading to a lower XRP. Another contribution of Chapter Two is the comparison of the survey-elicited risk premia with risk premia that result from statistical projections. Such

statistical projections use a set of macroeconomic variables to predict the risk premia demanded by market participants. However, these projections are clearly based on significant hypotheses, including the idea that risk premia can be perfectly described by a constant factor model and that one can formulate what the correct set of factors looks like. The chapter itself goes into some detail about this and highlights differences between the two approaches.

Studying fixed income and foreign exchange markets as a vehicle to understand the formation and effects of subjective beliefs is appealing because of their tremendous volume and, therefore, high degree of liquidity. Chapter One of this thesis, along with a large body of literature, emphasises that more liquidity usually helps market efficiency by reducing oligopolistic effects. A paper by Nagel and Xu (2022) implements a related empirical exercise on a cross-section of asset classes.

Given the huge volume in the fixed income and foreign exchange markets, economists have tried to develop models to explain their specificities for decades. The seminal models of Vasicek (1977), Cox et al. (1985), Campbell and Cochrane (1999), Bansal and Yaron (2004), and Lustig et al. (2011) have all been used to attempt to rationalise the UIP puzzle. The chapter finishes with a discussion of stylised facts that we observe about survey-implied risk premia and their compatibility with theoretical models of asset pricing and develops a framework to account for these features in the presence of subjective beliefs. To do this, it emphasises the difference between objective and subjective probability measures. Doing so, we provide a path for endogenising the difference between the way an econometrician would view the world and agents' opinions. Coibion and Gorodnichenko (2015) gives an overview of the efforts made in this regard in the presence of informational frictions. By incorporating belief distortion explicitly, we can quantify the impact of sub-optimal learning, behavioural biases, and informational frictions in the context of a fully-fledged asset pricing model.

A final contribution that this thesis makes is to extend the study of fixed income and foreign exchange risk premia by combining the UIP and EH into one unified, testable hypothesis.

Chapter Three complements the second chapter by studying the *long carry trade*, a commonly used trading strategy that combines the fixed income and foreign exchange markets. It works by selling a long-term bond (borrowing money) in a low-interest rate (domestic) currency and investing in a long-term bond in a high-interest-rate (foreign) currency. After a fixed holding period, say one year, an investor sells the bond in the foreign country and converts the proceeds back to the domestic currency. Not only is this trade economically relevant, but it has also been the interest of recent research, e.g., Lustig et al. (2019), because of its simultaneous connection to multiple sources of risk. Specifically, it is the additive combination of an exchange rate risk premium and the difference in bond risk premia between the foreign country and the domestic country. This divisibility allow for the identification of drivers of the long carry trade. Again, the analysis greatly benefits from a dataset consisting of monthly expectations on macroeconomic variables from expert forecasters. Conveniently, forecasters submit their expectations on both bond yields and exchange rates. This dual-market perspective sheds light on the consistency of expectations across markets and the dynamics that govern these relationships. Andrews et al. (2021) highlight the inner workings of this trade and find that before 2008, the currency risk dominates its returns. After 2008, however, the gain that investors make from trading the long end of the yield curve dominates the overall returns, a finding that Borri and Shakhnov (2021) confirm for emerging countries in particular. They use traditional, macroeconomic signals for implementing the two legs of the trade and do not consider survey expectations.

The chapter shows that the expected excess returns of the long carry trade are dominated by currency risk that stems from the need to repatriate the proceeds of the bond sale back to the domestic currency. In the second half of the sample, however, the bond risk premium differential gains significance. The sign of the overall subjective expected excess return changes over time and predicts realised ex-post returns. We also note that the forecast error that our consensus forecaster makes is predictable using standard macroeconomic variables. The expected excess return of the trade is also positively linked to proxies of the quantity of risk in

the market and, therefore, obeys a classical risk-return relationship. On top of this, the interplay of currency risk and interest rate risk is not trivial, and the correlation between the two sources of risk also changes sign over the time of our sample. We observe a change of sign in the correlation between the currency risk premia and the bond premium differentials around the Global Financial Crisis (GFC) in 2008 for almost all countries considered in our sample. This time period around the GFC is also the only point in time where the two components offset each other for the majority of countries. This finding confirms Andrews et al. (2021) but highlights the dimension of correlation which allows us to further pin-down the drivers of the trade's dynamics.

Furthermore, the chapter highlights similar issues with projection-based estimates of expected excess returns as Chapter Two and compares the forecast errors implied by surveys to those implied by in-sample statistical projections.

Another contribution of the chapter is a discussion on how the survey-elicited subjective risk premia can be used in trading strategies. We show that both strategies that exploit cross-sectional variation through a sorting approach and strategies that work along the time series generate significant returns. Just as we dissect the premium itself into its components, we analyse the return composition of the proposed trading strategies with respect to their currency components and bond differential components. Finally, the correlations between the expected excess returns across countries are almost all positive and significant, hinting at commonly perceived, international risk factors à la Lustig et al. (2011). We use this and the other stylised facts presented in the chapter to comment on the appropriateness of standard asset pricing models.

All three chapters taken together provide an overview of the theoretical and empirical landscape of asset pricing in relation to strategic market games and the elicitation of risk premia from survey data. Chapter one is co-authored with Dimitrios Tsomocos. Chapter two is co-authored with Ilaria Piatti and Paul Whelan. Chapter three is single-authored. Each of the chapters is self-contained.

1

The Impact of Beliefs on Price Formation

Joint work with Dimitrios P. Tsomocos

Abstract

This paper assesses subjective belief heterogeneity in an array of price formation mechanisms of classical strategic market games and matches several stylised facts of real-world markets.

We establish that the Shapley Window model generates more stable prices and wealth distributions and lower price volatility than Cournot-type models such as the Sell-All or Bid-Offer models when we increase belief disagreement since markets are decoupled in the latter models.

We also run several simulations that illustrate the dynamics of games' equilibria as we vary the numbers of agents, commodities and states, as well as the distribution of initial wealth. We demonstrate that increasing liquidity in the market for contingent commodities by introducing more players or more money helps to moderate the impact of heterogeneous beliefs.

1.1. Introduction

Thinking about the market for a perishable commodity, say coffee, one could refer to many different market architectures. Coffee beans are sold to wholesalers upon harvesting, sometimes as all at once, through futures contracts on exchanges, or simply over the counter. The inner workings of these markets can be studied using strategic market games. It is critical for both, traders and policymakers to understand the efficiency properties of these markets to ensure price stability. All three of the examples given at the beginning of the paragraph correspond to different, well-known strategic market games. All of these games do not allow for heterogeneity in agents' beliefs.

Belief disagreement among market participants is a complex phenomenon, and developing a comprehensive model to explain it is challenging. Nevertheless, empirical evidence consistently indicates that agents, including investors, traders, and fund managers, display diverse beliefs about the future state of the economy and asset prices. Numerous factors contribute to this heterogeneity, such as information asymmetry, differing prior beliefs, variations in risk appetite, and behavioural influences. By considering these diverse factors, researchers can better understand and account for the heterogeneity in beliefs observed among market participants.

Regardless of the source of belief heterogeneity, designing robust market mechanisms can help prevent spirals of optimism and pessimism as well as boom-bust cycles. The effects of such belief propagation have been well-analyzed by Simsek (2013), who documents how downside and upside optimism affected asset prices during the subprime mortgage crisis. Yan (2017) demonstrates that taking into account heterogeneous beliefs is crucial for explaining leverage decisions and credit cycles. He also sheds some light on the non-trivial relationship of belief heterogeneity in a fully-fledged general equilibrium model with incomplete markets. Several other studies make similar claims (e.g., Geerolf (2015), Fostel and Geanakoplos (2008)), but with limited consideration of the game-theoretic foundations of price formation itself.

This paper aims to fill exactly this gap by revisiting some well-established mechanisms of price formation that are derived from the literature on strategic games and adding heterogeneous beliefs. The concept of strategic market games that model price formation is a natural bridge between finance and economics because it describes the development of asset prices, derived from game-theoretic concepts. In Section 1.2, we will derive a natural way of incorporating subjective beliefs into the mechanisms.

We do all of this in a setting of complete markets. We consciously make this choice to place the emphasis on the impact of subjective beliefs on the dynamics of price formation without introducing the significant complexity of market incompleteness. We do, however, comment on the marginal effects of market completeness throughout the discussion.

In general, one can differentiate between two families of strategic market games and associated price formation mechanisms: Cournot-type models and Bertrand-type models. The former class of mechanisms was pioneered, among others, by Shapley and Shubik (1977) and Dubey and Shubik (1978) who introduced a model of trading posts. The strategies in models like the trading posts model of Shubik and Wilson (1977) are articulated in terms of quantities, resulting in the well-known Cournot result, i.e., that the equilibrium of these games cannot converge to a Walrasian equilibrium. The latter family of models was introduced and characterised by, among others, Dubey (1982) and Simon (1984). Here, strategies are formed in terms of prices and quantities. We argue that the first class of price formation mechanisms, those that we group into the Cournot family, are the more interesting to study. First, they result in more pronounced oligopolistic effects which makes the ranking of mechanisms in this family more relevant. Second, we argue that there are markets in the real-world like foreign exchange (FX) markets and commodity markets that implement price formation mechanisms that fall into the Cournot family. Finally, since all of the presented price formation mechansisms are related to one another, the framework to incorporate beliefs can remain constant throughout our analysis.

One further remark to make is that we will carefully circumvent the analysis of multiplicity and other market frictions. There is considerable work showing how to limit inefficiencies and how to remove indeterminacy that can arise, e.g., Ritzberger (2016) who shows that the double auction game with a limit order book eliminates the Cournot inefficiencies introduced by the mid-point rule that we will use.

Dubey and Shubik (1977b) have established a theoretical framework we leverage to analyse strategic market games with beliefs. While their setup is a sell-all game with compound futures, their setup is adaptable to other mechanisms of price formations. Their introduction of state-uncertainty involves symmetric beliefs about the probability of future states of the world across agents. In a follow-up paper, Dubey and Shubik (1977a) elaborate on their framework of state-uncertainty and discuss how strategic market games differ from the traditional Radner (1968) analysis. While their work is related to ours, they only consider one method of price formation and do not consider heterogeneity in agents' beliefs. Another paper that is related to ours is Giraud and Stahn (2008) who are concerned with the existence of equilibria in two-period strategic market games in a financial economy with uncertainty. Their analysis is somewhat more sophisticated than Dubey and Shubik (1977b,a) because they consider an economy with secured lending and financial assets that make for more complex, inter-temporal agent strategies. Our approach is different, we utilise Dubey and Shubik (1977b) in terms of notation and general setup but extend the analysis to various other price formation mechanisms and study the impact of belief changes.

One of our key findings is that Shapley's Window model is more robust to heterogeneous beliefs than all of the other classical price formation mechanisms. This is because of the high interconnectedness between markets which allows for the subjective beliefs to propagate into more markets, lowering the effect in any individual market.

This finding is in line with Huber et al. (2010), who conducted laboratory experiments in a similar setting and found that the non-cooperative equilibria

of Cournot-type models of price formation are further away from the respective competitive general equilibrium solutions.

Whether this is a desirable feature of mechanism design is not for us to judge. We merely compare and contrast different avenues of modelling price formation in strategic market games and their behaviour given subjective beliefs.

Beyond finance and economics, there is another discipline that makes regular use of strategic market games, machine learning. It so happens that traditional concepts, such as Shapley values, [1] that stem from mechanism design are used increasingly to enhance statistical modelling. However, this partnership is not a one-way street. Several machine learning scientists contribute to the academic literature in economics and finance by addressing traditional questions in a novel way. This increasing use of novel Machine Learning algorithms for policy design has garnered significant attention. Algorithms typically rely on simulating the impact of various policy alternatives to identify and contrast possible solutions. In many domains, they have demonstrated superior performance compared to both real-world policies and theoretical closed-form solutions found in economics. Most of these studies, such as Zheng et al. (2022), employ some form of double auction market to estimate the price impact of policy changes. Due to the substantial computational effort involved in simulating double auction markets with numerous agents, certain assumptions regarding the number of agents or assets are often made. However, little is known about how the choice of price formation mechanism might affect the predictions of such simulation-based models.

The rest of the paper is structured as follows: First, Section 1.2 presents the existing literature on price formation mechanisms. Section 1.3 then presents some theoretical results and adds a theoretical notion of beliefs to the presented models. Section 1.4 proceeds to simulate the implications of considering beliefs in price formation mechanisms and discusses how the simulation results match stylised

[1] The concept of Shapley values was first introduced in Shapley (1953) and described a distribution of surpluses that are earned by a coalition. The intuition behind the method was that some individuals are more important for the negotiation game between company and coalition. Today, the concept is widely used as one of the workhorses of interpretable artificial intelligence and can show which parts of a model are most important for a prediction.

facts in real markets. Finally, we conclude by highlighting potential avenues for future research in this area.

1.2. Five Price Formation Mechanisms

We outline the inner workings of the five predominant price formation mechanisms: These are the Sell-All model, the Bid-Offer model, the Complete Markets Model, the Shapley Window model and the Double Auction model. One of the advantages of all presented approaches is that they all characterise *how* exactly prices are formed. While conventional general equilibrium describes an economy at equilibrium, the strategic market game approaches also characterise off-equilibrium dynamics. Before diving into the mechanisms themselves, we will first describe the setup of the economy and how subjective beliefs are incorporated.

One of our contributions is to show a simple method of integrating subjective beliefs into well-known strategic market games. We concern ourselves with modelling the state-uncertainty of Arrow and Debreu (1954) which consists of a multiplicity of possible states of the world tomorrow. Agents in the economy naturally form expectations about the likelihood of these future states of the world and attempt to best insure themselves against consumption risks.

While there are some first attempts integrating the complexity of such multi-period games and state-uncertainty into price formation, there is little consensus in the literature. Giraud and Stahn (2008), for example, illustrate how a two-period economy can be modelled in the context of Shapley-Shubik games using financial assets.

However, by introducing such financial assets, they also significantly complicate the setup of the original Bid-Offer model of Dubey and Shubik (1978). As our paper aims to emphasise differences in the interplay of various standard mechanisms, we opt for the easiest modelling of state-uncertainty possible through contingent commodities. While Peck and Shell (1989), Weyers (1999) and others show that this is not necessarily equivalent to the case of financial assets in a non-competitive setting, we are able to illustrate some relevant points in this benchmark setting.

The process of expectation formation has been studied thoroughly by Coibion and Gorodnichenko (2015); Singleton (2021); Bordalo et al. (2019, 2020); Buraschi et al. (2022) and many others. The use of novel datasets like expert surveys has popularised a stream of literature that concerns itself with questions of rationality, consistency and predictability of beliefs.

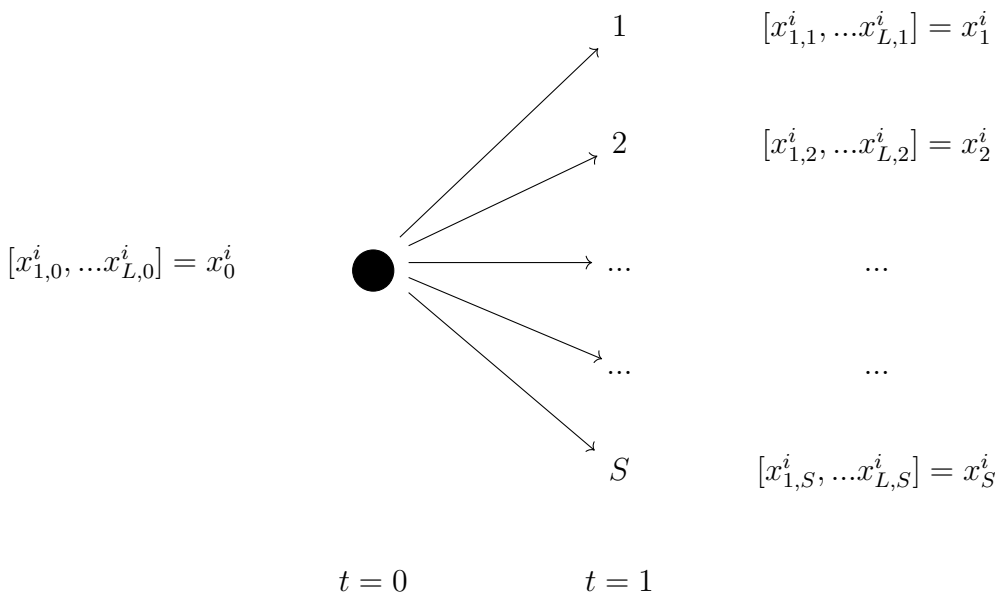


Figure 1.1. A two-period economy with state-uncertainty

Our setup is a two-period economy with S future states of nature and N agents indexed by a superscript i. There exist L physical commodities in the economy. We model a complete set of $L \times S$ contingent commodities indexed ls. This implies that agents are given the possibility to fully insure against consumption risk tomorrow. This setup is depicted in Figure 1.1 above. While we will proceed to explaining several mechanisms in relation to some general $S > 1$, we intentionally confine ourselves to $S = 2$ and $L = 2$ in the majority of Sections 1.3 and 1.4 for tractability and comparability. This procedure is not unusual and has been used by several papers to avoid unnecessary detail (e.g., Fostel and Geanakoplos (2008)).

Assuming markets to be complete is not necessarily realistic, but serves as a benchmark. Also, answering questions around existence, multiplicity and feasibility

is not trivial even when considering complete markets (Giraud and Stahn, 2008; Weyers, 1999; Peck and Shell, 1989), however the purpose of our paper is different.

1.2.1. Sell-All model

Arguably the most accessible mechanism of price formation is the Sell-All model. It was first presented in Shubik (1973) and later specified in Shapley and Shubik (1977). Conceptually, the Sell-All model tries formulate a reasonable mechanism of exchange while assuming a bare minimum of institutional detail. The Sell-All model is a general form of Cournot's model of competition whereby agents compete against each other. Since we usually think of Cournot competition in terms of quantities, it is useful to note that we will write strategies in terms of quantities of a commodity money.

Assume that there are n traders trading in $LS + L$ commodities. We assume the first commodity, $l = 1$, today, i.e., $s = 0$, to have a dual role: First, it can be consumed, i.e., it enters the utility function of the agents. Second, it can also be used as a *money* to facilitates trade. We call these goods commodity money [2].

We define the last LS commodities to be the LS state contingent commodities. The additional L commodities, the spot commodities, will act as the $t = 0$ consumable commodities that will allow us to capture wealth and price effects.

For convenience, we will define the following two sets:

$$M = \{1, ..., L\} \times \{0, ..., S\}$$

and

$$N = M \setminus \{(1, 0)\}$$

representing the commodity spaces with and without the spot commodity money.

We define an endowment vector across the $LS + L$ commodities as

$$a^i = \left(a^i_{10}, ..., a^i_{L0}, a^i_{1S}, \ldots, a^i_{LS}\right),$$

[2] As opposed to a *fiat money* which would not explicitly enter the agents' utility functions.

where $a^i_{ls} \geq 0$ for all $(l,s) \in M$ and $a^i_{ls} > 0$ for at least two traders. We also denote the aggregate endowment of some commodity ls as $\bar{a}_{ls} = \sum_{\forall i} a^i_{ls}$.

Furthermore, we assume that all n traders in the economy have well-behaved, concave utility functions u^i which depend on the holdings of commodities $\ell \in \{1, ..., L\}$ in all states $s \in \{0, ..., S\}$, $x_{\ell s}$. What follows is the verbal intuition of the mechanism: All of the n traders are forced to put their entire endowments up for sale on a common market. This assumption for the supply side of the economy is quite strict and not particularly realistic, but it yields a nice benchmark setting.[3] On the demand side of the market, the agents use their endowment in the monies, a^i_{l0}, to place simultaneously bids for all other available commodities. This yields a schedule of bids for each agent

$$b^i = \left(b^i_{20}, ..., b^i_{L0}, b^i_{1S}, \ldots, b^i_{LS}\right).$$

We denote aggregates similarly to aggregate endowments as $\left(\bar{b}_{20}, \ldots, \bar{b}_{LS}\right)$ where $\bar{b}_{ls} = \sum_{\forall i} b^i_{ls}$. It can also be noted that an agent is, via concavity, expected to buy back at least some share of their initial endowment but only after they put all of their commodities up for sale. The prices for the non-money goods can then be obtained by considering the overall market supply and demand for each good.

Shapley and Shubik (1977) describe this consideration as the use of m separate "trading posts" in the economy. All traders visit each of these trading posts, place bids, and deposit their entire endowments. Thus, the trading posts will be characterised by total supplies $(\bar{a}_{20}, \ldots, \bar{a}_{LS})$ and total demands $\left(\bar{b}_{20}, \ldots, \bar{b}_{LS}\right)$. These total demands are all expressed in terms of the commodity money that the traders are endowed with.

In this game, the strategy set of an individual trader i is a vector of bids

$$S^i := \left\{ b^i \in \mathbb{R}^{|N|}_+ \mid b^i_{ls} \geq 0, \forall (l,s) \in N, \sum_{(l,s)\in N} b^i_{ls} \leq a^i_{10} \right\}.$$

[3] The sell-all mechanism is akin to the mechanism of a representative agent model, Lucas (1972).

In the absence of credit, the individual bids across states and (contingent) commodities need to be non-negative, i.e., $b^i_{ls} \geq 0, \forall (l,s) \in N$, and sum up to the monetary endowment of a trader, $\sum_{(l,s)\in N} b^i_{ls} \leq a^i_{10}$. Without credit, this single budget set acts as a "cash in advance" constraint and can lead to inefficiencies if there is not enough money in the economy.

Given these assumptions, we can now define prices as the number of units of money offered per unit of commodity,

$$p_{ls} = \bar{b}_{ls}/\bar{a}_{ls}, \quad (l,s) \in N. \tag{1.1}$$

The sequence of events implies that traders need to submit their demand schedule without observing prices before. Shapley and Shubik (1977) argue that this assumption does not matter significantly in equilibrium because traders can rely on past prices. They also argue that the allocation of a portion of an agent's wealth should be not too different from them purchasing a known quantity.

Considering all commodities at once, we can write the proceeds from trade (consumptions) concisely as

$$x^i_{ls} = \begin{cases} b^i_{ls}/p_{ls} & \text{if } p_{ls} > 0, (l,s) \in N, \\ 0 & \text{if } p_{ls} = 0, (l,s) \in N, \\ a^i_{ls} - \sum_{(k,r)\in N} b^i_{kr} + \sum_{(k,r)\in N} a^i_{kr} p_{kr} & \text{if } l = 1 \wedge s = 0. \end{cases}$$

Definition 1. *The Sell-All Mechanism*

We define the Sell-All game, $\Gamma^{Sell-All}$, as a set of strategies, endowments, utility functions, beliefs and the price formation mechanism of Equation 1.1, $\left((S^i, a^i, u^i, \pi^i)_{i\in I}, p(\cdot)\right)$. The equilibrium of $\Gamma^{Sell-All}$ is a set of prices and quantities $\left(p, (x^i)_{i\in I}\right) \in \left(\mathbb{R}^{|N|}_+ \times \left(\mathbb{R}^{|M|}_+\right)^I\right)$ such that $u^i(x^i|\ddot{\imath}" \cdot \hat{S}) \geq u^i(\tilde{x^i}|\ddot{\imath}" \cdot \hat{S})$ where $\tilde{x^i}$ is the allocation resulting from agent i playing any $\tilde{s} \in S \neq S^i$ and $\hat{S}$ is the strategies played by all other agents.

1.2.2. Bid-Offer mechanism

While the Sell-All model offers an approachable insight into how prices can form, it assumes that traders act as buyers in all markets. The decision to sell is not part of the agents' strategy, S^i, as they are mandated to sell all their endowments.

To address this concern, Dubey and Shubik (1978) present an alternative method of price formation that allows traders to act as buyers, sellers, both, or even as silent observers. Again, we consider $LS + L$ commodities and consider the first one today as money which enters the utility function of the trader. That is each trader i is allowed to separately determine the quantities they want to buy and sell of each good j. Let the definitions for endowments and utility functions stay as before. Then we can rewrite the strategy set of each agent i in terms of bids, b^i_{ls}, and offers, q^i_{ls}, as:

$$S^i := \left\{ \left(b^i, q^i\right) \in \left(\mathbb{R}^{|N|}_+\right)^2 \mid q^i_{ls} \leq a^i_{ls}, b^i_{ls} \geq 0 \ \forall (l,s) \in N, \sum_{(l,s)\in N} b^i_{ls} \leq a^i_{10} \right\}$$

Verbally, the quantity of commodity ls offered for sale needs to be less than or equal to the endowment of said good. Also, the sum of all bids across physical and contingent commodities is bounded by the monetary endowment of the trader. Price formation then occurs by means of trading posts again. This time, both, bids and offers may be posted against trading posts. Given that there is more than one agent participating in the economy, the prices are then given by

$$p_{ls} = \frac{\bar{b}_{ls}}{\bar{q}_{ls}}, \quad (l,s) \in N. \tag{1.2}$$

where $\bar{b}_{ls}$ is defined as before. Similarly $\bar{q}_{ls}$ is defined as the per-commodity sum of offers, i.e., $\bar{q}_{ls} = \sum_{\forall i} q^i_{ls}$. As agents are now allowed to strategically choose how much of their endowments to put up for sale, the denominator of Equation 1.2 differs from Equation 1.1 in the denominator.

We can again deduce the final holdings of commodities and money given as

for trader i as:

$$x_{ls}^i = \begin{cases} a_{ls}^i - q_{ls}^i + \frac{b_{ls}^i}{p_{ls}} & \text{if } p_{ls} > 0, (l,s) \in N, \\ 0 & \text{if } p_{ls} = 0, (l,s) \in N, \\ a_{ls}^i - \sum_{(k,r)\in N} \left(b_{kr}^i - q_{kr}^i p_{kr}\right) & \text{if } l = 1 \wedge s = 0. \end{cases}$$

This idea of trading post-type models can be extended further. Postlewaite and Schmeidler (1978) extend the above setup by allowing for a fiat money instead of a commodity money.

Another interesting extension of Dubey and Shubik (1978) is to allow for the possibility of *non-money* commodities to be exchanged for each other without mediation. There are two ways of expanding the market setup of Dubey and Shubik (1978) to support multiple moneys. The first extension is presented in Amir et al. (1990) and introduces the ideas of multiple moneys with price formation rules that are independent between markets. The other one is presented in Sahi and Yao (1989) and is going back to an informal proposal of Shapley. Thus, their model is usually referred to as Shapley's "Window model".

Definition 2. *The Bid-Offer Mechanism*
We define the Bid-Offer game, $\Gamma^{Bid-Offer}$*, as a set of strategies, endowments, bids, offers, utility functions, beliefs and the price formation mechanism of Equation 1.2,* $\left((S^i, a^i, u^i, \pi^i)_{i\in I}, p(\cdot)\right)$.
The equilibrium of $\Gamma^{Bid-Offer}$ *is a set of prices and quantities* $\left(p, (x^i)_{i\in I}\right) \in \left(\mathbb{R}_+^{|N|} \times \left(\mathbb{R}_+^{|M|}\right)^I\right)$ *such that* $u^i(x^i|\hat{S}) \geq u^i(\tilde{x^i}|\hat{S})$ *where* $\tilde{x^i}$ *is the allocation resulting from agent* i *playing any* $\tilde{s} \in S \neq S^i$ *and* $\hat{S}$ *is the strategies played by all other agents.*

1.2.3. Complete markets mechanism

In departure from both, the vanilla Sell-All model of Shubik (1972) and Shapley (1976), and the Bid-Offer model of Dubey and Shubik (1978), the complete markets mechanism proposed in Amir et al. (1990) assumes that *all* pairwise commodity markets exist. In some sense there are L moneys in their economy. Modelling more than one money allows them to study more complex markets like the foreign

exchange market where pairwise prices are the norm. Also other real-world use cases like barter or bi- and multi-metallism can be modelled using this approach.

The definitions of initial endowments, a^i_{ls}, are similar as before. Utility functions are still assumed to be strictly concave, increasing, and continuously differentiable. The players moves are no longer vectors of length $|N|$, in case of Sell-All, or $2|N|$, in case of Bid-Offer. Instead, trader i now submits a $|M| \times |M|$ matrix of pairwise bids B^i such that:

$$S^i := \left\{ \left(b^i\right) \in \left(\mathbb{R}_+^{|M|}\right)^2 \mid b^i_{ls} \geq 0, \sum_{(k,r)\in M} b^i_{kr,ls} \leq a^i_{ls} \ \forall (l,s) \in M \right\}$$

Each entry of the matrix needs to be weakly positive to prevent short selling. Furthermore, the sum of all bids made in terms of a certain commodity money need to be less than or equal to the endowment of that commodity money. Prices emerge in form of a price matrix defined as

$$p_{kr,ls} = \begin{cases} \sum_i b^i_{kr,ls} / \left(\sum_i b^i_{ls,kr}\right) & \text{if } \sum_i b^i_{ls,kr} \neq 0 \\ 0 & \text{if } \sum_i b^i_{ls,kr} = 0, \end{cases} \tag{1.3}$$

while holdings are, similar to the other mechanisms, defined as

$$x^i_{ls} = a^i_{ls} - \sum_{(k,r)\in M} b^i_{ls,kr} + \sum_{(k,r)\in M} b^i_{kr,ls} p_{ls,kr}, \quad \forall (l,s) \in M.$$

Definition 3. *The Complete Markets Mechanism*
We define the Complete Markets game, $\Gamma^{Complete-Markets}$*, as a set of strategies, endowments, bids, offers, utility functions, beliefs and the price formation mechanism in Equation 1.3,* $\left((S^i, a^i u^i, \pi^i)_{i\in I}, p(\cdot)\right)$.
The equilibrium of $\Gamma^{Complete-Markets}$ *is a set of prices and quantities* $\left(p, (x^i)_{i\in I}\right) \left(\mathbb{R}_+^{|N|} \times \left(\mathbb{R}_+^{|M|}\right)^I\right)$ *such that* $u^i(x^i|\hat{S}) \geq u^i(\tilde{x^i}|\hat{S})$ *where* $\tilde{x^i}$ *is the allocation resulting from agent* i *playing any* $\tilde{s} \in S \neq S^i$ *and* $\hat{S}$ *is the strategies played by all other agents.*

There are several observations to be made. First, this mechanism allocates holdings in a similar fashion to the previous mechanisms. Second, the formations of

prices for each commodity pair, p_{ij} are decoupled. While this seems innocuous, this modelling feature warrants further consideration as it may result in inconsistency.

Definition 4 (No arbitrage)**.** *A price formation mechanism is consistent iff it yields prices p_{xy}, p_{yz}, and p_{xz} which satisfy $p_{xy}p_{yz} = p_{xz}$.*

Since the prices of all commodity pairs are modelled in isolation, it is possible that indeed $p_{xy}p_{yz} \neq p_{xz}$. In Amir et al. (1990), the authors give two examples under which this consistency property is violated. In Sahi and Yao (1989), the authors even claim that this departure from consistency is typical.

1.2.4. Shapley's "Window model" mechanism

Another way to allow for multiple moneys is the mechanism proposed in Sahi and Yao (1989). It is the formal summary of a discussion of Shapley, which earned the model the name of Shapley's "Window model". The root of the inconsistency in the model of Amir et al. (1990) is that prices in distinct markets are formed in isolation from another. Sahi and Yao (1989) address this by introducing a central clearing house that simultaneously solves a system of equations for all markets. Traders form strategies, much as before, as a matrix of pairwise bids to express their willingness to pay for a commodity kr in terms of a commodity ls,

$$S^i := \left\{ \left(b^i\right) \in \left(\mathbb{R}_+^{|M|}\right)^2 \mid \sum_{(k,r)\in M} b^i_{kr,ls} \leq a^i_{ls} \forall (l,s) \in M, b^i_{kr,ls} \geq 0 \ \forall (l,s),(k,r) \in M \right\}.$$

The clearing house which observes all agents' signals then calculates prices according to the following set of equations:

$$\sum_{(k,r)\in M} \left(\sum_{i=1}^{N} b^i_{kr,ls} p_{kr} \right) = p_{ls} \sum_{(k,r)\in M} \left(\sum_{i=1}^{N} b^i_{ls,kr} \right), \quad \forall (l,s) \in M. \tag{1.4}$$

Because the clearing house solves all of the prices simultaneously. Sahi and Yao (1989) give the intuition that by modifying the quantity of oranges you sent in exchange for apples, you not only modify the price of oranges relative to apples, but - almost always - all the other relative prices as well. Since the trading posts

are only ever considering the demand and supply dynamics for a given good, this observation would not hold for the other models outlined above.

Under this pricing regime we obtain final holdings of trader i as

$$x_{ls}^i := a_{ls}^i + \frac{1}{p_{ls}} \sum_{(k,r)\in M}^{S} p_{kr} b_{kr,ls}^i - \sum_{(k,r)\in M} b_{ls,kr}^i, \quad \forall (l,s) \in M.$$

Definition 5. *The Window model Mechanism*
We define the Window model game, $\Gamma^{Window-Model}$*, as a set of strategies, endowments, bids, offers, utility functions, beliefs and the price formation mechanism in Equation 1.4,* $\left((S^i, a^i, u^i, \pi^i)_{i\in I}, p(\cdot)\right)$.
The equilibrium of $\Gamma^{Window-Model}$ *is a set of prices and quantities* $\left(p, (x^i)_{i\in I}\right) \in \left(\mathbb{R}_+^{|N|} \times \left(\mathbb{R}_+^{|M|}\right)^I\right)$ *such that* $u^i(x^i|\ddot{\imath}" \cdot \hat{S}) \geq u^i(\tilde{x^i}|\hat{S})$ *where* $\tilde{x^i}$ *is the allocation resulting from agent* i *playing any* $\tilde{s} \in S \neq S^i$ *and* $\hat{S}$ *is the strategies played by all other agents.*

The distinctive difference between the price formation mechanisms described before and the one suggested by Sahi and Yao (1989) is that the Window model solves for all prices simultaneously. This allows prices to mediate all trades and not only the trades in the directly affected commodities while ensuring consistency in the sense of Definition 4. This feature comes at the cost that we need to assume a lot more market microstructure in form of a centralised clearing house.

These two extreme cases of price formation, namely fully-integrated and fully-isolated are viewed as two distinct types of mechanisms: the trading post mechanisms that allow prices to form separately for each commodity are referred to as TP-mechanisms. Similarly, the Window model is referred to as the W-mechanism.

An interesting observation made by Dubey and Sahi (2003) is that the most interesting real-world applications would need to implement a mechanism in between the TP- and W-mechanisms, namely guarantee local consistency while not assuming full integration. The authors give two key examples for this interesting middle ground. First, they establish that in foreign exchange markets not every two pairs of currencies are trades against each other. While currencies can usually be traded

on a liquid market against key currencies such as EUR and USD, they are rarely traded directly against AUD or NZD. This implies that the matrix of pairwise bids is restricted for some pairs. Second, they argue that contingent convertible bonds introduce a sense of direction to the mechanism. A convertible bond entitles the holder to the right, but not the obligation, to exchange their bond holding for an equity holding but not the other way around. Such a one-directional exchange would imply asymmetry of the pairwise matrix of bids in the Window model.

The natural consequence of these discussions is to describe the schedule of allowed trades using a directed graph, G, that captures the phenomena above and more. The axioms necessary for this graph are presented and discussed in Dubey and Sahi (2003). [4]

1.2.5. Double Auction model

There is one more general mechanism that characterises price formation, the double auction mechanism. The key departure from earlier games is that the double auction mechanism considers strategies in terms of prices and quantities as limit orders. Hence, traders submit the entire individual supply and demand schedules.

While the strategies of $\Gamma^{Sell-all}$ and $\Gamma^{Bid-Offer}$ are of dimensionality $|N|$ and $2|N|$, respectively, the double auction market introduces prices and quantities for both sales and purchases, leading to a strategy profile of dimensionality $4|N|$,

$$S^i := \left\{ \left(p^i, q^i, \tilde{p^i}, \tilde{q^i}\right) \in \left(\mathbb{R}_+^{|N|}\right)^4 \mid q^i \geq 0 \forall i, \tilde{q^i} \geq 0 \forall i, \tilde{q^i} \leq a^i, \sum_{(l,s)\in M} p_{ls}^i q_{ls}^i \leq a_{10}^i, \right\}.$$

The vector, p^i holds the maximum prices an agent is willing to pay in terms of the money for q^i units of commodities. On the flip side, the agent is demanding minimum prices of $\tilde{p^i}$ to sell $\tilde{q^i}$ units of commodities. We assume quantities to be positive, i.e., $q^i \geq 0 \ \forall i$ and $\tilde{q^i} \geq 0 \ \forall i$, because negative values are not insightful when modelling both sides of the market for each agent. We further restrict the

[4] Further discussion of this more general view on price formation mechanisms is given in Giraud (2003).

quantities put up for sale to be bounded by commodity endowments and the aggregate spending on purchases to be bounded by monetary endowments.

For the first time, prices and quantities are explicitly part of the agents' strategies. More specifically, agents are allowed to submit limit orders that implement upper and lower limits for prices. This is a desirable modelling assumption as it makes a tâtonnement process somewhat more believable.

It is explicitly allowed for an agent to take both sides of the market simultaneously. This general form of S^i leads to the dimensionality of $4|N|$ for the strategy set. Aggregating these strategies for multiple agents yields the familiar diagrams for demand an supply. Some cases of well-behaved strategies are visualised in Figure 1.2, which is a close replica of Dubey and Shubik (1980).

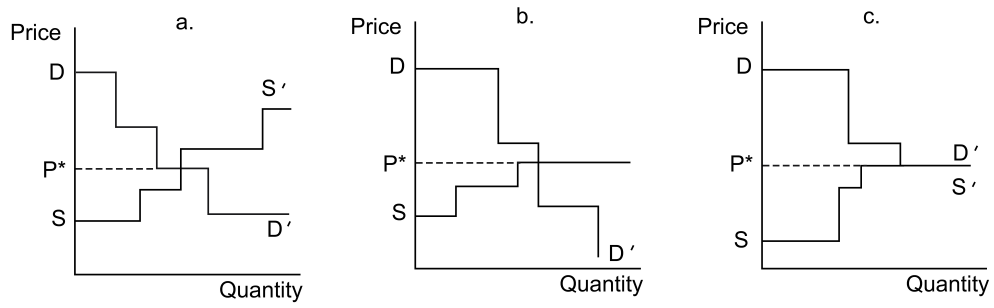


Figure 1.2. Interior Bid Offer Constellations
This figure displays three possible bid offer constellations that might emerge. In panel a, we observe a case where at p^*, there may be some buyers that are willing to pay the market price for whom there is not enough supply. In panel b, we see that suppliers are happy to sell more of the good at p^* than there is demand for. In panel c, there are multiple feasible quantities at p^* where demand is equal to supply.

One can generate the diagrams above by sorting the agents' order in terms of prices and cumulating the quantities. The aggregation of p^i and q^i across agents yield that downward sloping demand schedule while the aggregation of $\tilde{p^i}$ and $\tilde{q^i}$ for the same agents yields the upward sloping supply schedule. The space at which demand and supply are the same determines the equilibrium price, p^*. Agents that have strategies such that $\tilde{p^i} > p^*$ do not sell anything. Similarly, agents who have strategies such that $p^i < p^*$ do not buy anything. Figure 1.2a shows a

demand and supply schedule for which there is more demand than supply at p^*. In Figure 1.2b we see the opposite case, there is more supply than demand at p^*. Both of these indeterminacies are specific to the double auction mechanism and require further assumptions about the market's microstructure to be resolved. One could, for example, devise rules whereby orders are filled pro-rata or by random assignment to identify whose orders to clear.

There are, however, more serious issues with the double auction mechanism that are visualised in Figure 1.3.

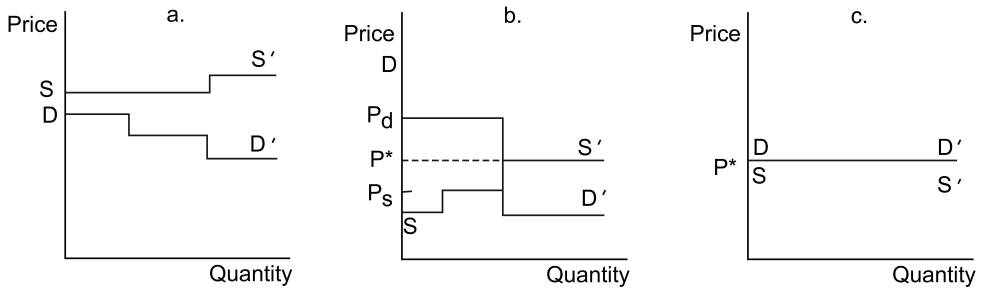


Figure 1.3. Corner Solution Bid Offer Constellations
This figure displays three possible indeterminate bid offer constellations that might emerge. In panel a, we observe a case where demand and supply do no overlap and there is no p^* at all. In panel b, we see the case where there is a range of admissible p^* for which demand equals supply. In panel c, we see the case where demand and supply are perfectly inelastic and demand equals supply for all q.

We specifically observe three cases that lead to problems in the double auction mechanism. Figure 1.3a depicts the case for which the highest bid across agents is less than the lowest ask across agents. In this case, the valuations of the commodity on the buy and sell sides are so different that they give rise to a no-trade equilibrium. Figure 1.3b shows the case for which there exists no unique p^*. Any price in the interval $[p_s, p_d]$ would allow demand and supply to clear. Again, one would need to stipulate some selection procedure, e.g., a midpoint rule, to determine p^*. Finally, Figure 1.3c shows the case of perfectly inelastic demand and supply. Regardless of the quantity, the price limits on the buy and sell sides is always constant.

It is evident that the double auction mechanism is a helpful tool to analyse and illustrate special configurations of demand and supply while largely avoiding

the tâtonnement critique. We will show, however, that as the number of agents grows to infinity, the double auction market will almost always coincide with the Bid-Offer model of Dubey and Shubik (1978).

1.3. Properties of Price Formation Mechanisms

In this section, we delve into the effects of heterogeneous beliefs on transformation mechanisms by exploring a simplified, yet insightful, two-commodity, two-state setting. By reducing the complexity of the model, we aim to distill the core elements that drive the interaction between heterogeneous beliefs and the various price-formation mechanisms.

We model two types, i and j. To motivate trade, these two types differ in their endowments of contingent commodities. Type i is endowed with one unit of commodity 1 in state 1 while type j is endowed with one unit of commodity 2 in state 1. Hence, because of concavity, they will attempt to obtain commodities 1 and 2, respectively, by bidding in the markets for the contingent commodities.

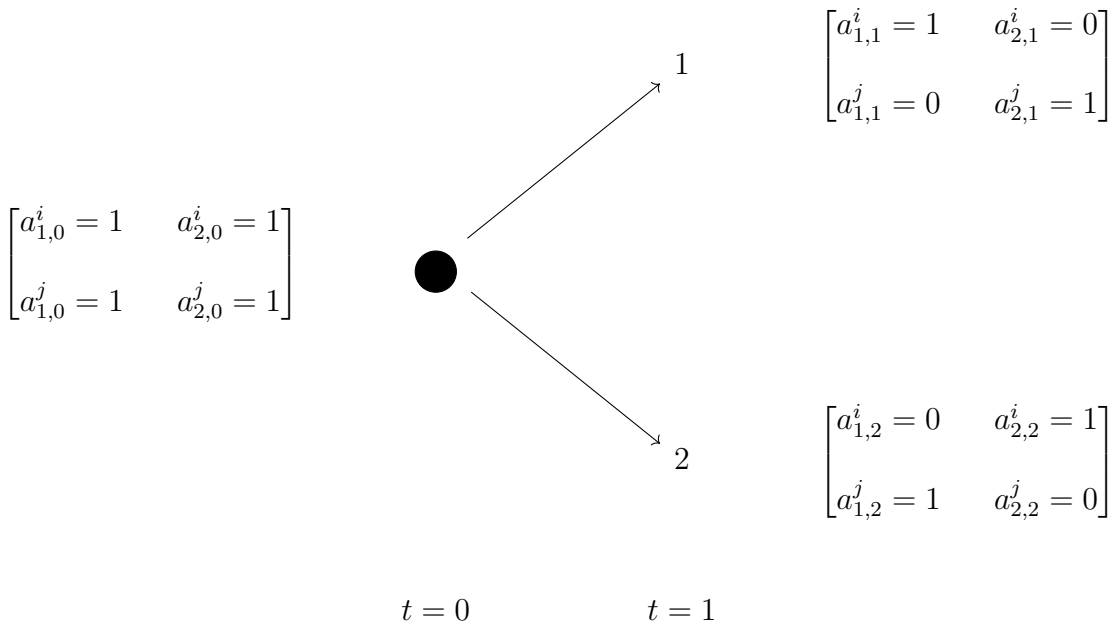


Figure 1.4. Visualisation of the two-period economy with state-uncertainty

In the Sell-All model, agents are able to choose their vector of bids b^{α} and b^{β} to maximise utility, obeying the strategy set defined above. To make analysis more tractable, we will parameterise the problem by introducing a specific utility function. While any concave utility would suffice for our argument, we choose to use a specification similar to Shubik and Yao (1990) and implement utility functions of the form:

$$u^i = log(x_0^i) + \mathrm{E}_{\pi^i}\left[x_s^i\right] = log(x_0^i) + \pi^i log(x_1^i) + (1-\pi^i) log(x_2^i). \tag{1.5}$$

As part of the simulations following in Section 1.4 we will study the impact of the price formation mechanisms on the volatility of prices across states. We formally define the volatility of the price of physical commodity l as:

$$\sigma_l = \sqrt{\sum_{\forall s>0} \ddot{\imath}" \cdot \left(p_{ls} - p_{l0}\right)^2} \tag{1.6}$$

To illustrate what this theory translates to, we compute an example that closely follows Shubik and Yao (1990) but adds subjective beliefs to the game.

Example 1. *Similar to the Sell-All case above, we have two types of agents, i and j, that simultaneously optimise their objective functions.*[5]
Type i optimises:

$$u^i = \pi^i log\left(a_1^i + \frac{b_i}{b} q\right) + (1-\pi^i) log\left(a_2^i - b_i\right)$$

while type j optimises:

$$u^j = \pi^j log\left(a_1^j - q_j\right) + (1-\pi^j) log\left(a_2^i + \frac{q_j}{q} b\right)$$

[5] In this proof, we assume no wash sales, i.e., to never buy and sell at the same time. This behaviour would, however, be supported by the strategy set of the game. We discuss the relevance of this "no wash sales" or "cashflow-minimisation" condition in Section 1.4.

Solving the first order conditions yields two optimality conditions:

$$\pi^i \frac{1}{a_1^i + q_j} \frac{q_j}{b_i} \frac{n-1}{n} = \frac{1}{a_2^i - b_i} \tag{1.7}$$

$$\pi^j \frac{1}{a_1^j + b_i} \frac{b_i}{q_j} \frac{n-1}{n} = \frac{1}{a_2^j - q_j} \tag{1.8}$$

which we can then combine to obtain the resulting b_i^* *and* q_j^**. To make the equations easier to read, we use the same endowment vector as Shubik and Yao (1990), i.e.,* $a_1^i = a_2^j = 10$ *and* $a_2^i = a_1^j = 30$*. Under this condition, we obtain*

$$b_i^* = q_j^* = \frac{10\left(9\pi^i\pi^j - 18n\pi^i\pi^j + n^2\left(-1 + \pi^i + \pi^j + 8\pi^i\pi^j\right)\right)}{n^2\left(1 + 3\pi^i - \pi^j\right) + 3\pi^i\pi^j - 2n\pi^i\left(2 + \pi^j\right)}$$

which is concavely increasing in π^i*.*

This results is the same as in Shubik and Yao (1990) for specific values of π [6]*.*

Motivated by this example, we can comment on several things. First, it shows that our expanded version of the Bid-Offer model collapses to the traditional version by choosing the belief parameters accordingly. Second, we find that there is a unique expression for $b_i^* = q_j^*$ which depends on the agents' beliefs and endowments as well as on the number of players participating in the game. Finally, this expression shows a concave relationship between $b_i^* = q_j^*$ and π^i which makes intuitive sense. The more likely player i deems state 1 to be, the more of contingent commodity 2 they are going to sell. Because of the assumed risk aversion of player i, we expect this relationship to be concave.

When moving our attention to Shapley's Window model, we find that agents are facing a significantly more involved optimisation problem. Specifically, they choose an $L \times L$ matrix, a^i whose $k\ell$-entry $a_{k\ell}^i$ indicates the amount of commodity

[6] Let $\Gamma^{Bid-Offer}$ be a Bid-Offer game with $2n$ agents. Let there be one type i of n of these agents with beliefs π^i and another type j of n agents with beliefs π^j. Then, using the endowment specifications of Shubik and Yao (1990) and as $n \to \infty$, the equilibrium will converge to:

$$\lim_{n\to\infty} b_i^* = q_j^* = 10\frac{8\pi^i\pi^j + \pi^i + \pi^j - 1}{4\pi^i - \pi^j + 1} \tag{1.9}$$

For the special case of $\pi^i = \pi^j$, this competitive equilibrium will have the solution $b_i^* = q_j^* = 10$ which is the solution of Shubik and Yao (1990).

k they are offering in exchange for commodity ℓ. In the two commodity case, the strategy set is still manageable. However, with increasing *windows*, the complexity of the strategy set grows exponentially. Crucially, the larger the commodity space, the more severe the impact of price contagion as can be seen in Equation 1.4. In the Window model, a given relative price will indirectly rely on all other relative prices. This is a key feature of W-mechanisms when compared to TP-mechanisms.

Remark 1. *Let* $\Gamma^{Bid-Offer}$ *be a Bid-Offer game and let* $\Gamma^{Double-Auction}$ *a double auction game as per the definitions in section 1.2. Let the aggregation of strategies over all agents be continuous, not constant and there exist positive gains from trade* [7]*. Then, as* $n \to \infty$*, the equilibrium of* $\Gamma^{Double-Auction}$ *will converge to the equilibrium of* $\Gamma^{Bid-Offer}$*.*

Proof. Consider two types of traders i and j. Both of them have strategies $\left(p^i, q^i, \tilde{p^i}, \tilde{q^i}\right)$ and $\left(p^j, q^j, \tilde{p^j}, \tilde{q^j}\right)$, respectively. Without loss of generality, further assume no wash sales, i.e., $\tilde{p^i} = \tilde{q^i} = 0$ and $p^j = q^j = 0$. Naturally, the strategies of the two types of traders simplify to (p^i, q^i) and $\left(\tilde{p^j}, \tilde{q^j}\right)$.

Since i and j denote types, the strategies of individual agents will follow a distribution with some p^i_{min}, q^i_{min} and $\tilde{p^j}_{max}, \tilde{q^j}_{max}$. As long as the distributions of (p^i, q^i) and $\left(\tilde{p^j}, \tilde{q^j}\right)$ are continuous, the aggregation of n agents strategies converges to a continuous function for demand and supply. Given the realistic assumption that $p^i_{max} > \tilde{p^j}_{min}$ and $p^i_{min} < \tilde{p^j}_{max}$, we know $\tilde{q^i}_{min}$ that a unique crossing point will exist by the Intermediate Value Theorem.

It is then left to show that the intersection of the strategy sets coincides with the bid-ask equilibrium, i.e., $p^* = \frac{B_k}{Q_k}$. Observe that B_k is nothing but the total customer expenditure, $\int_0^{q^*} p^* dq$, which is equal to $Q_k p^* = q^* p^*$. □

The idea for this proof is already hinted at in Dubey and Shubik (1980). They argue that the double auction market is characterised by discontinuous payoff functions that are obtained from aggregating the strategies of only few agents. As

[7] The intuition behind these assumptions becomes evident in Remark 2 in conjunction with Figure 1.3

the number of agents increases, this discontinuity decreases and vanishes in the limit. Once this discontinuity is gone, the special cases in Figure 1.3 no longer apply.

Remark 2. *It is possible to argue under which conditions this convergence breaks down to yield the corner solutions presented by Dubey and Shubik (1980) and illustrated in Figure 1.3. If* $p^a_{max} < \tilde{p^b}_{min}$*, we obtain the no-trade scenario of Figure 1.3 (a). If the distribution of the strategy parameters violates a non-satiation assumption such that the aggregation of strategies is non-continuous, we may obtain the indeterminacy problem of Figure 1.3 (b). If* p^a_i*,* q^a_i*,* $\tilde{p^b_i}$ *and* $\tilde{q^b_i}$ *are all equal to constant for all agents, we obtain price indeterminacy as shown in Figure 1.3 (c).*

1.4. Simulations

Up until now, we usually modelled agents to have asymmetric, yet similar, wealth across states to derive tractable results so that to articulate the differences between the equilibria of the presented price formation mechanisms. In the real word, however, agents are in no way identical and wealth is not distributed uniformly across states. In the following section, we will relax both of these assumptions. In a first instance, we will relax the assumption that wealth is distributed uniformly across agents and replace this with distributions of wealth that are skewed. Afterwards, we will also vary the endowment of agents across states to analyse the impact of aggregate uncertainty on the presented mechanisms of price formation.

We do so by introducing a series of discrete distributions, denoted θ, from which we sample the endowments of agents a_i. The procedure entails the following: First, we consider one of the discrete distributions outlined and visualised in Figure 1.7. We then determine its maximum and denote it as the endowment of a marginal agent n, a^n so as to consider the minimal impact of belief disagreement possible. Afterwards, we take the remainder of θ without a^n and reinterpret these values as the set $\{a^1, ..., a^{(n-1)}\}$. This marginal n-th agent is modelled to be an agent with differing beliefs. As an example consider a game of type $\Gamma^{Sell-All}$.

Agents $1, ..., n-1$ optimise:

$$u^i = \pi \log\left(\frac{b_1^i}{p_1}\right) + (1-\pi)\log\left(a_2^i - b_1^i + a_1^i p_1\right)$$

while agent n optimises:

$$u^n = \pi^n \log\left(\frac{b_1^n}{p_1}\right) + (1-\pi^n)\log\left(a_2^n - b_1^n + a_1^n p_1\right)$$

where $\pi \neq \pi^n$. Employing this methodology gives us a way to easily compare and contrast the impact of an irrational marginal agent in different regimes.

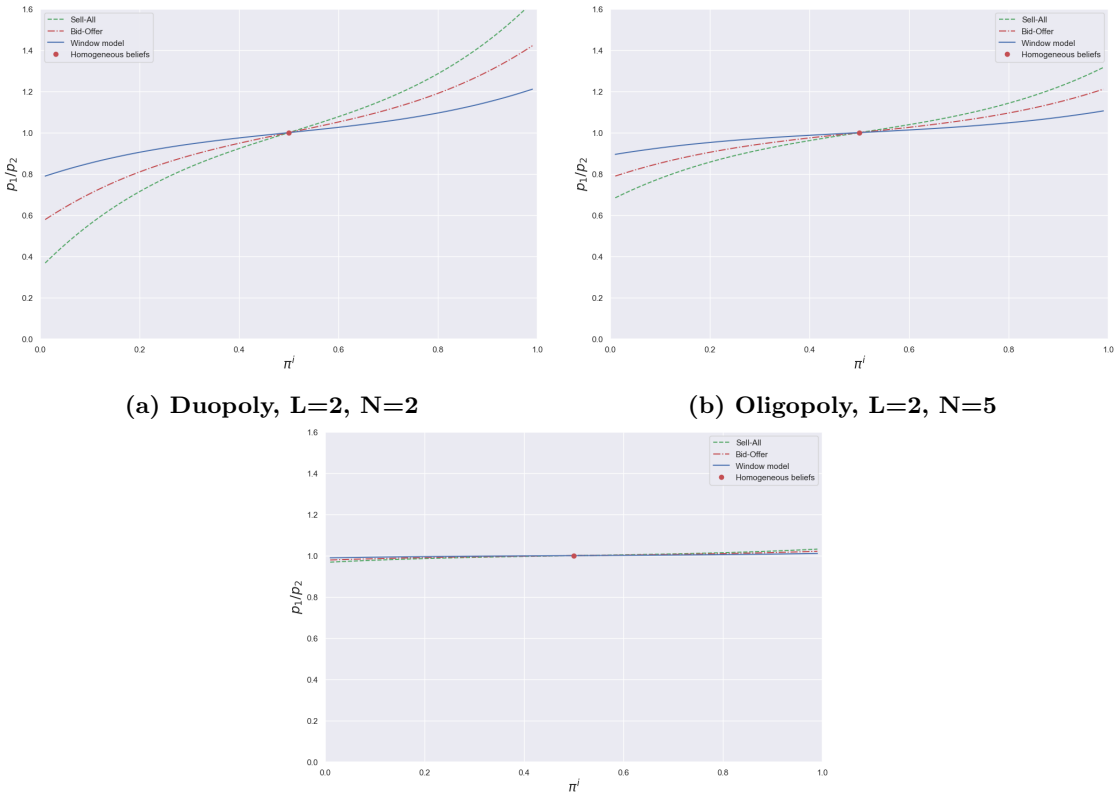
We note that studied mechanisms of price formation, in particular the Window model, generally, do not yield a unique equilibrium. In order to avoid the formulation and application of an extensive equilibrium selection routine, we place some restrictions on the bid matrix B in the Window model. Specifically, we impose the matrix B to be *irreducible*.

Definition 6 (Definition 2 in Sahi and Yao (1989))**.** *A non-negative square matrix A is said to be irreducible if for every pair $i \neq j$, there is a positive integer $k = k(i,j)$ such that $a_{ij}^{(k)} > 0$; where $a_{ij}^{(k)}$ denotes the i,j-th entry of the k-th power A^k of A.*

This property ensures that any two commodities are connected through some number of matrix self-multiplications. By ensuring that the bid matrix cannot be decomposed into smaller, independent parts, the solution of the model is simplified to the extent that the equilibrium will be unique [8]. While this assumption is trivial in our two commodity simulations, it is not obvious why it should hold in more general settings. Relaxing this assumption, however, leads to a problem of multiplicity and necessitates a discussion about equilibrium selection, e.g., Harsanyi and Selten (1988).

[8] The proof of this claim is presented as the proof of Lemma 2 in Sahi and Yao (1989).

1.4.1. More Players



(a) Duopoly, L=2, N=2

(b) Oligopoly, L=2, N=5

(c) Competition, L=2, N=50

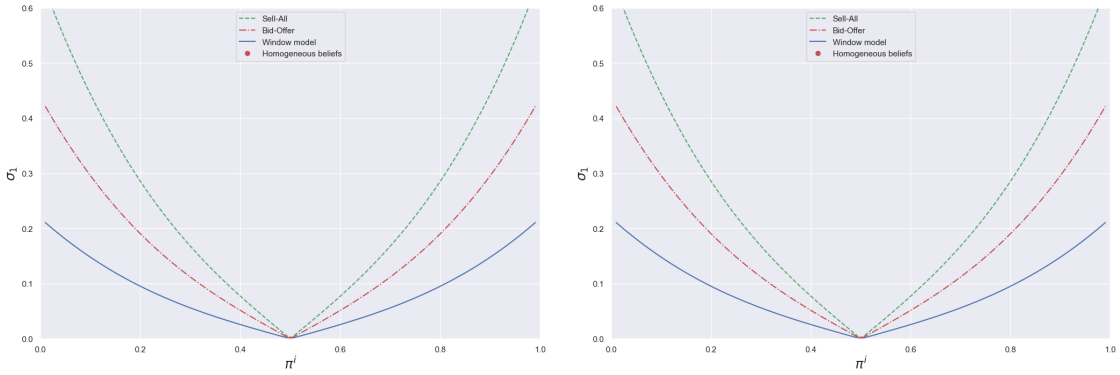
Figure 1.5. Impact of number of players on volatility:
Above are visualisations of the impact of the varying the number of players for each price formation mechanism, holding the number of contingent commodities constant.

Adding more players decreases the impact of the marginal individual on the price system. While their impact is significant in settings of duopoly and oligopoly in panels (a) and (b) of Figure 1.5, the effect almost vanished when increasing N to a larger number. In panel (c) of Figure 1.5 we observe that the impact of the marginal disagreement is almost zero for $N = 50$. For this first simulation, the intuition is that the importance of the one marginal agent diminishes as there are more players participating in the game. This is the kind of market liquidity that can "heal" or at least moderate market frictions in the sense of Dubey and Shubik (1978).

1.4.2. More Commodities

Up until now we have confined our simulations to scenarios with a fixed number of commodities and varying numbers of agents. While this is usual practice in literature, we demonstrate that the number of commodities is important for the level of price distortion, welfare and volatility. To illustrate the effects of adding more commodities, we will repeat the simulations above and increase the number of commodities while keeping the number of agents constant. Doing so, we do not consider the case of $N \to \infty$ and $L \to \infty$ simultaneously to isolate the impact of commodity space changes to the respective equilibria.

With more than two assets, the relative price of assets one and two is no longer a suitable metric to study. Instead, we now study the change in overall market volatility as defined in Equation 1.6 which is capable of capturing the effect of beliefs across several commodity markets.



(a) Uniform Wealth, L=2, N=2 **(b) Uniform Wealth, L=5, N=2**

Figure 1.6. Impact of number of commodities on volatility: Above are visualisations of the impact of the varying the number of commodities for each price formation mechanism, holding the number of agents constant at N=2.

The figure above illustrates the impact of increasing the number of commodities in all states of nature on market volatility. The horizontal axis illustrates the subjective belief that type i holds about the probability of state 1 while the vertical axis shows the level of volatility.

Remark 3. *Increasing the number of commodities does not affect the impact of heterogeneous beliefs on relative prices and their volatility given homothetic preferences.*

Proof. Consider a game $\Gamma^{Sell-All}$ with agents trading a set of commodities C. For simplicity, assume initially $|C| = 1$. The agent's maximisation problem is:

$$\begin{aligned} \max u &= \pi \log\left(\frac{b_1}{p_1}\right) + (1-\pi)\log\left(\frac{b_2}{p_2}\right) \\ \text{s.t.} \quad & b_1 + b_2 = a_1 p_1 + a_2 p_2. \end{aligned}$$

First order conditions imply:

$$b_1 = \pi(a_1 p_1 + a_2 p_2), \quad b_2 = (1-\pi)(a_1 p_1 + a_2 p_2)$$

Let T denote total wealth so that $T = a_1 p_1 + a_2 p_2$, so we have $b_1 = \pi T$ and $b_2 = (1-\pi)T$.

Now, increase the number of commodities, $|C'| = 2$. Homothetic preferences dictate the utility structure remains, and the new optimisation problem is:

$$\begin{aligned} \max u &= \pi \log\left(\frac{b_{11} + b_{21}}{p_{11} + p_{21}}\right) + (1-\pi)\log\left(\frac{b_{12} + b_{22}}{p_{12} + p_{22}}\right) \\ \text{s.t.} \quad & (b_{11} + b_{21}) + (b_{12} + b_{22}) = (a_{11} + a_{21})p_{11} + (a_{12} + a_{22})p_{12} \end{aligned}$$

The budget is still allocated across states as per π and $1-\pi$, resulting in:

$$b_{11} + b_{21} = \pi \sum_{i,j} a_{ij} p_{ij}, \quad b_{12} + b_{22} = (1-\pi) \sum_{i,j} a_{ij} p_{ij}$$

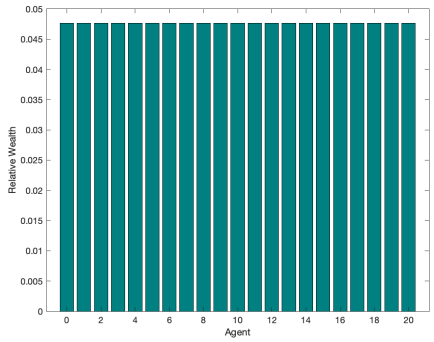
Let $T' = \sum_{i,j} a_{ij} p_{ij}$, thus $b_{11} + b_{21} = \pi T'$ and $b_{12} + b_{22} = (1-\pi)T'$.

Hence, relative expenditures across different states remain unchanged with the increase in the number of commodities. □

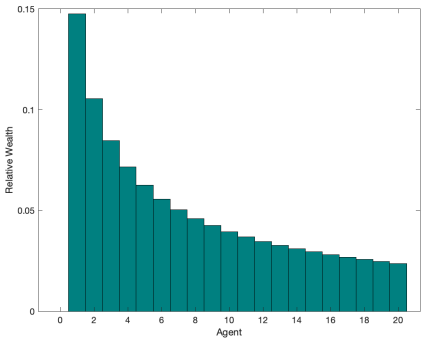
There is one special case in which the expansion of the number of commodities will indeed moderate the impact of the heterogeneous beliefs: disagreement about

the state space tomorrow. While Radner (1968) shows that the Arrow-Debreu view of contingent commodities does not yield a competitive-equilibrium price system, this must not be the case for strategic market games. [9] This observation is highly relevant to our example since an expansion of the commodity space that is identical across agents will indeed not change liquidity for hedging against a particular state. However, if we are in a regime where agents do not agree on the tree of possible states tomorrow as a whole, the introduction of new commodities may help moderate the impact of heterogeneous beliefs. Since infinite short selling is impossible in SMGs by construction, the effect of state disagreement may lead to more liquidity in the market for contingent commodities without losing equilibrium.

1.4.3. Wealth Distributions



(a) Uniform Distribution (Benchmark): All agents have an equal share of overall wealth.



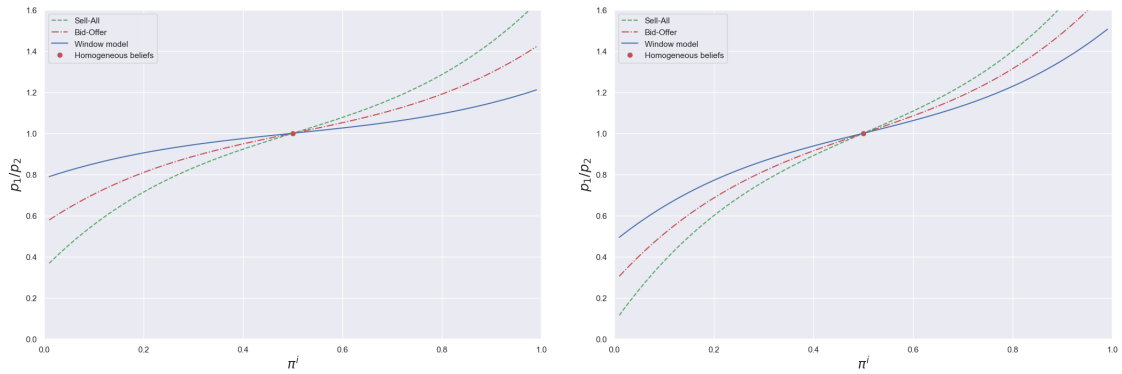
(b) Log-normal Distribution ($\mu = 5$, $\sigma = 3$): There are some differences between agents' wealths.

Figure 1.7. Wealth Distributions (no aggregate uncertainty): Above are the visualisations for two distributions of wealth. Each panel shows the relative share of wealth held by any agent on the vertical axis and a discrete counter for $N = 21$ agents. Panel (a) shows our benchmark case, wealth is uniformly distributed across agents. Panel (b) introduces some limited inequality by assuming a log-normal distribution of wealth.

To observe how a change in endowment wealth structure impacts the relative prices in a game, we simulate the strategic market games outlined in Section 1.2 for the various wealth distributions highlighted in Figure 1.7. Because the double

[9] The general proof is given in Dubey and Shubik (1978) who demonstrate that a noncooperative equilibrium will always exist, even when players disagree about the state space tomorrow.

auction mechanism requires significant assumptions about market microstructure and collapses to the Bid-Offer model under the conditions described in Remark 1, we choose to exclude it from the simulations. Crucially, we model the richest market participant as the marginal, irrational agent. This approach attempts to illustrate how a generally rational economy would react to one, marginal optimist/pessimist.



(a) Uniform Distribution (Benchmark): All agents have an equal share of overall wealth.

(b) Lognormal Distribution ($\mu = 5$, $\sigma = 2$): There are some differences between agents' wealths.

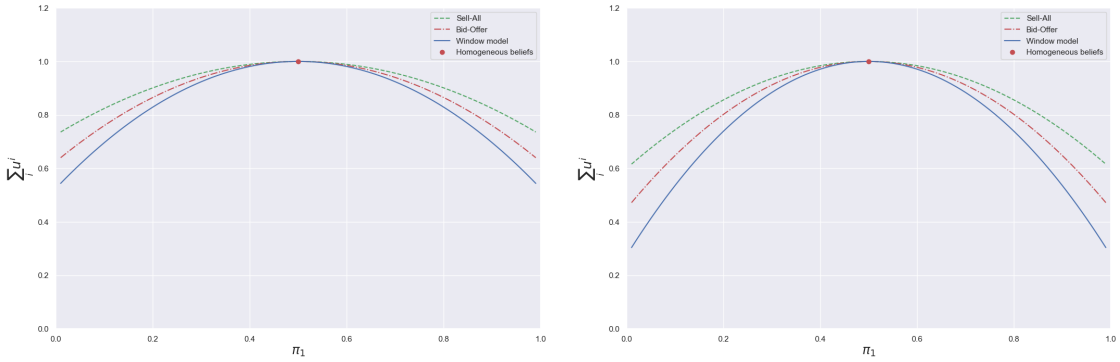
Figure 1.8. Impact of Wealth Distributions on Prices: Above are visualisations of the impact of the two distributions of wealth presented in Table 1.7 on relative prices for each price formation mechanism.

As can be seen from the figures above, there is a clear difference between the three price formation mechanisms studied. The left panel of Figure 1.8 shows that assuming uniformly distributed endowment wealth, the relative price of the impacted state changes most significantly under the Sell-All regime. The price impact of beliefs is more moderated in context of the Bid-Offer mechanism and the lowest in Shapley's Window model.

When inspecting the optimal bids and offers across different specifications of the Bid-Offer and Window models, it becomes apparent that the irrational agent chooses to market more of their endowment in subjectively less-likely states to spend it on consumption in subjectively more likely states. This behaviour means that the agents' beliefs collide with the market more than once (i.e., when selling and buying). In the Sell-All model, however, the agents are forces to sell all of their endowment so that beliefs are only revealed once - when buying contingent commodities from the market. Since we model the most wealthy agent to be the

agent with differing subjective beliefs, the impact of irrationality is amplified when modelling wealth to be distributed log-normally.

As illustrated in the figures below, there are notable differences in ex-post wealth distribution between the three price formation mechanisms studied. In the left panel of Figure 1.9, we observe that under the assumption of uniformly distributed endowment wealth, the distribution of ex-post wealth is most significantly impacted by the Sell-All regime. The effect of beliefs on ex-post wealth distribution is more moderate in the context of the Bid-Offer mechanism than in the Sell-All model and least pronounced in the Shapley Window model.



(a) Uniform Distribution (Benchmark): All agents have an equal share of overall wealth.

(b) Lognormal Distribution ($\mu = 5$, $\sigma = 2$): There are some differences between agents' wealths.

Figure 1.9. Impact of Wealth Distributions on Aggregate Wealth: Above are visualisations of the impact of the two distributions of wealth presented in Table 1.7 on aggregate wealth for each price formation mechanism.

The observations of the above simulations can be summarised as follows: more traders, more states and more liquidity all lead to thicker markets that can absorb the impact of heterogeneous beliefs more easily. While all of these levers ultimately contribute to the same effect, their efficiency varies and is moderated by the design of the market mechanism.

As shown in Dubey and Shubik (1978), a similar liquidity effect can be described by considering the mechanisms themselves. The expansion of the strategy set in the Bid-Offer game resulting from allowing wash sales leads to the paradoxical outcome that a market with heavy wash sales, i.e., an increase in liquidity that

diminishes the marginal effect of any individual trader on the price system, is preferred by all market participants.

We can also combine the contents of Figure 1.10 to illustrate the dynamics of the equilibria that result from varying subjective beliefs in terms of price volatility and welfare simultaneously.

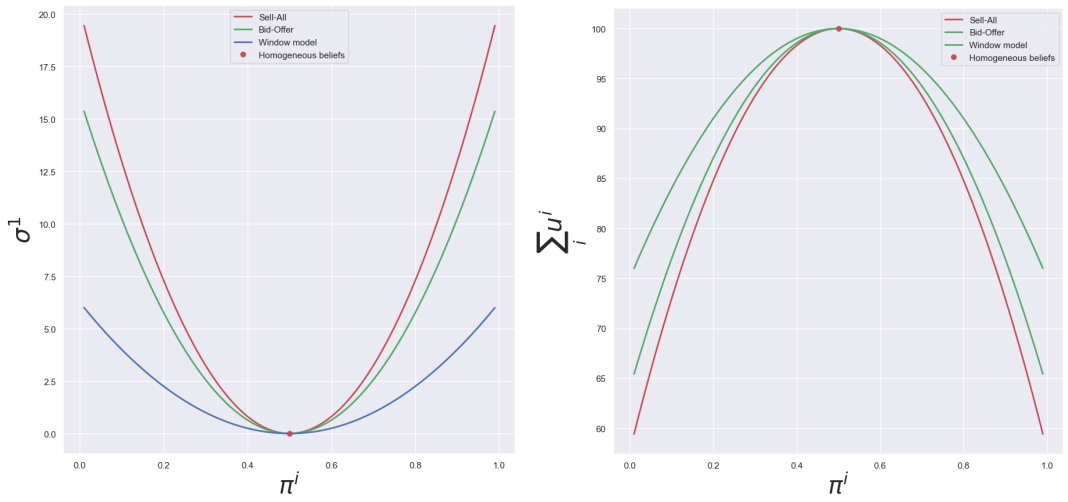


Figure 1.10. Impact of Disagreement jointly on Welfare and Price Volatility Figure displays the link between subjective beliefs of a marginal agent i on price volatility as defined in Equation 1.6 in the left panel. In the right panel, it visualises the link between subjective beliefs of a marginal agent i on overall ex-post utility. In both figures, a red reference point indicates the equilibrium under homogeneous beliefs. The two panels can be combined into a three-dimensional diagram (see Appendix).

We find that the dynamics of price volatility and ex-post welfare with respect to changes in subjective beliefs show exponentially increasing and decreasing behaviours, respectively. The intuition for the reduction of ex-post wealth is that the identification of seemly beneficial trades to smooth consumption across states grows as the subjective belief disparity grows. However, since the beliefs of agent i, π^i, do not materialise, these trades end up reducing the smoothness of the consumption plan. As utilities are concave, deviations from the optimal consumption plan grow in magnitude as the agent approaches a state-contingent consumption of zero in one of the states. While the other agent gains utility from these trades as they ex-post underpay for their consumption plan, their gain

cannot offset the loss of utility incurred by agent i, thus leading to an overall decrease of $\sum_i u^i$. A similar argument can be applied to the dynamics of prices volatilities and beliefs. As the subjective belief of agent i moves further away from 0.5, the compensation required by their counterparty increases convexly to offset the growing non-smoothness of their consumption plan. The result is the parabola-shaped dynamic depicted in the left panel of Figure 1.10.

1.4.4. Matching Stylised Facts

The simulations above allow for the comparison of theoretical findings to stylised facts observed in real markets. We present three of these stylised facts.

First, most trade happens on centralised exchanges. While this mode of trading has long been established practice for stocks and funds, it has become more prevalent in commodities and standardised derivatives. Most recently, even cryptocurrencies have been introduced to exchange-based trading. On the surface, this seems counterintuitive as cryptocurrencies are inherently over-the-counter products that aim to make a financial intermediary redundant. While there are several reasons for why one would want to trade these products on an exchange, one argument is that the double auction price formation mechanism implemented by an exchange reduces price volatility.

Second, commodity producers increasingly leverage financial innovation such as futures and derivatives to sell their produce (Prager et al., 2020). While this behaviour allows farmers to hedge against volatility in spot prices throughout the year, it also has a rationale that is anchored in price formation since they can move from a Sell-All price formation mechanism to a double auction price formation mechanism. This switch in price formation mechanism reduces the volatility that stems from market microstructure and, therefore, increased ex-post welfare.

Third, our simulations can rationalise and motivate regulations to promote interconnectedness of markets such as the National Best Bid and Offer (NBBO) in the United States. The NBBO ensures that brokers aggregate prices and quantities from multiple exchanges to give customers the best-possible prices. On the one

hand, regulations like the NBBO protect customers in financial markets by limiting the degree of freedom market makers have in fulfilling orders. On the other hand, this interconnection of markets effectively implements a Window model market microstructure that minimises price volatility due to belief heterogeneity. We observe a counterexample to this in the cryptocurrency market. Despite significant volume and low-friction, digital platforms, we see arbitrage opportunities, increased volatility and lower efficiency than in traditional markets (Makarov and Schoar, 2020). This is partially due to missing regulation and the market markets' incentives to (not) innovate on their own (Budish et al., 2019) and the resulting bid-offer market structure.

1.5. Conclusion

In conclusion, this paper has introduced a simple framework of subjective beliefs and demonstrating conditions under which these mechanisms converge to the competitive equilibrium. We show that the equilibrium of Shapley's Window model is closer to the competitive general equilibrium solution of the games than the equilibria of the Sell-All and Bid-Offer models.

We have gained insights into the impact of beliefs in the context of strategic market games, examining the effects of belief disagreement on relative prices, wealth distribution, and volatility.

Our findings suggest that price formation mechanisms that are theoretically closer to General Equilibrium, such as Shapley's Window model, exhibit greater resilience to belief disagreement compared to Cournot-type models. This highlights the importance of selecting appropriate price formation mechanisms when modelling markets and understanding the potential consequences of beliefs in different market structures.

Furthermore, this paper is relevant to any Machine Learning model that models trade. While the advances in hardware and Machine Learning software are pushing the boundaries of what can be simulated, one needs to consider what implications the choice of price formation mechanism has for the game's equilibrium.

Getting back to the first paragraph of the introduction, an analysis of the different markets for coffee beans needs to, among other things, consider which price formation mechanism is appropriate. Modelling the spot market for coffee during harvest using a Window model architecture may inadvertently predict less volatile prices and higher expected welfare than may be realistic. It is important to note that there are many confounding factors one would have to account for in fitting empirical data to the theory of strategic market games.

In future research, we aim to further expand our understanding of these price formation mechanisms, exploring additional market structures and investigating the role of other factors, such as more formalised asymmetric information and transaction costs, in shaping market outcomes. While this paper considered the properties of god-given price formation mechanisms, we do not consider how price formation mechanisms emerge in the context of financial intermediaries. Such a study could use the empirical emergence of dark pools to study how market makers incentives promote financial innovation. Additionally, we seek to refine our models to better capture the nuances of real-world markets and incorporate learning algorithms to enhance our understanding of how these mechanisms interact with artificial intelligence systems.

2

Subjective Risk Premia in Bond and FX Markets

Joint work with Ilaria Piatti & Paul K. Whelan

Abstract

This paper elicits subjective risk premia from an international survey dataset on interest rates and exchange rates. Survey-implied risk premia are (*i*) unconditionally negative for bonds, positive for investment currencies and negative for funding currencies, (*ii*) cyclical and correlated with subjective macro expectations; (*iii*) predict future realised returns with positive sign. Deriving a subjective belief decomposition, we estimate a multi-country asset pricing model with three probability measures: the risk-adjusted, physical and a subjective measure. The structural estimation quantifies the size of financial market belief distortions and demonstrates that subjective risk premia are well understood by a classical risk-return trade-off.

2.1. Introduction

Measuring expected returns is a long standing endeavour in financial economics; usually inferred by approximating conditional expectations with projections of realizations onto observable factors. Projection approaches are problematic for a number of well known reasons. For example, projections depend crucially on the researcher's choice of covariates. Moreover, proxying for expected returns with realized returns assumes that information surprises cancel out during the sample period studied, which in the context of trending interest rate markets is unlikely.[1]

This paper advocates an alternative approach. We extract subjective expected excess returns on government bonds and exchange rates from a panel of professional forecasters who are local specialists in their respective domestic markets.

The belief panel is available for the G10 countries at a monthly frequency for the sample period January 1995 - December 2020, and allows us to jointly measure (i) model free real-time risk premia; and (ii) expectation errors across both sovereign bond and exchange rate markets. The joint properties of subjective risk premia and expectation errors shed light on the validity of existing asset pricing models, and provide guidance for the design of future models that incorporate deviations from full information rationality.

Studying their empirical properties, we first show that subjective bond risk premia (BRPs) are negative on average but can change sign, being persistently positive. An unconditionally negative BRP is consistent with predictions from many leading equilibrium models. Interestingly, the signs of subjective BRP line up with those predicted by the capital asset pricing model (CAPM). To highlight this point, we compute stock-bond correlations ($corr_{SB}$), and show that the well-studied change from positive to negative $corr_{SB}$ that occurred in the late 1990's in the U.S. carries over to the cross-section of countries in our sample. This is an intriguing observation since observing $corr_{SB} < 0$ suggests bonds should also be hedges according to CAPM logic. Moreover, in the 1990s when $corr_{SB} > 0$ we

[1] For a detailed discussion of this point see the American Finance Association Presidential Address of Elton (1999).

do, in fact, observe BRPs which are largely positive. Thus, the sign of subjective BRPs is consistent with a risk based view of asset pricing.

Second, we explore the cyclical properties of *subjective* BRP through a series of panel regressions on *subjective* expected growth measures, also obtained from surveys, estimating a strong and robust negative relationship; thus, subjective BRP are counter-cyclical even if innovations to *realized* interest rates and macro quantities are largely unspanned. This is an important result since leading asset pricing models featuring priced long run risks (Bansal and Yaron, 2004), habit preferences (Campbell and Cochrane, 1999), and rare disasters (Wachter, 2013) predict that risk premia vary cyclically and are increasing in states of high marginal utility (low realized growth or expected growth).

Third, subjective BRPs are significantly positively linked to the realized volatility of bond returns; thus, survey expectations preserve the basic risk-return relation which predicts a tight link between quantities of risk and compensation for risk. This is an important take-away since detecting a link between realized returns and measures of volatility is notoriously difficult (see, e.g., Moreira and Muir, 2017 or Eraker, 2018).

Results for subjective exchange rate risk premia (XRP) are consistent with "carry trade" intuition. The average exchange rate risk premium for the standard "investment currencies" within the carry trade, i.e. Switzerland and Japan, is largely negative, while it is largely positive for Norway and Australia. For the remaining countries the average XRP is closer to zero, but it is still highly time-varying. Exchange rate risk premia display a large degree of co-movement, with all pairwise correlations being positive and significant, equal to 50% on average. The large cross-sectional correlation between individual countries risk premia, which we document in both fixed income and currency spaces, is consistent with highly globalized markets, in which sources of risk and risk compensations are common across countries. Subjective exchange rate risk premia also display strong cyclical properties shedding light on the long withstanding exchange-rate macro-disconnect puzzle, which grapples with the question "why are exchange rates so volatile yet apparently

disconnected from fundamentals ?".[2] We provide a potential resolution to this puzzle by showing that while volatile realized innovations are largely uncorrelated, subjective expectations of returns and macro aggregates are clearly related.

Next, we estimate a set of return predictability regressions in order to characterise the informational content of subjective risk premia as a signal about futures realized returns. We compare forecasts from subjective expectations to benchmark price based predictors, considering interest rate differentials and slopes of the yield curve, for the exchange rate and bond realized returns, respectively. Summarising, we show that ex-ante beliefs about returns significantly forecast realized returns, in both economic and statistical terms, and with the correct sign according to rational expectations. At the same time, survey expectations are "unspanned" by interest rate spreads which also significantly predict future excess returns. This implies that, while positively correlated with future realized returns, survey implied forecast errors are predictable by date t information, suggesting that beliefs are not fully rational consistent with existing literature (e.g., Frankel and Froot, 1987 and Cieslak, 2017).

Studying the properties of expectation errors we show that for all countries in our sample surveys over-predicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante. For exchange rates, the mean expectational error are close to zero, meaning that there is no systematic bias. Interestingly, forecast errors are highly correlated across countries. Finally, while errors display only mild persistence sampled at annual horizons, this translates into predictability by interest rate spreads, consistent with the 'lack of spanning' result discussed above.

Finally, we derive a subjective multi-country general equilibrium model in which investors fully optimize their consumption and investment choices conditional on beliefs formed under a subjective measure. We explicitly consider a (exogenous) distortion between the subjective and the physical measure, to allow for forecast

[2]This puzzle has been studied in various guises beginning from the seminal works of Meese and Rogoff (1983) in the context of a forecasting puzzle. Rogoff and Stavrakeva (2008) provide and survey of the literature. Related to our paper, Stavrakeva and Tang (2020) provide a complimentary analysis on the disconnect puzzle by exploiting subjective surprises obtained by combining subjective expectations and realized quantities.

errors that are not zero on average. Therefore, we model three probability measures in total: the subjective measure of the agents in the economy, the physical measure of the econometrician and the risk-neutral measure under which prices are computed, assuming no arbitrage and complete markets. Combining subjective risk premia, expectation errors and realized asset prices we estimate the model via simulated method of moments. The estimated model fits subjective risk premia and forecast errors well, with a relatively small distortion between subjective and physical probability measures, which suggests that subjective risk premia follow a classical risk-return trade off regardless of whether survey expectations are rational or not.

Related Literature: Our paper is related to a large literature that seeks to measure expected returns. In currency markets, Chernov et al. (2020) argue that expected returns can be summarised by a low dimensional set of signals (interest differential, trend, and mean reversion) while Kremens and Martin (2019) test a theory based expected currency return signal based on the prices of quanto index contracts. In bond markets, Bauer and Hamilton (2018) revisit much of the literature on expected bond return signals and argue that a low dimensional number of principal components span the vast majority of information about bond risk premia. Our paper compliments these works by proposing a summary measure for expected returns across currency and bond markets, extracted directly from survey expectations, as opposed to inferred from prices.

Piazzesi et al. (2015) is probably the first paper to construct subjective (bond) risk premia from surveys. These authors point out that while statistical measures of bond risk premia are volatile and countercyclical, subjective premia are less volatile and not 'very' cyclical. A related recent point is made by Nagel and Xu (2022) who analyse survey-based risk premia across different asset classes. These authors argue subjective risk premia are acyclical. We contribute to the debate on cyclicality, focusing on a cross-section of sovereign bond and currency markets, and demonstrate that subjective risk premia are indeed cyclical, if one considers measures of *subjective* macro growth rates also extracted from surveys.

Deriving a subjective belief decomposition we derive and study an equilibrium model with subjective beliefs. Thus, our paper relates to a growing literature that embeds subjective beliefs and belief distortions in equilibrium models (Brunnermeier and Parker (2005), Chen et al. (2020), Maenhout et al. (2021) and Bhandari et al. (2022)). In terms of the estimation, Chernov and Mueller (2012) also exploit information in the term structure of survey-based forecasts of inflation to estimate a term structure model that allows for differences between risk-adjusted, subjective, and physical probability measures.

2.2. Data

Survey Data: From Consensus Economics (CE) we collect professional financial market participants monthly forecasts of (*i*) spot exchange rates; and (*ii*) yields on 10 year government bonds for a variety of countries.[3] We focus on the most heavily-traded G10 currencies vis-a-vis the United States (USD): Australia (AUD), Canada (CAD), Switzerland (CHF), Europe (EUR), United Kingdom (GBP), Japan (JPY), New Zealand (NZD), Norway (NOK) and Sweden (SEK). CE reports projections for two horizons, 3 and 12 months, for both exchange rate and interest rate expectations. We focus on the 12-month forecasts which is the horizon where predictable variation in risk premia is most likely to arise. Forecasts begin in (*i*) 1990 for the USD, CAD, EUR, GBP, JPY; (*ii*) in 1995 for AUD, NZD and SEK; and (*iii*) in 1998 for NOK and CHF. All results in the paper, unless otherwise stated, are based on the period from January 1995 to December 2020, for a total of 312 monthly observations for all countries except NOK and CHF. In addition to forecasts of future asset prices CE covers projections for a large set of macro-economic variables. We focus on 1-year ahead real personal consumption growth and GDP growth, which are available for all G10 currencies.

CE has maintained a consistent questioning procedure over time and survey respondents face the same questions for each country. Forecasters receive the

[3] CE panellists provide par yield forecast which we treat as zero coupon forecasts. Section A.1 in the Online Appendix (OA) discusses the approximation error.

questionnaire in the first few days of the month, and survey forecasts are collected the second week of every month on Monday and then released by CE three days after on the Thursday of the same week. We sample all yields, spot rates and exchange rates on the date when the survey goes public, i.e. the release date, that is normally around the middle of the month, in order to avoid any look-ahead bias. Moreover, the survey focuses on experts for each region, with respondents generally located in the country for which they are asked to make predictions. Thus, the dataset is comparable across a large cross-section of countries and is available at monthly frequency for an extended sample period. Section A.1 in the (OA) provides additional details of the CE dataset.

Realized Data: We obtain monthly G10 FX spot and 3-month forwards from Refinitiv Eikon for the sample period January 1995 to December 2020. For the same panel, we obtain zero coupon bond yields, which are generally available for maturities 3, 6, 9 and 12-months, and 3, 5, 10 and 15 years from Bloomberg. Below we require country-specific yields for three bond maturities: the 12-month rate (risk-free), the 10-year rate, and the 11-year rate. While Bloomberg provides the former two series, it does not provide us with the latter. To remedy this, we fit a cubic spline to all available maturities and sample the desired yield. Realized second moments of bond and FX returns are measured at daily frequency by computing realized volatility from the sum of squared log returns between subsequent survey release dates, which are approximately $n = 22$ days apart. All volatility estimates are annualised. In addition, in foreign exchange markets, we exploit option-implied risk neutral variances constructed and discussed by Krohn et al. (2020).

We also use realized quarterly observations on real person consumption and consumer price inflation. These time-series are available for all G10 currencies for our sample period. Time-series originate from various sources but can be obtained directly from the "Trading Economics" archive.[4]

[4] www.tradingeconomics.com.

2.3. Framework and Notation

The price of an k-period bond satisfies the first order condition of a representative investor who forms their beliefs under a subjective measure ($\mathbb{S}$). This measure does not necessarily coincide with the objective measure ($\mathbb{S} \neq \mathbb{P}$), in which case the following decomposition holds

$$P_t^k = E_t^{\mathbb{S}}\left[M_{t+1}P_{t+1}^{k-1}\right] \tag{2.1}$$

$$= \underbrace{\frac{1}{R_t^f}E_t^{\mathbb{P}}[P_{t+1}^{k-1}]}_{\text{objective NPV}} + \underbrace{Cov_t^{\mathbb{S}}\left[M_{t+1}, P_{t+1}^{k-1}\right]}_{\text{subjective RP}} + \underbrace{\frac{1}{R_t^f}\left[E_t^{\mathbb{S}}[P_{t+1}^{k-1}] - E_t^{\mathbb{P}}[P_{t+1}^{k-1}]\right]}_{\text{forecast errors}}. \tag{2.2}$$

where M_{t+1} is the one period stochastic discount factor. The final line shows that subjective risk premia (studied in section 3.4) and forecast errors (studied in section 2.6) are flip sides of the same coin. In this section, we introduce notation and formulas used to compute subjective risk premia and later we study forecast errors.[5]

2.3.1. Risk premia in the fixed income market

Let $P_t^{(k)}$ be the time t price of a default risk-free zero-coupon bond of maturity k years. Spot k-year yields are then defined as $i_t^{(k)} = -\frac{\ln P_t^{(k)}}{k}$. The bond risk premium is defined as the expected excess return on the bond, so we start by computing the realized holding horizon return of a k-year bond, with a j-year holding horizon:

$$\ln \frac{P_{t+j}^{(k-j)}}{P_t^{(k)}} = -(k-j)i_{t+j}^{(k-j)} + ki_t^{(k)} \tag{2.3}$$

[5]The literature studying return predictability typically interprets expected excess log returns as risk premia. This is not quite correct since risk premia should be measured as expected excess simple returns. Assuming log-normality, arithmetic average returns differs from the geometric average returns by one half the variance (the Jensen's gap). Computing Jensen's terms from BlueChip Financial Forecasts, Buraschi et al. (2022) show that the Jensen's gap is tight using survey expectations on interest rates. Kyle and Todorov (2023) show that for currencies, the difference is indeed very small and that there are no higher-order risk premiums above the one on variance. In what follows we interpret expected log excess returns as risk premia.

The annualised expected excess return on a k-year bond with a j-year holding horizon is then:

$$E_t\left[rx_{t+j}^{(k)}\right] = -\frac{k-j}{j}E_t\left[i_{t+j}^{(k-j)}\right] + \frac{k}{j}i_t^{(k)} - i_t^{(j)}, \tag{2.4}$$

where continuously compounded yields are annualised and k and j are expressed in years. We denote the bond risk premium for maturity k and horizon j by

$$BRP_t^{(j,k)} \equiv E_t\left[rx_{t+j}^{(k)}\right]. \tag{2.5}$$

Note that under the expectation hypothesis (EH), $E_t\left[i_{t+j}^{(k-j)}\right] = f_t^{(j,k)}$, where $f_t^{(j,k)} = \frac{ki_t^{(k)} - ji_t^{(j)}}{k-j}$ is the forward rate for k periods starting from j periods from time t, so that the risk premium for investing in long-term bonds is zero, $BRP_t^{(j,k)} = 0$.

2.3.2. Risk premia in the foreign exchange market

Denote by x_{t+j} the log of the exchange rate, expressed in US Dollars per unit of foreign currency, and by Δx_{t+j} the change in the log exchange rate from time t to time $t+j$. Therefore, a positive Δx_{t+j} corresponds to a depreciation of the US Dollar relative to the foreign currency. The j-period interest rate in the foreign country is denoted by $i_t^{(j),f}$. The annualised log currency excess return is given by:

$$rx_{t+j}^{FX} = (i_t^{(j),f} - i_t^{(j)}) + \frac{1}{j}\Delta x_{t+j}. \tag{2.6}$$

The exchange rate risk premium is defined as the conditional expectation of the excess return in Equation (4), i.e., $XRP_t^{(j)} \equiv E_t\left[rx_{t+j}^{FX}\right]$:

$$XRP_t^{(j)} = (i_t^{(j),f} - i_t^{(j)}) + \frac{1}{j}\left(E_t\left[x_{t+j}\right] - x_t\right), \tag{2.7}$$

where $E_t\left[x_{t+j}\right]$ is the forecast of the exchange rate in j periods. $XRP_t^{(j)}$ can be interpreted as the annualised expected excess return of investing for j periods (using a j-period instrument, e.g. a 1-year bill if $j = 1$) in the foreign currency,

financing the investment selling the local currency (e.g. selling a 1-year T-bill). Also note that interest rates are annualised, so we also annualise the exchange rate change by dividing by j.

According to the uncovered interest rate parity (UIP), high interest rate countries are expected to experience an exchange rate depreciation to equalise expected exchange rate adjusted returns on assets. The idea behind UIP is that when the foreign interest rate is higher than the local interest rate, i.e., $i_t^{(j),f} > i_t^{(j)}$, the foreign currency will depreciate by the difference, i.e. x_{t+j} decreases by $i_t^{(j),f} - i_t^{(j)}$, so that in local currency terms the return on investing in the two countries is exactly the same, i.e. XRP is zero.

2.4. Subjective Risk Premia

We compute subjective bond risk premia and exchange rate risk premia for a 1-year forecast horizon j and a bond maturity k of *eleven years*. Since j and k are fixed, in the following we drop them and refer to the premia as BRP_t (bonds) and XRP_t (exchange rates).[6]

2.4.1. Subjective Bond Risk Premia

We compute bond risk premia from Equation (3). Figure 2.11 shows the time series of survey-based BRP_t and panel (a) of Table 3.34 reports summary statistics. Subjective bond risk premia are slightly negative on average, volatile and persistent. For example, the GBP BRP_t ranges between -8% around the year 2000 to +10% in the aftermath of the 2008 financial crisis. The JPY BRP_t is less volatile but still displays significant persistence ranging between -3% pre financial crisis to +7% post financial crisis.

[INSERT FIGURE 2.11 AND TABLE 3.34]

[6] CE Surveys provide us with the expected yield on a 10-year bond for a 1-year forecast horizon. In what follows, we compute subjective bond risk premia as the expected change in 11-year log bond prices above the 1-year risk free rate.

A negative BRP_t implies that default-free bonds are perceived as hedges, consistent with predictions from many structural models. Re-writing the definition for BRP_t above we have the following decomposition

$$BRP_t = 10 \times \left(i_t^{(11)} - E_t\left[i_{t+1}^{(10)} \right] \right) + \underbrace{\left(i_{t+1}^{(11)} - i_t \right)}_{\text{Term Spread}} . \tag{2.8}$$

On average, the second term is positive since the (nominal) term spread is typically positive; thus a negative BRP_t means that the first term in parenthesis is negative. Assume date t interest rate beliefs can be written as a random walk forecast plus an adjustment ϕ_t. The consensus belief can then be written as $E_t\left[i_{t+1}^{(10)} \right] = i_t^{(10)} + \phi_t$. Then, since empirically $i_t^{(11)} - i_t^{(10)} \sim 0$ this implies $\phi_t > 0$, i.e. conditional on the current level of the term structure, surveys expected long-term rates to rise. Actually, in turned out that in our sample there was an unprecedented decline in long term rates and survey forecasters made consistently biased forecasts. This point highlights the link between subjective risk premia and expectational errors, which we study in detail in sections 3.5 and 2.6.

Do forecasters believe bonds are genuinely hedges or is this an artefact of the sample period? Consider again Figure 2.11. In both first half and second half of the sample (when interest rates were relatively flat) we observe that $BRP_t < 0$ most of the time. Table A.10 in the OA confirms this point more formally by providing subsample statistics. Investigating this question further, Figure A.20 in the OA displays stock-bond correlations ($corr_{SB}$) computed as the rolling 200-day correlation between 10-year zero coupon bond returns and the corresponding major equity market indices in each country. The figure demonstrates that the well studied change from positive to negative $corr_{SB}$ that first occurred in the late 1990's in the U.S. is a common feature in the cross-section of G10 countries in our sample. This is an interesting observation since a negative $corr_{SB}$ suggests bonds should also be hedges according to a simple CAPM logic. Moreover, in the 1990s when $corr_{SB} > 0$ we do, in fact, observe BRPs which are largely positive. Thus, the average sign of subjective BRPs is consistent with a risk based view of bond pricing.

Another notable feature of our subjective measures of bond risk premia is their co-movement. Figure 2.11 displays a clear systematic pattern across countries, which is confirmed by a very high average cross-country correlation equal to 53%. All pairwise correlations are positive ranging from 19% between GBP and NOK, to 76% between EUR and AUD (Table A.12 in the OA).

In the time-series expected excess bond returns appear counter-cyclical, with risk premia being high in bad states such as the early 2000s and during the 2008 financial crisis, while in more recent years the average risk premia are lower, mainly negative and less volatile. We study the cyclical properties of subjective bond risk premia by estimating pooled OLS regressions of BRPs on *subjective* expectations of 1-year growth rates in real consumption and GDP, and panel (a) of Table 2.2 reports the findings. A constant is included in the regression but is not reported and we report 95% confidence intervals estimated using a circular block bootstrap with 1000 replications.[7]

[INSERT TABLE 2.2 HERE]

Considering specification (i) we find that lower expected real growth rates are strongly significantly linked to higher subjective bond risk premia with an $R^2 \approx 4\%$, while specification (ii) shows a similar relationship with expected GDP growth. Exploring the cyclical properties of subjective risk premia further, we estimate a regression of BPR_t on realized bond return variances $(\widehat{\sigma}_t^2)$ and realized excess bond returns measured over the past year, in specifications (iii) and (iv), respectively. Specification (iii)) shows a highly significant and positive link between subjective bond risk premia and the realized variance of bond returns, which is a simple proxy for the current quantity of risk in bond markets. Since quantities of risk are high in bad times this also demonstrates a link between survey implied risk premia

[7]The circular block bootstrap takes into account potential heteroskedasticity and autocorrelation. We use a block size of 12 observations, which is in the range of optimal block sizes for each of the countries' bond and exchange rate risk premia. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.18 of the OA. In a panel setting, $N \times T$ autocorrelated and spatial dependent observations contain less information than $N \times T$ independent observations. Driscoll and Kraay (1998) propose a nonparametric covariance matrix estimator that generates consistent standard errors which are robust to heteroskedasticity and general forms of temporal and spatial dependence.

and the state of the macro economy. Specification (*iv*) also shows a significant relationship to realized excess bond returns, which answers the question of whether subjective beliefs are extrapolative or not. The 95% confidence intervals do not display evidence of extrapolation. In fact, we find the opposite: past bond returns and consensus expectations of future returns are negatively correlated, consistent with predictions from benchmark asset pricing models with rational pricing of risk. Indeed, rational asset pricing models generally predicts that low prices due to discount rate variation generate high expected returns going forward.

2.4.2. Subjective Exchange Rate Risk Premia

We compute exchange rate risk premia using Equation (5). Figure 2.12 displays the dynamics of the XRPs and panel (b) of table 3.34 reports summary statistics. They are time-varying and volatile relative to their mean, with standard deviations ranging between 2.5% (CAD) and 5.6% (NZD). The average XRP_t is negative for Switzerland and Japan, equal to -1.7% and -2.8%, consistent with the idea that these are 'investment currencies' within the carry trade, and positive around 3.5% for Norway and Sweden. For the remaining countries, average exchange rate risk premia are smaller ($< 1.8\%$) but positive. All pairwise correlations between exchange rate risk premia are positive and their average is 50% (Table A.12). The XRP_t of Japan is less correlated to the remaining countries' XRP_t, in fact excluding Japan from the sample of countries the average pairwise correlation increases to around 63%. Interestingly, XPRs flip sign in a systematic fashion throughout the sample, being largely positive between 1995 – 2004 and oscillating between positive and negative values thereafter.

[INSERT FIGURE 2.12 HERE]

Next, we investigate the cyclicality of subjective exchange risk premia via pooled OLS regressions of XRP_t on *subjective* macro expectations, as we do for BRPs. From the perspective of a U.S. investor who sells USD and buys foreign currencies,

from a risk-sharing perspective, one should expect exchange rate risk premia to be correlated with both foreign growth and foreign growth relative to domestic growth.

Panel (b) of Table 2.2 shows the results of a panel regression of XRPs on subjective foreign country macro expectations. Specifications (i) and (ii) show that positive shocks to foreign country consumption and GDP growth rate expectations are associated with an increase in XRP_t, thus demonstrating that subjective expectations of business cycle dynamics are clearly linked to subjective exchange rate risk premia. Table A.22 in the OA shows that differences in subjective macro expectations defined as the foreign expected growth rate minus the U.S. expected growth rate are also, intuitively, associated with an increase in XRP_t. In bad times, the US Dollar is expected to appreciate due to its perceived safe-haven role, leading to a decrease in the expected exchange rate (expressed as US Dollars per foreign currency) and therefore a lower XRP.

Panel (b) of Table 2.2 also shows a positive and significant link between XRP_t and risk neutral return variance implied by FX options.[8] Finally, XRPs display evidence of mean-reversion around past realized returns, which contrasts with extrapolative beliefs that has been proposed as an explanation for return predictability by the behavioural finance literature.

2.4.3. Comparison with statistical risk premia

We now compare the dynamics of our survey-based bond risk premia (BRP) and exchange rate risk premia (XRP) with standard statistical models. Projection-based bond risk premia are obtained by regressing realized excess returns at time $t+1$ (one year holding period) on the slope of each yield curve at time t, defined as the spread between the 10-year and the 1-year yield. For exchange rate risk premia, we use the forecasts implied by 1-year interest rate differentials. Statistical forecasts are computed in sample so are subject to a look-ahead bias, contrary to the survey-based forecasts which are real-time. However, we do not attempt to

[8] We also find a positive and significant link to realized exchange rate variance although the statistical significance is smaller.

compute projection-based estimates in real time, i.e. out-of-sample, as our goal here is just to compare the time series dynamics of the survey and statistical-based foreign bond risk premia, not their predictive power.

Table A.11 in the OA reports summary statistics for risk premia based on statistical projections. Comparing summary statistics of subjective versus statistical projections reveals strong differences between the two sets of premia. For example, bond risk premia using statistical models are uniformly positive and do not switch sign, contrary to subjective BRP which are negative on average but frequently flip sign. The ranking of the country average exchange rate risk premia is clearly different than survey-based XRP. In particular, the negative exchange rate risk premia of SEK and NOK appear inconsistent with the standard intuition behind carry trade strategies.

Figure 2.13 displays time series of equally-weighted averages for each country's subjective risk premia against the same time series obtained with statistical models. The dynamics of the statistical model based premia are clearly very different in both panel (a) (bonds) and panel (b) (exchange rates) from the corresponding dynamics of the survey-based premia. In fact, their correlation is even slightly negative. In section 3.5, we evaluate the information content of statistical versus survey-based risk forecasts for future *realized* returns.

[INSERT FIGURE 2.13 HERE]

2.5. Predictability Regressions

We estimate a set of predictability regressions in order to characterise the informational content of survey-based risk premia as signals about future realized returns, and contrast these to statistical expected return signals. The benchmark signals we consider are the interest rate differential and slope of the yield curve,

which we denote[9]

$$IRD_t = i_t^{(1)} - i_t^{(1),f} \tag{2.9}$$

$$Slope_t \equiv i_t^{(10)} - i_t^{(1)}. \tag{2.10}$$

Figure A.24 plot time-series for the $Slope_t$ and IRD_t variables, and Table A.16 reports summary statistics.

2.5.1. Bond Risk Premia

Panel (a) of Table 2.3 reports estimates from the pooled OLS predictability regressions

$$rx_{t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t + \epsilon_{t+1}. \tag{2.11}$$

where confidence intervals are computed using circular block bootstrap.[10]

[INSERT TABLE 2.3 HERE]

The slope of the yield curve significantly positively predicts future realized returns, as expected. More importantly, survey-based expected excess bond returns are also positively and highly significantly linked to future realized excess bond returns, consistent with the results of Buraschi et al. (2022) for the U.S but contrary to what Greenwood and Schleifer (2014) document for the equity market. A common null for rational expectations would be $H_0 : b_2 = 1$. The estimates imply that the one-sided upper confidence interval is 0.77 which rejects but is not "a million miles away". There are, of course, many well known reasons for which this is not an appropriate null hypothesis for testing rationality. More importantly, we infer that $b_2 > 0$ with a high degree of statistical confidence. Interestingly, the predictive power of our survey-based measure does not disappear when adding the slope as an additional predictor, even if the estimated factor loading b_2 drops from 0.52 to

[9] Results that follow are unaffected by using the 11-year minus 1-year slope of the yield curve.

[10] For robustness, we report an equivalent table using the estimator of Driscoll and Kraay (1998) in the OA, see Table A.19.

0.30. This results suggests that survey forecasts contain valuable information to predict future bond returns, which is not completely spanned by the information in current interest rate term structures. This 'lack of spanning' also suggests that forecast errors are predictable by date t information since they do not subsume the predictive ability of the *Slope*. This suggests that bond market beliefs are not fully rational consistent with existing evidence (Cieslak, 2017). Section 2.6 studies forecast errors and predictability in expectation errors.

2.5.2. Exchange Rate Risk Premia

Beginning with Fama (1984) a vast literature has tested the UIP condition via a predictability regression of log spot exchange rate changes on lagged interest rate differentials (or forward-spot spreads). With the definition for IRD_t above, from Equation (4) the zero currency excess return condition says $IRD_t - \Delta x_{t+12} = 0$ which can be tested via the Fama regression

$$\Delta x_{t+1} = \alpha + \beta IRD_t + \epsilon_{t+1}. \tag{2.12}$$

The UIP condition predicts that $\alpha = 0$ and $\beta = 1$ so that earning positive carry from the perspective of a U.S. investor ($IRD_t < 0$) is be offset by a capital loss (a foreign currency depreciation, $\Delta x_{t+1} < 0$) when repatriating the initial investment. Subtracting $i_t^{(1)} - i_t^{(1),f}$ from both sides of the regression

$$\Delta x_{t+1} - IRD_t = \alpha + \underbrace{(\beta - 1)}_{b_1} IRD_t + \epsilon_{t+1}, \tag{2.13}$$

we see that considering a Fama regression with excess returns as a dependent variable, UIP implies a regression coefficient $b_1 = 0$. Testing the predictability of excess exchange rate returns, we estimate the following pooled OLS regressions

$$rx_{t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t+1}, \tag{2.14}$$

Panel (b) of table 2.3 shows that the coefficient for the usual UIP predictor, the interest rate differential, is significantly negative, consistent with the literature. The estimated b_1 is -1.56, meaning that not only do U.S. investors in high interest rate currencies earn positive carry but they also earn a capital gain when closing out their positions.

As above, a natural null hypothesis for the XRP_t coefficient is $H_0 : a = 0$ and $b_2 = 1$, i.e. if surveys were fully rational (in a full information sense) we should expect $rx^{FX}_{t+1} = XRP_t + \epsilon_{t+1}$. Testing this null, we do no reject at conventional levels based on bootstrap standard errors (nor based on Driscoll and Kraay (1998) standard errors in Table A.19). Moving to specification (*iii*) we find that both the interest rate differential and XRP_t are statistically significant. In fact, the point estimate of coefficient b_2 does not change much and remains statistically significant. Considering the measure's stability even when controlling for natural predictors, this results demonstrates a strong informational content. Notably, the R^2 of the single-variable regressions in columns (*i*) and (*ii*) almost add up to R^2 of the multivariate regression in column (*iii*). In summary, survey forecast of exchange rate returns not only significantly positively predict future realized excess returns but they do not *only* use information completely spanned by current interest rate spreads. As with the bond predictability regression above, while positively correlated with future realized returns their errors are predictable by the current observables (IRD_t in this case) suggesting that beliefs are not fully rational, consistent with existing literature (e.g., Frankel and Froot, 1987).

The findings of this section demonstrate that the information contained in survey-implied subjective risk premia are an important proxy for the underlying true unobservable time-varying (bond and FX) risk premia. However, we note that in all regressions above subjective risk premia are not completely spanned by date t observables. This result does not mean that forecasters ignore information in interest rate spreads. Indeed, they combine information in the term structures with other variables as well as their own judgement and intuition when they build projections. Identifying the exact information and models that survey forecasters

use is not an easy task, if at all feasible, since they likely assign time-varying weights to alternative models. For example, they might rely fully on their own judgement one month and instead follow a standard slope predictor another month. Moreover, individual forecasters probably follow different rules, and observing only the consensus of an unbalanced panel does not allow us to recover these individual rules. Overall however, our results show that we can use the consensus survey forecast as an aggregate, observable proxy of these unobservable and time-varying predictive information models.

2.6. Expectation Errors

We evaluate the forecast errors from surveys. We take consensus (arithmetic average) projections about a target variable y_{t+1} and compute forecast errors as follows:

$$y_{t+1} - y_t = E_t^{\mathbb{S}}\Big[y_{t+1} - y_t\Big] + FE_{t,t+1} \tag{2.15}$$

Note that by construction the errors of these projections are in real-time (out-of-sample).

Table 2.4 provides insights into the properties of forecast errors, by showing summary statistics of 10-year yield and exchange rate expectation errors for all countries in the sample. In particular the table displays the two drivers of the RMSE, i.e. the mean and standard deviation of the errors, as well as the minimum and maximum.[11]

For interest rates, the mean expectational error is always negative, meaning that all forecasts over-predicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante. For exchange rates, the mean expectational error are close to zero, meaning that there is no systematic bias.

[11] We discuss the potential extent of information frictions at the level of the consensus forecast using forecast errors and forecast revisions in the framework of Coibion and Gorodnichenko (2015) in Section A.3 of the OA.

[INSERT TABLE 2.4 HERE]

Studying systematic patterns in forecast errors, Figure 2.14 displays the time series dynamics of the 10-year yield forecast errors (left panel) and the exchange rate errors (right panel), for all countries in our sample. Interestingly, forecast errors across countries are highly correlated. Moreover, errors are volatile around zero and display only mild persistence, as evidenced by the AR(1) coefficients sampled at annual horizons (final row of each panel in Table 2.4).

[INSERT FIGURE 2.14 HERE]

2.6.1. Forecast Error Predictability

Next, we ask whether the mild persistence observed in the autocorrelation of $t+1$ errors results in predictability by factors observable at date t. Such a question is important since forecast error predictability suggests that consensus beliefs (measured from surveys) are not incorporating all available information. Table 2.5 shows pooled regressions of (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on the level and slope of the yield curve and the 1-year interest rate differential, respectively.

$$FE^i_{t,t+1} = a^i + b^i X^i_t + \eta^i_{t+1}, \tag{2.16}$$

where for interest rate forecast errors X_t includes the $Level_t = i^{(1)}_t$ and $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ and for exchange rates $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$.

Considering first panel (a) which displays interest rate forecast error predictability, we observe that the $Level_t$ of the yield curve is significant at 5%, while the $Slope_t$ of the yield curve is not significantly linked to future forecast errors. The point estimates on the $Level_t$ is positive, meaning that a negative shock to the date t interest rate causes agents to under predict future interest rates, consistent with the idea that agents generally perceive rate shocks to be *more* persistent than they turns out to be ex-post.[12]

[12] Piazzesi et al. (2015) document a related result using BlueChip Financial Forecasts.

Consider now panel (b) which analyzes the predictability of exchange rate forecast errors. Exchange rate errors are also predictable by IRD_t with a point estimate that is negative and statistically significant at 5%. The negative sign has the opposite interpretation than for interest rate errors: a negative shock to IRD_t causes agents to over predict future exchange rates, consistent with the idea that agents tend to perceive rate shocks to be *less* persistent than they turns out to be ex-post.

[INSERT TABLE 2.5 HERE]

These findings suggest agents do not exploit all available information when forming their beliefs, which is inconsistent with the basic idea of rational expectations. However, to the best of our knowledge it remains an open question whether statistical predictability can be exploited to correct errors in real-time. In the OA (Section A.3) we provide a partial answer to this question by designing an experiment which constructs fictitious expectations that correct for predictability in errors using date t observables. Summarising, we find that 'uncorrected' beliefs dominate their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements.

2.7. An Equilibrium Model with Subjective Beliefs

In this section, we adapt an off-the-shelf asset pricing model to incorporate a wedge between objective beliefs and subjective beliefs and use the model to shed light on the joint empirical properties of (*i*) subjective risk premia; (*ii*) expectation errors; (*iii*) physical and risk neutral FX and interest rate dynamics. We consider a 3-country world consisting of Australia, Switzerland and the U.S. and estimate the model via simulated methods of moments by exploiting beliefs from surveys and asset prices jointly. A full derivation is reported in Section A.4 of the OA.

2.7.1. Stylised Facts

The main empirical characteristics of subjective risk premia that we have outlined in Section 3.4 can be summarised as follows:

- **Risk Premia:** Subjective bond risk premia are negative on average but can also take positive values. This is consistent with the idea that bonds are usually hedges but can sometimes be risky bets. From the perspective of a U.S. investors, foreign currency risk premia display a clear carry trade pattern: large positive expected returns are earned on the Australian dollar and large negative expected returns are paid on the Swiss Franc.

- **Cyclicality:** Both subjective bond risk premia and exchange rate risk premia vary counter-cyclically with subjective macro-expectations of real growth rates and quantities of risk. Realized returns and realized growth rates instead are largely uncorrelated: the macro-finance disconnect.

- **Belief Distortions:** Ex-ante beliefs about returns significantly forecast realized returns and with the correct sign according to rational expectations. At the same time, survey expectations are not completely "spanned" by interest rate spreads, which also significantly predict future excess returns. This implies, and indeed we show, that while positively correlated with future realized returns, survey implied forecast errors are predictable by date t information; thus, there exists a distortion between subjective and physical measures.

Our model estimation aims at matching these empirical characteristics. We start with a standard equilibrium model in which risk premia are constant and we target unconditional moments of risk premia, as well as interest rates and forecast errors, in order to understand the magnitude and implications of the distortion between subjective and physical measures. Then, we extend the model introducing a stochastic volatility factor in order to rationalize the time series

dynamics of risk premia in the data and in particular the observed cyclicality and predictability properties.

2.7.2. Fundamentals and Preferences

Consider an economy where the state-variables are adapted to the filtered probability space $(\Omega, \mathcal{F}_{t\in T}, \mathbb{P})$ and $\mathbb{P}$ denotes the physical probability measure of the econometrician. The level of the aggregate consumption flow in this economy, C_t ($c_t = \ln C_t$), evolves according to the following dynamics, and its conditional mean is driven by a "long run risk" factor g_t:

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sigma_c dW_{c,t}, \tag{2.17}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g dW_{g,t}, \tag{2.18}$$

We assume a constant conditional mean Geometric Brownian Motion for the price level

$$\frac{dQ}{Q} = \pi dt + \sigma_Q dW_{Q,t} \tag{2.19}$$

and that all Brownian motions are independent.

A continuum of identical agents hold the Duffie and Epstein (1992a) parameterisation of the continuous time Epstein and Zin (1989) and Weil (1989) recursive preferences, where β is a time discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of inter-temporal substitution (EIS). As discussed in Bansal and Yaron (2004), recursive preferences allow for separation of time-and state-preferences for consumption.[13]

2.7.3. Bond Pricing

An approximate solution for asset prices in this economy follows from a standard log-linearisation of the consumption-wealth ratio. Denoting $h_1 = E\left[\frac{C_t}{W_t}\right]$ the diffusion

[13] When $\gamma \neq \psi$ agents have a preference for early (late) resolution of uncertainty when risk aversion is larger (smaller) than the inverse of EIS. A preference for early resolution combined with $\psi > 1$ implies that $\theta \equiv \frac{1-\gamma}{1-1/\psi} < 1$. In the limit $\psi \to \frac{1}{\gamma}$ we obtain power utility and for the special case $\psi \to 1$ we obtain a solution in which the consumption-wealth ratio is constant such that agents consume from their wealth at a rate equal to β.

for the nominal SDF is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{2.20}$$

$$r_t = r_0 + \frac{1}{\psi} g_t \tag{2.21}$$

$$\Theta_c = \gamma \sigma_c \tag{2.22}$$

$$\Theta_g = \left(\frac{1}{h_1 + \kappa_g}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \tag{2.23}$$

$$\Theta_Q = \sigma_Q \tag{2.24}$$

Bond prices are exponentially affine and given by

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] = E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right] = \exp\left(A_\tau + B_\tau g_t\right) \tag{2.25}$$

Under the $\mathbb{Q}$-measure the risk neutral diffusion for g_t is given by

$$dg_t = \kappa_g(\mu_g^{\mathbb{Q}} - g_t)dt + \sigma_g dW_{g,t}^{\mathbb{Q}}, \tag{2.26}$$

so that the unconditional mean $\mu_g^{\mathbb{Q}} = \mu_g - \Theta_g \frac{\sigma_g}{\kappa_g}$ is different under the physical and the risk-neutral measure.

The bond price factor loadings are obtained from standard risk-neutral pricing techniques

$$\frac{\partial A_\tau}{\partial \tau} = B_g(\tau)\mu_g^{\mathbb{Q}}\kappa_g + \frac{1}{2}B_g(\tau)^2\sigma_g^2 - r_0 \quad \text{with} \quad A_0 = 0 \tag{2.27}$$

$$B_\tau = -\frac{1}{\psi}\frac{1}{\kappa_g}\left(1 - e^{-\kappa_g \tau}\right) < 0 \tag{2.28}$$

so that higher real growth rates lower bond prices and increase interest rates. The term structure of interest rates is then given by $y_t^\tau = a_\tau + b_\tau g_t$ where the yield factor loadings are equal to the bond pricing factor loadings multiplied by $-\tau^{-1}$.

2.7.4. Subjective Measure and Belief Distortion

The state variables in the previous subsections are all adapted to the filtered probability space $(\Omega, \mathcal{F}_{t\in T}, \mathbb{P})$, where $\mathbb{P}$ represents the physical probability distribution. When investors only have a short sample of data available, learn sub-optimally, or suffer from some behavioural biases, their subjective measure $\mathbb{S}$ will not necessarily coincide with the physical (objective) measure $\mathbb{P}$ of an econometrician.

Formally, the relationship between the physical measure and the subjective measure is given by $E_t^{\mathbb{P}}\left[\frac{d\mathbb{S}}{d\mathbb{P}} z_s\right] = E_t^{\mathbb{S}}\left[z_s\right]$, where from Girsanov[14]

$$\frac{d\mathbb{S}}{d\mathbb{P}} = \exp\left(\int_0^t \phi_{z,u} dW_u^{\mathbb{P},z} + \frac{1}{2}\int_0^t \phi_{z,u}^2 du\right), \tag{2.29}$$

$$dW_t^{\mathbb{S},z} = dW_t^{\mathbb{P},z} - \phi_z dt. \tag{2.30}$$

We call ϕ_z a BELIEF DISTORTION. For simplicity and ease of interpretation we assume that the distortion is constant and that only the latent long run risk process g_t has different shocks under the subjective and physical measures, so that $\phi_g \neq 0$, while all other Brownian motions are the same under $\mathbb{S}$ and $\mathbb{P}$.[15] Potentially, a similar change of measure could hold for the other shocks and in different settings.

Recently, efforts have been devoted to endogenizing belief distortions. Microfoundations for belief distortions have been proposed in settings where agents suffer behavioural biases or are subject to information frictions (see Coibion and Gorodnichenko (2015) for an overview) or in settings where agents have preferences for statistical robustness (for recent examples, see Maenhout et al. (2021) and Bhandari et al. (2022)).

Abstracting from a specific mechanism for subjective belief formation, we have

[14] We assume $\mathbb{S}$ is absolutely continuous with respect to $\mathbb{P}$.

[15] This assumption is further supported by the empirical observation that the survey inflation forecast errors are on average very close to zero and do not display any significant pattern.

two changes of measure which satisfy

$$\begin{aligned} dW^{\mathbb{Q}}_{g,t} &= dW^{\mathbb{S}}_{g,t} + \Theta^{\mathbb{S}}_g dt, \\ dW^{\mathbb{Q}}_{g,t} &= dW^{\mathbb{P}}_{g,t} + \Theta^{\mathbb{P}}_g dt \\ dW^{\mathbb{S}}_{g,t} &= dW^{\mathbb{P}}_{g,t} - \phi_g dt \quad \rightarrow \quad \phi^g = \Theta^{\mathbb{S}}_g - \Theta^{\mathbb{P}}_g \end{aligned} \tag{2.31}$$

So, the SDF dynamics under the subjective measure is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta^{\mathbb{S}}_c dW^{\mathbb{S}}_{c,t} - \Theta^{\mathbb{S}}_g dW^{\mathbb{S}}_{g,t} - \Theta^{\mathbb{S}}_Q dW^{\mathbb{S}}_{Q,t} \tag{2.32}$$

$$\Theta^{\mathbb{S}}_c = \Theta^{\mathbb{P}}_c \tag{2.33}$$

$$\Theta^{\mathbb{S}}_g = \Theta^{\mathbb{P}}_g + \phi_g \tag{2.34}$$

$$\Theta^{\mathbb{S}}_Q = \Theta^{\mathbb{P}}_Q \tag{2.35}$$

This implies that only the market price of risk for the latent long-run risk factor g is different under the two measures and in particular it increases by the distortion ϕ_g. Under the subjective measure $\mathbb{S}$, the diffusion for g_t is given by

$$dg_t = \kappa_g(\mu^{\mathbb{S}}_g - g_t)dt + \sigma_g dW^{\mathbb{S}}_{g,t}, \tag{2.36}$$

where the unconditional mean of the long-run risk factor under the subjective measure is given by $\mu^{\mathbb{S}}_g = \mu_g + \frac{\sigma_g}{\kappa_g}\phi_g$.

Note that bond prices can equivalently be solved for under the subjective or physical measure via the changes of measure defined above:

$$\begin{aligned} P_\tau &= E^{\mathbb{P}}_t\left[\frac{\Lambda_T}{\Lambda_t}\right] = E^{\mathbb{P}}_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] \\ &= E^{\mathbb{P}}_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\frac{d\mathbb{S}}{d\mathbb{P}}\right] \\ &= E^{\mathbb{S}}_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\right] \\ &= E^{\mathbb{Q}}_t\left[e^{-\int_t^T r_s ds}\right] = \exp\Big(A_\tau + B_\tau g_t\Big). \end{aligned} \tag{2.37}$$

2.7.5. Subjective Bond Risk Premia and Yield Forecast Errors

The subjective price of risk outlined above also drives subjective expected excess bond returns. Namely, the subjective instantaneous bond risk premium on a T period bond is given by the negative of the covariance between SDF changes and bond returns:

$$BRP_t = -E_t^{\mathbb{S}}\left[\frac{d\Lambda_t^{\mathbb{S}}}{\Lambda_t^{\mathbb{S}}}\frac{dP_\tau}{P_\tau}\right] = \Theta_g^{\mathbb{S}} B_\tau \sigma_g \tag{2.38}$$

In this economy, the bond risk premium is constant and negative, consistent with the unconditional mean of the subjective bond risk premia observed in the data and for realistic parameter choices, we have $\Theta_g^{\mathbb{S}} > 0$ and $B_\tau < 0$.

Forecast errors on yields are given by

$$FE_{t+1}^\tau = y_{t+1}^\tau - E_t^{\mathbb{S}}\left[y_{t+1}^\tau\right] \tag{2.39}$$

where $y_t^\tau = \frac{-1}{\tau}\Big(A_\tau + B_\tau g_t\Big)$. Therefore, the forecast errors can be written as

$$FE_{t+1}^\tau = \frac{-B_\tau}{\tau}\left[g_{t+1} - E_t^{\mathbb{S}}(g_{t+1})\right], \tag{2.40}$$

where $E_t^{\mathbb{S}}(g_{t+1}) = g_t e^{-k_g} + \mu_g^{\mathbb{S}}(1 - e^{-k_g})$.

The physical expectation of the survey forecast errors therefore depends on the difference between the unconditional mean of the g process under $\mathbb{P}$ and $\mathbb{S}$, which is proportional to the belief distortion ϕ_g:

$$\begin{aligned} E_t^{\mathbb{P}}\left(FE_{t+1}^\tau\right) &= \frac{-B_\tau}{\tau}\left[E_t^{\mathbb{P}}(g_{t+1}) - g_t e^{-k_g} - \mu_g^{\mathbb{S}}(1 - e^{-k_g})\right] \\ &= \frac{-B_\tau}{\tau}\left[\left(1 - e^{-k_g}\right)\left(\mu_g^{\mathbb{P}} - \mu_g^{\mathbb{S}}\right)\right] \\ &= \frac{B_\tau}{\tau}\left[\left(1 - e^{-k_g}\right)\phi_g\frac{\sigma_g}{k_g}\right] \end{aligned} \tag{2.41}$$

In order to match the negative yield forecast errors in the data, since B_τ is negative, we need a positive belief distortion ϕ_g, so that the unconditional

mean of the long-run risk factor under the subjective measure is larger than under the physical measure.

2.7.6. Foreign Countries

In order to solve for the exchange rate risk premium we need to make an assumption about the foreign countries. Let us assume there are $N+1$ consumption goods and $N+1$ countries: "home" country (the U.S.) and N "foreign" countries. As in Colacito and Croce (2011), we assume that each country behaves as in autarky, in both consumption and financial assets (total home bias) so that a representative investor within each country only consumes the good which they are endowed.

We assume foreign country dynamics have the same structure as in (2.17) but with different shocks and potentially different parameters. We model cross-country correlation by assuming that shocks in the foreign country, i.e. $dW_{c,t}^{\mathbb{S},f}$, $dW_{g,t}^{\mathbb{S},f}$ and $dW_{Q,t}^{\mathbb{S},f}$, are correlated with the corresponding shocks in the home country, with correlations ρ_c, ρ_g and ρ_q, respectively. Cross-country correlation among the state variables potentially generates global and local factor pricing of risk as in Lustig et al. (2011). We also assume that the correlation between shocks in the foreign countries is completely driven by their exposure to the home country shocks.

Investors can trade in both domestic and foreign bond markets. Denote X_t the real exchange rate in the U.S good per the foreign good. When X_t goes up, the US dollar depreciates in real terms. If markets trade without frictions then exchange rates are pinned down by the ratio of the SDFs, i.e. $X_t = \Lambda_t^f \Lambda_t^{-1}$, where the dynamics of the foreign SDF Λ_t^f follows exactly the same structure as the domestic SDF in Equation (2.20) but with the country-specific parameters and shocks. The U.S. dollar price of the foreign bond ($P_{t,\tau}^f$) is given by

$$\Lambda_t X_t P_{t,\tau}^f = E_t^{\mathbb{S}}\left[\Lambda_{t+1} X_{t+1} P_{t+1,\tau-1}^f\right] \tag{2.42}$$

which implies that the exchange rate risk premium is obtained as the negative of the covariance between the SDF and exchange rate changes, which is equal

to the drift in the exchange rate dynamics plus the interest rate differential (equivalent to Equation (5)):

$$XRP_t = -\frac{1}{dt}Cov_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}, \frac{dX_t}{X_t}\right) = \frac{1}{dt}Var_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}\right) - \frac{1}{dt}Cov_t^{\mathbb{S}}\left(\frac{d\Lambda_t}{\Lambda_t}, \frac{d\Lambda_t^f}{\Lambda_t^f}\right) \quad (2.43)$$

$$= \Theta_c^{\mathbb{S}}\left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f}\rho_c\right) + \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right) + \Theta_Q^{\mathbb{S}}\left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f}\rho_Q\right) \quad (2.44)$$

As for the bond risk premium, in this simple setting the exchange rate risk premium is constant, but it can take both positive and negative signs, depending on the correlation across countries and their respective market prices of risk.

Forecast errors on log exchange rates are given by

$$FE_{t+1}^{FX} = x_{t+1} - E_t^{\mathbb{S}}\Big[x_{t+1}\Big] \quad (2.45)$$

The physical expectation of the survey forecast errors therefore depends on the difference between the $\mathbb{P}$ and $\mathbb{S}$ expectations of log exchange rates:

$$E_t^{\mathbb{P}}\left(FE_{t+1}^{FX}\right) = E_t^{\mathbb{S}}\Big[x_{t+1}\Big] - E_t^{\mathbb{S}}\Big[x_{t+1}\Big] \quad (2.46)$$

Using equation (5) we can rewrite this as the difference between the exchange rate risk premia under the physical and subjective measures:

$$\begin{aligned} E_t^{\mathbb{P}}\left(FE_{t+1}^{FX}\right) &= E_t^{\mathbb{S}}\Big[x_{t+1}\Big] - E_t^{\mathbb{S}}\Big[x_{t+1}\Big] \\ &= E_t^{\mathbb{S}}\Big[\Delta x_{t+1}\Big] - E_t^{\mathbb{S}}\Big[\Delta x_{t+1}\Big] \\ &= XRP_t^{\mathbb{P}} - IRD_t - (XRP_t^{\mathbb{S}} - IRD_t) \\ &= XRP_t^{\mathbb{P}} - XRP_t^{\mathbb{S}} \\ &= \Theta_g^{\mathbb{P}}\left(\Theta_g^{\mathbb{P}} - \Theta_g^{\mathbb{P},f}\rho_g\right) - \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right), \end{aligned} \quad (2.47)$$

which depends on the relative magnitudes of the belief distortions in the foreign and domestic country.

2.7.7. Estimation Results

We estimate the model in several steps. First, we estimate the parameters in the consumption process (Equations (2.17)-(2.18)) for each country separately using the simulated method of moments and targeting mean, standard deviation, skewness, kurtosis and autocorrelations (with 1, 5 and 10 lags) of the realized consumption processes. Tables A.31, A.32 and A.33 in the OA report estimation results for the US, Switzerland and Australia, respectively. Panel (a) shows parameter estimates alongside 95% confidence intervals and panel (b) reports the empirical moments as well as the model-implied moments and their confidence intervals.

Consistent with the literature on long-run risk models we match the properties of realized consumption processes in the three countries very well, with a very persistent long-run risk process g. The estimated mean reversion coefficient κ is around 0.05 for the US, 0.20 for Switzerland and 0.06. The unconditional mean of the log-run growth process is 1.3%, 1.6% and 3.2%, for the US, Switzerland and Australia, respectively.

Inflation parameters π and σ_Q (see Equation (2.19)) are set equal to the mean and volatility of inflation in each country.[16]

With these parameters at hand we then turn to the estimation of the preference parameters, i.e. the time preference parameter β, the coefficient of risk aversion γ, and the elasticity of inter-temporal substitution ψ. These preference parameters affect the term structures in each country, as well as the risk premia. However, the subjective bond risk premia depend on both the preferences and the distortion between physical and subjective beliefs. Therefore, we estimate the preference parameters and the distortion jointly using term structure moments and the average bond risk premia, as well as the average 10-year bond yield forecast errors. The preference parameters are assumed the same across countries, while the belief distortion ϕ_g could potentially be country-specific. Overall, we have 6 parameters to estimate (β, γ, ψ, ϕ_g^{US}, ϕ_g^{CHF}, ϕ_g^{AUD}) and 18 moment conditions (mean of the 3-month yield, standard deviation of 3-month yield changes, mean of 5-year yield,

[16] In our sample, $\pi^{US} = 2.29\%$, $\pi^{CHF} = 0.53\%$ and $\pi^{AUD} = 2.15\%$, while the unconditional volatility of inflation are $\sigma_Q^{US} = 0.81\%$, $\sigma_Q^{CHF} = 1.12\%$ and $\sigma_Q^{AUD} = 0.67\%$.

mean of 10-year yield, mean bond risk premium and average 10-year yield forecast error), for a total of 6 moments for each of the 3 countries.[17] Preference and distortion parameter estimates are summarised in Table 2.6, as well as the value of targeted moments in the model and in the data.

[INSERT TABLE 2.6 HERE]

The first thing to notice is that this simple model fits the average subjective bond risk premia and yield forecast errors pretty well. In particular, the estimated bond risk premia are all negative and around -1%, as in the data. The model also captures the standard deviation of yields and the level of the term structure. The estimated values of the risk aversion and intertemporal elasticity of substitution parameters are 6 and 1.4, respectively, implying a preference for early resolution of uncertainty. The main shortcoming of the model seems to be the fitting of the shape of the yield curves, with the model implying slightly downward sloping term structures while the observed term structures are all slightly upward sloping.

Interestingly, the estimated belief distortions for the latent factor shocks (ϕ_g) are all relatively small, with values of approximately 0.2, 1.4 and 0.1, for the US, Switzerland and Australia, respectively. In order to better understand the economic magnitude of these distortions, we can compare the unconditional mean of the latent risk factor g under the physical and the subjective measure. For the US, the physical mean is $\mu_g = 1.31\%$ while the subjective mean is $\mu_g^{\mathbb{S}} = 2.55\%$, which implies a 1.24 percentage points difference between the two. For Switzerland this difference is a little higher, at 1.84%, since we find $\mu_g = 1.55\%$ and $\mu_g^{\mathbb{S}} = 3.39\%$. For Australia instead the difference is a little smaller at 0.78%, with $\mu_g = 3.22\%$ and $\mu_g^{\mathbb{S}} = 4\%$. An alternative way to interpret the magnitude of the belief distortion is to look at the difference in the market prices of risk for the factor g under the two measures, i.e. $\Theta_g^{\mathbb{P}}$ and $\Theta_g^{\mathbb{S}}$, as well as the model-implied subjective and physical bond risk premia. Table 2.7 summarises these differences and confirms that, while

[17] In the estimation, instead of the instantaneous bond risk premium we compute the annualised expected excess bond return with a 1-year holding horizon, as in Equation (2), to be consistent with the empirical counterpart. We find that the model-implied values of the instantaneous and 1-year ahead bond risk premia are quite close.

the belief distortion helps matching the negative and relatively sizeable bond risk premia implied by survey forecasts, its magnitude in economic terms is quite small.

[INSERT TABLE 2.7 HERE]

The final step in our estimation procedure is to calibrate the missing parameters, i.e. the correlations between shocks in the US and in the foreign countries, by fitting the observed subjective exchange rate risk premia, while taking all other parameters as given from the previous steps. The correlations between consumption growth and inflation shocks, ρ_c and ρ_q, are observable, so we set them equal to the correlation between US and foreign consumption growth and inflation in the data (see Table A.16 in the Online Appendix). For Switzerland, we have $\rho_c = 0.97$ and $\rho_q = 0.89$, while for Australia we have $\rho_c = 0.76$ and $\rho_q = 0.95$. Panel (a) of Figure 2.15 shows the model-implied exchange rate risk premia for the two foreign countries as a function of the correlation between the unobservable long-risk risk factors, ρ_g, fixing all other parameters to their estimated values. Interestingly, the estimated model implies that XRP for AUD is always larger than the one for CHF, as in the data, even if we have still not used any information about the exchange rate dynamics or premia in the estimation. Note also that we can set the ρ_g parameters in order to exactly match the subjective exchange rate risk premia in the data, which are $XRP^{CHF} = -1.69\%$ and $XRP^{AUD} = 1.74\%$. The corresponding values of ρ_g are 0.364 for Switzerland and 0.922 for Australia.

[INSERT FIGURE 2.15 HERE]

The main driver of the sign of the XRP in the two countries is the relative magnitude of the market prices of risk for the long-run risk factor in the US versus the foreign country, weighted by the correlation ρ_g (from the middle term on the right hand side of (2.44), as the market prices of risk for the consumption and price level shocks are relatively smaller. Under the subjective measure, $\Theta_g^{\mathbb{S}}$ is very similar in Australia and in the US, leading to positive values of the exchange rate risk premia, while for Switzerland the market price of risk is much larger, so that XRP

can become positive if the correlation ρ_g is large enough. It is interesting to note that the ranking of the market prices of risk is very different under the physical measure, i.e. $\Theta_g^{\mathbb{P},CHF} < \Theta_g^{\mathbb{P},US} < \Theta_g^{\mathbb{P},AUD}$, which would imply a positive XRP for Switzerland and a possibly negative XRP for Australia.

It is interesting to note that average forecast errors on log exchange rates implied by the estimated model (from Equation (2.47), are approximately consistent with the observed ones, even though we have not used information from exchange rates forecast errors in the estimation, and more importantly we have not used information from exchange rates or exchange rate forecasts at all when estimating the belief distortion parameters. Panel (b) of Figure 2.15 shows the model-implied average log exchange rate forecast errors for the two foreign countries as a function of the correlation between the unobservable long-risk risk factors, ρ_g, fixing all other parameters to their estimated values. The values in the data are 1.01% for CHF and -0.34% for AUD, denoted by the horizontal dashed lines in the figure. Interestingly, the model-implied values are generally positive for CHF and negative for AUD, as in the data, and we can fit the observed value for CHF with a ρ_g not far from the one required to perfectly fit the exchange rate risk premium ($rho_g = 0.21$ vs $rho_g = 0.36$). The model cannot perfectly fir the FX forecast error for AUD but at least the model-implied value gets closer to zero in absolute value as rho_g increases, and the value of rho_g required to fit the exchange rate risk premium is indeed close to 1.

2.7.8. Introducing Stochastic Volatility

The model presented so far is able to match the unconditional mean of the subjective risk premia, in both fixed income and exchange rate markets, and allows us to clearly quantify and interpret the belief distortion between $\mathbb{P}$ and $\mathbb{S}$. However, it clearly cannot match the time series properties of the subjective risk premia, such as their volatility and cyclicality, as model-implied risk premia are constant. Therefore, in this subsection we look at the implications of introducing stochastic volatility in the model.

The only difference compared to the model above is that consumption growth and the long-run risk factor have stochastic volatility, so that Equations (2.17) and (2.18) are substituted by the following:

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sqrt{v_t}dW_{c,t}, \tag{2.48}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g\sqrt{v_t}dW_{g,t}, \tag{2.49}$$

$$dv_t = \kappa_v(\mu_v - v_t)dt + \sigma_v\sqrt{v_t}dW_{v,t}, \tag{2.50}$$

where v_t is the stochastic volatility factor, which follows a CIR process, and all Brownian motions are independent.

We assume that the preference structure is unchanged so that the nominal SDF satisfies the following process:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} - \Theta_v dW_{v,t}, \tag{2.51}$$

where the market prices of risk for consumption and the latent g and v processes are proportional to $\sqrt{v_t}$.

Bond prices are still exponentially affine but they also depend on the stochastic volatility factor:

$$\frac{dP_\tau}{P_\tau} = \mu_P(\tau)dt + B_g(\tau)\sigma_g\sqrt{v_t}dW_{g,t} + B_v(\tau)\sigma_v\sqrt{v_t}dW_{v,t}, \tag{2.52}$$

where $B_g(\tau) < 0$ and $B_v(\tau) > 0$ are the bond factor loadings.

As above, we assume that only the latent long run risk process g_t has different shocks under the subjective and physical measures, but instead of a constant distortion we assume that $\phi_g = \phi\sqrt{v_t}$, so that the market prices of risk for the long-run risk factor, Θ_g, have the same form under $\mathbb{P}$ and $\mathbb{S}$, i.e. they are both proportional to $\sqrt{v_t}$, just with a higher proportionality coefficient under $\mathbb{S}$. Note

that under the subjective measure $\mathbb{S}$, the diffusion for g_t is now given by

$$dg_t = [\kappa_g(\mu_g^{\mathbb{P}} - g_t) + \sigma_g \phi v_t]dt + \sigma_g dW_{g,t}^{\mathbb{S}}, \tag{2.53}$$

so that the distortion affects not only the unconditional mean of the long-run risk factor, but also its persistence.

The model-implied instantaneous subjective bond risk premium in this case is given by:

$$BRP_t = \Theta_g^{\mathbb{S}} B_g(\tau)\sigma_g\sqrt{v_t} + \Theta_v^{\mathbb{S}} B_v(\tau)\sigma_v\sqrt{v_t}. \tag{2.54}$$

Since both market prices of risk are proportional to $\sqrt{v_t}$, the subjective bond risk premium is proportional to v_t. So, in this economy, the bond risk premium is time varying with economic uncertainty. We have $\Theta_g^{\mathbb{S}} > 0$ and $B_g(\tau) < 0$, as well as $\Theta_v^{\mathbb{S}} < 0$ and $B_v(\tau) > 0$, so BRP is still negative and larger in absolute than in the base model, all else equal. This stochastic volatility version of the model still does not allow bond risk premia to switch sign, but it is straightforward to think about further extensions of the model that could gelp matching this feature of the data, such as priced inflation risk, credit risk, or allowing for (possibly stochastic) correlations across Brownian motions in the model.

The subjective exchange rate risk premium implied by the stochastic volatility model is given by:

$$XRP_t = \Theta_c^{\mathbb{S}}\left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f}\rho_c\right) + \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right) + \Theta_Q^{\mathbb{S}}\left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f}\rho_Q\right) + \Theta_v^{\mathbb{S}}\left(\Theta_v^{\mathbb{S}} - \Theta_v^{\mathbb{S},f}\rho_v\right), \tag{2.55}$$

so it has exactly the same form as in Equation (2.44) with an additional component that depends on the market prices of risk for the domestic and foreign volatility factors. Assuming that the stochastic volatility factor is global, i.e. common across all countries, the last term disappears (as we would have $\rho_v = 1$ and $\Theta_v^{\mathbb{S},f} = \Theta_v^{\mathbb{S}}$) and XRP_t is affine in v_t. The subjective exchange rate risk premium can be both

positive or negative, as in the main model, but it is also time-varying and could potentially switch sign depending on the relative sign and magnitude of the constant term in the expression (the $\Theta_Q^{\mathbb{S}}$ component) and the terms proportional to v_t.

In order to match the cyclicality of subjective bond risk premia we would need a negative covariance (under the subjective measure) between subjective bond risk premia and subjective expectations of consumption growth, i.e. $Cov^{\mathbb{S}}(BRP_t, g_t) < 0$. Since BRP_t is a negative linear function of v_t, this can be obtained with a positive correlation between v_t and g_t, i.e. between the latent volatility and expected growth factors. This same positive correlation is also consistent with the observed positive covariance between XRP and expected growth, at least for the countries in which the model-implied exchange rate risk premium is positive, that is the majority of countries in the data.

The predictability properties of subjective risk premia require a positive covariance between subjective risk premia and future realized excess returns, i.e. $Cov(BRP_t, rx_{t+1}) > 0$ for bonds. By construction, under the probability measure of the econometrician we have:

$$rx_{t+1} = BRP_t^{\mathbb{P}} + \epsilon_{t+1}, \tag{2.56}$$

where ϵ_{t+1} is a mean-zero error uncorrelated with excess returns. Therefore,

$$\begin{aligned} rx_{t+1} &= BRP_t^{\mathbb{S}} + (BRP_t^{\mathbb{P}} - BRP_t^{\mathbb{S}}) + \epsilon_{t+1} \\ &= BRP_t^{\mathbb{S}} - \phi B_g(\tau)\sigma_g v_t + \epsilon_{t+1}, \end{aligned} \tag{2.57}$$

from which

$$Cov(BRP_t, rx_{t+1}) = V\left(BRP_t^{\mathbb{S}}\right) - \phi B_g(\tau)\sigma_g Cov\left(v_t, BRP_t^{\mathbb{S}}\right)$$

where both $B_g(\tau)$ and $Cov\left(v_t, BRP_t^{\mathbb{S}}\right)$ are negative, so that the second term is positive and reduces the covariance between BRP and realized returns. This means that subjective bond risk premia can positively predict future excess returns if

the distortion between physical and subjective measure, ϕ, is not too large, as we have found in the base version of the model.

2.8. Conclusion

This paper exploits survey data on bond yields and exchange rates to jointly estimate risk premia in the foreign exchange and fixed income markets for the G10 countries. Subjective expected excess returns are obtained directly from a panel of investor forecasts allowing us to measure model free real-time risk premia on bonds and currencies.

We show that subjective risk premia are time varying and persistent, and highly correlated across countries. They are unconditionally negative for bonds, positive for investment currencies and negative for funding currencies. Importantly, we show that that subjective bond risk premia are countercyclical, meaning that they are significantly linked to subjective macro expectations, consistent with leading asset pricing models. Moreover, subjective risk premia are significantly positively linked to measures of quantity of risk, such as realised and implied volatility on bonds and exchange rates, consistent with the basic idea of a risk-return tradeoff.

Interestingly, we highlight that the subjective risk premia, in both fixed income and exchange rate markets, significantly positively predict future realized excess returns and the predictive power goes beyond that of standard predictors like the interest rate differential and the slope of the term structure. This finding suggests that survey forecasters use information beyond that spanned by current interest rate spreads when forming their expectations.

This last finding also implies that forecast errors based on survey expectations are predictable by yield curve factors, even if they are only mildly autocorrelated. We also show that average survey-based forecast errors on yields are positive, meaning that agents did not expect the strong downward trend in yields that we observed during our sample. However, survey forecast errors are unbiased for exchange rates, which are stationary instead. Overall, survey forecast errors do not display evidence of commonly studied information information frictions, such as extrapolation.

We argue that a significant link between subjective risk premia, subjective macro expectations and the quantity of risk, support asset pricing models that generate return predictability through cyclical variation in risk aversion, uncertainty, the likelihood of disasters or rational learning, but with the addition of a belief distortion between the probability measures of the econometrician ($\mathbb{P}$) and the subjective measure of the agents in the economy ($\mathbb{S}$), in order to take into account the properties of the forecast errors. Indeed, we conclude by deriving and estimating an equilibrium asset pricing model, with complete markets, long-run risk and recursive preferences. We estimate the model with the simulated method of moments exploiting information from the yield curves, subjective risk premia and forecast errors, to jointly estimate $\mathbb{Q}$, $\mathbb{P}$ and $\mathbb{S}$. The model implied unconditional moments line up closely with the empirical survey based unconditional moments, with a belief distortion that is relatively small economically, but still crucial to capture the properties of the forecast errors, as well as the unconditional mean of the subjective risk premia.

This demonstrates that subjective risk premia are well explained by a traditional model, irrespective of whether belief formation is rational or not, as long as we allow for a small distortion between the physical and subjective beliefs.

2.9. Figures

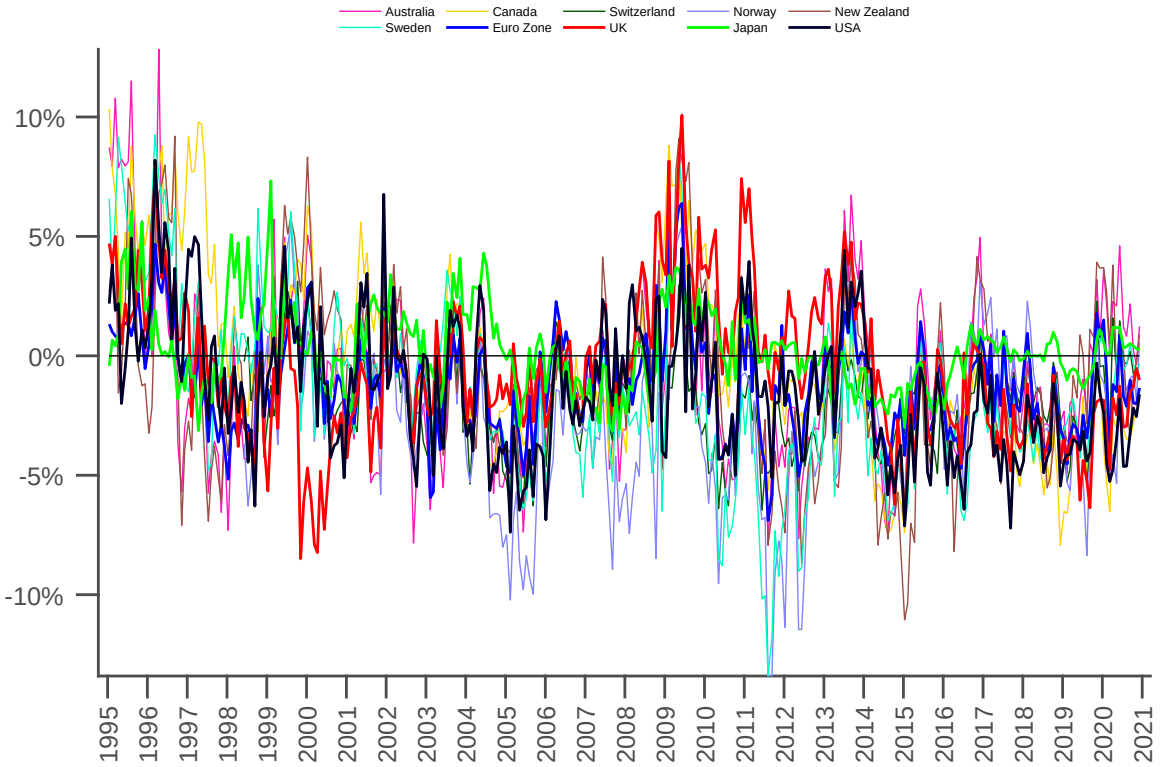


Figure 2.11. Subjective Bond Risk Premia

Figure displays real-time bond risk premia, defined as:

$$BRP_t^{11} = E_t^{\mathbb{S}}\left[rx_{t+1}^{(11)}\right] = E_t^{\mathbb{S}}\left[p_{t+1}^{(10)}\right] - p_t^{(11)} - i_t^{(1)},$$

where p_t^n are the log zero coupon bond prices for maturity n and i_t^1 is the continuously compounded one-year interest rate. The sample period is 1995.1 to 2020.12.

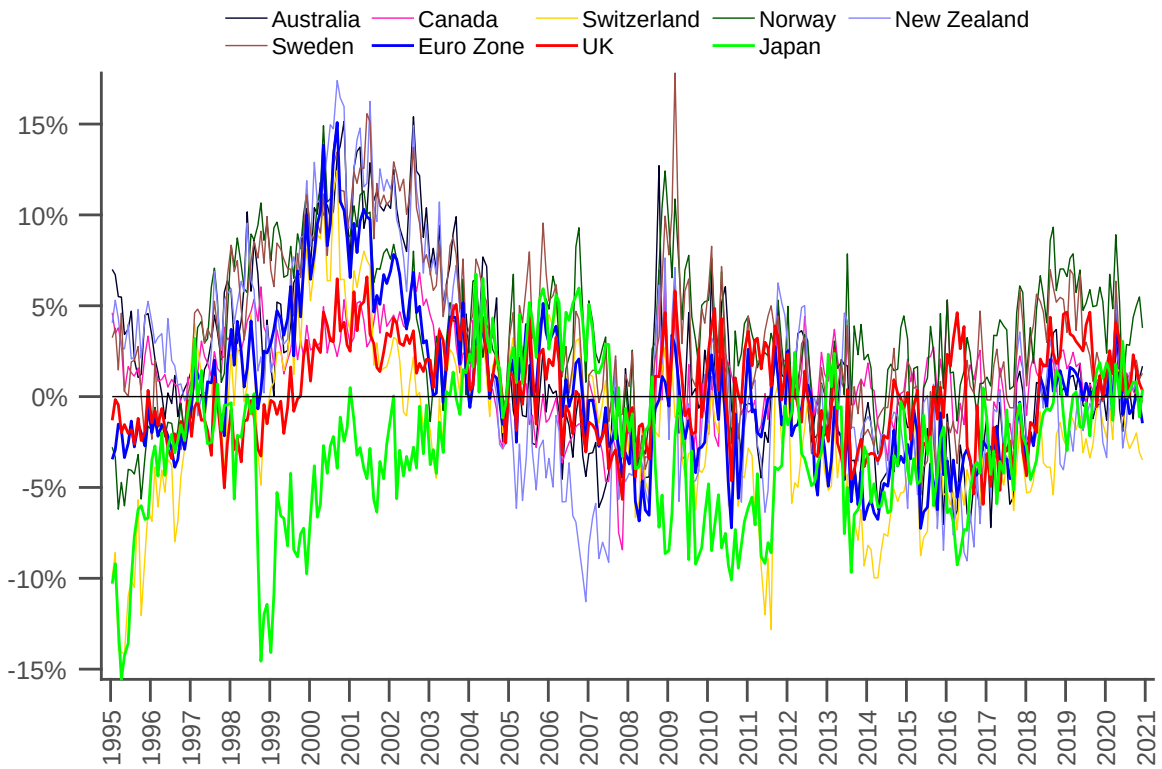
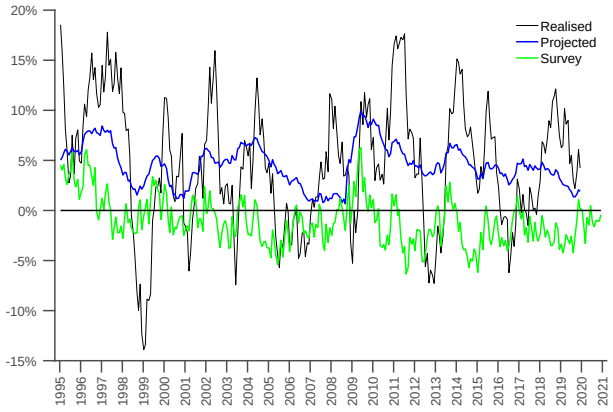


Figure 2.12. Subjective Exchange Rate Risk Premia

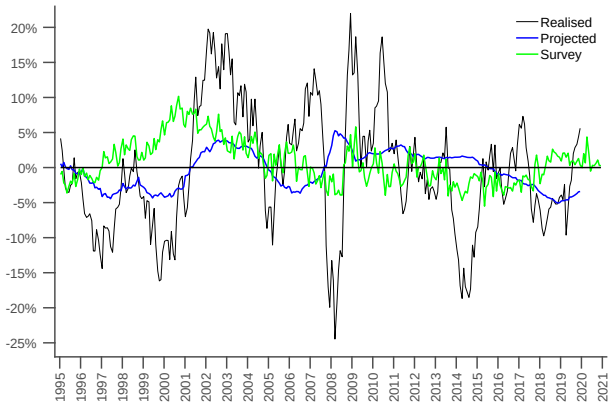
Figure displays real-time exchange rate risk premia, defined as:

$$XRP_t = E_t^{\mathbb{S}}\left[rx_{t+1}^{FX}\right] = (i_t^{(1),f} - i_t^{(1)}) + E_t^{\mathbb{S}}\left[\Delta x_{t+1}\right],$$

where the 1-year change in the log exchange rate is denoted Δx_{t+1}, and $i_t^{(1)}$ is the continuously compounded one-year interest rate in the foreign (f) versus domestic (U.S.) markets. The sample period is 1995.1 to 2020.12.



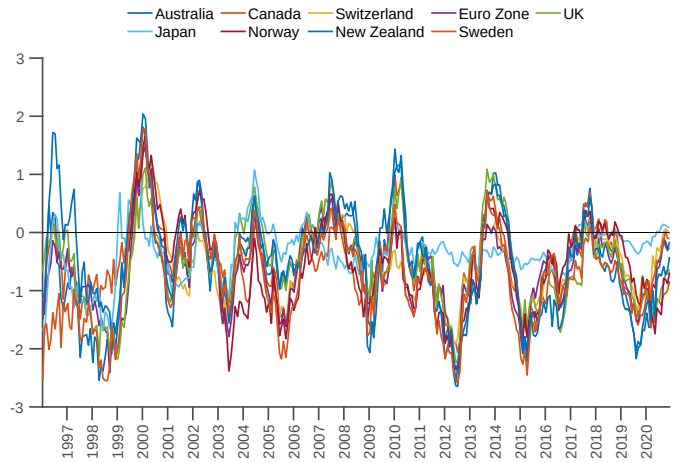
(a)



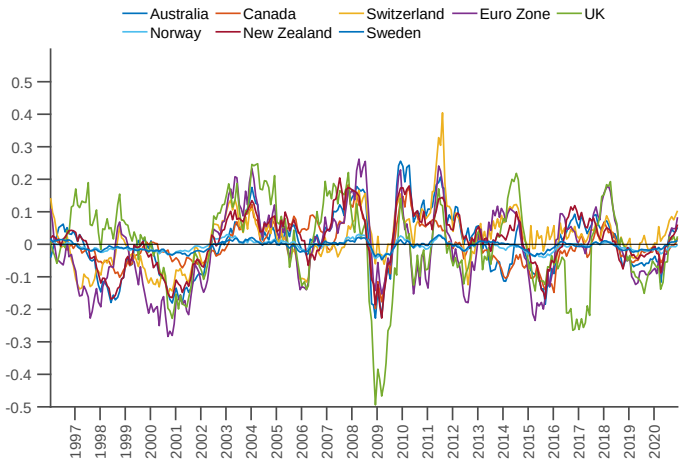
(b)

Figure 2.13. Survey vs Projected Risk Premia

The figure displays an equally-weighted average of subjective risk premia across countries. Blue lines represent the average projected risk premia, while green lines represent average survey-implied risk premia. In panel (a), projections for bond risk premia are obtained by regressing realized excess returns on the slope of the yield curve. In panel (b), projections for exchange rate risk premia are obtained by regressing realized excess returns on the interest rate differential between the foreign country and the home country. The sample period is 1995.1 to 2020.12 for survey forecasts and 1995.1 to 2019.12 for projection-based forecasts.



(a) Interest Rates



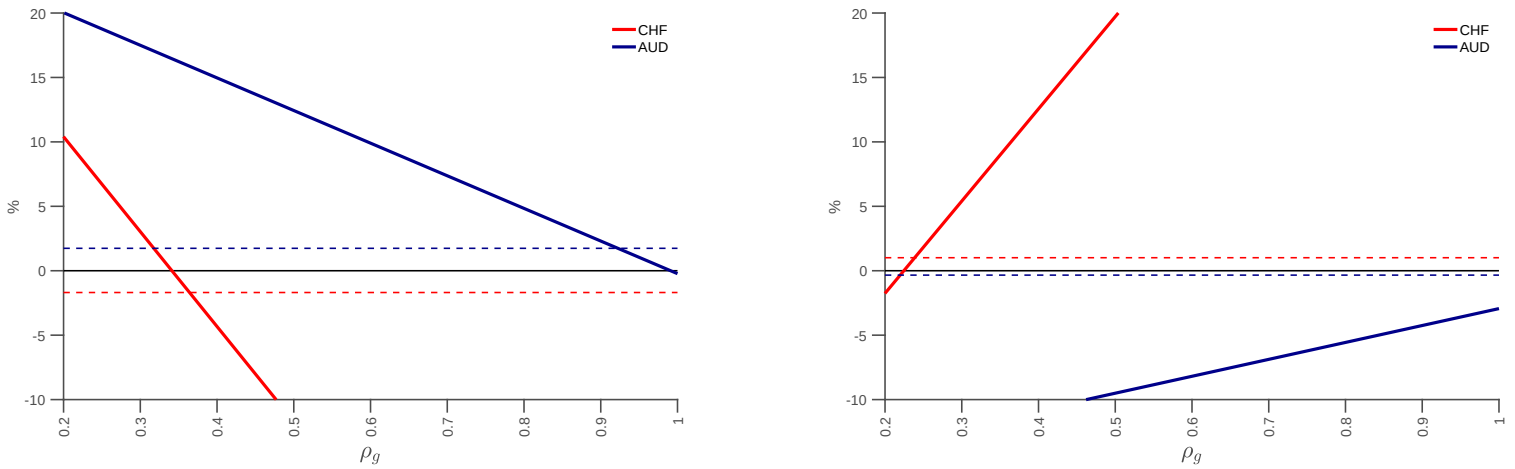
(b) Exchange Rates

Figure 2.14. Forecast Errors

Forecast errors are calculated as

$$y_{t+1} - y_t = E_t^{\mathbb{S}}\left[y_{t+1} - y_t\right] + \epsilon_{t+1}$$

and plotted for 10-year interest rates and exchange rates for a 1-year forecast horizon. Forecast errors are realized over the sample period 2001.1 to 2020.12. We do not include the forecast errors for the JPY/USD exchange rate in our plots due to the significant scale differences compared to the other currency pairs.



(a) XRP **(b)** FX FE

Figure 2.15. Model Implied Exchange Rate Risk Premia

This figure shows the model-implied exchange rate risk premia (Panel (a)) and average log exchange rate forecast errors (Panel (b)) for CHF and AUD as a function of the correlation between foreign and domestic long run risk factor, ρ_g, taking all other parameters as given from the previous steps of the model estimation. The horizontal dashed lines in both panels denote the corresponding values in the data.

2.10. Tables

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): BRP										
Mean	−0.83	−0.43	−1.91	−1.18	−0.55	0.29	−2.72	−0.94	−1.56	−1.41
Std	3.45	3.78	1.00	2.14	3.03	1.74	3.34	3.58	3.58	2.85
Skew	0.85	0.72	0.11	0.47	0.43	0.73	−0.30	0.32	0.31	0.46
AR(1)	0.06	0.45	−0.18	0.00	0.49	0.30	0.03	−0.02	0.38	0.20
Panel (b): XRP										
Mean	1.74	1.03	−1.69	0.17	0.20	−2.76	3.55	1.14	3.48	
Std	4.91	2.48	4.48	4.04	2.64	4.12	3.00	5.58	4.32	
Skew	0.63	−0.33	0.11	0.84	0.06	−0.24	0.11	0.58	0.52	
AR(1)	0.68	0.34	0.56	0.66	0.26	0.25	0.20	0.71	0.62	

Table 2.1. Descriptive Statistics for Subjective Risk Premia
This table presents the means, standard deviations, skewness, and AR(1) coefficients for subjective bond risk premia (BRP) and subjective exchange rate risk premia (XRP) as defined in Equations (3) and (5), respectively. The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

	(i)	(ii)	(iii)	(iv)
Panel (a): BRP				
E[con]	−0.57			
	[−0.92 −0.24]			
E[gdp]		−0.55		
		[−0.90 −0.12]		
BondVar			2.16	
			[0.96 3.14]	
$rx^{(11)}_{t-12,t}$				−0.09
				[−0.13 −0.05]
R^2 (%)	3.78	3.57	5.85	5.11
Panel (b): XRP				
E[con]	1.08			
	[0.54 1.76]			
E[gdp]		0.95		
		[0.39 1.81]		
FXVar			0.78	
			[0.35 1.55]	
$rx^{FX}_{t-12,t}$				−0.09
				[−0.18 −0.02]
R^2 (%)	6.36	4.87	3.58	4.81

Table 2.2. Cyclicality Regressions: Subjective Bond & FX Risk Premia
This table reports estimates from pooled OLS regressions of the forms

$$BRP_t = a + b^\top X_t + \epsilon_t, \text{ and}$$
$$XRP_t = a + b^\top X_t + \epsilon_t,$$

in panels (a) and (b), respectively. BRP_t is the survey-implied bond risk premium, XRP_t is the survey-implied currency risk premium, and X_t is a vector of explanatory variables. For the specifications in panel (a), X_t contains the 1-year *expected* consumption (*con*) growth and *gdp* growth, the realized bond variance ($BondVar$), and the excess return on an 11-year bond, realized between $t-1$ year and t. For the specifications in panel (b), X_t contains the 1-year *expected* consumption (*con*) growth and *gdp* growth in the foreign country, the FX option-implied risk neutral variance constructed and discussed by Krohn et al. (2020) ($FXVar$), and the excess currency return realized between $t-1$ year and t. A constant is included but not reported. Confidence intervals in [·] are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.18. The sample period is 1995.1 to 2020.12 except for specification (iii) in panel (b) which only contains data from 1999.1 to 2019.4.

	(i)	(ii)	(iii)
Panel (a): Bond Return			
Slope	2.81		2.49
	[1.96 3.69]		[1.53 3.46]
BRP		0.52	0.30
		[0.27 0.77]	[0.03 0.58]
R^2 (%)	13.85	4.96	14.79
Panel (b): FX Return			
IRD	−1.56		−1.41
	[−2.38 −0.45]		[−2.19 −0.36]
XRP		0.55	0.45
		[0.08 1.01]	[0.03 0.90]
R^2 (%)	8.89	5.43	12.54

Table 2.3. Bond & FX Return Panel Predictability Regressions
This table reports estimates from pooled OLS regressions of the forms

$$rx^{(11)}_{t,t+1} = a + b_1 Slope_t + b_2 BRP_t \epsilon_{t,t+1},$$
$$rx^{FX}_{t,t+1} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1},$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively. $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ is the slope of the domestic yield curve, $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$ is the one-year interest rate differential, BRP_t is the survey-implied bond risk premium, and XRP_t is the survey-implied currency risk premium. We also report univariate regressions with the same predictors in columns (i) and (ii). A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.19. The sample period is 1995.1 to 2020.12.

Panel (a): IR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Mean	-0.57	-0.55	-0.52	-0.64	-0.52	-0.33	-0.55	-0.45	-0.78	-0.60
Std	0.89	0.65	0.58	0.67	0.71	0.47	0.78	0.87	0.81	0.77
Min	-2.65	-2.06	-1.90	-2.41	-2.24	-1.75	-2.38	-2.58	-2.58	-2.37
Max	2.04	1.55	1.24	1.48	1.11	1.07	1.66	1.81	1.79	1.80
AR(1)	-0.18	-0.03	-0.08	-0.18	-0.10	-0.05	0.07	-0.10	-0.12	-0.23
Panel (b): XR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	
Mean	0.00	0.00	0.02	-0.01	0.00	0.00	-0.01	0.01	-0.01	
Std	0.10	0.06	0.08	0.12	0.13	0.00	0.02	0.08	0.01	
Min	-0.23	-0.20	-0.16	-0.28	-0.49	0.00	-0.04	-0.23	-0.05	
Max	0.26	0.18	0.40	0.26	0.25	0.00	0.03	0.20	0.03	
AR(1)	0.07	0.08	0.16	0.01	-0.05	0.23	0.01	0.02	-0.05	

Table 2.4. Summary Statistics: Expectation Errors

This table shows the means, standard deviations, minima and maxima of the forecast errors of survey forecasts. It also shows, at an annualised horizon, the 12 month autocorrelation. For each country in our sample, statistics are reported for long-term (10Y) interest rates, and for spot exchange rates w.r.t the US in panels (a) and (b), respectively. The reported AR(1) coefficients relate to the yearly autocorrelation. Realized forecast errors are computed for the sample period 2000.1 to 2020.12.

	Yields	FX
$Level_t$	0.08	
	[0.03 0.15]	
$Slope_t$	0.08	
	[−0.03 0.20]	
IRD_t		−0.95
		[−1.41 −0.45]
R^2 (%)	5.71	5.73

Table 2.5. Forecast Error Predictability

This table shows pooled OLS regressions of (a) 10-year yields and (b) foreign exchange forecast errors, both at an annual horizon, on either the level and slope of the yield curve or the 1-year interest rate differential:

$$FE_{t,t+1} = a + bX_t + \eta_{t+1},$$

where for interest rate forecast errors X_t includes $Level_t = i_t^{(1)}$ and $Slope_t = (i_t^{(10)} - i_t^{(1)})$ and for exchange rate forecast errors it includes $IRD_t = (i_t^{(1)} - i_t^{(1),f})$. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. We report an equivalent table using the estimator of Driscoll and Kraay (1998) in Table A.17. Individual country-by-country regressions are presented in Table A.21. The sample period is 1995.1 to 2020.12

Panel A: Distortions and Preferences

Parameter	Estimate	Lower	Upper
$\phi_{g,d}$	0.20	0.06	0.34
ϕ_{g,f_1}	1.40	0.81	1.99
ϕ_{g,f_2}	0.10	0.03	0.17
$\beta(\%)$	0.59	-0.23	1.40
γ	6.00	2.36	9.63
ψ	1.40	1.11	1.69

Panel B: Term Structure

Moment	Data	Model	Lower	Upper
		USD		
$E[y_t^{0.25}]$	2.33	3.56	2.69	4.44
std $\Delta y_t^{0.25}$	0.21	0.22	0.18	0.27
$E[y_t^5]$	3.37	3.41	2.54	4.29
$E[y_t^{10}]$	3.98	3.28	2.39	4.16
$BRP^{\mathbb{S}}$	-1.41	-0.87	-1.30	-0.43
FE^{10}	-0.60	-0.70	-1.13	-0.26
		CHF		
$E[y_t^{0.25}]$	0.82	2.19	1.35	3.02
std $\Delta y_t^{0.25}$	0.19	0.18	0.15	0.22
$E[y_t^5]$	1.42	2.16	1.33	2.99
$E[y_t^{10}]$	1.96	2.14	1.32	2.97
$BRP^{\mathbb{S}}$	-1.91	-1.09	-1.60	-0.57
FE^{10}	-0.52	-0.57	-0.83	-0.30
		AUD		
$E[y_t^{0.25}]$	4.13	4.55	3.55	5.56
std $\Delta y_t^{0.25}$	0.25	0.32	0.26	0.39
$E[y_t^5]$	4.65	4.26	3.31	5.21
$E[y_t^{10}]$	4.96	4.00	3.06	4.95
$BRP^{\mathbb{S}}$	-0.83	-1.25	-2.09	-0.40
FE^{10}	-0.57	-0.42	-0.71	-0.12

Table 2.6. Term Structure Estimation

This table presents the parameter estimates (Panel A) and moments (Panel B) from the SMM estimation of the distortion and preference parameters of the model presented in Section 3.6 of the paper. Panel A reports point estimates alongside 95% confidence intervals. Panel B reports moments in the data and model implied moments alongside 95% confidence intervals.

Panel A: Consumption growth

	USD	CHF	AUD
$\mu_g^{\mathbb{S}}$	2.55	3.39	4.00
$\mu_g^{\mathbb{P}}$	1.31	1.55	3.22

Panel B: Risk pricing

	USD	CHF	AUD
$\Theta_g^{\mathbb{S}}$	50.11	146.99	50.53
$\Theta_g^{\mathbb{P}}$	30.07	6.90	40.54
$BRP^{\mathbb{S}}$	-0.94	-1.23	-1.34
$BRP^{\mathbb{P}}$	-0.56	-0.06	-1.08

Table 2.7. Belief Distortion

Panel A reports the estimated unconditional mean of the long-run growth process g_t, i.e. μ_g, under the subjective and physical measures, for the three countries. Panel B reports the estimated market prices of risk for the g factor, Θ_g, and the model-implied bond risk premia, BRP, under the two measures for the three countries.

Subjective Risk Premia in Bond and FX Markets

Online Appendix

This online appendix is not intended for publication. Section A.1 contains additional details on the construction of our survey dataset. Section A.2 contains supplementary results to Section 3.4 in the main body, such as a principal component analysis of the constructed risk premia and comparison to statistical benchmark measures. Section A.3 provides supplementary material to Section 2.6 in the main body, such as an analysis of belief formation and rationality. Section A.4 reports the derivations of the subjective beliefs equilibrium model discussed in Section 3.6. Section A.5 provides details of the simulated method of moments estimation approach. Sections A.6 and A.7 contains supplementary tables and figures to the main body of paper.

A.1. Data Appendix

Integral part of this paper is the measurement of subjective beliefs. We use the Consensus Economics dataset to get these beliefs for a set of professional forecasters such as banks, funds, and economic advisors. In particular, we use the Consensus Forecasts - G7 & Western Europe and Asia Pacific Consensus Forecasts datasets. These two datasets contain a wide cross-section of countries - presented in Table A.8.

While a large cross-section of countries available, the number of forecasters participating for less developing economies is significantly smaller than the number of forecasters for developed economies. Furthermore, FX data is not available at all for some currencies and realized data is sparse for the smaller economies. As such we restrict our attention to the universe of countries outlined in Section 3.2 to maintain a satisfactory number of observations for each date.

Included in the two datasets are expectations about several macroeconomic variables, interest rates, and exchange rates. While we know the identity of forecasters predicting macroeconomic variables and interest rates, Consensus Economics only reports a consensus estimate for exchange rates. Figure A.16 displays the

number of forecasters that submitted estimates of future 10-year yields for each of the ten countries selected.

[INSERT TABLE A.8 AND FIGURE A.16 HERE]

A.1.1. Interest Rate Expectations

Consensus Economics asks its panellists to provide estimates of "yields on 10 year government bonds", without specifying what type of yield. However, it is generally understood that they are providing estimates of the on-the-run bond yield to maturity, which is effectively a par yield forecast. Since we only have two maturities available, we cannot bootstrap zero coupon bond yield estimates from the par yields provided. Therefore, in the main body of the text, we treat par yield forecasts as zero coupon forecasts. Moreover, the compounding frequency of the yields provided is also not explicitly stated, so we assume they are continuously compounded, i.e. log yields. This appendix shows the robustness of our results with respect to these assumptions, by comparing empirically par yields and zero-coupon bond yields for 10-year government bonds, as well as yields and bond returns based on different compounding frequency assumptions.

Panel A of Table A.9 displays the mean and standard deviation for (1) US 10-year par yields obtained from the Fed (H15), (2) US 10-year zero yields obtained from Bloomberg (BB), and (3) their differences. Figure A.17 shows the time series of the same par (H15) and zero (BB) yields, as well as their difference. We can see that the two series are extremely close and their difference is close to zero and insignificant. A similar picture arises when looking at the bond returns implied by par and zero yields.

Panel B of Table A.9 contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Again, both mean and standard deviation of the yields are extremely similar (see also Figure A.18).

[INSERT TABLE A.9 AND FIGURE A.17 HERE]

Summarising, we show that par yields and zero-coupon bond yields for 10-year government bonds are empirically very close and that the compounding frequency

has little impact on the bond yields, so our results would be practically unchanged if we assumed that yields are annually or semi-annually compounded instead of continuously compounded, and they are robust to our assumption that survey forecasters provide zero yields.

A.1.2. Macro Economic Expectations

In addition to interest rate and foreign exchange forecasts CE covers a large set of macro-economic variables. We focus on real GDP growth, industrial production growth, personal consumption growth, and the rate of unemployment. A complication with the survey projections is that respondents are asked to report expectations over the current and the next calendar year (except for interest rates, which are constant maturity forecasts); thus, the dataset represents a set of variable maturity events. For example, in July 2003 each contributor to the survey made a forecast for the percentage change in GDP for the remaining two quarters of 2003 (6 months ahead), and an average percentage change for 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induces a seasonal pattern in the survey. We compute an implied constant maturity forecast for each individual forecaster as in Buraschi and Whelan (2022) and Fendel et al. (2011). Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2..12$. A constant maturity expectation is formed taking as weight $(1 - \frac{j}{12})$, for the short term projection (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term projection (the forecast for the following year). Figure A.19 illustrates the weighting procedure visually.

[INSERT FIGURE A.19 HERE]

A.2. Supplementary material to Section 3.4

A.2.1. Are bonds risky bets or hedges?

Do forecasters believe bonds are genuinely hedges or is this an artefact of the sample period? Consider again figure 2.11. In both first half and second half of the sample (when interest rates were relatively flat) we observe that $BRP_t < 0$ most of the time. Table A.10 confirms this point more formally by providing subsample statistics. Investigating this question further, figure A.20 displays stock-bond correlations ($corr_{SB}$) computed as the rolling 200-day correlation between 10-year zero coupon bond returns and the corresponding major equity market indices in each country. The figure demonstrates that the well studied change from positive to negative $corr_{SB}$ that first occurred in the late 1990's in the U.S. is a common feature in the cross-section of G10 countries in our sample. This is an interesting observation since observing $corr_{SB} < 0$ suggests bonds should also be hedges according CAPM logic. Moreover, in the 1990s when $corr_{SB} > 0$ we do, in fact, observe BRPs which are largely positive. Thus, the average sign of subjective BRP_ts is consistent with a risk based view of bond pricing.

[INSERT TABLE A.10 and FIGURE A.20 HERE]

A.2.2. FX Factors

How many factors are needed to explain the cross-sectional variation in subjective risk premia? The answer to this question speaks to whether a risk based interpretation of subjective beliefs is supported in the data. To answer this question, we follow Lustig et al. (2011) and compute factors PC_t (principal components) from an eigenvalue decomposition of the covariance matrix of subjective exchange rate risk premia.

Time-series of factors are displayed in figure A.23 and variance explained by each factor is reported in table A.15. We find that 64% of the variation in subjective XRP is explained by a single factor (PC1), which as usual is a level factor. Estimating factors from portfolio returns sorted on interest rate differentials, as a proxy for expected returns, Lustig et al. (2011) call PC1 the 'dollar risk factor'.[18] Currencies

[18] We note that the JPY has a zero loading on $PC1$

are sorted via their exposure to $PC2$, which explains 14% of the variation in subjective XRP, and their loadings plotted in panel (a) of figure A.23. $PC2$ is clearly a 'slope' factor and has a straightforward economic interpretation inline with the risk based view of Lustig et al. (2011): the factor loads negatively on the JPY and CHF and positively on AUD and NZD and so generates a return spread between investment currencies and funding currencies. The factor itself became increasingly negative between 2003 and 2007 (see Panel (b) of Figure A.23) implying that agents believed the carry trade was falling apart but increased from 2007-2012 implying that subjectively they were forecasting increasing carry trade returns. Thus, complimenting the arguments Lustig et al. (2011), we show that there are common factors in subjective currency returns which supports a risk-based view of subjective asset pricing.

[INSERT FIGURE A.23 AND TABLE A.15 HERE]

A.3. Supplementary material to Section 2.6

A.3.1. How Rational are Survey Forecasts?

The main goal of this paper is to exploit surveys to obtain direct, forward-looking measures of risk premia in foreign exchange and fixed income markets. Exploiting surveys Section 3.6 derives and estimates a subjective general equilibrium model in which the representative agents first order condition is formed under a subjective probability measure $\mathbb{S} \neq \mathbb{P}$. Thus; we allow for the possibility that investors make forecast errors but we take beliefs as given and model the belief wedge between $\mathbb{S}$ and $\mathbb{P}$ exogenously.

In other words, we are silent on the micro-foundations for deviations from full information rationality. However, for the interested reader we do provide an analysis on the extent to which subjective *consensus* beliefs about financial variables display evidence of behavioural biases and/or information frictions.

A.3.2. Over-reaction (under-reaction) to news

An important stream of the literature investigate economies with rational but sticky-information (Mankiw and Reis, 2002) and with rational but noisy-information (Woodford, 2002, Sims, 2003, and Mackowiak and Wiederholt, 2009). In the first case, agents update their information sets infrequently as a result of a fixed costs of information acquisition and the degree of information rigidity is the probability of not acquiring new information each period. When agents are subject to noisy information, they rationally update their beliefs but, since they can never fully observe the true state, they use an optimal signal-extraction filter. If one were to aggregate these expectations, Coibion and Gorodnichenko (2015) (CG) show that average forecast errors are predictable by forecast revisions. Considering inflation expectations from the Survey of Professional Forecasters CG find that consensus forecast revisions positively predict forecast errors which in their framework corresponds to under-reaction due to information frictions. Exploiting the Consensus Economics dataset Bordalo et al. (2020) confirms the CG findings of under-reaction at the consensus level for macroeconomic series. [19]

While Singleton (2021) argues that predictability of the forecast errors is not necessarily evidence of irrationality, as even a purely Bayesian agent would exhibit serial dependence of the errors if factors are not fully observable, we still use the CG framework to examine the potential extent of information frictions at the level of the consensus forecast for the financial variables under consideration in this paper. Namely, we estimate regressions of forecast errors on forecast revisions

$$FE^i_{t,t+1} = a^i + b^i FR^i_{t-1,t} + \eta^i_{t+1}. \tag{A.58}$$

The rational expectations hypothesis predicts that forecast errors should be unpredictable conditional on the publicly available information filtration which includes forecast revisions; thus predicting $b^i = 0$. Instead if forecasters under-react to

[19] Bordalo et al. (2020) also study CG regressions at the individual forecast level and show that they typically overreaction which can be explained by a deviation from full information rationality due to what they call 'diagnostic expectations'. Note that Bordalo et al. (2020) do not use an annual forecasting horizon but a quarterly one.

information, because for example they are inattentive or hold adaptive expectations, we expect $b^i > 0$. Similarly, when forecasters over-react to information, because for example they are overly optimistic or pessimistic, we should expect $b^i < 0$.

[INSERT TABLE A.20 HERE]

Table A.20 presents parameter estimates with Newey-West adjusted t-statistics. First, consider panel (a) which displays estimates for 10-year interest rate survey forecasts. None of the point estimates for b are statistically different from zero and are generally small in economic terms. The estimated intercepts are, however, uniformly negative and statistically significant consistent with the bias in interest rate forecast errors in Table 2.4. Second, consider panel (b) which displays estimates for exchange rate survey forecasts. The results here are mixed: while almost all point estimates are positive only two out of nine of the point estimates (CAD and GBP) for b are at least marginally statistically significant consistent with the CG findings of under-reaction.

A.3.3. Forecast Error Predictability

Section 2.6 in the main body of the paper studies whether mild persistence observed in the autocorrelation of $t \to t+12$ errors results in predictability by observables at date t. The regression results show that interest rate and exchange rate expectation errors are predictable by interest rate levels and spreads. These findings suggests agents do not exploit all available information when forming their beliefs, which is inconsistent with the basic idea of rational expectations. However, to the best of our knowledge it remains an open question whether statistical predictability can be exploited to correct errors in real-time. In order to address this question we investigate the economic significance of the predictable component of forecast errors.

A.3.4. Economic Significance of Deviation from Rational Expectations

To study the economic significance of behavioural components in agents expectations, we design an experiment in which we construct fictitious expectations by correcting

the predictable errors using information available in date t observables. In this real-time experiment, we initialise a rolling regression with a window of w-years of data and recursively estimate a projection of realized errors on yields or interest rate differentials. The loadings available in the forecast error regression at date t can only be learned from errors realized one year earlier. These loadings are then applied to date t observables in order to build a 'corrected' beliefs from the following system

$$\widehat{FE}^{n}_{t-1,t} = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_{t-1} \tag{A.59}$$

$$\xi_t = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_t \tag{A.60}$$

$$\widehat{Y}_t = E_t^{\mathbb{S}}\Big[Y_{t+1}\Big] + \xi_t \tag{A.61}$$

The subscript t in the parameters $\hat{\alpha}_t$ and $\hat{\beta}_t^{\top}$ indicate that the correction is restricted to use only real-time information which is available at time t. The predictable component of forecast error is estimated using a rolling window to replicate real-life conditions of a trader. In unreported results we also consider expanding windows after a 5-year initial burn in period; the main message that follows is quantitively similar.

Panel (a) of figure A.27 compares the change in the $RMSE$'s (y-axis) implied by both the original forecasts and the corrected forecasts for various rolling window lengths (x-axis). We find that, although the initial regressions indicate the existence of predictability in the forecast errors, the $RMSE$ of the corrected beliefs are unambiguously *higher* than the uncorrected ones. For instance, using a rolling window of 5 years in the estimation of the correction parameters, the RMSE increase by around 96% for the 10-year bond. This shows that the expectations extracted from surveys cannot be easily improved using market based state-dependent information. In panel (b) of figure A.27 we show that if one were to correct interest rate expectations for a constant bias, obtained from the (ex-post) mean of the forecast errors, the RMSE of the forecast would decrease by about 11%. However, we note that this is due to the bias in forecasts coming from over-prediction in sample rather than agents omitting useful information from the term structure.

The result of replicating this experiment for exchange rate expectations yields similar results and is visualised in figure A.28.

Summarising, the findings of this section show that 'uncorrected' beliefs dominates their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements. This provides a possible explanation for why subjective expectation can be persistently different from the null implied by rational expectations.

[INSERT FIGURE A.27 AND A.28 HERE]

A.3.5. Note on the Driscoll and Kraay (1998) estimator

In Section 3.4, we discuss tables that show evidence about the explainability and predictability of survey-implied risk premia. For better readability, we choose to report point estimates as well as 95% confidence intervals. These intervals are estimated using a circular block bootstrap that uses a block length of 12 and 1000 bootstrap samples. As a robustness check to our predicability and cyclicality results, we also present confidence intervals using the estimator of Driscoll and Kraay (1998).

As shown in Tables A.12 and A.14, our panel dataset is characterised by significant cross-sectional correlation. We choose the Driscoll and Kraay (1998) estimator to account for this cross-sectional dependence in the estimation. Hoechle (2007) provides a discussion of the inner workings of the estimator and includes a STATA program called *xtscc* to run the estimation. In short, the estimator of Driscoll and Kraay (1998) applies a correction similar to Newey and West (1987) but adds robustness to cross-sectionally clustered standard errors. By combining these two properties, it yields a method to obtain standard errors that are not only robust to autocorrelation and heteroskedasticity but also consistent in settings of cross-sectional dependence. Another appealing property of the estimator is that it can, with a minor adjustment, handle unbalanced samples by applying the Newey and West (1987) correction at every time t to the moment conditions of individuals $N(t)$.

In terms of hypothesis testing, Tables A.18 and A.19 yield similar results as their counterparts (Tables 2.2 and 2.3) in the main body of the paper.

A.4. Asset Pricing Derivations

A.4.1. Economic Dynamics

Consider the following model. Aggregate consumption flow is given by C_t ($c_t = \ln C_t$) whose conditional mean is driven by a "long run risk' factor g_t

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sigma_c dW_{c,t}, \tag{A.62}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g dW_{g,t}, \tag{A.63}$$

$$= (a_{g0} + a_{g1}g_t)dt + \sigma_g dW_{g,t}, \tag{A.64}$$

where $\langle dW_{c,t}, dW_{g,t}\rangle = \rho_{cg}$ is potentially non-zero. However, in estimation we assume short run and long run risks are independent. We assume a constant conditional mean geometric Brown motion for the price level

$$\frac{dQ}{Q} = \pi dt + \sigma_Q dW_{Q,t} \tag{A.65}$$

with $dW_{\pi,t}$ independent of all other Brownian's. In the paper we set

A.4.2. Preferences

We employ the Duffie and Epstein (1992a) parameterisation of the continuous time Epstein and Zin (1989) and Weil (1989) recursive preferences

$$J_t = E_t\left(\int_t^\infty f(C_s, J_s)ds\right) \tag{A.66}$$

where $f(C, J)$ is the normalised Kreps and Porteus (1978) aggregator

$$f(C,J) = (1-\gamma)J\left(\frac{\beta}{1-\frac{1}{\psi}}\right)\left(\frac{C^{1-1/\psi} - [(1-\gamma)J]^{1/\theta}}{[(1-\gamma)J]^{1/\theta}}\right) \tag{A.67}$$

$$f(C,J) = \beta\theta J\left[\frac{C^{1-1/\psi}}{[(1-\gamma)J]^{1/\theta}} - 1\right] \tag{A.68}$$

$$= \beta\theta J\left[Z(C,J) - 1\right], \tag{A.69}$$

where we have defined

$$Z(C,J) \equiv \left[C^{1-1/\psi}\right]\left[(1-\gamma)J\right]^{-1/\theta} \tag{A.70}$$

$$\theta \equiv \frac{1-\gamma}{1-1/\psi}. \tag{A.71}$$

β is a time discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of intertemporal substitution (EIS). As discussed in Bansal and Yaron (2004), EZ preferences allow for separation of time-and state-preferences for consumption, a property unavailable in time-separable specifications where consumption profiles are governed by a single parameter, $\gamma = 1/\psi$. When $\gamma \neq \psi$ agents have a preference for early (late) resolution of uncertainty when risk aversion is larger (smaller) than the inverse of EIS. A preference for early resolution combined with $\psi > 1$ implies that $\theta < 1$. In the limit $\psi \to \frac{1}{\gamma}$ we obtain power utility

$$\lim_{\psi \to \frac{1}{\gamma}} Z(C,J) \equiv \left[C^{1-\gamma}\right]\left[(1-\gamma)J\right]^{-1} \tag{A.72}$$

$$\lim_{\psi \to \frac{1}{\gamma}} f(C,J) = \beta J\left(\left[C^{1-\gamma}\right]\left[(1-\gamma)J\right]^{-1} - 1\right) \tag{A.73}$$

$$= \frac{C^{1-\gamma}}{1-\gamma} - \beta J \equiv \beta \frac{C^{1-\gamma}}{1-\gamma} \tag{A.74}$$

To obtain limit $\psi \to 1$ first write

$$Z(C,J) = e^a e^b \tag{A.75}$$

$$a = \ln C - \psi^{-1} \ln C \tag{A.76}$$

$$b = -(1-\gamma)^{-1}(1-\psi^{-1}) \ln[(1-\gamma)J] \tag{A.77}$$

$$\lim_{\psi \to 1} \frac{\partial Z}{\partial \psi} = \lim_{\psi \to 1} e^a e^b \Big[\psi^{-2} \ln C - \psi^{-2}(1-\gamma)^{-1} \ln[(1-\gamma)J]\Big] \tag{A.78}$$

$$= \ln C - (1-\gamma)^{-1} \ln[(1-\gamma)J] \tag{A.79}$$

$$\lim_{\psi \to 1} \frac{\partial \beta\theta J}{\partial \psi} = \beta J \lim_{\psi \to 1} \frac{\partial \theta}{\partial \psi} = \beta J \lim_{\psi \to 1} \frac{1-\gamma}{1+\psi^{-2}} = 1-\gamma \tag{A.80}$$

$$\lim_{\psi \to 1} f(C,J) = \beta(1-\gamma)J\Big[\ln C - (1-\gamma)^{-1} \ln[(1-\gamma)J]\Big] \tag{A.81}$$

$$= \beta J\Big[(1-\gamma)\ln C - \ln[(1-\gamma)J]\Big] \tag{A.82}$$

Note that f_J and f_C are given by

$$f_J = \beta Z(C,J)(\theta - 1) - \beta\theta \tag{A.83}$$

$$f_C = \beta(1-\gamma)JC^{-1}Z(C,J). \tag{A.84}$$

and for the special case of $\psi \to 1$

$$f_J = \beta[(1-\gamma)\ln C - \ln[(1-\gamma)J] - \beta \tag{A.85}$$

$$f_C = \beta J(1-\gamma)C^{-1} \tag{A.86}$$

A.4.3. Bond Pricing

Duffie and Epstein (1992a) also show that

$$\Lambda_t^* = \beta e^{\int_0^t f_J(C_s,J_s)ds} f_c(C_t,J_t) \tag{A.87}$$

which defines the wealth process defined as the present value of future consumption

$$\Lambda_t^* W_t = E_t\left[\int_t^\infty \Lambda_s^* W_s ds\right] \tag{A.88}$$

The real risk-free interest rate is given by

$$r_t^* = -\frac{1}{dt}E_t\left(\frac{d\Lambda_t^*}{\Lambda_t^*}\right) \tag{A.89}$$

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = \frac{df_c}{f_c} + f_J dt \tag{A.90}$$

and prices of risk come from the diffusion terms in $\frac{df_c}{f_c}$. Solving for real bond prices, note that in what follows the logarithm of Λ_t^* follows the SDE

$$d\log\Lambda_t^* = \left(-r_t^* - \frac{1}{2}(\Theta_c)^2 - \frac{1}{2}(\Theta_g)^2\right)dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} \tag{A.91}$$

$$r_t^* = r_0 + r_g g_t. \tag{A.92}$$

This implies for $\tau = T - t$

$$P_\tau^* = E_t\left[\frac{\Lambda_T^*}{\Lambda_t^*}\right] = E_t\left[e^{\log\Lambda_T^* - \log\Lambda_t^*}\right] \tag{A.93}$$

$$= \mathbb{E}_t\left[e^{-\int_t^T\left[r_u^* du + \frac{1}{2}\Theta_c^2 + \frac{1}{2}\Theta_g^2\right]du - \int_t^T \Theta_c dW_{c,u} - \int_t^T \Theta_g dW_{g,u}}\right] \tag{A.94}$$

Notice that

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\frac{1}{2}\int_t^T\left[\Theta_c^2 + \frac{1}{2}\Theta_g^2\right]du - \int_t^T \Theta_c dW_{c,u} - \int_t^T \Theta_g dW_{g,u}\right) \tag{A.95}$$

is the Radon-Nikodym derivative between physical probability measure $\mathbb{P}$ and another measure risk-adjusted measure $\mathbb{Q}$. With the Radon-Nikodym derivative at hand we can re-write $P^\tau = E_t^{\mathbb{P}}[\dots]$ as

$$P_\tau^* = \frac{E_t\left[\frac{d\mathbb{Q}}{d\mathbb{P}}e^{-\int_t^T r_u^* du}\right]}{E_t^{\mathbb{P}}\left[\frac{d\mathbb{Q}}{d\mathbb{P}}\right]} = E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s^* ds}\right] \tag{A.96}$$

and from Girsanov theorem we know that this change of measure satisfies

$$W_{i,t}^{\mathbb{Q}} = W_{i,t} + \int_0^t \Theta_i^{\mathbb{P}} du. \tag{A.97}$$

Notice the SDF can be equivalently expressed as

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^* dt - \Theta_c\sqrt{1-\rho_{cg}^2}dW_{c,t}^\top - \Big(\rho_{cg}\Theta_c + \Theta_g\Big)dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.98}$$

where $dW_{c,t}^\top$ is an orthogonal Brownian. Under the $\mathbb{Q}$-measure the risk neutral diffusion for g_t is given by

$$dg_t = [a_{g0}^{\mathbb{Q}} + a_{g1}^{\mathbb{Q}} g_t]dt + \sigma_g dW_{g,t}^{\mathbb{Q}}, \tag{A.99}$$

$$a_{g0}^{\mathbb{Q}} = a_{g0} - (\rho_{cg}\Theta_c + \Theta_g)\sigma_g \tag{A.100}$$

$$a_{g1}^{\mathbb{Q}} = a_{g1} \tag{A.101}$$

Computing bond prices follows from the standard risk neutral pricing approach requiring a solution to

$$\frac{\partial P_\tau^*}{\partial \tau} = \Big[a_{g0}^{\mathbb{Q}} + a_{g1}^{\mathbb{Q}} g_t\Big]\frac{\partial P_\tau^*}{\partial g_t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 P_\tau^*}{\partial g_t^2} - r_t P_\tau^* \tag{A.102}$$

Conjecture the solution

$$P_\tau^* = \exp\Big(A^*(\tau) + B(\tau) g_t\Big). \tag{A.103}$$

Taking partials, substituting in the PDE above yields the system of equations

$$\frac{\partial A^*(\tau)}{\partial \tau} = B_g(\tau) a_{g0}^{\mathbb{Q}} + \frac{1}{2} B_g(\tau)^2 \sigma_g^2 - r_0 \tag{A.104}$$

$$\frac{\partial B(\tau)}{\partial \tau} = B(\tau) a_{g1} - r_g \tag{A.105}$$

The boundary conditional for the ODEs will have to be $A(0) = B(0) = 0$ which implies $P_0^* = 0$. $B(\tau)$ is a first order non-homogeneous linear DE, which can be solved using an integrating factor where

$$B_g(\tau) = \frac{r_g}{a_{g1}}\Big(1 - e^{a_{g1}\tau}\Big) \tag{A.106}$$

Note that $B_g(\tau) < 0$ because higher real growth rates increases short rates and lowers prices. Then $A(\tau)$ is an ugly expression that follows by direct integration. The specific preferences specifications that we consider below will generate differences in bond pricing solutions through the the risk adjustments $\{\Theta_c, \Theta_g\}$ and through the short rate loadings $\{r_0, r_g\}$. The nominal SDF is given by $\Lambda_t = d\Lambda_t^* Q_t^{-1}$

$$d\Lambda_t = d\Lambda_t^* Q_t^{-1} + \Lambda_t^* dQ_t^{-1} + d\Lambda_t^* dQ_t^{-1} \tag{A.107}$$

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{d\Lambda_t^*}{\Lambda_t^*} - \frac{dQ}{Q} - \sigma_q^2 dt - \frac{d\Lambda_t^*}{\Lambda_t^*}\frac{dQ_t}{Q_t} \tag{A.108}$$

Thus,

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.109}$$

$$r_t = r_t^* + \pi - \sigma_Q^2 \tag{A.110}$$

$$\Theta_Q = \sigma_Q \tag{A.111}$$

and the nominal bond pricing function is given by

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\frac{Q_t}{Q_T}\right] = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] E_t\left[\frac{Q_t}{Q_T}\right] = \exp(A^*(\tau) + B_g(\tau)g_t)\exp\left([\pi - \frac{1}{2}\sigma_Q^2]\tau\right) \tag{A.112}$$

$$= \exp(A(\tau) + B_g(\tau)g_t) \quad \text{where } A(\tau) = A^*(\tau) + [\pi - \frac{1}{2}\sigma_Q^2]\tau \tag{A.113}$$

The diffusion for bond returns follows as an application of itô's lemma

$$\frac{dP_\tau}{P_\tau} = \mu_\tau dt + \sigma_{\tau,g} dW_{g,t} \tag{A.114}$$

where

$$\mu_\tau = \frac{1}{\mu_{t,\tau}}\left[\frac{\partial P}{\partial g}\mu_g + \frac{1}{2}\frac{\partial^2 P}{\partial g^2}\sigma_g^2 + \frac{\partial P}{\partial \tau}\right] \tag{A.115}$$

$$= \left[B_g(\tau)(a_{g0} + a_{g1}g_t) + \frac{1}{2}B_g(\tau)^2\sigma_g^2 + A'(\tau) + B_g'(\tau)g_t\right] \tag{A.116}$$

$$\sigma_{\tau,g} = \frac{1}{P(\tau)}\left[\frac{\partial P}{\partial g}\sigma_g\right] = B_g(\tau)\sigma_g \tag{A.117}$$

Nominal instantaneous expected excess returns are given by

$$E_t\left[\frac{dP_\tau}{P_\tau}\right] - \frac{\partial P_\tau}{\partial \tau} - E_t^{\mathbb{Q}}\left[\frac{dP_\tau}{P_\tau}\right] = -E_t\left[\frac{d\Lambda_t}{\Lambda_t}\frac{dP_\tau}{P_\tau}\right] \tag{A.118}$$

$$\frac{1}{dt}BRP(\tau) = -E_t\left[\frac{d\Lambda_t}{\Lambda_t}\frac{dP_\tau}{P_\tau}\right] \tag{A.119}$$

$$= \Theta_c B_g(\tau)\sigma_g\rho_{cg} + \Theta_g B_g(\tau)\sigma_g \tag{A.120}$$

A.4.4. The Subjective Measure

Here, we use superscripts $\mathbb{P}$ to indicate the physical SDF which is that of an econometrician who has full information about the data generating process and the parameters which govern its distribution. The relationship between the econometricians measure and the subjective measure is given by $E_t^{\mathbb{P}}[z_s] = E_t^{\mathbb{S}}\left[\frac{d\mathbb{P}}{d\mathbb{S}} z_s\right]$where from Girsanov[20]

$$\frac{d\mathbb{P}}{d\mathbb{S}} = \exp\left(-\int_t^s \phi_z dW_{z,u}^{\mathbb{P}} - \frac{1}{2}\int_0^t \phi_z^2 du\right), \tag{A.121}$$

$$dW_{z,t}^{\mathbb{P}} = dW_{z,t}^{\mathbb{S}} + \phi_z dt \tag{A.122}$$

We call ϕ_z a BELIEF DISTORTION. Now we have two changes of measure which satisfy

$$dW_{z,t}^{\mathbb{Q}} = dW_{z,t}^{\mathbb{P}} + \Theta_z^{\mathbb{P}} dt \quad , \quad dW_{z,t}^{\mathbb{Q}} = dW_{z,t}^{\mathbb{S}} + \Theta_z^{\mathbb{S}} dt, \tag{A.123}$$

$$dW_{z,t}^{\mathbb{P}} = dW^{\mathbb{S},z} + \phi_z dt \quad \rightarrow \quad \phi^z = \Theta_z^{\mathbb{S}} - \Theta_z^{\mathbb{P}} \tag{A.124}$$

It what follows we assume that a belief distortion only exists on the distribution of g_t

$$dg_t = [a_{g0}^{\mathbb{S}} + a_{g1}^{\mathbb{S}} g_t]dt + \sigma_g dW_{g,t}^{\mathbb{S}}, \tag{A.125}$$

$$a_{g0}^{\mathbb{S}} = a_{g0} + \phi_g \sigma_g \tag{A.126}$$

$$a_{g1}^{\mathbb{S}} = a_{g1} \tag{A.127}$$

The subjective SDF is given by

$$\frac{d\Lambda_t^{\mathbb{S}}}{\Lambda_t^{\mathbb{S}}} = -r_t dt - \Theta_c^{\mathbb{S}} dW_{c,t}^{\mathbb{S}} - \Theta_g^{\mathbb{S}} dW_{g,t}^{\mathbb{S}} - \Theta_Q^{\mathbb{S}} dW_{Q,t}^{\mathbb{S}} \tag{A.128}$$

$$\Theta_c^{\mathbb{S}} = \Theta_c^{\mathbb{P}} \tag{A.129}$$

$$\Theta_g^{\mathbb{S}} = \Theta_g^{\mathbb{P}} + \phi_g \tag{A.130}$$

$$\Theta_Q^{\mathbb{S}} = \Theta_Q^{\mathbb{P}} \tag{A.131}$$

[20] We assume $\mathbb{S}$ is absolutely continuous with respect to $\mathbb{P}$.

Note that bond prices can equivalently be solved for under the subjective or physical measure via the changes of measure defined above:

$$P_\tau = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right] \tag{A.132}$$

$$= E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{P}}\right] \tag{A.133}$$

$$= E_t\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\frac{d\mathbb{S}}{d\mathbb{P}}\right] \tag{A.134}$$

$$= E_t^{\mathbb{S}}\left[e^{-\int_t^T r_s ds}\frac{d\mathbb{Q}}{d\mathbb{S}}\right]$$

$$= E_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right] = \exp\Big(A_\tau + B_\tau g_t\Big). \tag{A.135}$$

A.4.5. CRRA Solution

In the CRRA case f_J is zero and so from equation A.87 the real SDF is given by $\Lambda_t^* = f_C = \beta C_t^{-\gamma}$. The risk free rate is given by

$$r_t^* = r_0 + r_g g_t \tag{A.136}$$

$$r_0 = \beta - \frac{1}{2}\gamma(1+\gamma)\sigma_c^2 \tag{A.137}$$

$$r_g = \gamma \tag{A.138}$$

and the SDF follows

$$d\Lambda_t^*/\Lambda_t^* = -r_t^* dt - \Theta_c dW_{c,t} \tag{A.139}$$

$$\Theta_c = \gamma\sigma_c \tag{A.140}$$

and bond risk premium is given by

$$BRP(\tau) = \gamma\sigma_c\sigma_g\rho_{cg}B_g(\tau)dt \tag{A.141}$$

A.4.6. Recursive HJB equation

The HJB equation implied by the normalised aggregator is given by

$$\sup_{\{C\}} f(C,J) + \mathcal{D}J(\cdot) = 0 \tag{A.142}$$

which spelling out is equal to

$$\frac{1}{dt}\sup_{\{C\}} f(C,J)dt + J_g E(dg) + J_C E(dC\Big) + \frac{1}{2}\Big[J_{gg}d\langle g,g\rangle + J_{CC}d\langle C,C\rangle\Big] + J_{gC}d\langle g,C\rangle = 0 \tag{A.143}$$

which after inserting the states is given by

$$\sup_{\{C\}} f(C,J) + J_g(a_{g0}+a_{g1}g) + J_C C g + \frac{1}{2}\Big[J_{gg}\sigma_g^2 + J_{CC}C^2\sigma_c^2\Big] + J_{gC}C\sigma_g\sigma_c\rho_{cg} = 0 \tag{A.144}$$

and we denote the maximised aggregator f^*. In the following we will solve for an exact and an approximate solution based on the the following ansatz

$$J(W,g) = F(g)\frac{W^{1-\gamma}}{1-\gamma} \tag{A.145}$$

$$F(g) = \exp(a_{F0} + a_{Fg}g) \tag{A.146}$$

A.4.7. Exact Recursive Solution $\psi \to 1$

Using the envelope condition ($J_W = f_C$) we can obtain the consumption-wealth ratio as follows

$$J_W = \beta J(1-\gamma)C^{-1} \tag{A.147}$$

$$W = \beta^{-1}C \tag{A.148}$$

The value function written in terms of consumption is given by

$$J(C,g) = \beta^{\gamma-1} F(g) \frac{C^{1-\gamma}}{1-\gamma} \tag{A.149}$$

Taking partials the HJB becomes

$$f^* J^{-1} + a_{Fg}(a_{g0} + a_{g1} g) + (1-\gamma)g + \frac{1}{2}\Big[a_{Fg}^2 \sigma_g^2 - \gamma(1-\gamma)\sigma_c^2\Big] + a_{Fg}(1-\gamma)\sigma_g \sigma_c \rho_{cg} = 0 \tag{A.150}$$

Remembering that $f(C,J) = \beta J\Big[(1-\gamma)\ln C - \ln[(1-\gamma)J]\Big]$ we have

$$f^* J^{-1} = \beta\Big[(1-\gamma)\ln\beta - a_{F0}\Big] - \beta a_{Fg} g \tag{A.151}$$

and we are left with

$$\beta\Big[(1-\gamma)\ln\beta - a_{F0}\Big] - \beta a_{Fg} g + a_{Fg}(a_{g0} + a_{g1} g) + (1-\gamma)g + \frac{1}{2}\Big[a_{Fg}^2 \sigma_g^2 - \gamma(1-\gamma)\sigma_c^2\Big] \tag{A.152}$$

$$+ a_{Fg}(1-\gamma)\sigma_g \sigma_c \rho_{cg} = 0 \tag{A.153}$$

Via separation of variables we obtain

$$a_{F0} = \frac{1}{\beta}\Big[(1-\gamma)\beta\ln\beta + a_{Fg} a_{g0} + \frac{1}{2}[a_{Fg}^2 \sigma_g^2 + \gamma(\gamma-1)\sigma_c^2] \tag{A.154}$$

$$a_{Fg} = \frac{\gamma - 1}{a_{g1} - \beta} \tag{A.155}$$

Applying results from above we note that

$$f_J = (1-\gamma)\beta\ln\beta - \beta[a_{F0} + a_{Fg}] - \beta \tag{A.156}$$

$$f_C = \beta e^{a_{F0} + a_{Fg} g} C^{-\gamma} \tag{A.157}$$

and that the diffusions for f_C is given by

$$\frac{df_C}{f_C} = \left(-\gamma g + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + a_{Fg}(a_{g0}+a_{g1}g) + \frac{1}{2}a_{Fg}^2\sigma_g^2 - \gamma a_{Fg}\sigma_g\sigma_c\rho_{cg}\right)dt \tag{A.158}$$

$$- \gamma\sigma_c dW_{c,t} + a_{Fg}\chi\sigma_g dW_{g,t}. \tag{A.159}$$

The risk free rate is then given by

$$r_t^* = -\frac{1}{dt}E_t\left(\frac{df_C}{f_C}\right) - f_J \tag{A.160}$$

$$= \left(\gamma g_t - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - a_{Fg}(a_{g0}+a_{g1}g_t) - \frac{1}{2}a_{Fg}^2\sigma_g^2 + \gamma a_{Fg}\sigma_g\sigma_c\rho_{cg}\right) \tag{A.161}$$

$$- (1-\gamma)\beta\ln\beta + \beta[a_{F0}+a_{Fg}] + \beta \tag{A.162}$$

which for convenience we denote

$$r_t^* = r_0 + r_g g_t, \tag{A.163}$$

$$r_0 = \beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - a_{Fg}a_{g0} - \frac{1}{2}a_{Fg}^2\sigma_g^2 + \gamma a_{Fg}\sigma_g\sigma_c\rho_{cg} - (1-\gamma)\beta\ln\beta + \beta a_{F0} \tag{A.164}$$

$$r_g = \gamma - a_{Fg}[a_{g1} - \beta] = 1 \tag{A.165}$$

Notice in the limit $\psi \to \frac{1}{\gamma}$ we recover the CRRA solution above $r_x = \gamma$ and $r_0 = \theta\beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2$ since in this case $\theta = 1$ and $\chi = 0$. We now want the diffusion for the SDF which follows

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^* dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} \tag{A.166}$$

$$\Theta_c = \gamma\sigma_c \tag{A.167}$$

$$\Theta_g = -a_{Fg}\sigma_g = -\frac{\gamma-1}{a_{g1}-\beta}\sigma_g = \frac{1-\gamma}{a_{g1}-\beta}\sigma_g \tag{A.168}$$

Note the price of long run risk shocks can be positive or negative but for preference for early resolution $a_{Fg} < 0$ and so negative long run growth shocks are bad

news. The nominal SDF is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.169}$$

$$r_t = r_t^* + \pi - \sigma_Q^2 \tag{A.170}$$

$$\Theta_c = \gamma\sigma_c \tag{A.171}$$

$$\Theta_g = \frac{1-\gamma}{a_{g1}-\beta}\sigma_g \tag{A.172}$$

$$\Theta_Q = \sigma_Q \tag{A.173}$$

The bond risk premium is constant and given by

$$BRP_\tau = \Theta_c B_g(\tau)\sigma_g\rho_{cg}dt + \Theta_g B_g(\tau)\sigma_g dt \tag{A.174}$$

$$\frac{1}{dt}BRP_\tau^* = \gamma\sigma_c\sigma_g\rho_{cg}B_g(\tau) + \frac{1-\gamma}{a_{g1}-\beta}\sigma_g^2 B_g(\tau) \tag{A.175}$$

A.4.8. Log-linear solution

For $\psi \neq 0$ the consumption-wealth ratio again follows from the envelope condition

$$\frac{C}{W} = \beta^\psi F^{\frac{1-\psi}{1-\gamma}} = \beta^\psi F^\alpha \tag{A.176}$$

where we have defined

$$\alpha \equiv \frac{1-\psi}{1-\gamma} \tag{A.177}$$

The HJB becomes

$$f^* J^{-1} + a_{Fg}\psi(a_{g0}+a_{g1}g) + (1-\gamma)g + \frac{1}{2}\Big[(a_{Fg}\psi)^2\sigma_g^2 + (-\gamma)(1-\gamma)C^{-2}C^2\sigma_c^2\Big]$$
$$+ a_{Fg}\psi C^{-1}(1-\gamma)C\sigma_g\sigma_c\rho_{cg} = 0$$

From $f = \theta J(\beta Z - \beta)$ we can re-write the first term as

$$f^* J^{-1} = \theta J(\beta Z - \beta)J^{-1} = \theta\beta Z - \theta\beta = \theta\frac{C}{W} - \theta\beta \tag{A.178}$$

Where we obtain the last equality by nothing that $Z(C,J)$ can be re-written in terms of the consumption wealth ratio since

$$\begin{aligned} Z =&(1-\gamma)^{-1/\theta}\beta^{\psi-1}\exp\left([a_{F0}+a_{Fg}g]\left(\frac{1-\psi}{1-\gamma}\right)\left(\frac{\psi-1}{\psi}\right)\right)W_t^{\frac{\psi-1}{\psi}} \\ &\times\exp\left([a_{F0}+a_{Fg}g]\left(\frac{1/\psi-1}{\gamma-1}\right)\right)\frac{W_t^{1/\psi-1}}{(1-\gamma)^{-1/\theta}} \end{aligned} \tag{A.179}$$

$$Z =\beta^{\psi-1}\exp\left([a_{F0}+a_{Fg}g]\alpha\right)W_t^0 \tag{A.180}$$

$$\beta Z = \frac{C_t}{W_t} \tag{A.181}$$

Following Chacko and Viceira (2005), define $\mu_{cw} = E[c_t - w_t]$ where lowercases indicate logs, and take a Taylor expansion of the consumption wealth ratio around its long run mean

$$\frac{C}{W} = \exp(c-w) \sim \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})(c-w) = h_0 + h_1[a_{F0}+a_{Fg}g]\alpha \tag{A.182}$$

$$\frac{C}{W} \sim h_0 + h_1[a_{F0}+a_{Fg}g]\alpha \tag{A.183}$$

$h_0 = \exp(\mu_{cw})(1-\mu_{cw}+\psi\ln\beta)$ and $h_1 = \exp(\mu_{cw})$ are determined endogenously in equilibrium. Using $f = \theta J(\beta Z - \beta)$ we linearise the non-homogeneity in the Bellman equation. Then substituting into the Bellman gives

$$\begin{aligned} &\theta h_0 + \theta h_1\alpha(a_{F0}+a_{Fg}g) - \theta\beta + a_{Fg}\psi(a_{g0}+a_{g1}g) + (1-\gamma)g \\ &+\frac{1}{2}\Big[(a_{Fg}\psi)^2\sigma_g^2 + \gamma(\gamma-1)\sigma_c^2\Big] + a_{Fg}\psi(1-\gamma)\sigma_g\sigma_c\rho_{cg} = 0 \end{aligned}$$

Nothing that

$$\theta\alpha = \left(\frac{1-\gamma}{1-1/\psi}\right)\left(\frac{1-\psi}{1-\gamma}\right) = \frac{1-\psi}{1-1/\psi} = \frac{\psi(1-\psi)}{\psi-1} = -\psi \tag{A.184}$$

via separation of variables we have

$$a_{F0} = \frac{1}{\psi h_1}\left(\theta(h_0-\beta) + a_{Fg}\psi a_{g0} + \frac{1}{2}\left[(a_{Fg}\psi)^2\sigma_g^2 + \gamma(\gamma-1)\sigma_c^2\right] + a_{Fg}\psi(1-\gamma)\sigma_g\sigma_c\rho_{cg}\right)$$
$$a_{Fg} = \frac{1-\gamma}{\psi(h_1 - a_{g1})}$$

Now we see the effect of separating risk aversion with EIS. The sign of $h_1 a_{Fg}\alpha$ determines whether the consumption-wealth ratio (also the dividend-price ratio) rises or falls on good news (positive $dW_{g,t}$ shocks

$$\frac{\partial C/W}{\partial g} = h_1 \frac{1-\gamma}{\psi(h_1 - a_{g1})}\frac{1-\psi}{1-\gamma} \tag{A.185}$$
$$= \left(\frac{h_1}{h_1 - a_{g1})}\right)\frac{1-\psi}{\psi} \tag{A.186}$$

which is negative for $\psi > 1$ meaning that prices (wealth) rise faster than consumption. We now have to solve for h_1. Spelling things out explicitly

$$E\left[\frac{C}{W}\right] \sim \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})E[(c-w)] \tag{A.187}$$
$$= \exp(\mu_{cw})(1-\mu_{cw}) + \exp(\mu_{cw})\mu_{cw} \tag{A.188}$$
$$= \exp(\mu_{cw}) = h_1 \tag{A.189}$$
$$E\left[\frac{C}{W}\right] \sim h_1 \tag{A.190}$$

So

$$h_1 = E\left(\frac{C_t}{W_t}\right) = E\left[\beta^\psi \exp\left((a_{F0} + a_{Fg}g_t)\alpha\right)\right] \tag{A.191}$$
$$= \beta^\psi e^{a_{F0}\alpha} E\left[e^{a_{Fg}\alpha g_t}\right] \tag{A.192}$$
$$= \beta^\psi e^{a_{F0}\alpha} E\left[e^{u g_t}\right] \quad \text{where } u = a_{Fg}\alpha \tag{A.193}$$

We first need the *conditional* Laplace transform for g_t

$$\phi(t,h;u) = E_t\left(e^{u\cdot g_{t+h}}\right) \tag{A.194}$$

which Feynman-Kac says this function satisfies the following partial differential equation

$$0 \equiv \mathcal{D}\phi(t,h;u) + \frac{\partial \phi}{\partial t}(t,h;u) \tag{A.195}$$

with initial condition $\phi(t,0;u) = e^{u \cdot g_t}$ and where $\mathcal{D}$ is the infinitesimal generator for g_t. Spelling out the partials

$$\frac{\partial P}{\partial g}(a_{g0} + a_{g1} g_t) + \frac{1}{2}\frac{\partial^2 P}{\partial g^2}\sigma_g^2 + \frac{\partial P}{\partial t} = 0 \tag{A.196}$$

We conjecture an exponentially affine solution

$$\phi(t,h;u) = e^{A_{h_1}(h,u) + B_{h_1}(h,u) g_t} \tag{A.197}$$

with $A_{h_1}(0,u) = 0$ and $B_{h_1}(0,u) = u$. After substituting in we obtain

$$B_{h_1 g}(a_{g0} + a_{g1} g_t) + \frac{1}{2} B_{h_1 g}^2 \sigma_g^2 = \frac{\partial A_{h_1}}{\partial h} + \frac{\partial B_{h_1}}{\partial h} g_t \tag{A.198}$$

Collecting terms we obtain the system of equations

$$\frac{\partial A_{h_1}}{\partial \tau} = B_{h_1 g} a_{g0} + \frac{1}{2} B_{h_1 g}^2 \sigma_g^2 \tag{A.199}$$

$$\frac{\partial B_{h_1}}{\partial \tau} = B_{h_1 g} a_{g1} \tag{A.200}$$

whose solutions are given by

$$A_{h_1}(h,u) = u\frac{a_{g0}}{a_{g1}}\left[e^{a_{g1}h} - 1\right] + \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}\left[e^{2a_{g1}h} - 1\right] \tag{A.201}$$

$$B_{h_1}(h,u) = u e^{a_{g1}h} \tag{A.202}$$

Notice that $\lim_{h\to\infty} B_{h_1}(h,u) = 0$ only in the case that the parameter $a_{g1} < 0$ which is stationarity condition of the OU process. Under this condition the transition at horizon h tends to the stationary distribution (or marginal distribution, i.e., the distributions dependence on g_t has been marginalised out). The marginal

distribution admits the Laplace transform

$$E\left[e^{ug_t}\right] = \lim_{h\to\infty} e^{A_{h_1}(h,u)} = e^{-u\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}} \tag{A.203}$$

and so

$$h_1 = \beta^\psi e^{a_{F0}\alpha} E\left[e^{ug_t}\right] \tag{A.204}$$

$$= \beta^\psi e^{a_{F0}\alpha} e^{-u\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 u^2}{a_{g1}}} \tag{A.205}$$

$$= \beta^\psi e^{a_{F0}\alpha - a_{Fg}\alpha\frac{a_{g0}}{a_{g1}} - \frac{1}{4}\frac{\sigma_g^2 a_{Fg}^2 \alpha^2}{a_{g1}}} \tag{A.206}$$

A solution for h_1 follows from a fixed point iteration. Specifically, empirically speaking $\mu_{cw} = E[c_t - w_t] \sim -6 \to h_1 \sim 0.003$ so we take this as an initial value and update for a new h_1 by updating the parameters a_{F0} and a_{Fg} repeat this procedure until convergence.

A.4.9. Risk Free Rate

Above we derived

$$f_C = \beta(1-\gamma)JC^{-1}Z(C,J). \tag{A.207}$$

which we can re-express as

$$f_C = \beta^{\gamma\psi}\exp\left((a_{F0} + a_{Fg}g)\chi\right)C^{-\gamma} \tag{A.208}$$

where we have defined another constant

$$\chi = \psi + \alpha = \left(\frac{1-\psi\gamma}{1-\gamma}\right) \tag{A.209}$$

Similarly

$$f_J = (\theta-1)h_0 - \theta\beta - h_1\chi(a_{F0} + a_{Fg}g) \tag{A.210}$$

The diffusions for f_C is given by

$$\frac{df_C}{f_C} = \left(-\gamma g + a_{Fg}\chi(a_{g0} + a_{g1}g) + \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 + \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 - \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg}\right)dt \tag{A.211}$$

$$- \gamma\sigma_c dW_{c,t} + a_{Fg}\chi\sigma_g dW_{g,t} \tag{A.212}$$

The risk free rate is then given by

$$r_t^* = -\frac{1}{dt}E_t\left(\frac{df_C}{f_C}\right) - f_J \tag{A.213}$$

$$= \left(\gamma g_t - a_{Fg}\chi(a_{g0} + a_{g1}g_t) - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 + \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg}\right) \tag{A.214}$$

$$- (\theta-1)h_0 + \theta\beta + h_1\chi(a_{F0} + a_{Fg}g_t) \tag{A.215}$$

which for convenience we denote

$$r_t^* = r_0 + r_g g_t, \tag{A.216}$$

$$r_0 = +\theta\beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2 - a_{Fg}\chi a_{g0} - \frac{1}{2}a_{Fg}^2\chi^2\sigma_g^2 + \gamma a_{Fg}\chi\sigma_g\sigma_c\rho_{cg} - (\theta-1)h_0 + h_1\chi a_{F0} \tag{A.217}$$

$$r_g = \frac{1}{\psi} \tag{A.218}$$

Notice in the limit $\psi \to \frac{1}{\gamma}$ we recover the CRRA solution above $r_x = \gamma$ and $r_0 = \theta\beta - \frac{1}{2}\gamma(\gamma+1)\sigma_c^2$ since in this case $\theta = 1$ and $\chi = 0$. We now want the diffusion for the SDF which follows

$$\frac{d\Lambda_t^*}{\Lambda_t^*} = -r_t^* dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} \tag{A.219}$$

$$\Theta_c = \gamma\sigma_c \tag{A.220}$$

$$\Theta_g = -a_{Fg}\chi\sigma_g = \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \tag{A.221}$$

Note the price of long run risk shocks can be positive or negative but for preference for early resolution $a_{Fg} < 0$ and so negative long run growth shocks are bad news. The nominal SDF follows

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} \tag{A.222}$$

$$r_t = r_t^* + \pi - \sigma_Q^2 \tag{A.223}$$

$$\Theta_c = \gamma \sigma_c \tag{A.224}$$

$$\Theta_g = \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g \tag{A.225}$$

$$\Theta_Q = \sigma_Q \tag{A.226}$$

The nominal bond risk premium is constant and given by

$$BRP_\tau = \Theta_c B_g(\tau)\sigma_g \rho_{cg} dt + \Theta_g B_g(\tau)\sigma_g dt \tag{A.227}$$

$$\frac{1}{dt} BRP_\tau^* = \gamma \sigma_c \sigma_g \rho_{cg} B_g(\tau) + \left(\frac{1}{h_1 - a_{g1}}\right)\left(\gamma - \frac{1}{\psi}\right)\sigma_g^2 B_g(\tau) \tag{A.228}$$

The first term is the standard one arising in a CRRA Lucas tree economy: when $\langle dW_{c,t}, dW_{g,t}\rangle = \rho_{cg} > 0$ bonds have a hedging property against $dW_{c,t}$ shocks. The second term arises when agents have a preference for temporal resolution. The early resolution condition is $\gamma > \frac{1}{\psi}$ in which case $dW_{g,t} < 0$ are bad states of the world and bonds also hedge these shocks.

A.5. Simulated Method of Moments

We estimate the model via simulated method of moments (SMM) which is analogous to the generalized method of moments (GMM) estimator, but allows us to estimate the parameters even if latent volatility factors are not directly observable. Moreover, SMM avoids difficulties of computing analytical moment conditions which given the number of moment conditions is a tedious procedure.

We collect the structural parameters of interest in a $q \times 1$ parameter vector β. Given β, we simulate the counterpart $\tilde{x}(\beta)$ of the observed data sample x using

the model specification. Given a simulation of length $\tau \times T$ where $\tau > 1$ we obtain the $p > q$ vector of moments $M(\tilde{x}(\beta))$. Under the assumption that the model $\tilde{x}(\beta)$ is correctly specified and β_0 is the true structural parameter vector, the moment conditions $M(\tilde{x}(\beta_0))$ converge asymptotically to the sample moments $M(x)$. Thus, SMM proceeds in a similar fashion to GMM by choosing $\widehat{\beta}$ to minimize the weighted sum of squared moments errors $G_T(\hat{\beta}) = \left[M(\tilde{x}(\hat{\beta})) - M(x)\right]$

$$\widehat{\beta} = \min_{\hat{\beta}} \; G_T^\top \; W \; G_T \tag{A.229}$$

where W is a $q \times q$ positive definite weighting matrix. As in GMM, the optimal weighting matrix W is obtained from the inverse of the covariance matrix of the data moments. Under the regularity conditions set out in Duffie and Singleton (1993), $\widehat{\beta}$ is a consistent estimate of β_0. Moreover, when using the optimal weighting matrix $W^\star$, the asymptotic distribution of the parameters is given by

$$\sqrt{T}(\widehat{\beta} - \beta_0) \sim \mathcal{N}[0, (1 + 1/\tau)V^{-1}], \tag{A.230}$$

$$\text{where } V = d^\top W^\star d \quad \text{and} \quad d = E\left[\frac{\partial M}{\partial \beta}\right]. \tag{A.231}$$

To assess the model specification, we use the Chi-square test statistic proposed by Lee and Ingram (1991), which is the SMM analogue of the Hansen (1982) J_T-statistic:

$$J_T = T(1 + 1/\tau) \; G_T^\top \; W^\star \; G_T \xrightarrow{d} \chi^2_{(p-q)}. \tag{A.232}$$

We choose a set of moment conditions $p > q$ so that the system is over-identified.

Diffusions are discretized using a Milstein scheme (Kloeden and Platen (2013)). Our sample moments are estimated with $T = 20$ years of data, we set $\tau = 50$ and discard the first 5-years of each path to avoid sensitivity to initial conditions. Random number streams are held constant for each simulation path to avoid introducing sampling error. Moments $\widehat{M}_S$ are computed on data sampled at monthly frequency consistent with the sampling frequency of the empirical moment vector M_D. $\widehat{S}$ is computed using 2000 replications and a Newey and West (1987) with

lag length $K = 12$, and the Jacobian d is computed using 100 replications in line with the recommendations of Gouriéroux and Monfort (2000).

A.6. Tables

Asia Pacific	G7 & Western Europe	Latin America
Australia*	United States*	Argentina
China	Japan*	Brazil
Hong Kong	Germany	Chile
India	France	Mexico
Indonesia	United Kingdom*	Venezuela
Japan	Italy	Colombia
Malaysia	Canada*	Peru
New Zealand*	Euro zone*	Bolivia
Philippines	Netherlands	Costa Rica
Singapore	Norway*	Dominican Republic
South Korea	Spain	Ecuador
Taiwan	Sweden*	El Salvador
Thailand	Switzerland*	Guatemala
Bangladesh	Austria	Honduras
Pakistan	Belgium	Nicaragua
Sri Lanka	Denmark	Panama
Vietnam	Egypt	Paraguay
Myanmar	Finland	Uruguay
	Greece	
	Ireland	
	Israel	
	Nigeria	
	Portugal	
	Saudi Arabia	
	South Africa	

Table A.8. All countries included in the Consensus Economics datasets Highlighted countries (*) are selected to be part of this paper's main scope as they offer a rich cross-section of contributors.

Panel (a): Summary Statistics	Yields		Returns	
	Mean	Std Dev	Mean	Std Dev
Par	3.87	1.64	4.53	1.49
Zero	3.98	1.67	4.79	1.55
Difference	−0.13	0.09	−0.28	0.21

Panel (b): Frequency Comparison	Yields	
	Mean	Std Dev
Continuous	3.98	1.67
Annualy	4.07	1.74
Semi-Annual	4.02	1.70

Table A.9. Summary Statistics: Par Yields and Zero Yields
This table reports the means and standard deviations for (1) US 10-year par yields obtained from the Fed, (2) US 10-year zero yields obtained from Bloomberg, and (3) their differences. Panel (b) contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Sample period is monthly observations from between 1995.1 and 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): 1995.1 to 2007.12.										
Mean	−0.82	0.91	−1.61	−1.02	−0.86	0.58	−2.67	−0.29	−0.04	−0.73
Std	3.77	3.73	2.04	2.03	2.66	2.02	3.22	3.26	3.33	2.96
Skew	1.08	0.62	0.07	0.10	−0.03	0.58	−0.09	0.66	0.51	0.28
AR(1)	0.18	0.43	−0.17	0.03	0.49	0.15	0.63	0.10	0.53	0.23
Panel (b): 2008.1 to 2020.12.										
Mean	−0.85	−1.76	−2.13	−1.33	−0.24	−0.02	−2.78	−1.60	−3.08	−2.07
Std	3.10	3.34	1.93	2.23	3.33	1.34	3.43	3.76	3.13	2.56
Skew	0.41	0.96	0.10	0.77	0.56	0.37	−0.41	0.25	0.11	0.55
AR(1)	−0.27	0.30	0.04	−0.13	0.47	0.12	−0.04	−0.20	−0.08	0.03

Table A.10. Sub Sample Summary Statistics BRP
This table presents the means, standard deviations, skewness and AR(1) coefficients for subjective bond risk premia (BRP) for the sample periods 1995.1 to 2007.12. (panel A) and 2008.1 to 2020.12. (panel B). The reported AR(1) coefficients relate to the yearly autocorrelation.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): BRP projected										
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19	4.65
Std	2.99	3.21	1.88	2.98	2.82	2.57	3.44	2.49	3.77	1.86
Skew	0.04	0.40	0.29	0.27	0.24	0.44	−0.33	0.10	0.56	0.15
AR(1)	0.28	0.33	0.39	0.48	0.75	0.78	0.27	0.46	0.01	0.59
Panel (b): XRP projected										
Mean	1.43	0.29	−0.68	−1.32	−0.31	−3.00	−0.89	2.19	−1.20	
Std	4.19	1.15	4.24	3.53	1.40	3.18	3.37	3.79	3.65	
Skew	0.06	−0.11	0.02	0.13	0.25	−0.33	0.58	−0.42	0.29	
AR(1)	0.66	0.29	0.75	0.65	0.62	0.82	0.57	0.51	0.48	

Table A.11. Descriptive Statistics for Projected Risk Premia
This table presents the means, standard deviations, skewness, and AR(1) coefficients for exchange rate risk premia (XRP) and bond risk premia (BRP) where conditional expectations defined in Equations 3 and 5 are replaced by projections. These predictions are obtained by regressing realized ex-post excess returns on the slope of the yield curve (for bond risk premia) and the interest rate differential between the foreign country and the United States (for exchange rate risk premia). The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): *BRP*										
AUD	1.00	0.54	0.66	0.76	0.44	0.31	0.65	0.75	0.61	0.62
CAD	0.54	1.00	0.48	0.68	0.52	0.52	0.45	0.66	0.53	0.69
CHF	0.66	0.48	1.00	0.72	0.21	0.36	0.66	0.59	0.66	0.54
EUR	0.76	0.68	0.72	1.00	0.54	0.37	0.65	0.68	0.69	0.69
GBP	0.44	0.52	0.21	0.54	1.00	0.28	0.19	0.34	0.21	0.56
JPY	0.31	0.52	0.36	0.37	0.28	1.00	0.32	0.36	0.41	0.32
NOK	0.65	0.45	0.66	0.65	0.19	0.32	1.00	0.59	0.73	0.46
NZD	0.75	0.66	0.59	0.68	0.34	0.36	0.59	1.00	0.65	0.62
SEK	0.61	0.53	0.66	0.69	0.21	0.41	0.73	0.65	1.00	0.49
USD	0.62	0.69	0.54	0.69	0.56	0.32	0.46	0.62	0.49	1.00
Panel (b): *XRP*										
AUD	1.00	0.72	0.55	0.78	0.53	−0.07	0.54	0.91	0.78	
CAD	0.72	1.00	0.30	0.53	0.40	−0.17	0.47	0.71	0.58	
CHF	0.55	0.30	1.00	0.84	0.50	0.41	0.72	0.49	0.69	
EUR	0.78	0.53	0.84	1.00	0.67	0.15	0.76	0.73	0.88	
GBP	0.53	0.40	0.50	0.67	1.00	0.12	0.55	0.46	0.62	
JPY	−0.07	−0.17	0.41	0.15	0.12	1.00	0.06	−0.17	0.01	
NOK	0.54	0.47	0.72	0.76	0.55	0.06	1.00	0.48	0.77	
NZD	0.91	0.71	0.49	0.73	0.46	−0.17	0.48	1.00	0.69	
SEK	0.78	0.58	0.69	0.88	0.62	0.01	0.77	0.69	1.00	

Table A.12. Cross Country Correlations of Survey Premia
This table shows the correlation coefficients between the measures of subjective risk premia, as defined in Section 3.3 in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
					Panel (a): BRP_{proj}					
AUD	1.00	0.60	0.52	0.69	0.47	0.23	0.42	0.46	0.60	0.36
CAD	0.60	1.00	0.65	0.77	0.62	0.34	0.52	0.43	0.68	0.75
CHF	0.52	0.65	1.00	0.81	0.25	0.56	0.61	0.10	0.62	0.31
EUR	0.69	0.77	0.81	1.00	0.60	0.55	0.72	0.34	0.75	0.44
GBP	0.47	0.62	0.25	0.60	1.00	−0.10	0.52	0.60	0.46	0.72
JPY	0.23	0.34	0.56	0.55	−0.10	1.00	0.24	−0.29	0.28	−0.14
NOK	0.42	0.52	0.61	0.72	0.52	0.24	1.00	0.11	0.73	0.30
NZD	0.46	0.43	0.10	0.34	0.60	−0.29	0.11	1.00	0.40	0.61
SEK	0.60	0.68	0.62	0.75	0.46	0.28	0.73	0.40	1.00	0.36
USD	0.36	0.75	0.31	0.44	0.72	−0.14	0.30	0.61	0.36	1.00
					Panel (b): XRP_{proj}					
AUD	1.00	0.73	0.81	0.84	0.55	0.63	0.60	0.82	0.76	
CAD	0.73	1.00	0.69	0.73	0.44	0.57	0.71	0.56	0.79	
CHF	0.81	0.69	1.00	0.89	0.36	0.83	0.76	0.61	0.60	
EUR	0.84	0.73	0.89	1.00	0.69	0.60	0.86	0.78	0.76	
GBP	0.55	0.44	0.36	0.69	1.00	0.02	0.55	0.78	0.69	
JPY	0.63	0.57	0.83	0.60	0.02	1.00	0.46	0.36	0.32	
NOK	0.60	0.71	0.76	0.86	0.55	0.46	1.00	0.51	0.69	
NZD	0.82	0.56	0.61	0.78	0.78	0.36	0.51	1.00	0.76	
SEK	0.76	0.79	0.60	0.76	0.69	0.32	0.69	0.76	1.00	

Table A.13. Cross Country Correlations of Projected Premia
This table shows the correlation coefficients between the measures of projected risk premia, as defined in Section 3.3 in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): $BRP_{realized}$										
AUD	1.00	0.84	0.75	0.83	0.84	0.61	0.75	0.84	0.82	0.76
CAD	0.84	1.00	0.75	0.81	0.80	0.56	0.85	0.74	0.81	0.78
CHF	0.75	0.75	1.00	0.91	0.78	0.55	0.79	0.67	0.80	0.69
EUR	0.83	0.81	0.91	1.00	0.86	0.57	0.85	0.76	0.85	0.76
GBP	0.84	0.80	0.78	0.86	1.00	0.50	0.75	0.79	0.77	0.74
JPY	0.61	0.56	0.55	0.57	0.50	1.00	0.39	0.40	0.58	0.50
NOK	0.75	0.85	0.79	0.85	0.75	0.39	1.00	0.72	0.82	0.71
NZD	0.84	0.74	0.67	0.76	0.79	0.40	0.72	1.00	0.62	0.81
SEK	0.82	0.81	0.80	0.85	0.77	0.58	0.82	0.62	1.00	0.59
USD	0.76	0.78	0.69	0.76	0.74	0.50	0.71	0.81	0.59	1.00
Panel (b): $XRP_{realized}$										
AUD	1.00	0.86	0.55	0.66	0.58	0.39	0.80	0.93	0.80	
CAD	0.86	1.00	0.41	0.58	0.56	0.28	0.74	0.77	0.72	
CHF	0.55	0.41	1.00	0.84	0.45	0.42	0.72	0.59	0.71	
EUR	0.66	0.58	0.84	1.00	0.64	0.31	0.84	0.70	0.86	
GBP	0.58	0.56	0.45	0.64	1.00	−0.01	0.65	0.62	0.71	
JPY	0.39	0.28	0.42	0.31	−0.01	1.00	0.28	0.29	0.26	
NOK	0.80	0.74	0.72	0.84	0.65	0.28	1.00	0.80	0.84	
NZD	0.93	0.77	0.59	0.70	0.62	0.29	0.80	1.00	0.77	
SEK	0.80	0.72	0.71	0.86	0.71	0.26	0.84	0.77	1.00	

Table A.14. Cross Country Correlations of realized Premia
This table shows the correlation coefficients between the measures of realized risk premia, as defined in Section 3.3 in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

	PC1	PC2	PC3
BRP	58.75	11.76	8.42
XRP	63.96	14.34	6.55

Table A.15. PCA Analysis: Variance Explained (%)
Risk premium factors PC_t are formed from an eigenvalue decomposition of the covariance matrix of subjective risk premia $var(RP_t) = QDQ^\top$. This table displays the fraction of subjective risk premium variance due to the n'th factor which is computed from $D(n,n)/\sum_n D(n,n)$. Factors (principle components) are computed from linear combinations (rotations) of the input series via $PC_t = RP_tQ$. The sample period 1995.1 to 2020.12

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
					Panel (a): Δx_{t+1}				
Mean	0.09	0.85	−0.11	0.93	−0.26	−0.37	2.92	−0.99	3.70
Std	5.33	2.50	5.08	4.35	2.81	4.81	4.25	5.85	4.72
AR(1)	0.74	0.32	0.59	0.62	0.24	0.32	0.12	0.77	0.58
					Panel (b): IRD				
Mean	−1.64	−0.18	1.59	0.76	−0.47	2.39	−0.63	−2.13	0.22
Std	1.66	0.86	1.53	1.37	1.14	2.06	1.76	1.56	1.70
AR(1)	0.69	0.30	0.77	0.70	0.69	0.81	0.59	0.59	0.53
					Panel (c): $rx_{t+1}^{(11)}$				
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19
Std	8.94	6.65	5.80	6.69	7.33	4.46	7.20	8.29	8.68
AR(1)	−0.28	−0.03	−0.18	−0.30	−0.20	0.25	−0.08	−0.45	−0.15
					Panel (d): Slope				
Mean	0.78	1.26	0.98	1.35	0.93	1.09	0.80	0.55	1.27
Std	0.65	0.96	0.64	0.83	1.21	0.70	1.03	1.12	0.77
AR(1)	0.29	0.37	0.41	0.49	0.75	0.79	0.27	0.46	0.03

Table A.16. Descriptive Statistics Predictive Regressions
This table presents the means, standard deviations, skewness and AR(1) coefficient for log spot rate changes (Δx_{t+1}), interest rate differentials ($IRD_t = (i_t^{(1)} - i_t^{(1),f})$), realized excess bond returns ($rx_{t+1}^{(11)}$), and the slope of the yield curve ($i_t^{(10),f} - i_t^{(1),f}$). The reported AR(1) coefficients relate to the yearly autocorrelation. The sample period is 1995.1 to 2020.12.

	Yields	FX
$Level_t$	0.08	
	[0.01 0.15]	
$Slope_t$	0.08	
	[−0.06 0.22]	
IRD_t		−0.95
		[−1.48 −0.42]
R^2 (%)	5.71	5.73

Table A.17. Forecast Error Predictability

This table shows pooled OLS regressions of (a) 10-year yields and (b) foreign exchange forecast errors, both at an annual horizon, on either the level and slope of the yield curve or the 1-year interest rate differential:

$$FE_{t,t+1} = a + bX_t + \eta_{t+1},$$

where for interest rate forecast errors X_t includes $Level_t = i_t^{(1)}$ and $Slope_t = (i_t^{(10)} - i_t^{(1)})$ and for exchange rate forecast errors it includes $IRD_t = (i_t^{(1)} - i_t^{(1),f})$. Confidence intervals in $[\cdot]$ are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. Individual country-by-country regressions are presented in Table A.21. The sample period is 1995.1 to 2020.12

	(i)	(ii)	(iii)	(iv)
Panel (a): BRP				
E[con]	−0.57			
	[−0.98 −0.16]			
E[gdp]		−0.55		
		[−1.02 −0.09]		
BondVar			2.16	
			[0.87 3.45]	
$rx^{(11)}_{t-12,t}$				−0.09
				[−0.14 −0.04]
R^2 (%)	3.78	3.57	5.85	5.11
Panel (b): XRP				
E[con]	1.08			
	[0.32 1.83]			
E[gdp]		0.95		
		[0.14 1.75]		
FXVar			0.78	
			[0.20 1.36]	
$rx^{FX}_{t-12,t}$				−0.09
				[−0.19 0.01]
R^2 (%)	6.36	4.87	3.58	4.81

Table A.18. Cyclicality Regressions: Subjective Bond & FX Risk Premia
This table reports estimates from pooled OLS regressions of the forms

$$BRP_t = a + b^\top X_t + \epsilon_t, \text{ and}$$
$$XRP_t = a + b^\top X_t + \epsilon_t,$$

in panels (a) and (b), respectively. BRP_t is the survey-implied bond risk premium, XRP_t is the survey-implied currency risk premium, and X_t is a vector of explanatory variables. For the specifications in panel (a), X_t contains the 1-year *expected* consumption (*con*) growth and domestic *gdp* growth, the realized bond variance ($BondVar$), and the excess return on an 11-year bond, realized between $t-1$ year and t. For the specifications in panel (b), X_t contains the 1-year *expected* consumption (*con*) growth and foreign *gdp* growth, the FX option-implied risk neutral variance constructed and discussed by Krohn et al. (2020) ($FXVar$), and the excess currency return realized between $t-1$ year and t. A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. The sample period is 1995.1 to 2020.12 except for specification (iii) in panel (b) which only contains data from 1999.1 to 2019.4.

	(i)	(ii)	(iii)
Panel (a): Bond Return			
Slope	2.81		2.49
	[1.78 3.85]		[1.35 3.63]
BRP		0.52	0.30
		[0.22 0.82]	[−0.03 0.63]
R^2 (%)	13.85	4.96	14.79
Panel (b): FX Return			
IRD	−1.56		−1.41
	[−2.74 −0.38]		[−2.52 −0.31]
XRP		0.55	0.45
		[−0.03 1.12]	[−0.08 0.99]
R^2 (%)	8.89	5.43	12.54

Table A.19. Bond & FX Return Panel Predictability Regressions
This table reports estimates from pooled OLS regressions of the forms

$$rx_{t,t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t \epsilon_{t,t+1},$$
$$rx_{t,t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1},$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively. $Slope_t = (i_t^{(10)} - i_t^{(1)})$ is the slope of the domestic yield curve, BRP_t is the survey-implied bond risk premium, $IRD_t = (i_t^{(1)} - i_t^{(1),f})$ is the one-year interest rate differential, and XRP_t is the survey-implied currency risk premium. We also report univariate regressions with the same predictors in columns (i) and (ii). A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using the estimator of Driscoll and Kraay (1998) using 12 lags. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK	USD
FR	0.41	0.01	0.10	0.03	0.36	0.05	-0.01	0.32	0.25	0.08
	(0.84)	(0.03)	(0.24)	(0.06)	(0.90)	(0.16)	(-0.04)	(0.74)	(0.61)	(0.22)
a	-0.01	-0.01	-0.01	-0.01	-0.01	-0.00	-0.01	-0.00	-0.01	-0.01
	(-4.00)	(-5.21)	(-5.07)	(-5.99)	(-4.52)	(-4.54)	(-3.95)	(-3.17)	(-5.89)	(-4.97)
$R^2(\%)$	0.53	0.00	0.04	0.00	0.50	0.01	0.00	0.34	0.26	0.03

(a) Interest Rates

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK
FR	0.53	0.98	-0.12	0.09	0.21	1.13	0.15	0.82	0.18
	(0.88)	(1.73)	(-0.40)	(0.20)	(0.52)	(2.07)	(0.34)	(1.37)	(0.39)
a	0.00	-0.00	0.01	-0.01	-0.00	0.00	-0.01	0.01	-0.01
	(0.16)	(-0.44)	(1.13)	(-0.77)	(-0.20)	(0.04)	(-2.39)	(0.79)	(-2.54)
$R^2(\%)$	0.63	2.33	0.06	0.03	0.17	3.25	0.07	1.50	0.10

(b) Exchange Rates

Table A.20. Information Frictions: Coibion and Gorodnichenko (2015)

This table shows pooled OLS regressions of (a) 10-year yields and (b) foreign exchange forecast errors, both at an annual horizon, on corresponding forecast revisions defined as the monthly change in the respective forecasts:

$$FE^i_{t,t+1} = a^i + b^i FR^i_{t-1,t} + \eta^i_{t+1},$$

where t-statistics, reported in parentheses, are computed with Newey-West standard errors computed with 12-lags. The sample period 1995.1 to 2020.12

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK	USD
$Level_t$	0.14	0.13	0.17	0.09	0.12	-0.85	0.17	0.28	0.03	0.25
	(1.97)	(2.26)	(2.62)	(1.74)	(2.12)	(-3.16)	(2.28)	(4.69)	(0.51)	(3.92)
$Slope_t$	0.17	0.12	-0.04	-0.02	0.23	0.26	0.08	0.43	0.06	0.31
	(0.85)	(1.27)	(-0.25)	(-0.18)	(2.09)	(2.25)	(0.48)	(3.49)	(0.38)	(2.62)
a	-0.01	-0.01	-0.01	-0.01	-0.01	-0.00	-0.01	-0.02	-0.01	-0.02
	(-3.34)	(-4.48)	(-3.68)	(-3.75)	(-4.14)	(-3.96)	(-3.36)	(-5.82)	(-3.34)	(-5.25)
$R^2(\%)$	8.94	12.57	17.20	6.75	11.80	16.29	17.51	34.99	0.95	26.02

(a) Interest Rates

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK
IRD_t	-3.09	-0.88	-2.99	-3.14	-4.03	-0.02	-0.22	-1.91	-0.35
	(-4.20)	(-0.74)	(-4.86)	(-2.57)	(-2.46)	(-2.01)	(-1.70)	(-2.47)	(-2.65)
a	-0.05	-0.01	0.06	0.01	-0.02	0.00	-0.01	-0.03	-0.00
	(-2.74)	(-0.52)	(4.48)	(0.43)	(-1.07)	(1.51)	(-2.90)	(-1.43)	(-2.09)
$R^2(\%)$	30.13	1.36	33.37	13.91	12.65	10.33	6.44	13.28	14.20

(b) Exchange Rates

Table A.21. Forecast Error Predictability: Individual Country Regressions

This table shows individual country regressions of either (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on either the slope of the yield curve or the 1-year interest rate differential

$$FE^i_{t,t+1} = a^i + b^i X^i_t + \eta^i_{t+1},$$

where for interest rate forecast errors X_t includes the $Level_t = i^{(1)}_t$ and $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ and for exchange rates $IRD_t = (i^{(1)}_t - i^{(1),f}_t)$ and t-statistics, reported in parenthesis, are computed with Newey-West standard errors computed with 12-lags. The sample period 1995.1 to 2020.12

	(i)	(ii)
$E[con^f - con]$	0.72	
	[0.30 1.17]	
$E[gdp^f - gdp]$		0.76
		[0.17 1.35]
R^2 (%)	3.57	2.17

Table A.22. Cyclicality: Subjective FX Risk Premia (Macro Spreads) This table reports estimates from pooled OLS regressions of the forms

$$XRP_t = a + b^\top X_t + \epsilon_t.$$

where XRP_t is the survey-implied currency risk premium (XRP_t), and X_t is a vector of explanatory variables containing spreads in 1-year *expected* consumption *con* growth and *gdp* growth between the foreign countries and the US. A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
E[con]	−0.43	−0.14	−0.21	−0.22	−0.81	−0.51	−1.22	−0.60	−1.29	−0.23
	(0.46)	(0.71)	(0.35)	(0.43)	(0.45)	(0.31)	(0.49)	(0.48)	(0.60)	(0.31)
cons	0.00	−0.00	−0.02	−0.01	0.01	0.01	0.00	0.01	0.01	−0.01
	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.02)	(0.01)
$R^2(\%)$	1.19	0.09	0.31	0.53	9.87	5.57	13.43	2.05	10.97	0.54
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
E[gdp]	−0.94	0.08	−0.42	−0.67	−0.69	−0.61	−1.73	−0.69	−0.81	−0.21
	(0.41)	(0.82)	(0.27)	(0.30)	(0.51)	(0.18)	(0.47)	(0.58)	(0.33)	(0.27)
cons	0.02	−0.01	−0.01	−0.00	0.01	0.01	0.01	0.01	0.00	−0.01
	(0.01)	(0.02)	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
$R^2(\%)$	4.19	0.03	2.22	8.99	5.75	14.31	21.82	2.46	5.19	0.53

Table A.23. Cyclicality Regressions: Subjective Bond Risk Premia

This table reports estimates from OLS regressions of the form:

$$BRP_t = a + b^\top X_t + \epsilon_t,$$

where BRP_t is the survey-implied bond risk premium (BRP_t) in the given country, and X_t is either the 12-month *expected* consumption *con* and gdp growth *gdp* in the same country. A constant is included and reported. Standard errors are obtained as Newey-West standard errors computed with 12-lags. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
E[con]	1.53	−0.11	0.51	1.13	−0.11	0.92	−1.07	0.29	0.77
	(0.78)	(0.40)	(1.12)	(1.06)	(0.26)	(0.93)	(0.44)	(0.60)	(0.76)
cons	−0.03	0.01	−0.02	−0.01	0.00	−0.04	0.07	0.00	0.02
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
R^2(%)	7.30	0.12	0.39	4.01	0.23	3.25	8.38	0.20	2.69
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
E[gdp]	1.68	0.44	0.71	0.57	−0.15	0.92	−1.54	1.30	0.02
	(0.89)	(0.28)	(0.89)	(0.65)	(0.26)	(0.56)	(0.47)	(0.86)	(0.64)
cons	−0.03	−0.00	−0.02	−0.01	0.00	−0.04	0.07	−0.02	0.03
	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
R^2(%)	6.57	2.43	1.41	1.82	0.35	5.81	14.11	3.54	0.00

Table A.24. Cyclicality Regressions: Subjective Currency Risk Premia

This table reports estimates from pooled OLS regressions of the form:

$$XRP_t = a + b^\top X_t + \epsilon_t,$$

where XRP_t is the survey-implied currency risk premium (XRP_t) in the given country, and X_t is either the 12-month *expected* consumption *con* and gdp growth *gdp* in the same country. A constant is included and reported. Standard errors are obtained as Newey-West standard errors computed with 12-lags. The sample period is 1995.1 to 2020.12.

Panel (a)

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
BRP	0.56	0.58	0.46	0.51	0.68	0.17	0.13	0.26	0.84	0.88
	(0.20)	(0.25)	(0.36)	(0.25)	(0.24)	(0.25)	(0.21)	(0.20)	(0.36)	(0.30)
cons	0.05	0.05	0.04	0.06	0.05	0.04	0.03	0.04	0.08	0.06
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
$R^2(\%)$	4.75	11.16	2.53	2.74	8.08	0.47	0.35	1.30	12.48	9.63

Panel (b)

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
XRP	0.60	1.03	0.51	0.60	0.88	0.16	0.89	0.47	0.90
	(0.45)	(0.48)	(0.29)	(0.45)	(0.48)	(0.37)	(0.43)	(0.44)	(0.48)
cons	0.00	−0.01	0.00	−0.01	−0.00	−0.03	−0.04	0.02	−0.04
	(0.02)	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
$R^2(\%)$	5.42	10.48	5.85	5.78	7.31	0.36	9.94	3.98	10.83

Table A.25. Individual Predictability Regressions

This table reports estimates from OLS regressions of the form:

$$rx^{(11)}_{t,t+1} = a + b_1 BRP_t + \epsilon_{t,t+1} \quad \text{and} \quad rx^{FX}_{t,t+1} = a + b_1 XRP_t + \epsilon_{t,t+1},$$

in panels (a) and (b), respectively. BRP_t is the survey-implied bond risk premium and XRP_t is the survey-implied exchange rate risk premium. A constant is included and reported. Newey-West standard errors are computed with 12-lags. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
3m mean	4.13	2.52	0.82	1.70	2.92	0.12	3.18	4.68	2.18	2.33
3m delta std	0.87	0.76	0.66	0.64	0.78	0.25	1.20	1.04	0.74	0.73
5y mean	4.65	3.48	1.42	2.50	3.57	0.68	3.59	5.00	3.10	3.37
10y mean	4.00	4.01	1.96	3.16	3.97	1.28	4.00	5.25	3.62	3.98

Table A.26. Summary Statistics: Term Structure Moments
This table shows the means of 3 month, 5 year and 10 year bond yields for all countries in our sample. It also reports the annualised standard deviations of month-on-month differences in 3 month yields. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
mean	2.15	1.74	0.53	1.39	2.02	0.17	2.08	1.98	1.17	2.29
std	0.67	0.96	1.12	0.58	0.82	1.06	1.23	0.97	1.10	0.81
skew	1.89	−0.04	0.19	−0.44	0.56	1.38	−0.27	0.41	0.17	−2.08
kurt	11.65	3.17	3.15	3.80	3.88	8.02	5.93	4.50	4.03	12.35
AR(1)	0.26	0.03	−0.42	0.09	−0.06	−0.13	−0.12	0.21	−0.14	0.32
AR(5)	0.27	0.03	−0.53	−0.03	−0.15	−0.23	−0.12	0.04	−0.41	0.05
AR(10)	0.16	−0.26	0.44	−0.08	0.05	0.09	−0.01	0.03	0.27	−0.02

Table A.27. Summary Statistics: Inflation Moments
This table shows the annualised means, annualised standard deviations, skewness and kurtosis as well as AR(1), AR(5) and AR(10) coefficients of realized inflation for Australia, Switzerland and the United States. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
AUD	1.00	0.88	0.90	0.93	0.93	0.98	0.94	0.95	0.94	0.95
CAD	0.88	1.00	0.87	0.97	0.98	0.82	0.98	0.98	0.95	0.92
CHF	0.90	0.87	1.00	0.90	0.90	0.93	0.93	0.91	0.98	0.89
EUR	0.93	0.97	0.90	1.00	0.96	0.88	0.99	0.98	0.96	0.98
GBP	0.93	0.98	0.90	0.96	1.00	0.87	0.97	0.99	0.96	0.91
JPY	0.98	0.82	0.93	0.88	0.87	1.00	0.91	0.90	0.93	0.92
NOK	0.94	0.98	0.93	0.99	0.97	0.91	1.00	0.99	0.98	0.97
NZD	0.95	0.98	0.91	0.98	0.99	0.90	0.99	1.00	0.98	0.95
SEK	0.94	0.95	0.98	0.96	0.96	0.93	0.98	0.98	1.00	0.95
USD	0.95	0.92	0.89	0.98	0.91	0.92	0.97	0.95	0.95	1.00

Table A.28. Correlation of realized inflation
This table shows the correlation coefficients between realized inflation rates between any two countries. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
mean	3.29	2.87	1.57	1.21	2.24	0.59	3.05	3.43	2.38	1.36
std	0.99	0.95	0.56	0.00	1.57	1.99	1.88	1.66	1.34	0.67
skew	0.20	−0.56	0.00	−0.47	−0.36	−2.35	0.06	−0.62	0.11	−0.40
kurt	2.75	4.67	3.09	5.19	3.21	12.11	3.57	4.75	3.65	4.01
AR(1)	0.08	0.24	0.19	−0.17	0.19	−0.16	−0.11	−0.05	0.00	0.60
AR(5)	0.05	−0.11	0.10	−0.03	−0.07	−0.03	0.12	0.13	−0.07	0.25
AR(10)	0.06	0.00	−0.05	0.16	0.13	0.03	0.01	−0.03	−0.06	0.10

Table A.29. Summary Statistics: Growth Moments
This table shows the annualised means, annualised standard deviations, skewness and kurtosis as well as AR(1), AR(5) and AR(10) coefficients of realized consumption growth for Australia, Switzerland and the United States. Moments are computed for the sample period 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
AUD	1.00	0.91	0.87	0.73	0.92	0.60	0.96	0.94	0.93	0.76
CAD	0.91	1.00	0.99	0.93	0.97	0.87	0.95	0.99	0.98	0.96
CHF	0.87	0.99	1.00	0.96	0.94	0.90	0.92	0.96	0.97	0.97
EUR	0.73	0.93	0.96	1.00	0.89	0.98	0.84	0.90	0.92	0.98
GBP	0.92	0.97	0.94	0.89	1.00	0.83	0.98	0.99	0.98	0.90
JPY	0.60	0.87	0.90	0.98	0.83	1.00	0.75	0.82	0.84	0.96
NOK	0.96	0.95	0.92	0.84	0.98	0.75	1.00	0.98	0.98	0.84
NZD	0.94	0.99	0.96	0.90	0.99	0.82	0.98	1.00	0.99	0.91
SEK	0.93	0.98	0.97	0.92	0.98	0.84	0.98	0.99	1.00	0.91
USD	0.76	0.96	0.97	0.98	0.90	0.96	0.84	0.91	0.91	1.00

Table A.30. Correlation of realized consumption growth
This table shows the correlation coefficients between realized consumption growth rates between any two countries. Based on monthly observations from between 1995.1 to 2020.12.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.05	-0.11	0.21
$\theta_g(\%)$	1.31	1.02	1.60
$\sigma_c(\%)$	0.63	0.55	0.71

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	1.36	1.36	1.07	1.65
std (%)	0.67	0.67	0.59	0.75
skew	-0.40	-0.00	-0.00	0.00
kurt	4.01	2.90	2.88	2.92
AR(1)	0.60	0.12	0.06	0.19
AR(5)	0.25	0.08	0.03	0.14
AR(10)	0.10	0.07	0.02	0.11

Table A.31. US Consumption Growth Estimates
This table reports estimation results for the consumption growth process given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_c dW_{c,t},$$
$$dg_t = \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.70$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.07	-0.08	0.23
$\theta_g(\%)$	1.54	1.38	1.69
$\sigma_c(\%)$	0.53	0.47	0.59

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	1.57	1.57	1.42	1.73
std (%)	0.56	0.56	0.50	0.62
skew	0.00	-0.00	-0.00	-0.00
kurt	3.09	2.90	2.88	2.91
AR(1)	0.19	0.11	0.06	0.16
AR(5)	0.10	0.08	0.03	0.12
AR(10)	-0.05	0.06	0.03	0.09

Table A.32. Swiss Consumption Growth Estimates
This table reports estimation results for the consumption growth process given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_c dW_{c,t},$$
$$dg_t = \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.70$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

Panel A

Parameter	Estimate	Lower	Upper
κ_g	0.12	-0.12	0.35
$\theta_g(\%)$	3.24	2.95	3.53
$\sigma_c(\%)$	0.94	0.85	1.03

Panel B

Moment	Data	Model	Lower	Upper
mean (%)	3.29	3.29	2.99	3.59
std (%)	0.99	0.99	0.90	1.08
skew	0.20	-0.00	-0.00	-0.00
kurt	2.75	2.90	2.89	2.91
AR(1)	0.08	0.10	0.04	0.16
AR(5)	0.05	0.06	0.01	0.12
AR(10)	0.06	0.05	0.01	0.09

Table A.33. Australian Consumption Growth Estimates
This table reports estimation results for the consumption growth process given by

$$\frac{dC_t}{C_t} = g_t dt + \sigma_c dW_{c,t},$$
$$dg_t = \kappa_g(\mu_g - g_t)dt + \varepsilon \cdot \sigma_c dW_{g,t}.$$

where the Brownian shocks are orthogonal and we preset $\varepsilon = 0.70$. The model is estimated via simulated method of moments targeting the mean, standard deviation, skewness, kurtosis, and autocorrelations of quarterly realized consumption growth. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

A.7. Figures

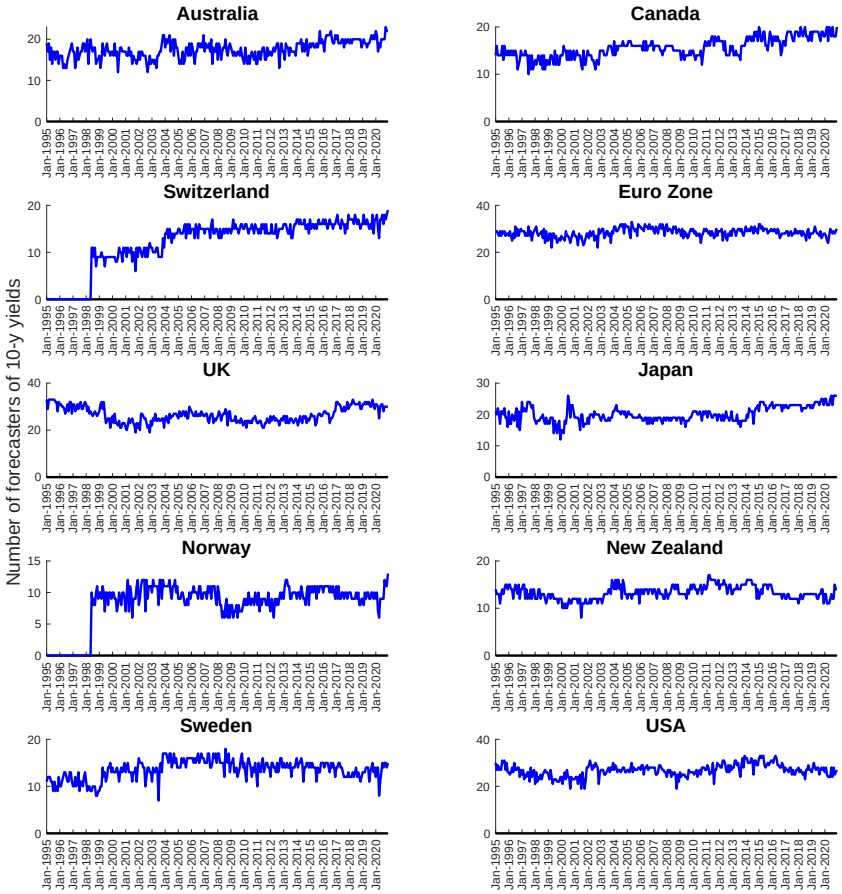
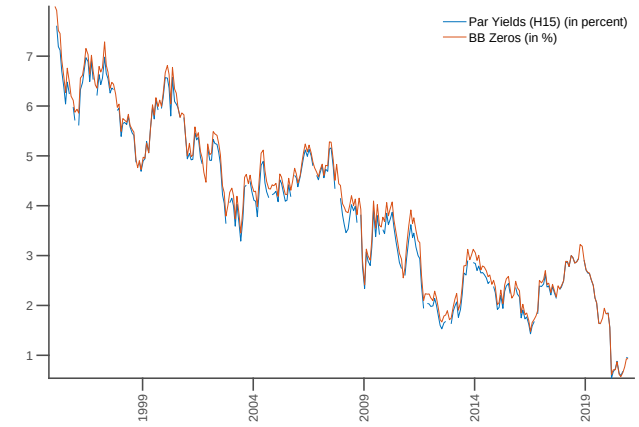
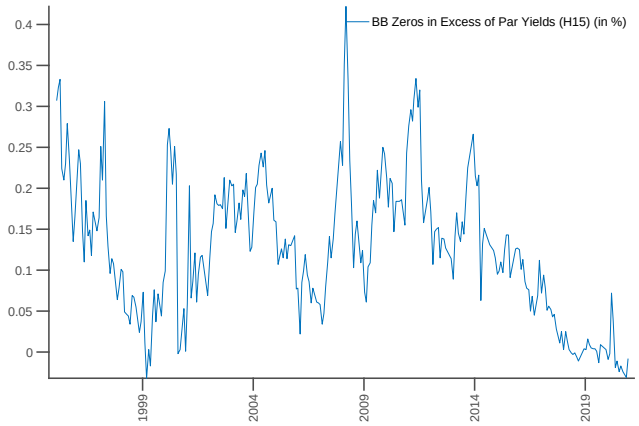


Figure A.16. Number of long-term bond yield forecasters by country
This figure displays number of forecasters predicting 10-year as part of the Consensus Economics surveys for the given country. The sample period is 1995.1 - 2020.12.



(a)



(b)

Figure A.17. Comparison of H15 and BB yields

The above figures show the time series of US 10-year par yields obtained from the Fed and US 10-year zero yields obtained from Bloomberg (Figure a) as well as the difference between the two series (Figure b). Data is available for the sample period 2000.1 to 2020.12.

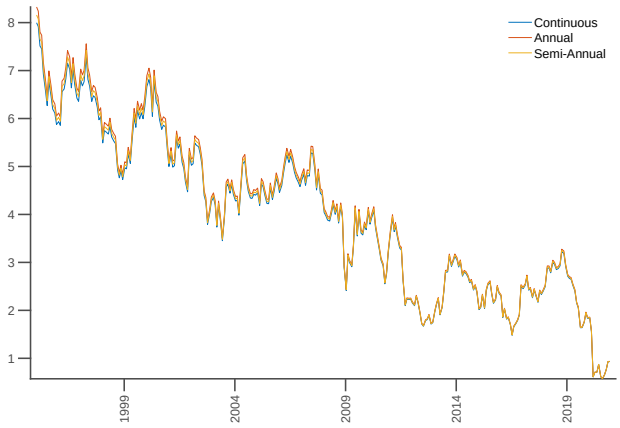


Figure A.18. Comparison of compounding frequencies
The above figure shows the time series of US 10-year zero log yields obtained from Bloomberg that have been generated assuming continuous, annual, and semi-annual compounding. Data is available for the sample period 2000.1 to 2020.12.

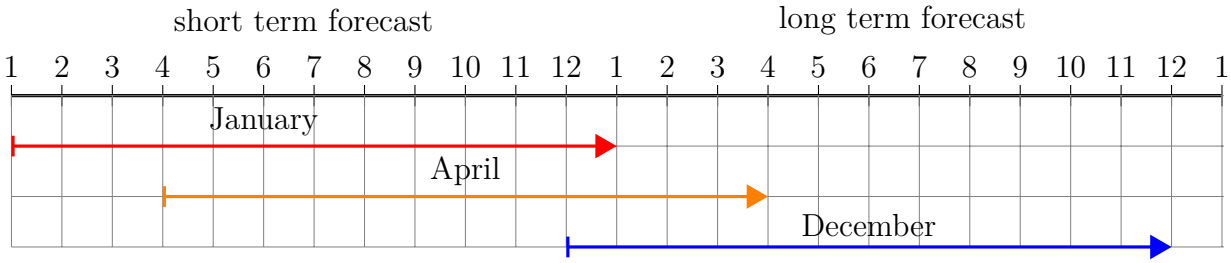


Figure A.19. Constant Maturity Macro Expectations
Figure displays a visual explanation to the construction of the constant maturity proxy. Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2..12$. A constant maturity expectation is formed taking as weight $(1 - \frac{j}{12})$, for the short term projection (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term projection (the forecast for the following year).

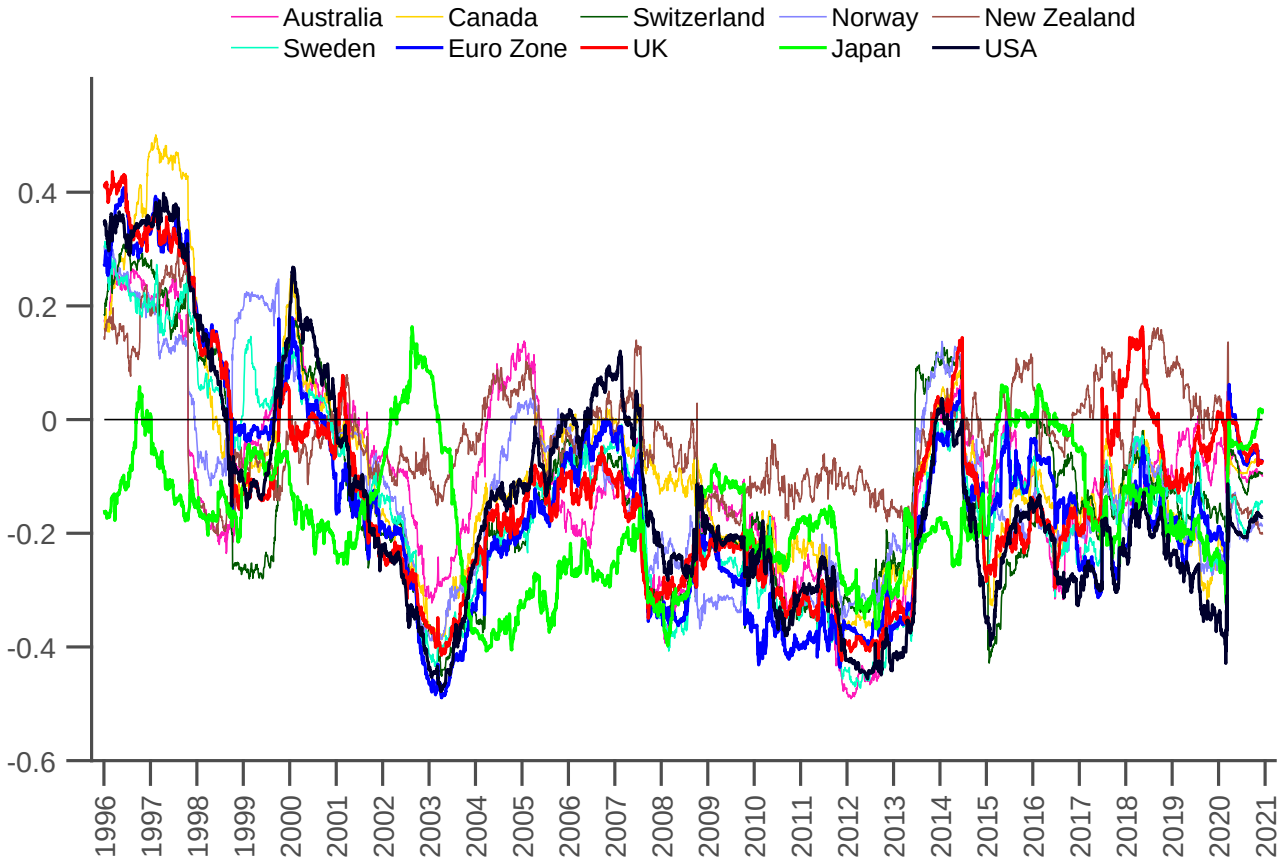
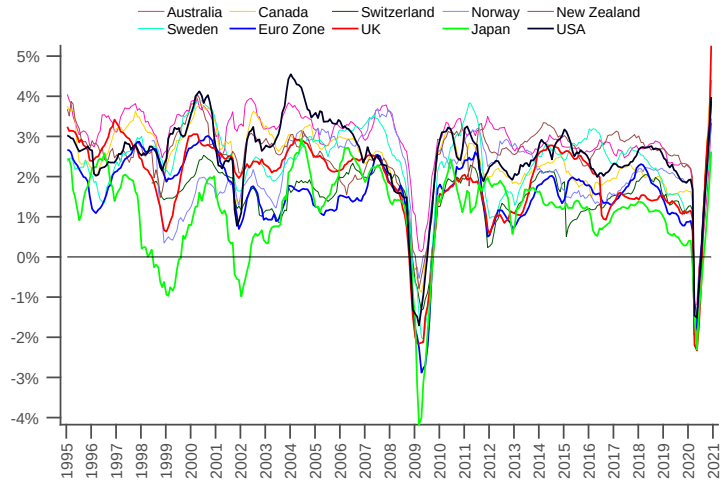
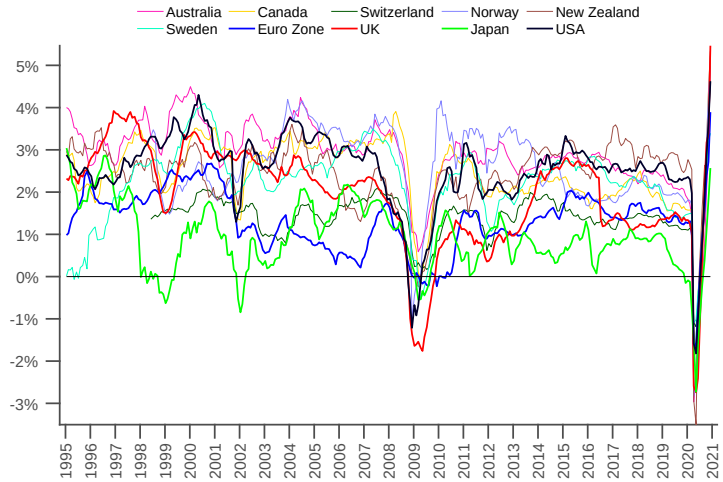
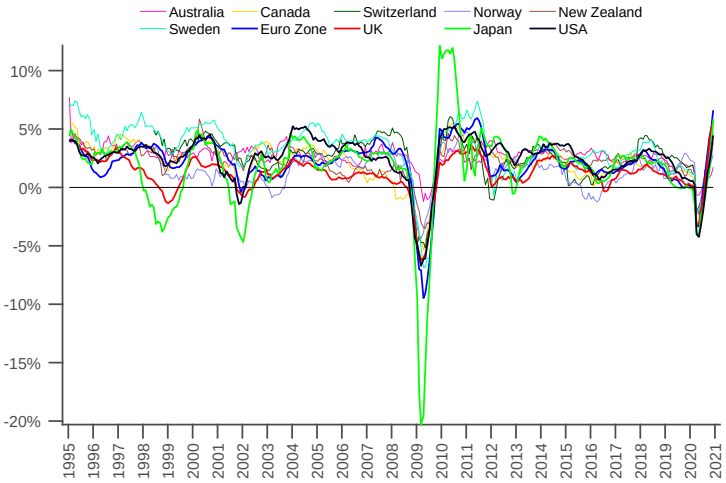


Figure A.20. Stock-Bond Correlation

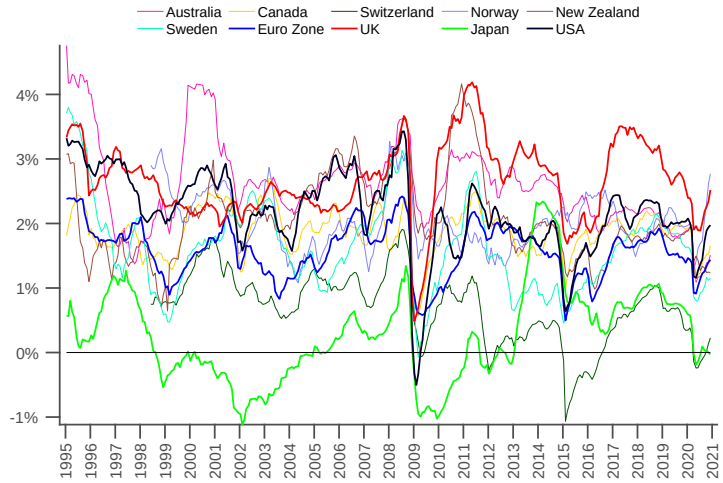
Figure displays the time series of rolling window correlations between ten-year log bond returns and log equity returns for a given country. We use the respective country-specific benchmark equity indices. The rolling window length is 222 daily observations. The sample period is 1995.1 to 2020.12.







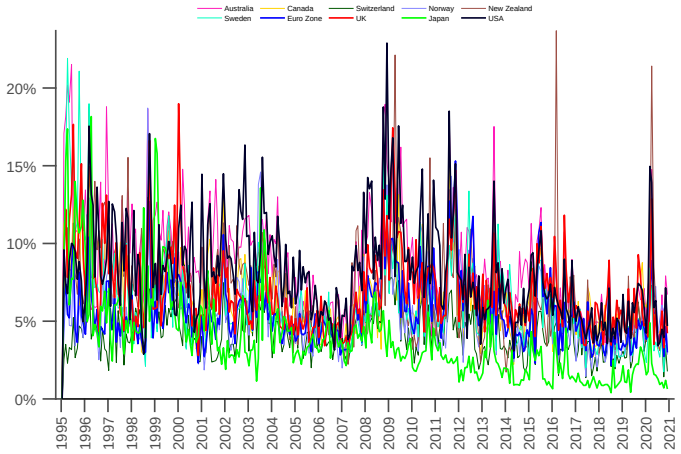
(c) Industrial Production



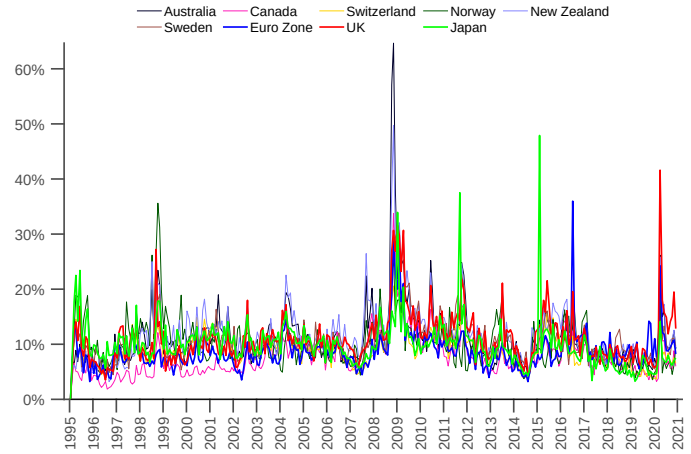
(d) Consumer Price Index

Figure A.21. Subjective Macro Expectations

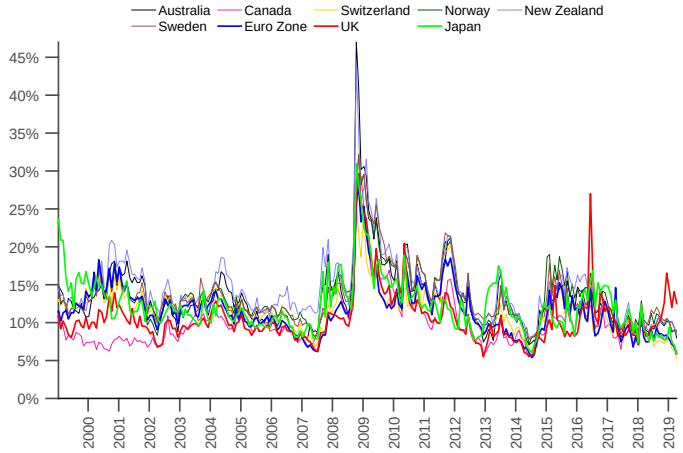
Figure displays subjective expectations of 12-month ahead GDP growth (%), real private consumption growth (%), real Industrial Production growth (%) and inflation (%) for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The sample period is 1995.1 to 2020.12.



(a) Bond returns



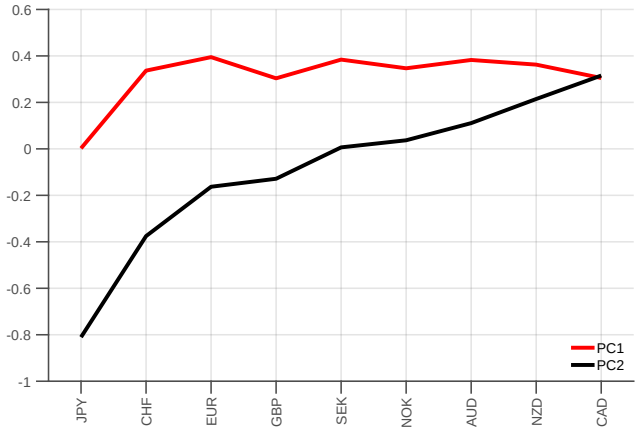
(b) Spot exchange rates



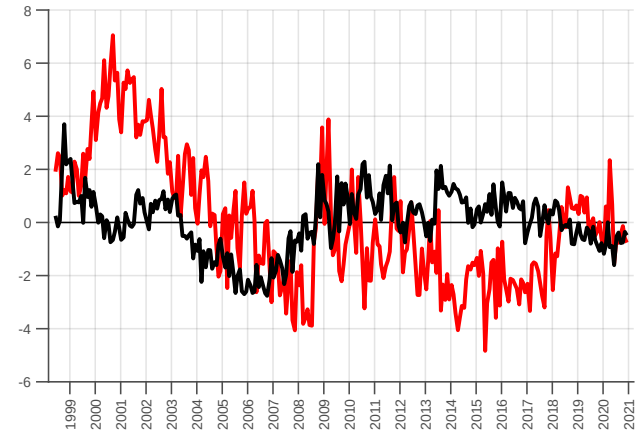
(c) Option-implied risk neutral exchange rate volatility

Figure A.22. Volatility of realized bond returns and realized and risk-neutral spot exchange rates

Panel (a) displays the volatilities of ten year sovereign bond returns, panel (b) displays the volatilities of spot exchange rates changes and panel (c) displays option-implied risk neutral exchange rate volatilities for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP (and USD for panel (a)). Volatilities in panels (a) and (b) are measured as the sum of squared differences of log prices in the 22 days preceding a sampled date. Dates are sampled as the survey dates of the Consensus Economics forecasts.

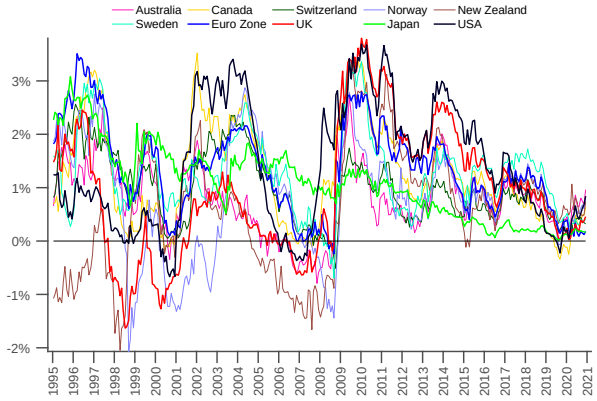


(a) Factor Loadings

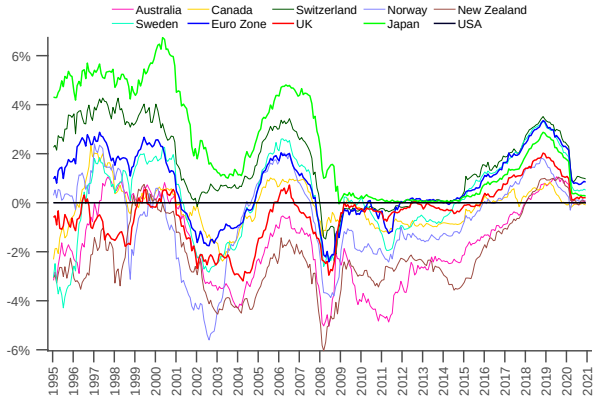


(b) Factor Time Series

Figure A.23. PCAs of Subjective Exchange Rate Risk Premia
Subjective exchange rate risk premia factors PC_t are formed from an eigenvalue decomposition of the covariance matrix of $var(XRP_t) = QDQ^\top$. The variance due to the n'th factor is computed from $D(n,n)/\sum_n D(n,n)$ which is displayed in the online appendix. Principle components (PCs) are computed from the rotation $PC_t = XRP_tQ$. Panel (a) displays the factor loadings (the columns of Q) and panel (b) displays the dynamics of PC1 and PC2. The sample period is 1995.1 to 2020.12.



(a) Slopes



(b) IRDs

Figure A.24. Interest Rate Spreads

Figure displays term structure slopes (panel a) and 1-year interest rate differentials (panel b) for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The slope of the yield curve is defined as the difference between the respective country's ten year bond yield and its one year bond yield. The sample period is 1995.1 to 2020.12.

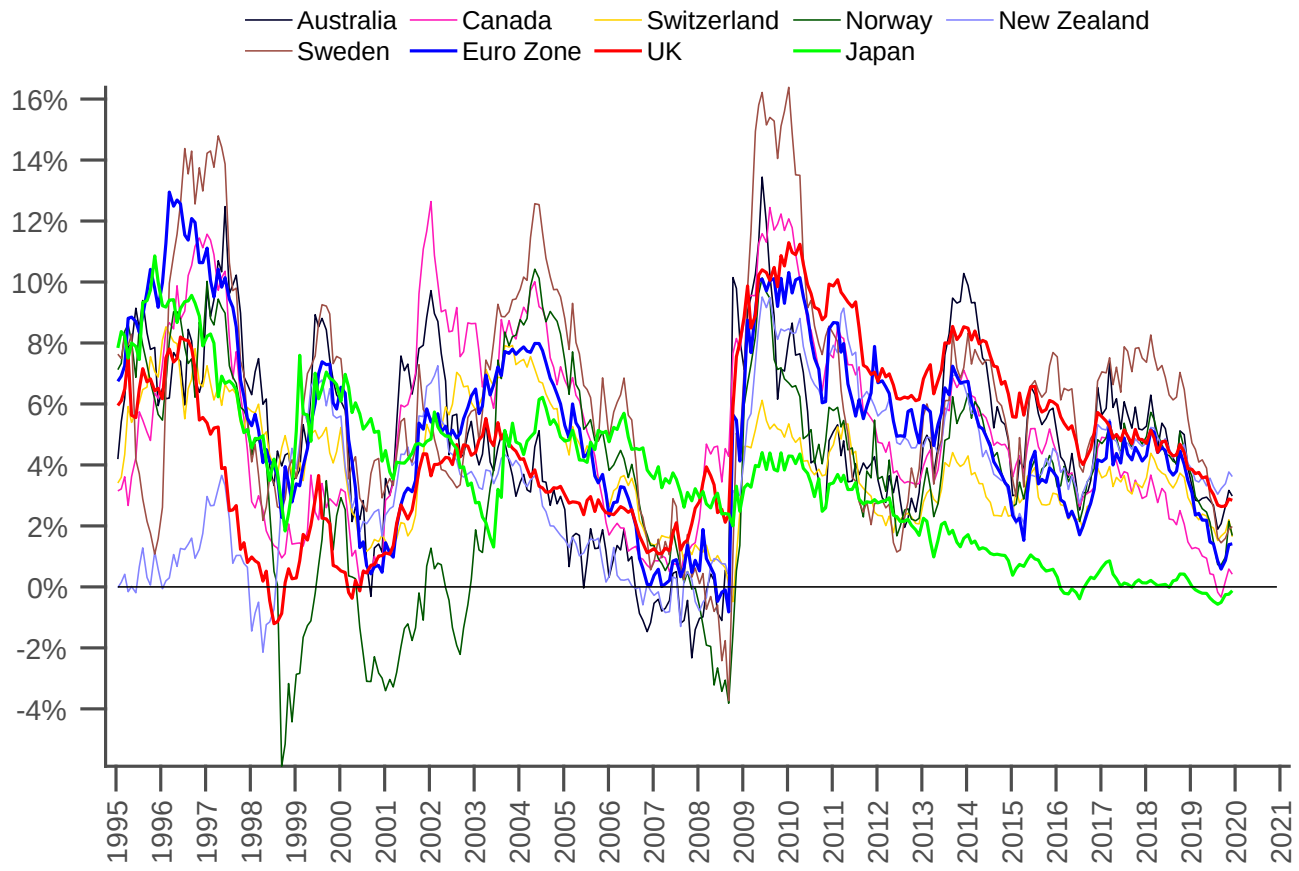


Figure A.25. Projected Bond Risk Premia

Figure displays projected bond risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projections for the bond risk premium are obtained by regressing realized ex-post premia on the slope of the yield curve and then forming 12-month ahead projections. Sample period is monthly observations from between 1995.12 and 2019.12.

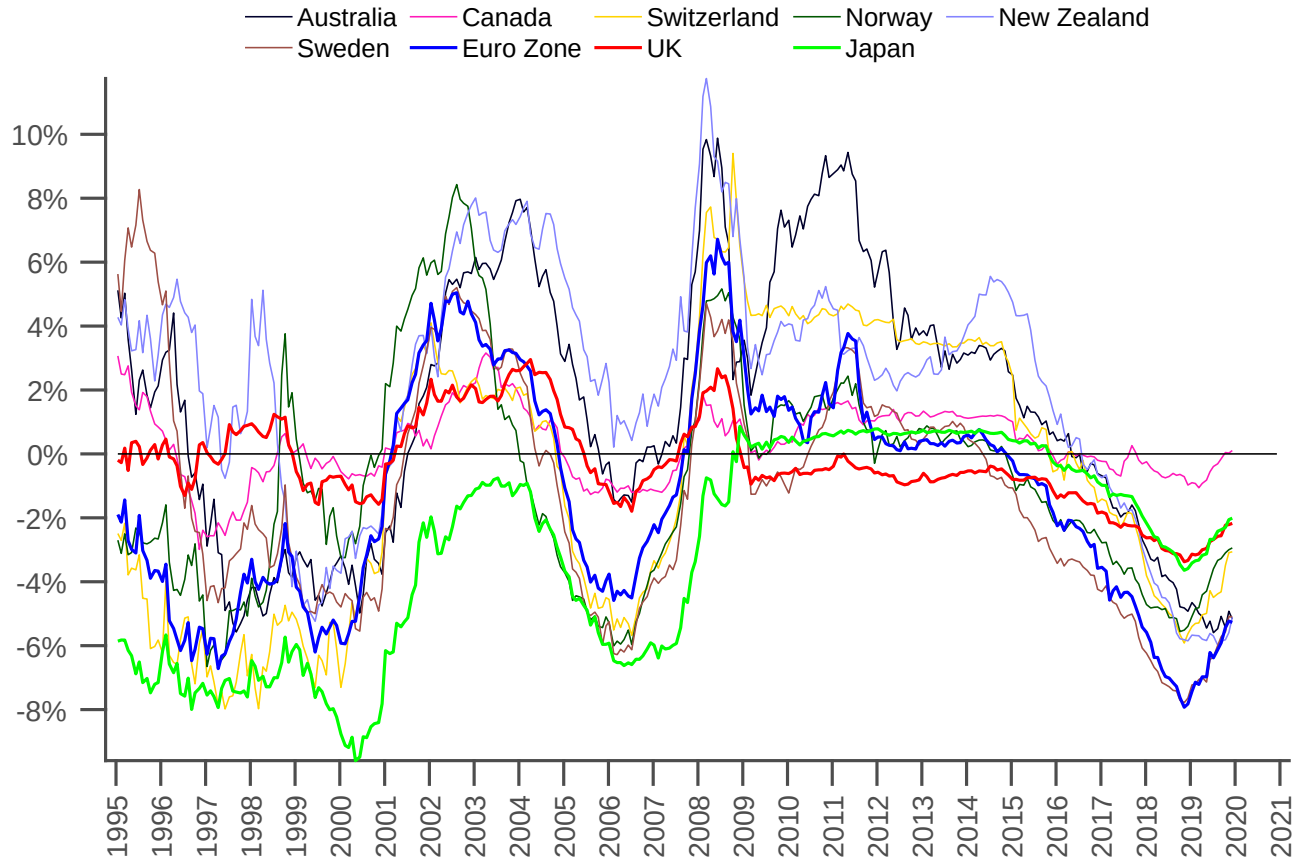
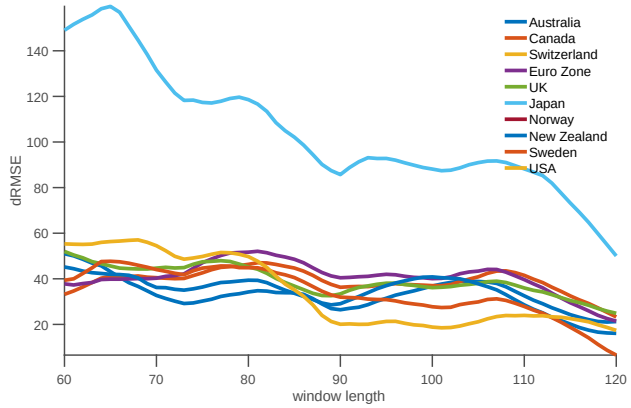
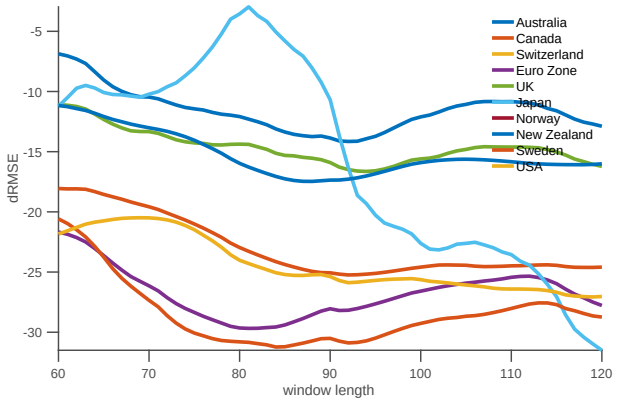


Figure A.26. Projected Exchange Rate Risk Premia

Figure displays subjective exchange rate risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projection for the bond risk premium are obtained by regressing realized ex-post premia on the interest rate differential between the foreign country and the United States and then forming 12-month ahead projections. The sample period is 1995.1 to 2020.12.

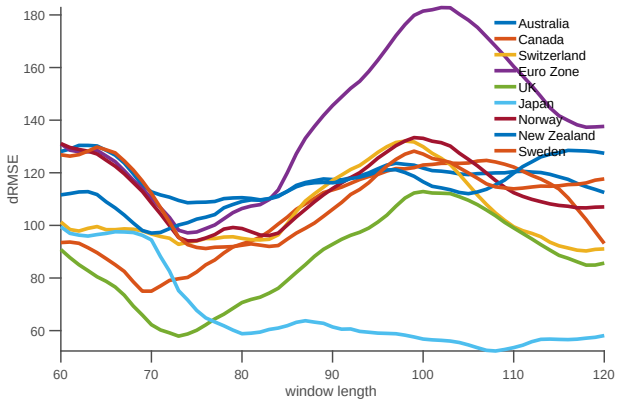


(a)

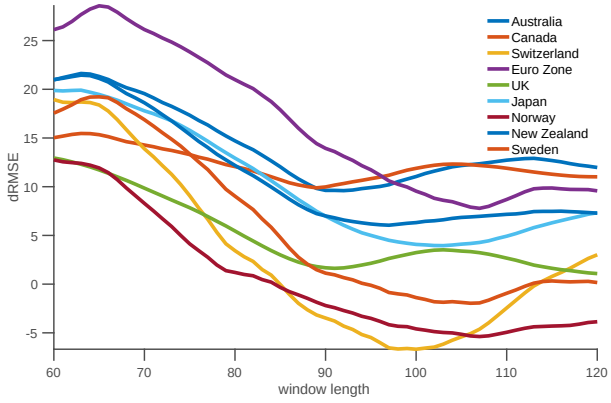


(b)

Figure A.27. Correcting Errors: Δ RMSE in the Fixed Income market Figure displays the exploitation of predictable forecast errors using Equation A.61. Panel A shows the difference in root mean squared error (Δ RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation 2.16 is used to predict the error term of Equation A.61. Panel B shows the same difference in Δ RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation A.61. The horizontal axes shows the behaviour of Δ RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the Δ RMSE in percent ($\frac{RMSE_{corrected}-RMSE_{survey}}{RMSE_{survey}} * 100$). Data is available for the sample period 2000.1 to 2020.12.



(a)



(b)

Figure A.28. Correcting Errors: Δ RMSE in the Foreign Exchange market Figure displays the exploitation of predictable forecast errors using Equation A.61. Panel A shows the difference in root mean squared error (Δ RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation 2.16 is used to predict the error term of Equation A.61. Panel B shows the same difference in Δ RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation A.61. The horizontal axes shows the behaviour of Δ RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the Δ RMSE in percent ($\frac{RMSE_{corrected}-RMSE_{survey}}{RMSE_{survey}} * 100$). Data is available for the sample period 2000.1 to 2020.12.

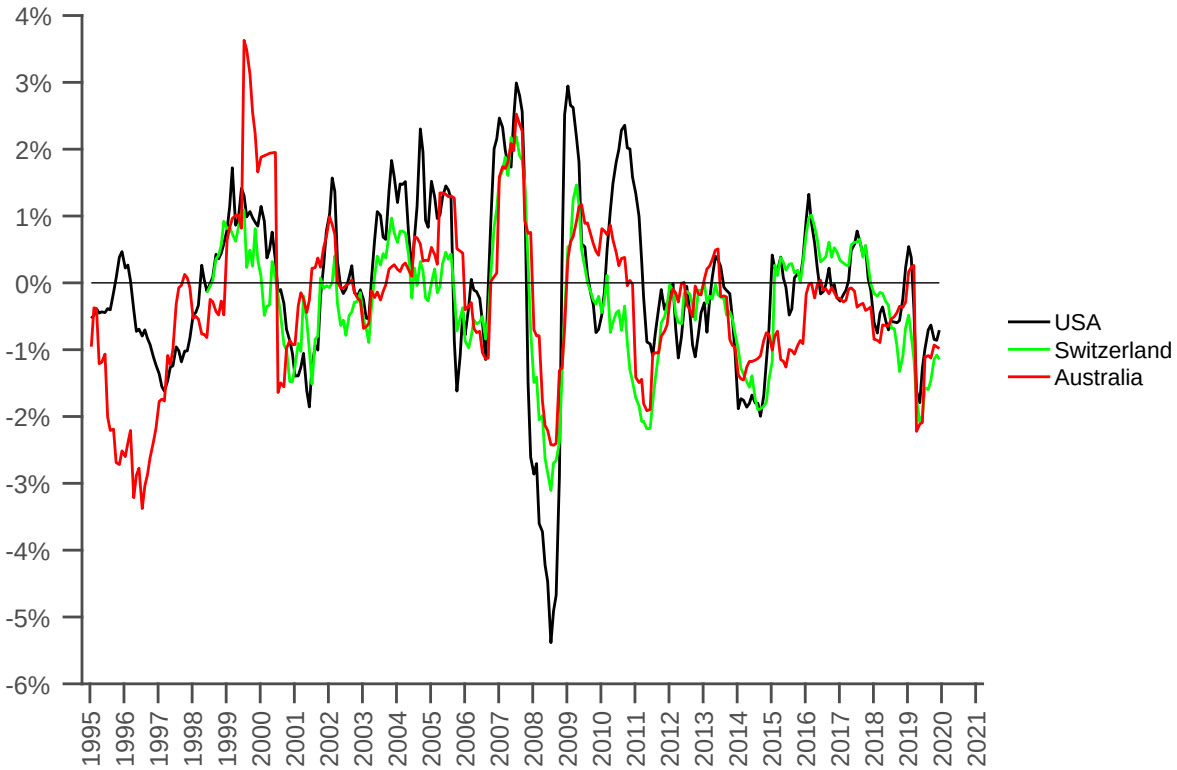


Figure A.29. Forecast errors of inflation expectations

Figure displays the difference between realized inflation and expected inflation as measured by the CPI for the US, Australia, and Switzerland. Expectations are obtained from Consensus Economics for a 12-month horizon while realisations are obtained from FRED, the Australian Bureau of Statistics (ABS), and the Federal Statistical Office of Switzerland, respectively. As the Australian Bureau of Statistics only reports CPI realisations on a quarterly frequency, we compute weighted averages of quarterly observations to obtain monthly estimates. The sample period is 1995.1 to 2020.12.

3

Subjective Risk Premia and the Long Carry Trade

Abstract

We establish that the expected excess return on the long carry trade as measured through surveys is non-zero, time-varying and positively predicts realisations. These observations differ from known results that use projections instead of surveys; we compare and contrast both approaches in this paper.

Furthermore, we highlight that the premium on this economically relevant trade are heavily intertwined with and driven by similar factors as exchange rate risk premia.

Finally, we highlight what empirical observations are unmatched by traditional economic models and how these can be extended to accommodate these features.

3.1. Introduction

The *standard carry trade*, a cornerstone arbitrage strategy in international finance, capitalises on interest rate differentials across countries to generate returns. Essentially, the strategy involves borrowing in low-yielding *funding* currencies and investing in high-yielding *investment* currencies. Naturally, any earnings made in foreign currencies must be converted back to one's home currency, introducing an exchange rate risk.

Financial theory posits that the expected return on this strategy should be zero, as any excess returns potentially generated by investing in the bond market should be counterbalanced by a depreciation of the respective foreign currency vis-à-vis the home currency. This relationship between interest rates and expected exchange rates is described by the uncovered interest rate parity (UIP). Empirically, at least for short horizons, it has been well-demonstrated that this is not the case, e.g., by Chinn and Meredith (2004); Fama (1984).

Nevertheless, the ongoing debate in the literature revolved around whether the realised excess returns on such a strategy are driven by unconditional risk premia in the foreign exchange market (Lustig et al., 2011; Hassan and Mano, 2019) or by time-varying, conditional risk premia (Lewis, 1995; Verdelhan, 2010; Bansal and Shaliastovich, 2013; Colacito and Croce, 2013). Recent papers like Pesch et al. (2024) have shed additional light on this relationship by leveraging survey data to analyse the dynamics of expected excess returns associated with trades of this nature.

This paper is concerned with a slight modification of this well-known strategy, termed the *long carry trade*. This strategy involves purchasing a foreign currency, investing in the long end of the foreign yield curve for a specified holding period, and then converting the proceeds back into one's home currency while financing the trade by selling the short end of the domestic yield curve. Implementing this trade has become especially appealing as hump-shaped yield curves have re-emerged in recent years.

[Insert Figure 3.30]

This strategy is economically significant as sovereign bond markets have become increasingly integrated in recent decades, implying that many investors are exposed to the risks embedded in such a trade. Indeed, in OECD countries on average, only about 60% of government debt is held domestically. Despite its economic significance, little is empirically known about the sources of risk and risk compensation for investing in foreign bond markets. This trade has also received recent academic attention, for instance, from Lustig et al. (2019), who argue that expected returns on this trade are close to zero and unpredictable, presenting a challenge for existing asset pricing models.

We encounter a similar no-arbitrage condition with respect to these expected returns as we did for the standard carry trade. However, now we are exposed to two types of risk simultaneously. On the one hand, we still face the same currency risk as before, which is covered by the UIP. On the other hand, we are exposed to a novel yield curve risk. Thus, risk compensation for this strategy is the sum of an exchange rate risk premium and a bond risk premium differential, which we denote as the Foreign Bond Dollar Risk Premium, LRP.

Given our later focus on this dollar risk premium of foreign bonds in excess of a US bond, our paper is related to Lustig et al. (2019), who argue that long carry trades are not profitable since the term premia differential and currency risk premia generally offset each other. Contrary to that, Andrews et al. (2021) suggest that the zero excess returns on this strategy result from a negative pre-crisis and a positive post-crisis return. Complementing these works, we study the properties of foreign bond risk premia using survey forecasts directly rather than projections of ex-post return realisations, and we uncover a number of novel insights. The use of surveys provides a new perspective in studying violations of economic hypotheses such as the UIP because they offer a real-time, model-free view of the world. Our dataset uniquely equips us to explore the intersection of the fixed income and foreign exchange markets united in the long carry trade, as we have data from the same source, covering the same cross-section of countries, over the same time frame and frequency.

In later sections of this paper, we document that survey-implied LRP are close to zero for all countries considered in our sample. However, they are highly time-varying, ranging between approximately -16% and +23% for all individual countries, and for the majority of countries, they display a clear pattern of being mostly positive in the first part of the sample and turning negative around the 2008 financial crisis. A decomposition of the overall dynamics into its components suggests that the switch in the sign is mainly driven by a corresponding flip in the exchange rate risk premia, which tends to drive a large fraction of the LRP variation. This contrasts with Lustig et al. (2019), who calculate risk premia based on projections of future realised returns on interest rate differentials and the slope of the term structure. They argue that projection-based bond risk premium differential and exchange rate risk premia are always negatively correlated, and that they offset each other leading to an insignificant risk premium on the long-term foreign bond dollar trade. As we are able to decompose the LRP into its components, the exchange rate risk premium and the bond risk premium differential, we can document that the correlation between the two subjective components varies between countries, is not strictly negative, and even switches sign.

Since XRP is the dominant source of variation in LRP, the loadings and factor dynamics largely carry over, with one factor explaining around 60% of the variation and a second factor that can be interpreted as a carry factor. In general, the dynamics of bond and exchange rate risk premia based on standard statistical models differ significantly from the corresponding dynamics of the survey-based premia. In fact, the correlation between an equally-weighted average of each country's LRP and the same time series obtained with benchmark statistical models is slightly negative, at around -0.13.

Furthermore, we investigate the predictability of realised excess returns using both, standard statistical predictors such as the slope of the domestic yield curve and the interest rate differential between foreign and domestic countries, and our subjective measure of expected excess returns. We find that both, traditional predictors and subjective expectations significantly predict realised foreign bond

dollar returns with the correct sign, even when used simultaneously. This has interesting implications for the predictability of forecast errors from surveys and the formulation of general equilibrium models in international finance and contradicts Greenwood and Schleifer (2014). Survey-implied subjective risk premia on the long carry trade also obey a risk-return tradeoff as they are correlated with proxies for the quantity of risk in the market.

The rest of the paper is structured as follows. In Section 3.2 we discuss our data sources for both, expectations and realisations. We then outline the theory underlying our analysis in Section 3.3, followed by the discussion of our empirical findings in Section 3.4. Finally, we investigate the predictability of realised returns in Section 3.5 and the implications of our work on model building in Section 3.6.

3.2. Data

Survey Data. Our survey data is provided by Consensus Economics (CE). Professional financial market participants submit monthly forecasts of (*i*) spot exchange rates; and (*ii*) yields on 10-year government bonds for a variety of countries.[1] We concentrate on the most heavily-traded G10 currencies vis-à-vis the United States (USD): Australia (AUD), Canada (CAD), Switzerland (CHF), Europe (EUR), United Kingdom (GBP), Japan (JPY), New Zealand (NZD), Norway (NOK), and Sweden (SEK). CE reports projections for two horizons, 3 and 12 months, for both exchange rate and interest rate expectations. In this paper, we focus on the 12-month forecasts, which are the horizon where predictable variations in risk premia are most likely to arise. Forecasts begin in (*i*) 1990 for the USD, CAD, EUR, GBP, JPY; (*ii*) in 1995 for AUD, NZD, and SEK; and (*iii*) in 1998 for NOK and CHF. All results in the paper, unless otherwise stated, are based on the period from January 1995 to December 2020, totaling 300 monthly observations for all countries except NOK and CHF.

CE has maintained a consistent questioning procedure over time, and survey

[1] CE panellists provide par yield forecasts which we treat as zero coupon forecasts. Section A.1 in the Online Appendix (OA) discusses the approximation error.

respondents are presented with the same questions for each country. Forecasters receive the questionnaire in the first few days of the month, and survey forecasts are collected on the second Monday of every month and then released by CE three days later on Thursday of the same week. We sample all yields, spot rates, and exchange rates on the date when the survey is made public, i.e., the release date, which is normally around the middle of the month, to avoid any look-ahead bias. Furthermore, the survey focuses on experts for each region, with respondents generally located in the country for which they are making predictions. Thus, the dataset is comparable across a broad cross-section of countries and is available at a monthly frequency for an extended sample period. Section A.1 in the Online Appendix (OA) provides additional details of the CE dataset.

Realised Data. We obtain monthly G10 FX spot and 3-month forwards from Thomson Reuters Eikon for the sample period January 1995 to December 2021. For the same panel, we obtain zero-coupon bond yields, which are generally available for maturities of 3, 6, 9, and 12 months, and 3, 5, 10, and 15 years from Bloomberg. Below we require country-specific yields for three bond maturities: the 12-month rate (risk-free), the 10-year rate, and the 11-year rate. While Bloomberg provides the first two series, it does not supply the latter. To address this, we fit a cubic spline to all available maturities and sample the desired yield.

3.3. Framework and Notation

In this section we introduce some notation and the formulas used to compute subjective risk premia on long term domestic bonds, currencies and foreign bonds.[2]

[2]The literature studying return predictability typically interprets expected excess log returns as risk premia. This is not quite correct since risk premia should be measured as expected excess simple returns. Assuming log-normality, arithmetic average returns differs from the geometric average returns by one half the variance (the Jensen's gap). Computing Jensen's terms from BlueChip Financial Forecasts, Buraschi et al. (2022) show that the Jensen's gap is tight using survey expectations. In what follows we interpret expected log excess returns as risk premia.

3.3.1. Risk premia in the fixed income market

Let $P_t^{(k)}$ be the time t price of a default risk-free zero-coupon bond of maturity k years. Spot k-year yields are then defined as $i_t^{(k)} = -\frac{\ln P_t^{(k)}}{k}$. The bond risk premium is defined as the expected excess return on the bond, so we start by computing the realized holding horizon return of a k-year bond, with a j-year holding horizon:

$$\ln \frac{P_{t+j}^{(k-j)}}{P_t^{(k)}} = -(k-j)i_{t+j}^{(k-j)} + ki_t^{(k)} \tag{1}$$

The annualised expected excess return on a k-year bond with a j-year holding horizon is then:

$$E_t\left[rx_{t+j}^{(k)}\right] = -\frac{k-j}{j}E_t\left[i_{t+j}^{(k-j)}\right] + \frac{k}{j}i_t^{(k)} - i_t^{(j)}, \tag{2}$$

where continuously compounded yields are annualised and k and j are expressed in years. We denote the bond risk premium for maturity k and horizon j by

$$BRP_t^{(j,k)} \equiv E_t\left[rx_{t+j}^{(k)}\right]. \tag{3}$$

Note that under the expectation hypothesis (EH), $E_t\left[i_{t+j}^{(k-j)}\right] = f_t^{(j,k)}$, where $f_t^{(j,k)} = \frac{ki_t^{(k)} - ji_t^{(j)}}{k-j}$ is the forward rate for k periods starting from j periods from time t, so that the risk premium for investing in long-term bonds is zero, $BRP_t^{(j,k)} = 0$.

3.3.2. Risk premia in the foreign exchange market

Denote by x_{t+j} the log of the exchange rate, expressed in US Dollars per unit of foreign currency, and by Δx_{t+j} the change in the log exchange rate from time t to time $t+j$. Therefore, a positive Δx_{t+j} corresponds to a depreciation of the US Dollar relative to the foreign currency. The j-period interest rate in the foreign country is denoted by $i_t^{(j)*}$. The annualised log currency excess return is given by:

$$rx_{t+j}^{FX} = (i_t^{(j)*} - i_t^{(j)}) + \frac{1}{j}\Delta x_{t+j}. \tag{4}$$

The exchange rate risk premium is defined as the conditional expectation of the excess return in Equation (4), i.e. $XRP_t^{(j)} \equiv E_t\left[rx_{t+j}^{FX}\right]$:

$$XRP_t^{(j)} = (i_t^{(j)*} - i_t^{(j)}) + \frac{1}{j}\left(E_t\left[x_{t+j}\right] - x_t\right), \tag{5}$$

where $E_t\left[x_{t+j}\right]$ is the forecast of the exchange rate in j periods. $XRP_t^{(j)}$ can be interpreted as the annualised expected excess return of investing for j periods (using a j-period instrument, e.g. a 3-month bill if $j = 0.25$) in the foreign currency, financing the investment selling the local currency (e.g. selling a 3-month T-bill). Also note that interest rates are annualised, so we also annualise the exchange rate change by dividing by j.

According to the uncovered interest rate parity (UIP), high interest rate countries are expected to experience an exchange rate depreciation to equalise expected exchange rate adjusted returns on assets. The idea behind UIP is that when the foreign interest rate is higher than the local interest rate, i.e. $i_t^{(j)*} > i_t^{(j)}$, the foreign currency will depreciate by the difference, i.e. x_{t+j} decreases by $i_t^{(j)*} - i_t^{(j)}$, so that in local currency terms the return on investing in the two countries is exactly the same, i.e. XRP is zero.

3.3.3. Risk premia on dollar-denominated foreign bonds

Denote by $rx_{t+j}^{(k),*}$ the annualised excess return of a k-year foreign bond with a j-year holding period, expressed in the foreign currency, and by $rx_{t+j}^{(k),\$}$ the equivalent annualized excess return of the foreign bond expressed in the local currency, i.e. the U.S Dollar. The j-period holding period return of the foreign bond expressed in local currency is:

$$\ln \frac{P_{t+j}^{(k-j),*}/S_{t+j}}{P_t^{(k),*}/S_t} = -(k-j)i_{t+j}^{(k-j),*} + ki_t^{(k),*} + x_{t+j} - x_t \tag{6}$$

This return is realized over a j period, so again if we want to annualise it we need to divide by j:

$$r_{t+j}^{(k),\$} = -\frac{k-j}{j} i_{t+j}^{(k-j),*} + \frac{k}{j} i_t^{(k),*} + \frac{1}{j} \Delta x_{t+j}. \tag{7}$$

The annualised expected excess return on a foreign k-year bond with a j-year holding horizon is then the sum of the foreign bond risk premium and the exchange rate risk premium:

$$\begin{aligned} E_t \left[rx_{t+j}^{(k),\$} \right] &= -\frac{k-j}{j} E_t(i_{t+j}^{(k-j),*}) + \frac{k}{j} i_t^{(k),*} + \frac{1}{j} \left(E_t \left[x_{t+j} \right] - x_t \right) - i_t^{(j)} \\ &= BRP_t^{(j,k),*} + XRP_t^{(j)}. \end{aligned} \tag{8}$$

The equivalent of the UIP condition that measures subjective risk compensation for investing in foreign countries with respect to the U.S is given by

$$\begin{aligned} LRP_t \equiv E_t \left[rx_{t+j}^{(k),\$} \right] - E_t \left[rx_{t+j}^{(k)} \right] &= XRP_t^{(j)} + BRP_t^{(j,k),*} - BRP_t^{(j,k)} \\ &= XRP_t^{(j)} + \Delta BRP_t^{(j,k),*} \end{aligned} \tag{9}$$

which is zero under the null hypothesis no risk compensation. Clearly, this can happen if all risk premia are equal to zero (the short term UIP holds and the EH also holds in both countries) or if risk premia in the foreign exchange and fixed income markets offset each other. LRP_t in Equation (9) is the dollar-denominated holding period expected excess return for selling a U.S Treasury bond and investing into a long-term foreign bond.

The next section derives empirical estimates of the expected excess return on the foreign bond trade as well as its components using survey forecasts to approximate the expected bond yield $E_t \left[i_{t+j}^{(k-j)} \right]$ and exchange rate change $E_t \left[x_{t+j} \right]$.[3]

[3]Note for interest rates we treat surveys as providing us with an approximation for log yields (continuously compounded yields) directly. For exchange rates we are making a different approximation, namely, that $\log E_t[X_t] = E_t[\log X_t] = E_t[x_t]$ i.e., we are assuming that Jensen's terms are small.

3.4. Subjective Risk Premia

In this section, we compute the bond risk premium $BRP_t^{(j,k)}$, exchange rate risk premium $XRP_t^{(j)}$, and the overall dollar risk premium on foreign bonds in excess of US bonds LRP_t, respectively, for a twelve-month forecast horizon j and a bond maturity k of *eleven years*. Since j and k are fixed, we simplify the notation by referring to the premia as BRP_t and XRP_t henceforth.[4]

3.4.1. Foreign Bond Dollar Risk Premia

Figure 3.33 illustrates the time series of the foreign bond dollar risk premium, which is clearly time-varying, displaying a pattern of being predominantly positive in the earlier part of the sample and turning negative around 2007. Moreover, the strong correlation in individual bond risk premia and exchange rate risk premia, demonstrated, for example, in Pesch et al. (2024), carries over into the cross-section of LRP_t with an average pairwise correlation of 45%. Japan is an outlier in this respect and is considered a special case in many regards.

[Insert Figure 3.33 here]

The strong positive correlation is evident from the time series plot alone, and Table 3.40 reinforces this point further. With the exceptions of Japan and the United Kingdom, almost all correlation coefficients between different LRPs exceed 0.5. In fact, the average pairwise correlation across all countries is 48.5%.

Comparing the dynamics of the subjective foreign bond dollar risk premia with their realised ex-post counterparts reveals several parallels between the two series. Firstly, both series roughly share the same scale, with most values falling within a range of -20% to $+20\%$. If anything, realised excess returns are slightly larger than the subjective premia. Secondly, both series share a global minimum in foreign bond dollar risk premia in 2008 followed by a peak in 2009. Thirdly, Japan remains an outlier in both series and is the only exception to an otherwise

[4] CE Surveys provide us with the expected yield on a 10-year bond for a 1-year forecast horizon. We compute subjective bond risk premia as the expected change in 11-year log bond prices above the 1-year risk-free rate.

heavily positively correlated set of cross-country premia. Conversely, the two series differ significantly in the early part of the sample; realised premia reached a low around 2000 while subjective premia peaked around that time. This peak only materialised in realisations about three years later.

[Insert Figure 3.32 here]

Unconditional average LRPs range from -1.40% (CHF) to 3.33% (SEK). The LRP for the JPY is also negative at -1.06%, suggesting that Swiss and Japanese bonds are, on average, hedges from the perspective of a domestic US investor. Conversely, US investors require a positive risk premium, typically between 2.32% and 3.33%, to purchase Australian, Norwegian, and Swedish bonds. The large negative values for Switzerland and Japan and the large positive values for the Scandinavian countries predominantly carry over from their corresponding subjective standard carry trade expectations (see panel B of Table 3.34).

Considering panels (a) and (b) of Table 3.34, it is evident that, with the exception of Japan, subjective BRPs are quite flat relative to subjective XRPs, which range from significantly negative (funding currencies) to significantly positive (investment currencies). From equation (9) and considering panel (c) of Table 3.34, it is clear that LRP is dominated by its exchange rate component. This dominance is also evident for projections, although the dynamics of subjective versus projection-based LRP differ significantly.

This, however, is not the end of the story. Since we have data from the same source for the same set of countries over the same period, we can display the time series of ΔBRP and XRP contributions to LRP for an average foreign bond dollar risk premium across countries. This analysis reveals that the XRP contribution was strongly positive and dominated the LRP between 1998 and 2005. From 2006 onwards, the contribution of the XRP begins to turn negative and is overshadowed by the ΔBRP component, which is almost negligible in the first part of the sample. The hypothesis that ΔBRP and XRP move in opposite directions, thereby leading to a LRP close to zero, is only verifiable between 2015 and 2017. This dynamic

aligns with observations that the standard carry trade largely ceased to be effective after 2005, as well-documented in the academic literature.

[Insert Figure 3.34 here]

A more detailed exploration of premium dynamics for the average foreign bond dollar risk premium is illustrated in the four panels of Figure 3.36. Panels (a) and (b) convey largely the same message as Figure 3.34. Panels (c) and (d), however, decompose the variation of LRP into the variations of ΔBRP and XRP. We observe that the variation of ΔBRP and XRP is largely constant across the five five-year periods considered. What changes, however, is the correlation between the two. Initially negative with values around -30%, the correlation becomes positive around the global financial crisis, peaking at $+35\%$, before turning negative again, reaching a low of -60% between 2010 and 2015. We discuss these results in depth for Australia and Switzerland but report them for all other countries in our sample in the appendix. All findings are consistent with a shift in the correlation between ΔBRP and XRP.

Panels (a) and (b) of Figures 3.37 and 3.38 depict a time-series decomposition of LRP into BRP and XRP components for the Australian Dollar (AUD) and Swiss Franc (CHF), respectively.

For the AUD, we display year-by-year medians of LRP, which are positive early in the sample, peaking at $+13\%$ in 2000, oscillating between positive and negative values (close to zero) between 2007 and 2018, and thereafter becoming strongly positive again. Panel (b) shows that over the full sample, the Australian XRP contributes more to the overall variation in LRP than ΔBRP does, and their effects often offset each other. From the perspective of US investors, most of the subjective risk of investing in foreign bonds originates from the exchange rate component of the trade. Panel (c) further explores this by providing a variance decomposition of LRP into ΔBRP and XRP contributions and their covariance for different subsamples. Here we see that the variance of bond risk premia differential contributes little to the variation of LRP and that the covariance between BRP and XRP changes sign throughout the sample.

Figure 3.38 displays comparable results for the CHF, whose XRP closely resembles that of the AUD pre-2007 but becomes strongly and persistently negative in the years post-credit crisis. As with the AUD, we find that the variation in XRP dominates the variation in LRP and the covariance term is mostly negative but switches sign in two sub-periods. The dynamics in the other sample countries are similar and are detailed in the appendix.

[Insert Figures 3.37 and 3.38 here]

To further shed light on the dynamics and drivers behind the foreign bond dollar risk premia, we also compute common factors, principal components PC_t, from an eigenvalue decomposition of the covariance matrix. This follows the approach of Lustig et al. (2011), who performed a similar exercise for a set of projection-based exchange rate risk premia. Table 3.38 shows how much variance of bond premia, exchange rate risk premia, and foreign dollar bond risk premia can be explained by the first three principal components, respectively. Since the three premia are connected through a linear relationship, it is unsurprising that the foreign dollar bond risk premium appears to be driven by the same factors as the ΔBRP and XRP. The first principal component of foreign dollar bond risk premia explains 58.87% of overall variation, while the second PC explains a further 14.53%. To understand what these two principal components signify, we need to inspect their time series, shown in panel (a) of Figure 3.39. Factor loadings for all nine countries are shown in panel (b) of the same figure. $PC1$ is a level factor with relatively equal, positive loadings on all countries. $PC2$ is a 'slope' factor, best understood through its loadings. Its highest loadings are on NZD, AUD, and NOK, while its lowest loadings are on JPY, GBP, and CHF. Therefore, it creates a wedge between low-interest rate currencies and high-interest rate currencies. In carry trade literature, the former are known as *funding currencies* while the latter are known as *investment currencies*. Inspecting the time series of $PC2$, we observe that it starts off positive at the beginning of our sample in 1998, turns negative around 2005, and stays negative until the GFC. From 2008 onwards, it remains close to zero. Both, Lustig et al. (2011) and Pesch et al. (2024), estimate a risk factor with

similar dynamics for exchange rate risk premia. Our analysis of the driving factors behind the LRP confirms several points. Firstly, it shows that the expected excess return is driven by a set of common factors across countries, supporting a risk-based view of asset pricing. Secondly, it confirms that the expected excess returns of the long carry trade are consistent with an investment and funding logic. Thirdly, it demonstrates that, while the 'slope' factor for the standard carry trade turned positive after the GFC as shown by Pesch et al. (2024), the subjective consensus is that the long carry trade ceased to be effective after the GFC.

3.4.2. Comparison with Statistical Models

We now compare the dynamics of our survey-based measures with standard statistical models. We consider the foreign bond dollar risk premium (LRP) as well as its components, the bond risk premia (BRP) and exchange rate risk premia (XRP).

Projection-based bond risk premia are obtained by regressing realised excess returns at time $t+1$, given our one-year holding period, on the slope of each yield curve at time t, defined as the spread between the 10-year and the 1-year yield. For exchange rate risk premia, we use the forecasts implied by 1-year interest rate differentials. For both projections, we also include a constant in the regression. We use yield curve slopes and interest rate differentials as predictors, respectively, because both economic theory and literature, such as Lustig et al. (2019), deem them as most appropriate.

Statistical forecasts are computed in-sample, so are subject to a look-ahead bias, unlike the survey-based forecasts, which are real-time. However, we do not attempt to compute projection-based estimates in real-time, i.e., out-of-sample, as our goal here is just to compare the time series dynamics of the survey and statistical-based foreign bond risk premia, not their predictive power. This method of constructing the statistical projections implies that the mean forecast errors shown in table 3.37 are equal to zero.

Table 3.35 reports summary statistics for risk premia based on statistical projections, and Figure 3.31 displays the time series of projected premia. Comparing

summary statistics between subjective risk premia and statistical projections reveals strong differences between the two sets of premia. For example, bond risk premia using statistical models are uniformly positive and do not switch sign, contrary to subjective BRP, which are negative on average but frequently flip sign. The ranking of the country average exchange rate risk premia is clearly different than survey-based XRP. In particular, the negative exchange rate risk premia of SEK and NOK appear inconsistent with the standard intuition behind carry trade strategies.

The overall foreign bond dollar risk premia based on statistical models (see panel c of Table A.11) are generally closer to zero and less volatile than their survey-based counterparts. This is consistent with Lustig et al. (2019), who argue that projection-based bond risk premium differential and exchange rate risk premia tend to offset each other, leading to an insignificant risk premium on the long term foreign bond dollar trade.

A shared feature of the two sets of premia is their high cross-sectional correlation, as illustrated in Tables A.12 and 3.41. This strong correlation, in the case of the projection-based premia, is driven by the high cross-sectional correlation of the predictors, i.e., yield curve slopes and interest rate differentials, across countries. Given such large cross-sectional correlation, to facilitate the comparison between our survey-based measures and statistical models, Figure 3.35 displays time series of equally-weighted averages for each country's subjective risk premia against the same time series obtained with statistical models.

An interesting observation is that projected and subjective expected excess returns somewhat co-move in the first half of the sample ($\rho = 0.21$) but then move in opposite directions in the second half of the sample ($\rho = -0.41$). Also, the projected mean premium is significantly less volatile than its subjective counterpart. This is not surprising since the projection is the result of an in-sample estimation procedure with relatively sticky state variables.

Comparing the accuracy of the two measures with respect to realised ex-post excess returns is not a fair exercise given the in-sample estimation procedure, but we explore some predictability and accuracy results in Section 3.5.

3.5. Forecast Errors and Predictability

In this section, we estimate a set of predictability regressions to characterise the informational content of survey-based measures of risk premia and contrast these measures with traditional expected excess return signals. The benchmark signals we consider are the interest rate differential and the slope of the yield curve, denoted as follows:

$$IRD_t = i_t^{(1)} - i_t^{(1)*} \tag{10}$$

$$Slope_t \equiv i_t^{(10)} - i_t^{(1)}. \tag{11}$$

3.5.1. Predicting Foreign Bond Dollar Risk Premia

Equation (9) delineates that LRP comprises two components: (1) the exchange rate risk premium; and (2) the differential between foreign and US bond risk premia. Given the informational content of both survey-based expected FX and local bond returns, an essential question arises: do survey-implied LRP predict their ex-post realised counterparts? We explore this by estimating the following pooled OLS regression with standard errors and confidence intervals computed accordingly:

$$rx_{t+1}^{(11),\$} - rx_{t+1}^{(11)} = a + b_1 IRD_t + b_2 Slope_t^* + b_3 \Delta LRP_t + \mu_i + \epsilon_{t+1}. \tag{12}$$

Additionally, we estimate univariate specifications of the above regression to analyse the varying informational value of the proposed predictors.

[Insert Table 3.39 here]

Specification (i) aims to explain the realised excess return on the long carry trade purely using interest rate differentials. The coefficient b_1 is negative (-1.58) and strongly statistically significant, indicating that increases in the difference between domestic and foreign interest rates, IRD_t, correlate with more negative excess returns. Therefore, when short-term 'carry' is large and positive (when $IRD_t < 0$), the expected excess return on long-term foreign bonds, denominated in

U.S. Dollars, is also substantial and positive. The R^2 in specification (i) is 8.31%, implying a reasonably large degree of predictability.

Specification (ii) employs $Slope_t$ as the sole predictor of realised ex-post returns, yielding a positive (1.93) and marginally significant estimate of b_2. This result suggests that a steeper foreign yield curve is associated with higher realised excess returns. The lower significance confirms our findings from Section 3.4 that the long carry trade is predominantly influenced by its exchange rate risk component, with a much lower R^2 statistic of 2.50% compared to specification (i).

Specification (iii) highlights the relevance of our subjective measure of expected excess returns in a univariate context. The estimated b_3 is positive (0.54) and borderline significant, with an R^2 of 6.13%, positioned between the previous two specifications and affirming that the subjective measure contains valuable predictive information.

The most interesting results, however, emerge from specification (iv), which combines macroeconomic state variables with the subjective measure of expected excess returns to examine their interplay. The estimate of b_1 closely mirrors specification (i), remains strongly significant, and becomes slightly more negative (-1.83). The estimate of b_2 gains significance compared to the univariate case (3.02) and is robustly significant. The estimate of b_3 remains relatively unchanged. Notably, the R^2 of this multivariate specification is 19.04%, significantly higher than any of the individual specifications, suggesting that the different predictors contain partially complementary information. While survey participants clearly consider interest rate differentials and slopes, their forecasts notably enhance the predictive accuracy of future realised excess returns.

The findings in this section demonstrate that the information contained in survey-implied subjective risk premia serves as a vital proxy for the underlying, true, unobservable time-varying bond and currency risk premia. However, it is important to note that subjective risk premia are not fully explained by observable data at time t. This observation does not imply that forecasters overlook information in interest rate spreads; rather, they amalgamate information from term structures

with other variables, alongside their judgement and intuition, when constructing projections. Pinpointing the exact information and models used by survey forecasters is challenging, as they likely assign time-varying weights to different models. For instance, they might rely entirely on their judgement one month and follow a standard slope predictor the next. Moreover, individual forecasters likely adopt diverse strategies, and observing only the consensus of an unbalanced panel does not allow us to discern these individual methodologies. Nevertheless, our results affirm that we can employ the consensus survey forecast as an aggregate, observable proxy for these elusive and dynamic predictive information models.

3.5.2. Errors in Forecasting Excess Returns

Table 3.36 outlines the forecast errors of the survey-implied foreign bond dollar risk premia and their components across various countries. The first panel reveals that mean subjective bond risk premium differentials are generally negative for most countries, though none are statistically significant. A similar pattern emerges in the second panel for currency risk premia, which also exhibit mostly negative means without statistical significance. It's noteworthy, however, that the range of currency risk premia is significantly broader than that of bond risk premium differentials. Combining the two components, the third panel presents the summary statistics for the LRP itself, which are uniformly negative and volatile.

Given that unconditional averages provide limited clarity regarding forecast errors, we turn to a visual exploration of these errors.

[INSERT FIGURE 3.40 HERE]

We observe that forecast errors are positively correlated across countries and exhibit notable peaks in 1996, 2004, 2010, 2014, and 2018, reaching highs of approximately 20%. Similarly, they share lows in 2000, 2008, 2015, and 2019, with minima of -25%. The time series for LRP in panel (c) of Figure 3.40 closely mirrors that for XRP in panel (b), except for a milder dip in 2005 and a non-existent peak in 1996 for XRP.

Table 3.39 further suggests an intriguing aspect: if traditional predictors remain statistically significant alongside the subjective risk premium as a predictor, this implies that forecast errors based on the subjective risk premium alone are predictable.

Table 3.42 presents results from a pooled regression of subjective forecast errors on IRD_t and $Slope_t$, the natural predictors for the two components of the long carry trade. The IRD_t, as defined in Equation 10, predicts the forecast error with a negative sign, while the $Slope_t$ predicts it with a positive sign. Both estimates are statistically significant, with an R^2 of this regression at 12.14%. This confirms that forecast errors are indeed explainable with statistical significance, leading to the natural question of the economic significance of this regression.

To explore this, we implement a rolling regression that adjusts the forecast errors in real time. This involves using w years' worth of observed subjective expectations and realisations to run a regression similar to that described in Table 3.42 to establish a predictive relationship. We then use this relationship to forecast the correction in $h = 12$ months. This method ensures that the predictions never access information unavailable to survey participants at the time of their submissions to Consensus Economics.

The error-corrected beliefs derive from the following equations:

$$\widehat{FE}^{n}_{t-h,t} = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_{t-h} \tag{13}$$

$$\xi_t = \hat{\alpha}_t + \hat{\beta}_t^{\top} X_t \tag{14}$$

$$\widehat{Y}_t = E_t^{\mathbb{S}}\left[Y_{t+h}\right] + \xi_t \tag{15}$$

The subscripts indicate that the error correction uses only real-time information, analogous to a real trader conducting monthly regressions and adjusting their predictions accordingly.

In Figure 3.42, we visualise the impact of the correction on forecast accuracy as measured by the root-mean-square error ($RMSE$). Panel (a) shows the difference between the $RMSE$ of the corrected forecasts and the $RMSE$ of uncorrected,

subjective expected excess returns. By definition, any $dRMSE > 0$ indicates that the real-time-corrected forecasts have a higher prediction error than the survey-implied forecasts. Across various values of w (60 to 120), $dRMSE > 0$ for all countries. Panel (b) presents a similar $dRMSE$ but corrects the time t forecast using the simple historical average forecast error. Again, except for the United Kingdom in some scenarios, the corrected forecast errors exceed the raw, survey-implied forecast errors. Using an expanding window for error correction does not substantially alter the results.

In summary, we highlight two key points: first, subjective forecast errors on the foreign bond dollar risk premium, as implied by expert surveys, are statistically significantly predictable by standard state variables. Second, however, real-time correction of these predictable forecast errors proves challenging, casting doubt on the economic significance of such corrections.

3.5.3. Trading Strategies

Another significant contribution of this work is the exploration of trading strategies related to the predictability of future realised excess returns on the long carry trade.

Potential trading strategies could be either cross-sectional or time-series based. A cross-sectional strategy, at each time t, would sort the nine countries based on subjective risk premia, going long on the top three countries and short on the bottom three. In this context, going long or short a country means either entering the long carry trade for that country or taking the counterposition. Such a strategy would involve exposure to six foreign countries at each time t.

Conversely, a time-series strategy would long all countries with a positive expected excess return and short the rest, thus maintaining exposure to all foreign countries at all times.

Figure 3.43 presents the outcomes of implementing a cross-sectional trading strategy for the long carry trade. Panel (a) of the figure delineates which countries are involved in the trade on the long (red) and short (blue) sides. Over time, the JPY, CHF, and EUR frequently serve as funding currencies, while the SEK, NOK,

and AUD are common investment currencies. However, there is notable month-to-month variation in portfolio composition, even for currencies that typically do not switch roles in standard carry trade scenarios. This variability stems from the volatile nature of the foreign bond dollar risk premia, as illustrated in Figure 3.33.

Panel (b) depicts the composition of cumulative carry returns (red) relative to the low (grey) and high (black) ends of the strategy. The high end generally exhibits increasing cumulative returns, while the low end shows decreasing returns. Since we are selling the low end of countries, the overall strategy return is calculated as the cumulative return from the high end minus the cumulative return from the low end.

Panel (c) provides a breakdown of long carry returns into bond risk premium differential and currency components. Both components yield positive unconditional cumulative returns, with the bond differential being significantly less volatile than the currency returns. Initially, the currency returns (i.e., the standard carry trade) outperform those generated from the bond risk premium differential. However, from around 2005, currency returns cease to yield positive results and consistently deliver negative returns from 2010 onwards.

Collectively, the strategy achieves a cumulative excess return of approximately 65% over the period from 1995 to 2020. With a monthly mean return of 0.0017 and a standard deviation of 0.0064, the annualised Sharpe ratio is calculated at 0.9512. It is somewhat surprising that the strategy manages to generate any return at all given the strong positive correlation between risk premia and the challenge of identifying a significant differential between countries.

Implementing a time-series strategy, which does not require balancing long and short positions, is outlined in Figure 3.44. Panel (a) shows continuous involvement in all countries, contrasting with the intermittent country composition in the cross-sectional strategy. The highs and lows are positively correlated and mimic a binary version of Figure 3.33. Panel (b) illustrates a division between returns from the high and low ends of the strategy, with the high end producing relatively stable profits until 2012 and then declining, while the low end consistently yields negative returns. The magnitudes are somewhat larger because the hypothetical trader is

more exposed to risk than in the cross-sectional strategy. Panel (c) reveals that the subjective long carry trade implemented in a multi-country time-series strategy also generates a positive cumulative excess return between 1995 and 2020. Both components of the trade, the bond risk premium differential and the currency risk premium, contribute positively to the overall return, with the currency risk premium component not producing significantly negative returns after 2010.

This time-series strategy yields a cumulative excess return of 103% over the period from 1995 to 2020, with a mean monthly return of 0.0023 and a standard deviation of 0.0071. The resulting annualised Sharpe ratio is 1.1386.

The objective of this section is not to assert the superiority of trading strategies based on surveys over all others. In fact, they obey a classical risk-return tradeoff that we document in Table 3.43 by regressing the expected excess return on an option-implied volatility metric proposed by Krohn et al. (2020). We find that the risk premia are positively correlated with this proxy for the quantity of risk in the foreign exchange markets.

However, several studies, such as Buraschi et al. (2022), suggest that surveys are not devoid of informational value. Since survey-based trading strategies both, generate meaningful excess returns over an extended period, and exhibit patterns consistent with carry trade literature, we provide further evidence of the economic significance of surveys.

3.6. Risk Premia Implied by an Equilibrium Model

Sections 3.4 and 3.5 document several stylised facts that are relevant for model-building.

- **Risk Premia:** Subjective foreign bond dollar risk premia are not constant over time and exhibit patterns that are consistent with a carry trade strategy: large positive excess returns are expected investing in Norwegian Krone and Australian Dollar and large negative returns are expected on the Swiss Franc and Japanese Yen.

- **Correlation of ΔBRP and XRP:** We document that the LRP does not only switch sign itself, but also that the sign of the correlation between its two constituent components, the currency risk premium and the bond risk premium differential, changes. This change in correlation is, similar to the premia themselves, significantly correlated across countries.

- **Belief Distortions:** Similar to Pesch et al. (2024), we document that subjective expected excess returns predict realised returns with the correct sign. There is however a residual which is predictable by date t macroeconomic data, suggesting that there exists a distortion between subjective and physical measures.

A model of international asset pricing would have to match the above characteristics of survey-implied foreign bond dollar risk premia. A useful starting point is the model presented in Section VI of Pesch et al. (2024), which is a simple equilibrium international asset pricing model with complete markets, recursive preferences and long-run risk. Pesch et al. (2024) show that this standard model with constant volatility is already able to match the unconditional means of subjective bond risk premia and exchange rate risk premia, as well as survey forecast errors, simply adding a small distortion between the physical and subjective probability measures. In particular, Pesch et al. (2024) estimate the model considering a three countries world including the U.S. and two foreign countries, i.e. Switzerland and Australia, and match the average negative exchange rate risk premium for CHF and negative for AUD, as well as the negative average bond risk premia, around -1% for all countries.

We consider here the stochastic volatility version of that model to allow for time variation in the risk premia, consistent with the stylized facts above, and we study the model-implied properties of the foreign bond dollar risk premium. The goal is to understand whether the model is able to capture the empirical properties of LRP and what are the key characteristics required in an equilibrium asset pricing model to match those properties.[5]

[5]For a full derivation see the Online Appendix of Pesch et al. (2024).

3.6.1. Model

Consider an economy where the state-variables are adapted to the filtered probability space $(\Omega, \mathcal{F}_{t\in T}, \mathbb{P})$ and $\mathbb{P}$ denotes the physical probability measure. The level of the aggregate consumption flow in this economy, C_t ($c_t = \ln C_t$), evolves according to the following dynamics, where its conditional mean is driven by a "long run risk" factor g_t and its conditional volatility by a "stochastic volatility" factor v_t:

$$dc_t = \left(g_t - \frac{1}{2}\sigma_c^2\right)dt + \sqrt{v_t}dW_{c,t}, \tag{16}$$

$$dg_t = \kappa_g(\mu_g - g_t)dt + \sigma_g\sqrt{v_t}dW_{g,t}, \tag{17}$$

$$dv_t = \kappa_v(\mu_v - v_t)dt + \sigma_v\sqrt{v_t}dW_{v,t}, \tag{18}$$

where all Brownian motions are assumed independent.

A continuum of identical agents hold the Duffie and Epstein (1992a) parameterisation of the continuous time Epstein and Zin (1989) and Weil (1989) recursive preferences, where β is a time discount factor, γ is the coefficient of risk aversion, and ψ is the elasticity of inter-temporal substitution (EIS). It is easy to show that the nominal SDF has the following dynamics:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_c dW_{c,t} - \Theta_g dW_{g,t} - \Theta_Q dW_{Q,t} - \Theta_v dW_{v,t}, \tag{19}$$

where the market prices of risk for consumption and the latent g and v processes are proportional to $\sqrt{v_t}$.

Bond prices are exponentially affine and depend on both the long-run risk and the stochastic volatility factors:

$$\frac{dP_\tau}{P_\tau} = \mu_P(\tau)dt + B_g(\tau)\sigma_g\sqrt{v_t}dW_{g,t} + B_v(\tau)\sigma_v\sqrt{v_t}dW_{v,t}, \tag{20}$$

where $B_g(\tau) < 0$ and $B_v(\tau) > 0$ are the bond factor loadings.

A key element in the model in order to match subjective risk premia and survey forecast errors is a wedge between objective and subjective beliefs. When investors

only have a short sample of data available, learn sub-optimally, or suffer from some behavioural biases, their subjective measure $\mathbb{S}$ will not necessarily coincide with the physical (objective) measure $\mathbb{P}$ of an econometrician. For simplicity, we assume that only the latent long run risk process g_t has different shocks under the subjective and physical measures. In particular, there are two changes of measure which satisfy

$$\begin{aligned} dW^{\mathbb{Q}}_{g,t} &= dW^{\mathbb{S}}_{g,t} + \Theta^{\mathbb{S}}_g dt, \\ dW^{\mathbb{Q}}_{g,t} &= dW^{\mathbb{P}}_{g,t} + \Theta^{\mathbb{P}}_g dt \\ dW^{\mathbb{S}}_{g,t} &= dW^{\mathbb{P}}_{g,t} - \phi_g dt \quad \rightarrow \quad \phi^g = \Theta^{\mathbb{S}}_g - \Theta^{\mathbb{P}}_g \end{aligned} \tag{21}$$

So, the SDF dynamics under the subjective measure is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta^{\mathbb{S}}_c dW^{\mathbb{S}}_{c,t} - \Theta^{\mathbb{S}}_g dW^{\mathbb{S}}_{g,t} - \Theta^{\mathbb{S}}_Q dW^{\mathbb{S}}_{Q,t} \tag{22}$$

$$\Theta^{\mathbb{S}}_c = \Theta^{\mathbb{P}}_c \tag{23}$$

$$\Theta^{\mathbb{S}}_g = \Theta^{\mathbb{P}}_g + \phi_g \tag{24}$$

$$\Theta^{\mathbb{S}}_Q = \Theta^{\mathbb{P}}_Q \tag{25}$$

This implies that only the market price of risk for the latent long-run risk factor g is different under the two measures and in particular it increases by the distortion ϕ_g. In order to preserve the form of the market prices of risk for the long-run risk factor, Θ_g under the subjective and physical measure, we assume that $\phi_g = \phi\sqrt{v_t}$. Namely, this assumption guarantees that both $\Theta^{\mathbb{P}}_g$ and $\Theta^{\mathbb{S}}_g$ are proportional to $\sqrt{v_t}$.

The model-implied instantaneous subjective bond risk premium is given by:

$$BRP_t = \Theta^{\mathbb{S}}_g B_g(\tau)\sigma_g\sqrt{v_t} + \Theta^{\mathbb{S}}_v B_v(\tau)\sigma_v\sqrt{v_t}. \tag{26}$$

Since both market prices of risk are proportional to $\sqrt{v_t}$, the subjective bond risk premium is proportional to v_t. So, in this economy, the bond risk premium is time varying with economic uncertainty. We have $\Theta^{\mathbb{S}}_g > 0$ and $B_g(\tau) < 0$, as well as $\Theta^{\mathbb{S}}_v < 0$ and $B_v(\tau) > 0$, so BRP is negative.

In order to solve for the exchange rate risk premium we need to make an

assumption about the foreign countries. Let us assume there are $N+1$ consumption goods and $N+1$ countries: "home" country (the U.S.) and N "foreign" countries. As in Colacito and Croce (2011), we assume that each country behaves as in autarky, in both consumption and financial assets (total home bias) so that a representative investor within each country only consumes the good which they are endowed. We assume foreign country dynamics have the same structure as above but with different shocks and potentially different parameters. We model cross-country correlation by assuming that shocks in the foreign country, i.e. $dW_{c,t}^{\mathbb{S},f}$, $dW_{g,t}^{\mathbb{S},f}$, $dW_{Q,t}^{\mathbb{S},f}$ and $dW_{v,t}^{\mathbb{S},f}$, are correlated with the corresponding shocks in the home country, with correlations ρ_c, ρ_g, ρ_q and ρ_v, respectively. We further assume that the correlation between shocks in the foreign countries is completely driven by their exposure to the home country shocks.

The subjective exchange rate risk premium implied by this model is given by:

$$XRP_t = \Theta_c^{\mathbb{S}}\left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f}\rho_c\right) + \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right) + \Theta_Q^{\mathbb{S}}\left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f}\rho_Q\right) + \Theta_v^{\mathbb{S}}\left(\Theta_v^{\mathbb{S}} - \Theta_v^{\mathbb{S},f}\rho_v\right). \tag{27}$$

The subjective exchange rate risk premium can be both positive or negative, it is time-varying (with v_t and v_t^f) and could potentially switch sign.

3.6.2. Model-Implied Foreign Bond Dollar Risk Premium

Using Equations (9), (26) and (27), the model-implied risk premium on on a foreign bond over the U.S. bond risk premium, in U.S. Dollars, is given by:

$$\begin{aligned} LRP_t \;=\; & \Theta_c^{\mathbb{S}}\left(\Theta_c^{\mathbb{S}} - \Theta_c^{\mathbb{S},f}\rho_c\right) + \Theta_g^{\mathbb{S}}\left(\Theta_g^{\mathbb{S}} - \Theta_g^{\mathbb{S},f}\rho_g\right) + \Theta_Q^{\mathbb{S}}\left(\Theta_Q^{\mathbb{S}} - \Theta_Q^{\mathbb{S},f}\rho_Q\right) + \Theta_v^{\mathbb{S}}\left(\Theta_v^{\mathbb{S}} - \Theta_v^{\mathbb{S},f}\rho_v\right) \\ & + \Theta_g^{\mathbb{S},f} B_g^f(\tau)\sigma_g^f\sqrt{v_t^f} + \Theta_v^{\mathbb{S},f} B_v^f(\tau)\sigma_v^f\sqrt{v_t^f} - \Theta_g^{\mathbb{S}} B_g(\tau)\sigma_g\sqrt{v_t} - \Theta_v^{\mathbb{S}} B_v(\tau)\sigma_v\sqrt{v_t}, \end{aligned} \tag{28}$$

that is the sum of the exchange rate risk premium and the bond risk premium differential. The estimation in Pesch et al. (2024) implies that the bond risk premium differential is on average close to zero. Therefore, in the model as in the data, LRP is mostly driven XRP and has similar properties. Namely, LRP is time-

varying and can switch sign. The ranking of the average LRP in different countries follows from the ranking of their corresponding model-implied $XRPs$ and depends on the relative ranking of market prices of risk. The model estimation in Pesch et al. (2024) suggests that we can match the large positive excess returns expected investing in Australian long-term bonds and large negative returns expected on Swiss long-term bonds, in excess of U.S. bonds.

Pesch et al. (2024) also show that the model is able to rationalise the cyclical properties of both bond and exchange rate risk premia, as well as their predictive power for future realised excess returns, simply allowing for a correlation between the latent long-run growth and volatility processes, and as long as the belief distortion is not too large. These properties clearly carry over to the foreign bond dollar risk premium, which is a combination of the risk premia in the fixed income and FX markets.

Finally, we analyse the correlation between the two elements of LRP, namely the exchange rate risk premium and the bond risk premium differential, as this is an important driver of the overall dynamics of LRP that is not studied in Pesch et al. (2024), and the data suggests that this correlation should be able to change sign. The analytical expression of this correlation in the model is long and not very easy to interpret, but intuitively it depends on the variances of the foreign and domestic volatility processes, as well as their covariance. Using the parameter estimates from Pesch et al. (2024), we simulate time series of model-implied exchange rate risk premia and bond risk premia differentials of the same length as in the data (300 monthly observations), for different values of the correlation between foreign and domestic volatility factors, ρ_v. Figure 3.45 shows the correlation between the two, for Switzerland and Australia, over rolling windows of 60 observations, for two values of $\rho_v = 0.5$ and $\rho_v = 0.95$. The correlation is positive unconditionally but can clearly switch sign, as in the data, in particular for large values of the correlation across volatility factors. Comparing these model-implied correlations with realised conditional correlations shown in Figure 3.46 reveals that a model of this sort can indeed match some of the features of survey-implied risk premia. Namely, we see that

peaks in correlation in the middle of the sample for AUD and the dips in correlation in the early sample for CHF can be nicely matched with higher values of ρ_v.

3.7. Conclusion

This paper exploits survey data on bond yields and exchange rates to jointly estimate risk premia in the foreign exchange and fixed income markets for the G10 countries. Subjective expected excess returns are obtained directly from a panel of investor forecasts allowing us to measure model free real-time risk premia on bonds and currencies, and then combine them to study the risk premium on an economically important investment strategy that buys a foreign long-term bond and sells a long-term U.S bond.

We show that subjective risk premia are highly correlated across countries, volatile, and tend to be countercyclical, consistent with leading asset pricing models. We also show that subjective risk premia are significantly positively linked to the realised volatility; therefore, are consistent with the basic idea of a risk-return tradeoff. Importantly, we show that the subjective risk premia, in both fixed income and exchange rate markets, significantly positively predict future realised excess returns and the predictive power goes beyond that of standard predictors like the interest rate differential and the slope of the term structure. This finding suggests that survey forecasters use information beyond that spanned by current interest rate spreads when forming their expectations.

We argue that a significant link between subjective risk premia, subjective macro expectations and the quantity of risk, support asset pricing models that generate return predictability through cyclical variation in risk aversion, uncertainty, the likelihood of disasters or rational learning. Indeed, we conclude by deriving and estimating an asset pricing model in which return predictability arises through time-variation in economic uncertainty. The model implied moments line up closely with the empirical survey based moments which demonstrates that subjective risk premia are well explained by a traditional model, irrespective of whether belief formation is rational or not.

3.8. Tables

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel A: BRP										
Mean	−0.83	−0.43	−1.91	−1.18	−0.55	0.29	−2.72	−0.94	−1.56	−1.41
Std	3.45	3.78	1.00	2.14	3.03	1.74	3.34	3.58	3.58	2.85
Skew	0.85	0.72	0.11	0.47	0.43	0.73	−0.30	0.32	0.31	0.46
AR(1)	0.71	0.86	0.68	0.66	0.79	0.75	0.75	0.75	0.81	0.67
Panel B: XRP										
Mean	1.74	1.03	−1.69	0.17	0.20	−2.76	3.55	1.14	3.48	
Std	4.91	2.48	4.48	4.04	2.64	4.12	3.00	5.58	4.32	
Skew	0.63	−0.33	0.11	0.84	0.06	−0.24	0.11	0.58	0.52	
AR(1)	0.87	0.70	0.85	0.88	0.73	0.83	0.81	0.88	0.85	
Panel C: ΔBRP										
Mean	0.58	0.98	−0.09	0.23	0.85	1.70	−0.90	0.47	−0.16	
Std	2.82	2.52	2.28	2.03	2.61	2.92	3.16	2.93	3.00	
Skew	0.23	0.69	−0.10	−0.18	−0.57	−0.41	−0.46	−0.09	−0.18	
AR(1)	0.70	0.76	0.61	0.68	0.74	0.70	0.74	0.70	0.77	
Panel D: LRP										
Mean	2.32	2.01	−1.40	0.40	1.06	−1.06	3.16	1.61	3.33	
Std	5.20	3.87	4.98	4.38	3.54	5.10	5.25	6.31	5.57	
Skew	0.55	−0.05	−0.03	0.58	0.13	0.01	−0.03	0.73	0.30	
AR(1)	0.81	0.75	0.81	0.84	0.73	0.83	0.80	0.87	0.86	

Table 3.34. Descriptive Statistics for Subjective Risk Premia
This table presents the means, standard deviations, skewness, and AR(1) coefficients for subjective bond risk premia (BRP), subjective exchange rate risk premia (XRP), subjective bond risk premia differentials (ΔBRP), and subjective bond dollar risk premia (LRP). The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel A: $\mathbf{BRP_{projected}}$										
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19	4.65
Std	2.99	3.21	1.88	2.98	2.82	2.57	3.44	2.49	3.77	1.86
Skew	0.04	0.40	0.29	0.27	0.24	0.44	−0.33	0.10	0.56	0.15
AR(1)	0.93	0.97	0.96	0.96	0.98	0.97	0.96	0.97	0.96	0.98
Panel B: $\mathbf{XRP_{projected}}$										
Mean	1.43	0.29	−0.68	−1.32	−0.31	−3.00	−0.89	2.19	−1.20	
Std	4.35	2.24	4.33	4.01	1.62	3.81	3.44	4.87	3.66	
Skew	−0.04	0.09	−0.01	−0.02	−0.21	−0.52	0.47	0.00	0.24	
AR(1)	0.98	0.98	0.99	0.98	0.98	0.98	0.99	0.98	0.98	
Panel C: $\mathbf{\Delta BRP_{projected}}$										
Mean	0.15	0.51	−0.67	0.56	0.11	−1.06	−1.01	−1.12	1.55	
Std	2.89	2.20	2.19	2.73	1.97	3.37	3.38	1.99	3.56	
Skew	0.21	1.32	0.22	0.88	−0.09	0.60	−0.35	−0.00	0.28	
AR(1)	0.94	0.97	0.97	0.97	0.97	0.98	0.97	0.96	0.96	
Panel D: $\mathbf{LRP_{projected}}$										
Mean	1.58	0.80	−1.36	−0.76	−0.21	−4.06	−1.90	1.07	0.35	
Std	4.20	4.02	2.37	2.27	2.33	1.29	2.56	4.75	4.02	
Skew	0.00	0.56	0.07	−0.15	0.39	0.03	0.22	0.38	0.62	
AR(1)	0.96	0.97	0.98	0.98	0.98	0.95	0.96	0.98	0.96	

Table 3.35. Descriptive Statistics for Projected Risk Premia
This table presents the means, standard deviations, skewness, and AR(1) coefficients for projected bond risk premia (BRP), projected exchange rate risk premia (XRP), projected bond risk premia differentials (ΔBRP), and projected bond dollar risk premia (LRP). The sample period is 1995.1 to 2020.12.

$FE_{\Delta BRP}$	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	-0.25	-0.51	-0.99	0.40	-0.73	-2.66	-0.63	-1.45	1.82
Std	6.12	4.60	5.16	5.23	5.16	7.07	5.22	5.36	6.83
Min	-14.52	-10.12	-18.46	-13.57	-14.12	-21.73	-14.53	-15.60	-15.11
Max	18.50	15.46	11.43	15.65	13.68	21.98	11.37	10.47	24.93
AR(1)	0.19	0.00	0.00	0.25	0.38	0.09	0.29	0.11	0.36
FE_{XRP}	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	-0.34	-0.75	1.01	-1.48	-0.47	-0.11	-4.41	1.02	-4.76
Std	12.75	7.58	9.60	10.07	8.33	11.42	10.81	13.50	11.26
Min	-30.85	-22.89	-24.64	-28.32	-29.20	-26.92	-28.93	-37.28	-36.42
Max	32.74	19.23	36.89	19.97	14.72	22.78	22.95	30.16	19.89
AR(1)	0.10	0.12	0.23	0.09	-0.07	0.19	0.10	0.08	0.01
FE_{LRP}	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	-0.60	-1.26	0.97	-1.08	-1.20	-2.77	-4.71	-0.44	-2.94
Std	11.80	9.01	10.39	10.67	9.49	11.24	13.06	13.18	12.66
Min	-34.81	-28.97	-32.98	-30.68	-36.11	-23.25	-34.69	-40.14	-41.95
Max	27.06	20.35	36.00	21.57	21.85	29.95	20.13	24.80	33.43
AR(1)	0.11	0.08	0.23	0.10	0.11	-0.01	0.26	0.17	0.21

Table 3.36. Summary Statistics Forecast Errors

This table presents the means, standard deviations, skewness, and AR(1) coefficients for the forecast errors on excess returns based on surveys. The forecast errors are defined as

$$FE_t = rx_t^* - E\left[rx_t^*\right]$$

The reported AR(1) coefficients relate to the yearly autocorrelation. Values are scaled in annualised percentages. The sample period is 1995.1 to 2020.12.

$FE^{proj}_{\Delta BRP}$	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	-1.71	-1.48	-0.64	0.73	-2.82	-0.65	0.08	-0.90	-2.43
Std	6.94	4.07	5.98	5.90	5.65	7.09	6.64	6.02	7.25
Min	-26.18	-12.00	-17.17	-13.31	-17.03	-21.26	-11.42	-22.46	-18.38
Max	10.63	12.14	17.56	15.13	11.96	16.64	22.27	14.59	16.20
AR(1)	0.05	-0.04	0.05	0.05	0.15	-0.08	0.17	0.03	0.10
FE^{proj}_{XRP}	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	1.61	-0.18	1.22	1.77	-0.63	-0.26	0.43	3.97	2.57
Std	15.14	9.37	13.16	16.10	10.36	12.69	14.31	15.19	14.06
Min	-34.28	-25.30	-31.61	-27.30	-35.72	-30.23	-32.18	-42.82	-38.45
Max	42.02	23.99	48.98	58.78	22.04	26.22	42.41	37.97	36.95
AR(1)	0.19	0.10	0.14	0.13	-0.01	0.14	0.03	0.13	0.11
FE^{proj}_{LRP}	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
Mean	-0.10	-1.66	0.58	2.50	-3.45	-0.91	0.51	3.07	0.14
Std	13.39	10.39	14.28	17.70	9.89	13.25	14.90	14.26	13.95
Min	-30.65	-31.45	-40.88	-23.38	-34.64	-39.56	-35.68	-36.18	-38.25
Max	39.80	27.64	57.11	69.24	22.34	32.46	45.98	43.93	37.43
AR(1)	0.04	0.06	0.17	0.18	-0.03	0.04	0.10	0.12	0.11

Table 3.37. Summary Statistics Projected Forecast Errors

This table presents the means, standard deviations, skewness, and AR(1) coefficients for the forecast errors on excess returns based on statistical projections. The forecast errors are defined as

$$FE_t = rx^*_t - E\left[rx^*_t\right]$$

The reported AR(1) coefficients relate to the yearly autocorrelation. Values are scaled in annualised percentages. The sample period is 1995.1 to 2020.12.

	PC1	PC2	PC3
ΔBRP	54.66	17.17	9.51
XRP	63.96	14.34	6.55
LRP	58.87	14.53	10.74

Table 3.38. PCA Analysis: LRP Variance Explained (%)
Risk premium factors PC_t are formed from an eigenvalue decomposition of the covariance matrix of subjective risk premia $var(RP_t) = QDQ^\top$. This table displays the fraction of subjective risk premium variance due to the n'th factor which is computed from $D(n,n)/\sum_n D(n,n)$. Factors (principle components) are computed from linear combinations (rotations) of the input series via $PC_t = RP_tQ$. The sample period 1995.1 to 2020.12

	(i)	(ii)	(iii)	(iv)
IRD	-1.58			-1.83
	[-1.78 -1.39]			[-2.02 -1.65]
$Slope$		1.93		3.02
		[1.56 2.32]		[2.71 3.39]
$FBDRP$			0.54	0.49
			[0.46 0.63]	[0.42 0.57]
R^2 (%)	8.31	2.50	6.13	19.04

Table 3.39. Predictability Regressions
This table reports estimates from pooled OLS regression of the form

$$rx_{t+12}^{(11),\$} - rx_{t+12}^{(11)} = a + b_1 IRD_t + b_2 Slope_t + b_3 LRP_t + \epsilon_{t,t+12},$$

in column (iv) where the dependent variables are the one-year excess returns on the long carry trade. $Slope_t = (i_t^{(10)} - i_t^{(1)})$ is the slope of the domestic yield curve, $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$ is the one-year interest rate differential and LRP_t is the survey-implied foreign bond dollar risk premium. We also report univariate regressions with the same predictors in columns (i) to (iii). A constant is included but not reported. Confidence intervals in $[\cdot]$ are bootstrapped with 1000 replications. The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.72	0.65	0.77	0.23	0.12	0.61	0.90	0.78
CAD	0.72	1.00	0.48	0.62	0.35	0.03	0.57	0.66	0.65
CHF	0.65	0.48	1.00	0.87	0.13	0.46	0.58	0.60	0.71
EUR	0.77	0.62	0.87	1.00	0.29	0.32	0.68	0.74	0.84
GBP	0.23	0.35	0.13	0.29	1.00	0.28	0.10	0.14	0.23
JPY	0.12	0.03	0.46	0.32	0.28	1.00	0.08	−0.05	0.23
NOK	0.61	0.57	0.58	0.68	0.10	0.08	1.00	0.59	0.75
NZD	0.90	0.66	0.60	0.74	0.14	−0.05	0.59	1.00	0.74
SEK	0.78	0.65	0.71	0.84	0.23	0.23	0.75	0.74	1.00

Table 3.40. Cross Country Correlations of Subjective Foreign Bond Dollar Risk Premia
This table shows the correlation coefficients between subjective foreign bond dollar risk premia. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
AUD	1.00	0.89	0.46	0.51	0.89	0.76	0.41	0.90	0.88
CAD	0.89	1.00	0.66	0.65	0.96	0.66	0.66	0.84	0.73
CHF	0.46	0.66	1.00	0.94	0.64	0.29	0.95	0.61	0.47
EUR	0.51	0.65	0.94	1.00	0.64	0.48	0.91	0.60	0.56
GBP	0.89	0.96	0.64	0.64	1.00	0.65	0.60	0.85	0.79
JPY	0.76	0.66	0.29	0.48	0.65	1.00	0.32	0.57	0.71
NOK	0.41	0.66	0.95	0.91	0.60	0.32	1.00	0.49	0.32
NZD	0.90	0.84	0.61	0.60	0.85	0.57	0.49	1.00	0.88
SEK	0.88	0.73	0.47	0.56	0.79	0.71	0.32	0.88	1.00

Table 3.41. Cross Country Correlations of Projected Foreign Bond Dollar Risk Premia
This table shows the correlation coefficients between projected foreign bond dollar risk premia. Based on monthly observations from between 1995.1 to 2020.12.

	FE_{LRP}
IRD_t	−1.75
	[−2.04 −1.46]
$Slope_t$	2.88
	[2.22 3.56]
R^2 (%)	12.14

Table 3.42. Forecast Error Predictability
This table shows pooled OLS regressions of LRP-implied forecast errors, at the annual horizon, on the slope of the yield curve and the 1-year interest rate differential

$$FE^i_{t,t+1} = a^i + b^i X^i_t + \eta^i_{t+1},$$

where X_t includes the $Slope_t = (i^{(10)}_t - i^{(1)}_t)$ and $IRD_t = (i^{(1)}_t - i^{*,(1)}_t)$. Confidence intervals in $[\cdot]$ are obtained through a circular block bootstrap with 1000 replications. The sample period is 1995.1 to 2020.12

	(i)	(ii)
FXVar	0.76	
	[0.03 1.47]	
$rx_{t-12,t}$		−0.14
		[−0.22 −0.07]
R^2 (%)	2.74	10.34

Table 3.43. Cyclicality Regressions: Subjective LRPs
This table reports estimates from pooled OLS regressions of the form

$$LRP_t = a + b^\top X_t + \epsilon_t.$$

LRP_t is the survey-implied bond risk premium and X_t is a vector of explanatory variables and contains FX option-implied risk neutral variance constructed and discussed by Krohn et al. (2020) ($FXVar$), and the excess foreign bond dollar return realized between $t-1$ year and t. A constant is included but not reported. Confidence intervals in $[\cdot]$ are estimated using a circular block bootstrap with 1000 replications. The sample period is 1995.1 to 2020.12 for specification (ii) 1999.1 to 2019.4 for specification (i) because of data availability.

3.9. Figures

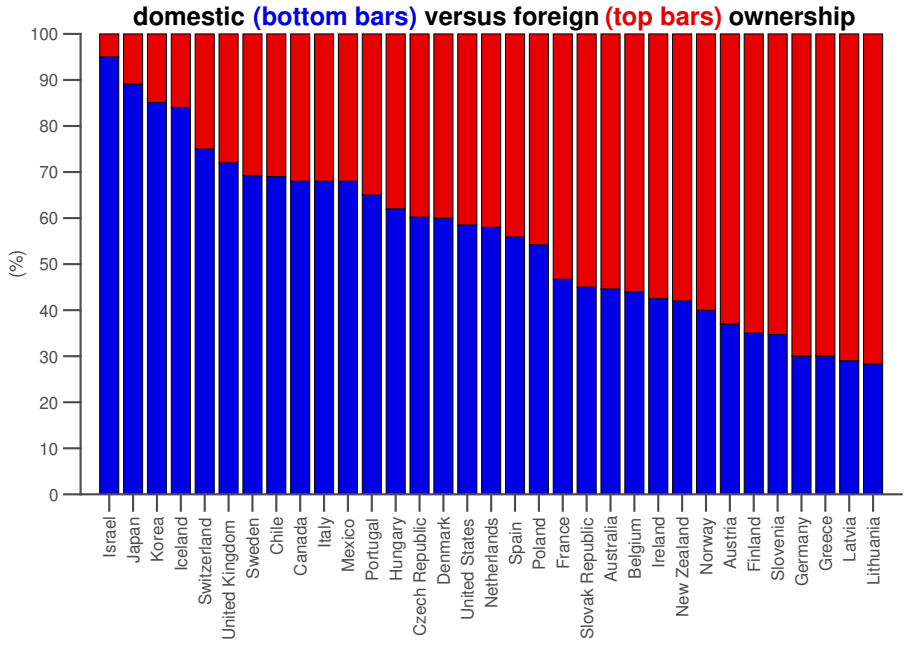


Figure 3.30. Motivation

Figure shows the shares of domestic and foreign ownership of government debt across OECD countries. The underlying data is from 2019.5 and originates from Deutsche Bank Securities.

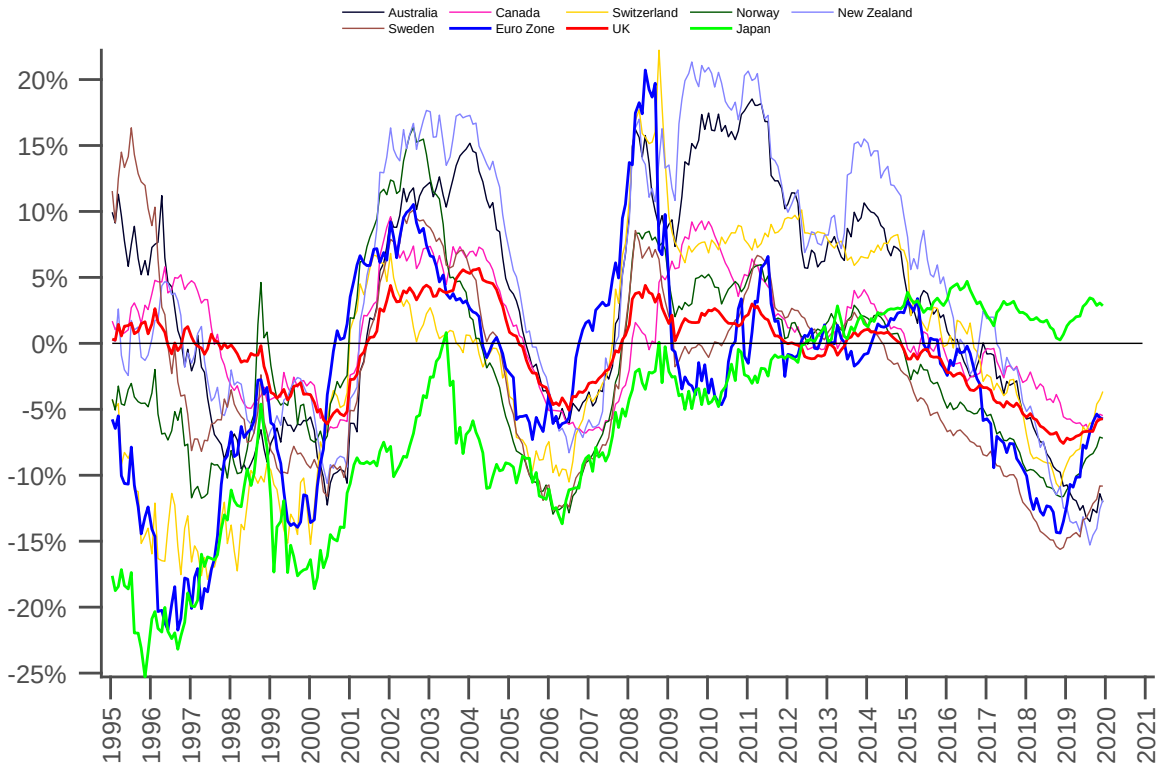


Figure 3.31. Projected Foreign Bond Dollar Bond Risk Premia

Figure displays real-time projected sovereign foreign bond dollar risk premia

$$LRP^{11}_{t,projected} = E_t^{projected}\left[rx_{t+j}^{(k),\$}\right] - E_t^{projected}\left[rx_{t+j}^{(k)}\right]$$

where subjective expectations are elicited from survey forecasts. The sample period is 1995.1 to 2020.12.

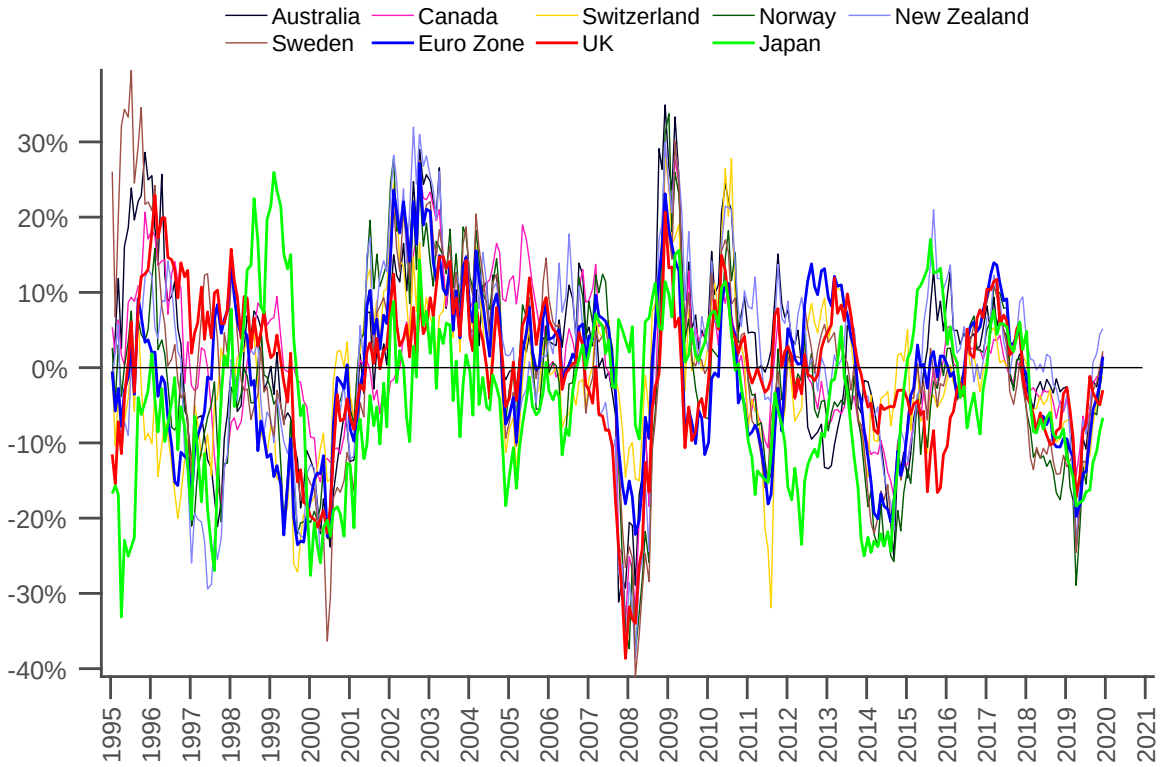


Figure 3.32. Realised Foreign Bond Dollar Bond Risk Premia

Figure displays real-time realised sovereign bond risk premia

$$LRP^{11}_{t,realised} = rx^{(k),\$}_{t+j} - rx^{(k)}_{t+j}$$

where subjective expectations are elicited from survey forecasts. The sample period is 1995.1 to 2020.12.

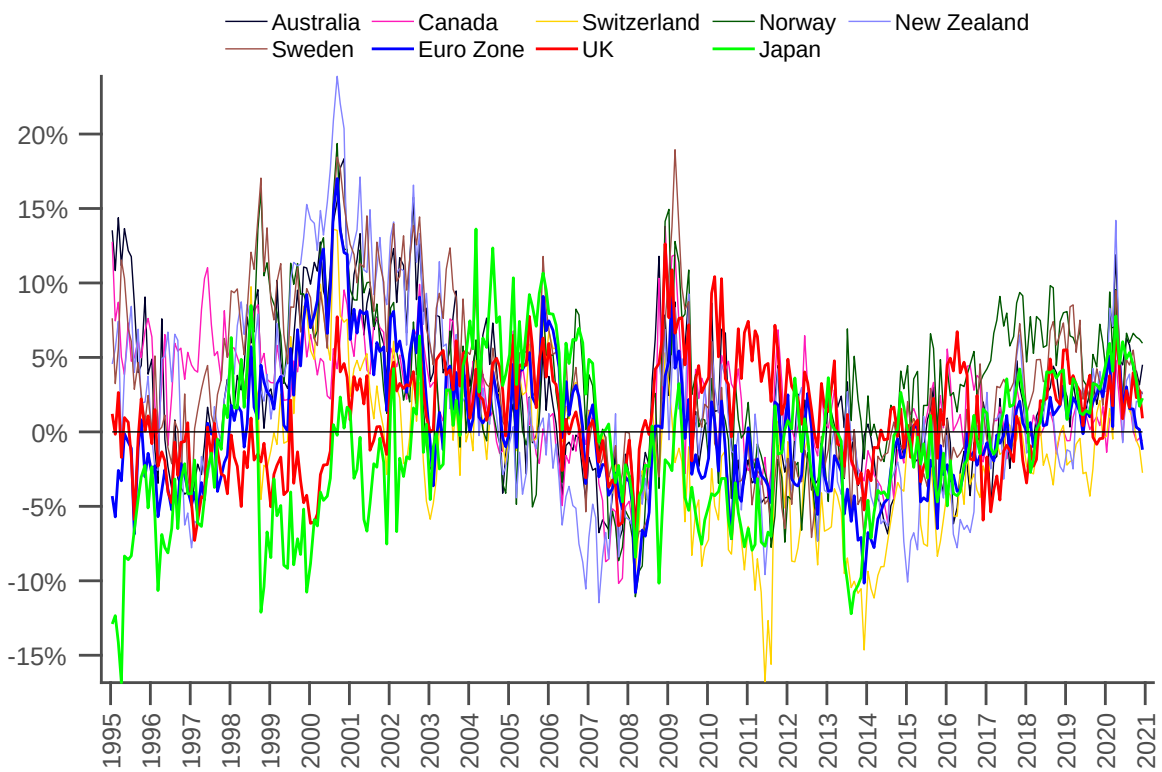


Figure 3.33. Subjective Foreign Bond Dollar Bond Risk Premia

Figure displays real-time subjective sovereign bond risk premia

$$LRP_t^{11} = E\left[rx_{t+j}^{(k),\$}\right] - \left[rx_{t+j}^{(k)}\right]$$

where subjective expectations are elicited from survey forecasts. The sample period is 1995.1 to 2020.12.

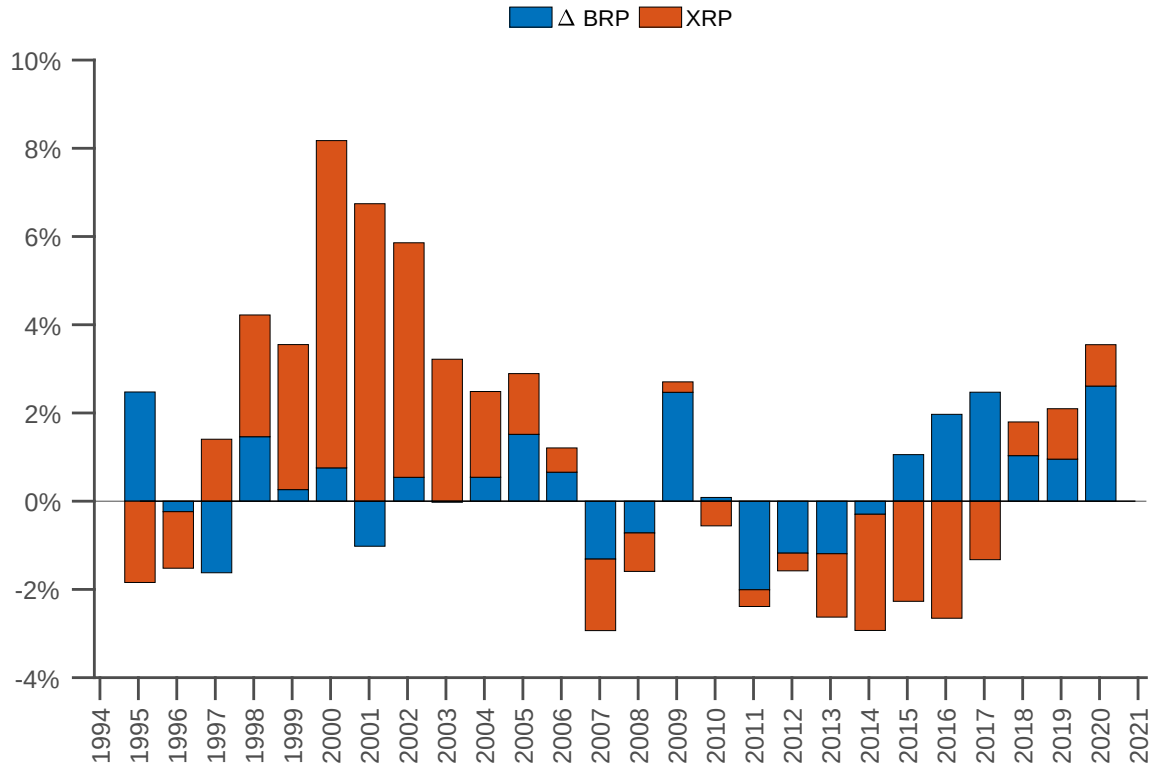


Figure 3.34. Subjective Long Term Dollar Bond Risk Premia Decomposed

Figure depicts a time series of the decomposition of mean LRP across our sample countries into its constituent components, ΔBRP and XRP. The sample period is 1995.1 to 2020.12.

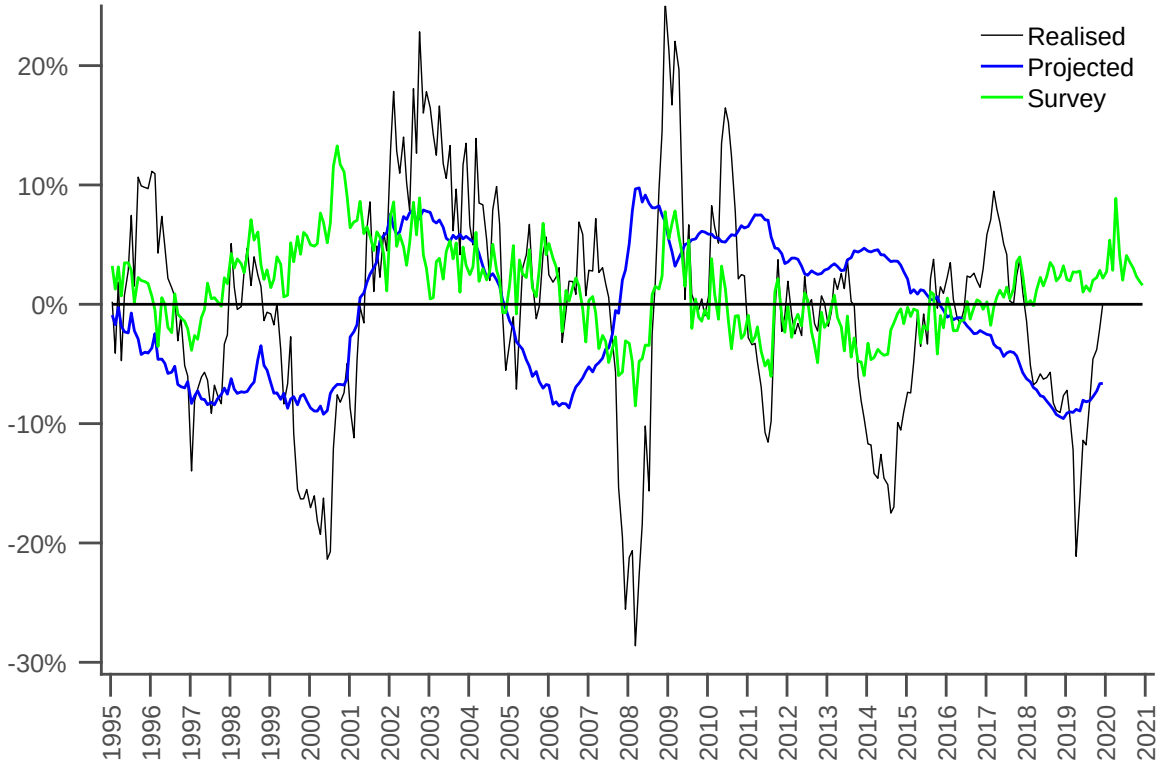
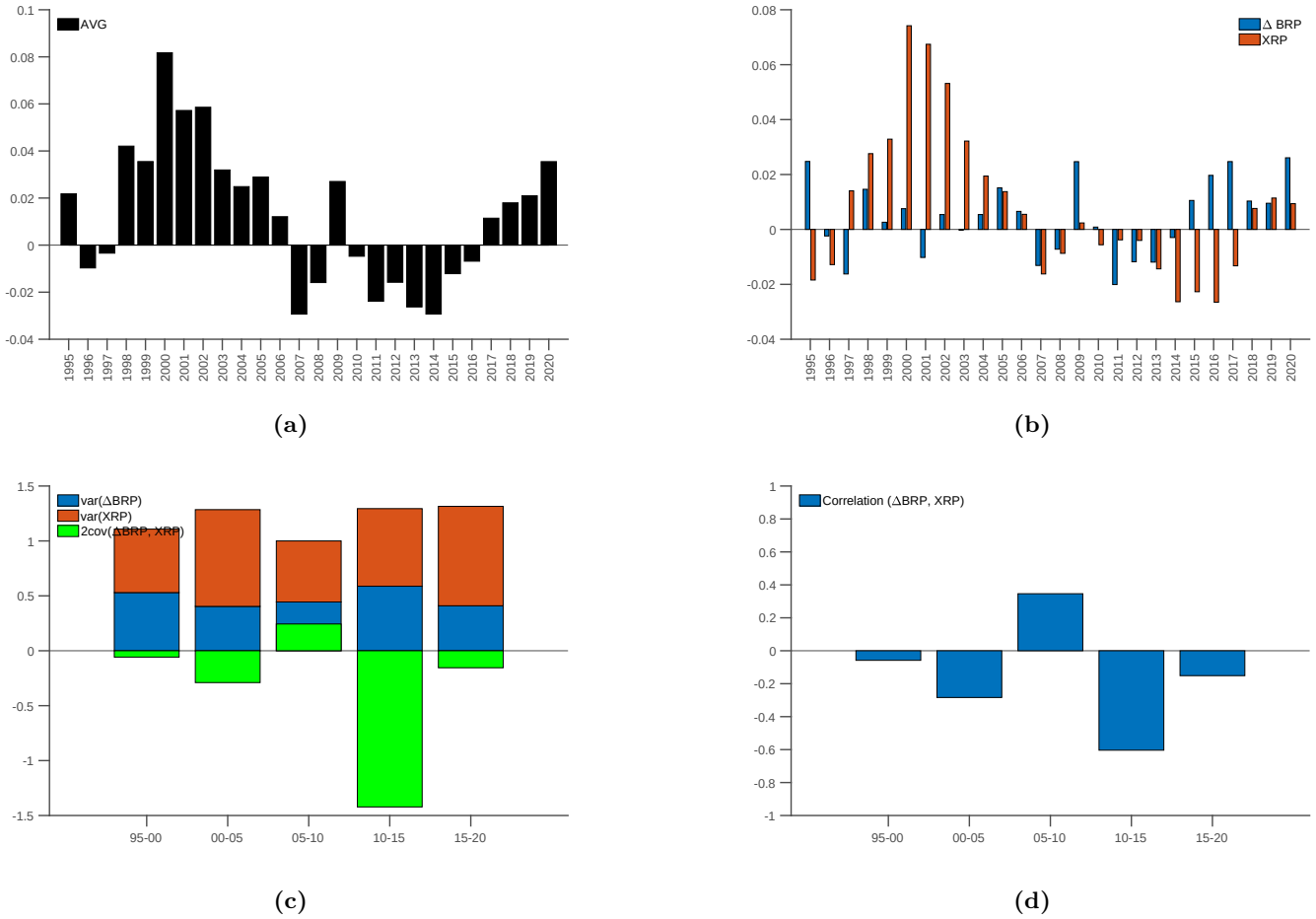


Figure 3.35. Comparison of subjective, projected and realised mean expected excess returns

Figure depicts a time series of mean subjective, projected, and realised LRP across our sample countries. The projection used is an in-sample projection on the $Slope_t = (i_t^{(10)} - i_t^{(1)})$, which is the slope of the domestic yield curve, and $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$, which is the one-year interest rate differential. The sample period is 1995.1 to 2020.12.



(a) **(b)** **(c)** **(d)**

Figure 3.36. $\Delta BRP^{\$}$ Decomposition: Average (AVG)

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

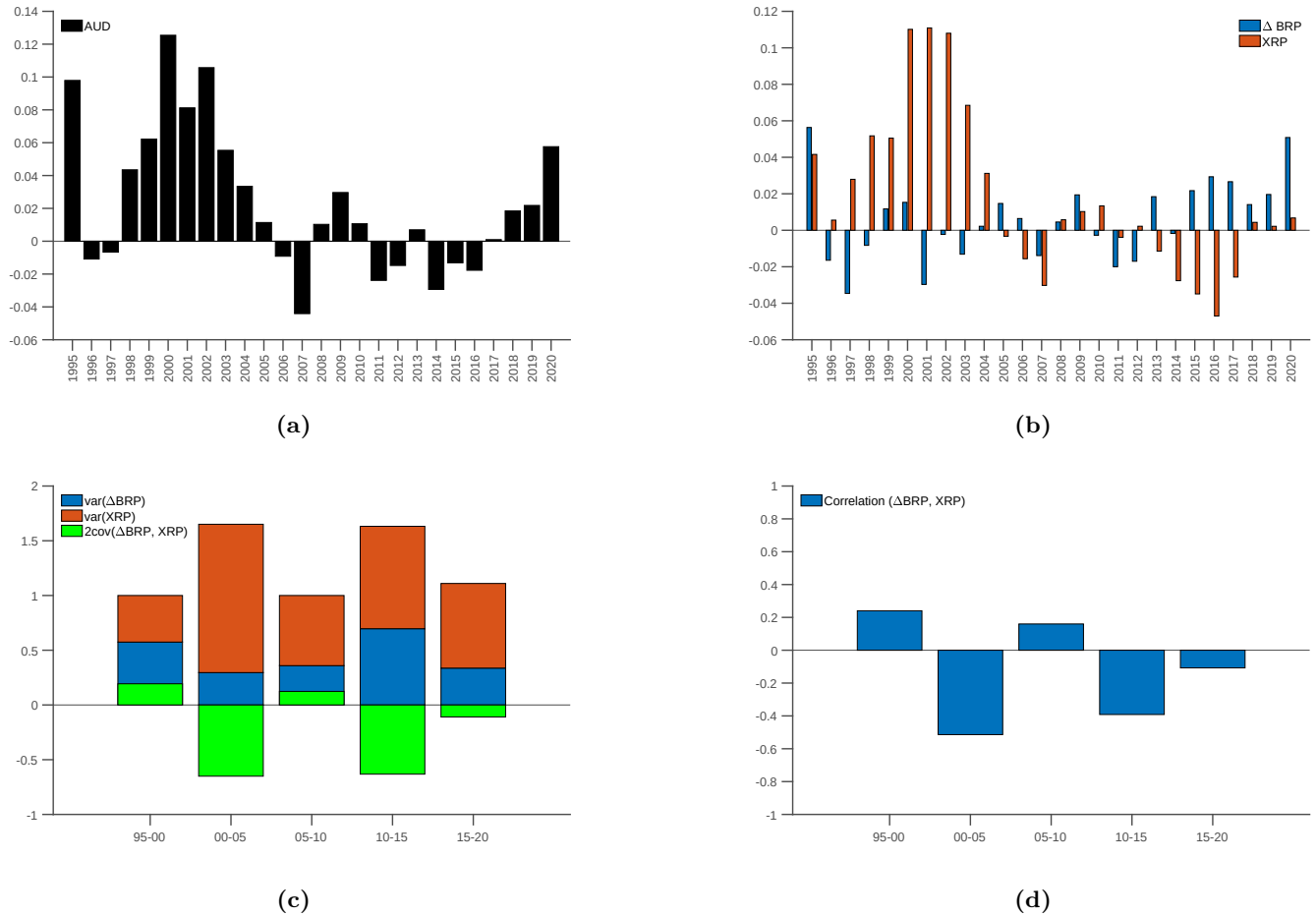
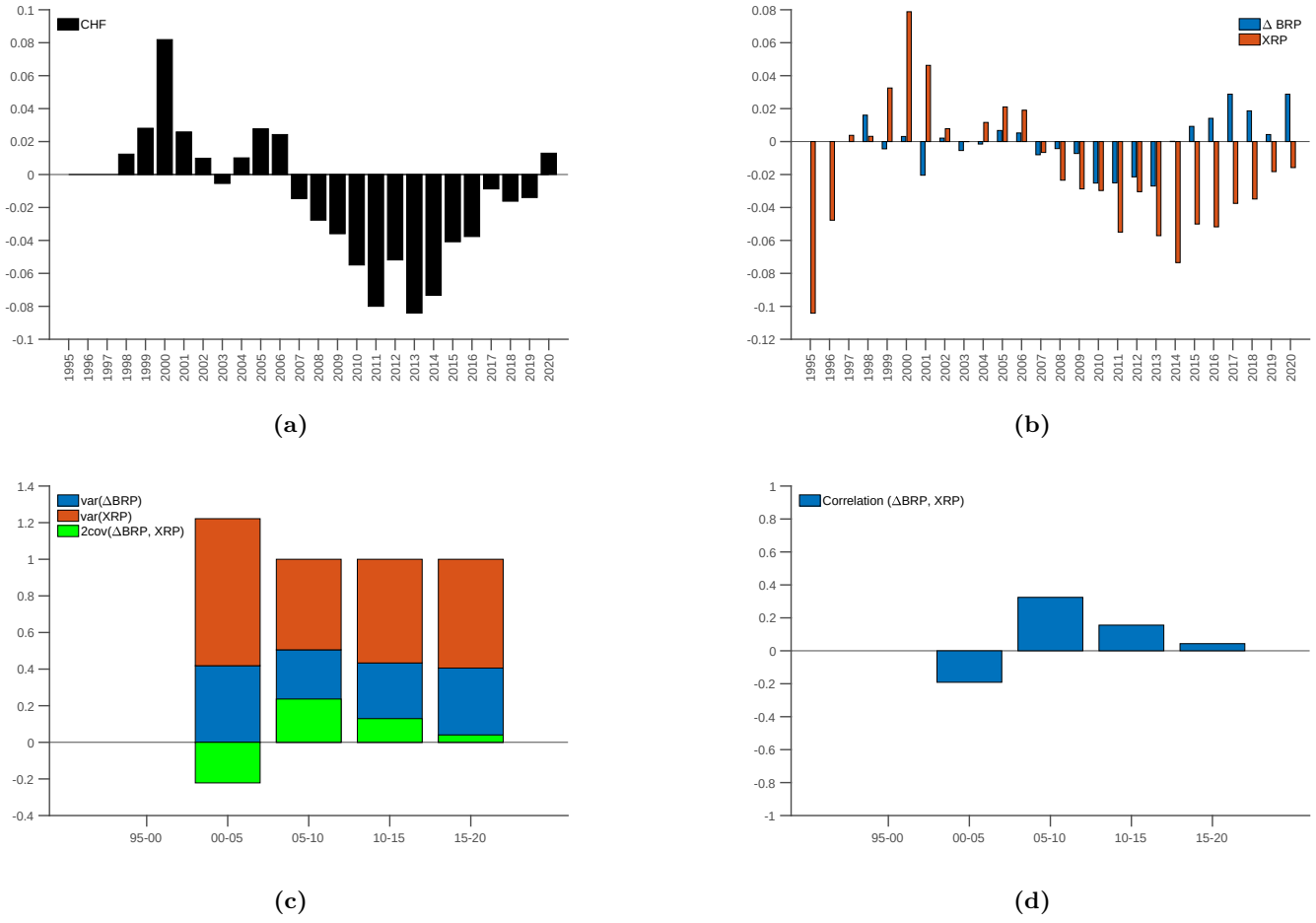


Figure 3.37. $\Delta BRP^{\$}$ **Decomposition: Australian Dollar (AUD)**

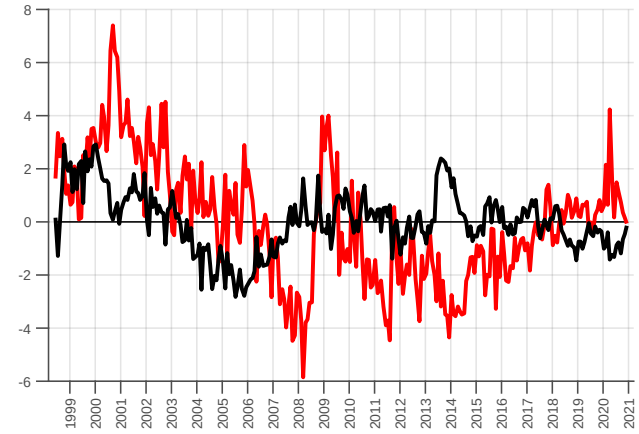
Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.



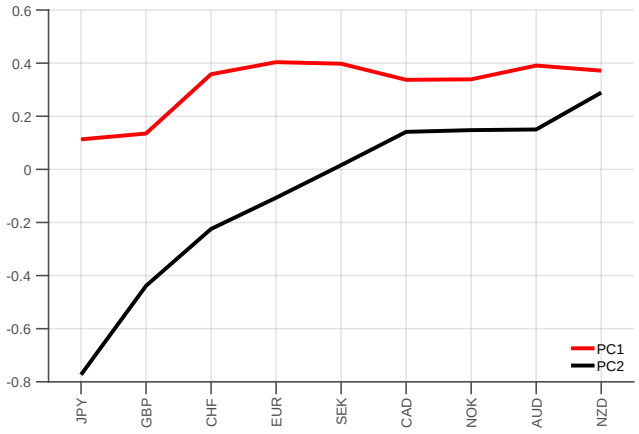
(a) **(b)** **(c)** **(d)**

Figure 3.38. $\Delta BRP^{\$}$ Decomposition: Swiss Franc (CHF)

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

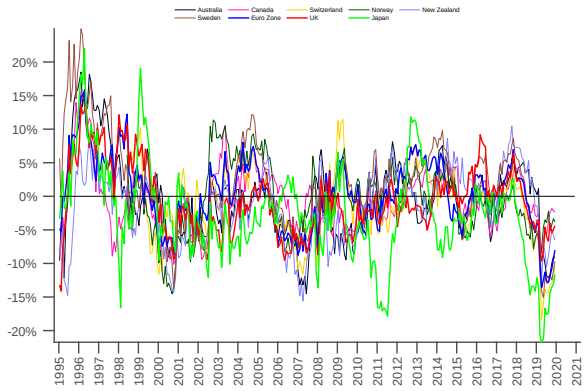


(a) Factor Time Series

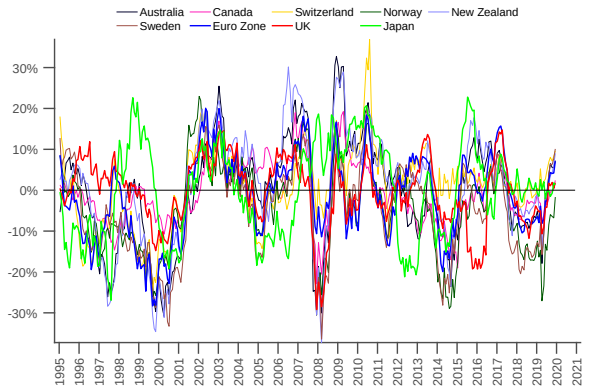


(b) Factor Loadings

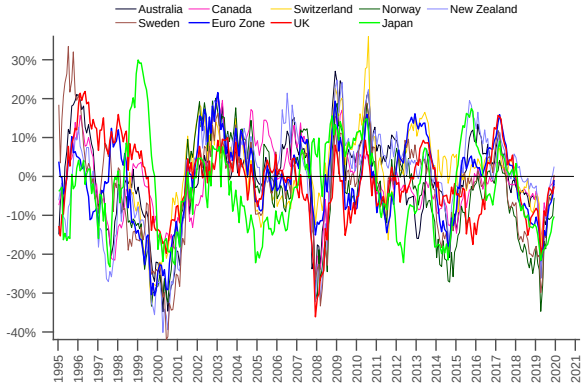
Figure 3.39. PCAs of Subjective Foreign Bond Dollar Risk Premia
Subjective exchange rate risk premia factors PC_t are formed from an eigenvalue decomposition of the covariance matrix of $var(XRP_t) = QDQ^\top$. The variance due to the n'th factor is computed from $D(n,n)/\sum_n D(n,n)$ which is displayed in the online appendix. Principle components (PCs) are computed from the rotation $PC_t = XRP_tQ$. Panel (a) displays the dynamics of $PC1$ and $PC2$ and panel (b) displays the factor loadings (the columns of Q). The sample period is 1995.1 to 2020.12.



(a) ΔBRP



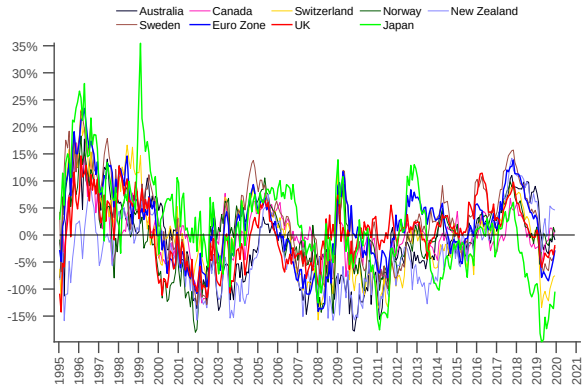
(b) XRP



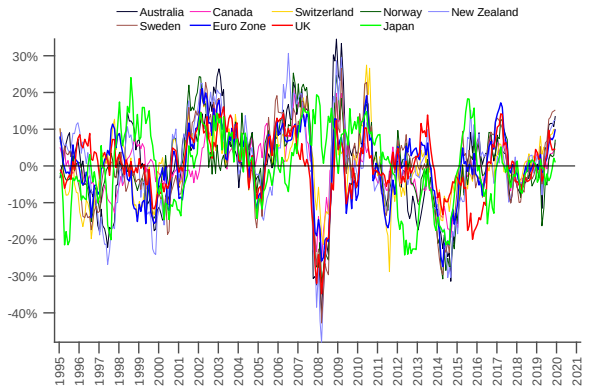
(c) LRP

Figure 3.40. Time Series of Forecast Errors of subjective risk premia

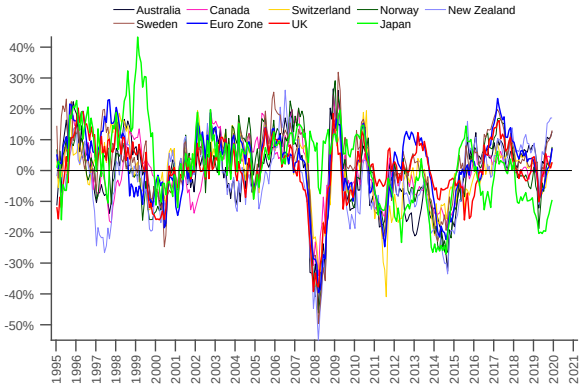
Figure depicts a time series of subjective LRP and its constituent components, ΔBRP and XRP, for each of our sample countries. The sample period is 1995.1 to 2020.12.



(a) ΔBRP



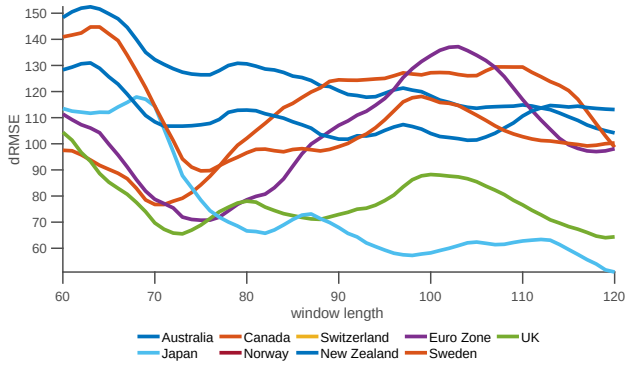
(b) XRP



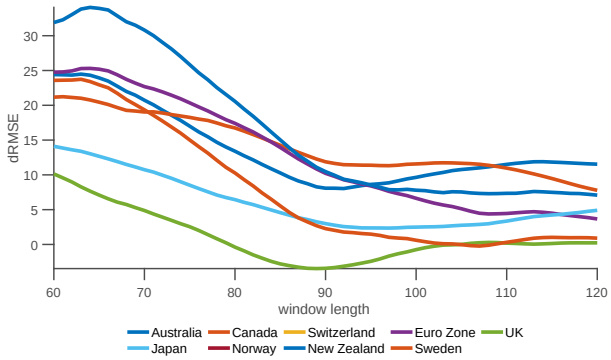
(c) LRP

Figure 3.41. Time Series of Forecast Errors of projected risk premia

Figure depicts a time series of projected LRP and its constituent components, ΔBRP and XRP, for each of our sample countries. The sample period is 1995.1 to 2020.12.



(a)



(b)

Figure 3.42. Correcting Errors: Δ RMSE in the long carry trade
Panel A shows the difference in root mean squared error (Δ RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model is used to predict the error term of Equation A.61. Panel B shows the same difference in Δ RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation A.61. The horizontal axes shows the behaviour of Δ RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the Δ RMSE in percent $(\frac{RMSE_{corrected}-RMSE_{survey}}{RMSE_{survey}} * 100)$. Data is available for the sample period 2000.1 to 2020.12.

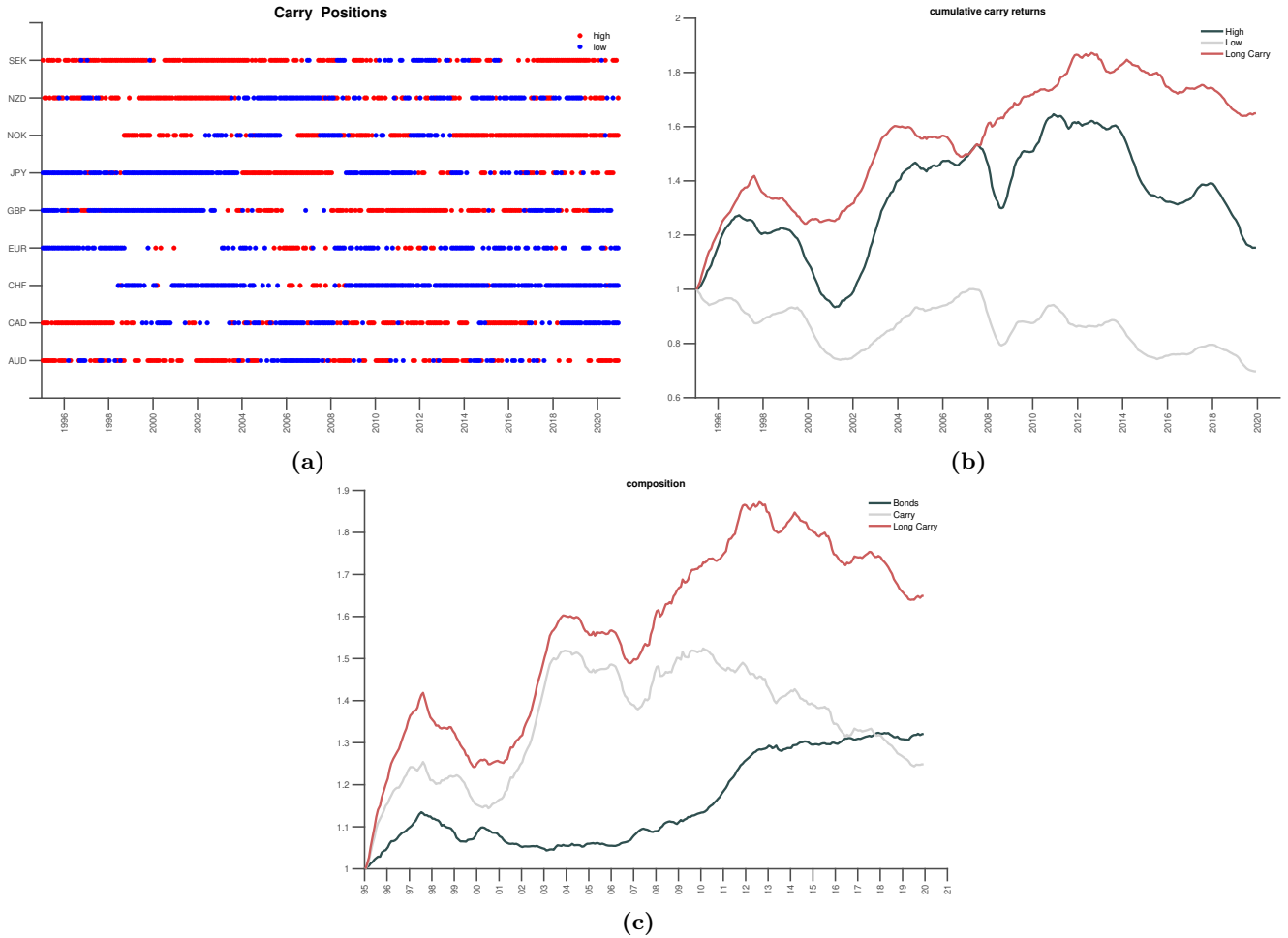


Figure 3.43. Cross-Sectional Trading Strategies using subjective expectations

Figure shows the implementation of a cross-sectional portfolio-sorting trading strategy on the long carry trade using surveys as signals. Panel (a) shows which countries are part of the long and high portfolios, respectively. Panel (b) shows the cumulative excess returns of the high and low portfolios. Panel (c) shows the cumulative strategy returns split into the ΔBRP and XRP components. The sample period is 1995.1 to 2020.12.

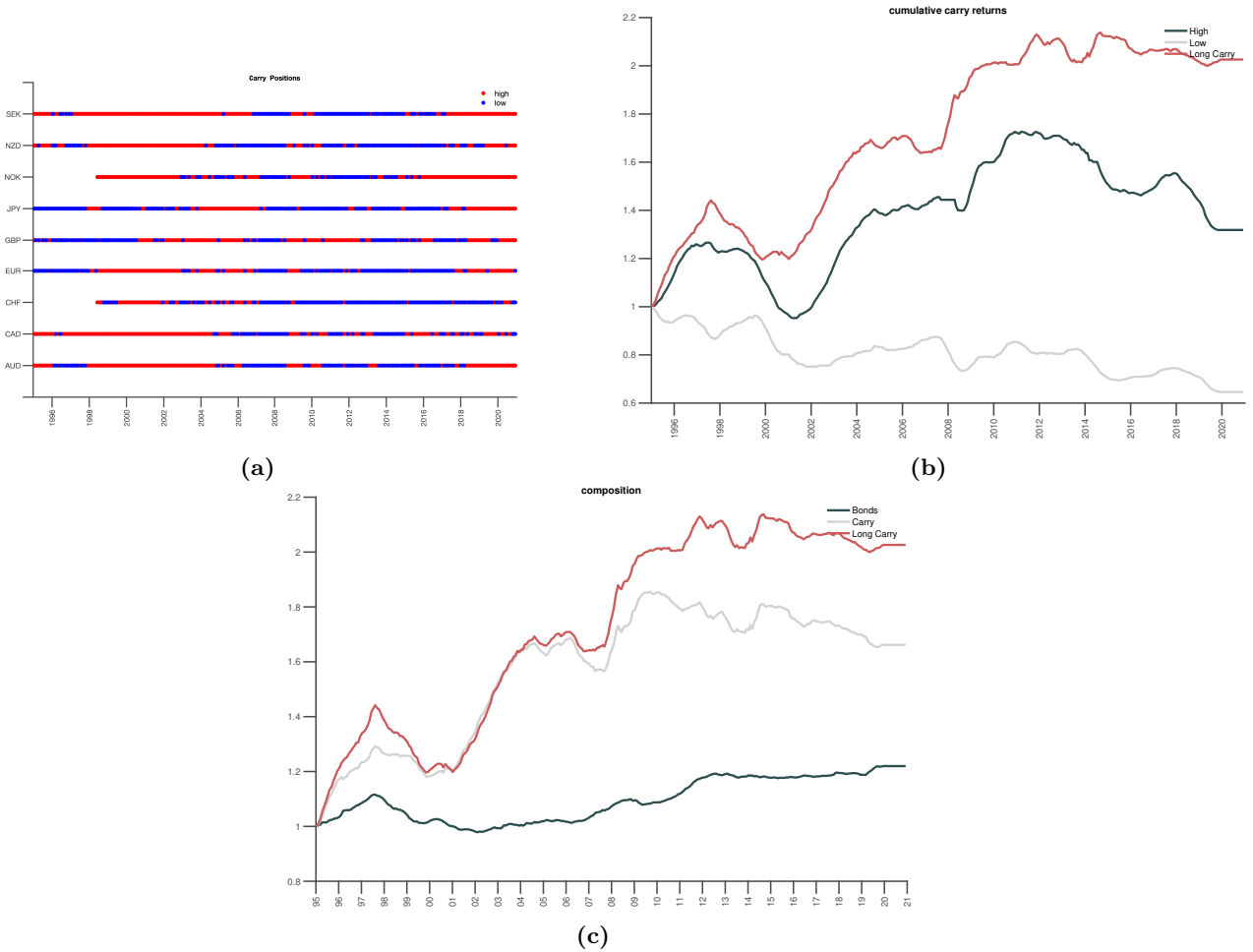


Figure 3.44. Time Series Trading Strategies using subjective expectations

Figure shows the implementation of a time-series trading strategy on the long carry trade using surveys as signals. Panel (a) shows which countries are part of the long and high portfolios, respectively. Panel (b) shows the cumulative excess returns of the high and low portfolios. Panel (c) shows the cumulative strategy returns split into the ΔBRP and XRP components. The sample period is 1995.1 to 2020.12.

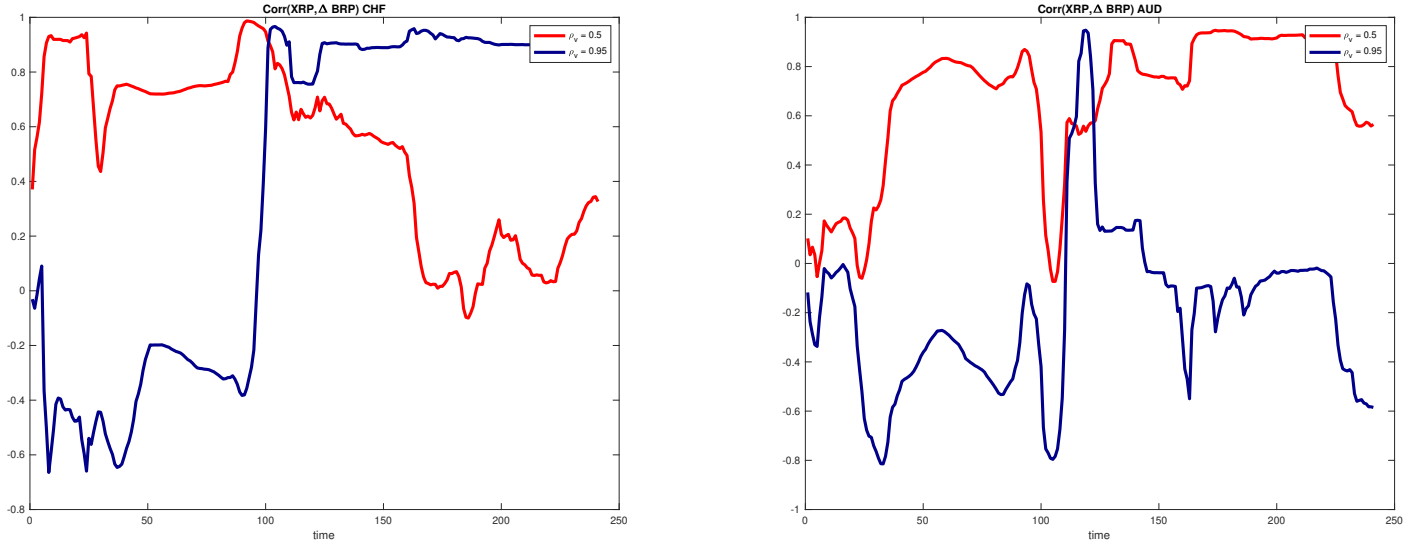


Figure 3.45. Model-implied correlations for CHF and AUD

This figure shows the model-implied correlations between exchange rate risk premia and the bond risk premium differentials for CHF (left) and AUD (right) for two distinct choices of ρ_g and holding everything else constant. Correlations are computed for rolling windows with 60 observations.

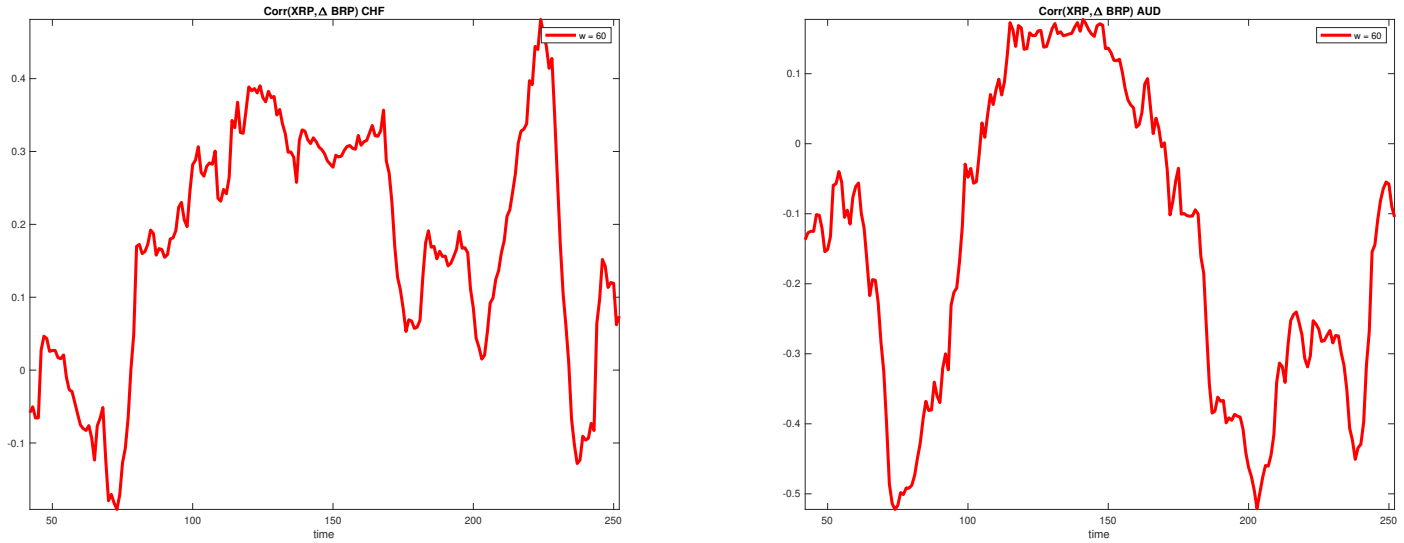


Figure 3.46. Realised correlations for AUD and CHF

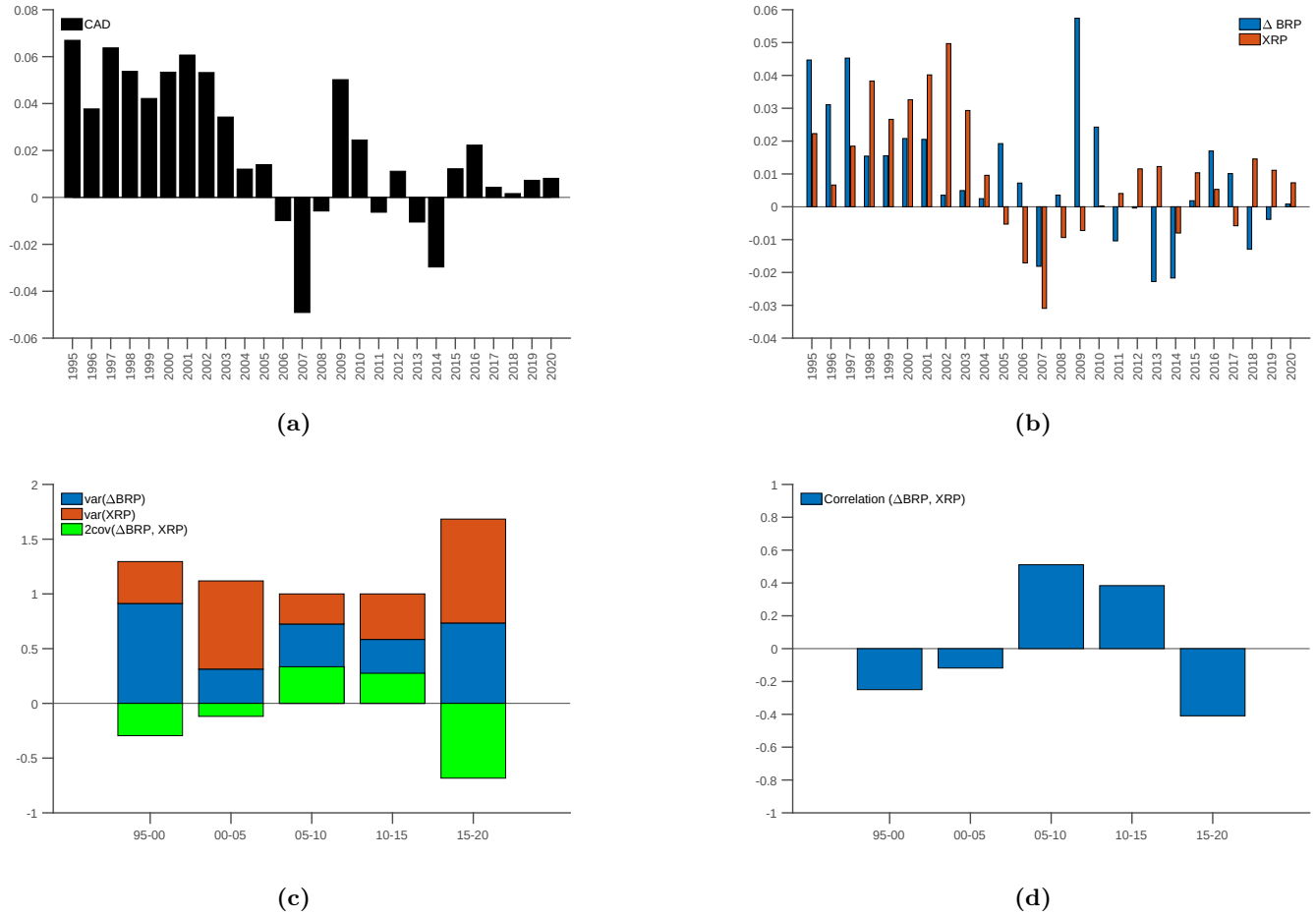
This figure shows the realised correlations between exchange rate risk premia and the bond risk premium differentials for CHF (left) and AUD (right). Correlations are computed for rolling windows with 60 observations. Realisations only start in 06.1998.

Subjective Risk Premia in Bond and FX Markets

Online Appendix

This online appendix is not intended for publication.

A.8. Figures



(a) **(b)**

(c) **(d)**

Figure A.47. $\Delta BRP^{\$}$ **Decomposition: Canadian Dollar (CAD)**

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

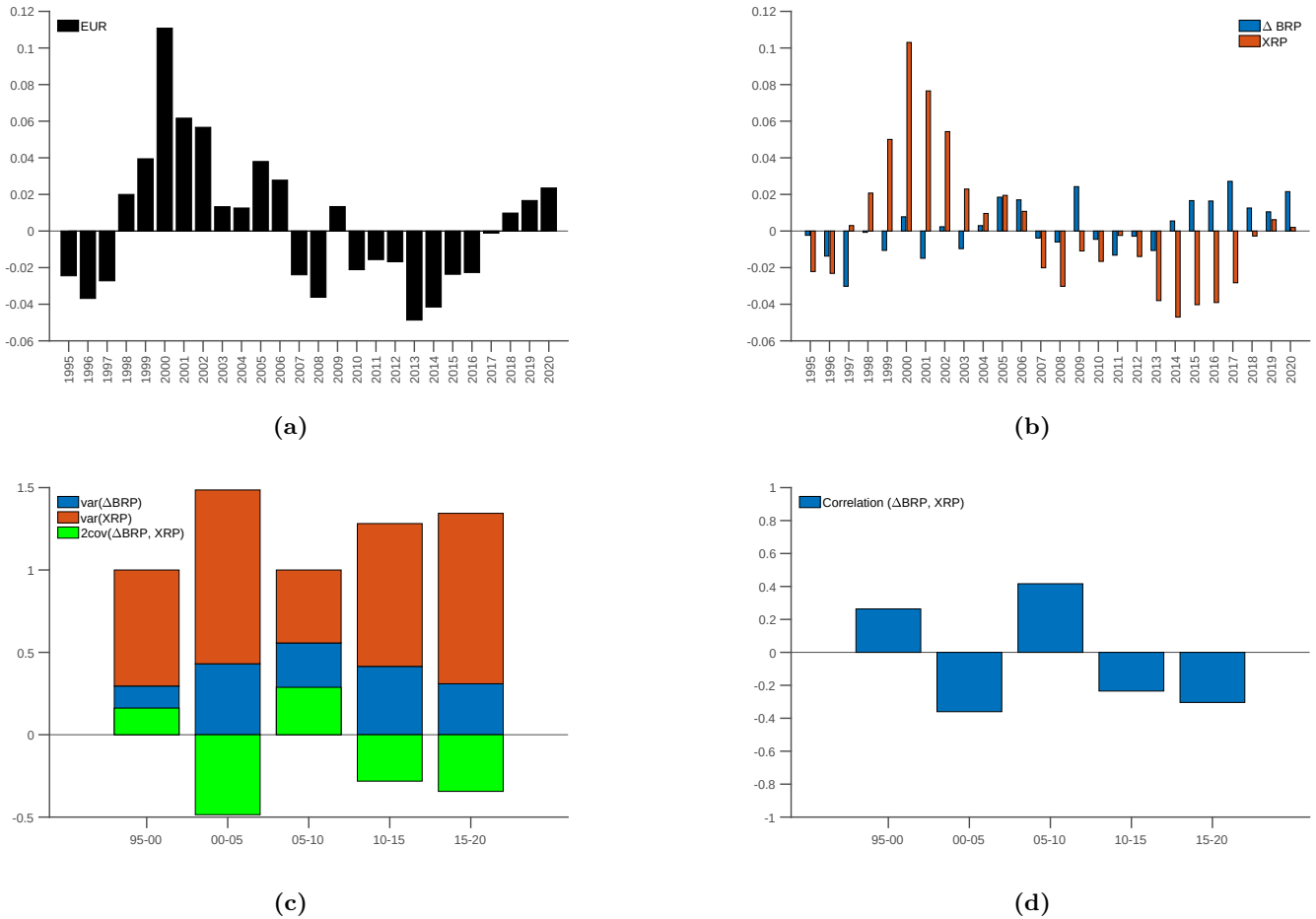
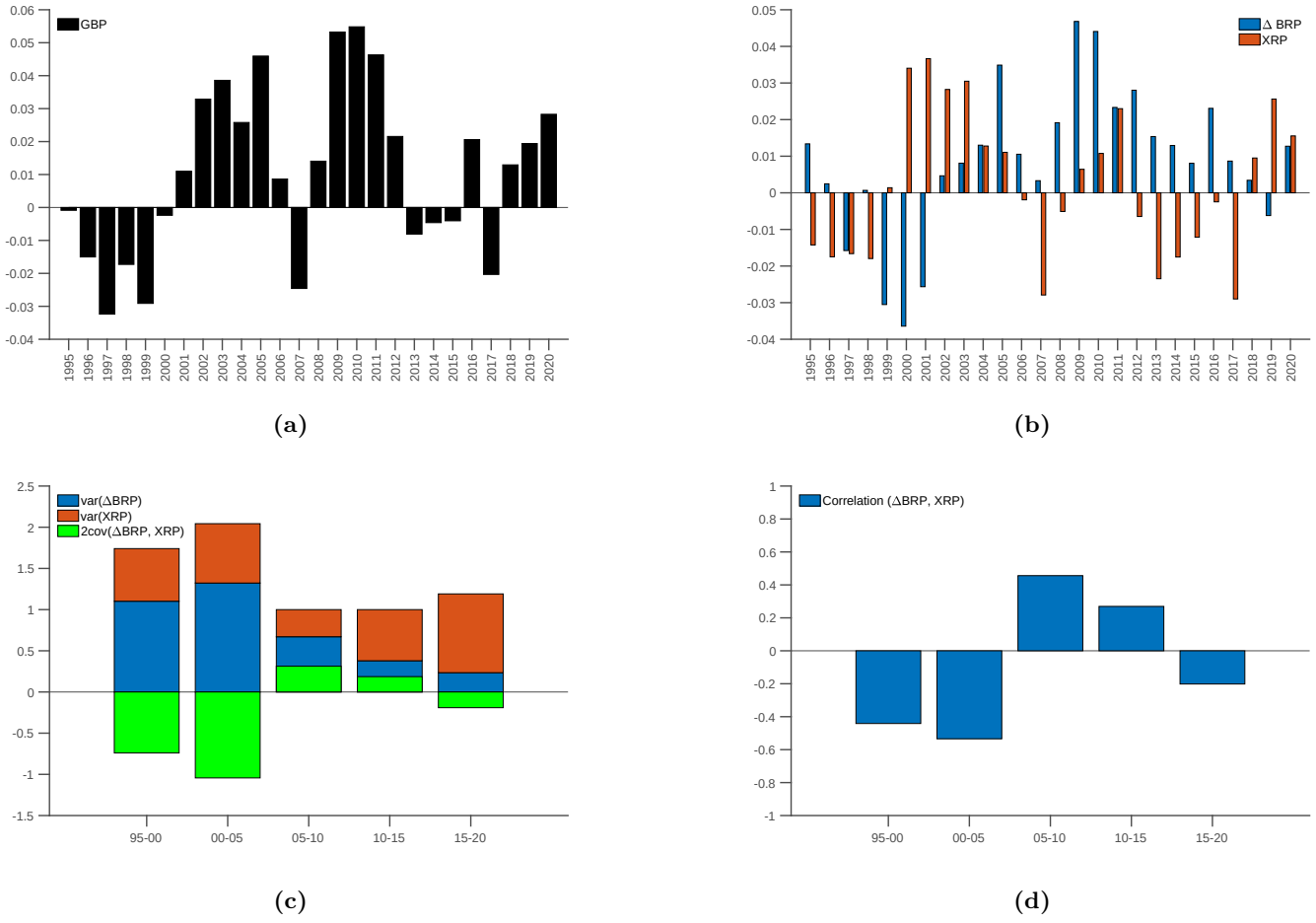


Figure A.48. $\Delta BRP^{\$}$ Decomposition: Euro (EUR)

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.



(a) **(b)**

(c) **(d)**

Figure A.49. $\Delta BRP^{\$}$ **Decomposition: British Pound (GBP)**

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

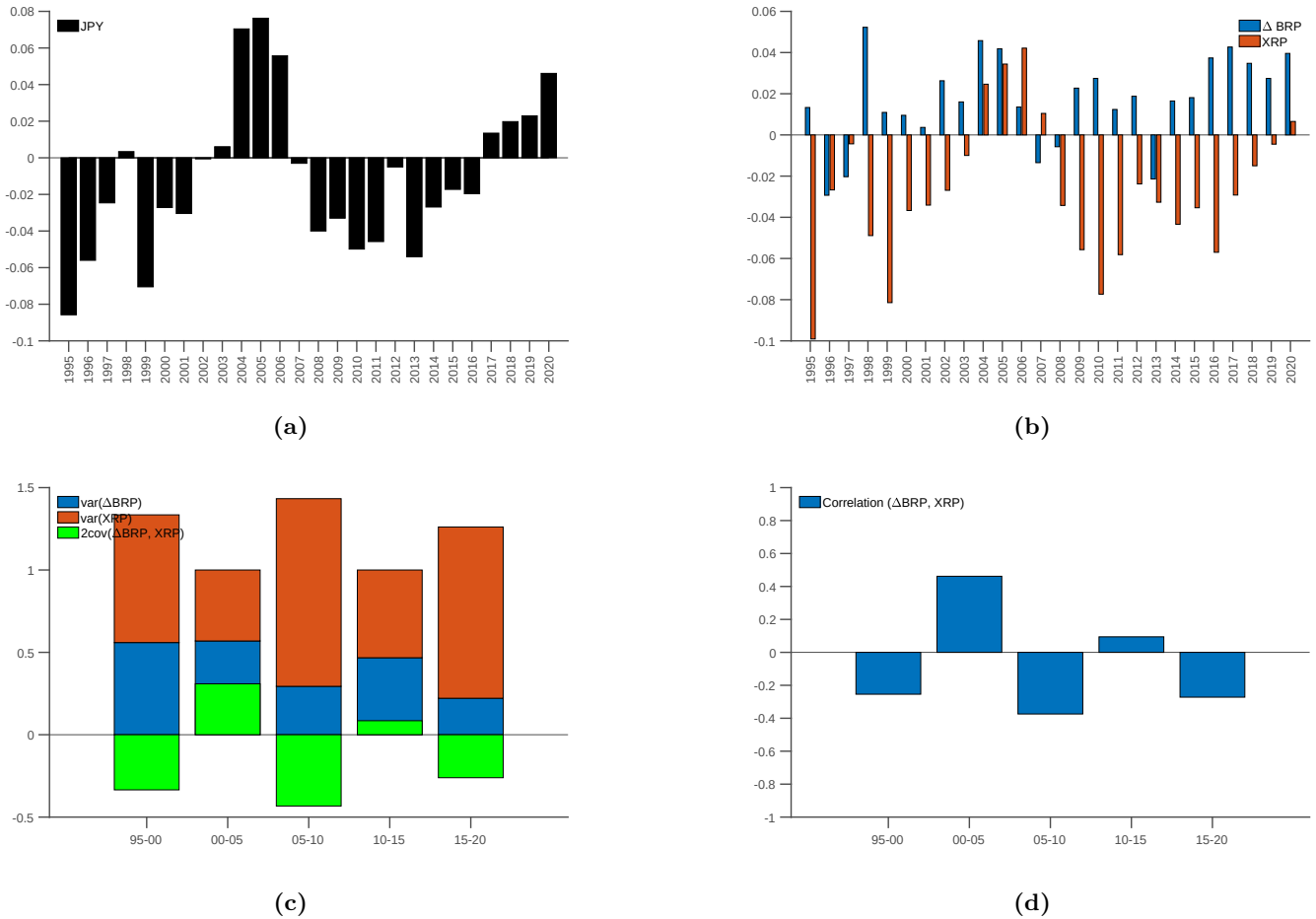
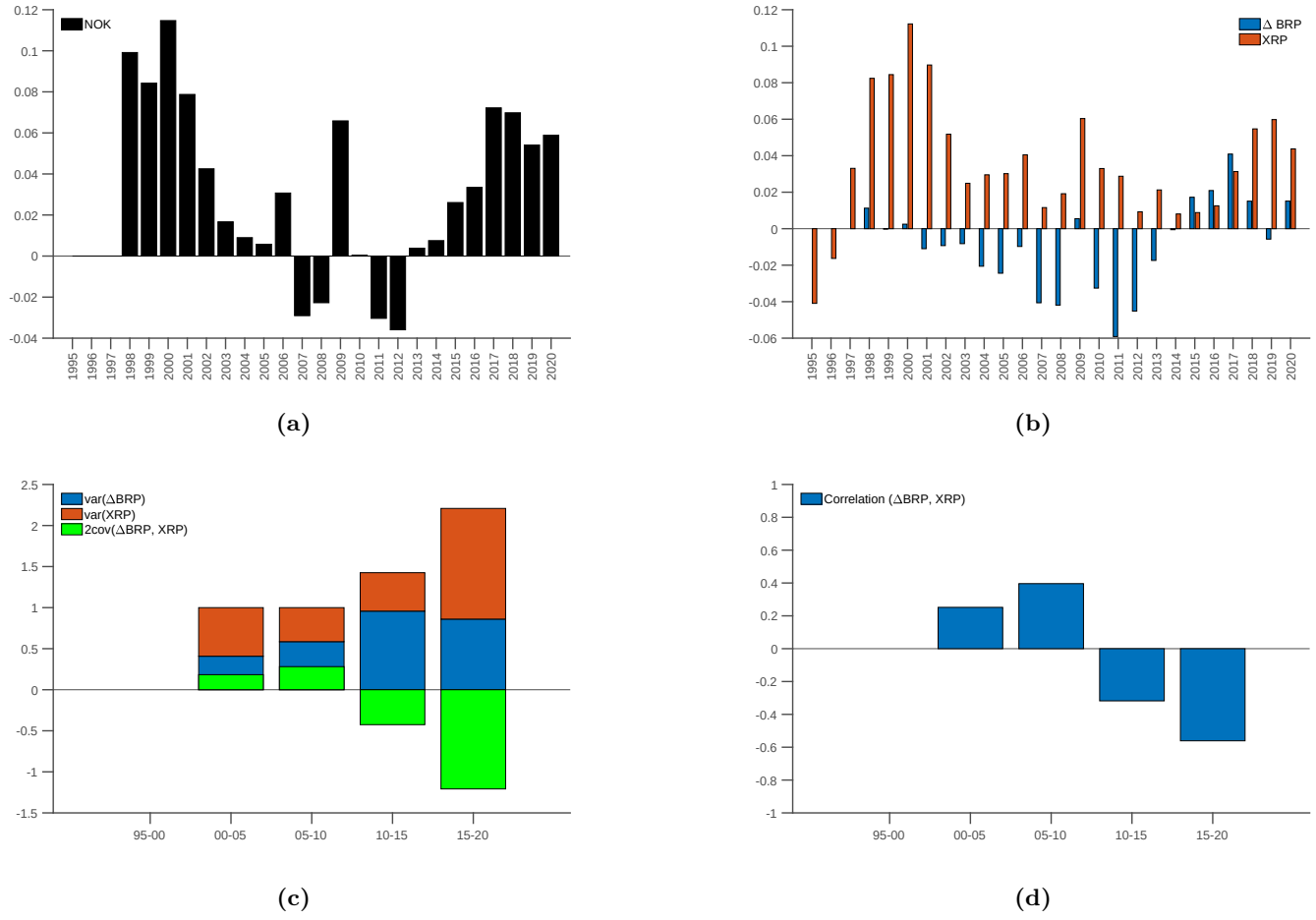


Figure A.50. $\Delta BRP^{\$}$ **Decomposition: Japanese Yen (JPY)**

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.



(a) **(b)**

(c) **(d)**

Figure A.51. $\Delta BRP^{\$}$ Decomposition: Norwegian Krona (NOK)

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

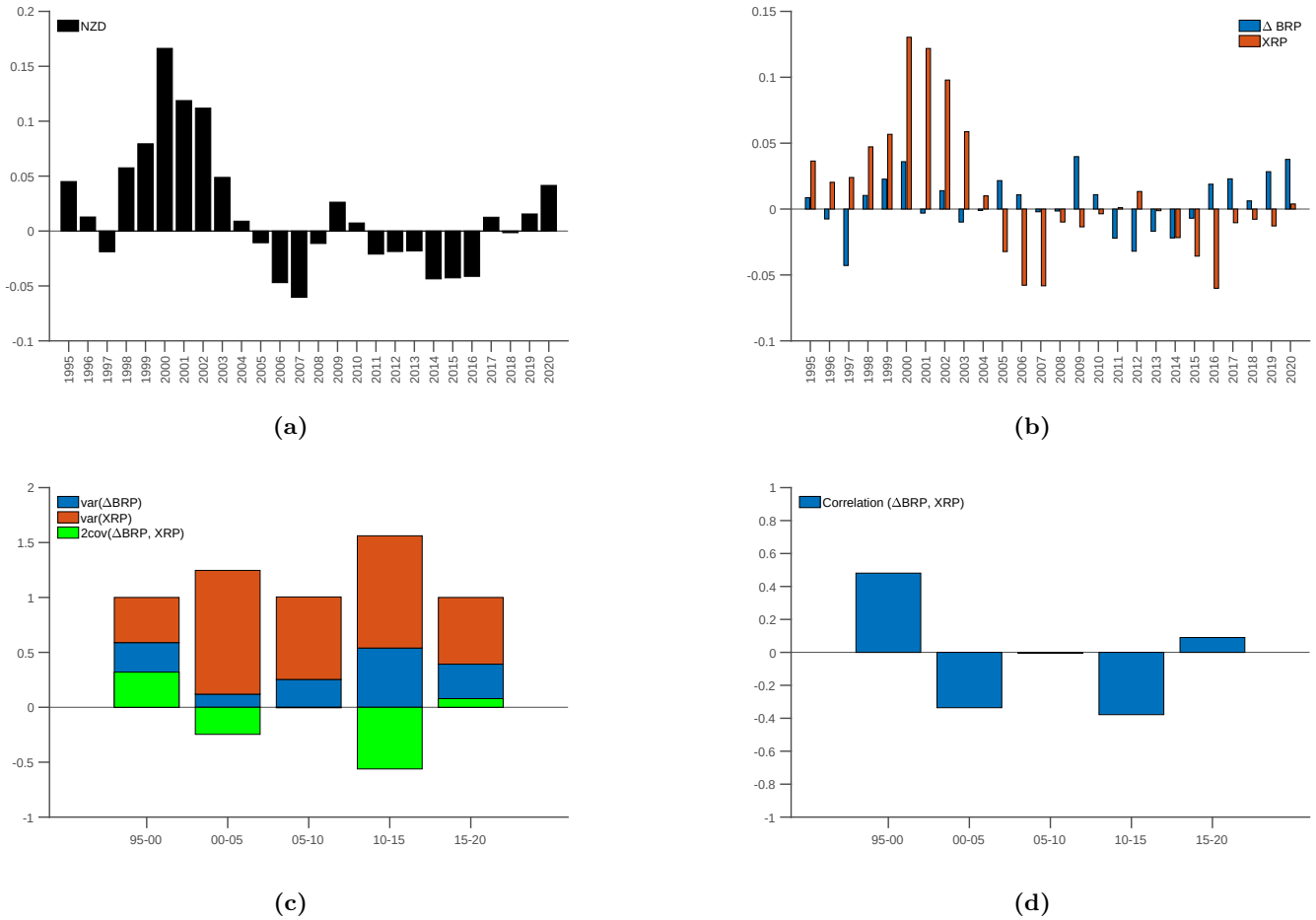
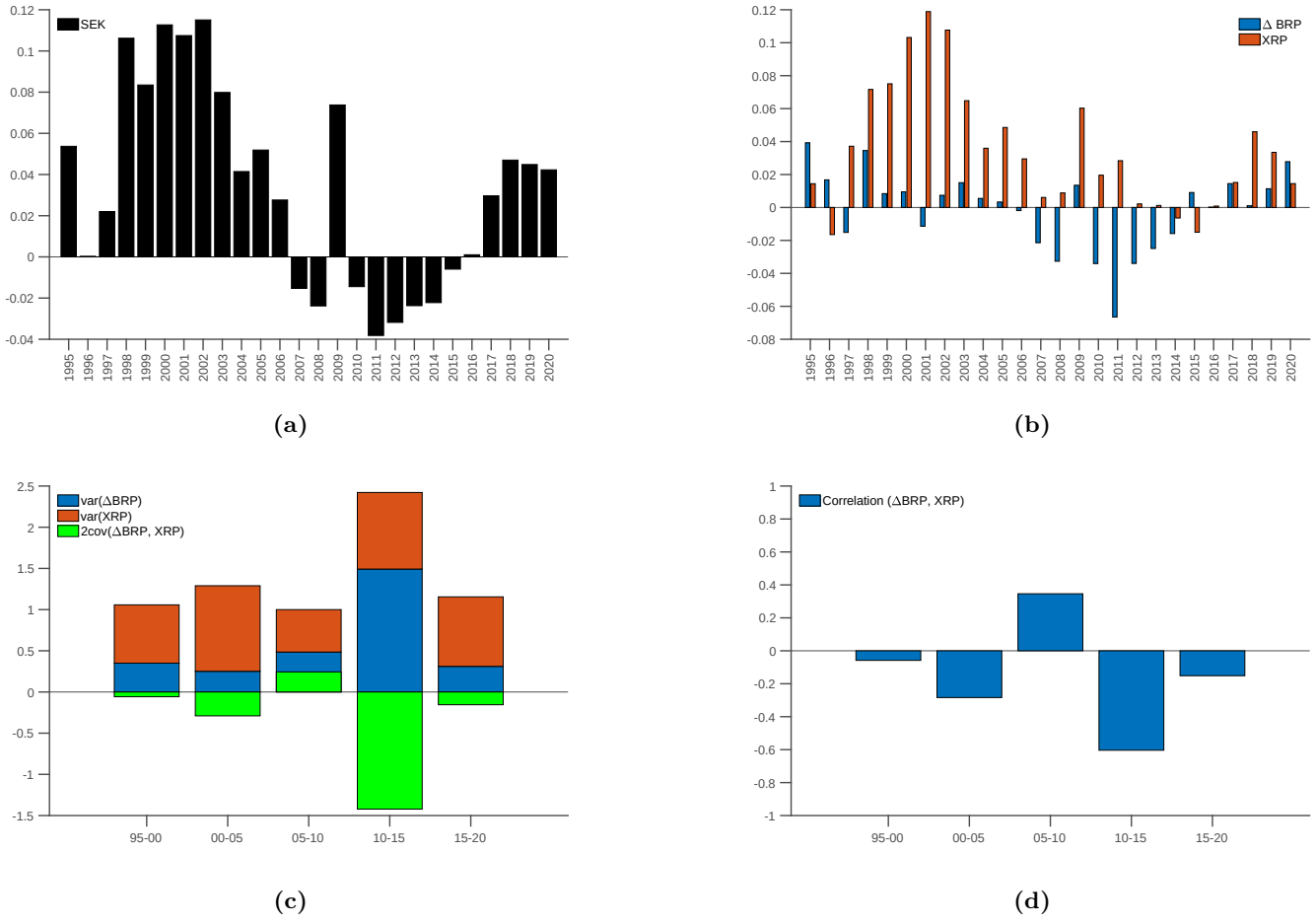


Figure A.52. $\Delta BRP^{\$}$ Decomposition: New Zealand Dollar (NZD)

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.



(a) **(b)** **(c)** **(d)**

Figure A.53. $\Delta BRP^{\$}$ **Decomposition: Swedish Krona (SEK)**

Figure displays the decomposition of yearly-averages of $\Delta BRP_t^{\$,11}$ (panel a) into its ΔBRP^{11} and XRP components (panel b), for the EUR. Panel (c) displays a subsample variance decomposition of $\Delta BRP_t^{\$,11}$ into its bond versus exchange rate risk premia components. The sample period is 1995.1 to 2020.12.

Conclusion

This thesis develops several ideas in relation to asset pricing and rational expectations.

Chapter One proposes a coherent theoretical framework for economists working on agent-based simulations. It introduces a notion of subjective beliefs into strategic market games and develops several key insights: First, it demonstrates that increased liquidity in the market for contingent commodities helps reduce belief-induced volatility. Second, it exemplifies the impact of heterogeneous, subjective beliefs on relative prices through simulations, varying the number of players, the number of commodities, and the distribution of initial wealth. Third, it rationalises the trend of trading financial assets on exchanges using the properties of strategic market games. Finally, it advocates for the use of strategic market games and game-theoretic price formation together with a theory on information transmission to explain why dark pools, i.e., anonymous and privately organised exchanges, have emerged.

Chapter Two considers subjective risk premia in fixed-income and foreign exchange markets. Using surveys, it examines traders' expectations about excess returns in real-time and establishes that these risk premia are conditionally non-zero and vary over time. Furthermore, it shows that the risk premia are correlated with subjective macro expectations and common metrics of volatility. The chapter concludes with the derivation of an asset-pricing model that incorporates risk-adjusted, physical, and subjective probabilities. This model allows for the quantification of observed belief distortions and reveals the risk premias' adherence to a classical risk-return trade-off.

Chapter Three establishes that the long carry trade can be decomposed into a standard carry trade and a slope trade, precisely the two trades considered in Chapter

Two. By combining two sources of risk, the long carry trade is particularly interesting from a subjectivity standpoint because expectational errors could potentially cancel each other out, a view supported by part of the literature. It is noted that not only does the foreign bond dollar risk premium change sign during the sample period, but its composition also changes. Finally, the chapter discusses what features of the foreign bond dollar risk premium need to be addressed in asset pricing models and provides a calibration that is able to match a switch in sign of the correlation between currency risk premia and the bond risk premium differential.

Taken together, the three chapters comprehensively cover the issues of subjective beliefs from both theoretical and empirical viewpoints. The three chapters are, of course, not exhaustive, and each presents avenues for further research.

Bibliography

Amir, Rabah, Siddharta Sahi, Martin Shubik, and Shuntian Yao, 1990, A strategic market game with complete markets, *Journal of economic theory* 51, 126–143.

Andrews, S., R. Colacito, M. M. Croce, and F. Gavazzoni, 2021, Concealed carry, *Working Paper* .

Arrow, K.J., and G. Debreu, 1954, Existence of an equilibrium for a competitive economy, *Econometrica: Journal of the Econometric Society* 265–290.

Bansal, R., and A. Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Bansal, Ravi, and Ivan Shaliastovich, 2013, A long-run risks explanation of predictability puzzles in bond and currency markets, *The Review of Financial Studies* 26, 1–33.

Bauer, M. D., and J. D. Hamilton, 2018, Robust bond risk premia, *The Review of Financial Studies* 31, 399–448.

Bhandari, Anmol, Jaroslav Borovička, and Paul Ho, 2022, Survey data and subjective beliefs in business cycle models, *Available at SSRN 2763942* .

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, Matthew O'Brien, and Andrei Shleifer, 2023, Long term expectations and aggregate fluctuations, Technical report, National Bureau of Economic Research.

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer, 2020, Overreaction in macroeconomic expectations, *American Economic Review* 110, 2748–82.

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer, 2019, Diagnostic expectations and stock returns, *The Journal of Finance* 74, 2839–2874.

Borri, Nicola, and Kirill Shakhnov, 2021, Global risk in long-term sovereign debt, *The Review of Asset Pricing Studies* 11, 654–693.

Brunnermeier, Markus K, and Jonathan A Parker, 2005, Optimal expectations, *American Economic Review* 95, 1092–1118.

Budish, Eric, Robin S Lee, and John J Shim, 2019, A theory of stock exchange competition and innovation: Will the market fix the market?, Technical report, National Bureau of Economic Research.

Buraschi, Andrea, Ilaria Piatti, and Paul Whelan, 2022, Subjective bond returns and belief aggregation, *The Review of Financial Studies* 35, 3710–3741.

Buraschi, Andrea, and Paul Whelan, 2022, Sentiment, speculation, and interest rates, *Management Science* 68, 2308–2329.

Campbell, J.Y., and J.H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of political Economy* 107, 205–251.

Chacko, George, and Luis M Viceira, 2005, Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets, *Review of Financial Studies* 18, 1369–1402.

Chen, Xiaohong, Lars Peter Hansen, and Peter G Hansen, 2020, Robust identification of investor beliefs, *Proceedings of the National Academy of Sciences* 117, 33130–33140.

Chernov, M., and P. Mueller, 2012, The term structure of inflation expectations, *Journal of financial economics* 106, 367–394.

Chernov, Mikhail, Magnus Dahlquist, and Lars A Lochstoer, 2020, Pricing currency risks, Technical report, National Bureau of Economic Research.

Chinn, M. D., and G. Meredith, 2004, Monetary policy and long-horizon uncovered interest parity, *IMF Staff Papers* 51, 409–430.

Cieslak, A., 2017, Short-rate expectations and unexpected returns in treasury bonds, *Working paper* .

Coibion, O., and Y. Gorodnichenko, 2015, Information rigidity and the expectation formation process: A simple framework and new facts, *American Economic Review* 105, 2644–2678.

Colacito, Riccardo, and Mariano M Croce, 2011, Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–181.

Colacito, Riccardo, and Mariano M Croce, 2013, International asset pricing with recursive preferences, *The Journal of Finance* 68, 2651–2686.

Cox, J.C., J.E. Ingersoll Jr, and S.A. Ross, 1985, An intertemporal general equilibrium model of asset prices, *Econometrica* 53, 363–384.

Driscoll, John C, and Aart C Kraay, 1998, Consistent covariance matrix estimation with spatially dependent panel data, *Review of Economics and Statistics* 80, 549–560.

Dubey, P, and M Shubik, 1977a, A closed economy with exogenous uncertainty, different levels of information, money, futures and spot markets, *International Journal of Game Theory* 6, 231–248.

Dubey, Pradeep, 1982, Price-quantity strategic market games, *Econometrica: Journal of the Econometric Society* 111–126.

Dubey, Pradeep, and Siddhartha Sahi, 2003, Price-mediated trade with quantity signals: an axiomatic approach, *Journal of mathematical economics* 39, 377–389.

Dubey, Pradeep, and Martin Shubik, 1977b, Trade and prices in a closed economy with exogeneous uncertainty, different levels of information, money and compound futures markets, *Econometrica: Journal of the Econometric Society* 1657–1680.

Dubey, Pradeep, and Martin Shubik, 1978, A theory of money and financial institutions. 28. The non-cooperative equilibria of a closed trading economy with market supply and bidding strategies, *Journal of Economic Theory* 17, 1–20.

Dubey, Pradeep, and Martin Shubik, 1980, A strategic market game with price and quantity strategies, *Zeitschrift für Nationalökonomie/Journal of Economics* 40, 25–34.

Duffie, Darrell, and Larry G. Epstein, 1992a, Asset pricing with stochastic differential utility, *Review of Financial Studies* 5, 411–436.

Duffie, Darrell, and Kenneth J. Singleton, 1993, Simulated moments estimation of markov models of asset prices, *Econometrica* 61, 929.

Elton, Edwin J, 1999, Presidential address: expected return, realized return, and asset pricing tests, *The Journal of Finance* 54, 1199–1220.

Epstein, Larry G., and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57, 937–969.

Eraker, B., 2018, Predictability puzzles, *Working paper, Wisconsin School of Business* .

Fama, Eugene F, 1984, Forward and spot exchange rates, *Journal of monetary economics* 14, 319–338.

Fendel, Ralf, Eliza M Lis, and Jan-Christoph Rülke, 2011, Do professional forecasters believe in the phillips curve? evidence from the g7 countries, *Journal of Forecasting* 30, 268–287.

Fisher, Irving, 1896, *Appreciation and Interest: A Study of the Influence of Monetary Appreciation and Depreciation on the Rate of Interest with Applications to the Bimetallic Controversy and the Theory of Interest*, volume 11 (American economic association).

Fostel, Ana, and John Geanakoplos, 2008, Leverage cycles and the anxious economy, *American Economic Review* 98, 1211–1244.

Frankel, J. A., and K. A. Froot, 1987, Short-term and long-term expectations of the yen/dollar exchange rate: evidence from survey data, *Journal of the Japanese and International Economies* 1, 249–274.

Froot, Kenneth A, and Jeffrey A Frankel, 1989, Forward discount bias: Is it an exchange risk premium?, *The Quarterly Journal of Economics* 104, 139–161.

Geerolf, François, 2015, Leverage and disagreement, *Working Paper, UCLA* .

Giraud, Gaël, 2003, Strategic market games: an introduction, *Journal of mathematical economics* 39, 355–375.

Giraud, Gaël, and Hubert Stahn, 2008, On shapley–shubik equilibria with financial markets, *Economic Theory* 35, 469–496.

Gouriéroux, Christian, and Alain Monfort, 2000, Simulation-based econometric method, *Econometric Theory* 16, 131–138.

Greenwood, R., and A. Schleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Hansen, Lars Peter, 1982, Large sample properties of generalized method of moments estimators, *Econometrica: Journal of the Econometric Society* 1029–1054.

Hansen, Lars Peter, and Robert J Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, *The Journal of Political Economy* 829–853.

Harsanyi, John C, and Reinhard Selten, 1988, A general theory of equilibrium selection in games, *MIT Press Books* 1.

Hassan, Tarek A, and Rui C Mano, 2019, Forward and spot exchange rates in a multi-currency world, *The Quarterly Journal of Economics* 134, 397–450.

Hicks, J.R., 1939, Value and capital: An inquiry into some fundamental principles of economic theory.

Hoechle, Daniel, 2007, Robust standard errors for panel regressions with cross-sectional dependence, *The Stata journal* 7, 281–312.

Huber, Juergen, Martin Shubik, and Shyam Sunder, 2010, Three minimal market institutions with human and algorithmic agents: Theory and experimental evidence, *Games and Economic Behavior* 70, 403–424.

Kloeden, Peter E, and Eckhard Platen, 2013, *Numerical solution of stochastic differential equations*, volume 23 (Springer Science & Business Media).

Kremens, Lukas, and Ian Martin, 2019, The quanto theory of exchange rates, *American Economic Review* 109, 810–43.

Kreps, David M., and Evan L. Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.

Krohn, Ingomar, Philippe Mueller, and Paul Whelan, 2020, Foreign exchange fixings and returns around the clock, *Available at SSRN 3521370* .

Kyle, A. S., and K. Todorov, 2023, Cumulant risk premium, *Working Paper* .

Lee, Bong-Soo, and Beth Fisher Ingram, 1991, Simulation estimation of time-series models, *Journal of Econometrics* 47, 197–205.

Lewis, Karen K, 1995, Puzzles in international financial markets, *Handbook of international economics* 3, 1913–1971.

Lucas, Robert E, 1972, Expectations and the neutrality of money, *Journal of economic theory* 4, 103–124.

Lustig, H., A. Verdelhan, and A. Stathopoulos, 2019, The term structure of currency carry trade risk premia, *American Economic Review* Forthcoming.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *The Review of Financial Studies* 24, 3731–3777.

Mackowiak, B., and M. Wiederholt, 2009, Optimal sticky prices under rational inattention, *American Economic Review* 99, 769–803.

Maenhout, Pascal J, Andrea Vedolin, and Hao Xing, 2021, Robustness and dynamic sentiment, *Available at SSRN 3798445* .

Makarov, Igor, and Antoinette Schoar, 2020, Trading and arbitrage in cryptocurrency markets, *Journal of Financial Economics* 135, 293–319.

Mankiw, N.G., and R. Reis, 2002, Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve, *Quarterly Journal of Economics* 117, 1295–1328.

Meese, Richard A, and Kenneth Rogoff, 1983, Empirical exchange rate models of the seventies: Do they fit out of sample?, *Journal of international economics* 14, 3–24.

Moreira, Alan, and Tyler Muir, 2017, Volatility-managed portfolios, *The Journal of Finance* 72, 1611–1644.

Muth, John F, 1961, Rational expectations and the theory of price movements, *Econometrica: Journal of the Econometric Society* 315–335.

Nagel, Stefan, and Zhengyang Xu, 2022, Dynamics of subjective risk premia, Technical report, National Bureau of Economic Research.

Newey, Whitney K, and Kenneth D West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Peck, James, and Karl Shell, 1989, On the nonequivalence of the arrow-securities game and the contingent-commodities game, *Economic complexity: chaos, sunspots, bubbles, and nonlinearity* 61–85.

Pesch, D., I. Piatti, and P. Whelan, 2024, Subjective risk premia in bond and fx markets, *Working Paper* .

Piazzesi, M., J. Salomao, and M. Schneider, 2015, Trend and cycle in bond premia, Technical report, Working paper.

Postlewaite, Andrew, and David Schmeidler, 1978, Approximate efficiency of non-walrasian nash equilibria, *Econometrica: Journal of the Econometric Society* 127–135.

Prager, Daniel, Christopher Burns, Sarah Tulman, and James MacDonald, 2020, Farm use of futures, options, and marketing contracts, Technical report.

Radner, Roy, 1968, Competitive equilibrium under uncertainty, *Econometrica: Journal of the Econometric Society* 31–58.

Ritzberger, Klaus, 2016, Order-driven markets are almost competitive, *The Review of Economic Studies* 83, 338–364.

Rogoff, Kenneth S, and Vania Stavrakeva, 2008, The continuing puzzle of short horizon exchange rate forecasting, Technical report, National Bureau of Economic Research.

Sahi, Siddhartha, and Shuntian Yao, 1989, The non-cooperative equilibria of a trading economy with complete markets and consistent prices, *Journal of Mathematical Economics* 18, 325–346.

Shapley, Lloyd, 1953, A value for n-person games, *Contributions to the Theory of Games* 307–317.

Shapley, Lloyd, and Martin Shubik, 1977, Trade using one commodity as a means of payment, *Journal of Political Economy* 85, 937–968.

Shapley, Lloyd S, 1976, Noncooperative general exchange, in *Theory and measurement of economic externalities*, 155–178 (Elsevier).

Shiller, Robert J, 1981, The use of volatility measures in assessing market efficiency, *The Journal of Finance* 36, 291–304.

Shubik, M, 1972, Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model, western econom, *J* 10, 24–38.

Shubik, Martin, 1973, Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model, *Economic Inquiry* 11, 24.

Shubik, Martin, and Charles Wilson, 1977, The optimal bankruptcy rule in a trading economy using fiat money, *Zeitschrift für Nationalökonomie/Journal of Economics* 37, 337–354.

Shubik, Martin, and Shuntian Yao, 1990, The transactions cost of money (a strategic market game analysis), *Mathematical Social Sciences* 20, 99–114.

Simon, Leo K, 1984, Bertrand, the cournot paradigm and the theory of perfect competition, *The Review of Economic Studies* 51, 209–230.

Sims, C. A., 2003, Implications of rational inattention, *Journal of Monetary Economics* 50, 665–690.

Simsek, Alp, 2013, Belief disagreements and collateral constraints, *Econometrica* 81, 1–53.

Singleton, Kenneth J, 2021, Presidential address: How much "rationality" is there in bond-market risk premiums?, *The Journal of Finance* 76, 1611–1654.

Stavrakeva, Vania, and Jenny Tang, 2020, A fundamental connection: Exchange rates and macroeconomic expectations .

Vasicek, Oldrich, 1977, An equilibrium characterization of the term structure, *Journal of financial economics* 5, 177–188.

Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, *The Journal of Finance* 65, 123–146.

Wachter, Jessica A, 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *The Journal of Finance* 68, 987–1035.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Weyers, Sonia, 1999, Uncertainty and insurance in strategic market games, *Economic Theory* 14, 181–201.

Woodford, M., 2002, *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, chapter Imperfect Common Knowledge and the Effects of Monetary Policy (Princeton University Press).

Yan, Ji, 2017, Belief heterogeneity, collateral and endogenous leverage, *Collateral and Endogenous Leverage (December 4, 2017)* .

Zheng, Stephan, Alexander Trott, Sunil Srinivasa, David C Parkes, and Richard Socher, 2022, The ai economist: Taxation policy design via two-level deep multiagent reinforcement learning, *Science Advances* 8, eabk2607.